Faculty Working Papers

CYCLICAL FACTORS IN CARTEL STABILITY: THEORY

Thomas S. Ulen, Assistant Professor,
Department of Economics

#572

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

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Summary:

The theory of cartels is extended over multiple time-periods on the assumptions that the cartel has provided itself with an internal policing mechanism and that price discounts below the collusive price occur only on units sold in excess of the cheater's quota. The amount to discount becomes a function of the probability of being caught and punished, which probability is itself a function of, inter alia, the extent of cheating. If the demand for the cartel's output is then allowed to expand and contract around a secular trend, it is shown that the incidence of cheating and, therefore, of cartel stability varies directly with changes in the demand for output. In the absence of entry, which compounds these factors, an internally-enforced cartel is likely to be successful in jointly-maximizing profits in the business upturn and to degenerate into the competitive solution in the downtrun.

Nothing works against the success of a conspiracy so much as the wish to make it wholly secure and certain to succeed. Such an attempt requires many men, much time and very favorable conditions. And all these in turn heighten the risk of being discovered. You see, therefore, how dangerous conspiracies are!

Francesco Guicciardini, Ricordi (1458-1530)

The success of the international oil cartel in maintaining a noncompetitive price for crude oil in world markets has caused a renewed interest in the problems of collusion.[1] Almost all of the recent work in this area has attempted to model cartel behavior in a static or comparative-static setting. While extremely useful insights have been provided by this work, the case may be made that this concentration of effort has yet to suggest the circumstances under which collusion succeeds in deterring cheating through internal enforcement devices. Indeed, the literature has either ignored the possibility that colluders could protect themselves against cheating or has dismissed it as being very unlikely. It is true that most modern successful cartels have employed the power of the state to enforce their collusion's contracts, and therefore, that it has been external enforcement rather than internal enforcement which has become associated with successful collusion.

From the point of view of a cartel seeking success an appeal to the state to enforce the cartel contract will be the recommended course

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[1] See, inter alia, D. K. Osborne, "Cartel Problems," American Economic Review, 66 (December, 1976); David E. Mills and Kenneth G. Elzinga, "Cartel Problems: Comment," American Economic Review, 68 (December, 1978); Robert S. Pindyck, "Cartel pricing and the structure of the world bauxite market," Bell Journal of Economics, v. 8, n. 2 (Autumn, 1977); Z. Mikdashi, "Collusion Could Work," Foreign Policy, No. 14 (1974); and Peter Asch and J. J. Seneca, "Is Collusion Profitable?", Review of Economics and Statistics, 58 (1976). The classic article is George J. Stigler, "A Theory of Oligopoly," Journal of Political Economy, 72 (1964).

of action if that is a cheaper way to insure success than are the alternatives. Among those alternatives are that of simply doing without any policing--the assumption of the simplest models--and that of privately-supplied enforcement among the colluders. The presence of vigorous antitrust enforcement policies has probably significantly raised the relative price of private enforcement, but it cannot be presumed to have made it completely out of the question for colluders.[2] Especially for cartels operating across national boundaries, as does the OPEC, and for those historical cartels which operated either before the passage of statutes defining anti-competitive practices[3] or during periods of lax enforcement of these statutes, the attractiveness of private enforcement may be and has been considerable.

The adoption by a cartel of internal policing mechanisms can lead to a cyclical pattern in cartel stability. Specifically, the members of the collusion appear to adhere to the joint-profit-maximizing price and output decisions when business is good and to cheat when business is bad. Not only has the OPEC obeyed this pattern;[4] other collusions of note have, too, e.g., the electrical equipment conspiracy[5] and late

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[2]See George J. Stigler and Gary Becker, "Law Enforcement, Malfeasance, and Compensation of Enforcers," Journal of Legal Studies, v. 3, n. 1 (January, 1974), and William Landes and Richard A. Posner, "The Private Enforcement of Law," Journal of Legal Studies, v. 4, n. 1 (January, 1975).

[3]Before the passage of the Sherman Act in 1890 the common law had typically found many anti-competitive practices illegal.

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nineteenth century U.S. railroad cartels.[6] It is this dynamic aspect of cartel behavior to which this paper is addressed. In section II a model of collusion is defined and compared to the existing models of cartel behavior. The success of the cartel in deterring cheating over the course of a simple expansion and contraction of the demand for the output of the cartel is examined in section III. The concluding section summarizes the findings and suggests their implications for a number of fields in economics.

II.

Assume that a cartel is established with the following strategies designed to prevent cheating: a governing agency is established to make rules, detect cheating, hear complaints, and impose punishments. Suppose now that an individual member, after the collusion is launched, considers the advisability of cheating. He must now realize that the decision to cheat is a decision to take a gamble. If he is discovered, he runs the risk of being fined or otherwise punished. Success implies secretly incrementing his share of cartel profits. Being a gamble, the decision to cheat or not to cheat will depend crucially on the probability of being caught.[7]

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[6] See Thomas S. Ulen, Cartels and Regulation: Late Nineteenth Century Railroad Collusion and the Creation of the Interstate Commerce Commission (Unpublished Ph.D. dissertation, Stanford, 1978). Other relevant examples are to be found in John S. McGee, "Ocean Freight Rate Conferences," University of Chicago Law Review, v. 27, n. 2 (Winter, 1960) and Kenneth G. Elzinga, "Predatory Pricing: The Case of the Gunpowder Trust," Journal of Law and Economics, v. 13 (1970). Also, see George W. Stocking and Myron W. Watkins, Cartels in Action (New York: The Twentieth Century Fund, 1947).

[7] And on the conditional probability that the fine can be enforced. For the remainder of the discussion, I make the strong assumption that the probability of enforcement, given that the firm has been caught, is equal to one.

Let us assume an industry in which there are n identical firms, where n is not too large. Each firm faces a linear demand curve, D_m, which is (1/n)th of the total industry demand. The industry, in the absence of collusion, is assumed to be monopolistically competitive so that each firm faces the usual second demand curve, D_i, which is more elastic than the market share demand curve and is defined so as to pass through each point on the first demand curve. I shall further assume that a cheater in this well-structured cartel practices a particular form of disloyalty which I shall call "cheating at the margin." By this I mean that he offers a discount only on those units he sells above his cartel quota. That is, he practices a form of price discrimination: up to his quota he charges the full cartel price; it is only to the new and stolen customers that he offers a price reduction.

This situation is indicated in the graph below, figure 1. The cheater offers the quantity $(q_i^* - q_m^*)$ at the discounted price, p_i^*. If he is not discovered, then his total profits are the sum of rectangles EFCG and ABCD, which is obviously greater than his share of cartel profits, EFCG.

The accepted theory of cartels assumes that cheating is done over both the member's quota and over the new and stolen customers. For comparison this strategy is also shown in the figure. The relevant marginal revenue curve, MR_h, now begins at the vertical axis since the cheater means to offer a lower price to all of his customers. If he then sells q_h^* at price p_h^*, his total profits are the rectangle IJLG. He has sacrificed the profits EFIK, which would have been his had he remained loyal, and got in return an increase in profits equal

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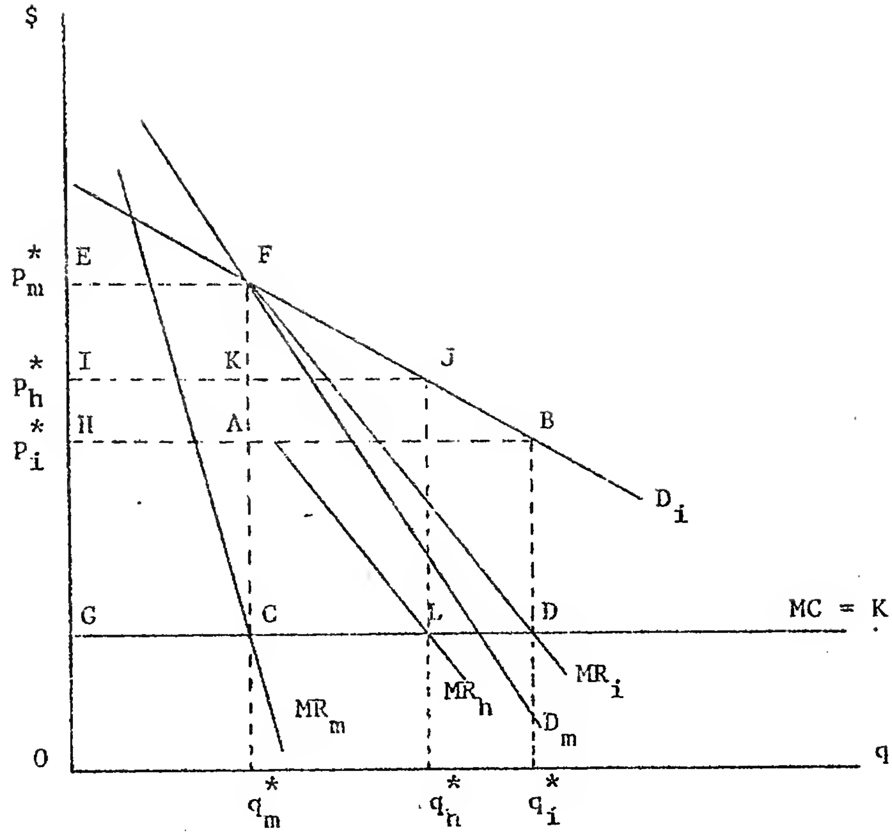

Figure 1

D_m = the market share demand curve

MR_m = the marginal revenue associated with D_m

p^*_m = the joint profit-maximizing price

q^*_m = the member's quota

D_i = the more elastic, cheater's demand curve

MR_i = the marginal revenue associated with D_i

MR_h = the marginal revenue associated with D_i if the cheater does not cheat at the margin but over his entire demand

$q^*_i - q^*_m$ = the amount offered at a discount by a margin cheater

p^*_h = the profit-maximizing price for non-margin cheating

q^*_h = the profit-maximizing quantity offered at a discount by a non-margin cheater to KJLC. It is evident that cheating will be irresistible so long as KJLC > EFIK.[8]

Since, in view of the precautionary measures taken by the cartel, the decision to cheat is a gamble, we are dealing in uncertain outcomes over which a probability density function is defined. If u(x) is a real-valued function defined on a set X and P(X) is a probability distribution defined over the same set, then we may write

$$E(u|P) = u(x)dP(x)$$

$$\text{and } E(u|P^*) = \max_{P_i} E(u|P_i), \qquad i = 1, 2, \ldots, n.$$

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[8] A word of explanation is in order for introducing "cheating at the margin" in addition to the sort of cheating which the accepted theory assumes. The reason for positing this new form is that is seems to capture one of two different kinds of cheating that afflict a cartel. Of those two kinds the first and less serious is the sniping which margin cheating implies. It is calculated to line the cheater's coffers but not to do mortal damage to the cartel. It can be seen that, in general, $(q_i^* - q_m^*)$ is not terribly large, at least, if the market share demand is large, nowhere near the quantity q_m^*. Since this margin cheating is not very big, relative to q_m^* and q_h^*, it does not involve stealing a great number of customers from the other (n-1) members. Especially if total market demand, D_m, is increasing, small losses in loyal members' customers will be less evident amid the influx of new customers. (See Stigler, op cit., supra n. 1, for the elucidation of the circumstances under which a cartel member may infer, from changes in the growth of his market share, if one of his co-conspirators is cheating.) The second kind of cheating is the whole-hog version. In what follows it will be seen to be associated with a complete collapse of the cartel. The analysis and a simulation will be developed for the margin cheating case. It should be evident that the whole-hog variety can be explained in the following model after appropriate substitution of variables. A point which will shortly be developed but which bears mentioning now is this: given the steps which the cartel has been assumed to have taken, the probability of detecting a discount over q_h^* is greater than that over $(q_i^* - q_m^*)$.

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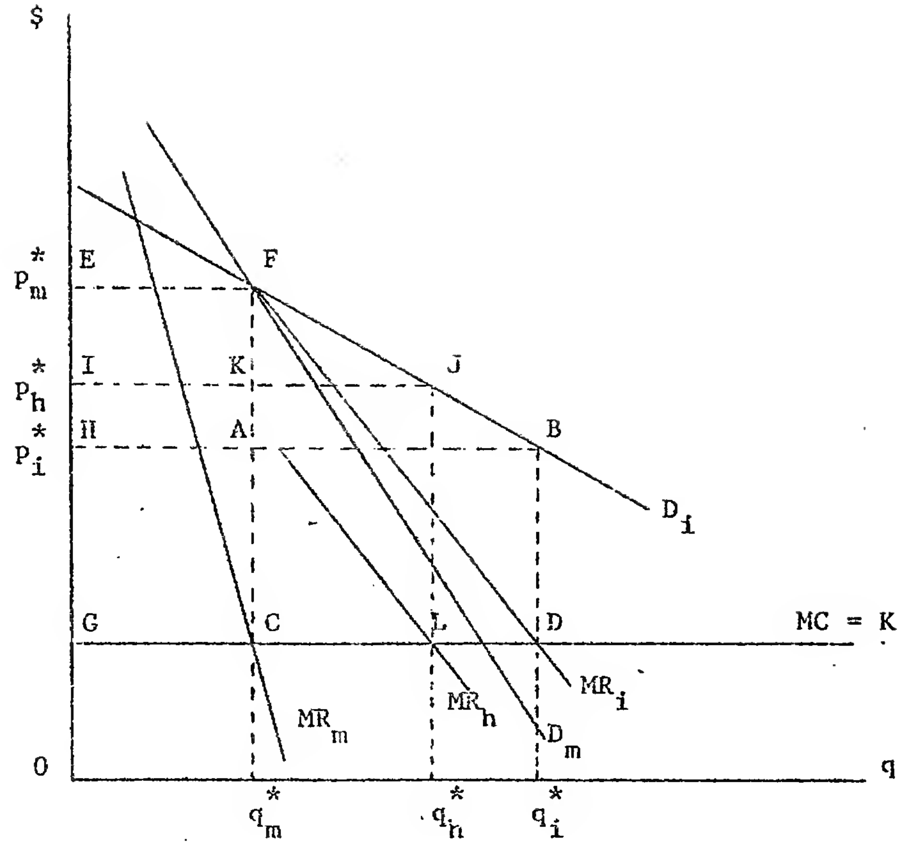

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It is assumed that $u'(x) > 0$, but there are the usual three possibilities for the second derivative. In what follows, I shall assume that the firms in the cartel are all risk-neutral.

The decision for each member of the collusion is whether to take the gamble—i.e., to cheat—or to remain loyal. Therefore, each firm must assess both the expected utility of loyalty and the expected utility of cheating. The first quantity is assumed to prevail with probability equal to one, if the firm remains loyal.[9] Thus,

$$(1) \quad u_{\ell} = u(\pi_m - X)$$

where π_m = the firm's share of joint maximum profits,

X = the firm's contribution to the cartel governing body, assumed constant.

The utility of cheating is a certainty equivalent:

$$(2) \quad u_c = p_d\, u(\pi_m - X - \delta\pi_i) + (1 - p_d)u(\pi_m - X + \pi_i)$$

where p_d = the probability of detection

π_i = the profits realized on the amount discounted a t the margin

[9]It should be noted that this certainty of loyalty profits is a special sort of assumption. Clearly if someone else is already cheating and the prospects for the continuation of the cartel are dim, the probability of loyalty profits is not one. In what follows I dismiss this complication by assuming that at any point in time the firm believes everyone else will remain loyal so that π_m can be had as a sure thing.

δ = a fine parameter, assumed to be strictly greater than 1.[10]

I shall assume that the firm is rational in such a manner that, if $u_\ell > u_c$, the firm will remain loyal to the cartel but that if $u_c > u_\ell$, it will cheat. Since all firms are assumed to be alike, $u_c > u_\ell$ is true for all n enterprises, and the cartel dissolves. The situation $u_\ell > u_c$ implies that the cartel remains together or, if apart before $u_\ell > u_c$ obtains, comes together in a collusion which endures as long as that condition holds.

In some expected utility problems the probability of success is assumed to be independent of the parameters of the utility function. This is certainly not the case with a cheater: the decisions he takes and those his fellows take affect the likelihood of his gamble's succeeding. The amount which the cartel as a whole spends policing

[10] Strictly speaking, there is a probability distribution over this fine parameter. The probability of enforcing a fine is really a conditional probability, given that cheating has been detected. We should expect that p_e would be relatively low in the downturn, when, we shall see, the cartel falls apart, and relatively high in the upturn. This latter point is hypothetical but rests on the supposition that when times are good, each firm is eager to continue sharing in the success of the cartel. Should a cheater be caught then, the prospect of being denied a fair quota in the future might persuade him to pay the fine and thereafter keep his nose clean. Refusal to pay the fine has a greater expected cost when the cartel is going along smoothly.

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itself directly affects p_d. Clearly, the greater the quantity which the cheater attempts to discount, the more likely he is to be discovered.[11]

Since I am assuming that the cheater is offering these discounts at the margin, there is another factor influencing the probability of detection, viz., the size of the market share. In offering a two-part tariff, the cheater must not only not let his business rivals hear of the discount; he must also take pains to insure that his customers who are paying the cartel price do not learn that there are some who are getting a lower price. If these customers hear of the discounts, they can threaten to reveal the disloyalty unless he gives them the same discount. The larger is q_m--the number of customers being charged the full cartel price under the assigned market share--the more likely it is that word of a price cut will be heard.[12]

Against all this, the cheater can reduce the probability of detection by taking special steps to guarantee secrecy.

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[11] Stigler gives a nice example of this in his "Theory of Oligopoly." He assumes that p is the probability that a rival will hear of a price reduction. Then $(1 - p)$ is the probability that the price cut will go undetected. If this reduction is offered to z customers, then the probability that the rival will not hear of this cheating is $(1 - p)^z$. Therefore, $1 - (1 - p)^z$ is the probability that the rival will hear of a price reduction given to z customers. Even if p is small, say 0.01, when $z = 100$, the probability of detection is 01634, and when $z = 1000$, it increases to 0.99996.

[12] I have not considered the different tactics which an old customer who learns of discounts might employ. It may matter very much whether this person goes to the authorities or to the cheater with his information. While this is an interesting problem in itself, I am positing the more simple case that, whichever happens, the size of market share is directly related to the probability of detection.

Let us assume that the cheater undertakes these evasion expenses and that the marginal cost associated with these evasion expenses is constant. Thus, the marginal cost of a discounted unit of output is $(K + E_i)$, where, as before, K is the constant marginal cost of production, and E_i is the constant marginal cost of maintaining secrecy.

We are now prepared to posit the following relationship:

(3) $p_d = p_d(q_m, q_i, nX, E_i)$

where q_m = the firms' quota calculated as Q_t/n where Q_T is the total industry demand

q_i = the amount of output offered at a discount

nX = the total amount spent by the cartel to govern itself

E_i = the marginal evasion expenses incurred by the cheater.

We also expect that the partial derivatives of p_d with respect to the arguments of the function to have the following signs:

$$\frac{\partial p_d}{\partial q_m} > 0, \quad \frac{\partial p_d}{\partial q_i} > 0, \quad \frac{\partial p_d}{\partial (nX)} > 0, \quad \text{and} \quad \frac{\partial p_d}{\partial E_i} > 0.$$

Let us examine the expected utility from cheating and see how it varies as some of the parameters vary. Consider first the partial of u_c with respect to q_i.

$$(4) \quad \frac{\partial u_c}{\partial q_i} = p_d[- \frac{\delta \partial \pi_i}{\partial q_i} u'(\pi_m - X)] + u(\pi_m - X) \frac{\partial p_d}{\partial q_i}$$

$$+ (1 - p_d) \frac{\partial \pi_i}{\partial q_i} u'(\pi_m - X) - u(\pi_m - X) \frac{\partial p_d}{\partial q_i}$$

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q_i = the amount of output offered at a discount

nX = the total amount spent by the cartel to govern itself

E_i = the marginal evasion expenses incurred by the cheater.

We also expect that the partial derivatives of p_d with respect to the arguments of the function to have the following signs:

$$\frac{\partial p_d}{\partial q_m} > 0, \quad \frac{\partial p_d}{\partial q_i} > 0, \quad \frac{\partial p_d}{\partial (nX)} > 0, \quad \text{and} \quad \frac{\partial p_d}{\partial E_i} > 0.$$

Let us examine the expected utility from cheating and see how it varies as some of the parameters vary. Consider first the partial of u_c with respect to q_i.

$$(4) \quad \frac{\partial u_c}{\partial q_i} = p_d[- \frac{\delta \partial \pi_i}{\partial q_i} u'(\pi_m - X)] + u(\pi_m - X) \frac{\partial p_d}{\partial q_i}$$

$$+ (1 - p_d) \frac{\partial \pi_i}{\partial q_i} u'(\pi_m - X) - u(\pi_m - X) \frac{\partial p_d}{\partial q_i}$$

itself directly affects p_d. Clearly, the greater the quantity which the cheater attempts to discount, the more likely he is to be discovered.[11]

Since I am assuming that the cheater is offering these discounts at the margin, there is another factor influencing the probability of detection, viz., the size of the market share. In offering a two-part tariff, the cheater must not only not let his business rivals hear of the discount; he must also take pains to insure that his customers who are paying the cartel price do not learn that there are some who are getting a lower price. If these customers hear of the discounts, they can threaten to reveal the disloyalty unless he gives them the same discount. The larger is q_m---the number of customers being charged the full cartel price under the assigned market share---the more likely it is that word of a price cut will be heard.[12]

Against all this, the cheater can reduce the probability of detection by taking special steps to guarantee secrecy.

---

[11] Stigler gives a nice example of this in his "Theory of Oligopoly." He assumes that p is the probability that a rival will hear of a price reduction. Then $(1 - p)$ is the probability that the price cut will go undetected. If this reduction is offered to z customers, then the probability that the rival will not hear of this cheating is $(1 - p)^z$. Therefore, $1 - (1 - p)^z$ is the probability that the rival will hear of a price reduction given to z customers. Even if p is small, say 0.01, when z = 100, the probability of detection is 01634, and when z = 1000, it increases to 0.99996.

[12] I have not considered the different tactics which an old customer who learns of discounts might employ. It may matter very much whether this person goes to the authorities or to the cheater with his information. While this is an interesting problem in itself, I am positing the more simple case that, whichever happens, the size of market share is directly related to the probability of detection.

$$= (-\ \delta p_d + 1 - p_d)\ \frac{\partial \pi_i}{\partial q_i}\ u'(\pi_m - X)$$

This will be less than zero if and only if, granting that $u'(\pi_m - X) > 0$ and that with q_i^* as an upper bound, $\frac{\partial \pi_i}{\partial q_i} > 0$, that is, if

(5) $-\ \delta p_d + 1 - p_d < 0$, or

(5') $p_d > \frac{1}{\delta + 1}$

The implication is that if $p_d < \frac{1}{\delta + 1}$, the firm will consider cheating. Since I have assumed that $\delta > 1$, no firm will consider adopting cheating unless the probability of detection is somewhat less than on-half. This relationship is represented in the following graph, figure 2.

There is, however, no reason for believing that if $p_d < \frac{1}{\delta + 1}$, cheating will be the recommended course of action. Whether it is, depends on the relative values of the expected utility of cheating and the expected utility of loyalty. For example, u_ℓ may look as it does in figure 2 with the implication that it will take a further drop in the probability of detection before u_c will dominate u_ℓ. For the firm considering cheating, the problem is that it can affect this point t, in the graph, by the amount it chooses to offer at a price cut and by the precautions it takes to cover up these discounts.

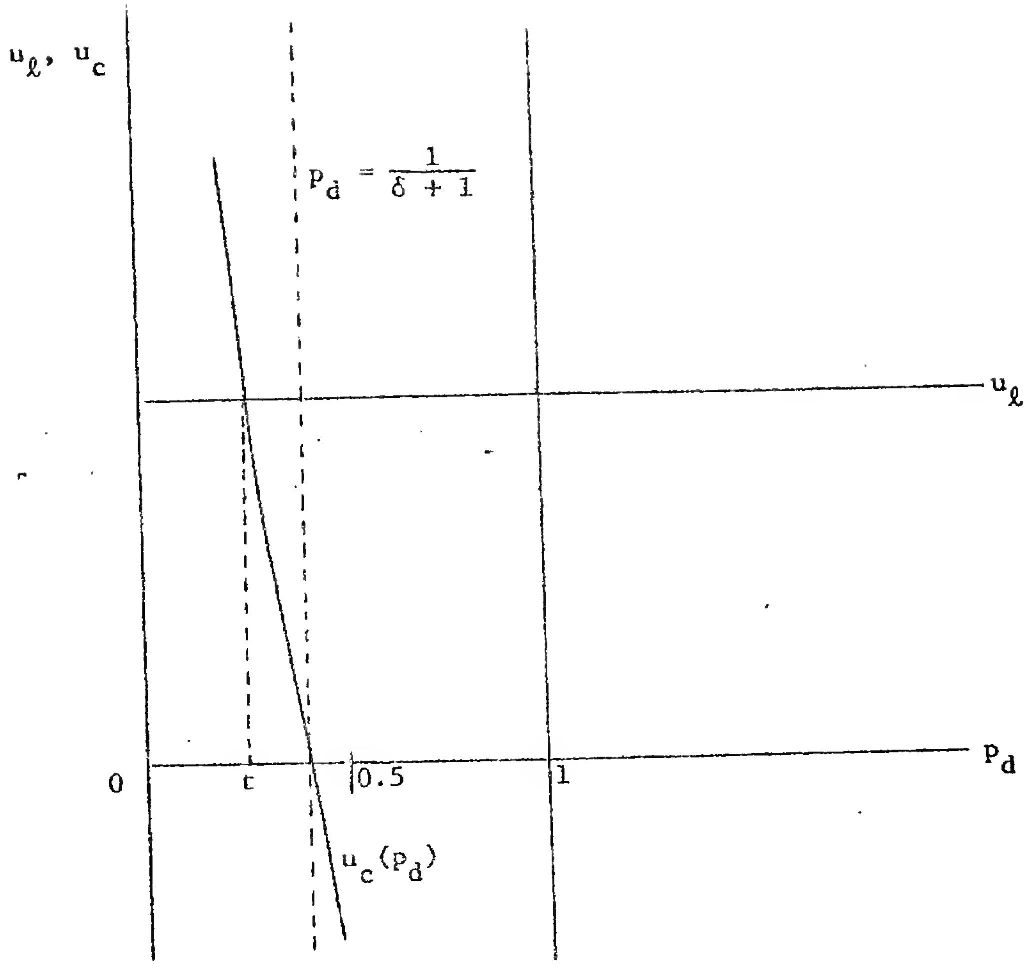

Figure 2

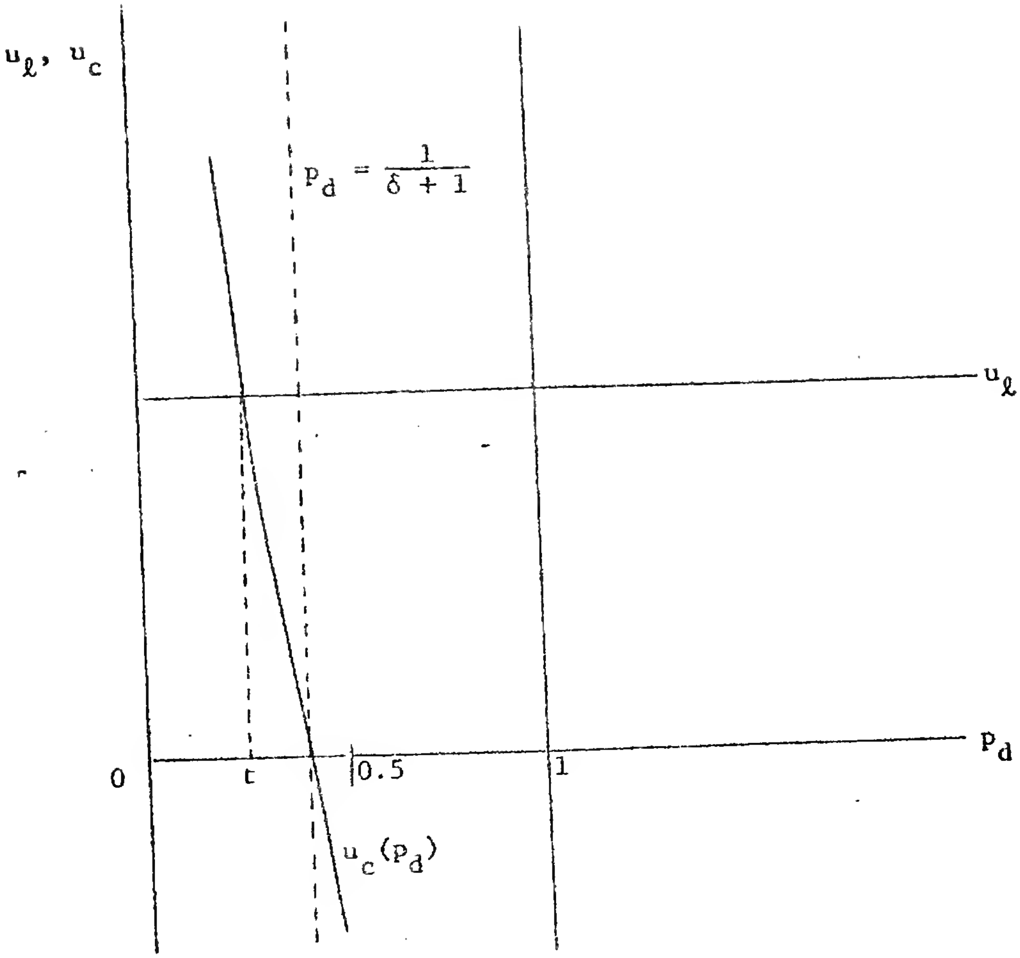

Figure 2

$$= (-\ \delta p_d + 1 - p_d) \frac{\partial \pi_i}{\partial q_i} u'(\pi_m - X)$$

This will be less than zero if and only if, granting that $u'(\pi_m - X) > 0$ and that with $q_i{}^*$ as an upper bound, $\frac{\partial \pi_i}{\partial q_i} > 0$, that is, if

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Let us now focus attention on the effects on the cartel of cyclical fluctuations in the demand for its output. This means the problem reduces to the following: given that $p_d < \frac{1}{\delta + 1}$, allow the cheater to choose q_i such that $\frac{\partial u_c}{\partial q_i} = 0$ and, therefore u_c is at a maximum.[13] Then allow business to contract and expand to see if $(u_\ell - u_{c,max})$ varies around zero. Under risk neutrality we may take the expected utility of cheating to be

$$(6)\quad u_c = p_d(\pi_m - X - \delta\pi_i) + (1 - p_d)(\pi_m - X + \pi_i)$$

$$= \pi_m - X + \pi_i[1 - p_d(\delta + 1)]$$

That is, we are simply talking about expected profit. We wish to see what q_i maximizes this expectation. Thus,

$$(7)\quad \frac{\partial u_c}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} - (\delta + 1)[\pi_i \frac{\partial p_d}{\partial q_i} + p_d \frac{\partial \pi_i}{\partial q_i}]$$

where π_i is defined in the appendix. We have only a general form for p_d, e.g.,

$$(8)\quad p_d = (1 + e^{\beta_0} q_m^{-\beta_1} q_i^{-\beta_2} (nX)^{-\beta_3} E_i^{-\beta_4})^{-1}$$

from which

$$(9)\quad \frac{\partial p_d}{\partial q_i} = \beta_2 Z q_i^{-\beta_2 - 1}(1 + e^{-\beta_0} q_m^{-\beta_1} q_i^{-\beta_2} (nX)^{-\beta_3} E_i^{-\beta_4})^{-2}$$

where

$$(10)\quad Z = e^{-\beta_0} q_m^{-\beta_1} (nX)^{-\beta_3} E_i^{-\beta_4}.$$

---

[13] It can easily be shown that the second-order condition is satisfied in what follows.

Therefore the solution for q_i^* is the solution of the polynomial

$$(11)\quad \frac{(\alpha - K - 2E_i)}{2} - 2\gamma q_i - (\delta + 1)[\{\frac{(\alpha - K - 2E_i)}{2} q_i - \gamma q_i^2\} (\frac{\beta_2 Z q_i^{-\beta_2 - 1}}{(1 + e^{-\beta_0} q_m^{-\beta_1} q_i^{-\beta_2} \ldots)^2})$$

$$+ \frac{1}{(1 + e^{\beta_0} q_m^{-\beta_1} q_i^{-\beta_2} \ldots)} (\frac{\alpha - K - 2E_i}{2} - 2\gamma q_i)] = 0$$

It is clear thus far that only in the extreme instance where $p_d = 0$ will the solution give the familiar $MR_i = MC$ solution, $q_i^* = \frac{(\alpha - K - 2E_i)}{4\gamma}$, from the full certainty situation, still allowing for evasion expenses. It is also certain that when $p_d \neq 0$, the solution will give some $q_i < q_i^*$. That is, the cheater will reduce the amount offered at a discount as the probability of detection increases. What, for example, will give the maximum utility when $p_d = 1$? In that case the optimal q_i is the solution to

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It can be seen that if $\beta_2 = 0$, i.e., if the amount offered at a discount has no effect on the probability of detection, then the optimal q_1 when $p_d = 1$, is again $q_1^* = \frac{\alpha - K - 2E_1}{4\gamma}$, the same value as when $p_d = 0$. For $\beta_2 \neq 0$ we suspect that $q_{1,opt} < q_1^*$.[14] In the absence of estimates, we won't know precisely what β_2 is. In order, however, to have some suggestions at hand, I shall make the rather strong assumption that $\beta_2 = 0$. This assures us, as we have seen, that no matter what the probability of detection, the optimal quantity to discount is q_1^*.

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[14]We can verify this, in the absence of an estimate, β_2, by picking some arbitrary value of β_2, say $\beta_2 = 1$. In that case the optimal value of q_1 is the solution to

$$(13)\quad 4\delta\gamma q_1^2 - q_1[\delta(\alpha - K - 2E_1) - 2\gamma(\delta + 1)Z] - (\delta + 1)(\alpha - K - 2E_1)Z = 0$$

Using the quadratic formula,

$$(14)\quad q_{1,opt} = \frac{\delta(\alpha-K-2E_1)-2\gamma(\delta+1)Z \pm \sqrt{[\delta(\alpha-K-2E_1)-2\gamma(\delta+1)Z]^2-16\delta\gamma(\delta+1)(\alpha-K-2E_1)Z}}{8\delta\gamma}$$

Since $q_1^* = \frac{\alpha - K - 2E_1}{4\gamma}$, it is clear that of the two roots, $q_{1,opt}^+$ and $q_{1,opt}^-$, $q_1^* > q_{1,opt}^-$. It is easily shown also that $q_1^* > q_{1,opt}^+$. It is also obvious, from inspection, that the larger is β_2, the smaller when $p_d = 1$, is the optimal amount of discount.

Later, I shall indicate what results need to be altered in the more likely event that $\beta_2 \neq 0$.

III.

Let us now see what happens to the decision to cheat as the probability of detection fluctuates, with other variables, over the business cycle. I shall assume that the business cycle as it affects the cartel is exogenously generated and takes the form of periodic fluctuations in the exogenous component of the demand curve, $p_m = \alpha - \beta q_m$, facing each firm in the cartel. Let this exogenous component, α, move according to the following:

(15) $\alpha = A\cos\omega t + rt + q_m(0)$,

where r = the secular growth rate of demand for the cartel output

and $q_m(0)$ = demand at time t = 0.

The period of the cycle is given by $T = \frac{2\pi}{\omega}$, the amplitude by A, as in figure 3.

To simplify matters, assume that r = 0, that is, there is no secular growth in demand, and that, as before, the utility functions are simply linear, so that we are dealing only in expected profits. Thus,

(16) $$u_\ell = \frac{(A\cos\omega t + q_m(0) - K)^2}{4\beta} - X$$

And similarly for the expected profit from cheating. In order to determine where these functions achieve maxima and minima, we need to know the first and second derivatives with respect to time. Begin with u_ℓ.

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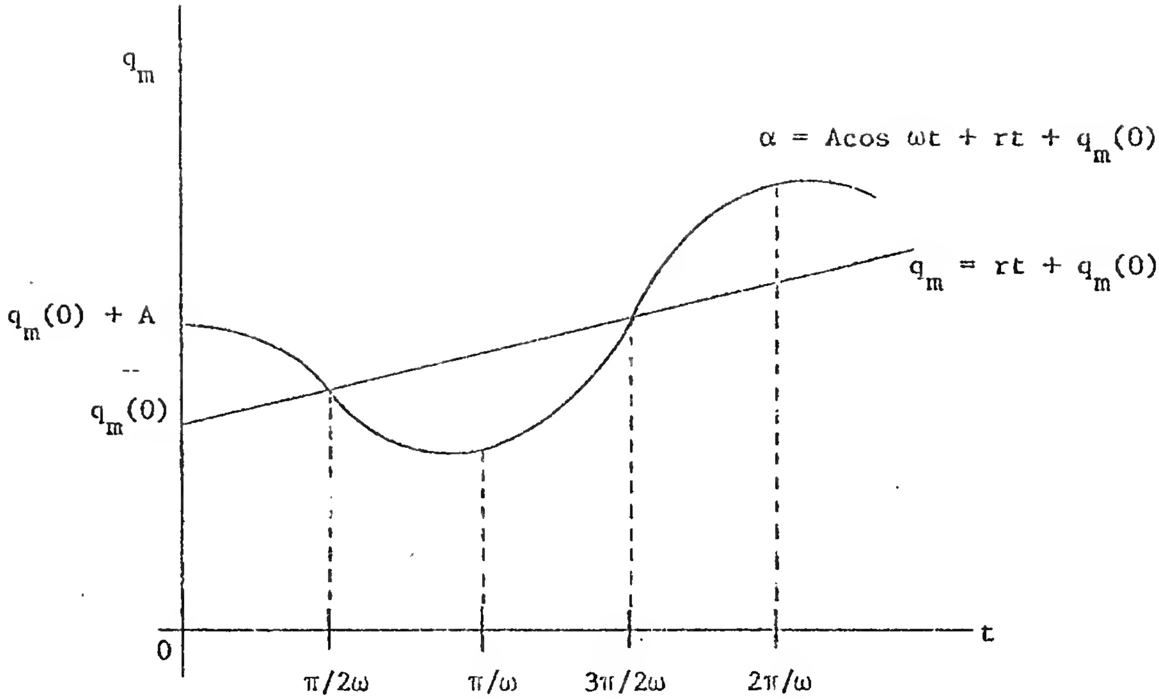

Figure 3

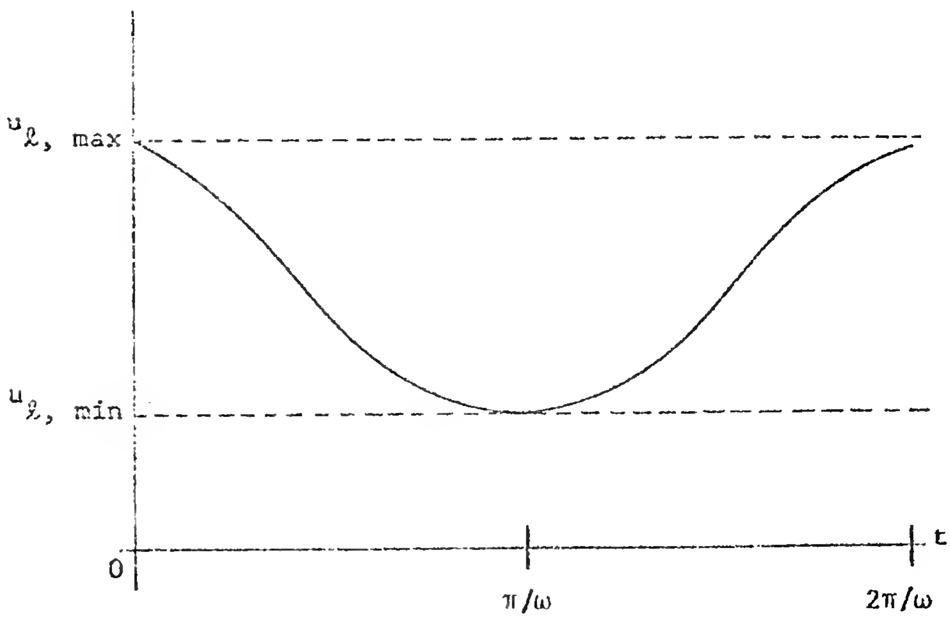

Figure 4

$$(17)\quad u_\ell' = \frac{\partial u_\ell}{\partial t} = \frac{-2A\omega \sin\omega t(A\cos\omega t + q_m(0) - K)}{4\beta}$$

At either a maximum or a minimum $u_\ell' = 0$, so that

$$(18)\quad \sin\omega t(A\cos\omega t + q_m(0) - K) = 0.$$

Clearly this is satisfied when either $\sin\omega t = 0$ or $A\cos\omega t + q_m(0) - K = 0$. The second possibility cannot be allowed since it implies that the share of monopoly profits is zero. Thus, values of t which satisfy $\sin\omega t = 0$ are the values which tell us about the extreme of u_ℓ. $\sin\omega t = 0$ for the series $t = 0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, \frac{3\pi}{\omega} \ldots$ Since the period of the oscillation is $\frac{2\pi}{\omega}$, we can distinguish all maxima and minima simply by evaluating the second derivative at $t = 0$ and $t = \frac{\pi}{\omega}$. It is easily shown that u_ℓ is at a maximum at $t = 0, \frac{2\pi}{\omega}$ and so on and at a minimum at $t = \frac{\pi}{\omega}, \frac{3\pi}{\omega}$ and so on. The path is seen in Figure 5.

Now the crucial question is how this time path compares with that of u_c. Unfortunately, we can tell in only the most schematized way. The time path of u_c depends on the changing weights assigned to success and failure, that is, to the values, over time, of the probability of detection. Without the values of the parameters $\beta_0, \ldots, \beta_4$, we cannot be certain how p_d moves. Note that since we have some idea about how q_m and q_1 move over the cycle, we know, through the priors we have on the partial derivatives of p_d with respect to the arguments, that p_d moves pro-cyclically. That is, it is high when business is good and low when business is bad. This means that u_c will be at its maximum when business is bad--the lesser weight being given to failure--and at

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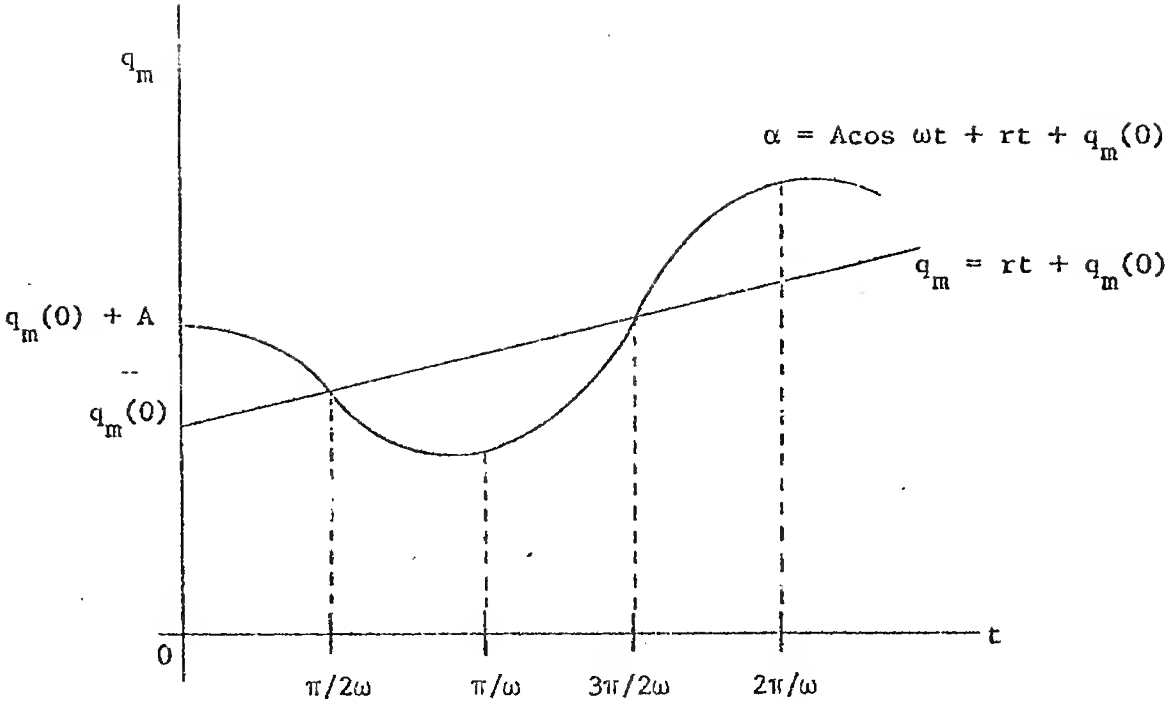

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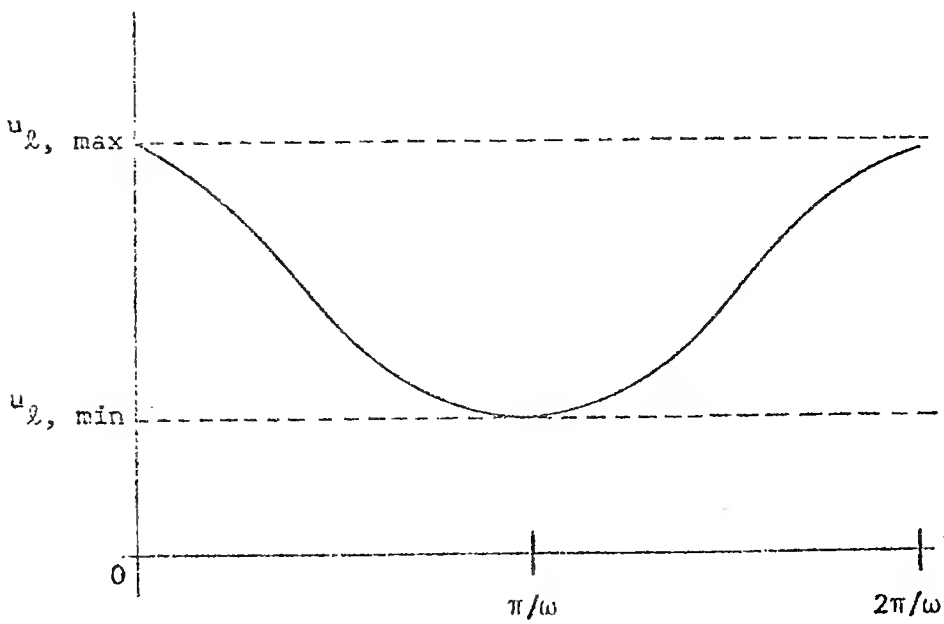


Figure 4

its minimum--the lesser weight being given to success--when business is good. First, rewrite u_c as follows:

$$(19)\quad u_c = p_d(\pi_m - X - \delta\pi_i) + (1 - p_d)(\pi_m - X + \pi_i)$$

$$= u_\ell + \pi_i[1 - p_d(\delta + 1)]$$

This formulation emphasizes the crucial role played p_d. Clearly if $\pi_i[1 - p_d(\delta + 1)] > 0$; $u_c > u_\ell$. And $u_\ell > u_c$ if $\pi_i[1 - p_d(\delta + 1)] < 0$. Since $\pi_i > 0$, the following is true,

$$(20)\quad u_l \gtreqless u_c \qquad \text{as} \qquad p_d \gtreqless \frac{1}{\delta + 1}$$

In order to see this more fully posit that p_d moves in the following way:

$$(21)\quad p_d = \frac{1}{2}(\cos\omega t + 1)$$

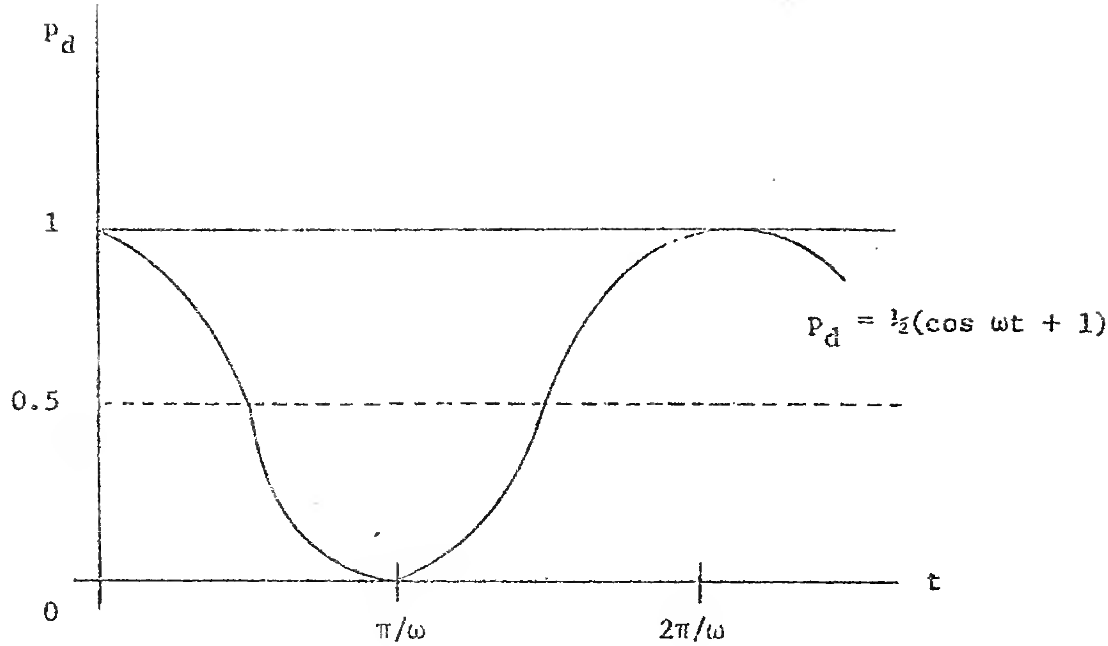
and further assume that the fine parameter $\delta = 1$. The first assumption certainly overstates the case. It implies that detection is a certainty at the peak of the cycle and an impossibility at the trough, as in figure 4. We can now simplify our expression for u_c,

$$(22)\quad 1 - p_d(\delta + 1) = 1 - \frac{1}{2}(\cos\omega t + 1)2 = -\cos\omega t.$$

Therefore,

$$(23)\quad u_c = \frac{(A\cos\omega t + q_m(0) - K)^2}{4\beta} - X - \frac{\cos\omega t(A\cos\omega t + q_m(0) - K)^2}{16\gamma}$$

And from this, Table I can be rendered, leading to the graph in figure 5.



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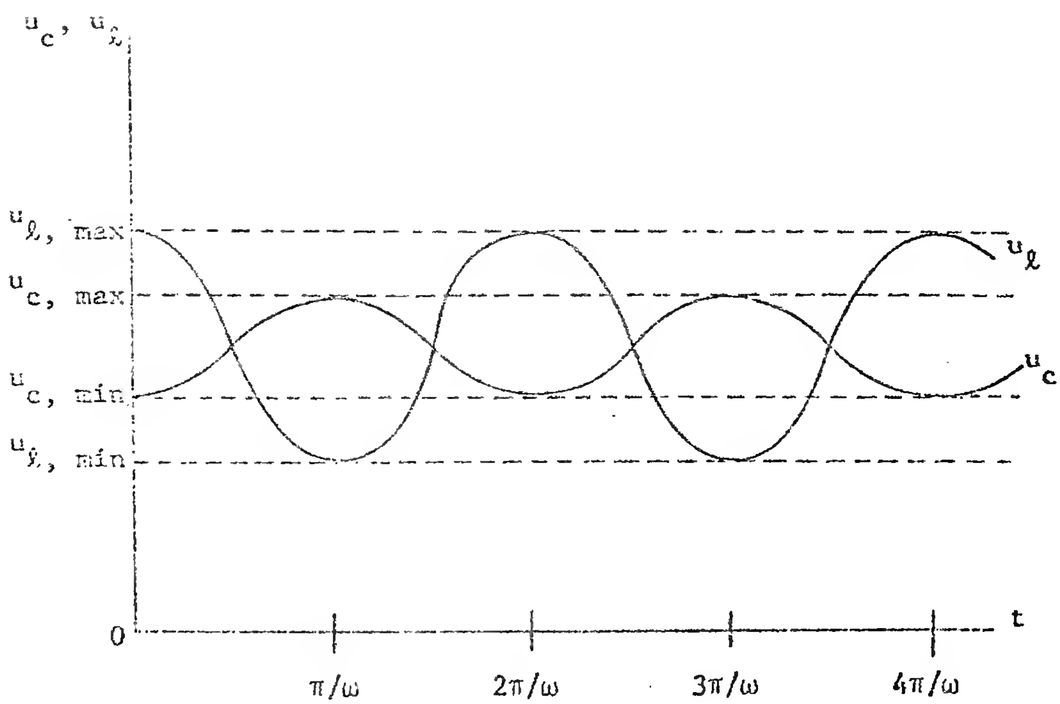

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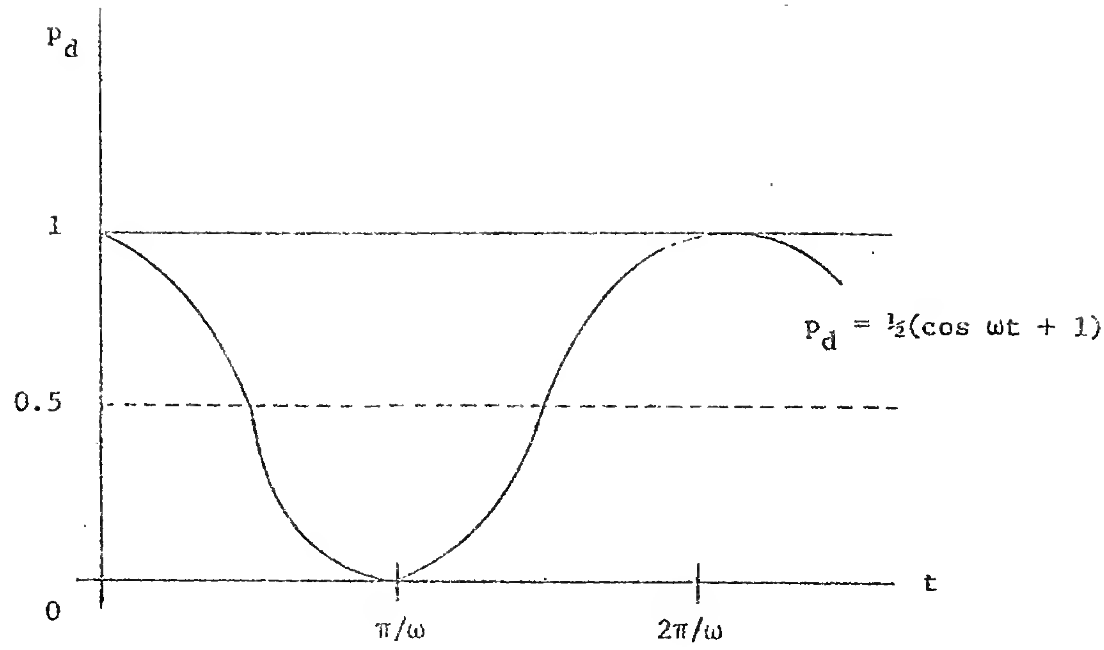

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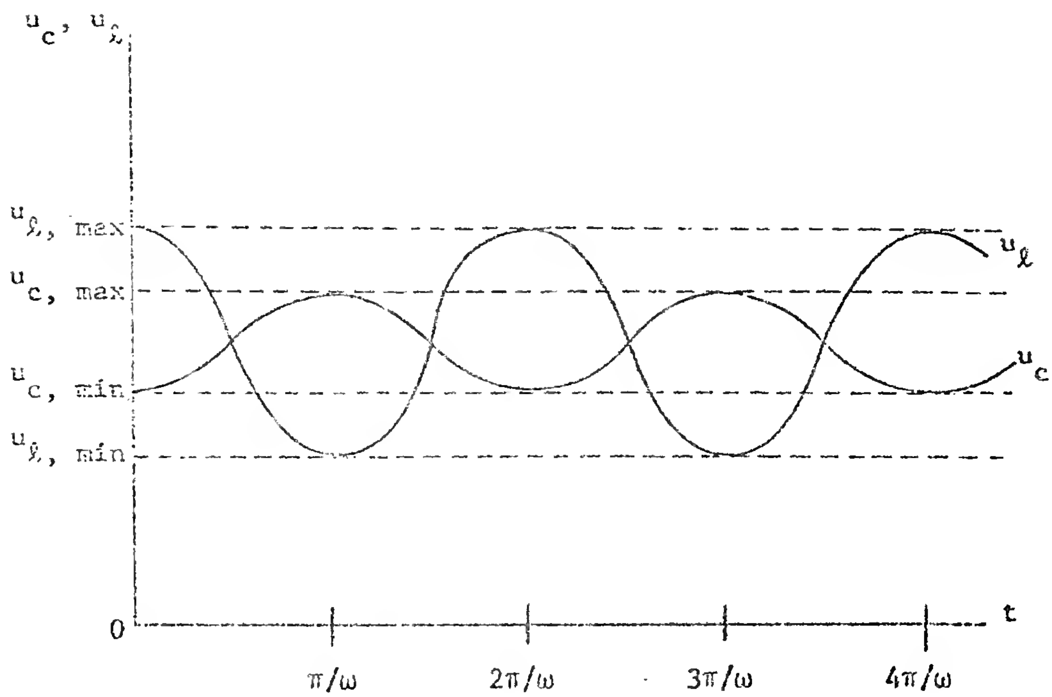

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$$(22)\quad 1 - p_d(\delta + 1) = 1 - \frac{1}{2}(\cos\omega t + 1)2 = -\cos\omega t.$$

Therefore,

$$(23)\quad u_c = \frac{(A\cos\omega t + q_m(0) - K)^2}{4\beta} - X - \frac{\cos\omega t(A\cos\omega t + q_m(0) - K)^2}{16\gamma}$$

And from this, Table I can be rendered, leading to the graph in figure 5.

Table 1

t	u_c	Notes
0	$\frac{(A + q_m(0) - K)^2}{4\beta} - X - \frac{(A + q_m(0) - K - 2E_i)^2}{16\gamma}$	u_c, min
$\frac{\pi}{2\omega}$	$\frac{(q_m(0) - K)^2}{4\beta} - X$	$u_c = u_\ell$
$\frac{\pi}{\omega}$	$\frac{(q_m(0) - K - A)^2}{4\beta} - X + \frac{(q_m(0) - K - 2E_i - A)^2}{16\gamma}$	$u_{c,max}$
$\frac{3\pi}{2\omega}$	$\frac{(q_m(0) - K)^2}{4\beta} - X$	$u_c = u_\ell$

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$\frac{\pi}{\omega}$	$\frac{(q_m(0) - K - A)^2}{4\beta} - X$	$u_{\ell,min}$
$\frac{3\pi}{2\omega}$	$\frac{(q_m(0) - K)^2}{4\beta} - X$	$u_\ell = u_c$

This suggests that an expected utility- or profit-maximizing firm will remain loyal to the cartel in the business upturn (0 to $\frac{\pi}{2\omega}$, $\frac{3\pi}{2\omega}$ to $\frac{5\pi}{2\omega}$, ...) and will find it more attractive to cheat in the downturn ($\frac{\pi}{2\omega}$ to $\frac{3\pi}{2\omega}$, ...).

I assumed previously that $\beta_2 = 0$, that is, that the amount offered at a discount did not affect the probability of detection. Figure 5 has been drawn on that assumption, and it is essential that we ask how these results would fare if, as we expect, $\beta_2 \neq 0$, i.e., $\frac{\partial p_d}{\partial q_i} \neq 0$. It has been shown that the q_i which maximizes the certainty equivalent u_c is less than q_i*. This means that by using q_i* we have understated the value of u_c. This is true at all points in figure 6 except at $\frac{\pi}{\omega}$, $\frac{3\pi}{\omega}$, ..., where, by assumption, $p_d = 0$. At those points, and, presumably, near-by, for $\beta_2 \neq 0$, $q_{i,opt} = q_i$*. Although we have no idea how much we have understated u_c by our assumption on β_2, we do know that the results are only marginally sensitive to relaxing that assumption. When $p_d = 1$, even if q_i is only slightly different from zero, u_c will still be less than u_ℓ. The alterations will have to be made in the region of $\frac{\pi}{2\omega}$, $\frac{3\pi}{2\omega}$, $\frac{5\pi}{2\omega}$, ..., where previously $u_\ell = u_c$ halfway between the peak and the trough of the cycle. For $\beta_2 \neq 0$, we shall have to push u_c up somewhat so that breakdowns in the cartel will occur sooner than before and the reformation of the cartel will take place later than previously.

It bears mentioning again that this is a highly schematized rendering of the complexities of any collusion. Nonetheless, we have established new principles for the examination of cartel behavior, and, in the model given here, they appear to conform to the pattern of cartel success and

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failure noted in the references in section I, viz., that stability occurs, if at all, in the upturn of the business cycle, and that instability is the result of the pressures of the cyclical downturn.

IV.

This model is not open, I think, to the same objections which may be directed at the accepted theory. It may, of course, be open to other objections. For example, the assumption that the probability of enforcing a fine on an offender is equal to or greater than one is overly strong. There is, however, this to be said for it as a first approximation: one would expect a firm to be more likely to pay a fine in an upturn, when prospects are good, than in the downturn. This is because not paying a fine may subject the offender to more lingering punishment which would deny him a share of the booming market. For example, in the Joint Executive Committee—a railroad cartel which flourished 1879-1893—a road which failed to heed cartel sanctions lost part or all of an annual good faith deposit and was denied the right to transfer freight with cartel members. This isolation—or threat of isolation—was usually sufficient to bring a miscreant back into the fold. A renegade who chose to fight the isolating decree was forced to contemplate immense capital expenses in order to build its own outlet to markets in the Midwest or on the seaboard. Given that there was usually a long lag before such a building project would be in service, the uncertain prospect in the market for transport service several years thence further tempered the incentive to fight enforcement.[15]

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[15] See Ulen, supra note 6, for a full description of the workings of this and other late 19th century rail d cartels.

Other objections are certainly possible to the model. One might wonder how the results would be altered if the marginal cost of production and evasion--or of either separately--was an increasing, rather than constant, function of output. The results derived in the previous section would, I think, remain the same under this alternative assumption. To see this, one can imagine what effect the imposition of a capacity constraint would have on the firm's output. Such a constraint will not eliminate cheating, but it will certainly mitigate it.[16] I believe it is obvious that the shifts in D_m which occur over the business cycle serve, in the upturn, to make this constraint more and more binding and thus, cheating less and less likely and vice versa in the downturn.

Because the accepted theory does not recognize the possibility of internally-generated stability in collusion in the short-run, it reduces the difference between the long-run and the short-run simply to the well-known difference between the competitive equilibria in those two time periods. If one recognizes the possibility that the joint-profit-maximum may be stable at certain points the short-run, as this model suggests it may, then naturally the question rises, "What will happen to the stable collusion in the long-run?" I suggest that the cartel will have severe trouble in maintaining the collective monopoly in the long-run because of entry into the industry, encouraged by that very

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short-run success which the cartel has previously enjoyed.[17] This points up the fact that there are two different problems facing a collusion in the two different periods. In the short-run, it is how to minimize the incentive to cheat. In the long-run it is how to prevent entry. Or alternatively, if entry can be prevented, it is how to prevent disproportionate investment in capacity by the various partners in the collusion. These are different sorts of problems, which, therefore, demand different sorts of solutions.

I have not attempted to add these long-run considerations to the model. There are some points worth mentioning in anticipation of further work in this area. Clearly, one way of minimizing the threat of entry is for the cartel to forego the joing maximum profits in favor of a limit price, still somewhere above the competitive price. It is not intutitively clear what effect this might have on the short-run decisions of the members. If they believe that charging the limit price greatly extends the ultimate life of the cartel, they may be more inclined to remain loyal than if they thought it would soon, because of entry, be every man for himself.

It is widely-held that the only sort of policing which will effectively maintain a collusion agreement is one backed by the power of the state. The instances of these external enforcement devices are so well-known as not to need rehearsing here. The implication of the model is that it is possible for internal enforcement devices to serve the

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