THE

COURSE OF STUDY

IN

ARITHMETIC

FOR THE

PUBLIC SCHOOLS OF PHILADELPHIA

PREPARED BY

EDWARD BROOKS

Superintendent of Public Schools, Philadelphia, Pa.

PHILADELPHIA:

BURK & MCFETRIDGE, PRINTERS, 306–308 CHESTNUT ST.

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Course of Instruction in Arithmetic

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Introduction.

Arithmetic is a science and an art. As a science it treats of the relations and principles of numbers. As an art it includes the various operations with numbers. In teaching arithmetic, the object is to make pupils familiar with both the science and the art of numbers.

The art of arithmetic and much of the science grow out of the language used in expressing numbers. This language is based upon the decimal system of grouping numbers and the Arabic system of writing numbers. This system of naming and writing numbers is purely conventional and has no necessary relation to numbers. Given another method of naming and writing numbers, and our arithmetic would be very materially changed.

The science and art of arithmetic are based upon four fundamental operations, viz.: addition, subtraction, multiplication, and division. Addition and subtraction are the two primary forms of synthesis and analysis; multiplication and division are the two derivative processes of synthesis and analysis. These processes, called the *fundamental operations* of arithmetic, involve certain elementary combinations which may be called "elementary

results." These "elementary results" are the sums of the "addition table," extending as far as "9 and 9," and the products of the multiplication table extending as far as "9 times 9." The corresponding elementary differences and quotients are immediately derived from these results. By means of these elementary results we are enabled to operate with large numbers, and to perform all the calculations of the fundamental operations. A clear conception of these facts will shape the methods of primary instruction in numbers.

I. *Ideas and Language.*—As thought and expression are so intimately allied, the methods of naming and writing numbers modify our conceptions of numbers. The oral language of arithmetic is based upon a principle peculiarly simple and ingenious. Instead of giving independent names to the different numbers, which would require more names to count even a million than one could acquire in a life-time, the method of classifying units into tens, hundreds, etc., has been adopted. This enables the mind to form clear and distinct ideas of large numbers which would otherwise be exceedingly difficult if not impossible. The method of writing numbers is based upon the ingenious device of place value, which is regarded as one of the happiest results of human intelligence. By this method the mind is enabled to conceive and to operate with numbers far beyond what would be possible without some such system. These facts with respect to the conception and expression of numbers, serve to shape the methods of primary instruction in arithmetic.

II. *Elementary Results.*—These elementary results, consisting of the sums of the addition table and the products of the multiplication table, are to be committed to memory. No progress can be made in the art of calculation until this is accomplished. The amount to be thus committed depends upon the systems of numeration and notation which have been adopted. If the base of the

method of naming numbers was smaller than *ten*, the time for learning these results would be lessened. If we had a binary system, as advocated by Leibnitz, the task could be accomplished in a few minutes. If our system were quarternary—that is, if it had *four* as the base, these elements could be learned in a few days. The elementary sums would include results only as far as 3+3; and the multiplication table would include the products only as far as "2 times 3" and "3 times 3." In the decimal system, these elements extend necessarily as far as "9 and 9" and "9 times 9," and it requires a child two or three years to master these tables of elementary results. It is thus seen that much of the time necessarily spent in teaching arithmetic is due, not to the nature of number itself, but to the system of arithmetical language adopted.

There are, or have been, at least three distinct ways of learning these elementary results of the addition and the multiplication tables. The old way was to have the child study the tables, repeating the combinations verbally until they were committed to memory. There was no illustration, no attempt to show the meaning or origin of the results, but merely a repeating of words until they became fixed in the mind. A newer way is the Grube Method, in which the four processes of addition, subtraction, multiplication and division are united in the study of each number in order from *one* to *one hundred*, the processes to be performed with objects and repeated until the child remembers the results. The third and still newer method, which may be called the Normal Method, is that which combines addition and subtraction, and afterward multiplication and division, but not the four processes at first. It illustrates the operations and results with objects, and also requires that special pains be taken to have pupils remember the elementary results.

In this discussion it will be noted that we have limited the elementary sums to "9 and 9" and the elementary

products to "9 times 9." This is really as far as the sums and products need to be committed for any practical purpose. It is, however, customary to extend them as far as "12 and 12" and "12 times 12," since the names of the numbers up to twelve suggest a duodecimal basis of numbering. Teachers, of course, are expected to follow this custom.

As previously stated, it is necessary to fix in the memory the elementary results of addition and multiplication. Pupils must know the elementary sums as far as "9 and 9," and then they can readily derive the sums of the larger numbers by using these elementary sums and the principle of the Arabic notation. So, also, pupils must know the elementary products as far as "9 times 9," and from these they can readily find the products of larger numbers. Special pains must, therefore, be taken to have pupils commit the elementary sums and products, so that they may spring up in the mind spontaneously and automatically when needed. The elementary differences are readily derived from the elementary sums and the elementary quotients from the elementary products, and thus no special pains need be taken to have them committed to memory.

III. *The Fundamental Operations.*—The elementary results having been learned, the pupil is prepared for the Fundamental Operations. These include written exercises in addition, subtraction, multiplication and division. The operations apply to large numbers expressed in the Arabic system of notation. These operations are mechanical in their nature, though involving some scientific principles; and it requires practice and drill to become familiar with them. The primary object with young pupils is to have them acquire facility and accuracy in these mechanical operations.

This is the art of arithmetic; and it will be remembered that an art is learned by practice. "We learn to

do by doing," is especially applicable here. It requires constant drill in these processes to attain quickness and accuracy in obtaining results. The operations require but little thought—a machine can be made to do them—but children should begin them early, and must be kept at them until they can "calculate" quickly and accurately.

IV. *Arithmetical Reasoning.* — Arithmetic is also a science—a science of interesting and beautiful relations; and we are to teach the science as well as the art. Thought is to be evolved and the power to judge and reason with numbers is to be developed. The child is to be made a thinker as well as a doer; an inventive intelligence rather than a mere arithmetical machine. Theory and practice must, therefore, be united in teaching arithmetic.

This thought-power is developed by the application of numbers to the various practical affairs of life, as well as by the pure science of arithmetic. Each distinct application usually requires some new process of reasoning, as in Interest, Banking, Insurance, etc. All these applications are to be taught intelligently to the student, so that he may understand them independently of the teacher or the text-book. He should be taught the underlying principles of each subject, and he will then be able to devise his own process or rule, and invent the shortest and best methods of solution.

One of the best means of developing the thought-power of the child, is that system of arithmetical reasoning known as "arithmetical analysis." This is a method of reasoning with numbers in which the various relations are determined by a reference to the unit. All numbers and their relations are understood by comparing them through their relation to unity. To and from and around the unit, the current of thought flows. The method is so simple and logical that the ordinary mind can follow it with ease and readily grasp what otherwise would be

complex and difficult. This method of arithmetical analysis is regarded as one of the most important of all the improvements of the New Education. It has been attributed to Pestalozzi, though in its development it is mainly a product of the thought of American teachers. Many of our educators, excepting those who have made a special study of teaching arithmetic, seem not to have understood or appreciated its value; but it is now steadily growing in favor with the leading educators of the country, and its more general introduction into our schools will do much to develop the powers of judgment and reasoning.

It is earnestly recommended that teachers bear clearly in mind the relation of the art to the science of arithmetic; that they drill their pupils until they have acquired facility in the operations of the art; and also that they take special pains to awaken and develop the thought-power of their pupils. Let the object be to make the pupils of our public schools skillful and ready in the practice of arithmetic, and at the same time clear, logical, and independent thinkers.

Primary and Secondary Grades.

General Suggestions to Teachers.

The primary and secondary grades embrace the first four years of a child's school life. During this time a thorough foundation in arithmetical knowledge and skill should be laid. A few general suggestions will be given in respect to the course of instruction in these grades and the methods of presenting it.

The Course of Instruction.—The course of instruction during the first four years should be as follows:—

First.—Pupils should attain a clear idea of numbers and the method of expressing them in words and figures.

Second.—They should learn how to unite and separate numbers by the general processes of addition and subtraction, and also by the derivative processes of multiplication and division.

Third.—They should fix in the memory the elementary results of the addition and multiplication tables, and be able to derive the corresponding differences and quotients from these sums and products.

Fourth.—They can also acquire a fair degree of skill in the mechanical operations of the fundamental rules.

Fifth.—Besides this, most of the denominate units can be learned, and a knowledge of the elements of fractions acquired.

Elementary Results.—One of the principal objects of the first two or three years' work is to give pupils clear

ideas of numbers and their relations as expressed by the primary facts of synthesis and analysis. These primary facts or results, as previously explained, must be fixed in the memory, as they lie at the foundation of the science and art of arithmetic. The first object should be to see that the pupils understand the nature and origin of these elementary results. This being assured, they should be led to memorize these elementary results or facts of number. These results or facts should be so thoroughly fixed in the mind that there will be no counting of fingers or marks when adding, or any mistakes or hesitation in the process of multiplying.

For this purpose there needs to be thorough and continued drill upon these primary facts. Pupils should be required to derive, repeat, and write the results until they are known. Intelligent repetition is the only way in which they can be committed to memory. Pupils should say the results and write the results, so that they may be fixed in the mind by both the oral and the visual memory. This repetition should be intelligent, however, and not the mere repeating of unmeaning words.

Fundamental Rules.—During the first four years pupils should acquire a fair degree of skill in the mechanical operations of arithmetic. For this purpose there should be a large amount of drill in the fundamental rules. It requires constant and unremitting labor to attain dexterity and accuracy in adding, subtracting, multiplying, and dividing. These processes are purely mechanical in their nature; but, like playing the piano, familiarity with them is acquired only by long and continued practice. It may seem like a waste of time to devote so much attention to the mechanics of arithmetic, but skill in these operations can be attained in no other way.

Denominate Numbers and Fractions.—During the first four years pupils can become familiar with the ordinary units of the weights and measures and their relation to

one another. Some skill can also be attained in the operations with these units and their practical application. A knowledge of the elements of fractions can also be acquired, including the several operations that pertain to them. The subjects now named are about all that should be embraced in the first four years' course.

The Use of Objects.—Much of this primary instruction should be given objectively. The child's ideas of numbers are most conveniently developed by means of objects. While numbers and things are not identical, the mind is aided in its early numerical ideas and operations by the use of visible and tangible objects. These ideas of numbers are usually developed by the process of "counting." In counting, however, we should not depend on the mere naming of numbers in succession; a child may do this and have no distinct idea of the meaning of the words used. Children are often able to run these words off glibly as far as a hundred and yet be unable to select a dozen objects from a collection. The names of numbers therefore should at first always be associated with the things numbered. The motto should be,—teach numbers rather than figures, ideas rather than abstract words.

The fundamental processes of addition, subtraction, multiplication, and division are also to be taught by means of objects. The child should unite and separate the objects that he may see clearly what is meant by the uniting and the separating of numbers. So strong is this necessity that if the teacher does not require the use of objects the child will count his fingers or use marks on the slate or blackboard, thus showing the natural demand of the mind for the use of concrete things in the primary exercises with numbers.

In teaching the elements of denominate numbers the measures are to be in the school-room for the pupils to handle and see. The "tables" are to be learned, not from the book, but by the use of the measures. In teaching

the elements of fractions also, the ideas are to be made clear to the mind of the learner by concrete illustrations. Concrete objects, and also lines, circles, and squares on the blackboard are to accompany all the elementary exercises in fractions.

In fact, all the processes of primary arithmetic should be illustrated concretely until they are thoroughly understood by the child. This suggestion cannot be too strongly emphasized, and no teacher should neglect it. The objects or "counters" used may consist of blocks, splints, balls, beans, etc. Each pupil should be furnished with a sufficient number of these counters to perform the operations required in each lesson. A numeral frame is an almost indispensable appliance in teaching primary grades. Every primary school in the country should be supplied with a large abacus, such as is usually found in the schools of Europe, and in many of our own schools.

A word of caution may be added,—not to carry the use of objects in arithmetic too far. A child soon learns to think independently of objects, and it is a mistake to fetter the mind with things when it is ready for abstract thoughts. The mind of the child is therefore to be led from the concrete to the abstract as soon as it is ready to make the transition. The teacher will remember that "language is the instrument of thought," and that pupils must be taught to think numerically through the beautiful and ingenious system of arithmetical language. It requires nice judgment on the part of the teacher to see just when the mind of the pupil is ready to dispense with visible or tangible illustrations, and care should be taken not to make the transition too early nor delay it too long.

General Suggestions.—The general order of instruction with young pupils may be indicated as follows: 1st, the operation with objects; 2d, the operation without objects; 3d, the representation of the operations with figures and

other symbols; 4th, the application of the process to concrete examples; 5th, a large amount of practice work for the acquisition of skill in the mechanical operations.

The work of these first four years should be carefully graded. The teacher should always bear in mind that she is dealing with children whose faculties are but slightly developed, and whose comprehension must be constantly aided. The processes should progress no faster than the minds of the learners develop; and concrete problems should be simple, and contain no more steps than the pupil can readily understand. No long and complicated problems should be given to the pupils. The concrete problems in first and second grades should contain only a single step, and those in third and fourth grades not more than two steps.

In the primary grades, especially, give short lessons and proceed slowly. Do not push ahead with the bright pupils who learn easily, and thus neglect the dull and slow pupils. Teach each individual of the class, and take special pains to stimulate the inattentive and backward pupils and to insure their progress day by day, that they may not drop behind the class so that the advancing instruction shall be lost to them.

For this purpose the classes should be divided into sections and each section taught separately. It is only in this way that each pupil can receive a proper share of attention. While one section is having an exercise with the teacher, the other sections may be doing written work in review of the previous exercises. There are many exercises, however, such as reviews and class drills, in which the entire class may be taught together.

Oral exercises, with small numbers, are to be given regularly in connection with the written work. Such exercises are useful both for rapid computation and illustration of principles and processes. A new process will be more readily seen by the use of small numbers than by large

ones. Besides the ability to reckon quickly "in the head" is valuable in the practical affairs of life.

In the processes of the fundamental operations, no rules are to be studied and committed to memory. Each process is to be illustrated and explained so that the pupils have an intelligent idea of it, and then they are to be drilled in doing the work until the process is fixed in the mind, and skill and accuracy are attained.

The written work on the board or slate or paper should be neat and in proper form. No carelessness in this respect should be permitted. The figures and signs should be carefully and neatly made, and all the work should express just what is intended to be expressed. Remember that accuracy of expression begets accuracy of thought.

Care should be taken that in all the exercises the language of the pupil is both correct and appropriate. The lesson in arithmetic affords an excellent opportunity for elementary exercises in language culture. The teacher must be careful, however, not to make these exercises formal and unnatural. No fixed forms of analysis or explanation which the pupils are required to commit to memory should be allowed. Such a practice begets the habit of using words without ideas, which should be carefully guarded against in the education of children. While there are some elementary thought-forms and forms of expression which the pupils will readily acquire, this is quite different from committing to memory set forms of reasoning or explanation.

Other suggestions will be presented in connection with the course of study in each of the several grades.

THE COURSE OF INSTRUCTION FOR PRIMARY AND SECONDARY GRADES.

First Grade—*Time, five months.*

Numbers from One to Ten, inclusive.

I. The Course of Instruction.

1. Ideas of numbers as far as *ten.*
   (*a*) Their names.
   (*b*) The figures.
2. Addition and subtraction as far as *ten.*
   (*a*) With objects.
   (*b*) Without objects.
   (*c*) With figures.
3. Multiplication and division as far as *ten.*
   (*a*) With objects.
   (*b*) Without objects.
   (*c*) No inexact division.
4. Signs of operation.
   $+$, $-$, and $=$

II. Methods of Teaching the Course.

1. Use objects in teaching the ideas and names of numbers.
2. Use objects in obtaining the "facts" of numbers.
3. Teach addition and subtraction together.
4. Teach multiplication and division together.
5. Do not combine the four processes at first.

6. Have no exercises in inexact division in this grade.

7. In this grade have but one class of problems in division; as "how many 3's in 6?"

8. Take special pains to have pupils first *understand* and then *fix in the memory* the elementary *sums* and *products*.

9. Have the pupils work out the "facts" with objects repeatedly, and also try to fix these facts in their minds by proper drilling.

10. Have pupils *see* the results and *say* the results, that they may learn them by both the visual and the verbal memory.

11. Do not allow any concert exercises in which pupils merely repeat results or processes without reference to objects.

12. Introduce the symbols $+$, $-$ and $=$ as they are needed and as substitutes for the words "and" or "added to," "less," etc. Do not introduce $\times$ and $\div$ in this grade.

13. Have written exercises in addition and subtraction; but no written exercises in multiplication or division in this grade.

14. Lead to the idea of multiplication by combining equal numbers, and to the idea of division by separating into equal numbers.

15. Besides the objects or "counters," make use also of such objects in the school-room as windows, doors, slates, desks, pictures, and also the parts of the body, as the eyes, hands, fingers, etc.

16. As soon as children can draw a little, have them illustrate the processes with simple pictures.

17. Teachers will be careful to distribute the work properly through the five months. Have no "more or less" examples in this grade. Use only two numbers in any exercise, except in deriving an elementary product or quotient.

18. Have pupils tell little "number stories" with the processes of addition and subtraction, but do not carry this exercise to excess.

19. Teachers will exercise their ingenuity in devising means to give variety to their methods of instruction.

20. Written work like the following affords a good exercise for this grade:—

4	5	6	7	8	9	10
2+2	2+3	3+3	6+1	6+2	7+2	6+4
3+1	4+1	2+4	5+2	5+3	5+4	8+2
1+3	3+2	5+1	4+3	4+4	6+3	7+3

Write the large figure and have the children write the parts below it.

SECOND GRADE.—*Time, five months.*

Numbers from One to Twenty, inclusive.

I. THE COURSE OF INSTRUCTION.

1. Ideas of numbers as far as *twenty*.
   (*a*) Their names.
   (*b*) Expression in figures.

2. Addition and subtraction as far as *twenty*.
   (*a*) With objects.
   (*b*) Without objects.
   (*c*) With figures.

3. Multiplication and division as far as *twenty*.
   (*a*) With objects.
   (*b*) Without objects.
   (*c*) With figures.
   (*d*) No inexact division.

4. Signs of operations.

   $+, -, \times, \div, =$.

5. Denominate numbers.

(*a*) Value:—1 cent, 1 dime.
(*b*) Length:—1 inch, 1 foot.
(*c*) Volume:—1 pint, 1 quart.

6. Fractional parts: one-half, one-fourth.

(*a*) Equal parts of a unit.
(*b*) Equal parts of numbers.
(*c*) No inexact fractional parts.

7. Simple exercises in concrete problems.

II. Methods of Teaching the Course.

1. Follow the directions given for the first grade.

2. Review the work of the first grade. Distribute the work carefully throughout the term.

3. Introduce the signs of multiplication and division, $\times$ and $\div$, in the second grade.

4. In this grade introduce both classes of problems in division; as "how many 4's in 8?" and "divide 8 into two equal parts," or "find one-half of 8."

5. In naming numbers from *ten* to *twenty*:—

(*a*) Form a group of ten objects, and combine with this group, *one*, *two*, *three*, etc.

(*b*) Require pupils to count "one and ten," "two and ten," "three and ten," etc., as far as "two tens."

(*c*) When pupils are familiar with these expressions (which show the principle of naming numbers), the common words, *eleven*, *twelve*, etc., may be used.

6. In writing numbers from *ten* to *twenty*:—

(*a*) Show the principle of notation somewhat as follows:—

That 15 expresses "5 and 10" : 12 expresses "2 and 10"
14 " "4 and 10" : 11 " "1 and 10"
13 " "3 and 10" : 10 " "0 and 10"
or simply 10.

(*b*) When pupils are familiar with these expressions (which show the principle of writing numbers), lead them to use the common words, *ten*, *eleven*, *twelve*, etc.

7. Have pupils write the elementary results in the form of tables, thus:—

$1+1=2$	$9-1=8$	$2\times1=2$	$2\div1=2$
$1+2=3$	$8-2=6$	$2\times2=4$	$4\div2=2$
$2+3=5$	$7-4=3$	$2\times3=6$	$6\div3=2$

8. Have written work also in the following form:—

$$\begin{array}{cccc} 4 & 8 & 6 & 2)\overline{8} \\ +\,2 & -\,5 & \times\,2 & \\ \hline & & & \end{array}$$

9. In teaching denominate numbers, show the pupils the real measures:—cent, dime, inch, foot, pint, etc.

10. In teaching fractional parts, use objects, circles, squares, pieces of paper, groups of objects, etc.

11. Drill the pupils on the elementary *sums* and *products* within the limits of twenty, until they are fixed in the memory.

12. Do not allow concert exercises in which pupils merely repeat processes or tables without reference to objects.

13. Have pupils derive the "elementary differences" from the "elementary sums" and the "elementary quotients" from the "elementary products."

14. Use only two numbers in any exercise except in obtaining an elementary product or quotient. Have no exercises of inexact division or of inexact fractional parts of numbers in this grade.

15. Have pupils illustrate the processes with little pictures on the slate and blackboard.

16. Have pupils tell little "number stories" with the processes, but do not carry the exercise to excess.

17. In concrete problems have oral and written work in addition and subtraction, but only oral work in mulipli-cation and division. Have no written work in fractions in this grade.

18. Use concrete illustrations as far as necessary, but do not continue to illustrate when no illustration is needed. Lead the mind of the child from the concrete to the abstract.

19. Have pupils analyze numbers into their parts, by the four processes. Thus take the number 8;—

$8 = 6 + 2$ $8 - 5 = 3$ $8 = 4 \times 2$ $8 \div 4 = 2$
$8 = 5 + 3$ $8 - 2 = 6$ $8 = 2 \times 4$ $8 \div 2 = 4$

20. An exercise in filling out blanks like the following will be found useful :—

$4 + ? = 6$ $8 - ? = 5$ $3 \times ? = 6$ $8 \div ? = 4$
$3 + ? = 8$ $10 - ? = 5$ $4 \times ? = 12$ $10 \div ? = 2$

NOTE.—To teach the naming and the writing of numbers, teachers need a few hundred splints or toothpicks or other objects and a few elastic bands.

THIRD GRADE.—*Time, five months.*

I. THE COURSE OF INSTRUCTION.

I. *Elementary results with numbers as far as fifty.*

1. Ideas of numbers as far as *fifty.*

   (*a*) Naming the numbers.

   (*b*) Expressing the numbers by figures.

   (*c*) Roman numerals to ten.

2. Addition and subtraction as far as 50.

(*a*) Orally (with and without objects).

(*b*) With figures.

(*c*) Commit elementary sums to 20.

(*d*) Derive sums beyond 20 from elementary sums.

3. Multiplication and division as far as 50.

(*a*) Orally (with and without objects).

(*b*) With figures.

(*c*) Commit elementary products to 50.

(*d*) Derive elementary quotients from elementary products.

(*e*) Simple exercises in inexact division to 20.

4. Simple oral and written exercises in concrete problems, no number or result to exceed fifty.

II. *Elements of denominate numbers and fractions.*

1. Denominate numbers.

(*a*) Value—coins from 1 cent to 1 dollar.

(*b*) Length—inch, foot, yard.

(*c*) Measures—pint, quart, gallon.

(*d*) Measures—pint, quart, peck.

2. Fractional parts, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{6}$.

(*a*) Equal parts of a unit.

(*b*) Equal parts of numbers.

(*c*) No inexact fractional parts.

(*d*) The work both oral and written.

3. Simple exercises in denominate numbers and fractions, both oral and written.

III. *Written work in the fundamental rules.*

1. Numeration and notation to a hundred.

(*a*) Naming numbers.

(*b*) Writing numbers.

2. Exercises in addition to one hundred.

(*a*) Add numbers of one term to numbers of two terms.

(*b*) Add single columns within the limit.

(*c*) Add two or more numbers of two terms.

3. Exercises in subtraction to one hundred.

(*a*) Subtract numbers of one term from numbers of two terms.

(*b*) Subtract numbers of two terms from numbers of two terms.

NOTE.—Limit the oral work and the concrete exercises in the fundamental rules to fifty.

II. METHODS OF TEACHING THE COURSE.

1. Follow the directions, which are applicable, given under the previous grades.

2. Review the work of previous grades. Be careful to distribute the work properly throughout the term.

3. Use sticks and other objects to form bunches or groups of tens, in obtaining the ideas and names of numbers up to one hundred.

4. In teaching pupils to name numbers up to 100, be careful to bring out the idea of groups, by naming numbers at first as follows:—

Two tens and one,	Three tens,	Four tens,
two tens and two,	three tens and one,	four tens and one,
etc., etc.	etc., etc.	etc., etc.

When this is clearly understood, then use the ordinary names, twenty-one, twenty-two, thirty, etc.

4. In teaching pupils to write numbers up to 100, be careful to bring out the idea of place value; that is that 35 is "3 tens and 5," 46 is "4 tens and 6," etc.

5. Drill the pupils until they know the "elementary sums" as far as "10 and 10 are 20."

6. Drill the pupils until they know the "elementary products" as far as 50. Derive the elementary quotients from the elementary products.

7. The sums of numbers beyond "10 and 10" are not to be committed to memory, but are to be derived by means of the "elementary sums."

8. Do not allow pupils to sing the multiplication table nor to repeat it in concert. Have each pupil make it, write it, study it, and recite it.

9. Require neat and accurate work. Do not allow children to make such mistakes in writing their work as $5 \div 2 = 2 + 1$.

10. In deriving the sums of numbers beyond "10 and 10," have the pupils think through the decimal system and the Arabic notation. The method of doing this is indicated in the next suggestion.

11. In adding numbers of one figure to numbers of two figures, take examples first where there is no "carrying," regularly graded, as 13 and 5, 23 and 5, 32 and 5, etc.; 12 and 6, 22 and 6, 32 and 6, etc.

Then take examples where there is "carrying," as 15 and 8, 25 and 8, 35 and 8, etc. Do these exercises with figures, and also orally.

12. In subtracting numbers of one term from numbers of two terms, take examples first in which there is no "borrowing," regularly graded, as 16 less 4, 26 less 4, 36 less 4, etc.

Then take examples where there is "borrowing," as 23 less 6, 33 less 6, 43 less 6, etc. Do these exercises with figures, and also orally.

13. In finding the "elementary results," teach addition and subtraction together, and also multiplication and

division together; but in the "fundamental rules" teach the four processes separately.

14. Teachers will exercise good judgment in combining properly the oral and the written work. Drill pupils on the written work to secure skill in the mechanical operations.

15. Have pupils invent concrete problems from written work placed on the board, thus:—

3 + 2 (problem) 4 × 3 (problem) ½ of 12 (problem)
7 — 3 (problem) 12 ÷ 4 (problem) 5 + 3 — 2 (problem)

16. In teaching fractional parts, use objects, circles and squares made of paper, and circles, squares and lines on the blackboard, etc.

17. In obtaining fractional parts, restrict the work to *one* of each of the equal parts, as *one*-third of 6, *one*-fourth of 8, etc.

18. In concrete problems have but two steps in problems involving addition and subtraction, and but one step in problems involving multiplication or division.

Fourth Grade.—*Time, five months.*

The Course of Instruction.

I. *Elementary results with numbers up to one hundred.*

1. Ideas and names of the numbers.
   (*a*) Naming and writing the numbers.
   (*b*) Roman numerals to fifty.
2. Addition and subtraction as far as one hundred.
   (*a*) With figures.
   (*b*) Without figures.
   (*c*) Derive sums beyond 20 from elementary sums.

3. Multiplication and division as far as one hundred.

(*a*) Orally (with and without objects).

(*b*) With figures.

(*c*) Commit elementary products to 100.

(*d*) Derive elementary quotients from elementary products.

(*e*) Simple exercises in inexact division.

4. Simple concrete exercises, both oral and written, no number or result to exceed one hundred.

II. *Elements of denominate numbers and fractions.*

1. Denominate numbers.

(*a*) Review the previous grades.

(*b*) Weight:—ounce, pound.

(*c*) Time:—day, week, month, year, hour, minute, second. Teach time from the clock.

(*d*) Counting:—dozen, score.

2. Fractional parts as far as one-tenth.

(*a*) Equal parts of a unit.

(*b*) Equal parts of numbers.

(*c*) No inexact fractional parts.

(*d*) Illustrate with lines, circles, squares, etc.

3. Simple exercises in denominate numbers and fractions, both oral and written.

NOTE.—Give only one number in changing to higher or lower denominations, and have no inexact divisions. Use the "analytic form" of solution.

III. *Written work in fundamental rules.*

1. Numeration and notation.

(*a*) Naming numbers to 1000.

(*b*) Writing numbers to 1000.

2. Written addition.

(*a*) Add single columns, the sum within 100.

(*b*) Add columns of two and three terms, within the limit.

3. Written subtraction.

(*a*) Subtract numbers of two and three terms in which no term of the subtrahend exceeds the corresponding term of the minuend.

(*b*) Subtract numbers of two and three terms in which a term of the subtrahend exceeds the corresponding term of the minuend.

4. Written multiplication.

(*a*) Multiplicand two and three terms, multiplier one term, and no "carrying."

(*b*) Multiplicand two and three terms, multiplier one term, and when there is "carrying."

5. Written division.

(*a*) Dividend two and three terms, divisor one term, and no partial remainders.

(*b*) Dividend two and three terms, divisor one term, and with partial remainders.

NOTE.—Limit the work in concrete problems, written as well as oral, to numbers not exceeding one hundred; also, to two steps in problems involving addition and subtraction, and to two steps in problems involving multiplication and division, one of these being addition or subtraction.

II. METHODS OF TEACHING THE COURSE.

1. Follow the directions under the previous grades which are applicable to the exercises of this grade.

2. In naming numbers do not neglect to use objects to illustrate the grouping by tens and hundreds.

3. Be sure the pupils know the elementary sums, and can use them in deriving the sums of larger numbers.

4. Be sure the pupils know the elementary products as far as "10 times 10." Have them understand, make and commit the multiplication table.

5. Be sure that pupils know how to derive the "elementary quotients" from the "elementary products."

6. The teacher will combine the oral and written exercises with careful judgment.

7. In the written work, take pains to secure neatness in writing and accuracy in results.

8. In the oral exercises in addition and subtraction beyond 10, train the pupils so that they may work with both the *visual* and the *oral* language.

9. The sums of numbers beyond 9 and 9 (or 10 and 10) need not be committed to memory, but are to be derived by means of the "elementary results."

10. In the concrete problems, use small numbers so as not to require much mental effort in the mechanical operations.

11. In the exercises in "fractional parts," restrict the work to *one* of each of the fractional parts, as one-fifth of 15, one-sixth of 24, etc.; also, to "elementary products" not exceeding 100.

12. Do not allow pupils to repeat the addition and multiplication tables in concert, or to sing them.

Fifth Grade.—*Time, five months.*

I. The Course of Instruction.

1. *Numeration and notation.*

   (*a*) Writing and reading numbers to *five places.*

   (*b*) Places of units, tens, hundreds, etc.

   (*c*) Roman notation, to one hundred.

II. *Fundamental operations.*

1. Addition.

(*a*) Rapid oral addition properly graded.

(*b*) Written exercises of two, three, and four terms.

(*c*) Concrete problems, oral and written.

(*d*) Introduce the terms *addition* and *sum*.

2. Subtraction.

(*a*) Rapid oral subtraction properly graded.

(*b*) Written exercises as far as four places.

(*c*) Concrete problems, oral and written.

(*d*) Introduce the terms *subtraction*, *minuend*, *subtrahend*, and *difference* or *remainder*.

3. Multiplication.

(*a*) The elementary products to "12 times 12."

(*b*) Multiplier of one and of two terms.

(*c*) Concrete problems, oral and written.

(*d*) Introduce the terms *multiplier*, *multiplicand*, and *product*.

4. Division.

(*a*) Drill on elementary quotients.

(*b*) Exercises in inexact division.

(*c*) Short division, no divisor exceeding 12.

(*d*) Long division, no divisor exceeding 50.

(*d*) Concrete problems, oral and written.

(*e*) Introduce the terms, *divisor*, *dividend*, *quotient*, and *remainder*.

NOTE.—In the oral exercises do not extend the work beyond 144. In concrete problems limit the work to three steps in problems involving addition and subtraction, and to two steps in problems involving multiplication and division.

III. *Denominate numbers.*

1. Review the measures of the previous grades.
2. Write and commit the tables of:—
   (*a*) United States money.
   (*b*) Dry measure.
   (*c*) Liquid measure.
   (*d*) Avoirdupois weight.
3. Simple exercises in changing to lower or higher denominations.

NOTE.—Limit the work to two denominations. Use the "analytic form" of written work. The measures will include bushels, gallons, tons, and the lower denominations; and in United States money, dollars and cents and the ordinary coins.

IV. *Common fractions as far as tenths.*

1. Write and read fractions.
2. Exact fractional parts of numbers.
3. Concrete problems involving fractional parts.
4. Illustrate with lines, circles, squares, objects, etc.

II. THE METHODS OF TEACHING THE COURSE.

1. Review the work of the previous grades.
2. Be sure that pupils know the elementary results of the "addition table."
3. Have frequent drills in the "multiplication table," and be sure that every pupil knows it.
4. Do not allow pupils to repeat the addition and multiplication tables in concert, or to sing them.
5. Have frequent oral exercises to secure rapidity and accuracy in the fundamental operations.
6. Oral exercises with small numbers should accompany all the written work.

7. Explain the reason for "carrying" in addition and multiplication and of "borrowing" in subtraction.

8. Take pains also to see that the steps in multiplication and division are understood.

9. The teacher will exercise ingenuity in devising methods of drill that will give variety and interest to the work.

10. The teacher will be careful to grade the exercises so as to adapt them to the knowledge and progress of the pupils.

11. In "long division" let the right digit of the divisor, at first, be small; as 13, 14, 21, 22, 23, 31, 32, 33, 41, 42, etc.

12. Have the weights and measures in the schoolroom to illustrate denominate numbers; and use objects, circles, squares, lines, etc., to illustrate fractions.

13. Illustrate concretely wherever such illustrations are applicable and useful. Do not hold the mind down to the concrete too long, but gradually lead to the abstract conception of the subject.

14. The exercises in fractions are to include no inexact fractional parts of numbers, and are to be *oral* and *written*. In this grade they should extend to more than one of the equal parts, as two-thirds of 12, three-fourths of 16, etc. All exercises limited to elementary results not exceeding 100.

15. Do not teach the definitions of the terms introduced, but lead the pupils to see their meaning by their use.

16. The teacher should exercise ingenuity in giving variety and interest to the drill in numbers. To illustrate, draw a circle on the board and write the digits in order from 1 to 12 as on the dial of a clock. Then write any number as 8 at the centre and require the pupils to begin at some

point and add 8 to the numbers in order around the circle. Have them also add 10 to the first number and obtain two sums, as follows:—One pupil says, "4 and 8 are 12, 14 and 8 are 22." The next pupil says, "5 and 8 are 13, 15 and 8 are 23." Require the pupils to keep the run of the exercise, so that the teacher need not indicate whose turn it is next. Devices like this, many of which the ingenious teacher will readily invent, give great interest to the mechanical drill.

17. The teacher will exercise ingenuity in making problems and in asking questions so as to cultivate the power of thought on the part of the pupils.

18. Do not teach "rules," but drill upon the *methods of operation* until they are understood and permanently acquired.

Sixth Grade.—*Time, five months.*

I. The Course of Instruction.

1. *Numeration and notation.*

(*a*) Writing and reading numbers to *six places.*
(*b*) Exercises in Roman notation to five hundred.

II. *Fundamental operations.*

1. Addition.
   (*a*) Rapid addition properly graded.
   (*b*) Adding numbers within the limit.

2. Subtraction.
   (*a*) Subtracting numbers within the limit.
   (*b*) With one or more ciphers in the minuend.

3. Multiplication.

(*a*) With two and three terms in the multiplier.

(*b*) With one or more ciphers in the multiplier.

4. Division.

(*a*) Short division in review.

(*b*) Long division, divisor not to exceed two figures.

(*c*) With one or more ciphers in the quotient.

5. Application of each to concrete problems.

6. Oral and sight exercises for rapidity and accuracy.

III. *United States money.*

1. Write and read United States money.

2. Addition and subtraction.

3. Multiplication and division.

4. Practical problems in United States money.

NOTE.—The divisor in this grade to be abstract.

IV. *Denominate numbers.*

1. Review measures of previous grades.

2. Write and commit the measures of:—

(*a*) Length:—inch, foot, yard, rod, mile.

(*b*) Time:—second, minute, hour, day, week, month, year, century.

(*c*) The number of days in each month.

3. Simple exercises in changing to lower or higher denominations.

NOTE.—Limit the work to two denominations. Use the "analytic form" of written work. Do not use 5½ in the operations.

V. *Common fractions as far as twelfths.*

1. Write and read fractions.

2. Exact fractional parts of numbers.

3. Simple concrete problems with fractional parts.

VI. *All necessary terms to be learned as needed.*

II. Methods of Teaching the Course.

1. Oral exercises in addition, subtraction, multiplication, and division.

2. Be sure that pupils know the elementary results of the addition table.

3. Have frequent drills on the multiplication table, and be sure that every pupil knows it. No concert exercises.

4. Do not permit the method of "long division" with a divisor of 12 or less.

5. Aim to make pupils quick and accurate in the fundamental rules.

6. No rules need be learned, but the pupils are to be drilled on the processes until they are familiar with them.

7 Awaken a spirit of emulation in respect to rapid and accurate work in the mechanical operations.

8. Exercise ingenuity in originating methods and devices that will give variety and interest to the mechanical work.

9. Illustrate fractions with objects, circles, squares, etc. Illustrate with concrete objects all such subjects as illustrations will simplify.

10. Explain the reasons for the various operations so far as they can be understood by pupils.

11. Have pupils invent problems for one another and solve one another's problems.

12. Have the units of length, inch, foot, yard and rod, represented concretely. The length of the rod can be marked off on the walls or the floor.

13. Have pupils measure the length of objects in the school-room, and also estimate lengths, heights, and distances, in feet, yards, etc.

14. Have pupils handle and estimate the weight of small objects in ounces and of larger objects in pounds and parts of pounds.

15. Have pupils estimate the contents of vessels in pints, quarts, gallons, pecks, etc. Give them concrete and practical ideas of all these measures.

16. The exercises in fractions are to be restricted to exact fractional parts, and are to be *oral* and *written*. They should extend to more than one of the equal parts, as 3-fourths of 16, 4-fifths of 20, etc. All exercises in fractions limited to elementary results not exceeding 144.

17. The practical examples should not include large numbers.

Seventh Grade—*Time, five months.*

I. The Course of Instruction.

I. *Numeration and notation.*

1. Read and write numbers to *seven places.*

2. Practice in Roman notation to one thousand.

II. *Fundamental rules.*

Addition and subtraction.

(*a*) Rapid addition of columns.

(*b*) Add and subtract numbers within the limit.

2. Multiplication.

(*a*) With three and four terms in multiplier.

(*b*) With ciphers at the right of multiplier and multiplicand.

3. Division.

(*a*) Divisor of two and three terms.

(*b*) With ciphers at right of divisor and dividend.

(*c*) Write the remainder in the form of a fraction.

4. Practical problems in fundamental rules.

5. Oral and sight exercises for rapidity and accuracy.

III. *United States money.*

1. Addition and subtraction.
2. Multiplication and division.
3. Simple problems in business transactions.

NOTE.—The divisor may be dollars, cents, etc.

IV. *Denominate numbers.*

1. Review previous measures.
2. Teach the following measures:—

(*a*) Counting:—dozen, gross, score.

(*b*) Surfaces:—square inch, square foot, square yard.

3. Reduction of denominate numbers.

(*a*) Descending, simple exercises.

(*b*) Ascending, simple exercises.

V. *Common fractions to twentieths.*

1. Write and read fractions.
2. Find exact fractional parts of numbers.
3. Simple concrete problems with fractions.
4. Reduction of fractions.
   (*a*) Mixed numbers to improper fractions.
   (*b*) Improper fractions to whole or mixed numbers.
   (*c*) To larger terms.
   (*d*) To smaller terms.

VI. *Arithmetical analysis.*

1. Pass from the unit to a collection.
2. Pass from a collection to the unit.
3. Pass from a collection to a collection.
4. Simple problems in addition and subtraction.

VII. *All necessary terms to be learned as needed.*

II. Methods of Teaching the Course.

1. Follow the suggestions of previous grades as far as they apply.

2. Have oral exercises with small numbers to accompany all the written work.

3. Have frequent drills on the multiplication table and be sure that every pupil knows it. No concert exercises.

4. Make the work in denominate numbers as concrete and practical as possible.

5. Drill on the fundamental rules as far as is necessary to insure rapidity and accuracy.

6. Illustrate the exercises in fractions with objects, circles, lines, squares, etc. Limit the exercises in fractions to small numbers.

7. Let the concrete work be related to practical life, the problems, as far as possible, being such as occur in the home, the shop, the store, etc.

8. The exercises in the reduction of denominate numbers should be simple, involving integers only. They should extend to three denominations. Both forms of written work, the "analytic" and the "operation," should be taught. Do not use $5\frac{1}{2}$ in these operations.

9. In this grade is first introduced "arithmetical analysis," which is regarded as the key to arithmetical reasoning. Much of the work up to this time has been mechanical, involving some judgment but little reasoning. These exercises will train the mind to logical habits of operating with numbers.

10. The exercises in analysis are indicated by such examples as the following:—

(*a*) If one orange costs 4 cents what will 3 oranges cost?

(*b*) If 3 oranges cost 12 cents what will one orange cost?

(*c*) If 4 oranges cost 24 cents what will 5 oranges cost?

11. The following suggestions are given for the work in "arithmetical analysis:"—

(*a*) The exercises should involve only small numbers so that the mind may be concentrated upon the thought.

(*b*) The thought should be simple, direct, and logical. The language will then also be simple and logical.

(*c*) While the thought must be determined by the language, teachers will be careful that the pupils think and express their thoughts, rather than merely go through a formula of expression mechanically.

NOTE.—The exercises in arithmetical analysis should be oral.

EIGHTH GRADE.—*Time, five months.*

I. THE COURSE OF INSTRUCTION.

I. *Notation and numeration.*

1. Write and read numbers to *eight places.*
2. Review Roman notation.

II. *Fundamental rules.*

1. Oral exercises for rapidity and accuracy.
2. Drill on the fundamental rules.
3. Require reasons for the different processes.
4. Concrete problems in fundamental rules.

III. *Factors, divisors, and multiples.*

1. Prime and composite numbers.
2. Principle of divisibility by 2 and 5.
3. Resolving numbers into their factors.
4. Greatest common divisor, by factoring.
5. Least common multiple, by factoring.

IV. *United States money.*

1. Review previous grade.
2. Practical business transactions.
3. Compute and make out bills.

V. *Denominate numbers.*

1. Review previous measures.

2. Teach the following measures:—(*a*) Solids: cubic inch, cubic foot, cubic yard.

3. Reduction, descending and ascending.
4. Addition and subtraction.
5. Multiplication and division.

NOTE.—These exercises should involve integers only, and should not extend beyond three terms. The divisor to be abstract.

VI. *Common fractions.*

1. Reduction of fractions.
   - (*a*) Mixed numbers to improper fractions.
   - (*b*) Improper fractions to whole or mixed numbers.
   - (*c*) To larger terms.
   - (*d*) To smaller terms.
   - (*e*) Parts of fractions.
2. Addition and subtraction.
   - (*a*) Denominators alike.
   - (*b*) Denominators unlike.
3. Multiplication.
   - (*a*) Fraction by an integer.
   - (*b*) Fraction by a fraction.
4. Division.
   - (*a*) Fraction by an integer.
   - (*b*) Fraction by a fraction.

NOTE.—In the exercises in fractions, no term of a given fraction is to exceed two figures; only simple fractions, with no term exceeding 20, are to be used in the operations of adding, subtracting, multiplying, and dividing. Not more than two fractions are to be used in the operations of adding, subtracting, multiplying, and dividing.

VII. *Arithmetical analysis.*

1. Pass from a number to a number.
2. Pass from a unit to a fraction.

3. Pass from a fraction to a unit.

4. Pass from a number to a fraction.

5. Pass from a fraction to a number.

VIII. *Necessary terms to be learned as needed.*

II. Methods of Teaching the Course.

1. Apply suggestions of previous grades so far as practicable.

2. Oral exercises with small numbers should accompany all the written work.

3. Give as much time to the fundamental rules as is necessary to insure rapidity and accuracy.

4. Have frequent drills on the tables, and see that every pupil knows them. No concert exercises.

5. Be sure that pupils can divide by short and long division up to the limit.

6. In denominate numbers be sure that the pupils have real, definite, and practical ideas of the measures and their uses.

7. In the reduction of denominate numbers, teach both forms of written work, the "analytic" and the "operation."

8. Illustrate the exercises in fractions with objects, and also with circles, lines, squares, etc., made of paper, and on the blackboard.

9. Let all the concrete work be related, as far as possible, to practical life, the problems being such as may actually occur in the school-room, household, store, shop, etc.

10. Have the pupils exercise their ingenuity in inventing problems, and also in originating solutions.

11. The exercises in factoring, common divisor, and common multiple should be restricted to numbers not exceeding 144.

12. Under arithmetical analysis, review the previous grades before beginning the exercises of this grade. Remember, also :—

(*a*) That the numbers should be small, so that the mind may be concentrated upon the thought.

(*b*) That the forms of thought should be simple, direct, and logical; and that the expression should be a reflection of the thought.

(*c*) Let the problems be properly graded, beginning with simple combinations and proceeding gradually to more complex ones.

(*d*) If any pupils do not seem to understand the analysis, lead them step by step to the form of thought by simple questions.

13. The exercises in analysis are indicated by such questions as the following :—

(*a*) If a yard of silk cost $6, what will $\frac{2}{3}$ of a yard cost?

(*b*) If $\frac{2}{3}$ of a yard of silk cost $4, what will 1 yard cost?

(*c*) If 2 yards of silk cost $12, what will $\frac{2}{3}$ of a yard cost?

(*d*) If $\frac{2}{3}$ of a yard of silk cost $4, what will 2 yards cost?

14. The exercises in arithmetical analysis should at first be given orally, but later in the term they may also be written. The "analytic form" of writing is preferred.

The Grammar Grades.

General Suggestions to Teachers.

The Grammar grades embrace the second four years of a pupil's school life. In these four years the subject of arithmetic is to be continued until it covers the most important business applications of the science. A few general suggestions will be given in respect to the methods of instruction in these grades, following which the course of study will be presented in detail.

1. The teacher will bear in mind that the object of the study of arithmetic is twofold—the development of mental power and the acquisition of skill in the use of numbers. The study of arithmetic is especially adapted to strengthen the mind and to cultivate the powers of attention, judgment, and reasoning. Skill in the use of numbers involves both the power to reason and facility and accuracy in the mechanical operations.

2. In the lower grades special pains were taken to secure facility in performing the various mechanical operations. This is the first requisite of an arithmetician. Such skill requires practice and is readily acquired in youth. In teaching arithmetic, difficult explanations are to be deferred until this mechanical skill is secured and the mind is sufficiently mature for the reasoning processes of the science. In the grammar grades, while drill exercises in the mechanical operations should be continued, more attention should be given to the reasoning powers of

the pupils. The teachers should aim to make of their pupils intelligent and independent thinkers.

3. Pupils in these grades should be gradually trained to depend upon themselves in their arithmetical work. In the previous grades the pupil has depended largely upon the teacher to aid him in understanding a subject; he should now begin to learn to depend more upon himself. The teacher will, therefore, exercise careful judgment to know just now far to throw the pupil upon his own responsibility. In presenting a new subject somewhat difficult, the teacher should explain it fully and clearly; when the subject is less difficult, a partial explanation or some suggestive questions will suffice, the pupil being required to work out the rest for himself. Subjects should often be assigned for pupils to prepare and bring to the recitation and thus show that they have made the preparation. The more a pupil does for himself the more power to do he acquires.

4. With this end in view, some home work or work outside the recitation, should be required in arithmetic. Certain subjects or problems under a subject, should be assigned for the pupil to prepare out of school hours, or when not reciting. This will not only cultivate industrious habits, but it will also serve to give the pupil confidence in his own ability to overcome difficulties without the assistance of the teacher. A sense of self-reliance and a spirit of ambition will be aroused that will prove of great advantage in after life.

5. Great care should be taken that all the work be properly graded. The difficulty pupils experience in arithmetic is often due to the large step between one subject and another. Where the exercises are so related that one point prepares the way for another, the work is usually easy and interesting. The teacher will, therefore, take great pains to see that the transitions in the different

lessons and exercises are natural and easy. In arithmetic, especially, each subject learned should be made a stepping-stone to the one that follows.

6. In entering upon a new subject, care should be taken that the pupil has a clear and definite conception of it. He should have a clear idea of its object, the meaning of its terms, and the practical business transaction of which it treats. The difficulty often experienced by pupils with a subject is due to their not understanding the terms used and the nature and object of the transaction involved. This is especially the case in the business applications of arithmetic which constitute so large a portion of the study to-day. Thus in Banking, Insurance, Discounting, etc., the main difficulty with the pupil consists in obtaining a definite idea of what these business operations actually are. The teacher should therefore take special pains to see that the language and nature of the subject under consideration are clearly understood.

7. Oral and written exercises should be combined at nearly every step of the work. Every subject should be first illustrated with oral exercises by the use of small numbers before the written work is attempted. Small numbers used orally enable the mind to be wholly concentrated upon the new idea or process, and thus the way is prepared for dealing with large numbers. These oral exercises, it will be noticed, are not necessarily, as is often supposed, identical with what is called arithmetical analysis.

8. Care should be taken also not to give long and complicated problems. The principle of a subject is exemplified in a short problem just as clearly as in a long one; and when the numbers are large and the conditions numerous the mind is led away from the principle by the labor of the mechanical operations. Large numbers may

be useful in acquiring mechanical skill in the fundamental rules; but they distract the mind of the pupil in the investigation of a new subject. Teach the new principle or method through the use of small numbers so that the mind may be concentrated upon it rather than upon the mechanical operations.

9. Pupils should gradually be required to give logical explanations of their work in the grammar grades. In the lower grades the teacher was satisfied if the child could do the work and if, by the answer of questions, he seemed to understand it. In the grammar grades the pupil should begin to express in a more formal and logical way his reasons for the various methods and operations. It is not only important to think but also to be able to express thought. Explanations both oral and written should therefore be combined with the operations.

10. These explanations, or "solutions," should be simple, direct, and logical. There is a simple way of thinking upon arithmetical subjects, and the pupil should acquire this simple and logical way of thinking. Taught to think in this way, the expression of thought will also be orderly, simple, and logical. The awkward habits of thought and expression that pupils often employ in discussing arithmetical subjects, are to be severely condemned. The wordy explanations that talk round and about a subject instead of saying just what is necessary to a clear conception of it, are pernicious in mental discipline. It is suggested, therefore, that careful attention be given to train pupils to simple and logical forms of thought and expression.

11. Under this head it may be well to guard teachers against permitting pupils to commit and repeat long explanations of problems or demonstrations of principles as given in a text-book. This would be to substitute language for thought, a most pernicious habit in education. The

pupil can often be aided in his comprehension of a subject by a written solution or demonstration, but when he explains he should think and express his own thought. If the mind is trained from the beginning in simple, logical forms of thinking, there will be little difficulty in this respect. Pupils will follow these simple thought-forms in all their investigations, and be enabled to originate new solutions and demonstrations.

12. In the lower grades but little attention should be given to definitions. The meaning of each technical term should be clearly understood, but the formal definition of it is of no value to the pupil. In the grammar grades, however, some attention should be paid to defining these terms. Every definition should be derived by illustrations before the idea is formulated into words; but when it is understood and formulated there is no objection to the pupils being required to state it. On the other hand, there is the advantage of the cultivation of the habit of the clear and definite expression of ideas. Great care should be taken, however, to avoid the common error of allowing pupils to repeat words without expressing ideas.

13. Very few, if any, rules need be learned by pupils. Processes are to be learned, but not rules. There should be such an intelligent drill on methods and processes that, in most cases, they will be remembered without any formal statement of them. No one would think of teaching addition, subtraction, multiplication and division by rules. Besides, pupils should be so familiar with principles that, in most cases, they can make their own rules. In a few cases, however, a rule may be fixed in the memory, as in the division of fractions or some short method of working interest. Such cases, however, are very rare, and the motto should be,—principles and processes rather than rules.

14. All the exercises in arithmetic should be as practical in their nature as possible. They should represent

the operations of real life, and not some abstract or fancied view of what these operations might be. In denominate numbers the measures should be applied to the actual uses of the store, the shop, the market, the household, etc. The problems in percentage should not merely illustrate the theoretical principle, but represent the actual business of the street or office. It would be well for the teacher to visit the bank, the insurance office, the tax office, the custom house, etc., and see what the real business transactions are. These forms of business may then be organized in the school-room, and the operations of real life be thus represented. In this way the pupil will obtain a practical knowledge of his arithmetic, and be prepared to step from the school into the business office.

15. Exercises requiring pupils to invent original problems will be found of great interest and value. These original problems may be either oral or written. They are to be assigned to the class for solution and also for the correction of any errors that they may contain. The making of these problems exercises the power of original thought, and at the same time tends to give pupils a clearer idea of the subject than the mere solution of problems already at hand.

16. Special attention should be given to "arithmetical analysis," or Mental Arithmetic, in the grammar grades. This is an exercise of great value for the training of the reasoning powers of the child. Taught in the correct way, as indicated in the following suggestions, it affords the finest discipline to the intellect, training it to habits of attention and logical forms of thought and expression.

The problems should be dictated by the teacher, then reproduced by the pupil, and then analyzed. The restatement by the pupil is regarded as important in insuring a clear conception of the conditions and the requirements of the problem. In order to cultivate attention, the problem should be announced but once, and the class be held

responsible for its repetition. The question should be stated before calling upon the pupil, in order to avoid any order of assignment that allows the class to anticipate whose turn comes next.

The analysis should be given orally, except in cases of review or examination, when it may also be written. This analysis should be simple, concise, and logical. All round-about, awkward, and wordy solutions are to be avoided. The thought should be straightforward and to the point, and the expression simple and explicit. Original solutions, even if they depart from the form preferred, are to be accepted and encouraged.

The problems should be carefully graded and adapted to the advancement of the class. They should at the beginning involve only one or two steps or conditions, and gradually pass to more complicated forms. They should be stated in simple and direct language, so as to be understood by the average pupil. All puzzling inversions of language and all so-called "catch questions," should be carefully avoided in the general exercises. With advanced classes, an occasional exercise with "arithmetical puzzles" may cultivate quickness and expertness of thought, but such exercises should be only occasional.

17. Every pupil leaving the public schools should have a practical knowledge of the elements of geometry. This is presented to the pupils of the Twelfth Grade under the head of Mensuration. The course includes the different geometrical figures and their parts, the area of plane surfaces, the circumference and area of the circle, and the surfaces and contents of the solid bodies. The rules for these measurements are to be derived concretely by illustrations with models, paper forms, blackboard, etc. The proper use of these illustrations will give pupils a very satisfactory idea of the reasons for the different rules. These "concrete demonstrations" will also pave the way

for a much clearer conception of the abstract demonstrations when the pupil is ready to study the science of geometry. There should be abundant practice in the different methods, until they are permanently fixed in the student's understanding and memory. In the practical application of the principles to measuring land, carpeting floors, papering rooms, etc., the teacher should be careful that the problems given represent the actual methods of practical life.

18. The judicious use of a text-book will be found a convenience in teaching arithmetic in the grammar grades. It economises the time of the teacher, presents things more definitely and with less liability of error than when copied from the blackboard, gives opportunity for the pupils to work out certain subjects for themselves, and thus cultivates a spirit of independence and self-reliance. It would seem unfortunate for a pupil of the grammar grades to leave school without knowing how to make use of an arithmetic.

The proper use of a text-book, however, requires as much skill on the part of the teacher as does teaching without a book. There should be no slavish dependence upon the book by either pupil or teacher. Much of the teaching should be independent of the text-book. A judicious selection and omission of the matter in the book should be made, and a large number of exercises should be given in addition to those found in the work used. In fine, the object should be to teach the subject rather than the book.

19. The recitation in written arithmetic requires great skill on the part of the teacher. As many pupils should be sent to the board as can be accommodated, and the rest of the class be required to work on slates or paper at their desks. All the pupils may be assigned the same problems, or different problems may be assigned to different members of the class. When the solution is completed, some one should be called on to explain a problem solved upon

the board, the others listening attentively and noticing any mistakes that may be made, and whether any shorter or better method of solution is possible. At the close of the explanation, questions should be asked by the pupils, corrections made, and suggestions given. The teacher, by appropriate questions, should test the knowledge of the class and lead them to see each step of the solution and the reason for it. Points not understood by any member of the class should be explained, and the effort made to see that every pupil has a clear conception of every part of the lesson.

20. The work on the board should be neat and accurate. No carelessness in writing out the solution should be allowed. The figures should be correctly made, the lines straight and generally parallel with the edge of the board. When the work is written on paper for examination or review, it is recommended that the problem be written out in full, and that the operation and explanation be written alongside of each other. The following problem and its solution will serve as an illustration:—

PROBLEM:—If 25 yards of cloth cost $156.25, what will be the cost of 36 yards of the same kind of cloth?

Solution.	*Operation.*	
If 25 yards of cloth cost $156.25, one yard will cost $\frac{1}{25}$ of $156.25, which is $6.25; and 36 yards will cost 36 times $6.25, or $225.00.	25)156.25(6.25 150 62 50 125 125	$6.25 36 3750 1875 $225.00
Answer, $225.00.		

This solution may also be presented in the "analytic form," as follows:—

If 25 yards of cloth cost $156.25,

1 yard of cloth will cost $\frac{1}{25}$ of $156.25=$6.25.

And 36 yards of cloth will cost $6.25×36=$225.00.

The solution of problems that require a simple analysis may be written in another form as follows:—

Problem.	*Solution.*
A has \$40, which is $\frac{2}{3}$ of B's money, and C has $\frac{3}{4}$ as much money as B; how much money have B and C separately?	If $\frac{2}{3}$ B's = \$40, $\frac{1}{3}$ B's = $\frac{1}{2}$ of \$40 = \$20 $\frac{3}{3}$ B's = \$20 × 3 = \$60 $\frac{1}{4}$ B's = $\frac{1}{4}$ of \$60 = \$15 $\frac{3}{4}$ B's = \$15 × 3 = \$45 Hence C's = \$45
Answer, B's \$60; C's \$45.	

21. These forms are merely suggestive of something systematic and definite in written work. Other forms similar to them may be used at the option of the teacher. These forms of writing the work should begin in the ninth grade, and be continued throughout the grammar grades. When the subject admits of it, a diagram neatly drawn to illustrate the problem should accompany the solution.

In the ordinary recitation, it is suggested that either the "analytic solution" or the "operation" be used, and not both by the same pupil. One part of the class may be required to write the operation and another part the solution. Occasionally, once a week or once in two weeks, the solution and operation may both be required written side by side somewhat in the form previously suggested.

22. In conclusion, it is suggested that teachers try to attain a high degree of excellence in teaching arithmetic. Let the aims be to awaken mind, arouse interest, train to habits of industry, develop skill in the use of numbers, and cultivate the power of clear, logical, and independent thinking. Properly taught, arithmetic can be made one of the most interesting and profitable studies of the public schools.

THE COURSE OF INSTRUCTION FOR THE GRAMMAR GRADES.

NINTH GRADE.—*Time, ten months.*

I. *Review of fundamental operations.*

1. Drill in reading and writing numbers.
2. Sight exercises for rapidity and accuracy.
3. Written exercises in fundamental operations.
4. Reasons for the different processes of the fundamental operations.

II. *Divisors and multiples.*

1. Factoring numbers.
2. Greatest common divisor.
   (*a*) By factoring.
   (*b*) By successive division.
3. Least common multiple.
   (*a*) By factoring.
   (*b*) By numbers in a line.
4. Exercises in cancellation.
   (*a*) Abstract exercises.
   (*b*) Concrete problems.
   (*c*) Operations with fractions.

III. *Common fractions.*

1. Reduction.
   (*a*) Numbers to fractions.
   (*b*) Fractions to numbers.
   (*c*) To larger and to smaller terms.
   (*d*) Compound fractions to simple ones.

2. Addition and subtraction.

3. Multiplication and division.

4. Relation of integers and fractions.

5. Principles of multiplying and dividing the numerator and the denominator.

6. Practical examples in fractions. Use no complex fractions.

IV. *Decimal fractions.*

1. Read and write decimals.

2. Addition and subtraction.

3. Multiplication and division.

4. Reduce common fractions to decimals.

5. Reduce decimals to common fractions.

6. Practical exercises in decimals.

7. Practice in making and computing bills and accounts.

V. *Denominate numbers.*

1. Review tables of previous grades.

2. English money and circular measure.

3. Exercises in reduction, addition, subtraction, etc.

4. Practical exercises in the measures. Use not more than four denominations. No denominate fractions.

VI. *Arithmetical analysis.*

1. Pass from a number to a number.

2. Pass from a fraction to a fraction.

3. Simple problems involving the relation of numbers.

4. Analysis of the following cases of fractions:—

(*a*) Mixed number to a fraction.

(*b*) Fraction to whole or mixed number.

(*c*) To higher terms.

(*d*) To lower terms.

(*e*) Compound fractions to simple ones.

5. Simple problems in denominate numbers.

6. Concrete problems involving the relation of integers and fractions.

TENTH GRADE.—*Time, ten months.*

I. *Review of previous grades.*

1. Notation and numeration.
2. Fundamental rules.
3. Divisors and multiples.
4. Bills and accounts.
5. Fractions and decimals. No complex fractions.
6. Exercises, oral and written, for facility and accuracy.
7. Reasons for these processes.

II. *Denominate numbers.*

1. Review previous measures.
2. Troy weight and apothecaries weight.
3. Exercises in these measures.
4. Exercises in denominate fractions and decimals.

III. *Percentage, without time.*

1. Concrete problems.
2. Profit and loss.
3. Commission.
4. Stocks and dividends.
5. Brokerage.

NOTE.—Make the pupils familiar with checks, bonds, stocks, par value, premium, discount.

IV. *Arithmetical analysis.*

1. Analyze all the cases of fractions.
2. Suitable exercises in denominate numbers.
3. Simple problems in "proportional parts."
4. Percentage and its applications.
5. Interesting problems suited to the grade.

ELEVENTH GRADE.—*Time, ten months.*

I. *Review of previous grades.*

1. Drill exercises (oral and written) for quickness and accuracy.

2. Review notation and numeration, fundamental rules, fractions, decimals and denominate numbers, percentage, etc.

3. Exercises in longitude and time. Simple forms of complex fractions.

4. Explanation of the reason of these processes.

II. *Business forms.*

1. Simple commercial transactions.

2. Bills, accounts, receipts, checks, notes, etc.

3. Terms:—*debtor*, *creditor*, *debt*, *credits*, *account*, *account current*, etc.

III. *Percentage, with time.*

1. Simple interest.
2. Interest on notes.
3. Banking and discount.
4. Compound interest.
5. Review the work of the previous grade.

NOTE.—Drill the pupils on notes, their indorsement and maturity, days of grace, protest, etc. In compound interest give only simple problems to illustrate the principle.

IV. *Proportion and partnership.*

1. Simple proportion. 2. Simple partnership.

NOTE:—Solve the problems in simple proportion by the method of proportion and also by analysis. Solve the problems in partnership by analysis only.

V. *Arithmetical analysis.*

1. Review fractions.
2. Exercises in "proportional parts."
3. Percentage and simple interest.
4. Interesting problems suited to the grade.

TWELFTH GRADE.—*Time, ten months.*

I. *A general review.*

1. Drill exercises, for facility and accuracy.
2. The method of operation in any previous subject.
3. Reasons for any process in any previous subject.
4. Definition and use of the terms presented.

II. *Percentage and its applications.*

1. Taxes
2. Customs.
3. Trade discounts.
4. Annual interest.
5. Insurance.
6. Partial payments.
7. Review of the work of the previous grades.

NOTE.—Drill thoroughly on business papers and methods. In partial payments, use only the United States rule. Give no problems requiring extended operations.

III. *Proportion and partnership.*

1. Compound proportion. 2. Compound partnership.

NOTE.—Let the problems be simple and solve them by analysis.

IV. *Involution and evolution.*

1. The powers of numbers.
2. Square root and its applications.
3. The right triangle.

V. *Mensuration of surfaces and solids.*

1. Geometrical figures, their names and parts.

2. Surfaces: rectangle, parallelogram, triangle, trapezoid, trapezium, and circle.

3. Solids: prism, parallelopiped, pyramid, cylinder, cone, and sphere.

NOTE.—Find the dimensions and the area of the surfaces, and the surface and the contents of the solids.

VI. *Application of mensuration to practical problems.*

1. Measurement of land.
2. Plastering and paving.
3. Carpeting and papering.
4. Capacity of cisterns, bins, etc.

NOTE.—Omit the denominations roods, chains, and links.

VII. *Arithmetical analysis.*

1. Review the analysis of previous grades.
2. Interesting problems suited to the grade.

NOTE.—All necessary terms in this and in all preceding grades to be learned as they are needed.