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University of Illinois at Urbana-Champaign

January 1984

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DECISION MODELS FOR
INTERNAL AUDIT FREQUENCIES

ABSTRACT

Increasing the internal audit frequency decreases the costs associated with being in the noncompliance state but increases the costs associated with audit processes. Relevance of normative decision models to resolve this trade-off is discussed. In the light of this discussion, four simple but not simplistic decision models are presented: (i) optimal policy resolves the foregoing trade-off by minimizing total cost criteria; (ii) periodic employs the same criteria but requires the intervals between two consecutive audits to be uniform; (iii) constant hazard policy also uses the same criteria but requires the failure rate between two consecutive audits to be uniform; (iv) sequential policy employs qualitative rather than pecuniary performance criteria. Managerial implications of models are discussed and numerical examples are provided.

DECISION MODELS

FOR INTERNAL AUDIT FREQUENCIES

1. INTRODUCTION

In this paper, two mutually exclusive groups are considered within an accounting control system of an organization, namely, auditors and the elements of the control system under audit. Henceforth, the latter will be referred to as the auditee. In general terms, the auditee's task consists of certain performance and internal control operations. The auditor is in charge of comparing the procedures used by the auditee, in the conduct of his internal control operations, with the procedures prescribed by the management. The auditee may be found in compliance or noncompliance with the management's procedures. In either case the auditor reports the situation to the management. This comparison and reporting process constitutes a compliance audit. This paper focuses on the problem of optimal scheduling of compliance audits, i.e., optimal frequency and timing of the audits of the accounting control system.

There is a striking resemblance between the problem setting briefly described above and the inspection, maintenance, and replacement problems associated with the machinery used for manufacturing or military purposes. The literature on inspection, maintenance and replacement decisions is extensive [Pierskalla and Voelker, 1976; Sherif and Smith, 1981]. The available normative decision models range from elaborate Markovian models with unobservable states to simplistic models employing the deterministic life-cycle cost concept.

The question is the applicability of these models to the audit scheduling problems. A conclusive answer is not possible in the absence of a general descriptive theory of the auditing process. An attempt is made below to give a partial answer.

Applicability of Normative Inspection Models to Audit Scheduling

The fundamental difference between inspecting a machine (or product of a machine) and auditing an accounting control system is behavioral. In machine systems the cost and performance-related consequences of any action taken are reasonably well-defined through physical laws. An accounting system, on the other hand, is normally embedded into intricate and not so well-defined behavioral issues of an organizational complex. This difference is obvious; what may not be so obvious is its pertinent implications.

For the purposes of developing and implementing decision models, Keen and Morton [1979] proposed the classification of decision problems into three groups⁺: structured, semi-structured, and unstructured. This classification scheme enables one to explain the implications of behavioral differences between problems. Normative models can be implemented to structured problems directly and, in many cases, in an automated fashion; whereas, unstructured problems are those which do not lend themselves to the analysis by normative

⁺Originally Simon [1960] classified decision problems into two categories: programmable and nonprogrammable. Keen and Morton extended and modified Simon's classification. Not to digress, in this paper definitions and justifications of these terms will not be repeated. An interested reader may refer to the lucid discussion given by Keen and Morton.

models. Semi-structured problems are described by Keen and Morton as follows:

"These are problems where managerial judgment alone will not be adequate, perhaps because of the size of the problem or the computational complexity and precision needed to solve it. On the other hand, the model or data alone are also inadequate because the solution involves some judgment and subjective analysis. Under these conditions the manager plus the model can provide a more effective solution than either along." [Keen and Morton, 1979, p. 86]

Applying this criteria to audit scheduling, the problem is obviously unstructured. The reason is mainly because of the behavioral and organizational factors whose interaction causes the unknown dimensions of this problem. On the other hand, the objections of the audit, the observability of the data, the methodology to be used, as well as the managerial judgments about some parameters in the model and its results provide a basis for classifying the audit scheduling problems as a semi-structured one for which normative decision models may be applied. The role of normative research in developing a body of knowledge in accounting has been remarkable. ~~On the other hand,~~ one can observe that normative accounting research [1960-1970] and often has had implications on the accounting standards setting. For example, the FASB # 33 about the disclosure of replacement cost information has benefitted from the discussion of normative accounting models presented by Edward and Bell [1961], Chambers [1966] and Sterling [1970]. In auditing normative model were introduced by Hughes [1977, 1980]. The use of normative models in auditing is justified by the complexity of the auditing process with insufficient descriptive theory.

The objective of this paper is to present a number of decision models for audit scheduling. The models presented include "cost-based models" for a regular audit scheduling and "non-cost models" for sequential or "as-you-go" audit decisions. The focus of this paper is on simple but not simplistic models. This choice is based on several considerations.

First, an elaborate model was introduced by Hughes [1977] which requires the definition of many parameters that may require significant amount (and cost) of data as well as skills in probability theory and dynamic programming for the application of the model. These observations may limit the practicality of the model. Second, in the absence of a complete descriptive theory for the auditing process, even the most complex models may not be able to capture all the variables in play, especially when dealing with human behavior in the auditing environment. Third, the model is used as a tool to aid in the managerial decisions regarding auditing and not to replace these judgments. Therefore, practicality, understandability, fewer but meaningful parameters, availability of data, and cost of application may constitute a criteria for usefulness that would suggest the use of simple yet meaningful models.

2. THE INSPECTION PROBLEM IN ACCOUNTING

The basic purpose of an inspection is to determine the condition of a system. The need for inspection arises for those systems which are aging in some statistical sense [Bryson and Siddiqi, 1969]. Some systems improve with age, when there is a learning process, but many

deteriorate over time. This deterioration may be expressed by a continuous or a binary index. In the former case, the state of the system is expressed by a continuous function, whereas, in the latter case, the system is said to be in a good or failed state.

It has been assumed that "...unless internal control receives constant attention, it tends to disintegrate" [Stettler, 1966, p. 47]. The American Institute of Certified Public Accountants states that "procedures (control procedures) may become inadequate because of changes in conditions and the degree of compliance with procedures may deteriorate" [Statement of Auditing Standards 1973]. At any given point in time an auditee's condition can generally be described by two states: compliance state or noncompliance state [Barefield, 1975] (in control or out of control [Gonedes, 1971]). Thus, similar to general inspection problems, an accounting control system deteriorates over time and its state can be described with a binary index. Furthermore, based on descriptive research findings [Barefield, 1975], the following plausible statements can be made:

- (i) If the audit effectiveness is assumed to be perfect, then (a) the auditor always determines the state of the auditee, and (b) once audited, the auditee always goes back to the state of compliance.
- (ii) Once an auditee reaches the noncompliance state, he remains there while the auditor is absent.

The existing descriptive theory on accounting control systems is inadequate to prove or refute the above conditions. However, these conjectures are commonly accepted and the models presented in this paper assume their validity.

The thesis of this paper is that when developing normative models for those systems with inadequate descriptive theories, it is desirable and realistic to start with simple cases. It is much easier to test a simple model and modify it step-by-step into a more elaborate one as more descriptive knowledge and experience accumulates. However, this paper tried to avoid oversimplifying the problem setting to a degree that the setting only remotely resembles any existing real system.

Inspection Cost Models

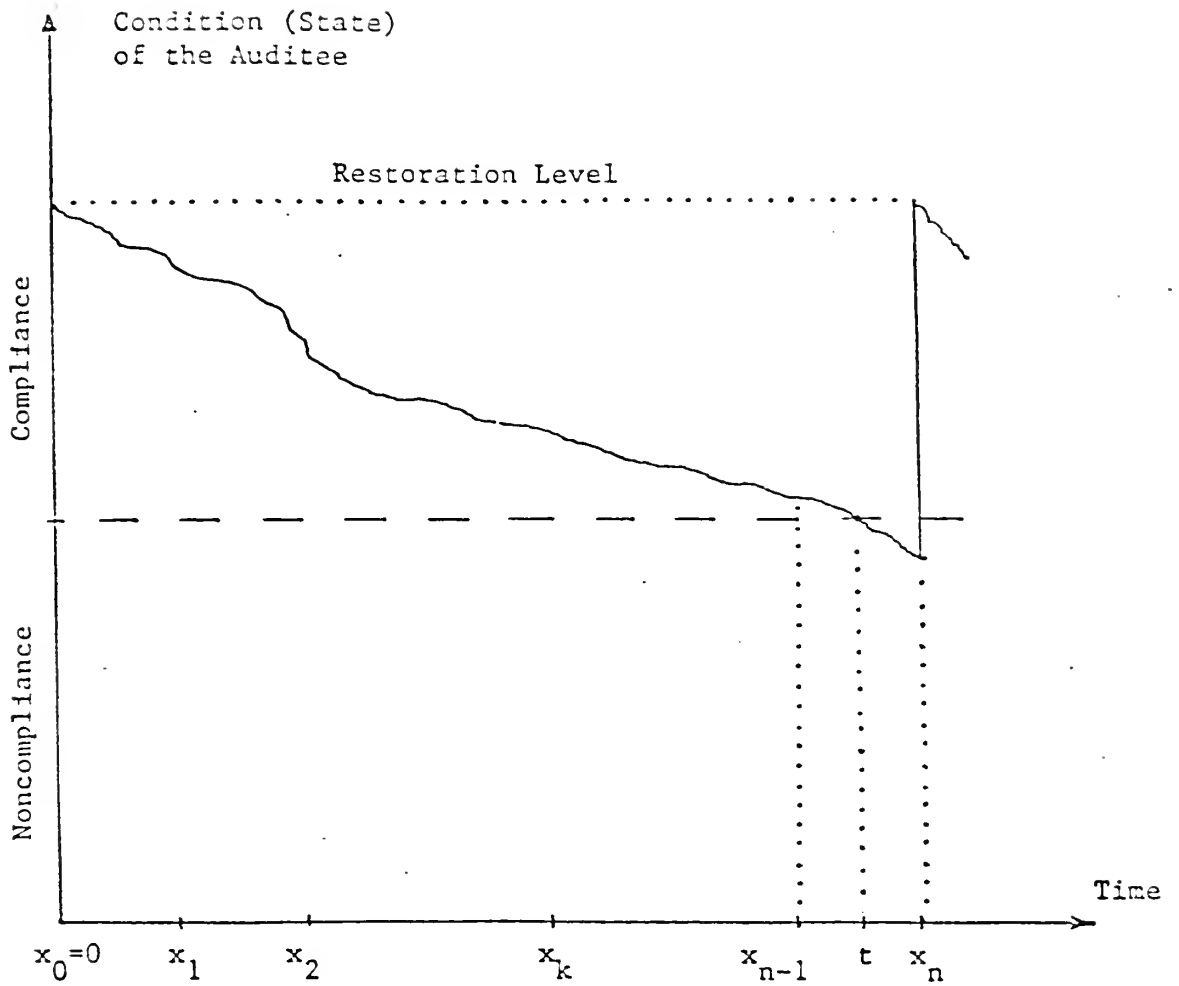
In general, the decision models for inspection scheduling attempt to resolve the trade-off between two types of costs: the cost of each inspection (c_1), and the cost of leaving the system in the non-compliance state per unit time (c_2). Suppose that at an arbitrary time, say zero, the system is inspected, found to be in the non-compliance state, and restored. The period between two consecutive such events is referred to as an inspection cycle. Clearly, any number of inspections, say n , may take place within an inspection cycle, but the first $n-1$ find the system in the compliance state (see Figure 1). Let T be a continuous random variable denoting the time to failure; and let t be a particular realization of T . Then, the total cost per inspection cycle is [Barlow, et. al., 1963]:

$$C_1(t;x) = c_1 n + c_2(x_n - t) \quad (1)$$

where $x = x_1, x_2, \dots$ is the sequence of inspection and n is such that

$$x_{n-1} < t \leq x_n, (x_0 = 0).$$

FIGURE 1
AN AUDIT CYCLE



In some situations there may be no way of knowing the actual time to failure, other than the fact that it is between x_{n-1} and x_n . In such cases the following model is proposed [Munford, 1981]:

$$C_2(t;x) = c_1n + c_2(x_n - x_{n-1}) \quad (2)$$

An optimal inspection frequency is specified once x^* is determined by minimizing the expected total cost per inspection, i.e., C_1 or C_2 . In the rest of this paper it is assumed both C_1 and C_2 are finite and pseudo-convex functions for all $x \in [0, \infty)$.

The accounting control system; for which the audits need to be scheduled, incur cost (loss), C_2 , when the process is "out of control." Auditing the system, on the other hand, will also incur cost, C_1 . The unit of time for the accounting control system can be defined the information (accounting) cycle at the end of which control reports are prepared. The audit scheduling problem is to define the time(s) at which the accounting control system should be audited such that the cost of audits do not exceed the cost of having the system in unacceptable condition (not functioning correctly). Thus, the foregoing models can describe the audit scheduling problem.

3. DECISION MODELS FOR AUDIT SCHEDULING

To compute the expected total cost, $E[C_1(T;x)]$, of each audit cycle, it is necessary to specify the probability distribution of the random variable T : the time it takes for the auditee (elements of the accounting control system) to go into the noncompliance state after restoration. Let the commulative probability distribution function

and probability density function of T be denoted $F(t) = P(T \leq t)$, and $f(t)$, respectively.

Optimal Solution

The optimal solution [optimum audit schedule $x^* = x_1^*, x_2^*, \dots$] based on the cost function in equation 1, and given an initial approximation of x_1 , is:†

$$x_{K+1} = x_K + \frac{F(x_K) - F(x_{K-1})}{f(x_K)} - \frac{C_1}{C_1} \quad (K=1,2,\dots) \quad (3)$$

Considering the cost function in equation 2, the optimal audit schedule is computed as:††

$$x_{K+1} - 2x_K + x_{K-1} = \frac{\overline{F}(x_{K+1}) - 2\overline{F}(x_K) + \overline{F}(x_{K-1}))}{f(x_K)} - \frac{C_1}{C_2} \quad (K=1,2,\dots) \quad (4)$$

Except for the case of exponential distribution, the determination of an optimal auditing schedule x^* by using C_1, C_2 has three main difficulties. First, the algorithm given for both cases applies only when certain conditions hold [Karlin, et. al., 1961] which are difficult to check in practice. Second, even if the conditions hold, the algorithm is computationally demanding and slow in convergence. Last, if the conditions do not hold then there is no theory based on which x^* may be computed from equations (2) and (4).

Considering these limitations, an easy to compute, theoretically justified, and an intuitively appealing optimal solution for the audit

†Computation algorithm for this solution is given in Barlow, et.al., [1963].

††See Munford [1981] for the derivation of this result.

scheduling problems may not be generally possible to obtain. Since cost parameters and failure distributions are estimates (and by no means exact) optimality of the solution may be impaired. Therefore optimality can be sacrificed for other practical considerations. The following models consider such proposition.

Suboptimal Solutions

Let $h(t) = f(t)/\bar{F}(t)$ and its integral $H(t) = \int_0^t h(\tau) d\tau$ be called the hazard rate function and hazard function, respectively. It should be noted that the hazard rate function is approximately equal to the conditional probability that the auditee will be in the noncompliance state during the interval $(t, t + \Delta t]$, for an infinitesimal dt , given that he was in compliance state at time t . In the following models it is assumed that if $f(t)$ has an increasing hazard rate then the audit intervals are decreasing over time. This assumption is valid for accounting control systems where there is an aging process in the statistical sense defined by Bryson and Siddiqi [1969].

i) Periodic Policy

A simple and commonly used policy in accounting practice [Hughes, 1977] is to audit at equal and predetermined intervals, θ . Thus, in periodic policy $x_k = k\theta$ ($k = 1, 2, \dots$), for some $\theta > 0$. It is already discussed in Section 3 that when T is exponentially distributed then the periodic policy is an optimal one for both cost models (1) and (2). Otherwise, it is suboptimal to restrict $x_k = k\theta$. Nevertheless, the optimal interval length θ for the restricted problem can be found by minimizing

$$E[C_1(T;x(\theta))] = (c_1 + c_2\theta)m(\theta) - c_2 E(T) \quad (5)$$

or

$$E[C_2(T;x(\theta))] = c_1m(\theta) + c_2\theta \quad (6)$$

where $m(\theta) = \sum_{j=0}^{\infty} F(j\theta)$ is the mean number of audits. Equation (5) and (6) can be minimized employing a numerical search procedure to determine the optimal interval between audits, θ^* . In many auditing situations, however, there is an easier procedure to determine θ^* . Suppose that in an accounting system the cost of each audit, c_1 , is not large relative to the cost that occurs when the auditee is in the noncompliance state, $c_2E(T)$. This implies that the mean number of audits, $m(\theta^*)$, is large, hence, the truncated Euler-MacLaurin summation formula (see, e.g., [Knopp, 1947]) can be used to approximate $m(\theta^*)$. If the probability that the accounting control system returns to the noncompliance state immediately after restoration by an audit is zero, which is normally the case, then (5) and (6) produce

$$\theta^* = \sqrt{2c_1E(T)/c_2} \quad (7)$$

and

$$\theta^* = \sqrt{c_1E(T)/c_2} \quad (8)$$

respectively. These formulas are analogous to the economic order quantity formulas in optimal inventory problems.

†See the complete derivation in Munford [1981].

Even though the periodic policy is convenient to implement and has an organizational appeal, it is a rather restrictive policy. Furthermore, success of the auditing process can be enhanced by introducing the "surprise" element, and is auditing at intervals which are not obvious to the auditee.

ii) Constant Hazard Policy

In periodic policy, the audits were assumed to be equally spaced over time. An alternate is to space them so that the probability of having the accounting control system at the noncompliance state be the same. This idea lends itself to the constant hazard policy [Munford and Shahani, 1973]. Specifically, let p denote the probability that the accounting control system will be in the noncompliance state during the k -th audit time (x_k) given that the system was in the compliance state during the $(k-1)$ st audit time (x_{k-1}), then

$$p = \frac{F(x_k) - F(x_{k-1})}{\bar{F}(x_{k-1})} \quad (k = 1, 2, \dots) \quad (9)$$

where $F(0) = 0$, hence $F(x_1) = p$. Equation (12) can be solved for x_k :

$$x_k = F^{-1}(1 - q^k) \quad (10)$$

where $q = 1 - p$ and $F^{-1}(\cdot)$ denotes the inverse of function F , that is, if $a = F(b)$ then $b = F^{-1}(a)$. Therefore, for a given p , one can easily compute $\{x_1, x_2, \dots\}$ from (13). Choice of p is based on minimizing the total expected cost. Expected values of C_1 and C_2 can be written as a function of p and $q = 1 - p$, as follows [Munford, 1981]:

$$E[C_1(T;x(p))] = \frac{c_1}{p} + c_2 \sum_{k=1}^{\infty} x_k q^{k-1} p - c_2 E(T) \quad (11)$$

and

$$E[C_2(T;x(p))] = \frac{c_1}{p} + c_2 p \sum_{k=1}^{\infty} x_k q^{k-1} p \quad (12)$$

After substituting x_k from (10) into (11) and (12), the latter two become functions of p only. Then, p is chosen such that either of these two functions is minimized.

Both of the methods discussed above, namely periodic policy and constant hazard policy, are not computationally difficult. Furthermore, the latter one especially has attractive managerial implications. For instance, the trade-off between the total expected cost and the probability that an auditee will be in the noncompliance state can be assessed relatively easily from equations (11) and (12). Furthermore, in minimizing (11) and (12) a manager may impose, without causing any computational difficulty, a lower bound on p , reflective of the standards of the management (i.e., immaterial level of p). These and other desirable features are obtained at a cost of suboptimality. Munford [1981] compared the performance of these models with optimal decision models for a number of probability distributions. In most cases both suboptimal models yield solutions within a few percent of the optimum; the constant hazard model appears to be performing better than the periodic model.

In spite of the desirable features of the suboptimal policies discussed above, a number of practical difficulties remain. Although

it may not be difficult to assign quantitative costs to the auditing process, it may be impossible in many cases to assign costs to the time during which the auditee remains in the noncompliance state. Even if these costs can be determined, they may not be commensurate. Furthermore, these costs may change over time. The method discussed below may apply when the foregoing difficulties exist.

Sequential Audit Policy (A Non-Cost Model)

Instead of choosing the complete audit schedule for each cycle, it may be more preferable to make the choice sequentially [Hayre, 1982]. That is, the management style may require that the decision of when to audit the accounting control system is made on an "as-you-go" basis. Specifically, suppose the k-th audit is carried out at time x_k and the auditee is found to be in the compliance state. At this point in time the next audit time x_{k+1} is to be scheduled. Suppose, the management requires the following events to occur with probability p: at the time of the next audit the accounting control system is either in the compliance state or in the noncompliance state for less than d time units prior to the audit. More specifically, it is required that x_{k+1} satisfies

$$P(T > x_{k+1} - d \mid T > x_k) = p \quad (16)$$

The audit times $\{x_1, x_2, \dots\}$ can be calculated recursively as follows. Let d_k and p_k be the values of d and p chosen after the k-th audit. Then, from (16)

$$\overline{F}(x_{k+1} - d_k) = p_k F(x_k) \quad k = 0, 1, \dots \quad (17)$$

which can be solved recursively, with $x_0 = 0$.

This method does not involve any optimization. Instead, more flexibility is given to the management. In choosing d and p values the manager is expected to use a judgmental procedure based on currently available data about the auditees and based on the current standards. For instance, if an external audit is expected in the near future, the management is expected to choose a large p and a small d values.

4. NUMERICAL EXAMPLES

The models discussed in the previous sections will now be illustrated and analyzed via numerical examples.

In the absence of empirical evidence as to which probability distribution best describes the behavior of an accounting control system, the Weibull and exponential distributions are used. The Weibull distribution is an empirical distribution capable of describing a wide variety of random processes. Furthermore, it is a general one in the sense that many commonly used distributions (e.g., normal, exponential) are special cases of the Weibull distribution. The exponential distribution, on the other hand, is generally agreed to be a good approximation for the time to failure of "...the complex system that consists of different components which are not necessarily alike and ...[which] may have different pattern[s] of failure and [for which] all components make up the failure pattern of the system as a whole." [Barlow and Proschan, 1965, p. 18]

The Weibull density function is:

$$f(t) = \mu \alpha t^{\alpha-1} \exp[-\mu t^\alpha] \text{ for } t > 0$$

and cumulative Weibull distribution is:

$$F(t) = 1 - \exp [-\mu t^\alpha]$$

where $\mu > 0$ and $\alpha > 0$ are scale and shape parameters, respectively. In the audit scheduling problem context, μ denotes the expected time it takes until the auditee goes into the noncompliance state. To be more general, the examples presented will use the Weibull distribution.

$$f(t) = \mu \exp [-\mu t]$$

and cumulative distribution

$$F(t) = 1 - \exp [-\mu t].$$

In the examples, the following values are used:

$$c_1 = \$ 5,000 \text{ per audit}$$

$$c_2 = \$20,000 \text{ per unit time (3 months)}$$

Weibull parameters:

$$\alpha = 2$$

$$\mu = 1$$

$$E(T) = \mu^{-1/\alpha} \Gamma\left(\frac{1}{\alpha} + 1\right) = 0.886+$$

Periodic Model

In the following $\hat{\theta}_i$ and θ_i^* denote, respectively, the approximate and actual nearly-optimal durations between two consecutive audits,

$$+\Gamma(n) = (n - 1)\Gamma(n-1), \text{ and } \Gamma(1/2) \equiv \pi$$

using cost functions given by equations (1) ($i = 1$) and (2) ($i = 2$). Furthermore, unit of θ is three months. Therefore, for example, $\theta^* = 0.5$ implies the optimal duration between audits is $0.5 \times 3 = 1.5$ months.

Since $c_1 = 5,000$ is not large relative to $c_2 E(T) = 17,720$, let us first use the approximations given by (7) and (8) (for cost functions given by equations (1) and (2), respectively) in order to determine the optimal duration between two consecutive audits. From (7) $\hat{\theta}_1 = 0.665$ and from (8), $\hat{\theta}_2 = 0.471$. Next, let us use equations (4) and (5), to determine the exact values of optimal θ_1 and θ_2 , for respective cost models. Noting that

$$\begin{aligned} m(\theta) &= \sum_{j=0}^{\infty} \overline{F}(j\theta) \\ &= \sum_{j=0}^{\infty} \exp[-(j\theta)^2] \end{aligned}$$

equations (6) and (7) can be computed iteratively. Results of these iterations are given in Table 1. It should be noted that in computing (6), $\theta_1 = 0.665$ is chosen as the initial trial point, which is the value determined above. Likewise, in computing (7) the initial trial value of θ_2 is 0.471. From Table 1, $\theta_1^* = .665$ and $\theta_2^* = .472$, which are remarkably close to the approximate values.

Constant Hazard Model

To solve (11) and (12), first x should be expressed as a function of p . For Weibull distribution:

TABLE 1

Computation of θ_1^* and θ_2^* from Equations (4) and (5)

From Equation (4)			From Equation (5)		
θ	$m(\theta)$	$E(C_1)$	θ	$m(\theta)$	$E(C_2)$
*.665	1.8326	\$15816.58	.471	2.3816	\$21328.0
.66	1.8427	15818.38	.475	2.3693	21346.45
.67	1.8227	15818.17	.468	2.3936	21328.41
.666	1.8306	15817.89	*.472	2.3775	21327.5
.664	1.8346	15817.93	.473	2.3736	21328.14
			.470	2.3856	21328.12

$$p = F(x_1)$$

$$= (1 - \exp(-\mu x_1^\alpha))$$

from which

$$x_1 = \left(-\frac{1}{\mu} \ln(1-p)\right)^{1/\alpha}$$

Using (18), from (12) it is easy to show by induction that

$$x_k = \left(-\frac{k}{\mu} \ln(1-p)\right)^{1/\alpha}$$

Substituting (14) for x_k in (11) and (12), respectively, we obtain

$$E[C_1] = c_1/p + c_2 \sum_{k=1}^{\infty} \left(-\frac{1}{\mu} \ln(1-p)\right)^{1/\alpha} q^{k-1} p - c_2 E(T)$$

and

$$E[C_2] = c_1/p + c_2 p \sum_{k=1}^{\infty} \left(-\frac{k}{\mu} \ln(1-p)\right)^{1/\alpha} q^{k-1} p$$

The p values minimizing $E[C_1]$ and $E[C_2]$, for this example, are+:

$p^* = 0.5850$ with $E[C_1^*] = \$14,402.67$ and $p^* = 0.4335$ with $E[C_2^*] = \$20,917.52$, respectively. Therefore, from (19), the optimal audit schedule

$$x_k^* = 0.9378\sqrt{k} \quad k = 1, 2, \dots$$

minimizes $E[C_1]$ and

+In order to avoid algorithmic digressions, the details of search procedures are omitted. Minimization of $E[C_1]$ and $E[C_2]$ require a line search algorithm; a particularly efficient one is the Fibonacci search [Bazaraa and Shetty, 1979, pp. 253-264].

$$x_k^* = 0.7538\sqrt{k} \quad k = 1, 2, \dots$$

minimizes $E[C_2]$.

Sequential Audit Policy

For $\bar{F}(t) = \exp[-\mu x_k^\alpha]$, equation (17) becomes

$$\exp[-\mu(x_{k+1} - d_k)^\alpha] = p_k \exp[-\mu x_k^\alpha] \quad k = 0, 1, 2, \dots$$

from which

$$x_{k+1} = d_k + (x_k^\alpha - \frac{1}{\mu} \ln p_k)^{1/\alpha} \quad k = 0, 1, 2, \dots$$

Suppose that the management does not wish the accounting system to stay more than 0.3 time units in the noncompliance state, 95 percent of the time. That is, $p_k = 0.95$ and $d_k = 0.3$ time units for all k . Then, for this example

$$x_{k+1} = 0.3 + (x_k^2 + 0.0513)^{1/2} \quad k = 0, 1, 2, \dots$$

is the desired audit schedule, with $x_0 = 0$.

6. SUMMARY

The primary objective of this paper is to present simple yet not simplistic decision models for the determination of appropriate (nearly optimal) audit schedule. Optimization involves resolving the trade-off between the cost of audit and the cost of the accounting control system being in the noncompliance state. Two cost models are considered. The first model (given by equation (1)) assumes that the time (between two consecutive audits) at which the auditees pass into

TABLE 2

Summary of the Example Solutions: The time (in months) from the beginning (time zero) to the first and k-th audits (i.e., x_1^* and x_k^*).

Distribution	Audit No.	Periodic Policy		Constant Hazard Policy		Sequential Decision Model
		Cost Model-1	Cost Model-2	Cost Model-1	Cost Model-2	
Weibull	1	2.00	1.42	2.81	2.26	1.58
	k	2.00k	1.42k	$0.9378\sqrt{k}$	$0.7538\sqrt{k}$	$0.9 + 3(x_{k-1}^2 + 0.513)^{1/2}$

From the summary of results presented in Table 2, in every case considered, the audit frequency for the cost model (2) is higher than that of the cost model (1). This is expected, since the cost model (2) is based on the assumption that the exact time of the beginning of noncompliance is not known, thus necessitating more frequent audits. This assumption appears to be true for most accounting systems.

the noncompliance state is known and computes the cost of the system being in this state from that time until the next audit time. The second cost model (given by equation (2)) assumes that the time at which the system went to the noncompliance state is not known and the whole interval between two consecutive audits is used in computing the cost of the system in the noncompliance state.

The optimal solution (schedule) for interval audits may have some theoretical or computational problems. Three nearly optimal models were presented to overcome such problems. Periodic policy, the first of these three, requires a uniform time interval between audits, which may provide a rationale for the current practice in accounting. The second one, the constant hazard policy, on the other hand, requires the probability of being in the noncompliance state in each interval to remain constant, but it may not produce uniform audit intervals. This model may be interesting to auditors and management since they do have a probability of failure (noncompliance) level beyond which they usually initiate an audit. This is the parameter on which this model is based. On the other hand, the non-uniform audit interval produced by this model may provide the surprise element in audit scheduling. However, both of these policies have intuitive and managerial appeal. Recently, Munford [1981] compared the performance of these two and other nearly-optimal policies. The two discussed above appear to yield results close to the optimal in most cases, the constant hazard policy being slightly superior.

The main difficulty in implementing the foregoing models involves the quantification of audit and noncompliance costs. This problem can

be avoided by the use of the third model, the sequential audit policy, which is based on the desirable performance of the auditee, not on the costs.

Numerical examples are provided to give insight into the types of results that can be obtained employing these models. Additionally, it is intended to serve as a brief tutorial.

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