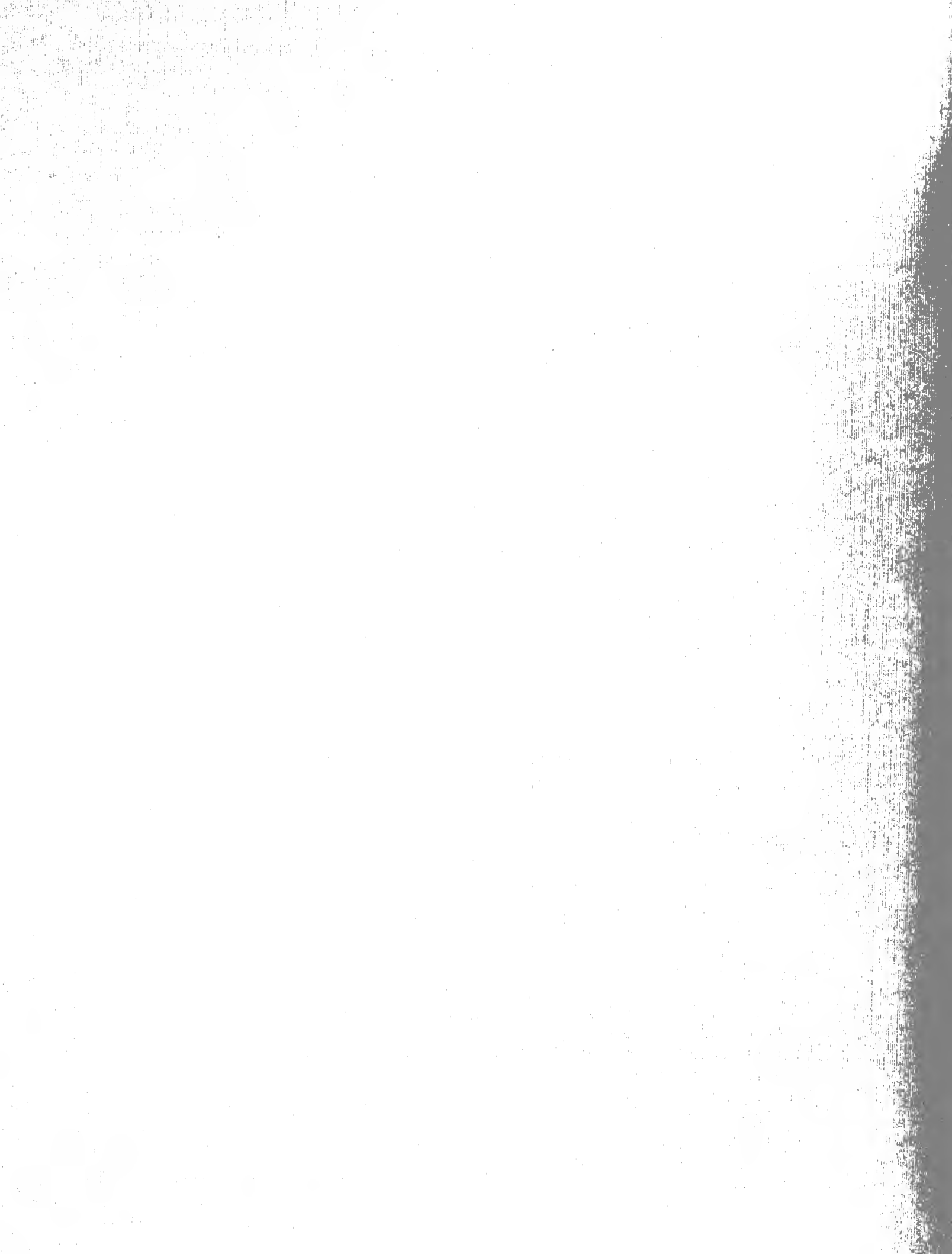


**UNIVERSITY OF  
ILLINOIS LIBRARY  
AT URBANA-CHAMPAIGN  
BOOKSTACKS**

324 323





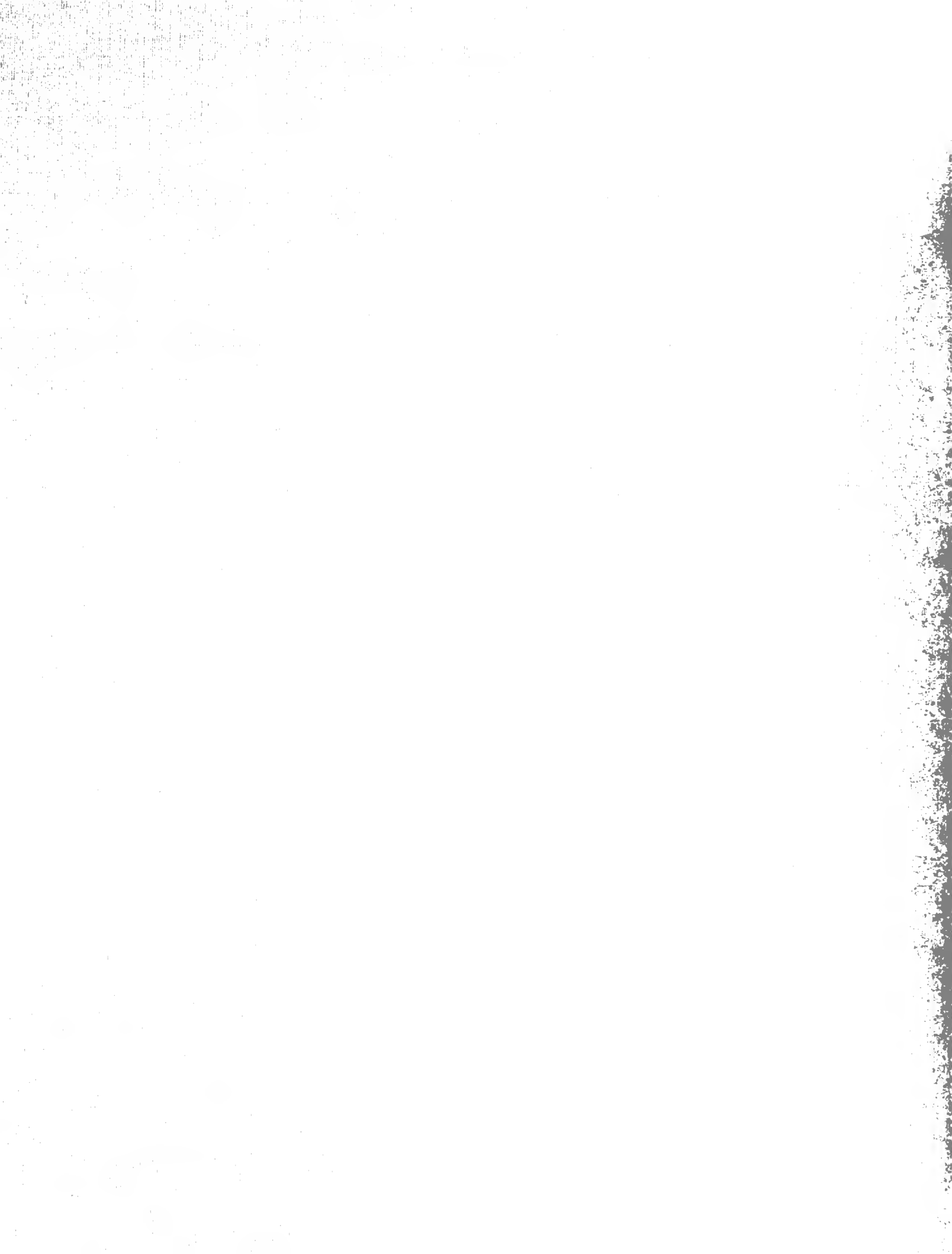
# BEBR

FACULTY WORKING  
PAPER NO. 1303

Danish Economic Theory—Were the Swedes Better?

*Hans Brems*

College of Commerce and Business Administration  
Bureau of Economic and Business Research  
University of Illinois, Urbana-Champaign



# **BEBR**

FACULTY WORKING PAPER NO. 1303

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

November 24, 1986

Danish Economic Theory-Were the Swedes Better?

Hans Brems, Professor Emeritus  
Department of Economics

Digitized by the Internet Archive  
in 2011 with funding from  
University of Illinois Urbana-Champaign

<http://www.archive.org/details/danischeconomicth1303brem>



# DANISH ECONOMIC THEORY—WERE THE SWEDES BETTER?

By HANS BREMS

## 102-WORD ABSTRACT

Inevitably the post-1870 neoclassical revolution spread to Scandinavia. In Denmark Westergaard and Birck introduced it in its Jevonian and Marshallian forms. But by confining themselves to do-it-yourself households and partial equilibria, respectively, Jevons and Marshall had both missed general equilibrium.

By skipping Jevons and Marshall and introducing general equilibrium in its Böhm-Bawerkian and Walrasian forms, respectively, Wicksell and Cassel gave Sweden a head start. Walras reached Denmark only with the second generation after Westergaard. Zeuthen was the first to use inequalities in general-equilibrium theory, thus formulating the primal half of what was to become the von Neumann duality.



History of Economics Society, Boston, June 1987

DANISH ECONOMIC THEORY--WERE THE SWEDES BETTER?\*

By HANS BREMS

I. INTRODUCTION

Sweden had Wickcell, Cassel, and Ohlin. Of Danish economic theory less has been heard. Were the Swedes better? The present paper will examine the record and see if they were and if so, why. The paper will confine itself to post-1870 microeconomic theory--our heartland as it were. As far as earlier Danish theory and its international roots are concerned, the reader is referred to Boserup's (1980) excellent "small-country case study." Danish contributions to the fiscal theory of old-age pensions are surveyed by Petersen (1986), to early econometrics by Kaergård (1984), and to early Keynesianism by Topp (1981), (1987).

## II. THE CHRONOLOGY

### 1. Westergaard (1853-1936)

Harald Ludvig Westergaard took degrees in mathematics (1874) and economics (1877) at the University of Copenhagen. He studied in Britain and Germany 1877-1878, and taught statistics and economics at his alma mater 1883-1924.

As an economist, Westergaard rejected English classical theory on two grounds. First, his warm heart reacted against a dismissal of social reform as futile. Like his German historical colleagues, Westergaard became an early champion of Sozialpolitik. Second, his keen mind reacted against a dismissal of the demand side. In his Indledning (1891) he introduced the post-1870 revolution of economic theory in the form received from his friend Jevons.

What was new in the post-1870 revolution? To come to grips with the allocation of resources, economics must make room for preferences. Jevons (1871) included them in a form which he must have thought of as tangible and tractable--so tangible and tractable that he expressed it in numbers and drew curves of it. The form was utility. If not yet practically measurable, utility was thought of as a measure. A measure

of what? A measure of human sensation. In all this Westergaard not only followed Jevons but went him one better. Westergaard's first article (1876), reported on by Davidsen (1986: 808-811), accepted cardinal utility to the point of making interpersonal utility comparisons: equalizing the distribution of income would maximize community utility.

But there was more to the post-1870 revolution than preferences. There was also the general-equilibrium vision. While the inclusion of preferences is a necessary condition for the formulation of general equilibrium, it is not a sufficient condition. Cournot (1838) and Marshall (1890) used preferences very effectively to build partial-equilibrium models, but both recoiled explicitly from any attempt to build general-equilibrium ones.

Implicitly, Jevons (1871) recoiled, too. His households were do-it-yourself households engaged in barter. Accordingly in his chapter IV on barter Jevons visualized at first persons each of whom initially possessed one single commodity some of which they would trade for other commodities. Strictly speaking such do-it-yourself households can have no income and can pay no prices. Yet Jevons [1871 (1931: 138)] supposed "that a person possesses one single kind of commodity, which we may consider to be money, or income." A general-equilibrium model in which inputs were supplied by households and demanded by industry would

have explained how such income was earned, but Jevons never did, nor could, mention its source. Furthermore, Jevon's chapter on barter contained numerous references to prices.

Such inconsistency forces us to rank Jevons third among the post-1870 pioneers: Menger, too, confined himself to do-it-yourself households but never referred to incomes earned or prices paid--and thus was guilty of no inconsistency.

In short, the post-1870 revolution found its way into Denmark in its weakest form. Two lone voices must, however, be remembered.

## 2. Bing (1839-1912) and Petersen (1839-1910)

In the very first issue of the very early Danish journal Nationaløkonomisk Tidsskrift two young mathematicians Bing and Petersen [1873 (1962)] offered a model of interest and wages. At the time they did not know Jevons's work--which was perhaps just as well, since Jevons [1871 (1931: 255)] conceived "that the returns to capital and labour are independent of each other." Bing and Petersen, by contrast, considered those returns interdependent and offered a marginal-productivity determination of them not unlike the determination safely established by von Thünen [1850 (1960: 249-264)] before indulging in his tombstone

formula [1850 (1960: 292-295)]--the only part of von Thünen's work referred to by Bing and Petersen, who correctly rejected it.

After Westergaard and the very brief Bing-Petersen interlude, the next Danish generation remained Jevonian with Marshall added, as we shall now see.

### 3. Birck (1871-1933)

Laurits Vilhelm Birck took his degree in economics at the University of Copenhagen in 1893, travelled in the United States in 1893 and in Britain and France in 1898-1899. He served as a member of parliament and was active in wartime price control and postwar royal commissions on financial collapse and the great depression. He taught economics and public finance at his alma mater 1903-1933.

Birck received the foundations of his theory of value (1902), (1922) from his teacher Harald Westergaard who, as we saw, had received them from Jevons. Jevonian households were do-it-yourself households engaged in barter, and to Birck their utilities remained cardinal to the end. To Jevons Birck added Marshall whom he considered the greatest name in our discipline. Marshall did separate industry from households. Once separated from households, industry demands inputs and supplies outputs; households demand outputs and supply inputs. Marshall's equilibria were

partial ones. First, Marshall kept the fraction of a household's budget spent on any single output small enough to enable him to ignore the effects of such expenditure upon the demand for other outputs. Second, Marshall kept his "representative firm" small enough to enable him to ignore the effects of its output upon market price. Third, Marshall kept an industry small enough to enable him to ignore the effects of its input or output upon other industries. By keeping his firms and industries small, he could justify a ceteris paribus assumption and consider the supply and demand curves of a competitive industry to be independent of the rest of the economy, hence of each other. The curves would intersect in two-dimensional, simple, and tidy partial equilibria. Birck applied such equilibria to case studies (1909), (1915) of twelve important commodities, i.e., coffee, flour, grain, kerosene, matches, meat, potash, potatoes, powder, salt, sugar, and tobacco. Applied to statistical and historical data, theory--however simple--came to life, and Birck was at his best.

An unmet challenge facing Marshall's partial-equilibrium method were the capital, labor, and land markets. By their very nature such factor markets are economy-wide. A treatment of them would require an aggregative general equilibrium like that of Böhm-Bawerk. Unable to go that far, Marshall tried to keep even his labor market small, e.g., his



market for plasterers. As a result, his treatment of the distributive shares in book vi was his weakest performance.

By skipping Jevons and Marshall and by introducing the post-1870 revolution in its Böhm-Bawerkian and Walrasian forms, Wicksell and Cassel gave Sweden a clear head start.

#### 4. Wicksell (1851-1926)

Knut Wicksell came to economics from mathematics. His inspiration was a general economic equilibrium in its Böhm-Bawerkian form. Two years before Marshall, Böhm-Bawerk had accomplished what Marshall's method would never accomplish, i.e., to determine simultaneously the period of production, the rate of interest, and the real wage rate. Böhm-Bawerk's capital was circulating capital in the form of a subsistence fund feeding labor for the period of production. In equilibrium the period of production would be long enough to absorb the entire available real capital stock and employ the entire available labor force. In such an equilibrium the profitability of the last extension of the period of production would determine the rate of interest. Böhm-Bawerk [1888 (1923: 401)] concluded: "We have, then, over the sphere of our investigation so far, to record three elements or factors which act as decisive determinants of the rate of interest: the Amount

of the national subsistence fund, the Number of workers provided for by it, and the Degree of productivity in extending production periods."

Having restated Böhm-Bawerk mathematically, Wicksell [1893 (1954)] began to wonder how a "natural" rate of interest thus determined was related to the rate of interest observed in markets where the supply of money met the demand for it. If commercial banks could create money in the form of drawing rights upon themselves, disposed of by checks, such a supply of money would be quite flexible. Would the "money" rate of interest determined by such supply coincide with the "natural" rate? If it didn't, would some equilibrating variable be set in motion and keep moving until the two rates coincided? Wicksell's answer [1898 (1936)] was the following.

The money rate of interest would not have to coincide with a Böhm-Bawerk "natural" rate of interest at all times. If it did not, nominal values would be changing. If, for example, the natural rate of interest were higher than the money rate of interest, entrepreneurs would be induced--and the money supply correspondingly expanded--to pay a higher money wage rate. Physically speaking, nothing would come of this, for when labor spent the higher money wage rate, prices would rise correspondingly and unexpectedly leave the real wage rate unchanged. There would be a cumulative process of inflation expected by nobody. Eventually, such inflation would drain the banks for cash, so the money

rate of interest would have to be raised to equality with the natural rate--thus stopping the expansion of credit. Vice versa if the natural rate of interest were lower than the money rate.

Wicksell's answer was made possible by a method fundamentally new in three respects. First, Wicksell's method was explicitly macro-economic, second, it was explicitly dynamic and, third, it was an explicit disequilibrium method based upon adaptive expectations whose disappointment constituted the motive force of the system.

5. Cassel (1866-1945)

Like Wicksell, Gustav Cassel came to economics from mathematics. His inspiration was general equilibrium in its Walrasian form. With it, Cassel (1899), (1918) could do what neither Jevons nor Marshall had ever done, and he did it as follows. Unlike the Walrasian one, Cassel's general equilibrium model assumed the factor endowments of all households to be fixed. Household income would then be the sum of the products of factor price and all factor endowments of that household. Furthermore, the model assumed the input-output coefficients of all goods to be fixed. The competitive price of a good would then be the sum of the products of factor price and all input-output coefficients of that good. Facing such household income and such competitive goods prices, every household would reveal its preference.

Goods-market equilibrium would require industry supply and such household demand to be equal for every good. Industry demand for a factor would be the sum of the products of such industry goods supplies and all input-output coefficients of that factor. Factor-market equilibrium would require household supply and such industry demand to be equal for every factor.

But Cassel [1923 (1932: 32-41 and 137-155)] went Walras one better by dynamizing the static Walrasian model into his "uniformly progressing state," thus inspiring John von Neumann [1937 (1968)] who, as Weintraub (1983: 4-5) has pointed out, knew the Walras system only in its Cassel version.

Inspired by Cassel, von Neumann went far beyond him in both rigor and substance. Like Walras, Cassel counted his equations and unknowns and was satisfied that equal numbers of them would "generally" guarantee the existence of a solution. Neither Walras nor Cassel discussed the properties of such a solution. Both failed to allow for substitution in production: Walras's "coefficients de fabrication" became Cassel's "technical coefficients," and it occurred to neither of them that with such fixed input-output coefficients, some inputs might become free goods. Indeed, like Walras, Cassel failed to treat the distinction between free and economic goods as endogenous, yet ironically offered [1923 (1932: 148)] an excellent example of a good that would be free under one technology but economic under another: when used merely to

generate mechanical power, the waterfalls of Scandinavia might be free goods. When used to generate electric power they might be economic ones. At this point we must return to Denmark. As we have seen, after Westergaard, the first Danish generation (Birck) remained Jevonian, and it took yet another generation (Zeuthen) for Walras to reach Denmark.

6. Zeuthen (1888-1958)

Frederik Ludvig Bang Zeuthen took a degree in economics at the University of Copenhagen in 1912 and spent the next eighteen years in the service of the Danish social-security system. But then, at 40, Zeuthen published his Fordeling (1928) in which was found, among other things, his use of inequalities in a Walras system as well as his theory of collective bargaining. The following year he published his article (1929) on product differentiation under monopolistic competition. Zeuthen's treatment of collective bargaining and monopolistic competition appeared in English (1930) with a preface by Schumpeter, who called it a "bold raid into new and difficult country." The new country would soon become part of mainstream economic theory. Also in English (1957), Zeuthen gave us his mature views on all this. He

taught theory, labor economics, and social security at his alma mater 1930-1958.

It all began with the seemingly pedestrian distinction between free and economic goods.

In a Walrasian system with fixed input-output coefficients, Zeuthen [1928: 27 (1932-1933: 2-3)] saw that feasibility would require the sum of all input demanded to be smaller than or equal to the sum supplied. By introducing a new variable, i.e., the unused portion, Zeuthen could then turn his inequality into an equality and say that either the unused portion of the input or the price of the input would be equal to zero. Amidst Great Depression and civil war a mathematical colloquium meeting regularly in Vienna was hospitable enough to devote its time to the very foundations of economic theory. Here, in a short paper Schlesinger [1935 (1968)] agreed with Zeuthen, but neither Zeuthen nor Schlesinger attempted to prove the existence of a general equilibrium. Wald [1935 and 1936 (1968)] made the attempt for a stationary economy, and von Neumann [1937 (1968)] succeeded for a growing one. Von Neumann formulated a primal and a dual problem. His primal problem was to maximize the rate of growth subject to the constraint that excess demand for any good must be nonpositive. That constraint was what Zeuthen and Schlesinger had seen.

Von Neumann's dual problem was this. We must minimize the rate of interest subject to the constraint that in any time-consuming process profits must be nonpositive. That constraint was seen by neither Zeuthen nor Schlesinger. Taking his primal and his dual together, von Neumann found his familiar existence proof in which the maximized rate of growth equaled the minimized rate of interest.

### III. CONCLUSION

Westergaard, Wicksell, and Cassel all came to economics from mathematics. But Westergaard introduced the post-1870 revolution in its weakest form, i.e., that of Jevons, whereas the Swedes introduced it in the superior forms of Böhm-Bawerk and Walras, respectively. Birck was handicapped by his complete lack of mathematical training and his confinement to the traditions of Jevons and Marshall. With their head start and their remarkable abilities to reduce a problem to its essence, Wicksell and Cassel went farther.

Wicksell inspired the Stockholm School: his short-run, macroeconomic, dynamic, disequilibrium method was just what was needed in the thirties. All that remained to be done was to add physical output as

an additional variable. Ohlin's (1934) feedback between physical output and aggregate demand unfolded in a cumulative process along a time axis and was a succession of disequilibria: expectations and plans were forever being revised in the light of new experience.

Cassel's microeconomic growth model inspired von Neumann (1937), his optimal depletion of mines came back with Hotelling (1931) 13 years later, his revealed preference with Samuelson (1938) 20 years later, his macroeconomic growth with Harrod (1948) 30 years later, and his dichotomy between nominal and real variables with Friedman (1968) 50 years later.

But Cassel was not alone in inspiring von Neumann. In the second generation after Westergaard, Zeuthen introduced the Walras tradition in Denmark and was the first to use inequalities in economic theory, thus formulating the primal half of what was to become the von Neumann duality. At long last, the Danes had caught up!



FOOTNOTE

\*The present paper utilizes the author's articles on Birck, Ohlin, Westergaard, and Zeuthen in the forthcoming New Palgrave, London: Macmillan, 1987, as well as passages from the author's (1986a), (1986b), (1986c), and (1986d). The paper has benefited from Kaergard's (1983) excellent survey of the marginalist breakthrough in Denmark and its men.

REFERENCES

Bing, Frederik Moritz, and Petersen, Peter Christian Julius, "Bestemmelse af den rationelle Arbejds løn samt nogle Bemaerkninger om Økonomiens Methode," Nationaløkonomisk Tidsskrift, 1873, 1, 296-320, translated as "The Determination of the Rational Wage Rate," International Economic Papers, 1962.

Birck, Laurits Vilhelm, Vaerditeori. En Analyse af Begrebet Efterspørgsel og Tilbud, Copenhagen: Sjøtofte, 1902.

\_\_\_\_\_, Sukkerets Historie. En handels- og finanspolitisk Studie, Copenhagen: Gad, 1909.

\_\_\_\_\_, Vigtige Varer. Deres Fremstilling, Forhandling og Beskatning, Copenhagen: Børsens forlag, 1915.

\_\_\_\_\_, The Theory of Marginal Value, London, Routledge, and New York: Dutton, 1922.

Böhm-Bawerk, Eugen von, Positive Theorie des Kapitaless, Innsbruck:

Wagner'sche Universitäts-Buchhandlung, 1888, translated as Positive Theory of Capital by William Smart, London and New York: Macmillan, 1891.

Boserup, Mogens, "The International Transmission of Ideas: A Small-Country Case Study," Hist. Polit. Econ., Fall 1980, 12, 420-433.

Brems, Hans, Pioneering Economic Theory, 1630-1980--A Mathematical Restatement, Baltimore: Johns Hopkins U. Press, 1986a.

\_\_\_\_\_, "Does It Matter Whether Capital is Fixed or Circulating?--A Wicksellian Exercise," Faculty Working Paper No. 1281, Urbana-Champaign: College of Commerce and Business Administration, Bureau of Economic and Business Research, University of Illinois, 1986b.

\_\_\_\_\_, "Gustav Cassel's Contributions to Economic Theory," Faculty Working Paper No. 1282, Urbana-Champaign: College of Commerce and Business Administration, Bureau of Economic and Business Research, University of Illinois, 1986c.

\_\_\_\_\_, "The Founding Fathers of the Swedish School: Wicksell and Cassel," Faculty Working Paper No. 1299, Urbana-Champaign: College of Commerce and Business Administration, Bureau of Economic and Business Research, University of Illinois, 1986d.

Cassel, Karl Gustav, "Grundriss einer elementaren Preislehre," Zeitschrift für die gesamte Staatswissenschaft, 1899, 55, 395-458.

\_\_\_\_\_, Theoretische Sozialökonomie, first edition, Leipzig, 1918, third edition, Leipzig: Deichertsche Verlagsbuchhandlung, 1923; translated from the fifth edition by S. L. Barron as The Theory of Social Economy, New York: Harcourt, Brace, 1932.

Cournot, Antoine Augustin, Recherches sur les principes mathématiques de la théorie des richesses, Paris: Hachette, 1838, translated as Researches into the Mathematical Principles of the Theory of Wealth by N. T. Bacon, New York, 1927.

Davidson, Thorkild, "Westergaard, Edgeworth and the Use of Lagrange Multipliers in Economics," Econ. J., Sep. 1986, 96, 808-811.

Jevons, William Stanley, The Theory of Political Economy, London:

Macmillan, 1871, second edition, London 1879, edited with notes and an extension of the bibliography by H. S. Jevons, London, 1931.

Kaergård, Niels, "Marginalismens gennembrud i Danmark og maendene bag,"

Nationaløkonomisk Tidsskrift, 1, 1983, 20-42.

\_\_\_\_\_, "The Earliest History of Econometrics: Some Neglected Danish Contributions," Hist. Polit. Econ., Fall, 1984, 437-444.

Marshall, Alfred, Principles of Economics, London: Macmillan, 1890.

Neumann, John von, "Ueber ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes," Ergebnisse eines mathematischen Kolloquiums, 1937, 8, 73-83, translated by G. Morgenstern (Morton) in William J. Baumol and Stephen M. Goldfeld (eds.), Precursors in Mathematical Economics: An Anthology, London: London School of Economics, 1968, 296-306.

Ohlin, Bertil, Penningpolitik, offentliga arbeten, subventioner och tullar som medel mot arbetslöshet--bidrag till expansionens teori, Unemployment Report II, 4, Stockholm: P. A. Norstedt & Söner, 1934.

Petersen, Jørn Henrik, "Three Precursors of Modern Theories of Old-Age Pensions: A Contribution to the History of Social-Policy Doctrines," Hist. Polit. Econ., Fall 1986, 18, 405-417.

Schlesinger, Karl, "Ueber die Produktionsgleichungen der ökonomischen Wertlehre," Ergebnisse eines mathematischen Kolloquiums, 1935, 6, translated by W. J. Baumol in W. J. Baumol and S. M. Goldfeld (eds.), Precursors in Mathematical Economics: An Anthology, London, London School of Economics and Political Science, 1968, 278-280.

Thünen, Johann Heinrich von, Der isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie, Hamburg: Perthes, 1826; second revised edition published as Erster Teil, Rostock: G. B. Leopold, 1842 and Zweiter Teil, Rostock 1850; published jointly, Jena: G. Fischer, 1930; partially translated by C. M. Wartenberg in P. Hall (ed.), Von Thünen's Isolated State, New York: Pergamon, 1966. The second part was fully translated by B. W. Dempsey, S. J., The Frontier Wage, Jesuit Studies, Chicago: Loyola U. Press, 1960.

Topp, Niels-Henrik, "A Nineteenth-Century Multiplier and its Fate:  
Julius Wulff and the Multiplier Theory in Denmark, 1896-1932,"  
Hist. Polit. Econ., Winter 1981, 13, 824-845.

\_\_\_\_\_, Udviklingen i de finanspolitiske ideer i Danmark 1930-1945,  
forthcoming, Copenhagen, 1987.

Wald, Abraham, "Ueber die eindeutige positive Lösbarkeit der neuen  
Produktionsgleichungen," Ergebnisse eines mathematischen Kolloquiums,  
1935, 6, translated by W. J. Baumol in W. J. Baumol and S. M.  
Goldfeld (eds.), Precursors in Mathematical Economics: An Anthology,  
London, London School of Economics and Political Science, 1968, 281-  
288.

\_\_\_\_\_, "Ueber die Produktionsgleichungen der ökonomischen Wertlehre,"  
Ergebnisse eines mathematischen Kolloquiums, 1936, 7, translated by  
W. J. Baumol in W. J. Baumol and S. M. Goldfeld (eds.), Precursors  
in Mathematical Economics: An Anthology, London, London School of  
Economics and Political Science, 1968, 289-293.

Weintraub, E. Roy, "On the Existence of a Competitive Equilibrium:  
1930-1954," J. Econ. Lit., Mar. 1983, 21, 1-39.

Westergaard, Harald, "Den moralske Formue og det moralske Haab,"  
Tidsskrift for Matematik, 1876.

\_\_\_\_\_, Indledning til Studiet af Nationaløkonomien, Copenhagen:  
Philipsen, 1891.

Wicksell, Knut, Ueber Wert, Kapital und Rente, Jena: G. Fischer, 1893,  
translated by S. H. Frohwein as Value, Capital and Rent, London:  
Allen & Unwin, 1954, reprinted 1970.

\_\_\_\_\_, Geldzins und Güterpreise, Jena: G. Fischer, 1898, translated  
by R. F. Kahn with an introduction by Bertil Ohlin as Interest and  
Prices, published on behalf of the Royal Economic Society, London:  
Macmillan, 1936.

Zeuthen, Frederik, Den økonomiske Fordeling, Copenhagen: Busck, 1928.

\_\_\_\_\_, "Mellem Konkurrence og Monopol," Nationaløkonomisk Tidsskrift,  
1929, 67, 265-305.

\_\_\_\_\_, Problems of Monopoly and Economic Welfare, with a preface by  
J. A. Schumpeter, London: Routledge, 1930.



\_\_\_\_\_, "Das Prinzip der Knappheit, technische Kombination und  
ökonomische Qualität," Zeitschrift für Nationalökonomie, 4,  
1932-1933, 1-24.

\_\_\_\_\_, Economic Theory and Method, Cambridge, Mass.: Harvard U.  
Press, 1957.







**HECKMAN**  
BINDERY INC.



**JUN 95**

Bound-To-Pleas<sup>®</sup> N. MANCHESTER,  
INDIANA 46962

UNIVERSITY OF ILLINOIS-URBANA



3 0112 005699951