THE CULMINATION

OF THE

SCIENCE OF LOGIC

WITH SYNOPSES OF

ALL POSSIBLE VALID FORMS OF CATEGORICAL REASONING IN SYLLOGISMS OF BOTH THREE AND FOUR TERMS.

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PREFACE.

The following are two chapters of a treatise now in course of preparation, and to be entitled "Logic as a Pure Science, illustrated only by means of symbols indefinite in material, but definite in logical signification, with synopses of all possible valid forms of categorical reasoning in syllogisms of both three and four terms."

The preparation of the treatise was undertaken with but little expectation that it, or any part of it, would ever be published; and certainly, with no thought of its resulting in any new contribution to the science.

The author had long thought an elementary treatise on Logic as a pure science, with matter wholly eliminated, a desideratum; and if any such has ever appeared, he is not aware of it. He acknowledges, however, that his acquaintance with the literature of the science is very limited. In writing the treatise, therefore, no concrete examples were employed, but only those with symbols indefinite as to matter, but made definite as to their logical signification.

The symbols adopted were the letters N, D, and J, to represent the Minor, Middle and Major terms of the Syllogism; they being the middle letters of these words respectively. S, M and P are usually employed, as the initials of Subject, Middle and Predicate, but S and P are objectionable, being equally applicable to the subject and predicate of the premises (as propositions), in each of which but one occurs in the statement of Syllogisms, and that one in its appropriate place in such representation in both premises, only in Syllogisms in the first figure; in one premise only, in the second and third figures; and in neither, in the fourth; and their dual possible representations tend to confusion. Distribution and non-distribution are signified by the use of capitals to represent terms distributed, and small letters, terms not distributed. Negation, in universal propositions, is indicated by crossing the capital letter representing the subject. The copula is expressed by the characters, "—" for "is," and "$\smile$" for "is not."

In translating the symbols and characters as employed in propositions into spoken language, the signification of the symbols should of course be *expressed* in respect to the *subject*, but *implied* in respect to the *predicate*, according to common usage and the well-known rules that all universal propositions (and no particular) distribute the subject, and all negative (and no affirmative) the predicate.

Thus the four propositions, A, E, I, O, when written in symbols and characters as above, should be read and understood as follows:

(A)	D — j	All D is j	(*meaning* All D is *some* J)
(E)	Đ — J	No D is J	(" No D is *any* J)
(I)	d — j	Some D is j	(" Some D is *some* J)
(O)	d ⌣ J	Some D is not J	(" Some D is not *any* J)

The consideration of Hypotheticals was reached in the preparation of the treatise, and in the course thereof, analyses of conditional propositions of both three and four terms, in all forms in which they can be expressed, were made; and the study of their results led to the gradual unfolding of the doctrine of Sorites contained in the second of the following chapters.

That doctrine is the culmination of the Science of Logic, which without it has hitherto been incomplete.

The treatise, up to this point, had been written wholly in short-hand, and to guard against the possibility that the discovery might be lost if the author should not live to finish it, and the notes should not be deciphered, these chapters were written out in full, and put in position where they would be found and published, in such contingency.

But, inasmuch as the work yet remains to be completed, and the notes to be written out (which can only be done by the author, his system of short-hand being in many respects peculiar), its appearance will be considerably delayed; and as the discovery, when made known, will, it is believed, not only be an occasion of interest from a scientific point of view, but will prove also to be of practical utility, the author has determined to publish these two chapters in advance. The chapter on Enthymemes is published as preliminary, and to exhibit the synopses therein contained (of which the last shows all valid simple Syllogisms [of three terms] at full length and in regular form), in connection with those contained in the chapter on Sorites (Syllogisms of four terms), thus bringing together, as it were in one view, *all* possible valid forms of categorical reasoning. To those for whose benefit they are thus published the chapters may seem to be unnecessarily diffuse and minute, but to condense them would involve very considerable labor, and they are therefore put forth in the form in which they were written to take their appropriate places in the full treatise, trusting that their minor defects and redundancies may be overlooked.

If the remainder of the treatise shall never appear from the author's pen, there will be little or nothing lost. The suggestion herein made, if it have any merit, will lead other and abler pens to supply the desideratum.

BROOKLYN, *January 14, 1888.*

OF ENTHYMEMES.

§ 1. We have hitherto considered the process of reasoning with three terms, categorically, in its full expression, and have examined all the possible forms of such expression. Such forms are seldom resorted to, either in common conversation or formal discourse, whether spoken or written, but abridged forms of argument are employed in which only part of the process is expressed, the remainder being implied, and being usually so obvious as not to require expression. We come now to consider such abridged forms.

They are called Enthymemes.

§ 2. An Enthymeme is a Syllogism of which but two propositions are expressed, the third being implied.

Enthymemes are of three orders;

1st. That in which the major premise is implied.

2d. That in which the minor premise is implied.

3d. That in which the conclusion is implied.

The following are examples.

Of the first:

$$\begin{array}{l} \text{N} - \text{d}; \\ \therefore \text{N} - \text{j}. \end{array}$$

Of the second:

$$\begin{array}{l} \text{D} - \text{j}; \\ \therefore \text{N} - \text{j}. \end{array}$$

Of the third:

D — j,
and N — d.

In each case the three terms requisite to make up a full Syllogism appear, and the implied premise or conclusion can be readily supplied.

Enthymemes of the first order are herein called *Minor*, and those of the second order *Major* Enthymemes, from the names of their expressed premises, respectively.

§ 3. As every Enthymeme, together with its implied premise or conclusion, is a Syllogism, it is evident that only such can be valid as are symbolized by the letters by which the expressed propositions are symbolized, in the combinations of vowels symbolizing the propositions of all allowable moods of categorical syllogisms, as hereinbefore shown.

By reference thereto, it will be found that all valid Enthymemes must consist of propositions of which the following are the symbols; namely,

Of the first order. (*Minor Enthymemes.*)	*Of the second order.* (*Major Enthymemes.*)	*Of the third order.*
—, A, A;	A, —, A;	A, A, —;
—, A, E;	A, —, E;	A, E, —;
—, A, I;	A, —, I;	A, I, —;
—, A, O;	A, —, O;	A, O, —;
—, E, E;	E, —, E;	E, A, —;
—, E, O;	E, —, O;	E, I, —;
—, I, I;	I, —, I;	I, A, —;
—, I, O;		
—, O, O.	O, —, O.	O, A, —.

The symbols of minor and major Enthymemes are the same, except that there is no valid major Enthymeme in I, O. There are no valid minors in E, O, except useless ones. Leaving the latter out of consideration, it will be found that A occurs four times as the symbol of the premise, and but once as the symbol of the conclusion in both minor and major Enthymemes; E once in minors and twice in majors as the symbol of the premise, and twice in each as the symbol of the conclusion; I twice in minors and once in majors as the symbol of the premise and twice in each as the symbol of the conclusion; and O once as the symbol of the premise and three times as the symbol of the conclusion in both minors and majors.

Minor Enthymemes are the most common, the suppressed major premise being usually a general rule, readily recognized and acquiesced in without being expressed.

Enthymemes of the third order are seldom employed, except in combination with one of the first or second order. They will be referred to when we come to the consideration of Sorites, and it will be found that they occur sometimes in the order of the symbols above shown, namely, major premise first, and minor second; and sometimes in the reverse order, minor first, and major second.

§ 4. To the three orders may be added a fourth; viz., an Enthymeme with but one expressed and two implied propositions. Every demonstrable categorical proposition, put forth independently as the expression of a judgment, is such an Enthymeme, being the conclusion of two implied premises. If the question is asked, "What is N?" the answer must be either a random expression in the form of a proposition, but meaningless, or the result of thought more or less deliberate, and therefore based upon some reason, which, as we have before seen, is a just (or assumed as just) ground of conclusion. This ground must be a mental comparison of the subject, N. with some other term, and of that again with the term predicated of the subject in the answer. The premises thus formed, but not expressed, must be obvious to the questioner, when the answer is given, and therefore admitted; or otherwise explanation would be demanded. Were this not so, there could be no reasoning without going back in every process to some indemonstrable proposition (*axiom* or *postulate*), or to the Great First Cause; with which or with Whom, when reached in the process of investigation, we must necessarily set out in retracing our steps in the deductive process of reasoning.

Such an Enthymeme may also consist, in so far as it is expressed, of a single proposition put forth as a premise (usually the major), the unexpressed premise and conclusion being left to be gathered from the attending circumstances or from the subject-matter under consideration.

§ 5. The middle term will of course be that term of the expressed premise, in minor and major Enthymemes, which is not common to both propositions, and in Enthymemes of the third order, that which is common to both; and will vary in position according to the figure, and the character of the premise, whether minor or major. In minor and major Enthymemes it may or may not be distributed, according to the mood, and character of the premise, whether minor or major; but in Enthymemes of the third order must be at least once distributed.

§ 6. It is manifest, that there are three, and can be but three, Enthymemes having two expressed propositions, viz., one minor, one major, and one of the third order, in each allowable mood of the syllogism; and as the number of such moods is twenty-four, including the useless ones, so the number of Enthymemes of each kind is limited to twenty-four.

The following are synopses of all possible valid forms of categorical Enthymemes of two expressed propositions, together with the implied premise or conclusion of each, as the case may be. On the first page of each of the two synopses of minor and major Enthymemes the forms of the expressed propositions are printed in full, each but once, in the order A, I, E, O, of the symbols of the conclusion, but on the second page they are printed in full throughout. Where they are repeated, they will be found to have in each case a different implied proposition. By counting, it will be found that there are fifteen forms of the expressed propositions of minor Enthymemes (of which four are useless) and twelve of majors. The capital letters in the names of the moods on each page of the synopses are the symbols of the proposition or propositions in the column next adjoining.

The synopsis of Enthymemes of the third order, will serve also as a synopsis of those of the fourth order, as first described, by considering the words "EXPRESSED" and "IMPLIED" as transposed in the headings over the columns of the propositions.

As arranged on page 17, and read across the page, it exhibits all possible valid forms of categorical reasoning with three terms, at full length and in regular form, in the order of the Moods of the Syllogism.

Synopsis of all Possible Valid Forms of Categorical Enthymemes of the First Order. (Minors.)

IN THE ORDER A, I, E, O, OF THE SYMBOLS OF THE CONCLUSIONS.

Symbols of Expressed Propositions.	Expressed Propositions. Minor Premise.	Conclusion.	Implied Proposition. Major Premise.	Moods of Syllogisms.
A, A.	N — d;	∴ N — j.	∵ D — j.	*bArbara.*
A, I.	N — d;	∴ n — j.	∵ D — j.	*A, a, i.* 1st fig.
	D — n;	∴ n — j.	∵ D — j.	*dArapti.*
	″	″	∵ d — j.	*dIsamis.*
	″	″	∵ J — d.	*brAmantip.*
	″	″	∵ j — d.	*dImaris.*
I, I.	n — d;	∴ n — j.	∵ D — j.	*dArii.*
	d — n;	∴ n — j.	∵ D — j.	*dAtisi.*
A, E.	N — d;	∴ $\not{N}$ — J.	∵ $\not{D}$ — J.	*cElarent.*
	″	″	∵ $\not{J}$ — D.	*cEsare.*
E, E.	$\not{N}$ — D;	∴ $\not{N}$ — J.	∵ J — d.	*cAmestres.*
	$\not{D}$ — N;	∴ $\not{N}$ — J.	∵ J — d.	*cAmenes.*
A, O.	N — d;	∴ n ‿ J.	∵ $\not{D}$ — J.	*E, a, o.* 1st fig.
	″	″	∵ $\not{J}$ — D.	*E, a, o.* 2d fig.
	D — n;	∴ n ‿ J.	∵ $\not{D}$ — J.	*fElapton.*
	″	″	∵ d ‿ J.	*bOkardo.*
	″	″	∵ $\not{J}$ — D.	*fEsapo.*
E, O.	$\not{N}$ — D;	∴ n ‿ J.	∵ J — d.	*A, e, o.* 2d fig.
	$\not{D}$ — N;	∴ n ‿ J.	∵ J — d.	*A, e, o.* 4th fig.
I, O.	n — d;	∴ n ‿ J.	∵ $\not{D}$ — J.	*fErio.*
	″	″	∵ $\not{J}$ — D.	*fEstino.*
	d — n;	∴ n ‿ J.	∵ $\not{D}$ — J.	*fEriso.*
	″	″	∵ $\not{J}$ — D.	*frEsison.*
O, O.	n ‿ D;	∴ n ‿ J.	∵ J — d.	*bAroko.*

Synopsis of all Possible Valid Forms of Categorical Enthymemes of the First Order. (Minors.)

IN THE ORDER OF THE MOODS OF CATEGORICAL SYLLOGISMS.

MOODS OF SYLLOGISMS.	EXPRESSED PROPOSITIONS. Minor Premise.		Conclusion.	IMPLIED PROPOSITION.	Major Premise.
barbArA.	N — d ;	∴	N — j.	∵	D — j.
celArEnt.	N — d ;	∴	$\cancel{N}$ — J.	∵	$\cancel{D}$ — J.
a, A, I.	N — d ;	∴	n — j.	∵	D — j.
darII.	n — d ;	∴	n — j.		
e, A, O.	N — d ;	∴	n ⌣ J.	∵	$\cancel{D}$ — J.
ferIO.	n — d ;	∴	n ⌣ J.		
cesArE.	N — d ;	∴	$\cancel{N}$ — J.	∵	$\cancel{J}$ — D.
camEstrEs.	$\cancel{N}$ — D ;	∴	$\cancel{N}$ — J.	∵	J — d.
e, A, O.	N — d ;	∴	n ⌣ J.	∵	$\cancel{J}$ — D.
festInO.	n — d ;	∴	n ⌣ J.		
a, E, O.	$\cancel{N}$ — D ;	∴	n ⌣ J.	∵	J — d.
barOkO	n ⌣ D ;	∴	n ⌣ J.		
darAptI.	D — n ;	∴	n — j.	∵	D — j.
disAmIs.	D — n ;	∴	n — j.	∵	d — j.
datIsI.	d — n ;	∴	n — j.	∵	D — j.
felAptOn.	D — n ;	∴	n ⌣ J.	∵	$\cancel{D}$ — J.
bokArdO.	D — n ;	∴	n ⌣ J.	∵	d ⌣ J.
ferIsO.	d — n ;	∴	n ⌣ J.	∵	$\cancel{D}$ — J.
bramAntIp.	D — n ;	∴	n — j.	∵	J — d.
camEnEs.	$\cancel{D}$ — N ;	∴	$\cancel{N}$ — J.	∵	J — d.
a, E, O.	$\cancel{D}$ — N ;	∴	n ⌣ J.		
dimArIs.	D — n ;	∴	n — j.	∵	j — d.
fesApO.	D — n ;	∴	n ⌣ J.	∵	$\cancel{J}$ — D.
fresIsOn.	d — n ;	∴	n ⌣ J.	∵	$\cancel{J}$ — D.

Synopsis of all Possible Valid Forms of Categorical Enthymemes of the Second Order. (Majors.)

IN THE ORDER A, I, E, O, OF THE SYMBOLS OF THE CONCLUSIONS.

Symbols of Expressed Propositions.	Expressed Propositions. Major Premise.	Conclusion.	Implied Proposition. Minor Premise.	Moods of Syllogisms.
A, A.	D — j;	∴ N — j.	∵ N — d.	*barbAra.*
A, I.	D — j;	∴ n — j.	∵ N — d.	*a, A, i.* 1st fig.
	"	"	∵ n — d.	*darIi.*
	"	"	∵ D — n.	*darApti.*
	"	"	∵ d — n.	*datIsi.*
	J — d;	∴ n — j.	∵ D — n.	*bramAntip.*
I, I.	d — j;	∴ n — j.	∵ D — n.	*disAmis.*
	j — d;	∴ n — j.	∵ D — n.	*dimAris.*
A, E.	J — d;	∴ N̶ — J.	∵ N̶ — D.	*camEstres.*
	"	"	∵ D̶ — N.	*camEnes.*
E, E.	D̶ — J;	∴ N̶ — J.	∵ N — d.	*celArent.*
	J̶ — D;	∴ N̶ — J.	∵ N — d.	*cesAre.*
A, O.	J — d;	∴ n ‿ J.	∵ N̶ — D.	*a, E, o.* 2d fig.
	"	"	∵ n ‿ D.	*barOko.*
	"	"	∵ D̶ — N.	*a, E, o.* 4th fig.
E, O.	D̶ — J;	∴ n ‿ J.	∵ N — d.	*e, A, o.* 1st fig.
	"	"	∵ n — d.	*ferIo.*
	"	"	∵ D — n.	*felApton.*
	"	"	∵ d — n.	*ferIso.*
	J̶ — D;	∴ n ‿ J.	∵ N — d.	*e, A, o.* 2d fig.
	"	"	∵ n — d.	*festIno.*
	"	"	∵ D — n.	*fesApo.*
	"	"	∵ d — n.	*fresIson.*
O, O.	d ‿ J;	∴ n ‿ J.	∵ D — n.	*bokArdo.*

Synopsis of all Possible Valid Forms of Categorical Enthymemes of the Second Order. (Majors.)

IN THE ORDER OF THE MOODS OF CATEGORICAL SYLLOGISMS.

Moods of Syllogisms.	Expressed Propositions. Major Premise.		Conclusion.		Implied Proposition. Minor Premise.
bArbarA.	D — j ;	∴	N — j.	∵	N — d.
cElarEnt.	~~D~~ — J ;	∴	~~N~~ — J.	∵	N — d.
A, a, I.	D — j ;	∴	n — j.	∵	N — d.
dArII.	D — j ;	∴	n — j.	∵	n — d.
E, a, O.	~~D~~ — J ;	∴	n ⁓ J.	∵	N — d.
fErIO.	~~D~~ — J ;	∴	n ⁓ J.	∵	n — d.
cEsarE.	~~J~~ — D ;	∴	~~N~~ — J.	∵	N — d.
cAmestrEs.	J — d ;	∴	~~N~~ — J.	∵	~~N~~ — D.
E, a, O.	~~J~~ — D ;	∴	n ⁓ J.	∵	N — d.
fEstinO.	~~J~~ — D ;	∴	n ⁓ J.	∵	n — d.
A, e, O.	J — d ;	∴	n ⁓ J.	∵	~~N~~ — D.
bArokO.	J — d ;	∴	n ⁓ J.	∵	n ⁓ D.
dAraptI.	D — j ;	∴	n — j.	∵	D — n.
dIsamIs.	d — j ;	∴	n — j.	∵	D — n.
dAtisI.	D — j ;	∴	n — j.	∵	d — n.
fElaptOn.	~~D~~ — J ;	∴	n ⁓ J.	∵	D — n.
bOkardO.	d ⁓ J ;	∴	n ⁓ J.	∵	D — n.
fErisO.	~~D~~ — J ;	∴	n ⁓ J.	∵	d — n.
brAmantIp.	J — d ;	∴	n — j.	∵	D — n.
cAmenEs.	J — d ;	∴	~~N~~ — J.	∵	~~D~~ — N.
A, e, O.	J — d ;	∴	n ⁓ J.	∵	~~D~~ — N.
dImarIs.	j — d ;	∴	n — j.	∵	D — n.
fEsapO.	~~J~~ — D ;	∴	n ⁓ J.	∵	D — n.
frEsisOn.	~~J~~ — D ;	∴	n ⁓ J.	∵	d — n.

Synopsis of all Possible Valid Forms of Categorical Enthymemes of the Third Order.

IN THE ORDER A, I, E, O, OF THE SYMBOLS OF THE CONCLUSIONS.

Symbols of Expressed Propositions.	Expressed Propositions. Major Premise.		Minor Premise.	Implied Proposition. Conclusion.	Moods of Syllogisms.
A, A.	D — j,	and	N — d.	∴ N — j.	*barbarA.*
A, A.	D — j,	"	N — d.	∴ n — j.	*a, a, I.* 1st fig.
A, I.	D — j,	"	n — d.	∴ "	*dariI.*
A, A.	D — j,	"	D — n.	∴ "	*daraptI.*
A, I.	D — j,	"	d — n.	∴ "	*datisI.*
I, A.	d — j,	"	D — n.	∴ "	*disamIs.*
A, A.	J — d,	"	D — n.	∴ "	*bramantIp.*
I, A.	j — d,	"	D — n.	∴ "	*dimarIs.*
E, A.	D̶ — J,	"	N — d.	∴ N̶ — J.	*celarEnt.*
E, A.	J̶ — D,	"	N — d.	∴ "	*cesarE.*
A, E.	J — d,	"	N̶ — D.	∴ "	*camestrEs.*
A, E.	J — d,	"	D̶ — N.	∴ "	*camenEs.*
E, A.	D̶ — J,	"	N — d.	∴ n ⌣ J.	*e, a, O.* 1st fig.
E, I.	D̶ — J,	"	n — d.	∴ "	*feriO.*
E, A.	D̶ — J,	"	D — n.	∴ "	*felaptOn.*
E, I.	D̶ — J,	"	d — n.	∴ "	*ferisO.*
O, A.	d ⌣ J,	"	D — n.	∴ "	*bokardO.*
A, E.	J — d,	"	N̶ — D.	∴ "	*a, e, O.* 2d fig.
A, O.	J — d,	"	n ⌣ D.	∴ "	*barokO.*
A, E.	J — d,	"	D̶ — N.	∴ "	*a, e, O.* 4th fig.
E, A.	J̶ — D,	"	N — d.	∴ "	*e, a, O.* 2d fig.
E, I.	J̶ — D,	"	n — d.	∴ "	*festinO.*
E, A.	J̶ — D,	"	D — n.	∴ "	*fesapO.*
E, I.	J̶ — D,	"	d — n.	∴ "	*fresisOn.*

Synopsis of all Possible Valid Forms of Categorical Enthymemes of the Third Order.

IN THE ORDER OF THE MOODS OF CATEGORICAL SYLLOGISMS.

MOODS OF SYLLOGISMS.	EXPRESSED PROPOSITIONS. Major Premise.		Minor Premise.	IMPLIED PROPOSITION. Conclusion.
bArbAra.	D — j,	and	N — d.	∴ N — j.
cElArent.	$\cancel{D}$ — J,	"	N — d.	∴ $\cancel{N}$ — J.
A. A, i.	D — j,	"	N — d.	∴ n — j.
dArIi.	D — j.	"	n — d.	∴ n — j.
E, A. o.	$\cancel{D}$ — J,	"	N — d.	∴ n –⌣ J.
fErIo.	$\cancel{D}$ — J,	"	n — d.	∴ n –⌣ J.
cEsAre.	$\cancel{J}$ — D,	"	N — d.	∴ $\cancel{N}$ — J.
cAmEstres.	J — d,	"	$\cancel{N}$ — D.	· $\cancel{N}$ — J.
E, A, o.	$\cancel{J}$ — D,	"	N — d.	∴ n –⌣ J.
fEstIno.	$\cancel{J}$ — D.	'	n — d.	∴ n –⌣ J.
A, E, o.	J — d,	"	$\cancel{N}$ — D.	∴ n –⌣ J.
bArOko.	J — d,	"	n –⌣ D.	∴ n –⌣ J.
dArApti.	D — j,	"	D — n.	∴ n — j.
dIsAmis.	d — j,	"	D — n.	∴ n — j.
dAtIsi.	D — j,	"	d — n.	∴ n — j.
fElApton.	$\cancel{D}$ — J,	"	D — n.	∴ n –⌣ J.
bOkArdo.	d –⌣ J,	"	D — n.	∴ n –⌣ J.
fErIso.	$\cancel{D}$ — J,	"	d — n.	∴ n –⌣ J.
brAmAntip.	J — d,	"	D — n.	∴ n — j.
cAmEnes.	J — d,	"	$\cancel{D}$ — N.	∴ $\cancel{N}$ — J.
A. E. o.	J — d,	"	$\cancel{D}$ — N.	∴ n –⌣ J.
dImAris.	j — d,	"	D — n.	∴ n — j.
fEsApo.	$\cancel{J}$ — D,	"	D — n.	∴ n –⌣ J.
frEsIson.	$\cancel{J}$ — D,	"	d — n.	∴ n –⌣ J.

§ 7. The following will serve as rules by which the implied proposition of every Enthymeme having two expressed propositions may be supplied, the first being applicable to those of either the first or second order, and the second to those of the third.

1st. The term of the conclusion of an Enthymeme of either the first or second order which is common to both expressed propositions determines the character of the expressed premise, whether minor or major, according as the same is either the subject (*minor term*) or predicate (*major term*) of the conclusion, and the implied premise may be found by comparing the other two terms.

2d. The term of the expressed minor premise of an Enthymeme of the third order not common to both premises is the subject, and that of the expressed major premise not common to both is the predicate, of the implied conclusion, which is universal or particular, and affirmative or negative, as called for by the premises.

OF SORITES.

§ 1. We come now to the consideration of reasoning with four terms, categorically; and we shall hereinafter find that that is the limit beyond which the human mind cannot go.

§ 2. If we set out to make an investigation concerning any subject, N, and, in the process of our investigation, become possessed of three judgments, which we put forth in the form of propositions, as follows:

N — d;
D — j;
J — x,

we may at once apply to such propositions the dictum of Aristotle, by extending it, as follows—

I first repeat the dictum:

"Whatever is predicated (*i. e.*, affirmed or denied) universally, of any class of things, may be predicated, in like manner (*viz.*, affirmed or denied), of any thing comprehended in that class."

As extended it will read:

Whatever is predicated (*i. e.*, affirmed or denied) universally, of any class of things, may be predicated, in like manner (*viz.*, affirmed or denied), of any *class* comprehended in that class; and, in like manner, of any thing comprehended in any *class* so comprehended.

We have in our last proposition predicated X (x) of the whole class J, and in the second proposition have shown that the *class* D is comprehended in the class J. X (x) may therefore be predicated of the *class* D. But we have also shown in the first proposition that N (which may be either a class, or some single thing) is comprehended in the *class* D. We are therefore warranted, by the dictum as extended, in predicating X (x) of N; viz.:

N — x.

Stating the propositions in their reverse order, and appending to them the proposition thus justified, with the word "therefore" prefixed, we shall have the following expression, which is a Sorites; viz.:

J — x;
D — j;
N — d;
∴ N — x.

But we may, without reversing the order of the propositions, append the new proposition, and will have *the same* Sorites, but in a different form; viz.:

N — d;
D — j;
J — x;
∴ N — x.

The conclusiveness of the reasoning in both forms is apparent.

§ 3. Thus we have a complete Syllogism (but in two different forms or figures) consisting of four propositions, composed of four terms.

Let us now proceed to analyze it, and in the course of the analysis I shall give new names to the terms and propositions, which will be used when referring to them as parts of the Sorites, so as to distinguish them from like parts of a simple Syllogism, which will be called, when referred to as such, by their old names.

And 1st, as to the terms.

The subject, N, with which we set out, is equivalent to the minor term as we have hitherto employed it. I call it the *magnus* term of the Sorites, in the sense of *holding a chief position;* it being the *principal* thing about which we are concerned.

The two terms, D and J, are each greater (*major*) than the magnus term in the forms above exhibited (which you will hereinafter find are the perfect forms), but one, D, is less (*minor*) than the other, J. They are both middle terms, and are each once distributed, and are compared, one with one of the other terms, and the other with the other, in the first and third propositions, and with each other in the second. They will be called, D, the *minor-middle*, and J, the *major-middle* terms.

The term X is equivalent to the major term as hitherto employed, but is greater than the major-middle term, and is the greatest of all the terms of the Sorites. It will therefore be called the *maximus* term.

The four terms, as in the case of a simple Syllogism, occur twice each, the magnus and maximus terms each once in the premises (first three propositions) and once in the concluding proposition, and the minor-middle and major-middle terms each twice in the premises.

N and X are letters in the words *magnus* and *maximus* respectively, and will serve to keep their logical significations in mind, in like manner as the letters N, D, and J, in the words *minor*, *middle*, and *major*, have hitherto served in respect to their logical significations: but they will not in their future use so serve invariably.

2d. As to the propositions.

Three are premises, and will be called from the names of the terms occurring in them respectively:

The *magnus* premise;

The *middle* premise (omitting the prefixes minor and major as unnecessary, there being no middle premise in a simple Syllogism);

The *maximus* premise.

The concluding proposition will hereinafter be found to be the ultimate one of two conclusions warranted by the premises; and to distinguish it as such, I shall call it the *ultima* (*conclusio* understood).

3d. As to the figure.

The figure of a simple Syllogism depends upon the positions of its terms, but that of a Sorites upon the positions of its magnus and maximus premises. It will be called the *configuration*. There are two, the first called *regressive*, in which the maximus premise is the first, and the magnus last; and the second, *progressive*, in which the magnus premise is the first and the maximus last. The progressive configuration was the only one known until about the beginning of the seventeenth century, when the regressive was discovered by a German logician named Goclenius; and it is called also Goclenian after him. It has been a subject of dispute among logicians as to which configuration should be called progressive, and which regressive, but the prevailing opinion is in favor of the names as herein used. They are generally treated of in the order as in the last sentence; but I have reversed it, exhibiting the regressive first, and the progressive last. The moods of each configuration, and their number, will hereinafter appear.

§ 4. If all Sorites, in respect to the positions of the terms, were in the forms hereinbefore given, and their conclusiveness were equally as apparent, I might at once proceed further to illustrate and comment upon them, and state the rules usually given in logical treatises concerning them, which are applicable only in such case; but such is not the case, and I defer further comment until I shall have exhibited them in another aspect in which they can be considered; viz., as complex expressions consisting of two Enthymemes.

The Sorites, so to be exhibited, will be the same as before given; and for the sake of brevity, I shall call the terms and propositions by the names hereinbefore given to them, in advance of exhibiting them under the new aspect.

§ 5. For the purpose of such consideration I repeat the three propositions with which we set out.

N — d;
D — j;
J — x.

If now, having possessed ourselves of these judgments, but failing to observe, from their perfect concatenation, that we may at once deduce from them the ultimate conclusion wrapped up in them, we proceed to syllogize with them by means of simple Syllogisms of three propositions, we shall naturally commence with the widest truth which we have discovered, viz., J — x; and we shall find our first Syllogism to be as follows:

$$\begin{array}{l} \text{J — x;} \\ \text{D — j;} \\ \therefore \text{D — x,} \end{array}$$

and, having thus become possessed of a new truth, viz., D — x, we shall put it forth as a premise, combining with it our first proposition, as yet unemployed, and produce a second Syllogism as follows:

$$\begin{array}{l} \text{D — x;} \\ \text{N — d;} \\ \therefore \text{N — x.} \end{array}$$

The conclusion of this second Syllogism is the ultima of the Sorites, as we have before seen it.

But if, in the course of our investigation, we had stopped after the discovery of the first two truths, viz.:

$$\begin{array}{l} \text{N — d;} \\ \text{D — j,} \end{array}$$

and had syllogized with them, we should in like manner have commenced with the widest truth then discovered, viz., D — j, and our first Syllogism would have been:

D — j;
N — d;
∴ N — j.

The question would then naturally have arisen, But what is J? and resuming the process of investigation, we should have discovered that J — x, and thereupon would have syllogized again:

J — x;
N — j;
∴ N — x,

and thus, by a second series of Syllogisms, we should have arrived at the ultima of the Sorites, as we have before seen it.

By the former process, we retraced our steps after having reached the summit of our investigation, and it is therefore properly called regressive; by the latter we have reasoned as we progressed, and it is therefore properly called progressive; but by both processes we have arrived at the same ultimate conclusion, illustrating the aphorism that "all truth is one."

The middle premise, as you will observe, is the minor premise of the first Syllogism in the first series, and the maximus premise the major; and the middle premise is the major premise of the first Syllogism in the second series, and the magnus premise the minor; and all the

Syllogisms are in *Barbara* in the first figure, which you have learned is the only perfect figure.

§ 6. But we may reason imperfectly, and that too, even when we have our judgments in a perfect concatenation, as they have thus far been exhibited; and, in such case, we shall find our Syllogisms to be in one or more of the imperfect figures. If, in the regressive process we begin to syllogize with the middle premise as the major premise of the first Syllogism (instead of the minor), and the maximus as the minor (instead of the major); and in the progressive process, with the middle premise as the minor premise of the first Syllogism (instead of the major), and the magnus premise as the major (instead of the minor), we can frame, or attempt to frame, two other series of Syllogisms, which I here exhibit, with the two Syllogisms of each series, side by side, as follows:

In the regressive process.

D — j;	x — d;
J — x;	N — d;
∴ x — d.	

In the progressive process.

N — d;	J — x;
D — j;	j — n;
∴ j — n.	∴ n — x.

In the latter series, only a particular ultimate conclusion is arrived at; in the former, no ultimate conclusion is warranted by reason of non-distribution of the middle term in the second attempted Syllogism.

Thus, as you will perceive, imperfect processes are followed by imperfect or no results.

§ 7. To recur now to the two principal series, and for the purpose of bringing the two Syllogisms of each together, in such a method of arrangement that you may at once see the connection between them, and the application of the remarks that are to follow, I repeat them, putting the two Syllogisms of each, side by side.

First, or regressive series.

J — x;	D — x;
D — j;	N — d;
∴ D — x.	∴ N — x.

Second, or progressive series.

D — j;	J — x;
N — d;	N — j;
∴ N — j.	∴ N — x.

By taking an Enthymeme of the third order from the first Syllogism, and one of the first order from the second Syllogism of the first series, and putting them together in one expression, and, by taking an Enthymeme of the third order from the first Syllogism of the second series. *but transposing the propositions so taken*, and one of the second order from the second Syllogism of the same series, and putting them together in one expression. we shall have the same Sorites, as before, in the two configurations, viz.:

Regressive Sorites from the first series.	*Progressive Sorites from the second series.*
J — x;	N — d;
D — j;	D — j;
N — d;	J — x;
∴ N — x.	∴ N — x.

The conclusion of the first Syllogism in each series is held in the mind (otherwise there were no Enthymeme), but carried forward mentally, and employed as a premise, still unexpressed, in connection with the Enthymeme taken from the second.

A Sorites considered as a complex expression as above shown is also called a Chain-Syllogism.

§ 8. The *middle premise* (being the proposition D—j in which the minor-middle and major-middle terms are compared) will always be the *second* proposition in every Sorites, *simple* (as hitherto shown) or *compound* (as to which latter you will hereinafter be instructed); and by expressing it, in connection with the ultima, every Sorites may be still further abridged, thus:

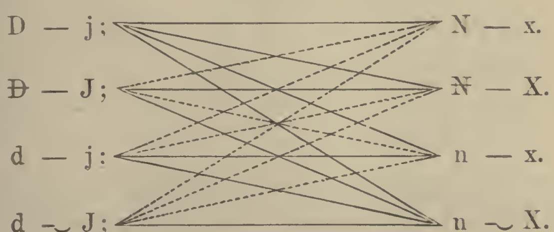
D — j;
∴ N — x.

All the four terms here appear, but each only once. Such an expression is *in the form of an Enthymeme* (but is not an Enthymeme, for that can have only three terms), and may properly be called an *Abridged Sorites.*

From the employment of the middle premise as the minor or major premise of the first Syllogism, I designate Sorites (considered as complex expressions) *minor* and *major* Sorites, respectively, for the purpose of classification as hereinafter shown. Either may be regressive or progressive; but we shall see that the proper division of Sorites is into regressives and progressives.

Observe, that in all major Sorites, but in no minors, the premises constituting the Enthymeme of the third order taken from the first Syllogism, are transposed.

§ 9. The Syllogisms of the two principal series (of Enthymemes of which the Sorites exhibited consist) are wholly in the first figure. But a little reflection will show that Sorites may also consist of Enthymemes taken from Syllogisms in any of the figures capable of combination in series, quantity and quality considered. And, as all Sorites may be abridged in the manner hereinbefore shown, it is also manifest that the range of possible abridged Sorites is limited to the number of possible combinations of two propositions composed of four terms, expressed in the same form as to the order of the terms throughout, but modified in respect to quantity and quality, as in the following scheme; and only such can be valid as are capable of being expanded into full Sorites, and from full Sorites into at least two series of Syllogisms. The propositions must be in one or another of the combinations shown by full lines in the scheme.



Considering the lines connecting the propositions, each as signifying "and therefore," there are sixteen different combinations. But of these, only nine will be found to be valid, and they are symbolized by the same symbols as those of valid Enthymemes of the first order, as hereinbefore shown, and may be expanded into full Sorites (the supplied premises varying in the order of the terms as well as in quantity and quality), and from full Sorites into two, three, or four series of Syllogisms, with the middle premise as either the minor or the major premise of the first Syllogism of one or more series, except in two cases, which will be hereinafter noted.

The number of valid full Sorites into which the nine abridged forms may be so expanded is one hundred and forty-four, of which one half are minors and one half majors, classified as such according to the combinations of the symbols of the abridged forms, as follows:

Symbols.	*Minors.*	*Majors.*
A, A.	1	1
A, E.	4	8
A, I.	16	10
A, O.	24	24
E, E.	4	4
E, O.	10	16
I, I.	4	4
I, O.	8	4
O, O.	1	1
	72	72

The following synopsis exhibits all possible valid categorical Sorites, in their abridged forms, as minors on the left-hand pages, and as majors on the right; together with the premises by which they may be expanded into valid full Sorites, and the names of the moods in which they can be further and fully expanded into series of Syllogisms. They are arranged in the order A, I, E, O, of the symbols of the ultima.

The abridged forms may be expanded into full Sorites by writing first, the first of the two supplied premises; secondly, the middle premise; thirdly, the second of the two supplied premises; and lastly, the ultima.

Preceding the synopsis are given two series of schemes, by which the different ways in which abridged Sorites may be expanded into full Sorites, and from full Sorites into series of Syllogisms, in all combinations of figures in which they are capable of being so expanded, may be clearly seen. The terms of the abridged Sorites are in capitals enclosed in circles connected by lines representing the copulas of the propositions. The curved lines (considered as copulas) above the propositions constituting the abridged Sorites, in connection with those propositions, indicate two expanded Sorites, and in connection also with the dotted straight line above, indicate two series of Syllogisms; and the lines below, two other expanded Sorites, and two other series of Syllogisms. The dotted straight lines show the unexpressed conclusions of the first Syllogisms, which in each case becomes one of the premises of the second. The modifications of the propositions of the abridged Sorites, in respect to quantity and quality, are indicated by the symbols above and below the lines representing their copulas respectively; those above referring to the Sorites and Syllogisms indicated above, and those below, to those below. The modifications of the other indicated propositions are also in like manner signified.

The symbols in connection with the lines are those only in which the Sorites and Syllogisms are valid in the figures shown.

It is not meant that each symbol in connection with each other will yield a valid Sorites, but that each, in connection with some one or more of the others, will be found valid. Thus, in the second scheme of minors, the maximus premise, A, will combine with the middle premise as E or O, and E with A or I, but not otherwise.

The designations of premises, written between parallel curved lines, refer to the propositions indicated by both lines; the symbols and number of the figure being on the other side of each line, respectively.

By marking all the lines with all the symbols, you will be able to make an exhaustive analysis of all possible ways in which it may be attempted to frame simple Sorites. In view of the number given on the next page, you may think the attempt formidable, but you will find it not so much so as it will at first appear, if you but consider and apply to the symbols the rules of the syllogism before proceeding to test them. The lines above the propositions constituting the abridged Sorites are marked with all the symbols of the propositions respectively, as they may be employed in single simple syllogisms, as hereinbefore shown, but those below, not; and if you first add to the latter the omitted symbols, making them to correspond with those above, you will find that such added symbols will, in all cases, yield no conclusion in the second of the Syllogisms, by reason of one or the other of the two faults, *undistributed middle* and *illicit process of the major*. If the remaining symbols be then added to each line, a violation of some one or more of the rules of the syllogism will be found in either the first or second Syllogism.

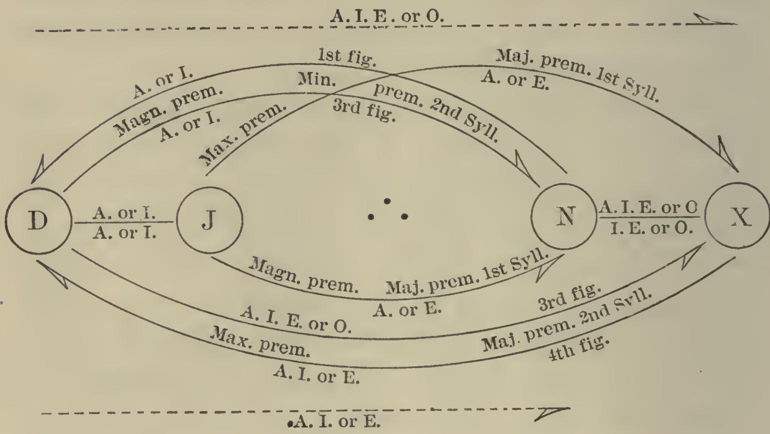
The total number of the ways in which it may thus be attempted to combine the four symbols A, E, I, O, according to the schemes is eight thousand one hundred and ninety-two, that being the product of the number of ways (256) in which the four symbols may be combined (all the same, or partly the same, or all different), multiplied by the number of combinations of propositions (4) indicated by each scheme, and again by the number of schemes (8)—($256 \times 4 \times 8 = 8192$).

The total number of valid Sorites without regard to their character as minor or major, or as regressive or progressive, will be hereinafter found to be forty-four.

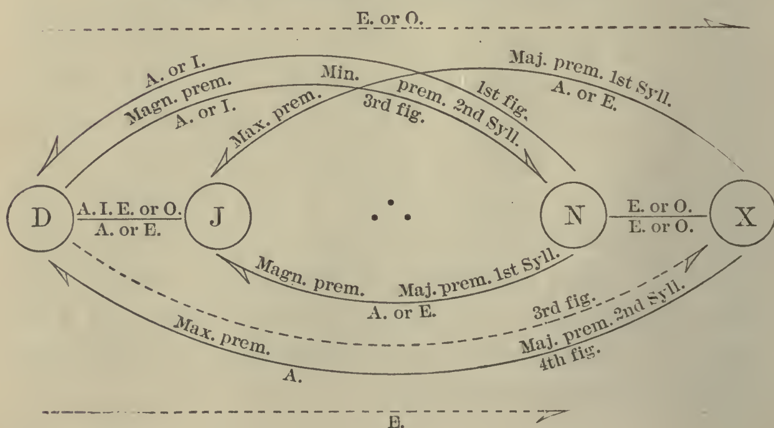
By examining each scheme, and comparing the Sorites and series of Syllogisms thereby indicated (those above with each other, and those below with each other), and by comparing each scheme with each of the others in all possible ways, the differences between, and correlations of, the several figures of the Syllogism and the two kinds of Sorites indicated by the schemes (that is, either minor or major), will also clearly appear, and the student cannot fail to be impressed with the harmony and symmetry of pure reasoning, in all its varied possible forms of expression.

SCHEMES OF MINOR SORITES.

FIRST SYLLOGISM IN FIRST FIGURE.

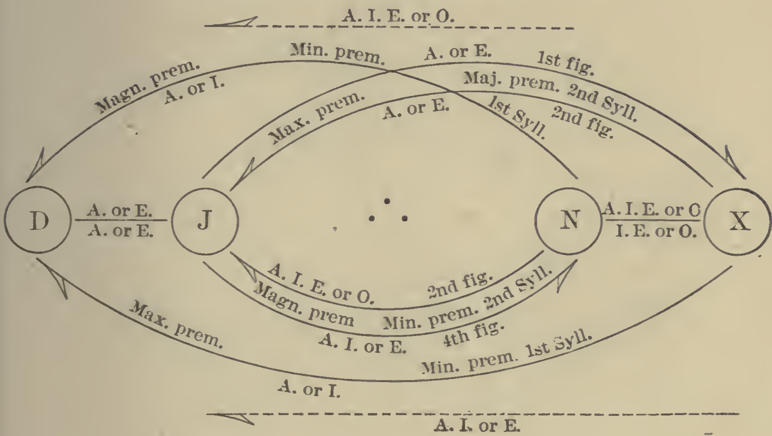


FIRST SYLLOGISM IN SECOND FIGURE.

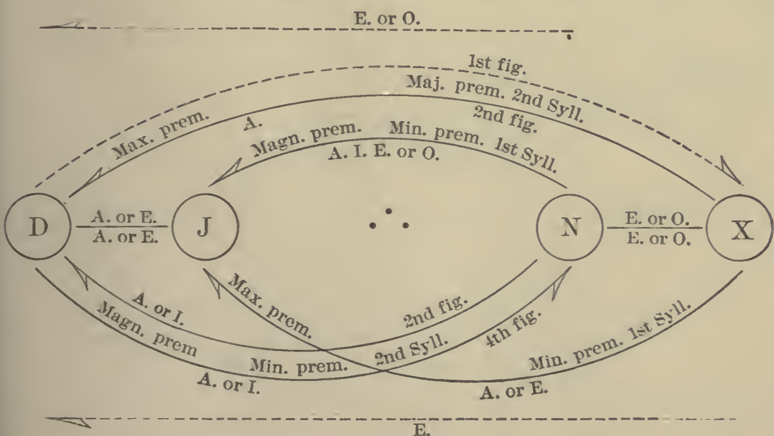


SCHEMES OF MAJOR SORITES.

FIRST SYLLOGISM IN FIRST FIGURE.

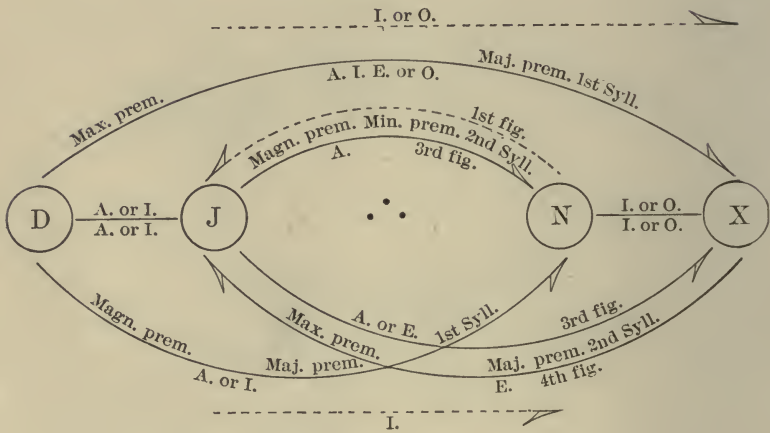


FIRST SYLLOGISM IN SECOND FIGURE.

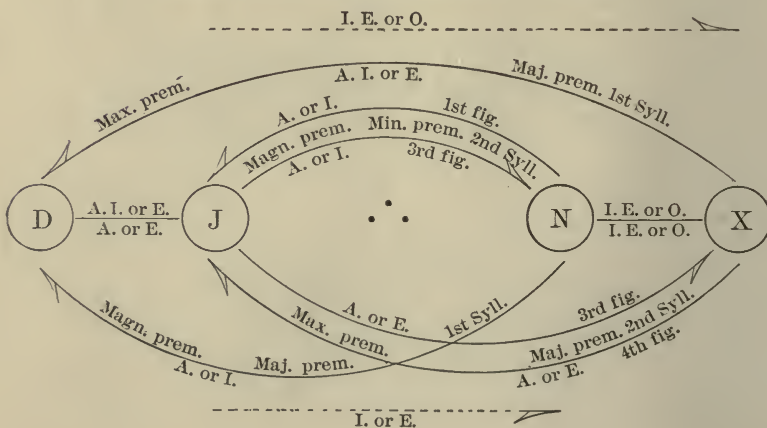


SCHEMES OF MINOR SORITES.

FIRST SYLLOGISM IN THIRD FIGURE.

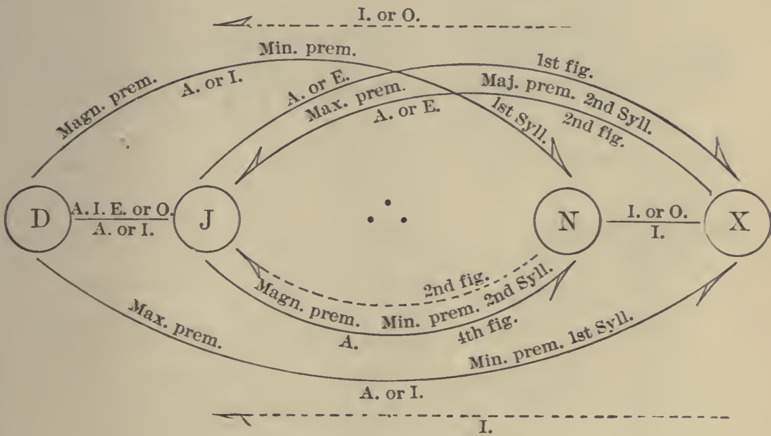


FIRST SYLLOGISM IN FOURTH FIGURE.

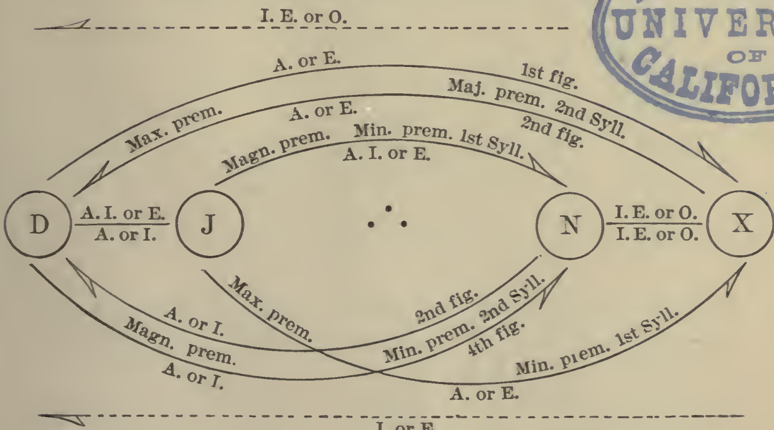


SCHEMES OF MAJOR SORITES.

FIRST SYLLOGISM IN THIRD FIGURE.



FIRST SYLLOGISM IN FOURTH FIGURE.



Synopsis of all Possible Valid Forms of

TOGETHER WITH ALL THEIR POSSIBLE MAGNUS AND MAXIMUS PREMISES,

SYMBOLS.	Nos.	ABRIDGED MINOR SORITES. Middle Premise.	Ultima.	MOOD OF FIRST SYLLOGISM.	MAJOR PREMISE OF FIRST SYLLOGISM.	MINOR OR MAJOR PREMISE OF SECOND SYLLOGISM.	MOOD OF SECOND SYLLOGISM.
A, A.	1	D—j ; ∴	N—x.	*bArbAra*	J—x	N—d	*barbArA*
A, I.	2	D—j ; ∴	n—x.	*bArbAra*	J—x	N—d	*a,A,I.* 1st fig.
	3	"	"	"	"	n—d	*darII*
	4	"	"	" or, *A, A, i*	"	D—n	*darAptI* *disAmIs*
	5	"	"	*bArbAra*	"	d—n	*datIsI*
	6	"	"	*brAmAntip*	N—d	J—x	*dAtisI*
	7	"	"	*dImAris*	n—d	"	"
	8	"	"	*dArApti*	D—n	"	"
	9	"	"	*dIsAmis*	d—n	"	"
	10	"	"	*bArbAra* or, *A, A, i*	J—n	D—x	*dAraptI* *dAtisI*
	11	"	"	*bArbAra*	"	d—x	*dIsamIs*
	12	"	"	"	"	X—d	*brAmantIp*
	13	"	"	"	"	x—d	*dImarIs*
	14	"	"	*dArApti*	D—x	J—n	*disAmIs*
	15	"	"	*dIsAmis*	d—x	"	"
	16	"	"	*brAmAntip*	X—d	"	"
	17	"	"	*dImAris*	x—d	"	"
I, I.	18	d—j ; ∴	n—x.	*dArIi*	J—x	D—n	*disAmIs*
	19	"	"	*dAtIsi*	D—n	J—x	*dAtisI*
	20	"	"	*dArIi*	J—n	D—x	"
	21	"	"	*dAtIsi*	D—x	J—n	*disAmIs*

Abridged Categorical Sorites.

AND MOODS OF SIMPLE SYLLOGISMS IN WHICH THEY CAN BE FULLY EXPANDED.

SYMBOLS.	NOS.	ABRIDGED MAJOR SORITES. Middle Premise.	Ultima.	MOOD OF FIRST SYLLOGISM.	MINOR PREMISE OF FIRST SYLLOGISM.	MAJOR OR MINOR PREMISE OF SECOND SYLLOGISM.	MOOD OF SECOND SYLLOGISM.
A, A.	1	D—j;	∴ N—x.	*bArbAra*	N—d	J—x	*bArbarA*
A, I.	2	D—j;	∴ n—x.	*bArbAra* or, *A*, *A*, *i*	N—d	J—x	*A. a. I* *dAriI*
	3	"	"	*dArIi*	n—d	"	"
	4	"	"	*dArApti*	D—n	"	"
	5	"	"	*dAtIsi*	d—n	"	"
	6	"	"	*brAmAntip*	J—x	D—n	*dimArIs*
	7	"	"	*dArApti*	D—x	J—n	"
	8	"	"	*dAtIsi*	d—x	"	"
	9	"	"	*bArbAra* or, *A*, *A*, *i*	X—d	"	*bramAntIp* *dimArIs*
	10	"	"	*dArIi*	x—d	"	"
	11	"	"	*brAmAntip*	J—n	D—x	*dAriI*
I, I.	12	d—j;	∴ n—x.	*dIsAmis*	D—n	J—x	*dAriI*
	13	"	"	*dImAris*	J—x	D—n	*dimArIs*
	14	"	"	*dIsAmis*	D—x	J—n	"
	15	"	"	*dImAris*	J—n	D—x	*dAriI*

Synopsis of all Possible Valid Forms of

TOGETHER WITH ALL THEIR POSSIBLE MAGNUS AND MAXIMUS PREMISES,

Symbols.	Nos.	Abridged Minor Sorites. Middle Premise.	Ultima.	Mood of First Syllogism.	Major Premise of First Syllogism.	Minor or Major Premise of Second Syllogism.	Mood of Second Syllogism.
A, E.	22	D—j ; ∴	$\cancel{N}$—X.	*cElArent*	$\cancel{J}$—X	N—d	*celArEnt*
	23	"	"	*cEsAre*	$\cancel{X}$—J	"	"
	24	"	"	*cElArent*	$\cancel{J}$—N	X—d	*cAmenEs*
	25	"	"	*cEsAre*	$\cancel{N}$—J	"	"
E, E.	26	$\cancel{D}$—J ; ∴	$\cancel{N}$—X.	*cAmEstres*	X—j	N—d	*celArEnt*
	27	"	"	*cAmEnes*	N—d	X—j	*cAmenEs*
	28	"	"	"	X—d	N—j	*celArEnt*
	29	"	"	*cAmEstres*	N—j	X—d	*cAmenEs*
A, O.	30	D—j ; ∴	n⌣X.	*cElArent*	$\cancel{J}$—X	N—d	{ *e, A, O.* 1st fig.
	31	"	"	"	"	n—d	*ferIO*
	32	"	"	{ " or, *E, A, o* }	"	D—n	{ *felAptOn* *bokArdO*
	33	"	"	*cElArent*	"	d—n	*ferIsO*
	34	"	"	*cEsAre*	$\cancel{X}$—J	N—d	{ *e, A, O.* 1st fig.
	35	"	"	"	"	n—d	*ferIO*
	36	"	"	{ " or, *E, A, o* }	"	D—n	{ *felAptOn* *bokArdO*
	37	"	"	*cEsAre*	"	d—n	*ferIsO*
	38	"	"	*cElArent*	$\cancel{J}$—N	X—d	{ *A, e, O.* 4th fig.
	39	"	"	*cEsAre*	$\cancel{N}$—J	"	"

Abridged Categorical Sorites. (Continued.)

AND MOODS OF SIMPLE SYLLOGISMS IN WHICH THEY CAN BE FULLY EXPANDED.

SYMBOLS.	NOS.	ABRIDGED MAJOR SORITES. Middle Premise.	Ultima.	MOOD OF FIRST SYLLOGISM.	MINOR PREMISE OF FIRST SYLLOGISM.	MAJOR OR MINOR PREMISE OF SECOND SYLLOGISM.	MOOD OF SECOND SYLLOGISM.
A, E.	16	D—j ;	∴ $\cancel{\text{N}}$—X.	*bArbAra*	N—d	$\cancel{\text{J}}$—X	*cElarEnt*
	17	"	"	"	"	$\cancel{\text{X}}$—J	*cEsarE*
	18	"	"	"	X—d	$\cancel{\text{J}}$—N	*camEnEs*
	19	"	"	"	"	$\cancel{\text{N}}$—J	*camEstrEs*
	20	"	"	*cAmEnes*	$\cancel{\text{J}}$—X	N—d	*cesArE*
	21	"	"	*cAmEstres*	$\cancel{\text{X}}$—J	"	"
	22	"	"	*cAmEnes*	$\cancel{\text{J}}$—N	X—d	*cAmestrEs.*
	23	"	"	*cAmEstres*	$\cancel{\text{N}}$—J	"	"
E, E.	24	$\cancel{\text{D}}$—J ;	∴ $\cancel{\text{N}}$—X.	*cElArent*	N—d	X—j	*cAmestrEs*
	25	"	"	*cEsAre*	X—j	N—d	*cesArE*
	26	"	"	"	N—j	X—d	*cAmestrEs*
	27	"	"	*cElArent*	X—d	N—j	*cesArE*
A, O.	28	D—j ;	∴ n‿X.	{ *bArbAra* or, *A*, *A*, *i* }	N—d	$\cancel{\text{J}}$—X	{ *E*, *a*, *O* *fEriO* }
	29	"	"	*dArIi*	n—d	"	"
	30	"	"	*dArApti*	D—n	"	"
	31	"	"	*dAtIsi*	d—n	"	"
	32	"	"	{ *bArbAra* or, *A*, *A*, *i* }	N—d	$\cancel{\text{X}}$—J	{ *E*, *a*, *O* *fEstinO* }
	33	"	"	*dArIi*	n—d	"	"
	34	"	"	*dArApti*	D—n	"	"
	35	"	"	*dAtIsi*	d—n	"	"
	36	"	"	*bArbAra*	X—d	$\cancel{\text{J}}$—N	{ *a*, *E*, *O*. 4th fig.
	37	"	"	"	"	$\cancel{\text{N}}$—J	{ *a*, *E*, *O*. 2d fig.
	38	"	"	"	"	n‿J	*barOkO*

Synopsis of all Possible Valid Forms of

TOGETHER WITH ALL THEIR POSSIBLE MAGNUS AND MAXIMUS PREMISES,

Symbols.	Nos.	Abridged Minor Sorites. Middle Premise.	Ultima.	Mood of First Syllogism.	Major Premise of First Syllogism.	Minor or Major Premise of Second Syllogism.	Mood of Second Syllogism.
A, O.	40	Ḋ—j ;	∴ n‿X.	*fElApton*	Đ—X	J—n	*bokArdO*
	41	″	″	*bOkArdo*	d‿X	″	″
	42	″	″	*fEsApo*	X̶—D	″	″
	43	″	″	{*bArbAra* or, *A*, *A*, *i*}	J—n	Đ—X	{*fElapt On* *fEris O*}
	44	″	″	*bArbAra*	″	d‿X	*bOkard O*
	45	″	″	{″ or, *A*, *A*, *i*}	″	X̶—D	{*fEsap O* *frEsis On*}
	46	″	″	*brAmAntip*	N—d	Ɉ—X	*fEris O*
	47	″	″	*dImAris*	n—d	″	″
	48	″	″	*dArApti*	D—n	″	″
	49	″	″	*dIsAmis*	d—n	″	″
	50	″	″	*brAmAntip*	N—d	X̶—J	*frEsis On*
	51	″	″	*dImAris*	n—d	″	″
	52	″	″	*dArApti*	D—n	″	″
	53	″	″	*dIsAmis*	d—n	″	″
I, O.	54	d—j ;	∴ n‿X.	*fErIo*	Ɉ—X	D—n	*bokArd O*
	55	″	″	*fEstIno*	X̶—J	″	″
	56	″	″	*fErIso*	Đ—X	J—n	″
	57	″	″	*frEsIson*	X̶—D	″	″
	58	″	″	*dAtIsi*	D—n	Ɉ—X	*fEris O*
	59	″	″	″	″	X̶—J	*frEsis On*
	60	″	″	*dArIi*	J—n	Đ—X	*fEris O*
	61	″	″	″	″	X̶—D	*frEsis On*

Abridged Categorical Sorites. (Continued.)

AND MOODS OF SIMPLE SYLLOGISMS IN WHICH THEY CAN BE FULLY EXPANDED.

SYMBOLS.	Nos.	ABRIDGED MAJOR SORITES. Middle Premise.	Ultima.	MOOD OF FIRST SYLLOGISM.	MINOR PREMISE OF FIRST SYLLOGISM.	MAJOR OR MINOR PREMISE OF SECOND SYLLOGISM.	MOOD OF SECOND SYLLOGISM.
A, O.	39	D—j ;	∴ n‿X.	*brAmAntip*	J—n	Đ—X	*fEriO*
	40	"	"	"	"	X̶—D	*fEstinO*
	41	"	"	*cAmEnes*	Ɉ—X	N—d	{ *e, A, O.* 2d fig.
	42	"	"	"	"	n—d	*festInO*
	43	"	"	"	"	D—n	*fesApO*
	44	"	"	"	"	d—n	*fresIsOn*
	45	"	"	*cAmEstres*	X̶—J	N—d	{ *e, A, O.* 2d fig.
	46	"	"	"	"	n—d	*festInO*
	47	"	"	"	"	D—n	*fesApO*
	48	"	"	"	"	d—n	*fresIsOn*
	49	"	"	{ *cAmEnes* or, *A, E, o* }	Ɉ—N	X—d	{ *A, e, O* *bArokO*
	50	"	"	{ *cAmEstres* or, *A, E, o* }	N̶—J	"	{ *A, e, O* *bArokO*
	51	"	"	*bArOko*	n‿J	"	"
I, O.	52	d—j ;	∴ n‿X.	*dIsAmis*	D—n	Ɉ—X	*fEriO*
	53	"	"	"	"	X̶—J	*fEstinO*
	54	"	"	*dImAris*	J—n	Đ—X	*fEriO*
	55	"	"	"	"	X̶—D	*fEstinO*

Synopsis of all Possible Valid Forms of

TOGETHER WITH ALL THEIR POSSIBLE MAGNUS AND MAXIMUS PREMISES,

SYMBOLS.	NOS.	ABRIDGED MINOR SORITES. Middle Premise.	Ultima.	MOOD OF FIRST SYLLOGISM.	MAJOR PREMISE OF FIRST SYLLOGISM.	MINOR OR MAJOR PREMISE OF SECOND SYLLOGISM.	MOOD OF SECOND SYLLOGISM.
E, O.	62	Đ—J ; ∴	n—‿X.	*cAmEstres*	X—j	N—d	*e,A,O.* 1st fig.
	63	"	"	"	"	n—d	*ferIO*
	64	"	"	{ " or, *A*, *E*, *o* }	"	D—n	{ *felAptOn* *bokArdO*
	65	"	"	*cAmEstres*	"	d—n	*ferIsO*
	66	"	"	*cAmEnes*	X—d	N—j	*e,A,O.* 1st fig.
	67	"	"	"	"	n—j	*ferIO*
	68	"	"	{ " or, *A*, *E*, *o* }	"	J—n	{ *felAptOn* *bokArdO*
	69	"	"	*cAmEnes*	"	j—n	*ferIsO*
	70	"	"	"	N—d	X—j	*A,e,O.* 4th fig.
	71	"	"	*cAmEstres*	N—j	X—d	*A,e,O.* 4th fig.
O, O.	72	d—‿J ; ∴	n—‿X.	*bArOko*	X—j	D—n	*bokArdO*

Abridged Categorical Sorites. (Concluded.)

AND MOODS OF SIMPLE SYLLOGISMS IN WHICH THEY CAN BE FULLY EXPANDED.

SYMBOLS.	NOS.	ABRIDGED MAJOR SORITES. Middle Premise.	Ultima.	MOOD OF FIRST SYLLOGISM.	MINOR PREMISE OF FIRST SYLLOGISM.	MAJOR OR MINOR PREMISE OF SECOND SYLLOGISM.	MOOD OF SECOND SYLLOGISM.
E, O.	56	Ð—J ; ∴	n‿X.	*cElArent* or, *E, A, o*	N—d	X—j	*A, e, O* *bArokO*
	57	"	"	*fErIo*	n—d	"	"
	58	"	"	*fE'Apton*	D—n	"	"
	59	"	"	*fErIso*	d—n	"	"
	60	"	"	*cEsAre* or, *E, A, o*	N—j	X—d	*A, e, O* *bArokO*
	61	"	"	*fEstIno*	n—j	"	"
	62	"	"	*fEsApo*	J—n	"	"
	63	"	"	*frEsIson*	j—n	"	"
	64	"	"	*cEsAre*	X—j	N—d	*e, A, O.* 2d fig.
	65	"	"	"	"	n—d	*festInO*
	66	"	"	"	"	D—n	*fesApO*
	67	"	"	"	"	d—n	*fresIsOn*
	68	"	"	*cElArent*	X—d	N—j	*e, A. O.* 2d fig.
	69	"	"	"	"	n—j	*festInO*
	70	"	"	"	"	J—n	*fesApO*
	71	"	"	"	"	j—n	*fresIsOn*
O, O.	72	d‿J ; ∴	n‿X.	*bOkArdo*	D—n	X—j	*bArokO*

§ 10. By examining the foregoing synopsis and testing the same, it will be found that

If the major-middle term (predicate of the middle premise) be the middle term of the first Syllogism, then if the Sorites be

MINOR;						*but if it be* MAJOR;					
The first Syllogism may be in figure;	*and the second may be in figure;*	*and the configurations of the Sorites, whether regressive or progressive, or both, and the number of each, will be as follows:*				*The first Syllogism may be in figure;*	*and the second may be in figure;*	*and the configurations of the Sorites, whether regressive or progressive, or both, and the number of each, will be as follows:*			
1	*1*	Reg.,	6								
1	*3*	Reg.,	6	Prog.,	6	*2*	*2*	Reg.,	6	Prog.,	6
1	*4*			Prog.,	6	*2*	*4*	Reg.,	4		
2	*1*	Reg.,	6			*4*	*1*			Prog.,	4
2	*3*	Reg.,	6			*4*	*2*	Reg.,	3	Prog.,	6
2	*4*			Prog.,	4	*4*	*4*	Reg.,	4		

But if the minor-middle term (subject of the middle premise) be the middle term of the first Syllogism, then, if, as secondly above, the figures of the Syllogisms may be, and the configurations of the Sorites and the number of each will be as follows:

MINOR;						MAJOR;					
						1	*1*			Prog.,	6
3	*3*	Reg.,	6	Prog.,	6	*1*	*2*	Reg.,	6	Prog.,	6
3	*4*			Prog.,	3	*1*	*4*	Reg.,	6		
4	*1*	Reg.,	3			*3*	*1*			Prog.,	6
4	*3*	Reg.,	6	Prog.,	4	*3*	*2*			Prog.,	6
4	*4*			Prog.,	4	*3*	*4*	Reg.,	3		
			39		33				32		40

Total minors, 72 ; Total majors, 72 ;

Grand total, 144.

But, by a careful examination of the synopsis, it will be found that fifty-six of the Sorites are both minors and majors. That number must therefore be deducted from the grand total, leaving eighty-eight different forms.

Each of the four figures occurs as the figure of the first Syllogism in both minor and major Sorites; but the second does not occur as the figure of the second Syllogism in minors, nor the third in majors. With these exceptions, all the figures occur also as figures of the second Syllogism.

The following is a synopsis of all the eighty-eight possible forms of valid simple Sorites arranged according to their configurations, regressives on the left-hand pages, and progressives on the right, and without regard to their being either minor or major, but showing in the columns on the left-hand side of each page, the moods of the Syllogisms in respect to which they are minors, and on the right, those in respect to which they are majors.

There will be found on the pages of regressives, seventeen, and on the pages of progressives, fifteen, in which the moods are only on one side, leaving twenty-seven regressives and twenty-nine progressives in which the moods are on both sides, and which together make the fifty-six alike on both sides of the preceding synopsis, as above stated. Two, namely, Nos. 25 and 38, are the exceptions hereinbefore referred to. No. 25 is a minor Sorites only, and No. 38 a major Sorites only, in both configurations.

As before, they are arranged in the order A, I, E, O of the symbols of the ultima.

Synopsis of all Possible Valid Forms

Series of Syllogisms in which the Middle Premise is Minor, and the Maximus Premise Major of the first.		Nos. of Moods.	Regressive Configuration.				Series of Syllogisms in which the Middle Premise is Major, and the Maximus Premise Minor of the first.	
			Maximus Premise.	Middle Premise.	Magnus Premise.	Ultima.		
bArbAra	*barbArA*	1	J—x	D—j	N—d	∴ N—x		
bArbAra	{ *a*, *A*, *I*. 1st fig. }	2	J—x	D—j	N—d	∴ n—x		
bArbAra	*darII*	3	J—x	D—j	n—d	∴ n—x		
bArbAra or, *A*, *A*, *i*	*darAptI* *disAmIs* }	4	J—x	D—j	D—n	∴ n—x	(2) (1) *brAmAntip*	(3) (4) *dimArIs*
bArbAra	*datIsI*	5	J—x	D—j	d—n	∴ n—x		
dArIi	*disAmIs*	6	J—x	d—j	D—n	∴ n—x	*dImAris*	*dimArIs*
dArApti	*disAmIs*	7	D—x	D—j	J—n	∴ n—x	*dArApti*	*dimArIs*
dAtIsi	*disAmIs*	8	D—x	d—j	J—n	∴ n—x	*dIsAmis*	*dimArIs*
dIsAmis	*disAmIs*	9	d—x	D—j	J—n	∴ n—x	*dAtIsi*	*dimArIs*
brAmAntip	*disAmIs*	10	X—d	D—j	J—n	∴ n—x	{ *bArbAra* or, *A*,*A*,*i*	*bramAntIp* *dimArIs*
dImAris	*disAmIs*	11	x—d	D—j	J—n	∴ n—x	*dArIi*	*dimArIs*
cElArent	*celArEnt*	12	$\cancel{J}$—X	D—j	N—d	∴ $\cancel{N}$—X	*cAmEnes*	*cesArE*
cEsAre	*celArEnt*	13	$\cancel{X}$—J	D—j	N—d	∴ $\cancel{N}$—X	*cAmEstres*	*cesArE*
		14	X—d	D—j	$\cancel{N}$—J	∴ $\cancel{N}$—X	*bArbAra*	*camEstrEs*
		15	X—d	D—j	$\cancel{J}$—N	∴ $\cancel{N}$—X	*bArbAra*	*camEnEs*
cAmEnes	*celArEnt*	16	X—d	$\cancel{D}$—J	N—j	∴ $\cancel{N}$—X	*cElArent*	*cesArE*
cAmEstres	*celArEnt*	17	X—j	$\cancel{D}$—J	N—d	∴ $\cancel{N}$—X	*cEsAre*	*cesArE*

of Simple Categorical Sorites.

Series of Syllogisms in which the Middle Premise is Minor, and the Magnus Premise Major of the first.		Nos. of Moods.	Progressive Configuration. Magnus Premise.	Middle Premise.	Maximus Premise.	Ultima.	Series of Syllogisms in which the Middle Premise is Major, and the Magnus Premise Minor of the first.	
							(2) (1)	(3) (4)
		1	N—d	D—j	J—x	∴ N—x	*bArbAra*	*bArbarA*
brAmAntip	*dAtisI*	2	N—d	D—j	J—x	∴ n—x	*bArbAra* or, *A, A, i*	*A, a. I* *dAriI*
dImAris	*dAtisI*	3	n—d	D—j	J—x	∴ n—x	*dArIi*	*dAriI*
dArApti	*dAtisI*	4	D—n	D—j	J—x	∴ n—x	*dArApti*	*dAriI*
dIsAmis	*dAtisI*	5	d—n	D—j	J—x	∴ n—x	*dAtIsi*	*dAriI*
dAtIsi	*dAtisI*	6	D—n	d—j	J—x	∴ n—x	*dIsAmis*	*dAriI*
bArbAra or, *A, A, i*	*dAraptI* *dAtisI*	7	J—n	D—j	D—x	∴ n—x	*brAmAntip*	*dAriI*
dArIi	*dAtisI*	8	J—n	d—j	D—x	∴ n—x	*dImAris*	*dAriI*
bArbAra	*dIsamIs*	9	J—n	D—j	d—x	∴ n—x		
bArbAra	*brAmantIp*	10	J—n	D—j	X—d	∴ n—x		
bArbAra	*dImarIs*	11	J—n	D—j	x—d	∴ n—x		
		12	N—d	D—j	~~J~~—X	∴ ~~N~~—X	*bArbAra*	*cElarEnt*
		13	N—d	D—j	~~X~~—J	∴ ~~N~~—X	*bArbAra*	*cEsarE*
cEsAre	*cAmenEs*	14	~~N~~—J	D—j	X—d	∴ ~~N~~—X	*cAmEstres*	*cAmestrEs*
cElArent	*cAmenEs*	15	~~J~~—N	D—j	X—d	∴ ~~N~~—X	*cAmEnes*	*cAmestrEs*
cAmEstres	*cAmenEs*	16	N—j	~~D~~—J	X—d	∴ ~~N~~—X	*cEsAre*	*cAmestrEs*
cAmEnes	*cAmenEs*	17	N—d	~~D~~—J	X—j	∴ ~~N~~—X	*cElArent*	*cAmestrEs*

Synopsis of all Possible Valid Forms

Series of Syllogisms in which the Middle Premise is Minor, and the Maximus Premise Major of the first.		Nos. of Moods.	Regressive Configuration.				Series of Syllogisms in which the Middle Premise is Major, and the Maximus Premise Minor of the first.	
			Maximus Premise.	Middle Premise.	Magnus Premise.	Ultima.		
cElArent	{ *e, A, O.* 1st fig. }	18	Ɉ—X	D—j	N—d	∴ n—‿X	(2) (1) *cAmEnes*	(3) (4) { *e, A, O.* 2d fig.
cElArent	*ferIO*	19	Ɉ—X	D—j	n—d	∴ n—‿X	*cAmEnes*	*festInO*
cElArent or, *E, A, o*	*felAptOn* *bokArdO* }	20	Ɉ—X	D—j	D—n	∴ n—‿X	*cAmEnes*	*fesApO*
cElArent	*ferIsO*	21	Ɉ—X	D—j	d—n	∴ n—‿X	*cAmEnes*	*fresIsOn*
fErIo	*bokArdO*	22	Ɉ—X	d—j	D—n	∴ n—‿X		
fElApton	*bokArdO*	23	Đ—X	D—j	J—n	∴ n—‿X		
fErIso	*bokArdO*	24	Đ—X	d—j	J—n	∴ n—‿X		
bOkArdo	*bokArdO*	25	d—‿X	D—j	J—n	∴ n—‿X		
fEsApo	*bokArdO*	26	X̶—D	D—j	J—n	∴ n—‿X		
frEsIson	*bokArdO*	27	X̶—D	d—j	J—n	∴ n—‿X		
cEsAre	{ *e, A, O.* 1st fig. }	28	X̶—J	D—j	N—d	∴ n—‿X	*cAmEstres*	{ *e, A, O.* 2d fig.
cEsAre	*ferIO*	29	X̶—J	D—j	n—d	∴ n—‿X	*cAmEstres*	*festInO*
cEsAre or, *E, A, o*	*felAptOn* *bokArdO* }	30	X̶—J	D—j	D—n	∴ n—‿X	*cAmEstres*	*fesApO*
cEsAre	*ferIsO*	31	X̶—J	D—j	d—n	∴ n—‿X	*cAmEstres*	*fresIsOn*
fEstIno	*bokArdO*	32	X̶—J	d—j	D—n	∴ n—‿X		

of Simple Categorical Sorites. (Continued.)

Series of Syllogisms in which the Middle Premise is Minor, and the Magnus Premise Major of the First.		Nos. of Moods.	Progressive Configuration.				Series of Syllogisms in which the Middle Premise is Major, and the Magnus Premise Minor of the First.	
			Magnus Premise.	Middle Premise.	Maximus Premise.	Ultima.		
							(2) (1)	(3) (4)
brAmAntip	*fErisO*	18	N—d	D—j	Ɉ—X	∴ n⌣X	{ *bArbAra* or, *A*, *A*, *i*	*E*, *a*, *O* *fEriO*
dImAris	*fErisO*	19	n—d	D—j	Ɉ—X	∴ n⌣X	*dArIi*	*fEriO*
dArApti	*fErisO*	20	D—n	D—j	Ɉ—X	∴ n⌣X	*dArApti*	*fEriO*
dIsAmis	*fErisO*	21	d—n	D—j	Ɉ—X	∴ n⌣X	*dAtIsi*	*fEriO*
dAtIsi	*fErisO*	22	D—n	d—j	Ɉ—X	∴ n⌣X	*dIsAmis*	*fEriO*
bArbAra or, *A*, *A*, *i*	*fElaptOn* *fErisO* }	23	J—n	D—j	Đ—X	∴ n⌣X	*brAmAntip*	*fEriO*
dArIi	*fErisO*	24	J—n	d—j	Đ—X	∴ n⌣X	*dImAris*	*fEriO*
bArbAra	*bOkardO*	25	J—n	D—j	d⌣X	∴ n⌣X		
bArbAra or, *A*, *A*, *i*	*fEsapO* *frEsisOn* }	26	J—n	D—j	Ӿ—D	∴ n⌣X	*brAmAntip*	*fEstinO*
dArIi	*frEsisOn*	27	J—n	d—j	Ӿ—D	∴ n⌣X	*dImAris*	*fEstinO*
brAmAntip	*frEsisOn*	28	N—d	D—j	Ӿ—J	∴ n⌣X	{ *bArbAra* or. *A*. *A*. *i*	*E*, *a*, *O* *fEstinO*
dImAris	*frEsisOn*	29	n—d	D—j	Ӿ—J	∴ n⌣X	*dArIi*	*fEstinO*
dArApti	*frEsisOn*	30	D—n	D—j	Ӿ—J	∴ n⌣X	*dArApti*	*fEstinO*
dIsAmis	*frEsisOn*	31	d—n	D—j	Ӿ—J	∴ n⌣X	*dAtIsi*	*fEstinO*
dAtIsi	*frEsisOn*	32	D—n	d—j	Ӿ—J	∴ n⌣X	*dIsAmis*	*fEstinO*

Synopsis of all Possible Valid Forms

Series of Syllogisms in which the Middle Premise is Minor, and the Maximus Premise Major of the first.		Nos. of Moods.	Regressive Configuration.				Series of Syllogisms in which the Middle Premise is Major, and the Maximus Premise Minor of the first.	
			Maximus Premise.	Middle Premise.	Magnus Premise.	Ultima.	(2)(1)	(3) (4)
cAmEnes	{ *e, A, O.* 1st fig. }	33	X—d	Đ—J	N—j	∴ n–‿X	*cElArent*	{ *e, A, O.* 2d fig
cAmEnes	*ferIO*	34	X—d	Đ—J	n—j	∴ n–‿X	*cElArent*	*festInO*
cAmEnes or, *A, E, o*	*felAptOn* *bokArdO* }	35	X—d	Đ—J	J—n	∴ n–‿X	*cElArent*	*fesApO*
cAmEnes	*ferIsO*	36	X—d	Đ—J	j—n	∴ n–‿X	*cElArent*	*fresIsOn*
		37	X—d	D—j	N̶—J	∴ n–‿X	*bArbAra*	{ *a, E, O.* 2d fig.
		38	X—d	D—j	n–‿J	∴ n–‿X	*bArbAra*	*barOkO*
		39	X—d	D—j	Ɉ—N	∴ n–‿X	*bArbAra*	{ *a, E, O.* 4th fig.
cAmEstres	{ *e, A, O.* 1st fig. }	40	X—j	Đ—J	N—d	∴ n–‿X	*cEsAre*	{ *e, A, O.* 2d fig.
cAmEstres	*ferIO*	41	X—j	Đ—J	n—d	∴ n–‿X	*cEsAre*	*festInO*
cAmEstres or, *A, E, o*	*felAptOn* *bokArdO* }	42	X—j	Đ—J	D—n	∴ n–‿X	*cEsAre*	*fesApO*
cAmEstres	*ferIsO*	43	X—j	Đ—J	d—n	∴ n–‿X	*cEsAre*	*fresIsOn*
bArOko	*bokArdO*	44	X—j	d–‿J	D—n	∴ n–‿X		

of Simple Categorical Sorites. (Concluded.)

Series of Syllogisms in which the Middle Premise is Minor, and the Magnus Premise Major of the first.		Nos. of Moods.	Progressive Configuration.				Series of Syllogisms in which the Middle Premise is Major, and the Magnus Premise Minor of the first.	
			Magnus Premise.	Middle Premise.	Maximus Premise.	Ultima.		
cAmEstres	{ *A*, *e*, *O*. 4th fig. }	33	N—j	Đ—J	X—d	∴ n⌣X	(2)(1) { *cEsAre* or, *E*, *A*.*o*	(3) (4) *A*, *e*, *O* *bArokO*
		34	n—j	Đ—J	X—d	∴ n⌣X	*fEstIno*	*bArokO*
		35	J—n	Đ—J	X—d	∴ n⌣X	*fEsApo*	*bArokO*
		36	j—n	Đ—J	X—d	∴ n⌣X	*frEsIson*	*bArokO*
cEsAre	{ *A*, *e*, *O*. 4th fig.	37	N̶—J	D—j	X—d	∴ n⌣X	{ *cAmEstres* or, *A*.*E*,*o*	*A*. *e*. *O* *bArokO*
		38	n⌣J	D—j	X—d	∴ n⌣X	*bArOko*	*bArokO*
cElArent	{ *A*, *e*, *O*. 4th fig.	39	J̶—N	D—j	X—d	∴ n⌣X	{ *cAmEnes* or, *A*.*E*,*o*	*A*, *e*, *O* *bArokO*
cAmEnes	{ *A*, *e*, *O*. 4th fig.	40	N—d	Đ—J	X—j	∴ n⌣X	{ *cElArent* or, *E*,*A*.*o*	*A*, *e*. *O* *bArokO*
		41	n—d	Đ—J	X—j	∴ n⌣X	*fErIo*	*bArokO*
		42	D—n	Đ—J	X—j	∴ n⌣X	*fElApton*	*bArokO*
		43	d—n	Đ—J	X—j	∴ n⌣X	*fErIso*	*bArokO*
		44	D—n	d⌣J	X—j	∴ n⌣X	*bOkArdo*	*bArokO*

§ 11. The number of forms of valid Sorites, shown in the foregoing synopsis, is eighty-eight, forty-four on each side; but a comparison of them, line by line, read across both pages of the synopsis, will show that, considered with respect to the propositions of which they are composed, without regard to the order of their statement, there are but forty-four; the first and third propositions in the regressive configuration changing places in each case, and becoming respectively third and first in the progressive throughout the whole series, the middle premise and ultima being the same in each case on both sides throughout. They are numbered from one to forty-four, on each side, to correspond.

To one or another of these forms, EVERY valid argument (expressed categorically) involving four terms, or, as will be hereinafter shown, involving any greater number of terms, MUST BE conformed.

§ 12. The moods, as determined by the quantity and quality of the propositions (indicated by their symbols), are twenty in number, of which fourteen occur in both configurations, three in the regressive only, and three in the progressive only.

The following table shows them, arranged in the order A, I, E, O of the symbols of the ultima, with their numbers in each configuration, as in the synopsis, repeated where they are both minor and major. The symbols are in capitals in the synopsis, the first two in the columns of Syllogisms, on the right-hand side of each page (*majors*) being transposed, as previously stated, and as shown by the figures over those columns.

Moods of Sorites.

Symbols.	Nos. in Regressive Configuration.		Nos. in Progressive Configuration.	
	Minors.	Majors.	Minors.	Majors.
A, A, A, A.	1.			1.
A, A, A, I.	2, 4, 7, 10.	4, 7, 10.	2, 4, 7, 10.	2, 4, 7.
A, A, I, I.	3, 5.		9, 11.	
I, A, A, I.	9, 11.	9, 11.	3, 5.	3, 5.
A, I, A, I.	6, 8.	6, 8.	6, 8.	6, 8.
E, A, A, E.	12, 13.	12, 13.	14, 15.	14, 15.
A, A, E, E.		14, 15.		12, 13.
A, E, A, E.	16, 17.	16, 17.	16, 17.	16, 17.
E, A, A, O.	18, 20, 23, 26, 28, 30.	18, 20, 28, 30.	37, 39.	37, 39.
A, A, E, O.		37, 39.	18, 20, 23, 26, 28, 30.	18, 20, 23, 26, 28, 30.
E, A, I, O.	19, 21, 29, 31.	19, 21, 29, 31.		
I, A, E, O.			19, 21, 29, 31.	19, 21, 29, 31.
E, I, A, O.	22, 24, 27, 32.			
A, I, E, O.			22, 24, 27, 32.	22, 24, 27, 32.
O, A, A, O.	25.			38.
A, A, O, O.		38.	25.	
A, E, A, O.	33, 35, 40, 42.	33, 35, 40, 42.	33, 40.	33, 35, 40, 42.
A, E, I, O.	34, 36, 41, 43.	34, 36, 41, 43.		
I, E, A, O.				34, 36, 41, 43.
A, O, A, O.	44.			44.

It is manifest that it would be a very difficult thing to classify Sorites in figures, according to the positions of the terms, and to devise names for the moods, analogous to those of simple Syllogisms; and, if it should be accomplished, the figures and names of the moods would be extremely burdensome to the memory. The different forms can be much more readily referred to by their numbers and the names of the configurations, as adopted, than by their symbols, or any names that could be devised for them. They will be hereinafter so referred to.

By counting the series of Syllogisms on the left (*minors*) and right (*majors*) of the synopsis in each configuration, there will be found to be :

Regressives.		*Progressives.*	
Minors.	*Majors.*	*Minors.*	*Majors.*
39.	32.	33.	40.

corresponding to the numbers shown in the table on page 46.

§ 13. Sorites, in the regressive configuration, may be expanded into series of Syllogisms in all combinations of figures, except those of the third and first, and third and second; and those in the progressive configuration, in all combinations, except those of the second and first, and second and third.

Such of them as can be expanded wholly in the first figure, are the only perfect forms. The series of Syllogisms, in which they can be so expanded, occur in the synopsis only on the left side of the regressives (*minors*), and on the right side of the progressives (*majors*); and the first figure occurs as the figure of the second Syllogism only on the same sides. Moods Nos. 10, 11, 15, 26, 27, 35, 36, and 39 cannot be expanded directly (that is, without conversion) except by the aid of the fourth figure; a fact which may tend in some measure to relieve that figure from the odium which has been cast upon it.

§ 14. There is a very remarkable and wonderful analogy between the forms of reasoning and the two simplest forms of geometrical figures, plane and solid (with plane surfaces); an analogy which is evidently something more than merely fanciful.

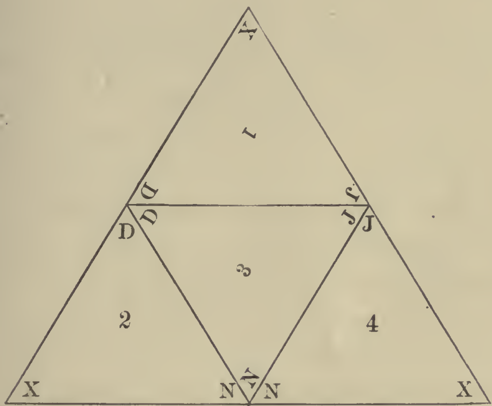
The Syllogism of logic and the triangle of geometry, and the Sorites and tetrahedron are, respectively, similar.

The triangle consists of three points, equivalent to the three terms of the Syllogism, connected by three lines, which answer to the copulas of the propositions. No plane surface can be represented by less points and lines, no argument by less terms and propositions. By means of the former, with the aid of the latter, all physical relations *in space* are determined, not only on the surface of the earth and within it, from those of the smallest subdivision to those of continents and oceans, but also in the heavens to the remotest star-depths, so far as the stars can be brought under observation; by the latter *all* relations are determined, not only of physical things, but also of the metaphysical and immaterial. But the analogy does not end here. In its very practical construction the triangle produces the equivalent of a perfect Syllogism in *Barbara*. If we are at any point, N, on the surface of the earth, from which we can see another point, J (either on the earth or in the heavens), which is inaccessible, and the distance to which we cannot therefore directly measure, we may select another point, D (either on the earth or its orbit), which is accessible, and from which the point, J, may also be seen; and first, carefully observing the directions from N to J, and from N to D, and thus determining the angle, we may then proceed to measure the distance between N and D in a straight line. The line thus laid down is equivalent to the first proposition, N — d, with which we set out in § 2 of this chapter. Arrived at D, we may then observe the direction from D to J, and determine the angle, and then, by means of the elements thus obtained, we may determine the distance in a straight line from D to J, and from N to J. The lines thus drawn, or supposed to be drawn, are the equivalents of the second proposition, D — j, with which we set out, and of the conclusion to be deduced from it and the first proposition, N — d, when put forth as premises of a Syllogism, namely, N — j.

The tetrahedron is the simplest form in which any solid with plane surfaces can be included, and is the analogue of the Sorites. Its four points answer to the four terms, its four planes (each in the form of a triangle) bounded by six lines (each being a boundary of two planes) to the four Syllogisms of the two principal series; each series with its six propositions. Each plane connects three points, each Syllogism three terms. Each of the four points is excluded from one of the planes, each of the four terms from one of the Syllogisms.

To illustrate by means of geometrical figures:

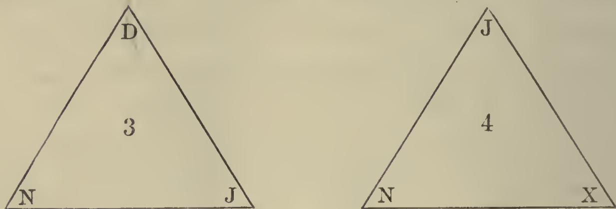
If we take a piece of card-board and, having cut it in the form of an equilateral triangle, inscribe therein another equilateral triangle, the lines of which terminate in the middle of the lines of the exterior one, and mark all the angles with letters, as follows:



we may then fold the card-board backward on the lines of the inscribed triangle so as to bring together the three points, X, X, X, and then fastening together the edges of the card-board so brought together, we shall have a regular tetrahedron, the very embodiment of a simple Sorites. Looked at from our present stand-point, we shall see only the inscribed triangle No. 3, and having its angles marked with the letters N, D, and J. The other triangles and the point X will not be seen. Turning the figure about, so as to bring its planes before us in the order in which they are numbered, and considering them in two series of two each, we shall find them as follows, beginning at the right hand with the first series, and reading backward, but from left to right, in the second.

First series.

Second series.



Observing that the letters at the apices of the triangles are the middle terms of the Syllogisms of the two principal series hereinbefore shown, and considering the lines of the triangles as copulas connecting the terms of propositions, and the lines at the bases as indicating conclusions, and beginning with the first series of triangles at the right hand and *regressing*, we can read as follows:

Because D is J and J is X, therefore D is X; and because N is D and D is X, therefore N is X,

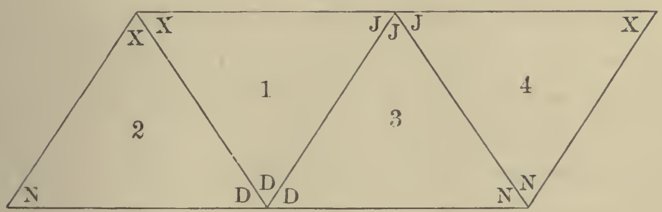
and then going to the second series, and beginning at the left hand and *progressing*, we can further read:

Because N is D and D is J, therefore N is J; and because N is J and J is X, therefore N is X.

The correspondence between the triangles and the Syllogisms is exact throughout, except that the premises in the latter are transposed, but the order of statement of the premises is a matter of no consequence, the terms determining their character.

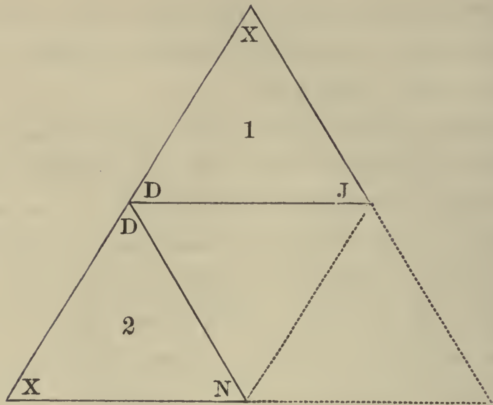
The middle terms D and J may, of course, be transposed in our original illustration, and in such case the numbers 2 and 4 would also have to be transposed, and the positions of all the letters and the numbers in triangles 1 and 3, relatively to the whole figure, would also require to be changed. The first series of triangles would then read forward and the second backward, but the series of Syllogisms would remain the same, the first regressive, and the second progressive.

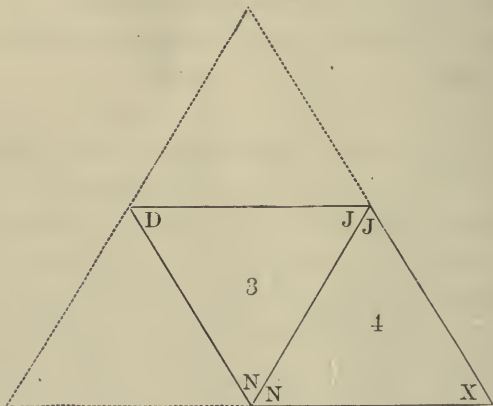
The four triangles may also be exhibited in the following form:



and may be folded on the interior lines with like result as before.

But the Sorites is superior to its analogue, the tetrahedron, in this, that its ultimate conclusion is reached by either process, regressive or progressive, but both are required to complete the tetrahedron. This will be apparent by the consideration of the two following forms.





If, in the first, beginning with N, we successively reach *by investigation* the points D, J, and X, and then commence to reason with the propositions which we enounce as the results of our investigation, we may by two Syllogisms, of which the two completed triangles 1 and 2 are analogues, arrive at the ultimate conclusion. But if, in the second, by the same process of investigation we reach only to the point J, and then commence to reason, we frame our first Syllogism, of which the triangle 3 is the analogue, resulting in the conclusion that N — J. We are thereupon, if we would advance further, obliged to resume investigation, and through it reach out to X, and are thence enabled to frame the second Syllogism, of which the triangle 4 is the analogue, arriving at the same ultimate conclusion. But in either case the tetrahedron is incomplete, and can only be completed by the union of the two. Each figure is the complement of the other, required to make the perfect figure, shown in our first illustration.

But again, the two different processes, regressive and progressive, in respect to argumentation by Syllogisms, are analogous to the two possible combinations of the two processes by which we may determine the length of the concluding line with which we enclose a triangle.

Leaving N, and going to D, we observe the direction in which we are traveling, and measure the distance traveled. Then observing the direction from D to J, and thus determining the angle, we go on from D to J, measuring the distance. If we then stop, we may, by the three elements thus obtained, viz., the two lines and the included angle, determine the distance and direction from N to J; then, having this distance and direction, and observing the direction of X from J, we go back to N, and observe its direction from X, and determine the angles, and then with the three elements thus secondly obtained, viz., the two angles and the included line from N to J, we may determine the distance from N to X. This is analogous to the progressive process.

But if, after reaching J, without stopping to determine its distance from N, we observe the direction therefrom to X, as in triangle 1, and going back to D, observe also its direction from X, and determine both angles, then with the three elements thus obtained (being like to those of the *second* three in the preceding process), we may determine the distance from D to X, and then, having the distances and directions from D to N, and from D to X (the figure being now considered as folded), and determining the included angle, we may by such elements (being like to those of the *first* three in the preceding process) determine the distance from N to X. This is analogous to the regressive process.

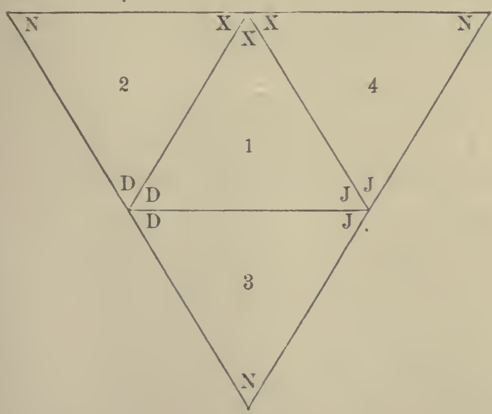
Surely, in all this wonderful accord there must be something more than mere coincidence. "The invisible things of God are clearly seen, being perceived through the things that are made."

§ 15. But the subject concerning which we set out to make investigation may be the summum genus instead of the infima species or individual, as hitherto, and in such case we shall find that the processes of both investigation and reasoning will be in the exactly opposite direction, and that the maximus term, instead of the magnus, as hitherto, will be the subject of the ultima, and the magnus term instead of the maximus will become the predicate.

Strictly speaking, the word "predicate" is not properly applicable to the last, but rather to the first term of propositions as they will be exhibited in this section, inasmuch as the species cannot be predicated of the genus, but the genus of the species. To change the names of the terms as they stand related to the propositions would, however, be confusing, and they will, therefore, be retained in their grammatical rather than in their strict, logical signification.

But we shall find it necessary to change the signification of the copula. As hitherto employed, such signification has been "is" or "is not" in the sense of "*is* (or *is not*) *comprehended in*," but as employed *in this section only*, the copula must be understood to signify "*comprehends*" or "*does not comprehend.*" The reason for this change, if not immediately obvious, will become clear as we progress. It will, however, be hereafter seen that in some cases the two significations are interchangeable, and either may be understood.

I shall have immediate recourse to illustration by means of geometrical figures, as thereby such illustration can be made much clearer, being exhibited to the eye as well as to the understanding; and I now give the following figure,



which you will observe is like to our original card-board figure on page 59, with triangles 1 and 3 remaining in the same position as therein, but with triangles 2 and 4 turned upward, each in a semicircle, on the points D and J as centres respectively.

The points X X X are now brought together in the figure, and N N N separated and become exterior. The points D D D and J J J retain their intermediate positions.

If now we begin to make investigation concerning X as the subject, we shall find ourselves proceeding in a descending instead of ascending direction, as before; and we shall also find that the notions which we discover as predicable (in the sense of the copula, as above changed), of our successive subjects, instead of being higher genera and comprehending the subjects, are lower species, and are wholly comprehended in the subjects respectively. The propositions in which we lay down our judgments will therefore necessarily be required to signify this difference, which may be done by putting the predicates in capitals instead of small letters, as before, and will be as follows:

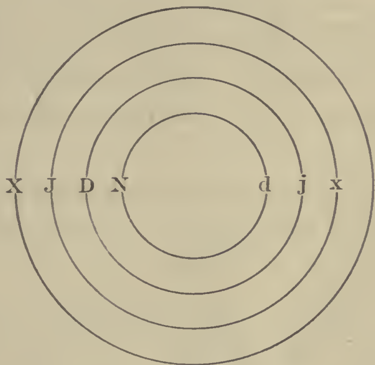
X — J (*meaning All* X *comprehends all* J);

J — D (*meaning All* J *comprehends all* D);

D — N (*meaning All* D *comprehends all* N).

The subject of each of the foregoing propositions is distributed. But it might have been undistributed in so far as relates to the manner of its representation, and the proposition still retain its character as universal.

To illustrate, I now reproduce the first combination of circles shown in the former part of this treatise when treating of simple Syllogisms, adding another circle to make it applicable to a Sorites, the letters being put on the lines of the circles, and to be considered as indicating the whole areas included in the circles respectively.



It will now be manifest, from mere inspection of the figure, that what we have predicated of X (viz., J) might also have been predicated of x. and in fact with more correctness, for J is comprehended *wholly* and *only* in *that part* of X which lies within the circle marked on one side J and on the other x. In like manner, what we have predicated of J (viz., D) might have been predicated of j, and what we have predicated of D (viz., N) might have been predicated of d.

The propositions may therefore be stated as follows:

X or x — J;
J or j — D;
D or d — N.

In either alternative the propositions must be regarded as universal. I shall hereafter make use only of that in which the subjects are represented by small letters, as apparently, but not in fact, undistributed. In reading the propositions, the words "All" and "Some" must be expressed, and it must be borne in mind that the word "Some" applies to a definite part of the term, and when in the process of the reasoning a term with that word prefixed shall be repeated, it must be read or understood as "The same some," or "The same definite part of."

The dictum of Aristotle, as applicable to the above propositions, will now have to be changed so as to read as follows.

Whatever definite term is affirmed or denied as comprehending any other definite term, may be affirmed or denied as comprehending any definite term comprehended in the definite term so comprehended, and in like manner of any definite term comprehended in the definite term so secondly comprehended, and so on *ad infinitum.*

Applying the dictum as thus changed to the above propositions, the two forms of the full Sorites warranted thereby will be as follows:

In the regressive configuration.	*In the progressive configuration.*
d — N,	x — J,
j — D,	j — D,
x — J;	d — N;
∴ x — N.	∴ x — N,

and the abridged form will be

∵ j — D; ∴ x — N.

All propositions put forth in the above form in the descending processes of investigation and reasoning, may be converted simply, provided the original signification of the copula be at the same time reinstated, and by simple conversion of the above, we shall have the two forms of Sorites as we have hereinbefore seen them.

But not only have the terms of all the propositions in the two forms changed places, but also the forms themselves, in respect to the configurations, the converse of that which before was regressive having become progressive and of that which was progressive, regressive. By examining our original card-board figure in connection with the figures on page 62, and the remarks on the latter, and comparing them with the first figure in this section, and applying such remarks to the configurations as herein given, it will be seen that such change is proper, triangles 3 and 4 in the latter figure being the analogue of the Sorites in the regressive configuration, and 1 and 2 of that in the progressive.

In like manner, it will be found that in all matters of form there will be continued inversions.

The Sorites herein given may be expanded into series of Syllogisms as follows :

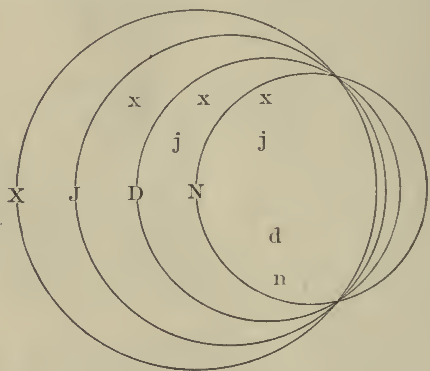
In the regressive process.

j — D,	x — J,
d — N ;	j — N ;
∴ j — N.	∴ x — N.

In the progressive process.

x — J,	x — D,
j — D ;	d — N ;
∴ x — D	∴ x — N.

All the propositions in the foregoing forms are universal, but they may all be particular in the manner of their representation (indicated by the apparent non-distribution of the predicate), provided the definiteness of the terms represented be kept in view. Thus, in the following figure, let the letters on the lines of the circles refer to the whole areas of the circles respectively as before, and those in areas only to the areas as bounded by lines respectively, but considering them where occurring more than once as to be taken together:



The Sorites exemplified will be as follows:

In the regressive configuration.	*In the progressive configuration.*
d — n,	x — j,
j — d,	j — d,
x — j;	d — n;
∴ x — n.	∴ x — n;

and may be expanded into series of Syllogisms, as follows:

In the regressive process.

j — d,	x — j,
d — n;	j — n;
∴ j — n.	∴ x — n.

In the progressive process.

x — j,	x — d,
j — d;	d — n;
∴ x — d.	∴ x — n.

Here apparently we have two anomalies—viz., Syllogisms having the middle terms undistributed in both premises, and Syllogisms in which conclusions are deduced from particular premises. But they are such only in appearance; all the propositions (the definiteness of the terms being kept in mind) being in fact universal, and the middle term distributed in each case in the major premise.

The terms of all the foregoing propositions may each be considered as comprising all the areas marked in the figure with the small letters representing them respectively, taken together respectively, or only those areas respectively, in which the letters representing both the subject and predicate appear, taken together. In the former case, the subjects will each be greater than their predicates respectively, and the copula must signify "*comprehends*," in the latter, the terms *of each proposition* will be co-extensive, and the copula may have either signification. But in the latter case, the major-middle term in the middle premise will narrow in signification to that of the minor-middle term, and the maximus term in the ultima will have a narrower signification than as employed in the maximus premise.

The middle premise, it will be seen, has become the major premise, and the magnus premise the minor of the first Syllogism of the series in the regressive process, and the middle premise has become the minor, and the maximus the major of the first Syllogism of the series in the progressive. The regressive Sorites in the descending process is therefore, in the forms above given, which you will find are the perfect forms, a major Sorites, instead of a minor, and the progressive a minor Sorites instead of a major as before. It will be also seen that the Enthymeme taken from the second Syllogism in the regressive series is of the second instead of the first order as before, and *vice versa* in the progressive. If a synopsis should be made, this would necessitate (to make it conform to the former) the transfer of the headings of the columns on each side of each page of the former from each page to the other, and their transposition after being so transferred.

All the Syllogisms are in the fourth figure, which in this process becomes the perfect figure, the first becoming imperfect. The second and third figures will also be found to have changed places, if indeed they and the first can have any place at all, in the new sense of the copula. One of the premises in each case in the second and third figures, and both in the first, would necessarily be in the inverse order, affirming or denying of the species that it comprehends or does not comprehend the genus, or else the original signification of the copula would have to be considered as reinstated in such premises, and the process would thereby lose its distinctive character as a process wholly in the descending direction, which only we are now considering. By examining the synopsis, it will be found that in all cases in which either of the involved Syllogisms in the columns on the left side of the regressives or right side of the progressives is in one of the imperfect figures, and in all cases of combinations of Syllogisms shown on the other side of each page respectively, the process of the reasoning partakes of both characters, being partly in the ascending and partly in the descending direction.

I shall not proceed further with the consideration of this subject, for the reason that propositions in the descending process are seldom, if ever, put forth in form as herein given, but in the converse. When you come to the study of Logic as illustrated by concrete examples (in which aspect it is, in respect to each such illustration, an applied science), you will find a distinction made in respect to the quantity of concepts (terms) as being either in *extension* or *intension*, the latter being called also *comprehension.* This distinction runs also into the propositions and syllogisms as treated of, according as the terms are considered as in one or the other quantity. You will find it, however, to be of no practical importance *in so far as the process of reasoning is concerned*, all reasoning being conducted on the lines of the process, as we have previously considered it, and being called reasoning in *extension*, in contradistinction to the process as shown in this section, which is called reasoning in *intension* or *comprehension.* The distinction, in so far as it relates to the terms (concepts), does not lie within the province of Logic as a Pure Science, and cannot be illustrated by means of symbols indefinite in material signification, but the illustration of the processes of investigation and reasoning wholly in the descending direction, given in this section, will serve to make it, as continued into the reasoning process, clearer and more easily understood.

The consideration of the subject matter of this section would perhaps have been more appropriately introduced when treating of simple Syllogisms, but it could not have been made as intelligible without as with geometrical illustration by combinations of triangles, and the latter has been more appropriately, and at the same time more effectively, introduced in this chapter, where it has been exhibited in one view, and to its full extent.

The copula must now be considered as returned to its original signification, and where the word "descending" shall be hereinafter used, it must be considered as applicable to the direction of the process of investigation, but not to the form of the propositions, which, in the perfect moods of the Sorites, will always be found in the converse of those herein given.

§ 16. Thus far the premises of the Sorites exhibited have consisted of propositions put forth independently as the results of investigation. They may, however, be the results of prior processes of reasoning, the premises of which may be required to be exhibited in connection with them, in order to a clear understanding of the principal argument. The full expression in such case will become complex, and may be in two forms, of which I first exhibit the following:

J — x, ∵ Z — x and Y — z and J — y.
D — j, ∵ B — j and D — b.
N — d. ∵ K — d and N — k;
∴ N — x.

Here each premise is the ultima or conclusion of a prior process of reasoning, the premises of which are *affixed*, with the word "because" preceding.

In the example, all the premises have supporting premises affixed. But any one, or two, only, may have such premises affixed, the other two, or one, as the case may be, being propositions put forth independently.

The whole expression, in either case, is called an Epicheirema, or Reason-rendering Syllogism (of either three or four terms). The principal argument, with reference to the supporting premises, is called an Episyllogism; and the supporting premises in each case, with reference to the premise proved, is called a Prosyllogism.

The second form is that in which the premises of the Prosyllogism are *prefixed*, those in relation to the first premise being stated antecedently to the whole principal expression: those in relation to the second or middle premise, interpolated between the first and middle, and those in relation to the last, interpolated between the middle and last.

If either of the first two be in such form, it will be found upon trial, that the principal expression has lost in forcibleness of statement or in perspicuity, and they may, therefore, be disregarded, but the third will be found to lead to greater perspicuity, and especially so if more than two new middle terms are called into requisition for the purpose of elucidation.

The first form (Epicheirema) is better adapted to the statement of arguments in which the premises are explained, the second to those in which either the first or last premise is disputed. It is seldom the case in any disputation, that more than one of the premises of the principal argument is called in question, and that one is generally the first or last, the middle premise being usually a general rule acquiesced in upon being stated; and if the disputed premise be the first, the principal argument, by changing the configuration, may be thrown into such form that it shall become the last.

I now proceed to consider Sorites as complex expressions, in the second form, but *only* as limited to those in which the *last premise* is disputed, and to distinguish them as such, shall call them *Compound Sorites*.

§ 17. A *Compound* Sorites, once compounded, *when fully expressed*, consists of a simple Sorites (herein called the *principal* Sorites) with two, or three, propositions interpolated between its *middle* and *last* premises; such propositions (if there be two) constituting the premises of a simple Syllogism of which the conclusion, or (if there be three) of a simple Sorites, of which the ultima is the *last premise* of the *principal* Sorites. The interpolated propositions will be herein called the *included* Enthymeme, if there be two, or Sorites, if there be three, giving the full name in the latter case, in default of one analogous to Enthymeme in the former. An included Sorites may in like manner have an Enthymeme or second Sorites included within it, and the second included Sorites may in like manner have an Enthymeme or third Sorites included within it, and so on *ad infinitum.* There can be but one included Enthymeme, and it will always be the last included expression. The reasoning in all such cases, while it will have the appearance of being very much involved, will in reality be very much clearer.

§ 18. But compound Sorites are seldom, if ever, *fully expressed* in *formal, prepared argumentation*, the *last premise* of the *principal* Sorites being *suppressed*, but, as will be hereinafter shown, in all cases *implied.* In this aspect a compound Sorites may be better defined as an argument consisting of more than four expressed propositions composed of as many terms as there are expressed propositions, including the ultima. Both definitions will be better understood by illustration.

Let us suppose the case of two disputants of whom one, the proponent, advances these propositions:

$$D — j;$$
$$\therefore N — x,$$

to which the other, the opponent, answers: I admit that D — j, but I deny that it follows that N — x.

The propositions, as you will observe, constitute the abridged form of the first mood.

The proponent replies, asserting, as the reason, the two propositions necessary to make up the expanded form, viz.:

$$\because J — x$$
$$\text{and } N — d,$$

and to this the opponent makes rejoinder: Admitting that J — x, I deny that N — d.

The issue is now clearly defined, and the whole case may be stated as follows:

J — x admitted,
D — j admitted,
N — d alleged but denied,
∴ N — x claimed but denied.

The proponent, to maintain the issue on his part, must establish that N — d, or must fail.

To do it, as the proposition to be established is A, he must find a middle term, with which both the terms N and D may be compared, so as to form, with the conclusion, a perfect Syllogism in *Barbara* (symbols AAA), or two middle terms, with one of which N may be compared and D with the other, and one of which may be predicated of the other, all in such manner as to constitute, with the ultima, a valid Sorites in the first mood (symbols AAAA).

Let the middle term, in the first case, be Y, and the two middle terms, in the second case, be Y and Z.

The Syllogism in the first case will be:

Y — d,
N — y;
∴ N — d.

But in the second case the two new terms are required to be compared, and either may be the subject of the proposition in which they are compared, viz., Y — z or Z — y. The abridged Sorites may therefore be either:

Y — z; ∴ N — d;
or, Z — y; ∴ N — d.

Let us take the first, and in order to expand it into a full Sorites, let us write down the first mood in the regressive configuration, as in the synopsis, and write under its second and fourth propositions the abridged Sorites thus taken, as follows:

J — x; D — j; N — d; ∴ N — x.
Y — z; ∴ N — d.

Then, by expressing the first and third implied propositions of the abridged Sorites (making them to correspond in respect to the terms employed), we shall have the expanded Sorites as follows:

Z — d; Y — z; N — y; ∴ N — d.

By taking from the Syllogism in the first case its two premises (constituting an Enthymeme of the third order), and from the Sorites in the second case, its three premises, and interpolating them (respectively) between the middle and last premises of the principal Sorites, we shall have, in each case, a compound Sorites fully expressed, as follows:

In the first case.	*In the second case.*
J — x,	J — x,
D — j,	D — j,
– – – –	– – – –
Y — d,	Z — d,
N — y;	Y — z,
∴ – – – –	N — y;
N — d,	∴ – – – –
and ∴ N — x.	N — d,
	and ∴ N — x.

The conclusion of the first Enthymeme of the principal Sorites, viz., D — x, is held in the mind ready to unite with the last premise, N — d (after the latter shall have been proved), in establishing the ultima, N — x.

§ 19. But there is a shorter and simpler process, and the one which is usually employed in *formal, prepared argumentation.* Instead of holding in the mind the conclusion of the first Enthymeme to unite with the last premise of the principal Sorites when proved, as above stated, we may at once employ it (mentally) as a premise in connection with the first of the new expressed propositions, and in like manner the unexpressed conclusion resulting from them as a premise in connection with the second new expressed proposition (and in the second case as above, the unexpressed conclusion thus resulting in connection with the third), and shall find that the *last premise* of the *principal* Sorites *will not appear.* Thus, in the two cases, the unexpressed conclusions being given in *italics:*

In the first case.	*In the second case.*
J — x,	J — x,
D — j, (∴ *D — x*).	D — j, (∴ *D — x*).
Y — d, (∴ *Y — x*).	Z — d, (∴ *Z — x*).
N — y;	Y — z, (∴ *Y — x*).
∴ N — x.	N — y;
	∴ N — x.

But the last premise of the principal Sorites will have been *implied*, as will be manifest from a comparison of the two forms in the second case put side by side, as follows:

First form in second case.

J — x,
D — j; (∴ *D—x, held in the mind*),
– – – –
Z — d,
Y — z,
N — y;
∴ – – – –
N — d;
and ∴ N — x. (∵ *D—x*).

Second form in second case.

J — x,
D — j,
Z — d,
Y — z,
N — y;
∴ N — x.

The second form is the simpler, but the first is the clearer, exhibiting the *entire* process of the reasoning.

The included Enthymeme in the first case, or Sorites in the second, serves only to *prove* the *last premise* of the principal Sorites, and forms *no part* of the *argument*, which is *wholly contained* in the *principal* Sorites.

§ 20. Both the principal and included Sorites in the examples are in the regressive configuration, but they may be in different configurations. If in the foregoing disputation the opponent in his rejoinder had admitted the magnus premise, N — d, but denied the maximus, J — x, the principal Sorites of the proponent would have been in the progressive configuration, and the included one could have still been in the regressive, viz.:

N — d,
D — j;
– – – –
Z — x,
Y — z,
J — y;
∴ – – – –
J — x,
and ∴ N — x.

The two configurations cannot be directly linked together in this example, as before shown in the second form, there being a break in the chain between the second and third propositions. But by considering the configuration of the included Sorites to be changed (as it may be by transposing the first and third premises thereof), the whole expression can be put in the second form as before, and the last premise of the principal Sorites, J — x, will not appear. It does not, however, follow that the two configurations cannot in any case be directly linked together. That they may be in some cases will be hereinafter seen.

§ 21. All the Syllogisms involved in all the foregoing examples are in *Barbara*, and the dictum of Aristotle, as, hereinbefore extended, may be directly applied to those in the second form, by extending it still further in like manner. But to those in the first form it would have to be twice applied, first to the included Sorites and secondly to the principal, and in that case would not require to be further extended, both the Sorites being simple.

§ 22. But if any of the involved Syllogisms are in any other figure, or combination of figures, they would have to be converted into Syllogisms in the first figure, before the dictum could be directly applied.

The following are examples of compound Sorites, the involved Syllogisms of which are in combinations of figures, as shown by the names of the moods given in connection with them. The conclusions proved, but not expressed, are also given in *italics* in connection with the names of the moods, except the ultima of the included Sorites (in the first forms), which is expressed as a premise below the second dotted line. The principal Sorites and the number of its mood and the configuration are given in advance of each example:—

6th Regressive Mood.

J — x, d — j, D — n; ∴ n — x.

First form.

J — x,
d — j; (∴ *d* — *x*, ***Darii***),
- - - -
D — z,
Z — y,
Y — n;
∴ - - - -
D — n;
and ∴ n — x. (∵ *d* — *x*, ***Disamis***).

Second form.

J — x,
d — j, (∴ *d* — *x*, ***Darii***),
D — z, (∴ *z* — *x*, ***Disamis***),
Z — y, (∴ *y* — *x*, ***Disamis***),
Y — n;
∴ n — x. (***Disamis***).

15th Progressive Mood.

$\not{J}$ — N, D — j, X — d; ∴ $\not{N}$ — X.

First form.

$\not{J}$ — N,
D — j; (∴ $\not{N}$ — *D*, ***Camenes***),
- - - -
Y — d,
Z — y,
X — z;
∴ - - - -
X — d;
and ∴ $\not{N}$ — X. (∵ $\not{N}$ — *D*, ***Camestres***).

Second form.

$\not{J}$ — N,
D — j, (∴ $\not{N}$ — *D*, ***Camenes***),
Y — d, (∴ $\not{N}$ — *Y*, ***Camestres***),
Z — y, (∴ $\not{N}$ — *Z*, ***Camestres***).
X — z;
∴ $\not{N}$ — X. (***Camestres***).

25th Regressive Mood.

d —⌣ X, D — j, J — n; ∴ n —⌣ X.

First form.

d —⌣ X,
D — j; (∴ *j* ⌣ *X*, *Bokardo*),
\- - - -
J — y,
Y — z,
Z — n;
∴ - - - -
J — n;
and ∴ n —⌣ X. (∵ *j* ⌣ *X*, *Bokardo*).

Second form.

d —⌣ X,
D — j, (∴ *j* ⌣ *X*, *Bokardo*),
J — y, (∴ *y* ⌣ *X*, *Bokardo*),
Y — z, (∴ *z* ⌣ *X*, *Bokardo*),
Z — n;
∴ n —⌣ X. (*Bokardo*).

§ 23. The included Sorites may have an Enthymeme or a second Sorites included within it, and the second included Sorites may have an Enthymeme or third Sorites included within it, and so on *ad infinitum*. Thus:

First form.

J — x,
D — j; (∴ *D — x*, *held in the mind*),
\- - - -
First included Sorites:
Y — d,
Z — y; (∴ *Z — d*, *held in the mind*),
\- - - -
Second included Sorites:
z — k,
K — q,
Q — n;
∴ - - - -
z — n;
∴ - - - - (∵ *Z — d*),
d — n;
and ∴ n — x. (∵ *D — x*).

Second form.

J — x,
D — j, (∴ *D — x*),
Y — d, (∴ *Y — x*),
Z — y, (∴ *Z — x*),
z — k, (∴ *k — x*),
K — q, (∴ *q — x*),
Q — n;
∴ n — x.

If the first included Sorites in the last example be put in the regressive configuration, its last premise will be Y — d instead of z — n, and the second included Sorites will be employed to establish the former instead of the latter, but of course by different premises. In such case we shall find that when we attempt to put the whole expression in the second form, the premises of the second included Sorites will take precedence of those in the first, and the latter will be transposed. Thus:

First form.

J — x,
D — j ; (∴ *D — x, held in the mind*),
- - - -
z — n,
Z — y ; (∴ *y — n, held in the mind*),
- - - -
Q — d,
K — q,
Y — k;
∴ - - - -
Y — d;
∴ - - - - (∵ *y — n*),
d — n ;
and ∴ n — x. (∵ *D — x*).

Second form.

J — x,
D — j, (∴ *D — x*),
Q — d, (∴ *Q — x*),
K — q, (∴ *K — x*),
Y — k, (∴ *Y — x*),
Z — y, (∴ *Z — x*),
z — n ;
∴ n — x.

The argumentation is supposed, of course, to have taken place on the lines of the process in the first form, and the second included Sorites did not therefore come into the process until the proposition, Y — d, was disputed. The illustration thus shows the superiority of the first over the second form, as exhibiting the *whole* process of the reasoning. The second could not have been framed until the first had been gone through with.

§ 24. Compound Sorites may, however, be exhibited in forms which at first sight may seem to be in contravention of what has been before laid down, but upon examination it will be found that such is not the case.

Thus, in the two following cases:*

(1.)	(2.)
$N - d,$	$N - d,$
$D - j,$	$D - j,$
$\cancel{J} - X,$	$J - x,$
$Y - x,$	$\cancel{Y} - X,$
$Z - y,$	$Z - y,$
$Q - z;$	$Q - z;$
$\therefore \cancel{N} - Q.$	$\therefore \cancel{N} - Q.$

Let us take the second and write in line with each premise (except the first and last) the implied conclusions:

$$
\begin{array}{l}
N - d, \\
D - j, \ (\therefore N - j), \\
J - x, \ (\therefore N - x), \\
\cancel{Y} - X, \ (\therefore \cancel{N} - Y), \\
Z - y, \ (\therefore \cancel{N} - Z), \\
Q - z; \\
\therefore \cancel{N} - Q.
\end{array}
$$

The expression, with the exception of the ultima, will be found, upon examination, to constitute the premises of two simple Sorites, of which the first is in the progressive configuration and the second in the regressive.

* Taken from Schuyler's Logic, p. 88.

By stating them successively with their implied ultimas, we shall have them in the following form:

$$
\begin{array}{l}
\mathrm{N} - \mathrm{d}, \\
\mathrm{D} - \mathrm{j}, \\
\mathrm{J} - \mathrm{x}; \quad (\therefore N - x, \textit{ held in the mind}), \\
- - - - \\
\cancel{\mathrm{Y}} - \mathrm{X}, \\
\mathrm{Z} - \mathrm{y}, \\
\mathrm{Q} - \mathrm{z}; \quad (\therefore \cancel{Q} - X, \textit{ held in the mind}), \\
- - - -
\end{array}
$$

and then,

$$
\begin{array}{l}
\because N - x, \\
\textit{and } \cancel{Q} - X; \ \therefore \cancel{\mathrm{Q}} - \mathrm{N};
\end{array}
$$

or,

$$
\begin{array}{l}
\because \cancel{Q} - X, \\
\textit{and } N - x; \ \therefore \cancel{\mathrm{N}} - \mathrm{Q}.
\end{array}
$$

I now proceed to show that Sorites, stated as above, fall within the definition of compound Sorites, as hereinbefore given.

The maximus premise, being the *last premise* of the *principal* Sorites involved in the foregoing examples, has not appeared, but has in all cases been *implied*. The *middle* premise is (as has been before stated to be the case in *all* Sorites, simple and compound) the *second*, and in the examples is D — j. Combining this with the ultima, the abridged form of the principal Sorites is therefore,

$$
\begin{array}{l}
\mathrm{D} - \mathrm{j}; \\
\therefore \cancel{\mathrm{N}} - \mathrm{Q}.
\end{array}
$$

Expanding this in the 12th progressive mood as in the synopsis, we shall have the full principal Sorites as follows:

N — d, D — j, Ɉ — Q; ∴ N̶ — Q;

and the compound Sorites will be as follows:

N — d,
D — j; (∴ *N — j, held in the mind*),
\- - - -
J — x,
Ɏ — X; (∴ *Ɉ — Y, held in the mind*),
\- - - -
Z — y,
Q — z;
∴ - - - -
Q — y;
∴ - - - - (∵ *Ɉ — Y*),
Ɉ — Q;
and ∴ N̶ — Q. (∵ *N — j*).

§ 25. But the magnus and maximus terms of the principal Sorites, at the ultima of which we first arrive, may not be the infima species and summum genus, and further investigation may bring into the process of the reasoning lower species or higher genera, and if in both directions, both; and the new term or terms, instead of being employed *interiorly* as middle terms as hitherto, will be employed *exteriorly*. In such case the new term, or terms, will constitute, if there be but one, a new magnus, or maximus term; or if there be two, obtained by investigation in both directions, both, and the displaced terms will become middle terms. We shall then find that there will be two new abridged and full principal Sorites in each case, one regressive and one progressive, but varying according as the new term, or terms, are applied to the original Sorites considered as both regressive and progressive. They will, however, be independent of each other, and each will have its correlative in their respectively opposite configurations. The displaced original term, if it shall have been the magnus, will become the minor-middle term of the new principal progressive Sorites, and will not appear in the new regressive, but if the magnus term be again displaced by bringing in another, then the displaced original term will become the major-middle; but if the displaced original term shall have been the maximus, then it will become the major-middle term of the new principal regressive Sorites, and will not appear in the new progressive, and if the maximus term be again displaced by bringing in another, the displaced original will become the minor-middle term.

But of the original premises in the case of one new term being brought in, one, or two, will still remain in each new principal Sorites, one in the regressive configuration, and two in the progressive, if the new term be maximus, and *vice versa*, if magnus. One original premise only will remain in each of the new principal Sorites in any case if two new terms, one magnus and one maximus, are brought in.

The original ultima will of course have disappeared in every case. But if two new terms are brought in, both having been discovered in a process of investigation in one direction only, the original ultima will reappear *as a* *premise* of one of the new principal Sorites, the regressive, if the investigation were in the ascending direction, and the progressive, if in the descending.

If the investigation shall be pursued so that more than two new terms shall be brought in, in each direction, every vestige of the original principal Sorites will have disappeared from the new principals, as they will then be constituted.

But all the premises of the original principal Sorites will, in all cases, be found to remain, either partly in the principal Sorites, and partly in the following included Enthymeme or Sorites, or in two of the included Sorites, or wholly in the last included Sorites, or partly in the Enthymeme, which is the final expression, and partly in the next preceding included Sorites, according as the new terms shall be brought in; and they will always be found together in their original order, either regressive or progressive, how far soever the process be continued, and this, also, whether the compound Sorites be in the first or second form, as hereinbefore shown.

The following examples illustrate all the foregoing remarks, except the last, as to compound Sorites in the second form, which can be verified by trial. All the involved Syllogisms are in the first figure throughout.

The original premises and ultima (employed as a premise) are printed in Roman letters, and those which remain in the principal Sorites in full-faced type. All other propositions are printed in *Italics*. The examples having the same number of new terms are so arranged, either on the same or opposite pages, that they may be readily compared.

With one new term, brought in in the ascending process of investigation, and therefore a new maximus term:

Regressive Configuration.	*Progressive Configuration.*
X — y,	N — d,
J — x ; $(\therefore J - y)$,	D — j ; $(\therefore N - j)$,
- - - -	- - - -
D — j,	J — x,
N — d;	X — y;
∴ - - - -	∴ - - - -
N — j;	J — y;
and ∴ N — y. $(\because J - y)$.	and ∴ N — y. $(\because N - j)$.

Full forms of new principal Sorites:

$X - y$, **J — x**, $N - j$; ∴ $N - y$.

N — d, **D — j**, $J - y$; ∴ $N - y$.

With one new term, brought in in the descending process, and therefore a new magnus term:

Regressive Configuration.	*Progressive Configuration.*
J — x,	K — n,
D — j; $(\therefore D - x)$,	N — d; $(\therefore K - d)$.
- - - -	- - - -
N — d,	D — j,
K — n;	J — x;
∴ - - - -	∴ - - - -
K — d;	D — x;
and ∴ K — x. $(\because D - x)$.	and ∴ K — x. $(\because K - d)$.

Full forms of new principal Sorites:

J — x, **D — j**, $K - d$; ∴ $K - x$.

$K - n$, **N — d**, $D - x$; ∴ $K - x$.

With two new terms, one brought in in the ascending process of investigation, and therefore a new maximus term, and the other brought in in the descending process, and therefore a new magnus term:

Regressive Configuration.	*Progressive Configuration.*
$X — y$,	$K — n$,
J — x; ($\therefore J-y$),	N — d; ($\therefore K-d$),
– – – –	– – – –
D — j,	D — j,
N — d,	J — x,
$K — n$;	$X — y$;
∴ – – – –	∴ – – – –
$K — j$;	$D — y$;
and ∴ $K — y$. ($\because J-y$).	and ∴ $K — y$. ($\because K-d$).

Full forms of new principal Sorites:

$X — y$, **J — x**, $K — j$; ∴ $K — y$.
$K — n$, **N — d**, $D — y$; ∴ $K — y$.

With two new terms, both brought in in the ascending process of investigation, and one therefore a new maximus term:

Regressive Configuration.

$$\begin{array}{l} Y - z, \\ X - y;\ (\therefore X - z), \\ - - - - \\ \mathrm{J} - \mathrm{x}, \\ \mathrm{D} - \mathrm{j}, \\ \mathrm{N} - \mathrm{d}; \\ \therefore - - - - \\ \mathrm{N} - \mathrm{x}; \\ \text{and} \therefore N - z.\ (\because X - z). \end{array}$$

Progressive Configuration.

$$\begin{array}{l} \mathrm{N} - \mathrm{d}, \\ \mathrm{D} - \mathrm{j};\ (\therefore N - j), \\ - - - - \\ \mathrm{J} - \mathrm{x}, \\ X - y, \\ Y - z; \\ \therefore - - - - \\ J - z; \\ \text{and} \therefore N - z.\ (\because N - j). \end{array}$$

Full forms of new principal Sorites:

$$Y - z,\quad X - y,\quad \mathbf{N - x};\quad \therefore\ N - z.$$
$$\mathbf{N - d},\quad \mathbf{D - j},\quad J - z;\quad \therefore\ N - z.$$

With two new terms, both brought in in the descending process, and one therefore a new magnus term:

Regressive Configuration.

$$\begin{array}{l} \mathrm{J} - \mathrm{x}, \\ \mathrm{D} - \mathrm{j};\ (\therefore D - x), \\ - - - - \\ \mathrm{N} - \mathrm{d}, \\ K - n, \\ Q - k; \\ \therefore - - - - \\ Q - d; \\ \text{and} \therefore Q - x.\ (\because D - x). \end{array}$$

Progressive Configuration.

$$\begin{array}{l} Q - k, \\ K - n;\ (\therefore Q - n), \\ - - - - \\ \mathrm{N} - \mathrm{d}, \\ \mathrm{D} - \mathrm{j}, \\ \mathrm{J} - \mathrm{x}; \\ \therefore - - - - \\ \mathrm{N} - \mathrm{x}; \\ \text{and} \therefore Q - x.\ (\because Q - n). \end{array}$$

Full forms of new principal Sorites:

$$\mathbf{J - x},\quad \mathbf{D - j},\quad Q - d;\quad \therefore\ Q - x.$$
$$Q - k,\quad K - n,\quad \mathbf{N - x};\quad \therefore\ Q - x.$$

With three new terms, of which two are brought in in the ascending process of investigation, and one of them therefore a new maximus term, and the third in the descending process, and therefore a new magnus term:

Regressive Configuration.

$Y - z,$
$X - y;$ $(\therefore X - z),$
$- - - -$
$\mathrm{J} - \mathrm{x},$
$\mathrm{D} - \mathrm{j};$ $(\therefore D - x),$
$- - - -$
$\mathrm{N} - \mathrm{d},$
$K - n;$
$\therefore - - - -$
$K - d;$
$\therefore - - - -$ $(\because D - x),$
$K - x;$
and $\therefore K - z.$ $(\because X - z).$

Progressive Configuration.

$K - n,$
$\mathrm{N} - \mathrm{d};$ $(\therefore K - d),$
$- - - -$
$\mathrm{D} - \mathrm{j},$
$\mathrm{J} - \mathrm{x};$ $(\therefore D - x).$
$- - - -$
$X - y,$
$Y - z;$
$\therefore - - - -$
$X - z;$
$\therefore - - - -$ $(\because D - x),$
$D - z;$
and $\therefore K - z.$ $(\because K - d).$

Full forms of new principal Sorites:

$Y - z,\ X - y,\ K - x;\ \therefore\ K - z.$
$K - n,\ \mathbf{N} - \mathbf{d},\ D - z;\ \therefore\ K - z.$

With three new terms, of which two are brought in in the descending process of investigation, and one of them therefore a new magnus term, and the third in the ascending process, and therefore a new maximus term:

Regressive Configuration.

$X - y,$
J — x; $(\therefore J - y),$
\- - - -
D — j,
N — d; $(\therefore N - j),$
\- - - -
$K - n,$
$Q - k;$
$\therefore$ - - - -
$Q - n;$
$\therefore$ - - - - $(\because N - j),$
$Q - j;$
and $\therefore Q - y.$ $(\because J - y).$

Progressive Configuration.

$Q - k,$
$K - n;$ $(\therefore Q - n),$
\- - - -
N — d,
D — j; $(\therefore N - j),$
\- - - -
J — x,
$X - y;$
$\therefore$ - - - -
$J - y;$
$\therefore$ - - - - $(\because N - j),$
$N - y;$
and $\therefore Q - y.$ $(\because Q - n).$

Full forms of new principal Sorites:

$X - y,$ **J — x**, $Q - j;\ \therefore\ Q - y.$
$Q - k,$ $K - n,$ $N - y;\ \therefore\ Q - y.$

With four new terms, of which two are brought in in the ascending process, and two in the descending:

Regressive Configuration.

$$
\begin{array}{rll}
 & Y - z, & \\
 & X - y; & (\therefore X - z), \\
 & - - - - & \\
 & \mathrm{J} - \mathrm{x}, & \\
 & \mathrm{D} - \mathrm{j}; & (\therefore D - x), \\
 & - - - - & \\
 & \mathrm{N} - \mathrm{d}, & \\
 & K - n, & \\
 & Q - k; & \\
\therefore & - - - - & \\
 & Q - d; & \\
\therefore & - - - - & (\because D - x), \\
 & Q - x; & \\
\text{and } \therefore & Q - z. & (\because X - z).
\end{array}
$$

Progressive Configuration.

$$
\begin{array}{rll}
 & Q - k, & \\
 & K - n; & (\therefore Q - n), \\
 & - - - - & \\
 & \mathrm{N} - \mathrm{d}, & \\
 & \mathrm{D} - \mathrm{j}; & (\therefore N - j), \\
 & - - - - & \\
 & \mathrm{J} - \mathrm{x}, & \\
 & X - y, & \\
 & Y - z; & \\
\therefore & - - - - & \\
 & J - z; & \\
\therefore & - - - - & (\because N - j), \\
 & N - z; & \\
\text{and } \therefore & Q - z. & (\because Q - n).
\end{array}
$$

Full forms of new principal Sorites:

$$Y - z,\ X - y,\ Q - x;\ \therefore\ Q - z.$$

$$Q - k,\ K - n,\ N - z;\ \therefore\ Q - z.$$

With eight new terms, of which four are brought in in the ascending process, and four in the descending:

Regressive Configuration.

$S - t,$
$Z - s;$ $(\therefore Z - t),$
$- - - -$
$Y - z,$
$X - y;$ $(\therefore X - z),$
$- - - -$
$\mathrm{J} - \mathrm{x},$
$\mathrm{D} - \mathrm{j};$ $(\therefore D - x),$
$- - - -$
$\mathrm{N} - \mathrm{d},$
$K - n;$ $(\therefore K - d),$
$- - - -$
$Q - k,$
$G - q,$
$H - g;$
$\therefore - - - -$
$H - k;$
$\therefore - - - -$ $(\because K - d),$
$H - d;$
$\therefore - - - -$ $(\because D - x),$
$H - x;$
$\therefore - - - -$ $(\because X - z),$
$H - z;$
and $\therefore H - t.$ $(\because Z - t).$

Progressive Configuration.

$H - g,$
$G - q;$ $(\therefore H - q),$
$- - - -$
$Q - k,$
$K - n;$ $(\therefore Q - n),$
$- - - -$
$\mathrm{N} - \mathrm{d},$
$\mathrm{D} - \mathrm{j};$ $(\therefore N - j),$
$- - - -$
$\mathrm{J} - \mathrm{x},$
$X - y;$ $(\therefore J - y),$
$- - - -$
$Y - z,$
$Z - s,$
$S - t;$
$\therefore - - - -$
$Y - t;$
$\therefore - - - -$ $(\because J - y),$
$J - t;$
$\therefore - - - -$ $(\because N - j),$
$N - t;$
$\therefore - - - -$ $(\because Q - n),$
$Q - t;$
and $\therefore H - t.$ $(\because H - q).$

Full forms of new principal Sorites:

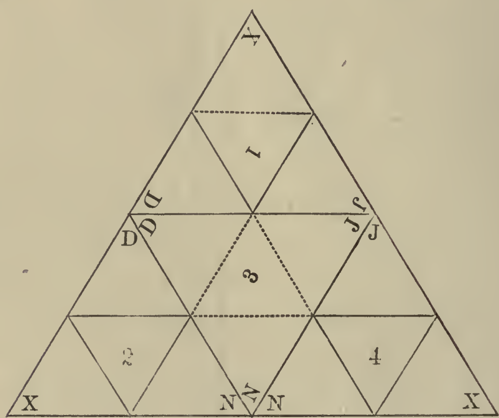
$S - t, \quad Z - s, \quad H - z; \quad \therefore \ H - t.$

$H - g, \quad G - q, \quad Q - t; \quad \therefore \ H - t.$

§ 26. To recur now to illustration by means of geometrical figures.

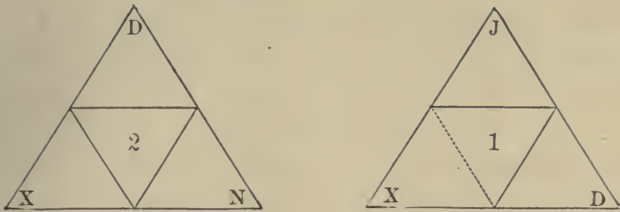
A regular tetrahedron may by four sections, beginning in the middle of each of its edges and made parallel to the opposite planes respectively, be divided into five figures, of which four will be regular tetrahedra, and the fifth and interior figure a regular octahedron.

Thus, by reproducing our former illustration on card-board before folding, and dividing it by lines which shall represent the four sections, we shall have the following :

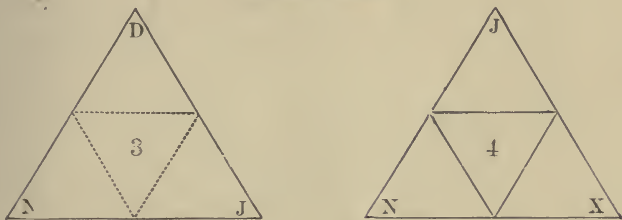


Now, assuming each interior dotted line to be the edge of an equilateral triangular plane, represented by card-board, projecting backward, divergingly, at the proper dihedral angles, from the plane of the one which we are supposed to have in hand, then, by folding the latter as before, we shall have a combination of five figures, as above stated, which will present to our eyes successively, as we turn it about as before, the following figures :

First Series.



Second Series.

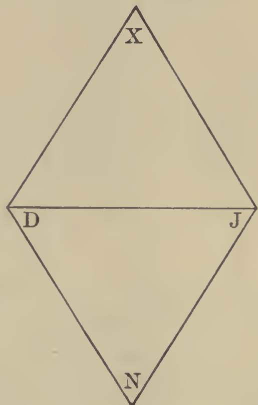


Each of the four tetrahedra having one original exterior point, and three visible and one invisible planes, will be found to have that point marked with one of the letters N, D, J, X on each visible plane; the fifth figure, the octahedron, having no original exterior point, and four visible, and four invisible planes, will be found marked on each visible plane with one of the numbers 1, 2, 3, 4. It is wholly included, and occupies all the space, between the invisible planes of the four tetrahedra and planes connecting their visible planes, and its volume is exactly equal to the sum of their volumes; and it may well be regarded as the analogue of the *ultima conclusio* of the Sorites, of which the abridged form is:

$$D - j; \ \therefore \ N - x.$$

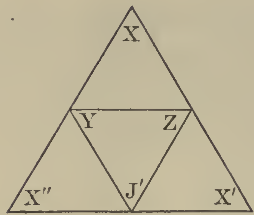
The analogy between a compound Sorites in which the original principal Sorites shall remain the principal, and a Sorites be interpolated as hereinbefore shown, and a tetrahedron divided by sections as represented in the foregoing illustration, cannot be exhibited as simply or as clearly as that between a simple Sorites and a tetrahedron considered as a unit, as in our former illustration, because the tetrahedron which is the analogue of the included Sorites is involved in and forms an indistinguishable, but, as must be regarded, separate, part of the included octahedron, having one of the visible planes of the octahedron as its only visible face. Its invisible faces cannot be brought to the surface in the following figures, but must be regarded as represented by the three triangles by which its visible face is bounded, the ultimate point of which will be found marked X in the figures. Its ultimate point will not be the point X as shown in the figures, but will lie in the perpendicular let fall from the point N upon the opposite plane of the original tetrahedron. We shall hereinafter find that perpendicular to be part of one axis of a sphere produced by the revolution of the tetrahedron, and that the pole of that axis opposite N should be marked X. The ultimate point of the indistinguishable tetrahedron which is the analogue of the included Sorites, may be at any point in the line of this axis within the octahedron, and let us assume that point to be in the centre of the invisible plane of the octahedron opposite its visible plane which is the visible face of the involved tetrahedron. The invisible faces of the latter will then be equal to the triangles by which its visible face is bounded in the figures.

Let us suppose that in the progressive process we have established the relation between N and J, as in the lower one of the following combination of triangles (which, observe, are the same as the triangles 1 and 3 in our original card-board illustration), and that the relation between J and X requires to be established.



We shall then have the upper triangle in which only the relation (length of line) between D and J is known, and let us suppose that the relation between each of those points and X is not capable of being immediately determined, but that there are two points (*middle terms*), one in each of the other two lines, capable of being successively reached from D or any point in the line D J except the point J, and the length of a straight line connecting them capable of being measured, and from both of which the direction of X can be observed, and the angles therefore determined.

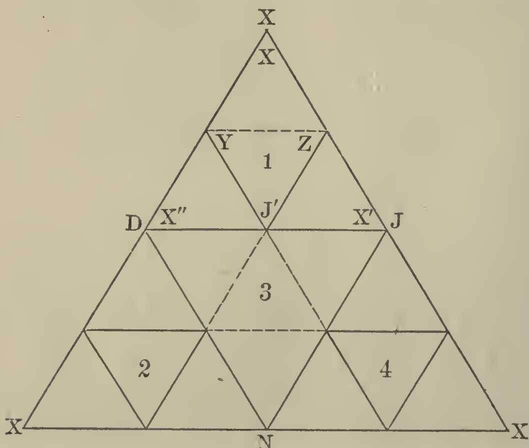
Reproducing the upper triangle and marking the middle point in the base line J′, and the two points at the extremities of the base X′ and X″, and the two new points Y and Z at the middle of each of the two lines connecting the extremes of the base with X, and connecting such new points, and each of them with J′, we shall have the following:



and we may now establish that J′ — X in the same manner as we have hereinbefore established that N — X.

But, the lines X′ X and X″ X are, by construction, equal to J X and D X in the upper triangle, on the preceding page, the middle points in which may be marked Z and Y. In the process we have found J′ Z equal to J′ Y, and X′ Z equal to J′ Z. X′ Z is therefore equal to J′ Y. But X′ Z is J Z. And as J′ Y is equal to J′ Z, it will, upon being applied to the latter, coincide with it, and the point Y will fall upon the point Z. J Z may therefore be called J Y, and is equal to J′ Y.

The whole combination of triangles will now be as follows, the original letters being put on the outside:



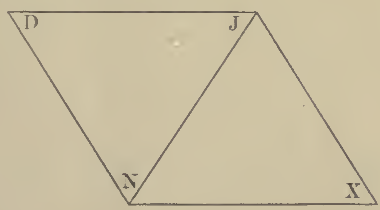
We can now express the full compound Sorites, exemplified by the foregoing illustration, as follows:

N — d,
D — j ; (∴ *N — j, held in the mind*),
\- - - -
J — y. (= *J Z* = *J′ Y, from which latter direction of X observed*),
Y — z, (= *Y Z, relation*, i. e., *length of line known*),
Z — x : (= *Z X, direction observed from former*),
∴ - - - -
J — x ;
and ∴ N — x. (∵ *N — j*).

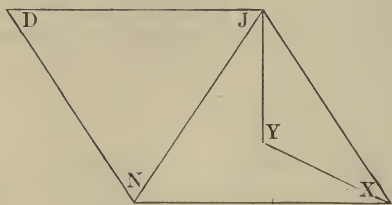
This is the same compound Sorites as that exhibited in § 20, on page 81, but with the included Sorites in the progressive, instead of the regressive. configuration.

But if the interpolated expression be an Enthymeme, the analogy will be much clearer, as the lines by which the Enthymeme will be represented will lie wholly in the surface and not involve any section of the original figure.

Thus, if in the following combination of triangles (which observe are the same as triangles 3 and 4 in our original card-board illustration):



we shall, in like manner as before, have established the relation between N and J (as in the upper, left-hand triangle), from which latter we can see X, but are unable immediately to determine its distance, without the knowledge of which we cannot establish the relation between N and X; we may select another mediate point, Y, which can be reached, and distance measured from J, and from which X may also be seen, and the angles therefore determined, as in the following figure:

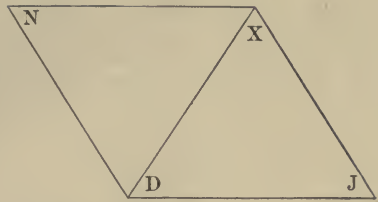


and then, by the elements thus obtained, we can determine the required distance from J to X, and by means thereof and the elements previously obtained, the distance from N to X.

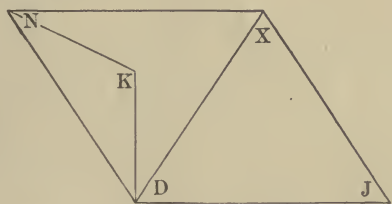
The compound Sorites exemplified by the foregoing illustration will be as follows:

N — d,
D — j; (∴ *N — j held in the mind*),
– – – –
Y — x,
J — y;
∴ – – – –
J — x,
and ∴ N — x. (∵ *N — j*).

But if, instead of having begun in the ascending direction, we shall have begun in the descending, and have established the relation between X and D, as in the lower, right-hand one of the following combination of triangles (1 and 2 in the figure on page 65):



and shall then, although able to see N from D, but not from X, be unable to determine its distance from D, without the knowledge of which, it would be impossible to determine its distance from X; we may, in like manner as before, select another mediate point K, which can be reached from D, and from which N can also be seen, as in the following figure:

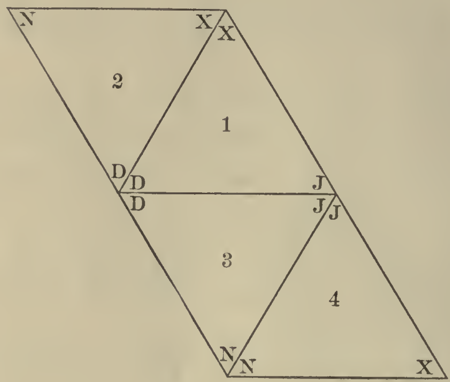


and then, as before, may determine the required distance from D to N, and by means thereof and the elements previously obtained, the distance from X to N.

The compound Sorites exemplified by the foregoing illustration, will be as follows:

X comprehends J,
J comprehends D; (∴ *X comprehends D, held in the mind*),
\- - - - - - -
D comprehends K,
K comprehends N;
∴ - - - - - - -
D comprehends N;
and ∴ X comprehends N. (∵ *X comprehends D*).

By putting together the first of each of the two sets of figures in the preceding illustrations, on the line D J, common to both, we shall have the following figure:



which is the same as that on page 61, but in a different position. By turning triangle 2 downward in a semicircle on the point D as a centre, we shall have our original card-board figure; or by turning triangle 4 upward to the like extent on the point J as a centre, we shall have the figure shown on page 65. Triangles 1 and 2 taken together and 3 and 4 taken together are analogues of progressive Sorites, 1 and 2, in the descending direction, and 3 and 4, in the ascending; but if 2 and 4 be both turned as above described, they will become analogues of regressive Sorites in the respectively opposite directions.

§ 27. All the four triangles in our original card-board illustration are equilateral and equal. The solid figure resulting from the folding of the card-board is a *regular* tetrahedron, which is defined as a solid having four faces, all equal equilateral triangles. But the triangles might have been all equal isosceles triangles, or partly equilateral and partly isosceles. Such can be exhibited in a plane figure bounded, by three, or four exterior lines, if the triangles are all equal, or by six, if they are partly equilateral and partly isosceles, and capable of being folded so that the exterior points shall meet in a *perfect*, but not *regular*, figure. But a *perfect* tetrahedron may have all its faces unequal, and in such case the faces may be spread out in an irregular plane figure having five exterior lines. In all cases the number of exterior lines will be found to be six, if bisected lines are counted each as two. All other plane figures having all the points exterior are *imperfect* and cannot be folded, so that the exterior points will meet, and their areas, and consequently the volume of space which they can be made respectively to inclose, can only be determined by means of the triangle. *Imperfect* Syllogisms and Sorites in logic must be reduced to the *perfect* figure before they can be submitted to the *dictum de omni et nullo.*

§ 28. On the other hand, a tetrahedron (regular or perfect) may be added to on the outside by superimposing on each of its faces another tetrahedron having a similar face, so that there shall be five tetrahedra in all. Four new points will have been added, all exterior to the original figure, the original points becoming interior, but their locations visible, the original figure having otherwise wholly disappeared from view.

Similarly, as we have before seen, in respect to a Sorites, when four new terms have been brought in exteriorly, two in each direction, the four propositions of the original principal Sorites will have disappeared from the *two new principals*, as they will then be constituted, but they will remain in the included Sorites, of which the inner tetrahedron is the analogue.

But in the figure, formed as above described, the four new points, which we will consider as marked K, Q, Y, and Z, will furnish only *one* new principal Sorites, as its analogue, which may be rendered in its abridged form thus:

$$\because\ Q — y;\ \therefore\ K — z.$$

But observe, the interior figure in the foregoing combination is a tetrahedron, not necessarily regular, but perfect; and if, instead of beginning with such a one, considered as a unit, we begin with a *regular* one considered as divided by four sections, as before shown, and superpose upon each of the visible planes of the included octahedron, a tetrahedron similar to each of the four resulting from such sections, we shall have a solid figure in the form of an eight-pointed star, the octahedron having entirely disappeared from view, except that the locations of its points will be visible. This eight-pointed star will be found to consist of two equal intervolved regular tetrahedra, to both of which the interior octahedron will be common, and its revolution about its centre will produce a sphere exactly equal to that produced by the revolution of the original tetrahedron. Four exterior points will have been added, but of these two are the opposite poles of the two original points marked N and X, and, having a common relation with them to the included octahedron, should be marked X and N respectively, leaving, in fact, but two new independent points, which may be marked Y and Z. The whole figure will then be the analogue of two independent full Sorites, of which one only is new, and that only in part, the abridged forms being:

$$\because\ D — j;\ \therefore\ N — x.$$
$$\because\ Y — z;\ \therefore\ N — x.$$

By comparing the foregoing illustrations with the Sorites having four new terms added exteriorly, given on page 96, the superiority of the Sorites over its analogue, the tetrahedron, will again be manifest.

§ 29. Thus everywhere, whether we go inwardly or outwardly, and in all things, metaphysical as well as physical, we find triniunity, and can thence proceed to quadriunity, but beyond that, except in composite forms, we cannot go.

§ 30. From the foregoing definitions and illustrations of Sorites, simple and compound, it seems manifest that the human mind is limited to reasoning concerning the relations of four terms. If other terms are brought in, they must relate to the terms of the principal argument, and in such case, if such relation be to the middle terms, they serve only to elucidate, but if to the magnus and maximus terms, then they supplant those terms; which, if there be one, or two successively of each (new magnus and maximus terms) respectively, become terms of the two new middle premises respectively, but if more than two of each, then are relegated to the subordinate position of middle terms employed only in elucidation. Otherwise they must be the terms of independent arguments.

§ 31. There remains but to say that I have not pointed out the characteristics of Sorites, nor given the rules in relation to them, as the same have been usually pointed out and given (or in part so) in logical treatises, and to which reference has been hereinbefore made; and I now refer to them only for the purpose of showing their inadequacy.

They have been written with reference to Sorites treated of as capable of being expanded *only* in Syllogisms *wholly in the first figure*, and without reference, of course, to the distinction between them as simple and compound, which has been hitherto unobserved. They relate,

1st. To the number of Syllogisms involved, as equal to the number of middle terms, and as ascertainable from the number of premises of the Sorites, less one.

2d. To the character of the premises of the involved Syllogisms, whether minor or major, and the number of each and their sequence, viz.: one only, and that the first, major, and all the following minor in a regressive Sorites; and *vice versa*, in a progressive.

3d. To the number and positions of particular and negative premises in the two configurations, viz.: that one only can be particular, and that the last, and one only negative, and that the first, in a regressive Sorites; and *vice versa* (in respect to positions) in a progressive.

The first is true of all Sorites, simple and compound, in respect to the number of Syllogisms involved being equal to the number of middle terms, and has been impliedly shown as true of all *simple* Sorites, in respect to such number being ascertainable from the number of premises less one, in that they have been described as having three premises, and as being capable of expansion into two Syllogisms; but in such latter respect it does not apply to *compound* Sorites when *fully expressed*.

The second, by an examination of the synopsis, will be found to hold good, of all regressive simple Sorites in respect to the moods in which they are minors, and not good in respect to those in which they are majors, and *vice versa* of all progressives.

The third is of course, and for obvious reasons, applicable to all *simple* Sorites (but not to all *compound*, when *fully expressed*), so far as the *number* of particular and negative premises is concerned, but to state it in respect to their *positions* as applicable to *all* Sorites capable of being expanded in Syllogisms *wholly in the first figure*, and also to *some* in combinations of figures, *either partly or not at all of that figure*, and then to point out the very numerous exceptions in *other like cases*, would tend rather to confuse than to enlighten; and I therefore leave the subject, and pass on to the consideration of Fallacies.