THE
UNIVERSITY OF MISSOURI
STUDIES

CONTRIBUTIONS TO A PSYCHOLOGICAL
THEORY OF MUSIC

BY

MAX MEYER, Ph. D.

Professor of Experimental Psychology

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VOLUME I

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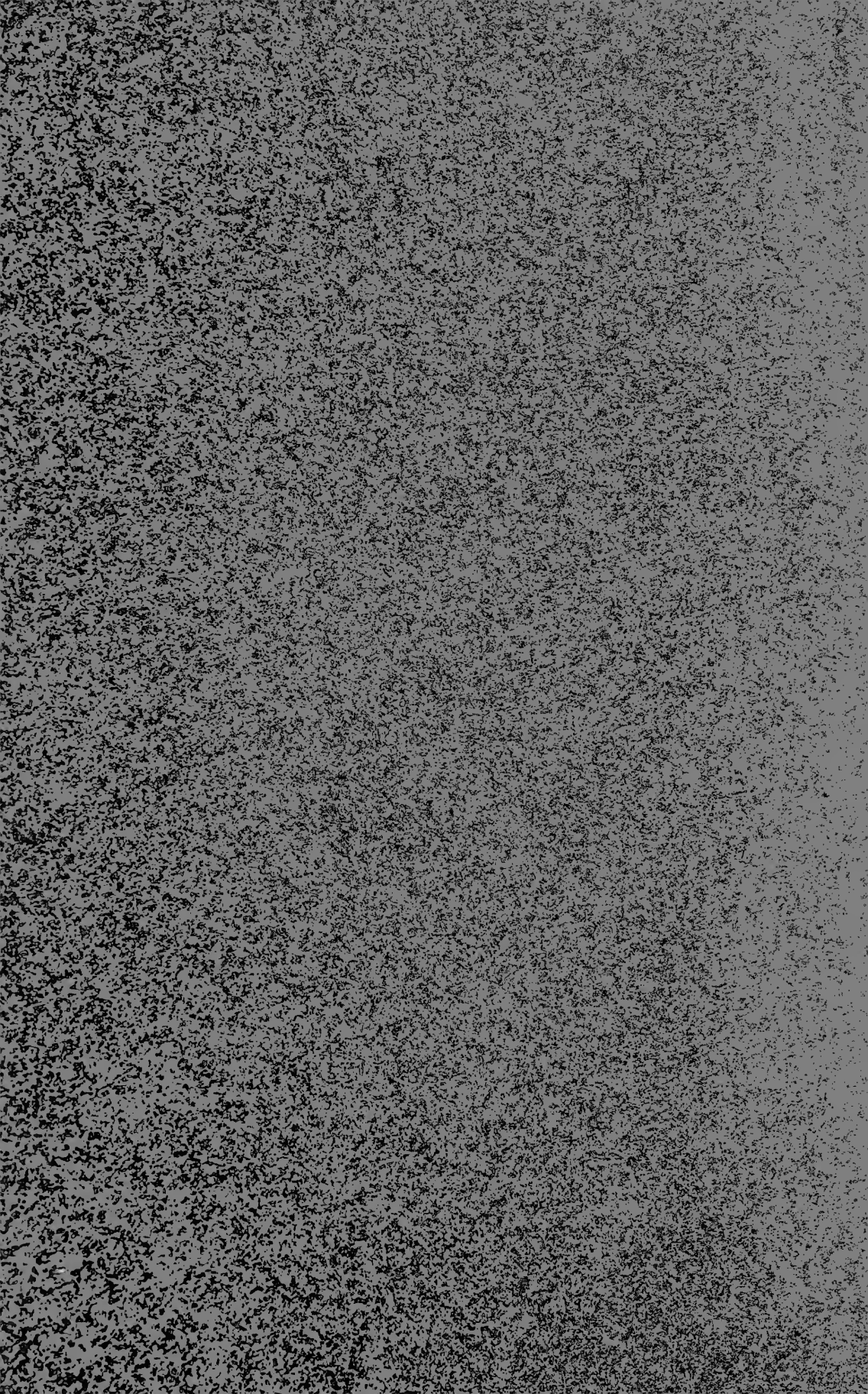
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INTRODUCTION

THAT musical theory, if it is to be regarded as a scientific theory, must be psychological, need hardly be emphasized. No theory of any department of aesthetics can be other than psychological; and musical theory is a department of aesthetics. Neither the physicist nor the physiologist can prove by physical or physiological laws, why we must enjoy certain combinations of tones. It is the psychologist's task to determine the aesthetic laws which describe the subjective as well as objective conditions of aesthetic enjoyment. That physical and physiological concepts are used in the formulation of these laws, is a matter of course. But the physical and physiological concepts cannot be the only constituents; psychological concepts must also enter into these laws. The aesthetic significance of the former consists only in their relation to psychological concepts.

Aestheticians have often overlooked this fact, particularly with respect to music. Some have tried to demonstrate that we must enjoy certain combinations of tones, because vibrating bodies produce overtones. Others have demonstrated the same proposition by means of physiological facts. Such methods, in aesthetics, are metaphysical, not scientific. The aesthetician's task is to determine that—not *why*—enjoyment of combinations of tones depends upon objective and subjective conditions, which may be—no one can tell *a priori*—psychological or partly psychological, partly physiological and partly physical.

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Scientific progress does not always consist in adding new notions to the existing ones; it very often consists in destroying an old system and constructing a new one. Of course, no sensible person will break down an old system unless he can substitute something better. We must accept all common terms which can be psychologically justified. If it is necessary, we must modify them, we must more accurately define their meaning. However, we must reject all terms that possess a purely historical value, but which cannot be justified by psychological observation. On the other hand, we must introduce new terms whenever psychological facts demand them.

The most important group of musical facts is the one referred to by the scientific term "melody." Music may be without rhythm; we find such music, e. g., among Oriental peoples. Music may be without harmony; no one doubts this, except perhaps a few theorists whose theories rest upon the contrary assumption; but the street boy who whistles his favorite tune contradicts them. There can be no music without melody. The usage of language does not permit us to speak of music when there is nothing but a single invariable chord, or when there is nothing but a rhythm beaten on a drum. Therefore we should take up first the theory of melody. In books on musical theory, however, one finds only elaborate discussions of harmonies. Melody, the essential part of music, does not seem to exist for the theorist. This applies to musicians as well as to psychologists. The only exception in psychological literature is Th. Lipps,[1] who tried to lay the foundations of a theory of melody, but was only partly successful. It is a common error in musical theory that the basis of all music is the so-called diatonic scale, represented by the numbers 24, 27, 30, 32, 36, 40, 45, 48. This view prevents the development of a scientific theory of music. Its adoption prevented Lipps from carrying his investigations farther. The same mistake prevented

[1] *Psychologische Studien*, Heidelberg, 1885.

Gurney (in his very good book, *The Power of Sound*) from solving the problem, and led him to the conclusion that it was "hopeless to think of penetrating music in detail," because he had found no way himself. This inability to reach positive results simply proves that he was on the wrong scent. Unable to explain the existence in music of those notes which have no theoretic existence in the diatonic scale, he attempted to explain them in a manner that has become quite common in recent years: he assumed that the place of such notes in a musical form, "relationship being in abeyance," was wholly due to "close propinquity."

The consequence of this theory would be that the tones corresponding to the numbers 10, 11, 12, 11, 10, would result in the formation of a melody, 10 and 12 being connected by relationship, 11 with either 10 and 12 by propinquity. Should the propinquity be not close enough, we might take the numbers 30, 31, 36, 31, 30. To listen to such a tune but once is enough to convince one of the absurdity of the theory. No melody can be formed by means of propinquity. No tone that appears in music is really without relationship. The fact that some one—without having sought for it—has not discovered any relationship between certain notes, does not prove that relationship is in abeyance.

Yet Gurney merely follows Helmholtz here, and Helmholtz has not found any relationship for tones which—though they appear in music—have no theoretic existence in the diatonic scale.[1] He says:[2] "It is nothing but a step intercalated between two tones, which has no relation to the scale, and only serves to render its discontinuous progression more like the gliding motion of natural speech, or weeping or howling." Now, I have not the least doubt that Helmholtz would gladly have relinquished the pleasure of hearing music "like the gliding motion of natural

[1] "Accidentals" these tones are called by the theorists. But in a real work of art there is nothing accidental.

[2] *Sensations of Tone*, Ellis's translation, 2d ed., p. 352 b.

speech, or weeping, or howling." That is the music of cats at midnight, not the music of mankind.

Stumpf's most recent publication on *Consonance and Dissonance*[1] shows that he believes in the dogma of the diatonic scale as firmly as anyone else.

My investigations have led me to the conclusion that one of the chief errors in musical theory is the *a priori* exclusion of the number 7. Herzogenberg,[2] it is true, tried to introduce the number 7 into the theory of music. But he obviously put it into the wrong place. Some years ago my investigations led me to conclude that the number 7 plays a part in music. I afterwards discovered in the English translation of Helmholtz's *Tonempfindungen,* that similar conclusions had been reached half a century before by H. W. Poole,[3] then an organ builder in Worcester, Mass. In the first two German editions of the *Tonempfindungen* Poole is not mentioned at all, in the third he is mentioned only as the inventor of a new keyboard, not as the author of a musical theory. It was left to Ellis to call attention to Poole's theory in his translation of the *Tonempfindungen,* although he did not accept it as true. However imperfect and inconsistent Poole's theory, as stated in his paper, may appear, he deserves the credit of having been the first to discover one of the most serious obstacles to the progress of musical theory. He seems to me to have made a mistake in so far as he attempted to introduce his immature theory into musical practice and spent his time in inventing new keyboards, which are very interesting indeed, but of little value in improving or developing the theory.

The so-called diatonic scale, which is the basis of all discussions in Helmholtz's *Tonempfindungen,* was introduced into the

[1] *Beiträge zur Akustik und Musikwissenschaft,* 1.

[2] *Vierteljahrsschrift für Musikwissenschaft,* vol. 10, pp. 133–145.

[3] *Silliman's American Journal of Arts and Sciences,* 1850, vol. 9, pp. 68-83, 199–216.

modern theory of music by Zarlino in 1558. It was accepted by Rameau in his *Traité de l'harmonie,* 1721. According to Rameau (and Helmholtz) no numbers play any part in music but 2, 3, and 5. This is certainly not a law derived inductively from observed facts, but a dogma, for the number 7 has a similar psychological effect to the numbers, 2, 3, and 5, whereas this is not the case with other prime numbers, as 11, 13, etc. Rameau constructed the scale in the following way. He started from a certain pitch, called tonic, to which, in the scale, he attributed the number 24. Now, because the tones of the intervals 2 : 3 and 3 : 4, besides the octave 1 : 2, have the closest relationship, he added to 24 the numbers 32 and 36, 24 : 36 = 2 : 3, 24 : 32 = 3 : 4. We shall see later that his mistake consisted in making harmony the starting point of his investigation, without noticing that in melody the form of succession, the way in which the tones follow each other, is a factor of the greatest importance. If Rameau had considered this, he would have found that the tonic of a melody can never be represented by the number 24, which contains 3. The same error has, so far as I know, entered into all theories down to this time. Even Poole, although he discovered the possibility of using the number 7 in melody, did not see that it is simply impossible to represent the tonic by a number containing 3, because he, too, made harmony the starting point. He stated that the old theory was right in many cases, whereas there is really not a single case in which the theory (representing a tonic by 3) cannot be demonstrated to be wrong.

We saw that Rameau concluded that the numbers 24, 32, and 36 were the most important in the scale. Now, since 24 : 30 : 36 and 32 : 40 : 48 and 36 : 45 : 54 each represent the chord 4 : 5 : 6, he constructed his complete scale with these numbers, using instead of 54 the half 27 ; *i. e.,* 24, 27, 30, 32, 36, 40, 45, 48.[1] From

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[1] Rameau's "explanation" of the aesthetic effect of the relation of pitches by the mere physical fact, that sounding bodies usually produce,

this scale he derived another scale for the so-called music in "minor" by beginning with 40 and multiplying the first numbers of the "major" scale by 2; *i. e.,* 40, 45, 48, 54, 60, 64, 72, 80. But neither Rameau, nor Zarlino before him, nor Helmholtz and his followers after him, have thought it necessary to tell their readers what facts observable in *melody* justified the use of this series of numbers as the basis of a theory of music in general—of melody as well as of harmony. We shall, in the following, make melody our starting point, because melody is the only essential of music.

Comparing different melodies with the so-called diatonic scale, we notice that some melodies contain fewer, others more notes than this scale. How do these facts agree with the theory that the diatonic scale is the basis of all music? To this question Helmholtz answers: In the first case, the composer has not made use of every possible note, in the second, he has added some notes in order to render the music more like howling. The only answer I know is this: To base any music theoretically on the diatonic scale is mere fancy.

For a long time the tempered scale of twelve equal intervals within the octave has been used in music. Rameau, who recommends this scale for practical purposes, claims that any interval of two successive notes in real music, equal to three-twelfths of the octave, *e. g., g—a*♯ on the piano, has to be regarded theoretically as the interval 5 : 6; any interval of two successive notes equal to four-twelfths, *e. g., a—c*♯ on the piano, as the interval 4 : 5, etc., although there are slight differences. The result of this assumption is the theory that music is made up of intervals, *i. e.,* that the intervals between the notes immediately following each other are the essentials of music, causing the aesthetic effect. This theory

not a single pitch, but several simultaneously (partial tones), does not concern us here any more than Helmholtz's "explanation" of the same effect by "identical partial tones:" 1. because these explanations do not explain anything, but contradict almost more facts than they confirm; 2. because our task in the present paper is only to describe observable facts by laws, not to deal with hypotheses.

is supported even by Helmholtz, although it obviously contradicts his theory of the diatonic scale and his doctrine of what he calls "just intonation."   An example of this theory of intervals is to be found in Helmholtz's statement that the aesthetic beauty of a melody depends on the number of Thirds and Sixths which it contains, a statement without any foundation, as Gurney has demonstrated.[1]   We shall see that the relative pitches do indeed cause the aesthetic effect, but not only the intervals of every two immediately following pitches.

There can be no doubt that the tempered scale cannot be made the basis of a theory of music, that theoretic conclusions based upon the intervals of the tempered scale have no scientific foundation.   A scientific theory of music can only be a *theory describing the laws of music performed in just intonation.*

[1] *Power of Sound*, p. 148.

CHAPTER I

The Aesthetic Laws of Melodies Containing only Two Different Notes

When we hear successively two tones, the vibration rates of which have the ratio 2 : 3, or briefly speaking, the tones 2 and 3, we notice something not describable, which I shall call the *relationship* of these tones.[1] To understand what is meant hereby, let the reader listen to the successive tones 7 and 11 or 11 and 10. He will notice that the two tones have no relation at all to each other. We may describe these facts in still another way, saying that in the first case (2-3) the two[2] tones *form a melody,* whereas in the other case (7-11) they do not. This expression means the same thing, although different words are used.

Besides the relationship, the hearer will observe in the case of 2 and 3 something else, namely, that after hearing 2 and then 3, he wishes to go back to 2, *i. e.,* to hear 2 once more. On the other hand, when we hear first 3 and then 2, we do not wish to hear 3 once more. If 3 and 2 are repeated several times in order to prolong the melody consisting of these tones, we are satisfied in the case where the melody ends with 2, dissatisfied in the case where the melody ends with 3. Save in a few instances, where a peculiar psychological effect is aimed at, no melody *that contains 2* can end

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[1] In the following I shall try to describe the facts in language as plain as possible, avoiding all flowery, but meaningless phrases, so common in musical aesthetics, and all the barbarous terms of the ancient and mediaeval and even modern theorists, which are a mere burden to the memory, and have no scientific value whatever.

[2] For the sake of simplicity let us first consider melodies of two different notes only.

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with any tone but 2. In the case of the tones 3 and 4, the melody must end with 4, not with 3; in the case of 4 and 5, with 4; 5 and 8, with 8; 7 and 8, with 8; 4 and 9, with 4; 15 and 16, with 16; *i. e.*, the *general law* is: *When one of two related tones is a pure power of 2, we wish to have this tone at the end of our succession of related tones, our melody.*[1]

When we hear a melody made up of the successive tones 3 and 5, or 5 and 7, we observe a *relationship* similar to that of the cases above, but we notice that it does *not* make any difference whether we hear, *e. g.*, 3 first and then 5, or 5 first and then 3. In neither case do we wish to hear the first tone once more; or rather, in case we do, the wish is not caused by the particular relationship of these tones. This psychological effect is indeed restricted to the powers of 2 (including $1 = 2^0$).

That no relationship at all is to be observed between tones represented by the prime numbers 11, 13, 17, 19, etc., leads to the conclusion that only tones represented by the prime numbers 1, 2, 3, 5, 7 and their composites possess that psychological property. That the number 7 cannot be excluded in the case of *two* different notes, does not, of course, decide whether it can be excluded in the case of more complicated melodies—as in real music. This question can be answered by observation only. So much is certain, however: there is no reason for excluding the number 7 from the theory of music on *a priori* grounds. We shall see later that there are comparatively few melodies which do not contain the number 7.

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[1] Lipps (*Psychologische Studien*, Heidelberg, 1885), who describes these facts in a similar, although not in the same way, has tried to explain them by the hypothesis that any sensation of tone is not really a continuous sensation, but a series of short sensations interrupted by short empty times. These single short sensations, each corresponding to one single vibration, are to be observed, according to Lipps, only in the case of very low tones. I have shown in my paper, "Ueber die Rauhigkeit tiefer Töne" (*Zeitschrift für Psychologie*, 13, p. 75), that there is not the slightest foundation for such a hypothesis. Doubtless, the future progress of the physiology of the nervous system will help us to explain the facts above stated.

The relationship which we observe is closer in some cases than in others; *e. g.,* very close between the tones represented by the numbers 1 and 2, or 3 and 8, less close between the tones 3 and 5, or 5 and 7, or 15 and 16. I have tried to bring the different relationships into a series. It seems to me that the order of relationship within one octave is the following: 1-2, 2-3, 4-5, 5-6, 4-7, 6-7, 8-9, 15-16, 5-7, 5-9. Whether there is a slight degree of relationship in the cases of 7-9 and 14-15, or no relationship at all, I was for some time unable to decide. It now seems to me that a succession of the two tones 7 and 9, or 14 and 15, does not sound melodious, *i. e.,* that there is no relationship in these cases, and I shall in the following assume that there is none. Transpositions by octaves do not, according to my observations, cause any difference, so that the relationship of 1-3 is of the same degree as that of 2-3, or 3-4. The relationship of 3-5 is of the same degree as that of 5-6, etc. We may therefore represent the relationship in all intervals of the ratios 2 : 3, 4 : 3, 1 : 6, 8 : 3, etc., by the one symbol 2-3; *i. e.,* in any number *composed of a power of 2 and a coefficient* we omit the power of 2 and use *simply the coefficient,* and instead of any *pure power of 2 (including $1 = 2^0$)* we use *simply 2.*[1] We may then substitute for the above series of relationships the following series of symbols representing the relationships without limitation as to distance of pitch: 2-2, 2-3, 2-5, 3-5, 2-7, 3-7, 2-9, 2-15, 5-7, 5-9. I do not assert that this series represents a quite accurate order of the relationships. It is in certain cases theoretically important to know whether a certain ratio represents closer relationship than another one. Yet it is often difficult to decide this question. For example, I do not feel able to decide which of the two ratios

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[1] I ask the reader, if he wishes to understand my theory, to bear in mind that according to this numeration 9:3 is identical with 3:2, although arithmetically 9:3=3:1. But in my theory it is necessary to regard 1 as a power of 2 and to represent 1—as all powers of 2—by the number 2. Similarly, 75:15 (which in absolute numbers may represent, *e. g.,* 150:240 or 75:30) is identical with 5:2.

3-7 and 2-9 represents the closer relationship. However, it is comparatively easy to arrange the ten ratios in three such groups as to leave no doubt concerning the closer or more remote relationship of a member of one group, when compared with any member of another group. I shall denote—whenever this is desirable—the ratios of the second degree of relationship by simple parentheses, the ratios of the third degree by double parentheses, in this way: 2-2, 2-3, 2-5, 3-5, (2-7), (3-7), (2-9), ((2-15)), ((5-7)), ((5-9)). The reader, most probably, will agree with me concerning the utility of this grouping. We may speak, then, of the first, second, and third degree of relationship, without meaning, however, that there are no differences within each group. I will emphasize, further, that when I speak of the *relationship* of two successive tones, the order of succession of the two tones in the melody is not regarded at all.

Similarly, we observe in those cases where one of the two tones is represented by a pure power of 2, that our wish to have this tone at the end of the melody is much stronger in some cases than in others; *e. g.* very strong in tones represented by the numbers 3 and 2, or 3 and 4, or 5 and 4, less strong in tones represented by the numbers 7 and 8, or 9 and 8. This seems to depend upon the degree of relationship of the tones.

One question of this kind we may still consider, namely, whether in the case of the octave, where the tones are represented by the numbers 1 and 2, the psychological effect of the melody's being closed by 1, is different from that when the melody is closed by 2. Lipps [1] asserts that of the two tones 1 and 2, we prefer to

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[1] *Psychologische Studien*, p. 132: "In jedem Ton ist der Rhythmus seiner tieferen Octave, nicht der der höheren, vollständig enthalten. Ist ein Ton gegeben, so bedarf es zum Vollzug der Empfindung eines nachfolgenden um eine Octave höheren Tones noch der selbständigen Zweitheilung des durch den gegebenen Ton vorgezeichneten Rhythmus. Diese Zweitheilung ist, wie oben gesagt, die einfachste rhythmische Leistung. Immerhin ist sie eine Leistung. Dagegen wird unserem Empfindungsvermögen gar nichts rhythmisch Neues zugemuthet, wenn die tiefere Octave folgt. Darnach muss der Octavenschritt von oben nach unten in geringerem Grade

have 1 at the end of the melody. I could not convince myself of the truth of Lipps's assertion, and I am inclined to believe that he was deceived when he made this observation, by having in mind at the same time a certain complicated melody which contains two tonics, but closes on the tonic below. I have to confess that, if I exclude all psychological effects of tones other than 1 and 2, I cannot detect any difference, whether 1 is at the end or 2. This justifies my regarding 1 simply as a power of 2.

I mentioned above that no melody can be formed by means of propinquity alone, that the result would be a howling, but not a melody. This does not mean, however, that more or less propinquity of related tones is perfectly indifferent. Observation teaches us that we have a decided preference for the *smallest* of all those intervals which possess a certain relationship. *E. g.*, of all intervals which possess the relationship 2-3 we greatly prefer the interval of the ratio 3-4 (the "Fourth") for constructing a melody of two tones. The aesthetic importance of this fact in *complex* melodies will soon be seen. ·

als der Fortschritt zu etwas Neuem erscheinen und in höherem Grade den Eindruck des sich in sich Beruhigenden, also endgültig Abschliessenden machen, als der von unten nach oben. Dass es in Wirklichkeit so ist, kann nicht bezweifelt werden." I regret that from my own observations I am compelled to doubt this.

CHAPTER II

The Complete Musical Scale

The complete musical scale is the series of all tones which may occur in one melody, however complex this may be. As soon as we know the aesthetic laws of melodies which contain only two different notes, it is very easy to construct this scale. Suppose a melody begins with the tone 5, followed by 3. Now, we know that if it results in a *melody* at all, the tone 3 can be followed only by a tone that is *related* either to 3 or perhaps to 5 or, if not related to either 3 or 5, perhaps to another tone that is itself related to either 3 or 5. It is impossible, therefore, that the third tone in this melody should be, *e. g.*, 19 or 33, because neither 19 nor 33 fulfils any of these three conditions. So the third tone can only be a product of powers of 2, 3, 5, and 7, since the numbers (2, 3, 5, 7, 9, and 15) representing the melodic relationships contain no other prime numbers than 2, 3, 5, and 7. The third tone may be, *e. g.*, 45, which is related to 5 as well as to 3; or 75, which is related to 5; or 135, if this is followed, *e. g.*, by 9, which is related to 135 as well as to 3 and 5. Generally speaking, since relationship of successive tones is observed only in the cases where the tones are represented by products of 2, 3, 5, and 7, we have to draw the conclusion that a complete musical scale cannot contain any numbers except powers of 2, 3, 5, and 7, and their products. Still, whereas in the case of only *two* different notes the numbers representing possible tones are—according to observation—*restricted* to the numbers of certain ratios enumerated in the above series of relationships, there can of course be no restriction if the melody can contain an unlimited number of different notes. *I. e., the complete*

musical scale is represented by the infinite series of all products of the powers of 2, 3, 5, and 7. We find the beginning of this series in the table.

COMPLETE MUSICAL SCALE

			1024	512	256	128	64	32	16	8	4	2	2	12. E. S.
XII	1015.9	-- -- --	1024	512	256	128	64	32	16	8	4	2		
		3^2 -- 7	1008	504	252	126	63						63	11.73
		-- 5^3 --	1000	500	250	125							125	11.58
XI	958.9	3^5 -- --	972	486	243								243	11.10
		3 5 --	960	480	240	120	60	30	15				15	10.88
		3^3 5 7	945										945	10.61
X	905.1	3^2 5^2 --	900	450	225								225	9.76
		-- -- 7	896	448	224	112	56	28	14	7			7	9.69
IX	854.3	3^3 -- --	864	432	216	108	54	27					27	9.06
		3 5 7	840	420	210	105							105	8.57
VIII	806.4	3^4 5 --	810	405									405	7.94
		-- 5^2 --	800	400	200	100	50	25					25	7.72
VII	761.1	3 -- --	768	384	192	96	48	24	12	6	3		3	7.02
		3^3 -- 7	756	378	189								189	6.75
		3 5^3 --	750	375									375	6.60
VI	718.4	3^6 -- --	729										729	6.12
		3^2 5 --	720	360	180	90	45						45	5.90
		-- 5^2 7	700	350	175								175	5.41
V	678.1	3^3 5^2 --	675										675	4.78
		3 -- 7	672	336	168	84	42	21					21	4.71
IV	640.0	3^4 -- --	648	324	162	81							81	4.08
		-- 5 --	640	320	160	80	40	20	10	5			5	3.86
		3^2 5 7	630	315									315	3.59
III	604.1	3 5^2 --	600	300	150	75							75	2.74
II	570.2	3^2 -- --	576	288	144	72	36	18	9				9	2.04
		3^4 -- 7	567										567	1.77
		-- 5 7	560	280	140	70	35	•					35	1.55
I	538.2	3^3 5 --	540	270	135								135	0.92
		3 5^2 7	525										525	0.43

Actually, of course, no music will ever make use of an *infinite* number of different notes, if for no other reason than that the life of a man would be too short for such a performance. We need not, therefore, continue the series farther than actually existing music requires. I have found the series to suffice when continued up to 1024. Besides, in the table, I have omitted of 5 all higher powers than 5^3, and of 7 all higher powers than 7 itself, because I have found no case where these omitted powers of 5 and 7 are used. This is interesting when compared with the fact that of 3

much higher powers are used, and that $9 = 3^2$ appears even in ratios representing direct relationship, and that of 2 *all* the powers are of absolutely equal value with respect to relationship.

Should the reader think it necessary to add the omitted products of powers of 3, 5, and 7, he is of course free to do so. Whether they are ever used can be decided only by experimenting upon special pieces of music—inductively, not deductively. Further, it may not be true that the series in the table goes far enough to permit the actual representation of all existing melodies. In this case it is easy to continue the series. Whoever desires to have the series continued further, may do this for himself, until either his paper, his ink, or his time fails him. But no one should tell me that I have no right to call the scale "complete," because, on paper, it is not infinite. We do not deny, *e. g.*, the mathematician's right to call a certain curve he has drawn a "parabola," although the curve on the paper is not infinitely extended, as a theoretical parabola is.

In order to represent the different relationships of *two* successive tones in a manner as simple as possible, I omitted the power of 2 in numbers composed of a power of 2 and a coefficient, and used simply the coefficient; and instead of any pure power of 2 I used simply 2. This is justified by the fact that transposition of one of the tones by octaves does not alter the relationship. In the same manner I shall omit in actual representation of music all powers of 2 contained in the numbers of the table. The numbers which I shall use, accordingly, as representatives of all numbers of each line in the table, are found in the second column from the right.

One notices at a glance that the ratio 3 : 4 occurs very often indeed among the numbers of the table. I mention this, because in a criticism of my theory I find the curious statement made that notes which constitute with other notes the interval represented by the ratio 3 : 4 are outlawed by my theory. As a matter of fact, the numbers 729 : 243 : 81 : 27 : 9 : 3 : 2, all of which are to be found in the "Complete Scale," represent a series of the ratio 3 : 2, wherein

3 and 2 signify, in my notation, any number that may be derived, respectively, from 3 or 2 by multiplying with any pure power of 2. Consequently the above series of the ratio 3 : 2 represents, among others, a series of the ratio of 3 : 4. I have given my system of numbers the name of a *complete* scale for the simple reason that there is no melodious combination of tones that is not represented within the infinite series of the Complete Scale. Besides, my critic's remark that some notes are outlawed by my theory loses sight of the fact that my theory is not a deductive theory, prescribing what should be, but an inductive theory, simply describing what has been observed.

The numbers of the table, which of course represent *relative* pitch, may be brought into connection with an *absolute* pitch by arbitrarily defining the absolute pitch of any one of them. If we substitute *tempered* intonation for the intonation theoretically required, each tempered tone (designated in the table by a Roman numeral) takes the place of several tones in perfect intonation. In order to show the magnitude of the difference between tempered and perfect intonation in vibrations, I have added to the Roman numerals the tempered vibration rates, making 640 identical in tempered and perfect intonation. Further, at the suggestion of Mr. Wead, of Washington, D. C., I have added to each tone of the Complete Scale the value which determines its distance from any other tone in units of equally tempered semitones (E. S.). We find, *e. g.*, that the interval between 21 and 5 is equal to 4.71—3.86 = 0.85 E. S. The interval between 3 and 5 is equal to 7.02—3.86=3.16 E. S. The interval between 9 and 15 is identical with this; the difference is 14.04—10.88=3.16. In the latter case we have to add 12 E. S. to 2.04, which represents 9. The addition of these values to the table increases its usefulness considerably.

The Complete Scale may be used in order to determine exactly the relative pitches which the composer *meant* in a certain melody. The common musical notation, because of its historical develop-

ment, does not give the pitches exactly, but approximately.   In order to determine the pitches meant by the composer, we must have an instrument that gives *all possible* pitches, so that we can choose between them.   The organs which have hitherto been built by Helmholtz and others in order to compare Helmholtz's "just intonation" with "tempered" intonation, cannot serve our purpose, because they do not contain all possible pitches, as represented in our table of the complete scale.   While at Clark University I constructed a reed organ that gives exactly the pitches of the highest four octaves (64-1024) in the table.   The absolute pitch of this organ was arbitrarily chosen and is identical with the numbers in the table representing the relative pitches, so that the lowest reed gives 64 (complete) vibrations in a second.   The organ does not possess any technical properties that make a detailed description of its construction desirable.   The only important question, how to tune the reeds, is easily answered by any expert in physiological acoustics or any professional tuner.   I selected a reed instrument for two reasons: 1.   It is extremely easy to tune reeds, in the simple ratios here required, with an accuracy that satisfies the highest demands.   2.   Reeds may be voiced so that their quality most closely resembles that of the tones of stringed instruments of the viol class, with which musical persons are well familiar.

When we have to determine the right intonation of a certain melody, we first note down the melody in intervals of twelfths of an octave, *i. e.,* in units of the equally tempered scale.   Now our table of the Complete Scale (and our organ) not only permits several slightly different intonations for each note of the tempered scale, but even leaves it absolutely open with which tone of the Complete Scale to begin.   Consequently we begin with any one of those tones which appear to possess a higher probability; for experience very soon teaches us that we are rarely successful when we begin with a very complex number.   We do not, therefore, begin with 729 or 675, but with 5, 3, or 15, or other rather simple numbers.   Now, beginning with a certain tone of

our instrument, we play one of those combinations of tones which are *objectively possible* according to our notation in intervals of "twelfths" and according to our Complete Scale. The result may be that all the combinations we try appear so utterly repugnant that we are subjectively convinced that no Bach or Mozart or Beethoven or Wagner could have aesthetically enjoyed them. In such a case, we begin with another tone and try those combinations of tones which are now possible, until we have succeeded in determining an intonation of the whole melody that appears satisfactory.

Usually one finds only *one* intonation of the whole melody which he feels inclined to attribute to the genius of a great composer. All other intonations, when compared with this one, appear devoid of beauty, *i. e.*, of aesthetic effect, and are therefore rejected. However, in some cases one notices that *two* different intonations of the whole melody surpass all others in aesthetic effect. Then, of course, we have to accept them both, at least temporarily, as long as there are no particular reasons for attributing the one or the other intonation to a special composer. I have never met with cases where a great many different intonations of a whole melody appear aesthetically effective. The expression "different intonations of the whole melody" may be misunderstood. Let me say, therefore, that I mean by this two (or more) intonations of the same ("the same" in intervals of twelfths) melody which are *not perfectly* identical, however great or small the differences may be.

Of course, the reeds in such experiments have to be voiced carefully, lest the observer should form a wrong judgment through peculiarities of the quality of certain reeds. Besides, one must be aware of the tendency to deviate a little from the numerically right intonation,[1] the cause of which has not yet been made quite clear.

[1] C. Stumpf and M. Meyer, *Maassbestimmungen über die Reinheit consonanter Intervalle. Zeitschrift für Psychologie*, XVIII, pp. 321–404; and Stumpf's *Beiträge zur Akustik und Musikwissenschaft*, Heft 2.

Such a tendency, however, is not likely to have affected my observations, because the smallest differences of pitch in the Complete Scale are considerably greater than the average error caused by that tendency.

In Chapter III we shall determine the intonation of some common melodies. I shall transpose some of them in certain ways, without regard to the notation (the "key") originally used by the composer. This transposition causes only a difference in the absolute pitch of the melody, a difference which does not at all concern us here. The reader will see at once that this transposition enables us to use in any example the same *musical name* for the same *number* representing the relative pitch. This is by no means necessary, however; it would even be misleading should it give rise to the view that a definite *absolute* pitch belongs to each number. I do not expect, therefore, to be consistent in this transposition.

CHAPTER III

ANALYSIS OF COMPLEX MELODIES

We are now sufficiently prepared to pass on to the analysis of complex melodies. By "complex melodies" I mean melodies which contain tones not related to each other, or, better, tones related to each other not directly, but by mediation of a third tone, so that, as we shall see, the melody must *theoretically be dissolved into partial melodies*. We shall analyze a number of well-known melodies and derive from them *inductively the laws of construction of complex melodies*. Before the theoretical analysis of complex melodies, however, I shall give in musical notation, with the theoretically corresponding numbers, an example of a simple melody, *i. e.*, a melody every tone of which is related to every other tone. Our example is taken from Beethoven's *Fidelio*.

I. BEETHOVEN, FIDELIO

Above the notes are set the theoretically corresponding numbers. According to my method of numeration two different pitches are represented by the one number 2. These *pitches represented by the number 2 I shall call "tonics."* So our melody starts with a tonic and passes to 3,[1] a movement which causes a strong wish to go back to a tonic. This wish is not granted, but the melody passes to a closely related tone 5, which strengthens the wish to go to a tonic 2. The melody then, indeed, goes over to 2, but only in order to leave it again and to pass through 5, which causes a rather strong wish to go to a tonic, to 3, which causes a

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[1] To this pitch 3 I do not give any name, least of all the name "dominant," because such a name cannot be scientifically justified.

very strong wish to go to a tonic. However, this wish is not grati-
fied. The melody passes again to 5, back to 3, and now to 2.   But

2 is left again for a short time and replaced by 3, whence the mel-
ody finally surrenders to 2.   This last movement over 3 back to 2
is repeated twice.   2 has won the battle, and the hearer is aestheti-
cally satisfied.

Such a simple melody as the above is not often used in music.
It serves, because of its simplicity, very well as a signal, for which,
indeed, it is used in Beethoven's opera.   A complex melody is the
following tune of Silcher to Uhland's poem, *Der gute Kamerad.*

2. SILCHER, DER GUTE KAMERAD

The melody starts with 3.   The hearer of course—granted
that the melody is unknown to him—can not guess whether the
pitch heard is a tonic or not.   From 3 the melody passes to 2.
Now the hearer wishes to remain on this pitch.[1]   But the melody
passes to 5 and from there to 9, which is related to 5 and 2, al-
though not very closely.   From 5 as well as from 9, more strongly
from the former, less from the latter, the hearer wishes to go to 2.
This wish is granted, but after some time has passed, 2 is left again
and is replaced by 3.   For a short moment the melody passes back
to 2 and thence to 9; from 9 not back to 2, but to the related tone

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[1] I hope that no one will raise the objection that the hearer will prob-
ably suppose the melody to be yet unfinished and so will certainly wish to
hear the rest.

5, and from 5 to the related tone 3. Although 3 is not 2, some rest on 3 causes the hearer a certain amount of satisfaction, because the 9, which he has met several times, is to 3 as 3:2 (according to my numeration). A tone like 3 in this case I call a *secondary tonic.* From 3 the melody passes to 21, which is to 3 as 7:2 (*i. e.,* 3 appears as a secondary tonic), so that the hearer wishes to go back to 3. Instead of such a movement the melody passes to 5, which is closely related to 3, but not at all to 21. We shall see that this is very common in music, *viz.,* that *the melody instead of passing to a pitch to which the hearer expects it to pass, passes to another pitch, but to one closely related to the expected pitch.* Now, with 5 the melody can not end, because the tonic 2 has been heard already, and in such a case the hearer expects the melody to end with a tonic; otherwise he would not be aesthetically satisfied. After a series of further movements the melody indeed ends with 2.

We may simplify this melody by reducing it to the relative pitches of the above melody of Beethoven, 2, 3, and 5. The first part of Silcher's melody is then represented by the numbers 3, 2, 5, 2, 3, 2, 5, 3, 5, forming by themselves an unfinished melody. From the remaining pitches combined with 3 we get the following melody: 3, 9, 3, 9, 3, 21, 3. Why I have added to the remaining pitches the last 3 will be seen at once. If we divide each number by 3, we get 2, 3, 2, 3, 2, 7, 2, a complete melody with tonics, ending with a tonic (which in the compound melody I call a *secondary tonic*). This second melody is interwoven with the first one, 3 in both cases being of identical pitch. We have to lay stress on the word "interwoven," because in books on musical aesthetics the impression is often made on one that a melody is always composed of smaller parts by simply filing them together. The psychological laws of relationship are, of course, valid even when the related tones are separated by other tones; though the psychological effect of the relationship becomes weaker and weaker as the time interval between the two related tones increases.

The possible ways of combining melodies such as those above, are of course almost infinite in number, and we cannot dis-

cuss all possibilities here. One point only I shall mention. I added to the second partial melody the last 3 in order that this melody should end with its tonic. In the compound melody it is not necessary that this last pitch should really be heard; it may be replaced by a closely related pitch (5), which must be a pitch of another partial melody (in our example: of the partial melody which carries the tonic of the whole.) Similar instances are innumerable in music. We shall see that there is a general law, that *the last tone of a partial melody may be left out, if it can be replaced by a tone closely related to the omitted tone.* This, however, is but a special case of the more general law mentioned on page 22.

Below the notes in our second example, are to be found the numbers which represent the same melody according to the old

theory (Zarlino-Rameau-Helmholtz). Yet here also the powers of 2 are omitted, in order to make the numbers more easily comparable. We notice that in the old theory a pitch is represented by 2, which is no tonic, but may even be left out entirely without changing greatly the character of the whole. But the tonic itself is represented by 3, and the melody ends with 3. According to that theory we have not several partial melodies, one interwoven in a highly artistic way with another; but we have simply a number of pitches, arbitrarily taken from a "scale" and combined not into a melody, of course, but into a succession of pitches, in the same manner as a bricklayer builds a wall out of bricks: here it is quite indifferent whether he takes first the one and then the other; or the reverse. I say "not into a melody," because *the most elementary psychological law of melodious succession is simply neglected by that theory, viz.,* the law, that no hearer is satisfied, if after having heard once or more often the tonic 2 he does not find 2 finally at the end of the melody.

When we compare the numbers of the new theory with those of the old one, we see that in the new theory the value of most of the numbers is one-third of that of the numbers of the old theory. In order to compare the intonation corresponding to the numbers of one theory, with the intonation corresponding to the numbers of the other, we may multiply all numbers of the new theory by 3. Then—in our second example—the numbers of both theories are identical, except in the case of *f,* which in the new theory is represented by 63, in the old theory by 64. Now, it is easy to make the experiment of playing the same melody with 63 (according to the new theory) and again with 64 (according to the old theory). In the first case the melody sounds all right. In the second case the hearer has an impression similar to that experienced when he looks at a painting totally misdrawn. As soon as one hears 2 (64) he expects this to be a tonic, but his wish to have the melody end with this tone or an octave of it, will never be satisfied. The succession of pitches does not end on the note that appeared by its

intonation to be a tonic, but on another one. And this is called "just intonation" by Helmholtz and his followers.[1]

3. Beethoven, Das Blümchen Wunderhold

The above tune of Beethoven begins with 3, 5, 2. Then follows 27, which has no relation to the preceding tonic 2, although

[1] Lipps (*Psychologische Studien*, p. 135) has gone still further and pretends to have derived from this "just intonation" a general psychological law of melody, *viz.*, the law that the tone 3 becomes in a higher degree the real aim of the movement, because it is preceded by 2. This would be like saying of Napoleon: "Elba" became in a higher degree his real aim, because he had previously been Emperor. In order to support his theory Lipps cannot bring forward any argument but the following: "Wenn zwei sich streiten, freut sich der Dritte." I am unable to see how this proverb applies here, since a struggle of sensations of tone has never been observed. Instead of deriving from a supposition such a strange theory, it would be better to give up the supposition and conclude that the Helmholtzian intonation is not a just one.

it does have to 3, the first note of the whole melody. This 27 following 2 is an enigma to the hearer, because of the lack of relationship. The solution is given at once by 9 and 3, which form a melody with 27 (3 being a secondary tonic) and which at the same time are closely related to the preceding melody 3, 5, 2. A detailed description of the following parts is scarcely necessary. The reader will find in them musical forms similar to those in the previous examples; similar forms, but not exactly the same, for even such a small number of different pitches allows an almost infinite number of melodious combinations.

4. IRISH FOLK SONG

The above melody may be easily analyzed by the reader, except that part where g♯ occurs. Our table shows for g♯ the numbers 25 and 405. We may play the melody with either of these tones, but we have a decided preference for 405. With the

succession of the tones 2, 15, 27, 405, 27, 15, 2 the melody obtains quite a peculiar color. From the first mentioned 2 the melody passes to the related tone 15, so that the hearer wishes to return to 2. The melody does not return, but passes from 15 to the related tone 27 (15:27=5:9), *i. e.*, to a tone that has no relation to the expected tone. Moreover, on 27 as a secondary tonic is based the partial melody 27, 405, 27 (separately 2, 15, 2), of which tones none has a relation to 2. The psychological effect of all this is similar to that of a dangerous situation from which one sees no escape. Yet very soon relief appears. From 27 the melody passes back again to the related tone 15, whence it returns to the tonic 2.

The melody is followed by the series of tones in musical notation of which it consists. The cross below g♯ means here, as in all cases where it occurs in the following, that the corresponding pitch—according to the old theory—has no relation to the melody, but "only serves to render the melody more like howling." A further proof of the worthlessness of that theory is to be found in the fact that this note—as we saw above—has a certain definite intonation, whereas a slight difference of intonation could not have any considerable effect, if the note were only to render the melody "more like howling."

A remarkable fact is that this melody contains no 7.

5. Mozart, Don Giovanni

In this case the melody begins with a tonic, but the first part (four bars) ends with 3. This in itself would not be at all remarkable, but the way in which the melody arrives at 3 shows that it originated in the mind of a master. From 2 the melody passes to 27, which has no relation to 2. This movement affects the hearer in the manner mentioned in our third example. After 27 we hear 9, which leads us nearer to the tonic. Now we might replace all the notes in the third bar by 9 and so pass directly from 9 to 3,

still nearer to the tonic. Here Mozart introduces the partial melody 15, 63, 9, 3 (not 15, 2, 9, 3, as the theory of the diatonic scale supposes), where 3 is a secondary tonic, so that this partial

melody may be separately represented by 5, 21, 3, 2. That the whole of the first four bars ends with the secondary tonic 3 of a partial melody, gives the hearer simultaneously some satisfaction and dissatisfaction: satisfaction, because of the partial melody ending with its tonic; dissatisfaction, because this part of the whole ends with a strongly accentuated tone that is not the tonic of the whole, but the tone 3, which causes a particularly strong wish to go to the tonic 2. Now only, is the final return to the tonic capable of producing the immense aesthetic effect of this melody of Mozart's. The old theory does not tell us anything of all this. According to that theory, the whole effect is caused simply by the pitches being taken from the diatonic scale. In that event it is difficult to understand why other melodies, the tones of which are also taken from that scale, are not just as beautiful.

6. BEETHOVEN, SONATA, OP. 14, No. 2

The first part 3, 3, ⁷5, 5, 15, 2 of this melody ends with a
tonic. It contains the partial melody ⁷5, 5 (15, 2), with 5 as a
secondary tonic. The second part is a partial melody (separately
represented by the numbers 3, 3, 9, ⁷5, 15, 2), based on 9 as a sec-
ondary tonic. This partial melody itself has to be analyzed again,
the melody 6⁷5, 135 (separately 5, 2) being a partial melody with
135 as a secondary tonic. The old theory represents the second
part of Beethoven's melody by 5, 5, 15, 2, †, 2⁷.

In the third part the melody, passing through a number of
smaller partial melodies, touches 2 several times and finally closes
with 2. The last notes are 3, 21, 15, 2. There we may question
why the 3 above the 2 has been used and not the 3 below the 2.

Some theorists have alleged that the mere movement of ascent and descent of the pitches, even without any relationship, is an important factor in the aesthetic effect of music. It is possible, indeed, that ascent and descent have a slight aesthetic effect. However, I have not observed this effect myself nor has any one else, and so I must regard it as unproved. What I have observed, is the following: Suppose the composer intends to form a melody of two different tones, one of which is absolutely given, *e. g.*, 2 in a certain pitch, while the other is determined only by its number, *e. g.*, 3, so that several pitches, differing by the interval of an octave, are possible. In such a case, unless there is some reason against it, that 3 is used which is *nearest to the given 2, i. e.*, the 3 below. Yet when not only the 2 is given, but another tone also, *e. g.*, the 5 just above the 2, and 3 (without determining pitch) has to be added, that 3 is added which is *nearest to both given tones; i. e.*, the 3 above the 2 is added. When in this latter case the 3 below 2 is used, the melodious unity of the three tones 2, 5, and 3 is dissolved, and two partial melodies (composed of 2 and 3 and of 2 and 5), interwoven with each other, are the result. When the composer desires a complex form, he has to use in this case the 3 below, when he desires a simple form, the 3 above.

If Beethoven, at the end of the present melody, had used the 3 below the 2, the whole of the last four notes would act upon us as a combination of the two partial melodies 3, 15, 2, and 3, 21, 3, the latter interwoven with the former. But in the present form the last four notes act upon us as a combination of the partial melodies 3, 2, and 21, 15, 2, because 3 has been separated from 15 and 21 by the greater distance. The second partial melody, 21, 15, 2, in this case is again composed of two partial melodies, *viz.*, 21, 15 (separately 7, 5), and 15, 2, so that, by this use of the 3 above, the whole form of these four notes has been changed.

Another case of this kind is the separation of the 21 from the *preceding* 9 in the third bar. The 21 forms a partial melody with the *following* 9 of the fourth bar.

Gurney says in his *Power of Sound*: "The ascent from the dominant to the tonic above, the descent to the tonic below, each seems right in its place, while in a form that was worth anything either would be resented as a substitute for the other." We have seen above that this fact is not so wonderful and beyond human understanding as Gurney assumes it to be. Of course, it is not impossible to exchange a 3 or *any tone* of another number *for an octave of it;* yet in some cases the melody ends better with a simpler, in others, with a more complex form.

7. BEETHOVEN, FOURTH SYMPHONY
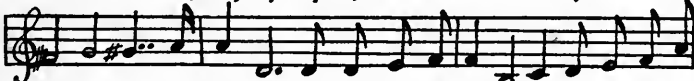
In this melody I wish to call the reader's attention to the melodious form 5, 45, 45, 3, 25 of the third and fourth bars, which, according to the old theory, would be nothing more than "howling." This form has to be regarded as composed of the
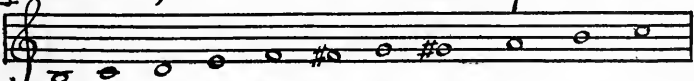
melodies 5, 45, 3, and 45, 25 (separately 9, 5). The following
27 is then connected with this form by the relation of 27 to 3. I
have tried to find out whether in the case of g♯ 25 sounds better
than 405, which would form a melody with the following 27 (sepa-
rately the melody 15, 2). 25 appears to me to yield a better aes-
thetic effect than 405.

8. Beethoven, Sixth Symphony

The first part of this melody is a partial melody (separately
represented by the numbers 2, 3, 5, 2, 3, 5, 2, 3, 5, 2, 3, 2) with 27

New theory   27   81   135   27   81   135   27   81   135   27   81   27

Old theory   5   15   †   5   15   †   5   15   †   5   15   5

27   9   9   27   9   9   27   9   9   15   9   9

5   27   27   5   27   27   5   27   27   45   27   27

5   2   3   5   2   3   2   5   3   5   21   21   9

15   3   9   15   3   9   3   15   9   15   2   2   27

21   9   3   5   2   5   9   27   15   3   3   2   2

2   27   9   15   3   15   27   5   45   9   9   3   3

New theory   9   81   3   27   15   2   135   9   5   21   3

Old theory   27   15   9   5   45   3   †   27   15   2   9

as a secondary tonic. The second part is a partial melody (separately 3, 2, 2, 3, 2, 2, 3, 2) with 9 as a secondary tonic, so that the movement from the first secondary tonic to the second is identical with a passage from 3 to 2. The next partial melody 9, 15, 9, 9 (separately 3, 5, 3, 3) is a melody without a tonic. From this we arrive at the last part, which contains the primary tonic 2. The first three bars of this part are composed very simply of 2, 3, and 5. Three times the melody touches 2; but when 2 is expected again for the fourth time, 21 is heard, which has no relation to 2. The psychological effect of such a movement has already been mentioned. Yet 21 is followed by 9, which is related to 21 as well as to 2. This melody of Beethoven's is an excellent means of convincing the reader of the truth of my theory. Let him play this melody in the intonation corresponding to the new theory and compare it with the same melody played in Helmholtz's "just intonation," which—when the hearer becomes aware of the intonation—destroys the whole aesthetic effect.

We shall now analyze *a second, somewhat different class of melodies, viz., melodies without a tonic.* Instances of such melodies have already been found in some partial melodies within the preceding examples. In this second class of music we find many instances of partial melodies containing a secondary tonic, but we do not find any primary tonic 2.

9. German Choral

In this melody there is no pure power of 2, *i. e.,* no primary tonic according to our definition of a tonic. But there are secondary tonics. The melody has been harmonically treated by mu-

sicians as if 63 were a tonic (identified with 64). However, the aesthetic effect of the melody itself is for the most part destroyed by this, although I will not deny that many a hearer may be much pleased by the successions of harmonies offered by the composer instead of the melody.

10. WAGNER, LOHENGRIN

This well known melody from Wagner's Lohengrin is another example of a melody without a (primary) tonic. The melody ends with 25. The previous melody ends with 15. We shall find other melodies ending with still other numbers. There is no law that a melody without a tonic must inevitably end with a certain number, as in the case of a melody with a primary tonic (one tonic or more than one in different octaves), where the last note of the melody must be a tonic.

The last two bars of this melody form the partial melody 15, 45, 25 (separately 3, 9, 5), all these tones being mutually related.

15   5   45   3   15   15   5   15   5   45   3   15   15   5   5

3   3   63   9   15   15   27   3   27   15   45   25

5   45   3   25   27   15   63   9
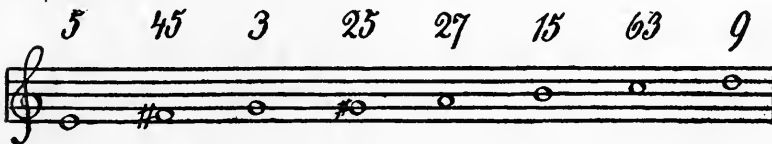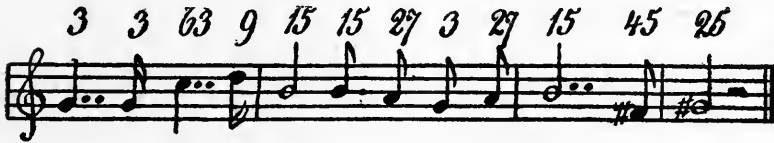
How the old theory of the diatonic scale could explain these two bars, I am unable to conceive.

11. OLD GERMAN SONG

This melody ends with 5. It is a melody without a primary tonic. The old theory, which calls the last note of any melody

15   5   45   3   5   75   5   45   5   15   3   45   3   5

5   75   5   45   5   75   5   45   5   15   3   45   3   5

15   75   5   45   3   15
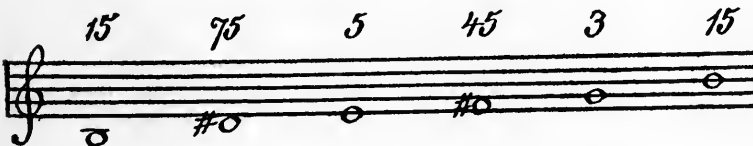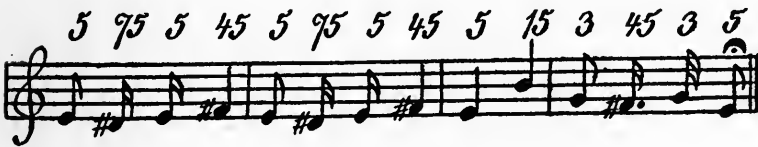
the "tonic" or "key note"—although there is no complete agreement on this matter—would regard 5 as the key note in this case; and since the Major Third (25) of 5 does not appear in the melody, but the Minor Third (3) does, the old theory would say that this melody is a melody "in minor." Now, according to the theory mentioned above, which Herzogenberg has brought forward, the interval of the Minor Third of the key note would have to be represented by 3 : 7. The whole melody then would be represented by the following numbers: 27, 9, 5, 21, 9, 135, 9, 5, 9, 27, 21, 5, 21, 9, 9, 135, 9, 5, 9, 135, 9, 5, 9, 27, 21, 5, 21 , 9. I have tried in this case as well as in others the intonation corresponding to Herzogenberg's theory. The result of this examination convinces me that Herzogenberg's theory cannot be accepted as a general theory.

12. LITHUANIAN FOLK SONG

I tried first to represent the above melody by numbers in such a manner that the last note corresponds to 2. Yet the melody sounds out of tune in that intonation. The intonation that corresponds to the numbers above the notes appears to me to be the right one. So this melody proves to be a melody without a tonic, ending with 9.

How the lack of a primary tonic in the melodies of the second class, the want of a tonal basis at the end of the melody, acts upon the hearer, is well known by every music lover. This pe-

culiarity of the aesthetic effect of these melodies is readily under-
stood from our theory.   It cannot be understood from the old
theory.
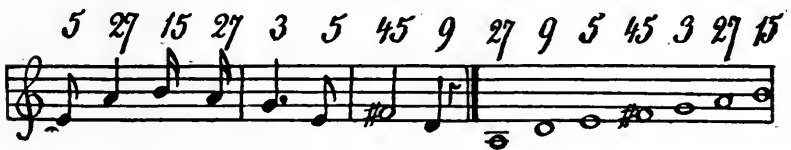
13.   CANON, SUMER IS ICUMEN IN

In the mediaeval canon, *Sumer is icumen in,* which I give
in musical notation including the bass, I determined the most
effective intonation as the one corresponding to the numbers above
the notes.   This then is a melody without a primary tonic.   Of
course, it is not *a priori* impossible to understand the melody (as
it is noted down by the composer) as a melody with a tonic (f=2).

In that case the beginning must be represented by the following
numbers: 2, 15, 27, 15, 2; 2, 15, 27, 3, 5, 5, 21, 9, 5, etc.   The
other intonation, however, appears to me by far preferable.   Very

noticeable is the aesthetic difference of the two intonations in the three bars XVII, XVIII, XIX (f, d, f), which are separated from the other parts of the melody by long pauses. In the one intonation we have 2, 27, 2, *i. e.,* tones not related. Such a succession of unrelated tones is only possible when both of them are related to a third tone that precedes or follows, *e. g.,* 3, or 15. 3 indeed precedes and 15 follows, but the time interval is very great. So this intonation gives very little aesthetic satisfaction. In the other intonation we have f, d, f represented by the closely related tones 9, 15, 9 (relationship 3-5). This causes a satisfactory aesthetic effect. We recognize now how confusing it is to speak here simply of a "diatonic scale." It is then quite impossible to distinguish between different intonations. Therefore we should not use the term "diatonic scale" at all, but should accurately express in numbers the intonation whenever we discuss a melody theoretically.

CHAPTER IV

Psychological Laws effective in the Historical Development of Melody

The existence of the "diatonic scale" in theory and practice, and the distinction between music in the "major" and music in the "minor" scale are doubtless important historical facts. Our psychological analysis of melodies, however, did not lead us to such "scales" at all, and furthermore it discovered some serious defects in those theories which make the "major" and "minor" scales the basis of the classification and description of music. This disagreement between our psychological analysis, based on observation, and the common, rather speculative, musical theory, does not prove, of course, that our analysis is erroneous. Our psychological analysis, on the contrary, explains the development of the old theory of the "major" and "minor" scales as well as of the older musical practice corresponding to that theory. Theory and practice, if our psychological theory is right, could not easily have developed otherwise than they did. Now, the development of musical practice and theory does not depend merely on those psychological laws which we derive inductively from melodies, but, like all historical development, on a great number of causes. Yet, it is probable that these psychological laws are such predominant causes, that the general direction of development has been determined by them, while the other, we may say "accidental," causes produced only temporary deviations from that direction. We shall see that such is indeed the case.

We shall neglect all other possible causes that may influence the historical development, and shall regard our psychological laws as the only effective ones. One is then led to assume that the

earliest music contains only *simple* melodies, *i. e.*, melodies every tone of which is related to every other tone. It is interesting to determine the greatest possible number of such tones that may compose a simple melody, either one with a tonic or one without a tonic. Let us examine the right hand column of our table (page 14) and, beginning with the smallest figure, mark those figures which fulfil the above condition of relationship. This will give us the tones 2, 3, and 5. Now the question arises, Should we add 7 to them? If we do so, we notice that this excludes according to our condition the following 9 and the following 15, since 9 and 15 are not related to 7 (according to the observations, from which we started). Consequently, we omit 7 and add the two tones 9 and 15, which are both related to every one of the tones 2, 3, and 5. No further tone, as 21, etc., fulfils the condition of being related to every accepted tone. The "scale" we reach in this manner—if we choose to give this series of numbers this name—is: 2, 3, 5, 9, 15; or arranged according to pitch within one octave: 15, 2, 9, 5, 3; or beginning and ending with the same number, in this case where there is a tonic, beginning and ending with the tonic: 2, 9, 5, 3, 15, 2.

The melodic advantage of omitting 7 and using 9 and 15, should not be measured simply by the fact that if we accept 7 we have a scale of four notes, if 9 and 15, five. We know that four elements permit only six combinations of two elements each (the mathematical formula is $\frac{n\,(n-1)}{1\,\cdot\,2}$) whereas five elements permit ten such combinations. This melodic advantage may be regarded as one of the psychological causes of the late and rare occurrence of the pure 7 in music. It is not the only psychological cause, however, as we shall see later.

In order to develop music to a higher stage, it is necessary to introduce further tones, even though these are not related to all other tones that are used, but only to some of them. The natural thing to do is to make 3 a secondary tonic, *i. e.*, to use 3 as the tonic of a partial melody. We must expect 3 rather than any

other tones of the above "scale" to be used in this way, since 3 is more closely related to the primary tonic 2 than either 5, 9, or 15.

2 appears in the following relationships: 2-3, 2-5, (2-7), (2-9), ((2-15)). If we make 3 a secondary tonic and use only the relationships of the first degree, we make use of the tones 9 and 15. We need not add 9 and 15, since our scale contains them already. But our aim is to introduce new tones. If we make use of the relationships of the second (but not of the third as yet) degree, viz., (2-7) and (2-9), we have to add to our "scale" the tones 21 and 27. We are now able to introduce partial melodies with 3 as a secondary tonic. These partial melodies may be composed of the five tones 3, 9, 15, 21, 27=3 (2, 3, 5, 7, 9), not only of 3 and the two tones we have added. We may now use 9 also for a secondary tonic in connection with 27. Further, 27 is related to 15, and 21 is related to 9 as well as to 15, so that we gain three more possibilities for partial melodies without a tonic. The addition of these two tones has therefore a very much greater significance than we had originally in view.

We thus arrive at the following conclusion. If we assume that the historical development of music follows, so to say, the line of least resistance, i. e., that closer relationships are used earlier than more remote ones; the stage of melody following directly the stage of "simple" melodies, must then be represented by melodies composed of the tones 2, 3, 5, 9, 15, 21, 27; or arranged according to pitch: 2, 9, 5, 21, 3, 27, 15, 2; or, if we use tempered intonation and express the intervals of two successive tones as twelfths of an octave: $1\frac{2}{12}$, $1\frac{2}{12}$, $1\frac{1}{12}$, $1\frac{2}{12}$, $1\frac{2}{12}$, $1\frac{2}{12}$, $1\frac{1}{12}$. This is the so-called diatonic scale in tempered intonation, represented, e. g., on the piano by the white keys. The fact that this scale actually represents a certain stage of music (folk song before the development of "modern" music) proves that the historical development is indeed, on the whole, in accordance with our assumption of the historical importance of the psychological laws of melody.[1]

[1] In using 3 as a secondary tonic, the relationships of the second degree,

Of all other circumstances which have probably influenced the fixation of the "diatonic scale," we may mention a few. One such circumstance is to be found in the fact that the maker of certain very old musical instruments, *e. g.,* the flute, detected some regularity in musical instruments employing the notes of the diatonic scale. We may regard the "scale" as divided into two equal parts by the middle interval. The first part begins with the tonic 2, the second with 3, the tone that is most closely related to the tonic. When now we look at the arrangement of the holes of a flute, we notice in going upwards in pitch that the holes of the first part of the scale are arranged in the same way as those of the second part; *viz.,* in both cases two larger distances are followed by a smaller one. The same regularity in the arrangement of the intervals of the scale may be recognized, of course, from a merely psychological (though not "musical") consideration. Two larger *distances of pitch* are in both parts of the scale followed by a smaller one. This fact was noticed and theoretically made use of even by the ancient Greek theorists. And it may, through the influence of the theorists, have determined, to some extent, the development of musical practice. It has hardly any significance, however, for a psychological theory of music.

Another accidental regularity of the scale, without significance for a correct (psychological) theory of music, but having a certain historical importance, is to be found in the numerical speculations of Zarlino and Rameau. When we represent the "diatonic scale," not by those numbers by which our experiments with folk songs teach us it must be represented, but by the numbers of Zarlino, 3, 27, 15, 2, 9, 5, 45, 3, which designate pitches differing but slightly from those of the right intonation, then the way for speculation is open. We may with pride point out that our "scale" does not contain any other numbers than products of 2, 3,

(2-7) and (2-9), are represented by the tones 21 and 27; the relationship of the third degree, ((2-15)), is represented by the tone 45. The tone 45 does indeed appear in some folk songs.

and 5.   But what scientific law prohibits the use of 7 in a "scale?"
And why is the tone 3 called "tonic" by Zarlino, Rameau, Helm-
holtz, and their followers?   One searches in vain the writings of
these theorists for a satisfactory answer to these questions.

We saw above that the historic development of music
"along the line of least resistance" explains why there is no pure
7—though there is $21 = 3 \cdot 7$—in the so-called diatonic scale, *viz.*,
the introduction of the 7 into "simple" melodies would have pro-
hibited the use of *two* other tones, 9 and 15, in such melodies. Yet
there is still another reason for the infrequency of 7 in music.
Suppose that one of the early, yet inexperienced, composers had
played or sung or simply imagined the following melody : 2, 2, 5, 3,
7, 3, 5, 2, 2 ; suppose, now, that he had happened to replace the last
2 by $\frac{4}{3}$.   He must at once have noticed the extraordinary change
of aesthetic effect, produced by so small an objective change as the
substitution of $\frac{4}{3}$ for the last 2.   The new melody must be repre-
sented, in order to avoid fractions, by 3, 3, 15, 9, 21, 9, 15, 3, 2.
How the great increase of beauty is caused, is easily seen.   In-
stead of a "simple" melody we have now a complex melody con-
taining a partial melody with a secondary tonic; besides, the
number of melodic relationships is greater than before because of
the relation of 2 to 9 and 15. A composer who has had this experi-
ence once, will hardly ever forget it.   Let the reader, who, I sup-
pose, possesses some musical experience, sing the first melody;   he
will easily notice the strong tendency of substituting the interval
3-2 (which may mean, *e. g.*, 3-4) for the last interval 2-2 and
thereby increasing the aesthetic effect.   This substitution causes
the change of 7 into 21.   The use of the pure 7 would give us a
melody of rather poor aesthetic effect, while the introduction of a
single new tone would give us a melody of comparatively very
great aesthetic effect.   The reader who will make these facts the
basis of further reflections, will no longer find any difficulty in
understanding why the pure 7 is not used in early music, al-
though the 21 is used.

We tried to make clear the influence of psychological laws upon the historical development of the "diatonic scale." Yet we have entirely neglected the fact that we find in the common musical theory *two* scales, the "major" and the "minor" scale. It is necessary to remember that there is a second class of melodies, those *without* a tonic, the historical development of which we shall consider now. There are more ways of constructing scales according to simplicity of development here than where one tone (2) dominates. Many series of tones (scales) may be derived with little difference in the proximity of their relationships. This corresponds to the statement, common among musical theorists, that the intervals of the minor scale are not so definitely fixed as those of the major scale. I must, however, warn the reader against *identifying* melody without a primary tonic with what is called by musicians melody of the minor scale. Melody of the minor scale is indeed invariably without a primary tonic. But melody without a primary tonic is not always melody of the minor scale. The Lithuanian folk song, *e. g.,* which I mentioned in the preceding chapter, I prefer to hear as a melody without a primary tonic. Nevertheless, musicians would call that song a melody of the *major* scale, because of the particular arrangement of the tones when noted in tempered intervals.

The simplest melody without a tonic, with relationship of the first degree only, consists of the tones 3 and 5. We may now use 3 or 5 or both of them as secondary tonics. In this way we derive smaller or greater series of tones which would be called "minor scales" by musicians. *E. g.,* in the folk song 11, which is a melody without a primary tonic, 5 is strongly emphasized by the composer by combining it with 15, 45, and 75, and so using it as a secondary tonic. By adding 3 to these four tones the peculiar aesthetic effect of a melody without a primary tonic is added to the aesthetic effect of 5 as a secondary tonic.

Another case where 3 and 5 are the chief tones of a melody, but where 3, not 5, is used as a secondary tonic, is the *Miserere* in

Verdi's *Trovatore*. The first part of it is the following twice re-
peated melody: 3, 45, 3, 27, 3, 5, 21, 5. This is a melody without
a primary tonic, but 3 is used as a secondary tonic in combination
with 45, 27, and 21.[1]

When we survey the Complete Scale in order to determine
the *greatest number* of directly *related* tones which may compose a
melody without a primary tonic, we must omit, of course, the 2,
this being the primary tonic. We may accept 3, 5, and 7. The
acceptance of 7, however, excludes the *two* following numbers, 9
and 15, since the tones 9 and 15 are not related to 7. We omit,
therefore, 7, and accept 9 and 15. The next number, 21, we cannot
accept, since it is not related to 5; nor can we accept 25, since
it is not related to either 3 or 9.   27 is not related to
5.   35 is not related to either 3 or 9.   45 we accept, it being
related to 3, 5, 9, and 15. None of the following numbers fulfils
the condition of relation. Consequently the greatest number of
tones which may compose a "simple" melody *without* a tonic is five,
the same number as in the case of a melody with a tonic; and the
five tones are represented by the numbers: 3, 5, 9, 15, 45; or ar-
ranged according to pitch: 9, 5, 45, 3, 15.

In order to develop music *without* a primary tonic from these
five tones to a higher stage, it is necessary to introduce further
tones into the above series of five. Into the series of tones repre-
senting simple melodies *with* a primary tonic we introduced fur-
ther tones by making 3 a secondary tonic, using 3 for a secondary
tonic since the tone 3 is more closely related to the primary tonic
2 than any of the rest. Now we must proceed in a different way,
there being no primary tonic at all. More ways than one are open
to us.

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[1] By the way, I wish to mention here that, if we represent the whole
Miserere by numbers, we must multiply all the above numbers of the first
part by 9. This, of course, does not alter the relative pitches. The first
part then ends with 45. The second part begins with 9. The second part
is made up of the tones 9, 5, 45, 3, 405, 27, 225, 15, 135.

We may add a sixth tone under the condition that, though this sixth tone cannot be related to each of the five, it shall be related to four of them. This condition is fulfilled by 27, which is related to 9, 45, 3, and 15; 5 being the only tone not related to 27. We have then the following series: 9, 5, 45, 3, 27, 15. (Compare in the preceding chapter the Lithuanian folk song, 12.)

We may add other tones under the following condition: Each of the five tones 9, 5, 45, 3, 15 shall be regarded as a secondary tonic, and the new tones shall be determined by the ratio 2:3, which signifies very close relationship.   This procedure yields two new tones, 27 and 135. The whole series arranged according to pitch is then this: 9, 5, 45, 3, 27, 15, 135. (Compare in the preceding chapter the Canon, 13.) Many other theoretical considerations may help us to develop the series farther. I will not continue, however, to do this at present. Let me say here that there is no difficulty in expressing the intervals of the last series, 9, 5, 45, 3, 27, 15, 135, by twelfths of an octave and in comparing the arrangement in which larger and smaller intervals succeed with the order of the intervals in the "tempered major scale." We notice at once that the two scales are identical. Such a comparison is quite common among musical theorists. Yet it is speculative rather than scientific.   From a scientific (psychological) standpoint it is impossible to compare the arrangement of twelfths of an octave in different cases, since a distance of one-twelfth (or two-twelfths or three-twelfths, etc.) may actually represent very different ratios.

The above tones (accepted under the second condition), 9, 5, 45, 3, 27, 15, 135, 9, when represented by intervals of twelfths of an octave, yield the same arrangement of such intervals as do the tones, 2, 9, 5, 21, 3, 27, 15, 2, namely $\frac{2}{12}, \frac{2}{12}, \frac{1}{12}, \frac{2}{12}, \frac{2}{12}, \frac{2}{12}, \frac{1}{12}$ although the intonation in the one case is by no means to be identified with that in the other, and the one series contains no tonic, whereas the other does.   This fact is one of the most serious causes of the present confusion in musical theory, *viz.,* all theor-

ists have identified these two series and—since Zarlino—repre-
sented the intonations by a *third* series, namely, 3, 27, 15, 2, 9, 5,
45, 3, which happens to yield, when represented in twelfths of an
octave, the same arrangement as these other two, $\frac{2}{12}$, $\frac{2}{12}$, $\frac{1}{12}$, $\frac{2}{12}$,
$\frac{2}{12}$, $\frac{2}{12}$, $\frac{1}{12}$. They named this third series the "diatonic scale" and
regarded 3 as the tonic, not knowing that this is psychologically
impossible (for the psychological tonic is 2), and not knowing that
many of the melodies which appear to be written in the diatonic
scale, are actually melodies *without* a primary tonic (*e. g.,* our
Lithuanian folk song and the Canon).
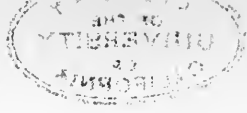
CHAPTER V

Further Comments concerning the Theory of Melody

1. The Seven in Musical Theory

In the preceding chapter (on the Historical Development) I mentioned some of the causes of the infrequency of the pure 7 in music (though $21=3\times7$ is quite common). Most theorists deny that the 7 has any significance in music at all; some, however, admit that the pure 7 ($21=3\times7$ they leave entirely out of consideration) is used by *modern* composers. Wead, in his discussion [1] of my theory of melody, makes the following remark concerning this question: "The Greek theorists were content to work with 2 and 3, their products and powers. Zarlino broadened the foundations, as the author points out, by introducing 5. * * * Poole and others have insisted on adding 7. * * * This question of the 7 is not one of mathematical jugglery or of musical interpretation by the hearer, but simply this: Did the composer mean to use at a certain point a natural seventh (written 7, having a pitch 0.31 E. S. below Bb in the scale of C), and score the passage for an instrument that could give it (horn, violin, etc.)?" I heartily agree with Mr. Wead that "the question of the 7 is not one of mathematical jugglery," but his other remarks do not fit into my system of thought. I have three methodological objections, which I shall mark a, b, and c.

    a. For what *instrument* a passage is scored, cannot decide any theoretical question; for, if that were the case, it would mean the theoretical exclusion of, not only 7, but also of 5 and 3, in all music written for the piano or the organ, since in these instruments the ratios are *irrational* numbers.

[1] *Psychological Review*, VII (4), p. 404.

b.   To speak of "a pitch 0.31 E. S. below Bb in the scale of C" is possible only on the (*a priori,* axiomatic) presupposition that there is such a thing as "a scale of C."   A melody containing no tonic, made up, *e. g.,* of the tones 9, 5, 45, 3, 27, 15 (I am using my own numerical notation; so the reader knows what these numbers signify) would be a theoretical enigma on that presupposition. Since neither 9, 5, 45, 3, 27, nor 15 is a tonic, there is no tone to be named C, and, without a C, no pitch can be determined as 0.31 E. S. below Bb.[1]   Wead's definition of "the question of the 7" *prohibits* any investigation into the use of the 7, save in melodies which contain a tonic.   But even if we restrict our investigation to melodies with a tonic, Wead's definition is too narrow.   The question of the 7 is *not* the question of the theoretical significance of the pure 7 alone, but the quite general question whether the 7, simple or in combination with a coefficient (as in 21, 35, etc.), has theoretical significance or not, just as Zarlino's use of the 5 does not mean simply the introduction into musical theory of the pure 5, but also of combinations of 5 with powers of (2 and) 3.   (Zarlino's diatonic scale, "3, 27, 15, 2, 9, 5, 45, 3," contains the products 15, 45!)   Consequently Wead's definition of "the question of the 7" must be rejected.

c.   Wead's distinction between "musical interpretation by the hearer" and "the composer's meaning," is hardly justified psychologically.   How can we know what Bach or Beethoven or Mozart meant, except by experimenting upon ourselves, assuming that the general psychological laws of melody applied to them as they do to us?   We cannot call up their spirits to answer our questions.   And why should we not assume the same general laws for them as for us?   That we are not great composers, not geniuses, as they were, is no objection.   Are not the general psychological laws of space perception the same in us as in Michelangelo and

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[1] We may, of course, arbitrarily name any tone "C," but Wead means to designate by "C" a *tonic* and by "Bb" a pitch of a definite relation to the tonic, the interval being two equally tempered twelfths of an octave.

Raffael? Why should we assume this to be different in auditory perceptions? We cannot *create* like those musical geniuses, but we can *enjoy* their creations as they did themselves. When a composer "means to use" a certain intonation, what is this other than "the musical interpretation by the hearer," only that "hearer" and "meaner" are the same person in this particular case? Had Beethoven been *born deaf*, he could not have been what he was: there is no "meaning" without "hearing." (I am perfectly aware of the fact that Beethoven *was* deaf in his later life; but "hearing," of course, includes auditory imagery.) The difference between Beethoven and us certainly does not consist in this, that the fundamental psychological laws of his nature differed from those of normal human beings. The identity of these laws is simply the *conditio sine qua non* of his greatness; for a man is great only so far as his influence reaches other men. Just as the composer "means" by hearing, so we find out his meaning by hearing. Only he who denies the identity in different human beings of the *fundamental* psychological laws can deny the possibility of finding the composer's meaning in this way. Of course, where we find more than one possibility of intonation, there we cannot always be sure which one the composer would have preferred.

2. TEMPERED OR JUST INTONATION?

Wead in his discussion of my theory of melody quotes the following remark of Professor Donkin,[2] of Oxford, "who was an accomplished musician and had a profound theoretical knowledge of the science of music" and wrote on Greek music for Smith's *Dictionary of Antiquities*: "The structure of modern music is founded on the possibility of educating the ear not merely to tolerate or ignore, but even in some degree to take pleasure in slight deviations from the perfection of the diatonic scale." By "diatonic scale" he means, of course, the intonation represented by the numbers of Zarlino: 3, 27, 15, 2, 9, 5, 45, 3, where 3 is sup-

[2] *Acoustics*, Oxford, 1870, pp. 25, V.

posed to represent a primary tonic (a psychological impossibility!). I certainly agree with Donkin that the ear takes pleasure in deviations from *this* scale, but I disagree with him when he attributes "perfection" to that scale. The ear takes pleasure in these deviations simply because only by such deviations is an aesthetically perfect intonation possible.

Wead also quotes these sentences from an article[1] on Wagner's methods: "His other innovation, which is not even yet acceptable to all ears, is to employ the chromatic scale of twelve equal semitones as a basis for melody instead of the diatonic scale. The whole of the music of *Tristan and Isolde* would be impossible under the old laws. I need only quote one example, and by no means an extreme one, of a passage impossible to sing or to listen to in anything but *strictly equal temperament* [he quotes the duet "Blissful Dreams"]. It is a constant wonder to me how singers trained upon diatonic scales can sing this and many still stranger passages with anything approaching bearable intonation."

I do not know who is the god that inspires the *Musical Times.* It is doubtless not the god of Science. I determined that pure intonation of the duet of Wagner in which, according to my judgment, it produces the greatest aesthetic effect, certainly a greater effect than in mistuned (equally tempered) intonation. I did not find any serious difficulty in doing this. Of course, it takes time— very much time indeed. But this is no reason for giving up the work and assuming that the melodies sung by Tristan and Isolde cannot be theoretically understood, that they do not possess any aesthetic structure, that they are not melodies at all, but merely a howling up and down in steps which for some reason or other are defined by dividing an octave into twelve equal parts—why twelve, no one knows. (One recalls the fact that according to some Oriental theorists the howling up and down proceeds in 17 steps.) In spite of the authority of the *Musical Times* who permits only "strictly equal temperament," I shall analyze the melodies and

[1]*Musical Times,* London, October, 1896, p. 652.

show their structure.   To me it is by no means a constant wonder that singers trained upon diatonic scales can sing the strange [?] passages of Wagner.   It simply proves that ultimately the psychological laws of melody come out victorious against misdirected training based on theoretical prejudices.   I shall not, however, analyze the melodies in this duet at present.   I wish to use the duet with its accompaniment as an example of the combined effects of melody and harmony, and shall therefore postpone the discussion of the melodies until I am able to take up, in a later chapter, the discussion of the harmonies also.

Let me mention here that just as little as I believe in tempered intonation of Wagner's music, do I believe in the theory that the music of Oriental peoples rests upon psychological laws fundamentally different from those of the European peoples.   I wish I could add to the musical examples analyzed in a previous chapter an Arabian melody containing a "quarter tone," since the quarter tone has been used as an argument in favor of the theory of a fundamental difference of the aesthetic laws.   Many Arabian "scales" with a quarter tone are to be found in different books. Yet scales alone are worthless for the science of music.   I have been unable to find and to analyze any real melody containing a quarter tone, although there is no doubt that such melodies are used in the Orient.   According to my theory there is nothing wonderful in the fact of a quarter tone in a melody.   *E. g.*, a melody made up of the tones 35, 9, 5, 21 may contain a "quarter tone" of the interval 35 :36.

3.   THE CLOSE ON THE TONIC

In Wead's discussion of my theory (p. 404) I read the following words: "The demand for a close on the tonic, on which he lays so much stress, is not commonly met with in Hindu music; though the final may be C, 'accidentals' (if Dr. Meyer will pardon the word for want of a better one) have often destroyed the feeling that the tune is in the key of C; and it is rare to find a

Hindu tune that seems to ordinary musicians to end right." I am far from denying the facts which are above referred to. Yet the words "on which he lays so much stress" seem to indicate a partial misunderstanding of my theory. On page 257 of my previous paper I made the following statement: "One of the most elementary psychological laws of melodious succession is, that no hearer is satisfied, if after having heard *the tonic 2* once or more often, he does not find 2 finally at the end of the melody." This I consider a natural law, though in a special case it may not be strictly true. To make this clearer by analogies: No one denies that Kepler's laws describing the movements of the planets are natural laws, and yet everyone knows that they are not strictly true, though the deviations are produced, not by chance, but by certain definite, but complicated conditions. In aesthetics, it is doubtless an important law that a painting must be in accordance with the principles of perspective. No one calls a painting perfect that is defective in this respect. Yet paintings which show defects of such a kind are regarded as works of art notwithstanding. Exceptions do not necessarily contradict the law. Why should we expect music *never* to neglect the above aesthetic law of melodious succession? I do not expect this and have not made such an assertion. Nevertheless the law is a natural law.

When a Hindu tune does not seem to an ordinary musician to end right, the question arises whether the melody as perceived by this musician is indeed the same as that perceived by the Hindu musician. We learned from several examples that a melody—if we mean by this term a melody as noted down in imperfect musical notation or played on an imperfectly tuned instrument or sung in impure intonation—may represent very different melodies in pure intonation and may, accordingly, act psychologically in very different ways upon different hearers. I will make this clearer by an analogy. It is well known that a certain design on a flat piece of paper may be perceived by one person as a flight of steps, by another (or the same person at another time) as an

overhanging piece of brick wall. So a certain "melody"—as above defined—may be perceived by one hearer as a melody with a tonic, by another as a melody without a tonic. Whether these Hindu tunes violate an aesthetic law or not, cannot be decided so easily. First we have to determine whether the tunes are melodies with a tonic, or—more probably—melodies without a primary tonic, misinterpreted by the biased European hearer, who is apt to as- similate a new experience to experiences with which he is already familiar. I regret that I have not had time enough to study Hindu music so closely as to express a decided opinion concerning this problem.

Of course, music which violates an aesthetic law most likely does it in a manner not particularly effective. I do not think it impossible that, *e. g.*, in a recitative of Wagner a pure 2 appears once and never again. The aesthetic effect of the movement from a related tone to 2 may be in such a case—among the great num- ber of simultaneous effects—comparatively too weak to leave an unrelieved dissatisfaction. Yet I am not quite sure whether cases of this kind may not be partly responsible for the dislike many persons have to Wagner's "endless melody."

4. SCHUBERT, HEIDENRÖSLEIN

It would doubtless be interesting to analyze in the same man- ner as I analyzed short melodies, a very long and complicated piece, *e. g.*, an opera. The limitation of space forbids this here. Yet Schubert's song "Heidenröslein," may serve as a brief example of this kind. My opinion as to the intonation of this song has been subject to some changes.[1] I shall here describe only the in-

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[1] Some readers (more in the habit of speculative than scientific thought, as it happens sometimes) may perhaps conclude: "He feels compelled to change his previous opinion; consequently his whole theory is useless." To them I have to say that the above change of opinion is no argument either against the general principles of my theory or against my method of investigation. It is simply a further demonstration of the fact that some- times two different intonations may produce very nearly the same aesthetic

tonation which at present I believe to be the most effective aesthetically.

The first part is a comparatively simple melody which ends with a tonic. It is represented by the numbers 5, 3, 21, 5, 9, 5,

21, 3, 2. In order to simplify the description, I do not repeat the numbers where the same note is repeated several times. The second part is a melody without a tonic, the intonation of which is represented by 5, 3, 45, 5, 9. The third part taken alone is identical with 2, 9, 2, 15, 2, 9, 5, 2. It is a melody ending on its tonic. The note, however, which is here called 2, is represented in the song by 3. We must therefore multiply all the above numbers by 3. The third part is then represented in the song by the numbers 3, 27, 3, 45, 3, 27, 15, 3, the 3 being the (secondary) tonic of the partial melody. The intonation of the fourth part is represented by 3, 15, 27, 3, 45, 5, 75, 5, 63, 45, 3. It is a melody without a tonic. It begins and ends with 3, so that it is closely related to the

effect, and that, in such a case, the actually preferable intonation can be found only by patiently experimenting, not by deducing from *a priori* reasons, not by forcing a special piece of music into the Procrustean bed of a "scale" of an *arbitrarily* limited number of tones (the "diatonic scale" of Zarlino, Rameau, Helmholtz).

third and fifth parts. The fifth part, represented by the numbers 9, 5, 21, 3, 27, 15, 2, leads back to a tonic 2. The sixth part I originally represented by 27, 63, 21, 27, 2, 5, 9, 2. I prefer now, after continued trials, to use for the last part the intonation 15, 9, 3, 15, 9, 45, 5, 9, instead of 27, 63, 21, 27, 2, 5, 9, 2; *i. e.*, the last part is a melody without a tonic. The tone 9 is identical with the tone represented in the preceding parts by 2. This must be avoided. The old theory often expresses identical tones by different numbers and different tones by identical numbers, and, in order to solve such contradictions, introduces the conception of "modulation," of "change of key." This conception of modulation, however, involves the theoretical acceptance of Zarlino's "diatonic scale" as the basis of all music. This "diatonic scale" being a psychological impossibility, we must reject the conception of modulation and must consequently, throughout the *whole* piece of music, represent different tones by different numbers, whatever the musical notation may be, and identical tones by identical numbers. We can easily satisfy this demand since our Complete Scale permits us to represent by numbers any undivided piece of music, be it so complex and extensive as Beethoven's Ninth Symphony or an opera of Wagner. So we must represent the undivided song of Schubert by numbers, in such a manner that different tones are represented by different numbers, identical tones by identical numbers. In the last part we had to substitute 9 for 2. Therefore we must substitute in all preceding parts 9 for 2, but without altering in the least the relative pitches, the relative intonation. (The absolute pitch is, of course, quite arbitrary, without any theoretical significance.) We do this by multiplying all numbers of the parts preceding the last by 9. The result is to be seen on a later page where I shall discuss Schubert's harmonization of this song. We now recognize that Schubert's song as a whole is a melody without a primary tonic, that it is to be divided into six parts the first of which is based on 9 as a secondary tonic, the second is a melody without a tonic, the third is based on 27 as a secondary tonic, the

fourth as a separate whole is again without a tonic, the fifth contains (and, of course, closes on) 9 as a secondary tonic, the sixth is a melody without a tonic. The first five parts taken as a whole are a melody that closes on its tonic, to which is added the last part as a melody without a primary tonic, but made up of tones related to the tones of the preceding parts.

Similar, yet still more complicated, is the structure of more comprehensive pieces of music, as sonatas, operas, etc. The analysis of this song, I believe, will show the reader how to proceed in analyzing a more complicated piece.

CHAPTER VI

AESTHETIC LAWS OF HARMONY [1]

Harmony—as compared with melody—is the totality of psychological effects observed in *simultaneous* tones. It is by no means such a simple phenomenon as it is often assumed to be. We shall see that we have to recognize at least two distinct aesthetic effects in harmony. When we hear the two tones, 3 and 5, or 1 and 2, simultaneously, we notice something very familiar, namely relationship. Since we found the same relationship in melody, and melody is the only essential of music, we may call it—in honor of melody—"melodic relationship" in simultaneous as well as in successive tones. There is this advantage of speaking of "melodic" relationship: by this adjective we express our opposition to the common theory, which assumes that relationship is to be found primarily in *simultaneous* tones. Those who adhere to this theory explicitly call relationship "harmonic," in order to express their opinion that relationship in melody is an imaginary psychological phenomenon only. According to them, the hearer knows merely that he would observe it if the tones were sounded simultaneously. Such a theory is that of Stumpf. (It is sufficiently plastic to serve as an evolutionary explanation of the *existence* of the relationships.) *Relationship is actually experienced in melody as well as in harmony.* If we choose to call it "melodic," we simply do so in order to honor melody. Since it is the same relationship in simultaneity as in succession, we may classify it in the same manner as we did above: 2-2, 2-3, 2-5,

[1] The critical reader, who compares this article with my earlier writings, will doubtless detect some inconsistencies concerning *terminology*. To him I am not ashamed to confess that I do not feel compelled to retain what now seems imperfect to me.

3-5, (2-7), (3-7), (2-9), ((5-7)), ((5-9)), ((2-15)).   The sec-
ond psychological phenomenon observed in successive tones has
no place in simultaneity. The specific aesthetic effect of the 2
when heard alternately with 3, 5, 7, 9, or 15, depends upon succes-
sion. Whether the 2 produces in simultaneity any specific aes-
thetic effect, instead of the one in melody just mentioned, I cannot
tell. I have not found any, and leave this question open.

Melodic relationship in succession as well as in simultaneity
of tones is a phenomenon conditioned by *two* tones. In chords
which contain more than two tones, we must consider the relation-
ship of every possible combination of two tones. *E. g.,* in the
chord, 5-3-7, we observe three relationships, 5-3, (3-7), and
((5-7)).   In the chord, 27-135-5-3, we observe relationship be-
tween 27 and 135, 27 and 3, 5 and 3; we notice the absence of rela-
tionship between 27 and 5, 135 and 5, 135 and 3.

I mentioned that the second psychological phenomenon in
successive tones, which depends upon one of two related tones
being a pure power of 2, is impossible in harmony, of course.
Still, we observe in harmony another phenomenon, which does
not exist in successive tones, *viz., consonance.* We may try to
make clear what is meant by the term consonance by saying that
there is in some cases of simultaneous tones a higher degree of
"unity" (other terms often used are "blending," "fusion," "Ver-
schmelzung," "harmoniousness") than in other cases. But de-
scription can never take the place of actual experience of this
phenomenon. We may best observe these different degrees of
consonance in making the following experiment. We take two
tuning forks, without resonance boxes, and keep one of them close
to the one ear, the other close to the other ear. This distribution
of the tones is necessary in order to avoid the disturbance caused
by beats, when both tones are heard with one ear. Now, we
notice, *e. g.,* in the tones 7 and 5 a higher degree of a certain
phenomenon than in the tones, say, 7 and 27, but a less degree
than in, say, 2 and 5. I shall use for this phenomenon the term

"consonance," not any of the other terms above mentioned, which are more or less ambiguous. The word "dissonance," commonly used by musicians, is not a necessary scientific term, though we may use the word in order to designate comparatively slight degrees of consonance.

Stumpf has paid more attention to the phenomenon of consonance than any previous writer. He sometimes uses the term "consonance," sometimes the term "fusion" (Verschmelzung) to designate the phenomenon in question. In his book on *Consonance and Dissonance* [1] he tried to explain every musical fact by reducing it to "consonance." He was, of course, not successful in this. His treatise is excellent in its criticism of older theories, but its positive statements simply add to the existing confusion. Stumpf's mistake consists in his utterly failing to distinguish between "melodic relationship" and "consonance" of simultaneous tones, as *two* distinct elements of what is commonly called "harmony."

While we can speak of melodic relationship only in pairs of simultaneous (or successive) tones, we have to recognize a degree of consonance of a compound sound, however great the number of the constituent tones may be. With respect to melodic relationship we found that there are close and more remote relationships, and that in many cases (*e. g.,* 5-11) there is no relationship at all. This is different with consonance. Here we can never say (according to Stumpf, with whom in this respect I agree) that there is no consonance at all. Dissonance means merely a comparatively slight degree of consonance. The investigations of consonance by Stumpf and others were made with duads of tones only. But few observations have as yet been published on the conditions of consonance of triads and chords of more than three constituents. All we know is that the degree of consonance depends in some manner upon simplicity of numerical relations, similarly as melodic relationship does. In duads we observe a high degree of consonance where we find a close relationship, a

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[1] *Beiträge zur Akustik und Musikwissenschaft,* Heft 1.

low degree of consonance where we find a remote relationship or no relationship at all. In triads, tetrads, and more complex chords there is no such simple relation between the consonance and the relationships. Here a low degree of consonance is often combined with comparatively close relationships. *E. g.,* the two triads 2-3-5 and 3-5-15 contain exactly the same relationships, namely, 2-3, 2-5, and 3-5; yet the consonance of the latter chord is much less than that of the former.

We must now ask what is the *relative importance in music* of the two elements of harmony, *viz.,* of melodic relationship in the several pairs of the chord tones and the degree of consonance of the chord as a whole.

This question is easily answered. Every one knows that the greatest beauty of, say, a triad of tones, is not to be found where the degree of consonance is the highest possible, *i. e.,* in the case of three tones in intervals of octaves. The aesthetic effect of, *e. g.,* the triad 2-5-3 is much greater than that of the triad 2-2-2. This fact has naturally caused much trouble to those who do not distinguish between "melodic relationship" in simultaneous tones and "consonance." It is the crux of their theories. For us there is nothing wonderful in that fact. It simply shows us that melodic relationship is just as important in harmony as consonance. In the triad 2-2-2 there is only the one relationship 2-2 three times. In the triad 2-5-3 there are three different relationships. It is no wonder that the greater variety makes the latter chord aesthetically more effective than the former, though the consonance of the latter is less than that of the former; yet the consonance of 2-5-3 is rather high still. We now also understand why the chord 5-3-15 is so much used in music, although it possesses a comparatively slight degree of consonance. Yet no other triad combines three different and *so close* relationships with such a comparatively slight degree of consonance, a combination which in particular cases is aesthetically very effective.

In all the preceding discussions of the aesthetic effects of

harmony I have supposed that the various chords are heard perfectly analyzed, *i. e.,* that all the constituent tones of a chord are noticed as different pitches. This, however, is an ideal condition, that is often not, or imperfectly, realized. It depends upon the practice of the hearer in analyzing, and this practice, of course, is different with different individuals. It depends also upon the simplicity or complexity of the chord that is to be analyzed. In some cases we do not expect the listener to hear a sound analyzed · at all. We know that the tones of musical instruments are composed of a fundamental tone and a greater or smaller number of overtones: yet the composer who writes a sonata for the piano does not expect the hearer to notice the different overtones as pitches. However, even the tones as noted down in the score are not noticed by every one at every time as pitches, as they should be in accordance with the composer's ideal. What then is our state of consciousness when not all of them are noticed as pitches? We may understand this best by discussing a special case as it occurs quite commonly. Imagine that a person hears the three simple tones of 300, 400, and 500 vibrations in a second, but notices only the pitch of 400. The tones 300 and 500, though not noticed as pitches, are psychologically not ineffective in such a case, but determine what is called the "quality" of the tone whose pitch is noticed, *i. e.,* here of the tone 400. (Some use instead of "quality" the French term, "timbre," or the German, "Klangfarbe.")

The confusion which exists concerning the psychological theory of "quality of tone" is so great that it is well to add a few remarks on this phenomenon. Most writers insist that a simple tone has no quality, but that a compound sound has. This theory is not based on observation, but on speculation. Stumpf seems to have been the first to emphasize that a simple tone possesses "quality" (Farbe, Tonfarbe) just as well as a compound sound (Klangfarbe). We may understand this from the following experiment. Sound the simple tone 100, add the weaker tone 200, but concentrate your attention upon 100. One then notices the pitch 100 and

a certain quality which may be theoretically best understood when compared with the visual sensation resulting from mixing white and black on a color wheel. The quality we notice is neither the quality of 100 nor that of 200, but a new quality similar to that of 100 as well as to that of 200. The relative intensity of the two tones is here effective. Now, when we make 200 weaker and weaker until it disappears entirely, we do not notice that a phenomenon which existed before has disappeared, that there was quality before, but that there is none now. That is not what we observe. We observe that the quality changes, but not that it disappears. Now add the tone 50, first very weak then with a little increasing intensity, but do not pay attention to its pitch, but only to the pitch of 100. One does not observe then that the quality of the pitch 100 reappears, but simply that it continues changing in the same direction. Another argument for distinguishing between pitch and quality of a simple tone is this: With very low tones (say, below 25 vibrations) and very high tones (say, above 12,000 vibrations) one is unable to recognize musical intervals; nevertheless one is able to judge that a tone of 20 vibrations is lower than one of 25, and a tone of 16,000 higher than one of 12,000. This may be theoretically explained by saying that the phenomenon of pitch is lacking in these extreme regions, but that the phenomenon of "quality" exists and permits us to judge. In a similar manner we may explain the fact that certain individuals possess in a high degree the so-called "memory for absolute pitch" and are able to name every tone on the piano, but are unable to sing musical intervals.[1] They possess a memory for "quality," but not for "pitch." This theory also makes it clear why certain individuals are able to name every tone on a familiar instrument, e. g., the piano, but find it extremely difficult to name the tones of another instrument or a voice. Their "memory for absolute pitch" seems to be much more a memory for quality than

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[1] Such cases are mentioned by Stumpf, *Tonpsychologie*, II, p. 555.

for pitch. However, further details concerning quality have no bearing on our present investigation.

We return now to our above example of the chord 300, 400, 500. We suppose that it is not heard analyzed, but that only the pitch of 400 is noticed and a mixture of the qualities of 300, 400, and 500. In this case we observe that the aesthetic effect of the three relationships is lacking. This effect depends upon "hearing analyzed." Yet the "consonance" of the chord is noticed; it does not depend upon analysis. This observation teaches how important practice in hearing chords analyzed is for the aesthetic enjoyment of music.

1. Drone Bass

Having discussed the aesthetic laws of harmony, we have to proceed to show that these laws govern, not only separate chords, but real music also, that their application to music enables us theoretically to understand music, without compelling us to *ignore* contradictions we cannot solve, or to make use of one of those "plastic" psychological explanations which are created for the special needs of the moment and consequently "explain" everything for the explanation of which they are created.

There is in music hardly a simpler form of simultaneity of tones than the drone bass, as produced by the bag pipe. That the drone bass is used in such an ancient instrument, that it is used in the simplest and oldest folk music, seems to indicate that it obeys a simple psychological law. Folk music easily gets rid of artificialities. We shall soon see why folk music makes use of a special tone for a drone bass. We saw that most folk songs are made up of the tones 2, 9, 5, 21, 3, 27, 15. Sometimes 45 is to be added to them, rarely another tone. Now we may ask: Is there any one tone that is melodically related in the first degree to all those seven tones? The answer is negative. We may further ask: Is there any one tone which is related to each of these seven tones either in the first or in the second (but not in the third) degree?

Here our answer is in the affirmative.   3 is this tone.   It is rather closely related to all the tones, 2, 9, 5, 21, 3, 27, and 15.   It is no wonder under these circumstances that 3 is so well suited to serve as a drone bass.

Yet not only melodies with a primary tonic, but also melodies without a primary tonic may be accompanied by a drone bass. E. g., melody 11 in a previous chapter is a folk song made up of the tones 15, 75, 5, 45, 3.   It is easily seen that in this case there is one tone that is related to all these five tones in the first degree, namely 15.   We may use 15 as a drone bass.   Now I am perfectly sure that everyone who has been trained in the common musical theory, will at once think that this demonstrates the usefulness of speaking of a "dominant," because the drone bass in our two cases has the relationship 3-2 with the last tone of the melody.   However, I cannot agree with those who introduce such a name.   This name does not help us in any respect.   The fact above mentioned is not general enough to be expressed by a special scientific term. Moreover, there is the danger that such a name will be thoughtlessly used as a startingpoint for speculation.   Science is not hunting for names, but for facts.

2. Canon

A canon is a piece of polyphonic music in which exactly the same melody appears in the several voices, but in various phases. I mentioned in one of the preceding chapters the melody of the mediaeval canon, "Sumer is icumen in."   In the following table the harmonies are given which correspond to the intonation that seems to me the most effective intonation of the melody in one voice.   One may easily convince himself that the same intonation is the most effective intonation of the canon when sung polyphonically.

This result may be understood theoretically.   If the reader will note down the melody in any other intonation and compare the harmonies obtained in that way, he will notice that no other

		I	II	III	IV	V	VI	VII	VIII
Canon	9-f″	9|135	|135	9|9|135					—|—|—
			15		15|27|27	45|45	3|5	45|45	
	9-f″				—|—|—|—				
		45|45	3|5	45|45		9|45	5|3	45|45	5|5|9
	9-f″					—|—|—			|—|—|—
		9|45	5|3	45|45	5|5|5|9	45|27	15|15	27|27	
	9-f″				—|—|—|—	9|9		9|9	—|—|—
		45|27	15|15	27|27			15|15		
Bass	9-f	27|27	3|3	27|27	—|—|—|—	9|9	5|5	9|9	5|45|45
	9-f	9|9	5|5	9|9	5|5|45|45	27|27	3|3	27|27	—|—|—

intonation yields in the harmonies such close relationships and such high degrees of consonance. In the harmonies, as found in the table, the chord 9, 27, 27, 45, 135, *e. g.*, is one of the few which possess only a slight degree of consonance. Yet the relationships are rather close: 2-2 occurs once, 2-3 three times, 2-5 three times, 3-5 twice, ((2-15)) once. There is no tone within this chord that is not related to every other tone. Moreover, this chord is not a long, accentuated, but a short, unaccentuated chord, so that the low degree of consonance is less effective. This is also the case with the other chords of less consonance. In other intonations we should meet with chords of less close relationships and still less consonance. The accentuated chords in the first, third, fifth, and seventh bars are all very consonant and of close relationships. The fourth bar contains no accentuated chord. The accentuated chords of the second and sixth bars are specimens of the kind I mentioned above as being so common in music. They are made up merely of the tones 5, 3, and 15, and consequently contain only very close relationships, though the consonance in these cases is of a moderate, yet not of a very slight degree.

The tones 9, 5, 45, 3, 27, 15, 135 seem to me to be particularly suited for a canon. It is quite natural that a melody without a

primary tonic is better suited for this peculiar species of music than a melody with a primary tonic, with a definite aim. However, it is not *a priori* impossible to construct a canon that contains a primary tonic. Yet I am inclined to believe that most canons were meant by their composers as melodies without a primary tonic.

The "octaves" in which the tones are used in this canon are represented in the table in this manner. I regard as the tones of one "octave" all the tones in the Complete Scale from 525 up to 2. All these tones are represented by the numbers on the *same level* in the table, and the (arbitrary) absolute pitch is designated by an arbitrary symbol on the same level. Whenever a number representing a tone of the same voice is set above the normal level, this means that it is the tone of the higher octave; when set below the normal level, it is the tone of the lower octave. *E. g.,* 5 followed by 3 on the same level means that this 3 is the 3 of the same octave, *i. e.,* the 3 a Minor Third higher than 5. 3 on a higher level than 5 means the interval of a Minor Third plus an Octave. 3 on a lower level than 5 means the interval of a Major Sixth. Movements which pass over an octave may easily be expressed by auxiliary lines. Of course, I do not propose to introduce this notation into musical practice. But for theoretical discussions, where the common musical notation is quite insufficient and the relative pitches must be represented by exact numbers, this seems to me the simplest way of designating the absolute pitch of the tones.

3. HEINRICH SCHÜTZ

I will now analyze a piece of polyphonic music in which the several voices are different from each other. I select this piece because it has been made the subject of an important psychological discussion by Planck[1]. Planck observed that a well trained

[1] Max Planck, *Die natürliche Stimmung in der modernen Vocalmusik, Vierteljahrsschrift für Musikwissenschaft,* 9, pp. 418–440. 1893.

Planck makes the following remarks: "Bei der ersten Probe pausierte

chorus singing this piece was in unison with a piano at the first chord.   But when the *a* of the piano was struck in the last chord, it was much higher than that sung by the chorus.   The same lowering of pitch occurred again and again, although the singers were aware of it and tried to avoid it.   Planck offers an explanation of this fact which I must reject as being psychologically unfounded.

Planck explains the lowering of pitch in this manner : The singers try to use in *melodic* succession *tempered* intervals, but alter this intonation a little in order to have the *chords* in *perfect* intonation. We start in this special case from the chord *c-e-g*, which—as a

das begleitende Klavier nach dem Beginn dieser Stelle oder wurde wenigstens so schwach, dass man es nicht hörte; als es dann am Schluss wieder einsetzte, war der Chor inzwischen so gesunken, dass der Dirigent abklopfte und die Stelle mit Klavier wiederholen liess.   Dabei war der Gesang aber keineswegs unrein gewesen, im Gegenteil hatten die consonanten Dreiklänge in dem zarten pianissimo ganz besonders gut geklungen. Diese Erscheinung zeigte sich nicht ein einziges mal, sondern wiederholte sich in der Folge jedesmal wieder, sodass keine einzige Probe vorüberging, ohne dass der Tenor daran erinnert wurde, das e im ersten und das h im vierten Takt recht hoch zu nehmen.   Denn offenbar [? M.] liegt es an der Grossen Terz des Tenors, der sich später der Zweite (bezw. der Erste) Sopran anschliesst, und die nachher zur Quinte wird, dass die natürliche Stimmung hier den ganzen Chor um ein Komma hinunterzwingt.''

chord—is sung in perfect intonation. The following chord *a-c-e* is again sung in perfect intonation. Now if each voice were using in melodic succession tempered intonation, the pitch of the *e* of the tenor and second soprano would be identical in both cases. In order to sing the chords in perfect intonation, however, the pitch of the *e* must be raised a little in the second chord as compared with the first. For in the second chord the *e* is a "Fifth," in the first it is a "Major Third," and the natural Fifth ($\frac{3}{2}$) is nearly identical with the tempered one, but the natural Major Third is much lower than the tempered one. Now, since the tenor and second soprano do not raise the *e* in the second chord,

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV
I Sopr...		27	15	15	15	63	15	15	135	135	135	225	15	225	225	15
II Sopr.		45	45	45	45	27	45	25	27	225	225	45	3	45	45	75
C' alto..		9	75	75	75	81	75	5	81	45	45		5	135	45	75
Tenor ..	27	45	45	45	15	45	45	45	5	135	135	135	45	135	135	135	15
Bass....		9	15	15	15	27	15	5	27	45	45	45	5	45	45	15

the whole chorus falls. Unfortunately, one of the psychological premises of this conclusion, namely, that there is a tendency of using tempered intonation in melody, is mere imagination. I perfectly agree with Planck in his assertion that the tendency of the singers to use *pure intonation* causes this lowering of the pitch on part of the whole chorus. Yet I do not agree with him in assuming that this is caused by the *pure intonation* of the chords in *harmony* conflicting with *tempered* intonation in *melodic* succession. I deny that there is such a thing as a singer's purposely intoning a melody in tempered intonation. The real cause of the lowering of the pitch we shall soon understand by a structural analysis of the piece of music in question.

In the table we find the whole piece represented by numbers, in such a manner that the numbers indicate what seems to me, after carefully experimenting, to be the aesthetically most effective intonation. All the chords, with the exception of XII, may be separately represented by much smaller numbers. I shall now discuss the structure of the whole piece and show that the simultaneous tones of each chord are connected by rather close relationship, and that the tones of each chord are closely related to the preceding or following chords, a connection which is the cause of the melodious effect of the whole piece. The chords I, II, III, and IV are all represented by 2, 3, 5, i. e., they possess very close relationships and a very high degree of consonance. V is represented by 3, 5, 7, 9. It is a less consonant chord, and the relationships are not so close. There is no relationship between 7 and 9, though 7 and 9 are not quite disconnected, since each of them is related to 3 and to 5. All the following chords from VI to XIV, with the exception of XII, are also represented by 2, 3, 5. The last chord, XV, is represented by 2, 5. XII is represented by 3, 5, 15, 135. This is a less consonant chord. The relationships between 3, 5, and 15 are very close. 135 is not related to either 3 or 5, but it is to 15, the relationship being (9-2). The harmonic structure being now clear, we may turn our attention to the melodic structure. We notice at a glance that, with two exceptions, each chord is connected with the preceding and following chord by identity of one or more tones. E. g., I and II contain the identical tones 45 (in various "octaves"). VIII and IX contain the identical tones 135. The exceptions are VII-VIII and XIV-XV. The latter chords, however, though not containing identical tones, are connected by rather close relationships. The relationships between 15 in XV and the tones of the chord XIV are 3-2, (9-2), and ((15-2)); the relationships between 75 in XV and the tones of XIV are 3-2, 3-5, and ((9-5)). The case of VII-VIII is different. Here most of the tones of the one chord are not related at all to the tones of the other; only 15 is connected with 135 and 27 by

the rather remote relationships (2-9) and ((5-9)). The chord VIII is much more closely related to the chord VI than to VII. We shall soon see the practical significance of this. On the whole we should expect—from our theory—a rather strong aesthetic effect from this piece in this intonation. And its aesthetic effect is indeed strong. Other intonations do not yield the same aesthetic effect, and, when represented by numbers, they do not show such close theoretical relationships in melody and harmony, nor such high degrees of consonance as the above table indicates. This is a very satisfactory agreement between theory and practice, and makes the following conclusions the more certain.

We may now turn to the problem of the lowering of the pitch on part of the chorus. The first important fact which no one can deny is this: the several voices, when sung separately, sound extremely poor, particularly the contralto and tenor. They are, indeed, not intended to be sung separately. This is not without effect upon the singers, who usually pay a little more attention to their own voice than to the others, and so separate to some extent their own voice from the rest. The contralto begins with 9. The intonation of the following 75 is determined by its relation to 5 in VII, which in turn is related to 9. The intonation of 81 in V is determined by its relation to 9 in I. However, none of these relationships is very close, so that, in this case, the singer is not very firm in his intonation unless he pays attention to the close relationship of his tones with those of the other voices. In VII he sings 5. But now, in the following chord VIII, he has not the slightest hesitation as to intonation. He finds in his music that he has to sing exactly the same tone in VIII as in VII. Consequently, he firmly intones 5 instead of 81 and drags the whole chorus down by the interval 81:80, i. e., 0.22 E. S. Now, we must remember that the tone 15 (a) in the chord XV is already 0.16 E. S. lower than the a on the piano, if the tone 9 (c) in I is identical with the c on the piano. Therefore the a as actually sung by the chorus is more

than $\frac{1}{3}$ E. S. (0.38) lower than that on the piano, a difference which is very conspicuous. No wonder the director interrupted the chorus. But, on the other hand, it is now not surprising to us that the advice he gave to the chorus, was quite ineffective, as Planck reports. The director told the tenor and second soprano to intone the *e* in the first chord too high, *i. e.*, to sing the harmonies *mistuned*. The singers, of course, did not do this, could not do this, but continued to intone as perfectly as their training permitted. The true cause of the lowering of pitch remained undiscovered, and it occurred again and again.

The reader may here justly raise the objection against this explanation that it does not seem likely that *one* voice should be able to direct the intonation of *four* other voices. This is generally true; but it is no valid objection in this special case. None of the other voices is by any means so sure of its intonation as the contralto, which finds in its music in VII and VIII identical notes. Furthermore, it is easily seen that the other voices themselves have a tendency, though not such a strong one as the contralto, to intone their notes in VIII too low; a tendency which causes them to yield readily to the confident intonation of the contralto. The tenor sings 5 in VII and 135 in VIII. If he knew that these tones were not related, but that 135 was related to 45 in VI (relationship 2-3), he would make no mistake in intonation. Yet he does not know this, but interprets the interval *d-b* as a "Minor Third" of the very close relationship 3-5. The interval 3-5 is 0.22 E. S. greater than the interval 5-135, as seen in the table of the Complete Scale. Consequently, the tenor has a tendency to intone the *b* 0.22 E. S. too low, and so he yields readily to the contralto. The case is the same with the bass. The interval 5-27 has no relationship; the interval 3-2 has very close relationship and is just 0.22 E. S. smaller than the interval 5-27. There is little doubt that the lowering of the pitch on part of the chorus could be avoided if we told the singers not, of course, to sing out of tune, as the director in Planck's report advised them, but to remember

the fact that the chord VIII is not related to VII, but very closely
to V and VI.   Unfortunately, I am unable to make this experi-
ment, having no chorus of well trained singers with me.

In certain musical theories which pretend to be psychological,
we find the term, "enharmonic confusion."   This term supposes
that a certain note in tempered intonation may be psychologically
effective in two different ways simultaneously.   *E. g.*, a certain
tempered tone in a piece of music, not exactly equal to 5 or to 81,
but of a pitch slightly differing from 5 and 81, is supposed to be
psychologically effective as 5 in relation to some tones, as 81 in
relation to some others.   This I must deny.   There is no doubt
that a tone slightly differing from 5 or 81 may be effective as 5 or
81, but it is not effective as 5 and as 81 simultaneously.   I shall try
to make clear what this means, by referring to an analogy.   Above
I mentioned the design which may be interpreted as a flight of
steps or as a piece of overhanging brick wall.   But one cannot see
such a design as a flight of steps and as an overhanging brick wall
simultaneously.   That we are able to analyze and theoretically
understand a piece of music like Schütz's, without the aid of an
accessory hypothesis like that expressed by the term "enharmonic
confusion," must be, I think, for any unprejudiced reader a strik-
ing demonstration of the superiority of our theory.

4. SCHUBERT

We now turn to a brief analysis of Schubert's "Heidenrös·
lein," including the accompaniment.   The whole song is repre-
sented below by numbers, according to the intonation which seems
to me the most effective aesthetically.   I have divided the song
into seven parts.   The numbers of the first five parts contain the
common divisor 9.   We may, therefore, when we consider these
five parts separately, use the smaller numbers given below the
numbers representing the actual intonation.

When we compare this song with a folk song accompanied by
a drone bass, with a canon, and with the above piece of Heinrich

Schütz, we observe the following facts: The drone bass alone does not possess any aesthetic effect, being one single tone. It

I

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r
Voice	45	45	45	45	27	189	189	45	81	81	81	81	45	189	27	27	9	—
		45		45		189	189		189			189			27	27		45
Accomp	9	9	9	9	9	9	81	81	9	81	135	81	189	135	81	9	135	9
	27			27			243	243		243		27		27		27		27
Divisor	9																	
Voice	5	5	5	5	3	21	21	5	9	9	9	9	5	21	3	3	2	—
		5		5		21	21		21			21			9	3		5
Accomp	2	2	2	2	2	2	9	9	2	9	15	9	21	15	9	2	15	2
	3			3			27	27		27		3		3		3		3

II

a	b	c	d	e	f	g	h	i	j
45	45	45	45	27	405	405	45	81	81
	45		45		405	405		405	
9	9	9	9	9		9	81	81	9	81
27			27			243	243		243
9									
5	5	5	5	3	45	45	5	9	9
	5		5		45	45		45	
2	2	2	2	2	2	9	9	2	9
3			3			27	27		27

III

a	b	c	d	e	f	g	h	i	j	k
27	27	243	243	27	405	27	243	135	27	27
135	27	567	243	243	81	81	405	405	45	27
81		729	729			567	567		63
9										
3	3	27	27	3	45	3	27	15	3	3
15	3	63	27	27	9	9	45	45	5	3
9		81	81			63	63		7

IV

a	b	c	d	e	f	g	h	i	j	k	l	m
27	135	243	27	405	45	675	45	567	567	405	27	27
63	63	27	27		9	9	243	243	81	405	405	27	27
		81	81			45	45		567	567
9												
3	15	27	3	45	5	75	5	63	63	45	3	3
7	7	3	3	2	2	27	27	9	45	45	3	3
		9	9			5	5		63	63

V

a	b	c	d	e	f	g	h	i
81	81	45	189	27	243	135	9	9
			81	27	27			45
27	81	189	135	45	9	9	9	9
	63		27		27	27		9
9								
9	9	5	21	3	27	15	2	2
			9	3	3			5
3	9	21	15	5	2	2	2	2
	7		3		2	2		3
								2

VI

a	b	c	d	e	f	g	h	i	j	k
15	9	3	15	9	45	5	9	9	9	9
	3		3	45	5	5				
3	9	15	9	9	63	63	9	45	3	27
	15			27	3	3	45	9	5	45
			3	27	27	27	9			

VII

a	b	c	d	e	f	g	h
15	9	3	15	9	5	45	9
				135	135	45
3	15	5	3	45	3	3	
				27	27	27	9

only serves to enrich the aesthetic effect of the tune. The tune, however, may be sung and aesthetically enjoyed without any accompaniment. The case is the same with Schubert's tune and accompaniment. This accompaniment, though incomparably richer than a drone bass, is not intended to have a separate existence. The case is different with a canon; here every voice has as much right to separate existence as every other voice, since there is only one tune in different phases. In the piece of Schütz *none* of the several voices has a right to separate existence; they all sound more or less poor when separated from the rest. We may say that the latter piece is one single melody compressed so that four or five tones sound simultaneously, but compressed in such a manner that these simultaneous tones, the harmonies, possess as close relationships and as high a degree of consonance as possible.

The accompaniment in Schubert's song serves three purposes. The first is a general one. The addition of the accompanying tones enriches the tune by increasing the number of relationships and adding the new element of consonance in various degrees. The two other purposes are more special. A certain tone in the accompaniment often increases the psychological effect of the same (or a closely related) tone in a preceding or following chord [1] in these ways: it either causes the hearer to recall this tone; or it prepares the hearer for this tone when it appears in a later chord.

The song begins with the highly consonant chords 2-5 and 2-3. In I f follows a dissonant chord 2-21 without any relationship. But 21 is melodically prepared for by the preceding 3. The tension caused by the lack of relationship between 21 and 2 is dissolved in g by replacing 2 by the related tone 9. The chord g

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[1] By "chord" I mean simply a plurality of (simultaneous) tones, two, three, four, or more. The restriction of the term to triads, which is common among musicians, cannot be psychologically justified. That triads, especially the triad 2-3-5, are particularly important in music, is a matter of course, if we admit that the aesthetic effect of a chord depends upon the variety of relationships, a high degree of the relationships, and a high degree of consonance.

is in smallest numbers 9, 3, 7; 7 as well as 9 are related to 3, though not related to each other. In l we have the chord 3-9-21-9 (*i. e.*, 2-3-7-3). This 21 prepares us for the 21 sung by the voice in n. The chord m is a dissonant chord without relationship. But 5 as well as 21 is closely related to the tones in l and n. I cannot, of course, describe every chord in detail. The reader can easily see from the table, how the composer has combined the various elementary aesthetic effects in order to produce the highest beauty. I shall only point out a few of the most interesting cases. The voice in II is a melody without a tonic. Yet it is well connected with the other parts of the voice, not only by the relationship of the melody, but also by the primary tonic 2 (of the parts I-V) being strongly emphasized in the accompaniment of II. III j is very consonant and has a relationship of the first degree. K, the last chord of this part, is less consonant. So is the first chord of IV. IV b is still less consonant and even without relationship, but it is melodically connected with the preceding chords through the close relationships of 3-15=2-5 and 7-7=2-2. The highly consonant chord IV d causes some solution of the tension, but only temporarily; the following chords are again rather dissonant and possess remote or no relationships. This part ends in m, in a manner similar to its beginning, with a combination of 3 and 7. In V b the 7 appears again without relationship with the simultaneous 9. The 21 in V c is related to 9 as well as to 7, but not to the simultaneous 5. In d we meet a chord of close relationships and a high degree of consonance. The 21 in this chord is well prepared for by the 21 in c and the 7 in b. VI contains a very interesting chord in f. 27-63-45 is in smaller numbers 3-7-5. The relationships which connect f with e are easily determined. VII f and g are quite interesting with regard to the 135, which leads to the 45 in the last chord through its relationship 135-45= 3-2.

5. WAGNER

I shall now describe what seems to me the most perfect intonation of Wagner's duet in *Tristan and Isolde*. Since the whole duet is too long, a few selected measures must suffice to show that there is absolutely no necessity of assuming that music like this must be played and sung in tempered intonation. The numbers represent the measures 66 to 76 of Act II, scene 2, in Bülow's piano edition (Breitkopf and Hartel). The first line represents the soprano, the second line the tenor. The chord I a is not very

```
       I                                          II
     a   b   c   d   e   f   g   h   i   j    a   b   c   d   e
   |45||  5| ..| ..| ..| ..| ..| ..| 25| 25| 25|| 25| 25| 45|   |   |
                                             || 75| 75| 75|135|135
   |15||135| 15|225|225| 15|135| 15|225|225| 15||135| 15|225|225| 15
   |45|| 25| ..| ..| ..| ..| ..| ..| ..| ...| 45| ..| ..| ..| ..
   |75||  5| ..| ..| ..| ..| ..| ..| ..| ...| 75| ..| ..| ..| ..
   |15|| 15| 15| 15|  5|  5| 25| 25| 25| 15| 15|| 15| ..| ..| ..| ..
   |15|| 15| 15| 15|  5|  5| 25| 25| 25| 15| 15|| 15| ..| ..| ..| ..

       II                      III
     f   g   h   i   j    a   b   c   d   e   f   g   h   i   j
    | 15| 15| 15| 45| 45||  5| ..| ..| ..| ..| ..| ..| 25| 15|
    |135| 15|225|225| 15||135| 15|225|225| 15|135| 15|225| 15|135
    | 45| ..| ..| ..| ..|| 25| ..| ..| ..| ..| ...| ..| ..| ..| ..
    | 75| ..| ..| ..| ..||  5| ..| ..| ..| ..| ...| ..| ..| ..| ..
    | 15| ..| ..| ..| ..|| 15| 15| 15|  5|  5| 25| 25| 25| 15| 15
    | 15| ..| ..| ..| ..|| 15| 15| 15|  5|  5| 25| 25| 25| 15| 15

   IV                                               V
 a  b  c  d  e  f  g   h   i  j   k   l  m  n  o   a   b   c   d   e   f   g   h   i   j
| 9| 9| 9| 9|15|15| 27| 27|27|27| 27|27| 3| 3| 3||         |   |   |   |   |   |   |
| 9|..|..|..|..|..| 9|  9| 9| 9|   |   |   |   |  | 5|  5|  5|135|135|15|15| 15| 27|27
| 9|15|27| 3| 9|75| 45|  5|75| 9|135|63|15|27| 3|| 5| 9|135|135|  9| 5| 9|135|135| 9
|15|                                              |45|..| ..| ..| ..| ..| ..| ..| ..| ..
| 3|                                              |63|..| ..| ..| ..| ..| ..| ..| ..| ..
| 9|                                              |        27| 27|27|..| ..| ..| ..| ..
| 3| 3| 3| 3|15|15|135|135| 9| 9| 45| 3|15|15|15|| 9|..| ..| ..| ..| ..| ..| ..| ..| ..

   VI                                              VII
 a  b  c  d  e  f  g   h   i  j   k   l  m  n  o   a   b   c   d   e   f   g  h  i  j
| 9| 9| 3|..|..|..|..| ..| ..| 9| 9| 15|15| 3| 3| 3||45|45| 45| 27| 27|25|25|25|15|15
|                                              | 5| 5|  5|135|135|15|15|15|25|25
| 9|15|27| 3| 9|75| 45|  5|75| 9|135|63|15|27| 3|| 5| 9|135|135|  9|21| 5|75|75| 5
|15|                                              |27|..| ..| ..| ..| 15|..|..|..|..
| 3|                                              |45|..| ..| ..| ..| 25|..|..|..|..
| 9|                                              |63|..| ..| ..| ..| 15|..|..|..|..
| 3| 3| 3| 3|15|15|135|135| 9| 9| 45| 3|15|15|15|| 9|..| ..| ..| ..|  9|..|..|..|..
```

VIII										IX										X				
a	b	c	d	e	f	g	h	i	j	a	b	c	d	e	f	g	h	i	j	a	b	c	d	e
27	27	27	63	63	15	15	15	9	9	135	..	..	..	..	..	..	..	..	..	63	..	..	..	..
63	63	63	27	27	9	9	9	15	15	81	..	..	..	..	..	..	..	..	..	45	..	..	..	..
3	45	81	81	45	27	3	45	45	3	15	27	405	405	27	15	27	405	405	27	15	27	405	405	27
63	..	..	..	..	9	..	..	..	..	81	..	..	..	..	..	..	..	..	..	45	..	..	..	..
27	..	..	..	..	15	..	..	..	..	135	..	..	..	..	..	..	..	..	..	63	..	..	..	..
	..	..	..	..	15	..	..	..	..	189	..	..	..	..	..	..	..	..	..	63	..	..	..	..
27	..	..	..	..	3	..	..	..	..	81	..	..	..	..	..	..	..	..	..	45	..	..	..	..
9	..	..	..	..	9	..	..	..	..	9	..	..	..	..	..	..	..	..	..	9	..	..	..	..

consonant, nor does it possess very close relationships only; some tones are not related at all. In I b the tone 135 is replaced by 15 (relationship 9-2). The chord b contains only the tones 5, 15, and 25 and is very consonant. In the following chords more and less consonance alternates. This is also the case in II. That the first chord of II and the last of I are bound together by rather close relationships, is seen at a glance. The chord II c would be very consonant if instead of 225 the 15 of the preceding chord b had been retained. However, Wagner's contempt for consonance is here clearly seen. The variety of relationships is much higher valued by him than a high degree of consonance. The same fact may be seen in the following measures again and again. It is so conspicuous that it would be tiresome to point it out each time. The tenor in II is melodically connected with the soprano by the close relationships 25-75=2-3 and 45-75=3-5. In IV the soprano again replaces the tenor; the melodic connection is again very close. Very close are the relationships in the successive tones of the accompaniment. Very close are for the most part the relationships of the simultaneous tones in the several chords. A variety of melodic relationships was sought after by the composer even at the expense of consonance. The latter is only of secondary importance for this composer.

One wonders after such a structural analysis how it is possible for a theorist to assert that nothing but "strictly tempered intonation" could be bearable in this piece of music. The chord VIII b is composed of 9, 27, 45, 63, or in smaller numbers 2, 3, 5, 7; why should this chord be replaced by one in tempered intonation?

I can not see any reason for this.   And why should all the other chords and melodic successions of tones be replaced by tempered chords and successions of intervals of equally tempered twelfths of an octave?

6. The Leading Chord

It is natural that music often closes with a chord composed of the tones 2, 3, and 5, since the relationships of these tones are close and the degree of consonance high.   This chord is very often preceded by the chord 3-9-15.   This succession of the chords 3-9-15 and 2-3-5 at the end of a piece of music is aesthetically very effective.   The musicians have invented a special name for the chord 3-9-15 in such a case.   They call it the "leading chord." How can this name be psychologically justified?

Helmholtz offers the following explanation of the particular aesthetic effect of the leading chord.   The tones d, g, and b, in a melody ending with c, are according to his theory "the most distant tones," *i. e.*, tones which have a very remote relationship to the tone c.   This, he thinks, makes it perfectly clear why the leading chord "leads" most naturally to the final chord.   Yet this is no scientific explanation by any means.   A scientific explanation is the reduction of a special case to a general law.   Now, is there any general psychological law according to which special sensations "lead" most naturally to other sensations if their relationship is as remote as possible?   Helmholtz does not mention such a general law.   I have not found any and have never heard of any.

The aesthetic effect of the leading chord is most easily understood from our theory, without any accessory hypothesis.   The most important tone of the final chord is 2.   This tone has been heard repeatedly in the melody.   If we sound in the leading chord any of the tones 3, 5, 7, 9, and 15, we strengthen the desire to end with 2, and this desire is fulfilled in the final chord.   However, if

we wish the leading chord to contain no tone not related to every
other tone, we must omit 7, which is not related to either 9 or 15.
Further, if we wish to hear only relationships of the first degree,
we must omit 5, which has a relationship of the third degree with
9. The triad 3-9-15, which then remains, does not only "lead" to
2, but contains relationships of the first degree only and is also
very consonant. Moreover, the tones 3, 9, and 15 do not only lead
to 2, but to 3 and 5 also, if the final chord contains these tones.
The relationships leading to 3 and 5 are 9-3=3-2, 15-3=5-2, and
15-5=3-2. No wonder that the tones 3, 9, and 15 are so com-
monly used in the "leading chord."