THE

CONSTRUCTION OF LOGARITHMS

WITH A

CATALOGUE OF NAPIER'S WORKS

THE CONSTRUCTION
OF THE WONDERFUL CANON OF
LOGARITHMS

BY

JOHN NAPIER
BARON OF MERCHISTON

TRANSLATED FROM LATIN INTO ENGLISH WITH NOTES

AND

A CATALOGUE

OF THE VARIOUS EDITIONS OF NAPIER'S WORKS, BY

WILLIAM RAE MACDONALD, F.F.A.

WILLIAM BLACKWOOD AND SONS
EDINBURGH AND LONDON
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To

The Right Honourable

FRANCIS BARON NAPIER AND ETTRICK, K.T.

descendant of

John Napier of Merchiston

this Translation of the

Mirifici Logarithmorum Canonis Constructio

is dedicated with much

respect.

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INTRODUCTION.

JOHN NAPIER* was the eldest son of Archibald Napier and Janet Bothwell. He was born at Merchiston, near Edinburgh, in 1550, when his father could have been little more than sixteen.

Two months previous to the death of his mother, which occurred on 20th December 1563, he matriculated as a student of St Salvator's College, St Andrews. While there, his mind was specially directed to the study and searching out of the mysteries of the Apocalypse, the result of which appeared thirty years later in his first published work, 'A plaine discovery of the whole Revelation of St John.'

Had he continued at St Andrews, his name would naturally have appeared in the list of determinants for 1566 and of masters of arts for 1568. It is not, however, found with the names of the students who entered college along with him, so that he is believed to have left

* See note, p. 84, as to spelling of name.

the University previous to 1566 in order to complete his studies on the Continent.

He was at home in 1571 when the preliminaries were arranged for his marriage with Elizabeth, daughter of Sir James Stirling of Keir. The marriage took place towards the close of 1572. In 1579 his wife died, leaving him one son, Archibald, who, in 1627, was raised to the peerage by the title of Lord Napier, and also one daughter, Jane.

A few years after the death of his first wife he married Agnes, daughter of Sir James Chisholm of Cromlix, who survived him. The offspring of this marriage were five sons and five daughters, the best known of whom is the second son, Robert, his father's literary executor.

Leaving for a moment the purely personal incidents of Napier's life, we may here note the dates of a few of the many exciting public events which occurred during the course of it. In 1560 a Presbyterian form of Church government was established by the Scottish Parliament. On 14th August 1561, Queen Mary, the young widow of Francis II., sailed from Calais, receiving an enthusiastic welcome on her arrival in Edinburgh. Within six years, on 24th July 1567, she was compelled to sign her abdication. The year 1572 was signalised by the Massacre of St Bartholomew, which began on 24th August; exactly three months later, John Knox died. On 8th February 1587 Mary was beheaded at Fotheringay, and in May of the year following the Spanish Armada set sail. The last event we need mention was the death of Queen Elizabeth

Elizabeth on 24th March 1603, and the accession of King James to the throne of England.

The threatened invasion of the Spanish Armada led Napier to take an active part in Church politics. In 1588 he was chosen by the Presbytery of Edinburgh one of its commissioners to the General Assembly. In October 1593 he was appointed one of a deputation of six to interview the king regarding the punishment of the "Popish rebels," prominent among whom was his own father-in-law. On the 29th January following, $159\frac{3}{4}$,* the letter which forms the dedication to his first publication, 'A plaine discovery,' was written to the king.

Not long after this, in July 1594, we find Napier entering into that mysterious contract with Logan of Restalrig for the discovery of hidden treasure at Fast Castle.

Another interesting document written by Napier bears date 7th June 1596, with the title, 'Secrett inuentionis, proffitable & necessary in theis dayes for defence of this Iland & withstanding of strangers enemies of Gods truth and relegion.'

The versatility and practical bent of Napier's mind are further evidenced by his attention to agriculture, which was in a very depressed state, owing to the unsettled condition of the country. The Merchiston system of tillage by manuring the land with salt is described in a very rare tract by his eldest son, Archibald, to whom a mono-

* 1593 old style, 1594 new style. Under the old style the year commenced on 25th March.

poly of the system was granted under the privy seal on 22d June 1598. As Archibald Napier was quite a young man at the time, it is most probable the system was the result of experiments made by his father and grandfather.

About 1603, the Lennox, where Napier held large possessions, was devastated in the conflict between the chief of Macgregor and Colquhoun of Luss, known as the raid of Glenfruin. The chief was entrapped by Argyll, tried, and condemned to death. On the jury which condemned him sat John Napier. The Macgregors, driven to desperation, became broken men, and Napier's lands no doubt suffered from their inroads, as we find him on 24th December 1611 entering into a contract for mutual protection with James Campbell of Lawers, Colin Campbell of Aberuchill, and John Campbell, their brother-german.

To the critical events of 1588 which, as we have already seen, drew Napier into public life, is due the appearance in English of 'A plaine discovery,' already mentioned. The treatise was intended to have been written in Latin, but, owing to the events above referred to, he was, as he says, 'constrained of compassion, leaving the Latin to haste out in English the present work almost unripe.' It was published in $159\frac{3}{4}$. A revised edition appeared in 1611, wherein he still expressed his intention of rewriting it in Latin, but this was never accomplished.

Mathematics, as well as theology, must have occupied Napier's attention from an early age. What he had done in

in the way of systematising and developing the sciences of arithmetic and algebra, probably some years before the publication of 'A plaine discovery,' appears in the manuscript published in 1839 under the title 'De Arte Logistica.' From this work it appears that his investigations in equations had led him to a consideration of imaginary roots, a subject he refers to as a great algebraic secret. He had also discovered a general method for the extraction of roots of all degrees.

The decimal system of numeration and notation had been introduced into Europe in the tenth century. To complete the system, it still remained to extend the notation to fractions. This was proposed, though in a cumbrous form, by Simon Stevin in 1585, but Napier was the first to use the present notation.*

Towards the end of the sixteenth century, however, the further progress of science was greatly impeded by the continually increasing complexity and labour of numerical calculation. In consequence of this, Napier seems to have laid aside his work on Arithmetic and Algebra before its completion, and deliberately set himself to devise some means of lessening this labour. By 1594 he must have made considerable progress in his undertaking, as in that year, Kepler tells us, Tycho Brahe was led by a Scotch correspondent to entertain hopes of the publication of the Canon or Table of Logarithms. Tycho's informant is not named, but is

* See note, p. 88.

generally believed to have been Napier's friend, Dr Craig. The computation of the Table or Canon, and the preparation of the two works explanatory of it, the Constructio and Descriptio, must, however, have occupied years. The Canon, with the description of its nature and use, made its appearance in 1614. The method of its construction, though written several years before the Descriptio, was not published till 1619.

Napier at the same time devised several mechanical aids to computation, a description of which he published in 1617, 'for the sake of those who may prefer to work with the natural numbers,' the most important of these aids being named Rabdologia, or calculation by means of small rods, familiarly called 'Napier's bones.'

The invention of logarithms was welcomed by the greatest mathematicians, as giving once for all the long-desired relief from the labour of calculation, and by none more than by Henry Briggs, who thenceforth devoted his life to their computation and improvement. He twice visited Napier at Merchiston, in 1615 and 1616, and was preparing again to visit him in 1617, when he was stopped by the death of the inventor. The strain involved in the computation and perfecting of the Canon had been too great, and Napier did not long survive its completion, his death occurring on the 4th of April 1617. He was buried near the parish church of St Cuthbert's, outside the West Port of Edinburgh.

It has been stated that Napier dissipated his means on

on his mathematical pursuits. The very opposite, however, was the case, as at his death he left extensive estates in the Lothians, the Lennox, Menteith, and elsewhere, besides personal property which amounted to a large sum.

For fuller information regarding John Napier, the reader is referred to the Memoirs, published by Mark Napier in 1834, from which the above particulars are mainly derived.

The 'Mirifici Logarithmorum Canonis Constructio' is the most important of all Napier's works, presenting as it does in a most clear and simple way the original conception of logarithms. It is, however, so rare as to be very little known, many writers on the subject never having seen a copy, and describing its contents from hearsay, as appears to be the case with Baron Maseres in his well-known work, 'Scriptores Logarithmici,' which occupies six large quarto volumes.

In view of such facts the present translation was undertaken, which, it is hoped, will be found faithfully to reproduce the original. In its preparation valuable assistance was received from Mr John Holliday and Mr A. M. Laughton. The printing and form of the book follow the original edition of 1619 as closely as a translation will allow, and the head and tail pieces are in exact facsimile. To the work are added a few explanatory notes.

The second part of the volume consists of a Catalogue

of the various editions of Napier's works, giving title-page, full collation, and notes, with the names of the principal public libraries in the country, as well as of some on the Continent, which possess copies. No similar catalogue has been attempted hitherto, and it is believed it will prove of considerable interest, as showing the diffusion of Napier's writings in his own time, and their location and comparative rarity now. Appended are notes of a few works by other authors, which are of interest in connection with Napier's writings.

It will be seen from the Catalogue that Napier's theological work went through numerous editions in English, Dutch, French, and German, a proof of its widespread popularity with the Reformed Churches, both in this country and on the Continent. The particulars now given also show that a statement in the Edinburgh edition of 1611 has been misunderstood. Napier's reference to Dutch editions was supposed by his biographers to apply to the German translation of Wolffgang Mayer, the Dutch translation by Michiel Panneel, being apparently unknown to them. His arithmetical work, Rabdologia, also seems to have been very popular. It was reprinted in Latin, and translated into Italian and Dutch, abstracts also appearing in several languages.

Rather curiously, his works of greatest scientific interest, the Descriptio and Constructio have been most neglected. The former was reprinted in 1620, and also in Scriptores Logarithmici, besides being translated into

into English. The latter was reprinted in 1620 only. This neglect is no doubt largely accounted for by the advantage for practical purposes of tables computed to the base 10, an advantage which Napier seems to have been aware of even before he had made public his invention in 1614.

For the completeness of the Catalogue I am very largely indebted to the Librarians of the numerous libraries referred to. I most cordially thank them for their kind assistance, and for the very great amount of trouble they have taken to supply me with the information I was in search of. To Mr Davidson Walker my hearty thanks are also due for assistance in collating works in London libraries.

I have only to add that any communications regarding un-catalogued editions or works relating to Napier will be gladly received.

W. R. MACDONALD.

1 FORRES STREET, EDINBURGH,
December 25, 1888.

THE

CONSTRUCTION OF THE WONDERFUL CANON OF LOGARITHMS;

And their relations to their own natural numbers;

WITH

An Appendix as to the making of another and better kind of Logarithms.

TO WHICH ARE ADDED

Propositions for the solution of Spherical Triangles by an *easier method: with Notes on them and on the above-mentioned Appendix by the learned* HENRY BRIGGS.

By the Author and Inventor, *John Napier*, Baron *of Merchiston*, &c., in Scotland.

Printed by ANDREW HART,
OF EDINBURGH;
IN THE YEAR OF OUR LORD, 1619.

Translated from Latin into English by William Rae Macdonald, 1888.

TO THE READER STUDIOUS OF THE MATHEMATICS, GREETING.

SEVERAL *years ago* (*Reader, Lover of the Mathematics*) *my Father, of memory always to be revered, made public the use of the Wonderful Canon of Logarithms; but, as he himself mentioned on the seventh and on the last pages of the Logarithms, he was decidedly against committing to types the theory and method of its creation, until he had ascertained the opinion and criticism on the Canon of those who are versed in this kind of learning.*

But, since his departure from this life, it has been made plain to me by unmistakable proofs, that the most skilled in the mathematical sciences consider this new invention of very great importance, and that nothing more agreeable to them could happen, than if the construction of this Wonderful Canon, or at least so much as might suffice to explain it, go forth into the light for the public benefit.

Therefore, although it is very manifest to me that the Author had not put the finishing touch to this little treatise, yet I have done what in me lay to satisfy their most honourable request, and to afford some assistance to those especially who are weaker in such studies and are apt to stick on the very threshold.

Nor do I doubt, but that this posthumous work would have seen the light in a much more perfect and finished state, if God had granted a longer enjoyment of life to the Author, my most dearly loved father, in whom, by the opinion of the wisest men, among other illustrious gifts this showed itself pre-eminent, that the most difficult matters were unravelled by a sure and easy method, as well as in the fewest words.

You have then (kind Reader) in this little book most amply unfolded the theory of the construction of logarithms, (here called by him artificial numbers, for he had this treatise written out beside him several years before the word Logarithm *was invented,) in which their nature, characteristics, and various relations to their natural numbers, are clearly demonstrated.*

It seemed desirable also to add to the theory an Appendix as to the construction of another and better kind of logarithms (mentioned by the Author in the preface to his Rabdologiæ) *in which the logarithm of unity is* 0.

After this follows the last fruit of his labours, pointing to the ultimate perfecting of his Logarithmic Trigonometry, namely certain very remarkable propositions for the resolution of spherical triangles not quadrantal, without dividing them into quadrantal or rectangular triangles. These propositions, which are absolutely general, he had determined to reduce into order and successively to prove, had he not been snatched away from us by a too hasty death.

We have also taken care to have printed some Studies on the above-mentioned Propositions, and on the new kind of Logarithms, by that most excellent Mathematician Henry Briggs, *public Professor at* London, *who for the singular friendship which subsisted between him and my father of illustrious memory, took upon himself, in the most willing spirit, the very heavy labour of computing this new Canon, the method of its creation and the explanation of its use being*

being left to the Inventor. Now, however, as he has been called away from this life, the burden of the whole business would appear to rest on the shoulders of the most learned Briggs, *on whom, too, would appear by some chance to have fallen the task of adorning this Sparta.*

Meanwhile (Reader) enjoy the fruits of these labours such as they are, and receive them in good part according to your culture.

Farewell,

Robert Napier, Son.

THE CONSTRUCTION OF *THE WONDERFUL CANON* OF LOGARITHMS; (HEREIN CALLED BY THE AUTHOR THE ARTIFICIAL TABLE) *and their relations to* their natural *numbers.*

1. A LOGARITHMIC TABLE *is a small table by the use of which we can obtain a knowledge of all geometrical dimensions and motions in space, by a very easy calculation.*

IT is deservedly called very small, because it does not exceed in size a table of sines; very easy, because by it all multiplications, divisions, and the more difficult extractions of roots are avoided; for by only a very few most easy additions, subtractions, and divisions by two, it measures quite generally all figures and motions.

It is picked out from numbers progressing in continuous proportion.

2. *Of continuous progressions, an arithmetical is one which proceeds by equal intervals; a geometrical, one which advances by unequal and proportionally increasing or decreasing intervals.*

Arithmetical progressions: 1, 2, 3, 4, 5, 6, 7, &c.; or 2, 4, 6, 8, 10, 12, 14, 16, &c. Geometrical progressions: 1, 2, 4, 8, 16, 32, 64, &c.; or 243, 81, 27, 9, 3, 1.

3. *In these progressions we require accuracy and ease in working. Accuracy is obtained by taking large numbers for a basis; but large numbers are most easily made from small by adding cyphers.* (zeros)

Thus instead of 100000, which the less experienced make the greatest sine, the more learned put 10000000, whereby the difference of all sines is better expressed. Wherefore also we use the same for radius and for the greatest of our geometrical proportionals.

4. *In computing tables, these large numbers may again be made still larger by placing a period after the number and adding cyphers.*

Thus in commencing to compute, instead of 10000000 we put 10000000.0000000, lest the most minute error should become very large by frequent multiplication.

5. *In numbers distinguished thus by a period in their midst, whatever is written after the period is a fraction, the denominator of which is unity with as many cyphers after it as there are figures after the period.*

Thus 10000000.04 is the same as $10000000\frac{4}{100}$; also 25.803 is the same as $25\frac{803}{1000}$; also 9999998.

0005021 is the same as $9999998\frac{5021}{10000000}$, and so of others.

6. *When the tables are computed, the fractions following the period may then be rejected without any sensible error. For in our large numbers, an error which does not exceed unity is insensible and as if it were none.*

Thus in the completed table, instead of 9987643.8213051, which is $9987643\frac{8213051}{10000000}$, we may put 9987643 without sensible error.

7. *Besides this, there is another rule for accuracy; that is to say, when an unknown or incommensurable quantity is included between numerical limits not differing by many units.*

Thus if the diameter of a circle contain 497 parts, since it is not possible to ascertain precisely of how many parts the circumference consists, the more experienced, in accordance with the views of Archimedes, have enclosed it within limits, namely 1562 and 1561. Again, if the side of a square contain 1000 parts, the diagonal will be the square root of the number 2000000. Since this is an incommensurable number, we seek for its limits by extraction of the square root, namely 1415 the greater limit and 1414 the less limit, or more accurately $1414\frac{604}{2828}$ the greater, and $1414\frac{604}{2829}$ the less; for as we reduce the difference of the limits we increase the accuracy.

In place of the unknown quantities themselves, their limits are to be added, subtracted, multiplied, or divided, according as there may be need.

8. *The two limits of one quantity are added to the two limits of another, when the less of the one is added to the*

less of the other, and the greater of the one to the greater of the other.

Thus let the line *a b c* be divided into two parts, *a b* and *b c*.

Let *a b* lie between the limits 123.5 the greater and 123.2 the less. Also let *b c* lie between the limits 43.2 the greater and 43.1 the less. Then the greater being added to the greater and the less to the less, the whole line *a c* will lie between the limits 166.7 and 166.3.

9. *The two limits of one quantity are multiplied into the two limits of another, when the less of the one is multiplied into the less of the other, and the greater of the one into the greater of the other.*

Thus let one of the quantities *a b* lie between the limits 10.502 the greater and 10.500 the less. And let the other *a c* lie between the limits 3.216 the greater and 3.215 the less. Then 10.502 being multiplied into 3.216 and 10.500 into 3.215, the limits will become 33.774432 and 33.757500, between which the area of *a b c d* will lie.

10. *Subtraction of limits is performed by taking the greater limit of the less quantity from the less of the greater, and the less limit of the less quantity from the greater of the greater.*

Thus, in the first figure, if from the limits of *a c*, which are 166.7 and 166.3, you subtract the limits of *b c*, which are 43.2 and 43.1, the limits of *a b* become 123.6 and 123.1, and not 123.5 and 123.2. For although the addition of the latter to 43.2 and

and 43.1 produced 166.7 and 166.3 (as in 8), yet the converse does not follow; for there may be some quantity between 166.7 and 166.3 from which if you subtract some other which is between 43.2 and 43.1, the remainder may not lie between 123.5 and 123.2, but it is impossible for it not to lie between the limits 123.6 and 123.1.

11. *Division of limits is performed by dividing the greater limit of the dividend by the less of the divisor, and the less of the dividend by the greater of the divisor.*

Thus, in the preceding figure, the rectangle *a b c d* lying between the limits 33.774432 and 33.757500 may be divided by the limits of *a c*, which are 3.216 and 3.215, when there will come out $10.505\frac{857}{3215}$ and $10.496\frac{2364}{3216}$ for the limits of *a b*, and not 10.502 and 10.500, for the same reason that we stated in the case of subtraction.

12. *The vulgar fractions of the limits may be removed by adding unity to the greater limit.*

Thus, instead of the preceding limits of *a b*, namely, $10{\cdot}505\frac{857}{3215}$ and $10{\cdot}496\frac{2364}{3216}$, we may put 10·506 and 10·496.

Thus far concerning accuracy; what follows concerns ease in working.

13. *The construction of every arithmetical progression is easy; not so, however, of every geometrical progression.*

This is evident, as an arithmetical progression is very easily formed by addition or subtraction; but a geometrical progression is continued by very difficult multiplications, divisions, or extractions of roots.

Those geometrical progressions alone are carried on

easily which arise by subtraction of an easy part of the number from the whole number.

14. *We call easy parts of a number, any parts the denominators of which are made up of unity and a number of cyphers, such parts being obtained by rejecting as many of the figures at the end of the principal number as there are cyphers in the denominator.*

Thus the tenth, hundredth, thousandth, 10000^{th}, 100000^{th}, 1000000^{th}, 10000000^{th} parts are easily obtained, because the tenth part of any number is got by deleting its last figure, the hundredth its last two, the thousandth its last three figures, and so with the others, by always deleting as many of the figures at the end as there are cyphers in the denominator of the part. Thus the tenth part of 99321 is 9932, its hundredth part is 993, its thousandth 99, &c.

15. *The half, twentieth, two hundredth, and other parts denoted by the number two and cyphers, are also tolerably easily obtained; by rejecting as many of the figures at the end of the principal number as there are cyphers in the denominator, and dividing the remainder by two.*

Thus the 2000^{th} part of the number 9973218045 is 4986609, the 20000^{th} part is 498660.

16. *Hence it follows that if from radius with seven cyphers added you subtract its* 10000000^{th} *part, and from the number thence arising its* 10000000^{th} *part, and so on, a hundred numbers may very easily be continued geometrically in the proportion subsisting between radius and the sine less than it by unity, namely between* 10000000 *and* 9999999; *and this series of proportionals we name the First table.*

Thus

First table.

10000000.0000000
1.0000000
9999999.0000000
.9999999
9999998.0000001
.9999998
9999997.0000003
.9999997
9999996.0000006
to be continued up to
9999900.0004950

Thus from radius, with seven cyphers added for greater accuracy, namely, 10000000.0000000, subtract 1.0000000, you get 9999999.0000000; from this subtract .9999999, you get 9999998.0000001; and proceed in this way, as shown at the side, until you create a hundred proportionals, the last of which, if you have computed rightly, will be 9999900.0004950.

17. *The Second table proceeds from radius with six cyphers added, through fifty other numbers decreasing proportionally in the proportion which is easiest, and as near as possible to that subsisting between the first and last numbers of the First table.*

Second table.

10000000.000000
100.000000
9999900.000000
99.999000
9999800.001000
99.998000
9999700.003000
99.997000
9999600.006000

Thus the first and last numbers of the First table are 10000000.0000000 and 9999900.0004950, in which proportion it is difficult to form fifty proportional numbers. A near and at the same time an easy proportion is 100000 to 99999, which may be continued with sufficient exactness by adding six cyphers to radius and continually subtracting from each number its own 100000^{th} part in the manner shown at the side; and this table

&c., up to	contains, besides radius which is the first, fifty other proportional numbers, the last of which, if you have not erred, you will find to be 9995001.222927.
9995001.222927	

[This should be 9995001.224804—see note.]

18. *The Third table consists of sixty-nine columns, and in each column are placed twenty-one numbers, proceeding in the proportion which is easiest, and as near as possible to that subsisting between the first and last numbers of the Second table.*

Whence its first column is very easily obtained from radius with five cyphers added, by subtracting its 2000th *part, and so from the other numbers as they arise.*

First column of Third table.

10000000.00000
5000.00000
9995000.00000
4997.50000
9990002.50000
4995.00125
9985007.49875
4992.50374
9980014.99501
&c., up to
9900473.57808

In forming this progression, as the proportion between 10000000.000000, the first of the Second table, and 9995001.222927, the last of the same, is troublesome; therefore compute the twenty-one numbers in the easy proportion of 10000 to 9995, which is sufficiently near to it; the last of these, if you have not erred, will be 9900473.57808.

From these numbers, when computed, the last figure of each may be rejected without sensible error, so that others may hereafter be more easily computed from them.

19. *The first numbers of all the columns must proceed from radius*

radius with four cyphers added, in the proportion easiest and nearest to that subsisting between the first and the last numbers of the first column.

As the first and the last numbers of the first column are 10000000.0000 and 9900473.5780, the easiest proportion very near to this is 100 to 99. Accordingly sixty-eight numbers are to be continued from radius in the ratio of 100 to 99 by subtracting from each one of them its hundredth part.

20. *In the same proportion a progression is to be made from the second number of the first column through the second numbers in all the columns, and from the third through the third, and from the fourth through the fourth, and from the others respectively through the others.*

Thus from any number in one column, by subtracting its hundredth part, the number of the same rank in the following column is made, and the numbers should be placed in order as follows :—

PROPORTIONALS OF THE THIRD TABLE.

First Column.	*Second Column.*
10000000.0000	9900000.0000
9995000.0000	9895050.0000
9990002.5000	9890102.4750
9985007.4987	9885157.4237
9980014.9950	9880214.8451
&c., continuously to	&c., descending to
9900473.5780	9801468.8423

Third Column.	*Thence 4th, 5th, &c., up to*	*69th Column.*
9801000.0000	&c., up to	5048858.8900
9796099.5000	&c., up to	5046334.4605
9791201.4503	&c., up to	5043811.2932
9786305.8495	&c., up to	5041289.3879
9781412.6967	&c., up to	5038768.7435
&c., descending to		finally to
9703454.1539	finally to	4998609.4034

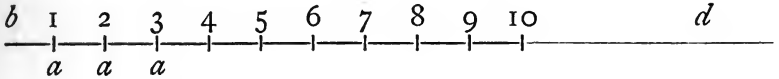
21. *Thus, in the Third table, between radius and half radius, you have sixty-eight numbers interpolated, in the proportion of* 100 *to* 99, *and between each two of these you have twenty numbers interpolated in the proportion of* 10000 *to* 9995; *and again, in the Second table, between the first two of these, namely between* 10000000 *and* 9995000, *you have fifty numbers interpolated in the proportion of* 100000 *to* 99999; *and finally, in the First table, between the latter, you have a hundred numbers interpolated in the proportion of radius or* 10000000 *to* 9999999; *and since the difference of these is never more than unity, there is no need to divide it more minutely by interpolating means, whence these three tables, after they have been completed, will suffice for computing a Logarithmic table.*

Hitherto we have explained how we may most easily place in tables sines or natural numbers progressing in geometrical proportion.

22. *It remains, in the Third table at least, to place beside the sines or natural numbers decreasing geometrically their*

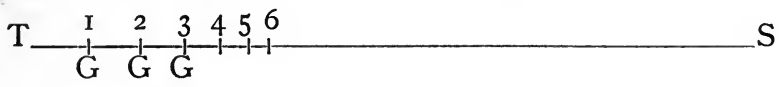
their logarithms or artificial numbers increasing arithmetically.

23. *To increase arithmetically is, in equal times, to be augmented by a quantity always the same.*



Thus from the fixed point *b* let a line be produced indefinitely in the direction of *d*. Along this let the point *a* travel from *b* towards *d*, moving according to this law, that in equal moments of time it is borne over the equal spaces *b* 1, 1 2, 2 3, 3 4, 4 5, &c. Then we call this increase by *b* 1, *b* 2, *b* 3, *b* 4, *b* 5, &c., arithmetical. Again, let *b* 1 be represented in numbers by 10, *b* 2 by 20, *b* 3 by 30, *b* 4 by 40, *b* 5 by 50; then 10, 20, 30, 40, 50, &c., increase arithmetically, because we see they are always increased by an equal number in equal times.

24. *To decrease geometrically is this, that in equal times, first the whole quantity then each of its successive remainders is diminished, always by a like proportional part.*



Thus let the line T S be radius. Along this let the point G travel in the direction of S, so that in equal times it is borne from T to 1, which for example may be the tenth part of T S; and from 1 to 2, the tenth part of 1 S; and from 2 to 3, the tenth part of 2 S; and from 3 to 4, the tenth part of 3 S, and so on. Then the sines T S, 1 S, 2 S,

3 S, 4 S, &c., are said to decrease geometrically, because in equal times they are diminished by unequal spaces similarly proportioned. Let the sine T S be represented in numbers by 10000000, 1 S by 9000000, 2 S by 8100000, 3 S by 7290000, 4 S by 6561000; then these numbers are said to decrease geometrically, being diminished in equal times by a like proportion.

25. *Whence a geometrically moving point approaching a fixed one has its velocities proportionate to its distances from the fixed one.*

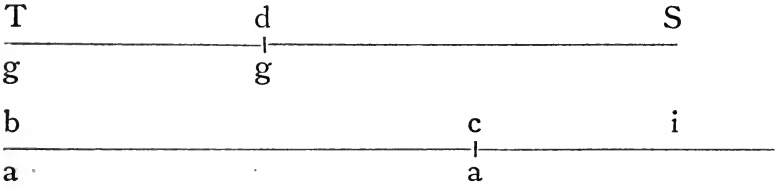
Thus, referring to the preceding figure, I say that when the geometrically moving point G is at T, its velocity is as the distance T S, and when G is at 1 its velocity is as 1 S, and when at 2 its velocity is as 2 S, and so of the others. Hence, whatever be the proportion of the distances T S, 1 S, 2 S, 3 S, 4 S, &c., to each other, that of the velocities of G at the points T, 1, 2, 3, 4, &c., to one another, will be the same.

For we observe that a moving point is declared more or less swift, according as it is seen to be borne over a greater or less space in equal times. Hence the ratio of the spaces traversed is necessarily the same as that of the velocities. But the ratio of the spaces traversed in equal times, T 1, 1 2, 2 3, 3 4, 4 5, &c., is that of the distances T S, 1 S, 2 S, 3 S, 4 S, &c.[*] Hence it follows that the ratio to one another of the distances of G from S, namely T S, 1 S, 2 S, 3 S, 4 S, &c., is the same as that of the velocities of G at the points T, 1, 2, 3, 4, &c., respectively.

[*] It is evident that the ratio of the spaces traversed T 1, 1 2, 2 3, 3 4, 4 5, &c., is that of the distances T S, 1 S,

1 S, 2 S, 3 S, 4 S, &c., for when quantities are continued proportionally, their differences are also continued in the same proportion. Now the distances are by hypothesis continued proportionally, and the spaces traversed are their differences, wherefore it is proved that the spaces traversed are continued in the same ratio as the distances.

26. *The logarithm of a given sine is that number which has increased arithmetically with the same velocity throughout as that with which radius began to decrease geometrically, and in the same time as radius has decreased to the given sine.*



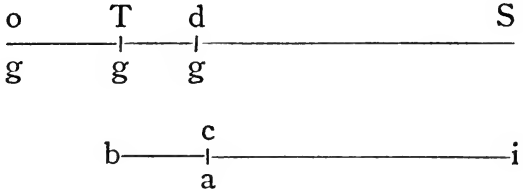
Let the line T S be radius, and d S a given sine in the same line; let g move geometrically from T to d in certain determinate moments of time. Again, let b i be another line, infinite towards i, along which, from b, let a move arithmetically with the same velocity as g had at first when at T; and from the fixed point b in the direction of i let a advance in just the same moments of time up to the point c. The number measuring the line b c is called the logarithm of the given sine d S.

27. *Whence nothing is the logarithm of radius.*

For, referring to the figure, when g is at T making its distance from S radius, the arithmetical point d beginning at b has never proceeded thence. Whence by the definition of distance nothing will be the logarithm of radius.

 28. *Whence*

28. *Whence also it follows that the logarithm of any given sine is greater than the difference between radius and the given sine, and less than the difference between radius and the quantity which exceeds it in the ratio of radius to the given sine. And these differences are therefore called the limits of the logarithm.*



Thus, the preceding figure being repeated, and S T being produced beyond T to o, so that o S is to T S as T S to d S. I say that b c, the logarithm of the sine d S, is greater than T d and less than o T. For in the same time that g is borne from o to T, g is borne from T to d, because (by 24) o T is such a part of o S as T d is of T S, and in the same time (by the definition of a logarithm) is a borne from b to c; so that o T, T d, and b c are distances traversed in equal times. But since g when moving between T and o is swifter than at T, and between T and d slower, but at T is equally swift with a (by 26); it follows that o T the distance traversed by g moving swiftly is greater, and T d the distance traversed by g moving slowly is less, than b c the distance traversed by the point a with its medium motion, in just the same moments of time; the latter is, consequently, a certain mean between the two former.

Therefore o T is called the greater limit, and T d

T d the less limit of the logarithm which b c represents.

29. *Therefore to find the limits of the logarithm of a given sine.*

By the preceding it is proved that the given sine being subtracted from radius the less limit remains, and that radius being multiplied into the less limit and the product divided by the given sine, the greater limit is produced, as in the following example.

30. *Whence the first proportional of the First table, which is* 9999999, *has its logarithm between the limits* 1.0000001 *and* 1.0000000.

For (by 29) subtract 9999999 from radius with cyphers added, there will remain unity with its own cyphers for the less limit; this unity with cyphers being multiplied into radius, divide by 9999999 and there will result 1.0000001 for the greater limit, or if you require greater accuracy 1.00000010000001.

31. *The limits themselves differing insensibly, they or anything between them may be taken as the true logarithm.*

Thus in the above example, the logarithm of the sine 9999999 was found to be either 1.0000000 or 1.00000010, or best of all 1.00000005. For since the limits themselves, 1.0000000 and 1.0000001, differ from each other by an insensible fraction like $\frac{1}{10000000}$, therefore they and whatever is between them will differ still less from the true logarithm lying between these limits, and by a much more insensible error.

32. *There being any number of sines decreasing from radius in geometrical proportion, of one of which the logarithm or its limits is given, to find those of the others.*

This necessarily follows from the definitions of arithmetical increase, of geometrical decrease, and of a logarithm. For by these definitions, as the sines decrease continually in geometrical proportion, so at the same time their logarithms increase by equal additions in continuous arithmetical progression. Wherefore to any sine in the decreasing geometrical progression there corresponds a logarithm in the increasing arithmetical progression, namely the first to the first, and the second to the second, and so on.

So that, if the first logarithm corresponding to the first sine after radius be given, the second logarithm will be double of it, the third triple, and so of the others; until the logarithms of all the sines be known, as the following example will show.

33. *Hence the logarithms of all the proportional sines of the First table may be included between near limits, and consequently given with sufficient exactness.*

Thus since (by 27) the logarithm of radius is 0, and (by 30) the logarithm of 9999999, the first sine after radius in the First table, lies between the limits 1.0000001 and 10000000; necessarily the logarithm of 9999998.0000001, the second sine after radius, will be contained between the double of these limits, namely between 2.0000002 and 2.0000000; and the logarithm of 9999997.0000003, the third will be between the triple of the same, namely between 3.0000003 and 3.0000000. And so with the others, always by equally increasing the limits by the limits of the first, until you have completed the limits of the logarithms of all the proportionals of the First table. You may in this way

way, if you please, continue the logarithms themselves in an exactly similar progression with little and insensible error; in which case the logarithm of radius will be 0, the logarithm of the first sine after radius (by 31) will be 1.00000005, of the second 2.00000010, of the third 3.00000015, and so of the rest.

34. *The difference of the logarithms of radius and a given sine is the logarithm of the given sine itself.*

This is evident, for (by 27) the logarithm of radius is nothing, and when nothing is subtracted from the logarithm of a given sine, the logarithm of the given sine necessarily remains entire.

35. *The difference of the logarithms of two sines must be added to the logarithm of the greater that you may have the logarithm of the less, and subtracted from the logarithm of the less that you may have the logarithm of the greater.*

Necessarily this is so, since the logarithms increase as the sines decrease, and the less logarithm is the logarithm of the greater sine, and the greater logarithm of the less sine. And therefore it is right to add the difference to the less logarithm, that you may have the greater logarithm though corresponding to the less sine, and on the other hand to subtract the difference from the greater logarithm that you may have the less logarithm though corresponding to the greater sine.

36. *The logarithms of similarly proportioned sines are equidifferent.*

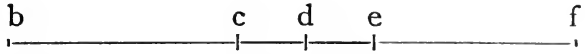
This necessarily follows from the definitions of a logarithm and of the two motions. For since by

these definitions arithmetical increase always the same corresponds to geometrical decrease similarly proportioned, of necessity we conclude that equidifferent logarithms and their limits correspond to similarly proportioned sines. As in the above example from the First table, since there is a like proportion between 9999999.0000000 the first proportional after radius, and 9999997.0000003 the third, to that which is between 9999996.0000006 the fourth and 9999994.0000015 the sixth; therefore 1.00000005 the logarithm of the first differs from 3.00000015 the logarithm of the third, by the same difference that 4.00000020 the logarithm of the fourth, differs from 6.00000030 the logarithm of the sixth proportional. Also there is the same ratio of equality between the differences of the respective limits of the logarithms, namely as the differences of the less among themselves, so also of the greater among themselves, of which logarithms the sines are similarly proportioned.

37. *Of three sines continued in geometrical proportion, as the square of the mean equals the product of the extremes, so of their logarithms the double of the mean equals the sum of the extremes. Whence any two of these logarithms being given, the third becomes known.*

Of the three sines, since the ratio between the first and the second is that between the second and the third, therefore (by 36), of their logarithms, the difference between the first and the second is that between the second and the third. For example, let the first logarithm be represented by the line b c, the second by the line b d, the third by the line b e, all placed in the one line b c d e, thus :—

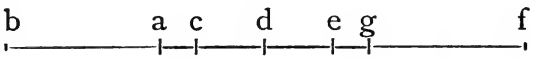
and



and let the differences c d and d e be equal. Let b d, the mean of them, be doubled by producing the line from b beyond e to f, so that b f is double b d. Then b f is equal to both the lines b c of the first logarithm and b e of the third, for from the equals b d and d f take away the equals c d and d e, namely c d from b d and d e from d f, and there will remain b c and e f necessarily equal. Thus since the whole b f is equal to both b e and e f, therefore also it will be equal to both b e and b c, which was to be proved. Whence follows the rule, if of three logarithms you double the given mean, and from this subtract a given extreme, the remaining extreme sought for becomes known; and if you add the given extremes and divide the sum by two, the mean becomes known.

38. *Of four geometrical proportionals, as the product of the means is equal to the product of the extremes; so of their logarithms, the sum of the means is equal to the sum of the extremes. Whence any three of these logarithms being given, the fourth becomes known.*

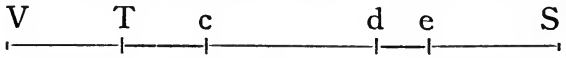
Of the four proportionals, since the ratio between the first and second is that between the third and fourth; therefore of their logarithms (by 36), the difference between the first and second is that between the third and fourth. Hence let such quantities be taken in the line b f as that b a



may represent the first logarithm, b c the second, b e the third, and b g the fourth, making the dif-

ferences a c and e g equal, so that d placed in the middle of c e is of necessity also placed in the middle of a g. Then the sum of b c the second and b e the third is equal to the sum of b a the first and b g the fourth. For (by 37) the double of b d, which is b f, is equal to b c and b e together, because their differences from b d, namely c d and d e, are equal; for the same reason the same b f is also equal to b a and b g together, because their differences from b d, namely a d and d g, are also equal. Since, therefore, both the sum of b a and b g and the sum of b c and b e are equal to the double of b d, which is b f, therefore also they are equal to each other, which was to be proved. Whence follows the rule, of these four logarithms if you subtract a known mean from the sum of the known extremes, there is left the mean sought for; and if you subtract a known extreme from the sum of the known means, there is left the extreme sought for.

39. *The difference of the logarithms of two sines lies between two limits; the greater limit being to radius as the difference of the sines to the less sine, and the less limit being to radius as the difference of the sines to the greater sine.*



Let T S be radius, d S the greater of two given sines, and e S the less. Beyond S T let the distance T V be marked off by the point V, so that S T is to T V as e S, the less sine, is to d e, the difference of the sines. Again, on the other side of T, towards S, let the distance T c be marked off by the point c, so that T S is to T c as d S, the greater sine, is to d e, the difference of the sines

sines. Then the difference of the logarithms of the sines d S and e S lies between the limits V T the greater and T c the less. For by hypothesis, e S is to d e as T S to T V, and d S is to d e as T S to T c; therefore, from the nature of proportionals, two conclusions follow :—

Firstly, that V S is to T S as T S to c S.

Secondly, that the ratio of T S to c S is the same as that of d S to e S. And therefore (by 36) the difference of the logarithms of the sines d S and e S is equal to the difference of the logarithms of the radius T S and the sine c S. But (by 34) this difference is the logarithm of the sine c S itself; and (by 28) this logarithm is included between the limits V T the greater and T c the less, because by the first conclusion above stated, V S greater than radius is to T S radius as T S is to c S. Whence, necessarily, the difference of the logarithms of the sines d S and e S lies between the limits V T the greater and T c the less, which was to be proved.

40. *To find the limits of the difference of the logarithms of two given sines.*

Since (by 39) the less sine is to the difference of the sines as radius to the greater limit of the difference of the logarithms; and the greater sine is to the difference of the sines as radius to the less limit of the difference of the logarithms; it follows, from the nature of proportionals, that radius being multiplied by the difference of the given sines and the product being divided by the less sine, the greater limit will be produced; and the product being divided by the greater sine, the less limit will be produced.

EXAMPLE.

THUS, let the greater of the given sines be 9999975.5000000, and the less 9999975.0000300, the difference of these .4999700 being multiplied into radius (cyphers to the eighth place after the point being first added to both for the purpose of demonstration, although otherwise seven are sufficient), if you divide the product by the greater sine, namely 9999975.5000000, there will come out for the less limit .49997122, with eight figures after the point; again, if you divide the product by the less sine, namely 9999975.0000300, there will come out for the greater limit .49997124; and, as already proved, the difference of the logarithms of the given sines lies between these. But since the extension of these fractions to the eighth figure beyond the point is greater accuracy than is required, especially as only seven figures are placed after the point in the sines; therefore, that eighth or last figure of both being deleted, then the two limits and also the difference itself of the logarithms will be denoted by the fraction .4999712 without even the smallest particle of sensible error.

41. *To find the logarithms of sines or natural numbers not proportionals in the First table, but near or between them; or at least, to find limits to them separated by an insensible difference.*

Write down the sine in the First table nearest to the given sine, whether less or greater. Seek out the limits of the table sine (by 33), and when found note them down. Then seek out the limits of the difference of the logarithms of the given sine

sine and the table sine (by 40), either both limits or one or other of them, since they are almost equal, as is evident from the above example. Now these, or either of them, being found, add to them the limits above noted down, or else subtract (by 8, 10, and 35), according as the given sine is less or greater than the table sine. The numbers thence produced will be near limits between which is included the logarithm of the given sine.

EXAMPLE.

Let the given sine be 9999975.5000000, to which the nearest sine in the table is 9999975.0000300, less than the given sine. By 33 the limits of the logarithm of the latter are 25.0000025 and 25.0000000. Again (by 40), the difference of the logarithms of the given sine and the table sine is .4999712. By 35, subtract this from the above limits, which are the limits of the less sine, and there will come out 24.5000313 and 24.5000288, the required limits of the logarithm of the given sine 9999975.5000000. Accordingly the actual logarithm of the sine may be placed without sensible error in either of the limits, or best of all (by 31) in 24.5000300.

ANOTHER EXAMPLE.

LET the given sine be 9999900.0000000, the table sine nearest it 9999900.0004950. By 33 the limits of the logarithm of the latter are 100.0000100 and 100.0000000. Then (by 40) the difference of the logarithms of the sines will be .0004950. Add this (by 35) to the above limits and they become 100.0005050 for the greater

limit, and 100.0004950 for the less limit, between which the required logarithm of the given sine is included.

42. *Hence it follows that the logarithms of all the proportionals in the Second table may be found with sufficient exactness, or may be included between known limits differing by an insensible fraction.*

Thus since the logarithm of the sine 9999900, the first proportional of the Second table, was shown in the preceding example to lie between the limits 100.0005050 and 100.0004950; necessarily (by 32) the logarithm of the second proportional will lie between the limits 200.0010100 and 200.0009900; and the logarithm of the third proportional between the limits 300.0015150 and 300.0014850, &c. And finally, the logarithm of the last sine of the Second table, namely 9995001. 222927, is included between the limits 5000. 0252500 and 5000.0247500. Now, having all these limits, you will be able (by 31) to find the actual logarithms.

43. *To find the logarithms of sines or natural numbers not proportionals in the Second table, but near or between them; or to include them between known limits differing by an insensible fraction.*

Write down the sine in the Second table nearest the given sine, whether greater or less. By 42 find the limits of the logarithm of the table sine. Then by the rule of proportion seek for a fourth proportional, which shall be to radius as the less of the given and table sines is to the greater. This may be done in one way by multiplying the less sine into radius and dividing the product by the greater. Or, in an easier way, by multiplying

multiplying the difference of the sines into radius, dividing this product by the greater sine, and subtracting the quotient from radius.

Now since (by 36) the logarithm of the fourth proportional differs from the logarithm of radius by as much as the logarithms of the given and table sines differ from each other; also, since (by 34) the former difference is the same as the logarithm of the fourth proportional itself; therefore (by 41) seek for the limits of the logarithm of the fourth proportional by aid of the First table; when found add them to the limits of the logarithm of the table sine, or else subtract them (by 8, 10, and 35), according as the table sine is greater or less than the given sine; and there will be brought out the limits of the logarithm of the given sine.

EXAMPLE.

THUS, let the given sine be 9995000.000000. To this the nearest sine in the Second table is 9995001.222927, and (by 42) the limits of its logarithm are 5000.0252500 and 5000.0247500. Now seek for the fourth proportional by either of the methods above described; it will be 9999998.7764614, and the limits of its logarithm found (by 41) from the First table will be 1.2235387 and 1.2235386. Add these limits to the former (by 8 and 35), and there will come out 5001.2487888 and 5001.2482886 as the limits of the logarithm of the given sine. Whence the number 5001.2485387, midway between them, is (by 31) taken most suitably, and with no sensible error, for the actual logarithm of the given sine 9995000.

44. *Hence it follows that the logarithms of all the proportionals*

tionals in the first column of the Third table may be found with sufficient exactness, or may be included between known limits differing by an insensible fraction.

For, since (by 43) the logarithm of 9995000, the first proportional after radius in the first column of the Third table, is 5001.2485387 with no sensible error; therefore (by 32) the logarithm of the second proportional, namely 9990002.5000, will be 10002.4970774; and so of the others, proceeding up to the last in the column, namely 9900473.57808, the logarithm of which, for a like reason, will be 100024.9707740, and its limits will be 100024.9657720 and 100024.9757760.

45. *To find the logarithms of natural numbers or sines not proportionals in the first column of the Third table, but near or between them; or to include them between known limits differing by an insensible fraction.*

Write down the sine in the first column of the Third table nearest the given sine, whether greater or less. By 44 seek for the limits of the logarithm of the table sine. Then, by one of the methods described in 43, seek for a fourth proportional, which shall be to radius as the less of the given and table sines is to the greater. Having found the fourth proportional, seek (by 43) for the limits of its logarithm from the Second table. When these are found, add them to the limits of the logarithm of the table sine found above, or else subtract them (by 8, 10, and 35), and the limits of the logarithm of the given sine will be brought out.

EXAMPLE.

THUS, let the given sine be 9900000. The proportional sine nearest it in the first column

column of the Third table is 9900473.57808. Of this (by 44) the limits of the logarithm are 100024.9657720 and 100024.9757760. Then the fourth proportional will be 9999521.6611850. Of this the limits of the logarithm, deduced from the Second table (by 43), are 478.3502290 and 478.3502812. These limits (by 8 and 35) being added to the above limits of the logarithm of the table sine, there will come out the limits 100503.3260572 and 100503.3160010, between which necessarily falls the logarithm sought for. Whence the number midway between them, which is 100503.3210291, may be put without sensible error for the true logarithm of the given sine 9900000.

46. *Hence it follows that the logarithms of all the proportionals of the Third table may be given with sufficient exactness.*

For, as (by 45) 100503.3210291 is the logarithm of the first sine in the second column, namely 9900000; and since the other first sines of the remaining columns progress in the same proportion, necessarily (by 32 and 36) the logarithms of these increase always by the same difference 100503.3210291, which is added to the logarithm last found, that the following may be made. Therefore, the first logarithms of all the columns being obtained in this way, and all the logarithms of the first column being obtained by 44, you may choose whether you prefer to build up, at one time, all the logarithms in the same column, by continuously adding 5001.2485387, the difference of the logarithms, to the last found logarithm in the column, that the next lower logarithm in the same column be made; or whether you prefer to com-

pute, at one time, all the logarithms of the same rank, namely all the second logarithms in each of the columns, then all the third, then the fourth, and so the others, by continuously adding 100503.3210291 to the logarithm in one column, that the logarithm of the same rank in the next column be brought out. For by either method may be had the logarithms of all the proportionals in this table; the last of which is 6934250.8007528, corresponding to the sine 4998609.4034.

47. *In the Third table, beside the natural numbers, are to be written their logarithms; so that the Third table, which after this we shall always call the Radical table, may be made complete and perfect.*

This writing up of the table is to be done by arranging the columns in the number and order described (in 20 and 21), and by dividing each into two sections, the first of which should contain the geometrical proportionals we call sines and natural numbers, the second their logarithms progressing arithmetically by equal intervals.

THE RADICAL TABLE.

First column.			*Second column.*	
Natural numbers.	Logarithms.		Natural numbers.	Logarithms.
10000000.0000	.0		9900000.0000	100503.3
9995000.0000	5001.2		9895050.0000	105504.6
9990002.5000	10002.5		9890102.4750	110505.8
9985007.4987	15003.7		9885157.4237	115507.1
9980014.9950	20005.0		9880214.8451	120508.3
&c., up to	up to		up to	up to
9900473.5780	100025.0		9801468.8423	200528.2

and

	69th column.	
	Natural numbers.	Logarithms.
and the others, up to	5048858.8900	6834225.8
	5046334.4605	6839227.1
	5043811.2932	6844228.3
	5041289.3879	6849229.6
	5038768.7435	6854230.8
	01 dn	01 dn
	4998609.4034	6934250.8

For shortness, however, two things should be borne in mind :—First, that in these logarithms it is enough to leave one figure after the point, the remaining six being now rejected, which, however, if you had neglected at the beginning, the error arising thence by frequent multiplications in the previous tables would have grown intolerable in the third. Secondly, If the second figure after the point exceed the number four, the first figure after the point, which alone is retained, is to be increased by unity : thus for 10002.48 it is more correct to put 10002.5 than 10002.4 ; and for 1000.35001 we more fitly put 1000.4 than 1000.3. Now, therefore, continue the Radical table in the manner which has been set forth.

48. *The Radical table being now completed, we take the numbers for the logarithmic table from it alone.*

For as the first two tables were of service in the formation of the third, so this third Radical

table serves for the construction of the principal Logarithmic table, with great ease and no sensible error.

49. *To find most easily the logarithms of sines greater than* 9996700.

This is done simply by the subtraction of the given sine from radius. For (by 29) the logarithm of the sine 9996700 lies between the limits 3300 and 3301 ; and these limits, since they differ from each other by unity only, cannot differ from their true logarithm by any sensible error, that is to say, by an error greater than unity. Whence 3300, the less limit, which we obtain simply by subtraction, may be taken for the true logarithm. The method is necessarily the same for all sines greater than this.

50. *To find the logarithms of all sines embraced within the limits of the Radical table.*

Multiply the difference of the given sine and table sine nearest it by radius. Divide the product by the easiest divisor, which may be either the given sine or the table sine nearest it, or a sine between both, however placed. By 39 there will be produced either the greater or less limit of the difference of the logarithms, or else something intermediate, no one of which will differ by a sensible error from the true difference of the logarithms on account of the nearness of the numbers in the table. Wherefore (by 35), add the result, whatever it may be, to the logarithm of the table sine, if the given sine be less than the table sine; if not, subtract the result from the logarithm of the table sine, and there will be produced the required logarithm of the given sine.

EXAMPLE.

EXAMPLE.

THUS let the given sine be 7489557, of which the logarithm is required. The table sine nearest it is 7490786.6119. From this subtract the former with cyphers added thus, 7489557.0000, and there remains 1229.6119. This being multiplied by radius, divide by the easiest number, which may be either 7489557.0000 or 7490786.6119, or still better by something between them, such as 7490000, and by a most easy division there will be produced 1640.1. Since the given sine is less than the table sine, add this to the logarithm of the table sine, namely to 2889111.7, and there will result 2890751.8, which equals $2890751\frac{4}{5}$. But since the principal table admits neither fractions nor anything beyond the point, we put for it 2890752, which is the required logarithm.

ANOTHER EXAMPLE.

LET the given sine be 7071068.0000. The table sine nearest it will be 7070084.4434. The difference of these is 983.5566. This being multiplied by radius, you most fitly divide the product by 7071000, which lies between the given and table sines, and there comes out 1390.9. Since the given sine exceeds the table sine, let this be subtracted from the logarithm of the table sine, namely from 3467125.4, which is given in the table, and there will remain 3465734.5. Wherefore 3465735 is assigned for the required logarithm of the given sine 7071068. Thus the liberty of choosing a divisor produces wonderful facility.

51. *All sines in the proportion of two to one have* 6931469.22 *for the difference of their logarithms.*

For since the ratio of every sine to its half is the same as that of radius to 5000000, therefore (by 36) the difference of the logarithms of any sine and of its half is the same as the difference of the logarithms of radius and of its half 5000000. But (by 34) the difference of the logarithms of radius and of the sine 5000000 is the same as the logarithm itself of the sine 5000000, and this logarithm (by 50) will be 6931469.22. Therefore, also, 6931469.22 will be the difference of all logarithms whose sines are in the proportion of two to one. Consequently the double of it, namely 13862938.44, will be the difference of all logarithms whose sines are in the ratio of four to one; and the triple of it, namely 20794407.66, will be the difference of all logarithms whose sines are in the ratio of eight to one.

52. *All sines in the proportion of ten to one have* 23025842.34 *for the difference of their logarithms.*

For (by 50) the sine 8000000 will have for its logarithm 2231434.68; and (by 51) the difference between the logarithms of the sine 8000000 and of its eighth part 1000000, will be 20794407.66; whence by addition will be produced 23025842.34 for the logarithm of the sine 1000000. And since radius is ten times this, all sines in the ratio of ten to one will have the same difference, 23025842.34, between their logarithms, for the reason and cause already stated (in 51) in reference to the proportion of two to one. And consequently the double of this logarithm, namely 46051684.68, will, as regards the difference of the logarithms, correspond to

to the proportion of a hundred to one; and the triple of the same, namely 69077527.02, will be the difference of all logarithms whose sines are in the ratio of a thousand to one; and so of the ratio ten thousand to one, and of the others as below.

53. *Whence all sines in a ratio compounded of the ratios two to one and ten to one, have the difference of their logarithms formed from the differences* 6931469.22 *and* 23025842.34 *in the way shown in the following*

Short Table.

Given Proportions of Sines.		Corresponding Differences of Logarithm.	Given Proportions of Sines.		Corresponding Differences of Logarithm.
Two	to one	6931469.22	8000	to one	89871934.68
Four	„	13862938.44	10000	„	92103369.36
Eight	„	20794407.66	20000	„	99034838.58
Ten	„	23025842.34	40000	„	105966307.80
20	„	29957311.56	80000	„	112897777.02
40	„	36888780.78	100000	„	115129211.70
80	„	43820250.00	200000	„	122060680.92
A hundred	„	46051684.68	400000	„	128992150.14
200	„	52983153.90	800000	„	135923619.36
400	„	59914623.12	1000000	„	138155054.04
800	„	66846092.34	2000000	„	145086523.26
A thousand	„	69077527.02	4000000	„	152017992.48
2000	„	76008996.24	8000000	„	158949461.70
4000	„	82940465.46	10000000	„	161180896.38

54. *To find the logarithms of all sines which are outside the limits of the Radical table.*

This is easily done by multiplying the given sine by 2, 4, 8, 10, 20, 40, 80, 100, 200, or any other proportional number you please, contained in the short table, until you obtain a number within the limits of the Radical table. By 50

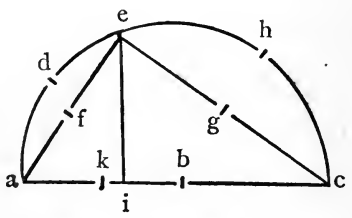
find the logarithm of this sine now contained in the table, and then add to it the logarithmic difference which the short table indicates as required by the preceding multiplication.

EXAMPLE.

IT is required to find the logarithm of the sine 378064. Since this sine is outside the limits of the Radical table, let it be multiplied by some proportional number in the foregoing short table, as by 20, when it will become 7561280. As this now falls within the Radical table, seek for its logarithm (by 50) and you will obtain 2795444.9, to which add 29957311.56, the difference in the short table corresponding to the proportion of twenty to one, and you have 32752756.4. Wherefore 32752756 is the required logarithm of the given sine 378064.

55. *As half radius is to the sine of half a given arc, so is the sine of the complement of the half arc to the sine of the whole arc.*

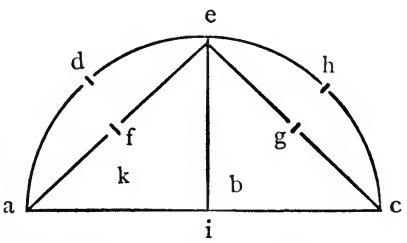
Let a b be radius, and a b c its double, on which as diameter is described a semicircle. On this lay off the given arc a e, bisect it in d, and from e in the direction of c lay off e h, the complement of d e, half the given arc. Then h c is necessarily equal to e h, since the quadrant d e h must equal the remaining quadrant made up of the arcs a d and h c. Draw e i perpendicular to a i c, then e i is



is the sine of the arc a d e. Draw a e; its half, f e, is the sine of the arc d e, the half of the arc a d e. Draw e c; its half, e g, is the sine of the arc e h, and is therefore the sine of the complement of the arc d e. Finally, make a k half the radius a b. Then as a k is to e f, so is e g to e i. For the two triangles c e a and c i e are equiangular, since i c e or a c e is common to both; and c i e and c e a are each a right angle, the former by hypothesis, the latter because it is in the circumference and occupies a semicircle. Hence a c, the hypotenuse of the triangle c e a, is to a e, its less side, as e c, the hypotenuse of the triangle c i e, is to e i its less side. And since a c, the whole, is to a e as e c, the whole, is to e i, it follows that a b, half of a c, is to a e as e g, half of e c, is to e i. And now, finally, since a b, the whole, is to a e, the whole, as e g is to e i, we necessarily conclude that a k, half of a b, is to f e, half of a e, as e g is to e i.

56. *Double the logarithm of an arc of* 45 *degrees is the logarithm of half radius.*

Referring to the preceding figure, let the case be such that a e and e c are equal. In that case i will fall on b, and e i will be radius; also e f and e g will be equal, each of them being the sine of 45 degrees. Now (by 55) the ratio of a k, half radius, to e f, a sine of 45 degrees, is likewise the ratio of e g, also a sine of



45 degrees, to e i, now radius. Consequently (by 37) double the logarithm of the sine of 45 degrees is equal to the logarithms of the extremes, namely radius and its half. But the sum of the logarithms of both these is the logarithm of half radius only, because (by 27) the logarithm of radius is nothing. Necessarily, therefore, the double of the logarithm of an arc of 45 degrees is the logarithm of half radius.

57. *The sum of the logarithms of half radius and any given arc is equal to the sum of the logarithms of half the arc and the complement of the half arc. Whence the logarithm of the half arc may be found if the logarithms of the other three be given.*

Since (by 55) half radius is to the sine of half the given arc as the sine of the complement of that half arc is to the sine of the whole arc, therefore (by 38) the sum of the logarithms of the two extremes, namely half radius and the whole arc, will be equal to the sum of the logarithms of the means, namely the half arc and the complement of the half arc. Whence, also (by 38), if you add the logarithm of half radius, found by 51 or 56, to the given logarithm of the whole arc, and subtract the given logarithm of the complement of the half arc, there will remain the required logarithm of the half arc.

EXAMPLE.

LET there be given the logarithm of half radius (by 51) 6931469; also the arc 69 degrees 20 minutes, and its logarithm 665143. The half arc is 34 degrees 40 minutes, whose logarithm

logarithm is required. The complement of the half arc is 55 degrees 20 minutes, and its logarithm 1954370 is given. Wherefore add 6931469 to 665143, making 7596612, subtract 1954370, and there remains 5642242, the required logarithm of an arc of 34 degrees 40 minutes.

58. *When the logarithms of all arcs not less than* 45 *degrees are given, the logarithms of all less arcs are very easily obtained.*

From the logarithms of all arcs not less than 45 degrees, given by hypothesis, you can obtain (by 57) the logarithms of all the remaining arcs decreasing down to 22 degrees 30 minutes. From these, again, may be had in like manner the logarithms of arcs down to 11 degrees 15 minutes. And from these the logarithms of arcs down to 5 degrees 38 minutes. And so on, successively, down to 1 minute.

59. *To form a logarithmic table.*

Prepare forty-five pages, somewhat long in shape, so that besides margins at the top and bottom, they may hold sixty lines of figures. Divide each page into twenty equal spaces by horizontal lines, so that each space may hold three lines of figures. Then divide each page into seven columns by vertical lines, double lines being ruled between the second and third columns and between the fifth and sixth, but a single line only between the others.

Next write on the first page, at the top to the left, over the first three columns, "0 *degrees*"; and at the bottom to the right, under the last

three columns, "89 *degrees*". On the second page, above, to the left, "1 *degree*"; and below, to the right, "88 *degrees*". On the third page, above, "2 *degrees*"; and below, "87 *degrees*". Proceed thus with the other pages, so that the number written above, added to that written below, may always make up a quadrant, less 1 degree or 89 degrees.

Then, on each page write, at the head of the first column, "*Minutes of the degree written above*"; at the head of the second column, "*Sines of the arcs to the left*"; at the head of the third column, "*Logarithms of the arcs to the left*"; at both the head and the foot of the third column, "*Difference between the logarithms of the complementary arcs*"; at the foot of the fifth column, "*Logarithms of the arcs to the right*"; at the foot of the sixth column, "*Sines of the arcs to the right*"; and at the foot of the seventh column, "*Minutes of the degree written beneath*".

Then enter in the first column the numbers of minutes in ascending order from 0 to 60, and in the seventh column the number of minutes in descending order from 60 to 0; so that any pair of minutes placed opposite, in the first and seventh columns in the same line, may make up a whole degree or 60 minutes; for example, enter 0 opposite to 60, 1 to 59, 2 to 58, and 3 to 57, placing three numbers in each of the twenty intervals between the horizontal lines. In the second column enter the values of the sines corresponding to the degree at the top and the minutes in the same line to the left; also in the sixth column enter the values of the sines corresponding to the degree

degree at the bottom and the minutes in the same line to the right. Reinhold's common table of sines, or any other more exact, will supply you with these values.

Having done this, compute, by 49 and 50, the logarithms of all sines between radius and its half, and by 54, the logarithms of the other sines; however, you may, with both greater accuracy and facility, compute, by the same 49 and 50, the logarithms of all sines between radius and the sine of 45 degrees, and from these, by 58, you very readily obtain the logarithms of all remaining arcs less than 45 degrees. Having computed these by either method, enter in the third column the logarithms corresponding to the degree at the top and the minutes to the left, and to their sines in the same line at left side; similarly enter in the fifth column the logarithm corresponding to the degree at the bottom and the minutes to the right and to their sines in the same line at right side.

Finally, to form the middle column, subtract each logarithm on the right from the logarithm on the left in the same line, and enter the difference in the same line, between both, until the whole is completed.

We have computed this Table to each minute of the quadrant, and we leave the more exact elaboration of it, as well as the emendation of the table of sines, to the learned to whom more leisure may be given.

Outline of the Construction, in another form, of a Logarithmic Table.

60. SINCE *the logarithms found by* 54 *sometimes differ from those found by* 58 (*for example, the logarithm of the sine* 378064 *is* 32752756 *by the former, while by the latter it is* 32752741), *it would seem that the table of sines is in some places faulty. Wherefore I advise the learned, who perchance may have plenty of pupils and computors, to publish a table of sines more reliable and with larger numbers, in which radius is made* 100000000, *that is with eight cyphers after the unit instead of seven only. Then, let the First table, like ours, contain a hundred numbers progressing in the proportion of the new radius to the sine less than it by unity, namely of* 100000000 *to* 99999999.

Let the Second table also contain a hundred numbers in the proportion of this new radius to the number less than it by a hundred, namely of 100000000 *to* 99999900.

Let the Third table, also called the Radical table, contain thirty-five columns with a hundred numbers in each column, and let the hundred numbers in each column progress in the proportion of ten thousand to the number less than it by unity, namely of 100000000 *to* 99990000.

Let the thirty-five proportionals standing first in all the columns, or occupying the second, third, or other rank, progress among themselves in the proportion of 100 *to* 99, *or of the new radius* 100000000 *to* 99000000.

In continuing these proportionals and finding their logarithms, let the other rules we have laid down be observed.

From

From the Radical table completed in this way, you will find with great exactness (by 49 *and* 50*) the logarithms of all sines between radius and the sine of* 45 *degrees; from the arc of* 45 *degrees doubled, you will find (by* 56*) the logarithm of half radius; having obtained all these, you will find the other logarithms by* 58. *Arrange all these results as described in* 59, *and you will produce a Table, certainly the most excellent of all Mathematical tables, and prepared for the most important uses.*

End of the Construction of the Logarithmic Table.

APPENDIX.

On the Construction of another and *better kind of* LOGARITHMS, *namely* one in which the Logarithm of unity is 0.

AMONG *the various improvements of* Logarithms, *the more important is that which adopts a cypher as the* Logarithm *of unity, and* 10,000,000,000 *as the* Logarithm *of either one tenth of unity or ten times unity. Then, these being once fixed, the* Logarithms *of all other numbers necessarily follow. But the methods of finding them are various, of which the first is as follows:—*

Divide the given Logarithm *of a tenth, or of ten, namely* 10,000,000,000, *by* 5 *ten times successively, and thereby the following numbers will be produced,* 2000000000, 400000000, 80000000, 16000000, 3200000, 640000, 128000, 25600, 5120, 1024. *Also divide the last of these by* 2, *ten times successively, and there will be produced* 512, 256, 128, 64, 32, 16, 8, 4, 2, 1. *Moreover all these numbers are logarithms.*

Thereupon let us seek for the common numbers which correspond

correspond to each of them in order. Accordingly, between a tenth and unity, or between ten and unity (adding for the purpose of calculation as many cyphers as you wish, say twelve), find four mean proportionals, or rather the least of them, by extracting the fifth root, which for ease in demonstration call A. *Similarly, between* A *and unity, find the least of four mean proportionals, which call* B. *Between* B *and unity find four means, or the least of them, which call* C. *And thus proceed, by the extraction of the fifth root, dividing the interval between that last found and unity into five proportional intervals, or into four means, of all which let the fourth or least be always noted down, until you come to the tenth least mean; and let them be denoted by the letters* D, E, F, G, H, I, K.

When these proportionals have been accurately computed, proceed also to find the mean proportional between K *and unity, which call* L. *Then find the mean proportional between* L *and unity, which call* M. *Then in like manner a mean between* M *and unity, which call* N. *In the same way, by extraction of the square root, may be formed between each last found number and unity, the rest of the intermediate proportionals, to be denoted by the letters* O, P, Q, R, S, T, V.

To each of these proportionals in order corresponds its Logarithm *in the first series. Whence* 1 *will be the* Logarithm *of the number* V, *whatever it may turn out to be, and* 2 *will be the* Logarithm *of the number* T, *and* 4 *of the number* S, *and* 8 *of the number* R, 16 *of the number* Q, 32 *of the number* P, 64 *of the number* O, 128 *of the number* N, 256 *of the number* M, 512 *of the number* L, 1024 *of the number* K; *all of which is manifest from the above construction.*

From these, once computed, there may then be formed both the proportionals of other Logarithms *and the* Logarithms *of other proportionals.*

For as in statics, from weights of 1, *of* 2, *of* 4, *of* 8, *and of other like numbers of pounds in the same proportion, every number of pounds weight, which to us now are* Logarithms, *may be formed by addition; so, from the proportionals* V, T, S, R, *&c., which correspond to them, and from others also to be formed in duplicate ratio, the proportionals corresponding to every proposed* Logarithm *may be formed by corresponding multiplication of them among themselves, as experience will show.*

The special difficulty of this method, however, is in finding the ten proportionals to twelve places by extraction of the fifth root from sixty places, but though this method is considerably more difficult, it is correspondingly more exact for finding both the Logarithms *of proportionals and the proportionals of* Logarithms.

Another method for the easy construction of the LOGARITHMS of composite numbers, when the LOGARITHMS of their primes are known.

IF *two numbers with known* Logarithms *be multiplied together, forming a third; the sum of their* Logarithms *will be the* Logarithm *of the third.*

Also if one number be divided by another number, producing a third; the Logarithm *of the second subtracted from the* Logarithm *of the first, leaves the* Logarithm *of the third.*

If from a number raised to the second power, to the third power, to the fifth power, &c., certain other numbers be produced; from the Logarithm *of the first multiplied by two, three, five, &c., the* Logarithms *of the others are produced.*

Also

Also if from a given number there be extracted the second, third, fifth, &c., roots; and the Logarithm *of the given number be divided by two, three, five, &c., there will be produced the* Logarithms *of these roots.*

Finally any common number being formed from other common numbers by multiplication, division, [*raising to a power*] *or extraction* [*of a root*]; *its* Logarithm *is correspondingly formed from their* Logarithms *by addition, subtraction, multiplication, by* 2, 3, *&c.* [*or division by* 2, 3, *&c.*]: *whence the only difficulty is in finding the* Logarithms *of the prime numbers; and these may be found by the following general method.*

For finding all Logarithms, *it is necessary as the basis of the work that the* Logarithms *of some two common numbers be given or at least assumed; thus in the foregoing first method of construction,* 0 *or a cypher was assumed as the* Logarithm *of the common number one, and* 10,000,000,000 *as the* Logarithm *of one-tenth or of ten. These therefore being given, the* Logarithm *of the number* 5 (*which is a prime number*) *may be sought by the following method. Find the mean proportional between* 10 *and* 1, *namely* $\frac{3162277660017}{1000000000000}$, *also the arithmetical mean between* 10,000,000,000 *and* 0, *namely* 5,000,000,000; *then find the geometrical mean between* 10 *and* $\frac{3162277660017}{1000000000000}$, *namely* $\frac{5623413325191}{1000000000000}$, *also the arithmetical mean between* 10,000,000,000 *and* 5,000,000,000, *namely* 7,500,000,000;

. . . . . .

In all continuous proportionals.

AS *the sum of the means and one or other of the extremes to the same extreme; so is the difference of the extremes to the difference of the same extreme and the nearest mean.*

A saving of half the Table of LOGARITHMS.

OF *two arcs making up a quadrant, as the sine of the greater is to the sine of double its arc, so is the sine of* 30 *degrees to the sine of the less. Whence the* Logarithm *of the double arc being added to the* Logarithm *of* 30 *degrees, and the* Logarithm *of the greater being subtracted from the sum, there remains the* Logarithm *of the less.*

The relations of LOGARITHMS & their natural numbers to each other.

[A] 1. LEt *two sines and their* Logarithms *be given. If as many numbers equal to the less sine be multiplied together as there are units in the* Logarithm *of the greater; and on the other hand, as many numbers equal to the greater sine be multiplied together as there are units in the* Logarithm *of the less; two equal numbers will be produced, and the* Logarithm *of the sine so produced will be the product of the two* Logarithms.

2. *As the greater sine is to the less, so is the velocity of increase or decrease of the* Logarithms *at the less, to the velocity of increase or decrease of the* Logarithms *at the greater.*

3. *Two sines in duplicate, triplicate, quadruplicate, or other ratio, have their* Logarithms *in double, triple, quadruple, or other ratio.*

4. *And two sines in the ratio of one order to another order, as for instance the triplicate to the quintuplicate, or the cube*

cube to the fifth, have their Logarithms *in the ratio of the indices of their orders, that is of* 3 *to* 5.

5. *If a first sine be multiplied into a second producing a third, the* Logarithm *of the first added to the* Logarithm *of the second produces the* Logarithm *of the third. So in division, the* Logarithm *of the divisor subtracted from the* Logarithm *of the dividend leaves the* Logarithm *of the quotient.*

6. *And if any number of equals to a first sine be multiplied together producing a second, just so many equals to the* Logarithm *of the first added together produce the* Logarithm *of the second.*

7. *Any desired geometrical mean between two sines has for its* Logarithm *the corresponding arithmetical mean between the* Logarithms *of the sines.*

8. *If a first sine divide a third as many times successively as there are units in* A ; *and if a second sine divides the same third as many times successively as there are units in* B ; *also if the same first divide a fourth as many times successively as there are units in* C ; *and if the same second divide the same fourth as many times successively as there are units in* D : *I say that the ratio of* A *to* B *is the same as that of* C *to* D, *and as that of the* Logarithm *of the second to the* Logarithm *of the first.* [B]

9. *Hence it follows that the* Logarithm *of any given number is the number of places or figures which are contained in the result obtained by raising the given number to the* 10,000,000,000th *power.* [C]

10. *Also if the index of the power be the* Logarithm *of* 10, *the number of places, less one, in the power or multiple, will be the* Logarithm *of the root.*

Suppose it is asked what number is the LOGARITHM of 2. I reply, the number of places in the result obtained by multiplying together 10,000,000,000 of the number 2.

But, you will say, the number obtained by multiplying together 10,000,000,000 of the number 2 is innumerable. I reply, still the number of places in it, which I seek, is numerable.

Therefore, with 2 as the given root, and 10,000,000,000 as the index, seek for the number of places in the multiple, and not for the multiple itself; and by our rule you will find 301029995 &c. to be the number of places sought, and the LOGARITHM of the number 2.

F I N I S.

SOME

SOME REMARKS
BY THE LEARNED
HENRY BRIGGS
On the foregoing APPENDIX.

The relations of LOGARITHMS and their natural *numbers to each other, when the* LOGARITHM *of unity is made* 0.

TWo *numbers with their* Logarithms *being given;* [A] *if both* Logarithms *be divided by some common divisor, and if each of the given numbers be multiplied by itself continuously, until the number of multiplications is exceeded, by unity only, by the quotient of the* Logarithm *of the other number, two equal numbers will be produced. And the* Logarithm *of the number produced will be the continued product of the quotients of the* Logarithms *and their common divisor.*

		Logarithms.
Let the given numbers be	$\left\{\begin{matrix}25118865\\39810718\end{matrix}\right.$	$\begin{matrix}4\\6\end{matrix}$

 Let

Let the common divisor be 1

The first multiplied by itself 5 times
The second „ „ 3 „ } makes $\frac{251188649}{1000000}$

Continued Proportionals.			Logarithms.
1	(0)		0
251188865	(1)	First power	4
63095737	(2)	Second power	8
158489331	(3)	Third power	12
39810718	(4)	Fourth power	16
100000000	(5)	Fifth power	20
251188649	(6)	Sixth power	24

Continued Proportionals.		Logarithms.
1	(0)	0
39810718	(1)	6
158489331	(2)	12
630957379	(3)	18
251188649	(4)	24

ANOTHER EXAMPLE.

		Logarithms.
Let the given numbers be	316227766	5
	50118724	7

Let the common divisor be 1

The first multiplied by itself 6 times
The second „ „ 4 „ } makes 316227766

		Logarith.
1	(0)	0
316227766	(1)	5
1000000000	(2)	10
100	(4)	20
1000	(6)	30
316227766	(7)	35

		Logarith.
1	(0)	0
50118724	(1)	7
2511886 49	(2)	14
630957376	(4)	28
316227766	(5)	35

It should be observed that if the common divisor be unity, as in both the preceding examples, the product of the given Logarithms *is the* Logarithm *of the number produced, because multiplication by unity does not increase the thing multiplied.*

THIRD EXAMPLE.

Let the given numbers be { 343 (Logarithms 2.53529412, Quotients 3) ; 823543 (5.91568628, 7) }

Let the common divisor be 84509804

Number of Places.			
	1	(0)	0
3	343	(1)	2.53529412
6	117649	(2)	5.07058824
8	40353607	(3)	7.60588236
11	3841287201	(4)	10.14117648
18	558545864083284007	(7)	17.74705884
6	823543	(1)	5.91568628
12	678223072849	(2)	11.83137256
18	558545864083284007	(3)	17.74705884

As the quotients of the given LOGARITHMS are 3 and 7, their product is 21, which, multiplied by 84509804 the common divisor, makes 17.74705884 the LOGARITHM of the number produced.

It should be observed that the cube of the second number, and its equal the seventh power of the first (which some call secundus solidus*), contain eighteen figures, wherefore its* Logarithm *has* 17. *in front, besides the figures following. The latter represent the* Logarithm *of the number denoted by the same digits, but of which* 5, *the first digit to the left, is alone integral, the remaining digits expressing a fraction added to the integer, thus* $5\frac{5854586408}{10000000000}$ *&c. has for its* Logarithm 74705884. *Again, if four places remain integral,* 3. *must be placed in front of the* Logarithm, *thus* $5585\frac{4586408}{10000000}$ *&c. has for its* Logarithm 3.74705884.

Hence from two given Logarithms *and the sine of the first we shall be able to find the sine of the second.*

Take some common divisor of the Logarithms, (*the larger the better*); *divide each by it. Then let the first sine multiply itself and its products continuously until the number of these products is exceeded, by unity only, by the quotient of the second* Logarithm; *or until the power is produced of like name with the quotient of the second* Logarithm. *The same number would be produced if the second sine, which is sought, were to multiply itself until it became the power of like name with the quotient of the first* Logarithm, *as is evident from the preceding proposition. Therefore*

Therefore take the above power and seek for the root of it which corresponds to the quotient of the first Logarithm; *thereby you will find the required second sine. Also the* Logarithm *of the power itself will be the continued product of the quotients and the common divisor.*

Thus let the given LOGARITHMS be 8 and 14, and the sine corresponding to the first LOGARITHM be 3. A common divisor of the LOGARITHMS is 2; this gives the quotients 4 and 7. If 3 multiply itself six times, you will have 2187 for the power which, in a series of continued proportionals from unity, will occupy the seventh place, and hence it may, without inconvenience, be called the seventh power. The same number, 2187, is the fourth power from unity in another series of continued proportionals, in which the first power, $6\frac{838521}{1000000}$, is the required second sine. The product of the quotients 4 and 7 is 28, which, multiplied by the common divisor 2, makes 56, the LOGARITHM of the power 2187.

Continued Proportionals.		Logarithms.
1	(0)	0
3	(1)	8
9	(2)	16
27	(3)	24
81	(4)	32
243	(5)	40
729	(6)	48
2187	(7)	56

Continued Proportionals.		Logarithms.
1	(0)	0
6838521	(1)	14
46765372	(2)	28
319805 98	(3)	42
2187	(4)	56

It will be observed that these Logarithms *differ from those employed in illustration of the previous Proposition;*

but they agree in this, that in both, the Logarithm *of unity is* 0; *and consequently the* Logarithms *of the same numbers are either equal or at least proportional to each other.*

[B] *If a first sine divide a third,*)

The first must divide the third, and the quotient of the third, and each quotient of a quotient successively as many times as possible, until the last quotient becomes less than the divisor. Then let the number of these divisions be noted, but not the value of any quotient, unless perhaps the least, to which we shall refer presently. In the same manner let the second divide the same third. And so also let the fourth be divided by each.

Thus let the { first sine be 2; second „ „ 4; third „ „ 16; fourth „ „ 64

The first, 2. *divides the third,* 16. *four times; and the quotients are* 8, 4, 2, 1. *The second,* 4. *divides the same third,* 16. *two times; and the quotients are* 4, 1. *Therefore* A *will be* 4, *and* B *will be* 2.

In the same manner the first, 2. *divides the fourth,* 64. *six times; and the quotients are* 32, 16, 8, 4, 2, 1. *The second,* 4. *divides the fourth,* 64. *three times; and the quotients are* 16, 4, 1. *Therefore* C *will be* 6, *and* D *will be* 3.

Hence I say that, as A, 4. *is to* B, 2. *so is* C, 6. *to* D, 3. *and so is the* Logarithm *of the second to the* Logarithm *of the first.*

If in these divisions the last and smallest quotient be everywhere unity, as in these four cases, the numbers of the

the quotients and the Logarithms *of the divisors will be reciprocally proportional.*

Otherwise the ratio will not be exactly the same on both sides; nevertheless, if the divisors be very small, and the dividends sufficiently large, so that the quotients are very many, the defect from proportionality will scarcely, or not even scarcely, be perceived.

Hence it follows that the logarithm) [C]

Let two numbers be taken, 10 *and* 2, *or any others you please. Let the* Logarithm *of the first, namely* 100, *be given; it is required to find the* Logarithm *of the second. In the first place, let the second,* 2. *multiply itself continuously until the number of the products is exceeded, by unity only, by the given* Logarithm *of the first. Then let the last product be divided as often as possible by the first number,* 10. *and again in like manner by the second number,* 2. *The number of quotients in the latter case will be* 100, (*for the product is its hundredth power; and if a number be multiplied by itself a given number of times forming a certain product, then it will divide the product as many times and once more; for example, if* 3 *be multiplied by itself four times it makes* 243, *and the same* 3 *divides* 243 *five times, the quotients being* 81, 27, 9, 3, 1.) *In the former case, where the product is continually divided by* 10, *it is manifest that the number of quotients falls short of the number of places in the dividend by one only. Therefore* (*by the preceding proposition*) *since the same product is divided by two given numbers as often as possible, the numbers of the quotients and the* Logarithms *of the divisors will be reciprocally proportional. But, the number of quotients by the second being equal to the* Logarithm *of the first, the num-*

ber of quotients by the first, that is the number of places in the product less one, will be equal to the Logarithm *of the second.*

Number of Places.		
	1	0
1	2	1
1	4	2
2	16	4
3	256	8
4	1024	10
7	1048576	20
13	1099511627776	40
25	1208925819614	80
31	1267650600228	100
61	16069379676	200
121	25822496318	400
241	66680131608	800
302	107150835165	1000
603	114813014767	2000
1205	131820283599	4000
2409	17316587168	8000
3011	19950583591	10000

Here we see that if we assume the LOGARITHM of 10 to be 10, the number of places in the tenth power is 4, wherefore the logarithm of 2 will be 3 and something over. The number of places in the hundredth power is 31; in the thousandth, 302; in the ten thousandth, 3011; and generally the more products we take the more nearly do we approach the true LOGARITHM sought for. For when the products are few, the fraction adhering to the

the last quotient disturbs the ratio a little; but if we assume the LOGARITHM of 10 to be 10,000,000,000, and if 2 be multiplied by itself continuously until the number of products is exceeded, by one only, by the given LOGARITHM; then the number of places, less one, in the last product, will give the LOGARITHM of 2 with sufficient accuracy, because in large numbers the small fraction adhering to the last quotient will have no effect in disturbing the proportion.

THE END.

SOME VERY REMARKABLE *PROPOSITIONS FOR THE* solution of spherical triangles *with wonderful ease.*

To solve a spherical triangle without dividing it into two quadrantal or rectangular triangles.

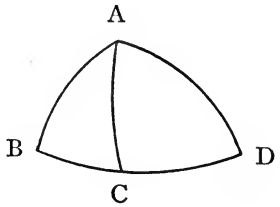
Prop. 1. GIVEN *three sides, to find any angle.*

And conversely,

2. *Given three angles, to find any side.*

This is best done by the three methods explained in my work on LOGARITHMS, *Book* II. *chap.* vi. *sects.* 8, 9, 10.

3.



Given the side A D, *& the angles* D *&* B, *to find the side* A B.

Multiply the sine of A D by the sine of D; divide the product by the sine of B, and you will have the sine of A B.

4. *Given*

4. *Given the side* A D, *& the angles* D *&* B, *to find the side* B D.

Multiply radius by the sine of the complement of D; divide by the tangent of the complement of A D, and you will obtain the tangent of the arc C D: then multiply the sine of C D by the tangent of D; divide the product by the tangent of B, and the sine of B C will result: add or subtract B C and C D, and you have B D.

5. *Given the side* A D, *& the angles* D *&* B, *to fina the angle* A.

Multiply radius by the sine of the complement of A D; divide by the tangent of the complement of D, and the tangent of the complement of C A D will be produced; whence we have C A D itself. Similarly multiply the sine of the complement of B by the sine of C A D; divide by the sine of the complement of D, and the sine of B A C will be produced; which being added to or subtracted from C A D, you will obtain the required angle B A D.

6. *Given* A D, *& the angle* D *with the side* B D, *to find the angle* B.

Multiply radius by the sine of the complement of D; divide by the tangent of the complement of A D, and the tangent of C D will be produced; its arc C D subtract from, or add to, the side B D, and you have B C: then multiply the sine of C D by the tangent of D; divide the product by the sine of B C, and you have the tangent of the angle B.

7. *Given* A D, *& the angle* D *with the side* B D, *to find the side* A B.

Multiply radius by the sine of the complement of D; divide the product by the tangent of the complement of A D, and the tangent of C D will be

produced; its arc C D subtract from, or add to, the given side B D, and you have B C. Then multiply the sine of the complement of A D by the sine of the complement of B C; divide the product by the sine of the complement of C D, and the sine of the complement of A B will be produced; hence you have A B itself.

Given A D, & the angle D with the side B D, to find the angle A.

This follows from the above, but the problem would require the "Rule of Three" to be applied thrice. Therefore substitute A for B and B for A, and the problem will be as follows:—

Given B D *&* D, *with the side* A D, *to find the angle* B.

This is exactly the same as the sixth problem, and is solved by the "Rule of Three" being applied twice only.

8. *Given* A D, *&* *the angle* D *with the side* A B, *to find the angle* B.

Multiply the sine of A D by the sine of D; divide the product by the sine of A B, and the sine of the angle B will be produced.

9. *Given* A D, *&* *the angle* D *with the side* A B, *to find the side* B D.

Multiply radius by the sine of the complement of D, divide the product by the tangent of the complement of A D, and the tangent of the arc C D will be produced. Then multiply the sine of the complement of C D by the sine of the complement of A B, divide the product by the sine of the complement of A D, and you have the sine of the complement of B C. Whence the sum or the difference of the arcs B C and C D will be the required side B D.

10. *Given*

10. *Given* A D, *& the angle* D *with the side* A B, *to find the angle* A.

Multiply radius by the sine of the complement of A D, divide the product by the tangent of the complement of D, and the tangent of the complement of C A D will be produced, giving us C A D. Again, multiply the tangent of A D by the sine of the complement of C A D, divide the product by the tangent of A B, and the sine of the complement of B A C will be produced, giving B A C. Then the sum or difference of the arcs B A C and C A D will be the required angle B A D.

11. *Given* A D, *& the angle* D *with the angle* A, *to find the side* A B.

Multiply radius by the sine of the complement of A D, divide the product by the tangent of the complement of D, and you have the tangent of the complement of C A D; C A D being thus known, the difference or sum of the same and the whole angle A is the angle B A C. Multiply the tangent of A D by the sine of the complement of C A D; divide the product by the sine of the complement of B A C, and you will have the tangent of A B.

12. *Given* A D, *& the angle* D *with the angle* A, *to find the third angle* B.

Multiply radius by the sine of the complement of A D, divide the product by the tangent of the complement of D, and the sine of the complement of B will be produced, from which we have the angle required.

Given A D, & the angle D with the angle A, to find the side B D.

This follows from the above, but in this form the problem would require the "Rule of Three" to be

three times applied. Therefore substitute A for D and D for A, and the problem will be as follows :—

Given A D *& the angle* A *with the angle* D, *to find the side* B A.

This is the same throughout as problem 11, and is solved by applying the "Rule of Three" twice only.

The use and importance of half-versed sines.

1. GIVEN *two sides & the contained angle, to find the third side.*

From the half-versed sine of the sum of the sides subtract the half-versed sine of their difference; multiply the remainder by the half-versed sine of the contained angle; divide the product by radius; to this add the half-versed sine of the difference of the sides, and you have the half-versed sine of the required base.

Given the base and the adjacent angles, the vertical angle will be found by similar reasoning.

2. *Conversely, given the three sides, to find any angle.*

From the half-versed sine of the base subtract the half-versed sine of the difference of the sides multiplied by radius; divide the remainder by the half-versed sine of the sum of the sides diminished by the half-versed sine of their difference, and the half-versed sine of the vertical angle will be produced.

Given the three angles, the sides will be found by similar reasoning.

3. *Given two arcs, to find a third, whose sine shall be equal to the difference of the sines of the given arcs.*

Let

Let the arcs be 38° 1′ and 77°. Their complements are 51° 59′ and 13°. The half sum of the complements is 32° 29′, the half difference 19° 29′, and the logarithms are 621656 and 1098014 respectively. Adding these, you have 1719670, from which, subtracting 693147, the logarithm of half radius, there will remain 1026523, the logarithm of 21°, or thereabout. Whence the sine of 21°, namely 358368, is equal to the difference of the sines of the arcs 77° and 38° 1′, which sines are 974370 and 615891, more or less.

4. *Given an arc, to find the Logarithm of its versed sine.* [a]

Let the arc be 13°; its half is 6° 30′, of which the logarithm is 2178570. From double this, namely 4357140, subtract 693147, and there will remain 3663993. The arc corresponding to this is 1° 28′, and the number put for the sine is 25595; but this is also the versed sine of 13°. ***

5. *Given two arcs, to find a third whose sine shall be equal to the sum of the sines of the given arcs.*

Let the arcs be 38° 1′ and 1° 28′; their sum is 39° 29′ and their difference 36° 33′, also the half sum is 19° 44′ and the half difference 18° 16′. Wherefore add the logarithm of the half sum, viz. 1085655, to the logarithm of the difference, viz. 518313, and you have 1603968; from this subtract the logarithm of the half difference, namely 1160177, and there will remain the logarithm 443791, to which correspond the arc 39° 56′ and sine 641896. But this sine is equal, or nearly so, to the sum of the sines of 38° 1′ and 1° 28′, namely 615661 and 25595 respectively.

6. *Given an arc & the Logarithm of its sine, to find the arc whose versed sine shall be equal to the sine of the given arc.*

Let the arc be 39° 56′, to which corresponds the logarithm 443791, the sine being unknown. To the logarithm 443791 add 693147, the logarithm of half radius, and you have 1136938. Halve this logarithm and you have 568469. To this corresponds the arc 34° 30′, which being doubled gives 69° for the arc which was sought. This is the case since the sine of 39° 56′ and the versed sine of 69° are each equal, or nearly so, to 641800.

[b] *Of the spherical triangle* A B D, *given the sides & the contained angle, to find the base.*

LEt the sides be 34° and 47°, and the contained angle 120° 24′ 49″. Half the contained angle is 60° 12′ 24½″, and its logarithm 141766. To the double of the latter, namely 283533, add the logarithms of the sides, namely 581260 and 312858, and the sum is 1177651. This sum is the logarithm of half the difference between the versed sine of the base and the versed sine of the difference of the sides; it is also the logarithm of the sine of the arc 17° 56′, which arc we call the "second found," for that which follows is first found.

Halve the difference of the sides, namely 13°, and you have 6° 30′, the logarithm of which is 2178570. Double the latter and you have 4357140 for the logarithm of the half-versed sine of 13°; it is also the logarithm of the sine of the arc 0° 44′, which arc we call the "first found."

The sum of the two arcs is 18° 40′, the half sum 9° 20′, and their logarithms 1139241 and 1819061 respectively. Also the difference of the two arcs is 17° 12′, the half difference 8° 36′, and their logarithms 1218382 and 1900221 respectively.

Now

Now add the logarithm of the half sum, namely 1819061,

either

to the logarithm 1218382, and the sum will be 3037443; from this subtract the logarithm 1900221 and there will remain 1137222.

or

to the logarithm of the complement of the half difference, namely 11307, and the sum will be 1830368; from this subtract 693147 and there will remain 1137221.

Halve the latter and you have the logarithm 568611, to which corresponds the arc 34° 30′, and double this arc is the base required, namely 69°.

Conversely, given the three sides, to find any angle. The solution of this problem is given in my work on Logarithms, Book II. chap. vi. sect. 8, *but partly by logarithms and partly by prosthaphæresis of arcs.*

It is to be observed that in the preceding and following problems there is no need to discriminate between the different cases, since the form and magnitude of the several parts appear in the course of the calculation.

Another direct converse of the preceding problem follows.—

[*Given the sides and the base, to find the vertical angle.*]

HALVE the given base, namely 69°, and you have 34° 30′, the logarithm of which is 568611. Double the latter and you have 1137222; corresponding to this is the arc 18° 42′, which note as the second found.

As before, take for the first found the arc 0° 44′, corresponding to the logarithm 4357140.

The complements of the two arcs are 89° 16′ and 71° 18′; their half sum is 80° 17′, and its logarithm

 14449;

14449; their half difference is 8° 59′, and its logarithm 1856956. Add these logarithms and you have 1871405; subtract 693147 and there remains 1178258. The arc corresponding to this logarithm is 17° 56′, which arc we call the third found.

From the logarithm of the third found, subtract the logarithms of the given sides, namely 581260 and 312858, and there remains 283533; halve this and you have 141766 for the logarithm of the half vertical angle 60° 12′ 24½″. The whole vertical angle sought is therefore 120° 24′ 49″.

Another rule for finding the base by prosthaphæresis.—

[*Given the sides and vertical angle, to find the base.*]

NOte the half difference between the versed sines of the sum and difference of the sides, and also the half-versed sine of the vertical angle. Look among the common sines for the values noted, and find the arcs corresponding to them in the table. Then write for the second found the half difference of the versed sines of the sum and difference of these arcs.

Also, as before, take for the first found the half-versed sine of the difference of the sides.

Add the first and second found, and you will obtain the half-versed sine of the base sought for.

Conversely—[*given the sides and the base, to find the vertical angle.*]

The first found will be, as before, the half-versed sine of the difference of the sides.

From the half-versed sine of the base subtract the first found and you will have the second found.

Multiply the latter by the square of radius; divide by

by the half difference between the versed sines of the sum and difference of the sides, and you have as quotient the half-versed sine of the vertical angle sought for.

Of five parts of a spherical triangle, given the three intermediate, to find the two extremes by a single operation. Or otherwise, given the base and adjacent angles, to find the two sides. [c]

(*) OF the angles at the base, write down the sum, half sum, difference and half difference, along with their logarithms.

Add together the logarithm of the half sum, the logarithm of the difference, and the logarithm of the tangent of half the base; subtract the logarithm of the sum and the logarithm of the half difference, and you will have the first found.

Then to the logarithm of the half difference add the logarithm of the tangent of half the base; subtract the logarithm of the half sum, and you will have the second found.

Look for the first and second found among the logarithms of tangents, since they are such, then add their arcs and you will have the greater side; again subtract the less arc from the greater and you will have the less side.

Another way of finding the sides.

ADd together the logarithm of the half sum of the angles at the base, the logarithm of the complement of the half difference, and the logarithm of the tangent of half the base; subtract the logarithm of the sum and the logarithm of half radius, and you will have the first found.

Again, add together the logarithm of the half difference, the logarithm of the complement of the half sum, and the logarithm of the tangent of half the base; subtract the logarithm of the sum and the logarithm of half radius, and you will have the second found.

Proceed as above with the first and second found, and you will obtain the sides.

Another way of the same.

MUltiply the secant of the complement of the sum of the angles at the base by the tangent of half the base.

Multiply the product by the sine of the greater angle at the base, and you will have the first found.

Multiply the same product by the sine of the less angle, and you will have the second found.

[d] Then divide the sum of the first and second found by the square of radius, and you will have the tangent of half the sum of the sides.

Also subtract the less from the greater and you will have the tangent of half the difference of the sides.

Whence add the arcs corresponding to these two tangents, and the greater side will be obtained; subtract the less arc from the greater and you have the less side.

Of the five consecutive parts of a spherical triangle, given the three intermediate, to find both extremes by one operation and without the need of discriminating between the several cases.

(*) OF the angles at the base, the sine of the half difference is to the sine of the half sum, as the sine of the difference is to a fourth which is the sum of the sines.

And

And the sine of the sum is to the sum of the sines as the tangent of half the base is to the tangent of half the sum of the sides.

Whence the sine of the half sum is to the sine of the half difference of the angles as the tangent of half the base is to the tangent of half the difference of the sides.

Add the arcs of these known tangents, taking them from the table of tangents, and you will have the greater side; in like manner subtract the less from the greater and the less side will be obtained.

FINIS.

SOME NOTES
BY THE LEARNED
HENRY BRIGGS
ON THE FOREGOING PROPOSITIONS.

[a] GIVEN *an arc, to find the logarithm of its versed sine.*

*To the end of this proposition *** I should like to add the following:—*

Conversely, given the logarithm of a versed sine, to find its arc.

Add the known logarithm of the required versed sine to the logarithm of 30°, *viz.*, 693147, *and half the sum will be the logarithm of half the arc sought for.*

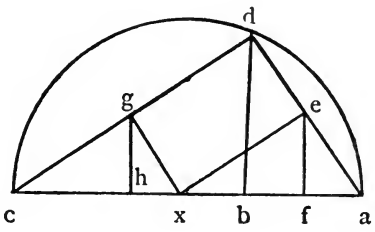
Thus let 35791 be the given logarithm of an unknown versed sine, whose arc is also unknown.

To this logarithm add 693147, and the sum will be 728938, half of which, 364469, is the logarithm of 43° 59′ 33″. The arc of the given logarithm is therefore 87° 59′ 6″, and its versed sine is 9648389.

Again, let a negative logarithm, say −54321, be the known logarithm of the required versed sine. To this logarithm

logarithm add, as before, 693147, and the sum, that is the number remaining since the sines are contrary, will be 638826, half of which, 319413, is the logarithm of 46° 36′ 0″. The arc of the given logarithm is therefore 93° 12′ 0″, the versed sine of which is 10558216, and since this is greater than radius it has a negative logarithm, namely −54321.

DEMONSTRATION.



c b, a b } versed sine of arc { c d, a d

x c, c g, c h	cont. pro- port.	x a, a e, a f	cont. pro- port.	½x c, sine of 30° 0′ c g, sine of ½ arc c d c b, double of line c h	cont. pro- port.

Later on I observed that the sixth proposition might be proved in an exactly similar way.

Of the spherical triangle A B D]

In finding the base we may pursue another method, namely:—

Add the logarithm of the versed sine of the given angle to the logarithms of the given sides, and the sum will be the logarithm of the difference between the versed sine of the difference of the sides and the versed sine of the base required. This difference being consequently known, add to it the versed sine of the difference of the sides, and the sum will be the versed sine of the base required.

For example, let the sides be 34° and 47°, their loga-

 rithms

rithms 581261 and 312858, and the logarithm of the versed sine of the given angle —409615. The sum of these three logarithms is 484504, which is the logarithm of the difference between the versed sine of the base and the versed sine of the difference of the sides.

Now the line corresponding to this logarithm, whether a versed sine or a common sine, is 6160057, and consequently this is the difference between the versed sine of the base and the versed sine of the difference of the sides. If to this you add the versed sine of the difference of the sides, that is 256300, the sum will be the versed sine of the base required, namely 6416357, and this subtracted from radius leaves the sine of the complement of the base, namely 3583643, which is the sine of 21°. Consequently the base required is 69°.

Conversely, given three sides, to find any angle.

If from the logarithm of the difference between the versed sine of the base and the versed sine of the difference of the sides you subtract the logarithms of the sides, the remainder will be the logarithm of the versed sine of the angle sought for.

As in the previous example, let the logarithms of the sides be 581261 and 312858. Subtract their sum, 894119, from the logarithm 484504, and the remainder will be the negative logarithm —409615, which gives the versed sine of the required angle 120° 24′ 49″.

[c] *Of five parts of a spherical triangle*]

This proposition appears to be identical with the one which is inserted at the end, and distinguished like the former by (*). *The latter proposition I consider much the superior. There are, however, three operations in it, the first two of which I throw into one, as they are better combined. Thus:—*

Let

Let there be given

the base 69°,

the angles at the base $\begin{cases} 42^\circ\ 29'\ 59'' \\ 31^\circ\ 6'\ 5'' \end{cases}$

73° 36′ 4″	sum.
36° 48′ 2″	half sum.
53° 11′ 58″	complement of ½ sum.
11° 23′ 54″	difference.
5° 41′ 57″	half difference.
84° 18′ 3″	compl. of ½ diff.

			Logarithms.
Proportion 1.	Sine half difference	5° 41′ 57″	23095560
	Sine half sum	36° 48′ 2″	5124410
	Sine difference	11° 23′ 54″	16213641
	Sum of sines		—1757509
Proportion 2.	Sine of sum	73° 36′ 4″	4153̇12
	Sum of sines		—1757509
	Tangent half base	34° 30′ 0″	3750122
	Tangent ½ sum of sides	40° 30′ 0″	1577̇301
Proportion 3.	Sine ½ sum of angles	36° 48′ 2″	5124410
	Sine ½ diff. of angles .	5° 41′ 57″	23095560
	Tangent ½ base . .	34° 30′ 0″	3750122
	Tangent ½ diff. of sides	6° 30′ 0″	21721272

40° 30′
6° 30′

47° 0′
34° 0′ } sides.

These are the operations described by the Author. But I replace the first two by another, retaining the third.

			Logarithms.
Proportion.	Sine compl. $\frac{1}{2}$ sum of angles	53° 11′ 58″	2222368
	Sine compl. $\frac{1}{2}$ diff. of angles	84° 18′ 3″	49553
	Tangent $\frac{1}{2}$ base . .	34° 30′ 0″	3750122
	Tangent $\frac{1}{2}$ sum of sides .	40° 30′ 0″	1577307

ANOTHER EXAMPLE.

Let there be given

the angle 47°,

the sides containing it { 59° 35′ 11″ / 31° 6′ 5″

90° 41′ 16″	sum.
45° 20′ 38″	half sum.
44° 39′ 22″	compl. of half sum.
28° 29′ 6″	difference.
14° 14′ 33″	half difference.
75° 45′ 27″	compl. of half diff.

			Logarithms.
Proportion 1.	Sine compl. $\frac{1}{2}$ sum of sides	44° 39′ 22″	3526118
	Sine compl. $\frac{1}{2}$ diff. of sides	75° 45′ 27″	312192
	Tan. compl. $\frac{1}{2}$ vert. angle	66° 30′ 0″	—8328403
	Tan. $\frac{1}{2}$ sum of angs. at base	72° 30′ 0″	—11452329
Proportion 2.	Sine $\frac{1}{2}$ sum of sides .	45° 20′ 38″	3406418
	Sine $\frac{1}{2}$ diff. of sides .	14° 14′ 33″	14023154
	Tan. compl. $\frac{1}{2}$ vert. angle	66° 30′ 0″	—8328403
	Tan. $\frac{1}{2}$ diff. of angs. at base	38° 30′ 0″	2288333

72° 30′
38° 30′

111° 0′
34° 0′ } angles at the base.

And these relations are all uniformly maintained, whether

whether there be given two angles with the interjacent side or two sides with the contained angle. In each operation the important point is what occupies the third place in the proportion. In the former it is the tangent of half the base, in the latter the tangent of the complement of half the vertical angle. In these examples, if the tangent or the sum of the sines be greater than radius, the logarithm is negative and has a dash preceding, for example —8328403.

Another way of the same]

Then divide the sum of the first and second found by [d] the square of radius, and you will have)

To make the sense clearer, I should prefer to write this as follows:—

Then divide both the first and second found by the square of radius, add the quotients, and you will have the tangent, &c.

This proposition is absolutely true, as well as the one preceding; but while the former may most conveniently be solved by logarithms, the latter will not admit of the use of logarithms throughout, as the quotients must be added and subtracted to find the tangents; for the utility of Logarithms is seen in proportionals, and therefore in multiplication and division, and not in addition or subtraction.

THE END.

NOTES

BY

THE TRANSLATOR

NOTES.

Spelling of the Author's Name.

THE spelling in ordinary use at the present time is Napier. The older spellings are various—for example, Napeir, Nepair, Nepeir, Neper, Nepper, Naper, Napare, Naipper. Several of these spellings are known to have been used by our author.

I adopt the modern spelling, which is that used by his biographers, and also in the 1645 edition of 'A Plaine Discovery.'

If, however, the claim of present usage be set aside, a strong case might be made out for Napeir, as this was the spelling adopted in 'A Plaine Discovery,' the only book published by Napier in English. In this work is a letter signed "John Napeir" dedicating the book to James VI., and as this letter is a solemn address to the King, we may infer that the signature would be in the most approved form. The work was first issued in 1593, and the same spelling was retained in the subsequent editions during the author's lifetime, as well as in the French editions which were revised by him. In the 1645 edition, as mentioned above, the modern spelling was introduced.

The form Nepair is used in Wright's translation of the Descriptio, published in 1616, but too much stress must not be laid on this, as very slight importance was attached to the spelling of names; thus although Briggs contributed a preface, his name is spelt in three different ways,—Brigs, Brigges, and Briggs.

In the works published in Latin the form Neperus is invariably used.

On

On some Terms made use of in the Original Work.

Napier's Canon or Table of Logarithms does not contain the logarithms of equidifferent numbers, but of sines of equidifferent arcs for every minute in the quadrant. A specimen page of the Table is given in the Catalogue under the 1614 edition of the Descriptio.

The sine of the Quadrant or Radius, which he calls *Sinus Totus*, was assumed to have the value 10000000.

Numerus Artificialis, or simply *Artificialis*, is used in the body of the Constructio for Logarithm, the number corresponding to the logarithm being called *Numerus Naturalis*.

Logarithmus, corresponding to which *Numerus Vulgaris* is used, is however employed in the title-page and headings of the Constructio, and in the Appendix and following papers. It is also used throughout the Descriptio published in 1614; and as the word was not invented till several years after the completion of the Constructio (see the second page of the Preface, line 12), the latter must have been written some years prior to 1614.

For shortness, Napier sometimes uses the expression logarithm of an arc for the logarithm of the sine of an arc.

The *Antilogarithm* of an arc, meaning log. sine complement of arc, and the *Differential* of an arc, meaning log. tangent of arc (see Descriptio, Bk. I., chap. iii.), are terms used in the original, but as they have a different signification in modern mathematics, we do not use them in the translation.

Prosthaphæresis was a term in common use at the beginning of the seventeenth century, and is twice employed by Napier in the Spherical Trigonometry of the Constructio as well as in the Descriptio. The following short extract from Mr Glaisher's article on Napier, in the 'Encyclopædia Britannica,' indicates the nature of this method of calculation.

> The "new invention in Denmark" to which Anthony Wood refers as having given the hint to Napier was probably the method of calculation called prosthaphæresis (often written in Greek letters προσθαφαίρεσις), which had its origin in the solution of spherical triangles. The method consists in the use of the formula $\sin a \sin b = \frac{1}{2}\{\cos(a-b) - \cos(a+b)\}$, by means of which the multiplication of two sines is reduced to the addition or subtraction of two tabular results taken from a table of sines; and as such products occur in the solution

of spherical triangles, the method affords the solution of spherical triangles in certain cases by addition and subtraction only. It seems to be due to Wittich of Breslau, who was assistant for a short time to Tycho Brahe; and it was used by them in their calculations in 1582.

In the spherical trigonometry the notation used in the original is either of the form 34 gr $24'$ $49''$ or $34 : 24' : 49''$, but in the translation the form of notation used is always $34° 24' 49''$.

References to delay in publishing the Constructio, and to a new kind of Logarithms to Base 10.

The various passages from Napier's works bearing on these points are given below.

The first two are referred to by Robert Napier in the first page of the Preface, line 5. They appeared in the Descriptio, published in 1614,—the first, entitled Admonitio, on p. 7 (Bk. I. chap. ii.), and the second, with the title Conclusio, on the 57th or last page of the work (Bk. II. chap. vi.)

The third passage, entitled Admonitio, is printed on the back of the last page of the Table of Logarithms published along with the Descriptio, but is omitted in many copies.

The fourth was inserted by Napier at p. 19 (Bk. I. chap. iv.) of Wright's translation, published in 1616.

The last is the passage referred to in the second page of the Preface, line 18. It is the opening paragraph in the Dedication of 'Rabdologiæ' to Sir Alexander Seton.

I. From DESCRIPTIO, Book I. Chapter II.

NOTE.

Up to this point we have explained the genesis and properties of logarithms, and we should here show by what calculations or method of computing they are to be had. But as we are issuing the whole Table containing the logarithms with their sines to every minute of the quadrant, we leave the Theory of their Construction for a more fitting time and pass on to their use. So that their use and advantages being first understood, the rest may either please the more if published hereafter or at least displease the less by being buried in silence.

silence. For I await the judgement and criticism of the learned on this before unadvisedly publishing the others and exposing them to the detraction of the envious.

II. From DESCRIPTIO, Book II. Chapter VI.

CONCLUSION.

It has now, therefore, been sufficiently shown that there are Logarithms, what they are, and of what use they are: for by their help without the trouble of multiplication, division, or extraction of roots we have both demonstrated clearly and shown by examples in both kinds of Trigonometry that the arithmetical solution of every Geometrical question may be very readily obtained. Thus you have, as promised, the wonderful Canon of Logarithms with its very full application, and should I understand by your communications that this is likely to please the more learned of you, I may be encouraged also to publish the method of constructing the table. Meanwhile profit by this little work, and render all praise and glory to God the chief among workers and the helper of all good works.

III. From the End of the TABLE OF LOGARITHMS.

NOTE.

Since the calculation of this table, which ought to have been accomplished by the labour and assistance of many computors, has been completed by the strength and industry of one alone, it will not be surprising if many errors have crept into it. These, therefore, whether arising from weariness on the part of the computor or carelessness on the part of the printer, let the reader kindly pardon, for at one time weak health, at another attention to more important affairs, hindered me from devoting to them the needful care. But if I perceive that this invention is likely to find favour with the learned, I will perhaps in a short time (with God's help) give the theory and method either of improving the canon as it stands, or of computing it anew in an improved form, so that by the assistance of a greater number of computors it may ultimately appear in a more polished and accurate shape than was possible by the work of a single individual.

Nothing is perfect at birth.

THE END.

IV. From WRIGHT'S TRANSLATION OF THE DESCRIPTIO, Book I. Chapter IV.

AN ADMONITION.

Bvt because the addition and subtraction of these former numbers [logs. of $\frac{1}{10}$ and its powers] may seeme somewhat painfull, I intend (if it shall please God)

God) in a second Edition, to set out such Logarithmes as shal make those numbers aboue written to fall upon decimal numbers, such as 100,000,000, 200,000,000, 300,000,000, &c., which are easie to bee added or abated to or from any other number.

V. From the DEDICATION OF RABDOLOGIÆ.

Most Illustrious Sir, I have always endeavoured according to my strength and the measure of my ability to do away with the difficulty and tediousness of calculations, the irksomeness of which is wont to deter very many from the study of mathematics. With this aim before me, I undertook the publication of the Canon of Logarithms which I had worked at for a long time in former years; this canon rejected the natural numbers and the more difficult operations performed by them, substituting others which bring out the same results by easy additions, subtractions, and divisions by two and by three. We have now also found out a better kind of logarithms, and have determined (if God grant a continuance of life and health) to make known their method of construction and use; but, owing to our bodily weakness, we leave the actual computation of the new canon to others skilled in this kind of work, more particularly to that very learned scholar, my very dear friend, Henry Briggs, public Professor of Geometry in London.

Notation of Decimal Fractions.

In the actual work of computing the Canon of Logarithms, Napier would continually make use of numbers extending to a great many places, and it was then no doubt that the simple device occurred to him of using a point to separate their integral and fractional parts. It would thus appear that in the working out of his great invention of Logarithms, he was led to devise the system of notation for decimal fractions which has never been improved upon, and which enables us to use fractions with the same facility as whole numbers, thereby immensely increasing the power of arithmetic. A full explanation of the notation is given in sections 4, 5, and 47, but the following extract, translated from 'Rabdologiæ,' Bk. I. chap. iv., is interesting as being his first published reference to the subject, though the above sections from the Constructio must have been written long before that date, and the point had actually been made use of in the Canon of Logarithms printed at the end of Wright's translation of the Descriptio in 1616.

From

From RABDOLOGIÆ, Book I. Chapter IV.

NOTE ON DECIMAL ARITHMETIC.

But if these fractions be unsatisfactory which have different denominators, owing to the difficulty of working with them, and those give more satisfaction whose denominators are always tenths, hundredths, thousandths, &c., which fractions that learned mathematician, "*Simon Stevin*, in his DECIMAL ARITHMETIC denotes thus—(1), (2), (3), naming them firsts, seconds, thirds: since there is the same facility in working with these fractions as with whole numbers, you will be able after completing the ordinary division, and adding a period or comma, as in the margin, to add to the dividend or to the remainder one cypher to obtain tenths, two for hundredths, three for thousandths, or more afterwards as required: and with these you will be able to proceed with the working as above. For instance, in the preceding example, here repeated, to which we have added three cyphers, the quotient will become 1993,273, which signifies 1993 units and 273 thousandth parts or $\frac{273}{1000}$, or, according to *Stevin*, 1993,$2'7''3'''$: further the last remainder, 64, is neglected in this decimal arithmetic because it is of small value, and similarly in like examples.

The preceding example:—division of 861094 by 432.

118
141
402
429
861094(1993$\frac{118}{432}$
432
3888
3888
1296

64
136
316
118,000
141
402
429
861094,000(1993,273
432
3888
3888
1296
864
3024
1296

Simon Stevin, to whom Napier here refers, was born at Bruges in 1548, and died at The Hague in 1620. He published various mathematical works in Dutch. The Tract on Decimal Arithmetic, which introduced the idea of decimal fractions and a notation for them, was published in 1585 in Dutch, under the title of 'De Thiende,' and in the same year in French, under the title of 'La Disme.'

We find Briggs, in his 'Remarks on the Appendix,' while sometimes employing the point, also using the notation $2\overline{5118865}$ for $2\frac{5118865}{10000000}$, distinguishing the fractional part by retaining the line separating the numerator and denominator, but omitting the latter. The form $2|\overline{5118865}$ has also been used. If we take any number such as $94\frac{1305}{10000}$, the following will give an idea of some of the different notations employed at various times:—

9 4 ⓪ 1 ① 3 ② 0 ③ 5 ④; 9 4 1 3 0 5;

9 4 $1'$ $3''$ $0'''$ $5''''$; $94\overline{1305}$; $94|\overline{1305}$; 94.1305.

Notwithstanding the simplicity and elegance of the last of these, it was long after Napier's time—in fact, not till the eighteenth century—that it came into general use.

The subject is referred to by Mark Napier in the 'Memoirs,' pp. 451-455, and by Mr Glaisher in the Report of the 1873 Meeting of the British Association, Transactions of the Sections, p. 16.

On the Occurrence of a Mistake in the Computation of the Second Table; with an Enquiry into the Accuracy of Napier's Method of Computing his Logarithms.

It is evident that a mistake must somewhere have occurred in the computation of the Second table, since the last proportional therein is given (sec. 17) as 9995001.222927, whereas on trial it will be found to be 9995001.224804.

This mistake introduced an error into the logarithms of the Radical table, as the logarithm of the first proportional in that table is deduced from the logarithm of the last proportional in the Second table by finding the limits of their difference. But these limits are obtained from the proportionals themselves, and, as shown above, one of these proportionals was incorrect: the limits therefore are incorrect, and consequently the logarithm of the first proportional in the Radical table.

We see the effect of this in the logarithm of the last proportional in the Radical table, which is given (sec. 47) as 6934250.8, whereas it should be 6934253.4, the given logarithm thus being less than the true logarithm by 2.6, or rather more than a three millionth part.

The logarithms as published in the original Canon are affected by the above mistake, and also, as mentioned in sec. 60, by the imperfection of the table of sines. It seems desirable, therefore, to enquire whether in addition any error might have been introduced by the method of computation employed.

Before entering on this enquiry, we should premise that in comparing Napier's logarithms with those to the base e^{-1} (which is the base required by his reasoning, though the conception of a base was not formally known to him), it must be kept in view that in making radius

10,000,000

10,000,000 he multiplied his numbers and logarithms by that amount, thereby making them integral to as many places as he intended to print. In this we follow his example, omitting, however, from the formulæ the indication of this multiplication.

In sec. 30, Napier shows that the logarithm of 9999999, the first proportional after radius in the First table, lies between the limits 1.00000010000001 0 etc., and 1.000000000000000 etc. And in sec. 31, he proposes to take 1.00000005, the arithmetical mean between these limits, as a sufficiently close approximation to the true logarithm; for, the difference of this mean from either limit being .00000005, it cannot differ from the true logarithm by more than that amount, which is the twenty millionth part of the logarithm. But there can be little doubt that Napier was able to satisfy himself that the difference would be very much less, and that his published logarithms would be unaffected.

We proceed to show the precise amount of error thus introduced into the logarithm of 9999999. If we employ the formula

$$\log_{e^{-1}}\left(1-\tfrac{1}{n}\right)=(-1)\left\{-\tfrac{1}{n}-\tfrac{1}{2n^2}-\tfrac{1}{3n^3}-\tfrac{1}{4n^4}-\text{ etc.}\right\},$$

substituting 10000000 for n, and multiplying the result by 10000000, as before explained, we have

1.00000005000000333333358 3 etc.

Again, if we take the arithmetical mean of the limits, carried to a similar number of places, we have

1.00000005000000500000050 0 etc.

The error introduced is consequently

.00000000000000166666691 6 etc.

or about a six hundred billionth part in excess of the true logarithm. It will be observed that besides being very much less, this error is in the opposite direction from that caused by the mistake in the Second table.

We have given above the analytical expression for the true logarithm, namely, $\frac{1}{n}+\frac{1}{2n^2}+\frac{1}{3n^3}+\frac{1}{4n^4}+\frac{1}{5n^5}+$ etc. The corresponding expression for the arithmetical mean is $\frac{1}{n}+\frac{1}{2n^2}+\frac{1}{2n^3}+\frac{1}{2n^4}+\frac{1}{2n^5}+$ etc. The latter, therefore, exceeds the true logarithm by $\frac{1}{6n^3}+\frac{1}{4n^4}+\frac{3}{10n^5}+$ etc., which multiplied by n gives $\frac{1}{6n^2}+$ etc., or $\frac{1}{6(10000000)^2}+$ etc., for the error in Napier's logarithm. So that up to the 15th place the logarithm

 obtained

obtained by Napier's method of computation is identical with that to the base e^{-1}. If, however, he had used the base $(1-\frac{1}{n})^n$, where $n=$ 10000000, then the logarithm of 9999999, multiplied by 10000000, as in the other two cases, would necessarily have been unity, or 1.000000000 etc., which would have agreed with the true logarithm to the 8th place only, and would not have left his published logarithms unaffected.

The small error found above in Napier's logarithm of 9999999 is successively multiplied on its way through the tables: thus, in the First table it is multiplied by 100, in the Second by 50, and in the Third by 20 and again by 69, or in all by 6900000; so that, multiplying the error in the first proportional by that amount, we should have for the error in the logarithm of the last proportional of the Radical table about .0000000115. The error, however, although continually increasing, yet retains always the same ratio to the logarithm, except for a very small disturbing element to be afterwards referred to, so that the true logarithm will always be very nearly equal to the logarithm found by Napier's method of computation less a six hundred billionth part.

Let us take, for example, the logarithm of 5000000 or half radius. When computed according to Napier's method, we find it comes out

6931471.805599464646O4 etc.

The true logarithm to the base e^{-1} is

6931471.805599453094 22 etc.

So that the difference between the two is

.000000011551 81 etc.

The six hundred billionth part of the logarithm is

.000000011552 45 etc.

The latter agrees very closely with the difference found above, and would have agreed to the last place given except for the small disturbing element referred to above, which is introduced in passing from the logarithms of one table to those of the next, or in finding the logarithm of any number not given exactly in the tables as in this case of half radius, but this element is seen to have little effect in modifying the proportionate amount of the original error.

From the above example we see that the error in the logarithm found by Napier's method amounts only to unity in the 15th place, so that his method of computation clearly gives accurate results far in excess of his requirements. But it is easy to show that Napier's method may be adapted

adapted to meet any requirements of accuracy. In sec. 60, Napier, in suggesting the construction of a table of logarithms to a greater number of places, proposes to take 100000000 as radius. The effect of this would be to throw still further back the error involved in taking the arithmetical mean of the limits for the true logarithm. Thus, using the formula given, substituting 100000000 for n, and multiplying the result by that amount as already explained, we should have for the true logarithm of 99999999, the first proportional after radius in the new First table,

1.000000005000000033333 etc.

If we take the arithmetical mean of the limits, we have

1.000000005000000050000 etc.

This brings out a difference of

.000000000000000016666 etc.,

or a sixty thousand billionth part of the logarithm. We see that the logarithms only begin to differ in the 18th place, and that thus to however many places the radius is taken, the logarithms of proportionals deduced from it will be given with absolute accuracy to a very much greater number of places.

To ensure accuracy in the figures given above, the three preparatory tables were recomputed strictly according to the methods described in the Constructio, fourth proportionals being found in all the preceding tables, and both limits of their logarithms being calculated, the work being carried to the 27th place after the decimal point.

As logarithms to base e^{-1} are now quite superseded, it is not worth while printing these preparatory tables. The following values (pp. 94-95), however, may be of service for comparison, and as a check to any one who may desire to work out for himself the tables and examples in the Constructio. The values given are the first proportional after radius, and the last proportional in each of the three tables, and also in the Third table, the last proportional in col. 1, and the first proportionals in col. 2 and 69. Opposite these are given their logarithms to base e^{-1}, computed, first, according to Napier's method, and second, by the present method of series which gives the value true to the last place, which is increased by unit when the next figure is 5 or more. The proportionals and logarithms are each multiplied by 10000000, as explained above.

Though the logarithms in the Canon of 1614 were affected by the

	PROPORTIONALS.
FIRST TABLE.	
First proportional after radius,	9999999.
The last proportional, . .	9999900.0004949983830039212174 71
SECOND TABLE.	
First proportional after radius,	9999900.
The last proportional, . .	9995001.22480402302788139889 7012
THIRD TABLE.	
Column 1.	
First proportional after radius,	9995000.
The last proportional, . .	9900473.57802328605019866742 4460
Column 2.	
The first proportional, . .	9900000.
Column 69.	
The first proportional, . .	5048858.88787069951905823800 6143
The last proportional, . .	4998609.40185318932503223381 1730
HALF RADIUS, ...	5000000.
ONE-TENTH OF RADIUS, . .	1000000.

mistake in the Second table, this was not the case with those in the Magnus Canon computed by Ursinus and published in 1624. The logarithm of 30° or half radius, for instance, is there given as 69314718 (see specimen page of his Table, given in the Catalogue), which is correct to the number of places given. But in a table of the logarithms of ratios (corresponding to the table in sect. 53 of the Constructio), which is given by Ursinus on page 223 of the 'Trigonometria,' the value is stated as 69314718.28, which exceeds the true value by .22. This example will explain how some of the logarithms at the end of the Magnus Canon are too great by 1 in the units place. Notwithstanding this,

LOGARITHMS COMPUTED BY NAPIER'S METHOD.	LOGARITHMS COMPUTED BY PRESENT METHOD.
1.00000005000000500000000500	1.000000050000003
100.00000500000050000050000	100.000005000000333
100.00050000333352500022500 2	100.0005000003333358
5000.02500016667625001125009 4	5000.025000166667917
5001.25041682298752773983923 1	5001.250416822979193
100025.00833645975055479678461 8	100025.008336459583854
100503.35853501457933263222632 0	100503.358535014411835
6834228.38038099139461899138979 1	6834228.380380980004813
6934253.38871745114517378817440 9	6934253.388717439588668
6931471.80559946464604196223636 7	6931471.805599453094225
23025850.92994049521466098915213 6	23025850.92994045684018 0

this, the Magnus Canon may safely be used to correct the figures in the text and in the Canon of 1614, as the latter is to one place less.

I find no reference by Ursinus to the discrepancies between the logarithms of the two Canons. The mistake in the Second table may possibly not have been observed by him, as the preparatory tables for the Canons were different.

The mistake was observed by Mr Edward Sang in 1865, when recomputing in full the preparatory tables of Napier's Canon to 15 places.

It had been previously pointed out by M. Biot, in his articles on Napier in the 'Journal de Savants' for 1835, p. 255. The following

 translation

translation of the passage is given in the 'Edinburgh New Philosophical Journal' for April 1836, p. 285:—

> It has been said, and Delambre repeats the remark, that the last figures of his [Napier's] numbers are inaccurate: this is a truth, but it would have been a truth of more value to have ascertained whether the inaccuracy resulted from the method, or from some error of calculation in its applications. This I have done, and thereby have detected that there is in fact a slight error of this kind, a very slight error, in the last term of the second progression which he forms preparatory to the calculation of his table. Now all the subsequent steps are deduced from that, which infuses those slight errors that have been remarked. I corrected the error; and then, *using his method*, but abridging the operations by our more rapid processes of development, calculated the logarithm of 5000000, which is the last in Napier's table, and consequently that upon which all the errors accumulate; I found for its value 6931471.808942, whereas by the modern series, it ought to be 6931471.805599; thus the difference commences with the tenth figure.

It has been shown in the foregoing pages that the difference referred to does not really commence until the fifteenth figure.

Numerical errata in the text.—In consequence of what is mentioned above, the figures in the text are in many places more or less inaccurate, but after careful consideration it is thought that the course least open to objection is to give them as in the original.

Different

Different Methods described in the Appendix for Constructing a Table of Logarithms in which Log. 1=0 and Log. 10=1.

I.

The first method of construction, described on pages 48–50, involves the extraction of fifth roots, from which we may infer that Napier was acquainted with a process by which this could be done. The inference is confirmed by an examination of his 'Ars Logistica,' at p. 49 of which (Lib. II., 'Logistica Arithmetica,' cap. vii.) he indicates a method by which roots of all degrees may be computed. This method of extraction is referred to by Mark Napier in the 'Memoirs,' p. 479 *seq.*, and a translation is there given of the greater part of the chapter above referred to. A method based on the same principles is given by Mr Sang in the chapter "On roots and fractional powers" in his 'Higher Arithmetic,' and these principles are also made use of by Mr Sang in his tract on the 'Solution of Algebraic Equations of all Orders,' published in 1829.

No general method of extracting roots was known at the time, and it does not appear that Napier had communicated his method to Briggs. At any rate, Briggs did not employ the first method described in computing the logarithms for his canon.

II.

The second method, described on page 51, is a method suitable for finding the logarithms of prime numbers when the logarithms of any two other numbers as 1 and 10 are given. This is done by inserting geometrical means between the numbers, and arithmetical means between their logarithms. The example given is to find the logarithm of 5, but as the example terminates abruptly after the second operation, I append the following table from the article on Logarithms in the 'Edinburgh Encyclopædia' (1830), which will sufficiently exhibit the method of working out the example, though it is not carried to the same number of places as that in the text.

THE TABLE.

Numbers.		Logarithms.	
A	1.000000	a	0.0000000
B	10.000000	b	1.0000000
C = √(AB) =	3.162277	c = ½(a + b) =	0.5000000
D = √(BC) =	5.623413	d = ½(b + c) =	0.7500000
E = √(CD) =	4.216964	e = ½(c + d) =	0.6250000
F = √(DE) =	4.869674	f = ½(d + e) =	0.6875000
G = √(DF) =	5.232991	g = ½(d + f) =	0.7187500
H = √(FG) =	5.048065	h = ½(f + g) =	0.7031250
I = √(FH) =	4.958069	i = ½(f + h) =	0.6953125
K = √(HI) =	5.002865	k = ½(h + i) =	0.6992187
L = √(IK) =	4.980416	l = ½(i + k) =	0.6972656
M = √(KL) =	4.991627	m = ½(k + l) =	0.6982421
N = √(KM) =	4.997240	n = ½(k + m) =	0.6987304
O = √(KN) =	5.000052	o = ½(k + n) =	0.6989745
P = √(NO) =	4.998647	p = ½(n + o) =	0.6988525
Q = √(OP) =	4.999350	q = ½(o + p) =	0.6989135
R = √(OQ) =	4.999701	r = ½(o + q) =	0.6989440
S = √(OR) =	4.999876	s = ½(o + r) =	0.6989592
T = √(OS) =	4.999963	t = ½(o + s) =	0.6989668
V = √(OT) =	5.000008	v = ½(o + t) =	0.6989707
W = √(TV) =	4.999984	w = ½(t + v) =	0.6989687
X = √(VW) =	4.999997	x = ½(v + w) =	0.6989697
Y = √(VX) =	5.000003	y = ½(v + x) =	0.6989702
Z = √(XY) =	5.000000	z = ½(x + y) =	0.6989700

III.

In the description of the third method, on pages 53-54, it is explained that when log. 1 = 0 and log. 10 is assumed equal to unit with a number of cyphers annexed, a close approximation to the logarithm of any given number may be obtained by finding the number of places in the result produced by raising the given number to a power equal to the assumed logarithm of 10. As an example, Napier mentions that, assuming log.

10 = 1000000000, the number of places, less one, in the result produced by raising 2 to the 1000000000th power will be 301029995. So that reducing these in the ratio of 1000000000, we have log. 10 = 1 and log. 2 = .301029995 &c. The process is explained by Briggs, pages 61-63, and the first steps in the approximation are shown in a tabular form. The table, extended to embrace Napier's approximation, is given below: in this form it will be found in Hutton's Introduction to his Mathematical Tables, with further remarks on the subject.

The method, it will be seen, is really one for finding the limits of the logarithm. These limits are carried one place further for each cypher added to the assumed logarithm of 10, but their difference always remains unity in the last place. Bringing together the successive approximations obtained in the table, we find—

When 2 is raised to the power	The greater limit of its logarithm is	And the less limit is
1	1.	0.
10	.4	.3
100	.31	.30
1000	.302	.301
10000	.3011	.3010
100000	.30103	.30102
1000000	.301030	.301029
10000000	.3010300	.3010299
100000000	.30103000	.30102999
1000000000	.301029996	.301029995

THE TABLE.

Powers of 2.	Indices of powers of 2.	Number of places in powers of 2.
2	1	1 ÷ 1 = log. 2
4	2	1 ,, 4
16	4	2 ,, 16
256	8	3 ,, 256
1024	10	4 ÷ 10 = log. 2
10486 etc.	20	7 ,, 4
10995 ,,	40	13 ,, 16
12089 ,,	80	25 ,, 256

THE TABLE—*continued.*

Powers of 2.	Indices of powers of 2.	Number of places in powers of 2.
12676 &c.	100	31 ÷100 =log. 2
16069 „	200	61 „ 4
25823 „	400	121 „ 16
66680 „	800	241 „ 256
10715 „	1000	302÷1000=log. 2
11481 „	2000	603 „ 4
13182 „	4000	1205 „ 16
17377 „	8000	2409 „ 256
19950 „	10000	3011÷10000=log. 2
39803 „	20000	6021 „ 4
15843 „	40000	12042 &c.
25099 „	80000	24083
99900 „	100000	30103
99801 „	200000	60206
99601 „	400000	120412
99204 „	800000	240824
99006 „	1000000	301030
98023 „	2000000	602060
96085 „	4000000	1204120
92323 „	8000000	2408240
90498 „	10000000	3010300
81899 „	20000000	6020600
67075 „	40000000	12041200
44990 „	80000000	24082400
36846 „	100000000	30103000
13577 „	200000000	60206000
18433 „	400000000	120411999
33977 „	800000000	240823997
46129 „	1000000000	301029996

A CATALOGUE
OF THE WORKS OF
JOHN NAPIER
of Merchiston

To which are added a Note of some Early Logarithmic Tables and other Works of Interest

Compiled by

WILLIAM RAE MACDONALD

N 3

PRELIMINARY.

Contents and Arrangement.

THE works of John Napier of Merchiston were published in the following order:—

A Plaine Discovery of the Whole Revelation of St John, published in English in 1593.

Mirifici Logarithmorum Canonis Descriptio, published in Latin in 1614, together with the Canon or Table of Logarithms.

Rabdologiæ, published in Latin in 1617, the year of the Author's death.

Mirifici Logarithmorum Canonis Constructio, published in Latin in 1619, two years after the Author's death, by his son, Robert Napier.

Ars Logistica, 'The Baron of Merchiston his booke of Arithmeticke and Algebra,' in Latin, edited by Mark Napier, and published in 1839.

These works naturally fall into three groups: the first contains the result of his early studies in Revelation by which he became famous among the Reformed Churches of Europe, as one of the most learned Theologians of the day; another contains his works on Logarithms, by which his fame as a Mathematician was established in the scientific world; between these two groups may be placed his other works, which were more or less preparatory to or suggested during the elaboration of his Logarithms. Accordingly, in the Catalogue we have arranged his works in the following order: I. A Plaine Discovery; II. Ars Logistica; III. Rabdologiæ; IV. The Descriptio and Constructio. As a supple-

 ment

ment are added particulars of the Logarithmic tables computed by Ursinus, Kepler, and Briggs, with a note of some other works of interest.

Collation.

The arrangement of the title-page in the original is indicated by placing an upright bar to mark the end of each line.

The symbols 4°, 8°, 12°, etc., indicate the number of leaves into which the sheet of paper was folded; but the number of leaves made up in the signatures sometimes differs from this: thus, for example, in the early editions of A Plaine Discovery, though the sheet is folded into 4 there are 8 leaves to each signature.

The measurement of the largest copy examined has been given, but in many cases the work in its original state must have been considerably larger, the copy having been cut down in rebinding.

The signatures in the editions described consist of the letters of the alphabet excluding J, U, and W, or 23 letters (in one or two instances J and U are used for I and V). To each letter belongs a bundle of leaves, 4, 8, 12, &c., as the case may be. The leaves in each bundle are usually numbered thus :—C, C2, C3, etc., but frequently the signatures are printed only on the first one or two leaves in each bundle. The signature is very rarely printed on a title-page. When a leaf is described as B3, for instance, both sides are included, $B3^1$ being used to signify the recto and $B3^2$ the verso.

Libraries.

To each entry in the Catalogue, under the head of *Libraries*, is appended a note of the principal public libraries in this country which possess copies, to these the names of a few foreign libraries are added. The following abbreviations are employed :—

Un. Ab.	University, ...	Aberdeen.
Un. Camb.	University, ...	Cambridge.
Trin. Col. Camb.	Trinity College, ...	do.
St John's Col. Camb.	St John's College, ...	do.
Trin. Col. Dub.	Trinity College, ...	Dublin.
Adv. Ed.	Advocates, ...	Edinburgh.
Sig. Ed.	Signet, ...	do.

Un. Ed.

Un. Ed.	University, ...	Edinburgh.
New Col. Ed.	New College, ...	do.
Act. Ed.	Faculty of Actuaries, . . .	do.
Un. Gl.	University, ...	Glasgow.
Hunt. Mus. Gl.	Hunterian Museum in the University buildings,	do.
Brit. Mus. Lon.	British Museum, ...	London.
Un. Col. Lon.	University College, ...	do.
Guildhall Lon.	Corporation or Guildhall, . .	do.
Roy. Soc. Lon.	Royal Society, ...	do.
Lambeth Pal. Lon.	Lambeth Palace, ...	do.
Sion Col. Lon.	Sion College, ...	do.
Act. Lon.	Institute of Actuaries, . . .	do.
Chetham's Manch.	Chetham's Library, ...	Manchester.
Bodl. Oxf.	Bodleian, ...	Oxford.
Qu. Col. Oxf.	Queen's College, ...	do.
Un. St And.	University, ...	St Andrews.
Kön. Berlin,	Königliche Bibliothek, . . .	Berlin.
Stadt. Bern,	Stadtbibliothek, ...	Bern.
Stadt. Breslau,	Stadtbibliothek, ...	Breslau.
Un. Breslau,	Königliche und Universitäts Bibliothek,	do.
Kön. Öff. Dresden	Königliche Öffentliche Bibliothek, .	Dresden.
Stadt. Frankfurt,	Stadtbibliothek, ...	Frankfurt a/M.
Pub. Genève,	Bibliothèque Publique, . . .	Genève.
Kon. Hague,	Koninklijke Bibliotheek, . . .	s'Gravenhage.
Un. Halle,	Königl. Universitäts-Bibliothek, .	Halle a/S.
Stadt. Hannover,	Stadtbibliothek, ...	Hannover.
Un. Leiden,	Bibliotheek der Rijks-Universiteit,	Leiden.
Maat. Ned. Let. Leiden,	De Maatschappij der Nederlandsche Letterkunde. Library in the University buildings,	do.
Un. Leipzig,	Universitäts-Bibliothek, . . .	Leipzig.
K. Hof u. Staats München,	K. Hof- und Staats-Bibliothek, .	München.
Astor New York,	Astor Library, ...	New York.
Nat. Paris,	Bibliothèque Nationale, . . .	Paris.
Soc. Prot. Fr. Paris,	Société de l'Histoire du Protestantisme Français,	do.
Min. Schaffhausen,	Ministerial Bibliothek, . . .	Schaffhausen.
Un. Utrecht,	Bibliotheek der Universiteit, . .	Utrecht.
Stadt. Zürich,	Stadtbibliothek, ...	Zürich.

Bibliographies.

As several works of this kind are mentioned in the Catalogue, a short note of the particular work and edition referred to is given below :—

Messkatalog.—Catalogus universalis pro nundinis Francofurtensibus autumnalibus, de anno MDCXI. Hoc est : Designatio omnium librorum, qui hisce nundinis autumnalibus vel noui vel emendatiores et auctiores prodierunt. Das ist : Verzeichnuss aller Bücher, so zu Franckfurt in der Herbstmess, Anno 1611 entweder gantz new oder sonsten verbessert, oder auffs new widerumb auffgelegt, in der Buchgassen verkaufft worden.
Francofurti, Permissu Superiorum, Typis Sigismundi Latomi.

The Frankfurt catalogues were issued for the half-yearly book fairs held in that city at Fastenmesse and Herbstmesse.

In these catalogues, and in bibliographical works founded on them, as those of Draudius, Lipenius, etc., the place and name given cannot be taken as the actual place of publication and name of publisher without corroborative evidence. Thus, for example, the editions of the 'Descriptio' 1614, 'Rabdologiæ' 1617, and the 'Constructio' 1619, which were published at Edinburgh by Andrew Hart, are sometimes given with the correct particulars, and again appear as issued at Amsterdam, the first by Iansonius, and the two others by Hondius. There is little doubt, however, that these were simply importers of the Edinburgh editions who supplied the German market. Similar remarks apply to the translations and other editions of Napier's works.

Draudius.—Bibliotheca Librorum Germanicorum Classica. Durch M. Georgium Draudium.
Franckfurt am Mayn, Balthasaris Ostern. 1625.

" Bibliotheca Classica sive Catalogus Officinalis. M. Georgiò Draudio. Fracofurti ad Moenum. Balthasaris Ostern. 1625.

" Bibliotheca Exotica sive Catalogus Officinalis Librorum Peregrinis Linguis usualibus scriptorum, videlicet Gallica . . . Anglica . . . &c., omnium, quotquot in Officinis Bibliopolarum indagari potuerunt, & in Nundinis Francofurtensibus prostant, ac senales habentur. A Frankfourt, Par Pierre Kopf. 1610.
Another edition, 1625.

Le

Le Long.—Bibliotheca Sacra, Jacobi Le Long.
Parisiis, Apud F. Montalant. 1723.

Freytag.—Analecta Literaria de Libris Rarioribus. Edita a Frider. Gotthilf Freytag, I.C. Lipsiæ, In Officina Weidemanniana. 1750.

Gerdes.—Florilegium Historico-criticum Librorum Rariorum. (By Daniel Gerdes.) Groningæ & Bremæ apud Haj. Spandaw & G. Wilh. Rump. 1763.

Rotermund.—Fortsetzung und Ergänzungen zu Christian Gottlieb Jöchers allgemeinem Gelehrten - Lexiko, worin die Schriftsteller aller Stände nach ihren vornehmsten Lebensumständen und Schriften beschrieben werden. Angefangen von Johann Christoph Adelung, und vom Buchstaben K fortgesetzt von Heinrich Wilhelm Rotermund, Pastor an der Domkirche zu Bremen. Fünster Band.
Bremen, bei Johann Georg Heyse. 1816.

Kayser.—Bucher Lexicon (1750-1832) von Christian Gottlob Kayser.
Leipzig. Ludwig Schumann. 1835.

Ebert.—A General Biographical Dictionary. Frederic Adolphus Ebert.
Oxford. University Press. 1837.

Lowndes.—The Bibliographer's Manual of English Literature, by William Thomas Lowndes. London. Henry G. Bohn. 1861.

Brunet.—Manuel du Libraire. Par Jacques Charles Brunet.
Paris. Firmin Didot, &c. 1863.

Graesse.—Trésor de Livres Rares et Précieux. Par Jean George Théodore. Graesse. Dresde. Rudolf Kuntze. 1863.

Laing Cat.—Catalogue of the Library of the late David Laing, Esq., LL.D., Librarian of the Signet Library (sold in four portions in Dec. 1879, in Apr. 1880, in Jul. 1880, and in Feb. 1881).

Memoirs.—Memoirs of John Napier of Merchiston. By Mark Napier.
Edinburgh. Wm. Blackwood. 1834.

A CATALOGUE

OF THE WORKS OF

JOHN NAPIER

of Merchiston.

I.—A Plaine Discovery of the whole Revelation of St John.

1. Editions in English.

A Plaine Dis-|couery of the whole Reue-|lation of Saint Iohn: set|downe in two treatises: The|one searching and prouing the| true interpretation thereof: The o-|ther applying the same paraphrasti-|cally and Historically to the text.|Set Foorth By|Iohn Napeir L. of|Marchistoun younger.|Wherevnto Are|annexed certaine Oracles|of Sibylla, agreeing with|the Reuelation and other places|of Scripture.|

Edinbvrgh|Printed By Ro-|bert Walde-graue, prin-|ter to the Kings Ma-|jestie. 1593.|Cum Priuilegio Regali.|

[On either side of the Title are well executed woodcuts of "Pax" and "Amor."]

4°. Size 7¼ × 5¼ inches. A1 is blank except for a capital letter 'A'. $A2^1$, *Title.* $A2^2$, Arms of Scotland and Denmark impaled, for James VI. and his Queen Anne of Denmark; at foot, "*In vaine are all earthlie conivnctions, vnles vve be heires together, and of one bodie, and fellovv partakers of the promises of God in Christ,*

Christ, by the Evangell." $A3^1$–$A5^1$, 5 pages, "*To The Right Excellent, High And Mightie Prince, Iames The Sixt, King Of Scottes, Grace And Peace, &c.*", 'The Epistle Dedicatorie,' signed "*At Marchistoun the 29 daye of Ianuar*, 1593. . . . *Iohn Napeir, Fear of Marchistoun.*" $A5^2$–$A7^2$, 5 pages, "*To the Godly and Christian Reader.*" $A8^1$, "*The booke this bill sends to the Beast,*|*Crauing amendment now in heast,*|" with 26 lines following, then "*Faults escaped.*", 16 lines. $A8^2$, "*A Table of the Conclusions introductiue to the Reuelation, and proued in the first Treatise.*" $B1^1$–$F3^1$, pp. 1-69, "*The First And Introdvctory Treatise, conteining a searching of the true meaning of the Reuelation, beginning the discouerie thereof at the places most easie, and most euidentlie knowne, and so proceeding from the knowne, to the prooving of the vnknowne, vntill finallie, the whole groundes thereof bee brought to light, after the manner of Propositions.*", 36 Propositions and Conclusion. $F3^2$, p. 70, "*A Table Definitive And Diuisiue of the whole Revelation.*" $F4^1$–$S7^1$, pp. 71-269, "*The Second And Principal Treatis, wherein (by the former grounds) the whole Apocalyps or Reuelation of S. Iohn, is paraphrasticallie expounded, historicallie applied, and temporallie dated, with notes on euery difficultie, and arguments on each Chapter*"; at the beginning of each chapter is "*The Argument.*", then follow "*The Text.*", "*Paraphrastical exposition.*", "*Anno Christi.*", and "*Historical application.*", the four subjects being arranged in parallel columns (in chapters 1 to 5, and 7, 10, 15, 18, 19, 21, and 22, there is no Historical application, in which case the columns for it and also for Anno Christi are omitted), at the end of each chapter "*Notes, Reasons, and Amplifications.*" are added. $S7^2$–$S8^2$, 3 pages, "*To the misliking Reader whosoeuer.*" $T1^1$–$T4^2$, 8 pages, "*Hereafter Followeth Certaine Notable Prophecies agreable to our purpose, extract out of the books of Sibylla, whose authorities neither being so authentik, that hitherto vve could cite any of them in matters of scriptures, neither so prophane that altogether we could omit them: We haue therefore thought very meet, seuerally and apart to insert the same here, after the end of this worke of holy scripture, because of the famous antiquitie, approued veritie, and harmonicall consentment thereof with the scriptures of God, and specially with the 18. chapter of this holy Revelation.*"

Signatures. A to S in eights + T in four = 148 leaves.

Paging. 16 + 269 numbered + 11 = 296 pages.

Errors in Paging. Page 26 numbered 62, and page 229 numbered 239.

The outside sheet (leaves 1, 2, 7, 8) of Signature B was set up a second time, with slight differences in the spelling and occasionally in the division into lines. Consequently copies may be found in which the title of the First treatise does not agree exactly with that given above. The Advocates' Library in Edinburgh has copies of the two varieties.

The following extract explains the circumstances under which this first work of Napier's was published. The passage begins at the second last line in the second page of the address 'To the Godly and Christian Reader.' (In the edition of 1611 the passage begins on line 7 of the third page.)

After

After the which, although (greatly rejoycing in the Lord) I began to write thereof in Latine: yet, I purposed not to haue set out the same suddenly, and far lesse to haue written the same also in English, til that of late, this new insolencie of Papists arising about the 1588 year of God, and dayly increasing within this *Iland* doth so pitie our hearts, seeing them put more trust in Iesuites and seminarie Priests, then in the true scripturs of God, and in the Pope and King of *Spaine*, then in the King of Kings: that, to preuent the same, I was constrained of compassion, leauing the Latine, to haste out in English this present worke, almost vnripe, that hereby, the simple of this *Iland* may be instructed, the godly confirmed, and the proud and foolish expectations of the wicked beaten downe, [purposing hereafter (Godwilling) to publish shortly, the other latin editiõ hereof, to the publike vtilitie of the whol Church.] Whatsoeuer therfore through hast, is here rudely and in base language set downe, I doubt not to be pardoned thereof by all good men.

The passage enclosed in square brackets is omitted in the edition of 1611 (also in that of 1645) and in its place is inserted the following passage.

And where as after the first edition of this booke in our English or Scottish tongue, I thought to haue published shortlie the same in Latine (as yet Godwilling I minde to doe) to the publike vtilitie of the whole Church. But vnderstanding on the one part, that this work is now imprinted, & set out diuerse times in the French & Dutch tongs, (beside these our English editions) & therby made publik to manie. As on the other part being aduertised that our papistical, adversaries wer to write larglie against the said editions that are alreadie set out. Herefore I haue as yet deferred the Latine edition, till hauing first seene the aduersaries obiections, I may insert in the Latin edition an apologie of that which is rightly done, and an amends of whatsoeuer is amisse.

We see from the above that in 1611 Napier still had the intention of publishing a Latin edition, but this idea had, no doubt, to be given up owing to the demands made on him by his invention of Logarithms.

Libraries. Adv. Ed. (both varieties); Sig. Ed.; Un. Gl.; Mitchell Gl.; Un. Ab.; Un. St And.; Brit. Mus. Lon.; Bodl. Oxf.; Qu. Col. Oxf.; Un. Camb.; Trin. Col. Camb.;

A Plaine|Discoverie Of|The Whole Revelation Of|Saint Iohn: Set Down In Two|Treatises: The one searching and proving| the true interpretation thereof. The other|applying the same Paraphrastically|and Historicallie to the text|Set Forth By Iohn Napeir|L of Marchistovn younger.|Wherevnto Are Annexed

Cer-|taine Oracles of Sibylla, agreeing|with the Revelation and other|places of Scripture.|Newlie Imprinted and corrected.|

Printed For Iohn Norton Dwel-|ling in Paules Church-yarde, neere vnto|Paules Schoole.| 1594.|

4°. Size $7\frac{3}{4} \times 5\frac{1}{4}$ inches.

This edition is very like that of 1593, only the ornamental Title-page has been superseded by a plainer one, the ornament appearing in 1593 at the head of the Epistle dedicatorie now doing duty at the head of the Title-page. The collation remains the same, except as regards the spelling, and also that on Signature $A8^1$ the 'Faults escaped' are now omitted, being corrected in the text. The type is the same, but has been reset, there being numerous differences in spelling and occasional slight differences in the division into lines. The headpieces employed are, with one exception, found in the edition of 1593, but they are less varied and are frequently used in different places. It seems highly probable that this edition was printed in Edinburgh by Waldegrave for John Norton.

Libraries. New Col. Ed.; Brit. Mus. Lon.; Bodl. Oxf.;

A|Plaine Disco-|very, Of The Whole|Revelation of S. Iohn: set|downe in two treatises: the one searching and|proving the true interpretation thereof:|The other applying the same para-|phrasticallie and Historicallie|to the text.|Set Foorth By Iohn Napeir|L. of Marchiston. And now revised, corrected|and inlarged by him.|With a Resolvtion Of|certaine doubts, mooved by some well-|affected brethren.|Wherevnto Are Annexed, Cer-|taine Oracles of Sibylla, agreeing|with the Revelation and other|places of Scripture.

Edinbvrgh,|Printed by Andrew Hart. 1611.|Cum Privilegio Regiæ Maiestatis.|

4°. Size $6\frac{3}{4} \times 5\frac{1}{4}$ inches. $A1^1$, *Title.* $A1^2$–$A4^1$, 6 pages, '*To the Godly . . . . Reader,*' and '*The book this bill . . . .*' $A4^2$, *Table* $B1^1$–$H2^2$, pp. 1-92, *The first Treatise.* $H3^1$, *Table.* $H3^2$–$Y8^1$, pp. 94-327, *The second Treatise.* $Y8^2$, blank. $Z1^1$–$Z2^2$, pp. 329-332, '*To the mislyking Reader . . . .*' $Z3^1$–$Bb3^2$, pp. 333-366, "*A Resolvtion, of certaine doubts, proponed by well-affected brethren, and needfull to be*

be explained in this Treatise," seyen Resolutions. $Bb4^{1}$–$Bb8^{1}$, pp. 367-375, *Oracles of Sibylla.* $Bb8^{2}$, blank.

Signatures. A & B in fours + C to Z and Aa to Bb in eights = 192 leaves.

Paging. 8 + 375 numbered + 1 = 384 pages.

Errors in Paging. Page 56 numbered 65, and page 299 not numbered.

In this edition the Arms, &c., on back of the title-page, and the Dedication to King James, are omitted, and for the first time the 'Resolution of Doubts' appears.

Libraries. Adv. Ed.; Sig. Ed.; Un. Camb.;

A | Plaine Disco- | very, Of The Whole | Revelation of S. Iohn: set | downe in two treatises: the one searching and | proving the true interpretation thereof: | The other applying the same para- | phrasticallie and Historicallie | to the text. | Set Foorth by Iohn Napeir | L. of Marchiston. And now revised, corrected | and inlarged by him. | With A Resolvtion Of | certaine doubts, mooved by some well- | affected brethren. | Wherevnto Are Annexed, Cer- | taine Oracles of Sibylla, agreeing | with the Revelation and other | places of Scripture. |

London, | Printed for Iohn Norton. 1611. | Cum Privilegio Regiæ Maiestatis.

4°. Size $7\frac{1}{16} \times 5\frac{3}{8}$ inches.

This edition is in every respect identical with the preceding, except that the last paragraph of the title-page has been reset, the four words "*Edinbvrgh. . . . by Andrew Hart*" being altered to "*London. . . . for Iohn Norton.*" The printing of both editions appears to have been done in Edinburgh by Andrew Hart; his type, head-pieces, &c., being employed in both. The two slight errors in pagination remain as before.

Libraries. Adv. Ed.; Sig. Ed.; Un. Ab.; Bodl. Oxf.; Astor New York;

A | Plaine Discovery | of the whole | Revelation | of St. John: | Set down in two Treatises: the one | searching and proving the true Interpreta- | tion thereof: the other applying the | same Para-

 phrastically

phrastically and Historically|to the Text.|By John Napier, Lord of Marchiston.|With a Resolution of certain doubts,|moved by some well affected brethren.|Whereunto are annexed certain Oracles of|Sibylla, agreeing with the Revelation,|and other places of Scripture.|And also an Epistle which was omitted in|the last Edition.|The fifth Edition: corrected and amended.|

Edinbvrgh,|Printed for Andro Wilson, and are to be sold at his|shop, at the foot of the Ladies steps. 1645.|

4°. **Size 7½ × 5¾ inches.** *Leaf* 1^1, *Title*, 1^2 blank. 2^1–3^1, 3 pages, *Dedication to King James.* 3^2–5^2, 5 pages, *To the Godly . . . . Reader.* 6^1, '*The Book this Bill . . . .*' 6^2, *Table.* $B1^1$–$I3^1$, pp. 1–61, *The first Treatise.* $I3^2$, blank. $I4^1$, p. 63, *Table.* $I4^2$, blank. $K1^1$–$Ii2^2$, pp. 65–244, *The second Treatise.* $Aaa1^1$–$Aaa2^1$, pp. 1–3, *To the misliking Reader.* $Aaa2^2$–$Ddd4^2$, pp. 4–32, *A Resolution of Doubts.* $Eee1^1$–$Eee4^2$, pp. 31–38 [33–40], *Prophecies of Sibylla.* (In some copies an additional sheet is inserted with list of *Errata*, see Note.)

Signatures. [A] in six (leaves 4 & 5 are an insertion)+B to Z and Aa to Hh in fours+Ii in two+Aaa to Eee in four=148 leaves.

Paging. 12+244 numbered+(38+2 for error=) 40 numbered=296 pages.

Errors in Paging. In pp. 1–244 there are 10 errors which do not affect the total; but in pp. 1–[40] the numbers 31 & 32 are twice repeated, so that numbers on all the subsequent pages are understated by 2.

In Glasgow University Library is a copy of this edition with an extra leaf inserted at the end containing "*Errata.—Curteous Reader thou art desired to correct these faults following, which chiefly happened through the absence of the Author and the difficulty of the Coppy. viz.*," this is followed by ten lines of corrections.

The author's name, it will be observed, is spelt on the title-page in the modern form, and the Dedication to King James is signed, "*John Napier, Peer of Marchiston.*" The substitution of Peer for Fear or Feuer of Merchiston seems to have been intentional. It is not noticed in the errata, but is of course a mistake.

This is the only edition in which "*The Text.*", "*The Paraphrasticall Exposition.*", and the "*Historicall Application.*", are printed successively and not in parallel columns. The "*Historicall Application.*", is printed in black letter. "*An. Chr.*" is printed on the margin of each page in both treatises.

Libraries. Adv. Ed.; Sig. Ed.; Un. Ed.; New Col. Ed. (2); Un. Gl. (2); Un. Ab.; Brit. Mus. Lon.; Sion Col. Lon.; Un. Camb.; Trin. Col. Camb.;

2. EDITIONS IN DUTCH.

Een duydelicke verclaringhe,|Vande Gantsche|Openbaringhe Joannis|Des Apostels.|T'samen ghestelt in twee|Tractaten: Het eene ondersoeckt ende|bewijst de ware verclaringhe der selver. Ende|het ander, appliceert ofte voeght, ende ey-|gentse Paraphrastischer ende Histo-|rischer wijse totten Text.|Wtghegeven by Johan Napeir,|Heere van Marchistoun, de Jonghe.| Nu nieuwelicx obergeset wt d'Engel-|sche in onse Nederlantsche sprake, Door|M. Panneel. Dienaer des H.|woort Gods, tot Middelburch.|

Middelburgh|By Symon Moulert, woonende op den|Dam inde Druckerije. Anno 1600.|

4°. **Size 7¼ × 5⅛ inches.** Black letter with exception of the pages from ∗ 1² to ∗ 3², and a few passages here and there. ∗ 1¹, *Title-page.* ∗ 1², "*Extract wt de Privilegie*" granted to M. Panneel for 10 years by "*De Staten Generael der vereenichde Nederlanden,*" signed at "*s'Graven-Haghe, den* 4. *Augusti.* 1600. . . . ." At the foot of the page are three lines of errata under the heading, "*Sommighe fauten om te veranderen.*" ∗ 2¹–∗ 3², 4 pages, "*Aende E. E. Wyse Ende Voorsienige Heeren, Myne Heeren, Bailliv Burghemeesteren, Schepenen, ende Raedt der vermaerder Coopstadt Middelburgh in Zeelandt,*" signed "*Tot Middelburgh in Zeelandt, desen* 20. *Julij, inden Jare Christi,* 1600. *V. E. E. Onderdanighe dienst-willige, M. Panneel.*" ∗ 4¹–∗ ∗ 1², 4 pages, "*Den Seer Wtnemenden hooghen ende Machtighen Prince Jacobo de seste Coninck der Schotten ghenade ende vrede, &c.,*" signed "*Tot Marchistoun Den* 29. *dagh Januarii* 1593, *wwe Hoocheyts seer ootmoedighe ende ghehoorsaem ondersaet* JOHAN NAPEIR. *Erfachtich Heer van Marchistoun.*" ∗ ∗ 2¹–∗ ∗ 4¹, 5 pages, "*Aen den Godtsalighen ende Christelijcken Leser.*" ∗ ∗ 4², "*T'boeck sent dit schrift de beeste, of zijt vvoude noteren,|Begeerende dats t'meeste, datse haestelijck vvil bekeeren.|*", followed by 26 lines. On a folding sheet preceding A1¹ is "*Een tafel vande inleydende sluytredenê deser openbaringe bewesen int e erste tractaet.*" A1¹–J2² pp. 1-68 (last 4 pages not numbered), "*De erste ende het inleydende Tractaet ofte handelinge Inhoudende een ondersoeck van den rechten sin ofte meeninghe der Openbaringhe Joannis d'openinghe van dien beginnende aende plaetsen die lichst om verstaen ende best bekent zijn ende also voortgaende vande bekende tot D'onbekende tot dat den gantschen grondt daer van eyndelinghe int licht ghebrocht werdt ende dat by maniere van Propositien.*" This Treatise contains the 36 Propositions, and on Aa1¹ is the "*Beslvyt*" or Conclusion. Aa1² [p. 1], "*Een verclarende en afdeelende Tafel vande gheheele openbaringhe.*" Aa2¹–Ggg3¹, pp. 2–237, "*Het tweede ende voornaemste Tractaet daer in (achtervolghende de voorgaende grontreden) t'geheele Apocalipsis ofte openbaringe des Apostels Joannis op paraphrastischer wijse wtgheleyt op historischer wijse toegheeygent en tijdelijck*

tijdelijck gedateert wort. Met aenwijsinghen op elcke swaricheyt ofte hinderinghe ende argument op elck Capittel.", the chapters commence with "*Het Argument,*" then follow in four parallel columns "*Den Text.*", "*Paraphrasis.*", "*Anno Christi.*", and "*Historie*" (the 3d and 4th columns are wanting in the chapters mentioned in the Edin. 1593 edition), at the end of the chapter are added "*Aenwijsinghen Redenen ende breeder Verclaringhen.*" Ggg3²–Hhh2¹, 6 pages, "*Tafel ofte Register der aenwistinghen Redenen ende breeder verclaringhen,*" an alphabetical index of the principal matters contained in the work. Hhh2²–Hhh3², 3 pages, "*Totten Leser,*" which appears to be a Glossary of certain words used in the work. Hhh4¹ "*Errata inde Propositien,*" followed by 15 lines of corrections. Hhh4², blank.

Signatures. * and * * and A to H in fours + J in two + Aa to Zz and Aaa to Hhh in fours = 166 leaves.

Paging. 16+68 numbered (except last 4)+237 numbered (except first 3)+11 =332 pages.

Errors in Paging. There are some 18 of these, mostly in the second part, but none of importance.

This translation by M. Panneel omits the address *To the Mislyking Reader*, and the *Oracles of Sibylla*, but otherwise it appears to be a full translation of the edition of 1593.

Graesse states that there is an edition, "*trad. en hollandais par M. Pannel: Amst.* 1600 *in* 8°." Most likely this is the edition referred to.

Libraries. Guildhall Lon.; Stadt. Zurich;

Een duydelijcke verclaringhe|Vande gantse Open-|baringe Ioannis des Apostels.|T'samen ghestelt in twee Trac-|taten: Het eene ondersoeckt ende bewijst de wa-|re verclaringe der selver. Ende het ander appliceert ofte|voecht, ende eyghentse Paraphrastischer ende|Historischer wijse totten Text.|Wt-ghegheven by Iohan Napeir, Heere|van Marchistoun, de Ionghe.| Over-gheset vvt d'Enghelsche in onse Nederlandtsche|sprake. Door|M. Panneel, vvijlent Dienaer des H. vvoords Godts|tot Middelburch.|Den tweeden druck oversien, ende in velen plaetsen verbetert.|Noch zijn hier by-ghevoecht vier Harmonien, &c. van nieus over-|gheset vvt het Fransche.|

Middelburch,|Voor Adriaen vanden Vivre, Boeck-vercooper,| woonende inden vergulden Bybel, Anno 1607.|Met Privilegie voor 10 Iaren.|

8°. Size

8°. Size 6¾×4½ inches. Black letter, except from ∗2 to ∗7. ∗1[1] *Title-page.* ∗1[2] blank. ∗2[1]–∗4[2], 6 pages, "*Aende E. E. VVyse Ende Voorsienige Heeren, ...*" ∗5[1]–∗7[2], 6 pages, "*Den seer wt-nemenden, Hooghen ende Machtighen Prince Iacobo . . . .*" ∗8[1]–∗∗4[1], 9 pages, "*Aen den Godtsalighen ende Christelijcken Leser.*" ∗∗4[2], "*T'boecks endt dit schrift der Beeste, ,en bidt dat sy't noteere,| Op dat sy haer (dit's t'meeste) soo't moghelijck is bekeere,|*" followed by 28 lines. First Table wanting. A1[1]–F5[1], pp. 1-89, "*Het eerste ende inleydende Tractaet ofte handelinghe . . . .*" the 36 propositions and the "*Beslvyt*" or conclusion. Second Table wanting. F5[2]–Aa4[2], pp. 90-376, "*Het tweede ende voornaemste Tractaet, daer in (achter volghende de voorgaende gront-reden) t'gheheele Apocalipsis ofte Openbaringhe des Apostels Ioannis, op Paraphrastischer vvijse vvtgeleydt, ende op Historischer vvijse ende nae de tijden der gheschiedenissen toe-gheeyghent vvordt: Verciert met aenvvijsinghen op elcke duystere plaetse, ende met Argument op elck Capittel.*" Aa5[1]–Aa8[1], pp. 377-383, "*Aen den Leser, wien dit werck mishaeght.*" Aa8[2], blank.

Vier Harmonien,|dat is,|Overeen-stemmin-|ghen over de Openbaringe Ioannis,|betreffende het Coninclijck, Priesterlijck, ende|Prophetisch ampt Iesu Christi.|Vervatende ooc ten deele de Prophe-|tien ende Christelijcke Historien, van de gheboorte Iesu Christi af, tot het eynde der VVeereldt toe, sonder|ontbrekinghe der ghesichten.|T'samen-ghestelt,|Door Greorgivm Thomson,|Schots-man.|Nu nieuvvelijcks wt de Fransche tale verduyscht.|Door G. Panneel.| M.DC.VII.

Bb1[1], *Title-page.* Bb1[2], blank. Bb2[1]–Bb3[2], 4 pages, "*Voorreden.*" signed "*Greorgivs Thomsson.*". Bb4[1]–Dd2[1], 29 pages, contain the *Vier harmonien.* Dd2[2]–Dd4[2], 5 pages, "*Tafel vande principaelste materien die int geheele Boec verhandelt werden soo in de Propositien als in de Aenwijsinghen achter yder Capittel.*" At the foot of the last page (Dd4[2]) is printed: "*Tot Middelbvrch,|Ghedruckt by Symon Moulert, Boeck-vercooper,|woonende op den Dam, inde Druckerije. Anno* 1607.|"

Signatures. ∗ in eight + ∗∗ in four + A to Z and Aa to Cc in eights + Dd in four = 224 leaves.

Paging. 24 + 383 numbered + 1 + 40 = 448 pages.

Errors in Paging. Pages 143, 187, 269, and 308 numbered in error 144, 189, 270, and 208 respectively.

On comparing this edition with that of 1600, we find that the address To the Mislyking Reader is now given, and there is also added a translation of the Quatre Harmonies, from the French editions of 1603 et seq. Further, we find, besides the usual differences in spelling, occasional alterations in the translation. For example, compare the wording, &c., in signatures ∗∗4[2] and F5[2] of the above collation with that corresponding in the signatures ∗∗4[2] and Aa2[1] of the collation of

the 1600 edition. From this it would appear that for this 1607 edition the translation of 1600 was revised, possibly by G. Panneel, the translator of the Quatre Harmonies. Both the Tables are wanting in the copy examined.

Libraries. Maat. Ned. Let. Leiden;

3. EDITIONS IN FRENCH.

Ovvertvre|De Tovs Les|Secrets De|L'Apocalypse|Ov Revelation|De S. Iean.|Par deux traités, l'vn recerchant & prouuant la vraye interpretation|d'icelle: l'autre appliquant au texte ceste interpretation|paraphrastiquement & historiquement,|Par Iean Napeir (c. a. d.) Nonpareil|Sieur de Merchiston, reueue par luimesme:|Et mise en François par Georges Thomson Escossois.|

Va, pren le liuret ouuert en la main de l'Ange. Apoc. 10. 8. | Hola Sion qui demeures auec la fille de Babylon, sauue-toi. Zach. 2. 7. | Ie te conseille que tu achetes de moy de l'or esprouué par le feu, afin que tu | deuiennes riche, | Et que tu oignes tes yeux de collyre, afin que tu voyes. Apoc. 3. 18. | Qui lit, l'entende. Matth. 24. 15.

A La Rochelle.|Par Iean Brenovzet, demeurant pres|la boucherie Neufue.|1602.|

4°. Size 9 × 6¼ inches. ã1[1], *Title.* ã1[2], blank. ã2[1]–ã3[1], 3 pages, "*A Treshavt Et Tres-pvissant Iaqves Sixiesme, Roy D'escosse. Gr. &c. P.*" signed "*Iean Nonpareil.*" ã3[2]–ẽ2[1], 6 pages, "*Av Lectevr Pievx Et Chrestien.*" ẽ2[2] & ẽ3[1], "*Avx Eglises Francoises Reformees Tant En La France Qv'aillevrs S.*", signed "*Georges Thomson.*" ẽ3[2], Poems—"*De Georgii Thomsonii Paraphrasi Gallica Ad Galliam. Ode,*" 40 lines; also "*Idem,*" 8 lines, signed "*Ioannes Duglassius Musilburgenus.*" Preceding A1[1] on a folding sheet is "*Table des propositions seruantes d'introduction à l'Apocalypse prouuees au premier Traité, lesquelles sont couchees en ceste table selon leur ordre naturel, mais au premier traité suiuant sont mises selon l'ordre de demonstration afin que chaque proposition soit prouuee par la precedante.*" A1[1]–G1[2], pp. 1-50, "*Le Premier Traité Servant D'introdvction, Contenant Vne recerche du vray sens de l'Apocalypse, commençant la descouuerture d'icelle par les points les plus aisés &c. manifestes, &c. passant d'iceux à la preuue des incognus, iusques à ce que finalement tous les points fondamentaux soyent esclaircis par forme de propositions.*" 36 Propositions

sitions and "Conclvsion." Before $G2^1$ on a folding sheet is "*Table difinissante & diuisante toute l'Apocalypse.*" $G2^1$–$Ff1^2$, pp. 51-234 [226], "*Le Second Et Principal Traité Avqvel (Selon Les Fondemens Desja posez) toute l'Apocalypse est paraphrastiquement interpretée, & appliquee aux matieres, selon leur histoire, & datee du temps, auquel chaque chose doit arriuer, auec annotations sur chaque difficulté, & argumens sur chaque chapitre.*"; at the beginning of each chapter is "*L'Argvment.*", followed by "*Le Texte*", "*L'Exposition Paraphrastique*", "*An de Christ*", and "*L'Application historique,*" in four parallel columns (the 3d and 4th of which are wanting in the chapters mentioned in the Edinr. 1593 edition), and at the end of each chapter are "*Annotations, Raisons, & Amplifications.*" $Ff2^1$–$Ff3^2$, pp. 235–238 [227–230], are "*Av Lectevr Mal-content.*" $Ff4^1$–$Ii1^1$, 19 pages, "*Table De Tovtes Les Matieres Principales Contenves, Tant au premier qu'au second Traité sur l'Apocalypse,*" arranged alphabetically; at foot of last page "*Fautes suruenues en l'impression.*" 4 lines. $Ii1^2$, blank.

Signatures. ã in four + ẽ in three (leaf ẽ4 being cut out), + A to Z and Aa to Hh in fours + Ii in one = 132 leaves.

Paging. 14 + (238–8 for error =) 230 numbered + 20 = 264 pages.

Errors in Paging. Numbers 81 to 90 omitted, and 98 & 137 twice repeated = − 10 + 2 = − 8.

The Tables are on two folding sheets which precede $A1^1$ and $G2^1$.

In all the French editions, the lines "*The Book this bill sends to the Beast . . .*" and '*The Oracles of Sibylla,*' are omitted. The addition here made to the title of the First Table appears in the English editions as a note at the end of the Table.

Libraries. Adv. Ed.; Nat. Paris;

Ovvertvre | De Tovs Les | Secrets De L'Apo- | calypse, Ov Reve- | lation de S. Iean. | En deux traités, l'vn recerchant & prouuant la vraye interpretation | d'icelle: l'autre appliquant au texte ceste interpretation | paraphrastiquement & historiquement. | Par Iean Napeir (C. A. D.) nompareil Sieur | de Merchiston, reueüe par luimesme: | Et mise en François par Georges Thomson Escossois. |

Va, pren le liuret ouuert en la main de l'Ange. Apoc. 10. 8. | Hola Sion, qui demeures auec la fille de Babylon, sauue-toi. Zach. 2. 7. | Je te conseille que tu achetes de moy de l'or esprouué par le feu, | afin que tu deuiennes riche. | Et que tu oignes tes yeux de collyre, afin que tu voyes. Apoc. 3. 18. | Qui lit, l'entende. Matth. 24. 15.

A La Rochelle, | Pour Timothee Iovan. | M. DC. II. |

This can in no sense be considered another edition, Brenouzet's title-page having simply been cut off half an inch from the back and the above substituted. This substituted title has an ornamental border round the type, whereas Brenouzet's has simply a line. The copy examined for this entry (from Bib. Pub. de Genève) differs, however, from that examined for the previous entry (from Adv. Lib. Ed.) in certain small points which may be noted, namely: on e3[1] the signature is omitted, on e3[2] three little ornaments are omitted, on p. 3 the number is omitted, and finally, the principal error in paging commences here with p. 80 being numbered 90 instead of as above, with p. 81 being numbered 91.

Libraries. K. Hof. u. Staats. München; Pub. Genève; Stadt. Bern;

Ovvertvre|Des Secrets|De L'Apocalypse,|Ov Revelation De|S. Iean.|En deux traités: l'vn recherchant &|prouuant la vraye interpretation|d'icelle: l'autre appliquant au|texte ceste interpretation|paraphrastiquement|& historique-|ment.|Par Iean Napeir (C. A. D.)|Nompareil, Sieur de Merchi-|ston, reueuë par lui-mesme.|Et mise en François par Georges|Thomson Escossois.|Edition seconde,|Amplifiee d'Annotations, & de quatre harmonies sur l'Apocalypse, par le|Translateur.|

Il te faut encores prophetizer à plusieurs|peuples, & gens, & langues, & Rois.| Apoc. 10. 11.|

A La Rochelle,|Par les Heritiers de H. Haultin.|M. DC III.|

8°. Size 7 × 4⅝ inches. ā1[1], *Title.* ā1[2], blank. ā2[1]–ā5[1], 7 pages, *Dedication to King James.* ā5[2]–ē3[2], 13 pages, *Av Lectevr pieux . . . .* ē4[1]–ē5[1], 3 pages, "*Avx Eglises Francoises . . . .*" ē5[2]–ē8[1], 6 pages, *Poems*, eight more being added to those in the 1602 edition. ē8[2], "*Aduertissement du Translateur au Lecteur.*" Before A1[1], *Table* on folding sheet. A1[1]–F6[1], pp. 1–91, *The first Treatise.* F6[2], blank. Before F7[1], *Table* on folding sheet. F7[1]–V8[2], pp. 93–318 [320], *The second Treatise.* X1[1]–X4[2], 8 pages, *Av Lectevr Mal-content.* X5[1]–Y7[1], 21 pages, "*Table Des Matieres Principales Contenves en ce livre.*" Y7[2], "*A L'Eglise. Sonnet.*" Y8, blank.

Qvatre|Harmonies|Svr La Revelation|De S. Iean: Tovchant La|Royavté Prestrise,|& Prophetie de Iesus|Christ.|Contenantes aussi

aussi la Prophetie & Histoire Chrestiene|aucunement depuis la naissance de Christ iusques|à la fin du monde, sans interruption| des visions.|Par G.T.E.|1603.|

$Z1^1$, *Title.* $Z1^2$, blank. $Z2^1$–$Z4^2$, 6 pages, "*La Preface.*" $Z5^1$–$Aa8^2$, pp. 1–24, *The Work itself.* At foot of p. 24 is printed, "*Acheué d'imprimer le premier iour de l'An* 1603."

Signatures. ã and ẽ and A to Z and Aa in eights = 208 leaves.

Paging. 32+(318+2 for error=) 320 numbered+32+8+24 numbered = 416 pages.

Errors in Paging. The numbers 143 and 144 are twice repeated.

The two Tables are on folding sheets which precede $A1^1$ and $F7^1$.

Libraries. Un. Ed.; Un. Gl.; Nat. Paris; Kön. Berlin;

Ovvertvre|De Tovs Les|Secrets De|L'Apocalypse,|Ov Revelation|De S. Iean.|En deux traités: l'vn recerchant & prouuant la|vraye interpretation d'icelle; l'autre appliquant|au texte ceste interpretation paraphrasti-|quement & historiquement.| Par Iean Napeir (c. a. d.) Nompareil, Sieur de Merchi-|ston, reueue par lui-mesme.|Et mise en François par Georges|Thomson Escossois.|Edition seconde,|Amplifiée d'Annotations & de quatre harmonies sur|l'Apocalypse par le Translateur.|

Il te faut encores prophetizer à plusieurs peuples,|& gens, & langues, & Rois.| Apoc. 10. 11.|

A La Rochelle,|Par Noel De la croix. 1605.|

8°. Size 7 × 4¼ inches. $ã1^1$, *Title.* $ã1^2$, blank. $ã2^1$–$ã4^1$, 5 pages, *Dedication to King James.* $ã4^2$–$ã8^2$, 9 pages, *Av Lectevr pieux . . . .* $ẽ1$, 2 pages, *Avx Eglises Francoises. . . . .* $ẽ2^1$–$ẽ4^1$, 5 pages, *Poems*, as in the edition of 1603. $ẽ4^2$, *Aduertissement. . . . .* Before $A1^1$, *Table* on folding sheet. $A1^1$–$F5^2$, pp. 1–90, *The first Treatise.* Before $F6^1$, *Table* on folding sheet. $F6^1$–$Cc3^2$, pp. 91–446 [406], *The second Treatise.* $Cc4^1$–$Cc6^2$, 6 pages, *Av Lectvr Malcontent.* $Cc7^1$–$Ee5^1$, 29 pages, *Table des Matieres. . . . .* $Ee5^2$, *Sonnet.*

Qvatre|Harmonies|Svr La Revelation|De S. Iean; Tovchant La|Royavté Prestrise,|& Prophetie de Iesus|Christ.|Contenantes aussi la Prophetie & Histoire Chrestienne|aucunement depuis la naissance de Christ iusques|à la fin du monde, sans interruption|des visions.|Par G.T.E.||1605.|

Ee6[1], *Title.* Ee6[2], blank. Ee7[1]–8[2], 4 pages, *La Preface.* Ff1[1]–Gg8[1], pp. 1–31, *The Work itself.* At foot of p. 31 is printed, "*Acheué d'imprimer le huictiesme iour de Iuin* 1605." Gg8[2], blank.

Signatures. ã in eight + ẽ in four + A to Z and Aa to Gg in eights = 252 leaves.

Paging. 24 + (446–40 for error=) 406 numbered + 36 + 6 + 31 numbered + 1 = 504 pages.

Errors in Paging. P. 15 is numbered 16, and there are several errors in signature E, but the only error affecting the last page, is p. 401 numbered 441, and so to the end.

The two Tables are on folding sheets which precede A1[1] and F6[1].

It will be observed that this is described as 'Edition seconde' as well as that of 1603.

Libraries. Adv. Ed.; Un Ed.; Un. Ab.; Brit. Mus. Lon.; Chetham's Manch.; Trin. Col. Dub.; Nat. Paris; Stadt. Frankfurt;

Ovvertvre | De Tovs Les Secrets | De | L'Apocalypse | Ov Revelation | De S. Iean. | En deux traités: l'vn recerchant & prouuant la | vraye interpretation d'icelle: l'autre appli- | quant au texte ceste interpretation pa- | raphrastiquement & histori- | quement. | Par Iean Napeir (c. à d. Nompareil) Sieur de | Merchiston: reueuë par lui-mesme. | Et mise en François par Georges | Thomson Escossois. | Edition troisieme | Amplifiee d'Annotations, & de quatre harmonies sur | l'Apocalypse par le Translateur. |

Il te faut encores prophetizer à plusieurs peuples, | & gens, & langues, & Rois. | Apoc. 10. 11. |

A La Rochelle, | Par Noel de la Croix. | cIↃ. IↃC. VII. |

8°. **Size 6⅞ × 4½ inches.** A1[1], *Title.* A1[2], blank. A2[1]–A4[1], 5 pages, *Dedication to King James.* A4[2]–A7[1], 6 pages, *Av Lectevr Pievx* . . . . . A7[2]–A8[1], 2 pages, *Avx Eglises Francoises.* . . . . . A8[2]–B2[2], 5 pages, *Poems*, as in the edition of 1603, also the *Aduertissement.* . . . . . Preceding B3[1], *Table* on folding sheet. B3[1]–G7[2], pp. 1-90, *The first Treatise.* Preceding G8[1], *Table* on folding sheet. G8[1]–Dd5[2], pp. 91-406, *The second Treatise.* Dd6[1]–Dd8[2], 6 pages, "*Av Lectevr Mal-content.*" Ee1[1]–Ee8[2], 16 pages, *Table des Matieres* . . . . , on the last page at the end is the *Sonnet.*

Qvatre | Harmonies | Svr La Revelation De | S. Iean: Tovchant La Royav- | te, Prestrise et Prophe- | tie de Iesus Christ. | Contenantes aussi la Prophetie & Histoire Chrestienne | aucunement depuis

depuis la naissance de Christ ius-|ques à la fin du monde, sans interru-|ption des visions.| Par G.T.E.|cIↃ. IↃC. VII.|

Ff1[1], *Title.* Ff1[2], blank. Ff2[1]–Ff3[2], 4 pages, *La Preface.* Ff4[1]–Hh3[1], pp. 1-31, *The Work itself.* Hh3[2]–Hh4[2], 3 pages, blank.

Signatures. A to Z and Aa to Gg in eights + Hh in four = 244 leaves.
Paging. 20 + 406 numbered + 22 + 6 + 31 numbered + 3 = 488 pages.
Errors in Paging. P. 397 numbered in error 367, and pp. 401-404 numbered in error 441-444. (Quatre Harm.) p. 3 numbered in error 5.

In the title-page of the Quatre Harmonies, the fifth line ends with "Prophe"; in the Oxford copy this is followed by a hyphen, but in the Breslau and Dresden copies the hyphen is wanting.

Libraries. Bodl. Oxf.; Stadt. Breslau; Un. Breslau; Kön. Öff. Dresden;

Ovvertvre|De Tovs Les Secrets|De|L'Apocalypse|Ov Revelation|De S. Iean.|En deux traités: l'vn recerchant & prouuant la|vraye interpretation d'icelle: l'autre appli-|quant au texte ceste interpretation pa-|raphrastiquement & histori-|quement.| Par Iean Napeir (c. à. d. Nompareil) Sieur de|Merchiston: reueuë par lui-mesme.|Et mise en François par Georges|Thomson Escossois.|Edition troisieme.|Amplifiee d'Annotations, & de quatre Harmonies sur|l'Apocalypse par le Translateur.|

Il te faut encores prophetizer à plusieurs peuples,|& gens, & langues, & Rois.| Apoc. 10. 11.|

A La Rochelle,|Par Noel de la Croix.|cIↃ. IↃC. VII.|

8°. Size 6¾ × 4½ inches. A1[1], *Title.* A1[2], blank. A2[1]–A4[1], 5 pages, *Dedication to King James.* A4[2]–A7[1], 6 pages, *Av Lectevr Pievx . . . .* A7[2]–A8[1], 2 pages, *Avx Eglises Francoises . . . .* A8[2]–B2[2], 5 pages, *Poems*, as in the edition of 1603, also the *Aduertissement . . . .* Preceding B3[1], *Table* on folding sheet. B3[1]–G5[2], pp. 1-86, *The first Treatise.* Preceding G6[1], *Table* on folding sheet. G6[1]–Cc6[1], pp. 87-391, *The second Treatise*, four lines are carried over to the top of the page following 391. Cc6[2]–Dd1[1], 6 pages, "*Av Lectevr Mal-content.*" Dd1[2]–Dd8[1], 14 pages, *Table des Matieres . . . .* Dd8[2], *Sonnet.*

Qvatre|Harmonies|Svr La Revelation De|S. Iean: Tovchant La Royav-|te, Prestrise Et Prophe-|tie de Iesus Christ.|Contenantes aussi la Prophetie & Histoire Chrestienne|aucunement

depuis la naissance de Christ ius-|ques à la fin du monde, sans interru-|ption des visions.|Par G.T.E.|cIↃ. IↃC. VII.|

Ee1[1], *Title.* Ee1[2], blank. Ee2[1]–Ee3[2], 4 pages, *La Preface.* Ee4[1]–Gg3[1], pp. 1–31, *The Work itself.* Gg3[2]–Gg4[2], 3 pages, blank.

Signatures. A to Z and Aa to Ff in eights + Gg in four = 236 leaves.
Paging. 20 + 391 numbered + 21 + 6 + 31 numbered + 3 = 472 pages.
Errors in Paging. These are numerous, especially in signatures L and S, but none affect the last page.
The two Tables are on folding sheets which precede B3[1] and G6[1].

For some reason the type for the Rochelle issue of 1607 was twice set up. In this variety it will be observed that the number of pages occupied by the body of the work is about four per cent less than in the variety described in the preceding entry. The above collation is from the Edinburgh copy. The Paris copy agrees with it, except that the word "Harmonies" in the seventeenth line of the first title-page commences with a small h, as in the previous entry.

An edition 'A Genève chez Iaques Foillet, 1607, in 8°,' is mentioned by Freytag in the Analecta Literaria, p. 1136. A similar entry, but omitting 'Genève,' is made by Draudius in the Bibliotheca Exotica, 1625 edition, p. 11. Possibly Foillet was only the introducer of the work at the Frankfurt Book Fair.

Le Long mentions an edition, Geneva, 1642, in 4°. An entry was found, in a library catalogue, under Napier's name, which appeared to substantiate Le Long's statement. In that case, however, the work proved to be the 'Ouverture des secrets de l'Apocalypse de Saint Jean, contenant tres parties . . . . par Jean Gros. Genève, Fontaine, 1642, in 4°.'

Libraries. Adv. Ed.; Soc. Prot. Fr. Paris; Stadt. Zurich;

4. EDITIONS IN GERMAN.

[*Note.*—In the German editions, the letters printed below as ä, ö, ü, are in the original printed aͤ, oͤ, uͤ, an earlier way of expressing the 'umlaut.']

Entdeckung aller Geheimnüssen in der | Apocalypsi oder Offenbarung S. Jo- | hannis begriffen. | Dariñen die | Zeiten vnd Jahren der Regierung desz Anti- | christs, wie auch desz Jüngsten Tages, so eygentlich | durch gewisse gegründete Vrsachen auszgerechnet, dasz man fast | nicht dran zweiffeln kan. | Zuvor zwar niemals gesehen noch gehört, wiewol von vielen vornehmen, gelährten vnnd erleuchteten Männern, wie | von dem seligen Mann D. Luthero selbsten, ge- | wündschet worden. | Von | Iohanne De Napeier, | Herrn de Merchiston, erstmals in Scotischer Sprache aus | Liecht gegeben. | Jetzt aber treuwlich verdeutschet, | Durch | Leonem De Dromna. |

Dan. 12. | Vnd nun Daniel verbirg diese Wort, (vom Reich vnnd Zeit desz | Antichrists, vnd desz Jüngsten Tages) vnd versiegele diese Schrifft, bisz auff die | bestimpte Zeit, so werden viel drüber kommen, vnd grossen Verstand finden. |

Gedruckt zu Gera, durch Martinum Spiessen. | Im Jahr 1611. |

[Printed in red and black.]

4°. Size 7½ × 6¼ inches. Black letter. *(:)* 1^1, *Title.* *(:)* 1^2, blank. *(:)* 2^1–*(:)* 3^2, 4 pages, The Preface "*An den guthertzigen Leser*" signed "*Im Jahr* 1611. *Leo de Dromna.*" *(:)* 4^1–)( )(2^2, 6 pages, "*Register aller vnnd jeder Propositionen, so in diesem Büchlein tractiert werden.*", being the titles of the 36 propositions contained in Napier's First treatise; at the end are added "*Errata Typographica,*" 18 lines. $A1^1$–$Y2^1$, pp. 1–171, *The first treatise.* $Y2^2$, blank.

Signatures. *(:)* in four +)( )(in two + A to X in four + Y in two = 92 leaves.

Paging. 12 + 171 numbered + 1 = 184 pages.

This edition contains a translation by Leo de Dromna of the 36 propositions of Napier's First Treatise, but without its Title and 'Conclusion.' The other parts of the original work are all omitted.

Le Long catalogues editions Leipsic 1611 and Gera 1612. Draudius also, in Bibliotheca Librorum Germanicorum Classica, p. 290, mentions a Leipsic edition of 1611. He gives the title exactly as above so far as the words 'zweiffeln kan' at the end of the eighth line,

after which he adds 'ausz Scotischer Sprach verteutscht durch Leonem de Dromna. Leiptzig bey Thoma Schürern, in 4. 1611.' These particulars are copied verbatim from the Frankfurter Messkatalog vom Herbst 1611, sheet $D1^2$. The appearance of Schürer's name may, however, imply simply that he brought the work to market and issued it at the Frankfurt Book Fair, not that there is an edition bearing on its title-page to have been issued by him at Leipsic.

An edition Gera 1661 in 4° is mentioned by Graesse. His particulars regarding German editions of 'A plaine discovery' appear to be copied from Rotermund, vol. v. p. 494, where the same date is given; but there is little doubt it is a misprint for 1611.

Libraries. Adv. Ed.; Stadt. Breslau; Stadt. Frankfurt; Min. Schafthausen;

Entdeckung aller Geheimnüssen in der | Apocalypsi oder Offenbarung S. Jo- | hannis begriffen. | Dariñen . . . . . [Same as preceding.]

Gedruckt zu Gera, durch Martinum Spiessen | Im Jahr 1612. |

This impression is identical in every particular with the foregoing, except that in the last line of the Title-page the date 1612 is substituted for 1611.

Libraries. Stadt. Breslau; Un. Breslau; Stadt. Zurich;

Johannis Napeiri, | Herren zu Merchiston, | Eines trefflichen Schottländischen | Theologi, schöne vnd lang gewünschte | Auszlegung der | Offenbarung Jo- | hannis, | In welcher erstlich etliche Propositiones | gesetzt werden, die zu Erforschung desz wahren Ver- | stands nothwendig sind: Demnach auch der gantze Text | durch die Historien vnd Geschichten der Zeit erklärt, vnnd | angezeigt wirdt, wie alle Weissagungen bisz daher | seyen erfüllt worden, vnd noch in das künfftig | erfüllt werden sollen. | Ausz

Ausz begird der Warheit, vnd der öffnung jrer Ge-|heimnussen, nach den Frantzösischen, Englischen vnnd | Schottischen Exemplaren, dritter Edition jetzund auch|vnserem geliebten Teutschen Verstand|vbergeben.|

Getruckt zu Franckfort am|Mayn, im Jahr 1615.|

[Printed in red and black.]

8°. Size 6¾ × 4 inches. Black letter. ✗1[1], *Title.* ✗1[2], blank. ✗2[1]–✗8[2], 14 pages, The Preface "*Den Gestrengen, Edlen, Ehrenvesten, Hochgelehrten, Frommen, Fürnemmen, Fürsichtigen, Ersamen vnd weisen Herrn*"; then follow the names, &c., of 2 "*Bürgermeistern*," the "*Statthaltern*," 2 "*Seckelmeistern*," the "*gewesenen Landvogt zu Louis*," and the "*Stattschreibern, &c. Sampt einem gantzen Ersamen Rath, löblicher Statt Schaffhausen, meinen gnädigen vnd gönstigen Herrn.*", signed "*Basel den 1 Augusti, Anno 1615. Ew. Gn. Vnderthäniger, dienstgeflissener Wolffgang Mayer, H. S. D. Diener am Wort Gottes daselbsten.*" First Table wanting. A1[1]–H6[1], pp. 1-123, "*Auszlegung vnd Erklärung der Offenbarung Johannis. Der erste Theil vnd Eyngang dieses Wercks, begreiffend ein ersuchung desz wahren Verstandts der Offenbarung ausz den Leuchtern, gewissen vnd bekanten Puncten, die vngewissere vnnd vnbekante Stück schlieszlich beweisende, bisz zu vollkommener Erklärung aller fürnembsten Puncten, in gewisse Propositiones abgetheilt.*", 36 propositions and "*Beschlusz*" or Conclusion. H6[2], [p. 124] is blank. On a folding sheet after H6[2] is "*Tabul, Erklärung vnd Abtheylung der gantzen Offenbarung S. Johannis.*" H7[1]–Kk8[2], pp. 125-528, "*Der ander vnd fürnembste Theyl, darinn (nach den hievorgesetzten Fundamenten vnnd Gründen) die gantze Offenbarung erklärt vnnd auszgelegt, vnd mit der Historien vnnd Geschicht der Zeit, wie sich die Sachen auff einander verlauffen vnd zugetragen, conferiert vnd verglichen wirdt, mit angehengter Verzeichnusz vnd Erklärung vber die Orth vnd Sprüch so schwerlich zu verstehen, vnd kurtzen Argumentis vnd Innhalt eines jeden Capitels.*"; the chapters commence with "*Argument oder Innhalt*", then follow in three parallel columns "*Auszlegung desz Texts*", the "*Jahr Christi*" and "*Historische Erklärung*", (the 2d and 3d columns are wanting in the chapters mentioned in the Edin. 1593 edition); at the end of each chapter are added "*Fernere Auszlegung vnd Erklärung der bezeichneten Oerter dieses Capituls.*" Ll1[1]–Nn2[2], 36 pages, "*Register Aller denckwürdigen Sachen, so in diesem Buch begrieffen, nach dem Alphabet ordentlich zu finden, auszgetheilet.*"

Signatures. ✗ and A to Z and Aa to Mm in eights + Nn in two = 290 leaves.

Paging. 16 + 528 numbered + 36 = 580 pages.

Errors in Paging. There are several, sheet X especially being in great confusion, but none of the errors affect the last page.

Following H6[2] on a folding sheet is the second Table.

This edition contains a translation by Wolffgang Mayer of the two Treatises, but without the Text in the second. All the other matter in the English editions is omitted. The additional matter consists of the Preface and the Alphabetical Index to the principal subjects referred to

in the book. The first Table is wanting in the copies both of this edition and that of 1627 in all the libraries noted.

Libraries. Kön. Berlin; Stadt. Breslau; Un. Breslau; Stadt. Frankfurt;

Johannis Napeiri, | Herren zu Merchiston, | Eines trefflichen Schottländischen | Theologi, schöne vnd lang gewünschte | Auszlegung der | Offenbarung Jo- | hannis. | In welcher erstlich etliche Propositiones | gesetzt werden, die zu Erforschung desz wahren Ver- | standts nothwendig sind: Demnach auch der gantze Text | durch die Historien vnd Geschichten der Zeit erklärt, vnd | angezeigt wirdt, wie alle Weissagungen bisz daher | seyen erfüllet worden, vnd noch in das künff- | tig erfüllt werden sollen. | Ausz begierdt der Warheit, vnnd der öffnung ihrer | Geheymnüssen, nach dem Frantzösischen, Englischen | vnnd Schottischen Exemplaren, dritter Edition, jetzund | auch vnserm geliebten Teutschen Ver- | standt vbergeben. |

Getruckt zu Franckfurt am | Mayn, Im Jahr 1627. |

[Printed entirely in black.]

8°. Size 6⅞ × 4 inches. Black letter. The type has been reset for this edition, and there are many differences in spelling. The collation, however, is the same as in the edition of 1615, to the end of the Second Treatise, after which we have Ll1[1]–Nn3[2], 38 pages, "*Register Aller denckwürdigen Sachen, . . .*" Nn 4, blank.

Signatures. ✠ and A to Z and Aa to Mm in eights, + Nn in four = 292 leaves.

Paging. 16 + 528 numbered + 38 + 2 = 584 pages.

Errors in Paging. Rather more numerous than in 1615, sheet X again in confusion, but, as before, the errors do not affect the last page.

The second Table, on a folding sheet, follows H6[2] as in the 1615 edition.

Libraries. Adv. Ed.; Stadt. Breslau; Stadt. Zurich;

II.—De Arte Logistica, in Latin.

De Arte Logistica|Joannis Naperi|Merchistonii Baronis|Libri Qui Supersunt.|

Impressum Edinburgi|M.DCCC.XXXIX.|

4°. Large paper. Size 11¼ × 8⅞ inches. There are 4 leaves at the beginning. The first is entirely blank, on the recto of the second is the single line "*De Arte Logistica.*", on the recto of the third is the *Title-page* as above, and on the recto of the fourth is the *Dedication to Francis Lord Napier of Merchiston.* The number and arrangement of these pages is slightly different in the Club copies—see note.

On the recto of a1 is the word "*Introduction.*" $a2^1$–$m3^2$, pp. iii–xciv, "*Introduction.*" by Mark Napier, dated 1 November 1839.

On the recto of m4 is the line "*De Arte Logistica.*", and on the recto of A1 is the title "*The* | *Baron Of Merchiston* | *His Booke Of Arithmeticke* | *And Algebra.* | *For Mr Henrie Briggs* | *Professor Of Geometrie* | *At Oxforde.* |" $A2^1$–$D1^2$, pp. 3-26, "*Liber Primus. De Computationibus Quantitatum Omnibus Logisticæ Speciebus Communium.*" $D2^1$–$L1^1$, pp. 27-81, "*Liber Secundus. De Logistica Arithmetica,*" $L1^2$ blank. $L2^1$–$L4^2$, pp. 83-88, "*Liber Tertius. De Logistica Geometrica.*" On the recto of M1 is the title "*Algebra Joannis Naperi* | *Merchistonii Baronis.* |" $M2^1$–$P2^1$, pp. 91-115, "*Liber Primus. De Nominata Algebræ Parte.*" $P2^2$, blank. $P3^1$–$X1^2$, pp. 117-162, "*Liber Secundus. De Positiva Sive Cossica Algebræ Parte.*" X2, blank.

Signatures. 4 leaves [see notes]+a to m and A to U in fours+X in two=134 leaves.

Paging. 8+XCIV numbered+2+162 numbered+2=268 pages.

There are also two plates, the one a portrait of Napier, the other a view of Merchiston Castle.

The collation is from a large-paper copy. Each page is enclosed in a double red line, the title-page being in part printed in red as well as the headings of chapters etc., throughout the work. Generally, however, the copies are printed entirely in black and are without the double line enclosing the type.

In his preface Mark Napier states that he was induced to publish the work "by the spirited interposition of the Bannatyne and Maitland Clubs of Scotland." The copies for members of these Clubs are printed entirely in black on their own water-marked paper, size

$10\frac{1}{2} \times 8\frac{1}{4}$ inches. They have not a blank leaf at the beginning, but have on the recto of a leaf after the title-page an extract from the minutes authorising the printing, followed by two leaves containing a list of the members of the Club; the Bannatyne Club having one hundred members and the Maitland Club ninety. The foregoing differences make the preliminary leaves six instead of four as in the collation.

The manuscript from which the work was published appears, from the following passage in the Memoirs (pp. 419, 420), to be the only one of Napier's papers which survives.

> Napier left a mass of papers, including his mathematical treatises and notes, all of which came into the possession of Robert as his father's literary executor. When the house of Napier of Culcreugh was burnt, these papers perished, with only two exceptions that I have been able to discover. The one is the manuscript treatise on Alchemy by Robert Napier himself; but the other is a far more valuable manuscript, being entitled, "The Baron of Merchiston, his booke of Arithmeticke, and Algebra; for Mr Henrie Briggs, Professor of Geometrie at Oxforde." . . . . . it is of great length, beautifully written in the hand of his son, who mentions the fact, that it is copied from such of his father's notes as the transcriber considered "orderlie sett doun."

The treatise on Alchemy is elsewhere stated (pp. 236, 237) to be contained in a thin quarto volume closely written in the autograph of Robert Napier, bearing the title "*Mysterii aurei velleris Revelatio; seu analysis philosophica qua nucleus veræ intentionis hermeticæ posteris Deum timentibus manifestatur. Authore R. N.*" and the motto—

> "*Orbis quicquid opum, vel habet medicina salutis,*
> *Omne Leo Geminis suppeditare potest.*"

In this connection the following entry may be mentioned which occurs in the sale catalogue of the first portion of the library of the late David Laing:—"*Lambye* (*J. B.*) *Revelation of the secret Spirit* (*Alchymie*) *translated by R. N. E.* (*Robert Napier Edinburgensis?*) 1623." The work sold for £7, 2s. 6d.

Libraries. Adv. Ed.; etc.

III.—Rabdologiæ.

III.—Rabdologiæ.

1. EDITIONS IN LATIN.

Rabdologiæ,|Sev Nvmerationis|Per Virgulas|Libri Dvo:|Cum Appendice de expeditissi-|simo Mvltiplicationis|promptvario.| Quibus accessit & Arithmeticæ|Localis Liber vnvs.|Authore & Inventore Ioanne|Nepero, Barone Mer-|chistonii, &c.|Scoto.| Edinbvrgi,|Excudebat Andreas Hart, 1617.|

12°. Size 5⅝ × 3¼ inches. ¶1[1], *Title.* ¶1[2], blank. ¶2[1]–¶4[1], 5 pages, "*Illustrissimo Viro Alexandro Setonio Fermelinoduni Comiti, Fyvœi, &c Vrqvharti Domino, &c. Supremo Regni Scotiæ Cancellario. S.*", signed "*Ioannes Neperus Merchistonii Baro.*" ¶4[2], Verses, viz.:—"*Avthori Dignissimo.*", 4 lines, unsigned; "*Lectori Rabdologiæ.*", 4 lines, signed "*Patricivs Sandevs*"; and "*Ad Lectorem.*", 6 lines, signed "*Andreas Ivnivs.*" ¶5[1]–¶6[1], 3 pages, "*Elenchvs Capitvm, et vsvvm totivs operis.*" ¶6[2], two lines in centre of page. A1[1]–B9[2], pp. 1-42, "*Rabdologiæ Liber Primvs De usu Virgvlarvm numeratricium in genere.*" B10[1]–D9[2], pp. 43-90, "*Rabdologiæ Liber Secvndvs De usu Virgularum Numeratricium in Geometricis & Mechanicis officio Tabularum.*" D10[1]–E8[2], pp. 91-112, "*De Expeditissimo Multiplicationis Promptvario Appendix.*", the "Præfatio" occupying the first page. E9[1]–G5[2], pp. 113-154, "*Arithmeticæ Localis, quæ in Scacchiæ abaco exercetur, Liber unus.*", the "Præfatio" occupying the first two pages. G6 blank.

Signatures. ¶ in six + A to F in twelves + G in six = 84 leaves.

Paging. 12 + 154 numbered + 2 = 168 pages.

There are 4 folding plates to face pages 101, 105, 106, and 130, which, with those on pages 6, 7, 8, 94, and 95, are copperplate.

Errors in Paging. None.

In one copy belonging to the Edinburgh University Library the signature B5 is printed in error A5, but in their other copy it is correct.

The word expeditissimo on the 5th and 6th lines of the title-page is in some copies correctly printed expeditis-|simo.

Libraries. Adv. Ed.; Sig. Ed.; Un. Ed. (2); Un. Gl. (2); Brit. Mus. Lon.; Un. Col. Lon.; Roy. Soc. Lon.; Bodl. Oxf. (5); Un. Camb.; Trin. Col. Camb.; Trin. Col. Dub.; Kön. Berlin; Stadt. Breslau; Un. Breslau; Kön. Öff. Dresden; Un. Halle; Un. Leiden; K. Hof u. Staats. München; Astor, New York; Nat. Paris; Un. Utrecht;

Rabdologiæ|Sev Nvmerationis|per Virgulas libri duo:|Cum Appendice de expe-|ditissimo Mvltiplicationis|promptvario.| Quibus accessit & Arithme-|ticæ Localis Liber unus.|Authore & Inventore Ioanne|Nepero, Barone Merchisto-|nij, &c. Scoto.| Lvgdvni.|Typis Petri Rammasenij.|M. DC. XXVI.|

12°. Size 5½ × 3¼ inches. †1[1], *Title.* †1[2], blank. †2[1]–†4[2], 6 pages, *Dedication to Alex. Seton, Lord Dunfermline.* †5[1], *Verses.* †5[2]–†6[2], 3 pages, *Elenchvs Capitvm*, with the 2 lines at end. A[1]–B9[2], pp. 1-42, *Rabdologiæ, Lib. I.* B10[1]–D6[2], pp. 43-84, *Lib. II.* D7[1]–E3[2], pp. 85-102, *Mult. promptuario.* E4[1]–G4[1], pp. 103-139, *Arith. Localis.* G4[2]-G6[2], 5 pages, all blank.

Signatures. † in six + A to E in twelves + F and G in sixes = 78 leaves.

Paging. 12 + 139 numbered + 5 = 156 pages.

There are 9 folding diagrams to face pages 49, 51, 59, 81, 94, 97, 98, 115, and 117; those facing pages 94, 97, 98, and 117, correspond to the 4 folding plates of the 1617 edition, the others are tables which in 1617 were printed in the text.

The numbering of the pages, though somewhat indistinct, seems to be correct throughout. In printing the signatures, however, C7 is numbered in error C6, and E3 has no signature printed.

This edition, published at Leyden, contains exactly the same matter as that of the Edinburgh edition of 1617. None of the plates, however, are engraved on copper. The decimal fractions are printed according to Simon Stevin's notation; thus, for example, on p. 41 we have 1994 ⓪, 9 ① 1 ② 6 ③ 0 ④ while in the 1617 edition it is printed 1994,9′ 1″ 6‴ 0⁗.

Libraries. Un. Ed.; Un. Ab.; Un. St And.; Greenock; Bodl. Oxf.; Chetham's Manch.; Trin. Col. Dub.; Kön. Berlin; Un. Breslau; Stadt. Frankfurt; K. Hof u. Staats. München; Astor, New York;

Rabdologiæ | Sev Nvmerationis | per Virgulas libri duo : | Cum Appendice de expe- | ditissimo Mvltiplicationis | promptvario. | Quibus accessit & Arithmeti- | cæ Localis Liber vnus | Authore & Inventore Ioanne Nepero Barone Merchisto- | nij, &c. Scoto. | Lvgd. Batavorvm. | Typis Petri Rammasenij. | M. DC. XXVIII. |

This

This edition is identical with that of 1626, described in the previous entry, but the original title-page has been cut out, and the above substituted. The only important change in this new title-page, besides the alteration of date, is the substitution of the name LVGD. BATAVORVM for LVGDVNI, and the object in printing a new title-page was probably to effect this change in name, as confusion may have arisen from the single word 'Lugduni' being used for Leyden, instead of the more common form Lugd. Batavorum, the word Lugdunum being the usual Latin form of Lyons, as, for example, in the 1620 edition of the 'Descriptio.'

Libraries. Adv. Ed.; K. Hof u. Staats. München; Nat. Paris;

2. EDITION IN ITALIAN.

Raddologia,|Ouero|Arimmetica Virgolare|In due libri diuisa;| Con appresso vn' espeditissimo|Prontvario Della Molteplicatione,|& poi vn libro di|Arimmetica Locale:|Quella mirabilmente commoda, anzi vtilissima|à chi, che tratti numeri alti;|Questa curiosa, & dileteuole|à chi, che sia d' illustre ingegno.|Auttore, & Inuentore|Il Baron Giovanni Nepero,|Tradottore dalla Latina nella Toscana lingua|Il Cavalier Marco Locatello;|Accresciute dal medesimo alcune consi-|derationi gioueuoli.|

In Verona, Appresso Angelo Tamo. 1623.|Con licenza de' Superiori.|

8°. Size $6\frac{1}{8} \times 4\frac{1}{8}$ inches. †1[1], *Title.* †1[2], blank. †2[1]–†3[1], 3 pages, "*Allo Illmo. & Eccmo. Sigre. Teodoro Trivvltio, Prencipe del Sac. Rom. Imperio, di Musocco, & della Valle Misolcina; Conte di Melzo, & di Gorgonzola; Signor di Codogno, & di Venzaghello; Caualier dell' Ordine di S. Giacomo, &c.*", dated and signed "*Di Verona li 12. Febraio* 1623. . . . *Marco Locatelli.*" †3[2], "*Al medesimo Sig. Prencipe Trivvltio L'istesso Locatelli.*", followed by 10 lines of verse. †4[1], "*Del Sig. Ambrosio Bianchi Co. Cau. e I. C. Coll. di Mil. Al Sig. Cau. Marco Locatelli.*", with 14 lines of verse. †4[2], "*Del Sig. Francesco Pona Med. Fis. & Acc. Filarm. Al medes. Sig. Cau. Locatelli.*", with 13 lines of verse.

†5[1]–†8[1], 7 pages, "*Racconto De' Capi di tutta l'Opera, Et de' Titoli più rileuanti in essi.*" On †8[2] is printed "*Imprimatur Fr. Siluester Inquisitor Veronæ. Augustinus Dulcius Serenissimæ Reip. Venetæ Secr.*" A1[1]–F4[1], pp. 1-95, "*Della Raddologia Libro*

Libro Primo. Dell' vso delle Virgole numeratrici in genere." $F4^2$, blank. $F5^1$–$K4^1$, pp. 97-159, "*Della Raddologia Libro Secondo. Dell'vso delle Virgole numeratrici nelle cose Geometriche, & Mecaniche, con l'aiuto di alcune Tauole.*" $K4^2$, blank. $K5^1$–$N3^2$, pp. 161-210, "*Prontvario Ispeditissimo Della Molteplicatione,*" the "Proemio" occupying the first 2 pages. $N4^1$–$Q8^1$, pp. 211-269, "*Arimmetica Locale, Che nel Piano dello Scacchiere si esercita.*", the "Prefatione" occupying the first 2 pages. On $Q8^2$, "*Il Fine di tutta l'Opera. In Verona, Appresso Angelo Tamo.* 1623. *Con licenza de' Superiori.*"

Signatures. † and A to Q in eights = 136 leaves + 7 diagrams interleaved and included in paging = 143 leaves.

Paging. 16 + 269 numbered + 1 = 286 pages.

There are 7 diagrams on interleaved and folded sheets, each of which counts as two pages; the sides containing the diagrams are numbered as pages 25, 36, 49, 63, 169, 179, and 233.

Errors in Paging. P. 75 not numbered, and pp. 251, 266, and 267 numbered in error 152, 264, and 165.

New Dedication and Complimentary Verses are substituted for those in the edition of 1617, and there are numerous notes throughout the work by the Translator, as well as additions and alterations. One of these may be mentioned. At the end of the work Napier adds these words, "Atque hic finem ARITHMETICÆ LOCALI imponimus. DEO soli laus omnis & honor tribuatur. FINIS.", but his Italian translator makes the champion of Protestantism say, "Con che a questa nostra ARIMMETICA LOCALE poniamo fine, a DIO, & alla Beatissima Vergine MARIA tutta la gloria, & l'honore attribuendo. Amen."

Of the four folding diagrams in the edition of 1617, the two facing pages, 101 and 130, are represented by the diagrams at pages 179 and 233, but the other two are not given in this edition.

Libraries. Un. Ed.; Brit. Mus. Lon.; Un. Col. Lon.; Trin. Col. Camb.; Nat. Paris;

3. EDITION IN DUTCH.

Eerste Deel | Vande Nievwe | Telkonst, | Inhovdende Verscheyde | Manieren Van Rekenen, Waer | door seer licht konnen volbracht worden de Geo- | metrische ende Arithmetische questien. | Eerst ghevonden van Ioanne Nepero Heer | van Merchistoun, ende uyt het Latijn overgheset door | Adrianvm Vlack. |

Waer achter bygevoegt zijn eenige seer lichte manieren van Rekenen

Rekenen | tot den Coophandel dienstigh, leerende alle ghemeene Rekeninghen | sonder ghebrokens afveerdighen. Mitsgaders Nieuwe Tafels | van Interesten, noyt voor desen int licht ghegeven. | Door Ezechiel De Decker, Rekenm^r. | Lantmeter, ende Liefhebber der Mathematische | kunst, residerende ter Goude. |

Noch is hier achter byghevoeght de Thiende van | Symon Stevin van Brugghe. |

Ter Govde, | By Pieter Rammaseyn, Boeck-verkooper inde corte | Groenendal, int Vergult ABC. 1626. | Met Previlegie voor thien Iaren. |

4°. Size 8⅝ × 6⅝ inches. ∗ 1[1] *Title-page.* ∗ 1[2], "*Copie Van De Previlegie,*" granted by the States-General to Adrian Vlack for ten years, signed at s'Gravenhaghe, 24 Dec. 1625. ∗2, 2 pages, The dedication, "*Toeeyghen-brief Aende Doorlvchtighe, Hooge Ende Mogende Heeren, mijn Heeren de Staten Generael vande Vereenighde Nederlanden. Mitsgaders De Edele, Erntfeste Ende Wyse Heeren, de Heeren Gecommitteerde Raden van Hollant ende Westvrieslant. Als Mede Aende Achtbare, Voorsienige Heeren, mijn Heeren Bailiu, Burghemeesteren, Schepenen, ende Vroetschap der Vermaerde Stadt Gouda.*" signed by Ezechiel de Decker at Gouda 4 Sep. 1626. ∗3[1]–∗4[1], 3 pages, the preface, "*Voor-reden tot den Goetwilligen ende Konstlievenden Leser,*" signed by Ezechiel de Decker at Gouda 4 Sep. 1626.

∗4[2], Three Latin verses : "*Ioanni Nepero | Avthore Dignissimo.* |," 4 lines ; "*Lectori Rabdologiæ.* |", signed Patricius Sandæus, 4 lines ; and "*Ad Lectorem.* |", signed Andreas Iunius, 6 lines. †1[1]–†2[1] 3 pages, The index, "*Register van alle de Hooftstucken, ende Ghebruycken deses gantschen Boeckx.*" †2[2] "*De Druck-fauten salmen aldus verbeteren.*", 21 lines of errata. A1[1]–E4[2], pp. 1–40, "*Ioannis Neperi Eerste Boeck, Vande Tellingh door Roetjes. Van Het Ghebruyck Der Telroetjes int ghemeen.*", in nine chapters. F1[1]–L4[1], pp. 41–87, "*Ioannis Neperi Tweede Boeck, Van de Tellingh door Roetjes. Van Het Ghebruyck Der Tel-Roeties in Meetdaden, ende Werckdaden, met behulp van Tafels.*", in eight chapters. L4[2], blank.

M1[1]–O4[2], pp. [89]–[112], "*Ioannis Neperi Aenhanghsel Van Het Veerdigh-Ghereetschap van Menighvvldigingh.*", in four chapters, the title is on p. [89] and the Preface on p. [90], the last page, O4[2], being blank.

P1[1]–T2[2], pp. [113]–148, "*Ioannes Nepervs Van de Plaetselicke Telkunst.*", in eleven chapters, the title is on p. [113] and the Preface on p. [114].

V1[1]–Rr 4[2], pp. [149]–308, "*Ezechiel De Decker Van Coopmans Rekeningen. Leerende Door Thiendeelighe Voortgangh sonder gebrokens met wonderlicke lichticheyt afveerdigen alle ghemeene Rekeninghen.*", in eight chapters, the title is on p. [149] and the Preface on p. [150].

a1[1]–q4[2], 128 pages. *Tables.*

De | Thiende. | Leerende Door | onghehoorde lichticheyt alle re-

re- | keninghen onder den Menschen noodigh val- | lende, afveerdighen door heele ghetal- | len, sonder ghebrokenen. | Door Simon Stevin van Brugghe. |

Ter Govde, | By Pieter Rammaseyn, Boeck- | vercooper, inde Corte Groenendal, int Duyts | Vergult ABC. | M. DC. XXVI. |

$A1^1$, *Title-page.* $A1^2$, blank, $A2^1$–$A3^1$, pp. 3–5, Preface "*Den Sterrekiickers, Landtmeters, Tapiitmeters, Wijnmeters, Lichaemmeters int ghemeene, Muntmeesters, ende allen Cooplieden, wenscht Simon Stevin Gheluck.*" $A3^2$, p. 6, "*Cort Begriip.*" A4, pp. 7 and 8, "*Het Eerste Deel Der Thiende Vande Bepalinghen.*" $B1^1$–$B4^1$, pp. 9–15, "*Het Ander Deel Der Thiende Vande Werckinghe.*" $B4^2$–$D2^1$, pp. 16–27, *Aenhanghsel.* $D2^2$, blank.

Signatures. ∗ in four and † in two (=6)+A to Z and Aa to Rr in fours, except T, Y, and Cc, which are in twos, (=154)+a to q in fours (=64)+A to C in fours and D in two (=14)=in all 238 leaves.

Paging. 12+308 numbered+128+27 numbered+1=476 pages.

Errors in Paging. The pages 111, 121, 218, 219, 270, 271, 274, are numbered 110, 221, 217, 218, 254, 255, 258, respectively, and the numbers are not printed on the following pages, 88–90, 103, 105, 112–114, 149, 150, 169–176, 187, 196, 222, 275–284, but the numbering of the last page, 308, is not affected.

Errors in Signatures. N3 is printed as N5, V2 as V, and Gg2 as Gg3.

The leaf K4 has been cut out and another substituted.

The translation of Rabdologiæ, extending from $*4^2$ to $T2^2$ and embracing 5 unnumbered and 148 numbered pages, appears to correspond exactly with the original Latin edition of 1617, except that Napier's dedication to Lord Dunfermline on the 5 pages $¶2^1$–$¶4^1$ and the two lines on the page $¶6^2$ of that edition are omitted. The translation is by Adrian Vlack, and was made at the request of De Decker, for this work.

Libraries. Un. Col. Lon.; Trin. Col. Camb.; Kön. Berlin; Kon. Hague; Nat. Paris;

IV.—Mirifici

IV.—Mirifici logarithmorum canonis descriptio *and*, Mirifici logarithmorum canonis constructio.

1. EDITIONS IN LATIN.

Mirifici|Logarithmorum|Canonis descriptio,|Ejusque usus, in utraque|Trigonometria; ut etiam in|omni Logistica Mathematica,|Amplissimi, Facillimi, &|expeditissimi explicatio.|Authore ac Inventore,|Ioanne Nepero,|Barone Merchistonii,|&c. Scoto.|

Edinbvrgi,|Ex officinâ Andreæ Hart|Bibliopôlæ, cIↃ. DC. XIV.|

[The title is enclosed in an ornamental border. A reproduction of the Title-page will be found at p. 374 of the Memoirs.]

4°. Size $7\frac{1}{2} \times 5\frac{3}{4}$ inches. $A1^1$, *Title.* $A1^2$, blank. A2, 2 pages, "*Illustrissimo, & optimæ spei Principi Carolo, Potentissimi, & Invictissimi, Iacobi D. G. magnæ Britanniæ, Franciæ, & Hiberniæ Regis, filio unico, Walliæ Principi, Duci Eboraci, & Rothesaiæ, magno Scotiæ Senescallo, ac Insularum Domino, &c. D. D. D.*", signed "*Ioannes Nepervs.*" $A3^1$, "*In Mirificvm Logarithmorum Canonem Præfatio.*" $A3^2$–$A4^1$, 2 pages, Verses:—"*Ad Lectorem Trigonometriæ studiosum.*", 12 lines signed "*Patricius Sandęus.*"; "*In Logarithmos D. I. Neperi.*", 10 lines; "*Aliud.*", 6 lines; "*Ad Lectorem.*", 4 lines signed "*Andreas Ivnivs Philosophiæ Professor in Academia Edinburgena.*" $A4^2$, "*In Logarithmos.*", 4 lines. $B1^1$–$D2^2$, pp. 1-20, "*Mirifici Logarithmorum canonis descriptio, eiusque usus in utrâque Trigonometria, ut etiam in omni Logistica mathematica, amplissimi, facillimi, & expeditissimi explicatio. Liber I.*" $D3^1$–$I1^1$, pp. 21-57, "*Liber Secvndvs. De canonis mirifici Logarithmorum præclaro usu in Trigonometria.*", on p. 57 after the "*Conclvsio.*" follow "*Errata ante lectionem emendanda.*", 7 lines; and the last line of the page is "*Sequitur Tabula seu canon Logarithmorum.*" $I1^2$ and $a1^1$–$m1^1$, 90 pages, *The Table.* $m1^2$, "*Admonitio*" or blank [see note].

Signatures. A to H in fours+I in one+a to l in fours+m in one=78 leaves.

Paging. 8+57 numbered+91=156 pages.

Errors in Paging. In some copies pp. 14 and 15 are numbered 22 and 23 [see note].

Gr. 30

30 min	Sinus	Logarithmi	Differentiæ +	Differentiæ −	logarithmi	Sinus	
0	5000000	6931469	5493059		1438410	8660254	60
1	5002519	6926432	5486342		1440090	8658799	59
2	5005038	6921399	5479628		1441771	8657344	58
3	5007556	6916369	5472916		1443453	8655888	57
4	5010074	6911342	5466206		1445136	8654431	56
5	5012591	6906319	5459498		1446821	8652973	55
6	5015108	6901299	5452792		1448507	8651514	54
7	5017624	6896282	5446088		1450194	8650055	53
8	5020140	6891269	5439387		1451882	8648595	52
9	5022656	6886259	5432688		1453571	8647134	51
10	5025171	6881253	5425992		1455261	8645673	50
11	5027686	6876250	5419298		1456952	8644211	49
12	5030200	6871250	5412605		1458645	8642748	48
13	5032714	6866254	5405915		1460339	8641284	47
14	5035227	6861261	5399227		1462034	8639820	46
15	5037740	6856271	5392541		1463730	8638355	45
16	5040253	6851285	5385858		1465427	8636889	44
17	5042765	6846302	5379177		1467125	8635423	43
18	5045277	6841323	5372499		1468824	8633956	42
19	5047788	6836347	5365822		1470525	8632488	41
20	5050299	6831374	5359147		1472227	8631019	40
21	5052809	6826405	5352475		1473930	8629549	39
22	5055319	6821439	5345805		1475634	8628079	38
23	5057829	6816476	5339137		1477339	8626608	37
24	5060338	6811516	5332471		1479045	8625137	36
25	5062847	6806560	5325808		1480752	8623665	35
26	5065355	6801607	5319147		1482460	8622192	34
27	5067863	6796657	5312488		1484169	8620718	33
28	5070370	6791710	5305831		1485879	8619243	32
29	5072877	6786767	5299177		1487590	8617768	31
30	5075384	6781827	5292525		1489302	8616292	30

59

There

There are two noticeable varieties of this edition, the one with an Admonitio printed on m1², the back of the last page of the table, the other with that page blank. In general the former variety has the error in paging before mentioned, while the latter has the paging correct. There are also, however, copies which want the Admonitio but have the error in paging, for instance, one of the copies in the Bodleian and the copy in University College, London. A translation of the Admonitio referred to above is given in the Notes, page 87.

A specimen page of the table is given opposite, and a full description of its arrangement will be found in section 59 of the Constructio.

Libraries.

(1.) *With Admonitio.* Sig. Ed.; Un. Ed. (2); Hunt. Mus. Gl.; Un. Ab.; Brit. Mus. Lon. (2); Roy. Soc. Lon.; Bodl. Oxf. (3); Un. Camb.; Trin. Col. Dub. (2);

(2.) *Without Admonitio.* Adv. Ed.; Un. Gl. (2); Un. Col. Lon. (see note); Bodl. Oxf. (see note);

Foreign Libraries, varieties not distinguished. Kön. Berlin; Stadt. Breslau; Un. Breslau; Stadt. Frankfurt; Pub. Genève; Un. Halle; Un. Leiden; Un. Leipzig; K. Hof u. Staats. München; Nat. Paris;

A reprint of the
Mirifici Logarithmorum Canonis Descriptio
is contained in

Scriptores Logarithmici;|or|A Collection|of|Several Curious Tracts|on the|Nature And Construction|of|Logarithms,|mentioned in Dr Hutton's Historical Introduction to his New|Edition of Sherwin's Mathematical Tables:|together with|Some Tracts on the Binomial Theorem and other subjects|connected with the Doctrine of Logarithms.|Volume VI.|

London.|Printed by R. Wilks, in Chancery-Lane;|and sold by J. White, in Fleet-Street.|MDCCCVII.|

The work, Scriptores Logarithmici, consists of six large quarto volumes, and was compiled by Baron Francis Maseres. The volumes appeared in the years 1791, 1791, 1796, 1801, 1804, and 1807 respect-

 ively.

ively. The reprint, which will be found on pages 475 to 624 of the sixth volume, gives the Descriptio and the Canon in full, with the Admonitio on its last page.

Graesse states that the edition of 1614 was "Réimpr. sous la même date dans les *Transact. of the Roy. Soc.*" He probably refers to this reprint as Baron Maseres was a member of the Royal Society.

Libraries. Adv. Ed.; etc.

Mirifici | Logarithmo-|rvm Canonis | Descriptio,| Ejusque usus, in utraque Trigonome-|tria; vt etiam in omni Logistica Ma-| thematica, amplissimi, facillimi,| & expeditissimi explicatio.| Accesservnt Opera Posthvma;|Primò, Mirifici ipsius canonis constructio, & Logarith-|morum ad naturales ipsorum numeros habitudines.| Secundò, Appendix de alia, eáque præstantiore Loga-| rithmorum specie construenda.| Tertiò, Propositiones quædam eminentissimæ, ad Trian-|gula sphærica mirâ facilitate resolvenda.| Autore ac Inventore Ioanne Nepero,| Barone Merchistonii, &c. Scoto.|

Edinbvrgi,| Excvdebat Andreas Hart.| Anno 1619.|

[The ornamental part of the Title-page is the same as in 1614, the type only being altered.]

4°. Size $7\frac{1}{2} \times 5\frac{3}{4}$ inches. [See note.]

Mirifici | Logarithmorvm | Canonis Con-| strvctio;| Et eorum ad naturales ipsorum numeros habitudines;| Vnà Cvm | Appendice, de aliâ eâque præstantiore Loga-| rithmorum specie condenda.| Qvibvs Accessere | Propositiones ad triangula sphærica faciliore calculo resolvenda:| Vnà cum Annotationibus aliquot doctissimi D. Henrici | Briggii, in eas & memoratam appendicem.| Authore & Inventore Ioanne Nepero, Barone | Merchistonii, &c. Scoto.|

Edinbvrgi,| Excudebat Andreas Hart.| Anno Domini 1619.|

$A1^1$, *Title.* $A1^2$, blank. A2, 2 pages, "*Lectori Matheseos Studioso S.*", signed "*Robertvs Nepervs, F.*" $A3^1$–$E4^1$, pp. 5–39, "*Mirifici Logarithmorvm Canonis Constrvctio; (Qvi Et Tabvla Artificialis ab autore deinceps appellatur) eorumque ad naturales ipsorum numeros habitudines.*" $E4^2$–$F3^1$, pp. 40–45, "*Appendix*" containing "*De alia eaque præstantiore Logarithmorvm specie construenda; in qua scilicet, vnitatis*

tatis Logarithmus est 0." ; "*Alius modus facilè creandi Logarithmos numerorum compositorum, ex datis Logarithmis suorum primorum.*" ; "*Habitudines Logarithmorvm & suorum naturalium numerorum invicem.*" $F3^2$–$G3^1$, pp. 46–53, "*Lvcvbrationes Aliqvot Doctissimi D. Henrici Briggii In Appendicem præmissam.*" $G3^2$,–$H3^2$, pp. 54–62, "*Propositiones Qvædam Eminentissimæ ad triangula sphærica, mirâ facilitate resolvenda.*" containing "*Triangulum sphæricum resolvere, absque eiusdem divisione in duo quadrantalia aut rectangula.*" ; "*De semi-sinuum versorum præstantia & vsu.*" $H4^1$–$I2^1$, pp. 63–67, "*Annotationes Aliqvot Doctissimi D. Henrici Briggii In Propositiones Præmissas.*" $I2^2$, blank.

Signatures. A to H in fours + I in two = 34 leaves.
Paging. 67 numbered + 1 = 68 pages.

The first title-page, given above, with a blank leaf attached, appears to have been printed in order that it might be substituted for the title-page of the 1614 edition of the Descriptio by those who desired to have the two works on logarithms bound together. In such cases the 1614 title-page is usually cut out and the new one pasted on in its place. Consequently, in these copies, we find the same varieties as mentioned in the preceding entry. In other copies, however, only the new title-page and blank leaf are inserted before the Constructio.

Libraries.

I. *Copies containing both the Descriptio and Constructio, with the new title-page substituted.*

1. *With Admonitio.* Un. Ed.;
2. *Without Admonitio.* Adv. Ed.; Sig. Ed.;

II. *Copies containing the Constructio only with the new title-page and blank leaf attached.* Un. Ed.; Un. Col. Lon.; Bodl. Oxf.; Un. Camb.; Trin. Col. Dub.;

Foreign Libraries, varieties not distinguished. Un. Halle;

Logarithmorvm | Canonis Descriptio, | Sev | Arithmeticarvm Svppvtationvm | Mirabilis Abbreviatio. | Eiusque vsus in vtraque Trigonometria, vt etiam in omni | Logistica Mathematica, amplissimi, facillimi & | expeditissimi explicatio. | Authore ac Inuentore Ioanne Nepero, | Barone Merchistonij, &c. Scoto. |

Lvgdvni, | Apud Barth. Vincentium. | M. DC. X X. | Cum Priuilegio Cæsar. Majest. & Christ. Galliarum Regis. |

[Printed in black and red.]

S 3 8°,

8°, printed as 4°. **Size 8½ × 5½ inches.** A1[1] *Title.* A1[2] blank. A2, 2 pages, *Dedication to Prince Charles* [see note]. A3[1] *Preface.* A3[2]–A4[2], 3 pages, *Verses.* B1[1]–D2[2], pp. 1-20 *The Descriptio, Lib I.* D3[1]–H4[2], pp. 21-56, *Lib. II.*

Signatures. A to H in fours = 32 leaves.

Paging. 8 + 56 numbered = 64 pages.

Errors in Paging. None, but sig. D3 is printed D5.

Seqvitvr | Tabvla | Canonis Loga- | rithmorvm seu | Arithmeticarvm | Svppvtationvm. | S'ensuit l'Indice du Canon des Logarithmes. | A Scavoir, | La Table de l'admirable inuention pour | promptement & facilement Abreger les sup- | putations, d'Arithmetique auec son vsage, en l'v | ne & l'autre Trigonometrie, & aussi en toute | Logistique Mathematique. |

Lvgdvni, | Apud Barthol. Vincentivm. | Cum priuilegio Cæsareo & Galliarum Regis. |

A1[1] *Title.* A1[2]–M2[1], *Table.* M2[2], '*Extraict*' or blank [see note].

Signatures. A to L in fours + M in two = 46 leaves.

Paging. 92 pages not numbered.

Mirifici | Logarithmorvm | Canonis Con- | strvctio ; | Et Eorvm Ad Natvrales | ipsorum numeros habitudines ; | Vna Cvm Appendice, De Alia | eâque præstantiore Logarithmorum specie condenda. | Quibus accessere Propositiones ad triangula sphæ- | rica faciliore calculo resoluenda : | Vnà cum Annotationibus aliquot doctissimi D. Henrici | Briggii in eas, & memoratam appendicem. Authore & Inuentore Ioanne Nepero, Barone | Merchistonii, &c. Scoto. |

Lvgdvni, | Apud Bartholomæum Vincentium, | sub Signo Victoriæ. | M. DC.XX. | Cum priuilegio Cæsar. Maiest. & Christ. Galliarum Regis. |

A1[1] *Title.* A1[2] blank. A2, pp. 3 & 4, "*Robertvs Nepervs Avctoris Filivs Lectori Matheseos Studioso. S.*" A3[1]–E2[1], pp. 5–35, *The Constructio.* E2[2]–F1[1], pp. 36–41, *The Appendix.* F1[2]–G1[1], pp. 42–49, *Lvcvbrationes* by Briggs [see note]. G1[2]–H1[1], pp. 50–57, *Propositiones Trigonometricæ.* H1[2]–H3[2], pp. 58–62, *Annotationes* by Briggs. H4[1], '*Extraict*' or blank [see note]. H4[2] blank.

Signatures. A to H in fours = 32 leaves.

Paging. 62 numbered + 2 = 64 pages.

Errors in Paging. None.

On the issue of the Edinburgh edition of 1619, Barth. Vincent would appear

appear to have at once set about the preparation of an edition for issue at Lyons, and, as will be seen from the next entry, had some copies printed with the date 1619 on the first title-page. The three parts are usually found together, but some copies contain only the Descriptio and Tabula. The Admonitio is omitted from the last page ($M2^2$) of the Tabula, but in many copies its place is taken by the "Extraict du Priuilege du Roy," at the end of which is printed "*Acheué d'Imprimer le premier Octobre, mil six cents dixneuf.*" The copies in the Advocates' Library, Edinburgh, and Astor Library, New York, have this Extraict on $M2^2$ of the Tabula, and have also on $H4^1$ of the Constructio the Extraict reset with the note at end altered to "*Mirifici Logarithmorum Acheué d'imprimer le* 31 *Mars* 1620."

The edition is a fairly correct reprint of the Edinburgh one, but the decimal notation employed by Briggs in his Remarks on the Appendix has not been understood, the line placed by him under the fractional part of a number to distinguish it from the integral part being here printed under the whole number. The only intentional alteration, besides the title-page, is in the Dedication to Prince Charles, where "Franciæ" is omitted from his father's title, "*magnæ Britanniæ, Franciæ, & Hiberniæ Regis.*"

Libraries. Adv. Ed.; Un. Ed.; Act. Ed.; Un. Gl.; Un. St. And.; Brit. Mus. Lon. (parts 1 and 2 only); Un. Col. Lon.; Roy. Soc. Lon.; Kön. Berlin; Un. Breslau (parts 1 and 2 only); Kön. Öff. Dresden; K. Hof u. Staats. München; Astor, New York; Nat. Paris; Un. Utrecht; Stadt. Zürich (parts 1 and 2 only);

Logarithmorvm | Canonis Descriptio, | Sev | Arithmeticarvm Svppvtationvm | Mirabilis Abbreviatio. | Eiusque vsus . . . . . [Same as preceding.]

Lvgdvni, | Apud Barth. Vincentium. | M. DC. X I X. | Cum Priuilegio Cæsar. Majest. & Christ. Galliarum Regis. |

[Printed in black and red.]

The only respect in which this entry differs from the preceding is in the date on the title-page. A possible explanation of this may be that the title-page was originally set up with the date M. DC. XIX., but

when it was found that the whole work could not be issued in that year, the date was altered to M. DC. X X., and a few copies may have been printed before the alteration. The only copy which we have found is in the Bibliothèque Nationale. The volume contains the three parts; the Tabula has $M4^2$ blank; the Constructio has on its title-page the usual date of 1620, and has on $H4^1$ the Extraict the same as in the Advocates' Library copy mentioned in the preceding entry.

Library. Nat. Paris;

Arcanvm | Svppvtationis | Arithmeticæ: | Quo Doctrina & Praxis | Sinvvm ac Triangvlorvm | mirè abbreuiatur. | Opvs Cvriosis Omnibvs, | Geometris præsertim, & Astronomis | vtilissimum. | Inuentore, nobilissimo Barone Merchistonio | Scoto-Britanno. |

Lvgdvni, | Apud Ioan. Anton. Hvgvetan, | & Marc. Ant. Ravavd. | M. DC. LVIII. |

[Printed in black and red.]

This issue is evidently not a new edition, but the remainder of the edition of 1620 with the following alterations. In the Descriptio signature A has been reprinted with title-page as above, and several other less important alterations. The Tabula is unaltered, still retaining the name of Barth. Vincent on the title-page. The Constructio has the first two leaves cut out so that the first page is numbered 5. The Extraict is often wanting on $M2^2$ of the Tabula, but in the copies examined is printed on $H4^1$ of the Constructio, exactly as in the Advocates' Library copy of 1620, the name of the work in the Extraict being that on the first title-page of the 1620 edition, and not that used in the title-page given above.

Libraries. Adv. Ed.; Un. Gl.; Kön. Berlin; Un. Breslau; Un. Halle; Stadt. Zürich;

2. EDITIONS IN ENGLISH OF THE DESCRIPTIO ALONE.

A | Description | Of The Admirable | Table Of Loga- | rithmes: | With | A Declaration Of | The Most Plentifvl, Easy, | and speedy vse

vse thereof in both kindes | of Trigonometrie, as also in all | Mathematicall calculations. | Invented And Pvbli- | shed In Latin By That | Honorable L Iohn Nepair, Ba- | ron of Marchiston, and translated into | English by the late learned and | famous Mathematician | Edward Wright. | With an Addition of an Instrumentall Table | to finde the part proportionall, inuented by | the Translator, and described in the end | of the Booke by Henry Brigs | Geometry-reader at Gresham- | house in London. | All perused and approued by the Author, & pub- | lished since the death of the Translator. |

London, | Printed by Nicholas Okes. | 1616. |

12°. Size $5\frac{7}{8} \times 3\frac{3}{8}$ inches. $A1^1$ *Title.* $A1^2$ blank. $A2^1$–$A3^1$, 3 pages, "*To The Right Honovrable And Right Worshipfvll Company Of Merchants of London trading to the East-Indies, Samvel Wright wisheth all prosperitie in this life, and happinesse in the life to come.*" $A3^2$–$A4^2$, 3 pages, "*To The Most Noble And Hopefvll Prince, Charles: Onely Sonne Of the high and mightie Iames by the grace of God, King of great Brittaine, France, and Ireland: Prince of Wales: Duke of Yorke and Rothesay: Great Steward of Scotland: and Lord of the Islands.*" signed '*Iohn Nepair.*' A5, 2 pages, "*The Authors Preface to the Admirable Table of Logarithmes,*". $A6^1$–$A8^2$, 6 pages, "*The Preface To The Reader By Henry Brigges.*", signed '*H. Brigges.*' A9, 2 pages, Lines, "*In praise of the neuer-too-much praised Worke and Authour the L. of Marchiston.*", 54 lines, "*By the vnfained louer and admirer of his Art and matchlesse vertue, Iohn Dauies of Hereford.*" A10 cut out in all copies. A11, 2 pages, Lines, "*In the iust praise of this Booke, Authour, and Translator.*", 49 lines, signed "*Ri. Leuer.*" $A12^1$, "*A View Of This Booke.*" $A12^2$, "*Some faults haue escaped in printing of the Table, ...*", 58 corrections are given. $B1^1$–$C3^1$, pp. 1-29, "*A Description Of The Admirable Table Of Logarithmes, With The Most Plentifvl, Easie, And Ready Vse thereof in both kindes of Trigonometrie, as also in all Mathematicall Accounts. The First Booke.*" $C3^2$–$E9^1$, pp. 30-89, "*The Second Booke.*" $E9^2$–$I\,6^1$, 90 pages, *The Table.* $I\,6^2$, blank. After $I\,6^2$, on a folding sheet, is an engraved diagram of the "*Triangular instrumentall Table.*" $I\,7^1$–$K2^2$, pp. 1-8, "*The Vse Of The Triangular Table for the finding of the part Proportionall, penned by Henry Brigges.*", also on p. 8, "*Errata in the Treatise.*", 8 corrections.

Signatures. A to H in twelves + I in eight + K in two = 106 − 1 = 105 leaves, A10 being cut out. Leaves E10 and E11 have also been cut out, but in their place two new leaves are inserted.

Paging. 22 + 89 numbered + 91 + 8 numbered = 210 pages. Also plate following $I\,6^2$.

Errors in Paging. None.

The Table is to one place less than the Canon of 1614, but the

logarithms of the sines for each minute from 89°–90° are given in full, the last figure being marked off by a point. This is, I believe, the earliest instance of the decimal point being used in a printed book. The Admonitio at the end of the Table is wanting.

The two words "and maintaine" in the last line of the first page of Briggs' preface ($A6^1$) are ruled out in ink in all copies both in this edition and in that of 1618.

Libraries. Adv. Ed.; Un. Gl.; Brit. Mus. Lon.; Bodl. Oxf.; Qu. Col. Oxf.;

A | Description | Of The Admirable | Table Of Loga- | rithmes: | With | A Declaration of the most Plenti- | full, Easie, and Speedy vse there- | of in both kinds of Trigonome- | try, as also in all Ma- | thematicall Calcu- | lations. | Inuented and published in Latine by that | Honourable Lord Iohn Nepair, Baron of | Marchiston, and translated into Eng- | lish by the late learned and famous | Mathematician, Edward | Wright. | With an addition of the Instrumentall Table | to finde the part Proportionall, intended | by the Translator, and described in the end of the | Booke by Henrie Brigs Geometry- | reader at Gresham-house in | London. | All perused and approued by the Authour, and | published since the death of the Translator. | Whereunto is added new Rules for the | ease of the Student. |

London, | Printed for Simon Waterson. | 1618. |

This edition is really that of 1616 with the title-page cut out and the above put in its place; there being also added at the end of the work ($A3^1$-$A10^2$, pp. 1-16) "*An Appendix to the Logarithmes, shewing the practise of the Calculation of Triangles, and also a new and ready way for the exact finding out of such lines and Logarithmes as are not precisely to be found in the Canons.*"

One of the copies in the Glasgow University Library has the new title-page with blank leaf attached, inside of which is placed the sig. A of the 1616 edition with its first leaf cut out, and also the new sig. A (A3-A10) containing the Appendix.

Signatures.

Signatures. As in 1616 edition 105 leaves + A, in eight (commencing with A3 and ending with A 10), = 113 leaves.

Paging. As in 1616 edition 210 pages + 16 numbered = 226.

Libraries. Un. Ed.; Un. Gl. (2); Roy. Soc. Lon.; Bodl. Oxf.; Un. Camb.; Trin. Col. Camb.;

The Wonderful|Canon Of Logarithms|or the|First Table Of Logarithms|with a full description of their ready use and easy|application, both in plane and spherical trigono-|metry, as also in all mathematical|calculations.|Invented and published by|John Napier,|Baron of Merchiston, etc., a native of Scotland, A.D. 1614.|Re-translated from the Latin text, and enlarged with a table of|hyperbolic logarithms to all numbers from 1 to 1201.|By Herschell Filipowski.

Published for the Editor|By W. H. Lizars|3 St. James' Square, Edinburgh.|1857.|

16°. Size $5\frac{1}{8} \times 3\frac{3}{4}$ inches. $a1^1$, *Title.* $a1^2$, blank. $a2^1$, "*This edition is inscribed to William Thomas Thomson, Esq.*, ..." $a2^2$, blank. A3, pp. v and vi, *Dedication to Prince Charles*, signed "*Iohn Nepair.*" a^4, pp. vii. and viii., *The Author's preface.* $a5^1$–$a6^2$, pp. ix-xii, "*The Preface To The Reader By Henry Briggs.*" $a7^1$–$b1^2$, pp. xiii-xviii, "*Translator's Preface.*" $b2^1$, p. xix, *Notes.* $b2^2$, *Errata.* $A1^1$–$B4^2$, pp. 1-24, *Book I.* $B4^2$–$F4^1$, pp. 24-71, *Book II.* $F4^2$, "*Note to Table II. by the Translator.*" $A1^1$–$F6^2$, 92 pages, "*Table I., Napier's Logarithms of Sines.*", the title occupies the first page, the last is blank, and the table occupies the intervening 90. $F7^1$–$G8^2$, 20 pages, "*Table II., Napier's Logarithms to Numbers, called also Hyperbolic Logarithms, from* 1·01 *to* 1200.", on first page is the title, then follow the table occupying 18 pages, and on the last page is printed "END" within an ornamental device.

Signatures. [a] in 8 + b in 2, + A, B, C and E in eights + F in four + A to G in eights = 102 leaves.

Paging. xx numbered + 72 numbered + 112 = 204 pages.

The numbers and logarithms in Table I. are those of the Canon of 1614, each divided by 10,000,000, so that the logarithms are strictly to base e^{-1}. The Admonitio at the end of the Table is wanting. The logarithms in Table II. are to base e.

Libraries. Act. Ed.; Brit. Mus. Lon.; Act. Lon. (2).

APPENDIX.

IN the preparation of the foregoing Catalogue, several works by other Authors were met with which have considerable interest from their connection with the works of Napier. It seemed desirable to preserve a record of them, and they are accordingly given below, with such particulars as were noted at the time.

Napiers|Narration:|Or,|An Epitome|Of|His Booke On The|Revelation.|Wherein are divers Misteries disclosed,|touching the foure Beasts, seven Vials, seven Trumpets,|seven Thunders, and seven Angels, as also a discovery of|Antichrist: together with very probable conjectures|touching the the time of his destruction, and|the end of the World.|A Subject very seasonable for these last Times.|

Revel. 22. 12.|And behold I come shortly, and my reward is with me, to give to every man|according as his worke shall be.|

London,|Printed by R. O. and G. D. for Giles Calvert, 1641.|

4°. Size inches. $A1^1$, *Title.* $A1^2$, blank. $A2^1$–$C3^2$, 20 pages, "*Napier's Narration Or An Epitome of his Booke on the Revelation.*" C4, 2 pages, blank.

Signatures. A, B, and C in fours = 12 leaves.

Paging. 2 + 20 + 2 = 24 pages, not numbered.

Errors in Signatures. C2 numbered in error C3.

This tract is written in the form of a dialogue, wherein Rollock is made the scholar and Napier the master: see Memoirs, p. 175.

Libraries. Brit. Mus. Lon.; Bodl. Oxf.;

The

The bloody Almanack :|To which England is directed, to foreknow what shall|come to passe, by that famous Astrologer, M. John Booker.|Being a perfect Abstract of the Prophecies proved out of Scripture,|By the noble Napier, Lord of Marchistoun in Scotland.|

London|Printed for Anthony Vincent, and are to be sold in the Old-Baily. 1643.|

[With large woodcut in centre of page containing symbolical designs.]

4°. Size inches. $A1^1$, *Title.* $A1^2$, blank. $A2^1$–$A4^2$, pp. 1-6, "*The bloody Almanack,*" containing "I. Concerning the opening of the seven Seales mentioned Revel. 6."; "II. Concerning the seven Trumpets mentioned chap. 8 & 9."; "III. Concerning the seven Angels mentioned Rev. 14."; "IIII. Concerning the Symboll of the Sabboth."; "V. Concerning the Prophesie of Elias."; "VI. Concerning the Prophecie of Daniel."; "VII. Concerning Christ's owne saying."

Signature. A in 4=4 leaves.

Paging. 2+6 numbered=8 pages.

Libraries. Brit. Mus. Lon.;

ANOTHER EDITION.

The bloody Almanack:|To which England is directed . . . . . |By the noble Napier, Lord of Marchistoun in Scotland.|With Additions.|

London|Printed for Anthony Vincent, and are to be sold in the Old-Baily. 1643.|

[The printing round the woodcut is slightly altered.]

The additions are on $A1^2$ "*A Table* . . . ." and "*M. J. Booker his Verses* . . . .", also on $A4^2$ at the end an added *Note.*

Libraries. Brit. Mus. Lon.;

A|Bloody Almanack|Foretelling|many certaine predictions which shall come to|passe this present yeare 1647.|With a calculation concerning the time of the day|of Judgement, drawne out and published by that famous|Astrologer.|The Lord Napier of Marcheston.|

[With symbolical woodcut surrounded by the signs and names of the zodiac.]

4°. Size inches. A1[1], *Title.* A1[2]–A2[2], 3 pages, Astrological prediction of events "*In Jannary,*" "*In February,*" etc. A3[1]–A4[2], 4 pages, "I. Concerning the seaven Angels mentioned Rev. 14."; "II. Concerning the Symboll of the Sabbath."; "III. Concerning the Prophesy of Elias."; "IV. Concerning the Prophecie of Daniel."; "V. Concerning Christ's own saying."; these contain the same matter as in the Almanack of 1643, but the first two subjects there treated of are here omitted.

Signature. A in 4=4 leaves.

Paging. 8 pages not numbered.

Libraries. Brit. Mus. Lon.; Bodl. Oxf.;

Le|Sommaire|Des Secrets De|l'Apocalypse, svy-|uant l'ordre des Chapitres.|Le tout conforme aux passages de l'Escriture saincte, tant|de la Doctrine des Prophetes, que des Apostres.| Par le Sieur de Perrieres Varin.|

Heureux sur qui le Soleil d'intelli-|gence se leue.|

Iouxte la coppie imprimee à Rouen.|

A Paris,|Chez Abraham le Feure, ruë sainct Ger-|main de Lauxerrois.|M.C.D.X.|Auec Priuilege du Roy.|

8°. Size 6½×4 inches. A1[1], *Title.* A1[2], blank. A2, pp. 3 & 4. Dedication "*A Tres-havt Et Tres-pvissant Seignevr, Messire Guillaume de Hautemer, sieur de Feruaques, ... & Baron de Manny; . . . .*" A3[1]–H3[2], pp. 5-62, "*Les Secretz De L'Apocalypse ouuerts et mis au iour.*" H4[1] [p. 63], "*Approbation,*" dated 20 June 1609. H4[2] [p. 64], "*Extraict du Priuilege du Roy,*" dated 27 March 1610.

Signatures. A to H in fours=32 leaves.

Paging. 64 numbered (except on first two and last two)=64 pages.

From the title-page it would appear that a previous edition had been published at Rouen. The work is written to confute Napier's interpretation of the Apocalypse, and commences thus:—

> Depvis quatre ou cinq ans, a esté veu vn liure intitulé, *L'ouuerture de l'Apocalypse*, mis en lumiere par Napeyr Escossois, duquel n'ay voulu publier les erreurs, aduerty que ses partisans mesme le desauouët, cõme plein de mensonges & impostures. . . . . Croy certainement qu'en son œuure Sathan a voulu iouër sa reste; Et voyant son temps si prés, nous enuoyer par ce docteur ses harmonyes Pythonissiennes, cauteleusement douces, & à la verité pleines d'attrait, pour nous pyper.

Libraries. Un. Ed.;

Le

Le | Desabvsement, | Svr Le Brvit Qvi Covrt | de la prochaine Consommation | des Siecles, fin du Monde, & du | Iour du Iugement Vniuersel. | Contre Perrieres Varin, qui | assigne ce Iour en l'année 1666. | Et Napier Escossois, qui le met en | l'année 1688. | Par le Sieur F. De Covrcelles. |

A Rouen, | Par Lavrens Mavrry, ruë neuve | S. Lo, à l'imprimerie du Louvre. | M. DC. LXV. | Avec Permission. |

12°.

Libraries. Brit. Mus. Lon.;

Aureum | Johannis Woltheri Peinensis Saxonis: | Das ist: | Gvlden Arch, Da- | rinn der wahre Verstand vnd Einhalt der | wichtigen Geheimnussen, Wörter vnd Zahlen, in der | Offenbahrung Johannis, vnd im Propheten Daniel, reichlich | vnd überflüssig gefunden wird, Wie dann auch eine bewerthe Prob aller | Propositionen, vnd auszführliche Wiederlegung, der vermeynten lang- | gewünscheten Auszlegung über diese Offenbarung Johannis, desz Treffli- | chen Schottländischen Theologi, Herrn Johannis Napeiri, durch | die Historien vnd Geschichten der zeit erkläret | vnd angezeigt. Mehr wird auch darinn vor Augen gelegt | vnd dargestellt, wie übel vnd böszlich M. Paulus Na- | gelius mit dem Propheten Daniel vnd der Offenbahrung Johannis vmbgehe, vnd was von seiner, vnd der andern Newen | Rosencreutzbrüder Astronomia gratiæ oder Apocaly- | ptica zu halten sey. | Letzlich werden auch erörtert H. Napeiri, Wolffgangi Mäyers, Leons de Dromna, vnd anderer Calvinisten | grobe Jrrthumbe von der Rechtfertigung eines armen | Sünders, auch anderen Glaubens Articuln | Nebenst auch einem kurtzen Discurs von den Kirchen- | Ceremonien, &c. |

Psal. 94. | Recht musz doch recht bleiben, vnd dem werden alle fromme Hertzen zufallen. |

Gedruckt zu Rostock, durch Mauritz Sachsen, In vorlegung | Johan Hallervordes Buchhändlers daselbst. 1623. |

[Printed in black and red.]

4°. Size 6 × 4 inches. Black letter. A1[1], *Title-page.* A1[2], blank. A2[1]–B3[1], 11 pages, *Dedication* to Kurfürst Georg Wilhelm Markgraf von Brandenburg, signed "*Datum Liechtenhagen den* 15 *Octobris des* 1621. *Jahres . . . . . E. Churf. Durchl. Gehorsamer Vnterthan Johannes Woltherus Pfarrherr daselbst.*" On B3[2] is the title of the German translation of 'A plaine discovery,' printed at Frankfurt in 1615. B4[1]–Nn3[2], pp. 1-272, *The work itself* [see note]. On Nn4[1] is printed "*Gedruckt zu Rostock* | *durch Moritz Sachsen,* | *Im Jahr Christi* | 1623. | " Nn4[2], blank.

Signatures. A to Z and Aa to Nn in fours = 144 leaves.
Paging. 14 + 272 numbered + 2 = 288 pages.

Johann Wolther, the Pastor of Liechtenhagen, was a zealous Lutheran, and adherent of the Augsburg Confession. In his work he reprints in full the 36 propositions of Napier's First treatise as given in the Frankfurt edition of 1615, pp. 1-122, omitting the 'Beschluz,' p. 123. To each proposition is appended a refutation of the same. These refutations, being much longer than the propositions, form the bulk of the book.

Libraries. Un. Breslau;

Künstliche Rechenstäblein | zu vortheilhafftiger vnd leichter ma- | nifaltigung, Theilungwie nicht | weniger | Auszziehung der gevierdten vnd Cubi- | schen Wurtzeln, allen Rechenmeistern, In- | genieuren, Bawmeistern, vnd Land- | messern, vber die masz dienlich. | Erstlich 1617. In lateinischer Sprach durch Herrn Johan Nepern, Freyherrn in Schottlandt, | beschrieben, nacher ausz anleytung, desz hochgelehrten | weitberühmbten Herrn v. Bayrn durch Frantz Keszlern zu Werck gericht. In Kurtz verfast, vnd zum Truck gefertigt. |

Gedruckt zu Straszburg bey Niclaus | Myriot, In verlegung Jacob von der Heyden Chal- | cographum | Anno MDCXVIII. |

4°. Size inches. Black letter.

Libraries. Stadt. Frankfurt; K. Hof u. Staats München;

Rhabdologia | Neperiana. | Das ist, | Newe, vnd sehr leichte | art durch etliche Stäbichen allerhand Zah- | len ohne mühe, vnd hergegen

hergegen gar gewisz, zu Multiplici- | ren vnd zu dividiren, auch die Regulam Detri, vnd beyderley ins | gemein vbliche Radices zu extrahirn: ohne allen brauch | des sonsten vb-vnnd nützlichen | Ein mahl Eins, | Alsz in dem man sich leichtlich | verstossen kan, | Erstlich erfunden durch einen vornehmen Schottlän- | dischen Freyherrn Herrn Johannem Neperum | Herrn zu Merchiston &c. | Anjtzo aber auffs kürtzeste, alsz jmmer müglich gewesen, | nach vorhergehenden gnugsamen Probstücken | ins Deutsche vbergesetzt, | Durch | M. Benjaminem Ursinum, Churf. Bran- | denburgischen Mathematicum. | Cum Gratia Et Privilegio. |

Gedruckt zum Berlin im Grawen Kloster, durch George Rungen, | Im Jahre Christi 1623. |

4°. Size 6¼ × 4½ inches. $A1^1$, *Title.* $A1^2$, blank. $A2^1$–$A3^1$, 3 pages, "*Vorrede an den guthertzigen Leser.*" $A3^2$–$C4^1$, 18 pages, "*Von der Stäbelrechnung,*" containing Cap. I. "*Von beschreibung vnd gebrauch der Stäblichen ins gemein.*"; Cap. II. "*Wie das Multipliciren mit hülff der Stäbichen verrichtet werde.*"; Cap III. "*Wie das Dividiren anzustellen sey.*"; Cap IV. "*Von erfindung einer jeden Zahlen quadrat Wurtzel.*"; Cap. V. "*Wie man mit hülffe des Blätichen Pro Cubica, vnd der Stäbichen einer jedern Zahl radicem cubicam erfinden solle.*" $C4^2$, "*Der Leser wisse, wo er der mühe die Stäbichen auffzutragen, wil vberhaben sein: das solche zierlich in einem subtilen Kästichen, aller notturff nach zugerichtet zubekommen sein. Vnd zufinden bey Martin Guthen, Buchhändlern zu Cölln an der Spree.*"

Signatures. A to C in fours = 12 leaves.
Paging. 24 pages not numbered.
Facing $A3^2$ on a folding sheet is a diagram of the rods.
Libraries. Brit. Mus. Lon.; Kön. Berlin; Stadt. Breslau; Nat. Paris;

ANOTHER EDITION.

Gedruckt im Jahr Christi, | Anno 1630. |

Libraries. Stadt. Zurich;

Manvale | Arithmeticæ & Geometriæ Practicæ: | In het welcke | Beneffens de Stock-rekeninghe ofte | Rhabdologia J. Napperi cortelick eñ duydelic t' ge- | ne den Landmeters eñ Ingenieurs, nopende 't Land- | meten en Sterckten-bouwen nootwendich is,

wort | geleert ende exemplaerlick aenghewesen. | Op een nieu verrijckt met een nieuwe inventie on alle ronde va- | ten hare wannigheden af te pegelen. | Door | Adrianum Metium. Med. D. & Ma- | thes. Profess. ordinar. binnen Franeker. |

Tot Amsterdam, | By Henderick Laurentsz, Boeckvercooper op 't | Water, int Schryfboeck, Anno 1634. |

8°. Size inches.
Paging. 16+246 numbered+8.

ANOTHER EDITION.

Gedruckt by Ulderick Balck, Ordi- | naris Landschaps ende Academiæ Boecke- | Drucker. Anno 1646. |

Paging. 8+377 numbered+11.

These two editions are catalogued by D. Bierens de Hann in his papers entitled 'Bouwstoffen voor de Geschiedenis der Wis- en Natuurkundige Wetenschappen in de Nederlanden,' communicated to the Amsterdam Academy—Verslag. xii., 1878 (Natuurk.), p. 19.

The Art of | Numbring | By | Speaking-Rods: | Vulgarly termed | Nepeir's Bones. | By which | The most difficult Parts of | Arithmetick, | As Multiplication, Division, and Ex- | tracting of Roots both Square | and Cube, | Are performed with incredible Cele- | rity and Exactness (without any | charge to the Memory) by Addi- | tion and Substraction only. || Published by W. L. ||

London; | Printed for G. Sawbridge, and are to be sold | at his House on Clerkenwell-Green, 1667. |

12°. Size $4\frac{3}{4} \times 2\frac{3}{4}$ inches. A1, blank? A2[1], *Title-page.* A2[2], blank. A3[1]–A6[1], 7 pages, "*The Argvment To The Reader.*" A6[2], blank. After A6[2] on a folding sheet is a diagram of the rods. B1[1]–E7[2], pp. 1-86, *The Work.* E8[1], "*Errata.*" E8[2]–E9[2], 3 pages, Advertisements. E10[1]–E12[2], 6 pages, blank?

Signatures. A in six+B to E in twelves=54 leaves.
Paging. 12+86 numbered+10=108 pages.

The author was William Leybourn, and the work contains a short description

description of the rods, with examples of their use in multiplication, division, and the extraction of square and cube roots.

Libraries. Un. Ed.; Brit. Mus. Lon. (2); Lambeth Pal. Lon.;

ANOTHER EDITION.

London, printed by T. B. for H. Sawbridge, at the | Bible on Ludgate-Hill. 1685. |

Libraries. Un. Ab.; Brit. Mus. Lon.;

Nepper's Rechenstäbchen, als Hülfsmittel bei d. Multiplication u. Division d. Zahlen- u. Decimalbrüche; hrsgg. v. F. A. Netto. Mit 100 Rechenstäbchen, Dresd. 1815. Arnold. 18*g*.

This entry is copied from C. G. Kayser's Vollständiges Bücher-Lexicon (1750-1832), published at Leipsic in 1835. The work apparently treats of 'Napier's Bones.'

Traité De La | Trigonometrie, | Povr Resovdre Tovs | Triangles Rectilignes | Et Spheriqves. | Avec Les Demonstrations Des | deux celebres Propositions du Baron de Merchiston, | non encores demonstrees. | Dediée | A Messire Robert Kar, Comte | d'Ancrame, Gentil-homme de la Chambre | du Roy de la Grand' Bretagne. |

A Paris, | Chez Nicolas et Iean de la Coste, au | mont S. Hilaire, à l'Escu de Bretagne, & en leur | boutique à la petite porte du Palais | deuant les Augustins. | M. DC. XXXVI. | Avec Privilege Du Roy. |

8°. Size $6\frac{1}{8} \times 3\frac{7}{8}$ inches. $\tilde{a}1^1$,–$\tilde{i}4^2$, 20 pages, *Preliminary matter.* $a1^1$–$p2^2$, pp. 1-116, "*Des Triangles Rectilignes.*" $A1^1$–$Y4^1$, pp. 1-193 [175] "*Des Triangles Spheriqves.*" $Y4^2$, woodcut.

In the first part, on a folio sheet facing p. 68, is a table of 'Racines de 10' and of their logarithms.

Paging. 20+116 numbered+(193–18 for error=) 175 numbered+1=312 pages.

Signatures. ã in 4, ẽ in 2 & ĩ in 4+a to o in 4 & p in 2+A to Y in 4=156 leaves.

Errors in Paging. In first part none of consequence. In second part 168 numbered 186, and so to the end, thus making an error in excess of 18.

Permission to print the work was given on 5th April 1635. The Dedication is signed by the Author 'IACOBVS HVMIVS, Theagrius Scotus.' On the last page (p. 116) of the first part will be found the passage relating to Napier's burial-place, &c., part of which is quoted at p. 426 of the Memoirs. The two celebrated propositions by Napier are Nos. 117 & 120 of the second part.

Libraries. Adv. Ed.; Un. Ed.; Roy. Soc. Lon.;

Primvs Liber | Tabvlarvm Directionvm. | Discentivm Prima Elemen-|ta Astronomiæ necessarius &|utilissimus.|His Insertvs Est Canon|fecundus ad singula scrupula qua-|drantis propagatus.| Item Nova Tabvla Clima-|tum & Parallelorum, item umbrarum.| Appendix Canonvm Secvndi | Libri Directionum, qui in Regiomontani | opere desiderantur. | Avtore Erasmo | Rheinholdo Salueldensi | Cum gratia & priuilegio Cæsareæ & | Regiæ Maiestatis. |

Tvbingæ Apvd Hære | des Vlrici Morhardi. Anno | M.D. LIIII. |

[Printed in black and red.]

4°. Size 8 × 5¾ inches.

In describing the formation of the Logarithmic Table, in section 59 of the Constructio, Napier says that Reinhold's common table of sines (or any other more exact) will supply the values for filling in the natural sines in columns 2 and 6, and the table of sines in this work ("*Canon Sinvvm Vel Semissivm Rectarvm In Circvlo Svbtensarvm.*", fol 114), was probably the one he made use of.

Libraries. Trin. Col. Dub.;

Benjaminis Ursini | Sprottavi Silesi | In Electorali Brandenburgico Gymnasio | Vallis Joachimicæ, | Cursus | Mathematici | Practici | Volumen Primum | continens | Illustr. & Generosi DN. | DN.

DN. Johannis Neperi | Baronis Merchistonij &c. | Scoti. | Trigonometriam Loga- | rithmicam | Usibus discentium accomoda- | tam. | Cum Gratia Et Privilegio. |

Typisq. exscriptam | Coloniæ sumtibus Martini Guthij, | Anno CIↃ IↃC XVIII. |

[Note. Colonia = Köln a.d. Spree = Berlin.]

8°. Size 4⅞ × 3 inches. $A1^1$, *Title.* $A1^2$–$A3^2$, 5 pages, Dedication to "*Illustri et generoso domino, Dn. Abrahamo lib. Baroni et Burggravio de Dohna. . . . . .*" signed "*in Valle nostra Ioachimica XVI. Kal. Jun. anni seculi hujus XVII. T. Illustr. Generos. humilimè addictus Cliens Benjamin Ursinus.*" $A4^1$–$C7^2$, 40 pages, "*Trigonometriæ Logarithmicæ J. Neperi, &c.*" C8, 2 pages, blank. $Aa1^1$, The title, "*Tabula Propor-* | *tionalis* | *Sequenti* | *Canoni* | *Logarithmo-* | *rum Inser-* | *viens.* |" $Aa1^2$–$Aa5^1$, 8 pages, *The Table.* $Aa5^2$–$Aa7^2$, 5 pages, "*Usus præcedentis tabulæ.*" Aa8, 2 pages, blank. $Bb1^1$, The title, "*J. Neperi* | *Baronis Mer-* | *chistonii, Sco-* | *ti, &c.* | *Mirificus* | *Canon Logarith-* | *morum.* |" $Bb1^2$–$Gg6^1$, 90 pages, *The Canon* to two places less than that of 1614. $Gg6^2$–$Hh2^2$, 9 pages, "*Lectori Benevolo,*" [errata.] The work should have ended on $Hh1^2$, but through an error in printing, the two pages, $Hh1^2$ and $Hh2^1$, have been left blank.

Signatures. A to C and Aa to Gg in eights + Hh in two = 82 leaves.

Paging. 48 + 16 + 91 + 9 = 164 pages not numbered.

Napier's Canon of 1614 is here reprinted, but is shortened two places.

Libraries. Brit. Mus. Lon.; Stadt. Breslau;

ANOTHER EDITION.

Coloniæ, Martinus Guthius, 1619.

Libraries. Bodl. Oxf.; Un. Camb.; Nat. Paris;

THE FIRST EDITION

is stated to have been published in 1617, which is no doubt correct, as the Dedication is dated 17th May 1617.

Beni. Ursini | Mathematici Electora- | lis Brandenburgici | Trigonometria | cum magno | Logarithmor. | Canone | Cum Privilegio |

 Coloniæ

Coloniæ | Sumptib. M. Guttij, tipys | G. Rungij descripta. | CIↃ IↃC XXV.|

[The above is engraved on a half-open door, forming the centre of a title-page elaborately engraved by Petrus Rollos.]

4°. Size $7\frac{5}{8} \times 5\frac{3}{4}$ inches.):(1^1, *Title.*):(1^2, blank.):(2^1–):(4^2, 6 pages, *Dedication* to Dn. Georgio Wilhelmo Marchioni Brandenburgico, dated 1624. $A1^1$–$Ll4^2$, pp. 1–272. *Trigonometria*, in three books. Liber I., "*De Triangulis, eorumq. affectionibus.*"; Sectio Prior, "*De Triangulis Planis.*"; Sectio Posterior, "*De Triangulis Sphæricis.*". Liber II., "*De Constructione Canonis Triangulorum; ejusq. usu in genere.*"; Sectio I., "*De Constructione Canonis Sinuum.*"; Sectio II., "*De Constructione Tabulæ Logarithmorum.*"; Sectio III., "*De usu Canonis Logarithmorum in genere.*". Liber III., "*De Usu Canonis Logarithmorum in utraq. Trigonometriâ.*"; Sectio I., "*De Mensuratione Triangulorum Planorum sive Rectilineorum.*"; Sectio II., "*De Trigonometriâ Sphæricorum.*"

Signatures.):(and A to Z and Aa to Ll in fours = 140 leaves.

Paging. 8+272 numbered = 280 pages.

Benjaminis Ursini | Sprottavi Silesi | Mathematici Electoralis Brandenburgici | Magnvs Canon | Triangulorum | Logarithmicvs; | Ex Voto & Consilio | Illustr. Neperi, p. m. | novissimo, | Et Sinu toto 100000000. ad scrupulor. | secundor. decadas | usq'. | Vigili studio & pertinaci industriâ | diductus. |

Keppler. Harmonic. Lib. IV. cap. VII. p. 168.—[followed by extract of 8 lines.]

Coloniæ, | Typis Georgij Rungij, impensis & sumtibus Martini Guttij | Bibliopolæ, Anno M. DC. XXIV.|

$A1^1$, *Title.* $A1^2$, blank. $A2^1$–$Lll2^2$, The *Table* occupying 450 pages. $Lll3^1$, "*Emendanda in Canone,*" 35 lines. On $Lll3^2$ is printed "*Berolini,* | *Excudebat Georgius Rungius Typographus,* | *impensis &ª sumtibus Martini Guttij* | *Bibliopolæ Coloniensis.* | Anno cIↃ IcↃ XXIV. |". $Lll4$, blank.

Signatures. A to Z and Aa to Zz and Aaa to Lll in fours = 228 leaves.

Paging. 2+450+4=456 pages not numbered.

Colonia, the place of publication, is Köln a. d. Spree or Berlin.

The second and third books of the Trigonometria deal with the subjects treated of in Napier's Descriptio and Constructio, these works being largely made use of by Ursinus, who speaks of Napier as a Mathematician without equal (see p. 131, l. 5). The references in the text are to the Lyons edition of 1620 (see p. 178).

The Magnus Canon contains the logarithms of sines for every 10″ in the quadrant. They are arranged in a similar way, and are of the same kind as those in Napier's Canon of 1614, but are carried one place further,

Grad. 30. + | —

M.S.	Sinus.	D.	Logarith.	D.	Different.	D.	Logarith.	D.	Sinus.	D.	
0 0	50000000		69314718		54930614		14384104		86602540		60
10	04199	99	06321	97	19417	97	86904	00	00116	24	50
20	08397	98	69297925	96	08222	95	89703	99	86597692		40
				95				2800		2425	
30	12595		89530		54897027		92503		95267		30
40	16794	99	81136	94	85833	94	95303		92842		20
50	20992	98	72743	93	74640	93	98103		90417		10
				92				01			
1 0	25190		64351		63447		14400904		87992		0 59
10	29387	97	55960	91	52255	92	03705		85567		50
20	33585	98	47570	90	41064	91	06506		83141	26	40
								02		25	
30	37783		39181	8389	29873		09308		80716	26	30
40	41980	97	30792		18682		12110		78290	27	20
50	46178	98	22405	87	07493	1118	14912		75863		10
		4197		86				03		26	
2 0	50375		14019		54796304	9	17715		73437	27	0 58
10	54572		05633		85115		20518		71010		50
20	58769		69197249	84	73928	87	23321		68583		40
				83				04			
30	62966		88866		62741		26125		66156		30
40	67163		80483		51554	86	28929		63729		20
50	71359	96	72101	82	40368		31733		61302		10
		97		81				05		28	
3 0	75556		63720		29182		34538		58874		0 57
10	79752	96	55340	80	17997	85	37343		56446		50
20	83949	97	46961	79	06813	84	40148		54018		40
		96		78		83					
30	88145		38583		54695630		42953	06	51590		30
40	92341		30206	77	84447		45759		49162		20
50	96537		21830	76	73265	82	48565		46733	29	10
				75				07			
4 0	50100733		13455		62083		51372		44304		0 56
10	04928	95	05081	74	50902	81	54179		41875		50
20	09124	96	69096708	73	39722	80	56986		39446		40
				72				08		30	
30	13320		88336		28542		59794	07	37016	29	30
40	17515	95	79964		17363	79	62601	08	34587	30	20
50	21710		71593	71	06184		65409		32157		10
				69		77					
5 0	25905		63224		54595007	78	68217	09	29727	31	0 55
10	30100		54855		83829		71026		27296	30	50
20	34295		46487	68	72652	77	73835		24866		40
				66		76		10		31	
30	38490		38121		61476		76645	09	22435		30
40	42685		29755		50301	75	79454	10	20004		20
50	46879	94	21390	65	39126		82264		17573		10
		95		64		74					
60	51074		13026		27952		85074		15142		0 54

Grad. 59. | S.M.

further, radius being made 100,000,000. The entire Canon was recomputed by Ursinus, and full details of its construction are given in Book II., sect. 2, of the Trigonometria. The methods employed are the same as those laid down in the Constructio with the modifications in regard to the preliminary tables proposed by Napier in sect. 60. A specimen page of the Table is given on the preceding page, and reference may also be made to my notes, pp. 94, 95.

Libraries. Un. Ed.; Bodl. Oxf.; Brit. Mus. Lon.; Stadt. Breslau; Nat. Paris;

Johann Carl Schulze | wirklichen Mitgliedes der Königl. Preussischen Academie der | Wissenschaften | Neue Und Erweiterte | Sammlung | Logarithmischer, | Trigonometrischer | und anderer | Zum Gebrauch Der Mathematik | Unentbehrlicher | Tafeln. || II. Band. ||

Berlin, 1778. | Bey August Mylius, Buchhändler | In Der Brüderstrasse. |

Size $8\frac{3}{8} \times 5$ inches.

In this work the logarithms of the Magnus Canon of Ursinus are reprinted to every 10 seconds in the case of the first four and last four degrees, being the same as in the original. The logarithms from 4° to 86° are given for every minute only. Ursinus' logarithms occupy half the lower portion of pp. 2-261 in Volume II., the title of the whole contents of these pages being :—

"*Tafel* | *der* | *Sinus, Tangenten,* | *Secanten* | *und* | *deren zustimmenden briggischen und hyperboli-* | *schen Logarithmen* | *für die vier ersten und vier letzten Grade von* 10 *zu* | 10 *Secunden* ; | *für den übrigen Theil des Quadranten aber von Minute zu* | *Minute, nebst dem 6ten Theile der Differenzen* | *berechnet.* | "

Joannis Kepleri | Imp. Cæs. Ferdinandi II. | Mathematici | Chilias | Logarithmorum | Ad Totidem Numeros | Rotundos, | Præmissâ | Demonstratione Legitima | Ortus Logarithmorum eorumq. usus | Quibus | Nova Traditur Arithmetica, Seu | Compendium, quo post numerorum notitiam | nullum nec admirabilius, nec

nec utilius solvendi pleraq. Problemata | Calculatoria, præsertim in Doctrina Triangulorum, citra | Multiplicationis, Divisionis Radicumq'. extractio- | nis, in Numeris prolixis, labores mole- | stissimos. | Ad | Illustriss. Principem & Dominum, | Dn. Philippvm | Landgravium Hassiæ, &c. | Cum Privilegio Authoris Cæsareo. |

Marpurgi, | Excusa Typis Casparis Chemlini. | cIↃ IↃc xxiv. |

4°. Size 8 × 6⅛ inches. $A1^1$, *Title.* $A1^2$, blank. Folding sheet with "* * * *Ad Postul* 2. *Exemplvm Sectionis*, · · · ·". $A2^1$, p. 3, *Dedication* by Kepler to Philip Landgrave of Hesse. $A2^2$–$F3^1$, pp. 4–45, "*Demonstratio Strvctvræ Logarithmorvm.*" in 30 propositions. $F3^2$–$G4^1$, pp. 46–55, "*Methodvs Compendiosissima construendi Chiliada Logarithmorum.*"

On $G4^2$ is the title "*Chilias* | *Logarithmorum* | *Joh. Kepleri, Mathem.* | *Cæsarei.* |" $H1^1$–$O2^2$, 52 pages occupied by the table, and at the foot of the last page "Errata," 10 lines.

Signatures. A to N in fours + O in two = 54 leaves.

Paging. 55 numbered + 1 + 52 = 108 pages, also folding sheet.

Signature O is distinctly in two, the work ending with p. 108, but the Supplementum assumes it to end with p. 112, which it would have done had sig. O been in four.

Joannis Kepleri, | Imp. Cæs. Ferdinandi II. | Mathematici, | Supplementum | Chiliadis | Logarithmorum, | Continens | Præcepta De Eorum Usu, | Ad | Illustriss. Principem et Dominum, | Dn. Philippum Land- | gravium Hassiæ, &c. |

Marpvrgi, | Ex officina Typographica Casparis Chemlini. | cIↃ IↃcXXV. |

$P1^1$, p.[113], *Title.* $P1^2$, blank. $P2^1$–$P3^2$, pp. 113 [115]–116 [118], 4 pages, "*Joannis Kepleri Supplementum Chiliadis Logarithmorum, Continens Præcepta De Eorum Usu. Lectori S.*" $P4^1$, p.[119] "*Correctio Figurarum post punctum in Logarithmis.*" $P4^2$, p.[120] "*Præterea in textu Demonstrationum jam impresso, notaviista, nondum à Typographo animadversa.*" 8 lines of corrections. $Q1^1$–$Dd4^2$, pp. 121–216. *The work* in 9 chapters. The pages are all headed "Joannis Kepleri Chiliadis Complement," not Supplement.

Signatures. P to Z and Aa to Dd in fours = 52 leaves.

Paging. P. [113] to p. 216 = 104 pages.

Errors in Paging. Pages 115 to 118 containing the Preface are numbered in error 113 to 116.

The first part of the work contains Kepler's demonstration of the structure of logarithms, which is in form geometrical, some of the Ger-

ARCUS *Circuli cum differentiis.*	SINUS *seu Numeri absoluti.*	*Partes vicesima quarta.*	LOGARITHMI *cum differentiis.*	*Partes sexagenaria.*
3. 55			206. 40	
29. 0. 45	48500. 00	II. 38. 24	72360.64	29. 6
3. 56			205. 97	
29. 4. 41	48600. 00	II. 39. 50	72154.67	29. 10
3. 56			205. 55	
29. 8. 37	48700. 00	II. 41. 17	71949.12	29. 13
3. 56			205. 13	
29. 12. 33	48800. 00	II. 42. 43	71743.99	29. 17
3. 57			204. 71	
29. 16. 30	48900. 00	II. 44. 10	71539.28	29. 20
3. 56			204. 29	
29. 20. 26	49000. 00	II. 45. 36	71334.99	29. 24
3. 57			203. 87	
29. 24. 23	49100. 00	II. 47. 2	71131.12—	29. 28
3. 57			203. 46	
29. 28. 20	49200. 00	II. 48. 29	70927.66	29. 31
3. 57			203. 05	
29. 32. 17	49300. 00	II. 49. 55	70724.61+	29. 35
3. 58			202. 63	
29. 36. 15	49400. 00	II. 51. 22	70521.98	29. 38
3. 57			202. 23	
29. 40. 12	49500. 00	II. 52. 48	70319.75+	29. 42
3. 57			201. 81	
29. 44. 9	49600. 00	II. 54. 14	70117.94—	29. 46
3. 58			201. 41	
29. 48. 7	49700. 00	II. 55. 41	69916.53—	29. 49
3. 57			201. 01	
29. 52. 4	49800. 00	II. 57. 7	69715.52	29. 53
3. 58			200. 60	
29. 56. 2	49900. 00	II. 58. 34	69514.92	29. 56
3. 58			200. 20	
30. 0. 0	50000. 00	12. 0. 0	69314.72	30. 0
3. 58			199. 80	
30. 3. 58	50100. 00	12. 1. 26	69114.92	30. 4
3. 59			199. 40	
30. 7. 57	50200. 00	12. 2. 53	68915.52—	30. 7
3. 58			199. 01	
30. 11. 55	50300. 00	12. 4. 19	68716.51+	30. 11
3. 59			198. 61	
30. 15. 54	50400. 00	12. 5. 46	68517.90+	30. 14
3. 59			198. 21	

man mathematicians, as he mentions in his Preface, not being satisfied with Napier's demonstration based on Arithmetical and Geometrical motion. The two parts together with the Table are reprinted in 'Scriptores Logarithmici,' vol. I. p. 1. At the beginning of the same volume is reprinted the Introduction to Hutton's Mathematical Tables, on p. liii of which will be found a "brief translation of both parts, omitting only the demonstrations of the propositions, and some rather long illustrations of them."

The logarithms in the Table are of the same kind as Napier's, but they are not affected by the mistake in the computation of the Canon of 1614.

The Tables of Kepler and Napier are differently arranged, and the numbers for which the logarithms are given are also different. In Napier's Canon the numbers in column "Sinus" are the values of sines of equidifferent arcs, while in this table the numbers or sines are equidifferent. For specimen page of the Table see preceding page. The arrangement is as follows:—

Column 2 contains 1000 equidifferent numbers, 10,000, 20,000, 30,000, . . . 9,980,000, 9,990,000, 10,000,000. It also has at the beginning the 36 numbers 1, 2, 3, to 9; 10, 20, 30 to 90; 100, 200 to 900; and 1000, 2000 to 9000.

Column 4 contains the logarithms of the numbers in column 2, with interscript differences.

The 2nd and 4th are the only columns containing entries for the first 36 numbers.

It will be observed that a point marks off the last two figures of the values in these two columns, but if it be left out of account the numbers and logarithms agree with those of the Canon of 1614, in being referred to a radius of 10,000,000. So that the values really represented are the ratios of the numbers there given to 10,000,000.

Taking as an example the first entry in the specimen page, the number in column 2 which is 4,850,000 represents the ratio 4,850,000 to 10,000,000 or a $\frac{4850000}{10000000}$th = a $\frac{485}{1000}$th part of radius. Similarly column 1 gives the arc, in degrees, minutes, and seconds, corresponding to a sine equal to the $\frac{485}{1000}$th part of the radius, with interscript differences;

Column 3 gives in hours, minutes, and seconds the $\frac{485}{1000}$th part of a day of 24 hours; and finally

Column 5 gives in minutes and seconds the $\frac{4\,8\,5}{1\,0\,0\,0}$th part of a degree of 60 minutes.

Libraries. Sig. Ed.; Un. Gl.; Hunt. Mus. Gl.; Bodl. Oxf.; Trin. Col. Dub.;

Tabvlæ Rudolphinæ. . . . Ioannes Keplerus. . . . Ulmæ. Jonæ Saurii. Anno M.DC.XXVII.

Folio. Size 13¾ × 9 inches.

The logarithms used in this work are those of Napier.

Libraries. Adv. Edin.; etc.

LOGARITHMORVM | CHILIAS PRIMA. |

Quam autor typis excudendam curauit, non eo con- | cilio, vt publici iuris fieret; sed partim, vt quorun- | dam suorum necessariorum desiderio priuatim satis- | faceret partim, vt eius adiumento, non solum Chilia- | das aliquot insequentes; sed etiam integrum Loga- | rithmorum Canonem, omnium Triangulorum cal- | culo inseruientem commodius absolueret. Habet e- | nim Canonem Sinuum, à seipso, ante Decennium, per | æquationes Algebraicas, & differentias, ipsis Sinu- | bus proportionales, pro singulis Gradibus & graduũ | centesimis, à primis fundamentis accurate extructũ: | quem vna cum Logarithmis adjvnctis, volente Deo, | in lucem sedaturum sperat, quam primum commode | licuerit. |

Quod autem hi Logarithmi, diversi sint ab ijs, | quos Clarissimus inuentor, memoriæ semper colendæ, | in suo edidit Canone Mirifico; sperandum, eius librũ | posthumum, abunde nobis propediem satisfactu- | rum. Qui autori (cum eum domi suæ, Edinburgi, | bis inuiseret, & apud eum humanissime exceptus, | per aliquot septimanas libentissime mansisset; eique | horum partem præcipuam quam tum absoluerat | ostendisset) suadere non destitit,

titit, vt hunc in | se laborem susciperet. Cui ille non | inuitus morem gessit. |

In tenui; sed non tenuis, structusve laborve.

8°. 16 pages.

The above short Preface occupies the first page of a small tract of sixteen pages, the remaining fifteen containing the natural numbers from 1 to 1000 with their logarithms, to base 10, to 14 places. The tract bears no author's name or place or date of publication, but the evidence which assigns it to Briggs, and fixes the place and date of its publication as, London, 1617, seems conclusive. The Table of Logarithms is the first published to a base different from that employed by Napier.

It is unnecessary here to refer to subsequent works on Logarithms of a different kind from those originally published by Napier.

Libraries. Brit. Mus. Lon.;

Note.—In the foregoing Catalogue the only collections of Napier's works referred to are in public libraries. The largest single collection, however, is that in possession of Lord Napier and Ettrick. Besides the editions more commonly met with, it embraces several not found in any of the public libraries of this country, as well as a copy of the rare '*Ephemeris Motuum Cœlestium ad annum* 1620,' which contains Kepler's letter of dedication to Napier, dated 27th July 1619.

SUMMARY OF CATALOGUE.

A Plaine Discovery.

In English.

In Dutch.

Translation by "MICHIEL PANNEEL, *Dienaer des Godelijcken vworts tot Middelborch.*"

In French.

Translation by GEORGES THOMSON.

In

In German.

Translation of the First Treatise only by LEO DE DROMNA.

Translation of the First and Second Treatises by WOLFFGANG MAYER.

De Arte Logistica.

In Latin.

Rabdologiæ.

In Latin.

In Italian.

Translation by "IL CAVALIER MARCO LOCATELLO."

In Dutch.

Translation by ADRIAN VLACK.

Works on Logarithms.

In Latin.

In English.

The Descriptio translated by EDWARD WRIGHT.

Retranslated by HERSCHELL FILIPOWSKI.

SUMMARY OF APPENDIX.

Traité