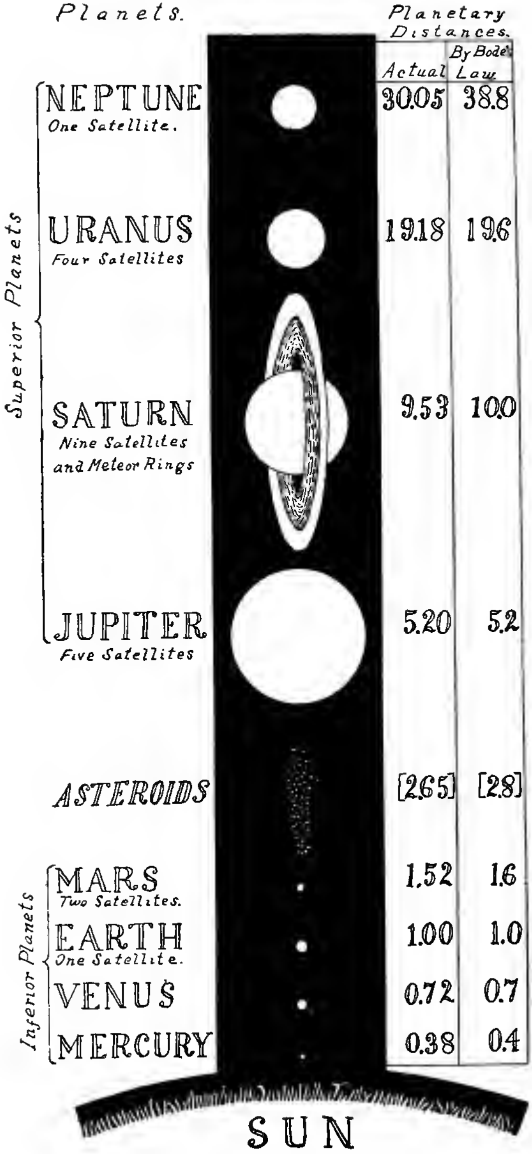
THE PLANETARY SYSTEM



FRONTISPIECE — Diagram showing the relative sizes of the planets and their arrangement with reference to the Sun. Distances given in astronomical units.

The PLANETARY SYSTEM

A Study of Its Structure and Growth

BY

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TO

MY BELOVED PARENTS

THIS BOOK

IS AFFECTIONATELY DEDICATED

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PREFACE.

Sir Isaac Newton solved the problem of the Moon's stability (the problem of three bodies) by what may be called the method of difference of attraction, and his solution of it by this method stands to-day as the only, and therefore as the accepted, solution. So little change has there been in theoretical astronomy since Newton's time that the current analysis of this problem is still in all essentials identical with his. If we glance back over the history of theoretical astronomy we find that this science seems to have come to a standstill with Newton. Although he was its illustrious founder, yet, excepting the magnificent results which he himself gave it, its progress has been slight.

Theoretical astronomy is securely founded upon Newton's laws of motion and his law of gravitation. This foundation has never been shaken and, so far as present knowledge enables us to see, it will never be; for it rests upon laws which seem to be universal and eternal. Newton's solution of the problem of *two* bodies partakes of the same high character as his laws: it appears to be complete and perfect. But because these things are or seem to be of truth eternal, it does not follow necessarily that Newton's solution of the problem of *three* bodies, or rather his attempted solution of it, belongs in the same high category. Indeed, his analysis of this problem has been singularly barren of fruitful results. This fact alone is sufficient reason for serious doubts as to its validity.

If Newton's analysis of this problem is correct it must be presumed that it shows the true mechanism of the Moon's stability, and unless this is something peculiar to the Moon and different from the mechanism of stability of the satellites of other planets, it must be capable of serving as a basis of generalization. For, as the Moon's stability is, so, with appropriate numerical differences in the values of the forces, ought the stability of the satellites of other planets to be, provided they are similarly related to their primaries; and it is evident that the *inner* satellite of each planet is, in fact, so related.

We are therefore warranted in concluding that if Newton's analysis is correct it ought to yield a general law for the stability of inner satellites. But it has yielded no general law and can not be made to yield one. Before it could do this, it would have to show that the Moon's stability is a definite thing for present conditions and can not be anything else. That is to say, it would have to show that the mean place of the Moon's orbit of stable revolution under present conditions is necessarily at a particular, *determinate* distance from the Earth and can not be at any other distance. With stability having this determinate quality, generalization becomes not only possible, but comparatively easy. Newton's analysis, however, does not reveal this quality and his method precludes the possibility of such a result. His analysis shows only *indeterminate* stability. An attempt to deduce a general law on the basis of this kind of stability yields only meaningless results.

The failure of Newton's analysis to serve in generalization explains why Bode's law of the planetary distances and the similar laws which govern the spacing of the satellites in their several systems still stand as merely empirical expressions without any physical foundation. These laws are, in truth, the laws of the structure of those systems, and until we know the physical basis upon which they rest, we can not hope to explain why the systems are so arranged. Moreover, it seems probable that if we were able to explain Bode's law and the laws of the satellite systems we might then find easy steps to an explanation of the growth of such systems. In any event, we can hardly hope to reach a full explanation of their growth until we have first explained their structure.

The present theory yields a general law which, in the hands of a competent mathematician, would have enabled him to predict the discovery of Jupiter's Fifth satellite before it was seen and to have calculated the place of its orbit and its period of revolution. The same can be done for Mars at the present time. For according to the present theory, there is now an undiscovered satellite revolving between Phobos and Mars. It will be hard to see, because it is so close to the planet, revolves so swiftly and is in all probability very small. Newton's analysis furnishes no basis for such calculations.

In the first five of the following chapters I endeavor to show that there is another way of solving the problem of three bodies, different from that employed by Newton, and that this way shows the Moon's stability to be determinate in its quality. The mechanism of determinate stability is described in chapters IV and V In chapters VI, VII and VIII, and especially in the last of these, the steps that lead to the general law of satellite stability are discussed and the law is stated. Chapters IX and X discuss the inclination and structure of satellite systems respectively, and chapters XI to XIV inclusive discuss the manner of their growth.

The principles deduced in Part I find a much larger field of application in Part II, where they are used in explanation of the structure and growth of the Planetary system. The structure and method of growth of the Planetary system are precisely the same as for satellite systems, so that if the new theory of stability and the principles deduced from it are correct, then these things are fully explained by a broader or more extended application of the Newtonian principles, without any reference whatever to the Nebular hypothesis in any of its forms. Hence, in a broad sense, these studies may be regarded as suggestions for the extension of theoretical astronomy toward the wider boundaries that properly belong to it. If the new theory of stability is true all that is accomplished by its use is simply so much added to the conquest previously made by Newton's laws.

Many attempts have been made in the past to promulgate new astronomical theories based on something which the authors have called « centrifugal force. » Such of these attempts as I have seen have appeared to be founded upon an erroneous conception of the nature of the force designated by this name. The fact that much use is made of this name in the following discussion will naturally lead to expectation of the same error, but I am sure that this expectation will not be realized.

It is well known that in a literal sense there is no such thing as « centrifugal force. » The term is a misnomer. The thing to which the name is applied is *inertia*—the inertia of a moving body toward a force which tends to deflect that body from motion in a straight course. The Moon's momentum as it revolves around the Earth tends at each instant to cause it to follow the tangent of that instant. If at a given instant the Earth's attraction should suddenly cease the Moon would not fly directly away from the Earth's center, but would follow the straight course of the tangent, and if it met no resistance nor any other attraction it would follow that course forever. Such a motion would carry the Moon farther and farther away from the Earth, but the force causing this motion would be momentum and not in a literal sense centrifugal force. The instant the Earth's attraction ceased to affect the Moon the so-called centrifugal force would cease to exist, for there would no longer be any force tending to bend the Moon's course out of a straight line. The force which would remain and cause the Moon to continue its forward motion would be momentum.

The mathematical expression for centrifugal force, considered abstractly, is $\frac{v^2}{r}$, or, when applied to concrete cases where it includes the idea of mass, it is $\frac{mv^2}{r}$. This expression shows with absolute clearness the real nature of the force. The identity of centrifugal force and inertia are briefly, but perhaps too briefly, stated on page 29. The term is used throughout this discussion in the sense defined above and in no other.

To some minds the conclusions and explanations of Part II may seem radical in the extreme. To such it may be of interest to note that astronomers are now approaching the same conclusions from a different direction. As will be shown in chapters XI and XII, Kirkwood and Miller especially have made recent advances which lead to the same theory of growth for satellite systems, and ultimately of necessity to the same theory of stability. If this book were not printed at all it is only a question of a few years before the same ideas would be brought to public notice by some one else.

These studies grew out of collateral reading in connection with a course in geology. In the later chapters it will be found that they return again to the borders of that science. If the theory presented here is true it promises to be as important to geology as to astronomy. Certain passages in the later chapters show some of the lines along which it may be expected to reveal the astronomical foundations of geology, but its relations to that science are discussed here only incidentally.

While this volume is not especially designed to be of a popular character, its technicalities are few and the discussion as a whole is, I believe, well within the reach of all who find pleasure in reading the current popular works on astronomy. Chapters IV and IX may seem a little heavy, but there is nothing mathematical in them; the diagrams are merely illustrative aids to discussion.

F. B. T.

MACKINAC ISLAND,
August 31, 1903.

PART I.

DETERMINATE STABILITY AND THE LAWS OF THE SATELLITE SYSTEMS.

DIVISION I.

DETERMINATE STABILITY.

CHAPTER I.

INTRODUCTION.

THEORETICAL astronomy has to do with the motions of the heavenly bodies. Some of these bodies, as the Earth, the Moon, the satellites of Mars and those of Jupiter and Saturn, revolve in elliptical orbits, which are nearly circular, and in which the revolving bodies keep constantly the same or very nearly the same relations to the centers around which they revolve. Excepting a few minor irregularities of relatively small importance, the revolutions of all the planets and satellites have proceeded in continual rounds time without human record. Hence it is that their revolutions are said to be stable.

Not only is it within the province of theoretical astronomy to investigate and analyze these revolutions with a view to determining the forces which make them stable, but this is by far the most important problem with which it has to deal. In practice, theoretical astronomy is profoundly mathematical. It rests securely upon Newton's laws of motion and his law of gravitation. But the application of these laws in analyses aiming at ultimate completeness and perfection demands more mathematical power than men can at the present time command. The greater problems, however, divested of minor complications and presented in their simplest forms, are capable of approximate solution. The motion of the Moon has been more minutely analyzed than that of any other body, and the mathematical theory by which it is expressed is one of the most profound and elaborate that has ever been devised by man.

It is supposed that the Moon's motion has been rightly analyzed and that the true mechanism of its stability of revolution is fully and accurately known, at least so far as concerns the main forces involved. Out of many lesser factors, the few remaining to be accounted for are regarded as neither essential to stability nor inimical to it, but as giving rise to nothing more than minute irregularities, most of which are in the long run compensatory. The mathematical theory is so nearly complete that it enables astronomers to predict the Moon's place with a marvelous degree of accuracy. Indeed, for all purposes, except those of the utmost refinement, it may be said to be perfection itself. And yet, if we would adhere to the narrow line of truth, it must be admitted that there is something still lacking. "There are still slow changes in the motion of our satellite which gravitation has not yet accounted for. We are, apparently, forced to the conclusion either that the motion of the moon is influenced by some other cause than the gravitation of the other heavenly bodies, or that these inequalities are only apparent, being really due to small changes in the earth's axial rotation, and in the consequent length of the day. If we admit the latter explanation, it will follow that the earth's rotation is influenced by some other cause than the tidal friction; and that, instead of decreasing uniformly, it varies from time to time in an irregular manner." (Newcomb's "Popular Astronomy," p. 101.) For the present purpose, however, we may believe that the chief forces — the controlling forces in the mechanism of stability — are known and correctly analyzed, and that the unexplained motions belong with the other smaller irregularities which are compensatory.

Assuming this to be true, there yet remain several large questions which the very perfection of the analysis suggests, but which have not been satisfactorily answered, so far as I am aware. Does this demonstration of the mechanism of the Moon's stability disclose any reason why the Moon has its *present* orbit rather than some other, nearer to, or farther from the Earth? Does it show any general law of stable satellite revolution, applicable to the satellites of other planets, as well as our own? Does it indicate that stability of satellite revolution is associated with and dependent upon determinateness of distance for the satellite orbit?

If these questions are to be answered in the negative, it is manifest that a very interesting field of astronomical study is still open. For one can hardly fail to see that they *might* be answered in the affirmative. Indeed, to my mind, there is an irresistible feeling that they *ought* to be so answered and that they would be if the whole truth were known. Irresistible feelings, however, are neither proofs nor arguments. But they have been the impelling motive in the prosecution of certain studies, some of the results of which are set forth in the following pages.

The Newtonian analysis, with the few and relatively unimportant additions which have been made to it since Newton's time, is the only analysis which we have, and must therefore form the basis of the answers to these questions. But, according to it, the answers are negative. For, we are led by it to conclude that the Moon's revolution would be as stable in one circular or nearly circular orbit around the Earth as in another—as stable in an orbit 60,000 miles away or in one 400,000 miles away as in its present orbit at 240,000 miles, provided in each case that the Moon were started toward the tangent with the velocity appropriate to circular revolution around the Earth at that distance. The truth of this statement will, I am sure, be made clear below, and will be readily granted on a little examination and reflection.

One who believes that the universe is governed by law is bound to anticipate its presence in the relation of satellites to their primaries. Few phenomena in nature are more perfectly amenable to law than the Moon's motion. The Lunar theory and its wonderful power of prediction is itself a most convincing proof of the reign of law in the Moon's motion as against chance or accident. There are possible events in the relations of the heavenly bodies which we might call accidents, such, for instance, as a direct collision between a cometary nucleus and a planet. But the orderly revolution of satellites in orbits adjusted in a particular way to their primaries, round after round many thousands of times without change, except for slight periodic inequalities due to unequal actions of the forces, can not be the result of mere chance. This seems apparent when we consider the nature of the forces which govern the Moon's motion. For the Moon's motion is affected by four great forces and no others, so far as we know; by *two attractions* — those of the Sun and the Earth — and *two centrifugal forces* — those which are brought into existence in consequence of the motions produced by the two attractions. Thus, there is the Earth's attraction and the centrifugal force due to the Moon's geocentric motion, and the Sun's attraction and the centrifugal force due to the Moon's heliocentric motion. The two attractions are in reality the positive forces which cause the Moon's motion, the centrifugal forces being merely secondary or responsive, and called into existence only as the attractions bend the Moon's motion from a straight line.

All these forces obey definite laws and are capable of the most accurate mathematical expression and calculation for any given conditions. That anything should have to be left to chance, if the analysis of the Moon's motion were wholly correct, seems incredible. There are many satellites in the planetary system revolving at various distances from their primaries. Why are their orbits where they are? Is it the happening of chance, or the ordering of law? It is to these inquiries that Part I of this discussion is directed.

CHAPTER II.

STABILITY AND ITS QUALITY ACCORDING TO THE CURRENT ANALYSIS OF THE MOON'S MOTION.

THE analysis of the Moon's motion around the Earth and of the Sun's perturbations of that motion, as set forth in the current text books of astronomy, are both substantially the same as the original analyses made by Sir Isaac Newton over two hundred years ago. The first is the problem of two bodies, and was propounded and completely solved by Newton himself. The second is the problem of three bodies, the difficulties of a complete solution of which transcend the present power of mathematical science. But although the general problem is so difficult, « all the special cases of it which arise in the consideration of the moon's motion and in the motions of the planets have been solved by special methods of approximation.» (Young.) My purpose here in reviewing briefly the current analysis of the Moon's motion is merely to bring out with greater clearness the quality of the stability which it gives to the Moon's revolution.

While the Moon's revolution around the Earth is universally recognized as *stable*, we are not accustomed to think of stability as a thing which may have different qualities. We can hardly use such terms as "stable stability" or «neutral or indifferent stability," or «unstable stability"; and yet a little reflection will show that stability will have one quality or another, according to the conditions under which it exists. Its possible qualities may be most clearly perceived by noting its relation to the different kinds of equilibrium. These are defined as follows in the Century dictionary:

> «When a body, being slightly moved out of its position, always tends to return to its position, the latter is said to be one of *stable equilibrium;* when a body, on the contrary, once removed, however slightly, from the position of equilibrium, tends to depart from it more and more, like a needle balanced on its point, its position is said to be one of *unstable equilibrium;* and when a body, being moved more or less from its position of equilibrium, will rest in any of the positions in which it is placed, and is indifferent to any particular position, its equilibrium is said to be *neutral or indifferent.*»

A body in motion may be in equilibrium as well as a body at rest, and the quality of its stability may correspond to any one of these kinds of equilibrium. A body having stable revolution around another body must be in some sort of equilibrium with respect to the forces which affect it. Let us consider a few supposable cases for the sake of illustration.

If the Earth and the Moon were the only bodies in the universe the Moon would describe an exact ellipse around the Earth, and it would continue to revolve in that same orbit forever. Or, more strictly speaking, each would revolve in an ellipse around their common center of gravity. If it were to begin its motion in a given circle or ellipse (and it is immaterial whether we suppose the circle to be great or small, or the ellipse more or less eccentric), it would continue to revolve forever in that same orbit. This is the simple case of two bodies, and the quality of stability which the Moon would have under these conditions corresponds to a *neutral or indifferent equilibrium.*

If the Moon's stability, on the other hand, is of such a nature that every time the Moon departs from its present orbit, either to take a path nearer to the Earth or farther from it, forces are brought into action which tend to carry the Moon farther and farther away from its present orbit, then the quality of the Moon's stability corresponds to that of an *unstable equilibrium.*

But if its stability is of such a nature that every departure from its present orbit brings into action forces which tend to carry it back to its present orbit, then the quality of its stability corresponds to that of a *stable equilibrium.*

Which of these qualities of stability is disclosed by the current analysis of the Moon's motion? Manifestly, we may at once reject that quality which corresponds to an unstable equilibrium as being inapplicable to the case of the Moon. The quality of the Moon's stability must therefore correspond to one of the two remaining kinds of equilibrium, either to the neutral or to the stable.

We have just seen above that the quality of stability in the simple case of two bodies is clearly that of a neutral equilibrium. Considered solely with regard to its quality, we may say that the stability of the Moon's revolution in such a case would be *indeterminate*. That is to say, the place or distance of its orbit (conveniently supposed to be circular) would be determined by no law. It might be near or far and the Moon's revolution be stable just the same. Its distance is a matter of indifference. Once started in an orbit at any given distance, Newton's laws show why the Moon, attracted by the Earth alone, would tend to be stable in that orbit and continue to revolve in it forever. But with the Earth and Moon revolving together around the Sun the case is quite different. Why is the Moon's present mean circular orbit of stable revolution 240,000 miles from the Earth, instead of 60,000 miles? Is there any law governing that fact? If there is none, and if the current analysis of the Moon's motion shows that the Moon would be as stable in one orbit as in another, and that its stability is therefore in no way dependent upon its distance from the Earth, then, as to its quality, the Moon's stability may be characterized as *indeterminate*.

On the other hand, if the place or distance of the Moon's mean circular orbit of stable revolution is governed by law—if its mean place or distance from the Earth was originally and is now continuously determined by forces which act in accordance with definite laws, so that under existing conditions the Moon could not have permanent or stable revolution in any other mean orbit than that in which it now moves, and would return to that orbit again if at any time temporarily perturbed out of it, then, as to its quality, the Moon's stability may be characterized as *determinate*. Of course, a perturbation might occur which would be so powerful as to destroy stability immediately, but I am supposing one of less power—one, for instance, which might cause the Moon to begin revolving around the Earth in an orbit at a mean distance of 60,000 miles. Assuming that a perturbation could occur which would produce so much change and no more in the Moon's orbit, the forces which tend to make stability determinate would gradually carry the Moon back to its present orbit as the only one in which all the forces affecting the Moon's motion can find that state of balance which is essential to permanent or stable revolution. Under such a law of stability, and under the present general conditions, the Moon would be unstable in an orbit at 60,000 miles and in all other orbits, except in its present mean orbit at 240,000 miles.

These are the meanings which I would convey by the terms *determinate* and *indeterminate stability*. Determinate stability has the quality of a stable equilibrium, while indeterminate stability has the quality of a neutral or indifferent equilibrium.

There needs no argument, it seems to me, to convince anyone that there is an immense difference between these two qualities of stability, and also that it is a matter of incalculable importance to theoretical astronomy to determine to which the Moon's stability actually corresponds. Nor is any argument required to show that it will make an enormous difference in our interpretations of the phenomena of the celestial bodies, according as we regard the Moon's stability as determinate or indeterminate.

Let us now note briefly the quality of stability disclosed by the current analysis. When Newton solved the problem of two bodies, he showed how the Moon is made to revolve around the Earth by the Earth's attraction alone, taking no account of the influence of the Sun. He showed that as the Moon moves in its orbit it is made to fall from the tangent in each second of time just as far as it would fall from a position of rest at the same distance and in the same time. Thus, in obedience to the force of gravitation, the Moon falls constantly toward the Earth, and yet, by moving at the same time with a certain uniform velocity toward the tangent, keeps constantly at the same distance from the Earth.

There is nothing in this demonstration to suggest that the Moon would not be just as stable in any other circular orbit around the Earth as in its present one, provided it started in the first place with the right velocity toward the tangent. It would be just as stable in an orbit at a distance of 60,000 miles or in one at 5000 or 500,000 miles as in its present orbit. The quality of stability in this case corresponds to that of a neutral or indifferent equilibrium, and is therefore *indeterminate*. Does the current solution of the problem of three bodies show a different quality of stability?

The Moon's motion around the Earth is not circular, nor even precisely elliptical, but shows many irregularities, some of them of considerable amount. Newton showed that these disturbances or perturbations are due to the unequal action of the Sun's attraction on the Earth and the Moon.

« The disturbing force of the attracting body depends upon the difference of its attraction upon the two bodies it disturbs ; difference either in amount or in direction, or in both.» (Young.) If the Sun attracted the Earth and the Moon with exactly equal force and along parallel lines its attraction would not affect or disturb their relative motions in any way. The Moon's *mean distance* from the Sun is the same as that of the Earth, and the Sun's *mean attraction* for both bodies is therefore the same. Hence, according to the current analysis, it is not from this equal or mean attraction that perturbations arise, but out of the *difference* of attraction or the unequal action of the forces which affect the two bodies. When the Moon is full, or in opposition, it is 240, 000 miles farther than the Earth from the Sun, and the Sun's attraction for the Moon is therefore less than the mean or its attraction for the Earth by an amount equal to $\frac{1}{90}$th of the Earth's attraction. This perturbing force tends in effect to draw the Moon away from the Earth, though in reality drawing the Earth away from the Moon. When the Moon is new, or in conjunction, it is 240,000 miles nearer than the Earth to the Sun and the Sun's attraction for the Moon is greater than it is for the Earth by an amount equal to $\frac{1}{89}$th of the Earth's attraction. This perturbation tends to draw the Moon away from the Earth and also toward the Sun.

As against the tendency of the perturbations due to difference of linear distance to separate the Moon from the Earth, there is another which tends to draw them together. When the Moon is at quadratures its angular distance from the Earth as seen from the Sun is at its greatest. In this position the Sun attracts the Earth and Moon equally, but on slightly converging lines, and so, as they fall toward the Sun, they are in a slight degree drawn together. This perturbation draws the Moon toward the Earth with a force equal to $\frac{1}{179}$th part of the Earth's attraction.

By a computation extending to every part of the Moon's orbit it is shown that, as a net result, the Earth's attraction on the Moon is diminished by about $\frac{1}{358}$th part. In consequence of this diminution the Moon revolves at a greater distance from the Earth, with a less angular velocity and in a longer time than it would if the Sun's perturbing force were absent. From this cause the month or period of the Moon's revolution is made about three hours longer than it would otherwise be. Having once been lengthened in this way, the month is supposed to continue ever afterward the same, except as the mean attraction of the Sun varies in value. The analysis by which this result is reached is given briefly and clearly in nearly all of the current text books and treatises. (See Herschel's «Outlines»; Loomis' «Treatise on Astronomy»; Young's «General Astronomy»; Proctor's «The Moon,» etc.)

This much do the solar perturbations modify the revolution of the Moon, and cause its orbit of stable revolution to differ from that which it would have if acted on by the Earth's attraction alone. But it does not appear that perturbations have any other effect or any other relation to stability. They are merely a modification superimposed upon a revolution which would be as stable without them as with them, and they neither destroy it nor change its nature.

Hence the conclusion seems plain that the only kind of stability which has been demonstrated for the Moon is that which is involved in the solution of the problem of two bodies as analyzed by Newton. But, as has been shown above, the quality of that stability corresponds distinctly to a neutral or indifferent equilibrium and is therefore *indeterminate*. That is to say, the *quality* of the Moon's present stability, as determined by the Newtonian analysis, is the same as it would be if the Sun were absent and the Moon revolved around the Earth alone and undisturbed. Or, in more general terms, the quality of stability growing out of revolution under the conditions affecting *three* bodies is set forth as being no different from that growing out of revolution under the conditions affecting *two* bodies. The perturbations due to the attraction of the third body serve only to modify the expression of stability and give the orbit a slightly greater distance, without in any way affecting or changing the *quality* of stability. If the Moon revolved in a smaller orbit than at present perturbations would play the same part as now, except that, in proportion as the differences of distance and attraction grew less, the power of the perturbing force would diminish and the month would be lengthened by a less amount. On the contrary, if the Moon revolved in a larger orbit than at present perturbations would be correspondingly more powerful, and probably in some orbit at a relatively great distance they would destroy stability. Within that outer limit, however, stability would exist, but, as before, it would be indeterminate in its quality.

So far then as current analyses and theories are concerned, theoretical astronomy appears to be committed to the proposition that the Moon's stability has the quality of a neutral or indifferent equilibrium and is therefore indeterminate. If it should be claimed that theoretical astronomy is not committed to this view of the Moon's stability it may be answered that there is a considerable amount of evidence, some direct and much more that is indirect, tending to show that indeterminate stability is the only kind contemplated in the current theory. This, I believe, can be shown by rigorous mathematical analysis, but of this method I am not able to avail myself. Among other ways, however, the same fact is shown by the open and friendly relations which are maintained between theoretical astronomy and certain well known hypotheses.

One of the most popular recent adjuncts of the Nebular hypothesis is Professor G. H. Darwin's hypothesis of tidal evolution for the Moon. According to his view, the Moon was a part of the mass of the Earth, when the latter body was still in a molten state. In consequence of excessively rapid axial rotation, presumably due to nebular condensation, the Earth developed a great equatorial bulge, which, by still further increase of rotation and the aid of the solar tidal force, finally gathered into a globular mass and separated from the Earth. This body continued to revolve around the Earth as its satellite, and thus the Moon is supposed to have originated.

From the laws of orbital revolution it follows that as the newly born Moon slowly withdrew from the Earth its velocity of revolution grew gradually less. The Earth, however, kept on rotating at first at the same rate as before. Being then so near to the Earth, the force of the Moon's attraction, and hence also the tidal force affecting the Moon, were much greater than now, and from this cause and the fact that the Earth was still in a molten condition, the Moon's attraction raised tremendous tides upon the Earth. But, because the Earth rotates faster than the Moon revolves, the maximum tidal wave on the side of the Earth nearest the Moon is carried beyond the line directly joining the two bodies, and so, from its forward position and its nearness as compared with the opposed tidal mass on the far side of the Earth, this wave has the effect of pulling the Moon forward a little in its orbit and accelerating it.

At the same time the tides on the Earth act as a friction brake upon its rotation and gradually reduce its rate. Thus, in consequence of tidal acceleration the Moon is supposed to be driven out farther and farther from the Earth into an orbit of constantly increasing radius, and the Earth in consequence of tidal friction rotates more and more slowly.

At length the Earth cools so as to acquire a solid crust, and after that the tides are restricted to the water and the air. But these are supposed to continue to operate in the same way until the Moon shall have receded to an orbit far outside of its present one, so that the month will become 55 days long instead of 27 days as at present. At this point the Moon's revolution and the Earth's rotation will have the same period and a condition of perfect and everlasting stability will have been reached, so far as the interactions of the Earth and Moon alone are concerned. But the solar tides will continue to slacken the Earth's rotation, and the lunar tides on the Earth will then begin to retard the Moon and drag it down again, and this is supposed to go on until the Moon finally becomes reunited with the Earth. (See « The Tides,» by Geo. H. Darwin. Also *Atlantic Monthly* for April, 1898.)

According to this hypothesis the Moon has already occupied orbits at all distances between actual contact with the Earth and its present place, and before its final extinction will occupy many more farther out from the Earth than now, and finally will pass through all of them again in its gradual return to the Earth. Throughout all this wide gamut of orbits the Moon's distance from the Earth during the expansion and contraction of its orbit in going out from the Earth and back again must be supposed to be under the immediate control of the tidal forces. Of course, the forces which hold the Moon in its orbit and give it stability in a primary sense are the attractions of the Sun and Earth. But, according to Professor Darwin's hypothesis, while these forces are constantly acting with their full power and in their normal way, the precise place of the Moon's orbit around the Earth is left to be determined solely by the attraction of the nearer tidal mass on the Earth, as though the Moon's orbit of stable revolution were adjustable to any distance with equal facility and indifference. It is an obvious corollary from Darwin's hypothesis that whenever the tidal force of the Earth ceases to act, the Moon's revolution immediately becomes stable and unchangeable in whatever mean orbit the Moon happens then to be, whether near to or far from the Earth. If Professor Darwin had in mind any other force or combination of forces, which he recognized as tending to control or determine the precise place of the Moon's stable revolution independently of the tidal force, it behooved him to point out that force, to show how it is related to his hypothesis, and how his controlling tidal force overcomes it. But he mentions no such force. Hence, by his hypothesis the place or distance of the Moon's orbit of stable revolution is a matter of entire indifference, so far as the original forces of stability are concerned, and are left to be determined solely by the tidal force. The stability which he attributes to the Moon is therefore the very ideal of indeterminate stability—stability having the quality of a neutral or indifferent equilibrium. Stability having this quality leaves no place for the operation of a governing law residing in the primary forces which affect the Moon's motion, but depends upon the intervention of some outside force.

That this quality of stability for the Moon stands approved or at least is not disapproved by most astronomers is made plain by the fact that nearly all current text books make mention of Darwin's

Tidal hypothesis in terms of more or less approval, and several of the foremost astronomers of the day have written of it in a popular vein with favor unreserved. (See « Birth and Death of the Moon,» by Professor E. S. Holden, *Harper's Monthly* for Aug., 1901; Young's « General Astronomy,» pp. 292 and 520–521; Todd's « New Astronomy,» pp. 338–339; Ball's « Story of the Heavens,» pp. 510–538; Miss Clerke's « History of Astronomy During the Nineteenth Century,» pp. 357–362.) In each of these references the language used makes clear the underlying assumption of indeterminate stability.

The foundation of Darwin's hypothesis seems unquestionable. The tides exist and it seems clear that they must exert a force tending to accelerate the Moon's motion. But the rest of his hypothesis can be true only on the assumption that the Moon's stability, so far as dependent upon non-tidal forces, is *indeterminate*. This is as clear as the existence of the tidal force itself.

Of indirect evidence the most important is that already mentioned, viz: that the current analysis of the Moon's motion has not disclosed a general law for the relation of satellites to their primaries.

Another kind of indirect evidence comprises a certain class of facts which remain stubbornly inexplicable on the basis of the current theory, but which the principles of determinate stability make clear.

There is also another consideration, which, while not possessing the character of proof, is nevertheless irresistible in its impress upon the mind. Knowing by observation that the Moon's revolution is stable at the present time, why must we be limited to the choice of a second rate quality of stability? Knowing that there are two possible kinds or qualities of stability corresponding to two kinds of equilibrium, why must we choose that one which possesses in the lesser degree the qualities of determinateness and steadfastness? Why must we choose the one that has the quality of indeterminateness and hence verges the more toward instability?

Finally, there appears to be a logical necessity that the Moon's stability shall be determinate and not indeterminate. The truth of this statement is apparent if we consider two things; the qualities of the two kinds of stability defined above and their relations to the manifest essentials of a satellite system.

Indeterminate stability is stability without a governing law of orbital place or distance. Under it the mean circular orbit of stable revolution has no fixed or determinate place for given conditions. Such stability is hardly worthy the name. Stability anywhere means stability nowhere. Such stability lacks the chief quality necessary for the organization and maintenance of a satellite system—*a governing law.*

Very different is determinate stability. By it, the Moon under existing conditions is compelled to revolve in its present mean orbit—to hover close to the mean circle of its present path—and it would not be stable in any other. The action of the forces which make stability is such that every departure of the Moon from its mean orbit brings into play forces which tend to carry it back again to that orbit as the only one in which it can be stable.

It is the function of law to limit and direct the action of force. No organized body or system of bodies can exist without it,—without some sort of limitation to the action of the forces which manifest themselves within it. The characteristics of a satellite system like that of the Earth or Mars or Jupiter are so simple and the forces involved are so few, that if the mass of the planet be given, then the only ways in which a law governing the structure or organization of the system can express itself are by limiting or determining either the distance of the satellite or the velocity with which it revolves around its primary. But distance and velocity are inseparable elements of orbital revolution and are related to each other according to a definite law. Knowing the one, the other is always readily determinable; so that there is after all only *one way* in which any governing law that may be supposed to exist can express itself, viz: *by the mean distance of the satellite from the planet.* Yet, according to the current theory, no such law is found, and we are led to suppose that satellite systems may be organized and maintained after organization without it.

The principles of the current analysis of the Moon's motion have been applied in analyzing the motions of the satellites of other planets, and a careful effort has been made to deduce from these studies, and from a comparison of them, a general law of satellite stability, but without success.

The foregoing discussion enables us to see why the search for a general law failed. It was because, following the Newtonian method of analysis, the investigation was founded on the assumption that the Moon's stability is indeterminate. With indeterminate stability there can be no general law. With no law to govern the distance of the Moon's orbit, it could hardly be expected that a general law would be found for other satellite systems.

It is to me an amazing fact that astronomers have been content, or seemingly so, to rest for over two hundred years without a general law for the revolution of satellites around their primaries. Considering the fact that all the planets and satellites revolve freely in space and under the dominion of the law of gravitation and the laws of motion, and under these alone, it seems impossible that there should be no general law governing the distance of the satellites from the planets. Such a law would seem to be as certain as the existence of the law of gravitation itself. The fact that it has not been deduced and stated seems to me to indicate that there is some important element of error in the current analysis.

CHAPTER III.

CONDITIONS PERTAINING TO DETERMINATE STABILITY.

§ 1. General conditions of the Moon's revolution. In order to get a clear view of the main facts relating to the Moon's motion and stability, it is desirable to confine attention at first to revolution under the simplest possible conditions. To this end, let it be supposed that the Earth revolves around the Sun in a circular orbit at its present mean distance of about 93 millions of miles. Its velocity with respect to the Sun is about 18½ miles per second, and the curvature of its orbit is therefore such that the Earth falls from the tangent about 0.116 of an inch in the same time; that is, the Earth's course is bent this much from a straight line while the Earth moves forward around the Sun 18½ miles.

In considering the Moon's revolution around the Earth we may assume conditions of like simplicity. Let it be supposed first that the Earth is at rest in space and that the Moon revolves around it in a circular orbit at its present mean distance of about 240,000 miles. Its velocity with respect to the Earth is about half a mile per second, and the curvature of its orbit is therefore such that the Moon falls from the tangent about 0.0534 of an inch in the same time; that is, the Moon's course is bent this much from a straight line while the Moon moves forward around the Earth half a mile.

The Earth, however, is not at rest in space, and the Moon's motion *with respect to the Sun* is therefore not so simple. For, while the Earth revolves in a circle around the Sun, the Moon revolves in a circle around the Earth, and the Moon's path with respect to the Sun is therefore the resultant of both revolutions. The wavy or undulating curve produced in this way is called an epicycle. «A point which moves uniformly round the circumference of a small circle whose center travels uniformly round the periphery of a large one, is said to describe an epicycle.» (Todd.)

As a matter of fact we know that the Sun also is moving through space. But for the present we may neglect this motion and consider the Sun to be at rest.

In order to complete the assumed simplified conditions, let it be supposed, further, that the plane of the Moon's orbit around the Earth is coincident with the ecliptic or plane of the Earth's orbit and that the Moon revolves in the direct order of motion or in the same direction around the Earth that the Earth revolves around the Sun.

From the fact that the Earth's velocity around the Sun is about thirty-seven times greater than that of the Moon around the Earth, it follows that the epicycle in which the Moon moves is a very open one. When plotted to scale, as in Fig. 1, it makes a slightly wavy or undulating curve, crossing and recrossing the path of the Earth—now inside, now out—but always concave toward the Sun. The undulations are quite inconspicuous, especially if the Earth's circular orbit about which the undulations are disposed as a mean be omitted. But although the undulations appear to be slight, they are sufficient to give the Moon a much more complicated relation to the Sun than obtains for the Earth, if we neglect the much smaller lunacentric revolution of the Earth, which gives its orbit around the Sun a precisely similar but much slighter undulation.

Fig. 1 is drawn to scale and shows the real character of the Moon's path around the Sun for the simplified conditions assumed. The broken line is a part of the Earth's assumed circular orbit around the Sun and represents about $\frac{1}{18}$th of its whole circumference, or the portion covered by the Moon in one month. The continuous heavy line is the Moon's epicyclic path and its very gentle undulations show how little the Earth causes the Moon to depart from a circular path around the Sun. The lines converging from *Q*, *O*, etc., produced to their intersection are supposed to meet at the Sun. *Q*, *Q′* and *Q″* are points of quadrature and *O* and *C* mark the points of opposition and conjunction respectively.

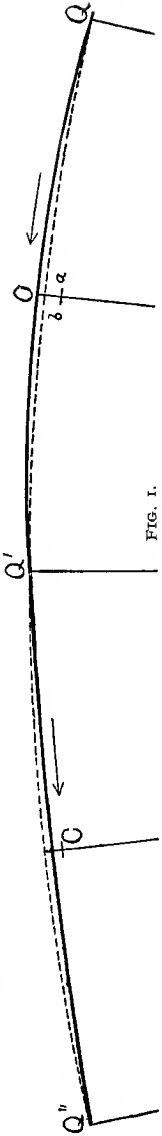

FIG. 1.

§ 2. **The epicycle and the forces which affect the Moon's revolution in it.** In studying a problem like that of the Moon's motion and stability, it is impossible to overestimate the importance and value of right method. Even if it be a little more cumbersome or perhaps more difficult, that method is safest which deals most directly with the forces and avoids unnecessary assumptions.

In the present case, there seems to me to be great advantage in approaching the problem by a direct study of the forces which affect the Moon as it follows the epicycle. For the epicycle is the Moon's real path around the Sun.

In the first place, it is to be remembered that the epicycle is not a thing separate from the geocentric circle. The two are really one and the same thing seen from different points of view. Seen from the north pole of the Earth and neglecting the Earth's lunacentric revolution, the Moon's path is a circle with the Earth in its center, but seen from a point toward the north pole of the ecliptic, it is a gently undulating epicycle around the Sun, like that shown in Fig. 1.

The mass of the Earth determines the velocity which the Moon or any other body must have in order to be stable in any given circular geocentric orbit, and a precise velocity is therefore fixed for every possible circle by the law of velocities, viz: « Velocities in circular orbits vary inversely as the square roots of the distances.» (Proctor.) Hence, in any given circle around the Earth the Moon can have but one particular velocity if its revolution is to be stable.

Supposing the Earth to revolve always in the same circle around the Sun, it follows that any particular circular orbit in which the Moon may be supposed to revolve can have but one particular epicycle corresponding to it, and the Moon can not depart from that epicycle by so much as a hair's breadth without at the same time leaving the circle. We may assume any number of circles around the Earth, any one of which might become the orbit of the Moon. Every such orbit has just one epicycle corresponding to it, and if the Moon is to have stable revolution in any one of these circles it must follow the exact curve of the corresponding epicycle. Just as the circles are all different, so the epicycles are all different. In general, the smaller the circle the shorter and more rapid are the undulations of the corresponding epicycle.

In following the epicycle the Moon undergoes a continually alternating round of changes in its relations to the Sun. The variable factors are (1) its direction of motion with reference to the Sun, (2) its angular distance from the Earth as seen from the Sun, (3) its linear distance from the Sun, (4) its velocity with respect to the Sun, (5) its curvature of path with respect to the Sun, (6) the varying force with which it is attracted toward the Sun and (7) the varying centrifugal force with which it resists that attraction. The variation of each one of these factors takes place about a mean, and the value of the mean is the same as that which affects the Earth. As the Moon's motion around the Earth is supposed to be in a circle, its relation to that body is always the same.

Among these variable elements there are only two primary forces: (1) *attraction*, which is a positive, original force and is the cause of the Moon's motion, and (2) *inertia* or *centrifugal force*, a merely responsive force by which the Moon resists the force of attraction, and which comes into action only as attraction tends to bend the Moon's course out of motion in a straight line. Velocity, curvature and direction of motion are resultants of the action of these forces, and angular and linear distances are conditions which affect the action and value of the forces respectively.

Fig. 2 is an epicycle shown in more convenient form for discussion than Fig. 1. In the current

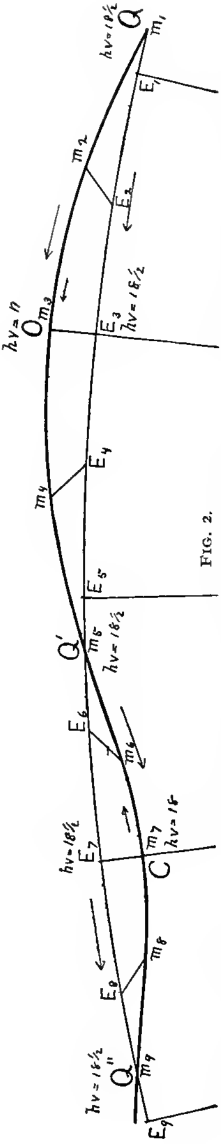

FIG. 2.

analysis the epicycle is not generally used, the discussion being presented by the aid of a figure showing only the Moon's circular orbit around the Earth, as in Fig. 3. The successive positions of the Moon in the epicycle may be seen by comparing Figs. 2 and 3. E_1 m_1 and E_2 m_2 in Fig. 2 mark the same positions as E m_1 and E m_2 in Fig. 3.

New Moon is at *C*, when the Moon is at the point of conjunction, and full Moon at *O*, when the Moon is at the point of opposition. The Moon's orbit is divided into quadrants according to the Moon's phases, the first quadrant extending from conjunction to quadrature (*C* to *Q''* in Fig. 2 or *C* to *Q* in Fig. 3), the second from quadrature to opposition (*Q* to *O*), the third from opposition to quadrature (*O* to *Q'*), and the fourth from quadrature to conjunction (*Q'* to *C*). In the middle of each quadrant are points called the octants, m_2, m_4, m_6 and m_8.

Whenever the Moon crosses the Earth's orbit, as at *Q*, *Q'* and *Q''* in Fig. 2, all the forces and conditions which affect its relation to the Sun are the same as those which affect the Earth, except two, which are relatively unimportant. The angular distance of the Moon from the Earth as seen from the Sun is then at its greatest, and the direction of the Moon's motion with reference to the Sun is then most divergent from coincidence with the direction of the Earth's motion.

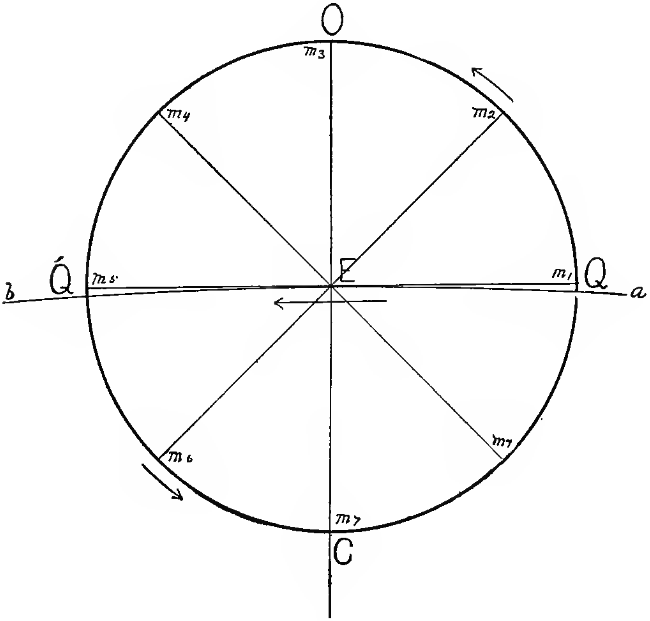

FIG. 3.

Beginning at *Q*, let us note the forces in the second quadrant. When the Moon is at *Q* its velocity and curvature of motion with respect to the Sun are the same as those of the Earth. But as it moves toward *O*, the Moon gains in velocity relatively to the Earth and at the same time passes outside of the Earth's orbit in a path more curved toward the Sun. The velocity and curvature both increase from *Q* to *O* and both attain their greatest values at *O*.

At *Q* the Earth's attraction pulls the Moon at a right angle to the Sun's pull, and hence no part of the Earth's attraction acts to either augment or diminish the Sun's pull upon the Moon. But when the Moon passes outside of the Earth's orbit the angle of the forces becomes less than a right angle and a part of the Earth's attraction begins to pull *with the Sun*, and so to augment the Sun's pull, and this tendency increases as the Moon moves forward, until at *O*, the whole power of the Earth's attraction pulls the Moon directly toward the Sun and therefore has the effect of augmenting the Sun's attraction.

In the third quadrant, from *O* to *Q′*, the reverse change takes place. The velocity and curvature decrease until at *Q′* they are again of the same value that they were at *Q*, and the augmenting component of the Earth's attraction also diminishes to zero.

Then in the fourth quadrant, from *Q′* to *C*, the Moon's velocity and curvature with respect to the Sun decrease until they reach their least values at *C*. After passing *Q′*, a component of the Earth's attraction begins to pull the Moon *away from the Sun*, and so, in effect, to diminish the Sun's attraction, until at *C* the whole power of the Earth's attraction pulls the Moon in a direction exactly opposite to that in which the Sun pulls it. At *C* the effective pull of the Sun is therefore diminished by the whole attraction of the Earth.

From *C* to *Q''* the values of velocity and curvature increase to the mean, or the same value as they had at *Q* and *Q'*, and the opposing component of the Earth's attraction also diminishes to zero.

That component of the Earth's attraction which either augments or diminishes the effective pull of the Sun on the Moon may be called its *heliocentric component*. When it augments, it may be called a positive component; when it diminishes, a negative one. It follows that the *effective heliocentric force*, which pulls the Moon toward the Sun, is not merely *the Sun's attraction*, except at the points of quadrature, but is *the Sun's attraction alternately augmented and diminished by the Earth's attraction or a part of it.*

When the Moon is at *O*, the Sun is about 400 times more distant than the Earth and in the same direction, so that, by the law of gravitation, the effect is the same as though the Earth were removed and the Sun's mass were for that moment *increased* by an amount equal to 160,000 times the mass of the Earth. At *C*, the effect is the same as though the Sun's mass were for the moment *decreased* by the same amount. This would in effect increase or decrease the Sun's mass nearly one-half. It would be the same if the Sun were for those moments removed and the Earth's mass increased or decreased by a mass equal to $\frac{1}{160000}$th of the Sun's mass.

According to Proctor, if the heliocentric attraction on the Moon at *Q* be represented by the number 15, then at *O* it will be represented approximately by 22 and at *C* by 8. Thus, as the Moon follows the epicycle, the value of the effective heliocentric force acting upon it goes through a continual round of variations in value, which may be expressed by the following series of numbers beginning at the right.

			22				22				22	
15		15		15		15		15		15		15
	8				8				8			

In this series the number 15 marks points of quadrature, 22 points of opposition and 8 points of conjunction, the Sun being toward the bottom of the page.

These variations in the effective heliocentric attraction seem quite large. But in order that the Moon shall have stable revolution in the epicycle, the centrifugal force which it develops by its motion with respect to the Sun must have the same values and variations and these must coincide exactly in time and place with the varying heliocentric attraction. The effective heliocentric attraction has a different value at every successive point in the epicycle; and yet, if the Moon is to revolve in a circle around the Earth, this force must be exactly balanced by centrifugal force at every point.

We have seen that, in the case of simple circular revolution, as of the Moon around the Earth at rest in space and free from disturbance by any other body, the balance of attraction and centrifugal force affecting the Moon must be constant and uniform to produce stability. In order to produce exactly the right value of centrifugal force, the Moon must have one particular velocity and no other, and it can not be accelerated or retarded in its orbit by the smallest amount without causing it to depart from the circular path and hence to change its distance from the Earth. Centrifugal force is generated only when the moving body is deflected from motion in a straight line, and it therefore depends upon both velocity and curvature.

This relation of forces, however, is not peculiar to stability in circular revolution. In an elliptical orbit the forces are balanced only as to their *mean* values in each whole revolution. But with the Moon revolving around the Sun in an epicycle, the conditions of stability are more like those of the simple circular orbit. For through all the undulations of the epicycle the velocity and curvature of the Moon with respect to the Sun must be exactly sufficient to develop the centrifugal force required to balance the widely varying power of the effective heliocentric attraction.

If the forces do not balance at every point, then the geocentric circle will become an ellipse or a modified ellipse, and the epicycle will be changed accordingly. But, as in the case of the simple ellipse, a balance must still be maintained between the *mean* values of the forces, or stability can not exist. In truth, the Earth's real path around the Sun is an ellipse slightly modified and the Moon's real orbit around the Earth is of the same character. Hence, the Moon's real path around the Sun is not a true epicycle, but an ellipse upon an ellipse, both slightly modified; that is, it is an epi-ellipse or *epellipse*, or rather *a slightly modified epellipse*, in which the mean values of the effective heliocentric centripetal and centrifugal forces affecting the Moon are at a balance. But for the purposes of the present discussion the previous assumption of the simpler epicyclic path is more convenient.

From these considerations it follows that the centrifugal force required to balance the varying heliocentric attraction affecting the Moon must be produced by the Moon's curvilinear motion with reference to the Sun; that is, from the varying combinations of heliocentric velocity and curvature which the Moon goes through in following the epicycle.

If we follow the Moon's motion in the epicycle and note the changes of curvature and velocity, and more particularly the relation which these changes bear to the coincident changes in the heliocentric attraction, we shall see one of those wonderful, smoothly running mechanisms by which nature shows so beautifully the constant reign of law. For, by reference to Fig. 2, it will be seen that when the Moon is at *Q*, the heliocentric attraction and centrifugal force which affect it are of precisely the same values as those which affect the Earth; but that as the Moon moves toward *O*, its velocity and curvature increase and hence also its centrifugal force with reference to the Sun, while at the same time the heliocentric component of the Earth's attraction also increases. Thus, both forces reach their greatest value at the same time and place — at the point *O*. In going from *O* to *Q'* they both decline to the mean, and thence both decrease to their least values at *C*, returning again to the mean at *Q''*.

Thus the two great forces which affect the Moon's stability go through a continual series of changes exactly in unison as to time, place and amount corresponding precisely with the undulations of the epicycle.

The relation of the opposing forces in this arrangement is highly significant, and is precisely what would be expected on philosophical grounds. By this I mean that the effective heliocentric attraction which urges the Moon toward the Sun is always related in a definite way to the centrifugal force, and in precisely that way which the laws of motion require for stability in the undulating path of the epicycle. For, when the Moon is at *O* and the effective heliocentric attraction is at its greatest value, the centrifugal force is then at its greatest value also; when the Moon is at *C* and the effective heliocentric attraction is at its least value, the centrifugal force is then at its least value also, and when the Moon is at quadratures and the heliocentric attraction is at its mean value, the centrifugal force is then at its mean value also. This is in accord with the laws of motion, for it is attraction which causes the Moon to move, and which deflects its motion from a straight line. The centrifugal force on the contrary, is of a different nature; it is a secondary or merely responsive force, and can have no existence, except as attraction causes the Moon to move out of a straight line; it is merely the inertia with which the Moon resists the deflecting force. Hence, when attraction or the deflecting force is greatest, the centrifugal force ought to be greatest also, and when it is least, the centrifugal force ought to be least; and this relation is in fact constantly maintained as the Moon follows the undulations of the epicycle. These are the forces whose immediate action determines the Moon's stability; and they are the forces of the epicycle; not those of the geocentric circle, nor of this circle modified by solar perturbations.

§ **3. The Earth's perturbation of the Moon's heliocentric motion.** It is the custom in current text books of astronomy to describe the Moon as primarily a satellite or dependent of the Earth, and its motion as a geocentric revolution perturbed by the Sun's attraction. For some purposes, this way of looking at the matter may not be objectionable. But for purposes of accurate thinking and analysis a much better understanding of the relation of the forces is obtained by regarding the Moon as a planet revolving around the Sun and perturbed in its motion by another larger planet, the Earth. These two planets, in truth, perturb each other as they revolve around the Sun, and it is only the imaginary point marking their common center of gravity that follows the path which either one of them would follow if it revolved alone. But, because the Moon perturbs the Earth so little, we may for present purposes disregard that perturbation. Astronomers sometimes object to this view of the Moon's relations, claiming that it reveals nothing new or different from what is shown in the current analysis. They claim that it is the same thing seen from a different point of view and is objectionable, because it is more difficult to handle in analysis. I think it can be shown, however, that this view is not the same as that of the current analysis, but quite different, and brings out clearly a different relation from any that is contemplated in that theory. Not that any serious objection is here made to the current theory of the Moon's inequalities of motion arising from differences of solar attraction, although some points might be explained differently. Considered merely as a theory explaining those inequalities, the current theory appears to be acceptable. But as a theory of stability it is not. More will be said later, however, on the relations of the current theory and difference of attraction to the theory here suggested.

The Moon and the Earth tend to revolve around the Sun in one and the same orbit. But this they can not do, because they perturb each other; so their present relation is a sort of compromise, and is the nearest they can come to realizing that tendency. They could not follow the same mean orbit and have the same annual period in any other way. The making of such a compromise implies and requires the operation of forces guided by a governing law, and the action of such forces tends to produce a definite result and yield a fixed or determinate relation between the two bodies for given conditions.

The Moon's perturbation of the Earth is so small that we may neglect it for the present purpose. But the Earth's perturbation of the Moon is relatively very large. If the Earth were absent and the Moon revolved alone around the Sun its orbit would be the present orbit of the Earth, omitting the lunar perturbation. It is the Earth's perturbation of the Moon's heliocentric motion which causes the latter's geocentric revolution. This view is quite different from that of current theory and leads to a different estimate of the forces affecting stability, and to a different method of analysis.

A certain value of force affecting the Moon's stability is produced by an Earth-perturbation of given magnitude; but a perturbation of another magnitude will produce a force of a different value for stability. If the mass of the Earth were increased to twice its present value, the Moon revolving at the same distance from the Earth as now, the attraction of the Earth upon the Moon would be twice as great as at present. The effective heliocentric centrifugal force, depending upon heliocentric velocity and curvature, would have different values from those now obtaining, being greater in opposition and less in conjunction. The undulations of the epicycle would be shorter and the whole character of the Moon's motion around the Sun would be changed. The effective heliocentric attraction would also change in the same order; that is, it would be increased in opposition and decreased in conjunction. In such a change as that supposed it seems plain that the two opposing forces of heliocentric attraction and heliocentric centrifugal force would have to balance exactly, or stability in the same geocentric orbit could not be maintained. It is of the very essence of the theory here presented that they would not be balanced after the Earth's increase of mass, and it is out of the relations of these antagonistic forces and the laws by which they attain a balanced adjustment under different conditions that we get the fundamental conditions of determinate stability.

It may be noted that in such a change as that supposed the solar difference of attraction would not be affected at all, except in its ratio to the Earth's attraction. The difference of attraction at *O* in Fig. 2 or Fig. 3 would be $\frac{1}{180}$th of the Earth's pull instead of $\frac{1}{90}$th as it is now.

If we regard the Moon as merely a satellite of the Earth perturbed by the Sun we are apt to overlook certain small but highly important factors in the great variations of the main forces which affect the Moon as it revolves around the Sun. Current text books have too little to say concerning the relation of the main forces to stability. That aspect of these forces is seldom given more than a bare mention. Everything is made to depend upon difference of attraction. But by regarding the Moon as a planet perturbed by the Earth, the main forces are brought more prominently before us, and any small differences in the rates of their variation under different conditions are not so likely to be missed.

The Earth, in fact, perturbs the Moon in the same general way that it does the planets Venus and Mars, but with this difference, that the Moon is very much nearer, and the Earth's attraction is correspondingly more powerful, so that the Earth is able to pull the Moon forward when it is behind, and backward when it is in front, with power enough each time to cause it to pass alternately from one relation to the other, and thus perform a continual series of complete geocentric revolutions.

Figs. 1 and 2 show how this perturbation takes place, but perhaps Fig. 4, on the plan of the geocentric circle, shows better to minds accustomed to that method of representation how the Earth alternately accelerates and retards the Moon's heliocentric motion. This figure omits the element of curvature as seen in the epicycle, and it omits the effects due to the angular motion of the Earth-Moon system around the Sun.

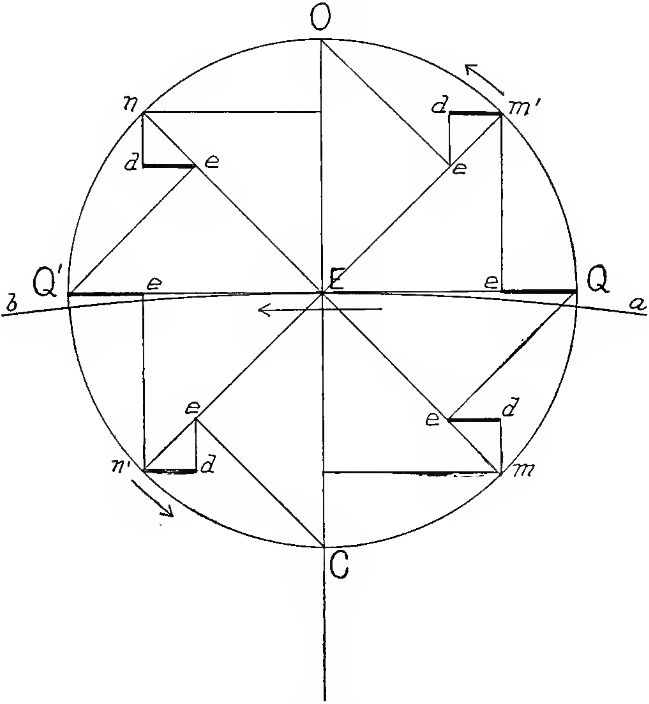

FIG. 4.

When the Moon is at *Q*, it is directly behind the Earth as the two move around the Sun, so that the whole of the Earth's attraction acts to accelerate the Moon's forward motion. When the

Moon reaches *m′*, it has been accelerated by the amount *Qe*, which is equal to the whole of its fall toward the Earth. But in going to *O* the component of the Earth's attraction which accelerates the Moon grows less, being *m′d* at *m′* and vanishing to zero at *O*. At the latter point the Earth's attraction neither accelerates nor retards the Moon's motion with reference to the Sun. But immediately on passing *O*, a component of the Earth's force begins to retard the Moon's heliocentric motion, and this increases until at *Q′* the whole force of the Earth's attraction acts in that way. From *Q′* to *C* the retarding component again decreases to zero. At *C* the Earth's pull neither retards nor accelerates, but at the next instant begins to accelerate and thence increases to a maximum at *Q*.

This relation affords a beautiful illustration of the fact that maximum forces and maximum effects do not or may not coincide in time and place. For while the accelerating force begins to act at *C*, it reaches its greatest power at *Q* and its greatest effect at *O*, where the Moon's velocity is half a mile per second more than that of the Earth, or 19 miles per second. At *O*, where the acceleration is greatest, the accelerating force has vanished, and at *C*, where retardation is greatest, the retarding force has vanished.

Thus it is that from *C* to *O* the Earth draws the Moon forward with sufficient cumulative power to carry it past the radius vector at *O*. But this is a relatively slight perturbation as compared with other planetary perturbations of satellites. Fig. 1

shows the relatively slight effect which it has upon the curvature of the Moon's path around the Sun. It may be noted that the greatest effect of acceleration is at *O*, just where the heliocentric attraction is greatest and where the greatest centrifugal force is needed, and the greatest effect of retardation is at *C*, just where the least centrifugal force is needed.

From this way of looking at the matter, it seems plain that the conditions of the Moon's revolution around the Earth, while the two revolve together around the Sun, are not the same as those which obtain for the simple case of two bodies. The same balanced state must be attained between the same opposing forces, but they are related to each other in a different way. In the case of two bodies all the forces involved have a constant and uniform value. But as the Moon follows the epicycle their values must keep equality while undergoing a continual series of large variations on opposite sides of the mean. They must be nicely offset against each other or stability in the geocentric orbit can not exist. By as much as these two forces are not perfectly balanced they will affect and disturb, one way or the other, the Moon's relation to the Earth. Hence, while the Moon's revolution around the Earth seems like the simple circular revolution of a satellite perturbed only by differences of solar attraction, immediately we turn attention to the Moon's relation to the Sun, this circular geocentric motion becomes a matter of relativity and takes on the aspect of a great perturbation produced by the Earth. The great advantage in this view of the matter is the clearer perception which it brings of the fact that the main forces — the effective heliocentric attraction and the Moon's heliocentric centrifugal force — undergo great variations, and that the Earth causes these variations. It is certain that a state of balance is at present being maintained between these varying forces, or the stability of the Moon's relation to the Earth could not be preserved.

From the foregoing considerations we see that the Earth perturbs the Moon's heliocentric motion and by that action brings forces into play which affect the Moon's relation to both the Sun and the Earth. So far as we have progressed, the solution of the problem of stability seems to depend upon an analysis of the effects of the Earth's perturbing action. As the perturbation is now, the Moon is stable. But would it be stable if the Earth's perturbation were greater than it is or less? It appears that astronomers are not accustomed to take this view of the Moon's relations and that the problem of stability has not been studied in this way.

CHAPTER IV

THE MECHANISM OF DETERMINATE STABILITY.

§ 1. General Statements. If the discussion in the foregoing pages has served the purpose intended it has led up to and suggests an analysis of the Moon's motion and stability by a different method from that used in the current theory. The problem presented is not new, except in the point of view and the manner of approach. For it is after all the same old problem — the famous problem of three bodies — which has stubbornly resisted solution by direct mathematical analysis and has been solved only by special methods of approximation — by a sort of mathematical cut-and-try.

Considering, therefore, the great difficulty of the problem, it will not be expected that a demonstrated solution of it will be attained by the very simple, unmathematical method here employed. But complete demonstration is not the only measure of utility and progress in the study of so great a problem. Much advancement may be made which falls short of that consummation. The defects of the current theory will never be removed until some one, who entertains honest doubts, makes an earnest effort to correct them, and a beginning in that direction, even if ever so meager, will be worth the effort if it gives a promise of disclosing new truth. If it can be shown that the current analysis is not a correct and complete demonstration of the mechanism of the Moon's stability; if the elements of a different and better method of analysis can be shown, even if the analysis itself be not made; if strong reasons can be shown why this new analysis may be expected to show that stability is determinate instead of indeterminate, and finally, if it can be shown that, by assuming stability to be determinate, not only are all of the facts now explained by the current analysis accounted for equally well, but that other things not explained by the current analysis, and some of them unanticipated, are also satisfactorily accounted for, then even without complete demonstration or analysis the conclusions reached may claim some degree of plausibility and the suggestions made may not be without some value.

Observations show that the Moon is stable in its present orbit around the Sun. That is to say, its heliocentric revolution is stable notwithstanding the relatively large perturbations which the Earth imposes upon it. From this fact it is obvious that the heliocentric centripetal and centrifugal forces which affect the Moon's motion at the present time are, in effect, balanced against each other. Although both of these forces act with different powers at each successive moment and are perhaps of slightly unequal value in some parts of the epicycle, they are, on the whole, evenly balanced against each other in each completed revolution. But while general statements like these are true, they disclose nothing as to the quality of stability. That can be determined only by finding upon what factor of force or combination of such factors stability depends, and how these factors vary under conditions differing in definable ways. The particular values which the forces now have depend upon the particular conditions now affecting the Moon's revolution. A change in these conditions, such as would result if the Moon be supposed to revolve in a widely different orbit around the Earth, would produce a change, not only in the values of these forces, but in all the conditions affecting stability. While this appears to make the problem highly complicated, it is perhaps not an absolute bar to profitable discussion in unmathematical terms. By supposing the simplest possible changes and following the effects separately, it may be possible to make out the general relations of the forces to stability.

§ 2. **The effective heliocentric centrifugal force which affects the Moon in the epicycle; its factors of excess and deficiency in opposition and conjunction.** Without undertaking anything in the nature of demonstration, a few simple diagrams with explanations may be helpful in discussing the relation of the forces. If the Earth were to move

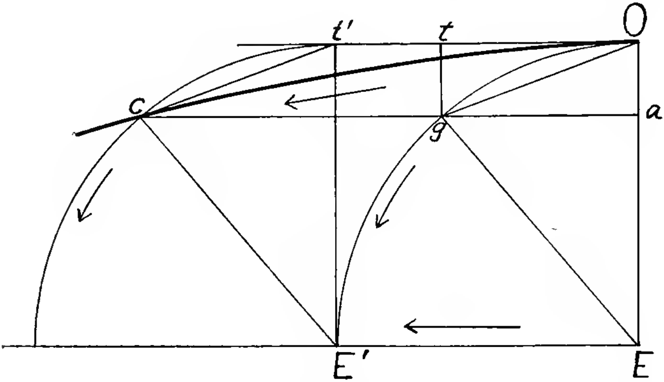

FIG. 5.

forward in space in a straight line and at a uniform rate with the Moon revolving around it in a circle, the path of the Moon, supposing no disturbing influence by a third body, would be an undulating curve closely resembling an epicycle. It would be like the epicycle in Fig. 1, except that the path of the Earth upon which it is superimposed would be a straight line. The Moon would then have two motions, a revolution in a circle around the Earth and a motion forward with the Earth. By per-forming both motions at once its path would be the resultant undulating curve just described. But the forward motion being in a straight line, would generate no centrifugal force and would in no way affect the stability of the Moon's circular revolution around the Earth. That would go on precisely as though the Earth were at rest. The generation of the Moon's resultant path in this case is illustrated by Fig. 5.

Let *EE'* be the straight line path of the Earth in space and *Og* a part of the Moon's circular orbit around the Earth. *Ot* is tangent to this orbit at *O*, and also parallel with *EE'*.

Suppose the Earth's attraction causes the Moon to fall from a state of rest at *O* to the point *a* in one second, and that the Moon's velocity around the Earth is sufficient to carry it from *O* to *t* on the tangent in the same time. By falling while it moves, the Moon will then follow the circular curve *Og* and reach *g* at the end of one second.

But while the Moon performs this motion, the Earth, carrying the Moon with it, advances from *E* to *E'*. This component of the Moon's motion is represented by the line *Ot'*, which is coincident in direction with the tangent *Ot*. *Og* and *Ot'* are therefore the two component motions from which the Moon's resultant path must be derived.

The curve *Og* is so slight that it may be regarded as a straight line. Then drawing *gc* parallel to *Ot'* and of equal length and *t'c* parallel to *Og*, we have the parallelogram *Ot'cg*. The diagonal curve *Oc*, is the resultant path which the Moon will follow to reach *c* in one second. By a precisely similar construction we might follow the Moon through a complete revolution around the Earth.

The case shown by Fig. 5, is one of extreme simplicity. It requires the action of only two forces—the attraction of the Earth, which deflects the Moon from a straight line as it revolves, and the centrifugal force which resists that deflection. The common motion in a straight line and the momentum which it produces are purely matters of relativity without effect upon the Moon's revolution. No force is developed by them which tends in any way to modify or change the relation of the Moon to the Earth.

A system like that shown in Fig. 5 is called an *equilibrium mobile.* It is, however, a *neutral* or *indeterminate* equilibrium mobile, because the mean distance of the Moon from the Earth and the character of the orbit it revolves in, whether circular or elliptical, are matters of entire indifference. The Moon would be exactly as stable in one orbit as in another.

If, however, the deflecting attraction of a third body, as that of the Sun, be introduced the number of forces in play will be doubled and their relations and the results they produce will be quite different. For the Moon will then perform *two revolutions* instead of *one revolution and a straight-line motion*, as in Fig. 5. There are in this case two attractions and two centrifugal forces to be reckoned with. The action of the forces is illustrated by the next figure.

In Fig. 6 the curve EE' is intended to represent a part of the Earth's circular orbit around the Sun, and Og, as before, a part of the Moon's circular orbit around the Earth. Let it be supposed that the Sun's attraction would cause the Earth to fall from a state of rest at E to the point r in one second. If the Earth be supposed to move at the same time with a velocity which would carry it to T' on the tangent, then at the end of one second it would reach E'.

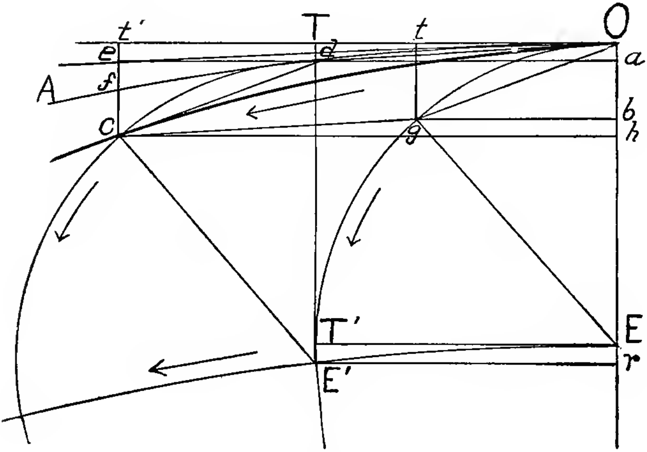

FIG. 6.

Let it be supposed also that the Sun's attraction on the Moon would cause it to fall from a state of rest at O to the point a in one second. The curve OA is a circular orbit around the Sun passing through the point O, and is therefore concentric with EE', the orbit of the Earth. By the law of velocities in circular orbits, the Moon's normal velocity around the Sun in OA would be a trifle less than that of the Earth in its orbit. But for the present purpose, the Moon's normal velocity may be regarded as the same as that of the Earth.

If the Moon, moving with velocity normal to *OA*, were deflected from the tangent by neither Sun nor Earth it would reach *T* at the end of one second. If deflected by the Sun alone it would fall the distance *Td*=*Oa*, and so would reach *d* instead of *T*. The Moon's resultant path in the first second, if the Moon were attracted by the Sun alone, would therefore be *Od*, which is part of *OA*.

But the Moon is at the same time attracted by the Earth, and by the Earth's attraction alone would be made to fall, we may suppose, from a state of rest at *O* to *b* in one second. If *Ot* be supposed to represent its velocity around the Earth, then by drawing *tg* perpendicular to *Ot* and *bg* parallel to *Ot*, the resultant path would be along the line *Og* and the Moon would reach *g* instead of *t* at the end of one second.

We have now two component motions or revolutions, one around the Sun in *Od* and one around the Earth in *Og*. The curvature of these lines is so slight that we may regard them as straight. Then by drawing *gc* parallel with *Od* and *dc* parallel with *Og*, we have the parallelogram *Odcg*, of which the diagonal *Oc* is the resultant path of the Moon from *O* to *c* in one second. By the same method of construction starting at *c*, we might follow the Moon through a whole revolution around the Earth. The Moon's actual orbit around the Sun is therefore the resultant of two revolutions, and may be called the *resultant path* or *curve.* The resultant curve *Oc* may be taken to represent the real path and velocity of the Moon in opposition. The Moon's *normal* velocity in the concentric circular orbit *OA* is represented by the resultant *Od*. These velocities marked off on the tangent are represented by *Ot'* and *OT* respectively. If *OT* be the normal velocity of the Moon in the Orbit *OA*, then the greater velocity *Ot'* is too great for stable revolution in *OA*. For if the Sun's attraction would pull the Moon from a state of rest at *O* only to *a* in one second, it can do no more while the Moon moves toward the tangent at any velocity. By producing *ad* parallel to *Ot'* to the line *t'c*, their intersection will fall at the point *e*, and *t'e* will therefore be equal to *Oa*, and will represent the distance which the Sun's attraction will cause the Moon to fall in one second from the tangent while moving from *O* to *e*.

But while *d* is on the curve of the concentric orbit *OA*, the point *e* is outside of it, and if, as assumed by construction, *OA* is a circle around the Sun, then *e* is farther from the Sun than *d*. For *OA* intersects *t'c* at *f*, and *d* and *f* are equidistant from the Sun. The line *ef* may therefore be taken as a measure of the amount by which the distance of *e* from the Sun exceeds that of *d* in consequence of the excess of velocity *Tt'*.

It follows that if the Moon started from *O* with the velocity *Ot'* it would move, if affected by the Sun's attraction alone, to *e*, instead of to *d*. But the Moon's actual heliocentric velocity as perturbed by the Earth *is Ot'*, and hence, so far as the Sun alone is concerned, the Moon tends to follow the curve *Oe* to the point *e*, and not *Od* or *Oc*.

If *Oa* represents the Sun's attraction, then *Td* represents the centrifugal force required to exactly balance that attraction, when the Moon moves with the velocity *OT*, which is the normal velocity for stable revolution in *OA*. But $t'e = Td$, and so represents the same centrifugal force. From this it follows that *ef* represents *an excess of centrifugal force due to excess of velocity over that which is required for stable revolution in OA.*

In Fig. 6, the line *t'c* represents the whole heliocentric centrifugal force affecting the Moon as it moves past *O* in Fig. 2. This line is made up of two factors, and one of these may be again subdivided. It is composed of *t'e*, which, as has been shown, is the heliocentric centrifugal force of normal revolution in *OA*, and of *ec*, which is the centrifugal force of the Moon's geocentric revolution as it passes through *O* in the epicycle (Fig. 2); or in other words, *ec*, which is equal to *tg*, is the heliocentric centrifugal force arising from that part of the Moon's motion with reference to the Sun which is due to its revolution around the Earth. Hence, *ec* is the heliocentric centrifugal force arising from the Earth's perturbation of the Moon's motion around the Sun, or, more exactly, the heliocentric centrifugal force of the Earth's perturbation in opposition.

The Sun by its unaided attraction can not support or counterbalance more centrifugal force in a body revolving in *OA* than is expressed by *Td*, which is equal to *t'e*. Hence the remaining portion, *ec*, must be balanced by the attraction of the Earth. The factor *ef*, as we have seen above, is the excessive centrifugal force arising from the excess of velocity Tt'—excess over OT, which is the normal velocity in OA. Obviously, this factor can not be counterbalanced by the Sun's attraction, but must be included as a part of ec which is balanced by the Earth's attraction. The curve Oc is a part of the epicycle in opposition. It follows that the Moon's course on leaving the point O must be inside of the concentric circle OA. Hence, to follow the path of the epicycle the Moon must always fall inside of OA, or it could never reach the point of quadrature (Q in Fig. 2) and perform a revolution. Oc in Fig. 6 is by construction a part of the curve of the epicycle, and the Moon follows this curve to c in one second. The line fc represents the increase of curvature which the Moon must take on in order to follow the epicycle instead of the circle OA. This factor is therefore *an excess of centrifugal force due to excess of curvature over that which is required for stable revolution in OA.*

Taken together, ef and fc represent the whole excess of heliocentric centrifugal force due to the excess of both velocity and curvature. For short, it may be called the *excessive centrifugal.* This force is disclosed by a study of the heliocentric aspect of the Earth's perturbation of the Moon. In a condition of stability this force must always be exactly equal to the Earth's attraction; that is, ec must be equal to tg which is equal to Ob (Fig. 6.). This force must always be present in opposition if the Moon revolves in the direct order with sufficient velocity to pass the radius vector of the Earth produced, and this whether the Moon revolves near the Earth and swiftly or far away and slowly.

We have seen that the Moon's greatest excess of heliocentric velocity and curvature is at *O*, and that from this point it diminishes to zero at *Q′* (Figs. 1, 2 and 3.). Hence, as would be expected, the value of *ec* is greatest at *O* and diminishes to zero at *Q′*, decreasing in value at the same ratio as the decrease of excess of velocity and curvature. At *Q′* the force *ec* vanishes, for at that point there is no excess either of velocity or curvature.

On passing the point of quadrature (*Q′*, Figs. 2 and 3) the relation of the heliocentric and geocentric velocities and curvatures changes. The Moon's motion around the Earth begins to be in a direction contrary to that of their common motion around the Sun, so that the Moon's heliocentric velocity is diminished by the geocentric motion. At *C* the Moon's velocity around the Sun is equal to the difference between the Earth's velocity in its orbit and the Moon's velocity around the Earth. Constructing Fig. 7 on the same principles as Fig. 6, we have *Ca* corresponding to *Oa*, *CT* to *OT* and *Cd* to *Od*. In this case the Earth is not on the same side of the Moon as the Sun, but on the opposite side. Hence, while *Cb* corresponds to *Ob*, it is differently placed, and the Moon moves with reference to the Earth from *C* to *g*. Constructing as before, we get the parallelogram *Cdcg*, with the diagonal *Cc* for the resultant path of the Moon. The Moon's velocity around the Sun is in this case reduced from the normal velocity *CT* to *Ct′*. But the Sun's attraction causes the Moon to fall from *C* to *a* in one second, and if this happens while the Moon moves to *t*, the Earth moving at the same time from E to E', the Moon will reach e instead of f. If the normal velocity CT produces just the right value of centrifugal force for stable revolution in the concentric circular orbit CB, then with the smaller velocity Ct' there will be a deficiency of centrifugal force and the Moon will not follow CB, but Ce. To have a deficiency of curvature in going the distance Ct', which shall be equal in value to the excess shown in Fig. 6, it is necessary for the Moon's path to curve *away from the Sun*, as shown in the curve Cc in Fig. 7. The line ec therefore represents *a deficiency of heliocentric centrifugal force* as compared with that required for stable revolution in CB. This may be called the *deficient centrifugal.* This force arises from the Earth's perturbation of the Moon in conjunction.

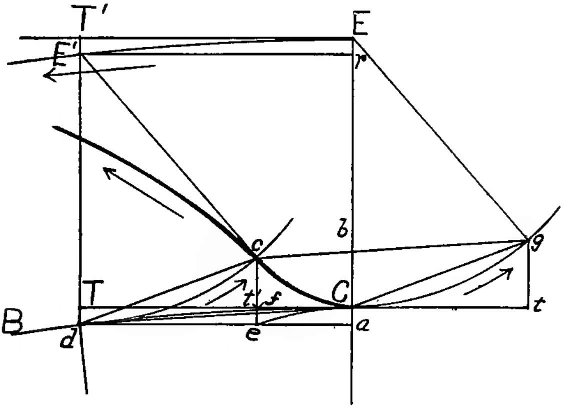

FIG. 7.

We have seen that the Moon's greatest deficiency of heliocentric velocity and curvature is at the point of conjunction. It follows that this force has its greatest value at that point, and it fades to zero at the quadratures. In the case of the Moon, the deficient centrifugal would be present in more or less value in every orbit the Moon could have around the Earth, from actual contact to the farthest limit of direct revolution. For there would always be a deficiency of heliocentric velocity and curvative in conjunction. This force has the same important relation to stability as the excessive centrifugal shown in Fig. 6.

Throughout this essay attention is centered mainly upon the case for opposition and the outer quadrants, it being deemed unnecessary to dwell further on the case for conjunction, since the reasoning and results are the same for stability, although the relations are slightly different.

The value of these factors depends upon the magnitude of the Earth's perturbation of the Moon's motion, being greater as the perturbation is greater and less as it is less. The magnitude of the perturbation in turn depends upon the power with which the Earth bends the Moon's course from a circular path around the Sun ; that is, upon the distance at which the Moon revolves around the Earth. In a smaller orbit the perturbation would be greater, in a larger orbit less, and the value of the excessive centrifugal varies in the same order. From these considerations it follows that if the Moon revolved in a smaller orbit than it does now the excessive centrifugal force would be greater than it is now, and if the Moon revolved in a larger orbit this force would be less.

§ 3. **The forces of the epicycle and their relation to geocentric stability.** It was pointed out above that the epicycle corresponding theoretically to any given circular orbit around the Earth is a thing definitely fixed by the Earth's mass and its orbital motion around the Sun. The mass determines the rate at which the Moon shall revolve in a given circle, and the orbital velocity of the Earth determines at what rate that circle shall be distributed along the Earth's curved orbit to form the epicycle. Remembering that the geocentric circle and the epicycle are really only two aspects of one thing, it is obvious that they must always vary coincidently, and never individually or separately. Nevertheless, it is convenient in discussion to speak of them as separate things.

Given the mass of the Earth, we may in imagination choose a circular orbit around it at any distance, and by the law of velocities in circular orbits we can calculate the velocity and periodic time which the Moon must have in order to revolve in that orbit. Then, knowing the Earth's distance from the Sun, we can construct in the most precise way the particular epicycle corresponding to the circle chosen. An epicycle constructed in this way is simply a geometrical representation of the distributed geocentric circle, the distribution following a precise rule. This theoretical epicycle may be called the *geometrical* or *absolute epicycle*, and it corresponds to that particular geocentric circle and to no other

We know by observation that the Moon's present revolution is stable. We may conclude therefore that the opposing forces which affect the Moon's motion in the epicycle are in equilibrium. The Earth perturbs the Moon's motion around the Sun by a certain amount and this perturbation engenders excessive heliocentric centrifugal force in opposition and a deficiency of force of the same kind in conjunction. But from the fact that stability exists, it must be true that these excessive and deficient factors are exactly counterbalanced by the heliocentric component of the Earth's attraction, augmenting the power of the Sun's pull on the Moon in opposition and diminishing it in conjunction. Being stable in the epicycle, the Moon's revolution in the geocentric circle is, of course, also stable.

There are an infinite number of circular orbits around the Earth in any one of which the Moon may be assumed to revolve. But its velocity and curvature and periodic time would not be the same in any two of them. Since an epicycle is only a distributed circle, and since the epicycle corresponding to a given circle must be distributed in a particular way, it follows that there are as many possible epicycles as there are possible circles, and the velocity and curvature characters and the period of each one are different from those of every other. From this fact it follows that the excess of velocity and curvature in opposition, and hence also the excessive centrifugal force, are different in each epicycle. By the law of gravitation the Earth's attraction for the Moon would have a different value in each circular orbit, and this means that the Earth would have a different power of perturbing the Moon's motion in every different epicycle. Being simply two views of one thing, the circles and the epicycles go together in inseparable pairs. A given circle can have only one epicycle corresponding to it for a given mass and heliocentric distance of the Earth from the Sun. Hence, if the Moon is to be stable in a given geocentric circle it must also be stable in the only epicycle which corresponds to that circle. The Moon can never revolve in one geocentric circle and at the same time be stable in the epicycle corresponding to some other circle, or in any epicycloidal curve which does not correspond in the mean to that particular circle.

The current theory has shown by its method of analysis and by many implications that the Moon would be as stable in one geocentric circle as in another. But although it assumes this to be the fact, it has not shown by explicit demonstration that the Moon would be as stable in one *epicycle* as in another. However, it is certain that if for any reason the Moon could not be stable in a given epicycle or series of epicycles, then it could not be stable in any of the geocentric circles corresponding to those epicycles. The Earth's mass has a definite value; its distance from the Sun and hence also its velocity and curvature with respect to that body have definite values. With these definite values as a foundation for the geocentric circle and the corresponding epicycle, we have an excellent basis of comparison for other orbits in which we may suppose the Moon to revolve. For, assuming the Earth to keep the same mass and distance from the Sun, any other geocentric orbit we may assume for the Moon will have the same basis of distribution in forming the corresponding epicycle.

Fig. 6 is assumed to represent a condition of stable revolution. That is to say, it is assumed that the curve *Oc* represents not only a part of the geometrical or absolute epicycle corresponding to the geocentric circle *Og*, but also the path which the Moon must follow in order to precisely satisfy the forces affecting it. It is assumed that if the Moon follows this curve the effective heliocentric centripetal and centrifugal forces affecting it will be at an exact balance.

If a figure like Fig. 6 were drawn, not by geometrical rules, but by a calculation of the path which the Moon would follow starting from *O*, when the opposing forces were exactly balanced, the path thus drawn would be in a certain sense a *dynamic* curve, and it might be considered and discussed independently of the geometrically constructed epicyclic curve *Oc*. It might agree with this curve exactly or it might not. If it did, then a condition of perfect stability would be indicated for the Moon; but if it did not, then a condition of instability would be indicated. If the two curves were the same the demonstration would show that the forces tend to make the Moon conform to the precise path which geometrical considerations require, and the Moon would then follow *Oc* and reach *c* at the end of one second. If the curves did not agree, then the demonstration would show that, although geometrical considerations would require the Moon to follow *Oc* to *c* in one second in order to be stable in the geocentric circle *Og*, yet that, under the action of the opposing forces, which are assumed in this case to be *not* exactly balanced against each other, the Moon would not follow *Oc*, but some other curve slightly inside or outside of it, and the Moon would not reach *c* in one second, but some point nearer to or farther from the Earth (*E′* in Fig. 6). If the centrifugal force overbalanced the centripetal the Moon would reach some point farther than *c* from *E′*; if the centripetal force overbalanced the centrifugal the Moon would reach some point nearer than *c* to *E′* In the first case the geocentric figure corresponding to the dynamic curve would be a slowly expanding spiral; in the second case it would be a slowly contracting spiral. In the first, the Moon would be expanding its geocentric orbit and gradually receding from the Earth; in the second, it would be contracting its orbit and drawing in nearer to the Earth.

The spiral is a symbol of unstable adjustment. The forces which cause spiral movement are not at a balance. A body following a spiral path is changing its relation to the center about which it is revolving, either receding from it or approaching it.

We know by observation that, as the term is generally used, the Moon's revolution around the Earth is stable. Hence the dynamic epicycloidal curve which the Moon follows coincides in the mean with the geometrical epicycle, and this in turn corresponds to the mean geocentric circle. There are only two other relations which the Moon could have to the Earth. If it is not following a mean geocentric circle, then it must be following a geocentric spiral. As a matter of exact truth, the Moon's motion is now undergoing an exceedingly slow acceleration, which is known as "the secular acceleration of the Moon's mean motion." This means that the Moon is not now following an orbit corresponding to a true mean circle, as is usually stated, but is following a contracting spiral in which the rate of contraction is exceedingly slow. To this extent the Moon's present geocentric revolution is unstable, but this small element of instability is usually neglected where stability is being discussed in general terms.

An *absolute demonstration* of the mechanism of determinate stability could be accomplished only by a mathematical analysis of the most profound nature. But the ground can be covered by a series of assumptions that will show the form which that analysis will have to take and the general relations of the forces involved.

CHAPTER V.

ALTERNATIVES AND COMPARISONS.

§ 1. **Alternative assumptions as to the solution of the problem of stability.** We are now prepared to make some comparisons which will of themselves bring up the fundamental question involved in the problem of stability. Let us consider, first, a supposable case in which the Moon revolves around the Earth at a distance of 60,000 miles instead of at its present distance of 240,000 miles. The Moon's present velocity around the Earth is about half a mile per second. By the law of velocities in circular orbits, its velocity in the smaller orbit would be twice as great or about one mile per second. Its periodic time or month would then be one-eighth as long as now. This would of course make a large difference in the character of the corresponding geometrical epicycle. For a complete revolution would be performed while the Earth moved one-eighth as far as during a present revolution. The power of the Earth's attraction and hence of its perturbation of the Moon would be sixteen times as great as now, and the heliocentric component of the Earth's attraction at O and C (Fig. 2) would therefore be increased by the same amount. The excess of velocity in opposition would be twice as great as at present, and the excess of curvature would also be greater — several times greater than now. It would be greater by such an amount that the total value of the excessive heliocentric centrifugal force in opposition would be *an exact or very nearly exact* balance for the heliocentric component of the Earth's attraction. That is to say, it would balance the Earth's perturbation *exactly* or *almost exactly*. Just here is the main question.

Would these opposing forces exactly balance each other in the smaller orbit so as to permit the Moon to follow the geometrical epicycle with the heliocentric forces in a state of exact equilibrium? If an exact balance would be attained, then stability in this epicycle and in the corresponding geocentric circle would be assured; if it would not, then stability could not exist under these conditions, and the Moon would have to follow a spiral path.

As was distinctly stated above, it is not within the intended scope of this essay to undertake a numerical solution of this problem. Precision in results can only be attained by mathematical treatment, but that method is not at my command. It is open to me to do no more than discuss this problem in common unmathematical terms. The only way in which I can approach it effectively is by the method of multiple hypotheses founded upon alternative assumptions as to the result of a correct solution of the problem stated above, or rather of that problem more fully stated, so as to cover the relations of the opposing heliocentric forces in every part of the epicycle; that is, at every point in it for one complete geocentric revolution.

We are bound to conclude that the Moon's stability in its present epicycle is attained by a perfect equilibrium between the opposing effective heliocentric forces; that is, that their *mean* values for each whole revolution are exactly balanced against each other. But this does not bind us to conclude that the same state of equilibrium would exist if the Moon revolved in an orbit 60,000 miles from the Earth.

As to their qualities, there are three results and only three, which might be attained by a mathematical solution of this problem. It might be shown, (1) that the opposing effective heliocentric centripetal and centrifugal forces would be exactly balanced; or (2), that the effective heliocentric centripetal force would be slightly stronger than the opposing heliocentric centrifugal force, or (3), that the effective heliocentric centrifugal force would be slightly stronger than the opposing heliocentric centripetal force.

Considered as abstract propositions, there is perhaps no reason to choose one of these results more than another. But one and only one of them can be the true result. Nor can it be a matter of indifference as to which one is the true relation. According as one or another of these results is the true one, stability is determinate or indeterminate or altogether impossible.

In the absence of a correct mathematical solution of the problem, there are two ways in which to make an intelligent choice of these results. Some progress toward a choice may be made by studying these alternatives in the light of sound theory, and still more may be accomplished by studying them in their relations to those phenomena which are manifestly dependent upon stability and its quality for their occurrence and arrangement.

If the first possible solution be true and the opposing heliocentric forces are exactly balanced, then the case for indeterminate stability is made good. If the Moon would be stable in an orbit 60,000 miles from the Earth the same as in its present orbit at 240,000 miles, the Earth's mass and distance from the Sun remaining the same, then we may conclude that it would be stable in orbits between these two and in others nearer the Earth. To test the matter still further, we might try another hypothetical case by supposing the

Moon to revolve in an orbit 400,000 miles from the Earth. At this greater distance we should find all the factors diminished in value, the power of the Earth's perturbation would be less and the power of the counterbalancing heliocentric centrifugal force would also be less. The same questions would arise here as before. Would the opposing forces be exactly balanced or not? And there would be the same three possible results as to the quality of the relation between the forces. If the forces were found exactly balanced, then the case for indeterminate stability would be made still stronger, and we may conclude that within certain limits the Moon would be stable in any orbit larger or smaller than its present one. This would be a complete verification of the current theory and of indeterminate stability.

If the second result were true and the heliocentric centripetal forces were the stronger, then stability in the orbit at 60,000 miles would not be possible. The Moon would follow a contracting spiral and hence would be drawn in nearer to the Earth at each round and would gradually contract its orbit nearer and nearer to the Earth, and in each smaller orbit the forces would tend still more strongly toward contraction. The end of this process would be collision with the Earth. But the forces affecting the Moon in its present orbit are at a balance. Such a relation would therefore indicate a difference in the rate of variation of the opposing forces. It would indicate that as the Moon took successively smaller orbits nearer to the Earth the centripetal force would increase at a slightly greater rate than the centrifugal force. If this were the case, the Moon could not be stable in any orbit smaller or nearer the Earth than that in which it now revolves. And further, supposing this to be the manner of variation of the controlling forces in smaller orbits, there is every reason to believe that the same manner of variation would hold for larger orbits also. In that case, the Moon revolving at a distance of 400,000 miles from the Earth would be affected by opposing forces of which the centrifugal factor would be slightly stronger than the centripetal. Stability could not be attained under such an adjustment, for the Moon would follow an expanding spiral and hence would move out a little farther from the Earth with each revolution. It would gradually expand its orbit, and as the orbit expanded the tendency to expand would gradually increase, and this would go on indefinitely or until the Moon drifted away and became permanently lost to the Earth. Such a relation of forces would indicate that the Moon would not be stable in any orbit larger or farther from the Earth than its present one. It appears then that with this manner of variation of the forces the Moon could not depart by the smallest amount from its present orbit, either to contract or expand it, without immediately falling into conditions in which its stability would be destroyed, and in which it would be acted on by forces that would tend to cause it to depart more and more from its present orbit. This kind of stability corresponds to the quality of an unstable equilibrium as defined above (page 9). It is an impossible mechanism for stability; indeed, it is the very ideal for instability, and may be rejected finally as inadequate and inapplicable. Such a scheme makes the Moon's present stability a most delicate uncertainty, like the balancing of a needle on its point.

The third result mentioned above was where the centrifugal factor of the two opposing forces would be slightly stronger than the centripetal factor, the Moon revolving in the smaller orbit 60,000 miles from the Earth. In this case the Moon's revolution would again be unstable, but the tendency to instability would be in the opposite direction. The Moon would follow an expanding spiral and hence would move out a little farther from the Earth at each round and expand its orbit, but as expansion proceeded the tendency to expand would grow gradually less and less until, having reached its present orbit at 240,000 miles, the Moon would find the forces exactly balanced, expansion would cease and stability would be attained. Manifestly, in this case the centrifugal force would vary at a slightly higher rate than the centripetal force.

Carrying this manner of variation as before out to the large orbit at 400,000 miles, we should find there that the centripetal force would be slightly stronger than the centrifugal, and the Moon's revolution would again be unstable. The Moon would follow a contracting spiral and hence would draw a little nearer to the Earth in each revolution and gradually contract its orbit. But as contraction went on the tendency to contract would grow gradually less until the Moon had reached its present place, when the forces would be exactly balanced, contraction would cease and stability would be attained. The relations in this case would be such that, *being stable in its present orbit, the Moon could not have stable revolution in any other orbit either smaller or larger, but if temporarily perturbed would be driven back to its present orbit.*

Once adjusted in a stable orbit of revolution around the Earth and in an epicycle in which the opposing effective heliocentric centripetal and centrifugal forces are at a balance, the Moon can not depart from that orbit either way, either to expand it or to contract it, without immediately bringing into action forces which tend to drive the Moon back to that orbit. So long as the Earth's mass remains the same, and so long as the Earth's distance from the Sun remains the same, so that its velocity and curvature with reference to that body remain unchanged, the Moon's place of stable revolution will be in its present orbit and it can not be stable in any other.

This, in my opinion, is determinate stability, and the relation of forces pointed out above constitutes its mechanism.

If the argument to this point is valid, then it seems plain that the gist of the problem of stability — which is the problem of three bodies — lies in the determination of the relative rates of variation in the values of the effective heliocentric centripetal and centrifugal forces which affect the Moon in the epicycles corresponding to orbits at different distances from the Earth, the Moon's revolution in its present orbit being regarded as stable.

Of the three possible solutions given above, one (the second) in which the centripetal forces were supposed to vary at a higher rate than the centrifugal, was found to correspond to an unstable equilibrium which is the mechanism of utter instability, and hence was rejected unconditionally. This leaves two other possible solutions. In one of these (the first of the three) the opposing forces were supposed to vary at exactly the same rate through all different distances of the Moon's orbit from the Earth. This relation corresponds to an indifferent equilibrium, which is the quality of indeterminate stability.

The third solution was one in which the centrifugal forces were supposed to vary at a higher rate than the centripetal. The quality of this relation corresponds to that of a stable equilibrium and this is the quality of determinate stability. So far as I am able to see, there is no other possible relation of the forces, and hence no other solution of the problem of stability. We are finally and irrevocably driven to a choice between determinate and indeterminate stability. Newton treated the problem in a way which led to indeterminate stability, and that has stood as the best possible solution down to the present day. But I believe that when the problem of stability is treated mathematically, with thoroughness and by right methods, determinate stability will prove to be the true stability. In the meantime, since I am unable to offer a mathematical discussion of the subject, I shall proceed upon the assumption that the truth of determinate stability has been demonstrated, and on that basis I shall endeavor to show its potency in the explanation of some of the well known facts of astronomy.

§ 2. Current theory and the Moon's present stability. According to the method of the current theory the analysis of the Moon's stability is made by a study of the forces which affect the Moon as it revolves in the geocentric circle, and a diagram of this circle is invariably used as the figure by the aid of which the demonstration is made. The analysis of the Moon's motion around the Earth at rest, urged by the Earth's attraction alone and without the disturbing attraction of any other body, is set forth as a demonstration of the mechanism of the Moon's stability as it would be if its motion were not disturbed by the Sun's unequal attraction on the Earth and the Moon. But the Sun's attraction must be reckoned with as "a disturbing force." It is pointed out in the Lunar theory, that it is not the Sun's *whole* attraction on the Moon which disturbs it, for that force is exerted alike and with equal power upon both the Earth and the Moon, but only the fractional part which is exerted *unequally* upon the two bodies. This part is the *difference of attraction* arising from differences of linear and angular distance respectively — differences in the distance and direction of the Earth and Moon from the Sun.

The general truth of these statements is readily granted, and also that of the principal conclusions drawn from them. These conclusions may be briefly stated as follows: 1. The solution of the problem of two bodies demonstrates that the Moon would revolve around the Earth forever in one and the same orbit, provided it were not disturbed by the attraction of any other body and met no resistance to motion in space; and it is immaterial whether the Moon's orbit be circular or elliptical, or large or small. 2. The analysis of the Sun's perturbations of the Moon's motion explains all or nearly all of the Moon's inequalities and shows that the perturbations in their total effect slightly modify the place of the orbit of stability, causing the Moon to revolve at a somewhat greater distance from the Earth, with less angular velocity and in a longer period than it otherwise would.

Neither one of these analyses nor both of them together shows stability to be determinate, nor do they in a positive way show it to be indeterminate. Indeed, they do not constitute a demonstration of the mechanism of stability at all, and seem to have no positive bearing upon its quality, because they do not reach the real fundamentals of the problem. *The current analysis of the Moon's motion does not touch the problem of stability at all, except by an assumption.* This assumption is a matter of much importance in the present discussion. For in the current theory, it stands in the place that ought to be occupied by a demonstration of the mechanism of stability.

Let us examine this assumption a little more closely. It is embodied in, or rather concealed behind, the statement that the Sun's whole attraction, viz: that part which is exerted equally upon both the Earth and the Moon, sustains them alike in their common motion around the Sun and hence can in no way disturb their mutual relations or motions, and that the disturbing force arises solely from the difference of attraction due to differences of linear and angular distance. While this statement is true, in part, it seems to leave the impression that there could be nothing in the relation of the main or original forces that could possibly affect the Moon's stability, except the difference of attraction, *even if the Moon revolved at a different distance from the Earth.*

This way of looking at the problem has been reached by a study of the *geocentric* aspect of the Moon's motion as perturbed by the Sun, to the neglect of its *heliocentric* aspect,—its motion in the epicycle. The conclusion of the current theory could not be entertained at all, except on the assumption that no force disturbing the Moon's relation to the Earth could possibly arise from the irregularities of the Moon's motion with respect to the Sun, even if it revolved at a different distance from the Earth. That is to say, it is assumed that the effective heliocentric attraction which acts upon the Moon as it follows the epicycle, and which continually varies between extreme values of 8 and 22, would always be exactly balanced by the effective heliocentric centrifugal force, which, to fulfill this requirement, would have to vary in exactly the same way. In other words, it is in effect assumed that the attraction of the Sun and Earth would at all times and in all orbits be exactly adequate to balance the effective *heliocentric centrifugal force* to which the Moon would be subjected in consequence of the *heliocentric motions* produced by those attractions. Or, to state it in still another way, *it is assumed that the heliocentric motion which the Earth's perturbation imparts to the Moon would always* *produce exactly that value of heliocentric centrifugal force which would be precisely balanced by the heliocentric component of the Earth's attraction.*

It is immaterial whether this assumption be explicitly stated in treatises on the current theory, or included only by unconscious implication; it is indirectly recognized as a legitimate inference just the same. In one respect the problem stands in a peculiar relation. So long as attention is directed solely to the conditions of the Moon's stability in its *present* orbit, statements like those made above seem almost superfluous. For it goes almost without saying and so without formal analysis or proof, that *the heliocentric centripetal and centrifugal forces which act on the Moon in its present epicycle are exactly balanced against each other in each completed revolution.* It seems certain that mathematical analysis would prove this to be true. We may assume that it would, and it may be granted that the assumption referred to is true and acceptable *as applied to the Moon's present revolution*, although a specific analysis of the forces in the epicycle, showing their balanced state, has not been made. Just here, however, the admissible application of this assumption ends. There is no warrant for applying it to the Moon's revolution in any other orbit than the one in which it now revolves. Granting for the sake of discussion that the balanced state of the forces affecting the Moon in its present orbit has been demonstrated, the extension of this state to the Moon's revolution in orbits at other distances and in different epicycles can not by any means be regarded as a part of the same demonstration. So far as this extension has been made, it has been by assumption only.

In discussing the Moon's stability in its present orbit the fact has constantly to be borne in mind that we are dealing with a case of revolution *which we know by observation to be stable*, and that any theory which explains the Moon's present stability, and which is at the same time capable of a happy expression in mathematical terms, may appear very plausible and still be wholly empirical and not the true explanation at all. The mechanism and quality of stability can not be finally determined by an analysis of the Moon's stability in its present orbit alone; it requires in addition a rigorous mathematical analysis of the forces affecting stability in a hypothetical case; i. e., in an orbit at a different distance, say at 60,000 miles from the Earth, where the form of the epicycle and all the effective heliocentric forces and the Earth's attraction and perturbing power would have different values from those which they now have. But this problem has not been solved. The fact is that the true mechanism of stability has never been demonstrated, and for that reason its quality has remained unknown. The analysis by which the problem may be solved must be made by a direct study of the forces of the epicycle and not of those of the geocentric circle, as is done in the current theory.

The conclusion seems plain therefore that the Moon revolves in its present orbit, not by chance or accident, not by the action of tidal forces, not merely because it was put there when it was first made out of a nebular ring and has been left without serious disturbance ever since; but because it moves now and always under the ceaseless urgency of forces whose rates of variation under different conditions are directed by laws in such ways as to make the place of its present orbit determinate. The Moon's stable revolution is a most delicate balance between opposing forces which can neither cease nor widely vary, so long as present conditions endure. The forces which maintain the Moon in stable revolution are the attractions of the Sun and Earth and the inertia which resides in the Moon itself and in virtue of which it resists the forces of attraction. The place of the Moon's particular orbit of stable revolution is determined by the mass and orbital revolution of the Earth, the latter being dependent upon the mass and distance of the Sun. No other condition has more than a relatively slight modifying influence. It follows also that, except as these conditions were different in the past, the Moon can never have had stable revolution nearer to nor farther from the Earth than now; nor can it in the future so long as present conditions endure. While it seems certain that the present tides of the Earth *tend* to accelerate the Moon's motion and drive it out, as Professor Darwin supposes, their power is so feeble as compared with the greater forces which make for determinate stability that they are rendered ineffectual as controlling factors and produce only minute modifications of the stable adjustment resulting from the action of the main forces, and in no appreciable degree affect stability.

The Moon's present stability is such that it may be said to have a certain quality of persistence by which it is enabled to offer more or less resistance to any disturbing force. It has a certain amount of flexibility by which, if disturbed within moderate limits, it is enabled to recover itself and return gradually to its former adjustment. When the heliocentric forces are at an exact balance, as we may presume they now are, the adjustment is extremely delicate and the force which maintains stability against any force that might disturb the adjustment is extremely feeble. But the sustaining force grows stronger, within certain limits, the farther the Moon departs from its present place. However, the forces causing expansion from a smaller orbit to the Moon's present place are more powerful than those causing contraction to it from a larger orbit. We shall see the evidence of this fact in a later part of this discussion.

In the Moon's present orbit, which is a modified epellipse, the effective heliocentric forces are so closely balanced that in its geocentric aspect the Moon is made to revolve around the Earth almost as perfectly as if the Earth were at rest. It is only now and then that the Moon departs widely enough from the theoretical epellipse, either by coming in or going out too far, to bring the restraining forces into play. At other times the forces are so perfectly balanced that the Moon revolves substantially as if the Earth were at rest, except for the slight irregularities produced by differences of solar attraction and other minute perturbations. The restraining forces act by occasional impulses so slight that their effects, even though cumulative, do not become certainly perceptible until at least ten or fifteen years have elapsed. Then it is found that the Moon does not agree exactly with its predicted place, and no reason accordant with the laws of gravitation as expounded in current theories can be assigned for the discrepance. These unexplained irregularities of the Moon's motion are probably due mainly to the action of exceedingly small unbalanced factors of one or the other of the heliocentric forces. On the whole, they tend by their action to keep the Moon in the orbit of perfect stability—that is, in conformity with the theoretical epicycle in which alone the forces are perfectly balanced.

§ 3. **Herschel and Proctor.** So far as I am aware, the only writer who has made even a remote approach to the idea of determinate stability presented here is Richard A. Proctor. He did not reach the conclusions stated in these pages, but he went a long way toward them; so far indeed, that it may almost be said that he narrowly missed them. It seems to me surprising that, after making such statements as I shall presently quote, he did not go a step or two farther. It is hard for me to believe that he would have failed to perceive the mechanism of determinate stability if he had followed the trend of his thoughts to their obvious goal.

Before quoting from Proctor, however, I will quote one of the standard authorities on the current theory, so that a comparison may be made between the two and with the statements made in preceding pages. For this purpose I quote from Sir John Herschel's « Outlines » as follows:

« Were there no other bodies in the universe but the sun and one planet, the latter would describe an exact ellipse about the former (or both round their common center of gravity), and continue to perform its revolutions in one and the same orbit forever; but the moment we add to our combination a third body, the attraction of this will draw both the former bodies out of their mutual orbits, and, by acting on them unequally, will disturb their relation to each other, and put an end to the rigorous and mathematical exactness of their elliptic motions, not only about a fixed point in space, but about one another. From this way of propounding the subject, we see that it is not the whole attraction of the newly-introduced body which produces perturbation, but *the difference* of its attractions on the two originally present.

« Compared to the sun, all the planets are of extreme minuteness; the mass of Jupiter, the greatest of them all, being not more than about one 1100th part that of the sun. Their attractions on each other, therefore, are all very feeble, compared with the presiding central power, and the effects of their disturbing forces are proportionally minute. In the case of the secondaries, the chief agent by which their motions are deranged is the sun itself, whose mass is indeed great, but whose disturbing influence is immensely diminished by their near proximity to their primaries, compared to their distances from the sun, which renders the *difference* of attractions on both extremely small, compared to the whole amount. In this case the greatest part of the sun's attraction, viz: that which is common to both, is exerted to retain both primary and secondary in their common orbit about itself, and prevent their parting company. Only the small overplus of force on one as compared with the other acts as a disturbing power. The mean value of this overplus, in the case of the moon disturbed by the sun, is calculated by Newton to amount to no higher a fraction than $\frac{1}{638000}$ of gravity at the earth's surface, or $\frac{1}{179}$ of the principal force which retains the moon in its orbit.» (pp. 412–413.)

In another paragraph Herschel describes the method of treating the problem of three bodies, which, of course, includes the problem of the Moon's motion and stability.

«In the treatment of the problem of three bodies, it is convenient, and tends to clearness of apprehension, to regard one of them as fixed, and refer the motions of the others to it as to a relative center. In the case of two planets disturbing each other's motions, the sun is naturally chosen as this fixed center; but in that of satellites disturbing each other, or disturbed by the sun, the center of their primary is taken as their point of reference, and the sun itself is regarded in the light of a very distant and massive satellite revolving about the primary in a *relative* orbit, equal and similar to that which the primary describes *absolutely* round the sun. Thus the generality of our language is preserved, and when, referring to any particular central body, we speak of an exterior and an interior planet, we include the cases in which the former is the sun and the latter a satellite; as, for example, in the Lunar theory.» (p. 415.)

It is plain that analysis by this method deals primarily with the forces which affect the Moon as it moves in the geocentric circle or ellipse, and not specifically with those which affect it as it moves in the epicycle. For the purposes of the current theory, this is, of course, a valid method. But it can not be said to solve what I take to be the real problem of stability, that is, its *quality*, because it omits the most important element of the problem—the relation of the effective heliocentric centripetal and centrifugal forces which act on the Moon as it follows the epicycle. In Herschel's description there is an implied assumption that a different method of analysis would make no difference in the result, because there is nothing that could disturb the Moon's motion around the Earth, except the difference of attraction, which is not lost or changed in any way by supposing the Earth to be at rest and transferring its annual revolution to the Sun in a relative orbit. But, obviously, this method completely eliminates the heliocentric centrifugal force which acts on the Moon in the epicycle; and this force is the controlling veriable factor of determinate stability.

Let us turn now to Mr. Proctor, and quote from his work on «The Moon.» In the first two paragraphs he seems to keep in line with the current view. After pointing out the fact that the Sun's influence on the Moon is more than twice as great as the Earth's, he says:

> «It may be asked, then, how it is that the moon does not leave the earth's company to obey the sun's superior influence? In particular it might seem that when the moon is between the earth and the sun (or as placed at the time of a total solar eclipse), our satellite being then drawn more than twice as forcibly

from the earth towards the sun as she is drawn towards the earth from the sun, ought incontinently to pass away sunwards and leave the earth moonless.

« The answer to this enigma is, simply, that the sun attracts the earth as well as the moon, and with almost the same degree of force, his pull on the earth sometimes exceeding, at others slightly falling short of, his pull on the moon, according as the distance of the moon or earth from him is greater at the moment. Thus the earth, in order to prevent the escape of her satellite, has not to overcome the sun's pull upon the moon, but only the excess of that pull over the pull he exerts upon the earth herself. This excess, as will presently appear, is always far less than the earth's own influence on the moon.» (p. 56.)

In the next paragraph he begins to see the real nature of the Moon's motion:

« But it may be noticed, that in considering the moon's course round the sun we recognize the inferiority of the earth's influence in a very evident manner. The moon seems well under the earth's control when we consider only the nature of the lunar orbit round the earth; but if for a moment we forget that the moon is circling round the earth, and consider only the fact that the moon travels as a planet round the sun, — with perturbations produced by the attractions of another planet, — our own earth, — we can readily test the extent of these perturbations.» (pp. 56–57.)

Then after discussing a figure designed to show the Moon's ins and outs with reference to the Earth's orbit as the two revolve around the Sun, Mr. Proctor goes on to say:

"Thus it will be readily understood that the curvature of the moon's path remains throughout concave towards S [the Sun], even when the convexity of the orbital path round the earth is turned directly towards the sun. In other words, as the moon travels in her orbit round the sun her course is continually being deflected inwards from the tangent line, or always towards the sun. It is to be noticed, however, that the earth's perturbing influence is an important element in determining the moon's real orbit. For when the earth and sun are on the same side of the moon, or at the time of full moon, the pull on the moon is the sum of the pulls of the earth and sun, or exceeds the sun's pull alone in the ratio 22 to 15; and on the other hand, when the earth and sun are on opposite sides of the moon, or at the time of new moon, the pull on the moon is the difference of the pulls of the sun and earth, or is less than the sun's pull alone in the proportion of 8 to 15. Thus at the time of full moon the moon is acted on by a force which exceeds that acting on her at the time of new moon in the ratio of 22 to 8 or 11 to 4. And though at the time of full moon the moon's actual velocity (that is, her velocity in her orbit round the sun) is at a maximum, being then the sum of her mean orbital velocity round the sun and of her velocity round the earth; yet this by no means counterbalances the effects of the greatly increased pull on the moon: so that the curvature of her path when she is 'full' greatly exceeds the curvature at the time of new moon." (pp. 58–59.)

And in a foot note he points out that

"The earth's velocity in her orbit being about 65,000 miles per hour, the extreme variation of the moon's motion in her orbit round the sun lies between the values in about the ratio of 110 to 103. But the attractive force on the moon varies in the ratio of 110 to 40, as above shown.» (p. 59.)

Mr. Proctor recognized clearly the fact that the Moon is in a physical sense a planet and that the Earth perturbs its motion around the Sun; and he seems to see that it is the Moon's excess of heliocentric centrifugal force (due to maximum velocity and curvature in opposition) that «counterbalances the effects of the greatly increased pull on the moon.» It seems incredible that he did not see that this «counterbalancing» of the heliocentric forces must be perfect or stability can not exist. If he had asked himself seriously whether these forces would exactly counterbalance if the Moon revolved in a smaller or a larger orbit than at present, it seems almost certain that he would have reached the same conclusions regarding the mechanism and quality of stability that I have endeavored to set forth in this writing. But Mr. Proctor did not see the point. For, without dwelling further on the interesting relations he had just pointed out, he goes on to say:

«In considering the moon's motion around the earth, however, we may leave out of consideration the common influence of the sun upon both these orbs, and need consider only the difference of his influence upon the earth and moon, since this difference can alone affect the moon's motion around the earth.» (p. 59.)

DIVISION II.

THE LAWS OF SATELLITE SYSTEMS.

CHAPTER VI.

SATELLITE STABILITY AS MODIFIED BY PLANETARY MASS.

THE idea of determinate stability as presented in the preceding pages constitutes the foundation of my theory. In attempting to make the matter clear by the simple method employed it seemed necessary to discuss it in considerable detail. But having presented the fundamental principles, their extension and application in what follows may be made with less elementary elaboration.

In all that has been said so far the Earth's mass and orbital revolution have been assumed to be the same as now and unchangeable. It remains to enquire what effect would be produced upon the Moon's stability by changes in these conditions. For instance, what would the effect be if the mass of the Earth were four times as great as it is, all other conditions remaining the same? In that case the Moon in its present orbit at 240,000 miles would have to revolve around the Earth at twice its present velocity or at the rate of about one mile per second. The geocentric circle would be the same as before and so would the revolution of the Earth around the Sun. But the corresponding epicycle would be very different; its undulations would be shorter and its curvature greater than before. The Moon's velocity with respect to the Sun at opposition would be 19½ miles per second, instead of 19 miles as now, and the month would be only half as long as it is now.

It is manifest that under these conditions the Moon in opposition in the epicycle would develop a greater value of excessive heliocentric centrifugal force than before, from increase of both velocity and curvature. The power of the Earth's perturbation would, of course, be quadrupled at the same time. Here again, the question arises as to how these forces would vary in the change — whether they would vary at the same rate or at different rates. But, as before, if they varied at the same rate and kept equality, then stability would be attained under the new conditions without change of the Moon's distance from the Earth. That would be indeterminate stability. This, however, would not be the case. The centrifugal factor, as before, would vary at a slightly higher rate than the attraction and this would cause the Moon to *expand* its orbit, but with a progressively diminishing tendency to expansion, until the opposing forces were again balanced and stability attained.

On the contrary, if the Earth's mass were one-quarter of what it is now the Moon in its present orbit would revolve around the Earth only half as fast as now, and this would require another adjustment for stability, for the month would be twice as long. The corresponding epicycle would have longer undulations and less curvature in opposition than the present one; the Moon would therefore develop less excessive centrifugal force in opposition and would have to *contract* its orbit and increase its velocity around the Earth until the excessive heliocentric centrifugal force developed such a value as to exactly balance the Earth's perturbation. This would require an increase of curvature and velocity in opposition — a change which could be accomplished only by a certain amount of contraction of the Moon's orbit around the Earth. Under the law of determinate stability this contraction would gradually take place at a diminishing rate, and the adjustment to stability under the new conditions would finally be attained.

In these adjustments due to different values of mass we come upon a new and important factor. In order to derive a given value of centrifugal force, the velocity must be relatively great if the curvature be small; but to derive the same value when the curvature is great the velocity must be relatively small. In the case of the Earth with four times its present mass, the expansion of the geocentric circle would lengthen the undulations of the epicycle so that the centrifugal force required to balance the attraction in the adjustment for stable revolution would be derived more than before from velocity, because the curvature would be diminished. On this account the Moon, in changing its orbit to find a new place of stability, would not simply expand until its velocity at opposition in the epicycle became reduced to 19 miles per second as now, but would stop somewhat short of that and become stable in an orbit in which its velocity would be a small fraction *more* than 19 miles per second.

So also in the contrary case, the mass of the Earth being one-quarter of what it is now, the Moon in changing its orbit to find a new place of stability would contract its orbit, not until its velocity at opposition in the epicycle became increased to 19 miles per second as now, but would stop somewhat short of that and become stable in an orbit in which its velocity would be a small fraction *less* than 19 miles per second. And this it would do, because the smaller geocentric circle would shorten the undulations of the epicycle and so increase the element of curvature that a correspondingly smaller part of the centrifugal force would be derived from velocity.

If these changes took place with absolute simplicity by variations of velocity alone, without the modifications due to changes of curvature, then the Moon's orbit of stable revolution would always be that geocentric circle in which its velocity around the Earth would be half a mile per second as now, no matter what the mass of the Earth might be. If the mass of the Earth were greater than now the Moon would find its half-mile orbit at a greater distance than now, and if the Earth's mass were less the half-mile orbit would be nearer than now. But the effects due to curvature modify this result a little, so that the Moon would have slightly less than this velocity for stability with smaller mass of the Earth and slightly more for greater mass.

In these considerations we see how the mass of the planet affects the adjustment of the forces which make stability determinate, and we shall see a little later what an important modification it makes in the adjustment of satellites to their primaries, and in the expression of the general law of satellite systems. The mass of a planet determines the *scale* of its satellite system.

CHAPTER VII.

SATELLITE STABILITY AS DETERMINED BY THE ORBITAL REVOLUTION OF THE PLANETS.

WE COME now to the consideration of the conditions which affect the stability of the satellites of other planets at different distances from the Sun. We have seen how the stability of the Moon's revolution in its present orbit is controlled by the forces of the epicycle, and how they give its stability the quality of a stable equilibrium. If, now, we turn to the satellite systems of the other planets it is manifest that their stability must be governed by the same general laws. For the forces which affect their motions are not different in kind or quality or number, but only in value or degree. The degrees by which these forces differ from those which affect the Moon's motion are capable of exact determination, and hence the orbit of stability for the satellites of those systems must be equally determinate. But as will be pointed out later, this law applies directly only to the *inner* satellites of the several systems; other conditions must be taken into account in considering the stability of satellites which revolve in orbits outside of the inner ones.

In approaching this part of the subject it is to be remembered first, that the Sun's attraction decreases with the square of the distance from its center, and that in consequence of this the several planets, in the order of their distances from the Sun, are attracted with less and less power. It is to be noted also that in a series of concentric circles the curvature grows less with increasing radius, and that in accordance with the law of velocities in circular orbits, the velocities with which the planets revolve around the Sun grow less with increasing distance. Hence, the attraction and centrifugal force affecting the planets are both decreased with increasing distance from the Sun. Supposing the planets to revolve in circles around the Sun and considering any one of them alone with the Sun as making an isolated pair, the conditions of revolution are those of the simple case of two bodies; the velocity of each around the Sun is constant and invariable, and the centrifugal force due to that motion is in each case of such value as to precisely balance the Sun's attraction at every instant. Thus, while the Earth at a distance of 93,000,000 miles revolves with a velocity of 18½ miles per second, Mars at 141,000,000 miles revolves with a velocity of 15 miles per second, and Jupiter at 483,000,000 miles revolves with a velocity of 8 miles per second. The curvature decreases with increasing distance from the Sun at such a rate that while the orbit of the Earth bends from the tangent 0.116 of an inch in one second, that of Mars bends only about 0.038, and that of Jupiter about 0.0046. In a certain sense we may say, therefore, that the *orbital intensity* of Mars is less than that of the Earth, and the orbital intensity of Jupiter less than that of Mars.

These conditions, which affect the revolution of the planets, are all capable of accurate mathematical determination and expression. As expressed in the orbital intensity of the primary, they constitute what may for present convenience be called the *primary base* or *foundation* to which the inner satellites must adjust their revolutions. The primary base of any inner satellite may be said therefore to have more or less intensity according as the planet revolves near to or far from the Sun. Mercury has the greatest intensity and Neptune the least. The satellite systems, on the other hand, have their intensities arranged in the inverse or complementary order, the least intensity being near the Sun and the greatest far away. Venus and Mercury have no satellites, so the Earth's system is the least intense and Neptune's the most intense. This is not dependent upon the mass of the planet, but upon its intensity. *Scale* and *intensity* are the two elements by which the structure of satellite systems is determined. Given the mass of a planet and its distance from the Sun, the intensity and scale of its satellite system are at once fixed. This means that the distance and velocity of revolution of the inner satellites are determinate and capable of exact calculation. An excess of half a mile per second in the velocity of the Earth, although proportionally a smaller excess, would nevertheless produce a greater value of excessive centrifugal force than the same excess in Mars, because the intensity of Mars is less than that of the Earth.

Thus far we have discussed mainly the forces acting upon the planets. But whatever effects would be produced on them by a given excess of velocity in their orbits would be produced just the same on the inner satellites of those planets at opposition in their epicycles, supposing the planets themselves to revolve in circles. If stability is now attained by the Moon with half a mile per second excess of velocity the same excess, producing a much lower value of heliocentric centrifugal force, could not give stability to Phobos, the inner satellite of Mars

In order to see more clearly the effects of planetary intensity upon satellite stability it is desirable to eliminate the complexities which arise from differences of planetary mass. If the mass of Mars were the same as that of the Earth, then Phobos would revolve half a mile per second at the same distance from Mars that the Moon does now from the Earth. But the corresponding geometrical epicycle of Phobos would have shorter undulations and hence somewhat sharper curves, because the velocity of Mars is only 15 miles per second, while that of the Earth is 18½ miles. Under the conditions assumed, the power of Mars to perturb Phobos would be the same as the Earth's present power to perturb the Moon. But the same perturbation by Mars would generate *less* excess of heliocentric centrifugal force in Phobos in opposition than is generated by the Moon, because the primary base of Phobos (orbital motion of Mars) is less intense. A given excess of velocity in opposition gives Phobos not only actually, but *relatively*, slightly less heliocentric centrifugal force than the same excess gives the Moon. Though less would be required to make Phobos stable, the amount generated would fall a little short of that required. Hence, the effective heliocentric attraction would slightly overbalance the heliocentric centrifugal force. From this it follows that Phobos would have to contract its orbit around Mars to one of smaller distance, swifter revolution and shorter period in order to find stability. The forces in the epicycle corresponding to the half-mile-per-second orbit would not balance, and the path of Phobos would therefore fall inside of the curve of the theoretical epicycle (*Oc* in Fig. 6), and Phobos would have to contract its orbit. The change brought about by the resulting contraction would reduce the maladjustment and bring the forces nearer and nearer to the balanced state in which alone stability can exist. These factors determine the stable orbit of Phobos to be in a much smaller orbit than that in which it would move half a mile per second around its primary. In fact, if the mass of Mars were the same as that of the Earth, Phobos would have to contract its orbit until it revolved around Mars with a velocity which would probably be somewhere near four miles per second — a value to be determined by mathematical analysis.

As a matter of fact, however, the mass of Mars is only about one-ninth of that of the Earth, so that the orbit in which Phobos would revolve with a velocity of half a mile per second is very much nearer the planet than the Moon's orbit is to the Earth, and its periodic time would be very much shorter. This would greatly shorten the undulations of the corresponding epicycle, as pointed out above, and cause a relatively small component of the heliocentric centrifugal force to be derived from velocity and a larger component from curvature. As a result, Phobos revolves around Mars in about seven and one-half hours and with a velocity of about one and one-third miles per second.

I have not made a mathematical calculation of the place of the determinate orbit of stability for Mars. It seems not improbable, however, that such a calculation will show that that orbit is nearer Mars than the present orbit of Phobos. If this were the case, then we might expect to find that there is another satellite between Phobos and the planet, and its period would be still shorter than the present seven-and-one-half hour period of Phobos.

If the mass of Mars were the same as that of the Earth the velocity of Phobos in its present orbit around Mars would be about three times what it is or about four miles per second. This would be approximately the place of the inner orbit of stability. But the mass of Mars being only one-ninth of that of the Earth, it follows, theoretically, that the inner limit of stable revolution is nearer Mars than the present orbit of Phobos—somewhere between Phobos and the planet. The reasons for this conclusion will be discussed more fully in the next chapter.

If the mass of Mars could be gradually increased until it became the same as that of the Earth, the inner orbit of stability would have to expand correspondingly, and as expansion progressed curvature would become a relatively less important factor and velocity a relatively more important factor in the generation of the heliocentric centrifugal force which would be required to balance the planet's perturbation.

The stability of the inner satellite of Jupiter may be analyzed in the same way. But since Jupiter is more than five times as far as the Earth from the Sun and its mass nearly 316 times greater than that of the Earth, the elements of stability for its inner satellite have widely different values from those of the Moon and Phobos. The factors dependent on the Sun's mass and distance are all of considerably less value, while those dependent on the planet's mass are much greater. Jupiter's orbital velocity is eight miles per second and the velocity of its inner satellite around it is more than sixteen miles per second. In this case the relatively great mass of Jupiter gives its inner satellite such a high velocity in a relatively large orbit that the component of heliocentric centrifugal force due to curvature, as the satellite moves at opposition in the epicycle, is greatly reduced and a correspondingly larger component is derived from velocity. The modification of the result due to planetary mass is therefore of the opposite order to that which applies in the case of Mars.

Thus, we may conclude that the primary base or foundation of stability for the inner satellite of each planet has all its factors precisely determined by the elements of the planet's revolution around the Sun. By themselves, however, these factors do not make a determinate orbit of stability. The mass of the planet must also be taken into account. This determines the velocity of the satellite in its revolution around the planet in any circle that may be chosen and, with the other elements depending upon the revolution of the planet around the Sun, fixes the epicycle. It also determines the planetary component of the effective heliocentric attraction. Given the elements of a planet's revolution around the Sun, the only other fact needed to establish the place of the orbit of stable revolution for its inner satellite is the mass of the planet. This makes the place of the orbit of stability determinate by giving it the character of a stable equilibrium. That the facts of observation appear to correspond closely with the conclusions reached by this method of analysis will be pointed out later.

We have now before us the principles for a determination of the place of the orbit of stable revolution for the inner satellite of any planet, provided the mass of the planet and its distance from the Sun be given.

CHAPTER VIII.

THE LAW OF THE ADJUSTMENT OF SATELLITES TO THEIR PRIMARIES.

IN THE foregoing pages an attempt has been made to show in a qualitative way, but without mathematical accuracy or demonstration, how the Moon's stability in its present orbit is made determinate. And, in the same way, how the revolution of the inner satellites of Mars and Jupiter are given the same quality of stability under circumstances in which the numerical values of all the conditions and forces are different from those that affect the Moon.

We have now reached a point from which it is apparent, supposing the general plan of the analysis given above to be correct, that the orbit of stability for the inner satellite is a determinate thing and may be calculated accurately for any planet whose mass and distance from the Sun are given. For all the factors upon which stability depends are capable of exact mathematical measurement and expression and all may be readily found if the two facts or elements mentioned above be given. By this method of analysis it is possible to show with precision the distance and velocity of the inner satellite of any planet, whether that satellite has ever been seen or not. It is only necessary that one satellite — any one belonging to that planet — shall have been observed so as to afford an accurate determination of the planet's mass. Even this would not be necessary if the mass of the planet could be accurately determined in some other way.

From the analysis given above it would be expected that the relation of the inner satellites to their primaries would show an orderly arrangement or regular progression corresponding with the order of the planetary distances from the Sun. The inner satellites would be expected to have their orbits of stability nearer and nearer to their primaries with increasing distance from the Sun, and they ought to revolve around their primaries with higher and higher velocities. But the regularity of the progression would depend largely upon the masses of the planets. It would hold accurately if their masses were all the same, and not too small nor too great. With their masses widely different, however, the progression would be made irregular in proportion to the magnitude of the differences of mass. A planet smaller than the mean would hold its inner satellite nearer than the mean and the satellite would revolve with less velocity and greater curvature than if the planet's mass were of mean value, while a planet larger than the mean would hold its inner satellite out farther than the mean and it would revolve faster and with less curvature. Beyond this, however, one can hardly go conveniently by the inexact method here employed. If we take the observed velocities per second of the following planets — the Earth 18½ miles, Mars 15, Jupiter 8, and Saturn 6, and note the relations of their inner satellites we find that their velocities around their primaries increase in the same order, or from the Earth to Saturn, and in a way that appears to be roughly complementary; that is, as the velocities of the planets grow less, those of the satellites grow greater, so as to keep the sum of the two nearly the same for all the planets named. The complementary relation just pointed out suggests the possibility of some kind of a constant of value, or possibly more than one. Or, if there is not exactly a constant, there must be a uniform variable which varies only slightly from a constant and in accordance with a definite law which may be accurately expressed by a mathematical formula. But whatever constants or uniform variables exist are much obscured by the effects of the different masses of the planets, and can be made to appear clearly as such only on the supposition of equal masses, and a calculation of the places of the orbits of their inner satellites on that supposition. To this point the method of reasoning employed in this study has been mainly deductive. Let us now note briefly some of the principal facts of observation.

By reference to the following table it will be seen that, by allowing for differences of planetary mass, the velocities of the inner satellites with reference to the Sun, when the satellites are at opposition in the epicycle, tend to a constant of value. This velocity is compounded of the velocity of the planet around the Sun with that of the satellite around the planet. For the Moon, it is $18\frac{1}{2}+.6=19.1$ miles per second. For Phobos, it is $15+1\frac{1}{3}=16\frac{1}{3}$ miles per second. But the mass of Mars is only about $\frac{1}{9}$th that of the Earth, so that a con-

RELATIONS OF PLANETS AND INNER SATELLITES.

Planet.	*Mean distance, millions of miles.*	*Mass, Earth = 1.*	*Velocity, miles per second.*	*Mean dist. of inner satellite from planet, thousands of miles.*	*Mean planetocentric velocity, inner satellite, miles per second.*	*Heliocentric velocity, inner satellite at opposition, miles per second.*	*Same when primary's mass=mass of Earth.*
Mercury.......	36.0	$\frac{1}{8}$?	29.				
Venus.........	67.2	0.78	22.				
Earth	92.9	1.00	18.5	240,000	0.6	19.1	19.1
Mars..........	141.5	$\frac{1}{9.34}$	15.	Phobos 5,800	Phobos 1.33	Phobos 16.3	inner moon 19.±
Ceres	257.1	$\frac{1}{70000}$?	11.				
Jupiter.........	483.3	316.0	8.	112,000	16.33	24.3	19.±
Saturn	886.0	94.9	6.	inner ring 44,000	inner ring 15.35	inner ring 21.3	inner ring 19.±
Uranus........	1781.9	14.7	4.2				
Neptune.......	2791.6	17.1	3.4				

The numbers in the first four columns are taken from Young's « General Astronomy.» In the other three columns the numbers have been obtained by rough calculation.

siderable amount would have to be added to the velocity of Phobos if the mass of Mars were the same as that of the Earth. For the inner or Fifth satellite of Jupiter it is 8+16.3=24.3 miles per second. This seems much too large. But when it is remembered that the mass of Jupiter is 316 times that of the Earth it is evident that a considerable amount would have to be subtracted from the velocity of the satellite if the mass of Jupiter were supposed to be the same as that of the Earth.

The case for Saturn, so far as defined by present knowledge, is perhaps a little less satisfactory, because the ill-defined, veil-like inner ring next to the planet leaves some uncertainty, on observational grounds, as to the precise distance of the inner satellite or its equivalent in the ring. If we take the faintly visible inner edge of this ring as the equivalent of an inner satellite in the same sense as the others we have just mentioned its velocity is $6+15\frac{1}{3}=21\frac{1}{3}$ miles per second. The mass of Saturn is about 95 times that of the Earth or less than one-third that of Jupiter. The slower velocity of the inner edge of the inner ring around Saturn, as compared with that of Jupiter's inner satellite around its primary, is partly due to the smaller mass of Saturn, but may also be due in part to the fact that the observed faint inner ring is slightly outside of the inner or determinate orbit of stability.

The last column of the table shows the velocity of the inner satellite with reference to the Sun when the satellite is at the point of opposition in the epicycle, assuming that the mass of each planet is the same as that of the Earth. The velocity seems to be 19 miles per second, or very close to that, in each case. The inexactness of the method here used, however, does not permit me to say positively that there is a *constant of velocity*, as seems roughly indicated. It may be so, or it may not. There is at least an apparent tendency in that direction. If this value is a constant it depends in part on a constant of mass for the planets, as pointed out above. There seems also to be a tendency to a *constant of value for the heliocentric centrifugal force at opposition* independently of the different masses of the planets.

In studying these relations it would seem better to adopt a different value from that of the Earth for the constant of planetary mass. For the Earth and Mars are too small and Jupiter and Saturn are too great for the best results. A planet having a mass, say, forty or fifty times the mass of the Earth would probably be best. For with a constant of velocity as suggested, and on the basis of the Earth's mass as a constant of mass, the Earth in the orbit of Jupiter could not have a satellite at all, because it could not prevent contraction of the satellite's orbit to the point of collision. It would absorb its satellite into its own mass; and so it would in any other orbit outside of that of Jupiter. On the other hand and for another reason, as we shall see later, Jupiter probably could not have a satellite if it revolved in the present orbit of the Earth, and certainly not if it revolved in the present orbit of Venus.

The ultimate dependence of the value of all the factors, except the masses of the planets, is, of course, upon the mass of the Sun. If the Sun's mass were one-fourth of what it is the planets would revolve in their present orbits with only half their present velocity; if it were four times what it is the planets would revolve twice as fast. A change in the mass of the Sun would produce a corresponding alteration of all the conditions affecting determinate stability throughout the whole Planetary system.

The law of the adjustment of the inner satellites to their primaries—that is, the general law of satellite stability—may be stated thus: (1) Each inner satellite is given a determinate orbit of stability around its primary by the balancing of the heliocentric centripetal and centrifugal forces of the epicycle, and this balance must be attained while the satellite follows the path of an epicycle corresponding geometrically to a particular geocentric circle. (2) The value of the excessive heliocentric centrifugal force developed by the satellite in consequence of a given excess of velocity and curvature in opposition decreases in a definable ratio with increasing distance of the planets from the Sun. (3) Beginning with that planet-satellite system which is nearest to the Sun, the velocities of the inner satellites around their primaries increase with increasing distance from the Sun at such a rate that their heliocentric velocities at opposition would have a constant or very nearly constant value if the masses of the planets were all the same. (4) But where the masses of the planets are of widely different values the resulting inner orbit of stability is modified; if the planet's mass be smaller than the mean the inner satellite will revolve closer to its primary and with less velocity than it otherwise would; but if the planet's mass be greater than the mean the inner satellite will revolve farther from its primary and with greater velocity than it otherwise would.

These are the laws that govern the structure and stability of the satellite systems. The form in which I have stated them is of course only an empirical expression. But when reduced by mathematical analysis to precise terms, they will have the character of exact physical laws.

The law of determinate stability is the fundamental law of the structure of satellite systems, but its expression is modified by the masses of the planets and their distances from the Sun. This law shows why the Moon's orbit of stability is just where it is rather than nearer to or farther from the Earth, and it furnishes a general law for the adjustment of the inner satellites to their primaries. Attempts to find such a law on the basis of the Newtonian analysis have been made, but they have failed, because analysis by the principles of difference of attraction does not find stability to be determinate.

CHAPTER IX.

THE ORTHOGONAL COMPONENT.

§ 1. **Inclination of plane of revolution.** The failure of the current or Newtonian analysis as a true exposition of the mechanism of stability is still more apparent in the analysis of the orthogonal component.

In the simplified conditions of revolution so far assumed the Moon has been supposed to revolve in the direct order and in the plane of the ecliptic. Let us now remove this limitation and consider how the forces of the epicycle affect the Moon's motion and stability when the plane of its orbit around the Earth is inclined to the plane of the ecliptic. With this modification, which is a nearer approach to the Moon's real motion, the path of the Moon in space becomes a very irregular one. Besides weaving in and out of the Earth's orbit as before, the undulations of the epicycle pass alternately above and below, or north and south of, the ecliptic plane. Twice in each revolution the Moon's path crosses the ecliptic, once going down and once coming up, and these points of intersection are called the descending and ascending nodes.

In the previous chapters especial emphasis was put upon the study of the forces of the epicycle rather than upon those of the geocentric circle. We come now to a branch of the subject in which the object of study is the plane of the Moon's motion around the Earth, and its changing relations to the forces of stability. As a matter of fact, the Moon's plane is never exactly the same in any two successive moments. In order to approach this subject by simple steps, it is necessary to eliminate the more seriously complicating conditions. It is relatively much easier to picture in the mind the position and changes of the Moon's geocentric plane regarded as a flat, circular disc, with the Earth in the center, than to picture it in its heliocentric projection along the epicycle. In the latter aspect it is hard to make its changes clear without an elaborate set of diagrams and explanations. I shall therefore use the geocentric circular plane in discussion, but it is to be remembered that in the last analysis of the forces they ought to be studied in epicyclic projection.

For the first case to consider, we may suppose the Moon to revolve in the direct order in an orbit inclined 45°, and we may suppose further that, as the Earth goes around the Sun, the plane of the Moon's orbit keeps constantly parallel to itself throughout the year. As a consequence of this relation, the Moon's plane undergoes a complete annual round of changes in its position relative to the Sun.

In two positions 180° apart on the Earth's orbit, the plane of the geocentric circle would face the Sun slantingly and would intersect the ecliptic on a line at right angles to the Earth's radius vector. In one of these positions the point of opposition would be above or north of the ecliptic and conjunction below, while in the other this relation would be reversed. In these positions the forces would have their strongest tendency to pull the Moon out of its geocentric plane. At two other points midway between these on each side of the Earth's orbit, the plane of the geocentric circle would stand edgewise to the Sun and the orthogonal force would disappear. This force would also vanish four times each month—twice when the Moon is at the nodes and twice when it is at quadratures.

Suppose the plane of the Moon's geocentric orbit to be tilted up from direct revolution in the plane of the ecliptic, which may be called *direct coincidence*, to an angle of 90° inclination, and let us note the action of the forces when the Moon's plane faces the Sun so as to form right angles with the Earth's radius vector. According to the Newtonian method, the Moon being then at the same distance as the Earth from the Sun at every point in its path, there would be no difference of attraction and hence no orthogonal force; nor would there be any perturbation, except that due to angular distance. But according to the method here suggested, the tilting force, instead of disappearing, would then be more powerful and effective than in any other position of less inclination.

In Fig. 8, *SE* is the plane of the ecliptic and *OC* the plane of the Moon's orbit, both planes being seen edgewise. The Earth and Moon in their common motion around the Sun are moving from the reader. *E* is receding at the rate of 18½ miles per second and the Moon at *O* moves in the same direction ½ mile per second faster or 19 miles per second. At *C* the Moon's geocentric motion is toward the reader, and its velocity with reference to *S* is therefore 18 miles per second from the reader. In the position *OC* the Moon's orbit is in direct coincidence and has no inclination to the ecliptic. In the position *oc* it is inclined 45° and in *O′C′* it is inclined 90°.

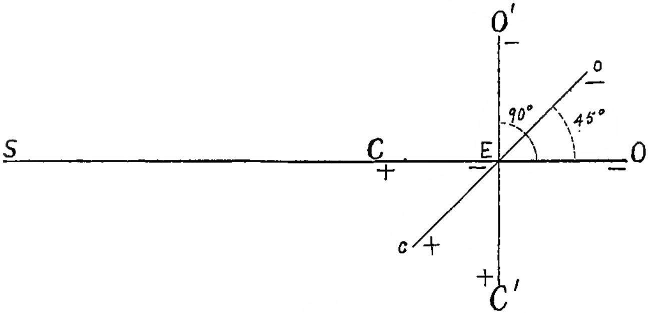

FIG. 8.

Now it is manifest that at *O′* the Moon would have the same excess of velocity with reference to the Sun that it had at *O* when its plane was coincident with the ecliptic. But in the position *O′C′* the relation of the forces is quite different from that in *OC*. For the Earth is no longer between the Moon and Sun, in effect augmenting the attraction of the latter. Instead of pulling together, the Earth and Sun now pull at right angles on the Moon, and the Moon's extra half mile per second of heliocentric velocity finds no augmented pull to counterbalance it. It follows that the centrifugal force due to the Moon's excess of velocity acts without restraint and must produce its legitimate effect. It therefore carries the Moon, not directly along the curve of *E*'s orbit, but nearer to the tangent, and so bends the Moon's plane a little from the perpendicular and reduces its inclination proportionally. At *C′* the effect on the inclination of the Moon's plane is precisely the same. For the deficient heliocentric velocity of the Moon allows it to fall toward *S* more than the Earth does, without any opposing pull by the Earth, thus reducing the inclination and bending the plane a little toward *C*. Perhaps in strict sense the tilting force at 90° inclination should not be called an «orthogonal» force. But I apply the name here indifferently to the tilting force in all degrees of inclination.

At 90° inclination, curvature of heliocentric path does not contribute to the excess of centrifugal force developed at *O′*, nor to the deficiency developed at *C′*. For in the position shown, the curvature of the Moon's path at *O′* is, on account of excess of velocity, less than that of the Earth in its orbit, and at *C′* it is more.

If the plane were tilted by throwing *O* down instead of up we should have relations just the same so far as the orthogonal force is concerned, except that the forces would be differently placed with regard to the ecliptic and its poles. For each direction of tilting the relation of the forces on the opposite side of the Sun, or 180° around the Earth's orbit (the Moon's plane keeping parallel to itself), are reversed in relation to the ecliptic, but tend to tilt the Moon's plane in the same direction. At two points 90° around the Earth's orbit from the position shown in Fig. 8, the Moon's plane would stand edgewise to the Sun and perpendicular to the ecliptic, and the orthogonal force in opposition would be derived from excess of curvature alone, velocity being normal.

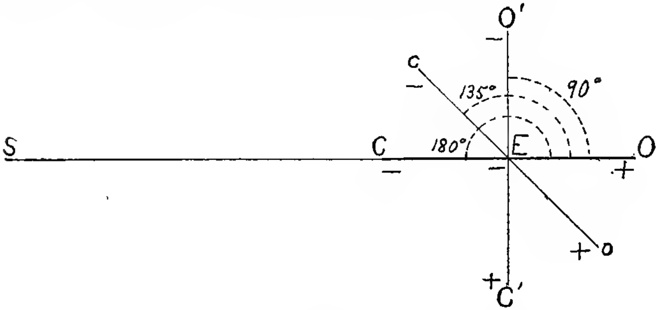

FIG. 9.

A tilted attitude of the Moon's plane, even to the slightest degree, is not characteristic of perfectly established conditions of stability. It indicates that there has been some change in the past which affected the position of the plane and that the Moon has not fully recovered from that event. Or, it may indicate the progress of a slow cause of tilting now going on, and to which the orthogonal has become adjusted in equilibrium;

or, the inclination may be due to both of these causes. But if due solely to the former we should expect the inclination to be now undergoing slow reduction, if solely to the latter we should expect it to keep a constant value.

If the Moon's orbit were tilted up through the north and turned over beyond 90° to an inclination of 135° from direct coincidence, as shown in Fig. 9, the forces would have a much more unstable relation than at 90°. For the Earth's attraction would then not only fail to augment the attraction of the Sun as before, but would pull in part *against* the Sun's attraction, and thus would in effect augment the excessive heliocentric centrifugal force due to excess of velocity and curvature. At *c* in Fig 9, the Moon would have an excess of velocity and a deficiency of curvature, though not so great a deficiency as at conjunction in direct coincidence (*OC* in Fig. 8). At *o*, the Moon would have a large deficiency of velocity with a considerable excess ot curvature. At this point, therefore, the Earth's attraction would augment the Sun's attraction and increase the deficiency of the heliocentric centrifugal. Both effects would tend to tip the plane up toward the perpendicular *O'C'*. In a system with a primary base of such high intensity as that of the Earth, stability at such an inclination would be impossible. The Moon's orbit so placed would turn over at a relatively rapid rate to its present place or to something near that.

The same would be the tendency if the tilting were to the south. From the action of the orthogonal force in these several positions we see that its tendency is to bring the Moon's plane back to direct coincidence from any and all degrees of inclination, whether revolution be direct or retrograde.

There remains one more attitude of the Moon's plane to be considered. This is where the revolution of the Moon is retrograde with its plane exactly in the plane of the ecliptic. This may be called *retrograde coincidence.* In this position there would be no orthogonal force, but the forces affecting the Moon's motion would tend strongly to instability. The curve *QOQ′C* in Fig. 10 shows the Moon's heliocentric path in retrograde coincidence; the broken curve shows its path in direct coincidence, as in Fig. 2.

The least velocity which the Moon would have with respect to the Sun in this relation would be at *O*, the point of opposition. The curvature there would be at its greatest, but the velocity would be at its least value or 18 miles per second. At the same time the Earth's attraction would join with and augment that of the Sun pulling in the same direction, just as at opposition in direct coincidence. Thus by deficiency of centrifugal force the Moon would tend to fall in toward the Earth, and the Earth's own attraction would largely augment and

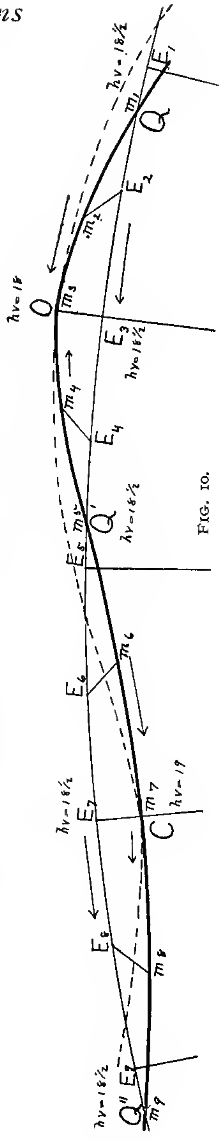

FIG. 10.

strengthen this tendency. At conjunction the Moon's heliocentric velocity would be 19 miles per second or half a mile per second more than that of the Earth. Its curvature with respect to the Sun would be less, and the excess of heliocentric centrifugal force would be augmented by the Earth's attraction, which would here act against the Sun's attraction. The result of such an adjustment of forces would be to cause the Moon to contract its orbit nearer and nearer to the Earth at every round, and the nearer it came to the Earth the more rapidly the orbit would contract. If the Moon kept constantly and exactly in the plane of the ecliptic it would finally come either into collision with the Earth, or, barely missing it, would dart close past it on the other side, and so pass from retrograde to direct revolution in a twinkling. But if in any stage of such retrograde contraction the Moon should depart even by the smallest amount from the plane of the ecliptic the orthogonal component would at that moment begin to exert a relatively powerful force to turn the plane over from retrograde inclination to direct coincidence, the danger of collision would cease and the tendency to orbital contraction would give place to tilting of plane.

From these considerations we see that, whatever the amount of inclination of the Moon's plane, the orthogonal force tends to bring it back to direct coincidence and keep it in that position, and that any force which operates to incline it either way from that position immediately brings into action the orthogonal component which tends to bring it back;

and the greater the degree of inclination the greater the power of this force. The forces of the epicycle tend constantly to control the degree of inclination of the Moon's plane, and their action tends to produce and maintain stability of plane in direct coincidence. Here again, then, stability as affecting the inclination of the plane of revolution has the quality of a stable equilibrium, and is therefore theoretically determinate.

I have not seen a discussion of the action of the orthogonal force in a case of inclination of 90° or more, on the basis of the current theory, though such a discussion must have been made for the retrograde satellites of Uranus and Neptune. But, following the example set in the discussion of the action of this force in the case of the Moon's present inclination, it would seem certain that the results would be quite different from those here outlined. On the principle of the difference of attraction the Moon with its plane inclined 90° and making right angles with the Earth's radius vector would not be affected by any orthogonal force arising from difference of distance from the Sun. For the distance of the Earth and Moon from that body would be the same. In retrograde revolution the result would again be different. The difference of attraction would tend to pull the plane down to retrograde coincidence instead of turning it over to direct coincidence.

§ 2. **Rotation of satellite planes.** Let us now pass from the consideration of the tendencies of the orthogonal force to some of its effects. One of the most important is the retrograde rotation of the plane of the Moon's orbit. It is to be remembered that the force acts, not on the Moon's plane, for that is imaginary, but on the Moon itself. Suppose the Moon's plane, as before, to be inclined 45° toward the Sun (the order of revolution being direct) so that its intersection with the ecliptic shall be along a line at right angles to the Earth's radius vector. The point of opposition would then be at the highest point above the ecliptic. As the Moon passes from opposition down to quadrature, the excessive centrifugal force, pulling it radially outward from the Sun, causes the Moon to pass through the plane of the ecliptic a little sooner than it otherwise would, so that the descending node is made to fall a little farther from the Sun and a little farther back on the plane of the ecliptic. Thus, the Moon's path is a little more steeply inclined to the ecliptic when the Moon reaches the node than would have been the case if the orthogonal force had not acted. But, on leaving the node and passing toward conjunction below the ecliptic, the Moon's path is bent the same amount the other way by a force acting radially toward the Sun, and the inclination of the plane is decreased in this quadrant just as much as it was increased in the previous one. Then in going from conjunction up to the ascending node the deficient centrifugal force allows the Sun to pull the Moon inward a little toward itself and this again steepens the Moon's plane at the node, and by increasing its inclination, causes the Moon to pass through the ecliptic a little sooner than it otherwise would and hence a little farther back on the ecliptic plane.

It follows that the nodes are less than 180° apart if measured in the order of their occurrence on the plane of the ecliptic, and the Moon's plane is in consequence made to rotate in retrograde order. In the position assumed the forces produce a periodic oscillation, the inclination always steepening toward the nodes and decreasing as the Moon departs from them. As the Moon is now adjusted these oscillations appear to have equal values and to balance each other. The forces act somewhat differently and with less power when the Moon's plane stands in other attitudes toward the Sun, and cause the retrograde rotation to be irregular. Its period is about 19 years. While the forces are balanced, as they appear to be at present, the effect of the orthogonal is merely to produce an oscillatory retrograde rotation of plane. The orthogonal force due to difference of attraction is represented as acting in the same way in direct revolution, and is well illustrated by diagrams in current text books.

The present value of the orthogonal affecting the Moon's revolution appears to produce only an oscillatory rotation of plane, the oscillations being compensatory. But if the orthogonal force were much more powerful, such for instance, as would be the case if the Moon's plane were tilted up to 80° inclination, then, although the same kind of oscillations would take place, it may be presumed that they would not be exactly compensatory. As they progressed there would be a gradual reduction of the degree of inclination. In the case of the satellites of the Earth, Mars, Jupiter and Saturn it can hardly be doubted that there are causes now acting which maintain their observed inclinations against the tendency of the orthogonal force to reduce them. The systems of Uranus and Neptune will be referred to later. With so high a degree of inclination at the outset, the decrease of inclination of the Moon's plane would go on until the orthogonal force became a small or feeble factor, so reduced in strength as to be balanced against the cause now tending to produce inclination of plane. Where the orthogonal force is strong, the adjustment to a state of equilibrium with the tilting force is likely to be associated with a small degree of inclination, but where the orthogonal is weak the adjustment may be associated with high degrees of inclination.

The Moon is the only inner satellite upon which the attraction of the Sun is greater than that of its primary. Hence a more powerful orthogonal force affects it than any other body in the planetary system. Under exceptional conditions to be pointed out presently, tending constantly to make the Moon's inclination greater than it is, its plane is nevertheless held down to the small inclination of about 5°.

It may be noted that by the present method of analysis, while the nodes would *regress* in direct revolution *they would continue in the same order in retrograde revolution;* that is, they would *advance* when considered with reference to the orthogonal projection of the Moon's orbit on the ecliptic. On the other hand, by the method of difference of attraction they would *regress* in retrograde revolution the same as in direct; that is, they would progress on the ecliptic in the opposite order from that which they take in direct revolution.

§ 3. **Persistence of satellite planes.** A solid body rotating upon its axis has *rotational momentum;* that is, once started, it not only tends to continue its rotation indefinitely, but it resists any force which tends to change the direction of its axis, or, what is equivalent to the same thing, the plane of its rotation, which is always perpendicular to the axis. The top and the gyroscope illustrate this principle. The greater the mass and the more rapid the rotation the greater the force required to change the plane of rotation.

For an analogous reason, a body revolving freely in space, as the Moon around the Earth, has the same tendency to persist in the plane of its revolution. It requires the action of force to change the position or inclination of the plane. The power of persistence depends in this case also partly on the rapidity of revolution. The inner satellites of the outer planets revolve swiftly and close to their primaries; on this account their planes have a greater power of persistence than the Moon which revolves slowly and far from the Earth. This persistence of plane, in connection with other factors, has had a large influence in giving the satellite systems their present diverse inclinations. We have now to consider the causes which have produced the various degrees of inclination.

§ 4. **Rotation of plane of Planetary system and its effect on the planes of the satellites.** Just as the planes of the satellites rotate in retrograde order, so we may suppose that the planes of the planets, or rather the so-called « invariable mean plane » of the Planetary system, rotates. By adopting this assumption we shall find that the origin of the inclinations of the various satellite planes, including those that are retrograde, are readily accounted for. The effects attributable to such a cause are exactly such as, taken in connection with persistence, will account clearly for the present facts of observation.

Let us suppose, first, that the Moon's orbit is in direct coincidence, and second, that, while it is in this attitude, the plane of the Earth's orbit around the Sun begins to undergo a slow tilting, such as must occur if it rotates. The forces tending to keep the Moon's plane in precise coincidence are extremely feeble. It is only when there is some degree of departure from coincidence that the orthogonal component begins to exert much force. When the Earth's plane begins to tilt and change its attitude in space, it might seem that the Moon's plane must necessarily go with it. But at first it would not, because the Moon's own revolution around the Earth gives it a certain power of persistence. As soon as coincidence ceases the orthogonal force begins to act. But not until the power of the orthogonal came to be exactly equal to the force producing inclination would the increase of the degree of inclination cease. This balance once established, the inclination would remain unchanged while the tilting of the Earth's plane continued. But let us suppose for the moment that the Moon's power of persistence in its plane is absolute, so that through all changes of attitude of the ecliptic or Earth's plane, the Moon's plane keeps constantly parallel to itself in space. In that case, as shown in Fig. 11, the Moon's plane would become inclined relatively to the Earth's plane as many degrees as the latter plane became tilted.

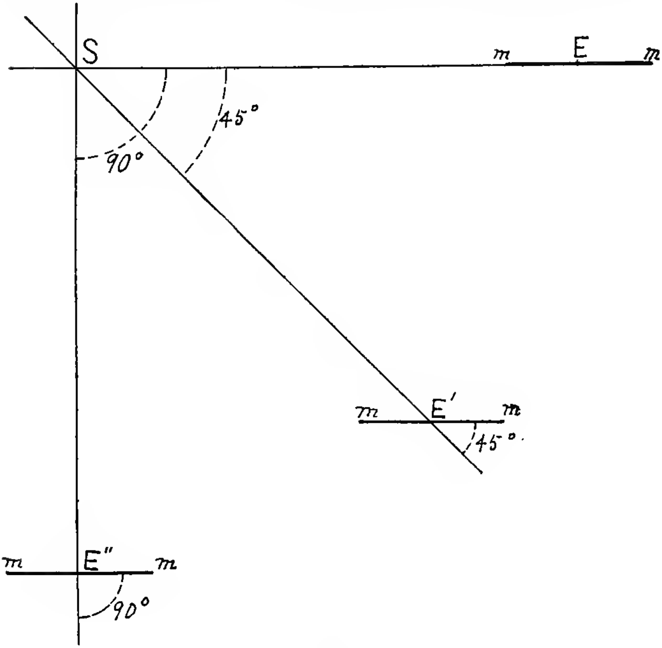

FIG. 11.

If the Earth's plane were tilted 90° from its original position the absolute persistence of the Moon's plane would give it an inclination of 90° to the ecliptic. If the tilting continued beyond 90° the Moon's plane would be correspondingly inclined and its revolution would become retrograde. This relation would hold if the Moon's persistence were absolute and the ecliptic were merely tilted without rotation.

If the ecliptic be supposed to have a motion of rotation that motion, combined with the persistence of the Moon's plane, would also operate to tilt the latter plane, but in a way different from that in Fig. 11. We may assume that as the Sun advances in space it moves in a plane which we may call *the Sun's plane*. Let it be supposed, further, that the ecliptic is inclined to the Sun's plane at an angle

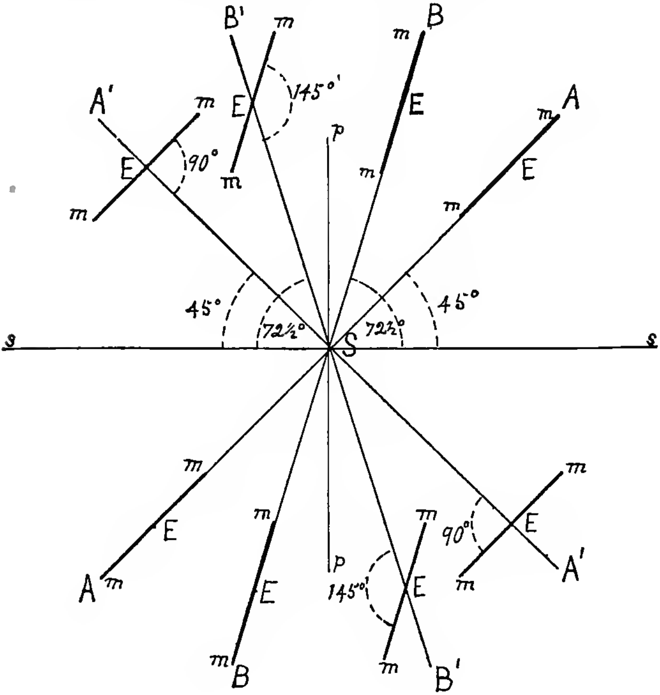

FIG. 12.

of 45° (*AA* in Fig. 12), and rotates in retrograde order. If at a given time the Moon's plane be in direct coincidence and its persistence absolute, then as the ecliptic rotates and turns away, the plane of the Moon will keep parallel to itself, and when the ecliptic has turned 90°, the Moon's plane will be inclined at an angle of 45°. When the ecliptic has turned through 180° the Moon's plane will be inclined 90° and will then be perpendicular to the ecliptic. (*A'A'* in Fig. 12.) After the ecliptic has turned 270°, the Moon's plane will again incline 45°, and when it has turned through 360° the Moon's plane will have returned to direct coincidence.

If the ecliptic were inclined at an angle of say 72½° to the Sun's plane, as shown by *BB* in Fig. 12, then when the former had rotated through 90° the Moon's plane would be inclined 72½°, and after turning through 180° the Moon's plane would be inclined 145°, as shown by *B'B'*, and its revolution would be retrograde. This happens to be the degree of inclination of the satellite of Neptune which is the most highly inclined of any.

We have thus far been supposing the persistence of the Moon's plane to be absolute, but in truth it is not, nor is that of any member of the planetary system. Their planes all rotate and they probably all change the degrees of their inclination from time to time, though only very gradually. As we have seen, the orthogonal force is greatest in satellite systems near the Sun and least in systems far away, while, on the contrary, persistence of plane is weakest in systems near the Sun and strongest in systems far away. Hence, the combination of both of these circumstances leads us to expect small degrees of inclination in systems near the Sun and greater degrees of inclination in the systems of the outer planets. In a general way, this agrees with the facts of observation. The planes of all the planets lie very close together, none departing more than three or four degrees from the mean plane, except Mercury whose inclination is 7°. The Moon's plane is inclined 5° to the ecliptic. The plane of the satellites of Mars is inclined nearly 25° to the plane of the planet's orbit; that of Jupiter is inclined 3°; that of the satellite system of Saturn about 27°; that of the system of Uranus nearly 98°, and that of Neptune about 145°. the last two being retrograde systems.

In the case of the Moon, the orthogonal force is relatively so powerful and persistence so weak that the Moon is not able to keep in the plane of the Earth's equator. This is quite significant, for the Earth's equatorial bulge tends to bring the Moon's plane into coincidence with the equator and keep it there. But even with this force to assist persistence the Moon's plane has been inclined only 5°. Excepting the outer members of the systems of Jupiter and Saturn, the Moon is the only satellite which does not revolve in or very nearly in the plane of its primary's equator, so far as known. It is the only *inner* satellite which has this relation. The system of Mars is so small and its satellites revolve in such short periods that persistence is much stronger and they keep in their primary's equatorial plane and at a higher degree of inclination than they would if Mars were of larger mass. The system of Jupiter is so great and so expanded that persistence is relatively weak, and since the orthogonal force is moderately strong, the inclination of the system is small. The system of Saturn is farther from the Sun, more compact and more inclined. In the system of Uranus the orthogonal has become relatively weak and persistence correspondingly strong, while in the system of Neptune persistence is at a maximum and orthogonal at a minimum; and both of the last named systems are retrograde.

The fact that the plane of each system of satellites conforms very nearly with the equatorial or rotation plane of its primary shows that there is a strong tendency for these planes to come together. The equatorial bulge of each planet tends to bring its satellites into the plane of its equator, and may do so, even where that plane is considerably inclined to the ecliptic. The orthogonal force tends to pull the satellite planes down toward direct coincidence, and it tends to hold them near the ecliptic while this plane rotates. To accomplish this the rotation plane of the planet as well as that of the satellites must change. The rotation planes of the planets have not persisted absolutely, but have changed as tilting progressed, and all the satellites have been kept in or nearly in their primaries' equatorial planes, except the Moon and Iapetus. Jupiter's persistence has been very slight.

Where a planet like Jupiter has a number of satellites, and the equatorial bulge is relatively great and the orthogonal force relatively powerful, this force, through its stronger control of the satellites, may be able to bring the equatorial plane of the planet quite near to the ecliptic and keep it there. But where the orthogonal is weak and persistence especially strong, as in the cases of Uranus and Neptune, their planes have persisted with relatively slight change through a very long period and while a considerable amount of rotation has taken place in the planetary plane. In these cases the orthogonal force has probably not been able to control the satellites against the influence of the equatorial bulges of their planets. Persistence has been dominant and they have probably kept together in spite of the orthogonal force.

One of the most readable and instructive of recent semi-popular works on astronomy is "The Solar System," by Professor Percival Lowell. (1903.) He shows by tables and a diagram how the planes of the satellites of the systems of Jupiter and Saturn increase their degrees of inclination to the equatorial planes with increasing distance from their primaries. The rings and inner satellites of Saturn are inclined only 12″ to the planet's equatorial plane, while Iapetus, the farthest well-determined satellite is inclined 9° 52′ 19″. In the less expanded system of Jupiter, Io is not sensibly inclined, while Callisto, the farthest satellite is inclined 24′ 35″. The inclinations in the two systems are represented by curves which are much alike. These results are quite in harmony with the causes here assigned. In the outer members persistence is weak and orthogonal relatively strong and the equatorial bulges of the planets have less power to control the degree of inclination.

Lowell points out also the lack of symmetry in the arrangement of the axial inclinations of the outer and inner planets. The degree of inclination increases rapidly in both groups going from the Sun.

OUTER PLANETS.	INCLINATION OF EQUATOR TO ORBITAL PLANE.
Neptune	145° (?)
Uranus	98° (?)
Saturn	27°
Jupiter	3°
INNER PLANETS.	
Mars	25°
Earth	23½°
Venus	0° (?)
Mercury	0°

This character of the two groups is due mainly to their large differences of mass. It seems certain that if the masses of the planets were all the same the inclinations of their axes would progress from Mercury to Neptune without the present break between Mars and Jupiter.

Thus we see that the present inclinations of the several satellite systems may be explained by supposing that the planes of the planets have been for indefinite ages in the past gradually rotating and turning away from the partially persisting planes of the satellites. The planes of those nearest the Sun have yielded most readily and lie nearest to direct coincidence with the ecliptic, while those farthest away have persisted most strongly and depart from direct coincidence most widely. The high inclinations of the outer systems are therefore a record of great change in the inclination of the planetary plane reaching back over long ages of the past. As records of that change they show only the minimum supposable degree of change; of the possible maximum they reveal nothing. Neptune's system shows that the present inclination of the planetary plane to the Sun's plane can hardly be less than 72½°, and it may be much more.

Of course the rotation of the planetary planes, or rather of the mean plane of the planetary system, is extremely slow, and while this goes on the planes of the individual planets must undergo many periodic and irregular shiftings and changes of minor magnitude in consequence of the mutual attractions of the planets themselves. These, however, are superimposed as mere irregularities on the main rotation of their mean plane.

If the planes of the planets rotate the results produced would certainly tend to be substantially identical with those observed. What could be more natural than to conclude that the observed inclinations of the satellite systems have really had their origin in this way, and that the dominant direct order of revolution, as well as the exceptional retrograde systems of Uranus and Neptune, are evidences of such rotation and of an orthogonal force of the nature described?

For the sake of comparison, I quote below a recent article by Professor W. H. Pickering of Harvard Observatory. (“Explanation of the Inclination of the Planetary Axes.” *Astronomical Journal*, No. 511, 1901, pp. 56–57.)

“Suppose a uniform spheroid to revolve in its orbit about the sun, and to present always the same face to a star. If this spheroid is covered with liquid an annual tide will be produced, which in the process of time will cause the spheroid to rotate upon its axis so as to present always the same face to the sun.

“Suppose now that this spheroid possesses an original rotation about its minor axis, and that this axis lies in the plane of its orbit, as is for instance approximately the case with the planet *Uranus*. We shall thus have two independent rotations about the two axes placed at right-angles to one another. When the motions are combined, however, as may be clearly illustrated by means of the gyroscope, the effect produced is to shift the minor axis of the planet out of its original plane, so that the plane of the planet's equator shall approach the plane of its orbit, and in such a manner that the rotation and revolution shall take place in the same direction.

“This shifting of the axis is not to be confused with that producing precession, which is due to a different cause and is periodic. The present shifting is continuous in its action and its direction lies at right-angles to that causing precession.

“According to Laplace's nebular hypothesis, as is well known, when the rings break up into planets, these should rotate in a retrograde direction. Owing to the tidal action above described, however, the plane of their rotation will gradually shift, so that from being at first nearly parallel to the plane of their orbits, it becomes later perpendicular to them, and finally again parallel, but this time with the rotation direct. Successive satellites formed by the contracting mass would thus originally revolve in different planes, but would all finally approach the plane of rotation of their primary through the attraction of their equatorial regions.

«Such a progressive change of plane is found in the orbits of the satellites of the four major planets. Thus, for *Neptune* the approximate angle is 145°, for *Uranus* 98°, for *Saturn* (inner satellites) 27°, and for *Jupiter* 2°. It is also found in the case of the four inner planets, as far as is known, as determined by the inclination of their equators to the plane of their orbits. Thus, for *Mars* the angle is 25°, for the *Earth* 23°, for *Venus* the angle is unknown, and for *Mercury*, while undetermined, it is certainly very small, as is indicated by the drawings of surface detail made at Milan, Arequipa and Flagstaff. While the force producing this change must at the present time be almost infinitesmal, yet such would not have been the case in the past, when the planets were perhaps one hundred or more times their present dimensions.

«If we take the great nebula in *Andromeda* as a type of our earlier existence, the planets being merely condensed masses revolving within and with the solar atmosphere, this atmosphere itself would excite a frictional force which would tend always to keep the same side of the planet towards its primary, while gravitation acting on the mass before it had separated itself from the spiral would tend to cause it to revolve in a retrograde direction. Thus in the earlier times it is very certain that these forces would be much more active than we find them to be at the present day.»

So far as I know, Professor Pickering's explanation is the best that has been given from the point of view of the current theory and associated hypotheses. Current theory alone has no explanation of the dominant direct revolution, nor of the exceptions to it, nor of the cause of the varying degrees of inclination. To complete his hypothesis, Professor Pickering relies upon the aid of the Nebular hypothesis of Laplace and the Tidal hypothesis of Darwin. If either of these possessed the value of demonstrated truth Professor Pickering's explanation would be formidable. But according to the theory presented here, it does not appear necessary to resort to either of them.

CHAPTER X.

THE SATELLITE ZONES.

§ 1. **The inner limit of satellite revolution.** We have seen above how the Moon's stability in its present orbit is made determinate. Under existing conditions of the Earth's mass and revolution the Moon could not be stable in any orbit either nearer to or farther from the Earth than that in which it now revolves.

In the cases of Mars, Jupiter and Saturn it was shown that the inner satellite of each revolves under the same relation of forces as the Moon and has its stability made determinate in the same way. But each of these planets has more than one satellite. How is this to be reconciled with the conclusions regarding the Moon's stability?

In the first place it will be remembered that the mechanism of determinate stability, as worked out for the Moon, was applied only to the *inner* satellites of the other planets on the assumption that each of these planets had only one satellite. In this way the case for each one was made the same as that of the Moon. We have now to remove this limitation and consider the conditions of stability where there is more than one satellite.

In discussing the mechanism of stability it was stated that those forces which would compel the Moon to expand its orbit from a smaller one to its present place are more powerful than those which would compel it to contract from a larger one to its present place. From this relation it follows that, as the Moon revolves in its ellipse around the Earth and passes alternately inside and outside of its mean circle, the forces which drive it out when it comes inside are slightly more powerful than those which drive it in when it goes outside; that is to say, the forces of expansion from a contracted orbit are stronger than those of contraction from an expanded orbit. From this fact it follows also that the forces which make stability determinate are better able to resist the action of any force which tends to cause contraction than they are to resist the action of a force which tends to cause expansion. Hence, where two or more satellites revolve around one planet, the inner one occupies the orbit of determinate stability, and revolves in the same orbit that it would have if the planet had but one satellite, or at least so nearly the same that the displacement due to the outer satellite is imperceptible. The outer members of a system revolve in orbits at determinate intervals of distance outside of the inner one. The inner satellite may therefore be said to revolve at the *inner limit of stability*, or at the *inner limit of satellite revolution.* No matter how many satellites there may be in a given system outside of the inner one, the inner one always occupies the fundamental orbit of stable revolution — the inner limit of stability — and this orbit may be called the fundamental or first orbit of the system.

§ 2. **Structure of a satellite system.** The determinate orbit of stability of the second satellite in a system of two or more depends upon three things: (1) Upon the primary base or intensity of the planet's revolution, which determines what value of excessive heliocentric centrifugal force the second satellite will develop in consequence of a given excess of velocity and curvature in opposition, and how much this will lack of furnishing the conditions of stability; (2) upon the mass of the planet, which determines the power of the planet's perturbation of the second satellite at a given distance, and (3) upon the frequency of the perturbative impulses which the second satellite receives from the first or inner satellite in augmentation of the perturbations of the planet.

On the basis of these principles we may conclude that if Mars and Jupiter had the same mass and such that they could both have at least two satellites, not only would the inner satellite of Jupiter revolve nearer to its primary and faster around it, as shown above, but the second satellite of Jupiter would revolve nearer to its first satellite than the second satellite of Mars would to its first satellite. In a series of satellite systems, say of four satellites each, the spacing between the four members would be widest in the system nearest to the Sun and grow less and less in systems farther away, supposing planets of equal mass. Such a system would therefore be more widely spaced for Jupiter than for Neptune.

In a system of two satellites the inner one occupies the determinate or first orbit of stability and the outer one revolves in an orbit at a determinate distance outside. The revolution of the outer satellite, however, is not made fully stable by the fundamental forces of determinate stability, but lacks a little of that consummation; indeed, this satellite revolves in an orbit in which it is unstable, except for the support which it receives from the inner satellite. If the inner satellite were taken away the outer satellite would immediately begin to contract its orbit and would continue to contract it until the outer satellite reached the place formerly occupied by the inner satellite, when its revolution would become stable and contraction would cease. The second satellite would then have taken the place of the first one at the inner limit of stability.

So long, however, as the inner satellite keeps its place, the orbit of the outer satellite can not contract. For every time the inner satellite passes between the outer one and the planet the effect is as though the attraction of the planet upon the outer satellite were temporarily increased by a small amount. By continual repetitions of these perturbative impulses the inner satellite keeps the outer one out to its place and counteracts its tendency to contract its orbit. *The perturbative impulses will make the second satellite stable in an orbit at that distance from the first satellite at which the value of the expansive impulses is precisely equal to the tendency of that satellite to contract its orbit.* The interval between the first and second satellites is determined in this way. Its width depends upon the mass and intensity of revolution of the planet and has a definite value for given conditions.

The effects of the perturbations of the outer satellite upon the inner one are counteracted by the original forces of determinate stability, which sustain the revolution of the inner satellite in its orbit. The adjustment is a very delicate one, but the inner satellite is able to hold its place against the perturbations of its outer companion. The outer satellite, however, can not hold its place against the tendency of the inner satellite to drive it out into a larger orbit until, at a certain determinate distance, the original tendency to contract is precisely balanced by the perturbative impulses received from the inner satellite. At this distance the opposing forces attain a balanced state and the second satellite becomes stable.

If there are more satellites the third one revolves in an orbit at a determinate distance outside of the second one and is sustained against contraction by the perturbative impulses received from the second, just as the second is sustained by the first. The fourth is sustained by the third, and so on to the last and farthest member of the system.

Thus it is that *in every satellite system the whole superstructure in a certain sense rests upon and is sustained by the inner member.* This member is nearest to the fountain head and source of stability, and being itself sustained in stable revolution in a determinate orbit by its primary, transmits to the other members of the system by perturbative impulses the force necessary to prevent the contraction of their orbits. The force of the impulses is relatively slight, but it is sufficient and is passed on successively from inner to outer members. By this action of the forces through the medium of the inner satellite, the place of stable revolution for the second satellite is made determinate, like that of the first one, and the orbits of all the others are also made determinate for given conditions.

The determining factors for the stability and spacing of the superstructure are two: (1) the primary base or intensity of the revolution of the planet, and (2) the mass of the planet. The first of these may be said to determine the *intensity* of the system, the second its *scale.*

That intensity is the chief factor controlling the spacing or arrangement of systems seems clearly shown by the fact that, within wide limits, the spacing of the members is independent of their masses. For the members of a system may have widely different masses and still be spaced at orderly intervals, showing clearly the control of a law having, within certain limits, little or no dependence upon mass. Beginning at the inner satellite of a system the power of the tendency to orbital contraction in larger and larger orbits must vary in every system according to a definite law. To make any member of the system stable, it is only necessary that it shall receive perturbative impulses from within adequate to balance its tendency to orbital contraction; and the efficiency of the impulses depends, within pretty wide limits, more upon their frequency in a given period of time than upon their strength.

For examples of scale, the systems of Mars and Jupiter may be regarded as two extremes. For examples of intensity, those of the Earth and Neptune would represent the two extremes, but a better comparison is represented by taking Saturn with its larger known family. **It is a general law that the intensities of the satellite systems vary inversely as the intensities of revolution of their primaries.** Thus, the Earth's revolution is much more intense than that of Saturn, but the intensity of Saturn's satellite system is just that much more intense than the revolution of the Moon.

The spacing of the satellites of a system at regular or precisely graduated intervals determined by the mass and orbital intensity of the primary is simply Bode's law, as expressed in its application to systems of small scale, like those that revolve around the planets. Concerning its expression in the Planetary system something more will be said later.

§ 3. Outer limit of satellite revolution. We have seen that the inner satellite of every system revolves in the first or fundamental orbit of determinate stability. It may be said therefore to occupy the inner limit of stable revolution. But there is also an *outer* limit of stable revolution for each system. This limit is fixed by the relation of the satellite's velocity of revolution around the planet to the rotation of the planet's radius vector. As a satellite expands its orbit and recedes from its primary it revolves more and more slowly. At the same time, points on the radius vector of the planet move with velocities which vary directly as their distances from the Sun. A satellite revolving in direct coincidence crosses the planet's radius vector at the point of conjunction, and again at opposition. From these relations it follows necessarily that, as a satellite expands its orbit and reduces its velocity around its primary, it will finally reach a place in which its velocity around the Sun will be slightly less than that of a point on the radius vector of the planet at that distance. Then the satellite will fail to pass the radius vector, either in opposition or in conjunction, and when this happens its relation to its primary becomes completely changed. It immediately ceases to revolve in the regular way and must either revolve henceforth in the retrograde order or leave the planet permanently. For a short time it may seem to come to a standstill near the radius vector, but that can only be a temporary relation. Neither can it have stable revolution in retrograde order if the planet be near the Sun. The consequence is that, around every planet which has one or more satellites, there is at some distance beyond the outer one an orbit in which the revolution of a satellite would have this relation to the radius vector, and this we may call *the outer limit of stability* or *the outer limit of satellite revolution.* The outer member of each system is of course nearest to this outer limit, but, except at critical stages of expansion, there is nearly always a safely wide interval between. It is conceivable that if a satellite could revolve constantly in a circle it might occupy an orbit barely inside of the outer limit, but in such a position any cause which tended to change its motion to an ellipse might give it such a degree of eccentricity that in the slower part of its orbit it could not pass the radius vector. Supposing the masses of the planets to be equal, it is obvious that the farther a planet is from the Sun the greater will the distance of the outer limit of revolution be from the planet, because points on the radius vector at a given distance from the planet move more slowly relatively to the planet the farther the planet is from the Sun.

There are some interesting notes on the « spheres of activity » of the planets, or the « limit of stability » by Professors Hall and Moulton in *Popular Astronomy*, Nos. 64 and 66, 1899. The «sphere of activity » and the « limit of stability » correspond with the *outer limit* of stable revolution as here defined. There is no hint of an *inner limit* of stability nor any suggestion of the idea of determinate stability. Professor Moulton's paper is also interesting as showing how fully Darwin's hypothesis of tidal evolution has been received into current astronomical theory; and it also shows quite clearly the freedom with which the principle of indeterminate stability is used.

§ 4. **The law of the satellite zones.** Between the inner and outer limits of stable satellite revolution as here defined there is a belt or zone around each planet in which stable satellite revolution is possible. This may be called the *satellite zone.* Going from the Sun, and with planets of equal mass, the inner limit of satellite revolution draws in nearer and nearer to the planets and the outer limit recedes farther and farther from them, resulting in the production of *widely expanded satellite zones for the outer planets and narrow ones for the inner planets.*

In this law we find the reason why the inner planets, Mercury and Venus, have no satellites. Their revolutions in their orbits are so intense that they have no satellite zones. Coming in toward the Sun from Neptune the satellite zone (supposing equal masses for the planets) grows rapidly narrower, until at the Earth it has become so narrow that it can hold but one satellite. In narrowing, the inner and outer limits of the zone both shift and approach each other. At some place between the Earth and Venus, or perhaps nearer the latter, they come together and the zone disappears. In all orbits nearer than this to the Sun, the zonal limits *pass each other* and change places, so that theoretically the outer limit is inside of the inner one and there is no satellite zone. The zonal limits are reversed in this way for Mercury and probably for Venus. There is no orbit around either of them in which a satellite could have stable revolution.

The law of the satellite zone points to many interesting conclusions. If Jupiter with its great mass revolved in the orbit of the Earth it could have no satellites. The scale of its system is so great that the outer limit would be very near if not inside of the inner limit. On the other hand, little Mars in the orbit of Jupiter or at any greater distance could not have a satellite in stable revolution, because the inner limit would be at or within the surface of the planet itself, so that any body that began to revolve around Mars would soon collide with the planet and be absorbed into its mass.

From these considerations it is easy to see that the peculiar relations of Phobos to Mars, being so near to the planet and revolving around it three times to one rotation of the planet on its axis, is not the result of any incident in the contraction of hypothetical nebular rings, nor is it in any way related to tidal perturbation or evolution, but is simply the place of the satellite as fixed by the factors and forces of determinate stability, and is the only orbit in which the first or inner satellite of Mars can have stable revolution under present conditions. This is supposing that Phobos is actually the inner satellite of Mars.

§ 5. **The Moon's annual inequality.** Throughout most of this discussion we have assumed for the sake of simplicity that the Earth's orbit around the Sun is a circle. The fact that in reality the Earth's orbit is an ellipse introduces certain periodic modifications in the adjustment of the Moon to the Earth. When the Earth is in perihelion it is nearer the Sun than at any other time, and when it is at aphelion it is farthest away. Since the Moon goes with the Earth, it has been determined in the current theory that the Sun's difference of attraction, and hence the solar perturbation affecting the Moon, is greater than the mean when the Earth is in perihelion and less than the mean when it is in aphelion. As a consequence, the Moon is drawn a little farther away from the Earth in perihelion, *expands* its orbit around the Earth by a small amount and falls a little behind its mean geocentric place. In aphelion it *contracts* its orbit by the same amount and gets a little ahead of its mean place. This is true as far as it goes, but it does not comprehend all the factors involved.

If the Earth revolved in a circle passing through its point of perihelion, instead of in its present mean circular orbit, its velocity and curvature with respect to the Sun would both be greater than in its present mean circle. In a circular orbit passing through aphelion these factors would be less. By the laws which make the stable orbit of the Moon determinate for given conditions, the place of this determinate orbit is farther from the Earth when the Earth is in perihelion than when it is in its present mean circle, and nearer when the Earth is in aphelion. This effect is independent of the solar difference of attraction, but is not taken into account in the current theory.

Both of these causes, viz: the forces of determinate stability and the forces of difference of attraction, operate to give the Moon an annual inequality in its geocentric motion. They cause the Moon to gradually expand its orbit from July to January and then to contract it from January to July. The current analysis of this inequality is in part empirical, because it does not take full account of all the factors involved. And the same is to be said of some of the Moon's other inequalities also. It remains for a revised mathematical analysis to show the true numerical values of the forces which cause the annual inequality. That analysis should also show whether the effects correspond in magnitude to the causes. Contraction and expansion require time for their accomplishment. It may be that on this account the Moon's annual inequality does not express the full range of variation in the value of the forces which cause it.

The inner satellites of the other planets ought to show more or less of the same inequality, dependent in each case upon the eccentricity of the planet's orbit. But in all of them it would probably be much less conspicuous, partly because of the reduced actual and relative strength of the factors of difference of attraction, and also because all of those satellites are so much nearer to their primaries that slighter degrees of expansion and contraction would satisfy the forces. The outer satellites of satellite systems with many members have the same tendency theoretically, but with them the stability forces are so feeble and the process becomes so slow that the inequality almost disappears.

If the Sun revolves in a great ellipse then the Planetary system must have a cyclic inequality corresponding to the Moon's annual inequality, and the whole Planetary system must be affected by it and undergo more or less periodic expansion and contraction, depending on the degree of eccentricity of the Sun's orbit. Early in the studies out of which the present theory has grown, I looked upon this inequality as a possible cause of glacial periods, the variations of terrestrial climate following the variations of the Earth's distance from the Sun. Further developments, however, seemed to show its inadequacy, because the cyclic period is many times too long. Along with this, I also entertained the idea that the same cause might account for the unexplained part of the «secular acceleration of the Moon's mean motion,» for, as the Earth slowly expands its orbit around the Sun, the Moon should *contract* its orbit around the Earth. But here again the rate of the Moon's acceleration seems far too rapid for the very long duration of the Sun's cyclic or orbital period. If the solar cycle were shorter by a large amount it would appear to account perfectly for both of these phenomena. All of the planets and satellites must be affected more or less in these ways by the cyclic inequality, but the changes are so slow that they have probably not become perceptible in the historic period. Something more will be said later upon the probable duration of the Sun's cyclic period.

DIVISION III.

THE GROWTH OF SATELLITE SYSTEMS.

CHAPTER XI.

THE ORIGIN OF CERTAIN OF THE SHORT-PERIOD COMETS.

DANIEL KIRKWOOD showed more than sixteen years ago that certain of the short-period comets are in all probability perturbed asteroids. He showed that in the asteroid ring there are certain gaps or vacant belts, where asteroids are few or entirely wanting, and that these gaps correspond to asteroid orbits that would have short commensurabilities with the revolution of Jupiter. He inferred from this that such asteroids as may have revolved in orbits lying within these gaps have been drawn out of their places by the perturbing action of Jupiter's attraction. From Kirkwood's work on « The Asteroids » I quote in full his section on « The Relation of Short-Period Comets to the Zone of Asteroids. »

« Did comets originate within the solar system, or do they enter it from without? Laplace assigned them an extraneous origin, and his view is adopted by many eminent astronomers. With all due respect to the authority of great names, the present writer has not wholly abandoned the theory that some comets of short period are specially related to the minor planets. According to M. Lehmann-Fihlès, the eccentricity of the third comet of 1884, before its last close approach to Jupiter, was only 0.2787. This is exceeded by that of twelve known minor planets. Its mean distance before this great perturbation was about 4.61, and six of its periods were nearly equal to five of Jupiter's,—a commensurability of the first order. According to Hind and Krueger, the great transformation of its orbit by Jupiter's influence occurred in May, 1875. It had previously been an asteroid too remote to be seen even in perihelion. This body was discovered by M. Wolf, at Heidelberg, September 17, 1884. Its present period is about six and one-half years.

« The perihelion distance of the comet 1867 II. at its return in 1885 was 2.073; its aphelion is 4.897; so that its entire path, like those of the asteroids, is included between the orbits of Mars and Jupiter. Its eccentricity, as we have seen, is little greater than that of Aethra, and its period, inclination, and longitude of the ascending node are approximately the same with those of Sylvia, the eight-seventh minor planet. In short, this comet may be regarded as an asteroid whose elements have been considerably modified by perturbation.

« It has been stated that the gap at the distance 3.277 is the only one corresponding to the first order of commensurability. The distance 3.9683, where an asteroid's period would be two-thirds of Jupiter's is immediately beyond the outer limit of the cluster as at present known; the mean distance of Hilda being 3.9523. The discovery of new members beyond this limit is by no means improbable. Should a minor planet at the mean distance 3.9683 attain an eccentricity of 0.3 — and this is less than that of eleven now known — its aphelion would be more remote than the perihelion of Jupiter. Such an orbit might not be stable. Its form and extent might be greatly changed after the manner of Lexell's comet. Two well-known comets, Faye's and Denning's, have periods approximately equal to two-thirds of Jupiter's. In like manner the periods of D'Arrest's and Biela's comets correspond to the hiatus at 3.51, and that of 1867 II. to that at 3.277.

« Of the thirteen telescopic comets whose periods correspond to mean distances within the asteroid zone, all have direct motion; all have inclinations similar to those of the minor planets; and their eccentricities are generally less than those of other known comets. Have these facts any significance in regard to their origin? » *

There can be little doubt, it seems to me, that the remarkable conclusion of Kirkwood is correct. But, whether it is demonstrably so or not on the basis of the current theory, his suggestion carries with it very important and far-reaching possibilities which fall directly into line with the present theory.

Since Kirkwood's publication, other facts of similar import have been found, the most important being the discovery of the remarkable so-called planet Eros, by Witt in 1898, and of the most

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* « The Asteroids, or Minor Planets Between Mars and Jupiter, » by Daniel Kirkwood, formerly professor of Astronomy in the University of Indiana; pp. 55 to 57. Philadelphia: J. B. Lippincott Co., 1888. See also Amer. Jour. of Sci., III, vol. XXXIII, Jan. 1887, p. 60.

eccentric asteroid, by Stewart in 1901. (*Popular Astronomy*, No. 91, 1902.)

In a popular address on the minor planets or asteroids, Professor J. K. Rees observes, "That the law of commensurability of orbits has greatly influenced the present distribution of the asteroids, as suggested by Kirkwood, there seems to be no room for doubt. The recent investigations of the German astronomers show that as the number of asteroids increases, this law exhibits itself more strikingly." (*Popular Astronomy*, No. 56, 1898.)

The asteroids appear to be little planets with solid stony bodies probably very much like the body of the Earth or the Moon, though they may be in general somewhat less dense than the Earth. If a planet of this character were to be perturbed out of the even way of its planetary orbit, what would become of it? It seems certain that it would become a wanderer and would henceforth revolve around the Sun in a more eccentric path, which would be in all respects like the orbit of a comet. If such a body became a comet it seems certain that its solid globe, whether visible under ordinary conditions from the Earth or not, would be the real nucleus, and that the tail would be formed in part of fragments separated in some way from it. So little has been learned by observation as to the nature of cometary nuclei that we are not debarred from accepting this conclusion, nor even from extending it, thus: that if the nuclei of *some* of the short-period comets are solid stony bodies, like small planets or satellites, then *all other comets* may, and probably do, have the same kind of nuclei. There is no distinction or classification of comets that precludes this generalization. And this in turn suggests that all comets may be lost asteroids, or lost satellites, or lost planets; and this may be true, whether they were formerly members of our own or of some other solar system.

All the periodic comets whose periods are less than one hundred years in length are divided into groups or families associated with the four outer or superior planets. Jupiter has thirty-two, Saturn two, Uranus three and Neptune six. This grouping with relation to the planets is exactly what would be expected if these comets are in fact lost satellites, or, especially in the case of Jupiter, lost asteroids. No doubt there are many members of Jupiter's comet family remaining to be discovered. Neptune's distance from the Sun is 30 astronomical units. In an interesting diagram of the perihelia of the periodic comets, Lowell shows that there are two perihelia near the distance of 50 astronomical units, and three near the distance of 75 units. These comets strongly suggest the presence of two trans-Neptunian planets to which they probably belonged formerly as satellites.

From these considerations we seem to have fair ground for the conclusion that the nuclei of all comets may be solid, stony, planet-like bodies. There is no proof to the contrary and there are many facts tending to support this view. When far from the Sun, comets give a faint, dull light which the spectroscope shows to be sunlight reflected from solid matter. Near the Sun they give forth bright intrinsic light, the spectrum of which shows bright bands, emanating mainly from hydrocarbon gas, and there is a faint continuous spectrum in the background. The tails of comets sometimes develop to a prodigious size and appear to fill enormous volumes of space; but they are so tenuous and diaphanous that stars seen through them and even through the brighter coma appear to be undimmed. Close to the nucleus, however, the star's rays become more or less obstructed, as though by dust or smoke.

These several evidences are supposed by some to show that comets are largely composed of hydrocarbon gas. But this does not appear to be a necessary conclusion. Sodium, magnesium, iron, nitrogen and probably oxygen have been found in comets. That the hydrocarbons are more active in giving forth light may be due to some peculiar condition or circumstance not now understood. Hydrocarbon gas is found occluded in meteorites in considerable quantities.

Probably most cometary nuclei are small, of the size of small asteroids or satellites, much too small to affect the planets appreciably by their perturbations and too small even to be seen through the telescope, surrounded and obscured, as they always are, by a cloud of dust and fragments, the wreckage of their own disintegration. Some, as has been said by Langley, are probably scarcely more than bowlders of gigantic size. Such a nucleus broken in two, becomes henceforth two separate, and to all appearances, perfect comets, as was apparently the case with Biela's comet in 1846. The great comet of

1882, and also Brooks's comet of 1889, were divided in the same way.

The comets of the families of Jupiter and Saturn all revolve in the direct order. Tempel's comet 1866 I and the Leonid meteors have a period of 33¼ years, revolve in retrograde order and belong to the family of Uranus. Halley's comet, having a period of 76 years and belonging to Neptune's family, also revolves in retrograde order. It is somewhat significant that these exceptions are associated with planets which have retrograde satellite systems. The idea that all comets originate outside of the Solar system and were captured by the various planets during visits to the Sun appears to be the most favorably received hypothesis of their origin at the present time. But when we note the distribution of the periodic comets in the several planetary families, Jupiter's thirty-two being several times as many as are possessed by any other planet, and the fact of Jupiter's great mass and his close proximity to the asteroids, it seems equally plausible to suppose that Jupiter's numerous family is made up mostly of lost asteroids perturbed out of their former orbits by the attraction of the giant planet. In the light of Kirkwood's discovery it seems to me that this interpretation is better than the other. For the purposes of this discussion, I shall consider the nuclei of all comets to be solid, stony, planet-like bodies resembling asteroids and satellites in all their physical properties.

CHAPTER XII.

ACCESSION OF SATELLITES BY THE CAPTURE OF COMETS.

SUPPOSE an asteroid to be perturbed out of its place in the ring by Jupiter's powerful attraction. Its orbit would henceforth be more eccentric than the average for those asteroids which remain undisturbed. It would then be a comet, so far as its orbital characters were concerned, and it might show the usual faint, short tail of short-period comets. Further perturbations might make its orbit still more eccentric, so that its path would be made to lie partly within and partly without the orbit of Mars. From this cause the perturbations affecting it would become more irregular than before, Mars being sometimes inside and sometimes outside of its orbit. Under such conditions it would be quite likely sometime or other to pass very close to the ruddy planet. Now if such an encounter were to take place, and if at the climax of the encounter the direction of motion of the comet relative to Mars were nearly that which a satellite of Mars would have in direct revolution, and if the comet's velocity were not too great, and the conditions were such that the perturbing attraction of Mars would turn the comet's course in the right way to swing it from its cometary path into a satellite orbit, then would the comet be captured and transformed into a satellite. Having once entered the satellite zone, the comet would not be able to get out again. In order to simplify the discussion, we may suppose this to be the first satellite to be acquired by Mars.

When first captured, a comet is likely — indeed, almost certain — to move in a planetocentric orbit which is crude and ill-adjusted. The plane of its motion around the planet may be inclined to the planet's equator at a high angle and its orbit may be much more eccentric than is characteristic of sober, well settled satellites. This is to be expected. It is a mere incident of the transition. But these irregularities of adjustment do not last. For the forces of determinate stability straightway set to work to reduce them.

Remembering that the only orbit of stable revolution for a single satellite is at the inner limit of stability, as defined above, and that this inner orbit is given a definite place and character by the original forces and conditions of determinate stability, we see that as soon as the comet is captured and begins revolving in a closed orbit around the planet, the forces of stability begin to reduce the size of its orbit and its eccentricity toward the circular orbit at the inner limit. The equatorial bulge of the planet also begins to pull the plane of the satellite into coincidence with the equatorial plane, and although these changes all progress very slowly, they go on continuously and in time bring the satellite into an almost perfectly circular orbit at the inner limit of stability and in the plane of the planet's rotation. At first, the satellite's periplaneta might be outside of the inner limit, but in the course of reduction about half of its orbit would come to lie inside and half outside of the inner limit. When this stage of adjustment is reached, the forces manifestly tend to cause the orbit to expand in periplaneta and to contract in apoplaneta. The continual alternation of minute factors of change acting in this way finally bring the satellite's orbit into an almost perfect adjustment.

Of course, the great outer planets with their very wide satellite zones stand far better chances of capturing comets than the smaller inner planets with their narrow zones. It is comparatively easy for Jupiter or Saturn to catch and retain comets that come near them, but the Earth's satellite zone is not over 380,000 miles broad between inner and outer limits. It is not broad enough for two satellites. For a comet to come into this relatively narrow zone with just such velocity that it shall stay in and not immediately dash out again and be permanently lost is a very delicate operation. Considering this fact, it can not be doubted that Mars and the Earth have both had many encounters with comets in which, as we may say, a capture was attempted, but failed. The intended victims got away; the grasp of the little planets was not strong enough to hold them. The satellite zone of Mars is only about 405,000 miles wide, which is but little wider than that of the Earth.

According to Moulton's outer limits, as given in the paper referred to above, the satellite zone of Jupiter is about 21,988,000 miles broad, and that of Saturn is over 5,000,000 miles broader. From these facts it is evident that captured comets may perform all kinds of antics within the limits of these great zones without being able to get out again. But it is not intended to convey the impression that every comet that enters the zones of the greater planets is captured and retained. The result always depends upon the comet's velocity and other circumstances of the encounter.

I have not had convenient access to the periodical literature of astronomy and my acquaintance with it is slight. I do not know who first suggested the idea that some of the satellites may have originated by the capture of perturbed asteroids. The idea seems like a not very remote sequence to Kirkwood's suggestion that perturbed asteroids become short-period comets, a suggestion which he published at least as early as 1887, as pointed out above.

The best and earliest discussion of the origin of satellites by the capture of perturbed asteroids that is known to me is that of Professor E. Miller of the University of Kansas. His paper is published in Nos. 38 and 39 of *Popular Astronomy*, 1897. My own adoption of this idea was eleven years earlier.

Discussing the origin of the satellites of Uranus and Neptune, Miller observes that "the comets moving along orbits that were at certain points close to the orbits of the planets, were so completely captured as to have their cometary orbits destroyed, and they themselves transformed into moons, ever after to exist as the moons or comet-satellites of Uranus and Neptune. In this manner comets transformed into moons or comet-satellites would, by the superior attraction of the planets and their proximity necessarily change the character of their orbits from that of the parabola to that of the ellipse."

Miller notes in his article that the orbit of the asteroid Aethra (132) lies very close to the orbit of Mars at one point and that there will come a time when they will be very near each other. "Then the attraction of Mars, at such close range, will be sufficient to change forever the direction of motion, and the plane of the orbit of the asteroid. What will then be the fate of Aethra? Under the conditions named, the asteroid will be transformed into a moon revolving about Mars as its primary, and so constituting one of the family of the planet."

Again, he observes that "The asteroidal belt may, for all we know, extend all the way from the orbit of Mars to that of Jupiter and the liability that some of them should be captured by their giant neighbors becomes almost, if not altogether, a certainty.

"Then if all of the foregoing be true, it may be affirmed with a reasonable degree of probability that the two moons of Mars, Deimos and Phobos, at one time in the far distant past, were members of the asteroid group. Their size also seems to indicate their origin, one of which, the larger, being not more than 16 miles in diameter, and possibly only seven miles. The time of revolution of Phobos the inner moon is 7 h. 39 m. That is to say it revolves about Mars a little more than three times every 24 hours, and presents all the different phases of new moon, first quarter, full moon and last quarter, at each revolution."

Professor Asaph Hall in his recent address as president of the American Association (*Science*, Jan. 2, 1903) refers to the probable capture of comets by planets and their transformation into satellites, and also to the intimate relation between asteroids and periodic comets.

Professor W. H. Pickering, in the prospectus of his forthcoming work on "The Moon," adopts an asteroid origin for the satellites of Mars, but rejects origin by capture for the Moon.

CHAPTER XIII.

THE ACCESSION OF THE SECOND SATELLITE.

THE discussion in the preceding chapter was confined to the case of a planet capturing its first comet and installing its first satellite. It was found that this satellite would gradually settle down to stable revolution in the first or inner orbit of stability. We have now to consider what will happen if the planet should capture another comet after the first satellite had been established. This problem raises some new and difficult questions. How will the new comet be taken? How will it adjust itself to the existing system? How will its advent affect the established satellite?

Suppose Mars had only one satellite. Phobos, the present inner satellite, revolves presumably in the orbit which a single satellite would take. The forces of determinate stability prescribe absolutely the place or distance at which the outer of two satellites will have to revolve, but they do not show whether the first satellite acquired or the second one acquired shall occupy that position. There appear to be only two alternatives open; the second comet must either adjust itself outside of the established satellite and settle directly into the second orbit of stable revolution, or, by pressing in between the planet and the first satellite, gradually press the latter out to second place, at the same time establishing itself in first place at the inner limit.

There is perhaps a very remote possibility that the second comet could successfully adjust itself outside of the established member, but such a method of induction into the family of a planet must be rare indeed. By this method the second comet would have to begin its revolution around the planet in almost perfect adjustment to the second orbit of stability at the very start. Velocity, direction, distance and plane would all have to be just right, and that too under conditions which make it almost absolutely certain that such would not be the case. As in the case of the first satellite, the beginning would nearly always be with relatively high eccentricity and ill-adjusted plane. So remote is the likelihood of a successful accession of a second satellite in this way that it may be put aside for the purposes of this discussion.

Besides, when we consider the usual great difference between the masses of the planets and their satellites, it is apparent that when a comet is first captured it is almost wholly domineered by the attraction of the planet. The influence of the established satellite upon the new arrival is at first extremely slight, both because its perturbative impulses are very feeble, and because they are at first unsystematically and therefore disadvantageously applied. The second comet approaches the planet substantially as though no earlier satellite were present. Ordinarily, it is only by a long continued series of perturbative impulses that the satellites affect each other in such a degree as to cause important changes in their mutual relations or in their relations to the planet.

It seems certain that the second comet, when first captured will begin to revolve in a highly inclined and relatively eccentric orbit, and this orbit is likely to lie at the start partly inside and partly outside of the orbit of the established satellite, which revolves at the inner limit of stability. It soon comes into this relation if it does not have it at first. Then the original forces of determinate stability, dependent on the mass and orbital revolution of the planet, domineer, and they immediately begin the process of reducing the orbit of the second comet down to the inner limit of stability just as they did for the first satellite. The forces which produce this change far outweigh all the influences which the established satellite can exert to counteract them.

Every time the second comet swings through periplaneta it passes inside of the established satellite's orbit, and many of those times *between* the planet and that satellite. Every time it does this it in effect produces a slight augmentation of the mass of the planet, and this, as we have seen, tends to drive the established satellite out a little from the planet and causes it to expand its orbit by that much. The effects are exceedingly slight at any one round, but by continual repetitions they finally produce important changes. Thus the first satellite, without being seriously jarred or disturbed in its even relations to the planet, is gradually driven out into a larger and larger orbit, while at the same time the original forces of stability are gradually reducing the eccentric revolution of the new member. Though the original forces tend to bring their planes together, the mutual perturbations of the two satellites tend to keep them apart so long as the apoplaneta of the second satellite lies outside of the orbit of the first one.

Obviously, as these changes go on, more and more of the second satellite's orbit comes to lie inside of the expanding orbit of the first one, until finally it lies wholly inside. Thenceforth the forces of determinate stability act to reduce the eccentricity and inclination of the second satellite's orbit and finally make it nearly a circle at the inner limit of stability, with its plane in the plane of the planet's rotation and coincident with that of the first satellite.

Thus, insidiously, the second comet worms and squeezes its way into a position between the previously established satellite and the planet, pressing the first satellite out to a larger orbit, and when the process is over, the original or first satellite is seen revolving in the second orbit from the planet, while the second comet has become established as a new satellite in the inner orbit of stability. So long as this process is going on, the planes of the two bodies can not coincide; they acquire coincidence later.

The conclusion seems clear, therefore, **that satellite systems grow by the accession of new members at their center, the last arrival always entering and establishing itself in the first orbit of stability next to the planet and pressing the pre-existing satellites out one step, causing them to expand their orbits.**

Supposing the planet's satellite zone to have a capacity for more than two satellites then the accession of the third and fourth and later ones must be by the same process. They must all enter by establishing themselves first in the inner orbit of stability next to the planet. Manifestly, it would be much more difficult for the third or a later satellite to catch on and establish itself outside of the existing members than for the second satellite. So in large systems, like those of Jupiter and Saturn, it is certain that when the system is full, newly captured comets can not attach themselves at the outer limit of the system; nor can they by any possibility squeeze themselves into the space between any two members of the established system. For although the spaces may seem wide enough, as in the outer part of Jupiter's system, the members are all very delicately adjusted to and dependent upon their next inner neighbors, and all rest in a certain delicate way upon the inner satellite, which is next to their common primary and is supported and given determinate stability by it.

CHAPTER XIV.

THE LOSS OF SATELLITES.

§ 1. **The expansion of systems already full.** When a great satellite system like that of Jupiter or Saturn is full and complete so that its satellite zone contains all the satellites which it can hold, it might be thought that no more comets could be captured. But such is not the fact. The next comet to come into the net is captured in precisely the same way as the second, third and later ones. This time, however, a new change takes place. The new arrival presses in at the center as before, and gradually drives all the previously established members out one step. The outer member was perhaps dangerously near to the outer limit of stability before the last capture was made, so that as expansion goes on this satellite is pressed out farther and farther, until at last it fails to pass the radius vector of the planet. Then it drifts away and is permanently lost to that planet. It falls out of its systematic relations and once more enters upon a cometary career. Thus comets may originate from the loss of satellites as well as by the perturbation of asteroids.

Satellite systems grow, therefore, by the accession of new members at their centers, by the coincident recession of all the previously established members into larger orbits and by the casting off of one member at the outer limit at each step of expansion. It follows that it is each time the oldest member that is lost, while the new arrival always enters at the center of the system.

The loss of satellites in this manner goes some way toward explaining the comet families so obviously related to the superior planets, though in all probability those that belong to Jupiter have been derived largely from the asteroids. It may be true, however, that on account of the nearness of the supply of asteroids the satellite system of Jupiter has undergone unusually rapid growth and that some of the comets of this planet's family are really lost satellites, although originally derived from the asteroid ring. It seems quite probable that Tempel and Halley's comets are lost satellites of Uranus and Neptune respectively. Their retrograde revolutions point strongly to this origin.

§ 2. **Origin of the comet Eros.** So far as known at present there is only one case in which special evidence points strongly to the identification of a lost satellite. This is the so-called new planet Eros. But Eros is in truth not a planet, for it has passed out of the systematic relations which belong to planets and has now the unsystematic character of a comet. Its path lies partly within and partly without the orbit of Mars. The diameter of Eros is supposed by some to be not over twenty-five miles and may be considerably less, which makes its size nearly the same as the present satellites of Mars.

Oppolzer discovered the variability of the light of Eros, and Andre and Professor E. C. Pickering have studied its peculiarities with much care. Dr. S. I. Bailey has given a good popular account of

Eros in the *Popular Science Monthly* for April, 1901. But in a later note in the same journal for August of the same year, the following additional facts of special interest are stated:

> «From a variable planet, having an extremely short period and larger range of variation, Eros recently became invariable. In Europe, soon after the discovery of its variability, its range was said to be two magnitudes, that is, it shone with about six times more light at maximum than at minimum.»

Deichmuller found the light period of Eros to be about two and one-half hours. But Andre and Pickering found the successive maxima to be unequal and Andre makes the period between equal maxima to be five hours and a quarter. The smaller maxima are nearly but not exactly midway between the larger ones.

Andre thinks Eros is a close double, and even calculates their orbits. He finds that their dimensions differ but little, and characterizes them as «very elongated ellipsoids» with marked meridional flattening. He notes also that their period is very near that of Phobos (seven hours and thirty-nine minutes.) But it has been pointed out by others that occulting doubles can reduce their light no more than one-half or 0.8 of a magnitude, while the variation observed is nearly two magnitudes. This seems fatal to Andre's interpretation. (*Popular Astronomy*, No. 85, 1901.) Pickering favors the view that Eros is a rotating, much elongated or cigar shaped body.

Through its slower changes of magnitude the short periodic variations appear to keep the same value. This may be regarded as a certain indication of either rotation by a single body or revolution by doubles. Unless Eros is a double, the slower irregular variation of magnitude seems to be inexplicable on the assumption that Eros is a planet. If, on the other hand, we regard it as a comet, which has thus far developed only faint rudiments of a tail, it is not hard to believe that the irregular variations are associated with that development. The irregular variations of magnitude would appear to have a periodic variation in the sight of the observer in consequence of axial rotation of the nucleus. This would be the case if the light is emitted, not from the whole surface of Eros, but from one or two limited areas or parts of its surface. Then if the light emitted from these parts underwent marked variations, say in a number of days or weeks or months, the combination of these variations with rotation would produce the effects observed.

Supposing Eros to be a single body, one of the most important things to account for is its appearance of axial rotation. There seems to be no possible way in which a small asteroid could acquire so high a rate of axial rotation. But there are excellent reasons why a lost satellite of Mars might have rotation in a very short period.

As the Moon revolves around the Earth it keeps one face constantly toward the planet, and therefore maintains an attitude of «fixed gaze.» This gives the Moon a rotation on its axis once in a month considered with reference to the stars. Other satellites are suspected of having the same attitude toward their primaries. If Phobos, the observed inner satellite of Mars, does this at the present time it must acquire rotation in a period of about seven and one-half hours.

The statement was made above (page 96) that there is a possibility of Mars having at the present time another satellite between Phobos and the planet. If there is such a satellite its period would be considerably shorter than that of Phobos, and if it acquired the attitude of fixed gaze toward Mars and kept this until it were lost it might show a shorter rotation period than that of Phobos; and this even if it lost a little of its original speed of rotation before the final separation. The shorter rotation period of Eros, supposing it to have been a satellite of Mars, seems to suggest an orbit nearer than that of Phobos. It is possible that when Eros, as the inner satellite of Mars, came to be forced out to second place by the capture of another comet, it kept the same rate of rotation that it had in first place. Then when it was forced away from Mars and lost, to start on a new cometary career, it would certainly take with it whatever rate of axial rotation it had at that time.

Further observations may prove, however, that Eros is merely a perturbed asteroid, but some of its characters certainly seem to point strongly to a former state in which it was a satellite of Mars.

If this idea of Eros is correct it seems to furnish some confirmation of the manner of growth of satellite systems outlined above, namely: by capture and installation at the center with expansion and loss at the outer limit.

The production of comets' tails is probably associated with something like volcanic eruption from the nucleus. We need only to look at the face of the Moon to see this. If such eruptions on Eros were on opposite sides, nearly 180° apart, and varied in intensity, they might account for the unequal maxima observed.

The general relations of Eros also incline strongly and independently to the view that it is a lost satellite of Mars. For its mean distance from the Sun is only 135 millions of miles, while that of Mars is 141.5 millions. How much more closely it is associated with Mars than with Jupiter may be inferred from the fact that its aphelion is 166 millions of miles from the Sun, while Jupiter's perihelion is 462 millions of miles with a mean distance of 483 millions. The aphelion of Mars is 148 millions of miles, which lacks only about 18 millions of miles of being as far out as the aphelion of Eros. Eros therefore comes nowhere near to Jupiter. The attitude of the comet's plane, however, is such that Eros does not come nearer to Mars than 20 millions of miles, while it comes within less than 14 millions of miles of the Earth. But this might easily have come about by perturbations affecting its plane subsequent to its loss from Mars.

On this view, then, Eros is now a comet, but sometime since was a satellite of Mars, and was its inner satellite for a time long enough to acquire the rate of fixed-gaze axial rotation belonging to that orbit. Before that, no doubt, it was an asteroid and may have been perturbed out of its orbit by either Jupiter or Mars. But whether Eros was formerly a satellite of Mars or not, it is now a comet and not a planet.

§ 3. **Retrospect and prospect.** We have now before us a discussion of the quality and mechanism of determinate stability, and a discussion of its application to satellites and to satellite systems. The theory which I have endeavored to present through the medium of this discussion rests solidly upon the same foundation as the current theory—upon Newton's laws of motion and his law of gravitation—and it does not require anything more than these. No new or mysterious force or combination of forces is invoked. The whole difference between the two theories grows out of differences in the method of analysis. The current theory proceeds by the method of difference of attraction, and as a result finds stability to be indeterminate, while the theory here set forth proceeds by a direct analysis of the opposing forces of the epicycle and finds stability to be determinate; that is, for a planet of given mass and distance from the Sun, the orbit of stable revolution is fixed at a determinate distance from the planet, where the opposing forces are at a balance. It finds, further, that these forces are not at a balance in any other orbit around that planet; that in the epicycles corresponding to orbits at different distances from the planet the heliocentric centrifugal force varies at a higher rate than the effective heliocentric attraction, thus causing instability with a tendency to contraction in orbits *larger* than the determinate orbit, and instability with a tendency to expansion in orbits *smaller* than the determinate orbit. This relation of forces gives stability in the determinate orbit the quality of a stable equilibrium. This is the foundation of the present theory.

The results growing out of the application of the two theories are vastly different, and in nothing do they differ more than in this, that the current theory in the hands of Newton and of all the philosophers, astronomers and mathematicians since his time has never yielded the first suggestion of a method of growth or origin for satellites or satellite systems. The recent suggestion by Miller and others of the origin of satellites by the capture of comets has not grown out of the current theory of stability, but out of certain suggestive facts of observation, which gave strong hints of such relations.

On the other hand, the present theory shows a quality of stability which of itself suggests a simple method of growth for satellite systems. For it shows the existence of the satellite zones with definite inner and outer limits within which any adventurer is likely to be captured and retained. We have seen, also, how further growth takes place by later captures and expansion, and how the outer limit of growth is defined.

Perhaps the road we have traveled from the beginning of this study will seem like a long and tiresome one, but it has been straight; and the while we have struggled against petty difficulties encountered by the way, we have been gradually uplifted — perhaps insensibly, but yet irresistibly — to a most inspiring eminence. Before us lies the Solar or Planetary system, — the Sun with all his magnificent retinue of attendant bodies — the planets, satellites, comets, meteors and meteor swarms. To explain the structure, origin and growth of this great system is indeed an inspiring theme. Yet, with the explanation of the growth of satellite systems before us as outlined above, we may, I think, now turn to this greater problem with an assurance of success hitherto unknown.

PART II.

THE STRUCTURE AND GROWTH OF THE PLANETARY SYSTEM.

DIVISION I.

THE STRUCTURE OF THE PLANETARY SYSTEM.

CHAPTER XV.

THE SUN'S GREAT ORBIT.

WE ARE now prepared to undertake the interpretation of larger things. The Sun is merely a planet grown great and the planets are its satellites. The general similarities of arrangement and of motion in the satellite systems and the Planetary system are so close and so complete that one can not fail to recognize the obvious inference that the mechanism of stability is the same for both. The logical basis for this conclusion appears to be unassailable. Compare, for instance, the system of Saturn with that of the Sun. Both have satellites of widely different masses and both have rings of lesser bodies — the asteroid ring of the Sun and the meteoroid ring of Saturn — though they are not quite the same in their relations to their respective primaries. Considered merely as primaries with their attendant secondary bodies revolving around them, there is no fundamental difference between the two systems. Such differences as exist are chiefly those of scale, due to the vastly greater mass of the Sun.

If the stability of the Moon and of Phobos and the inner satellite of Jupiter are made determinate, as set forth above, it seems an easy step, and indeed the only logical one, to conclude that the stability of Mercury with reference to the Sun is made determinate in precisely the same way. The mechanism of the Moon's stability is not special to the Moon's case, nor to that of the inner satellites of our planets; it is a universal type.

But if the stability of Mercury is made determinate in the same way as that of the Moon, then obviously, in order that the mechanism may be the same it is necessary to suppose, not merely that the Sun moves in space, as has been proved by observations, nor even that it moves in a straight line, nor in a path of aimless, irregular wandering, but that it moves in an elliptical orbit of great size and small eccentricity, probably nearly circular. Such motion can be ascribed only to the constant deflecting force of some great but very distant mass.

The conclusion therefore is that the Sun must revolve in an elliptical orbit of small eccentricity around some star or small cluster of stars. The radius of this orbit must be very great indeed, and the period of revolution in it must be extremely long. The Sun at the present time is in some part of that great ellipse and has a definite velocity and curvature as it moves. What these are we have no accurate means of knowing at the present time. A mathematical treatment of the problem however, ought to afford some idea—at least a rough approximation—of the elements or some of the elements of the Sun's great orbit. For the revolution of Mercury and the spacing of the planets furnish an expression for the *intensity* of the Planetary system from which, by the inverse, may be deduced the intensity of the Sun's motion through space. Present data are probably insufficient to give any accurate or reliable indication of either the direction or rate of curvature of the Sun's path in space; and the determinations of the Sun's velocity are of uncertain value,—probably too great—though the general direction of motion toward a point in the constellation Hercules seems well established. However, if an assumption be made as to the Sun's velocity, then the curvature necessary to produce that value of deflecting force, which corresponds to the deduced intensity of the Sun's motion and the observed intensity of the Planetary system, ought to be attainable, though it would still remain unknown as to which way the Sun's path curves among the stars. It ought to be possible to attain a fairly reliable solution of this problem by a comparison of the systems of Jupiter and Saturn with that of the Sun. The different estimates of the Sun's velocity in space range between about 5 and 27 miles per second, the later determinations tending to values less than the mean. There is no more pressing problem in astronomy than that of the Sun's motion in space. Repeated and careful determinations of the Sun's goal ought in the course of time to reveal, by its progressive displacement on the celestial sphere, the direction of curvature of the Sun's motion and the plane in which its orbit lies. Once this is accomplished, the identification of the Sun's companion star would be settled.

In Proctor's "Old and New Astronomy" the determinations of the Sun's goal plotted up to 1890 inclusive show a suggestive tendency to cluster into earlier and later groups. The earlier group, comprising those before 1880, center at a mean goal somewhere near long. 17 h. 20 m. and decl. N. 35°. The later series, made after 1880, center near long. 18 h. 50 m. and decl. N. 40°. But it is altogether impossible that the direction of the Sun's motion can have changed so much in so short a time. Indeed, it is not conceivable that the amount of change in that time could be detected at all. While the determinations made will some day be of great value, they appear at present to reveal nothing as to the direction of the Sun's curving.

Another most important question which would find solution at the same time is the relation of the mean plane of the planets to the plane of the Sun's great orbit—at what angle they are tilted and whether the revolution of the planets is direct or retrograde.

That the great orbit of the Sun is nearly circular, or at least of small or very moderate eccentricity, seems certain. For if the Sun revolved in a very eccentric orbit it could not retain the planets through even one revolution. The difference of the intensity of the Sun's revolution between the extremes of its orbit may be considerable. Yet, whatever its value, the whole Planetary system must undergo corresponding contraction as the Sun goes from periastron to apastron and expansion in the opposite order of change, in the manner of the Moon's present annual inequality. The very existence of the Planetary system is proof that the eccentricity is not large. For with high eccentricity the Sun would lose all its attendants every time it dashed through its periastron.

CHAPTER XVI.

BODE'S LAW.

IT IS quite the fashion with many astronomers at the present time to decry Bode's law, which expresses so well the orderly spacing of the planetary distances. They reject it as invalid, because Neptune departs widely from it and Uranus and Saturn in small degrees. But Bode's law is in truth the most important law of all. For it is a comprehensive expression for the mass and motion of the Sun, and more information is embodied in it than in any other single statement that can be made. This is because it is the expression of both the scale and intensity of the system. The distances of the planets from the Sun are shown in the frontispiece to this volume by the figures in the columns on the right.

The discrepancies for Uranus and Saturn are slight, and except for Neptune, the law holds very closely for all the rest of the system. It may be that the law as expressed by Bode is not precisely correct. But there is a law for the spacing of the planets and its expression is determined by the factors set forth above.

With the small or inferior planets inside, the great or superior planets outside and the thin ring of minute asteroids between, the Planetary system presents a very remarkable structure. The most striking thing is the wide differences of mass which characterize the various members or units of the system, and the fact that such differences can exist while the system still conforms to Bode's law. The fact that Jupiter and his great companions beyond receive the impulses necessary to their stability through the little planet Mars and the still smaller asteroids, shows how slight the requisite impulse is, how slight is Jupiter's tendency to orbital contraction and, within wide limits, how independent of mass is the spacing of the planets. It seems to suggest too that there may be a myriad of smaller asteroïds still undiscovered—enough altogether to compare in mass with Mars or the Earth.

The retrograde satellite systems of Uranus and Neptune were found above to be due to the rotation of the mean plane of the planets in combination with a greatly weakened orthogonal force and a very strong persistence of the planet's plane of rotation and of the satellite planes. It is also to be remembered that Mercury revolves at the relatively high mean velocity of 29 miles per second and has no satellite. The chief significance of these two facts and of Neptune's departure from Bode's law would be missed if we did not see that they all depend upon the relatively low intensity of the Sun's revolution in its great orbit. The Sun's intensity of revolution is low, because the deflecting force which bends it out of a straight line as it moves through space is a relatively feeble force; the deflecting body is very far away and the power of its attraction upon the Sun is greatly reduced by distance. It seems certain that the velocity of Mercury around the Sun must be many times greater than the Sun's velocity around the deflecting star.

It is quite probable that there is one or perhaps more than one undiscovered planet beyond Neptune. These most distant members, with Neptune and Uranus, form the peripheral portion of the system, where the forces of order become weak or fail almost entirely. Rigorous spacing by Bode's law would place Neptune so far beyond Saturn as to suggest that the perturbative impulses of the latter would not be sufficient to maintain stability. Hence, Neptune has contracted its orbit and come in enough nearer to receive the requisite support. If there are other planets outside of Neptune they are probably still more out of accord with Bode's law.

In studying the satellite systems we found that they are spaced according to their intensities and the masses of their primaries. The law of their spacing is simply Bode's law applied to the relatively small scale and various intensities of their systems. It is the same law as that which Bode found for the planets. The application of the law to the satellite systems would seem to suggest that there remain to be discovered many more members in Saturn's system, and probably nearly as many for those of Uranus and Neptune. By mathematical treatment it ought to be possible to make a nearly accurate count of the undiscovered members, with a fairly accurate placing of the orbit of each.

A new satellite of Saturn was discovered by Professor W. H. Pickering in 1899. "This satellite is three and a half times as distant from Saturn as Iapetus, the outermost satellite hitherto known. The period is about seventeen months and the magnitude of fifteen and a half." (*Popular Astronomy*, No. 64, 1899.) Iapetus is about 2,225,000 miles from Saturn and the distance of the new satellite is therefore about 7,787,500 miles, which is still very far from the outer limit of satellite revolution assigned by Moulton.

The system of Saturn is the only satellite system which seems to present facts out of harmony with Bode's law. But I think it may be confidently expected that the apparent gaps in this system will ultimately be filled by the discovery of more satellites. It may be that some of the gaps are occupied by meteoroid rings like the great rings, only too thin to be seen. The greater eccentricities of Titan and Iapetus and especially of Hyperion, and also the smaller inclination of Iapetus to the ecliptic, as compared with the eccentricities and inclinations of the five inner satellites and the rings, are quite in harmony with the weakening of the forces of stability toward the periphery of a very wide system. But it is possible that some of these irregularities have been caused by the perturbations of a large comet or comets which may have dashed through the outer part of Saturn's system without being captured.

CHAPTER XVII.

GROWTH BY CAPTURE AND EXPANSION.

MERCURY, then, is at the inner limit of stable revolution for planets—the determinate orbit of stability around the Sun. If the Sun had but one planet it would revolve in the present mean orbit of Mercury, and would therefore occupy first planetary position. Venus holds second planetary position, the Earth third, and so on. In short, the Planetary system has precisely the same mechanism and fundamental structure as was found for the satellite systems in the discussion of the satellite zones. Under the operation of the forces of stability as affecting the second, third and later satellites, we have the planets spaced according to Bode's law, and the Planetary system, at least as far out as Neptune, is full and complete. Considering the delicate nature of the mechanism, it is plain that the adjustment of the planets in the system is such that a new planet could by no possibility work its way into a permanent orbit between any two of the members now established. The possibility of the accession of new planets by additions at the extreme outer limit of the system is equally remote. The only place where a new planet could be received and adjusted in a permanent planetary orbit is at the center. The body which is to become a new planet must work its way in between Mercury and the Sun, and by gradually driving Mercury and all of his older companions out one step, take the place which Mercury holds at first planetary position—in the orbit of determinate stability. Thus, by the advent of a new planet at the center of the system, all the previously established planets are made to expand their orbits one step and take new positions farther from the Sun. The planetary zone has an outer limit corresponding to the outer limit of satellite zones, and when a full planetary system is made to expand by the inauguration of a new planet, the oldest or outer planet of the system is forced out and lost.

There is a remote possibility that a planet thus lost may re-enter the Planetary system. But if it does it must be on an even footing with lost asteroids and lost satellites; it can only re-enter as a comet, to be recaptured and inaugurated as a new planet at the center of the system as before.

Comets are the seed of planets just as they are also the seed of satellites. Planets originate by the capture of comets at the center of the Planetary system and they originate in no other way.

There is one other important source of comets. They enter the Solar system from the realms of outer space. Indeed, all of them are ultimately traceable to this source. But these comets are subject to the same laws as the others, and if any of them become satellites or planets it must be in the regular way.

Perhaps the possibility seems very remote that a comet like any of those now known should ever become a planet by such a process. But, as has been stated above, comets are not systematically related to the Sun and are therefore in reality in a condition of instability. In connection with the capture of comets by planets we have seen that the forces of stability tend to gradually reduce the orbit of every comet to the orbit of determinate stability—in the case of the Planetary system to Mercury's mean orbit. The process is a very slow one, and may be much delayed and interrupted by the perturbations of the planets, but it goes on nevertheless, and while many comets fail to attain planethood, there is one occasionally that succeeds in accomplishing the whole reduction of its orbit and finally becomes a planet.

At the present time apparently the most promising candidate for planethood is Encke's comet. This comet revolves in the direct order in a period of about three and a half years. Its aphelion lies among the outer asteroids and its perihelion barely inside of the orbit of Mercury, about 380 millions of miles and 31 millions of miles respectively from the Sun. Its eccentricity is 0.845, and its plane is inclined 13° to the ecliptic.

For comparison with Encke's comet the following distances are given. Mercury's mean distance from the Sun is 36 millions of miles, with extremes of 28.5 millions and 43.5 millions. The mean distance of Ceres, the largest asteroid, is 257 millions, while the mean distance of Jupiter is 483 millions with a least distance of 462 millions. (Young.) *Encke's comet is the only comet of short period which has its perihelion inside of Mercury's mean orbit.*

A peculiarity which has excited much interest in this comet has been the progressive shortening of its period. When several successive returns of Encke's comet had been observed, it was found that it did not come back to perihelion on its appointed or calculated time, but *always a little ahead.* At first it was calculated to come about two and a half hours ahead, but in 1868 new calculations seemed to show only half this much gain. Winnecke's comet was formerly supposed to show the same kind of change in its period, but later calculations seem to disprove this and, in fact, show such wide discrepancies in the determinations of its periodic time that the detection of differences so slight could hardly be relied upon. Todd says that nearly all of the short-period comets are invisible to the naked eye and that only about half of them have as yet been seen at more than a single return to perihelion. It requires several returns of a comet, each one observed with the greatest care, to afford a safe basis for determining whether it may not be shortening its periodic time a little at each round. The observations of the other comets are as yet too few for a fair comparison with Encke's in this respect. This means, of course, that Encke's comet is slowly contracting its orbit and drawing gradually nearer to the Sun. This change, small as it is, and slow as it would be in producing alterations of large magnitude, is nevertheless very suggestive and points to possibilities of the highest importance directly in the line of the present theory.

From this point of view the acceleration of Encke's comet is not due to a resisting medium in space nor to resistance by passage through a meteoric swarm, nor can it be due to perturbations of planets or other comets. It is the result of the action of the original forces of stability which tend to reduce the orbit of the comet gradually to the orbit of determinate stability—the present mean orbit of Mercury. The comet is at present in unstable adjustment. The perturbative impulses which it receives from the planets do not support it in its present orbit, as they do the planets outside of Mercury, but give it for the most part only minor irregularities. Having its plane inclined considerably more than the planes of the planets, it is in little danger of colliding with them. As reduction of orbit goes on it will finally bring its aphelion inside the orbit of Mars, then inside that of the Earth, then inside that of Venus and finally inside that of Mercury, all the while keeping its perihelion inside the orbit of Mercury. If it shall accomplish all this without mishap it may become a planet, provided its mass be sufficiently great to drive Mercury out to second planetary place. After getting wholly inside of Mercury's orbit, the inclination of its plane will be gradually reduced until it is the same or nearly the same as that of the other planets.

Now what may happen to Encke's comet in this way has occurred to other comets before Encke's, and they have become planets, each to go through the same steps of expansion and final loss at the outer limit of the Planetary system.

It will be assumed here, therefore, that comets do, in fact, wind their orbits down to first place and thus become captured and transformed into planets in the manner described. If they do not, then the analogy to the growth of satellite systems, otherwise complete, breaks down, and no other source of planets seems possible. Excepting for Encke's comet, the lack of a sufficient body of reliable data makes present discussion of the contraction of comets' orbits rather unsatisfactory and the conclusions tentative so far as they rest on observations.

CHAPTER XVIII.

Some of the Facts to be Explained.

According to the manner of growth here set forth, the oldest member of the Planetary system is that planet which is now farthest from the Sun, and the youngest member is nearest. Mercury is the youngest of the planets, Venus the next older, the Earth next, and so on out to Neptune, which is the oldest of all. With such a process of growth we can not think of the Planetary system as having been made once for all out of condensing nebular rings, either gaseous or meteoric, and as having remained ever since substantially *in statu quo;* nor can we think of it as continuing indefinitely as it is today. It has grown in the past and it will continue to grow in the future.

Before proceeding to apply this theory in explanation of the varied phenomena of the Planetary system, we may pass in brief review the chief characters to be explained. The following table shows the mean diameters of the planets in miles, their distances from the Sun in millions of miles, the eccentricities of their orbits, the times of their axial rotations in hours and minutes, and the inclination of their orbits to the ecliptic in degrees and minutes. The table is taken from Professor C. A. Young's "General Astronomy" (1888). Ceres is the largest of the asteroids.

PLANET.	DIAMETER.	DISTANCE.	ECCENTRICITY.	ROTATION.	INCLINATION.
Mercury.......	3,030	36.0	.20560	?	7° 00′
Venus.........	7,700	67.2	.00684	23h 21m	3 23
Earth.........	7,918	92.9	.01677	23h 56m	0 00
Mars..........	4,230	141.5	.09326	24h 37m	1 52
Ceres.........	100?	257.1	.07631	?	10 37
Jupiter........	86,500	483.3	.04825	9h 55m	1 18
Saturn........	71,000	886.0	.05607	10h 14m	2 29
Uranus........	31,900	1781.9	.04634	?	0 46
Neptune.......	34,800	2791.6	.00896	?	1 47

A more recent and probably more accurate determination of the diameters of the planets was made by Professor E. E. Barnard at Lick Observatory between 1891 and 1895. His measurements are given in the following table. (*Popular Astronomy*, No. 46, 1897, page 300.)

PLANET.	DIAMETER.
Mercury	2,765
Venus	7,826
Mars, equatorial	4,352
Mars, polar	4,312
Ceres	485
Jupiter, equatorial	90,190
Jupiter, polar	84,570
Saturn, equatorial	76,470
Saturn, polar	69,780
Uranus	34,900
Neptune	32,900

In these tables the great contrast between the diameters of the inferior and the superior planets is well shown. It will be noted that the later determinations of Barnard make Mercury and Neptune considerably smaller, while Ceres and Uranus especially are made considerably larger; Venus, Mars, Jupiter and Saturn appearing only slightly larger. The high eccentricity of Mercury is especially noticeable, being more than twice that of Mars, which is itself greater than that of Ceres; also the high inclination of Mercury's orbit — more than twice that of any other planet, excepting some of the asteroids.

After the great problems which include questions relating to the origin of the Sun, and after the problem of stability which includes the explanation of the cause of Bode's law, the greatest problem of the Planetary system is that of the origin of the *two great planet groups* and their relations to the asteroids and the Sun; that is, the distribution of mass in the Planetary system. Young gives the masses of the planets as follows, the mass of the Earth being 1.

Sun	331,100
Mercury	$\frac{1}{8}$
Venus	0.78
Earth	1.000
Mars	$\frac{1}{9.34}$
Ceres	$\frac{1}{70000}$
Jupiter	316
Saturn	94.9
Uranus	14.7
Neptune	17.1

The small planets, Mercury, Venus, the Earth and Mars constitute the *inner* or *inferior group.* This group stands next to the Sun. The great planets, Jupiter, Saturn, Uranus and Neptune constitute the *outer* or *superior group*, and between these two groups lies the belt of tiny, thinly scattered asteroids. Evidently the superior planets are the older of the two groups. Why are the groups thus arranged with the asteroids between? Why are the masses of the outer group so much greater than those of the inner?

Again, Jupiter stands next outside of the asteroids and is the greatest of the giant planets, Saturn is next in order and also the next smaller in size, Uranus and Neptune being still farther out and smaller. Why are the masses of the great planets thus arranged with reference to the asteroids?

Why has Jupiter the swiftest axial rotation, with that of Saturn next and both these much more rapid than those of the inferior group? What is the origin of Saturn's rings? Why has Mercury a more inclined and more eccentric orbit than any other planet except the asteroids? All of these questions find comparatively ready and simple answers, provided the growth of the Planetary system be interpreted on the theory outlined above.

What has been the probable origin and history of the Earth and Moon and the other inferior planets? These last questions can only be answered by two or more alternatives, but the possibilities are definitely limited to these alternatives.

The frontispiece to this volume shows the arrangement of the planets and their groups with reference to the Sun and the asteroids. It does not show the intervals of distance, but these intervals and the intervals according to Bode's law are set down in the two columns at the right. The Sun is at the center and Mercury is at the inner limit of stability, or, as we may say, *at the bottom.* For each of the other planets would gradually descend to that orbit if it were left with no other planet between it and the Sun.

Remembering these characteristics of the structure and arrangement of the Planetary system, let us now proceed to a brief sketch of the history of its growth.

DIVISION II.

A HISTORICAL SKETCH OF THE GROWTH OF THE PLANETARY SYSTEM.

CHAPTER XIX.

THE INSTALLATION OF THE SUPERIOR PLANETS.

ASSUMING that the system has grown by successive steps of expansion coincident with the accession of new planets at the center, it is evident that Neptune is the oldest of all the known planets. In imagination we may go back to that very remote time when Neptune was not yet a planet, but was a comet wandering among the planets of the then existing system. For there is no reason to suppose that the present planets are the only ones the Sun has ever had. There was in all probability a full retinue of planets filling the planetary zone of the Sun then the same as now. Neptune was a short-period comet in that system and was the planet-elect. Gradually it worked into first planetary position and pressed its elders out one step. Neptune was installed where Mercury is now and the next older planet had been pressed out to the present place of Venus.

Probably other comets tried to become planets while Neptune held first place, but failed. Of these no record now remains. After a time, however, another comet came and made a successful entrance into the planetary family and it pressed Neptune and the older planets out. This comet became Uranus. Neptune now occupied second place and Uranus first. Then another comet came into place to become the planet Saturn, and still another to become the planet Jupiter. At this stage Jupiter was at the inner limit of stability next the Sun, in an orbit corresponding to the present orbit of Mercury. Saturn held second place corresponding to Venus, Uranus third place corresponding to the Earth, and Neptune fourth, corresponding to Mars.

So far as we can tell from any evidence that now remains the accession of these four planets occurred without any exceptional incident. Up to this time apparently nothing happened that left any distinctive mark or peculiarity on these four planets. Or, if there were such events their marks have been overwhelmed by greater events of later date. The duration of time required for the installation of these four planets was of course enormously great.

It is perhaps worth while to inquire what sort of changes could have taken place that would leave permanent marks upon these planets — marks that would now be visible to us. Any great perturbation or other cause that might have destroyed or permanently displaced one or more of these planets would have left no such mark. We could have no means of knowing now that such an event had ever occurred. Mere perturbation in any degree short of destruction would be reduced and disappear in the course of time. The original forces of stability have the power to bring the planets back to their well ordered places after any moderate degree of perturbation. Nor can we conceive of any way in which the masses and sizes of these planets could have been notably reduced.

The only ways in which lasting marks visible to us now could be put upon these planets would be, (1) by large increase of mass and size, or, (2) by the production of a high rate of axial rotation, or, (3) by giving them peculiar or characteristic satellites to remain down to the present time. There is no reason to believe that any great event of this kind took place while the superior planets were being installed. But after Jupiter had become established in first planetary place the course of growth took a different turn which changed the appearance of the superior planets and greatly modified the general appearance, structure and future history of the Planetary system. It affected the superior planets in all three of the ways mentioned.

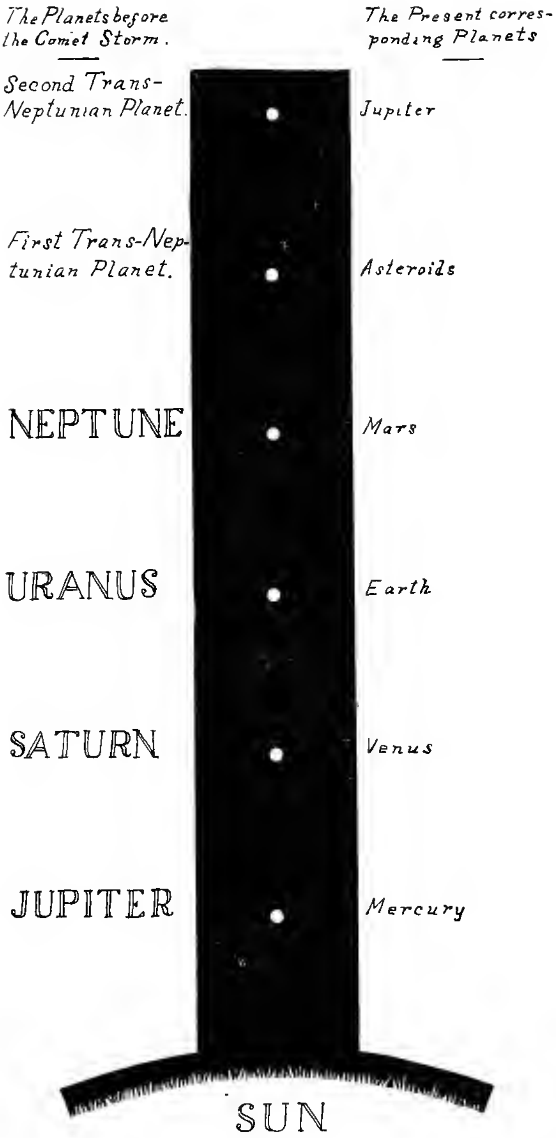
The Planets before the Comet Storm.
The Present corresponding Planets
Second Trans-Neptunian Planet.
Jupiter
First Trans-Neptunian Planet.
Asteroids
NEPTUNE
Mars
URANUS
Earth
SATURN
Venus
JUPITER
Mercury
SUN

FIG. 13.

CHAPTER XX.

THE GREAT COMET SWARM AND THE ACCESSION OF THE ASTEROIDS.

IN THE ordinary course of events it would be expected that the next accession to the Planetary system after Jupiter would be simply another comet, like the earlier ones, and that it would become a single new planet in first planetary position in the same way. But this was not the case.

At this point we come upon what I believe to be by far the most wonderful event in the whole history of the Planetary system. **The next candidate after Jupiter for planetary honors was not one comet, but a myriad host or swarm of comets. There were thousands of them, and they made a veritable storm of comets.** Their appearance in the sky was like a rain of fire. The whole expanse of the heavens was filled with them and they surged around the Sun with tremendous fury. No doubt most of them were small bodies like most of the present comets and asteroids. But what they lacked in individual size was amply made up in numbers.

Four of our present planets were witnesses of this awful spectacle. They were in the midst of it, but they survived the ordeal and they have preserved to the present day indisputable proofs of its indescribable magnitude and grandeur.

Like other comets, the individuals of this myriad swarm dashed swiftly through their perihelia as they revolved around the Sun, and each one underwent more or less of the usual disintegration, as it threw off the stony fragments which constitute in large part the tails or trains of comets. Probably a large portion of the comet swarm were totally destroyed in this way. But enough of them remained to make a most profound impression upon the structure and history of the Planetary system.

Besides the great influence of the asteroids themselves, there are to be reckoned with the more permanent and in some respects greater effects of the prodigious quantity of meteoric stones and dust which was produced by the disintegration of the comets before they became settled as minor planets or asteroids.

Through the tremendous assault of the comet swarm and the clouds of meteoric stones and dust that followed them, Jupiter, Saturn, Uranus and Neptune bravely held their places. The bombardment that fell upon these planets must have been terrific beyond all imagination. The four planets must have received many heavy blows, but, Spartan-like, they clung to their orbits and kept their appointed paths around the Sun. This of itself is proof of the elastic quality and power of determinate stability—the capacity to resist disturbing forces and to recover from their effects afterward.

The comets of the swarm all slowly contracted their orbits as Encke's comet is doing now, and at the same time, by continually dashing in between Jupiter and the Sun, began to drive Jupiter and his companions out. Though they withstood the meteoric storm, the planets gradually yielded to the pressure of the comets from within and slowly

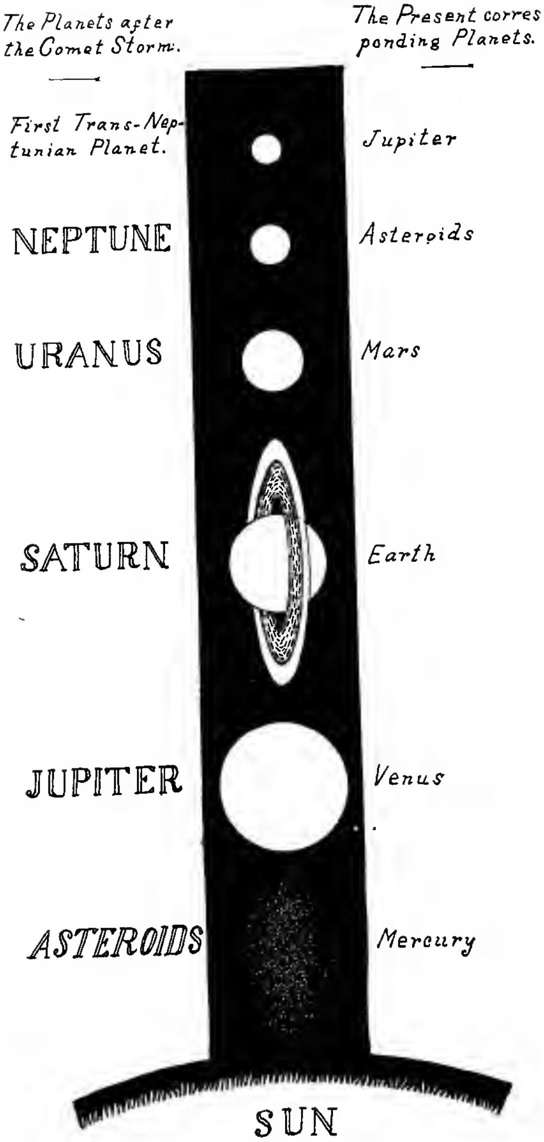
The Planets after the Comet Storm.
The Present corresponding Planets.
First Trans-Neptunian Planet.
Jupiter
NEPTUNE
Asteroids
URANUS
Mars
SATURN
Earth
JUPITER
Venus
ASTEROIDS
Mercury
SUN

FIG. 14.

expanded their orbits. In this way the planets were forced to let the comet swarm come in and establish itself in the first or inner orbit of stability. Instead of capturing one comet the Sun on this occasion captured a swarm of comets, and there followed, not the installation of *one planet of moderate size*, but of *a swarm of tiny planets.* And this is the origin of the asteroids. Their advent brought about a new order of things. The asteroids were now in first place, Jupiter in second, Saturn in third, and so on. Fig. 13 is an ideal representation of the Planetary system as it was *just before* the comet storm, when Jupiter was in the present place of Mercury, and none of the superior planets had yet grown great. All the planets shown are given the same size, about 10,000 miles in diameter. Their relative distances from the Sun are not represented, but only their order or arrangement.

CHAPTER XXI.

IMMEDIATE RESULTS OF THE GREAT COMET STORM.

§ 1. Augmentation of the masses of the superior planets. One of the most important consequences of the comet storm was the enormous augmentation of the masses of the superior planets. There is not the slightest reason to suppose that the masses and sizes of these planets differed very widely before the comet storm from the masses and sizes of the inferior planets as the latter appear today. The superior planets may have been some larger than the Earth, but there can be no doubt that they were very much smaller than they are now. Nor is there any reason to suppose that Jupiter and Saturn were any larger than Uranus and Neptune. The present characters of these planets are entirely due to the comet storm.

The meteor swarms of the comet storm were naturally thickest near the Sun. When the storm came on, Jupiter was in first planetary position about where Mercury is now; Saturn was in second place corresponding to Venus, Uranus in third place, where the Earth is now, and Neptune in fourth place or about in the present place of Mars. It follows that the opportunity for growth of mass by accretion of meteoric matter was greater for Jupiter than for Saturn and still less for Uranus and Neptune respectively. Obviously, Jupiter would be struck by many more cometary nuclei and would absorb a much greater quantity of meteoric stones and dust than Saturn or either of the other planets. If the size of Uranus or Neptune was originally much greater than that of Jupiter or Saturn, accretion must have been very great indeed to have obscured and reversed the order of their sizes. Accretion under such conditions would follow a definite law and would have a tendency to make Jupiter largest, Saturn next, Uranus next and Neptune smallest. That this is in fact the order of their sizes is in perfect accord with the supposition that they grew to their present states by meteoric accretion under the conditions assumed. Uranus was formerly supposed to be a trifle smaller than Neptune. But Barnard's recent determinations, referred to above, make Uranus a little larger, thus bringing all four planets into harmony with the law of accretion as it would affect the planets with Jupiter in first planetary place, Saturn in second, and so on.

The relative sizes, grouping and arrangement of the planets — the four greater outside, diminishing in size from Jupiter outward, the asteroids next inside of Jupiter and the lesser planets between the asteroids and the Sun — are stupendous facts. These features constitute the most profound characteristic of the Planetary system.

The theory of the comet storm appears to furnish a clear and simple explanation of all these peculiarities of arrangement of mass, — a result which neither the current theory nor the Nebular hypothesis has attained by the methods they employ. If it be supposed that the planets were produced by the condensation of nebular rings and have remained ever since in the same orbital relations which they had when they were formed it seems certain that the chief characters of the system must have originated then, and not have been acquired by any later process of growth or modification. But neither the original Nebular hypothesis nor any of its modified forms afford a satisfactory explanation of the planetary groups and the asteroids. Fig. 14 represents the Planetary system *just after* the comet storm. The asteroids have expanded the system and taken Jupiter's place. The superior planets have grown great and Saturn has acquired its meteor rings. The frontispiece to this volume shows the same system after the installation of four more planets, — the inferior group. The asteroids maintain their original relation to the superior planets, standing next inside of Jupiter.

It might be supposed that the superior planets could have grown great by meteoric accretion in the places where they now are, at some time subsequent to their original formation and perhaps by a comet storm like that here described. Such a conclusion, however, is rendered untenable when we remember that *meteoric accretion is necessarily greatest near the Sun.* If a comet storm should come now the inferior planets would receive greater accessions of mass than the outer group and Mercury would receive the most of all. So, in the past, if the planets had all been where they are now at the time of the comet storm, then the inner planets, not the outer ones, would now be of great size and Mercury, not Jupiter, would be the giant of the system. And such a supposition would still leave the origin of the asteroids unexplained. By combining the two theories here suggested, namely: the main theory of growth of the system by central capture and expansion, and the secondary theory of the comet storm in the time of Jupiter's infancy, the grouping and arrangement of the planets is fully explained. But both theories are indispensable.

The comet swarm probably drifted into the Solar system from outer space. When the storm came, Uranus was about where the Earth is now and was favorably situated to be inhabited by beings like ourselves. The comet storm was to them the end of the world, and it came by fire, that is, with the appearance of fire in the sky, and with immeasurable quantities of meteoric stones and occasional cometary nuclei like mountains falling on the surface of the planet. If such an event should come to pass again, while the Earth carries its present human freightage, the Scriptural descriptions of the last day would be vividly fulfilled.

When the manuscript of this study was nearly finished, Simon Newcomb's account of « The End of the World » appeared in *McClure's Magazine* for May, 1903. His picture is a lurid one. He makes the end to come in consequence of the falling of a so-called « dark star » into the Sun, greatly increasing its radiation of heat, so as to scorch the Earth and burn all living things that dwell upon its surface. The direct head-on collision pictured by Newcomb is an imaginary possibility which, as a reality, is *infinitely improbable*. But the probable consequences to the Earth might be something like those pictured if the « dark star » were a large enough mass, and provided the collision occurred head on exactly as supposed. That the event would result in expanding and transforming the Sun into a gaseous nebula, as Newcomb pictures it, seems at least a little doubtful. The « Professor of Physics » might have other ways of interpreting the outcome than in terms of the Nebular hypothesis.

The comet storm did not greatly augment the Sun's heat suddenly, but the sky was filled with comets and their great flame-like tails. The most destructive effect on the planets, however, would be the falling of meteoric matter upon them. Not that the comet storm came and went in a year or century or even in a millenium. It probably lasted for hundreds of thousands of years. Meteoric falls were probably occurring continually, but there may have been relatively long periods in which they were not very destructive especially on Uranus and Neptune. Now and then, however, would come periods in which the falls were so great as to overwhelm and bury the surfaces of the planets. A few falls like this would totally destroy all living inhabitants, or at least all those of the higher life forms. Jupiter, of course, suffered most severely and the other planets less with increasing distance from the Sun.

Seen through a good telescope from some neighboring star during the height of the comet storm, the Sun would probably have appeared hazy and nebulous, as some of the stars appear to us now, and with spectroscopic characteristics much like those now found in comets.

The members of the comet swarm were not large bodies, for the four planets kept their places in their orbits through the whole storm. By actual count the present members of the system that may be supposed to have been members of the comet swarm number nearly 500; and this allows nothing for undiscovered asteroids and satellites, nor for destruction by disintegration. These are only the survivors of myriads of others that came with them.

§ 2. **Augmentation of the mass of the Sun.** We get an adequate idea of the magnitude of the comet storm, however, only when we remember that *all the matter that fell into the planets was almost as* *nothing compared with that which must have fallen into the Sun*, largely increasing its mass, changing the scale of the Planetary system, and producing many other far-reaching effects.

It would be a matter of much interest to make such estimates as may be possible concerning the total mass of the comet swarm when it first entered the Solar system. It seems unlikely that any accurate results could be attained, but they might have considerable suggestive value.

Barnard's diameters for Uranus and Neptune are 34,900 and 32,900 miles respectively. This is the diameter of the sphere formed by the surface of clouds which fill the upper air of the planets. The denser central body is probably several hundreds of miles or perhaps a thousand or more miles less in diameter. The same considerations apply to Jupiter, Saturn and the Sun, in which the atmospheres are still greater. Some sort of assumption would have to be adopted as to the original sizes of the planets. It seems within safe bounds to say that the diameters of the superior planets were probably not at most over 10,000 miles, before the comet storm. On the basis of this assumption we might make a rough estimate showing what part of the whole mass of the comet swarm and its wreckage was captured and absorbed by the planets then existing, supposing the Planetary system to be full then as now.

Such an estimate would indicate that Jupiter had received an addition of solid matter 35,000 to 40,000 miles deep over its whole surface, and the other planets less amounts. The depth of solid matter added to the surface of the Sun must have been considerably greater, how much greater is a problem of great difficulty and complexity. However, it seems certain that such an augmentation of the Sun's mass must have occurred. For it is impossible that the conditions which furnished opportunities for such great growth to the planets of that time could have failed to give the Sun a great opportunity of the same kind. It is not likely that any large proportion of the comets and meteor swarms that entered at first were able to escape afterward and leave the Sun permanently.

Along this line of thought it is easy to see that *the total mass of the comet swarm, when it entered the Solar system, must have been many times greater than the present total mass of all the planets. The accession of such a mass to the Sun's globe must have changed the scale of the Planetary system.* From this it follows that the orbit of determinate stability, and hence Jupiter's orbit before the comet storm, was nearer the Sun than the same orbit has been since, as represented by Mercury's present mean orbit. Bode's law also had a different expression. The planets of the system were not spaced so far apart as now. They were set closer together, but still according to a regular law or progression. It was simply Bode's law with a different numerical expression.

The intensity of the Sun's motion in its great orbit and also the eccentricity of that orbit must have been affected more or less when the comet swarm was first captured, and this in turn would affect the intensity and the variations of intensity of the Planetary system.

§ 3. **Origin of Saturn's rings.** When the comet storm began, Jupiter was in first planetary place and Saturn in second, corresponding to the present orbit of Venus. Neither Mercury nor Venus has a satellite now, and it seems certain that Jupiter and Saturn had none then, nor could have any. But as the asteroids worked into place, Jupiter and Saturn were both compelled to expand their orbits, and when the asteroids had become fairly settled, Jupiter had retired to second place and Saturn to third, where the Earth is now. Jupiter was probably still unable to hold satellites. But Saturn had acquired a narrow satellite zone. Although the height of the storm had passed, there were probably some meteor swarms still revolving around the Sun, when most of the asteroids had become adjusted and the superior planets had taken their new places. From this circumstance it happened that, although Jupiter caught no bodies which it retained as satellites at that time, Saturn caught and retained large numbers of meteors. These little bodies formed themselves into a broad band or ring around the planet and revolved henceforth as a swarm of tiny satellites. These, or a considerable portion of them, the planet still holds.

The rings of Saturn are simply a lingering fragment of the great comet storm, and they testify to the prodigious quantity of meteors that surged around the Sun at that time.

Uranus and Neptune were both more favorably situated to retain meteoric rings, but not quite so well placed to catch large quantities of meteors, because at their greater distances from the Sun the swarms were not so thick. Still, it is more than probable that both of these planets formerly had rings like those of Saturn.

In one respect the rings of Saturn indicate a peculiar history. If their immediate origin had been the same as that of other satellites it seems certain that the rings would not now occupy the inner place in Saturn's system. They would either have been lost long ere this, or else they would now occupy a position near the outer limit of the system. A long time has elapsed since Saturn first acquired the rings and many changes must have taken place in Saturn's system. Some special circumstance must have led to the preservation of the rings and caused them to occupy their present place. With the progressive growth of Saturn's system, new satellites must always have been received at the center, and the tendency must have been to force the rings out into larger and larger orbits. The only thing that could save the rings from being forced out to the outer limit and being finally thrown off and lost would be to break up and fall in to the center again, before they had reached the outer limit. At the center they would reform into a new ring and begin a new period of expansion. It is not unlikely that this process of expansion, breaking up and reforming at the center has occurred several times since the rings were first formed. It seems doubtful, otherwise, whether the rings could have been retained until the present time.

While Saturn may have lost single satellites of larger size at the outer limit, it seems doubtful whether it has lost in this way any great part of the smaller meteorites which make up its rings. It is much more likely that the planet has absorbed into itself such of these bodies as have been lost, and it can hardly be doubted that a great number have been absorbed in this way. Indeed, considering the great lapse of time since the rings were acquired, it seems probable that the meteorites of the present rings are only a fraction of those originally captured. The mass of the rings may have been at first many times what it is now.

From this way of looking at their origin, we see that there is no need of invoking a « Roche limit » to account for Saturn's rings. The rings seem to show also that small bodies like the meteorites of which they are composed can not unite with each other to form larger bodies under such circumstances, because their individual attractions are relatively too feeble.

There is one alternative which requires consideration. It might be thought that Saturn has acquired its rings since the time of the comet storm, perhaps even since the planet came to its present place. There is nothing in the rings themselves to preclude this supposition, but the probabilities seem decidedly against such an origin. The suggestion, however, raises questions of considerable difficulty.

The supposition that there may have been a great meteoric storm like that described, but occurring since Saturn became settled in its present place, is made untenable by the fact that in such a storm the inferior planets would have grown to greater sizes than the superior. It is obvious, however, that the inferior planets have had no such opportunity of growth. The only way to account for Saturn's rings as a recent accession is to suppose that the planet captured some particular swarm of meteorites. Theoretically, this might have been accomplished at one encounter or possibly at several encounters. The probability of such a method of capture, however, is exceedingly remote, for the following reasons. In the first place, the meteorites at the time of capture would revolve around the planet in orbits differing widely as to eccentricity, inclination of plane, longitude of periplaneta, etc. It seems probable that at the start as many, or nearly as many, would move in retrograde order as in direct. From this cause probably a considerable portion of those first caught would be absorbed, either by Saturn or by its then existing satellites, before the meteorites could be adjusted to direct revolution at the inner orbit of stability. Besides, it is not conceivable that the planet could capture more than a fraction of a given meteor swarm at one encounter. Yet it seems certain that a great number would have to be taken in the first instance in order to form a ring of sufficient density and compactness to keep its place. In order to do this it would seem necessary for the swarm to be captured soon after its separation from its parent cometary nucleus, while the meteorites were still a close and compact body. But even on the best supposition admissible it seems certain that the number of meteorites captured would have to vastly exceed the compactness and numbers of any swarm known to observation, and that such rings as Saturn carries now could only be built up by many successive encounters of this sort. But all meteor swarms are necessarily dispersed with relative rapidity as they go through successive revolutions around the Sun. They are distributed along their mean orbit and grow less and less compact at each encounter.

There is another fact which adds some weight to the improbability of recent origin for Saturn's rings. Jupiter, at the present time, is in most ways a more favorable planet than Saturn for the capture and maintenance of rings. It is nearer the Sun and has a larger mass and more powerful attraction. The only point in which it is less favorable is the somewhat smaller breadth of its satellite zone.

There is perhaps a remote possibility that the rings are of recent origin. But at the close of the comet storm the two planets were so situated with reference to the Sun and to each other that *Jupiter could not acquire rings, although bombarded by immeasurable quantities of meteorites, because it had no satellite zone, while Saturn was at the same time in the most favorable situation possible for acquiring meteoric satellites in the form of rings.*

CHAPTER XXII.

THE INSTALLATION OF THE INFERIOR PLANETS.

WHEN the great storm of comets and meteorites had passed and the asteroids had become fully settled in first planetary place, the Planetary system appears to have returned to its normal, orderly manner of growth. Four new comets came in successively and were installed as new planets. The first became Mars, the second the Earth, the third Venus and the last Mercury. This completes the Planetary system and brings its growth down to the present time.

So far as is discoverable now, the comet storm is the only great exceptional event that occurred during the growth of the system, but it throws a beautiful light on the history and manner of that growth.

One of the most far-reaching consequences of the great comet storm was the formation of the asteroid ring itself. The installation of the asteroids changed the whole future of the Planetary system. With their advent a new and abundant source of planetary and satellite material was introduced. In all probability the growth and expansion of the system became henceforth comparatively rapid. There are a great number of asteroids, and they are continually being perturbed out of place. Now and then one of them becomes a planet, and the intervals between successful attempts have probably been much shorter since than they were before the comet storm. From this cause also it happens that since the storm each successive planet has probably occupied first planetary position a comparatively short time. It is in that place more than any other that the planets have opportunity to grow by meteoric accretion. We have seen how the giant planets and especially Jupiter improved this opportunity during the comet storm. But the inferior planets have experienced no such favoring conditions and so have not grown great. Their growth by meteoric accretion since they began their present planetary careers has apparently been slight. The meteoric growth of the Earth, for example, since the close of Cambrian time has certainly been exceedingly small. But it is a question whether there are not some discoverable geological evidences of such growth before that time. The Earth is the largest of the inferior planets, and while some of them may have undergone more growth, the difference can not have been great, for none of them has had a great growth.

Mars could not have had a satellite until its orbital expansion had reached nearly to third planetary place, where the Earth is now; nor did the Earth have a satellite until it had expanded its orbit nearly to its present distance from the Sun.

Except, probably, for a more rapid rate of growth, the history of the Planetary system after the comet storm appears to have been marked by no extraordinary events. The falling of the younger planets into their places and the occasional loss of a satellite or an asteroid, or the capture of a comet by a planet brings us down to the present day. But, as a result of the comet storm and the relatively rapid accession of planets since that event, the Planetary system as a whole exhibits a remarkable structure. The greater planets are outside and the smaller ones inside, the two groups being separated by the weak ring of the asteroids. The larger planets are above, as we may say, and the smaller ones below, holding them up. *The system is topheavy.*

CHAPTER XXIII.

MERCURY'S INEQUALITIES.

IF, AS supposed under the present theory, Mercury is the youngest of the planets and has only lately entered the planetary family, and if the process of transformation from comet to planet takes place as supposed, and further, if the transformation in the case of Mercury is not yet quite complete, then we should expect to find some evidence of that fact in the character of Mercury's orbit. *The significant characters are there.* In the table on page 187 it is shown that the eccentricity of Mercury's orbit is more than twice that of any other planet, excepting some of the asteroids. The inclination of its orbit to the ecliptic is also more than twice that of any other planet, excepting some asteroids. These characters are attributable to Mercury's *newness* as a planet. It has not yet fully settled down into its planetary orbit. These inequalities will probably be gradually reduced until its orbit is nearly circular and less inclined.

There is of course another possible explanation for Mercury's inequalities. Mercury may have been well settled in its orbit and been perturbed afterward by the near approach of some great comet. But although it is possible, the latter alternative seems much the less probable of the two. Such orbital characters resulting from cometary perturbations would be quite extraordinary, and there seems to be no reason to suspect planetary perturbations to be the cause in this case. The inequalities may well stand, however, for one stage of the transition through which every new planet must pass in the process of orbital reduction and transformation. Probabilities, it seems to me, incline strongly to the conclusion that Mercury's inequalities are transitional and not perturbative.

Leverrier found in 1859 that the perihelion of Mercury moves about 38′ faster in a century than can be accounted for by the attraction of all the known planets. This he attributed to the perturbation of a planet or a band of asteroids situated within the orbit of Mercury, that is, between Mercury and the Sun. It was thought at one time that such a planet had been found and it was named Vulcan. But this supposition was soon disproved. Still, it was thought possible that invisible asteroids might be present. Newcomb points out that if there are such they must revolve in the same plane as Mercury, because, otherwise, they would cause a motion of Mercury's nodes. But the nodes show no motion beyond that which is explained by the attractions of the known planets. (See Newcomb's "Popular Astronomy.")

The fact is, however, that there is no planet nor any permanent band of asteroids inside of Mercury's orbit. Cometary nuclei occasionally dash through that space, and meteorites must be continually passing, but there are too few of them to influence the planet perceptibly. The comets that pass are generally too small also.

This inequality of Mercury's motion is probably due to the action of the original forces of stability. When Mercury is in perihelion they operate to drive the planet out a little toward its mean orbit, and when it is in aphelion they drive it in a little toward the same orbit. The continual operation of these forces ought to produce an effect like that described.

CHAPTER XXIV.

The Axial Rotation of the Planets.

Neither the current theory nor the Nebular hypothesis has as yet furnished an adequate explanation of the cause of the axial rotation of the planets. In accordance with his hypothesis of condensing nebular rings, Laplace found that the planets should have *retrograde* rotation. The retrograde satellite systems of Uranus and Neptune seem to indicate a similar order of axial rotation for those planets and thus, by implication, confirm Laplace's idea. But his scheme does not fit the direct rotations of the other planets. Professor W. H. Pickering has suggested an ingenious way of accounting for a change of the order of rotation to direct from the supposed original retrograde order. His hypothesis is quoted on pages 129–130.

If the theory of determinate stability presented above be true axial rotation is not a residual phenomenon from the condensation of nebular rings, but is *an accumulated result of forces now in action. A force is acting on each planet today which tends to produce axial rotation in the direct order.* Its action is strongest on planets near the Sun and is almost nil on planets far away. At the present time, therefore, this force is acting more strongly upon Mercury than on any other planet and with the least strength on Neptune.

The analysis of the rotative force—the force that *produces* axial rotation—rests on the same foundation as that by which we have found determinate stability. The Moon has free motion in space as it revolves around the Earth and the forces affecting its stability are conditioned by that relation. While the different parts of the solid globe do not have free motion like the Moon, they are nevertheless affected by similar heliocentric forces which produce axial rotation in the direct order. While I am not able to prove the truth of my theory of the cause of axial rotation mathematically, yet I am strongly inclined to the belief that **axial rotation is a function of elasticity—of elastic motion in the body of a planet considered as solid and rigid from its surface to its center.** Mercury is nearest to the Sun, where the intensity of planetary revolution is at its maximum. A planet in Mercury's orbit may therefore be taken as the best case for consideration.

When a comet first becomes a planet, it may have more or less axial rotation left over from a previous period of planetary or satellite existence. Before it gets well settled as a planet it may be perturbed in irregular ways that leave its rotation in any one of all possible relations to the Sun. For the sake of discussion it is necessary to select some definite relation to begin with. The simplest case supposable appears to be that of a planet which keeps one face constantly toward a star. This is the attitude it would maintain if it had no axial movement whatever.

Suppose a planet to revolve around the Sun in this attitude. If we suppose it to revolve in a circle, then we may say that its center describes a circle around the Sun. For the sake of argument, let us suppose that the planet's diameter is 10,000 miles. At a given moment, then, a particle of matter in the peripheral part of the planet farthest from the Sun is 5,000 miles farther away from that body than is the particle at the planet's center; and the particle at the nearest point is 5,000 miles nearer. The Sun's attraction would be more powerful than the mean on the nearest particle and less powerful than the mean on the farthest particle. This difference of attraction would set up a certain amount of stress within the body of the planet.

But besides the stress arising from difference of attraction, there would be another arising from excess of heliocentric velocity in the particles of the far side of the planet, and from deficiency of heliocentric velocity in the particles of the near side. These particles may be said to be in opposition and conjunction respectively. In Fig. 3 the circle may be taken to represent the solid sphere of the planet, and particles in the planet's mass near *O* and *C* would correspond to those here named. As the planet revolved around the Sun under the conditions assumed, the particles at *O* and *C* would for a time move around the Sun with the same velocity as the particle at the center of the planet. (*E* in Fig. 3.) But the *normal* heliocentric velocities of particles revolving around the Sun in circular orbits passing through *O* and *C* are less and more respectively than the normal velocity of the particle at *E*. Hence the particles at *O* and *C* have an excess and deficiency of velocity respectively and tend to diverge from the two paths concentric with the path of the particle at *E*. In consequence of the stress due to these forces, the particles actually move out a little and diverge by an infinitesimal amount from the concentric circles through *O* and *C*. The amount of their motion is extremely minute and is presumably confined within the limits of the elasticity of the planet's sphere. Beyond a minute limit, this movement is effectively resisted or restrained by the attraction of the rest of the mass of the planet and by the cohesion of the matter composing the planet. Still, the very slight movement which takes place is a true heliocentric centrifugal movement in the opposition parts of the planet, and a heliocentric centripetal movement in the conjunction parts. The immediate tendency of these forces is to produce a minute deformation of the perfect sphere and change its shape to that of a prolate spheroid, with its longer axis on the radius vector of the planet. The centrifugal movement in the opposition half is restrained by the cohesion of the planet and by the attraction of the remainder of the planet's mass. On account of the orbital revolution of the planet, the maximum centrifugal movement of the particles is always a little back of the radius vector. The restraining forces are not exactly opposed to it, and a minute resultant forward movement is therefore produced. *This resultant constitutes the rotative force.*

In the conjunction half of the planet the restraint of the heliocentric centripetal movement acts in the opposite direction and produces an opposite rotative force. These two parallel forces, acting continuously on different parts of the planet and in opposite directions constitute a "couple," and their action tends to produce axial rotation in the direct order and in the plane of the ecliptic. As axial rotation becomes established and its speed increases, the period of the elastic movement of the particles becomes shorter and shorter. Mercury's period is about 88 days. If the Earth with its rotation period of 24 hours revolved in Mercury's orbit it would pass over 4° of its orbit in one day, or over 2° in half a day. The elastic motion of the particles in the opposition half of the planet would then have a period of only half a day. As the speed of rotation increases the rotative force grows less.

It follows that the rotative force depends upon the following definable factors: (1) *upon the Sun's mass*, (2) *on the planet's distance from the Sun and* (3) *upon the diameter of the planet.* Finally, *the speed of rotation established depends in part upon the duration of the time that the planet remains in first planetary place, for it is an accumulated result.*

If there are no great unsymmetrical aspects of mass in the planet it will gradually set up axial rotation. The rotative force, however, is not in a relative sense vastly great, or this condition would not modify the result. But if the mass is markedly unsymmetrical, as is probably the case with the Moon, an attitude of «fixed gaze» or isochronous rotation results; that is, a sidereal rotation is produced in synchronism with its revolution. If the Moon's plane were coincident with the ecliptic and its orbit around the Earth a circle the attitude of the Moon's unsymmetrical figure of mass would be absolutely constant toward the Earth. As it is, the elliptical orbit and the inclination to the ecliptic cause the Moon to have librations in both latitude and longitude. Isochronous or fixed-gaze rotation, like that of the Moon, is physically an entirely different thing from spinning or diurnal rotation, like that which gives the Earth day and night. The relations of the forces which produce them are quite different. Isochronous rotation exists only where the operation of the forces which cause spinning rotation are rendered ineffectual by unsymmetrical figure of mass.

There is need of keeping in mind a clear distinction here between the original rotative force, which starts rotation in the beginning, and the Earth's present axial motion with its vast rotational momentum and its oblate figure, which are not immediate results of the rotative force now acting, but are *accumulated results* of the action of this force through a very long period of time — the whole time that the Earth was in first planetary position. For it is in that place that the rotative force acts with its greatest strength.

According to the present theory, *each planet was for a time in first planetary position, approximately where Mercury is now, and it was while in that place that each one acquired its axial rotation.* If all the planets were of exactly the same size and mass and remained in first place the same length of time, and if after their removal from first place there were nothing to reduce their rotation appreciably, then we should expect the rotation periods to be all of the same length. But the real case is not so simple. In a general way it may be said that the greater the diameter of a planet the swifter the rotation it will acquire under given conditions, and also, within certain limits, the longer it remains in first place the swifter the rotation.

Of the planets whose rotations are known, or supposed to be known, Mercury has the smallest diameter, Mars next, Venus next and only a trifle smaller than the Earth, Saturn next and Jupiter the largest. If we note the rotation periods of these planets in the same order we find that the times of rotation bear in a general way an inverse relation to the diameters. (See table, page 187.)

This is as would be expected if rotation is acquired in first planetary place. Perhaps the rotation period of Venus is somewhat doubtful. Some recent observers claim to have discovered that Venus rotates only once in a revolution, thus keeping toward the Sun an attitude of fixed gaze just as the Moon does toward the Earth. I am strongly inclined, however, to adhere to the earlier observation as given above in Young's table, for I think there are good theoretical grounds for expecting Venus to have a rotation period nearly the same as that of the Earth. Venus and the Earth are very nearly of the same size and their periods ought theoretically to be nearly the same. By strict inference we should expect the period of Venus to be a little longer than that of the Earth, but the period given is a trifle shorter. The period of Mars, too, is somewhat shorter than would be expected theoretically in comparison with that of the Earth. These differences seem to suggest that both Venus and Mars may have remained longer than the Earth in first planetary position. If the histories of Saturn and Jupiter have been substantially as given here, then it seems probable that Jupiter acquired its rotation, not wholly in first place, but partly in the transition to and in second place, while Saturn's rotation bears the same relation to third place. For their rotation periods are evidently related in some way to their present great sizes, which would not have been the case if their rotations had been acquired solely before the comet storm.

The forces that produce rotation act on all the planets now, but with a power so slight as probably not to increase the rotation rates established when the planets were nearer to the Sun. The present rotations are probably largely residual. Much has been made of the idea that the friction of the tides is reducing the Earth's rate of axial rotation. But there seems to be nothing in the case of the Earth or of any other planet to indicate an appreciable factor of this kind, or even that its accumulation in all time past has had a measurable effect. The tendency to tidal retardation may be balanced by the forces that are now tending to produce rotation.

The rotation periods of the several planets seem in a general way to accord well with expectation based upon the present theory. If all the planets had actually acquired their axial motions in first planetary place their periods would not be widely different from what they now are. On account of its newness as a planet, Mercury is supposed to have only a very slow if any axial rotation, for it must take a very long time for the forces to overcome the inertia of a planet and set it to rotating. Indeed, from this fact it would not be surprising if Mercury were found to have a fixed gaze toward the Sun, as some recent observers claim.

We have now reached a point from which we can see better than before the probable relations of Uranus and Neptune to their retrograde satellite systems. These two planets probably have rotation periods somewhere between those of Saturn and the Earth, most likely a little shorter than that of the latter, say not far from twenty hours.

The original rotations of these planets were probably in the direct order and were acquired in first planetary place, though they were probably slightly shortened during and at the close of the comet storm, when the planets' masses were increased.

With the growth of the Planetary system, however, these planets have been forced out toward the outer limits of the system, where the forces of stability and especially the orthogonal force are very weak. And further, it can hardly be doubted that these planets have had satellites continuously since the time of the comet storm. At the close of that storm, when the asteroids had become settled, Uranus was where Mars is now and Neptune in the mean orbit of the asteroids. There is much reason to suppose that both their rotations and their satellites were then direct. On account of their fairly rapid rotations, both planets probably have equatorial bulges of considerable magnitude. If both planets have carried satellites ever since the comet storm those satellites have probably always kept their planes in or very nearly in the equatorial or rotational planes of their respective planets. The gradual turning away (rotation) of the plane of the Planetary system has left them persisting, not exactly in, but probably not far from, an attitude which they had long ago. Their satellites and rotation planes were at that time direct and not very highly inclined. Manifestly, it is the plane of the Planetary system which has turned away from them; not their planes which have turned away from it.

We have seen that the equatorial bulges of the rotating planets tend strongly to hold their satellites in the plane of rotation. The magnitude of the equatorial bulge depends in each case mainly upon the size of the planet and the rate of its rotation. We have seen too, that in planets far from the Sun the orthogonal force is relatively weaker in its effects on the planet than on the satellites, while persistence is much stronger. The satellites therefore tend to depart from the equatorial planes of their primaries in consequence of the action of the orthogonal force, but this tendency is resisted by the attraction of the equatorial bulge upon the satellites. Thus the satellites and their primaries contend against each other in an effort to control each other's planes, and the result is a compromise. The planes of the satellites persist less and are affected by the orthogonal force more than the equatorial planes of the planets, and in changing their inclination they strive to drag the equatorial planes of the planets after them and keep them inclined at the same angle. In the cases of Mars, Jupiter and Saturn the planes are in very close agreement.

The same principles apply in the case of the Sun, which rotates on its axis in a period of a little more than twenty-five days, and in a plane inclined to the ecliptic 7¼°. The Sun's rotation is relatively so slow that its effect in giving the Sun an equatorial bulge is imperceptible, so far as manifested in the diameter of the photosphere. But the photosphere is a cloud layer at the attenuated top of a deep atmosphere, so that there is probably some slight equatorial bulging of the unseen denser mass below. The small inclination of the Sun's equatorial plane to the ecliptic suggests this. But on the other hand, the existence of an inclination even as great as 7¼° shows that the influence of the Sun's equatorial bulge upon the planets is not great, and that the bulge is slight. It seems certain that the rate at which the stellar orthogonal force tends to reduce the inclination of the planetary planes is greater than that with which it tends to reduce the inclination of the Sun's rotational plane. In short, the planets are striving to bring the Sun's equatorial plane into coincidence with their own mean plane. But under present conditions the persistence of the Sun's rotational plane is strong enough to keep it inclined 7¼° against this force. The present position of the Sun's rotational plane is one in which the mean plane of the planets must have stood at some distant time in the past, but the Sun's plane was at that time in some other position.

In the case of the Earth-Moon system the difference of inclination of the two planes is probably in large part due to the Moon's newness as the Earth's satellite. The Moon under present conditions seems unable to acquire an inclination of more than 5° to the ecliptic. But the Earth's equatorial plane is inclined 23½°. The Earth's equatorial bulge strives to bring the Moon's plane to the same large inclination, while at the same time the Moon strives to reduce the inclination of the Earth's equatorial plane to the position of direct coincidence.

The Earth acquired its axial rotation in first planetary place before it had a satellite, and it continued moonless while in the present place of

Venus, where the rotational and orthogonal forces are both weaker. Meanwhile, the planetary orbital planes were slowly rotating and turning away from the plane in which the Earth had acquired its axial rotation. In this way the Earth's equatorial plane became more and more inclined, until the Earth captured the Moon. At the time of that event the Earth's equatorial plane was probably somewhat more inclined than it is now, for ever since then the Moon has been exerting a powerful force to reduce that inclination. How new the Moon is as the Earth's satellite is shown by the wide divergence which still remains between the two planes. The Moon appears as yet to have accomplished only a little of its task, although the conditions are perhaps more favorable for rapid reduction of inclination and final close approximation to direct coincidence than for any other satellite in the Planetary system.

CHAPTER XXV.

PROBABLE EARTH AND MOON HISTORY.

IN A planetary system which has grown in the way here outlined, there are many interesting possibilities as to the origin of its members. What, for instance, has been the origin of the Earth and the Moon? We have seen that the Moon was a comet before the Earth captured it, but what was the Moon before that? The only answer that can be made here is to name the several alternatives that seem open to us, without being able to choose decisively between them. All of the planets that now have satellites have probably had other earlier ones which they have now lost. These lost satellites all became comets and were then subject to capture by other planets. A satellite lost from one planet is particularly liable to be captured by some one of the other planets which are nearer to the Sun, if those planets have satellite zones. Thus if Mars were to lose a satellite that comet would be particularly liable, in the course of its subsequent orbital contraction, to be captured by the Earth.

The Moon was caught by the Earth in recent times, that is, since the Earth came to its present position in third planetary place. All the other planets including the asteroids were about where they are now, and probably all that have satellites now had them then. It seems certain that the Moon is too large to have been a satellite of Mars, but it may have belonged to Jupiter or Saturn. Probabilities, however, point strongly to the conclusion that it was an asteroid. This numerous and frail band of little planets is a fertile source of material for comets that may be turned into satellites or planets, and there is no source for the Moon so likely as this. Yet it is barely possible that the Moon was a satellite of some planet beyond Saturn, or that it came into the Solar system alone from the depths of space, or even that it is a lost trans-Neptunian planet recaptured. Nearly all of these alternatives, however, are only very remote possibilities. Even if the Moon was for a time a satellite of Jupiter it was almost certainly an asteroid before that.

On this view of the Moon's history, it was at one time a member of the comet swarm which drifted in from space far beyond the confines of our Planetary system. From what other system it came we do not know. Finally, the Moon entered our system and became a comet and probably developed a more or less magnificent tail as it dashed through its perihelia. After a long career as a comet, it settled down to the more quiet behavior of a planet as one of the asteroids. It was a planet with these associates for a long time, while the system grew by the accession of four new planets, and the asteroids in the meantime were expanded out to or nearly to their present place. Then the Moon-to-be was perturbed out of its place in the asteroids and once more became a comet, perhaps with a small tail like that displayed by Encke's comet, and after a time it was captured by the Earth. What we see when we look at the Moon is the face of a cometary nucleus which was suddenly interrupted in its activity. It is a dead comet which was before that a planet.

Mars is the oldest of the four inferior planets. It was the first planet to enter and establish itself between the asteroids and the Sun. Considering this fact it is easy to see how strong is the probability that Mars was one of the asteroids. When it was joggled out of its place it was a comparatively easy matter for it to slip in nearer the Sun, and drive the asteroids out so as to take possession of the inner planetary orbit. There are other possible origins for Mars, like those named for the Moon, but this is by far the most plausible.

Next came a comet which established itself as the Earth in the present place of Mercury, and after that came another and still another and these became Venus and Mercury respectively.

The probability that the Earth was an asteroid is almost as strong as it is in the case of Mars. Of course the Earth may have been a satellite of any of the superior planets or of a trans-Neptunian planet, or, it may have been a cast off single planet of the system recaptured, or, it may have entered the system as a lone wanderer from Siderea, the vast star-realm. But the probabilities are very great that it was an asteroid. If the Earth has ever been a satellite since it left the asteroid band it is certain that it must have been attached to Jupiter or some other one of the superior planets. It could not have been a satellite of Mars, for the mass of Mars is much less than that of the Earth. If size be taken as an indication of the Earth's former associations it might seem likely that it was a satellite of Jupiter or Saturn, for both of these planets have satellites which are as large or slightly larger than Mercury.

The Earth is the largest body that can with any notable degree of plausibility be identified as a probable former member of the original comet swarm. It was probably the largest of the planetoids that drifted into the Solar system and the largest comet in the great comet storm, and after the asteroids had become established as planets it was the largest of that band. No doubt the earliest asteroids to become settled in first place experienced some growth by accretion from the meteoric wreckage of those which were still swinging through cometary orbits and producing tails. The Earth may have acquired some part of its larger mass in this way.

The Earth was probably not perturbed out of the asteroid ring until after Mars had become well established as a planet. The asteroids were then in second place, where Venus is now, and Jupiter was where the Earth is. The Earth then was probably a planet during two earlier steps of the Planetary system's growth and all that time was, with its companion asteroids, the next inner neighbor of the great planet Jupiter. Finally, Jupiter dragged the Earth out of its place and the Earth was either captured and imprisoned by the giant as a satellite, or it went on a new but short cometary cruise at the end of which it sailed in between Mars and the Sun and took upon itself the honors of a single planet. **This was the beginning of the Earth's present planetary period.** It is interesting to think that the Earth probably had a *previous* period of the same kind, the two separated by a cometary epoch. Here is a problem for geologists and paleontologists to ponder. Do the rocks show any evidences of such stages in the Earth's history? To my mind there are evidences which favor this view. Some of them seem to me to constitute conclusive proofs. The possibilities of correlation between geological and astronomical history along these lines of interpretation seem to me to present one of the most attractive fields of investigation imaginable. Along other lines, too, these studies open up a vast prospect for geological researches relating to the fundamentals of the science. But these subjects do not come within the intended scope of this volume and are therefore left for a future occasion.

As to the origin of Venus and Mercury, pretty much the same probabilities attach to them as to the Earth. The most probable origin of both is in the asteroids.

CHAPTER XXVI.

METEORITES AND METEOR SWARMS.

WE HAVE now reached a point from which we can see more clearly the origin and history of the tiny meteoric bodies that circulate around the Sun — the meteorites and meteor swarms. When we consider the vast distance to the limit of the Sun's sphere of influence, a distance which Lowell puts at 114,000 astronomical units (114,000 × 93,000,000 miles), it becomes obvious that there is ample room within that sphere for comets with periods running certainly into the hundreds of thousands of years. The orbits of such comets would necessarily be sensibly parabolic, if the comets came near enough to the Sun to be seen by men, for no comet has ever been seen much beyond the distance of Jupiter. Moreover, it has been shown that with the Sun moving through space, the majority of comets entering the Solar system from without would be markedly hyperbolic. Only where the bodies were *overtaken* by the Sun would they be likely to have elliptical orbits and remain permanently with the Sun. According to Lowell, there is not a single comet which is clearly hyperbolic. All the orbits suspected of having this quality differ so slightly from ellipses which are sensibly parabolic, that they may in fact be such ellipses.

These considerations point to the conclusion that all the comets seen and recorded by men are probably now permanently within the Sun's domain, and that since meteor swarms and meteorites are only the wreckage of comets, or rather of their planetoid nuclei, these tiny bodies are also permanently within the system.

Proctor suggested that the ejectile power of some of the cyclonic storms of the Sun is probably great enough to throw meteoric stones permanently away from that body; and solar phenomena seem to give the idea much plausibility. But obviously, if *some* meteorites are fired permanently from the Sun in this way, it necessarily follows that a vastly greater number would fall somewhat short of permanent ejection. These would be thrown out toward the limit of the Sun's sphere of control to a greater or less distance, according as their initial velocities were great or small. But they would all fall directly back into the Sun, unless perturbed by other bodies — by planets, satellites or comets. In that case there ought to be two continuously active radiants for meteors, one for those coming from the Sun and one for those going toward it. But there are no indications of such radiants.

Moreover, if such a process is or has been going on it would result in the Sun's feeding the planets with meteoric matter, and a planet so near the Sun as Mercury should receive a much greater share than those far away. Venus, the Earth and Mars have each been in that place during and after their periods of installation, and unless we may suppose that a single body much smaller than any of these can become a planet, it does not seem likely that the amount of meteoric growth from this cause has been appreciable. Besides, the geological evidence is decidedly against appreciable meteoric growth, even from all sources combined,—at least since the beginning of Paleozoic time.

Hence the conclusion that **meteorites are not derived from the Sun, nor often from outer space directly, but substantially all come from the wreckage of cometary nuclei.**

When an asteroid or a satellite has been for a long period in a stable and nearly circular orbit, its outer parts become oxidized and silicified, and undergo other chemical changes, which result in covering their outer surfaces with a coating of what we call rocky or stony substances as distinguished from the metals. Then, when this satellite or asteroid is perturbed and starts off in a cometary orbit, it begins a process of eruptive disintegration from its surface and loses a part of its mass at each perihelion passage. At first this disintegration affects only the stony crust and produces only *stony* meteorites. But as the process continues, the meteorites are derived from deeper and deeper layers in the mass of the planetoid body, and the eruptions gradually pass below the stony crust and penetrate the metaliferous central mass or core. Then at first from the deeper parts we get meteorites composed of *stone and metal* the metal being chiefly iron, and when the core is more deeply penetrated, we get the typical *metallic meteorites of nickeliferous iron* with the Widmannstättian figures.

Meteorites, therefore, go some way toward showing the relative stages of disintegration to which their parent nuclei have progressed. **Stony meteorites are derived from new, recently launched comets; metallic meteorites in general from old, deeply worn and wasted comets.**

These considerations appear to afford an interesting view of the interior composition of planets and satellites. For if this idea of the relation of meteorites to comets be true, then we may fairly conclude that the central masses of all such bodies, including the Earth and the Moon and all the other planets and satellites, are metallic and are composed mainly of iron. At a depth of a few scores of miles, the Earth is probably of this composition and continues so to the center. The Ovifak and other similar irons may possibly be of terrestrial origin, as has been suggested. They may at least be profitably studied with this thought in mind. There is good reason for expecting the metallic core of the Earth to be nearer the surface in the Arctic regions than anywhere else.

Biela's comet split in two in 1846, or perhaps a little earlier, and the two pieces sailed henceforth as independent comets. Six years later the pair reappeared, still companions, but they had drifted farther apart. After that time they were not seen again, but a meteor swarm remained in the path of the main body and is known as the Bielids or Andromedes. During the shower of these meteors on November 27, 1885, a large iron meteorite fell at Mazapil in Mexico. From its behavior in splitting in two, it might have been guessed that the nucleus of Biela's comet was far gone in disintegration. The Mazapil iron is almost certainly a piece of that nucleus, and its character seems to confirm the idea that the nucleus in 1885 was only a last remnant of the core of what had been a much larger body.

Concerning the density of the Earth, Young says: "Since the average density of the earth's crust does not exceed three times that of water, while the mean density of the whole earth is about 5.58 (taking the average of all the most trustworthy results), it is obvious that at the center the density must be very much greater than at the surface,— very likely as high as eight or ten times that of water, and equal to the density of the heavier metals." ("General Astronomy," page 115.)

The density of pure iron is 7.75, so that so far as density is concerned, there appears to be ample room for the supposition that the central mass of the Earth is composed mainly of this metal. More can probably be learned concerning the composition of the Earth's interior by the study of meteorites than in any other way now available. Of course, such a method is indirect and depends wholly upon the validity of the assumption that the interior composition and conditions of cometary nuclei, and hence also of asteroids and satellites, are substantially the same as for the interior of the Earth. It seems to me that this assumption is not only strong and well founded, but that it is the only supposition which has any foundation in fact. The apparent smaller density of the Moon may well be due to a rocky, less dense crust which is relatively, though not actually, thicker than the rocky crust of the Earth. The metallic core of the Moon would then be very much smaller in proportion than in the case of the Earth, and the Moon's mean density consequently less.

It is interesting to note the bearing of meteor swarms upon the structure and conditions of the peripheral or outer, unseen parts of the Planetary system. Below is a list of meteor swarms and comets with their periods, associated planets (known or supposed) and orders of revolution. The last two are apparently associated with trans-Neptunian planets.

(1.) The Andromedes or Bielids and Biela's comet (late November meteors); period, 6½ years; belongs to Jupiter's comet family; *direct revolution.*

(2.) Tuttle's comet; period 13.7 years; belongs to comet family of Saturn; *direct revolution.*

(3.) The Leonids and Tempel's comet 1866, I, (early November meteors); period, 33¼ years; belongs to comet family of Uranus; *retrograde revolution.* This is the shortest period retrograde comet.

(4.) Halley's comet; period, 76 years; belongs to comet family of Neptune; *retrograde revolution.*

(5.) The Perseids and Tuttle's comet 1862, III, (August meteors); period, 120 years; probably belongs to comet family of first trans-Neptunian planet; *retrograde revolution.*

(6.) Lyrids and Comet I, 1861 (April meteors); period, 415 years; probably belongs to the comet family of another trans-Neptunian planet; *direct revolution.*

It is thus seen that the bodies associated with Uranus, Neptune and the first trans-Neptunian planet are *retrograde* in the order of their revolution. This is as would be expected if these bodies are lost satellites of those planets, and it suggests that the first trans-Neptunian planet has a retrograde system. It would not be surprising if that system were more highly inclined than that of Neptune. The last swarm named revolves in the direct order The significance of this fact is uncertain. The comet may have been perturbed from an original retrograde order of revolution, or possibly the direct order originated in the satellite system from which the comet came, through the accumulation of residual effects of tilting and persistence during a very long period of time.

CHAPTER XXVII.

FUTURE PROBABILITIES AND POSSIBILITIES.

LET US now turn about and face the other way and note briefly the probabilities that seem nearest at hand. In the immediate future history of the

Planetary system it seems probable that Encke's comet, Witt's comet (Eros), the Moon and perhaps Deimos (the outer satellite of Mars) will figure prominently. At the present time Encke's comet looks like the heir apparent to planethood. But there are many chances of failure. In the first place, its mass is probably much too small to admit of its becoming a single planet, so that even if it should get its aphelion reduced down close to Mercury's orbit, it might be unable to drive Mercury out. In that case it would try to revolve around the Sun in the same orbit as Mercury. But the ultimate end of an attempt to do that would be a perturbation by Mercury that would cause the comet to fall into the Sun.

But in the course of time Encke's comet may be captured by the Earth. Then we should lose the Moon, for the Earth can not keep two satellites. The Moon would then surely become a planet, for it is almost certainly large enough to succeed. Or, Encke's comet may be captured by Mars, in which case Mars would lose Deimos, its outer satellite, and this would then become the imminent disturber. The Earth might capture Deimos and lose the Moon as before with the same ultimate result.

From another point of view it is possible that Mars will capture Eros, in which case Deimos may act as before and rob us of the Moon. If Deimos, lost from Mars, should fail to displace the Moon then the little war dog would probably fail to become a planet, because it is too small.

But perhaps the greatest danger of this kind that threatens us is directly connected with the comet Eros. For there really seems to be a growing chance that the Earth will capture that comet.

In all of these nearest chances there seems to be a heavy weight of probability that the Moon will be the body to be installed as the next planet. So in looking at the Moon, it is interesting to bear in mind that we may be looking, not only upon an ex-planet and a dead comet, but probably also upon the next future planet.

Whenever the next planet takes its place, whether it be the Moon or some other body, the present system will expand one more step, and in that expansion the Earth will be carried out to the present place of Mars, and if the Earth has a satellite at that time, the satellite will contract its orbit and come down much nearer to the Earth than the Moon is now. We may say, therefore, that, except for such differences as depend on the larger size and mass of the Earth, our present environment is what that of the beings of Mars once was, and what theirs is now ours will sometime be.

The capture of the Moon was fraught with tremendous consequences to the Earth, and so will its loss be — but these themes belong to geology. It is a most fortunate thing that the Moon was captured by the Earth. Except for that event, the Moon would even now be approaching installation as the next new planet, if indeed it were not already settled. Its capture has surely given the Earth a much longer sojourn in third planetary place than it would otherwise have had. But while such changes affecting the Earth as the abode of man appear to be coming at some future time, it does not follow that they are coming soon. Indeed, everything we know indicates that it will be many thousands of years before any one of the possible events named will become really imminent.

It was noted above that in its present structure the Planetary system as a whole is topheavy. The giant planets are above and the small planets below holding them up, as it were, while between the two groups is the thin weak ring of the asteroids. The continuance of the system in its present arrangement depends, apparently for one thing, upon the asteroids maintaining their integrity as the equivalent of a planet competent to stand as a unit in the system. We saw also that Saturn's rings have probably broken in consequence of weakening, when expanded toward the outer limit of Saturn's system, and then returned to the center and reformed as new rings next to the planet. The expansion of a system, like that of the ring of the asteroids, has necessarily the effect of greatly weakening the bonds by which they are kept together in mutual dependence.

In the future of the Planetary system the application of this idea is fraught with momentous consequences. The asteroids have already been far expanded from the close band they formed in first planetary place and many individual members have been lost. The ring has become thin and scattered. How much farther can it be expanded and still keep its integrity as the equivalent of a planet satisfying the demands of Bode's law? It seems certain that it can not endure until it shall have expanded to the outer limit of the Planetary system and pressed all the giant planets out to final separation and loss from the system. To do that it would be necessary for the asteroids to keep together until they had expanded at least as far as the present place of Neptune and possibly farther. This seems a certain impossibility. Before that limit can be reached the asteroid ring will surely break up and all the asteroids then remaining will again become comets. This will be the beginning of another comet storm, like the one which occurred in Jupiter's infancy, but of much smaller proportions. The planets then next to the Sun will grow more rapidly than now by meteoric accretion, though not to the degree that Jupiter and his companions did in the storm that is past. If a comet storm should come now the Earth, besides increasing its size, would probably acquire rings like those of Saturn.

At the close of the future storm the comets will strive to form a new asteroid ring between the Sun and the nearest planet, and they will do it, unless their members are too few and their total mass too much reduced. If they fail, the effects they will produce will be just like those of the past storm on a smaller scale, except that there will be no expansion of the system to receive a new member. If they succeed, a new asteroid ring will be formed, but it will be a much weaker one than the present ring was at first and it will not endure so long. It will break up sooner and will again try to re-form at the center. Thus asteroid rings gradually wear themselves out expanding, breaking up, producing comet and meteoric storms and reforming again at the center. In this process they scatter the matter of which they are made among the other planets and contribute largely to the mass of the Sun at each break up. This process throws an interesting light on the community of matter composing the Sun and all of its attendant bodies; and indeed, also upon the same relation between the Solar system and some other system from whence the original comet swarm came.

These observations on the future of the asteroids suggest an interesting reflection on the past history of these same bodies. It is not impossible that they formed a greater asteroid ring in our own Planetary system before the storm that struck our superior planets. They may have been pressed out to some distant orbit from which they broke and dashed back to re-form at the center in the way just described. But I think that when we consider the great total mass of the bodies constituting the last storm, as shown by the growth of the superior planets, we can hardly suppose them to have held a place in our system before. It seems almost certain that they came to us then for the first time out of the abyss of space, probably by being overtaken and captured by the Sun.

But, to return to the grand catastrophe, when the present asteroid ring breaks, how will that affect the remaining planets? When the ring gives way, what will Jupiter and its huge companions do? That event will remove the band of tiny bodies which now keeps Jupiter in its proper place under the operation of Bode's law. The loss of the asteroids would not affect the stability of Mars directly, but it is not the same with Jupiter and the farther planets.

Perhaps there is a remote possibility that Jupiter could slowly contract its orbit until it reached the present place of the asteroids, where Bode's law would be satisfied and a new adjustment for stability would be attained; and the outer planets might follow Jupiter, all contracting their orbits one step. Such a change, however, seems to me impossible. With Mars staying where it is and Jupiter slowly contracting its orbit to the mean asteroid orbit, it seems inevitable that commensurabilities of the first order would be set up between these two bodies at some stage of the change. Could the little warrior stand that? I think not. He would almost certainly be perturbed out of his place and driven off as a comet.

The nature of Bode's law is such that in the normal expansion of the Planetary system no dangerous commensurabilities arise. But with an outer part of the system contracting to adjust itself to an inner part which remains unchanged, it seems certain that such dangers would arise.

But if Mars gave way, then the Earth would stand next, and Jupiter and his mighty companions would come thundering in upon us. The mass of the Earth is nine times greater than that of Mars, but could the Earth do what Mars failed to do? Could it check the giants in their mad descent? It does not seem likely. Nor is it any more probable that Venus or Mercury could check them.

When the asteroids give way, this unwieldy, top-heavy Planetary system will probably go to pieces.

There will be a general break up and a relatively rapid re-organization of the system. As an alternative, it is perhaps possible that on the breaking of the ring the superior planets will simply become huge comets and re-enter the system as new planets next to the Sun, without breaking up the systematic relations of the inferior planets. But this seems unlikely. More probably Jupiter will smash the system of the inferior planets. In that case the superior planets might re-enter the system as before, Jupiter being the first to enter, Saturn the next, Uranus the next and Neptune last. This would give the new system a peculiar structure. The arrangement of the masses of the superior planets with reference to their distances from the Sun would be reversed. If the Planetary system presented this arrangement now the discovery of the law of its growth would have been much more difficult.

If the asteroids succeeded in forming a new ring in advance of Jupiter and Jupiter's entrance did not disrupt it, and if that ring could remain intact during expansion, while Saturn, Uranus and Neptune were being installed, the planetary order beginning at the Sun would then be Neptune, Uranus, Saturn, Jupiter, the asteroids, outside of which would be another planet of large mass corresponding to the present Jupiter in its position and relation to that ring, and beyond that another large but somewhat smaller planet. The asteroids would then be between *two* great planets—a relation which would greatly weaken their bond of union. Such a ring would not last long. In such a catastrophe as the breaking up of the present asteroid ring and the re-organization of the Planetary system, what would become of the Earth? No doubt it would first become a comet, and then perhaps a planet again later. Or, there is a possible series of changes by which the Earth might become the satellite of some great planet, but that is unlikely. If it failed to do one or the other of these things the Earth could hardly escape falling into the Sun. Either to « fly off into space or fall into the Sun » is the Earth's manifest ultimate destiny.

In some such ways as these do planetary systems form and grow and pass away. Many planets are no doubt cast off at the outer limit of the system and pass away to other stars. But most of the matter circulating around the Sun as planets, satellites, comets and meteorites falls sooner or later into that body. It is to be noted that on the present theory neither the Earth nor any other of the Sun's attendants unless it be a few of the meteorites, were ever a part of the Sun's mass. We are all travelers from Siderea. We came from some other star system far away, we know not whence. It can hardly be doubted, however, that the Earth with the other asteroids was once a member of the planetary system of some other sun. This becomes apparent when we consider the probable ultimate origin of the asteroids.

Suppose the Sun's motion in space to be more intense than it is and the planetary zone narrower, as would naturally be the case. Suppose also that the asteroids were a much more numerous and compact body, so that they were able to keep together until they had forced all of their elders in the system beyond the outer limit. It would finally come the turn of the asteroids to take their leave. If such a band of planetoids should be cast off at the outer limit of our system and set adrift on some hyperbolic path in space, leaving our system permanently, it would be just such an aggregation of bodies as the asteroids were when they first entered the Planetary system. The comet swarm, therefore, was probably a lost asteroid ring from some other sun.

DIVISION III.

BRIEF VIEWS OF BROADER THEMES

CHAPTER XXVIII.

THE AGES OF THE SUN AND THE EARTH.

IF TRUTH lies along the path we have been following in the preceding pages, then certain other broad conclusions lie so near at hand that they may as well be stated, though they were not comprised in the original plan of this volume. The first relates to the Sun's manner of growth. I shall not go into the subject of the Sun's origin here, except to state the conclusions which seem plainly suggested by this discussion. The Sun, like the planets, has grown by the condensation of neither gaseous nor meteoric nebular matter as usually defined, but by meteoric accretion, markedly irregular in its rate, but mostly exceedingly slow. The age or duration of the Sun on this basis is indefinitely longer than the estimates which have been in vogue in the last half century—fifty to one hundred millions of years.

Along with this goes another conclusion, viz: that the Sun's heat is not due to the contraction of a gaseous mass, as held by Helmholtz and Kelvin, nor to the meteoric contractions supposed by Faye and Lockyer. The slowness of the growth by accretion renders such ideas inapplicable.

Consider these facts. We have seen the method by which the Planetary system has grown since the time that Neptune was a comet. The time consumed in that period of the system's growth must have been prodigiously long. And yet, what part is it of the whole life of the system and of the Sun? There is much reason to believe that it is only a very small fraction of that vast time. From Neptune to Mercury represents the growth of a full system as we know it. But how can we even guess at the number of planets that have entered at Mercury's place, passed through all the steps of expansion and been thrown off at the outer limit, before the time of Neptune's entrance? There may have been hundreds of them. The Sun's planetary zone has probably been filled, emptied and re-filled time after time by an endless procession of planets coming in at the center and going out at the periphery.

Those who have looked with favor on Mayer's theory of meteoric bombardment as the cause of the Sun's heat might be inclined to ascribe the Sun's present heat to the bombardment of the comets and meteors at the time of the comet storm. But such an idea is untenable. Such a cause would produce its greatest heat effects in the Sun's surface parts from which the heat would escape with relative rapidity. Moreover, even at the height of the comet storm the rate of bombardment was far too slow to account for a temperature in the Sun as high as the present, much less for a temperature high enough then to leave so high a temperature now, after cooling from that time to the present. No doubt a prodigious amount was produced, but it was produced too slowly to cause so great a rise of temperature, and even if the temperature was then so high it could not have lasted so long. At the present rate of bombardment, it may be doubted whether the Sun's temperature is appreciably affected. Some other way must be found to account for the heat of the Sun —some way that will abandon the ideas of contraction and bombardment and be consistent with a vastly greater age for the Sun than has been attributed to it hitherto.

On the present theory, the age of the Earth is also vastly greater than has been supposed. The Earth has been a member of the Planetary system since the time of the comet storm, when Jupiter was where Mercury is now. But this is only a part of its past life. It must have been an indefinitely long time drifting through space after it left its parent system and before it entered our system. Then also, in that other system it was in all probability a planetoid body, like one of our present asteroids, and must have been a member of a planetoid ring in that system at least during one whole period of expansion from center to periphery. Before that again it was probably a member of a comet storm in that system, and it may have had a long history before that. At this point there are one or two alternatives as to the ultimate origin of such bodies, but a consideration of these would carry us far beyond the intended scope of this discussion.

Great as is the age of the Earth, however, it is nothing like as old as the Sun. We have seen that the Moon was probably an asteroid and entered with the Earth in the comet swarm and participated in the comet storm. It is therefore probably of the same age as the Earth; and the same statement applies to all the present members of our system that were formerly members of the comet swarm.

To admit so great an age for the Earth, however, compels us to revise our ideas of the causes of the Earth's present physical condition along with those of its origin. There is every reason to believe that the Earth has grown solely by meteoric accretion. The growth was irregular in rate, but was mostly extremely slow. There is no reason to think that the Earth was ever gaseous or molten as a whole, or that it was ever hotter than it is now, except as it was nearer to our own or some other Sun and received more heat on its surface. Though heat is constantly escaping from the Earth, the globe is not growing cooler. Its temperature is sensibly constant and has been for a time indefinitely long.

But if the Earth is not cooling off, then it is not contracting, and the contraction theory, which has so long served to account for the wrinkles of the Earth's crust and for the formation of continents and mountains, must be given up. The phenomena must be accounted for in some other way. The globe is not molten liquid in the interior, nor has it ever been in that state. It is very hot and the rocks are so saturated with superheated steam and gases that, when the crust is penetrated by a deep crack, which temporarily removes the pressure on these hot steam-laden rocks, the expansive force of the steam reduces them instantly to a liquid and they flow out as lava, or are blown out as ash and scoria by violent explosions. Nor can the Earth's internal heat be accounted for by meteoric bombardment, for it has certainly not been subjected to any bombardment adequate for that purpose since the beginning of its present period of planetary history.

The same remarks regarding internal conditions apply to all the planets and planetoid bodies. Many false impressions have been given concerning the densities of the planets and of the superior planets especially. In getting the densities of these bodies, the thing measured—the apparent diameter—is the diameter of the cloud-sphere. The visible surface is composed of aqueous clouds floating in the upper heights of a great, deep atmosphere. These bodies are hotter than the Earth and hence are probably slightly less dense at equal depths below their subaerial surfaces. But the difference can not be great. Mercury is supposed to be more dense than the Earth. This might be expected, because Mercury is a freshly stripped cometary nucleus. It has probably lost most of its rocky coating and is mainly metallic and hence has a greater mean density. Both Mercury and Venus probably have relatively shallow atmospheres and are enveloped in perpetual mists.

CHAPTER XXIX.

PROBABILITIES AS TO THE SUN'S COMPANION STAR.

PRESENT knowledge relating to the Sun's motion in space is too meager to afford a basis for definite or precise conclusions. But there are certain general principles which it seems to me must guide the course of future thought and investigation on this subject, and it may perhaps be a matter of some interest to see what results follow an application of the principles of this theory to the present slender body of facts.

If it be granted that the existence of the Planetary system implies revolution by the Sun in an elliptical orbit of slight eccentricity, then, with the addition of a few other facts and one or two assumptions to be stated presently, we have the basis of a preliminary discussion of the Sun's great orbit. The conclusions reached will, of course, have only tentative value, but it may be that they will serve some good purpose by way of illustration and suggestion.

If the Sun's great orbit is nearly circular, then we are led by very simple steps to the conclusion that the deflecting force which sustains the Sun's revolution must reside in some one star or in the center of gravity of a small cluster of stars situated in a plane passing through the Sun and perpendicular or very nearly perpendicular to the path of its present motion in space. The deflecting force must lie in or very near to this plane. Excepting a belt some five or ten degrees wide on either side of this plane, all the rest of the celestial sphere is eliminated. By this limitation the problem is greatly simplified and the chances of correctly identifying the Sun's real companion are proportionally increased.

The plane referred to intersects the celestial sphere on a great circle, and it is interesting to note what stars lie near it. We may call this *the plane of the deflector.* Until the deflecting star is identified, the precise position of this plane in space will remain unknown; but when the companion is found, it will be defined nearly as accurately as the ecliptic is now.

One naturally inclines to the view that the nearest of the stars is probably the Sun's double—its *binary* companion—if the Sun be regarded as one of two stars forming a binary system. In looking for the Sun's companion the most important facts to be ascertained are the parallax or distance of the star and its position with regard to the theoretical plane of the deflecting force. It seems to be well settled that the Sun is moving toward a point in the constellation Hercules. Young gives its place as Right Ascension about 267° Declination about North 31°. If the Sun moved in a circle, the plane of the deflector would always pass through the Sun at a right angle to its path in space. But it is certain that the Sun's great orbit is at least slightly eccentric, so that the angle may be slightly more or less than a right angle, according to the amount of eccentricity and the Sun's place in its orbit.

It is certainly a significant coincidence that much the nearest of all the stars lies almost exactly in the plane of the deflector if set at a right angle. This star is Alpha Centauri, R. A. 14 h. 33 m. Declination S. 60° 25′. Alpha Centauri is a star of the first magnitude, low in the southern sky and invisible in north mid-latitudes. The bright star Sirius has often been mentioned as the possible companion of the Sun. But Sirius is twice as far from the Sun as Alpha Centauri and is not near the plane of the deflector, but near its south pole and the Sun's quit. Hence, if Sirius is the Sun's companion the Sun's orbit must have a very high eccentricity. The distance of Alpha Centauri from the Sun is given by Todd as 275,000 times the distance of the Earth from the Sun or about 25 trillions of miles (French system). The star 61 Cygni is 43 trillions of miles distant and Sirius 50 trillions. Procyon is 71 trillions and Altair 94 trillions. But none of these stars are near the plane of the deflector. Young gives the star Lalande 21185 a parallax equal to 6.6 light years or about 39 trillions of miles, and this star is close to the plane. The "runaway star," 1830 Groombridge, is also near the plane, but is 147 trillions of miles distant. But these two stars and 61 Cygni are all of less than the fifth magnitude. According to Todd, Alpha Centauri has a proper motion of 3.67 seconds of arc annually, and 61 Cygni 5.16 seconds. Todd gives two other stars with greater proper motions than Alpha Centauri, but they are four and six times farther from the Sun respectively. 1830

Groombridge is supposed to move over seven seconds of arc annually or at the rate of about 200 miles per second. A high rate of motion, however, is not a necessary attribute of the Sun's companion. Indeed, it would rather be expected to be slow, especially if the companion's mass is greater than that of the Sun, as seems probable. The first magnitude southern star Fomalhaut is also near the deflector's plane, and Capella of the same magnitude in the north might be included as one of those not far from it, but according to Todd Capella is 191 trillions of miles distant.

All things considered, there appears, on present knowledge, to be no other star that can compare favorably with Alpha Centauri in respect to nearness to the Sun and to the plane of the deflector. At a first glance, however, one is apt to be impressed with the idea that Alpha Centauri is much too far away to be the Sun's companion. But, as has been pointed out above, the relatively high intensity of the Planetary system indicates low intensity for the Sun's revolution and suggests a great distance for the deflecting center. We must therefore be prepared to expect much more magnificent distances than those with which we are familiar in the Planetary system.

Neptune is the farthest known planet and is 2862 millions of miles from the Sun or about 30 times as far as the Earth. Neptune moves 3.4 miles per second in its orbit, and by a rough calculation falls about 0.00013 of an inch toward the Sun in the same time. (The Earth falls about 0.116 of an inch in a second.) Now if Alpha

Centauri is 25 trillions of miles from the Sun, that is about 9167 times as far as Neptune. At that distance a body with unappreciable mass would, by rough calculation, fall toward the Sun about 0.000, 000,000,015 of an inch in one second of time. By the law of velocities in circular orbits a body revolving at that distance would move at a rate equal to about $\frac{1}{95.6}$ of the velocity of Neptune, which would be more than 3000 miles a day, 125 miles an hour, or nearly 185 feet per second. Taking the distance of Alpha Centauri as 275,000 times the distance of the Earth from the Sun (93 millions of miles), the circumference of its orbit would be roughly 160 trillions of miles, and its period of revolution about 144 millions of years. This is on the supposition that the Sun is at rest in space and that all the motion takes place in the other body at the distance of Alpha Centauri. It has been determined that the mass of Alpha Centauri is about twice that of our Sun. In that case their mutual revolution around their common center of gravity would be in a period of about 83 millions of years. Something like this seems to be indicated for the Sun's period in its great orbit. Flammarion and Gore in their « Popular Astronomy » have discussed the relation of the Sun to Alpha Centauri and 61 Cygni especially. But they do not recognize the limitations set by the plane of the deflector; they do not see that it is useless to discuss the possibility that stars like 61 Cygni and Sirius, which are far from that plane, may be the Sun's companion. All stars so far from the deflector's plane as these two are positively excluded on account of that relation.

The organization and intensity of the Planetary system appear to show the character properly belonging to the smaller component of a widely separated binary system. The relatively compact structure and great width of the Sun's attendant system and the relative feebleness of the forces affecting the outer planets—Uranus and Neptune—point to this interpretation independently of the indentification of any particular deflecting center.

We have already discussed the function of the orthogonal force as affecting the satellite systems, and in order to show the cause of their inclinations it was necessary to go into the matter there somewhat broadly. Their inclinations were shown to be mainly the composite result of the persistence of the satellite planes and of the rotation planes of the planets, and the rotation of the mean plane of the Planetary system. Following the tentative supposition that Alpha Centauri may be the Sun's companion, it is interesting to note the present relation of the mean planetary plane to the plane in which the Sun revolves if it revolves around this star.

If the Sun is in reality revolving around Alpha Centauri then its path in the sky must be gradually turning toward that star. It can hardly be supposed that the Sun's goal has been accurately determined as yet, but taking the place given by Young to be accurate (R. A. 267°, decl. N. 31°), then the Sun's path is now inclined to the ecliptic and the mean plane of the Planetary system something like 110° or 120°, and the planets therefore revolve in retrograde order. Considering the retrograde systems of Uranus and Neptune this result is not surprising. Indeed, a high degree of inclination would be expected on their account. While the high inclination of these satellite systems is hardly of the nature of proof positive, it is certainly very suggestive that the most plausible candidate among the stars for companionship with the Sun brings out this accordant result. If the relations suggested are in the line of truth it seems certain that the satellite systems of Uranus and Neptune have been an incalculably long time in acquiring their present high degrees of inclination.

If the Sun's goal is accurately located, it is interesting to note, further, the relation of the Sun's path to Alpha Centauri. The actual difference in inclination of the Sun's goal and Alpha Centauri is 91° and 25′, though some 4° difference in longitude adds a little to the angular distance between them. Thus, Alpha Centauri is almost exactly in the plane of the deflector set at 90° to the Sun's path.

Here again, it is easy to see an important relation if the Sun's companion is really identified. For, in going from periastron to apastron the angle of the radius vector connecting the Sun and its companion should form an angle something greater than 90° with the tangent to the Sun's path; in going toward periastron this angle should be less than 90°, while at these points it should be just 90°. Hence, the inference, that if the data are correct the Sun is to be regarded as at the present time drifting at a slowly slackening pace toward the apastron of its great orbit, and this means that for a long period in the past the Sun's revolution has been slowly decreasing in intensity and the Planetary system has therefore been contracting. This is leaving out of account expansion due to growth by accession of new planets at the center of the system.

It is also to be noted that the plane of this binary system lies roughly parallel with the plane of the Milky Way. However, the data for determining the Sun's companion are at present insufficient for final conclusions, and all that has been said above relating particularly to Alpha Centauri is necessarily tentative, and may be mistaken. Yet there is certainly a considerable degree of plausibility for this star. For if Alpha Centauri, the nearest and in all other respects the most favorably situated, be not the Sun's companion, to what other star shall we turn? There is reason to believe, as pointed out above, that the Sun's orbit is nearly circular and also that the Sun is the equal or lesser mass of a widely separated pair of suns. On the basis of present knowledge, the case for Alpha Centauri seems plausible, except for one thing. It has been determined by observations that the proper motion of Alpha Centauri is 3.67 seconds of arc per year. Such a velocity of revolution around the Sun at that distance in a circle would make its period 353,000 years—much too short to be attributed to the Sun's attraction. If this result must stand as the star's heliocentric velocity it seems like a serious objection, but possibly an explanation will be found which will not exclude companionship.

It is to be remembered, besides, that stellar parallaxes and proper motions are among the most difficult and delicate operations of measurement that are performed in astronomy. The sources of error are many and hard to eliminate. It may be that future studies will lessen or remove this apparent objection to the companionship of the Sun and Alpha Centauri.

CHAPTER XXX.

THE NEBULAR HYPOTHESIS.

IT SEEMS to me that anyone who has followed this discussion understandingly to this point ought to be prepared to accept the statement that the Nebular hypothesis, as usually applied in explanation of the origin of the Solar or Planetary system, is not true. I have not said much by way of direct attack upon that doctrine, but the outcome of this discussion seems to me to leave the fact apparent that there is no need of the Nebular hypothesis for the purpose named. If the present theory is valid it has affected the Nebular hypothesis by undermining its foundation rather than by direct and open attack. The Nebular doctrine has long impressed me as one of the most stupendous fallacies of modern science. It is founded on broad analogies which are of such a nature that it has seemed to me impossible to disprove them by direct attack, although their validity has seemed very doubtful and apparently can not be clearly established even by their most ardent advocates.

Probably the most successful attempt to disprove the Nebular hypothesis by direct attack is that recently led by Professor T. C. Chamberlin. Although his views are not the same, they appear at several points to approach quite closely the path we have been following in this discussion. His idea of slow growth for the planets by meteoric accretion and the consequent low temperature of the growing body are in close touch with the present theory. In a recent abstract of Professor Chamberlin's hypothesis he states these conditions thus:

> « In the former [gaseous and meteoroidal hypotheses] the aggregation is massive and relatively rapid; in the latter the aggregation is individual and relatively slow. In the gaseous hypothesis the temperatures are necessarily very high, and the planets are formed by detachments. In the meteoroidal conception of George Darwin, the conditions are practically the same, and in that of Lockyer they differ rather in degree and in detail than in essence. In the planetessimal conception the planets grew up separately by innumerable accretions of infinitesimal planetoids (planetessimals) and the external temperatures were not necessarily high, since the orbits of the planetessimals were normally direct and concurrent and the aggregation came about by overtakes in contradistinction to opposed collisions, and the frequency of these was limited by the concurrent direction of orbital movement.»

While I can not follow Professor Chamberlin in all the developments of his hypothesis, it is certainly true that from the point of view of the present theory it marks a very great advance over all other plans that have ever been suggested for the growth of the planets.

In all probability the Nebular hypothesis would never have existed but for the shortcomings of theoretical astronomy. Theoretical astronomy has failed to claim that which is really its own, and on this account it has not occupied fully the sphere that properly belongs to it. It is the proper province of this science to explain the motions and stabilities of the heavenly bodies. While it has explained many things and its apparent perfection is the admiration of all men, it has nevertheless left many other important things, which plainly lie within its proper sphere, unexplained. It was, in effect, to supplement this deficiency that the Nebular hypothesis was invented. The Nebular hypothesis has nothing to say of the motions and stabilities of the heavenly bodies as they are today, but keeps in the background of the distant past and future, hovering close to the confines of time, near to the beginning and end of things. Theoretical astronomy, on the other hand, has nothing to say of the origin, growth or destiny of the heavenly bodies, but confines itself to their present motions or to such of these in the near past or the near future as can be reached by calculation.

If Newton had found either theoretical or observational grounds for supposing transformations among the members of the Planetary system, as from asteroid to comet or from comet to planet or satellite, the idea of the possible growth of the present system through such changes would have been implanted in men's minds and the Nebular hypothesis would never have existed, or would at least have been confined in its application to a much narrower field. But Newton saw no such possibility, nor did his successors down to the time of Kirkwood, so that theoretical astronomy has continued to the present time barren of results so far as concerns the origin or growth of the Planetary system. It was left for Kirkwood to first establish the probability of such changes when he showed that asteroids may become periodic comets. But Kirkwood did not follow up his own brilliant discovery to its logical end, and it is only now beginning to bear the fruit it should have borne before. Miller took the next important step in laying the foundation of the scheme of growth for satellite and planetary systems, when he showed that the satellites of Mars were formerly asteroids, which Mars has captured and turned into satellites. But Miller, like Kirkwood, did not see the full import of his own conclusions. For he strove to reconcile his theory with the Nebular hypothesis by supposing the satellites of the superior planets to have originated by the condensation of residual wisps of a contracting, rotating nebula, though afterward becoming attached to their primaries by capture. Not having in mind a conception of the determinate quality of stability and of the consequent existence of a definite inner limit of stable satellite revolution for each planet, Miller was unable to see just how the first satellite captured would become adjusted to its primary, or how the second and later ones would be acquired and adjusted. It needed only this to give him a conception of the method by which satellite systems grow. The conclusions of Kirkwood and Miller are great advances over all earlier ideas of the origin of short-period comets and satellites.

The fact that all the planets revolve around the Sun in one direction and that all the satellites known to him revolve in the same direction around their primaries must have deeply impressed the mind of Newton. But his philosophy gave no explanation of this great fact. The continuation of the motions of the several bodies, and hence their stability taken in an immediate sense, he was able to explain. But he was unable to give any account of the source of the initial tangential impulse by which the bodies were started in their revolutions. Newton explained how the Moon, once started, continues to revolve around the Earth, but he appears to have had no conception of any way in which it could have been started. Although he recognized distinctly the necessity for an initial impulse, he left the question of its source unanswered, and it is still unanswered, except so far as it has been met by the Nebular hypothesis, and more recently by the suggestion of Miller, though Miller did not, in explicit terms, apply his theory in explanation of the origin of the Moon.

Theoretical astronomy does not show the Moon's stability to be determinate, and hence has yielded neither a true theory of stability nor a general law for the adjustment of satellites to their primaries. If all the forces affecting the Moon's motion had been known and rightly weighed in Newton's analysis the mechanism of stability would necessarily have been disclosed at the same time, and it would have been *determinate stability*. If the true mechanism of stability had been known we should then have been supplied with general laws applicable to all cases, and we should have been able to give a clear reason why the Moon's mean orbit around the Earth is at the distance of 240,000 miles; why that of Phobos around Mars is at 5800 miles; why Mercury and Venus have no satellites, and so on for every planet of the system. This general law would now be the law of the adjustment of inner satellites to their primaries and would enable us to determine in any hypothetical case whether a given planet could retain a satellite or not, and to predict the place of the orbit of the inner satellite in every case where the determining conditions were known. Earnest efforts have been repeatedly made along the lines of theoretical astronomy to discover some general law for the adjustment of satellites, but without success. The distribution and adjustment of satellites among the planets are problems which seem to be beyond the reach of theoretical astronomy as now constituted; nor has the Nebular hypothesis any adequate explanation to offer. Yet these matters are all clearly within the sphere of theoretical astronomy and ought to be fully explained by it.

The great demonstration of Lagrange that interplanetary perturbations can never destroy the Planetary system, while in reality not conclusive as formerly supposed, is acceptable so far as it goes, but it is only of negative value. It does not disclose the mechanism of stability, nor consequently its law. It is merely a guaranty of non-interference in destructive degree on the part of perturbations.

Probably the best attempt that has ever been made to account for the two planet groups and the greater masses of the superior planets was that of Kirkwood when he was a young man. His attempt attracted wide attention for a time, but under close analysis seemed to break down. It was known as « Kirkwood's Analogy.» (See « Proceedings of Am. Assoc. for Adv. of Sci.,» Vol. II, 1849, pp. 207-221 and 363-369.) But it was Kirkwood's great misfortune, as has been the case with many another man, that he accepted the Nebular hypothesis and strove to adapt his ideas to it. He was striving to improve and strengthen a scientific doctrine which was in reality dead at its birth.

Perhaps some men believe, as has often been justly said of other discarded or outgrown scientific hypotheses, that the Nebular hypothesis, even if now proved wrong, has served a good purpose in the past as a stepping stone in scientific progress. But while such a statement might be favored by some, I do not believe that it would be true. The Nebular hypothesis from its very inception has been a touchstone of blight and disaster. Its specious plausibility has led men to believe that, even if it be not the complete and final truth, it is nevertheless leading in the right direction and needs only refinement and development to become perfect. So, instead of going back to the beginning and devoting their efforts to a revision and correction of the foundations of theoretical astronomy, many men have devoted their best efforts — some of them their lives—to improve this false doctrine, which could never in any event have led them to the truth. Many astronomers and philosophers have contributed to its plausibility, either by the original advances which they have made or by the lucidity of the language in which they have expounded it. There is, I think, in the ultimate cosmic system of the universe a process which has some resemblance to that pictured in the Nebular hypothesis, but it has no immediate relation to the Solar or Planetary system and explains neither its growth nor any of its present characteristics.

Logicians say that when a theory formulated to explain a given limited set of facts is found upon careful trial to explain satisfactorily not only those facts, but also many other unanticipated facts of the same class, and further, if it explains, besides these, other large bodies of facts in other related classes, the presumption that the theory is true becomes very strong and may, with increasing breadth and comprehensiveness of scope, become irresistible. Even a theory like the present one, for which no mathematical proof is offered, may yet bear unmistakable evidences of its general truth, provided its potency in correlating and unifying large and diverse bodies of facts be great enough. On this ground, if at all, the present theory must stand. Not all that may be said for this theory has been said here, so that even if this presentation of it should seem impotent, yet would I rely upon its extension into the realms of sidereal astronomy upon the one hand, and into those of geology upon the other, to add greatly to its strength and scope and to show, perhaps, its fitness to stand as a world theory.

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