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PRINCIPLES OF LOGIC.

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CUSACK'S

PRINCIPLES OF LOGIC,

PREPARED EXPRESSLY TO MEET THE REQUIREMENTS

OF THE

SYLLABUS FOR CERTIFICATE STUDENTS.

BY

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"Grant that the phenomena of intelligence conform to laws; grant that the evolution of intelligence in a child also conforms to laws; and it follows inevitably that education cannot be rightly guided without a knowledge of those laws."

HERBERT SPENCER.

PREFACE TO THE FIRST EDITION.

The present Text Book has been prepared specially for students preparing for the final examination for the Elementary School Teacher's Government Certificate. Such students will find in it material amply sufficient to answer any question likely to be set on "The Processes of Reasoning," as in drawing it up it has been thought advisable to err on the side of *too much*, rather than on the side of *too little*, for the purpose in view. The book is the outcome of considerable experience in preparing students for the "Certificate" Examination.

The arrangement of the first few chapters is due in the main to the valuable criticisms and suggestions of Professor Cusack.

It is hoped that students other than those for whom it is specially prepared will find the book useful as a fairly complete introduction to the Study of Logic.

PREFACE TO THE SECOND EDITION.

In preparing the Second Edition of this Text Book, it was thought advisable, in the light of greatly extended experience in teaching the subject, to entirely rewrite and greatly enlarge it. The present Edition may therefore be regarded as a new book. The First Edition has proved useful to thousands of students. The hope of the writer is that in its present form the book will prove more useful to even a greater number.

S. B.

September 1st, 1899.

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PRINCIPLES OF LOGIC.

CHAPTER I.

Reasoning.

There has never been a time in the history of the human race when knowledge has been regarded as a possession of greater importance than it is at the present day. It is true, that in education mental training is said to be the object aimed at rather than the mere acquisition of knowledge, yet when it is remembered that mental training is essentially a fashioning of the mind so as to make it capable of more readily acquiring knowledge than would be possible without such training, it becomes clear that in education directly or indirectly the acquisition of knowledge is the main, if not the sole, object. The possession of greater knowledge on the part of the human race has brought it about that to-day "millions find support where once there was food only for thousands", and that to the meanest of these millions comforts and pleasures are possible such as formerly were unattainable by the most powerful.

We all are ready to allow the transcendent importance of knowledge and, at the same time, are possibly inclined

to take pride in the thought that our knowledge is far greater than that of our fathers.

"We think our fathers fools, so little did they know,"

says Pope, and we are quite ready to agree with him; and yet how few of us stop to ask ourselves the question, "How has this increase of knowledge been brought about?" Granted the supreme importance of knowledge, the question "How is knowledge acquired?" is surely one that deserves to be answered. It is clear that we learn that honey is sweet by tasting it, that iron is hard by feeling it, that a buttercup is yellow by looking at it, and so on.

In these cases we gain knowledge by the use of our senses. The mind is seated in the brain and has communication with the outside world only by means of the senses. These senses are means of ingress to the mind; they are, as indeed they have been called, "the portals of knowledge". Could there be such a thing as a human being endowed with none of the senses, his mind (if we could say he had a mind) would inevitably be a perfect blank. Hence the immense importance in education of training the senses.

A little thought will, however, serve to make it clear that some of the knowledge that we possess has not been acquired directly through the use of our senses. Thus, the sun is known to be a globe many times larger than the earth, but our sense of sight alone would lead us to believe that it is a ball having a diameter of, at most, a few feet; and mere observation would tell us that the moon is as large as the sun. If we trusted to our senses alone, we should believe that the stars are mere points, that the sun moves daily in a curve from east to west, that the surface

of the earth is flat, and many other things which, with our present knowledge, we know to be false.

Further, it is evident that many facts known to be true could not from their very nature be known by the use of the senses alone. Thus we know that we are mortal, but no amount of observation alone could tell us this, for no one could be known to be mortal until he died; and hence no one could ever know that he himself was mortal. The mind, then, must possess some other means of acquiring knowledge. What is this means? An example will make it clear. A child sees a dog and learns by the actual use of his sense of sight that the dog has four legs. His sight tells him the same fact of several other dogs. From this he concludes that all dogs have four legs.

In this case the child makes use of the facts supplied by his sense of sight, and out of them he evolves an entirely new fact, viz., that all dogs have four legs. It is quite clear the child would never be able to learn this fact by observation alone, for he would never be able to see all dogs. He has gone through a process altogether different from that of observation, he has *reasoned*. That power possessed by the human mind of taking certain given facts and from them deducing new facts is called the *Reasoning Faculty;* the process by which such new facts are reached is called *Thinking* or *Reasoning*. To reason, then, is to obtain additional facts out of given facts. *Sensation* and *Reasoning* are the two, and the only two, means by which the mind can possibly obtain knowledge.

It is evident from what is said above that in order to reason we must first possess knowledge from which to reason. Reason cannot produce knowledge without having

raw material to work upon. This raw material is ultimately derived from sensation, and when used by Reason to establish new truths, it is called the *data** or *premisses*. The truth reached by reasoning is said to be *inferred* from the given premisses. The inferred truth or inference is often called the *conclusion*.

When a conclusion follows properly from its premisses, that is, in such a manner that, granted the truth of the premisses, the conclusion is necessarily true, it is said to be *valid*. The meaning of this word should be carefully noticed, as students frequently fall into the error of regarding any conclusion as valid if they know it is true. The distinction between the words *valid* and *true* will become clear on reading the examples of reasoning in Chapter II. At present it is sufficient to note that a valid conclusion may be a false one, if one or more of the premisses be false; and that on the other hand, a conclusion may happen to be true and yet not be valid if it does not follow properly from the premisses.

Everybody is continually making inferences or drawing conclusions from given data or premisses: and if the inferences so made were always valid there would be no need for any one to trouble further about the matter. Experience, however, teaches us that nothing is more common than for people to draw conclusions in no way justified by the premisses. Most people at times find their calculations mislead them, which would be impossible were such calculations made validly from true data. Hence it becomes necessary to consider under what conditions we may draw conclusions from given premisses; to find out whether there are general rules which must be observed in reasoning, and if so what

* *Data* is the plural of *datum*, a Latin word meaning *something given*.

such rules are. If such rules can be found, they may be called the *Laws* or *Principles* of Reasoning. A *law* or *principle* is the statement of a general truth.

That there are laws of reasoning is evident. No one doubts the truth of the statement that "things which are identical with the same thing are identical with each other". This is a law which no one hesitates to use as a principle of reasoning. Thus, if I am told that A is identical in height with B, and that C is identical in height with B, I immediately conclude that A and C are of identically the same height. The study of such principles of reasoning is conveniently referred to as the study of *Logic*, and *Logic* may be defined as the "Science of the Principles of Valid Reasoning". *Science* is general systematized knowledge about some subject matter, or the knowledge of those general laws which hold good in any department of nature. If Logic be regarded merely as a means to enable us to reason correctly it is then more accurately defined as the "Art of Valid Reasoning". Art is the skilful application of scientific knowledge.

In the next chapter we shall consider various examples of Reasoning.

Résumé of Chapter I.

1. All allow the importance of knowledge.
2. All knowledge is ultimately derived from the senses.
3. Much of our knowledge is directly due to the Reasoning Faculty which elaborates the elements supplied by the senses into more complex knowledge.
4. Truths used to establish other truths are called data or premisses.

5. A truth inferred from others is called a conclusion.
6. Reasoning is the process by which the human mind infers truth from other truths.
7. A Law is the statement of a general truth.
8. The "Principles of Reasoning" are the laws that must be observed in reasoning if our inferences are to be trustworthy.
9. These general principles are the subject matter of the Science of Logic.

Questions on Chapter I.

1. Why have the senses been called the "Portals of Knowledge"?
2. What is Reasoning?
3. By what possible means can the mind acquire knowledge?
4. Explain *data* or *premisses*, and *conclusion*.
5. Point out the premisses and conclusion in the following:—

   All planets go round the sun in elliptic orbits;
   Therefore the earth goes round the sun in an elliptic orbit, for the earth is a planet.
6. What is meant by the "Principles of Reasoning"?

CHAPTER II.

Examples of Reasoning.

The following examples should make the student more clearly understand what is meant by reasoning and, at the same time, enable him to distinguish the different kinds of reasoning treated of in this book.

I.

1. If we know that the statements—

All coins are useful things,

and This shilling is a coin,

are true, can we from these as premisses infer the truth of a third statement different from either of these? Clearly we can.

This shilling is a useful thing,

is such a statement, which is certainly true if the preceding premisses are true and which is therefore a conclusion inferred from those premisses. This conclusion is a truth quite different from either premiss, although, of course, it is contained in the two premisses together. Here we have, then, obtained a fresh truth from known truths, that is, we have reasoned. Moreover, the reasoning is valid, for, granted the premisses are true, the conclusion is also certainly true. The reasoning in this example seems so simple that the student may be inclined to ask how there

can be any difficulty in such a process as this. Let him have patience and he will soon discover that to reach a valid conclusion is often by no means an easy matter.

2. Next consider the premisses—

All fish live in water,
All herrings are fish.

From these we can by a process of reasoning make the inference that

All herrings live in water.

This is, as clearly as in the first example, a valid conclusion.

3. Again, let the premisses be—

All quadrupeds are herbivorous ;
All tigers are quadrupeds.

From these we may apparently infer that

All tigers are herbivorous,

a conclusion which is not true, for tigers are carnivorous.

When the student can apply the rules for testing arguments, to be enunciated hereafter, he will find that the above violates none of such rules, and is therefore valid. If a conclusion is not true there must be something wrong either with the premisses or with the reasoning. These are the only possible sources of the error. If either of the premisses be false the conclusion may also be false and yet be valid. If we examine the premisses in the above argument we easily see that one of them, viz.—

All quadrupeds are herbivorous,

is false. Here, then, is the explanation of the falsity of the conclusion. The fault is not in the reasoning, but in the information given as premisses, and, therefore, such fault is no concern of the mere logician ; he is concerned only with the reasoning, and that is valid.

4. Consider one example more—

All coins are useful things,
All shillings are useful things,

therefore,

All shillings are coins.

The student may at first think this conclusion as valid as any of the preceding ; a little further thought will, however, convince him that it is invalid. There are many useful things which are not coins, and it may be, so far as we can tell from the premisses, that all shillings are among these useful things which are not coins. If instead of—

All shillings are useful things,

we take as our second premiss—

All pens are useful things,

then by the same argument we have as conclusion—

All pens are coins,

which is manifestly untrue ; and as both premisses are true, it must be that something is wrong with the reasoning, and the conclusion is invalid. But if this is invalid so is the previous conclusion, "All shillings are coins," for it is drawn from similar premisses in identically the same way. In treating of the rules of the syllogism we shall again recur to these or similar examples. The fact that we may draw a conclusion which is really invalid, although true and apparently valid, shows the need of rules by which such arguments may be tested.

Two things should be noticed with regard to all the above examples of reasoning.

1. The conclusion is drawn from two premisses.
2. One premiss in each case is a more general statement than the conclusion.

Thus, in Example 4 one premiss is a statement about *all coins*, and is, therefore, more general than the conclusion,

which is a narrower statement made of *all shillings*. The student should pay special attention to these two characteristics of the above arguments, as they are the two distinguishing marks of all *deductive reasoning*.

II.

Now consider another type of reasoning—

1. We gradually find out by observation such facts as the following :—

An apple tends to fall to the ground,
An orange does the same,
A book does the same,
&c., &c.,

And ultimately we draw the conclusion that

All bodies tend to fall to the ground.

That this is an inference is clear, for we certainly have not proved it by experience in every case. It applies to all bodies in all places and at all times—even to the future.

Whether such an inference is valid or not, it is often a matter of great difficulty to decide. Some of the later chapters of this book will be devoted to this form of reasoning, but in an elementary book like this it will be impossible to treat of it fully. We are constantly drawing such conclusions, and frequently they are invalid.

2. Thus, we meet a foreigner and find him hasty; we meet another who is also hasty; and we become acquainted with another of a similar disposition, and we begin to think that

All foreigners are hasty.

Such conclusion is, however, by no means validly drawn from our premisses. Consider another case of an invalid

inference in reasoning of this form. A foreigner in learning English is told

that *tough* is pronounced *tuff*,
that *rough* is pronounced *ruff*, and
that *sough* is pronounced *suff*,

and he straightway calls *dough duff*, having inferred that *ough* always has the sound *uff*.

Two things should be noticed about the examples of reasoning in II.

1. The conclusion is drawn from an indefinite number of premisses.
2. The conclusion is more general than the premisses.

These are the characteristic features of *inductive reasoning*.

III.

We have seen that in deductive reasoning the conclusion is drawn from two premisses and in inductive reasoning from an indefinite number of premisses; now suppose we have but one premiss given, as for instance,

All birds are bipeds,

can we make any inference from this? Clearly we can, for if

All birds are bipeds,

It is evidently true that

1. Some birds are bipeds, and
2. Some bipeds are birds.

And some readers may at first think that

3. All bipeds are birds,

is also a valid inference from the given premiss. It is, however, invalid, as will be fully shown later on.

When conclusions are drawn from a single premiss, as in III., 1, 2, 3, the process is known as *Immediate Inference*. Reasoning, *deductive*, *inductive*, and *immediate* are the forms of reasoning treated of in this book.

Résumé of Chapter II.

I. In *deductive reasoning* the conclusion
   1. Is drawn from two premisses, and
   2. Is less general than the premisses.

II. In *inductive reasoning* the conclusion
   1. Is drawn from an indefinite number of premisses.
   2. Is more general than the premisses.

III. In *immediate inference* the conclusion is drawn from one premiss.

Exercises on Chapter II.

1. Give three examples of valid deductive inference.
2. Refer the following to whichever of the three types of reasoning it belongs:—

   All planets go round the sun in elliptic orbits,
   The earth goes round the sun in an elliptic orbit,
   Therefore the earth is a planet.
3. Is the argument in Question 2 a valid one?
4. Give two original examples of inductive reasoning.
5. When is reasoning said to be immediate?
6. What inferences can be made from—

   All owls are birds?

CHAPTER III.

The Uses of Logic.

From the examples given in Chapter II., it is clear that in certain cases there is great danger of drawing conclusions which are not justified by the given premisses. How can such invalid conclusions be avoided? Whatever will give a clearer understanding of the processes and fundamental principles of valid reasoning will evidently be of great service in preventing and in detecting invalid reasoning, But Logic is the science of the principles of reasoning, hence one great and apparent use of Logic is that it points out what mistakes are of frequent occurrence in reasoning, and how to avoid them. The student should, however, guard himself against expecting too much from a knowledge of Logic. It is quite possible to know all the rules of Logic and yet to reason badly, and on the other hand, it is true that many people reason well who have never studied reasoning as a science.

The practised logician owes his ability to detect error in reasoning not so much to the rules of Logic as he does to the critical frame of mind which has been developed in him by the process of acquiring a knowledge of those rules. He is not so ready to take statements and conclusions for granted as are those who lack such a course of training; he is apt to look upon them with suspicion, and consequently

he detects errors which are passed over by others from lack of careful observation. It is, then, the mental discipline involved in acquiring a knowledge of Logic, rather than the knowledge itself, which gives increased power of reasoning. This mental discipline is, perhaps, the greatest advantage to be derived from the study of Logic. Man is distinguished from animals chiefly by the possession of a mind. The more he cultivates his mind, the farther does he remove himself from the animal state.

The student who subjects himself to the discipline of a course of study in Logic will enlarge his reasoning power and will thus enable himself to make greater and more rapid progress in any branch of study.

The relation between Logic and the different branches of science is an intimate one, for every branch of science consists in collecting facts, or observing phenomena, and making inferences from them; and, as we have already seen, inference is the subject matter of Logic.

A science is for the most part the practical application of Logic to some subject matter. Thus Geology is Logic applied to the study of the earth; Biology is Logic applied to the study of life, and so on. It is worth noticing that *-logy*, which is the termination of the names of so many sciences, is derived from the Greek word *logos* (discourse), the adjectival form of which, *logikos*, gives us our word *Logic*. It is because no science can be studied without the application of Logic that this latter has been called the "Science of Sciences".

For teachers a knowledge of Logic is of special importance. It is characteristic of modern ideas to regard education as consisting in mental training rather than in the mere imparting of information. The phase of mind

which most directly concerns the teacher is the intellect, the highest activity of which is reasoning. Hence, the most important part of mental training is the training of the reasoning power. It is clear, then, that the more the teacher knows about the laws of reasoning the better will he be able to train children to reason. In the words of Herbert Spencer:—"Grant that the phenomena of intelligence conform to laws; grant that the evolution of intelligence in a child conforms to laws; and it follows inevitably that education cannot be rightly guided without a knowledge of these laws".

Nothing is more productive of faults in reasoning than a loose and inaccurate use of words. The student may readily enough allow this, but, at the same time, think the remark does not apply to him. The number of people who are never misled by a wrong use of words is a very small one indeed. Now, one of the most important parts of Logic is that relating to words and the ambiguities to which they are liable; and no earnest student can study this subject without becoming in consequence more careful to use words in their exact meanings, and such care must re-act on his pupils, causing them to use more correct language and consequently to reason more accurately. Moreover, this is not a mere unconscious re-action. The teacher who has had his eyes opened to the dangers that lurk in words will constantly endeavour to enable his classes to avoid such dangers.

Résumé of Chapter III.

The Uses of Logic.

1. To avoid error in one's own reasoning.
2. To detect it in the reasoning of others.
3. As a means of mental discipline.
4. It is especially useful to teachers, for
   - *a.* It tends to make them use words in their exact meanings, and train their pupils to do likewise.
   - *b.* As being the Science of the Principles of Reasoning a knowledge of it gives teachers great help in developing the reasoning power in their pupils.

Exercises on Chapter III.

1. What is the greatest use to be derived from the study of Logic? Why?
2. Why should teachers especially have a knowledge of Logic?
3. Give the derivation of the word Logic.
4. Why has Logic been called the Science of Sciences?

CHAPTER IV.

The Parts of a Deductive Argument.

Before proceeding further it is advisable to examine more fully than was done in Chapter II. an example of deductive reasoning. Every such argument is a complex whole, consisting of certain essential elements. Before the validity of a deductive argument, then, can be tested, it is necessary to understand clearly what these essential elements are, so as to be able without hesitation to analyse the argument into its elements. Having found out what these elements are, it will be necessary to affix a name to each of them. Every art and every science has a language of its own, which it is essential that the learner should know, and Logic is no exception to this rule. It has its own technical terms, with which every student of the subject must make himself familiar. These technical terms are neither numerous nor difficult to learn. A few of them will be elucidated in this chapter.

As an example of the simplest form of the expression of a complete act of deductive thought, consider the argument—

All poets are philosophers,
Tennyson is a poet,
Therefore
Tennyson is a philosopher.

We have here three statements, or *propositions* in the language of Logic, which together form what is called a *syllogism*. This is the English form of the Greek word *syllogismos*, which means a *reckoning all together*. In Logic a syllogism may be regarded as the name applied to the expression of that form of mental activity in which three truths are brought together in one act of thought. A more technical definition will be given later on.

The syllogism is, then, a complex whole consisting of three propositions. Hence, to understand the syllogism the student must first clearly understand the nature of propositions. Now, in the above syllogism, the first two propositions are made use of to establish the truth of the third, and are therefore (as we have seen in Chapter I.), called the *premisses*.

The proposition—

Tennyson is a philosopher,

inferred from the other two, is called the *conclusion*.

The student should carefully notice that although the conclusion is, according to strict logical sequence, the third proposition of the syllogism, yet not infrequently it stands first, as for instance—

Brown is a man (conclusion)

For All heroes are men, } (premisses)
And Brown is a hero, }

Sometimes the conclusion occupies the second place in the syllogism, as—

All heroes are men—(first premiss)
Therefore Brown is a man—(conclusion)
For Brown is a hero—(second premiss)

Now we will consider more closely one of the propositions of our typical syllogism. If we examine the proposition—

Tennyson is a poet

we see that it is a complex whole consisting of three elements, namely two names ("Tennyson" and "a poet") and a verb connecting them. The names are called *terms* and the verb which connects them, with or without a negative particle, is called the *copula*. The first term in a proposition when in logical form is called the *subject*, being the name of that which forms for the moment the subject matter of the mind's thought; the second term is the *predicate*, being the name of something predicated of, or asserted to belong to, the subject.

The student must carefully note the difference between the *logical* and the *grammatical* use of the word predicate. The grammatical predicate of the above proposition is—*is a poet;* but in Logic *is* is a copula and forms no part of the predicate. The copula, in every proposition which is expressed in strict logical form, is some part of the present tense of the verb to be.

The student will notice that although each proposition of the above syllogism contains two terms, yet there are but three terms altogether, as each occurs twice. This is the case in every valid syllogism.

From the above analysis it is clear that a knowledge of terms is necessary to the proper understanding of propositions, and a knowledge of propositions to the understanding of the syllogism. Hence, in text books of Logic, terms and propositions are treated of before the syllogism.

As in every syllogism there are three terms, two only of which are contained in the conclusion, there will always be one term which does not occur in the conclusion. This is called the *middle term*. In the syllogism considered above the three terms are *poets*, *philosophers*, *Tennyson;* of these *poets* does not occur in the conclusion and is therefore the

middle term. The other two terms have also appropriate technical names. The subject of the conclusion is called the *minor term*, and the predicate of the conclusion is the *major term.* The reason for calling these terms by these names will be made clear later on.

If, then, the symbols S, M, P stand for the minor, middle, and major terms respectively, the syllogism can be represented in the form—

All M's are P,
All S's are M,

Therefore

All S's are P.

The symbols S, M, and P are used as being the initials of *subject* (of conclusion), *middle term*, and *predicate* (of conclusion). This method of representing syllogisms is frequently of use in testing their validity.

Terms, propositions, and syllogisms are properly applied to the language in which thought is expressed, and not to thought itself. It is useful to note what mental operations correspond to these parts of the language of thought.

The process by which the mind gains simple ideas, or ideas of individual objects, is called *perception* ; and the process by which it gains general ideas, or ideas of classes, is known as *conception.* A simple idea is a *percept* and a general idea is a *concept.* A term is the name of a percept or of a concept. Hence, terms may be regarded as corresponding to the mental processes of *perception* and *conception.* When the mind compares two ideas and decides whether or not such ideas agree it is said to *judge* ; the resulting product is a *judgment*, and the expression of this in words is a *proposition*, which is, therefore, the logical term corresponding to the mental operation of *judging.*

Finally, reasoning is the process by which one truth is inferred from others, the expression of such complete mental act being a syllogism whenever the reasoning is *deductive*. Hence we see that term, proposition, syllogism, are logical words corresponding to the psychological words perception and conception, judging, reasoning, respectively.

Résumé of Chapter IV.

I. Definitions.

1. A *term* is the name of an idea.
2. A *proposition* consists of two terms and a copula.
3. A *syllogism* consists of three propositions, two being premisses and one conclusion.

II. Terms in a syllogism.

a. The *Minor* term is the subject of the conclusion denoted by the symbol S.

b. The *Middle* term is the term which does not occur in the conclusion, denoted by the symbol M.

c. The *Major* term is the predicate of the conclusion, denoted by the symbol P.

III. Psychological words corresponding to term, proposition, syllogism.

1. *Perception* and *conception* supply ideas, the names of which are *terms*.
2. *Judging* results in a *judgment*, the *expression* of which is a *proposition*.
3. The expression of a complete act of deductive reasoning is a *syllogism*.

Exercises on Chapter IV.

1. Point out the *terms* and *propositions*, distinguishing premisses from conclusion in :—
   a. Socrates is mortal, for Socrates is a man, and all men are mortal.
   b. Diligent students pass their examinations, therefore A B will pass his examination, for he is a diligent student.
   c. S is P, for S is M and M is P.
2. What is the connection between terms and conception?
3. Why do text books of Logic treat of terms before propositions and of propositions before syllogisms?
4. Which is the middle term in the following syllogisms?
   a. All flowers are beautiful : therefore the rose is beautiful, for it is a flower.
   b. Logic is useful, for all science is useful, and Logic is a science.
   c. All coins are useful,
   A shilling is a coin, therefore
   a shilling is useful.
5. Give three original examples of valid syllogisms.

Also statement of general rules

CHAPTER V.

The Laws of Thought.

Before proceeding to treat of terms it will be well to consider somewhat more fully the Laws of Thought referred to in Chapter I. Underlying all valid reasoning there are certain general principles or laws which are usually referred to as "the fundamental laws of thought." They are "laws" of thought, for it is in accordance with them that all valid thought proceeds; and they are fundamental laws, for they are incapable of being derived from other simpler laws.

The fundamental Laws of Thought are three in number:—

1. The Law of Identity, *viz.*, Everything is what it is, or, A is A.
2. The Law of Contradiction, *viz.*, Nothing can both be and not be, or, A cannot both be B and not be B.
3. The Law of Excluded Middle, *viz.*, Everything must either be or not be, or, A either is or is not B.

These laws are the simplest possible expression of the principles underlying valid thought. They are so simple as to be incapable of proof (though of course they may be

illustrated) and as to make the discussion of them seem altogether unnecessary or possibly childish; and they are severally independent, that is, they are all individually incapable of being derived from the others. This must be the case, since each is a fundamental law.

The truth of the first of these laws is perfectly evident, and yet in practice it is constantly being violated. Every time that the same term in the *same* argument is allowed to have two meanings, the Law of Identity is violated. Later on we shall see how frequently this is the case. The student should further notice that this law not only includes such cases as A is A, where the identity is complete, but also such cases as A is B, where the identity is only partial, the identity existing amidst diversity. If we say apples are fruit, we do not mean that apples are identical with fruit, but that some of the qualities of apples are identical with some of the qualities of fruit.

The Law of Contradiction may, at first, appear less evident. It denies that anything can, *at the same time and same place*, both possess an attribute and not possess it, or, which is the same thing, that a statement can be both true and not true at the same time. The paper of this book may be partly white and partly not white (black); but in the same place it cannot both be white and not be white. It is inconceivable that of two contradictories both should be true. Immediate Inferences (see Chapters X., XI.) from affirmative propositions may be shown to depend, for their validity, on the Laws of Identity and of Contradiction. As the Law of Contradiction guards against the admission of two contradictories as true, that is, it preserves the condition of non-contradiction in argument, it would be better called the Law of Non-contradiction.

The Law of Excluded Middle is the least evident of the three laws. Some writers have even thought it untrue, but such thought has arisen from a misconception of its meaning. It has not been sufficiently noticed that the law is concerning *contradictories* and makes no assertion whatever about *contraries*. It asserts that A is or is not B; it does not assert that A is B or is the contrary of B. It is quite true that a piece of iron may be neither hot nor cold; but cold is the *contrary* and not the *contradictory* of hot, the true contradictory being not-hot, and it is inconceivable that a piece of iron, or anything else, should be neither hot nor not-hot—it must be one or the other.

Can we, in accordance with this law, assert that *honesty is, or is not, triangular?* Perhaps the simplest answer to this question is to be found in the fact that the Law of Excluded Middle is a law of *thought*, and has nothing to do with the meaningless, and such an assertion as the above is meaningless. But if we assert that honesty is not triangular are we necessarily to be taken to mean that honesty is not three-sided, but is of some other shape? Not necessarily so. Honesty has no shape at all and cannot therefore be triangular. Even in this case, then, the law holds good. The law has received the somewhat curious name of "excluded middle" from the fact that it asserts that any statement must either be true or not true, thus denying the possibility of the existence of a *middle course*—the possible existence of a middle course is excluded by the law.

These three laws are sometimes spoken of as the *necessary* Laws of Thought, because any one who thinks correctly, of necessity thinks in accordance with them.

The above are the only *fundamental* Laws of Thought; there is, however, another law of great importance, which

was first enunciated by Leibnitz and which is generally referred to as the "Law of Sufficient Reason." It may be stated thus:—"Nothing happens without a sufficient reason why it happens as it does, rather than otherwise." This, however, is not a law by which our thinking is regulated, but rather an assertion that thought is possible concerning any and every phenomenon. There are certain other principles, deducible from the fundamental Laws of Thought, which logicians have enunciated and have regarded as important because logical arguments can be tested by them more easily than by the fundamental Laws of Thought. They are usually spoken of as Canons or Axioms.

A *Canon* is a *rule*, and an *Axiom* is a statement which is so simple that it cannot be proved by reference to simpler statements, and which is, as soon as understood, necessarily regarded as true.

These canons are :—

1. "If two terms agree with one and the same third term, they agree with each other."
2. "If one term agrees and another disagrees with the same third term, these two disagree with each other."

Sometimes another canon is given, viz.,

3. "Two terms, both disagreeing with the same third term, may or may not agree with each other."

The student should compare Euclid's first axiom with the first of these canons. As an illustration of the first, consider the three terms *iridium*, *heaviest metal*, *rarest metal*. If *iridium* and *heaviest metal* both agree with the term *rarest metal*, then *iridium* and *heaviest metal* agree with

each other, or *iridium is the heaviest metal.* Symbolically, if A agrees with B, and C agrees with B, then A agrees with C.

As an illustration of the second canon, consider the terms *London, Metropolis, Cambridge.* The term *London* does, and the term *Cambridge* does not, agree with the same third term *Metropolis,* therefore they do not agree with each other, or, *London is not Cambridge.*

Again, consider the terms *ivory, elephants' tusks,* and *bone.* In this case the terms *ivory* and *elephants' tusks* both disagree with the term *bone,* and they do agree with each other, for *elephants' tusks* are *ivory;* but in the case of the terms, *Ely, Metropolis, Cambridge,* the first and third disagree with the second and they disagree with each other, for Ely is not Cambridge.

There is still one other rule in Logic that should be mentioned here, viz., Aristotle's *dictum de omni et nullo* (statement concerning all and none). The simplest form of this is:—"Whatever may be predicated distributively of a class may be predicated of every individual in that class, and whatever may be denied distributively of the class may be denied of every individual in it." The first part of this is the *dictum de omni* or *statement concerning all,* and the second part is the *dictum de nullo,* or *statement concerning none.* Thus, we may predicate of the class *material things* that they have weight, therefore the *dictum* says we may predicate of wood that it has weight, for it is part of the class *material things;* and of the class *material things* we may deny that they can reason, therefore we may deny of a *stone,* which is a *material thing,* that it can reason.

Résumé of Chapter V.

I. The fundamental Laws of Thought are —
   1. Law of Identity. Everything is what it is.
   2. Law of Contradiction. Nothing can both be and not be.
   3. Law of Excluded Middle. Everything must either be or not be.

II. The Law of Sufficient Reason is: — "Nothing happens without a sufficient reason why it happens so rather than otherwise."

III. Logical Canons.
   1. If two terms agree with one and the same third term, they agree with each other.
   2. If one term agrees and another does not agree with the same third term, these terms do not agree with each other.
   3. Two terms, both disagreeing with the same third term, may or may not agree with each other.

IV. Aristotle's *dictum de omni et nullo* is:—"Whatever may be predicated distributively of a class may be predicated of every individual in that class; and whatever may be denied distributively of a class may be denied of every individual in that class."

Exercises on Chapter V.

1. What is a law?
2. What is a fundamental law?
3. Enunciate and illustrate the Law of Contradiction.
4. Enunciate and illustrate the Law of Excluded Middle.

5. It has been said :—" A door may be neither open nor shut, but ajar, therefore the Law of Excluded Middle does not hold." Answer this objection.
6. Explain *canon* and *axiom*, giving examples.
7. Show, by using Aristotle's dictum, that the conclusion *Apples are useful* follows from the premisses, *apples are fruit*, and *all fruit is useful*.
8. Illustrate the first of the three logical canons.

CHAPTER VI.

Terms.

In Chapter IV. we saw that a proposition consists of two names or terms and a copula. In this and the following chapter we proceed to examine terms more closely.

The philosopher Hobbes, nearly two hundred and fifty years ago, defined a name or term in these words :—" A name is a word taken at pleasure to serve for a mark which may raise in our minds a thought like to some thought we had before, and which, being pronounced to others, may be to them a sign of what thought the speaker had or had not before in his mind." The words " or had not " may be omitted without injuring the definition. Hobbes introduced them in order to make the definition embrace the names of negative ideas, an explanation of which will be given shortly.

Note carefully the two purposes served by terms :—

1. As *marks* to recall ideas (for that is what Hobbes means by " thoughts ") which we have had previously in mind.
2. As *signs* to others of the ideas we have in mind.

The word *term* is merely a curtailed form of the Latin word *terminus*, an end, and was applied to the names of a proposition, because such names form the *ends* of the proposition. Strictly speaking, names are *terms* only when

they occur in propositions. Generally, however, all names are called terms, as being able to constitute the ends of propositions. A term may consist of any number of words. Thus in the proposition, "*Victoria, Queen of England, Ireland, Scotland and Wales, and Empress of India*, resides at Windsor," the twelve words in italics, constituting the subject of the proposition, form but one term. Clearly all words cannot stand alone as terms. We cannot say, "Very is honest". Words that can stand alone as terms are called *categorematic*; words that cannot, *syncategorematic*. If, however, a syncategorematic word be itself regarded as a thing, it can stand alone as a term. Thus we can say "Very is an adverb" meaning the word "very" is an adverb. This looking upon a word as a thing is known as *Suppositio materialis*.

Adjectives are to be classed as categorematic words, for in such a proposition as, *the day is warm*, *warm* is a term.

Since terms may consist of any number of words, they may be arranged in two classes.

1. Single-worded terms.
2. Many-worded terms.

A *single-worded term* consists of one word only, which must of course be a categorematic word. Thus, the terms in a proposition such as—

Coal is useful,

are single-worded terms.

A *many-worded term* is one consisting of more than one word, as, for instance, *the angles at the base of an isosceles triangle*.

Again terms are :—

1. Individual or Singular, and
2. General.

An *individual* or *singular* term is a name applicable in the same sense to only one thing, as *the Queen, my house.* Those names which in Grammar are known as proper nouns are logically individual terms, for though such a term as John may be applied to many different persons, yet when used it always refers definitely to one and it is incapable of being applied in the *same sense* to another. In the proposition—

He is quite a Milton,

the term Milton is the name of a class, as is clear from the fact that the article *a* goes with it; it is a *general* term and grammatically a common noun.

A *general term* is one that can be applied in the *same sense* to an indefinite number of things. Thus *pen* is a general term, for it is applicable in the same sense to an indefinite number of objects, all of which have certain common qualities which constitute the meaning of the term *pen.* Since these objects possess certain common qualities, we may regard them as forming a class which contains every object possessing these qualities. The name of this class is a general term, and every general term is a class name.

A general term is, from the psychological point of view, the name of a concept, which has been already defined as a general idea (see Chapter IV.). The above division of terms into the two classes of *individual* and *general* terms is exhaustive, that is, every term must belong to one or other of these classes. Both individual and general terms may be used in a *collective* sense, that is, as the name of a number of things regarded as forming one whole; when so used they are often called *collective terms.*

The collective use of *individual* terms is not of frequent occurrence. The *British Army, the Andes,* the *English*

Nobility, are examples. General terms are of much more frequent occurrence as collective terms; thus, *crowd*, *army*, *nation*, *mob*, *fleet*, etc., are examples of words that may be used as collective terms. The student should carefully note that it is impossible to say of any given term, such as *forest*, whether or not it is collective. It depends entirely on its use. Thus, if the word *forest* is used to denote a number of trees it is a collective term, but if used to mean *one of a number of forests* it is a general term. The collective term being the name of a number of objects, regarded as one, cannot be applied to the objects individually, but collectively only; on the other hand, the general term can be applied individually to the objects constituting the class of which it is the name.

Another division of terms is into

1. Concrete, and } terms.
2. Abstract }

A *concrete term* is the name of a thing, an *abstract term* is the name of an attribute of a thing, considered apart from the thing to which such attribute belongs. The word *thing*, it should be noticed, is used in its widest possible sense to denote every possible notion except those of attributes. There are non-material as well as material things. Thus, *mind*, *thought*, *will*, *ghost*, are names of things, although they are not material. The process by which the mind withdraws or abstracts its attention from all other qualities and fixes it on a certain one is called *abstraction*. The name of such quality contemplated by abstraction is an abstract term.

Thus, we talk of *honest people*. If we think of the quality from the possession of which they are called *honest*, and apply a name to it we have the abstract term

honesty. Adjectives are *concrete* terms, not *abstract*, for they always express some quality as belonging to an object and not considered as apart from such object; thus in the proposition "iron is heavy" it is not meant that iron is the quality denoted by the word heavy, but that it is a thing possessing such a quality; so again the proposition "Great is Diana of the Ephesians" does not mean that Diana of the Ephesians is the *quality* greatness, but that Diana is a great being or great goddess. Logically there is no distinction in kind between the term "great" in "Diana is great" and the term "goddess" in "Diana is a goddess". To every concrete term there is, or may be, a corresponding abstract term; thus *soapiness*, *inkiness*, *sweetness*, *nakedness*, are the abstract terms corresponding respectively to the concrete terms *soap*, *ink*, *sweet*, *naked*. Whether the abstract corresponding to a given concrete term exists in English or not is a mere accident of our language. If the need of the abstract has been frequently felt, someone is sure to have invented it and others to have used it; but if the need has never, or only occasionally, been felt, the corresponding abstract does not exist. Thus *orange*, *apple*, *potato*, *book*, *window*, are concrete terms to which corresponding abstract terms are not to be found in our dictionaries.

It is a fact deserving notice that an abstract term frequently, in course of time, becomes concrete. This arises from the difficulty of abstraction to the uneducated mind. Thus, *relation*, which properly denotes *the connection between relatives*, is frequently used concretely for *relative*; thus, we talk of our *friends and relations*, meaning *friends and relatives*.

To what a large extent this change of abstract to concrete terms has been carried on may be seen by noticing the

present meaning of terms whose endings indicate that originally they were abstract. It will be found that many, perhaps the majority, of such terms are now used as concrete general terms. Thus all words ending in -tion, -sion, were originally abstract; but now such words as benediction, proposition, proportion, addition, nation, and dozens of others, are generally used as concrete terms.

Another way of dividing terms is into the two classes—

1. Positive.
2. Negative.

A *positive term* is one which signifies the presence of some one or more qualities. In accordance with the Law of Excluded Middle, everything in the universe must either belong or not belong to the class denoted by any name. Thus, let the name be metal, then everything is, or is not, metal. If we frame a word to denote everything that is not metal (*non-metal*, say) then *metal* is a positive and *non-metal* the corresponding negative term. In many cases it is difficult to say which is the positive and which the negative term, and frequently it is a matter of little or no importance which is called positive, so long as the other is called negative.

The student will notice that certain prefixes are indicative of the absence of a quality, that is, of a negative term; such prefixes, for example, as *un-*, *in-*, *dis-*, *non-*, are of this nature. The form of a word is, however, often misleading; thus in form *inconvenient* is a negative term, but it seldom, if ever, denotes the mere absence of convenience, but rather implies the presence of actual trouble, and it should, therefore, be regarded as a positive term.

When a negative term implies the absence of a quality which was once present, or which might be expected to be

present, such term is called *privative*, *e.g.*, blind, dead, dumb, etc.

A positive and its corresponding negative are contradictory terms, each being the contradictory of the other. *Contradictory* terms must be carefully distinguished from what are called *contrary terms.* The *contrary* of a term is one denoting what is farthest removed from that term. Contrary terms are often called *opposite* terms. Thus *small* is the contrary or opposite of *great*, the strict contradictory of which is *not-great:* so *not-white* and *black* are the *contradictory* and *contrary* respectively of *white.* This is a distinction of great logical importance, and should, therefore, be carefully remembered.

Still one further way of dividing terms must be here noticed.

Terms are—1. Relative, or
2. Absolute.

A *relative term* is one which, besides denoting an object, implies the existence of another object without reference to which it is unintelligible; thus the term *king* can be understood only by reference to *subject*, and *subject* only by reference to *king*.

An *absolute* or *non-relative* term is one which denotes an object without any direct reference to the existence of another object, as, for instance, *sun*.

It is often impossible to say of a term apart from its context whether it is relative or absolute; thus *man* is a relative term when used in contra-distinction to *woman*, but it is absolute when meaning mankind.

Relative terms evidently go in pairs—as parent, child; king, subject; shepherd, sheep; teacher, pupil; every pair consisting of two *correlative terms.* That which constitutes

General term any 1 of whole class of object being applied in same sense to each particular object.
Collective names signify groups made of individuals
Distributives

the bond or connection between any pair of correlatives is called the *fundamentum relationis.* For instance, the *fundamentum relationis* in the case of the correlatives teacher, pupil, is the *giving of instruction* in the case of the teacher and the *receiving of such instruction* in the case of the pupil. In some cases a pair of correlatives have but one name ; thus, the correlative of *friend* is *friend*, of *foe* is *foe*, of *brother* is *brother*, though it might be *sister*.

Résumé of Chapter VI.

I. Words are—

1. Categorematic,
2. Syncategorematic.

The former can stand alone as terms, the latter cannot.

II. There are several ways of dividing terms, as into

1. *a.* Single-worded terms.
   b. Many-worded terms.

   A single-worded term is a categorematic word.
2. *a.* Individual or singular terms are those applicable to only one thing in the same sense.
   b. General terms are applicable in the same sense to every one of a class of things.
3. *a.* Terms used distributively.
   b. Terms used collectively, namely, when applied to a number of things regarded as one whole.
4. *a.* Concrete, being the names of things.
   b. Abstract, being the names of qualities.

   Adjectives are concrete. Corresponding to any given concrete an abstract may, but does not always, exist.

5. *a.* Positive terms implying the presence of qualities.
   b. Negative terms implying the absence of certain qualities.
   c. Privative terms implying the absence of qualities which might be expected to be present.
6. *a.* Relative terms. A relative term is one which is intelligible only by direct reference to another term.
   b. Absolute terms : An absolute term makes no direct reference to another term.

Exercises on Chapter VI.

In giving the logical characters of terms the student should say whether they are—

1. Single-worded or many-worded.
2. Singular or general.
3. Collective or distributive.
4. Concrete or abstract.
5. Positive, negative or privative.
6. Relative or absolute, *e.g.*—

   The Sun :—Many-worded, singular, concrete, positive, absolute.

   Blind :—Single-worded, general, concrete, privative, absolute. If there is any doubt as to the meaning of a term, it should be stated what such term is understood to mean, and its character given accordingly.

1. Give the logical characters of :—woman, king, school, church, army, honesty, moon, the moon, star, table, relation, relative, good, evil.

2. Explain the difference between contradictory and contrary terms.
3. Is the term *idle*, positive, negative, or privative? Give full reasons for your answer.
4. Give the negative of:—equal, great, London, good, tall, teacher.
5. Explain *fundamentum relationis.*
6. What is a collective term? Is the term *navy* collective?

Propositions contradictory when implies absence of quality which other implies presence of.
Contrary that things differ from one another by a wide margin.

CHAPTER VII.

Terms—(Continued).

We have now to consider an exceedingly important point with regard to terms, and one that will need the closest attention to be clearly understood.

Examine closely the meaning of any general term, such as *school*. This evidently includes, in its meaning, St. Paul's School, Westminster School, City of London School, and all buildings in the world which are rightly called "schools". All objects, then, which correctly bear the name *school*, are, in one sense, the meaning of the term *school*. They are the extent to which the term is applied, and form, therefore, what is sometimes called the *meaning in extension* of the term *school*. Similarly, every other general term has a meaning in extension, such meaning being all the objects forming the class of which the term is a name. But all general terms will be found to have another meaning. Consider further the term school. Before the term school can be applied to an object, such object must possess certain qualities: it must at least be a building in which teaching is carried on. Every general term is a name applied to all the individual objects constituting a class and any object can belong to such class only in consequence of the possession of those attributes on which the class is based. Hence, these attributes must be possessed by any object

before the class name or general term can be applied to it. The application of a general term to any object clearly, then, conveys certain information about that object, such information being, in one sense, the meaning of the term. This meaning is sometimes called *the meaning in intension*, the term *intension* being used in contra-distinction to *extension*. Thus, the meaning in extension of the term school is the number of things which are rightly called schools, and its meaning in intension is the *qualities* any object must possess before it can rightly be called a school. The meaning in extension is evidently the objects *denoted* by the term, and may, therefore, more conveniently be called the *denotation* of the term; as a correlative to this, the meaning in intension may be called *connotation*. These terms were first used by Mill, with these meanings. They are convenient, as we have the corresponding verbs to *denote*, and to *connote*, as well as the adjective *connotative*.

Moreover, their meaning is suggested by their etymology; to denote (Lat. *de*, down, and *noto*, I mark) is to mark down the objects to which the name may be applied, and to *connote* (Lat. *con*, along with, and *noto*, I mark) is to mark the qualities along with the objects.

There is a close connection between the denotation and the connotation of a term. Consider the term *book*. Now to *book* add *English*, so that the term becomes *English book*. Evidently, there are fewer *English books* than *books* in existence, and hence the denotation is decreased. By adding *English*, then, to the term *book* we obtain a new term, viz., *English book*, the denotation of which is less than that of book. The connotation of book is a certain number of qualities, all of which are connoted by the term *English book*, which also connotes the additional quality of

being printed in English. Hence, by adding English to the term book we obtain a new term, viz., *English book*, the connotation of which is greater than that of *book*.

If we make to *English book* an addition expressing another quality we shall still further increase connotation and decrease denotation. Thus, if we add *on logic*, we have the term *English book on logic*, the connotation of which is greater than that of *English book*, and the denotation is less, for clearly there are fewer *English books on logic* than there are *English books*. If the process were continued we should finally obtain a term the connotation of which would be so great that the term could be applied to only one object, which object would form the denotation of the term. *English book on logic which you are reading* is clearly such a term, for the connotation is such that it applies to only one object.

It is clear, from what has just been said, that if the denotation of a term be decreased the connotation will be increased, and that if the denotation be increased, the connotation will be decreased. This is sometimes expressed by saying that the *connotation increases as the denotation decreases*. It must, however, be noticed that the ratios between the connotation and denotation of two terms do not form a mathematical proportion. Thus, we must not suppose that the connotation of *English book* bears the same numerical relation to the connotation of *book* as the denotation of *book* bears to the denotation of *English book*. By a slight increase of connotation the denotation may be very greatly reduced. Thus, if to the term *mountain* we add *of altitude of above 29,000 feet*, the denotation is reduced to a minimum, for the term, *mountain of altitude of above 29,000 feet* denotes one

object only, viz., Mt. Everest. If, however, we add *of altitude of more than 3,000 feet*, the connotation is increased as much as in the previous case, although the denotation is decreased not nearly so much.

Now the question arises whether all terms have connotation or not. This question may be answered by the consideration of an example. If, on seeing a number of buildings in the distance, I ask what place it is, and I am told in reply that it is *Cambridge*, it is evident that such answer gives me no knowledge of the place itself. It tells me that there is a place called Cambridge, but it tells me none of the attributes of such place. I may possibly know that Cambridge is a *town* and *the seat of a University*, in which case, on hearing the term Cambridge, there instantly arises in my mind the fact that it is a town and the seat of a University. But the term *Cambridge* merely *suggests* such information by association of ideas; in no sense can such information be regarded as the *meaning* of the term. But if, in answer to my question, I am told that the place is a *town*, then the word itself does convey real information, viz., whatever is connoted by the term *town*. The term *Cambridge* has no connotation, but the term *town* has connotation. Terms possessing connotation are called *connotative* terms, and those not having connotation *non-connotative* terms. A connotative term denotes a thing and implies one or more attributes. It is clear that all general terms, all adjectives (which are general terms) and collective terms (except proper names, such as the Pyrenees) are connotative. A non-connotative term only denotes a thing or an attribute. All terms which grammatically are called proper nouns, are non-connotative. A proper noun is never the name of a class.

Of course, a proper noun may be used to designate a class, implying that every member of the class possesses the qualities characteristic of that to which such proper noun was first applied, but then in such use it is not a proper noun or individual term, but a class name or general term. In such a sentence as:—"He is quite a Shakespeare," the word Shakespeare is no more a proper noun than is the word poet in the sentence "He is quite a poet".

All *singular abstract terms* denote an attribute only and are, therefore, *non-connotative*. Abstract nouns, however, that can be used in the plural, such as, *colour*, *virtue*, *vice*, are the names of classes, that is, they are general terms and as such have connotation. The student is recommended to make himself thoroughly familiar with the terms denotation and connotation, as a thorough understanding of them is of the highest importance. Though the distinction between them is a perfectly definite and clear one, yet many students seem to find great difficulty in understanding it.

Résumé of Chapter VII.

I. 1. The denotation or meaning in extension of a term is the objects to which the term can be applied.

2. The connotation or meaning in intension is that quality or those qualities which an object must possess in order that such term may be rightly applied to it.

Denotation is *things*: *connotation* is *qualities*.

3. By increasing connotation, denotation is decreased, and by decreasing connotation, denotation is increased.

II. Terms are *connotative* or *non-connotative.*

1. A connotative term denotes a thing and implies one quality or more.
2. A non-connotative term denotes a thing only or an attribute only.
3. General terms, adjectives, and collective terms, are connotative.
4. Proper names and singular abstract terms are non-connotative.

Exercises on Chapter VII.

1. What is the denotation of a term?
2. What is the connotation of a term?
3. Give three examples of connotative and three of non-connotative terms.
4. What is the denotation of :—fish, man, book, servant, bird, table?
5. What is the connotation of :—triangle, line, book, animal?
6. Show that by increasing the connotation of the term *ship* its denotation is decreased.
7. Why have proper names no connotation?
8. Criticise the assertion that the denotation decreases as the connotation increases.

CHAPTER VIII.

Propositions.

The student has already from Chapter IV. gained some idea of the meaning of the word *proposition*; we proceed to discuss its meaning more fully. Consider the pair of terms —*sugar*, *sweet*. Do they agree? If so, we may say *sugar is sweet*. Do they not agree? If they do not we may say *sugar is not sweet*. So of the terms *silver*, *tenacious*. If we think they agree, we say *silver is tenacious*; but if we decide that they do not agree, we say *silver is not tenacious*. So again of the terms *gold*, *malleable*. We first decide as to whether or not they agree, and then we make the statement *gold is malleable* or *gold is not malleable*. What we decide in every such case is whether or not our ideas of the meanings of the two terms agree; and to express such decision we make use of what is called a *proposition*. A proposition may be defined, then, as *the assertion or the denial of agreement between two terms*.

A general term is, as already stated (Chapter VI.), the name of a *concept*. Thus, the term *tree* is to me a name of whatever is in my mind when I think of tree in general without special reference to any particular tree, that is, it is the name of my *concept* of tree.

The mental process by which we decide whether or not one concept may be regarded as connected with another

concept is what the psychologist means by *judging*, and the result of such process is a *judgment*, the verbal expression of which is a proposition. A proposition may, in fact, be defined as *the expression of a judgment*. The student will easily see that this definition is in reality the same as that given above, where a proposition is defined as the assertion or denial of agreement between two terms. It is clear, then, that every proposition is a sentence ; it is not true, however, that every sentence is a proposition. Those sentences only which unconditionally or conditionally assert facts are propositions.

Hence—

Mere exclamatory sentences Interrogative sentences Imperative sentences	are not propositions.

None of these express *thought*, which is the subject matter of Logic.

It has already (Chapter IV.) been pointed out that every proposition consists of two terms connected by a copula, which copula is always the *present* tense of the verb *to be*, affirmative or negative, and all sentences should be reduced to this form before attempting to apply any of the rules of Logic to them. Thus, the sentence, *Familiarity breeds contempt*, in the strict form of a logical proposition is, *familiarity is a thing which breeds contempt*, in which *familiarity* is the subject, *is* the copula, and *a thing which breeds contempt* the predicate. So the logical form of *birds fly* is *birds are things which fly*, the terms being, *birds* and *things which fly*. In Logic, it is always allowable to change the wording of a proposition, provided the meaning is not changed. A most important part of the usefulness of Logic as a means of mental discipline, is that which is involved

in the careful examination of common forms of expression and in the reduction of such to typical logical forms.

Propositions are of three kinds—

1. Categorical.
2. Conditional or hypothetical.
3. Disjunctive.

A proposition is (1) *categorical* when the predicate is simply, directly and unconditionally affirmed or denied of the subject, *e.g.*, *gold is yellow*, *silver is useful*, *rain is falling*.

(2). A conditional proposition is one in which the predicate is affirmed or denied of the subject under one or more conditions, *e.g.*, *if this is gold it is yellow; if he works hard he will pass his examination; if the wind is in the east it is cold.*

A proposition is (3) *disjunctive* when it asserts or denies the truth of some one of two or more facts but does not definitely assert or deny the truth of any given one; *e.g.*, *he is either a Spaniard or an Italian; the prisoner will be found guilty or he will be acquitted.*

Nothing further need be said at present on the subject of conditional and disjunctive propositions; we shall, however, discuss them more fully later on. For some time our whole attention will be given to categorical propositions.

Can categorical propositions be classified? The most evident way of classifying them is to arrange them in two classes according to the absence or presence of "not" or its equivalent in the copula. We have then—

I. *Affirmative propositions*, or those in which the copula does not include a negative particle, *e.g.*, *the man is honest.*

II. *Negative propositions*, or those in which the copula does include a negative particle, *e.g.*, *the man is not honest.*

This is a classification according to *quality*. By the Law of Excluded Middle every proposition must be of *affirmative* or of *negative* quality. All propositions that affirm the agreement of two terms are of affirmative quality, and all that deny such agreement are of negative quality.

Propositions may also be arranged in two classes by considering their *quantity*. The *quantity* of propositions depends upon whether the predicate is explicitly affirmed or denied of the whole of the subject or not.

Propositions in which the predicate is affirmed or denied definitely of the whole of the subject are said to be of *universal quantity*, or more shortly, *universal;* propositions in which the predicate is affirmed or denied of some indefinite part of the subject are said to be of *particular quantity*, or more briefly, *particular*. Thus, *All Englishmen are brave* is a universal proposition, because "bravery" is affirmed explicitly of the whole denotation of the subject; but in *some Englishmen are brave*, bravery is only affirmed of some indefinite part of the subject, and the proposition is, therefore, particular. If we combine this classification according to *quantity* with the classification according to *quality*, we shall have four classes of propositions—

I.	Universal.	1.	Affirmative.	Symbol	A.
	,,	2.	Negative.	,,	E.
II.	Particular.	3.	Affirmative.	,,	I.
	,,	4.	Negative.	,,	O.

These may be typically represented thus :—

Universal Affirmative,	All S's are P,	A
Universal Negative,	No S's are P,	E
Particular Affirmative,	Some S's are P,	I
Particular Negative,	Some S's are not P,	O

The letters A, E, I, O, are used for the sake of brevity as symbols of the four classes of propositions, respectively, after which they are placed. By an A proposition is meant a *universal affirmative*, by an E proposition a *universal negative*, by an I proposition a *particular affirmative*, and by an O proposition a *particular negative*. These symbols are very convenient and are constantly used in Logic; the student should, therefore, learn them thoroughly at once. As a help to remembering them it may be noticed that A and I, the symbols for affirmative propositions, are the first two vowels in *affirmo*, the Latin word for *I affirm;* and that E and O are the vowels in *nego*, the Latin word for *I deny*.

The four classes of propositions, A, E, I, O, may conveniently be symbolised by SaP, SeP, SiP, SoP, respectively, the vowel of each of these combinations denoting the quality and quantity of the proposition symbolised.

Hence the proposition :—

All S's are P	is represented by	either	A or SaP
No S's are P	,,	,,	E or SeP
Some S's are P	,,	,,	I or SiP
and Some S's are not P	,,	,,	O or SoP

This fourfold classification of propositions is exhaustive, that is, every categorical proposition belongs to one or other of the four classes. It is not, however, always easy to say at once to which class a proposition belongs; thus, in the case of the proposition, *Socrates is mortal*, it is not at once evident whether this is SaP or SiP. In this case and in all similar cases, the predication is made of the whole denotation of the subject, although that denotation extends to but one individual, and, therefore, it must be SaP.

Singular propositions, that is propositions having singular terms for their subjects, are always *universal*.

Some propositions do not by their form enable us to tell whether they are universal or particular, *e.g.*, *metals are heavy*. Such propositions are said to be *indefinite* or *indesignate*. Such propositions must be classified according to the meaning the students think they are intended to express. If the above is intended to mean *all metals are heavy*, it is of course SaP; if not, it is SiP. Until the meaning of indefinite propositions be settled they have no place in Logic.

It is most important to notice that in I and O propositions the *some* is altogether indefinite. Thus, when it is asserted that *some boys are clever*, it must not be taken to mean that some boys are and some are not clever. Cleverness is predicated of *some* boys, an *indefinite some*; of how many such predication is made it is altogether doubtful; it must be of one at least, and it may be of all.

One other way of classifying categorical propositions should be noticed, viz., into the classes *explicative* propositions and *ampliative* propositions. *Explicative* or *essential* propositions merely unfold the meaning of the subject term. They convey no knowledge beyond what is contained in the subject term if such term be understood.

A triangle is a figure bounded by three lines is an *explicative* proposition, as it merely unfolds the meaning of the term *triangle*. All definitions are explicative propositions. All propositions that really supply knowledge besides that contained in the meaning of the subject are called *ampliative* propositions, because they *amplify* or enlarge our knowledge. Thus the proposition, *roses are beautiful*, predicates of roses the quality (beauty), which is no part of the connotation of *rose*, and is therefore an ampliative proposition. This division of propositions is of historical interest rather than of logical importance.

Résumé of Chapter VIII.

I. Propositions.

1. Definition:—A proposition is the assertion or the denial of agreement between two terms; or, A proposition is the expression of a judgment.
2. Kinds of propositions.
   - *a.* *Categorical*, which simply and definitely assert something of the subject.
   - *b.* *Conditional or hypothetical*, which make an assertion under certain conditions.
   - *c.* *Disjunctive*, which affirm that one of two or more alternatives is true.

II. Categorical propositions.

1. The *quality* is determined by the absence or presence of the negative particle.
2. By the *quantity* of a proposition is meant whether it is *universal* or *particular*.
3. The classification of categorical propositions is based on the *quantity* and *quality*.
   - *a.* There are four classes—
     - *A.* Universal affirmative. All S's are P. SaP.
     - *E.* Universal negative. No S's are P. SeP.
     - *I.* Particular affirmative. Some S's are P. SiP.
     - *O.* Particular negative. Some S's are not P. SoP.
   - *b.* *Singular* propositions are classed as *universals*.
   - *c.* Indefinite propositions, while indefinite, do not belong to Logic.
4. "Some" in Logic has an indefinite meaning—one at least, it may be more, even *all*.

III. Categorical propositions may be classed as—

1. *Explicative* propositions, which merely unfold the meaning of the subject; and
2. *Ampliative* propositions, which convey information over and above what is contained in the meaning of the subject term.

Exercises on Chapter VIII.

1. Which of these propositions belong to Logic? Why?
   a. Go thou, and do likewise.
   b. Oh! that I had listened to you!
   c. The greater the knowledge, the greater the modesty.
   d. Shall I see you to-morrow?
   e. Cleanliness is next to godliness.
   f. To thine own self be true.
2. Analyse the following into subject, copula, and predicate.
   a. London is a large city.
   b. Hope springs eternal in the human breast.
   c. A thing of beauty is a joy for ever.
   d. Its loveliness increases.
   e. Great is Diana of the Ephesians.
   f. P struck Q.
3. Classify the following propositions according to quantity and quality, *i.e.*, say whether they are A, E, I or O.
   a. All metals are heavy.
   b. No money is useless.
   c. Some birds do not fly.
   d. Money is useful.
   e. Honesty is the best policy.
   f. London is a large city.
   g. Man is a rational animal.

4. Classify the propositions in question 3 into explicative and ampliative propositions.
5. Explain the meaning of *some* in *some men are rogues.*
6. Why is E written in the form *no S's are P*, and not in the form *all S's are not P?*
7. Express the following in strict logical form, adding the symbol to each:—

a. Men form theories.
b. Women jump to conclusions.
c. Teaching is wearying work.
d. Birds are not quadrupeds.
e. Anglo-Saxon is Old English.
f. Fish swim.
g. Man is mortal.
h. P struck Q.
i. Logic is difficult.
j. Shakespeare is many-sided.
k. It is better late than never.
l. I wish you many happy returns of the day.

CHAPTER IX.

Propositions and Distribution of Terms.

In the last chapter we have seen that a proposition is the assertion or the denial of an agreement between two terms; but terms have connotation and denotation. Is it the connotation or the denotation of the terms in a proposition that is most naturally suggested when the proposition is used? To answer this question is to give the *import* of propositions. The proposition—

Logic is useful

clearly affirms that *utility* is an *attribute* of Logic. The student will have no difficulty in understanding this, as the predicate is an adjective; it is equally true when the predicate is a noun. Thus if we say—

An apple is a fruit,

our meaning is, that whatever *attributes* belong to fruit in general belong to an apple; the predicate, although a noun, is really thought of as an adjective, but the subject of the proposition is regarded as a noun denoting a thing. That the predicate is really thought of as an adjective, even although it is grammatically a noun, may be seen from the fact that frequently the noun may be replaced by an adjective, without any material change of meaning.

Thus we may say—

Men are mortals or *men are mortal.*

Hence, in using a proposition, *things* are suggested to the mind by the subject, and *attributes* by the predicate. Logicians usually express this by saying that the *subject* of a proposition is read in *denotation*, but the predicate in *connotation*. The predicate in any categorical proposition expresses the qualities which we affirm or deny of some or all of the subject. Thus, in *Man is mortal*, the attribute *mortality* is predicated of all included in the class man.

Psychologically, then, the predicate of a proposition is to be regarded as an adjective; it is, however, for logical purposes, often convenient to regard the predicate as a class to which the subject class bears a certain relation, or one of a number of possible relations. This is of course always possible although it is not the most natural way of regarding the predicate of a proposition, as we have just seen. In the A proposition, all S's are P, it is asserted that the attributes which form the connotation of P belong to all the class denoted by S; hence, in such a statement, the whole denotation of S is contemplated, or the assertion is intended to apply to every S. When a term is thus referred to in its whole extent, it is said to be *distributed;* but when the whole denotation of a term is not explicitly referred to, the term is said to be *undistributed.*

The distribution of terms is a matter of some difficulty. The student should give his best attention to everything on the subject in this chapter.

From what is said above it is clear that the subject of an A proposition is *distributed*; in fact, it is because its subject is distributed that it is called *universal.*

We have seen that psychologically the predicate of a proposition is an adjective; for the application and illustration of logical rules, it must, however, be regarded as

a class having denotation as well as connotation. From this point of view, we may consider whether the predicate of a proposition is distributed or not. Is the predicate of SaP distributed? Clearly it is not. In the proposition—

All owls are birds,

the class birds is not referred to in its whole denotation, for there are many birds besides owls. In the proposition—

All boys and girls are children,

the predicate, *children*, as a matter of fact, is used in its whole denotation, for there are no *children* other than *boys and girls*. The question is, however, do we know this fact from the proposition itself? The proposition, clearly, no more tells us that *all children* are referred to than the proposition *all owls are birds* tells us that *all birds* are referred to, and, therefore, the predicate is no more distributed in the one case than in the other.

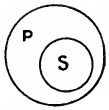
This may be illustrated by the diagrams which were first used by Euler, a German mathematician and logician. If we regard both subject and predicate as class names we may represent the subject class by one circle, and the predicate class by another.

Thus (S) may be regarded as a symbol for the whole denotation of the subject, and (P) as a symbol for the denotation of the predicate.

There are five possible relations between the two classes (S) and (P)

1. The case in which (S) is wholly included in

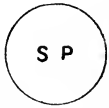
but is not the whole of 

. This case may be represented by 

.

2. The case in which

and

coincide. The symbol for this is 

.

3. The case in which

and

partly coincide, part of 

being outside of 

and 

being partly outside of 

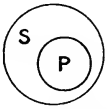
. The symbol is 

.

4. The case in which

is wholly included in 

but is not the whole of 

; the symbol is 

; and

5. The case in which 

is entirely outside of 

; this may be represented thus, 

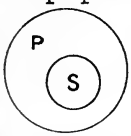
.

These are the only possible relations between the two classes 

and 

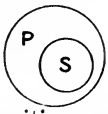
.

Every proposition, then, affirms some relation between subject and predicate which relation can be represented by some one, or more, of the above five diagrams. Now we are in a position to use these symbols to illustrate the A proposition.

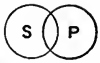
The proposition, *all owls are birds*, may be represented by 

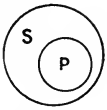
, where S stands for *owls*, and P for birds. This diagram shows at once that there are some P's which are not S's, or some *birds* which are not *owls*.

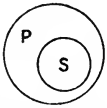
The proposition, *all boys and girls are children*, may be represented by 

, for subject and predicate in this case coincide. Now, the form of the proposition does not tell us which of these two diagrams should be used to represent it. Hence, to represent SaP fully, we must use both. The universal affirmative proposition then, must be represented by 

In the I proposition, some S's are P, the subject *some S's* is, as we have already seen, quite indefinite, the only thing definitely known of its extent is that it does not explicitly refer to the whole denotation of the class S. Hence, the subject of SiP is *undistributed*. The predicate also clearly is *undistributed*, for when we say *some apples are good to eat*, we certainly do not mean to assert that the only *things good to eat* are *apples*, there being many other edible things to which the predicate makes no reference. The following are all examples of I propositions, and are clearly represented by the diagrams as given to each.

1. Some fruits are edible 

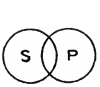
2. Some elements are metals 

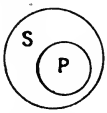
3. Some birds are bipeds 

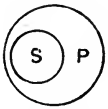
4. Some equilateral triangles are equiangular 

It will be noticed that the symbols affixed to 3 and 4 are the same as those used above to represent the A proposition. It is clear that if the relation expressed by SaP is true, the relation expressed by SiP must be true. If *all owls are birds*, it must also be true that *some owls are birds*. We shall return to this point in the following chapter.

Now, the form of the proposition is the same in each of the above four cases. Hence, the form of the proposition does not enable us to say which of the diagrams should be used to represent any given I proposition. This is only the same thing as saying that the meaning of *some* is altogether indefinite. Hence, SiP must be represented by







The first is the most typical symbol and is sometimes used alone, though inaccurately, to represent the I proposition.

In the E proposition, *no S's are P*, the attributes implied in P are denied of *all S's;* hence the subject is distributed.

This proposition may be represented by two separate circles 

, 

, representing that no part of the class S is the class P, and clearly showing also that no part of P is in S. Hence, the whole denotation of P is as explicitly referred to as is that of S. The predicate of SeP is, therefore, distributed as well as the subject. Note that only one symbol is needed to represent SeP; the reason of this is that both S and P are definitely referred to in their whole extent, that is, are *distributed.*

In the O proposition, *some S's are not P*, the subject clearly is not explicitly referred to in its whole extent and is, therefore, *not distributed.* That the subject of SiP and SoP is undistributed follows immediately from their definition as particular propositions.

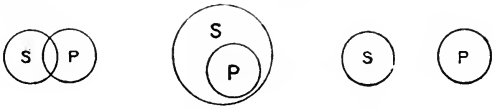
Is the predicate of SoP distributed? This is the most difficult point in connection with the distribution of terms and requires the student's closest attention. In the proposition—

Some elements are not metals,

the subject *some elements* is affirmed to be altogether excluded from the class *metals*; of *all* metals it is denied that any agree with the *some elements* of the subject. Hence the predicate of O is *distributed.*

Owing to the indefinite meaning of the word *some*, it may easily be seen that SoP does not definitely affirm any one of the five possible relations between subject and predicate, but merely affirms that of three of such relations one at least is true. The relations between S and P that the proposition, *some S's are not P*, may include are (3) (4) and (5) as enumerated on page 58.

Hence to represent SoP by Euler's diagrams we must use the three—



The proposition itself always leaves it doubtful as to which of these three diagrams would represent the actual relation between S and P ; this is owing to the indefiniteness of the *some* of the subject. These diagrams make it evident at once that the whole of P is excluded from at least a part of S—another way of seeing that the predicate of SoP is *distributed.*

Collecting the results of our examination of the quantity of the subject and predicate in each of the four classes of propositions, we have—

	Subject.	Predicate.
A or SaP	Distributed	Undistributed
E or SeP	Distributed	Distributed
I or SiP	Undistributed	Undistributed
O or SoP	Undistributed	Distributed

It will be seen at a glance that—

1. Universal propositions distribute their subjects.
2. Particular propositions have undistributed subjects.
3. Affirmative propositions have undistributed predicates.
4. Negative propositions distribute their predicates.

As much of the doctrine of the syllogism depends on the distribution of terms, it is of the greatest importance that the student should make himself thoroughly familiar with the results of this chapter.

The Mnemonic word *Asebinop** may help some students to remember which terms are distributed and which not. In this there are four vowels—A, E, I, and O, which are the symbols of the four classes of categorical propositions.

A is followed by *s*, which stands for *subject.*

E is followed by *b*, which stands for *both* subject and predicate.

I is followed by *n*, which stands for *neither* subject nor predicate.

O is followed by *p*, which stands for *predicate.*

Hence, *Asebinop* may help the student to remember that

A distributes its subject and not its predicate ; that

E distributes both subject and predicate ; that

I distributes neither subject nor predicate ; and that

O distributes its predicate.

Résumé of Chapter IX.

I. In a proposition the *subject* is read in *denotation*, the *predicate* in *connotation.*

II. *Distribution*—

1. A term is *distributed* when explicitly referred to in its whole extent.
2. A term is *undistributed* when the proposition does not make explicit reference to its whole denotation.
3. *a.* SaP distributes its *subject.*
   b. SeP distributes both *subject and predicate.*
   c. SiP distributes *neither subject nor predicate.*
   d. SoP distributes *predicate.*
4. Mnemonic for helping to remember the facts of distribution, AsEbInOp.

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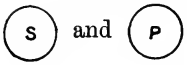
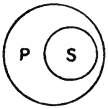
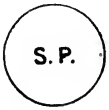
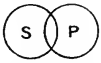
* Taken from *Picture Logic*, by Alfred Swinburne, M.A.

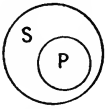
III. Euler's diagrams represent classes by circles.

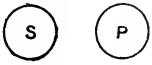
1. Five possible relations between the two classes  and which are represented thus:—

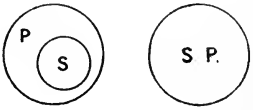
a. 

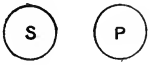
b. 

c. 

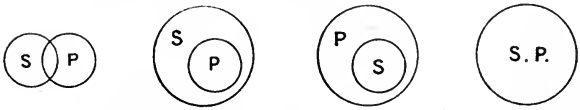
d. 

e. 

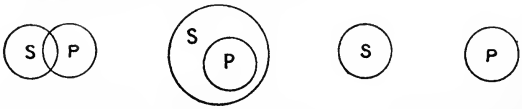
2. *a.* SaP is represented by 

b. SeP is represented by 

c. SiP is represented by



d. SoP is represented by



Questions on Chapter IX.

1. Explain the statement—"The subject of a proposition is read in denotation and the predicate in connotation."
2. What is meant by saying a term is *distributed?*
3. Show that negative propositions distribute their predicates.
4. Which terms are distributed in the following propositions. (First see that they are in logical form, then classify as A, E, I, O.)
   a. Some apples are sweet.
   b. Many students are clever.
   c. Nearly all clever and industrious students make good progress.
   d. These be none of Beauty's daughters.
   e. Honesty is the best policy.
   f. Virgil guided Dante.
5. Illustrate by Euler's diagrams all the possible relations between two classes, S and P.
6. Represent by Euler's diagrams :—
   a. Feathers are light.
   b. Glass is transparent.
   c. The penny post is a useful institution.
   d. Not all Englishmen are honest.
   e. Vain pomp and glory of this world, I hate ye.
   f. All that glitters is not gold.
8. Point out any ambiguities in :—
   a. Few birds do not fly.
   b. Every kind of fruit is not edible.
   c. All the angles of a triangle are equal to two right angles.
   d. All is not lost.

CHAPTER X.

Immediate Inference.—Opposition.

We have already seen (Chapter II.) that when a conclusion is drawn from a single premiss, the reasoning or inference is said to be *immediate*. Immediate inference makes explicit what is implicit in a single proposition. It should be at once carefully noticed that it is impossible by any form of immediate inference to evolve from a proposition what such proposition does not contain; immediate inference enables us to make clear what is involved in a given proposition.

There are two kinds of Immediate Inference, viz.—

1. Immediate Inference by *Opposition*; and
2. Immediate Inference by *Eduction*.

In this chapter we shall consider the former of these. *Opposition* is the technical term used to denote the relation existing between any two propositions which have the same subject and predicate but differ in quality, or in quantity, or in both. It should be carefully noticed that this is a somewhat different meaning from that which the word *opposition* has in ordinary discourse. Thus, technically SaP and SiP are *opposed*, though there is no incompatibility between them.

It is clear from the above definition of opposition that any one of the four propositions, SaP, SeP, SiP, SoP, is

opposed to all the others. *Immediate inference by Opposition* is an inference, from a given proposition, as to the truth or falsity of another proposition having the same subject and predicate as the given proposition. Hence the problem to be solved by *opposition* is:—granted the truth or falsity of any one of the propositions SaP, SeP, SiP, SoP, what can we know of the truth or falsity of the rest?

The opposition between
1. A and E is called *contrary*.
2. A and O, or E and I is called *contradictory*.
3. A and I, or E and O is called *subaltern*.
4. I and O is called *sub-contrary*.

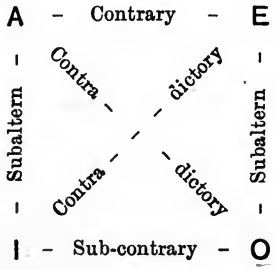
Contrary opposition, then, is the relation between two universals that differ in quality.

Sub-contrary opposition is the relation existing between two particular propositions that differ in quality.

Subaltern opposition is the relation between two propositions that differ in quantity only.

Contradictory opposition is the relation between two propositions that differ both in quality and quantity.

These various kinds of opposition can be most easily remembered by the help of the diagram known as the *square of opposition*.



This shows at a glance the name of the opposition between any one proposition and all the rest. The most important kind of opposition is the *contradictory*. What is its exact nature? What is, for example, the nature of the relation existing between the propositions—

All metals are heavy and *Some metals are not heavy.*

Clearly, if the first be true, the second cannot possibly be true, or we should have a violation of the Law of Contradiction, for in that case certain metals would be heavy and would not be heavy. Similarly, if the second be true, the first cannot be true. Hence, of two contradictories, both cannot be *true*. Can they both be *false?* Clearly not. If *all metals are heavy* is not true, then there are at least some metals of which heaviness cannot be predicated, this being the only conceivable circumstance that can make the statement, *all metals are heavy*, a false one. Therefore, if the first is false, the second is true. And similarly, if the second is false, the first is true. Hence, of two contradictories, both cannot be false. But to say of contradictories that both cannot be true and both cannot be false, is the same as saying that *one must be true and one false.* This is the characteristic of contradictory opposition, which might in fact be defined as the relation existing between two propositions, one of which must be true and one false. To *contradict* a statement is to say that such statement is *not altogether* true; if we go beyond this, and declare that the statement is *altogether false*, we are asserting the *contrary.* To disprove a universal proposition we need only establish the *contradictory*, which can be done by adducing one single instance which does not agree with the general assertion.

Thus, the proposition, *all birds fly*, may be *disproved* or *contradicted* by adducing the single case of the ostrich,

which does not fly ; for such disproof, there is no need to rush to the *contrary assertion* that *no birds fly*.

The student will easily see that of two contraries both may be *false*, but both cannot be *true*. Thus, *all birds fly*, and *no birds fly*, clearly cannot both be true, but both are, in fact, false.

If we consider the relation between I and O we see that they are in a sense contrary, for I and O differ in quality only as do A and E, but they clearly are not so contrary as are A and E, for these are *inconsistent*, whereas I and O are not. Thus

All swans are white, and *No swans are white*,

are utterly inconsistent, but *some swans are white* and *some swans are not white*, may be, and in fact are, both true. Of two sub-contrary propositions, then, both may be *true*. Can both be *false ?* Since the *contradictory* of I is E, it follows from what has been said above, that if I is false, E is true, and, clearly, if E is true, O must be true. If, then, I is false, O must be true, and similarly, if O is false, I must be true. Hence, I and O cannot both be false, but can both be true.

Of two propositions between which there is *subaltern* opposition, one, the *particular*, is called the *subaltern*, and the other, the universal, is called the *subalternant*. It is evident that the truth of the subaltern is contained in, and may be inferred from, that of the subalternant, but the truth of the subalternant cannot be inferred from that of the subaltern. Thus, if it is true that *all men are mortal*, it is evidently true that *some men are mortal ;* but, granted the truth of the latter, we should not be justified in inferring from it the truth of the former. From the proposition, *some metals float on water*, we cannot validly conclude that *all metals float on water*.

Granted the truth of SaP what immediate inferences by opposition can be made concerning the truth or falsity of SeP, SiP, and SoP?

1. The contradictory of SaP is SoP, and since two contradictories cannot both be true, from the truth of SaP we can infer the falsity of SoP.
2. The falsity of SeP, which is the contrary of SaP, may also be inferred, for two contraries cannot both be *true*.
3. The truth of SiP may be inferred, for SiP is the subaltern of SaP, and its truth is, therefore, contained in the truth of SaP.

Granted that SaP is false, we can infer—

1. The truth of its contradictory, SoP;
2. Nothing of the truth or falsity of its subaltern, SiP;
3. Nothing of the truth or falsity of its contrary, SeP.

Granted the truth of O there can be inferred—

1. The falsity of its contradictory, SaP.
2. Nothing of its sub-contrary, SiP.
3. Nothing of its subalternant, SeP.

Similarly, if SoP is false it follows—

1. That its contradictory, SaP, is true.
2. That its sub-contrary, SiP, is true.
3. That its subalternant, SeP, is false.

In the same way the student can easily obtain the inferences capable of being made by opposition from the *truth* and from the *falsity* of E and I.

The results may be collected in tabular form—

IF	A	E	I	O
SaP is true ...	t.	f.	t.	f.
SaP is false ...	f.	?	?	t.
SeP is true ...	f.	t.	f.	t.
SeP is false ...	?	f.	t.	?
SiP is true ...	?	f.	t.	?
SiP is false ...	f.	t.	f.	t.
SoP is true ...	f.	?	?	t.
SoP is false ...	t.	f.	t.	f.

In this table t, f, ?, stand for *true*, *false*, *doubtful*, respectively.

Résumé of Chapter X.

I. Immediate inference may be—
   1. By opposition or
   2. By eduction (explained in Chapter XI.).

II. *Opposition* is the relation existing between two propositions having the same subject and predicate, but differing in quality, or quantity, or both.

There are four kinds of opposition—
   1. *Contradictory*, viz., that between A and O, or E and I.
   2. *Contrary*, viz., that between A and E.
   3. *Sub-contrary*, viz., that between I and O.
   4. *Subaltern*, viz., that between A and I or E and O.

III. Immediate inference by opposition is an inference from the truth or falsity of a given proposition as to the truth or falsity of an opposed proposition. All such immediate inferences follow from the following--

1. Of two *contradictories*, one must be true and one false.
2. Of two *contraries*, both cannot be true, but both may be false.
3. Of two *sub-contraries*, both cannot be false, but both may be true.
4. The truth of the *subalternant* involves the truth of the subaltern.

IV. Contradiction is the most important logical opposition.

Exercises on Chapter X.

1. What is meant by saying that the contradictory of a proposition is an assertion that such proposition is not altogether true, and that the contrary of a proposition is an assertion that it is altogether false?
2. *a*. If I is true, what can be inferred of the truth or falsity of A, E, O?
   b. If I is false, what can be inferred of the truth or falsity of A, E, O?
   c. If O is true, what is inferrible of the truth or falsity of A, E, I?
   d. If O is false, what is inferrible of the truth or falsity of A, E, I?

3. Give the contradictories of—
   a. None but the brave deserve the fair.
   b. All quadrupeds are animals.
   c. Some stars are not suns.
   d. None of the guests have arrived.
   e. Some animals are tame.
   f. A stitch in time saves nine.
4. Give the contrary or sub-contrary of all the propositions in Question 3, and say which you give.
5. Give the subaltern or subalternant of all the propositions in Question 3, and say which you give.
6. What can be inferred by opposition from the following?
   a. All birds do not fly.
   b. Some birds do fly.
   c. All birds fly.
   d. No birds fly.

CHAPTER XI.

Immediate Inference—Eduction.

Eduction is immediate inference, from a given premiss, to the truth of another proposition in which the subject and predicate are not the same respectively as the subject and predicate of the premiss. If we have as a given premiss—

All men are mortal,

by *eduction* we may infer that—

No men are immortal ;

We can also make the inference—

Some men are mortal ;

but as the subject, *men*, and the predicate, *mortal*, are unchanged, the inference is, in this case, obtained by opposition—not by eduction.

There are various kinds of eduction, three of the most important of which, viz., *conversion*, *obversion*, and *contraposition*, we proceed to explain.

Conversion is the interchange of the subject and predicate of a proposition, such conversion being logical only when the resulting new proposition is a *valid* inference from the original proposition. The proposition which is converted is the *convertend*, the new proposition obtained by conversion is the *converse*. It is evident that in conversion both subject and predicate have to be regarded from the point of view of *denotation* ; hence, it is clearer to write P's rather

than P in the symbolic form of propositions. Thus A will, in treating of conversion, be written *all S's are P's*, rather than *all S's are P*.

Now consider the conversion of—

All S's are P's.

The converse is—

All P's are S's.

Is this the *logical* converse? Clearly not, for no valid immediate inference can give knowledge which is not implied in the premiss, and *all P's are S's* does give such knowledge, for it predicates a fact definitely of the whole of P, and the convertend, *all S's are P's*, does not do this, for the predicate is, as we have already seen, undistributed. This objection to the validity of—

All P's are S's,

as an inference from—

All S's are P's,

does not apply if we write the converse in such a form as to make the quantity of P indefinite, as it is in the convertend. We have, then, as a valid converse, the proposition—

Some P's are S's.

Two points may be noticed about this converse. In the first place, it is of the same quality (affirmative) as the convertend; secondly, the term *P's*, which is undistributed in the convertend, is undistributed in the converse.

These are the two essential points to be noticed in any case of conversion. In fact, they may be given as the two rules for conversion—

Rule I. The quality of the proposition must not be changed.

Rule II. No term may be distributed in the converse unless it is distributed in the convertend.

We may use these rules for testing the converses of E, I and O. Is *No P's are S's* the logical converse of *No S's are P's?*

Rule I. is observed, for the quality is not changed, converse and convertend being both negative. Nor is Rule II. violated, for both terms are distributed in the convertend as well as in the converse.

By applying the rules the student can easily prove that the logical converse of—

Some S's are P's, is, Some P's are S's.

The conversion of SoP presents a difficulty. If we simply interchange subject and predicate of Some S's are not P's we have as the converse Some P's are not S's. This clearly does not violate Rule I., but it does violate Rule II., for the term *S's* is distributed in the converse, being the predicate of a negative proposition, and it is undistributed in the convertend. It is, therefore, invalid. Nor can this violation be avoided, except by so altering the process that it ceases to be conversion at all. Hence, SoP has no logical converse.

These results should be remembered—

All S's are P's, or SaP, converts to *Some P's are S's, or SiP.*
No S's are P's, or SeP, converts to *No P's are S's, or SeP.*
Some S's are P's, or SiP, converts to *Some P's are S's, or SiP.*
Some S's are not P's, or SoP, is inconvertible.

In the case of E and I the converse is E and I, that is, is of the same form as the convertend, propositions being of the same form when they are of the same quality and quantity. Hence, if the converse be re-converted we obtain the convertend. Thus—

No S's are P's has as its converse
No P's are S's, and the converse of this is
No S's are P's, or the original convertend.

When this is the case the conversion is said to be *simple.*

In converting SaP we obtain as converse SiP. In this case, then, the converse is not of the same form as the convertend. If we again convert the converse we do not obtain the original convertend. The conversion in this case is said to be *per accidens*, or by *limitation.*

It should be carefully noticed that the logical converse of *all S's are P's* is always *some P's are S's*, and that this is the case even if we know that the *simple* converse *all P's are S's* is, as a matter of fact, true. Such converse is never *valid.* Thus, the logical converse of—

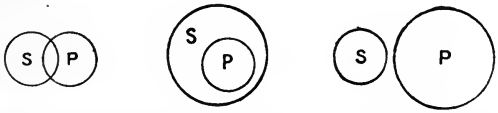
All equilateral triangles are equiangular, is
Some equiangular triangles are equilateral.

If we were to convert simply, we obtain as the converse—

All equiangular triangles are equilateral,

which is, as a matter of fact, true, but is logically invalid, as it cannot be inferred merely from the convertend.

The student should illustrate each case of conversion by means of Euler's diagrams. We will here illustrate the inconvertibility of SoP. SoP is represented by the three diagrams—



Now it is evident, to be valid, a converse must not be inconsistent with any possible case included in the convertend. If we convert SoP simply, we have *some P's are not S's*, which clearly is inconsistent with the second of the three diagrams representing SoP, and must, therefore, be an invalid inference. So any other form in which such converse can be written *as a converse* may be shown to be invalid.

Conversion is by far the most important form of eduction, and the student should make himself thoroughly understand it.

We proceed to *obversion*.

"Obversion is a process of immediate inference, in which from a given proposition we infer another, having for its predicate the contradictory of the predicate of the original proposition." (Keynes' *Logic*).

Consider the proposition—

All S's are P's.

To *obvert*, we must, according to the above definition, obtain a new proposition which shall have *non-P's* for its predicate, and which shall be inferrible from *all S's are P's*.

Clearly *no S's are non-P's* is such a proposition, which may be called the *obverse* of the original proposition, which is the *obvertend*. Three points should be noticed in this example.

1. The predicate is negatived.
2. The quality is changed, the obverse being negative, and the obvertend affirmative, and
3. The quantity is unchanged, both obverse and obvertend being universal. Hence, to obvert, we have merely to *negative the predicate, change the quality, and leave the quantity unchanged.* By applying this rule, we readily obtain the results shown in this table :—

Original Proposition or Obvertend.	Obverse.
A. All S's are P's.	E. No S's are non-P's.
E. No S's are P's.	A. All S's are non-P's.
I. Some S's are P's.	O. Some S's are not non-P's.
O. Some S's are not P's.	I. Some S's are non-P's.

The following are concrete examples of obversion:

A.	All birds are bipeds.	Obvertend.
E.	No birds are non-bipeds.	Obverse.
E.	No Europeans are Hottentots.	Obvertend.
A.	All Europeans are non-Hottentots.	Obverse.
I.	Some apples are sweet.	Obvertend.
O.	Some apples are not non-sweet.	Obverse.
O.	Some men are not honest.	Obvertend.
I.	Some men are non-honest.	Obverse.

Care is needed in negativing the predicate. If to the predicate of the obvertend there is an exact negative this can be used; thus, since *immortal* is the negative of *mortal*, we give—

No men are immortal,

as the obverse of—

All men are mortal.

More frequently than not, however, there is no exact negative term of the predicate of the obvertend, and whenever this is the case the negative should be formed by prefixing non- to the predicate term of the obvertend. Thus we must write the obverse of—

All apples are sweet, in the form

No apples are non-sweet.

The student might at first think that the obverse is—

No apples are sour,

reference to the definition of obversion as given above will, however, make it clear that it is not, for *sour* is not the *negative* but the *opposite* of sweet.

The last form of eduction that we have to consider is, *Contraposition.* Dr. Keynes defines contraposition as "A process of immediate inference in which, from a given

proposition, we infer another proposition, having the contradictory of the original predicate for its subject, and the original subject for its predicate."

If we *obvert* the proposition—

All S's are P's, we obtain

No S's are non-P's, as its obverse.

Now by *converting* this obverse we have—

No non-P's are S's,

and this is a proposition obtained by inference from—

All S's are P's,

and which has for its subject the contradictory of the predicate of the original proposition, and for its predicate the subject of the original proposition. Hence, according to the above definition, the whole inference must be a case of contraposition. The resulting proposition may be called the *contrapositive* of the original proposition, which *contrapositive* may, indeed, be defined as the *converse* of the *obverse*. The rule for contrapositing, then, is plain, *viz.*, *obvert and convert the obverse.*

The contrapositive of All S's are P's is, as we have just seen—

No non-P's are S's.

What is the contrapositive of E, *No S's are P's*?

The *obverse* is All S's are non-P's, and the *converse* of this is—

Some non-P's are S's,

which is therefore the *contrapositive* of—

No S's are P's.

If we attempt to contraposit I, *Some S's are P's*, we are met with a difficulty, for on obverting we obtain—

Some S's are not non-P's,

and this, being SoP, is incapable of conversion. Hence, no contrapositive can be obtained from an I proposition.

If we obvert O, Some S's are not P's, we obtain—

Some S's are non-P's,

the converse of which is

Some non-P's are S's,

which is therefore the contrapositive of

Some S's are not P's.

Our results may be tabulated thus—

Original Proposition.	Obverse.	Contrapositive.
SaP All S's are P's.	SeP No S's are non-P's.	SeP No non-P's are S's.
SeP No S's are P's.	SaP All S's are non-P's.	SiP Some non-P's are S's.
SiP Some S's are P's.	SoP Some S's are not non-P's	None.
SoP Some S's are not P's.	SiP Some S's are non-P's.	SiP Some non-P's are S's.

The following are concrete examples of contrapositives—

A. All Hottentots are Africans, Original proposition.
E. No non-Africans are Hottentots. Contrapositive.

E. No birds are quadrupeds. Original proposition.
A. All quadrupeds are non-birds. Contrapositive.

O. Some men are not honest. Original proposition.
I. Some non-honest beings are men. Contrapositive.

The student should illustrate contraposition by Euler's diagrams.

Résumé of Chapter XI.

I. *Eduction.*

1. Definition—Immediate inference from a given proposition to the truth of a new proposition, which differs in subject or predicate, or both, from the original proposition.
2. Example—*No men are immortal*, is an eduction from *all men are mortal.*
3. Kinds discussed—
   a. Conversion.
   b. Obversion.
   c. Contraposition.

II. *Conversion.*

1. Definitions—
   a. Conversion—the valid interchange of the subject and predicate of a proposition.
   b. The *convertend* is the proposition to be converted.
   c. The *converse* is the new proposition obtained by conversion.
   d. Conversion is *simple* when the converse is of the same form as the convertend.
   e. Conversion is *per accidens* when the converse is not of the same form as the convertend.
2. Rules for Conversion—
   a. Leave the quality unchanged.
   b. See that no term is distributed in the converse, unless it is distributed in the convertend.

3. Examples of Conversion—

a. A, *All S's are P's*, converts *per accidens* to I, *some P's are S's.*

b. E, *No S's are P's*, converts *simply* to E, *no P's are S's.*

c. I, *Some S's are P's*, converts *simply* to I, *some P's are S's.*

d. O, *Some S's are not P's*, is incapable of conversion.

III. *Obversion.*

1. Definitions—

a. Obversion is the inference, from a given proposition, of a new proposition, the predicate of which is the contradictory of the predicate of the original proposition.

b. The proposition to be obverted is the *obvertend.*

c. The proposition obtained by obverting is the *obverse.*

2. Rule for Obversion—Negative the predicate, change the quality and leave the quantity unaltered.

3. Types of Obversions—

a. A, *All S's are P's*, obverts to E, *no S's are non-P's.*

b. E, *No S's are P's*, obverts to A, *all S's are non-P's.*

c. I, *Some S's are P's*, obverts to O, *some S's are not non-P's.*

d. O, *Some S's are not P's*, obverts to I, *some S's are non-P's*

IV. *Contraposition.*

1. Definitions—
   a. Contraposition is the conversion of the obverse.
   b. The proposition obtained by contraposition is called the *contrapositive.*
2. Rule for contraposition :—*Obvert, and convert the obverse.*
3. Types of contraposition—
   a. A, *All S's are P's,* contraposits to E, *no non-P's are S's.*
   b. E, *No S's are P's,* contraposits to A, *all S's are non-P's.*
   c. I, *Some S's are P's,* is incapable of contraposition.
   d. O, *Some S's are not P's,* contraposits to I, *some non-P's are S's.*

Exercises on Chapter XI.

1. Distinguish between eduction and immediate inference by opposition.
2. Convert—
   a. Some stars are self-luminous.
   b. Some metals are not so heavy as water.
   c. All Europeans are white.
   d. All minerals are dug out of the earth.
   e. P struck Q.
   f. All that glitters is not gold.
3. Contraposit the above where possible.
4. Obvert the propositions in Question 2.
5. Illustrate the conversion of SaP, SeP, and SiP by Euler's diagrams.

6. Illustrate the contraposition of SaP by means of Euler's diagrams.
7. All equilateral triangles are equiangular. What can be inferred from this by conversion, by obversion, and by contraposition ?
8. What kinds of inferences are the following ? Say whether they are valid or not.
   a. Our knowledge must be gained from books, for books are a source of instruction.
   b. All triangles are trilateral, therefore, all trilateral figures are triangles.
   c. All that glitters is not gold, therefore, gold does not glitter.
   d. All wise students study hard, therefore, none who do not study hard are wise students.
   e. A met B, therefore, B met A.
9. Explain clearly why SiP cannot be contraposited.
10. Convert, obvert, and contraposit—
    Things which are equal to the same thing are equal to one another.

CHAPTER XII.

The Predicables.

In the last two chapters we have considered the most important relations existing between different propositions having the same or closely connected subjects and predicates; we have now to consider the different possible relations between the predicate and subject of the same proposition. The different classes into which predicates may be arranged by considering the relation in which they stand to their subjects, are known as the *predicables*. In every proposition something is affirmed or denied of the subject; what is so affirmed or denied is a *predicate*, what can be so affirmed or denied is a *predicable*. Aristotle (B.C. 384—322) arranged the predicables into *four* classes. His scheme has, however, completely given place to that of Porphyry (A.D. 234 to 304), who arranged the predicables under five heads.

In Porphyry's arrangement—

The Predicables are	1. Genus	Genus*
	2. Species	Species
	3. Difference, or	Differentia
	4. Property	Proprium
	5. Accident	Accidens.

* Singular, genus, species, differentia, proprium, accidens. Plural, genera, species, differentiæ, propria, accidentia.

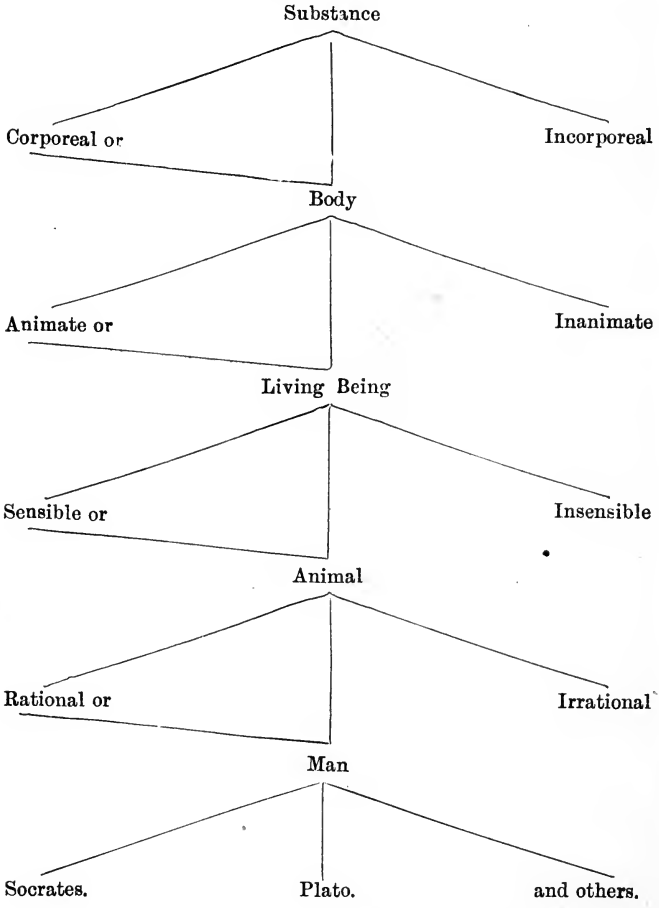
The second column gives the Latin names of the predicables. These are often used and should therefore be carefully remembered.

Any class regarded as consisting of two or more smaller classes is a *genus;* and the smaller classes which together constitute the *genus* are *species.* Thus, *triangle* is a *genus,* the *species* of which are scalene triangle, isosceles triangle, and equilateral triangle. In the proposition, *apples are fruit,* the predicate *fruit* is a *genus,* in relation to the subject *apples,* which is a *species,* in relation to the predicate *fruit.*

The term predicable itself is the name of a *genus,* the *species* of which are, *genus, species, difference, property* and *accident.* It is clear that *genus* and *species* are relative, for the definition of either involves that of the other; each is intelligible only in its relation to the other. It must be carefully noticed that the same term may at one time be the name of a genus and at another the name of a species, for whether a class is a genus or species depends entirely upon how we regard such class; regarded as a class consisting of sub-classes it is a genus, but regarded as a sub-class of a larger class it is a species. Thus, in *apples are fruit,* apples are a *species* of the *genus* fruit; but in *golden russets are apples,* the class *apples* is a genus, one species of which is *golden russets.*

A class which cannot be divided into sub-classes, but only into individuals, is called a *lowest species* or *species infima*; a *genus* which cannot be regarded as a species of any larger class is a *highest genus* or *summum genus.* Every genus must, of course, contain at least two species. All the species which together constitute the genus, are said to be *cognate.*

Any genus, with reference to the species into which it is immediately divided, is called a *proximate genus.* The following table was given by Porphyry to illustrate the connection between *genus* and *species*, and is consequently known as the Tree of Porphyry.



Here *substance* being regarded as a class including all other classes, and included in no other class, is the *summum genus; Man* being divided not into sub-classes, but into individuals, is the *species infima. Corporeal substance, or body*, and *incorporeal substance*, are *cognate* species of the *proximate* genus substance. So the *proximate genus* of rational animal and irrational animal is *animal.* Since the genus is a class consisting of smaller classes or species, the denotation of the name of the genus, or generic term, is greater than that of the name of the species, or specific term, and hence its connotation is less. Thus, *rational animal* is the name of a species of the genus *animal;* and, clearly, *animal* has greater denotation than *rational animal*, and has less connotation, for *rational animal* connotes all that *animal* does, and, in addition, it connotes the attribute of *rationality.* In fact, it is this additional connotation of the specific term that determines whether or not anything belonging to the genus belongs also to the species. It serves to differentiate the species from all other cognate species. This excess of connotation of the specific term over that of the generic term is the *differentia*, the third of the predicables. It is clear, then, that the connotation of the specific term equals the connotation of the generic term, together with the *difference.* If we remember that the terms are used with reference to connotation, this may be expressed thus :—

Species-name = genus-name + differentia.

Thus :—

Rational animal or species-name	=	animal or genus-name	+	rational or difference.

The word *differentia*, it will be noticed, is also a purely relative term, having no meaning apart from *genus* and *species*.

It may be useful to notice that the *genus* answers the question: What? The species: What kind? For example; what is a triangle? A figure (the genus). What kind of figure? One bounded by three lines (the species). A *proprium*, or *property*, is an attribute common to every member of a class which forms no part of the connotation of the class-name, but follows as a necessary consequence from it. For example, it follows as a necessary consequence from the connotation of *equilateral triangle*, as given in its definition, that an equilateral triangle is equiangular. *Equiangularity* is, then, a property of equilateral triangles. Similarly, the attribute of having the sum of its angles equal to two right angles is a *property* of triangles, for it forms no part of the connotation of *triangle*, but can be logically deduced from it.

Clearly, then, a property is an essential property in the sense that it is, and must be, possessed by every member of the class included under the class-name.

An *accidens* is a quality which forms no part of the connotation of the class-name, and cannot be deduced from it. It is not an essential quality, for its presence or absence in an object in no way affects the position of such object in the class. Thus, that the table at which I am writing is square is *an accident*, as is also the fact that it is made of mahogany. Both these qualities might be absent without the table ceasing to be a table.

Accidentia may be *separable* or *inseparable*.

A *separable* accident is one that does not belong to every individual of a class. Thus, whiteness is an *accident* of some swans, but a *separable* accident, for some swans are not white.

An *inseparable* accident is one that belongs to the whole class. Thus, it is an inseparable accident of pianos that some of their keys are black.

Résumé of Chapter XII.

Predicables.

I. Definitions.

1. A *predicate* is something affirmed or denied of a subject.
2. A *predicable* is whatever can be affirmed or denied of a subject.

II. Names of Predicables.

1. *Genus* is any class regarded as divided into two or more sub-classes.
2. *Species* is one of the sub-classes into which the genus is regarded as divided.
3. The *difference* (Latin *differentia*) is the excess of connotation of the name of the species over that of the name of the genus.
4. A *property* (Latin *proprium*) is an essential quality which forms no part of the connotation of the class-name, but is deducible from it.
5. An *accident* (Latin *accidens*) is any quality having no connection with the connotation of the class-name.

Exercises on Chapter XII.

1. Give six examples of species, mentioning in each case the proximate genus.
2. The species is part of the genus, and the genus is part of the species. Explain how this can be.
3. Refer the following predicates to their proper predicable—
   a. The horse is a quadruped.
   b. Socrates is a man.
   c. An equilateral triangle is bound by three equal sides.
   d. The angle in a semi-circle is a right angle.
   e. Some men are six feet high.
4. Give the proximate genera of those of the following which may be regarded as species—
   Man, teacher, Julius Cæsar, city, violin, table.
5. Specific name = Generic name + differentia. Explain this.
6. Is *laughter* a property or accident of man?

CHAPTER XIII.

Definition.

One of the most prolific sources of error in reasoning is the use of terms without there being in the mind of the speaker any definite idea of the connotation of such terms. We think we have an accurate knowledge of the language we use, but a very slight examination will convince most of us that such is not the case. When the student uses or hears used such familiar terms as *freedom*, *slavery*, *education*, *constitutional*, *honourable*, *definition*, let him but ask himself what their exact meaning is, and he will quickly discover that in many cases his ideas of their meaning are too vague to be put into words. Even in cases where the meaning is apparently perfectly known, it is frequently a matter of great difficulty to accurately express such meaning in words. Every general term has a certain connotation, and we should never rest satisfied with our knowledge of this connotation until we can accurately express it in words, that is, until we can *define the term*.

A *definition* is a proposition which gives the connotation of a term. Note carefully that definitions are of *terms*, not of *things*. By *connotation of a term*, it will be remembered, is meant those attributes, and those only, which must be possessed by an object before such term can rightly be applied to it. Thus, in *man is a rational animal*, we have a proposition setting forth the connotation of man, viz.,

the attribute implied by the term *animal*, together with *rationality*. If a term changes in meaning, as is frequently the case, then, of course, its definition also changes. Further, since a definition is an expression of the connotation of a term, it is evident *non-connotative* terms cannot be defined, for they have no connotation. We have already seen that proper names and singular abstract terms are non-connotative, and they, therefore, do not admit of definition.

The connotation of many terms is exceedingly complex, and a definition setting forth such connotation in simple attributes would be excessively unwieldy. Hence, a definition generally gives the greater part of the connotation by using a term expressive of a group of attributes, the meaning of such term being, of course, supposed to be known. Thus, if we define a *barrister* as a *lawyer who pleads at the bar*, all the connotation of *barrister*, with the exception of *pleading at the bar*, is given in the term *lawyer*. *Barrister* may be regarded as a species of the genus *lawyer*, and as we have already seen—

Connotation of species-name = connotation of genus-name + difference.

The above may be written thus—

Connotation of *barrister* = connotation of lawyer + pleading at the bar.

Hence, the term barrister is defined by giving the *proximate genus* (lawyer), and adding the *difference* (*pleading at the bar*). In defining, it is always best to proceed in accordance with the old rule that definition should be *per genus et differentiam*, *i.e.*, by means of the *genus* and the *difference*. Hence, if we wish to define any term, the first thing to do is to determine the class (genus) to which it belongs, and then add the difference.

The following are the rules usually given, to be observed in framing a definition—

1. The definition must contain neither more nor less than the connotation of the term defined.
2. It should be more intelligible than the term defined, and must not, therefore, be expressed in language the meaning of which is, from any cause, obscure.
3. It must not contain the term defined, nor any term synonymous with it.
4. It should not be negative if it can be affirmative.

To these may be added the old rule given above, *viz.*,

5. All definitions should be *per genus et differentiam.*

Rule 1 is not of much service in framing a definition. It is rather a test to be applied to the definition after it has been made. If more connotation be given than the term possesses, the definition is said to be too *narrow*, for it will not apply to all things bearing the name defined. Thus, if the term monarch is defined as a "man having supreme power in a country," such definition gives too much connotation, for it sets forth *sex*, which is no part of the connotation of monarch; it is, therefore, too *narrow*, as it excludes *women* who are monarchs.

The rule may be violated by giving too little, as well as by giving too much, connotation, in which case the definition is too *wide*, for it includes more *things* than rightly form the denotation of the term defined. Thus, if we were to define *hat* as *clothing for the head*, our definition would be too wide, for *clothing for the head* includes caps, turbans, bonnets, etc., as well as *hats.*

This rule is a very difficult one to avoid violating. It is often almost impossible to say precisely what the connotation of a term is, and especially is this the case if

the term is the name of a common object. To define may seem to be easy enough, but let the student try to frame accurate definitions of such common terms as *table*, *stool*, *chair*, *desk*, *hat*, *carriage*, and he will at once discover how difficult it is to avoid breaking the very first rule of logical definition.

The student should notice carefully that a definition gives *connotation*; if *properties*, or *accidents*, or both, are given, we have a *description* and not a *definition*.

Rule 2.—The reason for this rule is obvious. If connotation be given in terms less intelligible than the term defined, one object of the definition is defeated, for the meaning of the term is not made clearer by such a definition. Thus one's knowledge of the meaning of *soul* is not much improved by Aristotle's definition of it as "the first *entelecheia* of a natural body which has potential life". *Fluency* might be defined as the *exuberance of verbosity*, but such definition would hardly make clearer one's idea of fluency.

When this rule is violated the definition is said to define the *obscure* by the *more obscure (obscurum per obscurius)*, or the *unknown* by the *more unknown (ignotum per ignotius)*. This rule is also violated by the employment of metaphors in defining; thus, it is no definition of bread to say that it is the *staff of life*.

Rule 3.—The violation of this rule gives rise to what is known as a circle in defining (*circulus in definiendo*). Thus, if we define a metal as *one of the metallic elements*, we make no advance in knowledge, but rather travel in a circle and return to the place whence we started; if we do not know what a *metal* is neither do we know what *metallic* is. To define *life* as the *sum of the vital functions* is to commit

the same fault, for the word *vital* is from the Latin, *vitalis*, which, again, is from *vita*, the Latin word for *life*. Definitions in English are especially liable to this fault, as it frequently happens that a pure English word has a corresponding synonym derived from Latin.

Rule 4.—The definition of a purely negative term is naturally, negative. Thus, *alien* is a negative term, and its definition, which may be given as *one who does not belong to the British Empire*, is also negative.

Résumé of Chapter XIII.

I. Definitions are necessary—
   1. To make clear the meaning of terms.
   2. To fix such meanings.

II. Definitions.
   1. *Definition:*—A proposition setting forth the connotation of a term.
   2. Rules for defining.
      a. The definition must contain neither more nor less than the connotation of the term defined.
      b. It must be more intelligible than the term defined.
      c. It must not contain the term defined.
      d. It should be affirmative whenever possible.
      e. It should be *per genus et differentiam.*
   3. *Non-connotative* terms are incapable of being defined.

Exercises on Chapter XIII.

1. What is a definition ?
2. What is the difference between a *definition* and a *description* ?
3. What is meant by definition *per genus et differentiam* ? Give examples.
4. Which of the following terms cannot be defined and why ? School, college, honesty, Cæsar, horse, England.
5. Define the terms capable of definition in Question 4.
6. Criticise the following definitions taken from an English Dictionary—
   a. An auditor is one who audits accounts.
   b. An atlas is a collection of maps.
   c. A net is a reticulated fabric decussated at regular intervals.
   d. A burgess is an inhabitant of a borough.
   e. An apple is the fruit of the apple tree.
   f. An alley is a place along which one may go.

CHAPTER XIV.

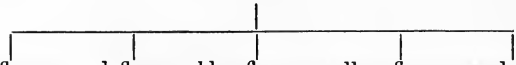
Logical Division.

Logical Division is closely connected with definition, bearing as it does, much the same relation to the *denotation* of a term that definition does to the *connotation*. As to *define* is to unfold connotation, so to *divide* is to unfold, or analyse in an orderly way, denotation. It is called *division*, for an orderly analysis of the denotation of a term can be made only by regarding the term as the name of a genus, and by dividing such genus into its constituent species. The genus to be divided is called the *totum divisum*, or *total divided*, and the species or sub-classes into which it is divided, the *membra dividentia*, or *dividing members*.

Thus, if the genus *animal* be divided into the two species *rational animal* and *irrational animal*, *animal* is the *totum divisum*, and *rational animal* and *irrational animal*, the *membra dividentia*.

Before any class can be divided into sub-classes, some attribute or attributes must be thought of, on the presence, or absence, or the varying degree, of which the division may be made to depend. Thus, if I wish to divide the genus *flowers*, I first think of some attribute, *e.g.*, colour, variation of which will enable me to divide *flowers* into species.

FLOWERS.



White flowers, red flowers, blue flowers, yellow flowers, and others.

Here *flowers* is the *totum divisum*, *white flowers*, *etc.*, are the *membra dividentia*, and *colour* is the attribute on which the division is based, or the *fundamentum divisionis*, as it is technically named.

If *plants* be divided into flowering-plants and non-flowering plants the *fundamentum divisionis* is the attribute *flower-producing*, the presence or absence of which serves as a basis of the division.

In the chapter on the *predicables* we have already seen that the species of any genus may themselves become genera for further division, which is then called *sub-division*. It is clear that for each step in the sub-division we shall require a fresh *fundamentum divisionis*. Thus, after having divided the genus *books* into the species *English books*, *French books*, etc., we may regard *English books* as a genus to be further divided into *species*, but we can no longer divide on the basis of the language in which printed, for all are printed in English ; a fresh *fundamentum divisionis* is needed, *e.g.*, size. We can then divide the genus *English books* into *English quarto books*, *English octavo books*, and so on.

There are certain processes to which the name *division* is applied, which should be carefully distinguished from *logical division*. *Physical division*, or *physical partition*, is a division of a unit into the parts of which such unit is formed, as, for instance, an individual house may be divided into the bricks, mortar, wood, etc., of which it is built. Another form of division is what is properly called *metaphysical division*, which by *abstraction* divides an individual object into the

qualities of which it may be said to consist. Thus, an orange may be regarded as an object consisting of a collection of qualities, such as yellow colour, a certain shape, a certain taste, etc. Logical division differs from both these in being applied to *classes* only, and not to *individuals*.

Certain rules have been given for carrying out a logical division; they are the following:—

1. Each step in the division must be founded upon one *fundamentum divisionis*.
2. The species when added together must equal the *proximate genus*.
3. The division must be gradual.

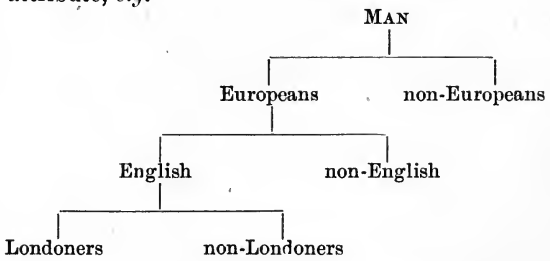
Rule 1.—The necessity for observing this is evident. If in dividing the genus *books* we choose size and subject matter as two *fundamenta divisionis*, we shall have two places in our division for each individual book. Where, for instance, shall we put an octavo book on Logic? Shall we put it in the *octavo class*, where it ought to go in virtue of its size, or in the class of Logic books, where it should go in virtue of its subject matter? We have what is called a *cross division*, which may be defined as the faulty form of division that results from its being based on more than one *fundamentum divisionis*.

The one *fundamentum divisionis* applies of course only to the *cognate* species. Every successive step in the division must have a fresh *fundamentum divisionis*. Thus, in dividing books according to subject matter, we should have as one of the cognate species *music books*. If now we regard this as a genus to be further divided, clearly we must have a fresh *fundamentum divisionis*, *e.g.*, size, in which case our division would result in *folio music books*, *quarto music books*, etc., as cognate species.

Rule 2.—If the sum of the cognate species be less than the genus, some part of the latter remains undivided, and the division is therefore not complete; and if it be greater we must have some things included in the species which do not belong to the genus at all. This rule does not help us much; it merely says that in dividing we should be careful to leave none of the *totum divisum* undivided.

Rule 3.—The old logicians gave this rule in the words *divisio non faciat saltum*, that is, *the division must not make a leap*. This rule will be kept if we always take care to make the species those of the proximate or next higher genus. Thus, it would be illogical to divide Europe at once into *parishes*; the proximate genus of *parish* might possibly be *union*, *barony*, or *county*, but is certainly not Europe.

The most perfect form of logical division is that in which at each step the genus is divided into two species, one species having a certain attribute and the other not possessing such attribute, *e.g.*—



Such a division is said to be by *Dichotomy*, from two Greek words meaning "a division into two."

In division by dichotomy cross division is impossible, for an object cannot both possess an attribute and not possess it, which it must do to belong to both the cognate species. Moreover, every object in the genus to be divided must, by

the Law of Excluded Middle, either possess any given attribute or not possess it, and hence every object will have its place in one or the other of the cognate species. Therefore, the second of the above rules will not be broken in a division by dichotomy if carefully carried out.

Division by dichotomy is, in many cases, however, too tedious. When we are certain of the number of the sub-classes it is better to divide the genus into such classes at once. It would be absurd to divide England into Middlesex and non-Middlesex, as cognate species; and then non-Middlesex into Surrey and non-Surrey, and so on. Since the number of counties is known, the rules may be kept by dividing at one step into counties.

The Tree of Porphyry given in the lesson on the predicables is a good example of logical division by dichotomy.

Résumé of Chapter XIV.

Division.

I. *Definitions.*

1. Logical division is an orderly analysis of the denotation of a term.
2. *Totum divisum* is the whole class to be divided.
3. *Membra dividentia* are the sub-classes in the division.
4. *Fundamentum divisionis* is the attribute, the presence or absence or varying degree of which determines the class of any object.

II. Processes to be distinguished from logical division.

1. *Physical division* is the division of an individual object into its material elements.

2. *Metaphysical division* is the division by abstraction of an individual object into the qualities which may be regarded as constituting it.

III. Rules for logical division.

1. Each step in the division must be founded on one *fundamentum divisionis.*
2. The species when added together must equal the proximate genus.
3. The division must be gradual.

IV. Division by dichotomy is that form of division in which each genus is divided into *two* species determined by the presence or absence of some attribute.

Exercises on Chapter XIV.

1. What is logical division?
2. Distinguish *logical* division from *physical* division, and from *metaphysical* division.
3. Explain *fundamentum divisionis.*
4. Divide book, teacher, school.
5. Criticise the following divisions—
   a. Books into folio, quarto, octavo, French and English.
   b. Quadrupeds into horses, ponies, mules, carnivora, and mammalia.
   c. Knowledge into useful, useless and harmful.
   d. Thieves into pickpockets, pilferers, highwaymen and pirates.
   e. Pictures into sacred, historical, imaginative, and realistic.
   f. Constitutions into good and bad.

CHAPTER XV.

The Syllogism.

We have already (Chapter IV.) seen what constitutes a syllogism. We have in the present chapter to consider under what circumstances we may rely on the conclusion of a syllogistic argument as being valid.

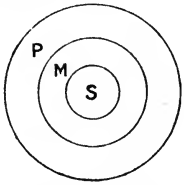
A syllogism has been defined as "a reasoning, consisting of three categorical propositions (of which one is the conclusion), and containing three, and only three, terms" (Keynes). Taking this as our definition of the syllogism, we see that every syllogism consists of three propositions, and contains three terms. These terms are the *middle term*, the *major term*, and the *minor term*. These have been already defined (Chapter IV.) and the student, if he has not already done so, must firmly fix in his mind that the *middle term* is the one which does not occur in the conclusion, that the *major term* is the predicate of the conclusion, and the *minor term* the subject of the conclusion. Consider the syllogism—

All M's are P,
All S's are M,
therefore All S's are P.

Here the proposition written last is the *conclusion*, for it is clearly the inference drawn from the other two, which are the *premisses*,

The three terms are S, M, P, each of which occurs twice. M is the middle term, for it is the term which does not occur in the conclusion; S is the minor term, being the subject of the conclusion; and P is the major term, being the predicate of the conclusion.

A syllogism like the above may be typically represented by the Eulerian diagrams, thus—



In this, for the sake of simplicity, only one diagram is used to represent SaP. According to this diagram S stands for a class *smaller* than the M class, P stands for a class *larger* than the M class, M denoting a class intermediate in size between the classes denoted by S and P. It is for this reason that they received the names, *minor*, *major* and *middle* respectively. These names are, however, used in the technical sense defined above, for the mutual relation implied by the names *minor*, *major* and *middle* does not always hold.

The premiss containing the *minor* term is called the *minor premiss*, and that containing the *major* term is called the *major premiss*. In the strict logical order of the syllogism the major premiss stands first, the minor second, and of course the conclusion last. This order is, however, in practice by no means always observed. Frequently the conclusion stands first as in the syllogism—

He is honest	(conclusion)
For he is an Englishman	(minor premiss)
and All Englishmen are honest	(major premiss)

The student should now be in a position to understand the rules of the syllogism. As usually given they are six in number, to which are added three additional ones deducible as corollaries from them.

The rules are—

1. Every syllogism contains three, and only three, terms.
2. Every syllogism consists of three, and only three, propositions.
3. The middle term must be distributed once at least, and must not be ambiguous.
4. No term may be distributed in the conclusion which was not distributed in one of the premisses.
5. From negative premisses nothing can be inferred.
6. If one premiss be negative, the conclusion must be negative, and conversely, to prove a negative conclusion one premiss must be negative.

The three supplementary rules deducible as corollaries from the above, are—

7. From two particular premisses, nothing can be inferred.
8. If one premiss be particular, the conclusion will be also particular.
9. From a particular major and a negative minor premiss, nothing can be inferred.

These rules are of such supreme importance, that they should be learned thoroughly at once. If any complete thought, when put into syllogistic form, is found not to violate any of these rules, the conclusion must necessarily be valid. Rules 1 and 2 are very important as a means of testing any given argument as to whether it is a syllogism or not; they are, however, of little or no practical use as tests of the validity of the syllogism.

The first part of Rule 3 is exceedingly important, and is constantly being violated by students who have not studied Logic. One of the commonest experiences of the teacher of Logic is, after having explained what a syllogism is, to have given him, as valid syllogisms, examples of syllogisms which violate Rule 3. Here are a few actually given by students—

All flowers are pretty,
All roses are pretty,
therefore All roses are flowers.

All children are playful,
Tommy is playful,
therefore Tommy is a child.

All quadrupeds are animals,
The horse is an animal,
therefore The horse is a quadruped.

In the first of these the middle term is *pretty*, and this is the predicate of both premisses, which are affirmative; it is therefore undistributed. When any rule of the syllogism is violated, a *fallacy* is said to be committed; in this case we have the *fallacy of the undistributed middle term*, or more briefly, the *undistributed middle*. Similarly, in both the second and third of the above syllogisms we have the fallacy of the undistributed middle. The student will now readily understand why the fourth syllogism given in Chapter II. is invalid. It is—

All coins are useful things,
All shillings are useful things,
therefore All shillings are coins.

The middle term *useful things* is not distributed in either premiss of the syllogism which, therefore, is guilty of the undistributed middle.

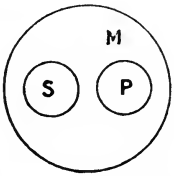
In all the above examples the conclusions are *true* and it is this which is so likely to mislead the student into thinking that they are valid. Express any of them in symbols and the fallacy is more apparent. Thus for "coins", "useful things", "shillings", the major, middle and minor terms respectively, put P, M, S, respectively.

The syllogism now is—

All P's are M,
All S's are M,
therefore All S's are P.

It is at once evident that it contains the undistributed middle.

This fallacy may be clearly illustrated by Euler's diagrams. The diagram—



clearly satisfies the premisses and is yet inconsistent with the conclusion of the syllogism, which is therefore invalid, for, in a valid syllogism, if the premisses are true the conclusion is also necessarily true.

The second part of Rule 3 is also important. It is easy for any one who has once understood what is meant by the *undistributed middle term* to avoid the violation of the first part of this third rule, but it is a much more difficult matter to avoid violating the second part of this rule. The violation of the second part of the third rule of the syllogism is known as the *fallacy of the ambiguous middle.*

A term is said to be ambiguous when in different parts of the same argument it has different meanings. Thus, in the syllogism—

Light dispels darkness ;
Feathers are light ;
therefore Feathers dispel darkness.

The term *light* has, evidently, two meanings, and is, therefore, *ambiguous.* Moreover, *light* is the middle term in the above syllogism, hence, the syllogism is invalid, containing the fallacy of the ambiguous *middle term.* In this case the ambiguity is easy enough to detect, and the student may think that the ambiguous middle can always easily be avoided. There is no fallacy that is more abstruse and more difficult to avoid falling into. Let the student try to discover the fallacy in the following syllogism, and he will at once see that the fallacy of ambiguous middle is not always easily detected—

A man deserves no credit for doing what he cannot possibly help doing ;
A benevolent man cannot possibly help relieving distress ;
therefore A benevolent man deserves no credit for relieving distress.

If we carefully examine the precise meanings of the terms we see that "cannot possibly help" is used in different senses in the two premisses. These words, moreover, form part of the middle term, hence, the above syllogism is guilty of the fallacy of *ambiguous middle term.*

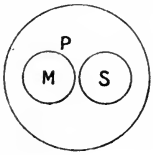
When a term is used in the same argument in two senses it is in reality *two terms* which happen to be spelled and pronounced alike, so that a syllogism containing an ambiguous middle term has really four terms and thus

violates the first rule. Every case of the fallacy of ambiguous middle may be regarded as a case of the *fallacy of four terms.*

The fourth rule of the syllogism is also an important one. The student will find no difficulty in detecting any cases of its violation. Consider the syllogism—

All M's are P,
No S's are M,
therefore No S's are P.

The major premiss in this is an A proposition and its predicate, which is the *major* term, is therefore undistributed; the conclusion, being an E proposition, distributes its predicate, which is the major term. Hence, the *major* term is distributed in the conclusion, but not in the major premiss, and the syllogism therefore violates the fourth rule. This is an example of the fallacy known as the *illicit process of the major term*, or more shortly, the *illicit major.* The above syllogism may be represented by the diagram—



which is in accordance with the premisses, but inconsistent with the conclusion. A concrete example of the *illicit major* is—

All apples are fruit,
No pears are apples,
therefore No pears are fruit.

The *invalidity* of the conclusion is, in this, at once noticed, because it is seen to be false. If however, we

have a syllogism guilty of the same fallacy, but having a conclusion known to be true, the fallacy is much more likely to be overlooked. Thus, the syllogism—

All Hottentots are Africans,
No English are Hottentots,
therefore No English are Africans,

is invalid, from its violating rule 4, although, knowing the *truth* of the conclusion, we are apt to conclude that it is valid.

Similarly we may have an *illicit process of the minor term.* Thus the syllogism—

All M's are P,
Some S's are M,
therefore All S's are P,

clearly has the minor term S undistributed in the minor premiss and distributed in the conclusion. The *illicit minor* is less frequent and more easily detected than the *illicit major.* A concrete example is—

All rogues deserve punishment
Some Englishmen are rogues,
therefore All Englishmen deserve punishment.

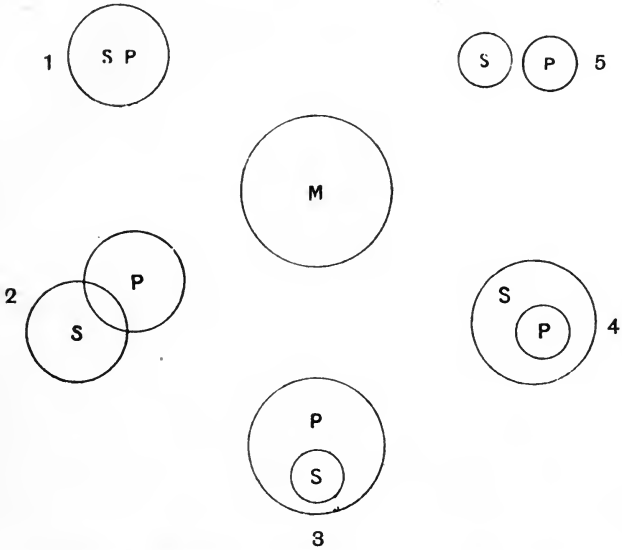
Rules 3 and 4 are the only ones that give any difficulty in applying. To be able to apply these two rules the student must have perfectly clear ideas on the subject of distribution of terms.

Rule 5 may be clearly illustrated by diagrams. To represent the premisses—

No P's are M,
and No S's are M,

we must be careful to make both the P circle and the S circle outside of the M circle, but the premisses give us no conditions as to the relation of S and P. In fact, the S and

P circles may have any one of the five relations possible between them. This may be shown thus—



the premisses allowing of all the five relations between S and P shown. Hence, we can no more make any inference from the premisses than from the mere mention of the terms S and P. If this is the case with *universal premisses*, still more so is it the case with *particular premisses*.

Rule 6. If one premiss is negative, the other must be positive, by Rule 5. Hence, in a syllogism having one premiss negative, of the *major* and *minor* terms, one is affirmed to agree wholly or partially with the middle term, and the other is affirmed to wholly or partially be separated from the middle term, and therefore the major and minor terms must be wholly or partially separate, that is, the conclusion in such a syllogism is negative.

Rule 7 may be deduced from the preceding six, thus :—If the two premisses are particular, they must be II, IO, OI, or OO. The premisses OO are excluded by Rule 5. II would clearly give undistributed middle, for I distributes neither its subject nor its predicate. Now IO or OI will give a negative conclusion, if any at all, by Rule 6. Therefore, the major term must be distributed to avoid *illicit major*; and the middle must be distributed to avoid undistributed middle. Hence, two terms must be distributed in the premisses. But OI or IO distribute only one term, *viz.*, the predicate of O; hence, any conclusion from them as premisses, must be obtained by means of an undistributed middle, or an illicit major. Therefore, two particular premisses give no conclusion.

Rule 8. The only possible pairs of premisses of which one is particular, are:—AI, AO, EI, EO, and of these, EO are excluded by the fifth rule. The premisses AI, distribute only one term between them, *viz.*, the subject of A. This must be the middle term, to avoid undistributed middle; hence, by Rule 4, no term can be distributed in the conclusion which must therefore be *particular*. The pairs of premisses, AO and EI, each distribute two terms. One must be the middle term, and the other the major term, as the conclusion is negative by Rule 6, and, therefore, distributes its predicate. Hence, the minor term cannot be distributed in the premiss, and, therefore, cannot be distributed in the conclusion, which is consequently a particular proposition.

Rule 9. This may be deduced from the first six rules, as follows :—The minor premiss being negative, the major must be affirmative by Rule 5, and it is particular by Rule 9 itself. Therefore, the major premiss can distribute no term,

and the *major* term consequently is undistributed in its premiss. The conclusion, however, being negative by Rule 6, distributes the major term, and we must have the *illicit major*. This is clearly seen in an example—

Some M's are P,
No S's are M,
therefore Some S's are not P,

which clearly is guilty of illicit major.

Résumé of Chapter XV.

I. Definitions.

1. A syllogism is a reasoning consisting of three propositions and containing three, and only three, terms.
2. The *middle* term is the term which does not occur in the conclusion.
3. The *major* term is the predicate of the conclusion.
4. The *minor* term is the subject of the conclusion.
5. The *minor* premiss is the one containing the minor term.
6. The *major* premiss is the one containing the major term.

II. Rules.

1. Six primary rules—
   a. Every syllogism has three, and only three, terms.
   b. Every syllogism consists of three, and only three, propositions.
   c. The middle term must be distributed once at least and must not be ambiguous.

d. No term may be distributed in the conclusion which is not distributed in its premiss.

e. From negative premisses nothing can be inferred.

f. If one premiss be negative, the conclusion must be negative. Conversely, to prove a negative conclusion, one premiss must be negative.

2. Three corollaries from the above.

a. From two particular premisses nothing can be inferred.

b. If one premiss is particular the conclusion is so also.

c. From a particular major and negative minor nothing can be inferred.

Exercises on Chapter XV.

1. Point out the major, minor, and middle terms in the following syllogisms and test their validity with reference to the rules of the syllogism.

a. All vegetables are useful,
All cabbages are vegetables,
therefore, All cabbages are useful.

b. All vegetables are useful,
All cabbages are useful,
therefore, All cabbages are vegetables.

c. The highest good is the end of life,
Death is the end of life,
therefore, Death is the highest good.

d. All animals can move,
Protococcus can move,
therefore, Protococcus is an animal.

e. All metals are elements,
Brass is a metal,
therefore, Brass is an element.

2. Put the following arguments into strict logical form and test their validity.

   a. The atmosphere has weight, for it is a material substance, and all material substances have weight.

   b. All planets move round the sun, therefore the earth moves round the sun, for it is a planet.

   c. David was mad, for he scribbled on the walls, and madmen do so.

3. Express the arguments in Question 2 in symbols.

4. Illustrate the arguments in Question 1 by Euler's circles.

5. Test the following—

   a. All P's are M: All S's are M: therefore All S's are P.

   b. All P's are M: All M's are S: therefore All S's are P.

   c. No P's are M: No M's are S: therefore No S's are P.

   d. No M's are P: All S's are M: therefore No S's are P.

6. If the major term of a syllogism is the predicate of the major premiss, what is known about the minor premiss?

7. Prove that when the minor term is the predicate of the minor premiss the conclusion cannot be A.

8. Prove that two particular premisses give no conclusion.

CHAPTER XVI.

Figure and Mood of the Syllogism.

In the preceding chapter we have discussed the rules by which syllogistic arguments may be tested. In the present chapter we shall determine all the various forms which a valid syllogistic argument may take.

The form of a syllogism, as determined by the quality and quantity of the propositions forming it, is called the *Mood* of the syllogism. Thus, the syllogism—

All M's are P,
All S's are M,
therefore All S's are P

is of the mood AAA, which simply means that all the three propositions constituting it are universal affirmative propositions. Similarly, EAE, AAI, etc., are moods. The student must carefully note that in giving the mood of a syllogism the order of the propositions is the logical order, *viz.*, major premiss, minor premiss, conclusion. Thus, the mood EAE means that the major premiss is E, the minor premiss A, and the conclusion E. By the *figure* of a syllogism is meant the form it takes as depending on the order of the terms in the premisses. We have seen (Chapter XV.) that the order of the terms in the conclusion is invariable, for in every syllogism the predicate of the conclusion is the major term and the subject of the conclusion

the minor term. Hence, the empty symbolic form of the conclusion is S–P. In the premisses, however, the terms may occur in any order. Since each term occurs twice in any syllogism, and the conclusion contains the major and minor terms, it is clear that the terms which must occur in the two premisses of any syllogism are S, P, M, M. The middle term cannot occur twice in one premiss, for if it did the other premiss would be S–P or P–S, and with this the conclusion S–P would be identical or be obtained from it by immediate inference. This limits us to *four* possible arrangements of the terms in the premisses. They are—

1	2	3	4
M–P	P–M	M–P	P–M
S–M	S–M	M–S	M–S

These are known as the *first*, *second*, *third* and *fourth* figures respectively.

We may add the conclusion to each and we then have—

First Figure	Second Figure	Third Figure	Fourth Figure
M–P	P–M	M–P	P–M
S–M	M–S	M–S	M–S
S–P	S–P	S–P	S–P

The student should learn thoroughly the position of the terms in each of the four figures. This is most easily done by learning the position of M in each figure, for when the place of M is known the syllogism may be easily completed.

The possible number of valid moods may be determined by first considering how many different pairs of propositions can possibly stand as premisses and then considering what conclusions are justified by each valid pair of premisses. The major premiss may be A, E, I, O. If the major is A,

the minor may also be A, E, I, or O, without violating Rules 5 and 7, the rules which refer to the premisses. If the major is E, the minor cannot be E or O, by rule 5, and can therefore be only A or I. If the major is I, the minor can be only A, O and I both being excluded by Rule 7 and E by Rule 9. If the major is O, the minor can only be A, as E would be excluded by Rule 5, and I and O by Rule 7. This gives altogether eight valid pairs of premisses.

AA, AE, AI, AO.
EA, EI.
IA.
OA.

The pairs in which each premiss is universal will give a universal conclusion, and, therefore, also, the subaltern to such conclusion.

AI and IA can each only give a particular affirmative as conclusion by Rule 8 ; and AO, EI, OA can each only give a particular negative conclusion by Rules 6 and 8. Hence, altogether, there are 11 possible moods, viz.—

AAA, AAI, AEE, AEO, EAE, EAO,
AII, AOO, EIO, IAI, OAO.

Each of these moods will give four syllogisms, for they may be of any figure. We have to determine, then, how many of these are valid in each of the four figures. This can easily be done by writing them in each figure and testing by rules 3 and 4. Thus, AAA in the four figures is—

1.	2.
All M's are P,	All P's are M,
All S's are M,	All S's are M,
∴ All S's are P.	∴ All S's are P,

3.	4.
All M's are P,	All P's are M,
All M's are S,	All M's are S,
∴ All S's are P.	∴ All S's are P,

Here, 1 is valid ; 2 violates rule 3, giving undistributed middle ; and 3 and 4 both violate rule 4, giving the illicit minor.

If we test in this way, we shall obtain as valid—

In Fig. 1. AAA, AAI, EAE, EAO, AII, EIO,
In Fig. 2. EAE, EAO, AEE, AEO, EIO, AOO,
In Fig. 3. AAI, IAI, AII, EAO, OAO, EIO,
In Fig. 4. AAI, AEE, AEO, EAO, IAI, EIO.

Five of these are rejected as useless. They are—

AAI, EAO, in Fig. 1.
EAO, AEO, in Fig. 2.
and AEO, in Fig. 4.

In each of these, it will be noticed that the conclusion is *particular*, although the universal is justified by the premisses. The conclusion in such syllogism is said to be *weakened*. Thus, in Fig. 1, the premisses EA justify either E or O as conclusion, and we are not likely to be satisfied with O as conclusion, when E is justified by the premisses. Granted the premisses—

No fish have lungs,
All salmon are fish,

the conclusion, *some salmon have not lungs*, is valid, but such conclusion is hardly satisfactory since the premisses justify

No salmon have lungs, as conclusion.

Omitting the five moods with weakened conclusions, we are left with 19 valid and useful moods. There is, in reality, no necessity to retain these 19 moods in memory,

as any syllogism may always be tested by the rules. To enable students easily to remember the moods valid in each figure, the following mnemonic lines were devised more than six centuries ago—

Bārbărā, Cēlārēnt, Dărĭī, Fĕrĭō que prioris;
Cēsărĕ, Cāmēstrēs, Fēstīnŏ, Bărōcŏ, secundae;
Tertia, *Dārāptī, Dĭsămīs, Dātīsĭ, Fĕlāpton,*
Bōcārdō, Fērīsŏn, habet; Quarta insuper addit,
Brāmāntīp, Căměnes, Dĭmăris, Fēsāpŏ, Frĕsīson.

These lines contain in italics nineteen words, the vowels of which denote the nineteen valid moods of the syllogism. These words are merely coined for the purpose for which they are here employed, and have no meaning other than the technical meaning here given to them. They are used as the names of the moods they severally stand for. Thus the mood AAA is usually called *barbara*, the mood AOO, *baroco*, and so on.

The words in the mnemonic lines not italicised are real Latin words. They are—

Que, meaning *and*; in Latin placed after the word which it joins to previous words.

Prioris, meaning *of the first* (*figure*).

Secundae, meaning *of the second* (*figure*).

Tertia . . . habet, meaning *the third* (*figure*) *contains.*

Quarta insuper addit, meaning the *fourth* (*figure*) *in addition adds.*

Résumé of Chapter XVI.

Figure and Mood.

I. Mood.

1. Definition—Mood is the form of the syllogism as determined by the quantity and quality of the propositions of which it consists.
2. In a mood symbol such as AEE, the letters always denote the major premiss, the minor premiss, and the conclusion respectively.
3. There are 11 possible moods.

II. Figure.

1. Definition—the form of the syllogism as determined by the order of the terms in the premisses.
2. There are four possible figures.

a. First figure.	M–P S–M
b. Second Figure	P–M S–M
c. Third Figure	M–P M–S
d. Fourth Figure	P–M M–S

III. Combinations of moods and figures.

1. The valid and useful moods of Fig. 1 are—
   Barbara, *Celarent*, *Darii*, and *Ferio*.
2. In Fig. 2 are—
   Cesare, *Camestres*, *Festino* and *Baroco*.
3. In Fig. 3 are—
   Darapti, *Disamis*, *Datisi*, *Felapton*, *Bocardo* and *Ferison*.
4. In Fig. 4 are—
   Bramantip, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

Exercises on Chapter XVI.

1. What is mood ?
2. What is figure ?
3. Test EIO in each of the four figures.
4. Test IAI in each of the four figures.
5. Why are there only negative conclusions in Fig. 2 ?
6. Why are there only particular conclusions in Fig. 3 ?
7. Give a concrete example of AEE in each figure. Which are valid ?
8. What rules are broken by *Bramantip* in the first three figures ?

CHAPTER XVII.

Reduction.

The oldest rule for testing the validity of syllogisms is Aristotle's *dictum de omni et nullo* (See Chapter V.). This rule is, however, directly applicable only to syllogisms in the first figure, which Aristotle regarded as the most perfect of the figures. Hence, before the *dictum* could be applied as a test, it was necessary to express in Fig. 1 all syllogisms not already in that figure. This was called the *reduction* of the figure. *Reduction* may be defined as the process of changing a syllogism from any mood and figure in which it may be, into another mood or figure. It is clear, from what has been said of conversion, that an argument stated in one figure may frequently be reduced to another figure, for the conversion of a premiss will often give an equivalent proposition, but such conversion necessarily changes the figure. Thus, take *Festino* of the second figure—

No P's are M,
Some S's are M,
therefore Some S's are not P.

By simple conversion of the major premiss, we have—

No M's are P,
Some S's are M,
therefore Some S's are not P,

and this is *Ferio* of the first figure.

If the student will again look at the mnemonic lines, containing the valid moods in the different figures as given in the last chapter; he will see that the only initials of the names of the different moods are B, C, D and F, and that these all occur in the first line, which gives the moods valid in Fig. 1. The initials of the moods in the other figures are the same as those of the first figure, to which they can be reduced. Thus, *Camestres* of the second figure, and *Camenes* of the fourth figure, on reduction to Fig. 1 become *Celarent.*

Further, the letters s, m and p have meaning. The letter *s*, when following a vowel, denotes that to reduce to Fig. 1, the proposition denoted by the preceding vowel must be *simply converted.* Thus, the *s* in *Festino* means, that to reduce *Festino* to *Ferio*, you simply convert the major premiss denoted by the preceding *e*. The letter *m* is the initial of the Latin *muta* which means *change*, and denotes that the premisses are to be interchanged. Thus, to reduce *Disamis* of Fig. 3, to *Darii* of Fig. 1, we have to simply convert the major premiss as indicated by *s*, to interchange the premisses, as indicated by *m*, and finally to simply convert the conclusion, as indicated by the last letter *s*. If we perform these operations on *Disamis*, that is, on—

Some M's are P,
All M's are S,
therefore Some S's are P,

it becomes—

All M's are S,
Some P's are M,
therefore Some P's are S,

or *Darii* of Fig. 1. In this it must be noticed that P is the minor and S the major term.

The letter *p* following a vowel indicates that the proposition denoted by that vowel has to be converted *per accidens*. Thus— All M's are P,
All M's are S,
therefore Some S's are P,

or *Darapti* of Fig. 3 becomes, if we convert the minor premiss, *per accidens*, as indicated by the letter *p*—
All M's are P,
Some S's are M,
therefore Some S's are P,

or *Darii* of Fig. 1.

All moods in the second, third, and fourth figures, with the exception of *Baroco* and *Bocardo*, can be reduced to the first figure by conversion and interchange of the premisses. To reduce *Baroco* we must *contraposit* the major and *obvert* the minor premiss.

The *major*, all P's are M, when contraposited, gives —No non-M's are P.

The *minor*, some S's are not M, by obversion, gives —Some S's are non-M.

The conclusion, some S's are not P, remains the same— Some S's are not P.

The resulting syllogism is *Ferio* in Fig. 1, having *non-M* as its middle term.

Bocardo of Figure 3 is—
Some M's are not P,
All M's are S,
therefore Some S's are not P.

By contrapositing the major premiss and interchanging the premisses, we obtain—
All M's are S,
Some non-P's are M,
therefore Some non-P's are S.

This is *Darii* of Fig. 1, the minor term being *non-P*. If we now simply convert the conclusion and then obvert, we have—

Some S's are not P,

which is the original conclusion of *Bocardo*.

No provision is made in the mnemonic lines for the reduction of *Baroco* and *Bocardo* in this way. They were reduced by a special method called *reductio per impossibile*, or *reductio ad absurdum*. The initial *B* indicates that in the cases both of *Baroco* and *Bocardo*, the reduction is to *Barbara*. The premisses of *Baroco* being granted, viz.—

All P's are M,
Some S's are not M,

the conclusion, some S's are not P, either is or is not necessarily true. If the conclusion is false, its contradictory must be true, that is—

All S's are P

is true. This, along with the major premiss of *Baroco*, gives us a new syllogism in *Barbara*, viz.—

All P's are M,
All S's are P,
therefore, All S's are M.

This conclusion contradicts the minor premiss of *Baroco*, which is known to be true. Hence, if we disallow the truth of the conclusion of *Baroco*, and at the same time allow the truth of its premisses, we fall into self-contradiction. The proof of *Bocardo* by this method may be left to the student.

Reductio per impossibile is a round-about method of proof applied to these two cases, because they do not readily admit of reduction to the first figure. It is not usually applied to the other moods, because they can be so readily reduced to Fig. 1, to which Aristotle's *dictum* is directly applicable.

The student will understand now how full of meaning the mnemonic lines are. They have been described as "the magic words which are more full of meaning than any that were ever made."

The student will do well to notice the following peculiarities of the different figures.

Fig. 1. This is the only figure in which A, E, I and O can all be proved. The most important mood is *Barbara*, this being the only mood which is capable of proving A. Further, this is the only figure in which the minor term is the subject of the minor premiss, and the major term the predicate of the major premiss.

Fig. 2. Only a negative conclusion can be established in this figure, for the middle term is the predicate in both premisses, one of which must consequently be negative, to avoid undistributed middle; and one premiss being negative, the conclusion must be negative also by Rule 6.

Fig. 3. In this figure, only *particular* conclusions can be established.

Fig. 4. This figure is somewhat unnatural and is seldom used; it is, in fact, rejected by some logicians.

Lambert, a logician of the eighteenth century, thus described the four figures: "The first figure is suited to the discovery or proof of the properties of a thing; the second to the discovery or proof of the distinction between things; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus."

Résumé of Chapter XVII.

I. *Reduction.*

1. Definitions—
   a. Reduction is the process of changing a syllogism from any mood and figure in which it may be into another mood or figure. Reduction is usually into Fig. 1 from another figure.
   b. *Reductio per impossibile* is a method of proving the conclusion of a syllogism, by showing that a denial of its truth results in a contradiction of one of the premisses.
2. All moods are reducible to Fig. 1 by conversion and transposition, except *Baroco* and *Bocardo.*
3. *Baroco* and *Bocardo* are reducible to Fig. 1 by the help of contraposition and obversion. They may be proved by *reductio per impossibile.*
4. Explanation of *s*, *m*, and *p* in the mnemonic lines.
   a. The letter *s* means *simply convert* the proposition denoted by the preceding vowel.
   b. The letter *m* means *interchange* the premisses.
   c. The letter *p* means *convert per accidens* the proposition denoted by the previous vowel.

II. Characteristics of the figures—

1. Fig. 1 proves A, E, I, and O.
2. Fig. 2 proves only negative conclusions.
3. Fig. 3 proves only particular conclusions.
4. Fig. 4 is unnatural and little used.

Exercises on Chapter XVII.

1. Which figure is of the greatest importance? Why?

2. How is Aristotle's *dictum de omni et nullo* connected with the subject of reduction.

3. What is *Reduction.*

4. Explain all the significant letters in the mnemonic lines.

5. Reduce *Darapti, Disamis, Datisi, Felapton,* and *Ferison.*

6. Reduce *Baroco* and *Bocardo.*

7. Prove *Baroco* and *Bocardo* by *reductio per impossibile.*

8. Why are the premisses of *Fesapo* and *Fresison* not transposed in reduction, like those of the other moods of the fourth figure?

CHAPTER XVIII.

Irregular Syllogisms.

Except in books on Logic one rarely meets with reasoning arranged in strict syllogistic form. This arises from various causes. Sometimes from the desire to be as concise as possible, and sometimes out of compliment to our hearers, we omit one of the premisses, taking it for granted that such premiss is well known to them. Sometimes, too, a premiss is omitted because it is not explicitly present as such to our own minds.

If we say:—*This is valuable, for it is gold*, we are reasoning syllogistically, but the syllogism is not fully expressed. In full our argument would stand—

All gold is valuable,
This is gold,
therefore *This is valuable.*

This is a syllogism of the mood *Barbara* in Fig. 1. The major premiss, *All gold is valuable*, is omitted from the expression of the syllogism in its first form, it being assumed that every one is well aware of the truth of what it asserts.

Such a syllogism is no violation of the rule that every syllogism consists of *three* propositions, for it does in reality consist of three propositions, though only two of these are expressed.

A syllogism in which one proposition is left unexpressed is called an *enthymeme*. An enthymeme is said to be of the *first*, *second*, or *third* order, according as the *major* premiss, the *minor* premiss, or the *conclusion*, respectively, is left unexpressed.

Thus, *he is honest, for he is an Englishman*, is an enthymeme of the *first* order, the unexpressed premiss being the *major*, viz.—

All Englishmen are honest.

An enthymeme of the *second* order is—

He is honest, for all Englishmen are honest,

the minor premiss, *he is an Englishman*, being implied, but left unexpressed.

The conclusion is less frequently omitted, but its omission is of frequent occurrence in witty sayings, the wit often consisting in making the conclusion so evident that it must be drawn by the hearer, and yet leaving it unexpressed.

All Englishmen are honest, and he is an Englishman,

is an enthymeme of the *third* order. The wonderful speech of Mark Antony over Cæsar's body (see *Julius Cæsar*, Act. III, scene 2) contains many enthymemes of the *third* order.

It should be carefully noticed that a fallacy is much more likely to escape detection in an enthymeme than in a fully-expressed syllogism. In testing an enthymeme, of course, the missing premiss must be supplied and the complete syllogism tested.

We not infrequently meet with trains of reasoning in which there is apparently but one conclusion drawn from many premisses. Thus, we may have such a train of reasoning as this—

All A's are B,
All B's are C,
All C's are D,
All D's are E,
therefore All A's are E.

This is in reality a number of connected syllogisms, all of which, except the last, are enthymemes of the third order. Such a series of syllogisms is known as a *sorites*, which means literally, a *heap* of syllogisms.

If we analyse the above we obtain the syllogisms—

1. All A's are B; All B's are C; therefore All A's are C.
2. All A's are C; All C's are D; therefore All A's are D.
3. All A's are D; All D's are E; therefore All A's are E.

The above is an example of the *ordinary* or *Aristotelian sorites*. In forming such a *sorites*, two rules should be observed—

1. *Only one premiss (the first) can be particular.*
2. *Only one premiss (the last) can be negative.*

It will form a useful exercise for the student to find out the reason for these rules.

It is possible to form a *sorites* having its premisses in the reverse order of the above, thus—

All D's are E,
All C's are D,
All B's are C,
All A's are B,
therefore All A's are E.

This is called the *Goclenian sorites*, after the name of Goclenius, a German philosopher (A.D. 1547 to 1628), who first enunciated it.

The special rules for the *Goclenian sorites* are—

1. *Only one premiss (the first) can be negative.*
2. *Only one premiss (the last) can be particular.*

The student can, by analysing any example of the *Goclenian sorites* into its consistent syllogisms, easily discover the reasons for these rules.

The above example of the ordinary *sorites* consists, as we have seen, of three simple syllogisms, the conclusion of the first being a premiss of the second, and the conclusion of the second a premiss of the third.

A syllogism, the conclusion of which stands as a premiss in another syllogism, is called a *prosyllogism*, and the one in which such conclusion is a premiss is called an *episyllogism*.

Thus the syllogism—

All A's are B,
All B's are C,
therefore All A's are C,

is a prosyllogism to—

All A's are C,
All C's are D,
therefore All A's are D,

which is an episyllogism with reference to the preceding prosyllogism.

Résumé of Chapter XVIII.

I. An *enthymeme* is an incompletely expressed syllogism.

1. An *enthymeme of the first order* is a syllogism having its major premiss unexpressed.
2. An *enthymeme of the second order* is a syllogism with an unexpressed minor premiss.
3. An *enthymeme of the third order* is a syllogism with an unexpressed conclusion.

II. *Sorites* is a number of syllogisms joined in a series.

1. Ordinary or Aristotle's sorites has the premiss which contains the subject of the conclusion stated first.
2. The Goclenian sorites has the premiss which contains the predicate of the conclusion stated first.

III. A *prosyllogism* is a syllogism, the conclusion of which is a premiss to another syllogism, this latter being called an *episyllogism*.

Exercises on Chapter XVIII.

1. Give two concrete examples of each order of enthymeme.
2. Analyse the following ordinary sorites into its constituent syllogisms—

I am a hard-working student,
All hard-working students pass their examinations,
All who pass their examinations gain good appointments,
All who gain good appointments have good incomes,
therefore
I shall have a good income.

3. Write the ordinary sorites in Question 2 in the form of the Goclenian sorites.
4. Why can the ordinary sorites have only one premiss, and that the first, particular?
5. Explain *prosyllogism* and *episyllogism*.
6. Why, in the Goclenian sorites, can there be only one premiss, and that the first, negative?

CHAPTER XIX.

Mixed Syllogism.

In all the syllogisms so far discussed, the premisses have both been *categorical* propositions. Syllogisms are frequently met with, however, in which one premiss is a *hypothetical* proposition, as also are those in which one premiss is a disjunctive proposition. Such syllogisms are generally called mixed syllogisms. A hypothetical proposition consists of two parts, the *antecedent* and the *consequent*. The antecedent is the part which states the condition, and is introduced usually by the word *if*, but not infrequently by other equivalent words or phrases, such as—*provided that*, *whenever*, *given that*, *granted that*, etc.; the consequent is the part of the hypothetical proposition which contains the statement made under the condition given in the antecedent. Thus, *if he studies, he will pass his examination*, is a hypothetical proposition, of which *if he studies* is the antecedent, and *he will pass his examination* is the consequent.

Since the truth of the consequent follows from the truth of the antecedent, it is clear that, in *thought*, the antecedent precedes the consequent; in *expression*, however, the consequent almost as frequently as not precedes the antecedent. There is no difference in meaning between—

If the temperature falls it will freeze,

and, *It will freeze if the temperature falls.*

The order *antecedent—consequent*, is the strictly logical order, and this order should be preserved in dealing with mixed syllogisms containing a hypothetical premiss.

The *mixed hypothetical syllogism* may be defined as a syllogism having for its major premiss a hypothetical proposition and for its minor premiss a categorical proposition: *e.g.*—

If it thundered, it lightened (hypothetical major)
But it did thunder, (categorical minor)
therefore, *It lightened.*

Since the minor premiss may affirm or deny the antecedent or the consequent of the hypothetical major premiss we have four *possible* types of mixed hypothetical syllogisms. The premisses of these may be represented in symbols thus:—

1. If A is B, C is D,
   But A is B.
2. If A is B, C is D,
   But A is not B.
3. If A is B, C is D,
   But C is D.
4. If A is B, C is D,
   But C is not D.

Now the question arises which of these four pairs of premisses will give a valid conclusion? It is easy to see that the first pair of premisses justify the conclusion *C is D*. Consider a concrete example of the same form, *e.g.*—

If this is bread, it is good to eat,
But it is bread,
therefore It is good to eat.

This is clearly true whenever the premisses are true; for if not we should have a conclusion, *it is not good to eat*, which with the minor premiss, *it is bread*, contradicts the

major premiss ; that is, it is possible to allow the truth of the major and minor premiss and deny the truth of the conclusion only by falling into contradiction. Hence, Case 1 gives a valid conclusion.

Take a concrete example of the form of the second pair of premisses, *e.g.*—

If this is bread, it is good to eat ;
But it is not bread.

This, clearly, will not give us the conclusion—

It is not good to eat,

for there are many things good to eat other than bread, and it may be one of these. The conclusion, *it is good to eat*, would be consistent with the premisses, as well as the conclusion, *it is not good to eat.*

Hence no conclusion can be drawn.

Consider the third case, of which this is a concrete example—

If this is bread, it is good to eat ;
But it is good to eat.

If any conclusion follows it must be—

It is bread.

But this does not follow, for it may be something other than bread and still be good to eat, in which case our premisses would be true and the conclusion, *it is bread*, false. The conclusion is, therefore, invalid.

This is a concrete example of the fourth type—

If this is bread, it is good to eat.
But it is not good to eat.

If these premisses give a conclusion it evidently is—

therefore *It is not bread.*

The premisses do justify this conclusion ; for if not, then the premisses can be true and the conclusion false. Assume such a case. Then, the conclusion being false, *it is bread*

must be true. From this and the major premiss, it follows (by Case 1) that *it is good to eat*, and this contradicts the minor. Hence, if the conclusion be not true, we fall into contradiction. Therefore, the conclusion, *it is not bread*, is a valid inference from the premisses.

Case 1, in which the minor premiss affirms the antecedent of the major, and Case 4, in which the minor denies the consequent of the major premiss, give valid conclusions, and they are the only ones which do so. Hence, we may use, as a test of the validity of mixed hypothetical syllogisms, the rule: *In mixed hypothetical syllogisms, either the antecedent must be affirmed, or the consequent denied.*

A syllogism, in which the minor denies the antecedent of the major premiss, contains the *fallacy of denying the antecedent*, as it is called; and a syllogism, the minor premiss of which asserts the truth of the consequent, contains what is called the *fallacy of affirming the consequent.*

Mixed hypothetical syllogisms can always be reduced to the form of a categorical syllogism, and can then be tested by the syllogistic rules. The first case considered above is equivalent to—

All things which are bread are good to eat,
This is bread,
therefore This is good to eat.

This is *Barbara* in Fig. 1.

The second case is equivalent to—

All things which are bread are good to eat,
This is not bread,
therefore This is not good to eat,

a syllogism containing the fallacy of the *illicit process of the major term*, of which the fallacy of *denying the antecedent* is the equivalent.

Case 3 in categorical form—

All things which are bread are good to eat,
This is good to eat,
therefore This is bread.

In this there is the fallacy of the *undistributed middle term*, of which the fallacy of affirming the consequent may be regarded as the equivalent.

In the mixed disjunctive syllogism, the major premiss is a disjunctive proposition (see Chapter VIII.), and the minor premiss a categorical proposition. There are two forms which may be written thus :—

1. A is either B or C,
But A is B,
therefore A is not C.
} This is called the *modus ponendo tollens.*

2. A is either B or C,
But A is not B,
therefore A is C.
} This is called the *modus tollendo ponens.*

The name given to the first, *modus ponendo tollens*, means, literally, the *mood which by affirming denies*, and although awkward, is appropriate enough, since in this, a minor premiss which *affirms* is followed by a conclusion which *denies.* The name given to the second form, *modus tollendo ponens*, means the *mood which by denying affirms*, and was given to this form because in it the minor which *denies* is followed by a conclusion which *affirms.*

The *modus ponendo tollens* is valid only if the disjunctives are mutually exclusive. Thus, *honesty* and *roguery* being mutually exclusive, the disjunctive syllogism—

A. B. is either an honest man or a rogue,
He is an honest man,
therefore He is not a rogue,

which is a case of the *modus ponendo tollens*, is valid. The case is different, however, with the following syllogism, which is of the same form—

He is either knave or fool,
He is a knave,
therefore He is not a fool.

In this, the conclusion is invalid, for he may be both knave and fool, *knavery* and *foolishness* not being mutually exclusive.

The *modus tollendo ponens* is always valid, whether the alternatives are mutually exclusive or not Thus, if we know that—

A. B. is either a soldier or a sailor,

and are further told that—

He is not a soldier,

we know certainly that—

He is a sailor.

Résumé of Chapter XIX.

Mixed Syllogisms.

I. Mixed hypothetical syllogisms are those having a hypothetical major and a categorical minor premiss.

1. Rule.—*Either the antecedent must be affirmed or the consequent denied.*
2. Types—

a. If A is B, C is D ; but A is B, therefore, C is D. —Valid.

b. If A is B, C is D ; but A is not B, therefore, C is not D. Invalid—commits the *fallacy of denying the antecedent.*

c. If A is B, C is D ; but C is D, therefore, A is B. Invalid—committing the fallacy of *affirming the consequent.*

d. If A is B, C is D ; but C is not D ; therefore, A is not B. Valid.

3. All mixed hypothetical syllogisms are reducible to the categorical form. By this reduction—

a. The fallacy of denying the antecedent becomes the *illicit process of the major term.*

b. The fallacy of affirming the consequent becomes the fallacy of the *undistributed middle term.*

II. Mixed disjunctive syllogisms are those in which the major premiss is a *disjunctive* and the minor a categorical proposition. There are two types—

1. The *modus ponendo tollens*, having an affirmative minor premiss and a negative conclusion. This is valid only when the disjunctives are mutually exclusive.

2. The *modus tollendo ponens*, having a negative minor premiss and an affirmative conclusion. This is always valid.

Exercises on Chapter XIX.

1. Explain *hypothetical proposition, antecedent, consequent.*
2. Test the arguments—
   a. If that plant is the deadly nightshade, it is poisonous.
   It is the deadly nightshade,
   therefore, It is poisonous.
   b. If he is guilty he will be hanged,
   But he is not guilty,
   therefore, He will not be hanged.
   c. If he is guilty he will be hanged,
   He will be hanged,
   therefore, He is guilty.
3. Reduce the syllogisms in Question 2 to the categorical form and test them by the syllogistic rules.
4. Explain carefully why *A is B* is not a valid inference from the premisses—
   If A is B, C is D,
   But C is D.
5. Is the following a valid argument ?—
   If there were no dew the weather will be foul,
   But there was dew,
   therefore, The weather will be fine.
6. What kind of argument is this—
   I cannot dig ; to beg I am ashamed.
7. Test—
   We shall have a tempest, for it is very warm.
8. Put the following argument into the form of a mixed hypothetical syllogism, and test it—
   "Since the laws allow everything that is innocent, and avarice is allowed, it is innocent."—(*London University.*)

CHAPTER XX.

The Dilemma.

There is still one kind of syllogism which we have not yet explained and which deserves explanation, if for no other reason than this, that its name is a word often used and frequently with no very clear idea of its meaning. It is the *dilemma*.

The dilemma is a syllogism in which the *major* premiss is a *compound hypothetical* proposition, and the *minor* premiss a *disjunctive* proposition. In accordance with the definition we shall obtain four types, as is clear from the following examples—

1. If A is B, C is D; and if E is F, C is D
   (compound hypothetical major premiss),
   But either A is B or E is F
   (disjunctive minor premiss),
   therefore C is D.
2. If A is B, C is D; and if A is B, E is F
   (compound hypothetical major),
   But either C is not D or E is not F
   (disjunctive minor),
   therefore A is not B.
3. If A is B, C is D; and if E is F, G is H
   (compound hypothetical major),
   But either A is B or E is F
   (disjunctive minor),
   therefore either C is D or G is H.

4. If A is B, C is D; and if E is F, G is H
(compound hypothetical major),
But either C is not D or G is not H
(disjunctive minor),
therefore either A is not B or E is not F.

In Example 1, it will be seen that the major premiss has two *antecedents*, but only one *consequent*. To obtain a valid conclusion in accordance with the rules for mixed hypothetical syllogisms, the minor must affirm the antecedent or deny the consequent. It cannot deny the consequent, for, if it did, it would be categorical, and the argument would not be a *dilemma*; the disjunctive affirmation of the two antecedents of the major premiss gives the necessary disjunctive minor.

Dilemmas with *affirmative* conclusions are called *constructive*, those with *negative* conclusions *destructive*; those with *categorical* conclusions are said to be *simple*, and those with *disjunctive* conclusions *complex*. Hence the first example is a *simple constructive* dilemma.

In Example 2, there is but one antecedent and two consequents in the major premiss, hence the only possible way to obtain a disjunctive minor premiss is to disjunctively deny the consequent. We thus obtain a categorical conclusion. This is the *simple destructive* dilemma.

In Example 3, the major premiss contains two antecedents and two consequents. Hence we can with this major premiss obtain two disjunctive minor premisses, for we may have as a minor premiss either the disjunctive affirmation of antecedent which gives Example 3, or the disjunctive denial of the consequent, which gives Example 4. Clearly, in both Examples 3 and 4, we must have a disjunctive conclusion.

Example 3 is known as the *complex constructive* dilemma, and Example 4 the *complex destructive* dilemma. The following table shows the principle on which dilemmas are named.

			Dilemma.
Conclusion.	I. Categorical	1. Affirmative	Simple constructive
		2. Negative	Simple destructive
	II. Disjunctive	3. Affirmative	Complex constructive
		4. Negative.	Complex destructive.

The following are concrete examples of the four kinds of dilemmas—

1. *Simple constructive dilemma.*—A cowardly soldier before battle, might argue:—If I run away I shall be killed for deserting the ranks, and if I stay I shall be killed in battle; therefore, in either case, I am sure to be killed.
2. *Simple destructive dilemma.*—If that ship catches the trade winds, it will have a short and prosperous voyage; but its voyage either will not be short or will not be prosperous: therefore, it will not catch the trade winds.
3. *Complex constructive dilemma.*—If a man is rich he is troubled by the care of his riches, and if he is poor he is anxious to obtain wealth; but either he is rich or poor: therefore, he is either troubled or anxious.
4. *Complex destructive dilemma.*—If our education improves, our commerce will increase, and if we have an efficient navy our commerce will be safe; but either our commerce is not increasing, or it is not safe; therefore, either our education is not improving or we have not an efficient navy.

The characteristic feature of the dilemma is that it allows a choice of two alternatives. In practice, the dilemma is

generally so contrived that, of the two alternatives, one must be accepted, although both are so disagreeable that the opponent would willingly reject both if possible. It is this difficulty of making a choice between equally unpleasant alternatives that is called *being on the horns of a dilemma.*

When a dilemmatic argument offers three alternatives, it is called a *Trilemma;* when four a *Tetralemma*, and when more than four a *Polylemma.*

The best way of escaping from "the horns of a dilemma" is to frame another, such that its conclusion contradicts the conclusion of the original dilemma and is equally evident. This is called *rebutting* the dilemma.

A very old example of a dilemma, and the way in which it may be rebutted, is the following:—

An Athenian mother said to her son:—"Do not enter public life, my son, for if you do and act justly men will hate you, and if in public life you act unjustly the gods will hate you!" The son replied "I will enter public life, for if in it I act justly the gods will love me, and if unjustly men will love me."

It will be seen in this case that the mother in the major premiss of her dilemma gives a one-sided view of the truth and that the apparent cogency of the conclusion depends upon a partial suppression of the truth. It is not the whole truth that her son will be hated by the gods if he acts unjustly. To give the whole truth she should have added, *but loved by men*: so his gaining the hate of men is only the partial consequence of his acting justly, for he will by so doing gain the love of the gods. The flaw in a dilemmatic argument should always be looked for in the major premiss.

Résumé of Chapter XX.

I. Definitions—

1. *Dilemma.*—A syllogism with a compound hypothetical major and a disjunctive minor premiss. The dilemma offers two alternatives.
2. *Trilemma.*—A dilemmatic argument offering three alternatives.
3. *Tetralemma.*—A dilemmatic argument offering four alternatives.
4. *Polylemma.*—A dilemmatic argument offering more than four alternatives.

II. Kinds of dilemmas—

1. *Simple constructive*, having a *categorical affirmative* conclusion.
2. *Simple destructive*, having a *categorical negative* conclusion.
3. *Complex constructive*, having a *disjunctive affirmative* conclusion.
4. *Complex destructive*, having a *disjunctive negative* conclusion.

III. To *rebut* a dilemma is to frame another dilemma having a contradictory but equally cogent conclusion.

Exercises on Chapter XX.

1. What is a dilemma? Can a dilemma offer more than two alternatives?
2. Give a symbolic and a material example of the *simple constructive dilemma.*

3. When is a dilemma said to be *destructive*? Give an example.

4. What kind of argument is this? Is it valid?

All existences are either mental or material; nothing is neither mental nor material; therefore nothing is not an existence. (Dr. Ray's *Logic*.)

5. What is meant by *rebutting a dilemma?*

Rebut:—If virtue were a habit worth acquiring, it must insure either power, or wealth, or honour, or pleasure; but virtue insures none of these; therefore, virtue is not a habit worth acquiring.

6. An old dilemma is the following:—

Protagoras teaches Euathlus rhetoric for a certain fee, half to be paid at once, and the other half when Euathlus wins his first law case. Euathlus undertaking no case, is sued for the remainder of the fee, and is confronted by Protagoras with this dilemma. "You must pay this fee, for if you lose this case you will have to pay me by order of the court, and if you gain it you must pay me in accordance with our agreement, for then you will have won your first case."

Can this be rebutted?

CHAPTER XXI.

Fallacies.

We have already had many cases of conclusions which were not justified by the premisses from which such conclusions were drawn. In all such cases a *fallacy* is said to be committed.

A *fallacy* may be defined as "any unsound mode of arguing, which appears to demand our conviction, and to be decisive of the question in hand, when in fairness it is not."

Since Logic treats of the principles of *valid* reasoning, it may at first appear strange to the student that text books of Logic always include a section on invalid forms of reasoning. It is, however, clear, that a knowledge of the commonest forms of invalid reasoning must be of great service in the effort to avoid invalid reasoning. It is an old and true saying that "the knowledge of contraries is the same." *Valid* and *invalid* reasoning may be regarded as relative terms, a knowledge of either involving a knowledge of the other. The rules of the syllogism are as much rules for avoiding invalid reasoning as they are for reasoning validly. Hence, in treating of *valid* reasoning, we necessarily at the same time, more or less, fully treat of *invalid* reasoning, and the fallacy committed in violating any rule of Logic is, perhaps, best discussed when such rule

is under consideration. It is, however, convenient to bring the various fallacies together into one section and treat of them together. Various attempts have been made to classify fallacies, none of which are entirely satisfactory. The simplest classification is Whately's, and his is a mere modification of Aristotle's. He first divided fallacies into those in the form of expression *(fallacies in dictione)*, and those in the *matter (fallacies extra dictionem)*. The fallacies in the form of expression he called *logical fallacies*. In the case of a logical fallacy the conclusion does not follow validly from the premisses: such fallacy can be detected by the mere logician without any knowledge of the subject matter of the argument. The fallacies in the matter or *material* fallacies are those arguments in which the conclusion does follow from the premisses, but yet is fallacious owing to some fault in the premisses. Such fallacies cannot be detected by the mere logician; for their detection a knowledge of the subject matter of the argument is needed. Material fallacy is in Whately's classification a *species infima*. He enumerates seven individual fallacies of this class. *Logical fallacy* is further divided into the two species—*purely logical fallacy* and *semi-logical fallacy*. A purely logical fallacy is a violation of any of the rules of the syllogism except the second part of Rule 3; a semi-logical fallacy is a violation of the second part of the third rule, or less frequently an ambiguous use of the major or minor term.

The following table shows these classes and the individuals constituting the *species infima*—

- Fallacies are
  - 1. Logical.
    - 1. Purely Logical.
      - *a.* Undistributed middle term.
      - *b.* Illicit process.
      - *c.* Negative premiss or affirmative conclusion from negative premiss.
      - *d.* Particular premisses or universal conclusion from a particular premiss.
      - *e.* Four terms.
    - 2. Semi-Logical.
      - *a.* Equivocation.
      - *b.* Amphibology.
      - *c.* Composition.
      - *d.* Division.
      - *e.* Accent.
      - *f.* Figure of speech.
  - 2. Material.
    - 1. Accident.
    - 2. Converse of accident.
    - 3. Irrelevant conclusion.
    - 4. Petitio Principii.
    - 5. Consequent.
    - 6. False cause
    - 7. Many questions.

The purely logical fallacies have been already discussed, in connection with the rules of the syllogism.

The semi-logical fallacies are nearly all cases of the ambiguous middle term. The rules of the syllogism enable us to tell that there is a fallacy as soon as we know the middle term is ambiguous; but to detect what the ambiguity is is impossible without a knowledge of the subject matter of the argument, hence the name *semi-logical* applied to these fallacies. We proceed to explain each of the semi-logical fallacies. The student should carefully

notice that the names given to the various fallacies are technical terms, and as such have a definite connotation which may differ from the meaning ordinarily attributed to them when used in common parlance.

By *equivocation* is meant the fallacy arising from the use of a term in two different senses in the same argument. This is, perhaps, the commonest of all fallacies, and is frequently very difficult to detect. The examples of ambiguous middle already discussed are cases of *equivocation.* Here is another example—

The end of life is the highest good,
Death is the end of life,
therefore Death is the highest good.

In this argument the middle term, *the end of life,* is used in different senses in the two premisses; in the major premiss it means the *object to be aimed at in life,* but in the minor premiss it means the *cessation of vital action.*

Another example is—

To call you an animal is to speak the truth,
To call you an ass is to call you an animal,
therefore To call you an ass is to speak the truth.

That there is a fallacy in this is evident, but what that fallacy is is not so apparent. The student may exercise himself in explaining it.

Amphibology is a fallacy arising from a loose grammatical structure. Many examples of this fallacy may be culled from newspaper advertisement columns. Here are two examples:—"A piano for sale by a lady about to cross the channel in oak case with carved legs."—"For sale—a large retriever; will eat anything, very fond of children."

Fallacies such as these may raise a laugh, but can scarcely mislead anyone. This is not, however, always the

case with fallacies of *amphibology*. It is impossible to tell from the words alone whether *twice two and three* are *seven* or *ten*.

Occasionally it is difficult or impossible to distinguish the subject from the predicate in a proposition. The Latin language is peculiarly liable to this ambiguity, and it is therefore, a language specially suited for giving oracular answers to replies about the future, answers which are sure to be right, however the event may turn out. If translated into English the ambiguity often disappears. A well-known example is—

Aio te Romanos vincere posse,

that is, in English—

I say that you the Romans can conquer.

Two good examples of (intentional) amphibology occur in Shakespeare. One of these occurs in *Henry VI.*, where it is said—

"The Duke yet lives that Henry shall depose,"

and the other in *Richard II.*, where York says—

"He loves you on my life and holds you dear,
As Harry, Duke of Hereford, were he here."

Let the student try to grammatically parse the words in these two cases and he will at once detect the ambiguity.

The fallacy of *composition* is that particular case of ambiguity of a term in which in one case of its occurrence the term is used in a *distributive* sense, and in the other in a *collective* sense. Thus, if we argue—

All the angles of a triangle are less than two right angles,
A, B, C, are all the angles of a triangle,
therefore
A, B, C, are less than two right angles,
we commit the fallacy of composition.

Other examples are—

A cannot lift this box, B cannot lift it,
therefore A and B cannot lift it.

I can afford to buy all I want, for I can afford to buy A, I can afford to buy B, I can afford to buy C, and A, B, C are all I want.

The fallacy of *division* is the converse of the fallacy of composition. It arises from first using a term in a *collective* sense and assuming that what is true of it in that sense, is also true of it when used in a distributive sense, *e.g.*—

A can sing a duet,
B can sing a duet,
for A and B can sing a duet.

All the angles of a triangle are equal to two right angles,
A is one of the angles of a triangle,
therefore A is equal to two right angles.

The fallacy of *accent* arises whenever greater importance is attributed to any word than is warranted by the context or was intended by the writer or speaker. In the First Book of Kings (Chapter XIII., v. 27), occur the words:—"And he spake to his sons, saying, 'Saddle me the ass.' And they saddled *him*". If in reading this passage the word "him" be strongly accented, as the fact of its being italicised might suggest it should be, we fall into the fallacy of accent, for we suggest a meaning to the word evidently different from that intended to be expressed by it. The translators of our Authorised Version of the Bible had words printed in italics to denote that words so printed had no corresponding words in the original Hebrew text; hence, in the Bible, italics do not denote special accent, but rather an absence of any accent at all.

It is an example of this fallacy: "when the country parson reads out, 'Thou shalt not bear false witness *against* thy neighbour' with a strong emphasis on the word 'against', his ignorant audience leap to the conclusion that it is not amiss to tell lies provided they be in favour of one's neighbour".*

Almost any sentence can be made to convey as many meanings as it contains words by emphasising each word in turn. Fallacies may, and often do, arise from taking for granted that words derived from the same root have corresponding meanings. Thus, it might be thought that he who tells a *lie* is a *liar*. This is not, however, necessarily so, for the word *liar* implies the *habit* of lying. Similarly, it is possible to *thieve*, without being a *thief*, to *drink* without being a *drunkard*, to follow a *craft* without being *crafty*, and so on. Fallacies arising from this cause are known as *fallacies of figure of speech*, or *fallacies of paronymous terms*.

A good example is—

Projectors are unfit to be trusted,
This man has formed a project,
therefore, This man is unfit to be trusted.

Mrs. Malaprop falls into this fallacy when she says she would have her daughter *artful*. Another example may be quoted from Shakespeare's *Richard II.*

Bolingbroke. Go, some of you convey him to the tower.
King Richard. O, good, convey? Conveyors are you all,
That rise thus nimbly by a true king's fall.

The word "conveyor" in Elizabethan English meant a *thief*.

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* St. George Stock's *Logic*.

Résumé of Chapter XXI.

I. Fallacy. Any unsound mode of arguing which appears to demand our conviction and to be decisive of the question in hand when in fairness it is not.

II. Fallacies are—

1. Logical. Subdivided into
   - *a.* Purely Logical.
     1. Undistributed middle.
     2. Illicit process.
     3. Negative premisses or affirmative conclusion where one premiss is negative.
     4. Particular premisses or universal conclusion with one premiss particular.
     5. Four terms.
   - *b.* Semi-Logical.
     1. Equivocation.
     2. Amphibology.
     3. Composition.
     4. Division.
     5. Accent.
     6. Figure of speech.

2. Material Fallacies (*See Résumé of Chapter XXII.*)

III. Definitions—

1. *Equivocation* is the use of one term in two senses in the same argument.
2. *Amphibology*, an ambiguity due to a faulty grammatical structure.
3. *Composition*, fallacy arising from a confusion of the *collective* and *distributive* use of a term.
4. *Division*, the converse of composition—a fallacy arising from a confusion of the *distributive* and *collective* use of a term.

5. *Accent*, a fallacy which consists in attributing to a word greater importance than it is intended to have.
6. *Figure of speech*, or the *fallacy of paronymous terms*, the fallacy which arises from assuming that words from the same root have corresponding meanings.

Exercises on Chapter XXI.

1. Give an example of the fallacy of *equivocation*.
2. Explain and illustrate the fallacy of *composition*.
3. Explain the distinction between fallacies *in dictione* and fallacies *extra dictionem*.
4. What fallacies (if any) are committed in the following arguments?
   a. Water is blue, this is a glass of water, therefore it is blue.
   b. To be acquainted with the guilty is a presumption of guilt. This man is acquainted with the guilty; therefore we may presume that he is guilty.
   c. Five is one number. Three and two are five; therefore three and two are one number.
   d. All the metals conduct heat and electricity, for iron, lead, and copper do so, and they are all metals.
   e. If there is a demand for education, compulsion is unnecessary.
   f. A man will become strong by drinking brandy, for brandy is a strong drink.

g. When Crœsus has the Halys crossed,
A mighty army will be lost.

h. Justice is the profit of others, therefore it is unprofitable to the just man to be just.

i. "Is a stone a body"? "Yes". "Well, is not an animal a body"? "Yes". "And are you an animal"? "It seems so". "Then you are a stone, being an animal".

CHAPTER XXII.

Material Fallacies.

The fallacies *extra dictionem*, or material fallacies, are, as we have seen—

1. The fallacy of accident.
2. The converse fallacy of accident.
3. The fallacy of the irrelevant conclusion.
4. The fallacy of the *petitio principii*.
5. The fallacy of the consequent, more frequently called *non sequitur*.
6. The fallacy of the false cause.
7. The fallacy of many questions.

These we proceed to explain.

The fallacy of accident is committed whenever we argue that a statement which is true as a general rule is true also in a particular case in which the rule, owing to special circumstances or *accidental* conditions, does not hold. A very old example is—

What we buy in the market we eat;
We buy raw meat in the market;
therefore We eat raw meat.

In this the major premiss gives the general rule, which is true in regard to the *substance*, but the minor premiss introduces a case having the *accidental* condition of *rawness* added; and this condition makes the general rule inapplicable.

Again if we argue—

Gooseberries are eatable,
This is a gooseberry,
therefore This is eatable,

our reasoning is fallacious, unless we take care that the middle term *gooseberry* is used exactly in the same sense. If the particular gooseberry referred to in the minor premiss is *unripe*, this accidental circumstance, not being contemplated by the general rule, renders such rule inapplicable to this case.

If, however, under the same circumstances, we argue—

This is not eatable, and it is a gooseberry,
therefore, Gooseberries are not eatable,

we commit the *converse fallacy of accidents*, for we are arguing to a general rule from a particular case, which owing to the accidental circumstance of its not being ripe is not a typical example.

Another example of the converse fallacy of accident, is—
He who hurts another deserves punishment,
The teacher who punishes a refractory pupil hurts another,
therefore, Such teacher should be punished.

In this, *he who hurts another* evidently means he who does so with *malicious intent*; hence, the major premiss makes the general statement *conditionally*, while the minor premiss uses the word *hurts* unconditionally.

This fallacy is often called *fallacia a dicto secundum quid ad dictum simpliciter* (the fallacy of making an unconditional assertion on the ground of an assertion made conditionally). This name should be remembered, although it is such a clumsy one, for it is the name of a very common fallacy.

Sometimes the fallacy of *accident* is called *fallacia a dicto simpliciter ad dictum secundum quid.* (Fallacy of arguing from an unconditional premiss to a conditional conclusion).

It is not always clear whether a fallacy is better explained as a case of the fallacy of accident or of its converse. Thus the fallacy in—

He who hurts another deserves punishment,
The teacher who punishes a refractory pupil hurts another,
therefore, Such teacher deserves punishment,

is explained as a case of the converse fallacy of accident. But if we choose to regard the word *hurts* in the major premiss as being used *unconditionally*, and in the minor premiss *conditionally* (the condition, *for the pupil's good*, being implied), we must regard it as an example of the fallacy of accident.

The fallacy of the *irrelevant conclusion* is often called the *ignoratio elenchi* (ignorance of the refutation). This consists in proving, instead of the proposition whose proof is required, another which is substituted for it. The argument in itself may be valid enough, but the conclusion is not to the point. There are teachers who argue that kindergarten methods are useless, for when children leave the kindergarten they are not so well advanced in reading, writing and arithmetic as they used to be before the introduction of kindergarten methods. But this is beside the point, for what has to be proved in order to establish the uselessness of kindergarten methods, is, that children, after going through the full kindergarten course, are no better trained than they were under the old system.

The street orator commits this fallacy when he dilates upon the great wealth of certain of the nobility, and then proceeds to take for granted that he has proved that such wealth should be distributed among his hearers. What is known as the *argumentum ad hominem* is a particular case of this fallacy. It consists in arguing upon any matter in

such a way that the conclusion reached is based on the character of those connected with it rather than on the merits or demerits of the case itself. If a man is known to be a thief, and is accused of a particular case of theft, it is a difficult matter to base our judgment as to whether he is guilty or not on the facts of the case, and not in any way allow ourselves to be influenced by our knowledge of such a man's character. In olden times, in trial by *Compurgation*,* this logical fallacy had a legal recognition. The only conclusion, however, logically valid, is one drawn *ad rem*, that is, one based on the facts of the case. Such a conclusion is the only one now recognised in English Law, according to which, at present, no evidence of previous crime is allowed until after the jury have given their verdict.

The *argumentum ad hominem* is not infrequently made use of in our Law Courts, where the counsel for the prosecution frequently tries to prejudice the jury against the prisoner by setting forth the heinousness of the crime of which he is accused, or by dilating upon the worthlessness of the prisoner's character.

The *argumentum ad populum* (argument with reference to a people) is of a similar nature, and consists in an appeal to the prejudices or passions of a people in order to warp their judgment.

The *petitio principii* consists in using the conclusion or its equivalent as a premiss from which such conclusion is drawn. It is often called *begging the proof*, and this aptly describes it. The fallacy is common, although it is difficult to give good *short* examples of it. This fallacy and the *ignoratio elenchi* are specially liable to occur in long

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* See any History of England.

argumentative speeches, and being enveloped in a multitude of words are by no means always easy of detection. The teacher of mathematics knows how easy it is for students to beg the proof in working exercises in geometry. The fallacy may be illustrated symbolically thus—

The syllogism M is P,
S is M,
therefore S is P,

is valid if we grant the truth of the premisses; but if we proceed to establish the truth of the minor premiss thus—

P is M,
S is P,
therefore S is M,

we evidently assume the truth of "S is P" which is exactly what has to be proved. Whately gives a good instance of this fallacy. He says, "Some mechanicians attempt to prove (what they ought to have laid down as a probable but doubtful hypothesis) that every particle of matter gravitates equally. 'Why'? 'Because those bodies which contain more particles ever gravitate more strongly, *i.e.*, are heavier'. 'But' (it may be urged) 'those which are heaviest are not always the most bulky'. 'No, but still they contain more particles, though more closely condensed'. 'How do you know that'? 'Because they are heavier'. 'How does that prove it'? 'Because all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space'."

In falling into the fallacy of the *petitio principii*, we are said to be *arguing in a circle*. This should be compared with the *circle in defining*. (See Chapter XIII.)

The fallacy of the *consequent*, or *non sequitur* (it does not follow), as it is often called, consists in drawing a conclusion which has very little or no connection at all with the premisses. Thus, it is evidently a *non sequitur* to conclude that London is a beautiful city, because it is a great city, and all great cities contain great wealth. De Morgan gives as an example of this fallacy—

Episcopacy is of Scripture origin,
The Church of England is the only episcopal church in England,
ergo The church established is the church that should be supported.

The fallacy of the *false cause* is sometimes known as *non causa pro causa* (what is not the cause for the cause). A good example of this fallacy was supplied by Whitefield when he gave as the reason of his being overtaken by a hailstorm the fact that he had not preached at the last town. When one event precedes another we are very likely to take for granted that it is the cause of the phenomenon it precedes. That such a method of argument is wrong is clear, since by it we should have to conclude that night is the cause of day as it always precedes day. This fallacy is denoted by the phrase *post hoc ergo propter hoc* (after this, therefore on account of this). Sometimes what is merely a sign of a phenomenon is regarded as the cause of it, as if it were to be supposed a falling barometer is a *cause* of rain. Not infrequently the effect is regarded as the cause, as when it is thought that much money in a country is the cause of a country's wealth, when it is, in fact, the effect of such wealth.

The fallacy of *many questions* consists in asking, as one question, two requiring different answers, so that the

disputant, if he is allowed to give only categorical replies, commits himself whatever answer he gives. When the counsel for the prosecution says to the prisoner, "How long is it since you ceased to be a forger"? there is involved a fallacy of *many questions*. The question quietly takes for granted that the prisoner was a forger at one time.

A celebrated example of this fallacy is the question put by Charles II. to the then newly-established Royal Society. "Why," he asked, "when a live fish is put into a vessel of water, do the vessel, the water and the fish not weigh more than the basin and water before fish is put in, when if the fish be dead this is not the case"? The learned members of the Society are said to have discussed the question, and formed several hypotheses by way of explanation of such a curious fact. After a considerable time it was suggested by one member that they should try if such were really the case, and after much opposition the experiment was tried, and it was found out that the weight was the same whether the fish was living or dead.

Résumé of Chapter XXII.

Material Fallacies.

1. The *fallacy of accident* or *fallacia a dicto simpliciter ad dictum secundum quid*, is the application of a general rule to a particular case which, owing to special circumstances, does not conform to the rule.

2. The *converse fallacy of accident*, or *fallacia a dicto secundum quid ad dictum simpliciter*, is the proceeding to a general from a particular case, which, owing to special circumstances, is not a typical case.
3. The *irrelevant conclusion*, or *ignoratio elenchi*, is the proof of a truth other than the one required, and the further assumption that what was required has been done. The *argumentum ad hominem* is a frequent form of this fallacy.
4. The *petitio principii*, or *begging the question*, is the taking for granted in the course of the argument the very proposition whose truth is being established.
5. The *fallacy of the consequent*, or *non sequitur*, is the fallacy of drawing a conclusion from premisses with which it has little or no connection.
6. The *fallacy of false cause*, or *non causa pro causa*, is the regarding as cause what is not the cause.
7. The *fallacy of many questions* is the fallacy of asking two questions in one and expecting one categorical answer.

Exercises on Chapter XXII.

1. Give three examples of the fallacy of *accident*.
2. Explain what is meant by a *non sequitur*.
3. Explain the *argumentum ad hominem*.
4. Give an example of the fallacy *a dicto secundum quid ad dictum simpliciter*.

5. What fallacies are committed in the following arguments?

a. A designing man is to be suspected,
This man gains his living by designing,
therefore, He is to be suspected.

b. The verdict of the jury is seldom wrong,
A is a member of the jury,
therefore, His verdict is seldom wrong.

c. The wind blew a gale yesterday because the barometer was very low.

d. When will you become careful?

e. "Shall thine anger burn like fire for ever?"

f. Opium produces sleep because it possesses a soporific quality.

g. A certain Member of Parliament refused to vote for a bill because it was introduced by one who was a bad man.

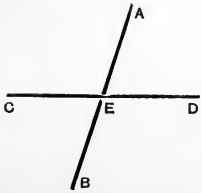
6. Examine the following:—

a. One number must win the prize,
My ticket is one number, and it will therefore win the prize.

b. All that glitters is not gold,
Tinsel glitters,
therefore, Tinsel is not gold. (Whately).

c. Nothing is heavier than platinum,
Feathers are heavier than nothing: therefore,
Feathers are heavier than platinum. (Whately).

Point out the *petitio* in the following:—



Given A B a straight line, and C E, D E two straight lines meeting A B in E, it is required to prove C E D a straight line.

By Euc. I. 15, angle C E B = angle A E D, and angle B E D = angle A E C.

Therefore angle C E B + angle B E D = angle A E D + angle A E C.

But all these angles are together equal to four right angles by Euc. I. 15 cor.

Therefore angles C E B, B E D are together equal to two right angles, and therefore by Euc. I. 14, C E D is a straight line.

CHAPTER XXIII.

Induction.

In this and the following chapters a few of the more important points, with regard to inductive reasoning, will be considered.

We have already seen (Chapter II.) that in an inductive argument the conclusion is more general than the premisses, that is, that the conclusion contains more knowledge than is contained in the premisses; but the premisses contain knowledge known before the inductive inference. Hence, in an inductive argument, we proceed from what is known to what is unknown. Induction has indeed been defined as *an inference from the known to the unknown.* Thus, when Shakespeare says "Murder will out" he is expressing an inductive inference. It had probably come within his experience many times, that murder had been discovered; this experience, together with what he had heard of the experiences of others, formed the premisses on which the general statement was based as an inductive inference.

This conclusion is clearly a much wider truth than all the premisses together, for it contains every case that has occurred since then, as well as every case that may occur in the future.

It is a curious fact that Aristotle, the great Greek philosopher, left Deductive Logic much as we have it now,

but left Inductive Logic in a very undeveloped state. Induction, as a science, was greatly advanced by Lord Bacon, but had to wait till the present century for anything like complete treatment. It first received full treatment at the hands of John Stuart Mill in his "System of Logic," which was published in A.D. 1843.

In an inductive argument there is always an extension of knowledge. How can we make sure that this extended knowledge is reliable? That is the question that Inductive Logic endeavours to answer. It formulates methods to be followed and rules to be observed in the pursuit of knowledge by induction if we wish our conclusions to be real knowledge. As Deductive Logic treats of the rules which must be observed in drawing valid conclusions syllogistically, so Inductive Logic discusses the rules to be observed in drawing inductive inferences.

The necessity for such rules may be more clearly understood by considering two or three cases of invalid inductive inferences. Previous to the discovery of Australia one might have argued, as no doubt many people did, that *all* swans were white, because every swan hitherto seen by Europeans was white. Such conclusion was proved to be an invalid one by the finding of *black* swans in Australia. Again we might argue—

Italy is a peninsula pointing to the south,
Malaya is a peninsula pointing to the south,
South America is a peninsula pointing to the south,
And so on of many others,

therefore—

All peninsulas point to the south.

Such conclusion is, however, invalidated by such peninsulas as Jutland and Yucatan, which point to the north.

Induction, as exemplified in these two instances, in which the conclusion is based on a greater or less number of individual cases taken at random, is known as *inductio per enumerationem simplicem* (induction by means of a simple enumeration of instances).

This is what Aristotle understood by induction. It was ridiculed by Bacon as being a childish thing and giving utterly untrustworthy conclusions. Here is an instance of *inductio per enumerationem simplicem* as carried out by a child of six years of age: he came to the conclusion that no brightly-coloured flowers are scented, because he had noticed of tulips, snapdragons, pansies, and of several other brightly-coloured flowers that they have no scent. The invalidity of this conclusion is evident.

Bacon saw that a small number of well-selected instances were of far greater weight as evidence of the truth of the general statement than an almost infinite number of haphazard instances.

Does *inductio per enumerationem simplicem* ever give reliable conclusions? There are certain cases in which it gives knowledge as certain as human knowledge can be. In the first place, it gives such knowledge wherever the premisses embrace the uncontradicted experience of the human race. How do we know that an object if unsupported falls to the earth? or that man is mortal? By induction, moreover, an induction *per enumerationem simplicem.*

But the facts of experience on which the conclusion in each of these cases is based, are facts supplied by the uncontradicted experience of the whole human race. Very seldom, however, does it happen that we can make use of premisses so numerous as in these cases.

There is another case in which induction gives sure conclusions and which is intimately connected with induction in its origin. The word *induction* means etymologically a *bringing in—a bringing in* of what? The particular facts contained in the general assertion. For example, A and B are engaged in an argument in which A says "All planets go round the sun". B, however, will not accept this statement. What can A do? He straightway makes use of *induction* to prove his general assertion, by bringing in the particular facts contained in the general statement and trying to get B to allow them one by one.

A. Well, Mercury is a planet and goes round the sun. You allow that?

B. Yes.

A. Venus is also a planet and goes round the sun. Do you allow that?

B. Yes.

A. The earth is a planet and goes round the sun?

B. Yes.

How far must A proceed in his bringing in of the particular cases, before he can be considered to have proved his original assertion? If the conclusion be reached, after bringing in only a part of the cases contemplated in the general assertion, we have an induction *per enumerationem simplicem*, and therefore B will be justified in rejecting the conclusion as invalid.

But if A proceeds to bring in every case of a planet, and B allows, in each case, that it goes round the sun, B is then bound to allow the truth of the assertion "All planets go round the sun", for this is nothing but a short way of expressing all the truths which he has individually accepted. A is said to have completed, or *perfected*, his induction, and

the process is what is known as *perfect induction*. The name must not be understood as meaning that this is a better method of induction than ordinary induction. It merely means that the induction has been *completed*, which is the meaning of the Latin word *perfecta*.

If we take as our definition of induction that it is *inference from the known to the unknown*, it is clear that *perfect induction* is not induction at all. Thus, if I argue—

January has not 32 days,
February has not 32 days,
March has not 32 days,
April has not 32 days,
May has not 32 days,
June has not 32 days,
July has not 32 days,
August has not 32 days,
September has not 32 days,
October has not 32 days,
November has not 32 days,
December has not 32 days,
therefore No month in the year has 32 days,

the conclusion is exactly co-extensive with the premisses, and there is, therefore, no proceeding from *the known to the unknown*. Hence, there is no real induction (according to our definition) in the process. Of course, the term induction may be so defined as to make *perfect induction* one of its constituent species.

Résumé of Chapter XXIII.

I. *Induction* is inference from the known to the unknown.

II. *Inductio per enumerationem simplicem.*

1. Definition.—That form of induction in which the premisses are taken at random without any consideration of their relative value.
2. Reliability.—Reliable only when the premisses are co-extensive with the experience of the human race.

III. *Perfect Induction.*

1. Definition.—The summation of a number of premisses into the form of a general assertion.
2. It contains no inference, and is, therefore, not induction at all.
3. The conclusion is as sure as the premisses.

Exercises on Chapter XXIII.

1. Give three examples of induction.
2. Explain *inductio per enumerationem simplicem.*
3. Explain and give an example of *perfect induction.*
4. Why is *perfect induction* not induction at all.
5. What was the first use of induction ?
6. Why was *induction* developed later than *deduction* ?

CHAPTER XXIV.

Cause, Law of Nature, Explanation.

Much has been written as to the exact meaning of the word *cause*, and much that has been written belongs to the subject of metaphysics rather than the domain of Logic. It is one of the most frequently used of technical terms in science, and is generally used with a definite, clearly intelligible meaning. An example will make its meaning clear.

A storm rages at sea, and during the storm a vessel sinks. In such a case, we say the storm is the *cause* of the vessel sinking, and that the sinking of the vessel is an *effect* of the storm.

According to such use, the *cause* is evidently a phenomenon which precedes the effect, and in the absence of which the effect would not have happened. It should, however, be noticed that usually many causes concur in producing the effect. Thus, a person may die from a complication of ailments, any one of which may be spoken of as the *cause* of his death. Indeed, it seldom or never happens that a phenomenon is the effect of only one cause. In fact, philosophically speaking, all the antecedent conditions of which a phenomenon is the outcome are its cause. Amongst all the antecedent circumstances, however, one is singled out as the *cause*, the one so chosen

generally being the new circumstance immediately antecedent to the effect, or a circumstance to which the result specially directs our attention. Thus, if a man falls into a river and is drowned, there are many circumstances which, in combination, produce the effect, *e.g.*, the circumstances causing his fall (his going out in a boat, possibly) his inability to swim, the fact that his nature is such that he cannot exist under water, the depth of the water, etc. But we probably choose the new circumstance of his going out in a boat, or his inability to swim, a circumstance to which his death calls special attention, and look upon it as the *cause* of his death.

Cause may be defined, then, as "that new circumstance or those new circumstances which precede the given effect, and without which such effect would not occur".

"Anything in the absence of which a phenomenon would not have come to pass as it did come to pass is a cause in the ordinary sense."* Given an effect to discover its cause or causes is the main problem of induction.

"Happy is he who has been able to discover the causes of things,"† wrote the Roman poet Virgil nearly two thousand years ago, and his words express a psychological truth, for now, as in his time, one of the keenest of intellectual pleasures is the discovery of the causes of given effects. In seeking for causes, it is always taken for granted—

1. That every event has a cause.
2. That the same cause always produces the same effect.

The former of these two assumptions is what is known as the *Law of Causation*. It would clearly be fatal to investigation did we not think ourselves justified in assuming that

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* Minto's *Logic*.

† Virgil's *Georgics*, II., 490.

the phenomenon in course of investigation has a cause, and therefore admits of explanation. It is really the assumption that no phenomenon is absolutely isolated from all other phenomena. Our knowledge of any phenomenon is in truth a knowledge of the relations between it and other phenomena. The second of the above two assumptions is what is known as the *Law of the Uniformity of Nature.* Any given natural cause produces the same effect to-day that it did a thousand years ago or that it will do a thousand years hence. It is only on this assumption and conditioned by it that science can foretell events. Thus, the astronomer foretells the exact time at which an eclipse of the moon or of the sun will occur, but he can do this only on the assumption that the course of nature will remain uniform. Should such uniformity cease the expected eclipse may not occur.

On what grounds do we rely for our belief in such uniformity of nature? It is a truth established by *inductio per enumerationem simplicem*, which in this case gives knowledge as certain as any knowledge possible to man, for it is the foundation of all knowledge. The simple enumeration is, however, an enumeration of facts in accordance with the unvarying experience of the whole human race.

The student is now in a position to understand clearly the meaning of the term *law* as used in science. A *law* of nature is a uniformity of nature. To say that it is a *law of nature* that all material things gravitate is merely saying that all material things always do gravitate, the law being merely the assertion of the general truth. The student should carefully distinguish this meaning of the term *law* from that intended when we talk of the *laws* of a

country. In this latter case it means a *command*. A *law of science* is expressed in the *indicative mood*, *a law of a country* in the *imperative mood*. Fallacies frequently arise from a confusion of these two senses of the term *law*. The primary meaning of the term *law* is a command ; and as the laws of the land produce in certain respects a *uniformity* of action on the part of the citizens, any uniformity came to be looked upon as the result of some *law*, and finally the statement of the uniformity came to be regarded as the *law* itself.

Science is mainly concerned in the discovery of the *causes* of phenomena. When the *cause* of any phenomenon is known, the particular uniformity of nature to which it belongs is also known. Thus, if I hear a loud noise and learn that it is caused by the firing of a gun, I am satisfied with what I learn only if I know that it uniformly happens, or is a law, that the firing of a gun always produces a loud noise. When we state the uniformity to which a phenomenon belongs, or, which is the same thing, the law under which it comes, we are said to *explain* it. Thus, why does a stone fall to the earth? Because all things when unsupported do so. This answer is an explanation of the phenomenon of the fall of any particular stone. The explanation consists in the assertion that the fall of the stone is not an isolated phenomenon, but is one of a class, or merely an instance of a uniformity. Of course, we may further want an explanation of this explanation and proceed to ask—Why do all things when unsupported fall to the earth? The answer to this is the statement of a wider uniformity. Because every material thing in nature tends to fall towards everything else. We may again ask for an explanation of this and receive the answer—because every particle of matter in the universe attracts every other particle.

Each of these difficulties is in turn explained by assigning it to a wider law or uniformity of nature till we reach the widest known, beyond which we cannot go. Newton's great law is the widest uniformity yet discovered, and it seems almost as if it is never to be explained as forming merely a part of a still wider law; at least at present no answer can be given to the question — Why does every particle of matter attract every other particle? Or if an answer is given it can only be—Because the Creator has made it so, and this is nothing but a confession of our ignorance. Newton's law, then, may at present be regarded as marking out the limit of human knowledge in that direction, beyond which man's intellect cannot pierce.

It may occur to the student to ask what advantage can there possibly be in substituting a wider uniformity for a narrower one, a greater mystery for a smaller one. The answer to this is, that when we know the cause of any effect, it is often possible to imitate nature's processes, and so to produce effects that may be exceedingly beneficial to mankind. Thus, in the passage of light from one medium to another there are certain uniformities of nature exhibited. The learning of these (both effects and causes) has enabled man to produce a great number of useful optical instruments, *e.g.*, spectacles, microscopes, telescopes, and so on. In fact, as Herbert Spencer puts it, "What we call civilisation could never have arisen had it not been for science. . . . . To the progress of science we owe it that millions find support where once there was food only for thousands".*

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* Herbert Spencer's *Education*, p. 46.

Résumé of Chapter XXIV.

I. *Cause.*

1. Definition.—The antecedent or antecedents in the absence of which the phenomenon would not have occurred.
2. Law of Causation. Every effect has a cause.
3. Law of Uniformity of Nature. The same cause always produces the same effect.
4. The Law of Causation and the Law of the Uniformity underlie all human knowledge.
5. Their truth is established by an *induction per enumerationem simplicem.*

II. *Law.*

1. A *law* of the land is an imperative.
2. A *law* of nature is the assertion of a uniformity.

III. *Explanation.*

1. An explanation is a statement of the law of nature, of which the phenomenon is an example.
2. The widest known laws are inexplicable.
3. Use of explanation.
   a. It is a satisfaction to the human mind.
   b. It is of practical value, as it often suggests valuable inventions.

Exercises on Chapter XXIV.

1. What is meant by the *cause* of a phenomenon? Give instances.
2. Night is an invariable antecedent of day, therefore it is its cause. Criticise this.

3. Explain the phrase the "Uniformity of Nature".
4. Point out two common meanings of the word *law*.
5. What is meant by the *explanation* of a phenomenon?
6. What is the Law of *Causation*? On what evidence does its truth rest?
7. What assumption is always made in predicting events?
8. "Happy is he who has been able to discover the causes of things". Comment on this.

CHAPTER XXV.

Observation, Experiment, Hypothesis.

In Chapter I. it was pointed out that all human knowledge depends ultimately on knowledge gained through the senses, that is, gained by experience. Knowledge derived solely from experience is said to be *empirical*. Empirical knowledge is the ultimate foundation of every induction. In order to make this empirical knowledge as extensive as possible, recourse is had to *observation* and *experiment*. To *observe* is to concentrate attention on phenomena as they occur in nature without any attempt to modify such phenomena in any way; to *experiment* is to modify the time, place, or circumstances generally of nature's processes in order to be able the more conveniently and the more effectively to carry on the observation of such processes.

By *observation* we gain what knowledge we can by attending to the phenomena as presented to us in nature, by *experiment* we first make the phenomena, and then *observe* it. Thus, if we attend to the phenomena of lightning and thunder as they occur in nature we are *observing*, but if by any means we generate and then discharge such a quantity of electricity that in the discharge a small flash of lightning and clap of thunder is produced we are *experimenting*. Of course, in this case we do not really create the phenomena of the lightning flash and the thunder clap; we bring

together those natural things from the close proximity of which these phenomena result. This is true in all cases of experiment. Again, the botanist *observes* all the natural phenomena of plant life as far as possible, but he also experiments by varying the conditions under which the plant grows.

It will be noticed that *experiment* without *observation* is worthless. *Experiment* always involves *observation.* The distinction between *observation* and *experiment* as here given should be carefully remembered, but, on the other hand, it should be noticed that no hard and fast line can be drawn between them. The man who takes the trouble to get into the most favourable position in order to observe phenomena may be said to *experiment* as well as he who varies the circumstances of such phenomena so as to render them more suited for observation. Evidently, *experiment* vastly enlarges the range of observation. Experiments with the spectroscope have revealed the elements of which the sun is composed; mere observation could never have accomplished this. By experiment the identity of the lightning flash and the electric discharge in the laboratory was established, and it is difficult to see how this could have been done by observation alone. Experiment possesses also this further vast advantage over observation: that by its means we can at will produce phenomena for the production of which by nature we might have to wait years, or even phenomena which nature might never repeat, or repeat them in such a place as to be out of the reach of human observation. For example, nature has at some past period of time produced diamonds. Possibly nature is producing them at the present time, but if so, such production is going on deep down in the earth where the process cannot be observed; by the help of experiment,

however, it is not at all improbable that nature's process may be so imitated that diamonds will be produced artificially. Indeed diamonds, though not valuable ones, have already been artificially made.

Again, consider how rapidly our knowledge of the phenomena of electricity has advanced of late years. This rapid advance is due entirely to the application of experiment. If for our knowledge of electricity we had to rely on observation alone, the times at which electrical phenomena could be studied would be limited to the occasions when thunderstorms or the phenomena of the Northern Lights occur. Progress in knowledge under such conditions cannot be other than very slow. By experiment, however, the phenomena of electricity can be produced at will in the laboratory. The importance of experiment is seen by the fact that the experimental sciences (those in which our knowledge is dependent mainly on experiment, such as chemistry and physics) have advanced more during the last hundred years than they did during the preceding two thousand years, and that it is during the last hundred years that experimental enquiry has been so vastly increased. Probably one great reason why the Greeks made so little progress in the physical sciences as they did, is that they made scarcely any use of experiment.

The following extract from Sir John Herschel's *Discourse on the Study of Natural Philosophy* puts the relation between observation and experiment in a very clear light. He says:—"Experience may be acquired in two ways: either, first, by noticing facts as they occur, without any attempt to influence the frequency of their occurrence, or to vary the circumstances under which they occur; this is *observation;* or secondly, by putting in action causes and

agents over which we have control, and purposely varying their combinations, and noticing what effects take place; this is *experiment*. To these two sources we must look as the fountains of all natural science. It is not intended, however, by thus distinguishing observation from experiment to place them in any kind of contrast. Essentially they are much alike, and differ rather in degree than in kind; so that, perhaps, the terms *passive* and *active* observation might better express their distinction; but it is, nevertheless, highly important to mark the different states of mind in enquiries carried on by their respective aids, as well as their different effects in promoting the progress of science. In the former, we sit still and listen to a tale, told us, perhaps, obscurely, piecemeal, and at long intervals of time, with our attention more or less awake. It is only by after rumination that we gather its full import; and often, when the opportunity is gone by, we have to regret that our attention was not more particularly directed to some point which at the time appeared of little moment but of which we at length appreciate the importance. In the latter, on the other hand, we cross-examine our witness, and by comparing one part of his evidence with the other, while he is yet before us, and reasoning upon it in his presence, are enabled to put pointed and searching questions, the answer to which may at once enable us to make up our minds. Accordingly it has been found, invariably, that in those departments of physics where the phenomena are beyond our control, or into which an experimental enquiry from other causes has not been carried, the progress of knowledge has been slow, uncertain and irregular; while in such as admit of experiment, and in which mankind have agreed to its adoption, it has been rapid, sure and steady."

Experiment is at a disadvantage as compared with observation in all those cases in which the phenomena dealt with are *effects*, whose *causes* are being sought. Granted a certain cause we can probably put it into action and learn what its *effect* is; but given an *effect* we cannot directly experiment to discover its *cause*. In such case we have first to guess a cause, and we can generally by experiment find out the effect of our supposed cause. If the effect so obtained be the same as the given effect the supposed cause is probably the true cause of the phenomenon. This, however, is not always the case, for different causes may, and often do, produce the same effect. Thus, let the death of a man be the given effect whose cause has to be discovered. By proving that a certain cause is capable of producing death as its effect we clearly do not prove that the death in question was due to this cause. When, in order to explain some effect, we assume a cause we are said to form a *hypothesis*. A *hypothesis* is a proposition whose truth is assumed for the purpose of deducing from it a conclusion in agreement with some observed fact. More briefly a hypothesis is an imagined cause of some known effect the real cause of which is unknown. We are all constantly making hypotheses, though we may not know that we are doing so. In fact, the human mind is so constituted that in the presence of a strange phenomenon it cannot resist the impulse to frame a hypothesis by way of explanation of such phenomenon.

Thus, on hearing of the sudden death of an acquaintance, we instantly begin to imagine causes, that is, to frame hypotheses, in order to account for it. When a hypothesis is proved to be true it is no longer to be regarded as a hypothesis, but rather as a valid induction. Newton's Law

of Gravitation was at first a mere hypothesis, but is now received as one of the most certain of inductions.

Hypotheses more frequently than not turn out to be false. It is said that Kepler, in order to explain the apparent movements of the planet Mars, made no fewer than nineteen hypotheses, all of which proved false, before he made the hypothesis that Mars moved in an elliptic orbit round the sun, which was at one focus, which hypothesis turned out to be the true one. In such cases the false hypotheses should not be looked upon as useless, for they, along with the efforts spent in testing them, often prove stepping-stones to the true hypothesis.

It should be noticed that *Imagination* is the chief faculty directly exercised in framing a hypothesis.

Résumé of Chapter XXV.

I. Experience is the foundation of all knowledge. It is gained
   1. By observation.
   2. By experiment.

II. Observation.
   1. Definition.—The attending to phenomena as they occur in nature without any attempt to modify the circumstances under which they occur.
   2. Mere observation is frequently of little use for advancing knowledge, owing to difficulties of time and of the place of the occurrence of phenomena.

III. *Experiment.*

1. Definition—Is the process of modifying the circumstances under which phenomena occur in order to facilitate the observation of them.
2. Advantages—
   a. It enlarges the range of observation.
   b. By it we can produce phenomena at will which nature produces only at rare intervals.
   c. By it we produce phenomena in situations convenient for observation.

IV. *Hypothesis.*

1. Definition.—Any supposition made to explain a phenomenon whose cause is unknown.
2. A false hypothesis often leads to the true one.

Exercises on Chapter XXV.

1. What is empirical knowledge?
2. What is the object of observation and experiment?
3. Explain the difference between observation and experiment.
4. Mention three sciences carried on mainly by *observation*, and three carried on mainly by *experiment*.
5. What advantages does experiment possess over observation?
6. Does observation possess any advantage over experiment?
7. Explain the term *hypothesis*. Give examples.
8. Show how a false hypothesis may prove useful.

CHAPTER XXVI.

Classification.

In the last chapter we saw that observation and experiment are necessary to give us empirical knowledge which we may use as a foundation for our inductions. To make inductions from this knowledge it is necessary first to arrange our facts so that those most closely connected may be closest together, that is, we must *classify* the facts given by experience. To classify is to arrange objects in groups in such a way that all the individuals in any group have certain attributes in common. We are in every-day life constantly carrying on this process. Thus we classify people into those we like and those we do not like, or into rich and poor, or into English and foreign, or into adults and those not adult, and in many other ways; books we, perhaps, classify into those that are interesting and those that are uninteresting; newspapers we arrange in classes according to the interval of time between successive issues; in fact, every time we use a general term we may be said to classify. Thus, the term *tree* suggests to the mind *trees*, as distinct from all other objects, that is, for the moment we classify all objects into *trees* and *non-trees*.

In many branches of knowledge at the present day, the number of known truths is so enormous that, without the aid of classification it would be impossible to deal with them.

It is said that botanists distinguish more than 300,000 different species of plants, and that zoologists distinguish about 2,000,000 species of animals. It is very evident that the immense amount of knowledge implied in these statements could never be learned by any individual unless he could deal with it in groups.

In forming a class a certain attribute is (or certain attributes are) chosen to constitute the connotation of the class name. All objects which possess such attribute (or attributes) are placed in the class, and all objects which do not are excluded. Hence, if we know that an object belongs to a certain class, we know also that it possesses those attributes which form the connotation of the class name.

The selection of the quality (or group of qualities) the presence or absence of which is to determine whether or not an object belongs to the class, is a most important matter. This selection settles the connotation of the name of the class. What quality is selected will depend mainly upon the purpose in view; thus, the length of life of a plant is an important matter for the gardener to consider, and for him the classification of plants into *annuals*, *biennials*, and *perennials*, is a convenient and useful one; but such a classification would be of little use to the botanist because the duration of the life of a plant has no very close connection with his purpose.

It may, in fact, be given as a fundamental principle in classifying, that the classification should be appropriate to the purpose in hand. A classification which is made to depend on the presence or absence of an attribute, or group of attributes, without reference to general similarity is said to be *artificial*. This is the kind of classification carried on

in every-day life. For example, a classification of books according to colour, or according to size, is artificial.

The scientist in classifying endeavours to arrange objects in classes according to their general similarity in such a way that members of a class may be known to have many attributes in common. A classification of this kind when properly carried out is said to be *natural*. The object aimed at then in a natural classification is the bringing nearest together those objects which are, taking all things into consideration, most alike. The Linnæan system of classifying plants was an artificial one, as the position of any plant in this system depended mainly upon the number of stamens it contained, and plants essentially unlike may contain the same number of stamens, and, further, flowers, even on the same plant, not infrequently differ in the number of their stamens. Hence, in such a system, flowers essentially alike might be far removed from one another, and flowers essentially unlike might be arranged side by side in the same class. This system has consequently been discarded, and plants are now classified upon a natural system of classification.

Flowering plants are divided first of all into two classes, *dicotyledons* and *monocotyledons*, the former containing all plants whose seeds are composed of two seed lobes or cotyledons, the latter all those the seeds of which contain but one cotyledon. The *naturalness* of this division lies in the fact that dicotyledons have many other important qualities in common besides the number of cotyledons in the seeds. Thus, in dicotyledons, the parts of the flower are arranged in multiples of *four* or *five*, the veins of the leaves are reticulate, and the stems increase in thickness by the addition of successive layers on the outside ; in

monocotyledons the parts of the flower are arranged in multiples of *three*, the veins of the leaves are more or less parallel, and the stems thicken from the inside. Hence, in the first step of this classification, plants which are essentially alike are brought together into the same class, and this is also carefully carried out throughout the classification.

It is evident that in any system of classification, natural or artificial, the classes and sub-classes could not be remembered unless they were named. The names of the classes are general terms, and connote all the attributes which determine the class. The names applied to the various classes in any classification are known as a *nomenclature*. *Dicotyledons*, *calycifloræ*, *rosaceæ*, *prunus*, *prunus spinosa* form part of the nomenclature of botany.

In addition to a nomenclature a large number of terms is usually required in any branch of science for designating the parts of, or describing, the individual objects. These terms constitute the *terminology* of the subject. Botany again supplies an excellent example of a terminology. Whewell says:—"The formation of an exact and extensive descriptive language for botany has been executed with a degree of skill and felicity which, before it was attained, could hardly have been dreamt of as attainable. Every part of a plant has been named, and the form of every part, even the most minute, has had a large assemblage of descriptive terms appropriated to it by means of which the botanist can convey and receive knowledge of form and structure as exactly as if each minute part were presented to him vastly magnified."

The following description of *Lonicera* (Honeysuckle) taken from Hooker's *Students' Flora of the British*

Islands will give some idea of the completeness of botanical terminology:—"Erect, prostrate, or climbing shrubs, with scaly buds. *Leaves*, opposite, entire, ex-stipulate, of the young shoots sometimes lobed. *Flowers* in peduncled cymes or heads, often connate in pairs by the ovaries, and subtended by connate bracheoles *Calx-tube* ovoid or sub-globose; teeth 5, often unequal. *Corolla*, tubular, funnel or bell-shaped; tube equal or gibbous at the base; limb oblique or 2-lipped, 5-lobed. *Stamens*, 5. *Disk*, humid; *Ovary*, 2-3-celled; style filiform, stigma capitulate; ovules, many in the inner angle of each cell. *Berry*, fleshy, 2-3-celled; cells few-seeded, sepha sometimes wanting. *Seeds*, ovoid or oblong, testa crustaceous".

To one ignorant of botany much of this may seem gibberish; but to the skilled botanist such a description causes a very distinct and accurate image of Lonicera to rise in the mind.

Résumé of Chapter XXVI.

I. Classification.

1. Definition.—Arrangement of objects in groups in such a way that those in the same group have certain common attributes.
2. Necessity.—Objects are so numerous as to be unmanageable without classification.
3. Rule for classification.—It must be appropriate to the purpose in hand.

II. Kinds of Classification.

1. Artificial. — When it depends simply on the presence or absence of a particular attribute.
2. Natural.—When it depends on general similarity, the most perfect example is the classification of plants.

III. Language of Classification.

1. Nomenclature.—The names of the classes in any system of classification.
2. Terminology. — The terms used in describing individual objects.

Exercises on Chapter XXVI.

1. What is classification? Why is it important?
2. How is classification an aid to induction?
3. How is the connotation of any general term connected with classification?
4. What is the difference between an *artificial* and a *natural* classification? Give examples.
5. What is meant by a *nomenclature?*
6. What is meant by a *terminology?*
7. What is the use of a full terminology?

CHAPTER XXVII.

The Canons of Inductions.

We have already seen that inductive inferences are frequently invalid, and consequently, that such inferences must be tested before they can be received as valid inductions.

Certain rules have been drawn up, setting forth the method of procedure that must be followed in order to establish valid inductions. These rules were first enunciated by Mill in his *System of Logic*, and were called by him, *canons of induction*. These we proceed briefly to explain.

The first of these canons Mill enunciated thus:—"If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon". If we pursue knowledge in accordance with this rule we are said to follow the *method of agreement*.

The method may be clearly illustrated by means of symbols. Let capital letters denote antecedents, and the corresponding small letters consequents.

Suppose the phenomena—

A,B,C,	are followed by the	phenomena	a,b,c
A,B,D,	,,	,,	a,b,d
A,B,E,	,,	,,	a,b,e
A,C,D,	,,	,,	a,c,d
A,C,E,	,,	,,	a,c,e

in accordance with the method of agreement we may assert that A and *a* are, to a high degree of probability, connected together by some causal relation.

Jevons expresses this canon more concisely in the words —"the sole invariable antecedent of a phenomenon is probably its cause". Thus, if we find out by observation or by experiment that the formation of ice is always preceded by a certain degree of cold, and that this is, as far as we can tell, the only invariable antecedent, we may conclude that almost certainly a certain degree of cold is the cause of the formation of ice. To take a more homely example, if a person notices in three or four successive years that at the end of August he is in unusually good health, and further, that the only invariable antecedent to this state of health, as far as he knows, is a month's holiday, he may conclude that in all probability the holiday is the cause of his unusually good health.

The method of agreement does not give absolutely safe conclusions. This arises from the fact already noticed that the *same* effect may be due to different causes. It is exceedingly difficult, or, in many cases impossible, to carefully observe *all* the conditions of a phenomenon, and there is always the possibility, if the method of agreement only is employed in the investigation, that the cause is one of the unobserved circumstances. In the symbolic illustration of the method given above it may be that the fact that in all the observed cases *a* is preceded by A is a mere coincidence, and that the real cause of *a* is Z, a circumstance which has escaped notice. Hence, when a probable conclusion has been reached by the method of agreement, a further test must be applied before the conclusion can be wholly relied on. This test is supplied by

Mill's canon of his method of difference. This is:—"If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause of the phenomenon".

This may be illustrated thus: suppose the set of antecedents A, B, C, D, be followed by the consequents *a*, *b*, *c*, *d*: and further that B, C, D, be followed by *b*, *c*, *d*, then this canon asserts that A and *a* are causally related. It is clear *a* may be the effect of A, for it follows it in time, and that *a* cannot be the effect of B, C, or D, for *a* is absent when these are present; further, supposing A, B, C, D, are the only antecedents of *a*, then *a* must be the effect of A, B, C or D; therefore it is the effect of A.

We know a candle burns in the air, and hence among the antecedents of the burning candle is the presence of oxygen. To prove that oxygen is the cause of the burning of the candle we must remove oxygen from the air so that the presence of oxygen may not be an antecedent of the burning of the candle. This can easily be done experimentally, and then it is found that the candle no longer burns. The first part of the process, the observation of the fact that whenever a candle burns oxygen is present, leads by the method of agreement to the probable conclusion that oxygen is the cause of the candle burning; the second part of the process, the elimination of oxygen from among the antecedents and the observation that the candle then fails to burn, is an application of the method of difference and gives us the sure conclusion that the presence of oxygen and the burning of the candle are causally connected.

The method of difference is essentially a method of experiment for in experiments we generally produce phenomena in such a way that any given antecedent may be totally or partially eliminated.

Mill's third canon is :—" If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of circumstances differ is the effect, or the cause, or an indispensable part of the cause of the phenomenon ".

Suppose we observe on many different occasions that A is always followed by *a* ; and, further, suppose we notice that sets of antecedents which do not include A are invariably followed by sets of consequents from which *a* is absent, then this canon says that almost certainly A is the cause of *a*.

Thus, if we observe on many occasions that the roads become exceedingly dry, consequent upon an east or north-east wind, and that they seldom dry (the canon strictly applied would require that they should *never dry*) so quickly when the wind is in any other quarter, we may feel pretty sure that the rapid drying of the streets and the blowing of the east wind are connected by some causal relation. This is merely a method of combining the method of agreement and the method of difference, and Mill consequently called it the *joint method of agreement and difference.*

The canon of the fourth method of inquiry is :—" Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents ". This is called the *method of residues.*

To illustrate this symbolically, suppose the antecedents A, B, C are found to be followed by the consequents a, b, c; suppose, further, that we have already proved B to be the cause of b, and C of c, then, in accordance with this canon, A is the cause of a. This method is largely followed in the pursuit of those branches of science to which mathematics has been applied. Especially is this the case with astronomy. Herschel, in his *Outlines of Astronomy*, says:—

"Almost all the greatest discoveries in astronomy have resulted from the consideration of what we have elsewhere termed *residual phenomena*, of a quantitative or numerical kind, that is to say, of such portions of the numerical or quantitative results of observation as remain outstanding and unaccounted for after subducting and allowing for all that would result from the strict application of known principles. It was thus that the grand discovery of the precession of the equinoxes resulted, as a residual phenomenon, from the imperfect explanation of the return of the seasons by the return of the sun to the same apparent place among the fixed stars".

Soon after the discovery of the planet Uranus, it was noticed by astronomers that this planet presented an irregularity or perturbation in its orbit which could not be accounted for by any or all of the known forces acting on it. The cause of this residual effect was surmised to be another planet whose position was calculated from the observed amount of the perturbation of the orbit of Uranus. Observation showed this calculation to be surprisingly accurate, for on turning his telescope to the point of the sky where it was calculated the unknown planet must be, Dr. Galle, of Berlin, immediately found the planet now

known as Neptune. The discovery of Neptune is a remarkable instance of the extension of knowledge possible as a result of reasoning methodically carried out.

The canon of the fifth method, the *method of concomitant variations*, Mill enunciated thus:—"Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation".

This method is applicable in those cases in which we cannot totally eliminate any given antecedent, but can cause such antecedent to vary in intensity. Thus, it is impossible to eliminate from a material substance *all* the heat contained therein, but the quantity of heat in such substance can be made to vary, and it is found that in every case, when the quantity of heat varies, there is an accompanying or concomitant variation in the volume of the substance. Hence this canon justifies the conclusion that increase of heat and increase of volume are connected by some causal relation.

Another application of this method of concomitant variations is what is frequently referred to as the *historical method*. This is applied mainly in ethical and political science, and has received the name *historical* because history is called upon to supply data which otherwise would be unattainable. "A certain institution, custom, or opinion is traced throughout various stages of society, and its growth or decline is connected with that of some other institution, custom, or opinion, or with the general state of civilisation prevalent throughout these periods, it being argued in the latter case that, as civilisation advances, the institution, custom, or opinion has grown or

declined as the case may be".* Many of our existing institutions have been explained by this method. Thus, the modern state has been shown to be a development and extension of the family government of patriarchal times.

The student who wishes for a fuller discussion of these canons of induction should read Mill's *Logic*, Book III., Chapters 8 and 9.

Résumé of Chapter XXVII.

The canons of the inductive methods are :—

1. The method of agreement.—If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.
2. The method of difference.—If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former circumstances in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause of the phenomenon.
3. The joint method of agreement and difference.—If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the

* Fowler's *Inductive Logic*, p. 201.

absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect or the cause, or an indispensable part of the cause of the phenomenon.

4. The method of residues.—Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

5. Method of concomitant variations. — Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.

Exercises on Chapter XXVII.

1. Explain why it is that a conclusion based on the method of agreement may be invalid.
2. Which of the inductive methods gives the most certain conclusions ?
3. Illustrate the joint method of agreement and difference.
4. Why is the method of concomitant variations indispensable in scientific inquiry ?
5. Illustrate the application of the method of concomitant variations.
6. Is *deduction* made use of in any of the inductive methods of inquiry ?

CHAPTER XXVIII.

Analogy.

Similarity or likeness is the ground of all reasoning. When we observe that many things agree in possessing certain attributes, and hence conclude that all the things of the same class possess those attributes, our conclusion is an induction; but when we compare two objects, and find that they have many attributes in common, and then, on finding that one of the objects has another attribute we infer that the other has also the same attribute, our argument is an *analogical* one. For example, knowing many points of resemblance between the Earth and Mars, and knowing further that the Earth is inhabited, we may infer from analogy that Mars also is inhabited. In this case, however, no one would place very great reliance on our conclusion.

An argument from analogy gives at best only probable conclusions. The degree of probability of any analogical inference depends on two things—

1. The relative number of known points of resemblance between the two objects to the number of known dissimilarities between them.
2. The importance of the resembling attributes in the two objects.

This second point is, however, difficult to determine and presupposes such a knowledge of the two objects as to leave

little room or need for any analogical argument from one to the other. Consequently in reasoning from analogy we have to rely mainly on the relation between the number of known similarities to the number of known dissimilarities between the two objects. In fact the fraction—

$$\frac{\text{Number of known points of resemblance between A and B}}{\text{Number of known points of disagreement between A and B}}$$

may be regarded as a more or less accurate measure of the probability of any conclusion regarding B based on an argument from analogy with A.

If we know that A resembles B in four ways and that A differs from B in four ways, we can infer nothing by analogy, but if A is known to agree with B in five points and to disagree in only three, we may infer from analogy that any additional fact *m* known to be true of A is more likely than not to be true of B also.

If our inference is to be purely one from analogy, the known common properties of A and B must not be known to be entirely unconnected with *m*, for if they are, their presence or absence can in no way affect the probability of the presence of *m*; nor must they be known to be connected with *m* by some causal relation, for in that case the argument ceases to be one from analogy and becomes inductive or deductive.

Thus, if we argue that because Mars resembles the Earth in having large patches of ice and snow round the poles, as well as in many other respects, therefore Mars resembles the Earth in having an atmosphere, our knowledge that Mars possesses an atmosphere is not due merely to analogy, for we know deductively that wherever there is snow there must be an atmosphere. Conclusions drawn from mere analogy are never to be trusted until such conclusions have

been tested by the method of induction. A man knows a mushroom is good to eat; he sees a fungus that resembles a mushroom very closely, and concludes from analogy that it is also good to eat; he eats it, and is poisoned.

The known points of resemblance between two objects are, however, sometimes so many and so great, that, knowing any fact of the one, we do not for a moment hesitate to assert the same fact of the other. Thus, suppose a new plant to be found presenting a thousand points of resemblance to the deadly nightshade, and only one known point in which it differed from it, the probability that its berries would be poisonous would be a thousand chances to one. Such a conclusion may well be compared with one reached by an *inductio per enumerationem simplicem*. The student should never rest satisfied with such inferences, but should in all cases endeavour to test them by the canons of induction. The great use of analogy is to suggest provisional conclusions to which to apply the canons of induction.

The term *analogy* has been used so far as connoting any resemblance whatever. It is, however, sometimes used in the restricted sense of a *resemblance of relations*. Thus, when a colony speaks of England as the *mother* country it is implied that the relation existing between the colony and England resembles, or is analogous to, the relation existing between a daughter and her mother. There is not much resemblance between a gardener and a teacher, or between a plant and a child; the relation, however, between the gardener and the plant is in some respects *analogous* to the relation between the teacher and child.

Résumé of Chapter XXVIII.

Analogy.

1. Definition.

   An argument from analogy is one based on any resemblance whatever.

   A resembles B in many respects, it probably does so in any other respect.

2. Difference between analogy and induction.

   a. Induction gives sure conclusions, analogy does not.

   b. Induction is reasoning that what is known of some members of a class is true of all the class ; inference by analogy is reasoning that an attribute belonging to one object also belongs to another object known to resemble it in certain respects. An analogical inference is frequently a mere preliminary to induction.

3. In reasoning from analogy the known similarities or dissimilarities must not be known to be connected or unconnected with the inferred fact.

4. Analogy is sometimes defined as a resemblance of relations.

Exercises on Chapter XXVIII.

1. Give an example of an argument from analogy, giving—

   a. A very doubtful conclusion.

   b. An almost certain conclusion.

2. What is the meaning of analogy ?

3. It is said that gold was discovered in Australia by a Californian who was struck with the apparent likeness between the rocks of Australia and the gold-bearing rocks of California. What kind of argument did he use? Answer fully.
4. Give an analogical argument to prove that the earth rotates on its axis.
5. On what does the strength of an argument from analogy depend?
6. What is a false analogy?

CHAPTER XXIX.

Relation between Induction and Deduction.

We have already (see Chapter XXIII.) seen that deduction depends on previous induction. In every syllogism the major premiss makes a general assertion. This general assertion may be the conclusion of another syllogism, and so depends for its truth on a wider statement, which again, may depend on a still wider fact; but if we trace back the foundation of the statement sufficiently far, we shall at length find that it ultimately is based on an assertion which cannot be the conclusion of a syllogism, and which must, therefore, be established by induction. Further, the premisses of an inductive argument are facts supplied by experience; hence, all our knowledge is ultimately derived from experience. Sir John Herschel says:—"We have thus pointed out to us, as the great, and indeed only ultimate, source of our knowledge of nature and its laws, *experience;* by which we mean, not the experience of one man only, or of one generation, but the accumulated experience of all mankind in all ages, registered in books or recorded by traditions".*

This explains why it is that the different branches of science tend to become more and more deductive. Fundamental

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* *Discourse on the Study of Natural Philosophy*, p. 67.

principles are first established by experience, and then these are used as premisses for deductive arguments. Those sciences in which experience has already supplied all the necessary fundamental principles, and the progress of which, therefore, is due entirely to deductive reasoning, are often called the *deductive sciences*, and if their fundamental principles are established so as to admit of no doubt they are then frequently spoken of as the *exact sciences*. The best type of an exact science is some branch of mathematics, *e.g.*, the science of geometry. Mathematics, since all reasoning in it is based upon a few fundamental facts established by experience, is entirely deductive; hence the gradual application of mathematical reasoning to any branch of science marks its progress towards the *deductive* or *exact* stage.

On the other hand those branches of science, the progress of which is due mainly to the gaining of additional facts by experience (by observation and experiment), are called the *inductive sciences* or the *experimental sciences*. The physical sciences, such as heat, light, sound, electricity, chemistry, are regarded as the inductive or experimental sciences *par excellence*; but in the last few years much of the advance made in these has been due to the application to them of mathematics; in other words, even the typically inductive sciences are no longer purely inductive.

When any branch of study is pursued inductively the *method of discovery* is said to be followed. The position of the student is in such a case analogous to that of the discoverer, seeing that he is in the midst of a mass of particular facts of experience from which he has to discover the general underlying truths. To accomplish this he must collect, arrange, and analyse his facts. To analyse

is to break up complex phenomena into simpler ones. Thus the inductive inquirer following the method of discovery tries to understand the complex phenomenon of vision. His first step is *analysis*. He analyses the phenomenon into several simpler phenomena which may be separately studied. In every case of vision the following points have to be considered—

1. The luminosity of the object seen.
2. The transmission of the effect from the object to the eye.
3. The action of the eye.
4. The action of the retina.
5. The transmission of the retinal disturbance to the brain.
6. The transition from a material brain disturbance to the mental phenomenon of vision.

He then proceeds to study these phenomena separately. One may compare his position to that of a man who wants to break a bundle of rods. So long as they are together in a bundle, to break them is impossible; but let the bundle be broken up (analysed) into the individual rods and each can separately be broken. This method is frequently referred to as the *à posteriori* method, for, by following it, we reason from what comes after, or from effects, to what goes before, or to causes.

The *method of discovery* may be regarded as a synonym of the *inductive method; analysis* is one of the steps taken in following that method; and the reasoning involved is *à posteriori*. The *deductive method* is sometimes called the *method of instruction*. It proceeds from general principles to particular cases, from causes to effects, from what proceeds to what follows. Hence, the reasoning is often called

à priori. An *à priori* argument is one based on some previous general principle assumed to be true. The danger in an *à priori* argument is that our ultimate assumption may be wrong. Conclusions reached *à priori*, are to be trusted only when we know that the assumptions involved are established inductions. Conclusions reached *à priori* should always be brought to the test of experience; when such conclusions are found to tally with the facts of experience, we may feel quite confident of their truth. Thus, taking for granted what psychology has to teach concerning the constitution and development of the human mind, we may, by an *à priori* argument, reach the conclusion that in teaching children the right method is to begin with the concrete and proceed thence to the abstract.

This *à priori* conclusion is found to agree with the conclusion reached by practical teachers from their own experience, or *à posteriori.* We may, therefore, regard the conclusion as a perfectly safe one. As the scientific inquirer in induction makes use of *analysis*, so in deduction he makes use of *synthesis.* By logical *synthesis* is meant the building up of complex structures of knowledge out of simple general principles. The best possible example of this is the Geometry of Euclid. Euclid starts with a few definitions and axioms, and proceeds to build up on these, as foundation, the whole structure of his Geometry.

Résumé of Chapter XXIX.

I. Relation between induction and deduction. The ultimate premisses of all deductive arguments depend for their truth upon induction.

II. Induction.

1. The method of inquiry, by means of induction, is called the *method of discovery*.
2. The inductive method proceeds *analytically*, that is, it breaks up complex phenomena into simpler phenomena for individual consideration.
3. The inductive method is an *à posteriori* one, that is, it proceeds from what is after to what goes before, from effect to cause.

III. Deduction.

1. The method of inquiry by means of deduction is called the *method of instruction*. It proceeds from cause to effect, from rule to example, or from general principle to illustration.
2. The deductive method proceeds *synthetically*, that is, it builds up simple facts into complex structures of knowledge.
3. The deductive method proceeds *à priori*, that is, from what goes before to what follows.

Exercises on Chapter XXIX.

1. Which is more important, induction or deduction? Why?
2. Explain what is meant by an *à priori* argument.
3. Explain what is meant by an *à posteriori* argument.
4. What is *analysis?* Give examples.
5. What is *synthesis?* Give examples.
6. What is meant by an *exact science?*

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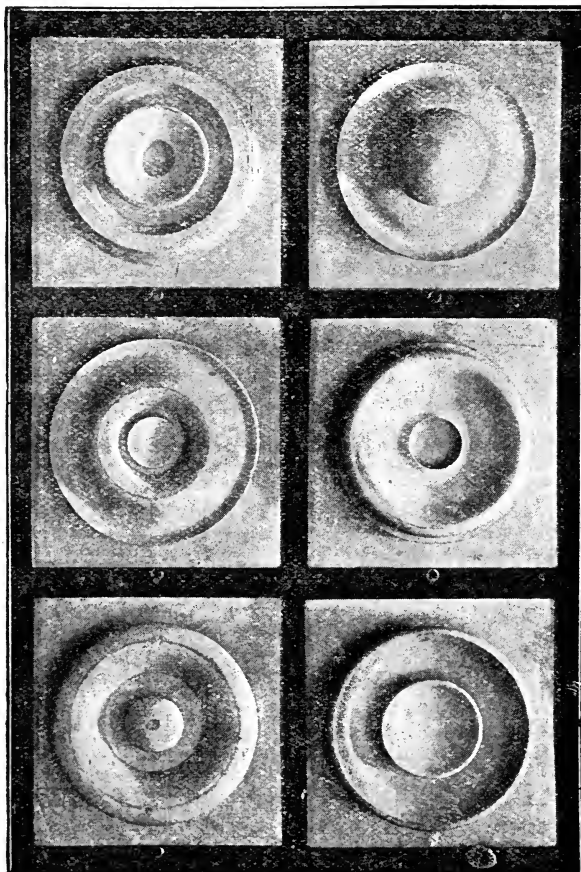

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