DAMS AND WEIRS

AN ANALYTICAL AND PRACTICAL TREATISE ON GRAVITY DAMS AND WEIRS; ARCH AND BUTTRESS DAMS; SUBMERGED WEIRS; AND BARRAGES

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ILLUSTRATED

AMERICAN TECHNICAL SOCIETY
CHICAGO
1915

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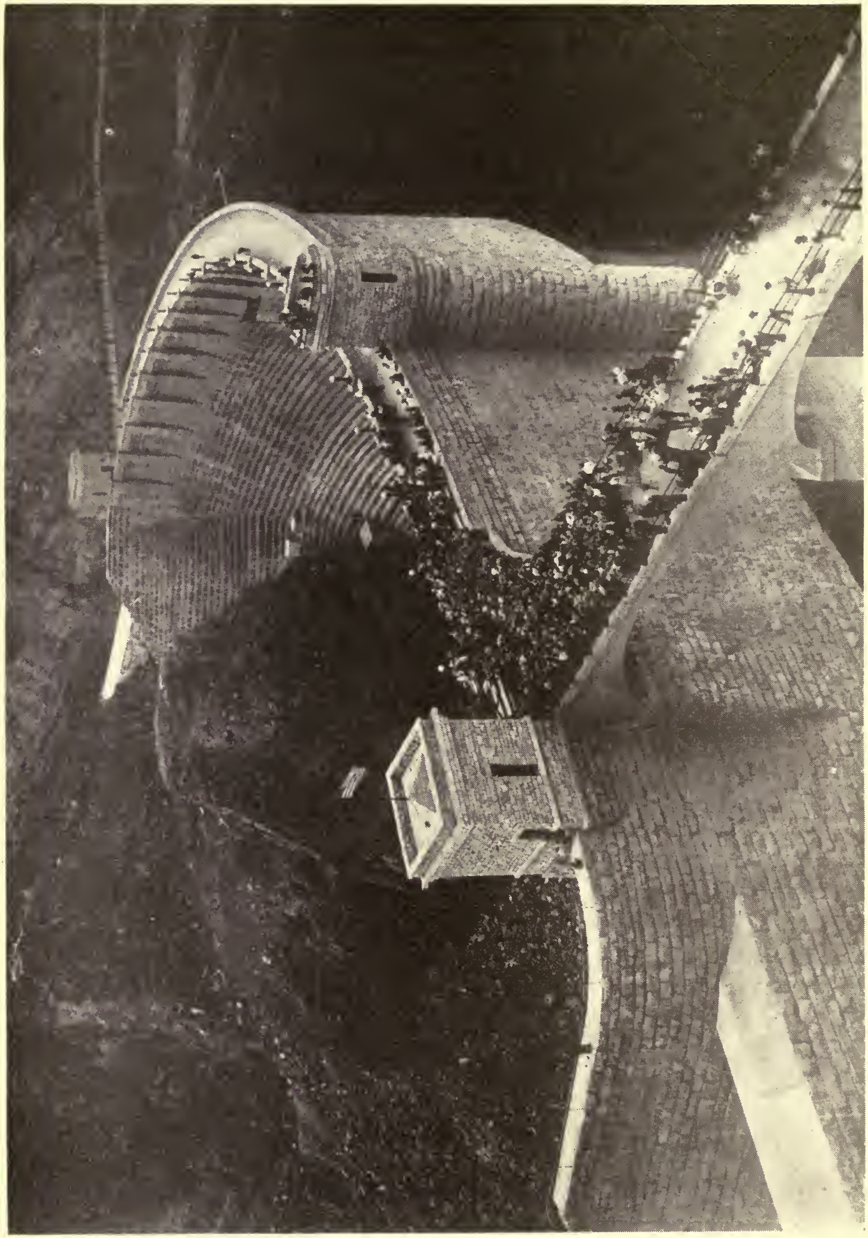


INTRODUCTION

AN unused waterfall, no matter how beautiful, appeals to an engineer mainly as an economic waste, and he fairly aches to throw a dam across the rushing torrents or to utilize the power of the water which glides gracefully over the falls and dashes itself into useless spray many feet below. His progress in the past years has, however, in no way measured up to his desires, but with the United States and other governments undertaking gigantic irrigation projects in order to reclaim vast areas of tillable lands and with hydroelectric companies acquiring the power rights of our great waterfalls, the last few years have witnessed wonderful progress in this type of engineering work. The use of reinforced concrete as a standard material and the solving of the many problems in connection with it has greatly simplified and cheapened the construction, thus avoiding the greater difficulties of masonry construction usually found in the older dams.

¶ All of this progress in the design of dams and weirs, however, has served to multiply the types of design and has increased the need for an authoritative and up-to-date treatise on the theoretical and practical questions involved. The author of this work has been a designing engineer for more than a generation and has built dams and weirs in India, Egypt, Canada, and this country. He is, therefore, abundantly qualified to speak, not only from the historic side of the work but from the modern practical side as well. In addition to a careful analysis of each different type of profile, he has given critical studies of the examples of this type, showing the good and bad points of the designs. A wealth of practical problems together with their solution makes the treatise exceedingly valuable.

¶ It is the hope of the publishers that this modern treatise will satisfy the demand for a brief but authoritative work on the subject and that it will find a real place in Civil Engineering literature.

DAMS AND WEIRS

PART I

INTRODUCTION

1. Definitions. A dam may be defined as an impervious wall of masonry, concrete, earth, or loose rock which upholds a mass of water at its rear, while its face or lower side is free from the pressure of water to any appreciable extent. The waste water of the reservoir impounded by the dam is disposed of by means of a waste weir, or by a spillway clear of the work, or in rare cases, by sluice openings in the body of the dam.

Weirs, or overfall dams, although often confounded with bulkhead dams, differ from the latter in the following points, first, that the water overflows the crest, and second, that the tail water is formed below the dam. These two differences often modify the conditions of stress which are applicable in the design of dams proper, and consequently the subject of weirs demands separate treatment.

2. Classification. Dams and weirs may be classified as follows:

1. Gravity Dams
2. Gravity Overfalls, or Weirs
3. Arched Dams
4. Hollow Arch Buttress Dams
5. Hollow Slab Buttress Dams
6. Submerged Weirs
7. Open Dams, or Barrages

The subjects of earth, rock fill, and steel dams will not be treated in this article, as the matter has been already dealt with in other volumes. Graphical as well as analytical methods will be made use of, the former procedure being explained in detail as occasion demands.

GRAVITY DAMS

GENERAL DISCUSSION OF DAMS

A gravity dam is one in which the pressure of the water is resisted by the weight or "gravity" of the dam itself.

3. Pressure of Water on Wall. The hydrostatic pressure of the water impounded by a wall or dam may be graphically represented by the area of a triangle with its apex at the surface and its base drawn normal to the back line of the dam, which base is equal or proportionate to the vertical depth. When the back of the wall is vertical, as in Fig. 1, the area of this pressure triangle will be $\frac{H^2}{2}$ H being the vertical height. When, as in Fig. 2, the back is inclined, this area will be $\frac{H'H}{2}$, H' being the inclined length of the exposed surface, which equals H cosec ϕ.

The actual pressure of water per unit length of dam is the above area multiplied by the unit weight of water. This unit

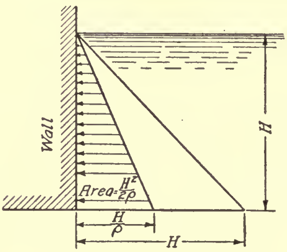


Fig. 1. Water Pressure Area with Back of Dam Vertical

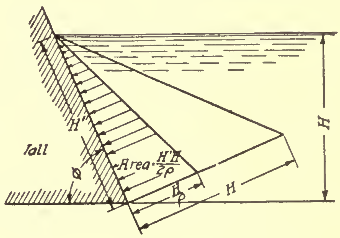


Fig. 2. Water Pressure Area with Back of Dam Inclined

weight is symbolized by w, which is 62.5 pounds, or $\frac{1}{32}$ short ton, or $\frac{1}{36}$ long ton, per cubic foot.

Unit Pressure. The pressure per square foot, or unit pressure on the wall at any point, is measured by the corresponding ordinate of the above triangle, drawn parallel to its base, and is evidently the same in both Figs. 1 and 2. The total pressure on the inclined back as represented by the triangle in Fig. 2 will, however, be greater than in Fig. 1.

4. Method for Graphical Calculations. For graphical calculations when forces of dissimilar unit weight or specific gravity are engaged, as in the case of water and masonry, or earth and masonry, it is the usual practice to reduce them to one common denominator by making alterations in the areas of one or the other, the weight of the masonry being usually taken as a standard. This result is effected by making the bases of the triangles of water pressure equal, not to H, but to $\frac{H}{\rho}$, ρ (rho) being the sign of the specific gravity of the solid material in the wall. The triangle thus reduced will then represent a weight or area of masonry 1 unit thick, equivalent to that of water. This device enables the item of unit weight, which is $w \times \rho$ to be eliminated as a common factor from the forces engaged, i. e., of the water pressure and of the weight of the wall. The factor thus omitted has to be multiplied in again at the close of the graphical operation, only, however, in cases where actual pressures in tons or pounds are required to be known.

Value of ρ. The values ordinarily adopted for ρ, the specific gravity of masonry or concrete, are $2\frac{1}{4}$ and 2.4, i. e., equivalent to weights of 141 and 150 pounds, respectively, per cubic foot, while for brickwork 2 is a sufficiently large value. The value of $w\rho$ in the former case will be .069 ton and in the latter .075, or $\frac{3}{40}$ ton.

In some cases the actual value of ρ mounts as high as 2.5 and even 2.7, when heavy granite or basalt is the material employed.

The reduction thus made in the water pressure areas has further the convenience of reducing the space occupied by the diagram. The areas of the reduced triangles of water pressure in Figs. 1 and 2 are $\frac{H^2}{2\rho}$ and $\frac{H'H}{2\rho}$, respectively.

5. Conditions of "Middle Third" and Limiting Stress. Sections of gravity dams are designed on the well-known principle of the "middle third." This expression signifies that the profile of the wall must be such that the resultant pressure lines or centers of pressure due first to the weight of the dam considered alone, and second with the external water pressure in addition, must both fall within the middle third of the section on any horizontal base. These two conditions of stress are designated, Reservoir Empty (R.E.) and Reservoir Full (R.F.). The fulfillment of this condition insures the following requirement: *The maximum compressive ver-*

tical unit stress (s), *or reaction on the base of a dam, shall not exceed twice the mean compressive unit stress,* or, stated symbolically,

$$s \leqq 2s_1$$

Now the mean vertical compressive unit stress s_1 is the weight of the structure divided by its base length- i.e.,

$$s_1 = \frac{W}{b}$$

Hence, s, the maximum vertical unit pressure, should not exceed $\frac{2W}{b}$. Further comments on the distribution of the reaction on the base of a dam will be made in a later paragraph.

6. Compressive Stress Limit. A second condition imposed is that of the internal compressive stress limit, that is: *The maximum permissible compressive unit stress which is developed in the interior of the masonry of the dam, must not be exceeded.* This value can be experimentally found by crushing a cube of the material employed, and using a factor of safety of 6 or 8. Cement concrete will crush at about 2000 pounds per square inch, equivalent to 144 tons (of 2000 pounds) per square foot. The safe value of s would then be $\frac{144}{8} = 18$ tons per square foot. For ordinary lime concrete as employed in the East, the limit pressure adopted is generally 8 "long" tons, equivalent to 9 tons of 2000 pounds. Ten "long" tons, or 11.2 "short" tons is also a common value.

DESIGN OF DAMS

7. Theoretical Profile. The theoretically correct profile of a so-termed "low" masonry dam, i.e., one of such height that the limit stress is not attained under the conditions above outlined, is that of a right-angled triangle having its back toward the water vertical, and its apex at the water surface. It can be proved that the proper base width b of this triangle is expressed by the formula

$$b = \frac{H}{\sqrt{\rho}} \tag{1}$$

This profile, shown in Fig. 3, will be termed the "elementary triangular profile", as on it the design of all profiles of dams is more or less based. In this expression, H is the vertical height. The base width of $\frac{H}{\sqrt{\rho}}$ insures the exact incidence of the vertical resultant (W) (R.E.) and of the inclined resultant R (R.F.) at the inner and outer edge, respectively, of the central third division of the base. The condition of the middle third is thus fulfilled in the most economical manner possible, a factor of safety of 2 against overturning is obtained, and further, the angle of inclination of the resultant R with regard to the base is usually such as to preclude danger of failure by sliding.

The fore slope or hypothenuse will be in the ratio $1:\sqrt{\rho}$ which, when $\rho=2\frac{1}{4}$, will equal 2:3, a slope very commonly adopted,

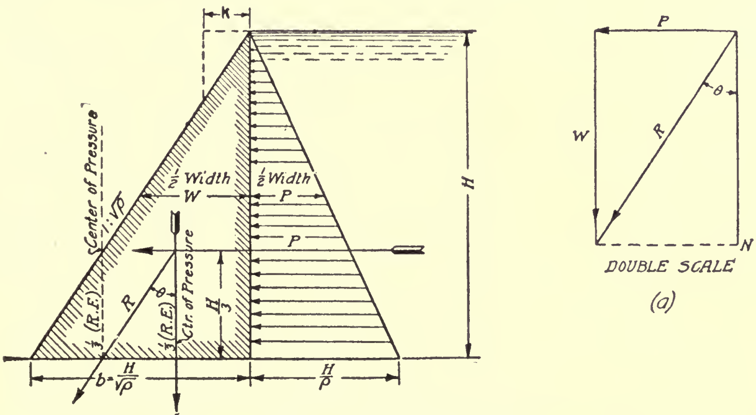


Fig. 3. Elementary Triangular Profile for "Low" Masonry Dam

and with $\rho=2.4$ the ratio will be 1:1.549. The area of the elementary triangle is $\frac{H^2}{2\sqrt{\rho}}$ while, as we have seen, that of the water pressure is $\frac{H^2}{2\rho}$. θ is the vertical angle between W and R, and

$\sec\theta = \frac{\sqrt{\rho+1}}{\sqrt{\rho}} = 1.187$ with $\rho=2.4$.

In Fig. 3 the resultant pressure lines are drawn to intersect the base so as to afford ocular proof of the stability of the section under the postulated conditions.

Graphical Method. The graphical procedure will now be briefly explained, and also in the future as fresh developments arise, for the benefit of those who are imperfectly acquainted with this valuable labor-saving method.

There are two forces engaged, P the horizontal, or, it may be P_1, the inclined water pressure acting through the center of gravity of its area normal to the back of the wall, and W the weight or area of the wall. Of these two forces the item $w\rho$, or unit weight, has already been eliminated as a common factor, leaving the pressures represented by superficial areas. As, however, the height H is also common to both triangles, this can likewise be eliminated. The forces may then be represented simply by the half widths of the triangular areas by which means all figuring and scaling may be avoided.

First, a force polygon has to be constructed. In Fig. 3a, P is first drawn horizontally to designate the water pressure, its length being made equal to the half width of its pressure area in Fig. 3. From the extremity of P, the load line W is drawn vertically, equal to the half width of the elementary triangular profile, then the closing line R according to the law of the triangle of forces will represent the resultant in magnitude and direction. Second, the lines of actual pressures reciprocal to those on the force polygon will have to be transferred to the profile. The incidence of the resultant water pressure on the back is that of a line drawn through the c.g. of the area of pressure, parallel to its base, in this case, at $\frac{H}{3}$, or one-third the height of the water-pressure triangle, above the base. Its direction, like that of the base, is normal to the back, in this case horizontal, and if prolonged it will intersect the vertical force W, which in like manner acts through the center of gravity of the elementary profile of the wall. From this point of intersection the resultant R is drawn parallel to its reciprocal in Fig. 3a. Both W and R are continued until they cut the base line, and these points of intersection will be found to be exactly at the inner and outer edges of the middle third division of the base. It will be seen that when the reservoir is empty the center of pressure on the base is at the incidence of W, when full it is shifted to that of R.

Analytical Method. The same proof can be made analytically as follows: The weight of the two triangles W and P can be represented by their bases which are $\frac{H}{\sqrt{\rho}}$ and $\frac{H}{\rho}$, respectively. If moments be taken about the outer edge of the middle third, the lever arm of the vertical force W is clearly $\frac{b}{3}$ or $\frac{H}{3\sqrt{\rho}}$ and that of P, the horizontal force, is the distance of the center of gravity of the triangle of water pressure above the base, viz, $\frac{H}{3}$. The equation will then stand

$$\left(\frac{H}{\sqrt{\rho}}\times\frac{H}{3\sqrt{\rho}}\right)-\left(\frac{H}{\rho}\times\frac{H}{3}\right)=0$$

or

$$\frac{H^2}{3\rho}-\frac{H^2}{3\rho}=0$$

If the actual values of R and of W were required, their measured or calculated lengths would have to be multiplied by H and by $w\rho$ in order to convert them to tons, pounds, or kilograms, as may be required. In many, in fact most, cases actual pressures are not required to be known, only the position of the centers of pressures in the profile.

Thus a line of pressures can be traced through a profile giving the positions of the centers of pressure without the necessity of converting the measured lengths into actual quantities. In the elementary triangle, Fig. 3, the value of the vertical resultant W is $\frac{H^2w\sqrt{\rho}}{2}$. That of R required in the older methods of calculation is $\frac{H^2w\sqrt{\rho+1}}{2}$. The following values relative to ρ will be found useful.

ρ or Specific Gravity	$\sqrt{\rho}$	$\frac{1}{\sqrt{\rho}}$	$\frac{1}{\rho}$	Pounds per Cubic Foot	Tons per Cubic Foot
2	1.414	.71	.5	125	.0625
$2\frac{1}{4}$	1.5	$\frac{2}{3}$	$\frac{4}{9}$	141	.07
2.4	1.55	.645	.417	150	$.075=\frac{3}{40}$
2.5	1.58	.633	.4	156	.078
2.7	1.643	.609	.37	168.7	.084

Profile with Back Inclined. If the elementary profile be canted forward so that its back is inclined to the vertical, it will be found that the incidence of R will fall outside the middle third while that of W will be inside. The base will, therefore, have to be increased above $\frac{H}{\sqrt{\rho}}$.

When the back is overhanging, on the other hand, R will fall inside and W outside the middle third. The vertically backed section is consequently the most economical.

8. Practical Profile. In actual practice a dam profile must be provided with a crest of definite width, and not terminate in the apex of a triangle. The upper part of a dam is subjected to indefinite but considerable stresses of an abnormal character, due to extreme changes in temperature, consequently a solid crest is a necessity. The imposition of a rectangular crest, as shown in dotted lines on

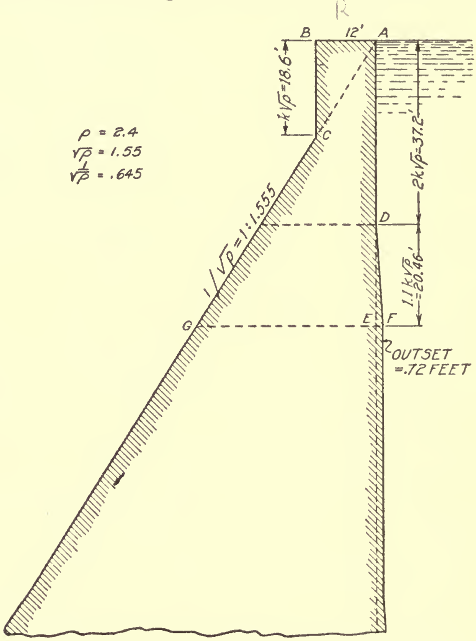


Fig. 4. Practical Pentagonal Profile for "Low" Masonry Dam

Fig. 3, transforms the triangular profile into a pentagon. This has the effect of increasing the stability of the section (R.F.) so that the base width can be somewhat reduced, at the same time the vertical resultant W (R.E.), falls outside the middle third, but to so small an extent that this infringement of the imposed condition is often entirely neglected. In order to provide against this, a strip of material will have to be added to the back of the plain pentagonal profile. Fig. 4 is a diagram explanatory of these modifications. The dimensions of this added strip, as well as its position, can be conveniently expressed in terms of (k) the crest width—i.e., AB in Fig. 4. The line of pressure (R.E.) will begin to leave the middle third at the depth AD, which is found by calculation which need not be produced here, to be $2k\sqrt{\rho}$. Below the point D, the divergence of the line of pressure will continue for a further depth DE, the point E, being close upon $3.1k\sqrt{\rho}$ below the crest, or $1.1k\sqrt{\rho}$ below D. Below point E, the line of pressure will no longer diverge outward, but will tend to regain its original position, consequently no further widening will be necessary, and the added

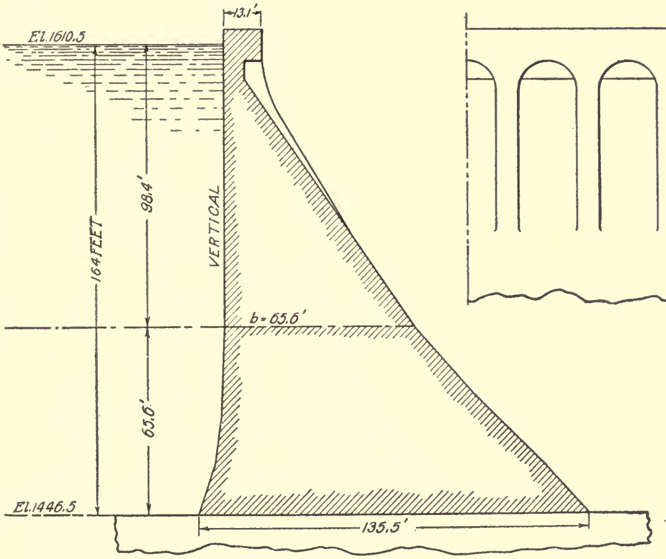


Fig. 5. Profile of Chartrain Dam Showing Crest with Overhang

strip will be rectangular in form down to the base. The points D and F being joined, this portion of the back will be battered. The width of this added strip EF will be, with close approximation, $\frac{k}{16}$ or $.06k$.

9. Crest Width. The crest width of a dam should be proportioned to its actual height in case of a "low" dam, and in the case of a "high" dam to the limiting height—i.e., to that depth measured below the crest at which the maximum stress in the masonry is first reached. Thus in "high" dams the upper part can always be of the same dimensions except where the requirements of cross communication necessitate a wider crest.

The effect of an abnormally wide crest can be modified by causing it to overhang the fore slope, this widening being carried by piers and arches. A good example of this construction occurs in the Chartrain dam, Fig. 5. The arches form a stiff but light finish to the dam and have a pleasing architectural effect. The same procedure, but in a less pronounced degree, is carried out in the Croton dam, Fig. 27.

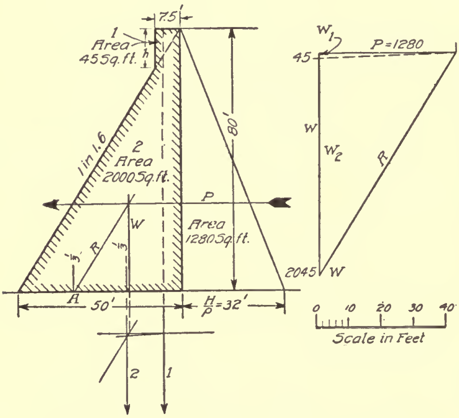


Fig. 6. Pentagonal Profile—Back Vertical

The formula for crest width can be expressed either in terms of the limiting height H_l, or of the base b, where the limiting height is not attained, and a good proportion is given by the following empirical rule:

$$k = \sqrt{H_l} \qquad (2)$$

or

$$k = .15b \qquad (2a)$$

This latter formula makes the crest width a function of the specific gravity as well as of the height, which is theoretically sound.

10. Rear Widening. Where the rear widening of a "low" dam is neglected or where a uniform batter is substituted for the arrangement shown in Fig. 4, the profile will be pentagonal in outline. When the back is vertical the two triangles composing the body of the dam are similar. If the ratio existing between the crest (k) and the base (b), or $\frac{k}{b}$ be designated by r, then $k = br$, and h, the depth of the vertical side in Fig. 6, $= Hr$ and $k \times h = Hbr^2$.

In order to find what value the base width b should have, so that the center of pressure (R.F.) will fall exactly at the edge of the middle third, the moments of all the forces engaged will have to be taken about this point and equated to zero. The vertical forces consist of W, the lower, and W_1 the upper triangle; the horizontal of P, the water pressure.

11. Method of Calculation. The pressures can be represented by the areas of the prisms involved, the triangle of water pressure being as usual reduced by dividing its base by ρ. A further elimination of common factors can be achieved by discarding $\frac{H}{2}$ which is common to all three forces, the area W_1 being represented by br^2 because the actual original value is $\frac{Hbr^2}{2}$. The forces then are W, represented by b; W_1 by br^2; and P by $\frac{H}{\rho}$; the actual value of the latter being $\frac{H^2}{2\rho}$. The lever arm distances of the c.g.'s of these three forces from A, the incidence of R, are as follows: of W, $\frac{b}{3}$, of W_1, $\frac{2}{3}(b-br)$, and of P, $\frac{H}{3}$. The equation will then stand, eliminating $\frac{1}{3}$,

$$b\times b+br^2(2b-2\times br)-\frac{H^2}{\rho}=0$$

or

$$b^2\,(1+2r^2-2r^3)-\frac{H^2}{\rho}=0$$

whence

$$b=\frac{H}{\sqrt{\rho}}\times\frac{1}{\sqrt{1+2r^2-2r^3}} \qquad (3)$$

The value of b thus obtained will prove a useful guide in deciding the base width even when the back of the wall is not vertical, as only a small increase will be needed to allow for the altered profile. When $\frac{k}{b}$ or $r=.15$ the reducing coefficient works out to $\frac{1}{1.019}$, the reciprocal of which is .981. Thus with a profile 80 feet high with $\rho=2.5$ and $r=.15$, the base width of the pentagonal profile will be $b=\frac{80}{\sqrt{2.5}}\times.981$ $=49.64$ feet; the decrease in base width below that of the elementary profile without crest will be $50.60-49.64=0.96$ feet. The crest width will be $49.64\times.15=7.45$ feet. In actual practice, the dimensions would be in round numbers, 50 feet base and $7\frac{1}{2}$ feet crest width as made on Fig. 6. The face of the profile in Fig. 6 is made by joining the toe of the base with the apex of the triangle of water pressure.

Graphical Process. The graphical processes of finding the incidences of W and of R on the base are self-explanatory and are shown on Fig. 6. The profile is divided into two triangular areas, (1), 45 square feet and (2), 2000 square feet. The two final resultants fall almost exactly at the middle third boundaries, W, as might be conjectured, a trifle outside. Areas are taken instead of $\frac{1}{2}$ widths, owing to H not being a common factor.

Analytical Process. The analytical process of taking moments about the heel is shown below:

	Area	Lever Arm	Moment
(1)	45	$\frac{7.5\times2}{3}$	225
(2)	2000	$\frac{50}{3}$	33333
W	2045		33558

The value 33,558, which is the total moment of parts equals the moment of the whole about the same point or

$$2045\times x=33558$$

$\therefore$

$$x=16.41 \text{ feet}$$

The incidence of W is therefore $\frac{50}{3}-16.41=.26$ ft. outside the middle third. To find that of R relative to the heel, the distance (see section 17) between W and R is $\frac{PH}{3W}=\frac{1280\times80}{3\times2045}=\frac{102400}{6135}=16.69$. The distance of R from the heel is therefore $16.69+16.41=33.10$ ft. The $\frac{2}{3}$ point is 33.33 feet distant, consequently the incidence of R is .23 foot within the middle third.

If the base and crest had been made of the exact dimensions deduced from the formula, the incidence of R would be exactly at the $\frac{2}{3}$ point while W would fall slightly outside the one-third point.

12. Variation of Height. The height of a dam is seldom uniform throughout; it must vary with the irregularities of the river bed, so that the maximum section extends for a short length only, while the remainder is of varying height. This situation will affect the relationship between the crest width and the height, and also the base width. To be consistent, the former should vary in width in proportion to the height. This, however, is hardly practicable, consequently the width of the crest should be based more on the average than on the maximum height, and could be made wider wherever a dip occurs in the foundation level.

13. High and Wide Crest. In case of a very high as well as wide crest, i.e., one carried much higher than the apex of the triangle of water pressure, it is not desirable to reduce the base width much below that of the elementary triangle. The excess of material in the upper quarter of a "low" dam can be reduced by manipulating the fore slope. This latter, which is drawn upward from the toe of the base, in Fig. 7, can be aligned in three directions. First, by a line terminating at the apex of the triangle of water pressure; second, it can be made parallel to that of the elementary profile, that is, it can be given an inclination of $\frac{1}{\sqrt{\rho}}$ to the vertical, and third, the slope or batter can be made flatter than the last. This latter disposition is only suitable with an abnormally high and wide crest and is practically carried out in the Chartrain dam, Fig. 5, where the base is not reduced at all below $\frac{H}{\sqrt{\rho}}$.

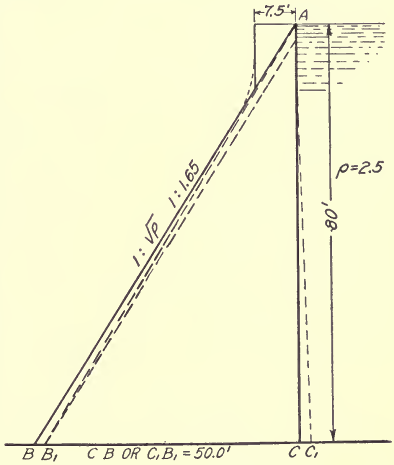


Fig. 7. Profile Showing Different Disposition of Fore Slope

Reduction to any large extent, of the neck of the profile thus effected is, however, not to be commended, as the upper quarter of a dam is exposed to severe though indeterminate stresses due to changes of temperature, wind pressure, etc., and also probably to masses of ice put in motion by the wind. The Cross River dam, to be illustrated later, as well as the Ashokan dam, are examples of an abnormally thick upper quarter being provided on account of ice. Whatever disposition of the fore slope is adopted, the profile should be tested graphically or analytically, the line of pressure, if necessary, being drawn through the profile, as will later be explained.

From the above remarks it will be gathered that the design of the section of a dam down to the limiting depth can be drawn by a few lines based on the elementary profile which, if necessary, can be modified by applying the test of ascertaining the exact position of the centers of pressure on the base. If the incidence of these resultants falls at or close within the edge of the middle third division of the base, the section can be pronounced satisfactory; if otherwise, it can easily be altered to produce the desired result.

Freeboard. The crest has to be raised above actual full reservoir level by an extent equal to the calculated depth of water passing over the waste weir or through the spillway, as the case may be. This extra freeboard, which adds considerably to the cost of a work, particularly when the dam is of great length and connected with long embankments, can be avoided by the adoption of automatic waste gates by which means full reservoir level and high flood level are merged into one.

In addition to the above, allowance is made for wave action, the height of which is obtained by the following formula:

$$h_w = 1.5\sqrt{F} + (2.5 - \sqrt[4]{F}) \qquad (4)$$

In this expression F is the "fetch", or longest line of exposure of the water surface to wind action in miles. Thus if $F=4$ miles, the extra height required over and above maximum flood level will be $(1.5\times 2)+(2.5-1.4)=4.1$ feet. If $F=10$ miles, h_w will work out to $5\frac{1}{2}$ feet. The apex of the triangle of water pressure must be placed at this higher level; the crest, however, is frequently raised still higher, so as to prevent the possibility of water washing over it.

14. Example. The working out of an actual example under assumed conditions will now be given by both graphical and analytical methods. Fig. 8 represents a profile 50 feet in height with crest level corresponding with the apex of the triangle of water pressure.

The assumed value of ρ is $2\frac{1}{4}$. The outline is nearly pentagonal, the crest width is made $.15b$ and the base width is the full $\frac{H}{\sqrt{\rho}}=\frac{2}{3}$ $\times 50=33.3$ feet, the crest width is thus 5 feet. The back slope is carried down vertically to the point e, a distance of 8 feet, and from here on, it is given a batter of 1 in 50. The outset at the heel beyond the axis of the dam, which is a vertical line drawn through the rear

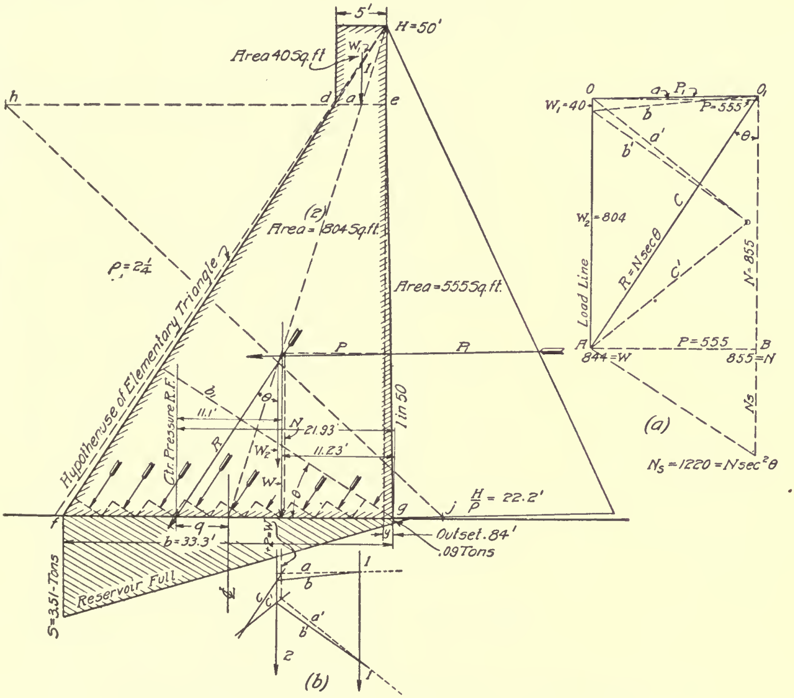


Fig. 8. Diagram Showing Suitable Profile for 50-Foot Dam

edge of the crest is therefore .84 foot. The toe is set in the same extent that the heel is set out. The face line of the body is formed by a line joining the toe with the apex of the water-pressure triangle. If the face line were drawn parallel to the hypothenuse of the elementary triangle, i.e., to a slope of $1:\sqrt{\rho}$, it would cut off too much material, the area of the wall being then but very little in excess of that of the elementary triangle, which, of course, is a minimum quantity. As will be seen later, the analysis of the section will show that the adopted base width could have been reduced below what has been provided, to an extent somewhat in excess of that given in formula (3).

15. Graphical Method. The graphical procedure of drawing the resultant lines W (R.E.) and R (R.F.) to their intersection of the base presents a few differences, from that described in section 7, page 6, with regard to Fig. 3. Here the profile is necessarily divided into two parts, the rectangular crest and the trapezoidal body. As the three areas (1), (2), and P_1, are not of equal height, the item H cannot be eliminated as a common factor, consequently the forces will have to be represented as in Fig. 6 by their actual superficial areas, not by the half width of these areas as was previously the case. In Fig. 8a the vertical load line consists of the areas 1 and 2 totaling 844 square feet, which form W. The water pressure P_1 is the area of the inclined triangle whose base is $\frac{H}{\rho}$. This is best set out graphically in the force polygon by the horizontal line P, made equal to the horizontal water pressure, which is $\frac{H^2}{2\rho}=\frac{2500\times2}{9}=555$ square feet. The water-pressure area strictly consists of two parts corresponding in depth to (1) and (2) as the upper part is vertical, not inclined, but the difference is so slight as to be inappreciable, and so the area of water pressure is considered as it would be if the back of the wall were in one inclined plane. In Fig. 8 the line P_1 normal to the back of the wall is drawn from the point of origin O and it is cut off by a vertical through the extremity of the horizontal line P. This intercepted length $O\,O_1$ is clearly the representative value of the resultant water pressure, and the line joining this point with the base of the load line W is R, the resultant of W and of P_1. If a horizontal line AB be drawn from the lower end of the load line W it will cut off an intercept (N) from a vertical drawn through the termination of P_1. This line $AB=P$, and N is the vertical component of R, the latter being the resultant of W and P_1 as well as of N and P. When the back is vertical, N and W are naturally identical in value, their difference being the weight of water overlying the inclined rear slope.

The further procedure consists in drawing the reciprocals of the three forces P_1, W, and R on the profile. The first step consists in finding the centers of gravity of the vertical forces 1 and 2 in which the hexagonal profile is divided. That of (1) lies clearly in the middle of the rectangle whose base is *de*. The lower division (2) is a trapezoid. The center of gravity of a trapezoid is best found by the following extremely simple graphical process. From *d* draw *dh* horizontally equal to the base of the trapezoid *fg* and from *g*, *gj* is set off equal to *de;* join *hj*, then its intersection with the middle line of the trapezoid gives the exact position of its center of gravity. Thus a few lines effect graphically what would involve considerable calculation by analytical methods, as will be shown later.

The next step is to find the combined c. g. of the two parallel and vertical forces 1 and 2. To effect this for any number of parallel or non-parallel forces, two diagrams are required, first, a so-termed force and ray polygon and, second, its reciprocal, the force and chord, or funicular polygon. The load line in Fig. 8a can be utilized in the former of these figures. First, a point of origin or nucleus of rays must be taken. Its position can be anywhere relative to the load line, a central position on either side being the best. The point O_1, which is the real origin of the force polygon at the extremity of P_1 can be adopted as nucleus and often is so utilized, in which case the force line P_1 and R can be used as rays, only one additional ray being required. For the sake of illustration, both positions for nucleus have been adopted, thus forming two force and ray polygons, both based on the same load line, and two funicular polygons, the resultants of which are identical. The force and ray polygon is formed by connecting all the points on the load line with the nucleus as shown by the dotted line *a, b,* and *c*, and *a′*, *b′*, and *c′*. Among the former, *a* and *c* are the force lines P_1 and R, the third, *b*, joins the termination of force (1) on the load line with the nucleus. These lines *a, b, c,* are the rays of the polygon. Having formed the force and ray diagram, in order to construct the reciprocal funicular polygon 8*b* the force lines (1) and (2) on the profile Fig. 8 are continued down below the figure. Then a line marked (*a*) is drawn anywhere right through (1) parallel to the ray *a*, from its intersection with the force (1), the chord (*b*) is drawn parallel to the ray (*b*) in Fig. 8b meeting (2); through this latter intersection the third chord (*c*) is drawn backward parallel to its reciprocal the ray *c*. This latter is the closing line and its intersection with the initial line (*a*), gives the position of the c.g. of the two forces.

A vertical line through this center of pressure, which represents W, i.e., W_1+W_2, is continued on to the profile until it intersects the inclined force P_1 drawn through the center of gravity of the water pressure area. This intersection is the starting point of R, drawn parallel to its reciprocal on the force polygon 8*a*. This resultant intersects the base at a point within the middle third. R is the resultant "Reservoir Full", while W, the resultant of the vertical forces in the masonry wall, is the resultant "Reservoir Empty". The intersection of the latter is almost exactly at the inner edge of the middle third—thus the condition of the middle third is fulfilled. The question of induced pressure and its distribution on the base will be considered later.

The incidence of N, the vertical component "Reservoir Full", on the base is naturally not identical with that of W, the resultant "Reservoir Empty", unless the back of the wall is vertical. The line R is the resultant of both P_1 and W, and of P and N. If it be required to fix the position of N on the profile, a horizontal line should be drawn through the intersection of P_1 with the back of the wall. This will represent the horizontal component of the water pressure P_1, and it will intersect R, produced upward. Then a line drawn vertically through this latter point will represent N, the vertical component (Reservoir Full). The position of N is necessarily outside of W, consequently if N is made to fall at the inner edge of the middle third of the base, W must fall within the middle third. This fact will later be made use of when the design of the lower part of a "high" dam comes under consideration.

16. Analytical Method. The analytical method of ascertaining the positions of the incidences of W and of R on the base, which has just been graphically performed, will now be explained.

The first step is to find the positions of the centers of gravity of the rectangle and trapezoid of which the profile is composed, relative to some vertical plane, and then to equate the sum of the moments of those two forces about any fixed point on the base, with the moment of their sum.

The most convenient point in most cases is the heel of the base; this projects a distance (y) beyond the axis of the dam, which axis is a vertical line passing through the inner edge of the rectangular crest.

As the areas of the divisions, whether of the masonry wall or of the water-pressure triangle, are generally trapezoids, the following enumeration of various formulas, whereby the position of the c.g. of a trapezoid may be found either with regard to a horizontal or to a vertical plane, will be found of practical utility. In Fig. 9, if the depth of the figure between the parallel sides be termed H, and that of the truncated portion of the triangle of which the trapezoid is a portion be termed d, and h be the vertical height of the c.g. above the base, then

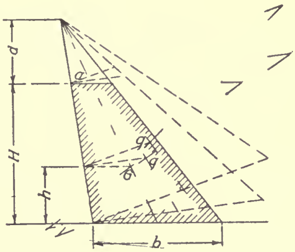


Fig. 9. Diagram Showing Centers of Gravity of Water Pressure Trapezoids

$$h = \frac{H}{3} \times \frac{H+3d}{H+2d} \qquad \textbf{(5)}$$

Thus, in Fig. 9, $H=13$ and $d=6$ feet, then

$$h = \frac{13}{3}\left(\frac{13+18}{13+12}\right) = 5.37 \text{ feet}$$

If the base of the triangle and trapezoid with it be increased or decreased in length, the value of h will not be thereby affected, as it is dependent only on H and d, which values are not altered. If, however, the base of the triangle be inclined, as shown by the

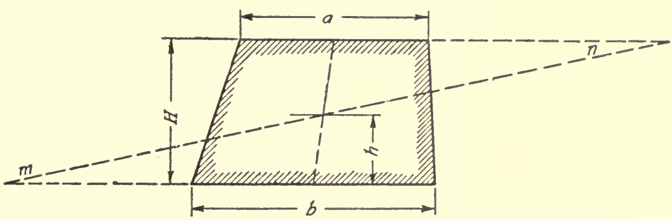


Fig. 10. Diagram Illustrating Height of c. g. Trapezoid above Base

dotted lines in Fig. 9, the center of gravity of the trapezoid will be higher than before, but a line drawn parallel to the inclined base through g, the c. g. will always intersect the upright side of the trapezoid at the same point, viz, one which is h feet distant vertically above the horizontal base.

The value of h can also be obtained in terms of a and b, the two parallel sides of the trapezoid, and is

$$h=\frac{H}{3}\left(\frac{b+2a}{a+b}\right) \tag{6}$$

For example, in Fig. 10, $H=12$, $a=10$, and $b=16$, then

$$h=\frac{12}{3}\left(\frac{16+20}{10+16}\right)=5.54 \text{ feet}$$

If the horizontal distance of the c.g. of a trapezoid from a vertical plane is required, as, for example, that of the trapezoid in Fig. 8, the following is explanatory of the working. As shown in Fig. 11, this area can be considered as divided into two triangles, the weight of each of which is equivalent to that of three equal weights placed at its angles; each weight can thus be represented by one-third of the area of the triangle in question, or by $\frac{aH}{6}$ and $\frac{bH}{6}$, respectively, H being the vertical depth of the trapezoid. Let y be the projection of the lower corner A beyond that of the upper one B. Then by equating the sum of the moments of the corner weights about the point A with the moment of their sum, the distance (x) of the c.g. of the whole trapezoid from A will be obtained as follows:

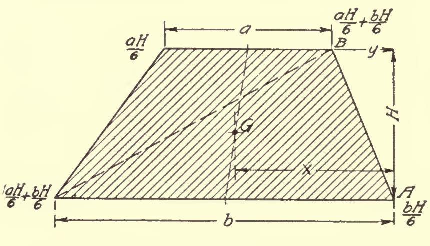


Fig. 11. Method of Finding Distance of Center of Gravity of a Trapezoid from Heel

$$\left(\frac{a+b}{2}\right)Hx=\frac{H}{6}\Big[b(a+b)+a(a+y)+y(a+b)\Big]$$

$\therefore$

$$x=\frac{1}{3}\left[(b+y)+a\left(\frac{a+y}{a+b}\right)\right] \tag{7}$$

where $y=0$, the formula becomes

$$x=\frac{1}{3}\left(b+\frac{a^2}{a+b}\right) \tag{7a}$$

For example, in Fig. 8, a or $de=5$ feet, $b=33.3$, and $y=.84$, whence

$$x=\frac{1}{3}\left(34.14+\frac{29.2}{38.3}\right)=11.63 \text{ feet}$$

The similar properties of a triangle with a horizontal base, as in Fig. 12, may well be given here and are obtained in the same way by taking moments about A, thus

$$\frac{bh}{2}\times x=\frac{bh}{6}\ (b+y)$$

$\therefore$

$$x=\frac{b+y}{3} \tag{8}$$

In Fig. 12, $b=14$ feet, $y=8$ feet, and $h=10$ feet, then

$$x=\frac{14+8}{3}=7\tfrac{1}{3}\text{ feet}$$

Reverting to Fig. 8, the position of the incidence of W on the base is obtained by taking moments about the heel g of the base as follows: Here W is the area of the whole profile, equal, as we have seen, to 844 sq. ft. The area of the upper component (1) is 40 sq. ft. and of (2) 804.

The lever arm of W is by hypothesis x, that of (1) is $2.5+.84=3.34$ feet, that of (2) by formula (7) has already been shown to be 11.63 feet. Hence, as the moment of the whole is equal to the sum of the moments of the parts, the equation will become

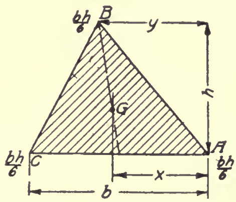


Fig. 12. Method of Finding Center of Gravity of a Triangular Profile

$$844x=40\times3.34+804\times11.63=9484.1$$

$\therefore$

$$x=11.23\text{ feet}$$

This fixes the position of the incidence of W relative to the heel. The position of the inner third point is $\frac{b}{3}$, or $\frac{33.3}{3}$ from the heel. The incidence of W is therefore $11.23-11.10=.13$ foot within the middle third, which complies with the stipulated proviso.

The next step is to find the position of R relative to the heel of the base. As in graphical methods, only horizontal and vertical forces are considered; the water-pressure area is split into two parts, one, P the horizontal component, the value of which is $\frac{H^2}{2\rho}$, or 555 feet, and w_3 the reduced area of water overlying the rear projection of the back. The latter area is a trapezoid of which the upper side

(a) is 8 feet long and b, the lower side, 50 feet, the depth being .84 foot, hence the distance of its c.g. inside the heel of the base will be by formula (6), $\frac{.84}{3}\frac{(50+16)}{58}=.32$ foot. Its actual area is $\frac{8+50}{2}\times$ $.84=24.4$ feet; this has to be divided by ρ or $2\frac{1}{4}$ to reduce it to a masonry base. The reduced area will then be 10.8 square feet, nearly. The distance of the incidence of W from the heel of the base has already been determined to be 11.23 ft. and that of w_3 being .32 ft., the distance of the c.g. of the latter from W will be $11.23-.32=10.9$, nearly. If the distance between the incidences of W and R be termed x, the equation of moments about the incidence of R, will stand thus:

$$P\frac{H}{3}=Wx+w_3(x+10.9)$$

or

$$555\times\frac{50}{3}=844x+10.8x+117.83$$

i.e.

$$x=\frac{9132.2}{854.8}=10.7 \text{ ft., nearly}$$

R is therefore $10.7+11.23=21.93$ ft. distant from the heel. The $\frac{2}{3}$ point being 22.2 ft. from the same point, R falls .3 ft. (nearly) within the middle third. This shows that a small reduction in the area of the profile could be effected.

17. Vertical Component. If the position of N, the vertical component of R and P_1, is required, as is sometimes the case, it is obtained by the equation $N\times x=(W\times 11.23)+(w_3\times .32)$, x being the distance from the heel of the base. Or in figures,

$$854.8x=(844\times 11.23)+(10.8\times .32)$$

$\therefore$

$$x=11.1 \text{ feet}$$

The incidence of N is, therefore, in this case, exactly on the limit of the middle third. This of course does not affect the condition of middle third, which refers to the resultant W (R.E.) not to the component N (R.F.) but, as will be seen later, when the lower part of a high dam comes to be designed, one condition commonly imposed is, that the vertical component N must fall at the inner edge of the middle third, in which case W will necessarily fall inside thereof.

It may here be noted that the space between the location of N and R, which will be designated (f), is $\frac{PH}{3N}$ because if moments are taken about the incidence of R, then $Nf=\frac{PH}{3}$; therefore $f=\frac{PH}{3N}$. The actual value of W in tons of 2000 pounds will be the superficial area, or 844 square feet multiplied by the eliminated unit weight, i.e., by $w\rho$, viz, $\frac{844\times 9}{32\times 4}=59.3$ tons, as $w=\frac{1}{32}$ ton. That of the inclined force R, is obtained from the triangle of forces PNR in which R, being the hypothenuse $=\sqrt{N^2+P^2}$. Here $N=855$ square feet, equivalent to 60 tons, nearly, and $P=555$ feet, equivalent to 39 tons, whence $R=\sqrt{60^2+39^2}=71.5$ tons.

18. Pressure Distribution. In the design of the section of a dam, pier, or retaining wall, the distribution of pressure on a plane in the section and the relations existing between maximum unit stress, symbolized by (s), and mean or average unit stress (s_1) will now be considered. The mean unit stress on any plane is that which acts at its center point and is in amount the resultant stress acting on the plane (the incidence of which may be at any point) divided by the width of the lamina acted on. Thus in Figs. 3 or 8 take the resultant W. This acts on the horizontal base and its mean unit stress s_1 will be $\frac{W}{b}$. In the same way, with regard to N, the vertical component of R the mean unit stress produced by it on the horizontal base will be $\frac{N}{b}$. The maximum unit stress occurs at that extremity of the base nearest to the force in question which is R. Thus the maximum unit stress due to W is at the heel while that due to a combination of P and N acting at the incidence of R is at the toe of the base b. It is evident that the nearer the incidence of the center of pressure is to the center point the less is the maximum stress developed at the outer edge of the section, until the center of pressure is actually situated at the center point itself. The maximum pressure at the outer part of the section then equals the average and is thus at a minimum value. The relation between maximum and mean unit stress or reaction is expressed in the following formula in which it is assumed that any tension at the heel can be cared for by the adhesion of the cementing material or of reinforcement anchored down:

$$s = s_1\left(1 + \frac{6q}{b}\right) \qquad \textbf{(9)}$$

or, letting m equal the expression in brackets,

$$s = ms_1 \qquad \textbf{(9a)}$$

In formula (**9a**), q is the distance between the center point of the base and the center of pressure or incidence of whatever resultant pressure is under consideration, and s_1 is the mean stress, or the resultant pressure divided by the base.

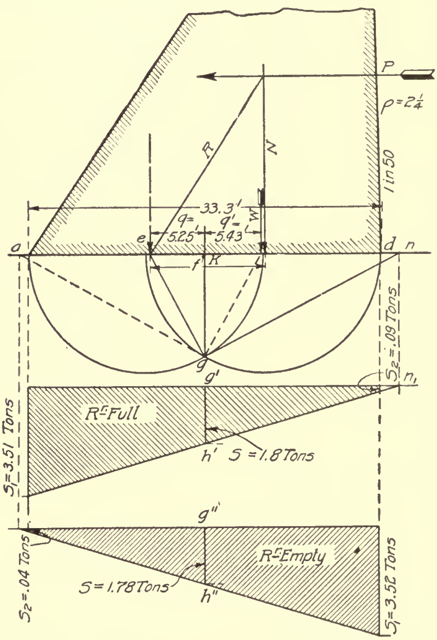


Fig. 13. Diagram Showing Pressure Distribution on a Dam with Reservoir Empty and Reservoir Full

In Fig. 8 as explained in section 16, the incidence of R, i.e., the center of pressure (R.F.), falls .3 ft. within the middle third of the base, consequently the value of q will be $\frac{b}{6} - .3 = \frac{33.3}{6} - .3 = 5.25$ ft., and in formula (**9a**) $m = 1 + \frac{6q}{b} = 1 + \frac{31.5}{33.3} = 1.95$. The maximum reaction at the toe always designated by $s = \frac{mN}{b} = \frac{1.95 \times 60}{33.3} = 3.51$ tons per sq. ft. For the reaction (R.F.) at the heel, $m = 1 - .95 = .05$, and $s_2 = \frac{.05 \times 60}{33.3} = .09$ tons. The distribution of pressure due to the vertical component of R is shown hatched in Fig. 8 as well as in Fig. 13.

From formula (**9**) the facts already stated are patent. When the incidence of the resultant force is at the center of the base,

$q=0$, consequently $m=1$ and $s=s_1$, that is, the maximum is equal to the mean; when at one of the third points, $q=\frac{b}{6}$, $m=2$, and $s=2s_1$; when at the toe, $m=4$, and $s=4s_1$, or $4\frac{W}{b}$.

If the material in the dam is incapable of caring for tensile strain, the maximum vertical compression, or s, obtained by formula (**9**) will not apply. Formula (**24**), section 86, should be used whenever R falls outside the middle third.

In designing sections it is often necessary to maneuver the incidence of the resultant stress to a point as close as possible to the center of the base in order to reduce the maximum stress to the least possible value, which is that of the mean stress. The condition of the middle third, insures that the maximum stress cannot exceed twice the mean, and may be less, and besides insures the absence of tensile stress at the base.

19. Graphical Method for Distribution of Pressure. The graphical method of ascertaining the distribution of pressure on the base of a masonry wall, which has already been dealt with analytically, is exhibited in Fig. 13, which is a reproduction of the base of Fig. 8. The procedure is as follows: Two semicircles are struck on the base line, having their centers at the third division points and their radii equal to $\frac{b}{3}$. From the point marked e, that of the incidence of R, the line eg is drawn to g, the point of intersection of the two semicircles. Again from g a line gn is set off at right angles to eg cutting the base or its continuation at a point n. This point is termed the antipole of e, or the neutral point at which pressure is *nil* in either sense—compressive or tensile. Below and clear of the profile a projection of the base is now made, and from g a perpendicular is let fall, cutting the new base in g' while, if the line be continued upward, it will intersect the base at K. This latter point will, by the construction, be the center point of the base. The line Kg is continued through g' to h', $g'h'$ being made equal to the mean unit pressure, $=1.8$ tons. A perpendicular is let fall from n cutting the new base line at n_1; the points n_1 and h' are then joined and the line continued until it intersects another perpendicular let fall from the toe of the base. A third perpendicular is drawn from the heel of the base, cutting off a corner of the triangle. The hatched trapezoid enclosed between the last two lines represents the distribution of pressure on the base. The maximum stress will scale close upon 3.51 and the minimum .09 tons. If W be considered, $s=\frac{W}{b}=\frac{59.3}{33.3}=1.78$ tons, the maximum stress at the heel will be 3.52 and the minimum .04, at the toe.

20. Examples to Illustrate Pressure Distribution. In Fig. 14 is illustrated the distribution of pressure on the base, due to the incidence of R, first, at the toe of the base, second, at the two-third point, third, at the center, and fourth, at an intermediate position. In the first case (R_1), it will be seen that the neutral point n_1 falls at the first third point. Thus two-thirds of the base is in compression and one-third in tension, the maximum in either case being proportional to the relative distance of the neutral point from the toe and heel of the base, the compression at the toe being four times, while the tension at the heel is twice the mean stress. In the second case (R_2) intersects at the two-third point, and the consequent position of n is exactly at the heel. The whole base is thus in compression, and the maximum is double the mean. In the third case (R_3), the line gn is drawn at right angles to fg. The latter is vertical and gn will consequently be horizontal. The distance to n is thus infinite and the area of pressure becomes a rectangle with a uniform unit stress s. In the fourth case (R_4), the neutral point lies well outside the profile, consequently the whole is in compression, the condition approximating to that of R_3.

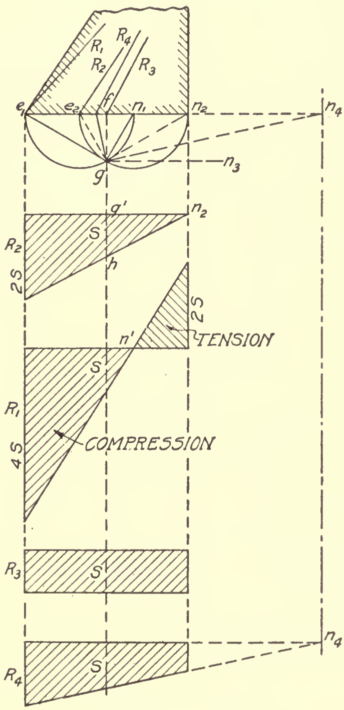


Fig. 14. Pressure Distribution on Base of Dam under Various Conditions

21. Maximum Pressure Limit. The maximum pressure increases with the depth of the profile until a level is reached where the limit stress or highest admissible stress is arrived at. Down to this level the design of the section of a dam, as already shown, consists simply in a slight modification of the pentagonal profile with a vertical back, the base width varying between that of the elementary profile or $\frac{H}{\sqrt{\rho}}$, or its reduced value given in formula (3). Beyond this limiting depth, which is the base of the so-termed "low" dam, the pentagonal profile will have to be departed from and the base widened out on both sides.

22. Formulas for Maximum Stress. The maximum unit stress in the interior of a dam is not identical with (s), the maximum vertical unit reaction at the base, but is a function of s_1. In Fig. 8, a representative triangle of forces is shown composed of N the vertical force (R.F.), P the horizontal water pressure, and R the resultant of N and P; therefore $R=\sqrt{N^2+P^2}=$ also $N \sec\theta$. If the back were vertical, N and W would coincide and then $R=\sqrt{W^2+P^2}$. Various views have been current regarding the maximum internal stress in a dam. The hitherto most prevalent theory is based on the assumption, see Fig. 8, that the maximum unit stress

$$c=\frac{mR}{b}=m\frac{\sqrt{N^2+P^2}}{b}=\frac{mN}{b}\sec\theta \qquad \textbf{(10a)}$$

Another theory which still finds acceptance in Europe and in the East assumes that the maximum stress is developed on a plane normal to the direction of the resultant forces as illustrated by the stress lines on the base of Fig. 8. According to this, the mean stress due to R would not be $\frac{R}{b}$ but $\frac{R}{b_1}$, and the maximum stress will be $\frac{mR}{b_1}$. But $\frac{R}{b_1}=\frac{R\sec\theta}{b}$ and $R=N\sec\theta$, consequently the maximum unit stress would be

$$c=\frac{mN}{b}\sec^2\theta \qquad \textbf{(10b)}$$

Recent experiments on models have resulted in the formula for maximum internal unit stress being recast on an entirely different principle from the preceding. The forces in action are the maximum vertical unit force or reaction s combined with a horizontal shearing unit stress $s_s = \frac{P}{b}$. The shearing force is the horizontal water pressure, or $\frac{H^2 w}{2\rho}$ symbolized by P, which is assumed to be equally resisted by each unit in the base of the dam; the unit shearing stress will thus be $\frac{P}{b}$. These forces being at right angles to each other, the status is that of a bar or column subject to compression in the direction of its length and also to a shear normal to its length. The combination of shear with compression produces an increased compressive stress, and also a tension in the material. The formula recently adopted for maximum unit compression is as follows:

$$c = \tfrac{1}{2}s + \sqrt{\tfrac{1}{4}s^2 + s_s^{\,2}} \qquad \textbf{(10)}$$

In this $s = ms_1, = \frac{mN}{b}$. As before $s_s = \frac{P}{b}$, substituting we have

$$c = \frac{mN}{2b} + \sqrt{\frac{(mN)^2}{4b^2} + \frac{P^2}{b^2}} = \frac{mN + \sqrt{(mN)^2 + 4P^2}}{2b} \qquad \textbf{(10}_1\textbf{)}$$

When $m = 2$, as is the case when the incidence of R is exactly at the outer boundary of the middle third

$$c = \frac{N + \sqrt{N^2 + P^2}}{b} = \frac{N}{b}(1 + \sec\theta) \qquad \textbf{(10}_2\textbf{)}$$

23. Application of All Three Formulas to Elementary Profile. In the case of elementary triangular profile which has a vertical back, $N = W$ and $\sec\theta = \frac{\sqrt{\rho + 1}}{\sqrt{\rho}}$ (section 7, page 5) and $m = 2$; then formula $(\mathbf{10_2})$ becomes

$$c = \frac{W}{b}(1 + \sec\theta) = \frac{W}{b}\left(1 + \sqrt{\frac{\rho + 1}{\rho}}\right)$$

Now

$$\frac{W}{b} = \frac{H^2}{2\sqrt{\rho}} \times \rho w \times \frac{\sqrt{\rho}}{H} = \frac{Hw\rho}{2}$$

$\therefore$

$$c = \frac{Hw\rho}{2}\left(1 + \sqrt{\frac{\rho + 1}{\rho}}\right) \qquad \textbf{(11)}$$

Example.

Let H in elementary triangle = 150 feet, $\rho = 2.4$, $w\rho = \frac{3}{40}$ ton.

When, according to (**11**), $c = \frac{150 \times 3}{2 \times 40}$ $(1+1.187) = 12.3$ tons per square foot.

Taking up formula (**10a**)

$$c = \frac{mW \sec\theta}{b} = \frac{2W \sec\theta}{b}$$

as above $\frac{W}{b} = \frac{Hw\rho}{2}$

∴

$$c = Hw\rho\sqrt{\frac{\rho+1}{\rho}} = Hw\sqrt{\rho}\sqrt{\rho+1} \qquad \textbf{(11a)}$$

Example with conditions as before

$$c = \frac{150 \times 1 \times 1.55\sqrt{3.4}}{32} = 7.26 \times 1.84 = 13.3 \text{ tons}$$

With formula (**10b**), $c = \frac{2W \sec^2\theta}{b}$, or in terms of H,

$$c = Hw\rho\left(\frac{\rho+1}{\rho}\right) = Hw(\rho+1) \qquad \textbf{(11b)}$$

Therefore, with values as above,

$$c = \frac{150 \times 1 \times 3.4}{32} = 15.9 \text{ tons}$$

From the above it is evident that formulas (**10b**) and (**11b**) give a very high value to c. Tested by this formula, high American dams appear to have maximum compressive unit stresses equal to 20 tons per square foot, whereas the actual value according to formula (**10**) is more like 14 tons. However, the stresses in the Assuan dam, the Periyar, and other Indian dams, as also French dams have been worked out from formula (**10b**) which is still in use.

24. Limiting Height by Three Formulas. The limiting height (H_l) of the elementary triangular profile forms a close guide to that obtaining in any trapezoidal section, consequently a formula will be given for each of the three cases in connection with formulas (**10**), (**10a**), and (**10b**). Referring to case (**10**), we have from formula (**11**)

$$c = \frac{Hw\rho}{2}\left(1+\sqrt{\frac{\rho+1}{\rho}}\right)$$

$$\text{Whence } H_l, \text{ the limiting height} = \frac{2c}{w\rho\left(1+\sqrt{\frac{\rho+1}{\rho}}\right)}$$

Example.

With $c=16$ tons and $\rho=2.4$, H_l, the limit height of the elementary profile will be $\frac{2\times16\times33}{2.4\ (2.187)}=\frac{1024}{5.254}=195$ feet.

Referring to case (**10a**), we have from formula (**11a**)

$$c = Hw\sqrt{\rho}\sqrt{\rho+1}$$

∴

$$H_l = \frac{c}{w\sqrt{\rho}\sqrt{\rho+1}}$$

Example.

With data as above $H=\frac{16\times32}{1.55\times1.84}=180$ feet, nearly.

Referring to case (**10b**), we have from formula (**11b**)

$$c = Hw(\rho+1)$$

∴

$$H_l = \frac{c}{w(\rho+1)}$$

Example.

With same data $H_l=\frac{16\times32}{3.4}=\frac{512}{3.4}=150$ feet.

Thus the new formula (**10**) gives much the same results as that formerly in general use in the United States (**10a**), while in the more conservative formula (**10b**) the difference is marked.

25. Internal Shear and Tension. We have seen that the combination of compressive and shearing stresses in a dam (R.F.) produces an increased unit compression. It further develops an increase in the shearing stress and also a tensile stress. The three formulas are given below.

Compression as before

$$c = \frac{1}{2}s + \sqrt{\frac{s^2}{4}+s_s{}^2} \quad \text{or} \quad \frac{mN+\sqrt{(mN)^2+4P^2}}{2b} \qquad \textbf{(10)}$$

Tension

$$t = \frac{1}{2}s - \sqrt{\frac{s^2}{4}+s_s{}^2} \quad \text{or} \quad \frac{mN-\sqrt{(mN)^2+4P^2}}{2b} \qquad \textbf{(12)}$$

Shear

$$s_h = \sqrt{\frac{s^2}{4} + s_s{}^2} \qquad \text{or} \quad \sqrt{\frac{(mN)^2 + 4P^2}{2b}} \qquad \textbf{(13)}$$

The tensile and shearing stresses are not of sufficient moment to require any special provision in the case of a gravity dam. The tension is greatest at the heel, diminishing toward the toe. This fact suggests that a projection of the heel backward would be of advantage. The direction (a) of c to the vertical is not that of R but is as follows:

$$\text{Tan } 2a = \frac{2s_s}{s} = \frac{2P}{b} \div \frac{mN}{b} = \frac{2P}{mN}$$

when $m = 2, \tan 2a = \frac{P}{N}$. In Fig. 8, $P = 555$ and $N = 855$. $\therefore \tan 2a = \frac{555}{855} = .649$ whence $2a = 33°\ 00'$ and $(a) = 16°\ 30'$. The inclination of R to the vertical, or θ, is $33°\ 50'$, i.e., twice as large as that of c. The direction of t is at right angles to that of c, while that of s_h the shear, lies at 45° from the directions of either c or t.

26. Security against Failure by Sliding or Shear. Security against failure by sliding depends on the inclination of W to P, i.e., on the angle θ between W and R. Thus $\tan \theta$ should be less than the angle of friction of masonry on masonry, or less than .7. This is the same as stating that the relation of W and P must be such that θ shall not be greater than 35°, or that the complement of θ be not less than 55°. The adoption of the middle third proviso generally insures this. With regard to sliding on the base, this can be further provided against by indentations in the base line or constructing it inclined upward from heel to toe.

27. Influence Lines. It is sometimes desirable for the purpose of demonstrating the correctness of a profile for tentative design, to trace the line of pressures corresponding to the two conditions of reservoir full and empty, through the profile of a dam. This is far better effected by the use of graphic statics.

There are two different systems of graphic construction that give identical results, which will now be explained and illustrated.

The first method, which is most commonly adopted, is exhibited in Fig. 15, which is the profile of a 100-foot high dam with specific gravity $2\frac{1}{4}$. It thus lies within the limiting depth, which for the elementary profile would be 190 feet.

The profile is pentagonal, with a vertical back, and has the full base width of the elementary profile, viz, $\frac{H}{\sqrt{\rho}}$ which in this case is $\frac{2}{3}\times 100 = 66.7$ feet. The crest k is $\sqrt{H} = 10$ feet wide. The water-pressure triangle has a base of $\frac{H}{\rho}$. The profile, as well as the water-pressure triangle, is divided into five equal laminas, numbered 1

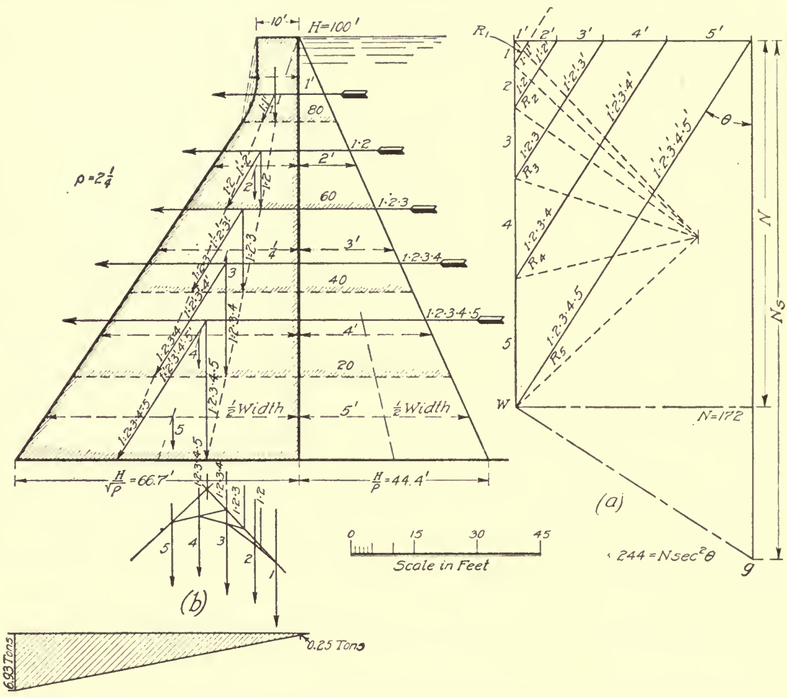


Fig. 15. Graphical Construction for Tracing Lines of Pressure on Dam of Pentagonal Profile

to 5 in one case and 1′ to 5′ in the other. The depth of each lamina, which is $\frac{H}{5}$ is, therefore, a common factor and can be eliminated as well as the item of unit weight, viz, $w\rho$. The half widths of all these laminas will then correctly represent their areas and also their weights, reduced to one denomination, that of the masonry. In Fig. 15a a force polygon is formed. In the vertical load line the several half widths of the laminas 1 to 5 are first set off, and at right angles to it the force line of water pressure is similarly set out with the half widths of the areas $1'$ to $5'$. Then the resultant lines of the combination of 1 with $1'$, 1, 2, with $1'\,2'$ and so on marked R_1 to R_5 are drawn. This completes the force polygon. The next step is to find the combinations of the vertical forces on the profile, viz, that of 1 and 2, 1, 2, and 3, etc. This, as usual, is effected by constructing a force and ray polygon, utilizing the load line in Fig. 15a for the purpose. Then the centers of gravity of the several individual areas 1 to 5 are found by the graphical process described in section 15, and verticals drawn through these points are projected below the profile. On these parallel force lines 1 to 5, the funicular polygon Fig. 15b is constructed, its chords being parallel to their reciprocal rays in Fig. 15a. The intersection of the closing lines of the funicular gives the position of the centroid of the five forces engaged. By producing each chord or intercept backward until it intersects the initial line, a series of fresh points are obtained which denote the centers of gravity of the combinations of 1 and 2; 1, 2, and 3, and so on. Verticals through these are next drawn up on the profile so as to intersect the several bases of the corresponding combinations, thus 1, 2, and 3 will intersect the base of lamina 3; and 1, 2, 3, and 4 will intersect that of lamina 4; and so on. These intersections are so many points on the line of pressure (R.E.). The next step is to draw the horizontal forces, i.e., their combinations on to the profile. The process of finding the centers of gravity of these areas is rendered easy by the fact that the combinations are all triangles, not trapezoids, consequently the center of gravity of each is at $\frac{1}{3}$ its height from the base. Thus the center of gravity of the combination $1'+2'+3'$ is at $\frac{1}{3}$ the height measured from the base of $3'$ to the apex, in the same way for any other combination, that of $1'$, $2'$, $3'$, $4'$, $5'$, being at $\frac{1}{3}$ the total height of the profile. The back being vertical, the direction of all the combined forces will be horizontal, and the lines are drawn through, as shown in the figure, to intersect the corresponding combinations of vertical forces. Thus $1'$ intersects 1, $1'\,2'$ intersects 1 2, and so on. From these several intersections the resultant lines R_1, R_2, to R_5 are now drawn down to the base of the combination to which they belong, these last intersections giving the incidence of R_1, R_2, etc., and are so many points on the line of pressures (R.F.). The process is simple and takes as long to describe as to perform, and it has this advantage, that each combination of forces is independent of the rest, and consequently errors are not perpetuated. This system can also be used where the back of the profile has one or several inclinations to the vertical, explanation of which will be given later.

28. Actual Pressures in Figures. In the whole process above described, it is noticeable that not a single figure or arithmetical calculation is required. If the actual maximum unit stress due to R or to W is required to be known, the following is the procedure. In Fig. 15a, N scales 174, to reduce this to tons it has to be multiplied by all the eliminated factors, which are $\frac{H}{5}=20$ and $w\rho=\frac{9\times 1}{4\times 32}$, that is, $N=\frac{174\times 20\times 9}{4\times 32}=244$ tons.

Assuming the incidence of R exactly at the third division, the value of q is $\frac{b}{6}$ and that of m is 2; P also scales 112, its value is therefore $\frac{112\times 20\times 9}{4\times 32}=157$ tons. Applying formula (**10_2**),

$$c=N+\frac{\sqrt{N^2+P^2}}{b}=\frac{244+\sqrt{244^2+157^2}}{66.7}=\frac{534}{66.7}=8 \text{ tons per sq. ft.,}$$

roughly. As 8 tons is obviously well below the limiting stress, for which a value of 16 tons would be more appropriate, this estimation is practically unnecessary but is given here as an example.

29. Analytical Method. The analytical method of calculation will now be worked out for the base of the profile only. First the position of W, the resultant vertical forces (R.E.) relative to the heel of the base will be calculated and next that of R. The back of the profile being in one line and vertical the whole area can be conveniently divided into two right-angled triangles, if the thickening of the curvature at the neck be ignored. As the fore slope has an inclination of $1:\sqrt{\rho}$ the vertical side of the upper triangle (1) is $k\sqrt{\rho}$ in length; its area will then be $\frac{k^2\sqrt{\rho}}{2}=\frac{100\times 1.5}{2}=75$ sq. feet. The distance of its c. g. from the heel of the base, which in this case corresponds with the axis of the dam, is $\frac{20}{3}$ feet $=6\frac{2}{3}$ feet.

The moment will then be $\frac{75 \times 20}{3} = 500$. With regard to the lower triangle, its area is $\frac{H^2}{2\sqrt{\rho}} = 5000 \times \frac{2}{3} = 3333.3$ sq. feet. The length of its lever arm is one-third of its base, or 22.2 feet. The moment about the axis will then be $3333.3 \times 22.2 = 74{,}000$. The moment of the whole is equal to the sum of the moments of the parts. The area of the whole is $75 + 3333 = 3408$. Let x be the required distance of the incidence of W from the heel, then

$$x \times 3408 = 74{,}500$$

$$\therefore \qquad x = \frac{74{,}500}{3408} = 21.9 \text{ feet}$$

The inner edge of the middle third is $\frac{b}{3}$ or 22.2 feet distant from the heel; the exact incidence of W is, therefore, .3 foot outside the middle third, a practically negligible amount. With regard to the position of R the distance (f) between the incidence of R and that of W is $\frac{PH}{3W}$; in this P the water-pressure area $= \frac{100 \times 44.4}{2} = 2220$. $\therefore f = \frac{2220 \times 100}{3 \times 3408} = 21.7$ feet. The total distance of R from the heel will then be $21.7 + 21.9 = 43.6$ feet; the outer edge of the middle third is 44.4 feet distant from the heel, consequently the incidence of R is $44.4 - 43.6 = .8$ foot within the middle third, then $q = \frac{b}{6} - .8 = \frac{66.7}{6} - .8 = 10.3$ ft., and $m = \left(1 + \frac{6q}{b}\right) = 1 + .93 = 1.93$. At this stage it will be convenient to convert the areas into tons by multiplying them by ρw, or $\frac{2.25}{32}$. Then N and W become 239.6, and P becomes 156.3 tons. Formula (**10**) will also be used on account of the high figures; then

$$c = \frac{s}{2} + \sqrt{\frac{s^2}{4} + s_s{}^2}$$

Here $s = \frac{mW}{b} = \frac{239.6 \times 1.93}{66.7} = 6.93$ tons, and $s_s = \frac{P}{b} = \frac{156.3}{66.7} = 2.34$ tons, therefore, $c = \frac{6.93}{2} + \sqrt{\frac{6.93^2}{4} + (2.34)^2} = 3.46 + \sqrt{17.48} = 7.64$ tons.

For s_2, or the compression at the heel, $m = 1 - \frac{6q}{b} = .07$. $s_2 = \frac{239.6 \times 0.07}{66.7}$ $= .251$ ton. The area of base pressure is accordingly drawn on Fig. 15.

If W (R.E.) be considered, $q = \frac{b}{6} + .3 = 11.42$, and $m = 1 + \frac{6 \times 11.42}{66.7} = 2.03$; therefore, $s = \frac{mW}{b} = \frac{239.6 \times 2.03}{66.7} = 7.30$ tons. The base pressure is therefore greater with (R.E.) than with (R.F.);

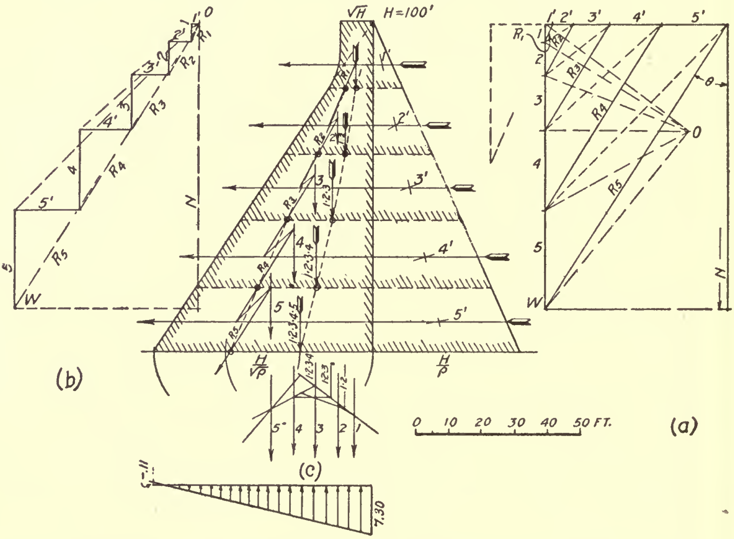


Fig. 16. Diagram Showing Haessler's Method for Locating Lines of Pressure on a Dam

there is also a slight tension at the toe of .11 ton, a negligible quantity. This pressure area is shown on Fig. 16.

30. Haessler's Method. A second method of drawing the line of pressures which is termed "Haessler's" is exhibited in Fig. 16, the same profile being used as in the last example. In this system, which is very suitable for a curved back, or one composed of several inclined surfaces, the forces are not treated as independent entities as before, but the process of combination is continuous from the beginning. They can readily be followed on the force polygon, Fig. 16a and are 1′ with 1 producing R_1; R_1 with 2′, i.e., 1′, 1, 2′, the last resultant being the dotted reverse line. This last is then combined with 2 producing R_2, and so on.

The reciprocals on the profile are drawn as follows: First the c.g.'s of all the laminas 1, 2, 3, etc., 1′, 2′, 3′, etc., are obtained by graphical process. Next the water-pressure lines, which in this case are horizontal, are drawn through the profile. Force line (1′) intersects the vertical (1), whence R_1 is drawn parallel to its reciprocal in Fig. 16a through the base of lamina (1), until it reaches the horizontal force line (2′). Its intersection with the base of (1) is a point in the line of pressure (R.F.). Again from the intersection of R_1 with (2′), a line is drawn backward parallel to its dotted reciprocal line in Fig. 16a until it meets with the second vertical force (2). From this point R_2 is then drawn downward to its intersection with the horizontal force (3′), its intersection with the base of lamina (2) giving another point on the line of pressures. This process is repeated until the intersection of R_5 with the final base completes the operation for (R.F.). It is evident that R_5 as well as all the other resultants are parallel to the corresponding ones in Fig. 15, the same result being arrived at by different graphical processes.

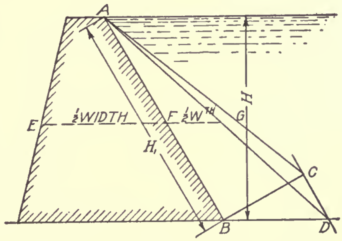


Fig. 17. Transformation of Inclined Pressure Area to Equivalent with Horizontal Base

31. Stepped Polygon. Fig. 16b is a representation of the so-called "stepped" polygon, which is also often employed; the form differs, but the principle is identical with that already described. Inspection of the figure will show that all the resultant lines are drawn radiating to one common center or nucleus (O).

The process of finding the incidence of W on the bases of the several lamina is identical with that already described with regard to Fig. 15, viz, the same combination of $1+2$, $1+2+3$, and so on, are formed in the funicular 16c and then projected on to the profile.

32. Modified Equivalent Pressure Area in Inclined Back Dam. When the back of a dam is inclined, the area of the triangle of water pressure ABC, in Fig. 17, will not equal the product of H, but of H_1 with its half width, which latter is measured parallel to the base, consequently the factor H cannot be eliminated. The triangle itself can, however, be altered in outline so that while containing the same area, it will also have the vertical height H as a factor

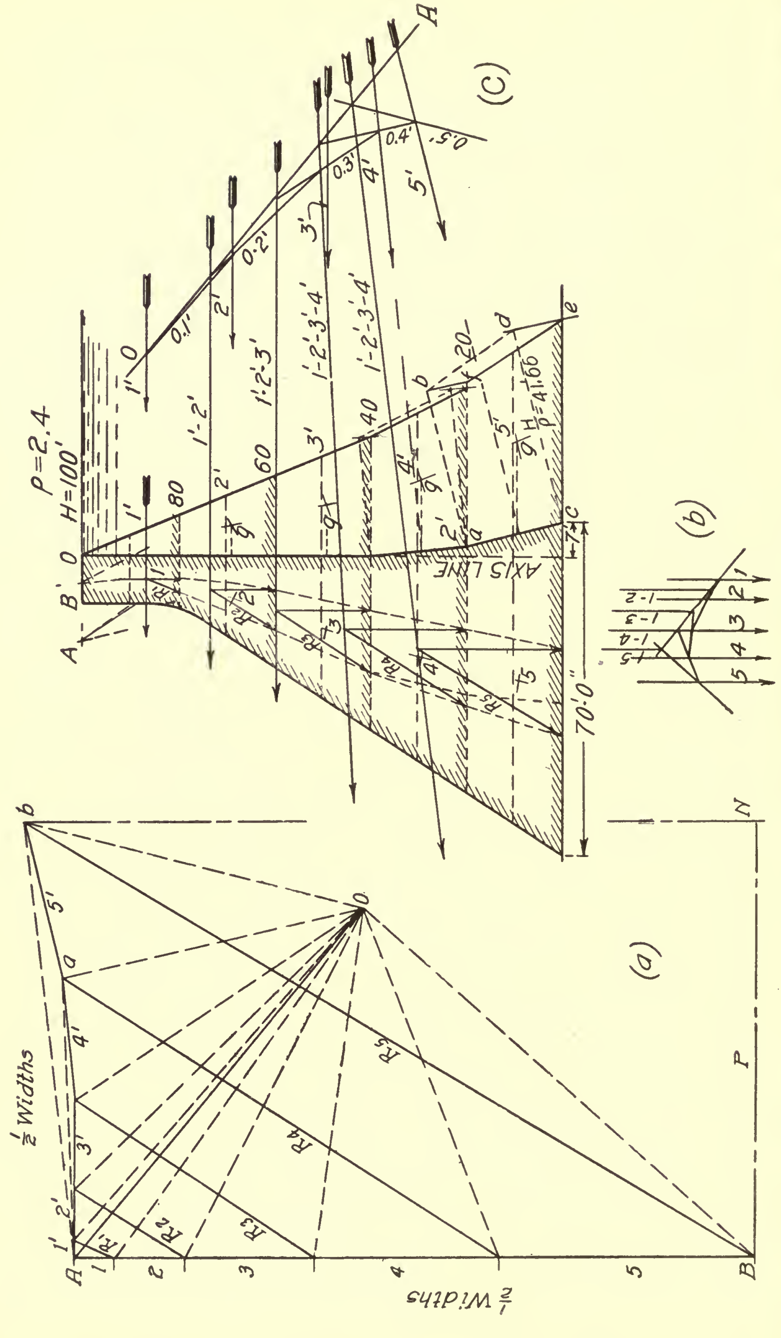


Fig. 18. Graphical Method of Fig. 15 Applied to Profile with Curved Back

in its area. This is effected by the device illustrated in Fig. 17, and subsequently repeated in other diagrams. In this figure ABC is the triangle of water pressure. By drawing a line CD parallel to the back of the wall AB, a point D is obtained on the continuation of the horizontal base line of the dam. A and D are then joined. The triangle ABD thus formed is equal to ABC, being on the same base AB and between the same parallels. The area of ABD is equal to $\frac{BD}{2} \times H$, and that of the wall to half width $EF \times H$. Consequently we see that the half width $\frac{BD}{2}$ of the triangle ABD can properly represent the area of the water pressure, and the half width EF that of the wall. The vertical height H may, therefore, be eliminated. What applies to the whole triangle would also apply to any trapezoidal parts of it. The direction of the resultant line of water pressure will still be as before, normal to the surface of the wall, i.e., parallel to the base BC, and its incidence on the back will be at the intersection of a line drawn through the c.g. of the area in question, parallel to the base. This point will naturally be the same with regard to the inclined or to the horizontally based area.

33. Curved Back Profiles. In order to illustrate the graphical procedure of drawing the line of pressure on a profile having a curved back, Figs. 18 and 19 are put forward as illustrations merely—not as models of correct design. In these profiles the lower two laminas of water pressure, $4'$ and $5'$, have inclined bases. Both are converted to equivalent areas with horizontal bases by the device explained in the last section. Take the lowest lamina *acdb;* in order to convert it into an equivalent trapezoid with a horizontal base, *de* is drawn parallel to *ac;* the point *e* is joined with A, the apex of the completed triangle, of which the trapezoid is a portion. When *af* is drawn horizontally, the area *acef* will then be the required converted figure, the horizontally measured half width of which multiplied by $\frac{h}{5}$ will equal the area of the original trapezoid *acdb;* $\frac{H}{5}$ can then be eliminated as a common factor and the weights of all the laminas represented in the load line in

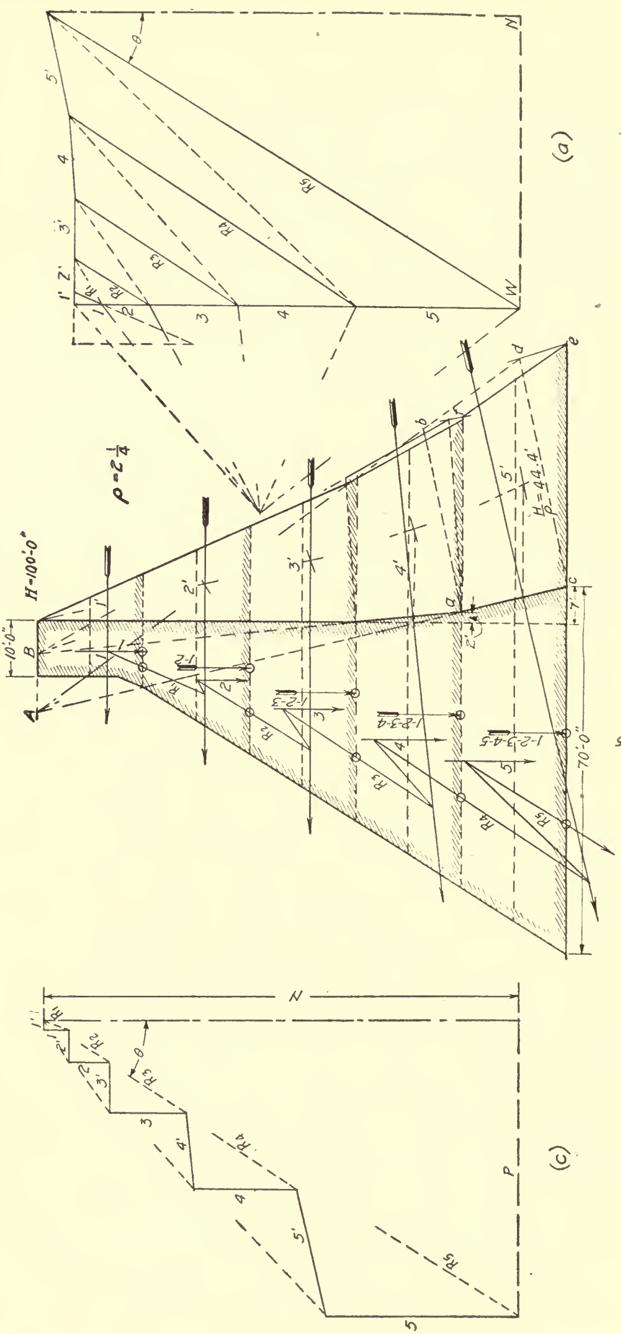


Fig. 19. Haessler's Method of Fig. 16 Applied to Curved Back

Fig. 18a, by the half widths of the several areas. The lamina 4′ is treated in a similar manner.

The graphical processes in Figs. 18 and 18a are identical with those in Fig. 15. In the force polygon 18a the water-pressure forces 1′, 2′, 3′, etc., are drawn in directions normal to the adjoining portion of the back of the profile on which they abut, and are made equal in length to the half widths of the laminas in question. The back of the wall is vertical down to the base of lamina 3, consequently the forces, 1′, 2′, and 3′, will be set out on the water-pressure load line in Fig. 18a from the starting point, horizontally in one line. In laminas 4 and 5, however, the back has two inclinations; these forces are set out from the termination of 3′ at their proper directions, i.e., parallel to their inclined bases to points marked *a* and *b*. The direction of the resultants of the combinations, 1′ and 2′, and 1′, 2′, 3′, will clearly be horizontal. If *Aa* and *Ab* be joined, then the directions of the combination 1′ 2′ 3′ 4′ will be parallel to the resultant line *Aa* and that of 1′ 2′ 3′ 4′ 5′ will be parallel to *Ab*. Thus the inclination of the resultant of any combination of inclined forces placed on end, as in the water-pressure load line, will always be parallel with a line connecting the terminal of the last of the forces in the combination with the origin of the load line.

34. Treatment for Broken Line Profiles. The method of ascertaining the relative position and directions of the resultants of water pressure areas when the back of the wall has several inclinations to the vertical is explained as follows: This system involves the construction of two additional figures, viz, a force and ray polygon built on the water-pressure load line and its reciprocal funicular polygon on one side of the profile. These are shown constructed, the first on Fig. 18a, the nucleus *O* of the vertical force and ray polygon being utilized by drawing rays to the terminations of 1′, 2′, 3′, 4′, and 5′. In order to construct the reciprocal funicular polygon, Fig. 18c, the first step is to find the c.g.'s of all the trapezoidal laminas which make up the water-pressure area, viz, 1′ to 5′. This being done, lines are drawn parallel to the bases of the laminas (in this case horizontal lines), to intersect the back of the wall. From the points thus obtained the force lines 1′, 2′, 3′, etc., are drawn at right angles to the portions of the back of the wall on which they abut. On these force lines, which are not all parallel, the chord polygon (18c) is constructed as follows: First the initial line AO is drawn anywhere parallel to its reciprocal AO, in Fig. 18a. From the intersections of this line with the force line $1'$ the chord marked $O1'$ is drawn parallel to $O1'$ in Fig. 18a and intersecting force line $2'$. Again from this point the chord $O2'$ is drawn intersecting force $3'$ whence the chord $O3'$ is continued to force $4'$, and $O4'$ up to the force line $5'$, each parallel to its reciprocal in Fig. 18a. The closing line is $O5$. The intersection of the initial and the closing lines of the funicular polygon gives the position of the final resultant line $1'\ 2'\ 3'\ 4'\ 5'$, which is then drawn from this point parallel to its reciprocal Ob in Fig. 18a to its position on the profile. The other resultants are obtained in a similar manner by projecting the several chords backward till they intersect the initial line OA, these intersections being the starting points of the other resultants, viz, $1'$-$4'$, $1'$-$3'$, $1'$-$2'$, and $1'$. These resultant lines are drawn parallel to their reciprocals in 18a, viz, $1'$-$4'$ is parallel to Aa, while the remainder are horizontal in direction, the same as their reciprocals.

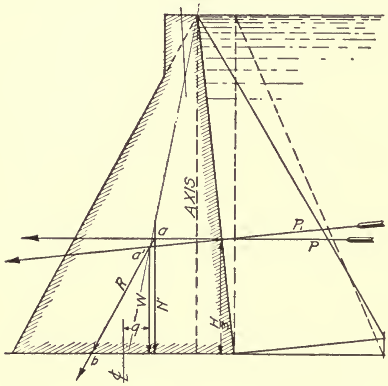


Fig. 20. Diagram Showing Third Method of Determining Water Pressure Areas

This procedure is identical with that pursued in forming the funicular 18b, only in this case the forces are not all parallel.

35. Example of Haessler's Method. In Fig. 19 the profile used is similar to Fig. 18, except in the value of ρ, which is $2\frac{1}{4}$, not 2.4 as previously. The graphical system employed is Haessler's, each lamina as already described with reference to Fig. 16 being independently dealt with, the combination with the others taking place on the profile itself. In this case the changes of batter coincide with the divisions of the laminas, consequently the directions of the inclined forces are normal to the position of the back on which their areas abut. This involves finding the c. g.'s of each of the water-pressure trapezoids, which is not necessary in the first system, unless the funicular polygon of inclined forces has to be formed. In spite of this, in most cases Haessler's method will be found the handiest to employ, particularly in tentative work.

36. Example of Analytical Treatment. In addition to the two systems already described, there is yet another corresponding to the analytical, an illustration of which is given in Fig. 20. In this the vertical and horizontal components of R, the resultants (R.F.), viz, N and P are found. In this method the vertical component of the inclined water pressure P_1 is added to the vertical weight of the dam itself, and when areas are used to represent weights the area of this water overlying the back slope will have to be reduced to a masonry base by division by the specific gravity of the masonry.

37. Relations of R. N. and W. The diagram in Fig. 20 is a further illustration showing the relative positions of R, P, P_1, N, and W. The line R starts from a, the intersection of the horizontal force P with N, the resultant of all the vertical forces, for the reason that it is the resultant of the combination of these two forces; but R is also the resultant of P_1 and W, consequently it will pass through a', the intersection of these latter forces. The points a and a_1 are consequently in the resultant R and it follows as well that if the position of R is known, that of N and W can be obtained graphically by the intersection of P or P_1 with R. These lines have already been discussed.

UNUSUALLY HIGH DAMS

38. "High" Dams. An example will now be given, Fig. 21, of the design of a high dam, i.e., one whose height exceeds the limit before stated. As usual the elementary triangular profile forms the guide in the design of the upper portion. We have seen in section 24 that the limiting depth with $\rho = 2.4$ and $c = 16$ tons = 195 feet, whence for 18 tons' limit the depth will be 219 feet. In Fig. 21 the tentative profile is taken down to a depth of 180 feet. The crest is made 15 feet wide and the back is battered 1 in 30; the base width is made $180 \times .645 = 116$ feet. The heel projects 6 feet outside the axis line. The graphical procedure requires no special explanation. It follows the analytical in dealing with the water pressure as a horizontal force, the weight of the water overlying the back being added to that of the solid dam. For purposes of

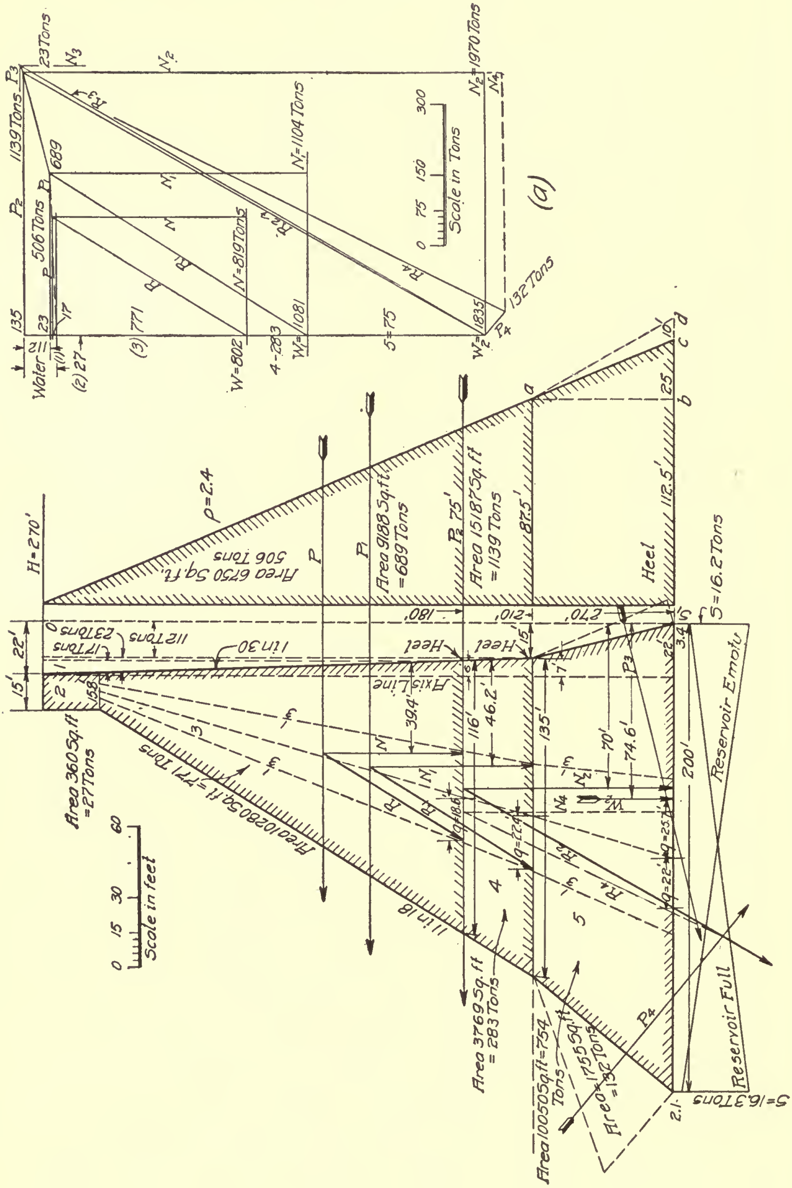


Fig. 21. Analytical Diagram for "High" Dam

calculation the load is divided into three parts (1) the water on the sloping back, the area of which is 540 sq. ft. This has to be reduced by dividing it by ρ and so becomes 225 sq. ft. As tons, not areas, will be used, this procedure is not necessary, but is adopted for the sake of uniformity in treatment to avoid errors. The c.g. of (1) is clearly 2 feet distant from the heel of the base of (3), about which point moments will be taken. That of the crest (2) is 13.3 feet and that of the main body (3) obtained by using formula (**7**) comes to 41.2 feet. The statement of moments is then as follows:

No.	Area	Tons	Lever Arm	Moment
1	225	17	2	34
2	360	27	13.3	359
3	10280	771	41.2	31765
Total	10866	815		32158

Then the distance of N from the heel will be $\frac{32158}{815}=39.4$ ft.; it thus falls $39.4-38.7=.7$ ft. within the middle third. The distance (f) between N and R is $\frac{PH}{3N}$. Now $P=$ the area of the right angle triangle whose base is $\frac{H}{\rho}$, or 75 feet, and is 6750 sq. ft. equivalent to $\frac{6750\times 3}{40}=506$ tons. The expression then becomes $f=\frac{506\times 180}{3\times 815}$ $=37.2$ feet. The incidence of R will then be $37.2+39.4=76.6$ feet distant from the heel of the base. The $\frac{2}{3}$ point being 77.3 from the heel, R falls .7 ft. within. Thus far the tentative profile has proved fairly satisfactory, although a slight reduction in the base width is possible. The position of W, or the resultant weight of the portions 2 and 3 of the dam is obtained from the moment table already given, and is the sum of the moments of (2) and (3) divided by (2) $+$(3) or $\frac{32124}{798}=40.2$ ft. This falls $40.2-38.7=1.5$ ft. within the middle third. The value of q (R.F.) is $\frac{b}{6}-.7=19.33-.7 = 18.6$ feet, and $m=\left(1+\frac{6q}{b}\right)=1+\frac{111.6}{116}=1.96.$

Then by formula (10_1), N being 815 and P, 506

$$c=\frac{1.96\times815+\sqrt{(1.96\times815)^2+4\ (506)^2}}{232}$$

$$=\frac{1597+\sqrt{2551367+1024144}}{232}$$

$$=15.03 \text{ tons per square foot}$$

Extension of Profile. This value being well below the limit of 18 tons and both resultants (R.F.) and (R.E.) standing within the middle third it is deemed that the same profile can be carried down another 30 ft. in depth without widening. The base length will now be a trifle over 135 feet. The area of the new portion (4) is 3769 sq. ft. =283 tons. The distance of its c.g. from the heel by formula (7) is found to be 63.4 feet. The position of W will be obtained as follows, the center of moments being one foot farther to right than in last paragraph.

No.	Tons	Lever Arm	Moment
2	27	14.3	386
3	771	42.2	32536
4	283	63.4	17942
Total	1081		50864

$$\therefore \qquad x=\frac{50864}{1081}=47.0 \text{ feet from heel of new base to } W$$

As $\frac{b}{3}$ is $\frac{135}{3}=45$ ft., the incidence of W_1 is 2.0 ft. within the base, which is satisfactory.

To find N_1, the moment of the water on whole back can be added to that of W first obtained. The offset from the axis being now 7 ft., the area will be $\frac{210\times7}{2}=735$ equal to an area of masonry of $\frac{735}{2.4}=306$ sq. ft. equivalent to 23 tons, nearly. The lever arm being $\frac{7}{3}$, or 2.3 feet the moment about the heel will be $23\times2.3=52.9$, say 53.0 ft.-tons. This amount added to the moment of W_1 will represent that of N_1 and will be $50{,}864+53=50{,}917$. The value of N_1 is that of W_1+ the water on back or, $1081+23=1104$ tons. The distance of N_1 from the heel is then $\frac{50917}{1104}=46.2$ feet. To obtain that of R_1 the value of $f=\frac{689\times210}{3\times1104}=43.7$ feet; this added to $46.2=89.9$ ft., the incidence of R_1 is therefore $\frac{135\times2}{3}-89.9=$ $90-89.9=.1$ ft., within the middle third boundary. Then $q=\frac{b}{6}$ $-.1=22.5-.1=22.4$ ft., and $m=1+\frac{6q}{b}=1+\frac{134.4}{135}=2.00$, nearly. To find c, formula (**10**) will be used, the quantities being less than in formula (**10**$_1$). Here $s=\frac{mN}{b}=\frac{2.00\times1104}{135}=16.4$ tons and s_s $=\frac{P}{b}=\frac{689}{135}=5.1$ tons. Then

$$c=\frac{16.4}{2}+\sqrt{\frac{(16.4)^2}{4}+(5.1)^2}=8.2+9.7=17.9\text{ tons}$$

The limit of 18 tons, being now reached, this profile will have to be departed from.

39. Pentagonal Profile to Be Widened. The method now to be adopted is purely tentative and graphic construction will be found a great aid to its solution. A lamina of a depth of 60 ft., will be added to the profile. It is evident that its base width must be greater than that which would be formed by the profile being continued down straight to this level. The back batter naturally will be greater than the fore. From examination of other profiles it appears that the rear batter varies roughly from about 1 in 5 to 1 in 8 while the fore batter is about 1 to 1. As a first trial an 8 ft. extra offset at the back was assumed with a base of 200 feet; this would give the required front projection. Graphical trial lines showed that N would fall without the middle third, and W as well; the stress also just exceeded 18 tons (R.E.). A second trial was now made in which the back batter was increased and the base shortened to 180 feet. In this case c exceeded the 18 ton limit.

Still further widening was evidently required at the heel in order to increase the weight of the overlying water, while it was clear that the base width would not bear reduction. The rear offset was then increased to 15 feet and the base width to 200 feet. The stresses now worked out about right and the resultants both fell within the middle third. By using formula (6) the distance of the c.g. of the trapezoid of water pressure, which weighs 112 tons, was found to be 7.2 feet from the heel of the base, and by formula (7) that of the lowest lamina (5) from the same point is found to be 91.8 feet; the weight of this portion is 754 tons. These two new vertical forces can now be combined with N_1 whose area and position are known and thus that of N_2 can be ascertained. N_1 is 46.2 ft. distant from the heel of the upper profile; its lever arm will, therefore, be $46.2+15=61.2$ feet. The combined moment about the heel will then be

	Weight	Lever Arm	Moment
Water	112	7.2	806
N_1	1104	61.2	67565
(5)	754	91.8	69217
Total	1970		137588

The incidence of N_2 is then $\frac{137588}{1970}=70$ feet from the heel; as the middle third boundary is $\frac{200}{3}=66.6$ feet distant from the same point, N_2 falls 3.4 feet within. The distance between N_2 and R_2 (viz, f) $=\frac{PH_2}{3N_2}$. Now P_2, or the horizontal water pressure, has a reduced area of 15187 feet, equivalent to 1139 tons, consequently $f=\frac{1139\times 270}{3\times 1970}$ $=52.0$ feet. This fixes the incidence of R_2 at $70+52.0=122.0$ feet distant from the heel; the $\frac{2}{3}$ point is 133.3 feet distant, consequently R_2 falls well within the middle third and as $q=122.0-\frac{200}{2}=22.0$ feet,

$$m=1+\frac{6\times 22.0}{200}=1.66,\ \text{and}\ s=\frac{mN}{b}=\frac{1.66\times 1970}{200}=16.3\ \text{tons.}$$

Now $$s_8=\frac{P_2}{b}=\frac{1139}{200}=5.7 \text{ tons}$$

Whence by formula (**10**),

$$c=\frac{16.3}{2}+\sqrt{\frac{265}{4}+32.5}=8.15+9.9=18.05 \text{ tons}$$

which is the exact limit stress.

The value of s_2 (the pressure at the heel) is obtained by the same formula, using the minus sign, viz, $m=1-\frac{130.8}{200}=.34$, therefore, $s_2=\frac{m\,N_2}{b}=\frac{.34\times1970}{200}=3.4$ tons, nearly. These vertical reactions are set out below the profile. With regard to W_2 it is composed of $W_1+(5)$. The table of moments is as follows:

	Weight	Lever Arm	Moment
W_1	1081	62.0	67022
(5)	754	91.8	69217
W_2	1835		136239

The distance of W_2 from the heel is $\frac{136239}{1835}=74.3$. W_2, therefore, falls 7.6 feet within the middle third; $q=\frac{200}{2}-74.3=25.7$ feet, and $m=1+\frac{6\times25.7}{200}=1.77$, and $m=1-.77=.23$; therefore, $s=\frac{mW_2}{b}=\frac{1.77\times1835}{200}=16.2$ tons, and $s_2=.23\times\frac{1835}{200}=2.1$ tons. These pressures are shown below the profile.

The value of θ in all three cases is less than 35° which is also one of the stipulations.

In continuing the profile below the 270-foot depth the probability is that for a further depth of 50 or 60 feet the same fore and rear batter would answer; if not, the adjustment is not a difficult matter to manipulate. As previously stated, the incidence of N should be fixed a little within the middle third when that of W and R will generally be satisfactory.

In the force diagram the water part of N is kept on the top of the load line W. This enables the lengths of the N series to be clearly shown. The effect is the same as if inclined water pressure lines were drawn, as has already been exhibited in several cases.

40. Silt against Base of Dam. In Fig. 21, suppose that the water below the 210-foot depth was so mixed with silt as to have a specific gravity of 1.4 instead of unity. The effect of this can be shown graphically without alteration of the existing work. In the trapezoid lying between 210 and 270 the rectangle on ab represents the pressure above 210 and the remaining triangle that of the lower 60 feet of water. The base of the latter, bc is, therefore, $\frac{H}{\rho}=\frac{60}{2.4}=$ 25 feet. Now the weight of the water is increased in the proportion of 1.4 : 1, consequently the proper base width will be $\frac{H'\times 1.4}{2.4}=$ $\frac{60\times 1.4}{2.4}=35$ feet. The triangle acd then represents the additional pressure area due to silt. The normal pressure on the back of the dam due to the presence of silt is shown graphically by the triangle attached, whose base $=cd=10$ feet; its area is 310 square feet, equivalent to 23 tons. This inclined force is combined with R_2 at the top right-hand corner of Fig. 21a and the resultant is R_3; on the profile the reciprocal inclined force is run out to meet R_2 and from this intersection R_3 can be drawn up toward P_2. This latter intersection gives the altered position of N_2, which is too slight to be noticeable on this scale. The value of c and the inclination of R are both increased, which is detrimental.

If the mud became consolidated into a water-tight mass the pressure on the dam would be relieved to some extent, as the earth will not exert liquid pressure against the back. Liquid mud pressure at the bottom of a reservoir can consequently be generally neglected in design.

41. Filling against Toe of Dam. Now let the other side of the dam be considered. Supposing a mass of porous material having an immersed s. g. of 1.8 is deposited on the toe, as is often actually the case. Then a pressure triangle of which the base equals $H\times\frac{1.8}{2.4}=$ 45 feet is drawn; its area will be 1755 and weight 132 tons; the resultant P_4 acting through its c. g. is run out to intersect R_2. At the same time from the lower extremity of R_2, in the force diagram, a reciprocal pressure line P_4 is drawn in the same direction equal in length 132 tons and its extremity is joined with that of P_2; the resulting line R_4 is then projected on the profile from the previous intersection until it cuts the force line P_2; this gives a new resultant R_4 and a new position for N, viz, N_4, which is drawn on the profile; W also will be similarly affected. The load on the toe of the dam increases its stability as the value of θ is lessened, the position of W is also improved, but that of R_4, which is nearer to the toe than R_3, is not. To adjust matters, the c. g. of (5) requires moving to the right which

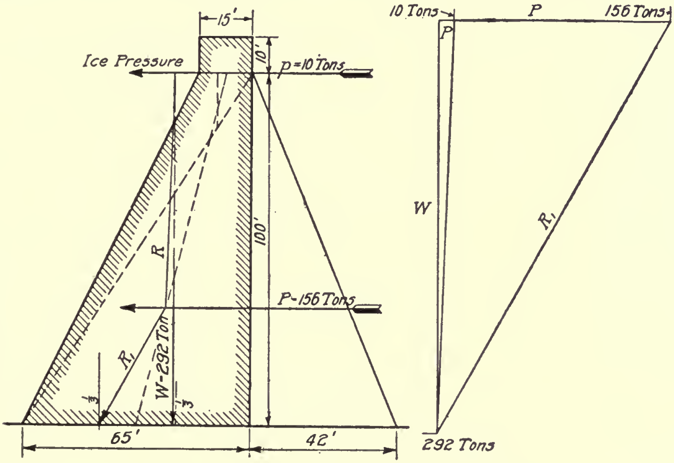


Fig. 22. Diagram Showing Effect of Ice Pressure

is affected by shifting the base line thus increasing the back and decreasing the front batter, retaining the base length the same as before.

42. Ice Pressure. Ice pressure against the back of a dam has sometimes to be allowed for in the design of the profile; as a rule, however, most reservoirs are not full in winter so that the expansive pressure is exerted not at the summit but at some distance lower down where the effect is negligible. In addition to this when the sides of a reservoir are sloping, as is generally the case, movement of ice can take place and so the dam is relieved from any pressure. In the estimates for the Quaker Bridge dam it is stated that an ice pressure of over 20 tons per square foot was provided for. No definite rules seem to be available as to what allowance is suitable. Many authorities neglect it altogether.

The effect of a pressure of ten tons per foot run on a hundred-foot dam acting at the water level is illustrated in Fig. 22. For this purpose a trapezoidal section has been adopted below the summit level. The crest is made 15 feet wide and 10 feet high. This solid section is only just sufficient, as will appear from the incidence of R' on the base. The area of this profile is 4150 sq. ft., while one of the ordinary pentagonal sections as dotted on the drawing would

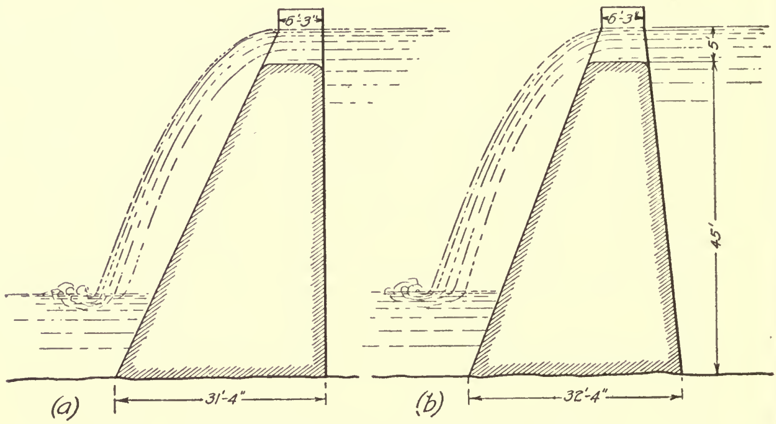


Fig. 23. Two Profiles for Partial Overfall Dams

contain but 3325 sq. ft. The increase due to the ice pressure is therefore 825 sq. ft. or about 25 per cent. The graphical procedure hardly needs explanation. The ice pressure p is first combined with W the weight of the dam and their resultant R cuts P at a point from which the final R_1 is run down to the base parallel to its reciprocal in the force diagram. It falls just within the middle third of the base. An actual example is given in section 56.

43. Partial Overfall Dams. It not infrequently happens that the crest of a dam is lowered for a certain length, this portion acting as a waste weir, the crest of the balance of the dam being raised above the water level. In such cases a trapezoidal outline is generally preferable for the weir portion and the section can be continued upon the same lines to form the upper part of the dam, or the upper part can be a vertical crest resting on the trapezoidal body. In a trapezoidal dam, if the ratio of $\frac{k}{b}$ be r, the correct base width is obtained by the following formula:

$$b=\frac{H}{\sqrt{\rho}}\frac{1}{\sqrt{1+r-r^2}} \qquad \textbf{(14)}$$

This assumes the crest and summit water level to be the same. In Fig. 23, ρ is taken as 2.4 and r as .2. The base width with a vertical back will then be $\frac{H}{\sqrt{\rho}}\times\frac{1}{\sqrt{1+.2-.04}}=50\times.645\times.935=31.3$ feet, and the crest width k will be $31.3\times.2=6.3$ feet. In the second figure the profile is shown canted forward, which is desirable in weirs, and any loss in stability is generally more than compensated for by the influence of the reverse pressure of the tail water which influence increases with the steepness of the fore slope of the weir. The base width is, however, increased by one foot in the second figure.

As will be seen in the next section, the crest width of a weir should not be less than $\sqrt{H}+\sqrt{d}$; in this case $H=45$ and $d=5$. This would provide a crest width of $6.7+2.2=9$ feet, which it nearly scales.

NOTABLE EXISTING DAMS

44. Cheeseman Lake Dam. Some actual examples of dam sections will now be exhibited and analyzed. Fig. 24 is the section of the Cheeseman Lake dam near Denver, Colorado, which is one of the highest in the world. It is built to a curvature of 400 feet radius across a narrow canyon. It is considered a gravity dam, however, and will be analyzed as such. The section can be divided into three unequal parts 1, 2, and 3, and the lines of pressure (R.F.) and (R.E.) will be drawn through the bases of these three divisions. Of the vertical forces (1) has an area of 756 sq. ft., (2) of 3840, and (3) of 13,356 the total value of W being 17,952 sq. ft., which is marked off on the load line in Fig. 24a. With regard to the water-pressure areas the most convenient method, where half widths are not used, which can only be done with equal divisions, is to estimate the areas of the horizontal pressures only and set them off horizontally, the values of the inclined pressures being obtained by construction. For this purpose the triangle of horizontal water pressure is shown adjacent to, but separate from, the profile. The three values of P

which are equal to $\frac{H^2}{2\rho}$ will be 270, 2631, and 7636, respectively, the total being 10,537 sq. ft. In this computation the value of ρ is assumed to be 2.4. These several lengths are now set out horizontally from the origin O in Fig. 24a, and verticals drawn upward intercept the chords, 1′, 2′, 3′, which latter are drawn from the origin O, parallel to their respective directions, i.e., normal to the adjacent parts of the wall. The rest of the process is similar to that already described, with reference to Figs. 16 and 18, and need not be repeated. In Fig. 24a N scales 19,450, equal to 1457 tons, and

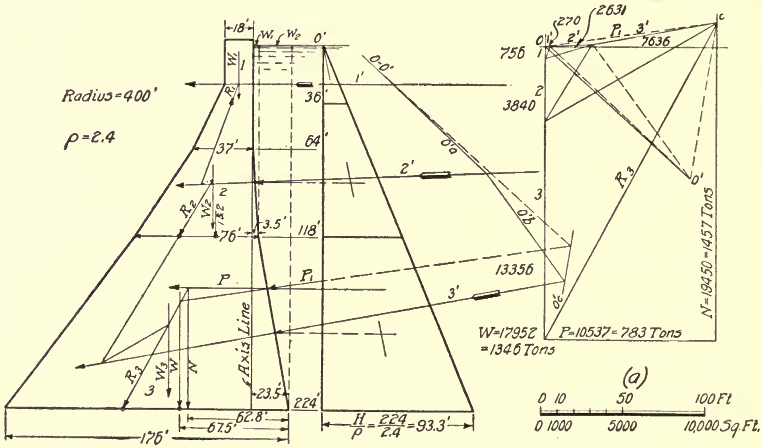


Fig. 24. Profile of Cheeseman Lake Dam

on the profile q scales 15 feet, therefore, in formula (**9**), $m=1+\frac{90}{176}=$ 1.51. Therefore, $s=\frac{mN}{b}=\frac{1.51\times1457}{176}=12.5$ tons, and $s_s=\frac{P}{b}=\frac{783}{176}=$ 4.45; then by formula (**10**)

$$c=\frac{12.5}{2}+\sqrt{\frac{(12.5)^2}{4}+(4.45)^2}=6.25+\sqrt{59}=13.9 \text{ tons, approx.}$$

With regard to W, q scales about 20 ft., m then works out to 1.7, nearly, and $s=\frac{mW}{b}=\frac{1.7\times1346}{176}=13.0$ tons.

As an exercise the inclined final resultant P is drawn on the profile. This line is parallel to Oc in Fig. 24a, its location is worked out by means of the funicular polygon, the construction of which need not be explained after what has gone before.

45. Analytical Check. In order to check this result analytically the procedure will be, first, calculate the position of the c.g. of the trapezoids (2) and (3) relative to the rear corner of their bases by formula (**7**) and also the positions of the resultants of the vertical components of the water pressure overlying the back with regard to the same points by formula (**6**). Second, convert the areas into tons by multiplying by $\frac{3}{40}$. The statement of moments about the heel of the base, with the object of finding the position of W is given below.

Moment of (1)	56.7 ×32.5	= 1843
Moment of (2)	288 ×47.9	= 13795
Moment of (3)	1001 ×75	= 75075
Total W =	1346 tons	90713

The distance of W from the heel will then be $\frac{90713}{1346}=67.5$ ft. In order to obtain N, the moments of the water weights will have to be added as below.

Moment of W	1346×67.5	= 90713
Moment of w_1	10×21.6	= 216
Moment of w_2	107×9	= 963
Total N=	1463 tons	91892

and

$$x=\frac{91892}{1463}=62.8 \text{ feet}$$

To find the incidence of R and its distance (q) from the center point, that from the known position of N must be computed from the formula $f=\frac{PH}{3N}=\frac{783\times 224}{3\times 1463}=40$ ft., therefore, $q=(62.8+40.0)-\frac{176}{2}=14.8$ feet. This is close to the value obtained graphically which was taken as 15 feet. The value of N is also seen to be close to that obtained graphically. The value of q with regard to W (R.E.) is as follows, $q=\frac{176}{2}-67.5=20.5$ feet, almost exactly what it scales on the diagram. In this profile the upper part is light, necessarily made up for in the lower part.

At the upper base line of (2) the incidence of W is exactly at the middle third edge, while R falls within it. At the final base the position of N is 62.8 distant from the heel and the inner third point is $\frac{176}{3} = 58.6$ distant, consequently the incidence of N lies 4.2 feet within the boundary.

If the position of N were made obligatory at the inner edge of the middle third, the value of W would be increased, but R would

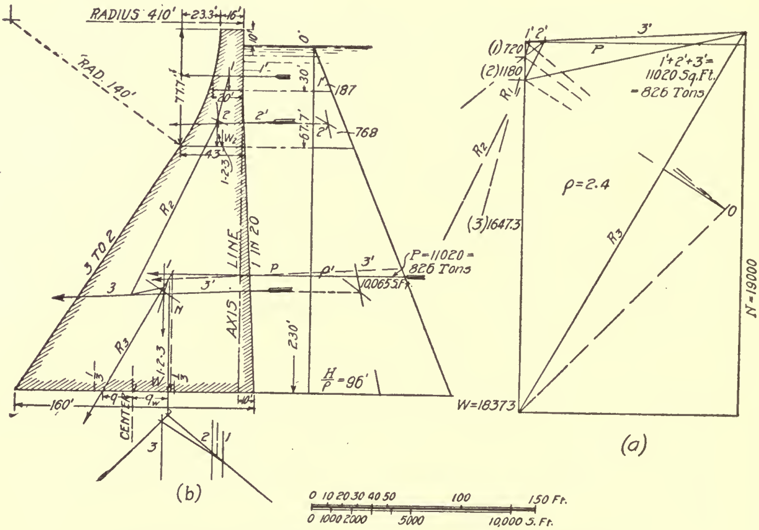


Fig. 25. Profile of Roosevelt Dam across Salt River, Arizona

be decreased. There may have been special reasons for limiting the maximum stress (R.E.). On Fig. 24 the position of N is obtained by the intersection of the horizontal resultant P with R prolonged upward. If the stress were calculated on the supposition that the structure was an arched dam, it would amount to $21\frac{1}{4}$ tons by the "long" formula, given in section 78, Part II.

46. Roosevelt Dam. In Fig. 25 is given the profile of the Roosevelt dam, and Fig. 26 is the site plan of that celebrated work. For some years, the Roosevelt dam was the highest gravity dam in existence. It spans a very deep canyon of the Salt River in Arizona and impounds the enormous quantity of $1\frac{1}{4}$ million acre-feet of water, which will be utilized for irrigation. This work is part of one of the greatest of the several large land reclamation projects undertaken by the U. S. Government for the watering and settling of arid tracts in the dry zone of the western states.

The profile is remarkable for the severe simplicity of its outline. It closely follows the elementary profile right down to its extreme base and forms a powerful advocate for this simple style of design. The graphical procedure is similar to that in the last example. The section is divided into three divisions. As the first two are comparatively small, the triangle of forces in Fig. 25a

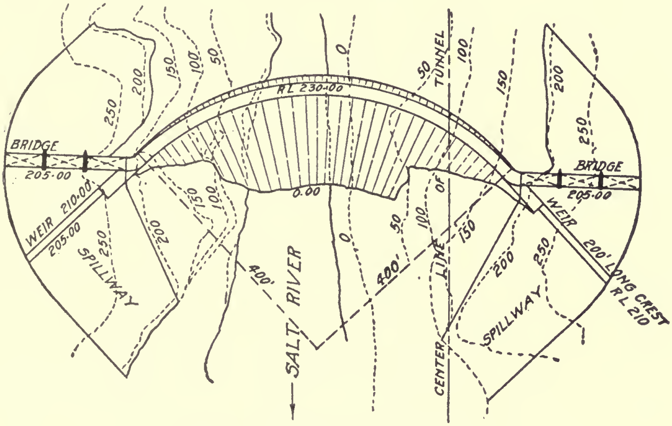


Fig. 26. Site Plan for Roosevelt Dam

is first plotted at a large scale in pencil and the inclinations of the resultants thus obtained are transferred to the profile; this accounts for the long projecting lines near the origin of the force diagram which also appear in some previous examples. A neater method for overcoming this difficulty is that adopted in the next figure, when the forces (1) and (2) are first amalgamated into one before being plotted on the force diagram.

In Fig. 25a, N scales roughly 19,000 sq. ft., equivalent to 1425 tons, q also measures approximately 20 ft., then $m=1+\frac{120}{160}=1.75$, and $s=\frac{mN}{b}=\frac{1425\times1.75}{160}=15.5$ tons. $s_s=\frac{P}{b}=\frac{826}{160}=5.1$ tons. By formula (10)

$$c=\frac{15.5}{2}+\sqrt{\frac{(15.5)^2}{4}+(5.1)^2}=7.75+\sqrt{86}=17 \text{ tons roughly}$$

With regard to W, q measures 23 ft. and m works out to 1.86 therefore $s=\frac{mW}{b}=\frac{1.86\times1378}{160}=16$ tons per sq. ft.

This dam is built on a radius of 410 feet, measured from the axis; if measured from the extrados of the curve at the base it will be 420 feet and the arch stress as calculated from the "long" formula used in "Arched Dams" will amount to 23.3 tons.

The site plan given in Fig. 26 forms an instructive example of the arrangement of spillways cut in the solid rock out of the shoulders of the side of the canyon, the material thus obtained being used in the dam. These spillways are each 200 feet wide and are excavated down to five feet below the crest of the dwarf waste weir walls which cross them. This allows of a much greater discharge passing under a given head than would be the case with a simple channel without a drop wall and with bed at the weir crest level. The heading up, or afflux, is by this means diminished and that is a matter affecting the height given to the dam crest.

47. New Croton Dam. The profile of the New Croton dam constructed in connection with the water supply of New York City is given in Fig. 27. This dam has a straight alignment and is 1168 feet long. Waste flood water is accommodated by an overfull weir 1000 ft. in length, which is situated on one flank forming a continuation of the dam at right angles to its axis. The surplus water falls into the Rocky River bed and is conveyed away by a separate channel. An elevation and plan of this work are given in Figs. 28 and 29.

The system of graphical analysis employed in this case is different from that in the last two examples and is that illustrated in Fig. 18, where independent combinations of vertical and inclined forces are used. The profile is divided into four divisions, the first being a combination of two small upper ones. The further procedure after the long explanations already given does not require any special notice except to point out that the directions of the combined forces $1'$, $1'+2'$, $1'+2'+3'$ etc., in (d) are drawn parallel to their reciprocal lines on Fig. 27a, namely to the chords Oa, Ob, Oc, and Od, respectively. The final resultants are R_4 (R.F.) and W (R.E.). The value of W is 1380 tons and that of N is 1484 tons, consequently applying formula (**10**), q in the first case scales 26 feet and m works out to 1.82 therefore $s=\frac{mW}{b}=\frac{1.82\times1380}{190}=13.2$ tons $=c$, as with W, s and c are identical.

With regard to N, q scales 7 feet, consequently $m=1+\frac{42}{190}=1.22$, $s=\frac{1.22\times1484}{190}=9.5$ tons only. As $P=10{,}010=750$ tons, $\frac{P}{b}=4$ tons; therefore $c=\frac{9.5}{2}+\sqrt{\frac{(9.5)^2}{4}+(4)^2}=11$ tons, which is very moderate. It is probable that other external pressures exist due

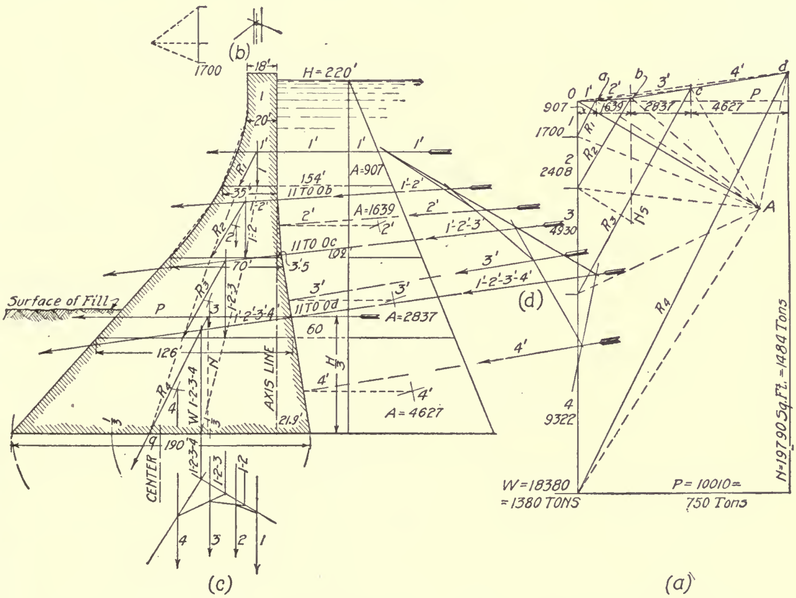


Fig. 27. Diagram of Profile of New Croton Dam Showing Influence Lines as in Fig. 18

to filling in front and rear, as also ice pressure, which would materially modify the result above shown. This dam, like the Cheeseman, is of the bottleneck profile, it is straight and not curved on plan.

48. Assuan Dam. The section, Fig. 30, is of the Assuan dam in Egypt, which notable work was built across the Nile River above the first cataract. As it stands at present it is not remarkable for

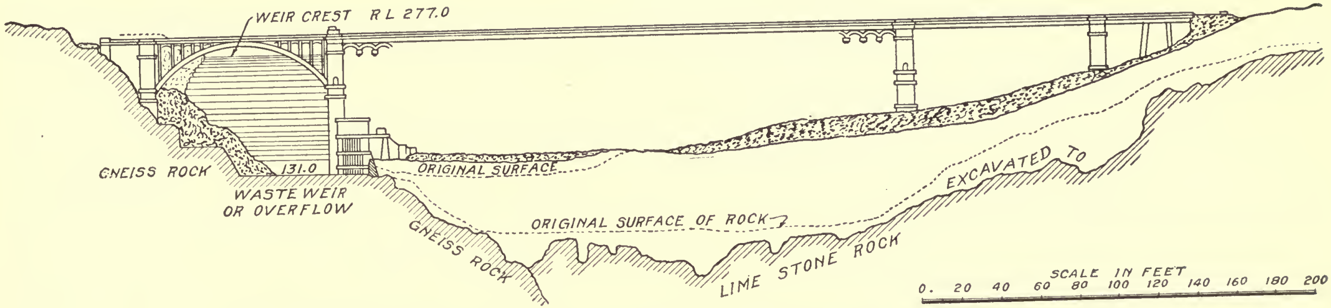


Fig. 28. Elevation of New Croton Dam Constructed in Connection with Water Supply of New York City

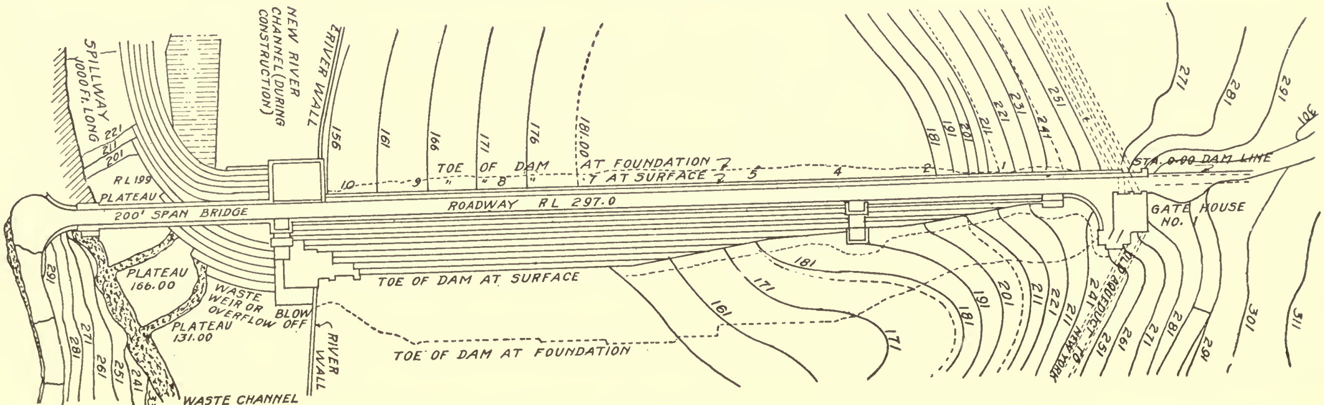


Fig. 29. Plan of New Croton Dam for New York City Water Supply

its height, but what it lacks in that respect, as in most eastern works, is made up in length, which latter is 6400 feet. No single irrigation work of modern times has been more useful or far-reaching in beneficial results upon the industrial welfare of the people than this dam. Its original capacity was 863,000 acre-feet and the back water extended for 140 miles up the river. The work is principally remarkable as being the only solid dam which passes the whole discharge of a large river like the Nile, estimated at 500,000 second-feet, through its body, for which purpose it is provided with 140 low and 40 high sluices. These are arranged in groups of ten, each

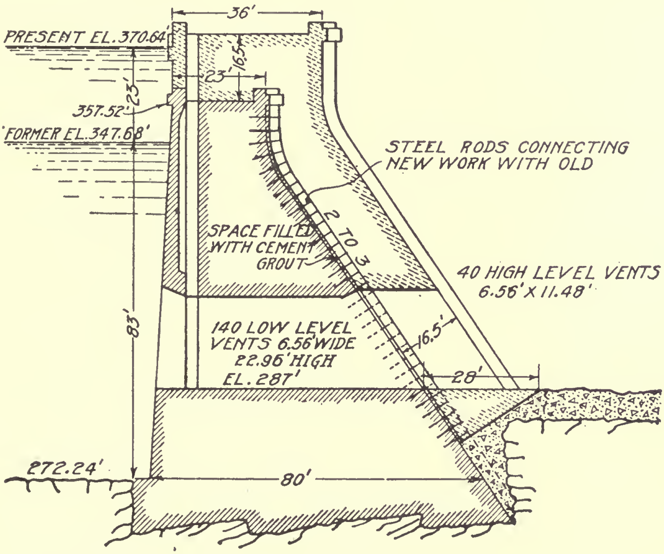


Fig. 30. Assuan Dam across the Nile Showing Old and New Profiles

low sluice is 23 feet deep by $6\frac{1}{2}$ feet wide with the dividing piers $16\frac{1}{2}$ feet wide. The diminution of the weight of the dam due to sluices necessitates an excess of width over what would be sufficient for a solid dam; in addition to which the maximum pressure in the piers is limited to the extremely low figure of 5 tons of 2000 pounds. The designers have thus certainly not erred on the side of boldness; the foundation being solid granite, would presumably stand, with perfect safety, pressure of treble that intensity, while the masonry, being also granite, set in cement mortar, is certainly capable of carrying a safe pressure of 15 tons, as many examples prove.

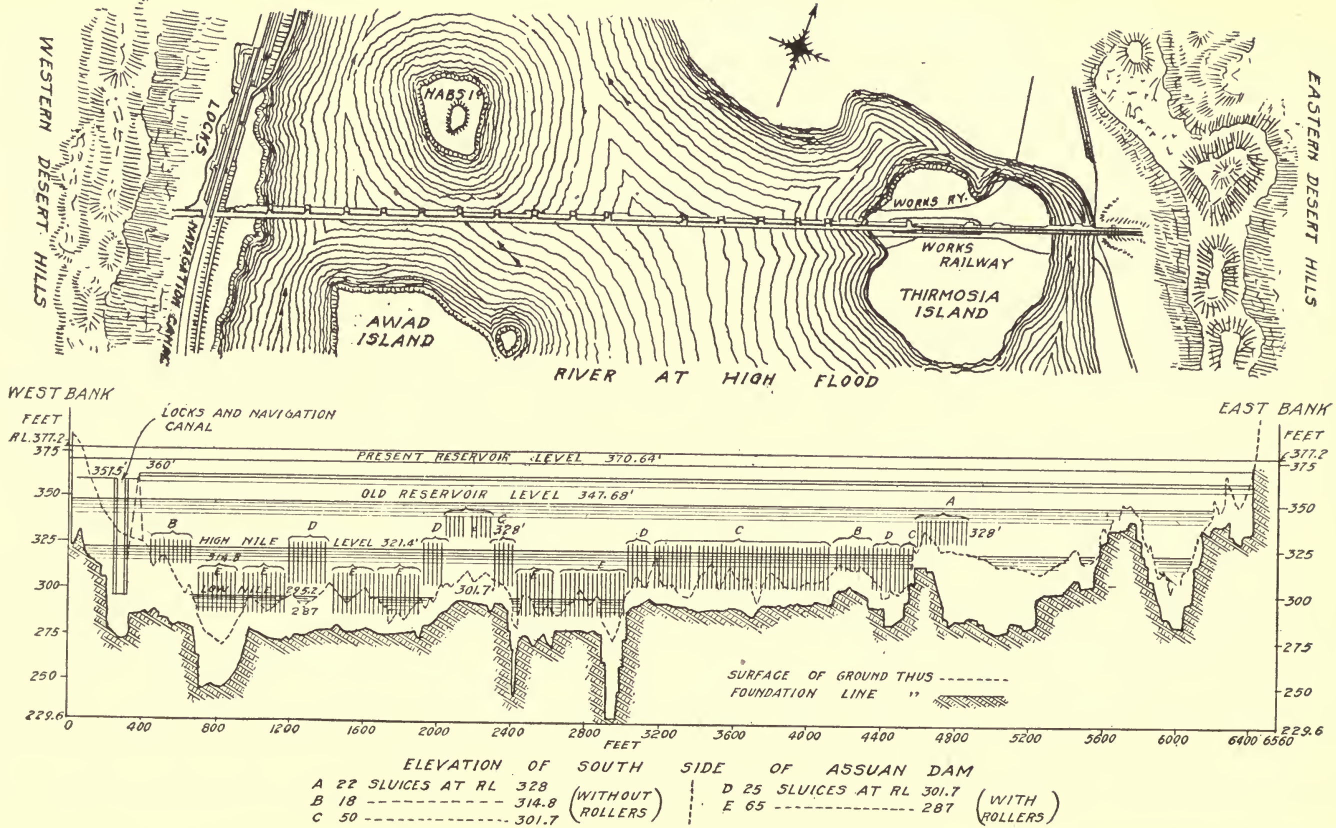


Fig. 31. Location Plan and Longitudinal Section of Assuan Dam

This dam has proved such a financial success that it has recently been raised by 23 feet to the height originally projected. The water thus impounded is nearly doubled in quantity, i.e., to over $1\frac{1}{2}$ million acre-feet; exceeding even that of the Salt River reservoir in Arizona. As it was decided not to exceed the low unit pressure previously adopted, the profile has been widened by $16\frac{1}{2}$ feet throughout. A space has been left between the new and the old work which has been subsequently filled in with cement grout under pressure, in addition to which a series of steel rods has been let

Fig. 32. View of Assuan Dam before Being Heightened with Sluices in Operation

into the old face by boring, and built into the new work. The enlargement is shown in the figure. The sluices are capable of discharging 500,000 second feet; as their combined area is 25,000 square feet this will mean a velocity of 20 feet per second. Owing, however, to the possibility of adjustment of level, by manipulation of the sluice gates, they will never be put to so severe a test.

A location plan and longitudinal section shown in Fig. 31, a view of the sluices in operation, Fig. 32, and a view of the new work in process, Fig. 33, will give a good idea of the construction features.

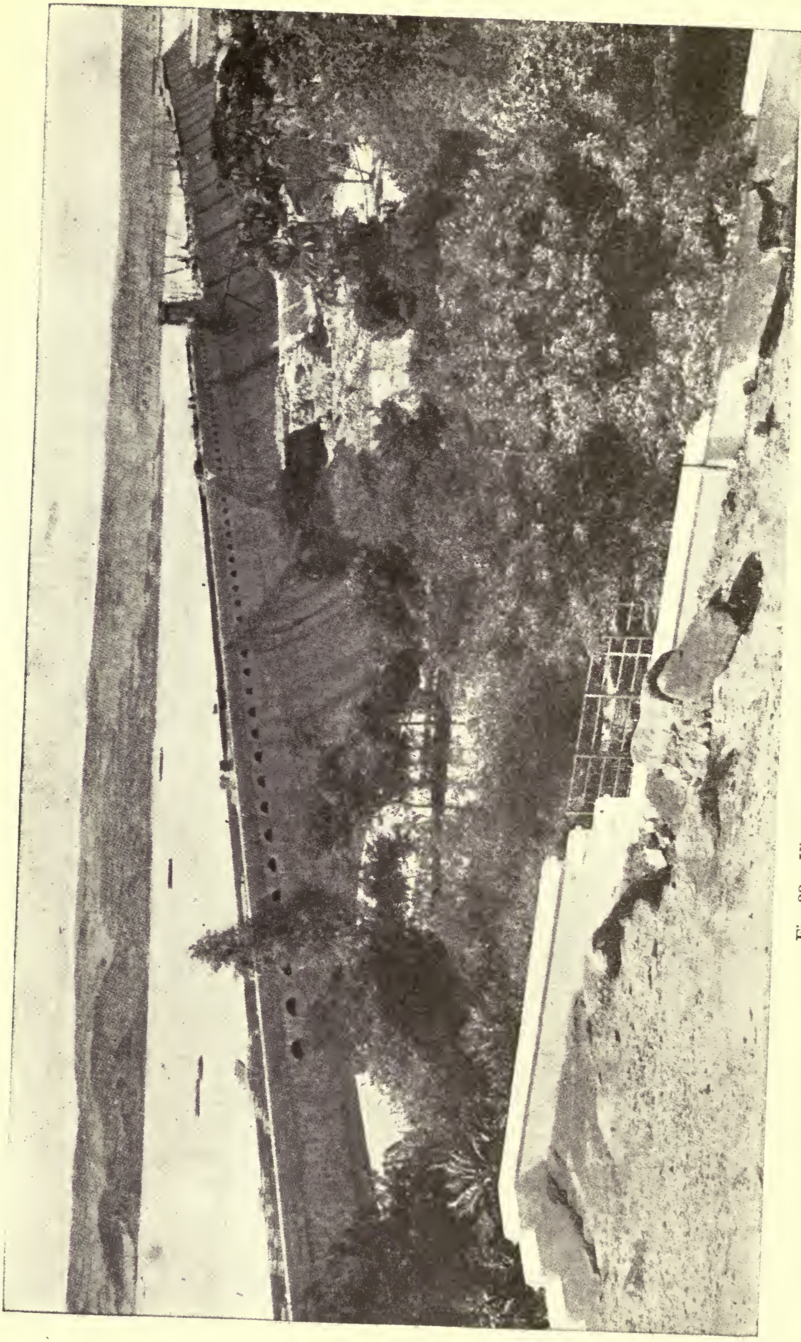


Fig. 33. View of Assuan Dam Showing New Work in Progress

49. Cross River and Ashokan Dams. Two further sections are given in Figs. 34 and 35, the first of the Cross River dam, and the second of the Ashokan dam in New York. Both are of unusually thick dimensions near the crest, this being specially provided to enable the dams to resist the impact of floating ice. These profiles are left to be analyzed by the student. The Ashokan dam is provided with a vertical line of porous blocks connected with two inspection galleries. This is a German innovation, which enables any

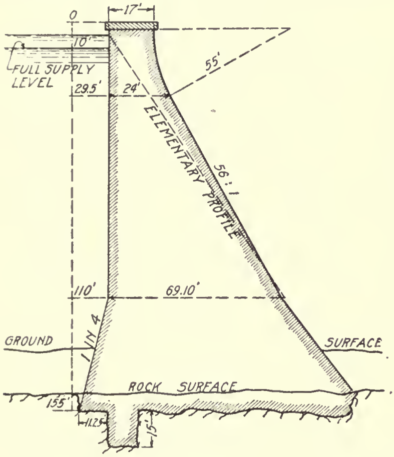


Fig. 34. Profile of Cross River Dam

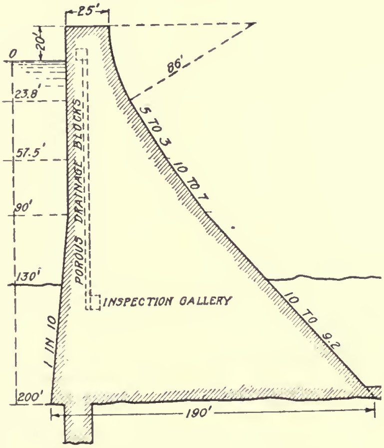


Fig. 35. Profile of Ashokan Dam

leakage through the wall to be drained off, thereby guarding against hydrostatic uplift. This refinement is now frequently adopted.

50. Burrin Juick Dam. The Burrin Juick dam in Australia, Fig. 36, which is generally termed "Barren Jack", is a close copy of the Roosevelt dam, Fig. 25, and is a further corroboration of the excellence of that profile. It is built across the Murrumbidgee River in New South Wales not far from the new Federal Capital. Its length is 784 feet on the crest, the maximum height being 240 feet. The fore batter is 3 vertical to 2 horizontal, and the back batter 20 vertical to 1 horizontal, both identical with those adopted in the Roosevelt dam; the crest width is 18 feet. It is built on a curve to a radius of 1200 feet. This dam will impound 785,000 acre-feet. The material of which the dam is composed is crushed sandstone in cement mortar with a plentiful sprinkling of large "plums" of granite. The ultimate resistance of specimen cubes was 180 "long" tons, per square foot; the high factor of safety of 12 was adopted, the usual being 8 to 10. The maximum allowable stress will, therefore, reduce to 15 "long" tons = 16.8 American short tons.

With regard to the maximum stresses, for *Reservoir Full*, $N = 16{,}100$, equivalent to 1210 tons, and q scales about 15 feet, conse-

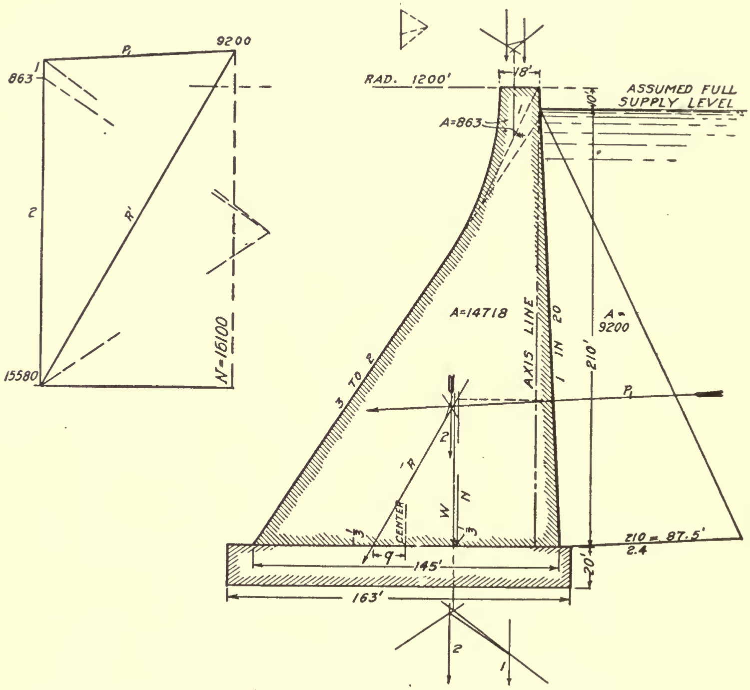


Fig. 36. Analytical Diagram Showing Profile of Burrin Juick Dam in Australia

quently m comes to 1.62, and $s = \dfrac{mN}{b} = \dfrac{1.62 \times 1210}{145} = 13.5$ tons, and $\dfrac{P}{b} = \dfrac{690}{145} = 4.8$ tons. Whence

$$c = \frac{s_1}{2} + \sqrt{\frac{s^2}{4} + s_s{}^2} = \frac{13.5}{2} + \sqrt{\frac{(13.5)^2}{4} + (4.8)^2} = 15 \text{ tons}$$

For Reservoir Empty, $W = 15{,}580$ feet or 1170 tons, $q = 24$, $m = 2$

$$\therefore \qquad s = \frac{mW}{b} = \frac{2 \times 1170}{145} = 16 \text{ tons, nearly}$$

The above proves that the stress (R.E.) is greater than that of (R.F.). Probably allowance was made for masses of porous filling lying at the rear of the dam, which would cause N and W to be shifted forward and so equalize the pressure. It will be noticed that the incidence of N, the vertical component (R.E.) falls exactly at the edge of the middle third, a condition evidently observed in the design of the base width.

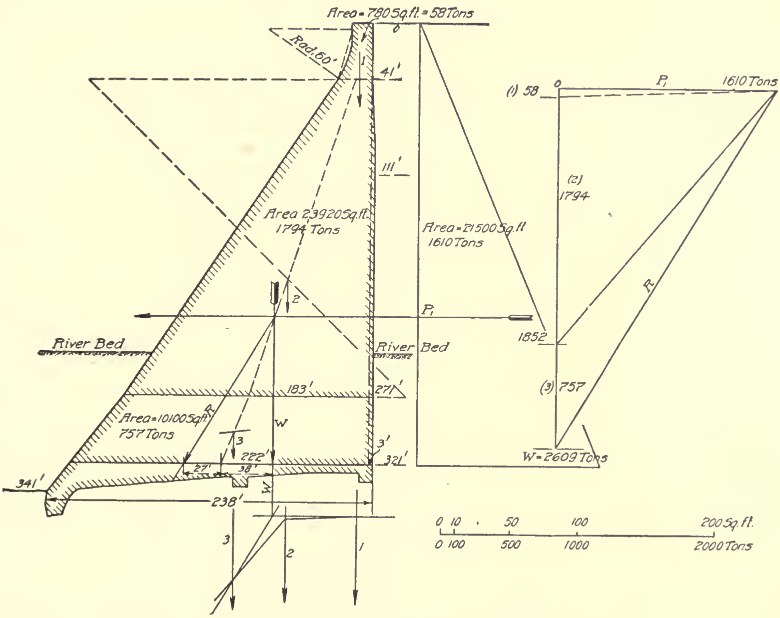


Fig. 37. Profile of Arrow Rock Dam, Idaho, Showing Incidence of Centers of Pressure on Base

The dam is provided with two by-washes 400 feet wide; the reservoir will be tapped by a tunnel 14×13 feet, the entrance sluices of which will be worked from a valve tower upstream, a similar arrangement to that in the Roosevelt dam. It is interesting to note that an American engineer has been put in charge of the construction of this immense work by the Commonwealth Government.

51. Arrow Rock Dam. The highest dam in the world now just completed (1915) is the Arrow Rock on the Boise, Idaho, project, a U. S. reclamation work. From the crest to the base the fore curtain is 351 feet. A graphical analysis of the stress in the base is given in Fig. 37. For *Reservoir Empty*, $W = 2609$ tons, and q measures 38 feet; therefore $m = 1 + \frac{228}{222} = 2$ nearly and $s = \frac{2W}{b} = 2 \times \frac{2609}{222} = 23.5$ tons. For *Reservoir Full*, $q = 27$, and $m = 1 + \frac{162}{222} = \frac{1.73 \times 2609}{222} = 20.2$ tons. $s_s = \frac{P}{b} = \frac{1610}{222} = 7.3$ tons, and $c = \frac{20.2}{2} + \sqrt{\frac{(20.2)^2}{4} + (7.3)^2} = 22.6$ tons. These values are, of course, but approximate.

Thus the compressive stresses (R.F.) and (R.E.) are practically equal, and the incidence of W and also of N is close to the edge of

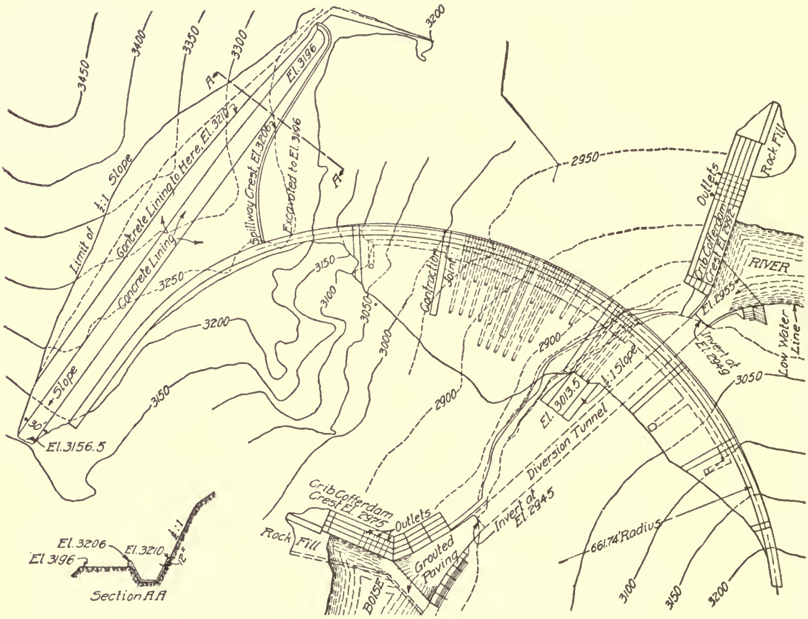


Fig. 38. Location Plan of Arrow Rock Dam
Courtesy of "Engineering Record"

the middle third. The dam is built on a radius of 661 feet at the crest. The high stresses allowed are remarkable, as the design is on the gravity principle, arch action being ignored. The curvature doubtless adds considerably to safety and undoubtedly tends to reduce the compressive stresses by an indeterminate but substantial amount. It is evident that formula (**10**) has been applied to the design. Reference to Figs. 38 and 39 will show that the dam is divided into several vertical sections by contraction joints. It is also provided with inspection galleries in the interior and vertical weeper drains 10 feet apart. These intercept any possible seepage, which is carried to a sump and pumped out. These precautions are

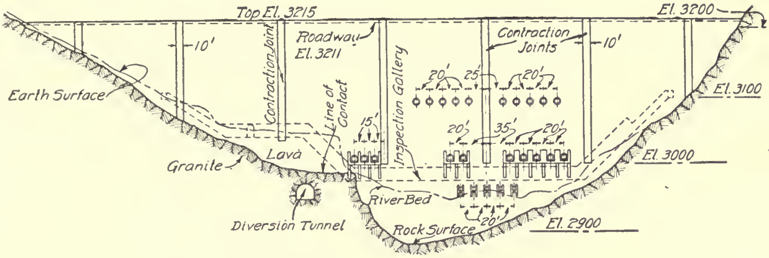


Fig. 39. Elevation of Arrow Rock Dam

to guard against hydrostatic uplift. The simplicity of the outline, resembling that of the Roosevelt dam, is remarkable.

SPECIAL FOUNDATIONS

52. Dams Not Always on Rock. Dams are not always founded on impervious rock but sometimes, when of low height, are founded on boulders, gravel, or sand. These materials when restrained from spreading, and with proper arrangements to take care of sub-percolation, are superior to clay, which latter is always a treacherous material to deal with. When water penetrates underneath the base of a dam, it causes hydrostatic uplift, which materially reduces the effective weight of the structure. Fig. 40 represents a wall resting on a pervious stratum and upholding water. The water has ingress into the substratum and the upward pressure it will exert at c against the base of the wall will be that due to its depth, in this case 30 feet. Now the point of egress of the percolation will be at b, and, as in the case of a pipe discharging in the open, pressure is nil at that point; consequently the uplift area below the base will be a triangle whose area equals $\frac{H \times b}{2}$. The diagram, Fig. 40, shows the combinations of the horizontal water pressure P with the hydrostatic uplift V and with the weight of the wall W. P is first combined with V, R_1 resulting, whose direction is upward. R_1 is then combined with W, R_2 being their resultant. The conditions without uplift are also shown by the dotted line drawn parallel to *dc* in Fig. 40. The line *ab* is termed the hydraulic gradient; it is also the piezometric line, i.e., a line connecting water levels in piezometer tubes, were such inserted.

Fig. 41 shows the same result produced on the assumption that the portion of the wall situated below the piezometric line is reduced in weight by an equal volume of water, i.e., the s.g. of this part may be assumed reduced by unity, i.e., from 2.4 to 1.4. The wall is

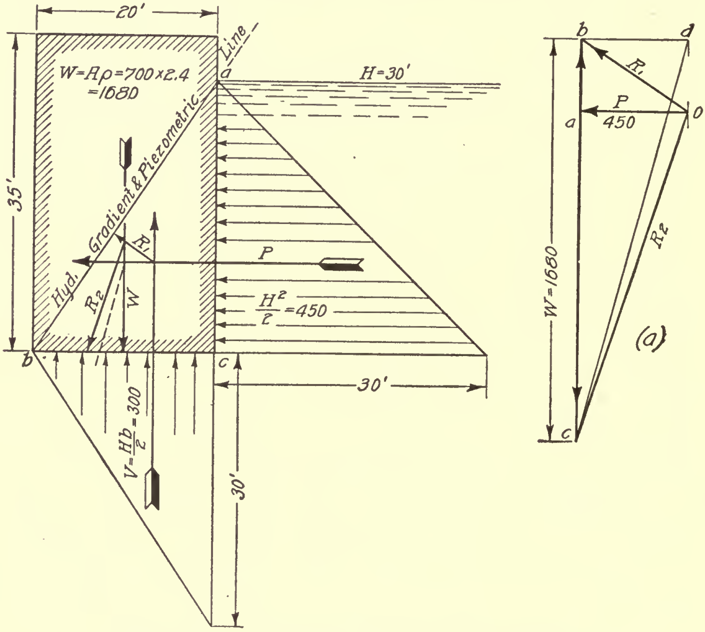


Fig. 40. Effect of Uplift on Dam Shown Graphically

thus divided diagonally into two parts, one of s.g. 2.4 and the other of s.g. 1.4. The combination of 1+2 with P is identical in result with that shown in Fig. 40. In the subsequent section, dealing with "Submerged Weirs on Sand", this matter of reduction in weight due to flotation is frequently referred to.

53. Aprons Affect Uplift. Fig. 42 is further illustrative of the principle involved in dams with porous foundations. The pentagonal profile *abc*, is of sufficient base width, provided hydrostatic uplift is absent. Supposing the foundation to be porous, the area of uplift will be a_1bc, in which ba_1, equals ab. This area is equal to abc, consequently practically the whole of the profile lies below the hydraulic gradient, may be considered as submerged, and hence loses weight; its s.g. can thus be assumed as reduced by unity, i.e., from ρ to $\rho-1$. The correct base width will then be found by making $b=\frac{H}{\sqrt{\rho-1}}$ instead of $\frac{H}{\sqrt{\rho}}$ The new profile will then be adb; the base width having been thus extended, the uplift is likewise increased in the same proportion. Now supposing an impervious apron to be built in front of the toe as must be the case with an overfall dam; then the area of uplift becomes ba_1e, and the piezo-

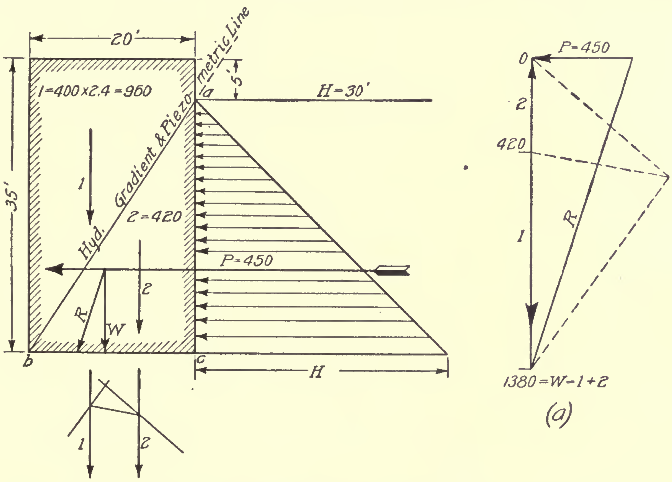


Fig. 41. Diagram Showing Identical Result If Weight Is Considered Reduced Due to Submersion

metric line and hydraulic gradient, which in all these cases happen to be one and the same line, is ae. Under these circumstances the comparatively thin apron is subjected to very considerable uplift and will blow up unless sufficiently thick to resist the hydrostatic pressure. The low water, or free outlet level, is assumed to be at the level e, consequently the fore apron lies above this level and is considered as free from flotation due to immersion.

54. Rear Aprons Decrease Uplift. Another case will now be examined. In Fig. 42 suppose the fore apron removed and a rear apron substituted. In this case the point of ingress of the percolating water is thrown back from b to b' the hydraulic gradient is $a'c$, the triangle of hydrostatic uplift is $b'a_2c$. This uplift from b' to b is more than neutralized by the rectangle of water $a'abb'$, which overlies the rear apron; the latter is therefore not subject to any uplift and, owing to its location, is generally free from erosion by moving water, consequently it can be made of clay, which in this position is water-tight as concrete masonry. A glance at Fig. 42 will demonstrate at once the great reduction of uplift against the base of the wall effected by the expedient of a rear apron, the uplift being reduced from a_1bc to fbc, more than one-half. Thus a rear

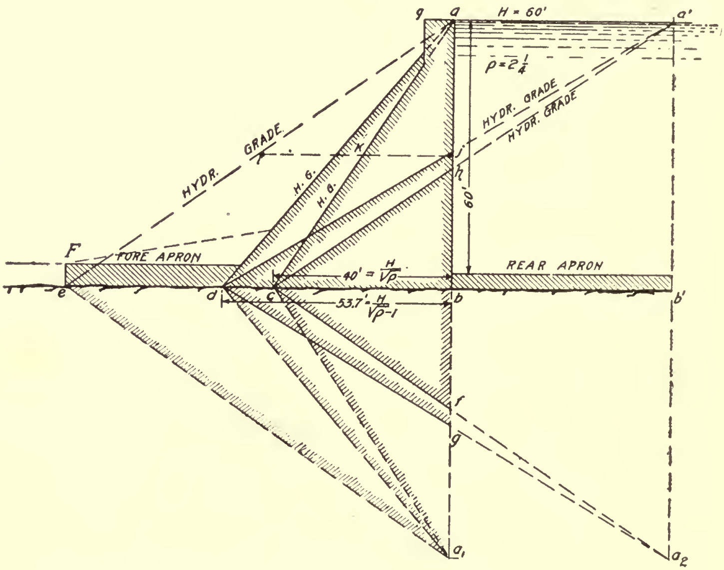


Fig. 42. Diagram Showing Uplift with and without Fore and Rear Aprons

apron is a sure remedy for uplift while the fore apron, if solid, should be made as short as possible, or else should be formed of open work, as heavy slabs with open joints. In the rear of overfall dams stanching clay is often deposited by natural process, thus forming an effective rear apron. Many works owe their security to this fact although it often passes unrecognized.

55. Rock Below Gravel. Fig. 43 represents a dam founded on a stratum of pervious material beneath which is solid rock. A fore curtain wall is shown carried down to the impervious rock. The conditions now are worse than those resulting from the impervious fore apron in Fig. 42 as the hydraulic gradient and piezometric line are now horizontal. The reduced area of vertical hydrostatic pressure is 1066 against which the wall can only furnish 1200; there is, therefore, an effective area of only 134 to resist a water pressure at the rear of 800, consequently the wall must fail by sliding or overturning as the graphical stress lines clearly prove. The proper position of a diaphragm curtain wall is at the heel, not at the toe of the dam; in this location it will effectively prevent all uplift. In the case where an impervious stratum does not occur at a reasonable depth the remedy is to provide a long rear apron which will reduce hydrostatic uplift to as small a value as may be desired, or else a combination of a vertical diaphragm with a horizontal apron can be

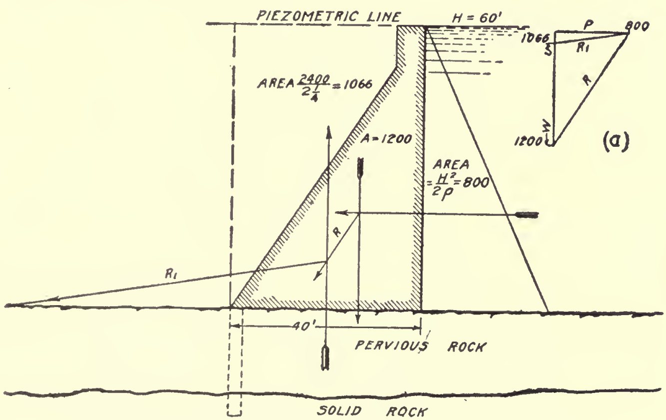


Fig. 43. Effect of Impervious Fore Curtain Wall on Uplift

used. In many cases a portion only of the required rear apron need be provided artificially. With proper precautionary measures the deposit of the remaining length of unfinished apron can safely be left for the river to perform by silt deposit, if time can be afforded for the purpose.

56. Gravity Dam Reinforced against Ice Pressure. This section will be concluded with a recent example of a gravity dam reinforced against ice pressure, which is given in Fig. 44, viz, that of the St. Maurice River dam situated in the Province of Quebec. The ice pressure is taken as 25 tons per foot run, acting at a level corresponding to the crest of the spillway, which latter is shown in Fig. 58. The profile of Fig. 44 is pentagonal, the crest has been given

the abnormal width of 20 feet, while the base is $\frac{3}{4}$ of the heigh
which is about the requirement, were ice pressure not considere
The horizontal ice pressure, in addition to that of the water uphel
will cause the line of pressure to fall well outside the middle thir
thus producing tension in the masonry at the rear of the sectio
To obviate this, the back of the wall is reinforced with steel rods
the extent of $1\frac{1}{2}$ square inches per lineal foot of the dam. If th
safe tensile strength of steel be taken at the usual figure of 16,00

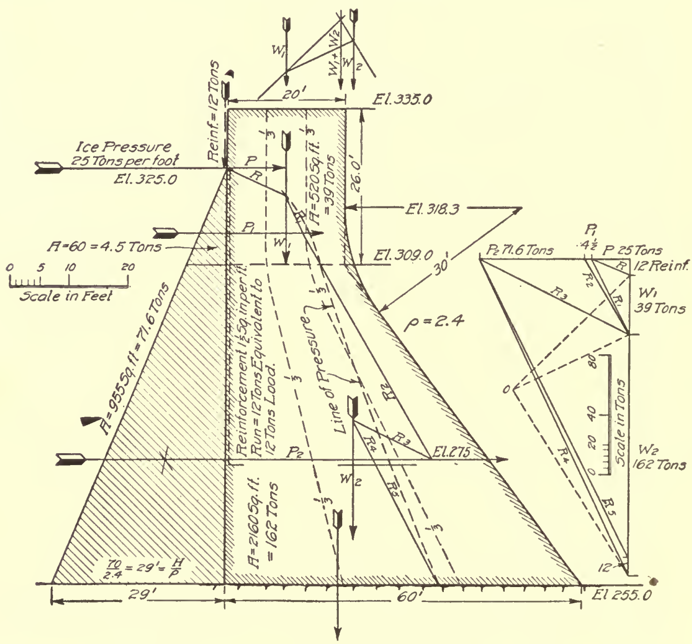


Fig. 44. Profile of Saint Maurice River Dam at Quebec

pounds, or 8 tons per square inch, the pull exerted by the reinforcement against overturning will be 12 tons per foot run. This force can be considered as equivalent to a load of like amount applied at the back of the wall, as shown in the figure. The section of the dam is divided into two parts at *El* 309 and the incidence of the resultant pressure at this level and at the base is graphically obtained. The line of pressure connecting these points is drawn on the profile. The line falls outside the middle third in the upper half of the section and within at the base, the inference being that the section would be improved by conversion into a trapezoidal outline with a narrower crest and with some reinforcement introduced as has been done in the spillway section, shown in Fig. 58.

It will be noticed that the reinforcement stops short at *El* 275. This is allowed for by assuming the imposed load of 12 tons removed at the base of the load line in the force polygon. The line R_5 starting from the intersection of R_4 with a horizontal through *El* 275.0 is the final resultant at the base. This example is most instructive as illustrating the combination of reinforcement with a gravity section in caring for ice pressure, thus obviating the undue enlargement of the profile.

GRAVITY OVERFALL DAMS OR WEIRS

57. Characteristics of Overfalls. When water overflows the crest of a dam it is termed an overfall dam or weir, and some modification in the design of the section generally becomes necessary. Not only that, but the kinetic effect of the falling water has to be provided for by the construction of an apron or floor which in many cases forms by far the most important part of the general design. This is so pronounced in the case of dwarf diversion weirs over wide sandy river beds, that the weir itself forms but an insignificant part of the whole section. The treatment of submerged weirs with aprons, will be given elsewhere. At present the section of the weir wall alone will be dealt with.

Typical Section. Fig. 45 is a typical section of a trapezoidal weir wall with water passing over the crest. The height of the crest as before, will be designated by H, that of reservoir level above crest by d, and that of river below by D. The total height of the upper still water level, will therefore, be $H+d$.

The depth of water passing over the crest should be measured some distance upstream from the overfall just above where the break takes place; the actual depth over the crest is less by reason of the velocity of the overfall being always greater than that of approach. This assumes dead water, as in a reservoir, in the upper reach. On a river or canal, however, the water is in motion and has a velocity of approach, which increases the discharge. In order to allow for this, the head (h) corresponding to this velocity, or $\frac{V^2}{2g}$

multiplied by 1.5 to allow for impact, or $h = .0233V^2$, should be added to the reservoir level. Thus supposing the mean velocity of the river in flood to be 10 feet per second $100 \times .0233$ or 2.3 feet would have to be added to the actual depth, the total being 15 feet in Fig. 45. The triangle of water pressure will have its apex at the surface, and its base will, for the reasons given previously, be taken as the depth divided by the specific gravity of the material of the wall. The triangle of water pressure will

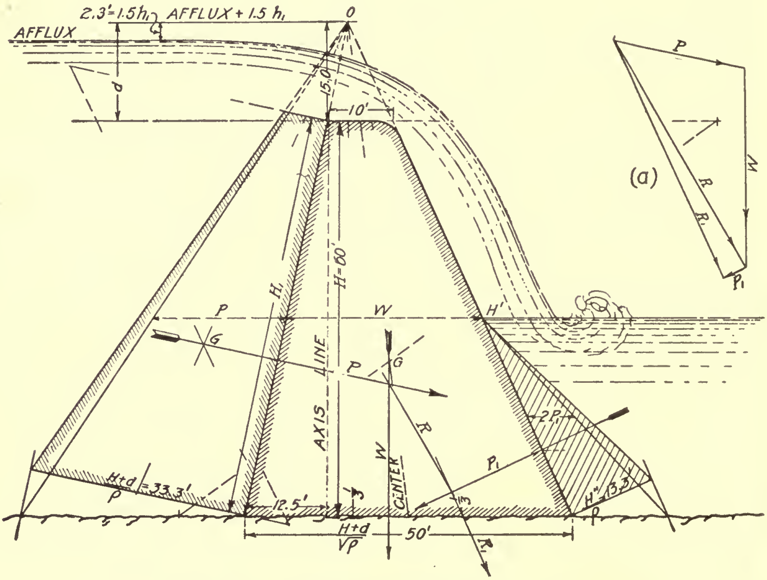


Fig. 45. Typical Section of Trapezoidal Weir Wall

be truncated at the crest of the overfall. The water pressure acting against the back of the wall will thus be represented by a trapezoid, not a triangle, whose base width is $\frac{H+d}{\rho}$ and its top width at crest level $\frac{d}{\rho}$. Its area therefore (back vertical) will be $\left(\frac{H+d}{\rho}+\frac{d}{\rho}\right)\frac{H}{2}$. If the back is inclined the side of the trapezoid becomes H_1. The general formula is therefore

$$A = (H \text{ or } H_1) \times \frac{(H+2d)}{2\rho} \tag{15}$$

H_1 being the inclined length of the back of the wall. The vertical distance of its point of application above the base according to formula (5) page 19 is $h=\frac{H}{3}\left(\frac{H+3d}{H+2d}\right)$ and will be the same whether the back is vertical or inclined.

58. Approximate Base Width. With regard to the drop wall itself, owing to the overfall of water and possible impact of floating timber, ice, or other heavy bodies, a wide crest is a necessity. A further strengthening is effected by adopting the trapezoidal profile. The ordinary approximate rule for the base width of a trapezoidal weir wall will be either

$$b=\frac{(H+d)}{\sqrt{\rho}} \tag{16}$$

or

$$b=\frac{(H+.6d)}{\sqrt{\rho}} \tag{16a}$$

The correctness of either will depend on various considerations, such as the value of d, the depth of the overfall, that of h_1 or velocity head and also of D, the depth of the tail water; the inclination given to the back, and lastly, whether the weir wall is founded on a porous material and is consequently subject to loss of weight from uplift. Hence the above formulas may be considered as approximate only and the base width thus obtained subject to correction, which is easiest studied by the graphical process of drawing the resultant on to the base, ascertaining its position relative to the middle third boundary.

59. Approximate Crest Width. With reference to crest width, it may be considered to vary from

$$k=\sqrt{H+d} \tag{17}$$

to

$$k=\sqrt{H}+\sqrt{d} \tag{18}$$

the former gives a width sufficient for canal, or reservoir waste weir walls, but the latter is more suitable for river weirs, and is quite so when the weir wall is submerged or drowned.

In many cases, however, the necessity of providing space for falling shutters or for cross traffic during times when the weir is not acting, renders obligatory the provision of an even wider crest. With a moderate width, a trapezoidal outline has to be adopted, in order to give the requisite stability to the section. This is formed by joining the edge of the crest to the toe of the base by a straight line, the base width of $\frac{(H+d)}{\sqrt{\rho}}$ being adopted, as shown in Fig. 45. When the crest width exceeds the dimensions given in formula (18), the face should drop vertically till it meets the hypothenuse of the elementary profile, as is the case with the pentagonal profile of dams. An example of this is given in Fig. 52 of the Dhukwa weir. The tentative section thus outlined should be tested by graphical process and if necessary the base width altered to conform with the theory of the middle third.

In Fig. 45 is given a diagram of a trapezoidal weir 60 feet high with $d=15$ feet. According to formula (17) the crest width should be $\sqrt{75}=8.7$ feet, and according to (18), $7.74+3.87=11.6$. An average of 10 feet has been adopted, which also equals $\frac{d.}{\rho}$ The profile therefore, exactly corresponds with the elementary triangle canted forward and truncated at the overfall crest.

60. Graphical Process. In graphical diagrams, as has already been explained, wherever possible half widths of pressure areas are taken off with the compasses to form load lines, thus avoiding the arithmetical process of measuring and calculating the areas of the several trapezoids or triangles, which is always liable to error. There are, however, in this case, three areas, one of which, that of the reverse water pressure, has an altitude of only half of the others. This difficulty is overcome by dividing its half width by 2. If one height is not an exact multiple, as this is of H, a fractional value given to the representing line in the polygon will often be found to obviate the necessity of having to revert to superficial measures. The application of the reverse pressure P_1 here exhibited is similar to that shown in Fig. 16; it has to be combined with R, which latter is obtained by the usual process. This combination is effected in the force polygon by drawing a line P_1 equal to the representative area, or half width of the back pressure, in a reverse direction to P. The closing line R_1 is then the final resultant. On the profile itself the force line P_1 is continued through its center of gravity till it intersects R, from which point R_1 is drawn to the base. If this portion of the face of the weir is very flat, as is sometimes the case, P_1 may be so deflected as to intersect R below the base altogether as is shown in Fig. 50. In such event, R_1 is drawn upward instead of downward to intersect the base. The effect of P_1 is to throw the resultant R_1 farther inward but not to any great extent. It improves the angular direction of R, however.

Reverse Pressure. A dam is usually, but not invariably, exempt from the effect of reverse pressure. This reverse water pressure is generally, as in this case, favorable to the stability of the weir, but there are cases when its action is either too slight to be of service or is even detrimental. This occurs when the face of the weir wall is much inclined, which points to the equiangular profile being most suitable. An example illustrative of the above remarks is given later in Fig. 50 of the Folsam dam.

As the moments of the horizontal pressure of water on either side of the weir wall vary almost with the cubes of their height, it is evident that a comparatively low depth of tail water will have but small influence and may well be neglected. When a vertical back is adopted, the slope is all given to the face; by which the normal reverse water pressure is given a downward inclination that reduces its capacity for helping the wall.

61. Pressures Affected by Varying Water Level. Calculations of the depths of water passing over the weir or rather the height of reservoir level above the weir crest, designated by d, and of the corresponding depth D in the tail channel, are often necessary for the purpose of ascertaining what height of water level upstream, or value of d, will produce the greatest effect on the weir wall. In low submerged or drowned weirs, the highest flood level has often the least effect, as at that time the difference of levels above and below the weir are reduced to a minimum. This is graphically shown in Fig. 46, which represents a section of the Narora dwarf weir wall, to which further reference will be made in section 124, Part II. In this profile two resultant pressures, R and R_1, are shown, of which R_1, due to much lower water level of the two stages under comparison, falls nearer the toe of the base.

The Narora weir, the section of the weir wall of which is so insignificant, is built across the Ganges River in Upper India at the head of the Lower Ganges Canal, Fig. 93. The principal part of this work, which is founded on the river sand, consists not in the low weir wall, although that is $\frac{3}{4}$ mile long, but in the apron or floor, which has to be of great width, in this case 200 feet.

As will be seen in Fig. 46 the flood level of the Ganges is 16 feet above the floor level, while the afflux, or level of the head water, is two feet higher. The river discharges about 300,000 second-feet when in flood. When full flood occurs, the weir is completely drowned, but from the diagrams it will be seen that the stress on the wall is less when this occurs than when the head water is

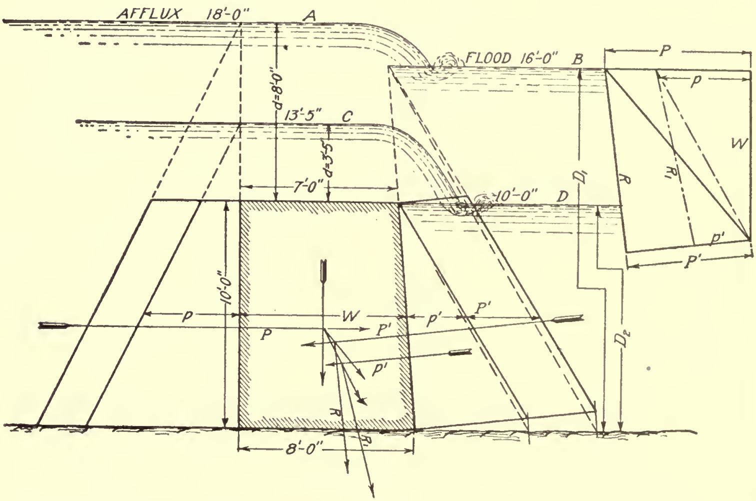


Fig. 46. Section of Narora Dwarf Weir Wall across Ganges River in Upper India

much lower. This result is due to the reverse pressure of the tail water.

The rise of the river water produces, with regard to the stress induced on the weir, three principal situations or "stages" which are enumerated below.

(1) When the head water is at weir crest level; except in cases where a water cushion exists, natural or artificial, the tail channel is empty, and the conditions are those of a dam.

(2) When the level of the tail water lies below weir crest level but above half the height of the weir wall. In this case the reciprocal depth of the head water above crest is found by calculation

(3) At highest flood level, the difference between the head and tail water is at a minimum. In an unsubmerged weir or overfall dam the greatest stress is generally produced during stage (3). In a submerged weir the greatest stress is produced during stage (2).

62. Moments of Pressure. The moments of the horizontal water pressure on either side of a wall are related to each other in proportion to the cubes of their respective depths. In cases where the wall is overflowed by the water, the triangle of pressure of the latter, as we have seen, is truncated at the weir crest. The moment (M) of this trapezoidal area of pressure will be the product of its area with h, or the product of the expressions in formula (1) and formula (5) as follows:

$$M = H\frac{(H+2d)}{2\rho}\times\frac{H}{3}\times\frac{(H+3d)}{(H+2d)}\times w$$

or

$$M = \frac{H^2 w}{6\rho}\ (H+3d) \qquad \textbf{(19)}$$

That of the opposing tail water will be $M = \frac{D^3 w}{6\rho}$, the difference of these two being the resultant moment. For example, in the case shown in Fig. 46, during stage (1) $H = 10$, $D = 0$, unbalanced moment $= \frac{10^3 w}{6\rho} = 166.6\frac{w}{\rho}$. In stage (2) $H = 10$, $d = 3.5$, and $D = 10$. Then the unbalanced moment will be $\frac{w}{6\rho}\,[(100\times 20.5) - 1000] = 175\frac{w}{\rho}$. In stage (3) $H = 10$, $D = 16$; $d = 8$, and $D - H = 6$ feet. There will thus be two opposing trapezoids of pressure, and the difference in their moments will be

$$\frac{w(100\times 34)}{6\rho} - \frac{w(100\times 28)}{6\rho} = 100\frac{w}{\rho}$$

Thus stage (2) produces the greatest effect, the least being stage (3). In this expression (w) symbolizes, as before, the unit weight of water, per cubic foot or, $\frac{1}{32}$ ton.

In spite of this obvious fact, many weir wall sections have been designed under the erroneous supposition that the overturning moment is greatest when the upper water is at crest level and the tail channel empty, i.e., at a time when the difference of levels above and

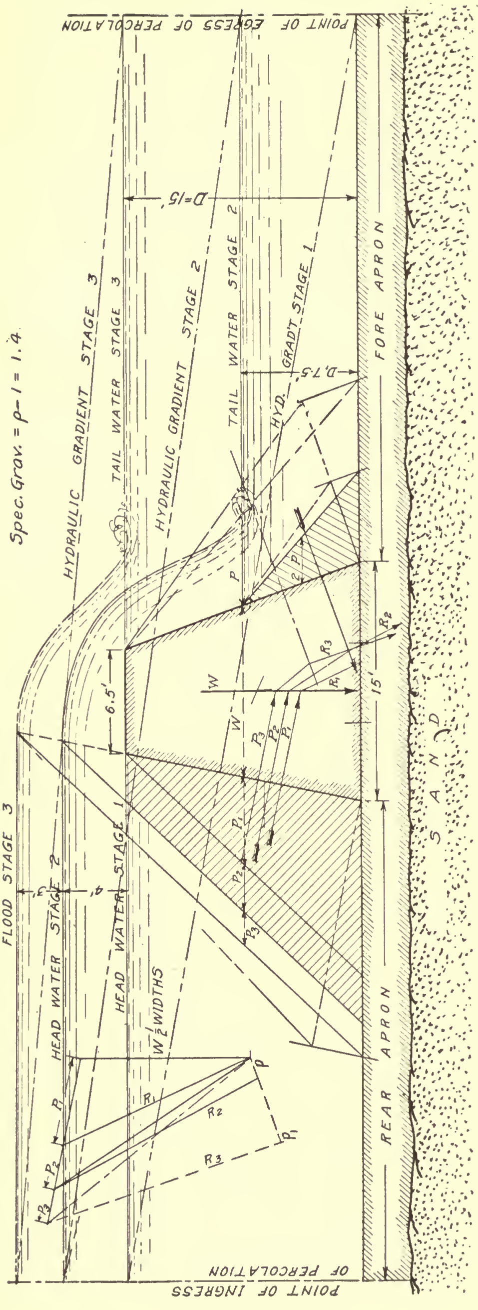


Fig. 47. Diagram Showing Analysis of Weir Wall Profile for Three Head-Water Levels

below the weir is at a maximum, or at full flood when the difference is at a minimum.

63. Method of Calculating Depth of Overfall. During the second stage of the river the value of d, the depth of the overfall, will have to be calculated. To effect this the discharge of the river must first be estimated when the surface reaches the crest level of the weir, which is done by use of the formula, $Q = Ac\sqrt{rs}$, given in section 35, page 47 of "Hydraulics, American School of Correspondence", A being the area, equal to d times length of weir (c) Kutter's coefficient, (r) the hydraulic mean radius, and s, the surface grade or slope of the river. The discharge for the whole river should now be divided by the length of the weir crest, the quotient giving the unit discharge, or that per foot run of the weir.

The depth required to pass this discharge with a free overfall is found by use of Francis' formula of $3.33d^{\frac{3}{2}}$ or a modification of it for wide crest weirs for which tables are most useful. See "Hydraulics", section 24, p. 30.

For example, supposing the river discharge with tail water up to crest level is 20 second-feet per foot run of the weir. Then $3.33d^{\frac{3}{2}}=20$. Whence $d^{\frac{3}{2}}=6$ and $d=\sqrt[3]{6^2}=3.3$ feet. This ignores velocity of approach, a rough allowance for which would be to decrease d by (h_1) the velocity head, or by $.0155V^2$.

64. Illustrative Example. Fig. 47 illustrates an assumed case. Here the weir is 15 feet high, 3 stages are shown:

(1) When head water is at crest level;

(2) When tail water is $7\frac{1}{2}$ feet deep, and the reciprocal depth of the head water is assumed as 4 feet; and

(3) With tail water at crest level and head water assumed 7 feet deep above crest.

The three resultants have been worked out graphically. From their location on the base the greatest stress is due to R_2, i.e., stage (2).

The hydraulic gradients of all three stages have been shown with an assumed rear and fore apron on floor. In (1) more than half the weir body lies below the piezometric line, which here corresponds with the hydraulic gradient, while in (2) nearly the whole lies below this line and in (3) entirely so.

Owing to this uplift it is well always to assume the s.g. of a weir wall under these conditions as reduced by immersion to a value of $\rho-1$. In these cases the triangles of water pressure are shown with their bases made $\frac{H}{\rho-1}$, or $\frac{H}{1.4}$, instead of $\frac{H}{2.4}$. Actually, however, the resistance of the weir wall to overturning relative to its base at floor level is not impaired by flotation, but as weight in these cases is a desideratum, the weir wall should be designed as if this were the case. The rear apron is evidently subject to no uplift, but the fore apron is, and its resisting power, i.e., effective weight, is impaired by flotation. See section 52 and also the later sections on "Submerged Weirs in Sand", Part II.

65. Examples of Existing Weirs. Some examples of existing weirs will now be given. Fig. 48 is a profile of the LaGrange overfall dam at the head of the Modesto and Tuolumne canals, Fig. 49. No less than 13 feet depth of water passes over its crest, 2 feet being

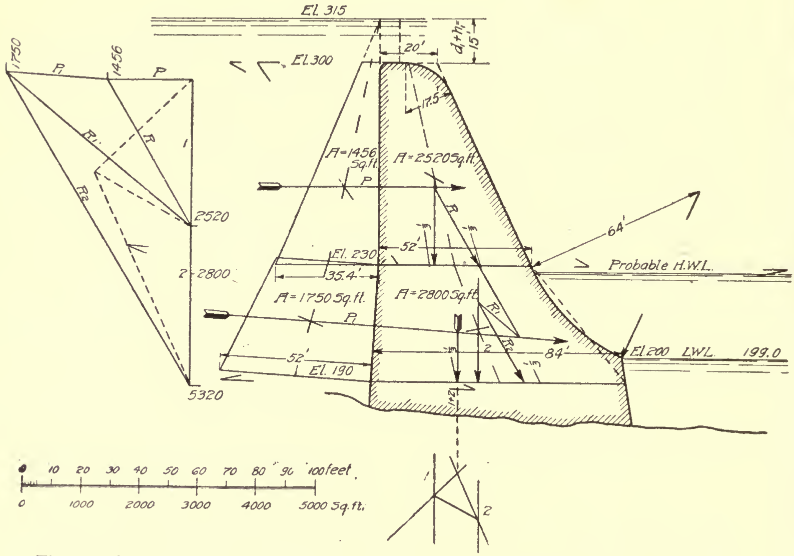


Fig. 48. Profile of LaGrange Overfall Dam at Head of Modesto and Tuolumne Canals

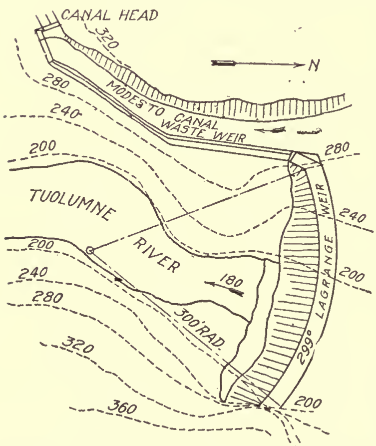


Fig. 49. Location Plan of LaGrange Weir

added to allow for velocity of approach. It is built on a curve of 300 feet radius. The graphical analysis of the section shows that the resultants (R.E.) and (R.F.) drawn on the profile fall within the middle third. In this process the reverse pressure due to tail water has been neglected. Its effect will be small.

It is a doubtful point whether the reverse pressure actually exercised is that due to the full depth of the tail water. The overflow causes a disturbance and probably more or less of a vacuum at the toe of the weir wall, besides which the velocity of impact causes a hollow to be formed which must reduce the reverse pressure. In some instances, as in the case of the Granite Reef dam, Fig. 55, the effective depth of the tail water is assumed as only equal to that of the film of overflow. This appears an exaggerated view. However, in a high overfall dam, the effect of the reverse is often so small that it can well be neglected altogether. In cases where the tail water rises to $\frac{3}{4}$ or more of the height of the dam its effect begins to be considerable, and should be taken into account.

66. Objections to "Ogee" Overfalls. Professional opinion seems now to be veering round in opposition to the "bucket" or curved base of the fore slope which is so pronounced a feature in American overfall dams. Its effects are undoubtedly mischievous, as the destructive velocity of the falling water instead of being reduced as would be the case if it fell direct into a cushion of water, is conserved by the smooth curved surface of the bucket. In the lately constructed Bassano hollow dam (see Figs. 84 and 85, Part II), the action of the bucket is sought to be nullified by the subsequent addition of baffles composed of rectangular masses of concrete fixed on the curved slope. The following remarks in support of this view are excerpted from "The Principles of Irrigation Engineering" by Mr. F. H. Newell, formerly Director of United States Reclamation Service. "Because of the difficulties involved by the standing wave or whirlpool at the lower toe of overflow dams, this type has been made in many cases to depart from the conventional curve and to drop the water more nearly vertically rather than to attempt to shoot it away from the dam in horizontal lines."

67. Folsam Weir. Fig. 50 is of the Folsam weir at the head of the canal of that name. It is remarkable for the great depth of flood water passing over the crest which is stated to be over 30 feet deep. The stress lines have been put on the profile with the object of proving that the reverse pressure of the water, although nearly 40 feet deep has a very small effect. This is due to the flat inclination given to the lower part of the weir, which has the effect of adding a great weight of water on the toe where it is least wanted and thus the salutary effect of the reverse pressure is more than neutralized. The section is not too heavy for requirements, but economy would undoubtedly result if it were canted forward to a nearly equiangular profile, and this applies to all weirs having deep tail water, and to drowned weirs. It will be noted that a wide crest allows but very little consequent reduction in the base width in any case.

The stress diagram in Figs. 50 and 50a are interesting as showing the method of combining the reverse pressures with the ordinary Haessler's diagram of the direct water pressure. The profile is divided into three parts as well as the direct water pressure, whereas the reverse pressure which only extends for the two lower divisions is in two parts. The stress diagrams present no novel features till R_2 is reached. This force on the profile comes in contact with reverse force 1″ before it reaches its objective 3^1. The effect of the reverse

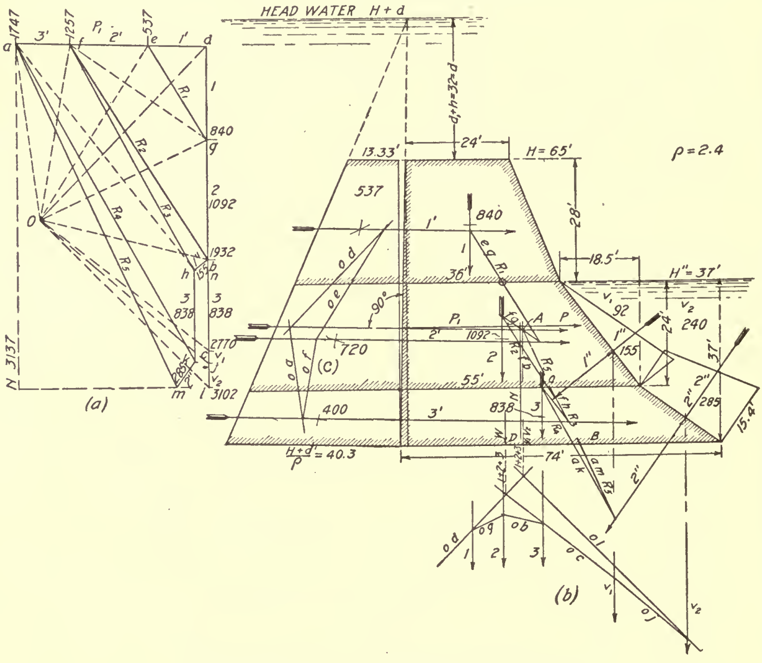


Fig. 50. Graphical Analysis of Folsam Weir

pressure is to deflect the direction of the resultant in the direction of R_3, which latter, as shown in the force polygon, Fig. 50a, is the resultant of 1″, set out from the point *b*, and of R_2. The new resultant R_3 continues till it meets 3^1. The resultant of R_2 and 3^1 is the reverse line drawn upward to meet the vertical force 3, parallel to its reciprocal in Fig. 50a, which is the dotted line joining the termination of 3^1, i.e., (*a*) with that of 1″.

Following the same method the resultant R_4 is next drawn downward to meet 2″, which latter in the force polygon is set out from the ternination of the vertical (3). The resultant of R_4 and $2''$ is the final R_5. This line is drawn upward on the profile intersecting the base at B. If the reverse pressure were left out of consideration, the force R_2 would continue on to its intersection with 3^1 and thence the reverse recovery line drawn to meet (3) will be parallel to *ba* (not drawn) in the force polygon. This reverse line will intersect the line (3) in the profile almost at the same spot as before.

The final line will be parallel to its reciprocal *ca* (not drawn in Fig. 50a) and will cut the base outside the intersection of R_5. To prevent confusion these lines have not been drawn on; this proves that the effect of the reverse pressure is detrimental to the stability of the wall, except in the matter of the inclination of R_5. If the profile were tilted forward this would not be so. If P_1 the resultant water pressure at the rear of the wall be drawn through the profile to intersect the resultant of all the vertical forces, viz, $1+2+3+v^1+v^2$, this point will be found to be the same as that obtained by producing the final R_5 backwards to meet P_1.

Determination of P_1. To effect this, the position of P_1 has to be found by the following procedure: The load line *db*, Fig. 50a, is continued to *l*, so as to include the forces 3, v_1, and v_2. The rays *oc*, *oj*, and *ol* are drawn; thus a new force polygon *dol* is formed to which the funicular, Fig. 50b, is made reciprocal. This decides the position of W, or of $1+2+3$, viz, the center of pressure (R. E.) as also that of $W+v_1+v_2$ which latter are the reverse pressure loads. The location of P_1 is found by means of another funicular polygon C derived from the force polygon *oad*, by drawing the rays *oa*, *of*, and *oe*; P_1 is then drawn through the profile intersecting the vertical resultant last mentioned at A. The line AB is then coincident with R_5 on Fig. 50. The vertical line through A is not N, i.e., is not identical with the vertical in Fig. 50, for the reason that N is the resultant of all the vertical forces, whereas the vertical in question is the centroid of pressure of all the vertical force less w_1, the weight of water overlying the rear slope of the wall. The location of N is found by drawing a horizontal P through the intersection of P_1 with a line drawn through the c.g. of the triangle of water pressure w, this will intersect the back continuation of BA at c. A vertical CD through this point will correspond with that marked

N in Fig. 50. The profile, Fig. 51, is a reproduction of that shown Fig. 50 in order to illustrate the analytical method of calculation or that by moments.

68. Analytical Method. The incidence of the resultant R is required to be as ascertained on two bases, one the final base and the other at a level 13 feet higher. The section of the wall as before, is divided into three parts: (1) of area 840 square feet, (2) of 1092, and (3) of 838 square feet. The position of the c.g. of (1) is found by formula (**7**) to be 15.15 feet distant from a the heel of the base and will be

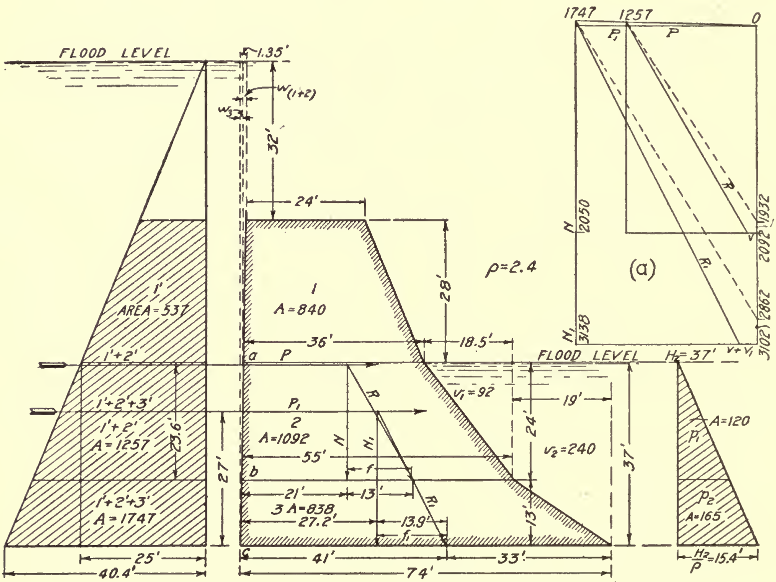


Fig. 51. Diagram of Folsam Weir Illustrating Analytical Method of Calculation

15.65 feet from b. That of (2) is 32.3 feet distant from its heel b. The reduced area of the water overlying the back down to b is estimated at 26 square feet and by formula (**6**) to be .5 feet distant from b. Again the reduced area of the reverse water overlying the fore slope v_1 is 92 square feet and the distance of its c.g. from b is $55 - \frac{18.5}{3}$ $= 48.8$ feet. The moments of all these vertical forces equated with that of their sum (N) about the point b will give the position of N relative to b.

Thus

$$\begin{array}{lrcl}
(1) & 840\times 15.65 & = & 13146 \\
(2) & 1092\times 23.3 & = & 25443 \\
(w) & 26\times .5 & = & 13 \\
(v_1) & 92\times 48.8 & = & 4490 \\
\hline
 & 2050\times x & = & 43092 = \text{Moment of } N \\
 & \therefore\quad x & = & 21 \text{ feet, nearly}
\end{array}$$

To obtain the distance f between N and R, $f=\dfrac{M_P-M_{p_1}}{N}$. Now the reduced area of $P=1257$ and the height of the c.g. of the trapezoid having its base at b, and its crest level with that of the wall is calculated by formula (**6**), to be 22.1 feet. Again the reduced area of the reverse water pressure triangle p_1 is 120 square feet, the height of its c.g. above base is 8 feet. Consequently:

$$f=\frac{(1257\times 22.1)-(120\times 8)}{2050}=\frac{26820}{2050}=13 \text{ feet}$$

For the lower base, the statement of moments about c is as follows, v_2 being 240, and its distance 65 feet by formula (**6**).

$$\begin{array}{lrcl}
 & (N)\ 2050\times(21+.3) & = & 43665 \\
 & (3)\ \ 838\times 32.3 & = & 27067 \\
 & (w)\ \ 10\times .15 & = & 2 \\
 & (v_2)\ \ 240\times 65 & = & 15600 \\
\hline
\text{Total} & 3138\times x & = & 86334
\end{array}$$

$$\therefore \qquad x-\frac{86334}{3138}=27.4 \text{ feet}$$

Now $f_1=\dfrac{M_{P_1}-M_{p_1+p_2}}{N_1}$, f_1 being the distance between N_1 and R_1. The value of P_1, the trapezoid of water pressure down to the base c, is 1747 square feet and the height of its c.g. by formula (**19**) or (**5**) is 27 feet, that of (p_1+p_2) is 285 square feet and its lever arm $\dfrac{37}{3}=12\frac{1}{3}$ feet. Then

$$f_1=\frac{(1747\times 27)-(285\times 12\frac{1}{3})}{3138}=\frac{47169-3514}{3138}=\frac{43655}{3138}=13.9 \text{ ft.}$$

The positions of N and N_1 being obtained, the directions of R and R_1 are lines drawn to the intersections of the two verticals

N and N_1 with two lines drawn through the c.g.'s of the trapezoid of pressure reduced by the moment of the reverse pressure, if any, or by $(P-p)$. This area will consist, as shown in the diagram, of a trapezoid superposed on a rectangle; by using formula (5) section 1, the positions of the c.g. of the upper trapezoid is found to be 12.58 feet above the base at a, while that of the lower is at half the depth of the rectangle, then by taking moments of these areas about b, the height of the c.g. is found to be 23.6 feet above the base at b, while the height for the larger area $[P_1-(p_1+p_2)]$ down to c is 27 feet.

In the graphical diagram of Fig. 51a the same result would be obtained by reducing the direct pressure by the reverse pressure area. Thus in the force diagram the vertical load line would remain unchanged but the water-pressure load line would be shorter being $P-p$ and $P_1-(p_1+p_2)$, respectively. This would clearly make no difference in the direction of the resultants R and R_1 and would save the two calculations for the c.g.'s of P and P_1.

This weir is provided with a crest shutter in one piece, 150 feet long, which is raised and lowered by hydraulic jacks chambered in the masonry of the crest so that they are covered up by the gate when it falls. This is an excellent arrangement and could be imitated with advantage. The shutter is 5 feet deep. The width at base of lamina 2 of this weir is 55 feet, or very nearly $\frac{H+d}{\sqrt{\rho}}$, formula (16).

69. Dhukwa Weir. A very similar work is the Dhukwa weir in India, Fig. 52, which has been recently completed.

This overfall dam is of pentagonal section. Owing to the width of the crest this is obviously the best outline.

The stress resultant lines have been drawn on the profile, which prove the correctness of the base width adopted. The tail water does not rise up to half the height of the weir. Consequently the formula $\frac{H+d}{\sqrt{\rho}}$ is applicable in stage 3. The effect of the tail water is practically nil. According to this formula the base width would be $63\times\frac{2}{3}=42$ feet, which it almost exactly measures—a further demonstration of the correctness of the formula. The crest width should be, according to formula (18), $\sqrt{50}+\sqrt{13}=11$ feet. The width of 17 feet adopted is necessary for the space required to work the collapsible gates. These are of steel, are held in position by struts connected with triggers, and can be released in batches by chains worked from each end. The gates, 8 feet high, are only 10 feet wide. This involves the raising and lowering of 400 gates, the weir crest being 4000 feet long. The arrangement adopted in the Folsam weir of hydraulic jacks operating long gates is far superior. An excellent feature in this design is the subway with occasional side chambers and lighted by openings, the outlook of which is underneath the waterfall, and has the advantage of relieving any vacuum under the falling water.

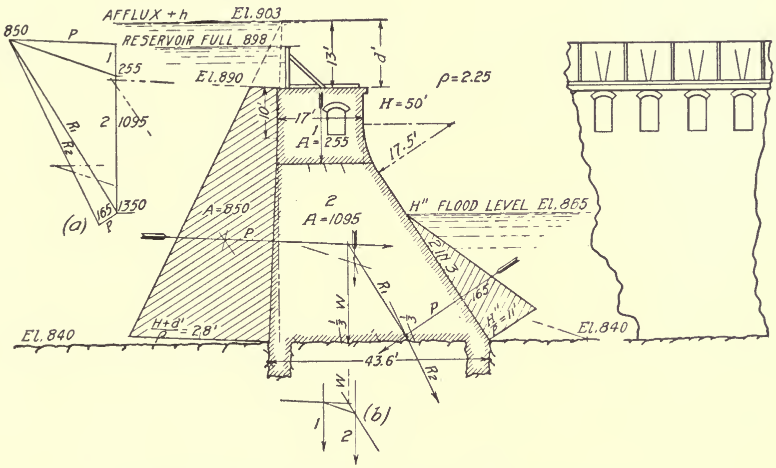


Fig. 52. Graphical Analysis of Profile of Dhukwa Weir in India

The subway could be utilized for pressure pipes and for cross communication, and the system would be most useful in cases where the obstruction of the crest by piers is inadvisable. The weir is 4000 feet long and passes 800,000 second-feet, with a depth of 13 feet. The discharge is, therefore, 200 second-feet per foot run of weir, which is very high. The velocity of the film will be $\frac{200}{13} = 15.4$ feet per second. With a depth of 13 feet still water, the discharge will be by Francis' formula, 156 second-feet per foot run. To produce a discharge of 200 feet per second, the velocity of approach must be about 10 feet per second. This will add 2.3 feet to the actual value of d, raising it from 13 to 15.3 feet which strictly should have been done in Fig. 52.

70. Mariquina Weir. Another high weir of American design, Fig. 53, is the Mariquina weir in the Philippines. It has the ogee curve more accentuated than in the LaGrange weir. The stress lines have been drawn in, neglecting the effect of the tail water which will be but detrimental. The section is deemed too heavy at the upper part and would also bear canting forward with advantage, but there are probably good reasons why an exceptionally solid

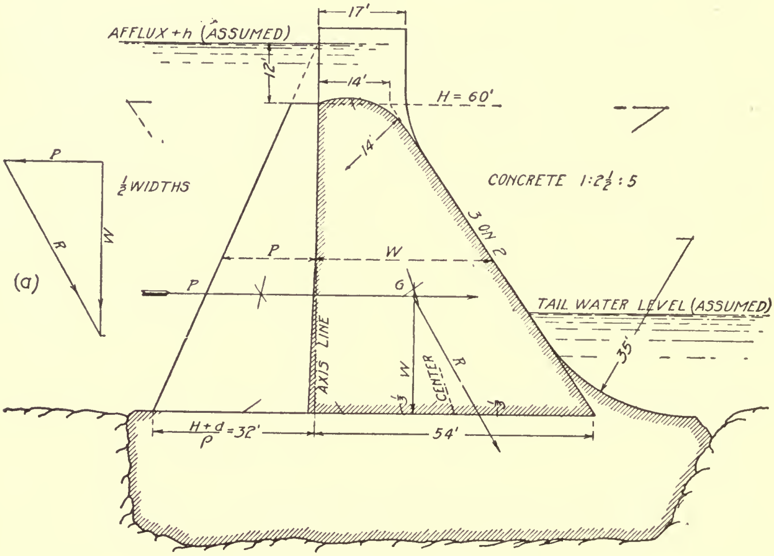


Fig. 53. Profile of Mariquina Weir in the Philippines

crest was adopted. The ogee curve also is a matter on which opinion has already been expressed.

71. Granite Reef Weir. The Granite Reef weir over the Salt River, in Arizona, Figs. 54 and 55, is a work subsidiary to the great Roosevelt dam of which mention was previously made.

It is founded partly on rock and partly on boulders and sand overlying rock. The superstructure above the floor level is the same throughout, but the foundations on shallow rock are remarkable as being founded not on the rock itself, but on an interposed cushion of sand. (See Fig. 54.) Reinforced concrete piers, spaced 20 feet apart, were built on the bedrock to a certain height, to clear all inequalities; these were connected by thin reinforced concrete side walls; the series of boxes thus formed were then filled level with sand, and the dam built thereon. This work was completed in 1908. The portion of the profile below the floor is conjectural. This construction appears to be a bold and commendable novelty. Sand in a confined space is incompressible, and there is no reason why it should not be in like situations. A suggested improvement would be to abandon the piers and form the substructure of two long outer walls only, braced together with rods or old rails encased in concrete. Fig. 55 is the profile on a boulder bed with rock below.

72. Hydraulic Conditions. The levels of the afflux flood of this river are obtainable so that the stresses can be worked out. In most cases these necessary statistics are wanting. The flood downstream has been given the same depth, 12 feet, as that of the film passing over the crest. This is clearly erroneous. The velocity of the film allowing for 5 feet per second approach, is quite 12 feet per second, that in the river channel could not be much over 5 feet, consequently it would require a depth of $\frac{12\times12}{5}=28$ feet. The dam would thus be quite submerged, which would greatly reduce the stress. As previously stated, the state of maximum stress would probably occur when about half the depth of flood passes over the crest. However, the graphical work to find the incidence of the resultant pressure on the base will be made dependent on the given downstream flood level. After the explanations already given, no special comment is called for except with regard to the reverse water pressure. Here the curved face of the dam is altered into 2 straight lines and the water pressure consists of two forces

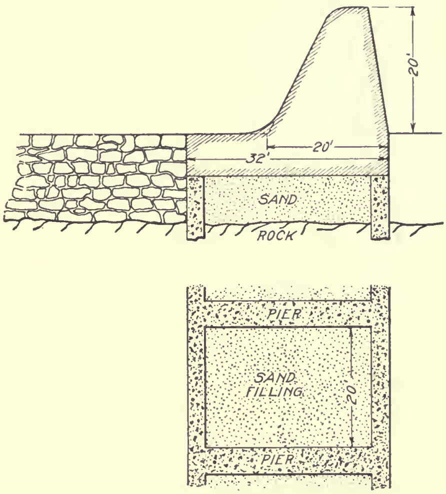


Fig. 54. Section of Granite Reef Weir Showing Sand Cushion Foundations

having areas of 17 and 40, respectively, which act through their c.g.'s. Instead of combining each force separately with the result-

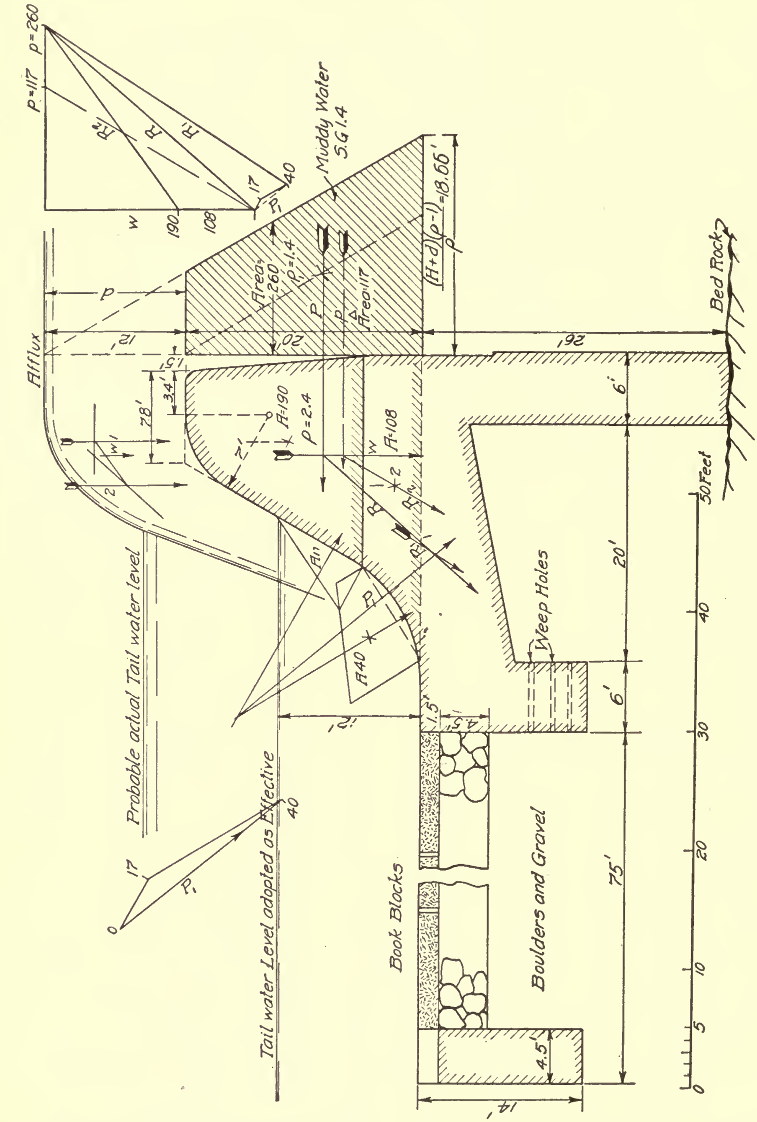


Fig. 55. Graphical Analysis of Profile of Granite Reef Dam over Salt River, Arizona

ant (R) it is more convenient to find their resultant and combine that single force with (R.) This resultant P_1 must pass through the intersection of its two components, thus if their force lines are run out backward till they intersect, a point in the direction of P_1 is found. P_1 is then drawn parallel to its reciprocal in the force polygon which is also shown on a larger scale at the left of the profile. The final resultant is R_1 which falls just within the middle third of the base. R_2 is the resultant supposing the water to be at crest level only. The water in the river is supposed to have mud in solution with its s.g. 1.4. The base length of the triangle of water pressure will then be $\frac{(H+d)\times(\rho-1)}{\rho}=\frac{32\times 1.4}{2.4}=18.66$. The other water-pressure areas are similarly treated. If the rear curtain reaches rock the dam should not be subject to uplift. It could, however, withstand sub-percolation, as the hearth of riprap and boulders will practically form a filter, the material of the river bed being too large to be disintegrated and carried up between the interstices of the book blocks. The effective length of travel would then be 107 feet; add vertical 52 feet, total 159 feet, H being 20 feet, $\frac{L}{H}$ works out to $\frac{160}{20}=8$ which ratio is a liberal allowance for a boulder bed. The fore curtain is wisely provided with weep holes to release any hydrostatic pressure that might otherwise exist underneath the dam. The Granite Reef dam has a hearth, or fore apron of about 80 feet in width. A good empirical rule for the least width for a solid or open work masonry fore apron is the following:

$$L=2H+d \qquad \textbf{(20)}$$

in which H is the height of the permanent weir crest above floor, and d is the depth of flood over crest. In this case $H=20$, $d=12$; least width of floor should then be $40+12=52$ feet. The Bassano dam is 40 feet high with 14 feet flood over crest, the width of hearth according to this formula should be 94 feet, its actual width is 80 feet which is admittedly insufficient. With a low submerged weir, formula (**34**), Part II, viz, $L=3\sqrt{cH}$, will apply. Beyond the hearth a talus of riprap will generally be required, for which no rule can well be laid down.

73. Nira Weir. Fig. 56 is of the Nira weir, an Indian work. Considering the great depth of the flood waterdown stream, the provision of so high a subsidiary weir is deemed unnecessary, a water cushion of 10 feet being ample, as floor is bed rock. The section of the weir wall itself, is considered to be somewhat deficient in base width. Roughly judging, the value of $H+d$, on which the base width is calculated, should include about 3 or 4 feet above crest level. This value of d, it is believed would about represent the height of head water, which would have the greatest effect on the weir. The exact value of d could only be estimated on a knowledge of the bed slope or surface grade of the tail channel. The above estimate would make $(H+d)=36$ feet, and with $\rho=2\frac{1}{4}$, $\dfrac{H+d}{\sqrt{\rho}}=24$ feet.

The top width, 8.3, is just $\sqrt{H}+\sqrt{d}$, in accordance with the rule given in formula (18).

A section on these lines is shown dotted on the profile. The provision of an 8-foot top width for the subsidiary weir is quite

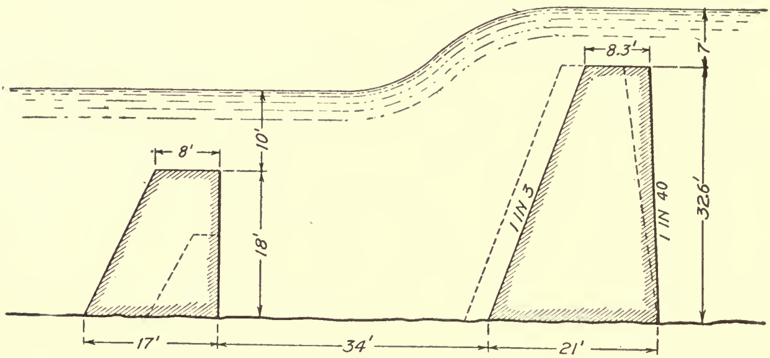


Fig. 56. Section of Nira Weir in India Showing Use of Secondary Weir

indefensible, while the base width is made nearly equal to the height, which is also excessive. For purposes of instruction in the principles of design, no medium is so good as the exhibition of plans of actual works combined with a critical view of their excellencies or defects. The former is obtainable from record plans in many technical works, but the latter is almost entirely wanting. Thus an inexperienced reader has no means of forming a just opinion and is liable to blindly follow designs which may be obsolete in form or otherwise open to objection.

74. Castlewood Weir. The Castlewood weir, Fig. 57, is of remarkable construction, being composed of stonework set dry, enclosed in a casing of rubble masonry. It is doubtful if such a section is any less expensive than an ordinary gravity section, or much less than an arched buttress dam of type *C*. Shortly after construction, it showed signs of failure, which was stated to be due to faulty connections with banks of the river; but whatever the cause it had to be reinforced, which was effected by adding a solid bank of earth in the rear, as shown in the figure. This involved lengthening the outlet pipes. In the overfall portion the bank must have been protected with riprap to prevent scouring due to the velocity of the approach current.

75. American Dams on Pervious Foundations. In the United States a very large number of bulkhead and overfall dams and regulating works, up to over 100 feet in height have been built on foun-

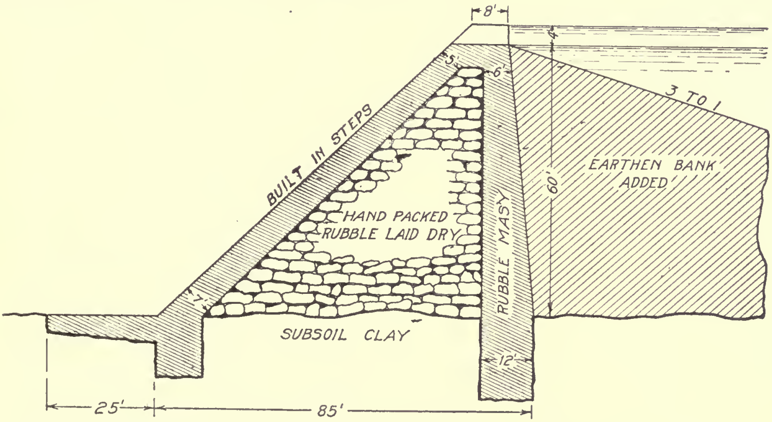


Fig. 57. Section of Castlewood Weir Showing Construction of Stone Work Set Dry, Enclosed in Rubble Masonry

dations other than rock, such as sand, boulders, and clay. Most of these, however, are of the hollow reinforced concrete, or scallop arch types, in which a greater spread for the base is practicable than would be the case with a solid gravity dam. Whenever a core wall is not run down to impervious rock, as was the case in the Granite Reef Overfall dam, Fig. 55, the matter of sub-percolation and uplift require consideration, as is set forth in the sections on "Gravity Dams" and "Submerged Weirs on Sand". If a dam 50 feet high is on sand or sand and boulders, of a quality demanding a high percolation factor of say 10 or 12, it is clear that a very long rear apron and deep rear piling will be necessary for safety.

All rivers bring down silt in suspension. When the overfall dam is a high one with a crest more than 15 or 20 feet above river-bed level, the deposit that is bound to take place in rear of the obstruction will not be liable to be washed out by the current, and additional light stanching silt will be deposited in the deep pool of comparatively still water that must exist at the rear of every high dam. For a low weir however this does not follow, and if deposit is made it will be of the heavier, coarser sand which is not impermeable.

The difficulty and expense of a long rear apron can be surmounted by the simple expedient of constructing only a portion of it of artificial clay, leaving the rest to be deposited by the river itself. To ensure safety the dam should be constructed and reservoir filled, in two or three stages, with intervals between of sufficient length to allow the natural deposit to take place. Thus only a fraction of the protective apron need be actually constructed. Many works are in existence which owe their safety entirely to the fortunate but unrecognized circumstance of natural deposit having stanched the river bed in their rear, and many failures that have taken place can only be accounted for from want of provision for the safety of the work against underneath scour or piping and also uplift. The author himself once had occasion to report on the failure of a head irrigation work which was designed as if on rock, whereas it was on a pervious foundation of boulders. When it failed the designers had no idea of the real cause, but put it down to a "treacherous river", "ice move", anything but the real reason, of which they were quite ignorant. Had a rear apron of sufficient width been constructed, the work would be standing to this day.

76. Base of Dam and Fore Apron. The fore apron and base of an overfall dam or weir must be of one level throughout its length, if the foundation is of any other material than rock. The foundation core walls may have to vary more or less with the surface of the river bed, which is deep in some places, and shallow in others, but the apron level should be kept at or about low water level throughout. When a horizontal wall as an overfall dam is built across a river bed it obliterates the depressions and channels in it, the discharge over the weir is the same at all points or nearly so, consequently the tendency will be to level the bed downstream by filling the hollows and denuding the higher parts.

Under these conditions it is evidently sheer folly to step up the apron to coincide with the section of the river bed, as the higher parts of the bed are bound to be in time washed out by the falling water and deposited in the deeper channels, and portions of the dam may easily be undermined. This actually occured in one case.

77. Section of Spillway of St. Maurice River Dam. Fig. 58 is a section of the spillway portion of the reinforced bulkhead gravity dam, illustrated in Fig. 44. Owing to the absence of the heavy crest of Fig. 44, the back of the spillway profile is provided with

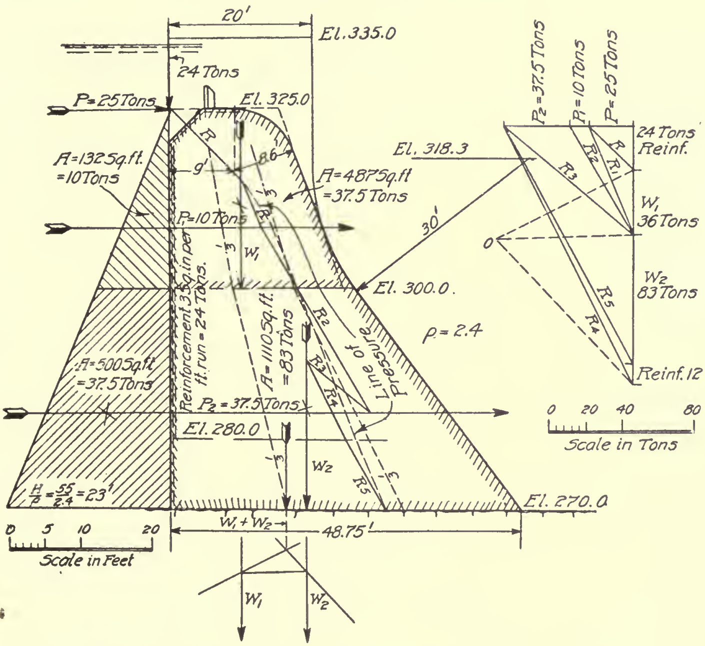
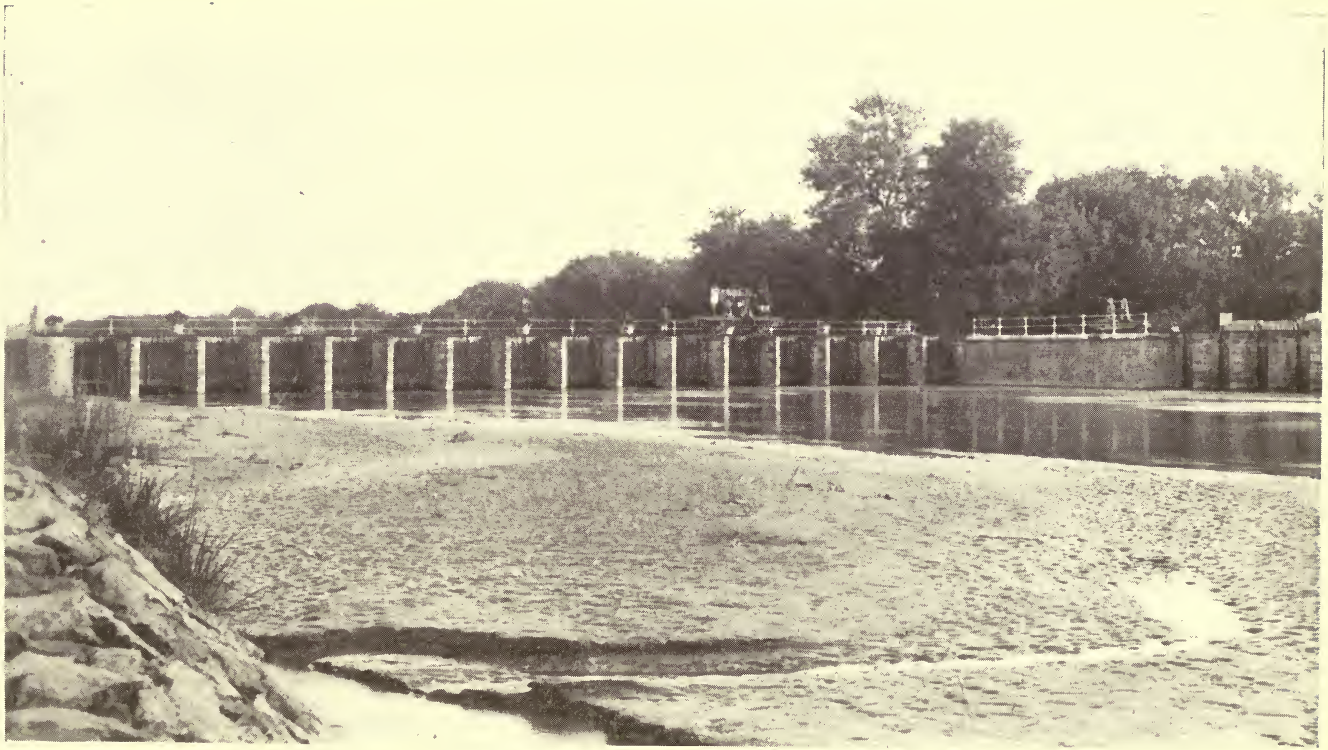


Fig. 58. Diagram Showing Profile of Spillway Portion of Saint Maurice River Dam (See Fig. 44)

double the amount of reinforcement shown in the former example. One half, viz, 1½ inches, extends right down to the base, while the other half stops short at *El* 280. This is arranged for in the stress diagrams in the same way as explained in section 55, R_5 being the final resultant on the base. The line of pressure falls slightly outside the middle third in the upper half of the section. The effect would be to increase the tension in the reinforcement somewhat above the limit of 8 tons per square inch. The adoption of a trapezoidal profile, would, it is deemed, be an improvement in this case as well as in the former.



OKHLA WEIR ON THE JUMNA RIVER, INDIA
The view is taken from the sand bar downstream

DAMS AND WEIRS

PART II

ARCHED DAMS

78. General Characteristics. In this type, the whole dam, being arched in plan, is supposed to be in the statical condition of an arch under pressure. As, however, the base is immovably fixed to the foundations by the frictional resistance due to the weight of the structure, the lowest portion of the dam cannot possess full freedom of motion nor elasticity, and consequently must act more or less as a gravity dam subject to oblique pressure.

However this may be, experience has conclusively proved that if the profile be designed on the supposition that the whole is an elastic arch, this conflict of stresses near the base can be neglected by the practical man. The probability is that both actions take place, true arch action at the crest, gradually merging into transverse stress near the base; the result being that the safety of the dam is enhanced by the combination of tangential and vertical stresses on two planes.

In this type of structure, the weight of the arch itself is conveyed to the base, producing stress on a horizontal plane, while the water pressure normal to the extrados, radial in direction, is transmitted through the arch rings to the abutments. The pressure is, therefore, distributed along the whole line of contact of the dam with the sides as well as the ground. In a gravity dam, on the other hand, the whole pressure is concentrated on the horizontal base.

Arch Stress. The average unit stress developed by the water pressure is expressed by the formula

$$s_1 = \frac{RHw}{b} \qquad \text{“Short” Formula} \quad \mathbf{(21)}$$

$$\therefore$$

$$b = \frac{RHw}{s_1} \qquad \text{“Short” Formula} \quad \mathbf{(21a)}$$

in which R is the radius of the extrados, sometimes measured to the center of the crest, H the depth of the lamina, b its width, and w the unit weight of water or $\frac{1}{32}$ ton. Into this formula ρ, the specific gravity of the material in the arch, does not enter. This simple formula answers well for all arched dams of moderate base width. When, however, the base width is considerable, as, say, in the case of the Pathfinder dam, the use of a longer formula giving the maximum stress (s) is to be preferred. This formula is derived from the same principle affecting the relations of s and s_1, or of the maximum and average stresses already referred to in Part I on "Gravity Dams". The expression is as follows, r being the radius of the intrados:

$$s = s_1 \frac{2R}{(R+r)} = \frac{RHw}{b} \times \frac{2R}{R+r}$$

or in terms of R and b

$$s = \frac{2Hw}{\frac{b}{R}\left(2 - \frac{b}{R}\right)} \qquad \text{"Long" Formula} \quad \mathbf{(22)}$$

also

$$b = R\left(1 - \sqrt{1 - \frac{2Hw}{s}}\right) \qquad \text{"Long" Formula} \quad \mathbf{(22a)}$$

79. Theoretical and Practical Profiles. In a manner similar to gravity dams, the theoretical profile suitable for an arched dam is a triangle having its apex at the extreme water level, its base width being dependent on the prescribed limiting pressure. Successful examples have proved that a very high value for s, the maximum stress, can be adopted with safety. If it were not for this, the profitable use of arched dams would be restricted within the narrow limits of a short admissible radius, as with a low limit pressure the section would equal that of a gravity dam.

The practical profile is a trapezoid, a narrow crest being necessary. The water pressure area acting on an arched dam, is naturally similar to that in a gravity dam, the difference being, however, that there is no overturning moment when reverse pressure occurs as in a weir. The difference or unbalanced pressure acting at any point is simply the difference of the direct and the reverse forces. The areas of pressure on both sides, therefore, vary with the squares of their respective depths.

The water pressure on an arch acts normally to the surface of its back and is radial in direction; consequently the true line of pressure in the arch ring corresponds with the curvature of the arch and has no tendency to depart from this condition. There is, therefore, no such tendency to rupture as is the case in a horizontal circular arch subjected to vertical rather than radial pressure. This property conduces largely to the stability of an arch under liquid pressure. This condition is not strictly applicable in its entirety to the case of a segment of a circle held rigidly between abutments as the arch is then partly in the position of a beam. The complication of stress involved is, however, too abstruse for practical consideration.

80. Correct Profile. As we have already seen, the correct profile of the arched dam is a triangle modified into a trapezoid with a narrow crest. With regard to arch stresses, the most favorable outline is that with the back of the extrados vertical. The reason for this is that the vertical stress due to the weight of the arch, although it acts on a different plane from the tangential stresses in the arch ring, still has a definable influence on the maximum induced stress in the arch ring. The vertical pressure produces a transverse expansion which may be expressed as $W \times E \times m$, in which E is the coefficient of elasticity of the material and m that of transverse dilation. This tends, when the extrados is vertical, to diminish the maximum stress in the section; whereas when the intrados is vertical and the back inclined, the modification of the distribution of pressure is unfavorable, the maximum stress being augmented. When the trapezoidal profile is equiangular, an intermediate or neutral condition exists. A profile with vertical extrados should, therefore, be adopted whenever practicable.

In very high dams, however, the pressure on the horizontal plane of the base due to the weight of the structure, becomes so great as to even exceed that in the arch ring; consequently it is necessary to adopt an equiangular profile in order to bring the center of pressure at, or near to, the center of the base, so as to reduce the ratio of maximum pressure to average pressure to a minimum. As stated in the previous section, when a vertical through the center of gravity of the profile passes through the center of the base, the maximum pressure equals the average, or $s = s_1$.

81. Support of Vertical Water Loads in Arched Dams. When the back of an arched dam is inclined, the weight of the water over it is supported by the base, the horizontal pressure of the water alone acting on the arch and being conveyed to the abutments. In the case of inclined arch buttress dams, however, a portion of the vertical load is carried by the arch, increasing its thrust above what is due to the horizontal water pressure alone. This is due to overhang, i. e., when the c. g. falls outside the base.

82. Crest Width. The crest width of arched dams can be safely made much less than that of gravity dams and a rule of

$$k=\frac{1}{2}\sqrt{H} \tag{23}$$

would seem to answer the purpose, unless reinforcement is used, when it can be made less.

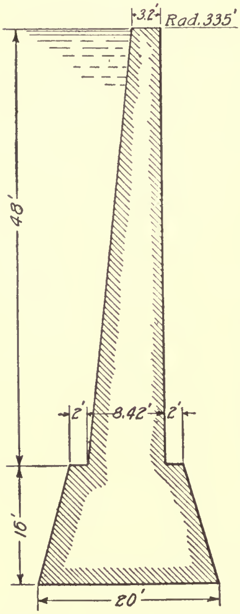


Fig. 59. Section of Old Bear Valley Dam

EXAMPLES OF ARCHED DAMS

The following actual examples of arched dams will now be given.

83. Bear Valley Dam. This small work, Fig. 59, is the most remarkable arched dam in existence and forms a valuable example of the enormous theoretical stresses which this type of vertical arch can stand. The mean radius being 335 feet according to formula (21) the unit stress will be

$$\frac{RHw}{b}=60 \text{ tons, nearly}$$

This section would be better if reversed. The actual stress is probably half this amount. This work has now been superseded by a new dam built below it, Fig. 77, section 103.

84. Pathfinder Dam. This immense work, Fig. 60, is built to a radius of 150 feet measured to the center of the crest. That, however, at the extrados of the base of the section is 186 feet and this quantity has to be used for the value of R in the long formula (22). The unit stress then works out to 18 tons, nearly. The actual stress in the lowest arch ring is undoubtedly much less, for the reason that the base must absorb so large a proportion of the thrust that very little is transmitted to the sides of the canyon. The exact determination of the proportion transmitted in the higher rings is an indeterminate problem, and the only safe method is to assume with regard to tangential arch stress that the arch stands clear of

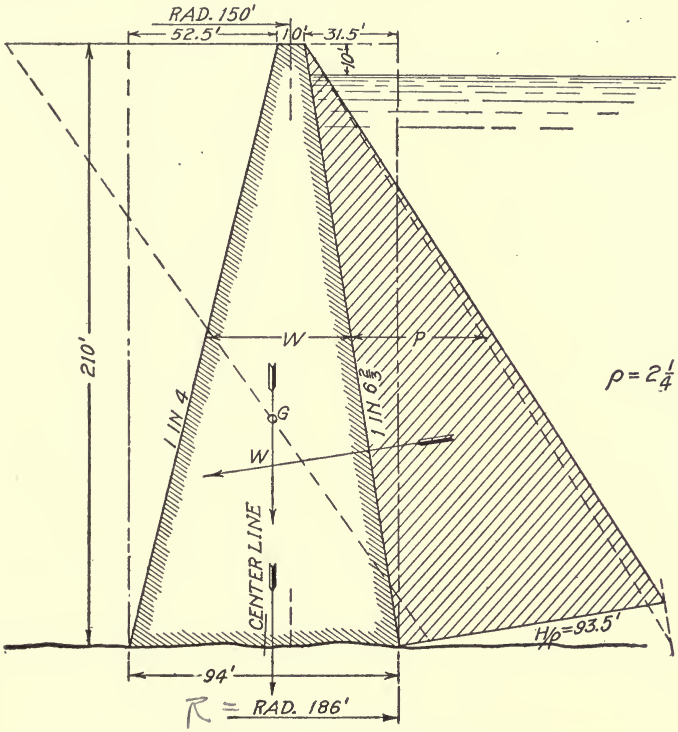


Fig. 60. Section of Pathfinder Dam

the base. This will leave a large but indeterminate factor of safety and enable the adoption of a high value for *s*, the maximum unit stress.

The profile of the dam is nearly equiangular in outline. This is necessary in so high a dam in order to bring the vertical resultants (*W*) *R. E.* and (*N*) R. F. as near the center as possible with the object of bringing the ratio of maximum to mean stress as low as possible.

The estimation of the exact positions of *W* and of *N* is made analytically as below.

There are only two areas to be considered, that of the water overlying the inclined back (v) and that of the dam itself (W). Dividing v by $2\frac{1}{4}$ (the assumed specific gravity of the material), reduces it to an equivalent area of concrete or masonry.

$$v=\frac{210\times 31.5}{2\times 2.25}=1470=103 \text{ tons}$$

$$W=\frac{104}{2}\times 210=\underline{10920}=\underline{768} \text{ tons}$$

$$\text{Total, or } N=12390=871 \text{ tons}$$

Using formula (7), Part I, the c.g. of W is 50.8 distant from the toe of the profile, then q or the distance of the incidence of W from the center point of the base is $50.8-\frac{94}{2}=3.8$.

The value of s_1, or the mean unit stress is $\frac{W}{b}$, or $\frac{768}{94}=8.1$ tons and $m=1+\frac{6q}{b}=1+\frac{6\times 3.8}{94}=1.24$; then $s=\frac{mW}{b}=1.24\times 8.1=10.1$ tons.

For Reservoir Full, to find the position of N, moments will be taken about the toe as follows

$$\begin{array}{lrl} \text{Moment of } v=103\times 83.5 & = & 8600 \\ \text{Moment of } W=\underline{768}\times 50.8 & = & \underline{39014} \\ \text{Total } N=871 & = & 47614 \end{array}$$

then $x=\frac{47614}{871}=54.6$; whence $q=54.6-\frac{94}{2}=7.6$ feet and $s_1=\frac{N}{b}=\frac{871}{94}=9.26$. By formula (9), Part I, $m=1+\frac{7.6\times 6}{94}=1.48$.

$$\therefore \qquad s=9.26\times 1.48=13.7 \text{ tons}$$

From this it is evident that the unit stress in the base, due to vertical load only, is a high figure. It could be reduced by still further inclining the back; on the contrary, if the back were vertical N would equal W. Let this latter case be considered. The distance of the c. g. of the profile from the heel will then be by formula (7a), Part I

$$x=\frac{1}{3}\left(b+\frac{a^2}{a+b}\right)=31.66$$

and the value of q will be $\frac{94}{2}-31.66=15.33$ feet

$$s_1 \text{ as before} = \frac{W}{b} = 8.1 \text{ tons}$$

Then

$$m = 1 + \frac{92}{94} = 1.98 \text{ and } s = 8.1 \times 1.98 = 16 \text{ tons}$$

This stress is greater than that of N in the previous working which proves that the forward tilt given to these high dams is necessary to reduce the maximum unit stress on the base to a reasonable limit. A more equiangular profile would give even better results.

85. Shoshone Dam. The Shoshone dam, Fig. 61, is designed on lines identical with the last example. It has the distinction of being the highest dam in the world but has recently lost this

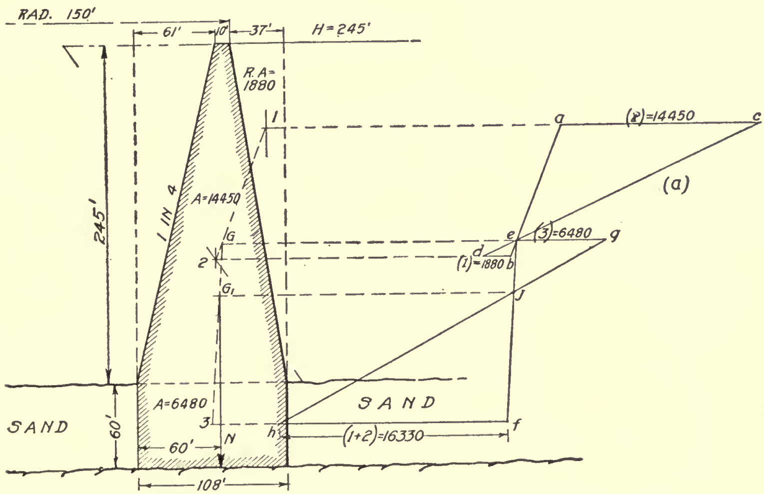


Fig. 61. Profile and Force Diagram for Shoshone Dam

preëminence, as the Arrow Rock, quite lately constructed, Fig. 37, Part I, is actually 35 feet higher. This work is also in the United States. The incidents of the resultants Reservoir Empty and Reservoir Full, which will be explained later, have been shown graphically, and the analytical computation is given below. The vertical forces taken from left to right are (1), area 6480; (2), 14,450; (3), water overlying back, reduced area 1880; total 22,810.

Taking moments about the toe of the base, the distance of (1) is 54 feet, of (2) calculated by formula (**7**), Part I, is 58.3, and of (3) is 95 feet, roughly.

Then $(6480 \times 54) + (14450 \times 58.3) + (1880 \times 95) = 22810 \times x$.

$\therefore \quad x = 60$ feet, nearly.

The value of q for N then is $60 - \frac{108}{2} = 6$ feet.

Now $s_1 = \frac{N}{b} = \frac{22810}{108} = 211$, and by formula (**9**), Part I, $s = 211 \times \frac{(108+36)}{108} = 281$ square feet $= \frac{281 \times 2.4}{32} = 21$ tons, nearly.

The maximum arch unit stress by formula (**22**) is as follows: the radius of the extrados of the base being 197 feet the fraction $\frac{b}{R} = \frac{108}{197} = .55$ and $H = 245$ therefore $s = \dfrac{2HW}{\frac{b}{R} \times \left(2 - \frac{b}{R}\right)} = \dfrac{2 \times 245 \times 1}{.55 \times 1.45 \times 32} = \frac{490}{25.5} = 19.2$ tons.

Below the level 60 ft. above base, the stress on the arch does not increase. The arch stress is less than that due to vertical pressure N. This base should undoubtedly have been widened, the battered faces being carried down to the base, not cut off by vertical lines at the 60-foot level.

Center of Pressure—New Graphical Method. In order to find the center of pressure in a case like Fig. 61, where the lines of forces (1) and (2) are close together, the ordinary method of using a force and funicular polygon involves crowding of the lines so that accuracy is difficult to attain. Another method now will be explained which is on the same principle as that of the intersection of cross lines used for finding the c. g. of a trapezoid.

In Fig. 61, first the c.g.'s of the three forces are found (1) the water pressure area divided by ρ or 2.4 which equals 1880 square feet, (2) the upper trapezoidal part of the dam area 14,450, and (3) the lower rectangular area 6480. Then (1) is joined to (2) and this line projected on one side in any location as at b in Fig. 61a.

From a, ac is set off horizontally equal to (2) or 14,450 and from b, bd is drawn equal to (1) or 1880; cd is then drawn and its intersection with ab at e gives the position of the resultant 1-2, which can now be projected on the profile at G. To obtain the resultant of the components (1-2) with (3) the line G-3 is drawn on the profile and a parallel to it drawn from e on Fig. 61a, intersecting the horizontal through (3) at f. From e, eg is laid off horizontally equal to (3) or 6480 and from f, fh equal to $(1+2)$ or $1880+14{,}450=16{,}330$. hg is then drawn and its intersection with ef at j is the centroid of the three forces, which projected on the profile to G_1 on the line G-3 gives the location of the vertical resultant of $1+2+3$.

86. Sweetwater Dam. The profile of the Sweetwater dam in California is given in Fig. 62. The original crest of the dam

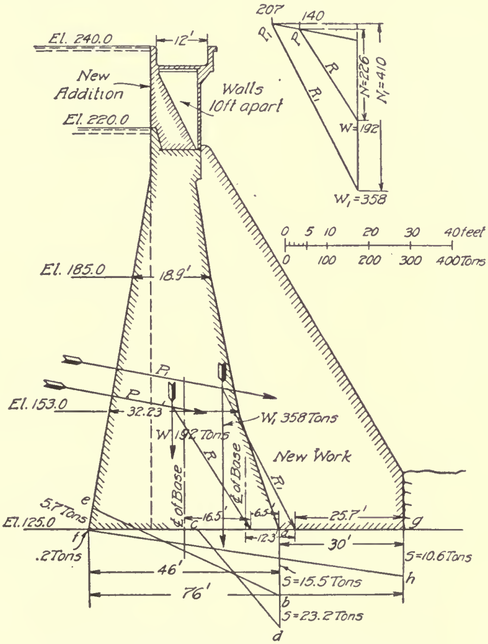


Fig. 62. Graphical Analysis of Sweetwater Dam, California

was at *El.* 220, or 95 feet above the base. Under these conditions the dam depended for its stability on its arched plan. If considered as a gravity dam with allowable tension at the heel, the vertical pressure area is the triangle abe, here $q=16.5$ and m works out to 3.15. $N=226$ tons and $b=46$ feet whence $s=\frac{mN}{b}=\frac{3.15\times 226}{46}=15.5$ tons which is set down from a to b.

The tension at the heel $= s_2-\frac{2N}{b}=15.5-9.82=5.7$ tons

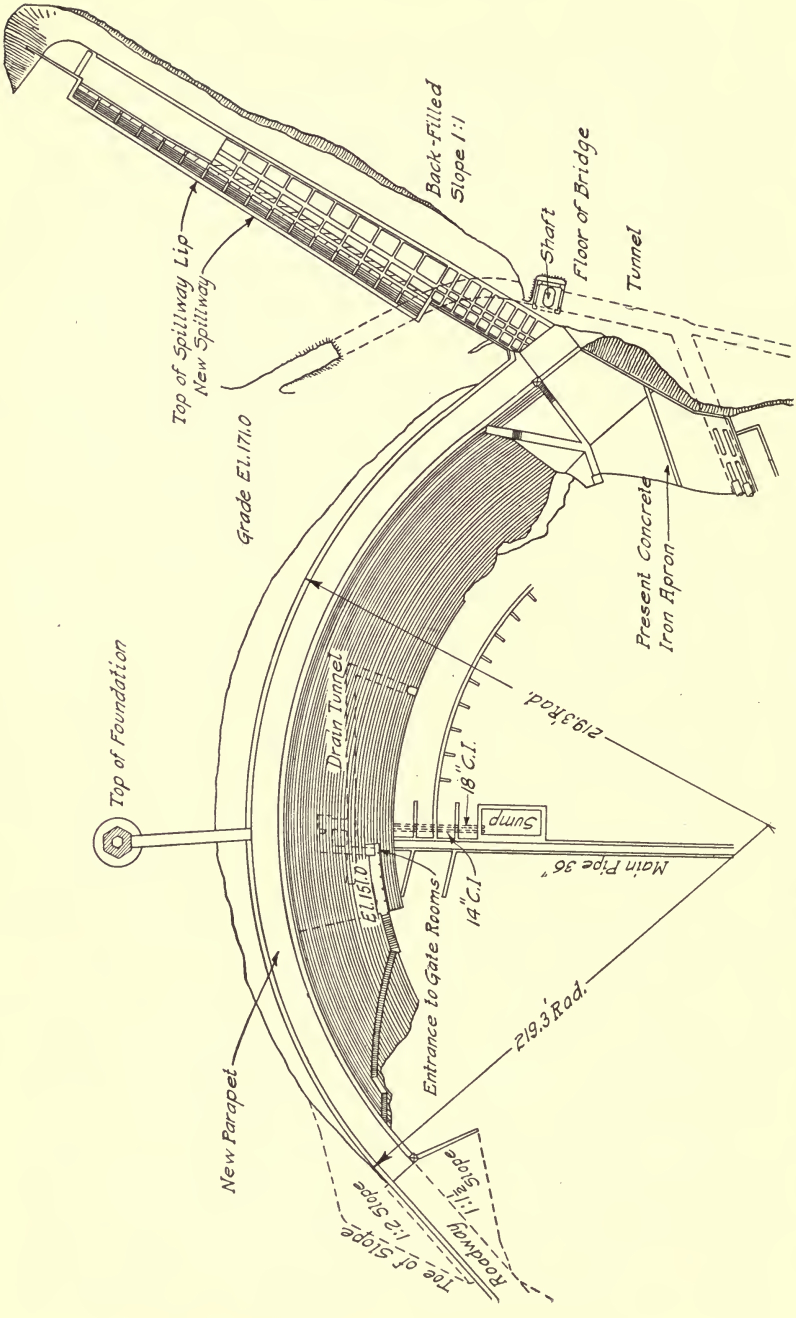


Fig. 63. Location Plan of Sweetwater Dam, Showing Alterations

Value of S When Heel Is Unable to Take Tention. If the heel is unable to take tension, the pressure triangle will then be *adc* in which $ac=3$ times the distance of the incidence of R from the toe, or $3\times6.5=19.5$ feet and s is obtained by the following formula

$$s=\frac{4}{3}\times\frac{N \text{ or } W}{b-2q} \qquad \textbf{(24)}$$

here $s=\dfrac{4\times226}{3\times46-33}=23.2$ tons

This dam has lately been raised to *El* 240, or by 20, feet and by the addition of a mass of concrete at the rear transformed into a gravity dam. The resultant due to this addition is R_1 on the diagram. s works out to 10.6 tons and there is no tension at the heel. Any bond between the new wall and the old has been studiously avoided. The new work is reinforced with cross bars and the rear mass tied into the superstructure. Fig. 63 is a plan of the dam as altered.

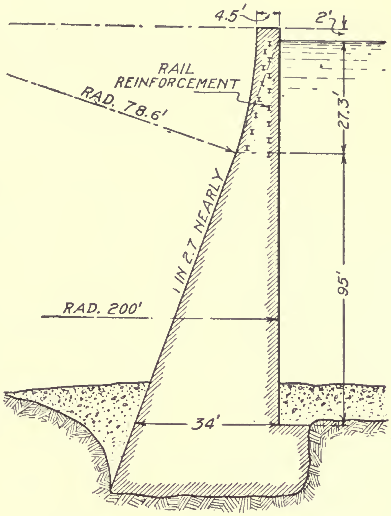


Fig. 64. Profile of Barossa Dam

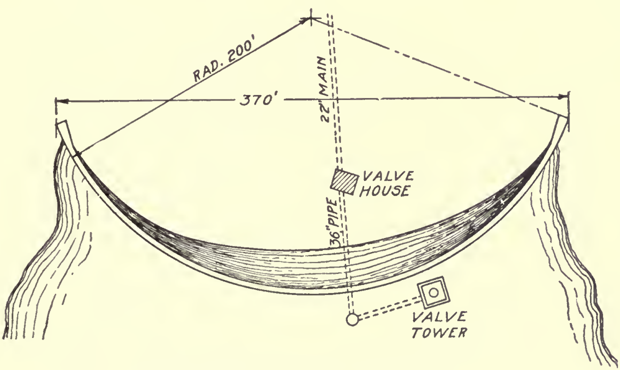


Fig. 65. Site Plan of Barossa Dam

87. Barossa Dam. This dam, Fig. 64, is an Australian work, and although of quite moderate dimensions is a model of good and bold design.

The back is vertical and the fore batter is nearly 1 in 2.7. The outline is not trapezoidal but pentagonal, viz, a square crest imposed on a triangle, the face joined with the hypothenuse of the latter by a curve. The crest is slender, being only $4\frac{1}{2}$ feet wide, but is strengthened by rows of 40-pound iron rails, fished together, built into the concrete. The maximum arch stress works out to $17\frac{1}{4}$ tons, the corresponding vertical stress on base to $6\frac{3}{4}$ tons. Fig. 65 is a site plan of the work.

88. Lithgow Dam. Another example very similar to the last is the Lithgow dam, No. 2, Fig. 66. The arch stress in this works out by the short formula to nearly 13 tons; the radius is only 100 feet, the vertical stress works out to 7 tons.

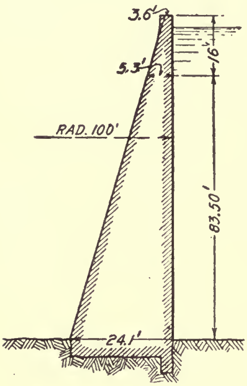


Fig. 66. Profile of Lithgow Dam

Arched dams abut either on the solid rocky banks of a canyon or else on the end of a gravity dam. In cases where a narrow deep central channel occurs in a river, this portion can advantageously be closed by an arched dam, while the flanks on which the arch abuts can be gravity dams aligned tangential to the arch at each end. The dam will thus consist of a central arch with two inclined straight continuations. The plan of the Roosevelt dam, Fig. 26, Part I, will give an idea of this class of work.

89. Burrin Juick Subsidiary Dam. In Fig. 67 is shown the profile of a temporary reinforced arched dam for domestic water supply at Barren Jack, or Burrin Juick, Australia. The reinforcement consists of iron rails. The unit arch pressure at the base works out to 21 tons, nearly. Reinforcement of permanent dams down to the base is not desirable, as the metal may corrode in time and cause failure, although the possibility is often stoutly denied. The main Burrin Juick dam is given in Part I, Fig. 36.

90. Dams with Variable Radii. The use of dams of the type just described, is generally confined, as previously noted, to narrow gorges with steep sloping sides in which the length of the dam at the level of the bed of the canyon is but a small proportion of that at the crest. The radius of curvature is usually fixed with regard to the length of chord at the latter level, consequently at the deepest level, the curvature will be so slight that arch action will be absent and the lower part of the dam will be subject to beam stresses, i.e., to tension as well as compression. In order to obviate this, in some recent examples the radius of curvature at the base is made less than that at the crest, and all the way up, the angle subtending the chord of the arc, which is variable in length retains the same measure throughout. This involves a change in the radius corresponding to the variable span of the arch. The further advantage is obtained, of reduction in the unit stress in the arch ring and in rendering the stress more uniform throughout. In very high dams, however, the base width cannot be much reduced as otherwise the limit stress due to the vertical loading will be exceeded. This arrangement of varying radii is somewhat similar to that used in the differential multiple arch given later.

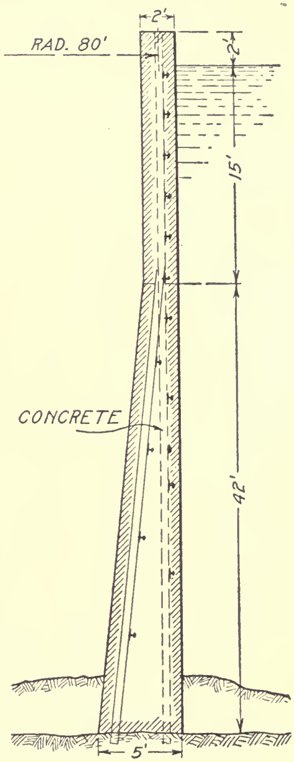


Fig. 67. Profile of Burrin Juick Subsidiary Dam

MULTIPLE ARCH OR HOLLOW ARCH BUTTRESS DAMS

91. Multiple Arch Generally More Useful Than Single Arch Dams. It is evident that a dam which consists of a single vertical arch is suitable only for a narrow gorge with rock sides on which the arch can abut, as well as a rock bed; consequently its use is strictly limited to sites where such conditions are obtainable. A rock foundation is also essential for gravity dams, the unit compression on the base of which is too high for any material other than rock.

The advantages inherent in the vertical arch, which are considerable, can however be retained by use of the so-termed multiple or scallop arched dam. This consists of a series of vertical or inclined arches, semicircular or segmental on plan, the thrust of which is carried by buttresses. The arrangement is, in fact, identical with that of a masonry arched bridge. If the latter be considered as turned over on its side, the piers will represent the buttresses. In the case of a wide river crossing, with a bed of clay, boulders, or sand, the hollow buttressed and slab buttressed dams are the only ones that can well be employed with safety. The wide spread that can be given to the base of the structure in these two types enables the unit pressure on the base to be brought as low as from 2 to 4 tons per square foot.

As has already been noticed in section 78, the arch is peculiarly well suited for economical construction. This is due to the fact that the liquid pressure to which the arch is subjected is normal to the surface and radial in direction. The pressure lines in the interior of the arch ring correspond with its curvature and consequently the arch can only be in compression; thus steel reinforcement is unnecessary except in a small degree near the crest in order to care for temperature stresses. In slab dams, on the other hand, the deck is composed of flat slabs which have to be heavily reinforced. The spacing of the buttresses for slabs is limited to 15 to 20 feet, whereas in hollow arch dams there is practically no limit to the spans which may be adopted. Another point is, that the extreme compressive fiber stress on the concrete in deck slabs is limited to five hundred to six hundred and fifty pounds per square inch; in an arch, on the other hand, the whole section is in compression which is thereby spread over a much greater area. For the reasons above given the arch type now under consideration should be a cheaper and more scientific construction than the slab type in spite of the higher cost of forms.

92. Mir Alam Dam. The first example given is that of the Mir Alam tank dam, Fig. 68. This remarkable pioneer structure was built about the year 1806, by a French engineer in the service of H. H. the Nizam of Hyderabad in Southern India. The alignment of the dam is on a wide curve and it consists of a series of vertical semicircular arches of various spans which abut on short buttress piers, Fig. 69. The spans vary from 83 to 138 feet, the one in Fig. 68 being of 122 feet. The maximum height is 33 feet. Water has been known to overtop the crest. The length of the dam is over 3000 feet.

On account of the inequality of the spans, the adoption of the semicircular form of arch is evidently a most judicious measure, for the reason that an arch of this form under liquid pressure exerts no lateral thrust at the springing. The water pressure being radial in direction, cross pressure in the half arches in the line of the springing is balanced and in equilibrium. Whatever thrust is exerted is not in the direction of the axis of the dam but that of the buttress piers. On the other hand, if the arches were segmental in outline the terminal thrust is intermediate between the two axes, and when resolved in two directions one component acts along the axis of

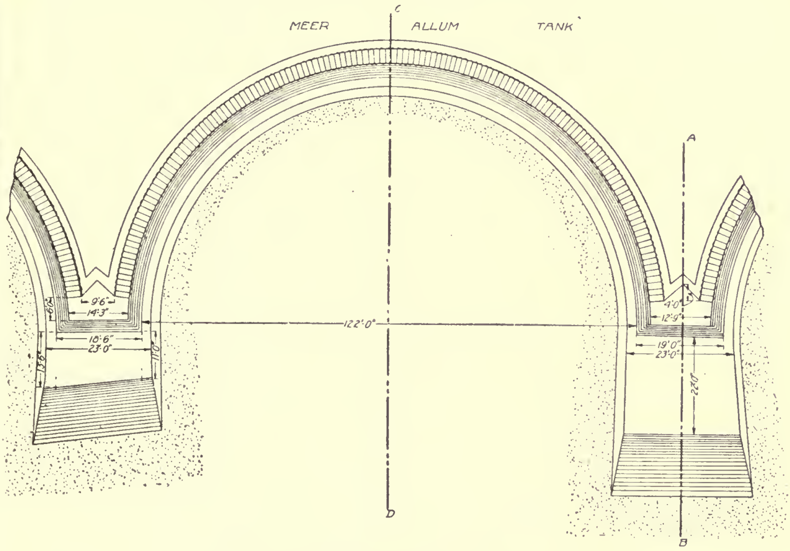


Fig. 68. Plan of One Arch of Mir Alam Dam
This remarkable pioneer dam was built in 1806, and consisted of 21 such arches.

the dam. This has to be met, either by the abutment, if it is an end span, or else by the corresponding thrust of the adjoining half arch. The other component is carried by the buttress; therefore, if segmental arches are used, the spans should be equal in order to avoid inequality of thrust. Longer buttresses will also be requisite. The whole of this work is built of coursed rubble masonry in lime mortar; the unit stress in the arch ring at the base, using the short formula (**21**), $\frac{(RHw)}{b}$ works out to $\frac{68\times33\times1}{14\times32}=5$ tons, nearly. The dam, therefore, forms an economical design.

The buttress piers are shown in section in Fig. 70, the section being taken through *AB* of Fig. 68. In this work the buttress piers are very short, projecting only 25 feet beyond the spring line of the arches, and being altogether only 35 feet long. This length and the corresponding weight would clearly be inadequate to with-

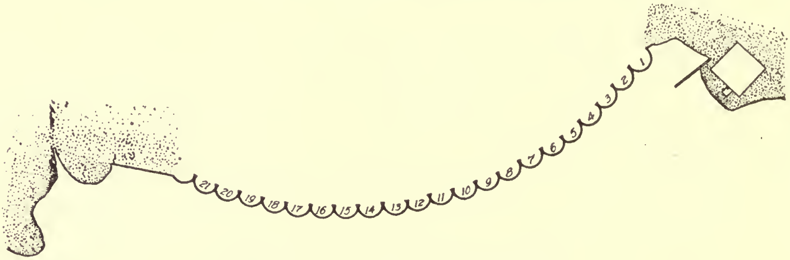


Fig. 69. Plan of Entire Mir Alam Dam

stand the immense horizontal thrust which is equivalent to $\frac{H^2}{2}lw = \frac{33^2 \times 1 \times 146}{2 \times 32} = 2500$ tons, nearly.

It is evident that if the buttress pier slides or overturns, the arches behind it must follow, for which reason the two half arches and the buttress pier cannot be considered as separate entities but as actually forming one whole, and consequently the effective length of the base must extend from the toe of the buttress right back to the extrados of the two adjoining arches. At, or a little in the

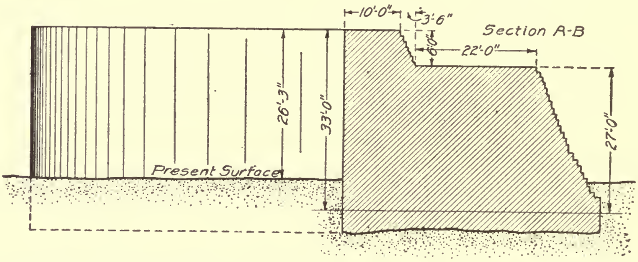


Fig. 70. Section of Buttress Pier of Mir Alam Dam Taken through *AB* of Fig. 68

rear of the spring line, the base is split up into two forked curved continuations. The weight of these arms, i.e., of the adjoining half arches, has consequently to be included with that of the buttress proper when the stability of the structure against overturning or sliding is estimated.

93. Stresses in Buttress. In the transverse section, Fig. 71, taken through *CD* of Fig. 68, the graphical calculations establish the fact that the resultant line R intersects the base, thus lengthened, at a point short of its center; the direction of the resultant R is also satisfactory as regards the angle of frictional resistance.

R_1 is the resultant on the supposition that the buttress is nonexistent. Its incidence on the base proves that the arch is stable without the buttress, which is therefore actually superfluous. With regard to sliding on the base, $P=2500$ and $W=6828$ tons.

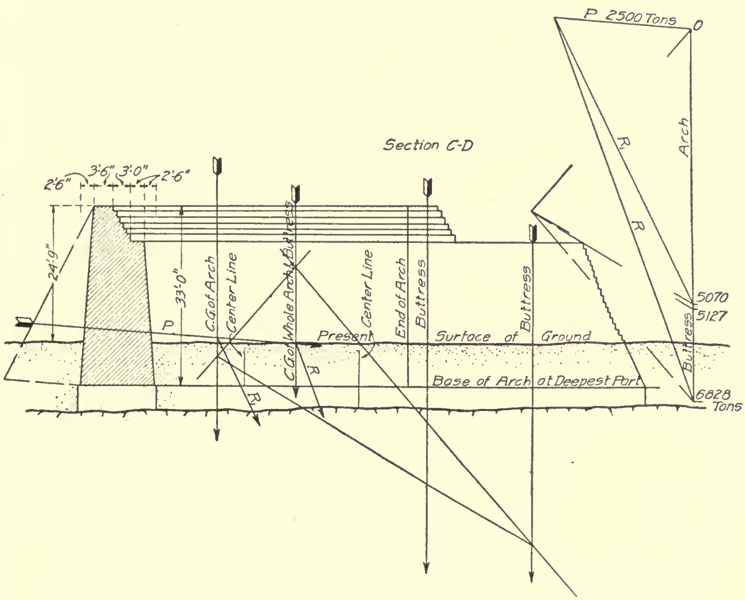


Fig. 71. Transverse Section of Mir Alam Dam Taken through *CD*, Fig. 68

The coefficient of friction being .7 the factor of safety against sliding is nearly 2. If the arch were altered on plan from a semicircle to a segment of a circle, the radius would of necessity be increased, and the stress with it; a thicker arch would, therefore, be required. This would not quite compensate for the reduced length of arch, but on the other hand, owing to the crown being depressed, the effective base width would be reduced and would have to be made good by lengthening the buttress piers. What particular disposition of arch and buttress would be the most economical is a matter which could only be worked out by means of a number of trial designs. The ratio of versed sine to span should vary from $\frac{1}{4}$ to $\frac{1}{2}$. Arcs subtending from 135 to 120 degrees are stated to be the most economical in material.

94. Belubula Dam. There are not as yet very many modern examples of arch buttress dams, but each year increases their num-

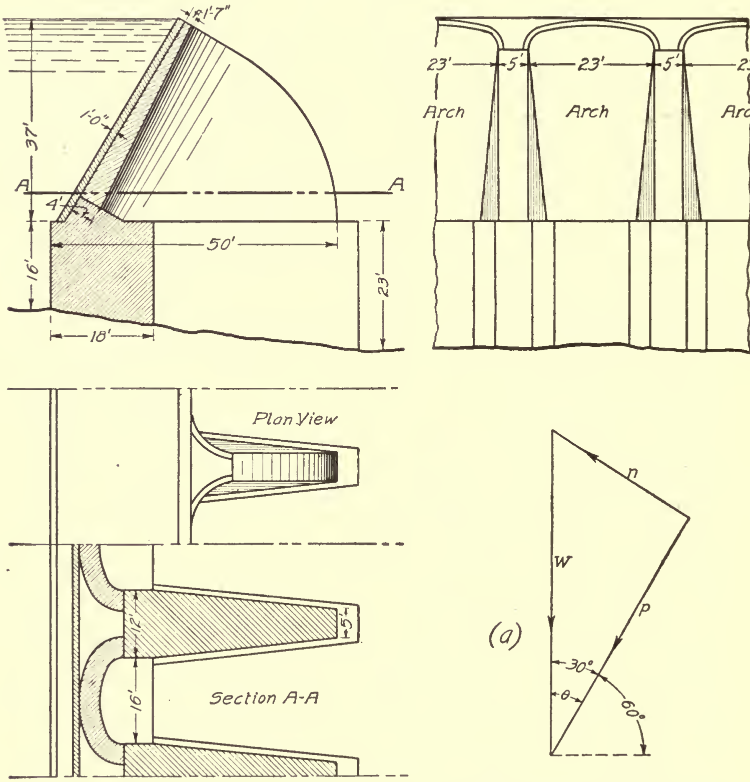


Fig. 72. Profile Sections and Force Diagram for Belubula Dam, New South Wales

ber. The Mir Alam dam has remained resting on its laurels without a rival for over 100 years, but the time has come when this type is being largely adopted. Fig. 72 shows an early example of a segmental panel arch dam. It is the Belubula dam in New South Wales. The arch crest is 37 feet above the base, very nearly the same as in the last example. The arches, which are inclined 60 degrees to the horizontal are built on a high solid platform which obliterates inequalities in the rock foundation. This platform is 16 to 23 feet high, so that the total height of the dam is over 50 feet. The spans are 16 feet, with buttresses 12 feet wide at the spring line, tapering to a thickness of 5 feet at the toe; they are 40 feet long. The buttress piers, which form quadrants of a circle in elevation, diminish in thickness by steps from the base up, these insets corresponding with similar ones in the arch itself. These steps are not shown in the drawing; the arch also is drawn as if in one straight batter. The arches are elliptical in form, and the spandrels are filled up flush with the crown, presenting a flat surface toward the water.

Some of the features of this design are open to objection: *First*, the filling in of the arch spandrels entirely abrogates the advantage accruing to arches under liquid pressure. The direction of the water pressure in this case is not radial but normal to the rear slope, thus exactly reproducing the statical condition of a horizontal arch bridge. The pressure, therefore, increases from the crown to the haunches and is parabolic, not circular, in curvature. The arches should have been circular, not elliptical, and the spandrels left empty to allow of a radial pressure which partly balances itself. *Second*, the stepping in of the intrados of the arch complicates the construction. A plain batter would be easier to build, particularly in concrete. *Third*, the tapering of the buttress piers toward the toe is quite indefensible; the stress does not decrease but with the center of pressure at the center of the base as in this case, the stress will be uniform throughout.

95. Inclination of Arch to Vertical. The inclination of the axis of the arch to the vertical is generally a desirable, in fact, a necessary feature when segmental arched panels are used; the weight of water carried is of value in depressing the final resultant line to a suitable angle for resistance to shearing stress. As noted in section 90, the weight of the water overlying the arch does not increase the unit stress in the arch ring. Consequently, any inclination of axis can be adopted without in any way increasing the unit stresses due to the water pressure.

When an arch is vertical it is clear that the water pressure is all conveyed to the abutments and the weight of the arch to its base. When an arch lies horizontally under water pressure both the weight of the water and that of the arch itself are conveyed to the abutment; when in an intermediate position part of the weight of the arch is carried to the base and part to the abutments.

With regard to water pressure, the thrust being normal to the extrados of the arch the whole is carried by the abutments. In the case of arches which do not overreach their base the weight of water overlying the inclined back is conveyed to the base. In any case the unit stress in the arch $\frac{(RHw)}{b}$ cannot exceed that due to horizontal thrust. The total water pressure is greater with an inclined back, as the length of surface acted on is increased. In the diagram, Fig. 72a, the vertical load line W represents the weight of one unit or one cubic foot of the arch ring which is equal to $w\rho$. This force is resolved in two directions, one p, parallel to the axis of the arch, and the other n, normal to the former. The force $n = W \sin\theta$, θ being the inclination of the arch axis to the vertical and $p = W \cos\theta$. The unit stress developed by the radial force n is similar to that produced by the water pressure which is also radial in direction and is $R_1 n$; but R_1, the radius in this case, is the mean radius, the pressure being internal, not external. The unit stress s_1 will then be

$$s_1 = R_1 w\rho \sin\theta \qquad \textbf{(25)}$$

When θ is 30° $\sin\theta = \frac{1}{2}$; when 45°, $\sin\theta = \frac{2}{3}$.

It will easily be understood that this unit stress due to n does not accumulate, but is the same at the first foot depth of the arch as it is at the bottom; the width of the lamina also does not affect it. However, the component p does accumulate, and the expression $w\rho \cos\theta$ should be multiplied by the inclined height H_1, lying above the base under consideration. As $H_1 = H \sec\theta$, the unit compressive stress at the base will be $\frac{Hw\rho b_1}{b}$, in which b_1 is the mean width of the arch. If the arch were a rectangle, not a trapezoid, s would equal $Hw\rho$ simply.

96. Ogden Dam. The Ogden dam, the profile and sectional details of which are shown in Fig. 73, is a notable example of the arch and buttress type. Its height is 100 feet. The inclination of the arches is less than ½ to 1, or about 25 degrees to the vertical.

The profile of the buttress is equinangular except for a small outthrow of the toe. On the whole it must be pronounced a good design, but could be improved in several particulars. For example, the arch is unnecessarily thick at the crest, and could well be reduced from 6 to 2 feet, thus effecting considerable economy. The designers were evidently afraid of the concrete in the arch leaking, and so overlaid the extrados with steel plates. A greater thickness of arch causing it to possess less liability to percolation under pressure, could have been provided by increasing the span and radius of the

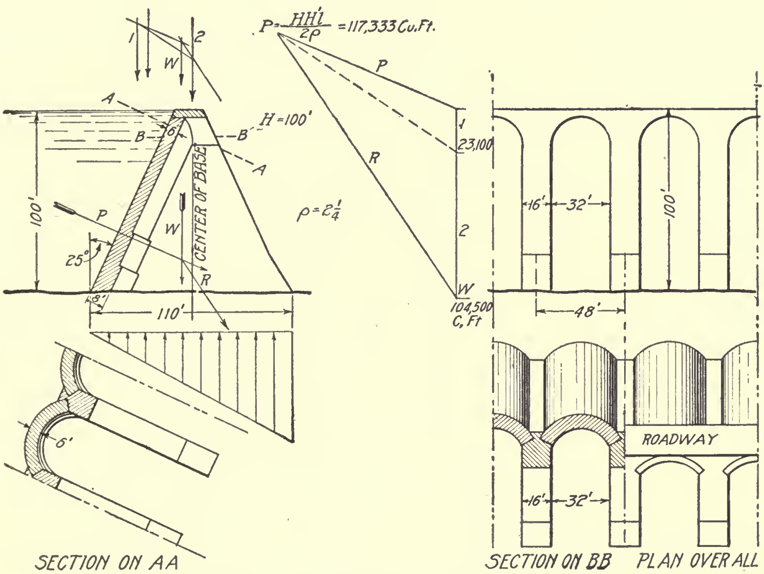


Fig. 73. Profile and Sections of Ogden Dam

arches. The design consequently would be improved by adopting larger spans, say 100 feet; buttresses, say, 25 feet thick, their length being dependent on the width of base required to provide sufficient moment of resistance; and further, the inclination of the arches might require increasing to bring the center of pressure at, or close to the center of the buttress. The finish of the crest by another arch forming a roadway is an excellent arrangement, and is well suited for a bulkhead dam; for an overfall, on the other hand, the curved crest is preferable on account of the increased length of overflow provided. The stress diagram shows that the value of the vertical load N is 155,000 cubic feet or 10,598 tons, ρ being taken at $2\frac{1}{4}$. The incidence of R on the base, is 5 feet from the center, whence $q=5$, and by formula (**9**), Part I

$$s=\frac{N}{A}\times\left(1+\frac{6q}{b}\right)=\frac{10598}{110\times16}\times\frac{140}{110}=8.91 \text{ tons}$$

the dimensions of A, the area of the base, being 110×16 feet. The pressure on the arch ring at the base by the short formula works out to $\frac{24\times100}{8\times32}=9.4$ tons.

The contents of the dam per foot run amounts to $\frac{104{,}500}{48}=$ 2,177 cubic feet; that of a gravity dam would be about 3,500 cubic feet per foot run, making a saving in favor of the arched type of nearly 30 per cent. With a better disposition of the parts as indicated above, the saving would be increased to 40 or 50 per cent. Actually the saving amounted to only 12 per cent; this was owing to the steel covering which, as we have seen, could have been dispensed with.

97. Design for Multiple Arch Dam. Fig. 74 is a design for a segmental arch panel dam, or rather, weir. The height of the crest is 64 feet above base with 5 feet of water passing over; the apex of the triangle of water pressure will then be 69 feet above the base. The inclination given the axis, which is coincident with that of the spring line and the intrados, is 60 degrees with the horizon.

In designing such a work, the following salient points first require consideration.

(1) *Width of Span.* This, it is deemed for economical reasons should be not less than the height of crest unless the state of the foundation requires a low unit stress. In the Mir Alam dam the span is over four times the depth of water upheld. In the present case it will be made the same, that is, 64 feet.

(2) *Thickness of Buttress Piers.* As with bridge piers, the width should be at least sufficient to accommodate the skew-backs of the two arches; a width of 12 feet or about $\frac{1}{5}$ span will effect this.

(3) *Radius and Versed Sine.* The radius will be made 40 feet; this allows a versed sine of $\frac{1}{4}$ span, or 16 feet, which is considered to be about the flattest proportion to afford a good curva-

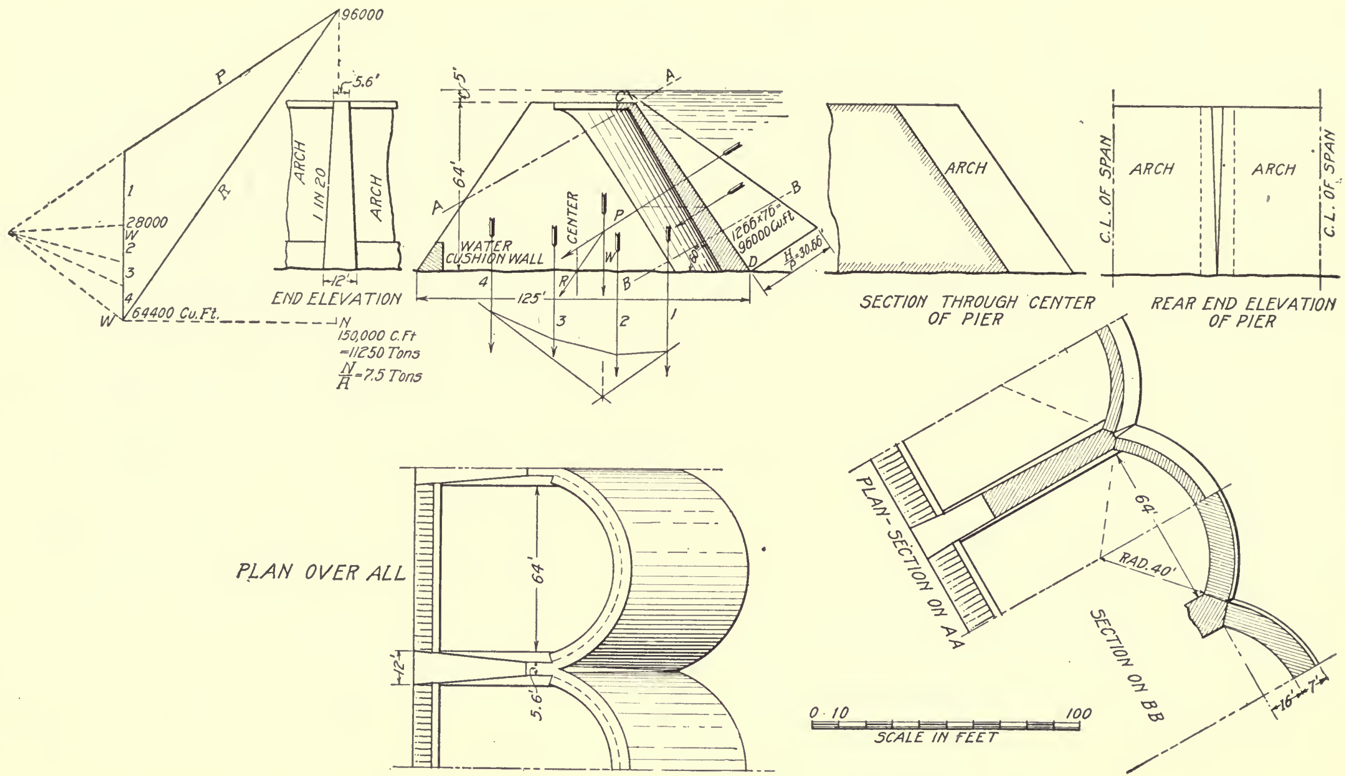


Fig. 74. Profile Plans and Sections of Segmental Arch Panel Dam

ture, the greater the length of the arc, the more its condition will approximate to that of a circular arch, under liquid pressure.

(4) *Thickness of Arch.* This must first be assumed, as its thickness depends on R, the radius of the extrados, as well as on the value assigned to s_1, the limiting pressure. This latter will be fixed at below 15 tons, a value by no means excessive for arches under liquid pressure. With a base width of 7 feet, the radius of the extrados will be 47 feet. The base will be considered, not at the extreme depth of 64 feet below crest, but at the point marked D, where a line normal to the base of the inclined intrados cuts the extrados of the arch. H will, therefore, be 60 feet, allowing for the reverse pressure. The stress due to the water pressure, using the short formula (**21**), section 78, will be

$$s_1 = \frac{RHw}{b} = \frac{47 \times 60 \times 1}{7 \times 32} = 12.6 \text{ tons}$$

To this must be added that due to the weight of the arch ring from formula (**25**), $s_1 = R_1 w\rho \sin\theta$ (the angle θ being 30° and its sine $= \frac{1}{2}$), which in figures will be $\frac{43.5 \times 3}{2 \times 40} = 1.6$ tons, the total stress being a trifle over 14 tons. The 7-foot base width will then be adopted. ρ is taken as 2.4 and $w\rho = \frac{3}{40}$ ton. The depth of water producing this pressure is taken as 60, not as 65, feet which is $(H+d)$, the reason being that the reverse pressure due to the tail water, which must be at least level with the water cushion bar wall, will reduce the effective depth to 60 feet, during flood conditions.

98. Reverse Water Pressure. The influence of the reverse pressure of water is much more considerable when hydrostatic pressure alone is exerted than is the case with overturning moment. In the case of an upright arch acting as an overfall weir the pressure of the tail water effects a reduction of the pressure to the extent of its area. Thus if A be the area of the upstream water pressure, and a that of the downstream, or tail water, the unbalanced pressure will be their difference, or $A - a$, and will vary as the square of their respective depths. When overturning moment is concerned, the areas have to be multiplied by a third of their depths to represent the moment on the base. The difference of the two will be in that case as the cubes of their respective depths.

99. Crest Width of Arch. The crest width of the arch, according to formula (**23**), should be $\frac{1}{2}\sqrt{H} = 3\frac{3}{4}$ feet, nearly. It will be made 3 feet, with a stiffening rib or rim of 3 feet in width. The crest width could be made proportional to the base width, say $.3b$, and if this falls below 2 feet, reinforcement will be required.

The length of the pier base is measured from the extrados of the arch, the two half arches forming, as already explained in section 92, a forked continuation of the buttress pier base.

The battering of the sides of the pier would clearly be a correct procedure, as the pressure diminishes from the base upward. A combined batter of 1 in 10 is adopted, which leaves a crest width of 5.6 feet. The length of the pier base, as also its outline, were determined by trial graphical processes, with the object of maneuvering the center of pressure as near that of the base as possible, so as to equalize the maximum and the mean unit stress as much as possible. This has been effected, as shown by the incidence of the final resultant on the elevation of the buttress pier.

100. Pressure on Foundations. The total imposed weight is measured by N in the force diagram, and is equivalent to 150,000 cubic feet of masonry, which at a specific gravity of 2.4 is equal to $\frac{150{,}000 \times 3}{40} = 11{,}250$ tons. The average pressure is this quantity divided by the area of the base, or by $125 \times 12 = 1500$ square feet, the quotient being $7\frac{1}{2}$ tons, nearly. The maximum pressure will be the same owing to the incidence of R at the center of the base. This $7\frac{1}{2}$ tons is a very moderate pressure for a hard foundation; if excessive, additional spread should be provided or else the spans reduced. It will be noticed that N greatly exceeds W. This is due to the added weight of water represented by the inclination given to the force line P, which represents the water pressure.

Economy of Multiple Arches. The cubic contents per foot run work out to $\frac{64{,}000}{76} = 850$ cubic feet, nearly, the denominator in the fraction being the distance apart of the centers of the buttress piers.

The contents of a gravity weir with base width $\frac{2}{3}(H+d)$ and top $\sqrt{H+d}$, works out to 1,728 cubic feet; the saving in material is therefore over 50 per cent.

101. Differential Arches. Fig. 75 is a study of a differential buttress arch weir. The principle of the differential arch consists in the radius increasing with the height of the arch, the unit stress is thus kept more uniform, and the stress area corresponds more closely with the trapezoidal profile that has necessarily to be adopted, than is the case when a uniform radius is adopted as in Fig. 74.

The arches are supposed to stand on a concrete or masonry platform ten feet high above the deepest part of the river bed, so that sluices if required could be provided below *L.W.L.* which is identical with the floor or fore apron level. The height is 35 feet to crest level. The depth of film passing over the crest is assumed at 5 feet and the reciprocal depth of tail water is 12 feet. Graphical analyses will be made at two stages, first, when water is at crest level and the river channel below is empty, second, at full flood. The inclination given to the intrados of the arch is 3 vertical to 2 horizontal. The buttresses are placed 31 feet centers, allowing a span of 25 feet at base, here they are 6 feet wide, tapering to 2 feet at crest. The span of the arch thus gradually widens from 25 to 29 feet. The versed sine of the arc is made 5 feet at base and $2\frac{1}{2}$ feet at crest. The radii at these positions are therefore 18.1 and 43.3, respectively, measured to the intrados of the arch. These radii are horizontal, not normal to the intrados as in Fig. 73, and thus vary right through from 18.1 to 43.3 corresponding to the altered versed sine which decreases from 5 to $2\frac{1}{2}$ feet, that half way up being 22 feet.

The thickness of the arch at base is made 2 feet.

Arch Unit Stress. Taking the base radius as 18.1, the base unit stress due to water pressure will be by formula (**21**), $s=\frac{RHw}{b}$; adding that due to the transmitted weight of the arch, formula (**25**), $s=Rw\left(\frac{H}{b}+\rho\sin\theta\right)$, sin θ being .6, the expression becomes $18.1w\left[\frac{35}{2}+(2.4\times.6)\right]$, ρ being taken at 2.4, w at $\frac{1}{32}$ ton, whence $s=10.4$ tons, a moderate stress for a vertical arch.

The real thickness of the arch is more like $2\frac{1}{2}$ feet than 2 feet as properly it should be measured horizontally, not normally.

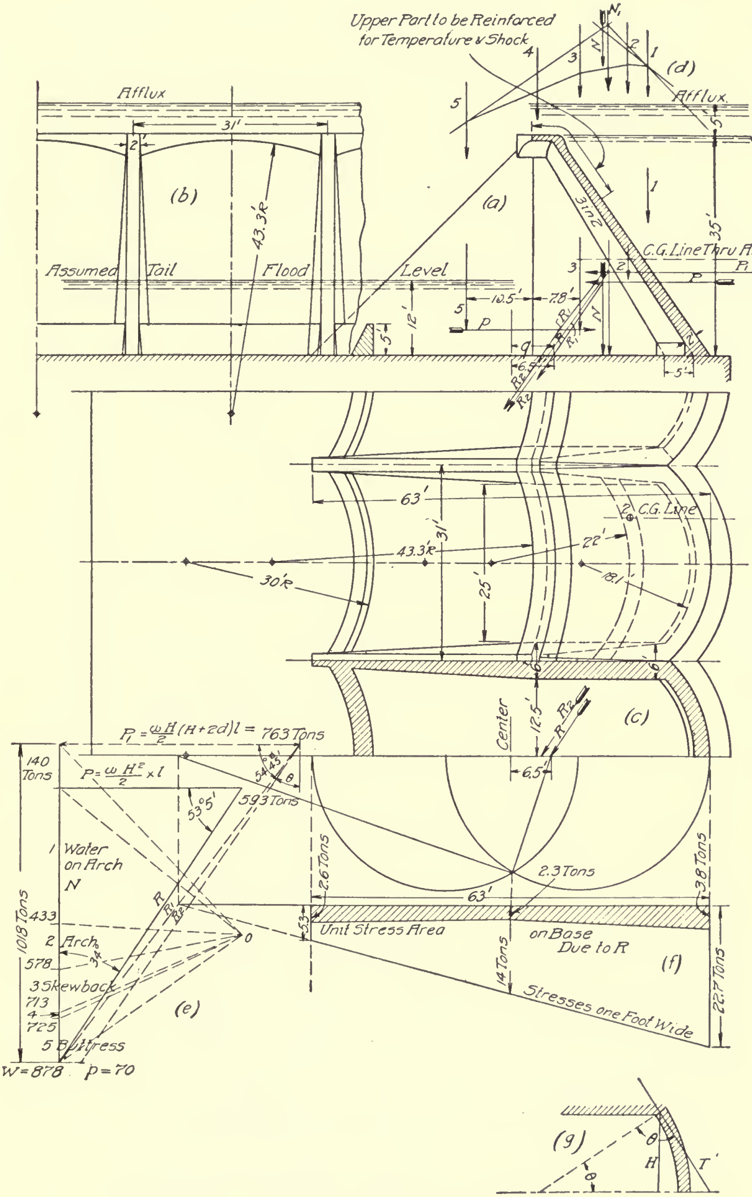


Fig. 75. Design Diagrams for Differential Buttress Arch Weir

Load Line. In the force polygon the load line is made up of five weights: (1) that of the overlying water has a content of $\frac{33\times24\times35}{2}=13860$ cubic feet, equivalent to 433 tons; (2) the arch 145 tons; (3) the contents of the pier underlying the arch is found by taking the contents of the whole as if the sides were vertical and deducting the pyramid formed by the side batters. Thus the contents of the whole is $\frac{22\times6\times35}{2}=2310$, that of the pyramid is $\frac{22\times2\times35}{3}=513$, difference 1797, or 135 tons; (4) the weight of the horizontal arch of the crest of the weir, 12 tons; (5) the contents of the buttress, by the prismoidal formula comes to $\frac{1}{6}\,l\,(A_1+4A_m+A_2)$ in which A_1 and A_2 are areas of the ends and A_m of the middle section. Here $A_1=0$, $A_m=\frac{35}{2}\times\frac{6}{2}=52.5$, and $A_2=35\times4=140$; therefore $(5)=\frac{1}{6}\times35\times[0+(4\times52.5)+140]=2042$ cubic feet equivalent to 153 tons.

The total load foots up to 878 tons.

P the horizontal water pressure $=w\frac{H^2}{2}\times l=\frac{1}{32}\times\frac{(35)^2}{2}\times31=593$ tons.

The position of the several vertical forces is obtained as follows: That of 1, a triangular curved prism is at $\frac{1}{3}$ its horizontal width; of 2 is found by formula (7), Part I, and by projection of this level on to the plan. The position of 3 has to be calculated by moments as below.

The lever arm of the whole mass including the battered sides is at $\frac{1}{3}$ width from the vertical end of $7\frac{1}{3}$ feet while that of the pyramidal batter is at $\frac{1}{4}$ the same distance, or $5\frac{1}{2}$ feet.

The statement is then

$$2310\times7\tfrac{1}{3}=(1797\times x)+(514\times5.5) \text{ whence } x=7.84 \text{ feet}$$

The position of 5, the battered sloping buttress is obtained by taking the center part 2 feet wide and the outer side batters separately. The c.g. of the former is at $\frac{1}{3}$ the length, $\frac{35}{3}=11\frac{2}{3}$ from the vertical end, and its contents are $\frac{35^2}{2} \times 2 = 1225$ cubic feet $= 92$ tons. The weight of the whole is 153 tons, so that the side batters will weigh $153 - 92 = 61$ tons, and be $\frac{35}{4} = 8.75$ feet distant from the end. Taking moments about the vertical end, we have

$$153x = (92 \times 11.67) + (61 \times 8.75)$$

$$x = \frac{1606}{153} = 10.5 \text{ feet}$$

Therefore, the incidence of the resultant on the base line measured 6.5 feet upstream from the center point.

In Fig. 75a, $N = 878$ tons and $\frac{N}{b} = 14$; b being 63 feet and $q = 6.5$ feet, whence $m = 1.62$ and the stress on the buttress, if only 1 foot wide, $= 14 \times 1.62 = 22.7$ tons. The compression at the toe $= \frac{2N}{b} - s = (28 - 22.7) = 5.3$ tons. These quantites have now to be divided by the base widths to obtain the unit stresses, which are as follows: at heel, $\frac{22.7}{6} = 3.8$; at center, $\frac{14}{6} = 2.3$; at toe, $\frac{5.3}{2} = 2.6$ tons. This stress area is shown hatched in Fig. 75f.

This stress diagram is useful as showing that owing to the incidence of R being behind the center point the total stress diminishes toward the toe of the buttress, consequently it should be tapered on plan, as has been done. In Fig. 74 it has been shown that the stress being uniform by reason of the incidence of R at the center point of the base, the buttress has been made rectangular in plan at its base. The indicated unit stresses are very light which is a great advantage on a bad foundation.

102. Flood Pressures. The second, or flood stage, will now be investigated. Here the vertical load line N in Fig. 75e is increased by 140 tons, the additional weight of water carried by the arch. The horizontal water pressure P_1 is now 763 tons and $N = 1018$, their resultant being R_1. The reverse pressure due to a depth of 12 feet of water is 70 tons, this combined with R_1, in Fig. 75a, results in R_2 the final resultant. The value of θ is 35° 15′ which is satisfactory. As q scales 5 feet, the unit stresses work out as follows:

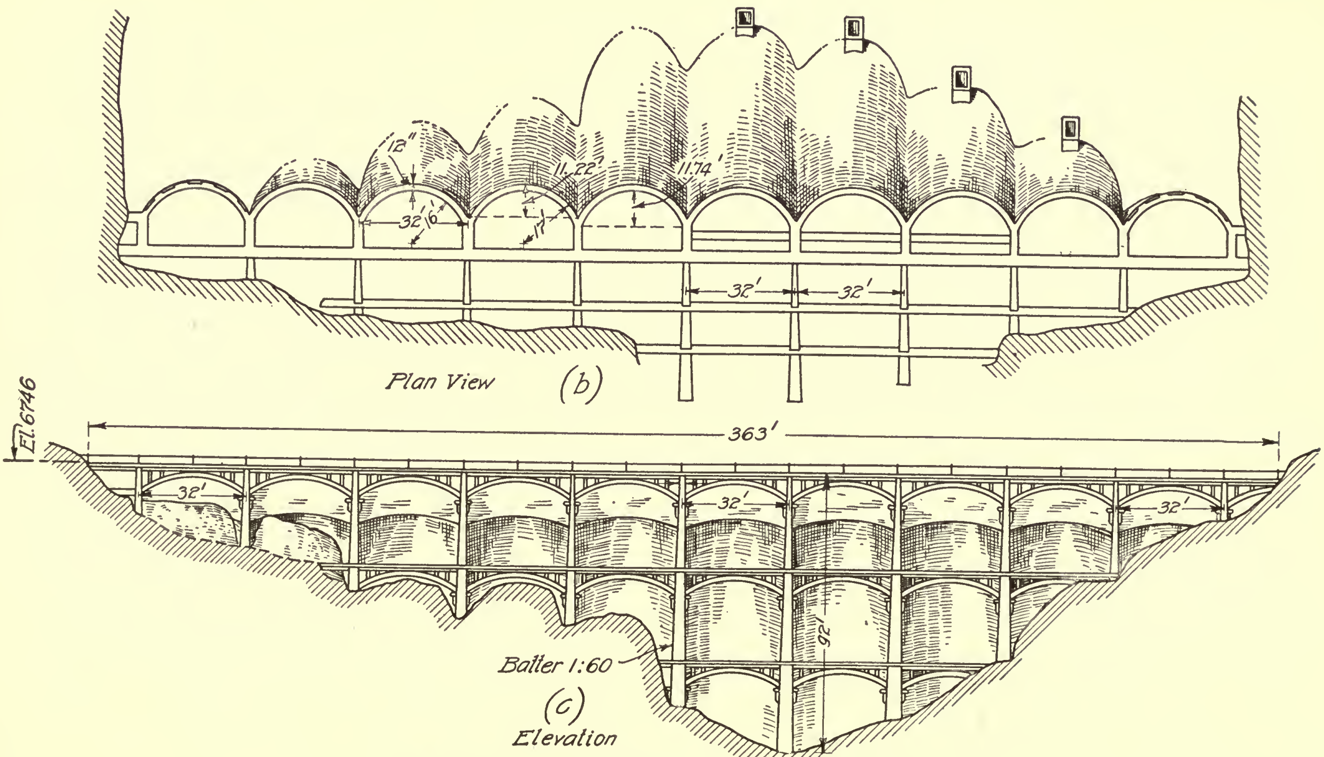


Fig. 76. Plan and Sectional Elevation of Big Bear Valley Dam

At heel 3.9 tons, at center 2.7, and at toe, 4.2 tons.

The stress in the arch under a head of 38 feet comes to 11.5 tons. Thus the stresses in stage 2 are higher than is the case with stage 1.

At the end of a series of these scallop arches near either abutment the thrust of the arch resolved axially with the weir has to be met either by tying the last two arches by a cross wall and reinforcing rods, or abutting the arch on an abutment supported by wall or a length of solid dam. This design would, it is considered, be improved if the versed sine of the arcs were made somewhat greater, as the arches are too flat near the crest.

The following remarks bear on the curvature of the arch mentioned in section 101. When a segmental arch is inclined, the spring line is at a lower level than the crown, consequently the water pressure is also greater at that level. But the thickness should vary with the pressure which it does not in this case. This proves the advisability of making the circular curvature horizontal, then a section at right angles to the inclined spring line will be an ellipse, while a horizontal section will be a segment of a circle. The reverse occurs with arches built in the ordinary way. There appears to be no practical difficulty in constructing forms for an inclined arch on this principle.

103. Big Bear Valley Dam. Fig. 76 is a plan and sectional elevation of the new Bear Valley reinforced concrete multiple arch dam which takes the place of the old single arch dam mentioned in section 83. The following description is taken from "Engineering News", from which Fig 78 is also obtained.

The new dam consists of ten arches of $30\frac{1}{2}$ feet, clear span at top, abutting on eleven buttresses. The total length of the dam is 363 feet on the crest; its maximum height from crest to base is 92 feet (in a pocket at the middle buttress only), although, as the elevation in Fig. 76 shows, the average height of the buttresses is much less than that figure. The water face of the structure and the rear edge of the buttresses are given such slopes as to bring the resultant of the water-pressure load and that of the structure through the center of the base of the buttresses at the highest portions of the dam, Fig. 79. The slope for the water face up to within 14 feet of the top is 36° 52′ from the vertical, and from that point to the crest is vertical. The slope of the downstream edges of the buttresses is 2 on 1 from the bottom to the top, the vertical top of the face arches giving the piers a top width of 10 feet from the spring line to the back edge. The buttresses are 1.5 feet thick at the top and increase in thickness with a batter of 0.016 feet per foot of height or 1 in 60 on each side to the base for all heights. The arch rings are 12 inches thick at the top and down to the bend, from which point they are increased in thickness at the rate of 0.014 feet per foot to the base, or 1 in 72.5

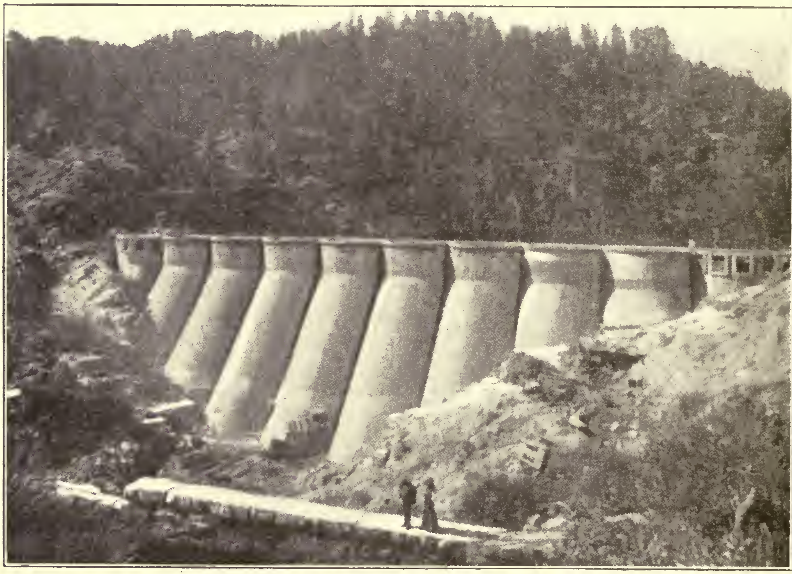


Fig. 77. View of Big Bear Valley Dam with Old Dam Shown in Foreground

The arc of the extrados of the arch ring is 140° 08′ from the top to bottom the radius being maintained at 17 feet and the rise at 11.22 feet. The extrados is, therefore, a cylindrical surface uniform throughout, all changes in dimensions being made on the intrados of the arch. Thus at the top, the radius of the intrados is 16 feet, the arc 145° 08′, and the rise 11.74 feet. At 80 feet from the top, Fig. 79, the thickness of the arch ring will be 2.15 feet, the radius of the intrados 14.85 feet (the radius of extrados less the thickness of the wall), the arc 140° 48′ and the rise 10.59

feet. In all cases of arch-dam design the clear span, radius, and rise of the intrados decrease from the top downward.

Fig. 78. Back View of Big Bear Valley Dam, Showing Construction of Multiple Arches and Buttresses
Courtesy of "Engineering News"

Strut-tie members are provided between the buttresses to stiffen and take up any lateral thrusts that might be set up by seismic disturbances or vibrations, these consisting of T-beams and supporting arches all tied together by heavy steel reinforcement. The T-beams are 12 inches thick and 2.5 feet wide, with a 12-inch stem, set on an arch 12 inches square at the crown and thickening to 15 inches toward the springing lines, with two spandrel posts on each side connecting the beam and arch, all united into one piece. There are provided copings for the arches and the tops of the buttresses with 9-inch projections, making the arch cope 2.5 feet wide and that on top of the buttresses 3 feet wide. The beam slab of the top strut members is built 4 feet wide to serve as an extra stiffener, as well as a comfortable footwalk across the dam. This footwalk is provided with a cable railing on both sides

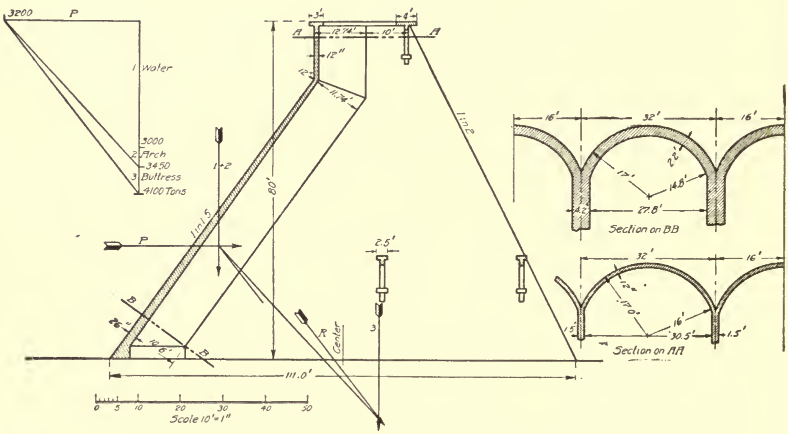


Fig. 79. Profile and Sections of Big Bear Valley Dam

to make it a safe place upon which to walk. To add to the architectural effect of the structure, the arches of the strut members terminate in imposts, built as part of the buttresses. The struts are reinforced with twisted steel rods, all being tied together and all being continuous through the buttress walls. The ends entering the buttresses are attached to other reinforcement passing crosswise into the buttress walls, forming roots by which the stresses in the beams may be transmitted to and distributed in the buttress walls. The ends of the strut members are all tied onto the granite rock at both ends of the structure by hooking the reinforcement rods into drill holes in the rock. The buttresses are not reinforced, except to be tied to the arch rings and the strut members, their shape and the loads they are to carry making reinforcement superfluous. The arch ribs are reinforced with $\frac{3}{4}$-inch twisted rods horizontally disposed 2 inches from the inner surface and variably spaced. These rods were tied to the rods protruding from the buttresses. For reinforcing the extrados of the arch ring ribs of $1\frac{1}{2}\times 1\frac{1}{2}\times \frac{3}{16}$-inch angles were used, to which "ferro-inclave" sheets were clipped and used both as a concrete form for the outer face and a base for the plaster surface.

104. Stress Analysis. On Fig. 79 a rough stress analysis is shown for 80 feet depth of water. As will be seen the resultant R cuts the base just short of the center point. The value of N is estimated at 4100 tons, the area of the base $A=110\times 4.2=460$ sq. feet whence $\frac{N}{A}=\frac{4100}{460}=9$ tons nearly, evenly distributed (m being taken as unity). The stress on the arch, 80 feet deep, neglecting its weight is $\frac{RHw}{b}-\frac{16\times 80\times 1}{2.2\times 32}=20$ tons, nearly. This shows the necessity for the reinforcement provided to take $\frac{1}{2}$ or $\frac{3}{8}$ of this stress. The tangent of $\theta=\frac{P}{N}=\frac{3200}{4100}=.78$. $\therefore\ \theta=39°$.

This is a large value, 35 degrees being the usual limit, 33 degrees better. If the arch thickness were doubled, reinforcement would not be necessary except near the crest and the additional load of about 320 tons would bring θ down to 35 degrees. If not, a greater inclination given to the arch would increase the load of water on the extrados. It is quite possible that a thicker arch without reinforcement would be actually cheaper. The downward thrust acting on the arch due to its own weight is on a different plane from the arch thrust. Its effect is to increase the unit stress to a certain extent, as is also the case with the combination of shearing and compressive stresses in the interior of a dam as explained in Part I. This increase can, however, be neglected. A considerable but undefined proportion of the water pressure near the base is conveyed to it and not to the buttresses; this will more than compensate for any increase due to vertical compression and consequently it can be ignored. The ribs connecting the buttresses form an excellent provision for stiffening them against buckling and vibration and are universally employed in hollow concrete dams. The buttresses in this instance are not reinforced.

HOLLOW SLAB BUTTRESS DAMS

105. Description of Type. There is a class of dam and weir similar in its main principles to the arch buttress type which is believed to have been first introduced by the Ambursen Hydraulic Construction Company of Boston. In place of the arch an inclined flat deck is substituted, which has necessarily to be made of reinforced concrete. For this reason, the deck slabs cannot exceed a moderate width, so numerous narrow piers take the place of the thick buttresses in the former type. A further development is a thin deck which covers the downstream ends of the buttresses or piers, forming a rollway. The enclosed box thus formed is occasionally utilized as a power house for the installation of turbines, for which purpose it is well suited.

The inclination given to the flat deck is such that the incidence of the resultant (R.F.) will fall as near the center of the base as possible and at the same time regulate the inclination of the resultant to an angle not greater than that of the angle of friction of the material, i.e., 30 degrees with the vertical. By this means any tendency to slide on the foundation is obviated.

Ellsworth Dam an Example. A good example of this style of construction is given in Fig. 80 of the Ellsworth dam in Maine. In this design the inclination of the deck is 45° or very nearly so; the piers are 15 feet centers with widened ends, so that the clear span of the concrete slabs is 9′ 1″ at the bottom.

The calculations necessary to analyze the thickness of the slabs and the steel reinforcement at one point, viz, at *El.* 2.5, will now be given. In this case the pressure of water on a strip of the slab, one foot wide, the unsupported span of which is 9′ 1″, is Hlw. Here $H=67$ feet and w is $\frac{1}{32}$ ton per cubic foot; therefore, $W=67\times9.1\times\frac{1}{32}=19$ tons. To this must be added the weight of the slab. As this latter lies at an angle with the horizontal its weight is partly carried by the base and is not entirely supported by the piers. The diagram in Fig. 80c is the triangle of forces. The weight of slab w is resolved in two directions, a and b, respectively, parallel and normal to face of slab. The angle being 45 degrees, $a=b=\frac{w}{\sqrt{2}}$. Consequently the thickness, 37 inches, can be considered as reduced to $\frac{3.1}{1.4}=2.2$ feet. The portion of the weight of the slab carried to the piers will, therefore, be $9.1\times 2.2\times\frac{3}{40}=1.5$ tons, the weight of the concrete being assumed at the usual value of 150 pounds per cubic foot. The total distributed load in the strip will then be $19+1.5=20.5$ tons.

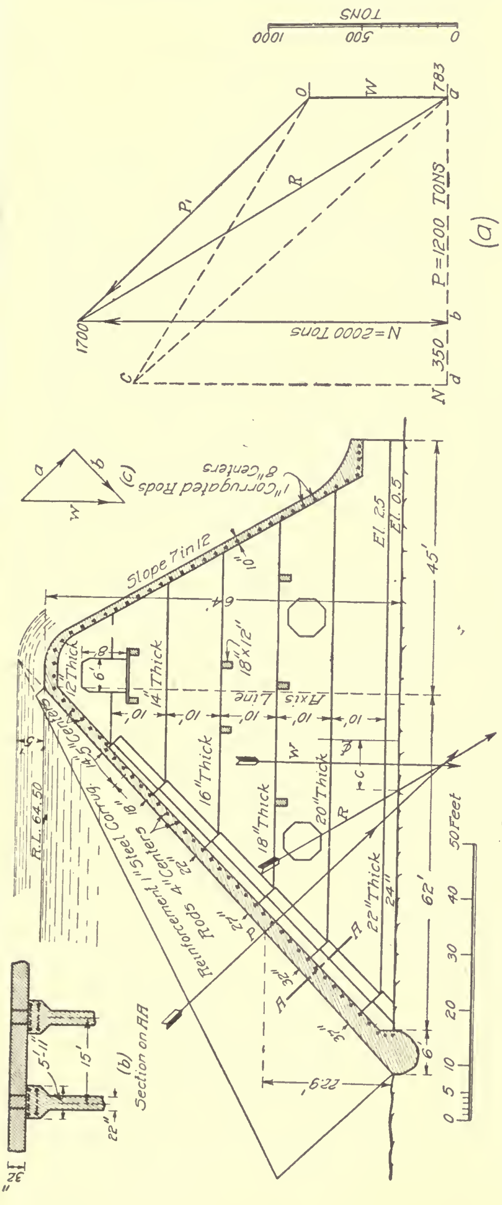


Fig. 80. Profile and Force Diagram of Ellsworth Dam

Now the moment of stress on a uniformly loaded beam with free ends is $\frac{Wl}{8}$, or $M=\frac{20.5\times 109}{8}=279$ inch-tons.

This moment must be equaled by that of the resistance of the concrete slab.

106. Formulas for Reinforced Concrete. For the purpose of showing the calculations in detail, some leading formulas connected with reinforced beams and slabs will now be exhibited:

$$bd^2=\frac{M_s}{f_s pj} \tag{26}$$

or, approximately,
$$bd^2=\frac{M_s}{\frac{7}{8}f_s p} \tag{26a}$$

$$bd^2=\frac{M_c}{\frac{1}{2}f_c kj} \tag{27}$$

or, approximately,
$$bd^2=\frac{M_c}{\frac{1}{6}f_c} \tag{27a}$$

From these are found d, the required thickness of a slab up to centroid of steel, or M_s+M_c the bending moments, in which b is width of beam in inches; d depth of centroid of steel below top of beam; M_c and M_s symbolize the moments of resistance of the concrete and steel, respectively; f_s safe unit fiber stress in steel, 12,000 to 16,000 lb., or 6 to 8 tons per square inch; f_c safe extreme stress in concrete 500, 600, or 650 lb., or .25, .3, or .325 ton per square inch; p steel ratio, or $\frac{A}{bd}$

$$\text{Ideal steel ratio } p=\frac{n}{2(r^2+rn)} \tag{28}$$

A area of cross-section of steel; k ratio of depth of neutral axis below top to depth of beam

$$k=\sqrt{2pn+(pn)^2}-pn \tag{29}$$

j ratio of arm of resisting couple to d

$$j=(1-\tfrac{1}{3}k) \tag{30}$$

n ratio $\frac{E_s}{E_c}$, E_s and E_c being the moduli of elasticity, ordinary values 12 to 15; r ratio $\frac{f_s}{f_c}$. As $p=\frac{A}{bd}$, when reinforced slabs are analyzed, formulas **(26)** and **(27)** can be transposed as below.

From **(26)** $M_s=f_sAjd$ **(31)**

From **(26a)** $M_s=\frac{7}{8}f_sAd$ Approximate **(31a)**

From **(27)** $M_c=\frac{1}{2}f_ckjbd^2$ **(32)**

From **(27a)** $M_c=\frac{f_c bd^2}{6}$ Approximate **(32a)**

In the case under review the reinforcement consists of three one inch square steel rods in each foot width of the slab. Using the approximate formulas (31a) and (32a), $f_s=8$ tons, $f_c=.3$ ton, $d=35$ inches and $b=12$ inches; then

$$M_s=8\times 3\times \frac{7}{8}\times 35=735 \text{ inch-tons}$$

$$M_c=\frac{3}{10}\times \frac{1}{6}\times 420\times 35=735 \text{ inch-tons}$$

the results being identical. As already noted the moment of stress is but 279 inch-tons. The end shear may have governed the thickness. Testing for shear the load on a 12-inch strip of slab is 20.5 tons of which one-half is supported at each end. Allowing 50 lb., or .025 ton, as a safe stress, the area of concrete required is $10.25 \div .025 = 410$ square inches the actual area being $37\times 12=444$ square inches.

107. Steel in Fore Slope. The reinforcement of the fore slope is more a matter of judgment than of calculation, this deck having hardly any weight to support, as the falling water will shoot clear of it. The piers are not reinforced at all, nor is it necessary, as the stresses are all compressive and the inclination of the upstream deck is such that the resultant pressure makes an angle with the vertical not greater than that of friction, i.e., 30 degrees. Fig. 80a is a force diagram of the resultant forces acting on the base at *El.* 0.00. The total weight of a 15-foot bay is estimated at 783 tons while that of P, the trapezoid of water pressure, is 1700 tons. The force line P in Fig. 80 drawn through the c.g. of the water pressure area intersects the vertical force W below the base line. From this intersection R is drawn upward parallel to its reciprocal in the force polygon, cutting the base at a point some 9 feet distant from the center point.

The maximum stress will occur at the heel of the base. $A=107\times 2=214$ sq. ft.; $\frac{N}{A}=\frac{2000}{214}=9.34$ tons; q being 9 ft., $m=\frac{107+54}{107}=1.5$ and $s=9.34\times 1.5=14$ tons. Formula (9), Part I. The horizontal component of $P=1200$ tons. The base being 2 ft. wide, $s_s=\frac{1200}{2\times 107}=5.6$ tons; therefore by formula (10), Part I, $c=7+\sqrt{49+31.4}=16.5$ tons, a decidedly high value. The usual limit to shearing stress is 100 lb. per sq. inch, equivalent to 7.2 tons per sq. ft., reinforcement is therefore not necessary and is not provided.

There appears to be no reason why a steeper slope should not have been given to the deck so as to bring the center of pressure up to the center of the base and thus reduce the unit stress. Possibly a higher river stage has been allowed for. The position of W as well as the weight of the structure were obtained from the section given in Schuyler's Reservoirs. Fig. 80 is of the so-termed "Curtain" type of dam. The "Half Apron" type, Fig. 82c, is sometimes used for overfalls, the main section of Fig. 82 illustrating the "Bulkhead" type.

108. Slab Deck Compared with Arch Deck Dam. The Ambursen dam, wherever the interior space is not required for installation of turbines, is undoubtedly a more expensive construction than the multiple arch type. This fact has at last been recognized and in one of the latest dams erected, scallop arches were substituted for the flat deck, thus obviating the expense of reinforcement. By

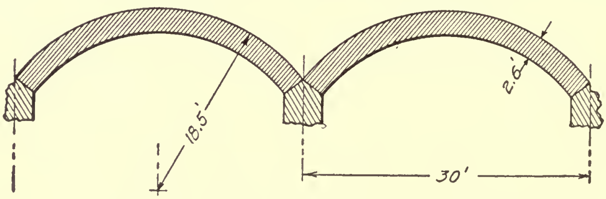


Fig. 81. Section of Arch with 30-Foot Span

increasing the width of the spans, the piers, being thicker in like proportion, will be in much better position for resisting compressive stress, as a thick column can stand a greater unit stress than a thin one. Another point in favor of the arch is that the effective length of the base of the piers extends practically to the crown of the arch. The arch itself need not be as thick as the slab. Owing to the liquid radial pressure to which it is subjected it is in a permanent state of compression and does not require any reinforcement except possibly at the top of the dam. Here the arch is generally widened, as in the case of the Ogden dam, Fig. 73, and thus greatly stiffened at the point where temperature variations might develop unforeseen stresses.

Fig. 81 is a sketch illustrative of the saving in material afforded by doubling the spans from 15 to 30 feet and conversion to multiple arch type. The radius of the extrados of the arches is 18.5 ft. H is 67 at elevation 2.50 and $w = \frac{1}{32}$ ton; hence the thickness of the arch by formula (21) (s_1 being taken as 15 tons), will be

$$b = \frac{RHw}{s} = \frac{18.5 \times 67 \times 1}{32 \times 15} = 2.6 \text{ feet}$$

It is thus actually thinner than the reinforced slab of one-half the span, or 15 feet. The greater length of the arch ring over that of the straight slab is thus more than compensated. The area of the arch, counting from the center of the pier, is $35\times2.6=91$ square feet, that of the slab is $30\times3.1=93$, that of the bracketing at junction with the piers, 13, giving a total of 106 square feet. The saving

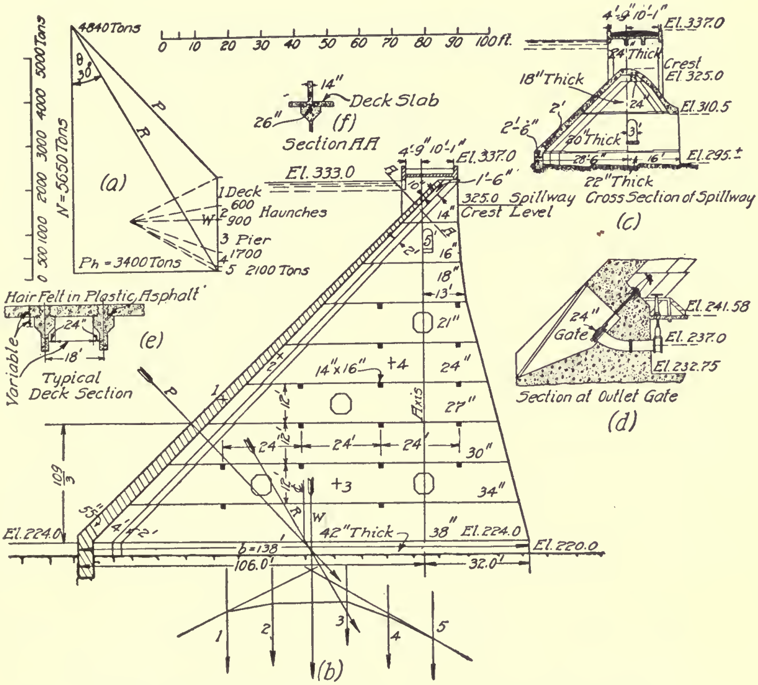


Fig. 82. Profile and Detailed Sections of Guayabal Dam, Porto Rico

due to decreased length of the piers is 25 square feet. Thus in the lower part of the dam over 40 cubic feet per 30′ bay per foot in height of concrete is saved, also all the steel reinforcement. If a rollway is considered necessary in the weir, the deck could be formed by a thin reinforced concrete screen supported on I-beams stretching across between the piers.

109. Guayabal Dam. Fig. 82 is a section of the Guayabal dam recently constructed in Porto Rico, its height is 127 feet and it is on a rock foundation. The following are the conditions governing the design; maximum pressure on foundation 10 tons per square foot; compression in buttresses 300 pounds per square inch or 21.6 tons per square foot; shear in buttresses 100 pounds per square inch, or 7.2 tons per square foot; shear in deck slabs 60 pounds, or .03 ton per square inch; f_c for deck slabs 600 pounds or .3 ton per square inch; f_s for deck slabs 14,000 pounds, or 7 tons per square inch.

The concrete in the slabs is in the proportion of 1:2:4, in the buttresses 1:3:6; $n=\frac{E_s}{E_c}$ is taken as 15 and $r=\frac{f_s}{f_c}=23.3$. The deck slab is 55 inches thick at *El.* 224, d is taken as 53, allowing 2 inches for covering the steel, bd or the area of the section one foot wide $=53\times12=636$ square inches. Now A the area of the steel $=pbd$. By formula (28), $p=\frac{n}{2(r^2+rn)}=\frac{15}{2(23.3)^2+23.3\times15}=.01044$, hence the required area of steel will be $636\times.01044=6.64$ square inches, provided d is of the correct value. The calculation will now be made for the thickness of the slab which is actually 55 inches. The load on a strip 12 inches wide is

$$\text{Water pressure } \frac{109\times13}{32}=44.3 \text{ tons}$$

To this must be added a portion of the weight of the slab which latter amounts to $\left(\frac{13\times55}{12}\right)\times\left(\frac{3}{40}\right)=4.5$ tons. Of this $\frac{4.5}{\sqrt{2}}=3.2$ tons must be added to the 44.3 tons above, $44.3+3.2=47.5$ tons. The bending moment M is $\frac{WL}{8}=\frac{47.5\times13\times12}{8}=927$ inch-tons. The depth of the slab can be estimated by using formulas (26) or (27) or the approximate ones (26a) and (27a). For the purpose of illustration, all four will be worked out. First the values of k and j will be found by formulas (29) and (30).

$$k=\sqrt{.313+.0245}-.156=.582-.156=0.426$$

$$j=\left(1=\frac{k}{3}\right)=1-.142=.858$$

By formula (26), $d^2=\frac{M_s}{12f_spj}=\frac{927}{12\times7\times.0104\times.858}=1234$

$$\therefore \qquad d=\sqrt{1234}=35.07 \text{ inches}$$

By formula (**27**), $d^2=\dfrac{2M_c}{12f_ckj}=\dfrac{927\times 2}{12\times .3\times .426\times .858}=1406$

$$\therefore \qquad d= 1406=37.5 \text{ inches}$$

Now the approximate formulas will be used. By (**26a**)

$$d^2=\frac{8\times 927}{7\times 12\times 7\times .0104}=\frac{1854}{1.53}=1210$$

$$\therefore \qquad d=\sqrt{1210}=34.8 \text{ inches}$$

by (**27a**)

$$d^2=\frac{6\times 927}{12\times .3}=\frac{5562}{3.6}=1542$$

$$\therefore \qquad d=\sqrt{1542}=39.3 \text{ inches}$$

The approximate formulas (**26a**) and (**27a**) give higher results than (**26**) and (**27**). The result to select is 37.5 inches, formula (**27**), which is higher than by (**26**). The depth of beam would then be 40 or 41 inches. It is actually 55. This discrepancy may be due to the water having been given a s.g. in excess of unity, owing to the presence of mud in suspension, say of 1.3 or 1.5, or shear is the criterion.

The corresponding steel area will be $A=pbd=.0104\times 12\times 37.5 =4.7$ square inches. $1\frac{7}{16}''$ round rods spaced 3 inches would answer. With regard to direct shear on the slab, W as before $=47.5$ tons of which half acts at each pier, viz, 23.7 tons. The safe resistance is $bd\times S_s=12\times 55\times .03=20$ tons, nearly. The shear $=\dfrac{23.7}{12\times 55}=.036$ ton $=72$ pounds per square inch. This figure exceeds the limit of 60 pounds. The deficiency is made up by adding the shear of the steel rods. The sectional area of this reinforcement is 4.7 square inches the safe shearing of which is over 20 tons. These rods are usually turned up at their ends in order to care for the shear.

Shear in Buttresses. With regard to shear in the buttresses, the horizontal component of the water pressure as marked on the force diagram is 3400 tons. The area of the base of the buttress at *El.* 224 is $138\times 3.2=441.6$, the shearing stress or s_s then $=\dfrac{3400}{441.6}=8$ tons per square foot, nearly. The allowable stress being only 7.2 tons the difference will have to be made good by reinforcing rods of which two of $\frac{3}{4}$-inch diameter would suffice.

Now with regard to compressive stresses on the buttresses the graphical working shows that the resultant R strikes the base at *El.* 224 almost exactly at the center, the angle θ also is 30 degrees. The value of N is 5650 tons; s_1 the mean and s the maximum stress will both equal $\frac{N}{A}$; and A, the area of the base, equals $138 \times 3.2 = 442$ sq. ft.; therefore, $s = \frac{5650}{442} = 12.78$ tons. The compression on the foundation itself, which is 4 feet lower will not be any less for, although the base width is greater, N as well as P are also increased. Thus the pressure on the foundation is in excess of the limit and widening to a further extent is required.

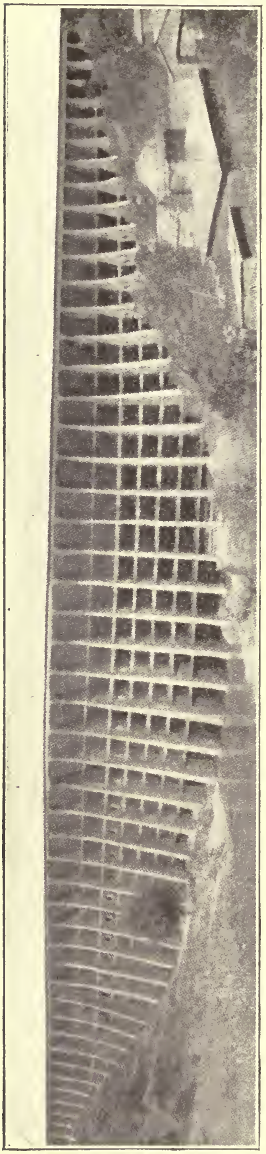


Fig. 83. Rear View of Bulkhead Section of Dam
This section has 51 spans with 18-foot centers, making a total length of 918 feet.

The maximum internal stress c, in the buttress at *El.* 224, will be by formula (10), Part I, $\frac{1}{2}s + \sqrt{\frac{s^2}{4} + s_s^2}$. Here $s = 12.8$ and s_s as we have seen is 8 tons, therefore, $c = 6.4 + \sqrt{\frac{164}{4} + 64} = 16.6$ tons. The limit compression in the buttress is 300 pounds per square inch, or 21.6 tons per square foot.

In the bulkhead portion of the dam, shown in Fig. 82b, every pier is run up 14 inches thick through the deck to form a support for a highway bridge, the spans of which are therefore 16 feet 10 inches in the clear; the roadway is carried on slabs which are supported by arches of reinforced concrete. The buttresses are laterally supported by several double reinforced sway beams, $16'' \times 14''$, and below the crest a through roadway is provided. The spillway section is shown on Fig. 82c. The

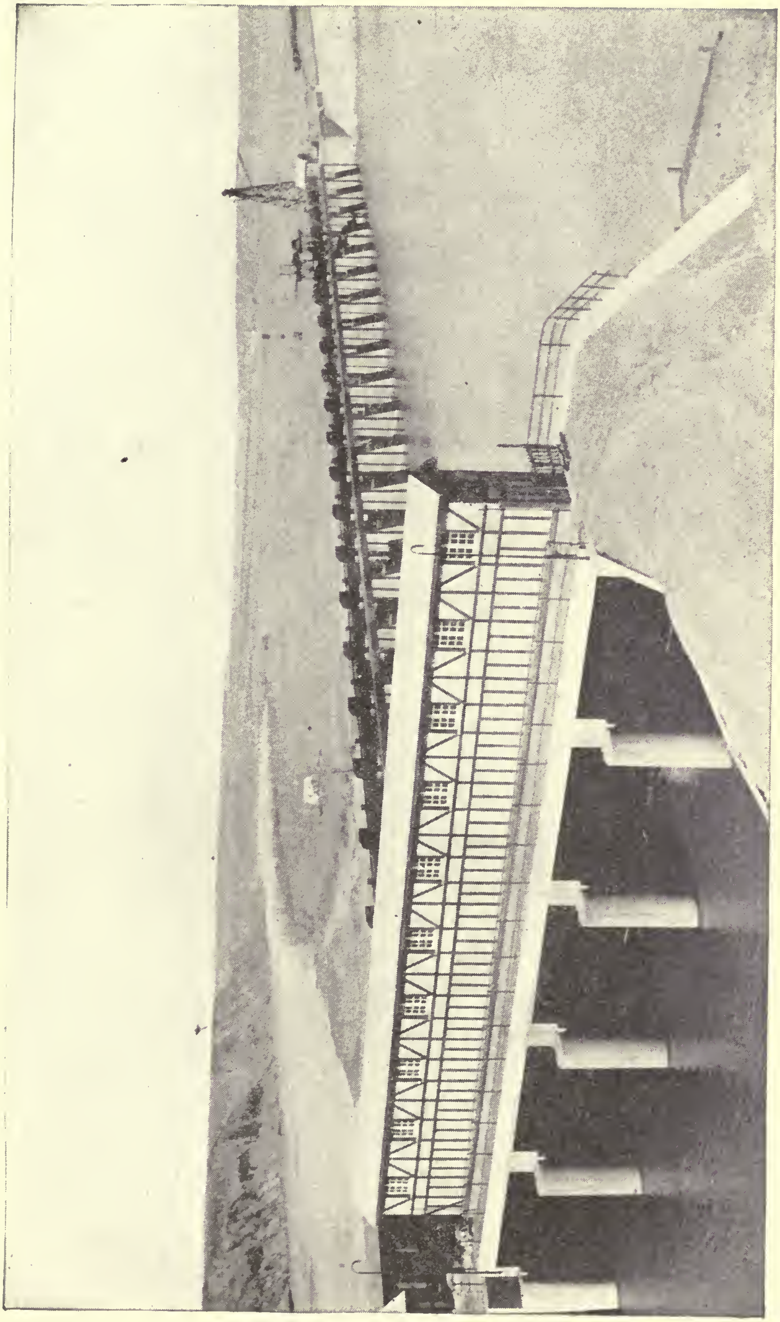


Fig. 84. View of Bassano Dam over the Bow River Taken Just after Water Was Turned into Can

ground level is here on a high bench at *El.* 295. The crest being *El.* 325, the fall is 30 feet. The spillway is of the "half apron type". The roadway here is carried on four reinforced concrete girders, a very neat construction; the piers are run up every alternate span and are therefore at 36-foot centers; they are beveled on both faces to reduce end contraction. The spillway will pass 70,000 second-feet; its length is 775 feet.

The bulkhead section of the dam (see also Fig. 83) has 51 spans of 18-foot centers, total length 918 feet; that of the spillway consists of 21 spans of 36-foot centers. The whole length is 1674 feet. The depth of the tail water is not known, it would probably be about 20 feet and its effect would be but trifling. This is one of the largest hollow dams ever constructed. The arrangement of the haunches or corbels of the buttresses is a better one than that in the older work of Fig. 80.

110. Bassano Dam. Another important work is the Bassano dam illustrated in Figs. 84 and 85. This is an overfall dam built over the Bow River at the head of the eastern section of the Canadian Pacific Railway Company's irrigation canal and is estimated to pass 100,000 second-feet of water at a depth of 14 feet. Though not so high nor so long as the Guayabal dam it presents several features of interest. First its foundations are on a thick blanket of clay some twelve feet deep which overlies boulders and gravel. This material is very hard blue clay of excellent quality. The great advantage of this formation, which extends over 1000 feet upstream from the work, is that it precludes all uplift, or very nearly so, consequently no special precautions have to be adopted, such as a long apron to ensure length of percolation, as would be necessary in case of a foundation composed of porous and loose materials. It has also disadvantages. The allowable pressure on the clay is limited to $2\frac{1}{2}$ tons per square foot. This influences the design necessitating a wide spread to the buttresses, laterally as well as longitudinally. The whole of the dam is an overfall and the general arrangements are very similar to those prevailing at Guayabal. The hearth or horizontal fore apron, a provision not necessary in the last example, is at *El.* 2512. The crest is at 2549.6 a height of 37.6 feet above the apron and corresponds with the level of the canal intake floor. Water is held up to eleven feet above crest level by draw gates

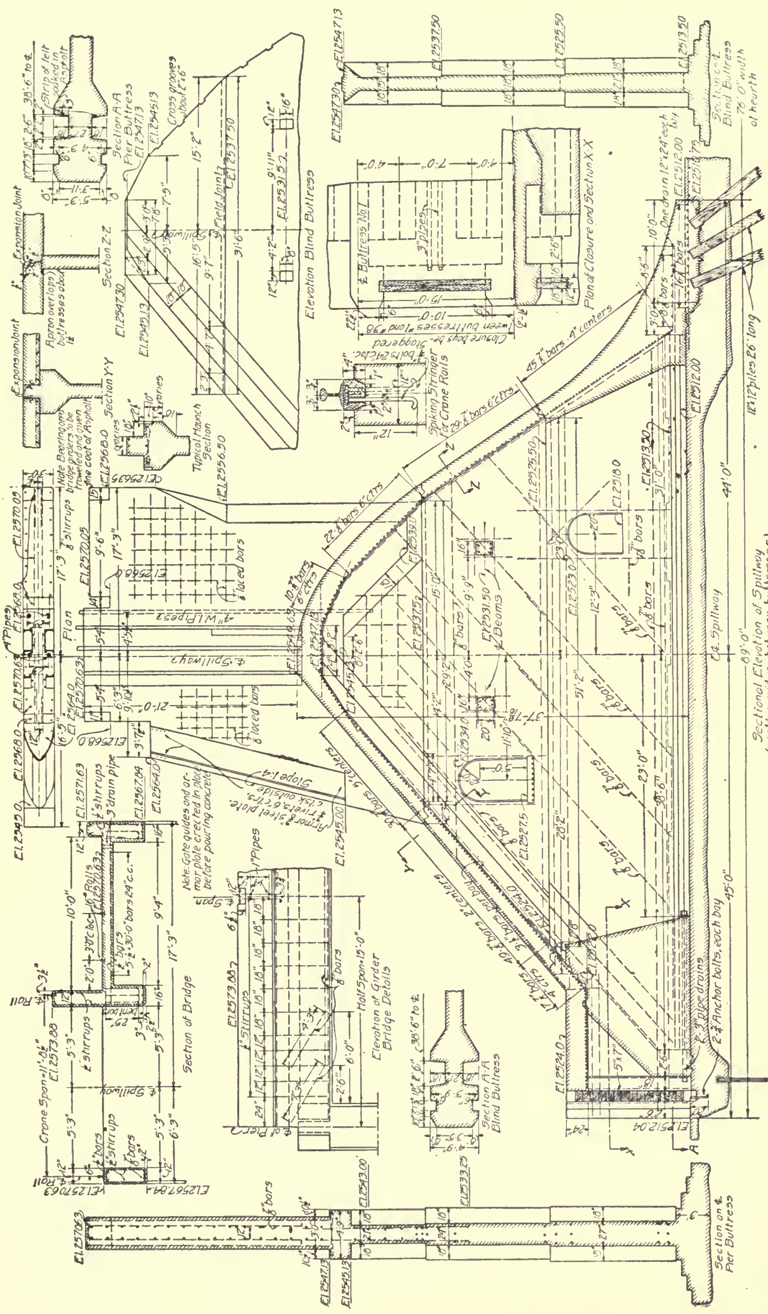


Fig. 85. Profile and Sections of Bassano Overfall Dam

eleven feet high, and this full supply level is three feet below that of the estimated afflux, which is fourteen feet above the crest.

For overturning moment the water-pressure area will be a truncated triangle with its apex at afflux level plus the height h or $1.5\,\frac{v^2}{2g}$ to allow for velocity of approach, as explained in section 57, Part I. This, in the Bow River with a steep boulder bed will be about 12 feet per second; h therefore will equal $1.5\times\frac{144}{64}=3.4$ feet and the apex of the truncated triangle will be at a point $14+3.4=17.4$ feet above the crest level. The depth of the tail water at full flood is not known, the ratio $\frac{d}{D}$ with a steep bed slope will not be under .5, consequently with $d=14$, D will have a value of about 25 to 28 feet, d being depth over crest and D that of tail water. The overturning moments direct and reverse can be represented by the cubes of the depths up- and downstream and the unbalanced moment by their difference. The upstream head is $37.0+14+3.4=55$ feet and the downstream head say 25 feet. Their cubes are 166,375 and 15,625 the difference being 150,750, thus the reverse pressure will not have much effect in assisting the stability of the structure. The corresponding representative moment when water is held up to 11 ft. above crest will be $49^3=117{,}649$, supposing the tail channel empty. This quantity is less than the 150,750 previously stated, consequently the afflux level is that which has to be considered when estimating the overturning moment. In the case of direct water pressure on the deck slabs, the acting head at full flood will be the difference of the flood level up- and downstream, which is 30 feet, as the tail water is allowed access to the rear of the deck slabs. This is less than the head, 49 feet, which exists when the gates are closed and water is held up to canal full supply, i.e., to *El.* 2560.6, consequently the head that has to be considered is that at this latter stage.

Analysis of Pressures on Bassano Dam. With this data the design can be analyzed, the procedure being identical with that explained in the last example, excepting that the reverse pressure might be taken into account as it will modify the direction and incidence of R in a favorable sense though not to any great extent. The limit stresses are those given in the last example with the following additions: Footings, compression in bending, 600 pounds per square inch, shear, 75 pounds per square inch.

Some explanation will now be given of the method of calculation of the footing to the buttresses and the Guayabal dam will be referred to, as the pressures on the base of the buttresses are known quantities. In section 109 the value of N is 5650 tons and $\frac{N}{b}=\frac{5650}{138}=41$ tons, nearly. This is the unit pressure per foot run on the base of the buttress. Supposing the limit pressure on the foundation was fixed at 3 tons per square foot, then the requisite base width of the footing would be $\frac{41}{3}=13.7$ feet. The footing consists of two cantilevers attached to the stem of the buttress. The bending moment M at the junction with the buttress of a strip 1 foot wide will be $\frac{Wl}{2}$. The buttress being 3.2 feet wide the projecting length of footing on each side will be $\frac{13.7-3.2}{2}=5.25$ feet.

The reaction on a strip one foot wide will be $5.25\times3\times1=$ 15.75 tons. The moment in inch-tons about the edge of the section of the buttress will be $\frac{12Wl}{2}=\frac{15.75\times5.25\times12}{2}=498$ inch-tons. According to formula (**27a**), $bd^2=\frac{6M}{f_c}$. $\therefore d=\sqrt{\frac{6\times498}{12\times.3}}=\sqrt{830}=$ 28.8 inches. Then $bd=28.8\times12=346$ and A the area of the steel at the base will be $pbd=.0104\times346=3.61$ inches, this in a 12-inch wide strip will take $1\frac{1}{4}$-inch bars 4 inches apart. When the weight on the buttress is considerable the depth of footing slab thus estimated becomes too great for convenience. In such cases, as in Fig. 85, the beam will require reinforcement in compression at the top. This complicates the calculation and cases of double reinforcement are best worked out by means of tables prepared for the purpose. The footings shown in Fig. 85, were thus double reinforced, in fact through bars were inserted at each step, the lower being in tension the upper ones in compression. The lower bars were continuous right through the base of the dam. This reinforcement of the footing is not shown on the blue print from which Fig. 85 is derived.

111. Pressure on Foundation Foredeck. A great many

Ambursen dams have been constructed on river beds composed of boulders and gravel, which require a pressure limit of about 4 tons per square foot. This can always be arranged for by widening the footing of the pier buttresses, the same can of course be done with arch buttressed dams. The base of the arch itself can be stepped out in a similar manner. In the Bassano dam the sloping fore deck is unusually thick and is heavily reinforced in addition; this is done with the idea of strengthening the structure against shock from ice, as well as from the falling water, and with the further idea of assisting the buttresses in carrying the heavy load of the piers and superstructure. It is doubtful if any calculations can well be made for this; it is a matter more of judgment than of estimation.

Buttresses. As with the Guayabal spillway, every alternate buttress is run up to form the piers of the superstructure, which latter consists of a through bridge which carries the lift gear for manipulating the draw gates. The so-termed blind buttresses—that is, those that do not carry a pier—are of thinner section and are apparently not reinforced. Both kinds of buttresses have cross-bracing as shown on the profile. In hollow dams the location of the center of pressure moves with the rise of water from the heel toward the center within the upstream half of the middle third. In solid dams, on the other hand, the movement is along the whole of the middle third division, consequently in hollow dams there is no tendency to turn about the toe as with solid dams, rather the reverse, namely, to upset backward. This latter tendency must cause tension in the buttresses which the cross-bracing is intended to care for.

Baffles. As noted already in section 66, baffles have been built on the curved bucket with the object of neutralizing this mischievous arrangement which it is hoped will soon become as obsolete in western practice as has long been the case in the East.

Hearth and Anchored Apron. The dam is provided with a solid horizontal fore apron or hearth 76 feet long and beyond this the device of an anchored floating apron of timber 30 feet in length has been added. The apron is undoubtedly too short and should have been made 100 feet or 2 $(H+d)$ in length, with cribbed riprap below it. The wooden sheet piling in the rear of the work is considered to be worse than useless; it merely breaks up the good clay blanket by cutting it in two. A wide solid curtain of concrete, not so deep as to penetrate the clay blanket, would have been a superior arrangement. The inclined piling below the bucket is provided to guard against sliding. This dam is provided with a number of sluice openings. Their capacity is such that one half will pass ordinary floods, allowing the other half of the dam to be cut off from the river by sheet piling during construction. On completion of the work these sluices were all closed from the inside by slabs of concrete deposited in position.

SUBMERGED WEIRS FOUNDED ON SAND

112. Description of Type. There is a certain type of drowned or submerged diversion weir which is built across wide rivers or streams whose beds are composed of sand of such depth that a solid foundation on clay is an impossibility. Consequently, the weir has to be founded on nothing better than the surface of the river bed, with perhaps a few lines of hollow curtain walls as an adjunct. Of this class of weir but one is believed to have been constructed in the United States, viz, the Laguna weir over the Colorado River at the head of the Yuma irrigation canals.

This type originated in India and in that country are found numerous examples of weirs successfully constructed across very large rivers of immense flood discharge. For instance, the Godaveri River in Southern India has a flood discharge of 1,200,000 second-feet and the weirs across it are nearly $2\frac{1}{2}$ miles in length. Not only is the length great, but as will be seen, the width has to be very considerable. The Okhla weir, Figs. 101 and 102, situated on the Jumna below the historic city of Delhi is 250 feet wide and $\frac{3}{4}$ mile long. The height of these submerged weirs is seldom over 12 feet, their rôle being purely diversion, not storage. No doubt more of this type of low diversion weirs will in the future have to be constructed in the United States or in Mexico, so that a knowledge of the subject is a necessity for the irrigation engineer.

Principles of Design. The principles underlying the successful design of these works are a comparatively recent discovery. Designs were formerly made on no fixed principles, being but more or less modified copies of older works. Fortunately some of these works failed, and it is from the practical experience thus gained that a knowledge of the hydraulic principles involved has at last been acquired.

A weir built on sand is exposed not only to the destructive influences of a large river in high flood which completely submerges it, but its foundation being sand, is liable to be undermined and worked out by the very small currents forced through the underlying sand by the pressure of the water held up in its rear. In spite of these apparent difficulties, it is quite practicable to design a work of such outline as will successfully resist all these disintegrating influences, and remain as solid and permanent a structure as one founded on bed rock.

113. Laws of Hydraulic Flow. The principle which underlies the action of water in a porous stratum of sand over which a heavy impervious weight is imposed is analogous to that which obtains in

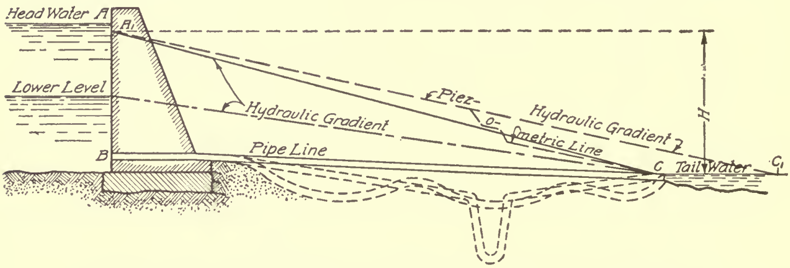


Fig. 86. Diagram Showing Action of Water Pipe Leading Out of Reservoir

a pipe under pressure. Fig. 86 exemplifies the case with regard to a pipe line BC, leading out of a reservoir. The acting head (H) is the difference of levels between A_1 a point somewhat lower than A, the actual summit level and C the level of the tail water beyond the outlet of the pipe. The water having a free outlet at C, the line A_1C is the hydraulic gradient or grade line. The hydrostatic pressure in the pipe at any point is measured by vertical ordinates drawn from the center of the pipe to the grade line A_1C. The uniform velocity of the water in the pipe is dependent directly on the head and inversely on the frictional resistance of the sides of the pipe, that is, on its length. This supposes the pipe to be straight, or nearly so.

114. Percolation beneath Dam. We will now consider the case of an earthen embankment thrown across the sandy bed of a stream, Fig. 87. The pressure of the impounded water will naturally cause leakage beneath the impervious earthen base. With a low depth of water impounded it may well be understood that such leakage might be harmless; that is, the velocity of the percolating under current would be insufficient to wash out the particles of sand composing the foundation of the dam. When, however, the head is increased beyond a safe limit, the so-termed piping action will take place and continue until the dam is completely undermined.

115. Governing Factor for Stability. The main governing factor in the stability of the sand foundation is evidently not the superimposed weight of the dam, as the sand is incompressible; although a load in excess of the hydraulic pressure must exercise a certain though possibly undefined salutary effect in delaying the disintegration of the substratum. However this may be, it is the

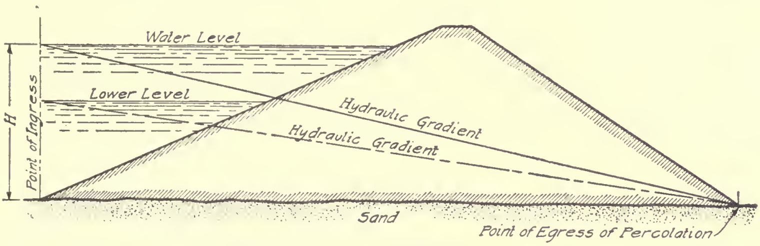


Fig. 87. Diagram Showing Effect of Percolation under Earthen Embankment across Stream

enforced length of percolation, or travel of the under current, that is now recognized to be the real determining influence.

In the case of a pipe, the induced velocity is inversely proportional to the length. In the case under consideration, the hydraulic condition being practically identical with that in a pipe, it is the enforced percolation through the sand, and the resulting friction against its particles as the water forces its way through, that effects the reduction of the velocity of the undercurrent, and this frictional resistance is directly proportional to the length of passage. In the case of Fig. 87, the length of enforced percolation is clearly that of the impervious base of the earthen dam. The moment this obstruction is passed the water is free to rise out of the sand and the hydrostatic pressure ceases.

116. Coefficient of Percolation. This length of enforced percolation or travel, which will be symbolized by L, must be some multiple of the head H, and if reliable safe values for this factor can be found, suitable to particular classes of sand, we shall be enabled to design any work on a sand foundation, with perfect confidence in its stability. If the percolation factor be symbolized by c, then L, or the length of enforced percolation, will equal cH, H being the head of water. The factor c will vary in value with the quality of the sand.

Fig. 88 represents a case similar in every respect to the last except, instead of a dam of earth, the obstruction consists of a vertical wall termed the weir or drop wall, having a horizontal impervious floor attached thereto, which appendage is necessary to prevent erosion of the bed by the current of falling water.

At the stage of maximum pressure the head water will be level with the crest, and the level of the tail water that of the floor; consequently the hydraulic gradient will be HB, which is also the piezometric line and as in the previous case of the pipe line, Fig.

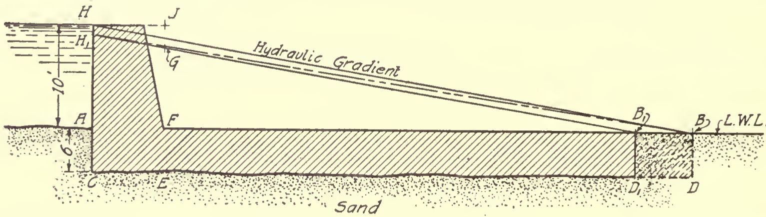


Fig. 88. Diagram Showing Design of Profile to Reduce Percolation

86, the ordinates of the triangle HAB will represent the upward hydrostatic pressure on the base of the weir wall and of the floor.

117. Criterion for Safety of Structure. The safety of the structure is evidently dependent on the following points:

First, the weir wall must be dimensioned to resist the overturning moment of the horizontal water pressure. This has been dealt with in a previous section. Second, the thickness, i. e., the weight of the apron or floor must be such that it will be safe from being blown up or fractured by the hydrostatic pressure; third, the base length, or that of the enforced percolation L, must not be less than cH, the product of the factor c with the head H. Fourth, the length of the masonry apron and its continuation in riprap or concrete book blocks must be sufficient to prevent erosion.

It is evident that the value of this factor c, must vary with the nature of the sand substratum in accordance with its qualities of fineness or coarseness. Fine light sand will be closer in texture, passing less water under a given head than a coarser variety, but at the same time will be disintegrated and washed out under less pressure. Reliable values for c, on which the design mainly depends, can only be obtained experimentally, not from artificial experiments, but by deduction from actual examples of weirs; among which the most valuable are the records of failures due to insufficiency in length of percolation. From these statistics a safe value of the relation of L to H, the factor c, which is also the sine of the hydraulic gradient, can be derived.

118. Adopted Values of Percolation Coefficient. The following values of c have been adopted for the specified classes of sand.

Class I: River beds of light silt and sand, of which 60 per cent passes a 100-mesh sieve, as those of the Nile or Mississippi; percolation factor $c = 18$.

Class II: Fine micaceous sand of which 80 per cent of the grains pass a 75-mesh sieve, as in Himalayan rivers and in such as the Colorado; $c = 15$.

Class III: Coarse-grained sands, as in Central and South India; $c = 12$.

Class IV: Boulders or shingle and gravel and sand mixed; c varies from 9 to 5.

In Fig. 88 if the sand extended only up to the level C, the length of percolation would be CD, the rise from D to B not being counted in. In that case the area of hydrostatic pressure acting beneath the floor would be the triangle HAB. As, however, a layer of sand from A to C interposes, the length will be ACD, and outline H_1B. The step HH_1 occurring in the outline is due to the neutralization of head symbolized by h, effected in the depth AC. Supposing AC to be 6 feet and the percolation factor to be 12, then the step in the pressure area, equal to h, will be $6 \div 12 = 6$ inches. The resulting gradient H_1B will, however, be flatter than 1 in 12; consequently the termination of the apron can be shifted back to B_1D_1, H_1B_1, being parallel to HB; in which case the area of hydrostatic pressure will be H_1AB_1. The pressure at any point on the base is represented by the ordinates of the triangle or area of pressure. Thus the upward pressure at E, below the toe of the drop wall, where the horizontal apron commences, is represented by the line

FG. Supposing the head HA to be 10 feet, then the total required length of percolation will be $cH = 12 \times 10 = 120$ feet. This is the length ACD_1. The neutralization of head, h, effected by the enforced percolation between H and G is represented by GJ, and supposing the base width of the drop wall CE to be 9 feet, AC being 6 feet, $h = \frac{6+9}{12} = 1\frac{1}{4}$ feet. The upward pressure FG is $(H-h) = 10 - 1\frac{1}{4}$ $= 8\frac{3}{4}$ feet.

The stepped upper line bounding the pressure area as has been noted in Part I, is termed the piezometric line, as it represents the level to which water would rise if pipes were inserted in the floor. It is evident from the above that when no vertical depressions occur in the line of travel that the piezometric line will coincide with the hydraulic gradient or virtual slope; when, however, vertical depressions exist, reciprocal steps occur in the piezometric line, which then falls below the hydraulic grade line. The piezometric line is naturally always parallel to the latter. The commencement of the floor at E is always a critical point in the design as the pressure is greatest here, diminishing to zero at the end.

119. Simplifying the Computations. In the same way that the water pressure is represented by the head producing it, the common factor w, or the unit weight of water, may also be eliminated from the opposing weight of the floor. The weight of the masonry, therefore, is represented by its thickness in the same way as the pressure, and if t be the thickness of the floor, $t\rho$ will represent its weight. Now the floor lies wholly below low water level. Consequently, in addition to the external hydrostatic pressure represented by H, due to the head of water upheld, there is the buoyancy due to immersion. The actual pressure on the base CD_1 is really measured by HC, not HA. Thus if a vertical pipe were inserted in the floor the water would rise up to the piezometric line and be in depth the ordinate of the pressure area plus the thickness of the floor. But it is convenient to keep the hydrostatic external pressure distinct from the effect of immersion. This latter can be allowed for by reduction in the weight of these parts of the structure that lie below L. W. L. See sections 52 and 53, Part I.

Effect of Immersion. When a body is immersed in a liquid it loses weight to the extent of the weight of the liquid displaced.

Thus the unit weight of a solid is $w\rho$. When immersed, the unit weight will be $w(\rho-1)$. As w is a discarded factor, the unit weight being represented only by ρ, the specific gravity, the weight of the floor in question will be $t(\rho-1)$ if immersed. We have seen that the hydrostatic pressure acting at F is $8\frac{3}{4}$ feet. To meet this the weight, or effective thickness, of the floor must be equal to $8\frac{3}{4}$ feet of water $+\frac{1}{3}$ for safety, or, in symbols, $t=\dfrac{H-h}{\rho-1}\times\dfrac{4}{3}$

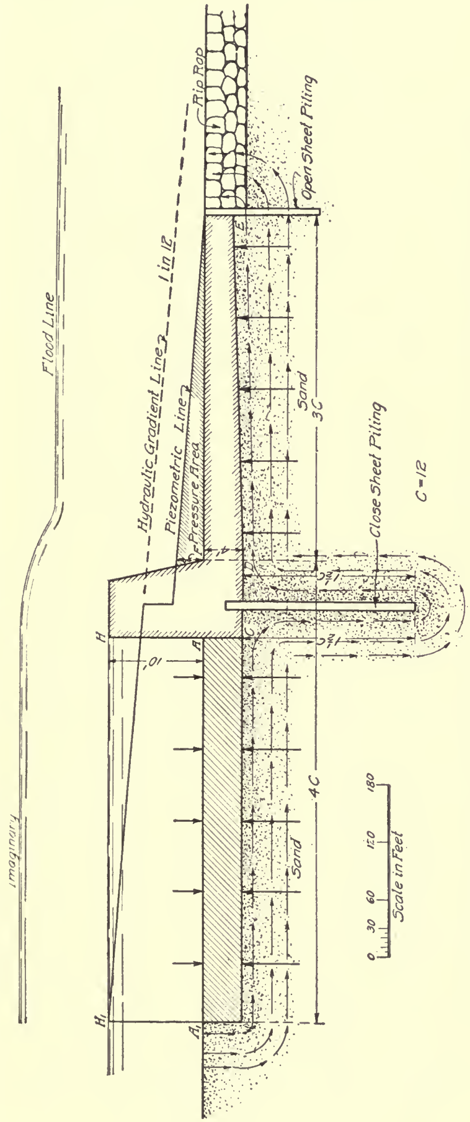


Fig. 89. Profile of Weir on Sand Showing Method of Providing Requisite Length of Travel of Percolation

Assuming a value for ρ of 2, the thickness required to counterbalance the hydrostatic pressure will be

$$t=8\tfrac{3}{4}\times\frac{4}{3}=11.6 \text{ feet}$$

The formula for thickness will then stand:

$$t=\frac{4}{3}\left(\frac{H-h}{\rho-1}\right) \qquad (33)$$

Uplift on Fore Apron. It is evident that in Fig. 88 the long floor is subjected to a very considerable uplift measured by the area HAB, the weight of the apron also is reduced in the ratio of $\rho:(\rho-1)$ as it lies below L.W.L., consequently it will have to be made as already noted of a depth of 11.6 feet which is a quite impossible figure. The remedy is either to make the floor porous in which case the hydraulic gradient will fall below 1 in 12, and failure will take place by piping, or else to reduce the effective head by the insertion of a rear apron or a vertical curtain wall as has been already mentioned in section 53, Part I. In these submerged weirs on large rivers and in fact in most overfall dams a solid fore apron is advisable. The length of this should however be limited to absolute requirements. This length of floor is a matter more of individual judgment or following successful precedent than one of precise estimation.

The following empirical rule which takes into account the nature of the sand as well as the head of water is believed to be a good guide in determining the length of fore apron in a weir of this type, it is

$$L=3\sqrt{cH} \qquad \textbf{(34)}$$

In the case of Fig. 89, the head is 10 feet and c is assumed at 12, consequently, $L=3\sqrt{120}=33$ feet, say 36, or $3c$. In Fig. 89 this length of floor has been inserted. Now a total length of percolation of 12 times the head, or 120 feet $=10c$ is required by hypothesis, of this $3c$ is used up by the floor leaving $7c$ to be provided by a rear apron and curtain. Supposing the curtain is made a depth equal to $1\frac{1}{2}c$, this will dispose of 3 out of the 7 (for reasons to be given later), leaving 4 to be provided for by the rear apron, the length of which, counting from the toe of the weir wall, is made $4c$ or 48 feet. The hydraulic gradient starts from the point H' which is vertically above that of ingress A. At the location of the vertical diaphragm of sheet piling, a step takes place owing to the sudden reduction of head of 3 feet, the obstruction being $3c$ in length counting both sides. From here on, the line is termed the piezometric line and the pressure area is the space enclosed between it and the floor. The actual pressure area would include the floor itself, but this has been already allowed for in reduction of weight, its s.g. being taken as unity instead of 2.

The uplift on the weir wall is the area enclosed between its base and the piezometric line. In calculating overturning moment, if this portion were considered as having lost weight by immersion it would not quite fully represent the loss of effective weight due to uplift, because above the floor level the profile of the weir wall is not rectangular, while that of the pressure area is more nearly so. The foundation could be treated this way, the superstructure above AF being given full s.g. and the uplift treated as a separate vertical force as was the case in Fig. 40, Part I.

120. Vertical Obstruction to Percolation. Now with regard to the vertical obstruction, when water percolates under pressure beneath an impervious platform the particles are impelled upward by the hydrostatic pressure against the base of the dam and also there is a slow horizontal current downstream. The line of least resistance is along the surface of any solid in preference to a shorter course through the middle of the sand, consequently when a vertical obstruction as a curtain wall of masonry or a diaphragm of sheet piling is encountered the current of water is forced downward and the obstruction being passed it ascends the other side up to the base line which it again follows. The outer particles follow the lead of the inner as is shown by the arrows in Fig. 89. The value of a vertical obstruction is accordingly twice that of a similar horizontal length of base. Valuable corroboration of the reliability of the theory of percolation adopted, particularly with regard to reduction of head caused by vertical obstruction, has been received, while this article was on the press, from a paper in the proceeedings of the American Society of Civil Engineers entitled "The Action of Water under Dams" by J. B. T. Coleman, which appeared in August, 1915. The practical value of the experiments, however, is somewhat vitiated by the smallness of the scale of operations and the disproportion in the ratio $H:L$ to actual conditions. The length of base of the dam experimented on should not be less than 50 feet with a head of 5 feet.

121. Rear Apron. The extension of the floor rearward is termed the rear apron. Its statical condition is peculiar, not being subject to any upward hydrostatic pressure as is the case with the fore apron or floor. Inspection of the diagram, Fig. 89, will show that the water pressure acting below the floor is the trapezoid enclosed between the piezometric line and the floor level; whereas the downward pressure is represented by the rectangle H_1A_1HA, which is considerably larger. Theoretically no weight is required in the rear apron, the only proviso being that it must be impervious and have a water-tight connection with the weir wall, otherwise the incidence of H may fall between the rear apron and the rest of the work, rendering the former useless. Such a case has actually occurred. It is, however, considered that the rear apron must be of a definite weight, as otherwise the percolation of water underneath it would partake of the nature of a surface flow, and so prevent any neutralization of head caused by friction in its passage through sand. Consequently, the effective thickness, or rather $t(\rho-1)$ should not be less than four feet. Its level need not be the same as that of the fore apron or floor. In fact, in some cases it has been constructed level or nearly so with the permanent crest

Fig. 90. View of Grand Barrage over Nile River

of the drop wall. But this disposition has the effect of reducing the coefficient of discharge over the weir and increasing the afflux or head water level, which is open to objection. The best position is undoubtedly level with the fore apron.

Another point in favor of the rear apron is the fact that it is free from either hydrostatic pressure or the dynamic force of falling water, to which the fore apron is subject; it can, therefore, be constructed of more inexpensive material. Clay consolidated when wet, i.e., puddle, is just as effective in this respect as the richest cement masonry or concrete, provided it is protected from scour where necessary by an overlay of paving or riprap, and has a reliable connection with the drop wall and the rest of the work. In old works these properties of the rear apron were not understood, and the stanching of the loose stone rear apron commonly provided, was left to be effected by the natural deposit of silt. This deposit eventually does take place and is of the greatest value in increasing the statical stability of the weir, but the process takes time, and until complete, the work is liable to excess hydrostatic pressure and an insufficient length of enforced percolation, which would allow piping to take place and the foundation to be gradually undermined.

122. First Demonstration of Rear Apron. The value of an impervious rear apron was first demonstrated in the repairs to the Grand Barrage over the Nile, Fig. 90, some time in the eighties. This old work was useless owing to the great leakage that took place whenever the gates were lowered and a head of water applied. In order to check this leakage, instead of driving sheet piling, which

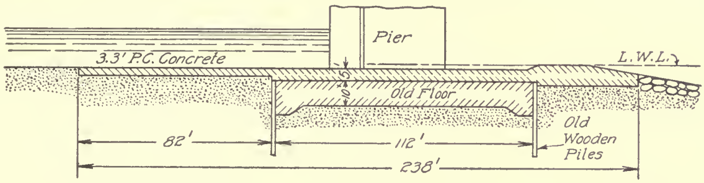


Fig. 91. Section Showing Repairs Made on the Grand Barrage

it was feared would shake the foundations, an apron of cement masonry 240 feet wide and 3.28 feet thick, Fig. 91, was constructed over the old floor, extending upstream 82 feet beyond it. This proved completely successful. By means of pipes set in holes drilled in the piers, cement mortar was forced under pressure into all the interstices of the rubble foundations, filling up any hollows that existed, thus completely stanching the foundations. So effectually was the structure repaired that it was rendered capable of holding up about 13 feet of water; whereas, prior to reconstruction, it was unsafe with a head of a little over three feet. The total length of apron is 238 feet, of which 82 feet projects upstream beyond the original floor and 44 feet downstream, below the floor itself, the latter having a width of 112 feet. The head H being 13 feet, and L being 238 feet, c, or the percolation factor is $\frac{238}{13} =$ 18, which is the exact value assigned for Nile sand in *Class I*, section 118. This value was not originally derived from the Grand Barrage, but from another work.

The utility of this barrage has been further augmented by the construction of two subsidiary weirs below it, see Figs. 92 and 106, across the two branches of the Nile delta. These are ten feet high and enable an additional height of ten feet to be held up by the gates of the old barrage, the total height being now $22\frac{1}{4}$ feet. The increased rise in the tail water exactly compensates for the additional head on the work as regards hydrostatic pressure, but the moment of the water pressure on the base of the masonry piers

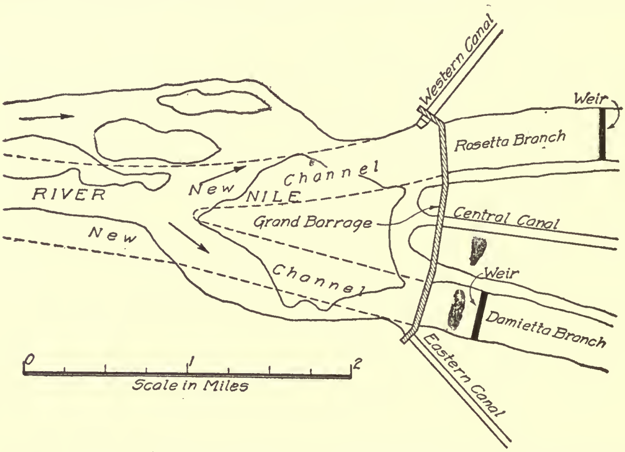


Fig. 92. Plan of Grand Barrage over Nile River Showing Also Location of Damietta and Rosetta Weirs

will be largely increased, viz, as from 13^3 to 22^3-10^3, or from 2197 to 9648.

The barrage, which is another word for "open dam" or bulkhead dam, is, however, of very solid and weighty construction, and after the complete renewal of all its weak points is now capable of safely enduring the increased stress put upon the superstructure. We have seen, in section 119, that the width of the impervious fore apron should be $L=3\sqrt{cH}$, formula (34). This width of the floor is affected by two considerations, first, the nature of the river bed, which can best be represented by its percolation factor c and second, by the height of the overfall including the crest shutters if any, which will be designated by H_a to distinguish it from H, which represents the difference between head and tail water and also

TABLE I

Showing Actual and Calculated Values of L_1 or Talus Width

Formula (35), $L_1 = 10c\sqrt{\frac{H_b}{10}} \times \sqrt{\frac{q}{75}}$

River	Name of Work	Type	c	H_b	q	Length L_1	
						Calculated	Actual
Ganges	Narora	A	15	10	75	150	140–170
Coleroon	Coleroon	A	12	$4\frac{1}{2}$	100	92	72
Vellar	Pelandorai	A	9	11	100	108	101
Tampra-parni	Srivakantham	A	12	6	90	102	106
Chenab	Khanki	B	15	7	150	182	170
Chenab	Merala	B	15	7	150	182	203
Jhelum	Rasul	B	15	6	155	160	135
Penner	Adimapali	B	12	$8\frac{1}{2}$	184	172	184
Penner	Nellore	B	12	9	300	228	232
Penner	Sangam	B	12	10	147	168	145
Godaveri	Dauleshwiram	B	12	13	100	158	217
Jumna	Okhla	C	15	10	140	210	210
Kistna	Beswada	C	12	13	223	236	220
Son	Dehri	C	12	8	66	100	96
Mahanadi	Jobra	C	12	100	140	163	143
Madaya	Madaya	C	12	8	280	207	235
Colorado	Laguna	C	15	10	below minimum	140	200

Type A has a direct overfall, with horizontal floor at L. W. L., as in Figs. 91, 93, and 95.

Type B has breast wall followed by a sloping impervious apron, Figs. 96 and 97.

Type C has breast wall followed by pervious rock fill, with sloping surface and vertical body walls, Figs. 101 to 106.

from H_b the height of the permanent crest above L. W. L. Taking the Narora weir as standard, a length of floor equal to $3\sqrt{cH}$ $=3\sqrt{15\times13}=42$ feet, is deemed to be the correct safe width for a weir 13 feet in height; where the height is more or less, the width should be increased or reduced in proportion to the square root of the height and that of the factor c.

123. Riprap to Protect Apron. Beyond the impervious floor a long continuation of riprap or packed stone pitching is required. The width of this material is clearly independent of that appropriate to the floor, and consequently will be measured from the same starting point as the floor, viz, from the toe of the drop wall. The formula for overfall weirs is

$$L_1 = 10c\sqrt{\frac{H_b}{10}} \times \sqrt{\frac{q}{75}} \tag{35}$$

For sloping aprons, type B, the coefficient of c will be 11
Then

$$L_1 = 11c\sqrt{\frac{H_b \times q}{10\ 75}} \tag{35a}$$

This formula is founded on the theory that the distance of the toe of the talus from the overfall will vary with the square root of the height of the obstruction above low water, designated by H_b, with the square root of the unit flood discharge over the weir crest q, and directly with c, the percolation factor of the river sand. The standard being these values; viz, 10, 75, and 10, respectively, in Narora weir. This height, H_b is equal to H when there are no crest shutters, and is always the depth of L. W. L. below the permanent masonry crest of the weir. This formula, though more or less empirical, gives results remarkably in consonance with actual value, and will, it is believed, form a valuable guide to design. Table I will conclusively prove this. As nearly all the weirs of this class have been constructed in India, works in that country are quoted as examples.

124. Example of Design Type A. Another example of design in type A will now be given, Fig. 93, the dimensions being those of an actual work, viz, the Narora weir over the Ganges River, the design being thus an alternative for that work, the existing section of which is shown in Fig. 95 and discussed in section 125. The data on which the design is based, is as follows: sand, class 2; percolation factor $c = 15$; H or difference between head and low water, the latter being always symbolized by L. W. L., 13 feet, unit dis-

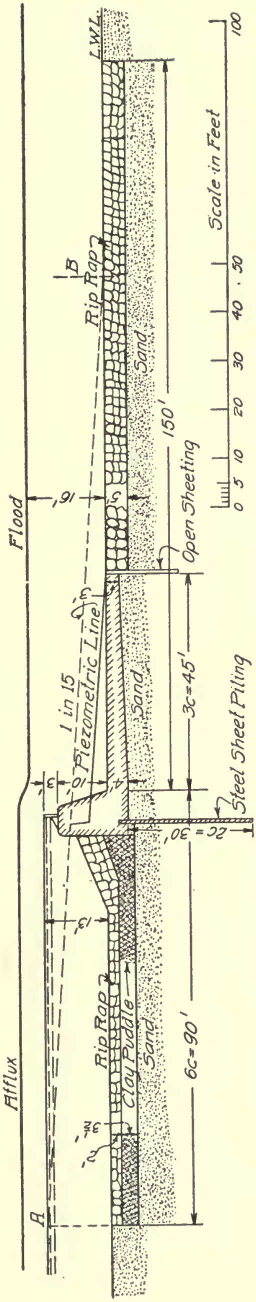


Fig. 93. Design of Weir on Sand Having Direct Overfall and Horizontal Flooring

charge over weir $q=75$ second-feet, the total length of the impervious apron and vertical obstructions will, therefore, have to be $L=cH=15\times 13=195$ feet.

The first point to be determined is the length of the floor or fore apron. Having fixed this length, the balance of L will have to be divided among the rear apron and the vertical sheet piling. It is essential that this minimum length be not exceeded, as it is clearly of advantage to put as much of the length into the rear apron as possible, owing to the inexpensive nature of the material of which it can be constructed. According to formula (**2**), $L=42$ feet, which is nearly equivalent to $3c$, or 45 feet, there thus remains $10c$ to be proportioned between the rear apron and the vertical curtain. If the latter be given a depth of $2c$, or 30 feet the length of travel down and up will absorb $4c$, leaving $6c$, or 90 feet for the rear apron. The measurement is taken from the toe of the drop wall. The neutralization of the whole head of 13 feet is thus accomplished. A second curtain will generally be desirable at the extremity of the fore apron as a precautionary measure to form a protection in case the loose riprap downstream from the apron is washed out or sinks. This curtain must have open joints to offer as little obstruction to percolation as possible. The outline of the pressure area, that is the piezometric line, is drawn as follows: $cH=195$ feet is measured horizontally on the base line of the pressure area, that is, at L. W. L. from a

Fig. 94. View of Narora Weir and Headworks

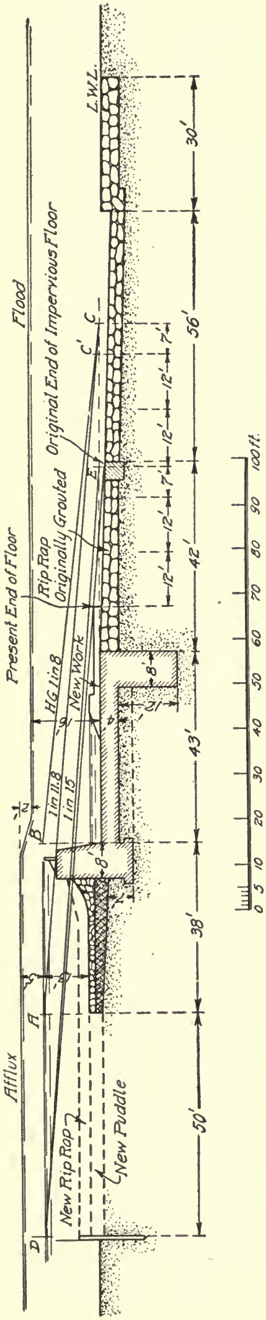


Fig. 95. Diagram of Narora Weir Showing Changes in Profile after Failure of Dam
The floor of this weir blew up due to uplift. Final repairs have proved satisfactory.

line through A to B. The point B is then joined with A on the head water level at the commencement of the rear apron. The hydraulic gradient will thus be 1 in 15. The intersection of this line BA with a vertical drawn through the first line of curtain is the location of a step of two feet equal to the head absorbed in the vertical travel at this point. Another line parallel to the hydraulic gradient is now drawn to the termination of the fore apron, this completes the piezometric line or the upper outline of the pressure area.

With regard to the floor thickness at the toe of the drop wall, the value of h, or loss of head due to percolation under the rear apron, is 6 feet, from the rear curtain, 4 feet; total 10 feet. $H-h$ is, therefore, $13-10=3$. The thickness of the floor according to formula (**33**) where $(H-h)=3$ feet comes to $\frac{4}{3}\times 3=4$ feet, the value of ρ being assumed at 2. The floor naturally tapers toward its end where the uplift is nil. The thickness at this point is made 3 feet which is about the minimum limit. There remains now the talus of riprap, its length from formula (**35**) is

$$L=10c\sqrt{\frac{H^b}{10}}\sqrt{\frac{q}{75}}=150\text{ feet}=10c$$

The thickness of the talus is generally four—often five feet—and is a matter of judgment considering the nature of the material used.

125. Discussion of Narora Weir. The Narora weir itself, Fig. 94, forms a most instructive object lesson, demonstrating what is the least correct base width, or length of percolation consistent with absolute safety, that can be adopted for sands of class 2. The system of analyzing graphically an existing work with regard to hydraulic gradient is exemplified in Fig. 95 under three separate conditions; *first,* as the work originally stood, with a hydraulic gradient of 1 in 11.8; *second,* at the time of failure, when the floor and the grouted riprap blew up. On this occasion owing to the rear apron having been washed away by a flood the hydraulic grade fell to 1 in 8; *third,* after the extension of the rear apron and curtailment of the fore apron had been effected. Under the first conditions the horizontal component of the length of travel or percolation L from A to E is 123 feet. The total length is made up of three parts: First, a step down and up in the foundation of the drop wall of 7 feet; second, a drop down and up of 12 feet either side of the downstream curtain wall; third, the horizontal distance 123 as above. The rise at the end of the floor is neglected. The total value of L is then $123+7+12+12=154$. This is set out on a horizontal line to the point C. AC is then the hydraulic gradient.

This demonstrates that the hydraulic gradient was originally something under 1 in 12, and in addition to this the floor is very deficient in thickness. The hydrostatic pressure on the floor at the toe of the drop wall is 8 feet. To meet this the floor has a value of $t\rho$ of only 5 feet. The specific gravity of the floor will not exceed 2, as it was mostly formed of broken brick concrete in hydraulic mortar. The value of $\rho-1$ will, therefore, be unity, the floor being submerged. In spite of this, the work stood intact to all external appearance for twenty years, when a heavy freshet in the river set up a cross current which washed out that portion of the rear apron nearest the drop wall, thus rendering the rest useless, the connection having been severed.

On this occurrence, failure at once took place, as the floor had doubtless been on the point of yielding for some time. In fact, this state of affairs had been suspected, as holes bored in the floor very shortly before the actual catastrophe took place showed that a large space existed below it, full, not of sand, but of water. Thus the floor was actually held up by the hydrostatic pressure; otherwise it must have collapsed. The removal of the rear apron caused this pressure to be so much increased that the whole floor, together with the grouted pitching below the curtain, blew up.

The hydraulic gradient BC is that at the time of the collapse. It will be seen that it is now reduced to 1 in 8. The piezometric line is not shown on the diagram.

In restoring the work the rear apron was extended upstream as shown dotted in Fig. 95, to a distance of 80 feet beyond the drop wall, and was made five feet thick. It was composed of puddle covered with riprap and at its junction with the drop wall was provided with a solid masonry covering. The puddle foundation also was sloped down to the level of the floor base to form a ground connection with the drop wall. At its upstream termination sheet piling was driven to a depth of twelve feet below floor level.

The grouted pitching in the fore apron was relaid dry, except for the first ten feet which was rebuilt in mortar, to form a continuation of the impervious floor. Omitting the mortar has the effect of reducing the pressure on the floor. Even then the uplift would have been too great, so a water cushion 2 feet deep was formed over the floor by building a dwarf wall of concrete (shown on the section) right along its edge. This adds 1 foot to the effective value of $t\rho$. It will be seen that the hydraulic gradient now works out to 1 in 15. A value for c of 15 has been adopted for similar light sands from which that of other sands, as *Classes I* and *III*, have been deduced.

It will be noticed that the crest of this weir is furnished with shutters which are collapsible when overtopped and are raised by hand or by a traveling crab that moves along the crest, raising the shutters as it proceeds. The shutters are 3 feet deep and some 20 feet long. They are held up against the water by tension rods hinged to the weir, and at about $\frac{1}{3}$ the height of the shutter, i.e., at the center of pressure.

126. Sloping Apron Weirs, Type B. Another type of weir, designated B, will now be discussed, in which there is no direct vertical drop, the fore apron not being horizontal but sloping from the crest to the L. W. L. or to a little above it, the talus beyond being also on a flat slope or horizontal.

In the modern examples of this type which will be examined, the height of the permanent masonry weir wall is greatly reduced, with the object of offering as little obstruction as possible to the passage of flood water. The canal level is maintained by means of deep crest shutters. In the Khanki weir, Fig. 96, the weir proper, or rather bar wall, is 7 feet high above L. W. L., while the shutters are 6 feet high. It, therefore, holds up 13 feet of water, the same as was the case with the Narora weir.

The object of adopting the sloping apron is to avoid construction in wet foundations, as most of it can be built quite in the dry above L. W. L. The disadvantage of this type lies in the constriction of the waterway below the breast wall, which causes the velocity of overfall to be continued well past the crest. With a direct overfall, on the other hand, a depth of 7 feet for water to churn in would be available at this point. This would check the flow and the increased area of the waterway rendered available

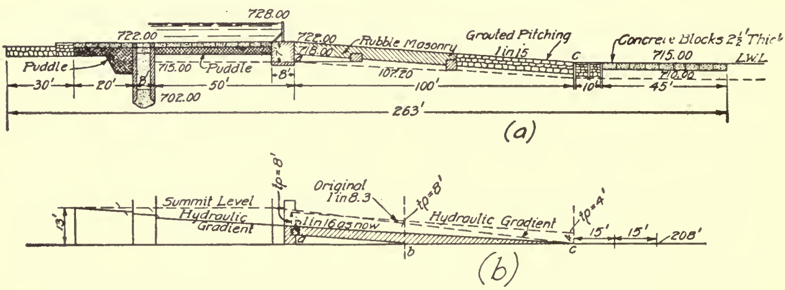


Fig. 96. Profile of Khanki Weir, Showing Restoration Work Similar to that of Narora Weir

should reduce the velocity. For this reason, although the action on the apron is possibly less, that on the talus and river bed beyond must be greater than in the drop wall of type A.

This work, like the former, failed for want of sufficient effective base length, and it consequently forms a valuable object lesson.

As originally designed, no rear apron whatever, excepting a small heap of stone behind the breast wall, was provided. The value of L up to the termination of the grouted pitching is but 108 feet; whereas it should have been cH or $15 \times 13 = 195$ feet. The hydraulic gradient, as shown in Fig. 96b, is only 1 in 8.3. This neglects the small vertical component at the breast wall. In spite of this deficiency in effective base width, the floor, owing to good workmanship, did not give way for some years, until gradually increased piping beneath the base caused its collapse.

Owing to the raised position of the apron, it is not subject to high hydrostatic pressure. At its commencement it is ten feet below the summit level and nine feet of water acts at this point. This is met by four feet of masonry unsubmerged, of s.g. 2, which almost balances it. Thus the apron did not blow up, as was the case with the Narora weir, but collapsed.

Some explanation of the graphical pressure diagram is required, as it offers some peculiarities, differing from the last examples. The full head, or H, is 13 feet. Owing, however, to the raised and sloping position of the apron, the base line of the pressure area will not be horizontal and so coincide with the L. W. L., but will be an inclined line from the commencement a to the point b, where the sloping base coincides with the L. W. L. From b where L. W. L. is reached onward, the base will be horizontal. With a sloping apron the pressure is nearly uniform, the water-pressure area is not wedge shaped but approximates to a rectangle. The apron, therefore, is also properly rectangular in profile, whereas in the overfall type the profile is, or should be, that of a truncated wedge.

127. Restoration of Khanki Weir. After the failure of this work the restoration was on very similar lines to that of the Narora weir. An impervious rear apron, seventy feet long, was constructed of puddle covered with concrete slabs, grouted in the joints. A rear curtain wall consisting of a line of rectangular undersunk blocks twenty feet deep, was provided. These additions have the effect of reducing the gradient to 1 in 16. The masonry curtain having regard to its great cost is of doubtful utility. A further prolongation of the rear apron or else a line of sheet piling would, it is deemed, have been equally effective. Reinforced-concrete sheet piling is very suitable for curtain walls in sand and is bound to supplant the ponderous and expensive block curtain walls which form so marked a feature in Indian works.

128. Merala Weir. Another weir on the same principle and quite recently constructed is the Merala weir at the head of the same historic river, the Chenab, known as the "Hydaspes" at the time of Alexander the Great.

This weir, a section of which is given in Fig. 97, is located in the upper reaches of the river and is subjected to very violent floods; consequently its construction has to be abnormally strong to resist

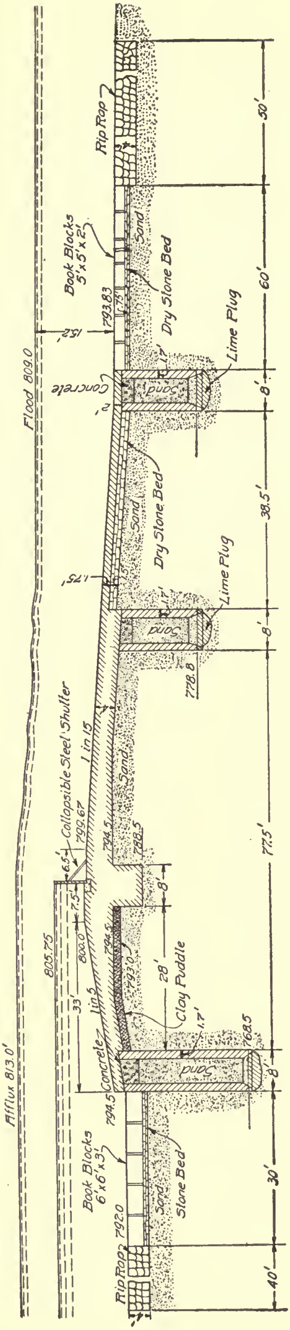


Fig. 97. Section of Merala Weir

the dynamic action of the water. This is entirely a matter of judgment and no definite rules can possibly be given which would apply to different conditions. From a hydrostatic point of view the two lower lines of curtain blocks are decidedly detrimental and could well be cut out. If this were done the horizontal length of travel or percolation will come to 140. The head is 12 or 13 feet. If the latter, c having the value 15 as in the Khanki weir, the value of L will be $15 \times 13 = 195$ feet. The horizontal length of travel is 140 feet and the wanting 55 feet will be just made up by the rear curtain. The superfluity of the two fore lines is thus apparent with regard to hydrostatic requirements. The long impervious sloping apron is a necessity to prevent erosion.

It is a question whether a line of steel interlocking sheet piling is equally efficient as a curtain formed of wells of brickwork 12×8 feet undersunk and connected with piling and concrete filling. The latter has the advantage of solidity and weight lacking in the former. The system of curtain walls of undersunk blocks is peculiar to India. In the Hindia Barrage, in Mesopotamia, Fig. 115, interlocking sheet piling has been largely employed in places where well foundations would have been used in India. This change is probably due to the want of skilled well sinkers, who in India are extremely expert and form a special caste.

The rear apron, in the Merala weir is of as solid construction as the fore apron and is built on a slope right up to crest level; this arrangement facilitates discharge. The velocity of approach must be very great to necessitate huge book blocks of concrete $6\times6\times3$ feet being laid behind the slope and beyond that a 40-foot length of riprap. The fore apron extends for 93 feet beyond the crest, twice as long as would be necessary with a weir of type A under normal conditions. The distance L of the talus is 203 feet against 182 feet calculated from formula (**35a**). That of the lower weir at Khanki is 170 feet. This shows that the empirical formula gives a fair approximation.

The fore apron in type B will extend to the toe of the slope or glacis. It is quite evident that the erosive action on a sloping apron of type B is far greater than that on the horizontal floor of type A; the

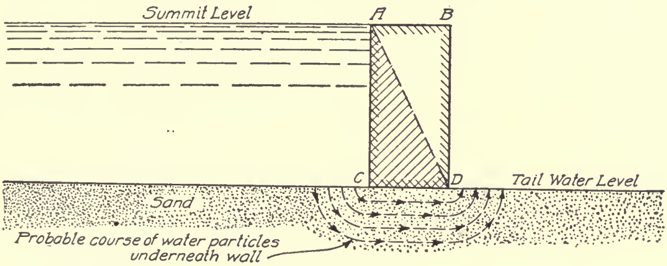


Fig. 98. Diagram Showing Effect of Percolation under a Wall Built on Sand

uplift however is less, consequently the sloping apron can be made thinner and the saving thus effected put into additional length.

129. Porous Fore Aprons. The next type of weir to be dealt with is type C. As it involves some fresh points, an investigation of it and the principles involved will be necessary. The previous examples of types A and B have been cases where the weir has as appendage an impervious fore apron which is subject to hydrostatic pressure. There is another very common type which will be termed C, in which there is no impervious apron and the material which composes the body of the weir is not solid masonry but a porous mass of loose stone the only impervious parts being narrow vertical walls. In spite of this apparent contrariety it will be found that the same principle, viz, that of length of enforced percolation, influences the design in this type as in the others.

Fig. 98 represents a wall upholding water to its crest and resting on a pervious substratum, as sand, gravel, or boulders, or a mixture of all three materials. The hydraulic gradient is AD; the upward pressure area ACD, and the base CD is the travel of the percolation. Unless this base length is equal to AC multiplied by the percolation factor obtained by experiment, piping will set in and the wall be undermined. Now as shown in Fig. 99, let a mass of loose stone be deposited below the wall. The weight of this stone will evidently have an appreciable effect in checking the disintegration and removal of the sand foundation. The water will not have a free untrammeled egress at D; it will, on the contrary, be compelled to rise in the interstices of the mass to a certain height EE determined by the extent to which the loose stones cause obstruction to the flow.

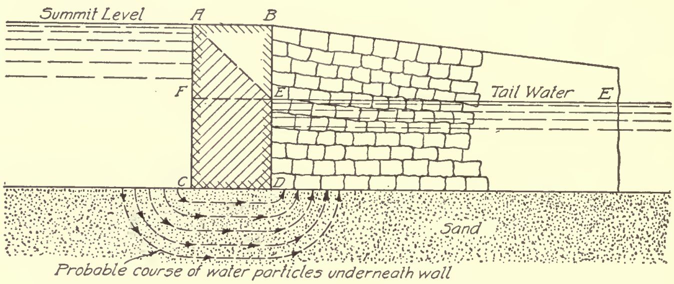


Fig. 99. Effect on Percolation Due to Stones below Weir Wall of Fig. 98

The resulting hydraulic gradient will now be AE—flatter than AD, but still too steep for permanency.

In Fig. 100, the wall is shown backed by a rear apron of loose stone, and the fore apron extended to F. The water has now to filter through the rear apron underneath the wall and up through the stone filling in the fore apron. During this process a certain amount of sand will be washed up into the porous body and the loose stone will sink until the combined stone and sand forms a compact mass, offering a greater obstruction to the passage of the percolating water than exists in the sand itself and possessing far greater resisting power to disintegration. This will cause the level of water at E to rise until equilibrium results. When this is the case the hydraulic gradient is flattened to some point near F. If a sufficiently long body is provided, the resulting gradient will be equal to that found by experiment to produce permanent equilibrium.

The mass after the sinking process has been finished is then made good up to the original profile by fresh rock filling. At F near the toe of the slope the stone offers but little resistance either by its weight or depth; so it is evident that the slope of the prism should be flatter than the hydraulic gradient.

The same action takes place with the rear apron, which soon becomes so filled with silt, as to be impervious or nearly so to the passage of water. But unless silt is deposited in the river bed behind, as eventually occurs right up to crest level, the thin portion of the rear slope, as well as the similar portion of the fore slope, cannot be counted as effective. Consequently out of the whole base length this part GF, roughly, about one-quarter, can be deemed inefficient as regards length of enforced percolation. As the consolidated lower part of the body of the weir gains in consistency, it can well be subject to hydrostatic pressure. Consequently, the value of $t\rho$ of the mass should be in excess of that of $H-h$, just as was the case with an impervious floor.

130. Porous Fore Aprons Divided by Core Walls. In Fig. 101 a still further development is effected by the introduction of vertical body or core walls of masonry in the pervious mass of the fore apron. These impervious obstructions materially assist the stability of the foundation, so much so that the process of underscour and settlement which must precede the balancing of the opposing forces in the purely loose stone mass need not occur at all, or to nothing like the same extent. If the party walls are properly spaced, the surface slope can be that of the hydraulic gradient itself and thus ensure equilibrium. This is clearly illustrated in the diagram. The water passing underneath the wall base CD will rise to

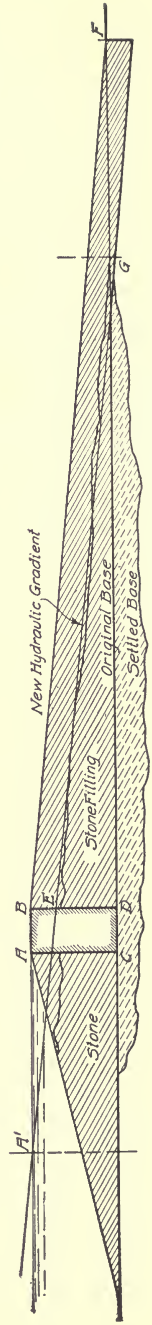


Fig. 100. Effect on Percolation When Weir Wall Is Provided with Rear Apron of Loose Stone and Extensive Fore Apron of Stone Filling

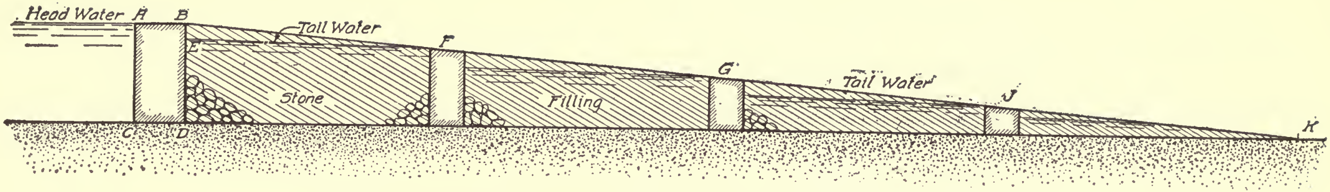


Fig. 101. Diagram of Weir Showing Porous Fore Apron Divided by Core Walls

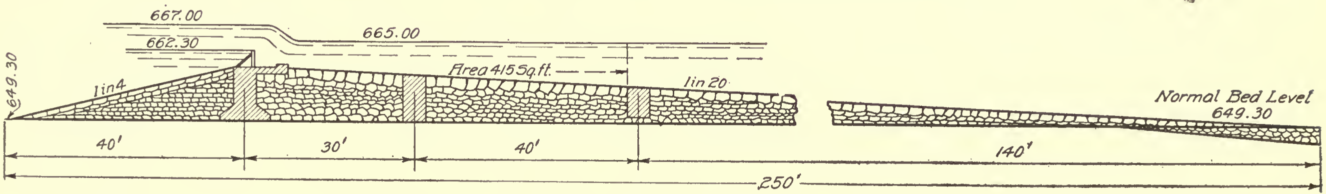


Fig. 102. Okhla Rock-Fill Weir

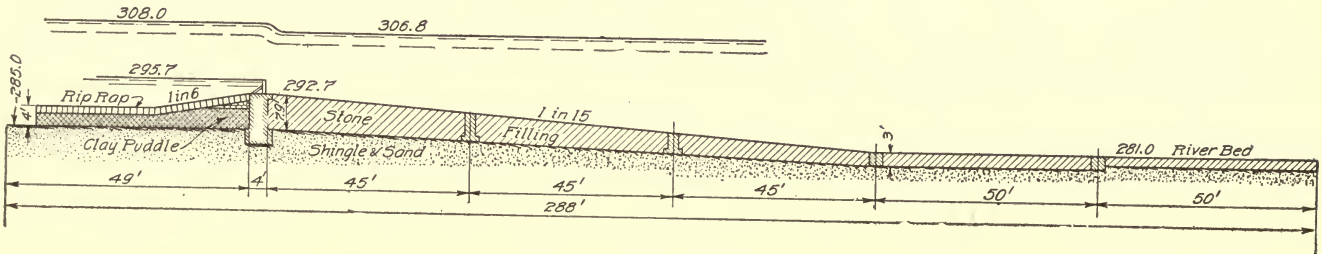


Fig. 103. Madaya Rock-Fill Weir

the level F, the point E being somewhat higher; similar percolation under the other walls in the substratum will fill all the partitions full of water. The head AC will, therefore, be split up into four steps.

Value of Rear Apron Very Great. The value of water tightness in the rear apron is so marked that it should be rendered impervious by a thick under layer of clay, and not left entirely to more or less imperfect surface silt stanching, except possibly in the case of high dams where a still settling pool is formed in rear of the work.

131. Okhla and Madaya Weirs. In Fig. 102 is shown a detailed section of the Okhla rock-filled weir over the Jumna River, India. It is remarkable as being the first rock-filled weir not provided with any lines of curtain walls projecting below the base line, which has hitherto invariably been adopted. The stability of its sand foundation is consequently entirely dependent on its weight and its effective base length. As will be seen, the section is provided with two body walls in addition to the breast wall. The slope of the fore apron is 1 in 20. It is believed that a slope of 1 in 15 would be equally effective, a horizontal talus making up the continuation, as has been done in the Madaya weir, Fig. 103, which is a similar work but under much greater stress.

The head of the water in the Okhla weir is 13 feet, with shutters up and weir body empty of water—a condition that could hardly exist. This would require an effective base length, L, of 195 feet; the actual is 250 feet. But, as noted previously, the end parts of the slopes cannot be included as effective; consequently the hydraulic gradient will not be far from 1 in 15. The weight of the stone, or $t\rho$, exactly balances this head at the beginning, as it is $10 \times 1.3 = 13$ feet. If the water were at crest level and the weir full of water, $t\rho$ would equal 8 feet, or rather a trifle less, owing to the lower level of the crest of the body wall. The head of 13 feet is broken up into four steps. The first is 3 feet deep, acting on a part of the rear apron together with 30 feet of the fore apron, say, 1 in 15; the rest are 1 in 20. A slope of 1 in 15 for the first party wall would cut the base at a point 40 feet short of the toe. Theoretically a fourth party wall is required at this point, but practically the riprap below the third dwarf wall is so stanched with sand as not to afford a free egress for the percolation; consequently the slope may

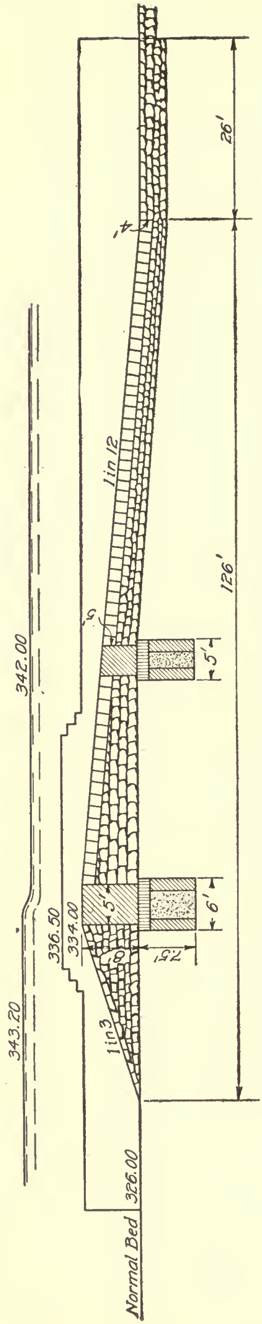


Fig. 104. Section of Dehri Weir Showing Steeper Gradient Than Fig. 103

be assumed to continue on to its intersection with the horizontal base. As already noted, material would be saved in the section by adopting a reliably stanch rear apron and reducing the fore slope to 1 in 15, with a horizontal continuation as was done in the Madaya weir.

Economy of Type C. This type C is only economical where stone is abundant. It requires little labor or masonry work. On the other hand, the mass of the material used is very great, much greater, in fact, than is shown by the section. This is owing to the constant sinking and renewal of the talus which goes on for many years after the first construction of the weir.

The action of the flood on the talus is undoubtedly accentuated by the contraction of the waterway due to the high sloping apron. The flood velocity 20 feet below the crest has been gaged as high as 18 feet per second. This would be very materially reduced if the A type of overfall were adopted, as the area of waterway at this point would be more than doubled.

132. Dehri Weir. Another typical example of this class is the Dehri weir over the Son River, Fig. 104. The value of L, if the apexes of the two triangles of stone filling are deducted and the curtain walls included, comes to about $12\,H$, 12 being adopted for this class of coarse sand. The curtain walls, each over 12,500 feet long, must have been enormously costly. From the experience of Okhla, a contemporary work, on a much worse class of sand, curtain protection is quite unnecessary if sufficient horizontal base width is provided. The head on this weir is 10 feet, and the height of breast wall 8 feet, $t\rho$ is, therefore, $1.3 \times 8 = 10.1$, which is sufficient, considering that the full head will not act here. The lines of curtains could be safely dispensed with if the following alterations were made: (1) Rear apron to be reliably stanched in order to throw back the incidence of pressure and increase the effective base length; (2) three more body walls to be introduced; (3) slope 1 in 12 retained, but base to be dredged out toward apex to admit of no thickness under five feet. This probably would not cause any increase in the quantities of masonry above what they now are, and would entirely obviate the construction of nearly five miles of undersunk curtain blocks.

133. Laguna Weir. The Laguna weir over the Colorado River, the only example of type C in the United States is shown in Fig. 105. Compared with other examples it might be considered as somewhat too wide if regard is had to its low unit flood discharge, but the inferior quality of the sand of this river probably renders this necessary. The body walls are undoubtedly not sufficiently numerous to be properly effective. The provision of an impervious rear apron would also be advantageous.

134. Damietta and Rosetta Weirs. The location of the Damietta and Rosetta subsidiary weirs, Fig. 106, which have been rather recently erected below the old Nile barrage, is shown in Fig. 92. These weirs are of type C, but the method of construction is quite

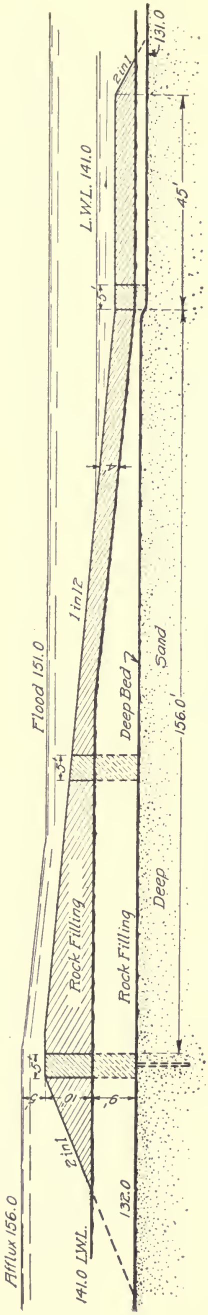


Fig. 105. Section of Laguna Weir over Colorado River

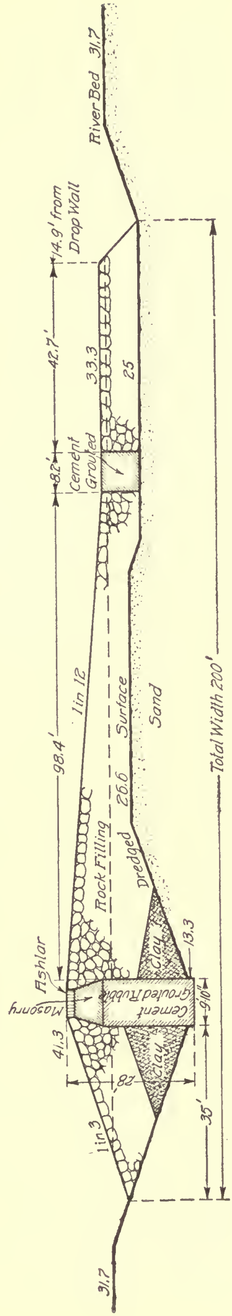


Fig. 106. Profile of Damietta and Rossetta Weirs

novel and it is this alone that renders this work a valuable object lesson. The deep foundation of the breast wall was built without any pumping, all material having been deposited in the water of the Nile River. First the profile of the base was dredged out, as shown in the section. Then the core wall was constructed by first depositing, in a temporary box or enclosure secured by a few piles, loose stone from barges floated alongside. The whole was then grouted with cement grout, poured through pipes let into the mass. On the completion of one section all the appliances were moved forward and another section built, and so on until the whole wall was completed. Clay was deposited at the base of the core wall and the profile then made up by loose stone filling.

This novel system of subaqueous construction has proved so satisfactory that in many cases it is bound to supersede older methods. Notwithstanding these innovations in methods of rapid construction, the profile of the weir itself is open to the objection of being extravagantly bulky even for the type adopted, the base having been dredged out so deep as to greatly increase the mean depth of the stone filling.

It is open to question whether a row or two rows of concrete sheet piles would not have been just as efficacious as the deep breast wall, and would certainly have been much less costly. The pure cement grouting was naturally expensive, but the admixture of sand proved unsatisfactory as the two materials of different specific gravity separated and formed layers; consequently, pure cement had to be used. It may be noted that the value of L here is much less than would be expected. At Narora weir it is $11c$, or 165 feet. Here, with a value of c of 18, it is but $8\frac{1}{2}c$, or 150 feet, instead of 200 feet, according to the formula. This is due to the low flood velocity of the Nile River compared with the Ganges.

135. The Paradox of a Pervious Dam. From the conditions prevailing in type C it is clear that an impervious apron as used in types A and B is not absolutely essential in order to secure a safe length of travel for the percolating subcurrent. If the water is free to rise through the riprap and at the same time the sand in the river bed is prevented from rising with it, the practical effect is the same as with an impervious apron. "Fountaining", as spouting sand is technically termed, is prevented and consequently also "piping". This latter term defines the gradual removal of sand from beneath a foundation by the action of the percolating under current. Thus the apparent paradox that a length of filter bed, although pervious, is as effective as a masonry apron would be. The hydraulic gradient in such case will be steeper than allowable under the latter circumstance. Filter beds are usually composed of a thick layer of gravel and stone laid on the sand of the river bed, the small stuff at the bottom and the larger material at the top. The ideal type of filter is one composed of stone arranged in sizes as above stated of a depth of 4 or 5 feet covered with heavy slabs or book blocks of concrete; these are set with narrow open intervals between blocks as shown in Figs. 96 and 97. Protection is thus afforded not only against scour from above but also from uplift underneath. Although the subcurrent of water can escape through a filter its free exit is hindered, consequently some hydrostatic pressure must still exist below the base, how much it is a difficult matter to determine, and it will therefore be left out of consideration. If the filter bed is properly constructed its length should be included in that of L or the length of travel. Ordinary riprap, unless exceptionally deep, is not of much, if any, value in this respect. The Hindia Barrage in Mesopotamia, Fig. 115, section 145, is provided with a filter bed consisting of a thick layer of stone 65.5 feet wide which occurs in the middle of the floor. The object of this is to allow the escape of the subcurrent and reduce the uplift on the dam and on that part of the floor which is impervious.

136. Crest Shutters. Nearly all submerged river weirs are provided with crest shutters 3 to 6 feet deep, 6 feet being the height adopted in the more recent works. These are generally raised by means of a traveling crane running on rails just behind the hinge of the gate. When the shutters are tripped they fall over this railway. In the case of the Merala weir, Fig. 97, the raising of the shutters is effected from a trolley running on overhead wires strung over steel towers erected on each pier. These piers or groins are 500 feet apart. The 6-foot shutters are 3 feet wide, held up by hinged struts which catch on to a bolt and are easily released by hand or by chains worked from the piers. On the Betwa weir the shutters, also 6 feet deep, are automatic in action, being hinged to a tension rod at about the center of pressure, consequently when overtopped they turn over and fall. Not all are hinged at the same height; they should not fall simultaneously but ease the flood gradually. The advantage of deep shutters is very great as the permanent weir can be built much lower than otherwise would be necessary, and thus offer much less obstruction to the flood. The only drawback is that crest shutters require a resident staff of experienced men to deal with them.

The Laguna weir, Fig. 105, has no shutters. The unit flood discharge of the Colorado is, however, small compared with that of the Indian rivers, being only 22 second-feet, whereas the Merala weir discharges 150 second-feet per foot run of weir, consequently shutters in the former case are unnecessary.

OPEN DAMS OR BARRAGES

137. Barrage Defined. The term "open dam", or barrage, generally designates what is in fact a regulating bridge built across a river channel, and furnished with gates which close the spans as required. They are partial regulators, the closure being only effected during low water. When the river is in flood, the gates are opened and free passage is afforded for flood water to pass, the floor being level with the river bed. Weir scouring sluices, which are indispensable adjuncts to weirs built over sandy rivers, belong practically to the same category as open dams, as they are also partial regulators, the difference being that they span only a portion of the river instead of the whole, and further are subject to great scouring action from the fact that when the river water is artificially raised above its normal level by the weir, the downstream channel is empty or nearly so.

Function of Weir Sluices. The function of weir sluices is twofold: First, to train the deep channel of the river, the natural course of which is obliterated by the weir, past the canal head, and to retain it in this position. Otherwise, in a wide river the low water channel might take a course parallel to the weir crest itself, or else one distant from the canal head, and thus cause the approach channel to become blocked with deposit.

Second, by manipulating the sluice gates, silt is allowed to deposit in the slack water in the deep channel. The canal is thereby preserved from silting up, and when the accumulation becomes excessive, it can be scoured out by opening the gates.

The sill of the weir sluice is placed as low as can conveniently be managed, being generally either at L. W. L. itself, or somewhat higher, its level generally corresponding with the base of the drop or breast wall. Thus the maximum statical head to which the work is subjected is the height of the weir crest plus that of the weir shutters, or H_1.

The ventage provided is regulated by the low-water discharge of the river, and should be capable of taking more than the average dry season discharge. In one case, that of the Laguna weir, where the river low supply is deficient, the weir sluices are designed to take the whole ordinary discharge of the river excepting the highest floods. This is with the object of maintaining a wide, deep channel which may be drawn upon as a reservoir. This case is, however, exceptional.

As the object of a weir sluice is to pass water at a high velocity in order to scour out deposit for some distance to the rear of the work, it is evident that the openings should be wide, with as few obstructions as possible in the way of piers, and should be open at the surface, the arches and platform being built clear of the flood level. Further, in order to take full advantage of the scouring power of the current, which is at a maximum at the sluice itself, diminishing in velocity with the distance to the rear of the work, it is absolutely necessary not only to place the canal head as close as possible to the weir sluices, but to recess the head as little

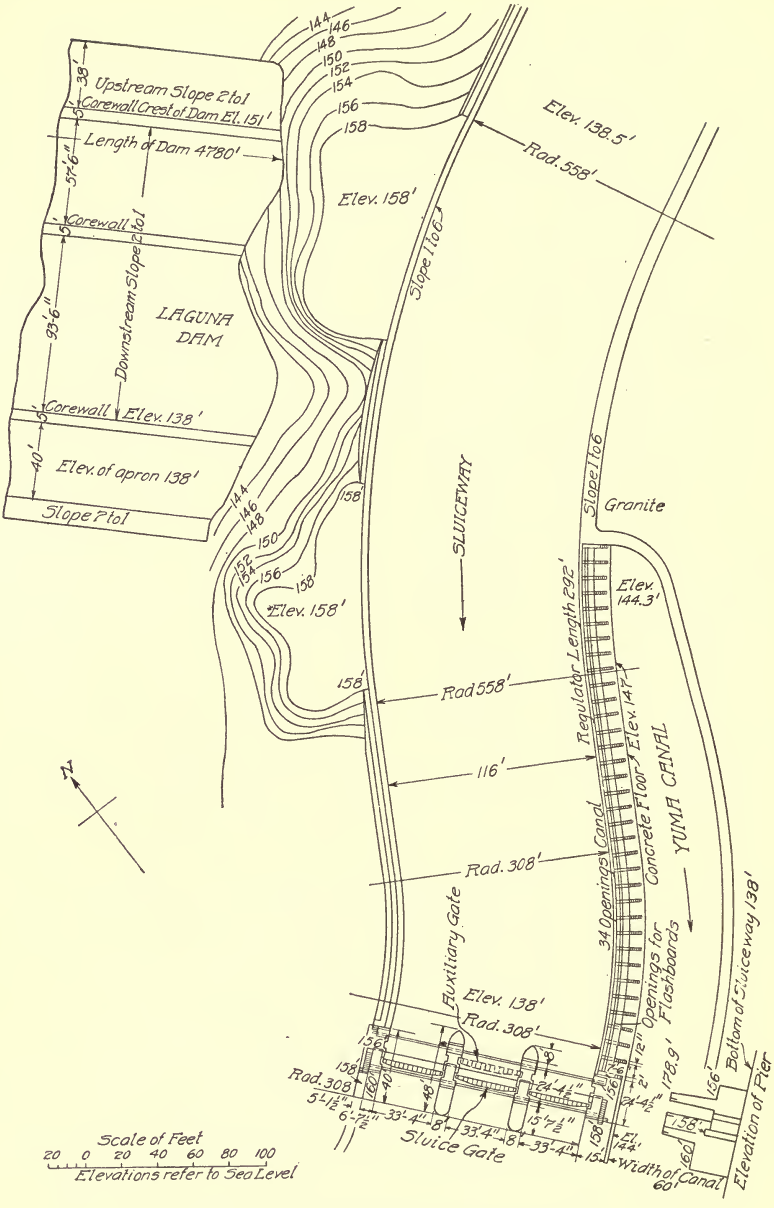


Fig. 107. Plan of Laguna Weir-Scouring Sluices

as practicable behind the face line of the abutment of the end sluice vent.

With regard to canal head regulators or intakes, the regulation effected by these is entire, not partial, so that these works are subjected to a much greater statical stress than weir sluices, and consequently, for convenience of manipulation, are usually designed with narrower openings than are necessary or desirable in the latter. The design of these works is, however, outside the scope of the subject in hand.

138. Example of Weir Scouring Sluice. Fig. 107 is an excellent example of a weir scouring sluice, that attached to the Laguna

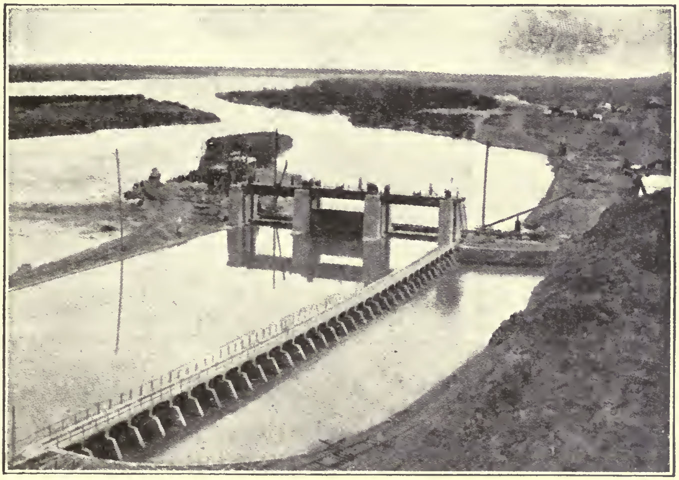


Fig. 108. View of Yuma Canal and Sluiceway Showing Sluice Gates under Construction

weir, the profile of which was given in Fig. 105. The Yuma canal intake is placed clear of the sandy bed of the river on a rock foundation and the sluiceway in front of it is also cut through solid rock independent of the weir. At the end of this sluiceway and just past the intake the weir sluices are located, consisting of three spans of $33\frac{1}{2}$ feet closed by steel counterweighted roller gates which can be hoisted clear of the flood by electrically operated winches. The gates are clearly shown in Fig. 108, which is from a photograph taken during the progress of the work. The bed of the sluiceway is at *El.* 138.0, that of the canal intake sill is 147.0, and that of the weir crest 151.0—hence the whole sluiceway can be allowed to fill up with deposit to a depth of 9 feet, without interfering with the

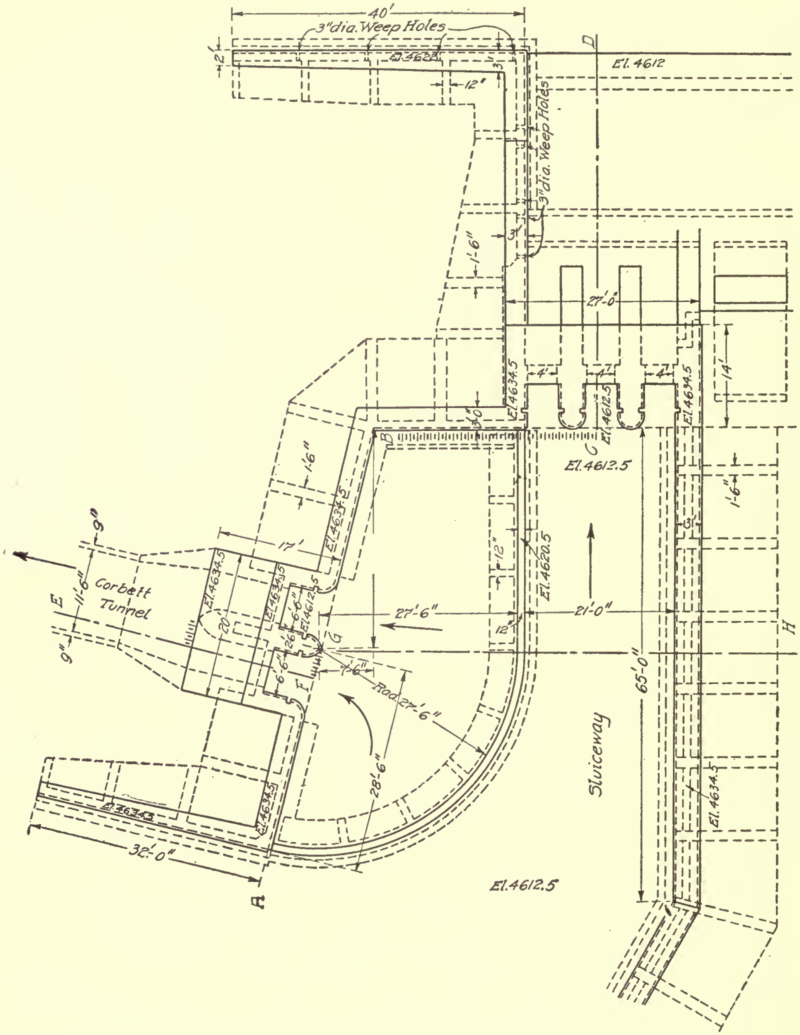


Fig. 109. Plan of Weir Sluices for Corbett Dam on Shoshone River, Wyoming

discharge of the canal, or if the flashboards of the intake are lowered the sluiceway can be filled up to *El.* 156 which is the level of the top of the draw gates, i.e., 18 feet deep. The difference between high

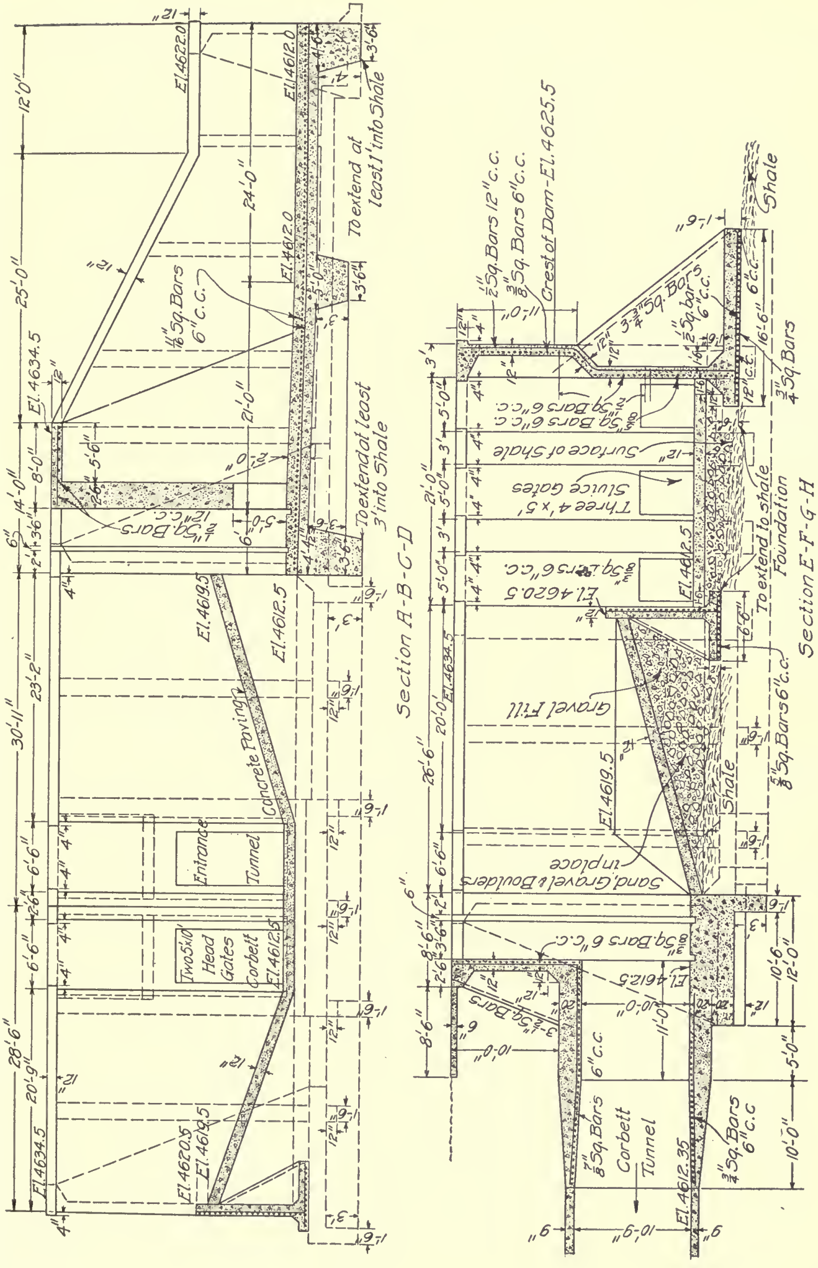


Fig. 110. Design Diagrams for Weir Sluices of Corbett Dam

Fig. 111. View of Corbett Dam on Shoshone River in Winter

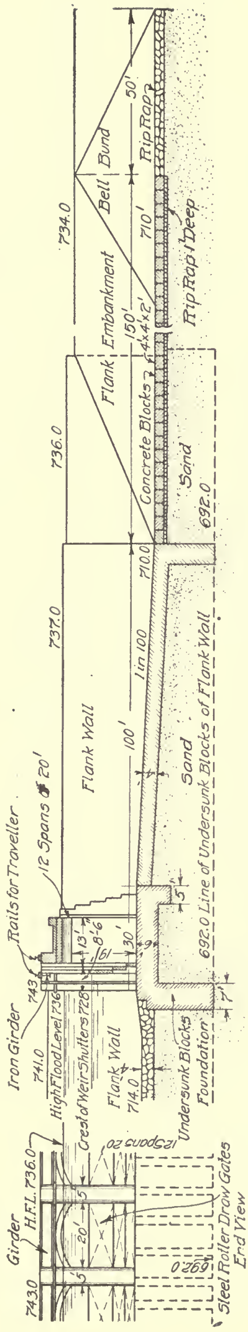


Fig. 112. Section of Undersluices of Khanki Weir over Chenab River, India

water above and below the sluice gates is 11 feet, consequently when the gates are lifted immense scour must take place and any deposit be rapidly removed. The sluiceway is in fact a large silt trap.

139. Weir Sluices of Corbett Dam. The weir sluices of the Corbett dam on the Shoshone River, Wyoming, are given in Figs. 109, 110, and 111.

The canal takes out through a tunnel, the head of which has necessarily to be recessed far behind the location of the weir sluices. Unless special measures were adopted, the space between the sluice gates and the tunnel head would fill up with sand and deposit and block the entrance.

To obviate this a wall 8 feet high is built encircling the entrance. A "divide" wall is also run out upstream of the weir sluices, cutting them off from the weir and its approaches. The space between these two walls forms a sluiceway which draws the current of the river in a low stage past the canal head and further forms a large silt trap which can be scoured out when convenient. Only a thin film of surface water can overflow the long encircling wall, then it runs down a paved warped slope which leads it into the head gates, the heavy silt in suspension being deposited in the sluiceway. This arrangement is admirable.

The fault of the weir sluices as built is the narrowness of the openings which consist of three spans of 5 feet. One span of 12 feet would be much more effective. In modern Indian practice, weir sluices on large rivers are built with 20 to 40 feet openings.

140. Weir Scouring Sluices on Sand. Weir scouring sluices built on pure sand on as large rivers as are met with in India are very formidable works, provided with long aprons and deep lines of curtain blocks. An example is given in Fig. 112 of the so-termed undersluices of the Khanki weir over the Chenab River in the Punjab. The spans are 20 feet, each closed by 3 draw gates, running in parallel grooves, fitted with antifriction wheels (not rollers), lifted by means of traveling power winches which straddle the openings in which the grooves and gates are located.

The Merala weir sluices of the Upper Chenab canal have 8 spans of 31 feet, piers $5\frac{1}{2}$ feet thick, double draw gates 14 feet high.

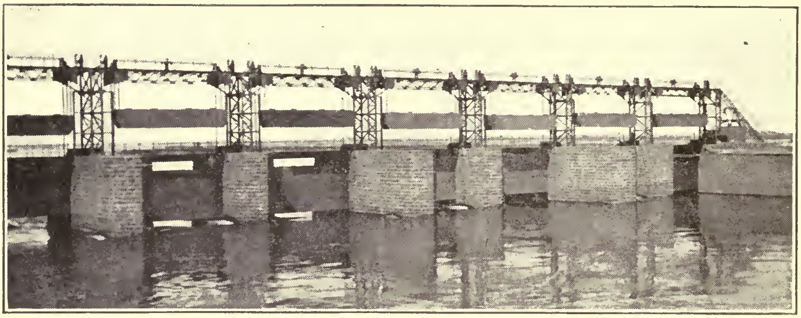


Fig. 113. View of Merala Weir Sluices, Upper Chenab Canal

These are lifted clear of the flood, which is 21 feet above floor, by means of steel towers 20 feet high erected on each pier. These carry the lifting apparatus and heavy counterweights. These gates, like those at Laguna weir, Fig. 108, bear against Stoney roller frames.

Fig. 113 is from a photograph of the Merala weir sluices. The work is a partial regulator, in that complete closure at high flood is not attempted. The Upper Chenab canal is the largest in the world with the sole exception of the Ibramiyah canal in Egypt, its discharge being 12,000 second-feet. Its depth is 13 feet. The capacity of the Ibramiyah was 20,000 second-feet prior to head regulation.

141. Heavy Construction a Necessity. In works of this description solid construction is a necessity. Light reinforced concrete construction would not answer, as weight is required, not only to withstand the hydrostatic pressure but the dynamic effects of flood water in violent motion. Besides which widely distributed

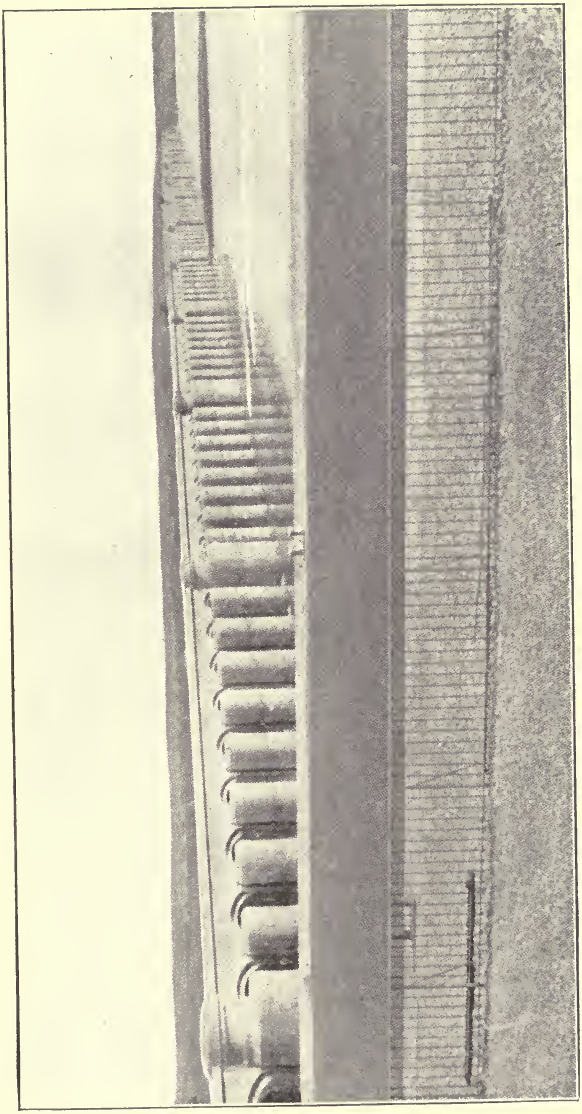


Fig. 114. View of the Assiut Barrage across the Nile

weight is undoubtedly necessary for works built on the shifting sand of a river bed, although this is a matter for which no definite rules can be formed.

The weir sluices at Laguna and also at the Corbett dam, are solid concrete structures without reinforcement.

In the East, generally, reinforced concrete is not employed nor is even cement concrete except in wet foundations, the reason being that cement, steel, and wood for forms are very expensive items whereas excellent natural hydraulic lime is generally available, skilled and unskilled labor is also abundant. A skilled mason's wages are about 10 to 16 cents and a laborer's 6 to 8 cents for a 12-hour day. Under such circumstances the employment of reinforced cement concrete is entirely confined to siphons where tension has to be taken care of.

In America, on the other hand, the labor conditions are such that reinforced concrete which requires only unskilled labor and is mostly made up by machinery, is by far the most suitable form of construction from point of view of cost as well as convenience.

This accounts for the very different appearance of irrigation works in the East from those in the West. Both are suitable under the different conditions that severally exist.

142. Large Open Dams across Rivers. Of open dams built across rivers, several specimens on a large scale exist in Egypt. These works, like weir sluices are partial regulators and allow free passage to flood water.

Assiut Barrage. In the Assiut barrage, Figs. 114 and 115,* constructed across the Nile above the Ibramiyah canal head in lower Egypt, the foundations are of sand and silt of a worse quality than is met with in the great Himalayan rivers. The value of c adopted for the Nile is 18, against 15 for Himalayan rivers. This dam holds up 5 meters of water, the head or difference of levels being 3 meters. Having regard to uplift, the head is the difference of levels but when considering overturning moment, on the piers, $\frac{H^3}{6}-\frac{h^3}{6}$ is the moment, H and h being the respective depths of water above and below the gates. It is believed that in the estimation of the length of travel the vertical sheet piling was left out of consideration. Inspection of the section in Fig. 115

* In Figs. 115 and 116 and in the discussion of these problems in the text, the metric dimensions used in the plans of the works have been retained. Meters multiplied by the factor 3.28 will give the proper values in feet.

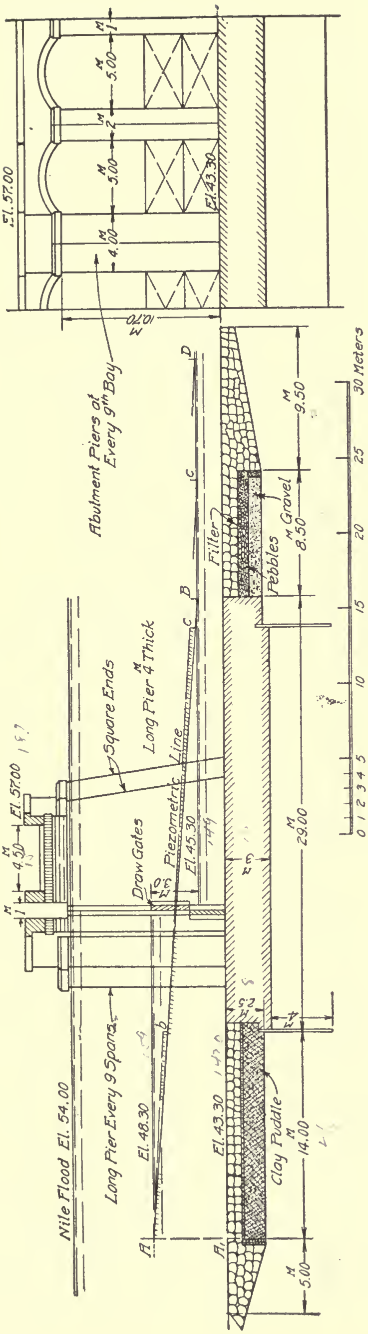


Fig. 115. Profile of the Assiut Barrage

shows that the foundation is mass cement concrete, 10 feet deep, on which platform the superstructure is built. This latter consists of 122 spans of 5 meters, or 16 feet, with piers 2 meters thick, every ninth being an abutment pier 4 meters thick and longer than the rest. This is a work of excessive solidity the ratio of thickness of piers to the span being .48, a proportion of .33 S would, it is considered be better. This could be had by increasing the spans to 6 meters, or 20 feet right through, retaining the pier thickness as it is at present.

143. General Features of River Regulators. All these river regulators are built on the same general lines, viz, mass foundations of a great depth, an arched highway bridge, with spring of arch at flood level, then a gap left for insertion of the double grooves and gates, succeeded by a narrow strip of arch sufficient to carry one of the rails of the traveling winch, the other resting on the one parapet of the bridge.

The piers are given a batter downstream in order to better distribute the pressure on the foundation. The resultant of the weight of one span combined with the horizontal water pressure must fall within the middle third of the base of the pier, the length of which can be manipulated to bring this about. In this case it does so even with increase of the span to 6 meters. This combined work is of value considered from a military point of view, as affording a crossing of the Nile River; consequently the extreme solidity of its construction was probably considered a necessity.

In some regulators girders are substituted for arches, in others as we have noted with regard to the Merala weir sluices, the superstructure above the flood line is open steel work of considerable height.

144. Stability of Assiut Barrage. The hydraulic gradient in Fig. 115, neglecting the vertical sheet piling, is drawn on the profile and is the line AB, the horizontal distance is 43 meters while the head is 3 meters. The slope is therefore 1 in $14\frac{1}{3}$. The uplift is the area enclosed between AB and a horizontal through B which is only 1.4 meters at its deepest part near the gates. Upstream of the gates the uplift is more than balanced by the weight of water overlying the floor. The horizontal travel of the percolation is from A to B plus the length of the filter as explained in section 135. The horizontal travel is therefore $51\frac{1}{2}$ meters and the ratio $\frac{L}{H}$, or c, is $\frac{51.5}{3} = 17.2$. The piezometric line has also been shown, including in this case the two vertical obstructions. Their effect on the uplift is very slight, owing to the fore curtain which raises the grade line. The slope in this case is obtained by adding the vertical to the horizontal travel, i.e., from B to D, BC and CD being 8 meters each in length, AD is then the hydraulic gradient which is 1 in 23. Steps occur at points b and c; for instance the line AB is part of AD, the line bc is parallel to AD drawn up from C, and the line cB is similarly drawn up from B forming the end step.

This work is the first to be built with a filter downstream, which has the practical effect of adding to the length of percolation travel irrespective of the hydraulic grade.

145. The Hindia Barrage. The Hindia barrage, quite recently erected over the Euphrates River near Bagdad, is given in Fig. 116. This work, which was designed by Sir William Willocks, bears a

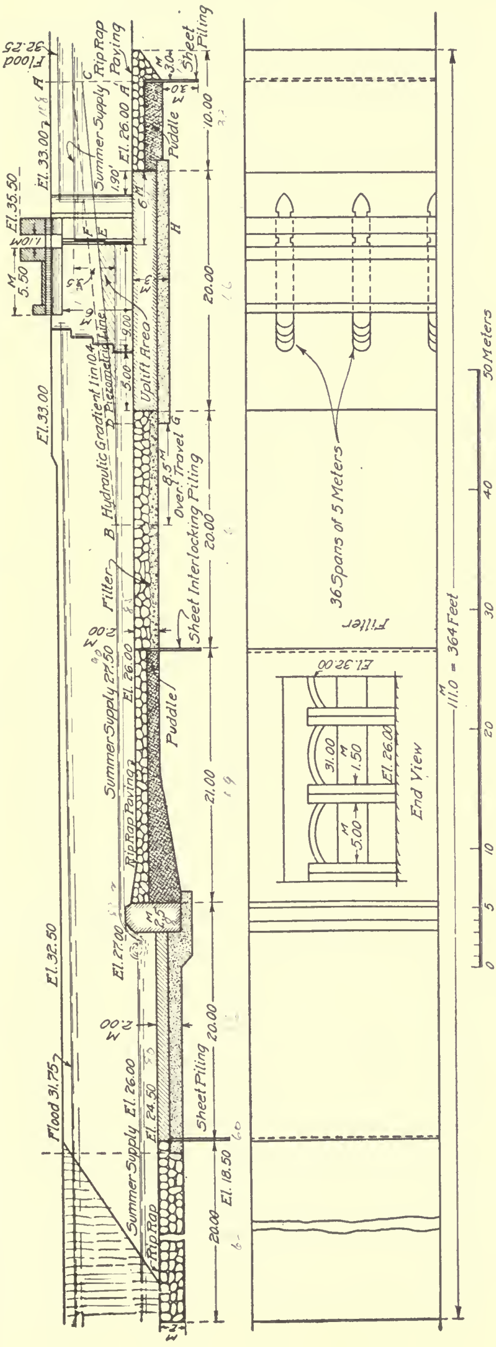


Fig. 116. Plan and Section of the Hindia Barrage across the Euphrates River

close resemblance to the Egyptian regulators, viz, the Assiut, the Zifta, and other works constructed across the river Nile. The piers are reduced to 1.50 meters from the 2-meters thickness in the Assiut dam, Fig. 115, and there are no abutment piers, consequently the elevation presents a much lighter appearance. The ratio of thickness to span is .3. In order to reduce the head on the work, a filter bed 20 meters wide is introduced just beyond the platform of the foundation of the regulating bridge. The upward pressure is thus presumed to be nil at the point *D*. The head is the distance between the summer supply level upstream, and that downstream above the subsidiary weir, this amounts to 3.50 meters. The length of compulsory travel from A to B including .50 meter due to the sheet piling is 36.50 meters. AB is then the hydraulic gradient, which is 1 in $\frac{36.5}{3.5}$ $=1$ in 10.4. The piezometric line DFC is drawn up from D parallel to AB. The area of uplift is $DGHEF$; that part of the uplift below the line DE is however accounted for by assuming all masonry situated below *El.* 27.50 as reduced in weight by flotation, leaving the area DEF as representing the uplift still unaccounted for.

Beyond the filter is a 21-meter length of impervious apron consisting of clay puddle covered by stone paving, which abuts on a masonry subsidiary weir. This wall holds the water up one meter in depth and so reduces the head to that extent, with the further addition of the depth of film passing over the crest at low water which is .5 meter, total reduction 1.50 meters.

This is the first instance of the use of puddle in a fore apron, or talus; its object is, by the introduction of an impervious rear apron 21 meters long, to prevent the subsidiary weir wall from being undermined. The head being $1\frac{1}{2}$ meters, the length of travel required, taking c as 18, will be $18 \times 1.5 = 27$ meters. The actual length of travel provided is vertical 15, horizontal 41, total 56 meters, more than double what is strictly requisite. The long hearth of solid masonry which is located below the subsidiary drop wall is for the purpose of withstanding scour caused by the overfall. Beyond this is the talus of riprap 20 meters wide and a row of sheet piling. The total length of the floor of this work is 364 feet, with three rows of sheet piles. That of the Assiut barrage is 216 feet with two rows of sheeting. The difference in head is half a meter only, so that certain unknown conditions of flood or that of the material in the bed must exist to account for the excess.

146. American vs. Indian Treatment. In American regulating works it is generally the fashion where entire closure is required to locate the draw gates and their grooves inside the panel or bulkhead wall that closes the upper part of the regulator above the sluice openings. Thus when the gates are raised they are concealed behind the panel walls. In Indian practice the gate grooves in the piers are generally located outside the bulkhead wall; thus when hoisted, the gates are visible and accessible. Fig. 117 is from a photograph of a branch head, illustrating this. The work is of reinforced concrete as can be told from the thinness of the piers. In an Indian work of similar character the pier noses would project well beyond the face wall of the regulator and the gates would be raised in front, not behind it.

The use of double gates is universal in Eastern irrigation works; they have the following unquestionable advantages over a single gate: *First,* less power for each is required to lift two gates than one; *second,* when hoisted they can be stacked side by side and so the pier can be reduced in height; *third,* where sand or silt is in suspension, surface water can be tapped by leaving the lower leaf down while the upper is raised; and *fourth,* regulation is made easier.

Fig. 117. Typical American Regulating Sluices in Reinforced-Concrete Weir

In the Khanki weir sluices, Fig. 112, 3 gates running in 3 grooves are employed.

147. Length of Spans. In designing open dams the spans should be made as large as convenient, the tendency in modern design is to increase the spans to 30 feet or more; the Laguna weir sluices are $33\frac{1}{2}$ feet wide and the Merala 31 feet. The thickness of the piers is a matter of judgment and is best expressed as some function of the span, the depth of water by which the height of the piers is regulated, forms another factor.

The depth of water upheld regulates the thickness more than the length. The length should be so adjusted that the resultant line of pressure combined of the weight of one pier and arch, or superstructure and of the water pressure acting on one span falls within the middle third of the base.

For example take the Assiut regulator, Fig. 115. The contents of one pier and span allowing for uplift is roughly 390 cubic meters of masonry, an equivalent to 1000 tons. The incidence of W is about 2 meters from the middle third downstream boundary.

The moment of the weight about this point is therefore $1000\times 2=2000$ meter tons. Let H be depth of water upstream, and h downstream, then the overturning moment is expressed by $\frac{(H^3-h^3)wl}{6}$. Here $H=5$, $h=2$ meters, $w=1.1$ tons per cubic meter, the length l of one span is 7 meters; then the moment $=\frac{(125-8)\times 1.1\times 7}{6}=150$ meter tons. The moment of resistance is therefore immensely in excess of the moment of water pressure. The height of the pier is however governed by the high flood level, the width by the necessity of a highway bridge. At full flood nearly the whole of the pier will be immersed in water and so lose weight. There is probably some intermediate stage when the water pressure will be greater than that estimated, as would be the case if the gates were left closed while the water topped them by several feet, the water downstream not having had time to rise to correspond.

148. Moments for Hindia Barrage. In the case of the Hindia barrage, Fig. 116, $H=5$ meters, $h=1.5$, then

$$M=\frac{(125-3.4)\times 1.1\times 6.50}{6}=145 \text{ meter tons}$$

The weight of one span is estimated at 180 tons. Its moment about the toe of the base is about $180\times 6.5=1170$ meter tons. The factor of safety against overturning is therefore $\frac{1170}{145}=8$. The long base of these piers is required for the purpose of distributing the load over as wide an area as possible in order to reduce the unit pressure to about one long ton per square foot. This is also partly the object of the deep mass foundation. The same result could doubtless be attained with much less material by adopting a thin floor say two or three feet thick, reinforced by steel rods so as to ensure the distribution of the weight of the superstructure evenly over the whole base. It seems to the writer that the Assiut barrage with its mass foundation having been a success

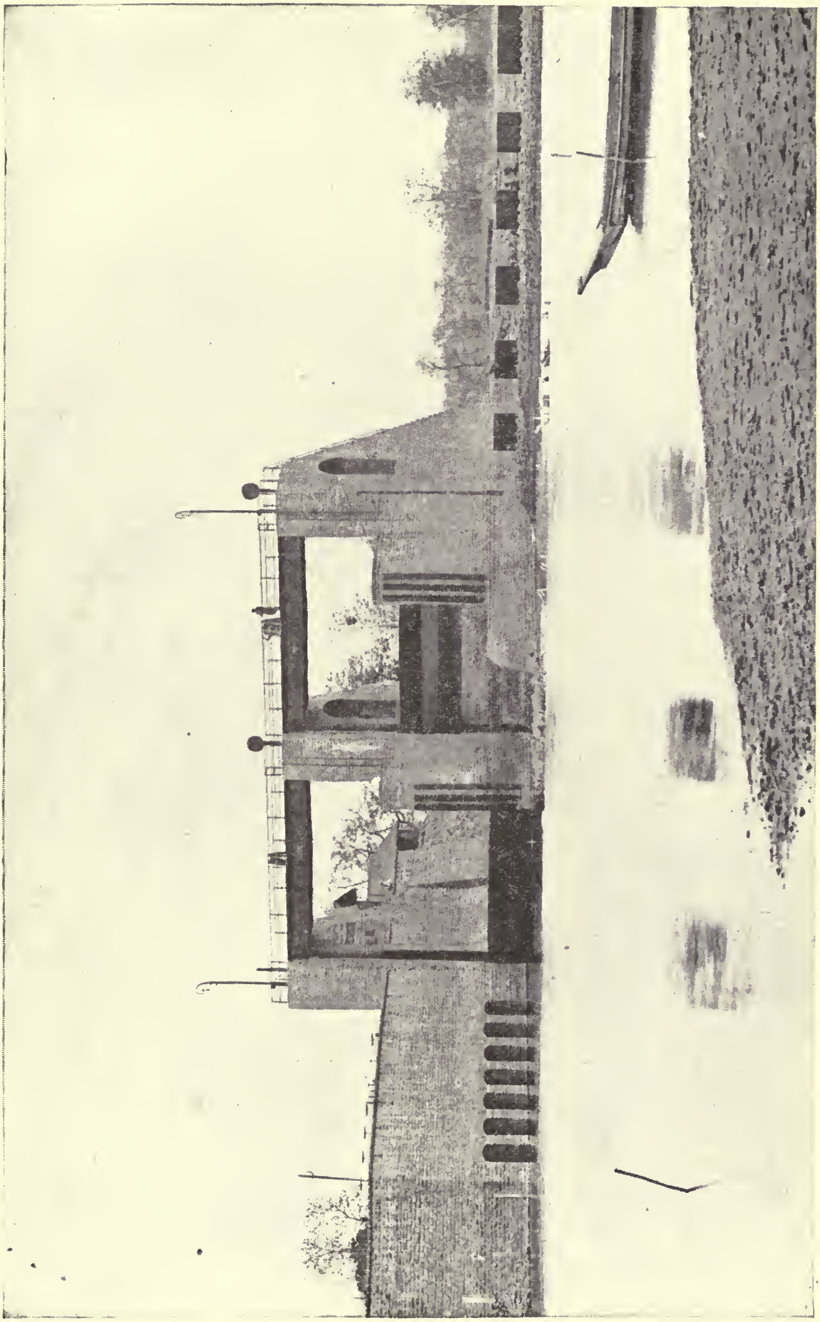


Fig. 118. Head Regulator and Undersluices of North Mon Canal in Burma, Showing Portion of Weir

TABLE II

Pier Thickness—Suitable for Open Partial Regulators and Weir Sluices

Span	Depths of Water							
	15 Feet		20 Feet		25 Feet		30 Feet	
	M.	*T.*	*M.*	*T.*	*M.*	*T.*	*M.*	*T.*
10 feet	.25	2.5	.27	2.7	.29	2.9	.31	3.1
15 feet	.24	3.6	.26	3.9	.28	4.2	.30	4.5
20 feet	.23	4.6	.25	5.0	.27	5.4	.29	5.8
25 feet	.21	5.3	.24	6.0	.26	6.5	.28	7.0

M is multiplier of span for thickness T.

as regards stability, is no reason why a heavy style of construction such as this should be perpetuated.

149. North Mon Canal. In Fig. 118 is shown the head works of the Mon right canal in Burma. The weir is of type A, with crest shutters and sluices of large span controlled by draw gates. In the canal head, the gates are recessed behind the face wall as in American practice.

150. Thickness of Piers. Table II, though purely empirical will form a useful guide of thickness of piers in open dams or partial regulators.

If reinforced, very considerable reduction can be made in the thickness of piers, say $\frac{2}{3}$, but for this class of river work a heavy structure is obligatory.

151. Advantages of Open Dams. Open dams have the following advantages over solid weirs, or combinations of solid and overfall dams: First, the river bed is not interfered with and consequently the heading up and scour is only that due to the obstruction of the piers, which is inconsiderable. This points to the value of wide spans. Second, the "river low" supply is under complete control. Third, a highway bridge across the river always forms part of the structure which in most countries is a valuable asset.

Open dams, on the other hand, are not suitable for torrential rivers as the Himalayan rivers near their points of debouchure from the mountains, or wherever such detritus as trees, logs, etc., are carried down in flood time.

152. Upper Coleroon Regulator. Fig. 119 is from a photograph of a regulating bridge on the upper Coleroon River in the Madras Presidency, southern India. Originally a weir of type A was constructed at this site in conjunction with a bridge. The constriction of the discharge due to the drop wall, which was six feet high, and the piers of the bridge, caused a very high afflux and great scour on the talus. Eventually the drop wall was cleared away altogether, the bridge piers were lengthened upstream and fitted with grooves and steel towers, and counterweighted draw gates some 7 feet deep

Fig. 119. View of Regulating Bridge on the Upper Coleroon River, Southern India

took the place of the drop wall. In the flood season the gates can be raised up to the level of the bridge parapet quite clear of the flood. The work was thus changed from one of a weir of type A, to an open dam. The original weir and bridge were constructed about half a century ago.

153. St. Andrew's Rapids Dam. Another class of semi-open dam consists of a permanent low floor or dwarf weir built across the river bed which is generally of rock, and the temporary damming up of the water is effected by movable hinged standards being lowered from the deck of an overbridge, which standards support

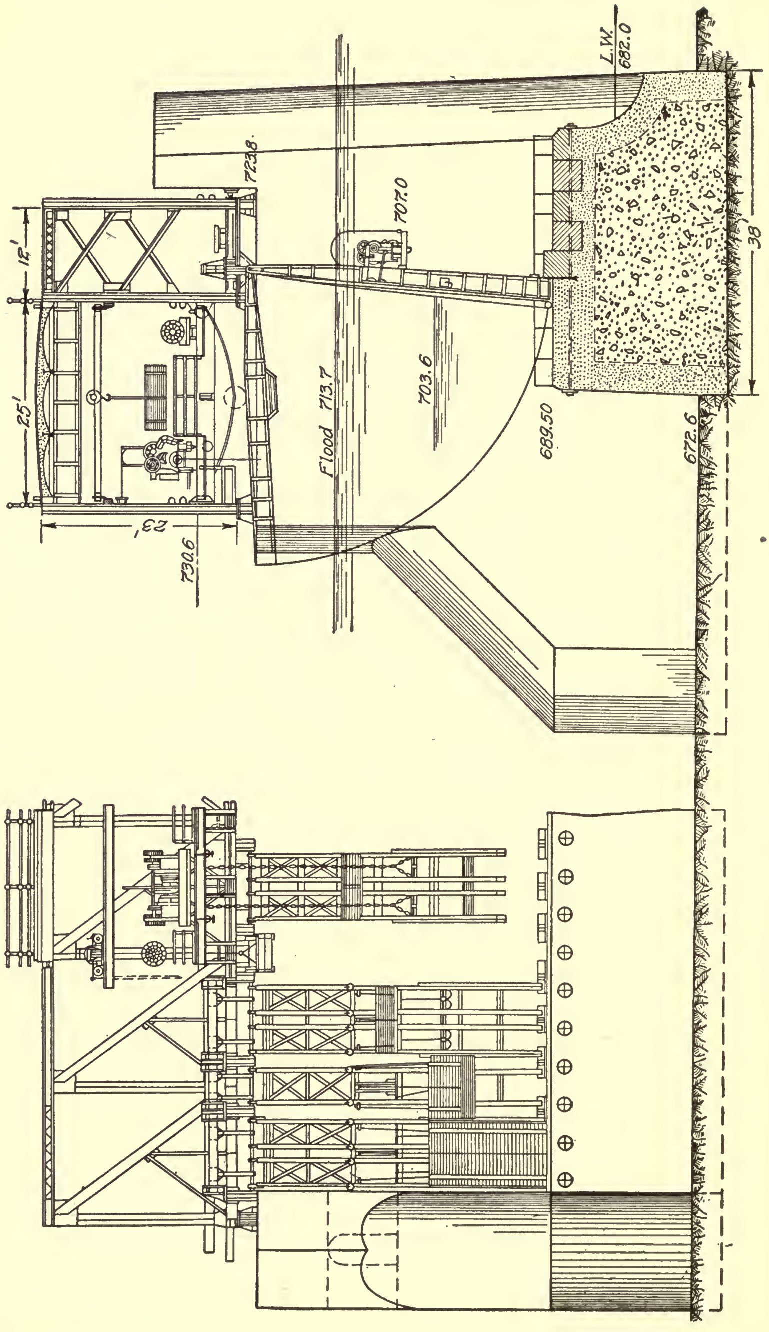


Fig. 120. Elevations and Part Section of St. Andrew's Rapids Semi-Open Dam

either a rolled reticulated curtain let down to cover them or else a steel sliding shutter mounted on rollers.

The St. Andrew's Rapids dam, Fig. 120, a quite recent construction, may be cited as an example. The object of the dam is to raise the water in the Red River, Manitoba, to enable steamboats to navigate the river from Winnipeg City to the lake of that name. To effect this the water level at the rapids has to be raised 20 feet above L. W. L. and at the same time, on account of the accumulation of ice brought down by the river, a clear passage is a necessity. The Red River rises in the South, in the State of North Dakota where the thaw sets in much earlier than at Lake Winnipeg, consequently freshets bring down masses of ice when the river and lake are both frozen.

Caméré Type of Dam. The dam is of the type known as the Caméré curtain dam, the closure being effected by a reticulated wooden curtain, which is rolled up and down the vertical frames thereby opening or closing the vents. It is a French invention, having been first constructed on the Seine. The principle of this movable dam consists in a large span girder bridge, from which vertical hinged supports carrying the curtain frames are let drop on to a low weir. When not required for use these vertical girders are hauled up into a horizontal position below the girder bridge and fastened there. In fact, the principle is very much like that of a needle dam. The river is 800 feet wide, and the bridge is of six spans of 138 feet.

The bridge is composed of three trusses, two of which are free from internal cross-bracing, and carry tram lines with all the working apparatus of several sets of winches and hoists for manipulating the vertical girders and the curtain; the third truss is mainly to strengthen the bridge laterally, and to carry the hinged ends of the vertical girders.

It will be understood that the surface exposed to wind pressure is exceptionally great, so that the cross-bracing is absolutely essential, as is also the lateral support afforded by a heavy projection of the pier itself above floor level.

In the cross-section it will be seen that there is a footbridge opening in the pier. This footbridge will carry winches for winding and unwinding the curtains, and is formed by projections thrown out at the rear of each group of frames. It will afford through communication by a tramway. The curtains can be detached altogether from the frames and housed in a chamber in the pier clear of the floodline.

The lower part of the work consists of a submerged weir of solid construction which runs right across the river; its crest is 7 feet 6 inches above L. W. L. at *El.* 689.50. The top of the curtains to which water is upheld is *El.* 703.6, or 14 feet higher. The dam actually holds up 31 feet of water above the bed of the river.

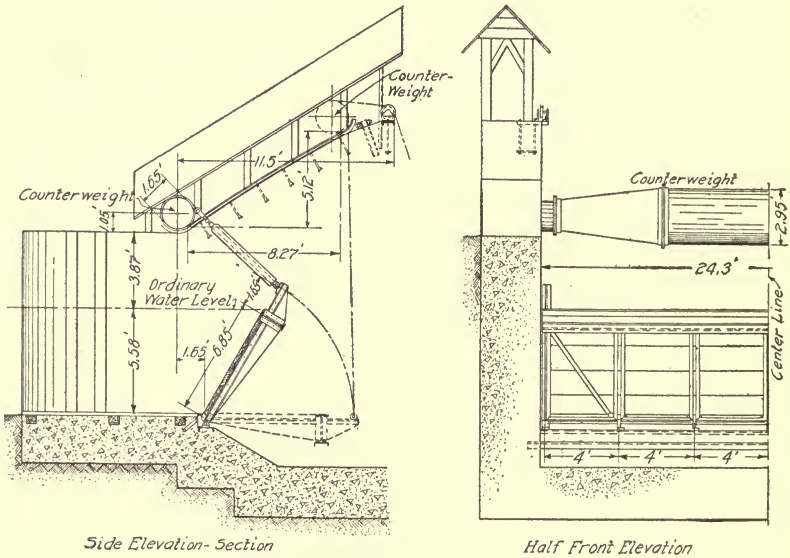


Fig. 121. Lauchli Automatic Sluice Gate

This system is open to the following objections: First, the immense expense involved in a triple row of steel girders of large span carrying the curtains and their apparatus; and second, the large surface exposure to wind which must always be a menace to the safety of the curtains.

It is believed that the raising of the water level could be effected for a quarter of the cost if not much less, by adopting a combination of the system used in the Folsam weir, Fig. 50, with that in the Dhukwa weir, Fig. 52, viz, hinged collapsible gates which could be pushed up or lowered by hydraulic jacks as required. The existing lower part of the dam could be utilized and a subway constructed through it for cross communication and accommodation for the pressure pipes, as is the case in the Dhukwa weir. This arrangement which is quite feasible would, it is deemed, be an improvement on the expensive, complicated, and slow, Caméré curtain system.

154. Automatic Dam or Regulator. Mr. Lauchli of New York, writing for Engineering News, describes a new design for automatic regulators, as follows:

In Europe there has been in operation for some time a type of automatic dam or sluice gate which on account of its simplicity of construction, adapt-

Fig. 122. View of Lauchli Automatic Dam Which Has Been for Several Years in Successful Operation in Europe

ability to existing structures, exact mathematical treatment, and especially its successful operation, deserves to attract the attention of the hydraulic engineer connected with the design of hydroelectric plants or irrigation works. Fig. 121 shows a cross-section and front elevation of one of the above-mentioned dams now in course of construction, and the view in Fig. 122 gives an idea of a small automatic dam of the same type which has been in successful operation for several seasons, including a severe winter, and during high spring floods.

Briefly stated, the automatic dam is composed of a movable part or panel, resting at the bottom on a knife edge, and fastened at the top to a compensating roller made of steel plate and filled with concrete. This roller moves along a track located at each of its ends, and is so designed as to take, at any height of water upstream, a position such as will give the apron the inclination necessary for discharging a known amount of water, and in so doing will keep the upper pool at a constant fixed elevation.

With the roller at its highest position the panel lies horizontally, and the full section is then available for discharging water. Any débris, such as trees, or ice cakes, etc., will pass over the dam without any difficulty, even during excessive floods, as the compensating roller is located high above extreme flood level.

The dam now in course of construction is located on the river Grafenauer Ohe, in Bavaria, and will regulate the water level at the intake of a paper mill, located at some distance from the power house. The dam has a panel 24.27 ft. long, 6.85 ft. high, and during normal water level will discharge 1400 cu. ft. per sec., while at flood time it will pass 3,530 cu. ft. per sec. of water. As shown in Fig. 121, the main body of the dam is made of a wooden plank construction laid on a steel frame. The panel is connected with the compensating roller at each end by a flexible steel cable wound around the roller end, and then fastened at the upper part of the roller track to an eyebolt. A simple form of roof construction protects the roller track from rain and snow. The panel is made watertight at each extremity by means of galvanized sheet iron held tight against the abutments by water pressure. This type of construction has so far proved to be very effective as to watertightness.

It may be needless to point out that this type of dam can also be fitted to the crest of overflow dam of ordinary cross-section, and then fulfill the duty of movable flashboards.

The probability is that this type will become largely used in the future. A suggested improvement would be to abolish the cross roller having instead separate rollers on each pier or abutment, working independently. There will then be no practical limit to the span adopted.

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