DAVIES' NEW ARITHMETIC.

NEW

SCHOOL ARITHMETIC

ANALYTICAL & PRACTICAL,

BY CHARLES DAVIES, LL.D.

.0001

.001

... Basis

of

100. .01

... Numbers

1. .1

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UNIT

1

INCREASED BY THE SCALE OF TENS. | DIVIDED, ACCORDING TO SCALE OF TENS.

&c. 10000 1000 100 10 | .1 .01 .001 .0001 .00001 &c.

1

INCREASED BY VARYING SCALES. | DIVIDED, ACCORDING TO VARYING SCALES.

		£	s.	d.	far.
		1	1	1	1
T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
1	1	1	1	1	1

&c., for all denominate numbers.

$\frac{1}{4}$ $\frac{6}{9}$ $\frac{8}{17}$ $\frac{9}{47}$ } Simple.

$\frac{1}{6}$ *lb.* $\frac{1}{8}$ *oz.* $\frac{1}{7}$ *cwt.* } Denominate.

PROPORTION OF **1** TO ALL NUMBERS.
and of the numbers to each other.

For explanation of the Diagram, see "GRAMMAR OF ARITHMETIC."

APPLICATIONS.

ARITHMETICAL DIAGRAM.

SCHOOL

ARITHMETIC.

ANALYTICAL AND PRACTICAL.

BY

CHARLES DAVIES, LL. D.

AUTHOR OF A FULL COURSE OF MATHEMATICS.

REVISED EDITION.

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The attention of Teachers is respectfully invited to the Revised Editions of

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The above Works, by Charles Davies, LL.D., Author of a Complete Course of Mathematics, are designed as a full Course of Arithmetical Instruction necessary for the practical duties of business life; and also to prepare the Student for the more advanced Series of Mathematics by the same Author.

PREFACE.

ARITHMETIC embraces the science of numbers, together with all the rules which are employed in applying the principles of this science to practical purposes. It is the foundation of the exact and mixed sciences, and the first subject, in a well-arranged course of instruction, to which the reasoning powers of the mind are directed. Because of its great practical uses and applications, it has become the guide and daily companion of the mechanic and man of business. Hence, a full and accurate knowledge of Arithmetic is one of the most important elements of a liberal or practical education.

Soon after the publication, in 1848, of the last edition of my School Arithmetic, it occurred to me that the interests of education might be promoted by preparing a full analysis of the science of mathematics, and explaining in connection the most improved methods of teaching. The results of that undertaking were given to the public under the title of " Logic and Utility of Mathematics, with the best methods of instruction explained and illustrated." The reception of that work by teachers, and by the public generally, is a strong proof of the deep interest which is felt in any effort, however humble, which may be made to improve our systems of public instruction.

In that work a few general principles are laid down to which it is supposed all the operations in numbers may be referred :

1*st.* The unit 1 is regarded as the base of every number, and the consideration of it as the first step in the analysis of every question relating to numbers.

2*d.* Every number is treated as a collection of units, or as made up of sets of such collections, each collection having its own base, which is either 1, or some number derived from 1.

3*d* The numbers expressing the relation between the different units of a number are called the SCALE ; and the employment of this term enables us to generalize the laws which regulate the formation of numbers.

4*th.* By employing the term "*fractional units*," the same principles are made applicable to fractional numbers ; for, all fractions are but collections of fractional units, these units having a known relation to 1

In the preparation of the work, two objects have been kept constantly in view:

1*st.* To make it educational; and,

2*d.* To make it practical.

To attain these ends, the following plan has been adopted:

1. To introduce every new idea to the mind of the pupil by a simple question, and then to express that idea in general terms under the form of a *definition.*

2. When a sufficient number of ideas are thus fixed in the mind, they are combined to form the basis of an analysis; so that all the principles are developed by analysis in their proper order.

3. An entire system of Mental Arithmetic has been carried forward with the text, by means of a series of connected questions placed at the bottom of each page; and if these, or their equivalents, are carefully put by the teacher, the pupil will understand the reasoning in every process, and at the same time cultivate the powers of analysis and abstraction.

4. The work has been divided into sections, each containing a number of connected principles; and these sections constitute a series of dependent propositions that make up the entire system of principles and rules which the work develops.

Great pains have been taken to make the work PRACTICAL in its general character, by explaining and illustrating the various applications of Arithmetic in the transactions of business, and by connecting as closely as possible, every principle or rule, with all the applications which belong to it.

I have great pleasure in acknowledging my obligations to many teachers who have favored me with valuable suggestions in regard to the definitions, rules, and methods of illustration, in the previous editions. I hope they will find the present work free from the defects they have so kindly pointed out.

Much more than a general acknowledgment is due to Mr. D. W. FISH, an able and distinguished teacher of Western New York, who has rendered me special and valuable aid in the preparation of this edition. His practical information and zealous labors have given additional value to many parts of the work.

FISHKILL LANDING, February, 1855.

CONTENTS.

FIRST FIVE RULES.

UNITED STATES MONEY.

DENOMINATE NUMBERS.

DECIMAL FRACTIONS.

ANALYSIS.

RATIO AND PROPORTION.

APPLICATIONS TO BUSINESS.

INVOLUTION.

EVOLUTION.

ARITHMETICAL PROGRESSION.

GEOMETRICAL PROGRESSION.

PROMISCUOUS QUESTIONS.

MENSURATION.

GAUGING.

APPENDIX.

ARITHMETIC.

DEFINITIONS.

1. A SINGLE THING is called *one* or a *unit.*

2. A NUMBER is a unit, or a collection of units. The unit is called the *base* of the collection. The *primary base* of every number is the unit one.

3. Each of the words, or terms, *one*, *two*, *three*, *four*, &c., denotes how many things are taken. These terms are generally called numbers; though, in fact, they are but the *names* of numbers.

4. The term, *one*, has no reference to the kind of thing to which it is applied: and is called an *abstract* unit.

5. The term, *one foot*, refers to a single foot, and is called a *concrete* or *denominate* unit.

6. An *abstract* number is one whose unit is abstract: thus, three, four, six, &c., are abstract numbers.

7. A *concrete* or *denominate* number, is one whose unit is *concrete* or *denominate:* thus, three feet, four dollars, five pounds, are denominate numbers.

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1. What is a single thing called?
2. What is a number? What is the unit called? What is the primary base of every number?
3. What does each of the words, *one, two, three*, denote? What are these words generally called? What are they, in fact?
4. Has the term one any reference to the thing to which it may be applied? What is it called?
5. What does the term, one foot, refer to? What is it called?
6. What is an abstract number? Give examples of abstract numbers.
7. What is a concrete or denominate number? Give examples of denominate numbers.

8. A SIMPLE NUMBER is a single collection of units, whether abstract or denominate.

9. QUANTITY is anything which can be measured by a unit.

10. SCIENCE treats of the properties and relations of things: ART is the practical application of the principles of Science.

11. ARITHMETIC treats of numbers. It is a *science* when it makes known the properties and relations of numbers; and an *art*, when it applies principles of science to practical purposes.

12. A PROPOSITION is something to be *done*, or *demonstrated*.

13. An ANALYSIS is an examination of the separate parts of a proposition.

14. An OPERATION is the act of doing something with numbers. The number obtained by an operation is called a *result*, or *answer*.

15. A RULE is a direction for performing an operation, and may be deduced either from an analysis or a demonstration.

16. There are five fundamental processes of Arithmetic: Notation and Numeration, Addition, Subtraction, Multiplication and Division.

EXPRESSING NUMBERS.

17. There are three methods of expressing numbers:

1st. By words, or common language;
2d. By letters, called the Roman method;
3d. By figures, called the Arabic method.

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8. What is a simple number?
9. What is quantity?
10. Of what does Science treat? What is Art?
11. Of what does Arithmetic treat? When is it a science? When an art?
12. What is a Proposition?
13. What is an Analysis?
14. What is an Operation? What is the number obtained called?
15. What is a Rule? How may it be deduced?
16. How many fundamental rules are there? What are they?
17. How many methods are there of expressing numbers? What are they?

BY WORDS.

18. A single thing is called	- - - -	*One.*
One and one more	- - - -	*Two.*
Two and one more	- - - -	*Three*
Three and one more	- - - -	*Four.*
Four and one more	- - -	*Five.*
Five and one more	- - - -	*Six.*
Six and one more	- - - -	*Seven.*
Seven and one more	- - - -	*Eight.*
Eight and one more	- - - -	*Nine.*
Nine and one more	- - - -	*Ten.*
&c.	&c.	&c.

Each of the words, *one*, *two*, *three*, *four*, *five*, *six*, &c., denotes how many things are taken in the collection.

NOTATION.

19. NOTATION is the method of expressing numbers either by letters or figures. The method by letters, is called *Roman Notation*; the method by figures is called *Arabic Notation.*

ROMAN NOTATION.

20. In the Roman Notation, seven capital letters are used, viz: I, stands for *one;* V, for *five;* X, for *ten;* L, for *fifty;* C, for *one hundred;* D, for *five hundred;* and M, for *one thousand.* All other numbers are expressed by combining the letters according to the following

ROMAN TABLE.

I. - - -	One.	LXX. - -	Seventy.
II. - - -	Two.	LXXX. -	Eighty.
III. - - -	Three.	XC. - -	Ninety.
IV. - - -	Four.	C. . - -	One hundred.
V. - - -	Five.	CC. - -	Two hundred.
VI. - - -	Six.	CCC. - -	Three hundred.
VII. - -	Seven.	CCCC -	Four hundred.
VIII. - -	Eight.	D. - - -	Five hundred.
IX. - - -	Nine.	DC. - -	Six hundred.
X. - - -	Ten.	DCC. - -	Seven hundred.
XX. - -	Twenty.	DCCC. -	Eight hundred.
XXX. - -	Thirty.	DCCCC. -	Nine hundred.
XL. - -	Forty.	M. - - -	One thousand.
L. - - -	Fifty.	MD. - -	Fifteen hundred
LX -	Sixty.	MM. - -	Two thousand.

NOTE.—The principles of this Notation are these

1. Every time a letter is repeated, the number which it denotes is also repeated.

2. If a letter denoting a *less* number is written on the right of one denoting a *greater*, their sum will be the number expressed.

3. If a letter denoting a less number is written on the *left* of one denoting a greater, their *difference* will be the number expressed.

EXAMPLES IN ROMAN NOTATION.

Express the following numbers by letters:

1. Eleven.
2. Fifteen.
3. Nineteen.
4. Twenty-nine.
5. Thirty-five.
6. Forty-seven.
7. Ninety-nine.
8. One hundred and sixty.
9. Four hundred and forty-one.
10. Five hundred and sixty-nine.
11. One thousand one hundred and six.
12. Two thousand and twenty-five.
13. Six hundred and ninety-nine.
14. One thousand nine hundred and twenty-five.
15. Two thousand six hundred and eighty.
16. Four thousand nine hundred and sixty-five.
17. Two thousand seven hundred and ninety-one.
18. One thousand nine hundred and sixteen.
19. Two thousand six hundred and forty-one.
20. One thousand three hundred and forty-two.

---

19. What is Notation? What is the method by letters called? What is the method by figures called?

20. How many letters are used in the Roman notation? Which are they? What does each stand for?

NOTE.—What takes place when a letter is repeated? If a letter denoting a less number be placed on the right of one denoting a greater, how are they read? If the letter denoting the less number be written on the left, how are they read?

21. What is Arabic Notation? How many figures are used? What do they form? Name the figures. How many things does 1 express? How many things does 2 express? How many units in 3? In 4? In 6? In 9? In 8? What does 0 express? What are the other figures called?

ARABIC NOTATION.

21. Arabic Notation is the method of expressing numbers by figures. Ten figures are used, and they form the *alphabet of the Arabic Notation.*

They are,	0	called zero, cipher, or		Naught,
	1	-	-	One.
	2	-	-	Two.
	3	-	-	Three.
	4	-	-	Four.
	5	-	-	Five.
	6	-	-	Six.
	7	-	-	Seven.
	8	-	-	Eight.
	9	-	-	Nine.

1	expresses a single thing, or the *unit* of a number.				
2	-	two things	-	-	or two units.
3	-	three things	-	-	or three units.
4	-	four things	-	-	or four units.
5	-	five things	-	-	or five units.
6	-	six things	-	-	or six units.
7	-	seven things	-	-	or seven units.
8	-	eight things	-	-	or eight units
9	-	nine things	-	-	or nine units.

The cipher, 0, is used to denote the absence of a thing Thus, to express that there are no apples in a basket, we write, the number of apples is 0. The nine other figures are called *significant figures*, or *Digits.*

22. We have no single figure for the number ten. We therefore *combine* the figures already known. This we do by writing 0 on the right hand of 1, thus :

10, which is read ten.

This 10 is equal to *ten* of the units expressed by 1. It is, however, but a *single ten*, and may be regarded as a *unit* the value of which is *ten times* as great as the unit 1. It is called a unit of the *second order.*

---

22. Have we a separate character for ten? How do we express ten? To how many units 1 is ten equal? May we consider it a single unit? Of what order?

23. When two figures are written by the side of each other, the one on the right is in the *place of units*, and the other in the *place of tens*, or of *units of the second order. Each unit of the second order is equal to ten units of the first order.* When units simply are named, *units of the first order are always meant.*

Two tens, or two units of the second order, are written	20
Three tens, or three units of the second order, are written	30
Four tens, or four units of the second order, are written	40
Five tens, or five units of the second order, are written	50
Six tens, or six units of the second order, are written	60
Seven tens, or seven units of the second order, are written	70
Eight tens or eight units of the second order, are written	80
Nine tens, or nine units of the second order, are written	90

These figures are read, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

The intermediate numbers between 10 and 20, between 20 and 30, &c., may be readily expressed by considering their tens and units. For example, the number twelve is made up of one ten and two units. It must therefore be written by setting 1 in the place of tens, and 2 in the place of units . thus, - - - - - - 12

Eighteen has 1 ten and 8 units, and is written - -	18
Twenty-five has 2 tens and 5 units, and is written -	25
Thirty-seven has 3 tens and 7 units, and is written -	37
Fifty-four has 5 tens and 4 units, and is written - -	54

Hence, any number greater than nine, and less than one hundred, may be expressed by two figures.

24. In order to express *ten units of the second order*, or *one hundred*, we form a new combination.

It is done thus, - - - 100

by writing two ciphers on the right of 1. This number is read, one hundred.

---

23. When two figures are written by the side of each other, what is the place on the right called? The place on the left? When units simply are named, what units are meant? How many units of the second order in 20? In 30? In 40? In 50? In 60? In 70? In 80? In 90? Of what is the number 12 made up? Also 18, 25, 37 54? What numbers may be expressed by two figures?

Now this one hundred expresses 10 *units of the second order*, or 100 *units of the first order.* The one hundred is but *an individual hundred*, and, in this light, may be regarded as a unit of the *third order.*

We can now express any number less than one thousand.

For example, in the number three hundred and seventy-five, there are 5 units, 7 tens, and 3 hundreds. Write, therefore, 5 units of the first order, 7 units of the second order, and 3 of the third; and read from the right, *units, tens, hundreds.*

huns.	tens.	units.
3	7	5

In the number eight hundred and ninety-nine, there are 9 units of the first order, 9 of the second, and 8 of the third; and is read, *units, tens, hundreds.*

huns.	tens.	units.
8	9	9

In the number four hundred and six, there are 6 units of the first order, 0 of the second, and 4 of the third.

huns.	tens.	units.
4	0	6

The right hand figure always expresses units of the first order; the second, units of the second order; and the third, units of the third order.

25. To express *ten* units of the third order, or one thousand, we form a new combination by writing three ciphers on the right of 1; thus, - - - 1000

Now, this is but *one single thousand*, and may be regarded as a unit of the fourth order.

Thus, we may form as many orders of units as we please:

a single unit of the first order is expressed by 1,
a unit of the second order by 1 and 0; thus, 10,
a unit of the third order by 1 and two 0's; 100,
a unit of the fourth order by 1 and three 0's; 1000,
a unit of the fifth order by 1 and four 0's; 10000;
and so on, for units of higher orders:

---

24. How do you write one hundred? To how many units of the second order is it equal? To how many of the first order? May it be considered a single unit? Of what order is it? How many units of the third order in 200? In 300? In 400? In 500? In 600? Of what is the number 375 composed? The number 899? The number 406? What numbers may be expressed by three figures? What order of units will each figure express?

26. Therefore,

1st. *The same figure expresses different units according to the place which it occupies :*

2d. *Units of the first order occupy the place on the right ; units of the second order, the second place ; units of the third order, the third place ; and so on for places still to the left :*

3d. *Ten units of the first order make one of the second ; ten of the second, one of the third ; ten of the third, one of the fourth ; and so on for the higher orders :*

4th. *When figures are written by the side of each other, ten units in any one place make one unit of the place next to the left.*

EXAMPLES IN WRITING THE ORDERS OF UNITS.

1. Write 3 tens.
2. Write 8 units of the second order.
3. Write 9 units of the first order.
4. Write 4 units of the first order, 5 of the second, 6 of the third, and 8 of the fourth.
5. Write 9 units of the fifth order, none of the fourth, 8 of the third, 7 of the second, and 6 of the first. *Ans.* 90876.
6. Write one unit of the sixth order, 5 of the fifth, 4 of the fourth, 9 of the third, 7 of the second, and 0 of the first *Ans.*
7. Write 4 units of the eleventh order.
8. Write forty units of the second order.
9. Write 60 units of the third order, with four of the 2d, and 5 of the first.
10. Write 6 units of the 4th order, with 8 of the 3d, 4 of the 1st.

---

25. To what are ten units of the third order equal ? How do you write it ? How is a single unit of the first order written ? How do you write a unit of the second order ? One of the third ? One of the fourth ? One of the fifth ?

26. On what does the unit of a figure depend ? What is the unit of the first place on the right ? What is the unit of the second place ? What is the unit of the third place ? Of the fourth ? Of the fifth ? Sixth ? How many units of the first order make one of the second ? How many of the second one of the third ? How many of the third one of the fourth, &c. When figures are written by the side of each other, how many units of any place make one unit of the place next to the left ?

11. Write 9 units of the 5th order, 0 of the 4th, 8 of the 3d, 1 of the 2d, and 3 of the 1st.

12. Write 7 units of the 6th order, 8 of the 5th, 0 of the 4th, 5 of the 3d. 7 of the 2d, and 1 of the 11th.

13. Write 9 units of the 7th order, 0 of the 6th, 2 of the 5th, 3 of the 4th, 9 of the 3d, 2 of the 2d, and 9 of the 1st.

14. Write 8 units of the 8th order, 6 of the 7th, 9 of the 6th, 8 of the 5th, 1 of the 4th, 0 of the 3d, 2 of the 2d, and 8 of the 1st.

15. Write 1 unit of the 9th order, 6 of the 8th, 9 of the 7th, 7 of the 6th, 6 of the 5th, 5 of the 4th, 4 of the 3d, 3 of the 2d, and 2 of the 1st.

16. Write 8 units of the 10th order, 0 of the 9th, 0 of the 8th, 0 of the 7th, 9 of the 6th, 8 of the 5th, 0 of the 4th, 3 of the 3d, 2 of the 2d, and 0 of the 1st.

17. Write 7 units of the ninth order, with 6 of the 7th, 9 of the third, 8 of the 2d, and 9 of the 1st.

18. Write 6 units of 8th order, with 9 of the 6th, 4 of the 5th, 2 of the 3d, and 1 of the 1st.

19. Write 14 units of the 12th order, with 9 of the 10th, 6 of the 8th, 7 of the 6th, 6 of the 5th, 5 of the 3d, and 3 of the first.

20. Write 13 units of the 13th order, 8 of the 12th, 7 of the 9th, 6 of the 8th, 9 of the 7th, 7 of the 6th, 3 of the 4th, and 9 of the first.

21. Write 9 units of the 18th order, 7 of the 16th, 4 of the 15th, 8 of the 12th, 3 of the 11th, 2 of the 10th, 1 of the 9th, 0 of the 8th, 6 of the 7th, 2 of the third, and 1 of the 1st.

NUMERATION.

27. Numeration is the art of reading correctly any number expressed by figures or letters.

The pupil has already been taught to read all numbers from one to one thousand. The Numeration Table will teach him to read any number whatever ; or, to express numbers in words.

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27. What is Numeration? What is the unit of the first period? What is the unit of the second? Of the third? Of the fourth? Of the fifth? Sixth? Seventh? Eighth? Give the rule for reading numbers.

NUMERATION TABLE.

6th Period. Quadrillions.			5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		
Hundreds of Quadrillions.	Tens of Quadrillions	Quadrillions	Hundreds of Trillions	Tens of Trillions	Trillions	Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands.	Tens of Thousands	Thousands	Hundreds	Tens	Units
																	6
																7	5
															8	7	9
														6,	0	2	3
													8	2,	3	0	1
												1	2	3,	0	8	7
											7,	0	0	0,	7	3	5
										4	3,	2	1	0,	4	6	0
									5	4	8,	0	0	0,	0	8	7
								6,	2	4	5,	2	8	9,	4	2	1
							7	2,	5	4	9,	1	3	6,	8	2	2
						8	9	4,	6	0	2,	0	4	3,	2	8	8
					7,	6	4	1,	0	0	0,	9	0	7,	4	5	6
				8	4,	9	1	2,	8	7	6,	4	1	9,	2	8	5
			9	1	2,	7	6	1,	2	5	7,	3	2	7,	8	2	6
		6,	4	0	7,	2	1	2,	9	3	6,	8	7	6,	5	4	1
	5	7,	2	8	9,	6	7	8,	5	4	1,	2	9	7,	3	1	3
9	2	0,	3	2	3,	8	4	2,	7	6	8,	3	1	9,	6	7	5

NOTES.—1. Numbers expressed by more than three figures are written and read by periods, as shown in the above table.

2. Each period always contains three figures, except the last which may contain either one, two, or three figures.

3. The unit of the first, or right-hand period, is 1; of the second period, 1 thousand; of the 3d, 1 million; of the fourth, 1 billion; and so, for periods, still to the left.

4. To quadrillions succeed quintillions, sextillions, septillions, octillions, &c.

5. The pupil should be required to commit, thoroughly, the names of the periods, so as to repeat them in their regular order from left to right, as well as from right to left.

RULE FOR READING NUMBERS.

I. *Divide the number into periods of three figures each, beginning at the right hand.*

II. *Name the order of each figure, beginning at the right hand*

III. *Then, beginning at the left hand, read each period as if it stood alone, naming its unit.*

EXAMPLES IN READING NUMBERS.

28. Let the pupil point off and read the following numbers —then write them in words.

1.	67	7.	6124076	13.	804321049
2.	125	8.	8073405	14.	90067236708
3.	6256	9.	26940123	15.	870432697082
4.	4697	10.	9602316	16.	1704291672301
5.	23697	11.	87000032	17.	3409672103604
6.	412304	12.	1987004086	18.	49701342641714

19.	8760218760541	23.	9080620359704567
20.	904326170365	24.	9806071234560078
21.	30267821040291	25.	30621890367081263
22.	907620380467026	26.	350673123051672607

NOTE.—Let each of the above examples, after being written on the black board, be analyzed as a class exercise; thus:

Ex. 1. How many tens in 67? How many units over?

2. In 125, how many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place? How many tens in the number?

3. In 6256, how many thousands in the thousands place? How many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place?

4. How many thousands in the number 4697? How many hundreds? How many tens? How many units?

5. How many thousands in the number 23697? How many hundreds? How many tens? How many units?

6. How many hundreds of thousands in 412304? How many ten thousands? How many thousands? How many hundreds? How many tens? How many units?

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28. Name the units of each order in example 9? In 10? In 15? In 20? Give the rule for writing numbers

RULE FOR WRITING NUMBERS OR NOTATION.

I. *Begin at the left hand and write each period in order, as if it were a period of units.*

II. *When the number in any period, except the left-hand period, is expressed by less than three figures, prefix one or two ciphers; and when a vacant period occurs, fill it with ciphers.*

EXAMPLES IN NOTATION.

29. Express the following numbers in figures:

1. One hundred and five.
2. Three hundred and two.
3. Five hundred and nineteen.
4. One thousand and four.
5. Eight thousand seven hundred and one.
6. Forty thousand four hundred and six.
7. Fifty-eight thousand and sixty-one.
8. Ninety-nine thousand nine hundred and ninety-nine.
9. Four hundred and six thousands and forty-nine.
10. Six hundred and forty-one thousand, seven hundred and twenty-one.
11. One million, four hundred and twenty-one thousands, six hundred and two.
12. Nine millions, six hundred and twenty-one thousands, and sixteen.
13. Ninety-four millions, eight hundred and seven thousands, four hundred and nine.
14. Four billions, three hundred and six thousands, nine hundred and nine.
15. Forty-nine billions, nine hundred and forty-nine thousands, and sixty-five.
16. Nine hundred and ninety billions, nine hundred and ninety-nine millions, nine hundred and ninety thousands, nine hundred and ninety-nine.
17. Four hundred and nine billions, two hundred and nine thousands, one hundred and six.
18. Six hundred and forty-five billions, two hundred and sixty-nine millions, eight hundred and fifty-nine thousands, nine hundred and six.

19. Forty-seven millions, two hundred and four thousands, eight hundred and fifty-one.

20. Six quadrillions, forty-nine trillions, seventy-two billions, four hundred and seven thousands, eight hundred and sixty-one.

21. Eight hundred and ninety-nine quadrillions, four hundred and sixty trillions, eight hundred and fifty billions, two hundred millions, five hundred and six thousands, four hundred and ninety-nine.

22. Fifty-nine trillions, fifty-nine billions, fifty-nine millions, fifty-nine thousands, nine hundred and fifty-nine.

23. Eleven thousands, eleven hundred and eleven.

24. Nine billions and sixty-five.

25. Write three hundred and four trillions, one million, three hundred and twenty-one thousands, nine hundred and forty-one.

26. Write nine trillions, six hundred and forty billions, with 7 units of the ninth order, 6 of the seventh order, 8 of the fifth, 2 of the third, 1 of the second, and 3 of the first.

27. Write three hundred and five trillions, one hundred and four billions, one million, with 4 units of the fifth order, 5 of the fourth, 7 of the second, and 4 of the first.

28. Write three hundred and one billions, six millions, four thousands, with 8 units of the fourteenth order, 6 of the third, and two of the second.

29. Write nine hundred and four trillions six hundred and six, with 4 units of the eighteenth order, five of the sixteenth, four of the twelfth, seven of the ninth, and 6 of the fifth.

30. Write sixty-seven quadrillions, six hundred and forty-one billions, eight hundred and four millions, six hundred and forty-four.

31. Write eight hundred and three quintillions, sixty-nine billions, four hundred and forty millions, nine hundred thousand and three.

32. Write one hundred and fifty-nine sextillions, four hundred and five billions, two hundred and one millions, three thousand and six.

33 Write four hundred and four septillions, nine hundred and three sextillions, two hundred and one quintillions, forty quadrillions, three hundred and four

ADDITION.

30. 1. John has two apples and Charles has three: how many have both?

Analysis.—If John's apples be placed with Charles's, there will be five apples.

The operation of finding how many apples both have is called *Addition.*

ADDITION TABLE.

2 and 0 are 2	3 and 0 are 3	4 and 0 are 4	5 and 0 are 5
2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 and 2 are 4	3 and 2 are 5	4 and 2 are 6	5 and 2 are 7
2 and 3 are 5	3 and 3 are 6	4 and 3 are 7	5 and 3 are 8
2 and 4 are 6	3 and 4 are 7	4 and 4 are 8	5 and 4 are 9
2 and 5 are 7	3 and 5 are 8	4 and 5 are 9	5 and 5 are 10
2 and 6 are 8	3 and 6 are 9	4 and 6 are 10	5 and 6 are 11
2 and 7 are 9	3 and 7 are 10	4 and 7 are 11	5 and 7 are 12
2 and 8 are 10	3 and 8 are 11	4 and 8 are 12	5 and 8 are 13
2 and 9 are 11	3 and 9 are 12	4 and 9 are 13	5 and 9 are 14
2 and 10 are 12	3 and 10 are 13	4 and 10 are 14	5 and 10 are 15
6 and 0 are 6	7 and 0 are 7	8 and 0 are 8	9 and 0 are 9
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 and 2 are 8	7 and 2 are 9	8 and 2 are 10	9 and 2 are 11
6 and 3 are 9	7 and 3 are 10	8 and 3 are 11	9 and 3 are 12
6 and 4 are 10	7 and 4 are 11	8 and 4 are 12	9 and 4 are 13
6 and 5 are 11	7 and 5 are 12	8 and 5 are 13	9 and 5 are 14
6 and 6 are 12	7 and 6 are 13	8 and 6 are 14	9 and 6 are 15
6 and 7 are 13	7 and 7 are 14	8 and 7 are 15	9 and 7 are 16
6 and 8 are 14	7 and 8 are 15	8 and 8 are 16	9 and 8 are 17
6 and 9 are 15	7 and 9 are 16	8 and 9 are 17	9 and 9 are 18
6 and 10 are 16	7 and 10 are 17	8 and 10 are 18	9 and 10 are 19

2. James has 5 marbles and William 7: how many have both?

3 Mary has 6 pins and Jane 9: how many have both?

4. How many are 4 and 5 and 3?

5. How many are 6 and 4 and 9?

6. How many are 3 and 7? 4 and 6? 2 and 8? 5 and 5? 9 and 1? 10 and 0? 0 and 10?

7. How many are 6 and 3 and 9? How many are 18 and 2? 18 and 3? 18 and 5?

8. James had 9 cents and Henry gave him eight more: how many had he in all ?

PRINCIPLES AND EXAMPLES.

31. James has 3 apples and John 4: how many have both ? Seven is called the *sum* of the numbers 3 and 4

The SUM *of two or more numbers is a number which contains as many units as all the numbers taken together.*

ADDITION *is the operation of finding the sum of two or more numbers.*

OF THE SIGNS.

32. The sign + is called *plus*, which signifies more. When placed between two numbers it denotes that they are to be added together.

The sign = is called the sign of equality. When placed between two numbers it denotes that they are equal; that is, that they contain the same number of units. Thus: $3 + 2 = 5$.

$2+3=$	how many ?
$1+2+4=$	how many ?
$2+3+5+1=$	how many ?
$6+7+2+3=$	how many ?
$1+6+7+2+3=$	how many ?
$1+2+3+4+5+6+7+8+9=$	how many ?

1. James has 14 cents, and John gives him 21 : how many will he then have ?

OPERATION

$$\begin{array}{r} 14 \\ 21 \\ \hline 35 \text{ cents.} \end{array}$$

ANALYSIS.—Having written the numbers, as at the right of the page, draw a line beneath them.

The first number contains four units and 1 ten, the second 1 unit and two tens. We write the *units* in one column and the *tens* in the column of tens.

---

31. What is the sum of two or more numbers ? What is addition ?
32. What is the sign of addition ? What is it called ? What does it signify ? Express the sign of equality ? When placed between two numbers what does it show ? When is a number equal to the sum of other numbers ? Give an example ?

We then begin at the right hand, and say 1 and 4 are 5, which we set down below the line in the units' place. We then add the tens, and write the sum in the tens' place. Hence, the sum is 3 tens and 5 units, or 35 cents.

2. John has 24 cents, and William 62: how many have both of them?

OPERATION.
```
 24
 62
 --
 86
```

3. A farmer has 160 sheep in one field, 20 in another, and 16 in another: how many has he in all?

OPERATION.
```
160
 20
 16
---
196
```

4. What is the sum of 328 and 171?

OPERATION
```
328
171
---
499
```

(5.)	(6.)	(7.)	(8.)
427	329	3034	8094
242	260	6525	1602
330	100	236	103
999			

9. What is the sum of 304 and 273?

10. What is the sum of 3607 and 4082?

11. What is the sum of 30704 and 471912?

12. What is the sum of 398463 and 401536?

13. If a top costs 6 cents, a knife 25 cents, a slate 12 cents: what does the whole amount to?

14. John gave 30 cents for a bunch of quills, 18 cents for an inkstand, 25 cents for a quire of paper: what did the whole cost him?

15. If 2 cows cost 143 dollars, 5 horses 621 dollars, and 2 yoke of oxen 124 dollars: what will be the cost of them all?

16. Add 5 units, 6 tens, and 7 hundreds.

Analysis.—We set down the 5 units in the place of units, the 6 tens in the place of tens, and the 7 hundreds in the place of hundreds. We then add up, and find the sum to be 765.

We must observe, that in all cases, *units of the same order are written in the same column.*

hundreds.	tens.	units.
		5
	6	
7		
7	6	5

RULE FOR WRITING DECIMALS.

Write the decimal as if it were a whole number, prefixing as many ciphers as are necessary to make it of the required denomination.

RULE FOR READING DECIMALS.

Read the decimal as though it were a whole number, adding the denomination indicated by the lowest decimal unit.

EXAMPLES.

Write the following numbers decimally:

(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{3}{100}$.	$\frac{16}{1000}$.	$\frac{17}{10000}$.	$\frac{32}{100}$.	$\frac{165}{10000}$.
(6.)	(7)	(8)	(9)	(10.)
$18\frac{3}{100}$.	$12\frac{9}{1000}$.	$16\frac{12}{1000}$.	$9\frac{565}{100}$.	$11\frac{121}{10}$.

Write the following numbers in figures, and then numerate them.

1. Forty-one, and three-tenths.
2. Sixteen, and three millionths.
3. Five, and nine hundredths.
4. Sixty-five, and fifteen thousandths.
5. Eighty, and three millionths.
6. Two, and three hundred millionths
7. Four hundred, and ninety-two thousandths.
8. Three thousand, and twenty-one ten thousandths.
9. Forty-seven, and twenty-one hundred thousandths,
10. Fifteen hundred, and three millionths.
11. Thirty-nine, and six hundred and forty thousandths.
12. Three thousand, eight hundred and forty millionths.
13. Six hundred and fifty thousandths.

---

203. Does the value of the unit of a figure depend upon the place which it occupies? How does the value change from the left towards the right? What do ten units of any one place make? How do the units of the places increase from the right towards the left? How may whole numbers be joined with decimals? What is such a number called? Give the rule for writing decimal fractions. Give the rule for reading decimal fractions.

UNITED STATES MONEY.

204. The denominations of United States Money correspond to the decimal division, if we regard 1 dollar as the unit.

For, *the dimes are tenths of the dollar, the cents are hundredths of the dollar, and the mills, being tenths of the cent, are thousandths of the dollar.*

EXAMPLES.

1. Express $39 and 39 cents and 7 mills, decimally.
2. Express $12 and 3 mills, decimally.
3. Express $147 and 4 cents, decimally.
4. Express $148 4 mills, decimally.
5. Express $4 6 mills, decimally.
6. Express $9 6 cents 9 mills, decimally.
7. Express $10 13 cents 2 mills, decimally.

ANNEXING AND PREFIXING CIPHERS.

205. Annexing a cipher is placing it on the right of a number.

If a cipher is annexed to a decimal it makes *one more decimal place*, and therefore, a cipher must also be *added to the denominator* (Art. 202).

The numerator and denominator will therefore have been multiplied by the same number, and consequently the value of the fraction will not be changed (Art. 161): hence,

Annexing ciphers to a decimal fraction does not alter its value.

We may take as an example, $.3=\frac{3}{10}$.

If we annex a cipher to the numerator, we must, at the same time, annex one to the denominator, which gives,

---

204. If the denominations of Federal Money be expressed decimally, what is the unit? What part of a dollar is 1 dime? What part of a dime is a cent? What part of a cent is a mill? What part of a dollar is 1 cent? 1 mill?

205. When is a cipher annexed to a number? Does the annexing of ciphers to a decimal alter its value? Why not? What does three tenths become by annexing a cipher? What by annexing two ciphers? Three ciphers? What does 8 tenths become by annexing a cipher? By annexing two ciphers? By annexing three ciphers?

$.3 = \frac{30}{100} = .30$ by annexing one cipher,
$.3 = \frac{300}{1000} = .300$ by annexing two ciphers,
$.3 = \frac{3000}{10000} = .3000$ by annexing three ciphers.

Also, $.5=\frac{5}{10}=.50=\frac{50}{100}=.500=\frac{500}{1000}$.

Also, $.8=.80=.800=.8000=.80000$.

206. Prefixing a cipher is placing it on the left of a number.

If ciphers are prefixed to the numerator of a decimal fraction, the same number of ciphers must be annexed to the denominator. Now, the numerator will remain unchanged while the denominator will be increased ten times for every cipher annexed; and hence, the value of the fraction will be *diminished* ten times for every cipher prefixed to the numerator (Art. 160).

Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher prefixed.

Take, for example, the fraction $.2=\frac{2}{10}$.

$.2$ becomes $\frac{02}{100} = .02$ by prefixing one cipher,
$.2$ becomes $\frac{002}{1000} = .002$ by prefixing two ciphers,
$.2$ becomes $\frac{0002}{10000} = .0002$ by prefixing three ciphers:

in which the fraction is diminished ten times for every cipher prefixed.

ADDITION OF DECIMALS.

207. It must be remembered, that only units of the same kind can be added together. Therefore, in setting down decimal numbers for addition, figures expressing the same unit must be placed in the same column.

---

206. When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? What effect then has it on the value of the fraction? What do .2 become by prefixing a cipher? By prefixing two ciphers? By prefixing three? What do .07 become by prefixing a cipher? By prefixing two? By prefixing three? By prefixing four?

207. What parts of unity may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

The addition of decimals is then made in the same manner as that of whole numbers.

1. Find the sum of 37.04, 704.3, and .0376.

Place the decimal points in the same column: this brings units of the same value in the same column: then add as in whole numbers: hence,

OPERATION.
37.04
704.3
.0376
741.3776

Rule.—I. *Set down the numbers to be added so that figures of the same unit value shall stand in the same column.*

II. *Add as in simple numbers, and point off in the sum, from the right hand, as many places for decimals as are equal to the greatest number of places in any of the numbers added.*

Proof.—The same as in simple numbers.

EXAMPLES.

1. Add 4.035, 763.196, 445.3741, and 91.3754 together.

2. Add 365.103113, .76012, 1.34976, .3549, and 61.11 together.

3. 67.407+97.004+4+.6+.06+.3

4. .0007+1.0436+.4+.05+.047

5. .0049+47.0426+37.0410+360.0039=444.0924.

6. What is the sum of 27, 14, 49, 126, 999, .469, and .2614?

7. Add 15, 100, 67, 1, 5, 33, .467, and 24.6 together.

8. What is the sum of 99, 99, 31, .25, 60.102, .29, and 100.347?

9. Add together .7509, .0074, 69.8408, and .6109.

10. Required the sum of twenty-nine and 3 tenths, four hundred and sixty-five, and two hundred and twenty-one thousandths.

11. Required the sum of two hundred dollars one dime three cents and 9 mills, four hundred and forty dollars nine mills, and one dollar one dime and one mill.

12. What is the sum of one-tenth, one hundredth, and one thousandth?

13. What is the sum of 4, and 6 ten-thousandths ?

14. Required, in dollars and decimals, the sum of one dollar one dime one cent one mill. six dollars three mills, four dollars eight cents, nine dollars six mills, one hundred dollars six dimes, nine dimes one mill, and eight dollars six cents.

15. What is the sum of 4 dollars 6 cents, 9 dollars 3 mills, 14 dollars 3 dimes 9 cents 1 mill, 104 dollars 9 dimes 9 cents 9 mills, 999 dollars 9 dimes 1 mill, 4 mills, 6 mills, and 1 mill ?

16. If you sell one piece of cloth for $4,25, another for $5,075, and another for $7,0025, how much do you get for all ?

17. What is the amount of $151,7, $70,602, $4,06, and $807,2659 ?

18. A man received at one time $13,25 ; at another $8,4 ; at another $23,051 ; at another $6 ; and at another $0,75 : how much did he receive in all ?

19. Find the sum of twenty-five hundredths, three hundred and sixty-five thousandths, six tenths, and nine millionths.

20. What is the sum of twenty-three millions and ten, one thousand, four hundred thousandths, twenty-seven, nineteen millionths, seven and five tenths ?

21. What is the sum of six millionths, four ten-thousandths, 19 hundred thousandths, sixteen hundredths, and four tenths ?

22. If a piece of cloth cost four dollars and six mills, eight pounds of coffee twenty-six cents, and a piece of muslin three dollars seven dimes and twelve mills, what will be the cost of them all ?

23. If a yoke of oxen cost one hundred dollars nine dimes and nine mills, a pair of horses two hundred and fifty dollars five dimes and fifteen mills, and a sleigh sixty-five dollars eleven dimes and thirty-nine mills, what will be their entire cost ?

24. Find the sum of the following numbers : Sixty-nine thousand and sixty-nine thousandths, forty-seven hundred and forty-seven thousandths, eighty-five and eighty-five hundredths, six hundred and forty-nine and six hundred and forty-nine ten-thousandths ?

SUBTRACTION OF DECIMALS.

208 Subtraction of Decimal Fractions is the operation of finding the difference between two decimal numbers.

1. From 3.275 to take .0879.

Note. — In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (Art. 205): hence,

OPERATION
3.2750
.0879
3.1871

Rule.—I. *Write the less number under the greater, so that figures of the same unit value shall fall in the same column.*

II. *Subtract as in simple numbers, and point off the decimal places in the remainder, as in addition.*

Proof.—Same as in simple numbers.

EXAMPLES.

1. From 3295 take .0879.
2. From 291.10001 take 41.375.
3. From 10.000001 take .111111.
4. From 396 take 8 ten-thousandths.
5. From 1 take one thousandth.
6. From 6378 take one-tenth.
7. From 365.0075 take 3 millionths.
8. From 21.004 take 97 ten-thousandths.
9. From 260.4709 take 47 ten-millionths.
10. From 10.0302 take 19 millionths.
11. From 2.01 take 6 ten-thousandths.
12. From thirty-five thousands take thirty-five thousandths.
13. From 4262.0246 take 23.41653.
14. From 346.523120 take 219.691245943.
15. From 64.075 take .195326.
16. What is the difference between 107 and .0007?
17. What is the difference between 1.5 and .3785?
18. From 96.71 take 96.709.

---

208. What is subtraction of decimal fractions? How do you set down the numbers for subtraction? How do you then subtract? How many decimal places do you point off in the remainder?

MULTIPLICATION OF DECIMAL FRACTIONS.

209. *To multiply one decimal by another.*

1. Multiply 3.05 by 4.102.

ANALYSIS.—If we change both factors to vulgar fractions, the product of the numerator will be the same as that of the decimal numbers, and *the number of decimal places will be equal to the number of ciphers in the two denominators:* hence,

OPERATION.

$$\begin{array}{r} \frac{305}{100} = 3.05 \\ \frac{4102}{1000} = 4.102 \\ \hline 610 \\ 305 \\ 12\,20 \\ \hline 12.51110 \end{array}$$

RULE.—*Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as there are decimal places in both factors; and if there be not so many in the product, supply the deficiency by prefixing ciphers.*

EXAMPLES.

1. Multiply 3.049 by .012.
2. Multiply 365.491 by .001.
3. Multiply 496.0135 by 1.496.
4. Multiply one and one millionth by one thousandth.
5. Multiply one hundred and forty-seven millionths by one millionth.
6. Multiply three hundred, and twenty-seven hundredths by 31.
7. Multiply 31.00467 by 10.03962.
8. What is the product of five-tenths by five-tenths?
9. What is the product of five-tenths by five-thousandths?
10. Multiply 596.04 by 0.00004.
11. Multiply 38049.079 by 0.00008.
12. What will 6.29 weeks' board come to at 2,75 dollars per week?
13. What will 61 pounds of sugar come to at $0.234 per pound?

---

209. After multiplying, how many decimal places will you point off in the product? When there are not so many in the product, what do you do? Give the rule for the multiplication of decimals.

14. If 12.836 dollars are paid for one barrel of flour, what will .354 barrels cost?

15. What are the contents of a board, .06 feet long and .06 wide?

16. Multiply 49000 by .0049.

17. Bought 1234 oranges for 4.6 cents apiece: how much did they cost?

18. What will 375.6 pounds of coffee cost at .125 dollars per pound?

19. If I buy 36.251 pounds of indigo at $0.029 per pound, what will it come to?

20. Multiply $89.3421001 by .0000028.

21. Multiply $341.45 by .007.

22. What are the contents of a lot which is .004 miles long and .004 miles wide?

23. Multiply .007853 by .035.

24. What is the product of $26.000375 multiplied by .00007?

CONTRACTIONS.

210. When a decimal number is to be multiplied by 10, 100, 1000, &c., the multiplication may be made by removing the decimal point as many places to the right hand as there are ciphers in the multiplier, and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers.

$$\text{Thus, } 6.79 \text{ multiplied by } \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}67.9\\679.\\6790.\\67900.\\679000.\end{array}\right.$$

$$\text{Also, } 370.036 \text{ multiplied by } \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}3700.36\\37003.6\\370036.\\3700360.\\37003600.\end{array}\right.$$

---

210. How do you multiply a decimal number by 10, 100, 1000, &c.? If there are not as many decimal figures as there are ciphers in the multiplier, what do you do?

DIVISION OF DECIMAL FRACTIONS.

211. Division of Decimal Fractions is similar to that of simple numbers.

1. Let it be required to divide 1.38483 by 60.21.

ANALYSIS.—The dividend must be equal the product of the divisor and quotient, (Art. 61); and hence must contain as many decimal places as both of them; therefore,

There must be as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor: hence,

OPERATION.

```
60.21)1.38483(23
      1.2042
      ------
       18063
       18063
       -----
  Ans. .023
```

RULE.—*Divide as in simple numbers, and point off in the quotient, from the right hand, as many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.*

EXAMPLES.

1. Divide 2.3421 by 2.11.
2. Divide 12.82561 by 3.01.
3. Divide 33.66431 by 1.01.
4. Divide .010001 by .01.
5. Divide 8.2470 by .002.
6. Divide 94.0056 by .08.
7. What is the quotient of 37.57602, divided by 3; by .3, by .03; by .003; by .0003?
8. What is the quotient of 129 75896, divided by 8; by .08; by .008; by .0008; by .00008?
9. What is the quotient of 187.29900, divided by 9; by .9; by .09; by .009; by .0009; by .00009?
10. What is the quotient of 764.2043244, divided by 6; by .06; by .006; by .0006; by .00006; by .000006?

NOTE.—1. When there are more decimal places in the divisor than in the dividend, annex ciphers to the dividend and make the decimal places equal; *all the figures of the quotient will then be whole numbers.*

---

211. How does the number of decimal places in the dividend compare with that in the divisor and quotient? How do you determine the number of decimal places in the quotient? If the divisor contains four places and the dividend six, how many in the quotient? If the divisor contains three places and the dividend five, how many in the quotient? Give the rule for the division of decimals.

EXAMPLES.

1 Divide 4397.4 by 3.49.

NOTE.—We annex one 0 to the dividend. Had it contained no decimal place we should have annexed two.

OPERATION.

```
3.49)4397.40(1260
     349
      907
      698
      2094
      2094
          Ans. 1260.
```

2. Divide 2194.02194 by .100001.
3. Divide 9811.0047 by .325947.
4. Divide .1 by .0001.
5. Divide 10 by .15.

6. Divide 6 by .6; by .06; by .006; by .2; by .3; by by .003; by .5; by .05; by .005.

NOTE.—2. When it is necessary to continue the division farther than the figures of the dividend will allow, we annex ciphers, and consider them as decimal places of the dividend.

When the division does not terminate, we annex the plus sign to show that it may be continued: thus .2 divided by .3=.666+.

EXAMPLES.

1. Divide 4.25 by 1.25.

ANALYSIS.—In this example we annex one 0, and then the decimal places in the dividend will exceed those in the divisor by 1.

OPERATION.

```
1.25)4.25(3.4
     3.75
      500
      500
   Ans. 3.4.
```

2. Divide .2 by .6.
3. Divide 37.4 by 4.5.
4 Divide 586.4 by 375.
5. Divide 94.0369 by 81.032.

NOTE.—3. When any decimal number is to be divided by 10, 100, 1000, &c., the division is made by removing the decimal point as *many places to the left as there are 0's in the divisor;* and if there be not so many figures on the left of the decimal point the deficiency is supplied by prefixing ciphers.

$$27.69 \text{ divided by } \left\{\begin{array}{l}10\\100\\1000\\10000\end{array}\right\} = \left\{\begin{array}{l}2.769\\.2769\\.02769\\.002769\end{array}\right.$$

$$642.89 \text{ divided by } \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}64.289\\6.4289\\.64289\\.064289\\.0064289\end{array}\right.$$

QUESTIONS IN THE PRECEDING RULES.

1. If I divide .6 dollars among 94 men, how much will each receive?

2. I gave 28 dollars to 267 persons: how much apiece?

3. Divide 6.35 by .425.

4. What is the quotient of $36.2678 divided by 2.25?

5. Divide a dollar into 12 equal parts.

6. Divide .25 of 3.26 into .034 of 3.04 equal parts.

7. How many times will .35 of 35 be contained in .024 of 24?

8. At .75 dollars a bushel, how many bushels of rye can be bought for 141 dollars?

9. Bought 12 and 15 thousandths bushels of potatoes for 33 hundredths dollars a bushel, and paid in oats at 22 hundredths of a dollar a bushel: how many bushels of oats did it take?

10. Bought 53.1 yards of cloth for 42 dollars: how much was it a yard?

11. Divide 125 by .1045.

12. Divide one millionth by one billionth.

13. A merchant sold 4 parcels of cloth, the first contained 127 and 3 thousandths yards; the 2d, 6 and 3 tenths yards; the 3d, 4 and one hundredth yards; the 4th, 90 and one millionth yards: how many yards did he sell in all?

14. A merchant buys three chests of tea, the first contains 60 and one thousandth pounds; the second, 39 and one ten thousandth pounds; the third, 26 and one tenth pounds: how much did he buy in all?

NOTE.—1. If there are more decimal places in the divisor than in the dividend, what do you do? What will the figures of the quotient then be?

2. How do you continue the division after you have brought down all the figures of the dividend? What sign do you place after the quotient? What does it show?

3. How do you divide a decimal fraction by 10, 100, 1000, &c.?

15. What is the sum of $20 and three hundredths; $4 and one-tenth, $6 and one thousandth, and $18 and one hundredth?

16. A puts in trade $504.342; B puts in $350.1965; C puts in $100.11; D puts in $99.334; and E puts in $9001.32: what is the whole amount put in?

17. B has $936, and A has $1, 3 dimes and 1 mill: how much more money has B than A?

18. A merchant buys 37.5 yards of cloth, at one dollar twenty-five cents per yard: how much does the whole come to?

19. If 12 men had each $339 one dime 9 cents and 3 mills, what would be the total amount of their money?

20. A farmer sells to a merchant 13.12 cords of wood at $4,25 per cord, and 13 bushels of wheat at $1,06 per bushel: he is to take in payment 13 yards of broadcloth at $4,07 per yard, and the remainder in cash: how much money did he receive?

21. If one man can remove 5.91 cubic yards of earth in a day, how much could nineteen men remove?

22. What is the cost of 8.3 yards of cloth at $5,47 per yard?

23. If a man earns one dollar and one mill per day, how much will he earn in a year of 313 working days?

24. What will be the cost of 375 thousandths of a cord of wood, at $2 per cord?

25. A man leaves an estate of $1473.194 to be equally divided among 12 heirs: what is each one's portion?

26. If flour is $9,25 a barrel, how many barrels can I buy for $1637,25?

27. Bought 26 yards of cloth at $4,37½ a yard, and paid for it in flour at $7,25 a barrel: how much flour will pay for the cloth?

28. How much molasses at 22½ cents a gallon must be given for 46 bushels of oats at 45 cents a bushel?

29. How many days work at $1,25 a day must be given for 6 cords of wood, worth $4,12½ a cord?

30. What will 36.48 yards of cloth cost, if 14.25 yard cost $21.375?

31. If you can buy 13.25*lb.* of coffee for $2,50, how much can you buy for $325,50?

212. *To change a common to a decimal fraction.*

The value of a fraction is the quotient of the numerator, divided by the denominator (Art. 148).

1. Reduce $\frac{5}{8}$ to a decimal.

If we place a decimal point after the 5, and then write any number of 0's, after it, the value of the numerator will not be changed (Art. 205).

If, then, we divide by the denominator, the quotient will be the decimal number: hence,

OPERATION.

$$\begin{array}{r} 8)\underline{5.000} \\ .625 \end{array}$$

RULE.—*Annex decimal ciphers to the numerator, and then divide by the denominator, pointing off as in division of decimals.*

EXAMPLES.

1. Reduce $\frac{635}{125}$ to its equivalent decimal.

We here use two ciphers, and therefore point off two decimal places in the quotient.

OPERATION.

$$\begin{array}{r} 125)635(5.08 \\ \underline{625} \\ 1000 \\ 1000 \end{array}$$

Reduce the following fractions to decimals:

1. Reduce $\frac{3}{7}$ to a decimal.
2. Reduce $\frac{15}{17}$ to a decimal.
3. Reduce $\frac{3}{35}$ to a decimal.
4. Reduce $\frac{1}{4}$ and $\frac{9}{1129}$.
5. Reduce $\frac{12}{480}$, $\frac{29}{39}$, and $\frac{3}{1000}$.
6. Reduce $\frac{1}{2}$ and $\frac{5}{1785}$.
7. Reduce $\frac{314957123}{210456891}$.
8. Reduce $\frac{8}{6}$, $\frac{1375}{8436}$, $\frac{3265}{4121}$.
9. Reduce $\frac{17}{20}$ to a decimal.
10. Reduce $\frac{3}{40}$ to a decimal.
11. Reduce $\frac{17}{125}$.
12. Reduce $\frac{7}{800}$.
13. Reduce $\frac{372}{1250}$.
14. Reduce $\frac{11}{1600}$.
15. Reduce $\frac{15}{1280}$.
16. Reduce $\frac{347}{2560}$.
17. Reduce $\frac{1}{10000}$.
18. Reduce $\frac{3476}{15625}$.

213. A decimal fraction may be changed to the form of a vulgar fraction by simply writing its denominator (Art. 202).

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212. How do you change a vulgar to a decimal fraction?
213. How do you change a decimal to the form of a vulgar fraction?

EXAMPLES.

1. What vulgar fraction is equal to .04 ?
2. What vulgar fraction is equal to 3.067 ?
3. What vulgar fraction is equal to 8.275 ?
4. What vulgar fraction is equal to .00049 ?

DENOMINATE DECIMALS

214. A denominate decimal is one in which the unit of the fraction is a denominate number. Thus, .5 of a pound, .6 of a shilling, .7 of a yard, &c., are denominate decimals, in which the units are 1 pound, 1 shilling, 1 yard.

CASE I.

215. *To change a denominate number to a denominate decimal.*

1. Change 9*d*. to the decimal of a £.

ANALYSIS.—The denominate unit of the fraction is 1£=240*d*. Then divide 9*d*. by 240: the quotient. .0375 of a pound is the value of 9*d*. in the decimal of a £ : hence,

OPERATION.

240*d*.=£1

240)9(.0375

Ans. £.0375

RULE.—*Reduce the unit of the required fraction to the unit of the given denominate number, and then divide the denominate number by the result, and the quotient will be the decimal.*

EXAMPLES.

1. Reduce 7 drams to the decimal of a *lb.* avoirdupois.
2. Reduce 26*d*. to the decimal of a £.
3. Reduce .056 poles to the decimal of an acre.
4. Reduce 14 minutes to the decimal of a day.
5. Reduce 21 pints to the decimal of a peck.
6. Reduce 3 hours to the decimal of a day.
7. Reduce 375678 feet to the decimal of a mile.
8. Reduce 36 yards to the decimal of a rod.
9. Reduce .5 quarts to the decimal of a barrel.
10. Reduce .7 of an ounce, avoirdupois, to the decimal of a hundred.

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214. What is a denominate decimal?

215. How do you change a denominate number to a denominate decimal?

CASE II.

216. *To find the value of a decimal in integers of a less denomination.*

1. Find the value of .890625 bushels.

OPERATION.

$$\begin{array}{r} .890625 \\ 4 \\ \hline 3.562500 \\ 8 \\ \hline 4.500000 \\ 2 \\ \hline 1.000000 \end{array}$$

ANALYSIS.—Multiplying the decimal by 4, (since 4 pecks make a bushel), we have 3.5625 pecks. Multiplying the new decimal by 8, (since 8 quarts make a peck), we have 4.5 quarts. Then, multiplying this last decimal by 2, (since 2 pints make a quart), we have 1 pint: hence,

Ans. 3*pk.* 4*qts.* 1*pt.*

RULE.—I. *Multiply the decimal by that number which will reduce it to the next less denomination, pointing off as in multiplication of decimal fractions.*

II. *Multiply the decimal part of the product as before; and so continue to do until the decimal is reduced to the required denominations. The integers at the left form the answer*

EXAMPLES.

1. What is the value of 002084*lb.* Troy?
2. What is the value of .625 of a *cwt.*?
3. What is the value of .625 of a gallon?
4. What is the value of £.3375?
5. What is the value of .3375 of a ton?
6. What is the value of .05 of an acre?
7. What is the value of .875 pipes of wine?
8. What is the value of .125 hogsheads of beer?
9. What is the value of .375 of a year of 365 days?
10. What is the value of .085 of a £?
11. What is the value of .86 of a *cwt.*?
12. From .82 of a day take .32 of an hour.
13. What is the value of 1.089 miles?
14. What is the value of .09375 of a pound, avoirdupois?
15. What is the value of .28493 of a year of 365 days?
16. What is the value of £1.046?
17. What is the value of £1.88?

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216. How do you find the value of a decimal in integers of a less denomination?

CASE III.

217. *To reduce a compound denominate number to a decimal or mixed number.*

1. Reduce £1 4s. $9\frac{3}{4}d$. to the decimal of a £.

ANALYSIS.—Reducing the $\frac{3}{4}d$. to a decimal (Art. 215), and annexing the result to the 9d., we have 9.75d. Dividing 9.75d. by 12, (since 12 pence = 1s.), and annexing the quotient to the 4s. we have 4.8125s. Then, dividing by 20 (since 20s = £1,) and annexing the quotient to the £1, we have £1.240625:

OPERATION.

$\frac{3}{4}d. = .75d.$

$9\frac{3}{4}d. = 9.75d.$

$12)9.75d$

$20)\overline{4.8125s.}$

Ans. £1 4s. $9\frac{3}{4}d$. = 1.240625£.

RULE.—*Divide the lowest denomination by as many units as make a unit of the next higher, and annex the quotient as a decimal to that higher: then divide as before, and so continue to do, until the decimal is reduced to the required denomination.*

EXAMPLES.

1. Reduce 4*wk.* 6*da.* 5*hr.* 30*m.* 45*s.* to the denomination of a week.

2. Reduce 2*lb.* 5*oz.* 12*pwt.* 16*gr.*, to the denomination of a pound.

3. Reduce 3 feet 9 inches to the denomination of yards.

4. Reduce 1*lb.* 12*dr.*, avoirdupois, to the denomination of pounds.

5. Reduce 5 leagues 2 furlongs to the denomination of leagues.

6. Reduce 4*bu.* 3*pk.* 4*qt.* 1*pt.* to the denomination of bushels.

7. Reduce 5*oz.* 13*pwt.* 12*gr.* to the decimal of a pound.

8. Reduce 15*cwt.* 3*qr.* $2\frac{1}{2}lb.$ to the decimal of a ton.

9. Reduce 5*A.* 3*R.* 21*sq. rd.* to the denomination of acres.

10. Reduce 11 pounds to the decimal of a ton.

11. Reduce 3*da.* $12\frac{3}{5}sec.$ to the decimal of a week.

12. Reduce 14*bu.* $3\frac{3}{5}qt.$ to the decimal of a chaldron.

13. Reduce 7*m.* 7*fur.* 1*r.* to the denomination of miles.

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217. How do you reduce a compound denominate number to a decimal?

ANALYSIS.

218. An analysis of a proposition is an examination of its separate parts, and their connections with each other.

The solution of a question, by analysis, consists in an examination of its elements and of the relations which exist between these elements. We determine the elements and the relations which exist between them, in each case, by examining the *nature* of the question.

In analyzing, we reason from a *given number* to its *unit*, and then from this unit to the *required* number.

EXAMPLES.

1. If 9 bushels of wheat cost 18 dollars, what will 27 bushels cost?

ANALYSIS.—One bushel of wheat will cost one ninth as much as 9 bushels. Since 9 bushels cost 18 dollars, 1 bushel will cost $\frac{1}{9}$ of 18 dollars, or 2 dollars; 27 bushels will cost 27 times as much as 1 bushel: that is, 27 times $\frac{1}{9}$ of 18 dollars, or 54 dollars: therefore, if 9 bushels of wheat cost 18 dollars, 27 bushels will cost 54 dollars.

OPERATION.

$$\frac{\$\overset{2}{\cancel{18}}}{1} \times \frac{1}{\cancel{9}} \times \frac{27}{1} = \$54; \quad \text{Or,} \quad \begin{array}{r|l} \cancel{9} & 18 \\ & \overset{3}{\cancel{27}} \\ \hline & 54 \; \textit{Ans.} \end{array}$$

NOTE.—1. We *indicate* the operations to be performed, and then cancel the equal factors (Art. 141).

219. Although the currency of the United States is expressed in dollars cents and mills, still in most of the States the dollar (always valued at 100 cents), is reckoned in shillings and pence; thus,

In the New England States, in Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, Mississippi and Texas, the dollar is reckoned at 6 shillings: In New York, Ohio and Michigan, at 8 shillings: In New Jersey, Pennsylvania, Delaware and Maryland, at 7*s*. 6*d*: In South Carolina, and Georgia, at 4*s*. 8*d*.: In Canada and Nova Scotia, at 5 shillings.

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218. What is an analysis? In what does the solution of a question by analysis consist? How do we determine the elements and their relations? How do we reason in analyzing?

NOTE.—In many of the States the retail price of articles is given in shillings and pence, and the result, or cost, required in dollars and cents.

2. What will 12 yards of cloth cost, at 5 shillings a yard, New York currency?

ANALYSIS.—Since 1 yard cost 5 shillings 12 yards will cost 12 times 5 shillings, or 60 shillings: and as 8 shillings make 1 dollar, New York currency, there will be as many dollars as 8 is contained times in $60=\$7\frac{1}{2}$.

OPERATION.

$$\overset{s.}{5}\times 12\div 8=\$7,50;\qquad \text{Or,}\qquad \begin{array}{r|l} & \cancel{12}\ ^{3} \\ 2\ \cancel{8} & 5 \\ \hline 2 & 15=\tfrac{15}{2}=\$7,50. \\ \hline & \$7,50. \end{array}$$

NOTE.—The fractional part of a dollar may always be reduced to cents and mills by annexing two or three ciphers to the numerator and dividing by the denominator; or, which is more convenient in practice, annex the ciphers to the dividend and continue the division.

3. What will be the cost of 56 bushels of oats at 3*s.* 3*d.* a bushel, New York currency?

OPERATION.

$$\begin{array}{r|l} & \cancel{56}\ ^{7} \\ 4 & 13 \\ \cancel{8} & \\ \hline 4 & 91 \\ \hline & 22,75. \end{array}\qquad \text{Or,}\qquad \begin{array}{r|l} 4 & \cancel{56}\ ^{7} \\ \cancel{12}\ \cancel{96} & \cancel{39}\ ^{13} \\ \hline 4 & 91 \\ \hline & \$22,75\ \textit{Ans.} \end{array}$$

NOTE.—When the pence is an aliquot part of a shilling the price may be reduced to an improper fraction, which will be the multiplier: thus. $3s.\ 3d.=3\frac{1}{4}s.=\frac{13}{4}s.$ Or: the shillings and pence may be reduced to pence: thus, $3s.\ 3d.=39d.$, in which case the product will be pence, and must be divided by 96, the number of pence in 1 dollar: hence,

220. *To find the cost of articles in dollars and cents.*

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219. In what is the currency of the States expressed? In what is the currency of the States often reckoned?

220. How do you find the cost of a commodity

Multiply the commodity by the price and divide the product by the value of a dollar reduced to the same denominational unit.

4. What will 18 yards of satinet cost at 3*s*. 9*d*. a yard Pennsylvania currency?

OPERATION.

$$\begin{array}{r|l} {}^{2}\cancel{4} & \cancel{18}\ {}^{9} \\ \cancel{15} & \cancel{15} \\ & \cancel{2} \\ \hline & \$9. \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} \cancel{2}\ \ \cancel{90} & \cancel{18}\ {}^{9} \\ & \cancel{45} \\ \hline & \$9\ \textit{Ans.} \end{array}$$

NOTE.—The above rule will apply to the currency in any of the States. In the last example the multiplier is 3*s*. 9*d*.$=3\frac{3}{4}s=\frac{15}{4}s.$ or 45*d*. The divisor is 7*s*. 6*d*.$=7\frac{1}{2}s.=\frac{15}{2}s.=90d.$

5. What will $7\frac{1}{2}lb.$ of tea cost at 6*s*. 8*d*. a pound, New England currency?

OPERATION.

$$\begin{array}{r|l} \cancel{2} & \cancel{15}\ \ \overset{5}{\cancel{10}}\ \ 5 \\ 3\ \ \cancel{3} & \cancel{20} \\ \cancel{6} & \\ \hline 3 & 25=\tfrac{25}{3}=\$8.333+ \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} 3\ \ \cancel{9}\ \ \cancel{2} & \cancel{15}\ \ \overset{5}{\cancel{10}}\ \ 5 \\ \cancel{72} & \cancel{80} \\ \hline 3 & 25 \\ \hline & \$8.333+\textit{Ans.} \end{array}$$

6. What will be the cost of 120*yds*. of cotton cloth at 1*s*. 5*d*. a yard, Georgia currency?

7. What will be the cost in New York currency?

8. What will be the cost in New England currency?

9. What will be the cost of 75 bushels of potatoes at 3*s* 6*d*., New York currency?

10. What will it cost to build 148 feet of wall at 1*s*. 8*d*, per foot, N. Y. currency?

11. What will a load of wheat, containing $46\frac{1}{2}$ bushels, come to at 10*s*. 8*d*. a bushel, N. Y. currency?

12. What will 7 yards of Irish linen cost at 3*s*. 4*d*. a yard, Penn. currency?

13. How many pounds of butter at 1*s*. 4*d*. a pound must be given for 12 gallons of molasses at 2*s*. 8*d*. a gallon?

OPERATION.

$$\begin{array}{c|c} & \overset{2}{12} \\ \cancel{3} & \cancel{8} \\ \cancel{4} & \cancel{3} \\ \hline & 24lb. \end{array} \qquad \text{Or,} \qquad \begin{array}{c|c} & \overset{2}{12} \\ \cancel{16} & \cancel{32} \\ \hline & 24lb. \end{array}$$

NOTE.—The same rule applies in the last example as in the preceding ones, except that the divisor is the price of the article received in payment, reduced to the same unit as the price of the article bought.

14. What will be the cost of 12*cwt.* of sugar at 9*d.* per *lb.* N. Y. currency?

NOTE.— Reduce the *cwts.* to *lbs.* by multiplying by 4 and then by 25. Then multiply by the price per pound, and then divide by the value of a dollar in the required currency, reduced to the same denomination as the price.

OPERATION.

$$\begin{array}{r|l} & \cancel{12} \\ 2 & \cancel{4} \\ \cancel{8}\ \cancel{96} & 25 \\ & 9 \\ \hline 2 & 225 \end{array}$$

Ans. \$112,50

15. What will be the cost of 9 hogsheads of molasses at 1*s.* 3*d.* per quart, N. E. currency?

16. How many days work at 7*s.* 6*d.* a day must be given for 12 bushels of apples at 3*s.*9*d.* a bushel?

17. Farmer A exchanged 35 bushels of barley, worth 6*s.* 4*d.*, with farmer B for rye worth 7 shillings a bushel: how many bushels of rye did farmer A receive?

18. Bought the following bill of goods of Mr. Merchant: what did the whole amount to, N. Y. currency?

$12\frac{1}{2}$	yards of	cambric	at 1*s.* 4*d.*	per yard.
8	"	ribbon	" 2*s.* 6*d.*	"
21	"	calico	" 1*s.* 3*d.*	"
6	"	alpaca	" 5*s.* 6*d.*	"
4	gallons	molasses	" 3*s.* 5*d.*	per gallon.
$2\frac{1}{2}$	pounds	tea	" 6*s.* 6*d.*	per pound.
30	"	sugar	" 9*d.*	" "

19. If $\frac{5}{8}$ of a yard of cloth cost \$3,20, what will $\frac{15}{16}$ of a yard cost?

ANALYSIS.—Since 5 eighths of a yard of cloth costs \$3,20, 1 eighth of a yard will cost $\frac{1}{5}$ of \$3,20; and 1 yard, or 8 eighths, will cost 8 times as much, or $\frac{8}{5}$ of \$3,20; $\frac{15}{16}$ of a yard will cost $\frac{15}{16}$ as much as 1 yard, or $\frac{15}{16}$ of $\frac{8}{5}$ of \$3,20=\$4.80.

OPERATION.

$$\overset{160}{\cancel{\$320}} \times \frac{1}{\cancel{5}} \times \frac{\cancel{8}}{1} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{16}}} = \$4,80. \quad \text{Or,} \quad \begin{array}{r|l} & \cancel{\$3,20}\ ^{1,60} \\ \cancel{5} & \cancel{8} \\ \cancel{2}\ \cancel{16} & \cancel{15}\ \ 3 \\ \hline & \$4,80. \end{array}$$

20. If $3\frac{3}{4}$ pounds of tea cost $3\frac{1}{3}$ dollars, what will 9 pounds cost?

NOTE.—Reduce the mixed numbers to improper fractions, and then apply the same mode of reasoning as in the preceding example.

21. What will $8\frac{1}{2}$ cords of wood cost, if $2\frac{3}{8}$ cords cost $7\frac{1}{8}$ dollars?

22. If 6 men can build a boat in 120 days, how long will it take 24 men to build it?

ANALYSIS.—Since 6 men can build a boat in 120 days, it will take 1 man 6 times 120 days, or 720 days, and 24 men can build it in $\frac{1}{24}$ of the time that 1 man will require to build it, or $\frac{1}{24}$ of 6 times 120, which is 30.

OPERATION.

$$120 \times 6 \div 24 = 30 \text{ days.} \quad \text{Or,} \quad \begin{array}{r|l} & \cancel{120}\ ^{30} \\ \cancel{4}\ \ \cancel{24} & \cancel{6} \\ \hline \textit{Ans.} & 30\ \textit{days.} \end{array}$$

23. If 7 men can dig a ditch in 21 days, how many men will be required to dig it in 3 days?

24. In what time will 12 horses consume a bin of oats, that will last 21 horses $6\frac{2}{7}$ weeks?

25. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of 7 dollars for 5 yards, and sold them at the rate of 11 dollars for 7 yards; and gained 200 dollars by the bargain: how many bales were there?

ANALYSIS.—Since he paid 7 dollars for 5 yards, for 1 yard he paid $\frac{1}{5}$ of \$7 or $\frac{7}{5}$ of 1 dollar; and since he received 11 dollars for 7 yards, for 1 yard he received $\frac{1}{7}$ of 11 dollars or $\frac{11}{7}$ of 1 dollar. He gained on 1 yard the difference between $\frac{7}{5}$ and $\frac{11}{7} = \frac{6}{35}$ of a dollar. Since his whole gain was 200 dollars, he had as many yards as the gain on one yard is contained times in his whole gain, or as $\frac{6}{35}$ is contained times in 200. And there were as many bales as $129\frac{17}{27}$, (the number of yards in one bale), is contained times in the whole number of yards $\frac{7000}{6}$; which gives 9 bales.

OPERATION.

$129\frac{17}{27}=\frac{3500}{27}$, number of yards in a bale;

$200\div\frac{6}{35}=\frac{7000}{6}$, whole number of yards:

$\frac{7000}{6}\div\frac{3500}{27}=9$ bales.

$$\begin{array}{r|l} \cancel{3}\quad \cancel{6} & \cancel{200}\ \cancel{2} \\ \cancel{100}\quad & \cancel{35}\quad 9 \\ \cancel{3500} & \cancel{27} \\ \hline \textit{Ans.} & 9\ \textit{bales.} \end{array}$$

26. Suppose a number of bales of cloth, each containing $133\frac{1}{3}$ yards, to be bought at the rate of 12 yards for 11 dollars, and sold at the rate of 8 yards for 7 dollars, and the loss in trade to be $100 : how many bales are there?

27. If a piece of cloth 9 feet long and 3 feet wide, contain 3 square yards; how long must be a piece of cloth that is $2\frac{2}{5}$ feet wide be, to contain the same number of yards?

28. A can mow an acre of grass in 4 hours, B in 6 hours, and C in 8 hours. How many days, working 9 hours a day, would they require to mow 39 acres?

Analysis.—Since A can mow an acre in 4 hours, B in 6 hours, and C in 8 hours, A can mow $\frac{1}{4}$ of an acre, B $\frac{1}{6}$ of an acre, and C $\frac{1}{8}$ of an acre in 1 hour. Together they can mow $\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{13}{24}$ of an acre in 1 hour. And since they can mow 13 twenty-fourths of an acre in 1 hour, they can mow 1 twenty-fourth of an acre in $\frac{1}{13}$ of 1 hour; and 1 acre, or $\frac{24}{24}$, in 24 times $\frac{1}{13}=\frac{24}{13}$ of 1 hour: and to mow 39 acres, they will require 39 times $\frac{24}{13}=\frac{936}{13}$ hours, which reduced to days of 9 hours each, gives 8 days.

OPERATION.

$\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{13}{24}$ hours.

$$\frac{\overset{8}{\cancel{24}}}{\cancel{13}}\times\frac{\overset{3}{\cancel{39}}}{1}\times\frac{1}{\underset{3}{\cancel{9}}}=8\text{ days. Or}\qquad \begin{array}{r|l} \cancel{13} & \quad 8 \\ \cancel{3}\quad \cancel{9} & \cancel{24}\ \cancel{13} \\ & \cancel{39} \\ \hline \textit{Ans.} & 8\text{ days.} \end{array}$$

29. A can do a piece of work in 4 days, and B can do the same in 6 days; in what time can they both do the work if they labor together?

30. If 6 men can do a piece of work in 10 days, how long will it take 5 men to do it?

Analysis.—If 6 men can do a piece of work in 10 days, 1 man will require 6 times as long, or 60 days to do the same work. Five men will require but one-fifth as long as one man, or $60\div5=12$ days.

OPERATION.

$10 \times 6 \div 5 = 12$ days.

$$\begin{array}{r|l} & \cancel{10}\ 2 \\ & 6 \\ \cancel{5} & \\ \hline \textit{Ans.} & 12 \text{ days.} \end{array}$$

31. Three men together can perform a piece of work in 9 days. A alone can do it in 18 days, B in 27 days; in what time can C do it alone?

32. A and B can build a wall on one side of a square piece of ground in 3 days; A and C in 4 days; B and C in 6 days: what time will they require, working together, to complete the wall *enclosing* the square?

33. Three men hire a pasture, for which they pay 66 dollars. The first puts in 2 horses 3 weeks; the second 6 horses for $2\frac{1}{2}$ weeks; the third 9 horses for $1\frac{1}{3}$ weeks: how much ought each to pay?

ANALYSIS.—The pasturage of 2 horses for 3 weeks, would be the same as the pasturage of 1 horse 2 times 3 weeks, or 6 weeks; that of six horses $2\frac{1}{2}$ weeks, the same as for 1 horse 6 times $2\frac{1}{2}$ weeks, or 15 weeks; and that of 9 horses $1\frac{1}{3}$ weeks, the same as 1 horse for 9 times $1\frac{1}{3}$ weeks, or 12 weeks. The three persons had an equivalent for the pasturage of 1 horse for $6+15+12=33$ weeks; therefore, the first must pay $\frac{6}{33}$, the second $\frac{15}{33}$, and the third $\frac{12}{33}$ of 66 dollars.

OPERATION.

$3 \times 2 = 6$;	then	$\$66 \times \frac{6}{33} = \$12$.	1st.
$2\frac{1}{2} \times 6 = 15$;	"	$\$66 \times \frac{15}{33} = \$30$.	2d.
$1\frac{1}{3} \times 9 = 12$;	"	$\$66 \times \frac{12}{33} = \$24$.	3d.

34. Two persons, A and B, enter into partnership, and gain \$175. A puts in 75 dollars for 4 months, and B puts in 100 dollars for 6 months: what is each one's share of the gain?

35. Three men engage to build a house for 580 dollars. The first one employed 4 hands, the second 5 hands, and the third 7 hands. The first man's hands worked three times as many days as the third, and the second man's hands twice as many days as the third man's hands: how much must each receive?

36. If 8 students spend \$192 in 6 months, how much will 12 students spend in 20 months?

ANALYSIS.—Since 8 students spend \$192, one student will spend $\frac{1}{8}$ of \$192, in 6 months; in 1 month 1 student will spend $\frac{1}{6}$ of $\frac{1}{8}$ of \$192=\$4. Twelve students will spend, in 1 month, 12 times as much as 1 student, and in 20 months they will spend 20 times as much as in 1 month.

OPERATION.

$$\frac{\overset{24}{\$\cancel{192}}}{1}\times\frac{1}{\cancel{8}}\times\frac{1}{\cancel{6}}\times\frac{\overset{2}{\cancel{12}}}{1}\times\frac{20}{1}=\$960.$$

$$\begin{array}{r|l} & \cancel{192}\ \ 48 \\ \cancel{4}\ \ \cancel{8} & \\ \cancel{6} & \cancel{12}\ \ \cancel{2} \\ & 20 \\ \hline & \$960.\ \textit{Ans.} \end{array}$$

37. If 6 men can build a wall 80 feet long, 6 feet wide, and 4 feet high, in 15 days, in what time can 18 men build one 240 feet long, 8 feet wide, and 6 feet high?

ANALYSIS.—Since it takes 6 men 15 days to build a wall, it will take 1 man 6 times 15 days, or 90 days, to build the same wall. To build a wall 1 foot long, will require $\frac{1}{80}$ as long as to build one 80 feet long; to build one 1 foot wide, $\frac{1}{4}$ as long as to build one 4 feet wide; and to build one 1 foot high, $\frac{1}{6}$ as long as to build one 6 feet high. 18 men can build the same wall in $\frac{1}{18}$ of the time that one man can build it: but to build one 240 feet long, will take them 240 times as long as to build one 1 foot in length; to build one 8 feet wide, 8 times as long as to build one 1 foot wide, and to build one 6 feet high, 6 times as long as to build one 1 foot high.

OPERATION.

$$\frac{15\times\cancel{6}}{1}\times\frac{1}{\cancel{80}}\times\frac{1}{\cancel{4}}\times\frac{1}{\cancel{6}}\times\frac{1}{\underset{3}{\cancel{18}}}\times\frac{\overset{3}{\cancel{240}}}{1}\times\frac{\overset{2}{\cancel{8}}}{\cancel{4}}\times\frac{\cancel{6}}{1}=30.$$

$$\begin{array}{r|l} & 15 \\ \cancel{80} & \cancel{6} \\ \cancel{4} & \cancel{240}\ \ \cancel{3} \\ \cancel{3}\ \ \cancel{6} & \cancel{8}\ \ 2 \\ \cancel{18} & \cancel{6} \\ \hline \textit{Ans.} & 30\ \text{days.} \end{array}$$

38. If 96*lbs.* of bread be sufficient to serve 5 men 12 days, how many days will 57*lb.* serve 19 men?

39. If a man travel 220 miles in 10 days, travelling 12 hours a day, in how many days will he travel 880 miles, travelling 16 hours a day?

40. If a family of 12 persons consume a certain quantity of provisions in 6 days, how long will the same provisions last a family of 8 persons?

41. If 9 men pay \$135 for 5 weeks' board, how much must 8 men pay for 4 weeks' board?

42. If 10 bushels of wheat are equal to 40 bushels of corn, and 28 bushels of corn to 56 pounds of butter, and 39 pounds of butter to 1 cord of wood; how much wheat is 12 cords of wood worth?

ANALYSIS.—Since 10 bushels of wheat are worth 40 bushels of corn, 1 bushel of corn is worth $\frac{1}{40}$ of 10 bushels of wheat, or $\frac{1}{4}$ of a bushel; 28 bushels are worth 28 times $\frac{1}{4}$ of a bushel of wheat, or 7 bushels: since 28 bushels of corn, or 7 bushels of wheat are worth 56 pounds of butter, 1 pound of butter is worth $\frac{1}{56}$ of $7=\frac{1}{8}$ of a bushel of wheat, and 39 pounds are worth 39 times as much as 1 pound, or $39\times\frac{1}{8}=\frac{39}{8}$ bushels of wheat; and since 39 pounds of butter, or $\frac{39}{8}$ bushels of wheat are worth 1 cord of wood, 12 cords are worth 12 times as much, or $12\times\frac{39}{8}=58\frac{1}{2}$ bushels.

OPERATION.

$$\frac{\cancel{10}}{1}\times\frac{1}{\underset{\cancel{4}}{\cancel{40}}}\times\frac{\cancel{28}}{1}\times\frac{1}{\underset{2}{\cancel{56}}}\times\frac{\overset{3}{39}}{1}\times\frac{\cancel{12}}{1}=58\tfrac{1}{2}\mathit{bush.}$$

$$\begin{array}{r|ll} \cancel{4} & \cancel{10} & \\ 2\ \ \cancel{40} & \cancel{28} & \\ \cancel{56} & 39 & \\ & \cancel{12} & 3 \\ \hline 2 & 117 & =58\tfrac{1}{2}\mathit{bush.} \end{array}$$

NOTE.—Always commence analyzing from the term which is of the same name or kind as the required answer.

43. If 35 women can do as much work as 20 boys, and 16 boys can do as much as 7 men: how many women can do the work of 18 men?

44. If 36 shillings in New York are equal to 27 shillings in Massachusetts, and 24 shillings in Massachusetts are equal to 30 shillings in Pennsylvania, and 45 shillings in Pennsylvania are equal to 28 shillings in Georgia; how many shillings in Georgia are equal to 72 shillings in New York?

PROMISCUOUS EXAMPLES IN ANALYSIS.

1. How many sheep at 4 dollars a head must I give for 6 cows, worth 12 dollars apiece?

2. If 7 yards of cloth cost \$49, what will 16 yards cost?

3. If 36 men can build a house in 16 days, how long will it take 12 men to build it?

4. If 3 pounds of butter cost $7\frac{1}{2}$ shillings, what will 12 pounds cost?

5. If $5\frac{1}{3}$ bushels of potatoes cost \$$2\frac{4}{5}$, how much will $12\frac{1}{2}$ bushels cost?

6. How many barrels of apples, worth 12 shillings a barrel, will pay for 16 yards of cloth, worth 9*s.* 6*d.* a yard?

7. If $31\frac{1}{2}$ gallons of molasses are worth \$$9\frac{3}{5}$, what are $5\frac{1}{4}$ gallons worth?

8. What is the value of $24\frac{3}{4}$ bushels of corn, at 5*s.* 7*d.* a bushel, New York currency?

9. How much rye, at 8*s.* 3*d.* per bushel, must be given for 40 gallons of whisky, worth 2*s.* 9*d.* a gallon?

10. If it take 44 yards of carpeting, that is $1\frac{1}{4}$ yards wide, to cover a floor, how many yards of $\frac{7}{8}$ yards wide, will it take to cover the same floor?

11. If a piece of wall paper, 14 yards long and $1\frac{1}{2}$ feet wide, will cover a certain piece of wall, how long must another piece be, that is 2 feet wide, to cover the same wall?

12. If 5 men spend \$200 in 160 days, how long will \$300 last 12 men at the same rate?

13. If 1 acre of land cost $\frac{1}{6}$ of $\frac{3}{7}$ of $\frac{4}{5}$ of \$50, what will $3\frac{1}{2}$ acres cost?

14. Three carpenters can finish a house in 2 months; two of them can do it in $2\frac{1}{2}$ months: how long will it take the third to do it alone?

15. Three persons bought 2 barrels of flour for 15 dollars The first one ate from them 2 months, the second 3 months and the third 7 months: how much should each pay?

16. What quantity of beer will serve 4 persons $18\frac{3}{4}$ days if 6 persons drink $7\frac{1}{5}$ gallons in 4 days?

17. If 9 persons use $1\frac{3}{8}$ pounds of tea in a month, how much will 10 persons use in a year?

18. If $\frac{1}{2}$ of $\frac{3}{4}$ of a gallon of wine cost $\frac{5}{8}$ of a dollar, what will $5\frac{1}{2}$ gallons cost?

19. How many yards of carpeting, $1\frac{3}{4}$ yards wide, will it take to cover a floor that is $4\frac{2}{3}$ yards wide and 6 and three-fifths yards long?

20. Three persons bought a hogshead of sugar containing 413 pounds. The first paid $\$2\frac{1}{2}$ as often as the second paid $\$3\frac{1}{3}$, and as often as the third paid \$4: what was each one's share of the sugar?

21. A, with the assistance of B, can build a wall 2 feet wide, 3 feet high, and 30 feet long, in 4 days; but with the assistance of C, they can do it in $3\frac{1}{2}$ days: in how many days can C do it alone?

22. If two persons engage in a business, where one advances \$875, and the other \$625, and they gain \$300, what is each one's share?

23. A person purchased $\frac{4}{7}$ of a vessel, and divided it into 5 equal shares, and sold each of those shares for \$1200: what was the value of the whole vessel?

24. How many yards of paper, $\frac{3}{4}$ of a yard wide, will be sufficient to paper a room 10 yards square and 3 yards high?

25. What will be the cost of 45*lbs.* of coffee, New Jersey currency, if 9*lbs.* cost 27 shillings?

26. What will be the cost of 3 barrels of sugar, each weighing 2*cwt.* at 10*d.* per pound, Illinois currency?

27. If 12 men reap 80 acres in 6 days, in how many days will 25 men reap 200 acres?

28. If 4 men are paid 24 dollars for 3 days' labor, how many men may be employed 16 days for \$96?

29. If \$25 will supply a family with flour at \$7,50 a barrel for $2\frac{2}{3}$ months, how long would \$45 last the same family when flour is worth \$6,75 per barrel?

30. A wall to be built to the height of 27 feet, was raised to the height of 9 feet by 12 men in 6 days: how many men must be employed to finish the wall in 4 days at the same rate of working?

31. A, B and C, sent a drove of hogs to market, of which A owned 105, B 75, and C 120. On the way 60 died: how many must each lose?

32. Three men, A, B and C, agree to do a piece of work, for which they are to receive \$315. A works 8 days, $10\frac{1}{2}$ hours a day; B $9\frac{3}{4}$ days, 8 hours a day; and C, 4 days, 12 hours a day: what is each one's share?

33. If 10 barrels of apples will pay for 5 cords of wood, and 12 cords of wood for 4 tons of hay, how many barrels of apples will pay for 9 tons of hay?

34. Out of a cistern that is $\frac{2}{3}$ full is drawn 140 gallons, when it is found to be $\frac{4}{7}$ full: how much does it hold?

35. If .7 of a gallon of wine cost \$2,25, what will .25 of a gallon cost?

36. If it take 5.1 yards of cloth, 1.25 yards wide, to make a gentleman's cloak, how much surge, $\frac{5}{8}$ yards wide, will be required to line it?

37 A and B have the same income. A saves $\frac{1}{8}$ of his annually; but B, by spending \$200 a year more than A, at the end of 5 years finds himself \$160 in debt: what is their income?

38. A father gave his younger son \$420, which was $\frac{3}{4}$ of what he gave to his elder son; and 3 times the elder son's portion was $\frac{1}{3}$ the value of the father's estate: what was the value of the estate?

39. Divide \$176,40 among 3 persons, so that the first shall have twice as much as the second, and the third three times as much as the first: what is each one's share?

40. A gentleman having a purse of money, gave $\frac{1}{2}$ of it for a span of horses; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder for a carriage; when he found that he had but \$100 left: how much was in his purse before any was taken out?

41. A merchant tailor bought a number of pieces of cloth, each containing $25\frac{5}{19}$ yards, at the rate of 3 yards for 4 dollars. and sold them at the rate of 5 yards for 13 dollars, and gained by the operation 96 dollars: how many pieces did he buy?

RATIO AND PROPORTION.

221. Two numbers having the same unit, may be compared in two ways:

1*st.* By considering *how much* one is greater or less than the other, which is shown by their difference; and,

2*d.* By considering *how many times* one is contained in the other, which is shown by their quotient.

In comparing two numbers, one with the other, by means of their difference, the less is always taken from the greater.

In comparing two numbers, one with the other, by means of their quotient, one of them must be regarded as a *standard* which *measures* the other, and the quotient which arises by dividing by the standard, is called the *ratio*.

222. Every ratio is derived from two terms: the first is called the *antecedent*, and the second the *consequent;* and the two, taken together, are called a *couplet.* The *antecedent* will be regarded as the *standard.*

If the numbers 3 and 12 be compared by their difference, the result of the comparison will be 9; for, 12 exceeds 3 by 9. If they are compared by means of their quotient. the result will be 4; for, 3 is contained in 12, 4 times: that is, 3 *measuring* 12, gives 4.

223. The ratio of one number to another is expressed in two ways:

1*st.* By a colon; thus, 3 : 12; and is read, 3 is to 12; or, 3 measuring 12.

2*d.* In a fractional form, as $\frac{12}{3}$; or, 3 measuring 12.

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221. In how many ways may two numbers, having the same unit, be compared with each other? If you compare by their difference, how do you find it? If you compare by the quotient, how do you regard one of the numbers? What is the ratio?

222. From how many terms is a ratio derived? What is the first term called? What is the second called? Which is the standard?

223 How may the ratio of two numbers be expressed? How read?

224. If two couplets have the same ratio, their terms **are** said to be proportional : the couplets

3 : 12 and 1 : 4

have the same ratio 4 ; hence, the terms are proportional, and are written,

3 : 12 : : 1 : 4

by simply placing a double colon between the couplets. The terms are read

3 is to 12 as 1 is to 4,

and taken together, they are called a *proportion* : hence,

*A proportion is a comparison of the terms of two equal ratios.**

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224. If two couplets have the same ratio, what is said of the terms ? How are they written ? How read ? What is a proportion ?

* Some authors, of high authority, make the consequent the standard and divide the antecedent by it to determine the ratio of the couplet.

The ratio 3 : 12 is the same as that of 1 : 4 by both methods ; for, if the antecedent be made the standard, the ratio is 4 ; if the consequent be made the standard, the ratio is one-fourth. The question is, which method should be adopted ?

The unit 1 is the number from which all other numbers are *derived*, and by which they are *measured.*

The question is, how do we most readily apprehend and express the relation between 1 and 4 ? Ask a child, and he will answer, " the difference is 3." But when you ask him, " how many 1's are there in 4 ?" he will answer, " 4." using 1 as the standard.

Thus, we begin to teach by using the standard 1 : that is, by dividing 4 by 1.

Now, the relation between 3 and 12 is the same as that between 1 and 4 ; if then, we divide 4 by 1, we must also divide 12 by 3. Do we, indeed, clearly apprehend the ratio of 3 to 12, until we have referred to 1 as a standard ? Is the mind satisfied until it has clearly perceived that the ratio of 3 to 12 is the same as that of 1 to 4 ?

In the Rule of Three we always look for the *result* in the 4th term. Now, if we wish to find the ratio of 3 to 12, by referring to 1 as a standard, we have

3 : 12 : : 1 : ratio,

which brings the result in the right place.

But if we define ratio to be the antecedent divided by the consequent, we should have

3 : 12 : : ratio : 1,

which would bring the ratio, or *required number*, in the 3d place.

What are the ratios of the proportions,

$$3 : 9 :: 12 : 36\,?$$
$$2 : 10 :: 12 : 60\,?$$
$$4 : 2 :: 8 : 4\,?$$
$$9 : 1 :: 90 : 10\,?$$

225. The 1st and 4th terms of a proportion are called the *extremes:* the 2d and 3d terms, the *means.* Thus, in the proportion,

$$3 : 12 :: 6 : 24$$

3 and 24 are the *extremes*, and 12 and 6 the *means:*

Since (Art. 224), $\frac{12}{3}=\frac{24}{6}$,

we shall have, by reducing to a common denominator,

$$\frac{12\times6}{3\times6}=\frac{24\times3}{6\times3}.$$

But since the fractions are equal, and have the same denominators, their numerators must be equal, viz;

$$12\times6=24\times3\text{; that is,}$$

In any proportion, the product of the extremes is equal to the product of the means.

Thus, in the proportions,

$1 : 6 :: 2 : 12$; we have $1\times12=2\times6$;
$4 : 12 :: 8 : 24$; " " $4\times24=12\times8$.

226. Since, in any proportion, the product of the extremes is equal to the product of the means, it follows that,

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In all cases, the *numerical value* of a quantity is the number of times which that quantity contains an assumed standard, called its *unit of measure.*

If we would find that numerical value, in its right place, we must say,

standard : quantity : : 1 : numerical value :

but if we take the other method, we have

quantity : standard : : numerical value : 1.

which brings the numerical value in the wrong place.

1st. *If the product of the means be divided by one of the extremes, the quotient will be the other extreme.*

Thus, in the proportion

$$3 : 12 : : 6 : 24, \text{ we have } 3 \times 24 = 12 \times 6;$$

then, if 72, the product of the means, be divided by one of the extremes, 3, the quotient will be the other extreme, 24: or, if the product be divided by 24, the quotient will be 3.

2d. *If the product of the extremes be divided by either of the means, the quotient will be the other mean.*

Thus, if $3 \times 24 = 12 \times 6 = 72$ be divided by 12, the quotient will be 6; or if it be divided by 6, the quotient will be 12.

EXAMPLES.

1. The first three terms of a proportion are 3, 9 and 12: what is the fourth term?

2. The first three terms of a proportion are 4, 16 and 15: what is the 4th term?

3. The first, second, and fourth terms of a proportion are 6, 12 and 24: what is the third term?

4. The second, third, and fourth terms of a proportion are 9, 6 and 24: what is the first term?

5. The first, second and fourth terms are 9, 18 and 48: what is the third term?

227. *Simple and Compound Ratio.*

The ratio of two single numbers is called a *Simple Ratio*, and the proportion which arises from the equality of two such ratios, a *Simple Proportion.*

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225. Which are the extremes of a proportion? Which the means? What is the product of the extremes equal to?

226. If the product of the means be divided by one of the extremes, what will the quotient be? If the product of the means be divided by either extreme, what will the quotient be?

227. What is a simple ratio? What is the proportion called which comes from the equality of two simple ratios? What is a compound ratio? What is a compound proportion?

QUESTIONS.

1. If 12 apples be equally divided among 4 boys, how many will each have?

ANALYSIS.—Since 12 apples are to be divided equally between 4 boys, one boy will have as many apples as 4 is contained times in 12, which is 3; therefore, if 12 apples be equally divided between 4 boys, each will have 3 apples.

2. If 24 peaches be equally divided among 6 boys, how many will each have? How many times is 6 contained in 24?

3. A man has 32 miles to walk, and can travel 4 miles an hour, how many hours will it take him?

4. How many yards of cloth, at 3 dollars a yard, can you buy for 24 dollars?

ANALYSIS.—Since the cloth is 3 dollars a yard, you can buy as many yards as 3 is contained times in 24, which is 8: therefore, you can buy 8 yards.

5. How many oranges at 6 cents apiece can you buy for 42 cents?

6. How many pine-apples at 12 cents apiece can you buy for 132 cents?

7. A farmer pays 28 dollars for 7 sheep: how much is that apiece?

ANALYSIS.—Since 7 sheep cost 28 dollars, one sheep will cost as many dollars as 7 is contained times in 28, which is 4; therefore, each sheep will cost 4 dollars.

8 If 12 yards of muslin cost 96 cents, how much does 1 yard cost?

9. How many lead pencils could you buy for 42 cents, if they cost 6 cents apiece?

10. How many oranges could you buy for 72 cents, if they cost 6 cents apiece?

11. A trader wishes to pack 64 hats in boxes, and can put but 8 hats in a box: how many boxes does he want?

12. If a man can build 7 rods of fence in a day, how long will it take him to build 77 rods?

13. If a man pays 56 dollars for seven yards of cloth, how much is that a yard?

14. Twelve men receive 108 dollars for doing a piece of work: how much does each one receive?

15. A merchant has 144 dollars with which he is going to buy cloth at 12 dollars a yard; how many yards can he purchase?

16. James is to learn forty-two verses of Scripture in a week: how much must he learn each day?

17. How many times is 4 contained in 50, and how many over?

PRINCIPLES AND EXAMPLES.

60. 1. Let it be required to divide 86 by 2.

Set down the number to be divided and write the other number on the left, drawing a curved line between them. Now there are 8 tens and 6 units to be divided by 2. We say, 2 in 8, 4 times, which being tens, we write it in the tens place. We then say, 2 in 6, 3 times, which being units, are written in the units' place. The result, which is called a *quotient*, is therefore, 4 tens and 3 units, or 43.

OPERATION.

Divisor. Dividend.
2) 86
43 quotie't.

2. Let it be required to divide 729 by 3.

ANALYSIS.—We say, 3 in 7, 2 times and 1 over. Set down the 2, which are hundreds, under the 7. But of the 7 hundreds there is 1 hundred, or 10 tens, not yet divided. We put the 10 tens with the 2 tens, making 12 tens, and then say, 3 in 12, 4 times, and write the 4 of the quotient in the tens' place; then say, 3 in 9, 3 times. The quotient, therefore, is 243.

OPERATION.

3)729
243

3. Let it be required to divide 466 by 8.

ANALYSIS.—We first divide the 46 tens by 8, giving a quotient of 5 tens, and 6 tens over. These 6 tens are equal to 60 units, to which we add the 6 in the units' place. We then say, 8 in 66, 8 times and 2 over; hence, the quotient is 58, and 2 over, which we call a *remainder*. This remainder is written after the last quotient figure, and the 8 placed under it; the quotient is read, 58 and 2 divided by 8.

OPERATION.

8)466
58–2 remain.

$58\frac{2}{8}$ quotient.

60 *Ex.* 1.—When you divide 8 tens by 2, is the unit of the quotient tens or units? When 6 units are divided by 2, what is the unit?

Analysis.—In the first example, 86 is divided into 2 equal parts, and the quotient 43 is one of the parts. If one of the equal parts be multiplied by the number of parts 2, the product will be 86, the number divided.

In the third example, 466 is divided into 8 equal parts, and two units remain that are not divided. If one of the equal parts, 58, be multiplied by the number of parts, 8, and the remainder 2 be added to the product, the result will be equal to 466, the number divided.

61. Division *is the operation of finding from two numbers a third, which multiplied by the first, will produce the second.*

The first number, or number by which we divide, is called the *divisor.*

The second number, or number to be divided, is called the *dividend.*

The third number, or result, is called the *quotient.*

The quotient shows how many times the dividend contains the divisor.

If anything is left after division, it is called a *remainder.*

62. There are three parts in every division, and sometimes four: 1st, the dividend; 2d, the divisor; 3d, the quotient; and 4th, the remainder.

There are three signs used to denote division; they are the following:

$18 \div 4$ expresses that 18 is to be divided by 4.
$\frac{18}{4}$ expresses that 18 is to be divided by 4.
4)18 expresses that 18 is to be divided by 4.

When the last sign is used, if the divisor does not exceed 12, we draw a line beneath, and set the quotient under it. If the divisor exceeds 12, we draw a curved line on the right of the dividend, and set the quotient at the right.

2.—When the seven hundreds are divided by 3, what is the unit of the quotient? To how many tens is the undivided hundred equal? When the 12 tens are divided by 3, what is the unit of the quotient? When the 9 units are divided by 3, what is the quotient?

3.—How is the division of the remainder expressed? Read the quotient. If there be a remainder after division, how must it be written?

61. What is division? What is the number to be divided called? What is the number called by which we divide? What is the answer called? What is the number called which is left?

62. How many parts are there in division? Name them. How many signs are there in division? Make and name them

SHORT DIVISION.

63. SHORT DIVISION is the operation of dividing when the work is performed mentally, and the results only written down. It is limited to the cases in which the divisors do not exceed 12.

Let it be required to divide 30456 by 8.

ANALYSIS.—We first say, 8 in 3 we cannot. Then, 8 in 30, 3 times and 6 over; then, 8 in 64, 8 times; then 8 in 5, 0 times; then, 8 in 56, 7 times: hence,

OPERATION.
8)30456
3807

RULE I.—*Write the divisor on the left of the dividend. Beginning at the left, divide each figure of the dividend by the divisor, and set each quotient figure under its dividend.*

II. *If there is a remainder, after any division, annex to it the next figure of the dividend, and divide as before.*

III. *If any dividend is less than the divisor, write 0 for the quotient figure and annex the next figure of the dividend, for a new dividend.*

IV. *If there is a remainder, after dividing the last figure, set the divisor under it, and annex the result to the quotient.*

PROOF.—Multiply the divisor by the quotient, and to the product add the remainder, when there is one; if the work is right the result will be equal to the dividend

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)
	3)9369	4)73684	5)673420	6)825467
Ans.	3123	18421	134684	$137577\frac{5}{6}$
	3	4	5	6
Proof.	9369	73684	673420	825467

5. Divide 86434 by 2.
6. Divide 416710 by 4.
7. Divide 64140 by 5.
8. Divide 278943 by 6.
9. Divide 95040522 by 6.
10. Divide 75890496 by 8.
11. Divide 6794108 by 3.
12. Divide 21090431 by 9.
13. Divide 2345678964 by 6
14. Divide 570196382 by 12.
15. Divide 67897634 by 9.
16. Divide 75436298 by 12.
17. Divide 674189904 by 9.
18. Divide 1404967214 by 11
19. Divide 27478041 by 10.
20. Divide 167484329 by 12

21. A man sold his farm for 6756 dollars, and divided the amount equally between his wife and 5 children : how much did each receive ?

22. There are 576 persons in a train of 12 cars : how many are there in each car ?

23. If a township of land containing 2304 acres be equally divided between 8 persons, how many acres will each have ?

24. If it takes 5 bushels of wheat to make a barrel of flour, how many barrels can be made from 65890 bushels ?

25. Twelve things make a dozen : how many dozens are there in 2167284 ?

26. Eleven persons are all of the same age, and the sum of their ages is 968 years : what is the age of each ?

27. How many barrels of flour at 7 dollars a barrel can be bought for 609463 dollars ?

28. An estate worth 2943 dollars, is to be divided equally between a father, mother, 3 daughters and 4 sons : what is the portion of each ?

29. A county contains 207360 acres of land lying in 9 townships of equal extent : how many acres in a township ?

30. If 11 cities contain the same number of inhabitants, and the whole number is equal to 3800247 : how many will there be in each ?

FRACTIONS.

64. 1. If any number or thing be divided into two equal parts, one of the parts is called *one-half*, which is written thus ; $\frac{1}{2}$.

2. If any number is divided into three equal parts, one of the parts is called *one-third*, which is written thus ; $\frac{1}{3}$; two of the parts are called *two-thirds*, and written thus ; $\frac{2}{3}$.

3. If any number is divided into four equal parts, one of the parts is called *one-fourth*, which is written thus ; $\frac{1}{4}$; two of the parts are called *two-fourths*, and are written thus ; $\frac{2}{4}$; three of them are called *three-fourths*, and written $\frac{3}{4}$; and similar names are given to the equal parts into which any number may be divided.

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63. What is short division ? How is it generally performed ? Give the rule. How do you prove short division ?

4. If a number is divided into five equal parts, what is one of the parts called? Two of them? Three of them? Four of them?

5. If a number is divided into 7 equal parts, what is one of the parts called? What is one of the parts called when it is divided into 8 equal parts? When it is divided into 9 equal parts? When it is divided into 10? When it is divided into 11? When it is divided into 12?

6. What is one-half of 2? of 4? of 6? of 8? of 10? of 12? of 14? of 16? of 18?

7. What is one-third of 3? What is two-thirds of 3?

ANALYSIS.—Two-thirds of three are two times one-third of three. One-third of three is 1; therefore, two-thirds of three are two times 1, or 2.

Let every question be analyzed in the same manner.

What is one-third of 6? 2 thirds of 6? One-third of 9? 2 thirds of 9? One-third of 12? two-thirds of 12?

8. What is one-fourth of 4? 2 fourths of 4? 3 fourths of 4? What is one-fourth of 8? 2 fourths of 8? 3 fourths of 8? What is one-fourth of 12? 2 fourths of 12? 3 fourths of 12? One-fourth of 16? 2 fourths of 16? 3 fourths?

9. What is one-seventh of 7? What is 2 sevenths of 7? 5 sevenths? 6 sevenths? What is one-seventh of 14? 3 sevenths? 5 sevenths? 6 sevenths? What is one-seventh of 21? of 28? of 35?

10. What is one-eighth of 8? of 16? of 24? of 32? of 40? of 56?

11. What is one-ninth of 9? 2 ninths? 7 ninths? 6 ninths? 5 ninths? 4 ninths? What is one-ninth of 18? of 27? of 54? of 72? of 90? of 108?

12. How many halves of 1 are there in 2?

ANALYSIS.—There are twice as many halves in 2 as there are in 1. There are two halves in 1; therefore, there are 2 times 2 halves in 2, or 4 halves.

13. How many halves of 1 are there in 3? In 4? In 5? In 6? In 8? In 10? In 12?

14. How many thirds are there in 1? How many thirds of 1 in 2? In 3? In 4? In 5? In 6? In 9? In 12?

15. How many fourths are there in 1? How many fourths of 1 in 2? In 4? In 6? In 10? In 12?

16. How many fifths are there in 1? How many fifths of 1 are there in 2? In 3? In 6? In 7? In 11? In 12?

17. How many sixths are there in 2 and one-sixth? In 3 and 4 sixths? In 5 and 2 sixths? In 8 and 5 sixths?

18. How many sevenths of 1 are there in 2? In 4 and 3 sevenths how many? How many in 5 and 5 sevenths? In and 6 sevenths?

19. How many eighths of 1 are there in 2? How many in 2 and 3 eighths? In 2 and 5 eighths? In 2 and 7 eighths? In 3? In 3 and 4 eighths? In 9? In 9 and 5 eighths? In 10? In 10 and 7 eighths?

20. How many twelfths of 1 are there in 2? In 2 and 4 twelfths how many? How many in 4 and 9 twelfths? How many in 5 and 10 twelfths? In 6 and 9 twelfths? In 10 and 11 twelfths?

21. What is the product of 12 multiplied by 3 and one-half, (which is written $3\frac{1}{2}$)?

ANALYSIS.—Twelve is to be taken 3 and one-half times (Art. 45). Twelve taken $\frac{1}{2}$ times is 6; and 12 taken three times is 36; therefore, 12 taken $3\frac{1}{2}$ times is 42.

22. What is the product of 10 multiplied by $5\frac{1}{2}$?

23. What is the product of 12 multiplied by $3\frac{1}{3}$?

24. What is the product of 8 multiplied by $4\frac{1}{2}$?

25. What will 9 barrels of sugar cost at $2\frac{2}{3}$ dollars a barrel?

ANALYSIS.—Nine barrels of sugar will cost nine times as much as 1 barrel. If one barrel of sugar costs $2\frac{2}{3}$ dollars, 9 barrels will cost 9 times $2\frac{2}{3}$ dollars, which are 24 dollars. For, 2 thirds taken 9 times gives 18 thirds, which are equal to 6; then 9 times 2 are 18, and 6 added gives 24 dollars.

26. What will 6 yards of cloth cost at $5\frac{2}{3}$ dollars a yard?

27. What will 12 sheep cost at $4\frac{1}{2}$ dollars apiece?

28. What will 10 yards of calico cost at $9\frac{2}{5}$ cents a yard?

29. What will 8 yards of broadcloth cost at $7\frac{5}{8}$ dollars a yard?

30. What will 9 tons of hay cost at $9\frac{3}{8}$ dollars a ton?

31. How many times is $2\frac{1}{2}$ contained in 10?

ANALYSIS.—Two and one-half is equal to 5 halves; and 10 is equal to 20 halves; then, 5 halves is contained in 20 halves 4 times: hence, *In all similar questions change the divisor and dividend to the same fractional unit.*

32. How many yards of cloth, at $3\frac{1}{2}$ dollars a yard, can you buy for 14 dollars? how many for 21 dollars?

33. If oranges are $3\frac{1}{3}$ cents apiece, how many can you buy for 20 cents?

34. If 1 yard of ribbon costs $2\frac{2}{5}$ cents, how many yards can you buy for 12 cents?

35. If 1 yard of broadcloth costs $3\frac{2}{3}$ dollars, how many yards can be bought for 33 dollars?

36. If 1 pound of sugar costs $4\frac{1}{2}$ cents, how many pounds can be bought for 36 cents?

37. How many times is $5\frac{1}{2}$ contained in 44?

38. How many times is $2\frac{2}{3}$ contained in 24?

39. How many lemons, at $2\frac{2}{3}$ cents apiece, can you buy for 32 cents?

40. How many yards of ribbon, at $1\frac{5}{7}$ cents a yard, can you buy for 12 cents?

LONG DIVISION.

65. Long Division is the operation of finding the quotient of one number divided by another, and embraces the case of Short Division, treated in Art. 63.

1. Let it be required to divide 7059 by 13.

Analysis.—The divisor, 13, is not contained in 7 thousands; therefore, *there are no thousands in the quotient.*

We then consider the 0 to be annexed to the 7, making 70 hundreds, and call this a *partial dividend.*

The divisor, 13, is contained in 70 hundreds, 5 hundreds times and something over. To find how much over, multiply 13 by 5 hundreds and subtract the product 65 from 70, and there will remain 5 hundreds, to which bring down the 5 tens, and consider the 55 tens a new *partial dividend.*

OPERATION.

```
    Thous. Hunds. Tens. Units.   Hunds. Tens. Units.
13)   7      0     5     9    (    5     4     3
      6      5
      -------
             5     5
             5     2
             ---------
                   3     9
                   3     9
                   -------
```

65. What is long division? Does it embrace the case of short division? What is a partial dividend?

Then, 13 is contained in 55 tens, 4 tens times and something over. Multiply 13 by 4 tens and subtract the product, 52, from 55, and to the remainder 3 tens bring down the 9 units, and consider the 39 units a new *partial dividend.*

Then, 13 is contained in 39, 3 times. Multiply 13 by 3, and subtract the product 39 from 39, and we find that nothing remains.

66. PROOF.—Each product that has arisen from multiplying the divisor by a figure of the quotient, is a partial product, and the sum of these products is the product of the divisor and quotient (Art. 51, NOTE). Each product has been taken, separately, from the dividend, and nothing remains. But, taking each product away in succession, leaves the same remainder as would be left if their sum were taken away at once. Hence, the number 543, when multiplied by the divisor, gives a product equal to the dividend: therefore, 543 is the quotient (Art. 61): hence, to prove division,

Multiply the divisor by the quotient and add in the remainder, if any. If the work is right, the result will be the same as the dividend.

67. Let it be required to divide 2756 by 26.

OPERATION.

```
26)2756(106
   26
   ---
    156
    156
    ---
```

We first say, 26 in 27 once, and place 1 in the quotient. Multiplying by 1, subtracting, and bringing down the 5, we have 15 for the first partial dividend. We then say, 26 in 15, 0 times, and place the 0 in the quotient. We then bring down the 6, and find that the divisor is contained in 156, 6 times.

If any one of the partial dividends is less than the divisor, write 0 for the quotient figure, and then bring down the next figure, forming a new partial dividend.

Hence, for Long Division, we have the following

RULE.—I. *Write the divisor on the left of the dividend.*

II. *Note the fewest figures of the dividend, at the left, that will contain the divisor, and set the quotient figure at the right.*

---

66. What is a partial product? What is the sum of all the partial products equal to? How do you prove division?

67. What do you do if any partial dividend is less than the divisor? What is the rule for long division?

III. *Multiply the divisor by the quotient figure, subtract the product from the first partial dividend, and to the remainder annex the next figure of the dividend, forming a second partial dividend.*

IV. *Find in the same manner the second and succeeding figures of the quotient, till all the figures of the dividend are brought down.*

NOTE 1.—There are five operations in Long Division. 1st. To write down the numbers: 2d. Divide, or find how many times: 3d. Multiply: 4th. Subtract: 5th. Bring down, to form the partial dividends.

2. The product of a quotient figure by the divisor must never be larger than the corresponding partial dividend: if it is, the quotient figure is too large and must be diminished.

3. When any one of the remainders is greater than the divisor, the quotient figure is too small and must be increased.

4. The unit of any quotient figure is the same as that of the partial dividend from which it is obtained. The pupil should always name the unit of every quotient figure.

EXAMPLES.

1. Divide 7574 by 54.

OPERATION.

```
54)7574(140
   54
   ---
   217
   216
   ---
    14
    00
    --
    14 Remainder.
```

PROOF.

```
 140 Quotient.
  54 Divisor.
 ---
 560
700
----
7560
  14 Remainder
----
7574 Dividend.
```

2. Divide 67289 by 261.

OPERATION.

```
261)67289(257
    522
    ----
    1508
    1305
    ----
     2039
     1827
     ----
      212 Remainder
```

PROOF.

```
  261 Divisor.
  257 Quotient.
 ----
 1827
1305
522
-----
  212 Remainder
-----
67289 Dividend.
```

3. Divide 119836687 by 39407.

OPERATION.	PROOF
39407)119836687(304	39407 Divisor.
118221	3041 Quotient.
161568	39407
157628	157628
39407	118221
39407	119836687 Dividend.

4. Divide 7210473 by 37.
5. Divide 147735 by 45.
6. Divide 937387 by 54,
7. Divide 145260 by 108.
8. Divide 79165238 by 238.
9. Divide 62015735 by 78.
10. Divide 14420946 by 74.
11 Divide 295470 by 90.
12. Divide 1874774 by 162.
13. Divide 435780 by 216.
14. Divide 203812983 by 5049.
15. Divide 20195411808 by 3012.
16. Divide 74855092410 by 949998.
17. Divide 47254149 by 4674.
18. Divide 119184669 by 38473.
19. Divide 280208122081 by 912314.
20. Divide 293839455936 by 8405.
21. Divide 4637064283 by 57606.
22. Divide 352107193214 by 210472.
23. Divide 558001172606176724 by 2708630425.
24. Divide 1714347149347 by 57143.
25. Divide 675437149567159 4 by 678957.
26. Divide 71900715708 by 37149.
27. Divide 571943007145 by 37149.
28. Divide 671493471549375 by 47143
29. Divide 571943007645 by 37149.
30. Divide 171493715947143 by 57007.
31. Divide 121932631112635269 by 987654321.

NOTES.—1. How many operations are there in long division? Name them.

2. If a partial product is greater than the partial dividend, what does it indicate? What do you do?

3. What do you do when any one of the remainders is greater than the divisor?

4. What is the unit of any figure of the quotient? When the divisor is contained in simple units, what will be the unit of the quotient figure? When it is contained in tens, what will be the unit of the quotient figure? When it is contained in hundreds? In thousands?

68. PRINCIPLES RESULTING FROM DIVISION.

NOTES.—1*st.* When the divisor is 1, the quotient will be equal to the dividend.

2*d.* When the divisor is equal to the dividend, the quotient will be 1.

3*d.* When the divisor is less than the dividend, the quotient will be greater than 1. The quotient will be as *many times* greater than 1, as the dividend is times greater than the divisor.

4*th.* When the divisor is greater than the dividend, the quotient will be less than 1. The quotient will be such a part of 1, as the dividend is of the divisor.

PROOF OF MULTIPLICATION.

69. Division is the reverse of multiplication, and they prove each other. The dividend, in division, corresponds to the product in multiplication, and the divisor and quotient to the multiplicand and multiplier, which are factors of the product: hence,

If the product of two numbers be divided by the multiplicand, the quotient will be the multiplier; or, if it be divided by the multiplier, the quotient will be the multiplicand.

EXAMPLES.

```
   3679 Multiplicand.          3679)1203033(327
    327 Multiplier.                 11037
  -----                             -----
  25753                               9933
  7358                                7358
 11037                               -----
 -------                              25753
 1203033 Product.                     25753
```

2. The multiplicand is 61835720, and the product 8162315040: what is the multiplier?

3. The multiplier is 270000; now if the product be 131517000000, what will be the multiplicand?

4. The product is 68959488, the multiplier 96: what is the multiplicand?

5. The multiplier is 1440, the product 10264849920 what is the multiplicand?

6. The product is 6242102428164, the multiplicand 6795634: what is the multiplier?

CONTRACTIONS IN MULTIPLICATION.

70. *To multiply by* 25.

1. Multiply 275 by 25.

ANALYSIS.—If we annex two ciphers to the multiplicand, we multiply it by 100 (Art. 55): this product is 4 times too great; for the multiplier is but *one-fourth* of 100; hence, to multiply by 25,

OPERATION.

$$4\underline{)27500}$$
$$6875$$

Annex two ciphers to the multiplicand and divide the result by 4.

EXAMPLES.

1. Multiply 127 by 25.
2. Multiply 4269 by 25.
3. Multiply 87504 by 25.
4. Multiply 704963 by 25.

71. *To multiply by* $12\frac{1}{2}$.

1. Multiply 326 by $12\frac{1}{2}$.

ANALYSIS.—Since $12\frac{1}{2}$ is *one-eighth* of 100, *Annex two ciphers to the multiplicand and divide the result by* 8.

OPERATION.

$$8\underline{)32600}$$
$$4075$$

EXAMPLES.

1. Multiply 284 by $12\frac{1}{2}$.
2. Multiply 376 by $12\frac{1}{2}$.
3. Multiply 4740 by $12\frac{1}{2}$.
4. Multiply 70424 by $12\frac{1}{2}$.

72. *To multiply by* $33\frac{1}{3}$.

1. Multiply 675 by $33\frac{1}{3}$.

ANALYSIS.—Annexing two ciphers to the multiplicand, multiplies it by 100: but the multiplier is but *one-third* of 100: hence.

Annex two ciphers and divide the result by 3.

OPERATION.

$$3\underline{)67500}$$
$$22500$$

EXAMPLES.

1. Multiply 889626 by $33\frac{1}{3}$.
2. Multiply 740362 by $33\frac{1}{3}$.
3. Multiply 5337756 by $33\frac{1}{3}$.
4. Multiply 2221086 by $33\frac{1}{3}$.

---

68. When the divisor is 1, what is the quotient? When the divisor is equal to the dividend, what is the quotient? When the divisor is less than the dividend, how does the quotient compare with 1? When the divisor is greater than the dividend, how does the quotient compare with 1?

69. If a product be divided by one of the factors, what is the quotient?

73. *To multiply by* 125.

1. Multiply 375 by 125.

ANALYSIS.—Annexing three ciphers to the multiplicand, multiplies it by 1000: but 125 is but *one-eighth* of one thousand: hence,

Annex three ciphers and divide the result by 8.

OPERATION.

$$\begin{array}{r} 8)\underline{375000} \\ 46875 \end{array}$$

EXAMPLES.

1. Multiply 29632 by 125.
2. Multiply 8796704 by 125.
3. Multiply 970406 by 125.
4. Multiply 704294 by 125.

74. By reversing the last four processes, we have the four following rules:

1. To divide any number by 25;

Multiply the number by 4, *and divide the product by* 100.

2. To divide any number by $12\frac{1}{2}$.

Multiply the number by 8, *and divide the product by* 100.

3. To divide any number by $33\frac{1}{3}$:

Multiply the number by 3, *and divide the product by* 100.

4. To divide any number by 125:

Multipy by 8, *and divide the product by* 1000.

EXAMPLES.

1. Divide 3175 by 25.
2. Divide 106725 by 25.
3. Divide 2187600 by 25.
4. Divide 2426225 by 25.
5. Divide 1762405 by 25.
6. Divide 4075 by $12\frac{1}{2}$.
7. Divide 3550 by $12\frac{1}{2}$.
8. Divide $59262\frac{4}{8}$ by $12\frac{1}{2}$.
9. Divide 880300 by $12\frac{1}{2}$.
10. Divide 22500 by $33\frac{1}{3}$.
11. Divide 654200 by $33\frac{1}{3}$.
12. Divide 7925200 by $33\frac{1}{3}$.
13. Divide 4036200 by $33\frac{1}{3}$.
14. Divide 93750 by 125.
15. Divide 3007875 by 125.
16. Divide 6758625 by 125.

---

70. What is the rule for multiplying by 25?
71. What is the rule for multiplying by $12\frac{1}{2}$?
72. What is the rule for multiplying by $33\frac{1}{3}$?
73. What is the rule for multiplying by 125?

CONTRACTIONS IN DIVISION.

75. Contractions in Division are short methods of finding the quotient, when the divisors are composite numbers.

CASE I.

76. *When the divisor is a composite number.*

1. Let it be required to divide 1407 dollars equally among 21 men. Here the factors of the divisor are 7 and 3.

ANALYSIS.—Let the 1407 dollars be first divided into 7 equal piles. Each pile will contain 201 dollars. Let *each* pile be now divided into 3 equal parts. Each part will contain 67 dollars, and the number of parts will be 21 : hence the following

OPERATION.

7)1407
3)201 1st quotient.
67 quotient sought

RULE.—*Divide the dividend by one of the factors of the divisor; then divide the quotient, thus arising, by a second factor, and so on, till every factor has been used as a divisor: the last quotient will be the answer.*

EXAMPLES.

Divide the following numbers by the factors ;

1. 1260 by $12=3\times4$.
2. 18576 by $48=4\times12$.
3. 9576 by $72=9\times8$.
4. 19296 by $96=12\times8$.
5. 55728 by $4\times9\times4=144$.
6. 92880 by $2\times2\times3\times2\times2$.
7. 57888 by $4\times2\times2\times2$.
8. 154368 by $3\times2\times2$.

NOTE.—It often happens that there are remainders after some of the divisions. How are we to find the *true* remainder?

74.—1. What is the rule for dividing by 25?
2. What is the rule for dividing by $12\frac{1}{2}$?
3. What is the rule for dividing by $33\frac{1}{3}$?
4. What is the rule for dividing by 125?

75. What are contractions in division? What is a composite number?

76. What is the rule for division when the divisor is a composite number?

77. Let it be required to divide 751 grapes into 16 equal parts.

$$4\times4=16\left\{\begin{array}{l} 4\underline{)751} \\ 4\underline{)187}\ .\ .\ .\ .\ 3 \text{ first remainder.} \\ \quad 46\ .\ .\ .\ .\ 3\times4=12 \\ \qquad\qquad\qquad\quad \underline{\ \ 3} \\ \qquad\qquad\qquad\quad 15 \text{ true rem.} \quad \textit{Ans. } 46\tfrac{15}{16}. \end{array}\right.$$

NOTE.—The factors of the divisor 16, are 4 and 4.

ANALYSIS.—If 751 grapes be divided by 4, there will be 187 bunches, each containing 4 grapes, and 3 grapes over. The unit of 187 is *one bunch;* that is, a unit 4 *times as great as* 1 grape.

If we divide 187 bunches by 4, we shall have 46 piles, each containing 4 bunches, and 3 bunches over: here, again, the unit of the quotient is 4 times as great as the unit of the dividend.

If, now we wish to find the number of grapes *not included* in the 46 piles, we have 3 bunches with 4 grapes in a bunch, and 3 grapes besides: hence, $4 \times 3 = 12$ grapes; and adding 3 grapes, we have a remainder, 15 grapes; therefore, to find the remainder, in units of the given dividend:

I. *Multiply the last remainder by the last divisor but one, and add in the preceding remainder:*

II. *Multiply this result by the next preceding divisor, and add in the remainder, and so on, till you reach the unit of the dividend.*

EXAMPLES.

1. Let it be required to divide 43720 by 45.

$$45=3\times5\times3\left\{\begin{array}{llr} 3\underline{)43720} & & \\ 5\underline{)14573} & .\ 1=\text{1st rem.} & 1\times5+3=8; \\ \ \ 3\underline{)2914} & .\ 3=\text{2d rem.} & 8\times3+1=25 \\ \qquad 971 & .\ 1=\text{3d rem.} & 25 \text{ true rem.} \end{array}\right.$$

Divide the following numbers by the factors, for the divisors:

2. 956789 by $7\times8=56$.
3. 4870029 by $8\times9=72$.
4. 674201 by $10\times11=110$.
5. 445767 by $12\times12=144$.
6. 1913578 by $7\times2\times3=42$
7. 146187 by $3\times5\times7=105$
8. 26964 by $5\times2\times11=110$
9. 93696 by $3\times7\times11=231$

7ʳ Give the rule for the remainder

CASE II.

78. *When the divisor is* 10, 100, 1000, *&c.*

Analysis.—Since any number is made up of units, tens, hundreds &c. (Art. 28), the number of *tens* in any dividend will denote how many times it contains 1 ten, and the units will be the remainder. The *hundreds* will denote how many times the dividend contains 1 hundred, and the tens and units will be the remainder; and similarly when the divisor is 1000, 10000, &c.; hence,

Cut off from the right hand as many figures as there are ciphers in the divisor—the figures at the left will be the quotient, and those at he right, the remainder.

EXAMPLES.

1. Divide 49763 by 10.
2. Divide 7641200 by 100.
3. Divide 496321 by 1000.
4. Divide 64978 by 10000.

CASE III.

79. *When there are ciphers on the right of the divisor.*

1. Let it be required to divide 673889 by 700.

OPERATION.

$$\begin{array}{l} 7|00)673|89 \\ \quad\quad 96\ .\ .\ 1 \text{ remains.} \\ \quad\quad\ \ 189 \text{ true remain.} \\ \quad\quad\quad \textit{Ans. } 96\tfrac{189}{700}. \end{array}$$

Analysis.—We may regard the divisor as a composite number, of which the factors are 7 and 100. We first divide by 100 by striking off the 89, and then find that 7 is contained in the remaining figures, 96 times, with a remainder of 1; this remainder we multiply by 100, and then add 89, forming the true remainder 189: to the quotient 96 we annex 189 divided by 700, for the entire quotient: hence, the following

Rule.—I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor, and annex to the remainder, if there be one, the figures cut off from the dividend: this will form the true remainder.*

EXAMPLES.

1. Divide 8749632 by 37000.

---

78. How do you divide when the divisor is 1 with ciphers annexed? Give the reason of the rule?

79. How do you divide when there are ciphers on the right of the divisor? How do you form the true remainder?

```
37|000)8749|632(236
       74
       134
       111
        239
        222
         17
```

Ans. $236\frac{17632}{37000}$

Divide the following numbers :

2. 986327 by 210000.
3. 876000 by 6000.
4. 36599503 by 400700.
5. 5714364900 by 36500
6. 18490700 by 73000.
7. 70807149 by 31500.

APPLICATIONS.

80. Abstractly, the object of division is to find from two given numbers a third, which, multiplied by the first, will produce the second. Practically, it has three objects :

1. Knowing the number of things and their entire cost, to find the price of a single thing :

2. Knowing the entire cost of a number of things and the price of a single thing, to find the number of things :

3. To divide any number of things into a given number of equal parts.

For these cases, we have from the previous principles (page 57), the following

RULES.

I. *Divide the entire cost by the number of the things : the quotient will be the price of a single thing.*

II. *Divide the entire cost by the price of a single thing : the quotient will be the number of things.*

III. *Divide the whole number of things by the number of parts into which they are to be divided : the quotient will be the number in each part.*

QUESTIONS INVOLVING THE PREVIOUS RULES.

1. Mr. Jones died, leaving an estate worth 4500 dollars, to be divided equally between 3 daughters and 2 sons : what was the share of each ?

---

80. What is the object of division, abstractly ? How many objects has it, practically ? Name the three objects. Give the rules for the three cases

2. What number must be multiplied by 124 to produce 40796?

3. The sum of 19125 dollars is to be distributed equally among a certain number of men, each to receive 425 dollars: how many men are to receive the money?

4. A merchant has 5100 pounds of tea, and wishes to pack it in 60 chests: how much must he put in each chest?

5. The product of two numbers is 51679680, and one of the factors is 615: what is the other factor?

6. Bought 156 barrels of flour for 1092 dollars, and sold the same for 9 dollars per barrel: how much did I gain?

7. Mr. James has 14 calves worth 4 dollars each, 40 sheep worth 3 dollars each; he gives them all for a horse worth 150 dollars: does he make or lose by the bargain?

8. Mr. Wilson sells 4 tons of hay at 12 dollars per ton, 80 bushels of wheat at 1 dollar per bushel, and takes in payment a horse worth 65 dollars, a wagon worth 40 dollars, and the rest in cash: how much money did he receive?

9. How many pounds of coffee, worth 12 cents a pound, must be given for 368 pounds of sugar, worth 9 cents a pound?

10. The distance around the earth is computed to be about 25000 miles: how long would it take a man to travel that distance, supposing him to travel at the rate of 35 miles a day?

11. If 600 barrels of flour cost 4800 dollars, what will 2172 barrels cost?

12. If the remainder is 17, the quotient 610, and the dividend 45767, what is the divisor?

13. The salary of the President of the United States is 25000 dollars a year: how much can he spend daily and save of his salary 4925 dollars at the end of the year?

14. A farmer purchased a farm for which he paid 18050 dollars. He sold 50 acres for 60 dollars an acre, and the remainder stood him in 50 dollars an acre: how much land did he purchase?

15. There are 31173 verses in the Bible: how many verses must be read each day, that it may be read through in a year?

16. A farmer wishes to exchange 250 bushels of oats at 42 cents a bushel, for flour at 7 dollars per barrel: how many barrels will he receive?

17. The owner of an estate sold 240 acres of land and had 512 acres left: how many acres had he at first?

18. Mr. James bought of Mr. Johnson two farms, one containing 250 acres, for which he paid 85 dollars per acre; the second containing 175 acres, for which he paid 70 dollars an acre; he then sold them both for 75 dollars an acre: did he make or lose, and how much?

19. A farmer has 279 dollars with which he wishes to buy cows at 25 dollars, sheep at 4 dollars, and pigs at 2 dollars apiece, of each an equal number: how many can he buy of each sort?

20. The sum of two numbers is 3475, and the smaller is 1162: what is the greater?

21. The difference between two numbers, 1475, and the greater number is 5760: what is the smaller?

22. If the product of two numbers is 346712, and one of the factors is 76: what is the other factor?

23. If the quotient is 482, and the dividend 135442: what is the divisor?

24. A gentleman bought a house for two thousand twenty-five dollars, and furnished it for seven hundred and six dollars; he paid at one time one thousand and ten dollars, and at another time twelve hundred and seven dollars: how much remained unpaid?

25 At a certain election the whole number of votes cast for two opposing candidates was 12672: the successful candidate received 316 majority: how many votes did each receive?

26. Mr. Place purchased 15 cows; he sold 9 of them for 35 dollars apiece, and the remainder for 32 dollars apiece, when he found that he had lost 123 dollars: how much did he pay apiece for the cows?

27. Mr. Gill, a drover, purchased 36 head of cattle at 64 dollars a head, and 88 sheep at 5 dollars a head; he sold the cattle at one-quarter advance and the sheep at one-fifth advance: how much did he receive for both lots?

28. Mr. Nelson supplied his farm with 4 yoke of oxen at 93 dollars a yoke; 4 plows at 11 dollars apiece; 8 horses at 97 dollars each; and agrees to pay for them in wheat at 1 dollar and a half per bushel: how many bushels must he give?

29. If a man's salary is 800 dollars a-year and his expenses 425 dollars, how many years will elapse before he will be worth 10000 dollars, if he is worth 2500 dollars at the present time?

30. How long can 125 men subsist on an amount of food that will last 1 man 4500 days?

31. A speculator bought 512 barrels of flour for 3584 dollars and sold the same for 4608 dollars: how much did he gain per barrel?

32. A merchant bought a hogshead of molasses containing 96 gallons at 35 cents per gallon; but 26 gallons leaked out, and he sold the remainder at 50 cents per gallon: did he gain or lose, and how much?

33. Two persons counting their money, together they had 342 dollars; but one had 28 dollars more than the other: how many had each?

34. Mrs. Louisa Wilsie has 3 houses valued at 12530 dollars, 11324 dollars, and 9875 dollars: also a farm worth 6720 dollars. She has a daughter and 2 sons. To the daughter she gives one-third the value of the houses and one-fourth the value of the farm, and then divides the remainder equally among the boys: how much did each receive?

35. A person having a salary of 1500 dollars, saves at the end of the year 405 dollars: what were his average daily expenses, allowing 365 days to the year?

36. Mr. Bailey has 7 calves worth 4 dollars apiece, 9 sheep worth 3 dollars apiece, and a fine horse worth 175 dollars. He exchanges them for a yoke of oxen worth 125 dollars and a colt worth 65 dollars, and takes the balance in hogs at 8 dollars apiece: how many does he take?

37. Mr. Snooks, the tailor, bought of Mr. Squire, the merchant, 4 pieces of cloth; the first and second pieces each measured 45 yards, the third 47 yards, and the fourth 53 yards; for the whole he paid 760 dollars: what did he pay for 35 yards?

38. Mr. Jones has a farm of 250 acres, worth 125 dollar per acre, and offers to exchange with Mr. Cushing, whose farm contains 185 acres, provided Mr. Cushing will pay him 20150 dollars difference: what was Mr. Cushing's farm valued at per acre?

39. The volcano in the island of Bourbon, in 1796, threw out 45000000 cubic feet of lava : how long would it take 25 carts to carry it off, if each cart carried 12 loads a day, and 40 cubic feet at each load ?

40. The income of the Bishop of Durham, in England, is 292 dollars a day ; how many clergymen would this support n a salary of 730 dollars per annum ?

41. The diameter of the earth is 7912 miles, and the diameter of the sun 112 times as great : what is the diameter of the sun.

42. By the census of 1850, the whole population of the United States was 23191876 ; the number of births for the previous year was 629444 and the number of deaths 324394 : supposing the births to be the only source of increase, what was the population at the beginning of the previous year ?

43. Mr. Sparks bought a third part of neighbor Spendthrift's farm for 2750 dollars. Mr. Spendthrift then sold half the remainder at an advance of 250 dollars, and then Mr. Sparks bought what was left at a further advance of 250 dollars : how much money did Mr. Sparks pay Mr. Spendthrift, and what did he get for his whole farm ?

44. George Wilson bought 24 barrels of pork at 14 dollars a barrel ; one-fourth of it proved damaged, and he sold it at half price, and the remainder he sold at an advance of 3 dollars a barrel : did he make or lose by the operation, and how much ?

45. A miller bought 320 bushels of wheat for 576 dollars, and sold 256 bushels for 480 dollars : what did the remainder cost him per bushel ?

46. A merchant bought 117 yards of cloth for 702 dollars, and sold 76 yards of it at the same price for which he bought it ; what did the cloth sold amount to ?

47. If 46 acres of land produce 2484 bushels of corn : how many bushels will 120 acres produce ?

48. Mr. J. Williams goes into business with a capital of 25000 dollars ; in the first year he gains 2000 ; in the second year 3500 dollars ; in the third year 4000 dollars ; he then invests the whole in a cargo of tea and doubles his money ; he then took out his original capital and divided the residue equally between his 5 children : what was the portion of each ?

UNITED STATES MONEY.

81. Numbers are collections of units of the same kind. In forming these collections, we first collect the lowest or primary units, until we reach a certain number; we then change the unit and make a second collection, and after reaching a certain number we again change the unit, and so on

In abstract numbers, we first collect the units 1 till we reach ten; we then change the unit to 1 ten and collect till we reach 10; we then change the unit to 100, and so on.

A SCALE expresses the relations between the orders of units, in any number. There are two kinds of scales, *uniform* and *varying*. In the abstract numbers, the scale is uniform, the units of the scale being 10, at every point.

82. United States money is the currency established by Congress, A. D. 1786. The names or denominations of its units are, Eagles, Dollars, Dimes, Cents, and Mills.

The coins of the United States are of gold, silver, and copper, and are of the following denominations:

1. Gold: Eagle, half-eagle, three-dollars, quarter-eagle, dollar.
2. Silver: Dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.
3 Copper: Cent, half-cent.

TABLE.

10 Mills	make	1 Cent, marked	*ct.*
10 Cents	- -	1 Dime, - -	*d.*
10 Dimes	- -	1 Dollar, - -	$.
10 Dollars	- -	1 Eagle, - -	*E.*

Mills.	Cents.	Dimes.	Dollars.	Eagles
10	= 1			
100	= 10	= 1		
1000	= 100	= 10	= 1	
10000	= 1000	= 100	= 10	= 1

81. What are numbers? How are numbers formed? How are simple numbers formed? What is the scale? What is the primary unit in simple numbers?

83. It is seen, from the above table, that in United States money, the *primary unit* is 1 mill; the units of the *scale*, in passing from mills to cents, are 10. The second unit is 1 cent, and the units of the scale in passing to dimes, are 10. The third unit is 1 dime, and the units of the scale in passing to dollars, are 10. The fourth unit is 1 dollar, and the units of the scale in passing to eagles, are 10. This scale is the *same as in simple numbers;* therefore,

The units of United States money may be added, subtracted, multiplied, and divided, by the same rules that have already been given for simple numbers.

NUMERATION TABLE.

Tens of dollars or Eagles.	Dollars.	Tens of cents or dimes.	Cents.	Mills.	
			5	7,	is read 5 cents and 7 mills, or 57 mills.
		1	6	4,	- - 16 cents and 4 mills, or 164 mills.
6	2,	1	2	0,	- - 62 dollars 12 cents and no mills.
2	7,	6	2	3,	- - 27 dollars 62 cents and 3 mills.
4	0,	0	4	1,	- - 40 dollars 4 cents and 1 mill.

The comma, or separatrix, is generally used to separate the cents from the dollars. Thus $67,256 is read 67 dollars 25 cents and 6 mills. Cents occupy the two first places on the right of the comma, and mills the third.

United States money is read in dollars, cents and mills.

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82. What is United States money? What are the names of its units? What are the coins of the United States? Which gold? Which silver? Which copper?

83. In United States money what is the primary unit? What is the scale in passing from one denomination to another? How does this compare with the scale in simple numbers? What then follows? What is used to separate dollars from cents? How is United States money read?

84. What is reduction? How many kinds of reduction are there? Name them. How may cents be changed into mills? How may dollars be changed into cents? How into mills?

REDUCTION OF UNITED STATES MONEY.

84. Reduction of United States Money is changing the unit from one denomination to that of another, without altering the value of the number. It is divided into two parts:

1st To reduce from a greater unit to a less, as from dollars to cents.

2d. To reduce from a less unit to a greater, as from mills to dollars.

85. *To reduce from a greater unit to a less.*

From the table it appears,

1st *That cents may be changed into mills by annexing one cipher.*

2d *That dollars may be changed into cents by annexing two ciphers, and into mills by annexing three ciphers.*

3d. *That eagles may be changed into dollars by annexing one cipher.*

The reason of these rules is evident, since 10 mills make a cent, 100 cents a dollar, and 1000 mills a dollar, and 10 dollars 1 eagle.

EXAMPLES.

1. Reduce 25 eagles, 14 dollars, 85 cents and 6 mills to the denomination of mills.

OPERATION.

25 eagles = 250 dollars,
add 14 dollars,
264 dollars = 26400 cents,
add - - 85 cents,
26485 cents = 264850 mills,
add - - - - - - 6 mills,
Ans. 264856 mills,

2. In 3 dollars 60 cents and 5 mills, how many mills?

3 dollars = 300 cents,
60 cents to be added,
360 = 3600 mills, to which add the 5 mills.

3. In 37 dollars 37 cents 8 mills, how many mills?
4. In 375 dollars 99 cents 9 mills, how many mills?
5. How many mills in 67 cents?
6 How many mills in $54 ?
7. How many cents in $125?
8. In $400, how many cents? How many mills?
9. In $375, how many cents? How many mills?
10. How many mills in $4 ? In $6 ? In $10,14 cents?
11. How many mills in $40,36 cents 8 mills?
12. How many mills in $71,45 cents 3 mills?

86. *To reduce from a less unit to a greater.*

1. How many dollars, cents and mills in 26417 mills?

ANALYSIS.—We first divide the mills by 10, giving 2641 cents and 7 mills over; we then divide the cents by 100, giving 26 dollars, and 41 cents over: hence, the answer is 26 dollars 41 cents and 7 mills: therefore,

OPERATION

10)2641|7

100)26|41

Ans. $26,417

I. To reduce mills to cents: *cut off the right hand figure*

II. To reduce cents to dollars: *cut off the two right hand figures:* and,

III. To reduce mills to dollars: *cut off the three right hand figures.*

EXAMPLES.

1. How many dollars cents and mills are there in 67897 mills?
2. Set down 104 dollars 69 cents and 8 mills.
3. Set down 4096 dollars 4 cents and 2 mills.
4. Set down 100 dollars 1 cent and 1 mill.
5. Write down 4 dollars and 6 mills.
6. Write down 109 dollars and 1 mill.
7. Write down 65 cents and 2 mills.
8. Write down 2 mills.
9. Reduce 1607 mills, to dollars cents and mills.
10. Reduce 170464 mills, to dollars cents and mills.
11. Reduce 8674416 mills, to dollars cents and mills.
12. Reduce 94780900 mills, to dollars cents and mills.
13. Reduce 74164210 mills, to dollars cents and mills.

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86. How do you change mills into cents? How do you change cents into dollars? How do you change mills to dollars?

87. One number is said to be an *aliquot* part of another, when it is contained in that other an exact number of times. Thus; 50 cents, 25 cents, &c., are aliquot parts of a dollar: so also 2 months, 3 months, 4 months and 6 months are aliquot parts of a year. The parts of a dollar are sometimes expressed fractionally, as in the following

TABLE OF ALIQUOT PARTS.

$1	=100 cents.	$\frac{1}{8}$ of a dollar	=$12\frac{1}{2}$ cents.
$\frac{1}{2}$ of a dollar	= 50 cents.	$\frac{1}{10}$ of a dollar	= 10 cents.
$\frac{1}{3}$ of a dollar	=$33\frac{1}{3}$ cents.	$\frac{1}{16}$ of a dollar	= $6\frac{1}{4}$ cents
$\frac{1}{4}$ of a dollar	= 25 cents.	$\frac{1}{20}$ of a dollar	= 5 cents.
$\frac{1}{5}$ of a dollar	= 20 cents.	$\frac{1}{2}$ of a cent	= 5 mills.

ADDITION OF UNITED STATES MONEY.

1. Charles gives $9\frac{1}{2}$ cents for a top, and $3\frac{1}{2}$ cents for 6 quills: how much do they all cost him?

2. John gives 1.37\frac{1}{2}$ for a pair of shoes, 25 cents for a penknife, and $12\frac{1}{2}$ cents for a pencil: how much does he pay for all?

ANALYSIS.—We observe that half a cent is equal to 5 mills. We then place the mills, cents and dollars in separate columns. We then add as in simple numbers.

OPERATION.

$1,375
,25
,125
$1,750

3. James gives 50 cents for a dozen oranges, $12\frac{1}{2}$ cents for a dozen apples, and 30 cents for a pound of raisins: how much for all?

OPERATION.

$0,50
,125
,30
$0,925

88. Hence, for the addition of United States money, we have the following

RULE.—I. *Set down the numbers so that units of the same value shall fall in the same column.*

---

87. What is an aliquot part? How many cents in a dollar? In half a dollar? In a third of a dollar? In a fourth of a dollar?

II. *Add up the several columns as in simple numbers, and place the separating point in the sum directly under that in the columns.*

PROOF.—The same as in simple numbers

EXAMPLES.

1. Add $67,214, $10,049, $6,041, $0,271, together.

(1.)	(2.)	(3.)
$ cts. m.	*$ cts. m.*	*$ cts. m.*
67,214	59,316	81,053
10,049	87,425	67,412
6,041	48,872	95,376
0,271	56,708	87,064
$83,575		$330,905

APPLICATIONS.

1. A grocer purchased a box of candles for 6 dollars 89 cents: a box of cheese for 25 dollars 4 cents and 3 mills; a keg of raisins for 1 dollar 12½ cents, (or 12 cents and 5 mills;) and a cask of wine for 40 dollars 37 cents 8 mills: what did the whole cost him?

2. A farmer purchased a cow for which he paid 30 dollars and 4 mills; a horse for which he paid 104 dollars 60 cents and 1 mill; a wagon for which he paid 85 dollars and 9 mills: how much did the whole cost?

3. Mr. Jones sold farmer Sykes 6 chests of tea for $75,641; 9 yards of broadcloth for $27,41; a plow for $9,75; and a harness for $19,674: what was the amount of the bill?

4. A grocer sold Mrs. Williams 18 hams for $26,497; a bag of coffee for $17,419; a chest of tea for $27,047; and a firkin of butter for $28,147: what was the amount of her bill?

5. A father bought a suit of clothes for each of his four boys; the suit of the eldest cost $15,167; of the second, $13,407; of the third, 12,75; and of the youngest, $11,047; how much did he pay in all?

---

88. How do you set down the numbers for addition? How do you add up the columns? How do you place the separating point? How do you prove addition?

6. A father has six children; to the first two he gives each $375,416; to each of the second two, $287,55; to each of the third two, $259,004: how much did he give to them all?

7. A man is indebted to A, $630,49; to B, $25; to C, 87½ cents; to D, 4 mills: how much does he owe?

8. Bought 1 gallon of molasses at 28 cents per gallon; a half pound of tea for 78 cents; a piece of flannel for 12 dollars 6 cents and 3 mills; a plow for 8 dollars, 1 cent and 1 mill; and a pair of shoes for 1 dollar and 20 cents: what did the whole cost?

9. Bought 6 pounds of coffee for 1 dollar 12½ cents; a wash-tub for 75 cents 6 mills; a tray for 26 cents 9 mills; a broom for 37 cents; a box of soap for 2 dollars 65 cents 7 mills, a cheese for 2 dollars 87½ cents; what is the whole amount?

10. What is the entire cost of the following articles, viz.: 2 gallons of molasses, 57 cents; half a pound of tea, 37½ cents; 2 yards of broadcloth, $3,37½ cents; 8 yards of flannel, $9,875; two skeins of silk, 12½ cents, and 4 sticks of twist, 8½ cents?

SUBTRACTION OF UNITED STATES MONEY.

1. John gives 9 cents for a pencil, and 5 cents for a top how much more does he give for the pencil than top?

2. A man buys a cow for $26,37, and a calf for $4,50: how much more does he pay for the cow than calf?

OPERATION.

NOTE.—We set down the numbers as in addition, and then subtract them as in simple numbers.

$26,37
4,50
$21,87

89. Hence, for subtraction of United States money, we have the following

RULE.—I. *Write the less number under the greater so that units of the same value shall fall in the same column.*

---

89. How do you set down the numbers for subtraction? How do you subtract them? Where do you place the separating point in the remainder? How do you prove subtraction?

II. *Subtract as in simple numbers, and place the separating point in the remainder directly under that in the columns.*

PROOF.—The same as in simple numbers.

EXAMPLES.

	(1.)		(2.)
From	$204,679	From	$8976,400
Take	98,714	Take	610,098
Remainder	$105,965	Remainder	$8366,302

(3.)	(4.)	(5.)
$620,000	$327,001	$2349
19,021	2,090	29,33
$600,979	$324,911	$2319,67

6. What is the difference between $6 and 1 mill? Between $9,75 and 8 mills? Between 75 cents and 6 mills? Between $87,354 and 9 mills?

7. From $107,003 take $0,479.

8. From $875,043 take $704,987.

9. From $904,273 take $859,896.

APPLICATIONS.

1. A man's income is $3000 a year; he spends $187,50: how much does he lay up?

2. A man purchased a yoke of oxen for $78, and a cow for $26,003: how much more did he pay for the oxen than for the cow?

3. A man buys a horse for $97,50, and gives a hundred dollar bill: how much ought he to receive back?

4. How much must be added to $60,039 to make the sum $1005,40?

5. A man sold his house for $3005, this sum being $98,039 more than he gave for it: what did it cost him?

6. A man bought a pair of oxen for $100, and sold them again for $75,37½: did he make or lose by the bargain, and how much?

7. A man starts on a journey with $100; he spends $87,57: how much has he left?

8. How much must you add to $40,173 to make $100?

9. A man purchased a pair of horses for $450, but finding one of them injured, the seller agreed to deduct $106,325: what had he to pay?

10. A farmer had a horse worth $147,49, and traded him for a colt worth but $35,048: how much should he receive in money?

11. My house is worth $8975,034; my barn $695,879: what is the difference of their values?

12. What is the difference between nine hundred and sixty-nine dollars eighty cents and 1 mill, and thirty-six dollars ninety-nine cents and 9 mills?

MULTIPLICATION OF UNITED STATES MONEY.

1. John gives 3 cents apiece for 6 oranges: how much do they cost him?

2. John buys 6 pairs of stockings, for which he pays 25 cents a pair: how much do they cost him?

3. A farmer sells 8 sheep for $1,25 each: how much does he receive for them?

OPERATION.

$$\begin{array}{r} \$1,25 \\ 8 \\ \hline \$10,00 \end{array}$$

Analysis.—We multiply the cost of one sheep by the number of sheep, and the product is the entire cost.

90. Hence, for the multiplication of United States money by by an abstract number, we have the following

Rule.—I. *Write the money for the multiplicand, and the abstract number for the multiplier.*

II. *Multiply as in simple numbers, and the product will be the answer in the lowest denomination of the multiplicand.*

III. *Reduce the product to dollars, cents and mills.*

Proof.—Same as in simple numbers.

EXAMPLES.

1. Multiply 385 dollars, 28 cents and 2 mills, by 8.

OPERATION.

$$\begin{array}{rr} & \$385,282 \\ & 8 \\ \hline \text{Product} & \$3082,256 \end{array}$$

(2.)

$$\begin{array}{rr} & \$475,87 \\ & 9 \\ \hline \text{Product} & \$4282,83 \end{array}$$

3. What will 55 yards of cloth come to at 37 cents per yard ?

4. What will 300 bushels of wheat come to at $1,25 per bushel ?

5. What will 85 pounds of tea come to at 1 dollar $37\frac{1}{2}$ cents per pound ?

6. What will a firkin of butter containing 90 pounds come to at $25\frac{1}{2}$ cents per pound ?

7. What is the cost of a cask of wine containing 29 gallons, at 2 dollars and 75 cents per gallon ?

8. A bale of cloth contains 95 pieces, costing 40 dollars $37\frac{1}{2}$ cents each : what is the cost of the whole bale ?

9. What is the cost of 300 hats at 3 dollars and 25 cents apiece ?

10. What is the cost of 9704 oranges at $3\frac{1}{2}$ cents apiece ?

NOTE.—We know that the product of two numbers contains the same number of units, whichever be used as the multiplier (Art. 48). Hence, we may multiply 9704 by $3\frac{1}{2}$ if we assign the proper unit (1 cent) to the product.

OPERATION.

$$\begin{array}{r} 9704 \\ 03\frac{1}{2} \\ \hline 4852 \\ 29112 \\ \hline \$339{,}64 \end{array}$$

11. What will be the cost of 356 sheep at $3\frac{1}{4}$ dollars a head ?

12. What will be the cost of 47 barrels of apples at $1\frac{3}{4}$ dollars per barrel ?

13. What is the cost of a box of oranges containing 450, at $2\frac{1}{2}$ cents apiece ?

14. What is the cost of 307 yards of linen at $68\frac{1}{2}$ cents per yard ?

15. What will be the cost of 65 bushels of oats at $33\frac{1}{3}$ cents a bushel ?

ANALYSIS.—If the price were 1 dollar a bushel, the cost would be as many dollars as there are bushels. But the cost is $33\frac{1}{3}$ cents $= \frac{1}{3}$ of a dollar: hence, the cost will be as many dollars as 3 is contained times in 65 = 21 dollars, and 2 dollars over, which is re-

OPERATION.

$$\begin{array}{r} 3)65{,}000 \\ \hline \$21{,}666\frac{2}{3} \end{array}$$

---

90. How do you multiply United States money ? What will be the denomination of the product ? How will you then reduce it to dollars and cents ? How do you prove multiplication ?

duced to cents by adding two ciphers, and to mills by adding three; then, dividing the cents and mills by 3, we have the entire cost: hence,

91. To find the cost, when the price is an aliquot part of a dollar.

Take such a part of the number which denotes the commodity, as the price is of 1 *dollar.*

EXAMPLES.

1. What would be the cost of 345 pounds of tea at 50 cents a pound?

2. What would 675 bushels of apples cost at 25 cents a bushel?

3. If 1 pound of butter cost $12\frac{1}{2}$ cents, what will 4 firkins cost, each weighing 56 pounds?

4. At 20 cents a yard, what will 42 yards of cloth cost?

5. At $33\frac{1}{3}$ cents a gallon, what will 136 gallons of molasses cost?

6. What will 1276 yds. of cloth cost at $1,25 a yard?

OPERATION.

4)$1276	cost at 1 dollar a yard.
319	cost at 25 cts. a yard.
$1595	cost at $1,25 a yard.

7. What would be the cost of 318 hats at $\$1,12\frac{1}{2}$ apiece?

8. What will 2479 bushels of wheat come to at $1,50 a bushel?

9. At $\$1,33\frac{1}{3}$ a foot, what will it cost to dig a well 78 feet deep?

10. What will be the cost of 936 feet of lumber at 3 dollars a hundred?

ANALYSIS.—At 3 dollars a foot the cost would be $936\times3=2808$ dollars; but as 3 dollars is the price of 100 feet, it follows that 2808 dollars is 100 times the cost of the lumber: therefore, if we divide 2808 dollars by 100 (which we do by cutting off two of the right hand figures (Art. 73), we shall obtain the cost.

OPERATION.

$$\begin{array}{r} 936 \\ 3 \\ \hline \$28,08 \end{array}$$

NOTE.—Had the price been so much per *thousand*, we should have divided by 1000, or cut off three of the right hand figures: hence,

---

91. How do you find the cost of several things when the price is an aliquot part of a dollar?

92. To find the cost of articles sold by the 100 or 1000 :

Multiply the quantity by the price ; and if the price be by the 100, *cut off* two *figures on the right hand of the product ; if by the* 1000, *cut off* three, *and the remaining figures will be the answer in the same denomination as the price, which if cents or mills, may be reduced to dollars.*

EXAMPLES.

1. What will 4280 bricks cost at $5 per 1000 ?
2. What will 2673 feet of timber cost at $2,25 per 100 ?
3. What will be the cost of 576 feet of boards at $10,62 per 1000 ?
4. What is the value of 1200 feet of lathing at 7 dollars per 1000 ?
5. *David Trusty*, *Bought of Peter Bigtree.*

2462	feet of	boards	at	$7,	per 1000.
4520	"	"	"	9,50	"
600	"	scantling	"	11,37	"
960	"	timber	"	15,	"
1464	"	lathing	"	,75	per 100
1012	"	plank	"	1,25	"

Received Payment,

Peter Bigtree.

6. What is the cost of 1684 pounds of hay at $10,50 per ton ?

ANALYSIS.—Since there are 2000*lb.* in a ton, the cost of 1000*lb.* will be half as much as for 1 ton: viz. $5,25, or 525 cents. Multiply this by the number of pounds (1684), and cut off three places from the right, in addition to the two places before cut off for cents : hence,

OPERATION.

```
2)10,50
   5,25  price of 1000lbs.
   1684
$8,84100 Ans.
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93. To find the cost of articles sold by the ton :

Multiply one-half the price of a ton by the number of pounds, and cut off three figures from the right hand of the product. The remaining figures will be the answer in the same denomination as the price of a ton.

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92. How do you find the cost of articles sold by the 100 or 1000 ?

EXAMPLES.

1. What will 3426 pounds of plaster cost at $3,48 per ton?
2. What will be the cost of the transportation of 6742 pounds of iron from Buffalo to New York, at $7 per ton?
3. What will be the cost of 840 pounds of hay at $9,50 per ton? at $12? at $15,84? at $10,36? at $18,75?

DIVISION OF UNITED STATES MONEY.

94. To divide a number expressed in dollars, cents or mills, into any number of equal parts.

RULE.—I. *Reduce the dividend to cents or mills, if necessary.*
II. *Divide as in simple numbers, and the quotient will be the answer in the lowest denomination of the dividend: this may be reduced to dollars, cents, and mills.*

PROOF.—Same as in division of simple numbers.

NOTE.—The sign + is added in the examples, to show that there is a remainder, and that the division may be continued.

EXAMPLES.

1. Divide $4,624 by 4; also, $87,256 by 5.

OPERATION.	OPERATION.
4)$4,624	5)$87,256
$1,156	$17,451$\frac{1}{5}$

2. Divide $37 by 8.

ANALYSIS.—In this example we first reduce the $37 to mills by annexing three ciphers. The quotient will then be mills, and can be reduced to dollars and cents, as before.

OPERATION.
8)$37,000
$ 4,625

3. Divide $56,16 by 16.
4. Divide $495,704 by 129.
5. Divide $12 into 200 equal parts.
6. Divide $400 into 600 equal parts.
7. Divide $857 into 51 equal parts.
8. Divide $6578,95 into 157 equal parts.

---

93. How do you find the cost of articles sold by the ton?
94. What is the rule for division of United States money? How do you prove division? How do you indicate that the division may be continued?

95. The quantity, and the cost of a quantity given, to find the price of unity (Art. 80).

Divide the cost by the quantity.

9. Bought 9 pounds of tea for $5,85; what was the price per pound?

10. Paid $29,68 for 14 barrels of apples: what was the price per barrel?

11. If 27 bushels of potatoes cost $10,125, what is the price of a bushel?

12. If a man receive $29,25 for a month's work, how much is that a day, allowing 26 working days to the month?

13. A produce dealer bought 3 barrels of eggs, each containing 150 dozens, for which he paid $63: how much did he pay a dozen?

14. A man bought a piece of cloth containing 72 yards, for which he paid $252: what did he pay per yard?

15. If $600 be equally divided among 26 persons, what will be each one's share?

16. Divide $18000 in 40 equal parts: what is the value of each part?

17. Divide $3769,25 into 50 equal parts: what is one part?

18. A farmer purchased a farm containing 725 acres, for which he paid $18306,25: what did it cost him per acre?

19. A merchant buys 15 bales of goods at auction, for which he pays $1000: what do they cost him per bale?

20. A drover pays $1250 for 500 sheep; what shall he sell them for apiece, that he may neither make nor lose by the bargain?

21. The dairy of a farmer produces $600, and he has 25 cows: how much does he make by each cow?

22. A farmer receives $840 for the wool of 1400 sheep: how much does each sheep produce him?

23. A merchant buys a piece of goods containing 105 yards, for which he pays $262,50; he wishes to sell it so as to make $52,50: how much must he ask per yard?

96. When the price of unity and the cost of a quantity are given, to find the quantity (Art. 80).

NOTE.—The divisor and dividend must both be reduced to the lowest unit named in either before dividing.

Divide the cost by the price.

24. If I pay $4,50 a ton for coal, how much can I buy for $67,50?

25. At $7 a barrel, how much flour can be bought for $178,50?

26. How many pounds of tea can be bought for $6,75, at 75 cents a pound?

27. What number of barrels of apples can be bought for $47,50, at $2,37½ a barrel?

28. At 44 cents a bushel, how many bushels of oats can be bought for $14,30?

29. At 34 cents a bushel, how many barrels of apples can I buy for $13,60, allowing 2½ bushels to the barrel?

30. If 1 acre of land cost $28,75, how much can be bought for $3220?

31. Paid $40,50 for a pile of wood, at the rate of $3,37½ a cord, how much was there in the pile?

32. How many sheep can be bought for $132, at $1,37½ a head?

33. At $4,25 a yard, how many yards of cloth can be bought for $68?

34. At $1,12½ a day, how long would it take a person to earn $157,50?

APPLICATIONS IN THE FOUR PRECEDING RULES.

NOTE.—See and repeat Rule—page 53: also the three rules—page 74.

1. If 1 yard of cloth costs 3½ dollars, what will 8 yards cost?

2. If 1 ton of hay costs $14½, what will 9 tons cost?

3. If 1 calf costs $4¼, what will 12 calves cost?

4. Mr. Jones bought 250 bushels of oats, for which he paid $156,25: how much did they cost him a bushel?

5 If 12 tons of hay cost 150 dollars, what does 1 ton cost? 8 tons? 50 tons?

6. If 9 dozen of spelling books cost $7,875, what will 1 dozen cost? 6 dozen? 8 dozen?

7. If 75 bushels of wheat cost $131,25, how much will 1 bushel cost? 8 bushels? 120 bushels?

8. If 320 pounds of coffee cost $44,80 cents, how much will 1 pound cost? What will 575 pounds cost?

9. Mr. James B. Smith bought 9 barrels of sugar, each weighing 216 pounds, for which he paid $116.64 : how much did he pay a pound?

10. If 40 tons of hay cost $580, how much is that per ton? What would 70 tons cost at the same rate?

11. If Mr. Wilson has $120 to buy his winter wood, and wood is $4 a cord, how many cords can he buy?

12. At 6 dollars a yard, how many yards of cloth can be bought for 24 dollars? How many for $36?

13. A farmer sold a yoke of oxen for $80,75 ; 6 cows for $29 each ; 30 sheep at $2,50 a head ; and 3 colts, one for $25, the other two for $30 apiece : what did he receive for the whole lot?

14. A merchant buys 6 bales of goods, each containing 20 pieces of broadcloth, and each piece of broadcloth contained 29 yards ; the whole cost him $15660 : how many yards of cloth did he purchase, and how much did it cost him per yard?

15. A person sells 3 cows at $25 each ; and a yoke of oxen for $65 ; he agrees to take in payment 60 sheep : how much do his sheep cost him per head?

16. A man dies leaving an estate of $33000 to be equally divided among his 4 children, after his wife shall have taken her third. What was the wife's portion, and what the part of each child?

17. A person settling with his butcher, finds that he is charged with 126 pounds of beef at 9 cents per pound ; 85 pounds of veal at 6 cents per pound ; 6 pairs of fowls at 37 cents a pair ; and three hams at $1,50 each : how much does he owe him?

18. A farmer agrees to furnish a merchant 40 bushels of rye at 62 cents per bushel, and to take his pay in coffee at 16 cents per pound : how much coffee will he receive?

19. A farmer has 6 ten-acre lots, in each of which he pastures 6 cows ; each cow produces 112 pounds of butter, for which he receives $18\frac{1}{2}$ cents per pound ; the expenses of each cow are 5 dollars and a half : how much does he make by his dairy?

20. Bought a farm of W. N. Smith for 2345 dollars, a span of horses for 375 dollars, 6 cows at 36 dollars each ; I paid him 520 dollars in cash, and a village lot worth 1500 dollars : how many dollars remain unpaid?

BILLS OF PARCELS.

(21.) New York, May 1st, 1854.

Mr. James Spendthrift,

Bought of Benj. Saveall.

16 pounds of tea at 85 cents per pound - - -
27 pounds of coffee at $15\frac{1}{2}$ cents per pound - -
15 yards of linen at 66 cents per yard - . - -

$

Received payment, *Benj. Saveáll.*

(22.) Albany, June 2d, 1854.

Mr. Jacob Johns,

Bought of Gideon Gould.

36 pounds of sugar at $9\frac{1}{2}$ cents per pound - -
3 hogsheads of molasses, 63 galls. each, at 27 cents a gallon - - - - - - - - -
5 casks of rice, 285 pounds each, at 5 cents per pound - - - - - - - - - - -
2 chests of tea, 86 pounds each, at 96 cents per pound - - - - - - - - - - -

Total cost, $

Received payment, For Gideon Gould,
Charles Clark.

(23.) Hartford, November 21st, 1854.

Gideon Jones,

Bought of Jacob Thrifty.

69 chests of tea at $55,65 per chest - - - -
126 bags of coffee, 100 pounds each, at $12\frac{1}{2}$ cents per pound - - - - - - - - -
167 boxes of raisins at $2,75 per box - - -
800 bags of almonds at $18,50 per bag - - -
9004 barrels of shad at $7,50 per barrel - - -
60 barrels of oil, 32 gallons each, at $1,08 per gallon - - - - - - - - - - -

Amount, $

Received the above in full, *Jacob Thrifty.*

DENOMINATE NUMBERS.

97. A SIMPLE NUMBER is a unit or a collection of units. The unit may be either abstract or denominate.

98. A DENOMINATE NUMBER is a collection of denominate units: thus, 3 yards is a denominate number, in which the unit is 1 yard.

99. Numbers which have the *same* unit, are of the *same* denomination: and numbers having *different* units, are of *different* denominations. If two or more denominate numbers, having different units, are connected together, forming a single number, such is called a *compound* denominate number.

100. There are eight different units in Arithmetic: 1st. The abstract unit: 2d. The unit of currency: 3d. The unit of length: 4th. The unit of surface: 5th. The cubic unit or unit of volume: 6th. The unit of weight: 7th. The unit of time: 8th. The unit of circular measure.

ENGLISH MONEY.

101. The units or denominations of English money are guineas, pounds, shillings, pence, and farthings.

TABLE.

4 farthings marked *far.*	make	1 penny, marked	*d.*
12 pence	- - -	1 shilling, -	*s.*
20 shillings	- - -	1 pound, or sovereign,	£
21 shillings	- - -	1 guinea.	

far.	*d.*	*s.*	£
4	= 1		
48	= 12	= 1	
960	= 240	= 20	= 1

NOTES.—1. The primary unit in English money is 1 farthing. The number of units in the *scale*, in passing from farthings to

---

97. What is a simple number?
98. What is a denominate number?
99. When are numbers of the same denomination? When of different denominations? If several numbers having different units are connected together, what is the number called?
100. How many units are there in Arithmetic? Name them.

pence, is 4; in passing from pence to shillings, 12; in passing from shillings to pounds, 20.

2. Farthings are generally expressed in fractions of a penny. Thus, 1 *far.*=¼*d.*; 2 *far.*=½*d.*; 3 *far.*=¾*d.*

3. By reading the second table from right to left, we can see the value of any unit expressed in each of the lower denominaions. Thus, 1*d.*=4*far.*; 1*s.*=12*d.*=48*far.*; £1=20*s.*=240*d.* =960*far.*

REDUCTION OF DENOMINATE NUMBERS.

102. Reduction is changing the unit of a number, without altering its value.

1. How many pence are there in 2*s.* 6*d.*?

ANALYSIS.—Since there are 12 pence in 1 shilling, there are twice 12, or 24 pence in 2 shillings: add the 6 pence: therefore, in 2*s.* 6*d.* there are 30 pence.

2. How many pence in 4 shillings? In 4*s.* 8*d.*? In 5*s.* 6*d.*? In 3*s.* 8*d.*? In 6*s.* 7*d.*?

3. How many shillings in £2? In £3 8*s.*, how many?

4. How many pence in £1? How many shillings in £2 8*s.*? How many in £3 7*s.*?

5. How many shillings are there in 48 pence?

ANALYSIS.—Since there are 12 pence in 1 shilling, there are as many shillings in 48 pence, as 12 is contained times in 48, which is 4: therefore, there are 4 shillings in 48 pence.

6. How many pounds in 40 shillings? In 60? In 80?

103. From the above analyses we see, that reduction of denominate numbers is divided into two parts:

1st. *To change the unit of a number from a higher denomination to a lower.*

2d. *To change the unit of a number from a lower denomination to a higher.*

---

101. What are the denominations of English money?
Notes. 1.—What is the primary unit in English money? Name the scales.
2.—How are farthings generally expressed?
3.—How is the second table read? What does it show?
102. What is Reduction?
103. Into how many parts is reduction divided? What are they?

PRINCIPLES AND EXAMPLES.

104. *To reduce from a higher to a lower unit.*

1. Reduce £27 6*s.* 8*d.* to the denomination of farthings.

ANALYSIS.—Since there are 20 shillings in £1, in £27 there are 27 times 20 shillings, or 540 shillings, and 6 shillings added, make 546*s.* Since 12 pence make 1 shilling, we next multiply by 12, and then add 8*d.* to the product, giving 6560 pence. Since 4 farthings make 1 penny, we next multiply by 4, and add 2 farthings to the product, giving 26242 farthings for the answer.

OPERATION.

£27 6*s.* 8*d.* 2*far.*
20
546*s.*
12
6560*d.*
4
26242 *Ans.*

NOTE.—The units of the scale, in passing from pounds to shillings, are 20; in passing from shillings to pence they are 12; and in passing from pence to farthings, 4.

Hence, to reduce from a higher to a lower unit, we have the following

RULE.—*Multiply the highest denomination by the units of the scale which connect it with the next lower, and add to the product the units of that denomination: proceed in the same manner through all the denominations, till the unit is brought to the required denomination.*

105. *To reduce from a lower unit to a higher.*

1. Reduce 3138 farthings to pounds.

ANALYSIS. — Since 4 farthings make a penny, we first divide by 4. Since 12 pence make a shilling, we next divide by 12. Since 20 shillings make a pound, we next divide by 20, and find that 3138 *far.*=£3 5*s.* 4*d.* 2 *far.*

OPERATION.

4)3138
12)784 - 2 *far.* rem.
2|0)6|5 - - 4*d.* rem.
3 - - - 5*s.* rem
Ans. £3 5*s.* 4*d.* 2 *far.*

Hence, to reduce from a lower to a higher denomination, we have the following

RULE.—I. *Divide the given number by the units of the scale*

---

104. How do you reduce from a higher to a lower unit?
105. How do you reduce from a lower to a higher unit? What will be the unit of any remainder? How do you prove reduction?

which connect it with the next higher denomination, and set down the remainder, if there be one.

II. *Divide the quotient thus obtained by the units of the scale which connect it with the next higher denomination, and set down the remainder.*

III. *Proceed in the same way to the required denomination, and the last quotient, with the several remainders annexed, will be the answer.*

NOTE.—Every remainder will be of the same denomination as its dividend.

PROOF.—After a number has been reduced from a higher denomination to a lower, by the first rule, let it be reduced back by the second; and after a number has been reduced from a lower denomination to a higher, by the second rule, let it be reduced back by the first rule. If the work is right, the results will agree.

EXAMPLES.

1. Reduce £15 7*s*. 6*d*. to pence.

OPERATION.	PROOF.	
15	12)3690	
20	2|0)30|7	. . . 6*d*. rem.
307	15	. . . 7*s*. rem.
12		
3690		*Ans*. £15 7*s* 6*d*.

2. In £31 8*s*. 9*d*. 3 *far*., how many farthings? Also proof
3. In £87 14*s*. $8\frac{1}{2}$*d*., how many farthings? Also proof.
4. In £407 19*s*. $11\frac{3}{4}$*d*., how many farthings? Also proof.
5. In 80 guineas, how many pounds?
6. In 1549 *far*, how many pounds, shillings and pence?
7. In 6169 pence, how many pounds?

LINEAR MEASURE.

106. This measure is used to measure distances, lengths, breadths, heights and depths, &c.

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106. For what is Linear Measure used? What are its denominations? Repeat the table. What is a fathom? What is a hand? What are the units of the scale.

TABLE.

12 inches	make	1 foot,	marked *ft.*
3 feet	- - - -	1 yard,	- - *yd.*
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	- -	1 rod, perch, or pole,	*rd.*
40 rods	- - - -	1 furlong,	- - *fur.*
8 furlongs or 320 rods	-	1 mile,	- - *mi.*
3 miles	- - - -	1 league,	- - *L.*
$69\frac{1}{2}$ statute miles (nearly) or 60 geographical miles,		1 degree of the equator,	*deg.* or °
360 degrees,	- - -	a circum'nce of the earth.	

in.	*ft.*	*yd.*	*rd.*	*fur.*	*mi.*
12	= 1				
36	= 3	= 1			
198	= $16\frac{1}{2}$	= $5\frac{1}{2}$	= 1		
7920	= 660	= 220	= 40	= 1	
63360	= 5280	= 1760	= 320	= 8	= 1

NOTES.—1. A fathom is a length of six feet, and is generally used to measure the depth of water.

2. A band is 4 inches, used to measure the height of horses.

3. The units of the scale, in passing from inches to feet, are 12; in passing from feet to yards, 3; from yards to rods, $5\frac{1}{2}$; from rods to furlongs, 40; and from furlongs to miles, 8.

1. How many inches in 5 feet? In 10 feet? In 16 feet?
2. How many yards in 36 feet? In 54 feet? In 96?
3. How many feet in 144 inches? In 96 inches? In 48?
4. How many furlongs in 3 miles? In 6 miles? In 8?

EXAMPLES.

1. How many inches in 6*rd.* 4*yd.* 2*ft.* 9*in.*

OPERATION.

$$\begin{array}{rl}
 & 6rd.\ 4yd.\ 2ft.\ 9in. \\
 & 5\frac{1}{2} \\ \hline
 3 & \\
 34 & \\ \hline
 37 & \text{yards.} \\
 3 & \\ \hline
 113 & \text{feet.} \\
 12 & \\ \hline
 1365 & \text{inches.}
\end{array}$$

2. In 1365 inches, how many rods?

OPERATION.

$$\begin{array}{ll}
12)1365 & \\ \hline
3)113 \text{ feet} & 9in. \\ \hline
5\frac{1}{2})37 \text{ yards} & 2ft. \\
11)74 & \\ \hline
6\text{rd. } 8 \text{ half yds.} = 4yd. &
\end{array}$$

Ans. 6*rd.* 4*yd.* 2*ft.* 9*in.*

NOTE.—When we reduce rods to yards, we multiply by the scale $5\frac{1}{2}$; that is, we take 6 rods 5 and one-half times. When we reduce yards to rods, we divide by $5\frac{1}{2}$, which is done by reducing the dividend and divisor to halves: the remainder is 8 half-yards, equal to 4 yards.

3. In 59*mi.* 7*fur.* 38*rd.*, how many feet?
4. In 115188 rods, how many miles?
5. In 719*mi.* 16*rd.* 6*yd.*, how many feet?
6. In 118°, how many miles?
7. In 54° 45*mi.* 7*fur.* 20*rd.* 4*yd.* 2*ft.* 10*in.*, how many inches?
8. In 481401716 inches, how many degrees, &c.?

CLOTH MEASURE.

107. Cloth measure is used for measuring all kinds of cloth, ribbons, and other things sold by the yard.

TABLE.

$2\frac{1}{4}$ inches, *in.*	make	1 nail, marked	*na.*
4 nails	- -	1 quarter of a yard,	*qr.*
3 quarters	- -	1 Ell Flemish,	*E. Fl.*
4 quarters	- -	1 yard, - -	*yd.*
5 quarters	- -	1 Ell English, -	*E. E.*

in.	*na.*	*qr.*	*E. Fl.*	*yd.*	*E. E.*
$2\frac{1}{4}$	= 1				
9	= 4	= 1			
27	= 12	= 3	= 1		
36	= 16	= 4	= $1\frac{1}{3}$	= 1	
45	= 20	= 5	= $1\frac{2}{3}$	= $1\frac{1}{4}$	= 1

NOTE.—The units of the scale, in this measure, are $2\frac{1}{4}$, 4, 3, 4 and 5.

1. In 9 inches, how many nails? How many nails in 1 yard? In 2 yards? In 6? In 8?

2. In 4 yards, how many quarters? How many quarters in 8 yards? In 7 how many?

3. How many quarters in 12 nails? In 16 nails? In 20 nails? In 36? In 40?

---

107. For what is cloth measure used? What are its denominations? Repeat the table. What are the units of the scales?

EXAMPLES.

1. How many nails are there in 35*yd.* 3*qr.* 3*na.*?

OPERATION.

35*yd.* 3*qr.* 3*na.*
4
143 quarters.
4
575 nails.

2. In 575 nails, how many yards?

OPERATION.

4)575
4)143 3*na.*
35 3*qr.*

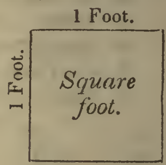
Ans. 35*yd.* 3*qr.* 3*na.*

3. In 49 *E. E.*, how many nails?
4. In 51 *E. Fl.*, 2*qr.* 3*na.*, how many nails?
5. In 3278 nails, how many yards?
6. In 340 nails, how many Ells Flemish?
7. In 4311 inches, how many *E. E.*?

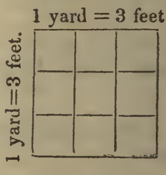
SQUARE MEASURE.

108. Square measure is used in measuring land, or anything in which length and breadth are both considered.

A square is a figure bounded by four equal lines at right angles to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot.*



If the sides of the square be each one yard, the square is called a *square yard.* In the large square there are nine small squares, the sides of which are each one foot. Therefore, the square yard contains 9 square feet.



The number of small squares that is contained in any large square is always equal to the product of two of the sides of the large square. As in the figure, $3 \times 3 = 9$ square feet. The number of square inches contained in a square foot is equal to $12 \times 12 = 144$.

---

108. For what is Square Measure used? What is a square? If each side be one foot, what is it called? If each side be a yard, what is it called? How many square feet does the square yard contain? How is the number of small squares contained in a large square found? Repeat the table. What are the units of the scale?

TABLE.

144	square inches, *sq in.*	make	1 square foot,	*Sq. ft.*
9	square feet	-	1 square yard,	*Sq yd*
$30\frac{1}{4}$	square yards	-	1 square rod or perch,	*P.*
40	square rods or perches	-	1 rood,	*R.*
4	roods	-	1 acre,	*A.*
640	acres	-	1 square mile,	*M.*

Sq. in.	*Sq. ft.*	*Sq. yd.*	*P.*	*R.*	*A.*
144	= 1				
1296	= 9	= 1			
39204	= $272\frac{1}{4}$	= $30\frac{1}{4}$	= 1		
1568160	= 10890	= 1210	= 40	= 1	
6272640	= 43560	= 4840	= 160	= 4	= 1.

NOTE.—The units of the scale are 144, 9, $30\frac{1}{4}$, 40, and 4. 106

1. How many square inches in 2 square feet? How many square feet in 3 square yards? How many in 6? In 8?

2. How many perches in 1 rood? In 3 roods? How many roods in 4 acres? In 8? In 12?

3. How many perches in an acre? How many in 2 acres? How many square yards in 81 square feet?

SURVEYORS' MEASURE.

109. The Surveyor's or Gunter's chain is generally used in surveying land. It is 4 poles or 66 feet in length, and is divided into 100 links.

TABLE.

$7\frac{92}{100}$ inches	make	1	link, marked	*l.*
4 rods or 66*ft.*	-	1	chain,	*c.*
80 chains	-	1	mile,	*mi.*
1 square chain	-	16	square rods or perches,	*P.*
10 square chains	-	1	acre,	*A.*

NOTE.—1. Land is generally estimated in square miles, acres, roods, and square rods or perches.

2. The unitss of the scale are $7\frac{92}{100}$, 4, 80, 1, and 10.

---

109. What chain is used in land surveying? What is its length? How is it divided? Repeat the table. In what is land generally estimated? What are the units of the scale?

1. How many rods in 1 chain? How many in 4? In 5?
2. How many chains in 1 mile? In 2 miles? In 3?
3. How many perches in 1 square chain? In 4? In 6?
4. How many square chains in 2 acres? How many perches in 3 acres? In 5? In 6?

EXAMPLES.

1. How many perches in 32*M*. 25*A*. 3*R*. 19*P*?

OPERATION.

```
  32M. 25A. 3R. 19P.
   640
 20505 acres.
     4
 82023 roods.
    40
3280939 perches.
```

2. How many square miles, &c., in 3280939*P*.?

OPERATION.

```
   40)3280939
      4)82023   19P.
   640)20505    3R.
           32   25A.
```

Ans. 32*M*. 25*A*. 3*R*. 19*P*.

3. In 19*A*. 2*R*. 37*P*., how many square rods?
4. In 175 square chains, how many square feet?
5. In 37456 square inches, how many square feet?
6. In 14972 perches, how many acres?
7. In 3674139 perches, how many square miles?
8. Mr. Wilson's farm contains 104*A*. 3*R*. and 19*P*.; he paid for it at the rate of 75 cents a perch: what did it cost?
9. The four walls of a room are each 25 feet in length and 9 feet in height and the ceiling is 25 feet square: how much will it cost to plaster it at 9 cents a square yard?

CUBIC MEASURE.

110. Cubic measure is used for measuring stone, timber earth, and such other things as have the three dimensions, length, breadth, and thickness.

TABLE.

1728 cubic inches, *Cu. in.*	make	1 cubic foot,	*Cu. ft.*
27 cubic feet,		1 cubic yard,	*Cu. yd*
40 feet of round or 50 feet of hewn timber,		1 ton	*T.*
42 cubic feet		1 ton of shipping,	*T.*
16 cubic feet		1 cord foot,	*C. ft*
8 cord feet, or 128 cubic feet,		1 cord,	*C.*

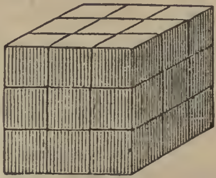
NOTE.—1. A cord of wood is a pile 4 feet wide, 4 feet high, and 8 feet long.

2. A cord foot is 1 foot in length of the pile which makes a cord.

3. A CUBE is a figure bounded by six equal squares, called *faces;* the sides of the squares are called *edges.*

4. A cubic foot is a cube, each of whose faces is a square foot; ts edges are each 1 foot.

5. A cubic yard is a cube, each of whose edges is 1 yard.



3 feet=1 yard.

6. The base of a cube is the face on which it stands. If the edge of the cube is one yard, it will contain $3\times3=9$ square feet; therefore, 9 cubic feet can be placed on the base, and hence, if the figure were 1 foot thick, it would contain 9 cubic feet; if it were 2 feet thick it would contain 2 tiers of cubes, or 18 cubic feet; if it were 3 feet thick, it would contain 27 cubic feet; hence,

The contents of a figure of this form are found by multiplying the length, breadth, and thickness together.

7. A ton of *round timber*, when square, is supposed to produce 40 cubic feet; hence, *one-fifth is lost by squaring.*

1. In 1 cubic foot, how many cubic inches? How many in 2? In 3?

2. In 1 cubic yard, how many cubic feet? How many in 2? In 4? In 6?

3. How many cord feet in 3 cords of wood? In 5? In 6?

4. How many cubic feet in 2 cords? In half a cord, how many? How many in a quarter of a cord?

5. How many cubic yards in 54 cubic feet? In 81?

6. In 120 feet of round timber, how many tons?

7. How many tons of shipping in 84 cubic feet? In 168?

8. How many cords of wood in 64 cord feet? In 96? In 128?

9. How many cubic feet in a stone 8 feet long, 3 feet wide and 2 feet thick?

---

110. For what is cubic measure used? What are its denominations? What is a cord of wood? What is a cord foot? What is a cube? What is a cubic foot? What is a cubic yard? How many cubic feet in a cubic yard? What are the contents of a solid equal to? Repeat the table. What are the units of the scale?

EXAMPLES.

1. In 15*cu. yd.* 18*cu. ft.* 16*cu. in.*, how many cubic inches?

OPERATION.

cu. yd.	*cu. ft.*	*cu. in.*
15	18	16
27		
113		
31		

$423 \times 1728 = 730960$ *cu. in.*

2. In 730960 cubic inches, how many cubic yards, &c.?

OPERATION.

1728)730960 *cu. in.*
27)423 *cu. ft.*
15 *cu. yd.*

	cu. yd.	*cu. ft.*	*cu. in.*
Ans.	15	18	16

3. How many small blocks 1 inch on each edge can be sawed out of a cube 7 feet on each edge, allowing no waste for sawing?

4. In 25 cords of wood, how many cord feet? How many cubic feet?

5. How many cords of wood in a pile 28 feet long, 4 feet wide, and 6 feet in height?

6. In 174964 cord feet, how many cords?

7. In 7645900 cubic inches, how many tons of hewn timber?

WINE OR LIQUID MEASURE.

111. Wine measure is used for measuring all liquids except ale, beer, and milk.

TABLE.

4 gills, *gi.*	make	1 pint,	marked	*pt.*
2 pints	- -	1 quart,	- -	*qt.*
4 quarts	- -	1 gallon,	- -	*gal.*
$31\frac{1}{2}$ gallons	- -	1 barrel,	-	*bar.* or *bbl.*
42 gallons	-	1 tierce,	- -	*tier.*
63 gallons -	- -	1 hogshead,	- -	*hhd.*
2 hogsheads	- -	1 pipe,	- -	*pi.*
2 pipes or 4 hogsheads		1 tun, -	- -	*tun.*

111. What is measured by wine or liquid measure? What are its denominations? Repeat the table. What are the units of the scale? What is the standard wine gallon?

gi.	*pt.*	*qt.*	*gal.*	*bar.*	*tier.*	*hhd.*	*pi.*	*tun.*
4	=1							
8	=2	=1						
32	=8	=4	=1					
1008	=252	=126	=31½	=1				
1344	=336	=168	=42		=1			
2016	=504	=252	=63		=1½	=1		
4032	=1008	=504	=126		=3	=2	=1	
8064	=2016	=1008	=252		=6	=4	=2	=1

NOTE.—The *standard unit*, or gallon of liquid measure, in the United States, contains 231 cubic inches.

1. How many gills in 4 pints? How many pints in 3 quarts? In 6 quarts? In 9? In 10?

2. How many quarts in 2 gallons? In 4 gallons? In 6 gallons? How many pints in 2 gallons? In 5?

3. How many barrels in a hogshead? How many in 4 hogsheads? In 6?

4. How many quarts in 3 gallons? In 5 gallons? In 20? In a barrel how many? In a hogshead how many?

EXAMPLES.

1. In 5 tuns 3 hogsheads 17 gallons of wine, how many gallons?

OPERATION.

```
5tuns 3hhd. 17gal.
 4
 23
 63
 76
139
1466 gallons.
```

2. In 1466 gallons, how many tuns, &c.?

OPERATION.

```
63)1466
  4)23    17 gal.
    5      3 hhd.
```

Ans. 5*tun.* 3*hhd.* 17*gal*

3. In 12 pipes 1 hogshead and 1 quart of wine, how many pints?

4. In 10584 quarts of wine, how many tuns?

5. In 201632 gills, how many tuns?

6. What will be the cost of 3 hogsheads, 1 barrel, 8 gallons, and 2 quarts of vinegar, at 4 cents a quart?

ALE OR BEER MEASURE.

112. Ale or Beer Measure is used for measuring ale, beer, and milk.

TABLE.

2 pints, *pt.*	make	1 quart,	marked	*qt.*
4 quarts	- -	1 gallon,	- -	*gal.*
36 gallons	- -	1 barrel,	- -	*bar.*
54 gallons	- -	1 hogshead,	-	*hhd.*

pt.	*qt.*	*gal.*	*bar.*	*hhd.*
2	= 1			
8	= 4	= 1		
288	= 144	= 36	= 1	
432	= 216	= 54	= $1\frac{1}{2}$	= 1

NOTE.—1 gallon contains 282 cubic inches.

1. How many pints in 3 quarts? How many in 5?
2. How many quarts in 3 gallons? In 4 gallons? In 9?

EXAMPLES.

1. How many quarts are there in 4*hhd.* 2*bar.* 29*gal.* 3*qt.*?

OPERATION.

4*hhd.* 2*bar.* 29*gal.* 3*qt.*
$1\frac{1}{2}$
―――
4
4
―――
8*bar.*
36
―――
57
26
―――
317*gal.*
4
―――
1271*qts.*

2. In 1271 quarts, how many hogsheads, &c.?

OPERATION.

4)1271
―――
36)317 3*qt.*
―――
$1\frac{1}{2}$)8 29*gal.*
―――
4 2*bar.*

Ans. 4*hhd.* 2*bar.* 2*gal.* 3*qt.*

3. In 47*bar.* 16*gal.* 4*qt.*, how many pints?
4. In 27*hhd.* 3*bar.* 25*gal.* 3*qt.*, how many pints?
5. In 55832 pints, how many hogsheads?
6. In 64972 quarts, how many barrels?

---

112. For what is ale or beer measure used? What are its denominations? Repeat the table What are the scales?

DRY MEASURE.

113. Dry Measure is used in measuring all dry articles, such as grain, fruit, salt, coal, &c.

TABLE.

2 pints, *pt.*	make	1 quart,	marked	*qt.*
8 quarts	-	1 peck,	-	*pk.*
4 pecks	-	1 bushel,	-	*bu.*
36 bushels	-	1 chaldron,	-	*ch.*

pt.	*qt.*	*pk.*	*bu.*	*ch.*
2	= 1			
16	= 8	= 1		
64	= 32	= 4	= 1	
2304	= 1152	= 144	= 36	= 1.

1. How many quarts in 2 pecks? In 5? In 8?
2. How many pecks in 24 quarts? In 32? In 64?
3. How many pecks in 6 bushels? In 8? In 12? How many bushels in 16 pecks? In 32? In 40?
4. How many bushels in 2 chaldrons? In 3? In 4?

NOTE.—The standard bushel of the United States is the Winchester bushel of England. It is a circular measure, $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains $2150\frac{2}{5}$ cubic inches, nearly.

2. A gallon, dry measure, contains $268\frac{4}{5}$ cubic inches.

EXAMPLES.

1. How many quarts are there in 65*ch.* 20*bu.* 3*pk.* 7*qt.*?

OPERATION.

```
65ch. 20bu. 3pk. 7qt.
 36
----
 390
 197
----
2360
   4
----
9443
   8
----
75551 quarts.
```

2. How many chaldrons, &c., in 75551 quarts?

OPERATION.

```
 8)75551
  4)9443    7qt.
 36)2360    3pk.
      65   20bu.
```

Ans. 65*ch.* 20*bu.* 3*pk.* 7*qt.*

---

113. What articles are measured by dry measure? What are its denominations? Repeat the table. What are the scales? What is the standard bushel? What are the contents of a gallon?

3. In 372 bushels, how many pints?
4. In 5 chaldrons 31 bushels, how many pecks?
5. In 17408 pints, how many bushels?
6. In 4220 pints, how many chaldrons?

AVOIRDUPOIS WEIGHT.

114. By this weight all coarse articles are weighed, such as hay, grain, chandlers' wares, and all metals except gold and silver.

TABLE.

16 drams, *dr.*	make	1 ounce,	marked *oz.*
16 ounces	-	1 pound,	- *lb.*
25 pounds	-	1 quarter,	- *qr.*
4 quarters	-	1 hundred weight,	*cwt.*
20 hundred weight		1 ton,	- - *T.*

dr.	*oz.*	*lb.*	*qr.*	*cwt.*	*T*
16	= 1				
256	= 16	= 1			
6400	= 400	= 25	= 1		
25600	= 1600	= 100	= 4	= 1	
512000	= 32000	= 2000	= 80	= 20	= 1

NOTES.—1. The standard avoirdupois pound is the weight of 27.7015 cubic inches of distilled water.

2. By the old method of weighing, adopted from the English system, 112 pounds were reckoned for a hundred weight. But now, the laws of most of the States, as well as general usage, fix the hundred weight at 100 pounds.

3. The units of the scale, in passing from drams to ounces, are 16; from ounces to pounds, 16; from pounds to quarters, 25; from quarters to hundreds, 4; and from hundreds to tons, 20.

1. In 2*oz.*, how many drams? In 3? In 4? In 5?
2. In 4*lb.*, how many ounces? In 3 how many? In 2?
3. In 6*qr.*, how many hundred weight? In 5*qr.*?
4. In 3*cwt.*, how many quarters? How many in 4*cwt.*?
5. In 60 hundred weight, how many tons? In 80?

---

114. For what is avoirdupois weight used? How is the table to be read? How can you determine, from the second table, the value of any unit in units of the lower denominations?

EXAMPLES.

1. How many pounds are there in 15*T.* 8*cwt.* 3*qr.* 15*lb.*?

OPERATION.

```
 15T. 8cwt. 3qr. 15lb.
  20
 ----
 308 cwt.
   4
 ----
1235 qr.
  25
 ----
6180      5 lb. added.
2471      1 ten added.
-----
30890 lb.
```

2. In 30890 pounds, how many tons?

OPERATION.

```
25)30890
   -----
  4)1235 qr.     15lb.
  20)308 cwt.    3qr.
     ----
      15 T.      8cwt.
```

15*T.* 8*cwt.* 3*qr.* 15*lb*

3. In 5*T.* 8*cwt.* 3*qr.* 24*lb.* 13*oz.* 14*dr.*, how many drams?

4. In 28*T.* 4*cwt.* 1*qr.* 21*lb.*, how many ounces?

5. In 2790366 drams, how many tons?

6. In 903136 ounces, how many tons?

7. In 3124446 drams, how many tons?

8. In 93*T.* 13*cwt.* 3*qr.* 8*lb.*, how many ounces?

9. In 108910592 drams, how many tons?

10. What will be the cost of 11*T.* 17*cwt.* 3*qr.* 24*lb.* of hay at half a cent a pound? How much would that be a ton?

11. What is the cost of 2*T.* 13*cwt.* 3*qr.* 21*lb.* of beef at 8 cents a pound? How much would that be a ton?

TROY WEIGHT.

115. Gold, silver, jewels, and liquors, are weighed by Troy weight.

TABLE.

24 grains, *gr.*	make	1 pennyweight, marked	*pwt.*
20 pennyweights	-	1 ounce - -	*oz.*
12 ounces -	-	1 pound - -	*lb.*

gr.	*pwt.*	*oz.*	*lb.*
24	= 1		
480	= 20	= 1	
5760	= 240	= 12	= 1

NOTES.—1. The standard Troy pound is the weight of 22.794377 cubic inches of distilled water. It is less than the pound avoirdupois.

2. The units of the scale, in passing from grains to pennyweights, are 24; from pennyweights to ounces, 20; and from ounces to pounds, 12.

1. How many grains in 2 pennyweights? In 3? In 4?
2. How many pennyweights in 48 grains? In 72?
3. How many ounces in 40 pennyweights? In 60?
4. How many ounces in 4 pounds? In 12? In 9? In 7?
5. How many pounds in 24 ounces? In 36? In 96?

EXAMPLES.

1. How many grains are there in 16*lb.* 11*oz.* 15*pwt.* 17*gr.*?

OPERATION.

```
  16lb. 11oz. 15pwt.
  12
 ----
  203  ounces.
   20
 ----
 4075  pennyweights.
   24
 -----
97817  grains.
```

2. In 97817 grains, how many pounds?

OPERATION.

```
24)97817
   20)4075 pwt.      17gr.
   12)2203 oz.       15pwt.
        16 lb.       11oz.
```

Ans. 16*lb.* 11*oz.* 15*pwt.* 17*gr*

3. In 25*lb.* 9*oz.* 20*gr.*, how many grains?
4. In 6490 grains, how many pounds?
5. In 148340 grains, how many pounds?
6. In 117*lb.* 9*oz.* 15*pwt.* 18*gr.*, how many grains?
7. In 8794 *pwt.*, how many pounds?
8. In 6*lb.* 9*oz.* 21*gr.* how many grains?
9. In 1*lb.* 1*oz.* 10*pwt.* 16*gr.*, how many grains?
10. A jewel weighing 2*oz.* 14*pwt.* 18*gr.*, is sold for half a dollar a grain: what is its value?

Notes. 1.—What is the standard avoirdupois pound?

2.—What is a hundred weight by the English method? What is a hundred weight by the United States method?

3. Name the units of the scale in passing from one denomination to another.

115. What articles are weighed by Troy weight? What are its denominations? Repeat the table. What is the standard Troy pound? What are the units of the scale, in passing from one unit to another?

APOTHECARIES' WEIGHT.

116. This weight is used by apothecaries and physicians in mixing their medicines. But medicines are generally sold, in the quantity, by avoirdupois weight.

TABLE.

20 grains, *gr.*	make	1 scruple, marked	℈.
3 scruples	- -	1 dram, - - -	ʒ.
8 drams	- -	1 ounce, - - -	℥.
12 ounces	- -	1 pound, - - -	℔.

gr.	℈	ʒ	℥	℔
20	= 1			
60	= 3	= 1		
480	= 24	= 8	= 1	
5760	= 288	= 96	= 12	= 1

NOTES.—1. The pound and ounce are the same as the pound and ounce in Troy weight.

2. The units of the scale, in passing from grains to scruples, are 20; in passing from scruples to drams, 3; from drams to ounces, 8; and from ounces to pounds, 12.

1. How many grains in 2 scruples? In 3? In 4? In 6?
2. How many scruples in 4 drams? In 7 drams? In 5?
3. How many drams in 5 ounces? How many ounces in 32 drams?

EXAMPLES.

1. How many grains in 9℔ 8℥ 6ʒ 2℈ 12*gr*.

OPERATION.

```
 9℔ 8℥ 6ʒ 2℈ 12gr.
 12
----
 116  ounces.
   8
----
 934  scruples.
   3
----
2804  drams.
  20
-----
56092 grains.
```

2. In 56092 grains, how many pounds?

OPERATION.

```
20)56092
   3)2804 ℈    12gr.
    8)934 ʒ    2℈
   12)116 ℥    6ʒ
         9℔    8℥
```

Ans. 9℔ 8℥ 6ʒ 2℈ 12*gr.*

3. In 27℔ 9℥ 6ʒ 1℈, how many scruples?
4. In 94℔ 11℥ 1ʒ, how many drams?
5. 8011 scruples, how many pounds?
6. In 9113 drams, how many pounds?
7. How many grains in 12℔ 9℥ 7ʒ 2℈ 18*gr.*?
8. In 73918 grains, how many pounds?

MEASURE OF TIME.

117. Time is a part of duration. The time in which the earth revolves on its axis is called a *day*. The time in which it goes round the sun is 365 days and 6 hours, and is called a *year*. Time is divided into parts according to the following

TABLE.

60 seconds, *sec.*	make	1 minute,	marked *m.*
60 minutes	- -	1 hour,	- - *hr.*
24 hours	- - -	1 day,	- - - *da.*
7 days	- - -	1 week,	- - *wk.*
4 weeks	- - -	1 month,	- - *mo.*
13*mo.* 1*da.* and 6*hrs.*, or 365*da.* 6*hr.*		1 Julian year,	- *yr.*
12 calendar months	-	1 year,	- - *yr.*

sec.	*m.*	*hr.*	*da.*	*wk.*	*yr*
60	= 1				
3600	= 60	= 1			
86400	= 1440	= 24	= 1		
604800	= 10080	= 168	= 7	= 1	
31557600	= 525960	= 8766	= $365\frac{1}{4}$	= 52	= 1

Notes.—1. The years are numbered from the beginning of the Christian Era. The year is divided into 12 calendar months, numbered from January: the days are numbered from the beginning of the month: hours from 12 at night and 12 at noon.

Names.	*No.*	*No. days.*	*Names.*	*No.*	*No. days*
January, - -	1st. - -	31	July, - - -	7th. - -	31
February, - -	2d. - -	28	August, - -	8th. - -	31
March, - - -	3d. - -	31	September, -	9th. - -	30
April, - - -	4th. - -	30	October, - -	10th. - -	31
May, - - - -	5th. - -	31	November, -	11th. - -	30
June, - - -	6th. - -	30	December, -	12th. - -	31

2. The length of the tropical year is 365*d.* 5*hr.* 48*m.* 48*sec.* nearly; but in the examples we shall regard it as 365*d.* 6*hr.*

3. Since the length of the year is 365 days and 6 hours, the odd 6 hours, by accumulating for 4 years, make 1 day, so that every fourth year contains 366 days. This is called Bissextile or Leap Year. The leap years are exactly divisible by 4 : 1852, 1856, 1860, are leap years.

4. The additional day, when it occurs, is added to the month of February, so that this month has 29 days in the leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

1. How many seconds in 4 minutes? How many in 6?
2. How many hours in 3 days? How many in 5? In 3?
3. How many days in 6 weeks? In 8, how many?
4. How many hours in 1 week? How many weeks in 42*da.*?

EXAMPLES.

1. How many seconds in 365*da.* 6*hr.*?

OPERATION.

```
365da. 6hr.
 24
----
1466
730
----
8766
  60
----
525960 × 60 = 31557600 sec.
```

2. How many days, &c. in 31557600 seconds?

OPERATION.

```
60)31557600
  ---------
  60)525960
   --------
   24)8766
     -----
      365     6hr.
```

Ans. 365*da.* 6*hr.*

3. If the length of the year were 365*da.* 23*hr.* 57*m.* 39*sec* how many seconds would there be in 12 years?

4. In 126230400 seconds, how many years of 365 days?

5. In 756952018 seconds, how many years of 365 days?

---

117. What are the denominations of time? How long is a year? How many days in a common year? How many days in a Leap year? How many calendar months in a year? Name them, and the number of days in each. How many days has February in the leap year? How do you remember which of the months have 30 days, and which 31?

6. In 285290205 seconds, how many years of 365*da.* 6*hr.* each?

7. How many hours in any year from the 31st day of March to the 1st day of January following, neither day named being counted?

CIRCULAR MEASURE.

118. Circular measure is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies.

The circumference of every circle is supposed to be divided into 360 equal parts, called degrees. Each degree is divided into 60 minutes, and each minute into 60 seconds.

TABLE.

60 seconds″	make	1 minute,	marked ′.
60 minutes	-	1 degree,	- - °.
30 degrees	-	1 sign,	- - s.
12 signs or 360°		1 circle,	- - c.

″	′	°	s.	c.
60	= 1			
3600	= 60	= 1		
108000	= 1800	= 30	= 1	
1296000	= 21600	= 360	= 12	= 1

1. How many seconds in 3 minutes? In 4? In 5?
2. How many minutes in 6 degrees? In 4? In 5?
3. How many degrees in 4 signs? In 6? In 7? In 8?
4. How many degrees in 240 minutes? In 720? How many signs in 90°? In 150°? In 180°?

EXAMPLES.

1. In 5s. 29° 25′, how many minutes?
2. In 2 circles, how many seconds?
3. In 27894 seconds, how many degrees, &c.
4. In 32295 minutes, how many circles, &c.
5. In 3 circles 16° 20′, how many seconds?
6. In 8s. 16° 25″, how many seconds?
7. In 8589 seconds, how many degrees, &c.

---

118. For what is circular measure used? How is every circle supposed to be divided? Repeat the table.

MISCELLANEOUS TABLES.

12 units, or things	make	1 dozen.
12 dozen	-	1 gross.
12 gross, or 144 dozen	-	1 great gross.
20 things	-	1 score.
100 pounds	-	1 quintal of fish.
196 pounds	-	1 barrel of flour.
200 pounds	-	1 barrel of pork.
18 inches	-	1 cubit.
22 inches, nearly	-	1 sacred cubit.
14 pounds of iron or lead	-	1 stone.
$21\frac{1}{2}$ stones	-	1 pig.
8 pigs	-	1 fother.

BOOKS AND PAPER.

The terms, *folio*, *quarto*, *octavo*, *duodecimo*, &c., indicate the number of leaves into which a sheet of paper is folded.

A sheet folded in 2 leaves	is called	a folio.
A sheet folded in 4 leaves	"	a quarto, or 4to.
A sheet folded in 8 leaves	"	an octavo, or 8vo.
A sheet folded in 12 leaves	"	a 12mo.
A sheet folded in 16 leaves	"	a 16mo.
A sheet folded in 18 leaves	"	an 18mo.
A sheet folded in 24 leaves	"	a 24mo.
A sheet folded in 32 leaves	"	a 32mo.

24 sheets of paper	make	1 quire
20 quires	-	1 ream.
2 reams	-	1 bundle.
5 bundles	-	1 bale.

MISCELLANEOUS EXAMPLES.

1. How many hours in 344*wk.* 6*da.* 17*hr.*?
2. In 6 signs, how many minutes?
3. In 15 tons of hewn timber, how many cubic inches?
4. In 171360 pence, how many pounds?
5. In 1720320 drams, how many tons?
6. In 55799 grains of laudanum, how many pounds?
7. In 9739 grains, how many pounds Troy?
8. In 59*lb* 13*pwt.* 5*gr.*, how many grains?
9. In £85 8*s.*, how many guineas?
10. In 346 *E. F.*, how many Ells English?

11. In 3*hhd.* 18*gal.* 2*qt.*, how many half-pints ?
12. In 12*T.* 15*cwt.* 1*qr.* 19*lb.* 12*dr.*, how many drams ?
13. In 40144896 square inches, how many acres ?
14. In 5760 grains, how many pounds ?
15. In 6 years (of 52 weeks each), 32*wk.* 6*da.* 17*hr.*, how many hours ?
16 In 811480″, how many signs ?
17 In 2654208 cubic inches, how many cords ?
18. In 18 tons of round timber, how many cubic inches ?
19. In 84 chaldrons of coal, how many pecks ?
20. In 302 ells English, how many yards ?
21. In 24*hhd.* 18*gal.* 2*qt.* of molasses, how many gills ?
22. In 76*A.* 1*R.* 8*P.*, how many square inches ?
23. In £15 19*s.* 11*d.* 3*far.*, how many farthings ?
24. In 445577 feet, how many miles ?
25. In 37444325 square inches, how many acres ?
26. If the entire surface of the earth is found to contain 791300159907840000 square inches, how many square miles are there ?

27. How many times will a wheel 16 feet and 6 inches in circumference, turn round in a distance of 84 miles ?

28. What will 28 rods, 129 square feet of land cost at $12 a square foot ?

29. What will be the cost of a pile of wood 36 feet long, 6 feet high and 4 feet wide, at 50 cents a cord foot ?

30. A man has a journey to perform of 288 miles. He travels the distance in 12 days, travelling 6 hours each day: at what rate does he travel per hour ?

31. How many yards of carpeting 1 yard wide, will carpet a room 18 feet by 20 ?

32. If the number of inhabitants in the United States is 24 millions, how long will it take a person to count them, counting at the rate of 100 a minute ?

33. A merchant wishes to bottle a cask of wine containing 126 gallons, in bottles containing 1 pint each : how many bottles are necessary ?

34. There is a cube, or square piece of wood, 4 feet each way : how many small cubes of 1 inch each way, can be sawed from it, allowing no waste in sawing ?

35. A merchant wishes to ship 285 bushels of flax-seed in casks containing 7 bushels 2 pecks each : what number of casks are required ?

36. How many times will the wheel of a car, 10 feet and 6 inches in circumference, turn round in going from Hartford to New Haven, a distance of 34 miles?

37. How many seconds old is a man who has lived 32 years and 40 days?

38. There are 15713280 inches in the distance from New York to Boston, how many miles?

39. What will be the cost of 3 loads of hay, each weighing 18*cwt.* 3*qr.* 24*lb.*, at 7 mills a pound?

ADDITION OF DENOMINATE NUMBERS.

119. Addition of denominate numbers is the operation of finding a single number equivalent in value to two or more given numbers. Such single number is called the *sum*.

How many pounds, shillings, and pence in £4 8*s.* 9*d.*, £27 14*s.* 11*d.*, and £156 17*s.* 10*d.*?

ANALYSIS.—We write the units of the same name in the same column. Add the column of pence; then 30 pence are equal to 2 shillings and 6 pence: write down the 6, carrying the two to the shillings. Find the sum of the shillings, which is 41; that is, 2 pounds and 1 shilling over. Write down 1*s.*; then, carrying the 2 to the column of pounds, we find the sum to be £189 1*s.* 6*d.*

OPERATION.

£.	*s.*	*d.*
4	8	9
27	14	11
156	17	10
£189	1*s.*	6*d.*

NOTE.—In simple numbers, the number of units of the scale, at any place, is always 10. Hence, we carry 1 for every 10. In denominate numbers, the scale varies. The number of units, in passing from pence to shillings, is 12; hence, we carry one for every 12. In passing from shillings to pounds, it is 20; hence, we carry one for every 20. In passing from one denomination to another, we carry 1 *for so many units as are contained in the scale at that place*. Hence, for the addition of denominate numbers, we have the following

RULE.—I. *Set down the numbers so that units of the same name shall stand in the same column;*

II. *Add as in simple numbers, and carry from one denomination to another according to the scale.*

PROOF.—The same as in simple numbers.

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119. What is addition of denominate numbers? How do you set down the numbers for addition? How do you add? How do you prove addition?

EXAMPLES.

(1.)

£	s.	d.
173	13	5
87	17	$7\frac{3}{4}$
75	18	$7\frac{1}{2}$
25	17	$8\frac{1}{4}$
10	10	$10\frac{1}{2}$
373	18	3

(2.)

£	s.	d.
705	17	$3\frac{1}{2}$
354	17	$2\frac{3}{4}$
175	17	$3\frac{3}{4}$
87	19	$7\frac{1}{2}$
52	12	$7\frac{3}{4}$

(3.)

£	s.	d.
104	18	$9\frac{1}{2}$
404	17	$8\frac{3}{4}$
467	11	$10\frac{1}{4}$
597	14	$4\frac{1}{4}$
22	18	5
18	6	5

TROY WEIGHT.

(4.)

	lb.	oz.	pwt.	gr.
Add	100	10	19	20
	432	6	0	5
	80	3	2	1
	7	0	0	9
	0	11	10	23
	0	0	8	9

(5.)

lb	oz.	pwt.	gr.
171	6	13	14
391	11	9	12
230	6	6	13
94	7	3	18
42	10	15	20
31	0	0	21

APOTHECARIES' WEIGHT.

(6.)

℔	℥	ʒ	℈	gr.
24	7	2	1	16
17	11	7	2	19
36	6	5	0	7
15	9	7	1	13
9	3	4	1	9

(7.)

℥	ʒ	℈	gr.
11	2	1	17
7	4	2	14
4	0	1	19
2	5	2	11
10	1	2	16

(8.)

ʒ	℈	gr
3	2	15
0	1	13
2	2	11
7	0	17
5	2	14

AVOIRDUPOIS WEIGHT.

(9.)

cwt.	qr.	lb.	oz.	dr.
14	2	0	14	9
13	2	20	1	15
9	3	6	7	3
10	0	18	12	11
7	3	2	3	2
6	1	19	8	1
4	3	0	15	5
12	2	0	0	13

(10.)

T.	cwt.	qr.	lb.	oz.
15	12	1	10	10
71	8	2	6	0
83	19	3	15	5
36	7	0	20	14
47	11	2	2	11
63	5	2	19	7
12	13	1	14	9
9	7	0	5	10

11. A merchant bought 4 barrels of potash of the following weights, viz.: 1st, 3*cwt.* 2*qr.* 0*lb.* 12*oz.* 3*dr.*; 2d, 4*cwt.* 1*qr.* 21*lb.* 4*oz.*; 3d, 4*cwt.*; 4th, 4*cwt.* 0*qr.* 2*lb.* 15*oz.* 15*dr*: what was the entire weight of the four barrels?

LONG MEASURE.

(12.)

L.	*mi.*	*fur.*	*rd.*	*yd.*	*ft.*
16	2	7	39	9	2
327	1	2	20	7	1
87	0	1	15	6	1
1	1	1	1	2	2

(13.)

rd.	*yd.*	*ft.*	*in.*
16	9	2	11
12	11	1	9
18	14	0	7
19	15	2	1

CLOTH MEASURE.

(14.)

E. Fl.	*qr.*	*na.*
126	4	4
65	3	1
72	1	3
157	2	3

(15.)

yd.	*qr.*	*na.*
4	3	2
5	4	1
6	1	0
25	2	2

(16.)

E. E.	*qr.*	*na.*	*in.*
128	5	1	3
20	3	1	2
19	1	4	1
15	3	1	2

LAND OR SQUARE MEASURE.

(17.)

Sq. yd.	*Sq. ft.*	*Sq. in.*
97	4	104
22	3	27
105	8	2
37	7	127

(18.)

M.	*A.*	*R.*	*P.*	*Sq. yd.*
2	60	3	37	25
6	375	2	25	21
7	450	1	31	20
11	30	0	25	19

19. There are 4 fields, the 1st contains 12*A.* 2*R.* 38*P.*; the 2d, 4*A.* 1*R.* 26*P.*; the 3d, 85*A.* 0*R.* 19*P.*; and the 4th, 57*A.* 1*R.* 2*P.*: how many acres in the four fields?

CUBIC MEASURE.

(20.)

Cu. yd.	*Cu. ft.*	*Cu. in.*
65	25	1129
37	26	132
50	1	1064
22	19	17

(21.)

C.	*S. ft.*
16	127
17	12
18	119
37	104

(22.)

C.	*Cord feet.*
87	9
26	7
16	6
19	5

WINE OR LIQUID MEASURE.

(23.)

hhd.	*gal.*	*qt.*	*pt.*
127	65	3	2
12	60	2	3
450	29	0	1
21	0	2	3
14	39	1	2

(24.)

tun.	*pi.*	*hhd.*	*gal.*	*qt.*
14	2	1	27	3
15	1	2	25	2
4	2	1	27	1
5	0	1	62	3
7	1	2	21	2

DRY MEASURE.

(25.)

ch.	*bu.*	*pk.*	*qt.*	*pt.*
27	25	3	7	1
59	21	2	6	3
2	1	2	7	1
5	9	1	8	2

(26.)

ch.	*bu.*	*pk.*	*qt.*	*pt.*
141	36	3	7	2
21	32	2	4	1
85	9	1	0	3
10	4	4	1	3

TIME.

(27.)

yr.	*mo.*	*wk.*	*da.*	*hr.*
4	11	3	6	20
3	10	2	5	21
5	8	1	4	19
101	9	3	7	23
55	8	4	6	17

(28.)

wk.	*da.*	*hr.*	*m.*	*sec.*
8	8	14	55	57
10	7	23	57	49
20	6	14	42	01
6	5	23	19	59
2	2	20	45	48

CIRCULAR MEASURE OR MOTION.

(29.)

s.	°	′	″
5	17	36	29
7	25	41	21
8	15	16	09

(30.)

s.	°	′	″
6	29	27	49
8	18	29	16
7	09	04	58

NOTE.—Since 12 signs make a circumference of a circle, we write down only the excess over exact 12's.

APPLICATIONS IN ADDITION.

1. Add 46*lb.* 9*oz.* 15*pwt.* 16*gr.*, 87*lb.* 10*oz.* 6*pwt.* 14*gr.*, 100*lb.* 10*oz.* 10*pwt.* 10*gr.*, and 56*lb* 3*pwt.* 6*gr* together.

2. What is the weight of forty-six pounds, eight ounces, thirteen pennyweights, fourteen grains ; ninety-seven pounds, three ounces ; and one hundred pounds, five ounces, ten pennyweights and thirteen grains?

3. Add the following together: 29*T.* 16*cwt.* 1*qr.* 14*lb.* 12*oz.* 9*dr.*, 18*cwt.* 3*qr.* 1*lb.*, 50*T.* 3*qr.* 4*oz.*, and 2*T.* 1*qr.* 14*dr.*

4. What is the weight of 39*T.* 10*cwt.* 2*qr.* 2*lb.* 15*oz.* 12*dr.*, 17*cwt.* 6*lb.*, 12*cwt.* 3*qr.*, and 2*qr.* 8*lb.* 9*dr.* ?

5. What is the sum of the following: 314*A.* 2*R.* 39*P.* 200*sq. ft.* 136*sq. in.*, 16*A.* 1*R.* 20*P.* 10*sq. ft.*, 3*R.* 36*P.* and 4*A.* 1*R.* 16*P.*?

6. What is the solid content of 64*ton* 33*ft.* 800*in.*, 9*ton* 1200*in.*, 25*ft.* 700*in.*, and 95*ton* 31*ft.* 1500*in.*

7. Add together, 96*bu.* 3*pk.* 2*qt.* 1*pt.*, 46*bu.* 3*pk.* 1*qt.* 1*pt.*, 2*pk.* 1*qt.* 1*pt.* and 23*bu.* 3*pk.* 4*qt.* 1*pt.*

8. What is the area of the four following pieces of land; the first containing 20*A.* 3*R.* 15*P.* 250*sq. ft.* 116*sq. in.* ; the second, 19*A.* 1*R.* 39*P.* ; the third, 2*R.* 10*P.* 60*sq. ft.* ; and the fourth, 5*A.* 6*P.* 50*sq. in.* ?

9. A farmer raised from one field 37*bu.* 1*pk.* 3*qt.* of wheat; from a second, 41*bu.* 2*pk.* 5*qt.* of barley ; from a third, 35*bu.* 1*pk.* 3*qt.* of rye ; from a fourth, 43*bu.* 3*pk.* 1*qt.* of oats ; how much grain did he raise in all?

10. A grocer received an invoice of 4*hhd.* of sugar; the first weighed 11*cwt.* 15*lb.* ; the second, 12*cwt.* 3*qr.* 15*lb.* ; the third, 9*cwt.* 1*qr.* 16*lb.* ; the fourth, 12*cwt.* 1*qr.* : how much did the four weigh?

11. A lady purchased 32*yds.* 3*qrs.* of sheeting ; 31*yds.* 1*qr.* of shirting ; 14*yds* 2*qrs.* of linen ; and 6*yds.* 2*qrs.* of cambric : what was the whole number of yards purchased ?

12. Purchased a silver teapot weighing 23*oz.* 17*pwt.* 11*gr.* ; a sugar bowl, weighing 8*oz.* 13*pwt.* 19*gr.* ; a cream pitcher, weighing 5*oz.* 11*gr.* : what was the weight of the whole?

13. A stage goes one day, 87*m.* 6*fur.* 24*rd.* ; the next, 75*m* 3*fur.* 17*rd.* ; the third, 80*m.* 7*fur.* 10*rd.* ; the fourth, 78*m.* 5*fur.* : how far does it go in the four days?

14. Bought three pieces of land ; the first contained 17 *acres* 1*R.* 35*rd.* ; the second, 36 *acres* 2*R.* 21*rd.* ; and the third. 46 *acres* 0*R.* 37*rd.* : how much land did I purchase?

SUBTRACTION OF DENOMINATE NUMBERS.

120. The difference between two denominate numbers is such a number as added to the less will give the greater.

SUBTRACTION is the operation of finding this difference.

1. What is the difference between £27 16*s.* 8*d.* and £19 17*s.* 9*d.*?

OPERATION.		
	20	12
£27	16*s.*	8*d.*
19	17	9
7	18	11

ANALYSIS.—We cannnot take 9*d.* from 8*d.*; we therefore add to the upper number as many units as are contained in the scale, and at the same time add 1, mentally, to the next higher denomination of the subtrahend. We then say, 9 from 20 leaves 11. Then, as we cannot subtract 18 from 16, we add 20 and say, 18 from 36 leaves 18. Now, as we have taken 1 pound=20 shillings, from the pounds, and added it to the shillings, there are but 26 pounds left. We may then say, 19 from 26 leaves 7, or 20 from 27 leaves 7. The latter is the easiest in practice.

The first step is called *borrowing*, the second, *carrying*: hence,

RULE.—I. *Set down the less number under the greater, placing units of the same value in the same column.*

II. *Begin with the lowest denomination, and subtract as in simple numbers, borrowing and carrying for each operation according to the scale.*

PROOF.—The same as in simple numbers.

EXAMPLES.

(1.)

	A.	*R.*	*P.*
From	18	3	28
Take	15	2	30
Remainder	3	0	38
Proof	18	3	28

(2.)

T.	*cwt.*	*qr.*	*lb.*
4	12	3	20
2	18	3	1
1	14	0	19
4	12	3	20

(3.)

	lb.	*oz.*	*pwt.*	*gr.*
From	273	0	0	0
Take	98	10	18	21
Remainder				

(4.)

lb.	*oz.*	*pwt.*	*gr.*
18	9	10	0
9	10	15	20

(5.)

	T.	cwt.	qr.	lb.	oz.
From -	7	14	1	3	6
Take -	2	6	3	4	11
Remainder					

(6.)

cwt.	qr.	lb.	oz.	dr.
14	2	12	10	8
6	3	16	15	3

(7.)

	T.	hhd.	gal.	qt.	pt.
From -	151	3	50	3	2
Take -	27	2	54	3	2
Remainder					

(8.)

yr.	wk.	da.	hr.	'	"
95	25	4	20	45	50
80	30	6	23	46	56

TIME BETWEEN DATES.

121. *To find the time between any two dates.*

1. What time elapsed between July 5th, 1848, and August 8th, 1850?

Note.—In the first date, the number of the year is 1848; the number of the month, 7, and the number of the day, 5 In the second date, the number of the year is 1850, the number of the month 8, and the number of the day, 8.

OPERATION.

yr.	mo.	da.
1850	8	8
1848	7	5
2	1	3

Hence, to find the time between two dates:

Write the numbers of the earlier date under those of the later, and subtract according to the preceding rule.

Note.—1. In finding the difference between dates, as in casting interest, the month is regarded as the twelfth part of a year, and as containing 30 days.

2. The civil day begins and ends at 12 o'clock at night.

2. What is the difference of time between March 2d, 1847, and July 4th, 1856?

3. What is the difference of time between April 28th, 1834, and February 3d, 1856?

4. What time elapsed between November 29th, 1836, and January 2d, 1854?

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120. What is the difference between two denominate numbers? Give the rule for subtraction. How do you prove subtraction?

121. Give the rule for finding the difference between two dates. How is the month reckoned? At what time does a civil day begin?

5. What time elapsed between November 8th, at 11 o'clock A.M., 1847, and December 16th, at 4 o'clock, P.M., 1850 ?

OPERATION.

yr.	mo.	da.	hr.
1850	12	16	16
1847	11	8	11
3	1	8	5

ANALYSIS.—The hours are numbered from 12 at night, when the civil day begins. The *numbers* of the years, months, days and hours, are used.

6. What time elapsed between October 9th, at 11 P.M., 1840, and February 6th, at 9 P.M., 1853 ?

7. Mr. Johnson was born September 6th, 1771, at 9 o'clock A.M., and his first child November 5th, 1801, at 9 o'clock P.M. : what was the difference of their ages ?

APPLICATIONS IN ADDITION AND SUBTRACTION.

1. From 38*mo.* 2*wk.* 3*da.* 7*hr.* 10*m.*, take 10*mo.* 3*wk.* 2*da.* 10*hr.* 50*m.*

2. From 176*yr.* 8*mo.* 3*wk.* 4*da.*, take 91*yr.* 9*mo.* 2*wk* 6*da.*

3. From £3, take 3*s.*

4. From 2*lb.*, take 20*gr.* Troy.

5. From 8℔, take 1℔ 1℥ 2ʒ 2℈.

6. From 9*T.*, take 1*T.* 1*cwt.* 2*qr.* 20*lb.* 15*oz.* 14*dr.*

7. From 3 miles, take 3*fur.* 19*rd.*

8. The revolution commenced April 19th, 1775, and a general peace took place January 20, 1783 : how long did the war continue ?

9. America was discovered by Columbus, October 11, 1492 : what was the length of time to July 25, 1855 ?

10. I purchased 167*lb.* 8*oz.* 16*pwt.* 10*gr.* of silver, and sold 98*lb.* 10*oz.* 12*pwt.* 19*gr.* : how much had I left ?

11. I bought 19*T.* 11 *cwt.* 2*qr.* 2*lb.* 12*oz.* 12*dr.* of old iron, and sold 17*T.* 13*cwt.* 2*qr.* 19*lb.* 14*oz.* 10*dr.* : what had I left ?

12. I purchased 101℔ 11℥ 7ʒ 2℈ 19*gr.* of medicine, and sold 17℔ 2℥ 3ʒ 1℈ 5*gr.* : how much remained unsold ?

13. From 46*yd.* 1*qr.* 3*na.*, take 42*yd.* 3*qr.* 1*na.* 2*in.*

14. Bought 7 cords of wood, and 2 cords 78 feet having been stolen, how much remains ?

15. A owes B £100: what will remain due after he has paid him £25 3*s.* 6½*d.*?

16. A farmer raised 136 bushels of wheat; if he sells 49*bu.* 2*pk.* 7*qt.* 1*pt.*, how much will he have left?

17. From 174*hhd.* 10*gal.* 1*qt.* 1*pt.* of beer, take 86*hhd.* 17*gal.* 2*qt.* 1*pt.*

18. A farmer had 576*bu.* 1*pk.* 2*qt.* of wheat; he sold 139*bu.* 2*pk.* 3*qt.* 1*pt.*: how much remained unsold?

19. A merchant bought 17*cwt.* 2*qr.* 14*lb.* of sugar, of which he sold at one time 3*cwt.* 2*qr.* 20*lb.*; at another 6*cwt.* 1*qr.* 5*lb.*: how much remained unsold?

20. Sold a merchant one quarter of beef for £2 7*s.* 9*d.*: one cheese for 9*s.* 7*d.*: 20 bushels of corn for £4 10*s.* 11*d.*; and 40 bushels of wheat for £19 12*s.* 8½*d.*: how much did the whole come to?

21. Bought of a silversmith a teapot, weighing 3*lb.* 4*oz.* 9*pwt.* 21*gr.*; one dozen of silver spoons, weighing 2*lb.* 1*oz.* 1*pwt.*; 2 dishes weighing 16*lb.* 10*oz.* 15*pwt.* 16*gr.*: how much did the whole weigh?

22. Bought one hogshead of sugar weighing 9*cwt.* 3*qr.* 2*lb.* 14*oz.*; one barrel weighing 3*cwt.* 1*qr.* 2*lb.*, and a second barrel weighing 3*cwt.* 0*qr.* 1*lb.* 4*oz.*: how much did the whole weigh?

23. A merchant buys two hogsheads of sugar, one weighing 8*cwt.* 3*qr.* 21*lb.*, the other 9*cwt.* 2*qr.* 6*lb.*; he sells two barrels, one weighing 3*cwt.* 1*qr.* 12*lb.* 14*oz.*, the other, 2*cwt.* 3*qr.* 15*lb.* 6*oz.*: how much remains on hand?

24. A man sets out upon a journey and has 200 miles to travel; the first day he travels 9 leagues 2 miles 7 furlongs 30 rods; the second day 12 leagues 1 mile 1 furlong; the third day 14 leagues; the fourth day 15 leagues 2 miles 5 furlongs 35 rods: how far had he then to travel?

25. A farmer has two meadows, one containing 9*A.* 3*R.* 37*P.*, the other contains 10*A.* 2*R.* 25*P.*; also three pastures, the first containing 12*A.* 1*R.* 1*P.*, the second containing 13*A.* 3*R.*, and the third 6*A.* 1*R.* 39*P.*: by how many acres does the pasture exceed the meadow land?

26. Supposing the Declaration of Independence to have been published at precisely 12 o'clock on the 4th of July, 1776, how much time elapsed to the 1st of January, 1833 at 25 minutes past 3, P. M.?

MULTIPLICATION OF DENOMINATE NUMBERS.

122. Multiplication of denominate numbers is the operation of multiplying a denominate number by an abstract number.

1. A tailor has 5 pieces of cloth each containing 6*yd.* 2*qr.* 3*na.*: how many yards are there in all?

Analysis.—In all the pieces there are 5 times as much as there is in 1 piece. If in 1 piece each denomination be taken 5 times, the result will be 5 times as great as the multiplicand. Taking each denomination 5 times, we have 30*yd.* 10*qr.* 15*na.*

OPERATION.

yd.	*qr.*	*na*
6	2	3
		5
30	10	15
33	1	3

But, instead of writing the separate products, we begin with the lowest denomination and say, 5 times 3*na.* are 15*na.*; divide by 4, the units of the scale, write down the remainder 3*na.*, and reserve the quotient 3*qr.* for the next product. Then say, 5 times 2*qr.* are 10*qr.*, to which add the 3*qr.* making 13*qr.* Then divide by 4, write down the remainder 1, and reserve the quotient 3 for the next product. Then say, 5 times 6 are 30, and 3 to carry are 33 yards: hence,

Rule.—I. *Write down the denominate number and set the multiplier under the lowest denomination.*

II. *Multiply as in simple numbers, and in passing from one denomination to another, divide by the units of the scale, set down the remainder and carry the quotient to the next product.*

Proof.—The same as in simple numbers.

EXAMPLES.

(1.)

£	*s.*	*d.*	*far.*
17	15	9	3
			6
106	14	10	2

(2.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*
	10	0	2	12
				7
3	10	0	19	4

(3.)

m.	*fur.*	*rd.*	*yd.*	*ft.*
9	3	20	3	2
				6

(4.)

s.	°	′	″
9	9	27	35
			3

122. What is multiplication of denominate numbers? Give the rule How do you prove multiplication?

(5.)

yr.	*mo*	*da.*	*hr.*
6	5	15	18
			5

(6.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
6	12	3	20	12	9
					8

7. A farmer has 11 bags of corn, each containing 2*bu.* 1*pk.* 3*qt.*: how much corn in all the bags?

8. How much sugar in 12 barrels, each containing 3*cwt.* 3*qr.* 2*lb.*?

9. In 7 loads of wood, each containing 1 cord and 2 cord feet, how many cords?

10. A bond was given 21st of May, 1825, and was taken up the 12th of March, 1831; what will be the product, if the time which elapsed from the date of the bond till the day it was taken up be multiplied by 3?

11. What is the weight of 1 dozen silver spoons, each weighing 3*oz.* 6*pwt.*?

12. What is the weight of 7 tierces of rice, each weighing 5*cwt.* 2*qr.* 16*lb.*?

13. Bought 4 packages of medicine, each containing 3 ℔ 4 ℥ 6 ʒ 1 ℈ 16*gr.*: what is the weight of all?

14. How far will a man travel in 5 days at the rate of 24*mi.* 4*fur.* 4*rd.* per day?

15. How much land is there in 9 fields, each field containing 12*A.* 1*R.* 25*P.*?

16. How many yards in 9 pieces, each 29*yd.* 2*qr.* 3*na.*?

17. If a vessel sails 5*L.* 2*mi.* 6*fur.* 36*rd.* in one day, how far will it sail in 8 days?

18. How much water will be contained in 96 hogsheads, each containing 62*gal.* 1*qt.* 1*pt.* 1*gi.*?

NOTE.—When the multiplier is a composite number, and the factors do not exceed 12, multiply by the factors in succession. In the last example $96 = 12 \times 8$.

19. If one spoon weigh 3*oz.* 5*pwt.* 15*gr.*, what is the weight of 120 spoons?

20. If a man travel 24*mi.* 7*fur.* 4*rd.* in one day, how far will he go in one month of 30 days?

21. If the earth revolve $0° 15'$ of space per minute of time, how far does it revolve per hour?

22. Bought 90*hhd.* of sugar, each weighing 12*cwt.* 2*qr.* 11*lb.*: what was the weight of the whole?

23. What is the cost of 18 sheep, at 5*s*. $9\frac{1}{2}$*d*. apiece ?

24. How much molasses is contained in 25*hhd*. each hogshead having 61*gal*. 1*qt*. 1*pt*. ?

25. How many yards of cloth in 36 pieces, each piece containing 25*yd*. 3*qr*. ?

26. A farmer has 18 lots, and each lot contains 41*A*. 2*R*. 11*P*. : how many acres does he own ?

27. There are three men whose mutual ages are 14 times 20*yr*. 5*mo*. 3*wk*. 6*da*. : what is the sum of their ages ?

28. Bought 90*hhd*. of sugar, each weighing 12*cwt*. 2*qr*. 14*lb*. ; what is the weight of the whole ?

29. If a vessel sail 49*mi*. 6*fur*. 8*rd*. in one day, how far will she sail in one month of 30 days ?

30. Suppose each of 50 farmers to raise 125 *bu*. 3*pk*. 6*qt*. of grain : how much do they all raise ?

31. If a steam ship, in crossing the Atlantic, goes 211*mi*. 4*fur*. 32*rd*. a day, how far will she go in 15 days ?

32. If 1 horse consume 2 *tons* 1*qr*. 20*lb*. of hay in a winter, how much will 36 horses consume ?

33. How much cloth will clothe a company of 48 men, if it takes 5*yd*. 3*qr*. 2*na*. to clothe one man ?

NOTE.—Each denomination may be multiplied by the multiplier, separately, and the results reduced and multiplied.

DIVISION OF DENOMINATE NUMBERS.

123. DIVISION of denominate numbers is the operation of dividing a denominate number into as many equal parts as there are units in the divisor.

1. Divide £25 15*s*. 4*d*. by 8.

ANALYSIS.—We first say 8 into 25, 3 times and £1 or 20*s*. over. Then, after adding the 15*s*. we say, 8 into 35, 4 times and 3*s*. over. Then, reducing the 3*s*. to pence and adding in the 4*d*., we say, 8 into 40, 5 times.

OPERATION.

8)£25 15*s*. 4*d*.

£3 4*s*. 5*d*.

123. What is division of denominate numbers ? Give the rule for division. How do you prove division ? How do you divide when the divisor is a composite number ? What will be the unit of each quotient figure ?

2. Divide 36*bu.* 3*pk.* 7*qt.* by 7.

OPERATION.

```
7)36bu. 3pk. 7qt.(5bu.
  35
  --
   1
   4
  --
7)7pk.(1pk.
   7
  ----
   0
   8
  ----
7)7(1qt.
  Ans. 5bu. 1pk. 1qt.
```

ANALYSIS.—In this example we find that 7 is contained in 36 bushels 5 times and 1 bushel over. Reducing this to pecks, and adding 3 pecks, gives 7 pecks, which contains 7, 1 time and no remainder. Multiplying 0 by 8 quarts and adding, gives 7 quarts to be divided by 7.

Hence, for the division of denominate numbers we have the following

RULE.—I. *Begin with the highest denomination and divide as in simple numbers:*

II. *Reduce the remainder, if any, to the next lower denomination, and add in the units of that denomination for a new dividend.*

III. *Proceed in the same manner through all the denominations to the last.*

PROOF.—Same as in simple numbers.

NOTES.—1. If the divisor is a composite number, we may divide by the factors in succession, as in simple numbers.

2. Each quotient figure has the same unit as the dividend from from which it was derived.

3. If the divisor is greater than 12 and not a composite number the operation is the same as long division.

EXAMPLES.

(1.)

		T.	*cwt.*	*qr.*	*lb.*
	7)	1	19	2	12
Quotient.			5	2	16

(2.)

	A.	*R.*	*P.*
9)	113	3	25
	12	2	25

(3.)

		L.	*mi.*	*fur.*	*rd.*
	8)	47	1	7	8
Quotient.					

(4.)

	bu.	*pk.*	*qt.*
11)	25	3	1

Divide the following:

5. 17*cwt.* 0*qr.* 2*lb.* 6*oz.* by 7.
6. 49*yd.* 3*qr.* 3*na.* by 9.
7. 131*A.* 1*R.* by 12.
8. £1138 12*s.* 4*d.* by 53.
9. 70*T.* 17*cwt.* 7*lb.* by 79.
10. 27*bu.* 3*pk.* 7*qt.* by 84.

11. Bought 65 yards of cloth for which I paid £72 14*s.* $4\frac{1}{2}$*d.*: what did it cost per yard?

12. If 15 loads of hay contain 35*T.* 5*cwt.*, what is the weight of each load?

13. If a man, lifting 8 times as much as a boy, can raise 201*lb.* 12*oz.*, how much can the boy lift?

14. If a vessel sail 25° 42′ 40″ in 10 days, how far will she sail in one day?

15. Divide 9*hhd.* 28*gal.* 2*qt.* by 12.

16. What is the quotient of 65*bu.* 1*pk.* 3*qt.* divided by 12?

17. In 4 equal packages of medicine there are 13℔ 7℥ 2ʒ 1℈ 4*gr.*; how much is there in each package?

18. In 25*hhd.* of molasses, the leakage has reduced the whole amount to 1534*gal.* 1*qt.* 1*pt.*: if the same quantity has leaked out of each hogshead, how much will each hogshead still contain?

19. In 9 fields there are 113*A.* 3*R.* 25*P.* of land: if the fields contain an equal amount, how much is there in each field?

20. If in 30 days a man travels 746*mi.* 5*fur.*, travelling the same distance each day, what is the length of each day's journey?

21. Suppose a man had 98*lb.* 2*oz.* 19*pwt.* 5*gr.* of silver; how much must he give to each of 7 men if he divides it equally among them?

22. When 175*gal.* 2*qt.* of beer are drank in 52 weeks, how much is consumed in one week?

23. A rich man divided 168*bu.* 1*pk.* 6*qt.* of corn among 35 poor men: how much did each receive?

24. In sixty-three barrels of sugar there are 7*T.* 16*cwt.* 3*qr.* 12*lb.*: how much is there in each barrel?

25. A farmer has a granary containing 232 bushels 3 pecks 7 quarts of wheat, and he wishes to put it in 105 bags how much must each bag contain?

26. If 90 hogsheads of sugar weigh 56*T.* 14*cwt.* 3*qr.* 15*lb.*, what is the weight of 1 hogshead?

27. One hundred and seventy-six men consumed in a week 13*cwt.* 2*qr.* 15*lb.* 6*oz.* of bread: how much did each man consume?

28. If the earth revolves on its axis 15° in 1 hour, how far does it revolve in 1 minute?

29. If 59 casks contain 44*hhd.* 53*gal.* 2*qt.* 1*pt.* of wine, what are the contents of one cask?

30. Suppose a man has 246*mi.* 6*fur.* 36*rd.* to travel in 12 days: how far must he travel each day?

31. If I pay £12 14*s.* 5*d.* 3*far.* for 35 bushels of wheat, what is the price per bushel?

32. A printer uses one sheet of paper for every 16 pages of an octavo book: how much paper will be necessary to print 500 copies of a book containing 336 pages, allowing 2 quires of waste paper in each ream?*

33. A man lends his neighbor £135 6*s.* 8*d.*, and takes in part payment 4 cows at £5 8*s.* apiece, also a horse worth £50: how much remained due?

34. Out of a pipe of wine, a merchant draws 12 bottles, each containing 1 pint 3 gills; he then fills six 5-gallon demijohns; then he draws off 3 dozen bottles, each containing 1 quart 2 gills: how much remained in the cask?

35. A farmer has 6*T.* 8*cwt.* 2*qr.* 14*lb.* of hay to be removed in 6 equal loads: how much must be carried at each load?

36. A person at his death left landed estate to the amount of £2000, and personal property to the amount of £2803 17*s.* 4*d.* He directed that his widow should receive one-eighth of the whole, and that the residue should be equally divided among his four children: what was the widow and each child's portion?

37. If a steamboat go 224 miles in a day, how long will it take to go to China, the distance being about 12000 miles?

38. How long would it take a balloon to go from the earth to the moon, allowing the distance to be about 240000 miles, the balloon ascending 34 miles per hour?

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* In packing and selling paper, the two outside quires of every ream are regarded as waste, and each of the remaining quires contains 24 perfect sheets: hence, in this example, the waste paper is considered as belonging only to the entire reams.

LONGITUDE AND TIME.

124. The circumference of the earth, like that of other circles, is divided into 360°, which are called *degrees of longitude.*

125. The sun apparently goes round the earth once in 24 hours. This time is called a *day.*

Hence, in 24 hours, the sun apparently passes over 360° of longitude; and in 1 hour over $360° \div 24 = 15°$.

126. Since the sun, in passing over 15° of longitude, requires 1 hour or 60′ of time, 1° will require $60' \div 15 = 4$ minutes of time; and 1′ of longitude will be equal to one sixtieth of 4′ which is 4″: hence,

15° of longitude require 1 hour.
1° of longitude requires 4 minutes.
1′ of longitude requires 4 seconds.

Hence, we see that,

1. *If the degrees of longitude be multiplied by* 4, *the product will be the corresponding time in minutes.*

2. *If the minutes in longitude be multiplied by* 4, *the product will be the corresponding time in seconds.*

127. When the sun is on the meridian of any place, it is 12 o'clock, or noon, at that place.

Now, as the sun apparently goes from east to west, at the instant of noon, it will be *past* noon for all places at the east, and *before* noon for all places at the west.

If then, we find the difference of time between two places and know the exact time at one of them, the corresponding time at the other will be found by *adding* their difference, if that other be *east*, or by subtracting it if *west.*

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124. How is the circumference of the earth supposed to be divided?

125. How does the sun appear to move? What is a day? How far does the sun appear to move in 1 hour?

126. How do you reduce degrees of longitude to time? How do you reduce minutes of longitude to time?

127. What is the hour when the sun is on the meridian? When the sun is on the meridian of any place, how will the time be for all places east? How for all places west? If you have the difference of time how do you find the time?

1. The longitude of New York is 74° 1′ west, and that of Philadelphia 75° 10′ west: what is the difference of longitude and what their difference of time?

2. At 12 M. at Philadelphia, what is the time at New York?

3. At 12 M. at New York, what is the time at Philadelphia?

4. The longitude of Cincinnati, Ohio, is 84° 24′ west: what is the difference of time between New York and Cincinnati?

5. What is the time at Cincinnati, when it is 12 o'clock at New York?

6. The longitude of New Orleans is 89° 2′ west: what time is it at New Orleans, when it is 12 M. at New York?

7. The meridian from which the longitudes are reckoned passes through the Greenwich Observatory, London: hence, the longitude of that place is 0: what is the difference of time between Greenwich and New York?

8. What is the time at Greenwich, when it is 12 M. at New York?

9. The longitude of St. Louis is 90° 15′ west: what is the time at St. Louis, when it is 3*h*. 25′ P. M. at New York?

10. The longitude of Boston is 71° 4′ west, and that of New Orleans 89° 2′ west: what is the time at New Orleans when it is 7 o'clock 12′ A. M. at Boston?

11. The longitude of Chicago, Illinois, is 87° 30′ west: what is the time at Chicago, when it is 12 M. at New York?

PROPERTIES OF NUMBERS.

COMPOSITE AND PRIME NUMBERS.

128. An Integer, or whole number, is a unit or a collection of *units.*

129. One number is said to be *divisible* by another, when the quotient arising from the division is a whole number. The division is then said to be *exact.*

NOTE —Since every number is divisible by itself and 1, the term *divisible* will be applied to such numbers *only*, as have other divisors.

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128. What is an integer?

130. Every divisible number is called a *composite* number (Art. 54), and any divisor is called a *factor*: thus, 6 is a composite number, and the factors are 2 and 3.

131. Every number which is not divisible is called a *prime* number: thus, 1, 2, 3, 5, 7, 11, &c. are prime numbers.

132. Every prime number is divisible by itself and 1; but since these divisors are common to all numbers, they are not called *factors.*

133. Every factor of a number is either prime or composite: and since any composite factor may be again divided, it follows that,

Any number is equal to the product of all its prime factors.

For example, $12=6\times2$; but 6 is a composite number, of which the factors are 2 and 3; hence,

$$12=2\times3\times2\text{; also, }20=10\times2=5\times2\times2.$$

Hence, to find the prime factors of any number,

Divide the number by any prime number that will exactly divide it: then divide the quotient by any prime number that will exactly divide it, and so on, till a quotient is found which is a prime number; the several divisors and the last quotient will be the prime factors of the given number.

NOTE.—It is most convenient, in practice, to use the least prime number, which is a divisor.

1. What are the prime factors of 42?

ANALYSIS.—Two being the least divisor that is a prime number, we divide by it, giving the quotient 21, which we again divide by 3, giving 7: hence, 2, 3 and 7 are the prime factors.

OPERATION.

$$\begin{array}{r} 2)\underline{42} \\ 3)\underline{21} \\ 7 \end{array}$$

$$2\times3\times7=42.$$

---

129. When is one number divisible by another? By what is every number divisible? Is 1 called a divisor?

130. What is a composite number? What is a factor?

131. What is a prime number?

132. By what divisors is every prime number divided?

133. To what product is every number equal? Give the rule for finding the prime factors of a number. What number is it most convenient to use as a divisor?

What are the prime factors of the following numbers?

1. Of the number 9?
2. Of the number 15?
3. Of the number 24?
4. Of the number 16?
5. Of the number 18?
6. Of the number 32?
7. Of the number 48?
8. Of the number 56?
9. Of the number 63?
10. Of the number 76?

NOTE.—The prime factors, when the number is small, may generally be seen by inspection. The teacher can easily multiply the examples.

134. When there are several numbers whose prime factors are to be found,

Find the prime factors of each and then select those factors which are common to all the numbers.

11. What are the prime factors common to 6, 9 and 24?
12. What are the prime factors common to 21, 63 and 84?
13. What are the prime factors common to 21, 63 and 105?
14. What are the common factors of 28, 42 and 70?
15. What are the prime factors of 84, 126 and 210?
16. What are the prime factors of 210, 315 and 525?

135. DIVISIBILITY OF NUMBERS.

1. 2 is the only even number which is prime.
2. 2 divides every even number and no odd number.
3. 3 divides any number when the sum of its figures is divisible by 3.
4. 4 divides any number when the number expressed by the two right hand figures is divisible by 4.
5. 5 divides every number which ends in 0 or 5.
6. 6 divides any even number which is divisible by 3.
7. 10 divides any number ending in 0.

GREATEST COMMON DIVISOR.

136. The *greatest common divisor* of two or more numbers, is the greatest number which will divide each of them, separately, without a remainder. Thus, 6 is the greatest common divisor of 12 and 18.

134. How do you find the prime factors of two or more numbers?

NOTE.—Since 1 divides every number, it is not reckoned among the common divisors.

137. If two numbers have no common divisor, they are called *prime with respect to each other.*

138. Since a *factor* of a number always divides it, it follows that the greatest common divisor of two or more numbers, is simply the *greatest factor* common to these numbers.

Hence, to find the greatest common divisor of two or more numbers,

I. *Resolve each number into its prime factors.*

II. *The product of the factors common to each result will be the greatest common divisor.*

EXAMPLES.

1. What is the greatest common divisor of 24 and 30?

ANALYSIS.—There are four prime factors in 24, and 3 in 30: the factors 2 and 3 are common: hence, 6 is the greatest common divisor.

OPERATION.

$24 = 2 \times 2 \times 2 \times 3$
$30 = 2 \times 3 \times 5$
$2 \times 3 = 6$ com. divisor

2. What is the greatest common divisor of 9 and 18?
3. What is the greatest common divisor of 6, 12 and 30?
4. What is the greatest common divisor of 15, 25 and 30?
5. What is the greatest common divisor of 12, 18 and 72?
6. What is the greatest common divisor of 25, 35 and 70?
7. What is the greatest common divisor of 28, 42 and 70?
8. What is the greatest common divisor of 84, 126 and 210?

139. When the numbers are large, another method of finding their greatest common divisor is used, which depends on the following principles:

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135. What even number is prime? What numbers will 2 divide? What numbers will 3 divide? What numbers will 4 divide? 5? 6? 10?

136. What is the greatest common divisor of two or more numbers?

137. When are two numbers said to be prime with respect to each other?

138. What is the greatest factor of two numbers? How do you find the greatest common divisor of two or more numbers?

1. *Any number which will divide two numbers separately, will divide their sum; else, we shauld have a whole number equal to a proper fraction.*

$$24 + 27 = 51$$

2. *Any number which will divide two numbers separately, will divide their difference; and any number which will divide their difference and one of the numbers, will divide the other; else, we should have a whole number equal to a proper fraction.*

$$51 - 27 = 24$$

1. What is the greatest common divisor of 27 and 51?

Divide 51 by 27; the quotient is 1 and the remainder 24; then divide the preceding divisor 27 by the remainder 24: the quotien is 1 and the remainder 3; then divide the preceding divisor 24 by the remainder 3; the quotient is 8 and the remainder 0.

OPERATION.

```
27)51(1
   27
   --
   24)27(1
      24
      --
       3)24(8
         24
```

Now, since 3 divides the *difference* 3, and also 24, it will divide 27, by principle 2d; and since 3 divides the *remainder* 24, and 27, it will also divide 51; hence, it is a common divisor of 27 and 51; and since it is the *greatest* common factor, it it their greatest common divisor. Since the above reasoning is as applicable to any other two numbers as to 27 and 51, we have the following rule:

Divide the greater number by the less, and then divide the preceding divisor by the remainder, and so on, till nothing remains: the last divisor will be the greatest common divisor.

EXAMPLES.

1. What is the greatest common divisor of 216 and 408?
2. Find the greatest common divisor of 408 and 740.
3. Find the greatest common divisor of 315 and 810.
4. Find the greatest common divisor of 4410 and 5670.
5. Find the greatest common divisor of 3471 and 1869.
6. Find the greatest common divisor of 1584 and 2772?

NOTE.—If it be required to find the greatest common divisor of more than two numbers, first find the greatest common divisor of

---

139. When the numbers are large, on what principles does the operation of finding the greatest common divisor depend? What is the rule for finding it?

two of them, then of that common divisor and one of the remaining numbers, and so on for all the numbers: the last common divisor will be the greatest common divisor of all the numbers.

7. What is the greatest common divisor of 492, 744, and 1044?

8. What is the greatest common divisor of 944, 1488, and 2088?

9. What is the greatest common divisor of 216, 408, and 740?

10 What is the greatest common divisor of 945, 1560, and 22683?

LEAST COMMON DIVIDEND.

140. The least *common dividend* of two or more numbers is the least number which they will separately divide without a remainder.*

Notes.—1. If a dividend is exactly divisible by a divisor, it can be resolved into two factors, one of which is the divisor and the other the quotient.

2. If the divisor be resolved into its prime factors, the corresponding factor of the dividend may be resolved into the same factors; hence, *the dividend will contain every prime factor of the divisor.*

3. The question of finding the least common dividend of several numbers, is therefore reduced to finding a number which shall contain all their prime factors and *none others.*

1. Let it be required to find the least common dividend of 6, 8 and 12.

OPERATION.

$$\begin{array}{ccc} 2\times3 & 2\times2\times2 & 2\times2\times3 \\ 6 \;\ldots\ldots & 8 \;\ldots\ldots & 12 \end{array}$$

Analysis.—We see, from inspection, that the prime factors of 6, are 2 and 3;—of 8; 2, 2 and 2;—and of 12; 2, 2 and 3.

Every number that is a prime factor must appear in the least common dividend, and none others; hence, it will contain all the prime

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140 What is the least common dividend of two or more numbers? State the principles involved in finding it. Give the rule for finding it. What is the dividend when the numbers have no common prime factors?

* The number which we call the least *common dividend* is generally called the least *common multiple.* We prefer the former for beginners

factors of any one of the numbers, as 8, and such other prime factors of the others, 6 and 12, as are not found among the prime factors of 8; that is, the factor 3: hence,

$$2 \times 2 \times 2 \times 3 = 24, \text{ the least common dividend.}$$

To find the least common dividend of several numbers,

I. *Place the numbers on the same line, and divide by any prime number that will exactly divide two or more of them, and set down in a line below the quotients and the undivided numbers.*

II. *Then divide as before until there is no prime number greater than* 1 *that will exactly divide any two of the numbers.*

III. *Then multiply together the divisors and the numbers of the lower line, and their product will be the least common dividend.*

NOTE—1. The object of dividing by any prime number that will divide two or more of the numbers, is to find common factors.

2. If the numbers have no common prime factor, their product will be their least common dividend.

EXAMPLES.

OPERATION.

1. Find the least common dividend of 3, 4 and 8.

Ans. $2 \times 2 \times 3 \times 1 \times 2 = 24$.

$$\begin{array}{r|lll} 2 & 3 & 4 & 8 \\ \hline 2 & 3 & 2 & 4 \\ \hline & 3 & 1 & 2 \end{array}$$

2. Find the least common dividend of 3, 8 and 9.

Ans. $3 \times 1 \times 8 \times 3 = 72$.

$$\begin{array}{r|lll} 3 & 3 & 8 & 9 \\ \hline & 1 & 8 & 3 \end{array}$$

3. Find the least common dividend of 6, 7, 8 and 10.
4. Find the least common dividend of 21 and 49.
5. Find the least common dividend of 2, 7, 5, 6, and 8.
6. Find the least common dividend of 4, 14, 28 and 98.
7. Find the least common dividend of 13 and 6.
8. Find the least common dividend of 12, 4 and 7.
9. Find the least common dividend of 6, 9, 4, 14 and 16.
10. Find the least common dividend of 13, 12 and 4.
11. Find the least common dividend of 11, 17, 19, 21, and 9

CANCELLATION.

141. CANCELLATION is a method of shortening Arithmetical operations by omitting or cancelling common factors.

1. Divide 24 by 12. First, $24=3\times8$; and $12=3\times4$.

ANALYSIS.—Twenty-four divided by 12 is equal to 3×8 divided by 3×4; by *cancelling* or striking out the 3's, we have 8 divided by 4, which is equal to 2.

OPERATION.

$$\frac{24}{12}=\frac{\cancel{3}\times8}{\cancel{3}\times4}=2.$$

142. The operations in cancellation depend on two principles:

1. *The cancelling of a factor, in any number, is equivalent to dividing the number by that factor.*

2. *If the dividend and divisor be both divided by the same number, the quotient will not be changed.*

PRINCIPLES AND EXAMPLES.

1. Divide 63 by 21.

ANALYSIS.—Resolve the dividend and divisor into factors, and then cancel those which are common.

OPERATION.

$$\frac{63}{21}=\frac{\cancel{7}\times9}{\cancel{7}\times3}=3.$$

2. In 7 times 56, how many times 8?

ANALYSIS.—Resolve 56 into the two factors 7 and 8, and then cancel the 8.

OPERATION.

$$\frac{56\times7}{8}=\frac{\cancel{8}\times7\times7}{\cancel{8}}=49.$$

3. In 9 times 84, how many times 12?
4. In 14 times 63, how many times 7
5. In 24 times 9, how many times 8?
6. In 36 times 15, how many times 45?

ANALYSIS.—We see that 9 is a factor of 36 and 45. Divide by this factor, and write the quotient 4 over 36, and the quotient 5 below 45. Again, 5 is a factor of 15 and 5. Divide 15 by 5, and write the quotient 3 over 15. Dividing 5 by 5, reduces the divisor to 1, which need not be set down: hence, the true quotient $4\times3=12$.

OPERATION.

$$\frac{\overset{4}{\cancel{36}}\times\overset{3}{\cancel{15}}}{\underset{\cancel{5}}{\cancel{45}}}=12.$$

---

141. What is cancellation?

142. On what do the operations of cancellation depend?

143. Therefore, to perform the operations of cancellation :

I. *Resolve the dividend and divisor into such factors as shall give all the factors common to both.*

II. *Cancel the common factors and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTES.—1. Since every factor is cancelled by *division*, the quotient 1 always takes the place of the cancelled factor, but is omitted when it is a multiplier of other factors.

2. If one of the numbers contains a factor equal to the product of two or more factors of the other, all the factors may be cancelled.

3. If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, such factors may be cancelled.

4. It is generally more convenient to set the dividend on the right of a *vertical* line and the divisor on the left.

EXAMPLES.

1. What number is equal to 36 multiplied by 13 and the product divided by 4 times 9?

ANALYSIS.—We may place the numbers whose product forms the dividend on the right of a vertical line, and those which form the divisor on the left. We see that $4 \times 9 = 36$; we then cancel 4, 9, and 36.

OPERATION.

$$\begin{array}{r|l} \cancel{4} & \cancel{36} \\ \cancel{9} & 13 \\ \hline \end{array}$$

Ans. 13.

2. What is the result of $20 \times 4 \times 12$, divided by $10 \times 16 \times 3$?

ANALYSIS.—First, cancel the factor 10, in 10 and 20, and write the quotients 1 and 2 above the numbers. We then see that $16 \times 3 = 48$, and that $4 \times 12 = 48$; cancel 16 and 3 in the divisor, and 4 and 12 in the dividend; hence, the quotient is 2.

OPERATION.

$$\begin{array}{r|l} \overset{1}{\cancel{10}} & \overset{2}{\cancel{20}} \\ \cancel{16} & \cancel{4} \\ \cancel{3} & \cancel{12} \\ \hline \end{array}$$

Ans 2.

3. Divide the product of $126 \times 16 \times 3$, by 7×12.

ANALYSIS.—We see that 7 is a factor of 126—giving a quotient of 18. We cancel 7, and place 18 at the right of 126. We then cancel 6, in 12 and 18, and write the quotients 2 and 3 at the right. We then cancel the factor 2, in 2 and 16, and set down the quotients 1 and 8. The product of 1×1 is the divisor, and the product of $3 \times 8 \times 3 = 72$, the dividend.

OPERATION

$$\begin{array}{r|l} \overset{1}{\cancel{7}} & \cancel{126}\ \overset{3}{\cancel{18}} \\ \cancel{1}\ \cancel{2}\ \cancel{12} & \cancel{16}\ 8 \\ & 3 \\ \hline \end{array}$$

Ans. $3 \times 8 \times 3 = 72$.

4. What is the quotient of $3 \times 8 \times 9 \times 7 \times 15$, divided by $63 \times 24 \times 3 \times 5$?

OPERATION.

$\cancel{63}$	$\cancel{3}$
$\cancel{24}$	$\cancel{8}$
$\cancel{3}$	$\cancel{9}$
$\cancel{5}$	$\cancel{7}$
	$\cancel{15}$

ANALYSIS.—The 63 is cancelled by 7×9; 24 by 3×8; 3 and 5, by 15; hence, the quotient is 1.

5. Divide the product of $6 \times 7 \times 9 \times 11$, by $2 \times 3 \times 7 \times 3 \times 21$.

6. Divide the product of $4 \times 14 \times 16 \times 24$, by $7 \times 8 \times 32 \times 12$.

7. Divide the product of $5 \times 11 \times 9 \times 7 \times 15 \times 6$, by $30 \times 3 \times 21 \times 3 \times 5$.

8. Divide the product of $6 \times 9 \times 8 \times 11 \times 12 \times 5$, by $27 \times 2 \times 32 \times 3$.

9. Divide the product of $1 \times 6 \times 9 \times 14 \times 15 \times 7 \times 8$, by $36 \times 126 \times 56 \times 20$.

10. Divide the product of $18 \times 36 \times 72 \times 144$, by $6 \times 6 \times 8 \times 9 \times 12 \times 8$.

11. Divide the product of $4 \times 6 \times 3 \times 5$, by $5 \times 9 \times 12 \times 16$

12. Multiply 288 by 16, and divide the product by $8 \times 9 \times 2 \times 2$.

13. In a certain operation the numbers 24, 28, 32, 49, 81, are to be multiplied together and the product divided by $8 \times 4 \times 7 \times 9 \times 6$: what is the result?

14. Multiply 240 by 18 and divide the product by 6 times 90.

15. Divide $16 \times 20 \times 8 \times 3$, by $30 \times 8 \times 6$.

16. How many pounds of butter worth 15 cents a pound, may be bought for 25 pounds of tea at 48 cents a pound?

17. How much calico at 25 cents a yard must be given for 100 yards of Irish sheeting at 87 cents a yard?

18. How many yards of cloth at 46 cents a yard must be given for 23 bushels of rye at 92 cents a bushel?

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143. Give the rule for the operation of cancellation.

19. How many bushels of oats at 42 cents a bushel must be given for 3 boxes of raisins each containing 26 pounds, at 14 cents a pound?

20. A man buys 2 pieces of cotton cloth, each containing 33 yards at 11 cents a yard, and pays for it in butter at 18 cents a pound: how many pounds of butter did he give?

21. If sugar can be bought for 7 cents a pound, how many bushels of oats at 42 cents a bushel must I give for 56 pounds?

22. If wool is worth 36 cents a pound, how many pounds must be given for 27 yards of broadcloth worth 4 dollars a yard?

23. If cotton cloth is worth 9 cents a yard, how much must be given for 3 tons of hay worth 15 dollars a ton?

24. How much molasses at 42 cents a gallon must be given for 216 pounds of sugar at 7 cents a pound?

25. Bought 48 yards of cloth at 125 cents a yard: how many bushels of potatoes are required to pay for it at 150 cents a bushel?

26. Mr. Butcher sold 342 pounds of beef at 6 cents a pound, and received his pay in molasses at 36 cents a gallon: how many gallons did he receive?

27. Mr. Farmer sold 1263 pounds of wool at 5 cents a pound, and took his pay in cloth at 421 cents a yard: how many yards did he take?

28. How many firkins of butter, each containing 56 pounds, at 18 cents a pound, must be given for 3 barrels of sugar, each containing 200 pounds, at 9 cents a pound?

29. How many boxes of tea, each containing 24 pounds, worth 5 shillings a pound, must be given for 4 bins of wheat, each containing 145 bushels, at 12 shillings a bushel?

30. A worked for B 8 days, at 6 shillings a day, for which he received 12 bushels of corn: how much was the corn worth a bushel?

31. Bought 15 barrels of apples, each containing 2 bushels, at the rate of 3 shillings a bushel: how many cheeses, each weighing 30 pounds, at 1 shilling a pound, will pay for the apples?

COMMON FRACTIONS.

144. The unit 1 denotes an entire thing, as 1 apple, 1 chair, 1 pound of tea.

If the unit 1 be divided into two equal parts, each part is called *one-half.*

If the unit 1 be divided into three equal parts, each part is called *one-third.*

If the unit 1 be divided into four equal parts, each part is called *one-fourth.*

If the unit 1 be divided into twelve equal parts, each part is called *one-twelfth ;* and if it be divided into *any number* of equal parts, we have a like expression for each part.

The parts are thus written :

$\frac{1}{2}$	is read, one-half.	$\frac{1}{7}$	is read, one-seventh.
$\frac{1}{3}$	- - one-third.	$\frac{1}{8}$	- - one-eighth.
$\frac{1}{4}$	- - one-fourth.	$\frac{1}{10}$	- - one-tenth.
$\frac{1}{5}$	- - one-fifth.	$\frac{1}{15}$	- - one-fifteenth.
$\frac{1}{6}$	- - one-sixth.	$\frac{1}{50}$	- - one-fiftieth.

The $\frac{1}{2}$, is an *entire half ;* the $\frac{1}{3}$, an *entire third ;* the $\frac{1}{4}$, an *entire fourth ;* and the same for each of the other equal parts hence, *each equal part is an entire thing*, and is called a *fractional unit.*

The unit, or whole thing which is divided, is called the *unit of the fraction.*

NOTE.—In every fraction let the pupil distinguish carefully between the *unit of the fraction* and the *fractional unit.* The first is the *whole thing* from which the fraction is derived ; the second, *one of the equal parts* into which that thing is divided.

145. Each fractional unit may, like the unit 1, become the base of a collection : thus, suppose it were required to express 2 of each of the fractional units : we should then write

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144. What is a unit? What is each part called when the unit 1 is divided into two equal parts? When it is divided into 3? Into 4? Into 5? Into 12?

How may the one-half be regarded? The one-third? The one-fourth? What is each part called?

What is the unit of a fraction? What is a fractional unit? How do you distinguish between the one and the other?

$\frac{2}{2}$	which is read			2 halves $=\frac{1}{2}\times 2$
$\frac{2}{3}$	"	"	"	2 thirds $=\frac{1}{3}\times 2$
$\frac{2}{4}$	"	"	"	2 fourths $=\frac{1}{4}\times 2$
$\frac{2}{5}$	"	"	"	2 fifths $=\frac{1}{5}\times 2$
	&c.,	&c.,	&c.,	&c.

If it were required to express 3 of each of the fractional units, we should write

$\frac{3}{2}$	which is read			3 halves $=\frac{1}{2}\times 3$
$\frac{3}{3}$	"	"	"	3 thirds $=\frac{1}{3}\times 3$
$\frac{3}{4}$	"	"	"	3 fourths $=\frac{1}{4}\times 3$
$\frac{3}{5}$	"	"	"	3 fifths $=\frac{1}{5}\times 3$
&c.,	&c.,	&c.,	&c.; hence,	

A FRACTION is a collection of one or more of the equal parts of a unit.

Fractions are expressed by two numbers, the one written above the other, with a line between them. The lower number is called the *denominator*, and the upper number the *numerator*.

The denominator denotes the number of equal parts into which the unit is divided; and hence, determines the *value of the fractional unit*. Thus, if the denominator is 2, the fractional unit is *one-half;* if it is 3, the fractional unit is *one-third;* if it is 4, the fractional unit is *one-fourth*, &c., &c.

The numerator denotes the *number* of fractional units taken. Thus, $\frac{3}{5}$ denotes that the fractional unit is $\frac{1}{5}$, and that 3 such units are taken; and similarly for other fractions.

In the fraction $\frac{3}{5}$, the base of the collection of fractional units is $\frac{1}{5}$, but this is not the *primary* base. For, $\frac{1}{5}$ is *one-fifth of the unit* 1; hence, *the primary base of every fraction is the unit* 1.

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145. May a fractional unit become the base of a collection? What is a fraction? How are fractions expressed? What is the lower number called? What is the upper number called? What does the denominator denote? What does the numerator denote? In the fraction 3 fifths, what is the fractional base? What is the primary base? What is the primary base of every fraction?

146. If we suppose a second unit of the same kind to be divided into equal parts, such parts may be expressed in the same collection with the parts of the first: thus,

$\frac{3}{2}$	is read	3 halves.
$\frac{7}{4}$	" "	7 fourths.
$\frac{16}{5}$	" "	16 fifths.
$\frac{18}{6}$	" "	18 sixths.
$\frac{25}{7}$	" "	25 sevenths.

147. A whole number may be expressed fractionally by writing 1 below it for a denominator. Thus,

3 may be written	$\frac{3}{1}$	and is read, 3 ones.
5 - - -	$\frac{5}{1}$	- - - 5 ones.
6 - - -	$\frac{6}{1}$	- - - 6 ones.
8 - - -	$\frac{8}{1}$	- - - 8 ones.

But 3 ones are equal to 3, 5 ones to 5, 6 ones to 6, and 8 ones to 8; hence, *the value of a number is not changed* by placing 1 under it for a denominator.

148. If the numerator of a fraction be divided by its denominator, the integral part of the quotient will express the number of entire units used in forming the fraction; and the remainder will show how many fractional units are over. Thus, $\frac{11}{3}$ are equal to 3 and 2 thirds, and is written $\frac{11}{3}=3\frac{2}{3}$: hence,

A fraction has the same form as an unexecuted division.

From what has been said, we conclude that,

1st. *A fraction is one or more of the equal parts of a unit*

2d. *The denominator shows into how many equal parts the unit is divided, and hence indicates the value of the fractional unit:*

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146. If a second unit be divided into equal parts, may the parts be expressed with those of the first? How many units have been divided to obtain 6 thirds? To obtain 9 halves? 12 fourths?

147. How may a whole number be expressed fractionally? Does this change the value of the number?

148. If the numerator be divided by the denominator, what does the quotient show? What does the remainder show? What form has a fraction? What are the seven principles which follow?

3d. *The numerator shows how many fractional units are taken :*

4th. *The value of every fraction is equal to the quotient arising from dividing the numerator by the denominator :*

5th. *When the numerator is less than the denominator, the value of the fraction is less than* 1.

6th. *When the numerator is equal to the denominator, the value of the fraction is equal to* 1.

7th. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

EXAMPLES IN WRITING AND READING FRACTIONS.

1 Read the following fractions ;

$$\tfrac{5}{12}, \tfrac{5}{9}, \tfrac{16}{7}, \tfrac{7}{10}, \tfrac{3}{8}, \tfrac{9}{50}, \tfrac{65}{117}.$$

What is the unit of the fraction, and what the fractional unit in each example? How many fractional units are taken in each?

2. Write 12 of the 17 equal parts of 1.

3. If the unit of the fraction is 1, and the fractional unit one-twentieth, express 6 fractional units. Express 12, 18, 16, 30, fractional units.

4. If the fractional unit is one 36th, express 32 fractional units ; 35, 38, 54, 6, 8.

5. If the fractional unit is one-fortieth, express 9 fractional units; 16, 25, 69, 75.

DEFINITIONS.

149. A Proper Fraction is one whose numerator is less than the denominator.

The following are proper fractions :

$$\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{3}{7}, \tfrac{5}{8}, \tfrac{9}{10}, \tfrac{8}{9}, \tfrac{5}{6}.$$

150. An Improper Fraction is one whose numerator is equal to, or exceeds the denominator.

Note.—Such a fraction is called *improper* because its value equals or exceeds 1.

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149. What is a proper fraction? Give examples.
150. What is an improper fraction? Why improper? Give examples.

The following are improper fractions :

$$\tfrac{3}{2},\ \tfrac{5}{3},\ \tfrac{6}{5},\ \tfrac{8}{7},\ \tfrac{9}{8},\ \tfrac{12}{6},\ \tfrac{14}{7},\ \tfrac{19}{7}.$$

151. A Simple Fraction is one whose numerator and denominator are both whole numbers.

Note.—A simple fraction may be either proper or improper.

The following are simple fractions :

$$\tfrac{1}{4},\ \tfrac{3}{2},\ \tfrac{5}{6},\ \tfrac{8}{7},\ \tfrac{9}{2},\ \tfrac{8}{3},\ \tfrac{6}{3},\ \tfrac{7}{5}.$$

152. A Compound Fraction is a fraction of a fraction, or several fractions connected by the word *of*.

The following are compound fractions :

$$\tfrac{1}{2} \text{ of } \tfrac{1}{4},\quad \tfrac{1}{3} \text{ of } \tfrac{1}{2} \text{ of } \tfrac{1}{3},\quad \tfrac{1}{6} \text{ of } 3,\quad \tfrac{1}{7} \text{ of } \tfrac{1}{8} \text{ of } 4.$$

153. A Mixed Number is made up of a whole number and a fraction.

The following are mixed numbers :

$$3\tfrac{1}{2},\qquad 4\tfrac{1}{3},\qquad 6\tfrac{2}{8},\qquad 5\tfrac{3}{5},\qquad 6\tfrac{5}{8},\qquad 3\tfrac{1}{7}.$$

154. A Complex Fraction is one whose numerator or denominator is fractional ; or, in which both are fractional.

The following are complex fractions :

$$\frac{\frac{1}{7}}{5},\qquad \frac{2}{19\frac{1}{2}},\qquad \frac{\frac{2}{3}}{\frac{4}{5}},\qquad \frac{45\frac{1}{8}}{69\frac{1}{7}}.$$

155. The numerator and denominator of a fraction, taken together, are called the *terms* of the fraction : hence, every fraction has two terms.

FUNDAMENTAL PROPOSITIONS.

156. By multiplying the unit 1, we form all the whole numbers,

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151. What is a simple fraction? Give examples. May it be proper or improper?

152. What is a compound fraction? Give examples.

153. What is a mixed number? Give examples.

154. What is a complex fraction? Give examples.

155. How many terms has every fraction? What are they?

156. How may all the whole numbers be formed? How may the fractional units be formed? How many times is one-half less than 1? How many times is any fractional unit less than 1?

2, 3, 4, 5, 6, 7, 8, 9, 10, &c.;

and by dividing the unit 1 by these numbers we form all the fractional units,

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, &c.

Now, since in 1 unit there are 2 halves, 3 thirds, 4 fourths, 5 fifths, 6 sixths, &c., it follows that the *fractional* unit becomes *less* as the denominators are *increased:* hence,

The fractional unit is such a part of 1, *as* 1 *is of the denominator of the fraction.*

Thus, $\frac{1}{2}$ is such a part of 1, as 1 is of 2; $\frac{1}{3}$ is such a part of 1, as 1 is of 3; $\frac{1}{4}$ is such a part of 1 as 1 is of 4, &c. &c.

157. Let it be required to multiply $\frac{5}{6}$ by 3.

OPERATION.

$$\frac{5}{6}\times 3=\frac{5\times 3}{6}=\frac{15}{6}$$

ANALYSIS.—In $\frac{5}{6}$ there are 5 fractional units, each of which is $\frac{1}{6}$, and these are to be taken 3 times. But 5 things taken 3 times, gives 15 things of the *same kind;* that is, 15 sixths; hence, the product is 3 times as great as the multiplicand: therefore, we have

PROPOSITION I.—*If the numerator of a fraction be multiplied by any number, the fraction will be increased as many times as there are units in the multiplier.*

EXAMPLES.

1. Multiply $\frac{3}{8}$ by 8.
2. Multiply $\frac{7}{5}$ by 5.
3. Multiply $\frac{1}{7}$ by 9.
4. Multiply $\frac{8}{19}$ by 14.
5. Multiply $\frac{7}{6}$ by 20.
6. Multiply $\frac{167}{81}$ by 25.

158. Let it be required to multiply $\frac{4}{6}$ by 3.

OPERATION.

$$\frac{4}{6}\times 3=\frac{4}{6\div 3}=\frac{4}{2}.$$

ANALYSIS.—In $\frac{4}{6}$ there are 4 fractional units, each of which is $\frac{1}{6}$. If we divide the denominator by 3, we change the fractional unit to $\frac{1}{2}$, which is 3 times as great as $\frac{1}{6}$, since the first is contained in 1, 2 times, and the second 6 times. If we take this fractional unit 4 times, the result $\frac{4}{2}$, is 3 times as great as $\frac{4}{6}$: therefore, we have

PROPOSITION II.—*If the denominator of a fraction be divided by any multiplier, the value of the fraction will be increased as many times as there are units in that multiplier.*

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157. What is proved in Proposition I?

EXAMPLES.

1. Multiply $\frac{3}{4}$ by 2, by 4.
2. Multiply $\frac{16}{32}$ by 2, 4, 8.
3. Multiply $\frac{8}{48}$ by 2, 4, 6.
4. Multiply $\frac{19}{84}$ by 2, 4, 6.
5. Multiply $\frac{37}{42}$ by 2, 6, 7.
6. Multiply $\frac{151}{200}$ by 5, 10.

159. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9 \div 3}{11} = \frac{3}{11}.$$

ANALYSIS.—In $\frac{9}{11}$, there are 9 fractional units, each of which is $\frac{1}{11}$, and these are to be divided by 3. But 9 things, divided by 3, gives 3 *things of the same kind* for a quotient; hence, the quotient is 3 elevenths, a number one-third as great as $\frac{9}{11}$; hence, we have

PROPOSITION III.—*If the numerator of a fraction be divided by any number, the value of the fraction will be diminished as many times as there are units in the divisor.*

EXAMPLES.

1. Divide $\frac{28}{15}$ by 2, by 7.
2. Divide $\frac{112}{160}$ by 56.
3. Divide $\frac{400}{117}$ by 25, by 8.
4. Divide $\frac{640}{530}$ by 8, 16, 10.

160. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9}{11 \times 3} = \frac{9}{33}.$$

ANALYSIS.—In $\frac{9}{11}$, there are 9 fractional units, each of which is $\frac{1}{11}$. Now, if we multiply the denominator by 3 it becomes 33, and the fractional unit becomes $\frac{1}{33}$, which is only $\frac{1}{3}$ of $\frac{1}{11}$, because 33 is 3 times as *great* as 11. If we take this fractional unit 9 times, the result, $\frac{9}{33}$, is exactly $\frac{1}{3}$ of $\frac{9}{11}$: hence, we have

PROPOSITION IV.—*If the denominator of a fraction be multiplied by any divisor, the value of the fraction will be diminished as many times as there are units in that divisor.*

EXAMPLES.

1. Divide $\frac{1}{2}$ by 2.
2. Divide $\frac{1}{5}$ by 7.
3. Divide $\frac{3}{16}$ by 4.
4. Divide $\frac{17}{15}$ by 8.
5. Divide $\frac{41}{65}$ by 17.
6. Divide $\frac{16}{175}$ by 45.

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158. What is proved in proposition II.?
159. What is proved in proposition III.?
160. What is proved in proposition IV.?

161. Let it be required to multiply both terms of the fraction $\frac{3}{5}$ by 4.

OPERATION.

$$\frac{3\times4}{5\times4}=\frac{12}{20}$$

ANALYSIS.—In $\frac{3}{5}$, the fractional unit is $\frac{1}{5}$, and it is taken 3 times. By multiplying the denominator by 4, the fractional unit becomes $\frac{1}{20}$, the value of which is $\frac{1}{4}$ times as great as $\frac{1}{5}$ By multiplying the numerator by 4, we increase the *number* of fractional units taken, 4 times; that is, we *increase the number just as many times* as we *decrease the value;* hence, the value of the fraction is not changed; therefore, we have

PROPOSITION V.—*If both terms of a fraction be multiplied by the same number, the value of the fraction will not be changed.*

EXAMPLES.

1. Multiply the numerator and denominator of $\frac{5}{7}$ by 7: this gives $\frac{5\times7}{7\times7}=\frac{35}{49}$.

2. Multiply the numerator and denominator of $\frac{7}{12}$ by 3, by 4, by 5, by 6, by 9.

3. Multiply each term of $\frac{75}{84}$ by 2, by 3, by 4, by 5, by 6.

162. Let it be required to divide the numerator and denominator of $\frac{6}{15}$ by 3.

OPERATION.

$$\frac{6\div3}{15\div3}=\frac{2}{5}.$$

ANALYSIS.—In $\frac{6}{15}$, the fractional unit is $\frac{1}{15}$, and is taken 6 times. By dividing the denominator by 3, the fractional unit becomes $\frac{1}{5}$, the value of which is 3 *times as great as* $\frac{1}{15}$. By dividing the numerator by 3, we diminish the number of fractional units taken 3 times: that is, we *diminish the number just as many times as we increase the value:* hence, the value of the fraction is not changed: therefore, we have

PROPOSITION VI.—*If both terms of a fraction be divided by the same number, the value of the fraction will not be changed.*

EXAMPLES.

1. Divide both terms of the fraction $\frac{8}{16}$ by 2: this gives

$$\tfrac{8}{16}\div\tfrac{2}{2}=\tfrac{4}{8}\ \textit{Ans.}$$

161. What is proved in proposition V.?
162. What is proved in proposition VI?

2. Divide both terms by 8: this gives $\frac{8}{16} \div \frac{8}{8} = \frac{1}{2}$.

3. Divide both terms of the fraction $\frac{32}{128}$ by 2, by 4, by 8, by 16.

4. Divide both terms of the fraction $\frac{60}{180}$ by 2, by 3, by 4, by 5, by 6, by 10, by 12.

REDUCTION OF FRACTIONS.

163. Reduction of Fractions is the operation of changing the fractional unit without altering the value of the fraction.

A fraction is in its *lowest terms*, when the numerator and denominator have no common factor.

CASE I.

164. *To reduce a fraction to its lowest terms.*

1. Reduce $\frac{70}{175}$ to its lowest terms.

Analysis.—By inspection, it is seen that 5 is a common factor of the numerator and denominator. Dividing by it, we have $\frac{14}{35}$. We then see that 7 is a common factor of 14 and 35: dividing by it, we have $\frac{2}{5}$. Now, there is no factor common to 2 and 5: hence, $\frac{2}{5}$ is in its *lowest terms*.

1st. operation.

$5)\frac{70}{175} = \frac{14}{35}$.

$7)\frac{14}{35} = \frac{2}{5}$.

The greatest common divisor of 70 and 175 is 35, (Art. 136); if we divide both terms of the fraction by it, we obtain $\frac{2}{5}$. The value of the fraction is not changed in either operation, since the numerator and denominator are both divided by the same number (Art. 162): hence, the following

2d operation.

$35)\frac{70}{175} = \frac{2}{5}$.

Rule.—*Divide the numerator and denominator by any number that will divide them both without a remainder, and divide the quotient, in the same manner until they have no common factor.*

Or: *Divide the numerator and denominator by their greatest common divisor.*

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163 What is reduction of fractions? When is a fraction in its lowest terms?

164. How do you reduce a fraction to its lowest terms?

EXAMPLES.

Reduce the following fractions to their lowest terms.

1. Reduce $\frac{12}{15}$.
2. Reduce $\frac{18}{24}$.
3. Reduce $\frac{27}{36}$.
4. Reduce $\frac{36}{144}$.
5. Reduce $\frac{84}{96}$.
6. Reduce $\frac{144}{864}$.
7. Reduce $\frac{288}{2592}$.
8. Reduce $\frac{85}{165}$.
9. Reduce $\frac{175}{375}$.
10. Reduce $\frac{104}{312}$.
11. Reduce $\frac{1049}{8392}$.
12. Reduce $\frac{275}{440}$.
13. Reduce $\frac{351}{795}$.
14. Reduce $\frac{172}{1118}$.
15. Reduce $\frac{63}{81}$.
16. Reduce $\frac{315}{405}$.

CASE II.

165. *To reduce an improper fraction to its equivalent whole or mixed number.*

1. In $\frac{59}{8}$ how many entire units?

ANALYSIS.—Since there are 8 eighths in 1 unit, in $\frac{59}{8}$ there are as many units as 8 is contained times in 59, which is $7\frac{3}{8}$ times.

OPERATION.

$$\begin{array}{r} 8)59 \\ \hline 7\frac{3}{8}. \end{array}$$

Hence, the following

RULE.—*Divide the numerator by the denominator, and the result will be the whole or mixed number.*

EXAMPLES.

1. Reduce $\frac{84}{4}$ and $\frac{67}{9}$ to their equivalent whole or mixed numbers.

OPERATION.

$$\begin{array}{r} 4)84 \\ \hline 21 \end{array}$$

OPERATION.

$$\begin{array}{r} 9)67 \\ \hline 7\frac{4}{9}. \end{array}$$

2. Reduce $\frac{99}{8}$ to a whole or mixed number.
3. In $\frac{19}{7}$ yards of cloth, how many yards?
4. In $\frac{51}{9}$ of bushels, how many bushels?

---

165. How do you reduce an improper fraction to a whole or mixed number?

5. If I give $\frac{1}{3}$ of an apple to each one of 15 children, how many apples do I give?

6. Reduce $\frac{327}{125}$, $\frac{3672}{153}$, $\frac{50287}{6941}$, $\frac{987625}{72301}$, to their whole or mixed numbers.

7. If I distribute 878 quarter-apples among a number of boys, how many whole apples do I use?

8. Reduce $\frac{62587}{3114}$, $\frac{4927}{109}$, $\frac{2641674}{278436}$, to their whole or mixed numbers.

9. Reduce $\frac{147254149}{4674}$, $\frac{145260}{108}$, $\frac{62015735}{7803}$, to their whole or mixed numbers.

CASE III.

166. *To reduce a mixed number to its equivalent improper fraction.*

1. Reduce $4\frac{4}{5}$ to its equivalent improper fraction.

ANALYSIS. — Since in any number there are 5 times as many fifths as units, in 4 there will be 5 times 4 fifths, or 20 fifths, to which add 4 fifths, and we have 24 fifths.

OPERATION.

$4 \times 5 = 20$ fifths.
add 4 fifths.
gives $\frac{24}{5} = 24$ fifths.

Hence, the following

RULE.—*Multiply the whole number by the denominator of the fraction: to the product add the numerator, and place the sum over the given denominator.*

EXAMPLES.

1. Reduce $47\frac{5}{6}$ to its equivalent fraction.
2. In $17\frac{3}{8}$ yards, how many eighths of a yard?
3. In $42\frac{9}{20}$ rods, how many twentieths of a rod?
4. Reduce $625\frac{4}{13}$ to an improper fraction.
5. How many 112ths in $205\frac{46}{112}$?
6. In $84\frac{17}{24}$ days, how many twenty-fourths of a day?
7. In $15\frac{142}{365}$ years, how many 365ths of a year?
8. Reduce $916\frac{450}{600}$ to an improper fraction.
9. Reduce $25\frac{9}{76}$, $156\frac{22}{60}$, to their equivalent fractions.

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166. How do you reduce a mixed number to its equivalent improper fraction?

CASE VI.

167. *To reduce a whole number to a fraction having a given denominator.*

1. Reduce 6 to a fraction whose denominator shall be 4.

ANALYSIS.—Since in 1 unit there are 4 fourths, it follows that in 6 units there are 6 times 4 fourths, or 24 fourths: therefore, $6 = \frac{24}{4}$: hence,

OPERATION.
$6 \times 4 = 24$.
$\frac{24}{4}$.

RULE.—*Multiply the whole number and denominator together, and write the product over the required denominator.*

EXAMPLES.

1. Reduce 12 to a fraction whose denominator shall be 9.
2. Reduce 46 to a fraction whose denominator shall be 15.
3. Change 26 to 7ths.
4. Change 178 to 40ths.
5. Reduce 240 to 114ths.
6. Change $54 to quarters.
7. Change 96*yd.* to quarters.
8. Change 426*lb.* to 16ths.

CASE V.

168. *To reduce a compound fraction to a simple one.*

1. What is the value of $\frac{3}{4}$ of $\frac{5}{7}$?

ANALYSIS.—Three-fourths of $\frac{5}{7}$ is 3 times 1 fourth of $\frac{5}{7}$; 1 fourth of $\frac{5}{7}$ is $\frac{5}{28}$ (Art. 160); 3 fourths of $\frac{5}{7}$ is 3 times $\frac{5}{28}$, or $\frac{15}{28}$: therefore, $\frac{3}{4}$ of $\frac{5}{7} = \frac{15}{28}$: hence,

OPERATION.
$$\frac{3 \times 5}{4 \times 7} = \frac{15}{28}$$

RULE.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE.—If there are mixed numbers, reduce them to their equivalent improper fractions.

EXAMPLES.

Reduce the following fractions to simple ones.

1. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{7}$.
2. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$.
3. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{15}$.
4. Reduce $2\frac{1}{4}$ of $6\frac{1}{2}$ of 7.
5. Reduce 5 of $\frac{1}{2}$ of $\frac{1}{7}$ of 6.
6. Reduce $6\frac{1}{3}$ of $7\frac{1}{4}$ of $6\frac{3}{21}$

METHOD BY CANCELLING.

169. The work may often be abridged by *cancelling* common factors in the numerator and denominator (Art. 143).

In every operation in fractions, let this be done whenever it is possible.

EXAMPLES.

1. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Here, $\frac{5}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{6}}\times\frac{\cancel{6}}{7}=\frac{5}{7}$, or,

$$\begin{array}{r|l} \cancel{3} & 5 \\ \cancel{6} & \cancel{3} \\ 7 & \cancel{6} \\ \hline 7 & 5=\frac{5}{7}. \end{array}$$

Note.—The divisors are always written on the left of the vertical line, and the dividends on the right.

2. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{15}$ to its simplest terms.

Here, $\frac{\overset{2}{\cancel{6}}}{\cancel{8}}\times\frac{\cancel{8}}{\cancel{9}}\times\frac{\cancel{9}}{\underset{5}{\cancel{15}}}=\frac{2}{5}$, or,

$$\begin{array}{rr|ll} & \cancel{8} & \cancel{6} & 2 \\ & \cancel{9} & \cancel{8} & \\ 5 & \cancel{15} & \cancel{9} & \\ \hline & 5 & 2=\frac{2}{5}. & \end{array}$$

Note.—Besides cancelling the like factors 8 and 8, and 9 and 9, we also cancel the factor 3, common to 15 and 6, and write over them, and at the right, the quotients 5 and 2.

3. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{5}{9}$ of $\frac{27}{100}$ of $\frac{5}{13}$ to its simplest terms.
4. Reduce $\frac{42}{110}$ of $\frac{3}{19}$ of $\frac{40}{108}$ of $\frac{3}{7}$ to its simplest terms.
5. Reduce $3\frac{3}{8}$ of $\frac{5}{9}$ of $\frac{27}{315}$ of 49 to its simplest terms.

CASE VI.

170. *To reduce fractions of different denominators to fractions having a common denominator.*

1. Reduce $\frac{1}{2}$, $\frac{7}{3}$ and $\frac{4}{5}$ to a common denominator.

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167. How do you reduce a whole number to a fraction having a given denominator?

168. How do you reduce a compound fraction to a simple one.

169. How is the reduction of compound fractions to simple ones abridged by cancellation?

OPERATION

$1 \times 3 \times 5 = 15$ 1st num.
$7 \times 2 \times 5 = 70$ 2d num.
$4 \times 3 \times 2 = 24$ 3d num.
$2 \times 3 \times 5 = 30$ denom.

ANALYSIS.—If both terms of the first fraction be multiplied by 15, the product of the other denominators, it will become $\frac{15}{30}$. If both terms of the second fraction be multiplied by 10, the product of the other denominators, it will become $\frac{70}{30}$. If both terms of the third be multiplied by 6, the product of the other denominators, it will become $\frac{24}{30}$. In each case, we have multiplied both terms of the fraction by the same number; hence, the value has not been altered (Art. 161): hence, the following

RULE.—*Reduce to simple fractions when necessary; then multiply the numerator of each fraction by all the denominators except its own, for the new numerators, and all the denominators together for a common denominator.*

NOTE.—When the numbers are small the work may be performed mentally. Thus,

$$\frac{1}{2},\ \frac{1}{4},\ \frac{2}{5} = \frac{20}{40},\ \frac{10}{40},\ \frac{16}{40}.$$

EXAMPLES.

Reduce the following fractions to common denominators.

1. Reduce $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{1}{7}$.
2. Reduce $\frac{2}{9}$, $\frac{4}{11}$, and $\frac{3}{5}$.
3. Reduce $\frac{5}{7}$, $\frac{1}{8}$, and $\frac{5}{9}$.
4. Reduce $2\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{1}{7}$.
5. Reduce $5\frac{1}{2}$, $\frac{6}{7}$ of $\frac{1}{3}$, and 4.
6. Reduce $3\frac{1}{6}$ of $\frac{1}{2}$ and $\frac{3}{9}$.
7. Reduce $\frac{7}{8}$, $\frac{135}{75}$, and 37.
8. Reduce 4, $\frac{31}{25}$, and $\frac{62}{2}$.
9. Reduce $7\frac{1}{2}$, $\frac{31}{18}$, $6\frac{1}{4}$.
10. Reduce $4\frac{1}{9}$, $8\frac{1}{7}$, and $2\frac{1}{2}$.

NOTE.—We may often shorten the work by multiplying the numerator and denominator of each fraction by such a number as will make the denominators the same in all.

10. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

OPERATION.

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{3} = \frac{2}{6}$

ANALYSIS.—Multiply both terms of the first by 3, and both terms of the second by 2.

11. Reduce $\frac{1}{5}$ and $\frac{1}{3}$.
12. Reduce $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{3}{4}$.
13. Reduce $\frac{3}{7}$, $\frac{8}{28}$, $\frac{4}{14}$.
14. Reduce $\frac{5}{8}$, $3\frac{5}{6}$, and $\frac{3}{4}$.
15. Reduce $6\frac{5}{12}$, $9\frac{1}{2}$, and 5.
16. Reduce $7\frac{5}{6}$, $\frac{4}{9}$, $\frac{1}{4}$, and $\frac{1}{9}$.

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170. How do you reduce fractions of different denominators to fractions having a common denominator? When the numbers are small how may the work be performed?

CASE VII.

171. *To reduce fractions to their least common denominator.*

The least common denominator is the number which contains only the prime factors of the denominators,

1. Reduce $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$, to their least common denominator.

OPERATION.

$(12 \div 3) \times 1 = 4$	1st Numerator.		$3)3 \;.\; 6 \;.\; 4$
$(12 \div 6) \times 5 = 10$	2d	"	$2)1 \;.\; 2 \;.\; 4$
$(12 \div 4) \times 3 = 9$	3d	"	$1 \;.\; 1 \;.\; 2$

$3 \times 2 \times 2 = 12$, least com. denom.

Therefore, the fractions $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$, reduced to their least common denominator, are $\frac{4}{12}$, $\frac{10}{12}$, and $\frac{9}{12}$.

Hence, the following

RULE.—I. *Find the least common dividend of the denominators* (Art. 140), *which will be the least common denominator of the fractions.*

II. *Divide the least common denominator by the denominators of the given fractions separately, and multiply the numerators by the corresponding quotients, and place the products over the least common denominator.*

NOTES.—1. Before beginning the operation, reduce every fraction to a simple fraction and to its lowest terms.

2. The expressions, $(12 \div 3) \times 1$, $(12 \div 6) \times 5$, $(12 \div 4) \times 3$, indicate that the *quotients* are to be multiplied by 1, 5, and 3.

EXAMPLES.

Reduce the following fractions to their least common denominator.

2. Reduce $\frac{4}{5}$, $\frac{8}{9}$, $\frac{3}{15}$.
3. Reduce $14\frac{5}{4}$, $6\frac{3}{8}$, $5\frac{1}{2}$.
4. Reduce $\frac{3}{15}$, $\frac{4}{24}$, $\frac{8}{9}$.
5. Reduce $\frac{67}{120}$, $\frac{6}{40}$, $\frac{5}{2}$.
6. Reduce $\frac{41}{50}$, $3\frac{6}{20}$, 4.
7. Reduce $3\frac{1}{8}$, $4\frac{4}{12}$ $8\frac{6}{18}$.
8. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
9. Reduce $2\frac{1}{2}$ of $\frac{1}{6}$, $3\frac{1}{7}$ of 2.
10. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$, and $\frac{7}{10}$.
11. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$.

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171. What is the least common denominator of several fractions? How do you reduce fractions to their least common denominator?

ADDITION OF FRACTIONS.

172. Addition of Fractions is the operation of finding the number of fractional units in two or more fractions.

1. What is the sum of $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$?

ANALYSIS.—The fractional unit is the same in each fraction, viz: $\frac{1}{2}$; but the numerators show how many such units are taken (Art. 148); hence, *the sum of the numerators written over the common denominator, expresses the sum of the fractions.*

OPERATION.
$$1+3+5=9.$$
$$Ans.\ \tfrac{9}{2}=4\tfrac{1}{2}$$

2. What is the sum of $\frac{1}{2}$ and $\frac{2}{3}$?

ANALYSIS.—In the first, the fractional unit is $\frac{1}{2}$, in the second it is $\frac{1}{3}$. These units, not being of the same kind, cannot be expressed in the same collection. But the $\frac{1}{2}=\frac{3}{6}$, and $\frac{2}{3}=\frac{4}{6}$, in each of which the unit is $\frac{1}{6}$: hence, their sum is $\frac{7}{6}=1\frac{1}{6}$.

OPERATION.
$$\tfrac{1}{2}=\tfrac{3}{6}$$
$$\tfrac{2}{3}=\tfrac{4}{6}$$
$$\tfrac{3}{6}+\tfrac{4}{6}=\tfrac{7}{6}=1\tfrac{1}{6}$$

NOTE.—*Only units of the same kind, whether fractional or integral, can be expressed in the same collection.*

From the above analysis, we have the following

RULE.—I. *When the fractions have the same denominator, add the numerators, and place the sum over the common denominator.*

II. *When they have not the same denominator, reduce them to a common denominator, and then add as before.*

NOTE.—After the addition is performed, reduce every result to its lowest terms.

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$, and $\frac{3}{2}$.
2. Add $\frac{1}{7}$, $\frac{5}{7}$, and $\frac{9}{7}$.
3. Add $\frac{3}{9}$, $\frac{4}{9}$, $\frac{6}{9}$, $\frac{13}{9}$, and $\frac{16}{9}$.
4. Add $\frac{3}{14}$, $\frac{8}{14}$, $\frac{9}{14}$, and $\frac{5}{14}$.
5. Add $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{2}{15}$.
6. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{8}$, and $\frac{3}{16}$.
7. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$.
8. Add $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{5}$, and $\frac{9}{10}$.
9. Add 9, $\frac{2}{3}$, $\frac{1}{15}$, $\frac{5}{6}$, and $\frac{2}{9}$.
10. Add $\frac{1}{2}$, $\frac{3}{7}$, $\frac{6}{7}$, $\frac{1}{8}$, and $\frac{4}{9}$.
11. Add $\frac{5}{12}$, $\frac{5}{8}$, $\frac{6}{10}$, and $\frac{3}{5}$.
12. Add $\frac{1}{7}$, $\frac{2}{9}$, and $\frac{5}{6}$.
13. Add $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$.
14. Add $\frac{6}{12}$, $\frac{3}{5}$, $\frac{4}{8}$, and $\frac{6}{30}$.

15. What is the sum of $19\frac{1}{7}$, $6\frac{2}{3}$, and $4\frac{4}{5}$?

OPERATION.

Whole numbers. $19+6+4=29$ *Fractions.* $\frac{1}{7}+\frac{2}{3}+\frac{4}{5}=\frac{169}{105}=1\frac{64}{105}$.

$$\text{Sum}=29+1\frac{64}{105}=30\frac{64}{105}.$$

173. Note.—When there are mixed numbers, *add the whole numbers and fractions separately, and then add their sums.*

Find the sums of the following fractions:

16. Add $3\frac{1}{4}$, $7\frac{9}{10}$, $12\frac{4}{5}$, $1\frac{7}{8}$.
17. Add 16, $9\frac{3}{4}$, $25\frac{7}{9}$, $\frac{11}{12}$.
18. Add $\frac{1}{2}$ of $\frac{2}{5}$, $\frac{4}{7}$ of 9, $14\frac{9}{10}$.
19. Add $2\frac{8}{11}$, $6\frac{1}{5}$, and $12\frac{19}{22}$.
20. Add $900\frac{1}{10}$, $450\frac{4}{5}$, $75\frac{19}{25}$.
21. Add $\frac{1}{3}$ of $\frac{3}{11}$ of $\frac{11}{12}$ to $\frac{1}{2}$ of $\frac{2}{9}$.
22. Add $17\frac{2}{3}$ to $\frac{4}{5}$ of $7\frac{5}{6}$.
23. Add $\frac{4}{5}$, $7\frac{1}{2}$, and $8\frac{3}{4}$.

24. What is the sum of $\frac{3}{4}$ of $12\frac{2}{3}$ of $7\frac{4}{5}$, and $\frac{8}{9}$ of 25?

25. What is the sum of $\frac{4}{25}$ of $9\frac{3}{5}$ and $\frac{4}{21}$ of $328\frac{2}{5}$?

174. 1. What is the sum of $\frac{1}{5}$ and $\frac{1}{6}$?

Note.—If each of two fractions has 1 for a numerator, the sum of the fractions will be equal to the sum of their denominators divided by their product.

OPERATION.

$$\frac{1}{5}+\frac{1}{6}=\frac{6}{30}+\frac{5}{30}=\frac{11}{30}.$$
$$\frac{1}{5}+\frac{1}{6}=\frac{5+6}{5\times 6}=\frac{11}{30}.$$

2. What is the sum of $\frac{1}{8}$ and $\frac{1}{9}$? of $\frac{1}{6}$ and $\frac{1}{10}$?

3. What is the sum of $\frac{1}{7}$ and $\frac{1}{13}$? of $\frac{1}{10}$ and $\frac{1}{12}$? of $\frac{1}{12}$ and $\frac{1}{5}$?

4. What is the sum of $\frac{1}{4}$ and $\frac{1}{12}$? of $\frac{1}{6}$ and $\frac{1}{9}$? of $\frac{1}{3}$ and $\frac{1}{15}$?

SUBTRACTION OF FRACTIONS.

175. Subtraction of Fractions is the operation of finding the difference between two fractions.

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172. What is addition of fractions? When the fractional unit is the same, what is the sum of the fractions? What units may be expressed in the same collection? What is the rule for the addition of fractions?

173. When there are mixed numbers, how do you add?

174. When two fractions have 1 for a numerator, what is their sum equal to?

175. What is subtraction of fractions?

1. What is the difference between $\frac{5}{8}$ and $\frac{3}{8}$?

ANALYSIS.—In this example the fractional unit is $\frac{1}{8}$: there are 5 such units in the minuend and 3 in the subtrahend: their difference is 2 eighths; therefore, 2 is written over the common denominator 8.

OPERATION.

$$\frac{5}{8}-\frac{3}{8}=\frac{2}{8}=\frac{1}{4}.$$

Ans. $\frac{1}{4}$.

2. From $\frac{15}{7}$ take $\frac{10}{7}$.
3. From $\frac{5}{9}$ take $\frac{2}{9}$.
4. From $\frac{125}{365}$ take $\frac{67}{365}$.
5. From $\frac{335}{105}$ take $\frac{169}{105}$.

6. What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

ANALYSIS.—Reduce both to the same fractional unit $\frac{1}{12}$: then, there are 10 such units in the minuend and 4 in the subtrahend: hence, the difference is 6 twelfths.

OPERATION.

$$\frac{5}{6}=\frac{10}{12}$$
$$\frac{1}{4}=\frac{4}{12}$$
$$\frac{10}{12}-\frac{4}{12}=\frac{6}{12}=\frac{1}{2}.$$

Ans. $\frac{1}{2}$.

From the above analysis we have the following

RULE.—I. *When the fractions have the same denominator, subtract the less numerator from the greater, and place the difference over the common denominator.*

II. *When they have not the same denominator, reduce them to a common denominator, and then subtract as before.*

EXAMPLES.

Make the following subtractions:

1. From $\frac{5}{7}$ take $\frac{4}{9}$.
2. From $\frac{6}{7}$ take $\frac{5}{8}$.
3. From $\frac{6}{13}$ take $\frac{5}{17}$.
4. From 1, take $\frac{67}{100}$.
5. From $\frac{1}{8}$ of 12, take $\frac{16}{17}$ of $\frac{1}{2}$.
6. F'm $\frac{3}{7}$ of $1\frac{1}{2}$ of 7, take $\frac{5}{4}$ of $\frac{3}{8}$.
7. From $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{1}{2}$ take $\frac{3}{11}$ of $\frac{2}{3}$ of 1.
8. From $\frac{5}{8}$ of $\frac{4}{5}$ of $6\frac{1}{3}$, take $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{5}{7}$.
9. From $\frac{4}{11}$ of $\frac{22}{18}$ of $\frac{1}{3}$, take $\frac{3}{12}$ of $\frac{3}{7}$.
10. What is the difference between $4\frac{1}{6}$ and $2\frac{1}{7}$?

OPERATION.

$$4\tfrac{1}{6}=\tfrac{25}{6}=\tfrac{175}{42}$$
$$2\tfrac{1}{7}=\tfrac{15}{7}=\tfrac{90}{42}$$
$$\tfrac{85}{42}=2\tfrac{1}{42}\ \textit{Ans.}$$

or,

$$4\tfrac{1}{6}=4\tfrac{7}{42}$$
$$2\tfrac{1}{7}=2\tfrac{6}{42}$$
$$2\tfrac{1}{42}\ \textit{Ans.}$$

176. Therefore: *When there are mixed numbers, change both to improper fractions and subtract as in* Art. 175; or, *subtract the integral and fractional numbers separately, and write the results.*

11. From $84\frac{7}{15}$ take $16\frac{1}{3}$. | 12. From $246\frac{3}{8}$ take $164\frac{1}{5}$.

13. From $7\frac{2}{7}$ take $4\frac{1}{3}$: $\frac{2}{7}=\frac{6}{21}$ and $\frac{1}{3}=\frac{7}{21}$.

NOTE.—Since we cannot take $\frac{7}{21}$ from $\frac{6}{21}$ we borrow 1, or $\frac{21}{21}$, from the minuend, which added to $\frac{6}{21}=\frac{27}{21}$; then $\frac{7}{21}$ from $\frac{27}{21}$ leaves $\frac{20}{21}$. We must now carry 1 to the next figure of the subtrahend and proceed as in subtraction of simple numbers.

OPERATION.

$$7\frac{2}{7}=7\frac{6}{21}$$
$$4\frac{1}{3}=4\frac{7}{21}$$
Ans. $2\frac{20}{21}$.

14. From $16\frac{5}{8}$ take $5\frac{8}{9}$.
15. From $26\frac{3}{7}$ take $19\frac{7}{9}$.
16. From $36\frac{3}{5}$ take $27\frac{8}{11}$.
17. From $400\frac{5}{12}$ take $327\frac{5}{8}$.

18. From $\frac{1}{8}$ take $\frac{1}{11}$.

NOTE.—When the numerators are 1, the difference of the two fractions is equal to the difference of the denominators divided by their product.

OPERATION.

$$\frac{1}{8}-\frac{1}{11}=\frac{11}{88}-\frac{8}{88}=\frac{3}{88}$$
$$\frac{1}{8}-\frac{1}{11}=\frac{11-8}{11\times 8}=\frac{3}{88}$$

19. What is the difference between $\frac{1}{6}$ and $\frac{1}{8}$? Between $\frac{1}{5}$ and $\frac{1}{12}$? $\frac{1}{9}$ and $\frac{1}{17}$? $\frac{1}{20}$ and $\frac{1}{37}$? $\frac{1}{11}$ and $\frac{1}{15}$? $\frac{1}{40}$ and $\frac{1}{112}$?

MULTIPLICATION OF FRACTIONS.

177. MULTIPLICATION of Fractions is the operation of taking one number as many times as there are units in another, when one of the numbers is fractional, or when they are both fractional.

1. If one yard of cloth cost $\frac{5}{8}$ of a dollar, what will 4 yards cost?

ANALYSIS.—Four yards will cost 4 times as much as 1 yard; if 1 yard costs 5 *eighths* of a dollar, 4 yards will cost 4 times 5 *eighths* of a dollar, which are 20 *eighths*: therefore, if 1 yard cost $\frac{5}{8}$ of a dollar, 4 yards will cost $\frac{20}{8}=2\frac{1}{2}$ dollars.

OPERATION.

$$\frac{5}{8}\times 4=\frac{5\times 4}{8}=\frac{20}{8}=2\frac{1}{2}.$$

---

176. When there are mixed numbers, how do you subtract? Explain the case when the fractional part of the subtrahend is the greater?
177. What is multiplication of fractions?

OPERATION.

$\frac{5}{8} \times 4 = \frac{5}{8 \div 4} = \frac{5}{2}$.

OR,

$$\begin{array}{r|l} 2 \quad \cancel{8} & 5 \\ & \cancel{4} \\ \hline 2 & 5 = \frac{5}{2} = 2\frac{1}{2}. \end{array}$$

2d. If we divide the denominator by 4, the fraction will be multiplied by 4 (Prop. II): performing the operation, we obtain, $\frac{5}{2}$ which $= 2\frac{1}{2}$: hence,

To multiply a fraction by a whole number:—*Multiply the numerator, or divide the denominator by the multiplier.*

EXAMPLES.

1. Multiply $\frac{37}{144}$ by 12.
2. Multiply $\frac{47}{49}$ by 7.
3. Multiply $\frac{175}{45}$ by 9.
4. Multiply $\frac{127}{15}$ by 5.
5. Multiply $\frac{369}{147}$ by 49.
6. Multiply $\frac{175}{273}$ by 26.

7. If 1 dollar will buy $\frac{5}{9}$ of a cord of wood, how much will 15 dollars buy?

8. At $\frac{5}{6}$ of a dollar a pound, what will 12 pounds of tea cost?

9. If a horse eats $\frac{3}{4}$ of a bushel of oats in a day, how much will 18 horses eat?

10. What will 64 pounds of cheese cost, at $\frac{3}{22}$ of a dollar a pound?

11. If a man travel $\frac{7}{8}$ of a mile an hour, how far will he travel in 16 hours?

12. At $\frac{3}{5}$ of a cent a pound, what will 45 pounds of chalk cost?

13. If a man receive $\frac{9}{10}$ of a dollar for 1 day's labor, how much will he receive for 15 days?

14. If a family consume $\frac{4}{7}$ of a barrel of flour in 1 month, how much will they consume in 9 months?

15. If a person pays $\frac{11}{12}$ of a dollar a month for tobacco, how much does he pay in 18 months?

181. *To multiply a whole number by a fraction.*

1. At 15 dollars a ton, what will $\frac{4}{5}$ of a ton of hay cost?

ANALYSIS.—1st. Four-fifths of a ton will cost 4 times as much as 1 fifth of a ton; if 1 ton cost 15 dollars, 1 fifth will cost $\frac{1}{5}$ of 15 dollars, or 3 dollars, and $\frac{4}{5}$ will cost 4 times 3 dollars, which are 12 dollars.

OPERATION.

$(15 \div 5) \times 4 = 12.$

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180. How do you multiply a fraction by a whole number?

OR: 2d. 4 fifths of a ton will cost 1 fifth of 4 times the cost of 1 ton; 4 times 15 is 60, and 1 fifth of 60 is 12.

$$15 \times 4 \div 5 = 12.$$

NOTE.—Both operations may be combined in one by the use of the vertical line and cancellation: hence,

$$\begin{array}{r|l} & \overset{3}{\cancel{15}} \\ \cancel{5} & 4 \\ \hline & 12 \text{ Ans.} \end{array}$$

Divide the whole number by the denominator of the fraction and multiply the quotient by the numerator;

Or: *Multiply the whole number by the numerator of the fraction and divide the product by the denominator.*

EXAMPLES.

1. Multiply 24 by $\frac{7}{8}$.
2. Multiply 42 by $\frac{11}{12}$.
3. Multiply 105 by $\frac{5}{7}$.
4. Multiply 64 by $\frac{15}{36}$.
5. What is the cost of $\frac{2}{3}$ of a yard of cloth at 8 dollars a yard?
6. If an acre of land is valued at 75 dollars, what is $\frac{7}{12}$ of it worth?
7. If a house is worth 320 dollars, what is $\frac{9}{16}$ of it worth?
8. If a man travel 46 miles in a day, how far does he travel in $\frac{4}{5}$ of a day?
9. At 18 dollars a ton, what is the cost of $\frac{9}{10}$ of a ton of hay?
10. If a man earn 480 dollars in a year, how much does he earn in $\frac{11}{12}$ of a year?

182. *To multiply one fraction by another.*

1. If a bushel of corn cost $\frac{3}{4}$ of a dollar, what will $\frac{5}{6}$ of a bushel cost?

OPERATION.

$$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}.$$

$$\begin{array}{r|l} \overset{2}{} \; \overset{4}{} & \cancel{3} \\ \cancel{6} & 5 \\ \hline \cancel{8} & 5 = \frac{5}{8}. \end{array}$$

ANALYSIS.—5-sixths of a bushel will cost $\frac{5}{6}$ times as much as 1 bushel, or 5 times 1 sixth as much: $\frac{1}{6}$ of $\frac{3}{4}$ is $\frac{3}{24}$, (Art. 180), and 5 times $\frac{3}{24}$ is $\frac{15}{24} = \frac{5}{8}$: hence,

181. How do you multiply a whole number by a fraction?

Multiply the numerators together for a new numerator and the denominators together for a new denominator.

NOTES.—1. When the multiplier is less than 1, we do not take the whole of the multiplicand, but only such a part of it as the multiplier is of 1.

2. When the multiplier is a proper fraction, multiplication does not imply *increase*, as in the multiplication of whole numbers. The product is the same part of the multiplicand which the multiplier is of 1.

EXAMPLES.

1. Multiply $\frac{7}{8}$ by $\frac{4}{5}$.
2. Multiply $\frac{9}{13}$ by $\frac{15}{27}$
3. Find the pro't of $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{15}$.
4. Find the pro't of $\frac{6}{7}$, $\frac{9}{14}$, $\frac{21}{27}$.

5. If silk is worth $\frac{9}{10}$ of a dollar a yard, what is $\frac{5}{6}$ of a yard worth?

6. If I own $\frac{6}{7}$ of a farm and sell $\frac{2}{3}$ of my share, what part of the whole farm do I sell?

7. At $\frac{4}{5}$ of a dollar a pound, what will $\frac{7}{16}$ of a pound of tea cost?

8. If a knife cost $\frac{5}{8}$ of a dollar and a slate $\frac{6}{7}$ as much, what does the slate cost?

9. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$ of $\frac{8}{9}$.

NOTE.—Before multiplying, reduce both fractions to the form of simple fractions.

OPERATION.

$5\frac{1}{4}=\frac{21}{4}$; $\frac{1}{6}$ of $\frac{8}{9}=\frac{8}{54}$.

$\frac{21}{4}\times\frac{8}{54}=\frac{7}{9}$

$$\begin{array}{r|l} & \quad 7 \\ 3\ \cancel{4} & \cancel{21} \\ \cancel{6} & 1 \quad 2 \\ 9 & \cancel{8} \\ \hline 9 & 7=\frac{7}{9}\ \textit{Ans.} \end{array}$$

GENERAL EXAMPLES.

1. Mult. $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{4}{5}$ by $\frac{9}{14}$.
2. Mult. $\frac{9}{10}$ by $\frac{2}{5}$ of $1\frac{1}{4}$.
3. Mult. $\frac{1}{8}$ of 3 by $\frac{1}{6}$ of $15\frac{1}{9}$.
4. Mult. 5 of $\frac{2}{3}$ of $\frac{3}{5}$ by $4\frac{1}{6}$.
5. Mult. 14 of $\frac{5}{6}$ of 9 by $6\frac{3}{7}$.
6. Mult. $\frac{3}{7}$ of 6 of $\frac{5}{4}$ by $\frac{8}{5}$ of 4.

183. *When the multiplicand is a whole and the multiplier a mixed number.*

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182. How do you multiply one fraction by another? When the multiplier is less than 1, what part of the multiplicand is taken? If the fraction is proper, does multiplication imply increase? What part is the product of the multiplicand?

7. What is the product of 48 by $8\frac{1}{6}$?

NOTE.—First multiply 48 by $\frac{1}{6}$, which gives 8; then by 8, which gives 384, and the sum, 392 is the product: hence,

OPERATION.

$$48 \times \tfrac{1}{6} = 8$$
$$48 \times 8 = 384$$
$$392$$

Multiply first by the fraction, and then by the whole number, and add the products.

8. Mult. 67 by $9\frac{1}{2}$.
9. Mult. $12\frac{2}{3}$ by 9.
10. Mult. 108 by $12\frac{4}{9}$.
11. Mult. $5\frac{6}{7}$ by $3\frac{1}{2}$.

12. What is the product of $6\frac{1}{5}$, $2\frac{7}{9}$ and $\frac{1}{4}$ of 12.

13. What will 24 yards of cloth cost at $3\frac{3}{4}$ dollars a yard?

14. What will $6\frac{2}{3}$ bushels of wheat cost at $3\frac{3}{4}$ dollars a bushel?

15. A horse eats $\frac{3}{14}$ of $\frac{7}{9}$ of 12 tons of hay in three months; how much did he consume?

16. If $\frac{2}{3}$ of $\frac{5}{8}$ of a dollar buy a bushel of corn, what will $\frac{7}{10}$ of $\frac{6}{11}$ of a bushel cost?

17. What is the cost of $5\frac{2}{3}$ gallons of molasses at $96\frac{1}{2}$ cents a gallon?

18. What will $7\frac{11}{12}$ dozen candles cost at $\frac{3}{16}$ of a dollar per dozen?

19. What must be paid for 175 barrels of flour at $7\frac{2}{5}$ dollars a barrel?

20. If $\frac{3}{9}$ of $\frac{6}{7}$ of 2 yards of cloth can be bought for one dollar, how much can be bought for $\frac{7}{8}$ of $13\frac{1}{3}$ dollars?

21. What is the cost of $15\frac{2}{3}$ cords of wood at $3\frac{6}{7}$ dollars a cord?

DIVISION OF FRACTIONS.

184. Division of Fractions is the operation of finding a number which multiplied by the divisor will produce the dividend, when one or both of the parts are fractional.

185. *To divide a fraction by a whole number.*

1. If 4 bushels of apples cost $\frac{8}{9}$ of a dollar, what will 1 bushel cost?

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183. *How may you multiply when the multiplicand is a whole and the multiplier a mixed number?*
184. What is division of fractions?
185. How do you divide a fraction by a whole number?

ANALYSIS.—Since 4 bushels cost $\frac{8}{9}$ of a dollar, 1 bushel will cost $\frac{1}{4}$ of $\frac{8}{9}$ of a dollar. Dividing the numerator of the fraction $\frac{8}{9}$ by 4, we have $\frac{2}{9}$ (Art. 159).

OPERATION.
$\frac{8}{9} \div 4 = \frac{2}{9}$.

Multiplying the denominator by 4 will produce the same result (Art. 160): hence,

$\frac{8}{9} \div 4 = \frac{8}{36} = \frac{2}{9}$.

Divide the numerator or multiply the denominator by the divisor.

NOTE.—By the use of the vertical line and the principles of cancellation (Art. 143), all operations in division of fractions may be greatly abridged.

$$\begin{array}{c|c} 9 & \cancel{8}\ 2 \\ \cancel{4} & \\ \hline 9 & 2 = \frac{2}{9}. \end{array}$$

EXAMPLES.

1. Divide $\frac{15}{11}$ by 6.
2. Divide $\frac{18}{37}$ by 9.
3. Divide $\frac{405}{19}$ by 15.
4. Divide $\frac{450}{531}$ by 75.
5. Divide $\frac{18}{19}$ by 6.
6. Divide $\frac{42}{91}$ by 12.
7. Divide $\frac{15}{33}$ by 20.
8. Divide $\frac{126}{651}$ by 27.

9. If 6 horses eat $\frac{9}{10}$ of a ton of hay in 1 month, how much will one horse eat?

10. If 9 yards of ribbon cost $\frac{6}{7}$ of a dollar, what will 1 yard cost?

11. If 1 yard of cloth cost 4 dollars, how much can be bought for $\frac{8}{9}$ of a dollar?

12. If 5 pounds of coffee cost $\frac{15}{16}$ of a dollar, what will 1 pound cost?

13. At \$6 a barrel, what part of a barrel of flour can be bought for $\frac{3}{5}$ of a dollar?

14. If 10 bushels of barley cost $3\frac{1}{3}$ dollars, what will 1 bushel cost?

NOTE.—We reduce the mixed number to an improper fraction and divide as in the case of a simple fraction.

OPERATION.
$3\frac{1}{3} = \frac{10}{3}$.

$\frac{10}{3} \div 10 = \frac{1}{3}$ *Ans.*

15. If 21 pounds of raisins cost $4\frac{2}{3}$ dollars, what will 1 pound cost?

16. If 12 men consume $6\frac{2}{5}$ pounds of meat in a day, how much does 1 man consume?

186. *To divide a whole number by a fraction.*

1. At $\frac{4}{5}$ of a dollar apiece, how many hats can be bought for 6 dollars?

ANALYSIS.—Since $\frac{4}{5}$ of a dollar will buy one hat, 6 dollars will buy as many hats as $\frac{4}{5}$ is contained times in 6; and as there are 5 times as many fifths as whole things in any number, in 6 there are 30 fifths, and 4 fifths is contained in 30 fifths $7\frac{1}{2}$ times: hence,

OPERATION.

$$6 \div \tfrac{4}{5} = 6 \times 5 \div 4 = 7\tfrac{1}{2}.$$

$$\begin{array}{r|l} & \overset{3}{\cancel{6}} \\ 2\ \cancel{4} & 5 \\ \hline 2 & 15 = 7\tfrac{1}{2}. \end{array}$$

Invert the terms of the divisor and multiply the whole number by the new fraction.

EXAMPLES.

1. Divide 14 by $\frac{7}{8}$.
2. Divide 212 by $\frac{15}{16}$.
3. Divide 63 by $\frac{18}{27}$.
4. Divide 420 by $\frac{9}{11}$.
5. At $\frac{11}{12}$ of a dollar a yard, how many yards of cloth can be bought for 9 dollars?
6. If a man travel $\frac{7}{8}$ of a mile in 1 hour, how long will it take him to travel 10 miles?
7. If $\frac{5}{8}$ of a ton of hay is worth 9 dollars, what is a ton worth?

187. *To divide one fraction by another.*

1. At $\frac{2}{5}$ of a dollar a gallon, how much molasses can be bought for $\frac{7}{8}$ of a dollar?

ANALYSIS.—Since $\frac{2}{5}$ of a dollar will buy 1 gallon, $\frac{7}{8}$ of a dollar will buy as many gallons as $\frac{2}{5}$ is contained times in $\frac{7}{8}$: one is contained in $\frac{7}{8}$, $\frac{7}{8}$ times; but $\frac{1}{5}$ is contained 5 times as many times as 1, or $\frac{35}{8}$ times; but 2 fifths is contained half as many times as $\frac{1}{5}$, or $\frac{35}{16}$ times, equal to $2\frac{3}{16}$ times: hence,

OPERATION.

$$\tfrac{7}{8} \div \tfrac{2}{5} = \tfrac{7}{8} \times \tfrac{5}{2} = \tfrac{35}{16} = 2\tfrac{3}{16}.$$

$$\begin{array}{r|l} 8 & 7 \\ 2 & 5 \\ \hline 16 & 35 = 2\tfrac{3}{16}. \end{array}$$

I. *Invert the terms of the divisor.*

II. *Multiply the numerators together for the numerator of the quotient, and the denominators together for the denominator of the quotient.*

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186. How do you divide a whole number by a fraction?

NOTES.—1. If the vertical line is used, the denominator of the dividend and the numerator of the divisor fall on the left, and the other terms on the right.

2. Cancel all common factors.

3. If the dividend and divisor have a common denominator, they will cancel, and the quotient of their numerators will be the answer.

4. When the dividend or divisor contains a whole or mixed number, or compound fractions, reduce them to the form of simple fractions before dividing.

EXAMPLES.

1. Divide $\frac{9}{10}$ by $\frac{15}{16}$.
2. Divide $\frac{4}{11}$ by $\frac{12}{13}$.
3. Divide $3\frac{7}{9}$ by $\frac{17}{21}$.
4. Divide $\frac{3}{4}$ of $\frac{7}{9}$ by $\frac{7}{12}$ of $1\frac{1}{2}$.
5. Divide $\frac{8}{9}$ of 21 by $\frac{4}{5}$ of $3\frac{2}{3}$.
6. Divide $6\frac{1}{8}$ by $2\frac{1}{3}$.

7. At $\frac{1}{8}$ of a dollar a pound, how much butter can be bought for $\frac{11}{16}$ of a dollar?

8. If 1 man consume $1\frac{1}{5}$ pounds of meat in a day, how many men would $8\frac{2}{5}$ pounds supply?

9. If 6 pounds of tea cost $4\frac{1}{2}$ dollars, what does it cost a pound?

10. At $\frac{4}{5}$ of a dollar a basket, how many baskets of peaches can be bought for $11\frac{1}{5}$ dollars?

11. If $\frac{5}{8}$ of a ton of coal cost $6\frac{2}{3}$ dollars, what will 1 ton cost, at the same rate?

12. How much cheese can be bought for $\frac{19}{24}$ of a dollar at $\frac{1}{9}$ of a dollar a pound?

13. A man divided $2\frac{4}{5}$ dollars among his children, giving them $\frac{7}{10}$ of a dollar a piece; how many children had he?

14. How many times will $\frac{11}{12}$ of a gallon of beer fill a vessel holding $\frac{1}{5}$ of $\frac{4}{5}$ gallons?

15. How many times is $\frac{1}{6}$ of $\frac{4}{9}$ of 27 contained in $\frac{7}{8}$ of $\frac{1}{2}$ of $42\frac{2}{3}$?

16. If $5\frac{1}{5}$ bushels of potatoes cost $2\frac{3}{5}$ dollars, how much do they cost a bushel?

17. If John can walk 21 miles in $\frac{7}{9}$ of a day, how far can he walk in 1 day?

18. If a turkey cost $1\frac{3}{8}$ dollars, how many can be bought for $12\frac{5}{6}$ dollars?

19. At $\frac{4}{5}$ of $\frac{1}{3}$ of a dollar a yard, how many yards of ribbon can be bought for $\frac{21}{25}$ of a dollar?

---

187. How do you divide one fraction by another?

REDUCTION OF COMPLEX FRACTIONS.

188. Complex Fractions are only other forms of expression for the division of fractions : thus ; $\frac{\frac{4}{7}}{\frac{2}{9}}$ is the same as $\frac{4}{7}$ divided by $\frac{2}{9}$; and may be written, $\frac{4}{7}\times\frac{9}{2}=\frac{36}{14}=2\frac{4}{7}$.

189. *To reduce a complex fraction to the form of a simple fraction.*

1 Reduce $\frac{2\frac{2}{5}}{4\frac{1}{2}}$ to its simplest form.

OPERATION.

$$\frac{2\frac{2}{5}=\frac{12}{5}}{4\frac{1}{2}=\frac{9}{2}}=\frac{12}{5}\div\frac{9}{2}=\frac{\overset{4}{\cancel{12}}}{5}\times\frac{2}{\underset{3}{\cancel{9}}}=\frac{8}{15}$$ *Ans.* ; hence,

RULE.—*Divide the numerator of the complex fraction by its denominator,*

Or : *Multiply the numerator of the upper fraction into the denominator of the lower, for a numerator ; and the denominator of the upper fraction into the numerator of the lower, for a denominator.*

NOTES.—1. When either of the terms of a complex fraction is a mixed number, or compound fraction, it must first be reduced to the form of a simple fraction.

2. When the vertical line is used, the numerator of the *upper* and the denominator of the *lower* numbers fall on the right of the vertical line, and the other terms on the left.

EXAMPLES.

Reduce the following complex fractions to their simplest form :

1. Reduce $\frac{\frac{4}{5}}{\frac{6}{7}}$.
2. Reduce $\frac{6\frac{2}{3}}{\frac{2}{3}}$.
3. Reduce $\frac{7\frac{11}{12}}{3\frac{5}{6}}$.
4. Reduce $\frac{3}{4}$ of $\frac{\frac{1}{2}}{\frac{3}{7}}$.
5. Reduce $\frac{\frac{2}{9}\text{ of }\frac{3}{5}}{\frac{1}{4}\text{ of }2\frac{4}{9}}$.
6. Reduce $\frac{25}{8\frac{3}{7}}$.
7. Reduce $\frac{14\frac{9}{10}}{\frac{7}{9}\text{ of }15}$.
8. Reduce $\frac{214\frac{3}{4}}{25\frac{11}{12}}$.
9. Reduce $\frac{\frac{21}{25}}{4\frac{9}{10}}$.
10. Reduce $\frac{\frac{4}{7}\text{ of }\frac{2}{3}\text{ of }5\frac{1}{4}}{\frac{15}{16}\text{ of }48}$.

DENOMINATE FRACTIONS.

190. A DENOMINATE Fraction is one in which the unit of the fraction is a denominate number. Thus, $\frac{5}{7}$ of a yard is a denominate fraction.

191. REDUCTION of denominate fractions is the operation of changing a fraction from one denominate unit to another without altering its value.

There are four cases:

1st. To change from a greater unit to a less, as from yards to inches:

2d. To change from a less unit to a greater:

3d. To find the value of a fraction in integers of lower denominations:

4th. To find the value of integers in a fraction of a larger unit.

These cases will be arranged in sets of two and two.

192. *To change from a greater unit to a less.*

1. In $\frac{5}{9}$ of a yard, how many inches?

OPERATION.

$\frac{5}{9} \times 3 \times 12 = \frac{180}{9} = 20$ inches.

ANALYSIS.—Since in 1 yard there are 3 feet, in $\frac{5}{9}$ yards there are $\frac{5}{9}$ times 3 feet $= \frac{15}{9}$ feet. And since in 1 foot there are 12 inches, in $\frac{15}{9}$ feet there are $\frac{15}{9}$ times 12 inches $= \frac{180}{9} = 20$ inch's: hence,

RULE.—*Multiply the fraction and the products which arise by the units of the scale, in succession, until you reach the unit required.*

193. *To change from a less unit to a greater.*

1. In 20 inches, how many yards?

OPERATION.

$20 \times \frac{1}{12} \times \frac{1}{3} = \frac{20}{36} = \frac{5}{9}$ yards.

ANALYSIS.—Since 12 inches make 1 foot, in 20 inches there are as many feet as 12 inches is contained times in 20 inches $= \frac{20}{12}$ feet; and as 3 feet make 1 yard, in $\frac{20}{12}$ feet there are as many yards as 3 feet is contained times in $\frac{20}{12}$ feet $= \frac{20}{36} = \frac{5}{9}$ yards: hence,

RULE.—*Divide the fraction, and the quotients which arise, by the units of the scale, in succession, until you reach the unit required.*

---

188. What are complex fractions?
189. How do you reduce complex to simple fractions?

NOTE.—It will be found most convenient in fractions, to perform the operations by cancellation: thus,

$$\begin{array}{r|l} \overset{3}{\cancel{9}} & 5 \\ & \cancel{3} \\ & \cancel{12}\ 4 \\ \hline & 20 \text{ inches.} \end{array} \qquad \begin{array}{r|l} & 5 \\ 3\ \cancel{12} & \cancel{20} \\ 3 & \\ \hline 9 & 5 = \tfrac{5}{9} \text{ yards.} \end{array}$$

EXAMPLES.

1. Reduce $\frac{3}{648}$ of a hogshead to the fraction of a quart.
2. Reduce $\frac{4}{320}$ of a bushel to the fraction of a pint.
3. Reduce $\frac{1}{8640}$ of a pound Troy to the fraction of a grain
4. What part of a foot is $\frac{2}{1980}$ of a furlong?
5. What part of a minute is $\frac{3}{5760}$ of a day?
6. Reduce $\frac{14}{25600}$ of a *cwt.* to the fraction of an ounce.
7. Reduce $\frac{4}{5}$ of a gallon to the fraction of a hogshead.
8. What part of a £ is $\frac{5}{8}$ of a shilling?
9. What part of a hogshead is $\frac{5}{9}$ of a quart?
10. What part of a mile is $\frac{9}{10}$ of a foot?
11. Reduce $\frac{3}{4800}$ of £ to the fraction of a farthing.
12. Reduce $\frac{3}{76}$ of an Ell Eng. to the fraction of a nail.
13. Reduce $\frac{5}{9}$ of a nail to the fraction of a yard.
14. Reduce $\frac{1}{3}$ of $\frac{4}{5}$ of a foot to the fraction of a mile.
15. Reduce $\frac{1}{3675}$ of a ton to the fraction of a pound.
16. Reduce $\frac{7}{8}$ of $3\frac{1}{3}$*pwt.* to the fraction of a pound Troy.
17. What part of a mile is $\frac{3}{4}$ of a rod?
18. What part of an ounce is $\frac{3}{10}$ of a scruple?
19. $\frac{3}{576}$ of a day is what portion of 10 minutes?
20. What part of $\frac{1}{8}$ of a foot is $\frac{2}{198}$ of a furlong?
21. Reduce $\frac{4}{9760}$ of a hogshead of ale to the fraction of a pint.

---

190. What is a denominate fraction?

191. What is reduction of denominate fractions? How many cases are there? Name them.

192. How do you change from a greater unit to a less?

193. How do you change from a less unit to a greater?

194. *To find the value of a fraction in integers of lower denominations.*

1. What is the value of $\frac{4}{5}$ of a pound Troy?

ANALYSIS.—$\frac{4}{5}$ of a pound reduced to the fraction of an ounce is $\frac{4}{5}\times12=\frac{48}{5}$ of an ounce, (Art. 177.), which is equal to $9\frac{3}{5}$ ounces: $\frac{3}{5}$ of an ounce reduced to the fraction of a pennyweight is $\frac{3}{5}\times 20=\frac{60}{5}$ of a *pwt.*, or 12*pwt.*

OPERATION.

```
Numer.   4
         12 oz.    pwt.
Denom.  5)48(9 . . . 12
          45
           3
          20
         5)60
          60
```

RULE.—I. *Multiply the numerator of the fraction by the number which will reduce it to the next lower denomination and divide the product by the denominator.*

II. *If there is a remainder, reduce it in the same manner, and so on, till the lowest denomination is obtained.*

195. *To find the value of integers in a fraction of a higher denomination.*

2. Reduce 9*oz.* 12*pwts.* to the fraction of a pound Troy.

ANALYSIS.—In 1 pound there are 240 pennyweights: 1 pennyweight is $\frac{1}{240}$ of a pound; and 9 ounces 12*pwts.* = 192*pwts.* is $\frac{192}{240}$ of a pound $=\frac{4}{5}$ of a pound.

OPERATION.

```
1   lb.            oz.   pwts.
12                  9 . . 12
12                 20
 20   Num.         192
240   Denom.       240  = 4/5 lb.
```

RULE.—I. *Reduce the given integers to the lowest denomination named, and the result will be the numerator of the required fraction.*

II. *Reduce 1 unit of the required denomination, to the denomination of the numerator, and the result will be the denominator of the required fraction.*

EXAMPLES.

3. What is the value of $\frac{7}{8}$ of a tun of wine?

4. What part of a tun of wine is 3*hhd.* 31*gal.* 2*qt.*?

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194. How do you find the value of a fraction in integers of lower denominations?

195. How do you find the value of integers in a fraction of a higher denomination?

5. What is the value of $\frac{9}{10}$ of a yard?
6. What is the value of $\frac{4}{5}$ of a month?
7. What is the value of $\frac{3}{8}$ of a chaldron?
8. What is the value of $\frac{7}{9}$ of a mile?
9. What is the value of $\frac{5}{32}$ of a ton?
10. What is the value of $\frac{6}{7}$ of 3 days?
11. What is the value of $\frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{2}{3}$ bushels of grain?
12. Reduce 3*gals.* 2*qts* to the fraction of a hogshead.
13. Reduce 2*fur.* 36*rd.* 2*yd.* to the fraction of a mile.
14. What part of a £ is 5*s.* $7\frac{1}{5}$*d.*?
15. What part of a pound Troy is 10*oz.* 13*pwt.* 8*gr.*?
16. 11*cwt.* 0*qr.* 12*lb.* 7*oz.* $1\frac{7}{9}$*dr.* is what part of a ton?
17. What part is 2*pk.* 4*qt.* of 1*bu.* 3*pk.*?
18. 24*lb.* 6*oz.* is what part of 3*qr.* 12*lb.* 12*oz.*?
19. Reduce 3*wk.* 1*d.* 9*h.* 36*m.* to the fraction of a month?
20. Reduce 2*R.* 32*rd.* 8*yd.* to the fraction of an acre.
21. Reduce 12*s.* 9*d.* $1\frac{1}{3}$*far.* to the fraction of a guinea.
22. What is the value of $\frac{7}{10}$*lb.* apothecaries' weight?
23. What part of an Ell English is 3*qr.* 3*na.* $1\frac{1}{4}$*in.*?
24. What is the value of $\frac{4}{5}$*hhd*?.
25. What is the value of $\frac{2}{9}$ of 3 barrels of beer?
26. What is the value of $\frac{7}{15}$ of a cwt.?
27. Reduce $3^\circ\ 15'\ 18\frac{3}{4}''$ to the fraction of a sign.
28. Reduce $3\frac{7}{9}$ inches to the fraction of a hand.
29. What is the value of $\frac{1}{30}$ of a hogshead of wine?
30. What is the value of $\frac{7}{13}$ of an acre of land?

ADDITION AND SUBTRACTION.

196. *To add or subtract denominate fractions.*

1. Add $\frac{2}{3}$ of a £ to $\frac{5}{6}$ of a shilling.

$\frac{2}{3}$ of a £ $=\frac{2}{3}$ of $\frac{20}{1}=\frac{40}{3}$ of a shilling.

Then, $\frac{40}{3}+\frac{5}{6}=\frac{240}{18}+\frac{15}{18}=\frac{255}{18}s.=\frac{85}{6}s.=14s.\ 2d.$

---

196. Give the rule for adding and subtracting denominate fractions.

Or, the $\frac{5}{6}$ of a shilling may be reduced to the fraction of a £: thus,

$$\tfrac{5}{6} \text{ of } \tfrac{1}{20} = \tfrac{5}{120} \text{ of a £} = \tfrac{1}{24} \text{ of a £}:$$

then, $\quad \frac{2}{3} + \frac{1}{24} = \frac{48}{72} + \frac{3}{72} = \frac{51}{72}$ of a £,

which being reduced, gives 14*s.* 2*d.* *Ans.*

2. Add $\frac{3}{5}$ of a year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day.

$\frac{3}{5}$ of a year $= \frac{3}{5}$ of $\frac{365}{1}$ days $= 31wk.\ 2da.$

$\frac{1}{3}$ of a week $= \frac{1}{3}$ of 7 days $= \text{-} \text{-}\ 2da.\ 8hr.$

$\frac{1}{8}$ of a day $= \text{-} \text{-} \text{-} \text{-} \text{-} = \text{-} \text{-} \text{-} \text{-} \text{-}\ 3hr.$

Ans. 31*wk.* 4*da.* 11*hr.*

3. From $\frac{1}{2}$ of a £ take $\frac{1}{3}$ of a shilling.

$\frac{1}{3}$ of a shilling $= \frac{1}{3}$ of $\frac{1}{20}$ of a £ $= \frac{1}{60}$ of a £.

Then, $\quad \frac{1}{2} - \frac{1}{60} = \frac{30}{60} - \frac{1}{60} = \frac{29}{60}$ of a £ $= 9s.\ 8d.$

4. From $1\frac{3}{4}lb.$ Troy weight, take $\frac{1}{6}oz.$

	lb.	*oz.*	*pwt.*	*gr.*
$1\frac{3}{4}lb. = \frac{7}{4}lb.$ of $\frac{12}{1}oz. = 21oz. =$	1	9		
$\frac{1}{6}oz. = \frac{1}{6}$ of $\frac{20}{1}$ of $\frac{24}{1}gr. = 80gr. =$	0	0	3	8
Ans.	1	8	16	16

RULE.—*Reduce the given fractions to the same unit, and then add or subtract as in simple fractions, after which reduce to integers of a lower denomination:*

Or: *Reduce the fractions separately to integers of lower denominations, and then add or subtract as in denominate numbers.*

EXAMPLES.

5. Add $1\frac{1}{4}$ miles, $\frac{7}{10}$ furlongs, and 30 rods.
6. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile.
7. Add $\frac{3}{4}$ of a *cwt.*, $\frac{42}{2}$ of a *lb.*, 13*oz.*, $\frac{1}{2}$ of a *cwt.* and 6*lb*
8. From $\frac{1}{2}$ of a day take $\frac{2}{3}$ of a second.
9. From $\frac{2}{3}$ of a rod take $\frac{3}{7}$ of an inch.
10. From $\frac{4}{15}$ of a hogshead take $\frac{6}{7}$ of a quart.
11. From $\frac{3}{5}oz.$ take $\frac{7}{8}pwt.$
12. From $4\frac{3}{7}cwt.$ take $4\frac{9}{10}lb.$

13. Mr. Merchant bought of farmer Jones $22\frac{1}{4}$ bushels of wheat at one time, $19\frac{5}{12}$ bushels at another, and $33\frac{5}{9}$ at another: how much did he buy in all?

14. Add $\frac{4}{7}$ of a ton and $\frac{9}{10}$ of a *cwt.*

15. Mr. Warren pursued a bear for three successive days; the first day he travelled $28\frac{3}{7}$ miles; the second $33\frac{1}{14}$ miles; the third $29\frac{1}{21}$ miles, when he overtook him: how far had he travelled?

16. Add $5\frac{5}{6}$ days and $52\frac{5}{19}$ minutes.

17. Add $\frac{4}{7}$*cwt.*, $8\frac{5}{6}$*lb.*, and $3\frac{9}{10}$*lb.*

18. A tailor bought 3 pieces of cloth, containing respectively, $18\frac{3}{4}$ yards, $21\frac{2}{3}$ Ells Flemish, and $16\frac{4}{5}$ Ells English: how many yards in all?

19. Bought 3 kinds of cloth; the first contained $\frac{1}{2}$ of 3 of $\frac{6}{2}$ of $\frac{2}{6}$ yards; the second, $\frac{1}{4}$ of $\frac{4}{5}$ of 5 yards; and the third, $\frac{1}{6}$ of $\frac{3}{2}$ of $\frac{1}{3}$ yards: how much in them all?

20. Add $1\frac{1}{7}$*cwt.* $17\frac{2}{3}$*lb.* and $7\frac{4}{5}$*oz.*

21. From $\frac{3}{8}$ of an *oz.* take $\frac{7}{8}$ of a *pwt.*

22. Take $\frac{1}{7}$ of a day and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of an hour from $3\frac{2}{3}$ weeks.

23. A man is $6\frac{1}{2}$ miles from home, and travels 4*mi.* 1*fur.* 24*rd.*, when he is overtaken by a storm: how far is he then from home?

24. A man sold $\frac{10}{34}$ of his farm at one time, $\frac{16}{34}$ at another, and $\frac{2}{17}$ at another: what part had he left?

25. From $1\frac{1}{2}$ of a £ take $\frac{3}{4}$ of a shilling.

26. From $1\frac{1}{5}$*oz.* take $\frac{7}{8}$*pwt.*

27. From $8\frac{6}{7}$*cwt.* take $4\frac{9}{10}$*lb.*

28. From $3\frac{1}{2}$*lb.* Troy weight, take $\frac{1}{6}$*oz.*

29. From $1\frac{1}{3}$ rods take $\frac{3}{7}$ of an inch.

30. From $\frac{49}{72}$ ℔ take $\frac{7}{16}$ ℥.

DUODECIMALS.

197. If the unit 1 foot be divided into 12 equal parts, each part is called an *inch* or *prime*, and marked ′. If an inch be divided into 12 equal parts, each part is called a *second*, and marked ″. If a second be divided, in like manner, into 12

equal parts, each part is called a *third*, and marked $'''$; and so on for divisions still smaller.

This division of the foot gives

$1'$ inch or prime - - - - - -	$= \frac{1}{12}$	of a foot.
$1''$ second is $\frac{1}{12}$ of $\frac{1}{12}$ - - -	$= \frac{1}{144}$	of a foot.
$1'''$ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ - -	$= \frac{1}{1728}$	of a foot.

NOTE.—The marks $'$, $''$, $'''$, &c., which denote the *fractional units*, are called *indices*.

TABLE.

$12'''$	make	$1''$	second.
$12''$	"	$1'$	inch or prime.
$12'$	"	1	foot.

Hence: Duodecimals are denominate fractions, in which the primary unit is 1 *foot*, and 12 the *scale* of division.

NOTE.—Duodecimals are chiefly used in measuring *surfaces* and *solids*.

ADDITION AND SUBTRACTION.

198. The units of duodecimals are reduced, added, and subtracted, like those of other denominate numbers. The *scale* is always 12.

EXAMPLES.

1. In $185'$, how many feet?
2. In $250''$, how many feet and inches?
3. In $4367'''$, how many feet?
4. What is the sum of 3*ft.* $6'$ $3''$ $2'''$ and 2*ft.* $1'$ $10''$ $11'''$?
5. What is the sum of 8*ft.* $9'$ $7''$ and 6*ft.* $7'$ $3''$ $4'''$?
6. What is the difference between 9*ft.* $3'$ $5''$ $6'''$ and 7*ft.* $3'$ $6''$ $7'''$?
7. What is the difference between 40*ft.* $6'$ $6''$ and 29*ft.* $7'''$?
8. What is the difference between 12*ft.* $7'$ $9''$ $6'''$ and 4*ft.* $9'$ $7''$ $9'''$?

---

197. If 1 foot be divided into twelve equal parts, what is each part called? If the inch be so divided, what is each part called? What are duodecimals? For what are duodecimals chiefly used?

198. How do you add and subtract duodecimals? What is the scale?

MULTIPLICATION.

199. Begin with the *highest* unit of the multiplier and the *lowest* of the multiplicand, and recollect,

1st. That 1 foot × 1 foot = 1 square foot (Art. 110).

2d. That a part of a foot × a part of a foot = some part of a square foot.

NOTE.—Observe that the unit is changed, by multiplication, from a linear to a *superficial unit.*

1. Multiply 6*ft.* 7′ 8″ by 2*ft.* 9′.

ANALYSIS.—Since a prime is $\frac{1}{12}$ of a foot and a second $\frac{1}{144}$,
$2\times8''=\frac{16}{144}$ of a square foot; which reduced to 12ths, is 1′ and 4″: that is, 1 twelfth, and 4 twelfths of $\frac{1}{12}$ of a square foot.
$2\times7'=14$ twelfths $=1ft.\ 2'$
$2\times6=12$ square feet,
$9'\times8''=\frac{72}{1728}$ of a square foot $=6''$
$9'\times7'=\frac{63}{144}=5'\ 3''$
$9\times6'=\frac{54}{12}=4\ 6'$

OPERATION.

	ft.		
	6	7′	8″
	2	9′	
2 × 8″ =		1′	4″
2 × 7′ =	1	2′	
2 × 6 =	12		
9′ × 8″ =			6″
9′ × 7′ =		5′	3″
9′ × 6 =	4	6′	
Prod.	18	3′	1″

RULE.—I. *Write the multiplier under the multiplicand, so that units of the same order shall fall in the same column.*

II. *Begin with the highest unit of the multiplier and the lowest of the multiplicand, and make the index of each product equal to the sum of the indices of the factors.*

III. *Reduce each product, in succession, to square feet, and 12ths of a square foot.*

NOTE.—The index of the unit of any product is equal to the sum of the indices of the factors.

EXAMPLES.

1. How many solid feet in a stick of timber which is 25 feet 6 inches long, 2 feet 7 inches broad, and 3 feet 3 inches thick?

---

199. Explain the method of multiplying duodecimals. Give the rule.

Beginning with the 2 feet, we say 2 times 6′ are 12′=1 square foot: then, 2 times 25 are 50, and 1 to carry are 51 square feet.

Next, 7′ times 6′ are 42″,=3′ and 6″: hen 7′ times 25=175′=14 7′: hence, the surface is 65 10′ 6″, and by multiplying by the thickness, we find the solid contents to be 214 1′ 1″ 6‴ cubic feet.

OPERATION.

ft.		
25	6′	length.
2	7′	breadth.
51	0′	
	3′ 6″	
14	7′	
65	10′ 6″	
3	3′	thickness
197	7′ 6″	
16	5′ 7″ 6‴	
214	1′ 1″ 6‴	

2. Multiply 9*ft.* 4*in.* by 8*ft.* 3*in.*
3. Multiply 9*ft.* 2*in.* by 9*ft.* 6*in.*
4. Multiply 24*ft.* 10*in.* by 6*ft.* 8*in.*
5. Multiply 70*ft.* 9*in.* by 12*ft.* 3*in.*
6. How many cords and cord feet in a pile of wood 24 feet long, 4 feet wide, and 3 feet 6 inches high?
7. How many square feet are there in a board 17 feet 6 inches in length, and 1 foot 7 inches in width?
8. What number of cubic feet are there in a granite pillar 3 feet 9 inches in width, 2 feet 3 inches in thickness, and 12 feet 6 inches in length?
9. There is a certain pile of wood, measuring 24 feet in length, 16 feet 9 inches high, and 12 feet 6 inches in width. How many cords are there in the pile?
10. How many square yards in the walls of a room, 14 feet 8 inches long, 11 feet 6 inches wide, and 7 feet 11 inches high?
11. If a load of wood be 8 feet long, 3 feet 9 inches wide, and 6 feet 6 inches high, how much does it contain?
12. How many cubic yards of earth were dug from a cellar which measured 42 feet 10 inches long, 12 feet 6 inches wide, and 8 feet deep?
13. What will it cost to plaster a room 20 feet 6′ long, 15 feet wide, 9 feet 6′ high, at 18 cents per square yard?
14. How many feet of boards 1 inch thick can be cut from a plank 18*ft.* 9*in.* long, 1*ft.* 6*in.* wide and 3*in.* thick, if there be no waste in sawing?

DECIMAL FRACTIONS.

200. There are two kinds of Fractions : *Common Fractions* and *Decimal Fractions.*

A Common Fraction is one in which the unit is divided into any number of equal parts.

A Decimal fraction is one in which the unit is divided according to the *scale of tens.*

201. If the unit 1 be divided into 10 equal parts, the parts are called *tenths.*

If the unit 1 be divided into one hundred equal parts, the parts are called *hundredths.*

If the unit 1 be divided into one thousand equal parts, the parts are called *thousandths*, and we have similar expressions for the parts, when the unit is further divided according to the scale of tens.

These fractions may be written thus :

Four-tenths, - - - -	$\frac{4}{10}$.
Six-tenths, - - - - -	$\frac{6}{10}$.
Forty-five hundredths, - - -	$\frac{45}{100}$.
125 thousandths, - - - -	$\frac{125}{1000}$.
1047 ten thousandths, - - -	$\frac{1047}{10000}$.

From which we see, that in each case the denominator indicates the fractional unit ; that is, determines whether the parts are tenths, hundredths, thousandths, &c.

202. The denominators of decimal fractions are seldom set down. The fractions are usually expressed by means of a period, placed at the left of the numerator.

Thus,	$\frac{4}{10}$	- is written - -	.4
	$\frac{45}{100}$	- - - - -	.45
	$\frac{125}{1000}$	- - - - -	.125
	$\frac{1047}{10000}$	- - - - -	.1047

---

200. How many kinds of fractions are there ? What are they ? What is a common fraction ? What is a decimal fraction ?

201. When the unit 1 is divided into 10 equal parts, what is each part called ? What is each part called when it is divided into 100 equal parts ? When into 1000 ? Into 10,000, &c. ? How are decimal fractions formed ? What gives denomination to the fraction ?

This method of writing decimal fractions is a mere language, and is used to avoid writing the denominators. The denominator, however, of every decimal fraction is always understood :

It is the unit 1 *with as many ciphers annexed as there are places of figures in the decimal.*

The place next to the decimal point, is called the place of *tenths*, and its unit is 1 tenth. The next place. to the right, is the place of *hundredths*, and its unit is 1 hundredth ; the next is the place of *thousandths*, and its unit is 1 thousandth ; and similarly for places still to the right.

DECIMAL NUMERATION TABLE.

Tenths.	Hundredths.	Thousandths.	Ten thousandths,	Hundred thousandths,	Millionths.	Ten millionths,		
.4							is read	4 tenths
.5	4						- -	54 hundredths.
.0	6	4					- -	64 thousandths.
.6	7	5	4				- -	6754 ten thousandths,
.0	1	2	3	4			- -	1234 hundred thousandths,
.0	0	7	6	5	4		- -	7654 millionths.
.0	0	4	3	6	0	4	- -	43604 ten millionths,

NOTE.—Decimal fractions are numerated from left to right; thus, *tenths*, *hundredths*, *thousandths*, &c.

---

202. Are the denominators of decimal fractions generally set down ? How are the fractions expressed ? Is the denominator understood ? What is it ? What is the place next the decimal point called ? What is its unit ? What is the next place called ? What is its unit ? What is the third place called ? What is its unit ? Which way are decimals numerated ?

203. Write and numerate the following decimals:

Four-tenths, - -	.4
Four hundredths, - -	.0 4
Four thousandths, - -	.0 0 4
Four ten thousandths, - -	.0 0 0 4
Four, hundred thousandths, -	.0 0 0 0 4
Four millionths, - - -	.0 0 0 0 0 4
Four ten millionths, - -	.0 0 0 0 0 0 4.

Here we see, that the same figure expresses different decimal units. according to the place which it occupies : therefore,

The value of the unit, in the different places, in passing from the left to the right, diminishes according to the scale of tens.

Hence, ten of the units in any place, are equal to one unit in the place next to the left ; that is, ten thousandths make one hundredth, ten hundredths make one-tenth, and ten-tenths, the unit 1.

This scale of *increase*, from the *right* hand towards the *left*, is the same as that in whole numbers ; therefore,

Whole numbers and decimal fractions may be united by placing the decimal point between them : thus,

Whole numbers.								Decimals.						
Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	Thousandths.	Ten thousandths,	Hundred thousandths,	Millionths.	Ten millionths,
8	3	6	3	0	6	4	1	.0	4	7	8	9	7	6

A number composed partly of a whole number and partly of a decimal, is called a *mixed number*.

RULE FOR WRITING DECIMALS.

Write the decimal as if it were a whole number, prefixing as many ciphers as are necessary to make it of the required denomination.

RULE FOR READING DECIMALS.

Read the decimal as though it were a whole number, adding the denomination indicated by the lowest decimal unit.

EXAMPLES.

Write the following numbers decimally:

(1.) $\frac{3}{100}$. (2.) $\frac{16}{1000}$. (3.) $\frac{17}{10000}$. (4.) $\frac{32}{100}$. (5.) $\frac{165}{10000}$.

(6.) $18\frac{3}{100}$. (7) $12\frac{9}{1000}$. (8) $16\frac{12}{1000}$. (9) $9\frac{565}{100}$. (10.) $11\frac{121}{10}$.

Write the following numbers in figures, and then numerate them.

1. Forty-one, and three-tenths.
2. Sixteen, and three millionths.
3. Five, and nine hundredths.
4. Sixty-five, and fifteen thousandths.
5. Eighty, and three millionths.
6. Two, and three hundred millionths
7. Four hundred, and ninety-two thousandths.
8. Three thousand, and twenty-one ten thousandths.
9. Forty-seven, and twenty-one hundred thousandths,
10. Fifteen hundred, and three millionths.
11. Thirty-nine, and six hundred and forty thousandths.
12. Three thousand, eight hundred and forty millionths.
13. Six hundred and fifty thousandths.

---

203. Does the value of the unit of a figure depend upon the place which it occupies? How does the value change from the left towards the right? What do ten units of any one place make? How do the units of the places increase from the right towards the left? How may whole numbers be joined with decimals? What is such a number called? Give the rule for writing decimal fractions. Give the rule for reading decimal fractions.

UNITED STATES MONEY.

204. The denominations of United States Money correspond to the decimal division, if we regard 1 dollar as the unit.

For, *the dimes are tenths of the dollar, the cents are hundredths of the dollar, and the mills, being tenths of the cent, are thousandths of the dollar.*

EXAMPLES.

1. Express $39 and 39 cents and 7 mills, decimally.
2. Express $12 and 3 mills, decimally.
3. Express $147 and 4 cents, decimally.
4. Express $148 4 mills, decimally.
5. Express $4 6 mills, decimally.
6. Express $9 6 cents 9 mills, decimally.
7. Express $10 13 cents 2 mills, decimally.

ANNEXING AND PREFIXING CIPHERS.

205. Annexing a cipher is placing it on the right of a number.

If a cipher is annexed to a decimal it makes *one more decimal place*, and therefore, a cipher must also be *added to the denominator* (Art. 202).

The numerator and denominator will therefore have been multiplied by the same number, and consequently the value of the fraction will not be changed (Art. 161): hence,

Annexing ciphers to a decimal fraction does not alter its value.

We may take as an example, $.3=\frac{3}{10}$.

If we annex a cipher to the numerator, we must, at the same time, annex one to the denominator, which gives,

---

204. If the denominations of Federal Money be expressed decimally, what is the unit? What part of a dollar is 1 dime? What part of a dime is a cent? What part of a cent is a mill? What part of a dollar is 1 cent? 1 mill?

205. When is a cipher annexed to a number? Does the annexing of ciphers to a decimal alter its value? Why not? What does three tenths become by annexing a cipher? What by annexing two ciphers? Three ciphers? What does 8 tenths become by annexing a cipher? By annexing two ciphers? By annexing three ciphers?

$.3 = \frac{30}{100} = .30$ by annexing one cipher,
$.3 = \frac{300}{1000} = .300$ by annexing two ciphers,
$.3 = \frac{3000}{10000} = .3000$ by annexing three ciphers.

Also, $.5 = \frac{5}{10} = .50 = \frac{50}{100} = .500 = \frac{500}{1000}$.

Also, $.8 = .80 = .800 = .8000 = .80000$.

206. Prefixing a cipher is placing it on the left of a number.

If ciphers are prefixed to the numerator of a decimal fraction, the same number of ciphers must be annexed to the denominator. Now, the numerator will remain unchanged while the denominator will be increased ten times for every cipher annexed; and hence, the value of the fraction will be *diminished* ten times for every cipher prefixed to the numerator (Art. 160).

Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher prefixed.

Take, for example, the fraction $.2 = \frac{2}{10}$.

$.2$ becomes $\frac{02}{100} = .02$ by prefixing one cipher,
$.2$ becomes $\frac{002}{1000} = .002$ by prefixing two ciphers,
$.2$ becomes $\frac{0002}{10000} = .0002$ by prefixing three ciphers:

in which the fraction is diminished ten times for every cipher prefixed.

ADDITION OF DECIMALS.

207. It must be remembered, that only units of the same kind can be added together. Therefore, in setting down decimal numbers for addition, figures expressing the same unit must be placed in the same column.

---

206. When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? What effect then has it on the value of the fraction? What do .2 become by prefixing a cipher? By prefixing two ciphers? By prefixing three? What do .07 become by prefixing a cipher? By prefixing two? By prefixing three? By prefixing four?

207. What parts of unity may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

The addition of decimals is then made in the same manner as that of whole numbers.

1. Find the sum of 37.04, 704.3, and .0376.

OPERATION.

Place the decimal points in the same column: this brings units of the same value in the same column: then add as in whole numbers: hence,

$$\begin{array}{r} 37.04 \\ 704.3 \\ .0376 \\ \hline 741.3776 \end{array}$$

RULE.—I. *Set down the numbers to be added so that figures of the same unit value shall stand in the same column.*

II. *Add as in simple numbers, and point off in the sum, from the right hand, as many places for decimals as are equal to the greatest number of places in any of the numbers added.*

PROOF.—The same as in simple numbers.

EXAMPLES.

1. Add 4.035, 763.196, 445.3741, and 91.3754 together.

2. Add 365.103113, .76012, 1.34976, .3549, and 61.11 together.

3. 67.407+97.004+4+.6+.06+.3

4. .0007+1.0436+.4+.05+.047

5. .0049+47.0426+37.0410+360.0039=444.0924.

6. What is the sum of 27, 14, 49, 126, 999, .469, and .2614?

7. Add 15, 100, 67, 1, 5, 33, .467, and 24.6 together.

8. What is the sum of 99, 99, 31, .25, 60.102, .29, and 100.347?

9. Add together .7509, .0074, 69.8408, and .6109.

10. Required the sum of twenty-nine and 3 tenths, four hundred and sixty-five, and two hundred and twenty-one thousandths.

11. Required the sum of two hundred dollars one dime three cents and 9 mills, four hundred and forty dollars nine mills, and one dollar one dime and one mill.

12. What is the sum of one-tenth, one hundredth, and one thousandth?

13. What is the sum of 4, and 6 ten-thousandths?

14. Required, in dollars and decimals, the sum of one dollar one dime one cent one mill, six dollars three mills, four dollars eight cents, nine dollars six mills, one hundred dollars six dimes, nine dimes one mill, and eight dollars six cents.

15. What is the sum of 4 dollars 6 cents, 9 dollars 3 mills, 14 dollars 3 dimes 9 cents 1 mill, 104 dollars 9 dimes 9 cents 9 mills, 999 dollars 9 dimes 1 mill, 4 mills, 6 mills, and 1 mill?

16. If you sell one piece of cloth for \$4,25, another for \$5,075, and another for \$7,0025, how much do you get for all?

17. What is the amount of \$151,7, \$70,602, \$4,06, and \$807,2659?

18. A man received at one time \$13,25; at another \$8,4; at another \$23,051; at another \$6; and at another \$0,75: how much did he receive in all?

19. Find the sum of twenty-five hundredths, three hundred and sixty-five thousandths, six tenths, and nine millionths.

20. What is the sum of twenty-three millions and ten, one thousand, four hundred thousandths, twenty-seven, nineteen millionths, seven and five tenths?

21. What is the sum of six millionths, four ten-thousandths, 19 hundred thousandths, sixteen hundredths, and four tenths?

22. If a piece of cloth cost four dollars and six mills, eight pounds of coffee twenty-six cents, and a piece of muslin three dollars seven dimes and twelve mills, what will be the cost of them all?

23. If a yoke of oxen cost one hundred dollars nine dimes and nine mills, a pair of horses two hundred and fifty dollars five dimes and fifteen mills, and a sleigh sixty-five dollars eleven dimes and thirty-nine mills, what will be their entire cost?

24. Find the sum of the following numbers: Sixty-nine thousand and sixty-nine thousandths, forty-seven hundred and forty-seven thousandths, eighty-five and eighty-five hundredths, six hundred and forty-nine and six hundred and forty-nine ten-thousandths?

SUBTRACTION OF DECIMALS.

208 Subtraction of Decimal Fractions is the operation of finding the difference between two decimal numbers.

1. From 3.275 to take .0879.

NOTE. — In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (Art. 205): hence,

OPERATION
3.2750
.0879
3.1871

RULE.—I. *Write the less number under the greater, so that figures of the same unit value shall fall in the same column.*

II. *Subtract as in simple numbers, and point off the decimal places in the remainder, as in addition.*

PROOF.—Same as in simple numbers.

EXAMPLES.

1. From 3295 take .0879.
2. From 291.10001 take 41.375.
3. From 10.000001 take .111111.
4. From 396 take 8 ten-thousandths.
5. From 1 take one thousandth.
6. From 6378 take one-tenth.
7. From 365.0075 take 3 millionths.
8. From 21.004 take 97 ten-thousandths.
9. From 260.4709 take 47 ten-millionths.
10. From 10.0302 take 19 millionths.
11. From 2.01 take 6 ten-thousandths.
12. From thirty-five thousands take thirty-five thousandths.
13. From 4262.0246 take 23.41653.
14. From 346.523120 take 219.691245943.
15. From 64.075 take .195326.
16. What is the difference between 107 and .0007?
17. What is the difference between 1.5 and .3785?
18. From 96.71 take 96.709.

---

208. What is subtraction of decimal fractions? How do you set down the numbers for subtraction? How do you then subtract? How many decimal places do you point off in the remainder?

MULTIPLICATION OF DECIMAL FRACTIONS.

209. *To multiply one decimal by another.*

1. Multiply 3.05 by 4.102.

ANALYSIS.—If we change both factors to vulgar fractions, the product of the numerator will be the same as that of the decimal numbers, and *the number of decimal places will be equal to the number of ciphers in the two denominators:* hence,

OPERATION.

$$\frac{305}{100} = 3.05$$
$$\frac{4102}{1000} = 4.102$$

```
      610
     305
   12 20
   12.51110
```

RULE.—*Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as there are decimal places in both factors; and if there be not so many in the product, supply the deficiency by prefixing ciphers.*

EXAMPLES.

1. Multiply 3.049 by .012.
2. Multiply 365.491 by .001.
3. Multiply 496.0135 by 1.496.
4. Multiply one and one millionth by one thousandth.
5. Multiply one hundred and forty-seven millionths by one millionth.
6. Multiply three hundred, and twenty-seven hundredths by 31.
7. Multiply 31.00467 by 10.03962.
8. What is the product of five-tenths by five-tenths?
9. What is the product of five-tenths by five-thousandths?
10. Multiply 596.04 by 0.00004.
11. Multiply 38049.079 by 0.00008.
12. What will 6.29 weeks' board come to at 2,75 dollars per week?
13. What will 61 pounds of sugar come to at $0.234 per pound?

---

209. After multiplying, how many decimal places will you point off in the product? When there are not so many in the product, what do you do? Give the rule for the multiplication of decimals.

14. If 12.836 dollars are paid for one barrel of flour, what will .354 barrels cost?

15. What are the contents of a board, .06 feet long and .06 wide?

16. Multiply 49000 by .0049.

17. Bought 1234 oranges for 4.6 cents apiece: how much did they cost?

18. What will 375.6 pounds of coffee cost at .125 dollars per pound?

19. If I buy 36.251 pounds of indigo at $0.029 per pound, what will it come to?

20. Multiply $89.3421001 by .0000028.

21. Multiply $341.45 by .007.

22. What are the contents of a lot which is .004 miles long and .004 miles wide?

23. Multiply .007853 by .035.

24. What is the product of $26.000375 multiplied by .00007?

CONTRACTIONS.

210. When a decimal number is to be multiplied by 10, 100, 1000, &c., the multiplication may be made by removing the decimal point as many places to the right hand as there are ciphers in the multiplier, and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers.

$$\text{Thus, } 6.79 \text{ multiplied by } \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{array}\right\} = \left\{\begin{array}{r} 67.9 \\ 679. \\ 6790. \\ 67900. \\ 679000. \end{array}\right.$$

$$\text{Also, } 370.036 \text{ multiplied by } \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{array}\right\} = \left\{\begin{array}{r} 3700.36 \\ 37003.6 \\ 370036. \\ 3700360. \\ 37003600. \end{array}\right.$$

---

210. How do you multiply a decimal number by 10, 100, 1000, &c.? If there are not as many decimal figures as there are ciphers in the multiplier, what do you do?

DIVISION OF DECIMAL FRACTIONS.

211. Division of Decimal Fractions is similar to that of simple numbers.

1. Let it be required to divide 1.38483 by 60.21.

ANALYSIS.—The dividend must be equal the product of the divisor and quotient, (Art. 61); and hence must contain as many decimal places as both of them; therefore,

There must be as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor: hence,

OPERATION.

```
60.21)1.38483(23
      1.2042
      ------
       18063
       18063
       -----
   Ans. .023
```

RULE.—*Divide as in simple numbers, and point off in the quotient, from the right hand, as many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.*

EXAMPLES.

1. Divide 2.3421 by 2.11.
2. Divide 12.82561 by 3.01.
3. Divide 33.66431 by 1.01.
4. Divide .010001 by .01.
5. Divide 8.2470 by .002.
6. Divide 94.0056 by .08.
7. What is the quotient of 37.57602, divided by 3; by .3, by .03; by .003; by .0003?
8. What is the quotient of 129 75896, divided by 8; by .08; by .008; by .0008; by .00008?
9. What is the quotient of 187.29900, divided by 9; by .9; by .09; by .009; by .0009; by .00009?
10. What is the quotient of 764.2043244, divided by 6; by .06; by .006; by .0006; by .00006; by .000006?

NOTE.—1. When there are more decimal places in the divisor than in the dividend, annex ciphers to the dividend and make the decimal places equal; *all the figures of the quotient will then be whole numbers.*

---

211. How does the number of decimal places in the dividend compare with that in the divisor and quotient? How do you determine the number of decimal places in the quotient? If the divisor contains four places and the dividend six, how many in the quotient? If the divisor contains three places and the dividend five, how many in the quotient? Give the rule for the division of decimals.

EXAMPLES.

1 Divide 4397.4 by 3.49.

OPERATION.

```
3.49)4397.40(1260
     349
     ---
      907
      698
      ----
      2094
      2094
      ----
```
Ans. 1260.

NOTE.—We annex one 0 to the dividend. Had it contained no decimal place we should have annexed two.

2. Divide 2194.02194 by .100001.
3. Divide 9811.0047 by .325947.
4. Divide .1 by .0001.
5. Divide 10 by .15.

6. Divide 6 by .6; by .06; by .006; by .2; by .3; by by .003; by .5; by .05; by .005.

NOTE.—2. When it is necessary to continue the division farther than the figures of the dividend will allow, we annex ciphers, and consider them as decimal places of the dividend.

When the division does not terminate, we annex the plus sign to show that it may be continued: thus .2 divided by .3=.666+.

EXAMPLES.

1. Divide 4.25 by 1.25.

OPERATION.

```
1.25)4.25(3.4
     3.75
     ----
      500
      500
      ---
```
Ans. 3.4.

ANALYSIS.—In this example we annex one 0, and then the decimal places in the dividend will exceed those in the divisor by 1.

2. Divide .2 by .6.
3. Divide 37.4 by 4.5.
4. Divide 586.4 by 375.
5. Divide 94.0369 by 81.032.

NOTE.—3. When any decimal number is to be divided by 10, 100, 1000, &c., the division is made by removing the decimal point as *many places to the left as there are 0's in the divisor;* and if there be not so many figures on the left of the decimal point the deficiency is supplied by prefixing ciphers.

$$27.69 \text{ divided by } \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array}\right\} = \left\{\begin{array}{l} 2.769 \\ .2769 \\ .02769 \\ .002769 \end{array}\right.$$

$$642.89 \text{ divided by } \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}64.289\\6.4289\\.64289\\.064289\\.0064289\end{array}\right.$$

QUESTIONS IN THE PRECEDING RULES.

1 If I divide .6 dollars among 94 men, how much will each receive?

2. I gave 28 dollars to 267 persons: how much apiece?

3. Divide 6.35 by .425.

4. What is the quotient of $36.2678 divided by 2.25?

5. Divide a dollar into 12 equal parts.

6. Divide .25 of 3.26 into .034 of 3.04 equal parts.

7. How many times will .35 of 35 be contained in .024 of 24?

8. At .75 dollars a bushel, how many bushels of rye can be bought for 141 dollars?

9. Bought 12 and 15 thousandths bushels of potatoes for 33 hundredths dollars a bushel, and paid in oats at 22 hundredths of a dollar a bushel: how many bushels of oats did it take?

10. Bought 53.1 yards of cloth for 42 dollars: how much was it a yard?

11. Divide 125 by .1045.

12. Divide one millionth by one billionth.

13. A merchant sold 4 parcels of cloth, the first contained 127 and 3 thousandths yards; the 2d, 6 and 3 tenths yards; the 3d, 4 and one hundredth yards; the 4th, 90 and one millionth yards: how many yards did he sell in all?

14. A merchant buys three chests of tea, the first contains 60 and one thousandth pounds; the second, 39 and one ten thousandth pounds; the third, 26 and one tenth pounds: how much did he buy in all?

NOTE.—1. If there are more decimal places in the divisor than in the dividend, what do you do? What will the figures of the quotient then be?

2. How do you continue the division after you have brought down all the figures of the dividend? What sign do you place after the quotient? What does it show?

3. How do you divide a decimal fraction by 10, 100, 1000, &c.?

15. What is the sum of $20 and three hundredths ; $4 and one-tenth, $6 and one thousandth, and $18 and one hundredth ?

16. A puts in trade $504.342 ; B puts in $350.1965 ; C puts in $100.11 ; D puts in $99.334 ; and E puts in $9001.32 : what is the whole amount put in ?

17. B has $936, and A has $1, 3 dimes and 1 mill : how much more money has B than A ?

18. A merchant buys 37.5 yards of cloth, at one dollar twenty-five cents per yard : how much does the whole come to ?

19. If 12 men had each $339 one dime 9 cents and 3 mills, what would be the total amount of their money ?

20. A farmer sells to a merchant 13.12 cords of wood at $4,25 per cord, and 13 bushels of wheat at $1,06 per bushel : he is to take in payment 13 yards of broadcloth at $4,07 per yard, and the remainder in cash : how much money did he receive ?

21. If one man can remove 5.91 cubic yards of earth in a day, how much could nineteen men remove ?

22. What is the cost of 8.3 yards of cloth at $5,47 per yard ?

23. If a man earns one dollar and one mill per day, how much will he earn in a year of 313 working days ?

24. What will be the cost of 375 thousandths of a cord of wood, at $2 per cord ?

25. A man leaves an estate of $1473.194 to be equally divided among 12 heirs : what is each one's portion ?

26. If flour is $9,25 a barrel, how many barrels can I buy for $1637,25 ?

27. Bought 26 yards of cloth at $4,37½ a yard, and paid for it in flour at $7,25 a barrel : how much flour will pay for the cloth ?

28. How much molasses at 22½ cents a gallon must be given for 46 bushels of oats at 45 cents a bushel ?

29. How many days work at $1,25 a day must be given for 6 cords of wood, worth $4,12½ a cord ?

30. What will 36.48 yards of cloth cost, if 14.25 yard cost $21.375 ?

31. If you can buy 13.25*lb.* of coffee for $2,50, how much can you buy for $325,50 ?

212. *To change a common to a decimal fraction.*

The value of a fraction is the quotient of the numerator, divided by the denominator (Art. 148).

1. Reduce $\frac{5}{8}$ to a decimal.

If we place a decimal point after the 5, and then write any number of 0's, after it, the value of the numerator will not be changed (Art. 205).

If, then, we divide by the denominator, the quotient will be the decimal number: hence,

OPERATION.

$$\begin{array}{r} 8)5.000 \\ \hline .625 \end{array}$$

RULE.—*Annex decimal ciphers to the numerator, and then divide by the denominator, pointing off as in division of decimals.*

EXAMPLES.

1. Reduce $\frac{635}{125}$ to its equivalent decimal.

We here use two ciphers, and therefore point off two decimal places in the quotient.

OPERATION.

$$\begin{array}{r} 125)635(5.08 \\ 625 \\ \hline 1000 \\ 1000 \end{array}$$

Reduce the following fractions to decimals:

1. Reduce $\frac{3}{7}$ to a decimal.
2. Reduce $\frac{15}{17}$ to a decimal.
3. Reduce $\frac{3}{35}$ to a decimal.
4. Reduce $\frac{1}{4}$ and $\frac{9}{1129}$.
5. Reduce $\frac{12}{480}$, $\frac{29}{39}$, and $\frac{3}{1000}$.
6. Reduce $\frac{1}{2}$ and $\frac{5}{1785}$.
7. Reduce $\frac{314957123}{210456891}$.
8. Reduce $\frac{8}{6}$, $\frac{1375}{8436}$, $\frac{3265}{4121}$.
9. Reduce $\frac{17}{20}$ to a decimal.
10. Reduce $\frac{3}{40}$ to a decimal.
11. Reduce $\frac{17}{125}$.
12. Reduce $\frac{7}{800}$.
13. Reduce $\frac{372}{1250}$.
14. Reduce $\frac{11}{1600}$.
15. Reduce $\frac{15}{1280}$.
16. Reduce $\frac{347}{2560}$.
17. Reduce $\frac{1}{10000}$.
18. Reduce $\frac{3476}{15625}$.

213. A decimal fraction may be changed to the form of a vulgar fraction by simply writing its denominator (Art. 202).

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212. How do you change a vulgar to a decimal fraction?
213. How do you change a decimal to the form of a vulgar fraction?

EXAMPLES.

1. What vulgar fraction is equal to .04 ?
2. What vulgar fraction is equal to 3.067 ?
3. What vulgar fraction is equal to 8.275 ?
4. What vulgar fraction is equal to .00049 ?

DENOMINATE DECIMALS

214. A denominate decimal is one in which the unit of the fraction is a denominate number. Thus, .5 of a pound, .6 of a shilling, .7 of a yard, &c., are denominate decimals, in which the units are 1 pound, 1 shilling, 1 yard.

CASE I.

215. *To change a denominate number to a denominate decimal.*

1. Change 9*d.* to the decimal of a £.

ANALYSIS.—The denominate unit of the fraction is 1£=240*d.* Then divide 9*d.* by 240: the quotient .0375 of a pound is the value of 9*d.* in the decimal of a £: hence,

OPERATION.

240*d.*=£1

240)9(.0375

Ans. £.0375

RULE.—*Reduce the unit of the required fraction to the unit of the given denominate number, and then divide the denominate number by the result, and the quotient will be the decimal.*

EXAMPLES.

1. Reduce 7 drams to the decimal of a *lb.* avoirdupois.
2. Reduce 26*d.* to the decimal of a £.
3. Reduce .056 poles to the decimal of an acre.
4. Reduce 14 minutes to the decimal of a day.
5. Reduce 21 pints to the decimal of a peck.
6. Reduce 3 hours to the decimal of a day.
7. Reduce 375678 feet to the decimal of a mile.
8. Reduce 36 yards to the decimal of a rod.
9. Reduce .5 quarts to the decimal of a barrel.
10. Reduce .7 of an ounce, avoirdupois, to the decimal of a hundred.

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214. What is a denominate decimal?

215. How do you change a denominate number to a denominate decimal?

CASE II.

216. *To find the value of a decimal in integers of a less denomination.*

1. Find the value of .890625 bushels.

	OPERATION.
ANALYSIS.—Multiplying the decimal by 4, (since 4 pecks make a bushel), we have 3.5625 pecks. Multiplying the new decimal by 8, (since 8 quarts make a peck), we have 4.5 quarts. Then, multiplying this last decimal by 2, (since 2 pints make a quart), we have 1 pint: hence,	.890625 4 3.562500 8 4.500000 2
Ans. 3*pk.* 4*qts.* 1*pt.*	1.000000

RULE.—I. *Multiply the decimal by that number which will reduce it to the next less denomination, pointing off as in multiplication of decimal fractions.*

II. *Multiply the decimal part of the product as before; and so continue to do until the decimal is reduced to the required denominations. The integers at the left form the answer*

EXAMPLES.

1. What is the value of 002084*lb.* Troy?
2. What is the value of .625 of a *cwt.*?
3. What is the value of .625 of a gallon?
4. What is the value of £.3375?
5. What is the value of .3375 of a ton?
6. What is the value of .05 of an acre?
7. What is the value of .875 pipes of wine?
8. What is the value of .125 hogsheads of beer?
9. What is the value of .375 of a year of 365 days?
10. What is the value of .085 of a £?
11. What is the value of .86 of a *cwt.*?
12. From .82 of a day take .32 of an hour.
13. What is the value of 1.089 miles?
14. What is the value of .09375 of a pound, avoirdupois?
15. What is the value of .28493 of a year of 365 days?
16. What is the value of £1.046?
17. What is the value of £1.88?

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216. How do you find the value of a decimal in integers of a less denomination?

CASE III.

217. *To reduce a compound denominate number to a decimal or mixed number.*

1. Reduce £1 4*s.* $9\frac{3}{4}d.$ to the decimal of a £.

ANALYSIS.—Reducing the $\frac{3}{4}d.$ to a decimal (Art. 215), and annexing the result to the 9*d.*, we have 9.75*d.* Dividing 9.75*d.* by 12, (since 12 pence = 1*s.*), and annexing the quotient to the 4*s.* we have 4.8125*s.* Then, dividing by 20 (since 20*s* = £1,) and annexing the quotient to the £1, we have £1.240625:

OPERATION.

$\frac{3}{4}d. = .75d.$

$9\frac{3}{4}d. = .9.75d.$

$12)9.75d$

$20)\overline{4.8125s.}$

Ans. £1 4*s.* $9\frac{3}{4}d.$ = 1.240625£.

RULE.—*Divide the lowest denomination by as many units as make a unit of the next higher, and annex the quotient as a decimal to that higher: then divide as before, and so continue to do, until the decimal is reduced to the required denomination.*

EXAMPLES.

1. Reduce 4*wk.* 6*da.* 5*hr.* 30*m.* 45*s.* to the denomination of a week.
2. Reduce 2*lb.* 5*oz.* 12*pwt.* 16*gr.*, to the denomination of a pound.
3. Reduce 3 feet 9 inches to the denomination of yards.
4. Reduce 1*lb.* 12*dr.*, avoirdupois, to the denomination of pounds.
5. Reduce 5 leagues 2 furlongs to the denomination of leagues.
6. Reduce 4*bu.* 3*pk.* 4*qt.* 1*pt.* to the denomination of bushels.
7. Reduce 5*oz.* 13*pwt.* 12*gr.* to the decimal of a pound.
8. Reduce 15*cwt.* 3*qr.* $2\frac{1}{2}lb.$ to the decimal of a ton.
9. Reduce 5*A.* 3*R.* 21*sq. rd.* to the denomination of acres.
10. Reduce 11 pounds to the decimal of a ton.
11. Reduce 3*da.* $12\frac{3}{5}sec.$ to the decimal of a week.
12. Reduce 14*bu.* $3\frac{3}{5}qt.$ to the decimal of a chaldron.
13. Reduce 7*m.* 7*fur.* 1*r.* to the denomination of miles.

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217. How do you reduce a compound denominate number to a decimal?

ANALYSIS.

218. An analysis of a proposition is an examination of its separate parts, and their connections with each other.

The solution of a question, by analysis, consists in an examination of its elements and of the relations which exist between these elements. We determine the elements and the relations which exist between them, in each case, by examining the *nature* of the question.

In analyzing, we reason from a *given number* to its *unit*, and then from this unit to the *required* number.

EXAMPLES.

1. If 9 bushels of wheat cost 18 dollars, what will 27 bushels cost?

ANALYSIS.—One bushel of wheat will cost one ninth as much as 9 bushels. Since 9 bushels cost 18 dollars, 1 bushel will cost $\frac{1}{9}$ of 18 dollars, or 2 dollars; 27 bushels will cost 27 times as much as 1 bushel: that is, 27 times $\frac{1}{9}$ of 18 dollars, or 54 dollars: therefore, if 9 bushels of wheat cost 18 dollars, 27 bushels will cost 54 dollars.

OPERATION.

$$\frac{\$\overset{2}{\cancel{18}}}{1}\times\frac{1}{\cancel{9}}\times\frac{27}{1}=\$54; \quad \text{Or,} \quad \begin{array}{r|l} \cancel{9} & 18 \\ & \overset{3}{\cancel{27}} \\ \hline & 54 \ \textit{Ans.} \end{array}$$

NOTE.—1. We *indicate* the operations to be performed, and then cancel the equal factors (Art. 141).

219. Although the currency of the United States is expressed in dollars cents and mills, still in most of the States the dollar (always valued at 100 cents), is reckoned in shillings and pence; thus,

In the New England States, in Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, Mississippi and Texas, the dollar is reckoned at 6 shillings: In New York, Ohio and Michigan, at 8 shillings: In New Jersey, Pennsylvania, Delaware and Maryland, at 7*s*. 6*d*: In South Carolina, and Georgia, at 4*s*. 8*d*.: In Canada and Nova Scotia, at 5 shillings.

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218. What is an analysis? In what does the solution of a question by analysis consist? How do we determine the elements and their relations? How do we reason in analyzing?

NOTE.—In many of the States the retail price of articles is given in shillings and pence, and the result, or cost, required in dollars and cents.

2. What will 12 yards of cloth cost, at 5 shillings a yard, New York currency?

ANALYSIS.—Since 1 yard cost 5 shillings 12 yards will cost 12 times 5 shillings, or 60 shillings: and as 8 shillings make 1 dollar, New York currency, there will be as many dollars as 8 is contained times in $60 = \$7\frac{1}{2}$.

OPERATION.

$$\overset{s.}{5} \times 12 \div 8 = \$7,50; \qquad \text{Or,} \qquad \begin{array}{r|l} & \cancel{12}^{\,3} \\ {}^{2}\cancel{8} & 5 \\ \hline 2 & 15 = \frac{15}{2} = \$7,50. \\ \hline \multicolumn{2}{c}{\$7,50.} \end{array}$$

NOTE.—The fractional part of a dollar may always be reduced to cents and mills by annexing two or three ciphers to the numerator and dividing by the denominator; or, which is more convenient in practice, annex the ciphers to the dividend and continue the division.

3. What will be the cost of 56 bushels of oats at 3*s*. 3*d*. a bushel, New York currency?

OPERATION.

$$\begin{array}{r|l} & \cancel{56}^{\,7} \\ 4 & 13 \\ \cancel{8} & \\ \hline 4 & 91 \\ \hline \multicolumn{2}{c}{22,75.} \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} 4 & \cancel{56}^{\,7} \\ \cancel{12}\,\cancel{96} & \cancel{39}^{\,13} \\ \hline 4 & 91 \\ \hline \multicolumn{2}{c}{\$22,75\ \textit{Ans.}} \end{array}$$

NOTE.—When the pence is an aliquot part of a shilling the price may be reduced to an improper fraction, which will be the multiplier: thus, 3*s*. 3*d*.$=3\frac{1}{4}$*s*.$=\frac{13}{4}$*s*. Or: the shillings and pence may be reduced to pence: thus, 3*s*. 3*d*.=39*d*., in which case the product will be pence, and must be divided by 96, the number of pence in 1 dollar: hence,

220. *To find the cost of articles in dollars and cents.*

---

219. In what is the currency of the States expressed? In what is the currency of the States often reckoned?

220. How do you find the cost of a commodity

Multiply the commodity by the price and divide the product by the value of a dollar reduced to the same denominational unit.

4. What will 18 yards of satinet cost at 3*s*. 9*d*. a yard Pennsylvania currency?

OPERATION.

$$\begin{array}{r|l} & \overset{9}{\cancel{18}} \\ \overset{2}{\cancel{4}} & \cancel{15} \\ \cancel{15} & \cancel{2} \\ \hline & \$9. \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} \overset{\cancel{2}}{\cancel{90}} & \overset{9}{\cancel{18}} \\ & \cancel{45} \\ \hline & \$9\ \textit{Ans.} \end{array}$$

NOTE.—The above rule will apply to the currency in any of the States. In the last example the multiplier is 3*s*. 9*d*.$=3\frac{3}{4}s$ $=\frac{15}{4}s.$ or 45*d*. The divisor is 7*s*. 6*d*.$=7\frac{1}{2}s.=\frac{15}{2}s.=90d.$

5. What will $7\frac{1}{2}lb.$ of tea cost at 6*s*. 8*d*. a pound, New England currency?

OPERATION.

$$\begin{array}{r|l} \cancel{2} & \cancel{15}\ \overset{5}{\cancel{10}}\ 5 \\ 3\ \cancel{3} & \cancel{20} \\ \cancel{6} & \\ \hline 3 & 25=\frac{25}{3}=\$8.333+ \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} \overset{3}{\cancel{6}}\ \cancel{2} & \cancel{15}\ \overset{5}{\cancel{10}}\ 5 \\ \cancel{72} & \cancel{80} \\ \hline 3 & 25 \\ \hline & \$8.333+\textit{Ans.} \end{array}$$

6. What will be the cost of 120*yds*. of cotton cloth at 1*s*. 5*d*. a yard, Georgia currency?

7. What will be the cost in New York currency?

8. What will be the cost in New England currency?

9. What will be the cost of 75 bushels of potatoes at 3*s* 6*d*., New York currency?

10. What will it cost to build 148 feet of wall at 1*s*. 8*d*. per foot, N. Y. currency?

11. What will a load of wheat, containing $46\frac{1}{2}$ bushels, come to at 10*s*. 8*d*. a bushel, N. Y. currency?

12. What will 7 yards of Irish linen cost at 3*s*. 4*d*. a yard, Penn. currency?

13. How many pounds of butter at 1*s*. 4*d*. a pound must be given for 12 gallons of molasses at 2*s*. 8*d*. a gallon?

OPERATION.

$$\begin{array}{c|l} \cancel{3} & 12 \\ & \cancel{8}\ 2 \\ \cancel{4} & \cancel{3} \\ \hline & 24lb. \end{array} \qquad \text{Or,} \qquad \begin{array}{c|l} & 12 \\ \cancel{16} & \cancel{32}\ 2 \\ \hline & 24lb. \end{array}$$

NOTE.—The same rule applies in the last example as in the preceding ones, except that the divisor is the price of the article received in payment, reduced to the same unit as the price of the article bought.

14. What will be the cost of 12*cwt.* of sugar at 9*d.* per *lb.* N. Y. currency?

NOTE.— Reduce the *cwts.* to *lbs.* by multiplying by 4 and then by 25. Then multiply by the price-per pound, and then divide by the value of a dollar in the required currency, reduced to the same denomination as the price.

OPERATION.

$$\begin{array}{r|l} 2 & \cancel{12} \\ \cancel{8} & \cancel{4} \\ \cancel{96} & 25 \\ & 9 \\ \hline 2 & 225 \end{array}$$

Ans. \$112,50

15. What will be the cost of 9 hogsheads of molasses at 1*s.* 3*d.* per quart, N. E. currency?

16. How many days work at 7*s.* 6*d.* a day must be given for 12 bushels of apples at 3*s.*9*d.* a bushel?

17. Farmer A exchanged 35 bushels of barley, worth 6*s.* 4*d.*, with farmer B for rye worth 7 shillings a bushel: how many bushels of rye did farmer A receive?

18. Bought the following bill of goods of Mr. Merchant: what did the whole amount to, N. Y. currency?

12½	yards of	cambric	at 1*s.* 4*d.*	per yard.
8	"	ribbon	" 2*s.* 6*d.*	"
21	"	calico	" 1*s.* 3*d.*	"
6	"	alpaca	" 5*s.* 6*d.*	"
4	gallons	molasses	" 3*s.* 5*d.*	per gallon.
2½	pounds	tea	" 6*s.* 6*d.*	per pound.
30	"	sugar	" 9*d.*	" "

19. If $\frac{5}{8}$ of a yard of cloth cost \$3,20, what will $\frac{15}{16}$ of a yard cost?

ANALYSIS.—Since 5 eighths of a yard of cloth costs \$3,20, 1 eighth of a yard will cost $\frac{1}{5}$ of \$3,20; and 1 yard, or 8 eighths, will cost 8 times as much, or $\frac{8}{5}$ of \$3,20; $\frac{15}{16}$ of a yard will cost $\frac{15}{16}$ as much as 1 yard, or $\frac{15}{16}$ of $\frac{8}{5}$ of \$3,20 = \$4.80.

OPERATION.

$$\overset{160}{\cancel{\$320}} \times \frac{1}{\cancel{5}} \times \frac{\cancel{8}}{1} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{16}}} = \$4,80. \quad \text{Or,} \quad \begin{array}{r|l} & \overset{1,60}{\cancel{\$3,20}} \\ \cancel{5} & \cancel{8} \\ 2\ \cancel{16} & \cancel{15}\ 3 \\ \hline & \$4,80. \end{array}$$

20. If $3\frac{3}{4}$ pounds of tea cost $3\frac{1}{3}$ dollars, what will 9 pounds cost?

NOTE.—Reduce the mixed numbers to improper fractions, and then apply the same mode of reasoning as in the preceding example.

21. What will $8\frac{1}{2}$ cords of wood cost, if $2\frac{3}{8}$ cords cost $7\frac{1}{8}$ dollars?

22. If 6 men can build a boat in 120 days, how long will it take 24 men to build it?

ANALYSIS.—Since 6 men can build a boat in 120 days, it will take 1 man 6 times 120 days, or 720 days, and 24 men can build it in $\frac{1}{24}$ of the time that 1 man will require to build it, or $\frac{1}{24}$ of 6 times 120, which is 30.

OPERATION.

$$120 \times 6 \div 24 = 30 \text{ days.} \quad \text{Or,} \quad \begin{array}{r|l} & \overset{30}{\cancel{120}} \\ 4\ \cancel{24} & \cancel{6} \\ \hline Ans. & 30\ days. \end{array}$$

23. If 7 men can dig a ditch in 21 days, how many men will be required to dig it in 3 days?

24. In what time will 12 horses consume a bin of oats, that will last 21 horses $6\frac{2}{7}$ weeks?

25. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of 7 dollars for 5 yards, and sold them at the rate of 11 dollars for 7 yards; and gained 200 dollars by the bargain: how many bales were there?

ANALYSIS.—Since he paid 7 dollars for 5 yards, for 1 yard he paid $\frac{1}{5}$ of \$7 or $\frac{7}{5}$ of 1 dollar; and since he received 11 dollars for 7 yards, for 1 yard he received $\frac{1}{7}$ of 11 dollars or $\frac{11}{7}$ of 1 dollar. He gained on 1 yard the difference between $\frac{7}{5}$ and $\frac{11}{7} = \frac{6}{35}$ of a dollar. Since his whole gain was 200 dollars, he had as many yards as the gain on one yard is contained times in his whole gain, or as $\frac{6}{35}$ is contained times in 200. And there were as many bales as $129\frac{17}{27}$, (the number of yards in one bale), is contained times in the whole number of yards $\frac{7000}{6}$; which gives 9 bales.

OPERATION.

$129\frac{17}{27}=\frac{3500}{27}$, number of yards in a bale :

$200\div\frac{6}{35}=\frac{7000}{6}$, whole number of yards :

$\frac{7000}{6}\div\frac{3500}{27}=9$ bales.

$\cancel{3}$ $\cancel{6}$	$\cancel{200}$ $\cancel{2}$
$\cancel{100}$	$\cancel{35}$ 9
$\cancel{3500}$	$\cancel{27}$
Ans.	9 *bales.*

26. Suppose a number of bales of cloth, each containing $133\frac{1}{3}$ yards, to be bought at the rate of 12 yards for 11 dollars, and sold at the rate of 8 yards for 7 dollars, and the loss in trade to be \$100 : how many bales are there?

27. If a piece of cloth 9 feet long and 3 feet wide, contain 3 square yards ; how long must be a piece of cloth that is $2\frac{2}{5}$ feet wide be, to contain the same number of yards?

28. A can mow an acre of grass in 4 hours, B in 6 hours, and C in 8 hours. How many days, working 9 hours a day, would they require to mow 39 acres?

ANALYSIS.—Since A can mow an acre in 4 hours, B in 6 hours, and C in 8 hours, A can mow $\frac{1}{4}$ of an acre, B $\frac{1}{6}$ of an acre, and C $\frac{1}{8}$ of an acre in 1 hour. Together they can mow $\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{13}{24}$ of an acre in 1 hour. And since they can mow 13 twenty-fourths of an acre in 1 hour, they can mow 1 twenty-fourth of an acre in $\frac{1}{13}$ of 1 hour; and 1 acre, or $\frac{24}{24}$, in 24 times $\frac{1}{13}=\frac{24}{13}$ of 1 hour: and to mow 39 acres, they will require 39 times $\frac{24}{13}=\frac{936}{13}$ hours, which reduced to days of 9 hours each, gives 8 days.

OPERATION.

$\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{13}{24}$ hours.

$$\frac{\overset{8}{\cancel{24}}}{\cancel{13}}\times\frac{\overset{3}{\cancel{39}}}{1}\times\frac{1}{\underset{3}{\cancel{9}}}=8 \text{ days. Or}$$

$\cancel{13}$	$\cancel{24}$ 8
$\cancel{3}$ $\cancel{9}$	$\cancel{39}$ $\cancel{13}$
Ans.	8 days.

29. A can do a piece of work in 4 days, and B can do the same in 6 days ; in what time can they both do the work if they labor together?

30. If 6 men can do a piece of work in 10 days, how long will it take 5 men to do it?

ANALYSIS.—If 6 men can do a piece of work in 10 days, 1 man will require 6 times as long, or 60 days to do the same work. Five men will require but one-fifth as long as one man, or $60\div5=12$ days.

OPERATION.

$10 \times 6 \div 5 = 12$ days.

$$\begin{array}{r|l} & \cancel{10}\ ^{2} \\ & 6 \\ \cancel{5} & \\ \hline \textit{Ans.} & 12 \text{ days.} \end{array}$$

31. Three men together can perform a piece of work in 9 days. A alone can do it in 18 days, B in 27 days; in what time can C do it alone?

32. A and B can build a wall on one side of a square piece of ground in 3 days; A and C in 4 days; B and C in 6 days: what time will they require, working together, to complete the wall *enclosing* the square?

33. Three men hire a pasture, for which they pay 66 dollars. The first puts in 2 horses 3 weeks; the second 6 horses for $2\frac{1}{2}$ weeks; the third 9 horses for $1\frac{1}{3}$ weeks: how much ought each to pay?

ANALYSIS.—The pasturage of 2 horses for 3 weeks, would be the same as the pasturage of 1 horse 2 times 3 weeks, or 6 weeks; that of six horses $2\frac{1}{2}$ weeks, the same as for 1 horse 6 times $2\frac{1}{2}$ weeks, or 15 weeks; and that of 9 horses $1\frac{1}{3}$ weeks, the same as 1 horse for 9 times $1\frac{1}{3}$ weeks, or 12 weeks. The three persons had an equivalent for the pasturage of 1 horse for $6+15+12=33$ weeks; therefore, the first must pay $\frac{6}{33}$, the second $\frac{15}{33}$, and the third $\frac{12}{33}$ of 66 dollars.

OPERATION.

$3 \times 2 = 6$;	then	$\$66 \times \frac{6}{33} = \$12$.	1st.
$2\frac{1}{2} \times 6 = 15$;	"	$\$66 \times \frac{15}{33} = \$30$.	2d.
$1\frac{1}{3} \times 9 = 12$;	"	$\$66 \times \frac{12}{33} = \$24$.	3d.

34. Two persons, A and B. enter into partnership, and gain \$175. A puts in 75 dollars for 4 months, and B puts in 100 dollars for 6 months: what is each one's share of the gain?

35. Three men engage to build a house for 580 dollars. The first one employed 4 hands, the second 5 hands, and the third 7 hands. The first man's hands worked three times as many days as the third, and the second man's hands twice as many days as the third man's hands: how much must each receive?

36. If 8 students spend \$192 in 6 months, how much will 12 students spend in 20 months?

ANALYSIS.—Since 8 students spend \$192, one student will spend $\frac{1}{8}$ of \$192, in 6 months; in 1 month 1 student will spend $\frac{1}{6}$ of $\frac{1}{8}$ of \$192 = \$4. Twelve students will spend, in 1 month, 12 times as much as 1 student, and in 20 months they will spend 20 times as much as in 1 month.

OPERATION.

$$\frac{\overset{24}{\$\cancel{192}}}{1}\times\frac{1}{\cancel{8}}\times\frac{1}{\cancel{6}}\times\frac{\overset{2}{\cancel{12}}}{1}\times\frac{20}{1}=\$960.$$

$$\begin{array}{r|l} & \cancel{192}\ \ 48 \\ \cancel{4}\ \ \cancel{8} & \\ \cancel{6} & \\ & \cancel{12}\ \ \cancel{2} \\ & 20 \\ \hline & \$960.\ \textit{Ans.} \end{array}$$

37. If 6 men can build a wall 80 feet long, 6 feet wide, and 4 feet high, in 15 days, in what time can 18 men build one 240 feet long, 8 feet wide, and 6 feet high?

ANALYSIS.—Since it takes 6 men 15 days to build a wall, it will take 1 man 6 times 15 days, or 90 days, to build the same wall. To build a wall 1 foot long, will require $\frac{1}{80}$ as long as to build one 80 feet long; to build one 1 foot wide, $\frac{1}{4}$ as long as to build one 4 feet wide; and to build one 1 foot high, $\frac{1}{6}$ as long as to build one 6 feet high. 18 men can build the same wall in $\frac{1}{18}$ of the time that one man can build it: but to build one 240 feet long, will take them 240 times as long as to build one 1 foot in length; to build one 8 feet wide, 8 times as long as to build one 1 foot wide, and to build one 6 feet high, 6 times as long as to build one 1 foot high.

OPERATION.

$$\frac{15\times\cancel{6}}{1}\times\frac{1}{\cancel{80}}\times\frac{1}{\cancel{4}}\times\frac{1}{\cancel{6}}\times\frac{1}{\underset{\cancel{3}}{\cancel{18}}}\times\frac{\overset{\cancel{3}}{\cancel{240}}}{1}\times\frac{\overset{2}{\cancel{8}}}{\cancel{1}}\times\frac{\cancel{6}}{1}=30.$$

$$\begin{array}{r|l} & 15 \\ \cancel{80} & \cancel{6} \\ \cancel{4} & \cancel{240}\ \ \cancel{3} \\ \cancel{6} & \cancel{8}\ \ 2 \\ \cancel{3}\ \ \cancel{18} & \cancel{6} \\ \hline \textit{Ans.} & 30\ \text{days.} \end{array}$$

38. If 96*lbs.* of bread be sufficient to serve 5 men 12 days, how many days will 57*lb.* serve 19 men?

39. If a man travel 220 miles in 10 days, travelling 12 hours a day, in how many days will he travel 880 miles, travelling 16 hours a day?

40. If a family of 12 persons consume a certain quantity of provisions in 6 days, how long will the same provisions last a family of 8 persons?

41. If 9 men pay \$135 for 5 weeks' board, how much must 8 men pay for 4 weeks' board?

42. If 10 bushels of wheat are equal to 40 bushels of corn, and 28 bushels of corn to 56 pounds of butter, and 39 pounds of butter to 1 cord of wood; how much wheat is 12 cords of wood worth?

ANALYSIS.—Since 10 bushels of wheat are worth 40 bushels of corn, 1 bushel of corn is worth $\frac{1}{40}$ of 10 bushels of wheat, or $\frac{1}{4}$ of a bushel; 28 bushels are worth 28 times $\frac{1}{4}$ of a bushel of wheat, or 7 bushels: since 28 bushels of corn, or 7 bushels of wheat are worth 56 pounds of butter, 1 pound of butter is worth $\frac{1}{56}$ of $7=\frac{1}{8}$ of a bushel of wheat, and 39 pounds are worth 39 times as much as 1 pound, or $39\times\frac{1}{8}=\frac{39}{8}$ bushels of wheat; and since 39 pounds of butter, or $\frac{39}{8}$ bushels of wheat are worth 1 cord of wood, 12 cords are worth 12 times as much, or $12\times\frac{39}{8}=58\frac{1}{2}$ bushels.

OPERATION.

$$\frac{\cancel{10}}{1}\times\frac{1}{\underset{4}{\cancel{40}}}\times\frac{\cancel{28}}{1}\times\frac{1}{\underset{2}{\cancel{56}}}\times\frac{\overset{3}{39}}{1}\times\frac{\cancel{12}}{1}=58\tfrac{1}{2}\textit{bush.}$$

$$\begin{array}{r|l} \overset{4}{\cancel{40}} & \cancel{10} \\ 2\ \cancel{56} & \cancel{28} \\ & 39 \\ & \cancel{12}\ 3 \\ \hline 2 & 117=58\tfrac{1}{2}\textit{bush.} \end{array}$$

NOTE.—Always commence analyzing from the term which is of the same name or kind as the required answer.

43. If 35 women can do as much work as 20 boys, and 16 boys can do as much as 7 men: how many women can do the work of 18 men?

44. If 36 shillings in New York are equal to 27 shillings in Massachusetts, and 24 shillings in Massachusetts are equal to 30 shillings in Pennsylvania, and 45 shillings in Pennsylvania are equal to 28 shillings in Georgia; how many shillings in Georgia are equal to 72 shillings in New York?

PROMISCUOUS EXAMPLES IN ANALYSIS.

1. How many sheep at 4 dollars a head must I give for 6 cows, worth 12 dollars apiece?

2. If 7 yards of cloth cost \$49, what will 16 yards cost?

3. If 36 men can build a house in 16 days, how long will it take 12 men to build it?

4. If 3 pounds of butter cost $7\frac{1}{2}$ shillings, what will 12 pounds cost?

5. If $5\frac{1}{3}$ bushels of potatoes cost \$$2\frac{4}{5}$, how much will $12\frac{1}{2}$ bushels cost?

6. How many barrels of apples, worth 12 shillings a barrel, will pay for 16 yards of cloth, worth 9*s.* 6*d.* a yard?

7. If $31\frac{1}{2}$ gallons of molasses are worth \$$9\frac{3}{5}$, what are $5\frac{1}{4}$ gallons worth?

8. What is the value of $24\frac{3}{4}$ bushels of corn, at 5*s.* 7*d.* a bushel, New York currency?

9. How much rye, at 8*s.* 3*d.* per bushel, must be given for 40 gallons of whisky, worth 2*s.* 9*d.* a gallon?

10. If it take 44 yards of carpeting, that is $1\frac{1}{4}$ yards wide, to cover a floor, how many yards of $\frac{7}{8}$ yards wide, will it take to cover the same floor?

11. If a piece of wall paper, 14 yards long and $1\frac{1}{2}$ feet wide, will cover a certain piece of wall, how long must another piece be, that is 2 feet wide, to cover the same wall?

12. If 5 men spend \$200 in 160 days, how long will \$300 last 12 men at the same rate?

13. If 1 acre of land cost $\frac{1}{6}$ of $\frac{3}{7}$ of $\frac{4}{5}$ of \$50, what will $3\frac{1}{2}$ acres cost?

14. Three carpenters can finish a house in 2 months; two of them can do it in $2\frac{1}{2}$ months: how long will it take the third to do it alone?

15. Three persons bought 2 barrels of flour for 15 dollars The first one ate from them 2 months, the second 3 months and the third 7 months: how much should each pay?

16. What quantity of beer will serve 4 persons $18\frac{3}{4}$ days if 6 persons drink $7\frac{1}{5}$ gallons in 4 days?

17. If 9 persons use $1\frac{3}{8}$ pounds of tea in a month, how much will 10 persons use in a year?

18. If $\frac{1}{2}$ of $\frac{3}{4}$ of a gallon of wine cost $\frac{5}{8}$ of a dollar, what will $5\frac{1}{2}$ gallons cost?

19. How many yards of carpeting, $1\frac{3}{4}$ yards wide, will it take to cover a floor that is $4\frac{2}{3}$ yards wide and 6 and three-fifths yards long?

20. Three persons bought a hogshead of sugar containing 413 pounds. The first paid \$$2\frac{1}{2}$ as often as the second paid \$$3\frac{1}{3}$, and as often as the third paid \$4: what was each one's share of the sugar?

21. A, with the assistance of B, can build a wall 2 feet wide, 3 feet high, and 30 feet long, in 4 days; but with the assistance of C, they can do it in $3\frac{1}{2}$ days: in how many days can C do it alone?

22. If two persons engage in a business, where one advances \$875, and the other \$625, and they gain \$300, what is each one's share?

23. A person purchased $\frac{4}{7}$ of a vessel, and divided it into 5 equal shares, and sold each of those shares for \$1200: what was the value of the whole vessel?

24. How many yards of paper, $\frac{3}{4}$ of a yard wide, will be sufficient to paper a room 10 yards square and 3 yards high?

25. What will be the cost of 45*lbs.* of coffee, New Jersey currency, if 9*lbs.* cost 27 shillings?

26. What will be the cost of 3 barrels of sugar, each weighing 2*cwt.* at 10*d.* per pound, Illinois currency?

27. If 12 men reap 80 acres in 6 days, in how many days will 25 men reap 200 acres?

28. If 4 men are paid 24 dollars for 3 days' labor, how many men may be employed 16 days for \$96?

29. If \$25 will supply a family with flour at \$7,50 a barrel for $2\frac{2}{3}$ months, how long would \$45 last the same family when flour is worth \$6,75 per barrel?

30. A wall to be built to the height of 27 feet, was raised to the height of 9 feet by 12 men in 6 days: how many men must be employed to finish the wall in 4 days at the same rate of working?

31. A, B and C, sent a drove of hogs to market, of which A owned 105, B 75, and C 120. On the way 60 died: how many must each lose?

32. Three men, A, B and C, agree to do a piece of work, for which they are to receive \$315. A works 8 days, $10\frac{1}{2}$ hours a day; B $9\frac{3}{4}$ days, 8 hours a day; and C, 4 days, 12 hours a day: what is each one's share?

33. If 10 barrels of apples will pay for 5 cords of wood, and 12 cords of wood for 4 tons of hay, how many barrels of apples will pay for 9 tons of hay?

34. Out of a cistern that is $\frac{2}{3}$ full is drawn 140 gallons, when it is found to be $\frac{4}{7}$ full: how much does it hold?

35. If .7 of a gallon of wine cost \$2,25, what will .25 of a gallon cost?

36. If it take 5.1 yards of cloth, 1.25 yards wide, to make a gentleman's cloak, how much surge, $\frac{5}{8}$ yards wide, will be required to line it?

37 A and B have the same income. A saves $\frac{1}{8}$ of his annually; but B, by spending \$200 a year more than A, at the end of 5 years finds himself \$160 in debt: what is their income?

38. A father gave his younger son \$420, which was $\frac{3}{4}$ of what he gave to his elder son; and 3 times the elder son's portion was $\frac{1}{3}$ the value of the father's estate: what was the value of the estate?

39. Divide \$176,40 among 3 persons, so that the first shall have twice as much as the second, and the third three times as much as the first: what is each one's share?

40. A gentleman having a purse of money, gave $\frac{1}{2}$ of it for a span of horses; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder for a carriage; when he found that he had but \$100 left: how much was in his purse before any was taken out?

41. A merchant tailor bought a number of pieces of cloth, each containing $25\frac{5}{19}$ yards, at the rate of 3 yards for 4 dollars. and sold them at the rate of 5 yards for 13 dollars, and gained by the operation 96 dollars: how many pieces did he buy?

RATIO AND PROPORTION.

221. Two numbers having the same unit, may be compared in two ways:

1*st.* By considering *how much* one is greater or less than the other, which is shown by their difference; and,

2*d.* By considering *how many times* one is contained in the other, which is shown by their quotient.

In comparing two numbers, one with the other, by means of their difference, the less is always taken from the greater.

In comparing two numbers, one with the other, by means of their quotient, one of them must be regarded as a *standard* which *measures* the other, and the quotient which arises by dividing by the standard, is called the *ratio.*

222. Every ratio is derived from two terms: the first is called the *antecedent*, and the second the *consequent;* and the two, taken together, are called a *couplet.* The *antecedent* will be regarded as the *standard.*

If the numbers 3 and 12 be compared by their difference, the result of the comparison will be 9; for, 12 exceeds 3 by 9. If they are compared by means of their quotient, the result will be 4; for, 3 is contained in 12, 4 times: that is, 3 *measuring* 12, gives 4.

223. The ratio of one number to another is expressed in two ways:

1*st.* By a colon; thus, 3 : 12; and is read, 3 is to 12; or, 3 measuring 12.

2*d.* In a fractional form, as $\frac{12}{3}$; or, 3 measuring 12.

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221. In how many ways may two numbers, having the same unit, be compared with each other? If you compare by their difference, how do you find it? If you compare by the quotient, how do you regard one of the numbers? What is the ratio?

222. From how many terms is a ratio derived? What is the first term called? What is the second called? Which is the standard?

223 How may the ratio of two numbers be expressed? How read?

224. If two couplets have the same ratio, their terms are said to be proportional : the couplets

3 : 12 and 1 : 4

have the same ratio 4; hence, the terms are proportional, and are written,

3 : 12 : : 1 : 4

by simply placing a double colon between the couplets. The terms are read

3 is to 12 as 1 is to 4,

and taken together, they are called a *proportion* : hence,

*A proportion is a comparison of the terms of two equal ratios.**

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224. If two couplets have the same ratio, what is said of the terms? How are they written? How read? What is a proportion?

* Some authors, of high authority, make the consequent the standard and divide the antecedent by it to determine the ratio of the couplet.

The ratio 3 : 12 is the same as that of 1 : 4 by both methods; for, if the antecedent be made the standard, the ratio is 4; if the consequent be made the standard, the ratio is one-fourth. The question is, which method should be adopted?

The unit 1 is the number from which all other numbers are *derived*, and by which they are *measured*.

The question is, how do we most readily apprehend and express the relation between 1 and 4? Ask a child, and he will answer, "the difference is 3." But when you ask him, "how many 1's are there in 4?" he will answer, "4," using 1 as the standard.

Thus, we begin to teach by using the standard 1: that is, by dividing 4 by 1.

Now, the relation between 3 and 12 is the same as that between 1 and 4; if then, we divide 4 by 1, we must also divide 12 by 3. Do we, indeed, clearly apprehend the ratio of 3 to 12, until we have referred to 1 as a standard? Is the mind satisfied until it has clearly perceived that the ratio of 3 to 12 is the same as that of 1 to 4?

In the Rule of Three we always look for the *result* in the 4th term. Now, if we wish to find the ratio of 3 to 12, by referring to 1 as a standard, we have

3 : 12 : : 1 : ratio,

which brings the result in the right place.

But if we define ratio to be the antecedent divided by the consequent, we should have

3 : 12 : : ratio : 1,

which would bring the ratio, or *required number*, in the 3d place.

What are the ratios of the proportions,

$$3 : 9 :: 12 : 36?$$
$$2 : 10 :: 12 : 60?$$
$$4 : 2 :: 8 : 4?$$
$$9 : 1 :: 90 : 10?$$

225. The 1st and 4th terms of a proportion are called the *extremes*: the 2d and 3d terms, the *means*. Thus, in the proportion,

$$3 : 12 :: 6 : 24$$

3 and 24 are the *extremes*, and 12 and 6 the *means:*

Since (Art. 224), $\frac{12}{3}=\frac{24}{6}$,

we shall have, by reducing to a common denominator,

$$\frac{12\times6}{3\times6}=\frac{24\times3}{6\times3}.$$

But since the fractions are equal, and have the same denominators, their numerators must be equal, viz ;

$$12\times6=24\times3; \text{ that is,}$$

In any proportion, the product of the extremes is equal to the product of the means.

Thus, in the proportions,

$1 : 6 :: 2 : 12$; we have $1\times12= 2\times6$;
$4 : 12 :: 8 : 24$; " " $4\times24=12\times8$.

226. Since, in any proportion, the product of the extremes is equal to the product of the means, it follows that,

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In all cases, the *numerical value* of a quantity is the number of times which that quantity contains an assumed standard, called its *unit of measure.*

If we would find that numerical value, in its right place, we must say,

standard : quantity :: 1 : numerical value :

but if we take the other method, we have

quantity : standard :: numerical value : 1.

which brings the numerical value in the wrong place.

1st. *If the product of the means be divided by one of the extremes, the quotient will be the other extreme.*

Thus, in the proportion

$$3 : 12 :: 6 : 24, \text{ we have } 3\times 24 = 12\times 6;$$

then, if 72, the product of the means, be divided by one of the extremes, 3, the quotient will be the other extreme, 24: or, if the product be divided by 24, the quotient will be 3.

2d. *If the product of the extremes be divided by either of the means, the quotient will be the other mean.*

Thus, if $3\times 24 = 12\times 6 = 72$ be divided by 12, the quotient will be 6; or if it be divided by 6, the quotient will be 12.

EXAMPLES.

1. The first three terms of a proportion are 3, 9 and 12: what is the fourth term?

2. The first three terms of a proportion are 4, 16 and 15: what is the 4th term?

3. The first, second, and fourth terms of a proportion are 6, 12 and 24: what is the third term?

4. The second, third, and fourth terms of a proportion are 9, 6 and 24: what is the first term?

5. The first, second and fourth terms are 9, 18 and 48: what is the third term?

227. *Simple and Compound Ratio.*

The ratio of two single numbers is called a *Simple Ratio*, and the proportion which arises from the equality of two such ratios, a *Simple Proportion.*

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225. Which are the extremes of a proportion? Which the means? What is the product of the extremes equal to?

226. If the product of the means be divided by one of the extremes, what will the quotient be? If the product of the means be divided by either extreme, what will the quotient be?

227. What is a simple ratio? What is the proportion called which comes from the equality of two simple ratios? What is a compound ratio? What is a compound proportion?

If the terms of one ratio be multiplied by the terms of another, antecedent by antecedent and consequent by consequent, the ratio of the products is called a *Compound Ratio.* Thus, if the two ratios

$$3 : 6 \text{ and } 4 : 12$$

be multiplied together, we shall have the compound ratio

$$3 \times 4 : 6 \times 12, \text{ or } 12 : 72;$$

in which the ratio is equal to the product of the simple ratios.

A proportion formed from the equality of two compound ratios, or from the equality of a compound ratio and a simple ratio, is called a *Compound Proportion.*

228. *What part one number is of another.*

When the standard, or antecedent, is greater than the number which it measures, the ratio is a proper fraction, and is such a part of 1, as the number measured is of the standard.

1. What part of 12 is 3 ? that is, what part of the standard 12, is 3 ?

$$\tfrac{3}{12} = \tfrac{1}{4}; \text{ or,}$$

$$12 : 3 : : 1 : \tfrac{1}{4};$$

that is, the number measured is one-fourth of the standard.

2. What part of 9 is 2 ?
3. What part of 16 is 4 ?
4. What part of 100 is 20 ?
5. What part of 300 is 200 ?
6. What part of 36 is 144 ?
7. 3 is what part of 12 ?
8. 5 is what part of 20 ?
9. 8 is what part of 56 ?
10. 9 is what part of 8 ?
11. 12 is what part of 132 ?

NOTE.—The standard is generally preceded by the word *of*, and in comparing numbers, may be named second, as in examples 7, 8, 9, 10 and 11, but it must always be used as a divisor, and should be placed first in the statement.

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228. When the standard is greater than the consequent, how may the ratio be compared ? What part is 3 of 1 ? 5 of 1 ? What part is 4 of 2 ? 12 of 3 ? 7 of 5 ?

SINGLE RULE OF THREE.

229. The Single Rule of Three is an application of the principle of simple ratios. Three numbers are always given and a fourth required. The ratio between two of the given numbers is the same as that between the third and the required number.

1. If 3 yards of cloth cost $12, what will 6 yards cost at the same rate?

NOTE.—We shall denote the required term of the proportion by the letter x.

ANALYSIS.—The condition, "at the same rate," requires that the *quantity* 3 yards must have the same ratio to the quantity 6 yards, as $12, the cost of 3 yards, to x dollars, the cost of 12 yards.

Since the product of the two extremes is equal to the product of the two means, (Art. 225), $3 \times x = 6 \times 12$; and if $3 \times x = 6 \times 12$, x must be equal to this product divided by 3: that is,

STATEMENT.

$$\begin{array}{ccccccc} yd. & & yd. & & \$ & & \$ \\ 3 & : & 6 & :: & 12 & : & x. \end{array}$$

OPERATION.

$$\begin{array}{r|l} \cancel{3} & 12 \quad 2 \\ x & \cancel{6} \end{array}$$

Ans. $x = \$24$.

The 4th term is equal to the product of the second and third terms divided by the first.

2. If 56 dollars will buy 14 yards of broadcloth, how many yards, at the same rate, can be bought for 84 dollars?

ANALYSIS.—Fifty-six dollars, (being the cost of 14 yards of cloth), has the same ratio to $84, as 14 yards has to the number of yards which $84 will buy.

NOTE.—When the vertical line is used, the required term, (which is denoted by x), is written on the left.

STATEMENT.

$$\begin{array}{ccccccc} \$ & & \$ & & yd. & & yd. \\ 56 & : & 84 & :: & 14 & : & x. \end{array}$$

OPERATION.

$$\begin{array}{r|l} \cancel{4} \quad \cancel{56} & \cancel{14} \quad 21 \\ x & \cancel{84} \end{array}$$

$x = 21$

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229. What is the Single Rule of Three? How many numbers are given? How many required? What ratio exists between two of the given numbers?

230. Hence, we have the following

RULE I. *Write the number which is of the same kind with the answer for the third term, the number named in connection with it for the first term, and the remaining number for the second term.*

II. *Multiply the second and third terms together, and divide the product by the first term:* Or,

Multiply the third term by the ratio of the first and second.

NOTES.—1. If the first and second terms have different units, they must be reduced to the same unit.

2. If the third term is a compound denominate number, it must be reduced to its smallest unit.

3. The preparation of the terms, and writing them in their proper places, is called the *statement*.

EXAMPLES.

1. If I can walk 84 miles in 3 days, how far can I walk in 11 days?

2. If 4 hats cost $12, what will be the cost of 55 hats at the same rate?

3. If 40 yards of cloth cost $170, what will 325 yards cost at the same rate?

4. If 240 sheep produce 660 pounds of wool, how many pounds will be obtained from 1200 sheep?

5. If 2 gallons of molasses cost 65 cents, what will 3 hogsheads cost?

6. If a man travels at the rate of 210 miles in 6 days, how far will he travel in a year, supposing him not to travel on Sundays?

7. If 4 yards of cloth cost $13, what will be the cost of 5 pieces, each containing 25 yards?

8. If 48 yards of cloth cost $67,25, what will 144 yards cost at the same rate?

9. If 3 common steps, or paces, are equal to 2 yards, how many yards are there in 160 paces?

10. If 750 men require 22500 rations of bread for a month, how many rations will a garrison of 1200 men require?

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235. Give the rule for the statement. Give the rule for finding the fourth term.

11. A cistern containing 200 gallons is filled by a pipe which discharges 3 gallons in 5 minutes; but the cistern has a leak which empties at the rate of 1 gallon in 5 minutes. If the water begins to run in when the cistern is empty, how long will it run before filling the cistern?

12. If $14\frac{1}{2}$ yards of cloth cost $\$19\frac{1}{3}$, how much will $19\frac{7}{8}$ yards cost?

NOTE.—First make the statement; then change the mixed numbers to improper fractions, after which arrange the terms, and cancel equal factors according to previous instruction.

STATEMENT.

$$\begin{array}{ccccccc} yd. & & yd. & & \$ & & \$ \\ 14\frac{1}{2} & : & 19\frac{7}{8} & :: & 19\frac{1}{3} & : & x \end{array}$$

$$\begin{array}{r|l} 14\frac{1}{2} & 19\frac{1}{3} \\ x & 19\frac{7}{8} \\ \hline \end{array} \quad \text{Or,} \quad \begin{array}{r|ll} \cancel{3} & \cancel{58} & 2 \\ 2\ \cancel{29} & \cancel{2} & \\ \cancel{8} & \cancel{159} & 53 \\ \hline 2 & 53 = \$26\frac{1}{2} & \end{array}$$

13. If $\frac{2}{5}$ of a yard of cloth cost $\frac{7}{9}$ of a dollar, what will $2\frac{1}{2}$ yards cost?

14. If $\frac{3}{16}$ of a ship cost £273 2*s.* 6*d.*, what will $\frac{5}{32}$ of her cost?

15. If $1\frac{4}{11}$ bushels of wheat cost $\$2\frac{5}{8}$, how much will 60 bushels cost?

16. If $4\frac{1}{2}$ yards of cloth cost $9,75, what will $13\frac{1}{2}$ yards cost?

17. If a post 8 feet high cast a shadow 12 feet in length, what must be the height of a tree that casts a shadow 122 feet in length, at the same time of day?

18. If 7*cwt.* 1*qr.* of sugar cost $64,96, what will be the cost of 4*cwt.* 2*qr.*?

19. A merchant failing in trade, pays 65 cents for every dollar which he owes: he owes A $2750, and B $1975: how much does he pay each?

20. If 6 sheep cost $15, and a lamb costs one-third as much as a sheep, what will 27 lambs cost?

21. If 2*lbs.* of beef cost $\frac{1}{8}$ of a dollar, what will 30*lbs.* cost?

22. If $4\frac{1}{7}$ gallons of molasses cost $\$2\frac{5}{9}$, how much is it per quart?

23. A man receives $\frac{3}{8}$ of his income, and finds it equal to $3724,16: how much is his whole income?

24. If 4 barrels of flour cost $\$34\frac{2}{3}$, how much can be bought for $\$175\frac{1}{5}$?

25. If 2 gallons of molasses cost 65 cents, what will 3 hogsheads cost?

26. What is the cost of 6 bushels of coal at the rate of £1 14*s*. 6*d*, a chaldron?

27. What quantity of corn can I buy for 90 guineas, at the rate of 6 shillings a bushel?

28. A merchant failing in trade owes $3500, and his effects are sold for $2100: how much does B. receive, to whom he owes $420?

29. If 3 yards of broadcloth cost as much as 4 yards of cassimere, how much cassimere can be bought for 18 yards of broadcloth?

30. If 7 hats cost as much as 25 pair of gloves, worth 84 cents a pair, how many hats can be purchased for $216?

31. How many barrels of apples can be bought for $114,33, if 7 barrels cost $21,63?

32. If 27 pounds of butter will buy 45 pounds of sugar, how much butter will buy 36 pounds of sugar?

33. If $42\frac{1}{2}$ tons of coal cost $206,21, what will be the cost of $2\frac{1}{2}$ tons?

34. If 40 gallons run into a cistern, holding 700 gallons, in an hour, and 15 run out, in what time will it be filled?

35. A piece of land of a certain length and $12\frac{1}{2}$ rods in width, contains $1\frac{1}{4}$ acres, how much would there be in a piece of the same length $26\frac{2}{3}$ rods wide?

36. If 13 men can be boarded 1 week for $39.585, what will it cost to board 3 men and 6 women the same time, the women being boarded at half price?

37. What will 75 bushels of wheat cost, if 4 bushels 3 pecks cost $10.687?

38. What will be the cost, in United States money, of 324 yards 3*qrs.* of cloth, at 5*s.* 4*d.* New York currency, for 2 yards?

39. At $\$1,12\frac{1}{2}$ a square foot, what will it cost to pave a floor 18 feet long and 12*ft.* 6*in.* wide?

CAUSE AND EFFECT.

231. Whatever produces *effects*, as men at work, animals eating, time, goods purchased or sold, money lent, and the like, may be regarded as *causes*.

Causes are of two kinds, simple and compound.

A simple cause has but a single element, as men at work, a portion of time, goods purchased or sold, and the like.

A compound cause is made up of two or more simple elements, such as men at work *taken in connection with time*, and the like.

232. The results of causes, as work done, provisions consumed, money paid, cost of goods, and the like, may be regarded as *effects*. A simple effect is one which has but a single element; a compound effect is one which arises from the multiplication of two or more elements.

233. Causes which are of the same kind, that is, which can be reduced to the same unit, may be compared with each other; and effects which are of the same kind may likewise be compared with each other. From the nature of causes and effects, we know that

1st Cause : 2d Cause : : 1st Effect : 2d Effect;

and, 1st Effect : 2d Effect : : 1st Cause : 2d Cause.

234. Simple causes and simple effects give rise to simple ratios. Compound causes or compound effects give rise to compound ratios.

NOTE.—Professor H. N. Robinson, author of a complete course of mathematics, first made a practical application of the terms "Cause and Effect," in the development of proportion, as published in his arithmetic. By his permission, I have used the same terms, but have somewhat varied the method and rule.

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231. What are causes? How many kinds of causes are there? What is a simple cause? What is a compound cause?

232. What are effects? What is a simple effect? What is a compound effect?

233 What causes are of the same kind? What causes may be compared with each other? What do we infer from the nature of causes and effects?

234. What gives rise to simple ratios?

DOUBLE RULE OF THREE.

236. The Double Rule of Three is an application of the principles of compound proportion. It embraces all that class of questions in which the causes are compound, or in which the effects are compound ; and is divided into two parts :

1*st*. When the compound causes produce the same effects ;

2*d*. When the compound causes produce different effects.

237. *When the compound causes produce the same effects.*

1. If 6 men can dig a ditch in 40 days, what time will 30 men require to dig the same ?

STATEMENT.

$$\left.\begin{array}{c} \textit{men.}\ \ \textit{men.} \\ 6 \ :\ 30 \\ \textit{days.}\ \textit{days.} \\ 40 \ :\ x \end{array}\right\} \begin{array}{ccc} & \textit{ditch.} & \textit{ditch.} \\ :: & 1 \ : & 1 \end{array}$$

$$240 \ :\ 30 \times x \ ::\ 1 \ :\ 1.$$

$$\begin{array}{c|c} \cancel{30} & \overset{2}{\cancel{6}} \\ x & \cancel{40} \\ \hline \multicolumn{2}{c}{x = 8 \text{ days.}} \end{array}$$

ANALYSIS.—The first cause is compounded of 6 men, and 40 days, the time required to do the work, and is equal to what 1 man would do in $6 \times 40 = 240$ days.

The second cause is compounded of 30 men and the number of days necessary to do the same work, viz : $30 \times x$.

But since the effects are the same, viz : the work done, the *causes must be equal ;* hence, *the products of the elements of the causes are equal.* Therefore, in the solution of all like examples,

Write the cause containing the unknown element on the left of the vertical line for a divisor, and the other cause on the right for a dividend.

NOTE.—This class of questions has generally been arranged under the head of "Rule of Three Inverse."

EXAMPLES.

1. A certain work can be done in 12 days, by working 4 hours a day : how many days would it require the same number of men to do the same work, if they worked 6 hours a day ?

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236. What is the double Rule of Three ? What class of questions does it embrace ? Into how many parts is it divided ? What are they ?
237. What is the rule when the effects are equal ? Under what rule has this class of cases been arranged ?

2. A pasture of a certain extent supplies 30 horses for 18 days: how long will the same pasture supply 20 horses?

3. If a certain quantity of food will subsist a family of 12 persons 48 days, how long will the same food subsist a family of 8 persons?

4. If 30 barrels of flour will subsist 100 men for 40 days, how long will it subsist 25 men?

5. If 90 bushels of oats will feed 40 horses for six days, how many horses would consume the same in 12 days?

6. If a man perform a journey of $22\frac{1}{2}$ days, when the days are 12 hours long, how many days will it take him to perform the same journey when the days are 15 hours long?

7. If a person drinks 20 bottles of wine per month when it costs 2*s.* per bottle, how much must he drink without increasing the expense when it costs 2*s.* 6*d.* per bottle?

8. If 9 men in 18 days will cut 150 acres of grass, how many men will cut the same in 27 days?

9. If a garrison of 536 men have provisions for 326 days, how long will those provisions last if the garrison be increased to 1304 men?

10. A pasture of a certain extent having supplied a body of horse, consisting of 3000, with forage for 18 days: how many days would the same pasture have supplied a body of 2000 horse?

11. What length must be cut off from a board that is 9 inches wide, to make a square foot, that is, as much as is contained in 12 inches in length and 12 in breadth?

12. If a certain sum of money will buy 40 bushels of oats at 45 cents a bushel, how many bushels of barley will the same money buy at 72 cents a bushel?

13. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 400 men?

14. The governor of a besieged place has provisions for 54 days, at the rate of 2*lb.* of bread per day, but is desirous of prolonging the siege to 80 days in expectation of succor: what must be the ration of bread?

238. *When the Compound Causes produce different Effects.*

In this class of questions, either a cause, or a single element of a cause may be required; or an effect, or a single element of an effect may be required.

1. If a family of 6 persons expend \$300 in 8 months, how much will serve a family of 15 persons for 20 months?

ANALYSIS.—In this example the second effect is required; and the statement may be read thus: If 6 persons in 8 months expend \$300, 15 persons in 20 months will expend how many (or x) dollars?

OPERATION.

$$\begin{array}{r|ll} \cancel{6} & 15 & \\ 2\ \cancel{8} & \cancel{20} & 5 \\ x & \cancel{300} & 25 \\ \hline \multicolumn{3}{l}{x=1875\ \textit{Ans}} \end{array}$$

STATEMENT.

1st Cause : 2d Cause : : 1st Effect : 2d Effect.

$$\left.\begin{array}{c}6\\8\end{array}\right\} : \left.\begin{array}{c}15\\20\end{array}\right\} : : \$300 : x;$$

Or, $6\times8 : 15\times20 : : 300 : x.$

2. If 16 men, in 12 days, build 18 feet of wall, how many men must be employed to build 72 feet in 8 days?

ANALYSIS.—In this example an element of the second cause is required, viz: the number of men. The question may be read thus: If 16 men, in 12 days, build 18 feet of wall, how many (or x) men, in 8 days, will build 72 feet of wall?

OPERATION.

$$\begin{array}{r|ll} \cancel{18} & \cancel{72} & 4 \\ \cancel{8} & \cancel{16} & 2 \\ x & 12 & \\ \hline \multicolumn{3}{l}{x=96\ \text{days.}} \end{array}$$

STATEMENT.

$$\left.\begin{array}{c}16\\12\end{array}\right\} : \left.\begin{array}{c}x\\8\end{array}\right\} : : 18 : 72;$$

Or, $16\times12 : x\times8 : : 18 : 72.$

3. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet thick, in 4 days, working 12 hours a day; how long a wall, that is 6 feet high, and 3 feet thick can 48 men build in 36 days, working 9 hours a day?

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238. When the compound causes produce different effects, what will always be required?

ANALYSIS.—In this example an element of the second effect is required, viz: the *length* of the wall, and the question may be read thus: If 32 men, in 4 days, working 12 hours a day, can build a wall 36 feet long, 8 feet high, and 4 feet thick, 48 men in 36 days, working 9 hours a day, can build a wall how many (or x) feet long, 6 feet high, and 3 feet thick?

OPERATION.

$$\begin{array}{r|ll} \cancel{32} & \cancel{48} & \\ \cancel{4} & 36 & 3 \\ \cancel{12} & \cancel{9} & 6 \\ \cancel{6} & \cancel{36} & \\ \cancel{3} & \cancel{8} & \\ x & \cancel{4} & \end{array}$$

$x = 648$ feet.

STATEMENT.

$$\left.\begin{array}{r} 32 \\ 4 \\ 12 \end{array}\right\} : \left.\begin{array}{r} 48 \\ 36 \\ 9 \end{array}\right\} :: \left.\begin{array}{r} 36 \\ 8 \\ 4 \end{array}\right\} : \begin{array}{c} x \\ 6 \\ 3 \end{array}$$

Or, $32 \times 4 \times 12 : 48 \times 36 \times 9 :: 36 \times 8 \times 4 : x \times 6 \times 3$.

239. Hence, we have the following

RULE.—I. *Arrange the terms in the statement so that the causes shall compose one couplet, and the effects the other, putting* x *in the place of the required element:*

II. *Then if* x *fall in one of the extremes, make the product of the means a dividend, and the product of the extremes a divisor; but if* x *fall in one of the means, make the product of the extremes a dividend, and the product of the means a divisor.*

EXAMPLES.

1. If I pay \$24 for the transportation of 96 barrels of flour 200 miles, what must I pay for the transportation of 480 barrels 75 miles?

2. If 12 ounces of wool be sufficient to make $1\frac{1}{2}$ yards of cloth 6 quarters wide, what number of pounds will be required to make 450 yards of flannel 4 quarters wide?

3. What will be the wages of 9 men for 11 days, if the wages of 6 men for 14 days be \$84?

4. How long would 406 bushels of oats last 7 horses, if 154 bushels serve 14 horses 44 days?

5. If a man travel 217 miles in 7 days, travelling 6 hours a day, how far would he travel in 9 days if he travelled 11 hours a day?

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239. What is the rule for finding the unknown part?

6. If 27 men can mow 20 acres of grass in $5\frac{5}{6}$ days, working $3\frac{3}{5}$ hours a day, how many acres can 10 men mow in $4\frac{1}{2}$ days, by working $8\frac{1}{2}$ hours a day?

7. How long will it take 5 men to earn \$11250, if 25 men can earn \$6250 in 2 years?

8. If 15 weavers, by working 10 hours a day for 10 days, can make 250 yards of cloth, how many must work 9 hours a day for 15 days to make $607\frac{1}{2}$ yards?

9. A regiment of 100 men drank 20 dollars' worth of wine at 30 cents a bottle: how many men, drinking at the same rate, will require 12 dollars' worth at 25 cents a bottle?

10. If a footman travel 341 miles in $7\frac{1}{3}$ days, travelling $12\frac{1}{2}$ hours each day, in how many days, travelling $10\frac{1}{3}$ hours a day, will he travel 155 miles?

11. If 25 persons consume 300 bushels of corn in 1 year, how much will 139 persons consume in 8 months, at the same rate?

12. How much hay will 32 horses eat in 120 days, if 96 horses eat $3\frac{3}{4}$ tons in $7\frac{1}{2}$ weeks?

13. If \$2,45 will pay for painting a surface 21 feet long and $13\frac{1}{8}$ feet wide, what length of surface that is $10\frac{2}{5}$ feet wide, can be painted for \$31,72?

14. How many pounds of thread will it require to make 60 yards of 3 quarters wide, if 7 pounds make 14 yards 6 quarters wide?

15. If 500 copies of a book, containing 210 pages, require 12 reams of paper, how much paper will be required to print 1200 copies of a book of 280 pages?

16. If a cistern $17\frac{1}{2}$ feet long, $10\frac{1}{2}$ feet wide, and 13 feet deep, hold 546 barrels of water, how many barrels will a cistern 12 feet long, 10 feet wide, and 7 feet deep, contain?

17. A contractor agreed to build 24 miles of railroad in 8 months, and for this purpose employed 150 men. At the end of 5 months but 10 miles of the road were built: how many more men must be employed to finish the road in the time agreed upon?

18. If 336 men, in 5 days of 10 hours each, can dig a trench of 5 degrees of hardness, 70 yards long 3 wide and 2 deep: what length of trench of 6 degrees of hardness, 5 yards wide and 3 yards deep, may be dug by 240 men in 9 days of 12 hours each?

PARTNERSHIP.

240. Partnership is the joining together of two or more persons in trade, with an agreement to share the profits or losses.

Partners are those who are united together in carrying on business.

Capital, is the amount of money employed :

Dividend is the gain or profit :

Loss is the opposite of profit :

241. The Capital or Stock is the *cause* of the entire profit :

Each man's capital is the *cause* of his profit :

The entire profit or loss is the *effect* of the whole capital :

Each man's profit or loss is the *effect* of his capital : hence,

Whole Stock : Each man's Stock
: : Whole profit or loss : Each man's profit or loss.

EXAMPLES.

1. A and B buy certain goods amounting to 160 dollars, of which A pays 90 dollars and B, 70 ; they gain 32 dollars by the purchase : what is each one's share ?

OPERATION.

160 : 90 : : 32 : A's share ; or,

$$\begin{array}{r|l} {}^{5}\ \cancel{160} & \cancel{32}\quad 18 \\ x & \cancel{90} \\ \hline \multicolumn{2}{c}{x=\$18.} \end{array}$$

160 : 70 : : 32 : B's share ; or,

$$\begin{array}{r|l} {}^{5}\ \cancel{160} & \cancel{32}\quad 14 \\ x & 70 \\ \hline \multicolumn{2}{c}{x=\$14.} \end{array}$$

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240. What is a partnership ? What are partners ? What is capita or stock ? What is dividend ? What is loss ?

241. What is the cause of the profit ? What is the cause of each man's profit ? What is the effect of the whole capital ! What is the effect of each man's capital ? What proportion exists between causes and their effects ? What is the rule !

Hence, the following

RULE.—*As the whole stock is to each man's share, so is the whole gain or loss to each man's share of the gain or loss.*

EXAMPLES.

1. A and B have a joint stock of $2100, of which A owns $1800 and B $300; they gain in a year $1000: what is each one's share of the profits?

2. A, B and C fit out a ship for Liverpool. A contributes $3200, B $5000, and C $4500; the profits of the voyage amount to $1905: what is the portion of each?

3. Mr. Wilson agrees to put in 5 dollars as often as Mr. Jones puts in 7; after raising their capital in this way, they trade for 1 year and find their profits to be $3600: what is the share of each?

4. A, B and C make up a capital of $20,000; B and C each contribute twice as much as A; but A is to receive one-third of the profits for extra services; at the end of the year they have gained $4000: what is each to receive?

5. A, B and C agree to build a railroad and contribute $18000 of capital, of which B pays 2 dollars and C, 3 dollars as often as A pays 1 dollar; they lose $2400 by the operation: what is the loss of each?

COMPOUND PARTNERSHIP.

242. *When the causes of profit or loss are compound.*

When the partners employ their capital for different periods of time, each cause of profit or loss is compound, being made up of the two elements of *capital* and *time*. The product of these elements, in each particular case, will be the cause of each man's gain or loss; and their sum will be the cause of the entire gain or loss: hence, to find each share,

Multiply each man's stock by the time he continued it in trade; then say, as the sum of the products is to each product, so is the whole gain or loss to each man's share of the gain or loss.

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242. When is the cause of profit or loss compound? What are the elements of the compound cause? What is the rule in this case?

EXAMPLES.

1. A and B entered into partnership. A put in \$840 for 4 months, and B, \$650 for 6 months; they gained \$363: what is each one's share?

OPERATION.

A. $\$840 \times 4 = 3360$
B. $\quad 650 \times 6 = 3900$

$$7260 : \begin{cases} 3360 : : \\ 3900 : : \end{cases} 363 : \begin{cases} \$168 \text{ A's.} \\ \$195 \text{ B's.} \end{cases}$$

2. A puts in trade \$550 for 7 months and B puts in \$1625 for 8 months; they make a profit of \$337: what is the share of each?

3. A and B hire a pasture, for which they agree to pay \$92,50. A pastures 12 horses for [illegible] weeks and B 11 horses for 7 weeks: what portion must each pay?

4. Four traders form a company. A puts in \$400 for 5 months; B \$600 for 7 months; C \$960 for 8 months; D \$1200 for 9 months. In the course of trade they lost \$750: how much falls to the share of each?

5. A, B and C contribute to a capital of \$15000 in the following manner: every time A puts in 3 dollars B puts in \$5 and C, \$7. A's capital remains in trade 1 year; B's $1\frac{2}{5}$ years; and C's $2\frac{3}{7}$ years; at the end of the time there is a profit of \$15000: what is the share of each?

6. A commenced business January 1st, with a capital of \$3400. April 1st, he took B into partnership, with a capital of \$2600; at the expiration of the year they had gained \$750: what is each one's share of the gain?

7. James Fuller, John Brown and William Dexter formed a partnership, under the firm of Fuller, Brown & Co., with a capital of \$20000; of whioh Fuller furnished \$6000, Brown \$5000, and Dexter \$9000. At the expiration of 4 months, Fuller furnished \$2000 more; at the expiration of 6 months, Brown furnished \$2500 more; and at the end of a year Dexter withdrew \$2000. At the expiration of one year and a half, they found their profits amounted to \$5400: what was each partner's share?

PERCENTAGE.

243. PERCENTAGE is an allowance made by the hundred.

The *base* of percentage, is the number on which the percentage is reckoned.

PER CENT means by the hundred : thus, 1 per cent means 1 for every hundred ; 2 per cent, 2 for every hundred ; 3 per cent, 3 for every hundred, &c. The allowances, 1 per cent, 2 per cent, 3 per cent, &c., are called *rates*, and may be expressed decimally, as in the following

TABLE.

1 per cent is	.01	7 per cent is	.07
3 per cent is	.03	8 per cent is	.08
4 per cent is	.04	15 per cent is	.15
5 per cent is	.05	68 per cent is	.68
6 per cent is	.06	99 per cent is	.99

ALSO,

100 per cent is 1. : for, $\frac{100}{100}$ is equal to 1.
150 per cent is 1.50 : for, $\frac{150}{100}$ is equal to 1.50
130 per cent is 1.30 : for, $\frac{130}{100}$ is equal to 1.30
200 per cent is 2. : for, $\frac{200}{100}$ is equal to 2.00
$\frac{1}{2}$ per cent is .005 : for, $\frac{1}{100} \div 2$ is equal to .005
$3\frac{1}{2}$ per cent is .035 : for, $3\frac{1}{2} = .03 + .005 = .035$
$5\frac{3}{4}$ per cent is .0575 : for, $5\frac{3}{4} = .05 + .075 = 075$

EXAMPLES.

Write, decimally, $8\frac{1}{2}$ per cent ; 9 per cent ; $6\frac{3}{4}$ per cent ; $65\frac{1}{2}$ per cent ; 205 per cent ; 327 per cent.

244. *To find the percentage of any number.*

1. What is the percentage of $320, the rate being 5 per cent ?

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243. What is per centage ? What is the base ? What does per cent mean ? What do you understand by 3 per cent ? What is the rate, or rate per cent ?
244. How do you find the percentage of any number ?

OPERATION.

$$\begin{array}{r} 320 \\ .05 \\ \hline \$16,00 \end{array} \ \textit{Ans}$$

ANALYSIS.—The rate being 5 per cent, is expressed decimally by .05. We are then to take .05 of the base (which is $320); this we do by multiplying $320 by .05.

Hence, to find the percentage of a number,

Multiply the number by the rate expressed decimally, and the product will be the percentage.

EXAMPLES.

1. What is the percentage of $657, the rate being $4\frac{1}{3}$ per cent?

OPERATION.

$$\begin{array}{rl} 657 & \\ .04\frac{1}{3} & \\ \hline 219 = \frac{1}{3} & \text{per cent.} \\ 2628 = 4 & \text{per cent.} \\ \hline \$28,47 = 4\frac{1}{3} & \text{per cent.} \end{array}$$

NOTE.—When the rate cannot be reduced to an exact decimal, it is most convenient to multiply by the fraction, and then by that part of the rate which is expressed in exact decimals.

Find the percentage of the following numbers:

1. $2\frac{1}{2}$ per cent of 650 dollars.
2. 3 per cent of 650 yards.
3. $4\frac{1}{2}$ per cent of 875*cwt.*
4. $6\frac{1}{3}$ per cent of $37,50.
5. $5\frac{3}{4}$ per cent of 2704 miles.
6. $\frac{1}{2}$ per cent of 1000 oxen.
7. $2\frac{3}{8}$ per cent of $376.
8. $2\frac{1}{10}$ per cent of 860 sheep.
9. $5\frac{2}{3}$ per cent of $327,33.
10. $66\frac{2}{3}$ per cent of 420 cows.
11. 105 per cent of 850 tons.
12. 116 per cent of 875*lb.*
13. 241 per cent of $875,12.
14. $37\frac{1}{2}$ per cent of $200.
15. $33\frac{1}{3}$ per cent of $687,24.
16. $87\frac{1}{2}$ per cent of $400.
17. $62\frac{1}{2}$ per cent of $600.
18. 308 per cent of $225,40.

19. A has $852 deposited in the bank, and wishes to draw out 5 per cent of it: how much must he draw for?

20. A merchant has 1200 barrels of flour: he shipped 64 per cent of it and sold the remainder: how much did he sell?

21. A merchant bought 1200 hogsheads of molasses. On getting it into his store, he found it short $3\frac{1}{2}$ per cent: how many hogsheads were wanting?

22. What is the difference between $5\frac{1}{3}$ per cent of $800 and $6\frac{1}{2}$ per cent of $1050?

23. Two men had each \$240. One of them spends 14 per cent, and the other $18\frac{1}{2}$ per cent: how many dollars more did one spend than the other?

24. A man has a capital of \$12500: he puts 15 per cent of it in State Stocks: $33\frac{1}{3}$ per cent in Railroad Stocks, and 25 per cent in bonds and mortgages: what per cent has he left, and what is its value?

25. A farmer raises 850 bushels of wheat: he agrees to sell 18 per cent of it at \$1,25 a bushel; 50 per cent of it at \$1,50 a bushel, and the remainder at \$1,75 a bushel: how much does he receive in all?

245. *To find the per cent which one number is of another.*

1. What per cent of \$16 is \$4?

OPERATION.
$\frac{4}{16}=\frac{1}{4}=.25$.
or 25 per cent.

ANALYSIS.—The question is, what part of \$16 is \$4, when expressed in hundredths: The standard is \$16 (Art. 228): hence, the part is $\frac{4}{16}=\frac{1}{4}=.25$; therefore, the per cent is 25: hence, to find what per cent one number is of another,

Divide by the standard or base, and the quotient, reduced to decimals, will express the rate per cent.

NOTE.—The standard or base, is generally preceded by the word of.

EXAMPLES.

1. What per cent of 20 dollars is 5 dollars?
2. Forty dollars is what per cent of eighty dollars?
3. What per cent of 200 dollars is 80 dollars?
4. What per cent of 1250 dollars is 250 dollars?
5. What per cent of 650 dollars is 250 dollars?
6. Ninety bushels of wheat is what per cent of 1800*bush.*?
7. Nine yards of cloth is what per cent of 870 yards?
8. Forty-eight head of cattle are what per cent of a drove of 1600?
9. A man has \$550, and purchases goods to the amount of \$82,75: what per cent of his money does he expend?

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245. How do you find the per cent which one number is of another?

10. A merchant goes to New York with \$1500 ; he first lays out 20 per cent, after which he expends \$660 : what per cent was his last purchase of the money that remained after his first ?

11. Out of a cask containing 300 gallons, 60 gallons are drawn : what per cent is this ?

12. If I pay \$698,23 for 3 hogsheads of molasses and sell them for \$837,996, how much do I gain per cent on the money laid out ?

13. A man purchased a farm of 75 acres at \$42,40 an acre. He afterwards sold the same farm for \$3577,50 : what was his gain per cent on the purchase money ?

STOCK, COMMISSION AND BROKERAGE.

246. A CORPORATION is a collection of persons authorized by law to do business together. The law which defines their rights and powers is called a *Charter*.

CAPITAL or STOCK is the money paid in to carry on the business of the Corporation, and the individuals so contributing are called *Stockholders*. This capital is divided into equal parts called *Shares*, and the written evidences of ownership are called *Certificates*.

247. When the United States Government, or any of the States, borrows money, an acknowledgment is given to the lender, in the form of a bond, bearing a fixed interest. Such bonds are called United States Stock, or State Stock.

The *par value* of stock is the number of dollars named in each share. The *market value* is what the stock brings *per share* when sold for cash.

If the market value is above the par value, the stock is said to be at a premium, or *above par* ; but if the market value is below the par value, it is said to be at a discount, or *below par*.

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246. What is a corporation ? What is a charter ? What is capital or stock ? What are shares ?

247. What are United States Stocks ? What are State Stocks ? What is the par value of a stock ? What is the market value ? If the market is above the par value, what is said of the stock ? If it is below, what is said of the stock ? What is the market value when above par ? What when below ?

Let $1 =$ par value of 1 dollar:

$1 +$ premium $=$ market value of 1 dollar, when above par:

$1 -$ discount $=$ maket value of 1 dollar when below par.

248. Commission is an allowance made to an agent for buying or selling, and is generally reckoned at a certain rate per cent.

The commission, for the purchase or sale of goods in the city of New York, varies from $2\frac{1}{2}$ to $12\frac{1}{2}$ per cent, and under some circumstances even higher rates are paid.

Brokerage is an allowance made to an agent who buys or sells stocks, uncurrent money, or bills of exchange, and is generally reckoned at so much per cent on the par value of the stock. The brokerage, in the city of New York, is generally one-fourth per cent on the *par value* of the stock.

EXAMPLES.

1. What is the commission on \$4396 at 6 per cent?

NOTE.—We here find the commission, as in simple percentage, by multiplying by the decimal which expresses the rate per cent.

OPERATION.

$$\begin{array}{r} \$4396 \\ .06 \\ \hline \textit{Ans. } \$263,76. \end{array}$$

2. A factor sells 60 bales of cotton at \$425 per bale, and is to receive $2\frac{1}{2}$ per cent commission: how much must he pay over to his principal?

3. A drover agrees to purchase a drove of cattle and to sell them in New York city for 5 per cent on what he may receive; he expends in the purchase \$4250, and sells them at an advance of 10 per cent: how much is his commission?

4. A commission merchant sells goods to the amount of \$8750, on which he is to be allowed 2 per cent, but in consideration of paying the money over before it is due, he is to receive $1\frac{1}{2}$ per cent additional: how much must he pay over to his principal?

5. A broken bank has a circulation of \$98000 and purchases the bills at 85 per cent: how much is made by the operation?

---

248. What is commission? What is brokerage?

6. Merchant A sent to B, a broker, \$3825 to be invested in stock; B is to receive 2 per cent on the amount paid for the stock: what was the value of the stock purchased?

ANALYSIS.—Since the broker receives 2 per cent, it will require \$1.02 to purchase 1 dollar's worth of stock; hence, there will be as many dollars worth purchased as \$1.02 is contained times in \$3825; that is, \$3750 worth.

OPERATION.

```
1.02)3825.00($3750 Ans.
     306
     ---
      765
      714
      ---
       510
       510
       ---
```

7. Mr. Jones sends his broker \$18560 to be invested in U. S. Stocks, which are 15 per cent above par; the broker is to receive one per cent; how many shares of \$100 each can be purchased?

ANALYSIS.—Since the premium is 15 per cent, and the brokerage 1 per cent, each dollar of par value will cost \$1 plus the premium plus the brokerage= \$1.16: hence, the amount purchased will be as many dollars as \$1.16 is contained times in \$18560.

OPERATION.

```
1.16)18560
     ------
     $16000 quotient,
or,   160     shares.
```

8. I have \$5000,89 to be laid out in stocks, which are 15 per cent below par: allowing 2 per cent commission, how much can be purchased at the par value?

ANALYSIS.—Since the stock is at a discount of 15 per cent, the market value will be 85 per cent; add 2 per cent, the brokerage, gives 87 per cent=.87. The amount purchased will be as many dollars as .87 is contained times in \$5000,89.

OPERATION.

```
.87)5000,89
    -------
    $5747.  Ans.
```

Hence, to find the amount at par value,

Divide the amount to be expended by the market value of \$1 *plus the brokerage; and the quotient will be the amount in par value.*

9. Messrs. Sherman & Co. receive of Mr. Gilbert \$28638,50 to be invested in bank stocks, which are $12\frac{1}{2}$ per cent above par, for which they are to receive one-fourth of one per cent commission: how many shares of \$127 each can they buy?

10. The par value of Illinois Railroad stock is 100. It sells in market at $72\frac{1}{2}$: if I pay $\frac{1}{4}$ per cent brokerage, how many shares can I buy for $5820 ?

PROFIT AND LOSS.

249. Profit or loss is a process by which merchants discover the amount gained or lost in the purchase and sale of goods. It also instructs them how much to increase or diminish the price of their goods, so as to make or lose so much per cent.

EXAMPLES.

1. Bought a piece of cloth containing 75*yd.* at $5,25 per yard, and sold it at $5,75 per yard: how much was gained in the trade?

OPERATION.

ANALYSIS.—We first find the profit on a single yard, and then multiply by the number of yards, which is 75.

$5,75 price of 1 yard.
$5,25 cost of 1 yard.
50*cts.* profit on 1 yard:
then, $\$0,50 \times 75 = \$37,50$.

2. Bought a piece of calico containing 56 yards, at 27 cents a yard: what must it be sold for per yard to gain $2,24?

OPERATION.

ANALYSIS.—First find the cost, then add the profit and divide the sum by the number of yards.

56 yards at 27 cents	=$15,12
Profit - - -	2,24
It must sell for -	$17,36.

56)17,36
31 cents.

250. *Knowing the per cent of gain or loss and the amount received, to find the cost.*

1. I sold a parcel of goods for $195,50, on which I made 15 per cent: what did they cost me?

ANALYSIS.—1 dollar of the cost plus 15 per cent, will be what that which cost $1 sold for, viz., $1,15: hence, there will be as many dollars of cost, as $1.15 is contained times in what the goods brought.

OPERATION.
1.15)195,50
$170 *Ans.*

---

249. What is loss or gain?

2. If I sell a parcel of goods for $170, by which I lose 15 per cent, what did they cost?

ANALYSIS.—1 dollar of the cost less 15 per cent, will be what that which cost 1 dollar sold for, viz., $0,85: hence, there will be as many dollars of cost, as .85 is contained times in what the goods brought.

OPERATION.

.85)170

$200 *Ans.*

Hence, to find the cost,

Divide the amount received by 1 *plus the per cent when there is a gain, and by* 1 *minus the per cent when there is a loss, and the quotient will be the cost.*

EXAMPLES.

1. Bought a piece of cassimere containing 28 yards at $1\frac{1}{4}$ dollars a yard; but finding it damaged, am willing to sell it at a loss of 15 per cent: how much must be asked per yard?

2. Bought a hogshead of brandy at $1,25 per gallon, and sold it for $78: was there a loss or gain?

3. A merchant purchased 3275 bushels of wheat for which he paid $3517,10, but finding it damaged, is willing to lose 10 per cent: what must it sell for per bushel?

4. Bought a quantity of wine at $1,25 per gallon, but it proves to be bad and am obliged to sell it at 20 per cent less than I gave: how much must I sell it for per gallon?

5. A farmer sells 125 bushels of corn for 75 cents per bushel; the purchaser sells it at an advance of 20 per cent: how much did he receive for the corn?

6. A merchant buys 1 tun of wine for which he pays $725, and wishes to sell it by the hogshead at an advance of 15 per cent: what must be charged per hogshead?

7. A merchant buys 158 yards of calico for which he pays 20 cents per yard; one-half is so damaged that he is obliged to sell it at a loss of 6 per cent: the remainder he sells at an advance of 19 per cent: how much did he gain?

8. If I buy coffee at 16 cents and sell it at 20 cents a pound, how much do I make per cent on the money paid?

---

250. Knowing the per cent of gain or loss and the amount received, how do you find the cost?

9 A man bought a house and lot for $1850,50, and sold it for $1517,41 : how much per cent did he lose?

10. A merchant bought 650 pounds of cheese at 10 cents per pound, and sold it at 12 cents per pound : how much did he gain on the whole, and how much per cent on the money laid out?

11. Bought cloth at $1,25 per yard, which proving bad, I wish to sell it at a loss of 18 per cent: how much must I ask per yard?

12. Bought 50 gallons of molasses at 75 cents a gallon, 10 gallons of which leaked out. At what price per gallon must the remainder be sold that I may clear 10 per cent on the cost?

13. Bought 67 yards of cloth for $112, but 19 yards being spoiled, I am willing to lose 5 per cent: how much must I sell it for per yard?

14. Bought 67 yards of cloth for $112, but a number of yards being spoiled, I sell the remainder at $2,216$\frac{2}{3}$ per yard, and lose 5 per cent: how many yards were spoiled?

15. Bought 2000 bushels of wheat at $1,75 a bushel, from which was manufactured 475 barrels of flour: what must the flour sell for per barrel to gain 25 per cent on the cost of the wheat?

INSURANCE.

251. INSURANCE is an agreement, generally in writing, by which an individual or company bind themselves to exempt the owners of certain property, such as ships, goods, houses, &c., from loss or hazard.

The POLICY is the written agreement made by the parties.

PREMIUM is the amount paid by him who owns the property to those who insure it, as a compensation for their risk. The premium is generally so much per cent on the property insured.

EXAMPLES.

1. What would be the premium for the insurance of a house valued at $8754 against loss by fire for one year, at $\frac{1}{2}$ per cent?

---

251. What is insurance? What is the policy? What is the premium? How is it reckoned?

2. What would be the premium for insuring a ship and cargo, valued at \$37500, from New York to Liverpool, at $3\frac{1}{2}$ per cent?

3. What would be the insurance on a ship valued at \$47520 at $\frac{1}{2}$ per cent; also at $\frac{1}{3}$ per cent?

4. What would be the insurance on a house valued at \$14000 at $1\frac{1}{2}$ per cent?

5. What is the insurance on a store and goods valued at \$27000, at $2\frac{1}{4}$ per cent?

6. What is the premium of insurance on \$9870 at 14 per cent?

7. A merchant wishes to insure on a vessel and cargo at sea, valued at \$28800: what will be the premium at $1\frac{3}{4}$ per cent?

8. A merchant owns three-fourths of a ship valued at \$24000, and insures his interest at $2\frac{1}{2}$ per cent: what does he pay for his policy?

9. A merchant learns that his vessel and cargo, valued at \$36000, have been injured to the amount of \$12000; he effects an insurance on the remainder at $5\frac{1}{2}$ per cent; what premium does he pay?

10. My furniture, worth \$3440, is insured at $2\frac{3}{8}$ per cent; my house, worth \$1000, at $1\frac{1}{4}$ per cent; and my barn, horses and carriages, worth \$1500, at $3\frac{1}{4}$ per cent: what is the whole amount of my insurance?

11. A man bought a house, and paid the insurance at $2\frac{1}{2}$ per cent, the whole of which amounted to \$1845: what was the value of the house and the amount of the insurance?

12. What would it cost to insure a store, worth \$3240, at $\frac{3}{5}$ per cent, and the stock, worth \$7515,75, at $\frac{3}{8}$ per cent?

13. A merchant imported 250 pieces of broadcloth, each piece containing $36\frac{1}{2}$ yards, at \$3,25 cents a yard. He paid $4\frac{1}{2}$ per cent insurance on the selling price, \$4,50 a yard. If the goods were destroyed by fire, and he got the amount of insurance, how much did he make?

14. A vessel and cargo, worth \$65000, are damaged to the amount of 20 per cent, and there is an insurance of 50 per cent on the loss: how much will the owner receive?

INTEREST.

252 INTEREST is an allowance made for the use of money that is borrowed.

PRINCIPAL is the money on which interest is paid.

AMOUNT is the sum of the Principal and Interest.

For example: If I borrow 1 dollar of Mr. Wilson for 1 year, and pay him 7 cents for the use of it; then,

1 dollar is the *principal*,
7 cents is the interest, and
$1,07 the amount

The RATE of interest is the number of cents paid for the use of 1 dollar for 1 year. Thus, in the above example, the rate is 7 per cent per annum.

NOTE.—The term per cent means, *by the hundred;* and *per annum* means *by the year.* As interest is always reckoned by the year, the term per annum is understood and omitted.

CASE I.

253. *To find the interest of any principal for one or more years.*

1. What is the interest of $1960 for 4 years, at 7 per cent?

ANALYSIS.—The rate of interest being 7 per cent, is expressed decimally by .07: hence each dollar, in 1 year, will produce .07 of itself, and $1960 will produce .07 of $1960, or $137,20. Therefore, $137,20 is the interest for 1 year, and this interest multiplied by 4, gives the interest for 4 years: hence, the following

OPERATION.

$1960	
.07	rate.
137,20	int. for 1*yr.*
4	No. of years.
$548,80	

RULE.—*Multiply the principal by the rate, expressed decimally, and the product by the number of years.*

252. What is interest? What is principal? What is amount? What is rate of interest? What does per annum mean?

253. How do you find the interest of any principal for any number of years? Give the analysis.

EXAMPLES.

1. What is the interest of $365,874 for one year, at $5\frac{1}{2}$ per cent?

ANALYSIS.—We first find the interest at $\frac{1}{2}$ per cent, and then the interest at 5 per cent; the sum is the interest at $5\frac{1}{2}$ per cent.

OPERATION.

$$\begin{array}{rl} \$365,874 & \\ .05\frac{1}{2} & \\ \hline 1,82937 & \frac{1}{2} \text{ per cent.} \\ 18,29370 & 5 \text{ per cent.} \\ \hline \textit{Ans.}\ \$20,12307 & 5\frac{1}{2} \text{ per cent.} \end{array}$$

2. What is the interest of $650 for one year, at 6 per cent?
3. What is the interest of $950 for 4 years, at 7 per cent?
4. What is the amount of $3675 in 3 years, at 7 per cent?
5. What is the amount of $459 in 5 years, at 8 per cent?
6. What is the amount of $375 in 2 years, at 7 per cent?
7. What is the interest of $211,26 for 1 year, at $4\frac{1}{2}$ per ct.?
8. What is the interest of $1576,91 for 3 years, at 7 per ct.?
9. What is the amount of $957,08 in 6 years, at $3\frac{1}{2}$ per ct.?
10. What is the interest of $375,45 for 7 years, at 7 per ct.?
11. What is the amount of $4049,87 in 2 years, at 5 per ct.?
12. What is the amount of $16199,48 in 16 yrs., at $5\frac{1}{2}$ per ct.?

NOTE.—When there are years and months, and the months are aliquot parts of a year, *multiply the interest for* 1 *year by the years and months reduced to the fraction of a year.*

EXAMPLES.

1. What is the interest of $326,50, for 4 years and 2 months, at 7 per cent?

2. What is the interest of $437,21, for 9 years and 3 months, at 3 per cent?

3. What is the amount of $1119,48, after 2 years and 6 months, at 7 per cent?

4. What is the amount of $179,25, after 3 years and 4 months, at 7 per cent?

5. What is the amount of $1046,24, after 4 years and 3 months, at $5\frac{1}{2}$ per cent?

CASE II.

254. *To find the interest on a given principal for any rate and time.*

1. What is the interest of \$876,48 at 6 per cent, for 4 years 9 months and 14 days?

ANALYSIS.—The interest for 1 year is the product of the principal multiplied by the rate. If the interest for 1 year be divided by 12, the quotient will be the interest for 1 month: if the interest for 1 month be divided by 30, the quotient will be the interest for 1 day.

The interest for 4 years is 4 times the interest for 1 year: the interest for 9 months, 9 times the interest for 1 month; and the interest for 14 days, 14 times the interest for 1 day.

OPERATION.

\$876,48		
.06		
12)52,5888 = int. for 1*yr.*	52,5888 × 4 = \$210,3552	4*yr.*
30)4,3824 = int. for 1*mo.*	4,3824 × 9 = \$ 39,4416	9*mo*
,14608 = int. for 1*da.*	,14608 × 14 = \$ 2,0451	14*da*
	Total interest, \$251,8442+	

Hence, we have the following

RULE.—I. *Find the interest for* 1 *year:*

II. *Divide this interest by* 12, *and the quotient will be the interest for* 1 *month:*

III. *Divide the interest for* 1 *month by* 30, *and the quotient will be the interest for* 1 *day.*

IV. *Multiply the interest for* 1 *year by the number of years, the interest for* 1 *month by the number of months, and the interest for* 1 *day by the number of days, and the sum of the products will be the required interest.*

NOTE.—In computing interest the month is reckoned at 30 days.

2. What is the interest of \$132,26 for 1 year 4 months and 10 days, at 6 per cent per annum?

3. What is the interest of \$25,50 for 1 year 9 months and 12 days, at 6 per cent?

---

254. How do you find the interest for any time at any rate?

2D METHOD.

255. There is another rule resulting from the last analysis, which is regarded as the best general method of computing interest.

RULE.—I. *Find the interest for* 1 *year and divide it by* 12: *the quotient will be the interest for* 1 *month.*

II. *Multiply the interest for* 1 *month by the time expressed in months and parts of a month, and the product will be the required interest.*

NOTE.—Since a month is reckoned at 30 days, any number of days is reduced to decimals of a month by dividing the days by 3.

EXAMPLES.

1. What is the interest of $327,50 for 3 years 7 months and 13 days, at 7 per cent?

OPERATION.

3*yrs.* = 36*mos.*	$327,50	
7*mos.*	.07	
13 *days* = .4⅓*mos.*	12)22.9250	= int. for 1 year.
Time = 43.4⅓*mos.*	1.9104 +	= int. for 1 month.
	43.4⅓	= time in months.
	.6368	
	76416	
	57312	
	76416	
	$82.97504 *Ans.*	

NOTE.—The method employed, and the number of decimal places used, in computing interest, may affect the mills, and *possibly*, the last figure in cents. It is best to use 4 places of decimals.

2. What is the interest of $1728,60, at 7 per cent, for 2 years 6 months and 21 days?

3. What is the interest of $288,30, at 7 per cent, for 1 year 8 months and 27 days?

4. What is the interest of $576,60, at 6 per cent, for 10 months and 18 days?

5. What is the interest of $854,42, at 6 per cent, for 3 months and 9 days?

6. What is the interest of $1153,20, at 6 per cent, for 11 months and 6 days?

---

255. How do you find the interest for years, months and days by the second method?

7. What is the interest of $2306,54, at 5 per cent, for 7 months and 28 days?

8. What is the interest of $4272,10, at 5 per cent, for 10 months and 28 days?

9. What is the interest of $1620, at 4 per cent, for 5 years nd 24 days?

10. What is the interest of $2430,72, at 4 per cent, for 10 years and 4 months?

11. What is the interest of $3689.45, at 7 per cent, for 4 years and 7 months?

12. What is the interest of $2945,96, at 7 per cent, for 7 years and 3 days?

13. What is the interest, at 8 per cent, of $675,89, for 5 years 6 months and 6 days?

14. What is the interest, at 8 per cent, on $12324, for 3 years and 4 months?

15. What is the interest, at 9 per cent, on $15328,20, for 4 years and 7 months?

16. What is the interest of $69450 for 1 year 2 months and 12 days, at 9 per cent?

17. What is the interest of $216,984 for 3 years 5 months and 15 days, at 10 per cent?

18. What is the interest of $648,54 for 7 years 6 months, at $4\frac{1}{2}$ per cent?

19. What is the interest of $1297,10 for 8 years 5 months, at $5\frac{1}{2}$ per cent?

20. What is the interest of $864,768 for 9 months 25 days, at $6\frac{1}{4}$ per cent?

21. What is the interest of $2594,20 for 10 months and 9 days, at $7\frac{1}{2}$ per cent?

22. What is the amount of $2376,84 for 3 years 9 months and 12 days, at $8\frac{1}{3}$ per cent?

23. What is the amount of $5148,40 for 7 years 11 months and 23 days, at $9\frac{1}{2}$ per cent?

24. What is the amount of $3565,20 for 3 years 9 months, at $10\frac{1}{2}$ per cent?

25. What is the amount of $125,75 for 1 year 9 months and 27 days, at 7 per cent?

26. What is the amount of $256 for 10 months 15 days, at $7\frac{1}{2}$ per cent?

27. What is the interest on a note of $264,42, given January 1st, 1852, and due Oct. 10th, 1855, at 4 per cent?

28. Gave a note of $793,26 April 6th, 1850, on interest at 7 per cent: what is due September 10th, 1852?

29. What amount is due on a note of hand given June 7th, 1850, for $512,50, at 6 per cent, to be paid Jan. 1st, 1851?

30. What is the interest on $1250,75 for 90 days, at 10 per cent?

31. What is the amount of $71,09 from Feb. 8th, 1848, to Dec. 7th, 1852, at $6\frac{3}{4}$ per cent?

32. What will be due on a note of $213,27 on interest after 90 days, at 7 per cent, given May 19th, 1836, and payable October 16th, 1838?

33. What is the interest of $426,54, from August 15th, 1837, to March 13th, 1840, at 7 per cent?

34. What is the interest of $2132,70, from Nov. 17th, 1838, to Feb. 2d, 1839, at $7\frac{1}{2}$ per cent?

35. What is the interest of $38463, from April 27th, 1815, to Sept. 2d, 1824, at 8 per cent?

36. What is the interest of $14231,50, from June 29th, 1840, to April 30th, 1845, at $8\frac{1}{4}$ per cent?

37. What is the interest of $426,50, from Sept. 4th, 1843, to May 4, 1849, at 9 per cent?

38. What is the interest of $4320, from Dec. 1st, 1817, to Jan. 22d, 1833, at $9\frac{1}{2}$ per cent?

39. What is the amount of $397,16, from March 23, 1824, to March 31st, 1835, at $10\frac{1}{2}$ per cent?

40. What is the amount of $328,12, from July 4th, 1809, to Feb. 15th, 1815, at 3 per cent?

41. What is the amount of $164,60, from Sept. 27th, 1845, to March 24th, 1855, at $1\frac{1}{2}$ per cent?

42. What is the amount of $1627,50, from July 4th, 1839. to August 1st, 1855, at 8 per cent?

CASE III.

256. *When the principal is in pounds shillings and pence.*

1. What is the interest, at 7 per cent, of £27 15*s.* 9*d.*, for 2 years?

OPERATION.

$$
\begin{array}{r}
£27\ 15s.\ 9d. = 27.7875 \\
.07 \\
\hline
1.945125 \\
2 \\
\hline
£3.890250
\end{array}
$$

$£.89025 = 17s.\ 9\frac{2}{4}d.$

Ans. £3 17*s.* $9\frac{2}{4}$*d.*

ANALYSIS.—The interest on pounds and decimals of a pound is found in the same way as the interest on dollars and decimals of a dollar: after which the decimal part of the interest may be reduced to shillings and pence: hence,

I. *Reduce the shillings and pence to the decimal of a pound and annex the result to the pounds.*

II. *Find the interest as though the sum were United States Money, after which reduce the decimal part to shilings and pence.*

2. What is the interest of £67 19*s.* 6*d.*, at 6 per cent, for 3 years 8 months 16 days?

3. What is the interest of £127 15*s.* 4*d.*, at 6 per cent, for 3 years and 3 months?

4. What is the interest of £107 16*s.* 10*d.*, at 7 per cent, for 3 years 6 months and 6 days?

5. What will £279 13*s.* 8*d.* amount to in 3 years and a half, at $5\frac{1}{4}$ per cent per annum?

PARTIAL PAYMENTS.

257. A PARTIAL PAYMENT is a payment of a part of a note or bond.

We shall give the rule established in New York (see Johnson's Chancery Reports, vol. i. page 17), for computing the interest on a bond or note, when partial payments have been made. The same rule is also adopted in Massachusetts, and in most of the other states.

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256. How do you find the interest when the principal is in pounds. shillings and pence?

RULE.—I. *Compute the interest on the principal to the time of the first payment, and if the payment exceed this interest, add the interest to the principal and from the sum subtract the payment: the remainder forms a new principal:*

II. *But if the payment is less than the interest, take no notice of it until other payments are made, which in all, shall exceed the interest computed to the time of the last payment: then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments: the remainder will form a new principal on which interest is to be computed as before.*

NOTE.—In computing interest on notes, observe that the day on which a note is dated and the day on which it falls due, are not both reckoned in determining the time, *but one of them is always excluded.* Thus, a note dated on the first day of May and falling due on the 16th of June, will bear interest but one month and 15 days.

EXAMPLES.

$349,998 *Buffalo, May 1st,* 1826.

1. For value received, I promise to pay James Wilson or order, three hundred and forty-nine dollars ninety-nine cents and eight mills, with interest at 6 per cent.

James Paywell.

On this note were endorsed the following payments:

Dec. 25th, 1826	Received	$49,998
July 10th, 1827	"	$ 4,998
Sept. 1st, 1828	"	$15,008
June 14th, 1829	"	$99,999

What was due April 15th, 1830?

Principal on int. from May 1st, 1826, - - - -	$349,998
Interest to Dec. 25th, 1826, time of first payment, 7 months 24 days - - - - - - -	13,649+
Amount - -	$363,647.

257. What is a partial payment? What is the rule for computing interest when there are partial payments?

Payment Dec. 25th, exceeding interest then due		$ 49,998
Remainder for a new principal - - - -		$313,649
Interest of $313,649 from Dec. 25, 1826, to June 14th, 1829, 2 years 5 months 19 days,		$ 46,4721
Amount - - - - -		$360,1211
Payment, July 10th, 1827, less than interest then due - - - - - -	$ 4,998	
Payment, Sept. 1st, 1828 - - - -	15,008	
Their sum less than interest then due	$20,006	
Payment, June 14th, 1829 - - - -	99,999	
Their sum exceeds the interest then due - -		$120,005
Remainder for a new principal, June 14, 1829,		$240,1161
Interest of $240,168 from June 14th, 1829, to April 15th, 1830, 10 months 1 day - - -		$ 12,0458
Total due, April 15th, 1830 -		$252,1619+

$3469,32 *New York, Feb.* 6, 1825.

2. For value received, I promise to pay William Jenks, or order, three thousand four hundred and sixty-nine dollars and thirty-two cents, with interest from date, at 6 per cent.

Bill Spendthrift.

On this note were endorsed the following payments :

May 16th, 1828, received $ 545,76.
May 16th, 1830, " $1276,00.
Feb. 1st, 1831, " $2074,72.

What remained due Aug. 11th, 1832 ?

3. A's note of $635,84 was dated September 5, 1817, on which were endorsed the following payments, viz. : Nov. 13th, 1819, $416,08 ; May 10th, 1820, $152,00 : what was due March 1st, 1821, the interest being 6 per cent ?

LEGAL INTEREST.

258. Legal Interest is the interest which the law permits a person to receive for money which he loans, and the laws do not favor the taking of a higher rate. In most of the States the rate is fixed at 6 per cent ; in New York, South Carolina and Georgia, it is 7 ; and in some of the States the rate is fixed as high as 10 per cent.

PROBLEMS IN INTEREST.

259. In all questions of Interest there are four things considered, viz. :

1*st*, The principal ; 2*d*, The rate of interest ; 3*d*, The time ; and 4*th*, The amount of interest.

If three of these are known, the fourth can be found.

I. Knowing the principal, rate, and time, to find the interest. This case has already been considered.

II. Knowing the interest, time, and rate, to find the principal.

Cast the interest on one dollar for the given time, and then divide the given interest by it—the quotient will be the principal.

III. Knowing the interest, the principal, and the time, to find the rate.

Cast the interest on the principal for the given time at 1 *per cent and then divide the given interest by it—the quotient will be the rate of interest.*

IV. Knowing the principal, the interest, and the rate, to find the time.

Cast the interest on the given principal at the given rate for 1 *year and then divide the interest by it—the quotient will be the time in years and decimals of a year.*

EXAMPLES.

1. The interest of a certain sum for 4 years, at 7 per cent. is $266 : what is the principal ?

2. The interest of $3675, for 3 years, is $771,75 : what is the rate ?

3. The principal is $459, the interest $183,60, and the rate 8 per cent : what is the time ?

4. The interest of a certain sum, for 3 years, at 6 per cent, is $40,50 : what is the principal ?

5. The principal is $918, the interest $269,28, and the rate 4 per cent : what is the time ?

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258 What is legal interest ?

259. How many things are considered in every question of interest ? What are they ? What is the rule for each ?

COMPOUND INTEREST.

260. Compound Interest is when the interest on a principal, computed to a given time, is added to the principal, and the interest then computed on this amount, as on a new principal. Hence,

Compute the interest to the time at which it becomes due; then add it to the principal and compute the interest on the amount as on a new principal: add the interest again to the principal and compute the interest as before; do the same for all the times at which payments of interest become due; from the last result subtract the principal, and the remainder will be the compound interest.

EXAMPLES.

1. What will be the compound interest, at 7 per cent, of $3750 for 2 years, the interest being added yearly?

OPERATION.

	$3750,000	principal for 1st year.
$3750 × .07 =	262,500	interest for 1st year.
	4012,500	principal for 2d "
$40 2,50 × .07 =	280,875	interest for 2d "
	4293,375	amount at 2 years.
1st principal	3750,000	
Amount of interest	$543,375.	

2. If the interest be computed annually, what will be the compound interest on $100 for 3 years, at 6 per cent?

3. What will be the compound interest on $295,37, at 6 per cent, for 2 years, the interest being added annually?

4. What will be the compound interest, at 5 per cent, of $1875, for 4 years?

5. What is the amount at compound interest of $250, for 2 years, at 8 per cent?

6. What is the compound interest of $939,64, for 3 years, at 7 per cent?

7. What will $125,50 amount to in 10 years, at 4 per cent compound interest?

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260. What of compound interest? How do you compute it?

NOTE.—The operation is rendered much shorter and easier, by taking the amount of 1 dollar for any time and rate given in the following table, and multiplying it by the given principal; the product will be the required amount, from which subtract the given principal, and the result will be the compound interest.*

TABLE.

Which shows the *amount* of $1 or £1, compound interest, from 1 year to 20, and at the rate of 3, 4, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	1.03000	1.04000	1.05000	1.06000	1.07000	1
2	1.06090	1.18160	1.10250	1.12360	1.14490	2
3	1.09272	1.12486	1.15762	1.19101	1.22504	3
4	1.12550	1.16985	1 21550	1.26247	1.31079	4
5	1.15927	1.21665	1.27628	1.33822	1.40255	5
6	1.19405	1.26531	1.34009	1.41851	1.50073	6
7	1.22987	1.31593	1.40710	1.50363	1.60578	7
8	1.26677	1.36856	1.47745	1.59384	1.71818	8
9	1.30477	1.42331	1.55132	1.68947	1.83845	9
10	1.34391	1.48028	1.62889	1.79084	1.96715	10
11	1.38423	1.53945	1.71033	1.89829	2.10485	11
12	1.42576	1.60103	1.79585	2.01219	2.25219	12
13	1.46853	1.66507	1.88564	2.13292	2.40984	13
14	1.51258	1.73167	1.97993	2.26090	2.57853	14
15	1.55796	1.80094	2.07892	2.39655	2.75903	15
16	1.60470	1.87298	2.18287	2.54035	2.95216	16
17	1.65284	1.94790	2.29201	2.69277	3.15881	17
18	1.70243	2.02581	2.40661	2.85433	3.37993	18
19	1.75350	2.10684	2.52695	3.02559	3.61652	19
20	1.80611	2.19112	2.65329	3.20713	3.86968	20

NOTE.—When there are months and days in the time, find the amount for the *years*, and on this amount cast the interest for the months and days: this, added to the last amount, will be the required *amount* for the whole time.

8. What is the amount of $96,50 for 8 years and 6 months, interest being compounded annually at 7 per cent?

9. What is the compound interest of $300 for 5 years 8 months and 15 days, at 6 per cent?

10. What is the compound interest of $1250 for 3 years 3 months and 24 days, at 7 per cent?

11. What will $56,50 amount to in 20 years and 4 months, at 5 per cent compound interest?

* The result may differ in the mills place from that obtained by the other rule

DISCOUNT.

261. Discount is an allowance made for the payment of money before it is due.

The face of a note is the amount named in the note.*

Note.—Days of grace are days allowed for the payment of a note after the expiration of the time named on its face. By mercantile usage a note does not *legally* fall due until 3 days after the expiration of the time named on its face, unless the note specifies *without grace.*

Days of grace, however, are generally confined to mercantile paper and to notes discounted at banks.

262. The present value of a note is such a sum as being put at interest until the note becomes due, would increase to an amount equal to the face of the note.

The *discount* on a note is the difference between the face of the note and its present value.

1. I give my note to Mr. Wilson for $107, payable in 1 year: what is the present value of the note, if the interest is 7 per cent? what the discount?

Analysis.—Since 1 dollar in 1 year, at 7 per cent, will amount to $1,07, the present value will be as many dollars as $1,07 is contained times in the face of the note: viz., $100: and the discount will be $107—$100=$7: hence,

OPERATION.

$107÷1,07=$100.

PROOF.

Int. $100 1*yr.*=$	7
Principal,	100
Amount,	$107
Discount,	7

Divide the face of the note by 1 dollar plus the interest of 1 dollar for the given time, and the quotient will be the present value: take this sum from the face of the note and the remainder will be the discount.

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261. What is discount? What is the face of a note? What are days of grace?

262. What is present value? What is the discount? How do you find the present value of a note?

* See Appendix, page 310.

EXAMPLES.

1. What is the present value of a note for $1828,75, due in 1 year, and bearing an interest of $4\frac{1}{2}$ per cent?

2. A note of $1651,50 is due in 11 months, but the person to whom it is payable sells it with the discount off at 6 per cent: how much shall he receive?

NOTE.—When payments are to be made at different times, *find the present value of the sums separately, and their sum will be the present value of the note.*

3. What is the present value of a note for $10500, on which $900 are to be paid in 6 months; $2700 in one year; $3900 in eighteen months; and the residue at the expiration of two years, the rate of interest being 6 per cent per annum?

4. What is the discount of £4500, one-half payable in six months and the other half at the expiration of a year, at 7 per cent per annum?

5. What is the present value of $5760, one-half payable in 3 months, one-third in 6 months, and the rest in 9 months, at 6 per cent per annum?

6. Mr. A gives his note to B for $720, one-half payable in 4 months and the other half in 8 months; what is the present value of said note, discount at 5 per cent per annum?

7. What is the difference between the interest and discount of $750, due nine months hence, at 7 per cent?

8. What is the present value of $4000 payable in 9 months, discount $4\frac{1}{2}$ per cent per annum?

9. Mr. Johnson has a note against Mr. Williams for $2146,50, dated August 17th, 1838, which becomes due Jan. 11th, 1839: if the note is discounted at 6 per cent, what ready money must be paid for it September 25th, 1838?

10. C owes D $3456, to be paid October 27th, 1842; C wishes to pay on the 24th of August, 1838, to which D consents; how much ought D to receive, interest at 6 per cent?

11. What is the present value of a note of $4800, due 4 years hence, the interest being computed at 5 per cent per annum?

12. A man having a horse for sale, offered it for $225 cash in hand, or $230 at 9 months; the buyer chose the latter: did the seller lose or make by his offer, supposing money to be worth 7 per cent?

BANK DISCOUNT.

263. BANK DISCOUNT is the charge made by a bank for the payment of money on a note before it becomes due.

By the custom of banks, this discount is the interest on the amount named in a note, calculated from the time the note is discounted to the time when it falls due; in which time the three days of grace are always included.

The interest is always *paid in advance.*

RULE.—*Add 3 days to the time which the note has to run, and then calculate the interest for that time at the given rate.*

EXAMPLES.

1. What is the bank discount of a note for $350, payable 3 months after date, at 7 per cent interest?

2. What is the bank discount of a note of $1000 payable in 60 days, at 6 per cent interest?

3. A merchant sold a cargo of cotton for $15720, for which he receives a note at 6 months: how much money will he receive at a bank for this note, discounting it at 6 per cent interest?

4. What is the bank discount on a note of $556,27 payable in 60 days, discounted at 6 per cent interest?

5. A has a note against B for $3456, payable in three months; he gets it discounted at 7 per cent interest. how much does he receive?

6. What is the bank discount on a note of $367.47, having 1 year, 1 month, and 13 days to run, as shown by the face of the note, discounted at 7 per cent?

7. For value received, I promise to pay to John Jones, on the 20th of November next, six thousand five hundred and seventy-nine dollars and 15 cents. What will be the discount on this, if discounted on the 1st of August, at 6 per cent per annum?

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263. What is bank discount? How is interest calculated by the custom of banks? How is the interest paid? How do you find the interest?

8. A merchant bought 175 barrels of flour at \$7,50 cents a barrel, and sells it immediately for \$9,75 a barrel, for which he receives a good note, payable in 6 months. If he should get this note discounted at a bank, at 6 per cent, what will be his gain on the flour?

264. *To make a note due at a future time, whose present value shall be a given amount.*

1. For what sum must a note be drawn at 3 months, so that when discounted at a bank, at 6 per cent, the amount received shall be \$500?

ANALYSIS.—If we find the interest on 1 dollar for the given time, and then subtract that interest from 1 dollar, the remainder will be the *present value* of 1 dollar, due at the expiration of that time. Then, the number of times which the present value of the note contains the present value of 1 dollar, will be the number of dollars for which the note must be drawn: hence,

Divide the present value of the note by the present value of 1 dollar, reckoned for the same time and at the same rate of interest, and the quotient will be the face of the note.

OPERATION.

Interest of \$1 for the time, 3*mo.* and 3*da.*=\$0,0155, which taken from \$1, gives present value of \$1=0,9845; then, \$500÷0,9845=\$507,872+=face of note.

PROOF.

Bank interest on \$507,872 for 3 months, including 3 days of grace, at 6 per cent=7,872, which being taken from the face of the note, leaves \$500 for its present value.

EXAMPLES.

1. For what sum must a note be drawn, at 7 per cent, payable on its face in 1 year 6 months and 15 days, so that when discounted at bank it shall produce \$307,27?

2. A note is to be drawn having on its face 8 months and 12 days to run, and to bear an interest of 7 per cent, so that it will pay a debt of \$5450: what is the amount?

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264. How do you make a note payable at a future time, whose present value shall be a given amount?

3. What sum, 6 months and 9 days from July 18th, 1856, drawing an interest of 6 per cent, will pay a debt of $674,89 at bank, on the 1st of August, 1856 ?

4. Mr. Johnson has Mr. Squires' note for $874,57, having 4 months to run, from July 13th, without interest. On the first of October he wishes to pay a debt at bank of $750,25, and discounts the note at 5 per cent in payment : how much must he receive back from the bank ?

5. Mr. Jones, on the 1st of June, desires to pay a debt at bank by a note dated May 16th, having 6 months to run and drawing 7 per cent interest : for what amount must the note be drawn, the debt being $1683,75 ?

6. Mr. Wilson is indebted at the bank in the sum of $367,464, which he wishes to pay by a note at 4 months with interest at 7 per cent : for what amount must the note be drawn ?

EQUATION OF PAYMENTS.

265. Equation of Payments is the operation of finding the mean time of payment of several sums due at different times, so that no interest shall be lost or gained.*

1. If I owe Mr. Wilson 2 dollars to be paid in 6 months, 3 dollars to be paid in 8 months, and 1 dollar to be paid in 12 months, what is the mean time of payment ?

OPERATION.

Int. of $2	for 6*mo.* = int. of $1 for	12*mo.*	$2 \times 6 = 12$
" of $3	for 8*mo.* = int. of $1 for	24*mo.*	$3 \times 8 = 24$
" of $1	for 12*mo.* = int. of $1 for	12*mo.*	$1 \times 12 = 12$
$6		48	48

Analysis.—The interest on all the sums, to the times of payment, is equal to the interest of $1 for 48 months. But 48 is equal to the sum of all the products which arise from multiplying each sum by the time at which it becomes due : hence, the sum of the products is equal to the time which would be necessary for $1 to produce the same interest as would be produced by all the principals.

* The mean time of payment is sometimes found by first finding *the present* value of each payment; but the rule here given has the sanction of the best authorities in this country and England.

17

If $1 will produce a certain interest in 48 months, in what time will $6 (or the sum of the payments) produce the same interest? The time is obviously found by dividing 48 (the sum of the products) by $6, (the sum of the payments.)

Hence, to find the mean time,

Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments: the quotient will be the mean time.

EXAMPLES.

1. B owes A $600; $200 is to be paid in two months, $200 in four months, and $200 in six months: what is the mean time for the payment of the whole?

ANALYSIS.—We here multiply each sum by the time at which it becomes due, and divide the sum of the products by the sum of the payments.

OPERATION.

$$
\begin{array}{r}
200 \times 2 = 400 \\
200 \times 4 = 800 \\
200 \times 6 = 1200 \\
\hline
6|00 \quad)24|00
\end{array}
$$

Ans. 4 *months.*

2. A merchant owes $600, of which $100 is to be paid in 4 months, $200 in 10 months, and the remainder in 16 months: if he pays the whole at once, in what time must he make the payment?

3. A merchant owes $600 to be paid in 12 months, $800 to be paid in 6 months, and $900 to be paid in 9 months: what is the equated time of payment?

4 A owes B $600; one-third is to be paid in 6 months, one-fourth in 8 months, and the remainder in 12 months: what is the mean time of payment?

5. A merchant has due him $300 to be paid in 60 days, $500 to be paid in 120 days, and $750 to be paid in 180 days: what is the equated time for the payment of the whole?

6. A merchant has due him $1500: one-sixth is to be paid in 2 months, one-third in 3 months, and the rest in 6 months: what is the equated time for the payment of the whole?

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265. What is equation of payments? How do you find th mean or equated time?

7 I owe \$1000 to be paid on the first of January, \$1500 on the 1st of February, \$3000 on the 1st of March, and \$4000 on the 15th of April : reckoning from the 1st of January, and calling February 28 days, on what day must the money be paid?

NOTE.—If one of the payments, as in the above example, is due on the day from which the equated time is reckoned, its corresponding product will be nothing, but the payment must still be added in finding the sum of the payments.

8. I owe Mr. Wilson \$100 to be paid on the 15th of July, \$200 on the 15th of August, and 300 on the 9th of September : what is the mean time of payment?

OPERATION.

From 1st of July to 1st payment 14 days.
" " " to 2d payment 45 days.
" " " to 3d payment 70 days.

Then by rule given above we have,

$$\begin{array}{rl} & 100 \times 14 = 1400 \\ & 200 \times 45 = 9000 \\ & 300 \times 70 = 21000 \\ \hline 600 & 6|00)\overline{314|00} \\ & \qquad 52\frac{1}{3} \end{array}$$

Hence, the equated time is $52\frac{1}{3}$ days from the 1st of July ; that is, on the 22d day of August.

But if we estimate the time from the 15th of July we shall have

From July 15th to 1st payment 0 days.
" " to 2d payment 30 days.
" " to 3d payment 54 days.

Then,

$$\begin{array}{rl} & 100 \times 0 = 000 \\ & 200 \times 30 = 6000 \\ & 300 \times 54 = 16200 \\ \hline 600 & 6|00)222|00 \\ & \qquad 37 \end{array}$$

Hence, the payment is due in 37 days from July 15th, or, on the 22d of August—the same as before.

Therefore: *Any day may be taken as the one from which the mean time is reckoned.*

NOTE.—If one payment is due on the day from which the time is reckoned, how do you treat it? Can you compute the time from any day?

9. Mr. Jones purchased of Mr. Wilson, on a credit of six months, goods to the following amounts :

15th of January,	a bill of $3750,
10th of February,	a bill of 3000,
6th of March,	a bill of 2400,
8th of June,	a bill of 2250.

He wishes, on the 1st of July, to give his note for the amount : at what time must it be made payable ?

10 Mr. Gilbert bought $4000 worth of goods : he was to pay $1600 in five months, $1200 in six months, and the remainder in eight months : what will be the time of credit, if he pays the whole amount at a single payment ?

11. A merchant bought several lots of goods, as follows :

A bill of $650,	June 6th,
A bill of 890,	July 8th,
A bill of 7940,	August 1st.

Now, if the credit is 6 months, how many days from December 6th before the note becomes due ? At what time ?

ASSESSING TAXES.

266. A tax is a certain sum required to be paid by the inhabitants of a town, county, or state, for the support of government. It is generally collected from each individual, in proportion to the amount of his property.

In some states, however, every white male citizen over the age of twenty-one years is required to pay a certain tax. This tax is called a poll-tax ; and each person so taxed is called a *poll.*

267. In assessing taxes, the first thing to be done is to make a complete inventory of all the property in the town on which the tax is to be laid. If there is a poll-tax, make a full list of the polls and multiply the number by the tax on each poll, and subtract the product from the whole tax to be

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266. What is a tax ? How is it generally collected ? What is a poll-tax ?

raised by the town: the remainder will be the amount to be raised on the property. Having done this, *divide the whole tax to be raised by the amount of taxable property, and the quotient will be the tax on* \$1. Then multiply this quotient by the inventory of each individual, and the product will be the tax on his property.

EXAMPLES.

1. A certain town is to be taxed \$4280; the property on which the tax is to be levied is valued at \$1000000. Now there are 200 polls, each taxed \$1,40. The property of A is valued at \$2800, and he pays 4 polls

B's at \$2400, pays 4 polls.	E's at \$7242, pays 4 polls.
C's at \$2530, pays 2 "	F's at \$1651, pays 6 "
D's at \$2250, pays 6 "	G's at \$1600,80 pays 4 "

What will be the tax on 1 dollar, and what will be A's tax, and also that of each on the list?

First, \$1,40 × 200 = \$280 amount of poll-tax.
\$4280 − \$280 = 4000 amount to be levied on property.
Then, \$4000 ÷ \$1000000 = 4 mills on \$1.
Now, to find the tax of each, as A's, for example,

A's inventory - - -	\$2800
	,004
	11,20
4 polls at \$1,40 each	5,60
A's whole tax - - -	\$16,80

In the same manner the tax of each person in the township may be found.

Having found the per cent, or the amount to be raised on each dollar, form a table showing the amount which certain sums would produce at the same rate per cent. Thus, after having found, as in the last example, that 4 mills are to be raised on every dollar, we can, by multiplying in succession by the numbers 1, 2, 3, 4, 5, 6, 7, 8, &c., form the following

267. What is the first thing to be done in assessing a tax? If there is a poll-tax, how do you find the amount? How then do you find the per cent of tax to be levied on a dollar? How do you then find the amount to be levied on each individual?

TABLE.

$		$	$		$	$		$
1	gives	0.004	20	gives	0.080	300	gives	1.200
2	"	0.008	30	"	0.120	400	"	1.600
3	"	0.012	40	"	0.160	500	"	2.000
4	"	0.016	50	"	0.200	600	"	2.400
5	"	0.020	60	"	0.240	700	"	2.800
6	"	0.024	70	"	0.280	800	"	3.200
7	"	0.028	80	"	0.320	900	"	3.600
8	"	0.032	90	"	0.360	1000	"	4 000
9	"	0.036	100	"	0.400	2000	"	8.000
10	"	0.040	200	"	0.800	3000	"	12.000

This table shows the amount to be raised on each sum in the columns under $'s.

EXAMPLES.

1. Find the amount of B's tax from this table.

B's tax on $2000 - -	is	-	$8.000
B's tax on 400 - -	is	-	$1.600
B's tax on 4 polls, at $1,40		-	$5.600
B's total tax - -	is	-	$15.200

2. Find the amount of C's tax from the table.

C's tax on $2000 - -	is	-	$8.000
C's tax on 500 - -	is	-	$2.000
C's tax on 30 - -	is	-	$0.120
C's tax on 2 polls - -	is	-	$2.800
C's total tax - - -	is	-	$12.920

In a similar manner, we might find the taxes to be paid by D, E, &c.

3. If the people of a town vote to tax themselves $1500, to build a public hall, and the property of the town is valued at $300,000, what is D's tax, whose property is valued at $2450?

4. In a school district a school is supported by a tax on the property of the district valued at $121340. A teacher is employed for 5 months at $40 a month, and contingent expenses are $42,68; what will be a farmer's tax whose property is valued at $3125?

COINS AND CURRENCY.

268. Coins are pieces of metal, of gold, silver, or copper, of fixed values, and impressed with a public stamp prescribed by the country where they are made. These are called specie, and are declared to be a legal tender in payment of debts. The Constitution of the United States provides, that gold and silver only shall be a legal tender.

269. Currency is what passes for money. In our country there are three kinds.

1*st*. The coins of the country:

2*d*. Foreign coins, having a fixed value established by law:

3*d*. Bank notes, redeemable in specie.

NOTE.—The foreign coins most in use in this country are the English shilling, valued at 22 cents 2 mills; the English sovereign, valued at \$4,84; the French franc, valued at 18 cents 6 mills; and the five-franc piece, valued at \$0,93.

Although the currency of the United States is in dollars, cents and mills, yet in some of the States accounts are still kept in pounds, shillings and pence.

In all the States the shilling is reckoned at 12 *pence, the pound at* 20 *shillings, and the dollar at* 100 *cents.*

The following table shows the number of shillings in a dollar, the value of £1 in dollars, and the value of \$1 in the fraction of a pound:

In English currency,	4*s*. 6*d*.	- £1=\$4,84,	and \$1=£$\frac{1}{4.84}$.
In N. E., Va., Ky., Tenn.,	6*s*.	- £1=\$$3\frac{1}{3}$,	and \$1=£ $\frac{3}{10}$.
In N. Y., Ohio, N. Carolina,	8*s*.	- £1=\$$2\frac{1}{2}$,	and \$1=£ $\frac{2}{5}$.
In N. J., Pa., Del. Md.,	7*s*. 6*d*.	- £1=\$$2\frac{2}{3}$,	and \$1=£ $\frac{3}{8}$.
In S. Carolina & Ga.	4*s*. 8*d*.	- £1=\$$4\frac{3}{7}$,	and \$1=£ $\frac{7}{30}$.
In Canada & Nova Scotia,	5*s*.	- £1=\$4,	and \$1=£ $\frac{1}{4}$.

268. What are coins? What are they called? What is made a legal tender?

REDUCTION OF CURRENCIES.

270. Reduction of Currencies is changing their denominations without changing their values.

There are two cases of the Reduction of Currencies:

1st. To change a currency in pounds shillings and pence, to United States currency.

2d. To change United States currency to pounds, shillings and pence.

271. *To reduce pounds, shillings and pence to United States currency.*

1. What is the value of £3 12*s*. 6*d*., New England currency, in United States money?

ANALYSIS.—Since £1 = \$3⅓, the number of dollars in £3 12*s*. 6*d*. = £3.625, will be equal to £3.625 taken 3⅓ times: that is, to \$12,08: hence,

OPERATION.

£3 12*s*. 6*d*. = £3.625
dolls. in £1 = $.3\frac{1}{3}$

1.208+
10.875

Ans. \$12.083+

Multiply the amount reduced to pounds and the decimals of a pound by the number of dollars in a pound, and the product will be the answer.

272. *To reduce United States money to pounds, shillings and pence.*

1. What is the value of \$375,87, in pounds, shillings and pence, New York currency?

ANALYSIS.— Since \$1 = £$\frac{2}{5}$, the number of pounds in \$375.87 will be equal to this number taken $\frac{2}{5}$ times: that is, equal to £150.348 = £150 6*s*. 11½*d*.: hence,

OPERATION.

$\$375.87 \times \frac{2}{5} = £150.348$
$= £150\ 6s.\ 11\frac{1}{2}d.$

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269. What is currency? How many kinds are there? What foreign coins are most used in this country? What are the denominations of United States currency? What denominations are sometimes used in the States?

270. What is reduction of currencies? How many kinds of reduction are there? What are they?

271. What is the rule for reducing from pounds, shillings and pence to United States money?

Multiply the amount by that fraction of a pound which denotes the value of $1, *and the product will be the answer in pounds and decimals of a pound.*

EXAMPLES.

1. What is the value of £127 18*s*. 6*d*., New England currency, in United States money?

2. What is the value of $2863.75 in pounds, shillings and pence, Pennsylvania currency?

3. What is the value of £459 3*s*. 6*d*., Georgia currency, in United States money?

4. What is the value of $973,28 in pounds, shillings and pence, North Carolina currency?

5. What is the value in United States money of £637 18*s*. 8*d*., Canada currency?

6. Reduce $102,85 to English money; to Canada currency; to New England currency; to New York currency; to Pennsylvania currency; to South Carolina currency.

7. Reduce £51 13*s*. 0$\frac{1}{4}$*d*. English money; £62 10*s*. Canada currency; £75 New England currency; £100 New York currency; £193 15*s*. Pennsylvania currency; and £58 6*s*. 7$\frac{3}{4}$*d*. Georgia currency, to United States money.

EXCHANGE.

273. EXCHANGE denotes the payment of a sum of money by a person residing in one place to a person residing in another. The payment is usually made by means of a bill of exchange.

A BILL OF EXCHANGE is an order from one person to another directing the payment to a third person named therein of a certain sum of money:

1. He who writes the open letter of request is called the *drawer* or *maker* of the bill.

2. The person to whom it is directed is called the *drawee*.

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272. What is the rule for reducing from United States money to pounds, shillings and pence?

273. What does exchange denote? How is the payment generally made? What is a bill of exchange? Who is the drawer? Who the drawee? Who the buyer or remitter?

3. The person to whom the money is ordered to be paid is called the *payee ;* and

4. Any person who purchases a bill of exchange is called the *buyer* or *remitter.*

274. A bill of exchange is called an *inland bill*, when the drawer and drawee both reside in the same country ; and when they reside in different countries, it is called a *foreign bill.*

Exchange is said to be at *par*, when an amount at the place from which it is remitted will pay an equal amount at the place to which it is remitted. Exchange is said to be at a *premium*, or *above par*, when the sum to be remitted will pay less at the place to which it is remitted ; and at a *discount*, or *below par*, when it will pay more.

EXAMPLES.

1. A merchant at Chicago wishes to pay a bill in New York amounting to \$3675, and finds that exchange is $1\frac{1}{4}$ per cent premium : what must he pay for his bill ?

2. A merchant in Philadelphia wishes to remit to Charleston \$8756,50, and finds exchange to be 1 per cent below par ; what must he pay for the bill ?

3. A merchant in Mobile wishes to pay in New York \$6584, and exchange is $2\frac{1}{2}$ per cent premium : how much must he pay for such a bill ?

4. A merchant in Boston wishes to pay in New Orleans \$4653,75 ; exchange between Boston and New Orleans is $1\frac{1}{2}$ per cent below par : what must he pay for a bill ?

5. A merchant in New York has \$3690 which he wishes to remit to Cincinnati ; the exchange is $1\frac{1}{4}$ per cent below par : what will be the amount of his bill ?

FOREIGN BILLS.

275. A Foreign Bill of Exchange is one in which the drawer and drawee live in different countries.

NOTE.—In all Bills of Exchange on England, the £ sterling is the unit or base, and is still reckoned at its former value of $\$4\frac{4}{9}$ $=\$4,4444+$, instead of its present value \$4,84.

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274. When is a bill of exchange said to be inland? When foreign? When is exchange said to be at par? When at a premium? When at a discount?

Hence,	£1 = \$4.4444 +
Add 9 per cent,	.3999
Gives the present value of £1	\$4.8443.

Hence, the true *par value* of Exchange on England is 9 per cent on the *nominal base.*

1. A merchant in New York wishes to remit to England a bill of Exchange for £125 15*s.* 6*d*: how much must he pay for this bill when exchange is at $9\frac{1}{2}$ per cent premium?

£125 15*s.* 6*d.* - - - - - -	= £125.775
Add $9\frac{1}{2}$ per cent - - - -	11.9486 +
gives amount in £'s, at $\$4\frac{4}{9} = \frac{40}{9}$.	£137.7236 +

NOTE.—The pounds and decimals of a pound are reduced to dollars by multiplying by 40 and dividing by 9—giving, in this case, \$612,105.

RULE.—I. *Reduce the amount of the bill to pounds and decimals of a pound, and then add the premium of exchange.*

II. *Multiply the result by* 40 *and divide the product by* 9 : *the quotient will be the answer in United States Money.*

2. A merchant shipped 100 bales of cotton to Liverpool, each weighing 450 pounds. They were sold at $7\frac{1}{2}$*d.* per pound, and the freight and charges amounted to £187 10*s.* He sold his bill of exchange at $9\frac{3}{4}$ per cent premium: how much should he receive in United States Money?

3. There were shipped from Norfolk, Va., to Liverpool, 85*hhd.* of tobacco, each weighing 450 pounds. It was sold at Liverpool for $12\frac{1}{2}$*d.* per pound, and the expenses of freight and commissions were £92 1*s.* 8*d.* If exchange in New York is at a premium of $9\frac{1}{4}$ per cent, what should the owner receive for the bill of exchange, in United States Money?

276. The *unit* or *base* of the French Currency is the French franc, of the value of 18 cents 6 mills. The franc is divided into tenths, called *decimes*, corresponding to our dimes, and into *centimes* corresponding to mills. Thus, 5.12 is read, 5 francs and 12 centimes.

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275. What is a foreign bill of exchange? In bills on England, what is the unit or base? What is the exchange value of the £ sterling? How much is the true value above the commercial value of the £ sterling? How do you find the value of a bill in English currency in United States money?

All bills of exchange on France are drawn in francs. Exchange is quoted in New York at so many francs and centimes to the dollar.

1. What will be the value of a bill of exchange for 4536 francs, at 5.25 francs to the dollar?

ANALYSIS.—Since 1 dollar will buy 5.25 francs, the bill will cost as many dollars as 5.25 is contained times in the amount of the bill: hence,

OPERATION.
5.25)4536($864 *Ans.*

Divide the amount of the bill by the value of $1 *in francs: the quotient is the amount to be paid in dollars.*

2. What will be the amount to be paid, United States money, for a bill of exchange on Paris, of 6530 francs,—exchange being 5.14 francs per dollar?

3. What will be the amount to be paid in United States money for a bill of exchange on Paris of 10262 francs, exchange being 5.09 francs per dollar?

4. What will be the value in United States money of a bill for 87595 francs, at 5.16 francs per dollar?

DUTIES.

277. Persons who bring goods or merchandise into the United States, from foreign countries, are required to land them at particular places or Ports, called Ports of Entry, and to pay a certain amount on their value, called a *Duty.* This duty is imposed by the General Government, and must be the same on the same articles of merchandise, in every part of the United States.

Besides the duties on merchandise, vessels employed in commerce are required, by law, to pay certain sums for the privilege of entering the ports. These sums are large or small, in proportion to the size or tonnage of the vessels. The moneys arising from duties and tonnage, are called *revenues.*

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276. What is the unit or base of the French currency? What is its value? How is it divided? In what currency are French bills of exchange drawn?

277. What is a port entry? What is a duty? By whom are duties imposed? What charges are vessels required to pay? What are the moneys arising from duties and tonnage called?

278. The revenues of the country are under the general direction of the Secretary of the Treasury, and to secure their faithful collection, the government has appointed various officers at each port of entry or place where goods may be landed.

279. The office established by the government at any port of entry is called a *Custom House*, and the officers attached to it are called Custom House Officers.

280. All duties levied by law on goods imported into the United States, are collected at the various custom houses, and are of two kinds, *Specific* and *Ad valorem.*

A *specific* duty is a certain sum on a particular kind of goods named; as so much per square yard on cotton or woollen cloths, so much per ton weight on iron, or so much per gallon on molasses.

An *ad valorem* duty is such a per cent on the actual cost of the goods in the country from which they are imported. Thus, an ad valorem duty of 15 per cent on English cloths, is a duty of 15 per cent on the cost of cloths imported from Engand.

281. The laws of Congress provide, that the cargoes of all vessels freighted with foreign goods or merchandise shall be weighed or gauged by the custom house officers at the port to which they are consigned. As duties are only to be paid on the articles, and not on the boxes, casks and bags which contain them, certain deductions are made from the weights and measures, called *Allowances.*

Gross Weight is the whole weight of the goods, together with that of the hogshead, barrel, box, bag, &c., which contains them.

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278. Under whose direction are the revenues of the country?

279. What is a custom house? What are the officers attached to it called?

280. Where are the duties collected? How many kinds are there, and what are they called? What is a specific duty? An ad valorem duty?

281. What do the laws of Congress direct in relation to foreign goods? Why are deductions made from their weight? What are these deductions called? What is gross weight? What is draft? What is the greatest draft allowed? What is tare? What are the different kinds of tare? What allowances are made on liquors?

Draft is an allowance from the gross weight on account of waste, where there is not actual tare.

On	112*lb.*			it is	1*lb.*
From	112	to	224	"	2,
"	224	to	336	"	3,
"	336	to	1120	"	4,
"	1120	to	2016	"	7,
Above	2016	any weight		"	9;

consequently, 9*lb.* is the greatest draft allowed.

Tare is an allowance made for the weight of the boxes, barrels, or bags containing the commodity, and is of three kinds: 1*st*, Legal tare, or such as is established by law; 2*d*, Customary tare, or such as is established by the custom among merchants; and 3*d*, Actual tare, or such as is found by removing the goods and actually weighing the boxes or casks in which they are contained.

On liquors in casks, *customary tare* is sometimes allowed on the supposition that the cask is not full, or what is called its *actual wants;* and then an allowance of 5 per cent for leakage.

A tare of 10 per cent is allowed on porter, ale and beer, in bottles, on account of breakage, and 5 per cent on all other liquors in bottles. At the custom house, bottles of the common size are estimated to contain $2\frac{3}{4}$ gallons the dozen.

Note.—For tables of Tare and Duty, see Ogden on the Tariff of 1842.

EXAMPLES.

1. What will be the duty on 125 cartons of ribbons, each containing 48 pieces, and each piece weighing 3*oz.* net, and paying a duty of $2,50 per pound?

2. What will be the duty on 225 bags of coffee, each weighing gross 160*lb.*, invoiced at 6 cents per pound; 2 per cent being the legal rate of tare, and 20 per cent the duty?

3. What duty must be paid on 275 dozen bottles of claret, estimated to contain $2\frac{3}{4}$ gallons per dozen, 5 per cent being allowed for breakage, and the duty being 35 cents per gallon?

4. A merchant imports 175 cases of indigo, each case weighing 196*lbs.* gross; 15 per cent is the customary rate of tare, and the duty 5 cents per pound: what duty must he pay on the whole?

ALLIGATION MEDIAL.

282. ALLIGATION MEDIAL is the process of finding the price of a mixture when the quantity of each simple and its price are known.

1. A merchant mixes 8*lb.* of tea, worth 75 cents a pound with 16*lb.* worth $1,02 a pound: what is the price of the mixture per pound?

ANALYSIS.—The quantity, 8*lb.* of tea, at 75 cents a pound, costs $6; and 16*lb.* at $1.02 costs $16,32: hence, the mixture, = 24*lb.*, costs $32,32; and the price of 1*lb.* of the mixture is found by dividing this cost by 24: hence, to find the price of the mixture,

OPERATION.

8*lb.* at 75*cts.* = $ 6,00
16*lb.* at $1,02 = $16,32
24   24)32,32
$0,93

I. *Find the cost of the entire mixture:*

II. *Divide the entire cost of the mixture by the sum of the simples, and the quotient will be the price of the mixture.*

EXAMPLES.

1. A farmer mixes 30 bushels of wheat worth 5*s.* per bushel, with 72 bushels of rye at 3*s.* per bushel, and with 60 bushels of barley worth 2*s.* per bushel: what should be the price of a bushel of the mixture?

2. A wine merchant mixes 15 gallons of wine at $1 per gallon with 25 gallons of brandy worth 75 cents per gallon: what should be the price of a gallon of the compound?

3. A grocer mixes 40 gallons of whisky worth 31 cents per gallon with 3 gallons of water which costs nothing: what should be the price of a gallon of the mixture?

4. A goldsmith melts together 2*lb.* of gold of 22 carats fine, 6*oz.* of 20 carats fine, and 6*oz.* of 16 carats fine: what is the fineness of the mixture?

5. On a certain day the mercury in the thermometer was observed to average the following heights: from 6 in the morning to 9, 64°; from 9 to 12, 74°; from 12 to 3, 84°; and from 3 to 6, 70°: what was the mean temperature of the day?

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282. What is Alligation Medial? What is the rule for determining the price of the mixture?

ALLIGATION ALTERNATE.

283. Alligation Alternate is the process of finding what proportions must be taken of each of several simples, whose prices are known, to form a compound of a given price. It is the opposite of Alligation Medial, and may be proved by it.

284. *To find the proportional parts:*

1. A farmer would mix oats at 3*s*. a bushel, rye at 6*s*., and wheat at 9*s*. a bushel, so that the mixture shall be worth 5 shillings a bushel: what proportion must be taken of each sort?

OPERATION

			A.	B.	C.	D.	E.
5	oats,	3	$\frac{1}{2}$	$\frac{1}{2}$	2	1	3
	rye,	6		1		2	2
	wheat,	9	$\frac{1}{4}$		1		1

Analysis.—On every bushel put into the mixture, whose price is *less* than the mean price, there will be a *gain;* on every bushel whose price is *greater* than the mean price, there will be a *loss;* and since there is to be neither gain nor loss by the mixture, the gains and losses must balance each other.

A bushel of oats, when put into the mixture, will bring 5 shillings, giving a gain of 2 shillings; and to gain 1 shilling, we must take half as much, or $\frac{1}{2}$ a bushel, which we write in column A.

On 1 bushel of wheat there will be a *loss* of 4 shillings; and to make a loss of 1 shilling, we must take $\frac{1}{4}$ of a bushel, which we also write in column A: $\frac{1}{2}$ and $\frac{1}{4}$ are called *proportional numbers.*

Again: comparing the oats and rye, there is a gain of 2 shillings on every bushel of oats, and a loss of 1 shilling on every bushel of rye: to gain 1 shilling on the oats, we take $\frac{1}{2}$ a bushel, and to lose 1 shilling on the rye, we take 1 bushel: these numbers are written in column B. Two simples, thus compared, are called a *couplet:* in one, the *price of unity is less than the mean price, and in the other it is greater.*

If, every time we take $\frac{1}{2}$ a bushel of oats we take $\frac{1}{4}$ of a bushel of wheat, the gain and loss will balance; and if every time we take $\frac{1}{2}$ a bushel of oats we take 1 bushel of rye, the gain and loss

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283. What is Alligation Alternate?
284. How do you find the proportional numbers?

will balance: hence, *if the proportional numbers of a couplet be multiplied by any number, the gain and loss denoted by the products, will balance.*

When the proportional numbers, in any column, are fractional (as in columns A and B), multiply them by the least common divisor of their denominators, and write the products in new columns C and D. Then, add the numbers in columns C and D, standing opposite each simple, and if their sums have a common factor, reject it: the last result will be the proportional numbers.

RULE.—I. *Write the prices or qualities of the simples in a column, beginning with the lowest, and the mean price or quality at the left.*

II. *Opposite the first simple write the part which must be taken to gain* 1 *of the mean price, and opposite the other simple of the couplet, write the part which must be taken to lose* 1 *of the mean price, and do the same for each simple.*

III. *When the proportional numbers are fractional, reduce them to integral numbers, and then add those which stand opposite the same simple: if the sums have a common factor, reject it: the result will denote the proportional parts.*

2. A merchant would mix wines worth 16*s*., 18*s*., and 22*s*. per gallon, in such a way, that the mixture may be worth 20*s*. per gallon: what are the proportional parts?

OPERATION.

		A.	B.	C.	D.	E.
20	16	$\frac{1}{4}$		1		1
	18		$\frac{1}{2}$		1	1
	22	$\frac{1}{2}$	$\frac{1}{2}$	2	1	3

PROOF.

1 gallon, at 16 shillings,	= 16*s*.
1 gallon, at 18 shillings,	= 18*s*.
3 gallon, at 22 shillings,	= 66*s*.
	5)100(20*s*., mean price.

NOTE.—The answers to the last, and to all similar questions, will be infinite in number, for two reasons:

1st. If the proportional numbers in column E be multiplied by any number, integral or fractional, the products will denote proportional parts of the simples.

2d. If the proportional numbers of *any couplet* be multiplied by any number, the gain and loss in that couplet will still balance, and the proportional numbers in the final result will be changed.

3. What proportions of tea, at 24 cents, 30 cents, 33 cents and 36 cents a pound, must be mixed together so that the mixture shall be worth 32 cents a pound?

4. What proportions of coffee at 16*cts.*, 20*cts.* and 28*cts.* per pound, must be mixed together so that the compound shall be worth 24*cts.* per pound?

5. A goldsmith has gold of 16, of 18, of 23, and of 24 carats fine. what part must be taken of each so that the mixture shall be 21 carats fine?

6. What portion of brandy, at 14*s.* per gallon, of old Madeira, at 24*s.* per gallon, of new Madeira, at 21*s.* per gallon, and of brandy, at 10*s.* per gallon, must be mixed together so that the mixture shall be worth 18*s.* per gallon?

285. *When the quantity of one simple is given:*

1. How much wheat, at 9*s.* a bushel, must be mixed with 20 bushels of oats worth 3 shillings a bushel, that the mixture may be worth 5 shillings a bushel?

ANALYSIS.—Find the proportional numbers: they are 2 and 1; hence, the ratio of the oats to the wheat is $\frac{1}{2}$: therefore, there must be 10 bushels of wheat.

RULE.—I. *Find the proportional numbers, and write the given simple opposite its proportional number.*

II. *Multiply the given simple by the ratio which its proportional number bears to each of the others, and the products will denote the quantities to be taken of each.*

EXAMPLES.

1. How much wine, at 5*s.*, at 5*s.* 6*d.*, and 6*s.* per gallon must be mixed with 4 gallons, at 4*s.* per gallon, so that the mixture shall be worth 5*s.* 4*d.* per gallon?

2. A farmer would mix 14 bushels of wheat, at $1,20 per bushel, with rye at 72*cts.*, barley at 48*cts.*, and oats at 36*cts.*: how much must be taken of each sort to make the mixture worth 64 cents per bushel?

3. There is a mixture made of wheat at 4*s.* per bushel, rye at 3*s.*, barley at 2*s.*, with 12 bushels of oats at 18*d.* per bushel: how much is taken of each sort when the mixture is worth 3*s.* 6*d.*?

4. A distiller would mix 40*gal.* of French brandy at 12*s.* per gallon, with English at 7*s.* and spirits at 4*s.* per gallon: what quantity must be taken of each sort that the mixture may be afforded at 8*s.* per gallon?

286. *When the quantity of the mixture is given.*

1. A merchant would make up a cask of wine containing 50 gallons, with wine worth 16*s.*, 18*s.* and 22*s.* a gallon, in such a way that the mixture may be worth 20*s.* a gallon: how much must he take of each sort?

ANALYSIS.—This is the same as example 2, except that the quantity of the mixture is given. If the quantity of the mixture be divided by 5, the sum of the proportional parts, the quotient 10 will show how many times each proportional part must be taken to make up 50 gallons: hence, there are 10 gallons of the first, 10 of the second, and 30 of the third: hence,

RULE.—I. *Find the proportional parts.*

II. *Divide the quantity of the mixture by the sum of the proportional parts, and the quotient will denote how many times each part is to be taken. Multiply this quotient by the parts separately, and each product will denote the quantity of the corresponding simple.*

EXAMPLES.

1. A grocer has four sorts of sugar, worth 12*d.*, 10*d.*, 6*d.* and 4*d.* per pound; he would make a mixture of 144 pounds worth 8*d.* per pound: what quantity must be taken of each sort?

2. A grocer having four sorts of tea, worth 5*s.*, 6*s.*, 8*s* and 9*s.* per pound, wishes a mixture of 87 pounds worth 7*s.* per pound: how much must he take of each sort?

3. A silversmith has four sorts of gold, viz., of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine; he would make a mixture of 42*oz.* of 17 carats fine; how much must be taken of each sort?

PROOF.—All the examples of Alligation Medial may be found by Alligation Alternate.

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285. How do you find the quantity of each simple when the quantity of one simple is known?

286. How do you find the quantity of each simple when the quantity of each mixture is known?

INVOLUTION.

287. A POWER is the product of equal factors. The equal factor is called the *root* of the power.

The *first power* is the *equal factor* itself, or the *root:*

The *second power* is the product of the root by itself :

The *third power* is the product when the root is taken 3 times as a factor :

The *fourth power*, when it is taken 4 times :

The *fifth power*, when it is taken 5 times, &c.

288. The number denoting how many times the root is taken as a factor, is called the *exponent* of the power. It is written a little at the right and over the root : thus, if the equal factor or root is 4.

$$4 = 4 \text{ the 1st power of 4.}$$
$$4^2 = 4 \times 4 = 16 \text{ the 2d power of 4.}$$
$$4^3 = 4 \times 4 \times 4 = 64 \text{ the 3d power of 4.}$$
$$4^4 = 4 \times 4 \times 4 \times 4 = 256 \text{ the 4th power of 4.}$$
$$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024 \text{ the 5th power of 4.}$$

INVOLUTION *is the process of finding the powers of numbers.*

NOTES.—1. There are three things connected with every power : 1st, The root ; 2d, The exponent ; and 3d, The power or result of the multiplication.

2. In finding a power, the root is always the 1st power : hence, the number of multiplications is 1 less than the exponent :

RULE.—*Multiply the number by itself as many times less 1 as there are units in the exponent, and the last product will be the power.*

EXAMPLES.

Find the powers of the following numbers :

1. Square of 1.
2. Square of $\frac{1}{2}$.
3. Cube of $\frac{1}{8}$.
4. Square of $\frac{3}{5}$.
5. Square of 9.
6. Cube of 12.
7. 3d power of 125.
8. 3d power of 16.
9. 4th power of 9.
10. 5th power of 16.
11. 6th power of 20.
12. 2d power of 225.
13. Square of 2167.
14. Cube of 321.
15. 4th power of 215.
16. 5th power of 906.
17. 6th power of 9.
18. Square of 36049.

EVOLUTION.

289. Evolution is the process of finding the factor when we know the power.

The *square root* of a number is the factor which multiplied by itself *once* will produce the number.

The *cube root* of a number is the factor which multiplied y itself *twice* will produce the number.

Thus, 6 is the square root of 36, because $6\times6=36$; and 3 is the cube root of 27, because $3\times3\times3=27$.

The sign $\sqrt{\ }$ is called the radical sign. When placed before a number it denotes that its square root is to be extracted. Thus, $\sqrt{36}=6$.

We denote the cube root by the same sign by writing 3 over it: thus, $\sqrt[3]{27}$ denotes the cube root of 27, which is equal to 3. The small figure 3, placed over the radical, is called the *index* of the root.

EXTRACTION OF THE SQUARE ROOT.

290. The *square root* of a number is a factor which multiplied by itself *once* will produce the number. To extract the square root is to find this factor. The first ten numbers and their squares are

1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

The numbers in the first line are the square roots of those in the second. The numbers 1, 4, 9, 16, 25, 36, &c. having *exact* factors, are called *perfect squares*.

A perfect square is a number which has two *exact factors*.

Note.—The square root of a number less than 100 will be less than 10, while the square root of a number greater than 100 will be greater than 10.

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287. What is a power? What is the root of a power? What is the first power? What is the second power? The third power?

288. What is the exponent of the power? How is it written? What is Involution? How many things are connected with every power? How do you find the power of a number?

289. What is Evolution? What is the square root of a number? What is the cube root of a number? How do you denote the square root of a number? How the cube root?

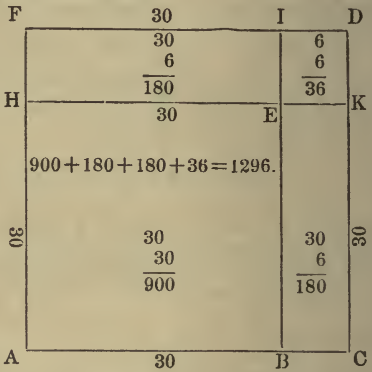
291. What is the square of $36=3$ tens$+6$ units?

ANALYSIS.—$36=3$ tens$+6$ units, is first to be taken 6 units' times, giving $6^2+3\times6$: then taking it 3 tens' times, we have $3\times6+3^2$, and the sum is $3^2+2(3\times6)+6^2$: that is,

$$\begin{array}{r} 3+6 \\ 3+6 \\ \hline 3\times6+6^2 \\ 3^2+3\times6 \quad\;\; \\ \hline 3^2+2(3\times6)+6^2 \end{array}$$

The square of a number is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

The same may be shown by the figure:



Let the line AB represent the 3 tens or 30, and BC the six units.

Let AD be a square on AC, and AE a square on the ten's line AB.

Then ED will be a square on the unit line 6, and the rectangle EF will be the product of HE, which is equal to the ten's line, by IE, which is equal to the unit line. Also, the rectangle BK will be the product of EB, which is equal to the ten's line, by the unit line B C. But the whole square on AC is made up of the square AE, the two rectangles FE and EC, and the square ED.

1. Let it now be required to extract the square root of 1296.

ANALYSIS.—Since the number contains more than two places of figures, its root will contain tens and units. But as the square of one ten is one hundred, it follows that the square of the tens of the required root must be found in the two figures on the left of 96. Hence, we point off the number into periods of two figures each.

---

290. What is the square root of a number? What are perfect squares? How many are there between 1 and 100?

291. Into what parts may a number be decomposed? When so decomposed, what is its square equal to?

We next find the greatest square contained in 12, which is 3 tens or 30. We then square 3 tens which gives 9 hundred, and then place 9 under the hundreds' place, and subtract; this takes away the square of the tens, and leaves 396, *which is twice the product of the tens by the units plus the square of the units.*

OPERATION.

```
 1̇2 9̇6(36
 9
 ---
66)396
   396
```

If now, we double the divisor and then divide this remainder, exclusive of the right hand figure, (since that figure cannot enter into the product of the tens by the units) by it, the quotient will be the units figure of the root. If we annex this figure to the augmented divisor, and then multiply the whole divisor thus increased by it, the product will be twice the tens by the units plus the square of the units; and hence, we have found both figures of the root.

This process may also be illustrated by the figure.

Subtracting the square of the tens is taking away the square AE and leaves the two rectangles FE and BK, together with the square ED on the unit line.

The two rectangles FE and BK representing the product of units by tens, can be expressed by no figures less than tens.

If, then, we divide the figures 39, at the left of 6, by twice the tens, that is, by twice AB or BE, the quotient will be BC or EK, the unit of the root

Then, placing BC or 6, in the root, and also annexing it to the divisor doubled, and then multiplying the whole divisor 66 by 6, we obtain the two rectangles FE and CE, together with the square ED.

292. Hence, for the extraction of the square root, we have the following

RULE.—I. *Separate the given number into periods of two figures each, by setting a dot over the place of units, a second over the place of hundreds, and so on for each alternate figure at the left.*

II. *Note the greatest square contained in the period on the left, and place its root on the right after the manner of a quotient in division. Subtract the square of this root from the first period, and to the remainder bring down the second period for a dividend.*

---

292. What is the first step in extracting the square root of numbers? What is the second? What is the third? What the fourth? What the fifth? Give the entire rule

III. *Double the root thus found for a trial divisor and place it on the left of the dividend. Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand figure, and place the quotient in the root and also annex it to the divisor.*

IV. *Multiply the divisor thus increased, by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Double the whole root thus found, for a new trial divisor, and continue the operation as before, until all the periods are brought down.*

EXAMPLES.

1. What is the square root of 263169?

OPERATION.

```
      2̇6 3̇1 6̇9(513
      25
  101)131
      101
 1023)3069
      3069
```

ANALYSIS.—We first place a dot over the 9, making the right-hand period 69. We then put a dot over the 1 and also over the 6, making three periods.

The greatest perfect square in 26 is 25, the root of which is 5. Placing 5 in the root, subtracting its square from 26, and bringing down the next period 31, we have 131 for a dividend, and by doubling the root we have 10 for a trial divisor. Now. 10 is contained in 13, 1 time. Place 1 both in the root and in the divisor: then multiply 101 by 1; subtract the product and bring down the next period.

We must now double the whole root 51 for a new trial divisor; or we may take the first divisor after having doubled the last figure 1; then dividing, we obtain 3, the third figure of the root.

NOTES.—1. The left-hand period may contain but one figure; each of the others will contain two.

2. If any trial divisor is greater than its dividend, the corresponding quotient figure will be a cipher.

3. If the product of the divisor by any figure of the root exceeds the corresponding dividend, the quotient figure is too large and must be diminished.

4. There will be as many figures in the root as there are periods in the given number.

5. If the given number is not a perfect square there will be a remainder after all the periods are brought down. In this case, periods of ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2 What is the square root of 36729?

OPERATION.

```
      3 67 29(191 64+,
      1
   29)267
      261
   381)629
       381
  3826)24800
       22956
 .38324)184400
        153296
         31104 Rem
```

In this example there are two periods of decimals, which give two places of decimals in the root.

293. *To extract the square root of a fraction*

1. What is the square root of .5?

OPERATION.

```
      .50(.707+
       49
   140)100
       000
  1407)10000
        9849
         151 Rem.
```

NOTE.—We first annex one cipher to make even decimal places. We then extract the root of the first period: to the remainder we annex two ciphers, forming a new period, and so on.

2. What is the square root of $\frac{4}{9}$?

NOTE.—The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

OPERATION.

$$\sqrt{\frac{4}{9}}=\frac{\sqrt{4}}{\sqrt{9}}=\frac{2}{3}.$$

3. What is the square root of $\frac{3}{4}$?

NOTE.—When the terms are not perfect squares, reduce the common fraction to a decimal fraction, and then extract the square root of the decimal.

OPERATION.

$\frac{3}{4}=.75$;

$\sqrt{\frac{3}{4}}=\sqrt{.75}=.8545+$

---

293. How do you extract the square root of a decimal fraction? How of a common fraction?

RULE.—I. *If the fraction is a decimal, point off the periods from the decimal point to the right, annexing ciphers if necessary, so that each period shall contain two places, and then extract the root as in integral numbers.*

II. *If the fraction is a common fraction, and its terms perfect squares, extract the square root of the numerator and denominator separately; if they are not perfect squares, reduce the fraction to a decimal, and then extract the square root of the result.*

EXAMPLES.

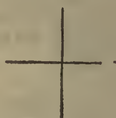
What are the square roots of the following numbers?

1. of 3?
2. of 11?
3. of 1069?
4. of 2268741?
5. of 7596796?
6. of 36372961?
7. of 22071204?
8. of 3271.4207?
9. of 4795.25731?
10. of 4.372594?
11. of .0025?
12. of .00032754?
13. of .00103041?
14. of 4.426816?
15. of $8\frac{3}{6}$?
16. of $9\frac{3}{4}$?
17. of $\frac{64}{125}$?
18. of $\frac{125}{729}$?
19. of $\frac{2304}{5148}$?
20. of $\frac{2704}{4225}$?

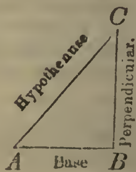
APPLICATIONS IN SQUARE ROOT.

294. A triangle is a plain figure which has three sides and three angles.

If a straight line meets another straight line, making the adjacent angles equal, each is called a right angle; and the lines are said to be perpendicular to each other.

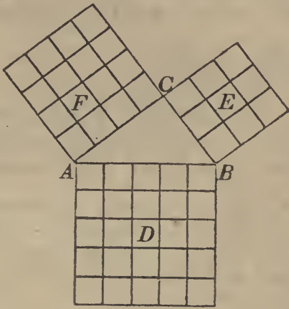


295. A right angled triangle is one which has one right angle. In the right angled triangle ABC, the side AC opposite the right angle B is called the *hypothenuse;* the side AB the *base;* and the side BC the *perpendicular*



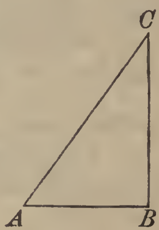
296. In a right angled triangle the square described in the hypothenuse is equal to the sum of the squares described in the other two sides.

Thus, if ACB be a right angled triangle, right angled at C, then will the large square, D, described in the hypothenuse AB, be equal to the sum of the squares F and E described on the sides AC and CB. This is called the carpenter's theorem. By counting the small squares in the large square D, you will find their number equal to that contained in the small squares F and E. In this triangle the hypothenuse AB=5, AC=4, and CB=3. Any numbers having the same ratio, as 5, 4 and 3, such as 10, 8 and 6; 20, 16 and 12, &c., will represent the sides of a right angled triangle.



1. Wishing to know the distance from A to the top of a tower, I measured the height of the tower and found it to be 40 feet; also the distance from A to B and found it 30 feet; what was the distance from A to C?

$$AB=30;\ AB^2=30^2=900$$
$$BC=40;\ BC^2=40^2=1600$$
$$AC^2=AB^2+BC^2=\overline{2500}$$
$$AC=\sqrt{2500}=50 \text{ feet.}$$



297. Hence, when the base and perpendicular are known and the hypothenuse is required,

294 What is a triangle? What is a right angle?
295 What is a right angled triangle? Which side is the hypothenuse?
296. In a right angled triangle what is the square on the hypothenuse equal to?

Square the base and square the perpendicular, add the results and then extract the square root of their sum.

2. What is the length of a rafter that will reach from the eaves to the ridge pole of a house, when the height of the roof is 15 feet and the width of the building 40 feet ?

298. *To find one side when we know the hypothenuse and the other side.*

3. The length of a ladder which will reach from the middle of a street 80 feet wide to the eaves of a house, is 50 feet : what is the height of the house ?

ANALYSIS.—Since the square of the length of the ladder is equal to the sum of the squares of half the street and the height of the house, the square of the length of the ladder diminished by the square of half the street will be equal to the square of the height of the house : hence,

Square the hypothenuse and the known side, and take the difference ; the square root of the difference will be the other side.

EXAMPLES.

1. If an acre of land be laid out in a square form, what will be the length of each side in rods ?

2. What will be the length of the side of a square, in rods, that shall contain 100 acres?

3. A general has an army of 7225 men : how many must be put in each line in order to place them in a square form ?

4. Two persons start from the same point ; one travels due east 50 miles, the other due south 84 miles : how far are they apart ?

5. What is the length, in rods, of one side of a square that shall contain 12 acres ?

6. A company of speculators bought a tract of land for $6724, each agreeing to pay as many dollars as there were partners : how many partners were there ?

---

297. How do you find the hypothenuse when you know the base and perpendicular ?

298. If you know the hypothenuse and one side, how do you find the other side ?

7. A farmer wishes to set out an orchard of 3844 trees, so that the number of rows shall be equal to the number of trees in each row: what will be the number of trees?

8. How many rods of fence will enclose a square field of 10 acres?

9. If a line 150 feet long will reach from the top of a steeple 120 feet high, to the opposite side of the street, what is the width of the street?

10. What is the length of a brace whose ends are each $3\frac{1}{2}$ feet from the angle made by the post and beam?

CUBE ROOT.

299. The Cube Root of a number is one of three equal factors of the number.

To extract the cube root of a number is to find a factor which multiplied into itself twice, will produce the given number.

Thus, 2 is the cube root of 8; for, $2 \times 2 \times 2 = 8$: and 3 is the cube root of 27; for $3 \times 3 \times 3 = 27$.

1,	2,	3,	4,	5,	6,	7,	8,	9.
1	8	27	64	125	216	343	512	729

The numbers in the first line are the cube roots of the corresponding numbers of the second. The numbers of the second line are called *perfect cubes.* By examining the numbers of the two lines we see,

1st. That the cube of units cannot give a higher order than hundreds.

2d. That since the cube of one ten (10) is 1000 and the cube of 9 tens (90), 81000, *the cube of tens will not give a lower denomination than thousands, nor a higher denomination than hundreds of thousands.*

Hence, if a number contains more than three figures, its cube root will contain more than one: if it contains more than six, its root will contain more than two, and so on; every additional three figures giving one additional figure in the root, and the figures which remain at the left hand, although less than three, will also give a figure in the root. This law explains the reason for pointing off into periods of three figures each

300. Let us now see how the cube of any number, as 16, is formed. Sixteen is composed of 1 ten and 6 units, and may be written $10+6$. To find the cube of 16 or of $10+6$, we must multiply the number by itself twice.

To do this we place the number thus		16=	10+	6
			10+	6
product by the units			60+	36
product by the tens		100+	60	
Square of 16		100+	120+	36
Multiply again by 16			10+	6
product by the units		600+	720+	216
product by the tens	1000+	1200+	360	
Cube of 16	1000+	1800+	1080+	216

1. By examining the parts of this number it is seen that the first part 1000 is the *cube of the tens;* that is,

$$10\times10\times10=1000.$$

2. The second part 1800 is *three times the square of the tens multiplied by the units;* that is,

$$3\times(10)^2\times6=3\times100\times6=1800.$$

3. The third part 1080 is *three times the square of the units multiplied by the tens;* that is,

$$3\times6^2\times10=3\times36\times10=1080$$

4. The fourth part is *the cube of the units;* that is,

$$6^2=6\times6\times6=216.$$

1. What is the cube root of the number 4096?

OPERATION.

$$\begin{array}{r}
\dot{4}\ 09\dot{6}(16 \\
1 \\
1^2\times3=3)3\ 0\ \ (9\text{–}8\text{–}7\text{–}6 \\
16^3=4\ 096.
\end{array}$$

ANALYSIS.—Since the number contains more than three figures, we know that the root will contain at least units and tens.

Separating the three right-hand figures from the 4, we know that the cube of the tens will be found in the 4; and 1 is the greatest cube in 4.

299. What is the cube root of a number? How many perfect cubes are there between 1 and 1000?

300. Of how many parts is the cube of a number composed? What are they?

Hence, we place the root 1 on the right, and this is the tens of the required root. We then cube 1 and subtract the result from 4, and to the remainder we bring down the first figure 0 of the next period.

We have seen that the second part of the cube of 16, viz 1800, is *three times the square of the tens multiplied by the units :* and hence, it can have no significant figure of a less denomination than hundreds. It must, therefore, make up a part of the 30 hundreds above. But this 30 hundreds also contains all the hundreds which come from the 3d and 4th parts of the cube of 16. If it were not so, the 30 hundreds, divided by three times the square of the tens, would give the unit figure exactly.

Forming a divisor of three times the square of the tens, we find the quotient to be ten; but this we know to be too large. Placing 9 in the root and cubing 19, we find the result to be 6859. Then trying 8 we find the cube of 18 still too large; but when we take 6 we find the exact number. Hence, the cube root of 4096 is 16.

301. Hence, to find the cube root of a number,

RULE.—I. *Separate the given number into periods of three figures each, by placing a dot over the place of units, a second over the place of thousands, and so on over each third figure to the left; the left hand period will often contain less than three places of figures.*

II. *Note the greatest perfect cube in the first period, and set its root on the right, after the manner of a quotient in division. Subtract the cube of this number from the first period, and to the remainder bring down the first figure of the next period for a dividend.*

III. *Take three times the square of the root just found for a trial divisor, and see how often it is contained in the dividend, and place the quotient for a second figure of the root. Then cube the figures of the root thus found, and if their cube be greater than the first two periods of the given number, diminish the last figure, but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period for a new dividend.*

IV. *Take three times the square of the whole root for a second trial divisor, and find a third figure of the root. Cube the whole root thus found and subtract the result from the first three periods of the given number when it is less than that number, but if it is greater, diminish the figure of the root; proceed in a similar way for all the periods.*

EXAMPLES.

1. What is the cube root of 99252847 ?

$$
\begin{array}{lrl}
 & & \dot{9}9\ \ 25\dot{2}\ \ 84\dot{7}(463 \\
 & 4^3 = & 64 \\
 & 4^2\times 3 = 48) & \overline{352} \quad \text{dividend.} \\
\text{First two periods} \quad - \quad - \quad - & & \dot{9}9\ \ 25\dot{2} \\
(46)^3 = 46\times 46\times 46 = & & 97\ \ 336 \\
 & 3\times(46)^2 = 6348\ ) & \overline{\ 19168\ } \ \text{2d dividend.} \\
\text{The first three periods} \quad - & & \dot{9}9\ \ 25\dot{2}\ \ 84\dot{7} \\
 & (463)^3 & = 99\ \ 252\ \ 848
\end{array}
$$

Find the cube roots of the following numbers :

1. Of 389017 ?
2. Of 5735339 ?
3. Of 32461759 ?
4. Of 84604519 ?
5. Of 259694072 ?
6. Of 48228544 ?

302. To extract the cube root of a decimal fraction,

Annex ciphers to the decimal, if necessary, so that it shall consist of 3, 6, 9, *&c., places. Then put the first point over the place of thousandths, the second over the place of millionths, and so on over every third place to the right; after which extract the root as in whole numbers.*

NOTES.—1. There will be as many decimal places in the root as there are periods in the given number.

2. The same rule applies when the given number is composed of a whole number and a decimal.

3. If in extracting the root of a number there is a remainder after all the periods have been brought down, periods of ciphers may be annexed by considering them as decimals.

EXAMPLES.

Find the cube roots of the following numbers ;

1. Of .157464 ?
2. Of .870983875 ?
3. Of 12.977879 ?
4. Of .751089429 ?
5. Of .353393243 ?
6. Of 3.408862625 ?

---

301. What is the rule for extracting the cube root ?

302. How do you extract the cube root of a decimal fraction ? How many decimal places will there be in the root ? Will the same rule apply when there is a whole number and a decimal ? If in extracting the root of any number you find a decimal, how do you proceed ?

303. To extract the cube root of a common fraction.

I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms.*

II. *Extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in the last case.*

EXAMPLES.

Find the cube roots of the following fractions:

1. Of $\frac{250}{686}$?
2. Of $31\frac{15}{343}$?
3. Of $\frac{324}{1500}$?
4. Of $\frac{4}{7}$?
5. Of $\frac{5}{9}$?
6. Of $\frac{2}{3}$?

APPLICATIONS.

1. What must be the length, depth, and breadth of a box, when these dimensions are all equal and the box contains 4913 cubic feet?

2. The solidity of a cubical block is 21952 cubic yards: what is the length of each side? What is the area of the surface?

3. A cellar is 25 feet long 20 feet wide, and $8\frac{1}{2}$ feet deep: what will be the dimensions of another cellar of equal capactiy in the form of a cube?

4. What will be the length of one side of a cubical granary that shall contain 2500 bushels of grain?

5. How many small cubes of 2 inches on a side can be sawed out of a cube 2 feet on a side, if nothing is lost in sawing?

6. What will be the side of a cube that shall be equal to the contents of a stick of timber containing 1728 cubic feet?

7. A stick of timber is 54 feet long and 2 feet square: what would be its dimensions if it had the form of a cube?

NOTES.—1. Bodies are said to be similar when their like parts are proportional.

2. It is found that the contents of similar bodies are to each other as the cubes of their like dimensions.

---

303. How do you extract the cube root of a vulgar fraction?

3. All bodies named in the examples are supposed to be similar.

8. If a sphere of 4 feet in diameter contains 33.5104 cubic feet, what will be the contents of a sphere 8 feet in diameter?

$$4^3 \; : \; 8^3 \; : : \; 33.5104 \; : \; Ans.$$

9. If the contents of a sphere 14 inches in diameter is 1436.7584 cubic inches, what will be the diameter of a sphere which contains 11494·0672 cubic inches?

10. If a ball weighing 32 pounds is 6 inches in diameter, what will be the diameter of a ball weighing 964 pounds?

11. If a haystack, 24 feet in height, contains 8 tons of hay, what will be the height of a similar stack that shall contain but 1 ton?

ARITHMETICAL PROGRESSION.

304. An Arithmetical Progression is a series of numbers in which each is derived from the preceding one by the addition or subtraction of the same number.

The number added or subtracted is called the *common difference.*

305. If the common difference is added, the series is called an *increasing series.*

Thus, if we begin with 2, and add the common difference, 3, we have

2, 5, 8, 11, 14, 17, 20, 23, &c.,

which is *an increasing series.*

If we begin with 23, and subtract the common difference 3, we have

23, 20, 17, 14, 11, 8, 5, &c.,

which is a *decreasing series.*

---

304. What is an arithmetical progression? What is the number added or subtracted called?

305. When the common difference is added, what is the series called? What is it called when the common difference is subtracted? What are the several numbers called? What are the first and last called? What are the intermediate ones called?

The several numbers are called the *terms* of the progression or series: the first and last are called the *extremes*, and the intermediate terms are called *means*.

306. In every arithmetical progression there are five parts:

1st, the first term;
2d, the last term;
3d, the common difference;
4th, the number of terms;
5th, the sum of all the terms.

If any three of these parts are known or given, the remaining ones can be determined.

CASE I.

307. *Knowing the first term, the common difference, and the number of terms, to find the last term.*

1. The first term is 3, the common difference 2, and the number of terms 19: what is the last term?

ANALYSIS.—By considering the manner in which the increasing progression is formed, we see that the 2d term is obtained by adding the common difference to the 1st term; the 3d, by adding the common difference to the 2d; the 4th, by adding the common difference to the 3d, and so on; *the number of additions being* 1 *less than the number of terms found.*

But instead of making the additions, we may multiply the common difference by the number of additions, that is, by 1 less than the number of terms, and add the first term to the product: hence,

OPERATION.

18	No. less 1
2	Com. dif.
36	
3	1st term.
39	last term.

RULE.—*Multiply the common difference by* 1 *less than the number of terms; if the progression is increasing, add the product to the first term and the sum will be the last term; if it is decreasing, subtract the product from the first term and the difference will be the last term.*

---

306 How many parts are there in every arithmetical progression? What are they? How many parts must be given before the remaining ones can be found?

EXAMPLES.

1. A man bought 50 yards of cloth, for which he was to pay 6 cents for the 1st yard, 9 cents for the 2d, 12 cents for the 3d, and so on increasing by the common difference 3: how much did he pay for the last yard?

2. A man puts out $100 at simple interest, at 7 per cent: at the end of the 1st year it will have increased to $107, at the end of the 2d year to $114, and so on, increasing $7 each year: what will be the amount at the end of 16 years?

3. What is the 40th term of an arithmetical progression of which the first term is 1, and the common difference 1?

4. What is the 30th term of a descending progression of which the first term is 60, and the common difference 2?

5. A person had 35 children and grandchildren, and it so happened that the difference of their ages was 18 months, and the age of the eldest was 60 years: how old was the youngest?

CASE II.

308. *Knowing the two extremes and the number of terms, to find the common difference.*

1. The extremes of an arithmetical progression are 8 and 104, and the number of terms 25: what is the common difference?

ANALYSIS.—Since the common difference multiplied by 1 less than the number of terms gives a product equal to the difference of the extremes, if we *divide* the difference of the extremes by 1 less than the number of terms, the quotient will be the *common difference:* hence,

OPERATION.

$$\begin{array}{r} 104 \\ 8 \\ \hline 25-1=24)96(4. \end{array}$$

RULE.—*Subtract the less extreme from the greater and divide the remainder by* 1 *less than the number of terms; the quotient will be the common difference.*

---

307. When you know the first term, the common difference, and the number of terms, how do you find the last term?

308. When you know the extremes and the number of terms, how do you find the common difference?

EXAMPLES.

1. A man has 8 sons, the youngest is 4 years old and the eldest 32 : their ages increase in arithmetical progression : what is the common difference of their ages?

2. A man is to travel from New York to a certain place in 12 days ; to go 3 miles the first day, increasing every day by the same number of miles ; the last day's journey is 58 miles: required the daily increase.

3. A man hired a workman for a month of 26 working days, and agreed to pay him 50 cents for the first day, with a uniform daily increase ; on the last day he paid $1,50 : what was the daily increase?

CASE III.

309. *To find the sum of the terms of an arithmetical progression.*

1. What is the sum of the series whose first term is 3, common difference 2, and last term 19 ?

Given series	3+ 5+ 7+ 9+11+13+14+17+19= 99
Same; order of terms inverted.	19+17+15+13+11+ 9+ 7+ 5+ 3= 99
Sum of both.	22 22 22 22 22 22 22 22 22=198

ANALYSIS.—The two series are the same ; hence, their sum is equal to twice the given series. But their sum is equal to the sum of the two extremes 3 and 19 taken as many times as there are terms ; and the given series is equal to half this sum, or to the sum of the extremes multiplied by half the number of terms.

RULE.—*Add the extremes together and multiply their sum by half the number of terms ; the product will be the sum of the series.*

EXAMPLES.

1. The extremes are 2 and 100, and the number of terms 22 : what is the sum of the series ?

ANALYSIS.—We first add together the two extremes and then multiply by half the number of terms.

OPERATION.

2	1st term.
100	last term.
102	sum of extremes.
11	half the number of terms.
1122	sum of series.

309. How do you find the sum of the terms?

2. How many strokes does the hammer of a clock strike in 12 hours?

3. The first term of a series is 2, the common difference 4, and the number of terms 9: what is the last term and sum of the series?

4. James, a smart chap, having learned arithmetical progression, told his father that he would chop a load of wood of 15 logs, at 2 cents the first log, with a regular increase of 1 cent for each additional log: how much did James receive for chopping the wood?

5. An invalid wishes to gain strength by regular and increasing exercise; his physician assures him that he can walk 1 mile the first day, and increase the distance half a mile for each of the 24 following days: how far will he walk?

6. If 100 eggs are placed in a right line, exactly one yard from each other, and the first one yard from a basket: what distance will a man travel who gathers them up singly, and places them in the basket?

GEOMETRICAL PROGRESSION.

310. A Geometrical Progression is a series of terms, each of which is derived from the preceding one, by multiplying it by a constant number. The constant multiplier is called the *ratio* of the progression.

311. If the ratio is *greater* than 1, each term is greater than the preceding one, and the series is said to be *increasing*.

---

310. What is a geometrical progression? What is the constant multiplier called?

311. If the ratio is greater than 1, how do the terms compare with each other? What is the series then called? If the ratio is less than 1, how do they compare? What is the series then called? What are the several numbers called? What are the first and last called? What are the intermediate ones called?

312. How many parts are there in every geometrical progression? What are they? How many must be known before the others can be found?

If the ratio is *less* than 1, each term is less than the preceding one, and the series is said to be *decreasing;* thus,

1, 2, 4, 8, 16, 32, &c.—ratio 2—increasing series:
32, 16, 8, 4, 2, 1, &c.—ratio $\frac{1}{2}$—decreasing series.

The several numbers are called *terms* of the progression. The first and last are called the *extremes*, and the intermediate terms are called *means.*

312. In every Geometrical, as well as in every Arithmetical Progression, there are five parts:

1st, the first term;
2d, the last term;
3d, the common ratio;
4th, the number of terms;
5th, the sum of all the terms.

If any three of these parts are known, or given, the remaining ones can be determined.

CASE I.

313. *Having given the first term, the ratio, and the number of terms, to find the last term.*

1. The first term is 3 and the ratio 2: what is the 6th term?

OPERATION.

$$2\times2\times2\times2\times2=2^5=32$$
$$3 \text{ 1st term}$$
Ans. 96

ANALYSIS.—The second term is formed by multiplying the first term by the ratio; the third term by multiplying the second term by the ratio, and so on; the number of multiplicators *being 1 less than the number of terms:* thus,

$3=3$ 1st term,
$3\times2=6$ 2d term,
$3\times2\times2=3\times2^2=12$ 3d term,
$3\times2\times2\times2=3\times2^3=24$ 4th term, &c.

Therefore, *the last term is equal to the first term multiplied by the ratio raised to a power* 1 *less than the number of terms.*

RULE.—*Raise the ratio to a power whose exponent is* 1 *less than the number of terms, and then multiply this power by the first term.*

EXAMPLES.

1. The first term of a decreasing progression is 192; the ratio $\frac{1}{2}$, and the number of terms 7: what is the last term?

NOTE.—The 6th power of the ratio, ($\frac{1}{2}$,) is $\frac{1}{64}$, and this multiplied by the first term 192, gives the last term 3.

OPERATION.
$$(\tfrac{1}{2})^6 = \tfrac{1}{64}$$
$$192 \times \tfrac{1}{64} = 3.$$

2. A man purchased 12 pears; he was to pay 1 farthing for the 1st, 2 farthings for the 2d, 4 for the 3d, and so on, doubling each time: what did he pay for the last?

3. The first term of a decreasing progression is 1024, the ratio $\frac{1}{2}$: what is the 9th term?

4. The first term of an increasing progression is 4, and the common difference 3: what is the 10th term?

5. A gentleman dying left nine sons, and bequeathed his estate in the following manner: to his executors $50; his youngest son to have twice as much as the executors, and each son to have double the amount of the son next younger: what was the eldest son's portion?

6. A man bought 12 yards of cloth, giving 3 cents for the 1st yard, 6 for the 2d, 12 for the 3d, &c.: what did he pay for the last yard?

CASE II.

314. *Knowing the two extremes and the ratio, to find the sum of the terms.*

1. What is the sum of the terms, in the progression, 1, 4, 16, 64?

---

313. Knowing the first term, the ratio, and the number of terms, how do you find the last term?

314. Knowing the two extremes and the ratio, how do you find the sum of the terms?

ANALYSIS.—If we multiply the terms of the progression by the ratio 4, we have a second progression, 4, 16, 64, 256, which is 4 *times as great* as the first. If from this we subtract the first, the remainder, 256−1, will be 3 *times* as great as the first; and if the remainder be divided by 3, the quotient will be the sum of the terms of the first progression. But 256 is the product of the last term of the given progression multiplied by the ratio, 1 is the first term, and the divisor 3 is 1 less than the ratio: hence,

OPERATION.

$$\begin{array}{rcl} 4+16+64+256 & = & 4 \text{ times.} \\ 1+4+16+64 & = & \underline{} \text{ once.} \\ \hline & & 256-1=3 \text{ times.} \end{array}$$

$$\frac{256-1}{3}=\frac{255}{3}=85 \text{ sum.}$$

RULE.—*Multiply the last term by the ratio; take the difference between the product and the first term and divide the remainder by the difference between* 1 *and the ratio.*

NOTE.—When the progression is *increasing*, the first term is subtracted from the product of the last term by the ratio, and the divisor is found by subtracting 1 from the ratio. When the progression is *decreasing*, the product of the last term by the ratio is subtracted from the first term, and the ratio is subtracted from 1.

EXAMPLES.

1. The first term of a progression is 2, the ratio 3, and the last term 4374: what is the sum of the terms?

2. The first term of a progression is 128, the ratio $\frac{1}{2}$, and the last term 2: what is the sum of the terms?

3. The first term is 3, the ratio 2, and the last term 192: what is the sum of the series?

4. A gentleman gave his daughter in marriage on New Year's day, and gave her husband 1*s.* towards her portion, and was to double it on the first day of every month during the year: what was her portion?

5. A man bought 10 bushels of wheat on the condition that he should pay 1 cent for the 1st bushel, 3 for the 2d, 9 for the 3d, and so on to the last: what did he pay for the last bushel, and for the 10 bushels?

6. A man has 6 children: to the 1st he gives $150, to the 2d $300, to the 3d $600, and so on, to each twice as much as the last: how much did the eldest receive, and what was the amount received by them all?

PROMISCUOUS EXAMPLES.

1. A merchant bought 13 packages of goods, for which he paid \$326: what will 39 packages cost at the same rate?

2. How many bushels of oats at $62\frac{1}{2}$ cents a bushel will pay for 4250 feet of lumber at \$7,50 per thousand?

3. Bought 2*hhd.* of sugar which weighed as follows: the 1st 5*cwt.* 1*qr.* 18*lb.*, the 2d 6*cwt.* 10*lb.*: what did it cost at 7 cents per pound?

4. How many hours between the 4th of Sept., 1854, at 3 P.M., and the 20th day of April, 1855, at 10 A.M.?

5. If $\frac{5}{6}$ of a gallon of wine cost $\frac{5}{8}$ of a dollar, what will $\frac{5}{9}$ of a hogshead cost?

6. What number is that which being multiplied by $\frac{2}{3}$ will produce $\frac{1}{4}$?

7. A tailor had a piece of cloth containing $24\frac{1}{2}$ yards, from which he cut $6\frac{5}{8}$ yards: how much was there left?

8. From $\frac{3}{4}$ of $\dfrac{6\frac{4}{5}}{12}$ take $\frac{1}{3}$ of $\dfrac{1\frac{7}{8}}{5}$.

9. What is the difference between $3\frac{3}{8}+7\frac{5}{6}$ and $4+2\frac{11}{12}$?

10. There was a company of soldiers, of whom $\frac{1}{8}$ were on guard, $\frac{1}{6}$ preparing dinner, and the remainder, 85 men, were drilling: how many were there in the company?

11. The sum of two numbers is 425, and their difference 1.625: what are the numbers?

12. The sum of two numbers is $\frac{5}{7}$, and their difference $\frac{1}{3}$: what are the numbers?

13. The product of two numbers is 2.26, and one of the numbers is .25: what is the other?

14. If the divisor of a certain number be $6.66\frac{2}{3}$, and the quotient $\frac{5}{8}$, what will be the dividend?

15. A person dying, divided his property between his widow and his four sons; to his widow he gave \$1780, and to each of his sons \$1250; he had been $25\frac{1}{2}$ years in business, and had cleared on an average \$126 dollars a year: how much had he when he began business?

16. A besieged garrison consisting of 360 men was provisioned for 6 months, but hearing of no relief at the end of five months, dismissed so many of the garrison, that the remaining provision lasted 5 months: how many men were sent away?

17. Two persons, A and B are indebted to C; A owes \$2173, which is the least debt, and the difference of the debts is \$571: what is the amount of their indebtedness?

18. What number added to the 43d part of 4429, will make the sum 240?

19. How many planks 15 feet long, and 15 inches wide, will floor a barn $60\frac{1}{2}$ feet long, and $33\frac{1}{2}$ feet wide?

20. A person owned $\frac{3}{5}$ of a mine, and sold $\frac{3}{4}$ of his interest for \$1710: what was the value of the entire mine?

21. A room 30 feet long, and 18 feet wide, is to be covered with painted cloth $\frac{3}{4}$ of a yard wide: how many yards will cover it?

22. A, B and C trade together and gain \$120, which is to be shared according to each one's stock; A put in \$140, B \$300, and C \$160: what is each man's share?

23. A can do a piece of work in 12 days, and B can do the same work in 18 days: how long will it take both, if they work together?

24. If a barrel of flour will last one family $7\frac{1}{2}$ months, a second family 9 months, and a third $11\frac{1}{4}$ months, how long will it last the three families together?

25. Suppose I have $\frac{3}{16}$ of a ship worth \$1200; what part have I left after selling $\frac{2}{5}$ of $\frac{4}{9}$ of my share, and what is it worth?

26. What number is that which being multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2}$, the product will be 1?

27. Divide \$420 between three persons, so that the second shall have $\frac{3}{4}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two?

28. What is the difference between twice five and fifty, and twice fifty-five?

29. What number is that which being multiplied by three-thousandths, the product will be 2637?

30. What is the difference between half a dozen dozens and six dozen dozens?

31. The slow or parade step is 70 paces per minute, at 28 inches each pace: how fast is that per hour?

32. A lady being asked her age, and not wishing to give a direct answer, said, "I have 9 children, and three years elapsed between the birth of each of them; the eldest was born when I was 19 years old, and the youngest is now exactly 19:" what was her age?

33. A wall of 700 yards in length was to be built in 29 days: 12 men were employed on it for 11 days, and only completed 220 yards: how many men must be added to complete the wall in the required time?

34. Divide \$10429,50 between three persons, so that as often as one gets \$4, the second will get \$6, and the third \$7.

35. A gentleman whose annual income is £1500, spends 20 guineas a week: does he save, or run in debt, and how much?

36. A farmer exchanged 70 bushels of rye, at \$0,92 per bushel, for 40 bushels of wheat, at $\$1,37\frac{1}{2}$ a bushel, and received the balance in oats, at \$0,40 per bushel: how many bushels of oats did he receive?

37. In a certain orchard $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ of them bear peaches, $\frac{1}{6}$ of them plums, 120 of them cherries, and 80 of them pears: how many trees are there in the orchard?

38. A person being asked the time, said, the time past noon is equal to $\frac{1}{5}$ of the time past midnight: what was the hour?

39. If 20 men can perform a piece of work in 12 days, how many men will accomplish thrice as much in one-fifth of the time?

40. How many stones 2 feet long, 1 foot wide, and 6 inches thick, will build a wall 12 yards long, 2 yards high, and 4 feet thick?

41. Four persons traded together on a capital of $6000, of which A put in $\frac{1}{2}$, B put in $\frac{1}{4}$, C put in $\frac{1}{6}$, and D the rest; at the end of 4 years they had gained $4728: what was each one's share of the gain?

42. A cistern containing 60 gallons of water has three unequal cocks for discharging it; the largest will empty it in one hour, the second in two hours, and the third in three hours: in what time will the cistern be emptied if they run together?

43. A man bought $\frac{5}{8}$ of the capital of a cotton factory at par; he retained $\frac{4}{5}$ of his purchase, and sold the balance for $5000, which was 15 per cent advance on the cost; what was the whole capital of the factory?

44. Bought a cow for $30 cash, and sold her for $35 at a credit of 8 months: reckoning the interest at 6 per cent, how much did I gain?

45. If, when I sell cloth for 8*s.* 9*d.* per yard, I gain 12 per cent, what will be the gain when it is sold for 10*s.* 6*d.* per yard?

46. How much stock at par value can be purchased for $8500, at $8\frac{1}{2}$ per cent premium, $\frac{1}{4}$ per cent being paid to the broker?

47. Twelve workmen, working 12 hours a day, have made, in 12 days, 12 pieces of cloth, each piece 75 yards long: how many pieces of the same stuff would have been made, each piece 25 yards long, if there had been 7 more workmen?

48. A person was born on the 1st day of Oct., 1801, at 6 o'clock in the morning: what was his age on the 21st of Sept., 1854, at half-past 4 in the afternoon?

49. A, can do a piece of work alone in 10 days, and B in 13 days: in what time can they do it if they work together?

50. A man went to sea at 17 years of age; 8 years after he had a son born, who lived 46 years, and died before his father; after which the father lived twice twenty years and died: what was the age of the father?

51. How many bricks, 8 inches long and 4 inches wide, will pave a yard that is 100 feet by 50 feet?

52. If a house is 50 feet wide, and the post which supports the ridge pole is 12 feet high, what will be the length of the rafters?

53. A man had 12 sons, the youngest was 3 years old and the eldest 58, and their ages increased in Arithmetical progression what was the common difference of their ages?

54. If a quantity of provisions serves 1500 men 12 weeks, at the rate of 20 ounces a day for each man, how many men will the same provisions maintain for 20 weeks, at the rate of 8 ounces a day for each man?

55. A man bought 10 bushels of wheat, on the condition that he should pay 1 cent for the 1st bushel, 3 for the 2d, 9 for the 3d, and so on to the last: what did he pay for the last bushel, and for the 10 bushels?

56. There is a mixture made of wheat at 4*s.* per bushel, rye at 3*s.*, barley at 2*s.*, with 12 bushels of oats at 18*d.* per bushel: how much must be taken of each sort to make the mixture worth 3*s.* 6*d.* per bushel:

57. What length must be cut off a board $8\frac{1}{2}$ inches broad to contain a square foot?

58. What is the difference between the interest of $2500 for 4 years 9 mo. at 6 per cent, and half that sum for twice the time, at half the same rate per cent?

59. A person lent a certain sum at 4 per cent per annum; had this remained at interest 3 years, he would have received for principal and interest $9676,80: what was the principal?

60. If 1 pound of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons, what is the value of a pound of tea, when a lemon is worth 2 cents?

61. A person bought 160 oranges at 2 for a penny, and 180 more at 3 for a penny; after which he sold them out at the rate of 5 for 2 pence: did he make or lose, and how much?

62. A snail in getting up a pole 20 feet high, was observed to climb up 8 feet every day, but to descend 4 feet every night: in what time did he reach the top of the pole?

63. A ship has a leak by which it would fill and sink in 15 hours, but by means of a pump it could be emptied, if full, in 16 hours. Now, if the pump is worked from the time the leak begins, how long before the ship will sink?

64. A and B can perform a certain piece of work in 6 days, B and C in 7 days, and A and C in 14 days: in what time would each do it alone?

65. Divide $500 among 4 persons, so that when A has $\frac{1}{2}$ dollar, B shall have $\frac{1}{3}$, C $\frac{1}{4}$, and D $\frac{1}{6}$.

66. A man purchased a building lot containing 3600 square feet, at the cost of $1.50 per foot, on which he built a store at an expense of $3000. He paid yearly $180,66 for repairs and taxes: what annual rent must he receive to obtain 10 per cent. on the cost?

67. A's note of $7851,04 was dated Sept. 5th, 1837, on which were endorsed the following payments, viz.: Nov. 13th, 1839, $416,98; May 10th, 1840, $152: what was due March 1st., 1841 the interest being 6 per cent?

68. A house is 40 feet from the ground to the eaves, and it is required to find the length of a ladder which will reach the eaves. supposing the foot of the ladder cannot be placed nearer to the house than 30 feet?

69. Sound travels about 1142 feet in a second; now, if the flash of a cannon be seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun?

70. A person dying, worth $5460, left a wife and 2 children. a son and daughter, absent in a foreign country. He directed that if his son returned, the mother should have one-third of the estate, and the son the remainder; but if the daughter returned, she should have one-third, and the mother the remainder. Now it so happened that they both returned: how must the estate be divided to fulfill the father's intentions?

71. Two persons depart from the same place, one travels 32, and the other 36 miles a day: if they travel in the same direction, how far will they be apart at the end of 19 days, and how far if they travel in contrary directions?

72. In what time will $2377,50 amount to $2852,42, at 4 per cent. per annum?

73. What is the height of a wall, which is $14\frac{1}{2}$ yards in length, and $\frac{7}{10}$ of a yard in thickness, and which has cost $406, it having been paid for at the rate of $10 per cubic yard?

74. What will be the duty on 225 bags of coffee, each weighing gross 160 lbs., invoiced at 6 cents per lb.; 2 per cent. being the legal rate of tare, and 20 per cent. the duty?

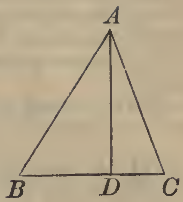
75. Three persons purchase a piece of property for $9202; the first gave a certain sum; the second three times as much; and the third one and a half times as much as the other two: what did each pay?

76. A reservoir of water has two cocks to supply it. The first would fill it in 40 minutes, and the second in 50. It has likewise a discharging cock, by which it may be emptied when full in 25 minutes. Now, if all the cocks are opened at once, and the water runs uniformly as we have supposed, how long before the cistern will be filled?

77. A traveller leaves New Haven at 8 o'clock on Monday morning, and walks towards Albany at the rate of 3 miles an hour: another traveller sets out from Albany at 4 o'clock on the same evening, and walks towards New Haven at the rate of 4 miles an hour; now, supposing the distance to be 130 miles, where on the road will they meet?

MENSURATION.

315. A triangle is a portion of a plane bounded by three straight lines. BC is called the *base;* and AD, perpendicular to BC, the *altitude.*



316. To find the area of a triangle

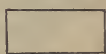
The area or contents of a triangle is equal to half the product of its base by its altitude (*Bk.* IV. *Prop.* VI).*

EXAMPLES.

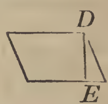
1. The base, BC, of a triangle is 40 yards, and the perpendicular, AD, 20 yards: what is the area?
2. In a triangular field the base is 40 chains, and the perpendicular 15 chains: how much does it contain? (Art. 110.)
3. There is a triangular field, of which the base is 35 rods and the perpendicular 26 rods: what are its contents?

317. A square is a figure having four equal sides, and all its angles right angles.

318. A rectangle is a four-sided figure like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.



319. A parallelogram is a four-sided figure which has its opposite sides equal and parallel, but its angles not right angles. The line DE, perpendicular to the base, is called the altitude.



320. To find the area of a square, rectangle, or parallelogram,

Multiply the base by the perpendicular height, and the product will be the area. (*Book* IV. *Prop.* V).

EXAMPLES.

1. What is the area of a square field of which the sides are each 33.08 chains?
2. What is the area of a square piece of land of which the sides are 27 chains?
3. What is the area of a square piece of land of which the sides are 25 rods each?

* All the references are to Davies' Legendre.

4. What are the contents of a rectangular field, the length of which is 40 rods and the breadth 20 rods?

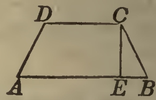
5. What are the contents of a field 40 rods square?

6. What are the contents of a rectangular field 15 chains long and 5 chains broad?

7. What are the contents of a field 27 chains long and 9 rods broad?

8. The base of a parallelogram is 271 yards, and the perpendicular height 360 feet: what is the area?

321. A trapezoid is a four-sided figure ABCD, having two of its sides, AB, DC, parallel. The perpendicular CE is called the altitude.



322. To find the area of a trapezoid.

Multiply the sum of the two parallel sides by the altitude, and divide the product by 2, *the quotient will be the area.* (*Bk.* IV. *Prop.* VII).

EXAMPLES.

1. Required the area of the trapezoid ABCD, having given
$AB=321.51ft.$, $DC=214.24ft.$, and $CE=171.16ft.$

2. What is the area of a trapezoid, the parallel sides of which are 12.41 and 8.22 chains, and the perpendicular distance between them 5.15 chains?

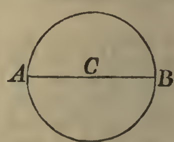
3. Required the area of a trapezoid whose parallel sides are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches.

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and the perpendicular distance between them 10.75 yards.

5. What is the area of a trapezoid whose parallel sides are 7.50 chains, and 12.25 chains, and the perpendicular height 15.40 chains?

6. What are the contents when the parallel sides are 20 and 32 chains, and the perpendicular distance between them 26 chains?

323. A circle is a portion of a plane bounded by a curved line, called the *circumference.* Every point of the circumference is equally distant from a certain point within called the *centre:* thus, C is the centre, and any line, as ACB, passing through the centre, is called a *diameter.*



If the diameter of a circle is 1, the circumference will be 3.1416. Hence, *if we know the diameter, we may find the circumference by multiplying by* 3.1416 *; or, if we know the circumference, we may find the diameter by dividing by* 3.1416.

EXAMPLES.

1. The diameter of a circle is 4, what is the circumference?
2. The diameter of a circle is 93, what is the circumference?
3. The diameter of a circle is 20, what is the circumference?
4. What is the diameter of a circle whose circumference is 78.54?
5. What is the diameter of a circle whose circumference is 11652.1944?
6. What is the diameter of a circle whose circumference is 6850?

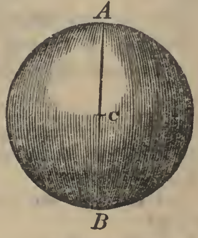
324. To find the area or contents of a circle,

Multiply the square of the diameter by the decimal .7854 (*Bk.* V. *Prop.* XII. *Cor.* 2).

EXAMPLES.

1. What is the area of a circle whose diameter is 6?
2. What is the area of a circle whose diameter is 10?
3. What is the area of a circle whose diameter is 7?
4. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet?

325. A sphere is a figure terminated by a curved surface, all the parts of which are equally distant from a certain point within called the centre. The line AB passing through its centre C is called the diameter of the sphere, and AC its radius.



326. To find the surface of a sphere,

Multiply the square of the diameter by 3.1416 (*Bk.* VIII. *Prop.* X. *Cor*).

EXAMPLES.

1. What is the surface of a sphere whose diameter is 12?
2. What is the surface of a sphere whose diameter is 7?
3. Required the number of square inches in the surface of a sphere whose diameter is 2 feet or 24 inches.

327. To find the contents of a sphere,

Multiply the surface by the diameter and divide the product by 6, *the quotient will be the contents.* (*Bk.* VIII. *Prop.* XIV. *Sch.* 3).

EXAMPLES.

1. What are the contents of a sphere whose diameter is 12?
2. What are the contents of a sphere whose diameter is 4?
3. What are the contents of a sphere whose diameter is 14*in.*?
4. What are the contents of a sphere whose diameter is 6*ft.*

328. A prism is a figure whose ends are equal plane figures and whose faces are parallelograms.

The sum of the sides which bound the base is called the *perimeter* of the base, and the sum of the parallelograms which bound the solid is called the *convex surface.*

329. To find the convex surface of a right prism,

Multiply the perimeter of the base by the perpendicular height, and the product will be the convex surface (*Bk.* VII. *Prop.* 1).

EXAMPLES.

1. What is the convex surface of a prism whose base is bounded by five equal sides, each of which is 35 feet, the altitude being 26 feet?

2 What is the convex surface when there are eight equal sides, each 15 feet in length, and the altitude is 12 feet?

330. To find the solid contents of a prism.

Multiply the area of the base by the altitude, and the product will be the contents (*Bk.* VII. *Prop.* XIV).

EXAMPLES.

1. What are the contents of a square prism, each side of the square which forms the base being 15, and the altitude of the prism 20 feet?

2. What are the contents of a cube each side of which is 24 inches?

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches?

4. How many gallons of water will a cistern contain whose dimensions are the same as in the last example?

5. Required the contents of a triangular prism whose height is 10 feet, and area of the base 350?

331. A cylinder is a figure with circular ends. The line EF is called the axis or altitude, and the circular surface the *convex surface* of the cylinder.

332. To find the convex surface,

Multiply the circumference of the base by the altitude, and the product will be the convex surface. (*Bk.* VIII. *Prop.* I).

EXAMPLES.

1. What is the convex surface of a cylinder, the diameter of whose base is 20 and the altitude 50?

2 What is the convex surface of a cylinder, whose altitude is 14 feet and the circumference of its base 8 feet 4 inches?

3. What is the convex surface of a cylinder, the diameter of whose base is 30 inches and altitude 5 feet?

333. To find the contents of a cylinder,

Multiply the area of the base by the altitude: the product will be the contents. (*Bk.* VIII. *Prop.* II).

EXAMPLES.

1. Required the contents of a cylinder of which the altitude is 12 feet and the diameter of the base 15 feet?

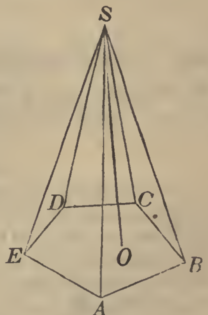
2. What are the contents of a cylinder, the diameter of whose base is 20 and the altitude 29?

3. What are the contents of a cylinder, the diameter of whose base is 12 and the altitude 30?

4. What are the contents of a cylinder, the diameter of whose base is 16 and altitude 9?

5. What are the contents of a cylinder, the diameter of whose base is 50 and altitude 15?

334. A pyramid is a figure formed by several triangular planes united at the same point S, and terminating in the different sides of a plain figure as ABCDE. The altitude of the pyramid is the line SO, drawn perpendicular to the base



335. To find the contents of a pyramid,

Multiply the area of the base by the altitude, and divide the product by 3 (*Bk.* VII *Prop.* XVII)

EXAMPLES.

1. Required the contents of a pyramid, of which the area of the base is 95 and the altitude 15.

2. What are the contents of a pyramid, the area of whose base is 260 and the altitude 24 ?

3. What are the contents of a pyramid, the area of whose base is 207 and altitude 18 ?

4. What are the contents of a pyramid, the area of whose base is 403 and altitude 30 ?

5. What are the contents of a pyramid, the area of whose base is 270 and altitude 16 ?

6. A pyramid has a rectangular base, the sides of which are 25 and 12: the altitude of the pyramid is 36 : what are its contents ?

7. A pyramid with a square base, of which each side is 30, has an altitude of 20 : what are its contents ?

336. A cone is a figure with a circular base, and tapering to a point called the *vertex*. The point C is the vertex, and the line CD is called the axis or altitude.

337. To find the contents of a cone,

Multiply the area of the base by the altitude, and divide the product by 3. (*Bk.* VIII. *Prop.* V).

EXAMPLES.

1. Required the contents of a cone, the diameter of whose base is 5 and the altitude 10.

2 What are the contents of a cone, the diameter of whose base is 18 and the altitude 27 ?

3. What are the contents of a cone, the diameter of whose base is 20 and the altitude 30 ?

4. What are the contents of a cone, whose altitude is 27 feet, and the diameter of the base 10 feet ?

5. What are the contents of a cone whose altitude is 12 feet, and the diameter of its base 15 feet ?

GUAGING.

338. The mean diameter of a cask is found by adding to the head diameter, two thirds of the difference between the bung and head diameters, or if the staves are not much curved, by adding six-tenths. This reduces the cask to a cylinder. Then, to find the solidity, we multiply the square of the mean diameter by the decimal .7854 and the product by the length. This will give the solid contents in cubic inches. Then, if we divide by 231, we have the contents in gallons. (Art. 114).

Multiply the length by the square of the mean diameter, then by the decimal $\cdot 7854$, and divide by 231.

OPERATION.

$$l \times d^2 \times \frac{\cdot 7854}{231} = l \times d^2 \times \cdot 0034.$$

If, then, we divide the decimal .7854 by 231, the quotient carried to four places of decimals is .0034, and this decimal multiplied by the square of the mean diameter and by the length of the cask, will give the contents in gallons.

339. Hence, for guaging or measuring casks, we have the following

RULE.—*Multiply the length by the square of the mean diameter; then multiply by* 34 *and point off four decimal places, and the product will then express gallons and the decimals of a gallon.*

1. How many gallons in a cask whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches?

We first find the difference of the diameters, of which we take two-thirds and add to the head diameter. We then multiply the square of the mean diameter, the length and 34 together, and point off four decimal places in the product.

OPERATION.

$$36 - 30 = 6$$
$$\tfrac{2}{3} \text{ of } 6 = 4$$
$$30 + 4 = 34$$
$$34^2 = 1156$$
$$1156 \times 50 \times 34 = 196.52 gal.$$

2. What is the number of gallons in a cask whose bung diameter is 38 inches, head diameter 32 inches, and length 42 inches?

3. How many gallons in a cask whose length is 36 inches, bung diameter 35 inches, and head diameter 30 inches?

4. How many gallons in a cask whose length is 40 inches, head diameter 34 inches, and bung diameter 38 inches?

5. A water tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes: the tap discharges at a medium, 40 gallons in 31 minutes. Now, supposing these to be left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5 shuts the tap, and is solicitous to know at what time the tub will be filled in case the water continues to flow.

APPENDIX.

FORMS RELATING TO BUSINESS IN GENERAL.

FORMS OF ORDERS.

MESSRS. M. JAMES & Co.

Please pay John Thompson, or order, five hundred dollars, and place the same to my account, for value received.

PETER WORTHY.

Wilmington, N. C., June 1, 1855.

MR. JOSEPH RICH,

Please pay, for value received, the bearer, sixty-one dollars and twenty cents, in goods from your store, and charge the same to the account of your

Obedient Servant,

JOHN PARSONS.

Savannah, Ga., July 1, 1855.

FORMS OF RECEIPTS.

Receipt for Money on Account.

Received, Natchez, June 2d, 1855, of John Ward, sixty dollars on account.

$60,00 JOHN P. FAY.

Receipt for Money on a Note.

Received, Nashville, June 5, 1856, of Leonard Walsh, six hundred and forty dollars, on his note for one thousand dollars, dated New York, January 1, 1855.

$640,00 J. N. WEEKS.

NOTES.

1. A NOTE, or as it is generally called, a promissory note, is a positive engagement, in writing, to pay a given sum at a time specified, either to a person named in the note, or to his order, or to the bearer.

2. By mercantile usage a note does not really fall due until the expiration of 3 days after the time mentioned on its face. The three additional days are called *days of grace*.

When the last day of grace happens to be Sunday, or a holiday, such as New Years, or the Fourth of July, the note must be paid the day before: that is, on the second day of grace.

3. There are two kinds of notes discounted at banks: 1st. Notes given by one individual to another for property actually sold—these are called *business notes*, or *business paper*. 2d. Notes made for the purpose of borrowing money, which are called *accommodation notes*, or *accommodation paper*. Notes of the first class are much preferred by the banks, as more likely to be paid when they fall due, or in mercantile phrase, "when they come to maturity."

FORMS OF NOTES.

No. 1. *Negotiable Note.*

$25,50 Providence, May 1, 1856.

For value received I promise to pay on demand, to Abel Bond, or order, twenty-five dollars and 50 cents.

REUBEN HOLMES.

Note Payable to Bearer.

No. 2.

$875,39. St. Louis, May 1, 1855.

For value received I promise to pay, six months after date, to John Johns, or bearer, eight hundred and seventy-five dollars and thirty-nine cents.

PIERCE PENNY.

Note by two Persons.

No. 3.

$659,27. Buffalo, June 2, 1856.

For value received we, jointly and severally, promise to pay to Richard Ricks, or order, on demand, six hundred and fifty-nine dollars and twenty-seven cents.

ENOS ALLAN.
JOHN ALLAN.

Note Payable at a Bank.

No. 4.

$20,25. Chicago, May 7, 1856.

Sixty days after date, I promise to pay John Anderson, or order, at the Bank of Commerce in the city of New York, twenty dollars and twenty-five cents, for value received.

JESSE STOKES.

REMARKS RELATING TO NOTES.

1. The person who signs a note, is called the *drawer* or *maker* of the note; thus, Reuben Holmes is the drawer of Note No. 1.

2. The person who has the rightful possession of a note, is called the *holder* of the note.

3. A note is said to be *negotiable* when it is made payable to A B, or order, who is called the payee (see No. 1). Now, if Abel Bond, to whom this note is made payable, writes his name on the back of it, he is said to *endorse* the note, and he is called the endorser; and when the note becomes due, the holder must first demand payment of the maker, Reuben Holmes, and if he declines paying it, the holder may then require payment of Abel Bond, the endorser.

4. If the note is made payable to A B, or bearer, then the drawer alone is responsible, and he must pay to any person who holds the note.

5. The time at which a note is to be paid should always be named, but if no time is specified, the drawer must pay when required to do so, and the note will draw interest after the payment is demanded.

6. When a note, payable at a future day, becomes due, and is not paid, it will draw interest, though no mention is made of interest.

7. In each of the States there is a *rate* of interest established by law, which is called the legal interest, and when no rate is specified, the note will always draw legal interest. If a rate *higher* than legal interest be taken, the drawer, in most of the States, is not bound to pay the note.

8. In the State of New York, although the legal interest is 7 per cent, yet the banks are not allowed to charge over 6 per cent, unless the notes have over 63 days to run.

9. If two persons jointly and severally give their note, (see No. 3,) it may be collected of either of them.

10. The words "For value received" should be expressed in every note.

11. When a note is given, payable on a fixed day, and in a specific article, as in wheat or rye, payment must be offered at the specified time, and if it is not, the holder can demand the value in money.

A BOND FOR ONE PERSON, WITH A CONDITION.

KNOW ALL MEN BY THESE PRESENTS, That I, *James Wilson of the City of Hartford and State of Connecticut, am* held and firmly bound unto *John Pickens of the Town of Waterbury, County of New Haven and State of Connecticut*, in the sum *of*

Eighty dollars lawful money of the United States of America, to be paid to the said *John Pickens*, *his* executors, administrators, or assigns: for which payment well and truly to be made *I* bind *myself*, *my* heirs, executors, and administrators, firmly by these presents. Sealed with *my* Seal. Dated the *Ninth day* of *March*, one thousand eight hundred and *thirty-eight*.

THE CONDITION of the above obligation is such, that if the above bounden *James Wilson*, *his* heirs, executors. or administrators, shall well and truly pay or cause to be paid, unto the above-named *John Pickens*, *his* executors, administrators, or assigns, the just and full sum of

[*Here insert the condition.*]

then the above obligation to be void, otherwise to remain in full force and virtue.

Sealed and delivered in
the presence of

John Frost,
Joseph Wiggins,

James Wilson. L.S.

NOTE. The part in Italic to be filled up according to circumstance.

If there is no condition to the bond, then all to be omitted after and including the words, "THE CONDITION, &c."

BOOK-KEEPING.

PERSONS transacting business find it necessary to write down the articles bought or sold, together with their prices and the names of the persons to whom sold.

BOOK-KEEPING is the method of recording such transactions in a regular manner.

COMMON ACCOUNT BOOK.

The following is a very convenient form for book-keeping, and requires but a single book. It is probably the best form of a common Account Book.

	J. BELL.	DR.			J. BELL.	CR.	
1846.		$	c.	1846.		$	c.
June 1	To 5 cords of wood,			July 6	By shoeing horse,	1	00
	at $1,75 per cord,	8	75	" 10	" mending sleigh,	3	25
" 6	To 1 day's work,	1	00	" 20	" ironing wagon,	5	12
July 9	To 4 *bu.* of rye, at 62			Aug. 1	" Cash to balance,	2	86
	cents per *bu.*	2	48				
		12	23			12	2[illegible]

ANSWERS.

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
24.	9	577	11	502616	13	43 cts.	15	$888
24.	10	7689	12	799999	14	73 cts.	—	——
25.	17	4083	19	9798	21	7032	23	559
25.	18	6846	20	8601	22	979	—	——
27.	5	12089	9	23272	13	175874		
27.	6	26901	10	233642	14	172775		
27.	7	28637	11	247481	15	98967		
27.	8	203933	12	1994439	16	10742750		
28.	20	787676921	23	26754	26	25687540		
28.	21	100570011	24	730528	27	297303078		
28.	22	15371781930	25	7047897	—	——		
29.	28	13115375	31	1819857171537	34	1118969		
29.	29	39428059	32	1105354	—	——		
29.	30	140700034	33	1079167	—	——		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
30.	1	365	2	5567	3	16375	4	421	5	392	6	34671660

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
31.	7	82869	10	4596-1199	12	1287462		
31.	8	2576406	11	$\left\{\begin{matrix}42390529\\4530902\end{matrix}\right.$	13	1665400		
31.	9	370			—	——		
32.	14	50994	17	5990267	20	23191876		
32.	15	143985	18	6644374	21	23191876		
32.	16	2728116	19	7685134	—	——		
37.	9	260822	13	99246591	17	4244083		
37.	10	2935621	14	999999	18	8013105		
37.	11	50391719	15	776462	19	52528		
37.	12	28443	16	18561747	—	——		
38.	1	10	4	234	7	62	10	175502
38.	2	45	5		8	785608	11	696
38.	3	$1115	6	67	9	37	12	2687
39.	13	250-$1500	17	239	21	190		
39.	14		18		22	$4020-1340		
39.	15	26	19	1759	23	2769818		
39.	16	1860805	20	55	24	94		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
40.	25	145	27	168	29	15914260	31	2769818
40.	26	168	28	137	30	20463760	—	——
40.	1	29045	2	$418	3	$714	4	$5795
41.	5	$390	8	$919	11	230-527	14	11854617
41.	6	$224980	9	55	12	19553068	—	——
41.	7	$1706	10	28223	13	$3818	—	——
47.	9	936	11	$298	13	$28511		——
47.	10	$1236	12	35688	14	$6578		——
49.	3	7913576	12	65948806	21	764819895290424		
49.	4	2537682	13	36914176	22	6241519790		
49.	5	4280822	14	85950000	23	105062176		
49.	6	19014604	15	3320863272	24	601380780		
49.	7	85564584	16	816515040	25	4984155396		
49.	8	2183178497	17	68959488	26	405768300		
49.	9	93939864472	18	35843685	27	800105244		
49.	10	395061696	19	267293339604	28	1227697160		
49.	11	393916488	20	214007086881	29	330445150		
51.	2	274032	4	15076944	6	7430778		
51.	3	19180896	5	50618898	7	553248		
52.	1	2540	7	214100	5	259175000000		
52.	2	64800	8	87200000	6	1960310474010		
52.	3	7987000	1	1833600	7	1484000		
52.	4	98400000	2	4368560000	8	109215040000		
52.	5	375000	3	148512000000	9	5210018850000		
52.	6	67040000	4	1315170000000	–	——		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
53.	1	480	2	4415	3	168	4	$291	5	2214-123	6	11680

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
54.	7	3087	12	349440	17	1620-2220
54.	8	18755	13	1057500	18	968710
54.	9	119568	14	150000	19	2720
54.	10	984072	15	131250000	20	408-2040
54.	11	24427326	16	53095	—	——
55.	21	$27625000	23	$19152500	25	$1211
55.	22	$636	24	$10368	26	$4044
60.	5	43217	8	46490-3	11	2264702-2
60.	6	104177-2	9	15840087	12	2343381-2
60.	7	12828	10	9486312	13	399946494

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
60.	14	47516365-2	17	74909989-3	20	13957027-5
60.	15	7544181-5	18	127724292-2	—	——
60.	16	6286358-2	19	2747804-1	—	——
61.	21	$1126	25	180607	29	23040
61.	22	48	26	88	30	345477
61.	23	288	27	87066-1	—	——
61.	24	13178	28	$327	—	——

63.	22	55	24	36	27	54	29	61
63.	23	40	26	34	28	94	30	$84\frac{3}{8}$

64.	32	4 and 6	34	5	36	8	38	9	40	7
64.	33	6	35	9	37	8	39	12	—	–

67.	4	194877-24	18	3097-33788
67.	5	3283	19	307140-121
67.	6	17359-1	20	34960078-346
67.	7	1345	21	80496-11707
67.	8	332627-12	22	1672940-165534
67.	9	795073-41	23	206008604-24
67.	10	194877 48	24	30001000-6347
67.	11	3283	25	9948157977-81605
67.	12	11572-110	26	1935468-14976
67.	13	2017-108	27	15395919-12214
67.	14	40367	28	14243757748-35411
67.	15	6704984	29	15395919-12714
67.	16	78795	30	3008292243-50442
67.	17	10110-9	31	123456789

68.	2	132	4	718328	6	918546
68.	3	4871000	5	7128368	–	——

69.	1	3175	1	3550	1	29654200
69.	2	106725	2	4700	2	24678733-1
69.	3	2187600	3	59250	3	177925200
69.	4	17624075	4	880300	4	74036200

70.	1	3704000	2	4269	7	284	12	237756
70.	2	1099588000	3	87504	8	4741	13	121086
70.	3	121300750	4	97049	9	70424	14	750
70.	4	88036750	5	70496-20	10	675	15	24063
70.	1	127	6	326	11	19626	16	54069

71.	1	105	3	133	5	387	7	1809
71.	2	387	4	201	6	1935	8	12864

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
72.	2	17085-29	5	3095-87	8	245-14
72.	3	67639-21	6	34171-2	9	405-141
72.	4	6129-11	7	1392-27		
73.	1	4976-3	3	496-321		
73.	2	76412	4	6-4978		
74.	2	4-146327	5	156557-344	1	900
74.	3	146	6	253-21700	–	—
74.	4	91-135803	7	2247-26649	–	—
75.	2	329	7	loses $26	12	75
75.	3	$45	8	$23	13	$55
75.	4	85	9	276	14	351
75.	5	84032	10	714-10	15	85-148
75.	6	$312	11	$17376	16	15
76.	17	552	21	4285	25	6178-6494
76.	18	lost $1625	22	4562	26	$42
76.	19	9	23	281	27	$3408
76.	20	2313	24	$514	28	794-2
77.	29	20	33	157-185	37	$140
77.	30	36	34	12923-13763	38	$60
77.	31	$2	35	$3	—	——
77.	32	g. 140cts	36	5	—	——
78.	39	3750	43	6000-9000	47	6480
78.	40	146	44	gained $12	48	8800
78.	41	886144	45	$1½	—	——
78.	42	22886826	46	$456	—	——
82.	3	37378	11	40368	7	$0,652
82.	4	375999	12	71453	8	$0,002
82.	5	670	1	$67,897	9	$1,607
82.	6	54000	2	$104,698	10	$170 464
82.	7	12500	3	$4096,042	11	$8674,416
82.	8	40000; 400000	4	$100,011	12	$94780,90
82.	9	37500; 375000	5	$4,006	13	$74164,21
82.	10	4000	6	$109,001	—	——
84.	1	$73,436	3	$132,475	5	$52,371
84.	2	$219,614	4	$99.11	–	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	
85.	6	$1843,94	8	$22,334	10	$14,405	
85.	7	$656,369	9	$7,952	—	——	
86.	6	$5,999. $9,742. $0,744. $87,345.	1	$2812,50	6	$24,625	
86.			2	$51,997	7	$12,43	
86.	7	$106,524	3	$2,50	8	$59,827	
86.	8	$170,056	4	$945,361	–	——	
86.	9	$44,377	5	$2906,961	–	——	
87.	9	$343,675	11	$8279,155		——	
87.	10	$112,442	12	$932,802		——	
88.	3	$20,35	7	$79,75	12	$82,25	
88.	4	$375	8	$3835,625	13	$11,25	
88.	5	$116,875	9	$975	14	$210,295	
88.	6	$22.95	11	$1157	—	——	
89.	1	$172,50	4	$8,40	8	$3718,50	
89.	2	$168,75	5	$45,333+	9	$104	
89.	3	$28,00	7	$357,75	–	——	
90.	1	$21,40	3	$6,117+	5	$105,026	
90.	2	$60,142+	4	$8,40	–	——	
91.	1	$5,961+	3	$3,99. $5,04. $6.6528. $4,3512. $7,8750.			
91.	2	$23,597					
91.	3	$3,51	5	$0,06	7	$16,803+	
91.	4	$5,342+	6	$0,666+	8	$41,904+	
92.	9	$0,65	14	$3,50	19	$66,666+	
92.	10	$2,12	15	$23,076+	20	$2.50	
92.	11	$0,375	16	$450	21	$24	
92.	12	$1,125	17	$75,385	22	$0,60	
92.	13	$0,14	18	$25,25	23	$3,00	
93.	24	15	28	$32\frac{1}{2}$	32	96	——
93.	25	$25\frac{1}{2}$	29	16	33	16	——
93.	26	9	30	112	34	140	——
93.	27	20	31	12	—	—	——
93.	1	$28	4	$0,625	7	$1,75. $14. $210.	
93.	2	$130,50	5	$12,50. $100. $625.			
93.	3	$51	6	$0,87$\frac{1}{2}$. $5,25. $7.	8	$0,14. $80,50.	
94.	9	$0,06	10	$14,50. $101,50.	11	30	

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
94.	12	4 yds. 6 yds.	15	$2,33$\frac{1}{3}$	18	155*lbs.*
94.	13	$414,75	16	$11000-5500	19	$547,92
94.	14	3480-$4,50	17	$23,16	20	$916
95.	21	$27,685	22	$290,82	23	$90277,70
99.	2	30183*far.*	4	391679*far.*	6	£1 12*s.* 3$\frac{1}{4}$*d.*
99.	3	84226*far.*	5	£84	7	£25 14*s.* 1*d.*
100.	1	60*in.* 120*in.* 192*in.*	3	12*ft.* 8*ft.* 4*ft.*		
100.	2	12*yd.* 18*yd.* 32*yd.*	4	24*fur.* 48*fur.* 64*fur.*		
101.	3	316767*ft.*	8	109° 21$\frac{1}{2}$*mi.* 7*fur.* 1*rd.*		
101.	4	359*mi.* 7*fur.* 28*rd.*		3$\frac{1}{2}$*yd.* 2*ft.* 8*in.*		
101.	5	3796602*ft.*	1	4*na.* 16*na.* 32*na.* 96*na.* 128*na.*		
101.	6	8201 *miles.*	2	16*qr.* 32*qr.* 28*qr.*		
101.	7	240700858*in.*	3	3*qr.* 4*qr.* 5*qr.* 9*qr.* 10*qr.*		
102.	3	980*na.*	6	28*El. Fl.* 1*qr.*		
102.	4	623*na.*	7	95*E. E.* 4*qr.*		
102.	5	204*yd.* 3*qr.* 2*na.*				
103.	1	288*in.* 432*in.*	2	40*P.* 120*P.* 640*R.* 1280*R.*		
103.		864*in.* 1152*in.*	3	160*P.* 320*P.* 9*yd.*		
104.	1	4*P.* 16*P.* 20*P.*	4	762300.		
104.	2	80*ch.* 160*ch.* 240*ch.*	5	260*sq. ft.* 16*sq. in.*		
104.	3	16*P.* 64*P.* 96*P.*	6	93*A.* 2*R.* 12*P.*		
104.	4	20*sq. ch.* 60*sq. ch.*	7	35*M.* 563*A.* 1*R.* 19*P.*		
104.		100*sq. ch.* 120*sq. ch*	8	$12584,25.		
104.	3	3157*P.*	9	$15,25.		
105.	1	1728*Cu. in.* 3456*Cu. in.*	5	2*C. yd.* 3*C. yd.*		
105.		5184*Cu. in.*	6	3*T.*		
105.	2	27*C. ft.* 54*C. ft.* 108*C. ft.*	7	2*T.* 4*T.*		
105.		162*C. ft.*	8	8*C.* 12*C.* 16*C.*		
105.	3	24*C. ft.* 40*C. ft.* 48*C. ft.*	9	48*C. ft.*		
105.	4	256*S. ft.* 64*S. ft.* 32*S. ft.*				
106.	3	592704.	6	21870 *cords*-4*C. ft.*		
106.	4	200*C. ft.* 3200*S. ft.*	7	88 *tons.* 24 *S. ft.*		
106.	5	5 *cords.* 2 *cord ft.*		1228 *S. in.*		
107.	1	16*gi.* 6*pt.* 12*pt.* 18*pt.* 20*pt.*	3	12602*pt.*		
107.	2	8*qt.* 16*qt.* 24*qt.* 16*pt.* 40*pt.*	4	10 *tuns* 2*hhd.*		
107.	3	2*bar.* 8*bar.* 12*bar.*	5	25 *tuns* 1*gal.*		
107.	4	12*qt.* 20*qt.* 80*qt.* 126*qt.* 252*qt.*	6	$36,64.		

P.	EX.	ANS.	EX.	ANS.
108.	1	6*pt.* 10*pt*	4	12734 *pt.*
108.	2	12*qt.* 16*qt.* 36*qt.*	5	129*hhd.* 13*gal.*
108.	3	13672*pt.*	6	451*bar.* 7*gal*
109.	1	16*qt.* 40*qt.* 64*qt.*	3	24*pk.* 32*pk.* 48*pk.* 4*bu.* 8*bu.* 10*bu.*
109.	2	3*pk.* 4*pk.* 8*pk.*		
109.	–	———	4	72*bu.* 108*bu.* 144*bu.*

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
110.	3	23808*pt.*	1	32*dr.*	5	3 *tons.*
110.	4	844*pk.*	2	64*oz.*		
110.	5	272*bu.*	3	1*cwt.*2*qr.*		
110.	6	1*ch.* 29*bu.* 3*pk.* 6*qt.*	4	12*qr.*		

P.	EX.	ANS.	EX.	ANS.
111.	3	2790366 *drams.*	7	6*T.* 2*cwt.* 4*lb.* 13*oz.* 14*dr.*
111.	4	903136*oz.*	8	2998128*oz.*
111.	5	5*T.*8*cwt.*3*qr.*24*lb.*13*oz.* 14*dr.*	9	212*T.* 14*cwt.* 1*qr.* 7*lb.*
111			10	$118,995-$10.
111.	6	28*T.* 4*cwt.* 1*qr.* 21*lb.*	11	$431,68-$160.
112.	1	48*gr.* 72*gr.* 96*gr.*	4	1*lb.* 1*oz.* 10*pwt.* 10*gr.*
112.	2	2*pwt.* 3*pwt.*	5	25*lb.* 9*oz.* 0*pwt.* 20*gr.*
112.	3	2*oz.* 3*oz.*	6	678618*gr.*
112.	4	48*oz.* 144*oz* 108*oz.* 84*oz.*	7	36*lb.* 7*oz.* 14*pwt.*
112.			8	38901*gr.*
112.	5	2*lb.* 3*lb.* 8*lb.*	9	6496*gr.*
112.	3	148340*gr.*	10	$657.
113.	1	40*gr.* 60*gr.* 80*gr.* 120*gr.*	3	40 ℥ 8 ℥.
113.	2	12 ℈ 21 ℈ 15 ℈.		
114.	3	8011 ℈.	7	73918*gr.*
114.	4	9113 ʒ.	8	12 ℔ 9 ℥ 7 ʒ 2 ℈ 18*gr.*
114.	5	27 ℔ 9 ℥ 6 ʒ 1 ℈.		
114.	6	94 ℔ 11 ℥ 1 ʒ.		
115.	1	240*sec.* 360*sec.*	3	379467108*sec.*
115.	2	72*hr.*120*hr.*72*hr.*	4	4*yr.* 1*da.*
115.	3	42*da.* 56*da.*	5	24*yr.* 1*da.* 26*m.* 58*sec.*
115.	4	168*hr.* 6*wk.*	–	———
116.	6	9*yr.* 14*da.* 17*hr.* 16*m.* 45*sec.*	7	6600*hr.*
116.			1	180*sec.* 240*sec.* 300*sec.*

P.	EX.	ANS.	EX.	ANS.
116.	2	360*m*. 240*m*. 300*m*.	3	7° 44′ 54″
116.	3	120° 180° 210° 240°	4	1c. 5s. 28° 15′
116.	4	4° 12° 3s. 5s. 6s.	5	3946800*sec.*
116.	1	10765′	6	921625*sec.*
116.	2	2592000″	7	2° 23′ 9″
117.	1	57953*hr.*	6	9℔ 8℥ 1ʒ 2℈ 19*gr.*
117.	2	10800′	7	1*lb.* 8*oz.* 5*pwt.* 19*gr.*
117.	3	1296000 *Cu. in.*	8	340157*gr.*
117	4	£714	9	81*g.* 7*s.*
117	5	3*T.* 7*cwt.* 20*lb.*	10	207*E. E.* 3*qr.*
118.	11	3320 *half pints.*	24	84*mi.* 3*fur.* 4*rd.*
118.	12	6539276*dr.*		3*yd.* 2*ft.*
118.	13	6*A.* 1*R.* 24*P.*	25	5*A* 3*R.* 35*P.* 3¼*yd.*
118.				2*ft.* 5*in.*
118.	14	1 *pound.*	26	197111025*M.*
118.	15	57953*hr.*	27	26880 *times.*
118.	16	7s. 15° 24′ 40″	28	$93024
118.	17	12 *cords.*	29	$27
118.	18	1244160 *Cu. in.*	30	4 *miles.*
118.	19	12096*pk.*	31	40 *yards.*
118.	20	377*yd.* 2*qr.*	32	5*mo.*3*wk.* 5*da.* 16*hr.*
118.	21	48976*gi.*	33	1008 *bottles*
118.	22	478602432*sq. in.*	34	110592
118.	23	15359*far.*	35	38 *casks.*
119.	36	17097$\frac{18}{126}$ *times.*	38	248 *miles*
119.	37	1013299200*sec.*	39	$39,879
120.	2	£1377 4*s.* 1¼*d.*	8	20ʒ 1℈ 10*gr.*
120.	3	£1616 7*s.* 6¾*d.*	9	79*cwt.* 2*qr.* 18*lb.*
120.	4	621*lb.* 8*oz.* 1*pwt.* 19*gr.*		15*oz.* 11*dr.*
120.	5	962*lb.* 6*oz.* 10*pwt.* 2*gr.*	10	340*T.* 5*cwt.* 2*qr.*
120.	6	104℔ 3℥ 3ʒ 2℈ 4*gr.*		20*lb.* 2*oz.*
120.	7	35℥ 7ʒ 2℈ 17*gr.*	—	——
121.	11	16*cwt.* 2*dr.*	16	184*E.E.* 4*qr.* 2*na.*
121.	12	432*L.* 2*mi.* 4*fur.*		1¼*in.*
121.		39*rd.* 4*yd.*	17	263*sq. yd.* 5*sq. ft*
121.	13	1*fur.* 34*rd.* 1½*yd.*		116*sq. in.*
121.		1*ft.* 4*in.*	18	27*M.* 277*A.* 1*R.*
121.	14	424*E.Fl.* 0*qr.* 3*na.*		0*P.* 24½*S. yd.*
121.	15	42*yd.* 3*qr.* 1*na.*	19	159*A.* 2*R.* 5*P.*

P.	EX.	ANS.	EX.	ANS.
121.	20	176*C. yd.* 18*C. ft.*	21	90*C.* 106*C. ft.*
121.		614*C. in.*	22	151*C.* 3*C. ft.*
122.	23	627*hhd.* 7*gal.* 1*qt.* 1*pt.*	28	50*wk.* 4*da.* 1*hr*
122.	24	50*tun* 0*p.* 1*hhd.* 38*gal.*		41*m.* 34*sec.*
122.		3*qt.*	29	1*cir.* 9s. 28° 33′
122.	25	94*ch.* 22*bu.* 3*pk.* 7*qt.*		59″
122.		1*pt.*	30	1*cir.* 10s. 27° 2′
122.	26	259*ch.* 12*bu.* 0*pk.* 0*qt.*		3″
122.		1*pt.*	1	291*lb.* 6*oz.* 15*p't.*
122.	27	172*yr.* 2*mo.* 1*wk.* 4*da.*		22*gr.*
122.		4*hr.*	—	——
123.	2	244*lb.* 5*oz.* 4*pwt.* 3*gr.*	8	45*A.* 3*R.* 31*P.*
123.	3	82*T.* 16*cwt.* 0*qr.*		38¾*Sq.ft.* 22*Sq.in.*
123.		16*lb.* 1*oz.* 7*dr.*	9	158*bu.* 0*pk.* 4*qt.*
123.	4	41*T.* 0*cwt.* 3*qr.* 17*lb.*	10	2*T.* 5*cwt.* 2*qr.* 21*lb.*
123.		0*oz.* 5*dr.*	11	85*yd.*
123.	5	336*A.* 1*R.* 31*P.*	12	3*lb.* 1*oz.* 11*pwt.* 17*gr*
123.		210*Sq.ft.* 136*Sq. in.*	13	322*mi.* 6*fur.* 11*rd.*
123.	6	170*T.* 11*C.ft.* 744*C.in.*	14	100*A.* 1*R.* 13*P.*
123.	7	168*bu.* 0*pk.* 2*qt.*	—	——
124.	3	174*lb.* 1*oz.* 1*pwt.* 3*gr.*	4	8*lb.* 10*oz.* 14*pwt.* 4*gr.*
125.	5	5*T.* 7*cwt.* 1*qr.* 23*lb.*	7	124*T.* 0*hhd.* 59*gal.*
125.		11*oz.*	8	14*yr.* 46*wk.* 4*da.*
125.	6	7*cwt.* 2*qr.* 20*lb.* 11*oz.*		20*h.* 58*m.* 54*sec.*
125.		5*dr.*	—	——
125.	2	9*yr.* 4*mo.* 2*da.*	4	17*yr.* 1*mo.* 3*da.*
125.	3	21*yr.* 9*mo.* 5*da.*	—	——
126.	6	12*yr.* 3*mo.* 26*da.* 22*hr.*	7	2*m.* 4*fur.* 21*rd.*
126.	7	30*yr.* 1*mo.* 29*da.* 12*hr.*	8	7*yr.* 9*mo.* 1*da.*
126.	1	27*mo.* 3*wk.* 0*da.*	9	362*yr.* 9*mo.* 14*da.*
126.		20*hr.* 20*m.*	10	68*lb.* 10*oz.* 3*pwt.* 15*gr.*
126.	2	84*yr* 11*mo.* 0*wk.* 5*da.*	11	1*T.* 17*cwt.* 3*qr.*
126.	3	£2 17*s.*		7*lb.* 14*oz.* 2*dr.*
126.	4	1*lb.* 11*oz.* 19*pwt.* 4*gr.*	12	84℔ 9℥ 4ʒ
126.	5	6℔ 10℥ 5ʒ 1℈		1℈ 14*gr.*
126.	6	7*T.* 18*cwt* 1*qr.* 4*lb.*	13	3*yd.* 2*qr.* 1*na.* ¼*in.*
126.		0*oz.* 2*dr.*	14	4 *cords* 50 *cubic feet.*

P.	EX.	ANS.	EX.	ANS.
127.	15	£74 16s. 5d. 2far.	22	16cwt. 0qr. 6lb. 2oz.
127.	16	86bu. 1pk. 0qt. 1pt.	23	12cwt. 0qr. 23lb. 12oz.
127.	17	87hhd. 46gal. 3qt.		
127.	18	436bu. 2pk. 6qt. 1pt.	24	14L. 1mi. 1fur. 15rd.
127.	19	7cwt. 2qr. 14lb.	25	11A. 3R. 18P.
127.	20	£27 0s. $11\frac{1}{2}$d.	26	56yr. 5mo. 27da. 3hr. 25m.
127.	21	22lb. 4oz. 6pwt. 13gr.		
128.	3	56mi. 5fur. 4rd.	4	27s. 28° 22′ 45″
129.	5	32yr. 3mo. 18da. 18hr.	14	122mi. 4fur. 20rd.
129.	6	53T. 3cwt. 2qr. 16lb. 4oz. 8dr.	15	111A. 2R. 25P.
129.			16	267yd. 0qr. 3na.
129.	7	25bu. 3pk. 1qt.	17	47L. 1mi. 7fur. 8rd
129.	8	2T. 5cwt. 0qr. 24lb.	18	95hhd. 6gal.
129.	9	8 cords 6 cord ft.	19	32lb. 9oz. 15pwt.
129.	10	17yr. 5mo. 3da.	20	746mi. 5fur.
129.	11	3lb. 3oz. 12pwt.	21	15°
129.	12	1T. 19cwt. 2qr. 12lb.	22	56T. 14cwt. 3qr. 15lb.
129.	13	13℔ 7℥ 2ʒ 1℈ 4gr.		
130.	23	£5 4s. 3d.	29	1493mi. 2fur.
130.	24	24hhd. 22gal. 1qt. 1pt.	30	6296bu. 3pk. 4qt.
130.	25	927yds.	31	3174 miles.
130.	26	748A. 0R. 38P.	32	72 tons 16cwt. 20lb
130.	27	286yr. 11mo. 2wk.	33	282 yds.
130.	28	56T. 17cwt. 2qr. 10lb.	—	——
131.	3	5L. 2mi. 6fur. 36rd.	4	2bu. 1pk. 3qt.
132.	5	2cwt. 1qr. 18lb. $3\frac{1}{7}$oz.	16	5bu 1pk. $6\frac{3}{12}$qt.
132.	6	5yd. 2qr. $0\frac{7}{9}$na.	17	3℔ 4℥ 6ʒ 1℈ 16gr
132.	7	10A. 3R. 30P.	18	61gal. 1qt. 1pt.
132.	8	£21 9s. 8d.	19	12A. 2R. 25P.
132.	9	17cwt. 3qr. $18\frac{60}{79}$lb.	20	24mi. 7fur. 4rd.
132	10	1pk. $2\frac{55}{84}$qt.	21	14lb. 0oz. 8pwt. 11gr
132.	11	£1 2s. 4d. 2far.	22	3gal. 1qt. 1pt.
132.	12	2T. 7cwt.	23	4bu. 3pk. 2qt.
122.	13	25lb. 3oz. 8dr.	24	2cwt. 1qr. 24lb.
132.	14	2° 34′ 16″	25	2bu. 7qt.
132.	15	49gal. 2qt. 1pt.	26	12cwt. 2qr. 11lb.
133.	27	7lb. 12oz. 2dr.	29	47gal. 3qt. $1\frac{8}{59}$pt.
133.	28	15′	30	20mi. 4fur. 23rd.

P.	EX.	ANS.	EX.	ANS.
133.	31	7*s.* 3*d.* 1*far.*	35	1 *T.* 1*cwt.* 1*qr.* 19*lb.*
133.	32	24 *r'ms* 5 *qrs.*	36	£600 9*s.* 8*d.*
133.		12 *sheets.*		£1050 16*s.* 11*d.*
133.	33	£63 14*s.* 8*d.*	37	$53\frac{128}{224}$*da.*
133.	34	1*hhd.* 19*gal.* 1*pt.*	38	$294\frac{712}{816}$*da.*

135.	1	1° 9′—*time* 4*m.* 36*sec.*	7	4*h.* 56*m.* 4*sec.*
135.	2	12*hr.* 4*m.* 36*sec.* *P.M.*	8	4*h.* 56*m.* 4*sec.* *P.M.*
135.	3	11*hr.* 55*m.* 24*sec.* *A.M.*	9	2*h.* 20*m.* 4*sec.* *P.M.*
135.	4	41*m.* 32*sec.*	10	6*h.* 0*m.* 8*sec.* *A.M.*
135.	5	11*hr.* 18*m.* 28*sec.* *A.M.*	11	11*h.* 6*m.* 4*sec.* *A.M.*
135.	6	10*hr.* 59*m.* 56*sec.* *A.M.*	—	——

137.	1	3×3	9	$7\times3\times3$
137.	2	3×5	10	$19\times2\times2$
137.	3	$3\times2\times2\times2$	11	3
137.	4	$2\times2\times2\times2$	12	3 *and* 7
137.	5	$3\times3\times2$	13	3 *and* 7
137.	6	$2\times2\times2\times2\times2$	14	2 *and* 7
137.	7	$3\times2\times2\times2\times2$	15	2 *and* 3 *and* 7
137.	8	$7\times2\times2\times2$	16	2, 3, 5, 7

138.	2	9	3	6	4	5	5	6	6	5	7	14	8	42

139.	1	24	2	4	3	45	4	630	5	267	6	396

140.	7	12	8	8	9	4	10	3	—	—	—	—	—

141.	3	840	5	840	7	78	9	1008	11	223839
141.	4	147	6	196	8	84	10	156	—	——

142.	3	63	4	126	5	27	6	12	—	—	—	—

144.	5	$1\frac{4}{7}$	9	$\frac{1}{8}$	13	7056	17	348*yds.*
144.	6	1	10	27	14	8	18	46*yds.*
144.	7	11	11	$\frac{1}{24}$	15	$5\frac{1}{3}$	—	——
144.	8	55	12	16	16	80 *pounds*	—	——

145.	19	26*bu.*	24	36*gal.*	29	58 *boxes*
145.	20	$40\frac{1}{3}$ *pounds*	25	40*bu.*	30	4*s.*
145.	21	$9\frac{2}{6}$*bu.*	26	57*gal.*	31	3 *cheeses.*
145.	22	300 *pounds*	27	15*yds.*	—	——
145.	23	500*yds.*	28	$5\frac{5}{14}$ *fir'ns.*	—	——

151.	1	$\frac{24}{8}$	3	$\frac{9}{7}$	5	$\frac{140}{6}$	——
151.	2	$\frac{35}{5}$	4	$\frac{112}{19}$	6	$\frac{4175}{81}$	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
152.	1	$\frac{3}{2}$ *and* 3	3	$\frac{8}{24}, \frac{8}{12}, \frac{8}{8}$.	5	$\frac{37}{21}, \frac{37}{7}, \frac{37}{6}$.
152.	2	1, $\frac{16}{8}, \frac{16}{4}$.	4	$\frac{19}{42}, \frac{19}{21}, \frac{19}{14}$.	6	$\frac{151}{40}, \frac{151}{20}$.

152.	1	$\frac{14}{15}, \frac{4}{15}$.	1	$\frac{1}{4}$.	5	$\frac{41}{1105}$.
152.	2	$\frac{2}{160}$.	2	$\frac{1}{35}$.	6	$\frac{16}{7875}$.
152.	3	$\frac{16}{117}, \frac{50}{117}$	3	$\frac{3}{64}$.	–	——
152.	4	$\frac{80}{530}, \frac{40}{530}, \frac{64}{530}$.	4	$\frac{17}{120}$.	–	——

153.	2	$\frac{21}{36}, \frac{28}{48}, \frac{35}{60}, \frac{42}{72}, \frac{63}{108}$.	3	$\frac{150}{168}, \frac{225}{252}, \frac{300}{336}, \frac{375}{420}, \frac{450}{504}$.
154.	3	$\frac{16}{64}, \frac{8}{32}, \frac{4}{16}, \frac{2}{8}$.	4	$\frac{30}{90}, \frac{20}{60}, \frac{15}{45}, \frac{12}{36}, \frac{10}{30}, \frac{6}{18}, \frac{5}{15}$

155.	1	$\frac{4}{5}$.	6	$\frac{1}{6}$.	11	$\frac{1}{8}$.	16	$\frac{7}{9}$.
155.	2	$\frac{3}{4}$.	7	$\frac{1}{9}$.	12	$\frac{5}{8}$.	2	$12\frac{3}{8}$.
155.	3	$\frac{3}{4}$.	8	$\frac{17}{33}$.	13	$\frac{117}{265}$.	3	$2\frac{5}{7}$*yds.*
155.	4	$\frac{1}{4}$.	9	$\frac{7}{15}$.	14	$\frac{2}{13}$.	4	$5\frac{2}{3}$*bu.*
155.	5	$\frac{7}{8}$.	10	$\frac{1}{3}$.	15	$\frac{7}{9}$.	—	——

156.	5	5 *apples.*	3	$\frac{849}{20}$ *rods*
156.	6	$2\frac{77}{125}$, 24, $7\frac{1700}{6941}$, $13\frac{47712}{72301}$.	4	$\frac{8129}{13}$.
156.	7	$219\frac{1}{2}$.	5	$\frac{23006}{112}$.
156.	8	$2\frac{307}{3117}$, $45\frac{22}{109}$, $9\frac{135750}{278436}$.	6	$\frac{2033}{24}$ *days.*
156.	9	$31504\frac{4453}{4674}$, 1345, $7947\frac{5294}{7803}$.	7	$\frac{5617}{365}$*yrs.*
156.	1	$\frac{287}{6}$.	8	$\frac{550050}{600}$.
156.	2	$\frac{139}{8}$*yds.*	9	$\frac{1909}{76}$. $\frac{9382}{60}$.

157.	1	$\frac{108}{9}$.	6	$\$\frac{216}{4}$.	3	$\frac{2}{5}$.
157.	2	$\frac{690}{15}$.	7	$\frac{384}{4}$*yds.*	4	$\frac{812}{8}$.
157.	3	$\frac{182}{7}$.	8	$\frac{6816}{16}$*lb.*	5	$\frac{15}{7}$.
157.	4	$\frac{819}{8}$.	1	$\frac{5}{42}$.	6	$\frac{106343}{324}$.
157.	5	$\frac{27360}{114}$.	2	$\frac{5}{7}$.	–	——
158.	3	$\frac{15}{416}$.	4	$\frac{2}{209}$.	5	$\frac{63}{8}=7\frac{7}{8}$.

159.	1	$\frac{63}{105}, \frac{70}{105}, \frac{15}{105}$.	7	$\frac{525}{600}, \frac{1080}{600}, \frac{22200}{600}$.	13	$\frac{12}{28}, \frac{8}{28}, \frac{8}{28}$
159.	2	$\frac{110}{495}, \frac{180}{495}, \frac{297}{495}$.	8	$\frac{200}{50}, \frac{62}{50}, \frac{1550}{50}$.	14	$\frac{15}{24}, \frac{92}{24}, \frac{18}{24}$.
159.	3	$\frac{360}{504}, \frac{63}{504}, \frac{280}{504}$.	9	$\frac{1080}{144}, \frac{218}{144}, \frac{900}{144}$.	15	$\frac{77}{12}, \frac{114}{12}, \frac{60}{12}$.
159.	4	$\frac{98}{42}, \frac{3}{42}$.	10	$\frac{518}{126}, \frac{1026}{126}, \frac{315}{126}$.	16	$\left\{\frac{282}{36}, \frac{16}{36}, \frac{9}{36}, \frac{4}{36}\right.$.
159.	5	$\frac{231}{42}, \frac{12}{42}, \frac{168}{42}$.	11	$\frac{3}{15}, \frac{5}{15}$.		
159.	6	$\frac{171}{108}, \frac{36}{108}$.	12	$\frac{2}{12}, \frac{1}{12}, \frac{9}{12}$.	—	—— ——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
160.	2	$\frac{36}{45}, \frac{40}{45}, \frac{9}{45}$.	6	$\frac{82}{100}, \frac{330}{100}, \frac{400}{100}$.	10	$\frac{36}{90}, \frac{60}{90}, \frac{50}{90}$,				
160.	3	$\frac{122}{8}, \frac{51}{8}, \frac{44}{8}$.	7	$\frac{225}{72}, \frac{312}{72}, \frac{600}{72}$.		$\frac{63}{90}$.				
160.	4	$\frac{72}{360}, \frac{60}{360}, \frac{320}{360}$.	8	$\frac{6}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}$.	11	$\frac{16}{48}, \frac{36}{48}, \frac{40}{45}$,				
160.	5	$\frac{67}{120}, \frac{18}{120}, \frac{300}{120}$.	9	$\frac{35}{84}, \frac{528}{84}$.	—	$\frac{2}{48}, \frac{33}{48}$.				
161.	1	$6\frac{1}{2}$.	4	$1\frac{11}{14}$.	7	$2\frac{5}{6}$.	10	$2\frac{179}{504}$.	13	$1\frac{181}{100}$.
161.	2	$2\frac{1}{7}$.	5	$1\frac{7}{30}$.	8	$2\frac{29}{40}$.	11	$2\frac{29}{120}$.	14	$1\frac{4}{5}$.
161.	3	$4\frac{2}{3}$.	6	$1\frac{11}{16}$.	9	$10\frac{71}{90}$.	12	$1\frac{25}{126}$.	—	—
162.	16	$25\frac{33}{40}$.	20	$1426\frac{33}{50}$.	24	$96\frac{29}{90}$.			4	$\frac{16}{48}$. $\frac{15}{54}$. [illegible]
162.	17	$52\frac{4}{9}$.	21	$\frac{7}{36}$.	25	$64\frac{232}{2625}$.			–	—
162.	18	$20\frac{17}{70}$.	22	$39\frac{14}{15}$.	2	$\frac{17}{72}$. $\frac{16}{60}$.			–	—
162.	19	$21\frac{87}{110}$.	23	$17\frac{1}{20}$.	3	$\frac{20}{91}$. $\frac{22}{120}$. $\frac{17}{60}$.			–	—
163.	2	$\frac{5}{7}$.	5	$\frac{166}{105}$.	3	$\frac{37}{221}$.	6	$4\frac{1}{32}$.	9	$\frac{31}{756}$.
163.	3	$\frac{3}{9}$.	1	$\frac{17}{63}$.	4	$\frac{33}{100}$.	7	$\frac{5}{33}$.	–	—
163.	4	$\frac{58}{365}$.	2	$\frac{13}{56}$.	5	$1\frac{1}{34}$.	8	$2\frac{17}{21}$.	–	—.
164.	11	$68\frac{2}{15}$.	13	$2\frac{20}{21}$.	15	$6\frac{41}{63}$.	17	$72\frac{19}{24}$.		
164.	12	$82\frac{7}{40}$.	14	$10\frac{53}{72}$.	16	$8\frac{48}{55}$.	19	$\frac{2}{48}$ $\frac{7}{60}$ $\frac{8}{153}$ $\frac{17}{740}$ $\frac{4}{165}$ $\frac{72}{4480}$		
165.	1	$3\frac{1}{12}$.	4	$42\frac{1}{3}$.	7	$8\frac{1}{3}$*cords*	10	\$$8\frac{8}{11}$.	13	\$$13\frac{1}{2}$.
165.	2	$6\frac{5}{7}$.	5	123	8	\$10	11	14*m.*	14	$5\frac{1}{7}$*bar.*
165.	3	35	6	$16\frac{182}{273}$.	9	$13\frac{1}{2}$*bu.*	12	27*cts.*	15	\$$16\frac{1}{2}$.
166.	1	21	3	75	5	\$$5\frac{1}{3}$.	7	\$180	9	\$$16\frac{1}{5}$.
166.	2	$38\frac{1}{2}$.	4	$26\frac{2}{3}$.	6	\$$43\frac{3}{4}$.	8	$36\frac{4}{5}$*m.*	10	\$440
167.	1	$\frac{7}{10}$.	4	$\frac{3}{7}$.	7	\$$\frac{7}{20}$.	2	$\frac{9}{20}$.	5	675
167.	2	$\frac{5}{13}$.	5	\$$\frac{3}{4}$.	8	\$$\frac{15}{28}$.	3	$\frac{17}{18}$.	6	$20\frac{4}{7}$.
167.	3	$\frac{7}{32}$.	6	$\frac{4}{7}$.	1	$\frac{9}{40}$.	4	$8\frac{1}{3}$.	–	—
168.	8	$636\frac{1}{2}$.	11	$20\frac{1}{2}$.	14	\$25	17	$546\frac{5}{6}$*cts.*	20	$6\frac{2}{3}$*yds.*
168.	9	114	12	$51\frac{2}{3}$.	15	2 *tons*	18	\$$1\frac{31}{64}$.	21	\$$60\frac{3}{7}$.
168.	10	1344	13	\$90	16	\$$\frac{7}{44}$.	19	\$1295.	—	—
169.	1	$\frac{5}{22}$.	4	$\frac{2}{177}$.	7	$\frac{1}{44}$.	10	\$$\frac{2}{21}$.	13	$\frac{1}{10}$*bar*
169.	2	$\frac{2}{37}$.	5	$\frac{3}{19}$.	8	$\frac{2}{279}$.	11	$\frac{2}{9}$*yd.*	15	\$$\frac{2}{9}$.
169.	3	$1\frac{8}{19}$.	6	$\frac{1}{26}$.	9	$\frac{3}{20}$ *ton.*	12	$\frac{3}{16}$*lb.*	16	$\frac{8}{15}$*lb*
170.	1	16	3	$94\frac{1}{2}$.	5	$9\frac{9}{11}$*yds.*	7	\$$14\frac{3}{5}$.		
170.	2	$226\frac{2}{15}$.	4	$513\frac{1}{3}$.	6	$11\frac{3}{7}$*hrs.*	–	—		

P.	EX	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
171.	1	$\frac{24}{25}$.	6	$2\frac{5}{8}$.	11	\$$10\frac{2}{3}$.	16	\$$\frac{1}{2}$.
171.	2	$\frac{13}{33}$.	7	$5\frac{1}{2}$*lb.*	12	$7\frac{1}{8}$*lb.*	17	27 *miles.*
171.	3	$4\frac{2}{3}$.	8	7 *men.*	13	4 *child'n.*	18	$9\frac{1}{3}$ *turkeys.*
171.	4	$\frac{2}{3}$.	9	\$$\frac{3}{4}$.	14	$5\frac{35}{48}$ *times.*	19	$3\frac{3}{20}$*yds.*
171.	5	$6\frac{4}{11}$.	10	14 *baskets.*	15	$9\frac{1}{3}$ *times.*	—	——

172.	1	$\frac{14}{15}$.	3	$2\frac{3}{46}$.	5	$\frac{12}{55}$.	7	$1\frac{97}{350}$	9	$\frac{6}{35}$.
172.	2	10	4	$\frac{7}{8}$.	6	$2\frac{57}{59}$.	8	$8\frac{89}{311}$	10	$\frac{2}{45}$.

174.	1	$1\frac{1}{6}$*qt.*	7	$\frac{4}{315}$*hhd*	13	$\frac{5}{144}$*yd.*	19	$\frac{3}{4}$.
174.	2	$\frac{4}{5}$*pt.*	8	£$\frac{1}{32}$.	14	$\frac{1}{19800}$*mi.*	20	$53\frac{1}{3}$
174.	3	$\frac{2}{3}$*gr.*	9	$\frac{5}{2268}$*hhd.*	15	$\frac{80}{147}$*lb.*	21	$\frac{54}{305}$*pt.*
174.	4	$\frac{2}{3}$*ft.*	10	$\frac{3}{17600}$*mi.*	16	$\frac{7}{576}$*lb.*	—	——
174.	5	$\frac{3}{4}$*mi.*	11	$\frac{3}{5}$*far.*	17	$\frac{3}{1280}$*mi.*	—	——
174.	6	$\frac{7}{8}$*oz.*	12	$\frac{15}{19}$*na.*	18	$\frac{1}{80}$	—	——
175.	3	3*hhd.* 31*gal.* 2*qt.*			4	$\frac{7}{8}$ *tun*	—	——

176.	5	3*qr.* $2\frac{2}{5}$*na.*	17	$\frac{5}{14}$.
176.	6	3*wk.* 1*da.* 9*hr.* 36*m.*	18	$\frac{65}{234}$.
176.	7	13*bu.* 2*pk.*	19	$\frac{4}{5}$*mo.*
176.	8	6*fur.* 8*rd.* 4*yd.* 2*ft.*	20	$\frac{849}{1210}$*A*
176.		8*in.*	21	$\frac{115}{189}$*g.*
176.	9	3*cwt.* 0*qr.* 12*lb.* 8*oz.*	22	8℥ 3ʒ 0℈ 12*gr.*
176.	10	2*da.* 13*hr.* 42*m.*	23	$\frac{7}{9}$*E. E.*
176.		$51\frac{3}{7}$*sec.*	24	50*gal.* 1*qt.* $1\frac{1}{5}$*pt.*
176.	11	2*bu.* 2*pk.*	25	24*gal.*
176.	12	$\frac{1}{18}$*hhd.*	26	1*qr.* 21*lb.* 10*oz.* $10\frac{2}{3}$*dr.*
176.	13	$\frac{4}{11}$*mi.*	27	$\frac{125}{1152}$*s.*
176.	14	£$\frac{7}{25}$.	28	$\frac{17}{18}$.
176.	15	$\frac{8}{9}$*lb.*	29	2*gal.* $3\frac{1}{5}$*gi.*
176.	16	$\frac{2503}{4500}$ *ton.*	30	2*R.* 6*P.* 4*yd.* 5*ft.* $127\frac{5}{18}$*in.*

177.	5	1*m.* 3*fur.* 18*rd.*	9	3*yd.* 1*ft.* $11\frac{4}{7}$*in.*
177.	6	7*fur.* 0*yd.* 2*ft.* 9*in.*	10	16*gal.* $2\frac{12}{35}$*qt.*
177.	7	1*cwt.* 2*qr.* 2*lb.* 13*oz.*	11	11*pwt.* 3*gr.*
177.	8	11*hr.* 59*m.* $59\frac{1}{3}$*sec.*	12	4*cwt.* 1*qr.* 12*lb.* 15*oz.* $5\frac{1}{35}$*dr.*

P.	EX	ANS.	EX	ANS.
178.	13	$75\frac{2}{9}$*bu.*	22	3*wk.* 4*da.* 12*h.* 19*m.*
178.	14	12*cwt.* 1*qr.* 7*lb.* 13*oz.*		$17\frac{1}{7}$*sec.*
178.		$11\frac{3}{7}$*dr.*	23	2*mi.* 2*fur.* 16*rd*
178.	15	$90\frac{23}{42}$*mi.*	24	$\frac{2}{17}$
178.	16	5*da.* 20*hr.* 52*m.*	25	£1 9*s.* 3*d.*
178.		$15\frac{15}{19}$*sec.*	26	1*oz.* 3*pwt.* 3*gr.*
178.	17	2*qr.* 19*lb.* 14*oz.* $0\frac{32}{105}$*dr.*	27	8*cwt.* 3*qr.* 5*lb.* 13*oz.*
178.	18	56*yd.*		$0\frac{16}{35}$*dr.*
178.	19	$2\frac{7}{12}$*yd.*	28	3*lb.* 5*oz.* 16*pwt.* 16*gr*
178.	20	1*c't.*1*qr.*7*lb.*7*oz.* $\frac{64}{105}$*dr.*	29	1*rd.* 1*yd.* 2*ft.* $5\frac{4}{7}$*in.*
178.	21	6*pwt.* 15*gr.*	30	7℥ 5ʒ 2℈ 10*gr.*

179.	1	15*ft.* 5′	4	5*ft.* 8′ 2″ 1‴	7	11*ft.* 6′ 5″ 5‴
179.	2	1*ft.* 8′ 10″	5	15*ft.* 4′ 10″ 4‴	8	7*ft.*10′ 1″ 9‴
179.	3	2*ft.* 6′ 3″ 11‴	6	1*ft.* 11′ 10″ 11‴	–	——

181.	2	77*ft.*	7	27*ft.* 8′ 6″	11	1*C.* 4*C. ft.*
181.	3	87*ft.* 1′	8	105*ft.*5′7″6‴		3 *cu. ft.*
181.	4	165*ft.* 6′ 8″	9	39*C.* 33*Cu.ft.*	12	158*c.yd.* 17*c.ft.* 4′
181.	5	866*ft.* 8′ 3″	10	46*yd.* 0*ft.*	13	$19,64
181.	6	2*C.* 5*C. ft.*		3′ 8″	14	84*ft.* 4′ 6″

185.	1	.3	7	12.009	3	5.09	9	47.00021
185.	2	.016	8	16.012	4	65.015	10	1500.000003
185.	3	.0017	9	9.565	5	80.000003	11	39.640
185.	4	.32	10	22.1	6	2.000300	12	.003840
185.	5	.0165	1	41.3	7	400.092	13	.650
185.	6	18.03	2	16.000003	8	3000.0021	—	——

188.	1	1303.9805	4	1.5413	7	246.067	10	494.521
188.	2	428.677893	5	444.0924	8	389.989	11	$641.249
188.	3	169.371	6	1215.7304	9	71.21	12	.111

189.	13	4.0006	17	$1033.6279	21	.560596
189.	14	$129.761	18	$51.451	22	$7.978
189.	15	$1132.365	19	1.215009	23	$417.563
189.	16	$16.3275	20	23001044.5000419	24	74435.0309

190.	1	3294.9121	7	365.007497	13	4238.60807
190.	2	249.72501	8	20.9943	14	126.831874057
190.	3	9.888890	9	260.4708953	15	63.879674
190.	4	395.9992	10	10.030181	16	106.9993
190.	5	.999	11	2.0094	17	1.1215
190.	6	6377.9	12	34999.965	18	.001

P.	EX	ANS.	EX	ANS.	EX	ANS.
191.	1	.036588	6	9308.37	11	3.04392632
191.	2	.365491	7	311.2751050254	12	$17.2975
191.	3	742.0361960	8	.25	13	$14.274
191.	4	.001000001	9	.0025	—	———
191.	5	.000000000147	10	.0238416	—	———
192.	14	$4.543944	18	$46.95	22	.000016
192.	15	.0036	19	$1.051279	23	.000274855
192.	16	240.1	20	.00025015788028	24	.00182002625
192.	17	$56.764	21	2.39015	—	———
193.	1	1.11		16.21987-1621.987		
193.	2	4.261	8	16219.87-162198.7		
193.	3	33.331		1621987.		
193.	4	1.0001		20.81100-208.1100		
193.	5	41 23.5	9	2081.100-20811.00		
193.	6	1175.07		208110.0-2081100.		
193.		12.52534		127.3673874-12736.73874		
193.		125.2534	10	127367.3874-1273673.874		
193.	7	1252.534		12736738.74-127367387.4		
193.		12525.34	—	———		
193.		125253.4	—	———		
194.	2	21940.		10.-100.-1000.-30.	3	8.3111+
194.	3	30100.	6	20.-2000.-12.-120.	4	1.563+
194.	4	1000.		1200.	5	1.160+
194.	5	66.666+	2	.3333+	-	———
195.	1	$.00638+		*.25 of* 3.26=.815	9	18.022+*bu.*
195.	2	$.10486+	6	*and* .034 *of* 3.04	10	$0.7909+
195.	3	14.941+		=.10336 .815÷	11	1196.172+
195.	4	16.119+		.10336=7.885+	12	1000.
195.	5	.08333+	7	.0470204+	13	227.313001*yds.*
195.	-	———	8	188*bu.*	14	125.1011 *p'ds.*
196.	15	$48.141	21	112.29*C.yd.*	27	15.68+*bar.*
196.	16	$10055.3025	22	$45.401	28	92*gal.*
196.	17	$934.699	23	$313.313	29	19.8*da.*
196.	18	$46.875	24	$0.75	30	$54.72
196.	19	$4070.316	25	$122.766+	31	1725.15*lb.*
196.	20	$16 63	26	177*bar.*	—	———
197.	1	.4285+	3	.08571+	5	.025-.7435-.003
197.	2	.88235+	4	.25-.00797+	6	.5-.0028+

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
197.	7	1.496+	11	.136	15	.01171875
197.	8	1.333+.162+.792	12	.00875	16	.135546875
197.	9	.85	13	.2976	17	.0001
197.	10	075	14	.006875	18	.222464

198.	1	$\frac{4}{100}$.	1	.02734375lb.	6	.125da.
198.	2	$\frac{3067}{1000}$.	2	£.108333+	7	71.151mi.+
198.	3	$\frac{8275}{1000}$.	3	.00035A.	8	6.5454rd.+
198.			4	.0097222da.+	9	.00396bar.
198.	4	$\frac{49}{100000}$.	5	1.3125pk.	10	.0004375cwt

199.	1	12.00384gr.	10	1s. 8d. $1\frac{3}{5}$far.
199.	2	2qr. 12lb. 8oz.	11	3qr. 11lb.
199.	3	2qt. 1pt.	12	19hr. 21m. 36sec.
199.	4	6s. 9d.	13	1mi. 28rd. 7ft. 11.04in.
199.	5	6cwt. 3qr.	14	1oz. 8dr.
199.	6	8P.	15	103da. 23hr. 59m. 12.48sec
199.	7	1hhd. 47gal. 1qt.	16	£1 0s. 11.04d.
199.	8	6gal. 3qt.	17	£1 17s. 7.2d
199.	9	136da. 21hr.	—	—

200.	1	4.889955wk.+	6	4.890625bu.	11	.42859226wk.
200.	2	2.4694lb.+	7	.472916lb.+	12	.39201ch.
200.	3	1.25yd.	8	.78875T.	13	7.878125M.
200.	4	1.046875lb.	9	5.88125A.	—	—
200.	5	5.0833L.+	10	.0055T.	—	—

203.	6	$36,428	8	$28,333+	10	$30,833$\frac{1}{3}$	12	$3,111+
203.	7	$21,25	9	$32,812+	11	$62.	13	24 pounds

204.	15	472,50	16	6 days' work.	17	31$\frac{2}{3}$bu.	18	$18,541+
205.	20	$8.	21	$25,50	23	49 men.	24	11wk
206.	26	18 bales.	27	11$\frac{1}{4}$ft. long.	29	2$\frac{2}{5}$ days.		

207.	31	54da.	34	A's g'n $58,33$\frac{1}{3}$	35	1st. $240 2d. $200
207.	32	10$\frac{2}{3}$ "		B's g'n $116,66$\frac{2}{3}$		3d. $140

208.	38	1$\frac{7}{8}$ days.	—	—	—	—				
209.	39	30da.	40	9da.	41	$96	43	72 wo'n.	44	42 Georgia.

210.	1	18 sheep	6	12$\frac{2}{3}$bar.	11	31$\frac{1}{2}$ft.= 10$\frac{1}{2}$yd.	15	1st. $2,50
210.	2	$112	7	$1,60				2d. $3,75
210.	3	48da.	8	$17,273+	12	100da.		3d. $8,75
210.	4	30s.	9	13$\frac{1}{3}$bu.	13	$10	16	22$\frac{1}{2}$gal.
210.	5	$6,5625	10	62$\frac{6}{7}$yds.	14	10mo.	17	18$\frac{1}{3}$lb.

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.
211.	18	$9,16$\frac{2}{3}$	21	28*da.*	25	$18	30	36 *men*
211.	19	17$\frac{3}{5}$*yd.*	22	$175	26	$83,33$\frac{1}{3}$	31	*A* 21
211.		1*st.* 105*lb.*		& $125	27	7$\frac{1}{5}$*da.*		*B* 15
211.	20	2*d.* 140*lb.*	23	$10500	28	3 *men*		*C* 24
211.		3*d.* 168*lb.*	24	160*yd.*	29	5$\frac{1}{3}$*mo.*		*hogs*

P.	EX.	ANS.	EX	ANS.	EX	ANS.
212.	32	*A's* $126. *B's*	35	$0,803+	39	1*st.* $39,20 2*d.*
212.		$117. *C's* $72.	36	10$\frac{1}{5}$*yd.*		$19,60 3*d.*117,60
212.	33	54*bar.*	37	$1344	40	$533,33$\frac{1}{3}$
212.	34	1470*gal.*	38	$5040	41	3 *pieces*

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.	EX	ANS.		
216.	1	36	2	60	3	12	4	2$\frac{1}{4}$	5	24	—	—

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.	EX	ANS.
217.	2	$\frac{2}{9}$.	4	$\frac{20}{100}=\frac{1}{5}$.	6	$\frac{144}{36}=4$.	8	$\frac{5}{20}=\frac{1}{4}$.	10	$\frac{9}{8}=1\frac{1}{8}$.
217.	3	$\frac{4}{16}=\frac{1}{4}$.	5	$\frac{200}{300}=\frac{2}{3}$.	7	$\frac{3}{12}=\frac{1}{4}$.	9	$\frac{8}{56}=\frac{1}{7}$.	11	$\frac{12}{132}=\frac{1}{11}$.

P.	EX.	ANS.	EX	ANS.	EX	ANS.
219.	1	308*mi.*	4	3300 *pounds.*	7 8	$243.75 $201,75
219.	2	$165	5	$61,425	9	106$\frac{2}{3}$*yd.*
219.	3	$1381,25	6	10955*mi.*	10	36000 *rations.*

P.	EX.	ANS.	EX	ANS.	EX	ANS.
220.	11	8*hr.* 20*m.*	17	81$\frac{1}{3}$ *ft.*	21	$1,87$\frac{1}{2}$
220.	13	$4,861+	18	$40,32	22	$0,154+
220.	14	£227 12*s.* 1*d.*+	19	*A's* $1787,50	23	$6206,93$\frac{1}{3}$
220.	15	$115,50		*B's* $1283,75	—	——
220.	16	$29,25	20	$22,50	—	——

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.
221.	24	20$\frac{14}{65}$*bar.*	28	$252	32	21$\frac{3}{5}$*lb.*	36	$18,27
221.	25	$61,425	29	24*yd.*	33	$12,13	37	$168,742+
221.	26	5*s.* 9*d.*	30	72 *hats*	34	28*hr.*	38	$108,25
221.	27	315*bu.*	31	37*bar.*	35	2$\frac{2}{3}$ *acres*	39	$253,125

P.	EX.	ANS.
223.	1	8 *days.*

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.	EX	ANS.
224.	2	27*da.*	5	20*hr.*	8	6*men*	11	16*in.*	14	1$\frac{7}{20}$*lb.*
224.	3	72*da.*	6	18*da.*	9	134*da.*	12	25*bu.*	—	——
224.	4	160*da.*	7	16*bot's*	10	27*da.*	13	10*da.*	—	——

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.	EX	ANS.
226.	1	$45	2	150*lb.*	3	$99	4	232*da.*	5	511$\frac{1}{2}$*mi.*

P.	EX.	ANS.	EX	ANS.	EX	ANS.	EX	ANS.
227.	6	13$\frac{31}{63}$*A.*	10	4$\frac{1}{31}$*da.*	14	15*lb.*	18	36*yd.*
227.	7	18*yr.*	11	1112*bu.*	15	38$\frac{2}{5}$*rea's*		*long*
227.	8	27 *weav's*	12	2$\frac{6}{7}$ *tons*	16	192*bar.*	—	——
227.	9	72 *men*	13	343$\frac{1}{8}$*ft.*	17	200 *more*	—	——

P.	EX.	ANS.	EX	ANS.
229.	1	$857,142$\frac{6}{7}$ *A's.* $142,857$\frac{1}{7}$ *B's.*	4	$1866,66$\frac{2}{3}$ *A's*
229.	2	$480 *A's.* $750 *B's.* $675 *C's.*		$1066,66$\frac{2}{3}$ *B's.*
229.	3	$1500 *Mr. W's.* $2100 *Mr J's.*		$1066,66$\frac{2}{3}$ *C's.*
229.	5	$400 *A's.* $800 *B's.* $1200 *C's.*	—	——

P.	EX.	ANSWERS.
230.	2	$77 *A's.* $260 *B's.*
230.	3	$54. *A's.* $38,50 *B's.*
230.	4	$60,777 + *A's.* $127,633 + *B's.* $233,387 + *C's.*
230.		$328,201 + *D's.*
230.	5	\$1666.66$\frac{2}{3}$ *A's.* \$3888.88$\frac{8}{9}$ *B's.* \$9444.44$\frac{4}{9}$ *C's.*
230.	6	*B's* = $273,365 *nearly.* *A's* = $476,635.
230.		*Fuller's* $1808,8669 +, *Brown's* $1596,0591 +
230.	7	*Dexter's* $1995,0738 +, *The remainders added*
230.		*will give the exact proof.*

232.	1	$16,25	7	$8,93	13	$2109,0392	19	$42,60
232.	2	19,50*yd.*	8	18,06 *sheep*	14	$75	20	432*bar.*
232.	3	39,375*cwt.*	9	$18.5487	15	$229,08	21	42*hhd.*
232.	4	$2,375	10	280 *cows*	16	$350	22	$24,25
232.	5	155,48*mi.*	11	892,5 *tons*	17	$375	—	——
232.	6	5 *oxen.*	12	1015*lb.*	18	$694,232	—	——

233.	23	$10,80	1	,25	5	,38$\frac{6}{13}$.	9	.15$\frac{1}{22}$.
233.	24	26$\frac{2}{3}$*per ct. left*	2	,50	6	,05	–	——
233.		= \$3333,33$\frac{1}{3}$.	3	,40	7	,01$\frac{1}{29}$.	–	——
233.	25	$1304,75	4	,20	8	,03	–	——

234.	10	.55	11	.20	12	.20	13	.12$\frac{1}{2}$	——

235.	2	$24862,50	3	$233,75	4	$8443,75	5	$14700

236.	9	200 *shares.*	237.	10	80 *shares.*	——

238.	1	\$1,06$\frac{1}{4}$.	3	$0,966 +	5	$112,50	7	$2,054
238.	2	$0,75 *loss.*	4	$1,00	6	$208,4375	8	25 *per ct.*

239.	9	18 *per ct.*	11	$1,025	14	19*yd.*
239.	10	$13 *whole g'n*	12	\$1,031$\frac{1}{4}$	15	\$9,21$\frac{1}{19}$.
239.		=20 *per ct.*	13	\$2,216$\frac{2}{3}$.	—	——

240.	1	$43,77	4	$210	8	$450	11	$1800—$45
240.	2	$1312,50	5	$607,50	9	$1320	12	$47,624 +
240.	3	$237,60	6	$1381,80	10	$142,95	13	$9558,437 +
240.		$158,40	7	$504	—	——	14	$6500

242.	2	$39	6	$427,50	10	$183,9705	2	$121,325
242.	3	$266	7	$9,5067	11	$4454,857	3	1315,389
242.	4	$4446,75	8	$331,1511	12	$30455,0224	4	221,075
242.	5	$642,60	9	$1158,0668	1	$95,229 +	5	1290,798

243.	2	$10,8012	3	$2,728 +	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.				
244.	2	$309,5634	4	$30,5598	6	$64,5792				
244.	3	$35,1485+	5	$14,0979	–	——				
245.	7	$76,2433	13	$190,148	19	$600,445				
245.	8	$194,6177	14	$3286,40	20	$44,2893				
245.	9	$328,32	15	$6322,8825	21	$167,001				
245.	10	$1004,6976	16	$7500,60	22	$3126,203				
245.	11	$1183,6935	17	$75,04	23	$9051,668				
245.	12	$1445,2388	18	$218,88	24	$4968,9975				
246.	25	$141,8136	31	$94,265	37	$217,5116				
246.	26	$272,80	32	$245,4896	38	$6214,14				
246.	27	$39,9274	33	$76,966	39	$856,686				
246.	28	$928,0686	34	$33,3232	40	$383,3808				
246.	29	$529,925	35	$28761,776	41	$188,0292				
246.	30	$31,2681	36	$5678,068	42	$2418,465				
247.	2	£15 2*s.* $8\frac{1}{2}$*d.*	4	£26 10*s.* 11*d.*		——				
247.	3	£24 18*s.* $3\frac{1}{4}$*d*+	5	£331 1*s.* 6*d.*		——				
249.	2	$860,4194	3	$167,983+	——	——				
250.	1	$950	2	7 *per ct.*	3	5*yr.*	4	$225	5	$7,33\frac{1}{3}$=7*yr.* 4*mo.*
251.	2	$19,101	4	$404,0625	6	$211,456				
251.	3	$36,50+	5	$291,60	7	$185,775				
252.	8	$171,5971	9	$118,528	10	$315,2438	11	$152,408		
253	1	$1750 *present value.*	2	$1565,402+ *pres. val.*						
254.	3	$9677,50 + *pres. val.*	8	$3869,407+ *pres. val.*						
254.	4	£223 5*s.*8*d.* *discount.*	9	$2109,236+ " "						
254.	5	$5620,176+ *pres. val.*	10	$2763,694+ " "						
254.	6	$702,485 " "	11	$4000 " "						
254.	7	$1,94 *difference.*	12	$6,473+ *loss.*						
255.	1	$6,3291	3	$15240,54	5	$3393,504	7	$122,81+		
255.	2	$10,50	4	$5,8408	6	$29.0096	–	——		
256.	8	$341.709+	1	$344,66+	2	$5734,32+				
257.	3	$695,64	4	$118,85+	5	$1740,60	6	$376,46+		
258.	2	12*mo.*	3	8*mo.* $22\frac{4}{23}$*da.*	4	9*mo.*	5	$137\frac{13}{31}$*da.*	6	$4\frac{1}{3}$*mo.*
259.	7	$67\frac{6}{19}$*da.* 9*th day of March.*	——	——						
260.	9	$60\frac{93}{114}$*da. or Aug.* 31*st.*	11	$49\frac{11}{158}$*da. or* *Jan.* 25*th*						
260.	10	6*mo.* 6*da.*								
262.	3	$12,25	4	$6,25	——	——	——			

P.	EX	ANS.	EX	ANS.
265.	1	$426,416	6	£21 5s.-£25 14s. 3d. £30
265.	2	£1073 18s. $1\frac{1}{2}$d.		17s. 1d.+-£41 2s. $9\frac{1}{2}$d.+
265.	3	$2033.489+		£38 11s. $4\frac{1}{4}$d.-£23 19s. $11\frac{1}{2}$d
265.	4	£389 6s. $2\frac{3}{4}$d.	7	$250-$250-$250-$250-$250
265.	5	$2551,733		$516,66⅔-$250.

266.	1	$3720,937	3	$6748,60	5	$3643,875
266.	2	$8668,935	4	$4583,94+	–	——

267.	2	$5944,791	3	$9226,061+	——	——
268.	2	$1270,428	3	$2016,11	4	$16975,775

270.	1	$2812,50	2	$423,36	3	$251,45+	4	$1457,75

271.	1	3*s.*	2	84*cts.*+	3	288+*cts.*	4	20⅔ *carats.*	5	73c

274.	4	1*lb.*-1*lb.*- 3*lb.*
274.	5	3 *of* 16. 2 *of* 18. 3 *of* 23. 5 *of* 24
274.	6	3*gal. at* 10*s.*-3 *at* 14*s.*-4 *at* 21*s.* 4 *at* 24*s.*
274.	1	4*gal. at* 5*s.*-8 *at* 5*s.* 6*d.*-8 *at* 6*s.*
274.	2	14*bu. W.* 28*bu. R.* 14*bu. B.* 28*bu. O.*
274.	3	96*bu. W.* 12*bu. R.* 12*bu. B.* 12*bu. O.*
275.	4	40*gal. F.* 80*gal. E.* 20*gal. spirits.*
275.	1	10 *of* 1*st.* 10 *of* 2*d.* 30 *of* 3*d.*
275.	1	36*lb. at* 4*d.* 36 *at* 6*d.* 36 *at* 10*d.* 36 *at* 12*d*
275.	2	$21\frac{3}{4}$ of each.
275.	3	4 *each of the* 1*st. three and* 30 *of* 15 *carats fine.*

276.	1	$1^2=1$	9	$9^4=6561$
276.	2	$\frac{1}{2}^2=\frac{1}{4}$.	10	$16^5=1048576$
276.	3	$\frac{1}{8}^3=\frac{1}{512}$.	11	$20^6=64000000$
276.	4	$\frac{3}{5}^2=\frac{9}{25}$.	12	$225^2=50625$
276.	5	$9^2=81$	13	$2167^2=4695884$
276.	6	$12^3=1728$	14	$321^3=33076161$
276.	7	$125^3=1953125$	15	$215^4=2136750625$
276.	8	$16^3=4096$	16	$=610437195439776$
276.		——	17	$9^6=531441$
276.			18	$36049^2=1299530401$

282.	1	1.73205+	6	6031	11	.05	16	3.12249
282.	2	3.31662+	7	4698	12	.01809	17	0.71554
282.	3	32.695+	8	57 19+	13	.0321	18	0.64599+
282.	4	1506.23+	9	69.247+	14	2.104	19	$\frac{2}{3}$.
282.	5	2756.22+	10	2.091+	15	2.91547+	20	$\frac{4}{5}$.

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
284.	2	25*ft.*	3	85	5	43.81*rd.*+
284.	1 2	12.649*rd.* 126.49*rd.*+	4	97.75*mi.*+	6	82 *partners*

285.	7	62 *trees*	8	160*rd.*	9	90*ft.*	10	4.94*ft.*

288.	1	73	3	319	5	638	1	.54	3	2.35	5	.707
288.	2	179	4	439	6	364	2	.955	4	.909	6	1.505

289.	1	$\frac{5}{7}$.	4	.289+	1	17	4	14.58*ft.*+	7	6*ft.*
289.	2	$3\frac{1}{7}$.	5	.822+	2	28-4704	5	1728	–	—
289.	3	$\frac{3}{5}$.	6	.873	3	16.197*ft.*+	6	12*ft.*	–	—

290.	8	268.0832	9	2*ft.* 4*in.*	10	2*ft.*	11	12*ft.*

292.	1	\$1,53	2	\$212	3	40	4	2	5	9*yr.*

293.	1	4*yr.*	2	5*mi.*	3	4*cents.*	——	——

294.	2	78*times*	3	34-162	4	\$1,35	5	175*mi.*	6	5*mi.* 1300*yd.*

296.	2	£2 2*s.* 8*d.*	3	4	4	78732	5	\$25600	6	\$61,44

297.	1	6560	3	381	5	\$196,83-\$295,24
297.	2	254	4	£204 15*s*	6	\$4800-\$9450

298.	1	\$978	6	$\frac{3}{8}$.	11	213,3125	15	\$3567
298.	2	51*bu.*	7	$17\frac{7}{8}$*yds.*		211,6875	16	288
298.	3	\$80,71	8	$\frac{12}{40}$.	12	$\frac{25}{42}$ *and* $\frac{11}{42}$.	17	\$4717
298.	4	5467*hr.*	9	$4\frac{7}{24}$.	13	9.04	18	137
298.	5	\$26,25	10	120 *men.*	14	$4\frac{1}{6}$.	—	

299.	19	$108\frac{7}{75}$ *planks.*	29	879000
299.	20	\$3800	30	792
299.	21	80*yds.*	31	1*mi.* 6*fur.* 33*rd.*
299.	22	*A's* \$28. *B's* \$60.		$15\frac{1}{2}$ *ft.*
299.		*C's* \$32.	32	62 *years*
299.	23	$7\frac{1}{5}$ *days.*	33	4
299.	24	3*mo.*	34	\$2450 1*st.*
299.	25	$\frac{37}{240}$ *left*		\$3681 2*d.*
299.		\$986,66$\frac{2}{3}$ *worth*		\$4294,50 3*d.*
299.	26	$2\frac{1}{4}$.	35	£408 *saves*
299.	27	1*st.* \$160. 2*d.* \$120.	36	$23\frac{1}{2}$*bu.*
299.		3*d.* \$140 ‖ 28 | 50.	37	2400

300.	38	3 *o'clock*	42	$32\frac{8}{11}$*min.*	48	52*yr.* 11*mo.*
300.	39	300 *men*	43	\$34782,608		20*da.* $10\frac{1}{2}$*hr*
300.	40	864	44	\$3,653	49	$5\frac{1}{2}\frac{5}{3}$.
300.	41	*A's* \$2364	45	$34\frac{2}{5}$ *per ct.*	50	111*yr.*
300.		*B's* \$1182	46	\$7816,091+	51	22500 *bricks*
300.		*C's* \$788	47	57 *pieces*	52	27.7*ft.*
300.		*D's* \$394	—	——	53	5 *years*

P.	EX.	ANS.	EX.	ANS.
301.	54	2250 *men*	62	4 *days*
301.	55	$196.83 *last term.*	63	240 *hours*
301.		$295,24 *whole am't*	64	*A* 21-*B* $8\frac{2}{5}$-*C* 42 *days.*
301.	56	94*bu. wheat.* 12*rye.*	65	*A's* = $194,80$\frac{40}{77}$.
301.		12 *barley.* 12 *oats.*		*B's* = $129,87$\frac{1}{77}$.
301.	57	$16\frac{16}{17}$*in.*		*C's* = $97,40$\frac{20}{77}$.
301.	58	356,25		
301.	59	$8640		*D's* = $77,92$\frac{16}{77}$.
301.	60	$1,20	66	$1020,66
301.	61	*lost* 4 *pence.*	67	$8925,544+

P.	EX.	ANS.	EX.	ANS.
302.	68	50*ft.*	73	4*yds.*
302.	69	9*mi.* 5*fur.* 34*rd.*+	74	$423,36
302.	70	*daughter* $780. *son*	75	$920,20 1*st.* $2760,60
302.		$3120. *wife* $1560.		2*d.* 5521.20 3*d.*
302.	71	76*mi.*-1292*mi.*	76	3*hr.* 20*m.*
302.	72	4*yr.* 11*mo.* 22.8*da.*	77	$69\frac{3}{7}$*mi. f'm N. Haven.*

303.	1	400*sq. yd.*	3	2*A.* 3*R.* 15*P.*	2	72*A.* 3*R.* 24*P.*
303.	2	30*A.*	1	109*A.* 1*R.* 28*P.*	3	3*A.* 3.*R.* 25*P.*

304.	4	5*A.*	8	32520*sq. yd.*	4	176.03125*sq. yd.*
304·	5	10*A.*	1	45849.485.	5	15*A.* 0*R.* 33.2*P.*
304.	6	7*A.* 2*R.*	2	5*A.* 1*R.* 9.95*P.*	6	67*A.* 2*R.* 16*P.*
304.	7	6*A.* 0*R.* 12*P.*	3	230*ft.* 5′ 7″ 6‴	–	——

305.	1	12.5664	1	28.2744	3	809.5616
305.	2	292.1688	2	78.5400	1	904.7804
305.	3	62.8320	3	38.4846	2	33.5104
305.	4	25.	4	1.069+	3	1436.7584
395.	5	3709	1	452.3904	4	113.0976*cu. ft*
305.	6	2180.41+	2	153.9384	–	——

306.	1	4550*Sq. ft.*	1	4500.	3	$21\frac{1}{9}$*cu. ft.*	5	3500*cu. ft.*
306.	2	1440 " "	2	13824*cu. ft.*	4	$157\frac{213}{231}$.	–	——

307.	1	3141.6	1	2120.58	4	1809.5616
307.	2	116.666*Sq. ft.*+	2	9110.64	5	29452.5
307.	3	5654.88*Sq. in.*	3	3392.928	–	——

308.	1	475	4	4030	7	6000	3	3141.6
308.	2	2080	5	1440	1	65.45	4	706.86
308.	3	1242	6	3600	2	2290.2264	5	706.86

309.	1	196.52 *gal.*	3	136*gal.*	5	6*hr.* 3*m.* $48\frac{114}{217}$
309.	2	185.0688*gal.*	4	182.844*gal.*	–	*sec.* ——