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CONCRETE-STEEL

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A TREATISE ON THE THEORY AND PRACTICE OF REINFORCED CONCRETE CONSTRUCTION

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WITH NUMEROUS ILLUSTRATIONS DIAGRAMS & TABLES

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PREFACE

THE object of this work is to present definite and reliable information relative to concrete-steel construction, a subject whose literature is scattered over articles in various technical journals, chiefly of foreign origin, and upon which no treatise of convenient form and dimensions has hitherto been published in the English language for the guidance of engineers, architects, and others who desire to make use of the new material.

In dealing with the subject, the author has endeavoured to preserve a strict continuity of treatment, commencing with the physical properties of concrete and steel, and the effects of their joint action. The principles underlying the theory of concrete-steel are then discussed, and in connexion with each of the chief types of members employed in construction, the rules necessary for correct design and for the precise calculation of strength are stated, and their uses demonstrated by practical examples.

The different chapters are divided into numbered articles for the purpose of facilitating reference; and, with the object of avoiding confusion, the various formulæ given are based, as far as practicable, upon a common notation, to which an index is given on the pages following the Table of Contents.

Less attention has been devoted to various patented systems of concrete-steel construction than to the discussion of principles in a manner which it is hoped will enable any engineer or architect to apply them with confidence to the requirements of his own practice.

The Author desires to express his indebtedness to Mr. Reginald Ryves, A.M.Inst.C.E., for valuable assistance in the correction of proofs and in checking the tables and calculations contained in this book.

W. NOBLE TWELVETREES.

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INDEX TO NOTATION

THE following is a list in alphabetical order of the ordinary symbols employed throughout this treatise. For the purpose of facilitating comparison and of avoiding confusion a uniform system of notation has been applied to all formulæ where practicable. A few special symbols are used in some of the formulæ, and the significations of these are explained in the body of the work.

As the precise manner in which the symbols are applied to the various equations is indicated in every case, it has not been thought necessary to give more than abbreviated definitions in this Index :

a = area.
b = breadth.
d = depth.
d = diameter.
E = coefficient of elasticity generally.
E_1 = ,, ,, excluding permanent set.
E_c = ,, ,, compression.
E_t = ,, ,, tension.
E' = ,, ,, of concrete, generally.
E_c' = ,, ,, ,, in compression.
E_s' = ,, ,, ,, in shear.
E_t' = ,, ,, ,, in tension.
E'' = ,, ,, steel, generally.
E_c'' = ,, ,, ,, in compression.
E_s'' = ,, ,, ,, in shear.
E_t'' = ,, ,, ,, in tension.
e = elastic limit of steel.
F_c' = compressive strength of concrete.
F_t'' = tensile strength of steel, and tension in steel.
f = extreme fibre stress in a beam, generally.
f_a = adhesion between concrete and steel.
f_c' = extreme fibre stress on concrete in compression, in a beam.
f_m' = mean unit ,, ,, ,, ,,
f_t' = extreme fibre ,, ,, tension ,,
f_c' = unit ,, ,, on steel in compression, in a beam.
f_r = modulus of transverse rupture.

f_t'' = unit fibre stress on steel in tension, in a beam.
h = distance of extreme fibres from the neutral axis of a beam.
h_x = ,, ,, in compression from the neutral axis.
h_y = ,, ,, in tension from the neutral axis.
h_v = ,, of the axis of tensile reinforcement from the neutral axis.
I = moment of inertia.
l = length.
M = bending moment.
m = $E'' \div E'$.
m_1 = $E_c' \div E_t'$.
m_2 = $E_t'' \div E_t'$.
N = number of bays in a braced girder.
n = numerical order of any bay in a braced girder.
n_1 = ,, ,, portion of flange in a braced girder.
P = normal stress.
P′ = ,, on concrete.
P″ = ,, on steel.
P_c = normal compression.
P_t = ,, tension.
p = intensity of normal stress.
p' = ,, ,, on concrete.
p'' = ,, ,, on steel.
p_c = ,, ,, compression.
p_t = ,, ,, tension.
Q = tangential stress.
q = intensity of tangential stress.
q' = ,, ,, on concrete.
q'' = ,, ,, on steel.
R = resistance of a column.
R = moment of resistance.
R_x = ,, ,, for failure in compression.
R_y = ,, ,, ,, tension.
S = reaction of abutments of a beam.
t = collective width of reinforcement in cross section of a beam.
W = total load.
w = unit load.
z = distance of the axis of tensile reinforcement from the bottom of a beam.

INTRODUCTION

In general building construction the materials employed and the methods followed are practically identical with those employed and followed for hundreds and even thousands of years. Stones or bricks are piled one above the other and lightly held in place by mortar or kindred material, so that the stability of structures so designed depends almost entirely upon the weight and compressive strength of the materials employed.

Conditions have arisen, however, demanding improved methods of construction in which the complex stresses encountered shall be resisted by materials so constituted and so employed that physical properties other than that of compressive strength may be utilised. The first evidence in recent times of this new demand came with the application of cast iron in buildings, to afford strength and stiffness with a minimum expenditure of material. Then came wrought iron, a better material; and finally steel, one better still.

But it happened, as a consequence of inherited views upon the question of building design, that the iron and steel members were not employed in the most scientific manner. Instead of being rigidly connected together and braced so as to form a complete system, of which the members should be capable of affording mutual aid one to another in resisting strain, the iron or steel columns, stanchions, and beams were loosely built into the brickwork. Thus the effect could be no more than local, the valuable properties of the new material could not be fully developed, and the general scheme of construction was not essentially improved.

To realise what science could do when unfettered by traditional art, we have only to turn to such works as the Crystal Palace, an early example of scientific construction in cast iron; and to numerous other structures, that need not be mentioned by name, as exemplifying the correct manner of employing wrought iron and steel for the satisfactory resistance of complex stresses.

Turning again to ordinary building construction, we find the fact soon became recognised that it was inadvisable to insert steel members in a casual and disjointed sort of way. Consequently there grew up the "steel cage" and "skeleton" systems of construction largely practised and highly developed in the United States, and employed to a considerable extent in this country.

While the preservation of steel properly embedded in concrete is beyond dispute, its endurance when merely cased in building stone, brick, or terra-cotta, is an unknown quantity, and the question is now being asked, even by engineers, whether steel is really the most suitable material for many of the purposes to which it has been applied during recent years. For the construction of railway bridges, steamships, exhibition palaces, market halls, water conduits, and other works where steel is openly and avowedly used, it is in every way suitable and appropriate. In the erection of houses, office buildings, warehouses, factories, churches, and public buildings generally, it may properly be used for multifarious purposes, but it is quite open to question whether it should form the skeleton in which is vested the strength of the building itself. The author himself asks this question in the hope that every one of his readers will consider it upon its merits, and without prejudice either against or in favour of steel.

The opinion has been expressed, and is one which appears to grow in favour, that a solution of the problem is to be found in the combination of the old with the new, in the union of concrete with steel so effected that the desirable qualities of each material may be supplemented by those of the other.

Concrete itself is by no means new to the building world. Early in the present era, Vitruvius Pollo wrote of lime concrete as one of the most valuable building materials known, and as one which had been used for many centuries in constructive works where strength and durability were most essential.

The dome of the Pantheon in Rome, more than 140 ft. in diameter, was, practically, built of concrete, and remains to this day after standing for nearly 2,000 years.

A concrete floor in the House of the Vestals with a span of 20 ft. and a thickness of 14 in., and the Aqueduct of Vejus are further examples of early concrete construction, while others are to be found in the remains of ancient buildings in Greece and Mexico.

The walls of Reading Abbey appear to have been built of stone with a concrete core, the concrete still remaining although the stone has long since disappeared.

As additional evidence of the durability of concrete it may be mentioned that the fortifications of Badajos still show the print of the boards used in moulding the concrete work.

If lime concrete, as employed in early times, shows such remarkable qualities, there is sufficient reason for confidence in the durability of concrete made with Portland cement of the excellent quality now available.

So far as steel is concerned there is every assurance as to strength, its only weak point being that of susceptibility to climatic influences. As the physical strength of steel is valuable for supplementing that of concrete in which it may be embedded, so what may be called the "con stitutional" strength of concrete is valuable in preserving steel from corrosion. Upon this aspect of the case there is ample evidence of a satisfactory nature.

Numerous instances are recorded of iron having been embedded in mortar for ages without appreciable change in condition. Iron clamps laid in mortar joints in the Parthenon have been uncovered and found to remain in good condition after a period of fully 2,000 years. Other examples are cited in which ironwork has been found in a perfect state of preservation after having been embedded in lime mortar for upwards of 500 years.

Among more recent experience may be mentioned that of an engineer who, having removed part of a cement floor laid some forty years ago on the iron hull of a ship, found the iron beneath the cement lining to be absolutely free from rust. After the demolition of the old *New York Herald* building, then thirty years old, and in which the ironwork was protected by concrete, search was made for specimens of badly-corroded iron. None such could be found, and in places where the mortar had been in absolute contact with the paint, this also was preserved.

In steel and concrete we have two materials, of which the most prominent features are respectively strength and durability. The problem is to effect their combination in such manner that they may be employed to satisfy the requirements of artistic design and of scientific construction. The solution is not to be found in the employment of the materials separately, as occurs when steel columns are erected and cased with concrete, or when steel girders and joists are laid and buried in the concrete which forms a floor between them. It is to be looked for rather in the union of concrete and steel in such a way as to produce what is practically a new material, permitting the retention of recognised architectural forms, and of the solidity that characterises masonry construction, while ensuring ample stability, strength, and stiffness—the three essential conditions of equilibrium. Compliance with these conditions is rendered possible by the use of the composite material described by the generic term "concrete-steel," and also known as steel-concrete, ferro-concrete, armoured concrete, reinforced concrete, and on the Continent as *béton armé*.

It is well known that the use of concrete alone is limited to classes of work where high compressive strength is one of the chief requirements, as the low tenacity of concrete makes it, like cast iron, an extremely uneconomical material of construction for many purposes.

Taking, as an example, the case of a concrete beam supported at the ends and loaded in the centre, we find the upper or compression side is capable of supporting at least ten times the load that would cause failure of the lower or tension side. This undesirable inequality of resistance can be adjusted by embedding steel so as to take the tension encountered in the lower portion of the beam, thus permitting the strength of the upper part to be fully utilised in supporting much greater loads than would otherwise be possible.

By the suitable incorporation of steel in vertical members equally satisfactory results can be obtained, and as the expansion and contraction of the two constituents under temperature changes differ very little, the combination acts in every way as a composite structure.

Further, it should be stated that concrete, when combined with steel in this way, acquires new and unexpected qualities.

For example, Considère has proved that in concrete-steel beams, the concrete on the tension side will submit without rupture to a proportionate distortion of twenty times that at which it would fail in a tension test of concrete alone.[1] He also shows that concrete can be reinforced in such a manner that the combination acquires high ductility and a crushing resistance considerably higher than the sum of the resistances of the two materials taken separately. These unexpected phenomena, show conclusively that by the employment of concrete-steel far greater economy may be secured than is possible in commonly adopted methods of using concrete and steel. Those who make use of such methods are sometimes aware of the fact that excessive quantities of metal are employed, and that, apart from the consequent waste, the excess is frequently distributed so as to constitute a source of probable weakness, by destroying the continuity of the concrete.

Finally, it may be remarked that while the theory of concrete-steel is now well developed, so that the material may be applied with perfect safety, its natural advantages have frequently been neutralised by indifferent methods of design and construction, and sometimes unwarranted liberties are taken by the adoption of dimensions that are far less than those consistent with the prolonged duration of the structures erected. This consideration constitutes one more reason why the whole subject should be studied by those who may be concerned with it in a professional capacity.

[1] *Report to the Académie des Sciences*, 1902.

CHAPTER I.

CONCRETE.

ALL Portland cement used by the designer of concrete-steel structures should be in compliance with the specification of the Engineering Standards Committee.[1]

1. Aggregate.—With regard to the nature of the aggregate it may be remarked that gravel is very generally favoured, but the sand should always be separated and added afterwards in the correct proportion. When crushed stone is employed, an adequate proportion of sand is added. Crushed brick is not a material to be recommended, chiefly because the strength of the resulting concrete is low. Coke breeze is unsuitable because of its porosity and insufficient strength. Cinder concrete, as one of the best non-conductors of heat, possesses recommendations from the point of view of fire protection, but from the Report of the Hamburg Commission, issued in 1895, it appears that cinders tend to cause corrosion of embedded iron. Hence, when cinder concrete is used, the metal should be completely coated with neat cement, a plan which is adopted in the Monier system. Whatever may be the character of the aggregate used, it must be sufficiently fine in order that the concrete may adhere closely to the metal, and protect it from moisture.

2. Voids in Aggregate.—It is a very important matter that all voids in the aggregate should be filled. The proportion of voids in broken stone can be satisfactorily determined by placing the material in a receptacle of known capacity, pouring in as much water as possible, and

[1] *British Standard Specification for Portland Cement*, 1904.

measuring the volume used. When applied to the measurements of voids in gravel mixed with sand, this method invariably gives results that are too low. The material is rarely dry, and it is quite probable that the voids measured by displacement will be under-estimated by fully 5 per cent. owing to the presence of moisture. With fine gravel and sand a serious error often results from the fact that all the air is not displaced by the water poured into the receptacle. These facts, considered in connexion with the variable proportions of sand and pebbles in mixed ballast, clearly point to the desirability of separating the constituent parts of such material, so that definite and predetermined proportions may be ensured. It is then easy to fill the voids with properly proportioned mortar.

3. **Sand.**—Some notes upon the most suitable description of sand for concrete-steel will be found in Article 5.

4. **Voids in Sand.**—The use of sufficient cement to fill the voids in the sand is absolutely necessary in concrete-steel work. A mortar, consisting of 1 part of Portland cement to 3 parts of sand, is strong and durable, and may be quite suitable for many purposes, although the cement cannot be expected to fill all voids in the sand. The most accurate method of determining voids in sand is that proposed by Mr. Allen Hazen,[1] and consists in determining the specific gravity first of the solid particles, and second of the material as a whole, including the voids. The percentage of space occupied by the solid particles is then obtained by dividing the specific gravity of the mass by the specific gravity of the particles, and the voids are represented by the difference between the whole volume and the space occupied by the particles. If the sand is not absolutely free from water, as is generally the case, a portion must be dried after the experiment to determine the amount of water, and a correction can then be applied. It is not advisable to dry a moist material before taking its specific gravity, as the closeness of packing will probably be different for wet and dry material.

5. **Impermeability.**—In making concrete for use with steel, it must always be remembered that impermeability is quite as important as mere strength. For ordinary con-

[1] *Proc. Am. Soc. C.E.*, vol. xxv., p. 488.

struction, so long as concrete is composed of an aggregate of sufficient hardness, voids are not objectionable within reasonable limits. But where concrete has to protect steel from corrosion by preventing the percolation of water, the greatest care must be taken to render it as solid as possible. Sir Alexander Binnie, whose experience as a waterworks engineer is valuable on this point, says that, "When attempting to make solid impermeable concrete, we must of necessity have water in superabundance, until the mass is made almost gelatinous; and then we get a perfect mixture."[1] It must not be forgotten, however, that, while some of the water used combines with the cement in the process of setting, the remainder will evaporate in the course of time, leaving a corresponding volume of voids. Hence it appears to be impossible to make an absolutely solid and impervious concrete, although the condition may be nearly approached by the adoption of correct proportions and by careful manipulation.

The opinion has been expressed that to make impermeable concrete, the sand should not be too sharp. Mr. Orange,[2] in a discussion upon waterworks in China and

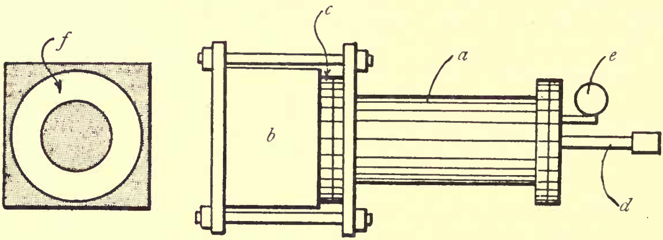


FIG. 1.

a=Cast iron cylinder.
b=Block of concrete.
c=India-rubber packing.
d=Connexion with pump.
e=Pressure gauge.
f=Cement filler.

Japan, said that "if the sand was very sharp, when cement was put with it the water ran through and carried it away.

[1] *Proc. Inst. C.E.*, vol. cvii., p. 125.
[2] *Proc. Inst. C.E.*, vol. c., p. 306.

It had to be a little thick, loamy, and close, in order to make a close concrete. That was why, when the sand was sharp, he always used a certain portion of the siftings from the stonebreaker, which was almost dust, to make the mixture close." This is quite contrary to practice in concrete-steel construction both in this country and in the United States. In a recent paper on the subject, Mr. A. de R. Galbraith [1] points out that "coarse sand, very gritty, and of the greatest cleanness must be used in preference to fine, but whatever the composition of the sand, it is absolutely necessary that it shall be entirely free from all traces of any argillaceous, or earthy matter." Again, to quote from an American source, one of the speakers at a meeting of the Engineers' Society of Western Pennsylvania, referring to a concrete-steel water tank, said that it was the practice in Europe not to use sand that was in the slightest degree dirty, and that he expected to make the tank watertight by giving what is termed the *glassé* finish. [2]

Some experiments were made by Mr. C. H. Colson [3] to ascertain the impermeability of concrete to be used under a constant head of about 46 ft., and in a position where this quality was of the utmost importance. The blocks tested were 24 in. by 24 in. by 18 in., and the apparatus, which is illustrated in Fig. 1, consisted essentially of a cast iron cylinder, connected at one end to the block to be tested, and at the other to a hydraulic pump. The blocks were prepared for testing by removing the top skin, and putting on a cement ring of the same diameter as the flanges of the cylinder, the surface inside the ring being kept quite clean. At a pressure of 20 lb. per square inch there was a slight appearance of dampness in some of the blocks, but in no case, even with the highest pressure, was there much percolation. The results of these tests are given in Table I. Although this table does not suggest any definite relationship between the age of the concrete and the depth to which percolation extended, it shows generally that permeability was less marked in the case of the richer mixtures of concrete subjected to examination.

[1] *Trans. Soc. Eng.*, 1902, p. 179.
[2] *Proc. Eng. Soc. Western Pa.*, vol. xix., p. 145.
[3] *Proc. Inst. C.E.*, vol. cvii., p. 178.

Table I.—Permeability Tests of Portland Cement Concrete Blocks under a pressure of 30 lb. per Square Inch. (Colson).

Proportions.			Age of Block.	Duration of Test at Maximum Pressure.	Depth of Percolation from Pressure side at end of Test.
Cement.	Sand.	Stone.			
Parts.	Parts.	Parts.	Days.	Hours.	Ft. In.
2	3	7	47	2	—
2	3	7	58	2	—
1½	3	7	44	1½	0 6
1½	3	7	48	2	0 7
1½	3	7	50	2	0 8
1½	3	7	52	2	—
1	3	7	33	2	0 10
1	3	7	53	2	0 10
1½	2	7	19	2	0 4
1½	2	7	21	2	—
1½	3	6	51	2	0 6
1½	3	6	57	2	—
1	3	6	48	2	1 6
1	3	6	42	2	1 6
1	3	6	50	2	1 0
2	4	8	50	2	0 10
2	4	8	56	2	1 0
2	3½	8	29	2	0 10

Some more recent experiments as to the watertightness of concrete were conducted in 1902 at the Thayer School of Engineering, in the United States.[1] The concrete consisted of 1 : 2 mortar, ranging from 30 to 45 per cent. of the concrete, and the specimens tested were 5 in. thick. The following results were noted:

Pressure per sq. in.	Number leaky.	Number not leaky.
80 lb.	11	7
40 lb.	5	7
20 lb.	4	0

It was curious that the poorest results were given by one brand of cement, furnished by the makers as being specially adapted for making watertight concrete.

[1] *Proc. Eng. Soc. Western Pa.*, vol. xix., p. 137.

Referring to this aspect of concrete, Mr. J. W. Sandeman,[1] of Newcastle-on-Tyne, says, "Judging from experience, the volume of mortar to insure watertightness should not be less than 50 per cent. of aggregates having 35 per cent. of interstices; adding, therefore, one-third of the volume of the mortar to give the volume of cement and sand, these would equal 66⅔ per cent. of the volume of the aggregates which should allow for all contingencies, and insure the stones or gravel being surrounded with mortar. Finally, to make certain that the mortar shall be watertight, the ratio of sand must be limited to about 2 parts of sand to 1 of cement."

These quotations serve to emphasise the necessity for filling all voids both in the sand and in the aggregate.

6. Proportions of Ingredients.—In cases where hydraulic pressure exists, as in water tanks and conduits, the proportions of the ingredients in concrete should not be less than 1 part of cement to 4 parts of ballast; or 1 part of cement, 1½ parts of sand, and 2½ parts of crushed stone. In works where percolation is not aided by hydraulic pressure, the concrete may be of poorer quality, but for outside work it would be wise to adopt the proportions of 1 to 4. The infiltration of water may be arrested by a rendering of 1 to 1 cement mortar and a facing of neat cement, while the steel may be further protected by being embedded in 1 to 1 cement mortar. With such precautions it may be allowable to adopt poorer concrete than would otherwise be necessary, especially for inside work. However, as absolute confidence in the preservation of the steel is an essential feature in concrete-steel construction, it will always be judicious to spend a little more money in cement than to run the slightest risk of corrosion. The most approved practice in dock and harbour works is to avoid reliance upon a mere facing, and to endeavour to make the entire mass impermeable. This example is certainly worthy of imitation in concrete-steel construction.

As will be shown later, the tensile resistance of concrete is not taken into account in the design of concrete-steel floors, beams, and other members subject to flexure. Consequently little is to be gained by increasing the proportion of cement in the concrete for those parts that are in tension.

[1] *Proc. Inst. C.E.*, vol. cxxi., pp. 219-220.

Table II.—Proportions of Concrete for Concrete-Steel.

Class of Work.	Cement.	Sand.	Stone.	Gravel with Sand.	Authority.
Columns, - -	1	—	—	4	Mr. A. Johnston.
,, - -	1	—	—	4½	M. Considère.
,, - -	1	—	—	3	,,
Walls, - -	1	—	—	6	Mr. A. Johnston.
Partitions, - -	1	—	—	3	,,
Beams, - -	1	—	—	4	,,
,, - -	1	2	4	—	Mr. Moisseiff.
,, - -	1	3	5	—	,,
,, - -	1	—	—	6	M. Considère.
,, Melan, -	1	2½	5	—	Prof. I. O. Baker.
,, Ransome,	1	3	6	—	Mr. Wason.
Floors, - -	1	—	—	4	Mr. A. Johnston.
Columbia, -	1	2½	5	—	Mr. Freitag.
Exp. Met., -	1	—	—	3-5	Newcastle Tests (1899)
,, -	1	2	5	—	Mr. Freitag.
Hennebique, -	1	—	—	4	Dr. W. Ritter.
Melan, - -	1	2	4	—	Mr. Freitag.
,, Upper ⅓rd,	1	2	—	—	Mr. Freitag.
Ransome,	1	3	6	—	Mr. Wason.
Roebling, -	1	2½	6	—	Mr. Freitag.
Arches, - -	1	2½	5	—	Mr. Buck.
,, - -	1	2	4	—	Mr. Moisseiff.
,, - -	1	3	5	—	Mr. Moisseiff.

But the opposite is the case in connexion with compression, which is resisted chiefly by the concrete. If the strength of concrete in the compression portions be increased with the idea of reducing the depth of floor panels, it does not follow that the cost will be correspondingly reduced. The moment of resistance for a rectangular beam being $R = fbd^2 \div 6$, it is evident that resistance is directly proportional to the square of the depth, and a small decrease of depth consequently means a comparatively large decrease of strength. At the same time, as the quantity of concrete decreases proportionately to the first power of the reduction in depth, the saving of concrete will be comparatively small. Further, the cost of concrete decreases less rapidly than the quantity, inasmuch as cost always includes expenses which are unaffected by a slight diminution of depth.

A further point worthy of mention, and confirmed by the recent investigations of M. Considère, is that the cracks appearing in exposed tension members are more serious in rich than in poor mixtures of concrete. In compression members, however, there are satisfactory reasons why richer proportions should be adopted. In the first place, the concrete is largely, if not entirely, relied upon for resistance; and in the second, it appears from the results obtained by Considère that as a consequence of the first load the elastic limit of the concrete increases very considerably with the proportion of cement. Consequently, where high compression resistances are necessary, it may frequently be desirable to use cement freely, especially as the increase of cost will not be great. Columns and foundations are of far more importance than beams and floor joists, for their failure may involve the collapse of an entire building, whereas the failure of a floor, although sufficiently undesirable, is of less consequence. Finally, the use of richer concrete for compression members may sometimes permit the adoption of a smaller cross section, thus reducing the space occupied by columns and stanchions, an advantage that will often be appreciated by those occupied with the design of buildings.

The proportions stated in Table II. are some of those adopted in various forms of concrete-steel construction.

7. Mixing—Whatever may be the proportions chosen, the value of the concrete will materially depend upon the thoroughness of mixing. In his *Treatise on Masonry Construction*, Professor Ira O. Baker says that: "It is possible to increase the strength of really good concrete 100 per cent. by prolonged trituration and rubbing together of its constituents." In practice it is frequently cheaper to use more cement or concrete and less labour, but for concrete-steel work no substitute can be accepted for the most thorough mixing of the ingredients. Good concrete can be secured either by hand or by machine mixing, but in the former case the difficulty is to get the work done properly. The material must be turned over thoroughly, the shovels must find their way to the boards, and the mixture must be picked up from the bottom and turned over and over again.

This is hard work, and, especially in warm weather, the men are inclined to become optimistic, and to believe that a batch of concrete is finished, when in reality it is only about half mixed. When machine mixing is adopted, no adverse human intellect opposes the completion of the process, and the result is far more likely to be satisfactory. Various forms of concrete mixers are available in the present day.

A simple and effective apparatus used on the new Vauxhall Bridge works consisted of a horizontal vessel of segmental form provided with a series of stirring blades on a central shaft. The receptacle had an open top, and could be rotated about the shaft so as to tip out the contents when required, while the revolution of the blades still continued. Ballast was brought in skips along a tramway ending opposite the hopper of the machine, each skip containing the exact quantity for a batch of concrete, and the necessary volume of cement was measured into a box on the opposite side of the hopper. The materials were tipped into the machine at the proper time, sufficient water being added from a tap in a convenient position. The entire apparatus was fixed below the level of the decking, so that it could be easily managed and the process observed without trouble.

Some useful data as to the relative strength of hand and machine-mixed concrete, recorded in the Report of the Watertown Arsenal for 1897,[1] relate to two series of concrete piers, one series being hand mixed and the other machine mixed. The average compressive strength of the former was 989 lb. per square inch, and of the latter 1,098 lb. per square inch, thus showing an advantage of 11 per cent. in favour of machine mixing.

Another test made at the same place showed an advantage of more than 25 per cent. in favour of machine mixing.

8. Ramming and Bonding.—Other essential points connected with the manipulation of concrete employed in combination with steel are uniformity and sufficiency of ramming, and the careful bonding of fresh and set concrete so that monolithic construction may be secured. Care

[1] *Report of the Watertown Arsenal*, 1897. "Tests of Metals and other Materials for Industrial Purposes," pp. 349-385.

must also be taken to ensure perfect contact between the concrete and the steel.

9. Physical Properties.—From what has already been said, it is clear that the widest differences are to be expected in records as to the strength of concrete. Even in iron and steel, which by comparison are simple in nature and constitution, considerable variations exist, but when either material is ordered from the maker subject to the provisions of a given specification there is every reason for anticipating more or less exact compliance with the expressed requirements.

Concrete cannot be purchased in the manufactured form, and the physical properties of the finished article vary from case to case, even when the ingredients are of apparently uniform quality, and when the greatest care is taken to ensure uniformity of treatment. Portland cement itself is now made according to scientific principles, and is carefully tested by all reputable makers; the tests, however, cannot be conducted upon the material in its manufactured state, but only after manipulation, the influence of which is a variable quantity. Sand and aggregate vary from place to place, and in the same place from day to day or hour to hour. There is little wonder, therefore, that the results of experiments upon concrete, conducted in scientific laboratories, do not afford the guidance desired by the engineer. Nevertheless, the data obtained by purely scientific investigation are of much value, for they serve to demonstrate the great possibilities of concrete construction, and lead practical men to think of improved methods by the aid of which higher standards of strength may be attained.

In the notes which follow some of the most reliable records, collected from various sources, are presented in a form which is intended to be convenient for comparison and reference.

10. Compressive Strength.—Part of the resistance of concrete to crushing is due to the frictional resistance of one piece of aggregate to motion along the face of another piece. It is also the fact that the cement adheres more tightly to the irregular surface of broken stone than to the smooth surface of pebbles.

Table III. gives some figures calculated from the results of a series of experiments made in 1897 by Mr. A. W. Dow, for the Engineering Department of the District of Columbia, U.S.A. The tests were made upon 12-in. cubes, and it is worthy of note that the gravel concrete shows an important increase of strength with age.

Table III.—Relative Compressive Strength of Gravel and Broken Stone Portland Cement Concrete.

Age when Tested.	Strength of Gravel Concrete. (Stone Concrete = 100.)	No. of Tests.
10 days.	76	2
45 ,,	91	2
3 months.	119	2
6 ,,	73	2
12 ,,	108	5

According to the results obtained from a series of forty-eight tests made in France upon 4 in. cubes,[1] it appears that the compressive strength of gravel concrete is less than 80 per cent. that of concrete made with broken stone. From these figures it may be inferred that an aggregate of broken stone is specially suitable for all members in which compressive strength is the chief factor for consideration.

Providing the volume of mortar is approximately equal to the voids in the aggregate, the strength of concrete increases with the quantity and the strength of the mortar used. This aspect of the case is demonstrated by Fig. 2, in which are plotted the mean results of a series of experiments made by Mr. G. W. Rafter[2] upon 12-in. cubes, the voids of the aggregate being practically filled with mortar varying from 1 : 1 to 1 : 6 in composition. Proportionately similar results were also evidenced by the French experiments to which reference has already been made.

[1] *Cements et Chaux Hydrauliques.* Paris, 1891.
[2] *Report of the New York State Engineer.* 1897.

Three varieties of Portland cement were used by Mr. Rafter, the sand was clean pure silica with 32 per cent. of voids, and the aggregate consisted of sandstone broken so as to pass through a 2 in. ring, and containing 37 per cent. of voids when rammed. In half the specimens tested the mortar used was rather more than enough to fill the

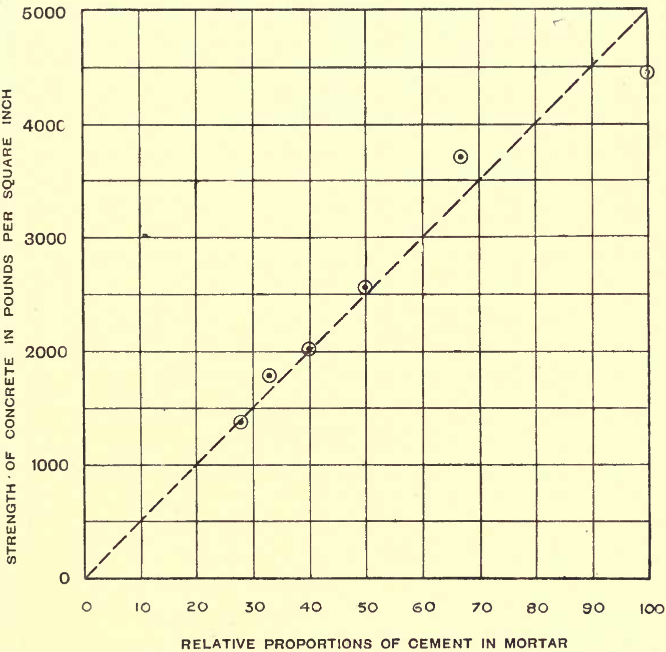
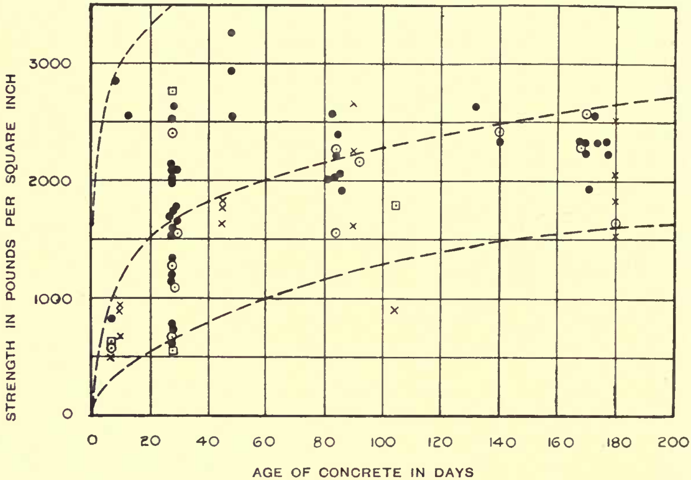


FIG. 2.

voids, and in the other half the proportion was equal to about 80 per cent. of the voids. The concrete was mixed about as "dry as damp earth."

It may be mentioned that companion cubes mixed with mortar of the "ordinary consistency" used by the average mason developed about 10 per cent. less strength than the cubes prepared with the drier mortar.

Some recent data as to the resistance of concrete to direct compression are summarised in Table IV., and the most useful results condensed in this table and relating to ages up to six months are plotted in Fig. 3.



⊡ = Results of tests upon 1 : 5 concrete.
• = " " 1 : 6 "
⊙ = " " 1 : 7 "
× = " " 1 : 8 "

FIG. 3.

The figures in the first three lines of Table IV. show clearly the increase of strength with age, and similar evidence is afforded by tests of concrete blocks made at various dates during the progress of the work at the Vyrnwy dam.[1] Table V. shows approximately the average crushing strength of blocks which measured about 9 in. cube and were tested at the end of 1885. Several of the blocks prepared with Parian cement on the top face gave rather higher results than the average, but others cut from the work and not so prepared gave even higher results.

[1] *Report as to the Vyrnwy Dam*, by George F. Deacon. 1885.

Table IV.—Compressive Strength of Portland Cement Concrete Cubes of Various Proportions and Ages, in pounds per square inch.

Proportions.	Age when Tested.					Size of Cube.	Authority.
	Four Weeks.	Six Weeks.	Three Months.	Six Months.	Twelve Months.		
Parts.	lb.	lb.	lb.	lb.	lb.	in.	
1 : 2 : 4	1,712	—	2,182	2,332	—	12	Mr. Costigan, Proc. Can. Soc. C.E., 1901-2.
1 : 2 : 5	991	—	1,997	2,240	—	12	
1 : 2 : 6	—	1,756	2,190	1,987	2,670	12	Mr. Dow, Eng. Dept., Washington, 1897.
1 : 2 : 3	2,783	—	—	—	—	12	Dr. Dycherkoff, in Der Portland Cement und seine Anwendungen in Bauwesen.
1 : 2 : 5	2,414	—	—	—	—	12	
1 : 3 : 5	1,661	—	—	—	—	12	
1 : 3 : 6½	1,534	—	—	—	—	12	
1 : 2 : 4	—	2,734	—	—	—	6	Dr. M'Kenna, Columbia University, U.S.A.
1 : 2 : 4	—	3,283	—	—	—	6	
1 : 2 : 3	—	—	—	—	1,751	6	Mr. M'Caustland, Proc. Am. Soc. C.E., vol. xxix.
1 : 2 : 5	—	—	—	—	1,090	6	
1 : 3 : 4	—	—	—	—	1,432	6	
1 : 3 : 5	—	—	—	—	1,092	6	
1 : 3 : 5	—	—	3,187	—	—	12	Mr. Wason, Proc. Am. Soc. C.E., vol. xxvii.
	—	—	4,263	—	—	12	
1 : 3	—	3,500	—	—	—	4	Report of Austrian Soc. Eng. & Archts., 1895.
1 : 2 : 4	2,350	2,450	3,000	3,800	—	12	Tests at Watertown Arsenal for the Boston Elevated Railway, 1899.
1 : 3 : 6	2,150	2,200	2,510	3,250	—	12	

Other experimental data not quite suitable for inclusion in Table IV. will be found in Table VI.

Table V.—Average Compression Strength of Portland Cement Concrete Blocks made at the Vyrnwy Dam.

Average Age when Tested.	Average Strength.
1.2 months.	1,772 lb. per sq. inch.
3.5 ,,	1,586 ,,
21.2 ,,	2,472 ,,
29.0 ,,	2,519 ,,
34.0 ,,	2,799 ,,

However satisfactory it may be to know that the strength of concrete grows with age, a matter of more immediate practical importance is to be acquainted with its properties, and to be able to predict the probable behaviour of concrete during the earlier stages of hardening. In most structural works this is the critical period when loads must be carried by the concrete before its maximum strength has been developed.

From the data now presented it is clear that the compressive strength of concrete is an extremely variable quantity, but that with adequate proportions and careful manipulation there should be no difficulty in securing a compressive strength of 2.500 lb. per square inch with 1 : 6 concrete at the age of two or three months. The figures indicate that considerably greater strength may be attained with richer proportions and under specially favourable conditions, but where concrete is mixed, as usual, on the site of the works to be executed, it cannot be recommended that the ultimate resistance should be estimated at more than 2,500 lb. per square inch. The limit suggested by the middle curve in Fig. 3 is 1700 lb. at one month and 2,200 lb. at three months, and by the bottom curve 700 lb. at one month and 1,200 lb. at three months; but it should be remembered that the results in the diagram refer principally to 1 : 6 and 1 : 8 concrete, and that the middle curve practically represents the average for the former

Table VI.—Compressive Strength of Portland Cement Concrete Blocks of Various Proportions and Ages, in pounds per square inch.

Proportions.			Age when Tested.	Lb. per sq. in.	Size of Block.	Authority.
Cement.	Sand.	Aggregate.				
1	1	voids filled	600 days	4,467	12-in. cube	Report of New York State Engineer, 1897.
1	2	voids filled	600 days	3,731	12-in. cube	Report of New York State Engineer, 1897.
1	3	voids filled	600 days	2,553	12-in. cube	Report of New York State Engineer, 1897.
1	3	6	28 days	2,455	8 in. by 8 in. by 12 in.	Mr. Wason, Proc. Am. Soc. C.E., vol. xxvii.
1	2	4	7 years	7,000-	4-inch cube	Mr. Falk, Proc. Am. Soc. C.E., vol. xxix.
1	3	5	7 years	9,000	4-inch cube	Mr. Falk, Proc. Am. Soc. C.E., vol. xxix.
1	2	4	9 days	2,880	12 in. by 8 in. dia.	Prof. Hatt, Purdue Univ., U.S.A., 1902.
			14 days	2,575		
1	—	3	39 days	2,850-3,175	3½ in. by 3½ in. by 10 in.	Report of Austrian Soc. Eng. and Archts., 1895.
1	2	4	10 days	1,575	12 in. cube	Tests at Watertown Arsenal for the Boston Elevated Railway, 1899.
1	2	4	20 days	2,000	12 in. cube	Tests at Watertown Arsenal for the Boston Elevated Railway, 1899.
1	3	6	10 days	1,450	12 in. cube	Tests at Watertown Arsenal for the Boston Elevated Railway, 1899.
1	3	6	20 days	1,750	12 in. cube	Tests at Watertown Arsenal for the Boston Elevated Railway, 1899.
1	2	4 (cinder)	8 days	502	12 in. by 8 in. dia.	Prof. Hatt, Purdue Univ., U.S.A., 1902.

mixture, and somewhat more than the average for the latter. Fig. 4 is a diagram combining the averages given in Tables IV., V., and VI. This diagram serves to show the gaps remaining to be filled by further experimental research, and to suggest that the compressive strength of well-made

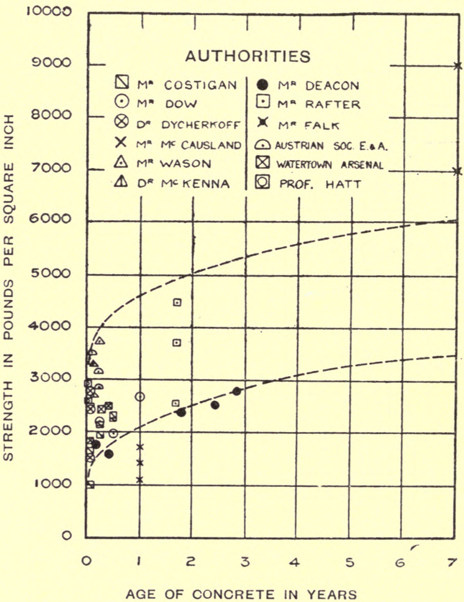


FIG. 4.

concrete is probably very much greater than has been supposed hitherto.

The compressive strength of coke concrete is said by Professor Ira O. Baker to vary from 600 lb. to 700 lb. per square inch, but this estimate appears to be somewhat high.

11. Tensile Strength.—Comparatively few experiments have been made for the purpose of determining the tensile strength of concrete, especially in Great Britain.

Table VII.—Tensile Strength of Portland Cement Concrete.

Proportions.	Age when Tested.	Tensile Strength lb. per sq. in.	Authority.
1 : 3	4½ months.	242	Report of Aust. Soc. Eng. and Archts., 1895.
1 : 3	,,	254	Report of Aust. Soc. Eng. and Archts., 1895.
1 : 4	—	356	Prof. Brik, 1901.
1 : 2 : 4	26 days	280	Prof. Hatt, Purdue Univ., 1902.
1 : 2 : 4	28 ,,	360	Prof. Hatt, Purdue Univ., 1902.
1 : 2 : 4	33 ,,	305	Prof. Hatt, Purdue Univ., 1902.
1 : 2 : 4	35 ,,	300	Prof. Hatt, Purdue Univ., 1902.
1 : 2 : 4 (cinder)	—	82	Prof. Hatt, Purdue Univ., 1902.

The results given in Table VII., as well as the investigations of Hartig and Durand-Claye, show that the tensile strength of concrete may safely be taken between 200 lb. and 300 lb. per square inch.

12. Transverse Strength.—Among tests of transverse strength those contained in Table VIII. represent the averages obtained from tests of concrete bars under various conditions and at various ages.

The investigation made by Mr. Bruce[1] covered 191 tests on concrete bars 30 in. long by 4 in. square, and was chiefly intended to ascertain the growth of transverse strength with age. Many of the tests were upon types of concrete unsuitable for use in combination with steel. The modulus of rupture (f_r) was calculated by the familiar formula

$$f_r = \frac{3}{2} \cdot \frac{Wl}{bd^2}.$$

Other recent experiments made with the object of determining the transverse strength of concrete do not add materially to the information contained in Table VIII., and in some cases the conditions are so variable that it is impossible to draw any general conclusions or to compare the results with the averages already given.

13. Shearing Strength.—According to M. Teret, Director of the Laboratory of the Ponts et Chaussées, Boulogne, the

[1] *Proc. Inst. C.E.*, vol. cxiii., p. 217; vol. cxviii., p. 388.

Table VIII.—Transverse Strength of Portland Cement Concrete.

Proportions.	Age when Tested.	Modulus of Rupture. lb. per sq. in.	Dimensions of Bars.	Authority.
1 : 3	6 months	654	Not stated	Prof. Hanisch and J. Spitzer.
1 : 3½	,,	518	,,	
1 : 4	,,	384	,,	
1 : 5	,,	256	,,	
1 : 6	,,	192	,,	
1 : 2 : 3	,,	303	30 in. by 4 in. by 4 in.	Mr. Bruce, Proc. Inst. C.E., vol. cxiii.
1 : 2½ : 5	,,	236		
1 : 3 : 5	,,	214		
1 : 2½ : 5	28 days	332	30 in. by 10 in. by 10 in.	Prof. C. Bach, Stuttgart, 1895.
1 : 2 : 4	7 years	296	19 in. by 4 in. by 4 in.	Mr. Falk, Proc. Am. Soc. C.E., vol. xxix.
1 : 3 : 5	,,	247	38 in. by 4 in. by 4 in.	
1 : 3 : 5	,,	208	19 in. by 4 in. by 4 in.	
1 : 2 : 4	28 days	540	80 in. by 8 in. by 8 in.	Prof. Hatt, Purdue Univ., U.S.A., 1902.
1 : 2 : 4	7 ,,	397	40 in. by 8 in. by 8 in.	
1 : 2 : 4	8 ,,	352	80 in. by 8 in. by 8 in.	
1 : 2 : 4 (cinder)	14 ,,	140	80 in. by 8 in. by 8 in.	
1 : 2 : 4 (cinder)	11 ,,	129	40 in. by 8 in. by 8 in.	

shearing resistance of concrete varies from 0.16 to 0.2 of its compressive strength.

14. Coefficients of Elasticity.—In the light of modern investigation it appears to be perfectly clear that the values hitherto assumed for the modulus of elasticity for concrete are much too low.

Examination of Tables IX. and X. will show the remarkable differences which exist in determinations of the coefficient of elasticity of concrete, both in compression and in tension. According to Professor Hatt and others, the value for E'_c is about double that for E'_t, while in the well-known tests of the Austrian Society of Engineers and

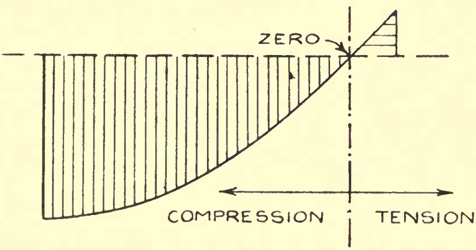


FIG. 5.

Architects, E'_t was shown to be about 20 per cent. greater in value than E'_c. In addition to affording another clue to the elastic properties of concrete, Hartig's experiments[1] serve to suggest a reason for the higher value of E'_t. Hartig found that at zero stress 3,775,410 lb. per square inch was the value of the coefficient both for tension and for compression. The value increased with tension, at a maximum of 549 lb. per square inch, up to 4,304,394 lb., and decreased with compression, at a maximum of 1,806 lbs. per square inch, to 1,901,214 lb. The latter results are graphically represented in Fig. 5.

In the course of an extended investigation at the Royal Testing Laboratory, Stuttgart, Professor C. Bach has shown

[1] *Civilingenieur*: "Das elastiche Verhalten der Moertel und Moertel bindematerialen." 1893.

Table IX.—Coefficient of Elasticity of Portland Cement Concrete in Compression.

Proportions.	Age when Tested.	Value of E_c lb. per sq. in.	Method of Computation.	Authority.
1 : 3	35-39 days	4,660,000 5,180,000	From total deformations.	Report of Austrian Soc. Eng. and Archts., 1895.
1 : 2 : 3	12 months	1,912,500	For elastic deformation at 500 lb. per sq. in.	Mr. M'Caustland, Proc. Am. Soc., C.E., vol. xxix.
1 : 2 : 5	,,	1,240,833		
1 : 3 : 4	,,	1,336,500		
1 : 3 : 5	,,	1,343,666		
1 : 3 : 6	35·47 days	2,500,000 2,870,000 4,545,000	At 700 lb. stress.	Mr. Wason, Proc. Am. Soc. C.E., vol. xxvii.
1 : 5 (gravel)	6 days	2,088,000	With regard to "set" after previous loads.	Prof. Hatt, Purdue Univ., 1902.
1 : 2 : 4 (stone)	9 days	4,702,000	At 750 lb. per sq. in.	
,,	9 ,,	3,940,000	,, 1,500 ,,	
,,	14 ,,	4,340,000	,, 750 ,,	
,,	14 ,,	3,680,000	,, 1,500 ,,	
1 : 7½	28 ,,	386,000	—	Prof. C. Bach, Stuttgart, 1895.
1 : 9	28 ,,	301,000		
1 : 2 : 4 (cinder)	8 ,,	580,500	With regard to "set" as above.	Prof. Hatt. Purdue Univ., 1902.

Table X.—Coefficient of Elasticity of Portland Cement Concrete in Tension.

Proportions.	Age when Tested.	Value of E_t lb. per sq. in.	Method of Computation.	Authority.
1 : 3	35-39 days	5,690,000	From total deformation.	Report of Austrian Soc. Eng. and Archts., 1895.
		6,220,000		
1 : 4	—		From deflection of floor at—	Professor Brik, 1901.
		2,490,000	199 lb. stress per sq. inch.	
		1,990,000	292 ,, ,,	
		1,422,000	328 ,, ,,	
		1,025,000	342 ,, ,,	
		600,000	356 ,, ,,	
1 : 5	2½-3 months	938,000	From deflection of floor plates.	Professor Melan, 1899.
1 : 2 : 4	35 days	2,700,000	With regard to "set" after previous loads.	Professor Hatt, Purdue Univ., 1902.
	33 ,,	2,400,000		
	28 ,,	1,400,000		
	26 ,,	1,900,000		
1 : 2 : 4	7 years	1,346,500	From elastic deflection of beams.	Mr. Falk, Proc. Am. Soc. C.E., vol. xxix.
1 : 3 : 5	,,	908,333		

that Hooke's law of the proportion between stress and strain does not hold good for concrete.

Further, as a result of tests at Watertown Arsenal,[1] Mr. Wason found that Hooke's law did not apply closely, except within the stress limit of 700 lb. per square inch.

These records help to explain some of the variations as to the value of the modulus of elasticity stated in Tables IX. and X. In the former table it will be seen that the stresses at which computations were made range from 500 lb. to 1,500 lb. per square inch, and in the latter the range is from 199 lb. per square inch upwards. The figures in Table X., relating to tests by Professor Brik, are particularly worthy of study, as they show great reduction in the value of E'_t, with increase of tensile stress, this result being the converse of that observed by Hartig. It is also worthy of notice that some results are based upon elastic, and others upon total deformation, thus constituting another reason for considerable differences.

As a matter of fact, some of the coefficients stated in Tables IX. and X. represent the modulus of deformation rather than of elasticity. The coefficient of deformation is certainly of service in concrete-steel design, but experimental results should in future be recorded so that no misconception may exist as to their interpretation. Differences of value are also due to the proportions of the concrete, and, generally, there is much need for additional and more precise data as to the elastic properties of this material.

Taking general averages of the figures in Tables IX. and X. we obtain the following approximations:

Elasticity of concrete in compression, 2,700,000 lb. per square inch.

Elasticity of concrete in tension, 2,200,000 lb. per square inch.

Engineers who are concerned with concrete-steel construction now frequently adopt the value of 2,800,000 lb. per square inch, for the modulus of elasticity, without complicating formulas by drawing any distinction between the values for tension and compression. In some cases it may be prudent to adopt considerably lower figures.

It may here be remarked that the variation of the value

[1] *Report of the Watertown Arsenal*, 1897.

of E, with increasing stress intensity, is of considerable importance to the designer of concrete-steel structures, for the more the concrete elongates per unit of stress, the greater is the proportion of stress carried by the steel.

15. Elastic Limit.—Although, strictly speaking, concrete has no elastic limit, there is a point in test diagrams beyond which a marked change in behaviour is noticeable, and it will be convenient to regard this point as denoting the elastic limit.

A careful study of all available records leads to the conclusion that what we may term the elastic limit of concrete in compression, with proportions not poorer than 1 : 3 : 6, may be placed at about 1,000 lb. per square inch. In the case of 1 : 2 : 4 mixtures, the elastic limit in compression may be as high as at 2,000 lb. per sq. in. Variations depend very much upon the proportion of cement used, and the recent investigations of M. Considère show that the elastic limit may be raised very materially by the imposition of test loads upon concrete members. It is well known that the elastic limit of iron and steel may be modified by the intermittent application of loads, but attention has not previously been directed to a similar phenomenon in connexion with concrete. (See Article 99.)

16. Coefficient of Expansion.—One constant which has received too little attention from investigators is the coefficient of expansion of concrete. It is generally assumed that this is of approximately equal value with the coefficient of expansion of iron and steel.

As general assumptions are not desirable in constructional work, we now give the results obtained from an investigation extending over two years, and conducted under the direction of Professor W. D. Pence in the School of Civil Engineering, Purdue University, U.S.A.[1] Three series of tests were made, each series including ten tests upon 1 : 2 : 4 concrete. In the first series the aggregate was hand-broken oölitic limestone; in the second it was limestone broken in a crusher, and a bar of the unbroken stone was also tested; and in the third series it was pit gravel of good quality. The following are the recorded results :

[1] *Journal of the Western Society of Engineers*, vol. vi., 1901.

Material tested.	Coefficient of expansion per deg. Fahr.
Gravel concrete, 1 : 2 : 4, - - -	0.0000054
Stone ,, 1 : 2 : 4, - - -	0.0000055
Limestone rock, - - - -	0.0000054

The above results indicate that the coefficient of expansion of broken stone concrete is not materially different from that of the solid rock from which the aggregate was crushed. From this investigation it may be concluded that the coefficient of expansion of concrete is about 0.0000055 per degree Fahrenheit, this being conveniently borne in mind as "five zeros fifty five."

More recent experiments show slightly higher results. Two bars of concrete tested by Mr. C. A. Lyford at the Worcester Polytechnic Institute, U.S.A., gave 0.0000056 and 0.0000064 as the mean values between 70 deg. and 120 deg. Fahr.

The values of the coefficient of expansion here stated for concrete should be compared with those given for steel in Article 23.

17. Contraction and Expansion during Setting.—Another characteristic of concrete which should be remembered is that, when hardened in air, it shows a decided tendency to shrink, causing internal stresses of considerable intensity. Evidence of this property is afforded by the experiments described by M. Considère in 1899 at a meeting of the French Academy of Sciences (see also Art. 96). With water-hardened concrete the case is different, and the material expands instead of contracting during the process of setting.

CHAPTER II.

STEEL.

18. Tensile and Compressive Strength and Elastic Limit. —From the report of the Steel Committee, 1868-70, it is clear that within the yield point of steel, the amount of lengthening from tension, or of shortening from compression, caused by equal forces per unit of area, is nearly the same, and further that the stress at which the material yields or breaks down is practically the same for tension and compression. The tests conducted by the committee were upon bars $1\frac{1}{2}$ inches in diameter and 10 feet in length between the measuring points, and owing to this unusual length the results obtained were particularly accurate. One noteworthy point is the uniformity of behaviour, within the elastic limit, of all the bars, irrespective of differences in manufacture. Some average values taken from the report of this Committee are recorded for reference in Table XI.

Table XI.—Tensile and Compressive Strength of Steel Bars.

Type and Mark of Steel.		Tension.		Compression.
		Ultimate Stress lb. per sq. in.	Yielding Stress lb. per sq. in.	Yielding Stress lb. per sq. in.
Bessemer	"SLW"	75,443	39,200	35,840
	"H"	76,585	44,800	47,398
	"K"	79,296	43,680	40,320
Cast	"31"	75,376	59,920	60,480
	"H"	93,744	60,480	58,240
	"46"	97,395	58,240	57,120
Crucible Cast	"KB"	77,145	45,920	53,760
	"NB"	114,262	57,120	58,598
	"SC"	118,182	58,240	58,979

The tensile strength and elastic limit in tension and compression of low carbon or mild steel, and of high carbon or hard steel, are shown in Table XII., which is calculated from the results of tests made by Professor Bauschinger at the Ternitz Steel Works.

Table XII.—Tensile Strength and Elastic Limit of Steel in Tension and in Compression.

Percentage of Carbon.	Ultimate Tensile Strength.	Elastic Limit in Tension.	Elastic Limit in Compression.
	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.
0.14 - -	62,944	41,955	39,536
0.19 - -	68,096	47,084	43,008
0.46 - -	75,712	49,056	48,944
0.51 - -	79,744	48,428	46,233
0.54 - -	79,072	49,616	48,944
0.55 - -	80,416	46,995	49,772
0.57 - -	79,744	47,084	48,944
0.66 - -	89,600	53,244	53,648
0.78 - -	92,064	53,312	53,648
0.80 - -	102,816	57,008	63,168
0.87 - -	104,608	61,017	56,000
0.96 - -	118,048	69,216	71,120

For the tension tests, the plates used measured 24 in. long by 2.8 in. wide by 0.48 in. thick, and for the compression tests, the specimens measured 3.6 in. long by 1.2 in. square. The percentage of carbon in steel for various purposes is stated below as an index to the percentages of carbon stated in Table XII., and other tables:

	Per cent.
Boiler plate, - - -	From 0.10 to 0.33
Wire ropes (ordinary), -	,, 0.15 to 0.30
,, (hard), - -	,, 0.45 to 0.75
Structural sections, - -	,, 0.20 to 0.25
Rails, - - - -	,, 0.30 to 0.60
Tool steel, - - -	,, 0.70 to 1.40

19. Shearing Strength.—The resistance of steel to shearing stress varies somewhat, according to the direction in which force is exerted with respect to the structure developed in rolling the plate or bar.

As a general rule, however, it may be taken that the ultimate shearing stress of steel is from 70 to 75 per cent. of the ultimate tensile stress. For mild steel the value is about 50,000 lb. per square inch, and for hard steel about 80,000 lb. per square inch.

20. Coefficients of Elasticity.—With regard to the coefficients of direct elasticity, it should be noted that, as in the case of concrete, values are calculated by some investigators from the total deformation, and by others from the elastic deformation only.

In Table XIII. are given the values of the coefficients according to the two modes of measurement, as ascertained by the tests of the Steel Committee. Here, E is the ordinary coefficient, and E_1 is the coefficient calculated after deducting permanent set.

Table XIII.—Coefficients of Elasticity of Steel in Tension and Compression at Different Loads.

Stress in lb. per square inch.	Values in 1,000,000-lb. units per sq. in.	
Tension.	E.	E_1.
15,232	28.7	28.7
17,785	28.6	28.6
20,316	28.6	29.0
22,870	28.9	27.4
25,401	28.5	29.1
27,955	28.5	29.4
30,486	28.2	29.3
38,102	26.6	30.0
Compression.		
23,251	29.7	30.1
31,001	29.5	30.3
34,227	29.2	30.8
36,176	28.9	30.6
38,774	28.0	30.5
41,350	24.0	30.1

Table XIV. shows the value of the coefficient E for different qualities of steel, and Table XV. indicates the variation of E, with the percentage of the carbon in the steel, and according to the nature of the test. The latter table is compiled from a series of records given by Professor Bauschinger.

Table XIV.—Coefficient of Elasticity of Steel of Different Kinds.

(Values of E in 1,000,000-lb. units per sq. in.).

Type of Steel.	E.	Percentage of Carbon.	Authority.
Soft cast,	30.3	—	Kupffer.
Hard file,	30.1	—	Kupffer.
Hammered Bessemer,	30.3	1.15	Knut Styffe.
Rolled cast,	31.2	1.22	Knut Styffe.
Krupp cast,	31.3	0.61	Knut Styffe.
Rolled puddled,	29.9	0.56	Knut Styffe.
Siemens-Martin,	29.9	—	Bauschinger.
Bessemer,	32.4	0.19 to 0.96	Bauschinger.

Table XV.—Coefficient of Elasticity of Steel with Different Percentages of Carbon under Different Tests.

Percentage of Carbon.	Values of E in 1,000,000-lb. units per sq. in.			
	Tension.	Compression.	Bending.	Mean.
0.19	30.8	37.0	29.1	32.9
0.46	32.0	32.7	29.3	31.8
0.54	30.6	36.1	28.8	32.4
0.57	30.7	32.0	29.2	31.0
0.66	32.4	35.7	32.1	33.6
0.78	33.5	32.4	30.1	32.4
0.80	30.5	32.3	32.9	31.7
0.87	31.0	31.5	30.4	31.1
0.96	30.9	32.7	29.2	31.2

From the foregoing tables it will be seen that the mechanical and elastic strength of steel is fairly uniform for the various qualities manufactured, and that the coefficient of elasticity is practically constant within the elastic limit.

21. Effect of High Temperatures.—An important matter in connection with steel used in structural work is the effect of temperature on its strength and ductility. The risk of fire has often to be considered, and it is very necessary for the designer to be able to estimate the probable influence of heat upon the metal.

Sir William Fairbairn's tests[1] upon iron plates and rivet iron showed that the strength of the former was not materially decreased until temperatures of over 1,200 deg. Fahr. were approached, and that the strength of the latter increased from 62,720 lb. per square inch at 60 deg. to 86,016 lb. at 435 deg. Fahr., while at 1,290 deg. the strength decreased to 35,840 lb. per square inch.

More precise experiments by Knut Styffe indicate that at temperatures between 212 deg. and 392 deg. Fahr. the strength of steel is practically the same as at normal temperatures.

According to the results obtained by Mr. C. Huston[2], in testing steel at temperatures ranging up to 932 deg. Fahr. there is gain of strength and loss of ductility with increase of temperature.

From a table calculated by Roelker[3] we take the following figures relative to Bessemer steel:

Temperature in Degs. Fahr.							
0	100	300	500	700	1,000	1,500	2,000
Percentage of Original Strength.							
100	100	100	98.5	68	31	12	5

Among the most important tests of this nature are those made for the Admiralty by Mr. Barnaby. Table XVI. embodies some of the results quoted by Professor Unwin,[4] the tensile strength being here stated in pounds per square inch. The bars were partly heated in oil and partly in sand, temperature being judged by the colour of the metal.

[1] *Report of the British Association*, 1857.
[2] *Proc. Inst. C.E.*, vol. liii., p. 304.
[3] *Proc. Inst. C.E.*, vol. lxvii., p. 437.
[4] *The Testing of Materials of Construction*, by W. C. Unwin, p. 300.

Table XVI.—The Influence of Temperature on the Tensile Strength and Elongation of Steel. (Elongation in a Length of 8 *in.)*

Type of Steel.	Temperature.	Tensile Strength.	Elongation.
	Degs. Fahr.	Lb. per sq. in.	Percentage.
Bessemer, - - - -	60	58,396	27.34
,, - - - - -	450	90,720	14.06
,, - - - - -	520	86,688	17.18
,, - - - - -	880	53,804	26.56
,, - - - - -	60	63,817	21.87
,, - - - - -	430	86,016	18.75
,, - - - - -	550	74,099	17.18
,, - - - - -	580	41,574	25.00
Siemens-Martin, - - -	60	65,184	25.78
,, - - - -	430	74,928	14.06
,, - - - -	490	77,280	18.75
,, - - - -	610	62,720	18.70
,, - - - -	630	69,059	18.74

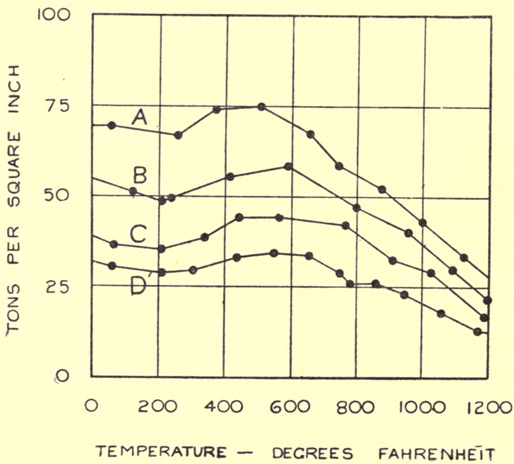


Fig. 6.

Other recent tests made by Mr. J. E. Howard at Watertown Arsenal, U.S.A., showed that tensile strength diminished as temperature was increased from 0 deg. to about 250 deg. Fahr. From the latter point upwards, strength increased with the rise of temperature until the maximum was reached between 500 deg. and 600 deg. Fahr. At higher temperatures strength again diminished. Curves are given in Fig. 6 for four bars containing the following percentages of carbon: A, 0.20; B, 0.37; C, 0.57; D, 0.97. Mr. Howard's investigation further showed that the coefficient of elasticity decreased as temperature increased, the decrease depending upon the percentage of carbon in the steel, as indicated in Table XVII.

Table XVII.—The Influence of Temperature on the Coefficient of Elasticity of Steel. (Values of E, in 1,000,000 *lb. Units per sq. in.).*

Percentage of Carbon.	Temp. Deg. Fahr.	E.	Temp. Deg. Fahr.	E.
0.20	74	29.2	470	26.6
0.37	68	29.7	475	27.5
0.57	80	30.0	465	28.1
0.97	80	29.1	455	27.7

An elaborate report by Professor Martens,[1] on the influence of heat upon the strength of iron and steel contains a great deal of useful information on this point, but it is only necessary to mention the following facts. Commencing at – 4 deg. Fahr. the tensile strength of mild steel diminished with increase of temperature to about 150 deg. Fahr. It then increased with the rise of temperature to about 480 deg., afterwards falling rapidly to about 13,440 lb. per sq. in. at the temperature of 1,100 deg. Fahr. The yield point was found to fall regularly with the rise of temperature.

The safety of steel structures may sometimes be influenced by the peculiar behaviour observed by Professor Martens in steel at 570 deg. Fahr. At that temperature it

[1] *Mittheilungen aus den Koeniglichen technischen Versuchsanstalten.* Berlin, 1890, p. 159.

appears that the strength of the material is greater, but its ductility is less than when in a cold state. Professor Martens says that fracture then occurs suddenly and without previous contraction, the metal exhibiting decided indications of brittleness. It may be doubted, therefore, whether steel at a temperature of 570 deg. Fahr. is capable of resisting repeated shocks or vibrations.

22. Repetitions and Alternations of Stress.—With regard to the endurance of steel subjected to repetitions of stress, a considerable quantity of data is available. Wöhler's well-known endurance tests included experiments upon steel subjected to torsion, tension, and bending.

Similar tests made by Sir Benjamin Baker included experiments on the endurance of bars subjected to repeated bending, and a large number of experiments have also been made by Professor Bauschinger. From the records thus made available we give in Table XVIII. a few of the lowest and highest results:

Table XVIII.—Endurance Tests of Steel subjected to Repetitions of Stress.

Character of Stress.	Maximum Stress.	Range of Stress, lb. per sq. in.	Number of Repetitions.
Torsion,[1] -	48,160	48,160	373,800
,, -	40,544	40,544	23,850,000[3]
Torsion,[2] -	25,760	51,520	859,700
,, -	23,520	47,040	19,100,000[3]
Tension, -	85,568	85,568	18,741
,, -	53,468	53,468	473,766
,, -	51,340	51,340	13,600,000[3]
,, -	58,688	58,688	40,000
,, -	35,840	35,840	11,030,000[3]
Bending, -	74,838	74,838	104,300
,, -	48,160	48,160	43,000,000[3]
,, -	44,128	88,256	12,240
,, -	34,496	34,496	3,145,020[3]

[1] In one direction. [2] In opposite directions alternately.
[3] Not broken.

From this table it is clear that the variation of stress—which in the experiments quoted ranged either from − to +, or from zero upwards—is of more importance than its actual intensity. A singular point is that a bar of steel subjected to so many repetitions of loading as to be on the verge of failure, gives no indication in the testing machine that its strength or its ductility has been altered. Nevertheless, the material actually breaks as if it were perfectly brittle. Taking this fact in conjunction with the ascertained increase of the elastic limit during repetition tests, it appears that the ductility of steel must be prejudicially affected by long continued vibration, although testing-machine observations seem to suggest that the alteration may be confined to planes of weakness. Further light is thrown upon the subject by the recent investigations of Professor J. O. Arnold, who has elicited the remarkable fact that the resistance of structural steel to rupture under rapidly alternating stresses is inversely proportional to the rate of alternation. Another important conclusion drawn from the same tests is that one side of a steel plate becomes brittle and the other tough under alternating stresses, and it also appears to be the fact that, when steel has once acquired distinct brittleness, its original qualities cannot be restored by any heat treatment short of re-melting.[1] The practical lesson to be learnt by the concrete-steel designer is that the metal incorporated should not be exposed to repetitions or alternations of stress beyond safe limits of stress intensity.

23. Coefficient of Expansion.—The coefficient of expansion of steel is variously stated between 0.0000056 and 0.00000695. It is now usually taken at 0.0000066 for mild steel, and 0.0000069 for hard steel (see Art. 16).

24. Forms of Steel used in Concrete-Steel Construction. Steel applied as reinforcement is generally in the form of round bars or rods, as these are obtainable from any metal merchant, and can be employed with a minimum expenditure of time in such manner as to comply with all essential requirements. Square and flat bars are also used, although for ordinary purposes they offer no advantages over round bars. Thin flat bars, or bands, are very useful and convenient for the application of reinforcement intended to protect the concrete against the effects of shearing force.

[1] *British Association, Cambridge Meeting*, 1904.

As mentioned in Article 26, twisted, corrugated, and other forms of bar are advocated by some patentees. Expanded metal and various types of latticed metal are also available, and may frequently be employed with advantage and economy. It should be remembered, however, that the adoption of specialties usually adds to the cost of construction and may involve risk of delays.

CHAPTER III.

THE GENERAL THEORY OF CONCRETE-STEEL BEAMS.

25. Reason for the Reinforcement.—Comparison of the data in Articles 10 and 11 makes evident the fact that the strength of concrete in compression is about ten times its strength in tension.

To make clear the effect of this difference of resistance, let us assume the case of a concrete beam supported at each end, and let us assume further that the neutral axis

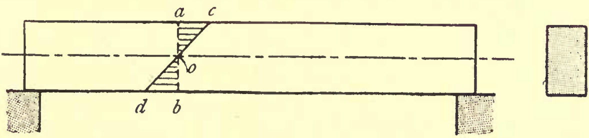


FIG. 7.

passes through the centre of gravity of the cross section of the beam. Then the stresses due to bending moment would be equal above and below the neutral axis, and their distribution would be represented in any section of the beam as in *aoc*, *bod*, Fig. 7. The maximum compressive stress is in the top fibres, and the maximum tensile stress in the bottom fibres of the beam, these stresses diminishing as the neutral axis is approached, where the stress reaches zero value. Owing to the difference between the compressive and tensile strength of concrete the lower fibres would fail in tension long before the upper fibres could fail in compression.

As a matter of fact, the neutral axis in a concrete beam does not pass through the centre of gravity of its cross section (see Art. 32).

A similar though less noticeable inequality of compressive and tensile strength characterises cast-iron, and the familiar section of a cast-iron beam illustrates one method of making good a deficiency of resistance in the tension area.

Treatment of the same kind would be out of the question in the case of concrete, for if the difference of resistance were completely adjusted, the dimensions of the beam would assume absolutely impracticable proportions. This view is

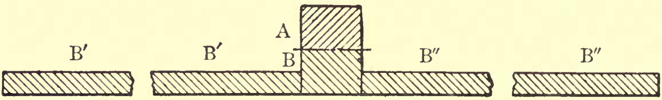


FIG. 8.

illustrated by Fig. 8, where A and B represent two halves of a concrete beam 8 in. wide by 12 in. deep. Assuming the neutral axis to be equidistant from the upper and lower surfaces, the compression area A would be 48 sq. in., and to provide a tension area of equal value, say $48 \times 10 = 480$ sq. in., it would be necessary to increase the width of the lower portion, B, by adding two 3 in. flanges B′ B′ and B″ B″, each 72 in. wide.

The final dimensions of the beam would be quite impracticable, and otherwise objectionable. We therefore see that concrete alone does not lend itself to the economical construction of members intended to resist tensile stress.

By the incorporation of steel in the tension area, the lacking element of strength can easily be supplied without altering the original dimensions of a concrete beam. The compressive strength of the concrete can then be fully utilised, and corresponding tensile strength be supplied by the steel.

If the steel were intended to act quite independently of the concrete, the principle involved would be as diagrammatically represented in Fig. 9, where the stippled upper area is supposed to consist of concrete entirely in compression, and the steel bar in the dotted lower area to be entirely in tension, these two elements being connected by an imaginary web. Assuming such an arrangement to be intended, it would only be necessary to provide a steel bar of any convenient section, and of sufficient area to withstand the calculated stress, and to connect the concrete and the steel by a suitable web. Taking the allowable

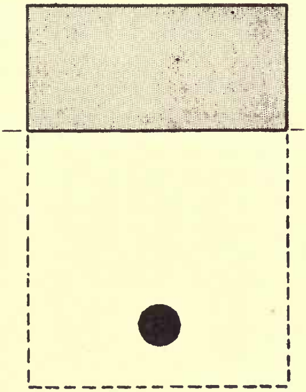


FIG. 9.

unit stress of concrete at 500 lb. per square inch, and of steel at 15,000 lb. per square inch, the relative areas of concrete and steel would be 30 sq. in. and 1 sq. in. respectively. The most suitable and convenient means of connexion would be to complete the beam with concrete, as indicated by broken lines in Fig. 9, thus incidentally affording protection to the metal against corrosion and fire. If the metal were free to slide within the enveloping concrete, the compression and tension elements of the beam, as represented by concrete and steel respectively, would still act independently, and the steel would also act independently of the concrete in the tension area. The concrete in the lower area would add comparatively little to the strength of the beam and would constitute an additional load to be supported.

In practice, however, the concrete would adhere firmly to the steel, and it follows that every variation in the volume or length of one material, whether caused by temperature or by stress following the application of a load, would be accompanied either by corresponding variation of the volume or length of the other material, or by internal stress due to the resistance of one material to the movement of the other.

26. Adhesion of Concrete to Steel.—Adhesion of the concrete to the steel, in spite of stresses caused by the load or by variations of temperature, is a matter of prime importance, and without it the distinctive qualities of concrete-steel could not be realised. The high resistance of the combination is largely due to the fact that the constituent parts adhere so strongly one to the other that the action is that of a homogeneous structure. Adhesion enables the concrete to offer resistance against sliding along the surface of the steel, and thereby facilitates the transference of forces from one material to the other. The result is uniform behaviour somewhat akin to that obtained by the employment of iron or steel plates in the ordinary flitch-beam. Should the resistance to sliding fall below the required amount, the materials will act independently, and the resistance of the structure will be greatly reduced.

Various experiments have been carried out with the object of determining the adhesion of concrete to steel.

Dr. Ritter[1] states the value to be from 568 lb. to 668 lb. per sq. in. Similar values are given by Professor Bauschinger and M. de Joly.

In the recent investigations of M. Considère it was found to vary from 71 lb., 157 lb., and 171 lb. per sq. in., according to the amount of water used in mixing the concrete; and in the case of steel upon which the mill scale had been left, it was 256 lb. per sq. in.

The general practice of engineers on the Continent and in the United States[2] is to adopt 570 lb. per sq. in. as the basis of calculation, but this is too high a value for plain rods or bars.

[1] *Schweizerische Bauzeitung*, 1899, pp. 41, 49, 59.
[2] *Proc. Eng. Soc., Western Pa.*, vol. xix., p. 147.

The statement is sometimes made that the bond of concrete and steel is not destroyed by thermal or other stresses. There is, however, satisfactory evidence that adhesion may be overcome by interior stresses following unequal expansion of the two materials when under the influence of heat.

The adhesion originally existing between the constituent parts of concrete-steel is also affected by working stresses, as well as by vibration and shocks such as are experienced in most structures. Hence, with the object of providing a reliable form of mechanical bond in addition to that given by adhesion of the concrete, some designers make use of corrugated and twisted bars, and others use network of various kinds. In some cases the network is formed by intertwining or otherwise disposing plain rods in the concrete during the course of construction, and in other cases previously manufactured netting is employed. For many forms of design expanded metal is an excellent type of reinforcement.

A corrugated bar made in St. Louis, U.S.A., consists of a square rod with a system of corrugations extending round it. This bar is of considerable merit, giving great mechanical adhesion with a proportionately small quantity of metal.

The Thacher bar, patented in the United States, is of circular section flattened at intervals, and it is claimed by the inventor that it never slips; also that the circular form does not cause incipient fracture, as may happen when square bars are employed.

The Ransome twisted bar, another American device, has been used with considerable success. Some experiments made to determine the strength of the bond so formed are thus described :—[1]

"Half-inch bars were embedded in blocks of concrete 6 in. square, and which varied in length from 12 in. to 28 in. The bars projected about 2 ft. from one end of the blocks. One face of the concrete bore against a plate, the bars passing through a hole in its centre. A direct pull was applied to the bar, and in each case, when the length exceeded 21 in., the bar broke outside the concrete without pulling therefrom. In these cases the ultimate stress on the steel averaged 81,835 lb. per square inch."

[1] *Proc. Am. Soc. C.E.*, vol. xxvii., p. 712.

From the data now presented it is possible to determine the exact length to which bars of all sizes should be embedded to secure a satisfactory bond.

The tensile force (P_t) required to break a round steel bar without breaking the bond can be calculated by the equation:

$$P_t = F_t''\frac{\pi d^2}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where F_t'' = ultimate tensile strength of the steel in pounds per square inch, and $\frac{\pi d^2}{4}$ = area of the bar in square inches.

The tensile force (P_t) to break the bond without breaking the bar is shown by the equation:

$$P_t = l\pi d \times f_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where l = length in inches of the embedded bar, $l\pi d$ being consequently the surface of steel in square inches in contact with the concrete, and f_a = adhesion in pounds per square inch.

By equating the foregoing expressions we obtain a simple rule for finding the length of steel that must be embedded in order to make the strength of the bond greater than that of the bar.

Thus:

$$l = \frac{F_t''}{4f_a} d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Table XIX.—The Adhesion of Concrete and Round Steel Bars.

(F_t'' = Tensile stress in pounds per square inch on steel.)

Adhesion lb. per sq. in. (fa).	Type of Bar.	Length in Diameters to assure Bond.		
		F_t'' = 80,000 lb.	F_t'' = 40,000 lb.	F_t'' = 20,000 lb.
71	Plain round, - -	281	140.5	70.25
157	,, - -	127	63.5	31.75
171	,, - -	116	58.0	29.00
256	,, - -	78	39.0	19.50
487	Twisted square, -	41	20.5	10.25
570	Plain round, - -	35	17.5	8.75
668	,, - -	29	14.5	7.25

The figures in the last three columns of Table XIX. have been calculated by this rule and represent in terms of the diameter, the minimum length for which a round steel bar should be embedded in concrete to ensure a satisfactory bond, the bases of computation being the data previously mentioned.

As the unit stress on steel would not exceed 15,000 lb. to 20,000 lb. in practice, it will be seen by Table XIX. that the embedded length necessary for security is always within moderate limits, and even if the lowest value for f_a be taken, the length for a $\frac{1}{2}$ in. diameter bar is less than 3 ft., or for a 1 in. diameter bar less than 6 ft. In the case of twisted bars, with a unit stress of 20,000 lb., the length for $\frac{1}{2}$ in. and 1 in. diameters are 5 in. and 10 in. respectively. In ordinary practice it is probably quite safe to take the value of f_a at between 250 lb. and 500 lb., according to circumstances, but the embedded steel should never be painted, as paint prevents proper adhesion of concrete to the metal.

27. Effect of Unequal Expansion on Adhesion.—The assumption is frequently made that the coefficients of expansion for concrete and steel are of equal value. This supposition is not quite correct, for taking the results of Professor Pence (Art. 16), the value of the coefficient for concrete is say 0.0000055, while that for mild steel (Art. 23) is 0.0000066, a difference of 0.0000011 per deg. Fahr. Hence the change of length for a given variation of temperature is about 17 per cent. less for concrete than for steel. Assuming the bond to be perfect, it is clear that temperature stresses must be caused in both materials. Increase of temperature above that prevailing during construction will cause tensile stress in the concrete, and, conversely, decrease of temperature will produce compressive stress.

Taking the coefficients of elasticity to be 2,800,000 lbs. for concrete and 28,000,000 lb. for steel, and assuming the sectional area of the steel to be 2 per cent. of the concrete section, the resulting stress per deg. Fahr. would be about 0.6 lb. per square inch. Under ordinary circumstances it is not likely that temperature will be increased much above that prevailing during construction, and there is no reason for anticipating abnormal tensile temperature stresses.

In the case of a fire it is not quite certain what might happen, for there are practically no records showing the precise extent of the influence exercised by concrete in retarding or minimising the expansion of metal embedded therein. If both the concrete and the metal were to acquire a temperature of 2,000 deg. Fahr., the resulting tensile stress on the concrete would reach an intensity of 1,200 lb. per square inch, but expansion would be equalised to some extent owing to protection of the metal on one hand and to exposure of the concrete to the direct action of flame on the other. As reinforced concrete is capable of sustaining without rupture a proportionate distortion of from ten to twenty times that sufficient to cause the failure of ordinary concrete, it does not follow that the inequality of expansion would necessarily be sufficient to cause serious cracks. This conclusion seems to be borne out by various fire tests of concrete-steel constructions in this country and abroad in which, although the effects of deflection and cold water are generally added, the damage caused rarely extends beyond the development of fine surface cracks.

During severe frost, it is not likely that the temperature of concrete-steel work will fall more than 50 deg. Fahr. below the temperature of construction. The effect then produced would be compressive stress, tending to counteract tension on the lower side of a beam, and its amount would be quite inconsiderable.

Of course, the precise effect of unequal expansion will depend in every case upon the value of the coefficients of elasticity of the materials, and also upon the percentage and distribution of the steel in the concrete.

Finally, it should be remembered that changes of temperature are always gradual owing to the heat-resisting properties of the concrete; and that internal stresses are to some extent relieved by infinitesimal slipping of the metal in the enveloping material. Hence, we may say that the secondary thermal stresses due to differences between the coefficients of expansion are never likely to be of great importance in ordinary practice.

28. Function of the Reinforcement.—The primary function of the steel used as reinforcement is to resist tensile stress as explained in Article 25. It has also to resist shearing stress (see Chapter V.), and sometimes compressive stress (see Art. 34, Ex. 4). But an important function of the reinforcement is to insure the thorough distribution of strain in every part of the concrete in which it is embedded, thus preventing the concentration of strain and consequent rupture at any point where there may be an element of weakness. In the case of a beam, the steel has chiefly to resist tensile stress, and if it be applied in large units—such as bars of T or I section—it will be impossible to attain the required result. Here we find the line of demarcation between ordinary "steel and concrete" construction, and concrete-steel construction. In the former, the steel beams and joists constitute the main source of strength, supporting the applied load and the weight of the concrete, receiving little or no aid from the concrete, and imparting no new qualities to that material. In the latter, the steel reinforcement is not expected to do more than half the work, as the concrete takes care of compressive stress. Further, it is known that the concrete actually resists part of the tensile stress, thus reducing still further the duty of the steel. At the same time, the steel increases the original resistance and elasticity of the concrete.

29. Disposition of the Reinforcement.—The exact disposition of the steel must be settled in each case in accordance with the distribution of stress, which can be ascertained by the usual analytical or graphical methods. With regard to subdivision of the steel, we may say that if absolute compliance with theory were practicable, the proper course for adoption would be to employ steel wire of the smallest possible gauge, distributed so that numberless hair-like filaments of metal might be present in every square inch of the concrete, for picking up and transmitting stress. Of course, ideal methods are not suited to the exigences of every-day work, but in concrete-steel as in other construction, the best results are always obtained when practice most faithfully represents theoretical principles. Hence the most successful designers of concrete-steel structures are those who make use of reinforcement in finely-divided forms, and apply it so that the stresses encountered may be adequately met.

30. Elastic Properties of Constituent Parts.—For the purpose of obtaining a clear conception of concrete-steel, the elastic properties of the constituents must be considered. The correct proportion of steel to concrete is directly contingent upon the ratio between the coefficients of elasticity of the two materials, and the whole theory of concrete-steel is dependent upon recognition of the values of these coefficients.

The coefficient of elasticity (E) represents the reciprocal of the extension or compression per unit stress and unit length. It represents the force that would, if such a thing were possible, stretch a bar of any material to double the original length, or compress it to half the original length.

Thus, the statement that the value of E for steel is 30,000,000 lb. per sq. in. implies that a force of 1 lb. would stretch or compress a bar of 1 in. area by $\frac{1}{30000000}$, or $0.00000003\dot{3}$ of its original length. Similarly, if the value of E for concrete is 3,000,000 lb. per sq. in., a force of 1 lb. corresponds with an extension or compression of $\frac{1}{3000000}$, or $0.0000003\dot{3}$ of the original length of the bar. Therefore the application of tensile force of any given intensity to a unit area of concrete will be followed by an extension ten times the length of the extension caused by the application of an equal force to a unit area of steel. Hence, for an equal elongation, the steel will carry ten times the stress that can be carried by concrete.

Further, it should be remembered that the greater the value of E, the less will be the extension or compression for a given stress, and that the smaller the value of E, the greater will be the extension or compression for a given stress.

We now see the importance of knowing in every case what are the actual values of the coefficients of elasticity for the materials to be used in concrete-steel. As evidenced by Tables XIII., XIV. and XV. the value of E for steel is a fairly constant quantity, but as indicated by Tables IX. and X., the value of E for concrete is extremely variable. The value varies with the proportions and materials adopted in making the concrete, and with the increase of unit stress. Tables may be used for the purpose of arriving at averages for ordinary work, but for important structures it is to be recommended that the elastic properties of the concrete should be specially determined.

Suitable allowance should always be made to compensate for the decrease of E with increase of unit stress. It is also necessary to decide the question whether calculations should be based upon the coefficient corresponding with total deformation, or upon that corresponding with purely elastic deformation.

Assuming the bond between the concrete and the steel to be perfect, the deformation of both materials must be equal, and all permanent deformation or "set" must, similarly, be equal. But permanent deformation is greater in the case of concrete than in that of steel. Hence, on the removal of load, the elastic contraction of steel previously in tension induces tensile stress in the steel and compressive stress in the concrete; and the reimposition of load causes stresses which are independent of the previously-created internal stresses. For these reasons, it appears that the coefficient of pure elasticity is of less service to the designer than the coefficient calculated from total deformation.

When the relationship between the coefficients of elasticity was first considered in connexion with concrete-steel, the value of E for concrete was assumed to be about 710,000 lb. per square inch, which is about one-fortieth the value of E for steel. Using the symbol m to represent the ratio of one coefficient to the other, we have in this case $m = 40$. Subsequently the opinion was formed that the value of E for concrete was 1,400,000 lb., giving $m = 20$; and more recently the value of E for 1 : 2 : 4 and 1 : 3 : 5 concrete has been shown to be from 2,800,000 lb. to 3,500,000 lb., so that $m = 10$ is an expression very generally used in the present day to indicate the relationship between the coefficients of elasticity for steel and concrete.

During the period when knowledge on the subject of concrete-steel was being acquired, the values for E for both steel and concrete really remained constant, and were in no way altered by the changing views of investigators. Consequently, structures designed upon the theoretical basis, $m = 40$, really embodied the basis, $m = 10$. Therefore in such cases the concrete was able to take, and actually did take, a greater share of the duty than was intended or known by the designers. The inference is that an excessive proportion of steel was used in former years.

One or two simple calculations will serve to indicate the importance of properly taking account of the values of E, in concrete-steel design.

With the object of avoiding unnecessary fractions we will take the value of E for steel at 30,000,000 lb. per sq. in. with a corresponding extension of $0.00000003\dot{3}$ per lb., and in each case we will consider a unit length of 1 in. so that extensions shall represent fractions of an inch. We will take three values for the ratio between the values of the coefficients of elasticity, viz.—$m = 40$, $m = 20$, and $m = 10$, corresponding with values of the coefficient for concrete, E = 750,000 lb., E = 1,500,000 lb., and E = 3,000,000 lb. The allowable unit stress for steel will be taken at 15,000 lb.

In Fig. 10 let the shaded area represent the cross section of a steel bar of 1 sq. in. area; and the outer ring a 1-in. area of concrete surrounding the steel.

(*a*) If a direct pull be now applied to the steel, so that the bar shall be under a stress of 15,000 lb., the extension for a unit length will be $15{,}000 \times 0.00000003\dot{3} = 0.0005$ in. The stress equivalent to an equal extension of the concrete will vary with the ratio of the coefficients of elasticity, and assuming stress to be strictly proportionate to strain, the results will be:—

		Stress in Concrete.
If $m = 40$,	$(0.0005 \times 750{,}000) =$	375 lb. per sq. in.
,, $m = 20$,	$(0.0005 \times 1{,}500{,}000) =$	750 ,, ,,
,, $m = 10$,	$(0.0005 \times 3{,}000{,}000) =$	1,500 ,, ,,

(*b*) We will now look at the matter from a different point of view, assuming the direct pull on the steel to be such that the stress in the concrete shall be only 50 lb. The elongation of the concrete, varying with the value of the coefficient of elasticity, will be

If $m = 40$, $(0.0000013\dot{3} \times 50) = 0.00006\dot{6}$ in.
,, $m = 20$, $(0.0000006\dot{6} \times 50) = 0.00003\dot{3}$,,
,, $m = 10$, $(0.0000003\dot{3} \times 50) = 0.000016\dot{6}$,,

For extensions of the steel, corresponding in amount to those just calculated for concrete, the stresses will be as follows:—

	Stress in Steel.
If $m = 40$, $(0.00006\dot{6} \times 30{,}000{,}000) =$	2,000 lb.
,, $m = 20$, $(0.00003\dot{3} \times 30{,}000{,}000) =$	1,000 ,,
,, $m = 10$, $(0.0000166 \times 30{,}000{,}000) =$	500 ,,

Comparing the results in paragraphs (*a*) and (*b*), we find in the first set of figures that the equivalent stress in the concrete is abnormally high for ordinary concrete, and in the second set of figures that the stress in the steel is ridiculously low, while the work left for the steel decreases with the increase of the value of E for concrete. In each

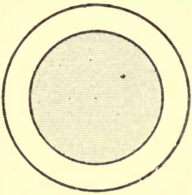


FIG. 10.

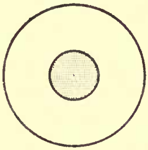


FIG. 11.

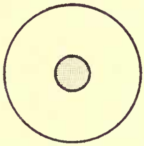


FIG. 12.

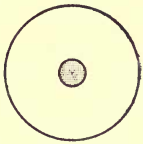


FIG. 13.

case the whole of the concrete is fairly within the sphere of influence of the steel, and, assuming the bond to be perfect, the elongation of the concrete must be equal to the elongation of the steel.

We will now see how the materials may be used to better advantage. As Hooke's law of the proportionality between stress and strain does not hold good for concrete, the elongation of 0.0005 in. would not involve stress to the extent shown in the first set of figures, and recent investigation has shown that the reinforced concrete should be capable of proportionate distortion of twenty times that at which simple concrete would fail in tension. Consequently the proportion of steel can be reduced, as shown in Figs. 11, 12, and 13, to $\frac{2}{15}$ sq. in., $\frac{1}{15}$ sq. in., and $\frac{1}{30}$ sq. in., according to the values, $m = 40$, $m = 20$, and $m = 10$, thereby raising the stresses for steel, from those found above, to 15,000 lb.

per square inch, without involving failure of the concrete. It should be noted that in each of the last mentioned diagrams the sectional area of concrete is 1 sq. in., as in Fig. 10.

The foregoing discussion is based upon the assumption that a direct pull is applied to the steel, and that the bond is theoretically perfect. Consequently, tensile stress is only communicated to the concrete through the steel, and we may say that the elongation of the concrete is directly caused by the elongation of the steel. In a beam the state of things is different. Tensile stress is then distributed over the area below the neutral axis in accordance with the laws governing flexure in beams. If no reinforcement be present, the whole of the stress will fall upon the concrete, the elongation of which will then be governed by the value of the coefficient E. If reinforcement be present, the tensile stress will come upon both the concrete and the steel in accordance with mechanical principles. But a portion of the total stress will be transferred to the steel. In the latter event, as steel elongates less than concrete for a given stress, and if the bond be perfect, the concrete will only be permitted to elongate to the same extent as the steel. The proportion of the total stress actually carried by the concrete will depend upon the proportion of steel to concrete. If the area of steel be proportionately great, most of the stress will be taken up by the steel; and if the area of steel be proportionately small, most of the stress will remain to be resisted by the concrete.

Therefore, we see that in the case of a simple pull applied to the steel, tensile stress is transferred from the steel to the concrete; while in the case of flexure, tensile stress, coming simultaneously upon the two materials, is transferred in part from the concrete to the steel. Notwithstanding this difference of action, the fact remains that the behaviour of concrete-steel is invariably governed by the ratio of the coefficients of elasticity of the two materials.

It is also evident from Figs. 10-13 that as the area of the steel diminishes in comparison with the area of the surrounding concrete, so the sphere of influence of the steel diminishes, and the transference of strain from one material to the other is rendered more difficult. Close to the steel the two materials will act more or less in unison, but as the distance increases, there will be a tendency for the concrete to act independently, because the reinforcing effect of the steel varies inversely as the square of the distance. If the steel be concentrated in the form of a single bar, it is certain that strain must be unequally distributed in the concrete, so that its ultimate strength may be reached in remote parts, while stress in the immediate neighbourhood of the steel may be well within permissible limits.

CHAPTER IV.

THE DESIGN OF BEAMS.

31. General Principles.—We have now sufficient data relative to the adherence, expansion, and proportionate elasticity of concrete and steel to enable us to form some idea of the principles governing concrete-steel design. Although it is the fact that concrete acquires greatly increased strength when combined with steel, a very usual practice is to allow nothing for the value of the concrete in the tension area of a beam, so far as concerns resistance to flexure.

It is known that the concrete is in tension, and as the value of every element in a structure should be determined and properly taken into account, this method of dealing with concrete below the neutral axis is not satisfactory from the theoretical standpoint.

A correct theory of concrete-steel design should offer a rational explanation of the relative work done by each material, and it should provide for utilisation of the augmented strength of the concrete up to the maximum permissible limit. Similarly it should take into account the increased compressive strength of concrete reinforced above the neutral axis. At the present time knowledge is inadequate for the establishment of a theory to which no objection might be taken, and many engineers prefer to wait for confirmation of the remarkable results that have been arrived at within recent years before setting aside rules that have been found satisfactory and safe in actual practice.

If the value of the concrete in tension be omitted from calculation, the proportion of cement adopted may certainly be much smaller than would otherwise be desirable for the lower portion of the beam; but, on the other hand, the concrete in the upper portion should be of superior quality, for the purpose of affording high resistance to compression. While variation of the proportions of concrete in this way is in strict accordance with theory, a practical objection is to be found in the difficulty of insuring a satisfactory bond between the two qualities of concrete.

In a symmetrical steel beam, where resistance to tension and compression is of equal value, the neutral axis must pass through the centre of gravity of the section. This is not so in a beam of simple concrete, for the position of the neutral axis is affected by the fact that the compressive strength of the material is greater than its tensile strength, and as stress increases, the neutral axis moves away from the axis through the centre of gravity of the section (see Art. 32).

Matters are further complicated by the facts that although it is usual to assume that the cross section of a beam remains plane after bending, this assumption is only approximately true, and that variations in the proportionality of stress and strain affect the form of stress diagrams, so that instead of being bounded by straight lines, the figures representing stresses are of more or less curved outline.

The ordinary rule for determining the stress in solid rectangular beams under uniformly distributed load is

$$f = \tfrac{3}{4}\,Wl \div bd^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

f = extreme fibre stress,

W = the weight or load,

l = length of beam between supports,

b = breadth of the beam,

d = depth of the beam.

This rule does not attempt to differentiate between stress at the upper and lower fibres, and it follows that the results thereby obtained can only apply to the case of beams in which the neutral axis passes through the centre of gravity of the section. Therefore, for materials in which the relations of stress and strain vary under tension and compression, formula (4) gives values that are too high for one kind of stress and too low for the other.

When dealing with beams of concrete-steel, where in addition to the effects of differences in the resistance of concrete to tension and compression we have to consider the effects due to the quantity and disposition of the steel, the formula is absolutely useless and some alternative method of calculation becomes necessary.

32. Position of Neutral Axis.—We will first inquire what is the probable position of the neutral axis in a beam where compressive stress is not limited by the low tensile resistance of the material.

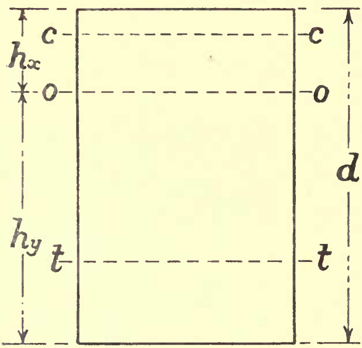


FIG. 14.

In Fig. 14, let h_x and h_y represent the distances of the top and bottom surfaces from the neutral axis *oo*, and *d* the total depth of the beam, the axis of compression *cc* passing through the centre of pressure of the upper stress area and the axis of tension *tt* passing through the centre of pressure of the lower stress area, in each case at two-thirds of the distance between the neutral axis and the extreme fibres.

Then, as the ratio 10 : 1 represents the compressive and tensile strength of concrete, the resistance of the compression area will be $10h_x \times 0.6\dot{6}h_x = 6.\dot{6}h_x^2$; and the resistance of the tension area will be $h_y \times 0.6\dot{6}h_y = 0.6\dot{6}h_y^2$. If these are equal, $6.\dot{6}h_x^2 = 0.6\dot{6}h_y^2$, or $10h_x^2 = h_y^2$. But as $h_x + h_y = d$, it follows that $h_x + h_x\sqrt{10} = d$,

$$h_x = d \div 4.16, \text{ and } h_y = d - h_x.$$

Thus the proportionate depths of h_x and h_y should be $0.24d$ and $0.76d$ respectively.

By adding steel in the tension area to such an extent as to make the resistance of steel + concrete ten times greater than the resistance of concrete in the compression area, we should get the result $h_x^2 = 10h_y^2$, and the proportionate depths of h_x and h_y should be $0.76d$ and $0.24d$. This would be the converse of the previous condition.

In one case the neutral axis would be near the top of the beam, and in the other near the bottom. Between these extremes any required variation may be effected by adjusting the relative resistance of the materials in the compression and tension areas, and the inference is that the position of the neutral axis in a concrete-steel beam depends largely upon the proportions of concrete and steel, and upon the disposition of the steel with respect to the neutral axis.

From the foregoing it is clear that the height of the neutral axis may be calculated for any beam after these details have been settled, or that the proportions of concrete and steel may be adapted to any chosen position for the neutral axis.

33. Rules derived from Beam Equations.—The following simple rules have been derived, from equations in ordinary use, for the purpose of making clear the general principles adopted in the design of concrete-steel beams. These rules do not involve any profound investigation, and are deficient in the respect that the relative values of the coefficients of elasticity of steel and concrete are not introduced as factors. This omission, however, does not very materially affect the accuracy of the calculated results.

The bending moment (M) developed in any beam must always be resisted and held in equilibrium by the moment of resistance (R) of the section. Therefore, in every case the bending moment must be equal to the moment of resistance; or $M = R$.

For a uniformly distributed load on a beam, of the length l, supported at each end, $M = Wl \div 8$, and the general expression for the moment of resistance is $R = f\,I \div h$.

As $I = bd^3 \div 12$, for a rectangular beam, and $d = (h + h)$, we get

$$R = fbh\tfrac{2}{3}h$$

where f = extreme fibre stress, at the distance h from the neutral axis; b = breadth of the beam section; and h = distance of the extreme fibres from the neutral axis.

Equating the expressions for M and R we have

$$\frac{Wl}{8} \times \frac{1}{fbh\frac{2}{3}h} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Whence
$$f = \frac{Wl}{5 \cdot \dot{3}bh^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and
$$W = \frac{5 \cdot \dot{3} fbh^2}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Formula (6) gives the extreme fibre stress at the top or bottom of a rectangular beam, but in the case of a steel bar used as reinforcement in the tension area, the resistance of the concrete is usually neglected, and the stress is regarded as being concentrated at the centre of the bar.

By eliminating the quantity bh from formulæ (6) and (7), and using F = total stress instead of f = intensity of extreme fibre stress, we should get

$$F = \frac{Wl}{5 \cdot \dot{3}h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and for the load
$$W = \frac{5 \cdot \dot{3}\, Fh}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But as the intensity of stress in a beam diminishes progressively from the extreme fibres towards the neutral axis, where its value = 0, the mean unit stress is $(f + 0) \div 2 = \frac{1}{2}f$. Hence formula (8) gives double the correct value for the total stress F, and formula (9) gives half the correct value for the corresponding load W. Therefore the equations must be modified as below.

Denoting total tensile stress considered to be concentrated at the centre of the reinforcement by the symbol F_t'', and the distance from the neutral axis to the bottom surface by the symbol h_y, we have

$$F_t'' = \frac{1}{2}\, \frac{Wl}{5 \cdot \dot{3}h_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and for the load
$$W = \frac{5 \cdot \dot{3}F_t''h_y}{l} \times 2. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

34. Examples.—The following Examples will serve to illustrate the use of the foregoing and kindred rules.

We have in each case taken a uniformly distributed load, but loads of different character can easily be dealt with by variation of the rules in accordance with the expression indicating the value of the corresponding bending moment.

The permissible unit stresses assumed are:—15,000 lb. per square inch for steel; 500 lb. per square inch for concrete in compression; and 50 lb. per square inch for concrete in tension.

Except where otherwise specified each example refers to the case of a beam 12 in. deep, with a clear span of 100 in. between the supports, and carrying a uniformly distributed load of 4,000 lb., the weight of the beam itself being disregarded in each case.

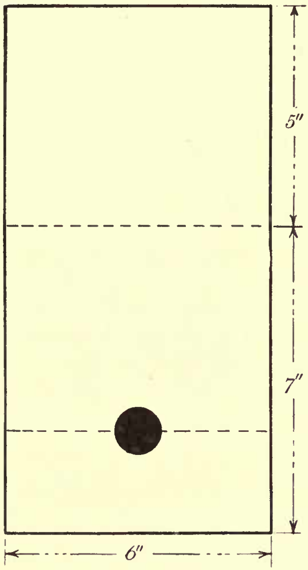


FIG. 15.

Example 1.—In the first place, we will assume the neutral axis at 7 in. above the bottom of the beam. The various measurements of height will, therefore, be in the proportions indicated in Fig. 15, the height of the steel above the bottom of the beam being $\frac{1}{3}h_y = \frac{1}{3} \times 7 = 2.\dot{3}$ in.

The total stress in the steel by formula (10) will be

$$F_t'' = \frac{1}{2}\,\frac{4{,}000 \times 100}{5\cdot\dot{3} \times 7} = 5{,}357\cdot14 \text{ lb.}$$

The permissible stress (f_t'') per square inch of steel being 15,000 lb., the requisite area (a) is

$$F_t'' \div f_t'' = 5{,}357\cdot14 \div 15{,}000 = 0.35 \text{ sq. in.}$$

The width of concrete section in the compression area can be found by an equation derived from formula (5).

Thus, denoting compressive fibre stress in the concrete by f_c', and the distance from the neutral axis to the top surface by h_x, we have

$$b = \frac{Wl}{5\cdot\dot{3} f_c' h_x^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

In this case the value of f_c' must be limited to 500 lb. per square inch, which is the permissible stress assumed for concrete in compression, and the width of the concrete by formula (12) must be

$$b = \frac{4{,}000 \times 100}{5\cdot\dot{3} \times 500 \times 5^2} = 6 \text{ in.}$$

The dimensions of the beam will consequently be 12 in. deep by 6 in. wide = 72 square inches area, as in Fig. 15, but the steel may be divided into two or more bars placed along the axis of tension, 2.$\dot{3}$ inches above the bottom of the beam.

To prove the accuracy of these proportions, the resistance of each area may be calculated by a formula derived from the expression for the moment of resistance.

Thus $$R = (f_t'' a \tfrac{2}{3} h_y) + (f_m' b h_x \tfrac{2}{3} h_x). \quad . \quad . \quad . \quad . \quad (13)$$

Where f_t'' = tensile stress per sq. in. on steel, a = area of steel in sq. in., and f_m' = mean stress per sq. in. on concrete in compression. In this example $F_t'' = f_t'' a$ has already been calculated at 5,357·14 lb., and $f_m' = (500 + 0) \div 2 = 250$ lb. per square inch.

Hence $$R = (5{,}357\cdot14 \times 4.\dot{6}) + (250 \times (6 \times 5) \times 3.\dot{3})$$
$$= (25{,}000) + (25{,}000) = 50{,}000 \text{ inch-pounds.}$$

Therefore the moments of the internal forces are in equilibrium, and as $M = Wl \div 8 = 400{,}000 \div 8 = 50{,}000$ inch-lb., the condition $M = R$ is satisfied.

It is often the case that the centre of the reinforcement is not placed at the same distance from the neutral axis of the section as the theoretical centre of pressure of the tension stress area. For use under such circumstances, formula (13) must be modified by substituting for the value of $\frac{2}{3} h_y$ the actual distance between the centre of the reinforcement and the neutral axis. If reinforcement be used in the compression stress area a similar modification must be made.

Example 1*a*.—We will now see what reduction might be made in the area of the reinforcement by taking account of the resistance offered by the concrete to tension.

The dimensions of the compression area remain as already determined, because the tensile resistance of concrete + steel will be exactly the same as the tensile resistance of the steel as calculated in Example 1.

The proportion of the load carried by the concrete in tension, and corresponding with an extreme fibre stress of 50 lb. per square inch, by formula (7), is

$$W = \frac{5\cdot\dot{3} \times 50 \times 6 \times 7^2}{100} = \frac{78{,}400}{100} = 784 \text{ lb.}$$

Consequently the load remaining to be carried by the steel is

$$4{,}000 - 784 = 3{,}216 \text{ lb.},$$

and the total stress on the steel, by formula (10), will be

$$F_t'' = \frac{1}{2}\,\frac{3{,}216 \times 100}{5\cdot\dot{3} \times 7} = 4{,}307.14 \text{ lb.}$$

The corresponding area of steel is

$$F_t'' \div f_t'' = 4{,}307 \div 15{,}000 = 0.28 \text{ sq. in.},$$

the reduction representing a saving of more than 20 per cent. of metal.

The dimensions of the beam may remain unaltered, being 12 in. deep by 6 in. wide = 72 square inches area as before.

Example 2.—In the next place, while assuming the neutral axis at 7 inches above the bottom of the beam, we will fix the axis of tension for the steel at 2 inches from the bottom.

Formula (10) will not apply to this case, and must be modified in the manner following :—

$$F_t'' = \frac{1}{2}\,\frac{Wl}{8(h_y - z)} \quad \dots \dots \quad (14)$$

Here z is the distance between the centre of the reinforcement and the bottom of the beam.

The total stress on the steel, by formula (14), is now

$$F_t'' = \frac{1}{2}\,\frac{4{,}000 \times 100}{8(7-2)} = 5{,}000 \text{ lb.}$$

Consequently, the required sectional area is

$$F_t'' \div f_t'' = 5{,}000 \div 15{,}000 = 0.3\dot{3} \text{ sq. in.}$$

Therefore economy is secured by increasing the leverage of internal forces in the tension area.

The final dimensions of the beam may again remain unaltered.

Example 2*a*.—Allowing for the resistance of concrete in the tension area, we find the proportion of the load that may be carried by concrete in tension is 784 lb. as before, while by formula (14) the total stress on the steel is reduced to

$$F_t'' = \frac{1}{2}\,\frac{3{,}216 \times 100}{8(7-2)} = 4{,}020 \text{ lb.}$$

Consequently, the required sectional area is

$$F_t'' \div f_t'' = 4{,}020 \div 15{,}000 = 0.268 \text{ sq. in.,}$$

the reduction representing a saving of more than 20 per cent. of metal, as in example 1*a*.

Example 3.—Further economy would be attained if the neutral axis were 1 inch higher, that is, 8 inches above the bottom of the beam. Keeping the reinforcement at 2 inches from the bottom, formula (14) gives for steel in tension

$$F_t'' = \frac{1}{2}\,\frac{4{,}000 \times 100}{8(8-2)} = 4{,}166 \text{ lb.}$$

This is equal to a sectional area of

$$F_t'' \div f_t'' = 4{,}166 \div 15{,}000 = 0.27 \text{ sq. in.}$$

For concrete in compression the required width, according to formula (12), is

$$b = \frac{4{,}000 \times 100}{5.\dot{3} \times 500 \times 4^2} = 9.375 \text{ inches.}$$

Therefore the compression area measures $4 \times 9.375 = 37.5$ square inches, but the tension area need not be wider than is necessary for the satisfactory enclosure of the steel, say $4.\dot{3}$ inches, giving a section of $4.\dot{3} \times 8 = 34.6$ square inches, and making a total area of concrete for the beam of about 72 square inches. These dimensions give a T-shape to the beam, and if incorporated in a floor, the wide flanges would practically cost nothing, as they would form part of the floor panelling.

Example 3a.—Allowing for the resistance of the concrete in the tension area of $34.\dot{6}$ square inches, by formula (7) we have

$$W = \frac{5 \cdot 3 \times 50 \times 4 \cdot \dot{3} \times 8^2}{100} = \frac{73{,}955}{100} = 739 \cdot 55 \text{ lb., say, } 740 \text{ lb.}$$

The total stress on the steel by formula (14) will be

$$F_t'' = \tfrac{1}{2}\, \frac{(4{,}000 - 740) \times 100}{8 \times (8 - 2)} = \tfrac{1}{2}\, \frac{326{,}000}{48} = 3{,}395 \text{ lb.,}$$

and the area of steel will be reduced to

$$F_t'' \div f_t'' = 3{,}395 \div 15{,}000 = 0.226 \text{ sq. in.}$$

Hence the saving of metal is over 15 per cent.

Example 4.—Keeping the neutral axis, as in Example 3, at 8 inches above the bottom of the beam a uniform width of 6 inches can be given to the beam by incorporating steel bars in the compression area. The sectional area of steel in tension will again be 0.27 square inch, or allowing for the tensile resistance of the concrete, the area of steel will be thus reduced :—

By formula (7) the proportion of the load carried by the concrete in tension will be

$$W = \frac{5 \cdot \dot{3} \times 50 \times 6 \times 8^2}{100} = \frac{102{,}400}{100} = 1{,}024 \text{ lb.}$$

By formula (14) the total stress on the steel will be

$$F_t'' = \tfrac{1}{2}\, \frac{(4{,}000 - 1{,}024) \times 100}{8(8 - 2)} = \tfrac{1}{2}\, \frac{297{,}600}{48} = 3{,}100 \text{ lb.}$$

Consequently the reduced area of steel becomes

$$F_t'' \div f_t'' = 3{,}100 \div 15{,}000 = 0.206 \text{ sq. in.}$$

Taking now the compression area, the proportion of the load that may be carried by the concrete is, by formula (7),

$$W = \frac{5 \cdot \dot{3} \times 500 \times 6 \times 4^2}{100} = \frac{256{,}000}{100} = 2{,}560 \text{ lb.}$$

The total stress F_c'' to be taken by steel in compression, situated 2 inches from the top surface of the beam, can be calculated by a rule essentially similar to formula (14). Thus

$$F_c'' = \frac{1}{2}\,\frac{Wl}{8(h_x - x)}. \quad . \quad . \quad . \quad . \quad (14a)$$

In this example

$$F_c'' = \frac{1}{2}\,\frac{(4{,}000 - 2{,}560) \times 100}{8(4-2)} = \frac{1}{2}\,\frac{144{,}000}{16} = 4{,}500 \text{ lb.}$$

Hence the area of steel in compression must be

$$F_c'' \div f_c'' = 4{,}500 \div 15{,}000 = 0.3 \text{ sq. in.}$$

Here f_c'' = permissible stress per sq. in., on steel.

It should be noted that if the steel were placed at $\frac{1}{3}h$ below the upper surface of the beam, the required area would be less, but owing to the close proximity of the metal to the surface, this course would be undesirable from the point of view of fire protection unless the beam were covered by concrete flooring. Adopting this position for the axis of compression the total stress in the steel can be calculated by a rule similar to formula (10), namely:

$$F_c'' = \frac{1}{2}\,\frac{Wl}{5{\cdot}\dot{3}h_x}. \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

Whence,

$$F_c'' = \frac{1}{2}\,\frac{(4{,}000 - 2{,}560) \times 100}{5{\cdot}\dot{3} \times 4} = \frac{1}{2}\,\frac{144{,}000}{21.\dot{3}} = 3{,}375 \text{ lb.}$$

and the area of steel is

$$F_c'' \div f_c'' = 3{,}378 \div 15{,}000 = 0.225 \text{ sq. in.}$$

35. Comparison of Results.—Table XX. is a summary of the results obtained in the foregoing calculations, and it will be observed that considerable advantage follows the higher position of the neutral axis, and the lower position of the steel reinforcement.

It is further noticeable that much less advantage is derived from the incorporation of steel in the compression area than in the tension area. This is quite natural, because the steel in tension has the effect of utilizing advantageously an excess of compressive strength in the concrete, whereas steel in compression merely counter-balances the strength of part of the steel added in

Table XX.—Calculated Proportions of Concrete-Steel Beams.

(Uniformly Distributed Load 4,000 lb. Span, 100 inches.)

Example No.	Dimensions of Cross Section of Beam.		Height of Neutral Axis above Bottom of Beam.	Height of Steel Axis above Bottom of Beam.	Area of Steel. No Allowance made for Tension in Concrete.	Area of Steel, allowing for Stress in Concrete.
	b	*d*				
	In.	In.	In.	In.	Sq. in.	Sq. in.
1	6	12	7	$2.3\dot{3}$	0.35	—
1a	6	12	7	$2.3\dot{3}$	—	0.28
2	6	12	7	2.	$0.3\dot{3}$	—
2a	6	12	7	2.	—	0.268
3	{ 9.37[1] / $4.3\dot{3}$ }	12	8	2.	0.27	—
3a	{ 9.37[1] / $4.3\dot{3}$ }	12	8	2.	—	0.226
4	6	12	8	2.	0.27	{ 0.206 / 0.30[1] }

[1] Compression Area.

the tension area. Nevertheless, the employment of steel in this manner is very frequently necessary, as it enables the designer to carry out work that would otherwise be impracticable. The saving to be effected by allowing for the strength of concrete in tension is small, but the influence of the strength so contributed ought always to be taken into account, whether any reduction in the area of steel be made or not.

36. Further Examples.—In Examples 1 to 4, the depth, load, and span of the beam were definitely fixed, and the height of the neutral axis was assumed for each case, the object being to find the required areas of steel and concrete to comply with the stated conditions. But the equations already stated are equally available for determining the proportions of a beam to suit any load and span if the essential factors are known.

Example 5.—For instance, we can calculate the height of the compression area from the load, span, and width of the section, and the unit stress of the concrete.

Let $Wl = 400{,}000$ inch-lb., $b = 10$ in., and $f_c' = 500$ lb.

Then by derivation from formula (5) we get

$$h_x^2 = \frac{Wl}{5 \cdot \dot{3}\, bf_c'} \text{ or } h_x = \sqrt{\frac{Wl}{5 \cdot \dot{3}\, bf_c'}}. \quad . \quad . \quad (15)$$

Thus

$$h_x = \sqrt{\frac{400{,}000}{5 \cdot \dot{3} \times 10 \times 500}} = 3.87 \text{ inches.}$$

Example 6.—Similarly, taking the width at 6 inches, the value of h_x is found to be

$$h_x = \sqrt{\frac{400{,}000}{5 \cdot \dot{3} \times 6 \times 500}} = 5 \text{ inches.}$$

Example 7.—If the compression area is to be square in section, it follows from formula (5) that

$$bh^2 = \frac{Wl}{5 \cdot \dot{3} f}.$$

But in this case $bh^2 = b^3$, whence

$$b = \sqrt[3]{\frac{Wl}{5 \cdot \dot{3} f_c'}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Using this formula we can obtain the dimension of any side of the square. Thus

$$b=\sqrt[3]{\frac{400,000}{5\cdot 3\times 500}}=5\cdot 313 \text{ inches.}$$

In the tension area, the required distance of the reinforcement from the neutral axis will depend upon the area of steel used; and, conversely, the area of steel will be governed by its distance from the neutral axis.

Denoting this distance by the symbol h_v, we can determine the required value for any case by a modification of formula (14), adding the factor a = area of steel.

Thus
$$h_v=\tfrac{1}{2}\frac{Wl}{8f_t''a} \quad \ldots \ldots \quad (17)$$

Example 8.—Calculating upon the basis of 1 square inch of steel, placed 2 inches above the bottom of the proposed beam, and taking the unit stress at 15,000 lb. per square inch, as before, we have by formula (17)

$$h_v=\tfrac{1}{2}\frac{400,000}{8\times 15,000}=1.6\dot{6} \text{ inches,}$$

and as in this case the bar is 2 inches above the bottom the height of the tension area is $1.6\dot{6}+2=3.6\dot{6}$ inches.

Example 9.—If we reckon upon a bar of $\frac{3}{4}$ square inch area, also placed 2 inches above the bottom of the proposed beam, we have by formula (17)

$$h_v=\tfrac{1}{2}\frac{400,000}{8\times 11,250}=2.2\dot{2} \text{ inches;}$$

and the height of the tension area becomes

$$2.2\dot{2}+2=4.2\dot{2} \text{ inches.}$$

Example 10.—Similarly allowing for a bar of $\frac{1}{2}$ square inch area placed 2 inches above the bottom of the beam, we have by formula (17)

$$h_v=\tfrac{1}{2}\frac{400,000}{8\times 7,500}=3\cdot 3\dot{3} \text{ inches,}$$

and the height of the tension area will be

$$3\cdot\dot{3}+2=5\cdot\dot{3} \text{ inches.}$$

Example 11.—Making the area of the steel $\frac{1}{3}$ square inch, and placing it 2 inches above the bottom of the beam, we have by formula (17)

$$h_v = \frac{1}{2}\,\frac{400{,}000}{8 \times 5{,}000} = 5 \text{ inches,}$$

and the height of the tension area will be 7 inches.

37. Summary of Results.—In Table XXI. the results obtained in the Examples 5 to 11 are summarised, and the final dimensions of the beams are shown for various areas of steel reinforcement. It will be understood that the width of the tension area need not necessarily be the full width of the compression area, the chief point for consideration being the provision of ample width for the proper spacing of the steel bars. In many cases, however, no practical advantage would be gained by making the width of the tension area less than that of the compression area, as the additional cost of moulds would more than counterbalance any saving due to reduction in the quantity of concrete.

38. Professor Hatt's Theory.—A theory of the strength of reinforced concrete beams, by Professor Hatt of Purdue University, U.S.A., is based upon experimental results which are discussed in a paper read before the American Society for Testing Materials.[1]

The beams tested were 8 in. square, and the clear span between supports was 80 in., the load being applied at the centre. Some of the beams were plain and others were reinforced.

From several load deformation diagrams, presented in Professor Hatt's paper, it appears that the curve of deflection followed a nearly straight line up to a load varying from 1,500 lb. to 3,000 lb. for the beams tested. At higher loads the deflection increased more rapidly than the load, but the curve again straightened, so that the deflection once more increased uniformly with the load until a crack appeared at the lower surface of the beam. This point was reached with a load ranging from 4,000 lb. to 10,000 lb., and beyond it the deflection continued uniformly with the load until the reinforcement reached its elastic limit.

[1] *Proc. Am. Soc. Testing Materials*, vol. ii. 1902.

Table XXI.—Calculated Proportions of Concrete-Steel Beams.

(Uniformly Distributed Load 4,000 lb. Span of 100 inches.)

Compression Area.		Tension Area, height for Areas of Steel stated below.				Dimensions of Beam ($b \times d$) for Areas of Steel stated below.							
Width.	Height.	1 sq. in.	¾ sq. in.	½ sq. in.	⅓ sq. in.	1 sq. in.		¾ sq. in.		½ sq. in.		⅓ sq. in.	
						b	*d*	*b*	*d*	*b*	*d*	*b*	*d*
In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
10.	3.87					10.	7.53	10.	8.09	10.	9.20	10.	10.87
6.	5.	$3.6\dot{6}$	$4.2\dot{2}$	$5.3\dot{3}$	7.0	6.	$8.6\dot{6}$	6.	$9.2\dot{2}$	6.	$10.3\dot{3}$	6.	12.00
5.3	5.31					5.31	8.97	5.31	9.53	5.31	10.64	5.31	12.31

Thereafter the deflection increased rapidly without any corresponding increase of load.

It should be noted that three characteristic points are evidenced by these diagrams:—(A) that at which the curve first deviates from the straight line, (B) that denoting the first crack in the concrete, and (C) that denoting the elastic limit of the reinforcement. (See Fig. 16.)

In none of the beams made with stone aggregate was any indication afforded that the compressive strength of the concrete had been reached at the point where the reinforcement failed. It should be said, however, that the metal used was iron, and the results would have been more interesting if steel bars had been adopted permitting the fuller development of the strength of the concrete in compression.

According to these tests, the ratio of the coefficients of elasticity of concrete in compression and in tension was 2.17, this ratio being ascertained with respect to stresses of 750 lb. per square inch in compression, and 300 lb. per square inch in tension. Similarly, the ratio of the coefficients of the iron and the concrete in tension was 13.9, these values applying at the point A.

We have previously mentioned the influence of reinforcement in augmenting the extensibility of concrete, and it is interesting to observe that, according to the results obtained by Professor Hatt, while the concrete in non-reinforced beams was ruptured with an average elongation of one part in 7,000 parts the reinforced concrete endured an average elongation of one part in 1,140 parts before failure. The explanation suggested for this phenomenon is that the reinforcement distributes the total elongation over the whole length of the concrete, so that it is not confined to the fractured section, as in the case of simple concrete.

Assuming the tensile strength of the concrete at 300 lb. per square inch, and the elongation at 1 : 1,000, the value of E_t for the reinforced concrete in tension was 300,000 lb. per square inch. The value of E_c' for concrete in compression at 1,500 lb. per square inch was found to be about 3,840,000 lb. per square inch, and the value of E_t' for the iron bars was 29,000,000 lb. per square inch.

Hence at the point B, the ratio of the coefficients of elasticity of concrete in compression and in tension was 12.8, and the ratio of the co-efficients of the iron and the concrete in tension was 96.6.

Among other matters, enquiry was directed to the influence of varying percentages and positions of the reinforcement. The subjoined notes summarise the more important conclusions to be drawn from the records :—

(1) One per cent. of metal in the sectional area of beam placed 1 in. from the bottom raises the load-carrying capacity to more than 3 times, and flexibility to fully 10 times that of a simple concrete beam.

(2) Two per cent. of metal in the sectional area of beam placed 1 in. from the bottom increases the load-carrying capacity to 5 times, and flexibility to about 15 times that of a simple concrete beam.

(3) One per cent. of metal in the sectional area of beam placed 2 in. from the bottom raises the load-carrying capacity to nearly $2\frac{1}{2}$ times, and flexibility to 12 times that of a simple concrete beam.

(4) Two per cent. of metal in the sectional area of beam placed 2 in. from the bottom raises the load-carrying capacity to about $4\frac{1}{2}$, and flexibility to 18 times that of a simple concrete beam.

The foregoing observations are only useful in a relative sense, for they do not indicate the increase of strength that might have been obtained by the employment of steel instead of iron reinforcement.

Professor Hatt's theory is intended to account for the three characteristic points, A, B, and C, which are diagrammatically represented in Fig. 16, and the main assumptions upon which it is based are the following :—

1. That the cross-sections of a beam remain plane surfaces after bending.

2. That the forces are applied perpendicularly to the neutral surface of the beam.

3. That the values of the coefficients of elasticity obtained by tests for simple compression and tension will apply to the material when under stress in the beam.

4. That the bond between the materials is perfect.

5. That there are no initial stresses in the beam due to contraction of the concrete during setting.

Of course, it is necessary in connexion with every theory to make certain assumptions, but it is generally unlikely that the bases assumed will be entirely realised in practice. This is so in the case of the conditions now stated, but the resulting differences appear to be so small that they do not affect the substantial accuracy of the calculations.

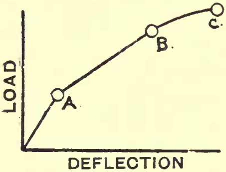


FIG. 16.

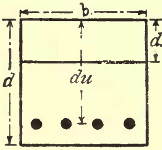


FIG. 17.

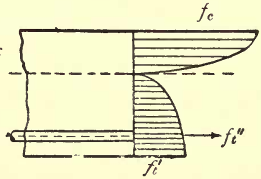


FIG. 18.

Fig. 17 is the cross-section of a concrete-steel beam, typical of those tested by Professor Hatt, and Fig. 18 is a part longitudinal section, to which are applied typical curves, as ascertained by Professor Hatt, representing compression and tension in the concrete at different distances from the neutral axis and tension in the iron. For the purpose of avoiding unnecessary complication it is assumed that the curves are parabolic arcs.

39. Formulæ.—For the sake of uniformity we have adopted for Professor Hatt's formulæ the system of nomenclature elsewhere followed in this treatise, except in the case of two or three special factors. The following is a list of the symbols used:

b = breadth of cross section.

d = depth of cross section.

$du = d \times u$ = distance of the upper fibres from the centre of gravity of the reinforcement.

$dx = d \times x$ = distance of the upper fibres from the neutral axis.

E_c' = co-efficient of elasticity of the concrete in compression.

E_t' = co-efficient of elasticity of the concrete in tension.

E_t'' = co-efficient of elasticity of the reinforcement in tension.

f_c' = maximum fibre stress of concrete in compression.
f_t' = maximum fibre stress of concrete in tension.
f_t'' = tensile stress in reinforcement.
l = length of span.
$m_1 = E_c' \div E_t'$.
$m_2 = E_t'' \div E_t'$.
ρ = ratio of the cross-sectional areas of the metal and of the beam.

The values of E_c' and E_t' are measured at the stresses represented by f_c' and f_t' respectively. The values of x, u, and ρ are ratios; u and ρ can be settled by the designer, but x is dependent upon the values of ρ, u, m_1, and m_2, the two last factors being variable, according to the quality of the materials, and changing under stress, as previously pointed out, with the varying values of f_t'', f_c', and f_t'. In practical design, however, it is sufficient to use constant values of m_1 and m_2, corresponding with the points A and B.

In the development of his formulæ, Professor Hatt proceeds first to determine the ratio of f_c' to f_t', and of f_t'' to f_t'.

Thus
$$f_c' = f_t' m_1 x \div (1 - x), \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$
$$f_t'' = f_t' m_2 (u - x) \div (1 - x). \quad . \quad . \quad . \quad . \quad . \quad (19)$$

To ascertain the position of the neutral axis, which is given by the value of x, the forces of tension and compression on the cross section of the beam are equated.

Thus
$$\tfrac{2}{3} f_c' x = \tfrac{2}{3} f_t'(1 - x) + \rho f_t''. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Next, inserting the values of f_c' and f_t'' as ascertained by formulæ (18) and (19) the equation is obtained
$$\tfrac{2}{3} x^2 m_1 = \tfrac{2}{3}(1 - x)^2 + \rho m_2 (u - x).$$

Being solved, this gives $x =$
$$\frac{-\frac{1}{2}(4 + 3\rho m_2) + \sqrt{4m_1 + \frac{9}{4}\rho^2 m_2^2 + \rho[6m_2\{u(m_1 - 1) + 1\}]}}{2(m_1 - 1)}. \quad (21)$$

When the value of x is thus found, f_c' and f_t'' are calculated, and finally the moment of resistance of the whole section is computed.

Thus

$$R = f_t' b d^2 \left\{ \frac{5}{12}(1 - x)^2 + \frac{5 m_1 x^3}{12(1 - x)} + \rho \frac{(u - x)^2}{1 - x} m_2 \right\}. \quad (22)$$

The above formulæ apply to the computation of the load carried by a beam up to the point A. At the point B, the stress curve loses its parabolic form to some extent, and for very accurate results, the equations should be altered, but by adopting suitable values of m_1 and m_2, as given above, the error in computation will be small. If it be desired to calculate the strength of the beam after the crack has developed and extended through the tension area, it becomes necessary to modify the formulæ by omitting the effect of the tensile force resulting from the resistance of concrete in that portion of the section.

The position of the neutral axis is then found by the rule

$$\tfrac{2}{3} f_c' x = \rho f_t'', \text{ or } \rho \frac{E_t''}{E_c'}(u - x) = \tfrac{2}{3} x^2. \quad . \quad . \quad (23)$$

When f_t'' is assumed to represent the elastic limit of the reinforcement, the value of f_c' may be calculated. The moment of resistance of the section is found by the following modification of the original rule:

$$R = b d^2 \left\{ \frac{5}{12} f_c' x^2 + \rho f_t''(u - x) \right\}. \quad . \quad . \quad . \quad (24)$$

With the object of testing the correctness of his theory, Professor Hatt made a series of calculations, using values representing the actual strength of the constituent materials, as found by tests, and the previously stated values of m_1 and m_2.

The tensile strength of the concrete was taken at 300 lb. per sq. in., and the elastic limit of the iron at 36,000 lb. per sq. in. A summary of calculated and measured results is given in table XXII., by which it will be seen that the agreement between theory and experiment is fairly close.

Table XXII.—Theoretical and Experimental Results from Reinforced Concrete Beams under Concentrated Centre Loads. (Hatt.)

(Dimensions 8 in. by 8 in. Span 80 in.)

Note.—All calculated figures are printed in italics.

Index to Results.	Reinforcement.			
	1 per cent.		2 per cent.	
	Height from bottom.		Height from bottom.	
	2-in.	1-in.	2-in.	1-in.
At Point A.	lb.	lb.	lb.	lb.
Centre load, - - -	2,000	2,500	2,000	3,000
	2,103	*2,266*	*2,400*	*2,648*
Stress in iron (tension), -	*2,000*	*2,700*	*1,950*	*2,760*
,, concrete (compression),	*457*	*474*	*408*	*516*
At Point B.				
Centre load, - - -	5,000	6,500	5,750	10,000
	4,925	*6,232*	*7,123*	*9,680*
Stress in iron (tension), -	*17,320*	*22,000*	*16,550*	*21,570*
,, concrete (compression),	*1,556*	*1,680*	*1,937*	*2,188*
At Point C.				
Centre load, - - -	5,500	7,300	10,250	12,000
	5,470	*6,705*	*10,840*	*12,515*
Stress in concrete, - -	*1,699*	*1,810*	*3,050*	*2,710*

It must not be forgotten that the point C, although representing the failure of the beam, does not represent the ultimate strength of the concrete in compression, for, as already mentioned, the iron failed before the concrete gave any indication that its compressive strength had been reached.

Table XXIIa gives the calculated heights of the neutral axis from the bottom of the beam. We state the figures in this way instead of giving the distance *dx*, as they will be more convenient to compare with previous references to the neutral axis. It should be particularly noted that the height of the neutral axis varies, not only with the amount and disposition of the steel, but also with increase of load.

Table XXIIa.—Calculated Positions of Neutral Axis and Values of x for Beams in Table XXII.

(Depth of beam, 8 in.)

Reinforcement.		Height of Neutral Axis and Value of x.					
		Point A.		Point B.		Point C.	
Percentage.	Distance from bottom.	Neutral Axis.	x	Neutral Axis.	x	Neutral Axis.	x
	In.	In.		In.		In.	
1	2	4.536	0.433	5.584	0.302	5.824	0.272
1	1	4.472	0.441	5.456	0.318	5.616	0.298
2	2	4.368	0.454	5.168	0.354	5.304	0.337
2	1	4.296	0.463	4.976	0.378	4.944	0.382

Professor Hatt's formulæ themselves may appear to be somewhat intricate for everyday use, but in many cases their direct employment might be avoided by the preparation of diagrams or tables from which the proportions of beams for various loads and spans could be found without the necessity for individual calculations.[1]

40. Examples.—In practical computations m_1, m_2, u, and ρ, are settled; x is then calculated from formula (21), f_c' and f_t'' are determined by the aid of formulae (18) and (19), and finally the moment of resistance is ascertained from formula (22). As the moment of resistance is always equal to the bending moment, it is easy to ascertain the load corresponding with any conditions of loading and span.

Example 12.—For the purpose of practically illustrating the application of the formulæ, we will now calculate the

[1] Since this chapter was written, Professor Hatt has published a simplified formula with explanatory diagrams. (*Proc. Am. Railway Eng. and Maintenance of Way Assoc.*, 1904.)

load at the point A, for one of the beams tested by Professor Hatt (Art. 38, p. 70), selecting a proportionate reinforcement of one per cent. placed two inches above the bottom of the beam. We will take the following values for the ratios, viz.:

$$m_1 = 2, \ m_2 = 12, \ u = (6 \div 8) = 0.75, \text{ and } \rho = 0.01.$$

The first two of these differ fractionally from the ratios found by Professor Hatt, and the results will consequently be slightly different from those in Tables XXI. and XXII.

The first step is to compute the value of x, serving to settle the height of the neutral axis.

Thus by formula (21) we obtain

$$x = \frac{-\frac{1}{2}(4 + 3 \times .12) + \sqrt{(8) + (\frac{9}{4} \times .0144) + .01[72\{.75 + 1\}]}}{2(2-1)}$$

$$= \frac{-\frac{1}{2}(4.36) + \sqrt{(8 + .0324) + .01[72\{1.75\}]}}{2}$$

$$= \frac{0.863}{2} = 0.434.$$

As in this case $d = 8$ in., we have $dx = 8 \times 0.434 = 3.472$ in., and the height of the neutral axis above the bottom of the beam is $d - dx = 4.528$ in.

Calculations for the determination of x might be materially simplified for concrete of constant quality, and further simplification would result from the preparation of separate forms of equation (21) for the four ratios stated, or for any desired values of ρ.

As an example we append a condensed form of the equation, in which the values of m_1 and m_2, for the quality of concrete before considered, and ρ, for one per cent. of reinforcement, are treated as constants.

Thus

$$m_1 = 2\ ; \ m_2 = 12\ ; \text{ and } \rho = 0.01.$$

For all cases complying with these conditions, the condensed formula then becomes:

$$x = \frac{-2.18 + \sqrt{8.0324 + 0.72(u+1)}}{2}. \quad . \quad . \quad (25)$$

Here the only variable factor is the value of u which remains to be settled by the designer as may best suit his convenience. Similarly simplified forms of the equation can be prepared for any other proportions of reinforcement and qualities of material.

To resume the present calculation, we have next to compute the values of f_c', the maximum compressive fibre stress in the concrete; and f_t'', the tensile stress in the reinforcement.

For the purpose of finding the value of f_c', we take f_t', the limiting fibre stress of concrete in tension at 300 lb. per square inch. Then by formula (18) we have

$$f_c' = (300 \times 2 \times 0.434) \div (1 - 0.434) = 460 \text{ lb.};$$

also by formula (19)

$$f_t'' = \{300 \times 12(0.75 - 0.434)\} \div (1 - 0.434) = 2{,}009 \text{ lb.}$$

Next, we determine the moment of resistance R, of the section, by formula (22).

Thus

$$R = 300 \times 8 \times 8^2 \left\{ \frac{5}{12}(1 - 0.434)^2 + \frac{5 \times 2 \times 0.434^3}{12 \times (1 - 0.434)} + 0.01 \frac{(0.75 - 0.434)^2}{(1 - 0.434)} 12 \right\}$$

$$= 153600\{0.13348166 + 0.12035 + 0.02117088\}$$

$$= 153600 \times 0.27501 = 42{,}241.53 \text{ inch-pounds,}$$

say, 42,240 inch-pounds.

Finally we find the theoretical concentrated load in the following manner:

As R = M, it follows that M = 42,240 inch-pounds.

But for a concentrated centre load, $M = \dfrac{Wl}{4}$, and as in this case $l = 80$, we have

$$M = 20 \text{ W inch-pounds.}$$

Hence

$$W = 42{,}240 \div 20 = 2{,}112 \text{ lb.}$$

By reference to Table XXII., it will be seen that this result very nearly agrees with that given for the load at point A, the small difference between the calculated loads being due to our slight modification of the values for m_1 and m_2.

Similarly, the theoretical distributed load would be thus ascertained:

As $M = \frac{Wl}{8}$ it follows that $M = 10$ W inch-pounds.

Hence

$$W = 42,240 \div 10 = 4,224 \text{ lb.}$$

Example 13.—For the purpose of preparing a table or diagram of beams, considerable time will be saved by calculating the moment of resistance per inch width of the section. Then for the beam considered in *Example* 12, we have values of x, f_c', and f_t'', as before, and the moment of resistance per inch of width becomes by formula (22)

$$R = 300 \times 1 \times 64\left\{\frac{5}{12}(0.566)^2 + \frac{5 \times 2 \times 0.434^3}{12 \times (1 - 0.434)} + 0.01\frac{(0.75 - 0.434)^2}{(1 - 0.434)}12\right\}$$

$$= 19200\{0.13348166 + 0.12035 + 0.02117088\}$$

$$= 19200 \times 0.27501$$

$$= 5,280.19 \text{ inch-pounds, say } 5,280 \text{ inch-pounds.}$$

From this value of R, the corresponding concentrated load is found to be

$$W = 5,280 \div 20 = 264 \text{ lb.},$$

and the distributed load to be

$$W = 5,280 \div 10 = 528 \text{ lb.}$$

The corresponding load for any width of a beam 8 in. deep, and otherwise complying with the conditions stated in *Example* 12, can now be ascertained by a simple multiplication sum.

Example 14.—We will next take a beam similar to that for which the essentials are stated in *Example* 12, but with a depth of 12 in.

In this case, the reinforcement still being 2 in. from the bottom, the ratio $u=(10 \div 12)=0.833$. Substituting this value in equation (25) we have

$$x=\frac{-2.18+\sqrt{8.0324+0.72(0.833+1)}}{2}$$

$$=\frac{-2.18+\sqrt{8.0324+1.32}}{2}$$

$$=\frac{-2.18+3.058}{2}=0.439.$$

This means that $hx=(0.439 \times 12)=5.268$ in., and that the height of the neutral axis above the bottom of the beam is $(12-5.268)=6.732$ in.

For the moment of resistance per inch in width of the section we have, by formula (22)

$$R=300 \times 1 \times 12^2\left\{\frac{5}{12}(1-0.439)^2+\frac{5 \times 2 \times 0.439^3}{12(1-0.439)}+0.01\frac{(0.833-0.439)^2}{1-0.439}12\right\}$$

$$=43{,}200\{0.131133+0.1257+0.033\}$$

$$=43{,}200 \times 0.289833=12{,}520 \text{ inch-pounds.}$$

The corresponding loads found in the manner previously indicated, are

(*a*) For a concentrated load $W=12{,}520 \div 20=626$ lb.

(*b*) For a distributed load $W=12{,}520 \div 10=1{,}252$ lb.

As before, the load for any width of a beam, 12 in. deep and otherwise complying with the stated conditions, can now be found by multiplication.

Example 15.—In all the foregoing calculations, we have been dealing with a span of 80 in., that being the distance between the supports in the case of the beams tested by Professor Hatt. (Art 38, p. 70).

But, having the moment of resistance of any given section, the load can easily be computed for a unit span, and from the values so obtained the load for any other span can be found by a simple division sum.

By *Example* 13, we found the moment of resistance per inch width of the 8 in. deep beam, with 80 in. span, to be $R = 5,280$ inch-pounds, and as $R = M$, $M = 5,280$ inch-pounds.

But for a concentrated load $M = \frac{Wl}{4}$.

Hence for a unit span of 1 inch.

$$M = \tfrac{1}{4} W \text{ inch-pounds.}$$

Whence

$$W = 5,280 \div \tfrac{1}{4} = 21,120 \text{ lb.}$$

For a distributed load

$$M = \frac{Wl}{8},$$

Hence for a unit span of 1 in.

$$M = \tfrac{1}{8} W \text{ inch-pounds.}$$

Whence

$$W = 5280 \div \tfrac{1}{8} = 42,240 \text{ lb.}$$

These loads are per unit width and unit span of one inch in each case, and to test their accuracy we may compare the results to be obtained therefrom with the calculated loads in *Example* 12.

Thus for a clear span of 80 in. the concentrated load becomes $21,120 \div 80 = 264$ lb. per unit width of beam, and for a width of 8 in. the load is $264 \times 8 = 2,112$ lb. as previously found.

Similarly the distributed load is

$$42,240 \div 80 = 528 \text{ lb. per unit width,}$$

and for a width of 8 in. the load is

$$528 \times 8 = 4,224 \text{ lb. as before.}$$

Example 16.—By *Example* 14, we found the moment of resistance of the 12 in. deep beam with 80 in. span to be $R = 12,520$ inch-pounds. From this we obtain loads per unit width and span as follow :

For a concentrated load, $W = 12,520 \div \tfrac{1}{4} = 50,080$ lb.

For a distributed load, $W = 12,520 \div \tfrac{1}{8} = 100,160$ lb.

If we wish to find the concentrated load for a 12 in. × 6 in. beam with a clear span of 80 in., the result is soon obtained. Thus

$$W = (50{,}080 \div 80) \times 6 = 3{,}756 \text{ lb.}$$

And the distributed load for the same beam is

$$W = (100{,}160 \div 80) \times 6 = 7{,}512 \text{ lb.}$$

This case shows incidentally the advantage derived from increasing the depth of a beam in proportion to its width, as the load here carried is more than 75 per cent. greater than the load for the 8 in. square beam previously considered, while the sectional area of concrete is about $12\frac{1}{2}$ per cent. greater.

41. The Preparation of Tables of Beams.—By the aid of calculations similar to those already made the essential units may be determined for beams varying in depth over any desired range, and by adopting values of ρ, for different percentages of reinforcement, a series of tables can readily be prepared for office use, so that the proportions of a beam for any required duty may be found without the least trouble or delay.

If tables of the kind were prepared they would only apply to the particular quality of concrete and proportions of reinforcement taken as the basis of calculation, but if in practice only one type of concrete were employed, there would be no reason for inaccuracy. By adopting steel reinforcement instead of wrought iron, the altered value of the coefficient of elasticity would naturally require a proportionate modification of the factors.

All the foregoing calculations refer to point A, where the curve first deviates from the straight line, and the load in every case includes the weight of the beam. On reference to Table XXII, it will be seen that the load at this point may be taken, for 1 per cent. of reinforcement, at about one-third of the load at the point C representing failure; and the load at the point of A, for 2 per cent. of reinforcement, may be taken at about one-fourth of the load at failure. As compared with the load at point B, corresponding with the first crack in the tension flange, the load at point A may be put roughly at two-fifths in the case of 1 per cent. of reinforcement, and one-third in the case of 2 per cent. of reinforcement. For all practical purposes we may regard the first crack as representing failure, and it is evident that some further factor of safety should be applied to results based on point A conditions. No further calculation is necessary in such cases, if tables of beams have been prepared, as the estimated load may be increased by any advisable proportion, and a beam can then be selected to suit the total load so given.

By adding 50 per cent. to the estimated load, a total factor of safety of about 4 will be given for 1 per cent. reinforcement, or of about 6 for 2 per cent. reinforcement; and by doubling the estimated load the factors become 6 and 8 respectively.

The proper factor for adoption must, of course, depend upon the character of the work to be executed.

42. Mr. Johnson's Theory.—Another series of rules for calculating the strength of concrete steel beams is that formulated by Mr. A. L. Johnson.[1]

The formulæ are perfectly suitable for general use, as they have no relation to any of the specific styles of concrete-steel construction favoured by various patentees. They are based upon the assumption that the concrete in tension is of no value in the moment of resistance of the cross-section. The following notation is uniform with that employed throughout this treatise:

a = Total area of reinforcement;

$\frac{a}{s}$ = Area of reinforcement per inch in width of beam;

b = Width of beam section;

d = Total depth of the beam, in inches;

E_c' = Coefficient of elasticity of concrete in compression;

E_t'' = Coefficient of elasticity of steel in tension;

e = Elastic limit of steel;

F_c' = Compressive strength of concrete;

h_x = Distance between the top of the beam and the neutral axis, in inches;

h_v = Distance between the middle plane of the reinforcement and the neutral axis, in inches;

λ_1 = Unit strain of extreme fibre in compression;

[1] *Proc. Am. Soc. C.E.*, vol. xxvii., no. 6.

λ_2 = Unit strain of steel in tension;
M = Bending moment of external forces;
P′ = Total stress on concrete in width b;
P″ = Total stress on metal in width b;
R_x = Moment of ultimate resistance of cross-section, for failure in compression;
R_y = Moment of ultimate resistance of cross-section for failure in tension;
s = Spacing of reinforcement;
W = Total load on beam;
w = Uniformly distributed load, per square foot;
z = Distance from the bottom of the concrete to the middle plane of the reinforcement;

As at the maximum load, the stress developed in the metal cannot be much greater than its elastic limit, the employment of steel is advocated with an elastic limit as high as may be consistent with the ductility desirable in any particular construction.

Although it is possible to break the metal reinforcement in a beam, it should be remembered that the intensity of stress thereby involved cannot be developed until the construction as a whole has far exceeded its maximum load carrying capacity. Under these circumstances it is quite correct to assume that the coefficient of elasticity of the steel is constant up to the elastic limit, and when this point is reached the construction as a whole has practically attained its maximum resistance.

A similar condition does not obtain so far as concerns the concrete in compression, for the maximum carrying capacity is not reached until the extreme fibre stress becomes equal to the compressive strength of the material. Whether the maximum capacity can be so attained depends upon the strength and amount of the reinforcement.

In the case of the beams tested by Professor Hatt, to which reference is made in Article 38, the reinforcement was not adequate for developing the maximum strength of the concrete. Mr. Johnson shows that when such development has been attained, the stress diagram above the neutral axis is as shown in Fig. 20, and he states that this representation was deduced from the examination of numerous stress diagrams for both stone and cinder concretes of average proportions. On this diagram the total stress P′, equal to the area *ogk* is about 25 per cent. greater than the area of the shaded part *ojk*.

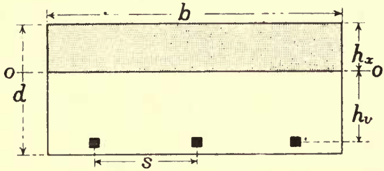


FIG. 19.

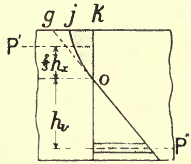


FIG. 20.

Similarly it was ascertained that the coefficient of elasticity as represented by the tangent of the angle *jok* is only two-thirds the value of the original coefficient for stone concrete, and only one-half the value of the original coefficient for cinder concrete, the original value being represented by the tangent of the angle *gok*.

43. Demonstration and Formulæ.—The following demonstration relates to Mr. Johnson's formulæ for single-span beams supported at the ends, and reference to Figs. 19 and 20 will assist the reader in following the line of reasoning pursued.

In the first place, the total tensile stress on the metal in width b is

$$P'' = \frac{eab}{s};$$

when the unit strain or extension of the metal is

$$\lambda_2 = \frac{e}{E_t''},$$

and

$$P'' = E_t''\lambda_2\frac{ab}{s}.$$

Taking next the compression side, the unit strain of the concrete is

$$\lambda_1 = \lambda_2\frac{h_x}{h_v},$$

with the following results:—

$$F_c' = \lambda_1\frac{2E_c'}{3} = \lambda_2\frac{h_x}{h_v}\frac{2E_c'}{3} \text{ for stone concrete;}$$

and

$$F_c' = \lambda_1\frac{E_c'}{2} = \lambda_2\frac{h_x}{h_v}\frac{E_c'}{2} \text{ for cinder concrete.}$$

The total stress on the concrete in width b is $P' = \frac{5F_c'bh_x}{8}$ for both stone and cinder concretes.

Then $$P' = \frac{5bh_x^2\lambda_2 E_c'}{12h_v} \text{ for stone concrete,}$$

and $$P' = \frac{5bh_x^2\lambda_2 E_c'}{16h_v} \text{ for cinder concrete.}$$

As, however, $P' = P''$, it follows for stone concrete that

$$E_t''\lambda_2 \frac{ab}{s} = \frac{5bh_x^2\lambda_2 E_c'}{12h_v};$$

whence $$h_x^2 = \frac{12ah_vE_t''}{5sE_c'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Similarly, for cinder concrete it follows that

$$E_t''\lambda_2 \frac{ab}{s} = \frac{5bh_x^2\lambda_2 E_c'}{16h_v}$$

whence $$h_x^2 = \frac{16ah_vE_t''}{5sE_c'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

In computing the moments of ultimate resistance of the cross-section, the axis of compression is taken in the position that it would occupy if the shaded area jok, in Fig. 20 were a triangle. Although not precisely accurate this assumed position leads to no appreciable error.

The result is that for the moment of resistance for failure in tension

$$R_y = P''h_v + \frac{2P'h_x}{3}$$

$$= \frac{P''}{3}(3h_v + 2h_x)$$

$$= \frac{eab}{3s}(3h_v + 2h_x) \quad . \quad . \quad . \quad . \quad (28)$$

and for the moment of resistance for failure in compression,

$$R_x = P''h_v + \frac{2P'h_x}{3}$$

$$= \frac{P'}{3}(3h_v + 2h_x)$$

$$= \frac{5bF_c'h_x}{24}(3h_v + 2h_x) \quad . \quad . \quad . \quad (29)$$

The total depth of the beam is

$$d = h_x + h_v + z. \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The distance represented by z can, of course, be varied by the designer according to the thickness and character of the metal employed, and with due regard to the position and intended purpose of the construction. In using the formulæ (28) and (29) the smaller of the two values for the moment of resistance should be employed, but in designs where the quantity of metal can be varied, it is always desirable that the construction should be of equal strength in tension and compression, so that $R_y = R_x$, or that,

$$\frac{eab}{3s} = \frac{5bF_c'h_x}{24}$$

From this equation is obtained the area of reinforcement per inch in width of the beam, or

$$\frac{a}{s} = \frac{F_c'h_x}{1.6e}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The combination of formulae (26) and (31) gives for stone concrete

$$h_v = \frac{2eE_c'h_x}{3F_c'E_t''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

and the combination of formulæ (27) and (31) gives for cinder concrete

$$h_v = \frac{eE_c'h_x}{2F_c'E_t''}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

These values introduced into formula (29) give for stone concrete

$$R_y = R_x = \left[\frac{5beE_c'}{12E_t''} + \frac{5bF_c'}{12}\right]h_x^2. \quad . \quad . \quad . \quad . \quad (34)$$

and for cinder concrete

$$R_y = R_x = \left[\frac{5beE_c'}{16E_t''} + \frac{5bF_c'}{12}\right]h_x^2. \quad . \quad . \quad . \quad . \quad (35)$$

The construction should always be designed of equal strength in tension and in compression, or so that $R_y = R_x$. Then if it be desired to adjust the moment of resistance of a beam, so that it may be equal to a predetermined bending moment we have $R_y = R_x = M$, and from formula (34) for stone concrete

$$h_x = \pm\sqrt{\frac{M}{\frac{5beE_c'}{12E_t''} + \frac{5bF_c'}{12}}} \quad . \quad . \quad . \quad . \quad (36)$$

and from formula (35) for cinder concrete

$$h_x = \pm\sqrt{\frac{M}{\frac{5beE_c'}{16E_t''} + \frac{5bF_c'}{12}}} \quad . \quad . \quad . \quad . \quad (37)$$

The foregoing equations can be considerably simplified for general use when the physical properties of the reinforcement and the concrete have been settled, and in the case of (36) and (37) the factors are readily reducible to numerical fractions. In using the formulæ for beams of equal strength above and below the neutral axis, the value of h_x is determined from (36) or (37); h_c from (32) or (33); and $\frac{a}{s}$ from (31).

The thickness of the concrete and the spacing of the bars then depends upon the size of the bars used.

Let us assume e, the elastic limit of the steel to be used, at 54,000 lb. per square inch; E_c', the coefficient of elasticity of concrete, at two-thirds the original value, or, say $\frac{2}{3} \times 3{,}000{,}000$ lb. $= 2{,}000{,}000$ lb. per square inch: E_t'', the coefficient of elasticity of steel at 30,000,000 lb. per square inch; and F_c', the compressive strength of stone concrete, at 2,400 lb. per square inch.

Then formula (36) for stone concrete may be reduced thus

$$h_x = \pm\sqrt{\frac{M}{\frac{5b \times 54{,}000 \times 2{,}000{,}000}{12 \times 30{,}000{,}000} + \frac{5b \times 2{,}400}{12}}}$$

$$= \pm\sqrt{\frac{M}{(b \times 1{,}500) + (b \times 1{,}000)}} \quad . \quad . \quad . \quad . \quad . \quad (38)$$

44. Examples.—For the purpose of illustration we will apply these formulæ to the case of a beam 8 inches wide supported at each end, with a clear span of 80 inches, and subjected to a concentrated centre load of 10,000 lb.

Example 17.—We have $b = 8$ inches, and for the bending moment

$$M = \frac{Wl}{4} = \frac{10{,}000 \times 80}{4} = 200{,}000 \text{ inch-pounds.}$$

Inserting these values in formula (38) we obtain

$$h_x = \sqrt{\frac{200{,}000}{(8 \times 1{,}500) + (8 \times 1{,}000)}} = \sqrt{10} = 3.16 \text{ in.}$$

This is the distance of the extreme fibre in compression from the neutral axis as represented in Fig. 19.

Example 18.—To find the distance between the neutral axis and the centre of the reinforcement, h_v (Fig. 19) we use formula (32), and inserting the values stated above, and the value of h_x, it becomes

$$h_v = \frac{2 \times 54{,}000 \times 2{,}000{,}000 \times 3.16}{3 \times 2{,}400 \times 30{,}000{,}000} = 3.16 \text{ in.}$$

Example 19.—From formula (31) the amount of metal in square inches per inch in width is obtained.

Thus

$$\frac{a}{s} = \frac{F_c' h_x}{1.6\,e} = \frac{2{,}400 \times 3.16}{1.6 \times 54{,}000} = 0.0877 \text{ sq. in.}$$

This is equal to a total area of steel reinforcement for the width b amounting to $8 \times 0.0877 = 0.7016$ square inch, and if we adopt a spacing of 2 inches there will be four bars, each with an area of $0.7016 \div 4 = 0.1754$ square inch.

Example 20.—The depth of concrete below the centre line of the reinforcement may be fixed at about 1.5 inch, or say 1.68 inch, which, by rule (30), will give for the total depth of the beam

$$d = h_x + h_v + z = 3.16 + 3.16 + 1.68 = 8 \text{ in.}$$

Example 21.—Finally, the moment of ultimate resistance may be calculated by formula (34) after inserting the proper values, thus:

$$R_y = R_x = \left[\frac{5 \times 8 \times 54{,}000 \times 2{,}000{,}000}{12 \times 30{,}000{,}000} + \frac{5 \times 8 \times 2{,}400}{12}\right] \times 10$$

$$= [12{,}000 + 8{,}000] \times 10 = 200{,}000 \text{ inch-pounds.}$$

Therefore, the moment of resistance agrees with the stated bending moment.

45. Note on Results.—The general result of the foregoing calculations indicate that 10,000 lb. is the maximum centre load for a concrete-steel beam, 8 inches square, with a clear span of 80 inches, containing about 1.1 per cent. of steel reinforcement, and otherwise conforming with the stated conditions.

Making due allowance for the relatively high elastic strength of the steel here considered, and also for the percentage and position of the reinforcement, this result is fairly in agreement with the records as to the ultimate strength of the beams of similar dimensions tested by Professor Hatt. (See Table XXII., p. 77.)

In using the foregoing formulæ any desired factor of safety may be applied at the discretion of the designer to suit the requirements of the work to be carried out. In Examples 1 to 11, the general factor of safety is about 5, the unit stresses of concrete and steel being taken at one-fifth the average ultimate strength. For many purposes it would be desirable to increase the factor of safety by further limitation of the unit stresses upon which computations are based.

An important point to be noted in this connexion is that the elastic limit, and not the ultimate strength, of the reinforcement really governs the resistance of the construction.

CHAPTER V.

SHEARING STRESSES IN BEAMS.

46. Necessity for Considering Shearing Stresses.—All the formulæ hitherto discussed are based on the assumption that if sufficient steel be employed for resisting bending moments, shearing stresses may safely be left to the concrete. No doubt the concrete is perfectly capable of taking shearing stresses up to a certain limit, within which the safety of the beam may be fully assured, but greater strength is attainable by more complete conformity with theoretical requirements, and greater economy may be secured thereby.

Shearing force in a beam is developed by the downward action of the load, and the upward reaction of the supports. In the case of a beam with a uniformly distributed load, and supported at the ends, the maximum shearing force has the value, $-wl \div 2$ at the left hand end, decreasing uniformly to zero at the middle of the span, thence increasing uniformly to $wl \div 2$ at the right hand end. If the load be concentrated at the centre of the beam, the shearing force will be constant throughout, and equal to $W \div 2$, but the force will be *negative* along the left half of the beam, and *positive* along the right half.

It is known that when a test specimen of concrete is crushed, failure is due wholly or in part to shearing action on surfaces inclined to the direction of the force exerted. Further, it is known that beams frequently fail in consequence of tensile stresses resulting from shear. These facts show that it is very necessary for the designer of concrete-steel beams to consider the nature and influence of shearing stresses, and to make due provision for them in all works to be executed.

47. Definition of Simple Shear.—A state of simple shearing stress, or, as it is alternatively called, a state of simple shear, occurs when two simple stresses of equal intensity act in opposition. Under such a condition there is no normal stress on a plane inclined at 45 deg. to the two directions, because the normal component of one force is equal and opposite to that of the other, and nothing remains but shearing, or tangential, stress.

Hence the state of simple shear is one in which there are two principal stresses only, giving rise to a stress that is wholly tangential on the two planes inclined at 45 deg. to the axes of principal stress, and that is equal in intensity to the intensity of either of the principal stresses.

The same conclusion may be reached by a different line of reasoning. Thus, if to a cube of unit dimensions, represented by the rectangle in Fig. 21, tangential stresses QQ,

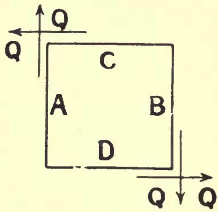


FIG. 21.

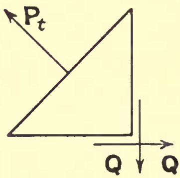


FIG. 22.

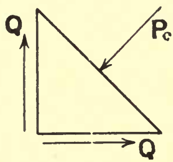


FIG. 23.

be applied to the pair of opposite faces A and B, and equal tangential stresses QQ, to the pair of opposite faces C and D, the effect will be to create a state of simple shear. There will then be only tangential stress on all planes parallel to A and B, and to C and D, the intensity of the stress on each system of planes being equal throughout to the intensity applied to the face of the cube.

In order to show the relation between these two modes of defining a state of simple shear, we will consider the equilibrium of two triangular prisms, Fig. 22 representing the half DB of the cube in Fig. 21, and Fig. 23 the half DA of the same cube.

To balance the stresses QQ (Fig. 22) we must have normal tension P_t, on the diagonal plane. As the diagonal is at 45 deg. to the horizontal, its length compared with the length, l, of any side of the cube is: $l \sec\theta$, or $l\sqrt{2}$, and the amount of the tension is: $P_t = Q\sqrt{2}$. As the area acted over by P_t is $\sqrt{2}$ times the area over which Q acts, it follows that the intensity of P_t is equal to the intensity of Q.

Taking the opposite prism (Fig. 23) similar conditions prevail, but normal compression P_c, is here evidenced, the value of which is: $P_c = Q\sqrt{2}$, and, for the reason already given, the intensity of P_c is equal to the intensity of Q.

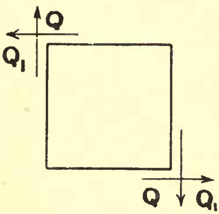


FIG. 24.

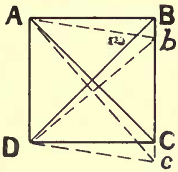


FIG. 25.

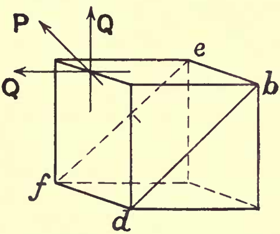


FIG. 26.

Therefore the state of simple shear admits of analysis into two equal and opposite principal stresses, one tensile and the other compressive, acting in directions at right angles to one another, and inclined at 45 deg. to the directions of the shearing stress

It is important to remember that tangential stress cannot exist in one direction unless accompanied by an equal intensity of tangential stress in a direction at right angles to the first direction.

The proof of this principle may be found by considering the equilibrium of a small cube of unit dimensions, represented by the rectangle in Fig. 24, and having one pair of sides parallel to the directions of the stresses QQ. These stresses, acting on opposite sides, produce a couple tending to rotate the cube. The couple cannot be balanced by any arrangement of normal stresses on any of the sides of the cube, and the only way in which the couple can be balanced is by equal and opposite stresses Q_1, Q_1, acting at right angles to the first directions, thus producing a couple that resists the tendency of the cube to rotate under the action of the first couple.

In this case it will be seen that the cube is subjected to equal shearing stresses on two pairs of opposite faces, and the resultant stresses form a system of forces in equilibrium. The cube will be distorted into the form of an oblique prism, without perceptible change of volume. Hence, as in Fig. 25 it will assume a form such as A*bc*D. The material must be lengthened in the direction AC, and shortened in the direction DB, so that the diagonals become A*c* and D*b* respectively, causing tension in one case and compression in the other. And it can be shown that the lengthening and shortening along the face of the diagonals of a cube are each equal to half the sliding that takes place on such inclined planes.

Let us next consider the diagonal plane, *befd* (Fig. 26) in a cube of unit dimensions, and forming part of a beam whose axis is parallel with the front of the cube.

Representing the area of one face of the cube by a, we have for the area of the diagonal plane: $a\sqrt{2}$.

As previously demonstrated, the forces QQ, to the left of this plane have a resultant $P_t = Q\sqrt{2}$, normal to *befd*.

Hence, there is on the plane a tensile stress of unit intensity

$$p_t = (Q\sqrt{2}) \div (a\sqrt{2}) = Q \div a. \qquad (39)$$

If Q represent the whole shearing force, the unit intensity of shearing stress is

$$q = Q \div a.$$

Hence

$$p_t = q = Q \div a. \qquad (40)$$

Similarly, on a diagonal plane at right angles to *befd*, there is a compressive stress of unit intensity

$$p_c = (Q\sqrt{2}) \div (a\sqrt{2}) = Q \div a.$$

Whence

$$p_c = q = Q \div a. \qquad (41)$$

It is now clear that tensile stress p_t of intensity equal to the intensity of vertical or horizontal shearing stress q, acts over the whole of the diagonal plane *befd*, and that compressive stress p_c of intensity equal to the intensity of the vertical or horizontal shearing stress q acts over the whole of a diagonal plane at right angles to the plane *befd*.

48. Distribution of Shearing Stress.—We have shown that the shearing stress at any point in the vertical cross-section of a beam is accompanied by a shearing stress of equal intensity in a horizontal plane through the same

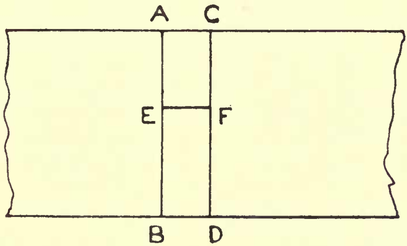


FIG. 27.

point. Therefore, if AB and CD (Fig. 27) represent two vertical sections separated by any distance δx, the intensity of shearing stress in the plane of EF is equal to the intensity of the shearing stress at the point E in the section AB.

If q equal the intensity of vertical shearing stress at E, or at any other point in AB, and b the width of the beam at the same point, the intensity of horizontal shearing stress equals q, and the total horizontal shearing stress Q, in the plane EF, or in any plane corresponding to a point selected in the section AB, is $Q = qb\delta x$.

Again, considering an infinitesimal area in the section AB at any distance y, from the neutral axis, the intensity of shearing stress q, on the area may be expressed thus

$$q = \frac{Q}{2I}(h^2 - y^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where Q equals the total shear on the section; I equals the moment of inertia of the section about the neutral axis;

and h equals the distance of the extreme fibre from the neutral axis.

Hence, it is seen that for any vertical cross-section, q is at a maximum when y equals 0; or, in other words, that for any given vertical cross-section the intensity of the shearing stress is greatest for points on the neutral axis, diminishing to zero at the top and bottom of the section.

The further condition is obvious that for any point at a given distance y from the neutral axis, q is greatest where Q is greatest.

From the foregoing consideration it is evident that the intensity of the tensile and compressive stress on planes inclined at 45 deg. to the horizontal, is greatest for points on the neutral axis in the plane of maximum shear, and that at any such point it is equal to the vertical or the horizontal shear at the same point.

In Fig. 28 let (a) represent a uniformly loaded beam supported at each end, and (b) the diagram of shearing force for the same beam. Then the diagonal stresses of tension and compression will vary in intensity from point to point along the length of the beam as suggested in (c), which represents the diagram of shearing force in an altered form.

The final result is indicated in (d), where the uniformly varying intensity of tensile and compressive stress is represented on the vertical longitudinal section of the beam, it being understood, of course, that the greatest intensity in every plane occurs at the neutral axis, here assumed to be along the middle of the section.

In the case of a steel beam of I section with wide flanges and a thin web, the intensity of shearing stress may be regarded as uniform over the whole of the web, although varying from point to point along the length with a uniform load, and it is much greater in the web than in the flanges, owing to the comparative thinness of the web. Again, for rectangular beams of steel, wrought iron, and wood, it is the fact that safety against shear will usually be assured if the design be such as to afford safety against the results of bending moment.

When beams of concrete steel are in question, however, the conditions are by no means similar.

In the first place, we have two different materials which must adhere so firmly together as to behave similarly to one homogeneous material, and unless this condition be secured, the object of the designer will be frustrated.

In the second place, the concrete in the tension area is not capable of resisting the tensile stress caused within the area in question, and as a general rule is not relied upon for the resistance of any part of the tension.

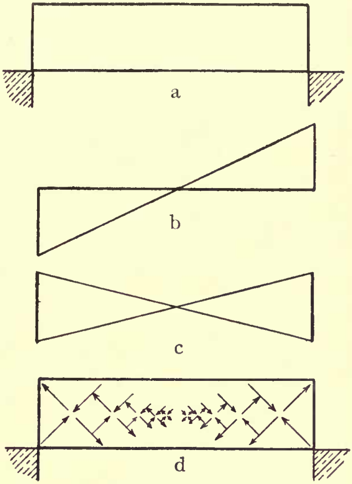


FIG. 28.

Consequently, it is desirable to ascertain whether or not the shear in the horizontal plane of junction between the concrete and the steel reinforcement is sufficient to overcome the adhesion between the concrete and the steel; and whether the intensity of the tensile and compressive stress on planes inclined at 45 deg. to the horizontal is within the safe, or permissible, limits of tensile stress for the concrete employed.

49. Resistance of Concrete-Steel to Horizontal Shear.—We will next consider a horizontal longitudinal section of a beam in connexion with the influence of horizontal shear in the plane of junction between the concrete and the steel reinforcement, with the view of throwing light upon the adequacy of adhesion between concrete and steel for the resistance of horizontal shear.

Let the symbols E_s' and E_s'' represent the co-efficients of shearing elasticity of concrete and steel respectively, and let q' and q'' represent respectively the intensities of shearing stress at any point on the concrete and the steel reinforcement in a beam.

Then, assuming that the concrete and the steel act together as if they were one homogeneous substance, and that plane sections remain plane after flexure, it follows that

$$\frac{q'}{q''} = \frac{E_s'}{E_s''}.$$

Similarly, if p' and p'' represent the intensities of normal stress on concrete and steel respectively, at any point in a plane section inclined at 45 deg. to the horizontal, we have

$$\frac{p'}{p''} = \frac{E'}{E''}.$$

We have already shown that the intensity of normal stress on diagonal planes is equal to the vertical or horizontal shearing stress acting at the same point.

Consequently $$\frac{E_s'}{E_s''} = \frac{E'}{E''}.$$

Let us now take a horizontal longitudinal section of a beam through the centre of the reinforcement, which is assumed to consist of horizontal bars. Let b represent the width of the beam, and t the collective width of the reinforcement, each measurement being taken across the plane of the section.

Then in a portion of the length δx in the horizontal plane,

the area of concrete is $(b - t)\,\delta x$,

and the area of steel is $t\delta x$.

Hence, if q represent the intensity of horizontal shearing stress on the portion $b\delta x$ of the longitudinal section, we get

$$qb\delta x = q'[(b - t)\,\delta x + q''\,t\delta x],$$

and the intensity of horizontal shear on the concrete is

$$q' = \frac{qbE'}{E'(b - t) + E''t}. \quad . \quad . \quad . \quad . \quad (43)$$

If the ratios expressing the relative values of the coefficients of elasticity for concrete and steel be determined for the materials employed in any given construction, this equation can be simplified.

Thus, if the values of the coefficients be $E' = 3{,}000{,}000$

lb., and $E'' = 30,000,000$ lb., corresponding with the ratio $m = 10$, we have for concrete

$$q' = \frac{qb}{(b - t) + 10t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The agreement of this rule with the principles governing the distribution of shearing stress (Art. 48) may be seen by taking a horizontal longitudinal section through the concrete just above the reinforcement. Then $t = 0$, and $m = 0$, and the result is

$$q' = \frac{qb}{b} = q.$$

In this case the concrete takes the whole of the horizontal shear, but when steel reinforcement is present, most of the shear comes upon that material, and the value q' is proportionately reduced.

Moreover, the intensity of vertical and consequently of horizontal shearing stress diminishes with its distance with the neutral axis. Therefore, in concrete-steel beams under ordinary loading there is very little probability that the shear in the horizontal plane of junction between the concrete and the steel will be sufficient to overcome the adhesion between the two materials.

On reference to Table XIX. it will be found that the average adhesion may be taken at about 350 lbs. per square inch, and unless the greatest value of q', obtained by equation (43) or (44), exceed the safe adhesive strength of the combined material, the beam will be perfectly safe, so far as the effect of horizontal shear is concerned.

In this connexion it may be mentioned that, in various experiments made upon concrete-steel beams, fracture has resulted without any evidence being given of slipping of the concrete on the steel. Still, in short and heavily-loaded beams, and in long beams where a high percentage of reinforcement permits heavy loading with a relatively small sectional area of concrete, the shear may become excessive, and in such cases it is important to ascertain whether the intensity of horizontal shear is likely to overcome the adhesion of the materials.

50. Resistance of the Concrete to Normal Tension.—The next question is suggested by the resolution of shearing force, first, into four equal tangential forces, as in Fig. 21, and, second, into two equal forces acting normally to diagonal planes at right angles to each other, as in Figs. 22 and 23.

Taking any diagonal cross section of a beam, such section being inclined at 45 deg. to the horizontal and subject to normal tensile stress, we have to ascertain whether or not the intensity of such tensile stress, normal to the diagonal plane, is within the permissible limit for concrete. It will, of course, be understood that the diagonal lines in Fig. 28 representing tension and compression, merely illustrate the relative intensities of stress at particular points in the length of the beam, and that there is no arithmetical or other limit to the number of similar hypothetical planes.

Nevertheless, as tension and compression exist in opposite diagonal directions, it is necessary to base calculations upon the intensity of stress existing at assumed positions in the length of a beam.

As the concrete is fully capable of withstanding any normal compressive stress likely to be developed, we need only devote special attention to normal tensile stress.

51. Calculation of Tension on Diagonal Planes.—For the purpose of illustrating the calculation of tension on

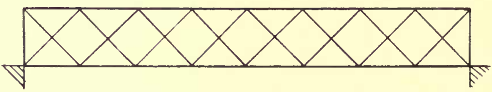


FIG. 29.

diagonal planes, we may regard a concrete-steel beam as approximately resembling a lattice girder (Fig. 29) which includes two systems of the Warren type, and is subject to tensile and compressive stress in the diagonals of the web, somewhat as we have found such stresses to exist in the part of a concrete-steel beam that acts as the web. As the stresses in the intersecting diagonals of a lattice girder of this type are unequal unless verticals be introduced between the apices of the two systems, our object will be better served by considering the tensile stress resulting from shear to be concentrated along the lines occupied by the ties in a Linville or "N" girder.

As an example we will take the case of a concrete-steel beam (Fig. 30) under a uniformly distributed load, and having horizontal reinforcement sufficient to ensure safety against the effects of bending moment. As we shall disregard concrete below the axis of the reinforcement, the lower part of the beam is drawn in broken lines. Let

b = breadth of the beam = 5 in.
d = depth from top of beam to axis of reinforcement = 10 in.
l = clear span between supports = 80 in.
δx = distance between assumed diagonals = 10 in.
w = load per inch of length = 100 lb.

The load applied upon the top of the beam is practically equivalent, so far as concerns stress on the assumed

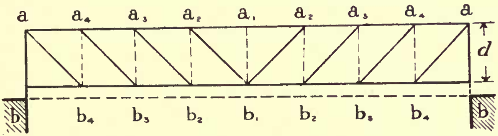


FIG. 30.

diagonals, to downward vertical forces equal to wx, acting at each of the points, a_1, a_2, a_3, a_4, and downward forces equal to $\frac{1}{2}wx$, acting at the points a, a. The total tensile stresses along the diagonals may be ascertained by adding together the stresses due to the different weights, or by the simple rule,

$$P_t = \frac{(N - 2n) + 1}{2} w\delta x \sec\theta. \quad . \quad . \quad . \quad . \quad (45)$$

Where

P_t = total tensile stress in any diagonal,
N = number of bays formed by assumed diagonals,
n = numerical order of any bay from the abutment,
θ = inclination of assumed diagonals.

In this case the inclination of the diagonals to the horizontal is 45 deg., and the value of $\sec\theta = 1.41421$, say 1.414. The value of $w\delta x = 100 \times 10 = 1{,}000$ lb.

Calculating the tensile stresses in the diagonals for one half of the beam, we have

$$a\,b_4,\ P_t = \frac{(8 - 2 \times 1) + 1}{2} \times (1{,}000 \times 1.414) = 4{,}949 \text{ lb.};$$

$$a_4b_3,\ P_t = \frac{(8 - 2 \times 2) + 1}{2} \times (1{,}000 \times 1.414) = 3{,}535 \text{ lb.};$$

$$a_3b_2,\ P_t = \frac{(8 - 2 \times 3) + 1}{2} \times (1{,}000 \times 1.414) = 2{,}121 \text{ lb.};$$

$$a_2b_1,\ P_t = \frac{(8 - 2 \times 4) + 1}{2} \times (1{,}000 \times 1.414) = 707 \text{ lb.}$$

Dividing each of these results by the area of the diagonal plane $= (10 \times 5 \times \sec\theta) = 70.7$ square in., we have the intensity of tensile stress per square in., (p_t) for each diagonal. Thus

$a\,b_4$, $p_t = 70$ lb. per square inch.
a_4b_3, $p_t = 50$ lb. " "
a_3b_2, $p_t = 30$ lb. " "
a_2b_1, $p_t = 10$ lb. " "

These figures show that in the case of ab_4 the stress is beyond the limit of 50 lb. per square inch, generally taken as the allowable unit stress for concrete. Still, as the ultimate resistance of the concrete would probably be between 250 lb. and 300 lb., it is clear that the beam might be safe without diagonal reinforcement, although one or two ties would be desirable.

Under more severe conditions of loading than those assumed, and ranging up to the ultimate resistance of the beam, we should have proportionately greater intensity of tensile stress on diagonal planes, and, unless suitable reinforcement were added, the beam might fail in consequence of tensile stress arising out of shear.

Reverting to the conditions now under discussion, we may point out that as some designers leave the effects of shear to be resisted by the concrete alone, it would be perfectly reasonable to take into account the resistance of the concrete when settling the sectional area of the reinforcement. The other alternative is to disregard altogether the strength contributed by the concrete, and to provide sufficient steel to resist the whole of the tension due to shear.

In the first case, limiting the tensile stress on the concrete to 50 lb. per square inch, and taking the area of the diagonal planes at $(10 \times 5 \times \sec\theta) = 70.7$ square in., we have for the resistance of the concrete

$$50 \times 70.7 = 3{,}535 \text{ lb.},$$

leaving $(4{,}949 - 3{,}535) = 1{,}414$ lb. to be met by steel reinforcement in the first bay, while in the remaining portions of the beam, between the abutment and the centre, tension can be met by the concrete alone.

If the limit of tensile stress for the steel be fixed at 15,000 lb. per square inch, the sectional area of the steel tie along the diagonal ab_4, will be

$$1{,}414 \div 15{,}000 = 0.0942 \text{ square inch.}$$

It would perhaps be more correct to calculate the area of the concrete by taking the effective depth between the axes of compression and tension. Assuming the position of the neutral axis to be 5 in. above the centre of the reinforcement, the effective depth would be $(5 + 3.\dot{3}) = 8.\dot{3}$ in., or, say, 8 in. Then, proceeding as before, the area would be

$$(8 \times 5 \times \sec\theta) = 56.56 \text{ square inches,}$$

and the resistance of the concrete

$$(56.56 \times 50) = 2{,}828 \text{ lb.}$$

This would leave

$$(4{,}949 - 2{,}828) = 2{,}121 \text{ lb.}$$

to be met by the first diagonal tie, ab_4, the sectional area of which would be

$$2{,}121 \div 15{,}000 = 0.141 \text{ square inch.}$$

In the second place, disregarding the value of the concrete altogether, the areas of the diagonal ties should be as follows:

$a\,b_4$, $4{,}949 \div 15{,}000 = 0.3299$ square inch.
a_4b_3, $3{,}535 \div 15{,}000 = 0.2356$ „
a_3b_2, $2{,}121 \div 15{,}000 = 0.1414$ „
a_2b_1, $707 \div 15{,}000 = 0.0471$ „

As it would be inconvenient in practice to use round rods of different diameters, or thin strips of different gauges, the nearest commercial section to the average area of 0.1885 square inch might be adopted in conjunction with a suitable adjustment of the spacing, the ties being placed near together at the abutment, and the distance progressively increased towards the centre of the beam.

Another method of calculating the sectional area of reinforcement in the form of ties, or "stirrups," for resisting tension on planes inclined at 45 deg. to the horizontal, is given by Dr. Slocum, of Cincinnati University, in the following terms: "The maximum intensity of tensile stress is first found for points midway between successive stirrups. The average of these two intensities is taken and multiplied by the width of the beam times the distance between two successive stirrups measured on a horizontal line, not greater than the depth of the beam. If this tension is divided by the safe unit stress for steel, it will give the area of the steel required in a cross section of the stirrups."

Stated algebraically, this method gives the equation

$$P_t = \max. p_t \, b \delta x, \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

where P_t = total tension on any tie. Denoting the safe unit stress for steel by f_t'', the area of steel is $P_t \div f_t''$.

The practice of some engineers is to place the reinforcement for resisting shear at right angles to the horizontal, but from the preceding discussion it is evident that the ties or stirrups should be perpendicular to the planes subject to normal tensile stress, and should be inclined at an angle of 45 deg. to the horizontal.

CHAPTER VI.

BRACED GIRDERS.

In the preceding chapters we have dealt with beams in which the main reinforcement consists of steel rods or bars placed horizontally in the tension area at a suitable distance above the lower surface of the concrete. Reference has also been made to the necessity for similar reinforcement in the compression area when the magnitude of the load and limitation of the beam dimensions involve compressive stress beyond the permissible intensity. We have further shown that auxiliary reinforcement may be desirable, and sometimes absolutely necessary, for resisting shearing stress.

52. General Arrangement of the Reinforcement.—Assuming the case of a beam in which the three elements of reinforcement are present, we have what is essentially a braced girder. Compressive stress is resisted by longtitudinal steel bars and concrete, forming the top flange; tensile stress is similarly resisted by steel bars and concrete, forming the bottom flange; while compressive and tensile stresses arising out of shear are resisted by the concrete and steel, constituting the web that connects the flanges.

The precise design and arrangement of the steel ties included in the web may be varied, and the number of the ties may be small or great, but, as we have already shown, the ties should be placed diagonally so that they may be the better able to withstand tensile stress coming on diagonal planes in the concrete. Compressive stress, due to shear, in the web is resisted by the body of the concrete, which, so far as concerns resistance to compression, resembles the continuous plate forming the web of an ordinary rolled steel joist, or of a built-up plate girder.

The three elements of reinforcement combined with the compressive strength of the concrete give all the essentials of a braced girder, and therefore suggest variations in the design of concrete-steel beams.

We will take two types of construction for the purpose of illustrating the possibilities open to the designer who has made himself familiar with the essential principles governing the application of concrete-steel to structural engineering.

53. Diagonal Bracing for a Solid Beam.—Let us first assume a solid rectangular beam to be the most convenient form for adoption in a particular situation, and that no reinforcement is necessary other than horizontal bars in the tension area, and diagonal reinforcement in the connecting area, or, as we may briefly term it, the web of the beam.

When discussing the general theory of concrete-steel we showed that, in order to develop to the utmost extent the distinctive qualities of the new material, it is desirable to subdivide the reinforcement, as far as may be practically possible, with the object of bringing the whole area of concrete within the sphere of influence of the steel.

When this is done we have no longer "concrete and steel," but the remarkable combination "concrete-steel." Not only the longtitudinal reinforcement, but the diagonal reinforcement also, should be finely subdivided.

The former may be taken to consist of thin rods disposed about the axis of tension in the lower part of the beam, to resist the effects of bending movement in an adequate manner, and the latter must satisfactorily protect every part of the concrete from tensile stresses due to shear.

If, as generally happens in building construction, the load be uniformly distributed, the effects of shearing force will be relatively great at the ends of the beam, gradually diminishing to zero at the centre.

But as the central plane is purely geometrical, no part of the beam can possibly be free from shearing stress. Consequently, the most efficient form of reinforcement for resisting tension on diagonal planes must be one in which the planes of reinforcement are equal in number with the planes subject to tension.

(*a*) *Continuous Plate.*—As the number of such planes is necessarily infinite, it might seem that a continuous plate of metal would form the most adequate type of reinforcement. If such a plate were of graduated thickness in accordance with the progressive diminution of shearing stress towards the centre of the beam, it would satisfy some theoretical requirements. But it would not afford a satisfactory bond between the concrete and the steel, and it would not ensure that co-operation which is invariably demanded between the two materials. These objections might be overcome in part by punching holes, with projecting rims, all over the plate, by cutting triangular tongues to project into the surrounding concrete, or by any device of similar nature.

(*b*) *Metal Network.*—A better plan would be to employ steel netting, or expanded metal, preferably the latter.

By taking a flat sheet of expanded metal, of suitable mesh and thickness, and bending it into a trough-like shape to suit the dimensions of the beam, the whole length in a longitudinal section of the beam could be adequately protected against shear, in whatever way its influence might be directed. Moreover, every part of the longitudinal section would be brought within the influence of the metal reinforcement. Towards the centre of the beam the shearing stress would be so small as to render reinforcement unnecessary, but considerable advantage would follow the employment of a continuous trough from one end of the beam to the other, and the saving to be expected from the omission of a few feet of metal would be quite insignificant. (See Fig. 34, p. 115.)

In small beams the most simple course would be to use a single sheet of expanded metal for forming the trough-like reinforcement, selecting a gauge and mesh appropriate to the maximum shear.

In large beams, where the amount of metal might form a considerable item of expenditure, the proper course for adoption would be to employ two or more troughs of varying length, and fitted one inside the other, so as to give the required graduation of resisting power, somewhat after the manner in which engineers graduate the plates of a built-up girder.

The form of reinforcement here suggested for protecting a beam against the vertical, horizontal, or diagonal effects of shear, could be cheaply and easily applied. It would clearly be more efficient, and more in accordance with theory than reinforcement consisting of ties or stirrups at more or less widely separated intervals. Tension on diagonal planes could be calculated at the option of the designer to be met entirely, or in part, by the metal, which would at the same time provide against compression on diagonal planes at right angles to those subject to tension.

This type of construction represents a simple and effective form of braced girder, and the calculations required for its design are perfectly free from complexity.

54. Design of Braced Girders.—We will next assume that it is not necessary to have a solid rectangular beam, and that the top and bottom flanges may be connected by any convenient number of members, after the manner of a "Warren" or other variety of braced girder. Of course, the ties and struts so employed must be partly or entirely formed of concrete, in order to come within the type of construction under consideration, and to ensure safety against fire and corrosion. The arrangement of the web members may be varied at the discretion of the designer, and while those intended to resist tension must be reinforced, those subject to compression may be of concrete alone.

When diagonal ties are employed for resisting tension due to shear, the concrete intervening between the ties is of no value, and therefore unnecessary.

In small beams the saving to be expected from the omission of unnecessary material is too small to be taken into account. In larger beams the construction of ties and struts as individual members, securely built up and connected with the flanges, may often be advantageous for the purpose of saving unnecessary weight and expense.

Up to the present time very little use has been made of the type of design here indicated, but M. Visintini, a Swiss architect, has availed himself of its advantages by devising a complete system of construction which is now being introduced both on the Continent and in this country. As a general rule the "Warren" type of girder is adopted, with a single system of triangulation, but use is also made of the "Linville" or N truss.

Although it is frequently the case that the compression flange of a concrete-steel girder requires no reinforcement, steel rods are employed in this member by M. Visintini, because a simple and effective means of connexion is thereby afforded for the ties. Moreover, the thickness of concrete in the top flange can be reduced, and the two flanges need not contain much more concrete than the amount necessary for the efficient protection of the metal. The rods serving to reinforce the ties are connected by bending their ends round the horizontal rods in the flanges, and they are held firmly in position by the enclosing mass of concrete.

The struts—whether diagonal, as in the "Warren" girder, or vertical, as in the "Linville" truss—are simply of concrete, but if circumstances rendered such a course necessary they could be suitably reinforced.

We are now concerned with general questions of beam design, and it would be inappropriate to go into full details relative to the application of this system of braced girder construction. It may be remarked, however, that the spaces between the web members may be filled up solid, if thought desirable; or when a series of beams is employed in floor construction the flues formed by the spaces can be left for use as ducts for pipes, wires, and for air in connexion with heating or ventilation apparatus.

55. Calculation of Proportions.—In order to make clear the method of calculating the proportions of braced girders of concrete-steel, the following rules are given by M. Visintini.

The symbols have been altered so as to accord, as far as possible, with the system of notation adopted in this book.

Denoting the reaction of the abutments by the symbol S, we have—

$$S = (N - 1)\frac{w\delta x}{2}. \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Where N = number of bays formed by the diagonals.

w = load per unit length.

δx = distance between diagonals, or length of bay.

For the moment of resistance in any member of the top flange we have

$$R = \frac{w(\delta x)^2}{2}[(N+1)(n_1 - \tfrac{1}{2}) - n_1^2], \quad . \quad . \quad . \quad (48)$$

where n_1 = numerical order of any member of the top flange from the abutment.

For compression in any member of the top flange we have

$$P_c = \frac{w(\delta x)^2}{2d}[(N+1)(n_1 - \tfrac{1}{2}) - n_1^2], \quad . \quad . \quad . \quad . \quad (49)$$

where d = distance between the top and bottom flanges.

As the top flange is built up uniformly, it is only necessary to calculate the moment of resistance for a cross section at the middle of the beam. This, of course, gives the maximum value, and the corresponding compression

$$P_c = R \div d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

From this the requisite area of concrete can easily be computed, due allowance being made for the resistance afforded by the steel reinforcement.

For the moment of resistance in any member of the bottom flange we have

$$R = \frac{w(\delta x)^2}{2} n_1(N - n_1). \quad . \quad . \quad . \quad . \quad . \quad (51)$$

For tension in any member of the bottom flange

$$P_t = \frac{w(\delta x)^2}{2d} n_1(N - n_1). \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Having found the amount of tension, it is easy to proportion the reinforcement for the bottom flange, and as the beam is of uniform dimensions, calculations will be made at the centre of the beam, as in the case of the top flange. Following the usual practice, no account is taken of the resistance to tension afforded by the concrete.

In calculating tensile and compressive stresses on the diagonals, the method adopted is very similar to that already discussed in Article 51.

Tension is calculated for any diagonal by the equation

$$P_t = \frac{w\delta x}{2\cos\theta} \times (N - 2n + 1). \quad . \quad . \quad . \quad (53)$$

For small beams it would be inconvenient and scarcely necessary to employ rods of different diameters to suit the diminution of tension towards the centre of the beam, and it will be sufficient to calculate the stress for the first diagonal from the abutment. The sectional area of steel suitable for the stress so found may then be adopted for all the ties. For larger beams, calculations should be made for successive diagonals until the tension becomes so small that further diminution in the area of the rods would be attended with no practical advantage. Here, again, no account is taken of the tensile strength of the concrete.

Compression on diagonal struts is similarly calculated by the equation stated above, but compressive stress is resisted by the concrete without the aid of reinforcement. For small beams a uniform sectional area of concrete may be adopted, while for large beams the sectional area of the struts can be proportioned to the amount of stress to be resisted.

If the struts be vertical, as in the case of a "Linville" or "N" girder, the rule requires modification, and becomes

$$P_c = \frac{w\delta x}{2} \times (N - 2n + 1). \quad . \quad . \quad . \quad . \quad (54)$$

CHAPTER VII.

TYPICAL FORMS OF BEAM DESIGN.

DETAILED discussion of the principal types of the almost endless variations that could be made in the design of concrete-steel beams is scarcely necessary. If we have succeeded in the endeavour to make clear the fundamental principles underlying the theory of concrete-steel, and some of the rules suitable for use in general practice, the reader should have no difficulty in dealing with any type of concrete-steel beam hitherto mentioned, or in designing beams of the same material in other types that are fully discussed in text-books on constructional steelwork.

56. Summary of Different Types of Construction.—Before leaving the present part of our subject, however, it will be useful to add a few notes relative to different primary forms of design suitable for general adoption.

For the purpose of making these notes more intelligible, we now give a series of diagrams representing different methods of applying reinforcement. Some of these methods have already been discussed, and others have not yet been mentioned.

In Fig. 31 we have the simplest possible form for a sup-

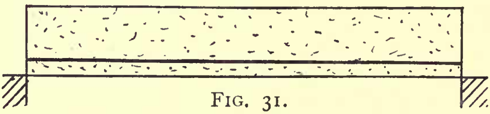


FIG. 31.

ported concrete-steel beam, with horizontal reinforcement consisting of one or more bars placed near the bottom of the beam, for resisting tensile stress. Compressive and shearing stresses are taken by the concrete without assistance.

In Fig. 32 a similar beam is represented, but shearing stress is here resisted wholly or in part by vertical ties or

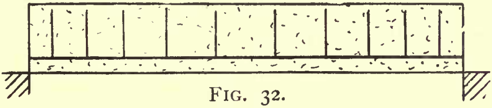


FIG. 32.

stirrups. This auxillary reinforcement can be formed of round rods, or of flat strips of steel, as may be preferred.

In Fig. 33 the main reinforcement is the same as before, but the auxilliary reinforcement is placed diagonally, for reasons that are fully discussed in Chapter V., and its func-

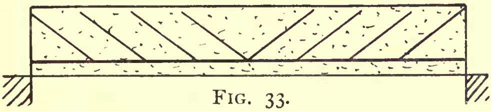


FIG. 33.

tion is to withstand tension resulting from shearing stress on diagonal planes in the concrete, while compression similarly resulting from shearing stress is resisted by the concrete alone.

In Fig. 34 we have a suggested method of applying expanded metal for taking stresses arising from shear. The metal can be applied in the form of separate sheets, the

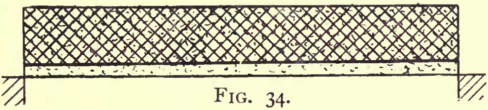


FIG. 34.

lower edges wrapped round the outer rod of the reinforcement on each side of the beam; or in the form of a trough, the bottom of which would lie under the longitudinal reinforcing rods. In the latter arrangement a considerable part of the tension due to bending moment would be carried by the expanded metal. The main reinforcement is disposed similarly to that in previous examples.

In Fig. 35 the beam is designed on the principle of the "Warren" girder. Both the top and the bottom flanges are reinforced, the latter with more metal than the former; the diagonal web members in tension are reinforced; but the

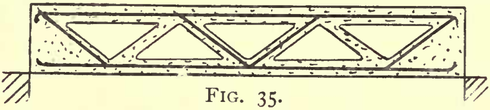


FIG. 35.

diagonals in compression rely upon the concrete for the necessary resistance. This beam is a framed structure, consisting of clearly defined flanges and diagonals as shown in the drawing. If made solid it would be practically similar to the beam represented in Fig. 33.

In Fig. 36 is shown another type of framed beam, designed on the principle of the Linville truss, and it should be noted that the spacing of the tension members of the web

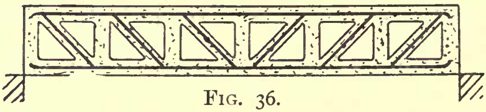


FIG. 36.

is one-half that of similar members in the "Warren" type. If this beam were made solid, it would, like Fig. 35, become equivalent to the beam illustrated in Fig. 33.

In Fig. 37 is a type of design not yet considered. Here we have longitudinal horizontal reinforcement, consisting of

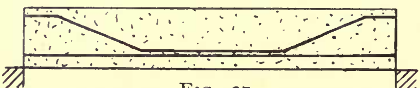


FIG. 37.

one or more bars near the bottom of the beam, and additional reinforcement consisting of one or more bars bent in an upward direction near the supports. The horizontal part of the latter reinforcement is shown above the first-mentioned bars for the sake of clearness, but in practice it is usually situated in the same horizontal plane. The concrete, acting in conjunction with the steel bars, practically forms a trapezoidal, or queen-post, truss. This type of construction is most suitable for beams with fixed ends, and for continuous beams (see pp. 118-120).

It may be sufficient in many cases to rely upon the concrete to take the place of the usual struts, and to bind together the different elements of such a beam, but more satisfactory results will be obtained by the use of vertical reinforcement for this purpose. The diagonal ends of the upper bars here take a portion of the tension due to shearing stress, and if vertical or diagonal ties be used, it will probably be safe to calculate that one-half of the tension is already carried by the bent ends of the second series of longitudinal bars.

In Fig. 38 we have a development of the simple triangular truss, sometimes described as the "Bollman" truss. This

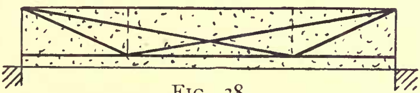


FIG. 38.

form of braced girder is less suitable for concrete-steel than some other types, because the ties are necessarily of unequal length, the only exception being in one pair of ties in a similar girder having two ties meeting at the centre. Consequently, any extension of the metal, whether resulting from strain on the bars or from temperature changes, must be of unequal degree, the longer bars being extended more than the shorter ones. This naturally tends to induce secondary stresses of prejudicial character. The example is given chiefly for the purpose of showing the discrimination that must be exercised in applying types of steel-girder construction to concrete-steel design.

There are many other varieties of the braced girder in which similar defects do not exist, and some of them are quite worth consideration for special examples of concrete-steel construction. As a rule, however, the more simple methods of applying reinforcement are those which should be adopted in general practice.

In Fig. 39 we approach another method of reinforcing concrete beams, in which the reinforcement is shown bent into a curve similar to that forming one boundary of the bending moment diagram of a supported beam under a

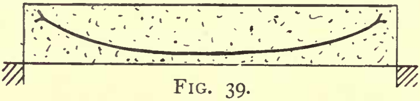


FIG. 39.

uniformly distributed load. Although theoretically correct, so far as bending moment is concerned, this arrangement does not lend itself conveniently to the application of ties for resisting shearing stress, and in practice it will be found to involve more expense and trouble than the use of other forms of reinforcement.

In Fig. 40 the last mentioned system is shown in connexion with a beam having its ends firmly fixed in the walls of a building.

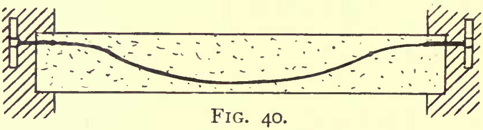


FIG. 40.

In such a case, if the condition of fixity be fully satisfied, the maximum bending moment occurs near the ends of the beam, and has the value

$$M = Wl \div 12, \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

and the bending moment at the middle of the beam is

$$M = Wl \div 24. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

At the distance of about one-fourth of the length from each end of the beam, or, to be exact, $l \times 0.211$, there is a point of contrary flexure where the bending moment becomes zero.

Such a beam may be regarded as consisting of three parts—two cantilevers, and a supported beam between the points of contrary flexure.

It must further be remembered that between the supports and the points of contrary flexure the upper fibres of the beam are in tension, while the same condition is evidenced in the lower fibres of the central portion. Hence the reinforcement must be of the curved form shown in the figure, and to secure the maximum advantage it should be secured at each end by anchors, as indicated in the diagram.

As the maximum bending moment in the middle part of the beam is only one-third the value of that in a beam merely supported at the ends, the depth may be proportionately reduced, and as the maximum bending moment at the ends is only two-thirds the value of that developed in a similar beam supported, but not fixed, at the ends, the depth of concrete may be proportionately less.

The most economical profile is consequently that of a beam with parallel top and bottom faces between the points of contrary flexure and arched haunches at the ends, as Fig. 48, p. 134. Such a profile has the further advantage of providing a considerable thickness of concrete in cross sections where shearing stress approaches its maximum intensity.

In Fig. 41 is shown another system of reinforcement, providing in a very simple manner for the incidence of tensile

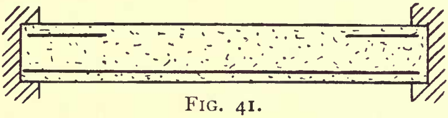


FIG. 41.

stress in a beam fixed at the ends, and although this mode of treatment does not conform so closely with theoretical requirements as the method described above, it can be applied easily and cheaply.

In Fig. 42 we have part of a continuous beam, where the conditions generally resemble those in a beam with fixed

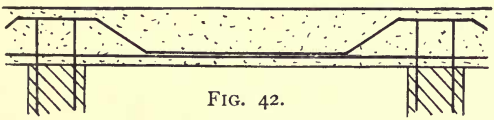


FIG. 42.

ends, and in this diagram we have shown vertical reinforcement in the supports, connected to the bent ends of the bars serving to resist tension up to the points of contrary flexure and afterwards in the central portion of the beam.

The foregoing examples are sufficient to suggest the principal features embodied in concrete-steel beams, and, as each type illustrated is susceptible of further modification and development, it will be seen that an almost infinite variety of design is available for the structural engineer.

57. Tubular Beams.—A further modification of design is suggested by the application of the tubular system to beam construction. In this way the usefulness of a specified quantity of concrete can be increased, or for a specified duty the quantity of concrete can be reduced.

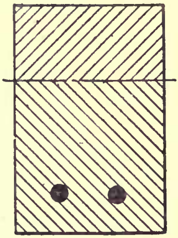


FIG. 43.

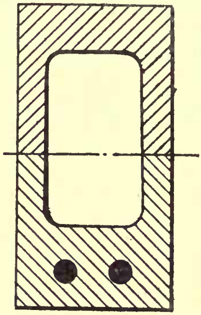


FIG. 44.

Figs. 43 and 44 show diagrammatically the manner in which a given area of concrete, in a vertical cross-section of a beam, may be advantageously employed for the purpose of gaining increased strength.

As a matter of fact, it will be found by measurement that the solid section represented in Fig. 43 has a somewhat greater area of concrete than the hollow section in Fig. 44, while, owing to the greater distance of the axis of compression from the neutral axis, the resistance of the second section is considerably greater than that of the first section.

The area of concrete removed, below the neutral axis, would have been of comparatively little value for resisting tension, and if, as usual, the tensile strength of concrete is not taken into account, the theoretical loss is nothing. On the other hand, the omission of concrete above the neutral axis does not materially affect the strength of the compression area. The general result, so far, is an increase of resistance to compressive stress and a negligible diminution of resistance to tensile stress.

The balance of strength between the two areas must be adjusted by using more metal in the reinforcement of the section in Fig. 44 than in that of Fig. 43. The hollow section will then possess much greater resistance to the effects of bending moment than the solid section, and the only sources of extra cost are to be found in the necessary moulds for forming the central cavity, and in the weight of additional steel reinforcement.

So far as concerns shearing stress, it is obvious that the resistance of a tubular beam must be proportionately less than that of a solid beam (see Art. 48), but this feature can very easily be met by the employment of rods or strips of steel placed diagonally, or, better still, by using sheets of expanded metal in the manner already proposed.

A very simple and effective way of finishing the upper edges of the expanded metal would be to turn them over two thin steel rods in the compression area, thus conducing to the thorough connexion of the whole construction.

The tubular system is more especially suitable for large beams, and, if worked out in accordance with the principles governing all beam design, it may very frequently be of much convenience and advantage to the engineer.

58. Notes on the Application of Reinforcement.—With regard to the form of steel section used for reinforcement, the first essential is to avoid the concentration of metal, and always to use thin rods in preference to heavy bars (see Art. 29). We have previously referred (Art. 26) to the advantages possessed by some of the patented bars intended to add mechanical adhesion to the adhesion existing between the concrete and the steel, and in works where vibrations and shocks are likely to be severe, the employment of special forms of reinforcing bars may be thought necessary. As, however, recent investigation has shown that plain round bars afford satisfactory results, they may be used with confidence in structural engineering. A very great advantage is that such bars can always be purchased from stock without the delay that would probably attend the delivery of patented or other special sections. The merits of expanded metal have already been shown in connexion with resistance to shearing stresses (see Art. 53); but it may now be added that this material will also be found of great value when used horizontally, either alone or in conjunction with longitudinal bars, as it will then serve to strengthen the beam laterally, and to bind together the longitudinal bars.

In every case the reinforcement should be securely anchored by bending or splitting the ends of bars and by bending the edges of sheets. Where, owing to the length of a beam or to other causes, a joint has to be made between two bars, the ends should be bent or hooked, or a slip socket, formed of iron tubing, may be used. Cement mortar can then be applied to complete the joint. Sheets of expanded metal can be joined by laying them to overlap, and the concrete will bind them securely together.

Whatever form of section be selected, the great thing is to dispose the metal in those places where an analysis of the stresses shows it to be most required. If proper study be made of the principles governing concrete-steel construction, there is not the slightest reason why structures should not be designed in this material as readily and as successfully as in stone, timber, and brick.

Among the different systems of construction advertised in the present day, there is scarcely one that has not good features: but our advice to the designer is that he should not feel bound to confine himself to any particular system. It is far better to reserve complete liberty of action, and, after carefully studying the requirements of any given constructional work, to apply such forms of design as will most adequately fulfil the conditions that are known to be essential.

CHAPTER VIII.

FLOOR DESIGN.

59. Ordinary Concrete and Steel Floors.—No detail of modern building construction is more intelligible than the simple floor of concrete supported by steel girders and joists. However extensive such a floor may be, it is readily analysed into units, each of which has to carry a certain definite proportion of the total load, and the dimensions of each unit are calculated by the simple rules governing the design of beams and slabs.

Thus, in the case of a floor intended to carry a uniformly distributed load, the weight per square foot of the external loads is first taken into account in settling the necessary thickness of each floor slab, and sufficient margin is allowed for the weight of concrete in the slab itself. The total load per panel so obtained serves as the basis of calculations by which the dimensions of the floor joists are determined, and the collective weight of slabs and floor joists, transmitted to the girders, represents the external load to be carried by the latter members. Their proportions are then calculated with due regard to the dead weight of the material, and the general design of the floor is finished, apart from practical considerations and details leading to complications of any kind.

It would be perfectly easy to adopt a similar process in connexion with the design of concrete-steel floors, but the inevitable result would be a very considerable excess of strength and consequent waste of material.

The ordinary steel and concrete floor is so constructed that the concrete is practically split up into a number of detached and non-continuous units, none of which can be treated more favourably than as a beam supported at each end, or as a slab more or less efficiently supported along each side. Similarly, the steel members have to be separately considered as individual details.

60. Origin of the Concrete-Steel Floor.—In a properly designed concrete-steel floor the conditions are quite different. Such a floor is really a homogeneous structure, and may be said to constitute a single unit, or one continuous slab—with stiffening ribs—extending throughout the whole area covered.

As a matter of fact, however, if the steel skeleton of an ordinary concrete floor is efficiently filled in and covered with concrete at the top and bottom, a noticeable increase of strength is secured.

This effect was observed in the early days of concrete-steel construction, and a step towards the more economical design of floors was made by employing T-bars instead of I-bars for joists, and in placing these joists so that the two arms of each T-bar formed a tension flange, situated a few inches below the under surface of the intervening concrete. The projecting parts of the joists were, of course, completely covered with concrete during the construction of the floor, and the result was a continuous floor slab with a number of projecting beams. By adopting this type of construction it was found that a considerable saving of metal could be effected, and that the thickness of the concrete between the beams could be materially reduced.

Subsequent experiments, and experience gained in practical work, ultimately led to the development of still better designed concrete-steel floors, whose successors flourish so abundantly in the present day.

61. Evolution of a Concrete-Steel Floor.—Before discussing the distinctive features of any of the types of concrete-steel flooring at the disposal of the designer, we will briefly trace the evolution of a complete and self-contained floor from a collection of simple beam forms.

In the first place, let us consider a beam of simple concrete in which the neutral axis is assumed to be midway between the top and bottom surfaces.

Then, in order to preserve a correct balance between the resistance of the compression and tension areas, the sectional area of concrete below the neutral axis must have ten times the sectional area of the concrete above the neutral axis, to compensate for the lower tensile strength of concrete as compared with its compressive strength.

Next, let us assume that instead of increasing the area of concrete in the tension portion of the beam, we add steel reinforcement near the bottom surface. By the employment of a suitable proportion of metal, the resistance of the compression and tension areas can be exactly adjusted, and the cross-section will have the form of a four-sided rectangular figure.

Proceeding a step further, we will assume that the amount of reinforcement added is greater than that necessary for balancing the tensile and compressive resistance of the two areas in the cross-section of the beam.

In such a case the balance of strength can only be restored in one of two ways:

(1) By placing reinforcement near the top of the beam; or

(2) By increasing the area of concrete above the neutral axis.

It is evident, however, that these two alternatives might be combined.

Thus, let us take a beam in which the reinforcement in tension is such that, for the proper balancing of resistance, the concrete in compression must be extended in the form of a wide flange, giving a T-shape to the cross-section of the beam.

If we assume the further addition of reinforcement in the tension area it might be inconvenient or impracticable to increase the width of the compression flange to the required extent, but a similar effect would be obtained by the application of reinforcing bars near the top of the beam. In the latter event we should have a combination of the two alternatives suggested above.

A little consideration will show that a very wide upper flange, whether of concrete alone, or of concrete with longitudinal rods, would be weak laterally, and might tend to crack or to break off from the main body of the beam. Hence, transverse reinforcement could be added with advantage.

The resulting beam would consequently be of T-section with longitudinal reinforcement near the bottom and a network of rods placed longitudinally and transversely, near the top.

It is evident, then, that the general proportions of the beam and the width of the compression flange can be varied at pleasure for a given load by corresponding adjustment of the reinforcement. The latitude so given is of considerable value in the design of flooring systems.

We will now consider the effect of employing beams designed in this manner as parts of a homogeneous floor structure.

Let us take one floor panel, in which the main girders are 8 ft. apart, and the transverse girders, or joists, are 4 ft. apart.

We assume that, by suitable adjustment of the reinforcement, the compression flanges of the main girders have been made 4 ft. wide and, say, 4 in. thick. Then, between the centres of two consecutive girders, spaced 8 ft. apart, there will be an interval of only 4 ft., as each of the compression flanges projects 2 ft.

Similarly, it is assumed that the top flanges of the joists are made 2 ft. wide. Consequently the interval between successive joists will only be 2 ft., as the compression flanges of the joists account for the other 2 ft. of the distance.

As shown in Fig. 45, the general result is that the assumed floor panel is complete, with the exception of a series of rectangular spaces, such as those represented by the shaded areas in the diagram, each space measuring 4 ft. by 2 ft., and the number of which depends upon the span of the panel. These spaces can be filled in with concrete, 4 in. thick, and the floor is thereby finished.

It will be understood that in practice the spaces would be closed concurrently with the formation of the top flanges of the girders and joists. Practically, therefore, the whole of the floor slab forms the compression flange of the connected system of beams.

Further, assuming the top flanges to be reinforced by longitudinal and transverse rods, it is a natural step to conclude that all these rods should be laid continuously from end to end of the span, and from side to side in the width of each panel.

The additional cost of the metal would be quite inconsiderable, and the cost of labour would probably be reduced because the rods would not have to be cut, and the number of separate pieces would be smaller. These rods would then occur at predetermined intervals in the flanges of the

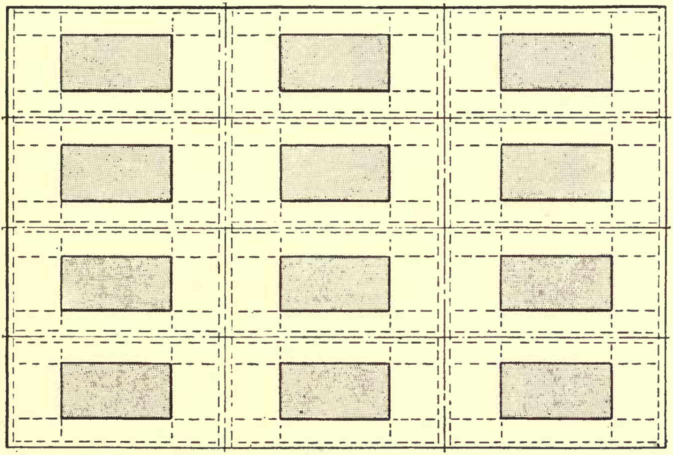


FIG. 45.

girders and joists, and would also continue across the concrete filling the parts of the floor system not covered by the previously defined widths of the flanges.

Here we have an additional feature of strength due to the reinforcement of the whole of the floor slab by steel rods running in two directions, and thus binding the construction still more firmly together.

It will also be seen that in a floor designed in the manner here outlined, each of the joists acts as a stiffener for the flanges of the adjoining girders, and that the entire structure is so interconnected that an exact calculation of its strength is practically impossible.

But we have a further element of strength, arising from the action of the various spans after the manner of continuous beams. To develop this action to its fullest extent, reinforcement should be applied as shown by Fig. 42, p. 119, and further strength may be added by the formation of concrete haunches on the girders and joists, as in Fig. 48, p. 134.

No one has yet developed an exact theory of the strength of concrete-steel floors, and the distribution of stresses is so complicated, and the inter-relation of the various constituents is so involved, that it seems unlikely that anything more than approximate rules will ever be available for the guidance of the designer.

62. The Stiffness of Concrete-Steel Floors.—One result following the method of construction outlined above is that floors so made are very much stiffer than steel-girder floors with ordinary concrete filling. In this connexion, it may be interesting to give the result of two tests conducted by the engineering department of the Paris and Orleans Railway Company at the Austerlitz and the Quai d'Orsay stations in Paris.

The first test was conducted upon a concrete-steel floor of the Hennebique type, built in the electric sub-station at the Austerlitz railway station. It was calculated to carry a machinery load of 280 lb. per square foot, the clear span was 16 ft., and the floor was subjected for a width of 17 ft. to a uniformly distributed test-load of 420 lb. per square foot. The resulting deflection was then found to be only one eighth of an inch, without any permanent set.

With the object of comparing the resistance of this floor to shocks with that of a steel-girder floor, it was subjected to severe blows from falling weights, and similar tests were applied to a floor at the Quai d'Orsay railway station. The latter floor, built of steel girders and brick arches, was of the same span and had been calculated for the same load as the floor at the Austerlitz station.

A weight of 220 lb. falling upon the concrete-steel floor from a height of 13 ft. caused a maximum vibration of one-sixteenth of an inch, lasting five-sevenths of a second, whereas a weight of 112 lb. falling from a height of 6 ft. 6 in. upon the steel and brick floor produced vibrations measuring five-sixteenths of an inch and lasting two seconds.

Hence, although the weight falling upon the concrete-steel floor was nearly twice the value of that falling upon the steel girder floor, and fell from twice the height, the resulting deflection was only one-fifth of the amount recorded for the latter construction, and the vibrations lasted for less than one-third of the time.

Considering these figures in connexion with the dead weights of the floors—62 lb. per square foot for concrete-steel and 100 lb. for steel and brick—it will be seen that a very great advantage is possessed by the former type of construction.

No specific mention has been made in this article of ties for withstanding shear, but our readers will, of course, understand that reinforcement of the kind is equally desirable for beams incorporated in floors as for simple beams of concrete-steel.

63. Resistance of Concrete-Steel Floors.—Very few records have been published relative to the resistance of concrete slabs, supported on four sides, and not merely acting as simple beams supported at each end. Some experiments upon slabs were conducted by Colonel Seddon in 1874, and the results then obtained are recorded in a paper read two or three years ago before the Northern Architectural Association. These tests are rather old, and they do not apply, except in a very general way, to slabs of concrete-steel. Moreover, the strength of a simple slab affords very little indication as to the strength of a complex floor system.

If we consider a floor constructed on the lines described in the preceding article, it becomes evident that it contains no parts acting as separate slabs. It is true that the spaces between the main girders and joists are occupied by moderately thin layers of concrete-steel, but we have seen that such layers constitute a continuous compression flange common to all the girders and joists. Hence they cannot be regarded as isolated slabs. The entire floor, doubtless, acts as a slab supported along four sides, but its complicated construction precludes the possibility of devising a general formula that will yield precise results for different types of construction, or even for different examples of the same type.

Speaking generally, we may safely say that a slab of specified thickness, when incorporated in a floor, will carry double the load that it will carry when used as a beam simply supported at the ends. Further, in spans not exceeding 4 ft., slabs forming part of a floor will support more than three times the load that would be carried if the slab were merely tested as a beam. This increased ratio is attributable to arching action caused in the material by a uniformly distributed load.

Bearing in mind the general disposition of the reinforcement, and the sectional outline of the concrete in the different members of a concrete-steel floor of scientific design, we may obtain an approximately correct idea of the conditions which lead to augmented resistance in such constructions.

64. Continuous-Girder Action.—We are justified in considering that the effect of judiciously placed reinforcement, and of haunches on the girders and joists, is to give a continuous-girder action. This means that the bending moment in either span of a girder continuous over two spans will only be about one-half of the bending moment that exists under equal conditions of loading in a span of the same length where the ends are free.

65. Interior Arching Effect.—We may take it that interior arching effect, due to the action of adjoining floor panels, adds about one-third to the area of the compressive stress diagram, the additional compression being taken up by adjoining floor panels, instead of by the reinforcement.

66. Arching Effect due to Load.—Something may be safely allowed for the arching effect produced by a uniformly distributed load. No allowance for arching effect should be made in the case of loads that are free to "flow" like corn and other loose materials.

The arching action necessarily varies with the deflection and with the details of design, but the result of practical experience is to show that the increased resistance thereby given to a span of 4 ft. may be generally stated at 50 per cent. Hence, taking 50 per cent. at 4 ft. span as the starting point, we are justified in arguing that for a zero span the arching action is infinite, and that for an infinite span it is zero.

Consequently, the percentage to be added for a span of any length may be conveniently obtained by the use of a suitable diagram. In Fig. 46, we produce a diagram[1] containing a hyperbolic curve passing through the percentage of 50 per cent. at the span of 4 ft., and having the axes as asymptotes. From this diagram the percentage for a span of any length can be read off.

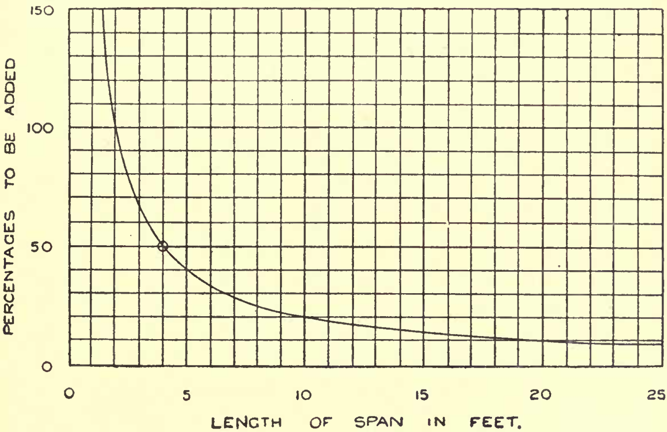


FIG. 46.

Taking into account continuous-girder action and interior arching effect, we arrive at the conclusion that the beams forming part of a floor system are capable of carrying double the load that could be carried by beams of similar construction and dimensions when acting separately as simple beams. Further, we have to allow for arching due to the load.

67. Simple method of Calculating Proportions of Floor Beams.—Bearing in mind the points mentioned in the three preceding articles, we arrive at the following simple method of determining the proportions of beams used in floor construction:

1. Calculate the values of the moments of resistance by any of the rules previously stated for simple beams.

[1] *Proc. Am. Soc. C.E.* vol. xxvii., No. 6.

2. Double the values so obtained, to allow for continuous girder action and the arching action of adjoining floor panels.

3. Add a percentage from Fig. 46, corresponding with the length of span, to allow for arching effect caused by a uniformly distributed load. The advantage of this method of procedure is that it is generally applicable to concrete-steel floor construction, and does not tie the user down to any one kind of design.

68. Hennebique Method of Computation.—For the purpose of making clear the method followed in connexion with the Hennebique system of floor construction, we select as an example one of the floors in a school building, erected in Paris from the designs of M. Leclerc. The dimensions and general method of treatment are taken from a *mémoire* by M. Boileau.[1]

Fig. 47 is a part plan and Fig. 48 a section of the floor, which has a length of 38 metres (124.6 ft.), and a span of 7.5 metres (24.6 ft.). The girders are spaced 3.8 metres (12.46 ft.), centre to centre, and their ends are carried on concrete-steel stanchions, 0.27 metre (10.6 in.) square, the connexions being strengthened by the formation of concrete haunches (see Fig. 48). The floor surfaces, or slabs, between the beams are only 0.08 metre (3.149 in.) thick, but they are reinforced by joists of 0.08 metres by 0.12 metre (3.149 in. by 4.724 in.), spaced 1.28 metres (4.2 ft.) from centre to centre.

The following calculations show the manner in which the strength of the various portions of the floor was determined. Metrical measurements are retained so as to avoid tedious fractions, and to ensure the agreement of the results with the actual dimensions.

Three elements of the floor are considered, in the following order:

(1) A portion of the floor surface, or slab.
(2) A main girder.
(3) A floor joist.

1. The portion of the floor slab (see Fig. 49) is calculated as a beam of 1.20 metres span, 1 metre wide, and 0.08 metre deep.

[1] *Le Ciment Armé*, G. Delarme, Paris, 1897.

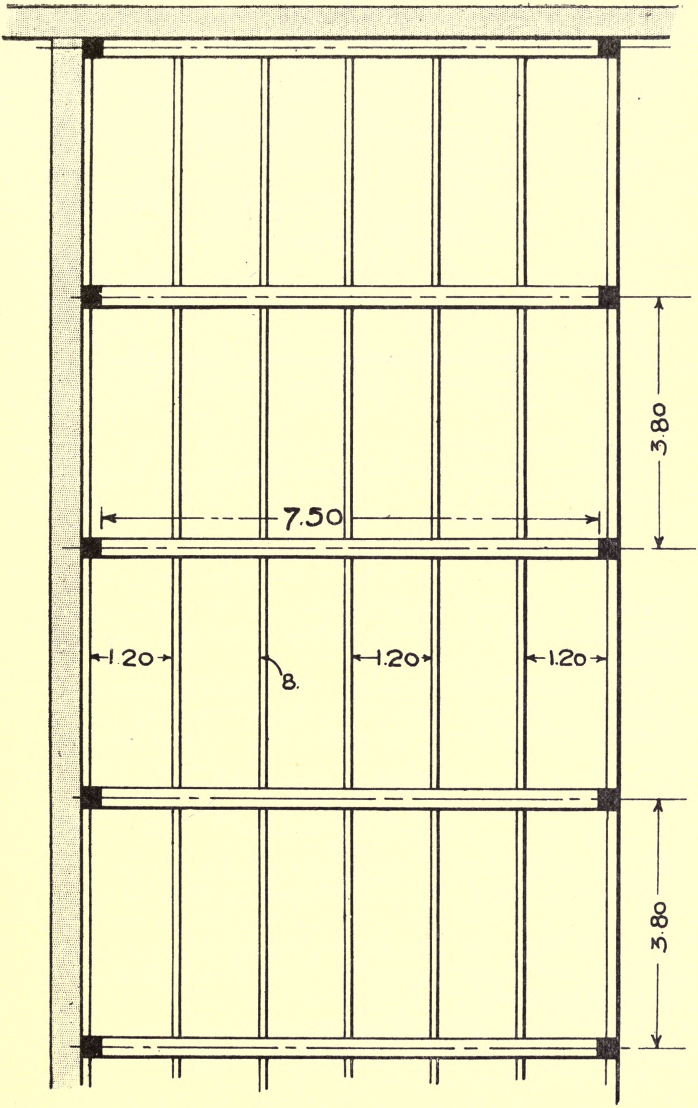


FIG. 47.

The external load being in this case 400 kilo. per square metre, and the weight of the concrete being 2,500 kilo. per cubic metre, we have:

External load

$$W_e = (1.2 \times 1) \times 400 = 480 \text{ kilo.}$$

Weight of concrete

$$W_c = (1.2 \times 1) \times 0.08 \times 2500 = 240 \text{ kilo.}$$

Total $(W_e + W_c) = 720$ kilo.

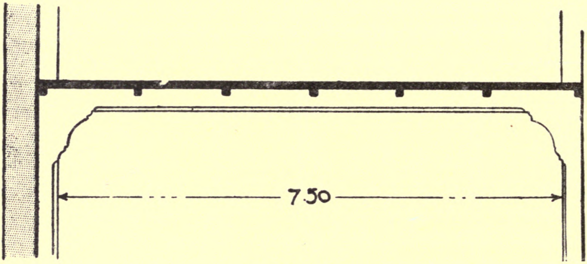


FIG. 48.

Owing to the intimate connexion of the slab with the main girders and joists, it may be considered as a beam fixed at the ends.

Consequently, in calculating the bending moment the ordinary formula $M = \frac{Wl}{8}$ is altered to $M = \frac{Wl}{10}$, or adding W_c to represent the dead weight of the concrete,

$$M = \frac{(W_e + W_c) \times l}{10}. \quad . \quad . \quad . \quad . \quad (57)$$

Hence we get $M = \frac{720 \times 1.2}{10} = 86.4$ kilogrammetres.

To calculate the necessary depth of concrete in compression, and the position of the neutral axis, the following method is adopted:

The depth of the compression area is denoted by the expression $2H$, and the distance between the centre of compression and the neutral axis, by the symbol H. The width (b) of the section is already known to be 1 metre, and the compressive strength of the concrete (F_c') is taken at 25 kilo. per square centimetre.

Then considering that

$$b \times 2H \times H \times F_c' = \frac{M}{2},$$

it is deduced that

$$(2H)^2 = \frac{M}{b \times F_c'}. \quad . \quad . \quad . \quad . \quad . \quad (58)$$

In the present case we have

$$(2H)^2 = \frac{86.4}{1 \times 250{,}000} = 346 \text{ sq. mm.}$$

Whence $2H = \sqrt{0.000346} = 19$ millimetres.

This result indicates the depth of the compression area, and immediately below this is the neutral axis of the slab.

The sectional area of steel in tension is thus determined:

Let a denote the area of steel, H' the distance from the centre of the reinforcement to the neutral axis, and F_t'' the tensile strength of the steel per square millimetre.

Then $$a = \frac{M}{2H' \times F_t''}. \quad . \quad . \quad . \quad . \quad . \quad (59)$$

Assuming the distance of the reinforcement from the lower surface of the concrete to be 0.025 metre, the distance $H' = 0.08 - (0.019 + 0.025) = 0.036$ m., and, taking F_t'' at 10 kilo, we have

$$a = \frac{86.4}{2 \times .036 \times 10{,}000{,}000} = 120 \text{ sq. mm.},$$

a sectional area that will be amply provided by spacing three round bars of 8 millimetres diameter, in the metre width of slab here considered.

2. In calculating the proportions of the main girders, the floor slabs are regarded as compression flanges. Consequently every main girder of the floor now in question may be considered as having a T-section of the following dimensions (see Fig. 50).

Compression flange,

3.80 m. wide by 0.08 m. deep.

Vertical leg of T,

0.27 m. wide by 0.35 m. deep.

Every girder of the series is 7.50 metres long, and has to carry the following loads:

(1) The weight of the top slab, with loads of 400 kilo. (W_e), and 200 kilo. (W_c) per sq. m. respectively, $=(7.5 \times 3.8) \times 600$.................. = 17,100 kilo.

(2) The weight of ten floor joists at 41.2 kilo. each............... .... = 412 „

(3) The weight of the vertical leg of the girder itself

$(7.5 \times 0.27 \times 0.35) \times 2{,}500$.......... = 1,771.87 „

Total $(W_e + W_c) = 19{,}283.87$ „

or, say, 19,284 kilogrammes.

Notwithstanding the efficient nature of the connexion between the stanchions and the girder, the latter is regarded as if the ends were simply resting upon supports.

Hence, for the bending moment, we have

$$M = \frac{(W_e + W_c)l}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

This gives

$$M = \frac{19{,}284 \times 7.5}{8} = 18{,}079 \text{ kilogrammetres.}$$

As the compression flange is more than sufficient for its required duty, it is not necessary to occupy space by detailed calculations.

The position of the neutral axis is found by a formula similar to that deduced for the floor slab.

Let H denote the distance between the centre of compression and the neutral axis, and e the thickness of the compression flange, or floor slab.

Then $$2H = \frac{M}{b \times e \times F_c'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

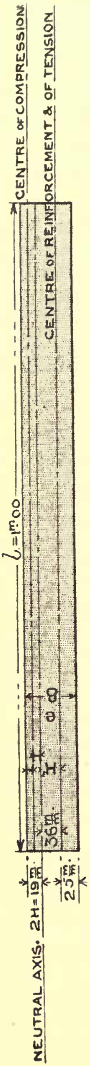


FIG. 49.

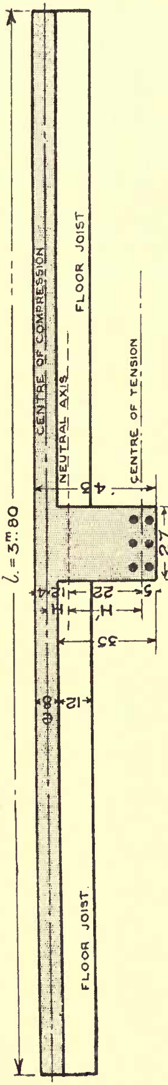


FIG. 50.

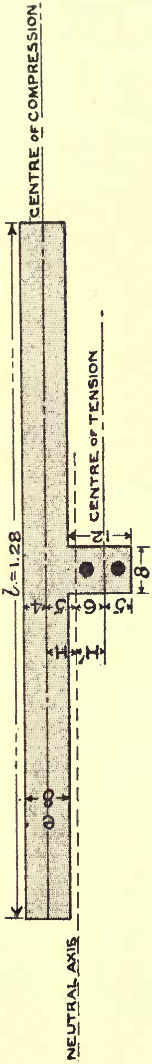


FIG. 51.

Thus $$2H = \frac{18,079}{3.8 \times 0.08 \times 250,000} = 0.238 \text{ cm.}$$

Whence $$H = 0.238 \div 2 = 12 \text{ centimetres.}$$

The centre of compression is assumed to be in the middle of the flange, that is, at 4 centimetres below the upper surface.

Consequently the distance of the neutral axis below the upper surface is

$$0.04 + 0.12 = 16 \text{ centimetres.}$$

In determining the sectional area of steel required, the centre of the reinforcement is settled at 5 centimetres above the lower surface of the girder.

The distance H′ from the centre of tension to the neutral axis is

$$H' = 0.35 + 0.08 - (0.04 + 0.12 + 0.05) = 22 \text{ cm.}$$

The total section of steel (a), calculated as before, is

$$a = \frac{18,079}{2 \times 0.22 \times 10,000,000} = 4,109 \text{ sq. mm.}$$

Six round bars of 30 millimetres diameter, will amply comply with this requirement, and in order to avoid the necessity for increasing the width of the lower portion of the girder, the reinforcement can be arranged in two rows of three bars, one above the other.

3. Each floor joist (see Fig. 51) is considered as having a T-section, consisting of a compression flange, 1.28 metres wide by 0.08 metre thick, formed by two halves of the adjoining floor slabs, and the vertical leg of the T, measuring 0.08 metre wide by 0.12 metre deep.

It is quite evident that the quantity of concrete in compression is in considerable excess, and calculations, performed in the manner described above, show that the neutral axis of these joists would pass within the slab forming the top flange. The effect of this would be to cause tension in the lower portion, and at the same time to diminish the amount of reinforcement required in the vertical leg.

Such a development is extremely undesirable, because unreinforced concrete in tension is very apt to crack, and the saving of metal is inconsiderable.

For the purpose of adjusting the height of the neutral axis, its position is assumed in the present case to be 1 centimetre below the under surface of the floor slab. The sectional area of steel is then calculated upon this assumption.

The following values are found by calculation as before:

$$(W_e + W_c) = 2{,}788 \text{ kilo.}$$
$$M = 1{,}004 \text{ kgm.}$$
$$H' = (0.20 - 0.14) = 0.06 \text{ m.},$$

and for the area of steel we have

$$a = \frac{1{,}004}{2 \times 0.06 \times 10{,}000{,}000} = 836 \text{ sq. mm.}$$

For this requirement, two bars of 24 millimetres diameter may be used, placed one above the other.

From the last calculations, it becomes evident that the floor slab, regarded as a flange for the joists, is relatively too thick for the depth of the vertical leg. As the total depth of the joists is fixed by structural considerations, the correct balance can only be attained by increasing the amount of reinforcement, or by reducing the width of each span by increasing the number of joists. But the latter alternative implies a diminution in the thickness of the slab, which would detract from the general solidity of the floor. Therefore it is better to increase the reinforcement as already indicated.

Under more favourable conditions, the joists are calculated in precisely the same manner as that adopted for the main girders.

We may here remark that the addition of reinforcing bars, bent upwards towards the ends, would permit the application of the modified formula $M = \frac{Wl}{10}$, to the girder and joists.

In the foregoing calculations, this modified form of the rule is only applied to the floor slab, because the reinforcement in the floor consists simply of horizontal bars.

No details are given by M. Boileau relative to the calculation of the stirrups for withstanding shear, but this part of the reinforcement can be settled by the rules we have previously given.

Shortly after completion, a complete panel of the floor was tested in the presence of several engineers and architects, under an external load of 700 kilos. per square metre.

The maximum deflection registered was 2.8 millimetres for the main girder, and 2.4 millimetres for the centre joist. On removal of the load, the permanent set was shown to be less than two-tenths of a millimetre.

69. Examples and Tests of Concrete-Steel Floors.—In the following notes we give particulars relative to the dimensions and general construction of some typical floor slabs and complete floor systems, and in nearly every case are appended the results obtained from actual tests.

Expanded Metal Slabs.—Table XXIII. contains particulars of experiments conducted by Messrs. Fowler & Baker, MM.Inst.C.E., upon concrete-steel slabs with reinforcement of expanded steel.

The slabs were tested as beams simply supported at the ends, therefore the results may be assumed to be about two-thirds those which would have been obtained if the slabs had been supported along all four sides, or one-half those to be expected if the slabs had been incorporated in a floor system.

At the same time tests were made upon slabs of concrete without reinforcement with the object of ascertaining whether by the addition of greater quantities of cement a slab could be produced of approximately the same strength and cost as that of an ordinary 1 : 3 concrete slab made with expanded metal, which in this series of tests consisted of very mild steel with an ultimate resistance of 22 tons per square inch.

It was very soon perceived, as the experiments proceeded, that, so far as regarded strength, the slabs made without reinforcement did not approach those made with expanded metal, and that even with a neat cement slab, which would cost as much as the concrete slab with expanded metal, the strength was much less.

Comparison of the results showed that slabs made without reinforcement gained considerably by increasing the proportion of cement, but the increase of strength was not more than one-fourth of that given by the use of expanded metal in the case of 3 ft. 6 in. spans, and a little more than one-fourth in the case of 6 ft. 6 in. spans.

Table XXIII.—Expanded Metal Floor Slabs tested as Beams. Proportions of Concrete, Portland cement, 1 : sand, 1 : Thames ballast, 2 : water, 0.5. (Fowler & Baker.)

No. or Mark on Slab.	Expanded Steel.		Slab.			Breaking Load.		Centre Deflection. (Total Breaking load = 1.00.)			
	Mesh.	Strand.	Clear Span.	Width.	Age in Days.	Total.	Per sq. ft.	0.25.	0.5.	0.75.	1.00.
	in.		ft. in.	ft. in.		lb.	lb.	in.	in.	in.	in.
1	3	$\frac{7}{32} \times \frac{3}{32}$	6 6	2 0	63	6,720	516.32	—	$\frac{7}{16}$	$\frac{13}{16}$	$1\frac{1}{8}$
2	3	$\frac{7}{32} \times \frac{3}{32}$	3 6	2 0	63	9,408	1,344.00	$\frac{1}{16}$	$\frac{11}{64}$	$\frac{1}{4}$	$\frac{9}{32}$
3	3	$\frac{7}{32} \times \frac{5}{32}$	6 6	2 0	63	7,392	568.96	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{11}{16}$	$1\frac{1}{8}$
6	3	$\frac{7}{32} \times \frac{5}{32}$	3 6	2 0	63	13,440	1,919.68	—	$\frac{3}{32}$	$\frac{3}{16}$	$\frac{9}{32}$
A	3	$\frac{7}{32} \times \frac{3}{32}$	6 6	2 0	77	6,720	516.32	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{13}{16}$	$1\frac{3}{16}$
B	3	$\frac{7}{32} \times \frac{3}{32}$	3 6	2 0	77	15,456	2,209.76	$\frac{3}{32}$	$\frac{7}{32}$	$\frac{3}{8}$	$\frac{17}{32}$
C	3	$\frac{7}{32} \times \frac{5}{32}$	3 6	2 0	77	16,800	2,400.16	$\frac{1}{16}$	$\frac{5}{32}$	$\frac{1}{4}$	$\frac{13}{32}$
D	3	$\frac{7}{32} \times \frac{5}{32}$	6 6	2 0	77	7,392	567.84	$\frac{1}{4}$	$\frac{9}{16}$	1	$1\frac{3}{16}$

Table XXIV. shows the deflection of a concrete-steel slab as reported by Mr. A. T. Walmisley, M. Inst. C.E.[1] This slab measured 12 ft. 6 in. by 11 ft. by 5 in. thick, and was composed of 1 : 4 clinker concrete, with expanded metal reinforcement near the under surface. The metal had a 3 in. mesh, and $\frac{1}{4}$ in. by $\frac{3}{16}$ in. strands, and was laid in four sheets with a 12 in. lap, the overlapping meshes being secured by steel clips. A 3 in. support was provided along the four sides of the slab.

Table XXIV.—Test of Expanded Metal Floor Slab (supported on four sides). Proportions of Concrete:—Portland Cement, 1*; Fine Clinker,* 4*; Age,* 49 *days. (Walmisley.)*

Distributed Load per sq. ft.	Deflection at Centre.	Remarks.
lb.	in.	
112	—	No deflection.
140	$\frac{1}{8}$	——
224	$\frac{1}{2}$	——
280	$\frac{3}{4}$	——
336	$1\frac{1}{4}$	Measured after an interval.
392	—	Slight crack on underside extending from corner towards centre.
448	$1\frac{3}{4}$	——
532	—	Crack across centre in direction of longest span.
588	—	Load carried $1\frac{1}{2}$ hours, slab then collapsed.

Expanded Metal Floor for Office Building.—A series of useful tests upon an expanded metal floor was conducted in 1900 by Dr. Bovey and Mr. Kennedy at La Presse Building, Montreal. The floor slab was essentially a sheet of concrete about 3 in. thick, reinforced by expanded steel embedded about 1 in. above the lower surface. The sheet rested upon steel girders and floor joists encased in concrete, both connected with the same material in the floor slab.

It will be seen that although the slab was of concrete-steel, the floor as a whole does not represent true

[1] *Proc. British Association*, 1900.

concrete-steel construction. For this reason we do not state any of the results obtained from test No. 1 upon the complete floor.

Test No. 2 related to one panel of the slab, and is suitable for our present purpose. This panel measured 13 ft. 2 in. by 7 ft. 4 in. between the supports, and 3 in. thick. A layer of sand was spread about 12 in. wide over the middle of the panel along the 13 ft. 2 in. span. Over this a pine board 1 ft. wide was laid for supporting the load.

The uniformly distributed load upon the panel was 240.3 lb. per square foot, and the maximum concentrated load was 120 lb. per square foot spread over the longitudinal axis for a width of 12 in.

Table XXV. gives the deflection under the several loads applied.

After removal of the external load, the deflection was found to be 0.02 in. at the left support, 0.04 in. at the right support, and 0.02 in. at the centre of the span.

Table XXV.—Test of Expanded Metal Floor Panel under Concentrated Loads. (Bovey & Kennedy.)

Total Load in pounds.	Deflection in inches.		
	At Supports.		At Centre.
	Left.	Right.	
2,500	0.045	0.08	0.07
5,000	0.100	0.13	0.14
8,000	0.140	0.19	0.22
10,000	0.180	0.24	0.28
11,580	0.220	0.28	0.34

Test No. 3 related to a panel measuring 14 ft. by 6 ft. by 3 in. thick, under a heavy distributed load.

A thin layer of sand was spread over the slab, and on this 1 in. pine boards were laid, upon which pig iron and bags of sand were placed. Table XXVI. shows the central deflection under the various loads.

Table XXVI.—Test of an Expanded Metal Floor Panel under Heavy Distributed Loads. (Bovey & Kennedy.)

Total Load in pounds.	Time.	Load in lbs. per sq. ft.	Deflection at centre in inches.
0	10.45 a.m.	—	—
12,300	10.55 a.m.	0.120	128.6
22,800	11.5 a.m.	0.285	238.5
32,800	—	0.505	343.1

After removal of the load, the recovery was as follows:

Total load, pounds -	27,800	25,300	0
Central deflection, inches	0.500	0.480	0.085

A little later the deflection at the centre became 0.08 in., and it was thought that further recovery would be evidenced after a longer interval had elapsed.

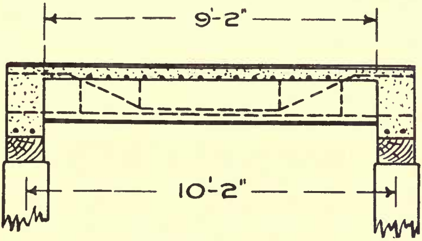
Ransome Floor Panel (*experimental*).—Comparative tests of full-size floor panels were made in September, 1902, for the Building Department of the Borough of Richmond, New York. These experiments were intended to throw light upon the actual relation of theory to practice in commercial construction, and to satisfy the officials as to the strength of the Ransome system of twisted steel reinforcement. (See Art. 26.)

The concrete in the first floor tested was mixed in the proportions of 1 part of Portland cement, 2 parts of clean sharp sand, and 4 parts of gravel varying from $\frac{1}{4}$ in. to $\frac{3}{4}$ in. diameter, and sufficient water was used to flush readily to the surface in tamping.

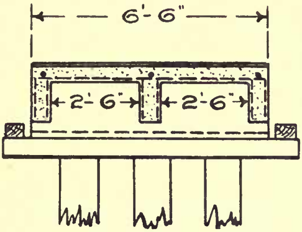
Deflection was measured, by means of an engineer's level, on a rule set upon the middle of the centre joist, and also on top of the concrete over the supports. The differences of these readings permitted correction for any errors due to settlement of the supports. The total displacement was also checked by means of a bevelled gauge moved between a horizontal straight-edge and the lower side of the centre joist.

Longitudinal and transverse sections of the floor are given in Fig. 52, and the following statement shows the details of construction:

Top Slab.—9 ft. 2 in. long between main girders, 6 ft. 6 in. wide, $3\frac{1}{2}$ in. thick with $\frac{3}{4}$ in. granolithic finish, reinforced with $\frac{1}{4}$ in. square twisted steel bars near the lower surface, and spaced 6 in. centre to centre.



LONGITUDINAL SECTION.



TRANSVERSE SECTION.

FIG. 52.

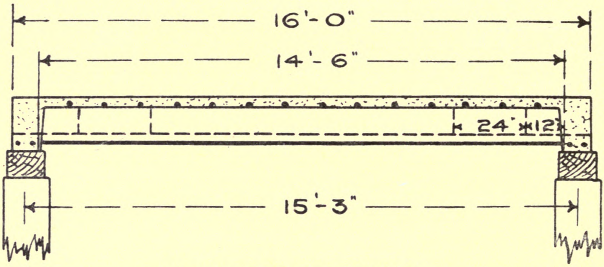
Main Girders.—6 ft. 6 in. long by 24 in. deep by 12 in. wide, spaced 10 ft. 2 in. centre to centre, each reinforced with two $\frac{7}{8}$ in. square twisted steel bars 2 in. above the lower surface.

Floor Joists.—9 ft. 2 in. long by $13\frac{1}{2}$ in. deep below floor slab, by 6 in. wide, spaced 3 ft. centre to centre, reinforced with one $\frac{1}{2}$ in. square twisted steel bars placed horizontally, one $\frac{1}{2}$ in. square twisted bars bent upwards towards the ends of the joists and anchored into the main girders, and four $\frac{3}{8}$ in. U-shaped stirrups placed vertically.

From this description it will be seen that the complete panel formed a structure comprising a slab, two girders and two joists forming an outer frame, and one joist acting as a central rib.

It should be noted, however, that continuous supports were fixed below the two end girders, and that these members were not subjected to any direct load.

Loading was effected by piling pig iron over the middle joist on a width of 2 feet between the end girders, and when the load of 110,218 lb. or 4,007 lb. per square foot, was reached, the central deflection was about $\frac{1}{16}$ in.



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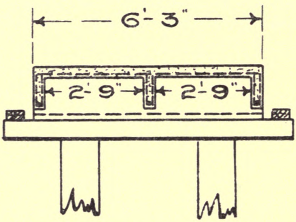


TRANSVERSE SECTION.

FIG. 53.

As no more pig iron was then available, it became necessary to abandon the test. Consequently, no theoretical deductions could be established, but the result quoted is quite sufficient to show the strength and rigidity of the construction.

Two other floors were afterwards tested, one of concrete made as stated above, and the other of concrete mixed in the proportions of 1 part Portland cement, 2 parts sand, and 4 parts $\frac{3}{4}$ in. screened trap rock.

Fig. 53 contains longitudinal and transverse sections which apply to both of these floors. The main details were as follows:

Top Slab.—14 ft. 6 in. long between main girders, $2\frac{1}{2}$ in. thick, reinforced with $\frac{1}{4}$ in. square twisted steel bars near the lower surface, and spaced 12 in. centre to centre.

Main Girders.—6 ft. 3 in. long by 18 in. deep by 9 in. wide, spaced 15 ft. 3 in. centre to centre, each reinforced with two $\frac{3}{8}$ in. square twisted steel bars 2 in. above the lower surface.

Floor Joists.—14 ft. 6 in. long by 9 in. deep below floor slab, by 3 in. wide, spaced 3 ft. centre to centre, reinforced with one $\frac{3}{4}$ in. square twisted steel bar placed horizontally, and anchored into the main girders, and four $\frac{1}{4}$ in. U-shaped vertical stirrups.

Tables XXVII. and XXVIII. give the centre deflections under various distributed loads for these two floor panels, the loads being applied on a width of 2 feet, as in the other case, between the end girders, which were supported on timber frames resting on piles.

Table XXVII.—Test of Ransome Floor Panel (supported at each end). Gravel Concrete-Steel—14 ft. 6 in. Span.

Distributed Load.		Centre Deflection in inches.
Total in pounds.	Lb. per sq. ft.	
9,800	225	$\frac{1}{16}$
11,046	253	$\frac{2}{16}$
13,523	310	$\frac{3}{16}$
16,642	382	$\frac{5}{16}$
19,578	450	$\frac{6}{16}$
22,696	521	$\frac{7}{6}$
25,501	586	$\frac{9}{16}$
28,652	658	$\frac{12}{16}$
32,680	752	$\frac{15}{16}$
35,800	822	$\frac{21}{16}$

Table XXVIII.—Test of Ransome Floor Panel (supported at each end). Stone Concrete-Steel—14 ft. 6 in. Span.

Distributed Load.		Centre Deflection in inches.
Total in pounds.	Lb. per sq. ft.	
9,559	219	$\frac{1}{16}$
11,228	258	$\frac{2}{16}$
13,900	319	$\frac{3}{16}$
16,528	379	$\frac{4}{16}$
19,879	456	$\frac{5}{16}$
22,925	527	$\frac{7}{16}$
25,477	585	$\frac{9}{16}$
28,756	661	$\frac{12}{16}$
31.097	712	$\frac{17}{16}$
32,784	753	$\frac{19}{16}$

In the gravel concrete panel, the first sign of failure was observed between loads of 28,652 lb. and 30,952 lb. Fine vertical cracks then appeared at regular intervals near the centre of the span, diagonal shearing cracks appeared at 30,952 lb. load, about 2 ft. from the supports, and these cracks increased in size until at the maximum load the openings were about $\frac{1}{4}$ in. wide. Cracks were also developed at the junction of the floor slab with the supporting girders.

In the stone concrete panel, the first evidence of failure occurred between loads of 25,477 lb. and 28,756 lb., when diagonal shearing cracks commenced, and increased proportionately with the loading.

Both panels showed a satisfactory degree of elasticity, but the concrete made with gravel proved to be stronger than that made with broken stone.

Failure resulted from shear, and was not due to lack of strength on the part of the steel in tension or of the concrete in compression. This result clearly points to the desirability of using reinforcement disposed so as to withstand shearing stresses.

Ransome Floor for a Foundry.—This floor was built in Paterson, New Jersey, for a foundry having a main building

100 ft. long by 88 ft. wide, and a wing 100 ft. long by 22 ft. wide. The floor was built at street level, and was proportioned for a load of 250 lb. per square foot. It was made in panels 22 ft. long by 5 ft. 6 in. wide, the material being 2 : 3 : 5 Portland cement stone concrete. The beams, joists, and floor slab are reinforced with twisted square steel bars in accordance with the Ransome system.

The floor slab is 4 in. thick, with a $\frac{3}{4}$ in. finish of 1 part of cement and 2 parts of sand with 50 per cent. of $\frac{1}{2}$ in. gravel, and is reinforced with $\frac{1}{4}$ in. square bars spaced 6 in. apart.

The slab is carried by concrete-steel joists, 22 ft. long by 7 in. wide at the top, tapering down to 6 in. at the bottom, and projecting 12 in. below the slab. The joists are spaced 5 ft. 6 in. apart, and are reinforced at the bottom with two $1\frac{1}{4}$ in. square bars.

The joists are carried by concrete-steel girders, supported on cast-iron columns, which are spaced 11 ft. apart in rows 22 ft. apart. The girders are 14 in. deep by 9 in. wide at the top, tapering down to 8 in. wide at the bottom, and are reinforced by two $1\frac{1}{4}$ in. square bars.

Both the girders and the joists are monolithic with the floor slab, and each section of 22 ft. by 11 ft. was completed in one continuous operation. At the walls, the girders are supported on steel I-beams rigidly connected to the columns.

Since the completion of the floor, core ovens have been built upon it, heavy castings have been moulded and poured at any convenient part, and 20 ton loads have been wheeled across it on the flanges of a four-wheeled truck with a 6 ft. by 2 ft. wheel base. In spite of these severe practical tests, no perceptible deflection has been noticed, and no cracks or chippings from impact have been observed. The shafts for an electric motor and a blower are suspended from the floor by bolts in holes drilled through the concrete, and the bearings are said to be quite firm and solid, while there is no sign of difficulty from vibration.

Ransome Floor for a Power House.—A floor similar to that described above is in use in the power house of the Edison Electric Lighting Company, Paterson, New Jersey. The floor slabs are 10 ft. by 4 ft. by 4 in. thick, and are reinforced by $\frac{1}{4}$ in. square twisted steel bars spaced 12 in. apart. The floor joists are 10 ft. long by 4 in. wide by 15 in. deep, and are reinforced with single $\frac{7}{8}$ in. square twisted steel bars near the bottom. The main girders are 12 ft. long by 9 in. wide by 18 in. deep, and are reinforced with three 1 in. square twisted steel bars near the bottom.

Although this floor was only calculated for a load of 250 lb. per square foot, it showed no sign of permanent set or injury under a test load of 500 lb. per square foot applied when the concrete was forty days old, and the elastic deflection proved to be less than $\frac{1}{32}$ in.

Hennebique Warehouse Floor.—Another illustration of the suitability of concrete-steel for warehouse buildings is afforded by a floor built on the "Hennebique" system at Messrs. Whitbread & Co's storage depôt in Tottenham, London. This floor is in an extension of a previously existing warehouse, and covers an area measuring 123 ft. 2 in. long by 25 ft. 7 in. wide. The new building comprises a basement and three floors, the ground floor being that to which attention is now directed. Support for the main beams is furnished by the brick walls of the building, and as the inner, or south, wall is only 14 in. thick, a continuous beam, or lintel, of concrete-steel has been inserted below the ends of the main beams, for the purpose of distributing the load equally over the entire length of the brickwork. Blocks of stone are let into the outer walls to perform a similar duty on a less extended scale. The lintel is 14 in. wide by 10 in. high, and the reinforcement includes three pairs of $\frac{7}{8}$ in. dia. steel bars, three bars near the upper surface and three near the lower surface, each pair of bars being joined vertically by "stirrups" of steel, 2 in. wide, the thickness varying from No. 12 to No. 15 S.W.G. according to requirement. These stirrups are for the purpose of resisting shear, and the lintel is further strengthened by $\frac{1}{2}$ in. dia. transverse rods. The main floor beams, spaced 11 ft. 3 in. centre to centre, are 14 in. wide, and project 22 in. below the underside of the floor slab, which is 5 in. thick and finished on top with cement paving $1\frac{1}{2}$ in. thick. As the slab is continuous in every direction, it practically constitutes an upper flange for the whole of the beams. The steel reinforcement in each beam consists of four pairs of

$1\frac{3}{8}$ in. dia. bars in the tension area, stirrups being placed round each pair and continued to the upper portion for withstanding shearing stress. Above each beam, and embedded in the floor slab, is a series of $\frac{5}{16}$ in. dia. rods, 5 ft. long, and spaced 12 in. apart, for the transmission of stress in such manner as to avoid any element of weakness at the junction between the beams and the remainder of the floor. The floor joists, spaced 5 ft. 3 in. centre to centre, extend from beam to beam, and measure 5 in. wide by 9 in. deep below the underside of the floor, the steel reinforcement consisting of two 1-in. dia. bars in the tension area, with stirrups as before. The floor slab is suitably reinforced, and the steel generally is so disposed that the several elements of the floor are combined to form a single structure. The floor has a clear span of 25 ft. 7 in. between the walls, and no supporting columns are required.

These are the main features of the floor, of which one complete bay of 288 sq. ft. area was tested in the presence of the author on July 30, 1903.

The floor was calculated for a superimposed load of 4 cwt. per square foot, therefore the normal test load was 57 tons 12 cwt. The test was conducted in the following manner: Instruments for the measurement of deflection were fixed in position below the main beam, one instrument at each end, 1 ft. away from the wall to indicate compression of the supports, and one instrument at the centre to show the maximum deflection of the beam. At 11.25 A.M. loading of the floor was commenced, the weight being distributed along the axis of the beam until the calculated superimposed load of 4 cwt. per square foot was reached. Readings were taken for increments of load up to 50 tons, and again when the load was equal to 4 cwt. per square foot. The total deflection for the normal load, measured in millimetres, was 2.9 mm. at the centre, the average of the deflection of the piers being 0.85 mm., giving a real deflection of the beam of 2.05 mm. or $\frac{1}{3800}$ of the span. An additional load of 2 cwt. per square foot was then imposed, and further records were taken. The total load of 86.7 tons, equal to 6 cwt. per square foot, was completed at 3 P.M. and allowed to remain in position for one hour, when the total deflection was found to be 4.8 mm., the average of the deflections of the piers being 1 mm., thus giving a real deflection of the beam of 3.8 mm , equal to $\frac{1}{2052}$ of the span.

At 4 P.M. the removal of the load was commenced. This operation was finished by 5.25 P.M., when the deflection was 0.35 mm. at the centre, 0.65 mm. at the north pier, and 0.05 mm. at the south pier, or an average of 0.35 deflection at the supports, showing that the deformation of the beam was entirely elastic. At 7.25 P.M., two hours after the

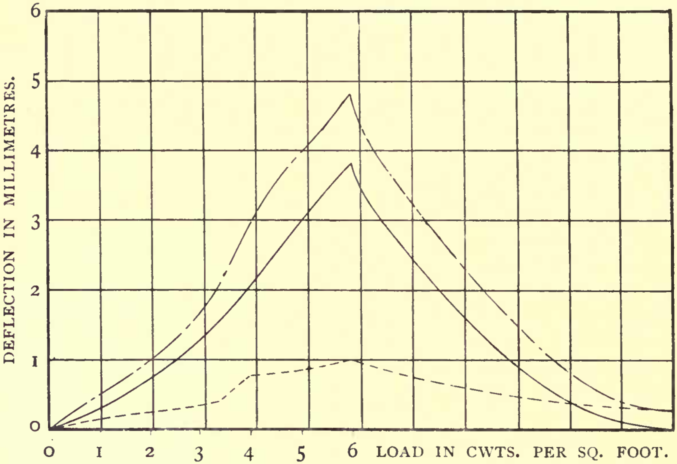


FIG. 54.

removal of the load, the readings were: Centre, 0.3 mm.; north pier, 0.6 mm.; south pier, 0.0 mm. It was therefore evident, not only that the beam had entirely recovered its original form, but that there was no permanent deformation of the support strengthened by the concrete-steel lintel. No cracks or fissures were observed during the course of the tests.

The history of the test is shown from beginning to end by Fig. 54. The top curve gives the total deflection as read from the centre instrument, the bottom curve indicates the average deflection of the supports, and the middle curve represents the actual deflection of the beam itself. The vertical lines in the diagram intersect the curves at intervals of 15 tons total load on the bay of 288 sq. ft. area, but for convenient reference we have also shown the load in cwts. up to the maximum of 6 cwt. per square foot.

It may be interesting to mention that the front wall of the new warehouse is supported by a concrete-steel beam or lintel extending in a single span from side to side of the building, the weight carried by this lintel being about 95 tons. Its chief dimensions are 25 ft. 7 in. span, by 16 in. wide, by 18 in. deep. There are four pairs of $1\frac{7}{8}$ in. dia. reinforcing bars in the compression area, and four pairs of $1\frac{3}{4}$ in. bars in the tension area, with stirrups 2 in. wide by 12 S.W.G. for affording resistance to shear.

The concrete used throughout is of the following proportions: cement, 6 cwt.; sand, 0.5 cubic yard; screened gravel, 1.0 cubic yard; making 31 cubic feet of concrete in position after ramming.

CHAPTER IX.

WORKING STRESSES AND BUILDING RULES FOR BEAMS AND FLOORS.

70. Working Stresses.—The permissible unit stresses on concrete and steel should be determined on a basis such that the loads coming on any given structure shall not cause stresses likely to endanger its prolonged existence.

Considered alone, the static load does not afford a sufficient indication as to the required strength, which must be determined with due regard to the greatest stresses that, even if frequently repeated, will not endanger the safety of the structure.

Recent investigations by M. Considère have shown that such stresses are caused by repetitions of loads equal to about two-thirds of the greatest static load. His results, therefore, are in substantial agreement with those obtained by Wöhler's well-known experiments on wrought iron and steel.

The consideration of frequent repetitions of load is the more important because it is probable that the adhesion between the concrete and the steel may become weakened in course of time by frequently repeated loads, especially in structures liable to severe shocks and vibrations.

Another point for attention is the possibility of danger from cracks in the concrete, as the weakening of a section where cracks have occurred is necessarily followed by displacement of the neutral axis, as well as by sudden changes of stress.

Under the action of live loads, the deformation induced at such points may lead to unexpectedly high unit stress on the steel reinforcement.

Further, the existence of cracks in a structure facilitates the admission of moisture, and, in exposed positions, frost may have a tendency to disintegrate the construction.

For these reasons it is very desirable that the proportions of concrete-steel beams should be such as to preclude the establishment of stresses likely to cause cracks of any kind.

A very general rule in practice is to adopt the following permissible unit stresses:

Concrete in compression, -	500 lb. per sq. in.
Concrete in tension, - -	50 lb. per sq. in.
Steel in tension, - -	15,000 lb. per sq. in.

These stresses, however, are only applicable to materials of such qualities as provide for an ample factor of safety, and they would not be justifiable in the cases of poor concrete and steel of low tensile strength.

Professor Brik recommends that the allowable unit stresses should be fixed on the following basis:

Concrete in compression,	One-seventh the ultimate compressive strength.
Concrete in tension due to bending,	One-half the ultimate bending strength.
Steel in tension, - -	14,200 lb. per sq. in.

Professor Brik further advises that the tensile bending stresses should first be determined, and that in no case should they be allowed to exceed the permissible unit stresses.

For the computation of the reinforcement he proposes that the tensile resistance of the concrete should be neglected, and that the area of steel should be computed on the basis of assumed cracks in the section under consideration.

In all calculations he considers that an addition of from 20 to 50 per cent. should be made to the estimated value of the load to provide for shocks and vibrations.

M. Christophe[1] recommends that the ratio of the coefficients of elasticity of the concrete and the steel should

[1] *Le Béton Armé et Ses Applications.* 1902.

be taken at 1 : 10, and proposes two standards for the unit stresses:

	Safety of 1st Degree. lb. per sq. in.	Safety of 2nd Degree. lb. per sq. in.
Concrete in compression, -	427	711
Steel in tension, - - -	12,800	21,400

In each case the tensile resistance of the concrete is to be neglected. The allowable shearing stress on concrete is to be taken at 21 lb. to 35 lb. per square inch.

71. Extracts from Continental Building Codes.

Berlin.—In the design of floors the static computation must prove that concrete-steel structures are capable of carrying ten times the specified load, including their own weight, and that the steel is able to resist the entire tensile stresses.

Dresden.—Calculations must be based upon the following maximum working stresses:

Concrete in compression, -	356 lb. per sq. in.
Steel in tension, - -	12,500 lb. per sq. in.
Steel in shear, - - -	10,000 lb. per sq. in.

Dusseldorf.—The proportions of the concrete must be such as to provide for a compressive resistance of 2,140 lb. per square inch at the age of twenty-eight days. The permissible compressive stress on concrete is fixed at 427 lb. per square inch.

If a higher compressive resistance is proved to be possessed by the concrete a proportionate increase of the working stress is permitted.

The ultimate tensile resistance of the concrete is taken at 570 lb. per square inch, but this is not taken into account in settling the reinforcement.

The maximum working stresses on the reinforcement are as follows:

Steel bars in compression,	12,500 lb. per sq. in.
Steel bars in tension, -	12,500 lb. per sq. in.
Steel bars in shear, - -	10,000 lb. per sq. in.
Steel wire in compression,	14,200 lb. per sq. in.
Steel wire in tension, -	14,200 lb. per sq. in.
Steel wire in shear, - -	11,400 lb. per sq. in.

The reinforcement must be so designed that in addition to taking the calculated tensile stresses it shall also provide for any shearing stresses for which no other provision has been made.

Frankfort.—Concrete-steel floors must be able to carry their own weight and ten times the specified load without perceptible deformation. Compressive stress in the concrete must not exceed 356 lb. per square inch, and all tensile stresses must be taken by the steel reinforcement.

Hamburg.—The permissible working stresses for concrete and steel are not to exceed the following intensities:

Concrete in direct compression,	427 lb. per sq. in.
Concrete in compression due to bending, - - - -	356 lb. per sq. in.
Concrete in tension, - -	0 lb. per sq. in.
Concrete in shear, - - -	22 lb. per sq. in.
Steel in compression, - -	12,500 lb. per sq. in.
Steel in tension, - - -	12,500 lb. per sq. in.
Steel in shear, - - -	10,000 lb. per sq in.

The steel must be designed to withstand the shearing stresses. For floors liable to vibration 20 per cent. is to be added to the specified loads.

72. Extracts from the New York Building Code.—The following notes from the regulations issued in 1903 by the Bureau of Buildings of the Borough of Manhattan, New York, will be found generally serviceable in the design of concrete-steel beams and floors.

Concrete-steel is defined as an approved concrete mixture reinforced by steel of any shape, so combined that the steel will take up the tensional stresses and assist in the resistance to shear.

Concrete-steel construction will be approved only for buildings which are not required by the Building Code to be fireproof, unless satisfactory fire and water tests shall have been made under the supervision of the bureau, and in accordance with the regulations fixed by the bureau and conducted as nearly as practicable in the same manner as prescribed for fireproof floor fittings in the Building

Code. Any builder offering concrete-steel construction for fireproof buildings must submit such construction to a fire and water test.

Before permission to erect any concrete-steel structure is issued, complete drawings and specifications must be filed with the Superintendent of Buildings, showing all details of the construction, the size and position of all reinforcing rods and stirrups, and giving the composition of the concrete.

Gravel or broken stone concrete is to be used, mixed in the proportions of 1 : 2 : 4; or the proportions may be such that the resistance of the concrete to crushing shall not be less than 2,000 lb. per square inch after hardening for 28 days. The tests to determine this value must be made under the direction of the Superintendent of Buildings. The concrete used in concrete steel construction must be what is usually known as a wet mixture.

Only high-grade Portland cements shall be permitted in concrete-steel construction. Such cements, when tested neat, shall, after one day in air, develop a tensile strength of at least 300 lb. per square inch; and after one day in air and six days in water shall develop a tensile strength of at least 500 lb. per square inch; and after one day in air and 27 days in water shall develop a tensile strength of at least 600 lb. per square inch. Tests as to fineness, constancy of volume, and other properties, made in accordance with the standard method prescribed by the American Society of Civil Engineers' Committee may, from time to time, be prescribed by the Superintendent of Buildings.

The sand to be used must be clean, sharp grit sand, free from loam or dirt, and shall not be finer than the standard sample of the Bureau of Buildings.

The stone used in the concrete shall be a clean, broken trap rock, or gravel, of a size that will pass through a $\frac{3}{4}$-inch ring. In case it is desired to use any other material or other kind of stone than that specified, samples of the same must be first submitted to and approved by the Superintendent of Buildings.

Concrete-steel shall be so designed that the stresses in the concrete and the steel shall not exceed the following limits:

Extreme fibre stress on concrete in compression, - - -	500 lb. per sq. in.
Shearing stress in concrete, - -	50 lb. per sq. in.
Concrete in direct compression, -	350 lb. per sq. in.
Tensile stress in steel, - -	16,000 lb. per sq. in.
Shearing stress in steel, -	10,000 lb. per sq. in.

The adhesion of concrete and steel is assumed to be not greater than the shearing strength of the concrete.

The ratio of the co-efficients of elasticity of the concrete and the steel is taken as 1 : 12.

The following assumptions are to be taken for guidance in the determination of the bending moments due to external forces. Beams and girders shall be considered as simply supported at the ends, no allowance being made for continuous construction over supports. Floor plates, when constructed continuous and when provided with reinforcement at the top over the supports may be treated as continuous beams, the bending moment for uniformly distributed loads being taken at not less than $Wl \div 10$; but the bending moment may be taken at $Wl \div 20$ in the case of square floor-plates which are reinforced in both directions and supported on all sides. The floor-plate, to the extent of not more than ten times the width of any beam or girder, may be taken as part of that beam or girder in computing its moment of resistance.

The moment of resistance of any concrete-steel construction under transverse loads shall be determined by formulae based on the following assumptions:

(*a*) That the bond between the concrete and the steel is sufficient to make the two materials act together as a homogeneous solid.

(*b*) That the strain in any fibre is directly proportionate to the distance of that fibre from the neutral axis.

(*c*) That the modulus of elasticity of the concrete remains constant within the limits of the working stresses fixed in these regulations.

(*d*) That the tensile strength of the concrete shall not be considered.

When the shearing stresses developed in any part of a concrete-steel construction exceed the safe working strength of concrete, as fixed in these regulations, a sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance to shear is overcome.

When the safe limit of adhesion between the concrete and the steel is exceeded, some provision must be made for transmitting the strength of the steel to the concrete.

The contractor must be prepared to make load tests on any portion of a concrete-steel construction within a reasonable time after erection, and as often as may be required by the Superintendent of Buildings. The tests must show that the construction will sustain, without any sign of failure, a load of three times that for which it was designed.

CHAPTER X.

FOUNDATIONS.

73. Foundation Slabs.—Concrete-steel is of especial value in foundation work of all kinds. By its use the amount of excavation required for ordinary foundations is considerably reduced, and there is a corresponding reduction in the mass and weight of the foundations necessary for the support of buildings and other structures.

Apart from the saving so effected, the further advantage follows that there is little interference with the hard crust frequently overlying soft and unstable strata.

Foundation slabs are often desirable for ground of low or variable bearing power. In one case there would be a general settlement with ordinary foundations subjected to uniform loading, and in the other case there would be local settlement in the specially weak points which are not infrequently found in unsuspected places.

By forming a foundation slab, or "raft," the superstructure may be "floated," so that adequate support will be afforded over the entire site.

Assuming the ground to possess uniform bearing power, the tendency must be for columns or walls to depress the portions of the slab immediately below them, and to force up the intermediate portions, as shown diagrammatically in Fig. 55. In a case of this kind the effect is the converse of that produced in a continuous beam carrying a uniformly distributed load. Beneath, and for a short distance on each side of each column or wall, the lower fibres are in tension, as also the upper fibres in the middle portion of each span. To meet these conditions, the steel reinforcement of the slab might be disposed after the manner indicated in Fig. 56.

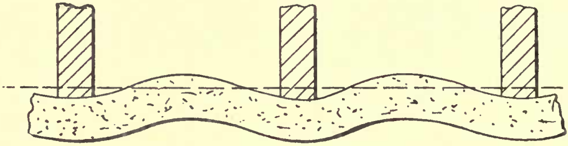


FIG. 55.

But there is no guarantee that the ground will act in the uniform manner here suggested. It may subside at some entirely unexpected part, as shown in Fig. 57. The effect will then be to disorganise any arrangement of the

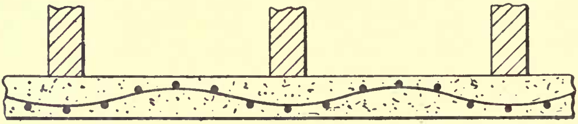


FIG. 56.

reinforcement such as that proposed in Fig. 56, for the whole of the lower fibres in the unsupported portion of the foundation slab will be in tension, and in sections on each side the whole of the upper fibres will be in tension.

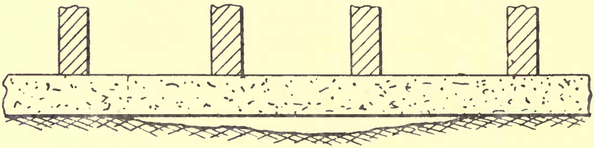


FIG. 57.

Consequently we arrive at the conclusion that in all cases where the uniformity of the ground cannot be absolutely relied upon it is a necessary precaution to employ reinforcement continuously in the upper and the lower fibres of foundation slabs.

This is a point of vital importance which is sometimes overlooked in concrete-steel construction.

It may frequently be quite safe to apply all the reinforcement near the lower surface of the slab, but as the absolute stability of the foundation is a most essential factor in all building construction, we think it would be wise to apply reinforcement near both surfaces as a matter of precaution against unforeseen developments.

Economy is always desirable, but an extra ton or two of steel is much less costly than a new building, and a little extra expense in the foundations may generally be regarded as an insurance premium on account of contingencies.

It will, of course, be understood that the reinforcement must be disposed in longitudinal and transverse lines in all foundation slabs, thereby forming a network capable of withstanding stresses in any direction.

74. Hennebique Foundation for Mill.—As an example of a continuous foundation slab, we may quote one constructed on the Hennebique system in a large granary and flour mill at Swansea.

The granary measures 127 ft. long by 48 ft. 6 in. wide, the total height from the foundations to the roof being about 112 ft., or 130 ft. to the roof of the tower. This building is designed for the storage of more than 7,000 tons of grain.

The adjoining flour mill measures 80 ft. long by 40 ft. wide, and is 112 ft. high from foundation to roof, the latter being constructed so as to form a storage reservoir capable of holding about 100 tons of water.

The foundations of both buildings are upon soil consisting of sand and rubbish brought as ships' ballast and deposited on the soft mud of the estuary at Swansea.

In order to ensure perfect stability, a continuous foundation slab was formed for the two buildings, and, as the buildings are entirely of concrete-steel, they constitute a monolithic structure in which all the members afford mutual support to each other, and local settlement is practically impossible.

75. Hennebique Foundation for Warehouse.—A somewhat similar example is to be found in the Co-operative Wholesale Society's warehouse at Newcastle-on-Tyne.

This building measures about 125 ft. long by 90 ft. wide, the height from foundation to roof being 95 ft.

The ground is said to be composed of slush, peat, and quicksand to a depth of more than 60 ft., and here again a continuous foundation slab of concrete-steel was laid down, calculated to resist a reaction of the ground to the extent of about $2\frac{1}{2}$ tons per square foot, and to distribute the weight of the building uniformly over the whole area of the site.

This building, also, is constructed entirely of concrete-steel, and with its foundation slab practically represents a huge box girder in which no local displacement is at all likely to occur.

76. Expanded Metal in Foundation Work.—Expanded metal is a very suitable form of reinforcement for foundation work of the kind mentioned above. It affords an excellent bond, and owing to its continuity in every direction gives to every part of a foundation adequate resistance to local settlement.

77. Cottancin Foundation on Site of Salt Lake.—Foundations constructed on the Cottancin system have been used in France and elsewhere with considerable success. The distinctive characteristic of this type of concrete-steel is a woven core of metal rods forming a continuous network in which longitudinal and transverse wires are interwoven after the manner of basket-work. Foundations on the caisson principle constitute a feature of this system. The foundation slab is of concrete or brick reinforced in the usual way by a network of steel rods, and is further stiffened by walls built on the underside, so as to form a series of cellular compartments. Foundations so designed have been found very useful in the case of treacherous soils.

As an example showing the suitability of the Cottancin system under such conditions, we may mention the experiments conducted by engineers of the Ponts et Chaussées, at Tunis, for the purpose of ascertaining the most suitable type of foundation for the extension of the city over the silt-covered area of an ancient salt lake. Ordinary foundations were found to be incapable of supporting a load exceeding 32 lb. per square foot without unequal settlement, while those of the Cottancin type carried a load of

777 lb. per square foot, with a uniform subsidence of only 0.08 in., and when the load was increased to 1,711 lb. per square foot the settlement was still uniform and was less than 1.19 in.

78. Cottancin Foundation for Boiler House.—An example in a more accessible locality is to be found in the boiler-house foundation recently constructed in St. James's Park, Westminster, for H.M. Office of Works. The soil in this district is of alluvial character, and in early times the whole area was more or less covered with water. Those of our readers who have had occasion to erect buildings in Westminster, or who have observed the operations conducted by others, are probably aware that deep and expensive foundations are generally necessary to insure stability. The Cottancin foundation to which we refer was designed for the purpose of supporting some heavy steam boilers for the pumping-station in St. James's Park, and as a depth of 12 in. was found to be amply sufficient, the advantage presented by this form of concrete-steel foundation is quite evident.

In this instance the walls of the caissons, or cellular compartments, were built of steel-cored brick with a cover, or foundation slab, of reinforced concrete, the whole being tied together so as to constitute a single self-contained structure.

79. Cottancin Foundation for 1,000 H.P. Gas Engine.—One argument advanced in favour of the Cottancin system is the non-communication of vibrations from heavy machinery through the soil. Satisfactory confirmation of this contention was furnished by the foundation built for the 1,000 horse-power Cockerill gas-engine at the Paris Exposition of 1900. This machine was said to cause a shock equal to 250 tons every tenth of a second. The foundation, constructed on the artificial soil of the Champ de Mars, consisted of brick-steel caissons with a concrete-steel covering slab, and the holding-down bolts of the engine were taken through cement-steel tubes to the bottom of the brick walls.

80. Footings for Columns.—Footings for columns are frequently constructed as shown in Fig. 58, which includes a section and half plan of a footing on the Hennebique system, but, with the exception of the stirrups for resisting shear, the design is very similar to that generally adopted by engineers.

A footing of this character may be regarded as consisting of an infinite number of cantilevers connected together and radiating from a central abutment, and subject to

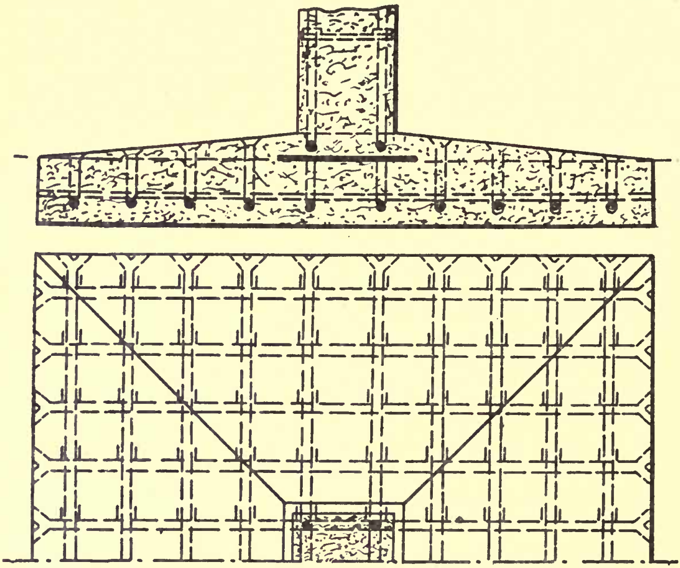


FIG. 58.

bending stress from uniformly-distributed loads acting in an upward direction, while the central abutment carries an almost concentrated load acting in a downward direction.

Hence, as the upper portion of the footing is in compression and the lower portion in tension, the main reinforcement must be near the lower surface.

The horizontal plate in Fig. 58 is intended to distribute the vertical pressure of the column, of which a small part is shown in the section.

Footings of very considerable size have been made on this principle, the largest hitherto constructed having a side of about 70 ft.

81. Ransome Foundation for Tall Office Building.— Concrete-steel foundations, substantially as described in the preceding article, were employed for all the columns in a large sixteen-story office-building recently erected at Cincinnati, U.S.A.

This building, which is designed in accordance with the Ransome system of concrete-steel, is rectangular in plan, and occupies a site measuring approximately 100 ft. by 50 ft. It is built on a stratum of gravel and sand, and the supporting columns are spaced so as to require main girders some of 16 ft. and others of 33 ft. span, and secondary girders with spans of 16 ft. and 18 ft.

The footings vary in size according to their position and loading. Along the two street frontages seven footings are built separately, one for each column; at the back, which adjoins a four-story building, there are six foundations each carrying two columns; and along one side, next to a six-story building, one continuous foundation carries four columns.

The dimensions of the single-column foundations are as follows:

Area of base - - -	12 ft. 9 in. square.
Area of top - - -	4 ft. $9\frac{1}{4}$ in. square.
Height (base to top) - -	3 ft. 1 in.
Reinforcement, longitudinal	27 bars, $\frac{3}{4}$ in. square, 11 ft. 3 in. long.
,, transverse -	26 bars, $\frac{3}{4}$ in. square, 11 ft. 3 in. long.

The two-column foundations have sloping sides and one sloping end, the other end being connected with the back wall of the building.

Above the sloping sides a rectangular longitudinal rib is formed running the whole length of the foundation, at each end of which one column is fixed, and at the junction with the column away from the party wall additional concrete is placed to fill up the angle formed by the sloping sides and the upper rib of the footing.

The main dimensions of the two-column foundations are as follows:

Length of base	20 ft. 10 in.
Width of base	13 ft. 10 in.
Height (base to end of batter)	3 ft. 1 in.
Height (base to top of rib)	6 ft. 0 in.
Length of longitudinal rib	19 ft. 10$\frac{7}{8}$ in.
Width of longitudinal rib	4 ft. 5 in.
Height of longitudinal rib	2 ft. 11 in.
Reinforcement in lower portion, transverse.	49 bars, 1 in. square, 12 ft. 0 in. long
Reinforcement in upper portion, longitudinal.	2 bars, $\frac{3}{4}$ in. square, near top surface in middle.
" "	10 bars, 1$\frac{1}{4}$ in. square, spaced equally across section just below $\frac{3}{4}$ in. bars.
" "	9 bars, 1$\frac{1}{4}$ in. square spaced equally across section just below the other 1$\frac{1}{4}$ in. bars, and having the ends bent down to form a truss.
Stirrups in upper portion, vertical.	6 $\frac{1}{2}$ inch. inverted U-bars, in cross section. (8 such sets of stirrups in longitudinal section.)

The four-column foundation extends below and beyond the external face of the party-wall at one side of the building, and the base of the footing projects about 4 ft. beyond the front end of the wall. The width of the foundation varies from 7 ft. 6 in. to 8 ft. 10 in., the other dimensions and the reinforcement being generally similar to those of the two-column foundations.

The footings are all built independently of the columns, for which cast-iron base plates are provided. In one sense this seems to be an undesirable feature, as it breaks the continuity of the concrete-steel construction. As, however, the loads are large—ranging from 500 tons to 750 tons per column—the adoption of cast-iron column bases is a wise precaution for insuring the adequate distribution of the load transmitted.

82. **Foundation for Railroad Subway.**—Fig. 59 represents an interesting example of concrete-steel construction designed for the subway of the Philadelphia Rapid Transit Company. The section shows rather more than one-half of a four-track subway, and the drawings serve to illustrate floor as well as foundation design, for the roof of the subway is essentially the same as a floor under a uniformly distributed load.

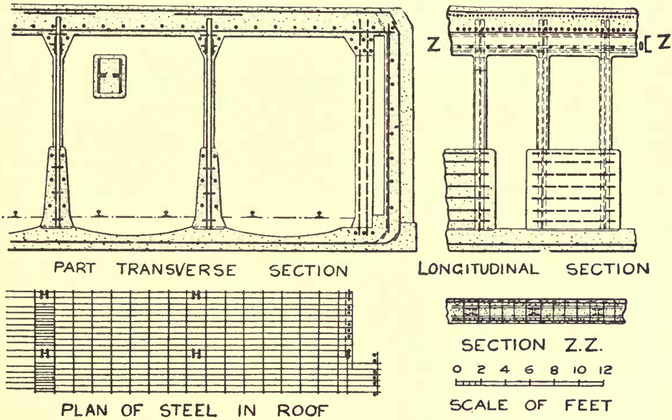


FIG. 59.

It should be remarked that the outer black line in the foundation, wall and roof is intended to represent a waterproof layer of asphaltic mastic $\frac{1}{2}$ in. thick. In other respects the drawings are self-explanatory.

83. **Concrete-Steel Piles.**—Notwithstanding the general convenience and efficiency of concrete-steel foundations as described above, cases often occur in which either the load or the soil, or both, are such that it would be unwise to rely upon "floating" a building on a slab foundation. Three alternatives then present themselves for consideration, (1) to construct piers; (2) to sink cylinder foundations; and (3) to increase the supporting power of the soil by piling.

The construction of concrete or masonry piers for a heavy building on soil of low bearing capacity involves deep and costly excavation, and the sinking of cylinder foundations is an equally expensive operation.

The driving of timber piles is quite easy, and timber is an inexpensive material. Further, the use of piles in the place of deep and difficult piers saves expense, obviates any risk of disturbing the foundations of neighbouring structures, and avoids any possible trouble from subsoil water or quicksand.

Timber piles are generally connected with the superstructure by means of a steel grillage, or a foundation slab of concrete or concrete-steel, the object in each case being to distribute the weight of the building equally over the heads of the piles.

When any such type of foundation is adopted, it is always desirable that, for their preservation, the piles should be permanently saturated with ground water. Hence it often becomes necessary to cut down the piles and to commence the masonry at a low level, thus involving excavation that may be difficult, and is sure to be expensive.

Piles formed of concrete-steel possess all the advantages offered by timber piles and none of their disadvantages. By the combination of concrete with suitable steel reinforcement, piles are produced that can be driven in any moderately soft earth, and that are equally durable in dry or wet positions. Moreover, they offer exceptional facilities for through connexion with foundation slabs or superstructures of almost any kind. The principles governing the application of steel reinforcement to piles are essentially the same as those involved in the design of columns, a subject which is fully discussed in Chapter XI.

84. Concrete-Steel Piles in Harbour and River Work.—In various classes of foundation work not coming under the head of ordinary building construction, piles are practically indispensable. Thus, in the building of docks, wharves, piers, and jetties, piling is always necessary, except in the case of structures founded upon rock of solidity sufficient to support the weight transmitted through ordinary footings or piers.

Timber piles are undoubtedly very satisfactory for works of such nature, their chief foe being the "teredo," but iron piles are liable to corrosion, and must be continually painted.

Concrete-steel piles cannot be attacked by worms, and require no painting. The only questions that can be raised as to their desirability are :

(1) Whether they are likely to prove durable in sea-water, and

(2) Whether they are strong enough to withstand fracture if unsuspected strata or masses of stone should be encountered during the process of driving.

With regard to (1), we know that the action of sea-water upon cement and concrete is a source of trouble, and sometimes of anxiety, to harbour engineers. But the fact remains that these materials continue to be used in all kinds of marine engineering work, and, on the whole, with considerable satisfaction.

So far as embedded in the ground, concrete-steel piles are practically indestructible, and any parts exposed to sea-water and air can be repaired as may be found necessary, just as simple concrete walls are repaired. Therefore there seems no reason on the score of durability for rejecting concrete-steel as a material for the construction of piles to be employed in harbours, docks, and coast protection works generally.

With regard to (2), we think the details of construction given in Art. 85 should afford sufficiently convincing evidence as to the capacity of concrete-steel piles to withstand any shocks that could be borne by timber piles during driving.

85. Concrete-Steel Piles in Bridge Building.—As the chief objection urged against concrete-steel piles is the assumed difficulty of driving, it may be useful to refer to the experience of Dr. F. von Emperger[1] in driving piles for the pier foundations of a small bridge in Elsass-Lothringen.

The pier was supported on eleven piles from 14 ft. to 16 ft. long and 16 in. square, each pile being moulded horizontally in one length. The soil was of marshy character to a depth of about 6 ft. 6 in., and below that was a stratum of coarse gravel.

[1] *Zeitschrift des Oesterreichischen Ingenieure und Architecten Vereins.* Nov. 7, 1902.

The ram of the pile-driver weighed 8,800 lb., and, with a drop of 20 in., it drove the piles from 8 in. to 12 in. through the soft soil. On reaching the gravel the drop was from 3 ft. 4 in. to 4 ft. 3 in., driving the piles about 2 in. at first, and finally to refusal by from 80 to 120 blows, with a penetration varying between 0.3 in. to 0.4 in.

From a lengthy report made by Dr. von Emperger as to the behaviour of the piles it is clear that good workmanship is absolutely necessary if satisfactory results are to be obtained.

Very little trouble was experienced from fracture of the concrete in driving. In only one of the piles was the concrete broken away at the head, a result attributed to the lack of proper precautions.

86. Hennebique Piles.—The details of a concrete-steel foundation pile are shown in Fig. 60, which illustrates in section and elevation the Hennebique method of construction. The cross-section below is drawn to a larger scale for the sake of clearness.

These piles are made in vertical moulds, and the concrete is reinforced by longitudinal bars connected at short intervals by transverse ties or stirrups.

The pile is armed at the point with a steel shoe having four side tongues, the extremities of which are bent inwards at right angles and bedded into the concrete as shown in the vertical section.

It will be noticed that the head of the pile is of smaller sectional area than the body, the object of this being to allow clearance between the heads of adjoining piles to facilitate driving.

In order to distribute the force of the blow uniformly over the entire head, and to prevent local injury to the concrete, a cap of cast-steel is used during driving. This cap is filled with dry sand, and the joint at its lower end is made by plugging clay into the space between it and the pile head, afterwards caulking it with hemp or spun yarn. In this manner a sand cushion is formed above and around the head, which insures completely uniform distribution of the force applied by the pile driving ram.

Another advantage secured by the use of the sand cap is that the longitudinal reinforcing bars can be allowed to pro-

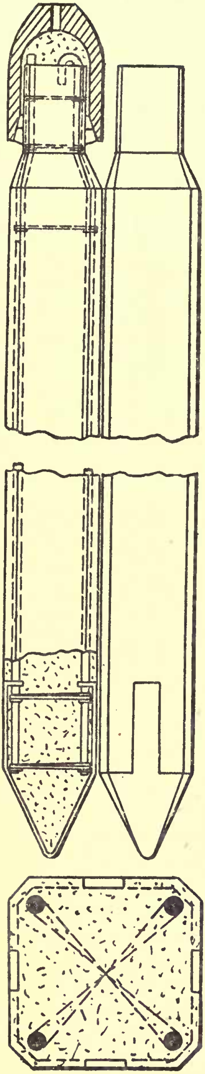


FIG. 60.

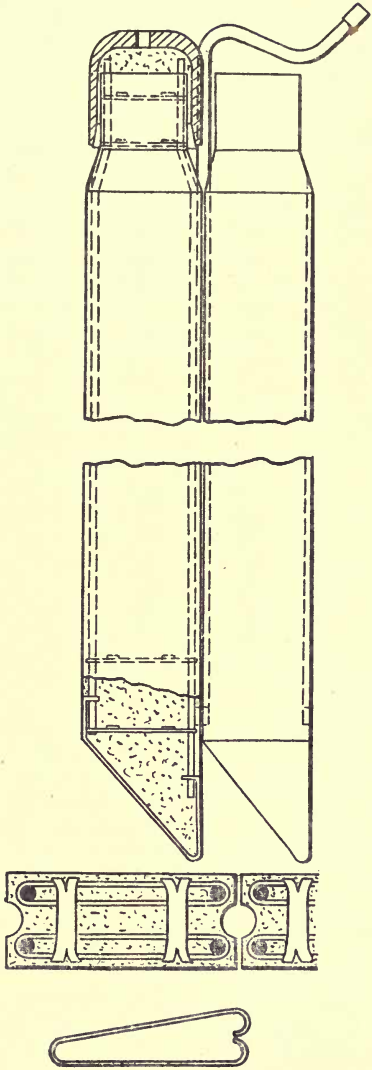


FIG. 61.

ject above the top of the concrete, as shown in the section, for connexion with other members of the structure, or the ends of one or more bars may be bent over to form eyes for the convenient attachment of hoisting tackle.

Fig. 61 represents two Hennebique sheet piles in elevation and partly in section, the small drawings beneath showing respectively a transverse section and one of the ties, both to a larger scale. The piles are reinforced by four longitudinal steel bars, connected at intervals with ties formed of steel wire, and these are braced by flat bars with split ends to insure a good bond with the concrete.

The point of each pile is protected by a steel shoe with tongues turned into the concrete as before.

As may be seen by the transverse section, the two narrow sides of each pile are moulded with semi-circular grooves.

The groove in the longer of these two sides extends from the shoulder of the pile down to a small projection with a semi-circular face, shown by a small square in the part sectional elevation, the remainder of this side and the whole of the shorter side of the pile being grooved. In ramming, the projection on one pile is fitted into the groove of the pile previously driven.

A cast-iron cap similar to that already described is fitted to the head of the pile, and a device is provided for facilitating the operation of driving by means of a water jet.

This arrangement consists of an iron pipe fitting the circular space formed by the grooves of the last-driven pile and of the pile to be driven. The pipe is connected by means of a flexible tube with a pump or tank at sufficient elevation to furnish water at the necessary pressure. The pipe also serves as a guide, and the water admitted through it forces out any entering sand that might have a tendency to choke or jam the grooves.

When the pile has been sunk to the desired depth, the pipe is withdrawn, and the circular hole is filled with cement grout, thus ensuring a good joint, and keying the piles together so that the entire series constitutes a continuous watertight wall.

87. Method of Moulding.—Hennebique piles are formed in vertical timber moulds, well white-washed or soaped, and supported in timber racks. The working face is left open, care being taken to see that the mould is fixed in a truly vertical position. The steel shoe is then placed in the bottom of the mould, and the vertical rods, together with their connecting stirrups, are next put in position and adjusted by gauges, so that they will be about 1 in. inside the surface of the concrete. Concreting is then commenced, and the working face of the mould is closed, as the work proceeds, with boards from 4 in. to 6 in. high, which are fixed into grooves on the side of the mould. The concrete is thoroughly rammed as it is put in, about a bucketful at a time.

About thirty-eight hours after the concreting has been finished, the mould is stripped, and the pile is seasoned for a period of three weeks in summer time, and six weeks in spring and autumn, before use.

Vertical moulding is considerably more costly, as well as more troublesome, than horizontal moulding, but it ensures far better results. Piles are frequently moulded horizontally, but when this method is followed care must be taken to obtain the proper consistency of concrete, and to avoid, as far as possible, inequalities of density in the piles.

Even with the greatest care, it is impossible to prevent the lower portion of the concrete in the horizontal moulds from being denser than the upper portion, and, moreover, when gravel concrete is used, the pebbles do not adjust themselves in the most suitable direction for resisting axial thrust.

The vertical reinforcement generally consists of round steel bars, varying in diameter from 1 in. to $1\frac{1}{2}$ in., and the stirrups are of $\frac{3}{16}$ in. steel wire. The spacing of the stirrups varies from 2 in. at the bottom to 10 in. at the middle of the pile, and the proportion of the reinforcement varies from $2\frac{1}{2}$ per cent. to 5 per cent. Piles of this description are made in various lengths, the maximum length hitherto used being about 60 ft.

88. Details of Piles at Southampton.—Among other extensive buildings in which Hennebique pile foundations have been adopted, we may mention the Southampton Cold Storage Warehouse, a six-story building, measuring 400 ft. long by 120 ft. wide, and some cargo sheds at Southampton Docks, 800 ft. long by 150 ft. wide.

The following are the particulars of the piles used at Southampton :

Cross Section.	Reinforcement.		
12 in. by 12 in.	4 bars,	$1\frac{1}{2}$ in. dia.	5 per cent.
14 in. by 14 in.	,,	$1\frac{1}{4}$,,	$2\frac{1}{2}$,,
15 in. by 15 in.	,,	$1\frac{3}{8}$,,	$2\frac{1}{2}$,,
16 in. by 12 in.	,,	$1\frac{5}{8}$,,	$4\frac{1}{4}$,,

The piles were driven by a Lacour pile-driver with a ram weighing about 3,350 lb., and a 6 ft. drop.

In some cases it is possible to sink the piles entirely by the hydraulic method, and in others a combined process may be best.

At Southampton Old Extension Quay, for instance, concrete-steel piles were sunk recently through 22 ft. of gravel and hard sand by the water jet in conjunction with the ordinary pile driver, as it was found impracticable to carry out the work by the hydraulic method alone. Water was taken from the hydraulic mains through a $\frac{3}{4}$ in. pipe, with a $\frac{3}{8}$ in. nozzle at the point of the shoe, and the working pressure was reduced to about 300 lb. per square inch, each pile being sunk to the required depth in less than one hour.

89. Details of Piles at Rotherhithe.—Fig. 62 includes elevations of the broad and narrow sides of some piles designed by Mr. Andrew Johnston, M. Inst. C.E., for use on the Thames at Rotherhithe.

In this case the longitudinal reinforcement is formed by steel angle bars, 2 in. by 2 in. by $\frac{3}{8}$ in., connected horizontally with $1\frac{1}{4}$ in. by $\frac{3}{16}$ in. flat bars, as shown in Fig. 62*a*. Diaphragm bars of $\frac{1}{8}$ in. steel wire are also provided as shown in Fig. 62*b*. The lower end of the pile is furnished with a securely attached steel point.

These piles are from 40 ft. to 50 ft. in length, the cross section measures 1 ft. 6 in. by 12 in., and the reinforcement is 2 in. from the surface of the concrete.

90. Piled Foundations in New York.—Figs. 63 and 63*a* show the details of the concrete-steel piles used in the foundations of the Hallenbeck Building, New York. This is a ten-story steel frame structure, having wall columns carried on I-beam grillage foundations, embedded in the

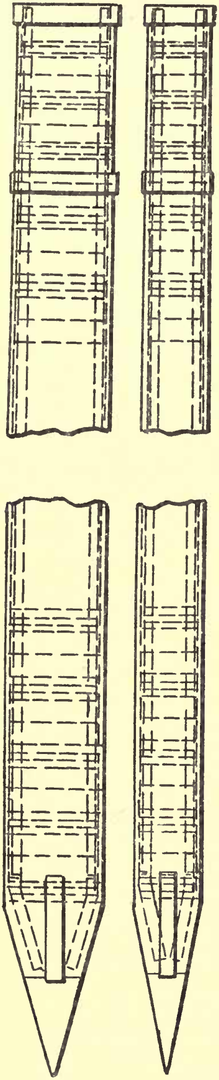


FIG. 62.

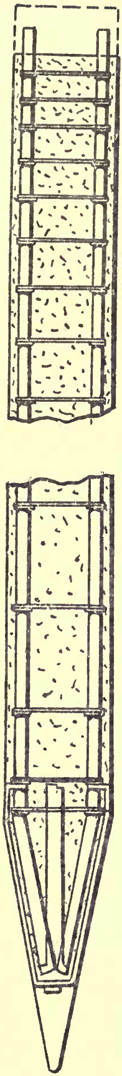


FIG. 63.

concrete caps of timber piles. The building has lately been extended by the erection of an addition measuring 102 ft. by 20 ft. in area.

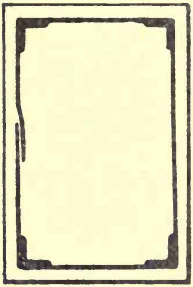


FIG. 62 *a*.

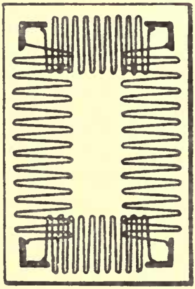


FIG. 62 *b*.

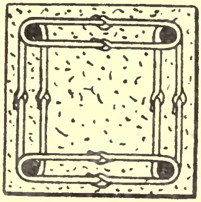


FIG. 63 *a*.

After several test borings, it was found that the use of wooden piles for the extension would necessitate excavation below the wall footings of an adjoining building, and that

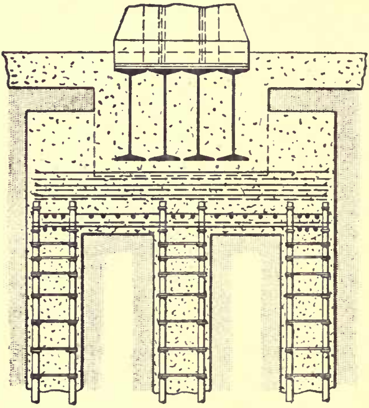


FIG. 64.

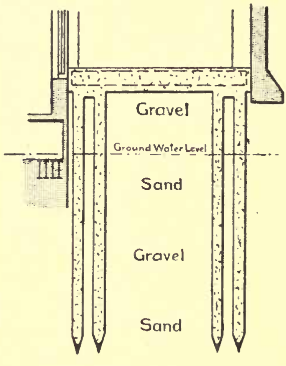


FIG. 65.

it would be essential to underpin the foundations and to carry them down to a lower depth. The cost of this work, together with that of the timber piles, would have been greater than the expenditure involved in driving concrete-steel piles, for which no trenching, no extra excavation, and no underpinning of the adjoining structure would be required.

Fig. 64 is a section showing the upper portion of the concrete-steel piling for one column, the concrete-steel slab connecting the series of three piles, and the manner in which the girder grillage and column are fixed. Fig. 65 is from a transverse section of the extension building and shows the general arrangement of the new concrete-steel foundation. The latter consists of reinforced concrete piles extending to a height of about 6 ft. above the ground water level, where they are connected with the concrete-steel floor slab so as to form a monolithic structure. The piles are 28 ft. long by 12 in. square, and are made with Portland cement concrete in the proportions of 1 : 2 : 4, rammed longitudinally into wooden moulds.

Each pile was calculated to carry a load of 80,000 lb., which includes a load of 36,000 lb., per square foot on the concrete as permitted by the New York Building Regulations for concrete piers, and an additional load of 44,000 lb. assumed to be directly supported by the reinforcement, consisting of four $1\frac{17}{16}$ in. diameter vertical steel bars, the same regulations permitting this material to work in compression up to 7,000 lb. per square inch.

CHAPTER XI.

CONCRETE-STEEL COLUMNS.

In view of the fact that increasing importance attaches to the reliability and endurance of columns and stanchions in modern building construction, such members deserve the most careful consideration. It is not enough for the designer to know that the strength of the columns is adequate to support the calculated loads; he must also be satisfied as to their durability and fire-resisting qualities.

91. Corrosion of Steel Columns.—The possibility of corrosion in steel columns erected in the ordinary way, protected by nothing better than one, two, or three coats of paint, and simply enclosed in the walls of a building, ought to cause more uneasiness than appears to be experienced by those who follow this undesirable practice. Even if the metal were thoroughly cleaned before painting—which is seldom the case—no one can reasonably expect the original coating of paint to be sufficiently durable to insure the safety of a structure for all time. It is universally admitted that steelwork exposed to climatic influences must be repainted at regular intervals for the purpose of preventing corrosion, but steel columns encased in brickwork cannot be treated in the same way, and those responsible do not always reflect that the porosity of brick, and the presence of fissures or other internal spaces, may permit moisture to gain access to the protecting paint. Considering these points, together with the admitted fact that corrosion may take place underneath the paint itself, it becomes evident that columns applied in the manner indicated, cannot be considered as really durable members of a structure. They may be moderately well protected against fire, but interior columns that are not encased in brickwork are certainly not.

92. Concrete as a protection against Corrosion and Fire.—The excellent practice of enclosing isolated columns in terra-cotta or other casings affords considerable protection from fire, but not necessarily from corrosion, and the next step in advance is to cover all isolated columns with concrete, at the same time filling with the same material any interior cavities that may exist. Similar treatment is also advisable in the case of columns built into the walls.

Practical experience, and notably the recent examination of steel work taken from some buildings demolished for the construction of the New York Rapid Transit Subway,[1] have sufficiently proved the preservative effect of concrete so far as corrosion is concerned, while the service rendered by concrete as a fire-resisting material is too well known to require comment.

We are brought in this way to the conclusion that the joint use of steel and concrete is most desirable in column construction, as conducing to durability and fire-resisting qualities.

93. Scientific Combination of Concrete and Steel.—The next point to consider is whether the materials can be employed to better advantage than they are when the central core consists of steel and the outer shell is composed of concrete.

This question is soon answered, for the laws governing the design of columns tell us that metal is of far less value in the centre than at the outside of such members. As a hollow cylinder undoubtedly possesses the best and most economical section, the proper place for the steel would be outside the concrete if nothing but strength had to be considered. But we have also to insure safety from corrosion and fire, and this may be done by adding an outer casing of concrete.

Thus we arrive at a form of column in which there are three concentric parts, (1) a central core of concrete, (2) a middle zone of steel, and (3) an outer zone of concrete.

The steel is applied so that its maximum strength may be developed, while the concrete protects the inner and outer

[1] *Proc. Am. Soc. C.E.*, vol. xxix., No. 3.

surfaces of the metal, and contributes to the resistance of the column.

In numberless cases the hollow interiors of cast-iron and steel columns have been filled with concrete, sometimes with and at others without an exterior covering, and it is quite a common practice to employ foundations for bridges and other structures, where concrete is filled into steel cylinders.

But we believe that engineers rarely calculate upon the strength so gained, and it is not generally known that the additional resistance actually is very great, or that the concrete used in this way acquires new qualities of truly surprising character.

Engineering text-books have nothing to say upon these points, which belong to the new and comparatively unknown domain of concrete-steel.

94. The Effect of Tubular Reinforcement.—The internal forces acting in solid bodies may be placed in two categories. Cohesion is the force by which the molecules are held together, this force varying in proportion to the distance of the molecules from each other up to the elastic limit. Friction is the resistance to the relative motion of bodies, and it exerts an action in the interior of bodies similar to that exerted on their surface.

Concrete is a body in which both cohesion and friction are evidenced, and these forces unite to resist compression.

Sand may be taken as an illustration of a cohesionless body, in which resistance to compression is afforded by friction alone.

When sand is filled into a tube and subjected to pressure by a piston accurately fitting the inside of the tube, friction between the particles resists compression to a certain degree, but as the height of the sand column is decreased there is a tendency to bulge outwards, causing pressure on the interior of the tube. As this pressure is resisted by the surrounding metal, the longitudinal shortening of the column is prevented to a very considerable extent, and the sand is effectively reinforced.

When concrete is placed in a tube and subjected to axial pressure, the resistance to longitudinal shortening is increased by molecular cohesion, and proportionately less pressure is exerted laterally on the metal. Consequently, the thickness of the latter can be much less than that required for the reinforcement of a sand column.

Assuming the coefficients of friction to be of equal value, the difference between the resistance of the sand and of the concrete indicates the effect due to cohesion, and the difference in question may be termed the specific resistance of concrete.

The efficiency of a tube for reinforcing sand or concrete in the manner suggested, depends largely upon the fact that the lateral bulging at any cross section is small as compared with the total longitudinal shortening. Consequently the intensity of tensile stress in the metal is far less than the intensity of the stresses that would result from the application to the metal of axial pressure, equal in amount to the pressure applied at the top of the sand or concrete.

The steel is applied in a form that permits its tensile strength to be utilised in the most advantageous manner, and the concrete, prevented from extending laterally, is most materially assisted in withstanding axial pressure, a duty for which it is well fitted by its comparatively great compressive strength.

The foregoing argument leads naturally to the conclusion that exceptionally favourable results may be realised by confining, or, in other words, by reinforcing the concrete so as to minimise lateral bulging. This applies to the design of concrete-steel piles, as well as of concrete-steel columns.

So far as we are aware, the only investigation bearing upon this treatment of concrete is one conducted by M. Considère, and reported by him in two communications to *L'Académie des Sciences*. As the scope of the *Comte Rendus* did not permit a fully detailed account of the entire investigation to be given, the information therein contained has been supplemented by articles published simultaneously in two technical journals.[1]

One remarkable experiment made by M. Considère was the following: A metal tube, $7\frac{1}{2}$ in. diameter, was filled with concrete, and then bent, so that the radius of curvature of the central line was 21.6 in. After this, the tube was cut and removed.

[1] *Le Génie Civil*, 1902, Paris: *Beton & Eisen*, Vienna, 1902-3.

Notwithstanding the considerable deformation undergone by the concrete, it was not broken, only a few cracks were found on the compressed side, and subsequent tests showed that it still possessed very considerable strength.

This test does not throw light upon the compressive resistance of concrete reinforced by a surrounding case of metal, but it is sufficient to reveal the very unexpected fact that a material usually regarded as extremely brittle can be made to acquire considerable ductility.

95. Circumferential v. Longitudinal Reinforcement.— The relative effect of different methods of reinforcement for columns may be made clear by considering a cylinder of simple concrete not long enough to develop flexure when

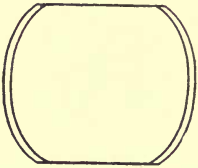


FIG. 66.

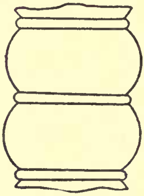


FIG. 67.

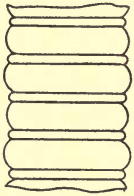


FIG. 68.

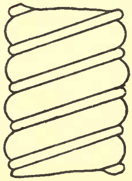


FIG. 69.

under compression. The application of pressure will result in longitudinal shortening and lateral bulging.

Let us assume that to such a cylinder four thin steel rods be applied for longitudinal reinforcement. On the application of pressure to concrete and steel alike, the concrete will be compressed at the top and swelled out to a barrel-like shape, as in Fig. 66.

The steel will bend outward owing to the lateral thrust exerted by the concrete, and also in consequence of the load imposed, for while the concrete cylinder does not bend like a column, the steel does so bend, owing to the small diameter of the rods in proportion to their length.

These rods give very little support to the concrete, and they do not to any appreciable extent help to restrain it from extending laterally. This is an extreme case, and the conditions are purposely exaggerated.

Now let us assume a cylinder of concrete armoured with welded steel hoops or rings, as in Fig. 67, which shows part of the cylinder. Here the concrete is prevented from bulging except between the hoops, and a certain amount of assistance is thereby given.

In Fig. 68, the hoops are closer together, and there is a still further reduction in the amount of lateral swelling, with a proportionate increase of resistance.

In Fig. 69 spiral coils are applied instead of detached hoops, and the effect is naturally more continuous throughout the length of the column.

These illustrations are purely diagrammatic, and are merely intended to suggest in a general way the influence of reinforcement applied in the modes stated.

It is clear that by hooping or completely surrounding the concrete with steel, considerably higher resistance to compression can be obtained than by simple longitudinal reinforcement. But between these two types of reinforcement many intermediate varieties or combinations are possible.

Engineers rarely make use of longitudinal rods alone, and columns are almost always designed on lines generally similar to those adopted in the construction of concrete-steel piles, of which Figs. 60-63 (pages 173 and 177) may be taken as fairly representative examples. The longitudinal rods are tied together by a more or less complex system of transverse rods or wires, and, in addition to keeping the vertical reinforcement in position, these transverse ties are of some assistance in preventing flexure of the metal, and in retarding the lateral swelling or bulging of the concrete.

Reinforcing wires, bars, and plates are generally applied in this way by designers of concrete-steel columns and other members intended to withstand compression, but the value of the transverse reinforcement varies with its disposition, as well as with its spacing measured along the axis of the column.

As a general rule it may be taken that in concrete-steel columns, reinforced by longitudinal rods and transverse ties placed too far apart to afford adequate resistance to lateral swelling of the concrete, the resistance to direct compression is little more than the sum of the resistance offered by the concrete and the steel.

It has been generally acknowledged hitherto that test specimens of stone, concrete, and similar materials, subjected to direct compression, always fail by shearing along planes inclined to the direction of the stress.

Recent investigations conducted in Germany by M. Foeppel, and in France by M. Mesnager, seem to prove, however, that this particular mode of failure is due to friction between the planes of the test specimens and the plates transmitting pressure. And by the introduction of a greased surface for reducing friction to a sufficient extent, it was found by these experimentalists that failure took place along planes parallel to the direction of the stress. If we accept the accuracy of this result an additional reason is furnished for the comparatively small value of longitudinal reinforcement, for rods placed so as to be parallel to the planes of rupture cannot have much influence in helping the concrete to hold together.

Further, whether concrete in compression fails along planes inclined to, or parallel with, the direction of the force, it is perfectly clear that circumferential reinforcement must be beneficial. The tendency to slide along oblique planes would be resisted by any bars intersecting such planes, and rupture along planes parallel to the direction of the force would be most effectively resisted by circumferential reinforcement.

96. Internal Stresses due to Shrinkage. The precise influence of longitudinal reinforcing rods is somewhat difficult to define owing to the complicated nature of the stresses involved. It is known that the shrinkage of concrete when setting in air causes internal stresses of great intensity in concrete-steel.

Experiments made in 1902 at L'Ecole des Ponts et Chaussées served to throw valuable light upon the effects resulting from shrinkage in large specimens of concrete-steel in which the concrete was mixed in the proportions of 1 : 3 : 6.

In the case of specimens, 6 ft. 6 in. long by 4 in. square, each reinforced by four $\frac{1}{4}$ in. diameter rods near the surface, measurements showed that after three months the shrinkage of the concrete had caused a compressive stress in the steel equal to 6,540 lb. per square inch.

Similarly, in specimens, 13 ft. long by 8 in. by 16 in., each reinforced by four $\frac{7}{8}$ in. bars about 1.3 in. from the surface, the compressive stress in the steel varied from 10,800 lb. to 14,220 lb. per square inch.

No reflection is required to show the extreme importance of internal stresses such as these in the reinforcement of concrete-steel columns.

Internal compressive stress is of less importance in the steel reinforcement of beams, because the bars are generally situated in the tension area, and the effect of loading is first to relieve the compression due to shrinkage of the concrete and second to establish tensile stress, to resist which is the special function of the reinforcement. Hence in beams, the initial stress may be an advantage; at any rate so far as the steel is concerned.

But in the case of columns the conditions are different, for the compression due to loading is a more important factor for consideration than tension caused by flexure. Unless this point is thoroughly recognised by designers, serious undercalculation of strength must inevitably follow.

For example, let us assume a quality of concrete that is able to withstand without failure a reduction of length equal to 0.1 per cent. Compression to this extent in a concrete-steel specimen will be sufficient to account for a stress in the reinforcement of 29,000 lb. per square inch if the coefficient of elasticity be 29,000,000 lb.

Taking the initial internal stress at 14,000 lb. per square inch, we have a total compressive stress of 43,000 lb. per square inch, which is perilously near, or even above, the elastic limit of ordinary structural steel.

Hence, before failure of the concrete, the reinforcement would be practically stressed up to its elastic limit, beyond which no addition would be permissible in practice.

Of course, the figures here used do not apply to all qualities of material, and the internal stress due to air-hardened concrete may not be so high as 14,000 lb., but the fact remains that very considerable compressive stress exists in concrete-steel, set in air, which should never be overlooked by the designer. [See also Article 100.]

97. Hooped Concrete.—In consequence of the remarkable results obtained from the experiments mentioned in

Article 94, it was decided to make similar tests upon concrete reinforced with a spiral winding of iron wire, a type of reinforcement termed "hooping" by M. Considère.

Accordingly, a specimen was prepared, consisting of a concrete prism, 4 ft. 3 in. long, reinforced with longitudinal rods and spiral hooping, the concrete being mixed in the proportion of 840 lb. of Portland cement to 1 cubic yard of gravel and sand. The prism was subjected to a pressure of 7,940 lb. per square inch of original section, and was bent into the shape of a letter S, with a greatest versed sine of 0.4 in. in a length of 13 in. Although the prism was so much bent that the radius of curvature was 24 in., the outer surfaces of the concrete showed no transverse cracks.

Hence the deformation which occurred was not due so much to extension of the outer fibres as to compression of the inner fibres, which on calculation was found to be as much as 17 per cent.

Upon removal of the reinforcement the concrete was sufficiently cohesive to permit of its being handled without breaking.

It was then placed on two supports about 3 ft. 7 in. apart, and required a weight of 55 lb. to break it by bending.

One half of the prism which was less bent than the other, but had been subjected to the pressure of 7,940 lb. per square inch, was next placed on two supports 20.5 in. apart, and being loaded did not break across until a weight of 428 lb. had been applied.

The diameter of the concrete, after the reinforcement had been taken off, was about 4.75 in., and the tensile resistance, judged by the last-mentioned test, was calculated at 205 lb. per square inch, or very little below the original tensile strength of the concrete.

A second test was made with greater precautions on a hooped specimen, mixed in the proportion of 630 lb. of Portland cement to 1 cubic yard of gravel and sand, and this withstood a pressure of 10,270 lb. per square inch, with a longitudinal shortening which averaged 2.4 per cent., and did not exceed 2.8 per cent. in any part.

After removal of the reinforcement, the concrete core sustained a pressure of 9,700 lb, or, as the sectional area was about 10.5 in., nearly 924 lb. per square inch.

Compression tests made on specimens of hooped concrete, short enough to be loaded without causing flexure, showed longitudinal shortening of about 3 per cent. before failure.

Some other hooped specimens were tested under compression. The first, made of concrete mixed in the proportions stated above, bent under a pressure of 6,970 lb. per square inch, and was shortened 0.6 per cent.

After removal of the reinforcement, the average compressive resistance of the plain concrete was 1,420 lb. per square inch, but its real resistance was considerably more, as the pressure was not applied quite accurately.

The foregoing examples are selected from a large number of similar tests, and serve to demonstrate the remarkable qualities acquired by concrete when bound with spiral coils of wire.

Table XXIX. gives the results obtained by M. Considère in 1901 from specimens of cement mortar, 1.6 in. in diameter, reinforced by a binding of fine iron wire and without longitudinal reinforcement. The mortar was mixed in the proportions of 675 lb. of cement per cubic yard of sand, with the exception of one prism, in which the proportion of cement was 730 lb. per cubic yard. The iron wire was drawn cold and had no definite elastic limit, but the stress of 78,200 lb. per square inch corresponded with what was virtually the yield point.

The compressive strength of the specimen in the last column is very remarkable, and it is instructive to compare the resistance with that of an iron prism of equal weight. The specific gravity of the mortar was 2.4, and as the average specific gravity for iron is 7.65, the ratio of the densities is about 3.2. If we take the compressive strength of ordinary wrought iron at 44,800 lb. per square inch and divide by 3.2 we have 14,000 lb. as the resistance per square inch for comparison with an equal weight of concrete. But as the average resistance of a riveted iron section is not more than 75 per cent. of a solid section, the compressive resistance of iron-hooped cement mortar comes out at the same value as that of an equal weight of iron. It is true that the strength of the iron wire used in the experiments quoted was higher than 78,200 lb. per square inch; but

Table XXIX.—Properties of Hooped Cement Mortar (Considère).

	(1)	(2)	(3)	(4)	(5)
Cement per cu. yd. of sand, lb.,	675	675	675	675	730
Age of mortar, days,	8	14	22	23	100
Percentage of reinforcement in cross section,	2	3	4	2	3.4
Crushing strength of total section, lb. per sq. in.,	4,870	6,540	7,360	4,930	10,500
Crushing strength of mortar only, lb. per sq. in.,	569	711	853	853	2,420
Increased strength due to hooping,	4,301	5,829	6,507	4,077	8,080
Calculated resistance of iron as longitudinal reinforcement,	1,564	2,346	3,128	1,564	2,658
Value of hooping (longitudinal reinforcement = 1),	2.7	2.5	2.1	2.6	3.0

even allowing for this high resistance, the results are sufficiently remarkable.

Advantageous as hooping is shown to be with regard to crushing resistance, it is less serviceable than longitudinal reinforcement for resisting flexure, and further experiments were made upon specimens provided with both types of reinforcement. With the co-operation of M. Hennebique, of Paris, a number of prisms were formed, of octagonal section 5.9 in diameter, and of two lengths—1.64 ft. for testing resistance to crushing and 4.25 ft. for studying the elastic and ductile properties of the combination. The proportions of the reinforcement were varied, the concrete consisting of 1,000 lb. of Portland cement to 45 cubic feet of gravel and 22.5 cubic feet of sand in some specimens, and in others of 2,000 lb. cement to the quantities of gravel and sand before stated.

As fully 1,200 observations were taken, it is impossible to refer to them in detail, but a few remarks on the more important results may be useful. The following notes apply to one set of six specimens:

No. 1.—Formed of simple concrete,[1] and crushed under a load of 1,050 lb. per square inch.

No. 2.—Reinforced with helicoidal spirals of $\frac{1}{4}$ in. diam. iron wire, the coils spaced 1.18 in. centres.[1] This failed by crushing under a load of 5,120 lb. per square inch of total section.

No. 3.—Reinforced with helicoidal spirals of 0.17 in. diam. iron wire, spaced 0.59 in. centres.[1] This was not crushed by a pressure of 5,400 lb. per square inch. The testing machine was not suitable for greater pressure.

No 4.—Same as No. 2, with the addition of eight longitudinal $\frac{1}{4}$ in. rods inside the spiral coils.[2] This failed as a column under a load of 4,550 lb. per square inch.

No. 5.—Same as No. 3, with the addition of longitudinal rods, as in No. 4.[2] Failed as a column under a load of 4,700 lb. per square inch.

No. 6.—Reinforced with eight longitudinal rods 0.35 in. diam., and transverse ties of 0.17 in. diam. wire spaced 3.15 in. centres.[2] Failure took place at a load of 2,420 lb. per square inch.

[1] *Length*, 1.64 *ft.* [2] *Length*, 4.25 *ft.*

The result given by No. 6 is conclusive as to the inferiority of ordinary transverse reinforcement to spiral hooping. Some of the observations made during these tests are very instructive. No. 1, of simple concrete, ruptured suddenly without any premonitory sign, and No. 6 failed almost as suddenly, its breaking load being only about 7 per cent. more than the load causing the first cracks perceptible. The longitudinal rods bent outwards between the transverse ties and the concrete failed by crushing. These results show that simple concrete, and concrete reinforced by longitudinal rods, must be regarded as materials which fail suddenly with little accompanying deformation.

Nos. 2, 3, 4, and 5 behaved very differently. At the commencement of testing they showed, as did Nos. 1 and 6, very little deformation under small pressures, but the end of this elastic condition was not immediately followed by failure. Shortening was seen to increase rapidly, and cracks appeared in the concrete covering the spirals, the cracks growing gradually wider. Approximate measurements indicated that deformations of as much as 3 per cent of the length took place before failure.

From these notes it is clear that hooped concrete is able to sustain without crushing much heavier loads than simple concrete, or concrete reinforced with longitudinal rods and transverse ties, and, further, that it does not fail for some time after cracks have appeared in the outer surface, and considerable deformation has been evidenced. Hence this type of concrete-steel gives ample warning of approaching failure—a most valuable characteristic of any material used in structural work.

98. Spacing of Spirals for Hooped Concrete.—For guidance as to the spacing of the spirals, the following observations, from the results stated in the preceding Article, should be of service. While specimen No. 2 was under test, cracks began to appear under the comparatively small pressure of 1,730 lb. per square inch, a little later the concrete commenced to flake off, and failure ultimately took place between the coils. Failure was due to rupture of the concrete, there being no indication that the metal had reached its elastic limit. Specimen No. 3 showed no cracks until the pressure of 2,480 lb. per square inch had been attained, and flaking occurred much later; while, as recorded above, the prism did not fail under the maximum load imposed. In Nos. 4 and 5 cracks appeared under pressures of 2,900 lb. and 3,360 lb. per square inch, the higher figures being no doubt due to the addition of longitudinal reinforcement.

Considering these facts in connexion with the spacing and other details of the reinforcement, and with observations made during other series of experiments, M. Considère arrived at the conclusion that the most suitable spacing of the coils when auxiliary longitudinal rods are employed is from one-seventh to one-tenth of the diameter.

Numerous and varied experiments have demonstrated the fact that this ratio holds good almost independently of the absolute values of the dimensions.

99. Effects of Repeated Loading on Hooped Concrete.—We will now deal with further experimental data obtained by M. Considère, throwing valuable light upon the behaviour of concrete-steel columns with spiral reinforcement.

For this purpose we take the results afforded by the testing of three specimens. In all these the concrete was in the proportions of 840 lb. of cement per cubic yard of aggregate, the form of cross-section was octagonal, the diameter 5.9 in., the length 4.27 ft., and the reinforcement consisted of longitudinal rods and helicoidal spirals. Further details of the reinforcement are given below in tabular form.

Specimen.	Spirals.		Longitudinals.	
	Diameter.	Spacing.	Diameter.	No.
7	0.25 in.	0.79 in.	0.3125 in.	8
8	0.25 in.	0.79 in.	0.3125 in.	8
9	0.25 in.	0.79 in.	0.2760 in.	20

In the conduct of the tests the specimens were first loaded up to a certain point, pressure was then relaxed, and the specimens were again loaded.

Table XXX. shows the loads in pounds per square inch, and the consequent shortening in decimals of an inch.

As the spirals in these specimens were of the same thickness and the same spacing, and the concrete was mixed in the same proportions, it would be only reasonable to expect very similar results, especially in the case of specimens 7 and 8, where the longitudinal rods were equal both in diameter and in number.

Calculations made from actual measurements indicated that in the case of specimen No. 7 the coefficient of elasticity for pressures below 2,845 lb. per square inch was 7,111,000 lb., while in the case of No. 8 the coefficient was only 2,845,000 lb.

This remarkable difference appears to have been due to the amounts of water used in preparing the concrete for the two specimens. The quantity of water used in mixing the first batch was correct, but for the second batch an excessive quantity was added.

From a general study of the deformation curves of these and similar specimens of concrete-steel, it is evident that a marked change of inclination takes place beyond a certain stage of the loading, and the point corresponding with this change may be regarded as indicating the elastic limit, although, strictly speaking, concrete has no true elastic limit.

Specimens of similar concrete appear to show far less irregularity with regard to the elastic limit than to the coefficient of elasticity.

Thus the elastic limit of specimens 7, 8, and 9 was not found to vary to a greater extent than between 4,830 lb. to 5,400 lb. per square inch, while, as already stated, the coefficients of elasticity for Nos. 7 and 8 were widely divergent. Further, the elastic limit depends very much upon the proportion of cement in the concrete, a factor which has little influence on the coefficient of elasticity.

Other observations of considerable practical utility are to be made from examination of the results afforded by the same tests.

In the first place, permanent deformation is caused by the first loading, as might be anticipated, and the deformation increases with the second loading, but in smaller and

Table XXX.—Effect of a Repeated Loading on Hooped Concrete (Considère).

First Loading and Unloading.				Second Loading.			
Load. lb. per sq. in.	Shortening, inches.			Load. lb. per sq. in.	Shortening, inches.		
	No. 7.	No. 8.	No. 9.		No. 7.	No. 8.	No. 9.
1,053	.0047	.0173	.0118	1,850	—	.0540	.0335
1,850	.0095	.0276	.0205	3,300	.0449	.0642	.0406
2,375	.0130	.0350	.0264	4,490	.0540	.0745	.0540
3,330	.0158	.0437	.0350	4,890	.0645	.0772	.0583
3,825	.0260	.0567	.0437	5,150	.0662	.0934	.0630
4,490	.0480	.0709	.0449	5,420	.0685	.0945	.0760
3,825	.0480	.0658	.0445	5,600	.0827	—	.0875
3,300	.0378	.0615	.0400	6,210	.1356	—	—
2,375	—	—	.0354	6,740	.1940	—	—
1,850	—	.0496	.0335				
1,053	—	.0480	—				
0	.0146	.0248	.0102				

decreasing measure. Consequently an appreciable increase in the value of the coefficient of elasticity takes place with repetitions of the load.

In the second place, it appears that the coefficient of elasticity increases with the pressure in the reloading instead of decreasing with increasing pressure as it does during the application of the first loading.

The latter point is of much practical importance to the designer of columns. In actual practice, columns are of such length in proportion to their transverse dimensions that the stresses due to flexure are the main points for consideration. Hence it is an unfavourable circumstance that in concrete, steel, and other materials of construction, the coefficient of elasticity should decrease with the increase of the load. This condition applies equally to concrete-steel, however reinforced, under the first loading; but, as we have seen, it does not apply to hooped concrete which has already been subjected to a first loading, providing the second load does not exceed the first in value. So far as we are aware, the fact has never been observed in connexion with any other variety of material.

The phenomenon is of such importance that we think it well to give some results obtained by M. Considère at the laboratory of the Ponts et Chaussées.

The tests were conducted upon a specimen in which the concrete was mixed in the proportion of 1,000 lb. of cement to 32 cubic feet of gravel, varying from 0.2 in. to 1 in. diameter, and 10.7 cubic feet of sand passed through a sieve with 0.2 in. holes. The form of cross-section was octagonal, the diameter 4.3 in., the length 51.18 in., and the reinforcement consisted of eight longitudinal rods 0.17 in. diameter, and helicoidal spirals of 0.17 in. diameter wire spaced 0.82 in. centres, the average diameter of the coils being 3.75 in.

In Table XXXI. will be found the shortenings consequent upon three repetitions of load.

The concrete surrounding the spiral reinforcement was removed before the experiments were commenced because difficulty was anticipated in determining the precise value of the effects produced thereby. Consequently the net diameter of concrete was 3.41 in. As will be seen from

Table XXXI.—Effect of Several Repeated Loadings on Hooped Concrete (Considère).

First Loading and Unloading.		Second Loading and Unloading.		Third Loading and Unloading.		Fourth Loading and Unloading.	
Load. lb. per sq. in.	Shortening. inches.	Load. lb. per sq. in.	Shortening. inches.	Load. lb. per sq. in.	Shortening. inches.	Load. lb. per sq. in.	Shortening. inches.
128	.0	1,180	.0504	1,620	.2790	1,620	.553
441	.0047	1,620	.0544	3,170	.3105	4,720	.614
810	.0142	1,990	.0615	4,720	.3400	6,340	.638
1,180	.0205	2,360	.0693	5,530	.3580	7,100	.658
1,620	.0299	3,170	.1088	6,340	.3980	7,525	.669
1,990	.0457	3,990	.1560	7,525	.6590	7,910	.684
2,360	.0630	4,720	.2240	7,910	.6600	8,710	.768
1,620	.0606	5,530	.3600	7,525	.6575	10,290	—
1,180	.0599	5,160	.3590	7,100	.6580	9,280	.950
810	.0528	4,720	.3386	6,340	.6540	7,910	.950
441	.0394	3,170	.3245	4,720	.6410	6,340	.934
128	.0299	1,620	.2980	1,620	.5830	128	.795
—	—	128	.2410	128	.5480	—	—

Table XXXI., the first load attained the maximum value of 2,360 lb. per square inch, and the three succeeding loads amounted to 5,530 lb., 7,910 lb., and 10,290 lb. per square inch. The shortening was measured, with one exception, under each pressure during the loading and unloading, the only exception being in the case of the 10,290 lb. load, when flexure prevented the measurement from being taken. The coefficients of elasticity corresponding with the various pressures were carefully calculated, and were found to confirm the conclusions previously stated.

Even after the pressure of 10,290 lb. per square inch had been imposed and taken off, it was ascertained that the test specimen possessed a coefficent of elasticity equal in value to that determined after the imposition of the lightest loads. It was found, further, that the imposition of a first load had the effect of improving the concrete proportionately to its original deficiency of resistance.

Summing up the conclusions drawn from the foregoing and similar experiments, we find:

1. That the imposition of a first load on a specimen of hooped concrete, however high may be the value of the load, has the effect of raising the elastic limit up to the value of that load.

2. That the coefficient of elasticity developed by hooped concrete under all variations of load between the lowest and the previously applied load is higher than the highest coefficient of elasticity possessed by the specimen before the first loading, and which even then corresponded only with low pressures.

3. That the increase in the value of the coefficient of elasticity after the first loading is inversely proportionate to the richness and original strength of the concrete. That is to say, the increase of value is greater for poor and weak concrete than for rich and strong concrete.

The properties observed by M. Considère resemble those exhibited by iron and steel in respect of increase in the value of the elastic limit, but they are quite novel in so far as they relate to the increasing values of the coefficient of elasticity under loads of smaller value than the first load imposed.

It should be particularly noted that in this respect hooped concrete behaves quite differently from reinforced concrete in tension, as in the latter case the coefficient of elasticity decreases with deformation, and the greater the deformation the greater is the decrease in the value of the coefficient.

100. Laws governing the Elasticity and Resistance of Hooped Concrete.—Some further experiments were conducted with the object of throwing light upon the laws governing the elasticity and resistance of hooped concrete.

Two methods were adopted for the purpose of ascertaining experimentally the effects of hooping.

The first was to prepare two specimens as nearly identical as possible, and to correct for differences.

The second was to test a specimen with its spirals and afterwards without them.

In applying the first method, two specimens were carefully made of the same proportions and dimensions. These were first tested to ascertain the differences in their elastic behaviour so as to afford data for correction of the results obtained. The exterior sheathing of concrete was then removed from each specimen, and further tests were made, one upon a specimen so prepared and one upon the other specimen after removal of the spiral coils.

In applying the second method three specimens were made, and each of them was subjected to seven successive loadings and unloadings. Afterwards the concrete core, stripped of its spirals, was tested in a similar manner, and, making due allowances for the different conditions, the results obtained by the first method were practically confirmed. After comparison and correction of the results, it was found (1) that the increase in the value of the coefficient of elasticity due to hooping was equal, for a first loading, to 90 per cent. of the coefficient of elasticity of longitudinal rods having the same weight of metal, and (2) that the increase in the value of the coefficient of elasticity due to hooping was equal, for a subsequent loading or subsequent loadings, to nearly double the value of a similar weight of metal applied in the form of longitudinal reinforcement.

It was observed, further, that the hooping did not produce its normal effect on the elastic behaviour of the concrete until the load approached the value of about 220 lb. per square inch, and that of the total deformation due to the first loading a permanent bulging of the concrete was left between the coils after unloading. This was sufficient to cause an effective contact between the concrete and the spirals, so that the latter were able to exercise their full effect during subsequent loadings.

These facts appear to account for an important difference between the influence of spiral and longitudinal reinforcement.

We have previously stated (see Art. 96) that in air-hardened concrete the longitudinals are compressed by shrinking of the concrete, and consequently that the elastic limit of the metal is soon reached. In the case of steel this point is reached when the load has caused a longitudinal shortening equal to 0.06 per cent. of the length of the test specimen. On the other hand, the hoops compressed by the setting of the concrete must first be relieved of compression before they can be subjected to tension, and tensile stress does not reach a critical value until longitudinal shortening has passed beyond 0.06 per cent. of the length.

Hence we come to the conclusion that the spirals only begin to suffer serious stress under the first loading of specimens hardened in air after the longitudinals have passed their elastic limit, and have almost reached their ultimate strength.

Extension of the metal forming the spirals results simply from lateral bulging of the concrete. This is comparatively small, being only about 0.3 to 0.4 of the longitudinal shortening. Notwithstanding the comparatively slow development of stress in the spirals, these produce most important effects, which account for the exceedingly great deformation that may be caused in hooped concrete without injury to the reinforcement or to the concrete itself.

In concrete-steel hardened in water the concrete expands, causing tension in the hoops and in the longitudinals. Hence the former will at once begin to take tension on the application of load, and the latter will first be relieved of tension before suffering compressive stress.

These considerations suggest that advantage may be derived from keeping hooped members in water or moist air for a sufficient period of time before exposing them. We have seen that the efficiency of the hoops is most marked after a preliminary loading, and it is probable that a similarly beneficial result might be obtained by hardening the concrete either in water or in a humid atmosphere.

The last experiments to which we will refer had for their object the determination of the elasticity of the concrete in hooped members. To effect this, the necessary measurements were made upon specimens when hooped and upon the same specimens after the hoops had been removed.

One fact evidenced by these experiments is that the compressive resistance of concrete which has been hooped, after having reached a certain maximum value, remains constant within wide limits, notwithstanding the increase of deformation. A similar phenomenon, it may be mentioned, is also exhibited by concrete-steel in tension and subject to elongation exceeding the elastic limit.

Another point worthy of notice in connexion with the same experiments is that while the resistance of concrete cores, tested after the hooping had been removed, reached the value of 1,565 lb. per square inch, the same quality of concrete which had not been hooped gave a resistance varying from 832 lb. to 1,170 lb. per square inch.

Hence we may conclude that preliminary loading in conjunction with spiral reinforcement has the effect of increasing the resistance of the concrete by 50 per cent. or more.

101. Basis of Computations for Hooped Concrete Columns.—From the foregoing discussion the following assumptions may be derived as the basis of computations relating to the practical strength of hooped concrete columns:

1. For a first loading, the coefficient of elasticity of a hooped concrete column is equal to the sum of the coefficients of elasticity of the concrete, of the longitudinal reinforcement, and of imaginary longitudinal reinforcement with a volume equal to 90 per cent. of the actual spiral reinforcement.

2. For a second loading, in which the pressure is less than the maximum pressure of the first loading, the coefficient of elasticity of a hooped concrete column is equal to the sum of the coefficients of elasticity of the concrete as increased by the first loading of the longitudinal reinforcement, and of imaginary longitudinal reinforcement

with a volume equal to double that of the actual spiral reinforcement.

3. For a first loading, the elastic limit of a hooped concrete column is equal to the original elastic limit of the concrete plus the resistance of the reinforcement, computed on the basis of a shortening of from 0.08 per cent. to 0.13 per cent., this shortening representing the range of the practical elastic limit of hooped concrete.

4. For subsequent loadings, the value of the elastic limit is not an important consideration, for we have already seen that it can be raised at will, even to the ultimate compressive strength of the concrete.

5. The compressive resistance of a hooped concrete column is fully equal to the sum of the original resistance of the concrete at the elastic limit of the longitudinal reinforcement, and of the resistance at the elastic limit of imaginary longitudinal reinforcement with a volume equal to 2.4 times that of the actual spiral reinforcement.

102. Column Formulæ.—It is well known that in calculating the strength and proportions of practical columns their resistance to flexure must be taken into account, as well as their resistance to direct compression.

Any reliable column formula may be used for determining the ratio of length to transverse dimensions necessary for ensuring adequate strength and stability. Practically, all trustworthy column formulæ are based more or less directly upon Euler's well-known equation for long columns, but these formalæ are so disguised and so qualified by empirical coefficients that at first sight it is sometimes difficult to recognise their common origin and to account for the extremely variable results they give.

One of the most rational column theories is that evolved by Mr. J. Mitchell Moncrieff, M.Inst.C.E., and this, together with its resulting formula, has been fully discussed by the author in another treatise.[1] There are two general forms of the Moncrieff formula—one applying to the failure of round-ended and fixed-ended columns by compression, and the other to the failure of flat-ended columns by incipient tension. In ordinary constructional work it is extremely doubtful whether the condition of fixity is ever

[1] *Structural Iron and Steel*, p. 150. London: Whittaker & Co., 1900.

attained in a measure sufficient to justify the application of the formula for columns with fixed ends. In concrete-steel construction, however, the condition of fixity is comparatively easy of attainment, and should be attained in most cases, as a concrete-steel building, when properly designed, is a monolithic structure. Hence, under these conditions, the formula for fixed ends may be used with safety.

This consideration in itself points to one reason for the relatively great strength of concrete-steel columns, and for the comparatively small proportions that are necessary.

M. Considère, in communications to the Congress on Methods of Construction, 1889, and to the Commission on Methods of Testing, 1892, showed that the Euler's formula was correct for iron columns of any dimensions if the coefficient of elasticity were taken at the value possessed by a column under flexure, and not at the value corresponding with light loads. Such a variation of Euler's rule will not directly give the resistance (which we will here denote by the symbol R) of a column, because the value of the coefficient of elasticity E is itself a function of the unknown quantity R. The formula can be used, however, to determine the usual ratio $\frac{r}{l}$, as stated by M. Considère in the form—

$$\frac{r}{l}=\frac{1}{\pi}\sqrt{\frac{R}{E}}.$$

It then gives the value of the ratio $\frac{r}{l}$ (where l = length and r = radius of gyration), that will ensure the resistance R of the column.

In applying this formula any value for R can be introduced, with the corresponding value for E, and by a succession of calculations a table or diagram can be constructed for general reference.

This variation of Euler's formula is based on the assumption that a loaded column has an indefinitely small curvature, and, further, that it is in equilibrium under the action of an axial load. This implies equilibrium between the moment of resistance and the bending moment.

The moment of resistance consists of two component parts, one represented by the increase of pressure on one side of the axis, and one by the decrease of pressure on the other side of the axis, such decrease being due to the relief afforded by flexure.

For one component part the value of the coefficient of elasticity of the column is that of the material under a first loading, and for the other part the value is that for a decreasing load. These two values of E are practically equal under light loads, but they vary in proportion to the increase of load, and when the elastic limit is passed the divergence is very considerable.

M. Considère gives a rule by which, when the ratio of the two different values for E is known, the correct proportional value may be calculated for insertion in his variation of Euler's formula, and when the value of this ratio is near to unity he says that the average value of the coefficients may be introduced. It is quite evident that the use of his formula involves more information as to the properties of hooped concrete than is generally available at the present time, and for this reason no useful purpose would be served by giving numerical examples. We may add however, that after discussing his method of calculation, M. Considère considers the question of the factor of safety to be used for hooped concrete columns, coming to the conclusion that a factor of from 3 to 3.5 will be found ample.

From examples worked out according to the formula, we find that for a first loading a factor between these limits would involve stresses of about 1,000 lb. per square inch on the concrete in hooped columns, and that for subsequent loadings a similar factor would involve stresses of about 2,000 lb. per square inch on the concrete in hooped columns previously tested at the place of manufacture.

These figures are high, but they seem to be justified by the remarkable results obtained in the course of the investigation to which we have already referred at some length.

Even if engineers entertain the opinion that it is not prudent to apply the niceties of M. Considère's rule in the course of ordinary practice until further data are available, the results already obtained by M. Considère are sufficiently clear as an indication that considerably higher values may be set on some of the factors employed in formulæ of the usual kind.

So far as concerns the calculation of concrete-steel columns with the ordinary longitudinal reinforcement and transverse ties, the maximum compressive strength to be taken into account is represented by the resistance of the concrete plus the resistance of the reinforcement, and ordinary column rules may be employed for computing the requisite proportions.

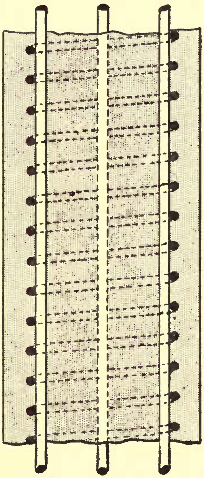


FIG. 70.

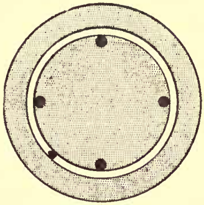


FIG. 71.

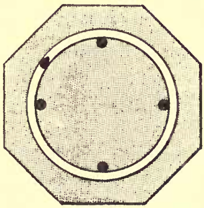


FIG. 72.

103. Practical Columns of Hooped Concrete.—Subsequent to the conduct of the experiments mentioned in the preceding articles, M. Considère has applied his system to the construction of practical columns, and quite recently has made arrangements for its introduction into this country. Fig. 70 is a vertical section showing the arrangement of the reinforcement in a "Considère" column, and Figs. 71 and 72 are typical cross sections. It is claimed by M. Considère that the manufacture of hooped concrete columns is cheaper and requires less skilled labour than the systems at present in use, and that the permissible unit stress for hooped concrete may be taken at 2,000 lb. per sq. in., as against the usual limit of about 500 lb. per sq. in. for concrete in compression.

104. Building Regulations for Concrete-Steel Columns.— The building rules of some German cities provide for the proportions of concrete-steel columns, which are assumed generally to be constructed with longitudinal rods and transverse ties as in the Hennebique and other systems.

For instance, the Berlin code says that to prevent the buckling of Hennebique columns the concrete must have dimensions sufficient to guarantee the avoidance of dangerous stresses. The columns must be calculated with due regard to flexure as well as to direct compression, and the following equations are to be used:

For axial loads - - - - $I = 60Pl^2$.

For slightly eccentric loads - - $I = 100Pl^2$.

Where I = the least moment of inertia of the column section, P = the load, and l = the unsupported length.

Similar rules are in force at Dusseldorf and Hamburg, but in Frankfort the employment of concrete-steel columns is interdicted.

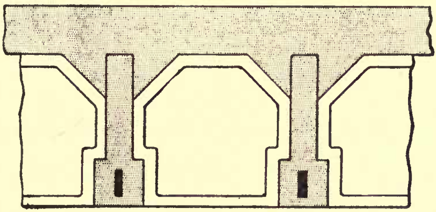
In the New York regulations governing the use of concrete-steel it is enacted that concrete-steel may be used for columns in which the ratio of length to the least side or diameter does not exceed twelve. The reinforcing rods must be tied together at intervals of not more than the width of the least side or the diameter of the column.

These stipulations must be read in accordance with others which specify the composition and proportions of the concrete, the quality of the steel, and which limit the extreme fibre stress to 350 lb. per square inch on concrete in direct compression. The permissible compressive stress on steel is not stated, but as tensile stress in steel is limited to 16,000 lb. per square inch, we presume the intention is that this limit shall also apply to compression. Engineers must carry out any required tests showing that the construction is capable of carrying, without any sign of failure, a load equal to three times the calculated amount.

APPENDIX.

A Self-Centring Floor System.—In the construction of concrete-steel floor panels, of the customary type, it is necessary to erect moulds over the whole area to be covered, the construction practically constituting a temporary floor of timber. Upon this the reinforcing bars are laid and fixed in position, and the floor is completed by depositing the necessary thickness of concrete. The chief objections to this method are the cost of timber, for the staging, and of the time occupied in its erection and removal, and delays in the execution of structural work.

The "Armocrete" floor, described on this page, obviates these disadvantages, inasmuch as it can be constructed on any story of a building without centring of any kind. The accompanying section will serve to make clear



SECTION OF ARMOCRETE FLOOR.

the essential features of the invention. The vertical members, termed "webs," consist of concrete, reinforced by flat steel bars placed vertically. The hollow members, termed "tubes," are of terra cotta or concrete as may be preferred, and the remainder consists of a concrete filling forming the surface of the floor, which can be finished by a layer of cement or any suitable covering. The webs are made of three standard sizes, 6 in., 8 in., and 10 in., in depth, and in any length up to 25 ft., the reinforcement being proportioned to the load to be carried. The tubes are made in 9 in. lengths of terra cotta, stoneware, or they can be made upon the site of concrete, with cinders or other light material as aggregate.

In constructing a floor upon this system, the webs are first placed on their permanent supports, at intervals of 9 in. apart. Then the tubes are dropped into place, and the result is a rough floor strong enough to bear the weight of workmen and building materials, and to serve as centring for the upper layer of concrete. As soon as a sufficient area has been covered by webs and tubes, a gang of men commences to fill in the upper layer of concrete, the thickness of which varies from 2 in. to 4 in. according to circumstances. During construction, the webs afford more than ample resistance to tensile stress, and sufficient resistance to compressive stress, and when thoroughly hardened the concrete is ready to take its share of the compressive stress that will be developed when the floor is subjected to its full load. The webs can be moulded on the site of the works, and being of small section, are readily handled by two men. In these respects the system is superior to others in which tubular or other beams, manufactured away from the site, are used for forming floor panels, for apart from the cost of packing and freight, the handling of complete beams involves considerable trouble and expense.

ALPHABETICAL LIST OF TABLES.

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