RETAINING WALLS

THEIR DESIGN AND CONSTRUCTION

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BY

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PREFACE

The presentation of another book on retaining walls is made with the plea that it is essentially a text on the design and construction of retaining walls. The usual text on this subject places much emphasis upon the determination of the lateral thrust of the retained earth; the design and construction of the wall itself is subordinated to this analysis. Without gainsaying the importance of the proper analysis of the action of earth masses, it is felt that such is properly of secondary importance in comparison with the design of the wall itself and the study of the practical problems involved in its construction.

It is the purpose of the first chapter to present the existing theories of lateral earth pressure and then to attempt to codify such theories evolving a simple, yet well-founded expression for the thrust.

An attempt is made to continue this codification throughout the theories of retaining wall design so that a direct and continuous analysis may be made of a wall from the preliminary selection of the type to the finished section. Such mathematical work as is presented is given with this essential object in view.

Under Construction advantage is taken of a classic pamphlet on Plant issued by the Ransome Concrete Plant Co. (which pamphlet should be in the possession of every construction engineer) to illustrate the principles of proper plant selection.

A retaining wall is a structure exposed to public scrutiny and must, therefore, present a pleasing, but not necessarily ornate appearance. Since, in the case of concrete walls, the appearance of the wall is dependent upon the character of the concrete work, it is essential that the edicts of good construction be observed. For this reason the modern development of concreting is presented fully with frequent extracts from some of the recent important reports of laboratory investigators.

It is hoped that proper credit has been given to the authors of all such quoted passages, as well as to other references used. A vast amount of literature exists on the subject of retaining walls and earth pressure (see bibliography at the end of the book), and in view of the absence of a proper collation of all this material there is, of course, much duplication of the analysis. It is hoped that before future studies are made of earth pressure phenomena, an attempt will be made to examine existing literature and that a due appreciation will be had of the subordinated importance of the determination of lateral pressure.

I must take this opportunity to thank Mr. Arthur E. Clark, Member, Am. Soc. C. E., for his patient reading of the text and his many helpful hints.

To Mr. F. E. Schmitt, Associate Editor of the Engineering News-Record, I am deeply grateful for encouragement and aid in preparing the book and in arranging the subject matter in a logical and clear manner.

THE AUTHOR.

NEW YORK,
Feb, 1919.

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PART I

DESIGN

CHAPTER I

THEORY OF EARTH PRESSURE

The Development of the Theory of Earth Pressure.[1]—A search through engineering and other scientific archives fails to yield any evidence that prior to 1687 an attempt had been made to analyze the action of earth pressure upon a retaining wall. Undoubtedly, rough methods of computing wall dimensions existed back in prehistoric times, since the art of constructing retaining walls is as old as building art itself. In 1687 GENERAL VAUBAN,[2] a French military engineer gave some rules for figuring walls, but presented no theoretical basis for these rules. It is questionable whether such existed. In 1691 BULLET[3] advanced a rather primitive method, assuming that the angle of sliding (see Fig. 1) is 45°. The weight of this sliding

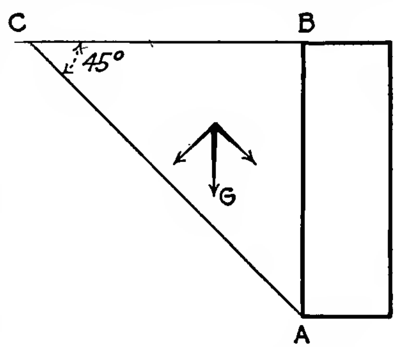


FIG. 1.—Method of Bullet.

[1] The facts in the historical outline are taken from "Neue Theorie des Erddruckes," E. WINKLER, Wien, 1872.

[2] Traite de la defense des places.

[3] Traite d'architecture practique.

wedge ABC is resolved into components parallel and normal respectively to the plane of slip. The former component was the only one considered, and by taking moments about A, proper wall dimensions are found to resist this thrust. COUPLET in 1727 makes the plane of cleavage pass through the outer edge of the wall (see Fig. 2) at D. The prism $ACFE$ is resisted by AED, the remaining portion of the wall $EBID$ supporting the wedge EFB. As before, the weight of this latter wedge EFB is resolved into parallel and normal components and the former is applied directly to the portion of the wall concerned. To get the angle that the plane of cleavage makes with the vertical, he followed the method of MAYNIEL,[1] taking this angle equal to that of the slope of a uniformly built pile of shot, the tangent of which angle is $\sqrt{8}$.

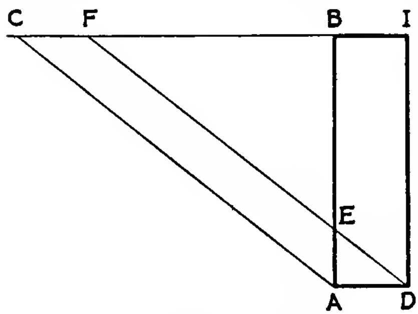


FIG. 2.—Method of Couplet.

SALLONMEYER and RONDELET (1767) follow the method of Couplet, save that the plane of cleavage starts from the back of the wall. BELIDOR,[2] an architect formulated a method in which the action of friction is considered. Proceeding as in the above methods, he arbitrarily assumes that one-half of the wedge weight is consumed in overcoming friction, the balance, properly resolved into parallel and normal components, acting upon the wall.

COULOMB in 1774, presented the first rational *theory* making proper allowance for friction and then determining the wedge of maximum thrust. Following him, NAVIER and finally PONCELET developed the theory into its present form, the elegant graphical method of determining the amount of thrust being due to the latter.

It was to be expected that the brilliant school of the English and French mathematical physicists of the middle of the last century would attempt to analyze the action of earth pressure. Levy, Boussinesq and Resal of France and Rankine of England,

[1] Traite de la pousee des terres. Memoire publiee dans l'histoire de l'academie des sciences, 1728.

[2] La Science des Ingenieurs L. I., 1729.

applied the methods of the theory of elasticity of solids to granular masses with varying degrees of success. Rankine's results are best known. Utilizing the so-called ellipse of stress (the stress quadric of elastic theory) he developed his theory of conjugate pressures. His results are probably the most universally applied of all the varied methods.

Later analysts of earth pressure have attempted to include in the theory the elements of friction between the earth and the back of the wall and that of cohesion in the mass. Such attempts leave intricate expressions of decidedly questionable practical value.

The want of agreement between theory and experiment has led to many attempts to establish empiric relations between the width of the wall base and the height without determining the earth thrust. Sir Benjamin Baker, the illustrious English engineer, under whose supervision the London tubes and outlying extensions were built, advocated a value of this ratio of about 0.4, one which Trautwine warmly seconds in his handbook. Such empiric constants were of value when walls were of the rectangular section, or verging upon the revetment type. With the modern development of the concrete walls, both gravity and reinforced sections, the use of such empiric relations is decidedly questionable and good engineering practice requires that a rational method of ascertaining the wall pressures be used in determining the proper dimensions of a retaining wall.

Exact Analysis of the Action of Earth Masses.—The correct interpretation of the character, distribution and amount of pressures throughout an earth mass typical of ordinary engineering construction, cannot be expressed by exact mathematical analysis. The usual earth mass retained by a wall contains so many uncertain elements (see page 4) that can neither be anticipated nor determined by typical tests, that it becomes very hard to assemble sufficient data for a premise upon which to found any satisfactory conclusion. To analyze an earth mass an ideal material must first be assumed. The divergence in properties between that of the actual material and the ideal material determines, in a more or less exact degree, the approximation of the results found theoretically.

Under such uncertain circumstances and with a consequent skepticism of mathematical results, the natural query is—why attempt a refined mathematical analysis? There are several praiseworthy reasons. The general action of earth pressures may be indicated and reasonable theories may be advanced as to the probable character of pressures to be anticipated. A good framework may be built upon which to hang modifications experimentally determined. The several mathematical modes of treatment may indicate a common and possibly a simple expression for the pressures, of easy and safe application to most of the conditions occurring in actual practice. Finally, the analysis of the ideal earth mass may show the maximum pressures that can exist in the usual fills, which pressures the actual ones may approach as the character of the fill approaches that of the ideal one assumed. Thus the probable maximum value of earth pressures may be established; an important function and an indication of the probable factor of safety so far as the amount of the earth thrust is concerned.

The Ideal Earth and the Fill of Actual Practice.—The mathematical discussions of the action of earth masses premise a granular, *homogeneous* mass, devoid of any *cohesion* (see page 20) and possessing frictional resistance between its particles. In addition, the surface along which sliding is impending is assumed to be a plane. Such a fill is rarely found in practice. Fills, ordinarily, are made either from balanced cuts for street or railroad grading, or depend upon local excavations. In the usual city work, materials for fill may be expected from other local improvements, public or private, which may be prosecuted simultaneously, or which may be induced to be prosecuted because of the expected place of disposal for spoil. In out of town improvements special steps, such as the employment of borrow pits, may become necessary to provide the needed material. It becomes evident that the character of the fill may vary greatly, containing any one or several types of earth, and including, usually, a large proportion of excavated rock.

The construction of the embankment itself may be carried out in widely different manners. It may be built up from a temporary railroad trestle, the materials dumped from cars and against the wall, if it be already built. Ordinary teams, or motor trucks may dump materials upon the ground, riding over the fill, or may dump over the slope of the fill already formed. Little homogeneity can be expected from either of these methods. Attempts to puddle a fill to give it eventual compactness and increased density make it difficult to team over the puddled portion and are usually abandoned on this account. While specifications often require the construction of an embankment in thin well-rammed layers, this requirement is observed more often in the breach than in the observance. It is a costly time-consuming expedient and unless required by special types of design (see page 21) may safely be ignored.

Rarely then, in either the type of the earth, or in the mode of utilizing it to make a fill, can the engineer make any definite assumptions as predicated for the ideal earth, nor would he be justified, from the standpoint of economy, in limiting the selection of materials for fill to such as approach the character of the ideal material, especially in view of the uncertainty of local geologic conditions. Obviously, refinements in the theory of earth pressures and attempts to predict with any degree of exactness the angles of repose become matters of more or less academic interest only.

Bearing in mind these limitations placed upon the ideal material assumed in the following analysis and that the mathematical work is developed solely as a means toward an end, as was pointed out in a previous page, a proper appreciation will be had of the relative value of the discussion in the next sections.

The Two Theories.—The theoretical treatment of the action of earth pressures follows along two fairly distinct lines. The RANKINE method is an analytic one, starting with an infinitesimal prism of earth and leading to expressions for the thrust of the entire earth mass upon a given surface. The COULOMB method, or the method of the *maximum wedge of sliding* is essentially a graphical one, as finally shaped by PONCELET and treats the mass of earth in its entirety, finding by the principle of the sliding wedge, the maximum thrust upon a given surface. It will be noticed that the final algebraic expressions for the thrust, as determined by either method, are similar in form, and, when certain reasonable modifications (introduced by Prof. William Cain) are placed upon the Coulomb method, are approximately alike in value also.

The Rankine Theory.—The angle of internal friction (approximately equal to the angle of repose) of an ideal earth as defined above, is the angle ϕ, (see Fig. 3) which the resultant force R makes with the normal to the plane when sliding along this plane is just about to start.

In a mass of earth unlimited in extent, select a minute triangular prism, whose section parallel to the page, is a right angle triangle, as shown in Fig. 3. In addition, let the prism be so selected that only normal stresses exist upon its arms. These stresses are then termed principal stresses, and the planes to which these stresses are normal, are termed principal planes. The existence and location of such planes are found by simple methods given in the text books on applied mechanics. For earth masses, whose upper bounding surfaces are planes, Rankine has shown that the principal planes are parallel and normal, respectively to the upper boundary plane.

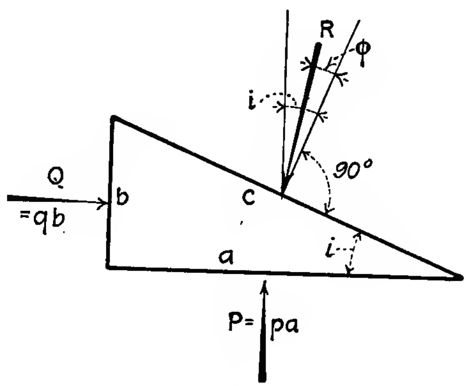


FIG. 3.

p and q are, respectively, the normal stress intensities upon the principal planes shown in Fig. 3.

Since there is a limiting value of the angle ϕ, which limiting value is the angle of repose, or better termed, the angle of internal friction, and since the angle i of the triangular prism may vary, it is possible to determine a maximum value of ϕ for some value of the angle i. The ratio between the principal stress intensities p and q may be shown to be independent of the angle i[1] and can be denoted by some constant. With the value of the angle ϕ thus defined, it is possible to express it in terms of the ratio p/q, since the angle i may be eliminated after its value rendering ϕ a maximum is found. Knowing the maximum value of ϕ, from the physical properties of the earth in question, it is thus possible to express the stress intensity ratio in terms of the

[1] See Howe's "Retaining Walls," 5th Ed.

angle ϕ. This work may be carried out by utilizing the statics of the force system as given in Figure 3.

From the statics of Fig. 3

$$\tan (i - \phi) = Q/P$$

and, since $Q = qb$, and $P = pa$,

$$Q/P = q \tan i/p; \qquad \text{since } b/a = \tan i.$$

Place the ratio of the intensities $q/p = n$.

The above equation then becomes,

$$\tan (i - \phi) = n \tan i \tag{1}$$

Denote $\tan i$ and $\tan \phi$ by x and y respectively, and expand $\tan (i - \phi)$ by the formula

$$\tan (i - \phi) = \frac{\tan i - \tan \phi}{1 + \tan i \tan \phi}$$

Equation (1) becomes

$$y = \frac{x(1 - n)}{1 + nx^2} \tag{2}$$

By the principle of the theory of maxima and minima, this expression is found to have a maximum value when $x = 1/\sqrt{n}$. The expression for y, or rather, $\tan \phi$, for this value of x is

$$\tan \phi = \frac{1 - n}{2\sqrt{n}} \tag{3}$$

To reduce this to the form as finally given by Rankine, note that

$$\sin \phi = \frac{\tan \phi}{\sqrt{(1 + \tan^2 \phi)}}$$

which trigonometric relation reduces (3) to

$$\sin \phi = \frac{1 - n}{1 + n}$$

and similarly

$$n = \frac{q}{p} = \frac{1 - \sin \phi}{1 + \sin \phi} \tag{4}$$

This gives the fixed relation between the principal intensities of stress when the maximum angle of friction ϕ is given, and the upper surface is a horizontal plane. The value of the principal intensity p upon the horizontal plane, is easily seen to be the weight of the earth mass above this plane. If the depth to this plane is h and the unit weight of the material is w then $p = wh$ and (4) becomes

$$q = wh \frac{1 - \sin \phi}{1 + \sin \phi} \qquad (5)$$

which is the classic relation between the vertical and horizontal pressures as first given by Rankine.

This is the fundamental equation of the Rankine method and the following theorems are deduced directly from it:[1]

(*a*) The direction of the resultant earth pressure against a vertical plane is parallel to the free upper bounding surface and is independent of the interposed wall.

(*b*) For an earth mass whose upper bounding plane makes an angle a with the horizontal (see Fig. 4), the intensity of pressure parallel to CA is

$$t = wh \cos a \frac{\cos a - \sqrt{\cos^2 a - \cos^2 \phi}}{\cos a + \sqrt{\cos^2 a - \cos^2 \phi}} \qquad (6)$$

This expression may be simplified by placing $\cos \phi / \cos a = \sin u$ whence

$$t = wh \cos a \tan^2 (u/2) \qquad (7)$$

Note that, in this expression, t is a linear function of the depth of earth h, so that the value of the entire thrust upon a plane AB of depth h is

$$T = th^2/2. \qquad (8)$$

and the point of application of this thrust is at one third the distance h above B.

(*c*) The final resultant thrust upon the back of the wall BC is compounded of the above thrust and the vertical weight G of the prism ABC (see Fig. 4).

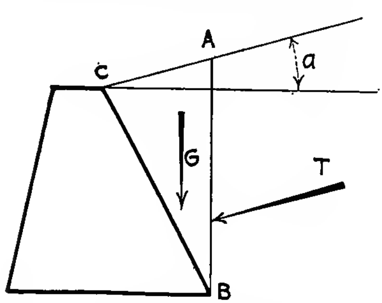


FIG. 4.—The Rankine method of determining the thrust.

It is to be noted that no allowance is made for any frictional resistance that may exist between the back of the wall and the earth mass immediately adjacent to it. The upper surface must be free, *i.e.*, the mathematical treatment excludes external loading upon the upper bounding surface. J. Boussinesq has

[1] HOWE, "Retaining Walls," 5th Ed., p. 11 *et seq.*

attempted to extend the theory of Rankine to include frictional action between the earth and wall.[1] The complexity of his analysis and the arbitrary premises although of the utmost elegance, preclude its acceptance by engineers. In fact, it is quite doubtful whether the Rankine method can be extended much beyond that set forth above.

The average earth fill has an angle of repose approximately equal to 30°. As pointed out on page 4, no refinements in the selection of this angle are justified by practical conditions. The expression for the thrust upon a vertical plane with this value of ϕ becomes with $t = wh/3$

$$T = w\frac{h^2}{6} \tag{9}$$

Taking the value of w as 100 pounds per cubic foot, this becomes

$$T = 16\,h^2 \tag{10}$$

For a wall with sloping back (the usual form of wall), as shown in Fig. 5 the thrust is found by combining the thrust upon the vertical plane AB with the weight of the earth over the batter of the back.

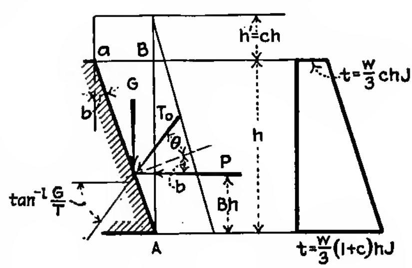


Fig. 5.—Typical loading Rankine method.

The upper bounding surface shown in Fig. 5 is that typical of the usual composite fill and surcharge equivalent loads (see later pages in the chapter for a full discussion on surcharges). Most retaining walls support an embankment of this type. For upper surfaces of varying types, a detailed analysis is given on pages 25 to 31.

The angle of friction is taken at 30°, with the consequent simplification of the Rankine formula. The ratio of the height h' to h is denoted by c, whence the total depth of fill acting upon the plane AB, Fig. 5, is $h(1 + c)$. The thrust acting upon this plane is then

$$P = wh^2\,(1 + c)^2/6.$$

[1] See an admirable resume of his work in this direction in a series of articles by him in the *Annales Scientifiques de L'Ecole Normale Superiere,* 1917 and reprinted in pamphlet form by Gauthier-Villars, Paris, 1917.

The ratio c is small, generally less than one-third, whence it is permissible to substitute $1 + 2c$ for $(1 + c)^2$. The expression for P takes the form

$$P = wh^2 \frac{1 + 2c}{2} \frac{1}{3} \qquad (11)$$

Note here, that if a trapezoid be drawn as shown in Fig. 5 with ordinates at the top and bottom of the wall the earth pressure intensities at these points, the area of this trapezoid becomes

$$wh^2 \frac{1 + 2c}{2} \frac{1}{3}$$

and the center of gravity lies at a point Bh above the base, where B has the value

$$B = \frac{1}{3} \frac{1 + 3c}{1 + 2c} \qquad (12)$$

From (11), the area of this trapezoid may be taken equivalent to the thrust upon the plane, and consequently, equivalent to the horizontal component of the resultant thrust upon the back of the wall AB. The thrust is located at the center of gravity of this trapezoid as found above.

The weight of the earth mass superimposed upon the back of the wall is

$$G = w\left(h'h \tan b + \frac{h^2 \tan b}{2}\right) = wh^2 \tan b \frac{1 + 2c}{2} \qquad (13)$$

This is the vertical component of the resultant thrust upon the back of the wall and the value of the thrust T is

$$T = \sqrt{(P^2 + G^2)}$$
$$= wh^2 J \frac{1 + 2c}{2} \qquad (14)$$

where J is equal to $\frac{1}{3} \sqrt{(1 + 9 \tan^2 b)}$ (15)

TABLE 1

$b°$	J	$\theta°$	$b°$	J	$\theta°$
0	0.33	0	14	0.42	23
2	0.34	4	16	0.44	25
4	0.34	8	18	0.47	26
6	0.35	11	20	0.49	27
8	0.36	15	22	0.52	28
10	0.38	17	24	0.56	29
12	0.40	21			

To aid in the computation of the thrust when the height of wall and the amount of surcharge is given, as well as the slope of the back of the wall, Table 1 has been prepared covering a number of values of J for the varying values of the angle b.

The angle which the thrust T makes with the normal to the back of the wall is (see Fig. 5)

$$\theta = \tan^{-1}(G/P) - b = \tan^{-1}(3 \tan b) - b \qquad (16)$$

from equations (11) and (13) above.

For a basis of comparison with the formulas developed later, a table of values of θ for the several values of the angle b is given in Table 1.

To summarize briefly the results above, it may be said that equation (14) is the Rankine expression for the thrust of an earth with an angle of repose of 30° whose upper surface is a horizontal plane. The former remarks upon the usual nature of embankments as found in actual practice justify a blanket assumption of 30° for this angle of repose and the resulting simplification of the thrust expression strengthens the reasons for the selection of that particular value of the angle of repose. For a wall with sloping back retaining a fill of shape shown in Fig. 5 equation (14) gives the expression for the thrust. The computation of this thrust is to be aided by the use of Table 1.

Coulomb Method of Maximum Wedge of Sliding.—The same assumptions as to the properties of the ideal earth mass are made as were made in the preceding theory. Referring to Fig. 6 any

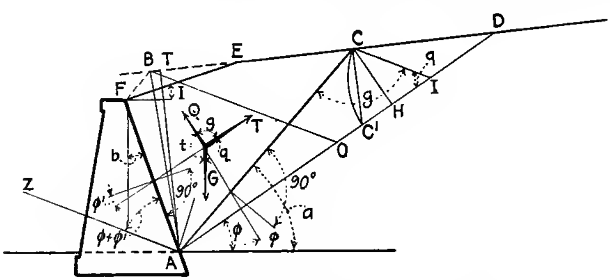


FIG. 6.—Method of maximum wedge of sliding.

prism of earth AFC, where AC makes an angle a with the horizontal, which is greater than the angle of repose ϕ, will tend to slough away from the remaining earth bank and will therefore require a retaining wall with back AF to hold it. In this prism of earth the forces acting upon it are its weight G, the reaction of the thrust T upon the wall and the reaction of its pressure Q upon the remaining bank. As different wedges of possible sliding are selected, some one wedge will produce the maximum thrust upon the wall AF, which is the actual thrust sought.

From the equilibrium of the figure, the forces T, G and Q, are concurrent, *i.e.*, must meet in a common point. From the law of concurrent forces

$T/\sin t = G/\sin g = Q/\sin q$. t, g and q are the angles as shown in the figure.

G is the weight of the irregular prism $AFEC$ and is resolved by the methods of equivalent figures (any elementary text in plane geometry) into the triangular prism ABC. If a slice of earth of unit thickness is taken and its unit weight denoted by w, the value of G is

$$G = w\frac{AT \times BC}{2} \tag{17}$$

AT is normal to BC

From the sine relation above shown

$$T = \frac{w}{2} AT \times BC \frac{\sin t}{\sin g} \tag{18}$$

To obtain the maximum value of this expression, it is necessary to separate its factors into those which remain constant as various planes of sliding are selected, and those which vary with the different planes of sliding. This is effected as follows:

Draw, in Fig. 6, what may be termed a base line AZ making an angle $\phi + \phi'$ with the normal to the back of the wall. (The explanation of the angle ϕ' will be given later.) Parallel to this line draw BO and CI. In ACI, from the law of sines

$$CI/AI = \sin t/ \sin g.$$

(Note in the figure that the angles g, t and q and their supplements are denoted by the same letters.

In similar triangles CID and BOD

$$CI/ID = BO/OD \quad \text{and} \quad BC/BD = OI/OD.$$

Inserting these values in (18)

$$T = \frac{w}{2} AT \frac{BD}{OD} OI \frac{CI}{AI} = \frac{w}{2}\left(\frac{AT \times BD \times BO}{OD^2}\right) \frac{ID \times OI}{AI} \tag{19}$$

In this expression all factors are invariant for the figure except the factor $\frac{ID \times OI}{AI}$ and to obtain the maximum value of the thrust T, it is sufficient to find the maximum value of this variable factor. Upon placing $AI = x$, $AD = a$ and $AO = b$, introducing these values in this factor and then proceeding to find the maximum value by the differential calculus, this maximum value is found to occur when

$$x = \sqrt{(ab)} \tag{20}$$

In other words the maximum thrust exists upon the back of the wall when AI is a mean proportional between AO and AD. Fig. 7 shows a simple method of finding a mean proportional by geometric construction. The value of T as given in (19), with this new value of the term $\frac{ID \times OI}{AI}$ may further be simplified by noting that triangles DTA and CHD are similar, whence $AT/CH = AD/CD$; $BO/OD = CI/ID$; $BD/OD = CD/ID$. Substituting these values in that expression for the thrust, there is the simple form

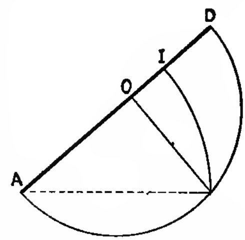


FIG. 7.—Geometric construction for mean proportional.

$$T = \frac{w}{2}(CH \times CI) \tag{21}$$

If, with I as a center, an arc CC' is described, the area of triangle $CC'I$, multiplied by the unit weight of the earth is equivalent to the maximum thrust T.

The direction of the thrust is assumed, in the original method, to be normal to the back of the wall, but Prof. Cain has modified this so that the direction of the thrust makes an angle ϕ' with the normal to the back of the wall. The angle ϕ' is the angle of friction between the earth back of the wall and the wall masonry. (See page 19 for a discussion of this frictional action between earth and wall.)

The above method as outlined is essentially a graphical one and in order to make a comparison between the results of this method and the results of the Rankine method, it will be necessary to obtain an algebraic expression for the thrust. To avoid needless complications, the profile of the earth surface will be assumed to have the shape shown in Fig. 8. Without entering into the tedious but quite simple steps in reducing the geometric substitutions above to algebraic ones, the thrust is finally found to have the form

$$T = \frac{w}{2} h^2 L \left[1 + c - p\sqrt{(1 + c)^2 - c^2 f}\right]^2 \tag{22}$$

where

$$L = \frac{\cos (\phi' + b)}{\cos^2 (\phi' + \phi + b)}. \qquad p = \sqrt{m \sin \phi}$$

$$n = \frac{vd}{\sin \phi}$$

$$f = n/m \qquad m = \frac{u + v \tan b}{\sin \phi} \qquad d = \tan b + \cot i$$

$$u = \frac{\sin (\phi' + \phi + b)}{\cos (\phi' + b)} \qquad v = -\frac{\cos (\phi' + \phi + b)}{\cos (\phi' + b)}$$

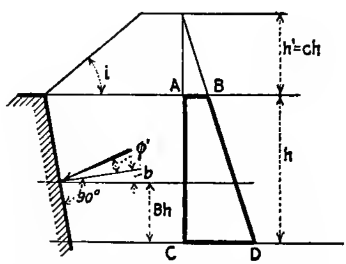


FIG. 8.—Typical loading Coulomb-Cain Method.

TABLE 2

$b°$	K			$b°$	K		
	$\phi' = 0°$	$\phi' = 15°$	$\phi' = 30°$		$\phi' = 0°$	$\phi' = 15°$	$\phi' = 30°$
0	0.33	0.30	0.29	15	0.45	0.42	0.43
3	0.36	0.32	0.32	18	0.48	0.45	0.47
6	0.38	0.34	0.34	21	0.51	0.48	0.50
9	0.40	0.37	0.37	24	0.54	0.52	0.57
12	0.43	0.40	0.39				

When the back of the wall is vertical, *i.e.*, $b = 0$, and the uppe, surface is horizontal and at the level of the top of the wall, *i.e.* $c = i = 0$, the expression for the thrust reduces to

$$T = \frac{w}{2} h^2 \frac{1 - \sin \phi}{1 + \sin \phi} \tag{23}$$

which agrees with the expression obtained on page 7 using the Rankine method, and there is the important note that *the Thrust* *upon a Wall with Vertical Back Due to a Fill Whose Upper Surface is Horizontal and Level with the top of the Wall is found to Have the Same Expression in Both Rankine and Coulomb Methods.*

In the equation for the thrust (22), the term c^2f may be neglected and as before the term $(1 + c)^2$ may be replaced by $1 + 2c$, whence the expression takes the form

$$T = wh^2K\frac{1 + 2c}{2} \tag{24}$$

$K = L\,(1 - p)^2$. K is finally reduced by substituting the above values of m and p in it and, without introducing the trigonometric steps, is given by

$$K = \frac{\cos(\phi' + b)}{\cos^2(\phi' + \phi + b)}\left[1 - \sqrt{\frac{\sin\phi\sin(\phi' + \phi)}{\cos b\cos(\phi' + b)}}\right]^2 \tag{25}$$

To compare the values of this constant K with the constant of parallel meaning J found on page 10, Table 2 has been prepared covering a range of values of b and ϕ'. As before the value of the angle of repose ϕ has been taken as 30°.

Note that if in Fig. 8, the trapezoid $ABCD$ be drawn with base $Kwh\,(1 + c)$ and ordinate at A $Kgch$, its area is

$$wh^2K\frac{1 + 2c}{2}$$

which is equivalent to the value of the thrust as found in equation (24). A comparison of these two expressions for the thrust, found by the Rankine and by the Coulomb method and a study of the tabular values of J (Table 1) and K (Table 2) shows the following points:

The form of the expression giving the thrust is the same by either method.

For values of the angle b less than 5°, K with ϕ' equal zero is the same, approximately, as J.

For values of the angle b greater than 5°, K with ϕ' equal to 30° is the same, approximately, as J.

For the values of ϕ' as noted in the preceding the directions of the thrusts are approximately alike using either theory.

From the above comparative study (also see examples at the end of this chapter giving numerical comparisons of thrust computation by either method) it is seen that, with the limitations as shown above (see pages 19 and 20 for a discussion of the proper values of the angle of friction to be assumed between the back of the wall and the earth) either of equations (14) or (24) may be used to obtain the value of the thrust. As a matter of fact the expression as deduced from the Rankine equation (14) will be used to obtain the thrust, and the Coulomb form of the thrust given in (24) will only be used where its form lends itself more readily to the analysis of the special problem at hand.

To recapitulate: The thrust upon any wall with sloping back, and earth profile as shown in Fig. 5, is to be found from

$$T = Jwh^2 \frac{1 + 2c}{2}$$

where J is the earth pressure constant to be taken from the values of J found in Table 1, c is the surcharge ratio, and w is the unit weight of the earth. The point of application of the thrust is located at a distance Bh above the base of the wall, where the values of the ratio B, is to be found from Table 3.

TABLE 3

c	B	c	B	c	B
0.0	0.33	0.5	0.42	1.0	0.44
0.1	0.36	0.6	0.42	1.5	0.46
0.2	0.38	0.7	0.43	2.0	0.47
0.3	0.40	0.8	0.44	Infinite	0.50
0.4	0.41	0.9	0.44		

Admittedly, neither theory meets rigorously the application of actual conditions, nor are they confirmed, experimentally (see page 18 for some experimental data on earth pressures) to any great degree of exactness. It follows, then, since refinements are not only unnecessary but superfluous in earth pressure theories, that such assumptions and approximations as have been noted and applied above, should suffice for all retaining wall design.

It is essential that simplicity of thrust calculation be kept in mind, as it is by far more important that a standard method of such thrust determination be had, than that the refinements of such analysis be noted. The emphasis upon retaining wall design must be placed upon the actual design of the wall itself and not merely upon the derivation of the thrust.

As a matter of interest, several of the other methods of thrust determination are given in the following section.

Various Methods of Thrust Calculation.—Most of the empirical expressions for the thrust have the form

$$T = ch^2 \tag{26}$$

with various assumptions as to the value of c. On page 9 above, the value of c, from Rankine and from Coulomb, when the angle of repose ϕ is taken as 30°, was found to be 16.

In an interesting series of discussions of earth pressures[1] this value of c, namely 16, met with considerable approval.

The analogy between lateral and hydrostatic pressures has been utilized in some formulas by assuming the earth to be a fluid with unit weight varying from 25 to 62 pounds per cubic foot, the latter amount supposedly used to insure a satisfactory factor of safety. These assumed weights would give to c in the above empiric equations a value varying from 12.5 to 31.

C. K. Mohler, in the *Journal* of the Western Society of Engineers, Vol. 15, gives a modified form of hydrostatic pressure in the compromise formula

$$T = wh^2(1 - \sin \phi)/2 \tag{27}$$

where w is the unit weight of the material and ϕ is the so-called "angle of flow." He states that the lateral earth pressures due to earth surcharges is probably insignificant and illustrates this by an ingenious arrangement of cylinders. Considerable skepticism, however, is shown in regard to this latter statement in the discussions on his paper, and doubtlessly, the author of the paper has not credited a correct effect to such surcharges.

In Vol. 19 of the same *Journal*, a modified form of the Rankine formula is given and is urged as a true expression for both lateral and vertical pressures.

To summarize the various comments upon the methods of deriving an expression for the earth thrust, it may be stated that although objections are raised to practically every suggested mode of treating such pressures, it is generally conceded that retaining wall failures are not due to weaknesses in the theory of pressures, but are primarily due to faulty design and construction. This is a vital conclusion and is a further justification for the adoption of the simple, and mathematically sound, expressions given in the

[1] Western Society of Engineers, Vol. 16, 1911.

preceding pages. Examples at the end of the chapter will illustrate the application of the various formulas and will show the simplicity of application as well as the approximate correctness of these concise expressions.

It may be stated that rule of thumb methods, both for the computation of the earth thrust and for the relations between the wall dimensions are undesirable, are of questionable professional practice and, in the case of reinforced concrete walls, are not only inapplicable, but even dangerous.

Experimental Data.—The various attempts to determine earth pressure values experimentally, have been quite disappointing, so far as definite results are concerned; but they have led to several important conclusions. The results of two such series of experiments are given here, and are of value, not only for the conclusions reached in the papers themselves, but also because of the summary of previous experiments given therein.

In a paper by E. P. Goodrich, "Lateral Earth Pressures and Related Phenomena," *Trans.* A.S.C.E., Vol. liii, p. 272, the following may be quoted as of some bearing:

Sir Benjamin Baker has pointed out that the coarser the materials the less the lateral pressure.

A. A. Steel.[1] For dry and moist earth the lateral pressure is from 1/5 to 1/3 the vertical and, in saturated materials is practically equal to it.

Some of Mr. Goodrich's important conclusions are as follows:

(*a*) The point of application of the resultant thrust is above the 1/3 point, usually about 0.4 of the height of the wall.

(*b*) Rankine's theory of conjugate pressures is correct *when the proper angle of friction is found* (the italics are mine), and probable adaptations of his formulas will be of most practical value.

(*c*) Angles of internal friction and not of surface slope must be used in all formulas which involve the sliding of earth over earth. (Such tables are to be found in the author's paper.)

It must be emphasized that the experiments mentioned above were performed upon a more or less homogeneous material. The actual composition of fills has been described on page 4.

In a paper[2] by William Cain, the conclusions, after analyzing

[1] Engineering News, Oct. 19, 1899.

[2] "Experiments of Retaining Walls and Pressures on Tunnels," *Trans.* A. S. C. E. Vol. lxxii, p. 403.

some experiments performed by the author and analyzing also the extensive experiments carried on in the past, are:

"1. When wall friction and cohesion are included, the sliding wedge theory is a reliable one, when the filling is a loosely aggregated granular material, for any height of wall.

"2. For experimental walls, from 6 to 10 feet high, and greater, backed by sand or any granular material possessing l ttle cohesion, the influence of cohesion can be neglected in the analysis. Hence further experiments should be made only on walls 6 feet and preferably 10 feet high.

"3. The many experiments that have been made on retaining walls less than one foot high have been analyzed by their authors on the assumption that cohesion could be neglected. This hypothesis is so far from the truth that the deductions are very misleading.

"4. As it is difficult to ascertain accurately the coefficient of cohesion, and as it varies with the amount of moisture in the material, small models should be discarded altogether, in the future experiments and attention should be confined to large ones. Such walls should be made as light, and with as wide a base as possible. A triangular frame of wood on an unyielding foundation seems to meet the conditions for precise measurements.

"5. The sliding wedge theory, *omitting cohesion, but including wall friction*, is a good practical one for the design of retaining walls backed by fresh earth, when a proper factor of safety is used."

Clearly, experimental data verifies neither of the above theories with any degree of exactness, yet does indicate that either of the two theories may form a rational basis for a working formula. Equation (14) may again be brought forward as the practical formula to be used in obtaining the thrust upon a wall, due to the usual type of embankment loading.

The above work has frequently discussed the items of *wall friction* and *cohesion* and these two factors will be taken up in the following sections.

Wall Friction.—The question, whether frictional resistance between the back of a retaining wall and the adjacent earth is, or is not, a permissible factor to be included in the computation of the thrust and in the determination of its direction, plays an important role in various theories of earth pressure. Since the earth backing exerts a pressure upon the wall, then by the elementary theories of physics, there must be friction between the two surfaces in contact. The angle of friction cannot be assumed larger than the angle of friction of the earth material, since if it is larger, and this is quite possible, the effect is that a layer of earth will adhere to the wall and slipping will take place between this layer and the remainder of the earth bank. If allowance is made for such frictional resistance, it is customary to take the angle of such friction (ϕ') the same as the angle of repose. This angle has been taken as 30°, and ϕ' may therefore be given the same value.

The question of lubrication between the earth and wall due to the presence of water, must be taken into account and generally the more vertical the wall is, the greater will be the effect of this lubrication upon the angle of wall friction. The use of equation (14) founded upon the Rankine method, automatically provides for this condition, as was pointed out in the comparison between the Rankine and Coulomb method on page 15.

It will be seen later, in analyzing the various types of walls, that in finding the proper dimensions of a gravity wall to safely withstand a given thrust, quite an economy in the necessary section of the wall is effected by a favorable consideration of wall friction. It is to good advantage, then, that the back of the wall be stepped or roughened so as to fully develop such wall friction.

It seems better engineering practice to make allowance for such a force than to ignore it and assume that a factor of safety of unknown value is thereby added to the wall. Such uncertain conditions as exist in wall design may more properly be allowed for in a final factor of safety of some assumed value, than to merely add blind factors by ignoring forces which must surely exist.

The question of wall friction plays an unimportant role in the design of reinforced walls (whose backs are usually nearly vertical) and as its neglect simplifies the calculation of the wall, it is permissible to ignore it—not on the basis that it does not exist, but because it has no effect upon the attendant analysis.

Cohesion.—Cohesion, as it exists in an earth mass, is rather a loosely applied term, which had better be called cohesional friction. Prof. William Cain, has defined its action:[1]

"The term 'cohesive resistance' of earth may properly apply either to its tensile resistance or to its resistance to sliding along a plane in the earth, dependent on the viewpoint. However, as the tensile resistance of the earth is rarely called for, the term 'cohesive resistance of earth' from Coulomb's time to the present, has been generally restricted to mean the resistance to sliding as affected by cohesion * * *."

[1] *Proc.* A. S. C. E. Vol. xlii, August, 1916, p. 969.

To properly appreciate the effect of this cohesional friction, it must be borne in mind that it exists to some extent, varying from a slight amount to a very large amount, in all earth masses. It is the one element that probably accounts for the large divergence between theoretically determined and experimentally determined thrusts. It is least for dry granular masses, and reaches a maximum value in the plastic clays.

In the ordinary fills as found in engineering practice (and over 90 per cent. of walls retain embankments of fresh fill) its presence is a highly uncertain one and in view of the mixed character of such a fill containing boulders, cinders and other miscellaneous material, its existence as a definite resisting force to sliding must be ignored. General practice while admitting that cohesion does exist in earth masses, has taken the very wise step, to ignore its action. While this may increase the amount of thrust upon a wall, it is very possible that, due to vibrations, or other disturbances, the cohesive action in the earth is destroyed, temporarily at least making the actual thrust approach very closely, in value, the theoretical thrust. The conclusions of Prof. Cain, quoted on page 19 may again be noted, where the method of the sliding wedge, *ignoring cohesion*, is recommended as one properly determining the thrust.

Under certain conditions, where a direct effort is made to obtain and preserve a cohesive effect in the earth mass, it is within reasonable practice to take advantage of the force. When a wall retains an old embankment, where only a thin wedge of new fill is placed between the old fill and the back of the wall, there is good justification for assuming that cohesion will be a permanent force. Again, by carefully placing and ramming in thin layers a specially selected fill, cohesion is practically assured and the design of the wall, may safely include this factor. The retaining walls of the approach to the Hell Gate Arch[1] over the East River, New York contain a fill placed with extreme care and the determination of the thrust included the factor of cohesion, permitting the construction of a fairly thin wall, where, under ordinary granular theory, a wall of prohibitive section would have been required.

The effect of cohesion may be interpreted in two manners. It has been noticed that the bank of a freshly cut trench will keep its vertical slope for quite a period, and then as it sloughs away will gradually approach a parabolic shape, with the upper portion

[1] *Engineering News*, Vol. 73, p. 886.

more or less vertical. It will be remembered that the granular theories above discussed have assumed that the surface of rupture is a plane. To allow for the cohesive action as described, a much steeper angle of slope for the material may be assumed than its ordinary angle of repose would warrant, in that way approaching the parabolic curve or it may be assumed that for a certain distance below the surface of the ground there is no lateral pressure, the surface of rupture being a vertical plane, and below this critical point the material observes the ordinary laws of the granular materials.

The first method is an empiric one and seems a rather perilous one to adopt, in view of the uncertainty of cohesive action. The above mentioned retaining walls of the Hell Gate Arch Approach were designed on this basis, the fill taking a very steep slope.[1]

TABLE 4

Material	c in lbs. per sq. ft.
Dry sand	1.5
Wet sand	8.3
Very wet sand	6.4
Clayey earth	23.1
Damp fresh earth	18.5
Clay of little consistency	39.5

A theoretical discussion of cohesion[2] indicates that the latter method is founded on more logical a basis. The effect of cohesion is to lower the "head" of earth pressure so that a soil possessing cohesion exerts no lateral pressure until a certain vertical pressure has been reached, corresponding to a depth x in the earth. The value of x is given by the expression

$$x = \frac{2c}{w} \tan\left(45° + \frac{\phi}{2}\right) \tag{28}$$

c is the coefficient of cohesion for the material and may be taken from Table 4. w is the unit weight of the material and ϕ is the usual angle of repose of the material. Below this depth x, the earth pressures follow the ordinary laws of non-coherent earths (see Fig. 9). An application of the above formula to ordinary

[1] See previously quoted article in *Engineering News*.

[2] CAIN, "Earth Pressure, Walls and Bins," p. 182 *et seq.*

earth with some cohesion shows that this lowering of the head is but a slight one and for all practical purposes may be ignored. For a densely compacted material, approaching a plastic clay this lowering of the head reaches a value that has a marked effect upon reducing the amount of the thrust.

In an interesting paper on the lateral and vertical pressure of clay[1] a set of formulas for the stress system in a coherent earth mass was given, after a careful experimental study of the necessary coefficients. While of limited application (they are primarily for the clayey materials) they are worthy of quotation and may prove of service in interpreting the action of materials

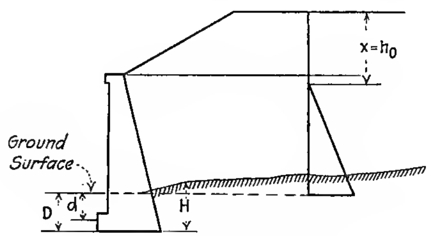


Fig. 9.—Coherent earth.

of that nature. Before presenting these equations it may be well to note the character of some of the stresses. In a material more or less plastic there is a tendency for the surface adjacent to an applied loading to heave and raise. This may be shown by a mathematical discussion of the stress distribution in a material of that character[2] and is clearly demonstrated by experiment. Under a retaining wall the pressure is generally non-uniformly distributed, having a maximum value at the toe and a minimum value at the heel. From the foregoing note it is clear that when the wall bears on a plastic coherent soil, there must be a certain minimum downward pressure at the heel to compensate for the upward heaving pressure caused by the soil loading. This is given below. The loading which a soil can stand without excessive yielding is usually termed its passive stress, as distinguished from the stress which it exerts (the lateral stress) and which is termed its active stress. The passive stress is frequently called the ultimate bearing value of the soil.

[1] Bell, "Minutes of the Proceedings of the Institute of Civil Engineers," Vol. cxcix, p. 233.

[2] See Howe, 5th Ed., "Retaining Walls."

TABLE 5

Character of clay	k tons, sq. ft.	a
Very soft puddle clay	0.2	0°
Soft puddle clay	0.3	3°
Moderately firm clay	0.5	5°
Stiff clay	0.7	7°
Very stiff boulder clay	1.6	16°

The retaining wall is subjected to a lateral pressure from the coherent material of intensity p_1, which is given by the equation

$$p_1 = wh \tan^2 \left(\frac{\pi}{4} - \frac{a}{2}\right) - 2k \tan \left(\frac{\pi}{4} - \frac{a}{2}\right)$$

(See Fig. 9.) a and k are the constants of the coherent material, and may be taken from Table 5. From the above expression it is to be noted that within a given distance x below the surface, there is no intensity of pressure. This value of x,

$$x = \frac{2k}{w} \cot \left(\frac{\pi}{4} - \frac{a}{2}\right) \tag{29}$$

may be compared to the similar value of x given in equation (28) on page 22.

If p_2 is the *minimum* permissible intensity of downward pressure on the foundation at the *heel* of the wall, where the depth is H

$$p_2 = wH \tan^4 (\pi/4 - a/2) - 2k \tan^3 (\pi/4 - a/2) - 2k \tan (\pi/4 - a/2) \tag{30}$$

The retaining wall rests in a trench and its footing butts against the forward part of the trench when the earth pressure acts upon the wall. The maximum intensity of horizontal resistance in front of a wall at any depth d (note that this is a passive stress) is

$$r_1 = wd \tan^2 (\pi/4 + a/2) + 2k \tan (\pi/4 + a/2) \tag{31}$$

The maximum permissible intensity of downward pressure on the foundation at the toe of the wall, where the depth is D (note that this is a passive stress, usually termed the safe bearing value of the soil) is

$$r_2 = wD \tan^4 (\pi/4 + a/2) + 2k \tan^3 (\pi/4 + a/2) + 2k \tan (\pi/4 + a/2) \tag{32}$$

While the above series of equations are intended primarily for the clays, they are applicable to all materials upon proper adjustment of the values of the coefficients. Thus for non-coherent or ordinary granular masses, the cohesion coefficient k is zero and the angle ϕ replaces the angle a.

In a discussion upon the results given by Bell, Prof. Cain has noted, that if A is the value of a unit area, then the relation between the k given here and the c of his material is $k = cA$.

In the analysis of the walls in the following chapters and in the application of the results of the text to specific problems the action of cohesion will be entirely ignored, the formulas given in equations (14) and (24) being used to obtain the thrust upon the wall.

In determining the strength of an existing wall retaining a well-settled and aged embankment, there is little doubt of the existence of cohesion, and with the aid of the preceding equations a proper determination of the load carrying capacity of the wall may be obtained. Whether to increase the load upon the wall, by addition of a surcharge, because of the lowered lateral pressure, is a matter of judgment and in view of the uncertain character of cohesion and the possibility of its absence for some unforeseen reason, a careful engineer may sacrifice apparent economy to an easier conscience.

Surcharge.—While a surcharge denotes an earth mass above the level of the top of the wall, it is customary to reduce applied loadings on the upper surface to equivalent surcharges. In the theory of the distribution of stress through elastic solids, it has been proven[1] that such distributions are independent of the manner of the local loading except for points fairly close to such loads and it is permissible to substitute the resultant load for this distribution, or conversely a distributed loading for a series of concentrated loads.

It seems quite justifiable to extend this law to granular masses and, in fact, it is generally accepted that applied loadings may be reduced to a distributed earth surcharge equivalent. The reduction of dynamic loadings is, possibly more involved than that of the reduction of still loadings. Nevertheless, it would seem that in view of the comparative inelastic properties of a granular mass and of the large amounts of voids in the material,

[1] See for example, J. Boussinesq, "On the Applications of the Potential," etc.

the vibrations are completely "dampened" before they reach the wall. If this is conceded, no distinction need be made between static and dynamic loads. In any event, impact coefficients of as great value as are applied to elastic solids should not be applied to the earth mass.

While there may be some question as to whether a surcharge loading produces a lateral pressure of intensity proportionate to the fill proper, below the level of the top of the wall a theoretical analysis gives no foundation for such doubt, and there is as tangible a basis for assuming the full proportionate effect of the surcharge upon the wall as there is for the other theoretical assumptions of earth pressures.

When the surcharge is uniformly distributed over the top of the embankment and extends to the back of the wall, equations (14) and (24) give the amount and Table 3 gives the location of the resultant thrust. When the surcharge is not of uniform distribution, or does not extend to the back of the wall, the conditions require special analysis. The following treatment of such surcharges is given primarily for the same reasons as in the treatment of earth pressures in general and is to be used in the same sense.

When an external loading upon an embankment has been reduced to a uniformly distributed loading equivalent to the same weight of earth, a new profile has been given to the top of the embankment. It must be noted here, however, that when a wedge of earth is about to slide along some plane in the fill proper, this plane cannot extend at the same slope throughout the surcharge, but must be directed vertically upwards after reaching the surface of the ground upon which the surcharge rests (see Fig. 10). The method of the maximum wedge of sliding is most easily applied to the discussion of this case and a simple graphical analysis follows.[1]

Let the equivalent surcharge extend to *v*. Draw a line parallel to the upper surface and at a distance $2h'$ above it. Draw *bn* parallel to *ov*. Connect *o* and *n*. The intersection *s* of this line with the ground surface is the usual base point to construct the equivalent thrust triangle. Thus through *s*, let *sa* be parallel to the base line *oz*. Locate *d* as the mean proportional between *oA* and *oD*, and locate *c* by drawing through *d* a line parallel

[1] Taken from MEHRTENS "Vorlesungen * * * * * Baukonstructionen" as translated by G. M. PURVER, *Engineering & Contracting*, Nov. 2, 1910.

to the base line. Through c draw uk parallel to no. With d as a center describe an arc cm. The thrust on the wall due to earth and the surcharge is the area of the triangle udm multiplied by the unit weight of the earth. It is shown[1] that this triangle is equivalent to the area of cdm multiplied by the ratio $(h+2h')/h = 1+2c$ where c is the usual surcharge ratio. The triangle cdm is the measure of the thrust upon a wall, with

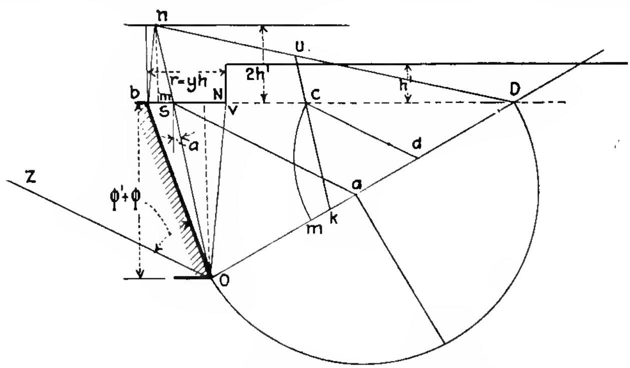


FIG. 10.—Surcharge not extending to back of wall.

no surcharge, whose back is the line so, making the angle α with the vertical. The thrust may then be expressed algebraically by

$$T = \frac{gh^2(1 + 2c)}{2} K \tag{33}$$

with K as given in (25) with the value of $b = \alpha$. When the surcharge extends to the back of the wall, then the b of the wall is equal to α and the form for the thrust in this case is the same as that given in (24), which is a measure of the approximation of that formula.

To determine α denote the distance vb by r and let this be equal to yh. Let the angle voN be β. $h \tan \beta = r - h \tan b$ or $\tan \beta = y - \tan b$. $bm = 2h' \tan \beta = 2ch \tan \beta$. $Nv = h \tan \beta$. $mN = r - bm - nv = h[y - (1+2c) \tan \beta]$. $\tan \alpha =$

$$\frac{mN}{h(1+2c)} = \frac{y-(1+2c)(y-\tan b)}{1+2c} = \tan b - \frac{2c}{1+2c}y. \tag{34}$$

It is to be noted that α may be negative. For K then see Table 13.

The application of the wedge of maximum thrust to the case

[1] *Ibid.*

of isolated loads on the surface, is quite lengthy and involves considerable geometric construction. It is discussed fully in the lectures mentioned previously. For ordinary practice it seems quite sufficient to replace it by its equivalent uniform spread over the surface and then to apply the wedge theory to a surface of broken contour, as shown in Fig. 10.

An effective and simple manner of treating this case has been devised by the Design Bureau, Public Service Commission, 1st district N. Y. and is as follows:

In Fig. 11 there is a concentration of L/a as shown, a surcharge of h', and the earth back of the wall. For some plane of rupture

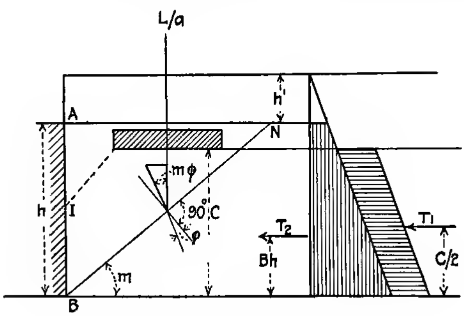


FIG. 11.—Surcharge concentrations.

BN all three exert a maximum thrust upon the wall. A few trials are ample to determine this plane with sufficient accuracy.[1] Let the plane of maximum thrust make an angle m with the horizontal. The thrust T_1 due to the concentrated load is $\frac{L}{a}$ $\tan(m - \phi)$. The thrust T_2 due to earth and surcharge is $\frac{gh^2(1+c)^2}{2}$ $\cot m \tan(m - \phi)$ and the total thrust is $\frac{L}{a}\tan(m - \phi) + \frac{gh^2}{2}(1 + c)^2 \tan(m - \phi) \cot m$ the maximum value of this is found either graphically as noted above or by equating the derivative of this last expression to zero, whence, upon placing the ratio of L/a to $\frac{gh^2(1+c)^2}{2} = r$

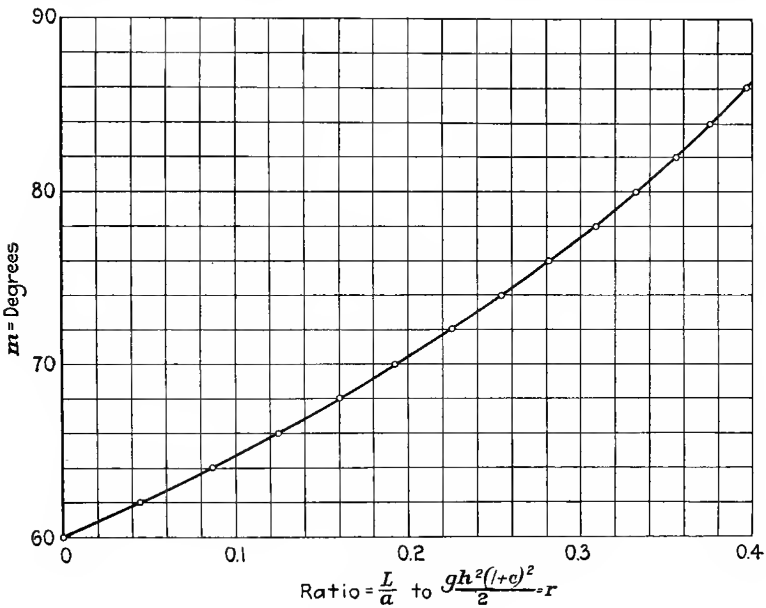
$$r = \frac{\sin(m - \phi)\cos(m - \phi)}{\sin^2 m} - \cot m \qquad (35)$$

[1] See CAIN, "Earth Pressure, Walls and Bins," p. 43 for an excellent graphical solution of this case.

Assuming $\phi = 30°$ and simplifying the expression

$$r = \frac{\sin (2m - 120°)}{2 \sin^2 m} \tag{36}$$

The relation between m and r is shown on Curve Plate 1. When the value of m brings the wedge of thrust inside the distribution of the loading L, it is reasonably certain, unless L is small, that the maximum thrust upon the wall occurs when the plane of



Curve Plate No. 1.

slip just encloses the spread of the load L. Where the back of wall is battered, the above method may be applied to the vertical plane through the heel of the wall, and this thrust may be combined with the superimposed weight of the wall over the back.

The application of the earth and surcharge thrust, if, as before, $(1 + c)^2$ is replaced by $1 + 2c$, (see page 15) is at the center of gravity of the trapezoid of loading, or at a distance Bh above the bottom of wall, with B as given in Table 3. The thrust due to the isolated load may be assumed to be distributed uniformly along the back of the wall, from the base of such load to the bottom of wall. As shown in Fig. 10 its lever arm is then $C/2$.

A simple method of reducing isolated concentrated loads to a uniformly distributed surcharge, making the standard thrust equations (14) and (24) applicable is as follows. The concentrated load is assumed to be transmitted along slope lines making an angle of 30° with the vertical. (See the following pages of this chapter for the experimental justification of this assumption.) At the point I, where this distribution strikes the line AB, see Fig. 11, determine the intensity of vertical pressure. With this as the new surcharge equivalent, employ the above equations to determine the thrust. This method is, of course, quite approximate, and should be used more as a method of confirming the results obtained in the more exact construction above, than as a primary method of getting the thrust. An example at the end of this chapter will illustrate the two methods.

The preceding discussion of surcharge loadings has confined itself to the *lateral* effect of such loadings upon a retaining wall. It may be of interest to determine the vertical intensity of such loadings at distances below the upper bounding surface. The intensity diminishes as the distance from the upper surface increases and its spread may be said to be confined, roughly, within the surface of a cone. Several expressions are given for the intensity at any plane below the upper surface.

In Vol. 20, *Journal* of the Western Society of Engineers, Mr. Lacher has given the following expression for the vertical live-load intensity at any depth h below the surface (due to locomotive wheel loads)

$$P = \frac{11000}{8 + 2hx}$$

where x is the inclination of the spread planes in fractions of a foot per foot of depth.

The distribution of pressure through soil has been experimentally determined[1] and for depths of over 3 feet there is a spread of fairly uniform intensity extending within slope planes making an angle of 30° with the vertical.

An empiric expression given by Prof. Melvin L. Enger in the *Engineering Record* Jan. 22, 1916, p. 107, for the intensity of

[1] *Proc.* Am. Soc. Testing Materials, Vol. 17, part 2, 1917.

vertical pressure at any depth as experimentally determined is as follows:

$$A = pB$$

where A is the intensity of pressure at a depth h in inches, B is the surface intensity of pressure and p is the percentage of the surface intensity given by the following

$$p = 91\ d^{1.86}/h^{1.95}$$

The authors of the paper doubt whether the above expression has general application. It would show, roughly, however, that such transmitted pressure varies as the inverse square of the distance below the loaded surface. A. E. H. Love has shown[1] that the transmitted pressure through an isotropic solid, at a distance h below the loaded surface and directly below the loaded point is

$$B = \frac{3W}{2\pi}\,\frac{1}{h^2}$$

so that there is a striking agreement in the variation of transmitted pressure in solid and granular masses. For an interesting treatise on the distribution of pressure through solids for any character of surface loading, See "Application des Potentials" by J. Boussinesq, pp. 276 *et seq.*

Pressure on Cofferdams.—A cofferdam retaining earth is in a sense, a retaining wall subject to the ordinary theory of lateral pressures. The cofferdam itself is an assembly of sheeting, waling pieces, or rangers and braces, the design of which follows the ordinary theory of the design of timber structures. Mr. F. R. Sweeny[2] has presented a thorough investigation of the loadings upon such a structure together with a study of the economics of its design.

His design has been predicated upon the assumption that the ratio of the unit lateral pressure to the unit vertical pressure is given by a constant c (corresponding to the earth pressure coefficients K and J of the preceding pages). The unit weight of the material outside the sheeting is denoted by w. To quote the author:

"The values of w and c are not easily determined being largely matters of mature judgment. In any event, it is important to look into the

[1] "A Treatise on the Mathematical Theory of Elasticity," 1st Ed., p. 270.

[2] *Engineering News-Record*, April 10, 1919, pp. 708 *et seq.*

matter of possible saturation of the soil to the point where hydrostatic pressure will be developed and superimposed upon the earth pressure."

The economic proportions and the best dimensioning of the timbers and sheeting (wood and steel) are given in the article and the entire design is exhaustively treated.

Pressures of Saturated Soils.—With the presence of water in a soil, an additional lateral pressure is exerted from the plane of the water surface to the bottom of the wall. An interesting paper by A. G. Husted[1] discusses in detail this important question. The following quotations from the paper cover the salient features of the treatment.

"Formulas giving the lateral pressure of earth against vertical walls may be found in many text books and hand books. These formulas, however, usually refer to dry earth and not to earth which is saturated with water. The writer has had occasion when designing structures, wholly or in part below water level to calculate the lateral pressure of saturated earth, and being unable to find a satisfactory method for computing these pressures has worked out the method herein set forth."

The writer of the paper states that he will apply the RANKINE relation between the lateral and vertical intensities as given by equation (14).

"As has been noted before, the formula assumes that the lateral pressure at any point bears a definite relation to the vertical pressure, this relation depending entirely upon the angle of repose. It will thus be seen that the second part of the equation can be divided into two parts, wh representing the unit vertical pressure and $(1 - \sin \phi)/(1 + \sin \phi)$ representing the relation between lateral and vertical pressures.

"Two methods of applying this formula to cases involving saturated earths have been and are still in quite general use. One of these methods consists in computing the total lateral pressure in the usual way using for w the weight of dry earth and for ϕ the angle of repose of dry earth. To this pressure, then, is added full hydrostatic pressure below the plane of saturation. This method may quite often give results close enough to actual conditions for ordinary purposes of design, but it appears to the writer to be at variance with the fundamental formula. In the first place, no allowance is made for the fact that saturated earth has a smaller angle of repose than dry earth, and in the

[1] *Engineering News-Record*, Vol. 81, p. 441 *et seq.*

second place it is assumed that earth weighs the same in water as it does out of water.

"Another method of calculating lateral earth pressures consists in computing the total lateral pressure in the ordinary way and adding to this, partial hydrostatic pressure below the plane of saturation. The amount of the partial hydrostatic pressure is determined by taking the difference between full hydrostatic pressure and lateral earth pressure for an equivalent depth. This method, however, can easily be proved erroneous by applying it to a fill of completely saturated earth. In this case the partial hydrostatic pressure to be added will be the difference between full hydrostatic pressure and lateral earth pressure for the total depth of earth. It can thus be seen that the total lateral pressure at the bottom would be exactly equal to full hydrostatic pressure. This is absurd.

"In order to correct the errors in the above mentioned methods, a method has been worked out which the writer believes to be theoretically correct. In this method the following assumptions are made:

Lateral earth pressure varies directly with the vertical earth pressure for earth with any given angle of repose and is equal to the vertical pressure multiplied by $(1 - \sin \phi)/(1 + \sin \phi)$.

Water exerts full hydrostatic pressure laterally as well as vertically regardless of the amount of the space occupied by earth.

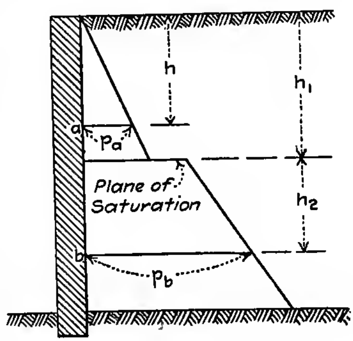


FIG. 12.

"It is a well known fact that the angle of repose of earth in water is less than the angle of repose of dry earth. Therefore the ratio of lateral pressure to vertical pressure is greater below the plane of saturation than above. On page 580 of Merriman's "American Civil Engineers' Pocket Book" the angle of repose of dry earth is given as 36°53′ while that of soil under water is given as 15°57′.

"Above the plane of saturation the lateral pressure is computed in the usual manner. Below the plane of saturation the lateral pressure is obtained by multiplying the total vertical pressure less the buoyant effect of water by $(1 - \sin \phi)/(1 + \sin \phi)$ and adding to this the full hydrostatic pressure. For example, in Fig. 12 the unit lateral pressure p_a at point a which is above the plane of saturation is $w_1 h(1 - \sin \phi)/(1 + \sin \phi)$. w_1 is the weight of the dry earth per cubic foot, h is the distance of the point a below the surface and ϕ is the angle of repose of dry earth. Likewise the unit lateral pressure p_b at point b below the plane of saturation is $(w_1 h_1 + w_2 h_2)\ (1 - \sin \phi)/(1 + \sin \phi) + 62.5\, h_2$.

w_1 as above is the weight of the dry earth per cubic foot, h_1 is the distance from the ground surface to the plane of saturation, w_2 is the weight per cubic foot of earth under water, h_2 is the distance of the point b below the plane of saturation and ϕ_2 is the angle of repose of earth under water.

"It will be noticed that in this method, for points below the plane of saturation, hydrostatic pressure and earth pressure are separated; that full hydrostatic pressure is allowed; that the vertical pressure is obtained by adding the total weight of earth above the plane of saturation to the net weight (weight under water) of earth below the plane of saturation; that the lateral earth pressure is obtained by multiplying the vertical pressure by $(1 - \sin \phi_2)/(1 + \sin \phi_2)$; that the total lateral pressure is obtained by adding the hydrostatic pressure to this lateral earth pressure.

"It can be readily seen that if a smaller angle of repose is assumed for saturated earth than for dry earth, there will be a decided increase in the unit lateral pressure at the plane of saturation. In other words, the unit lateral pressure an infinitesimal distance below the plane of saturation will be much greater than that at an infinitesimal distance above the plane of saturation.

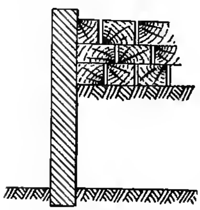


Fig. 13.

"At first thought this appears absurd, but it can be seen that it should be so. It can perhaps be best illustrated by an exaggerated example. Take the case of a retaining wall supporting a bank of earth loaded with timbers (Fig. 13), the lateral pressure of the timbers against the wall is zero, but at an infinitesimal distance below the surface of the earth the pressure is a considerable amount due to the load that is superimposed.

"The difference is plainly due to a difference in the angle of repose."

While the preceding analysis is a correct mathematical interpretation of the action of saturated, *homogeneous* material, devoid of *cohesion*, and may be used with the same degree of freedom as any of the carefully worked out theories of earth pressure, it is open to the same vital objections as were stated on the pages preceding. However, as long as a proper appreciation is had of the limitations of theory in general and if the lateral pressures are computed as suggested on page 16 and as given by the equations there shown the method presented by Mr. Husted is a practical one and should be followed provided a safe lateral thrust of saturated soils is sought.

Sea Walls.—A sea wall is essentially a retaining wall with a fill of varied character behind it, composed, usually of rip-rap, earth, cinders and the like, and subject to a hydrostatic pressure varying with the tide. An analysis of the pressure to which sea walls are subjected is given in an article by D. C. Serber, *Engineering News*, August 23, 1906, excerpts of which are quoted below. Walls with vertical backs are the only type treated. The Rankine method, as applied in the previous pages, is used in this treatment, the thrust intensity being given by equation (5). It is assumed in the paper that the fill varies by strata, a horizontal plane separating the fills of different character. If the fill back of the wall is assumed to be composed of two such materials, of weights w_1 and w_2, respectively and separated from each other by a horizontal plane, h_2 above the bottom of the wall and h_1 below the top of the wall Mr. Serber notes the following important conclusion (theoretically deduced):

"The total pressure on the lower section of the wall (*i.e.*, below the plane of separation) is entirely independent of the angle of natural repose of the material above the plane of separation."

If the angle of repose of the upper material, of weight w_1 is ϕ_1 and that of the lower material, of weight w_2, is ϕ_2 and if, for the sake of simplifying the resulting expression there is put

$m = h_1/h_2$; $n = w_1/w_2$ and $a_1 = \frac{1}{2}(90° - \phi_1)$ $a_2 = \frac{1}{2}$ $(90° - \phi_2)$

the total pressure P_2 on the back of the wall is

$$P = \frac{w_2 h_2^2}{2}[m^2 n \tan^2 a_1 + (2mn + 1) \tan^2 a_2]$$

An ingenious graphical method of obtaining the total pressure of two or more layers of different fill is presented in the paper founded upon the reduction of the different weights in terms of one of the weights.

The effect of surcharge upon a sea wall is discussed as follows:

"Merchandise, cranes and other loads of considerable weight are apt to be stored temporarily or permanently on the sea wall and the backing immediately behind it. The Department of Docks and Ferries of New York City assumes a uniform vertical load of 1000 pounds per square foot, * * *. When the bottom is very soft mud of considerable depth and a pile foundation is to be resorted to, the normal difficulties of sustaining a retaining wall are so great that it becomes highly desirable to avoid the additional thrust due to the surcharge. In such cases a platform may be built * * * supported on an independent foundation sufficient to carry the surcharge, thus relieving the wall of the thrust * * *."

The inclusion of hydrostatic pressure upon this wall may be dealt with in the manner outlined in the preceding section, the formulas of Mr. Serber being readily adaptable to the principles given in that section.

It must be emphasized that a sea wall is a structure of peculiar importance in the design of which the paramount question is not one of ascertaining how great the thrust upon its back is, but how can its foundation carry the loads brought upon it. Accordingly due appreciation to this question must be given before attempting refinements in the calculation of the thrusts that may be induced in the wall by the fills deposited behind it.

A number of problems have been prepared at the end of this and the succeeding chapters to illustrate the application of the several tables, curves and equations given in the text immediately preceding. They will also serve to demonstrate, numerically, the points discussed in the chapter, bringing home more forcibly the truths quoted than the literal equations.

Problems

1. A wall with a back sloped to a batter of one on four and 30 feet high supports a level fill subject to a surcharge loading of 600 pounds per square foot. What are the thrusts, by both Rankine's and Coulomb's methods (*a*) when there is no surcharge; (*b*) when the surcharge extends to the wall *a* (see Fig. 5); (*c*) when the surcharge extends up to the point *b*, directly over the heel of the wall.

The angle that the back makes with the vertical is $\tan^{-1}(\frac{1}{4}) = 14°$. For the condition of no surcharge, from (14) and Table 1 with $J = 0.42$ for $b = 14°$.

$$T = \frac{100 \times 30^2}{2} \times 0.42 = 18{,}900 \text{ pounds.}$$

From Table 1, $\theta = 23°$ and the angle that the thrust makes with the horizontal is $23° + 14° = 37°$.

From (25) and Table 2 for $\phi' = 0°$, $15°$ and $30°$, $K = 0.44$, 0.41 and 0.42 respectively and the values of the thrusts are accordingly, 19,800, 18,500 and 18,900 pounds.

For the condition of the surcharge extending to the back of the wall, the constants remain as above and since $c = \frac{6}{30} = 0.2$, the thrusts are each increased by $(1 + 2c)$ or by 1.4. The thrust, using Rankine's method is then $1.4 \times 18{,}900 = 26{,}500$ pounds. The three thrusts, employing the method of the sliding wedge method become, respectively 27,800, 25,900 and 26,500 pounds as the angle of friction between wall and earth is taken as 0° 15° or 30°.

When the surcharge extends to *b* the condition under which the method of Rankine is used must receive special investigation, since equation (14) no longer applies. From (11) with $c = 0.2$, the thrust is

$$T = \frac{100 \times 30^2 \times 1.4}{2 \times 3} = 21{,}000$$

The weight of the triangle G is, since $ab = 30 \times 1/4 = 7.5$, 11,250 pounds and the resultant thrust upon the wall is

$$T_o = \sqrt{(21{,}000)^2 + (11{,}250)^2} = 23{,}700 \text{ pounds.}$$

The angle which this final thrust makes with the horizontal is

$$\tan^{-1}(11{,}250/21{,}000) = 28°.$$

With the expression given in (33), the method of the sliding wedge may be employed, after the proper value of a has been found. The value of the ratio y is $7.5/30 = 0.25$. From (34) $\tan a = 0.25 - \frac{0.4}{1.4}0.25 = 0.18$, from which $a = 10°$ and the corresponding values of K for the angles of friction 0°, 15° or 30° are 0.42, 0.39 or 0.39 giving for T the corresponding values 23,500, 24,500 or 24,500 pounds.

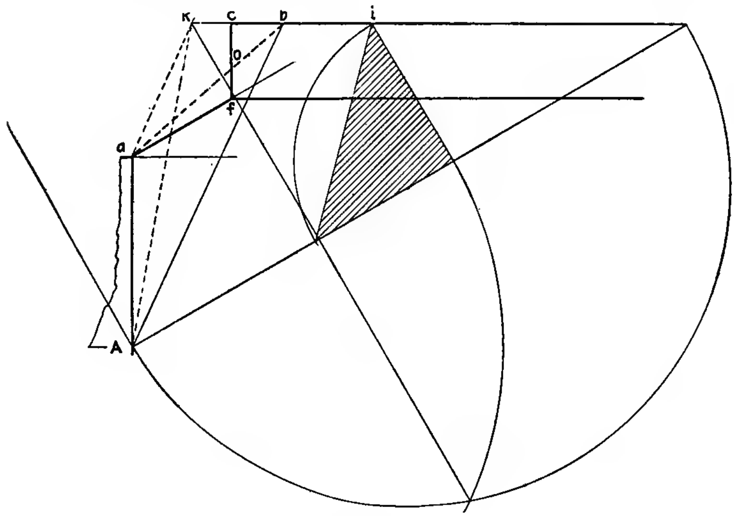


FIG. 14.

Allowing for friction between the back of the wall and the retained earth, a close agreement is again to be noted between the two methods of computing the thrust.

2. A wall with vertical back 20 feet high supports an embankment as shown in Fig. 14 subject to a surcharge of 800 pounds per square foot. Determine the thrust for the two conditions of no surcharge and surcharge.

For the condition of no surcharge, equation (22) may be used. Here $h' = 6$ feet approximately and c is then $6/20 = 0.3$. The angle $b = 0°$ and f the friction between wall and earth is ignored (which is advisable when the back of the wall is vertical, as it is in this problem) ϕ' is also zero. Again the angle of repose and the angle i are both equal to 30°. The various factors in the expression then take the following values:

$L = 1/\cos^2 \phi = 4/3$. $d = \cot\ i = \cot \phi$. $u = \sin \phi$ and $v = -\cos \phi$. $n = -\frac{\cos \phi \cot \phi}{\sin \phi} = -\cot^2 \phi$. $m = \sin \phi / \sin \phi = 1$, and $f = -\cot^2 \phi = -3$. $p = \sin \phi = 1/2$.

$$T = \frac{gh^2}{2} \times \frac{4}{3}\left(1.3 - \frac{1}{2}\sqrt{1.3^2 + 3 \times 0.09}\right)^2$$
$$= 9{,}600 \text{ pounds.}$$

If the expression in (24) had been used with $K = 1/3$ and with the same value of $c = 0.3$, the value of the thrust thus found would be

$$\frac{100 \times 400 \times 1.6}{2 \times 3} = 10{,}700$$

The latter method, or rather, equation (24) is apparently sufficiently exact for the conditions under which the problem was analyzed.

For the surcharge of 800 pounds per square foot, as shown in the figure, the graphical construction of Poncelet is employed to determine the thrust.

Draw *aob*, making the triangles *aof* and *cob* of equivalent area. (A few trials will determine the location of this line. In fact the accuracy of the problem is easily satisfied by locating the line *aob* by inspection.) Draw *Ab*, then *ak* parallel to it and proceed as before with this method. The thrust is then the area of the thrust triangle *inm*, multiplied by the unit weight of the earth 100 pounds per cubic foot and is then equal to

$$\frac{16.7^2 \times 100}{2} = 13{,}900 \text{ pounds.}$$

As a check upon this method, note that the line *aob* makes an angle of 41° with the horizontal. The method, using equation (22) may be employed with the new surface *abi*. . . With the same scheme of substitution as employed in the first part of the problem, with $i = 41°$, $n = \cot \phi \cot i = 2.0$ and $c = 14/20 = 0.7$. The thrust is then found from the expression

$$T = \frac{100 \times 20^2 \times 4}{2 \times 3}\left(1.7 - \frac{1}{2}\sqrt{1.7^2 + 2 \times 0.49}\right)^2 = 13{,}700$$

affording a satisfactory check upon the graphical calculation.

3. A material is so densely compacted and well drained upon being placed behind a retaining wall that it is safe to take its angle of slope as 45°. Derive an expression for the thrust against a vertical wall and also against a wall with a batter of one in four.

With the surface horizontal and against a vertical wall the expression for K in both the Rankine and Coulomb method is

$$\frac{1 - \sin \phi}{1 + \sin \phi}$$

which becomes for a value of $\phi = 45°$, closely one-sixth. The thrust for this material is then one-half of the normal thrust against a vertical wall, the normal thrust being that produced by a material with a slope angle of 30°.

The value of the slope angle is 14°. From (14) the expression for the thrust becomes, using the above value of ϕ and ¼ for tan b

$$T = 0.3 \frac{gh^2(1 + 2c)}{2}$$

the value of J now being 0.3, which may be compared to the value 0.42 for $\phi = 30°$.

The corresponding values for the thrust as determined by the method of the sliding wedge are easily found by proper substitution of the value of $\phi = 45°$ in the constant K, in the expression as given in (25). This arithmetic work need not be given here.

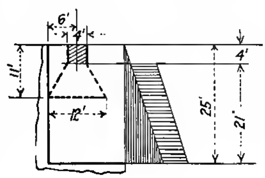


FIG. 15.

4. A building wall running parallel to a retaining wall, as shown in Fig. 15 carries a load of one ton per square foot and has a spread of four feet at a base four feet below the top of the retaining wall. The retaining wall is subject to no surcharge load other than that produced by the bearing wall. What is the total thrust upon the wall and where is it located?

Referring to Fig. 15, the value of L/a is four tons or 8000 pounds per lineal foot of wall. There is no surcharge and with $h = 25$ feet

$$\frac{gh^2(1 + c)^2}{2} = \frac{100 \times 625}{2} = 31{,}250 \text{ pounds.}$$

The ratio L/a to $gh^2(1 + c)^2/2$ is 0.256. This is the value of the ratio r. With this value entering curve plate No. 1, the value of m for a maximum wedge of sliding is 74°. It is observed that this plane will intersect the footing and accordingly the maximum plane of slip is made to pass through the inner edge of the base. This gives a value of 69° for m.

The thrust due to the concentrated load is

$$8000 \tan(69° - 30°) = 6480 \text{ pounds.}$$

That due to the earth wedge is

$$\frac{100 \times 625}{2} \cot 69° \tan(69° - 30°) = 9700 \text{ pounds.}$$

The point of application of the thrust due to the concentrated load is 10.5 feet above the base of the vertical wall. That of the earth wedge is one-third of the distance up or 8.33 feet. The total thrust is then 6500 + 9700 = 16,200 pounds and is located

$$\frac{6500 \times 10.5 + 9700 \times 8.33}{6500 + 9700} = 9.2 \text{ feet above the base of the wall.}$$

Assuming that the transmitted pressure of the bearing wall is contained within planes making an angle of 30° with the vertical, at a point approximately 11 feet below the surface the distribution of the load would strike the back of the retaining wall. With a uniform distribution of the load at this plane, the intensity of the transmitted pressure is $^{8000}/_{12} = 670$ pounds per square foot. If this is treated as a surcharge at the surface and equation (24) is employed to obtain the thrust, c is then $^{6.7}/_{25} = 0.27$. With K taken as ⅓

$$T = \frac{100 \times 625 \times 1.54}{2 \times 3} = 16{,}050 \text{ pounds.}$$

From Table 3 the point of application of this thrust is located $\frac{1.81 \times 25}{3 \times 1.54}$ = 9.8 feet above the base of the wall. See page 30 for a discussion of the use of this method of analysis as a check upon the prev ous method.

As a problem illustrative of the action of saturated earth the author of the paper on page 32 has given the following example:[1]

"Take for example a wall supporting ten feet of earth the lower 6 ft. of which are below water level and hence saturated. Assume dry earth at 100 pounds per cubic foot and earth under water at 70 pounds per cubic foot. Assume a natural slope for dry earth of 1.5 to 1 ($\phi_1 = 33°41'$) and for earth under the water of 2.5 to 1 ($\phi_2 = 21°48'$).

"Lateral pressure at the plane of saturation due to dry earth = $100 \times 4 \times (1 - \sin \phi_1)/(1 + \sin \phi_1) = 114.4$ lbs. per square foot.

"Lateral pressure at the plane of saturation due to saturated earth = $100 \times 4 \times \frac{1 - \sin \phi_2}{1 + \sin \phi_2} = 183.2$ lbs. per square foot.

"Lateral earth pressure at the bottom

$$(100 + 4 + 70 \times 6)\frac{1 - \sin \phi_2}{1 + \sin \phi_2} = 374.6 \text{ lbs. per sq. ft.}$$

"Hydrostatic pressure at the bottom = $62.5 \times 6 = 375$ lbs. per square foot.

"Total lateral pressure at the bottom = $374.6 + 375 = 749.6$ lb. per sq. ft.

"Total resultant lateral pressure above the plane of saturation per foot length of wall is $114.4 \times 0.5 \times 4 = 228.8$ lb. This is applied at a point 1⅓ ft. from the plane of saturation or 7⅓ ft. from the bottom of the wall.

"Total resultant lateral pressure below the plane of saturation is $0.5 (183.2 + 749.6) \times 6 = 2798.4$ lb. This is applied at a distance of $\frac{6(749.6 + 2 \times 183.2)}{3(749.6 + 183.2)}$ or 2.4 feet from the bottom.

"The resultant lateral pressure against the wall per foot of length is then $228.8 + 2798.4 = 3027.2$ lb. This is applied at a distance of $\frac{228.8 \times 7.3 + 2798.4 \times 2.4}{3027.2} = 2.77$ feet from the bottom."

Bibliography

For an exhaustive bibliography on the various theories and experiments upon earth pressures, both active and passive see Howe, "Retaining Walls," 5th Ed. (see also Appendix).

[1] A. G. Husted, *Engineering News-Record*, Vol. 81, p. 442.

The following is a list of interesting papers upon the subject matter of the chapter.

Earth Pressures: A practical comparison of theory and experiments, CORNISH, *Trans.* A. S. C. E., lxxxi, p. 191.

Cohesion in Earth: CAIN, *Trans.* A. S. C. E., lxxx, p. 1315.

Earth Pressure Lateral: *Cornell Civil Engineer*, April, 1913.

Lateral Pressure of Clay: W. L. COOMBS, *Journal* Western Society of Engineers, Vol. 17, p. 746.

Retaining Wall Theories: PERRY, *Journal* Western Society of Engineers, Vol. 19, p. 113.

Retaining Walls: Based entirely upon the theory of friction, P. DOZAL, Buenos Aires. Translated.

CHAPTER II

DESIGN OF GRAVITY WALLS

Location and Height of Wall.—The need for a retaining wall arises from the construction of a cut or an embankment, whose side banks are not permitted to take their natural slopes. Where the amount of land necessary for the construction of such a fill or cut is, to all intents, unlimited, the wall may be located at any point where economy dictates that a wall of the necessary height and section is cheaper than the additional cut or fill which it replaces. Thus in Fig. 16 the wall replaces all fill shown cross-hatched. A comparative estimate, taking into consideration the cost of masonry, of embankment, or excavation for the wall footing, will show, after a few trials as to location, at what point the wall should be placed to obtain the minimum cost.

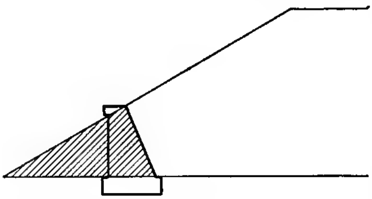


FIG. 16.

If the wall, however, is to run along a highway or other fixed property line, then, this at once determines its location. Again,

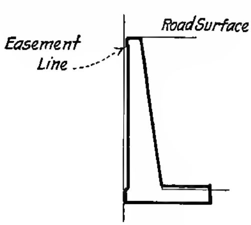


FIG. 17.

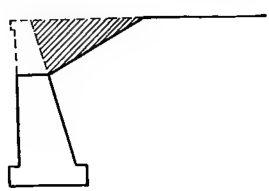


FIG. 18

in railroad work through cities, especially grade elimination and track elevation work, easements are costly and are generally restricted by the municipalities which grant them, so that it is necessary to get the wall as close to the tracks as possible, whence a wall is placed as shown in Fig. 17. Even in the case where easements are cheap and unlimited, an eye to future development and consequent increased trackage may make it desirable to so construct a wall, that the additional fill necessary for the future tracks may easily be placed. In Fig. 18 the wall may be so built, that, with placing a new top above A, the section will be ample to take care of the new fill and live load, or the wall may be built to the future required height at once. This latter may, however, prove unsightly.

General Outlines of the Wall.—The section of a wall should be so chosen that, at a minimum cost, it yields a maximum area for the improvement work. When this work runs through valuable property acquired at high cost, so that every square foot possible must be made available for the roadway or tracks, the front face, on the property line, should be made vertical as shown in Fig. 17 and placed as close to the line as the details of the coping and footing will permit. To insure no possible encroachment at a future date, due to settlement of the wall, surveying or construction errors and the like, it is better to place the coping a few inches back from the line. The coping usually projects a few inches beyond the face of the wall.

Before entering into a discussion of the relative merits of walls with various outlines, it is necessary that the principles upon which the walls are designed, be first explained. This will be done in the following pages. The section of the wall may be controlled not only by these general principles, but also by specific limitations foreign to the actual stress system existing in the wall. Architectural treatment may determine the shape of the wall, when the wall is part of some general landscape scheme. The selection of a type of wall that will suit peculiar foundation conditions is discussed in detail in later chapters. Generally speaking, however, that section of wall is chosen which can be most economically and expeditiously built.

The Two Classes of Retaining Walls.—Retaining walls fall into two broad classes. The walls which retain an earth bank wholly by their own weight are termed *gravity* walls. This type is discussed in the present chapter. Those which utilize the weight of the earth bank in sustaining the pressures of the bank form the *reinforced concrete* type of walls. This latter class, because of the mobile character of reinforced concrete has an infinite variety of shapes. The following chapters will take up in detail the analysis of the shapes occurring in ordinary construction work.

Since the active element of support in the gravity wall is the material out of which it is composed, the wall may be made of other materials besides concrete. The reinforced walls are made of concrete and steel.

Fundamental Principles of Design.—A retaining wall, in supporting an earth bank must successfully withstand the following possible modes of failure:

(*a*) The overturning moment caused by the earth thrust may exceed the stability moment of the weight of the wall, or in the case of the cantilever type, of the combined weight of the wall and relieving earth weights. Thus in Fig. 19 the thrust moment Tt is greater than the stability moment Gg, and the wall will

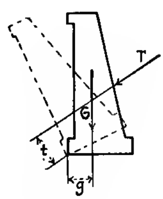


FIG. 19.—Criterion of overturning.

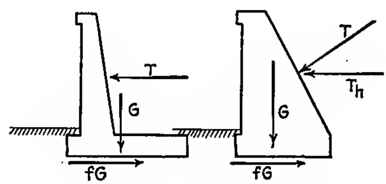


FIG. 20.—Criterion of sliding.

rotate about its toe. To remedy this, the weight G or the lever arm g is increased by adding to the dimensions of the wall, usually by widening the base.

(*b*) The pressure on the toe caused by the resultant forces of the thrust and weight of wall and earth may exceed the bearing power of the soil at that point, crushing the ground and causing the wall to tilt forward and, in the extreme case, topple over. The remedy lies in a wall properly shaped and dimensioned to insure safe soil pressures, or where dimensions alone will not suffice the preparation of a proper foundation either by further excavation to a better bottom or by the use of timber or pile foundations.

(*c*) The frictional resistance between the wall base and the foundation may be insufficient to overcome the horizontal component of the thrust and the wall will slide forward along the base. In Fig. 20 fG is less than T_h. f is the coefficient of friction, a table of which for various materials, is shown here (Table 6). T_h is the horizontal component of the thrust. With a wall properly proportioned against failure through overturning or excessive bearing on the foundation, this condition rarely exists. It is most likely to occur on a clay bottom, if water is present, since the wet clay acts as a lubricant. To remedy a condition of this kind, the base may either be widened, increasing the weight on the wall, or a bottom may be prepared offering mechanical as well as frictional resistance to sliding. If narrow trenches are dug in the foundation, projections will be formed which will materially increase the resistance. Again, the bottom may be tilted upwards towards the toe, giving a horizontal component of resis-

FIG. 21.—Types of bottoms to increase resistance against sliding.

tance in addition to the frictional (see Fig. 21 for both cases). Filling the foundation trench completely with masonry, so that the front of the wall butts against the original earth of the trench (not any backfill) may also prove efficacious.

TABLE 6

Character of foundation	Coefficient
Dry clay	.50
Wet or moist clay	.33
Sand	.40
Gravel	.60
Wood (with grain)	.60
Wood (against grain)	.50

These are, then, the potential modes of failure of a retaining wall, and the wall satisfying most economically these criteria against failure has been properly designed.

To recapitulate, the following equations must be satisfied:

(*a*) Gg must be greater than Tt.

(*b*) S_1 must be less than S (where S_1 is the toe pressure actually induced and S is the permissible soil pressure.)

(*c*) fG must be greater than Th.

Concrete or Stone Walls.—In spite of the well-nigh universal adoption of concrete as a retaining wall material, many yards of stone wall are still being built. Under certain conditions, this type of wall is the more economical one. The cut stone walls, however, with their ashlar or coursed masonry faces are much more costly than the concrete walls and are only used when necessitated by architectural treatment. With the development of the artistic treatment of concrete faces and with the ability to duplicate practically every cut-stone effect in concrete, the need of stone walls for even this purpose is rapidly diminishing. The rubble walls, both mortar and dry, do have an important application and where local stone cuts are available, are far the cheapest material out of which to build the wall.

When a wall is to be built adjacent to property, to which no access is permissible, even during construction, thus preventing the placing of the bracing and concrete forms, a stone wall becomes a very convenient type of wall to build. Rubble walls were so used in the track elevation of the Philadelphia, Germantown, and Norristown Railroad through Philadelphia.[1]

The dry rubble wall is frankly a temporary expedient, awaiting further local improvements, upon the arrival of which, the need for the wall itself is either removed or else the walls are replaced by those of more permanent and pleasing effect. The word "temporary" should be used most qualifiedly, for many dry rubble walls have existed for long periods of time, exceeding, by far their expected duration of life. In municipal improvements, as for, example the grading of a highway, leaving surrounding unimproved property below the future grade, it is customary to place a dry rubble wall along the highway with the expectation that when the adjacent property is improved or graded, the wall will either be removed or buried (see Plate 1, Fig. 1*a*).

The cement rubble wall is of as permanent a nature as the concrete wall. Its face, unless more or less screened is not as pleasing as a concrete face when viewed at close range. At comparatively small distances away, however, it presents quite a pleasing effect, the variegated coloring of the local stone showing to advantage (see Plate 1, Fig. 1*b*).

The stone walls require a distinct class of labor, familiar with the work. Stone masons are not always available and because of the diminishing amounts of stone walls built, are becoming fewer in number. The universal adaptability of concrete, its independence of local material conditions and the large amount

[1] See S. T. Wagner, *Trans.* A.S.C.E., Vol. lxxvi.

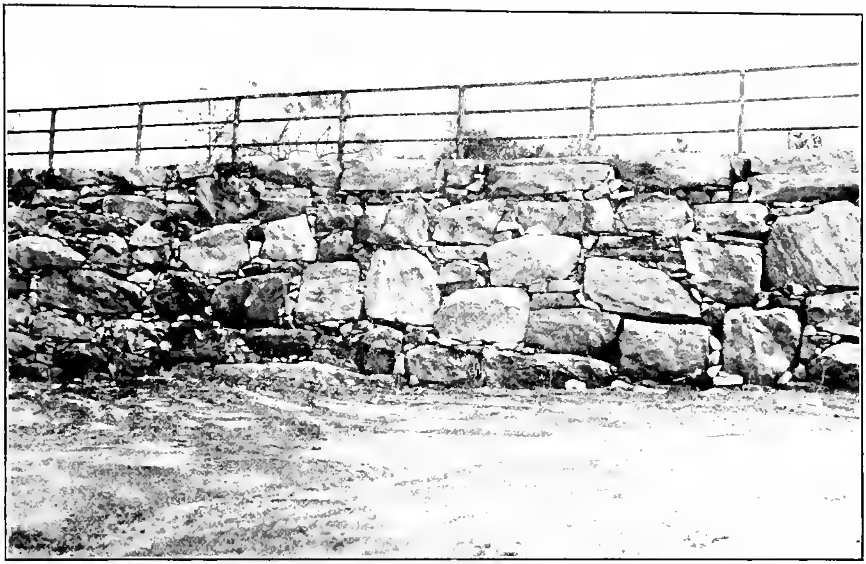


Fig. A.—Dry rubble wall along highway.

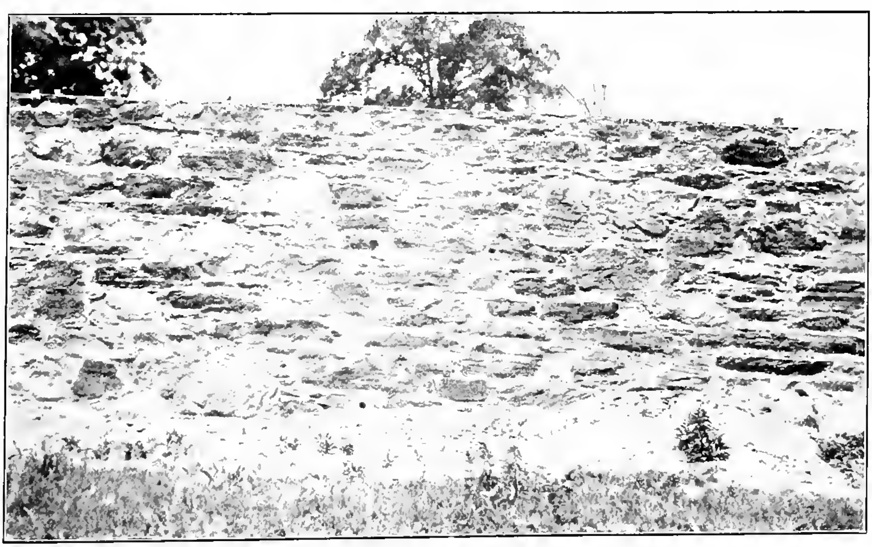


Fig. B.—Characteristic appearance of cement rubble wall.

(*Facing page* 46)

of concrete laborers and foremen all tend to explain the waning popularity of stone masonry.[1]

Where the selection of the material out of which the wall is to be built is governed solely by economic reasons, then, with labor and material conditions of equal weight the costs of the dry rubble wall, the cement rubble wall and the concrete wall stand in the order one, two and three, *i.e.*, the cost of the cement rubble wall is twice that of the dry rubble wall and the concrete wall three times that of the dry rubble wall. It is understood that there are available local stone quarries for the rubble wall. A very long haul for the stone makes the cost of the wall far too high to permit a serious consideration of its construction.

When using a dry wall, care must be taken to allow for the voids in assuming the weight of the masonry. The voids may vary from 15 to 40 per cent. of the section. A problem at the end of this chapter brings out this in some detail.

Thrust and Stability Moments.—The method of determining the thrust upon the back of a gravity wall follows the recommended form of procedure given on page 16. The thrust T upon the back of the wall is located at a point Bh above the bottom of the wall, where the value of B is found from Table 3. The standard type of surcharge loading of height h' is used (see Fig. 5) and the ratio h'/h is denoted by c. The amount of the thrust is

$$T = Jgh^2 \frac{1 + 2c}{2}$$

where J is the adopted earth pressure coefficient to be taken from equation (14) or from Table 1. The unit weight of earth is g (replacing w in the original equation to avoid confusion with a more natural form of lettering used in the following algebraic work).

If, under special conditions (see problems at the end of this chapter) it is decided to use the method of the maximum wedge of sliding, with the equation 24 on page 15, the thrust is

$$T = Kgh^2 \frac{1 + 2c}{2}$$

where K is the earth pressure coefficient of this method corresponding to J above and is to be taken from equation (24) or

[1] See *Engineering News-Record,* Vol. 81, p. 890 for a description of the use of dry rubble walls to retain the Hetch-Hetchy Railroad. The cuts for the highway afforded large amounts of stone.

from Table 2. Unless the back of the wall has a small batter (less than 5°) it is recommended that a value of $\phi' = 30°$ be used in finding the value of K.

Following are some general relations between the wall factors and the thrust, covering all shapes of gravity walls and all varieties of earth pressures.

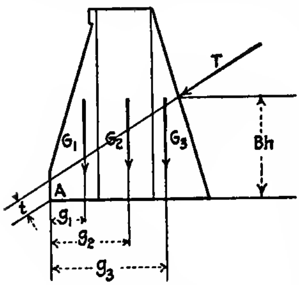


FIG. 22.—Stress system in gravity wall.

Let Fig. 22 represent a general section of gravity wall. Assume that the thrust has been found, in value T and located at a point Bh vertically above the base. The weight of the wall G is usually found by breaking up the figure as shown into triangles and rectangles. Algebraically then, by taking moments about some convenient point, as, for example, at the toe A, both the thrust moment Tt and the stability moment $G_1g_1 + G_2g_2 + G_3g_3$ are easily found. Graphically by means of an equilibrium polygon it is a simple matter to locate the resultant of the forces both in amount and in point of application. In the above algebraic method it is necessary to proceed further to obtain the resultant in both location and in amount. Fig. 23 shows the

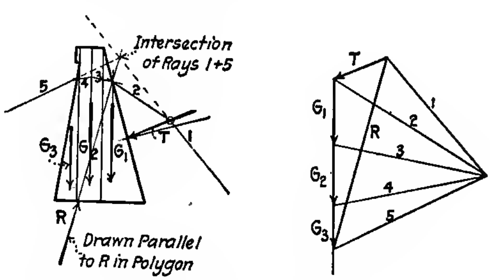


FIG. 23.—Graphical analysis of gravity wall stresses.

method of applying the thrust polygon to the determination of the stability of the wall.

The wall is on the verge of overturning when the stability moment is equal to the thrust moment or what is the same thing when the resultant just intersects the toe of the wall. For this condition the factor of safety is one.

As long as the stability moment exceeds the thrust moment, or as long as the point of application of the resultant falls within the base, the wall is safe against overturning. The proper location of the resultant depends not only upon the factor of safety thought desirable but also upon the question of a satisfactory foundation pressure. Before entering upon a discussion of a safety factor against overturning, it may be well to discuss the matter of foundations.

Foundations, those most vexing problems of engineering practice, are of paramount importance in both wall design and construction. Generally a correct foundation design demands a uniform distribution of load as its most important premise. Unfortunately, the economics of retaining walls usually forbid the fulfillment of this premise. The wall is considered satisfactorily designed so long as the resultant of the pressure on the base falls within the middle third of the base, and more often at the outer edge of this middle third, so that the pressure intensity on the base varies from nothing at the heel to the maximum at the toe.

For foundations varying from rock to hard soils, such as coarse sands and gravels or loamy soils, *i.e.*, a mixture of gravelly sand and clay, the relative settlements due to the varying loads is small and a non-uniformly distributed load may safely be placed upon them. For the finer sands, wet soils, reaching down to the plastic bottoms, it is imperative to have a uniform distribution of pressure and foundations must be designed to secure this or recourse must be had to special types of walls, such as the cellular and similar types (see later pages).

There is no intention of entering into a detailed analysis of the proper selection and preparation of a foundation.[1] A brief description only of the various types of bottoms will be given. Various phases of this subject, however, will be taken up under the headings of "Varied Types of Walls," "Settlement," etc.

Rock is an elastic term, embracing all the types from a disintegrated product, that can easily be picked and shovelled to the hard gneiss, trap and granite which prove so costly to drill bits. The poor rocks, when stripped of a one or two foot layer usually present a bottom sufficiently strong to take as heavy a load as the safe crushing strength of the wall material will permit, and this is, of course, the maximum pressure that can

[1] See texts by JACOBY & DAVIS; PATTON; FOLWELL, etc.

be allowed on any masonry foundation. Under these conditions, the resultant may intersect the outer edge of the middle third with a triangular distribution of base loading. Occasionally the resultant is permitted to fall outside the middle third, so that the wall bears on only part of the foundation. While, theoretically, tension must then exist between the base and the foundation towards the heel of the wall, the rock is unyielding, so that there can be no opening at the heel while the criteria of overturning and safe bearing loads are satisfied. In the gravity walls, when this type of foundation is adopted, care must be taken that the tension then developed in the back of the wall at the base does not exceed the tensile strength of the masonry. If it does, it is necessary to reinforce the back with rods.

With a rock bottom well cleaned, left in the usual rough condition, and, with a good bond secured between it and the base of the wall, there is ample resistance to sliding.

Shales, cementatious gravels, coarse sand and gravel, in similar fashion present but little difficulty and it is customary, here also, to permit a triangular distribution of soil pressure. Shading off into the finer sands, dry clays and bottoms of like type with moderately yielding propensities, a theoretical discussion[1] of passive earth pressures seems to indicate that in yielding soils there is an upward heaving of the soil adjacent to the downward loads, so that, to counteract this tendency, there must be a minimum downward pressure on the base. For this reason, the resultant of the pressures should strike the base within the middle third, giving a trapezoidal distribution of pressure.

Coming down to the plastic bottoms, there must be a uniform distribution along the base not to exceed the safe bearing value of the soil in question. If this is not possible it is necessary to place piles. It is highly desirable that the piles carry equal loads. If the base pressure is not uniform a uniform pile loading may, nevertheless, be secured, by proper spacing of the piles.

Distribution of Base Pressures.—The analysis of the loadings upon the wall determines, finally, the location and amount of the resultant pressure upon the base of the wall. Since this resultant force is eccentrically placed upon the base, it is necessary to obtain the manner of the distribution of the pressure due to

[1] Howe, "Retaining Walls, Earth Pressures and Foundations."

this resultant. The vertical component of the resultant is analyzed here; the horizontal component affecting only the frictional resistance between the wall and the earth.

Referring to Fig. 24, let R be the vertical component of the resultant of all the pressures upon the base. S_1 and S_2 are the extreme pressure at the toe and heel respectively. With these limiting intensities found all the necessary data for the footing is had.

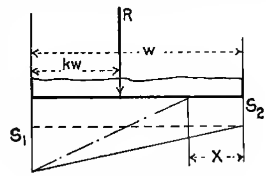


FIG. 24.—Foundation pressures.

Take moments about (the heel)

$$kwR = \frac{S_2 w^2}{2} + \frac{(S_1 - S_2)w^2}{6}$$

and

$$S_1 + 2S_2 = 6kR/w \tag{37}$$

Again, since the area of the trapezoid is equivalent to the value of the resultant R

$$S_1 + S_2 = 2R/w \tag{38}$$

Solving these simultaneous equations, there is

$$S_1 = \frac{2R}{w}(2 - 3k) \tag{39}$$

$$S_2 = \frac{2R}{w}(3k - 1) \tag{40}$$

When $k = \frac{1}{3}$, *i.e.*, when the resultant intersects at the outer edge of the middle third — a very common condition, $S_1 = 2R/w$ and $S_2 = 0$. When $k = \frac{1}{2}$, *i.e.*, when there is a uniform distribution of pressure along the base $S_1 = S_2 = R/w$.

Note that when, k is less than one-third, there is pressure along only a portion of the base. The point of zero intensity is given by

$$x = \frac{w}{3}\,\frac{1 - 3k}{1 - 2k} \tag{41}$$

where x is the distance from the *heel* to the point of zero intensity.

Table 7 gives the permissible intensities of soil pressures as allowed by the various codes.

TABLE 7.—PERMISSIBLE SOIL PRESSURES IN TONS PER SQUARE FOOT

Soil	*A*	*B*	*C*	*D*	*E*
Quicksand, silt	½–1	1			
Clay, soft	½–2	2	1	1	1
Clay and sand	2–4	..	2	2	†2
Sand, clean, dry	2–4	4	3	3	
Sand compacted, well cemented	4–6				
Gravel and coarse sand	6–8	6	6	4	6
Gravel and coarse sand well compacted	8–10	10	10		
Clay, hard, moderately dry	4–6				
Clay, hard, dry	6–8	4	4		4
Rock, soft to hard	5–200	75*	8–40		12–20

A. Prof. Cain.
B. Public Service Commission, 1st District, New York City.
C. Building Code, New York City.
D. Building Code, Dist. of Washington.
E. Building Code, Baltimore.
* Sound ledge rock.
† Clay or clay mixed with sand, firm and dry. 3 tons.

Proper Centering for Piles.—Since the retaining wall brings a non-uniform distribution of loading upon the base, a uniform spacing of piles would produce unequal loading upon them. This is not a desirable type of loading for piles. The following is a method of so spacing the piles as to secure a uniform loading.

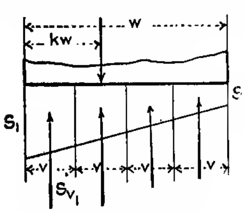


FIG. 25.—Pile spacing. Case I.

The piles may be spaced either in rows parallel to the face of the wall, or in rows perpendicular to the face of the wall. A graphic and an analytic method are outlined below for either of these two methods of spacing the piles.

Let *P* be the safe bearing value per pile. In Fig. 25 divide the base into a series of strips of equal width *v*. From the eccentric position of *R* determine the extreme bearings, S_1 and S_2 and lay these off to scale. The soil pressure in any strip *v*, S_v, is readily obtained by scaling the figure. vS_v then gives the total load on the *v* strip taken for a unit width of wall. Dividing *P* by this product determines the spacing necessary in that strip. The minimum spacing of piles is about three feet, so that, when the spacing in a strip is found to be less than this minimum, it is necessary to take the strips closer together. When this fails the base must be widened by placing a toe extension.

The piles may be spaced perpendicularly to the paper at equal intervals, but at varying distances along the base of the wall (see Fig. 26). Assume that a width of wall is taken (perpendicular to the sheet) equal to the permissible or desirable spacing of piles. The values of R, S_1 and S_2, as found above are increased accordingly. Making a scale layout as above, trial irregular widths are taken decreasing in width towards the toe, each being equivalent to the safe bearing of one pile. The following is an analytic discussion of the two cases.

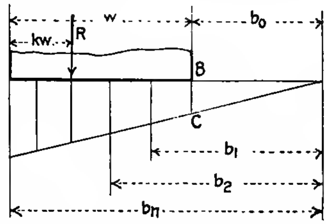


FIG. 26.—Pile spacing. Case II.

Case I.—From the geometry of Fig. 25 the total pressure in any width v of the base (a unit's thickness of wall is assumed) is

$$vS_{vi} = S_2v + v\frac{\frac{v}{2} + (i-1)v}{w}(S_1 - S_2)$$

i is the number of the division, counting from the back of the wall.

Replacing S_1 and S_2 by their values in terms of R and k

$$S_{vi} = \frac{2R}{w}(3k-1) + 3\frac{v}{w}\frac{2i-1}{2}(1-2k) \tag{42}$$

Since the pile can take P as a safe load, the required spacing of piles in the "i" the row is, then

$$a = \frac{P}{vS_{vi}} \tag{43}$$

Case II.—Let it be assumed that the rows of piles, parallel to the page, are spaced m feet apart. The total vertical load on the foundation is then mR and if, as above, P is the safe load per pile, the number of piles required in each thickness m of the wall is $mR/P = n$ and this is the required number of spaces of equal area into which it is required to divide the trapezoid, in Fig. 26. Complete the triangle as shown and let the area of BCO be P_0. The area of any other triangle, bound, say, by the vertical side b_i, as base, is $P_0 + iP$, where i is the number of divisions, or of piles, from the back of the wall. Since the areas of similar triangles are to each other as the square of their homologous sides

$\frac{b_i^2}{b^2_{i-1}} = \frac{P_0 + iP}{P_0 + (i-1)P}$, then $\frac{b_1^2}{b_0^2} = \frac{P_0 + P}{P_0}$, Similarly $b_2^2 = b_1^2 \frac{P_0 + 2P}{P_0 + P} = b_0^2 \frac{P_0 + 2P}{P_0}$

Extending this result to the general case

$$b_i = b_0\sqrt{\frac{P_0 + iP}{P_0}} \tag{44}$$

Let l_i be the distance from B to the corresponding i line then $l_i = b_i - b_0 = b_0\left(\sqrt{\frac{P_0 + iP}{P_0}} - 1\right)$ *Since* $P_0 = \frac{b_0 S_2}{2}$, $b_0 = \frac{wS_2}{S_1 - S_2}$, $P = \frac{w}{2n}(S_1 + S_2)$ $l_i = \frac{wS_2}{S_1 - S_2}\left(\sqrt{1 + \frac{i}{n}\frac{S_1^2 - S_2^2}{S_2^2}} - 1\right)$ and if, finally, S_1 and S_2 are replaced by their values in terms of R and k

$$l_i = wF\left(\sqrt{1 + \frac{i}{n}H} - 1\right); F = \frac{3k-1}{3(1-2k)}; H = \left(\frac{2-3k}{3k-1}\right)^2 - 1 = \frac{1}{(3k-1)F} \tag{45}$$

That the distance between the two piles adjacent to the toe shall not be less than a specified amount a (usually about three feet) it may be necessary to extend the base by means of a toe With sufficient exactness the distance a may be taken as one-half the distance between the toe and the point l_{n-2}. Then

$$w_0 - l_{n-2} = 2a$$

Replacing l_{n-2} by its value from (45), simplifying the resulting equation and eliminating the radical and putting $2a/w_0 = \lambda$

$$\frac{N}{3k-1} = \frac{(1-\lambda)^2}{F} + 2(1-\lambda), \qquad N = \frac{n-2}{n}$$

and solving for k

$$k = \frac{N - (1-\lambda)(1-3\lambda)}{6\lambda(1-\lambda)} \tag{46}$$

If the width including the toe extension is w_0, and the width without the toe extension is w, letting $2a/w = \lambda'$ and noting that $w_0 = w(1+i)$ and $\lambda' = \lambda(1+i)$ also $k = \frac{i+e}{1+i}$ (see Fig. 24).

Equation (46) becomes a cubic in $(1 + i)$ or u, $A = 3\lambda' [\lambda' + 2(1 - e)]$, $B = 6\lambda'^2 (1 - \mu)$;

$$\frac{2}{n} u^3 + 2\lambda' u^2 - Au + B = 0. \tag{47}$$

In view of the fact that i is small in comparison with unity, (it cannot exceed ⅓ for a valid solution), it is permissible to replace u^3 by $1 + 3i$, and u^2 by $1 + 2i$, which makes (47) linear in i and gives the relation

$$i = \frac{A - B - 2\lambda' - 2/n}{6/n + 4\lambda' - A} \tag{48}$$

This apparently complicated analysis together with the entire mathematical treatment of pile loading is given with the idea of affording a direct solution of pile spacing problems for eccentric distributions of loading. The problems at the end of the chapter will bring to bear the arithmetic application of the literal equations just developed. The work just shown of determining the proper offset to maintain the minimum pile spacing replaces a rather tedious method of trial and error. In all the above work it is understood that a *uniform* loading of the several piles used is the result sought.

For the special case of $k = ⅓$, *i.e.* the resultant intersects the base at the outer edge of the middle third, and (45) becomes

$$l_i = w\sqrt{\frac{i}{n}} \tag{49}$$

Table 8 gives values of F and H.

Since either method, theoretically, must give the same density of piles, it is immaterial, from the standpoint of the number of piles required, which method is adopted. Practically, however, it seems simpler to use the latter method of distribution since the piles are lined up in both directions. In the former, they are in line longitudinally, only, *i.e.* parallel to the face of the wall making the work in the field a little more cumbersome than in the latter method.

TABLE 8

k	F	H
.36	.10	131.0
.37	.14	65.0
.38	.19	36.8
.39	.26	23.0
.40	.33	15.0
.41	.43	10.0
.42	.54	7.11
.43	.69	5.00
.44	.89	3.51
.45	1.20	2.50
.46	1.58	1.66
.47	2.30	1.10
.48	3.67	.62

Occasionally eccentric bearing is allowed on piles, the piles then being unequally loaded. This practice is far from commendable, since a pile is, by its very nature, a yielding support (unless driven to absolute refusal) and unequal settlement is unavoidable. Pile foundations, and, in fact, all foundations, demand most mature engineering judgment in their planning and construction and time and money spent in consulting experienced men on this part of the work is an ideal assurance towards a safe and well-appearing wall.

A problem at the end of this chapter illustrates the application of the above analysis to a concrete case.

Factor of Safety.—It has been seen that, as long as the resultant intersects the base inside the toe, there is no danger that the wall will overturn. Since the thrust is computed from the maximum load possible or anticipated upon the wall, a factor of safety but little greater than one seems ample. However, to insure that there will be no tension in the back of the wall, the resultant should intersect within the middle third.

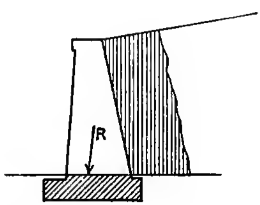


Fig. 27.—The retaining wall and its foundation.

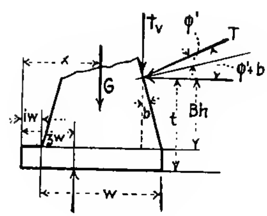


Fig. 28.

The wall may be divided into two parts; that portion (see Fig. 27) above the ground surface, retaining the fill; and the foundation course. At the junction of these two parts, that is, at the surface of the ground, the resultant should intersect at the outer edge of the middle third. This insures the most economical wall above the surface and at the same time prevents any tension in the wall. The dimensions of the footing are then solely governed by the permissible soil pressures.

The ratio between the moment tending to resist the overturning of the wall and the moment tending to overturn the wall, has been termed the factor of safety against overturning. Referring to Fig. 28 the overturning moment is $T_h t$ and the resisting moment is $Gx + T_v[(1 + i)w - Bh \tan b]$. Denoting the factor of safety by n

$$Gx + T_v[(1 + i)w - Bh \tan b] = nT_ht$$

Taking moments about the point where the resultant intersects the base $G(x - zw) = T_ht - T_v[(1 + i - z)w - Bh \tan b]$ Placing $A = T_v[(1 + i)w - Bh \tan b]$ the two equations become $Gx + A = n\,T_h\,t$; $Gx - Gzw = T_h\,t + T_v\,zw - A$. Combining these two equations and solving for n

$$n = \frac{T_ht + zw\,(G + T_v)}{T_ht} = 1 + \frac{zw(G + T_v)}{T_ht} \qquad (50)$$

and conversely

$$z = \frac{(n - 1)\,T_ht}{(G + T_v)w}. \qquad (51)$$

Prof. Cain[1] advocates designing a wall for a definite factor of safety and recommends the following values of n for walls subjected to vibratory loadings, such as walls adjacent to passing trains:

Walls less than 10 feet high	$n = 3.5$
Walls from 10 to 20 feet high	$n = 3$
Walls around 50 feet high	$n = 2.5$

Prof. Hool[2] recommends a factor of safety of 2 for the average retaining wall.

To assign a definite, integral factor of safety against overturning locates the position of the resultant upon the base without regard to the character of the distribution of the pressure upon the soil that seems most desirable. Walls fail because of foundation weakness (see pages 160–163) rarely because the overturning moment exceeds the stability moment. An integral factor of safety reverses this order of importance and makes the less usual potential mode of failure the more important criterion. It is better procedure to decide upon the location of the resultant of the pressures and then to learn what factor of safety is to be had following the method given on page 56. It is assumed, in figuring the factor of safety against overturning, that the wall will revolve about its toe as a fulcrum. This is possible only upon an unyielding soil; for the other soils, as the wall tends to turn on

[1] *Trans.* A. S. C. E., Vol. lxxii.

[2] "Reinforced Concrete Construction," Vol. 2.

its toe, the ground in the immediate vicinity of the toe will crush so that the conditions under which the factor of safety was computed will no longer be valid.

It is doubtful whether, in actual practice this factor against overturning is ever predetermined or subsequently ascertained. It is well, however, as an additional precautionary measure, to find its value in the manner outlined before.

Footing.—The retaining wall proper may be considered to end at the bottom of the fill retained, or at the natural ground surface (see Fig. 27). It is then necessary to design a footing that will properly distribute upon the soil the pressures brought to it from the retaining wall. If the base of this wall proper is projected vertically downwards, and if the values of S_1 and S_2 as found on page 51 in equations 39, 40 are within the allowable pressures as shown in Table 7 no extension of the base is necessary. When these values exceed the permissible ones a toe extension becomes necessary. This may be found as follows: In Fig. 29 let ew locate the position of the resultant pressure and let S be the permissible soil pressure. The offset iw is that necessary to make the value of S_1 approach as nearly as possible the allowable value S. Referring to equation (39), the value of k is now

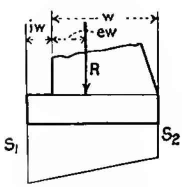


FIG. 29.—Toe extension.

$$k = \frac{(i + e)w}{(1 + i)w} = \frac{i + e}{i + 1} \tag{52}$$

The value of S_1 is

$$S_1 = \frac{2R}{w(1 + i)}\left(2 - 3\frac{i + e}{i + 1}\right) \tag{53}$$

Place

$$w\,S_1/2R = r \tag{54}$$

and the above equation becomes

$$r = \frac{2 - 3e - i}{(1 + i)^2} \tag{55}$$

which is a quadratic in i, which when solved gives

$$i = \frac{\sqrt{12\,r(1 - e) + 1} - (2r + 1)}{2r} \tag{56}$$

The usual value, and the one most properly taken for e is $\frac{1}{3}$. This makes (56)

$$i = \frac{\sqrt{(8r + 1)} - (2r + 1)}{2r} \tag{57}$$

which determines the necessary offset for the base when the resultant is given in amount and location and the value of the soil pressure intensity has been assigned. To aid in the determination of the offset when the value of r is given, Table 9 has been prepared giving the values of i for a range of values of r. Some examples at the end of the chapter illustrate the application of Table 9 to specific problems.

A less frequent requirement, but one which may possibly exist (see problems at end of chapter) is the determination of a toe offset to give a minimum intensity S_2 at the heel. With the value of k as in equation (52) and from (40) after placing

$$s = wS_2/2R \tag{58}$$

$$s = \frac{2i + 3e - 1}{(1 + i)^2} \tag{59}$$

There is obtained a value of i

$$i = \frac{1 - s - \sqrt{1 - s(2 - 3e + 1)}}{s} \tag{60}$$

For $e = \frac{1}{3}$, this becomes

$$i = \frac{1 - s - \sqrt{(1 - 2s)}}{s} \tag{61}$$

Table 10 has been prepared giving a range of values of i for the possible variations in the ratio s.

TABLE 9

r	i
1.00	.00
.9	.04
.8	.08
.7	.12
.6	.18
.5	.24
.4	.31
.375	.33

TABLE 10

s	i
.00	.00
.05	.02
.10	.05
.15	.09
.20	.13
.25	.17
.30	.22
.35	.29
.375	.33

The toe extension is a cantilever beam and must be so dimensioned as to satisfy the shear and bending moment requirements of such a beam. Let the thickness of the toe be d. Since the extension is usually small in comparison with the rest of the footing, the distribution of soil pressure may be taken as uniformly spread over the toe and equal in intensity to S_1, per unit of length. If f_c is the concrete stress allowed in compression, the external moment equated to the resisting moment gives $S_1 i^2 w^2/2 = f_c d^2/6$ and $d = kiw$, with $k = \sqrt{(3S_1/f_c)}$.

It is necessary here to locate the principal planes to determine along what plane there exists a maximum tension, *i.e.*, the plane of weakness of the step. The stresses on the principal planes are given by the expression $f = c/2 \pm \sqrt{(c^2/4 + p^2)}$. c is the unit compressive stress and p the unit shearing stress found in the body with the axes corresponding to the axes of loading of the body, *i.e.*, as in the sketch, vertical and horizontal. In slightly altered form, this may be written $f = \frac{c}{2} - \frac{c}{2}\sqrt{1 + \frac{4p^2}{c^2}}$. For concrete c is large in comparison with p and in developing the radical by the binomial theorem it will be permissible to stop with the second term, whence $f = p^2/c$, or $p = \sqrt{(fc)}$. The unit shear is then a geometric mean[1] between the tension and compression as exerted along the vertical and horizontal planes of the body. In the first expression for the principal stresses, the minus sign was taken since the principal tension was sought.

The angle between the principal tension plane and the vertical plane is given by $\tan^{-1}(-2p/c)$, or using the approximate relation between p and c is equal to $\tan^{-1} 2\sqrt{\frac{f}{c}}$. Upon the recommendation of the special concrete committee of the A.S.C.E. (a summary of which is given later in a section on "Reinforced Concrete") the ratio f/c is to be taken as $1/16$, and this angle becomes $\tan^{-1}(-1/2)$ or the ratio of the extension to the depth is one-half.

The maximum tension then exists along a plane making a slope of one to two with the vertical. Again, it has been demonstrated that the transmission of loading through a solid is contained with-

[1] In "Reinforced Concrete" by MORSCH, as translated by E. P. GOODRICH, this theorem is established by somewhat different an analysis.

in planes making an angle of about 30° with the vertical. For both these reasons, good practice would demand that, wherever possible the ratio of step to depth for a foundation offset be one to two.

The maximum pressure that can be brought to bear upon a foundation is limited by the permissible bearing on the masonry, usually taken at about thirty tons per square foot or about 400 pounds per square inch. From the preceding formula for the depth of step as required because of the bending moment, k is then less than 2, so that a step of 1 to 2 will always satisfy the bending moment requirements with the above maximum loading. The shear on the plane where the toe joins the footing is $S_1iw/d = S_1/k$. If the shearing stress is taken as 75 pounds per square inch, then as long as S_1 does not exceed 150 pounds per square inch or about ten tons per square foot, a value of $k = 2$, is good. When the soil pressure does exceed this amount, it will be necessary to reinforce the base.

For all ordinary soil pressures, then, a step of one to two is satisfactory and should be adopted for the toe extension.

A Direct Method of Designing the Wall Proper.—In the ordinary course of design of a gravity wall, a tentative section, governed by the judgment and experience of the designer, is selected. This is analyzed in accordance with the methods outlined in the preceding pages. It has been pointed out that the usual goal of the designer is to select such a section of wall that the resultant intersects exactly at the outer edge of the middle third. As the tentative section does not, at first choice, fulfill this condition, one or more succeeding sections are chosen until the final one does meet this criterion. By using the criterion that the resultant must intersect at the outer edge of the middle third and by giving the thrust the standard form of expression on page 16, it is possible to effect a direct solution of the required dimensions of the wall. The analysis following develops an equation, predicated upon these assumptions, from which Table 12 has been prepared. This table covers the usual range of the factors controlling the wall section and is to be used in place of the method of trial and error as stated above. The numerical application of the table and of the equations upon which it is based is to be found in the problems at the end of the chapter.

The general gravity type of wall is shown in Fig. 30. The rectangular wall, the wall with a vertical front face and the wall with a vertical rear face are, of course, but special cases of this general type.

In taking moments about the outer edge of the middle third, *i.e.*, about the point I, the moment of the thrust must be equal to the wall moment. These moments are found as follows:

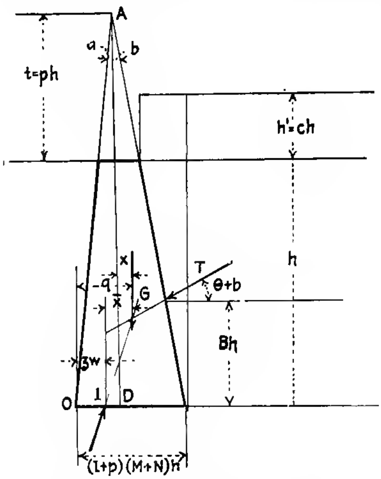


FIG. 30.—Design of gravity wall.

Extend the sides of the wall to their intersection at A roject the point A vertically down upon the base, meeting the base at the point D. The vertical distance that A is above the top of the wall is t. Let the ratio t/h be put equal to p. The front face of the wall makes an angle a with the vertical; the rear face (the face adjacent to the earth embankment) an angle b. Place tan a and tan b equal to M and N respectively. Taking moments about the point D, the location of the point of application of the weight of the wall with respect to D is x, where

$$x = (N - M)\frac{h}{3}\,\frac{1 + 3p + 3p^2}{1 + 2p} \tag{62}$$

The distance of G from the point O, *i.e.*, from the toe of the wall is

$$(1 + p)Mh + x \tag{63}$$

and from 62 this becomes

$$\frac{h}{3}\left(\frac{2 + 6p + 3p^2}{1 + 2p}\,M + \frac{1 + 3p + 3p^2}{1 + 2p}\,N\right) \tag{64}$$

This expression, locating the center of gravity of a general type of gravity wall with respect to the toe may be further simplified by putting the ratio of the upper to the lower base equal to u. Then

$$u = p/(1 + p). \tag{65}$$

Calling the distance of the center of gravity of the wall from the toe, q, from (63) and (64)

$$q = \frac{h}{3}(U_1M + U_2N) \qquad (66)$$

where $$U_1 = \frac{2 + 2u - u^2}{1 - u^2} \qquad U_2 = \frac{1 + u + u^2}{1 - u^2}$$

Table 11

u	U_1	U_2
.0	2.00	1.00
.1	2.21	1.12
.2	2.46	1.29
.3	2.76	1.53
.4	3.14	1.86
.5	3.67	2.33
.6	4.44	3.06
.7	5.70	4.30
.8	8.22	6.78
.9	15.73	14.27

Table 11 has been prepared giving the values of these coefficients for the range of values of u. The table, and the above formulas for the center of gravity with respect to the toe are applicable to any method of analyzing the wall, not only the special method now being followed.

The distance from the outer third point I to the point of application of the force G is $\bar{x}$, where

$$\bar{x} = (1 + p)Mh + x - \frac{1}{3}(1 + p)(M + N)h \qquad (67)$$

When simplified this value becomes

$$\bar{x} = \frac{h}{3}\left(\frac{1 + 3p + 3p^2}{1 + 2p}M + \frac{p^2}{1 + 2p}N\right) \qquad (68)$$

If the unit weight of the masonry is m pounds per cubic foot, then the value of G is

$$G = mh^2\frac{(1 + 2p)(M + N)}{2} \qquad (69)$$

and its moment about the outer third point I is $G\bar{x}$, or

$$G\bar{x} = \frac{mh^3}{6}\{M(1 + 3p) + (M + N)p^2\}(M + N) \qquad (70)$$

To determine the thrust moment resolve the thrust into its horizontal and vertical components as shown on page 10. The horizontal component is T_h and its value is

$$T_h = gh^2(1 + 2c)/6 \qquad (71)$$

The vertical component is T_v and its value is

$$T_v = gh^2(1 + 2c)N/2 \qquad (72)$$

Taking moments about the outer edge of the middle third I, and letting the thrust moment be M_0.

$$M_0 = T_hBh - T_v\left[\frac{2}{3}(1 + p)(M \quad N)h - BhN\right]$$

$$= \frac{gh^3}{6}(1 + 2c)\{B - N[2(1 + p)(M + N) - 3BN]\} \qquad (73)$$

Equating this thrust moment to the stability moment of the wall, putting the ratio of the unit weight of the earth g to the unit weight of the masonry m equal to s, and writing the equation in the form of a quadratic in $p(M + N)$,

$$(M + N)^2p^2 + Ip(M + N) + H = 0 \qquad (74)$$

$$I = 3M + 2sN(1 + 2c);\ H = M(M + N) - \frac{s}{3}[1 - 6MN - 3N^2 + 3c(1 - 4MN - N^2)]$$

It will be noticed that the quantity $p(M + N)$ is the ratio of the width of the top of the wall to the height of the wall. Table 12 has been prepared based upon equation 74, giving the ratios

TABLE 12

M	$N = 0.0$			$N = 0.1$			$N = 0.2$			$N = 0.3$			$N = 0.4$			$N = 0.5$		
	c			c			c			c			c			c		
	0	.2	.4	0	.2	.4	0	.2	.4	0	.2	.4	0	.2	.4	0	.2	.4
	.47	.60	.70	.40	.50	.58	.33	.41	.47	.25	.33	.37	.17	.23	.28	.07	.17	.23
0	.47	.60	.70	.50	.60	.68	.53	.61	.67	.55	.63	.69	.57	.63	.68	.57	.67	.73
	.33	.46	.56	.26	.36	.44	.19	.27	.34	.10	.18	.24	.02	.09	.14		.05	.08
.1	.43	.56	.66	.46	.56	.64	.49	.57	.64	.50	.58	.64	.52	.59	.64		.65	.68
	.22	.34	.44	.15	.24	.32	.07	.15	.22		.06	.11			.02			.02
.2	.42	.54	.64	.45	.54	.62	.47	.55	.62		.56	.61			.62			.72
	.13	.24	.33	.05	.14	.22		.05	.11			.01						
.3	.43	.54	.63	.45	.54	.62		.55	.61			.61						
	.05	.15	.23		.05	.12			.01									
.4	.45	.55	.63		.55	.62			.61									
		.07	.15			.03												
.5		.57	.65			.63												

of the top and bottom widths of the wall to the height of the wall for a sufficient range of values to determine very closely the required dimensions of any gravity type of wall, assuming that the ratio of the weight of the earth to masonry is $\frac{2}{3}$ (*i.e.*, $s = \frac{2}{3}$) and that the resultant intersects the base of the wall proper at the outer edge of the middle third.

With both M and N zero, the wall is the rectangular type. With M zero, the wall is the vertical front and battered back type, a very popular type forming a large percentage of all gravity types built and very efficient where maximum trackage and minimum easements are wanted (see page 42). With N zero there is the less usual type, but a most economical one with vertical back and battered face. A slight face batter and a larger back batter make a wall of economical section and pleasing appearance. It is understood in selecting the dimensions of the wall that a proper footing is to be developed as shown on the preceding pages, to give the correct distribution of pressure upon the foundation.

The converse problem, given the section of a retaining wall, to locate the position of the resultant pressure upon the base may be solved as follows: Referring to Fig. 30, with the weight of the wall G a distance q from the toe and the point of application of the resultant pressure a distance zw from the toe where $zw = OI$, as in Fig. 23, take moments about I

$$G(q - zw) + T_v(w - BhN - zw) = T_hBh$$

and solving this expression for z,

$$z = \frac{Gq + T_v(w - BhN) - T_hBh}{(G + T_v)w} \qquad (75)$$

The value of q and of the thrust components may be taken from the appropriate equations and tables given in the preceding work.

Revetment Walls.—The wall leaning toward the earth bank which it supports, as shown in Fig. 31, is termed a revetment wall. It is more of historic than of present interest. Prof. Cain has shown[1] that when the angle b is less than 10°, the ordinary theory of earth pressure as given by the method of the wedge of maximum thrust (see pages 11–15), may safely be applied to determine the thrust.

[1] "Earth Pressure, Walls and Bins," pp. 96, 97.

That the wall be self-sustaining while under construction, it is necessary that its center of gravity projected down, always falls within the base. To effect this, denote the ratio of the width of base to height of wall (a parallelogram is the only type of section discussed in detail here) by k. That the wall be self-sustaining, it is necessary that k be greater than $\tan b$.

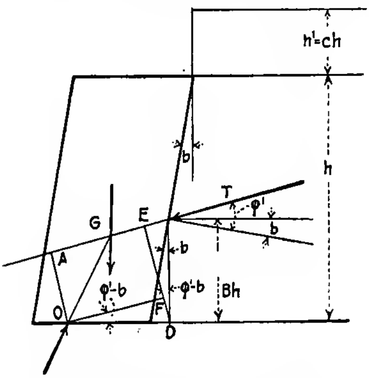


FIG. 31.—Design of revetment wall.

As in the former pages, a direct method of determining upon the ratio k for any character of loading, predicated upon the resultant intersecting at the outer edge of the middle third may be found for this type of wall. In the following work the earth pressure coefficient is K, defined by equation (25). In view of the fact that the angle b is now negative, Table 13 has been prepared giving the values of this coefficient K for negative values of the angle b.

The thrust moment is

$$T \times \overline{AO} \tag{76}$$

From (24)

$$T = gh^2K\frac{1 + 2c}{2}$$

$$AO = EF = ED - FD.$$
$$ED = Bh\cos(\phi' - b)$$
$$FD = (Bh\tan b + \frac{2}{3}kh)\sin(\phi' - b)$$

and (76) becomes

$$\frac{1}{6}Kh^3(1 + 2c)[3B\cos(\phi' - b) - (3B\tan b + 2k)\sin(\phi' - b)]$$

The stability moment of the wall (both of the moments are taken about the outer edge of the middle third, *i.e.* 0) is

$$mkh^2\left(\frac{h}{2}\tan b + kh/2 - kh/3\right) = m\frac{kh^3}{6}(3\tan b + k)$$

Equating these two moments, and writing the resulting equation as a quadratic in k

$$k^2 + Rk = S \tag{77}$$

where

$$R = 3 \tan b + 2s(1 + 2c) \sin (\phi' - b) \quad (78)$$

$$S = s(1 + 3c) \frac{\cos \phi'}{\cos b} \quad (79)$$

s is the ratio $\frac{gK}{m}$

TABLE 13

b	$\phi' = 0°$	$\phi' = 15°$	$\phi' = 30°$
0°	.33	.30	.29
5°	.30	.27	.26
10°	.27	.24	.23

TABLE 14

c	$\phi' = 0$		$\phi' = 15°$		$\phi' = 30°$	
	$b = 5°$	$b = 10°$	$b = 5°$	$b = 10°$	$b = 5°$	$b = 10°$
0	.35	.25	.30	.21	.23	.17
.2	.47	.37	.40	.29	.31	.23
.4	.57	.46	.48	.37	.37	.29

Table 14 gives a series of values of the ratio, k, based on the above equation for several values of b and ϕ'. Revetment walls are usually built of stone masonry, presenting quite a rough surface adjacent to the earth bank, and it is therefore safe to allow the usual value of ϕ' (about 30°). Revetment walls, because of construction difficulties are rarely built of concrete. If concrete should be used, its smooth surface, together with the possibility of lubrication due to water, makes it inexpedient to allow for any frictional resistance between the wall and the adjacent earth.

Problems: Gravity Walls and Foundations

NOTE.—A comparative study of various sections of walls, with illustrative plates, is given in a pamphlet published by the *Engineering News*, 1913, entitled "Comparative Sections of Thirty Retaining Walls and Some Notes on Design," by E. H. CARTER.

1. A wall with a slight face batter and battered back, 25 feet high, supports a fill level with its top and subject to a uniformly distributed load of 600 pounds per square foot. What is the necessary width of the base assuming that the top width is taken as 2′ 6″ wide? Determine the offset of its footing that the toe pressure shall not exceed 6000 pounds per square foot. What is the factor of safety of the wall? If the method of the maximum wedge of sliding is used where is the point of application of the resultant located and what is the factor of safety (*a*) when the angle of friction is assumed as 30° (*b*) and when it is assumed as 0° between earth and back of wall?

The equivalent surcharge to a load of 600 pounds per square foot is six feet, whence the value of c is $6/25$, or 0.24. The ratio of top width to height is 2.5/25 or 0.10. By interpolation in Table 12 the values $M = 0.067$ and

$N = 0.5$ satisfy the given arguments and the resulting width of base is $h(0.1 + 0.5 + 0.067) = 16.7$ feet. The face batter is ¾″ to the foot and the rear 6″ to the foot.

To obtain the proper soil distribution, the weight of the wall (taking the masonry unit weight 150 pounds per cubic foot) is 35.9 kips (*i.e.* a kip is a 1000 pound unit). The vertical component of the thrust is (Eq. 72) $T_v = 23.1$. The vertical component of the resultant pressure upon the base is the sum of these two forces or is equal to $35.9 + 23.1 = 59.0$ kips. From (54) $r = 0.85$ and from Table 9, $i = 0.057$, whence the necessary projection is iw or 1′ 0″. Since the wall foundations are carried down about four feet to prevent frost action and surface water erosion, the step of one foot to four feet is a satisfactory one.

From (50) referring to Fig. 28, B from Table 3, is 0.39 whence $t = 0.39 \times 25 + 4.0 = 13.75$. $zw = \frac{1}{3}$ of $16.7 + iw = 6.56$ and the horizontal component of the thrust from (71) is 15.4 whence the factor of safety $= 1 + 1.8 = 2.8$, a satisfactory one from Prof. Cain's recommendations, page 57, but clearly without significance, unless taken in conjunction with the location of the resultant and with the manner of the distribution of pressure upon the soil.

By the sliding wedge method the horizontal component of the thrust is $T \cos (b + \phi')$, with T as given in (24). For $N = 0.5$, $b = 26° 34'$ and from (25) $K = 0.60$. T_h and T_v are then 15.4 and 23.3 respectively. (Cf. corresponding values by other method.) The location of the weight of the wall G is obtained from (66) and Table 11 with $u = 0.10/0.567 = 0.18$. $q = \frac{25}{3}(2.41 \times 0.067 + 1.25 \times 0.5) = 6.55$, whence from (75) $z = 0.364$, not at large variance with the value of $i + e$ in the Rankine's method. The toe pressure is from (53) 6.4 kips, approximating with sufficient exactness the result obtained in the suggested standard method of obtaining the thrust.

If the frictional resistance between earth and masonry is ignored, $K = 0.64$ and T_h, T_v are respectively 26.5 and 13.2. With the revised values, $z = 0.163$, a very unsatisfactory result. If the section of wall is changed to give a value of $z = 0.333$ by the last method, a much heavier section of wall results, showing the costly effect of omitting the consideration of frictional action of the earth upon the back of wall. All the standard sections exhibited in the above-mentioned pamphlet would develop high tension at the heel of the wall and a high bearing at the toe leading to the disfiguration, if not destruction of the wall were they designed in accordance with the maximum wedge of sliding, ignoring frictional action between the earth and wall. The sections are all extensively used in actual practice with excellent results.

Allowing for frictional resistance between earth and wall the factor of safety is 3; ignoring such action the factor becomes 1.5, *i.e.*, such favorable consideration doubles the factor of safety.

2. A standard wall for highways, is to be built, with a face batter of 1½″ to the foot and a back batter of 4″ to the foot. Give a section with the proper tabular dimensions. Also prepare plans for the proper foundation dimensions for (*a*) coarse sand and clay, well compacted, permissible bearing

4 tons per square foot, (*b*) coarse sand, permissible bearing 3 tons per square foot (*c*) fine sand, where a maximum intensity of toe pressure is 2 ton per square foot and a minimum intensity of heel pressuie is 0.5 tons per square foot. Also give a pile foundation section, allowing twenty tons per pile.

With the batters as given, $M = 0.125$, and $N = 0.333$. For highways, an average uniformly distributed load of 500 pounds per square foot will safely provide for the heavy surface loadings. Then for $h = 15$, $c = 0.33$; for $h = 20$, $c = 0.25$; for $h = 25$, $c = 0.20$; ior $h = 30$, $c = 0.17$. From data obtained from Table 12, the following table of top and bottom widths of wall has been prepared. (*d* is the top width, *b* the base width.)

h	*d*	*b*
15	2′ 5″	9′ 3″
20	2′ 8″	11′ 10″
25	3′ 0′	14′ 5″
30	3′ 4″	17′ 0″

Following the preparation of this table, a similar one may be prepared, giving the data necessary to compile the required toe extensions for the several allowable pressuies.

h	*G*	T_h	T_v	*R*	$S_1 = 4$ tons			$S_1 = 3$ tons			$S_1 = 2$ tons		$S_2 = 0.5$ ton		
					r	*i*	*iw*	*r*	*i*	*iw*	*r*	*i*	*s*	*i*	*iw*
15	13.1	6.2	6.2	19.3			*	...	*	*	.96	.02	.24	.17°	1′–6″
20	21.7	10.0	10.0	31.7			*	...	*	*	.75	.10	.19	.13°	1′–6″
25	32.7	14.6	14.6	47.3			*	.90	.035	0′–6″	.61	.18°	.15	.09	2′–6″
30	45.7	20.1	20.1	65.8			*	.78	.08	1′–4″	.52	.23°	.13	.08	4′–0″

* No offset necessary. °This value governs.

For the coarse sand and clay bottom (4 tons per square foot) no toe extension is necessary.

In preparing typical pile foundation plans, it is assumed that the piles will be in line both transversely and longitudinally (Case II).

$h = 15'$. Assume two piles to a section. If the rows are *m* feet apart, and with a beaiing value of 40 kips each, the necessary spacing of the rows is $80/R = 4.15$; therefore space these rows on four foot centers. The total load on each row is then $4R = 4 \times 19.3 = 77K$. With a value of $k = \frac{1}{3}$, Eq. 49 is applicable and $l_1 = w\sqrt{0.5} = 6.55$. The location of the pile is at the center of gravity of this triangle or at a distance $\frac{2}{3} \times 6.55$ from the heel. The pile is, accordingly 4′ 4″ from the heel. The other pile is at the center of gravity of the trapezoid bounded by the toe and the line l_1. The center of gravity of the trapezoid may be found in a manner similar to the location of the center of gravity of the earth pressure triangle Fig. 5. The value of *c* is here $\frac{6.55}{9.25 - 6.55} = 2.4$ and the value of *B* from Table 3 is 0.47. The center of gravity is then $\frac{47}{100}$ of the distance 2.70 from the toe, or approximately 1′ 3″ from the toe. It is safe, generally to take the pile at the center of the trapezoid, the erroi being one of a few inshes only.

$h = 20'$. Assuming two piles in a row here, with the value of $R = 31.7$ gives a spacing between rows of 2.5′ which is too close to space the piles; therefore three piles are taken. With this value $m = 3.8$ and may be taken as 4′. To ascertain whether a toe extension is necessary to permit a minimum spacing of 3′ between the piles adjacent to the toe, the value of i from equation (48) with $\lambda' = 6/11.83 = 0.506$; $e = \frac{1}{3}$; and $n = 3$, is found to be 0.073. The required toe extension is thus $0.073 \times 11.83 = 0.86$ or 10″. The corresponding value of k is $\frac{i + e}{1 + i} = 0.37$. From Table 8, F and H are respectively 0.14 and 65.0. Applying equation (45)

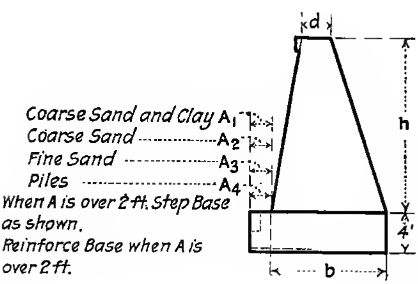
$$l_1 = (11.83 + 0.83)0.14(\sqrt{1 + 22} - 1) = 6.75$$
$$l_2 = 12.66 \times 0.14(\sqrt{1 + 44} - 1) = 10.2$$

The pile adjacent to the heel is 4′ 0″ from the heel, and bearing in mind the remarks previously made, the other two piles are 8′ 6″ from heel and 1′ 2″ from toe respectively.

$h = 25'$. Here $R = 47.3$. With an assumed number of piles, 4 to a row, the required spacing between rows is found to be 3′ 6″. To get the toe extension, $\lambda' = 6/14.42 = 0.414$. Accordingly $i = 0.16$ and the toe extension is $0.16 \times 14.42 = 2' \ 4''$. For simplicity make this 2′ 0″. k is then $\frac{0.14 + 0.33}{1.14} = 0.41$ and F and H are 0.43 and 10.00 respectively.

From (45)

$$l_1 = (14.42 + 2.0) \times 0.43 \times 0.87 = 6.11$$
$$l_2 = 16.42 \times 0.43 \times 1.45 = 10.2$$
$$l_3 = 16.42 \times 0.43 \times 1.92 = 13.5$$



h	d	b	A_1	A_2	A_3	A_4
15	2′-5″	9′-3″	0	0	1′-6″	0
20	2′-8″	11′-10″	0	0	1′-6″	1′-0″
25	3′-0″	14′-5″	0	0-6″	2′-6″	2′-0″
30	3′-4″	17′-0″	0	1′-4″	4′-0″	3′-0

FIG. 32.

The pile adjacent to the heel is 4′ 0″ from the heel; the next is 8′ 0″; the third 12 0″ and the face pile is 1′ 6″ from the toe, this spacing closely approximating the centers of the several pressure trapezoids.

$h = 30'$. $R = 65.8$. With 5 piles in a row, the required spacing between rows is found to be 3′. e is 0.353 and $i = 0.2$. The toe extension may be taken as 3′ 0″. The value of k is 0.42 and F and H are 0.54 and 7.11 respectively. Then $l_1 = 6.0$, $l_2 = 10.4$; $l_3 = 13.9$; $l_4 = 17.2$. The piles are spaced 3′ 0″; 8′ 0″; 12′ 0″; and 15′ 6″ from the heel and the face pile 1′ 6″ from the toe.

Figs. 32 and 33 show the wall proper and its foundations. It is understood, of course, that in preparing actual plans for construction that the plans will cover a much closer variation in the heights.

3. A wall of "quaker" section, 25 feet high is to rest upon a rock bottom. A surcharge of 500 pounds per square foot extends to the back of the wall. It will be permissible to let the resultant intersect at the outer ¼ point. Any tension developed in the wall because of this location of the resultant must be carried by steel reinforcement.

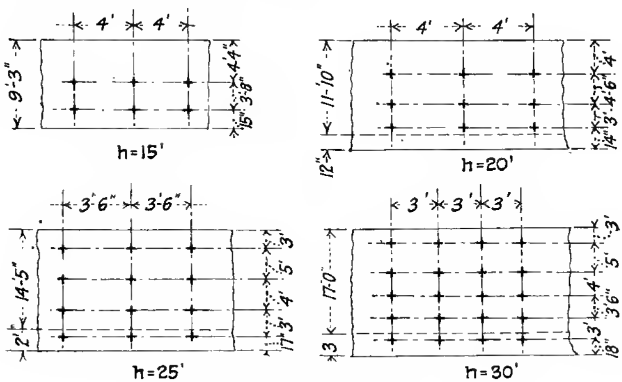


Fig. 33.—Pile layout.

In order to effect a direct design of a wall of this section, when the position of this resultant is at the outer quarter point, it will be necessary to proceed as in the present chapter. Referring to Fig. 30 and equation (62) with $M = O$,

$$x = \frac{h}{3}\,\frac{1 + 3p + 3p^2}{1 + 2p}\,N$$

From the quarter-point to D is

$$\frac{(1 + p)hN}{4}$$

and G from (69) is

$$\frac{mh^2(1 + 2p)N}{2}$$

The lever arm of G about the quarter-point is then

$$\frac{h}{3}\,\frac{1 + 3p + 3p^2}{1 + 2p}\,N - \frac{hN}{4}\,(1 + 2p) = \frac{hN}{12}\,\frac{1 + 3p + 6p^2}{1 + 2p}$$

And the moment of G about this point becomes

$$m\,\frac{h^3N^2}{24}\,(1 + 3p + 6p^2)$$

The horizontal and vertical components of the thrust are respectively from (71, 72)

$$\frac{gh^2(1 + 2c)}{6},\quad \frac{gh^2(1 + 2c)N}{2}$$

The lever arm of the horizontal component is simply Bh and that of the vertical component is

$$\tfrac{3}{4}(1 + p)hN - BhN = \frac{hN}{4}[3(1 + p) - 4B]$$

The overturning moment due to the thrust is

$$\frac{gh^2(1 + 2c)}{6}Bh - \frac{gh^2(1 + 2c)N}{2} \cdot \frac{hN}{4}[3(1 + p) - 4B]$$
$$= \frac{g(1 + 2c)h^3}{24}\left\{4B - 3N^2[3(1 + p) - 4B]\right\}$$

Equating the stability and overturning moments

$$mN^2(1 + 3p + 6p^2) = g(1 + 2c)\{4B - 3N^2[3(1 + p) - 4B]\}$$

and replacing, as before g/m by s

$$6p^2N^2 + 3pN^2 + N^2 = \frac{4s(1 + 3c)}{3} - 9s(1 + 2c)N^2 - 9s(1 + 2c)pN^2 + 4s(1 + 3c)N^2$$

or

$$6p^2N^2 + 3pNI + J = 0$$

where
$$I = N[1 + 3s(1 + 2c)]$$
$$J = N^2[1 + s(5 + 6c)] - 4s(1 + 3c)/3$$

Solving the quadratic

$$pN = \tfrac{1}{12}\{\sqrt{9I^2 - 24J} - 3I\}$$

TABLE 15

c \ N	0	0.1	0.2	0.3	0.4
		.31	.23	.16	.08
0	.39	.41	.43	.46	.48
		.38	.30	.21	.11
.2	.49	.48	.50	.51	.51
		.46	.35	.25	.14
.4	.57	.56	.55	.55	.54

To establish a table (see Table 15) take the ratio s at its usual value $\tfrac{2}{3}$.

To apply the results of the above to the problem at hand note that $c = 5/25 = 0.2$. Let the coping width be placed at 2 feet. From the variation of the top and base ratios as seen in the table the base width may be taken as 0.5×25 or 12.5 feet.

To determine the character of the stresses in the wall it becomes necessary to locate the line of resultant pressures, or thrusts in the wall. This is best done graphically. The wall is divided up into sections five feet high. The weight and thrust upon each section is determined as shown in Fig. 34. The points of application of each of these forces are found as follows: the center of gravity of the masonry trapezoids is taken from equation (66) and table 11, where $q = hU_2N/3$, or, since $N = 0.42$ and h is constant and equal to 5 for each section,

$$q = 0.7U_2$$

For the five sections starting from the top the ratios of the upper to lower base (u) are respectively 0.50; 0.66; 0.74; 0.79 and 0.83 and the corresponding values of q are then 1.64; 2.60; 3.60; 4.45 and 5.7. The weights of these sections are respectively 2.3; 3.8; 5.4; 7.0 and 8.6. The centers of gravity of the thrust triangles are found most easily from table 3, using the proper value of c. Since the surcharge is 5 feet, the respective values of c to be used in determining the values of B to locate the point of application of the thrusts are 1; 2; 3; 4 and 5 and the point of application above the base of

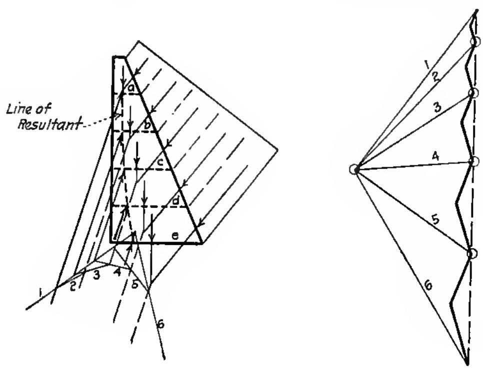


FIG. 34.

each trapezoid is 2.2; 2.35; 2.38; 2.41 and 2.42. For the sake of simplicity and to reduce the number of lines to be drawn the resultant of each of these two forces will be used. To determine the line of thrusts it is most easy to apply the principles of the funicular polygon. The load polygon, at the right of the figure is first drawn. The direction of each of the resultants is found to be the same and parallel to the total resultant at the base of the wall. The pole of the polygon is taken at convenience and the rays are drawn to the individual resultants. The funicular polygon is drawn in the usual manner and the location of the resultant thrust upon each section is determined by the intersection of the corresponding ray with ray 1, extended when necessary. A line through this intersection parallel to the direction of the resultant shown in the load polygon determines this location of the resultant thrust. The vertical components of the resultant pressure upon the base of each section is scaled from the load polygon.

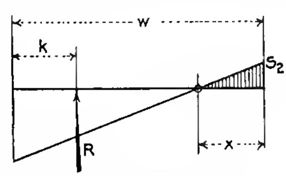


FIG. 35.—Amount of tension in wall.

Whenever the point of application of the resultant thrust lies within the outer third there is tension developed at the rear of the wall and it is necessary to determine this amount and supply sufficient steel rod reinforcement to take care of this tension, it being assumed that the wall shall take no tension whatsoever. From an inspection of the figure it is seen that above the line a the resultant pressure lies within the middle third and there is consequently no tension in the concrete above this point.

From Fig. 35, the steel area necessary to take the tensile stresses developed is that required by the shaded portion. The area of this portion is $x\,S_2/2$. From (41),

$$x = \frac{w}{3}\,\frac{1 - 3k}{1 - 2k}$$

and S_2 from (40) is

$$S_2 = \frac{2R}{w}(1 - 3k),\ \text{disregarding the negative sign.}$$

The area is then

$$\frac{R}{3}\,\frac{(1 - 3k)^2}{1 - 2k} = VR$$

where

$$V = \frac{1}{3}\,\frac{(1 - 3k)^2}{1 - 2k}$$

Table 16 gives a list of the values of V for several values of k less than $\frac{1}{3}$.

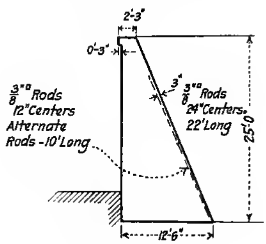


FIG. 36.

TABLE 16

k	V
.33	.00
.30	.01
.25	.04
.20	.09
.15	.14
.10	.20
.05	.27

The values of R as determined from the load polygon for each of the sections b, c, d and e are, respectively 10.3; 19.2; 31 and 45.5. The corresponding values of k (by scaling) are $8/30$; $10/41$; $12/52$ and $15/60$. (Note that this last value of k affords a check upon the algebraic method of obtaining the dimensions of the wall; having assumed the location of the resultant at the outer quarter point) or 0.26; 0.25; 0.23 and 0.25 for which the values of V are 0.04, 0.04; 0.06 and 0.04. The total area of the sections, or rather, the total tension that must be taken by the steel are respectively 410; 770; 1860 and 1820. Assuming that the steel rods can take 16,000 pounds per square inch, a $\frac{3}{8}$ inch square rod every 12″ will afford sufficient section to take the maximum stress. Since it is not necessary to have this amount of metal to the plane b, the rods will be spaced at 12″ centers to the plane c and at 24″ centers to the plane a. The rods will be placed 3″ from the back of the wall. Figure 36 shows the final wall section.

4. A dry rubble wall, 35 feet high with front face battered one-inch to the foot and rear face battered $4\frac{1}{2}$ inches to the foot weighs 125 pounds per cubic foot. The earth surface is horizontal and is subject to a live load of 500 pounds per square foot. The soil pressure must not exceed 3 tons per square foot. Determine the proper wall and footing dimensions.

For this problem $M = 1/12$ and $N = 3/8$. Referring to (74), $s = 100/125 = 0.8$ and $c = 5/35 = 1/7$.

$I = 3/12 + 2 \times 0.8(1 + 2/7)3/8 = 1.02$

$$H = \frac{1}{12}\left(\frac{1}{12} + \frac{3}{8}\right) - \frac{0.8}{3}\left\{1 - 6 \times \frac{1}{12} \times \frac{3}{8} - 3 \times \frac{3^2}{8^2} + \frac{3}{7}\left(1 - 4 \times \frac{1}{12} \times \frac{3}{8} - \frac{3^2}{8^2}\right)\right\} = -0.15$$

The quadratic now becomes, after putting $p(M + N) = x$

$$x^2 + 1.02x - 0.15 = 0$$

From which $\qquad x = 0.13$

The base ratio is $0.13 + (M + N) = 0.59$. The top and base width of the wall are then $0.13 \times 35 =$ 4 feet 6 inches and $0.59 \times 35 =$ 20 feet.

Note that for a wall of concrete or rubble masonry weighing 150 pounds per cubic foot the top and base ratios for the same conditions as the wall in the problem are, from table 12, 0.07 and 0.53 or the widths become 2′6″ and 18′6″ respectively. The area of this latter wall is 85 per cent. of the area required of the dry rubble wall. That is, 15 per cent. more area is required when the unit weight of the masonry is decreased 15 per cent.—a result quite obviously expected.

The vertical component of the thrust is from (72)

$$g\frac{(1 + 2c)h^2}{2}N$$

and in the variables of this problem

$$100 \times 1.286 \times 35^2 \times 0.375/2 = 29.5$$

The weight of the wall is

$$35 \times 125 \times \frac{4.5 + 20}{2} = 53.5 \text{ kips.}$$

The total vertical component of the resultant pressure upon the base is 83 kips. The permissible soil pressure intensity is 6 kips per square foot. From (54) $r = wS_1/2R = 0.723$ and from Table 9 with this value of r, the necessary value of i is 0.11. The toe extension is $0.11 \times 20 = 2'3''$. As indicated on page 61 the depth of footing will be 4′6″. The complete section of the wall is shown in Fig. 37. In conformity with the usual practice the coping is made of concrete and carried back 2′6″.

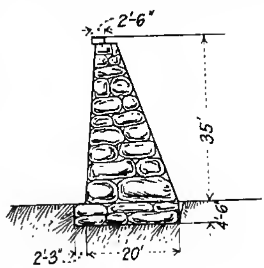


FIG. 37.—Dry rubble wall.

5. A rectangular wall is to line a rock cut twenty feet high and may be subjected to hydrostatic pressure up to one-half of the full water pressure. Determine the necessary wall thickness. To avoid the necessity of placing steel in the wall the point of application of the resultant should lie at the outer third point.

One-half fluid pressure is 31 pounds per cubic foot. For a wall with vertical back the lateral earth pressure has an intensity of 1/3 of the vertical and with earth at 100 pounds per cubic foot (the usual value) this intensity is 33 pounds per cubic foot. The problem is then merely to find a wall satisfying an earth pressure thrust as given in (14) with $c = 0$ and $K = \frac{1}{3}$. From Table 12 with $N = M = 0$ and $c = 0$, the required ratio of base to height is 0.47. The necessary thickness of the wall for the conditions of the problem is 9′6″.

6. A wall, whose resultant brings a vertical component of 35 kips per linear foot of wall located at the outer third point must have a uniform distribution of loading. The base of the wall proper is 12 feet wide. Design the foundation.

For a uniform distribution the resultant must be at the center of the footing. Since, under the conditions of the problem the location of the point of application of the resultant is 8 feet away from the heel, the footing must be 16 feet wide, necessitating a four-foot toe extension. The uniform intensity of pressure is then $^{35}/_{16}$ or 2.2 kips per linear foot. The shear at the cantilever support, see Fig. 38, is $4 \times 2.2 = 8.8$ kips. Since the usual depth of footing is four feet, to bring the base of the wall below the frost line, the step will be made 4 feet high as shown in figure.

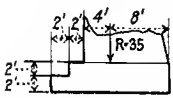


Fig. 38.—Footing for uniformly distributed base load.

The unit shear is $8800/(48 \times 12) = 15$ pounds per square inch. The cantilever moment at the same point is $8800 \times 24 = 211,000$ inch pounds. The section modulus is $bd^2/6 = 8600$. The tension at the lower edge of the base is then 211,000/8600 or 24 pounds per square inch. Clearly no reinforcement is necessary. For the economy of the material the step will be made in two sections of like dimensions. The shear is now $4400/(24 \times 12) = 15$ pounds per square inch and the moment is $220 \times 24 = 53,000$. The section modulus is $12 \times 24^2/6 = 1150$. The unit tension is $53,000/1150 = 46$ pounds per square inch. The safe value is slightly less than this (40 pounds per square inch) but this variation from the safe stress is a permissible one and no reinforcement will be added.

7. In the wall of problem 3 an opening is to be placed as shown in Fig. 39. Determine whether it is necessary to reinforce the section of the wall to make it span safely the opening.

The resultant load per lineal foot of the wall was found to be 45.5 kips per foot. The span in the clear is 20 feet.

The wall is on a rock footing, so that settlement is improbable and it seems reasonably safe to take the wall as a fixed beam, with moment $wl^2/12$ at the support. However, since the wall may crack near the supports for some reason unforeseen, it is better to investigate the stresses at the center of the span on the assumption that the beam is a simple one, and to make provision for stresses at the support in accordance with the assumption of a fixed beam. As a simple beam the moment is $45.5 \times 400/8 = 2275$ kip feet. As a fixed beam the moment is $45.5 \times 400/12 = 1520$ kip feet. The total moment is then $45.5 \times 400/12 = 1520$ kip feet. The shear is 455

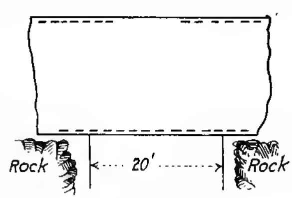


Fig. 39.

kips. The area of the wall is 26,100 square inches giving a unit shear o 455,000/26,100 = 17 pounds per square inch.

The apex of the section (produced) is about 5 feet above the top of the wall. Analogous to the location of the center of gravity of the thrust triangle the center of gravity of the beam section is located a distance Bh above the base, where with $c = 0.2$, $B = 0.38$ from Table 3 and the center of gravity of the section is located 0.38 × 25 or 9.5 feet above the base.

From the "Carnegie Handbook" (p. 137) the moment of inertia of the section about its center of gravity axis is given by an expression

$$I = \frac{d^3(b^2 + 4bb_1 + b_1^2)}{36(b + b_1)}$$

where d is the depth corresponding to h here and b and b_1 are respectively the lower and upper bases.

Using foot⁴ units, the moment of inertia of the given section is

$$I = \frac{25^3(2^2 + 4 \times 12.5 \times 2 + 12.5^2)}{36(2 + 12.5)} = 7800.$$

To the extreme fibre in tension at the center of the span, the distance is 9.5 feet, and the section modulus becomes 7800/9.5 or 820.

The unit tension per square *foot* is then 2,275,000/820 = 2780 or 19 pounds per square inch. No reinforcement is then necessary. Over the supports the maximum tension occurs at the top of the wall. The distance of the extreme fibre is now 25 − 9.5 = 15.5 and the corresponding section modulus is 7800/15.5 = 500. The unit tension per square foot is 1,520,000/500 = 3040 and the unit tension per square inch is 21 pounds.

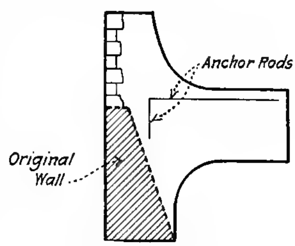


FIG. 40.—Reconstruction of gravity wall.

While no steel is necessary theoretically, a prudent engineer may specify light reinforcement over the supports, at the top of the wall and along the bottom of the wall from support to support (see Fig. 39).

Some Examples in Recent Practice

1. Wall, Reinforced on Bottom on Account of Threatened Settlement, *Engineering Record*, Vol. 64, p. 715.
2. Wall Across Marsh, on Piles, *Engineering Record*, Vol. 61, p. 242.
3. Wall on Piles, *Engineering Record*, Vol. 66, p. 132.
4. Heavy Gravity Section, Railway Improvement, *Engineering Record*, Vol. 66, p. 720.
5. Wall, 33 Feet High, on Piles, *Journal* Western Society of Eng., Vol. 16 (1911), p. 970.

Walls to Meet Special Conditions

1. Retaining Wall as Beam over Arch, *Engineering Record*, Vol. 64, p. 715.
2. Raising Existing Wall (see Fig. 40), *Journal* W. S. E., Vol. 16, p. 970.

Section avoided the necessity of deep excavation, with consequent heavy shoring of adjacent tracks. The abutting private property made it impossible to place face forms for a concrete wall, and a rubble masonry wall was built instead, backed by concrete. The author adds an interesting note: "It has occurred to the writer, that there is one feature of this type of wall, that might frequently be employed as a measure of economy. That is the saving in excavation and masonry effected by setting the foundation of the heel higher than the foundation of the toe. There are usually but two reasons for carrying the foundations of a retaining wall lower than the surface of the ground. The first is to reach a material that will sustain a greater pressure and the second, to get the foundation below the action of frost. The first is usually only necessary at the toe of the wall, for almost any good soil will sustain the heel pressure. The second, also, is only necessary under the toe for the heel is protected from frost by the embankment."

CHAPTER III

DESIGN OF REINFORCED CONCRETE WALLS

General Principles.—Reinforced concrete retaining walls form a class of walls in which the weight of the earth sustained is the principal force in the stability moment. Typical sections of this class of wall are shown in Fig. 41. The same fundamental principles governing the general outlines of the gravity wall, as given in the preceding chapter, likewise govern the outlines of this type of wall and the same criteria against impending failure must be satisfied. The actual section of the wall, once the forces upon it are known, is determined from the principles of design of reinforced concrete, a brief outline of which principles is given in this chapter.

As in the case of gravity walls, the stress system, soil pressures and other wall functions are known only when the final section

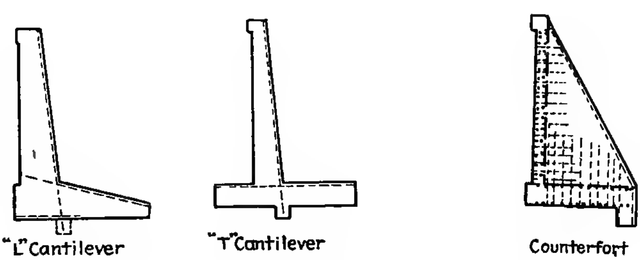


FIG. 41.—Typical reinforced concrete sections.

of wall is known. This, of course, necessitates a process of trial and error until a wall section has been found satisfying most economically all the necessary requirements of the data at hand. On the other hand, assuming the standard type of loading as shown in Fig. 5 and using the standard thrust equation as given in (14), and adding a few approximate conditions, a tentative section may be chosen from appropriate tables, varying but little from the final section of wall.

Preliminary Section.—The masonry composing the wall proper of a reinforced concrete section plays but a minor role in controlling the final wall section. The difference in its weight and the weight of the earth retained may thus be ignored and a skeleton section of wall treated as shown in Fig. 42. The thickness of the vertical arm of the wall is that demanded by the stresses existing within it (for a certain minimum thickness because of construction limitations, see the following pages) and whatever batter is given to the back of the arm is that necessary to take care of the increasing moments and shears in going toward the base of the wall. This is comparatively a small batter, and for a tentative design may be ignored. The back of the wall is then taken vertical and the thrust upon it is assumed to have a horizontal direction. The value of the earth pressure coefficient J is, for this condition $\frac{1}{3}$ (see Table 1).

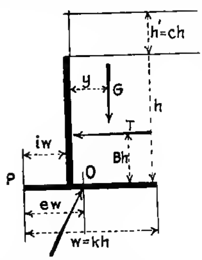


FIG. 42.—Skeleton wall.

The required outline of the wall is satisfactorily determined when the ratio between the width of base and height of wall is known. This ratio is denoted in the following work by k. Controlling the determination of this ratio are the location of the point of application of the resultant pressure, the toe extension, if any is assumed, the maximum permissible intensity of pressure upon the soil at the toe and the factor of safety. The value of the determination of this factor has been discussed on page 57.

The approximate assumption as to a skeleton outline of wall in addition to the adoption of the standard forms of loading and thrust makes it possible to determine directly the value of the ratio k depending upon the various functions enumerated above. While this section is not to be taken as the final one, it is sufficiently correct a section upon which to base estimates of cost and to determine the limitations of the various types of the walls to the peculiar conditions at hand.

Based upon the above assumptions the following relations between the various criteria affecting the wall section are found. Refer to Fig. 42. This is known as the "T" type cantilever wall and is together with its modified "L" shape wall, the type of most frequent occurrence. The thrust T is found from equation (14) and is located at a distance Bh above the base, where B has been defined by equation (12) and may be found from Table 3. The moment of the thrust about the toe P is then

$$TBh$$

and if these quantities are replaced by their values as taken from the equation mentioned, the thrust moment is

$$M_o = Jg \frac{1 + 2c}{2} \frac{1}{3} \frac{1 + 3c}{1 + 2c} h^3$$

$$= \frac{1}{6} Jg(1 + 3c)h^3 \qquad (80)$$

as before g is the unit weight of the retained earth, and is ordinarily taken as 100 pounds per cubic foot.

The stability moment of the wall, M_s is

$$M_s = Gy \qquad (81)$$

Since, as per the adopted approximation, the difference in weight between the masonry comprising the wall and the weight of the retained fill is ignored, the value of G is

$$G = gh\,(1 + c)\,w\,(1 - i) \qquad (82)$$

i is the ratio between the length of the toe extension and the entire width of the base. The value of the lever arm is

$$y = w - \frac{w(1 - i)}{2} = \frac{w(1 + i)}{2} \qquad (83)$$

Let the ratio between the width of base, w, and the height of wall, h, be denoted by k. If the factor of safety of the wall is taken to mean the ratio between stability moment and the overturning moment, and is denoted by n,

$$M_s = nM_o \qquad (84)$$

From (81), (82) and (83)

$$M_s = \frac{1}{2} gk^2(1 + c)(1 - i^2)\, h^3 \qquad (85)$$

From equations 80, 84, 85,

$$gn \frac{1}{6} J(1 + 3c)h^3 = \frac{1}{2} gk^2(1 + c)(1 - i^2)h^3$$

and finally

$$k = \sqrt{\frac{J(1 + 3c)n}{3(1 + c)(1 - i^2)}} \qquad (86)$$

expressing the ratio between the width of the base and the height of the wall in terms of factor of safety assumed and the width of toe extension. The surcharge ratio c and the earth pressure coefficient J are, for the purposes of the problem, independent of the functions of the wall outlines.

To establish the base ratio k in terms of the location of the point of application of the resultant and the toe extension (and these are the two functions generally known, or easily found in advance), take moments about the point of application of the resultant. M_o, the thrust moment remains the same as before and is given by equation (80). The new stability moment M_s is related to that found in equation (81) in the ratio of the respective lever arms of the force G, or if M'_s denotes the new stability moment

$$M'_s/M_s = \frac{\frac{1+i}{2} - e}{\frac{1+i}{2}} = 1 - \frac{2e}{1+i} \tag{87}$$

Taking moments about the point O, $M'_s = M_o$ and from (87) and since $M_s = nM_o$

$$n = \frac{1+i}{1+i-2e} \tag{88}$$

A relation between the factor of safety, the location of the point of application of the résultant and the toe extension ratio. Inserting this value of n in (86)

$$k = \sqrt{\frac{J(1+3c)}{3(1+c)(1-i)(1+i-2e)}} \tag{89}$$

which may be written

$$k = \sqrt{\frac{J(1+3c)}{3(1+c)}}\sqrt{\frac{1}{(1-e)^2-(i-e)^2}} \tag{90}$$

Inspecting this last expression, it is seen that k is a minimum when the factor $(i - e)$ in the denominator vanishes, or for $i = e$.

For a given location of the resultant pressure the most economical width of base is had when the vertical arm is placed over the assumed point of application of the resultant pressure.

When the back of the wall is vertical, as is assumed in the present analysis, J has the value $1/3$, which should be inserted in expressions (86) and (90). Again, introducing this value of

J and also, the economical criterion established above (90) becomes

$$k = \frac{1}{3}\frac{1}{1-e}\sqrt{(1+3c)/(1+c)} \qquad (91)$$

The application of these equations to specific problems is shown at the end of the chapter.

Distribution of Base Pressures.—The manner of the distribution of pressure on the base is again controlled by the type of soil upon which the wall will rest, with an advantage over the gravity type of wall in that, any tension developed in the wall may be taken care of by proper reinforcement. Continuing the approximations given above, further guidance may be had in shaping the wall to meet the anticipated soil conditions.

The total load upon the base of the wall is G. From (39) of Chapter II and from (82) above

$$S_1 = \frac{2G}{w}(2-3e) = \frac{2ghw(1+c)(1-i)}{w}(2-3e) \qquad (92)$$

Place $H = h(1+c)$; that is, H is the total depth of fill plus the depth of surcharge. Solve the equation for e, taking the unit weight g of the earth as 100 pounds per cubic foot and expressing both this weight and the soil pressure intensity S_1 in tons. There is

$$e = \frac{2}{3} - \frac{10S_1}{3H(1-i)} \qquad (93)$$

When the maximum soil pressure intensity S is given as well as the toe extension ratio i, this equation may be used to locate the point of application of the resultant pressure upon the base. When this value of e has been found, equation (90) is then applied to find the value of the base width ratio k.

Conversely when the point of application of the resultant is assigned (and with a foundation known in advance, the location of the point of application of the resultant is usually indicated) the toe extension necessary to give this resultant location is found from

$$i = 1 - \frac{10S_1}{(2-3e)H} \qquad (94)$$

If, in equation (93), i is put equal to e (the economy criterion), and the resulting equation is solved for e

$$e = \frac{5}{6} - \frac{1}{6}\sqrt{\left(1 + \frac{120S_1}{H}\right)} \qquad (95)$$

Under the above conditions, given H and S_1, the toe extension ratio i is determined at once. The conditions under which the location of the stem is governed solely by the economy of the wall have been previously touched upon (see pages 42–44) and will be discussed in more detail further on. Clearly, if no limitation is placed upon the location of the vertical arm, it should be placed where the economy criterion dictates: directly over the indicated position of the point of application of the resultant upon the base.

Tables and Their Use.—Tables are readily founded upon the preceding equations and simplify the necessary calculation of the wall outlines. From the relation existing between the location of the point of application of the resultant, the factor of

TABLE 17.—VALUES OF e

i \ n	2	3	4	5
0	.25	.33	.38	.40
.1	.27	.37	.41	.44
.2	.30	.40	.45	.48
.3	.33	.43	.49	
.4	.35	.46		
.5	.38	.50		

safety and the amount of toe projection, equation (88), Table No. 17 has been prepared. With a given location of the resultant and an assigned factor of safety, the required toe projection is taken from the table. Again, for an assigned location of the point of application of the resultant and a given toe projection, the factor of safety may be taken from the same table. For the criterion of economy *i.e.* $i = e$, this relation becomes

$$e = \frac{n-1}{n+1} \qquad n = \frac{1+e}{1-e} \tag{96}$$

TABLE 18.—VALUES OF k

c	$i=0$				$i=\frac{1}{4}$					$i=\frac{1}{3}$					$i=\frac{1}{2}$				
	$e=.1$	$e=.2$	$e=.3$	$e=.4$	$e=.1$	$e=.2$	$e=.3$	$e=.4$	$e=.5$	$e=.1$	$e=.2$	$e=.3$	$e=.4$	$e=.5$	$e=.1$	$e=.2$	$e=.3$	$e=.4$	$e=.5$
0	.37	.43	.53	.74	.37	.42	.48	.57	.77	.38	.42	.48	.56	.70	.41	.45	.50	.56	.67
.1	.42	.49	.60	.85	.43	.48	.54	.65	.88	.44	.48	.54	.64	.81	.47	.51	.57	.64	.76
.4	.47	.54	.66	.94	.47	.53	.60	.72	.97	.48	.53	.60	.70	.89	.52	.57	.63	.71	.84

A general table, Table 18 has been prepared, giving the value of k, as found from equation 90, for a range of values of c, e and i. The earth pressure constant J, has been taken as $\frac{1}{3}$.

With the general outlines of the wall approximately established by aid of the foregoing, it is possible to proceed with the actual design of the several members composing the reinforced concrete retaining wall. While it is not the purpose of the preceding analysis to replace a careful, exact analysis of the wall, its prime intent is to permit an intelligent selection of a wall without a tedious process of trial and error. It should be pointed out, that the approximations consist in ignoring factors which have proven negligible in controlling the wall dimensions, so that even though the selection of the wall outlines are finally determined by these approximations, no serious error has been committed. However, a careful and painstaking designer will analyze the completed wall, to see whether the stress system in it checks with the one first determined.

Theory of the Action of Reinforced Concrete.—The assumptions in the design of reinforced concrete beams are those of the ordinary beam theory, namely: the Bernoulli—Euler theory of flexure. The fundamental premise is that a plane section before bending, remains a plane section after bending, with the further assumption that Hooke's Law, *i.e.* the stress is proportional to the strain, is true.

Although the brilliant researches of Barre de St. Venant, have shown that plane sections do not remain plane during bending, the error becomes appreciable when the ratio of depth of beam to span exceeds one-fifth. Since for such ratios, stresses, other than those induced by bending moment, usually govern the required reinforcement and depth of beam *e.g.* the unit shear and adhesion, these assumptions of plane sections may be taken as valid, so long as the stresses induced by the bending moment govern the required depths and amounts of steel reinforcement. The concrete is assumed to take no tension.

The excellent report of the Special Committee on Concrete of the A.S.C.E., has set the seal of approval on this mode of figuring the action of reinforced concrete after most thorough investigation, both from a theoretical and experimental standpoint, and the engineer may accept this method, with no fear of beam failure ensuing, so long as care has been taken of all the stress criteria.

Under load, the distribution of stress across a section normal to the axis of the beam is shown in Fig. 43. Adopting the recommended nomenclature as suggested in the above report, E_s is the steel modulus, E_c the concrete modulus, and n the ratio of the steel modulus to the concrete modulus. A_s and A_c are the areas of the steel and concrete in the section respectively. f_c and f_s are, respectively, the unit concrete and steel stresses. Let e_c be the displacement of the section at a and e_s that at b. From the assumption that a plane section remains a plane section after bending, and from Hooke's Law

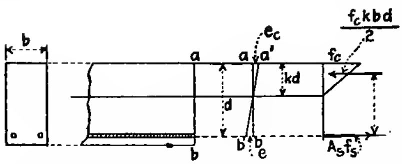


Fig. 43.—Theory of reinforced concrete.

$$\frac{e_c}{e_s} = \frac{kd}{(1-k)d} = \frac{k}{1-k} \tag{97}$$

$$f_c = e_c E_c;\ f_s = e_s E_s A_s;\ A_s = pA_c = pbd$$

and, by summation of all the horizontal forces

$$\frac{f_c\ kbd}{2} = e_s E_s A_s, \text{ or } \frac{E_c e_c kbd}{2} = e_s E_s pbd$$

$$\frac{ke_c E_c}{2} = e_s E_s p$$

whence

$$\frac{e_c}{e_s} = \frac{2pn}{k} \tag{98}$$

and equating this to (97)

$$\frac{2pn}{k} = \frac{k}{1-k} \tag{99}$$

and finally

$$k^2 + 2kpn - 2pn = 0.$$

Solving this for k

$$k = \sqrt{p^2n^2 + 2pn} - pn \tag{100}$$

which locates the position of the neutral axis, once the ratio of the two moduli are adopted and the percentage of steel assumed. It is to be noticed that it is a function of these two quantities only.

The resisting moment of the section may be expressed with either the steel force or the concrete force as the force factor in the couple. If M_c and M_s are the concrete and steel resisting moments respectively,

$$M_c = f_c \frac{k\left(1 - \frac{k}{3}\right)}{2} bd^2; \qquad M_s = f_s A_s \left(1 - \frac{k}{3}\right) d = f_s p \left(1 - \frac{k}{3}\right) bd^2$$

$1 - k/3 = j$ and is the effective lever arm of the couple, corresponding to the effective depth of homogeneous beams. The moments may be expressed as

$$M_c = k_c bd^2; \; M_s = k_s bd^2 \tag{101}$$

where

$$k_c = f_c kj/2; \; k_s = f_s pj \tag{102}$$

Ordinarily, the most economical section is that one in which the concrete and the steel are each stressed to their permissible limits. The percentage of steel to satisfy this condition may be found as follows:

Since, from the summation of horizontal components of stress intensities across the right section of the beam, the total concrete stress must be equal to the total steel stress

$$A_s f_s = pbdf_s = kbd\, f_c/2$$

from which equality

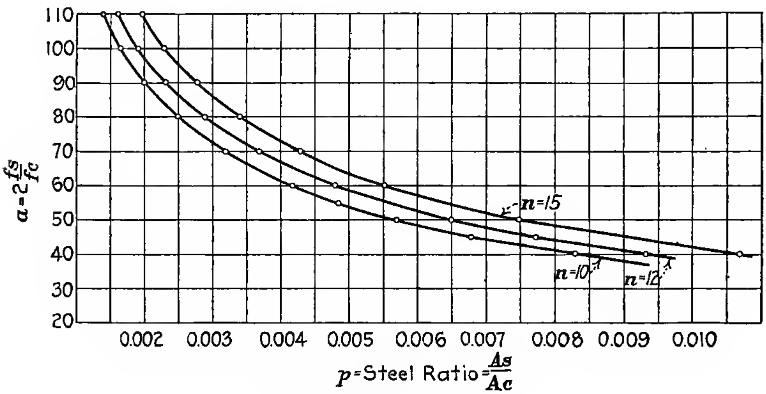
$$k = 2p\frac{f_s}{f_c} \tag{103}$$

Equating this value of k to that found in equation (100) and replacing the ratio $2f_s/f_c$ by a, and solving the equation for p

$$p = \frac{2n}{(n + a)^2 - n^2} = \frac{2n}{a(2n + a)} \tag{104}$$

If in the ratio a, the unit stresses are those allowed for the material at hand, than this value of p proves to be the most economical one to use.

The above analysis is of course, predicated upon the assumption that the section is controlled by the bending moment. Other stresses may determine the percentage of steel or the depth of the section. When the percentage of steel is above that necessary for the economical steel ratio as given by (104), then the concrete stress in the section will determine the resisting moment to be used and the section constant is found from k_c, as defined above. With this value of k_c, the proper percentage of steel is to be taken from Table 19. Again the depth of the section may be greater than required by the bending moment, and accordingly the percentage of steel to satisfy the bending moment will be less than that required by equation (104). The steel stress will be the governing stress in the section and the section constant to be used will be k_s as defined in equation (102). The proper percentage is found from Table 19 with this value of k_s. The conditions under which these constants control are best illustrated by specific problems as given at the end of the chapter.



Curve Plate No. 2.
Economical steel percentage.

To simplify the use of equation (104), Curve Plate No. 2 has been drawn from which the proper value of p may be taken once the value of n and of the ratio a are known.

In the report of the Special Concrete Committee, mentioned above, the following values of n are suggested, depending upon the ultimate strengths of concrete:

$n = 15$. Ultimate strength equal to or less than 2200 lbs. per sq. in.
$n = 12$. Ultimate strength between 2200 and 2900 lbs. per sq. in.
$n = 10$. Ultimate strength greater than 2900 lbs. per sq. in.

Table No. 19 is a compilation of the values of the several functions entering into the computation of a concrete-stee section. It is noticed that the terms are not carried out to the usual degree of refinement. In view of the approximation in both the theory and in the experimental determination of the concrete constants, it does not seem good practice to carry the work out to any greater degree of exactness than shown here.

Table 19.—Reinforced Concrete Constants

	$n = 10$				$n = 12$				$n = 15$			
p	k	j	$\frac{1}{2}jk$	pj	k	j	$\frac{1}{2}jk$	pj	k	j	$\frac{1}{2}jk$	pj
.002	.18	.94	.09	.002	.20	.93	.09	.002	.22	.93	.10	.002
.004	.25	.92	.12	.004	.26	.91	.12	.004	.29	.90	.13	.004
.006	.29	.90	.13	.005	.31	.90	.14	.005	.34	.89	.15	.005
.008	.33	.89	.14	.007	.35	.88	.15	.007	.38	.87	.17	.007
.010	.36	.88	.16	.009	.38	.87	.16	.009	.42	.86	.18	.009
.012	.38	.87	.17	.010	.41	.86	.18	.010	.45	.85	.19	.010
.014	.40	.87	.17	.012	.44	.85	.19	.012	.47	.84	.20	.012
.016	.43	.86	.19	.014	.46	.85	.20	.014	.49	.84	.21	.013
.018	.45	.85	.19	.015	.48	.84	.20	.015	.51	.83	.21	.015
.020	.47	.85	.20	.017	.49	.84	.21	.017	.53	.82	.22	.016
.025	.50	.83	.21	.021	.53	.82	.22	.020	.57	.81	.23	.020
.030	.53	.82	.22	.025	.56	.81	.23	.025	.60	.80	.24	.024

In addition to determining the resisting moment of a section, it is necessary to find the unit shear and the unit adhesion, each of which stresses may demand more resisting material than that required by the moment.

Analagous to a steel or other section of homogeneous material the shear over any section is assumed distributed over the effective depth (jd) of the section, so that, if s is this unit shear, and V is the total shear over the section

$$s = \frac{V}{jbd} \tag{105}$$

The unit adhesion corresponds again, to the horizontal shear, and since the unit vertical shear is equal to the unit horizontal shear, the periphery of the steel embedded in the concrete per unit length must carry the unit horizontal shear (or its equivalent, the unit vertical shear.)

If r is the periphery of the rods per unit length, and q is the permissible adhesion stress,

$$q = \frac{V}{jdr} \tag{106}$$

TABLE 20.—STANDARD ULTIMATE STRENGTHS OF AGGREGATES AS SUGGESTED BY THE SPECIAL COMMITTEE ON CONCRETE A. S. C. E.

Aggregate	1 : 1 : 2	1 : 1½ : 3	1 : 2 : 4	1 : 2½ : 5	1 : 3 : 6
Granite, trap	3300	2800	2200	1800	1400
Gravel, limestone	3000	2500	2000	1600	1300
Soft limestone	2200	1800	1500	1200	1000
Cinders	800	700	600	500	400

The following are the percentages of the above ultimate stresses that may be allowed:

Bearing.—Compression applied to surface twice the loaded area, 32.5 per cent.

Axial Compression.—Where the length is not greater than twelve diameters, 22.5 per cent.

Compression Extreme Fibre.—32.5 per cent.

Shear and Diagonal Tension.—Beams, with horizontal bars (*i.e.*, bars parallel to the longitudinal axis of the beam only) no web reinforcement, 2 per cent.

Bond.—4 per cent. In case of wires 2 per cent.

Upon the recommendation of the above Committee, Table 20 was compiled, giving the standard ultimate strengths for the several combinations of the different aggregates, and then the percentages of these ultimate loads to be used for the different type of stresses.

Bending and Anchoring Rods.—Rods are anchored in the concrete by (1) carrying them beyond the theoretical end, a distance sufficient to develop, in bond, its tensile stress; (2) hooking the end of the rod around a rod at right angles to it; (3) threading the end of the rod and bolting it to a steel washer or other steel device buried in the concrete (4) making a U turn in the rod. The first and last methods are the usual ones because of cheapness of these details. The second and third are used only where lack of room makes such details necessary. Bending rods around another rod, and threading and bolting rods are expensive details to be avoided as far as possible.

If the unit adhesion is q and f_s is the steel stress, then, if L is the length necessary to carry the rod beyond its theoretical end

$$4qtL = f_s t^2 \text{ and } L = \frac{f_s}{4q} t. \qquad (107)$$

The value of this fraction varies from 40 to 50 (the unit stresses taken from Table 20) and the rod is carried passed the theoretical end, this number of thicknesses.

If a rod is twisted about another rod then the twist should be at least one complete turn (360°) and carried beyond about six inches, not only to satisfy the theoretical requirements, but to aid the work in the field. In bending a rod care must be taken that the radius to which the rod is bent is sufficiently large that the bearing induced on the concrete will be within the allowable limits. For a rod bent to a circular arc, carrying a tension of T at either end, the condition is similar to that of a hoop (see any text on applied mechanics) and the compressive stress upon the concrete per linear unit of the rod is

$$C = T/R$$

Where R is the radius of the bend. If c is the permissible *unit bearing* on concrete and f_s is the permissible steel unit stress, then introducing these factors in this last equation

$$R = \frac{f_s}{c}t$$

t is the thickness of the rod. The ratio f_s/c has a value of about 30 and in the work that follows this proportion will be used in determining the proper radius to turn the rod.

To get the area of a washer necessary to hold the bar, with A the area of the washer and c the unit concrete bearing, let d be the side of the square (if a square washer be used) and with the same units as before, the total bearing is Ac. Since $Ac = c\,(d^2 - t^2)$

$$d = t\sqrt{(f_s/c + 1)} \qquad (108)$$

With the usual unit stresses, d is about six thicknesses of the bar. If d is the diameter of a round washer

$$d = 2t\sqrt{\left(\frac{f_s + c}{\pi c}\right)} \qquad (109)$$

With the usual values, the diameter of a round washer should be about seven and one-half thicknesses of the bar.

Vertical Arm.—The vertical arm of a reinforced concrete wall as shown in Fig. 42 and as tentatively analyzed on pages 80 and 82 is a cantilever beam, subjected to a horizontal load of T, located at a point Bh above the base. In the skeleton wall, the basis for the approximate analysis, h is measured from the bottom of the wall. In the actual final section, the correct value of h must be used, namely the height of the vertical wall above the top of the footing. The discrepancy in the assumed and correct h may be ignored in the tentative selection of the thicknesses of the arm and footing.

As above shown the cantilever moment in the arm is TBh, and if T is replaced by its value in (14), and B by its value in (12) then

$$M = \frac{1}{6}Jg(1 + 3c)h^3 \qquad (110)$$

The value of J is taken as one-third (see page 80). g is the unit weight of earth and c is the ratio of the surcharge height to the actual height of wall assumed. The standard type of loading as shown in Fig. 5 is to be used.

While the shear and the unit adhesion may, and frequently do, control the depth of beam required, this depth will not vary much from that required by the bending moment depth and it is safe in this preliminary analysis to work with the depth required by the bending moment. The resisting moment has been given by (101) and equating this to the external moment given in (110), and solving for d

$$d = h^{3/2}\sqrt{\frac{Jg(1 + 3c)}{6k_c}} \qquad (111)$$

J may be given the value $\frac{1}{3}$ as above. g is taken at the usual weight 100 lb. per cubic foot. If the economy criterion of (104) is used, and if in accordance with general practice a 1:2:4 concrete is specified with the resulting permissible stresses as given in Table 20, from Curve Plate No. 2 with $n = 15$, the steel ratio p is 0.0075. From Table 19, $0.5kj$ is, for this value of p, 0.17 and since f_c, in conformity with the other terms of (111) is to be expressed in units of pounds per square foot, the bending moment constant k_c from equation (102) is about 16,000. With these values equation (111) becomes

$$d = 0.0185h^{3/2}\sqrt{(1 + 3c)} \qquad (112)$$

The depth d necessary to satisfy the bending moment due to the earth thrust may be closely approximated from this equation and the same expression may be used to find the required depth at any point on the cantilever arm, by using the proper values of c and h.

To determine the depth to satisfy the shear requirements, apply equation (105). V is the thrust T and j may be safely taken, for the purposes at hand, at ⅞. With the same concrete constants as assumed above, the shearing value for a simply reinforced beam is $s = 40$ pounds per square inch or 5760 pounds per square foot. The required value of d is

$$d = T/5040 = Jgh^2(1 + 2c)/10080 = 0.0033h^2(1 + 2c) \quad (113)$$

Comparing this equation with (112), the shearing stress will control the required depth of the arm, whenever the value of d as found from (113) is greater than that value as found from (112). Solving this inequality, the shearing stress will determine the necessary depth when

$$h > \frac{31(1 + 3c)}{(1 + 2c)^2} \quad (114)$$

TABLE 21

c	h_1
.0	31
.1	28
.2	25
.3	23
.4	21
.5	19
.6	18
.7	17
.8	16
.9	15
1.0	14

This may be termed the "critical" value of h and Table 21 gives the values of the "critical" value of h for several values of surcharge ratio c. Its use is explained in the problems at the end of the chapter. The above equations suffice to determine, approximately, the thickness of the arm to satisfy the stresses induced by the earth thrusts. While such thicknesses are fairly accurate (the problems at the end of the chapter are illustrative of this) it is better practice to take the wall thus approximately outlined as the tentative section and design finally by the more exact methods the required dimensions of the wall.

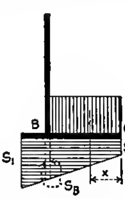


FIG. 44.—Loading on footing.

Footing.—The footing, see Fig. 44, is again a cantilever, with its maximum moment at the foot of the vertical arm B. Its loading is the net difference between the downward weight of the retained fill and the upward thrust of the soil pressures. The soil pressure intensity at B is

$$S_B = S_1 + (S_1 - S_2)\frac{(1 - i)w}{w} = (1 - i)S_1 + iS_2 \quad (115)$$

Taking moments at B

$$M_B = Gp/2 - S_2p^2/2 - (S_B - S_2)p^2/6 = Gp/2 - (2S_2 + S_B)p^2/6 \quad (116)$$

From (115)

$$2S_2 + S_B = (1 - i)\,S_1 + (2 + i)\,S_2$$

and from (39) and (40) of chapter 2

$$2S_2 + S_B = 6\,\frac{G}{w}\Big[e - i(1 - 2e)\Big]$$

The expression (116) for the bending moment now becomes

$$M_B = \frac{Gp}{2}\left[1 - \frac{2p}{w}\left\{e - i(1 - 2e)\right\}\right] \qquad (117)$$

Note that $p = w(1 - i)$ and that

$$G = gh(1 + c)\ (1 - i)w; \quad \text{and} \quad w = kh$$

Using the value of k as found in equation (90), the expression for the bending moment (117) is finally

$$M_B = Ig(1 + 3c)h^3/18 \qquad (118)$$

where, $$I = \frac{(1 - i)\{1 - 2(1 - i)[e - i(1 - 2e)]\}}{1 + i - 2e} \qquad (119)$$

Comparing the value of this moment as given in equation with that of the vertical arm, as given in equation (110), it is seen that the footing moment is I times the arm moment with I varying from one to one-half. Table 22 gives a series of values of I.

TABLE 22

i	$e = 0$		$e = 0.4$		$e = 0.5$	
	I	Q	I	Q	I	Q
.0	1.00	.00	1.00	.00	1.00	.00
.1	.96	.03	.95	.05	.90	.10
.2	.88	.11	.85	.14	.80	.20
.3	.76	.19	.72	.25	.70	.30
.33	.72	.22	.69	.28	.60	.33
.4	.64	.27	.62	.34	.60	.40
.5	.50	.33	.50	.43	.50	.50

As before, the shearing stresses and the adhesion stresses must be found. The complicated type of loading upon the footing makes it impossible to find an easily applied expression for these stresses and resort must be had to specific problems to illustrate the effects of these stresses. Some problems at the end of this chapter bring out in detail these points.

Toe Extension.—The approximate design of the toe extension of the footing, if such an extension is used, follows along lines similar to those of the preceding paragraphs. Referring again to Fig. 44 with the value of the soil intensities as previously found S_B is taken the same as in the design of the heel extension. For the exact analysis, the moments for the heel and the toe are taken at the intersection of the rear and face planes of the vertical arm respectively. For the approximate solutions now sought this refinement is unnecessary and taking moments about B

$$M'_B = S_B\, i^2k^2h^2/2 + \frac{S_1 - S_B}{3}\, i^2k^2h^2 = (S_B + 2S_1)\,\frac{i^2k^2h^2}{6} \quad (120)$$

and again replacing the soil intensities and k by their values,

$$M'_B = \frac{Qgh^3J(1 + 3c)}{6} \quad (121)$$

where

$$Q = \frac{2i^2[2 - 3e - 2i(1 - 2e)]}{1 + i - 2e} \quad (122)$$

The toe footing moment is thus Q times the arm moment, with Q varying from zero to one-half. Table 22 gives a set of values for Q.

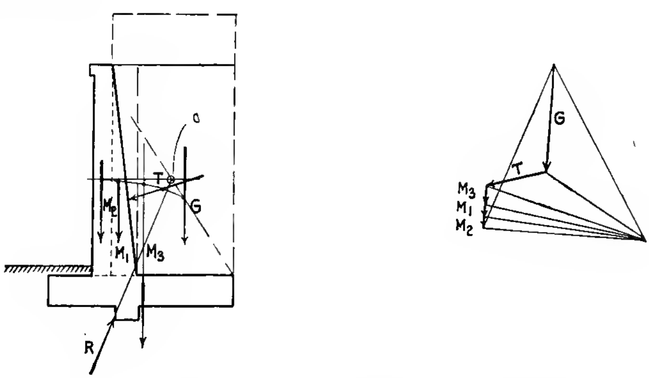


Fig. 45.—Graphical analysis of reinforced concrete wall.

It is again necessary to emphasize the fact that the shearing and adhesion stresses must be ascertained.

The dimensions of the wall are thus approximately determined, and with the outlines of the wall previously found, it is possible to proceed with the definite final design. Laying out the wall in accordance with these dimensions, the thrust may be found by the graphical methods or may, once more, be taken with J = one-third as urged in Chapter I and then combined with the vertical weight of the earth on the projection of the back of the arm (if the arm be battered from the minimum practical width at the top to the required width at the base). With the thrust determined, the location of the resultant and the soil pressure intensities are found and checked with the location and intensities of pressure assumed originally. This is best found graphically as shown in Fig. 45, where the properties of the funicular polygon are utilized. Several problems at the end of this chapter develop in greater detail the methods sketched here.

Counterfort Walls.—A study of the expressions determining the thicknesses of the members of the cantilever walls discussed in the preceding sections, will show, that as the walls increase in height, the required thicknesses of these members become very large. To reduce the sizes of the arm and of the footing, supporting walls are introduced between these members, termed loosely, counterforts. See Fig. 46. These serve a function similar to that performed by the gusset plate on a through girder, anchoring the wall and base slab to each other.

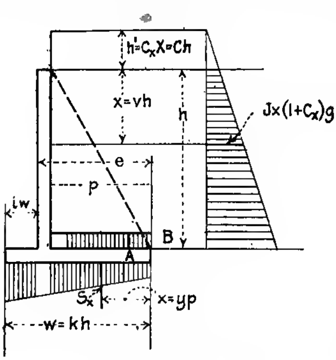


FIG. 46.—Stresses in a counterforted wall.

This combination of counterfort, wall and footing, forms a structure quite difficult to analyze exactly and, generally, no such exact analysis is attempted. The usual modes of treating the wall and base slabs of the counterfort wall are as follows:

(*a*) The wall and the footing slabs are treated as composed of a series of independent longitudinal strips, freely supported at the ends, *i.e.*, at the counterforts. The bending moment is then $WL/8$. W is the total weight acting upon the strip in question.

(*b*) The wall and footing are treated in strips as above, but the supports are taken as fixed at the counterforts. Although, exactly speaking, for this condition, the moment at the support is $WL/12$, and that at the center of the beam is $WL/24$, the moment is assumed alike at the center and at the support and of value $WL/12$.

Method (*b*) is the one generally used in the design of the slabs forming the counterfort wall and will be used in the present text.

The design of the counterfort itself is a matter of much controversy and practice is far from uniform here.[1] It may be taken as a tension brace, simply anchoring, by means of the rods contained in it, the base slab and the wall slab to each other, the concrete merely acting as a protection to the steel; as a cantilever beam, anchored at the base and receiving its load from the wall slab, or as the stem of a "*T*" beam. In the following work the counterfort will be treated as a cantilever beam. Prof. Cain has made an exact analysis of a beam of this wedge shape (see his "Earth Pressures," etc.) but the theory of retaining walls and of earth pressures does not seem to justify such refinements of design.

Not only are all of the methods of stress computation above discussed approximate, but it is difficult to make an estimate as to their degree of exactness. If the slabs are designed as outlined under (*a*) and (*b*) the relieving action of the portion of the slab adjacent to the strip under question is ignored. That is, no account is taken of the plate action that may exist in the slab. Toward the junction of the base and the arm, the two members tend to mutually stay each other, reducing the possible deflection and thus the resulting stress. It is clear that there is considerable latitude permissible in making stress assumptions and here again, simplicity of design should dictate the formulas to be used rather than an intricate analysis of questionable accuracy.

While attention has been paid only to bending moments in discussing stresses, it is understood that the other stresses, such as shear and adhesion are likewise to be ascertained, and, in fact, it will be seen that these latter stresses may more often control the required dimensions than the bending moment stress.

Face Slab.—The same assumptions as to standard character of loading, of amount of earth thrust etc. will obtain here as have obtained in the former work on the design of the walls. The intensity of earth pressure upon any horizontal strip (see Fig. 46) at a depth x below the top of the wall is

$$Jx(1 + c_x)g \qquad (123)$$

[1] See E. Godfrey, *Trans.* A.S.C.E., Vol. lxx, p. 57, and accompanying discussion.

where J is to be taken at its usual value $\frac{1}{3}$; c_x is the ratio of the surcharge height h' to x and g is the unit weight of the earth. If m is the counterfort spacing, and if the moment is as above defined $WL/12$, then

$$M = x(1 + c_x)gm^2/36 \tag{124}$$

Placing $x = vh$, so that v is the ratio between the distance from the top of the wall to the point in question and the total height of the wall, then $c_x = h'/x = c/v$; where c is the standard ratio between the surcharge height h' and the total height h. The moment may now be placed

$$M = h(c + v)\ gm^2/36 \tag{125}$$

As before (see page 92), the resisting moment of the slab, for a condition of balanced reinforcement may be placed equal to $k_c d^2$. Equating this to the external moment (125), and solving for d

$$d = \frac{m}{6}\sqrt{gh(c + v)/k_c} \tag{126}$$

Ordinarily this depth is less than a certain minimum necessary for good construction and a minimum depth of from 12 to 18 inches is usually specified to make the working conditions favorable for good concrete work (see later sections).

The shear (see equation 123), is found to be

$$V = \frac{1}{2}J \times m(1 + c_x)g = \frac{1}{6}\,mh(c + v)g$$

From (105) the necessary value of d is

$$d = \frac{mgh(c + v)}{6js} \tag{127}$$

Since the beams are comparatively short (the counterforts are generally spaced about 8 to 10 feet apart) it is quite likely that the unit adhesion stress will be high, and may, in fact, control the thickness of the concrete and the spacing of the reinforcement.

The use of the preceding formulas, and the relative value of the several stresses and their effect upon the dimensions of the member are illustrated in some problems at the end of the chapter.

Footing.—The loading upon the base slab is the net difference between the downward weight of the retained fill and the upward soil pressure. (In this work the weight of the slab itself is neglected, since its downward weight is practically reflected in the upward soil pressure intensity caused by this weight.) The load distribution upon the slab is quite problematical, and the net difference as stated above does not exactly give the actual loads. The distribution of soil pressures is of course conditioned upon the deflection of the base slab,[1] so that at those portions, where there is a maximum stiffness of base there will be less pressure (other things being equal). Accordingly for the counterfort walls, the maximum deflection of the base slab will occur midway between the counterforts and toward the heel and the minimum at the counterforts and toward the junction of the arm and footing slabs. These niceties of pressure distribution will not enter into the following treatment of the design of the base slab but they should be borne in mind, and it is permissible to let the true state of affairs color, more or less, the computations involved in the design of this slab. Essentially, however, the following analysis, gives a simple method of design, with probably a stronger section of base than is actually required, but not enough stronger to justify a highly refined analysis. It may again be emphasized, that a little excess section may be sacrificed to simplicity of analysis.

So long as there is not a uniform distribution of soil pressure, the minimum upward pressure occurs at the heel. Since the downward load is, to all intents, uniformly distributed, the maximum net intensity of load occurs at the heel. Again, the maximum soil pressure occurs at the toe, and since its intensity will be larger than the downward intensity of pressure, there will likely be a net difference of pressure upon the base of considerable magnitude and directed in an opposite direction to the net pressure at the heel. This may be brought out algebraically as follows (see Fig. 46):

The unit downward load is $gh(1 + c)$

where the variables have the usual meaning as defined in the preceding pages. The soil pressure intensity, S_x, at a point x from the heel is, from (39) after makng the proper substitutiens,

$$S_x = 2gh(1 + c)(1 - i)\left[3e - 1 + 3(1 - 2e)\frac{x}{w}\right] \quad (128)$$

[1] A discussion of this point is given in Cain "Earth Pressures, Walls and Bins," p. 157.

making the net downward load at the point x, P_x

$$P_x = gh(1 + c)J_x \tag{129}$$

where

$$J_x = 1 - 2(1 - i)\left[3e - 1 + 3(1 - 2e)\frac{x}{w}\right] \tag{130}$$

The maximum net downward pressure, at the heel, P_1, is, with $x = 0$

$$P_1 = gh\,(1 + c)J_1 \tag{131}$$

where

$$J_1 = 1 - 2(1 - i)(3e - 1) \tag{132}$$

and the maximum upward net pressure, at the toe, P_2, with $x = w$

$$P_2 = gh\,(1 + c)\,J_2 \tag{133}$$

with

$$J_2 = 1 - 2(1 - i)(2 - 3e) \tag{134}$$

When the point of application of the resultant falls within the outer third of the base, the soil intensity at the end of the heel is zero and

$$P_1 = gh(1 + c) \tag{135}$$

The above equations determine the loads to be used in designing the longitudinal strips of the base slab and with m the distance between the counterforts, the moment is

$$M = Pm^2/12 \tag{136}$$

where the proper value of P from the preceding equations is to be used. The shear is $P/2$.

Similarly to the design of the face slab, the required depth of the slab, due to the bending moment is

$$d = m\sqrt{J_x gh(1 + c)/12} \tag{137}$$

A theoretical comparison, based upon the bending moment requirements only, may be had between the depths of the base and of the arm slabs. The depth of the face slab is governed by the thickness required at the base of the arm; that of the base slab by the thickness required by the maximum value of J. When the resultant falls at the outer third point, or within the outer third the value of J_x is 1. Denoting the respective required thicknesses of face and base slabs by d_v and d_b respectively, comparing equations (126) and (137), after placing $v = 1$, there is

$$d_v = d_b\sqrt{(1/3J_x)} \tag{138}$$

and with $J_x = 1$, this relation becomes

$$d_v = 0.58d_b \tag{139}$$

This relation, however, is more of academic than practical interest, since it will be found that the thicknesses of these slabs are controlled by factors other than the bending moments. Later on this relation will serve a fairly useful purpose in obtaining relative economy of the several wall types, for which purpose it is of some practical application.

Counterfort.—The counterfort is designed as a simple cantilever beam, with effective depth e as shown in Fig. 46. For the reasons given on the preceding pages no other refinement is desirable in treating this member. To anchor the slabs to the counterfort, rods are placed as shown in Fig. 51 of a section sufficient to hold the stresses induced by the loadings. For the face slab the necessary rod area to hold a strip of face bounded by the two horizontal lines x_1 and x_2 from the top of the wall, with m the distance between the counterforts, and taking the earth pressure coefficient J as $\frac{1}{3}$ is (see Fig. 46).

$$A_s = mg\frac{(x_1 + 2h' + x_2)(x_2 - x_1)}{6f_s} \tag{140}$$

f_s is the permissible unit steel stress, and g the weight of the earth per cubic foot. Using a value of 16,000 pounds per square inch for f_s and 100 pounds per cubic foot for g, this last equation takes the form

$$A_s = \frac{m}{960}(x_1 + 2h' + x_2)(x_2 - x_1) \tag{141}$$

To anchor the base slab to the counterfort it is noticed (see Fig. 46) that beyond the point A the slab and the counterfort are in compression. It is therefore necessary to provide anchorage for the portion of the base between A and B, only. The point A is located as follows: The soil intensity at A is found from (128). At the point A this intensity is equal to the downward intensity $gh(1 + c)$. Forming this equality, and solving for x

$$x = w\frac{1 - 2(1 - i)(3e - 1)}{6(1 - i)(1 - 2e)}$$

or

$$x = Dw \tag{142}$$

To facilitate the computation of D, Table 23 has been prepared covering a range of values of i and e. The total rod area necessary to hold the portion of the slab AB to the counterfort is then that area required to hold the net difference in the upward and downward loadings between these two points. Two conditions exist (see Fig. 47): when the point of application of the resultant force lies within the outer third, and when it lies without the outer third. For the former case, the point of zero soil intensity has been given by equation (41) of Chapter 2, and the net difference in loading is

TABLE 23.—VALUES OF "D"

$e \backslash i$	0	.1	.2	.3	.4	.5
0	.50	.52	.54	.57	.61	.67
.1	.50	.52	.55	.59	.63	.71
.2	.50	.53	.57	.62	.68	.78
.3	.50	.55	.60	.68	.77	.92
.33	.50	.56	.62	.72	.83	1.00
.4	.50	.59	.71	.86	.95	1.00

$$T = mghw\,(1 + c)\,\frac{D + \frac{1}{3}\frac{1 - 3e}{1 - 2e}}{2} \tag{143}$$

which may be written, simply

$$T = mghw(1 + c)E, \tag{144}$$

where E represents the fraction in the above equation.

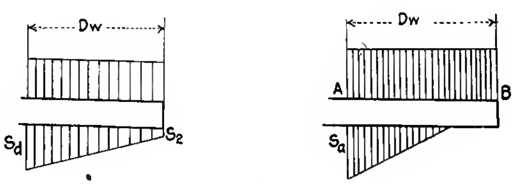


FIG. 47.

Again, when the point of application of the resultant pressure is without the outer third, *i.e.*, when the soil distribution is a trapezoidal one, the value of T may be given by

$$T = mghw(1 + c)E'$$

where

$$E' = D\{1 - (1 - i)\,[2(3e - 1) + 3(1 - 2e)D]\} \tag{145}$$

Table No. 24 gives the values of E for a range of values of i and e.

The application of the above expressions to specific problems is given at the end of the chapter.

The required rod area to hold the load T is

$$A_s = T/f_s \tag{146}$$

where f_s is the permissible unit steel stress.

The preceding analysis, involving as it does a series of mathematical expressions, is not to be taken as interpreting with exactness the stress system in the counterfort wall. The difficulty of attaining such exact statement has been pointed out above. The work as given is to be used as a logical step-by-step process of taking care in as simple a way as possible the stresses that are indicated by a general study of the wall. The equations together with the tables based upon them are readily applied to numerical problems (as given at the end of this chapter) and cover in sufficient detail the necessary work in determining the wall dimensions and the size and distribution of the rod system.

TABLE 24.—VALUES OF "E"

0	.1	.2	.3	.4	.5
.42	.43	.44	.45	.47	.50
.40	.41	.42	.44	.46	.50
.36	.38	.40	.42	.45	.50
.29	.31	.34	.38	.43	.50
.25	.28	.31	.36	.42	.50
.15	.19	.24	.31	.40	.50

Rod System.—The anchorage of the rod system into the wall members forms the vital part of the design of the counterforted wall. While it may seem a simple feat to anchor such rods to the face slab (note that, in what follows, particular stress is laid upon the face slab; the thickness of the base slab is such that ample room is had for anchorage of the tie-rods by simple extension of their length and no further treatment is thus required) by threading their ends and bolting through steel plates or washers or even to assembled steel sections; or by bending around rods at right angles to the anchoring rods, such details involve costly field work, the use of an expensive class of labor and slow up to a considerable extent the progress of the work. Simple details are essential. In a problem, discussed in some detail at the end of the current chapter, a detail is given showing such anchor rods bent into Us of a radius large enough to prevent crushing of the concrete and lying in a vertical plane. Rods of small thickness are usually used because of the greater total surface presented for adhesion.

Problems

1. A wall, of height 25 feet, retains an ordinary railroad fill subject to a surcharge of 600 pounds per square foot. It is placed along the easement line, beyond which no encroachment is permissible. The soil is a sandy loam on which four tons per square foot is allowable (see Table 7). A design as a "L" shaped cantilever, and as a counterforted reinforced concrete wall is desired.

With the above data $c = 6/25$ or 0.24; $i = 0$. From (93), page 83, with $H = 31$ and $i = 0$, the location of the resultant is

$$e = 2/3 - 40/93 = 0.24$$

With this value of e and with $i = 0$, the factor of safety against overturning is 2 (Table 17), a satisfactory one according to Hool, but less than the 2.5 suggested by Cain. See page 57. Adopting this value of e, from Table 18 the required value of k is 0.57 and accordingly the base will be made 14 ft. wide.

From Table 21 the shear and the bending moment require about the same depth. Using the shear equation (113)

$$d = 0.0033 \times 25^2 \times 1.48 = 3.05$$

and the thickness of the base will be taken as three feet. At a point halfway up the wall for which $c = 6/12.5 = 0.48$, the moment determines the depth at this point, as can be seen from Table 21, and from (112)

$$d = 0.0185 \times 44.2 \times \sqrt{(1 + 1.44)} = 1.28$$

For the sake of simplicity of forms, bracing and rods, the wall will be given an unbroken batter from the coping to the base, with the top width a minimum practical width of one foot.

At the midpoint just investigated, the thickness will then be two feet, in place of the required 1.28 feet. In the final design of the wall, the rod section will be diminished to allow for the decreased moment.

Footing.—From (111,118) the required depth will be $\sqrt{I}$ times the depth necessary for the arm (since the arm depth here is that practically demanded by the moment). From Table 22, since $i = 0$, $I = 1$, and the depth will be the same as that required of the arm at its base, namely 3′ 0″ This thickness will be maintained to the end of the heel.

Enough data has now been gathered to prepare an exact and final design. From table 3, for $c = 0.27$, $B = 0.40$, whence $Bh = 0.40 \times 22 = 8.8$. Note here that the exact length of the arm is now considered, proper allowance having been made for the thickness of the footing. The batter of the back is two feet in twenty-two feet, or $b = \tan^{-1} (1/11) = 5° 40' = 6°$. From Table 1, $J = 0.345$ and $\theta = 9°$. The value of the thrust T is, from (14), 16 kips, and is inclined at an angle of 15° $(\theta + b)$ to the horizontal. The weight G of the superimposed earth on the footing is $22 + 6 = 28 \times 11 \times 0.1 = 30.8$ kips. The weights of the footing and of the rectangular and triangular portions of the arm are respectively, 6.3, 2.2 and 2.2 kips (see Fig. 48). Graphically, the resultant is found to intersect the base 3.5 feet from the toe or exactly 1/4 of the distance from the toe, checking the first assumption. The horizontal and vertical components of the resultant found graphically are respectively 15.8 and 46.5 kips. With the latter value and using equation (39) $S_1 = \frac{93}{14}(2 - 0.75) = 8.3$ kips, a permissible variation from the 4 tons or 8 kips assumed.

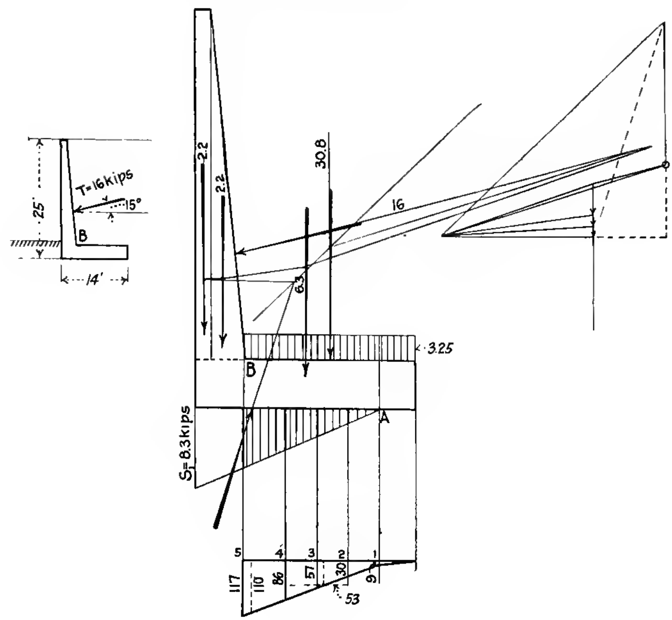


FIG. 48.

Resistance to Sliding.—The coefficient of friction between sandy loam and concrete is about 0.5 (an average between sand and gravel), see Table 6. The sliding resistance is then $0.5 \times 46.5 = 23.2$ kips. The horizontal component is 15.8 kips, giving a factor of safety against sliding of $23.2/15.8 = 1.5$ which is ample.

Design of the Vertical Arm.—The actual loading on the arm extends to the top of the footing and for the arm, h is 22′ and $c = 0.27$. From (110) with $J = 1/3$, $M = 106.5$ kip ft. and the shear is 12.4 kips. Taking, as before, the steel ratio for balanced reinforcement, or $p = 0.75$ per cent., $0.5kj = 0.16$ and $k_c = 105$. With $b = 12''$, the required depth d in inches is

$$d^2 = \frac{106{,}500 \times 12}{12 \times 105}$$

whence $d = 32''$. From (113) the depth required on account of shear is

$$d = \frac{12{,}400}{0.89 \times 12 \times 40} = 28''$$

(From Table 20 with $c = 0.27$, the shearing stress governs, when h is greater than 27′.)

The steel area required at B is $0.0075 \times 32 \times 12 = 2.88$ square inches. This is a rather heavy reinforcement, necessitating great expense in handling and placing bars. If a thicker wall is assumed, *e.g.*, $d = 40''$, then, from the properties of the section $k_s = \frac{106{,}500 \times 12}{12 \times 40^2} = 66$. $pj = 66/16{,}000 = 0.004$ and the required percentage of steel is 0.4 per cent. The steel area is, then, $0.004 \times 40 \times 12 = 1.92$ square inches and 1 inch square bars on 6″ centers will give the necessary area. The unit adhesion is

$$\frac{12{,}400}{.89 \times 40 \times 8} = 43$$

The permissible stress is 80 pounds per square inch. To determine at what point it is possible to stop one half of these rods, *i.e.* to space the rods 12″ apart, note that the external moment is given by the expression (110) or

$$M = 66.7(1 + 3c)x^3$$

Since the coping width is taken as 12″, the effective thickness at any point of the wall x is

$$d = 9'' + \frac{x}{22}(40 - 9) = 9'' + 1.5x.$$

The resisting moment is given by $M = k_s bd^2$, where $k_s = f_s pj$. For small values of p, pj may be taken equal to p, and with $f_s = 16{,}000$ pounds per square inch, and noting that since the area of steel is to be one square inch, $p = 1/bd$, the resisting moment becomes

$$M = 16{,}000d = 16{,}000\ (9 + 1.5x)$$

Equating the resisting moment to the external bending moment and replacing c by its value $6/x$, there results a cubic in x

$$x^3 + 18x^2 - 360x - 2160 = 0$$

which is satisfied by $x = 15$. Accordingly, at a point 15′ below the top of the arm the rods will be spaced 12″ apart. Since a further reduction in the spacing would place the rods 24″ apart, which is not good practice, the 12″ spacing will be continued to the top of the arm.

Footing.—To analyze the footing stresses, a moment diagram has been drawn in Fig. 48. Note that the moment at B is very nearly equal to the arm moment at B, affording another check upon the approximate method. With $M_b = 110$ foot kips, and for balanced reinforcement, the required depth is 34″. The necessary amount of steel is $0.0075 \times 34 \times 12 = 2.73$. This again demands too heavy a reinforcement for efficient handling, and a thicker concrete will be assumed. With $d = 40$, $k_s = \frac{110{,}000 \times 12}{12 \times 40^2} = 69$, and $pj = 69/16{,}000 = 0.0043$. The steel area is then 2 square inches and one inch bars spaced 6″ apart will be used. To determine, again, at what point it will be possible to reduce the steel section to one-inch bars at 12″ spacing, the resisting moment of such a steel section, since the thickness of the base is kept constant, is found to be, with $p = 1/(12 \times 40) = 0.0021$.

$M = 0.0021 \times 16{,}000 \times 12 \times 40^2 = 635$ inch kips or 53 foot kips. Plotting this value upon the moment diagram of the footing, it is found that at a point 6 feet from the heel it is possible to reduce the rod section to one inch bars 12″ apart. For the reasons outlined above, there will be no further increase in this spacing.

To develop the adhesion in the vertical and horizontal rods, which must be carried out 50 thicknesses or 4′ beyond the point of maximum moment, it is necessary to place a 6″ projection at the toe and into the footing as shown in Fig. 50.

The spacing of the secondary rod system for shrinkage, settlement and temperature will be discussed in a later chapter.

Counterforted Wall.—Adopting the economical[1] spacing of ten feet for the counterforts; from (126), with $v = 1$, $k_c = 16{,}000$, $h = 25$,

$$d = 0.7 \text{ feet.}$$

It is impractical to pour concrete in a wall this thick for the height as given and a minimum thickness of 12″ will be adopted.

From (139) the required thickness of the footing slab is $\sqrt{3}$ times that required of the vertical slab. It will be seen later that this thickness will be controlled by a thickness necessary to get a practical spacing of rods for adhesion. The dimensions of the separate members as now found are less than those of the cantilevered wall, and since that wall as finally designed agreed with the approximate dimensions it is clear that the counterforted wall, will likewise agree and it will not be necessary to recheck the outline dimensions of the section.

In selecting rod systems, both spacings and sizes, and wall thicknesses, it must be borne in mind that there must be sufficient working space to pour the concrete; that small sizes of rods are relatively more expensive than the larger sizes; that many variations in both length and spacing tend to cause confusion in construction. This limitation of the economical section on paper by field conditions, is discussed more in detail in the following chapter.

The moment at the base of the vertical slab (here $h = 23.5$ feet), with $c = .265$, and $P = \dfrac{100 \times 29.5}{3} = 0.98$ kips, is from (125) 8.3 kip feet. As before the depth for this moment, with balanced reinforcement is 0.73 feet, but, for reasons, outlined above the thickness will be taken as 12″. With a wall of this thickness the utmost care must be exercised in pouring concrete into it. See Chapter VIII for the precautions to be used to insure a well mixed and rammed concrete.

With a depth to steel of 10″, $k_c = 8300 \times 12/12 \times 100 = 83$ and $p = 0.004$, which gives a required area of 0.48 square inches, which ¾″ rods on 12″ centers satisfies.

The total shear is $980 \times 5 = 4900$ pounds, and the unit shear from (105) is

$$\frac{4900}{0.89 \times 10 \times 12} = 45 \text{ pounds per square inch}$$

which is so slightly in excess of the permissible stress of 40 pounds, that the section will be maintained as assumed. The area required for adhesion is from (106)

$$r = \frac{4900}{0.89 \times 10 \times 80} = 6.9 \text{ square inches}$$

[1] See problem, Chapter IV, p. 150.

The adhesion stress thus governs the spacing of the rods and ¾″ rods spaced 5″ apart will give the required periphery of section.

At $h = 15$, the moment is 5.8 kip feet and the shear is $700 \times 5 = 3.5$ kips. The area required for the bending moment is accordingly 0.18 square inches, while that required for adhesion is found to be 5 inches.

At $h = 10$ feet, the periphery required for adhesion is 3.8 and at $h = 5$ feet, the required periphery for adhesion is 2.6 inches.

It is seen that the adhesion stress will determine the spacing of the rods throughout the arm. At $h = 15$, since 5 inches are required for adhesion the spacing at the base will be maintained beyond this point. At $h = 10$ feet, since $r = 3.8$ the rods may be spaced on 10″ centers. At $h = 5'$ the value of r required will not permit a further reduction in the spacing of the rods. There will thus be ¾″ square rods spaced on 5″ centers from the base to $h = 10$ feet and ten inch spacing from there to the top of the wall. To take care of the equal but negative moment at the counterfort, with the corresponding adhesion stresses, the same spacing will be maintained on the inner face of the vertical slab. Since the rods must be carried beyond the point of zero moment (approximately the quarter point) the rods on the inner face will be made five feet long centered at the counterforts.

Footing.—The net weight on the footing excluding the excess weight of the masonry over the earth, is 3100 pounds. As before a depth to satisfy the bending moment, is from (138) 18″. For adhesion

$$r = \frac{3100 \times 5}{.88 \times 16 \times 80} = 13.7$$

which cannot be readily and practically provided. Conversely since it is desirable to use a rod not exceeding the section of ¾″ rod whose minimum spacing is 5″ on center, d is found

$$d = \frac{3100 \times 5}{0.88 \times 7.2 \times 80} = 30''$$

and the total depth of the footing slab is thus $30 + 2'' = 32''$.

The point where the upward and downward intensities balance each other is, from (142) and Table No. 23 with $i = 0$ and $e = \frac{1}{4}$, at the midpoint or seven feet from the end of the heel. To avoid many changes in the spacing of the rods, the ¾-inch square rods will be spaced on 5-inch centers to a point 3.5 feet from the heel and thence, to the midpoint on 10 inch spacing.

For the portion between the midpoint and the vertical arm it is reasonable to assume that the slab is supported on three edges—the counterfort edges and the vertical arm—and that such support is uniformly distributed along such edges.[1] From (134) $J_2 = 1 - 2(2 - \frac{3}{4}) = -1.5$. $P_2 = 1.5 \times 100 \times 28 = 4200$ lb. The total net load between the counterforts reacting upward upon the slab is then, since the intensity is zero at the midpoint, $4.2 \times 7 \times 9/2 = 132$ kips. The total length of supporting edge is $2 \times 7 +$

[1] For an interesting discussion of this modification of plate theory it may be well to consult Prof. Eddy's brilliant little book on the "Theory of Rectangular Plates."

9 = 23 feet and the shear per linear foot is 5.7 kips. For a 30″ slab the unit shear is then $\frac{5700}{.89 \times 12 \times 30}$ = 18 lb. considerably below the allowable and the periphery of rod required for adhesion is $\frac{5700}{.89 \times 80 \times 30}$ = 2.7 square inches. It is then sufficient to carry the ¾″ rods on 10″ spacing to the toe of the base. The rod spacing will be duplicated on the opposite face to take care of the negative moment and reversed stresses. Thus from the midpoint out to the heel the rods on the lower face will be carried full length and those on the upper face five feet beyond the counterfort. From the midpoint to the heel the rods on the lower face will be carried for the full length and those on the upper face will be extended five feet on either side of the counterfort.

Counterfort.—Designed as a cantilever beam, the moment at the base is then TBh, with the thrust taken for a length m of the wall. $T = 10 \times 13 = 130$ kips. With $c = 6/22 = 0.27$; $B = 0.39$ and $h = 22$,

$$M = 130 \times 0.39 \times 22 = 1{,}140 \text{ kip feet}$$

The depth e of the cantilever is 14′. Assume, tentatively, its thickness as 1.0 feet.

$k_c = 41$, making $p = 0.001$ and the required area of steel in square inches is 2 square inches. Therefore two inch square bars are ample to take care of the moment in this counterfort. Investigating the unit adhesion, it is found that, with a value of $V = 130{,}000$ pounds, the unit adhesion is 110 pounds. If two 1¼″ bars are used, the unit adhesion is found to be 85 pounds per square inch, a permissible variation from the allowable 80 pounds. To anchor these rods into the base it is necessary to carry them fifty thicknesses or about five feet into the foundation. For this reason an extension will be built into the foundation two feet below the slab and carried six inches on either side of the counterfort. The radius to which the rods must be bent in going into the base slab is 30 × 1¼ = 3′ 0″.

To anchor the face slab to the counterfort, since the thickness of the face slab does not permit a straight extension of the rods into it, it will be necessary to adopt the expedient of bending the rods into a U, with the radius of the curve $30t$.

From (141) for the top five feet of the wall

$$A_s = \frac{10}{960}(5 + 12) \times 5 = 0.89$$

Therefore two ½″ Us give sufficient bond for this length. The bars must be bent to a radius of 15″. For the next five feet the required amount of steel is 1.4 and two ⅝″ rods bent to a U with radius of 18″ provide the requisite bond. For the five feet below this section $A_s = 1.9$, and three ⅝″ Us as shown in Fig. 51 satisfy the requirements of this portion. The remaining space from 15′ to 22′ is divided into two parts, the area of the first part is found to be 1.6, of the lower part 1.9. Therefore three ⅝″ Us as previously detailed will provide the remaining bond rods.

To get the necessary rod area to anchor the heel portion of footing to the counterfort (the portion from the midpoint to the heel) from (144) and

Table 24 with $E = 0.33$ the total load to be held by these rods is $10 \times 100 \times 28 \times 0.33 \times 14 = 129$ kips. The steel area is then, from (146) $129/16 = 8.1$ square inches. Using ½ square rods, one on either side of the counterfort 32 are required. With 6″ spacing 15 spaces will carry the rods beyond the midpoint. The depth of the footing is ample to develop these rods in adhesion without any special detail and they will be carried to two inches from the bottom, of the footing. Theoretically they need be carried into the counterfort the same distance, but it seems better practice to carry the rods for the full height of the portion of the counterfort affected (see Fig. 51).

2. Modify the preceding problem to carry a railroad track system with wall track 8 feet away from the face of the wall and the other tracks on 12.5 foot spacing. Assume that all tracks but the wall track are loaded; then assume no tracks loaded. In what way is the pressure upon the footing affected, and do any of the stresses exceed those for the case of all tracks loaded (the former case)?

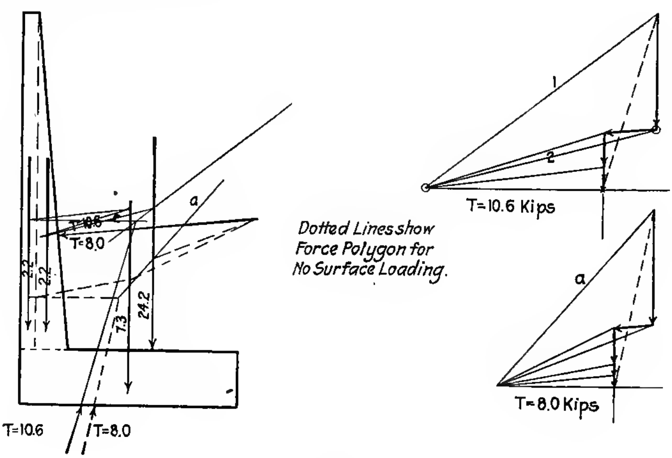


FIG. 49.

For this case, see Fig. 10, the surcharge extends to 14 feet from the wall face. As above $b = 6°$ and from (32) of Chapter I

$$T = gh^2\frac{(1+2c)K}{2}$$

The proper value of a to use in determining the coefficient K is from (34) with $y = 14/22 = 0.64$

$$\tan a = \tan 6° - \frac{0.54}{1.54}0.64 = -0.119$$

whence

$$a = -7°$$

From table No. 13, allowing no friction upon the back of the wall, $K = 0.285$ and the thrust is then 10.6 kips. Fig. 49 shows the force system on the wall for this case.

For the second condition, no loading upon the surface, the thrust becomes, with $K = 0.33$ and $c = 0$, $T = 8$ kips. Fig. 49 shows the force system for this case.

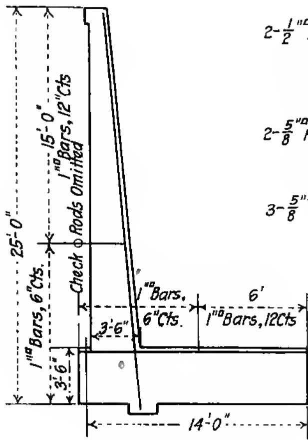


Fig. 50.—Cantilever wall.

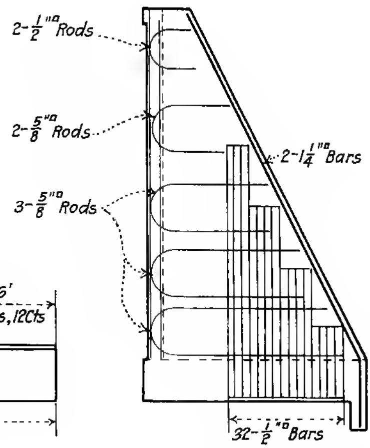


Fig. 51.—Counterfort wall.

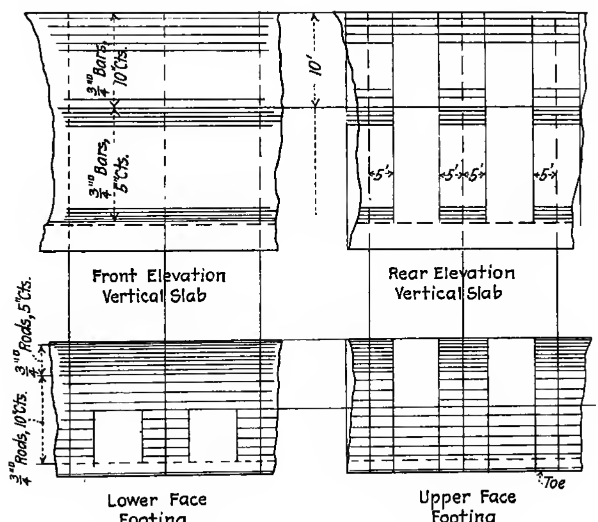


Fig. 52.—Rod layout counterfort wall.

From Fig. 49, e for the former condition is 0.28 and $R = 37.5$. For the second condition $e = 0.35$ and $R = 37$. From (38) S_1 for the former is 6750 pounds per square foot and for the latter is 5000 pounds per square foot. It is obvious that the analysis of the first problem will require no modification of stress distribution because of these latter conditions.

Fig. 50 gives the detailed layout of the "L" shaped cantilever. Fig. 51 gives the rod layout of the counterfort and Fig. 52 of the vertical and base slabs. In neither of the sketches are the temperature and check rods shown. A later chapter will indicate such distributions.

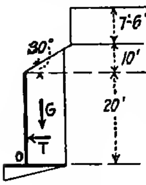


FIG. 53.

3. A "T" shaped cantilever wall is to be built, retaining an embankment as shown in Fig. 53. The embankment is subject to a surcharge live load of 750 pounds per square foot. The foundation pressure must not exceed 5000 pounds per square foot. Determine the proper wall dimensions and details.

For the condition of no surcharge, both the exact and the approximate expressions for the thrust, as given on page 14 may be employed. Exactly, with the angle $i = 30°$, $b = \phi' = 0$ and $\phi = 30°$, $L = 1/\cos^2\phi = 4/3$; $u = \sin\phi$; $v = -\cos\phi$; $d = \cot\phi$; $m = 1$; $n = -\cot^2\phi = -3$; $c = 0.5$; $p = \sin\phi = \frac{1}{2}$ and $f = -3$. The expression for the thrust is then

$$T = \frac{gh^2}{2} \times \frac{4}{3} \times \left[1.5 - 0.5\sqrt{1.5^2 + 0.25 \times 3}\right]^2$$
$$= 10.7$$

The approximate method, which since $c = 0.5$, is not to be used (see page 15), gives a value

$$T = \frac{gh^2}{6}(1 + 2c) = 13.3.$$

A variation from the true value too excessive to permit of its use.

For the condition of a live load surcharge, in place of the graphical method of obtaining the thrust, the compromise, algebraic geometric method outlined in the problem at the end of Chapter 1, may be used. The value of i is determined graphically, the line forming the equivalent triangles as shown in Fig. 54. With aoe making an angle of 35°, the triangles afo and obe are equivalent. With this value the thrust may be determined as above. From Eq. 22 $L = 1/\cos^2\phi$; $u = \sin\phi$; $v = -\cos\phi$; $n = -\cot 35° \cot\phi = -2.43$; $p = \sin\phi = 0.5$; $m = 1$ and $f = -2.43$.

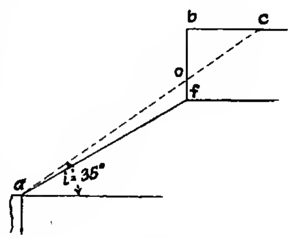


FIG. 54.

$$T = \frac{100 \times 400}{2} \times \frac{4}{3}\left[1.875 - \frac{1}{2}\sqrt{1.875^2 + 0.761 \times 2.43}\right]^2$$
$$= 13.6$$

Refer to Figs. 42 and 53 assuming, as the condition of economy, that $i = e$. In addition, assume that the resultant intersects the base at the outer third point, *i.e.* $i = \frac{1}{3}$. Noting that $g = 100$; $h = 20$ and $\tan 35° = 0.7$ the weight G has the value

$$G = g(1 - i)wh + \frac{g(1 - i)^2 w^2 \tan 35°}{2}$$
$$= 0.67\, w(2 + 0.023w) \qquad (A)$$

Taking moments about 0, and noting that without serious error the point of application of the weight may be taken at the middle of the base

$$G(1 - i)w/2 = Th/3.$$

Introducing the values above, this equation becomes

$$G = \frac{181.5}{(1 - i)w}$$

and with $i = 1/3$

$$G = 273/w \qquad (B)$$

Equating (A) and (B), there results a cubic in w

$$410 = 2w^2 + 0.023w^3$$

which is satisfied by the root, $w = 13.5$. With this value of w, $G = 20.2$ and from (39)

$$S_1 = 2G/w = 40.4/13.5 = 3 \text{ kips.}$$

The projection of the toe beyond the face of the wall is 4′ 6″. Assume tentatively that the thickness of the base and of the vertical ar 1 at its base is two feet. The thrust, for the purposes at hand may be assumed to vary as the square of h. Since the effective height of the wall, so far as the arm is concerned is 18 feet,

$$T = \frac{18^2}{20^2} \times 13.6 = 11.$$

and its point of application is one-third of h or 6 feet above the top of the footing. The bending moment is then $11 \times 6 = 66$ and with $k = 16{,}000$ for balanced reinforcement, the required depth on account of moment is

$$d = \sqrt{(66/16)} = 2.03$$

The shear is 11,000 pounds and the depth to satisfy this amount is

$$d = 11{,}000/5040 = 2.18$$

The thickness of the vertical arm at its base may be taken as 2′ 6″. The back will be battered to a top thickness of one foot.

Footing.—The face of the vertical arm is, on the assumptions previously made at the third point or 4′ 6″ from the end of the toe. The moment of the heel cantilever is then taken at a point 4′ 6″ + 2′ 6″ from the toe or 6′ 6″ from the end of the heel. At this point, since S_1 is 3000 pounds, the soil intensity is $\frac{6.5}{13.5} \times 3 = 1.44.$

Taking the approximate value of G as 20.2 and again assuming that it is directed over the center of the heel cantilever, the bending moment becomes

$$20.2 \times 3.25 - \frac{1.44 \times 6.5}{2} \times 2.2 = 55.4$$

The shear is

$$20.2 - 1.44 \times 6.5/2 = 15.5.$$

Evidently the shear will control the depth required and

$$d = 15{,}500/5040 = 3.08$$

Whence take 3′ as the required thickness of base.

It is now possible to proceed with the exact design. (See Fig. 55.) The thrust is found from equation (22), with $c = 17.5/17 = 1.03$ and

$$T = \frac{100 \times 17^2}{2} \times \frac{.4}{3}\left[2.03 - \frac{1}{2}\sqrt{2.03^2 + 1.06 \times 3}\right]^2 = 8.9$$

This will be applied at a point 17/3 or 5.65 feet above the top of footing. The weights of the earth has been divided up into the triangles $abc = G_1$; $ade = G_3$ and the rectangle $dcfe = G_2$. The weight of the masonry has been divided into the triangle G_5 and the rectangles G_4 and G_6. The weights are:

$$G_1 = 9 \times 4.75 \times 100/2 = 2.14 \text{ kips.}$$
$$G_2 = 6 \times 17 \times 100 = 10.2 \text{ kips.}$$

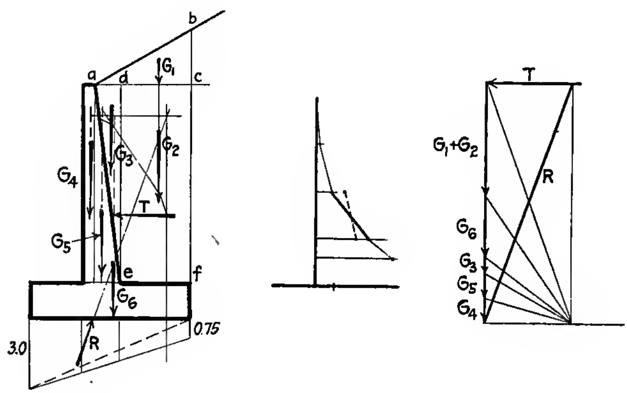


Fig. 55.

Note that the two above act in practically the same vertical line, so that the two may be added and treated as one force

$$G_1 + G_2 = 12.3$$
$$G_3 = 2 \times 17 \times 100/2 = 1.7$$
$$G_4 = 1 \times 17 \times 150 = 2.55$$
$$G_5 = 1 \times 17 \times 150 = 2.55$$
$$G_6 = 3 \times 13.5 \times 150 = 6.07$$

With the forces as above found the polygon is drawn in the usual manner, see Fig. 55, and the location and amount of the resultant pressure is found. The actual value of k is $5.5/13.5 = 0.4$ and $R = 25.5$.

$$S_1 = \frac{51}{13.5}(2 - 0.12) = 3.00 \text{ and } S_2 = \frac{51}{13.5}(1.2 - 1.0) = 0.75$$

Vertical Arm.—The moment of the thrust is $8.9 \times 5.65 = 50.4$ and the depth to satisfy this moment is

$$d = \sqrt{(50.4/16)} = 1.78$$

The shear is 8900 and the corresponding depth required is

$$d = 8900/5040 = 1.77$$

The required depths are thus identical and the total thickness of slab at the base of the arm will be 2′ 0″, allowing 3″ for a protective concrete coat. Since, for balanced reinforcement the steel ratio is 0.0075, the amount steel required is

$$A_s = 0.0075 \times 21 \times 12 = 1.89.$$

Spacing 1 inch square bars (deformed) 6″ apart will furnish the necessary section. Assuming that there is a triangular distribution of pressure,[1] the moment diagram is shown in Fig. 55. To obtain the thrusts for the moment, note that at the points 15′, 10′ and 5′ from the top of the wall the corresponding values of the surcharge ratio are 1.17; 1.75 and 3.5. The values of the thrust are then

$$T_{15} = \frac{100 \times 15^2}{2} \times \frac{4}{3}\left[2.17 - \frac{1}{2}\sqrt{2.17^2 + 3 \times 1.17^2}\right]^2 = 6.9$$
$$T_{10} = \frac{100 \times 10^2}{2} \times \frac{4}{3}\left[2.75 - \frac{1}{2}\sqrt{2.75^2 + 3 \times 1.75^2}\right]^2 = 3.3$$
$$T_5 = \frac{100 \times 5^2}{2} \times \frac{4}{3}\left[4.5 - \frac{1}{2}\sqrt{4.5^2 + 3 \times 3.5^2}\right]^2 = 0.9$$

The moments are, assuming again that the thrusts are ⅓ of the distance above the point in question,

$$M_{15} = 6.9 \times 5 = 34.5$$
$$M_{10} = 3.3 \times 3.3 = 10.9$$
$$M_5 = 0.9 \times 1.67 = 1.5$$

At some intermediate point along this arm, it will be found that one half of the rods are sufficient to carry the stress; *i.e.*, the rods from this point on may be carried on 12 inch spacing. As before the width of the wall at the coping will be taken as 12 inches. With a spacing of 12 inches for the one-inch rods at

$$h = 15; \quad d = 19'' \quad \text{and} \quad p = 1/(19 \times 12) = 0.0044$$
$$h = 10' \quad d = 16'' \quad p = 1/(16 \times 12) = 0.0052$$

The corresponding values of pj are 0.0042 and 0.0047, and the resisting moments are then, expressed in foot-pound units,

$$M_{15} = 144 \times 0.0042 \times 16{,}000 \times 1.59^2 = 24.3$$
$$M_{10} = 144 \times 0.0047 \times 16{,}000 \times 1.33^2 = 19.2$$

Plotting these two values on the moment diagram, Fig. 55, it is seen that the resisting moment of one-inch rods on twelve-inch centers, is equal to the external bending moment at a point approximately 4.5 feet above the footing. The six-inch spacing will then be stopped at a point 5′ above the top of the base slab. As previously explained, this spacing will be continued to the top of the arm.

[1] While this is, strictly speaking, incorrect, since the thrust is not a linear function of h, which condition is the necessary one that there be a triangular distribution of pressure, the ease of handling the problem with that assumption counterbalances the slightly excessive pressures thus found.

Footing.—The force acting upon the base slab over the heel is $G_1 + G_2$ or 12.3 kips. The weight of the base slab (maintaining the thickness first found) is $6.5 \times 3 \times 150 = 2.9$ kips. The total downward load upon the heel is 15.2 kips. The upward soil pressure is $\frac{1.3 \times 6.5}{2} = 4.22$ kips. The moment for the heel is thus

$$15.2 \times 3.25 - 4.22 \times 4.33 = 31.2 \text{ kip feet}$$

The shear is $15.2 - 4.2 = 11$ kips. The required depth, for shear is 2.18, which clearly, is greater than that required for the moment. With a protective concrete over the rods the thickness of the heel slab will be taken as 30″. With the net depth (effective) of 27″, $k_s = 31.2/2.25^2 = 42.5$ and $pj = 42.5/16{,}000 = 0.003$. The steel ratio is then 0.003 and the necessary section of rods becomes $0.003 \times 27 \times 12 = 0.97$ square inches. One-inch rods spaced twelve inches apart will provide the requisite steel area and this spacing will be carried out to the end of the heel.

Toe.—At the toe the cantilever moment is

$$M = \frac{(3 + 2.3)4.5^2}{2 \times 2} = 26.8 \text{ kip feet}$$

and the shear is

$$(3 + 2.3)4.5/2 = 11.9 \text{ kips}$$

As before the shear requirement will control the depth of the section

$$d = 11.9/5040 = 2.33$$

The same thickness of both heel and toe will be used, which in view of the usual manner of pouring the wall is practically mandatory.

$$k_s = \frac{26{,}800 \times 12}{12 \times 27^2} = 37$$

and $pj = 37/16{,}000 = 0.0023$. The steel ratio is then 0.0023 and the area required is $0.0023 \times 27 \times 12 = 0.83$. 1-inch bars spaced twelve inches apart will provide the steel reinforcement.

Since a 1″ bar requires four feet to develop its tension by adhesion, the heel rods will be carried four feet beyond the rear face of the vertical arm and the toe rods four feet beyond the front face of the vertical arm. For the reinforcement of the vertical arm, an extension 1′ 0″ wide and 1′ 6″ deep will be built into the foundation to provide the required length.

Fig. 56 shows the complete section of wall. The rods necessary for shrinkage and temperature stresses have not been shown.

4. In the wall of problem 1, it will be necessary, for a given stretch to provide a foot-walk as shown in Fig. 57. Without changing the outlines or the design of the wall proper, design the bracket to carry this walk, subject to a live load of 100 pounds per square foot.

Assuming that the concrete bracket will be 6″ thick, the dead load will be 75 pounds per square foot, making the entire load upon the bracket 175 pounds per square foot. For balanced reinforcement

$$d = \sqrt{(790/16{,}000)} = 0.22, \text{ or } 3'' \text{ thick.}$$

With 2″ protective concrete over the rods the total thickness of slab is 5″.

The required steel area is $0.0075 \times 3 \times 12 = 0.27$, and ½ inch square rods, 12″ apart will provide the required steel section. The unit shear is

$$s = \frac{525}{.89 \times 3 \times 12} = 16 \text{ pounds}$$

The unit adhesion, with $r = 2$, is

$$q = \frac{525}{.89 \times 3 \times 2} = 98 \text{ pounds per square inch.}$$

This latter value is excessive and the depth of section must be increased at this point. If at the cantilever junction between the wall and bracket a fillet is placed as shown in Fig. 57, the unit adhesion at the point D is ⅔ of that above found or 70 pounds per square inch. To provide the necessary bond the ½″ rods will be bent as shown and carried into the vertical arm.

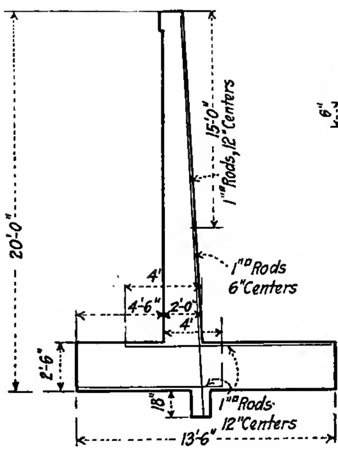


Fig. 56.

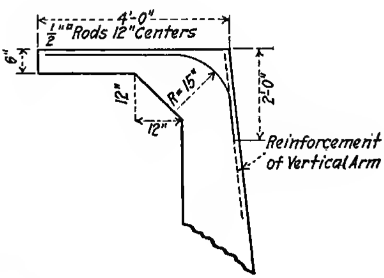


Fig. 57.

5. A counterforted wall, resting upon a rock bottom, is to take a surcharge of 500 pounds per square foot. The easement does not permit a toe extension. Determine the general wall outlines from the approximate formulas given and design a counterfort made up of a steel truss.

With $i = 0$, and the foundation rock e may be taken equal to ¼, giving a value of k from Table 18 of 0.51. The width of base is thus $0.51 \times 50 = 25' \, 6''$. From Table 17 the factor of safety is found to be two. Assume that the counterforts will be spaced ten feet apart. The pressure at the base of the vertical slab is $Jgh(1 + c) = 0.33 \times 50 \times 1.1 \times 0.1 = 1.83$ kips per square foot. From (126)

$$d = 10\sqrt{\frac{1}{3}\,\frac{100 \times 50 \times 1.1}{12 \times 16{,}000}} = 1.0$$

The depth for shear is

$$d = \frac{1.83 \times 5}{5.04} = 1.83$$

It will be found, later that the thickness of the face slab at the base will be controlled by the necessary dimensions of the member composing the vertical arm of the truss. The thickness of the base slab is controlled by the depth necessary for the adhesion stresses. If 1″ square bars, spaced 6″ apart are to be used, then the depth necessary to satisfy the limiting adhesion stress of 80 pounds per square inch is

$$d = \frac{5500 \times 5}{80 \times 0.89 \times 8} = 49''.$$

To avoid the use of so heavy a slab throughout the base, a fillet of concrete will be placed at the junction of the base and counterfort, dimensioned as shown in Fig. 58. The main body of the slab will then be taken as 2′ 9″ thick.

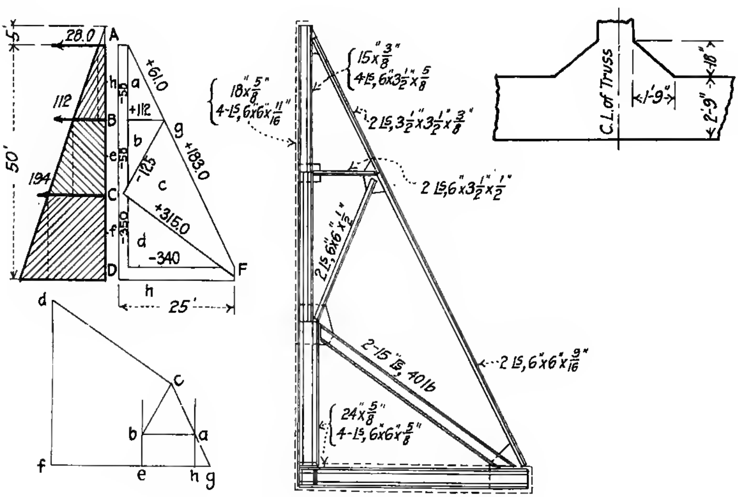


FIG. 58.—Counterfort wall.

The design of the counterfort proper (note that a final check of the dimen sions just found is omitted—in actual practice such omission is poor design) is most conveniently made by graphical methods. The skeleton outline of the truss is shown in Fig. 58. The loads at the panel points A, B, C are, allowing for the ten foot spacing of counterforts:

$$P_a = \frac{1.83 \times 16}{2} + \frac{5 \times 16}{6} = 28$$

$$P_b = \frac{7 \times 16}{2} + \frac{5.5 \times 16}{6} + \frac{1.83 \times 16}{2} + \frac{2 \times 5 \times 16}{6} = 112$$

$$P_c = \frac{12.5 \times 15}{2} + \frac{5.5 \times 15}{6} + \frac{7 \times 16}{2} + \frac{2 \times 5.5 \times 16}{6} = 194.$$

The stress polygon is drawn as shown and the stresses are denoted plus or minus as they are, respectively tension or compression. The vertical members of the face and the horizontal member of the base, must carry the moment induced by the slab reactions. These moments are

$$M_{ab} = \frac{4.3 \times 16^2}{8} = 138 \text{ ft. kips}$$

$$M_{bc} = \frac{10 \times 16^2}{8} = 320 \text{ ft. kips}$$

$$M_{cd} = \frac{15 \times 15^2}{8} = 424 \text{ ft. kips}$$

The unit stress in tension will be assumed to be 16,000 pounds per square inch. That in compression, long column, 12,000 pounds per square inch. The vertical arm and the base arm are buried in concrete. It is the practice, for members thus stressed, to let the concrete take the load from the steel member by adhesion so that the member carries only the bending load. Such practice will be adopted here.

Where deductions from gross section are necessary because of rivet holes, $^{13}\!/_{16}$ inch open holes will be assumed. The actual work of the design is not shown here.

ag.	$S = 61.$	$A = {}^{61}\!/_{16} = 3.8$	2 *Ls* $3.5 \times 3.5 \times {}^{3}\!/_{8}$
ab.	$S = 112.$	$A = {}^{112}\!/_{16} = 7.$	2 *Ls* $6 \times 3.5 \times {}^{1}\!/_{2}$
bc	$S = -125.$	$A = {}^{125}\!/_{12} = 10.4$	2 *Ls* $6 \times 6 \times {}^{7}\!/_{16}$
cg	$S = 183$	$A = {}^{183}\!/_{16} = 11.4$	2 *Ls* $6 \times 6 \times {}^{9}\!/_{16}$
cd	$S = 315$	$A = {}^{315}\!/_{16} = 19.7$	2 Channels 15″ 40#

Since the member *AB* is subject to bending only,

AB, $M = 138$ Sect. Modulus $138 \times {}^{12}\!/_{16} = 103$
Web plate $15 \times {}^{3}\!/_{8}$; 4 *Ls* $6 \times 3.5 \times {}^{5}\!/_{8}$
EB $M = 320.$ *S. M.* $= 320 \times {}^{12}\!/_{16} = 240$
Web plate $18 \times {}^{5}\!/_{8}$; 4 *Ls* $6 \times 6 \times {}^{11}\!/_{16}$
FD $M = 424.$ *S. M.* $= 424 \times {}^{12}\!/_{16} = 318$
Web plate $24 \times {}^{5}\!/_{8}$; 4 *Ls* $6 \times 6 \times {}^{5}\!/_{8}$.

The details are not given of the connections, etc.

It will be assumed that the truss work is either encased, member by member in concrete, or is coated with gunite, or other preparation of similar nature.

6. A counterforted wall, 24 feet high, subject to a surcharge of 6 feet, is to rest upon a soil capable of holding not more than 5000 pounds per square foot. Determine the general wall outlines and design the toe extension.

From (95), with $S_1 = 2.5$ tons and $H = 30$ feet, and $i = e$

$$e = \frac{5}{6} - \frac{1}{6}\sqrt{1 + \frac{300}{30}} = 0.28$$

From Table 17, for $e = 0.25$, $k = 0.56$, and the width of the base is $0.56 \times 24 = 13' \, 6''$. The toe projection is $0.28 \times 13.5 = 3.8$ or $4' \, 0''$. Without attempting to design the separate sections of the wall and then redetermining these general outlines from the more exact data, let it be assumed that these preliminary outlines will remain in the final analysis.

The loading upon the toe extension is shown in Fig. 59. $R = 30 \times 9.5 \times 0.1 = 28.5$ kips. From (39) $S_1 = 4.9$ kips checking the first assumption. From (41), the location of the point of zero intensity of soil pressure is found at $x = \frac{w}{3}\frac{1 - 3k}{1 - 2k} = 4.5(0.16/0.44) = 1.63$ feet from the heel. The center of gravity of this loading may be found by aid of Table 3, noting that the value of c is $7.8/4.0 = 2$ approx., whence $B = 0.47$ and the location of the force is 1.88 from the toe, little error would have resulted in taking the center of gravity at the center of the load. The total load is $\frac{5 + 3.3}{2} \times 4 = 16.6$. For shear $d = 16,600/5040 = 3.3$. The moment requirement is less and the depth chosen will be that required by the shear. The total thickness of the toe, including the protective concrete over the steel rods will be 3′ 6″.

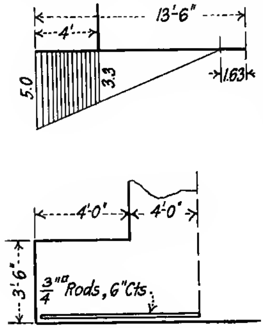


FIG. 59.

$$k_s = \frac{16,600 \times 2.12 \times 12}{12 \times 39^2} = 23$$

and $pj = 23/16,000 = 0.0014$. This is substantially the steel ratio p. The area of steel required becomes $0.0014 \times 39 \times 12 = 0.66$ square inches. Taking j again as 0.89, the periphery of steel necessary for the proper adhesion stress, namely 80 pounds per square inch, is

$$r = \frac{16,600}{0.89 \times 39 \times 80} = 6 \text{ sq. inches.}$$

This latter requirement controls the selection of the reinforcement and ¾ inch square bars spaced on 6″ centers will be used. Since, to develop the stress (and in accordance with the principle of the proper detailing of structures, the section as used is developed and not merely the stress existing in it) the bars will be carried by the face of the vertical arm for $50 \times ¾ = 4$ feet.

The toe as finally laid out is shown in Fig. 59.

It must be again emphasized that in none of the preceding problems have the secondary rod systems, for temperature, etc., been shown. In a later chapter these rod systems will be completely detailed, with reference to these problems.

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The following is a list of articles on reinforced concrete walls:

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Walls for Yale Bowl, Maximum Height 42 Feet, *Engineering News,* Vol. 72, p. 997.

Counterforted Walls with Structural Steel Frame, *Engineering News,* Vol. 73, p. 776.

The Design of Counterforted Walls, E. GODFREY, *Engineering & Contracting,* Vol. xxxiv, Dec. 21, 1910.

(See Also Bibliography in Appendix.)

CHAPTER IV

VARIOUS TYPES OF WALLS

The types of walls discussed in the previous chapters are those generally used in engineering practice. Occasionally, conditions are such that these general types are inapplicable and it becomes necessary to devise special types to meet the peculiarities of the given environment. Such walls are described briefly below.

Cellular Walls.—A type of wall insuring a light foundation pressure approaching a uniform distribution is shown in Fig. 60. It is essentially a gravity type, the interior concrete replaced by

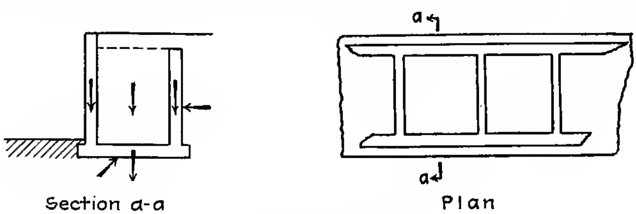


Fig. 60.—Cellular wall.

an earth fill. The principles governing its outlines are thus identical with those governing the outlines of the rectangular gravity walls, with the correct allowance made for the reduced stability moment. In a finished wall, complete with the fill outside and inside, the rear wall is under no pressure. To insure no possibility of failure during construction or at some later date in consequence of an adjacent excavation, it is well to make the rear wall like the face wall. Theoretically the wall may be built without a base. Practically, to insure an even distribution of pressure upon the bottom, and to avoid unsightly settlement, a base is generally used.

The design of the separate members is identical with the method used in the design of the several members composing the counterforted wall. For the base, when such is used, the slab should be designed for the net difference between the upward and downward loads. A description of a wall of this type is given in *Engineering & Contracting*, Vol. 35, p. 530, by J. H. Prior.

Hollow Cellular Walls.—To insure even lighter soil pressures than given by the type previously discussed, a hollow cellular wall may be used, as described in Fig. 61. Its stability is

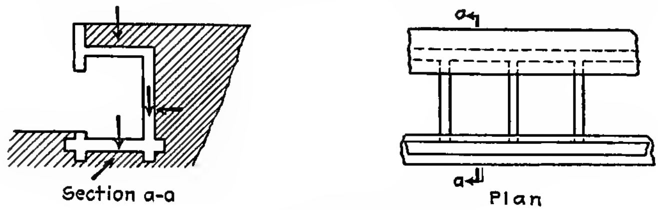


FIG. 61.—Hollow cellular wall.

furnished by the small amount of earth fill resting immediately upon it and by the weight of the track ballast, in addition to the weight of the separate members composing the cells. It is essential, because of the light weight of the wall that adequate attention should be paid to its tendency to slide forward. The face of the lower part of the wall should abut against the firm ground, and, if possible, extensions should be built into the bottom to add to the sliding resistance. Two interesting types of the wall are described here. The former, as shown in Fig. 61,

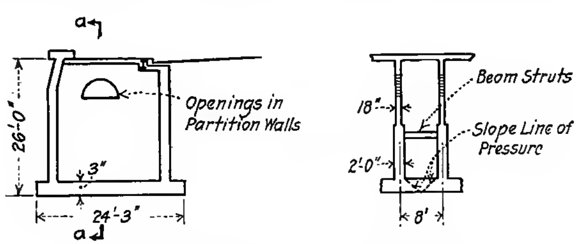


FIG. 62.—Cellular wall on timber cribbing.

termed the "Lacher" wall is described in detail in an article by J. H. Prior.[1] While this was the most expensive type of five types analyzed for the track elevation work of the Chicago, Milwaukee and St. Paul (gravity, "L" shape, counterfort "L," cellular as described previously and the hollow cellular) it was the only type insuring a safe permissible pressure on the soil encountered in the work. The maximum soil intensity was two

[1] *Engineering & Contracting*, Vol. 35, p. 530.

tons per square foot. This type also permitted a full use of the easement for tracks. It was not feasible to use piles.

The second type, shown in Fig. 62, was used in supporting the Speedway, a highway along the west bank of the Harlem River, New York City. It is described in the *Engineering Record*, Vol. 66, p. 22. A good foundation could be had upon a timber cribbing already in place, below mean high water, giving promise of little future settlement. The wall is about square in section and the sidewalk forms the upper slab of the cell. The walls are thinned down towards the top and a circular segment is cut out of the transverse walls, to diminish the load upon the base. The distribution of the pressure is practically a uniform one. To quote from the article:

"The transverse walls are so spaced that their weight is evenly distributed upon the foundation cribs by the 3 foot concrete flooring. It was assumed that the line of thrust at the base of these walls due to their weight and the weight of the sidewalks which they carry, would be at an angle of 45°. Upon this basis, the lines of thrust from the bottoms of successive transverse walls intersect just at the base of the 3 foot concrete floor, causing a uniform application of the loads upon the foundation cribs." See Fig. 62.

Timber Cribbing.—Walls have been constructed of old ties, forming practically cellular walls. The transverse ties are spiked to the stretcher ties forming the rear and front faces. See Fig. 63. Such a wall was used in Chicago by the Chicago, Rock

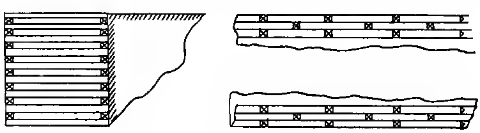


FIG. 63.—Timber crib.

Island and Pacific Railroad for heights varying from four to twenty feet. There is an interesting discussion on the use of this type of wall in the *Joural* of the Western Society of Engineers, Vol. 20, 232 *et seq.*

Concrete Cribbing.—In exactly identical fashion with the use of timber cribs, concrete cribbing may be used, the members constructed in units of a shape similar to a tie and reinforced at the four corners. A description of the use of such cribbing in Oregon along a highway is given in the *Engineering News-Record*, Vol. 81, p. 763. It is pointed out in this article that the life of timber cribs is so short that their use is not economical. Concrete cribs, would not be open to this objection.

Walls with Land Ties (or Backstays).—This is a practically obsolete type of wall, but is occasionally used for small light walls usually along the water front. A typical wall of such character is described in *Engineering and Contracting,* Vol. 37, p. 328. It is shown in Fig. 64. Its design follows from the ordinary

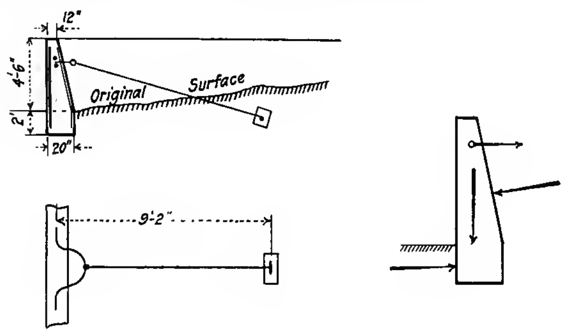


FIG. 64.—Wall with land ties.

principles of statics and the force system is shown in Fig. 64. If the tie is a metal one, there is danger of its gradual destruction by rust. It should be encased in concrete, which adds considerably to the expense of the wall. On a fair foundation and for a small wall, this type may prove economical. The theory of such walls is given by Rankine 23rd Ed., 1907, pp. 410, 411.

Walls with Relieving Arches.—This is another type of historical interest rarely used now. As constructed of brick with

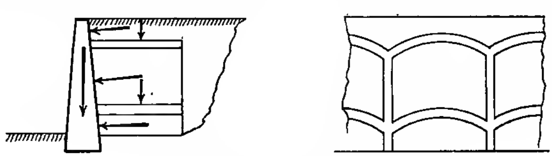


FIG. 65.—Wall with relieving arches.

cheap labor it afforded an economical type of substantial construction. The theory of such a wall is given by Rankine, in his 23rd Ed., p. 412. Fig. 65 shows a typical view of such a wall.

An interesting example of a wall of this kind is given on p. 353 Handbuch Für Eisenbetonbau III Band. The relieving arches are of cast iron and the wall masonry of brick. The section of the wall is shown in Fig. 66.

A novel type of wall is shown in Fig. 67, and is a compromise between a cellular and cantilever type. It is taken from the handbook on concrete quoted above.

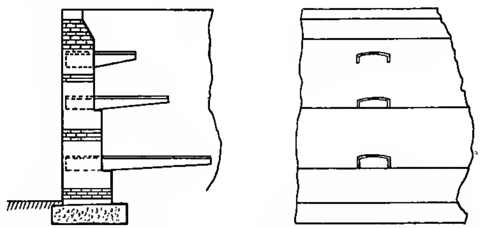


FIG. 66.—Brick wall with cast iron relieving arches.

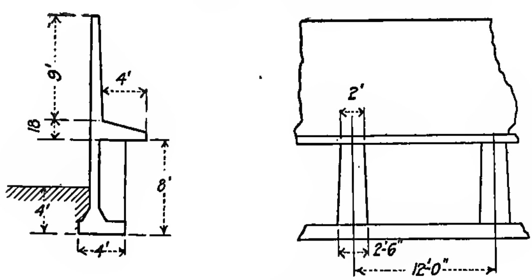


FIG. 67.—Special shape wall.

Euorpean Practice.—Some very interesting types of walls, mostly of European origin are given in the Handbuch Für Eisenbetonbau III Band, pp. 369 to 402. The intricate rod systems and complicated form details necessary in the construction of these walls would preclude their use in America. It is notable to see the latitude allowed individual engineering talent in the adoption of the various designs and such freedom of thought should prove, in the long run, very fruitful in useful wall sections.

Embankments Bounded by Two Walls.—The construction of embankments through narrow easements, requiring retaining walls on either side of the fill makes it possible to utilize the mutual action of the two walls to effect quite a reduction in the section of each wall required. The wall thus built is in effect a modification of the counterforted wall and so far as the actual design of the wall itself, the theory as previously given is sufficient to design this wall. Two interesting examples of this type of construction are given here.

Retaining Walls, New York Connecting Railroad, Hell Gate Arch

Approach.—The embankment to be retained was practically of square section, 60 feet wide and high. The ordinary theory of earth pressure would have necessitated enormous sections. A carefully specified embankment well drained and compacted made it possible to reduce the thrusts (see page 21). The walls were divided into ten foot square panels, at each corner of which a tie rod 2¼ inch diameter extended between the walls and was anchored to a steel channel embedded in the face walls (see Fig. 68). Every fifty-feet, a partition wall ran between the face walls giving additional stability to the section, and especially stiffness against wind stresses prior to the placing of the fill within the wall. A most careful system of drainage was placed at every row of tie rods to prevent the accumulation of water with a consequent increased pressure.

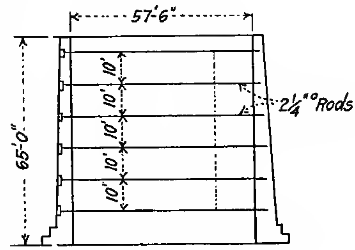


Fig. 68.—Walls of Hell Gate arch approach.

Interboro Rapid Transit Railroad, Eastern Parkway Improvement

The walls here were about 25 feet high and tied to each other at intervals of 20 feet by reinforced concrete partition walls (see Fig. 69).

In both examples it is to be noticed that no bottom slab is used, forming the true cellular wall as described by Lacher in the previously mentioned issue of the *Journal* of the Western Society of Engineers. The interesting details in connection with the use and non-use of expansion joints are discussed in the following chapter.

The widening of an existing right of way prior to its final completion (White Plains Rd. Extension, Interboro Rapid Transit Co.) made it possible to adopt an unusual expedient of anchoring the new wall directly to the existing wall. Structural steel frames were anchored through the existing wall as shown in Fig. 70 (See Plate II, Fig. 26). The new face wall consisted of slabs supported by upright channels. To insure the permanence of the anchors they were embedded in concrete partition walls. In placing the fill care was observed to carry up the fill levels at the same rate on either side of these partition walls to prevent

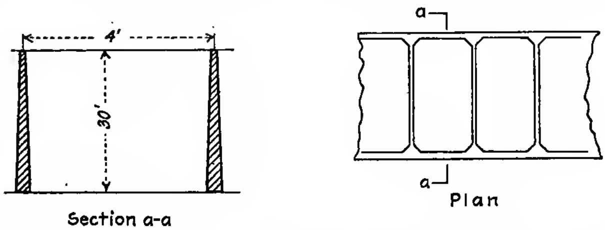


Fig. 69.—Walls Eastern Parkway Extension Interboro Rapid Transit R. R.

placing an earth pressure upon them. The thickness of the face slabs was the minimum width it was found practicable to construct in the field with the equipment at hand.

Abutments.—The design of the abutment differs from that of the ordinary retaining wall, merely in that an extra dead or dead and alive load, is superimposed upon the wall and serves to counteract the overturning moment of the earth pressure. This

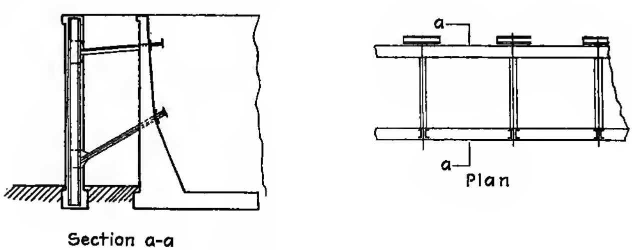


Fig. 70.—Anchoring new wall to old wall.

additional load, resting upon the abutment is assumed to be uniformly distributed along the abutment and is, thus, treated, mathematically, as an additional masonary surcharge. The variable conditions of loading make it necessary to investigate all possible states of loading, in order to ascertain the maximum forces upon the wall.

The following combinations of dead and live loads are all possible ones and each is worthy of investigation. The accompanying Fig. 71 may serve to give a better idea of these combinations as listed below.

(*a*) The earth backing in place, but no span construction set. The abutment is a plain retaining wall.

(*b*) The crane to be used in erecting the span is in place behind the abutment. Here the abutment is a retaining wall with a surcharge load due to the erecting crane.

(*c*) The construction complete. Live load approaching the span. The abutment has the full earth and surcharge load, but only the dead load of the span as a relieving load.

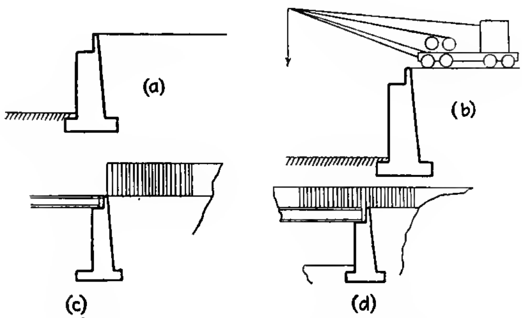


FIG. 71.—Conditions of Abutment loading.

(*d*) The live load is on both the span and back of the abutment. There is here the maximum earth pressure and maximum relief. This latter case gives the greatest total loading upon the base. The others, however, may give a greater toe intensity.

In connection with the conditions of loading subsequent to the completion of the structure, the span construction, in addition to the relief afforded by its weight upon the wall also exerts a horizontal relieving action, forming a beam out of the abutment with both a top and bottom support. Such relief, however, is most difficult to compute, due to the uncertainty of the action of the roller bearings and had better be neglected in the design of the wall.

The designer should, of course, govern the design of the wall by the above four conditions and not attempt to control the field conditions, such as the sequence of operations in the placing of embankment and erection of the bridge, by his design. It is, of course, within the province of the experienced engineer to determine how best to adapt the design to take care of the construction loadings. The factor of safety against sliding and overturning may be temporarily lowered to take into account the conditions prior to final completion, but it does not seem advisable to permit the soil intensity under any combination of loading, temporary or otherwise, to exceed the safe allowable pressure.

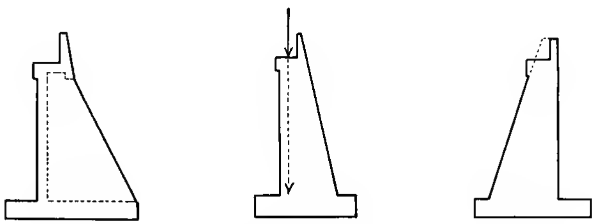


FIG. 72. FIG. 73. FIG. 74.

Abutment types.

The location of an abutment is usually transverse to the right of way, permitting the footing to encroach upon the crossing, whether public or private. It is thus possible to secure the best type of soil pressure distribution, keeping, at the same time, an economical section of wall. Since the abutment is a combination of a retaining wall and an ordinary pier subject to vertical loads only, it is customary to extend both the heel and toe (see Figs. 73, 74, 75).

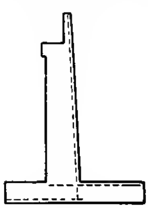


FIG. 75.—Reinforced-concrete abutment.

Abutments may be either composed of plain masonry or of reinforced-concrete, as economy or other factors dictate. The flexibility of reinforced-concrete in permitting slender walls with projecting heel and toe indicates that for practically every condition a reinforced-concrete type of wall may be found that will prove more economical than the gravity masonry walls. The counterforted retaining walls may readily be adapted to form an abutment, by placing a cap over the top to form the girder seat (see Fig. 72). Several of the usual types of abutments are shown in Figs. 73, 74 and 75.

Wing-walls.—The wing walls attached to the abutments are ordinary retaining walls and are so designed. Their location is governed by the conditions of the intersection and may either be in line with the abutment, following the slope of the fill, or if the condition of the easement does not permit may make an angle with the abutment determined by the economical limitations. The combination of wing wall and abutment, makes it possible to devise ingenious schemes to effect an economy of material used. The walls and abutment may form a U-band of constant cross-section as described in *Engineering News*, Mar. 8, 1917, p. 393, the walls partially buried in the fill and holding, by friction, the abutment portion of the U. Cellular abutments have also been used.

Occasionally an abutment is supported by a stem buried in the retained embankment, forming a T (see Fig. 76).

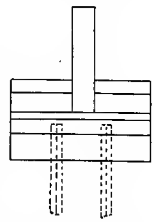


FIG. 76.—Plan of "T" abutment.

An exhaustive analysis of abutments and wing walls, with a wealth of practical hints, is given by J. H. Prior in the American Railway Engineering Association, Vol. 13, p. 1085.

C. K. Mohler,[1] Consulting Engineer, has pointed out the economy effected by turning back the wing wall in place of merely extending it in the line of the abutment to follow the slope of the retained embankment. E. F. Kelly has pointed out[2] that for minimum wing length, the face of the wing should bisect the angle between the shoulder of the fill (sometimes termed the berm) and the face of the abutment produced. This assumes that the end of the wing wall becomes a line, in place of, as in actual practice, the wall being cut off at a convenient height. Since the end of the wall has no serious effect upon the entire amount in question, such approximation has but negligible effect. To take into account such practical factors, the author of the paper has prepared curves giving the actual angle required when the character of the end detail is taken into account together with the character of the junction of the wing with abutment at the shoulder. It is emphasized[3] that where minimum volume, rather than minimum length is sought, the above rule and curves do not hold. For minimum volume the wing wall carried out directly in the plane of the abutment face gives the least volume until the angle between the wing and the axis of the retained embankment exceeds a right angle.

[1] *Engineering News-Record*, Vol. 80, p. 168.

[2] Ibid, p. 785.

[3] Ibid, p. 1243.

For track elevation, where full trackage on a limited easement is essential, the abutment frames into the two parallel retaining walls on either side of the embankment forming a box-like structure. Other details are made to fit into the special circumstances of the given location.

A number of examples of the varied types of gravity and reinforced concrete abutments is given in the Handbuch fur Eisenbetonbau iii Band, pp. 415 to 422.

For ordinary highway abutments it is possible to compile standard sections to cover practically all the cases expected. Thus H. E. Bilger in a paper read before the Illinois Society of Engineers and Surveyors[1] states: For walls up to 25 feet in height:

(*a*) For ordinary earth bottoms, the base is ⅓ the height;

(*b*) For rock or shale bottoms the base is ¼ the height.

The footing is 18 inches thick and is offset 9 inches at the heel and toe. The back of the wall is vertical. Gravity walls are generally used because the character of local labor does not permit the use of the reinforced concrete sections.

Box Sections Subject to Earth Pressures.—The section, shown in Fig. 77, subjected to earth pressure, both horizontal and vertical requires an intricate analysis, if designed as a monolith. Since such structures, though otherwise designed, are actually rigid frames, it is quite desirable to learn the true stresses existing in them.

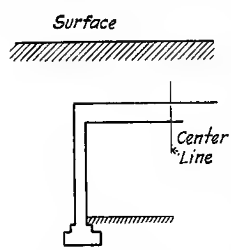


FIG. 77.—Sub-surface structures.

The principles of the theory of least work applicable to the problem in question may be stated as follows:

(*a*) The work performed by the shear and thrust is negligible in comparison with the work done by the moment.

(*b*) The work performed by the moment between any two points s_1 and s_2 is given by the expression:

$$w = \frac{1}{2}\int_{S_1}^{S_2} \frac{M^2 dx}{EI} \qquad (147)$$

(*c*) The derivative of this expression with respect to a force that does no work *i.e.*, a force whose point of application is at a fixed point, is zero.

[1] Given in *Engineering Record*, Vol. 63, p. 205.

Corollary: It is permissible to differentiate the expression under the integral sign with respect to a variable other than the variable of the integrand, thus

$$\frac{\partial}{\partial H}\int_{S_1}^{S_2} F(H,x)dx = \int_{S_1}^{S_2}\frac{\partial F(H,x)}{\partial H}dx \qquad (148)$$

Finally, it shall be arbitrarily taken that a moment which causes compression in the outside of the member is positive.

In Fig. 78 the moments between the following points are:

$$\begin{aligned} &C \text{ to } a: M &&= -M_1 + Hx \\ &a \text{ to } A: &&= -M_1 - W(x-a) + Hx \\ &A \text{ to } B: &&= -M_1 - W(h-a) + Hh \\ &B \text{ to } D: &&\text{same as } c \text{ to } A \end{aligned}$$

The total work is, with I_1 and I_2 the moments of inertia of the roof and sidewalls respectively, and E the modulus of elasticity

$$w = \frac{1}{EI_2}\left\{\int_o^a (-M_1 + Hx)^2dx + \int_a^h [-M_1 - W(x-a) + Hx]^2dx\right\} + \frac{1}{2EI_1}\int_o^b [-M_1 - W(h-a) + Hh]^2dx \qquad (149)$$

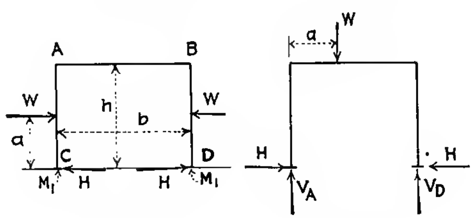


FIG. 78. FIG. 79.
Loads on sub-surface frame.

The forces M_1 and H shall be taken as the forces with respect to which the partial derivatives of the work are zero. The points C and D are taken as fixed. From the corollary and since

$$dw/dH = dw/dM_1 = 0$$

$$\frac{1}{EI_2}\left\{\int_o^a 2(-M_1 + Hx)xdx + \int_a^h [-M_1 - W(x-a) + Hx]xdx\right\} + \frac{1}{2EI_1}\left\{\int_o^b 2[-M_1 - W(h-a) + Hh]hdx\right\} = 0$$

$$\frac{1}{EI_2}\left\{\int_o^a - 2(- M_1 + Hx)dx + \right.$$

$$\int_a^h - 2[- M_1 - W(x - a) + Hx]dx\Bigg\} +$$

$$\frac{1}{2EI_1}\left\{\int_o^b - 2[- M_1 - W(h - a) + Hh]dx\right\} = 0$$

Solving these two simultaneous equations for H and M_1

$$M_1 = \frac{Wa(h - a)[hI_1(h - a) + bI_2(2h - a)]}{h^2(hI_1 + 2bI_2)} \quad (150)$$

$$H = \frac{W(h - a)^2[h(h + 2a)I_1 + b(2h + a)I_2]}{h^3(hI_1 + 2bI_2)} \quad (151)$$

In similar fashion,[1] referring to Fig. 79, the base moment and horizontal thrust due to concentrated load upon the roof is found to be

$$H = W\frac{3I_2a(b - a)}{2h(hI_1 + 2bI_2)} \quad (152)$$

$$M_1 = \frac{I_2}{2}a(b - a)W\left\{\frac{1}{hI_1 + 2bI_2} - \frac{b - 2a}{b(6hI_1 + bI_2)}\right\} \quad (153)$$

Using these four equations as a foundation, it is possible to establish some general conditions of loading on either roof, side-walls or upon both simultaneously. For a uniformly distributed load on the roof of w per foot, replace in (152) a by x, W by w, multiply the expression by dx and integrate between the limits O and b. The expressions for the thrust H and the mo ent M_1 are

$$H = \frac{wb^2}{4h}\frac{bI_2}{hI_1 + 2bI_2} \quad (154)$$

$$M_1 = \frac{wb^2}{12}\frac{bI_2}{hI_1 + 2bI_2} \quad (155)$$

For a uniformly distributed loading p on the side walls, in similar manner integrate the expressions given in (150) and (151) between the limits O and h. The thrust and moment are then

$$H = \frac{ph}{4}\frac{2hI_1 + 5bI_2}{hI_1 + 2bI_2} \quad (156)$$

$$M_1 = -\frac{ph^2}{12}\frac{hI_1 + 3bI_2}{hI_1 + 2bI_2} \quad (157)$$

Again for a triangular distribution of loading on the side wall, with maximum base intensity q, the expressions become

$$H = \frac{qh}{20}\frac{7hI_1 + 16bI_2}{hI_1 + 2bI_2} \quad (158)$$

$$M_1 = -\frac{qh^2}{60}\frac{3hI_1 + 8bI_2}{hI_1 + 2bI_2} \quad (159)$$

[1] See Hiroi, "Statically Indeterminate Structures."

Denote the ratio $\frac{bI_2}{hI_1}$ by e and let $1/(1 + 2e) = Z_1$; $(2 + 5e)/(1 + 2e) = Z_2$. Then $(1 + 3e)/(1 + 2e) = Z_2 - 1$; $(7 + 16e)/(1 + 2e) = 3 + 2Z_2$; $(3 + 8e)/(1 + 2e) = 1 + 2Z_2$. Table 25 gives the values of Z_1 and Z_2 for several values of the ratio e.

With the above substitutions the expressions given in (154 to 159) become

For uniform loading on roof.

$$H = \frac{wb^2}{4h} Z_1, \qquad M_1 = \frac{wb^2}{12} Z_1 \tag{160}$$

For uniform loading on side wall.

$$H = \frac{ph}{4} Z_2, \qquad M_1 = -\frac{ph^2}{12}(Z_2 - 1) \tag{161}$$

For triangular loading on side wall.

$$H = \frac{qh}{20}(3 + Z_2), \qquad M_1 = -\frac{qh^2}{60}(1 + 2Z_2) \tag{162}$$

To apply these expressions to a subsurface structure subject to earth pressure upon roof and sidewalls, let the loading above the roof line be treated as a surcharge, with the usual terminology that c is the ratio of this surcharge height to the full wall height h. The roof loading w is then gch and the side wall pressure is compounded of a uniform intensity $p = Jgch$ at the top of the side wall, and a triangular loading with base intensity $q = Jgh$. For a loading upon the roof alone the respective thrust and moment are

TABLE 25

e	Z_1	Z_2
0	1.00	2.00
.2	.72	2.14
.4	.56	2.22
.6	.45	2.27
.8	.38	2.31
1.0	.33	2.33
1.5	.25	2.37
2.0	.20	2.40
Infin.	.0	2.50

$$H = \frac{gcb^2}{4} Z_1 \tag{163}$$

$$M_1 = \frac{gcb^2h}{12} Z_1 \tag{164}$$

For a loading upon the side wall alone the thrust and moment are

$$H = \frac{Jgh^2}{20}[3 + (2 + 5c)Z_2] \tag{165}$$

$$M_1 = -\frac{Jgh^3}{60}[1 - 5c + (2 + 5c)Z_2] \tag{166}$$

For a simultaneous load upon roof and sidewall the two above expressions are added to give the total thrust and moment. It is possible, of course to have a different surcharge for the roof than for the sidewall, since there may be no surface load over the roof and a surface load whose weight will affect the sidewall pressure. This is taken care of by giving the proper values to the surcharge ratio c in the above expressions.

With the thrust H and the base moment M_1 known the moment at any other point of the frame can easily be found by the ordinary principles of statics.

Fig. 89 is a typical section of such a structure analyzed by the above method. A radically different distribution of stress exists in this structure when analyzed exactly as above than when it is treated as an assembly of independent units. It is the very essence of the design of such structures, usually subsurface, that they be waterproof. Any cracks developed in the structure due to ignored stresses are fatal to the integrity of the structure. It is patent that regardless of what method is employed in designing such structures, provision must be made for stresses as found above.

The theory as above outlined and the formulas as given are ample to analyze any subsurface structure subject to lateral and vertical pressures.

The mutual effect of the members upon each other makes it essential that such conditions be combined as will produce the maximum stresses at the separate points of the structure.

It may be interesting to note, while treating sub-surface structures that a very thorough analysis, both theoretical and practical, of stresses in large sewer pipe is given in *Bulletin* No. 31, issued by the Engineering Experimental Station of the Iowa State College of Agriculture and Mechanic Arts. See also for a comparison between theoretical and actual stresses "Analysis and Tests of Rigidly Connected Reinforced-Concrete Frames" by Mikishi Abe, *Bulletin* No. 107. Engineering Experiment Station, University of Illinois.

Economy of the Various Types.—Broadly speaking, the selection of a given type of wall is governed by one, or more of the following reasons: economy of section; character of foundation; demands of the environment, in which latter may be included the relation between walls and property line; architectural treatment, the wall entering into a part of some general landscape scheme; the availability of materials necessary for its construction and the character of the labor to be had in the vicinity of the work.

So far as the economy of the section is involved, it must be noted that the relative economy of gravity and reinforced concrete walls is not that given merely by a parallel comparison of materials required for the finished wall. The reinforced concrete wall has thinner members, requiring more form work per cubic yard of concrete. The slenderness of this wall, together with the net-work of rods within it, makes it more difficult to properly place and distribute the concrete, necessitating more skillful labor and more competent foremanship. The gravity walls are more capacious within the forms, the laborers have, consequently, more room to move about and can thoroughly spade and turn over the mix, giving better assurance of a flawless wall. This is a very important item and one too frequently overlooked. A concrete gang of the average type, *i.e.*, a class of men just a shade above the common excavators, will tackle a gravity section of wall and turn out a good looking section. Upon attempting to pour a reinforced concrete wall, a very inferior piece of work is constructed. Before preparing plans for a thin reinforced concrete wall, it is essential to insist upon a capable contractor, equipped with the proper labor gangs to do such work. With a policy of awarding the work to the lowest bidder where competitive bids are asked, it is necessary that the engineer adapt the type of wall to one that can safely be built by the general run of low bidders.

Unsuspected variations in the character of foundations, may demand an abrupt change in the section of wall. For a reinforced concrete wall the rods are usually ordered some time in advance of the actual construction of the wall. It is necessary that the section of the wall be determined at the time of ordering the rods[1]. Despite careful boring made at the site of the work, the soil encountered at the proposed bottom of the wall may prove to be different from that assumed and it may thus become necessary to excavate deeper to obtain the desired character of bottom, or even to change the type of wall. Since the rods have been ordered, the wall design is inflexible and if a new section is

[1] While it is possible to get shipments from local markets at short notice, quite a premium must be paid for this material and such orders are given only when economy must be sacrificed to urgency.

ordered, it may mean delay awaiting mill shipments of the new lengths needed, costly orders of rods from stock supplies, the undesirable splicing of rods or the placing of a plain concrete base to bring the actual bottom level up to the theoretical one—all expensive and undesirable expedients. For this condition the gravity wall is the more flexible type and the section may be changed without any additional trouble should soils at variance with the originally assumed ones, be encountered.

On the other hand, where the character of the soil is assured, the reinforced concrete type of wall may be molded to adapt themselves to any distribution of soil pressure desirable. This has been shown in the previous work.

It has been pointed out[1] . . . for walls of the height required for track elevation and track depression a gravity wall, will under ordinary conditions be cheaper than the reinforced concrete types.

Again, in the same issue of the *Journal* in discussing the relative demerits and merits of the cellular types it was pointed out[2] in connection with track elevation work, that such a wall, with the bottom left out offers great resistance to sliding and overturning and "occupies the right of way so as to afford little opportunity for encroachment. It permits of ready driving of a pile trestle right over it." On the other hand "it occupies considerable space before filling and may thus interfere with the use of the tracks. Settlement may also give an unpleasing appearance."

So far as the actual amounts of materials involved, both during construction (forms, etc.) and in the permament structure it is possible to determine the more economical wall by comparison of two types or by mathematical and tabular methods as given at the end of this chapter. It is understood that the proper weight is given to the indeterminate factors of cost as above mentioned *i.e.* the construction limitations of the several types.

It must be emphasized that wall details should be simple. Shapes that apparently make for economy may prove exceedingly difficult to pour in the field. Thus for example, a section of a cantilever wall as shown in Fig. 80 (see also Photo Plate No. 4*a*) with a net work of obstructing rods at *A* makes it very hard to get a good concrete at and below that point. The break in the form work is also objectionable because of the added labor and

[1] *Journal* of Western Society of Engineers, Vol. 20, p. 653.
[2] P. 232, *et seq.*

difficulty of pouring the concrete. When a shape, such as just shown is much more economical than the straight battered back, it will be found that the counterforted wall will prove even more economical, and should therefore be adopted.

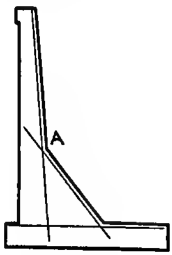


FIG. 80.

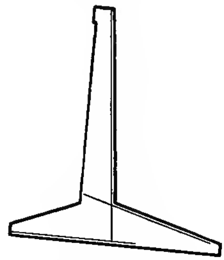


FIG. 81.

Sloping the footing as shown in Fig. 81 may prove troublesome and more costly in the end than the plain rectangular section. Much, of course, depends upon the ability of the contractor to carry out the niceties of the design and it is thus incumbent upon the engineer planning an intricate section of wall to see that its execution is placed in the proper hands.

One is tempted, in designing counterforted walls to mold corners and make steel details as shown in Fig. 82, in order to effect a thorough bond between the slab and the counterfort. These

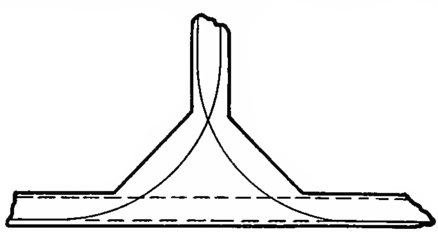


FIG. 82.

details, again, demand extra form work, steel work and labor and should therefore be employed with due appreciation of the possibility of their added expense.

On the whole, that wall is most effectively and economically designed which is most compactly and simply shaped.

With the rapid development of thin slab construction as markedly shown in the construction of concrete ships and barges, there is excellent promise of the extension of such work to retaining walls. If the construction of thin slabs and intricate details becomes commercially applicable, then a vast field is opened to economic wall design, permitting the shape to follow every peculiarity of the environment and to take advantage of whatever economies the site may offer. At present the practical limitations of construction have restricted retaining walls to but few types which in turn are limited in economic thickness by field conditions.

Problems

1. An abutment is to carry two tracks as shown in Fig. 83. Each of the stringers, under full load brings a reaction of 50 tons upon the abutment. Determine the necessary dimensions of both a gravity and a reinforced concrete "T" wall.

An abutment is a combination of a retaining wall and a pier. Its economical design is affected not only by the type adopted, but also by the assumed location of the girder reaction. In the case of a gravity wall, the

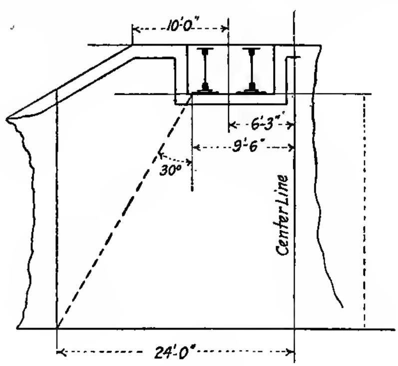


Fig. 83.

vertical girder reaction, while assisting in the stability of the wall, may by the location of its point of application, induce tensile stresses in the back of the wall. Thus in Fig. 73, the girder load falls within the outer third, violating an essential requirement of gravity walls. The selection of a type as shown in Fig. 74 brings the girder reaction towards the center of the wall and assists quite materially in the stability moment of the wall.

The distribution of these girder loads may be assumed to follow within planes making an angle of 30° with the vertical as shown in Fig. 83. The abutment should be made long enough to permit the distribution to follow along these planes. In addition, it is assumed that (for reasons given in the following chapter) the abutment is independent of the adjacent structures, so that the span loads will be confined within the abutment proper as shown in Fig. 83.

Since the reaction from each girder is 100 kips, the area for bearing upon the concrete, allowing 0.5 kip per square inch, is 200 sq. in. A plate 12″ × 18″ provides this bearing area. The plate will be placed as shown in Fig.

84, where the remaining details of the girder seat are shown. As shown in Fig. 83, the distribution of the loads spreads between a distance of 48′, making the load per linear foot at the foot of the abutment $^{400}/_{48} = 8.3$ kips. As a retaining wall, prior to the setting of the steel, the height is 30′ (above the footing) without any surcharge. From Table 12, a face batter of 5″ to the foot will give the necessary dimensions for stability, and will also satisfy the details of the girder seat.

The crane load is taken equivalent to 500 pounds per square foot. The cases are lettered and discussed in the same order as on page 129. The graphical analysis is shown in Fig. 85.

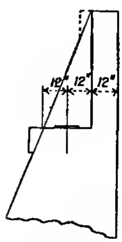


FIG. 84.

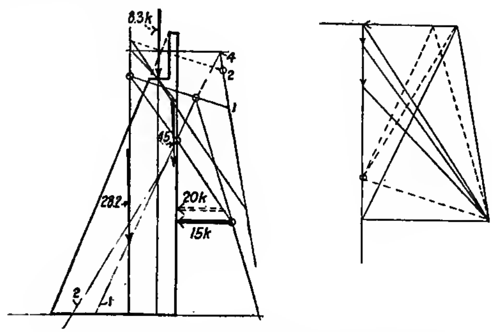


FIG. 85.—Graphical analysis of abutment.

(*a*) The resultant intersects the third point (Checking the tabular value) and $R = 28.2 + 4.5 = 32.7$

$S_1 = 65/13.5 = 4.8$ kips per square foot. The permissible soil intensity in this and the following work is taken as 4 tons per square foot.

(*b*) The resultant intersects at the $^5/_{27}$ point, and from (39)

$S_1 = \frac{66}{13.5}(2 - 3 \times 0.185) = 7$ kips per square foot; which is within the permissible value.

$S_2 = \frac{66}{13.5}(-0.44) = 2120$ pounds per square foot, or 15 pounds per square inch. This tensile stress in the concrete, developed under a crane load prior to the setting of the span, is a permissible stress.

(*c*) This condition is quite similar to the preceding one, with the exception that the indeterminate factor of the frictional resistance between the girder bearing and the abutment, together with the dead weight of the span add to the wall stability.

(*d*) For this case (that of full loading) the resultant is found to intersect exactly at the third point. $R = 42$ kips

$$S_1 = 84/13.5 = 6.2 \text{ kips per square foot.}$$

The section, then, satisfies all the necessary conditions of design and construction.

Reinforced concrete section. Assume, as in the case of the ordinary reinforced concrete retaining wall, the criterion of economy, $i = e$. Let the total toe pressure not exceed 7 kips per square foot, leaving a margin for the toe pressure caused by the girder load. Note here, that since a skeleton section of wall is assumed, with the point of application of the resultant located at the vertical stem of the wall, the girder load, which is at the same point, can have no effect upon the wall dimensions, and merely increases the intensity of the soil distribution. From (95), with $S_1 = 3.5$ tons per square foot, $H = 38'$ (taking the thickness of footing 3 feet) allowing for a five foot surcharge:

$$e = \frac{5}{6} - \frac{1}{6}\sqrt{1 + \frac{120 \times 3.5}{38}} = 0.26$$

Take the point of application of the resultant, and the location of the face of the abutment at the quarter point of the base. From Table 18 with this value of e and i, $k = 0.50$ and the base width w is, accordingly 16.5 feet. With a girder load of 8.3 at the quarter point, from (39)

$$S_1 = \frac{2 \times 8.3}{16.5}(2 - 0.75) = 1.25$$

and the total toe pressure is 8.25 kips, a permissible excess over the allowable 4 tons per square foot.

The height of the vertical stem is 30′, and from Table 21 the critical height, above which the shear controls the thickness of the stem is less than 30′. The thrust for the given surcharge is 20 kips, located 11.4 feet above the top of the footing. From (113), the thickness of wall because of shear is

$$d = 20/5.04 = 3.95$$

A thickness of 4′ will be used at the base.

The footing moment is found to be 119 ft. kips and the depth for balanced reinforcement is, from (101)

$$d = \sqrt{(^{119}/_{16})} = 2.75$$

requiring a thickness of 3 feet. If no special stirrup reinforcement is placed to take care of the diagonal tension, an excessive depth will be required for

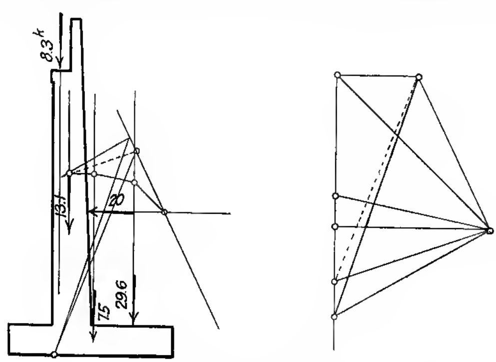


Fig. 86.—Graphical analysis of abutment.

the shear (24.5 kips). For this reason it will be assumed that such reinforcement is employed here and the depth of the slab adopted will be that required by the bending moment. The thickness of the toe extension will also be taken as 3 feet, bearing in mind that the thickness of the footing, both heel and toe, must, for construction reasons, be kept the same. The introduction of concrete fillets at the junction of the footing and arm would obviate the need for web rods and a comparative estimate may prove that the fillets, with the extra work involved, are cheaper than the complicated rod details of web reinforcement.

Discussing the separate cases of loading, treated graphically in Fig. 86, for the case of total loading (Case *d*) the point of application of the resultant is at $e = 4.75/16.5 = 0.288$; whence from (39), with $R = 59$ kips, $S_1 = 8100$ pounds per square foot.

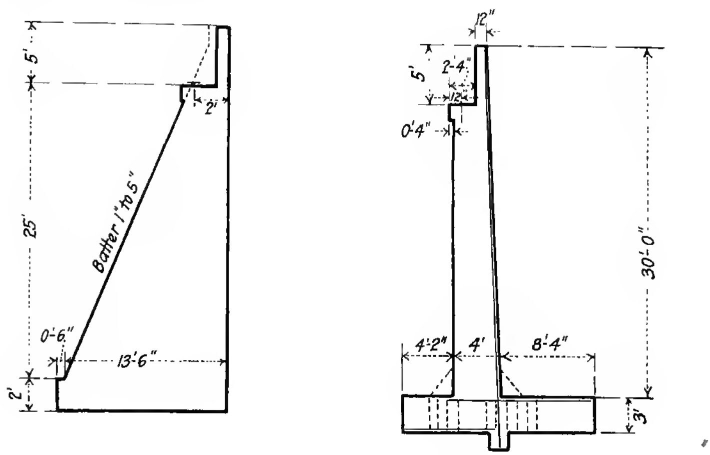


Fig. 87.

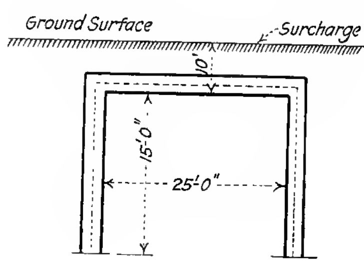


Fig. 88.

Omitting the span load (Cases *b* and *c*) the point of application of the resultant is at $e = 4.5/16.5 = 0.273$ and with $R = 51$ $S_1 = 7.3$ kips per square foot.

The section as shown therefore satisfies the governing conditions. The wall should be recalculated, using the dimensions and loadings as actually found.

Fig. 87 shows the sections of the gravity and reinforced concrete walls.

2. Find the stresses, moments, etc., in a box section as shown in Fig. 88. It is necessary to make a preliminary assumption in order to proceed with the analysis of this section under the theory of least work. For this reason, it will be assumed, tentatively, that the moments of inertia of the side-walls and roof are equal. Adding two feet to b and one foot to h, gives the dimensions along the gravity axes of the section. The value of e is now $^{27}\!/_{16}$ = 1.69. From Table 25, $Z_1 = 0.23$ and $Z_2 = 2.38$. The value of $c = {}^{14}\!/_{16}$ $=0.875$. J is then taken at its usual value $\frac{1}{8}$.

For roof loading alone

$$H = \frac{.1 \times .875 \times 27^2}{4} \times 0.23 = 3.7 \text{ kips}$$

$$M = \frac{.1 \times .875 \times 27^2 \times 16}{12} \times 0.23 = +19.7 \text{ ft. kips.}$$

For side-wall loading alone

$$H = \frac{.1 \times 16^2}{3 \times 20}(3 + 6.38 \times 2.38) = 7.8 \text{ kips}$$

$$M = -\frac{.1 \times 16^3}{3 \times 60}(1 - 4.38 + 6.38 \times 2.38) = -27.0 \text{ kips.}$$

For simultaneous loading

$$H = 7.8 - 3.7 = 4.1 \text{ kips, directed outwards.}$$

$$M_1 = -27 + 20 = -7 \text{ kip feet.}$$

At any point x, above the base, where $x = kh$, the moment is

$$M_x = -7 + Hkh - \frac{gh^3}{18}[3(1 + c - k)k^2 + 2k^3]$$

$$= -7 + 66k - 22.7k^2(5.6 - k)$$

For the various values of k, M_x has been tabulated as shown in accompanying table. The roof moment at any point y, where $y = pb$, is, taking the last found value of M_x as given in the table, -46,

k	M_x
0	−7
.1	−2
.2	+1
.3	+2
.4	0
.5	−3
.6	−8
.7	−16
.8	−24
.9	−34
1.0	−46

$$M = -46 + 510p(1 - p)$$

A table has been similarly prepared for a set of values of p, up to the center of the span.

p	M
0	−46
.1	0
.2	36
.3	61
.4	76
.5	82

The assumption that the roof and sidewalls are simultaneously loaded does not, necessarily give the maximum moments. During construction it is quite possible that the side walls will be loaded up to the roof line, before any load is placed upon the roof. The only roof load is then its dead weight, which, with the assumption that the roof is two feet thick, gives a load of 0.3 kips per foot. There is a triangular distribution of pressure along the side wall, with a value of $q = 1600/3 = 0.53$ kips.

For roof loaded alone, from (160)

$$H = \frac{.3 \times 27^2 \times .23}{4 \times 16} = 0.8 \text{ kips}$$

$$M = \frac{.3 \times 27^2 \times .23}{12} = 4.2 \text{ kip feet}$$

For side wall loaded alone, from (162)

$$H = \frac{.533 \times 16}{20}(3 + 2.38) = 2.3$$

$$M = -\frac{.533 \times 16^2}{60}(1 + 4.76) = -13.1 \text{ kip feet}$$

Under the simultaneous loading

$H = 1.5$ directed outwards.

$M_1 = -$ 9kip feet.

As before, $x = kh$, and $c = 0$

$$M_x = -9 + 24k - 22.7k^2(3.6 - k)$$

A table of values of M for the side wall is given here.

k	M
0	-9
.1	-7
.2	-7
.3	-8
.4	-11
.5	-15
.6	-19
.7	-24
.8	-32
.9	-37
1.0	-44

The roof moment is, with p the same as above,

$$M = -44 + 111p(1 - p)$$

A table of these moments up to the center is given here.

p	M
0	-44
.1	-34
.2	-26
.3	-21
.4	-17
.5	-16

A further condition of loading may be anticipated. With time the effect of cohesion may materially reduce the side-wall pressure, or due to a variety of conditions, the side wall pressure may be considerably less than that assumed. Let this state of loading be analyzed upon the assumption of a full roof loading and a sidewall pressure as given in the work immediately preceding.

For roof loading alone, from before

$H = 3.7$; $M = 19.7$ ft. kips

For the side wall loading as assumed

$H = 2.3$ and $M = -13.1$ ft. kips

The net thrust due to both loadings is 1.4 directed outwards, and the moment is +6.6 ft. kips.

$$M_x = 6.6 - 22k - 22.7k^2(3.6 - k)$$

The tabular values for the moments in the sidewall are again shown in the accompanying table.

k	M
0	$+7$
.1	$+4$
.2	$+1$
.3	-7
.4	-14
.5	-22
.6	-31
.7	-40
.8	-52
.9	-63
1.0	-74

The roof moment is

$$-74 + 510p(1 - p)$$

The values for this moment up to the center of the span are given in the table.

p	M
0	-74
.1	-28
.2	$+8$
.3	$+33$
.4	$+48$
.5	$+53$

The structure is designed to satisfy the maximum moments shown in the diagrams. The maximum roof moment is 82 with practically an equal but opposite moment at the fixed corner. The thickness for balanced reinforcement is found to be 2.25 feet. The steel ratio 0.0075, requires 2.4 square inches per linear foot; too heavy a reinforcement. A thickness of 33″, or 3 feet overall is finally adopted, which requires a steel reinforcement of 1 inch square bars spaced 6″. The maximum side wall moment will occur about at $k = 0.9$ (since the roof is 3′ thick), whence $M = -63$ ft. kips. Again, although balanced reinforcement needs a 2′ slab, to keep the rod weight within reasonable limits a 27″ slab will be used, with an overall dimension of 2′ 6″. For this condition 1″ bars 6″ apart are required.

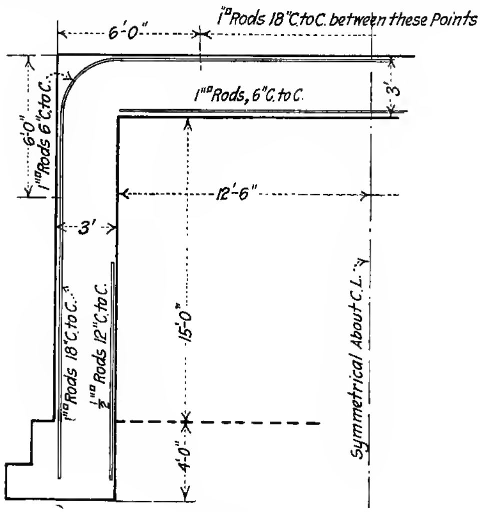


Fig. 89.

The moments of inertia of these sections, it is noticed, do not fulfill the assumed condition. To take the ratio as found for the sections above, will again prove slightly incorrect in the final analysis, and for this reason an intermediate value of the moment of inertia ratio, between that first assumed and that now found will be used. The moments of inertia of rectangular sections, of the same width are to each other as the cubes of their depths. The ratio $I_2/I_1 = 15.6/27 = 0.58$. The average of this value and the value 1, first taken is 0.79. The value of e is now 1.3, making Z_1 and Z_2 0.28 and 2.35 respectively.

In tabular form the moments at the three important points, for the three conditions discussed above are

Condition of Loading

	C	A	Center of roof
Full roof and sidewall....................	− 2	−56	+71
Dead weight roof and light wall..........	− 5	−50	−25
Full roof and light wall..................	+11	−83	+44

It is seen that quite a large variation in the assumed values of the moment of inertia ratio has but sluggish effect upon the moments and it is probably safe to take both the roof and sidewalls of the same thickness, subject to a bending moment of 70 foot kips at the center of the roof and at the upper fixed corners, and to a negative moment of −25 foot kips at the center of the roof.

The final section must take care of the moments throughout the frame detailed in accordance with the adhesion requirements and bent in accordance with the bearing formulas given in the preceding chapter. Fig. 89 gives a layout of the section, with the rod layouts as indicated by the previous work.

It must again be emphasized that the stresses existing in a structure of this character are quite different from those which are found upon analyzing the structure into its separate members and when a subsurface structure is built as shown above, provision must be made for the distribution of stresses as given by the analysis just made.

The Selection of an Economical Type.[1]—While, clearly, for some given height, a counterforted wall becomes cheaper than a cantilever wall, a search of pertinent literature fails to yield any method of obtaining such a height, save by actual comparison of two completed designs. It may be well worth while to establish some method of obtaining this "critical" height.

It is true, extraneous factors may control the selection of types of walls and the dimensions of the component members, but generally, a wall is so designed as to satisfy, most economically, its stresses.

Again, the bending moment, shear, or bond stress, may each in turn control the necessary thickness of the several parts of the wall, as the height is varied. It is to be noted that, with few exceptions, such several stresses usually require about the same thickness of section, though probably, a greater variation in the amount of reinforcement required. In assuming that the wall dimensions follow the theoretical requirements a large percentage of actual cases are covered and, if, further, these dimensions are taken in accordance with the stress of simplest expression, no serious error results. With this in mind, the various thicknesses of both the cantilever and the counterforted walls are those selected in accordance with the bending-moment requirements.

In the work that follows, since it is a comparative estimate of the cost of the two types that is sought, it is justifiable to select as a type for the present analysis, that involving the least mathematical analysis. It is quite clear that variations in the toe

[1] Reprinted from *Engineering and Contracting*, Feb. 26, 1919.

length or in the assumed position of the resultant, will not affect, to any material extent, the comparative estimate. For this reason, the condition for economy as given on page 82 is adopted here, with a further provision, that $e = \frac{1}{3}$, the usual soil pressure distribution. With these conditions (91) then becomes

$$k = \frac{1}{2}\sqrt{\frac{1+3c}{1+c}}$$

The dimensions for the "*T*" cantilever are taken as follows: the thickness of the base of the vertical arm, from (112) is

$$d_v = 0.0185\, h^{3/2}\sqrt{1+3c} = C_v h^{3/2}$$

and the thickness of the top of the arm is taken at its usual minimum value one foot. For the footing, from (119) I is about 0.7 and the required thickness of the footing slab is then $\sqrt{.7}$ or 0.84 times the arm base thickness. For the counterfort wall, from (126) with the usual value of the constants the thickness of the vertical slab is

$$d'_v = 0.0132m\sqrt{h(1+c)} = C'_v m\sqrt{h}$$

and that of the footing, from (138) is

$$d'_b = \sqrt{3d'_v}$$

The counterfort itself is usually one foot thick and will be so taken here.

The cost of the steel rods is a small part of the total cost of the wall and the relative difference of the cost of the steel rods in the two types of walls would thus be negligible.

The amount of face and rear forms for the vertical arm of both types is substantially the same and will not enter into the comparative estimate. The variable factors in the comparative estimate are then: the amount of concrete in either type and the forms required for the counterfort itself.

Let L be the total length of wall under consideration, r be the cost of placing concrete into the forms (the cost is practically the same for both types) and let t be the cost of the form work and necessary bracing, per square foot of concrete face supported. For the counterforted wall the amount of concrete is

$$L(d'_v h + khd'_b) + \frac{L}{m}\frac{kh^2}{2}$$

and its total cost

$$Lrh\left\{d'_v(1 + k\sqrt{3}) + \frac{hk}{2m}\right\}$$

The cost of the face forms for the counterfort is

$$t\frac{L}{m}\frac{kh^2}{2}.2$$

making the total variable cost of the counterfort wall

$$Lrh\left\{d'_v(1 + k\sqrt{3}) + \frac{hk}{2m}\left(1 + 2\frac{t}{r}\right)\right\} \tag{167}$$

The volume of the "T" cantilever is

$$L\left(\frac{1 + d_v}{2}h + khd_b\right) = Lh\left[\frac{1}{2} + d_v\left(\frac{1}{2} + 0.84k\right)\right]$$

and its total cost

$$Lhr\left\{\frac{1}{2} + d_v\left(\frac{1}{2} + 0.84k\right)\right\} \tag{168}$$

Equating (167) and (168)

$$d'_v(1 + k\sqrt{3}) + \frac{hk}{2m}\left(1 + 2\frac{t}{r}\right) = 0.5 + d_v(0.5 + 0.84k)$$

Replacing the thicknesses of the sections by their values given above

$$C'_v m\sqrt{h}(1 + k\sqrt{3}) + \frac{hk}{2m}\left(1 + 2\frac{t}{r}\right) = 0.5 + C_v h^{3/2}(0.5 + 0.84k) \tag{169}$$

Later it will be shown that the economic spacing of the counterforts is given by

$$m = 3.1\ Rh^{1/4}$$

where

$$R = \sqrt{1 + 2\frac{t}{r}}$$

With this value (169) becomes

$$C_2 h^{3/2} - RC_1 h^{3/4} + \tfrac{1}{2} = 0$$

a quadratic in $h^{3/4}$

with $C_2 = .0132\sqrt{1 + c} \quad 3.1\ (1 + k\sqrt{3}) + \frac{k}{6.2}$

and $C_1 = .0186\sqrt{1 + 3c}\ (0.5 + 0.84k)$

The value of $h^{3/4}$ is $\dfrac{RC_1 + \sqrt{R^2C_1^2 - 2C_2}}{2C_2}$

Table 26 gives a series of values of this critical height h for several values of the cost ratio t/r and the surcharge ratio c.

TABLE 26

c \ t/r	¼	½	¾	1
0	15	22	28	33
¼	11	17	22	27
½	10	15	19	23

Economic Spacing of Counterfort.—To determine the spacing of the counterforts to give the most economic wall sections, it is seen that (167) is the required expression for the variable cost of the counterforted wall as the spacing of the counterforts change. If, by the theory of Maxima and Minima, the derivative of this expression with respect to m, is put equal to zero, there results, after replacing the several thicknesses by their values as previously found

$$m = \sqrt{\frac{k\sqrt{h}(1 + 2t/r)}{2C'_v(1 + k\sqrt{3})}} = h^{1/4}\sqrt{\frac{1}{2}\left(1 + 2\frac{t}{r}\right)\cdot\frac{k}{.0132\sqrt{1 + c}(1 + k\sqrt{3})}}$$

With R as given above, and noting that the expression

$$\sqrt{\frac{k}{(1 + k\sqrt{3})\sqrt{1 + c}}}$$

after using the value of k as given in (91) is practically constant and equal to ½, this expression becomes

$$m = 3.1Rh^{1/4}$$

Table 27 gives a series of values of m for the several values of t/r and the height.

TABLE 27

t/r \ h	15	20	25	30	35	40	50
¼	7.5	8.1	8.6	8.9	9.3	9.6	10.2
½	8.6	9.3	9.8	10.2	10.7	11.0	11.6
¾	9.6	10.4	11.0	11.5	12.0	12.4	13.1
1	10.6	11.4	12.0	12.6	13.1	13.5	14.3

It is reasonable to expect that the laws governing the theory of probabilities hold here and that, therefore, the small errors introduced in the above approximations are fairly compensatory.

FIG. A.—Crack in reenforced concrete wall at junction of wing wall and abutment.

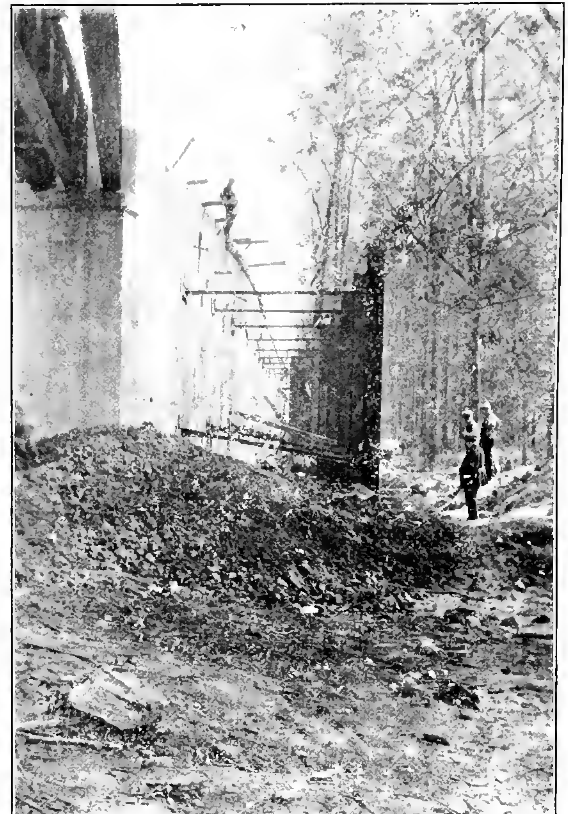


FIG. B.—Structural steel supports for special type retaining wall.

(Facing page 150)

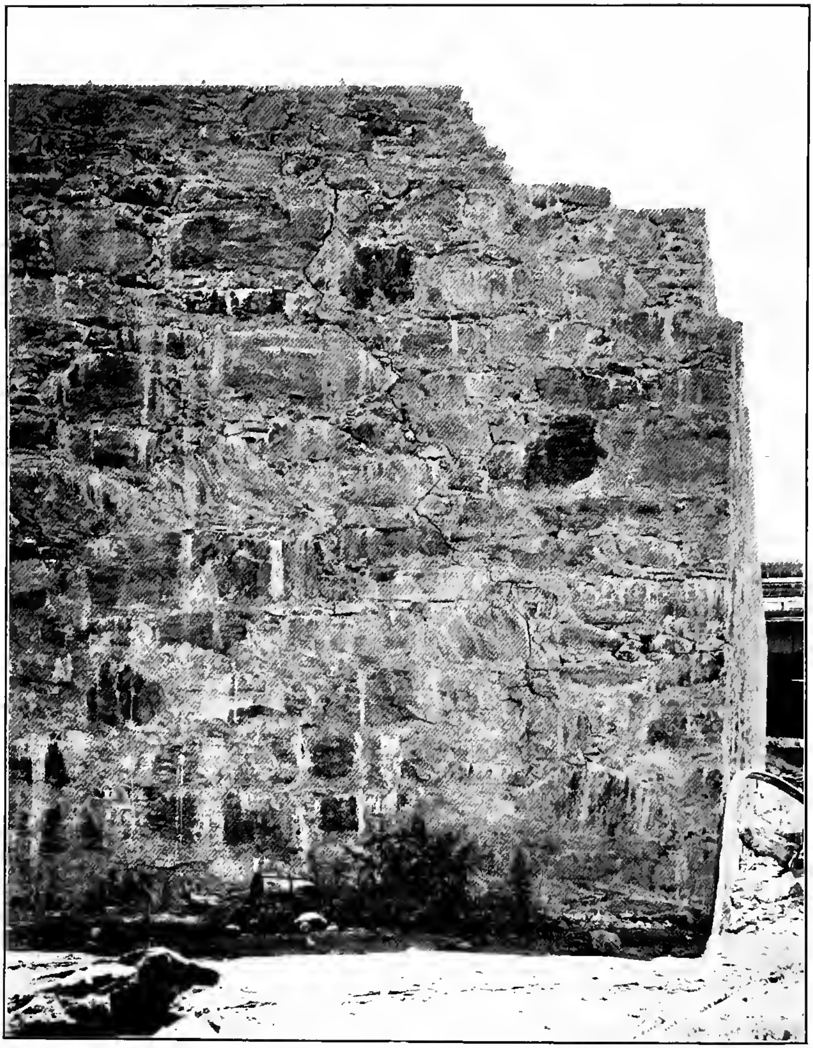


FIG. C.—Crack at sharp corner of wall due to tension component of thrust.

CHAPTER V

TEMPERATURE AND SHRINKAGE STRESSES, EXPANSION JOINTS, WALL FAILURES

In the setting and curing of concrete and in the seasonal variations in temperature, stresses are induced in retaining walls which, because of the longitudinal continuity of the wall, must be resisted by the material itself. Plain concrete monoliths, unreinforced, will crack at well defined intervals because of failure of the material through tension. It is quite difficult, despite the insertion of rods to prevent cracks. It is possible, however, by properly introducing rods, to concentrate the tendency to cracking at assigned intervals and then, to avoid unsightly breaks, to place an actual joint at such places. Reinforced walls are at times built without any joints and seem to have such proper reinforcement that no cracks are apparent.

A theoretical discussion of the temperature changes that may be expected within masonry masses may be interesting as indicating the expected amount of stresses to be anticipated by rod reinforcements.

It is patent, that the further from the exposed surface a point is within the mass, the smaller will be the variation of temperature at that point for any given surface range of temperature. Experiments have been made to determine this range at various points, covering quite long periods of time[1] and in recent masonry dam construction, automatic temperature recording devices have been incorporated in the work so that an exhaustive record of the variation of temperature is available.

It seems desirable to attempt to express, mathematically, this distribution of temperature and, in view of the fact that the theoretical results so obtained are reasonably in accord with the experimental results, they should prove of service in making provision for temperature stresses in masonry structures.

[1] *Trans.* A.S.C.E., Vol. lxxix, p. 1226.

The variation of seasonal temperatures at the surface may be given by an expression of the form,

$$u = A + B \cos \frac{2\pi}{T} t \tag{170}$$

in which u is the temperature, A and B are constants, T is the period of change and t is the time.

In the distribution of heat through large masses, where the temperature at the surface is a function of the time, it can be shown[1] that the temperature u at any distance x from the surface at the time t is

$$u = A + Be^{-kx} \cos (2\pi/T - kx) \tag{171}$$

in which e is the base of natural logarithms and $k = \frac{1}{a}\sqrt{\frac{\pi}{T}}$. a^2 is known as the coefficient of thermal diffusivity, which, for concrete (Smithsonian Physical Tables) is 0.0058 in the C.G.S. system.

The maximum range of temperature occurs between t equal any integer say n and $t = n + \frac{1}{2}$. At the surface this range becomes, from (170) $2B$; at any point x from the surface the range is from (171) $2Be^{-kx} \cos kx$. The ratio of the range at any point x to that at the surface is

$$e^{-kx} \cos kx = I_x \tag{172}$$

and if U is the surface range, that at any plane x away from the surface is UI_x.

In discussing seasonal changes, the period T is one year, which must be expressed in seconds in accordance with the diffusivity constant a^2. For this period, and for concrete $k = 0.00413$. Table 28 shows a comparison with the results from the formula and those experimentally found in the records quoted above.[2]

The daily range may in itself be taken as periodic and expressed by (170) and (171). For this period, one day expressed in seconds $k = 0.079$. Table 29 gives a parallel comparison between the theoretical and the experimentally determined range.

It is seen, from a study of the daily variation of temperatures that the surface range is rapidly decreased a few inches from the surface. In designing masonry structures it is sufficient, in making provision for the temperature range to take a seasonal range based on about weekly averages. For climates in the

[1] W. E. Byerly, "Fouriers Series and Spherical Harmonics," p. 89.

[2] Tables for ϵ^{-x} are to be found in Pierce, "A Short Table of Integrals."

Middle Atlantic States, this range is about 40° either way from the mean.

TABLE 28

x	I	Theoretical range	Actual range
0.0	1.00	75	75
1.0	.87	65	
2.0	.76	57	
3.5	.57	43	32
5.0	.42	31	
10.0	.09	7	12
20.0	.04	3	0

TABLE 29

x	I_x	Theoretical range	Actual range
0.	1.00	50	50
.25	.45	22	
.50	.11	5	
1.0	.07	3	2
1.5	.02	1	
2.0	.01	1	1
2.5	.002	0	
3.0	.000		
3.5			0

If the unit stress developed by a change of one degree in the temperature is s and if the surface range is U, then the stress at any x is sUI_x and the total stress across a section of thickness w and unit width is

$$sU\int_0^w I_x dx = sU\int_0^w e^{-kx}\cos kx dx = sUcw, \tag{173}$$

where

$$cw = \frac{1}{2k}\left\{e^{-kw}(\sin kw - \cos kw) + 1\right\} \tag{174}$$

and the average unit stress over the section is csU. Table 30 gives the value of c for various values of w.

TABLE 30

w	Seasonal change	
	c	j
1	.95	.48
2	.87	.47
3	.82	.46
4	.75	.43
5	.70	.42
6	.65	.41
7	.60	.39
8	.55	.37
9	.51	.35
10	.47	.33

If E denotes the modulus of elasticity for masonry and n the coefficient of expansion,

$$s = nE \tag{175}$$

For concrete this value of s is about ten pounds per square inch, for every degree change in temperature (Fahrenheit).

Replacing w in (173) by the area of the concrete section A_c, the total stress across a section is

$$csUA_c. \tag{176}$$

Let the range of temperature where the steel rod is to be placed be U' and let the area of steel be A_s, with the ratio of steel to concrete area, as before p. The stress developed in the steel by a change of one degree is s' and will be ns, with n the ratio of the two moduli (see page 86). The total stress across a section because of a surface range of U is then

$$csUA_c + A_s s'U'. \tag{177}$$

The concrete can take f_c pounds per square inch before failure and the steel can take f_s pounds per square inch up to its elastic limit. The resisting section to the above temperature stress is thus

$$f_sA_s + f_cA_c = f_spA_c + f_cA_c \tag{178}$$

Equating (177) and (178) and solving for p

$$p = \frac{csU - f_c}{f_s - s'U'}. \tag{179}$$

For example, take a range from the mean, as above of 40°, and average slab thickness of two feet, $f_c = 200$ pounds, and $f_s = 45{,}000$ pounds. From the Table 30 $c = 0.87$, and since for a cantilever wall, where the vertical rods are at the rear face it is customary to likewise place the check rods (for convenience of construction) at the rear face from Table 28 $I_x = 0.76$, whence $U' = 0.76 \times 40° = 30°$. The required ratio of steel is then, from (179) with $s' = 15 \times 11 = 165$

$$p = \frac{0.87 \times 10 \times 40 - 200}{45{,}000 - 165 \times 30°} = .0037$$

Specifications usually require about ⅓ of one per cent. of steel for temperature reinforcement, which agrees fairly well with the above value just found. It is seen that a steel of high elastic limit should be specified. The expansion coefficients of both steel and concrete are fairly alike so that there is no stress induced between steel and concrete because of this temperature change.

Shrinkage.—Unlike temperature stresses, the stress due to shrinkage is induced in the steel by the action of the concrete in curing and drying out. While there is little definite regarding the theory of shrinkage experimental data has shown[1] that the shrinkage of concrete is about 0.0004 of the length. In the same paper the stress due to the shrinkage is given by the expression

$$f_c = CE_c \frac{np}{1 + np} \tag{180}$$

C is the coefficient of shrinkage (given above) E the concrete modulus, n and p the usual concrete functions. The stress induced in the steel is then

$$f_s = f_c/p \tag{181}$$

With the amount of reinforcement as specified for temperature stresses, the concrete stress is seen to be, from (181) 40 pounds per square inch and the corresponding steel stress about 12,000 pounds per square inch.

To provide for temperature and shrinkage stresses the rods should be placed at right angles to those put in to take care of the earth pressure stresses. Since the maximum temperature ranges occur at the surface, it is desirable but not necessary that the rods be placed at the surface. It has been seen that for the cantilever walls it is not feasible to place the rods at the face. Generally these rods are woven in with the vertical stress rods.

Settlement.—The settlement of a wall is intimately connected with the character of its foundation. From the discussion on foundations in Chapter 2, it was seen that certain types of soil require a distinct distribution of loading; the more yielding the soil was, the more urgent it became that the distribution of soil pressure be a uniform one. It is generally agreed, that, within reasonable limits (these limits determined by the structures adjacent to or supported by the wall) a uniform settlement of the wall is harmless, since, with a proper spacing of expansion joints, or with carefully distributed reinforcement, no cracking will occur in the wall body. Unequal settlement produces

[1] See *Bulletin* No. 30, Iowa State Agricultural College.

cracks, which not only prove unsightly, but may indicate incipient failure.

Unequal settlement may be expected on yielding soils where the distribution of pressure is not a uniform one; where the character of the soil changes, one type yielding more than the other type; at junctions of new and old work, the old work having settled with the soil, the new, in gradually taking up its settlement, necessarily destroying the bond between the new and old work. The remedies for these are quite obvious. For the first case it has been sufficiently emphasized that there must be a uniform distribution of pressure. A joint should be placed in the wall wherever the character of the soil changes and especially between a yielding and non-yielding soil. Joints should also be placed between new and old work. It is a good detail, where settlement is expected, to reinforce the bottom of the footing with longitudinal rails or rods as shown in Fig. 90. Such reinforcement will tend to distribute any impending movement and thus prevent a crack.

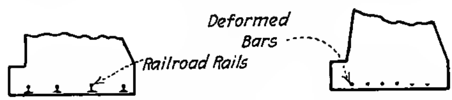


FIG. 90.—Bottoms reinforced because of threatened settlement.

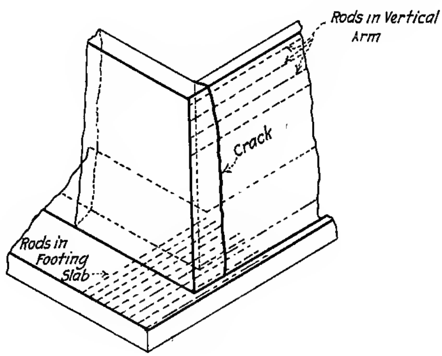


FIG. 91.

While of common occurrence it is poor practice to make a wing wall monolithic with the abutment, save on unyielding soils. The character of loading for each type is radically different making unequal settlement inevitable. Reinforcement across the junction of the two walls is uncertain and cracking may occur despite such rods. A photograph (Plate No. 2*a*) and Fig. 91 are given illustrative of this.

While settlement is an uncertain problem, careful attention to the foregoing points will reduce to a minimum the chances of cracks on these accounts. Where the face of the wall is to receive special treatment or is to be panelled, it is vital that every precaution be taken against unsightly cracks. As in the case of foundations, the provisions to be made against expected settlement demand most mature engineering judgment. A large crack in a wall is usually an indication of lack of engineering foresight and where such work is adjacent to public highways, becomes unpardonable.

Expansion Joints.—Where movement is expected in a wall, due to any of the interior or exterior changes discussed in the foregoing pages, it is customary to attempt to localize such movement to small sections of the wall. For this purpose, vertical joints are placed in the wall at regular intervals and are constructed so that no movement can be carried vertically or longitudinally across them. Since it is desirable that a wall be kept in good line, the joints are usually so built to prevent transverse movement.

In a monolithic gravity wall, joints are essential and are customarily spaced at from 30 to 50 feet intervals. This makes ample provision for temperature and shrinkage stresses and makes it possible to have complete concrete pours from joint to joint. An excellent detail of such a joint is shown in Fig. 92, giving

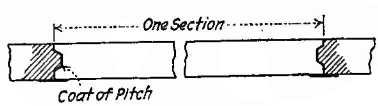


Fig. 92.—Expansion joints.

freedom of movement in every direction except a transverse one. One section of wall is poured completely between the joints. After the joints are given a coat of some tar or asphalt preparation the adjoining sections are then poured. To prevent seepage of water into the joint, several layers of fabric and tar are placed over the back of the joint and extend about 1½ feet on either side of it and from the row of weep holes at the bottom of the wall up to the top of the wall.

While, theoretically, steel-concrete walls can so be reinforced that expansion joints are unnecessary, such implicit confidence in the theoretical action of such rods is not wholly warranted and expansion joints are usually placed with about the same frequency as in plain concrete walls. The check rod system then distributes all movement to these joints and the wall is surely safe against cracking. Mr. Gustav Lindenthal[1] has stated that expansion joints are a source of danger because of the possible accumulation of water in them with a threatened wedge action due to ice formation. Accordingly, in the walls of the New York Connecting Railroad, described on page 127, no joints were used, full dependence having been placed in ½ per cent. of reinforcement to take up whatever secondary stresses were induced by temperature changes, shrinkage and settlement. General engineering practice is, however, not in accord with this view and expansion joints are almost universally used in reinforced concrete walls.

The details of an expansion joint for the cantilever wall are simple and may be made the same as the detail for the gravity wall shown in Fig. 92. For the counterforted and other slab types of wall, a break cannot be made in the face without providing a special detail. It is, of course, possible, in the case of counterforted walls, to build two adjoining counterforts with the

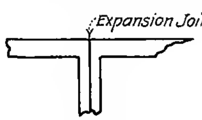


FIG. 93.

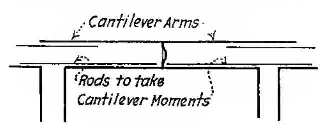


FIG. 94.

joint immediately between them as shown in Fig. 93, but such a detail is necessarily a costly one and to be avoided. Generally the joint is made midway between the two buttresses and the slab in between is made up of two cantilevers as shown in Fig. 94. The bottom slab, buried in the ground can usually be made continuous and the expansion joint need only extend to the bottom of the vertical slab. This applies equally well to the cantilever type of wall.

In stone masonry walls it is inexpedient to place any joints in the wall, but where the stones have carefully been bedded any

[1] *Engineering News*, Vol. 73, p. 886.

movement is usually taken up and distributed by the mortar joints. It is essential, of course, that there be the proper ratio of headers to stretchers to effectively distribute all such movements.

Construction Joints.—Any break in the continuity of pouring a wall, other than at an expansion joint, leaves a joint in a wall, which is usually termed a construction joint. It is not generally possible to pour a section of a wall between expansion joints completely in one continuous operation. It is impractical, usually, to, indicate such construction joints in advance, due to the exigencies of field conditions. The steps in pouring are generally: the bottom slab is poured; the vertical is later poured in as few operations as possible. While such a sequence does not give the ideal location for such joints, by the proper keying and cleaning of the construction joints, the strength of a wall may be satisfactorily maintained. It may be interesting to note a series of tests on the efficiency of various modes of treating a construction joint to insure a proper bond between the old and new work.

H. St. G. Robinson, Minutes of the Proceedings, Inst. of C. E., Vol. clxxxix, 1911–1912, Part III, p. 313, has performed the following series of tensile tests taking the efficiency of a solid prism as 100 per cent. A series of five tests upon this solid prism gave an average ultimate strength, in tension, of 329 pounds per square inch.

For the abutting faces (new and old) merely wetted, the efficiency of such a joint was 38.3 per cent. of the solid. A series of five tests gave an average ultimate strength of the joint of 126 pounds per square inch.

For the abutting faces roughened and wetted the efficiency was 56.2 per cent. of the solid. A series of six tests gave an average ultimate strength of the joint of 185 pounds per square inch.

For the abutting faces treated with acid the efficiency of the joint was 82 per cent. of the solid. An average of six tests gave an ultimate strength of 270 pounds per square inch.

For the abutting faces roughened and grouted the efficiency of the joint was 85.5 per cent. of the solid. An average of four tests gave an ultimate strength of the joint of 281 pounds per square inch.

From the above it is evident, that by cleaning and grouting the surface on which the new concrete is to rest almost the full efficiency of the joint will be attained.

It must be noted that construction joints in the face of a wall leave a permanent, and often unsightly mark. This matter is discussed somewhat in detail in a later chapter.

It is now possible to complete the reinforced concrete design of Chapter 3. The secondary rod system for temperature, shrinkage and settlement may now be added to the sections shown in that chapter. For simplicity of construction the rods are usually attached to the primary system of the wall. In the "L" and "T" walls the rods are horizontal as shown in Fig. 95. If the distance between expansion joints is too large, or if there are no expansion joints, it becomes necessary to splice these rods. The rods are carried beyond the point of splice each a distance sufficient to develop the rod in adhesion.

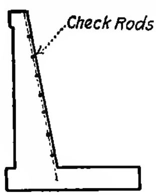


Fig. 95.

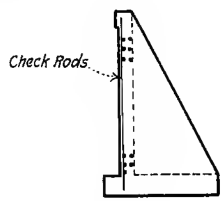


Fig. 96.

While strictly, such rods are unnecessary in the footing, they will act as a distributing system in case of threatened settlement.

For the counterfort and other slab sectioned walls, the check rods are vertical and placed at the outer face, see Fig. 96.

Small size rods are desirable for this secondary system, both on account of the adhesion area and because of the ease in handling the long lengths. A high elastic limit steel should be specified (see specifications at end of book).

Wall Failures.—It was a famous maxim of Sir Benjamin Baker, that no engineer could claim to be experienced in the design and construction of retaining walls until he had several failures to his credit. Such, however, is not the viewpoint of the modern engineer. It is to-day clearly apparent that walls, when they do fail, fail for definite reasons that can generally be anticipated and for which provision can be made. It is necessary, not only to find a proper foundation for a wall, but also to take extreme precaution that such a foundation will be maintained permanently in its proper condition. It is essential to guard against possible saturation of the bottom and against erosion of the soil beneath the toe by streams of water which, if long continued, reduce the bearing capacity of the soil and lead to subsequent failure. A majority of partial and complete wall failures are clearly attributable to foundation weakness developed subsequently to the construction of the wall.

Cases of failure due to excess of overturning moment over stability moment are rare. It is possible that in placing the fill behind the wall, material may be dropped from some height, either striking the wall or setting up vibrations in the retained mass that may exert an excessive action upon the wall. A failure of a barge canal wall in New York State[1] is alleged to be due to this cause. The fill behind the wall was saturated and in a quaking condition. The material was dropped behind the wall by a clam shell, from considerable height, setting up heavy vibrations in the mushy mass, which eventually destroyed the wall.

Care should be observed in dropping big stone from trestles or from the partially built embankment against the back of the wall. While complete failure is unlikely, small cracks, due to the impact may be developed. At first not serious, later, due to frost and other weathering action, they become unsightly, marring the face and eventually develop erosive gullies.

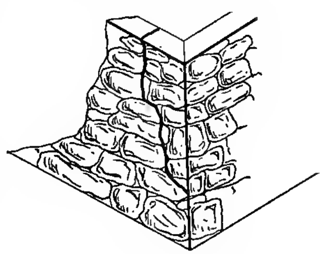


FIG. 97.

The improper and insufficient attention to drainage (discussed in a later chapter) may permit the accumulation of water behind a wall increasing the pressure to such a degree as to push the wall out of line.

Among minor instances of possible causes of failure, complete or partial, may be mentioned the following.

Lack of expansion joints, or joints spaced too far apart.

The junction of radically different types of walls without a proper joint. Thus a wing wall to an abutment; a very light section wall to a heavy section wall. Walls on different foundations. Walls carrying a building load. A sharp angle in a gravity wall, so that there is a component of the earth pressure acting in tension (see Fig. 97, and Photograph Plate No. 3*a*).

[1] *Engineering News*, Vol. 67, p. 384.

In the *Trans.* Engineer's Society of Western Pennsylvania, Vol. 26, it was noted in gravity walls, where the base varied from ⅓ to ½ the height, that:

"Such failures as have occurred have been due, to the most part to poor construction and lack of drainage."

In discussing the action of clay, both as a fill and as a foundation material, Bell, Minutes of the Proceedings, Inst. C. E., Vol. cxcix, 1914–5, Part 1, p. 233, notes that:

"It was disquieting to note the high percentage of failures in works constructed in clay. Taking all the available records of works subject to earth pressure, which had failed, it appears that 70 to 80 per cent. referred to works constructed in clay. While every one recognizes that clay is a treacherous material and that it will always claim a substantial percentage of total failures, still this preponderance is remarkable and would perhaps of itself indicate that there is something wrong with existing methods."

Some Wall Failures.—Chas. Baillarge[1] has pointed out that the life of the retaining walls in Quebec has been but a brief one. They were designed upon the assumption of a dry granular fill and the base, accordingly was made from one-fifth to one-third the height. Subsequently the filling became waterlogged and since no weep holes or other drainage had been provided to dispose of such accumulations of water, the excessive pressures developed caused the failure of the walls.

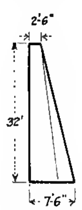


FIG. 98.

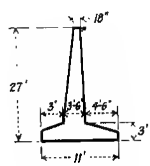


FIG. 99.

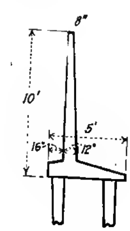


FIG. 100.

Mr. Lindsay Duncan[2] has described the tilting and settling of an abutment prior to the setting of the span upon it. The section of the abutment is shown in Fig. 98. The wall rested upon an adobe foundation and surface waters gradually softened the

[1] *Engineering News*, Vol. 45, p. 96.
[2] *Engineering News*, Vol. 55, p. 386.

adobe, causing the wall to tip forward. An ingenious method of reinforcing the wall and bringing it back to line is described in the above article.

Due to the failure of a dam[1] the foundation of a wall shown in Fig. 99 was washed out, and a section of the wall between two expansion joints was moved out.

A wall of section shown in Fig. 100 was placed in an old creek bed.[2] The freshet from a spring thaw undermined the foundation washing away the soil adjacent to the piles. Excessive loads developed on the piles, and these failed causing the wall to settle about two feet.

A wall failure due to excessive overturning moment is described in the *Engineering Record*, Vol. 41, p. 586 (see Fig. 102). A wall of rectangular shape, of small stone rubble, supported a

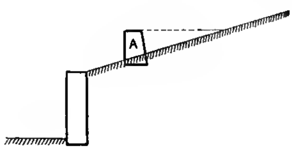


FIG. 101.

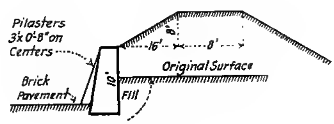


FIG. 102.

fill slightly surcharged. It had already given evidence of incipient failure by bulging in several places. In grading an adjacent lot, an additional fill supported by the wall "A" was placed upon the old embankment, followed by the complete failure of the wall.

A wall shown in Fig. 101, supported a reservoir embankment adjacent to a roadway.[3] The brick pavement lining the road was taken up, and the wall slid forward from one to two feet, and in several places tilted out of line about 6 in. This seems to be an instance of insufficient frictional resistance between the footing and the wall—the brick pavement supplying the necessary resistance to prevent the forward movement of the wall.

[1] *Engineering News*, Vol. 63, p. 285.
[2] *Engineering News*, Vol. 61, p. 503.
[3] *Engineering Record*, Vol. 44, p. 7.

PART II
CONSTRUCTION

CHAPTER VI

PLANT

Plant Expenditure.—With the exception of very small construction jobs amounting to but a few hundred dollars in value, it is necessary to employ tools, machinery and other implements to supplement and replace manual labor. Such auxiliary appliances are termed *plant.*

There are no fixed relations between the amounts to be expended on plant and the total value of the work contemplated. The principal factors of a general nature determining the amount of plant required are, the yardage of concrete wall, the time given in which to build the wall and the manner of the distribution of the wall over the work. Few jobs are exactly alike or sufficiently similar that the plant requirements become identical and it is a matter of economy to so buy plant that its cost less its salvage value, if any, at the completion of the job, is carried by this job alone. This permits a careful study of the field conditions and insures a selection of plant most fitted for this work. It is a slogan of most contractors, that if a job is not worth the plant, the job is not worth having.

"Inasmuch[1] as plant is in reality but a substitute for labor, it would seem obvious that no more should be invested in plant than will yield a good return. This relation between plant and labor is apparently ignored in many instances, and plant charges are incurred out of all proportion to the volume of work to be done. The ultimate comparison, whether made directly or indirectly, between hand labor and the proposed plant, or between this and that plant, must be made if the selection is to stand the test of experience.

"The selection of plant, the purchase of this or that machinery, has to a large extent been more or less haphazard. Contractors and engi-

[1] From "Concrete Plant" issued by Ransome Concrete Machinery.

neers, experienced and successful men, have been slow to awake to the possibilities for loss or gain afforded by plant selection; but it is nevertheless deserving of careful study.

"There seems to be a strong tendency toward excess in plant expenditure and a fact worthy of note is the tendency toward simplicity in plant upon the part of engineers and contractors whose experience and success in the field entitles them to be considered as leaders.

"In estimating plant cost, various elements other than first cost of plant must be carefully considered. Cost of installation, including freight, cartage, labor, etc., cost of maintenance, cost of removal, interest upon the investment, must be considered on the one hand, as against the resultant saving in labor and salvage value of the plant on the other.

"In general the plant best suited to the work is cheapest, regardless of whether or not it costs a few dollars more than something less suited to the conditions. First cost is perhaps less important in influence on final results than cost of operation and maintenance. In many cases a higher salvage return will offset to a large degree higher first cost. First cost, too, is a definite constant. It can be positively assessed and proper allowance made for it in estimating, in this respect differing from maintenance, which is an unknown quantity subject to great variations."

Standard Layouts.—There are certain types of work, again, generally speaking, for which the plant layouts are obvious. Thus a concrete wall in a compact area, all within strategetic reach of a center not exceeding some maximum distance away, calls for a central mixing plant and a tower system of distribution. In track elevation work, to eliminate grade crossings, the availability of a track adjacent to the proposed wall, permits the use of a compact concreting train. Usually conditions are not so typical and local topographical conditions, as well as the character of the work play an important role in determining the character of the plant best suited for the job.

Arrangement of Plant.—It may be stated as almost axiomatic, that, that wall is most economically built which, other things being equal, is most expeditiously built. This necessitates a certain degree of flexibility in the plant that little time may be lost in bringing concrete to the forms awaiting it.

"The character[1] and arrangement of plant depend to a large extent upon local conditions, such as contour of ground. The general layout of the work, while the manner in which the materials are to be delivered to the site, whether in cars or in wagons, regularly or irregularly, has an important bearing upon the type of plant. Similarly, the matter of

[1] *Ibid.*

total yardage to be placed, of time limit set for the work, of bonus or penalty, will have a bearing upon plant selection.

"Other considerations which may affect materially the selection is the amount of ground available for material storage, and the time of the year during which the operation must be carried on, winter work requiring very different plant arrangement from summer work.

"Contour of ground is principally effective in determining the location of the plant with respect to the work and the storage of materials. For example, a steep slope will often make advisable a system of overhead bins with gravity feed, which under other conditions would not be advisable.

"The general layout of the work will usually be the determining factor in the adoption of means for handling mixed concrete, subject, of course, to modifications imposed by total yardage, etc. It may make for the adoption of two or more separate installations rather than one central plant or it may cause the adoption of a portable plant rather than a stationary one.

"Delivery of materials is principally effective in determining the arrangement for the storage of raw materials.

"Total yardage, time limit, etc. are generally the controlling factors in determining the amount available for plantage."

Subdivision of Work.—It seems natural to divide the plant necessary for concrete retaining walls into three subdivisions: (1) the plant to bring the materials to the mixer; (2) the mixer, (3) the plant to bring the materials from the mixer and place it in the forms.

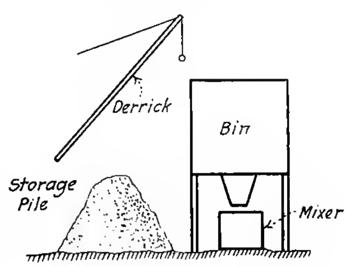


FIG. 103.—Loading bin by derrick from storage pile of aggregate.

1. When the layout of the work is such that one or a few central plants may be used, this problem is comparatively simple. The material is dumped alongside a storage bin and is fed to this bin as required, the bin having a hopper to drop material into the mixer. See Fig. 103. It may be possible, due to the advantageous location of this bin below the delivery point, that the material cars or wagons may unload directly into the bin. This requires a regular and reliable delivery system to keep the bin constantly supplied, since, with sporadic delivery of material the concrete work would frequently be delayed. Usually the material is allowed to accumulate in a

storage pile near the bin and is fed from this pile to the hopper bin by a derrick, with preferably a clam shell, to save the labor of loading the skips.

When a central plant is not used, the material is distributed along the site of the work in small piles. It must be remembered that when the material is distributed in this fashion, there is considerable loss due to rehandling, to the gathering of foreign matter such as dirt, etc., and to the inevitable loss of the bottom portion of the pile on the ground. If the material is to be on the ground for some time then a large portion of it may be lost on account of the weather. Such losses may amount to quite a large percentage of the material ordered and proper allowance must be made to determine the final net cost of the material in the concrete.

For this latter mode of the distribution of material the mixer is usually fed by wheelbarrow from the nearest pile. Other modes of getting the material to the mixer are easily determinable from the local environment.

Mixers.—The selection of a proper mixer is comparatively simple. The requirements of good concreting (as described in a later chapter) should be noted and a type of mixer chosen that will make it possible to carry out these requirements. The necessary capacity of the mixer is readily determined from the expected daily output required to prosecute the work within the assigned time limit. Naturally a mixer attached to a central mixing plant if run continuously will have a greater output than one of like capacity carried about the work. The catalogues of the manufacturers of the various types of mixers can be consulted to good advantage and, with the advice of their experienced salesmen, a type most suitable for the work can readily be selected.

"It[1] is true that one mixer may have an excess of power with resultant acceleration of the various operations going to complete the mixing cycle, one machine may be quicker in mixing or discharging than another; but these differences will influence the final result less than a defective organization. For example, it is common practice to employ extra men to fill wheelbarrows, a practice which increases the cost of this work twenty-five to thirty-five per cent. according to whether or not the wheeler helps fill his own barrow. Similarly it is common practice to handle mixed concrete in small wooden or iron barrows holding an average of two cubic feet. By furnishing substantial runways and the adoption

[1] *Ibid.*

of carts an average load of 4.5 cubic feet can easily be handled. It is to such elements of organization that attention should be directed, if you would cut down the cost of operation. Properly handled, concrete plant becomes an important factor in setting the pace for the work.

"Cost of installation includes freight, cartage and erection, elements varying with the character of the plant, location of the work, with respect to the source of supply, etc. * * *.

* * * "No other class of machinery is subjected to the severe usage imposed on concrete machinery. The nature of the materials handled make for excessive wear, to which should be added the fact that the machinery is ordinarily handled by a class of labor not calculated to give it the intelligent care and attention to which it is properly entitled. It is to long experience upon the part of the manufacturer in this special field that the purchaser must look for protection against failure, under the severe conditions which actually prevail in the field. The history of success in this line of work is a history of constant changes in design, a story of heavier, stronger parts, of adapting the machine to the character of the work by reducing parts to a minimum.

"The fewer parts your machine has, the less likely it is to get out of order, and the more readily the operator of ordinary capacity can keep it in working order.

"Considered broadly, mixers may be divided into Drum Mixers, Trough Mixers, Gravity Mixers, Pneumatic Mixers.

"Drum Mixers may again be divided into Tilting Mixers (Smith Type) and Non-Tilting (Ransome Type). In the former class the mixing drum is mounted on a swinging frame, and the discharge of the mixed materials is accomplished by a tipping of the frame and drum. In the latter class mixed materials are drawn out through a chute inserted in the drum.

"Trough mixers, as a whole, may be designated as Paddle Mixers, though the paddles may vary in form from a broken worm, through the various stages, to the continuous worm and the conveyor flight may be single or double, of varying or uniform pitch.

"Gravity Mixers are of the same general characteristics, depending for success upon a series of deflectors, chains, pegs, or conical hoppers, for the mixing action. They are not adapted to building work in any case and do not deserve serious consideration here.

"Pneumatic Mixers include the various types of pneumatic mixers developed during the past two or three years by Wm. L. Canniff, A. W. Ransome, McMichael, Eichelberger. In the Ransome and Canniff mixers, the materials are first mixed by air in a container, and the mixed concrete then forced out through pipes to its ultimate destination. In the McMichael and Eichelberger machines the materials are assembled in a container and forced through pipes without premixing. These latter machines depend for successful results upon such mixing action as may take place in transit through the pipe. Pneumatic mixers are all expensive to operate and cannot be used to advantage except in special cases."

Distributing Systems.—There is greater latitude in the selection of plant for a distributing system than in the selection of plant for the two prior operations and since this portion of the work is the most costly of the three, greater care should be spent upon the proper selection of the necessary plant.

A retaining wall covers, generally, a long narrow strip, making a compact, single distributing system from a single central plant usually out of the question. Nevertheless, heavy walls, with large concrete yardages within fairly restricted areas may permit, economically the use of one or more central distributing plants.

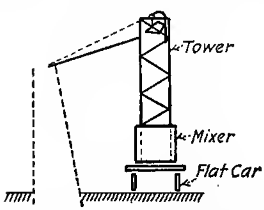


FIG. 104.—Pouring concrete by tower and mixer mounted on flat car.

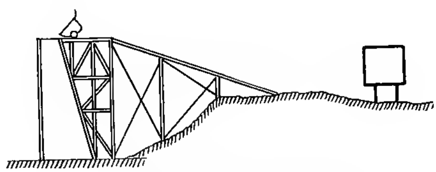


FIG. 105.—Pouring concrete from platform erected on trestle.

The greater mass of the wall lying above the ground surface, the concrete must be raised to permit its placement within the forms. This is accomplished by several methods. The mixer, a travelling one, may be raised and its contents spouted directly, by gravity, into the form. The mixer may remain on the ground and its contents raised and delivered into the form. Following are some possible methods of this latter mode of distribution.

(*a*) The mixer is on a flat car, with a tower and hoist (see Fig. 104).

(*b*) The mixer is on the ground and the concrete taken from it by cars, or barrows and run over platforms along the top of the form into the wall (see Fig. 105).

(*c*) A derrick takes the bucket from the mixer and dumps its contents either directly into the form or into a spouting device leading to the form.

(*d*) Tower distribution.

(*e*) Cableway distribution.

(*f*) Pneumatic distribution.

"The handling[1] of concrete through spouts or chutes is of comparatively recent development, and as in many other similar developments, there has been a tendency to overdo. Spouting systems have been installed on many buildings where the distribution might have been better done by barrow or cart.

"The installation of a spouting system is expensive, and should not be undertaken blindly, nor with expectations of abnormal savings.

"Spouting plants may be grouped under Boom plants, Guy line plants, Tripod plants. In the former, the spouting is mounted on a swivelled bracket at the tower end, and the outer end supported by a boom moves freely about the work. A second length of spout ordinarily completes the unit. This type of plant has a greater freedom of movement than either guy line or tower plants, but is not as free moving as might be desired.

"Many means have been tried to facilitate ready moving of the free end, none of them, however, proving entirely satisfactory. A suggestion has been made to counterbalance the free end, but this has not, as yet, been tried out thoroughly.

"In guy line plants, the spouting is suspended by ordinary blocks and falls from guy lines or from special cables set up for the purpose. In some cases the outer end of the cable is mounted on a portable tower or "*A*" frame and the blocks and falls are preferably arranged so that necessary adjustments in the line may be made from the ground.

"In Tripod plants movable towers are used to support the ends of various sections of spouting.

"It has been found by practical experience that concrete, thoroughly mixed and of proper consistency will flow on a slope of eighteen degrees, with the best results obtained at twenty-three degrees. These slopes, however, are based upon a rigidly supported chute. Where the spouts are supported from guy lines, the slope must be a little steeper, preferably from twenty-seven to thirty degrees. By proper consistency is meant a mixture with approximately one and a quarter to one and a half gallons of water to the cubic foot of material. There should be just as much water, as the material can carry without separation, so that the stone particles will be carried in suspension in the mass. There should be a sliding of materials down the spout rather than a rolling.

"Various types of spouting have been tried, ranging from round pipe to rectangular troughs. Best results have been secured from the use of 5-inch pipes, or 10-inch open troughs, the latter having the preference for flat slopes and the former where there is necessity for varying pitch, some of steeper pitch than named above.

"With open spouting, the use of line hoppers in connection with

[1] "Concrete Plants," Ransome Concrete Machinery, p. 23.

flexible spouting accomplishes satisfactorily the necessary changes in pitch. The greatest items of expense in spouting plants are first cost, installation and maintenance.

"Maintenance charges are particularly heavy. The ordinary stock spouting which is made of No. 14 gage metal will seldom handle more than two thousand yards without renewal. This is due to the abrasive action of the material, especially as affecting the rivets which join the various sections.

"In general we would say that whether or not you can use spouting to advantage must be carefully considered for each job. Where the work is light and scattered any attempt to spout concrete into place is foredoomed to failure."

"The economy[1] of distributing concrete through properly designed chuting plants has been demonstrated time after time, on all kinds of construction and it has been conclusively shown that properly proportioned, thoroughly mixed concrete may be conveyed to any mechanically practical distance without disintegrating the mass.

"Concrete should flow at a uniform speed of from seventy-five to one-hundred feet per minute. The best results are attained with the chute line pitched with a fall of one foot in four up to 150 feet radius. For longer distances the fall should be about one in three, starting with one foot in four and increasing the grade towards the discharging point."

When it is remembered that a cableway mode of distribution moves in but two dimensions *i.e.* in a vertical plane only and that its cost rapidly increases, and the amount of load to be carried, decreases with an increase in span, its use as a distributing system is usually discarded for the methods of distribution previously mentioned.

Below are given a series of descriptions of various plants used. While it is impractical to attempt to make a standard classification of construction problems, the illustrations selected are thought to be more or less typical and the character of the plant used probably the most fitted for the environment and character of the work at hand.

(A) TOWER DISTRIBUTION

Railroad station at Memphis * * * Ill. Cent. R. R. (see Fig. 106) for the skeleton layout of the work), *Engineering News*, Vol. 72, p. 629.

"The construction of the retaining walls and subway bridges was hampered by the necessity of providing for traffic. There were about

[1] *Bulletin* No. 23, The Lakewood Engineering Co.

60 trains daily, the heaviest traffic being from 7 A.M. to noon and 3 to 5 P.M. The only freight movements over this part of the line were in switching service. The great difficulties encountered were the limited space available, the handling of concrete while keeping clear of the trains and the inability of the contractor to get certain parts of the site delivered to him for work at the time desired. For all work * * * the storage space for materials was limited and it was necessary to regulate shipments of all kinds so as to be able to use the material upon arrival.

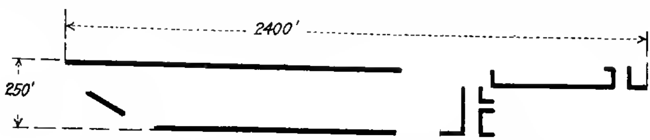


FIG. 106.—Layout of retaining walls and abutments.

"The concrete was delivered in place by spouting from elevator towers, using self-supporting trussed chutes. Two stationary plants with 100′ towers and one portable plant with a 50′ tower were used, each of the former being set up twice (in different locations) and the latter being shifted as required. Each had its mixer, and, in order to work to full capacity, a two-compartment material bin or hopper was erected over the mixer, holding about 30 cubic yards of stone and 15 cubic yards of sand. The materials were brought in railway cars and unloaded direct to the mixer bin or to small storage piles, there being little room for storage. A derrick with clam shell bucket took the material from the car or storage pile and dumped it into 1½ cubic yard cars, which were hauled up a cable incline and dumped into the material hopper. The incline had a four rail track in the lower portion and a three rail track at the top. * * * * * The maximum output per day was 550 cubic yards. The entire concrete yardage was 30,000 cubic yards."

(B) CONCRETE TRAINS

As has been previously noted, railway improvement work, such as track elevation or depression, permits the use of a compact concrete train. A typical piece of work is the track elevation work of the Rock Island lines, described in *Engineering News* Vol. 73, p. 670; Vol. 74 p. 1275 and Vol. 74 p. 890. The concrete plant which placed the necessary 30,000 yards of concrete for this improvement is described as follows:

"Concrete train consists of a mixer car, four to seven stone cars and two to four cars of sand. * * * * * * The mixer car is a thirty-five foot flat car, equipped with a ⅔ yard Smith non-tilting mixer 10 h.p.

vertical engine, 20 h.p. vertical boiler, 700 gallon storage tank and 60 gallon feed tank for the mixer. The machinery is housed the roof of the car being higher at the discharging hopper than at the ends of the car, thus forming an easy incline from the runways on the tops of the gondola cars to the charging hopper above the mixer. The mixer is located about 8′ from one end of the car and faces the end. It discharges the concrete into a swivelling chute which may be swung to discharge the end or either side of the car. This arrangement of pouring from different angles or from either end of the train eliminates the necessity of turning the mixer car (as required with the other types) and makes a considerable saving in working train space.

"The chute has intermediate openings, so that concrete can be discharged at different points. A man on top of the car regulates the charging of the mixer, the supply of water and the dumping of the concrete (see Fig. 107). Usually the mixer train stands on trestles

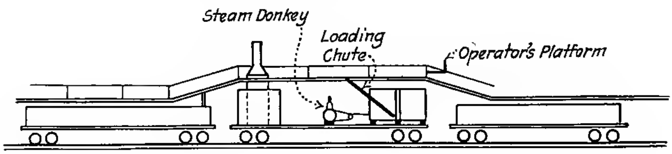


Fig. 107.—Connecting train.

and the concrete is spouted to the form beneath. For the upper part of the piers, it has been necessary to elevate the concrete, a crane and bucket being used to place the concrete in the forms.

"The mixer is designed to carry a tower and hoisting engine if required. * * * * * * * A valuable feature of the car is a powerful winch-head for a cable, which is anchored ahead. This enables the mixer car to move the train along as the work progresses, thus dispensing with the constant attendance of locomotive and crew.

"Each train is placing at the rate of 20 to 30 cubic yards per hour, with a monthly total for both trains of 11,000 yards of concrete."

Other instances of the use of similar work trains are mentioned below.

Engineering News, Vol. 75, p. 634. In filling in an old trestle, and building the neeessary retaining walls, a eoncrete train of three cars one mixing, one stone and one sand, were used.

Engineering News, Vol. 75, p. 1192. The interesting feature of the work train here was the fact that the hoist was operated by steam from the locomotive.

Engineering News, Vol. 75, p. 494. Fort Wagner Track Elevation. The concrete train worked on a temporary operating trestle, the track being out of eommission while the concrete train was on it.

Engineering Record, Vol. 70, p. 240. Chicago, Milwaukee and St. Paul.

The concrete train operated upon a trestle. A cableway on the concrete train took materials from the intermediate cars to the bins. This proved cheaper than tower cars and hoist cranes.

Cableway.—The use of a cableway for pouring the concrete walls of a viaduct is described as follows in the *Engineering News*, Vol. 72, p. 930 (see Fig. 108).

"Concrete material was delivered in cars on a siding and unloaded unto stock piles by a stiff-leg derrick mounted (with its engine and hoist) on a tower or platform some 15′ high. The same derrick and clam shell bucket handled the material from the stock piles to the 200 yard bins over the one-yard concrete mixing plant which was located just east of the structure and on the north side of the tracks.

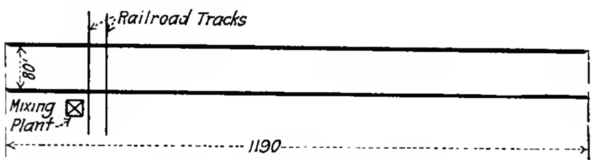


FIG. 108.—Layout for cableway.

"The cableway was 800 feet long with an 80 foot tower at the mixer end and a single bent 60 feet high at the further end. It was placed over each wall in turn and was shifted laterally 80 feet, from one wall to the other without being dismantled; this was done by placing timber dollies under the tower. Handling the 12,500 yards of concrete by cableway was economical as the amount of concrete at the ends of the walls is small and wheeling it in buckets would have been slow and expensive."

An interesting comparative analysis of the use of several different plant layouts for a series of similar pieces of work is described by Mr. Armstrong in the *Journal* of the Western Soc. of Engineers, Vol. 16. New Passenger Terminal: C. & N. W. R. R. The retaining walls enclosed a rectangular layout, bounded by two street crossings and the parallel easement lines.

The plant layouts to pour the walls were as follows:

(*a*) A cableway, placed on movable trucks was used, permitting the shifting of the towers to pour each of the walls. This plant did not prove economical and was of low capacity. The best run was 24 yards per hour.

(*b*) A runway with rails ran around the top of the wall forms. A derrick hoisted the buckets of concrete to a hopper which dumped into cars running along the form runway. This was cheaper than the cableway and had a capacity of about 33 yards per hour.

(*c*) In place of the derrick as above a short tower was used with a hoisting engine. The best average was 37 yards per hour. The dump cars ran as much as 500 feet away from the tower.

(*d*) A mixer, elevator and a hoist were mounted on a car and ran around the forms. This proved very unwieldy and could not get close to the forms. Less labor was needed here, however, since the dump cars were eliminated. The best results with this plant were about 25 yard of concrete per hour.

The following is a trite recommendation by the author of the above paper:

"It might be stated as a general principle in the design of plant that the capacity of the mixer should be made the determining factor in the output. The charging hoisting and conveying appliances should be designed with such a degree of flexibility as to preclude the possibility of retarding the mixing process by delay in charging the mixer or delay in removing the discharged concrete. The most economical mixer, other things being equal, is the one which discharges its mixed batch and receives its new batch in the shortest time."

Tower and Trestle.[1]—In concreting a high wall, 50 feet in height, the following description is given of the plant used.

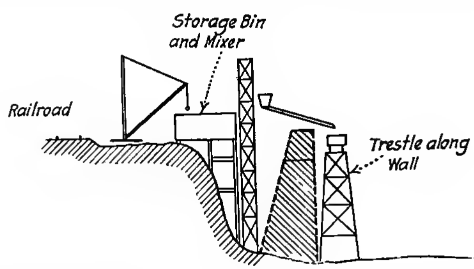


FIG. 109.—Central mixing plant. Combined tower and trestle distribution.

"For concreting the wall a very efficient plant was installed. A Hains gravity mixer was located about the center of the length of the wall, where it was easily loaded by derrick, from the adjacent high level railway. Concrete from the bottom or delivery end of this mixer was run into an elevator whence it was lifted to be dumped into a hopper and chute leading to another hopper with a bottom dump located on a frame just outside of the wall forms. All of the preceding equip-

[1] *Engineering News*, Vol. 73, p. 776.

ment was stationary, but alongside of the wall was a trestle which took concrete from the last noted hopper and dumped it through another chute to its proper place in the forms (see Fig. 109). The number of chutings given each batch should be especially noted."

In pouring a retaining wall for the Baltimore and Ohio Improvements[1] the inaccessibility of the site made it necessary to use a gantry crane device with a platform and stiff leg derrick, as shown in Fig. 110. A narrow gage railroad ran alongside the roadway and brought the concrete from a central mixing plant about one-half a mile from the work. The gantry served also to support the wall forms. (This work is also described on page 211 under winter concreting.)

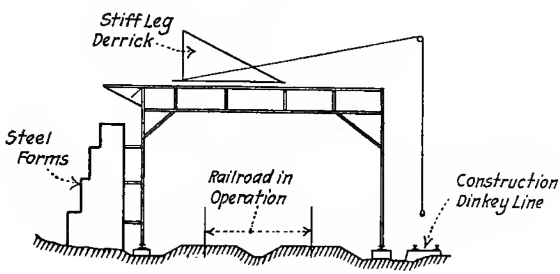


Fig. 110.

The following is an interesting description of several methods of handling the material on a bridge abutment job.[2]

"Hopper cars, derrick skips, elevator buckets and inclined chutes were combined in placing 3360 cu. yds. of concrete in abutments and approach retaining walls for a steel highway bridge across the Chicago & Northwestern Ry. at Wheaton, Ill. To give increased headway the bridge is at a higher elevation than the old span parallel to it, so that long inclined approaches were required, practically at right angles to the bridge, as shown by the accompanying plan (see Fig. 111). Each approach has a retaining wall on one side, and the wall on the south side of the railway is about 600 feet along.

"A concrete-mixing plant was located beyond the end of the cut. Sand and gravel were unloaded from cars into stock piles on the side of the adjacent fill, and the stone was loaded into an elevated bin by a derrick with a grab bucket. The sand was wheeled to the loading chute. The mixer discharges the concrete into a sidegate hopper car.

[1] *Engineering News*, Vol. 76, p. 269.
[2] *Engineering News-Record*, March 13, 1919, p. 553.

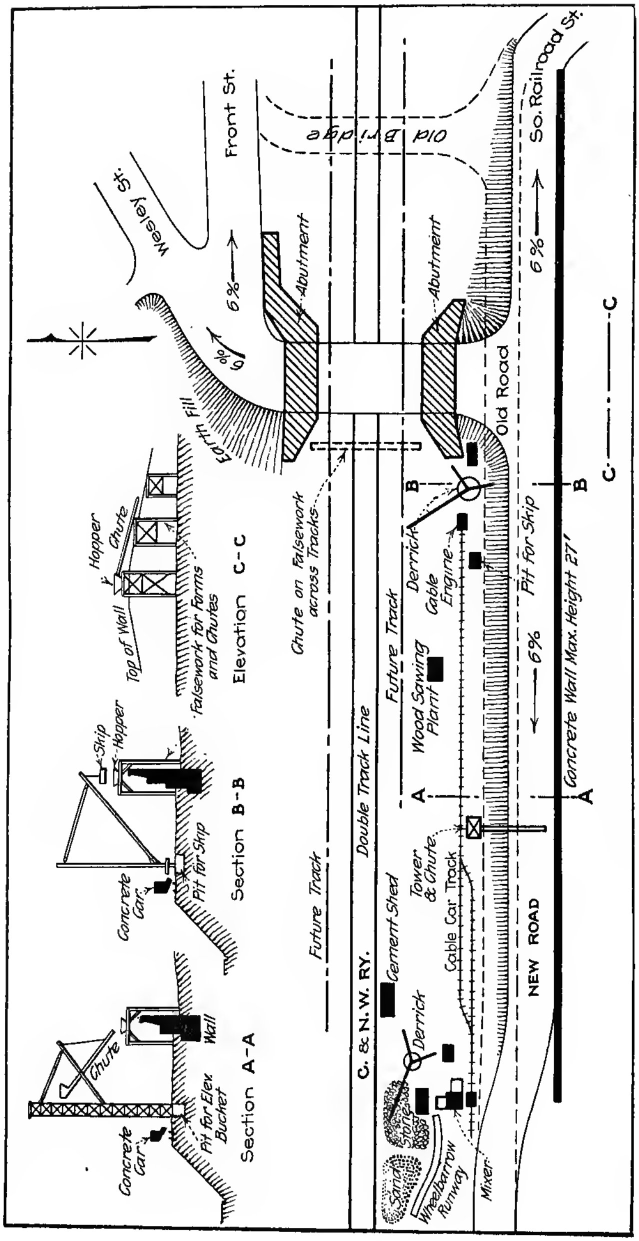


Fig. 111.—(*From Engineering News-Record, Mar.* 13, 1919, *p.* 553.)

"Between this plant and the bridge site an elevator tower with a chute was erected, while beyond this and close to the abutment was a guyed derrick, both tower and derrick being on the narrow strip between the old road and the top of the cut. A narrow-gage track with one automatic siding extended from the mixer plant to the tower and derrick. This was operated by an endless cable with a hoisting engine placed near the derrick and on it the concrete was handled in the hopper cars mentioned above.

"At first the concrete was delivered to the elevator buckets and spouted to the forms. The tower chute or spout extended across the road and delivered the concrete into lateral chute supported directly above the forms by falsework. This sufficed for about one-half the length of the wall.

"For the remainder of the work the cars ran up to the derrick and discharged the concrete into a home-made wooden skip which was placed in a pit at the side of the cable track and was handled by the derrick. A movable gate was fitted to one end of the skip, with inclined boards on the inside to guide the concrete to the opening and to prevent it from being pocketed in the corners. The skip was dumped into a feed hopper at the summit of the inclined chutes carried along and above the forms for falsework.

"Concrete for the abutment on this side of the railway was placed directly by the derrick and skip. For the abutment and short wall on the opposite side and inclined chute was extended across the tracks, having a feed hopper at its upper end within reach of the derrick. At its lower end was a vertical drop line leading to the head of the chutes over the abutment form, these being shifted to deliver the concrete in the desired portions of the form.

"Baffles were used at the discharge ends of the long chutes to prevent segregation of the concrete as it was deposited in place. In some cases these were short troughs secured to the trench bracing or form struts, being placed opposite the end of the chute and sloping in the opposite direction, so that the direction of the concrete was reversed just before its final discharge."

Conclusion.—To summarize, plant is employed solely to effect an economy in the construction of a wall. To use plant that does not, in the final analysis, show a saving because of its employment, is unjustifiable. It is understood, of course, that all economies accomplished are legitimate ones; not such as are made at the expense of good construction.

Bearing in mind that most jobs are unique in character, plant should be bought for the sole requirements of the work at hand and in proportion to the total cost of the work. Such illustrations of actual construction work as have been cited may furnish an idea of general plant layouts—but each piece of work contemplated must be studied out individually that advantage may be taken of all local situations, such as topography, railroad and highway location and the like.

Naturally some pieces of plant are standard. A good mixer, hoists, derricks and small plant such as barrows, carts, shovels, etc., may survive a job and be easily fitted to other work. This is a matter of judgment. Little mistake is made, however, if plant is procured for one job and charged off to that one job. The cost accounting and the preparation of bids for new work are thus vastly simplified and each job carries itself, the ideal contracting condition.

In the following chapters some stress is laid upon the requirements of good form work and of good concrete work. To secure the proper results as indicated in those chapters requires a coördination between the plant and the methods used and plant that will make it difficult to secure the desired results should not be employed. It is only just to add that plant manufacturers are keenly alive to the demands of modern construction and strive to coöperate with the engineer and contractor to supply machinery that will aid in turning out flawless work.

Plant Literature

Ransome Concrete Machinery Co., "Concrete Plant."
Hool, "Reinforced Concrete," Vol. II.
Taylor and Thompson, "Concrete Costs," pp. 376–380.
"Handbook of Construction Plant," R. T. Dana.
"Concrete Engineers Handbook," Hool and Johnson, "Concreting Plant."

CHAPTER VII

FORMS

Panels.—Form work for concrete walls may be divided into two parts. (*a*) the form panel proper, consisting of the lagging with the supporting joists and (*b*) the necessary bracing to hold the form panel in place. With the exception of very small jobs or of intricate and varying shaped walls, forms are usually designed to be used several times. To insure maximum economy, then, it is necessary that the panels be stoutly built, yet of such dimensions that they be easily set up, stripped and carried about. The details should be such that the panels can be assembled, put in place and made grout tight with a minimum of carpentry work.

Concrete Pressure.—That the form panel be properly designed, it is necessary that some attempt be made to determine the amount of the concrete pressure. Both theoretically and experimentally, it has been found exceedingly difficult to formulate the action of wet concrete upon the form. At the instant it is placed in the form, its pressure approximates closely a fluid pressure, the fluid weighing 150 pounds per cubic foot. Soon afterwards, both on account of the setting action and of the solids contained in the concrete, the pressure drops away from the linear fluid pressure law. For a thin wall with the concrete level rising with a fair degree of rapidity, this linear law ($p = wh$) is a good approximation. For a wall of heavy section, such as a gravity wall and the like, this linear law would give excessive pressures.

Concrete pressures are quite often underestimated with the result that the forms yield, or give way entirely, spoiling much work and entailing an expense far in excess of that required by the increased amount of material to hold the concrete properly.

Probably the most extensive series of experiments upon concrete pressures and the one most frequently quoted, were those performed by Major Shunk.[1] His conclusions are as follows:

[1] A résumé of these experiments is given in *Engineering News*, Vol. 62, p. 288.

The pressure of concrete follows the linear law

$$p = wh \tag{182}$$

with w equal to 150 lb. per cubic foot, until a time T has elapsed, in minutes,

$$T = c + 150/R \tag{183}$$

where c is a constant depending upon the temperature of the mix (see Table 31) and R is the rate of pouring *i.e.* the rate at which the concrete is rising in the form, in feet per hour. A series of charts giving the pressure after the time T has elapsed is given in the résumé of the report quoted above.

TABLE 31.—CONCRETE PRESSURE CONSTANTS

Temp.	c
80	20
70	25
60	35
55	42
50	50
40	70

A series of experiments upon the pressure of liquid concrete has been given by Hector St. George Robinson. See Minutes of the Proceeding of the Institute of Civil Engineers, Vol. clxxxvii, 1911–1912, Part I, "The Lateral Pressure of Liquid Concrete" excerpts of which are quoted here:

"Numerous experiments were made on different types of concrete structures. In heavy walls, large piers and other members of fair size the lateral pressure exerted was found to be fairly uniform and practically constant for equal heads; but in reinforced concrete columns of small dimensions, thin walls and other light concrete work, the effect of friction between the more or less rough timber forms and the concrete, together with the arching action, was found to reduce the pressure considerably.

"The first series of tests were made during the building of a long wall about three feet thick, constructed of concrete weighing 140 pounds per cubic foot and composed of slow-setting cement, sand and crushed granite in the proportions of 1 : 3 : 6 by volume. In mixing sufficient water was used to bring it to a thoroughly plastic condition, requiring little or no tamping to consolidate. The concrete was laid more rapidly than is usual in this class of work, being carried up as rapidly as the mixing and placing would permit to a height of 8 feet above the center of the pressure face, during which time a light iron bar with a turned up end was used for churning the semi-liquid mass.

"The second series was carried out on large piers, four feet square, the concrete in this case being a 1 : 2 : 4 mixture of cement, sand and Thames ballast, weighing about 145 lbs. per cubic foot. The conditions as to mixing and laying were similar to those of the first tests and the concrete was carried up to a height of 10 feet above the center of the pressure face.

"In the first series the temperature was fairly uniform throughout, while in the second considerable variation was experienced; but the effects of the differences in temperature on the lateral pressure cannot be traced and would appear to be very small.

"The general conclusions to be drawn from these and other experiments is that the lateral pressure of concrete for average conditions is equivalent to that of a fluid weighing 85 pounds per cubic foot. * * * For concrete in which little water is used in mixing, the pressure is rather less, having an equivalent fluid value as low as 70 lbs. per cubic foot in very dry mixtures."

There is apparently a large divergence of pressures as experimentally obtained and until more extensive experimentation has been performed it is hardly justifiable to use other than an empiric table of pressures; guided, however, by the results of the above quoted work. A simple code may be used as indicated below wherein the pressure is obtained from the equation

$$p = wh$$

with p the lateral pressure in pounds per square foot, h is the concrete head in feet, and w is to be used as follows:

For heights of concrete less than 5′,	$w = 150$
For concrete 5 to 10 feet,	$w = 100$
For concrete 10 to 20 feet,	$w = 75$
For concrete over 20 feet,	$w = 50$

These are all safe values and insure, when used, a form that will not yield.

A comparison of the pressures obtained by using the results as tabulated by Major Shunk and by using the suggested series of values just given show quite a divergence in numerical values. The pressures using the values given by Major Shunk (the curves giving the maximum pressure for a given C and T are to be found on p. 448, "American Civil Engineers Pocket Book") are far lower than those found by the latter method. In view of the fact, however, that concrete pressures are not readily formulated and that form failures have demonstrated that such pressures do reach a high value, it seems better to follow the scheme of pressure intensities given above. The forms should be designed then, using these values in preference to using the experimental m aximum pressure.

The extra cost of the stronger forms thus obtained is far less than the expense entailed in remedying the result of a form failure.

At the end of the chapter a problem is given illustrating the application of the preceding formulas to a specific example.

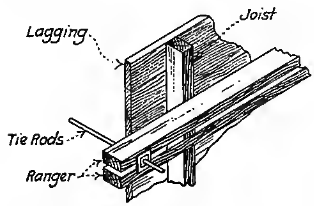


Fig. 112.—Typical form assembly.

Since a form panel may be placed at any point of the face of the wall, it should be designed for the maximum pressure that can come upon it. The concrete pressure is carried by the lagging to the joists, which in turn carry it to the longitudinal rangers. These carry the load to tie rods, or where such rods are not used, to shores placed against the rangers (see Fig. 112).

Lagging.—Generally tongue and grooved lumber is specified for the sheeting. The boards are continuous over the joists and with the support of the tongue and grooving, it is possible to treat the panel as a plate. Ordinarily, no reliance should be placed on such plate action and the boards should be designed as either smple or fixed beams. Another most important fea-

Table 32.—Safe Load per Square Foot on Lagging

$L \backslash h$	1(⅞)	1¼ (1⅛)	1½ (1⅜)	1¾ (1⅝)	2(1⅞)	2¼(2⅛)	2½(2⅜)	2¾(2⅝)	3(2⅞)
12	1,000	1,700	2,500	3,500	4,700	5,950	7,500	9,200	11,000
14	750	1,250	1,850	2,600	3,450	4,450	5,550	6,750	8,100
16	600	950	1,400	2,000	2,650	3,400	4,250	5,200	6,200
18	450	750	1,100	1,550	2,100	2,650	3,350	4,100	4,900
20	350	600	900	1,250	1,700	2,150	2,700	3,300	3,950
22	300	500	750	1,050	1,400	1,800	2,250	2,750	3,300
24	250	400	650	900	1,200	1,500	1,900	2,300	2,750
26	200	350	550	750	1,000	1,300	1,600	2,000	2,350
28	175	300	450	650	850	1,100	1,400	1,700	2,050
30	160	275	400	550	750	950	1,200	1,500	1,800
33	135	225	350	450	600	800	1,000	1,200	1,500
36	110	200	300	400	500	650	850	1,000	1,200
39	100	150	250	350	450	550	700	850	1,050
42	85	135	200	300	400	500	600	750	900
45	75	125	175	250	350	450	550	650	800
48	65	100	160	225	300	400	450	600	700

TABLE 33.—SAFE TIMBER STRESSES FOR FORM LUMBER

(Taken from A. R. E. A., railroad timber stresses, the stresses increased 50 per cent. because of the nature of the loading and the temporary character of the work.)

Douglas fir	1800
Longleaf pine	2000
Shortleaf pine	1600
White pine	1350
Spruce	1500
Norway pine	1200
Tamarack	1350
Western hemlock	1600
Redwood	1350
Bald cypress	1350
Red cedar	1200
White oak	1600

TABLE 34.—SAFE LOADS ON RANGERS AND JOISTS IN KIPS

	2′-0″						3′-0″						4′-0″					
h \ b	2	4	6	8	10	12	2	4	6	8	10	12	2	4	6	8	10	12
2	0.4	0.9	1.3	1.8	2.2	2.7	0.3	0.6	0.9	1.2	1.5	1.8	0.2	0.4	0.7	0.9	1.1	1.3
4	1.8	3.5	5.3	7.1	8.8	10.6	1.2	2.4	3.6	4.7	5.9	7.1	0.9	1.8	2.7	3.6	4.4	5.3
6	4.0	8.0	12.0	16.0	20.0	24.0	2.7	5.3	8.0	10.7	13.3	16.0	2.0	4.0	6.0	8.0	10.0	12.0
8	7.1	14.2	21.2	28.3	35.4	42.5	4.7	9.5	14.2	19.0	23.6	28.5	3.6	7.1	10.7	14.2	17.8	21.3
10	11.1	22.1	33.2	44.4	55.3	66.2	7.4	14.8	22.2	29.6	37.1	44.5	5.6	11.1	16.7	22.2	27.8	33.3
12	16.0	32.0	48.0	64.0	80.0	96.0	10.7	21.4	32.0	42.7	53.4	64.0	8.0	16.0	24.0	32.0	40.0	48.0

	5′-0″						6′-0″						7′-0″					
2	0.2	0.4	0.5	0.7	0.9	1.1	0.1	0.3	0.4	0.6	0.7	0.9	0.1	0.3	0.4	0.5	0.6	0.8
4	0.7	1.4	2.1	2.8	3.6	4.3	0.6	1.2	1.8	2.4	3.0	3.6	0.5	1.0	1.5	2.0	2.5	3.0
6	1.6	3.2	4.8	6.4	8.0	9.6	1.3	2.7	4.0	5.3	6.7	8.0	1.1	2.3	3.4	4.5	5.7	6.8
8	2.8	5.7	8.5	11.4	14.2	17.0	2.4	4.7	8.1	9.5	11.9	14.2	2.0	4.1	6.1	8.1	10.2	12.2
10	4.4	8.9	13.4	17.8	22.2	26.7	3.7	7.4	11.1	14.8	18.7	22.2	3.2	6.3	9.5	12.7	15.8	19.0
12	6.4	12.8	19.2	25.6	32.0	38.4	5.3	10.7	16.0	21.3	26.7	32.0	4.5	9.1	13.7	18.3	22.8	27.4

	8′-0″						10′-0″					
2	0.1	0.2	0.3	0.4	0.6	0.7	0.1	0.2	0.3	0.4	0.5	0.6
4	0.4	0.9	1.3	1.8	2.2	2.7	0.4	0.7	1.1	1.4	1.8	2.1
6	1.0	2.0	3.0	4.0	5.0	6.0	0.8	1.6	2.4	3.2	4.0	4.8
8	1.8	3.6	5.3	7.1	8.9	10.7	1.4	2.8	4.3	5.7	7.1	8.5
10	2.8	5.6	8.3	11.1	13.9	16.7	2.2	4.4	6.7	8.9	11.1	13.3
12	4.0	8.0	12.0	16.0	20.0	24.0	3.2	6.4	9.6	12.8	16.0	19.2

ture is the amount of defection permissible. It is well to keep the deflection of the panel within one-eighth of an inch.

Table 32 gives the load per square foot to be carried by a board 12 inches wide, L feet long (L the distance between joists) and h inches thick. The unit timber stress taken is 1,000 pounds per square inch. The boards are designed as simple beams. Should the permissible stress be greater than that used here the load may be increased in direct proportion to the new stress. Again, if the board is to be treated as a fixed beam the load to be carried may be increased 50 per cent. That the deflection may not exceed one-eighth of one inch, for simple span.

$$L \text{ must be less than } 25 \sqrt{h}$$

and for a fixed span

$$L \text{ must be less than } 45 \sqrt{h}$$

Table 33 gives a range of unit timber stresses for several woods.

Table 34 gives the maximum loads to be carried by the joists for various spacing. The thickness of the joist is b inches and its depth h inches. The loads may again be increased in the same proportion for a permissible unit stress greater than one thousand pounds per square inch and again when the beam is assumed as fixed in place of simply supported. This same table may also be used to design the rangers supporting the panels.

Tie-rods.—The diameter of the tie-rod depends upon the size of the panel supported and its position in the form. The concrete pressures may be taken from the empiric scheme given on

TABLE 35.—LOADS IN LBS. ON TIE RODS

Rod diameter	Permissible unit stresses			
	12,000	16,000	20,000	25,000
1/8	150	200	250	300
1/4	600	800	1,000	1,200
3/8	1,320	1,750	2,200	2,700
1/2	2,350	3,150	4,000	4,900
5/8	3,700	4,900	6,100	7,700
3/4	5,300	7,100	8,800	11,000
7/8	7,200	9,650	12,000	15,000
1	9,400	12,700	15,700	19,600
1 1/4	14,700	19,700	24,500	30,700

page 183. The unit stress in the steel is usually taken at 16,000 lb. per square inch. Small diameter rods may be pulled out and this should be borne in mind in selecting the rod spacing. Table 35 gives the load on tie rods for a range of unit steel stresses.

A simple detail carrying the tie rod load is shown in Fig. 112. This obviates the necessity of boring a large timber to allow the rod to pass through. The tie rods may be threaded on the end and fastened to the rangers by nuts and washers, or a patented support, such as the universal clamp (*Universal Clamp Co.*) may be used on a plain round bar.

Rangers.—The rangers themselves may be designed as simple or fixed beams, with spans between tie rods and carrying the joists. If the ranger is to be b inches wide and h inches deep, with span between tie rods L, then

$$WL/I = pbh^2/6 \text{ and } bh^2 = \frac{6WL}{pI} \qquad (184)$$

I may be taken as 8 or 12, depending upon the assumption that the beam is a fixed or simple one; and p may be taken as the safe permissible unit stress in the timber.

Form Re-use.—If the panels are built in stout units, carefully put together, they may be used several times. When the lagging becomes splintered marring the face of the concrete and making it very difficult to strip the form, the form should be abandoned. With care in placing and stripping the forms, a panel may be used from 3 to 10 times. Two inch tongue and grooved sheeting makes a good strong form but its weight limits it to small panels. If plant is available to handle these units, this objection is removed and as large sections may be used as is found convenient to assemble. Usually a limiting section would be about 8′ by 10′.

Form Work.—It is essential that a careful study be made of the form work, taking into consideration the expected daily output of concrete and the time the forms must remain in place. It must be remembered that forms of simple shape, quickly assembled, put in place and stripped, make for large economy on the work. Skilled carpenters will prepare excellent, well-fitting forms of long duration. It is a poor economy to substitute for such labor the ordinary wood butcher, a most competent man in his sphere.

In this connection the use of a portable machine saw, propelled electrically or by gasoline is a marvellous labor and time saver and few jobs, however small, can afford to be without one.

The rear and face forms of the wall are kept the proper distance from each other by means of wooden separators called spreaders. When the tie rods are placed or wire used in place of tie rods and tension put on them the spreaders are held in place without any further details. As the concrete is poured and reaches the lever of a spreader, the spreader is knocked out. The tie rods and wires must remain until the concrete has set (see later chapter).

Bracing.—Bracing, or shoring is necessary to take care of unbalanced pressures and the possible overturning of the form due to the vibrations and shocks set up during the pouring of the concrete. Such stresses are obviously not to be computed and experience alone dictates the proper amount of bracing to be used. They are made usually of 4 inch by 4 inch, or 4 in. by 6 in. stock, nailed to the rangers and held against foot blocks or stakes in the ground (see Fig. 113). Where concrete is to be poured against a permanent mass, requiring forms on one side only, no tie rods or wires can be used through the concrete and the bracing on the one side must take the full concrete pressure and are to be designed accordingly. When walls of some height are to be poured in several lifts, an overlapping of the joists may render bracing unnecessary above the lower lift.

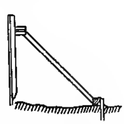


Fig. 113.—Form brace.

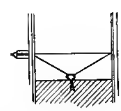


Fig. 114.—Holding forms by bolt in concrete.

Occasionally the environment is such that bracing cannot be used on either side. It is possible here to concrete eye bolts into the bottom lift and into each succeeding lift and to anchor the forms to these (see Fig. 114).

Generally an excessive amount of bracing is used, with a resulting forest of timber and making it impossible to run plant close to the forms. Form-work is a fertile field of study for the engineer and the designing and detailing of such work is worthy of as serious attention as the design and construction of the wall itself.

Stripping Forms.—It is essential that forms be stripped as soon as it is possible to do so. To keep a form in place longer than is required makes it impossible to get the full economical reuse of the form and makes it very difficult to finish and repair the concrete surface. In the warm summer months the forms may be stripped after 24 hours. In the spring and fall months they should be left in place from 48 to 72 hours. When in doubt as to the hardness of the concrete a small portion of the form may be taken off and a thumbnail impression made. If there is no indentation, it is safe to take off the balance of the forms. If it is possible to remove the tie-rods (rods ½ inch or less in diameter may be economically recovered; rods of larger diameter are usually left in the wall) these should be taken out before the forms are stripped. Patent rod pullers[1] may be used to take out the rods. Where the rods are left in the wall, they should be cut back an inch to an inch and a half and the face of the wall plastered at these points. Wires are rarely recovered and are cut off in the same fashion as the rods. The sooner after stripping these rods and wires are cut, the easier it is to repair and finish the face of the wall (see later chapter of wall finish).

From ten days to two weeks of favorable, warm weather should elapse before the fill is placed behind the wall. If the fill is to be placed at a rapid rate, *e.g.*, by dump cars from a temporary trestle and the like, a greater period of time should elapse. This is especially important for the reinforced concrete walls, where the concrete will receive the full load immediately after the completion of the embankment.

Oiling and Wetting Forms.—A dry form will absorb the water from the concrete, in the process of curing, leaving a peculiar pock-marked appearance of the concrete face due to the honeycombing of the surface. The forms should be wetted by pail or hose immediately before the concrete pour is started. To aid in the stripping of the form, the inside face of the form is usually oiled, with a heavy oil, termed a form oil, which is a heavy sludge. Although this stains the concrete face, the rubbing and washing of the concrete surface easily removes the oil marks.

Patent Forms.—For a wall of large yardage and of fairly constant outline, permitting many reuses of the form panel, the use of some of the patent forms may show quite an economy, both in the construction of the form and in the labor of setting up and stripping the forms.

The two best known types of such forms are the Hydraulic Pressed Steel Form and the Blaw Form.

[1] An excellent rod puller is sold by the Universal Clamp Co. of Chicago.

The Hydraulic Pressed Steel Form consists of two parts: the bracing and the form panel. The bracing is formed of upright Us spliced as necessary and held together by tie rods and spacers or liners. Fig. 115 shows a sketch of the bracing and its details.

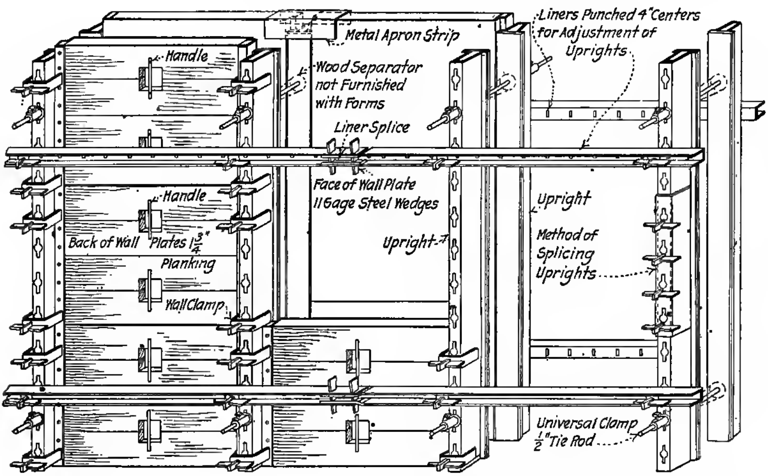


FIG. 115.—Hydraulic Pressed Steel Co. form assembly of liners and plates.

The form panel consists of a sheet metal (all metal used in these forms both panel and uprights are no. 11 gage, *i.e.*, one-eighth inch metal) backed by 2″ boards. Around the periphery of the panel a U steel edge is put, to which the boards are screwed (see Fig. 116). The panels are held in place against the uprights by means of stout Us spaced about one foot apart (see Fig. 115).

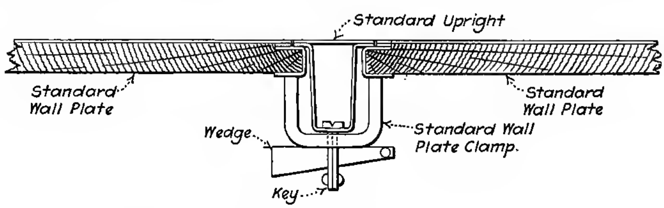


FIG. 116.—Section of Hydraulic Pressed Steel Co. form.

It is claimed by the company that the panels may be reused about 300 times before wearing out. Where a job will permit a reuse of the form panel exceeding twenty or thirty times, they maintain that their form will prove cheaper than the wood form ordinarily built.

FIG. A.—Unsatisfactory rod detail for concrete pouring.

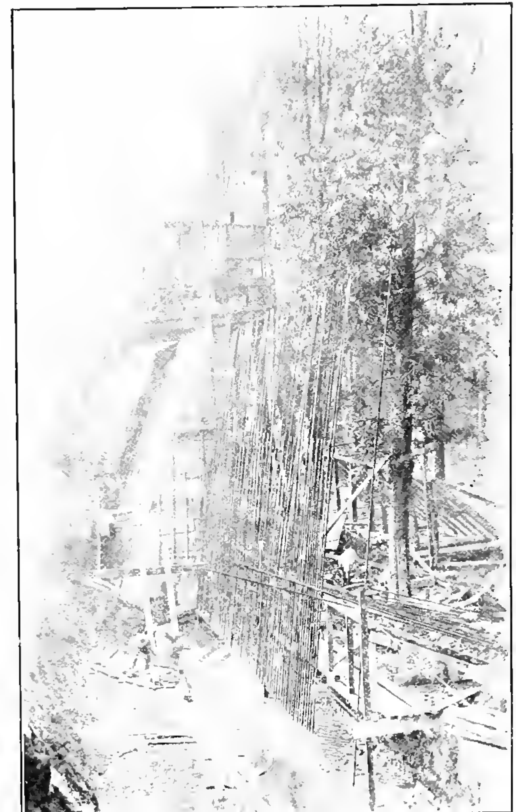


FIG. B.—Holding vertical rods in place before concrete is poured.

(Facing page 191)

The advantages of the form are quite obvious. The uprights may be built up to the top of the wall. After the lower lift of the wall is poured no further bracing becomes necessary, since the form is now anchored against the lower half of the wall. The panels may be removed after twenty-four hours, the uprights and liners remaining as much longer as is necessary before the wall is self-supporting.

The panels need only be put in as the concrete comes near their level, thus permitting a thorough spading and tamping of the mass: quite a vital point where the wall is thin or has a special shape.

Blawform.—The Blawform consists, essentially of a steel panel, reinforced with angle on the back and held in place with a steel assembly of joists and rangers. By an ingenious travelling gantry device, the form panels are braced against this traveller, which runs on rails alongside the work. A large number of instances of their use for both heavy and light retaining walls are given in their Catalogue 16.

Supporting the Rod Reinforcement.—Since most of the rod system in a reinforced concrete wall must be in place before the concrete pouring is started, some means of support must be provided. In the "L" or "T" shaped cantilevers, the heavy rod system of the vertical arm extends into the footing and must, therefore, be set up and in place before the wall forms are up. Many simple devices may be used for this purpose. Fig. 117, (See Fig. B, Plate IV) shows a typical and efficient method of taking care of these rods. When the footing has been poured, thereby anchoring these rods, the wall forms are set in place and the rods are wired and held the required distance away from the concrete face. The horizontal rod system is wired to the vertical rods and helps to maintain the proper spacing of these vertical rods. Patent wire clips may be used to wire the horizontal and vertical rods together.

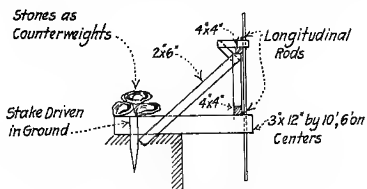


FIG. 117.—Supporting rod reinforcement of cantilever wall.

The horizontal rods in the footing, itself, are laid in the concrete when the proper level has been reached. It is preferable to wire a net of these rods together before placing in the wet concrete to make sure that the proper spacing as called for on the plans, will be kept.

The rod systems of the other types of the reinforced concrete walls are supported and placed in similar fashion. The problem of supporting the rods extending into the footing for the slab types of wall is comparatively a simple one, since these rods are the light system and therefore need little framework to carry them. The main system (particular stress is placed upon the counterfort and box types of wall) is suspended to the forms in the usual manner and kept at the proper distance away from the face by means of small wooden spreaders which are removed in pouring as quickly as the concrete reaches their level. The tie rods form a good support for the horizontal rods and are generally so used.

It is important that, whatever method of support is employed, the rods should be held firmly in place. Spading and spouting of concrete are liable to shift the rods unless they are stoutly supported. It is understood that in the design of walls involving intricate rod systems (see Chapter 4) proper consideration has been given to the practicability of the rod layout and to the feasibility of supporting the rods and of pouring the concrete. Simplicity of rod design insures an easy concrete pour and leaves the engineer with a reasonable assurance that the rods are finally placed where they were originally designed to go.

The rod system has, presumably, been carefully and economically designed and no variations in spacing should be permitted in the field, except in isolated instances, where a proper attempt should then be made to reinforce the weak spots resulting. Leaving openings in the walls for construction reasons, as, for example, to permit placing timbers through the wall, or to place large pipe etc., will result, when the wall is finally patched in portions being without the proper reinforcement. The rods should be bent around these openings as shown in Fig. 118.

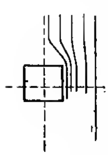


FIG. 118.

Undoubtedly walls are at times designed with excessive reinforcement due to indifference or carelessness and the knowledge of such excessive strength has encouraged the engineers in the field and the contractors constructing such walls to alter the rod spacing to accommodate minor construction exigencies. Such acts are, in the main, unfortunate and designs which can safely permit many such liberties are to be deplored. Walls should be designed as economically as possible with due consideration for all contingencies and when a design has left the hands of a competent, conscientious engineer, no changes should be permitted in the field save with the concurrence of the man responsible for the design.

Travelling Forms.—*Engineering News*, Vol. 73, p. 67. Track Elevation Rock Island Lines Chicago.

"The walls are built in travelling forms which straddle the site of the wall and are carried by wheels on either side. Both wood and steel forms of this type are used, each being long enough for a 35 foot section and having grooved wheels riding upon two lines of rails. * * * The abutments are built in fixed forms of the usual type. Plank sheeting is used in both cases and the two lines of sheeting are held together by tie-rods instead of wires. The rods are plain bars, not threaded, and are fitted with clamps instead of nuts. When a clamp is in place, a set screw jams the rod against a V slot in the clamp, securing it rigidly in position. (*Engineering News*, Sept. 10, 1914). Each rod is imbedded in a tin tube, so that it can be withdrawn readily, the holes being then packed with stiff cement grout at each end.

"The retaining walls are built in alternate sections of 35 feet with the travelling forms. It takes about six hours to fill the form, which is then left in place about 15 hours. It then takes about 20 hours to release the travelling form move seventy feet forward and adjust them and the sheeting ready for the concrete. The use of the travelling forms has enabled the work to be done in about 25 per cent. of the time required with the ordinary forms (from the building to the removal of the form) and at about 50 per cent. of the cost (including erecting, pouring and dismantling)."

New Passenger Terminal, C. & N. W. R. R. Armstrong, *Journal* of the Western Society of Engineers, Vol. 16.

"The forms were built in sections 30 feet long. The footings were first built and allowed to set. The forms for the super walls were then built. It was required that an entire section of superwall should be poured at one continuous run of the mixing plant, in order that no horizontal joints might occur in the walls. The forms were constructed of 4-inch by 6-inch studding and 2-inch by 8-inch dressed and matched sheeting. The two sides were tied together with ⅞-inch rods which were passed through iron pipes consisting of old boiler flues. The rods were drawn out when the forms were removed, but the pipes were left in place, the opening in the face of the wall being filled with mortar."

Forms Built in Central Yard.—*Engineering and Contracting*, June 11, 1913, p. 649. Track Elevation, Chicago, Milwaukee and St. Paul R. R. For this work the forms were built in a central yard and were shipped out to the work as required on flat cars. They were taken from the cars and set in place by means of locomotive cranes.

Erecting Forms on Curves.—R. H. Brown, *Engineering Record*, Vol. 61, p. 714.

"There is nothing more unsightly in concrete work than to see the impression of the forms running out of level. A great deal of pains is taken to produce smooth surfaces by spading, but very little attention is given to the mold itself. This is very noticeable in massive work. On a straight wall there is no excuse for this, but in building forms on curves of short radius there is great difficulty in making a symmetrical surface and eliminating the segmental effect. If the following method is carried out a piece of concrete will be produced which is a true curve in every foot of its length.

"Take a wire about the size of that used in telephone lines and upon a smooth level surface strike on the board an arc of the radius of the center-line of the wall or dam. Arc of radius of 150 feet can easily be handled. Care must be used in doing this that the wire is always straight. This template is now sawed out on a band saw in about ten-foot lengths. The rear and face templates can be struck from this one by means of a T-square.

"Run out the center line of the wall in chords of 10 feet and put in permanent plugs at these points. Erect a well-braced series of batters around the curve and set the top ledger board at the exact crest of the wall. Place the center-line templates on these boards and plumb them over the plugs, cleating them together as fast as they are put in correct position. With this center to work from, the outside and inside curves can be set.

"Make two boards four feet long, one edge straight and the other bevelled to the batter of the front and rear faces respectively. The studding can now be set, using a carpenter level. The upper end will rest against the template, the lower end following the inequalities of the ground.

"Start the bottom board as low as possible and run it along the curve on both sides making it absolutely level. The rest of the boarding can now be nailed to the studding, springing each one carefully into place. The purlins (Rangers) are put in and rods run through and tightened. After everything is well braced, remove the batter boards used in lining up. When the forms are removed a true curve is presented to the eye."

Problems

It is required to design and construct a set of forms for a wall 30 feet high above the footing with expansion joints 40 feet apart, of section shown in Fig. 119. It is figured that the mixer can pour 100 yards of concrete in an 8-hour shift, this to govern the lift of concrete poured.

The portion of the wall requiring forms contains a volume between expansion joints of 93 cubic yards. It is thus possible to complete the pouring of the section in one continuous pour within the time specified—the ideal arrangement. The forms will be designed upon this basis.

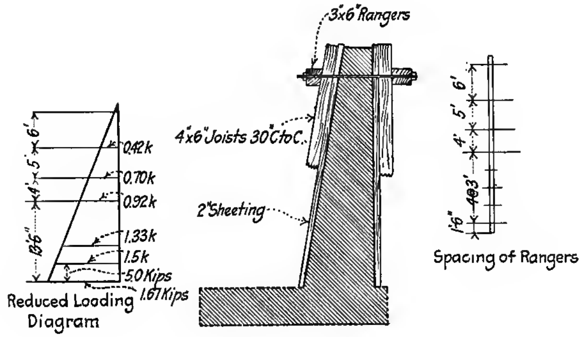


Fig. 119.

Concrete Pressures.—On the basis of Major Shunk's experiments, the concrete pressure at the base is determined as follows: (It is assumed that the concrete enters the form at the temperature of 70°.) Since the concrete form is 30 feet high and is filled in 8 hours, the rate of filling per hour is 3.75 feet, the value of R to be used in the work following. From Table 31 with the temperature of 70°, $c = 25$ and from (183)

$$T = 25 + 150/3.75 = 65 \text{ minutes} = 1.1 \text{ hrs.}$$

The maximum pressure that can occur is found by employing the curves of Major Shunk, which can be found in the American Civil Engineers' Pocket Book, page 448. This maximum pressure, with the value of c and T as above found is 850 pounds per square foot. Using the empiric rule given on page 183, the pressure function to use is 50 lb. per square foot, which would give, at the base of the wall 30 × 50 or 1500 lb. per square foot. The average pressures found by Robinson, page 182, of 85 lb. per square foot intensity would give a base pressure of 85 × 30 = 2550 lbs. far in excess of both of the pressures just found. The experimental value of 85 lbs. is based upon heads not exceeding 10 feet—and is therefore of little application to the case at hand. Again, the experiments of Major Shunk, while most admirably and extensively performed cannot be made the final basis for concrete pressure determination. It is therefore logical to employ, awaiting more experimental data, the empiric table suggested in the previous pages and the form work of the given problem will be designed upon the table quoted.

In line with the recommendations of the text, 2-inch tongue and grooved sheeting will be used. North Carolina spruce dressed all sides will permit a working stress for the form work of 1500 lbs. per square inch. The sheeting will be treated as continuous, so that the product kp of Table 32 is $1500 \times 12 = 18000$. Since the loads on the sheeting of Table 32 employ the constant 8000, to use the table directly, the above load of 1500 pounds per square inch will be reduced in the ratio of $^{18000}/_{8000}$, or will become 670 lb. per square inch. For 2″ material, the dressed thickness is 1⅞″ and the table shows that a load of 670 pounds will permit the joists to be spaced 30 inches apart. In view of the fact that the forms are to be used several times, the panels may be set at any position in the form, and will therefore all be constructed alike, and of the heaviest dimensions required.

The rangers are set after the panels are in place and may therefore be spaced to accommodate the concrete pressures. A good working size for a joist is a 4-inch by 6-inch stick. Fig. 119 gives the load layout for the 30-inch spacing of the joists. The loads have been divided by the constani 2.25 *i.e.*, the ratio of $^{18000}/_{8000}$, to permit a direct use of the Tables. Table 34 is to be used in the design of the joists. Let the lower ranger carry a three-foot panel of sheeting. From the figure, the lower three feet bring a tabular equivalent load of 4.8 kips. Table 34 permits a three-foot spacing of this size joist and accordingly the first ranger having been placed as close to the bottom as is feasible, the next will be spaced three feet above it. A similar study of the loading above the lower panel shows that, to maintain the same size of joist, the next four rangers must be spaced on three feet centers. The remainder of the spacing is shown on Fig. 119.

The rangers will be made up of two 3-inch by 6-inch sticks, a handy merchantable size. The safe load span upon these two pieces will determine the tie-rod spacing. From equation (184) page 187, with $I = 12$; $p = 1500$ as before, $b = 6$ and $h = 6$,

$$WL = 648{,}000$$

or if w is the load per linear foot upon the ranger and L is the length expressed in feet

$$wL^2 = 54{,}000$$

The lower ranger will carry 4500 lbs. per linear foot (the actual loads are used here), whence

$$L = 3'\ 6''$$

The tie rods will accordingly be spaced 3′ 6″ apart at the lower lift of rangers. The panel load that a tie-rod will be called upon to carry is

$$3.5 \times 3 \times 1500 = 15{,}700 \text{ lb.}$$

To avoid using large size tie-rods, which cannot be recovered two tie rods will be used together at the lower lift. From Table 35 with a unit stress of 16,000 pounds per square inch for steel, two ¾-inch rods will be used. The other tie rod spacing, and the necessary rod section are both found by identical means.

CHAPTER VIII

CONCRETE CONSTRUCTION

Water Content.—Recent years have noted a marked increase in the knowledge of the proper mode of selecting and mixing the aggregates necessary to produce good, strong concrete masonry. Not only must the various aggregates be put in the correct proportions, but the amount of water used is vitally important. The excess or deficiency of water seriously affects the strength of the concrete.

Each element entering into a concrete mix performs a definite and separate function and each is, accordingly, capable of affecting favorably or unfavorably the strength of the concrete. Concrete is usually so proportioned that each finer material fills, more or less completely, the voids in the coarser aggregate (see following pages on Prof. Abrams demonstration that the strength of the concrete does not require, prima facie, this condition). The action of water is in part a solvent and in part a chemical one. The results of Mr. Nathan C. Johnson[1] and other laboratory investigators have strikingly demonstrated the vital importance of the correct amount of water and it has been shown that concrete failures, both partial and complete are attributable to excess of water. The evaporation of this excess amount of water leaves pockets and crevices in the concrete, materially reducing the effective area capable of resisting stress. The widely varying results of concrete tests and the necessary high factors of safety are thus quite obviously explained.

Prof. Talbot[2] has made a series of timely pointers on concrete, some of which may, with profit, be quoted here.

"The cement and the mixing water may be considered together to form a paste; this paste becomes the glue which holds the particles of the aggregate together.

[1] *Engineering News Record,* June 26, 1919, p. 1266. Also "Better Concrete—The Problem and Its Solution," N. C. Johnson, Journal Engineer's Club, Philadelphia, Pa.

[2] *Engineering News-Record,* May 1, 1919 for a résumé of his remarks at the annual convention of the American Railway Engineering Association.

"The volume of the paste is approximately equal to the sum of the volume of the particles of the cement and the volume of the mixing water.

"The strength given this paste is dependent upon its concentration—the more dilute the paste the lower its strength; the less dilute the greater its strength.

"The paste coats or covers the particles of aggregate partially or wholly and also goes to fill the voids of the aggregate partially or wholly. Full coating of the surface and complete filling of the voids are not usually obtained.

"The coating or layer of paste over the particles forms the lubricating material which makes the mass workable; that is, makes it mobile and easily placed to fill a space compactly.

"The requisite mobility and plasticity is obtained only when there is sufficient paste to give a thickness of film or layer of paste over the surface of the particles of aggregate and between the particles sufficient to lubricate these particles.

"Increase in mobility may be obtained by increasing the thickness of the layer of paste; this may be accomplished either by adding water (resulting in a weaker paste) or by adding cement up to a certain point (resulting in a stronger paste).

"Factors contributing to the strength of concrete are then, the amount of cement, the amount of mixing water, the amount of voids in the combination of fine and coarse aggregate and the area of surface of the aggregate.

"For a given kind of aggregate the strength of the concrete is largely dependent upon the strength of the concrete paste used in the mix, which forms the gluing material between the particles of the aggregate.

"For the same amount of cement and same voids in the aggregate, that aggregate (or combination of fine and coarse aggregates) will give the higher strength which has the smaller total area of surface of particles, since it will require the less amount of paste to produce the requisite mobility and this amount of paste will be secured with a smaller quantity of water; this paste being less dilute will therefore be stronger. The relative surface area of different aggregates or combination of aggregates may readily be obtained by means of a surface modulus calculated from the screen analysis of the aggregate.

"For the same amount of cement and the same surface of aggregate, that aggregate will give the higher strength which has the less voids, since additional pore space will require a larger quantity of paste and therefore more dilute paste.

"Any element which carries with it a dilution of the cement paste may in general be expected to weaken the concrete. Smaller amounts of cement, the use of additional mixing water to secure increased mobility in the mass, increased surface of aggregate, and increased voids in the aggregate all operate to lower the strength of the product.

"In varying the gradation of aggregate a point will be reached, however, when the advantage in the reduction of surface of particles is offset by increased difficulty in securing a mobile mass, the voids are greatly increased, the mix is not workable and less strength is developed in the concrete. For a given aggregate and a given amount of cement, a decrease in the amount of mixing water below that necessary to produce sufficient paste to occupy most of the voids and provide the lubricating layer will give a mix deficient in mobility and lower in strength.

"A certain degree of mobility is necessary in order to place concrete in the forms in a compact and solid mass, the degree varying considerably with the nature of the work and generally it will be found necessary to sacrifice strength to secure the requisite mobility. It is readily seen, however, that the effort should be made to produce as strong a cementing layer of paste as practicable by selecting the proper mixture of aggregate and by regulating the amount of mixing water.

"More thorough mixing not only mixes the paste and better coats the particles, but it makes the mass mobile with a smaller percentage of mixing water and this less dilute paste results in higher strength. Any improvement in methods of mixing which increases the mobility of the mass will permit the use of less dilute paste and thereby secure increased strength."

In connection with the above remarks by the Dean of Concrete Investigators, there may be quoted the conclusions of a classic report prepared by the Bureau of Standards.[1]

"1. No standard of compressive strength can be assumed or guaranteed for concrete of any particular proportions made with any aggregate unless all the factors entering into its fabrication are controlled.

"2. A concrete having a desired compressive strength is not necessarily guaranteed by a specification requiring only the use of certain types of materials in stated proportions. Only a fractional part of the desired strength may be obtained unless other factors are controlled.

"3. The compressive strength of concrete is just as much dependent upon other factors, such as careful workmanship and the use of the proper amount of water in mixing the concrete as it is upon the use of the proper quantity of cement.

"4. The compressive strength of concrete may be reduced by the use of an excess of water in mixing to a fractional part of what it should attain with the same materials. *Too much emphasis cannot be placed upon the injurious effect of the use of excessive quantities of water in mixing concrete.* [The italics are mine.]

[1] *Technology Papers* of the Bureau of Standards, No. 58.

"5. The compressive strength of concrete may be greatly reduced if, after fabrication, it is exposed to the sun and wind or in any relatively dry atmosphere in which it loses its moisture rapidly, even though suitable materials were used and proper methods of fabrication employed.

"6. The relative compressive strengths of concretes to be obtained from any given materials can be determined only by an actual test of those materials combined in a concrete.

"7. Contrary to general practice and opinion the relative value of several fine aggregates to be used in concrete can not be determined by testing them in mortar mixtures. They must be tested in the combined state with the coarse aggregate.

"8. Contrary to general practice and opinion the relative *value* of several coarse aggregates to be used in concrete cannot be determined by testing them with a given sand in one arbitrarily selected proportion. They should be tested in such combination with the fine aggregate as will give maximum density, assuming the same ratio of cement to total combined aggregate in all cases.

"9. No type of aggregate such as granite, gravel or limestone can be said to be generally superior to all other types. There are good and poor aggregates of each type.

"10. By proper attention to methods of fabricating and curing, aggregates which appear inferior and may be available at the site of the work may give as high compressive strength in concrete as the best selected materials brought from a distance, when the latter are carelessly or improperly used.

"11. Density is a good measure of the relative compressive strength of several different mixtures of the same aggregates with the same proportion of cement to the total aggregate. The mixture having the highest density need not necessarily have the maximum strength but it will have a relatively high strength.

"12. Two concretes having the same density but composed of different aggregates may have widely different compressive strength.

"13. There is no definite relation between the gradation of the aggregates and the compressive strength of the concrete which is applicable to any considerable number of different aggregates.

"14. The gradation curve for maximum compressive strength, which is usually the same as for maximum density, differs for each aggregate.

"15. With the relative volumes of fine and coarse aggregate fixed, the compressive strength of a concrete increases directly, but not in a proportionate ratio as the cement content. An increase in the ratio of cement to total fine and coarse aggregates when the relative proportions of the latter are not fixed does not necessarily result in an increase in strength, but may give even a lower strength.

"16. The compressive strength of concrete composed of given materials, combined in definite proportions and fabricated and exposed under given conditions can be determined only by testing the concrete actually prepared and treated in the prescribed manner.

"17. *The results included in this paper would indicate that the compressive strength of most concretes, as commercially made can be increased* 25 *to* 100 *per cent. or more by employing rigid inspection which will insure proper methods of fabrication of the materials.*"

In a striking report on how to properly design a concrete mixture to obtain the utmost strength from the aggregate at hand by Prof. Duff A. Abrams[1] it is shown how little the present day standard methods of proportioning concrete make for concrete strength. The importance of the report and its vital conclusions justify the rather lengthy excerpts below.

The general problem of concrete mixtures has been defined in the report as follows and some of the principles following a series of 50,000 tests are noted therein.

"The design of concrete mixtures is a subject of vital interest to all engineers and constructors who have to do with concrete work. The problem involved may be one of the following:

"1. What mix is necessary to produce concrete of proper strength for a given work?

"2. With given materials what proportion will give the best concrete at minimum cost?

"3. With different lots of materials of different characteristics which is best suited for the purpose?

"4. What is the effect on strength of concrete from changes in mix, consistency or size and grading of aggregate?

"Proportioning concrete frequently involves selection of materials as well as their combination. In general, the question of relative costs is also present."

Of the different methods of proportioning concrete, Prof. Abrams has noted the following as among the most important:

"1. Arbitrary selection, such as 1 : 2 : 4 mix, without reference to the size or grading of the fine and coarse aggregate;

"2. Density of aggregates in which the endeavor is made to secure an aggregate of maximum density;

"3. Density of concrete in which the attempt is made to secure concrete of maximum density;

[1] Design of Concrete Mixtures, *Bulletin* 1, Structural Materials Research Laboratory, Lewis Institute, Chicago.

"4. Sieve analysis, in which the grading of the aggregates is made to approximate some predetermined sieve analysis curve which is considered to give the best results;"

"5. Surface area[1] of aggregates.

"It is a matter of common experience that the method of arbitrary selection in which fixed quantities of fine and coarse aggregates are mixed without regard to the size or grading of the individual materials, is far from satisfactory. Our experiments have shown that the other methods mentioned above are also subject to serious limitations. We have found that the maximum strength of concrete does not depend on either an aggregate of maximum density or a concrete of maximum density, and that the methods that have been suggested for proportioning concrete by sieve analysis of aggregates are based on an erroneous theory. All of the methods of proportioning concrete which have been proposed in the past have failed to give proper attention to the water content of the mix. *Our experimental work has emphasized the importance of the water in concrete mixtures, and shown that the water is, in fact, the most important ingredient, since very small variations in water content produce more important variations in the strength and other properties of concrete than similar changes in the other ingredients.*

After performing a series of over 50,000 tests, covering a period of three years, Prof. Abrams has established the following important principles in regard to the correct design of a concrete mix.

"1. With given concrete materials and conditions of test the quantity of mixing water determines the strength of the concrete, so long as the mix is of workable plasticity.

"2. The sieve analysis furnishes the only correct basis for proportioning aggregates in concrete mixtures.

"3. A simple method of measuring the effective size and grading of an aggregate has been developed. This gives rise to a function known as the "fineness modulus"[2] of the aggregate.

"4. The fineness modulus of an aggregate furnishes a rational method for combining materials of different size for concrete mixtures.

"5. The sieve analysis curve of the aggregate may be widely different in form without exerting any influence on concrete strength.

"6. Aggregate of equivalent concrete-making qualities may be produced by an infinite number of different gradings of a given material.

"7. Aggregates of equivalent concrete-making qualities may be produced from materials of widely different size and grading.

[1] See end of chapter for a definition of Surface Area.

[2] See end of chapter for a complete definition of the fineness modulus.

"8. In general, fine and coarse aggregates of widely different size or grading can be combined in such a manner as to produce similar results in concrete.

"9. The aggregate grading which produces the strongest concrete is not that giving the maximum density (lowest voids). A grading coarser than that giving maximum density is necessary for highest concrete strength.

"10. The richer the mix, the coarser the grading should be for an aggregate of given maximum size; hence, the greater the discrepancy between maximum density and best grading.

"11. A complete analysis has been made of the water requirements of concrete mixes. The quantity of water required is governed by the following factors:

"(*a*) The condition of "workability" of concrete which must be used—the relative plasticity or consistency;

"(*b*) The normal consistency of the cement;

"(*c*) The size and grading of the aggregate—measured by the fineness modulus;

"(*d*) The relative volumes of cement and aggregate—the mix;

"(*e*) The absorption of the concrete;

"(*f*) The contained water in aggregate.

"12. There is an intimate relation between the grading of the aggregate and the quantity of water required to produce a workable concrete.

"13. The water content of a concrete mix is best considered in terms of the cement—water-ratio.

"14. The shape of the particle and the quality of the aggregate have less influence on the concrete strength than has been reported by other experimenters."

Prof. Abrams has experimentally determined the relation between the water content and the strength of the concrete and reports the following most important conclusions together with an empiric relation between the two.

"It is seen at once that the size and grading of the aggregate and the quantity of cement are no longer of any importance except in so far as these factors influence the quantity of water required to produce a workable mix. This gives us an entirely new conception of the function of the constituent materials entering into a concrete mix and is the most basic principle which has been brought out in our studies of concrete.

"The equation of the curve is of the form

$$S = \frac{A}{B^x}$$

where S is the compressive strength of the concrete and x is the ratio of the volume of water to the volume of cement in the batch. A and B are constants whose values depend on the quality of the cement used, the age of the concrete, curing conditions, etc.

"This equation expresses the law of the strength of concrete so far as the proportions of materials are concerned. It is seen that for given concrete materials the strength depends upon only one factor—the ratio of water to cement. Equations which have been proposed in the past for this purpose contain terms which take into account such factors as quantity of cement, proportions of fine and coarse aggregate, voids in aggregate, etc., but they have uniformly omitted the only term which is of any importance; that is, *the water*.

"A vital function entering into the analysis is the so-called 'fineness modulus' which may be defined as follows:

"The sum of the percentages in the sieve analysis of the aggregate divided by 100.

"The sieve analysis is determined by using the following sieve from the Tyler standard series: 100, 48, 28, 14, 8, 4⅜, ¾ and 1½ in. These sieves are made of square-mesh wire cloth. Each sieve has a clear

TABLE 36.—METHOD OF CALCULATING FINENESS MODULUS OF AGGREGATES

The *sieves* used are commonly known as the Tyler standard sieves. Each sieve has a *clear opening* just double that of the preceding one.

The *sieve analysis* may be expressed in terms of volume or weight.

The *fineness modulus* of an aggregate is the sum of the precentages given by the sieve analysis, divided by 100.

Sieve size	Size of square opening, in.	Size of square opening, mm.	Sieve analysis of aggregates per cent. of sample coarser than a given sieve: Sand, Fine (*A*)	Sand, Medium (*B*)	Pebbles, Coarse (*C*)	Pebbles, Fine (*D*)	Pebbles, Medium (*E*)	Pebbles, Coarse (*F*)	Concrete aggregate (*G*)*
100-mesh....	.0058	.147	82	91	97	100	100	100	98
48-mesh....	.0116	.295	52	70	81	100	100	100	92
28-mesh....	.0232	.59	20	46	63	100	100	100	86
14-mesh....	.046	1.17	0	24	44	100	100	100	81
8-mesh....	.093	2.36	0	10	25	100	100	100	78
4-mesh....	.185	4.70	0	0	0	86	95	100	71
⅜-in......	.37	9.4	0	0	0	51	66	86	49
¾-in.......	.75	18.8	0	0	0	9	25	50	19
1½-in.......	1.5	38.1	0	0	0	0	0	0	0
Fineness modulus..........			1.54	2.41	3.10	6.46	6.86	7.36	5.74

* Concrete aggregate G is made up of 25 per cent. of sand B mixed with 75 per cent. of pebbles E. Equivalent gradings would be secured by mixing 33 per cent. sand B with 67 per cent. coarse pebbles F; 28 per cent. A with 72 per cent. F, etc. The proportion coarser than a given sieve is made up by the addition of these percentages of the corresponding size of the constituent materials.

opening just double the width of the preceding one. The exact dimensions of the sieves and the method of determining the fineness modulus will be found in Table 36. It will be noted that the sieve analysis is expressed in terms of the percentages of material by volume or weight coarser than each sieve."

Prof. Abrams notes that there is a direct relation between the fineness modulus as above defined and the compressive strength of the concrete, after noting that the "fineness modulus simply reflects the changes in water-ratio necessary to produce a given plastic condition." This is, of course, consistent with his main thesis that the water-ratio is the all important function in determining the concrete strength. It is stated that the relation between the compressive strength of the concrete, as brought out by tests and the fineness modulus is to all intents a linear one, *i.e.* an increase in the fineness modulus has a proportionate increase in the compressive strength.

With an assigned compressive strength of concrete, it is now possible to proceed directly to assemble an aggregate to meet this strength. The water-ratio forming the fundamental basis of the process, the empiric relation above mentioned is employed to determine the proper value of x, when S is given and A and B are known. The details following, showing the method of obtaining the values of the constants, of the fineness modulus and of the several combinations possible to satisfy most economically the strength requirements of the concrete are given with elegance and clearness in the *Bulletin* just quoted. The novelty of the method and its apparent intricacy (and such intricacy is only apparent) and the fact that concrete mixes usually just "grow" and are not scientifically developed may make Prof. Abrams' procedure seem very cumbersome. A little study of his methods will show that the contrary is true and that the correct design of a concrete mix predicated upon his assumptions (and these assumptions are assuredly based on most valid premises) is a matter of very simple analysis.

The further comments on the design of a concrete mix, given at the conclusion of the *Bulletin* are worthy of quotation here:

"The importance of the water-ratio on the strength of concrete will be shown in the following considerations:

"One pint more water than necessary to produce a plastic concrete reduces the strength to the same extent as if we should omit 2 to 3 lb. of cement from a one-bag batch.

"Our studies give us an entirely new conception of the function performed by the various constituent materials. The use of a coarse well-graded aggregate results in no gain in strength unless we take advantage of the fact that the amount of water necessary to produce a plastic mix can thus be reduced. In a similar way we may say that the use of more cement in a batch does not produce any beneficial effect except from the fact that a plastic workable mix can be produced with a lower water-ratio.

"The reason a rich mixture gives a higher strength than a lean one is not that more cement is used, but because the concrete can be mixed (and usually is mixed) with a water-ratio which is relatively lower for the richer mixtures than for the lean ones. If advantage is not taken of the fact that in a rich mix relatively less water can be used, no benefit will be gained as compared with a leaned mix. In all this discussion the quantity of water is compared with the quantity of cement in the batch (cubic feet of water to one sack of cement) and not to the weight of dry materials or of the concrete as is generally done.

"The mere use of richer mixes has encouraged a feeling of security, whereas in many instances nothing more has been accomplished than wasting a large quantity of cement, due to the use of an excess of mixing water. The universal acceptance of this false theory has exerted a most pernicious influence on the proper use of concrete materials and has proven to be an almost insurmountable barrier in the way of progress in the development of sound principles of concrete proportioning and construction.

"Rich mixes and well-graded aggregates are just as essential as ever, but we now have a proper appreciation of the true function of the constituent materials in concrete and a more thorough understanding of the injurious effect of too much water. Rich mixes and well-graded aggregates are, after all, only a means to an end; that is, to produce a plastic, workable concrete with a minimum quantity of water as compared with the cement used. Workability of concrete mixes is of fundamental significance. This factor is the only limitation which prevents the reduction of cement and water to much lower limits than are now practicable.

"The above considerations show that the water content is the most important element of a concrete mix, in that small variation in the water cause a much wider change in the strength than similar variations in the cement content or the size or grading of the aggregate. This shows the absurdity of our present practice in specifying definite gradings for aggregates and carefully proportioning the cement, then *guessing at the water*. (The italics are mine.) It would be more correct to carefully measure the water and guess at the cement in the batch.

"The grading of the aggregate may vary over a wide range without

producing any effect on concrete strength so long as the cement and water remain unchanged. The consistency of the concrete will be changed, but this will not affect the concrete strength if all mixes are plastic. The possibility of improving the strength of concrete by better grading of aggregates is small as compared with the advantages which may be reaped from using as dry a mix as can be properly placed.
* * * * * * * * * *

"Without regard to actual quantity of mixing water the following rule is a safe one to follow: *Use the smallest quantity of mixing water that will produce a plastic or workable concrere.* The important of any method of mixing, handling, placing and finishing concrete which will enable the builder to reduce the water content of the concrete to a minimum is at once apparent."

Practical Application.—Some of the details of these copious excerpts may eventually prove without adequate experimental basis; yet the fundamental truth conveyed in all the foregoing must be recognized—namely, the role of the water content of a concrete mix. The question of paramount importance is the manner and means of applying these truths to actual concrete work in the field. Stone, gravel, sand and cement companies have been educated to furnish products meeting with the requirements of long continued experimental and field research. These products are naturally much costlier than are aggregates unrestricted as to nature, impurities, grading and size. It is essential then that this added cost be not squandered without any benefit through oversight of some simple principles.

The proper mixing of the ingredients is conditioned upon the plant used, both for mixing and for distributing. The character of such plant has been described both generally and in some detail in a previous chapter on plant. The average mixer, while a more or less efficient machine has some difficulty in producing a well mixed batch of low water content in a short-timed mix. A little patience in educating the mixer operator to keep the water contents low and an insistence that the concrete be not dumped until a specified time of mixing has elapsed, will go a long way towards meeting the experimental requirements of good concrete. Clearly, it is of no avail to go to the bother, expense and the possible delay of securing specified concrete materials, if little attention is paid to the final steps in concrete mixing.

A batch of concrete must be in the mixer a certain minimum time before the aggregate has been properly transformed into concrete. What this time is depends upon the character of the machine and the number of revolutions it makes per minute. This time can not be specified in advance nor can good concrete be expected merely from long time mixing. In this connection see the *Engineering News-Record*, Nov. 28, 1918, p. 966, and Jan. 23, 1919, p. 200. The average time of mixing a batch is about one minute. A little care and study of the particular machine at hand will determine the correct time for a batch mix. Careful inspection will then insure that each batch of concrete will receive this length of time for its proper mix.

In the use of small mixers, the so-called one or two bag batch mixers, it is exceedingly hard to get a uniform water ratio for all the batches. Variations in the piling of the stone and sand in the barrows; in the dryness of the aggregate all make it impossible to apply a constant amount of water and turn out the same consistency of mix. However, by a careful attention to the piling of the carts and by an insistence that water be used in measured quantity only—preferably from an overhead tank attached to the machine and certainly not by an indiscriminate use of the hose or pail—a concrete can be obtained meeting with a fair degree of success the water requirements of workable plastic concrete.

It should be definitely predicated that the principles of good concrete should determine the plant and not, conversely, the plant determine the mode of concreting (see chapter on Plant).

Concrete Methods.—The question of competent labor proves a most irritating one. It may be set down as axiomatic that common labor, however willing, and in spite of competent leadership cannot mix and place good concrete. A trained concrete force is necessary for this work. The use of incompetent labor on concrete work is a most short-sighted policy and here, as in every other industrial enterprise, the best is decidedly the cheapest in the end.

The use of poor materials and the employment of lax and indifferent methods together with incompetent labor are dependent upon the laxity of inspection and, unfortunately, the minimum requirements of the engineer form the maximum goal of the average contractor and, to use the colloquialism of the field, the construction superintendent will "get away with" as much as he can. True, there are many exceptions, but the engineer does well to prepare for the worst.

To specify a good concrete, especially in light of the above researches, is, comparatively an easy matter. To assign proper inspection, tempered by practical judgment and equipped with a thorough knowledge of good concrete, so that in matters of field decision the concrete is given the benefit of the doubt, is a far more difficult matter.

As the details of the requirements of good concrete become more generally known undoubtedly the common welfare of the concrete interests, contractors, engineers, plant manufacturers and the like, will promote a coöperation that will make it a much simpler matter to secure the maximum strength of concrete from a given assembly of materials. At present it is necessary to specify in detail the desired concrete aggregates and the methods by which these are to be mixed and, in addition, to make ample provision for carrying out the intent and letter of the specifications.

Distributing Concrete.—Concrete, properly mixed, must likewise be properly distributed. Poor distribution will nullify the beneficial results of good mixing. The concrete mix is an aggregate of solids in a fluid vehicle and, when transported in any but a vertical direction, will tend to separate in accordance with natural laws. The distributing system must aid in overcoming this separation tendency. For this reason concrete should be dropped *vertically* into the forms and spread by shovels and hoes into thin layers. Spouting a concrete into a form in any direction but the vertical is a serious offence. The mix will separate and any subsequent hoeing, shovelling or spading will prove ineffectual. Upon stripping the forms the inevitable pouring streaks will appear; evidence of poor workmanship and presenting a most unpleasing appearance.

With a concrete of workable plasticity, properly delivered into a form, but little additional work should be necessary to bring it to its final place in the form. The concrete should be spaded at the form to permit the grout to collect at the face, insuring a smooth face and should also be spaded at the rods to aid in getting a firm grout bond between the steel and the concrete.

The distributing systems have been discussed in detail in the preceding chapter on plant, which chapter should be read again in the light of the present observations upon the requirements of good concrete.

Keying Lifts.—If the day's pour is finished before reaching the top of the wall, the concrete surface should be brought to a rough level and a long timber to form a longitudinal key should be imbedded in the top. Dowels may be inserted instead, made up either of steel rods, or of stones and carried about one foot into each of the layers. At the pouring of the next layer, the timber key, if used, is to be removed, the surface to be thoroughly cleaned and the fresh concrete then placed upon it. For the efficiency of various treatments of this joint see "Construction Joints," page 159.

Use of Cyclopean Concrete.—In large concrete walls, it is permissible to place stones over 12 inches in diameter wherever the thickness of the concrete mass exceeds 30 inches. The stones are kept about 12 inches apart and about 6 inches from the face of the wall. They should be sound, hard rock, well-cleaned and should be placed by hand into the concrete and not dumped indiscriminately from a bucket or thrown in at random. A little care in placing the stone will permit a larger number to be used and thus cut down the cost of the wall by economizing on the amount of concrete aggregate required.

In reinforced concrete walls it is questionable whether the use of such "plums" should be permitted. The rod system makes it difficult to place the stones, even though the wall exceeds 30 inches in thickness. Since the concrete in this wall is highly stressed in compression, sound rock must be used. With a carefully specified aggregate for the concrete, it seems a little inconsistent then to permit the use of an indeterminate material. Local conditions will generally indicate whether good stones are available. As a general rule, however, for the usual type of cantilever and counterforted walls, the use of plums is inadvisable.

Winter Concreting.—Quite often the urgent need of a concrete retaining wall makes it imperative that its construction proceed despite winter weather. As the temperature drops, the setting time of concrete increases. The setting action stops when the concrete is frozen and does not continue until the concrete has thawed. It is doubtful whether frost injures a concrete permanently. This much, however, is certain—a frozen concrete must thaw out completely and then be given ample time to set, before the forms are stripped or any load placed upon the wall. It is highly desirable and it is generally so specified that concrete be mixed in such a manner that it reaches the form at a favorable setting temperature and is then to be suitably protected against frost until it is thoroughly set.

Concrete should not reach the forms at a temperature less than 45° (Fahrenheit). The aggregate and the water should be heated when the temperature drops below this mark. While, ordinarily, concreting is permitted without heating the materials until the temperature drops below the freezing point, the above temperature should preferably be the controlling one.

A simple method of heating the aggregate is to pile it around a large metal pipe (a large diameter metal flue, or a water pipe is just the thing) and have a fire going within the pipe. Old form lumber is an excellent and cheap fuel for this fire. Another, similar method is to pile the material on large metal sheets resting on little stone piers, and beneath which sheets fires are kept burning. In both the methods care must be taken not to burn the material next to the metal, and not to use such material if it does become burned. The water may be heated in large containers over fires, or by passing live steam through the water, either directly in it or through coils.

An interesting description of a winter concreting job is given here:[1]

"The sand and crushed stone used in making the wall concrete were heated by diffusion of steam from perforations in a coil of a 2″ pipe placed at the bottom of the storage pile. The bottoms of the charging bin above the mixer were also fitted with perforated piping so that the heat might be retained in the materials.

"The water used in mixing was maintained at about 100° F. by a live steam jet discharging at the bottom of a 3000 gallon tank, or reservoir kept constantly full. The overflow from the tank discharged into a 50 gallon measuring barrel, being heated to scalding temperature by another jet of superheated steam.

"The walls forms were insulated with straw and plank on the back and covered with tongue and grooved flooring on the face, retaining a 2″ space between the steel (metal forms were used) and the wood, through which low pressure steam from one of the boilers on the deck was diffused by a perforated 1″ pipe. This pipe was at the bottom of the form and ran longitudinally the entire length connecting with the boiler by a *T* connection and vertical pipe at about the middle of the section.

"A stationary mixing plant was installed adjacent to the main line of the railway about half a mile west of the wall site. The concrete was conveyed to the wall in buckets on cars drawn by a dinkey on narrow gage."

[1] Retaining Walls, Baltimore & Ohio Railroad, *Engineering News*, Vol. 76, p. 269.

A general note on winter concreting on Miami Conservancy Work is given here as of interest in connection with the present topic.[1]

"Concreting has been carried on through the winter in the dam construction work of the Miami Conservancy District, Ohio, with only occasional interruption. As the nature of the enterprise demands that progress be rapid and according to schedule, and as it is important to keep the working organization intact to avoid losses and delays, it became necessary to plan reducing the interruptions of concreting to a minimum.

"Study of the extra costs involved in heating materials and protecting deposited concrete led to the conclusion that the greater part of the extra cost is incurred only at temperatures below 20°, and a general rule was therefore made that work through the cold season is to be continued until the thermometer drops below 20°.

"Provision for heating aggregates by steam coils built in the bins has been made at all three of the dams where concreting has been going on * * * . Means have also been provided for protecting the surfaces from freezing by tarpaulins and salamanders, or, in some instances by steam coils (where steam was available because it was used for other purposes).

"Care is taken that no fresh concrete is placed on frozen foundations. With a view to reducing the liability of freezing also, the amount of water used in the mixing is closely regulated."

Concrete work in winter, observing the necessary precautions to prevent freezing, is, of course, more costly, than work at the seasonable temperatures. Whether this extra cost is less than the loss involved in the break in the continuity of the work and the delay in receiving the finished structure, is a matter to be disposed of uniquely for each piece of work. If the work is to proceed regardless of the weather, the specifications must so be drawn, that the precautions to be used when the temperature falls below a given point (which must be clearly noted) are emphatically set forth. General specifications as to heating are unsatisfactory—the details should be given.

Acceleration of Concrete Hardening.—The quicker a concrete sets, other things being equal, the quicker the forms can be stripped and the sooner can the fill be deposited behind the wall. Under natural conditions, the warmer the concrete is the quicker it sets. Therefore work in the summer can proceed at a faster

[1] *Engineering News-Record*, Vol. 82, p. 618.

rate than work at the other seasons. Some cements are more quickly setting than others. It is possible, by adding certain chemicals to accelerate the hardening of the concrete. The effect of the addition of calcium chloride has been noted as follows:[1]

"As the result of some experiments made by the Bureau of Standards to develop a method to accelerate the rate at which concrete increases in strength with age, it was found that the addition of small quantities of calcium chloride to the mixing water gave the most effective results. A comprehensive series of tests was inaugurated to determine further the amount of acceleration in the strength of concrete obtained in this manner and to study the effect of such additions on the durability of concrete and the effect of the addition of this salt on the liability to corrosion of iron or steel imbedded in mortar or concrete.

"The results to date indicate that in concrete at the age of two or three days, the addition of calcium chloride up to 10 per cent. by weight of water to the mixing water results in an increase in strength, over similar concrete gaged with plain water, of from 30 to 100 per cent., the best results being obtained when the gaging water contains from 4 to 6 per cent. of calcium chloride.

"Compressive strength tests of concretes gaged with water containing up to 10 per cent. calcium chloride, at the age of one year gave no indication that the addition of this salt had a deleterious effect on the durability of the concrete.

"Corrosion tests that have been completed indicate that the presence of calcium chloride, although the amount used is relatively small, in mortar slabs exposed to the weather, causes appreciable corrosion of the metal within a year. This appears to indicate that calcium chloride should not be used in stuccos and warns against the unrestricted use of this salt in reinforced concrete exposed to weather or water."

Concrete Materials.—Concrete aggregates and cement have been so well classified and placed under standard specifications that any typical specification will serve as a model for the character of the material to enter into the construction of a retaining wall. A brief description may be given of the essential requirements of these concrete constituents. It may be well to read once more the previous pages upon the bearing of the type of the aggregate on the concrete strength and the relative importance of the character and proportions of the aggregates (including water) as compared with the methods of preparation

[1] *Engineering News Record*, Vol. 82, p. 507.

and distributing. The amounts of the material required depend upon the proportions specified. Table 37 is given here based upon the standard proportion and shows the amount of cement, sand and stone required for the various mixes. These are the theoretical requirements. It must be borne in mind that the method of distributing the material, whether in central bins or in local piles (see chapter preceding on "Plant") will involve a certain amount of wastage which must be taken into consideration in ordering the aggregate. Properly constructed shacks for the storage of cement will reduce to a minimum the loss of cement through accidental weathering, etc.

Table 37.—Proportions for Mixing Concrete

Mixtures			Yardages of materials for one cubic yard of concrete in the form					
			Specification stone up to 2 in.			Gravel, ¾ in. size		
Cement	Sand	Stone	Cement, bbls.	Sand, yds.	Stone, yds.	Cement, bbls.	Sand, yds.	Stone, yds.
1	1.0	2	2.6	.4	.8	2.3	.4	.7
1	1.0	3	2.1	.3	.9	1.9	.3	.9
1	1.5	3	1.9	.4	.8	1.7	.4	.8
1	1.5	4	1.6	.4	1.0	1.5	.3	.9
1	2.0	3	1.7	.5	.8	1.5	.5	.7
1	2.0	4	1.5	.4	.9	1.3	.4	.8
1	2.0	5	1.3	.4	1.0	1.2	.4	.9
1	2.5	5	1.2	.5	.9	1.1	.4	.8
1	3.0	4	1.3	.6	.8	1.2	.5	.7
1	3.0	6	1.0	.5	.9	.9	.4	.8
1	3.5	5	1.1	.6	.8	1.0	.5	.8
1	3.5	7	0.9	.5	.9	.8	.4	.9
1	4.0	6	0.9	.6	.8	.8	.5	.8
1	4.0	8	0.8	.5	.9	.7	.4	.9

Cement.—(Portland cement, alone is discussed here.) It is usual to specify that cement will meet the requirements of the Committee of the American Society of Civil Engineers on "Uniform Tests of Cement." It is usual to insist that the brand of cement used is one that has been employed on large engineering works for at least five years.

Portland cement has been defined as the finely pulverized product resulting from the calcination to incipient fusion of the properly proportioned mixture of argillaceous and calcareous materials to which no addition greater than 3 per cent. has been made subsequent to calcination.

Its fineness shall be determined and limited as follows: The cement shall leave by weight a residue of not more than 8 per cent. on a No. 100 sieve and not more than 25 per cent. on a No. 200 sieve, the wires of the sieve being respectively 0.0045 and 0.0024 of an inch in diameter.

The time of setting shall be as follows: The cement shall develop initial sct in not less than 30 minutes, and shall develop hard set in not less than 1 hour, nor more than 10 hours.

The minimum requirements for tensile strength for briquettes one inch square in minimum section shall be as follows:

Age	Strengtn
Heat Cement	
24 hours in moist air	175 lb.
7 days (1 day in moist air, 6 days in water)	500 lb.
28 days (1 day in moist air, 27 days in water)	600 lb.
One Part Cement, Three Parts Standard Sand	
7 days (1 day in moist air, 6 days in water)	170 lb.
28 days (1 day in moist air, 27 days in water)	225 lb.

Neat briquettes shall show a minimum increase in strength of 10 per cent. and sand briquettes 20 per cent. from the tests at the end of 7 days, to those at 28 days.

Tests for constancy of volume will be made by means of pats of neat cement about 3 inches in diameter, ½ inch thick at the center and tapering to a thin edge. These pats to satisfactorily answer the requirements shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

The cement shall contain not more than 1.75 per cent. of anhydrous sulphuric acid (SO_3), or more than 4 per cent. of magnesis (MgO).

The cement shall have a specific gravity of not less than 3.10 nor more than 3.25 after being thoroughly dried at a temperature of 212°F. The color shall be uniform, bluish gray, free from yellow or brown particles.

Sand.—Sand for concrete shall bc clean, containing not more than 3 per cent. of foreign matter. It should be reasonable free from loam and dirt. When rubbed between the palm the hand should be left clean. It should be well graded from coarse to fine. No grains should be left on a ¼-inch sieve and not more than 6 per cent. should pass through a 100 mesh sieve. Fine sand is undesirable and its presence in a quantity greater than that just specified will materially weaken the concrete. A coarse smooth-grained sand is not objectionable and will produce, with other things being equal, an effective and strong concrete. In connection with the selection of the aggregate and the proportioning of the coarse and fine particles, a note in the appendix is given on the selection and mixing of aggregates by the surface area method and by the fineness modulus method and the relation between these two modes of selection and the strength of the concrete.[1]

Crushed Stone and Gravel.—Crushed stone should be made from trap or limestone. Stone from local quarries, or from rock cuts encountered in the work should be used only after tests have been made on concrete containing this stone. For ordinary gravity walls, the size of the crushed stone or of the gravel may vary from 3/8 inch to 1 3/4 inch in diameter. For the thin reinforced concrete walls the stone should not exceed 3/4 inch in size.

Occasionally the sand and the stone are delivered already mixed in the required proportions. Parallel to this method, the run of a gravel bank may be taken, including the gravel with the finer sands. Either method of supplying the aggregate is far from ideal and does not lend itself well to a conscientious proportioning of the materials. It is preferable to supply the coarse and the fine aggregates separately and mix them in the required proportions in the mixer.

A résumé of the above methods of selecting the aggregates and cement is presented in the appendix in the shape of a standard specification for retaining walls, including the proper specifying of the materials entering into its composition.

Fineness Modulus of Aggregate.[2]—The experimental work carried out in the laboratory has given rise to what we term the fineness modulus of the aggregate. It may be defined as follows: The sum of the percentages in the sieve analysis of the aggregate divided by 100.

The sieve analysis is determined by using the following sieves

[1] See preceding pages on the fineness modulus; also *Engineering News-Record*, June 12, 1919, pp. 1142 to 1149.

[2] *Bulletin* No. 1, Structural Materials Research Laboratory, Lewis Institute, Chicago, D. A. Abrams.

from the Tyler standard series: 100, 48, 28, 14, 8, 4, ⅜-in., ¾-in. and 1½-in. These sieves are made of square-mesh wire cloth. Each sieve has a clear opening just double the width of the preceding one. The exact dimensions of the sieves and the method of determining the fineness modulus will be found in Table 36. It will be noted that the sieve analysis is expressed in terms of the percentages of material by volume or weight coarser than each sieve.

A well-graded torpedo sand up to No. 4 sieve will give a fineness modulus of about 3.00; a coarse aggregate graded 4–1½-in. will give fineness modulus of about 7.00; a mixture of the above materials in proper proportions for a 1:4 mix will have a fineness modulus of about 5.80. A fine sand such as drift-sand may have a fineness modulus as low as 1.50.

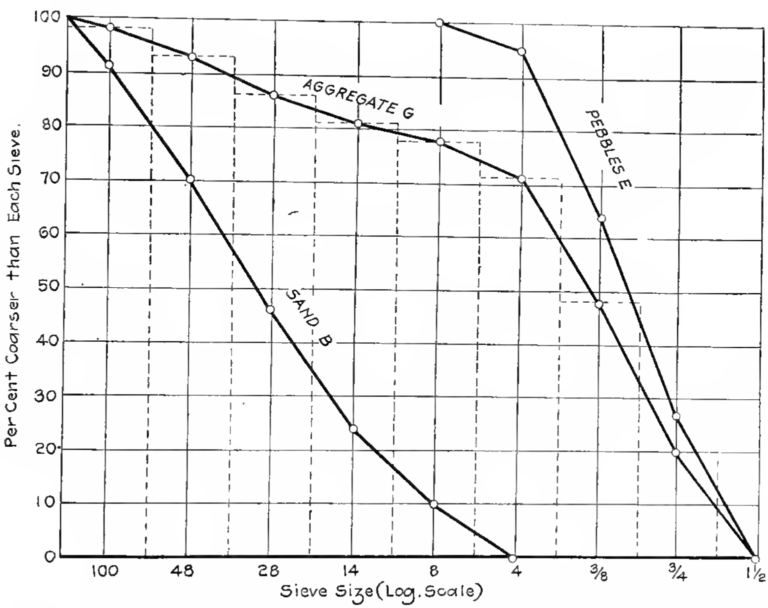


FIG. 120.—From Bulletin No. 1. D. A. Abrams, Structural Materials Research Laboratory, Lewis Institute, Chicago.

Sieve Analysis of Aggregates.—There is an intimate relation between the sieve analysis curve for the aggregate and the fineness modulus; in fact, the fineness modulus enables us for the first time to properly interpret the sieve analysis of an aggregate.

If the sieve analysis of an aggregate is platted in the manner indicated in Fig. 120 that is, using the per cent. coarser than a given sieve as ordinate, and the sieve size (platted to logarithmic scale) as abscissa, the fineness modulus of the aggregate is measured by the area below the sieve analysis curve The dotted rectangles for aggregate "*G*" show how this result is secured. Each elemental rectangle is the fineness modulus of the material of that particular size. The fineness modulus of the graded aggregate is then the summation of these elemental areas. Any other sieve analysis curve which will give the same total area corresponds to the same fineness modulus and will require the same quantity of water to produce a mix of the same plasticity and gives concrete of the same strength, so long as it is not too coarse for the quantity of cement used.

The fineness modulus may be considered as an abstract number; it is in fact a summation of volumes of material. There are several different methods of computing it, all of which will give the same result. The method given in Table 38 is probably the simplest and most direct.

TABLE 38.—TABLES SHOWING MIXTURES OF TEST MORTARS

Test Series No. 1. Cement Content—1 G.: 13 Sq. In.

Sand letter	Surface area per 1000 g., sq. in.	Cement, *g*	Water, cc.	Ratio of cement to aggregate by weight
A	5,856.6	450.5	128.0	1:2.22
B	5,106.1	392.0	111.5	1:2.55
C	7,683.7	591.0	134.5	1:1.69
D	6,758.4	520.0	148.0	1:1.92
E	12,816.4	986.0	280.5	1:1.12
F	6,769.1	521.0	148.0	1:1.92
G	4,182.0	321.5	91.5	1:3.11
H	6,564.6	505.0	143.5	1:1.98
I	6,564.6	505.0	143.5	1:1.98
Test Series No. 2. Cement Content—1 G.: 10, 15, 20 and 25 Sq. In.				
F	6,769	677.0	183.0	1:1.47
	6,769	451.0	132.5	1:2.21
	6,769	338.5	105.5	1:2.95
	6,769	270.5	92.5	1:3.61

Some of the mathematical relations involved are of interest. The following expression shows the relation between the fineness modulus and the size of the particle:

$$m = 7.94 + 3.32 \log d$$

Where m = fineness modulus
d = diameter of particle in inches

This relation is perfectly general so long as we use the standard set of sieves mentioned above. The constants are fixed by the particular sizes of sieves used and the units of measure. Logarithms are to the base 10.

This relation applies to a single-size material or to a given particle. The fineness modulus is then a logarithmic function of the diameter of the particle. This formula need not be used with a graded material, since the value can be secured more easily and directly by the method used in Table 36. It is applicable to graded materials provided the relative quantities of each size are considered, and the diameter of each group is used. By applying the formula to a graded material we would be calculating the values of the separate elemental rectangles shown in Fig. 120.

Proportioning Concrete by Surface Areas of Aggregates.[1]—Volumetric proportioning of concrete is notoriously unsatisfactory. Many investigators have been studying other proportioning methods which will at the same time be practical and will insure a maximum strength of concrete with any given material. The latest of such methods and one which in the tests gives promise of some success is that devised by Capt. L. N. Edwards, U.S.E.R., testing engineer of the Department of Works, Toronto, Ontario, which was explained in some detail in a paper entitled 'Proportioning the Materials of Mortars and Concrete by Surface Areas of Aggregates," presented to the American Society for Testing Materials at its annual meeting in June.

Briefly, Captain Edwards' principle is that the strength of mortar is primarily dependent upon the character of the bond existing between the individual particles of the sand aggregate, and that upon the total surface area of these particles depends the quantity of cementing material. Reduced to practical terms, this means that a mixture of mortar for optimum strength is a

[1] *Engineering News-Record,* Aug. 15, 1918, p. 317 *et seq.*

function of the ratio of the cement content to the total surface area of the aggregate regardless of the volumetric or weight ratios of the two component materials. As a corollary to his investigations, Captain Edwards also lays down the principle that the amount of water required to produce a normal uniform consistency of mortar is a function of the cement and of the surface area of the particles of the sand aggregate to be wetted. Some of the tests deduce the fact, already demonstrated in a number of previous tests, that strength of mortars and concrete is a definite function of the amount of water used in the mix.

In demonstrating the cement-surface area relation, the test procedure was as follows: First, a number of different sands were graded through nine sieves, varying from 4 to 100 meshes per inch, and the material passing one sieve and retained on the next lower was separated into groups. From each group, then, an actual count was made of the average number of particles of sand per gram. For the larger sizes 8 to 10 grams or more, medium sizes 3 to 5 grams, and for the smallest sizes ¼ to 1 gram were counted. For six sands counted, including a standard Ottawa which is composed of grams passing a 20 and retained on a 30-mesh sieve, the following averages were obtained for the number of sand particles per gram:

Passing 4, retained on 8	14
Passing 8, retained on 10	55
Passing 10, retained on 20	350
Passing 20, retained on 30	1,500
Passing 30, retained on 40	4,800
Passing 40, retained on 50	16,000
Passing 50, retained on 80	40,000
Passing 80, retained on 100	99,000

With a specific gravity of sand of 2.689, which had been determined by a number of tests, the average volume per particle of sand was determined for each group, and assuming that the shape of the particles of sand was spherical, which is approximately correct, the surface area per gram of sand was determined for each group. The results are shown in Fig. 121. This gave a basis of surface areas for the various groups of sand in hand.

The sands were then regarded to different granulometric analyses in order to get representative and different kinds of aggregate for the tests. Using these sands for the aggregate, numerous briquets and cylinders were made up and tested in tension and in compression, varying the mix according to the ratio of the weight of cement to the surface area of the sand aggregate. The basis of the ratio of grams of cement to square inches of surface area were 1:10, 1:15, 1:20 and 1:25. The consistency throughout was controlled so that the water content would not affect the relative strengths of the different specimens.

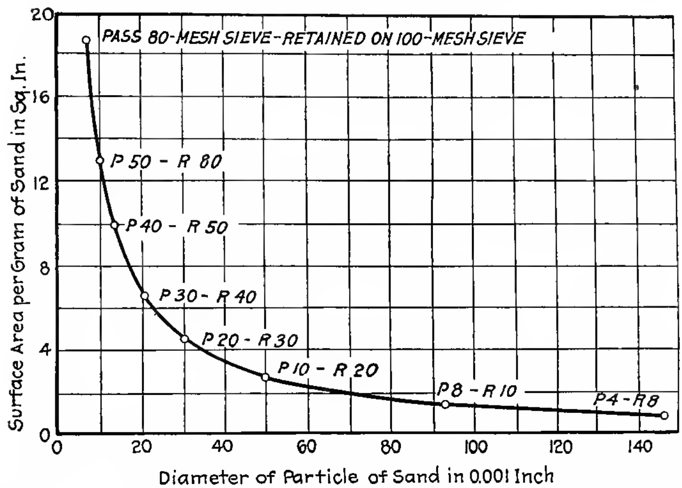


FIG. 121.—Capt. Edwards' method of surface areas. (*From Engineering News-Record, Aug.* 15, 1918, *p.* 317.)

Test mortars were then made, first, by keeping the cement-surface area ratio constant and varying the kinds of sand; second, by varying the ratio and using the same and. These two series are shown in the accompanying table. As will be noted from Table 38, in test series No. 1 the cement content is one gram for thirteen square inches of surface area, but the sand has such a different grading and therefore total surface area that the ratio o cement to aggregate by weight varys from 1:1.12 to 1:3.11. In spite of this wide variation in weight and therefore in volumetric relation of the cement to the aggregate, the strength values, as shown in Fig. 122, were markedly constant. In series No. 2 the cement constant varied from 1 gram to 10 sq. in. to 1 gram to 25 sq. in. of sand surface, and, as shown in Fig 123, the strength curves are proportionate te the cement-area ratio.

Further tests were made by Captain Edwards extending this investigation to concrete, and while these showed the same general results, the tests were not sufficiently elaborate to warrant an abstract of them here.

It might seem offhand that there is no practical occupation to the method. Certainly, the very considerable labor involved in counting 125,000 sand grams for one sieve group alone would deter anyone from contemplating such a program for practical

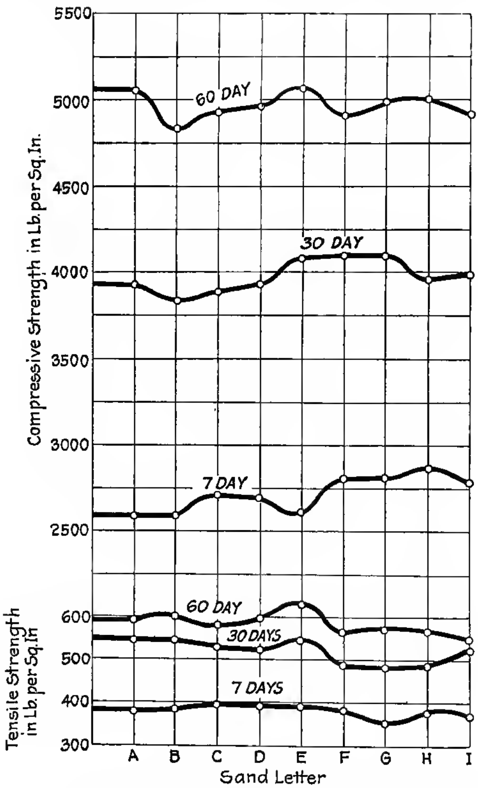


FIG. 122.—Capt. Edwards' method of surface areas. (*From Engineering News-Record, Aug.* 15, 1918, *p.* 317.)

work, if such a count had to be made very often. However, Captain Edwards points out that this elaborate counting is required only as a preliminary to his method and once done need not be repeated. He says:

"The adaptation of the surface area method of proportioning mortars and concretes to both laboratory investigation and field construction operation presents no serious difficulty. The outstanding feature of this method, insofar as its practical application is concerned, is the importance of knowing the granulometric composition of the aggregate. The securing of this all important information involves a comparatively small amount of labor and by way of equipment the use of only the necessary scales, standard sieves and screens. The time element involved is comparatively negligible, since the computation work of determining areas and quantities of cement may be largely reduced to the most simple mathematical operation by the use of tables and diagrams."

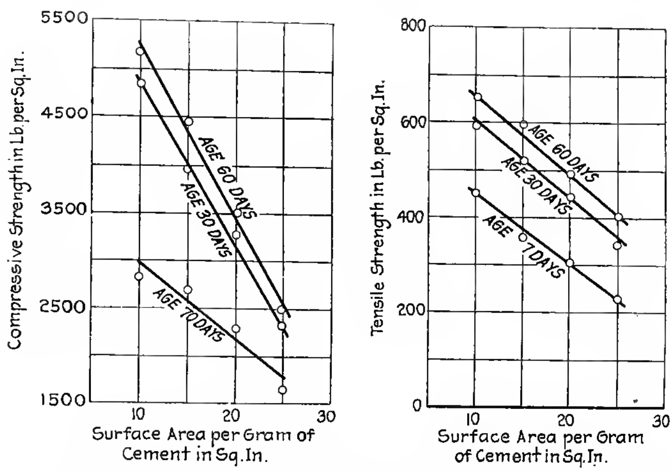
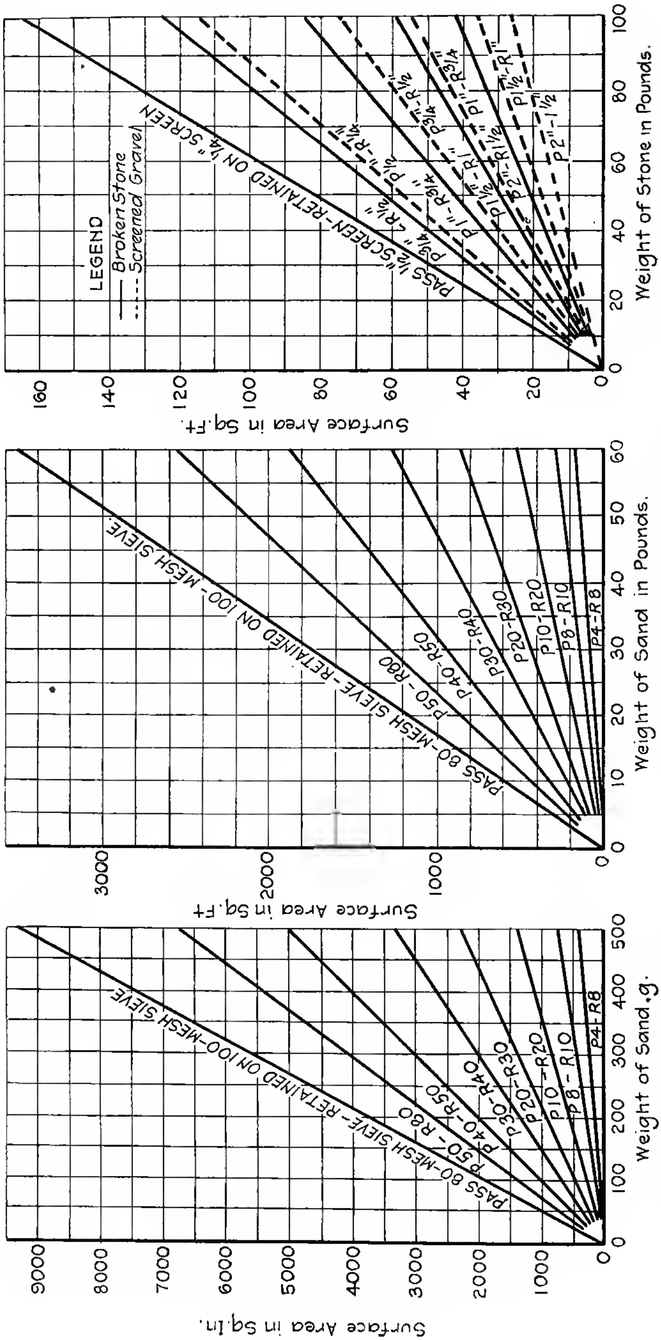


FIG. 123.—Capt. Edwards' method of surface areas. (*From Engineering News-Record, Aug.* 15, 1918, *p.* 317.)

Diagrams for Laboratory and Field Use.—For use in the laboratory and in the field, diagrams drawn to a large scale increase accuracy and reduce labor. Fig. 124 is designed for use in determining the surface area of sand aggregate. It is intended for laboratory use. Fig. 125 is the same sort of diagram intended for both laboratory and field use. The diagrams are derived from information obtained in the tests. Fig. 126 is designed for use in determining the surface of stone aggregate, and is intended for both field and laboratory use, and Fig. 127 shows the conversion diagram for determining the relative quantity of cement in pounds per 100 lb. of sand, and the corresponding relation of cement in grams to the surface area of 1,000 grams of sand, and *vice versa.* The author then gives the following example of how the diagrams shown in Figs. 124–127 may be used:



FIGS. 124, 125 and 126.—Capt. Edwards' method of surface areas. (*From Engineering News-Record, Aug. 15, 1918.*)

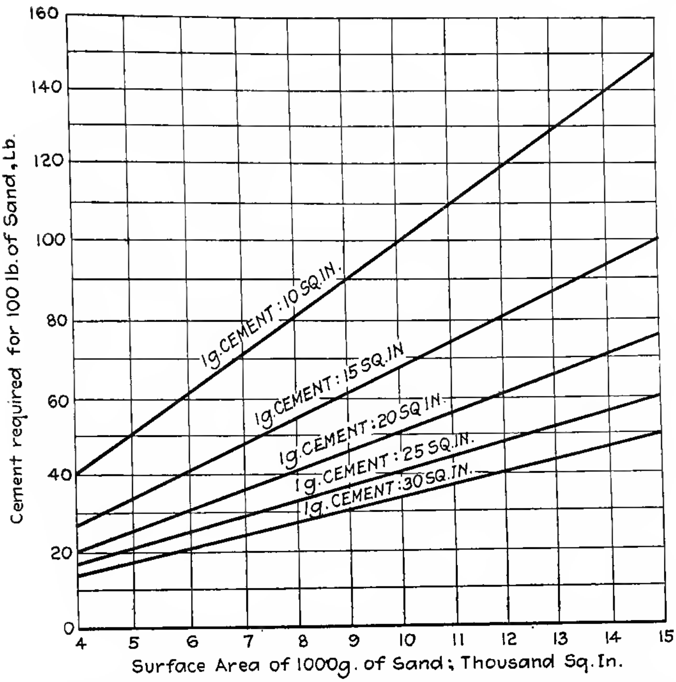


FIG. 127.—(*From Engineering News-Record, Aug. 15, 1918. Capt. Edwards.*)

Example No. 1.—Required to find the composition of a batch of mortar using 1,000 g. of sand A and a cement content proportioned: 1 g. cement to 15 sq. in. sand area.

SAND AREA

Sieve	Grading, per cent.	Weight, g.	Area (Fig. 4), sq. in.
P 4-R 8	15.0	150	142
P 8-R 10	5.0	50	75
P 10-R 20	25.0	250	694
P 20-R 30	15.0	150	676
P 30-R 40	15.0	150	997
P 40-R 50	10.0	100	992
P 50-R 80	10.0	100	1,348
P 80-R 100	5.0	50	932
Totals	100.0	1,000	5,856

$$\text{Cement (g.)} = \frac{5856}{15} = 390.5$$

$$\text{Water (c.c.)} = \left\{ 390.5 \times 22.25 \text{ per cent. (normal consistency)} \right\} + \frac{5856}{210} = 115$$

The author does not give anywhere what he considers to be proper ratio of the cement to the sand surface area. That would presumably have to be determined by investigations of the aggregates involved in any case.

Ratio of Fine to Coarse Aggregate Basis for Concrete Mixture.[1]—Another method of proportioning concrete mixtures is proposed by R. W. Crum, in a paper, read before the American Society for Testing Materials, and entitled "Proportioning of Pit-Run Gravel for Concrete." The method was devised for and is specially applicable to Middle Western gravels which occur in assorted gradings. By its use a proper concrete can be had with any pit gravel by the addition of the correct amount of cement, to be determined by the method. Basically, the author's scheme rests on the assumption that the ratio of cement to air and water voids is an indication of strength. In other words, the nearer the cement content approaches the volume of the voids the greater is the strength of the concrete. He assumes that for certain classes of concrete—that is, for concrete to be used under certain conditions—there is an optimum sand-aggregate ratio. In that ideal mix the cement-void ratio is computed and the amount of cement necessary to bring the actual mix up to that ratio is found. This gives the best mixture—reducing to loose volume—for that particular aggregate. Although the author states that the proper grading depends upon the consistency or amount of water in the mixture, and although he says specifically that one must get a concrete which will yield a workable mixture for the conditions under which it is placed, he does not tell in the paper just what degree of workability is reached by his method nor the standard of consistency or workability which was used in making the tests. He claims that the method gives results about midway between the fineness-modulus method of Abrams and the surface-area method of Edwards. Analyses of prospective aggregates may be readily made in the field for the method, inasmuch as it requires only to be known the gradings above and below a No. 4 sieve.

[1] *Engineering News-Record*, July 10, 1919.

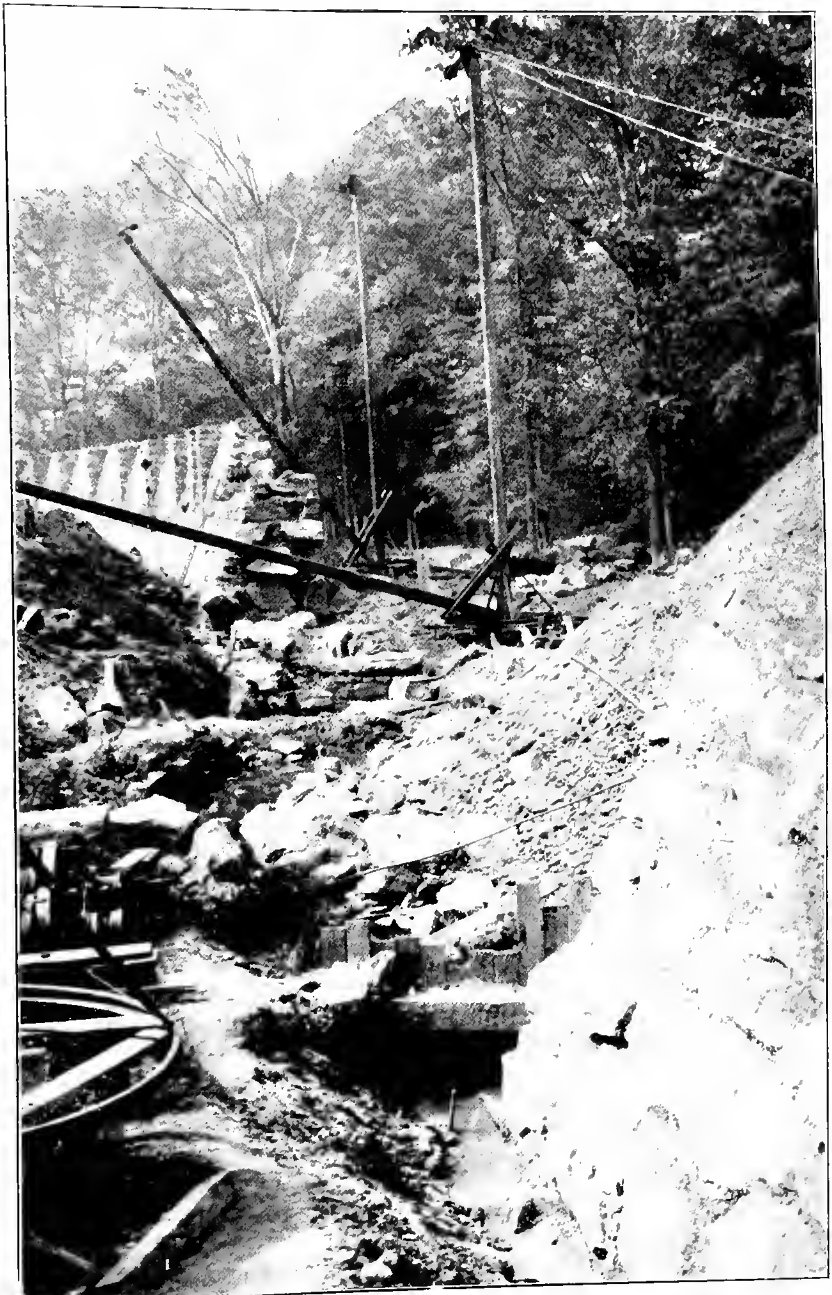


FIG. 1.—Method of laying stone wall by series of derricks.

(Facing page 227)

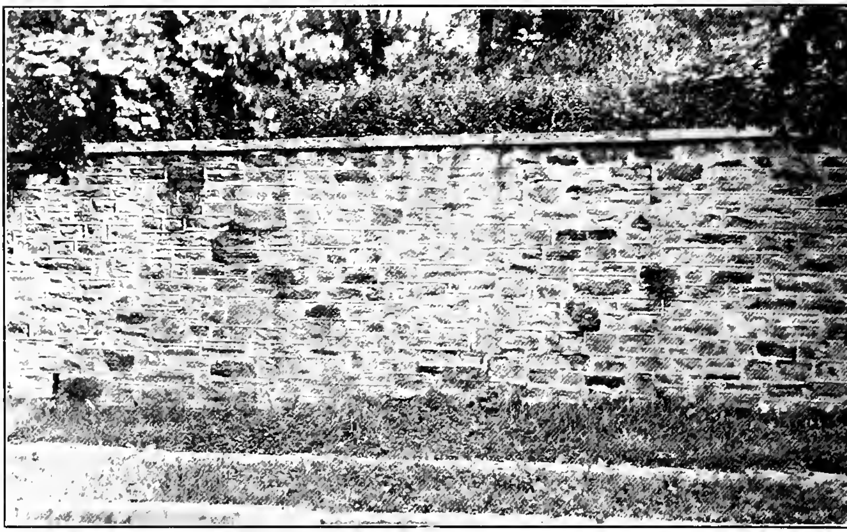


FIG A.—Uncoursed rubble wall with coursed effect given by false pointing.

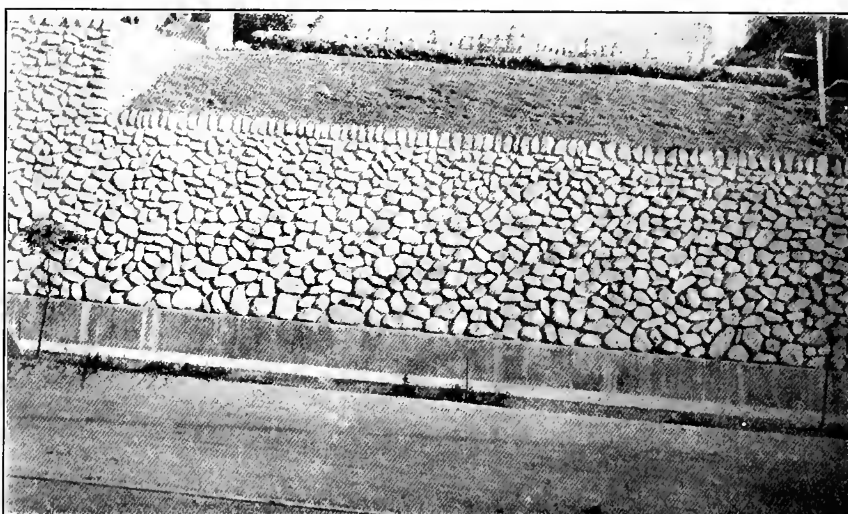


FIG. B.—Rubble wall (Los Angeles) with face formed by nigger-heads.

CHAPTER IX

WALLS OTHER THAN CONCRETE

Plant.—Rubble and cut-stone walls up to 5 or 6 feet in height are built of stone of such size that they are easily raised and set by hand. No special plant is therefore required and the wall is built entirely by hand labor. As the walls increase in height, good construction requires the use of larger stone, to insure a wall properly bonded together and it becomes necessary then to employ plant to raise and set the large stone. A derrick, either a guy or a stiffleg, is probably the most serviceable and efficient piece of plant to use in setting stone walls. It is operated by a hoist run by steam, electricity or air. A guy derrick is possibly preferable in that it permits a greater swing of the boom. It is limited however by the fact that it requires ample room to anchor its guys, room not always available, especially in city work. A stiffleg derrick is a self contained unit, the weight of the hoist and power plant providing the necessary anchorage.

In setting a derrick care should be observed that it is placed back from the wall a distance sufficient to ensure 'topping' out the wall. When the yardage of masonry permits, it is most economical and proves most time saving to set up a chain of derricks at such intervals that no gaps are left in the wall. This continuity of the work will obviate the tendency to cracks caused by joining up new work with old work (see Chapter V, "Settlement"). The derricks, when set up in sequence, are easily dismantled and set up in their new positions by aid of the adjoining derricks. Photographic Plate No. V, Fig. *A* shows the method of constructing high rubble walls (over 32 feet high) by means of such plant.

Mortar.—The mortar for use in the rubble masonry walls is mixed alongside the wall and is delivered to the working gangs in bucket loads as required. The usual mortar is mixed in proportions of one cement to three sand. For work of large size conveniently located, it may prove economical to mix the mortar by machine in a central plant and deliver by cart or otherwise over the work. Usually, however, it has proven most efficient to mix the mortar by hand for each gang, or for two adjacent gangs. The cement required for a rubble masonry wall of fairly large size (varying from twelve to forty feet in height) will average about one and one-half bags to the finished yard of wall. Due care in dressing the stone and chinking up the interstices with spalls will help to keep the amount of cement required to a minimum. With mortar mixed in the proportion of one cement to three sand, the finished wall should contain from 15 to 20 per cent. mortar.

Construction of Wall.—In constructing the wall the largest stone should be placed at the bottom course. If the soil is a slightly yeilding one the stones may be dropped from a height of two to three feet to insure their thorough imbedment. The bottom course may consist of a lean concrete in place of the rubble stone. The wall should have a proper proportion of headers (stones lying transversely) usually about 1/5 of the total yardage. The stones should be most carefully bedded, and all the interstices filled with spalls and if the wall is a mortar one, finished off with the cement mortar. The construction of a rubble masonry wall, both dry and cement requires a most conscientious coöperation between the engineer and the contractor and it is only by such mutual aid that a good masonry wall can be built. When a section of wall is to be finished some time before the adjoining section is to be built it is well to "rack" back the sides to insure a good bond between the old and new work. It must be remembered that a masonry wall has no expansion joints and that all movement of the wall, must be taken up by the masonry itself. Cracks will therefore a pear along the plane of weakness and unless great care has been exercised in the laying of the wall, these cracks will become very disfiguring.

The stone should be good, sound stone, thoroughly cleaned and roughly dressed to take off the soft and cracked edges. It should be wet before setting, especially in hot weather. Friable and soft stone should not be used. An excellent example of rubble masonry specifications, is quoted here.[1]

[1] Track Elevation, Philadelphia, Germantown and Norristown R. R., S. T. WAGNER, *Trans.* A.S.C.E., Vol. lxxvi, p. 1833.

"Third-class masonry shall be formed of approved quarry stone of good shape and of good flat beds. No stone shall be used in the face of the walls less than 6 inches thick or less than 12 inches in their least horizontal dimension.

Headers shall generally form about 1/5 of the faces and backs of the walls with a similar proportion throughout the mass when they do not interlock, and the face stones shall be well scabbed or otherwise worked so that they may be set close and chinking with small stone avoided.

In walls five feet thick or less, the stones used shall average 6 to 8 cubic feet in volume and the length of the headers shall be equal to two-thirds the thickness of the wall. In walls more than five feet in thickness the stones used shall average 12 cubic feet in volume and the headers shall not be less than four feet long. Generally no stones shall be used having a less volume than four cubic feet except for filling the interstices between the large stones.

In no case shall stones be used having a greater height or build than 30 inches and these stones must bond the joints above and below at least 18 inches; in all other cases the smaller stone must bond the joints above and below at least 10 inches.

The stones in the foundation shall generally not be less than 10 inches in thickness and contain not less than 10 square feet of surface. The foundation shall consist of 1 : 3 : 6 concrete, if so directed by the Chief Engineer."

Coping.—The wall, either dry or cement, is usually topped with a coping. Expansion joints in this coping should be placed at intervals of about five to ten feet. The sections may be separated by plain paper or may be tarred. The coping should preferably be placed after the wall has been constructed for some time. This permits settlement to take place and where definite cracks appear in the wall, expansion joints may be placed, to avoid unsightly cracking of the coping itself. When built of concrete, the coping should be about one foot thick and offset from the face of the wall about 3 or 4 inches. The form for the coping should be well built and carefully lined. Any carelessness in lining the coping forms shows in a wavy broken coping line and proves unsightly. The forms should be built of 2 inch stock, carefully wired and braced. This will prevent the bulging of the coping face and the thickness of the form will permit a frequent reuse of the form.

If a stone coping is desired, a blue stone flagging from 4 to 8 inches in thickness makes an effective top finish for the wall

Face Finish.—The face of a rubble or other masonry wall, receives such treatment as the environment of the wall requires (see Chapter X on "Architectural Treatment"). With care in the selection of face stone and with a fair attempt to dress these stones, the wall needs but little other work upon it except some pointing of the joints. As the demand for special face treatment increases, more attention must be paid to the selection of face stones and to the pointing of the joints. Face treatment may, roughly be divided into the following classifications: Rough pointing; special or false pointing; selection of special face stone; plaster finishes.

Rough Pointing.—After laying the wall, the stones are cleaned of whatever mortar has accidentally dropped upon them. The joints are raked and then brought to the rough face plane with mortar. For walls as generally built in the outlying districts, this type of treatment is sufficient.

False Pointing.—To obtain a somewhat more pleasing and decorative effect, rough, uncoursed masonry is pointed falsely, to give the appearance of coursed masonry. After cleaning the face stones the face of the wall is brought to a rough plane and is then coursed with the trowel into rectangles. Work of this nature is not of great permanence, the mortar slowly spalling off with the weather. (To secure the coursed masonry effect, more surface of the wall must receive a mortar coat than is necessary otherwise.) It is of questionable taste to attempt to mask the nature of the wall by such face treatment. This mode of treatment is usually limited to small walls forming the street walls of residential plots. A photograph of this class of wall is shown here (Photo Plate VI, Fig. *A*).

Special Stone.—The character of the masonry comprising the wall body may be completely masked by forming the face of the wall with specially selected stone. The rough masonry may then be considered a backing for the selected stone masonry. For walls entering into a costly and decorative scheme of landscape work, the face may be made an ashlar, or other coursed masonry effect, using limestone, sandstone or granite. When the walls are of considerable thickness it is usual to build them thus, with the expensive stone at the surface only. Walls of this type are the most costly of all walls, yet present the most imposing and pleasing types of masonry construction. The details of construction of these walls are thoroughly discussed in a number of standard text-books (*e.g.* Baker's "Masonry Construction") and need not be mentioned here.

A very pleasing effect secured by the use of boulders or "nigger heads" is shown here (see Photo Plate VI, Fig. *B*) (used extensively in Los Angeles). Various modifications of work of this kind are readily adapted to local environments with exceptionally pleasing results.

Plaster Coats.—This is probably the least desirable of surface finishes, both in effect and in duration of life. Because of its limited permanence great care must be exercised in applying these coats to the face of rough masonry walls. Plaster or stucco coats, when applied to the face of a wall, are rough cast or stippled. No trowelling is done upon the face, the mortar being placed with the usual wooden mortar board. To insure permanence some form of wire mesh or other netting should be fastened to the face of the wall to hold the plaster coat. The netting may be attached to wooden plugs inserted in the mortar while the wall is in the course of construction.

Cost Data.—The following is an analysis of the cost of a wall 36 feet high, averaging about 13 cubic yards to the running foot. It is merely a labor charge and does not include the cost of obtaining the stone, etc.

Cement Rubble Wall. 2750 Cubic Yards

Foreman, 114 days at $6.00 per day	$684.00
Masons, 167 days at $4.50 per day	751.50
Hoistrunner, 113 days at $6.00 per day	678.00
Signalman, 90 days at $2.50 per day	225.00
Laborers, 625 days at $2.50 per day	1562.50
Total cost	$3901.00

The average cost per yard, exclusive of all overhead, insurance, plant charges, materials, etc., is $1.42 per yard.

CHAPTER X

ARCHITECTURAL DETAILS, DRAINAGE, WATERPROOFING

Architectural Treatment.—Concrete retaining walls form a class of engineering structures for which ornate decorations are of questioned taste. Occasionally, however, some special face treatment becomes necessary to permit the wall to enter into the general landscape improvement involving a particular architectural scheme. Thus, for example, retaining walls forming an approach to a bridge, especially a concrete arch are usually made to follow the general viaduct architecture. Walls for a railroad station, where the main line is on the fill, must be in keeping with the architectural motive of the building itself. Walls in parks must receive such treatment as will make them harmonize with the park landscape work. In general, however, simplicity of treatment is essential, to conform with good taste.

Concrete walls are finished on top with a coping; usually about one foot thick and projecting 3 to 6 inches beyond the face of the wall. In addition a hand rail, picket fence, or concrete parapet wall is placed on top of the wall of plain or ornamental effect as conditions indicate. The face of the wall receives such treatment as will remove the unavoidable blemishes of construction.

Face Treatment.—The concrete face of the retaining wall may either be rubbed, tooled or receive a special composition surface. Preliminary to applying the face treatment, the tie rods, wires, etc. are cut back, and the face patched where necessary, employing a stiff mortar for this purpose. To insure a successful surface finish, it is imperative that the wall be well built. A surface finish cannot conceal poor work and poor work will eventually destroy the best surface finish. The less a wall is patched or otherwise repaired, the more certain it is that the surface treatment will be of pleasing and permanent character. Board marks are left after the forms are stripped which may be more or less masked by careful treatment. It may be set down as almost axiomatic that board marks can never be entirely eradicated, no matter what face treatment is applied. For this reason care must be taken in the continued use of the same set of forms, so that no panel is used in the face after its edges become splintered or frayed.

It has been pointed out in a previous chapter that construction joints leave a distinct cleavage mark. To make sure, for walls that will occupy a position of more or less architectural prominence, that there shall be no construction joints, it is specified that the section of wall between the expansion joints shall be poured completely in one operation. This is a praiseworthy mandate and is worthy of adoption for all character of work, regardless of merely the insistence of an architectural finish. The distance between expansion joints may be made such that it is practicable to pour a section complete with ordinary plant in one pour.

Defective concrete work appearing at the surface must be removed immediately upon stripping the forms and a rich mortar concrete inserted. Haphazard patchwork will not do. It is but a temporary expedient and the patch will soon spall off leaving a disfigured wall. A photograph of a wall so treated is shown here (See Fig. *A*, Plate VII) and is eloquent of the effects of poor concrete work and poor patch work.

If the forms are not held tight, or are not carefully caulked above work already completed, the yielding of the form, even to a minute degree, will permit the grout to run down coating and disfiguring the concrete work.

Briefly stated, conscientious vigilance in the observance of the edicts of good concrete work is the price of a good surface finish and using the analogy of pathology, diseases of the concrete body of a wall are usually exhibited by symptoms of facial blemishes.

Rubbing.—The face of concrete mirrors most faithfully the inside face of the form, bringing out the delineations of the board marks, the lips of the panels, etc. Immediately upon stripping the forms, and after cutting the rods and wires where necessary, and after making such patches as are indicated, the face is rubbed down with an emery block, and a thin grout wash is applied at the same time. The fresher the concrete, the easier it is to remove the facial blemishes by rubbing and it is therefore imperative that the forms be stripped as soon as good construction permits. For the average environment, and over 90 per cent.

of retaining walls are built in such environment, rubbing a wall presents finally a surface that is sufficiently pleasing.

In applying the grout wash, care must be taken to use a constant proportion of the cement and water. It is quite possible, where the rubbing is not done on one day, to use grout mixes of different strengths leaving the surface finished in two shades.

Tooling.—If the cement skin of a concrete wall is removed by sharp bits, the abraded surface gives a rough stone appearance quite pleasing in effect. This skin may be removed by hand with an ordinary wedge bit, or with special two, four and six edged bits. If there is a large amount of surface to be so treated, it is a matter of economy to use an air drill to work the hammer. The hammer is passed lightly over the surface, applied just long enough to remove the grout skin, care being taken not to start ravelling the stone. A gravel concrete seems to give a better appearance than a broken stone concrete, the sparkling effect of the pebbles presenting an excellent appearance, especially in the direct sunlight. When broken stone is used, the size of the stone should be limited to $\frac{3}{4}$ inch stone, the ordinary commercial stone. With larger stone it is difficult, in tooling the wall to prevent ravelling.

It is understood that tooling is much more expensive than rubbing (roughly about ten to fifteen times) and, ordinarily is only specified to effect a special architectural feature.

As in the case of rubbing, the concrete wall must be carefully patched and construction devices, such as rods, wires, etc., removed or cut back several inches from the surface.

It is usual to finish the edges of a tooled surface by means of a rubbed border of one or more inches in width. Care must be taken not to tool too near an edge as the concrete may be broken off.

Special Finishes.—To enhance the architectural appearance of a retaining wall, a special face finish is applied to the wall, masking its construction finish. An ordinary plaster coat may be applied to the wall, or a granolithic or other fine grit finish may be placed upon its surface. In applying such a coat it is essential that due appreciation should be had of the proper bond between the wall and the coat. To apply a coat of mortar or other finish after the forms have been stripped and the wall set gives little assurance of a permanent finish. The coefficients of expansion between the wall concrete and the rich mortar are unlike, producing eventually voids between the wall and coat. The action of frost and the other destructive elements finally cause the coat to spall. It is therefore usually specified that the finish coat shall be applied simultaneously with the pouring of the wall, so that the coat is a part of the wall itself, and is therefore more or less immune to the weathering actions. An excellent specification for a granolithic coat is quoted here and may be used as a model clause for all grit finishes.[1]

"Surface of concrete exposed to the street shall be composed of one part cement, two parts coarse sand or gravel and two parts granolithic grit, made into a stiff mortar. Granolithic grit shall be granite or trap rock crushed to pass a 1/4 inch sieve and screened of dust. For vertical surfaces the mixture shall be deposited against the face forms to a least thickness of one inch by skilled workmen, as the placing of concrete proceeds and thus form a body of the work. Care shall be taken to prevent the occurrence of air spaces or voids in the surface. The face forms shall be removed as soon as the concrete has sufficiently hardened and any voids that may appear shall be filled up with the mixture.

"The surface shall then be immediately washed with water until the grit is exposed and rinsed clean and protected from the sun and kept moist for three days. For horizontal surfaces the granolithic mixture shall be deposited on the concrete to a least thickness of 1.5 inches immediately after the concrete has been tamped and before it has set and shall be trowelled to an even surface and after it has set sufficiently hard shall be washed until the grit is exposed.

"All concrete surfaces exposed to the street shall be marked off into courses in such detailed manner as may be directed by the Chief Engineer."

Finishes of various colors may be secured by the use of properly colored grit. A red finish may be secured by the use of brick grit; a gray by bluestone screenings, etc. Below is a method of obtaining still another type of surface finish.[2]

"A surface finish for concrete, whereby a sand coating is applied may be secured by the following method, outlined by Mr. Albert Moyer of the Vulcanite Portland Cement Co. Erect forms of rough boards in courses of three feet or less and plaster the insides with wet clay worked to a plastic consistency. While the clay is wet apply evenly loose buff, red or other colored sand and then pour in the concrete.

[1] S. T. WAGNER, Track Elevation, Philadelphia, Germantown and Norristown Railroad, *Trans.* A.S.C.E., Vol. lxxvi, p. 1836.

[2] *Engineering Record*, Vol. 61, p. 454.

After removing the forms, wash off the clay with water and if necessary scrub lightly with a brush. The sand, Mr. Moyer states will adhere to the concrete and givs a surface of pleasing color and texture.

The following table gives the proportion of coloring matter to use to secure a desired shade of concrete finish. The table is taken from "Concrete Construction for Rural Communities," by Roy A. Seaton, page 148.

Color of hardened mortar	Mineral to be used	Pounds of color to each bag of cement
Gray	Germantown lamp-black	½
Black	Manganese dioxide	12
Black	Excelsior carbon black	3
Blue	Ultramarine blue	5
Green	Ultramarine green	6
Red	Iron oxide	6
Bright red	Pompeian or English red	6
Brown	Roasted iron oxide or brown ochre	6
Buff	Yellow ochre	6

"Colors will usually be considerably darker while the concrete is wet than after it dries out and the colors are likely to grow somewhat lighter with age. Hence considerably more pigment should be used than is necessary to bring wet concrete or mortar to the desired shade."

Artistic Treatment of Concrete Surfaces in General.—The treatment of concrete surfaces of all types is ably discussed in a book by Lewis and Chandler, "Popular Hand Book for Cement and Concrete Users" (see chapter "Artistic Treatment of Concrete Surfaces"). The various methods of finishing a concrete surface are classified as follows:

"1. Spading and trowelling the surface.
"2. Facing with Stucco.
"3. Facing with Mortar.
"4. Grouting.
"5. Scrubbing and washing.
"6. Etching with Acid.
"7. Tooling the Surface with Bush-hammers or other tools.
"8. Surfacing with gravel or pebbles.
"9. Tinting the surface.
"10. Panelling, Mosaic, carving etc."

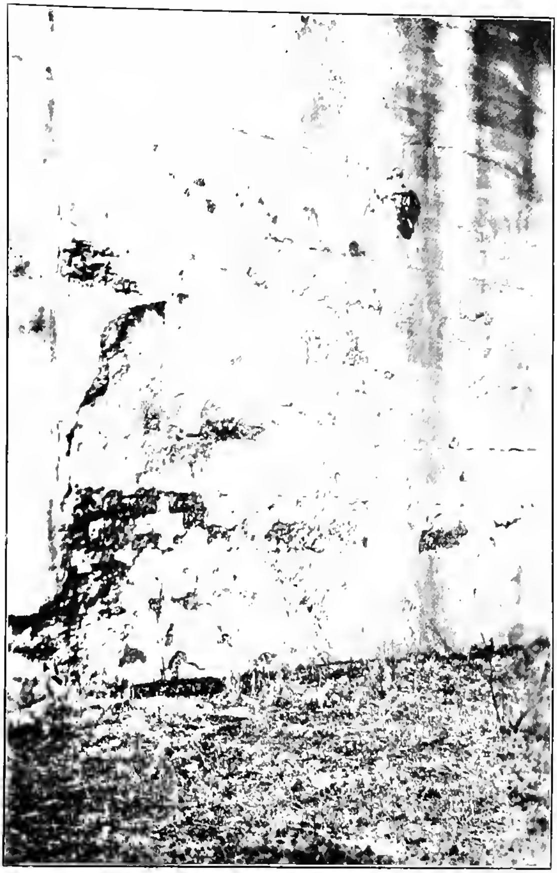


FIG. A.—Showing effects of poor concrete work

FIG. B.—Ornamental parapet wall. Tooled with rubbed borders.

(Facing page 236)

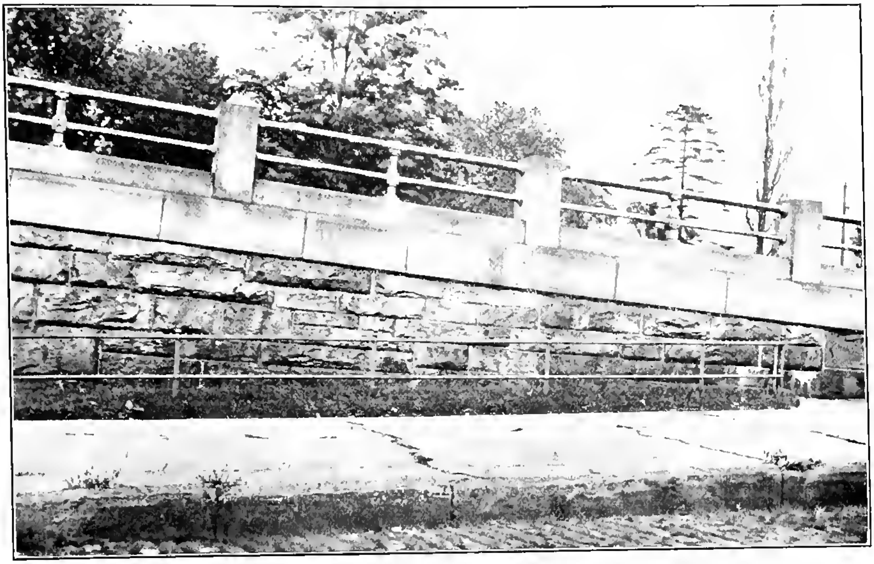


Fig. A.—Ornamental handrail—approach to viaduct.

Fig. B.—Picket fence. Wall lining open cut approach to depressed street crossing.

(Facing page 236)

FIG. A.—Ornamental concrete handrail approach to concrete arch.

The methods specially applicable to retaining walls have been analyzed in detail in the present chapter.

In connection with the artistic treatment of retaining wall surfaces, it may prove of interest to note that an exhaustive study of a special surface was made by John J. Earley, *Proceedings* American Concrete Institute, 1918, in a paper entitled "Some Problems in Devising a New Finish For Concrete." The wall under discussion was built in Meridian Hill Park, Washington, D.C. The original plans called for a stucco-finished wall. A sample of wall with such a finish was built. "The result was a plaster wall, nothing more * * * * * * * * * * the wall was without scale. It did not give the appearance of strength or size equal to its task as a retaining wall." It was finally decided to strip the forms of the wall as soon as possible after pouring (from 24 to 48 hours) and scrub the surface with steel brushes "until the aggregate was exposed as evenly as possible."

"This method of treating the surfaces at once supplied the sense of strength and size that was lacking before. The wall was no longer a plastered one, but was reinforced concrete and nothing else, and it seemed big and strong enough to suit all demands that would be made upon it."

The face was panelled and the piers were treated differently, to afford a contrast to the tooled surfaces.

Hand Rails.—To prevent accidents and trespassing or to lend a pleasing finish to a retaining wall a railing of some kind is built into the coping of the wall, of a character in conformity with the needs of the environment. When a wall retains an embankment rising above the surrounding country, the railing is required as a protection to those walking along the edge of the embankment. If the environment demands a railing more ornate in character, the railing may be made of concrete, stone, concrete blocks, etc. Some photographs of railings of this latter character are shown here (see Plates VII, Fig. *B*, VIII, Fig. *A* and IX, Fig. *A*). To prevent trespassing, by climbing over low walls, or walls which line cuts along a highway, it is usual to build a picket fence. A photograph of a standard type of such fence is shown on Plate VIII, Fig. *B*.

The metal railings are anchored to the wall by bolts. Holes are drilled in the wall coping to fit the railing bolts and the bolts are fastened in by means of grout, lead or sulphur. To properly and securely fasten concrete railings to the wall reinforcing rods should be incorporated in the coping while it is being poured and should project a distance above the top of the coping to obtain a good bond to the hand rail. For all types of railing provision should be made for the expansion due to temperature changes.

Drainage.—The presence of water in a retained fill increases the earth thrust in an uncertain but considerable amount. Again, to insure a well founded roadbed, water must not be permitted to accumulate in the fill. For these reasons means are provided for the removal of any water that may collect in the fill behind the wall. The simplest method of accomplishing this is to insert pipes in the walls at frequent intervals, permitting the water to drain through them and out on the surrounding ground. To insure ample provision for the run-off of the water and to prevent the pipe from silting up, a large size pipe, about 4 inches in diameter has proven to be most satisfactory as a weep-hole drain. The pipes should be spaced from twenty-five to ten-feet intervals depending upon the anticipated conditions of water accumulation. That water may be permitted to reach these openings in the wall, some rough drainage must be placed at the back of the wall. A well planned wall will provide for a layer of broken stone, from 6-inches to a foot in thickness upon the back of the wall and extending down to the level of the weep-holes. If this method is considered too expensive, or unnecessary for the conditions at hand, a layer of broken stone may be placed immediately around the weep-hole, preventing the silt from accumulating at the opening and permitting the water to drain off. Under no circumstances should the fill be placed immediately against the wall drains.

It is sometimes objectionable or impossible to dispose of the water through drains leading out from the face of the wall, because of private property, or important public thoroughfares adjoining and a regular sewerage system must be installed to dispose of the water through the neighboring sewers. For example in the track elevation work of the Rock Island Lines.[1]

"An unusual feature is the provision of drainage wells in the ends of the retaining walls adjacent to the abutments at the subway bridges. These are 3 feet by 3 feet and extend to the bottom of the wall (see Fig. 128). There are no weep holes through the retaining walls, but along

[1] *Engineering News*, Vol. 73, p. 671.

the backs of the walls are laid inclined drains of 6-inch porous tile on a grade of 0.5 per cent. extending from subgrade level to 6-inch pipes, which are imbedded in the rear part of the walls and discharge to the drainage wells. Each well has an 8-inch connection to the catch-basin of a city sewer as shown."

Again in the track elevation work of the Philadelphia, Germantown and Norristown R. R.[1] The walls were on private property and a layer of loose stone made up in sizes varying from 3/4 inch to two feet were placed along the back of the wall. A 6-inch vitrified tile pipe was laid along the bottom of the wall below this stone layer, on a 1 per cent. grade, with open joints and led to sewers on the cross streets.

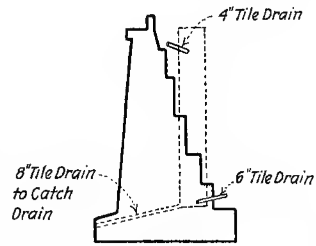


FIG. 128.—Drainage of retained fill, carried to sewer system.

Another efficient method of securing a well drained fill is to place wells of broken stone at each weep hole extending from the subgrade of the fill to the weep holes. In the construction of the retaining walls for the Hell Gate Arch Approach (see page 127) it was vital that no water be allowed to accumulate in the fill and wells were built at each weep hole to insure the drainage of the earth work.

Waterproofing.—The presence of water in the wall body, aside from that left originally from the concrete mix, has a harmful effect both on the concrete mass itself and upon the face of the wall. Generally it is specified that some means shall be taken to keep the water out of the wall. Retaining walls are not made of very rich mixes so that the wall cannot be said to be inherently water-proof. It is an easy matter to coat the back of the wall with tar or asphalt preparation. While it is exceedingly difficult to get an intact skin and to keep it intact, care exercised in placing the waterproofing and in preserving it from accidental abrasion after it has been placed will give a membrane of sufficient integrity to save the face of the wall. It is much better practice to place two coats of waterproofing upon the back of the wall, thus insuring that there are no bare spots on the wall back.

[1] *Trans.* American Society of Civil Engineers, Vol. lxxvi, S. T. WAGNER.

Before placing the membrane of tar, it is absolutely necessary that the wall be dry, free from frost and well cleaned. After the tar has been placed the fill should be deposited with care and large boulders should not be permitted to roll down and against the back of the wall. Where a mixed fill, rock and earth is used, it is good practice to carry up the soft fill against the back of the wall (unless a stone drainage well has been placed against the back of the wall) to act as a cushion for the rock fill.

Where expansion joints occur, several layers of fabric coated with hot tar are placed across the joint to insure its water-tightness, extending about a foot or two on either side of the joint.

Sub-surface walls and walls whose exterior face receive special architectural treatment to which any moisture is damaging, must, of course, receive more detailed waterproofing, involving the extensive use of fabrics, of brick laid in an asphaltic mastic, or the possible additions of chemicals to the concrete mix itself (the integral method of waterproofing) all of which fall without the province of the present text.

A typical and well-tried specification for a tar coating for the back of the wall, may read as follows:

Coal-tar shall be straight-run pitch containing not less than twenty-five percentum (25%) and not more than thirty-two percentum (32%) of free carbon, and shall soften at approximately 70° F., and melt at 120° F., determined by the cube (in water) method, being a grade in which distillate oils distilled therefrom shall have a specific gravity of 1.05.

Asphalt shall consist of fluxed natural asphalt, or asphalt prepared by the careful distillation of asphaltic petroleum and shall comply with the following requirements:

The asphalt shall contain in its refined state not less than ninety-five percentum (95%) of bitumen soluble in cold carbon disulphide, and at least ninety-eight and one-half percentum (98.5%) of the bitumen soluble in the cold carbon disulphide shall be soluble in cold carbon tetrachloride. The remaining ingredients shall be such as not to exert an injurious effect on the work.

The asphalt shall not flash below 350 degrees Fahr., when tested in the New York State Closed Oil Tester. When twenty (20) grams of the material are heated for five (5) hours at a temperature of 325 degrees F., in a tin box two and one-half inches in diameter it shall lose not over five percentum (5%) by weight nor shall the penetration at 77 degrees Fahr. after such heating be less than one-half of the original penetration.

The melting point of the material shall be between 115 degrees and 135 degrees Fahr., as determined by the Kraemer and Sarnow method.

The consistency shall be determined by the penetration which be between 75 and 100 at 77° F.

A briquette of solid bitumen of cross-section of one square centimeter shall have a ductility of not less than twenty centimeters at 77° F., the material being elongated at the rate of five (5) centimeters per minute. (Dow moulds.)

The penetrations indicated herein refer to a depth of penetration in hundredth centimeters of a No. 2 cambric needle weighted to one hundred grams at 77° F., acting for five seconds.

CHAPTER XI

LINES AND GRADES. COMPUTATION OF VARIOUS SECTIONS. ISOMETRIC WORKING SKETCHES. COST DATA

Surveying.—As an engineering structure, a retaining wall requires but little more special field work than other masonry structures. The trenches within which the wall is to rest must be staked out, the face of the wall must be laid out on the concrete bottom of the wall in its correct location with respect to the property, or other governing line, and finally the forms must be checked as to correct section and location. As the wall is essentially a longitudinal strip, a preliminary line, parallel to the face, or other important line of the wall, is staked out. This forms the base line of the wall location work, and the accuracy with which this line is laid out determines all the accuracy of the lines subsequently staked out from this line. The degree of exactness which must be employed in laying out the wall is conditioned upon several factors. The presence of adjacent structures, the nearness of the wall to important easement lines, either public or private, the necessity of tying other structures to the retaining wall (or abutment), the proposed permanence of the wall, will each control the permissible error in the field-work. An allowable error of one in 25,000 is sufficiently exact for any type of wall, regardless of the degree of exactness required and larger error factors should be used for less important structures.

The importance of the base line with reference to the field work which follows and is dependent upon it makes it necessary that it be laid out at a distance away from the work that will keep it safely out of the construction way, and yet close enough that it can readily be employed as a reference line. If th location of the work permits the line should be about 25 feet away from the wall line and referenced at frequent intervals to fixed land marks. It should be tied in to other important lines of permanent nature, such as city monument lines, the main railroad survey lines and such lines as control more or less, the location of the easement lines of the wall.

In conjunction with the location of the base line, a run of benches is made, safely established, so that the progress of construction will not disturb them. The accuracy of this run need not be high, unless steel structures are to be tied into the wall (*e.g.* abutments supporting steel bridges; retaining walls carrying building walls upon them, etc.).

It is patent, that in the establishment of both the base line and the bench run, points must be selected that can readily be found and used for the construction work. This is a matter of judgment, tempered by much field experience and vexatious delays must occur through poor selection of important surveying points.

Construction Lines.—The base line as above described is not used directly to stake out the construction work. It is customary to place a line about five feet from the face of the wall, and where possible, another line ten feet from the face, and both parallel to the base line, which lines are directly employed by the mechanics to lay out the excavation lines and the concrete lines. As the lines are destroyed in the ordinary course of construction, they may easily be restored, where necessary, by recourse to the permanent base line. On tangent walls, net line stakes (*i.e.* the actual wall lines) may be placed at twenty to twenty-five foot intervals. On curves, they should be placed close enough, that the chords do not diverge more than the permissible limit from the true arc of the wall. For the excavation lines, rough work is, of course permissible. For the concrete lines more refinement is required. To determine the proper chord length, L, to be used in staking out the wall, so that its middle ordinate will not exceed the permissible allowance a (see Fig. 129), note that from the approximate parabolic relation that the offset y to an arc, at a distance x from the point of tangency is given by the formula

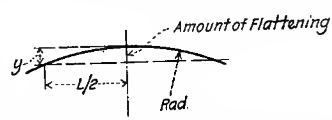


FIG. 129.—Length of chord for permissible amount of flattening.

$$y = x^2/2R$$

where R is the radius of the arc. Employing this formula in the present case

$$a = L^2/8R$$

It is generally specified that the flatness of the wall shall not exceed ⅛ of an inch, or 0.01 feet. This last equation when solved for L, using the value 0.01 for a is then

$$L = \frac{\sqrt{2R}}{5}$$

To aid in the use of this equation, Table No. 39 is given herewith showing the necessary chord length to be used for any assigned radius of arc, that the chord offset shall not exceed one-eighth of an inch. For example, by reference to the table, a radius of 800 feet makes it necessary to stake out the wall in eight-foot chord lengths, while a radius of 8,000 feet permits the use of twenty-five-foot chords.

Table 39.—Maximum Chord Lengths

R	L	R	L	R	L	R	L	R	L
50	2.0	325	5.1	700	7.5	1250	10.0	4500	19.0
75	2.5	350	5.3	750	7.8	1300	10.2	5000	20.0
100	2.8	375	5.5	800	8.0	1350	10.4	5500	21.0
125	3.2	400	5.7	850	8.2	1400	10.6	6000	21.9
150	3.5	425	5.8	900	8.5	1450	10.8	6500	22.8
175	3.7	450	6.0	950	8.7	1500	11.0	7000	23.7
200	4.0	475	6.2	1000	9.0	2000	12.7	7500	24.5
225	4.2	500	6.3	1050	9.2	2500	14.2	8000	25.0
250	4.5	550	6.6	1100	9.4	3000	15.5		
275	4.7	600	6.9	1150	9.6	3500	16.7		
300	4.9	650	7.2	1200	9.8	4000	17.9		

The bottom of the wall, whether of concrete or other masonry should not necessarily fill the trench unless this has been trimmed with unusual care. If the net-line stakes have been lost in the excavation, these should be restored, and the proper bottom lines given for the masonry footing. For grades, stakes may be driven into the side of the cut at the required elevation, or at a stated distance above this line. For the elevation of the bottom of the wall the same stakes may be employed.

Forms.—With the bottom concrete in, it is necessary to give some line to commence the form work. A very serviceable method is that of nailing a molding strip to the concrete bottom, marking the inside of the lagging of the form (see Fig. 130). With this line in place the face forms may be set and the rear forms placed at the required distance away as specified on the plans. After the forms are assembled, wired and braced, they may be rechecked from the reference line, and then plumbed to see that the section meets that theoretically required. The proper grades at which to make the breaks in the wall section, if there be any, and the grade for the top of the wall, are most commonly given by nails driven in the side of the form at these elevations.

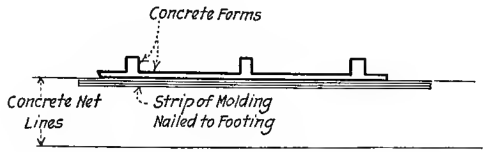


FIG. 130.—Method of lining in concrete forms.

Computation of Volumes.—When the section of a retaining wall remains constant between two given points, its volume is the product of the area of the section by the distance between the two points. Generally the section of the wall varies, the top of the wall following a given grade. Breaks in the width of the wall, or in other but the vertical dimensions, are made at the expansion joints, so that between two adjacent expansion joints the width of wall at the coping and at the base remain constant. The volume of a wall, whose coping and base widths are respectively a and b, and whose heights at the beginning and end of the section are h_1 and h_2, respectively, is

$$V = \frac{L}{4}(a + b)(h_1 + h_2)$$

To get the volume of sections of the wall which are irregular because of breaks in the wall, or because of intersections with other walls, it is essential that a careful and detailed drawing be made. It is difficult to show clearly the volume in question when the drawing merely gives a two-dimension section. For this reason isometric drawings may serve to bring out clearly and exactly all the dimension necessary to obtain the volume of the portion sought. To make the isometric drawing correct to scale and to be able to interpret mathematically the lengths scaled from the isometric drawing the following matter gives some formulas and tables which should serve to make the isometric layout as easy to handle as the plane detail drawing.[1]

[1] See *Engineering News-Record*, April 3, 1919, p. 661.

It is assumed that the isometric taxes of the figure have so been chosen that all the important lines of the figure lie in planes parallel to the axes. The following theorems apply solely to such lines. Lines parallel to the axes are shown correctly to length by the principles of isometric projection. Lines not parallel are not shown correctly to length. To obtain the angles which these lines make in actual space, and the actual lengths of such lines and conversely, the lengths of such lines in isometric projection and the angles which they make with the isometric axes, refer to Figs. 131 and 132. In Fig. 131, the line L has pro-

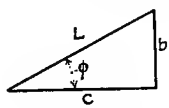


FIG. 131.

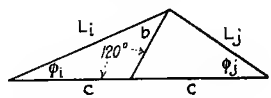


FIG. 132.

The plane and isometric triangles.

jections b and c and makes an angle ϕ with the projection c. In isometric projection the length L becomes either L_i or L_j depending upon whether it subtends an angle of 120° or 60°. The angle ϕ is again ϕ_i or ϕ_j in isometric projection. The lengths b and c remain unchanged. $\tan \phi = b/c$. Referring to Figure 132, by the law of sines

$$b/c = \sin \phi_i/\sin (180° - 120° - \phi_i)$$
$$b/c = \sin \phi_j/\sin (180° - 60° - \phi_j)$$

From which two equations,

$$\phi_i = \cot^{-1}\left(\frac{1 + 2 \cot \phi}{\sqrt{3}}\right)$$
$$\phi_j = \cot^{-1}\left(\frac{2 \cot \phi - 1}{\sqrt{3}}\right)$$

Table 40 gives the values of ϕ_i and ϕ_j for the several values of ϕ. Referring to Figs. 131, 132.

$$L^2 = b^2 + c^2$$
$$L_i^2 = b^2 + c^2 - 2bc \cos 120° = b^2 + c^2 + bc$$
$$L_j^2 = \qquad\qquad = b^2 + c^2 - bc$$
$$b = L \cos \phi \qquad \text{and} \qquad c = L \sin \phi$$

Substituting these values in the preceding equations there is finally

$$L_i = kL; \quad L_j = jL$$

where $k^2 = 1 + \sin \phi \cos \phi; \quad j^2 = 1 - \sin \phi \cos \phi.$

Table 40 gives a series of values of k and j for the run of values of ϕ.

TABLE 40.—ISOMETRIC FUNCTIONS

ϕ	ϕ_i	ϕ_j	k	j
0°	0°	0°	1.00	1.00
5	4	5	1.04	0.95
10	8	10	1.09	0.90
15	12	15	1.12	0.87
20	15	21	1.15	0.82
25	18	28	1.18	0.79
30	21	35	1.20	0.75
35	24	43	1.21	0.73
40	27	51	1.22	0.71
45	30	60	1.23	0.71

Fig. 133 gives an illustration of some wall details shown isometrically and properly scaled and dimensioned (all dimensions

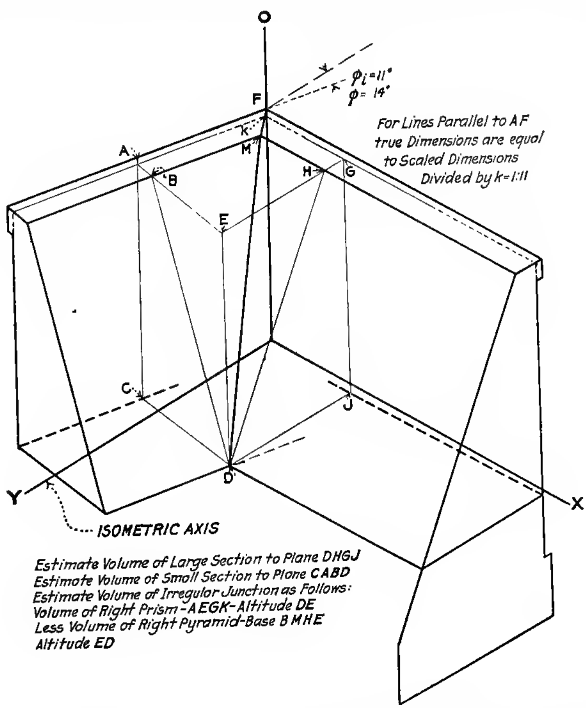


FIG. 133.—The isometric detail and its application to the computation of volumes.

shown are the true ones, the isometric lengths as shown having been corrected by means of the tables above.

Cost Data.—The compilation of worth-while cost data is conditioned upon the proper valuation of the relative operations involved in the piece of work under analysis as well as a correct understanding as to how much of the work is standard in connection with retaining wall construction and how much is peculiar to the individual piece of work in question. Merely gathering cost statistics without an intelligent interpretation of the operations affecting or controlling costs is a valueless and time wasting procedure.

Cost analysis in general may be said to serve two purposes. It furnishes an accounting of work already done, in order that proper disbursements may finally be made and a correct financial history compiled of the job in question. In this sense it is properly an accounting job, based upon payroll and material forms prepared by the timekeeper. It may also be an anticipatory analysis of work to be done and then comes within the province of an engineer preparing such an estimate. Proper attention to the former purpose of cost data is of course essential that the latter purpose may be efficiently carried out and the more voluminous the files of cost accounts (intelligently kept) the better able is the engineer to make a scientific prediction of the cost of future work.

That a true comparison may be made of the relative value of the various types of retaining walls, it is apparent that the elements entering into the cost data must be properly weighted, so that items of cost unique to a peculiar environment be disregarded. For this purpose, it is best that cost data be reduced, as far as is practicable, to fundamental and elemental operations, independent, more or less, of the peculiar character of any piece of construction.

Cost may be divided into several general subdivisions: Labor cost; material cost; plant cost and general administrative expenses. The first item, the labor cost, is the uncertain item, and one requiring experience and judgment in its proper determination. Material costs are simple, are easily compiled; can easily be anticipated and with a proper allowance for the wastage involved in the several operation are estimated with a high degree of accuracy. Plant cost, while possibly not so easily compiled or anticipated as material cost, should not, at least to the engineer with a moderate amount of experience, prove difficult of computation. In a previous chapter the character and the distribution of plant employed for a number of pieces of typical retaining wall construction may furnish a good working clue to the type most suited to the work under analysis. General administrative expenses will cover office expenses, salaries of the executives, insurance upon the labor, miscellaneous casualty and public liability insurance, minor expenses in connection with the prosecution of the work, such as telephone, fares, taxes, etc. This item is usually termed the overhead of the work and is spread over all the items entering into the construction of a wall. While of an indefinite character, it must be properly ascertained or anticipated in order to be included in the estimated cost. It must be remembered that it is a constant charge carried continuously, regardless of the weather or other delays and in work of long duration, may effect materially the cost of the operations. Blanket percentages added to cover items of this nature, while excusable in small work, are apt to work hardships upon large work unless the percentage factor so applied is the result of data compiled from several jobs of similar nature. Naturally the number of items of uncertain amount appearing in an estimate of future work will be in inverse proportion to the amount of experience of the engineer preparing such estimates.

Labor Costs.—Without entering into a detailed analysis of the various labor elements involved in wall construction,[1] some general labor costs may be presented to guide an estimator in preparing a bid for contemplated work. Before employing such data it is well to read again (chapter on "Plant") the important bearing of plant selection and arrangement upon the cost of labor. A good bid is not one that contains merely a carefully and detailed analysis of the cost of the labor. It must plan a scheme of the work together with the amount of plant to be had and the character of the labor to operate it. Haphazard bidding or snap judgment estimates are unpardonable in all but the most experienced of contractors and engineers, and must eventually lead to financial disaster. Such figures and quoted estimates of the cost of work as are given below must be used in light of the above remarks.

The material for the wall is taken from the point of delivery

[1] See Dana, "Cost Data," Gillette "Handbook of Cost Data;" Taylor and Thomson, "Concrete Costs."

and brought to the site of the work either at a contracted price per yard (which price may be ascertained at the time of preparing the bid) or if delivered F.O.B. nearest railroad station or lighterage dock may be hauled by hired team or auto truck. With the latter method, the length of haul will determine the average number of trips that the trucks can make, and knowing the load that can be carried, the price per yard for delivering the material can be computed with no great difficulty. An analysis of the cost of several pieces of work, follows. The files of the *Engineering Press* may be used to examine the cost of numerous pieces of work.

From Taylor and Thompson "Concrete Costs," p. 16:

Cantilever wall, 16 feet high, 250 feet long; common labor $2.00 per day, carpenters $3.82 per day. Concrete yardage 277 cubic yards.

Cost of labor of forms per cubic yard of concrete....	$2.75
Total cost of forms per cubic yard of concrete.......	$3.91
Cost of material per cubic yard of concrete..........	$3.57
Cost of mixing and placing concrete per cubic yard..	$1.35
Total cost of concrete in place (including superintendence).....................................	$12.03

Cantilever wall 16 feet high. Labor 20 cents per hour; carpenters 50 cents per hour.

Total cost of forms per cubic yard of concrete.......	$3.60
Cost of concrete material per cubic yard............	4.75
Cost of mixing and placing the concrete............	1.25

Cantilever wall 8 feet high. Labor 20 cents per hour; carpenters 50 cents per hour.

Total cost of forms per cubic yard of concrete.......	$6.23
Total cost of material per cubic yard of concrete....	4.75
Cost of mixing and placing per cubic yard..........	1.25

A resume of the total labor cost of pouring retaining walls of both gravity and reinforced "L" type, averaging about 35 feet in height is as follows:

Gravity Type 1935, cubic yards of concrete. Plant used was two small batch mixers, the concrete wheeled to the forms and poured in.[1]

[1] See "Enlarging an Old Retaining Wall," for a detailed description of the methods and plant used, *Engineering News*, Sept. 8, 1915.

The forms were used on the average about four times.

Foreman, 175 days at $5.00 per day	$875.00
Carpenters, 190 days at $3.50 per day	$665.00
Engineer, 46 days at $5.00 per day	230.00
Laborers, 926 days at $1.75 per day	1620.50
Teams, 21 days at $3.50 per day	73.50
Timbermen, 20 days at $3.00 per day	60.00
Masons, 37 days at $4.00 per day	148.00
Riggers, 14 days at $3.00 per day	42.00
Watchmen, 33 days at $1.00 per day	33.00
Total labor cost	$3747.00

This makes the labor cost per yard, exclusive of all overhead insurance, plants charges etc., $1.94 per cubic yard of concrete.

A similar detailed labor cost to pour a "L" shaped cantilever wall, involving a yardage of 1697 cubic yards is:

Foremen, 197 days at $5.00 per day	$985.00
Carpenters, 503 days at $3.50 per day	1760.50
Engineer, 37 days at $5.00 per day	185.00
Riggers, 24 days at $3.00 per day	72.00
Laborers, 1197 days at $1.75 per day	2094.75
Masons, 55 days at $4.00 per day	220.00
Teams, 51 days at $3.50 per day	178.50
Watchmen, 124 days at $1.00 per day	124.00
Total labor cost	$5619.75

The unit labor cost per cubic yard for pouring this type of wall, exclusive of all overhead charges as above enumerated is $3.31 per cubic yard.

While endless data might be furnished of the cost of existing work, conditions are usually too unique to make such data of general usefulness. Unit costs as quoted above may fill in uncertain data in a bid, when properly altered to take care of changed labor rates. The labor cost on a retaining wall, roughly, averages about one-quarter the total cost of the wall. Barring unforseen contingencies an estimator with a fair knowledge of construction work should be able to anticipate the labor cost within 20 per cent. of its correct final value. Should the discrepancy amount to the limiting value of 20 per cent., in the final data it will amount to merely 5 per cent. of the total cost of the work. Estimates of work can hardly be expected to reach a higher degree of accuracy than this.

As an example of the analysis of a proposed piece of work, let it be required to determine the cost of constructing a retaining wall about 1,000 feet long, 40 feet high, with a yardage of about 10,000 cubic yards. One year is the allotted time in which to construct the wall. The wall is a cantilever type.

Plant.—A mixer of about 100 yards per day capacity (a ⅓ to ½ yard batch mixer will easily satisfy this requirement) should pour the required yardage of concrete with an ample time margin. This mixer should be obtained in the neighborhood of about $1,000. The other plant requirements, such as wheelbarrows, shovels, etc.; shanties for storing cement and tools, for temporary offices; lumber for runways for pouring the concrete etc., should not cost more than an additional $1,000 making the total plant charge $2,000.

Materials.—Assuming that the wall is a 1:2.5:5 mixture of concrete, there will be required about 1.2 barrels of cement for each yard of concrete placed. Theoretically about 10,000 yards of stone and 5,000 yards of sand will be required. To allow for wastage of all kinds these quantities will be increased 10 per cent. It will be assumed that the materials will be delivered on the job, where required for the following unit prices; cement $3.50 per barrel (net, no allowance for bags); stone for $2.50 per yard and sand for $2.00 per yard. The material totals are then

13,200 bbls. cement at $3.50	$46,200
11,000 yards stone at $2.50	27,500
5,500 yards sand at $2.00	11,000
The total material will cost	$84,700

Form Lumber.—Assume that 2 inch tongue and grooved sheeting will be used to make the form panels. Allow about 20 per cent. wastage of forms each time the forms are stripped (this is equivalent to a form use of five times). The area of wall surface that must be covered with new form lumber is then (allowing a footing thickness of four feet)

$$\frac{36 \times 2 \times 1000}{5} = 14{,}400 \text{ square feet.}$$

To allow for the joists, rangers, bracing etc., and to allow for wastage in material due to cutting it to required lengths, it is customary to double the board feet required for the sheeting. (Exactly, the forms may be designed as outlined in the chapter on FORMS, and detailed as shown in the problem accompanying the chapter, and the required amount of timber taken from these estimates. An estimate of the cost of the work, does not, however, justify such refinement, and it is better to use the rule of thumb method just stated.) Since the sheeting is to be 2 inches thick, the total lumber requirements are 4 board feet for every square foot of new lumber surface. With a price of $75 per *M* for timber delivered on the job, tongue and grooved, the timber cost is

$$14.4 \times 4 \quad \text{at} \quad \$75 = \$4{,}320$$

Labor Costs.—To get the total labor costs on the wall, the analysis of the cost of the reinforced concrete wall at last outlined may be used with the following revised rates of labor: Foreman $8.00 per day, Engineer and Carpenters, $7.00 per day; laborers $4.00 per day and the other items in keeping. This will practically double the unit cost of labor as given. The unit cost is then about $6.75 per yard, or the total cost is $67,500. To this must be added the item of insurance, amounting to about 10 per cent. of the labor total, or $6,750.

Overhead.—The work will require the employment of a superintendent for one year ($4,000) and a timekeeper ($1,500) Miscellaneous expenses around the work should not exceed $1,000, making the field overhead about $6,500.

The office overhead is indeterminate, depending upon the number of jobs going on at one time. This factor will be omitted here.

The rods are usually quoted at a separate unit price and are not mentioned here.

To summarize:

Plant	$2,000
Materials	84,700
Lumber	4,320
Labor (and Ins.)	74,250
Overhead	6,500
Total	$171,770

With an allowance for profit the wall will be estimated in the neighborhood of $200,000, or at a unit cost of $20.00 per cubic yard.

SPECIFICATIONS

General Layout of Work.—The retaining walls to be constructed under this contract are shown on Plans Nos. to inclusive. These specifications and the plans are intended to be consistent and where any apparent inconsistency appears the interpretation shall convey the intent of the best work and construction.

Classes of Work.—The retaining walls shall be classified for payment as follows:

Class A.—Walls without reinforcement, marked *A* on the plans, of whatever height indicated.

Class B.—Reinforced concrete walls up to but not including twenty (20) feet in height from subgrade to top of coping.

Class C.—Reinforced concrete walls from twenty (20) feet up to but not including thirty (30) feet from subgrade to top of coping.

Class D.—Reinforced concrete walls over thirty (30 feet) in height from subgrade to top of coping.

Class E.—Walls of cement rubble masonry of whatever height indicated.

Payment.—Payment for the walls as indicated shall include the furnishing of all labor and materials necessary, including the cost of all scaffolding, forms and the cost of removing the same; also the cost of finishing the face of the wall where a rubbed finish is indicated.

Concrete Proportions.—Concrete for class *A* walls shall be mixed in the proportions of one part cement, two and one-half parts of sand and five parts of stone or gravel, by volume.

Concrete for reinforced concrete walls (classes *B*, *C* and *D*) shall be mixed in the proportions of one part cement, two parts sand and four parts of stone or gravel, by volume.

Cement.—The cement shall be Portland Cement of a brand that has been on the market for the last five years.

(Insert here the details of the properties of cement as has been given on pages 214 to 215.)

Sand.—Sand for use in making the concrete shall be clean and well graded, not exceeding ¼ inch in size. Not more than six per centum (6%) by weight shall pass a 100 mesh screen. It shall contain not more than three per centum (3%) by weight of foreign matter.

Broken Stone.—Stone for concrete shall be a clean sound, hard broken limestone or trap rock and graded from three-eighths (⅜) of an inch in diameter up to one and three-quarters (1¾) inches in diameter. Where the thickness of the concrete wall is twelve inches or less in thickness the size of the stone shall not exceed three-quarters (¾) of an inch in diameter. It shall be screened and washed to remove all impurities and shall be carefully stored along the site of the work to prevent the gathering of any foreign matter in it.

Gravel.—Gravel shall be screened, cleaned and graded in the same manner as the broken stone.

Use of Large Stone.—In Class *A* walls (and in these walls only) where the thickness of the wall exceeds thirty (30) inches the contracter will be permitted to imbed stones of at least 12 inches in thickness not closer than four (4) inches to the face of the form and not closer than six (6) inches to each other. The stones shall be sound, clean stones and shall be carefully placed in the concrete.

Concrete.—Concrete shall be mixed by machine. In case of emergency it shall be within the discretion of the Engineer to state whether the mixing shall proceed by hand.

It is the very essence of these specifications that the water content of the concrete mix by kept low. No machine mixer shall be used that is not equipped with a tank or other device for supplying a measured amount of water to each batch of concrete and a competent operator shall be in attendance upon the machine.

The Engineer, or his duly authorized representative shall decide upon the length of time each batch shall be mixed and upon the amount of water that shall go into each batch.

The contractor shall permit the Engineer to take samples of the concrete mix to be tested and no charges shall be made for material taken for such purposes.

The use of a continuous mixer is forbidden and a mixer that is found incapable of delivering a concrete in conformity with the specifications shall be removed from the work and a mixer substituted for it that is capable of mixing concrete in accordance with these specifications.

Concrete shall be conveyed to the forms in water-tight conveyances and shall be dropped vertically into the forms. It shall then be shovelled into place and thoroughly compacted and rammed to insure a concrete of uniform density.

Spades or other special tools shall be used on the concrete to insure a free circulation of the grout around the reinforcing bars and against the face of the forms.

Forms.—The forms for concrete shall be made of stout tongue and grooved sheeting, properly supported and braced and of strength sufficient to meet the concrete pressures. If so required the contractor shall submit to the engineer plans of the form work and bracing.

Before pouring the forms shall be oiled, or thoroughly wetted and before reusing shall be cleaned of all adhering cement, dirt, etc., to insure a smooth face on all exposed concrete work.

The joints shall be water-tight and shall be carefully inspected while the pouring is in progress to prevent the escape of any grout.

Concrete shall set at least twenty-four (24) hours before the tie-rods are loosened or any of the sheeting removed. This time shall be increased when the temperature of the air drops below sixty (60) degrees Fahr. Forms shall be stripped in the presence of the Engineer, if the contractor is so directed.

Placing Fill.—No fill shall be deposited behind the walls until ten days have elapsed since the walls were poured and not until the assent of the Engineer or his duly authorized representative has been obtained.

Defective Work.—If upon stripping the forms there is evidence of any defective work, such defective work shall immediately be repaired and the surface of the wall finished in a manner that will present as little evidence of such defective work as possible.

Evidence of extensive defective work shall be sufficient cause to order the contractor to remove portions of the work showing such defective work and all such repairs and reconstruction work shall be made at the contractor's own expense.

Concrete Work in Winter Weather.—When the temperature of the air drops below 45 degrees Fahr. it shall be within the discretion of the Engineer to order the contractor to heat the concrete materials before pouring them into the forms.

No concrete shall be deposited in the forms in freezing temperature that has not been mixed with materials heated by means of suitable appliances so that the temperature of the concrete upon being placed in the forms shall not be less than 60 degrees F. Concrete deposited in freezing weather shall be protected while setting by means of salt hay, tarpaulin, canvas, or by other devices which will maintain the temperature of the concrete above freezing until it has set.

No concrete shall be deposited in the forms when the temperature drops below 20 degrees Fahr., unless such forms have been constructed in a manner approved by the Engineer, to prevent freezing of the concrete mix.

Joints.—Where a break occurs in the day's pour, no additional concrete shall be deposited on such a joint when work is subsequently started until the joint has been thoroughly scrubbed to remove all laitance and other foreign matter. If so directed a layer of cement grout shall be deposited upon the joint immediately before placing fresh concrete.

It is the intent of these specifications to secure a section of wall between expansion joints free of all joints as above and the contractor shall use plant of such capacity that a section can be poured complete in a regular day's operation. When, due to an emergency, such a construction joint is unavoidable, the Engineer, or his duly authorized representative shall instruct the Contractor as to what details of construction must be adopted to obtain the full efficiency of such a joint and to prevent, as far as possible, any unsightly appearance of the face of the wall after the forms have been stripped.

Drains.—There shall be incorporated in the wall, tile drains of spacing and diameter shown on the plans. Immediately back of the drains shall be placed one cubic yard of broken stone.

Waterproofing.—The back of the retaining walls shall be given two coats of hot asphalt or pitch. The back of the wall, before the tar is applied shall be thoroughly dried and free of all frost.

(Insert specifications for tar as given on page 240.)

Extreme care shall be exercised in placing the fill back of the wall so that the coats of tar shall not be abraded.

If, after the fill has been in place the face of the wall shows evidence of water leaking through it, the contractor, if so directed by the Engineer, shall excavate back of the wall to the indicated position of the defective waterproofing and shall make such repairs as are necessary, no additional payment to be made for this work

Concrete Finish.—Where no special face finish is indicated, the Contractor shall, immediately upon removing the forms, remove all wires, rods, etc., or cut them back to about two inches from the face of the wall. He shall then point up these places with a rich mortar or concrete. The face of the wall will then be rubbed down with suitable appliances as approved by the Engineer and the entire surface given a coat of thin grout wash.

Reinforcing Bars.—Reinforcing bars shall be placed in the concrete walls of dimensions and spacing as shown on the plans. Payment for these rods includes all labor and material required for their installation as indicated.

Rods shall be deformed as approved by the Engineer. Plain bars may not be used.

Rods shall be bent to radii as indicated and shall generally be delivered in the full length as required on the plans.

Rods shall be made by the open hearth process with the following maximum impurities:

Phosphorus, not more than 0.04 per cent.

Sulphur, not more than 0.05 per cent.

The elastic limit or yield point shall not be less than 40,000 pounds per square inch.

Test specimens for bending shall be bent under the following conditions without fracture on the outside of the bent portion:

Around twice their own diameter.

1 in. in diam., 80 degrees.
3/4 in. in diam., 90 degrees.
1/2 in. in diam., 110 degrees.

Around their own diameter.

1/4 in. in diam., 130 degrees.
3/16 in. in diam., 140 degrees.
1/8 in. or less in diam., 180 degrees

Retaining Walls, Including Lateral Earth Pressure[1]

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[1] From Report Spec. Comm. on Soils A.S.C.E.

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