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CONSTRUCTION OF A REAL WORLD BILEVEL LINEAR PROGRAM OF THE HIGHWAY NETWORK DESIGN PROBLEM

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The formulation of the Highway Network Design Problem (NDP) as a Bilevel Linear Program (BLP) allows more realistic solutions taking into account the reaction of the users to the improvements made by the system. In this paper, a conceptual framework for the optimization of the investment on the inter-regional highway networks in developing countries is proposed. The model is applied to the formulation of a real world case study based on empirical data for Tunisia. Much effort was ended to make the implementation as realistic as possible, taking into consideration travel time, operating costs, accidents costs, improvement costs, conservation laws, effect of intra-regional flow... A new formulation allowing the incorporation of any improvement cost functions, including non-convex and non-concave functions, is introduced.

1. INTRODUCTION

This paper is concerned with the construction of a Bilevel Linear Program (BLP) for optimizing the investment in the interregional highway network of a developing country. The notion of development is related, in this study, only to rural highway transportation. A country or region is considered to be developing if most or all of its inter-city traffic is carried on two-lane highways or on a lower quality of roads.

The study is based on actual data for Tunisia. Tunisia (Figure 1) is a 164,000 square kilometer country located in North-Africa and bounded on the north and east by the Mediterranean Sea, on the south-east by Libya and on the west by Algeria (The American University 1979 pp.ix-xvii). Its population was 6,966,173, with 47 % rural and less than 15 % living in the south, as reported by the census of March 30, 1984 (Institut National de la Statistique 1984 p.24 and pp.269-270). The largest cities are Tunis (the capital), Banzart (Bizerte), Safaqis (Sfax), Susah (Sousse), Qayrawan (Kairouan), and Qabis (Gabès). The per capita income is about US$ 1,100 and the inflation averaged nearly 10.5 % in 1983-1984 (Embassy of Tunisia 1987). The currency in Tunisia is the Tunisian Dinar (TD); 1 TD is equivalent to 1.24 US$ as of November 4, 1987 (Bank of America Global Trading 1987).

The highway network in Tunisia consists of 18,142 kilometers (KM) of roads (Direction de l'Entretien et de l'Exploitation Routière 1984 pp.1-2); 53 % of them are paved. The average width of paved roads is 5.5 meters (M); 17 % are narrower than 4.5 M. The total number of vehicles in the country was estimated at 385,600 on December 31, 1982 (Direction de l'Entretien et de l'Exploitation Routière 1982 p.78).

The choice of a developing country seeks to motivate wider applications of operations research models to solve transportation design problems in those regions where sophisticated decision making techniques are not sufficiently introduced. The allocation of the budget among road links in developing countries is often based on simple rules, such as the comparison of the increases of daily traffic on the links to be improved or the comparison of the differences between benefits and costs for the projected improvements. The high cost of highway improvements makes it necessary to elaborate more powerful optimization models to utilize better the assigned budget and to take into account the specific characteristics of the transportation networks in those countries. Such characteristics are very important especially in rural networks where the difference between developed and developing is so significant that the models elaborated for one category can hardly be applied to the other. For example, models measuring capacity in number of lanes do not make sense in a developing country where it is uncommon to find a rural highway with more than two lanes; even worse, a high proportion of the roads are not paved.

While a great effort has been made to make the presentation as realistic as possible, considerable liberties were taken with the data, both to fill in gaps and to simplify the model. Some of the data used were recent; others were old. For some parts of the model, no data existed at all; in this case information was obtained from other countries and adjusted, or sensible subjective estimates were used.

2· RURAL HIGHWAY NETWORKS IN DEVELOPING COUNTRIES

The highway network in developing countries basically consists of two-lane paved and unpaved roads. Two-lane roads are in principle undivided, meaning that the passing of slower vehicles requires the use of the opposing lane where sight distance and gaps in the opposing traffic stream permit. The traffic flow in one direction influences the flow in the other direction, which requires the inclusion of the total flow in both directions in all cost functions.

A significant part of the analysis is built on the data from the Highway Capacity Manual (Transportation Research Board 1985 pp.8·1-8·32). The Manual uses the concept of levels of service (LOS), a qualitative measure describing operational conditions within a traffic stream and their perception by the travellers. Six levels of service are defined from A to F; level of service A represents the best operating conditions (free flow) and level of service F the worst (forced or breakdown flow). Level of service E represents operating conditions at or near capacity.

The Highway Capacity Manual (HCM) provides (Table 8·1 p. 8·5) maximum values of the ratio of flow to capacity measured under ideal conditions (ideal capacity); ideal conditions, as defined by the HCM, are nonrestrictive geometric, traffic, or environmental conditions; specifically they include: 1) design speed is greater than or equal to 60 miles per hour (97 KM/H), 2) roadway width is greater than or equal to 24 feet (7.3 M), 3) width of both usable shoulders is greater than or equal to 12 feet (3.7 M), 4) "no passing zones" are not used on the highway, 5) all vehicles are passenger cars, 6) directional split of traffic is 50/50, 7) there are no impediments to through traffic due to traffic control or turning vehicles, and 8) terrain is level.

Whenever ideal conditions are not satisfied, adjustments are made using the following relationship:

$$X_i = 2800 R_i D W H, \tag{1}$$

where X_i is the flow in both directions for prevailing roadway and traffic conditions for level of service i in vehicles per hour, R_i is the ratio of flow to ideal capacity for level of service i, obtained from Table 8·1 in the HCM, D is an adjustment factor for directional distribution of traffic, obtained from Table 8·4, W is an adjustment factor for narrow lanes and restricted shoulders width, obtained from Table 8·5, H is an adjustment factor for the presence of heavy vehicles in the traffic stream, computed as:

$$H = [1 + p_T(e_T - 1) + p_R(e_R - 1) + p_B(e_B - 1)]^{-1}$$

where p_T is the proportion of trucks in the traffic stream, p_R is the proportion of recreational vehicles, p_B is the proportion of buses, e_T is the passenger-car units (PCU) equivalent of one truck (passenger-cars refer to all vehicles having exactly four wheels contacting the road, including light vans and pickup trucks), e_R is the PCU equivalent of one recreational vehicle, and e_B is the PCU equivalent of one bus; e_T, e_R, and, e_B are obtained from Table 8·6 in the HCM. Under ideal conditions and level of service E, relationship (1) gives an ideal capacity of 2800 PCU; for different conditions the actual capacity can be found easily by simple application of the formula.

The data suggested by the HCM might be applied to developing countries; however, some extensions are necessary. Table 8·5 does not consider roadways narrower than 5.5 M, whereas 4 M roadways are very common in developing countries. Moreover, the HCM ignores the condition of the road surface, even though the surface has an important impact on many factors such as capacity and operating costs. Surfaces can be classified into three categories: asphaltic concrete overlay (referred to in this study as asphaltic), surface treatment (referred to in this study as treatment), and unpaved. Every road in each surface category can be ranked as good, fair, or poor, which gives a total of nine states of surface.

The surface of the shoulders as well as the roadway can be included in a capacity analysis. The surface of the shoulders is always of lower quality; when the surface is the same, the road is considered to consist of a roadway and no shoulders, as is usually the case for unpaved roads. According to Table 8·5 in the HCM, widening of shoulders beyond 12 feet (3.7 M) has no effect on increasing capacity. This result is applicable to roads with wide roadways (> 5.5 M); however, when roads with narrower roadways are considered, the effect of wider shoulders is significant and must be included in the Table. The quality of the surface can be given by two other tables, one for the roadway and the other for the shoulders. It is important to notice that the effect of the surface of the shoulders depends on their width: the narrower the shoulders, the less significant the effect of their surface. Relationship (1) can be modified to become:

$$X_i = 2800R_iDWHPS, \qquad (2)$$

where P is an adjustment factor for the effect of the quality of the roadway, and S is an adjustment factor for the effect of the quality of the shoulders. An empirical analysis is necessary to extend Table 8·5 in the HCM to narrower widths and to provide realistic estimates for the coefficients P and S. Two more ideal conditions need be added to those listed in the HCM: 9) the surface of the roadway is asphaltic concrete (or a similar quality) and in good condition, 10) the shoulders are treatment and in good condition.

3· THE DECISION VARIABLES

The two decision variables that are related to every (existing or added) link in the network are the *flow* and the *added capacity*. The flow is usually measured with respect to the design hour. The selection of an appropriate hour for design purposes is a compromise between providing an adequate level of service for every (or almost every) hour of the year and economic efficiency. Customary practice is to base design on an hour between the 10^{th} and the 50^{th} highest hour of the year. For rural highways, the 30^{th} highest hour is commonly used (Transportation Research Board 1985 p.2·10). The flow during the design hour is not a constant value; however, it is often agreed that for rural highways the flow during the 30^{th} highest hour is about 12 % of the average daily flow (Wohl and Martin 1967 pp.164-179; Transportation Research Board 1985 pp.2·7-2·12). This coefficient, called the design hour ratio, is used to convert daily flow into hourly flow, or vice-versa.

The capacity is assumed to be continuous to permit small link improvements. One convenient unit of measure of capacity is the PCU per hour. Since the traffic involves other vehicles such as buses and large trucks, those heavy vehicles are converted to equivalent PCU (Table 8·6, HCM). Capacity refers to the maximum PCU allowed in the road in both directions, without leading to heavily congested flow (high delays and low speeds). Likewise, the added capacity is evaluated in PCU added to the capacity of the road, and the flow is also measured in PCU.

For the study of the Tunisian highway network, we found it suitable, given the data available, to partition Tunisia into 19 regions (Figure 1). The study examines the transportation network connecting those regions. Each region is both an origin and a destination, and is represented by a centroid node labeled from 1 to 19. The network includes also 39 intermediate nodes labeled from 20 to 58. The 112 links included in this study, described in Table 1, are defined by their end nodes, length, terrain (1 for level, 2 for rolling and 3 for mountainous), roadway width, shoulder width, roadway surface quality and shoulder quality (1 for asphaltic-good, 2 for asphaltic-fair, 3 for asphaltic-poor, 4 for treatment-good, 5 for treatment-fair, 6 for treatment-poor, 7 for unpaved-good, 8 for unpaved-fair and 9 for unpaved-poor). The data about the terrain are given by the Army Map Service (NSPE 1957), whereas most of the other data are provided by the publications of the Direction de l'Entretien et de l'Exploitation Routière (detailed maps, Infrastructure du Réseau Routier pp.7-51 and map of Recensement Général de la Circulation). However, little information is available about the width and the quality of the roadway surface and no information was available about the shoulders. The following assumptions are made: 1)if the road is national (referred to as GP) its width is 6.5 M or larger; if it is regional (referred to as MC) its width is 4.5 M or larger but less than 6.5 M; and if it is local (referred to as RVE) its width is less than 4.5 M; 2)the real width of the roadway is proportional to the width of the road on the detailed maps and is also proportional to its average flow; 3)the sum of the widths of roadway and shoulders of all roads is at least 10 M (SETEC 1982 pp.An·13·2·1-An·13·2·10); 4)the pavement is asphaltic when the roadway is at least 8 M width; otherwise, the pavement is treatment (SETEC 1982 p.13·25); 5)the surface quality of the pavement of all links in a given region is equal to the average quality of all pavements of that region, rounded to the nearest integer; 6)if a link connects two regions, its quality is the average of their qualities; 7)the quality of the surface of all unpaved roads is fair; 8)the quality of the shoulders is unpaved-fair for paved roads; 9)there are no shoulders for unpaved roads.

4. FORMULATION OF THE NETWORK DESIGN PROBLEM (NDP) AS A BLP

Bilevel Linear Programming (BLP) is similar to a standard Linear Programming (LP), except that the constraint region is modified to include a linear objective function; BLP can be visualized as an organizational hierarchy in which two decision makers seek to improve their strategies from a jointly dependent set S, $S=\{(x,y): Ax+By\leq b,\ x,y\geq 0\}$. The upper decision maker, who has control over x, makes his decision first, hence fixing x before the lower decision maker selects y (Bialas and Karwan 1982 and 1984, Bard 1983, and Ben-Ayed 1988). The optimization problem being formulated in this paper is concerned with the optimal allocation of investment among the links of the transportation network, by adding new arcs or improving existing ones.

While investment costs are controlled and allocated in an optimal way from the system's perspective, travel costs depend on traffic flows, which are determined by users' route choice (Ben-Ayed, Boyce and Blair 1988). Since users are assumed to make their choices so as to maximize their individual utility functions, their choices do not necessarily coincide, and may conflict, with the choices that are optimal for the system. The system can influence users' choices, however, by improving some links and making them more attractive than others. In deciding on these improvements, the system tries to influence the users' preferences in minimizing total system costs. The partition of the control over the decision variables between two ordered levels requires the formulation of the Network Design Problem (NDP) as a Bilevel Program, in which the system is the upper-level decision maker and the user is the lower-level decision maker.

LeBlanc and Boyce (1986) first gave an explicit formulation of the NDP as a BLP; their formulation, however, requires the unrealistic assumption of linear improvement cost functions. Ben-Ayed et al. (1988) gave a similar, but more general, formulation that allows convex as well as concave improvement cost functions. Next, we propose a new formulation that has the ability to incorporate any piecewise linear function, including non-convex and non-concave functions.

Consider any piecewise linear function of Z defined as:

$$f(Z) = b_m Z + d_m, \text{ for all } Z \in [q_{m-1}, q_m],\ m=1,..,J.$$

The function f can be equivalently stated as:

$$\begin{aligned} f(Z) &= (b_1 q_1 + d_1) + \Sigma_{j=1,m-1}\, b_{j+1}(q_{j+1} - q_j) + b_m(Z - q_{m-1}) \\ &= (b_m Z + d_1) + \Sigma_{j=1,m-1}\, q_j(b_j - b_{j+1}) \end{aligned}$$

for all $Z \in [q_{m-1}, q_m]$, m=1,..,J. By adding and subtracting $\Sigma_{j=1,m-1}\, Z(b_j - b_{j+1})$, we obtain:

$$\begin{aligned} f(Z) &= (b_1 Z + d_1) + \Sigma_{j=1,m-1}(b_{j+1} - b_j)(Z - q_j) + \Sigma_{j=m,J-1}(b_{j+1} - b_j)0 \\ &= (b_1 Z + d_1) + \Sigma_{j=1,J-1}\, (b_{j+1} - b_j)\text{MAX}\{(Z - q_j),\ 0\}. \end{aligned}$$

If we denote by W_j the maximum of $(Z-q_j)$ and 0, function f can be formulated as a BLP as follows:

$$\begin{gathered} \text{MIN } (b_1 Z + d_1) + \Sigma_{j=1,J-1}\, (b_{j+1} - b_j)W_j \\ \text{where } W_j \text{ solve:} \\ \text{MIN } \Sigma_{j=1,J}\, W_j \\ W_j \geq Z - q_j \\ W_j \geq 0. \end{gathered}$$

Using the above concepts for improvement cost functions and building on Ben-Ayed et al., the following formulation of the NDP as a BLP can be obtained:

$$\mathrm{MIN}_{Za}\ \Sigma_a\ \{C_a + \tau[(b_{1a}Z_a + d_{1a}) + \Sigma_{m=1,Ja-1}\ (b_{m+1,a} - b_{ma})W_{ma}]\}$$
$$\text{where } C_a,\ \underline{C}_a,\ W_a,\ X_a \text{ and } X_{ad} \text{ solve:}$$
$$\mathrm{MIN}\ \Sigma_a\ \{(\underline{C}_a + \varepsilon C_a) + \Sigma_{m=1,Ja-1}\ W_{ma}\}$$
$$\text{such that:}$$
$$\text{for each node } n \text{ and each destination } d:$$
$$\Sigma_{a\varepsilon An}\ X_{ad} - \Sigma_{a\varepsilon Bn}\ X_{ad} = u_{nd}$$
$$\text{for each link } a:$$
$$C_a - r_{ma}X_a + g_{ma}Z_a \geq s_{ma}, \qquad m=1,..,M_a$$
$$\underline{C}_a - \underline{r}_{ma}X_a + \underline{g}_{ma}Z_a \geq \underline{s}_{ma}, \qquad m=1,..,\underline{M}_a$$
$$W_{ma} - Z_a \geq -q_{ma}, \qquad m=1,..,J_a-1$$
$$W_{ma} \leq b_a - q_{ma}, \qquad m=1,..,J_a-1$$
$$\Sigma_d\ X_{ad} - X_a = 0$$
$$Z_a,\ C_a,\ \underline{C}_a,\ W_{ma},\ X_a,\ X_{ad} \geq 0$$

where Z_a is the number of units of capacity added to link a; X_a is the flow on link a; X_{ad} is the flow on link a with destination d; C_a is the approximation of the system travel cost function on link a; $\underline{C}_a$ is the approximation of the cumulative user cost function on link a; d_{1a} is the intercept of the first piece of the improvement cost function; b_{ma} is the slope of the piece m delimited by $q_{m-1,a}$ and q_{ma}; W_{ma} is the maximum of (Z_a-q_{ma}) and 0; τ is a factor to convert improvement cost to the same units as travel time; M_a, $\underline{M}_a$, and J_a are the numbers of segments in the approximations of system travel cost, cumulative user travel cost, and improvement cost of link a, respectively; ε is a positive scalar sufficiently small so that the optimum to the above problem is the same as the one with ε equal to zero; A_n and B_n are the sets of links pointing out of and in node n, respectively; u_{nd} is the required number of trips between node n and destination d; r_{ma} and $\underline{r}_{ma}$ are the slopes of the linear pieces in the travel cost approximations; s_{ma} and $\underline{s}_{ma}$ are the intercepts of the same pieces; and g_{ma} and $\underline{g}_{ma}$ are the effects of improvement on reducing s_{ma} and $\underline{s}_{ma}$, respectively.

It should be emphasized that the NDP is always concerned with the costs of the system to the community as a whole; the objective function to be minimized is the *total system costs* consisting of *system travel costs* as a function of the flows and the links improvements, and *improvements costs* as a function of the links improvements. The lower objective, which is a constraint, consists of the cumulative user travel cost, or integral of the average travel cost with respect to flow, and the sum of the W_{ma} required by the non-convex improvement functions.

5. THE OBJECTIVE FUNCTIONS

Travel Time Functions

To our knowledge, all travel time functions in the literature are function of directional flow, and therefore are implicitly intended for divided highways only; no analytical representation is available for two-lane highways. An empirical travel time function is now proposed based on data provided by the HCM. Table 8·1 in the HCM gives the minimum average speeds versus the maximum values of the ratio of flow to capacity. For example under ideal conditions the speed at level of service A is greater than or equal to 93 KM/H. Equivalently the travel time spent per KM is less than or equal to .643 minute (60/93). On the other hand, the ratio of flow to capacity is less than or equal to .15 which means that the flow is less than or equal to 420 PCU. Therefore, a flow less than or equal to 420 PCU corresponds to a travel time less than or equal to .6428 minute per KM. It seems reasonable to assume *equality*. Similarly for the other levels of service, a flow equal to 756 PCU corresponds to a travel time of .678 minute/KM, 1204 corresponds to .717, 1792 corresponds to .746 and 2800 corresponds to .829. A sixth point can also be obtained; free flow corresponds to the design speed, or equivalently, travel time is .621 minute/KM when speed is 97 KM/H. For any other value of the flow between 0 and capacity, a linear interpolation is used; an average travel time function is obtained by connecting the six points, as shown in Figure 2.

For every link, an average travel time function can be obtained by applying relationship (2) to the corresponding entries of Table 8·1. For rolling and mountainous terrain, the design speeds need be known to find the travel time corresponding to free flow. Given any geometric, traffic, environmental, and surface condition, the average travel time obtained using the above procedure is monotonously increasing with respect to the flow.

Level of service F was not considered so far because of its instability. This level is distinguished by its high density, or number of vehicles occupying a given length or roadway. The density corresponding to the capacity is called the critical density; level of service F occurs when the density exceeds the critical density. At this level queues are formed which are characterized by stop-and-go movement. Since at level of service F the vehicles are delayed, the flow is small. A small flow can correspond to either a high travel time if the flow is unstable, or a low travel time if the flow is stable. Therefore, each flow can correspond to two travel times, which violates the definition of a function. This problem can be overcome by assuming that the flow can exceed the capacity, but for those flows greater than the capacity the travel time increases sharply.

For our case study, some of the data needed for the formulation of the travel time functions are not available; the following assumptions are added: 1)the data of Tables 8·1, 8·4, 8·5 and 8·6 in the HCM apply to the Tunisian case; 2)the directional split is 60/40 for all links (HCM p.8·13); 3)the percentages of no passing zones are 20 % in level terrain, 40 % in rolling terrain and 60 % in mountainous terrain (HCM p.8·13); 4)the design speeds are 95 KM/H for level terrain, 90 KM/H for rolling terrain and 75 KM/H for mountainous terrain (personal experience); 5)additional data can be obtained from any table by using either interpolation or extrapolation; interpolation is linear while extrapolation is based on regression analysis; 6)the vehicle mix, for all links, consists of 83.5 % passenger cars, 13.9 % trucks and 2.6 % buses (Direction de l'Entretien et de l'Exploitation Routière 1982 p.22); 7)the factors P and S in Tables 4 and 5 apply to the Tunisian case; 8)when the flow exceeds the capacity, the slope of the average travel time is an arbitrarily high number that is the same for all links. Based on those assumptions, Tables 2 and 3 are constructed as extensions of the original tables 8·1 and 8·5 in the HCP.

For each link, an average travel time function is found by applying the procedure explained at the beginning of this section, and using the new Tables. To illustrate, consider a typical road in Tunisia; it is a 6.5 M width paved road where the pavement type is treatment and the condition is fair, the shoulders are unpaved-fair and 5 M wide, and the terrain is level. The average travel time function obtained for this link is shown in Figure 3. The system travel time function is shown in Figure 4; the system travel time at a flow X is equal to the product of X by the average travel time at flow X. The cumulative user travel time function is shown in Figure 5; the cumulative user travel time at a flow X is equal to the area below the average travel time and above the abscissa axis, and delimited by 0 and X.

In developing countries, travel time cannot be the only component of the travel cost function because of the low value of time of travellers. Other factors such as operating costs and accidents costs must be included.

Operating Costs

Operating costs include ownership costs and running costs. Ownership costs, such as purchase costs, insurance, and economic ageing costs can be neglected because they are independent of the decision variables; so only running costs, such as fuel, grease, oil, tires, and depreciation (technical ageing) are considered. Steenbrink (1974 p.209) emphasized that vehicle operating costs be taken without taxes "because the share of taxes in the market-prices of these costs are very high". The inclusion of taxes in the social costs is misleading since what is relevant for the community is not what is paid by the individual for a liter of gasoline, for example, but what the value of that liter is. In developing countries, besides the products being highly taxed where the consumer pays more than the economic value, there are other products that are subsidized by the government and their prices are less than their economical values. As far as the social objective function is concerned, the economic values must be considered without taking into account the price paid by the individual; but when the objective of the user equilibrium is concerned, the price paid by the individual is the one to be considered.

Since running costs differ from one car to another, a standard car representing the average cost must be considered. The average running cost of this car should be computed for different conditions including surface, flow, capacity, and terrain. As was noted by Moyer and Winfrey (1949), the differences in operating costs on stabilized surfaces, bituminous, portland cement concrete, and brick surfaces are small and difficult to measure; but when compared to operating costs on untreated gravel and earth, the differences are very marked. The connection of the flow and the capacity to the running costs is visualized by the fact that in high traffic densities, queues are formed and then breaking and accelerating are more often required, causing extra use of fuel, extra wear of tires and extra wear on the gearbox (Steenbrink 1974 p.210). Assuming that drivers operate their vehicles at the maximum speed permitted by prevailing flow and road design, high speeds made possible by low traffic densities also result in extra running costs (e.g. increases in fuel consumption). As long as the flow is *stable*, it seems reasonable to assume that the effect of the flow on operating costs is reflected by the effect of speed. To obtain running costs as a function of the flow, the inverse function of average travel time is used, since travel time is the inverse of speed. However, for flow corresponding to level of service F, operating costs must increase sharply, as was the case for travel time. While the effect of flow and capacity is assumed to be reflected by the effect of speed, this assumption does not hold for surface and terrain; operating costs are higher in mountainous and unpaved roads than in level and paved roads, even when speed remains the same.

The formulation of the running costs in Tunisia is based on the data provided by SETEC (1982 pp.7·10-7·16). Table 6 gives the average running costs for 1980 in TD/100 KM of an average passenger car in Tunisia, when the speed is 80 KM/H, the terrain is level and the surface is paved and fair. Tables 7 and 8 provide the adjustment factors for the effects of the surface condition and the terrain, respectively. The running costs (tax excluded) for speeds ranging from 24 KM/H to 112 KM/H are given in Table 9. The Tables need some modifications to better fit our study. For this purpose, the following assumptions are added: 1)running costs on asphaltic are the same as on treatment; 2)the economical value of 1 TD in a given year is the same as 1.1 TD in the next year (based on the inflation rate); 3)taxes in 1990 are the same as in 1980; 4)running costs, tax excluded, in 1990, at a given speed, are equal to running costs, tax excluded, in 1980, at the same speed, multiplied by a constant; 5)when flow exceeds capacity, running costs increase as sharply as travel time.

Let RCTE be the running costs in TD/100 KM, tax *excluded*, in 1990, at 80 KM/H; RCTE is obtained by multiplying the values of the first four entries of the first row of Table 6 by $(1.1)^{10}$, then multiplying the obtained values by the corresponding row of Table 7 depending on the surface. The fuel entry also needs to be multiplied by the corresponding adjustment factor of the terrain from Table 8; the sum of the calculated values of the components gives RCTE. Let RCTI be the running costs in TD per 100 KM, tax *included*, in 1990 at 80 KM/H. Every component of RCTE is multiplied by the corresponding tax, obtained from the original Table 6, and the sum gives RCTI. The running costs as a function of the flow are computed from Table 9; after the speed is substituted by its equivalent travel time, the inverse function of the average travel time function is applied to convert the travel times into the corresponding flows; columns with negative flows are discarded. Also, the costs corresponding to unstable flows are replaced by higher values; the values in the original Table 9 were measured for test speeds and therefore are not applicable to unstable flows. An adjustment factor, AF, is obtained by dividing RCTE by the entry of the second row of Table 9 corresponding to the 80 KM/H column. The average running costs, tax excluded, as a function of the flow are obtained after the multiplication of the second row of Table 9 by the calculated adjustment factor AF and the connection of the points. To have the same function with tax included, we just multiply the costs, tax included, by (RCTI/RCTE).

Figure 6 shows the function of the average running costs, tax excluded, of the road described above; this function is the basis of the system running costs functions (Figure 7). In contrast, the cumulative user running costs function (Figure 9) is based on the average running costs, tax included (Figure 8).

Accident Costs

The costs of accidents include the social costs caused by road accidents of corporal damages, such as deaths and hospitalization, and material damages to vehicles and environment. Hence, they are included in the system objective function, but not in the user objective; the costs are not direct costs for the individual. The number and severity of accidents depend on many factors, such as congestion (flow), signalization, width, surface (capacity), illumination, time (day, night), condition of the car, the driver, and speed. Some of those factors are related to the decision variables of the model, and hence can be controlled by the model; but some others are beyond the scope of those variables because of their independence with flow and capacity.

The number of accidents is usually defined with respect to a unit of length. Depending on the data available, accidents are classified into different categories and a function is evaluated to relate the costs of each category to the causal factors included in the decision variables of the optimization problem. A natural way to find the relationships is a regression model. Its coefficient of determination R^2 is not expected to be very high since only the factors controlled by the optimization problem are included in the regression model.

The accident data available for this study pertain to 20 regions in Tunisia comprising for each region the average number of accidents in 1983 (without any details on the severity), the average flow per day, the average width of paved roads, the average quality of the surface of the roadway, and the length of the included links (Direction de l'Entretien et de l'Exploitation Routière 1984 pp.7-51, and 1982 pp.46-47 and p.82). Four independent variables are chosen (Ben-Ayed 1988 Table 4·14): 1)average flow per hour, 2)width of paved roads in M, 3)surface condition of the roadway, 4)capacity in PCU using the relationship (2). Surface condition is quantified by introducing a scale measure (10 for good, 5 for fair and 0 for poor). One observation having an exceptionally high number of accidents was discarded to avoid misleading results. Based on the remaining 19 observations, the regression analysis shows that accidents depend mainly on flow and width. However, those two variables are significantly correlated (R=.69). Since the number of accidents depends more on flow (R=.88) than on width (R=.56), the following relationship is retained (Figure 10):

Number of Accidents per 100 KM = .2677(Flow per hour) - 5.6466

the adjusted coefficient of determination R^2 is 76 %.

The insignificant correlation between the number of accidents and the remaining two variables (quality of roadway surface and capacity) may reflect that good roads attract more vehicles, which results in more accidents due to congestion. However, more accurate data that include unpaved roads may give different results. Nineteen observations are too few to generate a reliable regression model. Moreover, those observations are averages for each region; they are not specific to any link of the network.

Improvement and Maintenance Costs

Link improvement has two different aspects; the first is its cost, and the second is its effect on reducing travel costs. For this reason, the choice of the improvement cost functions is critical in the model. Unfortunately, no accurate data are available about improvement costs for Tunisia. Subjective estimates given by Tollié (1987) and adjusted by Thompson (1987) are used.

The improvements considered in this study do not include adding new arcs, nor widening the roadway beyond the existing width of the roadway and the shoulders. This restriction is made to avoid consideration of the costs of land acquisition and earthwork. Land acquisition costs vary dramatically from one region to another, but no data are available. The solution of the optimization problem may suggest further study of some links to investigate the possibility of adding more lanes or even upgrading the road to a divided highway. In this study, however, the types of improvements are limited to maintenance costs, roadway widening, roadway surface improvement, and shoulders surface improvement, which excludes some major periodic costs such as the construction of the road-bed and bridges.

The costs related to each type of improvement depend on several factors other than the length of the improved road. The cost of resurfacing per unit of length, for example, depends on the width of the roadway, the type of the surface, and the condition of the existing one. For each link, it is necessary to have one specific improvement cost depending on the specific conditions of that link. The data needed can be summarized in two tables, one giving the cost of improving the existing surface (of shoulders or roadway) from the existing state to any other feasible state, and the other giving the effect of width on that cost. Widening of the roadway is considered to be an improvement of a portion of the shoulder, because the widening of the roadway is always made at the expense of the shoulders. For the same reason, the effect of the terrain is ignored since the costs of resurfacing are not considerably affected by terrain.

Table 10 gives the costs of possible surface improvements considered in this study. The costs given in the Table apply if the surface to be improved is at least 4 M wide. If the width is only .5 M, the cost per M^2 is assumed to double. For widths between .5 and 4 M, a linear interpolation is used. It is usually required that widening of the roadway be accompanied by resurfacing. To avoid double counting fixed costs, we assume that the costs of widening and resurfacing at the same time is 90 % the sum of their costs if done separately. We also assume that if the existing surface is good and if the new surface is in the same category (asphaltic, treatment or unpaved), the cost of resurfacing is 30 % of the cost of upgrading from poor to good in that category.

An ideal improvement cost function for our model is one that gives the cost of every PCU of added capacity. However, the capacity can be increased in many ways with different costs and effects on travel costs. Resurfacing the roadway, for example, may increase the capacity by the same amount as improving the shoulders, but the two alternatives do not necessarily have the same cost nor have the same effect on reducing the travel cost. Even for the same improvement, the effect varies from one flow to another. The formulation of the investment function, therefore, involves serious difficulties because of the large number of the decision variables, which is equal to the number of possible improvements, their overlapping, their technical requirements, their qualitative nature, their dependency on the flow, and their different effect on travel costs. Following the analysis of the maintenance costs and the additivity of the cost functions, a procedure is proposed to formulate *implicitly* all details related to each type of improvement and yield the cost and the effect of each added PCU.

The analysis of the maintenance costs is based on the data provided by SETEC (1982 pp.13·19-13·26). Those annual costs depend on the type of road (Table 11). Based on the following assumptions, Tables 10 and 11 can be used after updating the values of the costs to 1990 and deducting the taxes: 1)Table 10 applies to Tunisia for the year 1987; 2)taxes on investment and maintenance costs are 20 % of the total cost (SETEC 1982 p.13·13); 3)the data about earth surface apply to all unpaved roads.

Additivity of the Cost Functions

The upper and lower objective functions for each link are obtained by adding the components of the system cost functions and the components of the cumulative user cost functions, respectively. However, such an addition is possible only when the components are expressed in the same *units* and for the same *period* of time. For this study, we choose the unit to be *one thousand TD* and the period to be *one year*. We assume that this study is intended to allocate the optimal interregional highway investment for a five year period from 1988 to 1992; the year 1990 is assumed to be an average year. The choice of the unit and the period is not relevant to the model because their change is equivalent to the multiplication of the objective functions by a positive constant which does not affect the solution of the optimization problem.

In contrast, the choice of the conversion factors is significant. The value of time is provided by SETEC (1984 p.24). This value is estimated to be .250 TD/hour in 1986 with an increase of 2.8 % per year, which gives a value of .279 TD/hour in 1990; however, since the average number of passengers per vehicle in Tunisia is 4.38 (SETEC 1982 p.5·8), this value becomes 1.217 TD. For the costs of accidents, no Tunisian data are found; therefore, the cost per accident in 1985 provided by the National Safety Council (1986 pp.2-49) for the United States is applied to Tunisia. To take into account the differences in the standards of living, we multiply the costs of wage loss, medical expense and insurance administration (29.3 billions of dollars) by the ratio of the

Tunisian gross national product per capita to the U.S. gross national product per capita (1,420/12,820 according to the World Tables 1986). To convert the value to 1990 we multiply by $(1.028)^5$ which corresponds to the increase of salaries in Tunisia during that period (SETEC 1984 p.24). Then we add the motor-vehicle property damage (19.3 billions of dollars) multiplied by the inflation factor $(1.1)^5$, and divide the computed total cost by the total number of accidents in the U.S. in 1985 (19,300,000). We obtain a cost of \$ 1,832 or 1,573 TD per accident. The number of accidents in 1990 is assumed to be the same as in 1983.

The travel time function has units of time per hour because flow is defined with respect to the hour. To convert those functions to the year, we divide by .12, to obtain the travel time per day, as explained in Section 3, then we multiply by 365. We assume that the flow is uniform during the 365 days of the year (Direction de l'Entretien et de l'Exploitation Routière 1982 p.74). On the other side, improvement costs cover a period that is usually much longer than one year. Let C_1 be the actual expenditure in the first year, n the lifetime of the investment and C_n the equivalent annual expenditure over the n-year period. Assuming an instant year, C_n is given by (Morlok 1978 pp.345-369):

$$C_n = [i(1+i)^n C_1]/[(1+i)^n - 1]$$

where i is the interest rate. The interest rate retained for Tunisia is 10 % (SETEC 1982 pp.15·1-15·18). The lifetime of each investment (Table 12) is assumed to be 1.5 times the period given by SETEC for what they call "periodical maintenance costs" (p.13·25).

Using the above information, the total system travel costs and the total cumulative user costs for the illustrative link can be obtained (Figures 11 and 12 respectively). As shown by the figures, those functions are basically composed of two pieces corresponding to stable and unstable flows. This result was confirmed for all other links of the data.

Piecewise Linear Approximation of the Cost Functions

To have a finite number of possible added widths, we assume that the widening of the roadway can be done only by adding to the existing width a multiple of one-half meter, up to the maximum width of the road. Since the number of all possible states of surface is limited to nine, the total number of possible improvements of a given road is finite. For the illustrative link, the roadway can be widened by 0, .5, 1, 1.5, ... up to 5 M (11 possibilities). The pavement can be either improved to asphaltic-good or treatment-good or kept as it is; however, if there is widening there must be resurfacing. The shoulders can be left as they are, improved to unpaved-good, or, in case the roadway is upgraded to asphaltic, they can be improved to treatment-good. This gives a total number of 54 possible improvements (7 possibilities if the roadway is not widened, 2 if it is widened by 5 meters, and 45 otherwise). Those improvements, including the possibility of no improvement, represented by their added capacities versus their costs, are shown in Figure 13.

For each link, all possible improvement points are enumerated and the cost for each improvement is evaluated using Tables 11 and 12. Whenever one improvement has a lower cost and a higher added capacity than another improvement, the latter one is dominated by the first and eliminated. For the illustrative link, 34 possible improvements are dominated and eliminated. The remaining 20 points are shown in Figure 16. For each of the possible and non-dominated improvements, new travel cost functions are obtained, as shown by Figures 14 and 15. Those figures confirm that only two pieces have to be considered with the breakpoints at the capacity level.

To linearize the above functions, piecewise linear approximation is used. For the improvement cost functions, we assume a maximum of three pieces; each piece is obtained by using linear regression and the best approximation is chosen to be the one that minimizes the error, defined as the sum of the squares of the differences between the actual costs and the approximated costs. In order to ensure that we get the best approximation, we take all possible combinations. Every segment is the best linear fit of an ordered set of points where the ending point could be any point that exceeds the ending point of the set of the previous segment, and the starting point could be any point preceding its own ending point.

There are two special cases; the set of the first segment must start with the 0 improvement point and that of the third segment must end with the last point, the most expensive improvement. It is required that the approximated cost given by the first segment at zero improvement be nonnegative and no greater than twice the actual cost; when this condition is not satisfied for a given approximation, the exact value of that point is imposed. For each of the 112 links of the study, the best approximation consists of a three-piece nonconvex function, thereby requiring BLP formulation for all 112 links and introducing a great complexity to the problem to be solved. The other alternative, which is the convex and/or linear formulation, is much more efficient from a computational point of view, but is often much less accurate; the best convex formulation increases the error for 10.71 % of the links by more than 12 times as compared to the nonconvex approximation. Fortunately, there are other links for which the difference is small enough to allow convex approximation.

In dealing with such a tradeoff between accuracy and computational efficiency, we decided to use concave approximation only when this latter decreases the error by more than 50 % as compared to convex approximation. This restriction permitted the number of nonconvex improvement cost functions to be reduced to 36 (all 36 are three-piece nonconvex-nonconcave); the remaining 76 consist of 8 linear and 68 two-piece convex. For our illustrative link, the best approximation is three-piece nonconvex-nonconcave (Figure 16); this approximation was selected after considering 16,816 combinations.

With regard to the total travel cost functions, a unique piece is involved when the flow is stable. Since the costs are defined in our case by breakpoints, their approximation is obtained by applying linear regression, with zero intercept, to all breakpoints corresponding to stable flow for all possible and non-dominated improvements. Figures 17 and 18 show the plots of those points and their approximations for the illustrative link. The effect of the added capacity on the travel cost is to shift the breakpoint separating stable and unstable flows.

To obtain the approximation of the total travel costs for unstable flow, we have to take into account two facts; 1)the costs depend on the flow X and the added capacity Z; 2)the points to be considered are the ones at levels E and F for all possible and non-dominated improvements. Let c_0 and c_1 be the slopes of the (system or cumulative user) total travel costs at stable and unstable flows, respectively, and d_1 the intercept at unstable flow. Let X_E be the flow at LOS E before improvement and X_E+Z the flow at LOS E after improvement. We have:

$$\text{Decrease in intercept} = c_1(X_E + Z) + d_1 - c_0(X_E + Z) = (c_1 - c_0)Z$$

because $c_1X_E + d_1 = c_0X_E$. The total travel cost function T is formulated as a piecewise linear function:

$$\text{MIN } T$$
$$\text{such that:}$$
$$T \geq c_0 X$$
$$T \geq c_1 X - (c_1 - c_0)Z + d_1.$$

Let us call Z_i one possible improvement, X_i the flow at the new capacity (X_{Ei}) and/or beyond the new capacity (X_{Fi}), and T_i the corresponding travel cost. As for stable flow, by enumerating all possible Z_i an accurate linear approximation of the travel cost at unstable flow can be obtained by using regression analysis to obtain the parameters c_1 and d_1 of the following relationship:

$$T_i = c_1(X_i - Z_i) + (c_0 Z_i + d_1).$$

A simpler way would take one specific value of Z_i, such as the one corresponding to the existing situation (added capacity equal to zero) and find the line that connects the two points (X_{E0}, T_{E0}) and (X_{F0}, T_{F0}); c_1 is assumed to be the same for any improvement (see Figures 14 and 15) and d_1 varies linearly with Z as shown above. In our case, assuming an unstable flow caused by a flow 10 % larger than capacity, the following linear approximations are obtained for the travel costs at unstable flow:

System:	3,708.94X - 5,049,167.84
User:	308.22X - 387,532.36

Another way to deal with the total travel costs at unstable flow is to eliminate them by imposing strict capacities:

$$\text{MIN } c_0 X$$
$$\text{such that:}$$
$$X \leq \text{Existing Capacity} + Z$$

which means that instead of assigning an arbitrarily high cost for unstable flow, this flow is simply not allowed in the model. Such a simplification is not always possible; sometimes it causes infeasibility, specifically when the number of travellers to be carried from all origins to all destinations is higher than the sum of the capacities after improvement of all links.

6. THE CONSTRAINTS

The constraints treated in the previous section are needed just to complete the formulation of the objective functions. In this section, different types of constraints are considered.

The Conservation of Flow Conditions

One way to state the conservation of flow conditions is:

$$\Sigma_{r \varepsilon Rod} X_{r,od} = u_{od}, \text{ for each origin-destination pair od} \qquad (3)$$

where R_{od} is the set of all routes r going from origin-node o to destination node d, $X_{r,od}$ is the flow of passengers (per unit time) from o to d using route r, and, u_{od} is an element of the trip-matrix giving the required number of vehicle trips (per unit time) from o to d.

An equivalent formulation of those conditions is:

for each destination d and each node n other than d:

$$\Sigma_{a \varepsilon An} X_{ad} - \Sigma_{a \varepsilon Bn} X_{ad} = u_{nd} \qquad (4)$$

where X_{ad} is the flow on link a with destination d, and, u_{nd} is the required number of trips between node n and destination d.

A choice has to be made between relationship (3) and relationship (4) in order to minimize the numbers of constraints and variables involved in the formulation of the conservation of flow conditions. Since the network consists of 19 origin-destination nodes, 39 intermediate nodes and 224 directed links, relationship (4) requires 4,256 variables (19 times 224) and 1,083 equality constraints (19 times 57). Relationship (3), however, requires at most 342 equality constraints (19 times 18), but many more variables. Besides its fewer number of constraints, the latter relationship has a very important advantage, as compared to the first one; the numbers of variables and constraints can be considerably decreased as there are origin-destination pairs with zero entries in the trip matrix. If an origin-destination pair has a zero entry in the trip matrix, it means that it does not belong to the set of od pairs; therefore there is no reason to have any variable or constraint associated with it.

In attempting to test the efficiency of such a property in the use of relationship (3), we had to limit the number of variables. The following three assumptions are made: 1) every traveller going from origin o to destination d chooses his route among the 100 first shortest routes from o to d, 2) no traveller chooses a route where there is a node visited more than once, 3) no traveller chooses a route that is more than twice as long as the shortest route. Based on those assumptions, for every origin-destination pair, the 100 shortest paths are computed, then all routes with nodes visited more than once and all routes with length more than twice that of the first shortest path are excluded. Using the data provided by Table 1 (columns 2, 3 and 4) after its modification to include directed links, the number of variables was limited to 11,900. The computation of those routes is based on Shier's Double-Sweep Method (1974).

SETEC (1982 pp.5·9-6·1) provided a 29 by 29 trip matrix for 1977 and predicted numbers for 1986. We retained the same rates of increase to find the predicted numbers of required trips between each origin-destination pair for 1990. Some subregions had to be grouped to obtain the 19 by 19 trip matrix; flows between subregions belonging to the same region were discarded. The entries of the matrix are converted into average annual trips per hour (in PCU) to be consistent with the definition of the flow. The trip matrix obtained is shown in Table 13. All origin nodes are also destination nodes (refer to them as origin-destination nodes), which explains the symmetry of the trip matrix in Table 13. Among the 342 entries of the Table, 24 have a value of zero which means that the 342 constraints required by relationship (3) can be decreased to 318. Among the 11,900 variables, 1006, corresponding to those 24 od pairs with zero entry, are redundant, thereby decreasing the number of variables to 10,894. Those numbers can be decreased by much more when the following two facts are recognized.

First, in many cases all trips from origin o to destination d have to go through a third origin-destination node x. In such a case, the trip matrix can be modified to include zero trips from o to d. Call t, m and n the required numbers of trips from o to d, o to x, and, x to d, respectively; if all routes from o to d include the node x, then it is equivalent to say that m+t are required to go from o to x, n+t are required to go from x to d, and zero trips are required to go from o to d. Applying this fact to the trip matrix shown in Table 13, the number of entries with zero value is increased to 102 which decreases the number of od constraints to 240 and the number of routes to 7,655.

The second fact is even more important; it applies only to two-lane highways for which all objective functions depend on the sum of the flows in both directions. Since the trip matrix is symmetrical, we kept the directional split constant; the cost is the same if we consider 2t trips going from o to d and zero trips from d to o, instead of t trips from o to d and t trips from d to o. Also, because of the symmetry of the trip matrix, the conservation of flow conditions are still satisfied if the entries of the upper right triangle of the matrix are changed to zero, and the values of the entries of the lower left triangle are multiplied by two. This fact, by itself, increases the number of zero entries to 183.

When both facts are used, the number of zero entries becomes 222 (Table 14), yielding 3,824 variables and 120 constraints. Those numbers clearly dominate those obtained by relationship (4), namely 4,256 for the variables and 1,083 for the constraints. Therefore, for our case, relationship (3) is used for the formulation of the conservation of flow conditions. Furthermore, many od pairs are far away from each other, which results in a large number of routes and a small number of required trips between the od pairs. In other words, those pairs are introducing a huge number of variables that can be discarded without having a considerable effect on the problem. For this purpose, we limit the number of routes corresponding to each od pair to twice the number of required trips between the nodes of that pair. This limitation allows the elimination of 1,729 more routes leaving a final number of flow variables equal to 2,095.

Intra-Regional Flow Effect

The links included in the study are used by inter-regional flow as well as by intra-regional flow. Intra-regional flow includes all trips between intermediate nodes or between any other places inside the regions. Those flows have to be considered by the model because of their effect in congesting the roads and thereby increasing the travel costs to the inter-regional flow. A convenient way to include intra-regional flows is to assume that they reduce the capacity of the road used by the interregional flow. Assuming that the intra-regional flow is 40 % of the existing capacity, when strict capacity is used the constraint $X_a \le k_a$, where k_a is the existing capacity of a, is replaced by $X_a \le .6k_a$. For the case of nonstrict capacity, we substitute $(X_a+.4k_a)$ in the objective functions for X_a, and we subtract the constant cost of $.4k_a$:

$$\begin{gathered} \text{MIN } (C_a - .4c_{0a}) \\ C_a \ge c_{0a}(X_a + .4k_a) \\ C_a \ge c_{1a}(X_a + .4k_a) - (c_{1a} - c_{0a})Z_a + d_{1a}. \end{gathered}$$

7. FINAL FORMULATION OF THE PROBLEM

The empirical analysis described above results in the following final formulation:

$$\begin{gathered} \text{MIN}_{Za}\ \Sigma_{a=1,N}\ \{(C_a - .4c_{0a}k_a) + (b_{1a}Z_a + b_{0a}) \\ + \Sigma_{m=1,Ma}(b_{m+1,a} - b_{ma})W_{ma}\} \\ \text{where } \{X_a,\ C_a,\ \underline{C}_a,\ W_{ma},\ a=1,112,\ m=1,M_a\} \text{ and } \{Y_r,\ r=1,2095\} \text{ solve:} \\ \text{MIN } \Sigma_{a=1,N}\ \{\underline{C}_a + \Sigma_{m=1,Ma}\ W_{ma}\} \\ \text{subject to:} \\ \Sigma_{a=1,N}\ \{b_{1a}Z_a + \Sigma_{m=1,Ma}\ (b_{m+1,a} - b_{ma})W_{ma}\} \le \beta - \Sigma_a\ b_{0a} \\ \text{for each origin-destination pair od:} \\ \Sigma_{r\varepsilon Rod}\ Y_r = u_{od} \\ Y_r \ge 0, \text{ for all } r\varepsilon R_{od} \\ \text{for each link a:} \\ X_a - \Sigma_{r=1,2095}\ \delta_{ar}Y_r = 0 \end{gathered} \qquad (5)$$

$$C_a \geq c_{0a}(X_a + .4k_a)$$
$$C_a \geq c_{1a}(X_a + .4k_a) - (c_{1a} - c_{0a})Z_a + d_{1a}$$
$$\underline{C}_a \geq \underline{c}_{0a}(X_a + .4k_a)$$
$$\underline{C}_a \geq \underline{c}_{1a}(X_a + .4k_a) - (\underline{c}_{1a} - \underline{c}_{0a})Z_a + \underline{d}_{1a}$$
$$W_{ma} - Z_a \geq -q_{ma} \qquad m=1,M_a$$
$$Z_a \leq q_{3a}$$
$$Z_a,\ X_a,\ C_a,\ \underline{C}_a,\ W_{ma} \geq 0 \qquad m=1,M_a$$

where N is the number of links in the network, c_{0a} and c_{1a} are the slopes of the system travel cost at stable and unstable flows, respectively; $\underline{c}_{0a}$ and $\underline{c}_{1a}$ are the slopes of the cumulative user travel cost at stable and unstable flows, respectively; d_{1a} and $\underline{d}_{1a}$ are the intercepts of the travel cost at unstable flow of the system and the cumulative user, respectively; M_a is the number of breakpoints in the improvement cost function of link a; M_a is 0 when the curve is linear, 1 when the curve has 2 pieces and 2 when it has 3 pieces; b_{0a} is the intercept of the first piece of the improvement cost function on link a, b_{ma} is the slope of the piece delimited by $q_{m-1,a}$ and q_{ma}, $m=1,M_a+1$, q_{0a} being equal to zero and q_{Ma+1} equal to q_{3a}, the maximum improvement allowed by the model; k_a is the existing capacity of link a, X_a is the flow on link a, C_a is the system total travel cost on link a, $\underline{C}_a$ is the cumulative user total travel cost on link a, Z_a is the PCU added to the capacity of link a, W_{ma} is the maximum of (Z_a-q_{ma}) and 0, Y_r is the flow on route r, ß is the amount of budget available, R_{od} is the set of all routes from origin o to destination d, u_{od} are the entries of the trip matrix, δ_{ar} is binary number equal to 1 when link a belongs to route r, and equal to 0 otherwise.

The values of the coefficients of the above formulation are provided by Tables 5·1, 5·2, 5·3 and Appendix C in Ben-Ayed 1988. More details about the formulation and the data can be found in the same reference.

8· THE SOLUTION OF THE EMPIRICAL PROBLEM

An algorithm based on the structure of the empirical problem formulated above is described by Ben-Ayed, Blair and Boyce (1988). There are two reasons for having a BLP formulation in (5); first the user-optimized flow requirement (user-equilibrium), and second the nonconvex improvement functions. The algorithm deals with each of the two lower problems separately; at each iteration we try to find a better compromise with the user, while including the smallest possible number of nonconvex improvement functions to get the exact solution with the minimum computation effort. The algorithm is an iterative procedure that tries at each iteration to reduce the gap between ideal solution and incumbent solution. The procedure terminates when the gap is brought below a desired accuracy value, or when the number of iterations exceeds a fixed limit. About 15.25 minutes CPU time and 1.4 million words (64 bits per word) computing storage space were required to solve the problem; the computation was conducted on the supercomputer CRAY X-MP/24 of the National Center for Super-computing Applications at the University of Illinois at Urbana-Champaign. A fairly accurate solution, with a gap between upper and lower bounds decreased to as low as 2.56 %, was obtained despite the complexity involved in the problem (see Ben-Ayed and Blair 1988 for a discussion on the computational difficulties of BLP). The complete presentation of the solution is provided in Appendix E of Ben-Ayed (1988).

The links to be improved in this solution are shown on the map in Figure 19. Almost all improvements are on the roads connecting Tunis, the capital, to its major neighboring cities. This result can be intuitively predicted by examining the trip matrix; the trips originating or ending in Tunis are 56 % of all the trips of the matrix. A similar recommendation was made by SETEC (1982 pp. 6-8) stating that the roads corresponding to the exits of Tunis will be highly congested by the year 2000 if no improvements are made. In contrast, the entries of the trip matrix corresponding to regions in the south are very low, which results in no improvement even for unpaved roads.

Table 15 gives for each link to be improved the interregional flow, the existing capacity available to this flow (60 % of the total existing capacity), the added capacity, its cost and the maximum added capacity allowed by the formulation. The improvement required for link 14 is very small; a slight improvement of the surface of this link gives it exactly the same capacity as the adjacent link 79 which does not need improvement although it is closer to the capital. All other improvements are more significant and three of them, namely those on links 3, 20 and 26, require the maximum added capacity allowed by the formulation. For each of the links the flow is higher than the new capacity; more detailed study is needed to include the possibility of upgrading them to divided four-lane highways. Link 90 is also congested; however, the solution for the system does not add more capacity although it has the possibility to do so. The reason is that the congestion on those roads is less expensive than the addition of more capacity.

An analysis of the results obtained (Ben-Ayed 1988) draws the attention to several possible improvements of the empirical formulation. First, the travel times at the different levels of service A to E are so close, according to the HCM, that the resulting system and cumulative user costs at stable flow turned out to be one-piece linear functions that do not depend on the added capacity. The only instrument in the model for the system to influence the user's choice of the routes is by adding capacity if flow is unstable; therefore, the ability of the system to affect the user-optimized equilibrium is almost negligible. More detailed empirical data about travel costs are needed to represent the effects of congestion on user costs.

Second, the data about land purchase should be included in the study. Their unavailability meant that new links could not be considered and restricted investments to improvement of existing links without allowing the added width to go beyond the existing shoulders. This limitation resulted in low improvement costs. It can be seen from Table 15 that improvement costs are about fifty times smaller than travel costs. The introduction of the sophisticated BLP formulation of the investment costs did not give much saving as compared to the simpler LP formulation because the costs are already low according to the data of the problem.

9. CONCLUSIONS

This paper is an attempt to go beyond theoretical formulations and the small illustrative examples to much more interesting real world problems. The study was devoted to the construction of a Bilevel Linear Programming formulation, a major step in the formal optimization of the inter-regional highway network of a developing country. The Tunisian case study gave an illustration for the application of some advanced operations research techniques, such as Bilevel Linear Programming, in real situations.

The reliability of any formulation is conditioned by the realism of the formulation and the tractability of the resulting problem to be solved. The problem formulated in this paper has been shown to be tractable from the computational point of view (Ben-Ayed 1988 and Ben-Ayed, Blair and Boyce 1988). The complexity of the formulation does not present a barrier to solving the problem. However, the refinement of the formulation and the success to solving the resulting optimization problem do not necessarily guarantee the credibility of the solution. The most important dilemma is the availability of the data. Careful and detailed theoretical and empirical studies are needed to give more representative data, including better travel cost functions and better improvement cost functions for two-lane and unpaved highways.

Finally, our fixed demand, user-equilibrium route choice formulation is somewhat artificial. A further study could include stochastic route choice and estimation of travel demand. Moreover, transportation improvements often have a considerable impact on regional location decisions, potentially implying income redistribution effects of considerable importance. It is therefore hardly surprising that motives of income redistribution have a way of appearing implicitly or explicitly in the rationale for many public transportation investments (Meyer and Straszheim 1971). In fact, transportation investments can result directly in improved productivity and expanded employment over the long run; transportation infrastructure can affect ways in which regions and communities develop (National Transportation Policy Commission 1979). Therefore, the study could be expanded to optimize societal objectives such as a more balanced economic growth among the regions. One simple way to include those concepts in the optimization model is to extend the same problem we solved, modifying it to introduce higher trip matrix entries to the less developed regions, so that more budget is allocated to the roads in those regions in order to help activate their development.

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link number	from node	to node	length (km)	terrain (1,2,3)	roadway width(m)	shoulders width (m)	roadway surface(1-9)	shoulders surface(4-9)
1	1	2	65	1	8.0	5.	1	8
2	1	20	65	1	7.0	5.	4	8
3	1	23	37	1	10.0	5.	1	8
4	1	24	23	1	7.0	5.	4	8
5	1	25	31	1	7.0	5.	4	8
6	1	26	53	1	6.5	5.	4	8
7	2	20	37	1	6.5	5.	4	8
8	3	20	64	2	6.5	5.	5	8
9	3	21	38	2	4.5	7.	5	8
10	3	28	47	1	6.5	5.	5	8
11	3	29	37	2	4.0	7.	6	8
12	3	31	22	1	6.5	5.	6	8
13	4	5	47	2	6.5	5.	6	8
14	4	29	68	2	6.5	5.	6	8
15	4	33	60	3	6.5	5.	5	8
16	4	48	52	3	6.5	5.	5	8
17	4	49	59	3	4.5	7.	5	8
18	5	30	24	2	6.5	5.	6	8
19	5	31	23	1	6.5	5.	6	8
20	6	23	30	1	7.5	5.	4	8
21	6	27	14	1	8.0	5.	1	8
22	7	25	28	1	4.5	7.	4	8
23	7	26	13	1	5.5	6.	4	8
24	7	32	27	1	4.5	7.	4	8
25	7	35	41	1	4.5	7.	4	8
26	8	10	23	1	10.0	5.	1	8
27	8	11	54	1	7.0	5.	4	8
28	8	35	44	1	8.0	5.	1	8
29	8	38	31	1	6.5	5.	4	8
30	8	41	12	1	8.0	5.	1	8
31	8	39	26	1	6.5	5.	4	8
32	9	14	104	1	4.5	7.	5	8
33	9	39	25	1	4.5	7.	4	8
34	9	40	41	1	4.5	7.	4	8
35	10	39	21	1	6.5	5.	5	8
36	10	41	25	1	6.5	5.	5	8
37	11	12	93	1	4.5	7.	5	8
38	11	37	80	1	6.5	5.	5	8
39	11	38	36	1	6.5	5.	5	8
40	11	44	24	1	6.5	5.	4	8
41	11	47	17	1	6.5	5.	4	8
42	12	29	51	2	5.5	6.	5	8
43	12	32	67	2	6.5	5.	5	8
44	12	33	34	2	6.5	5.	5	8
45	13	16	112	2	6.5	5.	4	8
46	13	34	27	2	6.5	5.	4	8
47	13	49	66	3	6.5	5.	4	8
48	14	40	62	1	7.5	5.	2	8
49	14	42	82	1	7.0	5.	5	8
50	14	43	75	1	6.5	5.	5	8
51	14	50	61	1	6.5	5.	5	8
52	14	58	69	1	5.5	6.	5	8
53	15	46	16	2	5.5	6.	5	8
54	15	51	55	1	4.5	7.	5	8
55	15	55	19	1	5.5	6.	5	8
56	15	56	23	2	5.5	6.	5	8
57	16	17	143	1	6.5	5.	4	8
58	16	19	97	1	6.5	5.	4	8
59	16	51	77	2	6.5	5.	4	8
60	16	56	66	2	6.5	5.	4	8
61	17	18	77	1	6.5	5.	4	8
62	17	42	51	1	6.5	5.	5	8
63	17	52	125	1	6.5	5.	4	8
64	17	53	42	2	4.0	7.	4	8
65	18	53	73	2	10.0	0	8	8
66	19	52	99	1	4.0	7.	4	8
67	20	21	70	1	6.5	5.	5	8
68	20	24	48	1	5.5	6.	4	8
69	21	22	34	2	6.5	5.	6	8
70	22	30	43	2	6.5	5.	6	8
71	23	27	26	1	8.0	5.	1	8
72	24	25	22	1	5.5	6.	4	8
73	24	28	34	1	7.0	5.	5	8
74	25	28	35	1	7.0	5.	5	8
75	25	32	29	1	7.0	5.	4	8
76	26	27	29	1	7.5	5.	1	8
77	26	36	18	1	6.5	5.	4	8

Table 1   Definition of Links

link number	from node	to node	length (km)	terrain (1,2,3)	roadway width(m)	shoulders width (m)	roadway surface(1-9)	shoulders surface(4-9)
78	27	36	16	1	7.5	5.	1	8
79	28	29	43	2	6.5	5.	5	8
80	28	32	45	2	4.5	7.	5	8
81	29	31	38	2	4.0	7.	6	8
82	30	31	37	2	4.0	7.	6	8
83	32	35	58	1	4.0	7.	4	8
84	32	37	28	2	6.5	5.	4	8
85	33	44	58	3	5.5	6.	5	8
86	33	48	25	3	6.5	5.	5	8
87	34	45	28	2	4.0	7.	4	8
88	34	46	23	2	6.5	6.	4	8
89	34	48	66	3	4.0	5.	4	8
90	35	36	21	1	8.0	5.	1	8
91	35	38	24	1	7.0	5.	4	8
92	37	38	41	1	10.0	0	8	8
93	39	40	41	1	4.5	7.	5	8
94	39	41	47	1	5.0	6.	4	8
95	40	41	52	1	8.0	5.	1	8
96	40	58	48	1	5.5	6.	5	8
97	42	50	71	1	10.0	0	8	8
98	42	57	42	1	10.0	0	8	8
99	43	50	16	1	10.0	0	8	8
100	43	54	38	1	4.5	7.	5	8
101	43	55	36	1	6.5	5.	5	8
102	44	45	54	1	6.5	5.	5	8
103	44	47	19	1	4.0	7.	5	8
104	45	46	22	1	6.5	5.	5	8
105	46	56	29	1	6.5	5.	5	8
106	47	54	17	1	6.5	5.	5	8
107	47	55	69	1	10.0	0	8	8
108	48	49	35	3	10.0	0	8	8
109	50	57	32	1	4.5	7.	5	8
110	51	57	24	2	4.5	7.	5	8
111	52	53	81	1	10.0	0	8	8
112	54	58	33	1	5.5	6.	5	8

Table 1 (Continued) Definition of Links

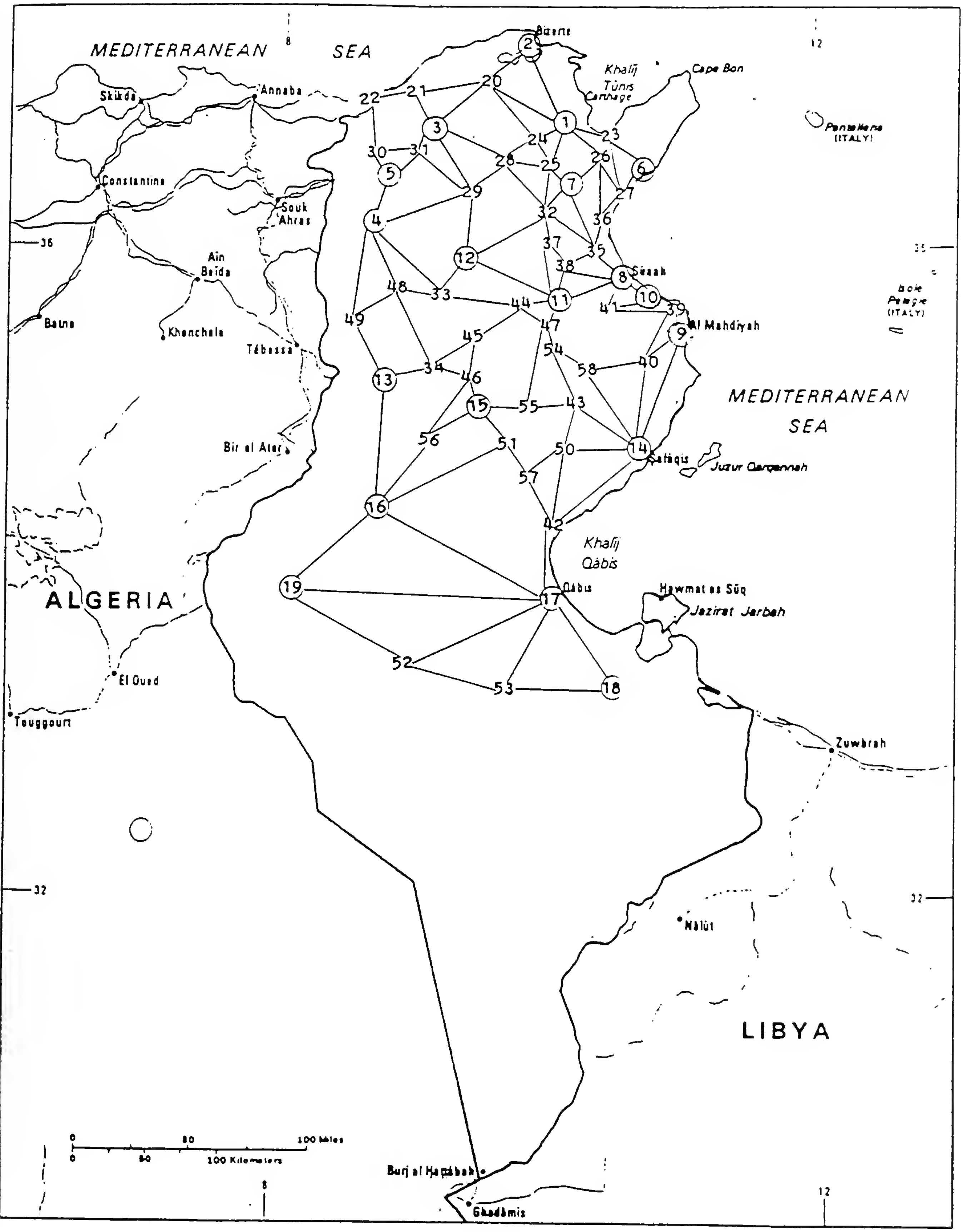

Figure 1 Tunisian Inter-Regional Highway Network

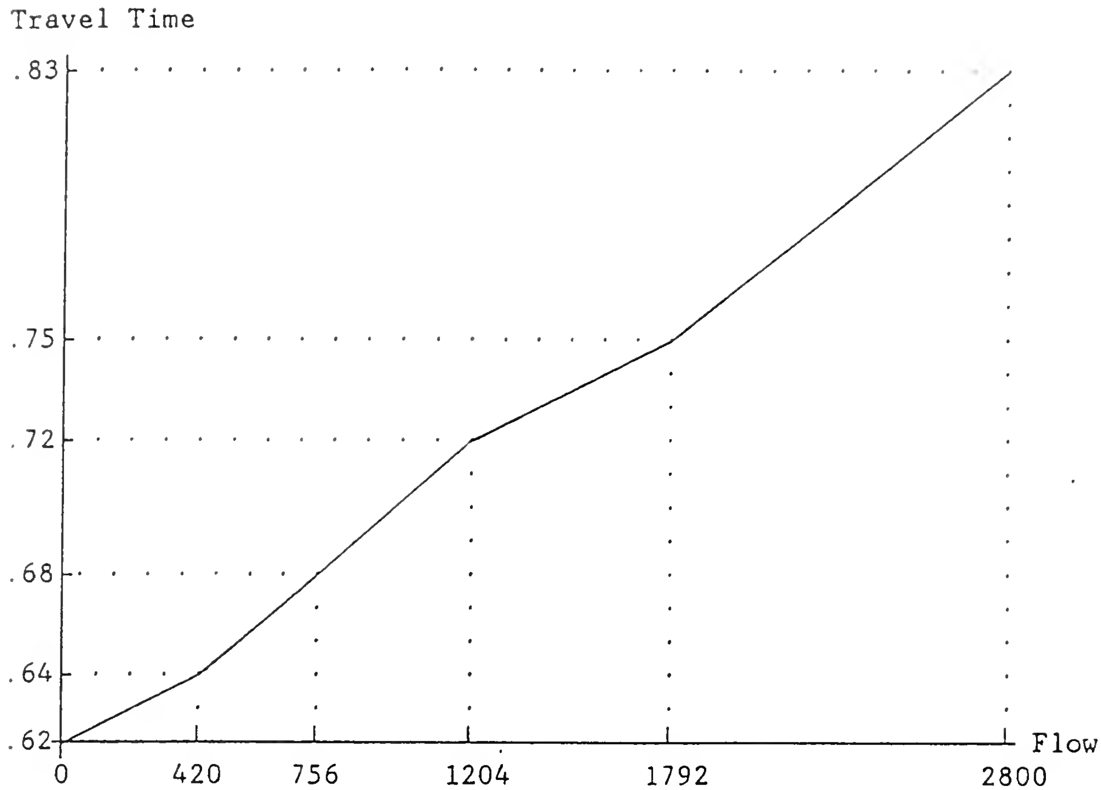

Figure 2 Average Travel Time Function

Maximum Ratios R_i of Flow to Ideal Capacity (Both Directions)						
Flow (LOS)	0 FLOW	LOS A	LOS B	LOS C	LOS D	LOS E
	Level Terrain					
Average Speed in KM/H	95	92	87	82	79	71
Ratio R_i	0	.12	.24	.39	.62	1.00
	Rolling Terrain					
Average Speed in KM/H	90	86	81	77	74	60
Ratio R_i	0	.07	.19	.35	.52	.92
	Mountainous Terrain					
Average Speed in KM/H	75	70	68	61	56	44
Ratio R_i	0	.04	.13	.23	.40	.82

Table 2

Adjustment Factors W for the Combined Effect of Roadway and Shoulder Width								
Width in Meters of Both Usable Shoulders	Width in Meters of Both Lanes							
	10.0		7.5		7.0		6.0	
	LOS A-D	LOS E	LOS A-D	LOS E	LOS A-D	LOS E	LOS A-D	LOS E
≥ 4.0	1.28	1.28	1.02	1.02	.97	.97	.82	.85
2.5	1.20	1.25	.94	.99	.89	.95	.75	.83
1.0	1.04	1.19	.81	.94	.76	.89	.65	.78
0	1.00	1.10	.72	.90	.67	.85	.57	.74
Width in Meters of Both Usable Shoulders	Width in Meters of Both Lanes							
	5.5		5.0		4.5		4.0	
	LOS A-D	LOS E	LOS A-D	LOS E	LOS A-D	LOS E	LOS A-D	LOS E
≥ 7.0	.70	.76	.65	.71	.60	.66	.54	.60
4.0	.70	.76	.62	.70	.53	.62	.42	.53
2.5	.66	.74	.58	.68	.48	.60	.38	.52
1.0	.56	.70	.50	.63	.41	.55	.32	.47
0	.49	.66	.43	0.59	.36	.52	.28	.43

Table 3

Adjustment Factors P for the Surface of the Roadway			
Quality	Good	Fair	Poor
Asphaltic	1.0	.8	.5
Treatment	.9	.7	.4
Unpaved	.5	.4	.2

Table 4

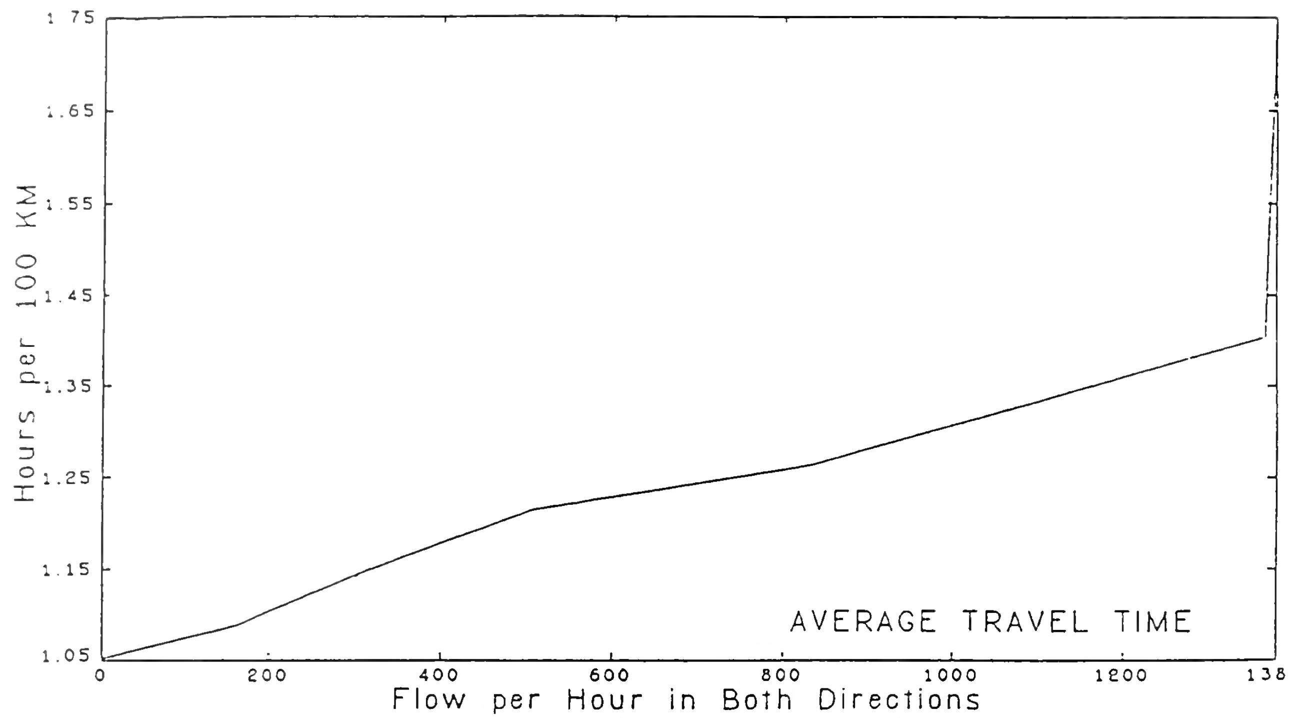

Figure 3

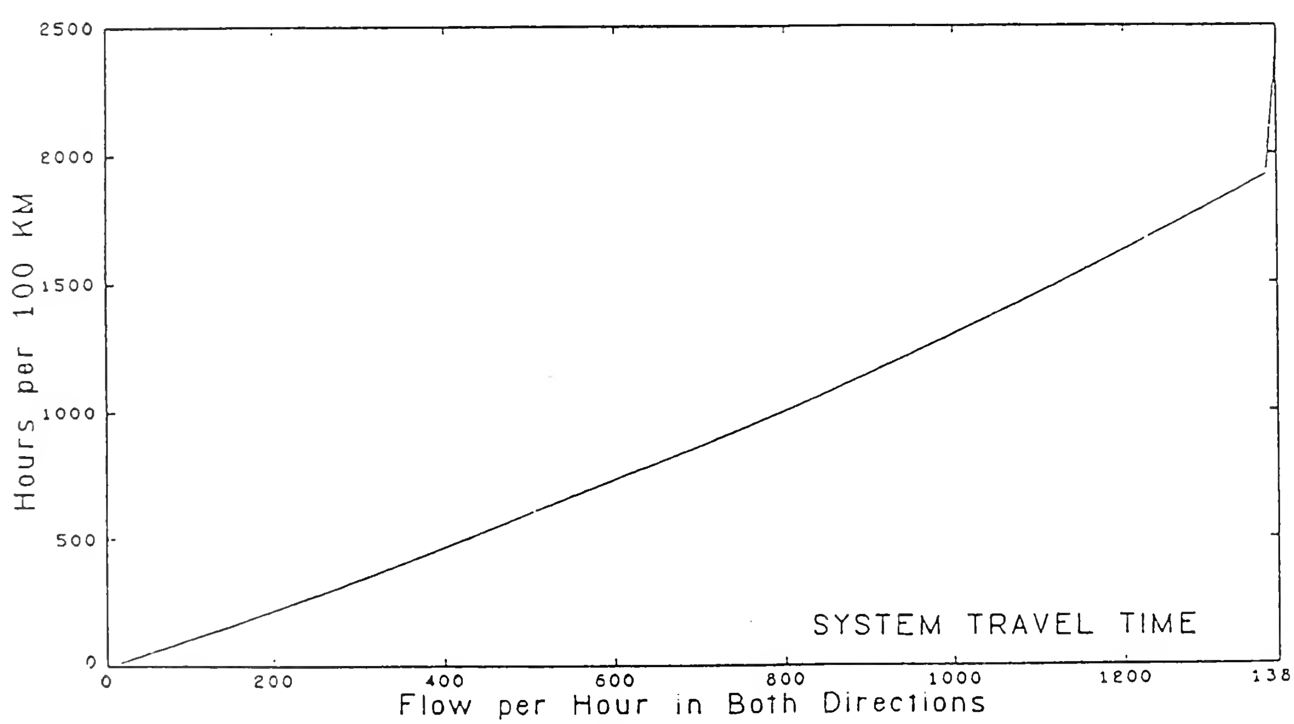

Figure 4

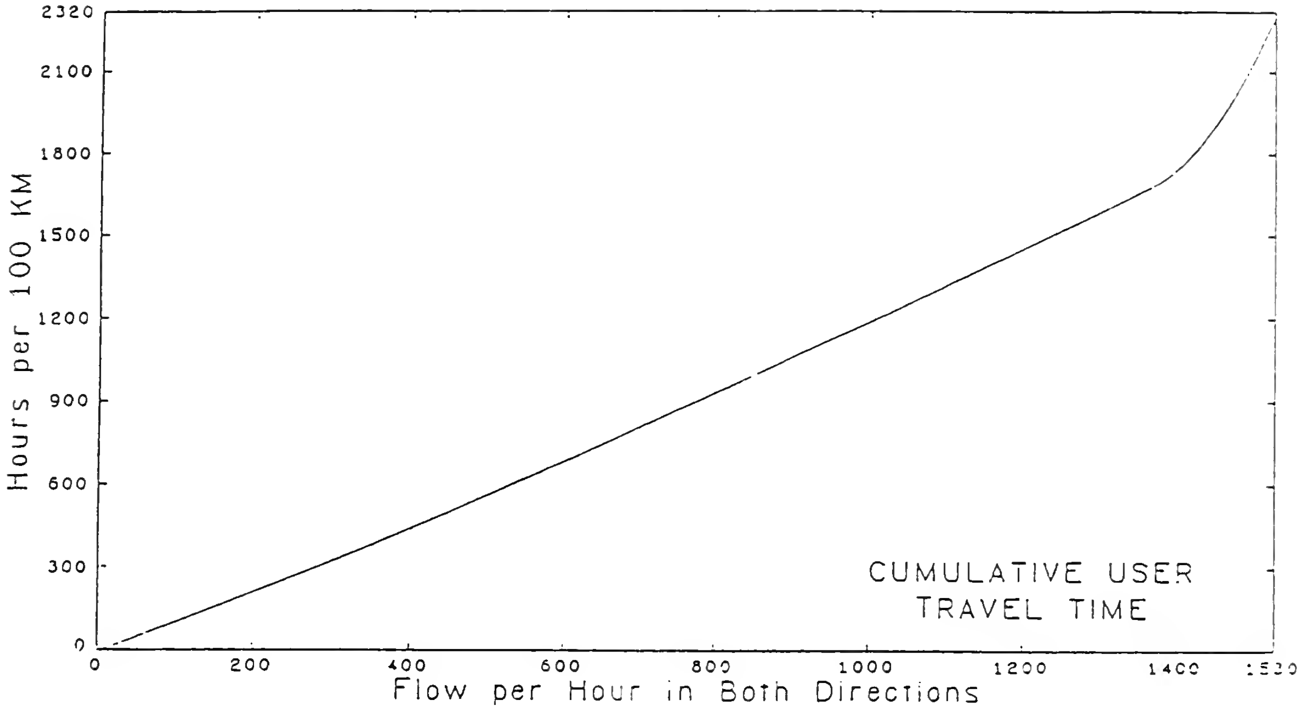

Figure 5

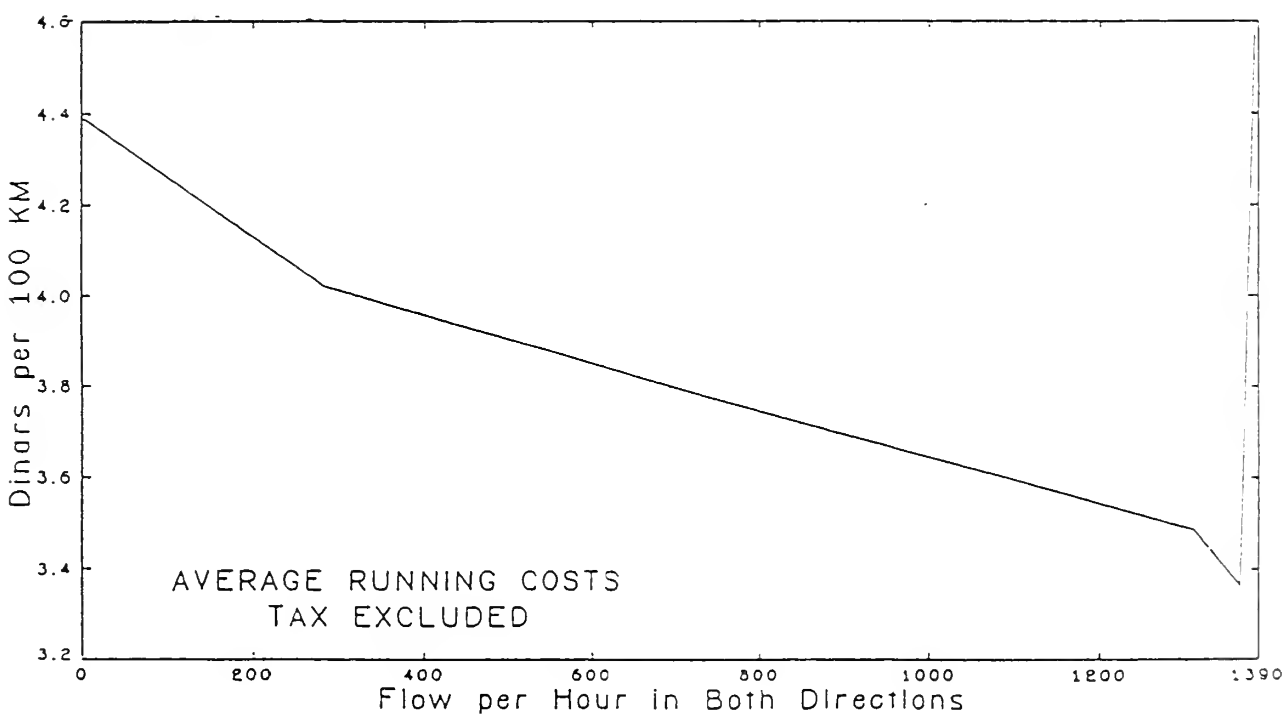

Figure 6

Adjustment Factors S for the Quality of the Shoulders				
Shoulders Quality	Treatment Good	Treatment Poor	Unpaved Good	Unpaved Poor
≥ 4 M width shoulders	1.00	.95	.97	.90
0 M width shoulders	1.00	1.00	1.00	1.00

Table 5

Average Running Costs in TD per 100 KM for 1980					
Tax	Fuel	Grease-Oil	Tires	Depreciation	Total
Excluded	.8623	.0299	.1503	.4139	1.4564
Included	1.7500	.0447	.2322	.4967	2.5236

Table 6

Adjustment Factors for the Effect of the State of the Surface on the Running Costs				
State of the Surface	Fuel	Grease-Oil	Tires	Depreciation
Paved and Good	.96	.94	.92	.80
Paved and Fair	1.00	1.00	1.00	1.00
Paved and Poor	1.04	1.06	1.08	1.20
Unpaved and Good	1.20	1.11	2.41	1.40
Unpaved and Fair	1.26	1.25	3.73	1.68
Unpaved and Poor	1.37	1.39	5.06	1.96

Table 7

Adjustment Factors for the Effect of the Terrain on the Fuel Consumption			
Terrain	Level	Rolling	Mountainous
Adjustment	1.000	1.022	1.041

Table 8

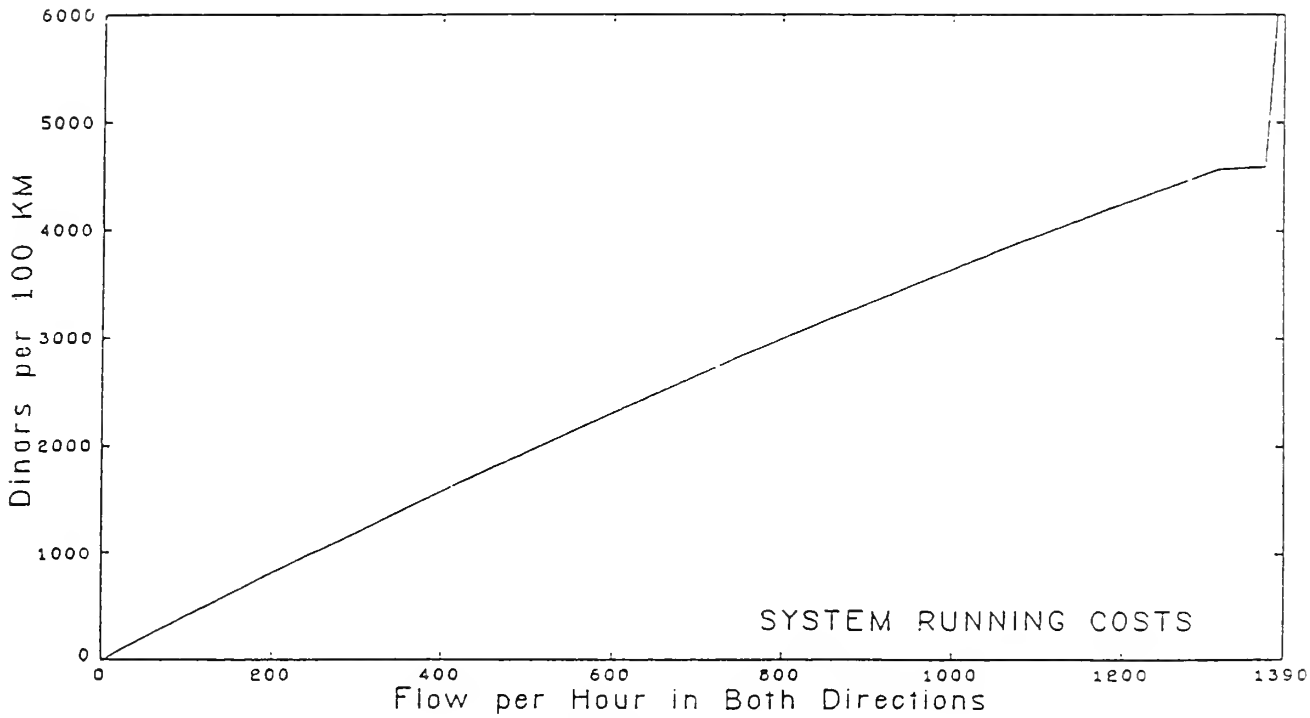

Figure 7

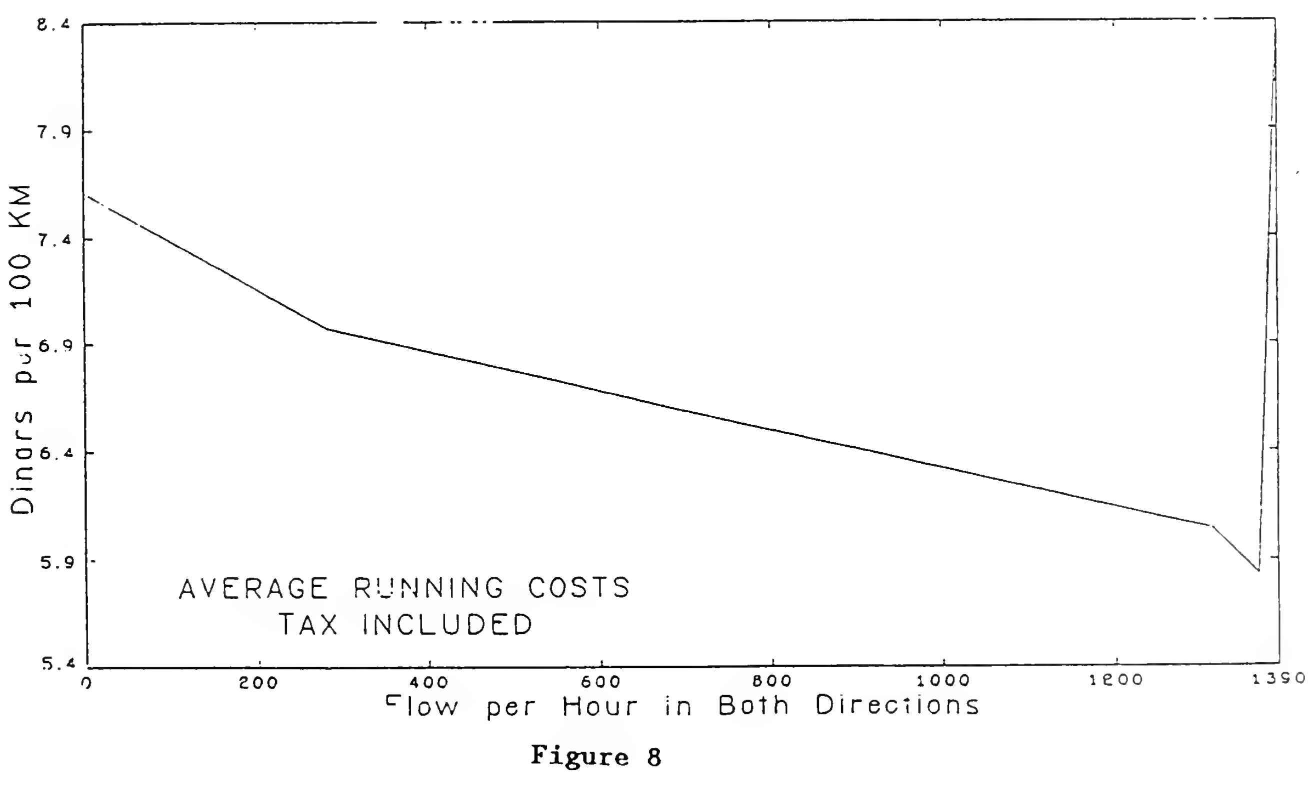

Figure 8

Running Costs (Tax Excluded in TD per 100 KM) as a Function of the Speed in KM/H for 1980						
Speed	24	32	40	48	56	64
Cost	1.241	1.204	1.148	1.174	1.200	1.291
Speed	72	80	86	96	104	112
Cost	1.344	1.456	1.551	1.717	1.903	2.241

Table 9

Costs (Tax Included) of Surface Improvement in TD per M^2			
Existing Surface	Asphaltic Good	Treatment Good	Unpaved Good
Asphaltic-Fair	6.0	-	-
Asphaltic-Poor	9.0	-	-
Treatment-Good	9.5	-	-
Treatment-Poor	11.0	5.5	-
Unpaved-Good	12.0	6.5	-
Unpaved-Poor	13.0	7.5	2.5

Table 10

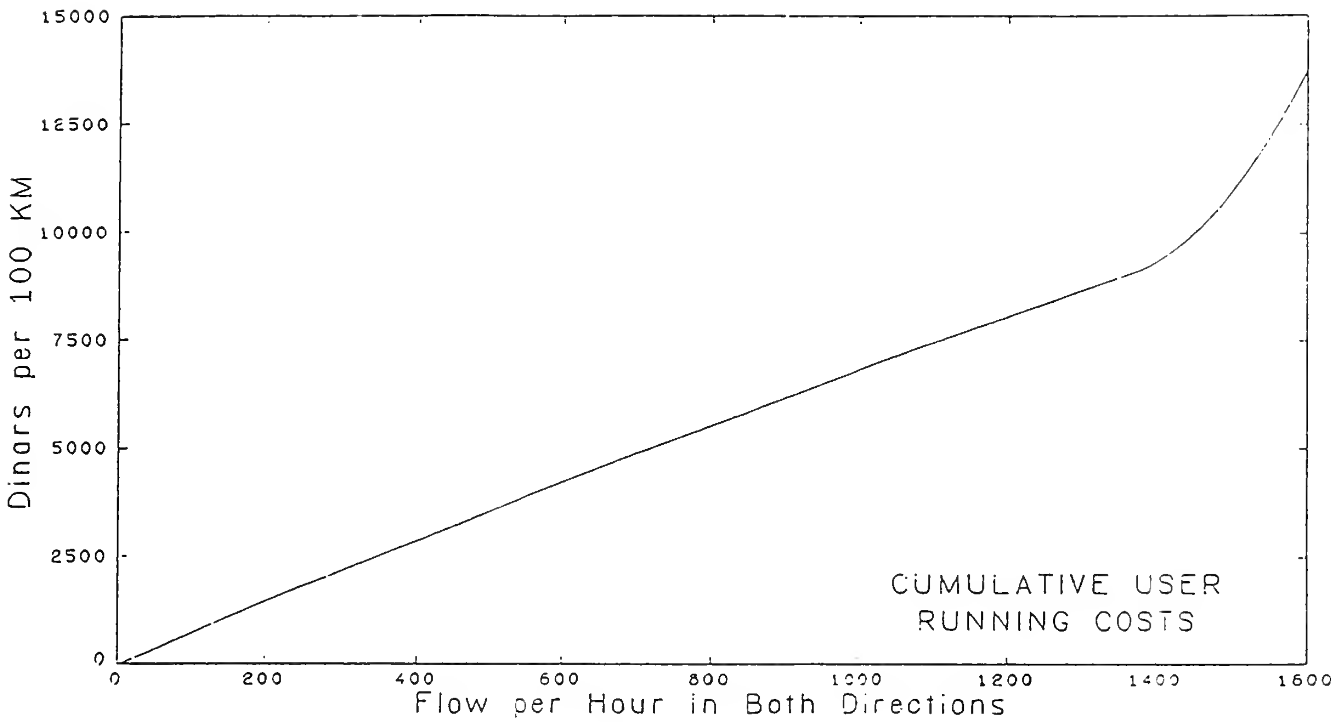

Figure 9

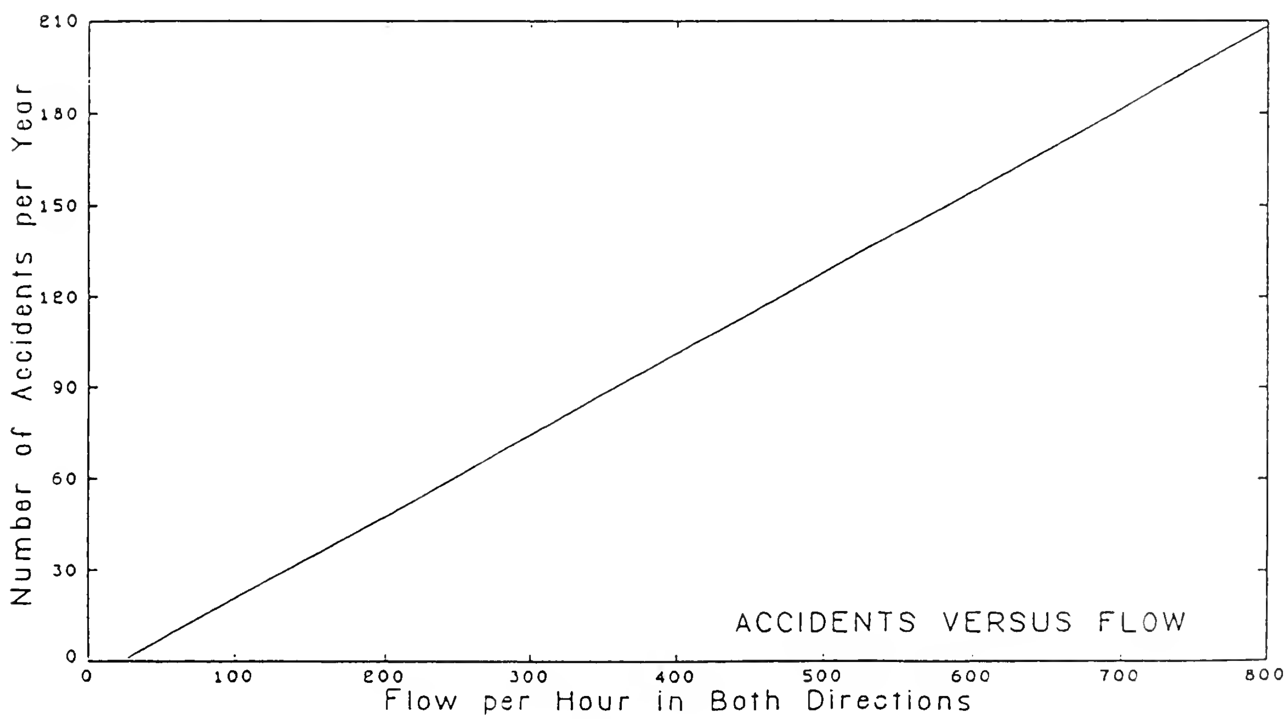

Figure 10

Annual Fixed Maintenance Costs in TD per KM for 1981				
Type of Road	Earth	Paved 4-5 M	Paved 6-7 M	Paved 9-10M
Annual Costs	120	220	260	330

Table 11

Lifetime of Investment			
Type of Road	Earth	Treatment	Asphaltic
Lifetime(Years)	7.5	10.5	18

Table 12

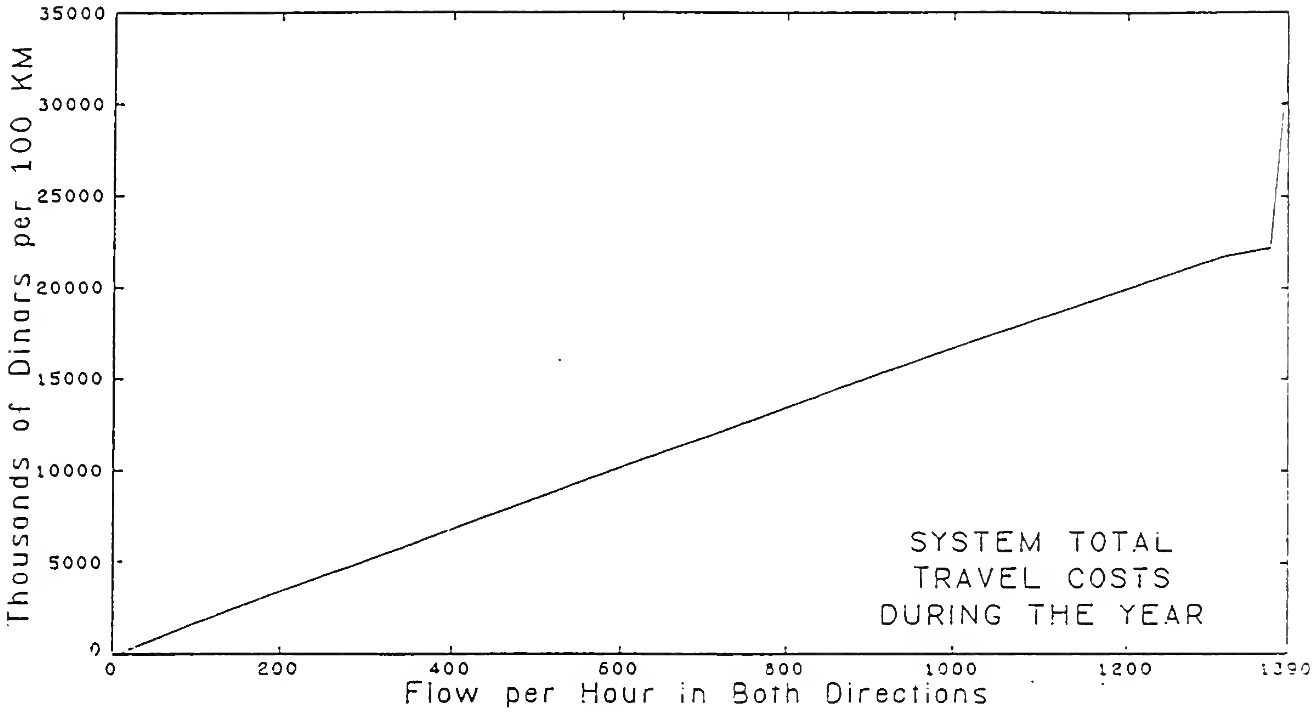

Figure 11

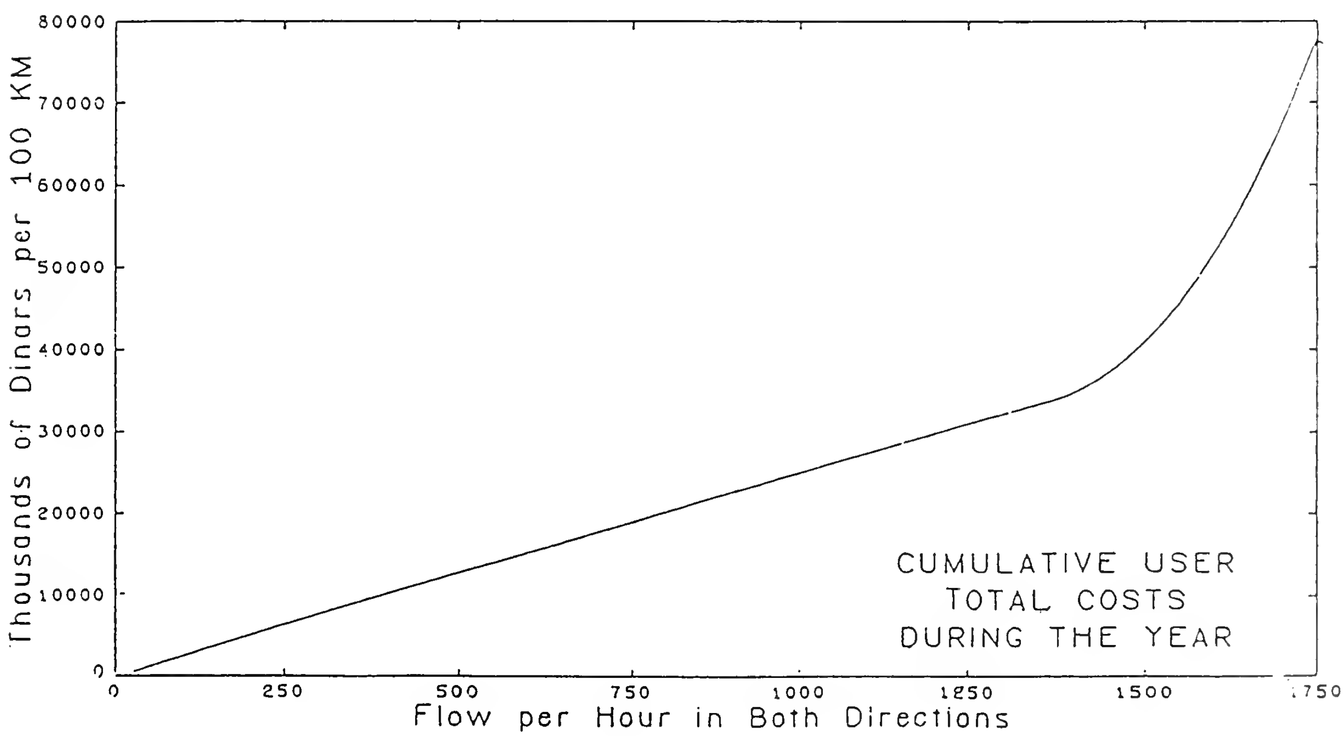

Figure 12

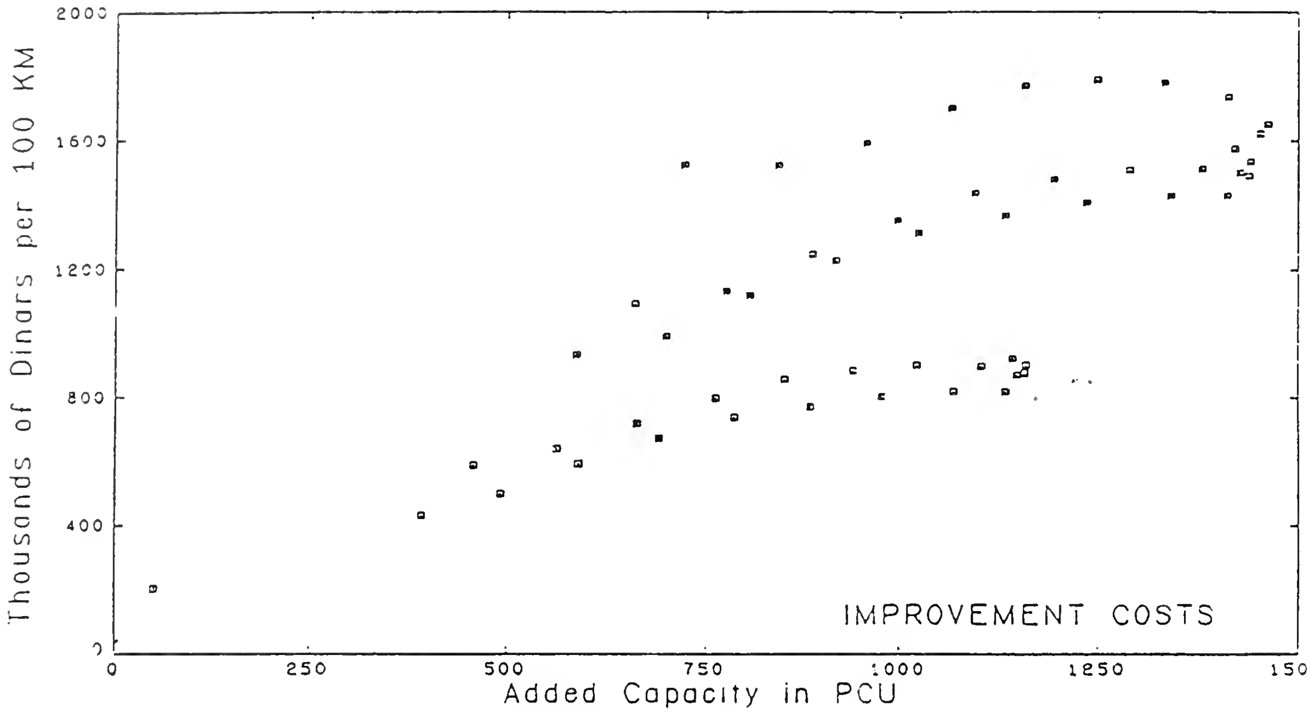

Figure 13

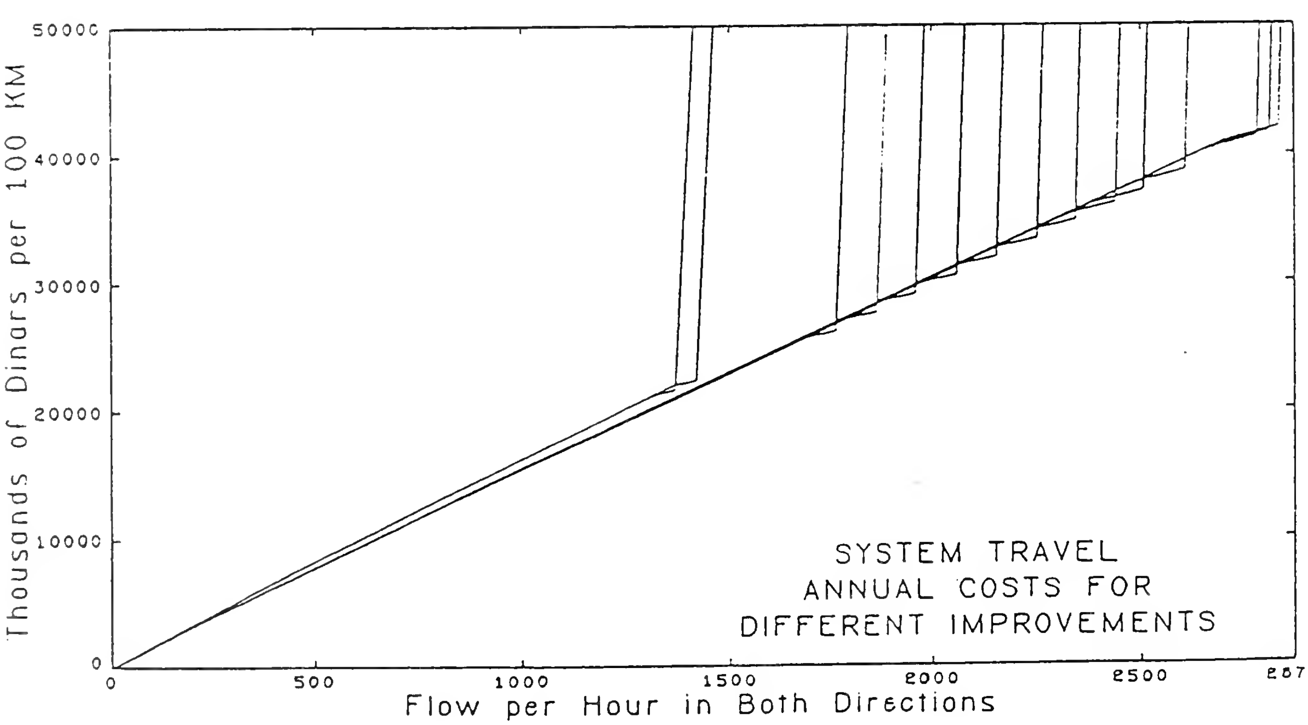

Figure 14

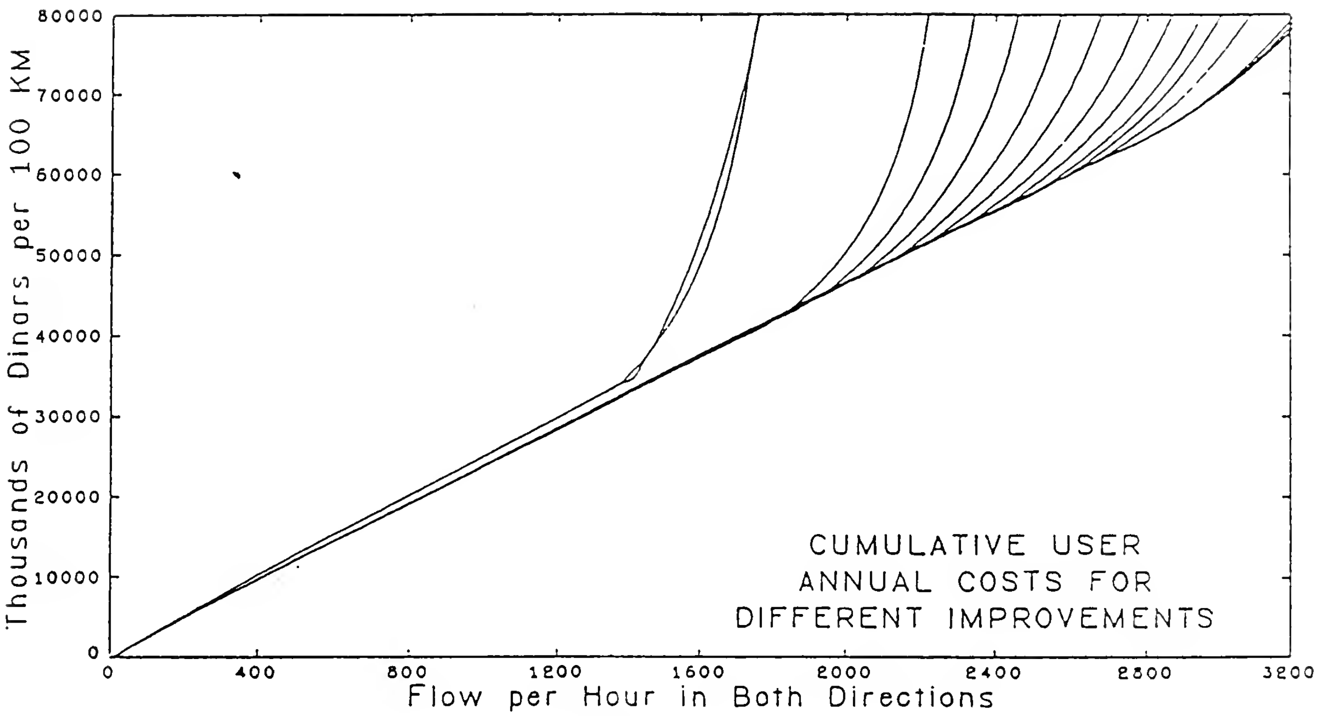

Figure 15

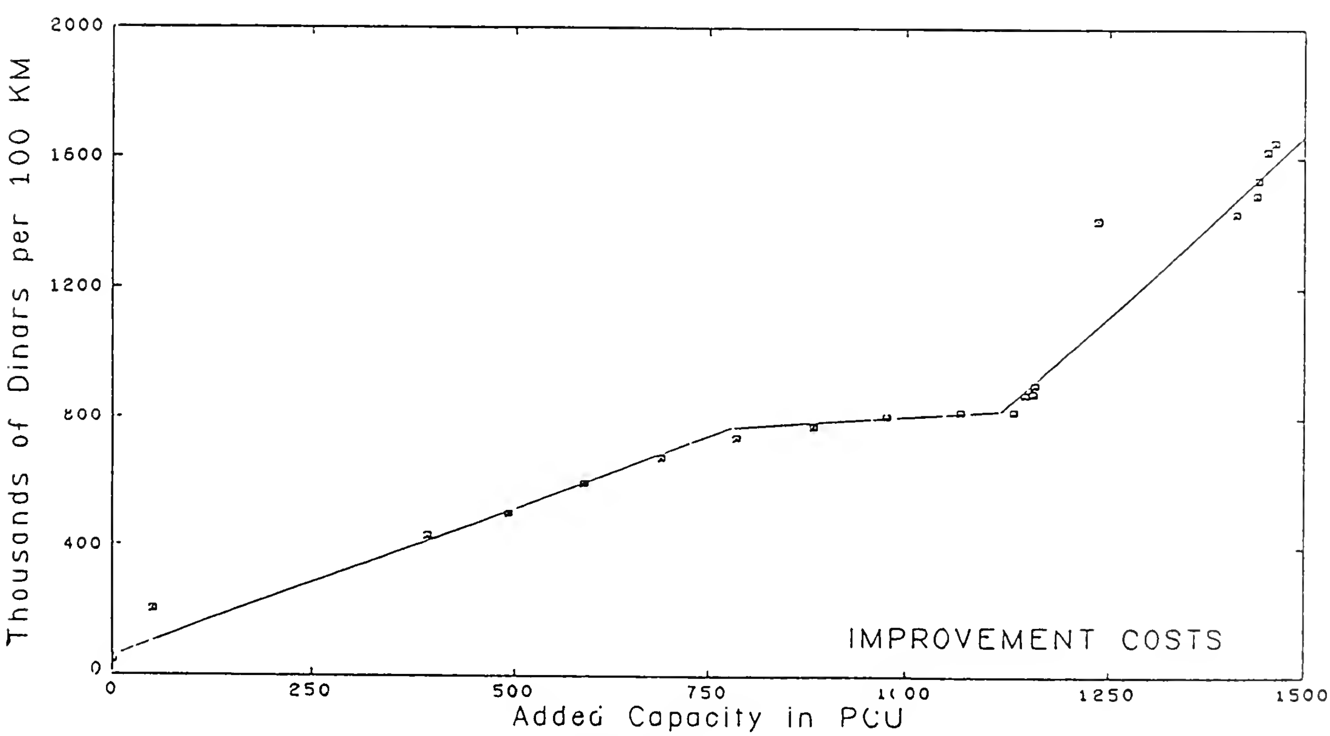

Figure 16

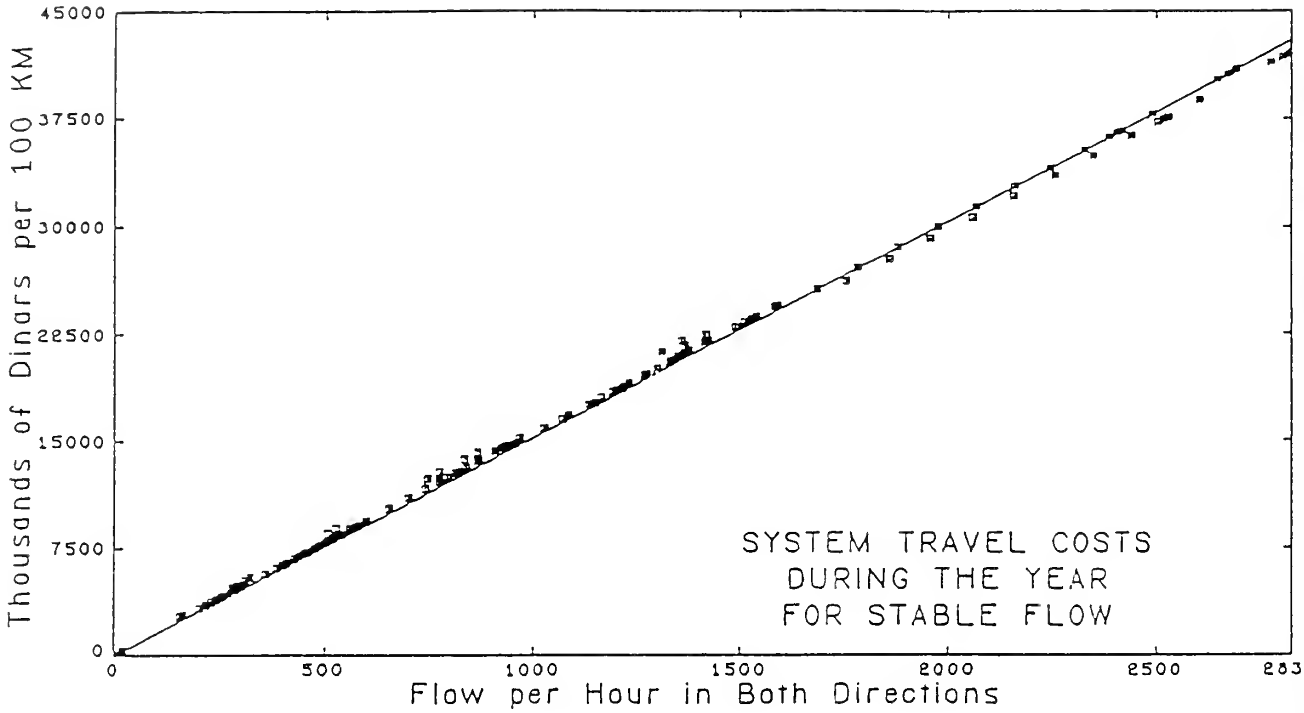

Figure 17

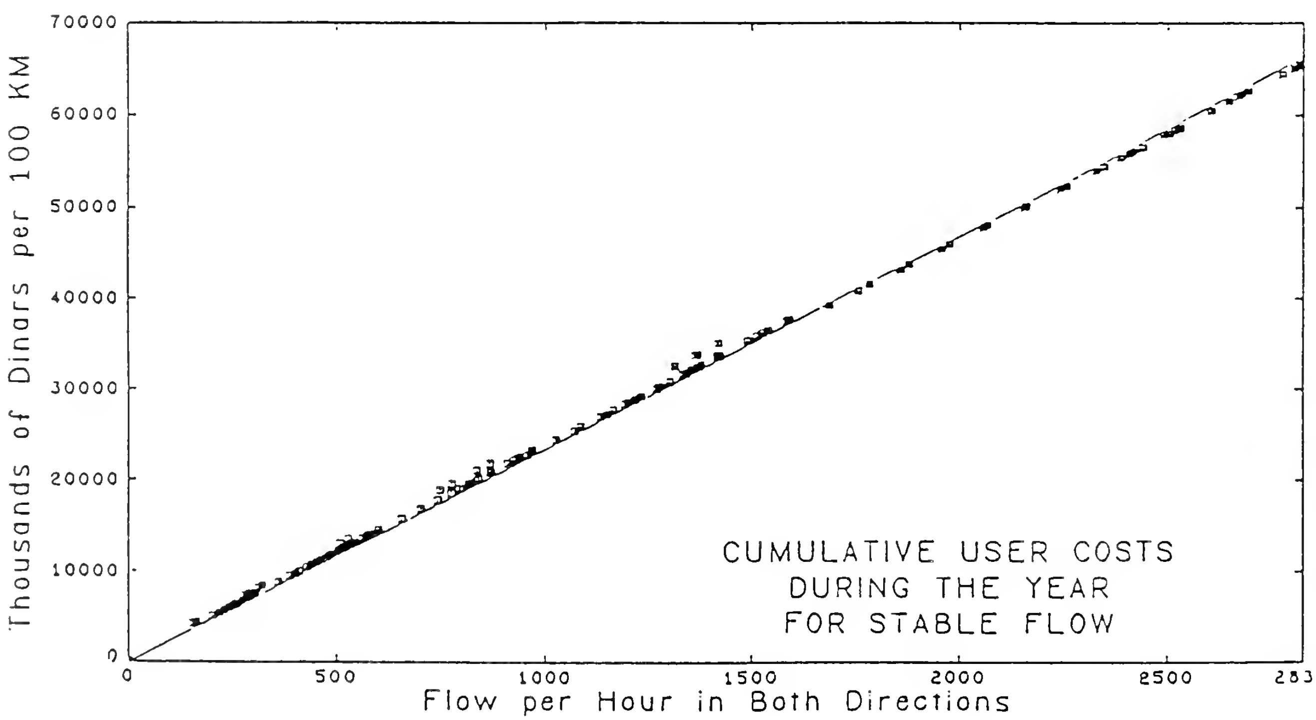

Figure 18

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	total
1		916	491	146	419	1330	116	361	67	100	86	62	24	251	9	14	48	15	16	4471
2	916		51	7	8	60	28	22	3	15	1	2	3	23	0	1	2	1	0	1143
3	491	51		25	40	28	15	8	3	8	3	22	4	3	2	1	4	0	1	709
4	146	7	25		49	4	3	9	3	9	9	35	37	6	1	1	1	0	1	346
5	419	8	40	49		13	5	4	2	2	4	2	2	5	1	2	2	0	1	561
6	1330	60	28	4	13		20	50	6	29	15	4	4	33	1	1	3	0	1	1602
7	116	28	15	3	5	20		18	6	16	17	12	1	5	1	2	1	0	1	267
8	361	22	8	9	4	50	18		91	1409	183	11	8	67	6	2	12	2	1	2264
9	67	3	3	3	2	6	6	91		162	24	3	4	170	5	2	5	1	1	558
10	100	15	8	9	2	29	16	1409	162		22	15	5	31	2	1	3	1	1	1831
11	86	1	3	9	4	15	17	183	24	22		23	9	37	25	7	3	0	2	470
12	62	2	22	35	2	4	12	11	3	15	23		8	3	4	3	1	0	1	211
13	24	3	4	37	2	4	1	8	4	5	9	8		14	8	16	2	0	4	153
14	251	23	3	6	5	33	5	67	170	31	37	3	14		81	9	39	3	5	785
15	9	0	2	1	1	1	1	6	5	2	25	4	8	81		16	10	0	9	181
16	14	1	1	1	2	1	2	2	2	1	7	3	16	9	16		28	0	16	122
17	48	2	4	1	2	3	1	12	5	3	3	1	2	39	10	28		31	12	207
18	15	1	0	0	0	0	0	2	1	1	0	0	0	3	0	0	31		1	55
19	16	0	1	1	1	1	1	1	1	1	2	1	4	5	9	16	12	1		74
total	4471	1143	709	346	561	1602	267	2264	558	1831	470	211	153	785	181	122	207	55	74	16010

Table 13 Original Trip Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	total
2	1832																		1832
3	982	102																	1084
4	292	14	50																356
5	838	16	80	98															1032
6	2660	120	56	8	26														2870
7	232	56	30	6	10	40													374
8	1056	80	38	36	12	170	80												1472
9				6	4			352											362
10								3260	324										3584
11	172	2	6	18	8	36	34	454	48										778
12	124	4	44	70	4	8	24	52	6		46								382
13	48	6	8	74	4	8	2		8		44	16							218
14	502	46	6	12	10	66	10	134	340	62	74	6	28						1296
15	18		4	2	2		2		10	4	64	8	16	162					292
16	60	2	4	4	6		6		6	4	28	8	40	18	50				236
17	126	6	8	2	4	6	2	28	12	8	6	2	4	84	20	56			374
18																	108		108
19														10		112	24	2	148
total	8942	454	334	336	90	334	160	4280	754	78	262	40	88	274	70	168	132	2	16798

Table 14 Equivalent Trip Matrix

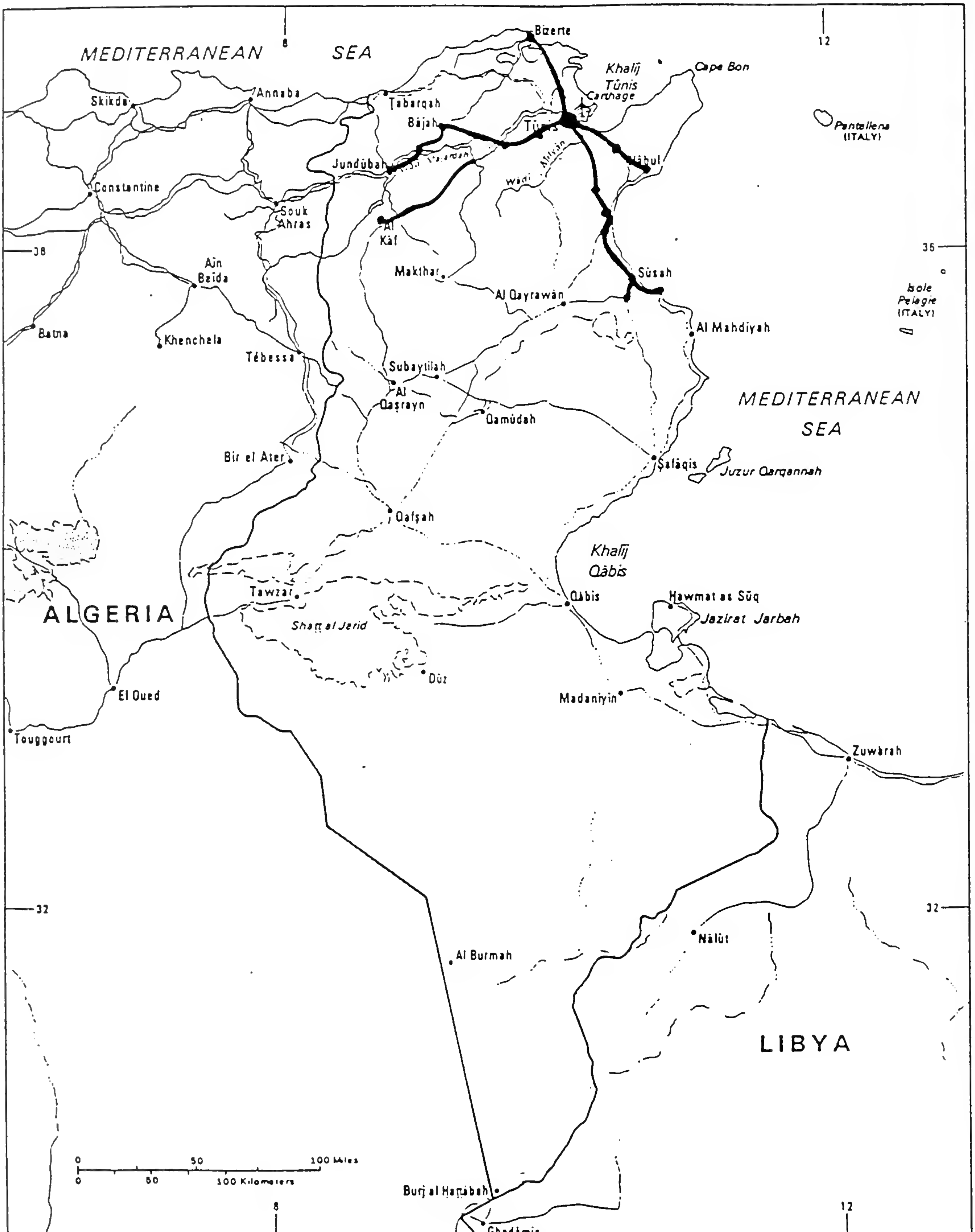

Figure 1 Links to Be Improved

Links to Be Improved					
Link	Flow	Existing Capacity	Added Capacity	Improvement Cost	Limit on Added Capacity
1	2144	1370	774	592	887
3	2537	1637	899	394	899
4	2112	1115	997	297	1083
6	2015	1055	960	615	1070
10	1980	814	1166	484	1460
12	1052	448	604	84	2046
14	306	271	35	22	1354
19	1056	447	609	89	2046
20	2275	1173	1101	463	1101
26	2537	1637	899	245	899
28	2010	1370	640	362	887
30	1517	1370	147	60	887
73	2112	862	1250	394	1496
77	1772	1055	717	156	1070
90	2058	1370	687	179	887

Table 15