WEAD

Contributions to the
History of Musical Scales

SMITHSONIAN INSTITUTION.

UNITED STATES NATIONAL MUSEUM.

CONTRIBUTIONS TO THE HISTORY OF MUSICAL SCALES.

BY

CHARLES KASSON WEAD,

Examiner, U. S. Patent Office.

From the Report of the United States National Museum for 1900, pages 417-462, with ten plates.

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I. INTRODUCTION.

In the development of musical scales four stages may be recognized:

1. The stage of primitive music, where there is no more indication of a scale than in the sounds of birds, animals, or of nature. Students of the origin of music may give free rein to their fancy in this period, and the uncertain musical utterances of living primitive peoples may be construed in accordance with almost any prepossession of the hearer.

2. The stage of instruments mechanically capable of furnishing a scale. This stage has been almost entirely overlooked by students and is the special subject of the following paper.

3. The stage of theoretical melodic scales—Greek, Arab, Chinese, Hindu, Mediæval, etc. All the original treatises concerning these scales imply that a stage of development has been reached far in advance of the second. Thousands of pages have been written on this stage, largely polemical and lacking in insight, for the subject has been a dark one; but Ellis and Hipkins's work of 1885 has thrown a flood of light on it.

4. The stage of the modern harmonic scale and its descendent, the equally tempered scale, which are alike dependent both on a theory and on the possibility of embodying it in instruments. The relation of this scale to the present study will be noticed later.

These four stages, of course, overlap even in the same locality; they correspond in a rough way to the recognized four culture stages, namely: the savage, barbarous, civilized, and enlightened.

At the outset it should be recognized that the only working hypothesis the physicist can use is that of the instrumental origin of scales. Helmholtz's view that the harmonics in the voice and in the tones of instruments were influential in settling the positions of the notes of our scale is obviously consistent with this hypothesis; and his opinion that this influence acted on other scales need not be wholly rejected, though some of his historical authorities were untrustworthy, and some of his coincidences between other scales and the harmonic scale can be explained in other and simpler ways. Writers less careful than Helmholtz have made the assumption that these harmonics and the constitution of the ear must have guided primitive musicians to a substantially harmonic scale; and one writer has even maintained that instruments corrupted the taste of men. But as yet there has been no such body of facts collected in support of this assumption as need delay one following out the other theory. Of course the knowledge of the scales is only a stepping-stone to the understanding of the music and something of the life of a people; so some day the materials worked into shape by the physicist may be built into a fairer structure by the psychologist.

The broad fact which underlies all study of scales was recognized by the Greek musician Aristoxenus three centuries before the Christian era. He pointed out that the voice, in speaking, changes its pitch by insensible gradations, while in singing it moves mostly by leaps. We recognize the same fact when we say that a singer follows a *scale*, but do not say it of a speaker. The one, to use the common figure, ascends or descends a ladder or staircase; the other follows a continuous slope, and may never step twice in the same place. Now, it is quite possible that in a song the voice may always move by leaps, and in repeating the song always take the same leaps as closely as can be observed, yet never strike a note which it has struck before; just as one may toss a stone up and down on a hillside, marking each time where it lands, and after a hundred tosses finds it had not landed twice at quite the same level, or in striding up and down hill may never plant his foot twice at the same level. I think this was the character of the songs of the first stage and of much primitive song to-day, though the evidence is too scanty to be conclusive.

However this may be, it is certain that most peoples who have attained any moderate degree of civilization have attempted to limit the number of steps to be taken by the voice in any song between the highest and lowest note, and to fix these steps by rules, so that many men may learn them and be in substantial agreement. Various old writers give the rules in vogue among Greek theorists; in the last century Amiot described the Chinese rules, while in the last two decades the rules of Arab, Hindu, Japanese, and Siamese musicians have been made accessible. The most familiar rules, as is well known, depend on that law of vibrating strings which is followed by a violinist in his fingering—namely, that the frequency of vibration of parts of any stretched string is inversely as the length of the parts, provided the tension does not change. Our latest rule, historically derived from one of the many Greek and Arab rules by subdividing the whole tones, so giving twelve steps to the octave, is embodied on the neck of a guitar or mandolin; here it is obvious that the successive stopping points as marked by frets get closer and closer together as the pitch rises. All musicians know that this number of notes, twelve, is found confusingly great for ordinary playing, and know the principles by which the player selects certain notes for any tune. But this multiplicity of notes has an important bearing on all studies on nonharmonic music made by harmonic musicians. For every sound within the compass of the instrument comes very near to some one of the twelve notes and may readily be represented thereby, owing to the difficulty the hearer has in estimating deviations from the familiar series and in noting them down. The results of this approximation are to mask all deviations from the twelve-tone piano scale, whether intentionally or accidentally made, and to make it appear to musicians, first, that nearly all the music of the world is performed substantially in our scale; and second, that any other theoretical scales, such as those found among Orientals, or described by our European ancestors, are merely mathematical jugglery and of as little significance as proposals for a change that occasionally appear in modern musical or scientific journals.

It is the purpose of this paper first to describe several types and forms of instruments widely used, each embodying a principle of scale building distinctly unlike ours, though sometimes giving a result that seems surprisingly familiar. Nearly all these instruments, it will be noted, belong to what was called above the second or barbarous stage, though a few of them come from countries where musicians have reached the third and fourth stages. A second purpose is to present a new and generic principle of primitive scale-building applicable to the various types of instruments discussed.

But before going further it must be recognized that the word "scale" has many meanings. Perhaps the lowest and loosest is—the series of sounds used in any musical performance, arranged in order of pitch. The one that will most closely fit the present needs is—the series of sounds produced upon a particular instrument; while the most exact definition, but one applicable only where musical principles are well developed is this:

A scale is an independently reproducible series of sounds arranged in order of pitch, recognized as a standard and fitted for musical purposes.

While the last two definitions imply an instrument in which the scales are embodied, the limitation is in appearance only, for there is no evidence that any musicians do have a standard series of tones, unless they have one or more instruments embodying it, and have learned the series directly or indirectly from such an instrument.

II. STRINGED INSTRUMENTS.

In sharp contrast to that widely used division of a string which we know on the guitar, showing decreasing distances between the frets as the pitch rises, we find many instances of a uniform spacing of the frets through a considerable distance. Instances from four countries may here be cited:

1. The well-known architect, Viollet-le-Duc,[1] gives a figure (fig. 1) of a mandolin from the end of the sixteenth century which shows frets for the first seven semitones pretty uniformly spaced; the frets for the next five to complete the octave are again uniform, though closer than before, and the following five are also uniformly spaced and still closer. Figures in other books[2] of European lutes, viols, etc., very often show a similar equal spacing. These are too numerous to be lightly treated as artists' blunders. Two instruments in the United States National Museum are illustrated in Plate 1.

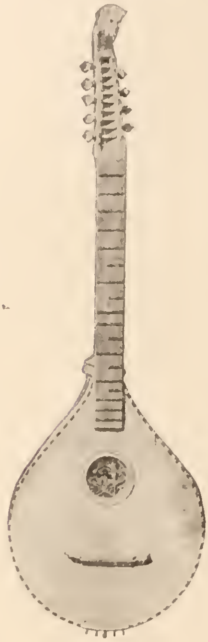

Fig. 1.
EUROPEAN MANDOLIN.
After Viollet-le-Duc.

2. Among the Greek rules given by Ptolemy is one for the division called *Diatonon homalon*, in which the whole string being twelve units long the points for stopping would be at 11, 10, 9, and 8, giving C, a note between D♭ and D, E♭, F, and G. Here it will be noticed the intervals get larger and larger as the pitch rises. Again, Carl Engel[3] refers to Drieberg's drawing of the ancient Greek guitar in the Berlin Museum, which has "seven frets at equal distances," but objects to it as it does not give a diatonic scale. The tracing of this drawing furnished by Professor Howard, of Harvard, adds to Engel's data the fact that the whole compass of the six intervals is slightly more than an octave (fig. 2).

Fig. 2.
GREEK GUITAR.
After Drieberg.

3. Among the instruments described in the Arabic treatise of the

[1] Dictionnaire raisonné du mobilier français, II, 1871, pl. LI.

[2] M. Praetorius, Syntagma Musicum, II, 1618. Reprint, 1894. Plates V, fig. 3; VI, fig. 1; XVI, fig. 1; XVII, fig. 4; XX, figs. 1, 3.

Bonanni, Description des instruments harmoniques. 2d. ed. Rome, 1776. Plates LII, LVII, LX, LXXI.

J. Ruhlmann, Geschichte der Bogeninstrumente, 1882. Plates IX, figs. 2, 5, 6, 13; X, fig. 16; XIII, figs. 3, 8.

[3] Music of the Most Ancient Nations, 1864, p. 205.

famous Al Farabi,[1] who died 950 A. D., is the short-necked *tanbour* of Bagdad, usually having two strings: on this a fret was first placed at one-eighth the length of the string from the upper end, and this space then divided into five equal parts. As the compass on each string was but little over a whole tone, each step was about a quarter-tone. These ligatures or frets are called "heathen" or "pagan," and the tunes played on them "heathen airs," clearly indicating that there was a scale native to the people whom the Mohammedan armies had conquered, a scale utterly different from either that of the lute or the *tanbour* of Khorassan, with their resemblances to Greek scales. Three hundred years later, or about 1250 A. D., Safi-ed-din,[2] a famous musician of Bagdad, wrote for his pupil, the son of the Vizier, a Treatise on Musical Ratios. He based them on string lengths, and in discussing instruments gives a figure of the frets on the neck of the lute, and it is noteworthy that these are equally spaced over a distance of a quarter length of the string. Further, he explains how of the ten frets in this short distance, located by various rules, five were fixed by arithmetical bisection or halving of the space between two frets already fixed; one of these, midway between what we should call D and E, if the open string gives C, was called the "Persian middle," and was very much in use in his time. Safi-ed-din[3] further describes, in two connections, a division of the Fourth, like the Greek one already quoted, where the string lengths are 12, 11, 10, 9, saying it is consonant and much used; in fact it is preferred to one that is substantially like the theoretical diatonic scale; still it should be added that when he comes to arrange intervals to make up two octaves he puts our arrangement along with the most agreeable half dozen genera.

4. In India there has been in modern times a curious reversion from an elaborate historical scale of twenty-two steps to the octave, of which no modern Hindu or European knows the theory, to an equal linear division;[4] one-half of the string on the *sitar* is bisected; the first or end quarter-length is then divided into nine parts, each marked by a fret, and the second quarter-length into thirteen parts similarly marked. Out of the twenty-three tones within the octave the player selects a limited number, five, six, or seven, rarely eight, for any particular tune. Most of the notes used are found on calculation to be deceptively close to the notes of our chromatic scale, and so may be easily confounded with them by European hearers.

5. This arithmetical division has been advocated by European

[1] Land's translation in Travaux de la 6e session du Congrès internationale des Orientalistes à Leide, 1883, pp. 107–114.

[2] Carra de Vaux's translation in Journal Asiatique, XVIII, 1891, p. 330.

[3] Idem, pp. 308–317.

[4] Tagore, Musical Scales of the Hindus, Calcutta, 1884, supplement. Partly quoted by C. R. Day, Music . . . of Southern India, 1891, and Ellis, Journal Society of Arts, XXXIII, 1885, p. 502.

theorists, as by Jamard[1] in a treatise of 1759, a copy of which is in the Lenox Library, New York; and Fétis[2] in his brief account of this author refers to others who maintained similar views.

III. INSTRUMENTS OF THE FLUTE TYPE.

The simple flutes are instruments of a type more primitive and more widely distributed than fretted stringed instruments. These instruments are sometimes side-blown, as is the case with the modern flute; or end blown, as one blows into a key or pan's pipe; or blown with a whistle mouthpiece, like the flageolet; or blown with a weak reed, as the oboe. For the purposes of this discussion the mode of exciting the vibration is immaterial. All of them embody the law that the frequency of vibration of a column of air in a tube depends mainly on its length, and the variation in length of the air column so as to produce several sounds from one tube is produced by opening holes in the sides of the tube. In practice these holes never can open so freely to the outside air that the portion of the tube beyond them may be considered as removed (the possibility or necessity of cross-fingering proves this to the player), so the proper location and diameter of the holes to produce the notes of our scale of even quality are fixed, not by a simple law as the frets on the guitar are located, but by laborious experimenting to get a standard instrument which is then reproduced with Chinese fidelity.

Now, as one looks over a collection of wind instruments, like the splendid one in the U. S. National Museum, or examines flutes figured in books, it will be easy to recognize that there are two principal types—(A) those having the holes spaced at sensibly equal distances, and (B) those having two groups each of three equally spaced holes, the interval between the nearest holes of the two groups being obviously greater than that between the holes of each group. As the common primitive method of making the holes is by burning, the holes are generally more uniform in diameter than those on European flutes of a century ago.

Illustrations of flutes of type A are found in Engel's Musical Instruments, some of which are copied on Plate 2. Dr. Wilson's paper on Prehistoric Art[3] has many more illustrations, as the figures of bone flutes from Costa Rica and British Guiana, of pottery flutes from Mexico and the Zuñi Indians, of tubes with a simple reed from Egypt and Palestine, of wooden flutes brought from Thibet by Mr. Rockhill, and a wooden flute from the Kiowa Indians. Fétis[4] has a cut of the staghorn flute from the stone age with three equidistant holes, referred

[1] Jamard, Recherches sur la Théorie de la Musique.
[2] Fétis, Biographie universelle des Musiciens.
[3] Report of the U. S. National Museum for 1896, pp. 325–664.
[4] Histoire générale de la musique, I, p. 26.

to by Wilson (p. 526). So far as is known not one of the peoples from whom these instruments have come has any musical theory, but some of them do have a principle of instrument construction; for a partly educated young Kiowa Indian, in Washington a few years ago, in a party under charge of Mr. James Mooney, showed the writer how the holes on a flute on which he played were located by measuring three finger-breadths from the lower end to the lower hole, and then taking shorter but equal spaces for the succeeding holes. The interpreter added that he had seen the holes spaced by cutting a short stick as a measure. The late Mr. F. H. Cushing has furnished the additional fact that measurement by finger-breadths is very common among Indians; and Dr. Fewkes[1] gives a figure to show how the prayer sticks, used by the Hopi Indians in the Snake ceremonials at Walpi, are measured off into seven parts by the distances from creases on the hand to the tip of the finger. On the Kiowa flute (Plate 4, No. 2) the distance between the centers of the holes is 32 mm., which is two medium finger-breadths. Some instruments of this type belonging to the U. S. National Museum are shown in Plates 3 and 4.

But it is not only among primitive and prehistoric peoples that such a succession of holes is found. The common military fife has it. The bagpiper recently seen on the streets of Washington used a chaunter (oboe), the holes of which were at sensibly equal distances, so conforming to the well-known fact that the bagpipe scale is intentionally unlike the harp scale. A Japanese *Fouye* with 7 holes figured in the catalogue of the Kraus collection at Florence shows to the eye holes at nearly equal spaces, and has, as reported, the steps of the scale increasing in length as the pitch rises. From Egypt[2] there have come twenty-five 3- and 4-hole ancient flutes, or more exactly, oboes, and a few of 5, 6, and more holes. One of the 4-holed instruments from a tomb of about 1100 B. C. shows the holes 35 mm. apart and the lowest hole twice this distance from the bottom. Villoteau's[3] plates of modern Egyptian instruments show various types of tubes with equally spaced holes.

Flutes of the second or B type with two groups of equal-spaced holes were sold in quantities at the Java village at the World's Fair held in Chicago in 1893 (Plate 6, No. 1). No two of the instruments seemed to have the same length or location of holes, but this grouping was unmistakable. Of this type is also a curious ancient Chinese instrument, the *Tche*, described by Amiot,[4] closed at both ends with

[1] Journal of American Ethnology and Archaeology, IV, 1894, p, 25–26.

[2] Loret, Journal Asiatique, 8th ser., XIV, 1889, pp. 111, 197. Musical Times, London, XXXI, 1890, pp. 585, 713.

[3] Description de l'Egypt, État moderne, II, 1809, plate cc.

[4] Mémoires concernant l'histoire . . . . des Chinois, VI, 1780, p. 76, pl. vi, fig. 42. Mahillon, Brussels Conservatory Catalogue I, No. 865.

an embouchere at the middle and holes symmetrically placed on each side dividing the whole length into thirds, quarters, and sixths; so, if the whole length is called 12, the mouth hole is at 6 and the finger holes at 2, 3, 4, 8, 9, and 10. Mahillon copied the instrument, but did not close the ends, and reports the scale as a chromatic one from E to A♯. Most of the old European wood wind instruments figured by Prætorius[1] (1618) are conspicuously of this type, as the appended Plate 5 shows without necessity of description, and various similar instruments of the Museum collections are figured in Plate 6.

IV. INSTRUMENTS OF THE RESONATOR TYPE.

1. The next group includes a variety of instruments of the resonator type, a type that is widely distributed and conforms to a law hitherto unrecognized as capable of furnishing a scale; though Sondhaus in 1850 stated the law and tried a few rough experiments. The mathematicians[2] have proved that a mass of air in a confined space with a very small nearly circular opening, as a short-necked bottle or a whistle, has a frequency of vibration proportional to the square root of the fraction which expresses the diameter of the hole divided by the volume of the cavity; and if there are two such openings so placed that the flow of air through one does not interfere with that through the other, the numerator of the fraction will be the sum of the two diameters. Now extend the same principle, and one may have a series of sounds rising in pitch as one after another of several holes in the wall is opened; and provided the character of the vibration is not essentially changed, the frequency of vibration of these notes will increase as the square root of the sum of the diameters of the holes opened. Suppose, for example, that a vessel has one mouth-hole of diameter 2 and several properly placed finger-holes of diameter 1; then on successively opening these a scale may be produced having vibration frequencies in the ratio of the square roots of 2, 3, 4, 5, etc. A moment's consideration will show that in such a scale the intervals between successive sounds become less and less as the pitch rises, instead of becoming greater as is the case with strings or flutes where the spacing of frets or holes is uniform.

The most elaborate and beautiful illustrations of instruments of this type are from graves in Central and South America. (See Plate 7.) The United States National Museum has many whistles from Chiriqui in Colombia, most of them giving but a single high note; these differ substantially, it will be noticed, from stopped organ pipes, since in the latter the mouth extends the full width of the tube. Whistles with one or two finger-holes have come from Mexico and San Salvador, but the most complete and perfect are from Costa Rica. Of these the one

[1] Syntagma Musicum, pls. IX and X.

[2] Rayleigh, Theory of Sound, II, 1878, Chap. XVI.

bearing the catalogue number 59970 (Plate 7, fig. 1) has served as the type specimen, and is the instrument which led to this investigation. It has a globular body with bird's head, a mouthpiece about in the position of a bird's tail, and four finger-holes on the back symmetrically placed; these holes seem to be precisely equal in diameter, and equivalent in musical effect, so the order of fingering is a matter of indifference, and all the tones are clear and distinct; in Dr. Wilson's paper,[1] Mr. Upham, who is a violinist, notes them as F, A, C, D, E. On measurement the volume was found to be 36.0 cc., the equivalent diameter of the trapezoidal mouth hole 1 cm., and the diameter of the finger holes .65 cm.; these diameters, however, need a correction on account of the thickness of the walls, since the air can not pass freely through the rather thick wall. The final result of the calculation is to give, with all finger-holes closed, the note F on the highest line of the treble staff, to within half a semitone, and on opening the finger-holes in any order to give the succession of intervals 4, 3, 2, and 2 equal semitones, with a mean error of only one-eighth E. S. According to the theory the series of intervals depends only on the ratio between the diameters of the holes and the mouth hole, in this case 1 to 1.62; so the series of tones has vibration frequencies approximately as the square roots of 1.6, 2.6, 3.6, 4.6, 5.6, or of 1, 1.62, 2.24, 2.86, 3.48; but the pitch of all depends on the quotient of the radius of the mouth-hole by the volume. Although the theoretical correction for thickness of wall can not be quite precise, it affects all the holes to nearly the same extent, and the greatest probable error that can be assumed will not change the whole compass more than half a semitone; so the calculated scale would still be substantially what the ear confirms—F, A, C, D, E, or in syllables *do*, *mi*, *sol*, *la*, *si*.

The Museum has several other Costa Rican instruments also of pottery quite similar in appearance to this, but not capable of giving such clear tones, or quite so perfect in the equality of the holes. If the holes are unequal in diameter, in thickness of wall, or in location with reference to the vibrating mass of air, the order of pitch will depend on which holes are opened instead of merely on how many; with five holes sixteen combinations are possible; but of the eleven instruments in the Museum eight give only five notes each, two give seven notes, and one gives nine notes. If the finger-holes are small relatively to the mouth hole, the compass is small, so one high-pitched whistle has a compass of only six semitones—G to C♯—and another runs from B to E; three have a compass of seven E. S., that is, a musical fifth, and two each have, respectively, eight, nine, and eleven semitones.

Still other National Museum instruments, similar in principle, but ruder in workmanship and more grotesque in form, have come from Chiriqui, Columbia, and are figured in Dr. Wilson's report, pages 628

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[1] Report of United States National Museum for 1896, p. 617.

to 646. In other museums similar instruments are to be found. A few from Chiriqui were briefly described forty years ago as belonging to the American Ethnological Society.[1]

In the American Museum of Natural History in New York, as reported by Prof. F. W. Putnam, half a dozen such three- and four-hole whistles from the region of Santa Marta, Colombia, are to be seen; while under his charge at Cambridge, Mass., there are a number from the Uloa Valley, Central America;[2] of those figured, three have three finger-holes and are said to give five notes each.

In the Brussels Conservatory Collection[3] there are twenty-five terra cotta instruments from Mexico; two of them are clearly of this resonator type, giving five notes and having a compass, respectively, of eight and eleven E. S. (fig. 3). Lastly, a similar instrument described and figured by Dr. Walter Hough, in the Report on the Columbian Historical Exposition at Madrid, 1892–1893, has the small compass of six E. S. The point should again be emphasized that with these instruments the notes get closer and closer together as the pitch rises; for instance, on the type instrument the successive intervals are in whole numbers 4, 3, 2, 2, E. S.; on the Brussels instruments, 3, 2, 2, 1, and 4, 3, 2, 2; on the Madrid specimen, 2, 2, 1, 1. A chart (Plate 10) will show more accurately what the four intervals are with any specified ratio of holes, and whether there is appreciable error in expressing the interval in whole numbers. Of course the calculations assume uniformity in the blowing, for it is easy for the performer to vary the notes by a considerable amount. Still, it is a surprise to find how well these simple scales satisfy the ear.

Fig. 3.
TERRA COTTA WHISTLE.
After Mahillon.

A sort of stone flageolet from Costa Rica appears to be connected with these instruments in principle (Plate 7, fig. 8). This is closed at one end and has a small mouth opening and four finger-holes arranged in pairs; its scale of seven notes from five holes proves that the holes are not acoustically equivalent, but the two of each pair are found to be nearly equivalent; so on trial it appears that the square root formula may be applied, by giving to the mouth-hole the value 5, to each of the nearer holes the value 1, and to the other holes the value 2; then the vibration frequencies will be as the square roots of the numbers 5 to 11. The calculated intervals from the lowest note are 1.6, 2.9, 4.1, 5.1, 6.0, 6.8 E. S.; the observed intervals are 2, 3, 4, 5, 6, and 7 E. S.

[1] Magazine of American History, IV, 1860, pp. 144, 177, 240, 274.
[2] Memoirs of the Peabody Museum, I, No. 4, pl. IX.
[3] Mahillon's Catalogue, II, Nos. 852, 853.

2. A pottery whistle found in the ruins of Babylon, dating probably from about 500 B. C., is in the Museum of the Royal Asiatic Society, London[1] (fig. 4). Rowbotham[2] says this is similar to the reindeer joint used by the cave men. Its extreme length is 3 inches and it has two finger holes. The three notes are stated to be C (of 525 d. v.), E, and G; but the holes not being quite equal, the E from one of them is a quarter of a tone flat. By blowing hard the G can be carried up to A. The chart (Plate 10) shows that if the interval C–G is exact, with equal holes the intermediate note E will be a very little sharp of the piano note, but the difference is only about 1 per cent, one-fifth of a semitone, and so is utterly negligible in notes of such uncertain intonation.

Fig. 4.
BABYLONIAN WHISTLE
After Engel.

3. Striking comparisons have sometimes been made, and especially by the late Prof. Terrien de la Couperie, between the Assyrian and early Chinese civilizations. Whatever their relations may have been, it is curious that the only instrument of the resonator type, having several finger holes and coming from a people who had a musical theory, is the *Hsüan* (Van Aalst)[3] or *Hiuen* (Amiot)[4] of the Chinese, said to have been invented some 2,700 years before our era, and still used in the Confucian ceremonies, though very rarely seen. It is described as a hollow cone of baked clay about 3½ inches high, having a mouth-hole at the top, three equal finger-holes on one side, and two equal holes on the other. The descriptions available are inconsistent and incomplete, but that given by Amiot a century ago is the fullest. He reports the scale as *re, fa, sol, la, do, re,* and as he gives a cut (fig. 5), also the diameters of holes and the external measures, an approximate calculation can be made of the scale by the laws of resonators. The pitch of the fundamental comes out D above middle C, and the other notes, F, G, and A, for one side; then starting anew for the other side we get C and D, all within a quarter of a semitone. This, it will be noticed, is a five-

Fig. 5.
CHINESE RESONATORS.
After Amiot.

[1] Engel, Music of the Most Ancient Nations, p. 75.
[2] History of Music, II, p. 628.
[3] Chinese Music, Shanghai, 1884, p. 82.
[4] Mémoires concernant l'histoire . . . des Chinois, p. 225.

step scale, like most of the theoretical Chinese scales. The agreement between the mathematical theory and observation is strikingly close.

4. All the cases thus far referred to have been of prehistoric or very ancient instruments. But some curious little instruments of this type are figured by Frobenius[1] (fig. 6) as "a splendid parallel between the cultures" of some West African tribes and the natives of New Pomerania. These are little whistles made out of gourds (a, b, d) or pottery (c). They have the mouth-hole and two, three, or four finger-holes. No dimensions are given. Kraus, of Florence, figures and describes[2] a similar instrument from Melanesia made of a gourd 6 cm. in diameter, having three finger-holes close to the mouth-hole (fig. 7). The scale is stated to be A, B, C♯, E, F, but no further measures are given. However, this series is easily obtained by assuming the diameter of the mouth-hole to be 1.0, of one hole 0.3, and of the others 0.6; apparently D♯ is omitted.

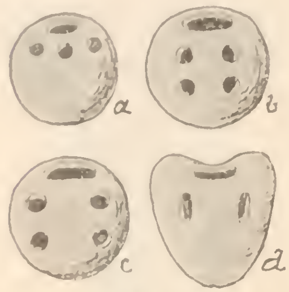


Fig. 6.
GLOBULAR WHISTLES.
After Frobenius.

In the Finsch Collection in the American Museum of Natural History, in New York City, there are several similar gourds of different sizes having three finger-holes. They are labeled "Bläsekugeln," "used by women."

5. In Europe there have been many instruments depending on the same general principle of resonance in a nearly closed cavity (in distinction from the open or closed organ-pipe principle), but not conforming to the simple law already set forth. Prætorius[3] in his famous book of 1618 gives figures and descriptions of several such instruments, along with the recorders, flutes, violins, etc., that one reads of more frequently; for instance, he says the fagotti are sometimes closed at the extreme end, but have a side hole; the Cornamuse has the end closed and holes in the side. Besides these he describes various instruments having stopped bodies on reeds—the rankett, bear pipes, etc., and similar forms on the organ. These things have all gone out of use along with the other delicate and weak-toned instruments of their times. To-day musicians demand tones more powerful and richer in harmonics than instruments of this type can give. But a curious survival or revival of this earlier type occurred in the middle of this century, which is told of in Groves's Dictionary of Music. A blind peasant, named Picco, gave public performances in London on a

Fig. 7.
GLOBULAR WHISTLE.
After Kraus.

[1] Der Ursprung der Afrikanischen Kulturen, 1898, p. 150.
[2] Archivio per L'Antropologia e la Etnologia, XVII, 1887, pp. 35–41, fig. 5.
[3] Syntagma Musicum II, pp. 44, 48, 85.

flageolet 2 inches long and having only three holes. By partially or wholly closing the end of the tube with his hand he made use of the resonator principle to lower the pitch of his notes; so he obtained a compass of more than two octaves. The instrument is similar to Prætorius's *schweigel*[1] except that it is shorter, and the accuracy of the notes performed would depend almost wholly on the performer. Later a traveling troupe appeared in European cities with seven instruments called ocarinas. These are familiar to us, being on sale everywhere. They are properly resonators, but the holes are more numerous than in the instruments already considered and vary widely in size. The scale, which the instruments furnish with more or less precision, is not dependent on any simple principle, but is adjusted by the maker by varying the sizes of the holes so as to conform to a scale fixed on other instruments.

V. THE INFLUENCE OF THE HAND.

All the instruments of the three groups now discussed are "fingered;" that is, the acoustical dimensions of the vibrating string or mass of air are varied as the player manipulates the fingers of one or both hands. These instruments therefore involve a feature not associated with drums and other instruments of percussion, or with primitive harps. Instead of using the hand as a whole, the more delicate fingers are utilized separately; so the simple instrument becomes in a peculiar sense a part of the player's means of self-expression and is specially responsive to his own moods, as many legends of the power of music testify. But leaving to the musical writers such comparisons between instruments, it is important to the physicist to recognize that the dimensions of the human hand have fixed absolutely some dimensions of these instruments.

The first thing to strike one, considering the hand from this point of view, is the fact that only with difficulty can the five digits be brought into line, so the thumb is not used on primitive instruments for fingering, so far as observed. In the more highly developed flutes there may be a hole for it on the back side, while on our own flutes, clarinets, etc., it governs one or more keys. Similarly, the little finger does not readily fall in line with the three longer ones, and, besides, is much weaker. The remaining three fingers on a hand of medium size can be brought into a space of about 1 cm., or spread to span perhaps 12 cm. (5 inches). To fix one's ideas before comparing these limits with measures on some actual instruments, it will be convenient to recall that on piano keyboards the distance between key-centers an octave apart is 165 mm. (6½ inches), the same as on a spinet of 1602; but on the physiologically designed Janko keyboard, with the octave distance

[1] Syntagma Musicum, p. 39, pl. ix.

140 mm. (5½ inches), an ordinary hand can readily span an octave and a Fifth, because the fingers are not forced into line.

Examining first some string instruments, it is found that on a guitar of New York make (No. 55690, U.S.N.M.) the distance between frets ranges from 33 to 14 mm. The greatest distance noticed between frets is on the large Siamese *Kra Chappee* (No. 27310, U.S.N.M.), where there are three spaces, respectively, of 71, 73, and 77 mm. A similar instrument examined at the World's Fair held in Chicago had the corresponding spaces 60, 60, and 67 mm. The string lengths to the first frets were, respectively, 878 and 740 mm. The smallest distance observed between frets is the above-cited 14 mm., except that the Syrian lute, *Bizug* (No. 95144, U.S.N.M.), has two spaces of 12 and 13 mm. On most instruments the frets cease when the limit of 20 to 25 mm. is reached. It is obvious that these and similar data for fretted instruments are not of much importance unless one can know that the hand was not shifted from one fret to another.

With our instruments shifting is notoriously common, but the histories of the violin report that two or three centuries ago it was a notable thing for a player to shift. The usual theory of the old many-stringed instruments, of which the Arab lute is a particularly good example, required the strings to be tuned in Fourths, and the string lengths were not too great for the four fingers to govern all the frets within this range—that is, in a quarter-length of the string—so a shift would be unnecessary. On the Arab lute[1] there were sometimes ten very unequally spaced frets in this space, but for any one tune only a few of them were used, and in the principal modes, *'Ochaq* and *Rast*, one fret each for the index and ring fingers sufficed to give substantially our diatonic scale.

With simple wind instruments the case is quite different, for several fingers must be used simultaneously to cover holes, so the hand can not be shifted. In the Kiowa flute referred to above the uniform distance between holes is 32 mm.; in the stone whistle from Mexico, 20 mm.; in the four Egyptian flageolets and oboes figured by Villoteau (his Plate c c) the intervals are, respectively, 12, 15, 15, and 36 mm. These distances require only a convenient spread of the fingers. Many other measures can readily be obtained from the accompanying figures with their appended scales.

If the musician has a theory demanding that the holes be so near together or so far apart as to make direct fingering inconvenient or impossible, keys with long or short levers are added, as on modern flutes and clarinets, while among the Romans extra holes were bored to provide for several genera, the holes not needed for any tune being closed by plugs or rotating rings.

In a few cases wind instruments are found so long that the player's

[1] Land, Travaux de la 6e Congrès des Orientalistes, 1883, pp. 107–114, or Ellis, Journal of the Society of Arts, XXXIII, 1885, p. 502.

arm is too short to reach the lower end. Then, necessarily, the holes to be fingered are located at the middle or upper end of the tube, but the holes are so small that the pitch of the resulting notes is much lower than the position of the holes would suggest, so the discrepancy to the ear is not as great as to the eye. In other cases the length is misleading, for the holes are bored obliquely or holes are bored in the tube below the holes to be fingered, thereby raising and adjusting the pitch of the lowest note, as Mahillon shows in the Brussels catalogue (Nos. 830, 1039, 1117, 1119, and 1123) and Villoteau shows on his Plate c c, No. 1. This is a possible explanation of the superfluous holes in the flute on the statue from the ruins of Susa (Plate 2, fig. 1), if the figure be accepted as archæologically correct. In modern instruments, as is well known, the distant holes are controlled by covers at the ends of long levers.

The relation of the instruments of the resonator type to the hand is too obvious to need discussion; the objects must be of such size and shape as to be held by the hand or by two hands while the fingers are manipulated, and the holes must be conveniently located and small enough to be closed by the tips of the fingers, or in the Chinese *hiuen* also by the thumbs.

It is rather surprising to see how little the thumb is used in playing upon the instruments under consideration. Although from its anatomical structure the thumb has a peculiar independence in its movements, yet most of its services are rendered by cooperation with the other fingers; and the natural training of these, as in grasping, sewing, weaving, or the most delicate savage industries, appears likewise to call for their cooperation, not for independent action. It is only in playing instruments like the lyre and harp (whose tuning depends on principles outside the instrument, and so they do not belong to the present discussion) that one sees a grasping action requiring two or more fingers at once. But in the guitars, flutes, etc., under consideration, the thumb is constantly occupied in merely supporting the instrument, so any variation in the pitch of the sound can come only as the other fingers become independent in action. When we remember how difficult it is for a civilized piano-player or typewriter to-day to acquire a satisfactory independence in movement of all the fingers, especially of the third and fourth, and recall that the early instruction-books for the harpsichord required the use of but two fingers on each hand, we shall have a higher respect for the technique of primitive musicians, and shall not wonder that primitive wind instruments have so few holes. Presumably the index finger first gained independence, and then it marked a long advance when two fingers could act independently of one another. So the four-hole flute or resonator, requiring the action of two fingers from each hand, and giving a scale of five tones, is a monument commemorating an important stage both in the development of the hand and in the extension of musical resources.

VI. COMPOSITE INSTRUMENTS.

Each of the instruments thus far examined is capable of furnishing several notes of approximately constant pitch, but the general principle before us may be embodied in composite instruments, where each note has its own vibrating body; thus

1. Various forms of harps and dulcimers show strings of regularly decreasing length; here, of course, difference of tension may nullify the scale due to the lengths. One form is shown on Plate 8.

2. Pan's pipes are sometimes seen with regularly decreasing lengths; it is true that this regularity is not very common, but it is the only principle of scale building (except the Chinese cycle of fifths) yet recognizable in these primitive instruments. (Plate 9.)

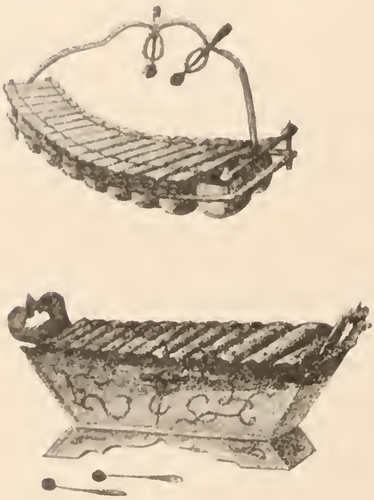


Fig. 8.
XYLOPHONES.
After Kraus.

3. Instruments of the bar type are found frequently in our orchestras and bands under various names, as *xylophone;* they are familiar in children's toys and are widely distributed in savage and half-civilized lands under the names of *marimba*, *balafong*, *harmonicon*, etc. (Plate 8 and fig. 8.) The law of the uniform bar is that the frequencies of vibration of a series of bars of the same material are proportional to the quotients of the thickness divided by the square of the length; the breadth is immaterial if it is uniform. So if one takes a series of *uniform* bars of the same thickness and regularly decreasing length he may obtain a series of ascending notes. Thus, let the first bar be 24 units long (for example 24 cm.), the successive bars decreasing by one unit; the eighth bar will be 17 units long, and the fifteenth bar 10 units; the series of frequencies would then be as the reciprocals of the squares of 24, 23, etc., so giving to the ear a series of increasing intervals; with these proportions bar No. 8 would give the Octave of the first, but bar No. 15 would give the Twelfth of bar No. 8. The simplicity of the rule, however, frequently disappears, either because of variations in the thickness, as when a savage splits a bamboo stem and then cuts his bars so that the shorter ones are also thinner, or because of the attachment of lumps of wax or clay to the bars to tune them to some other instrument; or because of the hollowing of the center, as is done by modern Japanese; so at present one can not affirm that this

theoretical principle of scale determination is certainly and consciously embodied in any instrument anywhere; but some instruments in the National Museum and some drawings in books make the assumption seem plausible, that the primitive type of this instrument is a series of bars, supported at points about one-fifth or one-fourth of their length from their ends, and decreasing in length by equal linear amounts.

It is evident that these composite instruments are of minor importance in this study; but in the light of the theoretical laws here suggested perhaps travelers may learn something of the intention of a savage who cuts his Pandean pipes or bars to form a musical instrument.

VII. CONCLUSIONS.

There have now been considered all the types of instruments in which several notes of different pitch are produced from the same vibrating body—whether string, column of air, or mass of air.

(1*a*.) There have been found examples from various parts of the world of the intentional location of the stopping points of a vibrating string at equal linear distances; since with all stringed instruments the fingering will cause a slight increase of tension, the equivalent length of the string is less than the actual length.

(1*b*.) There have been found numerous examples of wind instruments pierced with holes in one or two groups spaced at equal linear distances; since these holes are never sufficiently large to allow the air to flow through them with perfect freedom (unless in some Chinese flutes) the equivalent length of the vibrating column of air is greater than the actual distance from the mouthpiece to the hole.

(2.) There have been found instruments of the Marimba type with bars of regularly decreasing lengths.

(3.) There have been found many forms of instruments of the resonator type embodying a series of equal and similarly-located holes; in these, thickening the wall is equivalent acoustically to making the holes smaller; while locating the hole nearer the point where the vibrating air has its maximum change of density is equivalent to enlarging the hole.

Three simple laws give to the first approximation the scales of these several instruments, namely:

(1) The law of inverse lengths.
(2) The law of inverse squares of lengths.
(3) The law of the square roots of a series of numbers proportional to sums of diameters.

The first and second laws give scales whose intervals increase as the pitch rises; scales based on the third law have decreasing intervals. Some results are shown in a table in the appendix and graphically in Plate 10.

From these it is evident that whichever type of instrument one may take, there will be some intervals that very closely agree with intervals of our familiar scale. In a few cases this comes about because our scale is principally derived from the Greek theorists, who based their scales on proportional string-lengths; so, if the unit of equal distance on a simple guitar chances to be an aliquot part of the length of the string from the bridge to the nut, some of the resulting notes will belong to our scale. (However, the divisor must not have a prime factor greater than five.) But whatever the instrument, on any doctrine of chances, there will be some approximate coincidences; and these coincidences, as judged by the ear, will be found much closer and more numerous than when judged mathematically or graphically; for the training of modern musicians, as has often been recognized, not only allows but compels them to ignore deviations from their standard scale—deviations amounting sometimes to more than half a semitone. So one is forced to conclude that the recognition, even by a musically educated ear, of a series of notes as agreeing substantially with our diatonic scale or with any other known scale, does not afford any adequate ground for judging of the principles underlying the series; in fact, the failure to note the deviation may prevent the recognition of the underlying principle.

The type Costa Rican four-hole whistle is the most striking example of a series agreeing closely with notes of our scale, yet based on an absolutely different principle; for the mean computed deviation from the piano intervals is only one-eighth of a semitone.

Further, the whole discussion makes it evident that the people who made and used these instruments, or any single type of them, had not that idea of a scale which underlies all our thinking on the subject, namely: A series either of tones or of intervals recognized as a standard, independent of any particular instrument, but to which every instrument must conform. Modern Europeans for the sake of harmony have nearly banished all scales but one, and seldom know by what rules the instruments are tuned to furnish this. But for these people the instrument is the primary thing, and to it the rule is applied, while the scale is a result, or a secondary thing; and the same rule applied a hundred times may possibly give a hundred different scales. Naturally one does not expect to find much concerted music among people in this stage of development.

The various rules discussed above may be united in a generic one, namely:

The primary principle in the making of musical instruments that yield a scale is the repetition of elements similar to the eye; the size, number, and location of these elements being dependent on the size of the hand and the digital expertness of the performer.

This principle shows itself in the occasional equal spaces on the neck or table of a stringed instrument, and conspicuously in the series of holes on flutes and primitive oboes, while a sense of balance and symmetry added to the repetition appears in the two groups of holes on the flutes, etc., and especially in the resonators, and appears in a different way in the trapezoidal forms of dulcimers, Pan's pipes, and marimbas. The pitch-determining elements are therefore primarily decorative. In fact no one can examine any collection of primitive wind instruments, or drawings of them, without being struck by the way in which the holes often cooperate in the decoration; while they are not found interfering with the artistic design (see fig. 3, page 430; Plate 2, figs. 1 and 2; Plate 3, fig. 2).

Simple decoration involving only repetition and symmetrical placing or grouping of similar parts is not only found among living primitive peoples everywhere that musical instruments embodying a scale can be found, but is prehistoric. The prehistoric flutes are believed to come from the neolithic age, and the pottery from this age shows a multitude of geometrical designs, some of which are collected in Wilson's Plates 19 and 20. The paleolithic age has furnished few geometrical designs and no flutes or many-holed resonators. In applying such decoration to the hollow bones of animals or human enemies, to the hollow reeds that Lucretius says whistle in the wind, or to gourds and simple pottery, nothing can be more natural than sometimes to perforate the walls and to get a several-toned musical instrument as the result. So although no conclusions regarding the mental operations of prehistoric man can be absolutely certain, one feels a strong conviction that, as with immature minds among us, art appealed first to the eye and later to the ear; that beauty of material form incidentally furnished series of sounds that could be repeated, and could give to the ear and the mind the idea of the definite leaps or steps that Aristoxenus, countless ages afterward, called the characteristic of music. (Of course rhythm in movement and in sound are independent of the structure of an instrument.) Any influence that may have been exerted on the establishment of scales by the songs of birds, by the recognition of overtones in the sounds of the human voice, or by the production of harmonics on the horn must have been limited and trivial. The principle here presented is at any rate a *vera causa*, and explains facts hitherto unexplained; further, (1) it is extremely simple both in theory and practice; (2) it is flexible, allowing of multifarious results in practice; (3) it is suggested by prehistoric instruments, supported by the instruments of many living primitive peoples and repeatedly confirmed by its survival in several instruments of peoples in an advanced stage of musical culture.

It only remains to add, in order to prevent misunderstanding, that the principle here set forth never appears as the dominating one among peoples who are known to have had a theory of the scale. The Greek theoretical scales, diatonic and nondiatonic, are doubtless its direct descendants, though at present it is not known what the influence was that so transformed them and made them depend on ratios, not on difference of lengths. Possibly the theory of numbers bewitched musicians then as it has sometimes since, though the converse speculation is a plausible one—that the recognized musical ratios gave a mystical meaning to numbers. It is curious to note that Aristoxenus had somehow got far enough to complain that flutes distort most of the intervals (p. 42, Mb.), and if his lost treatise on boring flutes should be found it might throw light on this history. The Arab "step by step" method is apparently a late descendant of the equal linear divisions, appearing after men had learned to recognize the equality of intervals as well as of spaces. But the Chinese cycle of fifths must be explained and determined on entirely different physical principles, and the various European scales as defined by theorists or rendered by the best violinists or fixed by good tuners, when properly examined, reveal elements as diverse as the elements of our language or our population. The principle in question is therefore presented only as the simplest, earliest, and most primitive principle of scale-building.

APPENDIX.

The laws briefly stated on page 437 for the several kinds of instruments discussed in the paper may be expressed more accurately by the following formulæ:

Let N = number of complete vibrations per second.
l = length of string or column of air or bar.
a = diameter of mouth-hole of resonator, corrected for thickness of wall.
b = diameter of finger-holes of resonator, corrected for thickness of wall.
n = number of finger-holes opened on resonator.
t = thickness of bar.
K = constant, depending on material and units of measurement.

Assuming centimeter-gram-second units and ordinary temperatures,

$K^{i} = \sqrt{\text{Tension in dynes} \div \text{mass in grams per cm.}}$ = velocity; e. g., in piano strings 17,000 to 40,000 cm.-sec.; in violin strings from 13,000 for the covered string to 43,000 for the gut E-string; in weak primitive instruments probably much less.

K^{ii} = 34,000 cm.-sec., the velocity of sound in air.

K^{iii} = 520,000 cm.-sec. for iron bars; 340,000 to 520,000 for wood bars supported as usual in a xylophone.

K^{iv} = 5,500.

Then, corresponding with the brief laws,

(1a) For strings: $N = \frac{K^{i}}{2l-} = \frac{1}{2\times l} \sqrt{\text{tension} \div \text{linear density}}$.

(1b) For columns of air: $N = \frac{K^{ii}}{2l+} = \frac{17,000}{l+}$.

(2) For bars: $N = K^{iii}\frac{t}{l^2} = 340,000 \text{ to } 520,000\ \frac{t}{l^2}$.

(3) For resonators: $N = K^{iv}\frac{\sqrt{a + \text{sum of } b}}{\sqrt{\text{volume}}} = \frac{5,500}{\sqrt{\text{volume}}}\sqrt{a\left(1 + n\frac{b}{a}\right)}$.

These constants are sufficiently accurate for the general purposes of the anthropologist and musician. But the results should be expressed in musical terms. The French standard pitch, now adopted by the Piano Makers' Association, gives A = 435 d. v., or C = 258.7 d. v., and the ratio for any interval of p piano semitones is $2^{\frac{p}{12}}$. In most cases it is much more convenient to have intervals than ratios; and incomparably the most convenient unit of intervals is the piano semitone, of which 12 by definition make an octave; these can readily be grouped by anyone with slight musical knowledge into larger intervals, Thirds, etc., and the musical value of any whole number of them can instantly be found on a well-tuned piano.

Since the reduction of ratios to intervals can not ordinarily be done without logarithms, a short table has been calculated and is appended by the use of which the reduction may be done by inspection in most practical cases. This table gives the logarithm of every whole number from 1 to 40, and the product of these by 40, less one three-hundredth, together with the successive differences; these are in semitones; for the factor is so chosen that when the logarithm of the ratio 2:1 is multiplied by it the product will be 12, which is the number of semitones corresponding to the ratio of the octave. Much more elaborate tables, but without the column of differences, have been published by Prony and by Ellis. In using the table it is well to remember that the average uncertainty in pitch of public performers in Berlin was found to be about one-tenth of a semitone.

As illustrations of the use of the table, find the successive intervals of the scale of the Hindu *Sitar*, the string being stopped successively at 36, 35, 34, . . 27; the corresponding differences in column 4 of the table are 0.49, .50, .52, ...63 E. S., the sum being 4.98 E. S., as required by Tagore's rule (p. 425 above). To complete the octave, the space 27 to 18 is to be divided into 13 equal parts; substitute for the ratio 27:18, or 3:2, 39:26, and use the table again; the differences are now 0.45, .46, ...65, the sum being 7.02 E. S., which added to 4.98 gives 12 E. S., or the octave.

If the law be that of the square roots, as with resonators, the table is to be used in precisely the same way, but the final results are to be divided by 2; for example, in the type resonator, calling the equivalent radius of the mouth hole 1.0, that of the finger holes is 0.6 (more accurately, 0.62); the series of terms will therefore be 1.0, 1.6, 2.2, 2.8, 3.4; multiply all by 10, and take the corresponding numbers from column 3 of the table; divide the differences by 2, and add the quotients to the fundamental pitch. The results are as follows:

	E. S.			
10	39.86			F
16	48.00	8.14	4.07	A + .07 E. S.
22	53.51	13.65	6.83	C — .17.
28	57.69	17.83	8.92	D — .08.
34	61.05	21.19	10.60	E — .40.

If 0.62 had been taken the results would have been slightly higher in pitch.

Plate 10 has been plotted to give directly the intervals of resonator scales for any number of open holes up to 5, and for ratios of radii between 0 and 1. The dotted line corresponds to the type resonator.

The table may also be used for bars; the only change is that the differences are to be doubled instead of halved. Thus, with a series of uniform bars whose lengths are 24, 23, etc., to 17, the compass will be 2 × (55.02 — 49.05) = 11.94 E. S., which is practically an octave, as stated on page 436.

Table for computing musical intervals.

N.	Log. N.	E. S.	Dif.	N.	Log. N.	E. S.	Dif.
1	0.0000	....	....	21	1.3222	52.70	0.84
2	.3010	12.00	12.00	22	.3424	53.51	.81
3	.4771	19.02	7.02	23	.3617	54.28	.77
4	.6021	24.00	4.98	24	.3802	55.02	.74
5	.6990	27.86	3.86	25	.3979	55.73	.71
6	.7782	31.02	3.16	26	.4150	56.41	.68
7	.8451	33.69	2.67	27	.4314	57.06	.65
8	.9031	36.00	2.31	28	.4472	57.69	.63
9	.9542	38.04	2.04	29	.4624	58.30	.61
10	1.0000	39.86	1.82	30	.4771	58.88	.58
11	.0414	41.51	1.65	31	.4914	59.45	.57
12	.0792	43.02	1.51	32	.5051	60.00	.55
13	.1139	44.41	1.39	33	.5185	60.53	.53
14	.1461	45.69	1.28	34	.5315	61.05	.52
15	.1761	46.88	1.19	35	.5441	61.55	.50
16	.2041	48.00	1.12	36	.5563	62.04	.49
17	.2304	49.05	1.05	37	.5682	62.52	.48
18	.2553	50.04	0.99	38	.5798	62.98	.46
19	.2788	50.98	0.94	39	.5911	63.43	.45
20	.3010	51.86	0.88	40	.6021	63.86	.43

EXPLANATION OF PLATE 1.

STRINGED INSTRUMENTS.

Fig. 1. Small Turkish Tamboura.
(Cat. No. 95312, U. S. N. M.)

Fig. 2. Medium Colascioni (Italian).
(Cat. No. 95307, U. S. N. M.)

Note.—The scale shown on this and most of the following plates is 20 centimeters long.

STRINGED INSTRUMENTS.

EXPLANATION OF PLATE 2.

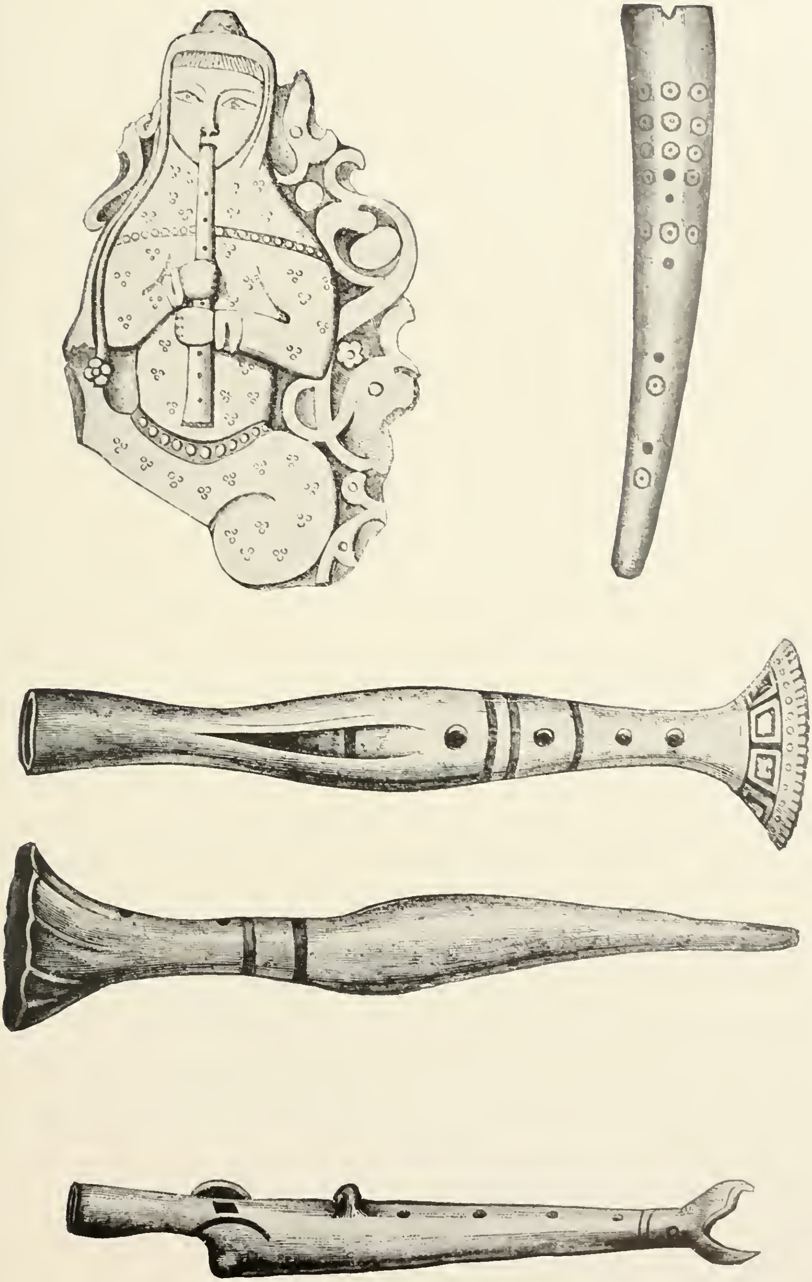
FLUTES WITH EQUAL-SPACED HOLES, TYPE A.

Fig. 1. Pipe from Susa. Engel, Music of the Most Ancient Nations, p. 77.

Fig. 2. Bone Flute, about 6 inches long, disinterred at Truxillo, Peru. British Museum. Engel, Musical Instruments, p. 64.

Figs. 3, 4. Aztec Pipes, called by Mexicans *pito;* usual form; scale, a, b, c♯, e, f♯. Engel, Musical Instruments, p. 62.

Fig. 5. Aztec Pipe; unusual form. Engel, Musical Instruments, p. 62.



FLUTES WITH EQUAL-SPACED HOLES.

EXPLANATION OF PLATE 3.

FLUTES WITH EQUAL-SPACED HOLES, TYPE A.

Fig. 1. DOUBLE FLAGEOLET. Mexico.
(Cat. No. 197173, U. S. N. M. Report 1896, fig. 250b.)

Fig. 2. AZTEC FLAGEOLET (*pito*). Mexico.
(Cat. No. 172819, U. S. N. M. Report 1896, fig. 252.)

Fig. 3. STONE FLAGEOLET. Mexico.
(Cat. No. 98948, U. S. N. M.)

Fig. 4. BONE FLAGEOLET. Costa Rica.
(Cat. No. 18108, U. S. N. M. Report 1896, fig. 273.)

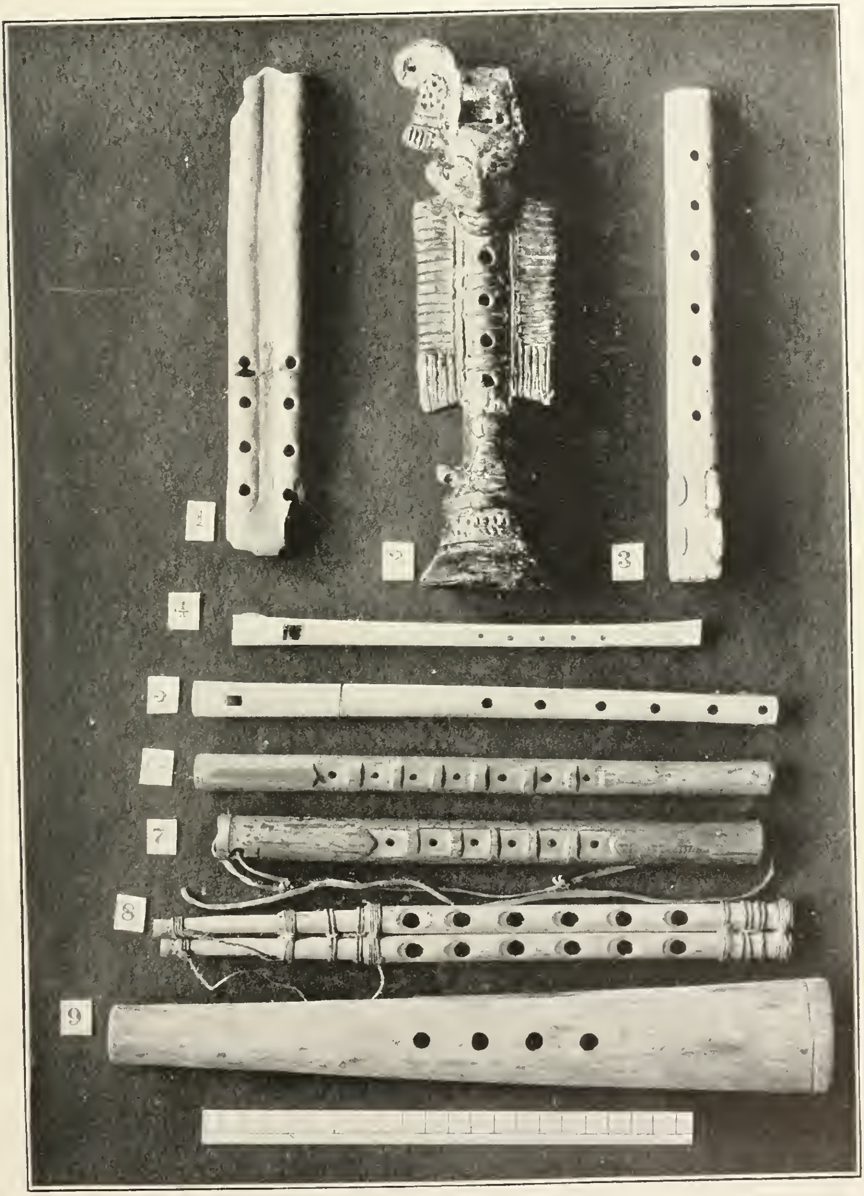
Fig. 5. BONE FLAGEOLET. Amazon.
(Cat. No. 5719, U. S. N. M.)

Fig. 6. BAMBOO WHISTLE. Thibet.
(Cat. No. 167165a, U. S. N. M. Report 1896, plate 69.)

Fig. 7. BAMBOO WHISTLE. Thibet.
(Cat. No. 167165b, U. S. N. M. Report 1896, plate 69.)

Fig. 8. SHEPHERD'S PIPE, WITH REED. Arabia.
(Cat. No. 93555, U. S. N. M.)

Fig. 9. HORN (*Soittotorri*). Finland.
(Cat. No. 95686, U. S. N. M.)



FLUTES WITH EQUAL-SPACED HOLES.

EXPLANATION OF PLATE 4.

FLUTES WITH EQUAL-SPACED HOLES, TYPE A.

Fig. 1. DIRECT FLUTE. Peru.
(Cat. No. 95904, U. S. N. M.)

Fig. 2. FLUTE OR FLAGEOLET. Kiowa Indians.
(Cat. No. 153584, U. S. N. M.)

Fig. 3. FLUTE OR FLAGEOLET. Mohave Indians.
(Cat. No. 107535, U. S. N. M.)

Fig. 4. FLUTE OR FLAGEOLET. Dakota Indians.
(Cat. No. 23724, U. S. N. M.)

Fig. 5. TRANSVERSE FLUTE (*Ti-tzu*). China.
(Cat. No. 130446, U. S. N. M.)

Fig. 6. TRANSVERSE FLUTE (*Koma Fuye*). Japan.
(Cat. No. 93205, U. S. N. M.)

Fig. 7. OBOE (*Pee Chawar*). Siam.
(Cat. No. 27313, U. S. N. M.)

Fig. 8. FLAGEOLET (*Sopilka*). Little Russia.
(Cat. No. 96466, U. S. N. M.)

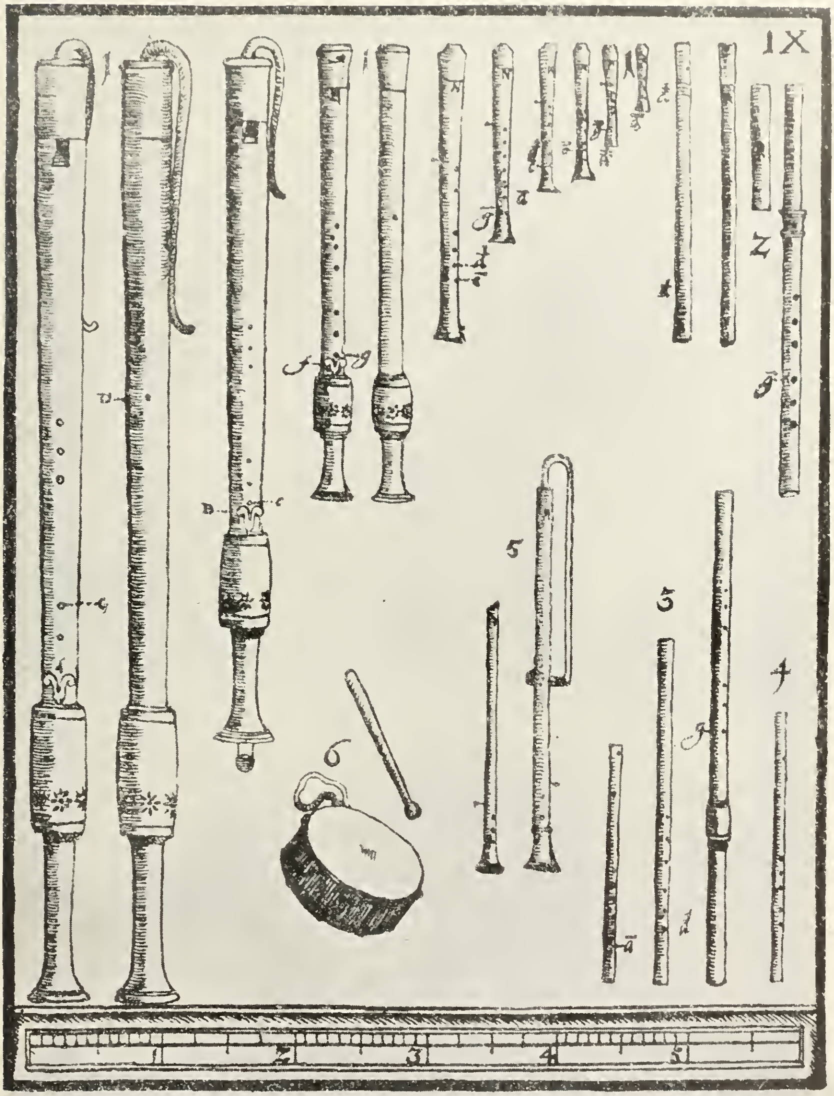
Fig. 9. DOUBLE FLAGEOLET. Thibet.
(Cat. No. 95816, U. S. N. M.)

FLUTES WITH EQUAL-SPACED HOLES.

EXPLANATION OF PLATE 5.

FLUTES WITH HOLES IN TWO GROUPS, TYPE B.

From Praetorius's Syntagma Musicum of 1618, to show finger-holes grouped in two sets.



1 Blockflöten, ganz Stimmwerk. 2. Doltzflöt [illegible] 3 Querflöten, ganz Stimmwerk 4 Schweitzer Pfeiff.
5 Stamentienbass und Discant 6 Klein Päuklin zu den Stamentien Pfeifflin zu gebrauchen

FLUTES WITH HOLES IN TWO GROUPS.

EXPLANATION OF PLATE 6.

FLUTES WITH HOLES IN TWO GROUPS, TYPE B.

Fig. 1. Flageolet (*Souling*). Java.
(Cat. No. 95669, U. S. N. M.)

Fig. 2. Direct Flute. Ceylon.
(Cat. No. 95727, U. S. N. M.)

Fig. 3. Direct Flute (*Manjairah*). Syria.
(Cat. No. 95150, U. S. N. M.)

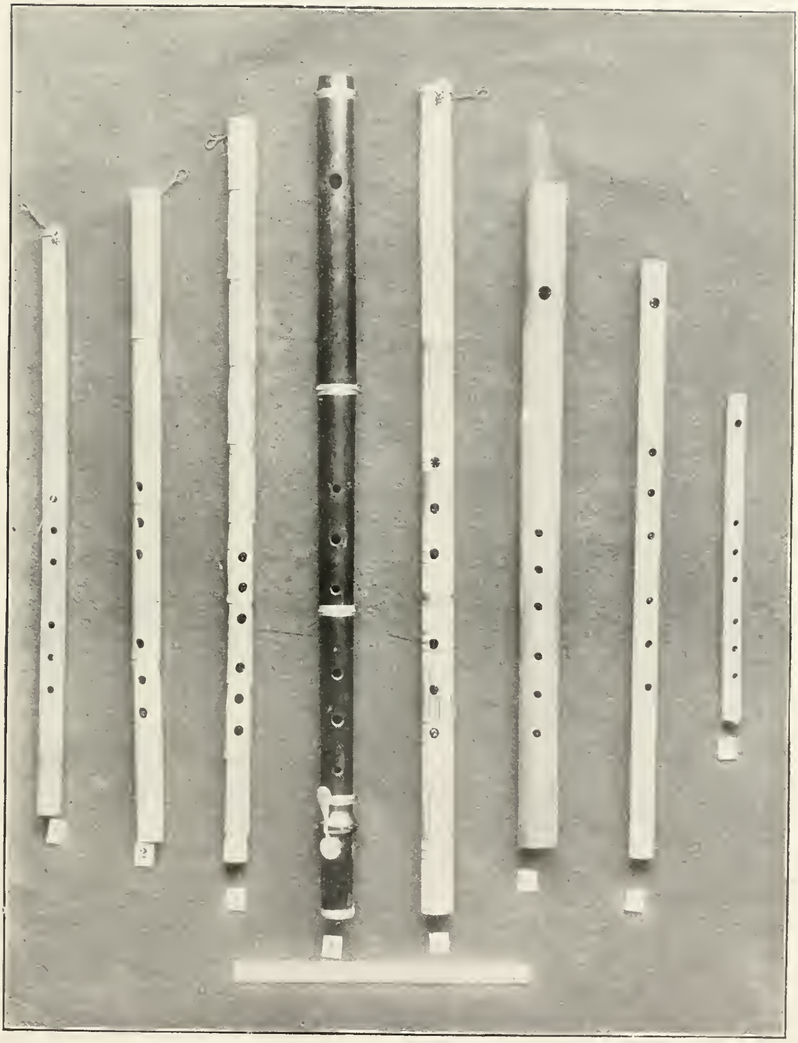
Fig. 4. German D Flute. New York.
(Cat. No. 55624, U. S. N. M.)

Fig. 5. Flageolet (*Souling*). Java.
(Cat. No. 95666, U. S. N. M.)

Fig. 6. Transverse Flute (*Murali*). Bengal.
(Cat. No. 92707, U. S. N. M.)

Fig. 7. Transverse Flute. Manila.
(Cat. No. 95061, U. S. N. M.)

Fig. 8. Transverse Flute. Manila.
(Cat. No. 95060, U. S. N. M.)



FLUTES WITH HOLES IN TWO GROUPS.

EXPLANATION OF PLATE 7.

CENTRAL AMERICAN RESONATORS OR WHISTLES.

Fig. 1. Costa Rica.

(Report U. S. Nat. Mus., 1896, p. 617. Scale: f, a, c, d, e. Cat. No. 59970, U. S. N. M.)

Fig. 2. Costa Rica.

(Report U. S. Nat. Mus., 1896, fig. 263. Scale: d, e, f♯, g, a. Cat. No. 59969, U. S. N. M.)

Fig. 3. Costa Rica.

(Report U. S. Nat. Mus., 1896, fig. 262. Scale: g♭, b♭, b, c, d♭, d, e♭, e, f. Cat. No. 28952, U. S. N. M.)

Fig. 4. Costa Rica.

(Report U. S. Nat. Mus., 1896, fig. 269. Scale: f, g, a, b♭, c. Cat. No. 28956, U. S. N. M.)

Fig. 5. Costa Rica.

(Report U. S. Nat. Mus., 1896, p. 617. Scale: d♭, f, g♭, a♭, b♭. Cat. No. 60045, U. S. N. M.)

Fig. 6. Panama, Chiriqui.

(Report U. S. Nat. Mus., 1896, figs. 304–5. Holmes, Report Bureau Ethnol., 1884–5, figs. 245–246. Scale: end closed, f, g, a♭, b♭; open, f♯, g♯, a♯, b. Cat. No. 109682, U. S. N. M.)

Fig. 7. Costa Rica.

(Report U. S. Nat. Mus., 1896, p. 614. Scale: g♭, b♭, c♭, d♭, e♭. Cat. No. 28954, U. S. N. M.)

Fig. 8. Costa Rica.

(Report U. S. Nat. Mus., 1896, fig. 270. Scale: a♭, b♭, b, c, d♭, d, e♭. Cat. No. 6423, U. S. N. M.)

CENTRAL AMERICAN RESONATORS, OR WHISTLES.

EXPLANATION OF PLATE 8.

COMPOSITE INSTRUMENTS.

Fig. 1. Pan's Pipes. Cairo, Egypt.
(Cat. No. 94653, U. S. N. M.)

Fig. 2. Kantele. Finland.
(Cat. No. 95691, U. S. N. M.)

Fig. 3. Mokkin. Japan. Two bars turned edgewise to show their form.
(Cat. No. 96841, U. S. N. M.)

The paper scale is 20 centimeters long.

COMPOSITE INSTRUMENTS.

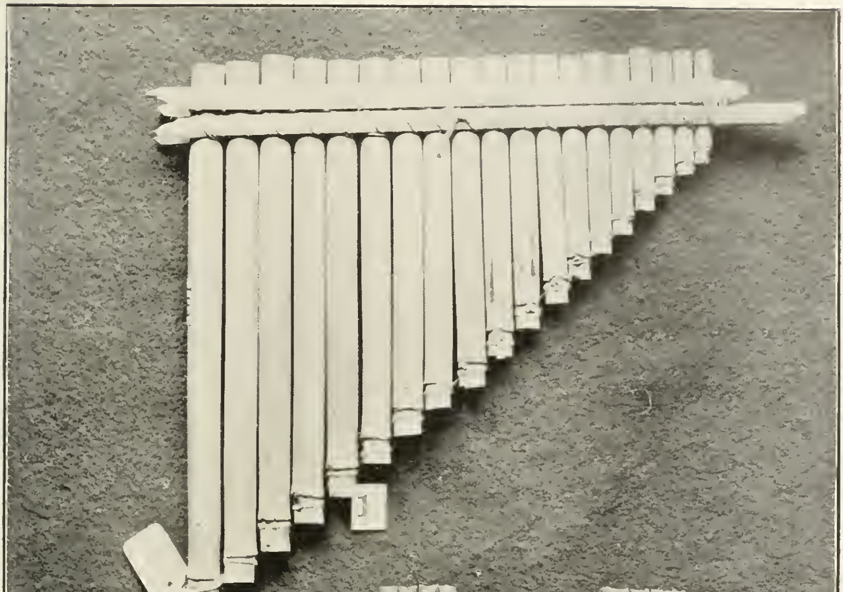
EXPLANATION OF PLATE 9.

PAN'S PIPES.

Fig. 1. Pan's Pipes (*Safafir*). Egypt.
(Cat. No. 94653, U. S. N. M.)

Fig. 2. Pan's Pipes. Fiji Archipelago.
(Report U. S. Nat. Mus., 1896, p. 559; Cat. No. 23942, U. S. N. M.)

Fig. 3. Pan's Pipes (*Huayra Puhura*). Peru, from an ancient grave.
(Cat. No. 136869, U. S. N. M.)



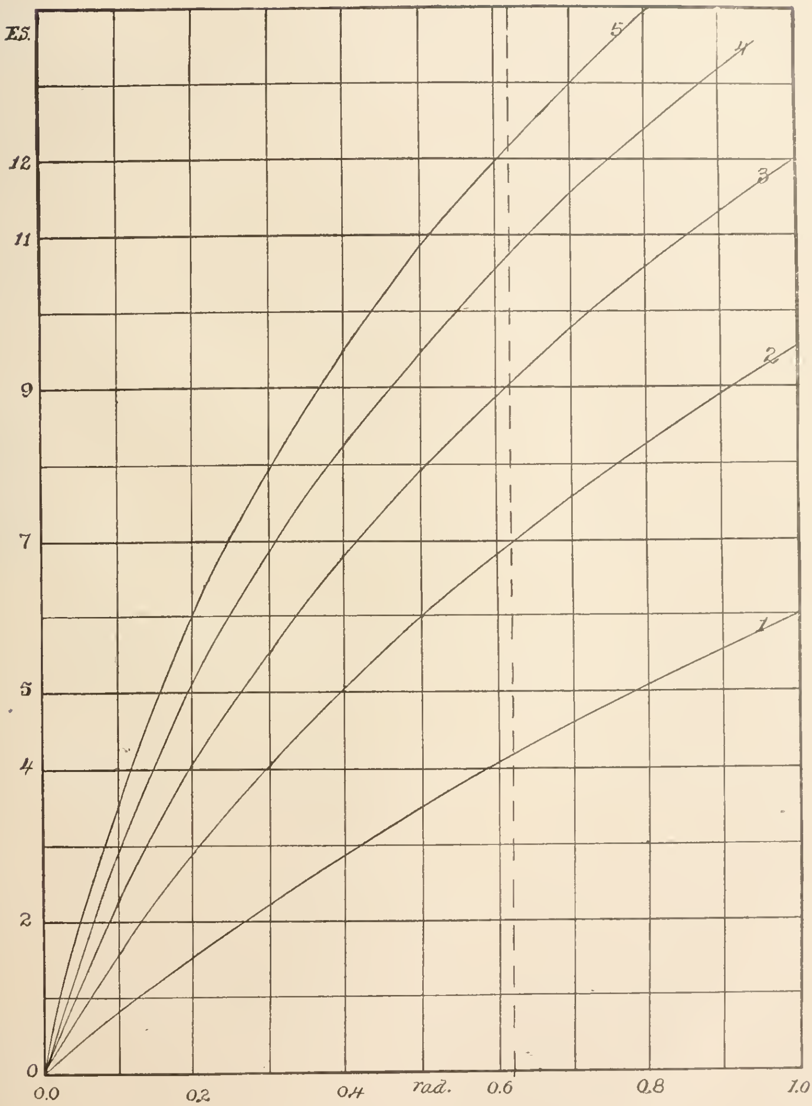
PAN'S PIPES.

EXPLANATION OF PLATE 10.

SCALES GIVEN BY RESONATORS.

The construction of this chart has been explained in the appendix. To use it, find in the base line the number which expresses the radius of the finger holes, that of the mouth hole being considered 1.0, and erect a perpendicular therefrom; the heights of the points of intersection with the successive curves, measured on the left-hand scale, give the pitch of the successive notes produced as the holes 1, 2, 3, etc., are opened, expressed in equally tempered semitones, E. S. The dotted line corresponds to the position on the chart of the type resonator. The chart shows clearly how the successive intervals become smaller as the number of open holes increases, and how the total compass is small if the finger holes are relatively small.

Use may be made of the chart for many ready calculations of intervals other than those due to equal differences, and by doubling the readings in E. S. the result may be applied to string ratios; e. g., find the interval corresponding to the ratio 5 : 4, or 1 + 0.25; the chart gives directly 1.9; the double of which is 3.8 E. S. The table in the appendix gives more accurately 3.86 E. S., showing that the just Third is 0.14 E. S. flatter than the piano Third.



SCALES GIVEN BY RESONATORS.