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A Decision Theory Model of Standards Setting

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Abstract

The role of cost accounting is defined as providing useful information for managerial planning and control decisions. The conventional approach treats the cost estimation and planning decisions as related but separable problems. This note considers both issues as related parts in one integrated decision problem of opportunity cost minimization. An example is provided to demonstrate that the conventional approach does not always yield the optimum result, thus showing that the cost estimation and planning decisions are not separable in general.

"Decision Theory Model of Standards Setting"

Multitude of cost accounting texts start out by defining the role of cost accounting as providing useful information for managerial planning and control decisions. Then, the role and value of information in decision making context is discussed along with various techniques for cost estimation (usually least squares regression) and short term planning (Linear Programming). Variance analysis is used as the core concept for control phase of management.[1] Unfortunately, these topics are presented independently of each other, providing little integration of topics. The purpose of this note is to provide an illustration of planning and control activities consistent with the overall objective of profit maximization.

Linear Programming technique will be used as the short-term planning tool of production quantities and product mix. Certain standard production costs are used as inputs to the planning problem. The significance of deviations of actual costs from the standards will be evaluated based on the "opportunity" costs incurred by the firm due to the prediction error. This formulation is unique in two aspects:

(1) the goodness of the estimates of standard costs are evaluated based on the impact on the planning decisions, rather than based on the measure of deviations of the actuals from the estimates.

(2) the significance of the variances, deviations from the planned results are assessed based on the opportunity cost of the production decisions.

A numerical example will be used to facilitate the discussion and the theoretical issues will be summarized later in the note.

A Planning Decision Problem:

A company's short run objective is to maximize her total contribution margin by deciding on the production quantities of various products subject to the production costs, productive capacities and the market demand constraints.

Assume the following facts

Products	Price	Material	Labor*	Machine Hours	Demand
A	$28	$6.30	2 Hrs	1	5000
B	$22	$4.25	1.5	1	4200
C	$45	$4.25	4	1.5	3500
TOTAL AVAILABLE			26,000	12,000	

*Variable Overhead = [(Machine Hours + Labor Hours)/2] x Labor Rate
Direct Labor cost per standard hour is assumed to be $5.30 per hour.

Then, the short run decision can be modeled in the Linear Programming framework.

$$\text{Max} \quad 3.15A + 3.175B + 4.975C$$

$$\begin{aligned}
\text{S.T.} \quad 2A + 1.5B + 4C &\leq 26{,}000 \\
1A + 1B + 1.5C &\leq 12{,}000 \\
A &\leq 5{,}000 \\
B &\leq 4{,}200 \\
C &\leq 3{,}500 \\
A,\ B,\ C &\geq 0
\end{aligned}$$

The final tableau of the above problem is presented below:

Row	A	B	C	SLACKS LABOR	MACHINE	A	B	C	RHS
Obj	0	0	0	0	3.15	0	.025	.25	38,780
Labor	0	0	0	1	-2.00	0	.50	-1.00	600
A	1	0	0	0	1	0	-1	-1.5	2,550
Dem A	0	0	0	0	-1	1	1	1.5	2,450
B	0	1	0	0	0	0	1	0	4,200
C	0	0	1	0	0	0	0	1	3,500

From the tableau we can obtain following information:

OBJECTIVE FUNCTION VALUE

1) 38780

VARIABLE	VALUE
A	2550
B	4200
C	3500

ROW	SLACK OR SURPLUS	DUAL PRICES
Labor)	600	0.
Mach)	0	3.150
Dem. A)	2450	0.
Dem. B)	0	.025
Dem. C)	0	.250

RANGES IN WHICH THE BASIS IS UNCHANGED

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	3.150	.025	3.150
B	3.175	INFINITY	.025
C	4.975	INFINITY	.250

		RIGHTHAND SIDE RANGES	
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
LABOR	26000	INFINITY	600
MACHINE	12000	300	2550
DEM A	5000	INFINITY	2450
DEM B	4200	2550	1200
DEM C	3500	600	1633.33

Given the above sensitivity information, we can assess the impact of small changes in one variable. We can, for example, see that if the material cost for item B should increase by any amount greater than \$.025, then the product mix should change in order to be optimal under the new situation.

However, the traditional sensitivity analysis technique cannot handle simultaneous changes in more than one variable such as a change in average labor rate which would affect the contribution margins of all three products. Analysis of observed deviation, the difference between the actual and budgeted labor rate, should reveal the consequence of non-optimal decisions made due to the inaccuracy of data used in planning. Parametric programming is a tool for evaluating the impact of systematic changes of the objective function coefficients and/or the resource and demand constraints.[2]

To illustrate, let the difference between the actual and standard labor rate be θ. The objective function coefficients, or the contribution margins, change to:

$$(3.15 - 3.5\theta)A + (3.175 - 2.75\theta)B + (4.975 - 6.75\theta)C$$

These changes can be worked into the final tableau and after proper arithmetical operations to make all objective row coefficients of the basic variable columns to be zero are made, we can proceed to determine the range in which the basis remain unchanged. Tables 1-1 through 1-6 in Appendix show the details of the parametric programming steps.

The results are summarized below to indicate the steps of significant changes and the optimal production schedules, as well as the total contribution margin as a function of the deviation θ. We can proceed

Insert Table 1 and Figure 1 about here

to calculate the opportunity cost of planning at one level of labor cost when the actual labor cost is at another level as shown in table 2.

Insert Table 2 and Figure 2 about here

The opportunity cost is the theoretically "correct" measure of the consequence of the actual labor rate deviating from the standard rate used in planning. A significant implication of this interpretation is that the standards are to be set so as to minimize the expected value of the opportunity costs rather than a purely statistical measure of deviation such as sum of the squared errors or sum of the absolute deviations. Also, the significance of observed deviation from the standard is to be assessed based on the opportunity cost function rather than on the magnitude of the deviation in the labor rate itself.[3] For example, an actual labor rate between \$5.27 and \$5.46 is not significantly different from the standard rate \$5.30 since the production and resource allocation decision would have been the same had we known the "actual" labor rate prior to the planning decisions (Case 2). However, any labor rate outside the range would have lead to a different production schedule

(Cases 1, 3, 4, & 5). The magnitude of the opportunity cost is a piecewise linear function of the deviation θ. Opportunity costs exists even when the actual labor rate is lower than the standard (Case 1). When the actual labor rate decreases by more than 3 cents, \$5.26 or lower, the realized contribution margin is greater than the budgeted amount. Yet, it could have been even higher, had the manager known the actual labor rate and adjusted the production plan accordingly, thus the opportunity cost. The significance of labor rate variance in this system is based on neither the magnitude nor the direction of the deviation but on the opportunity cost. That is, a deviation is considered significant if the deviation measure would have lead to a different optimal production decision. Similar analysis can be made on the systematic changes on resource availability/market demand constraints.

Setting Standards:[4]

We can now take a step back and consider the decision problem of setting labor standards to be used for planning purposes. Given the appropriate assumptions, we can state the objective as to select a measure that minimizes the expected opportunity cost. The measure need not be the expected value, the median or the mode of the distribution of the labor rates. A significant implication of this formulation is that the standard setting is a decision problem rather than an inference problem, and any analysis of data dealing only with the deviation measure θ is at best incomplete. There may exist a function of θ which can be used as a surrogate of the opportunity cost function for wide variety of decision situations, yet it is only a surrogate measure computationally convenient but not a theoretically correct one. The theoretically correct measure would be a measure s such that

$$\min_{s \in S} \int_x L(x,s)\ f(x)dx$$

where

x: actual labor rate

s: standard labor rate

f(x): density function of x

L(x,s): opportunity cost of planning based on s when the actual is x; $0[(a^*|x),x] - 0[(a^*|s),x]$.

Within the example given above we can look into the standard setting problem. For ease of computation we will assume a triangular distribution of x. Three different distributions will be considered and the optimal standard under each distribution will be estimated. Figure 2 shows the distributions of x superimposed on the opportunity loss measures. Given an assumed distribution of the labor rate we can calculate the expected opportunity cost. The results are shown in table 3 below.

Insert Table 3 about here

The most important finding is that, for planning purposes the standard with least opportunity cost is the one in the range between $5.267 and $5.467, under each of the three distributions of x. Especially note distribution 3 where none of the central tendency measures of x was in the optimal planning value region. Of course this result is specific to the given decision problem and the probability distributions of the example used. Yet, the point to be made is that a loss function based only on the deviation measure, θ, cannot yield general solutions. The techniques illustrated in the cost accounting texts tends to dwell on the estimation of the "average" rate in the system. Even the sensitivity analysis of the obtained results is performed in non-decision context. Even when this conceptual deficiency is pointed out, a common reply is a question as to whether the OLS estimates are significantly different from the optimal standards set with explicit consideration of the decision problem. Implicit in this question is an assertion that unless the numbers are significantly different there is no reason to study the cumbersome process of expected opportunity cost minimization. This argument puts the cart before the horse. The proper way is to establish a correct procedure, then look for a surrogate that is efficient and effective. The example provides an instant where any of the central tendancy measures of a distribution is not a good estimate to be used in planning (decision making). An additional implication is that for planning purposes the managers may intentionally, yet properly, use "inaccurate" state description (e.g., labor rate).

The issue addressed in this note can be summarized as below:

Approach	Inference	Decision
Variable of Interest	x	a
Measure of error	$g(x-\tilde{x})$	$H(x,\hat{x}) = O[(a^*\|x),x]-O[(a^*\|\hat{x}),x]$
Estimate	1) $\underset{\tilde{x}\in X}{\text{Min}}\int g(x-\tilde{x})f(x)dx$	1) $\underset{\hat{x}\in X}{\text{Min}}\int L(x,\hat{x})f(x)dx$
	2) $\underset{a^*\in A}{\text{Max}}\int O(a^*\|\tilde{x})f(x)dx$	$= \underset{\hat{x}\in X}{\text{Min}}\int\{[O(a^*\|x),x]-O[(a^*\|x),\hat{x}]\}f(x)dx$

The key concern was whether the decision based on inference approach is the same as the one based on decision approach. And if they are not, which is the proper one? Given a managerial decision context, the decision theory approach provided in this paper considers the information system and production planning issues as one integrated problem. The conventional approach separates this problem into data generating phase and alternative choosing phase, and this paper has shown that the two phases are not separable in general.

TABLE 1

Various Deviation Ranges (θ) and Optimal Production Values ($Q^*|\theta$)

	1		2		3		4		5	
	$\theta < -.0333$		$-.0333 < \theta < .16667$		$.16667 < \theta < .737037$		$.737037 < \theta < .9$		$.9 < \theta < 1.1545$	
	Q*	CM	Q*	CM	Q*	CM	Q*	CM	Q*	CM
A	3750	3.2666	2550	2.567	5000	.5704	5000	0	0	-.891
B	3000	3.2666	4200	2.7167	42000	1.1481	4200	.7	4200	0
C	3500	5.20	3500	3.85	1866.7	0	0	-1.1	0	-2.818
TOTAL	\$38,750-45,000θ		\$38,780-44,100θ		\$38,371.66-41,650θ		\$29,085-29,000θ		\$13,335-11,500θ	

TABLE 2

Opportunity Cost of Planning with Errornous Contribution Margin Information

Actual / Plan	1 DL < 5.2667	2 5.2667 < DL < 5.4667	3 5.4667 < DL < 6.037	4 6.037 < DL < 6.20	5 6.20 < DL < 6.4545
1	0	900θ + 30	3350θ - 378.34	15,950θ - 9665	33,450θ - 25,415
2	-900θ - 30	0	2450θ - 408.34	15,050θ - 9695	32,550θ - 25,445
3	-378.34 - 3350θ	408.34 - 2450θ	0	12,600θ - 9286.66	30,100θ - 25,036.66
4	9665 - 15,950θ	9695 - 15,050θ	9286.66 - 12,600θ	0	17,500θ - 15,750
5	25,415 - 33,450θ	25,445 - 30,205θ	25,036.66 - 30,100θ	15,750 - 17,500θ	0
	θ < -.0333	-.03333 < θ < .16667	.16667 < θ < .737037	.737037 < θ < .9	.9 < θ < 1.1545

TABLE 3

Expected Opportunity Cost of Various Production Plans for Different Distributions of Labor Rate

DIST	Mean	Med	Mode	1 5.26667 > S	2 5.26667 < S < 5.46667	3 5.46667 < S < 6.037037
1	5.3	5.3	5.3	1399	1371 *	1777.37
2	5.4666	5.53208	5.8	2063	1803 *	1911.638
3	5.1333	5.0679	4.8	1051.4217	1021.4059 *	1854.5336

Distributions of Labor Rate

(1) $f(\theta) = \begin{cases} 4/9\ (1.5-\theta) & \theta > 0 \\ 4/9\ (1.5+\theta) & \theta < 0 \end{cases}$

(2) $f(\theta) = \begin{cases} 2/3\ (1.5-\theta) & \theta > .5 \\ 1/3\ (1.5+\theta) & \theta < .5 \end{cases}$

(3) $f(\theta) = \begin{cases} 1/3\ (1.5-\theta) & \theta > -.5 \\ 2/3\ (1.5+\theta) & \theta < -.5 \end{cases}$

$$EOC = \int_{\theta} L(\theta,S)f(\theta)d\theta$$

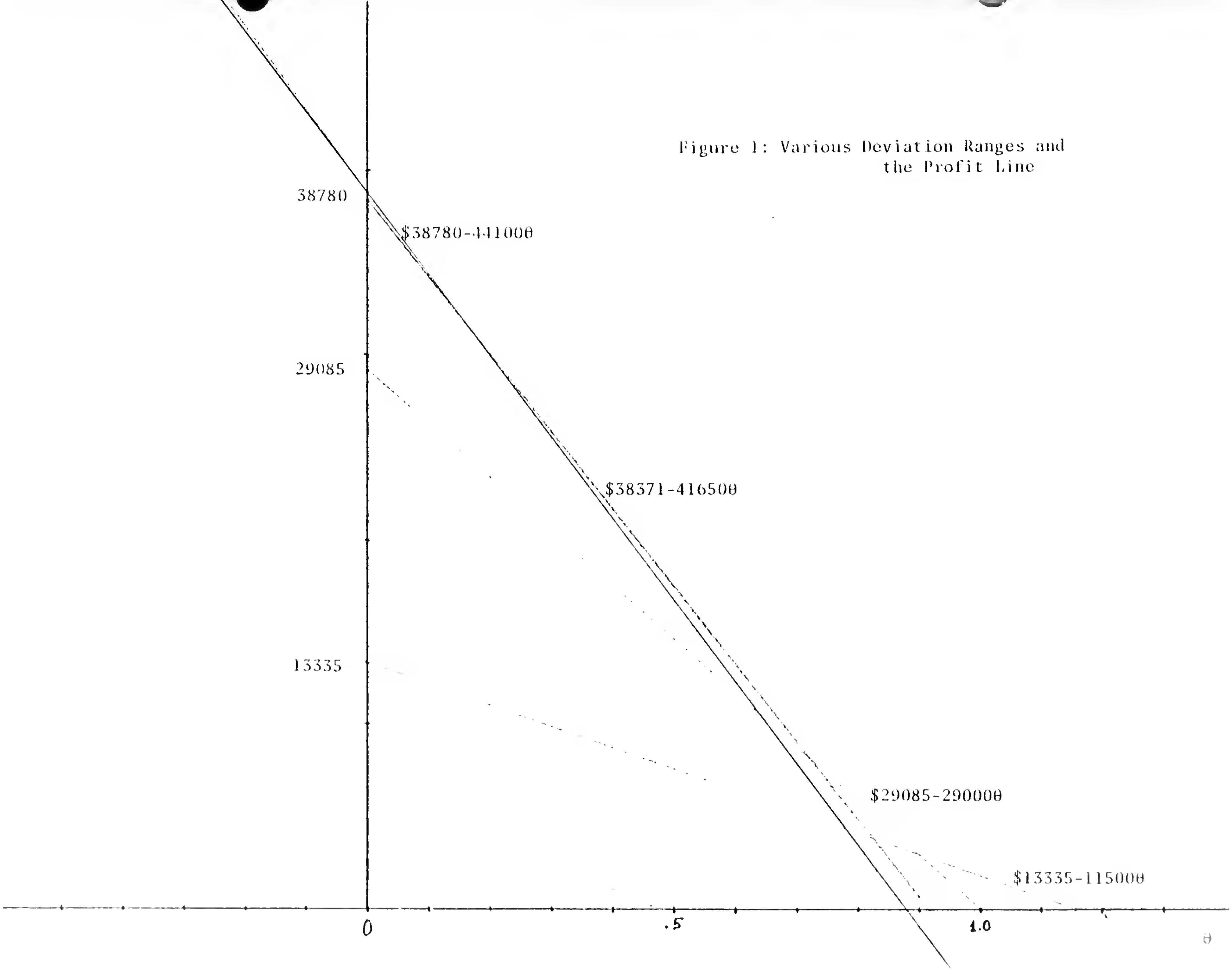

Figure 1: Various Deviation Ranges and the Profit Line

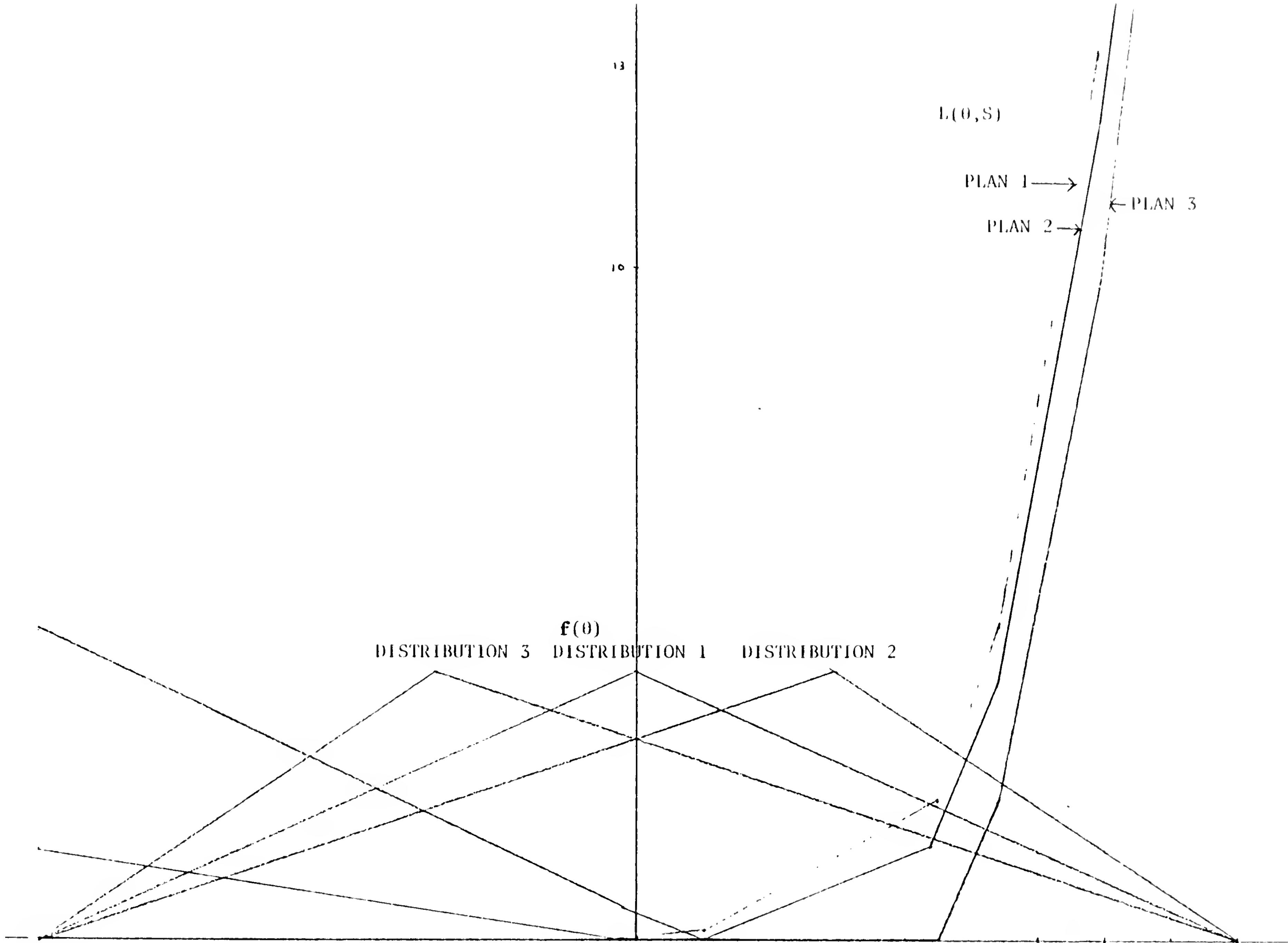

Figure 2

Opportunity Loss of Various Production Plans and the Probability Distributions

Notes

[1]Below are some examples of coverage in cost accounting texts (numbers in the table represent chapters in the texts).

Text	Role of Cost Accounting	Value of Information	Cost Estimation	L.P.	Var. Anal.
Deakin & Maher	1	26	10	13	19,25
Dopuch et al.	1	1	3	4	7,8
Horngren	1	21	24	23	25
Morse	1		6	8	13

[2]See Cooper & Steinberg [1974, pp. 271-74], Taha [1971, pp. 74-94] and Hillier & Lieberman [1980, pp. 689-94] for more detailed descriptions of the parametric programming technique.

[3]We are not considering the role of deviation measure in the in-control vs. out-of-control state determination. In this paper we assume the deviations were uncontrollable. See Demski [1976] for an example of opportunity cost based model.

[4]In this note we only consider the issue of setting standard labor rate for planning purposes. The actual labor rate is assumed to be independent of the performance target set by the management. That is, the motivational effect of the labor standards is ignored.

References

1. Cooper, Leon and David Steinberg, Methods and Applications of Linear Programming, W. B. Saunder Company, Philadelphia, 1974.

2. Edward Deakin and Maher, Michael, Cost Accounting, Richard D. Irwin, Inc., 1984.

3. Joel Demski, "An Accounting System Structured on a Linear Programming Model," The Accounting Review, October 1976.

4. Nicholas, Dopuch, Birnberg, Jacob, and Demski, Joel, Cost Accounting: Accounting Data for Management's Decisions, Third Edition, Harcourt Brace Jovanovich, New York, 1982.

5. Frderick S. Hillier and Lieberman, Gerald, Introduction to Operations Research, 3rd edition, Holden-Day, Inc., 1980.

6. Charles Horngren, Cost Accounting: A Managerial Emphasis, Fifth Edition, Prentice-Hall, Englewood Cliffs, N.J., 1982.

7. Hamdy Taha, Operations Research, MacMillan, New York, 1971.

Appendix

Parametric Programming

Parametric programming is a sensitivity analysis tool where some systematic changes are introduced into the system.

In this note, a change in labor rate affects the profitability (contribution margins) of all three products.

Briefly, the procedure is to introduce the change and relative impact on the products into the objective row of the final tableau of the original problem. Then, the basic variables are no longer basic and the objective row should be cleared up through some raw operations as shown in Table 1-1.

Once the changes (θ) are incorporated into the problem we can proceed to assess the impact of various levels of changes using simplex methods.

For example, as shown in Table 1-2, for θ less than .16667, the original production schedule remains optimal. Once the change in labor rate exceeds this level the production schedule needs to be adjusted resulting in Table 1-3 and so on. Table 1-6 shows the ease of $\theta < 0$.

Table 1-1

			SLACKS					
A	B	C	LABOR	MACHINE	A	B	C	RHS
-3.5θ	-2.75θ	-6.75θ	0	3.15	0	.025	.25	38780
0	-2.75θ	-6.75θ	0	3.15-3.5θ	0	.025+3.5θ	.25+5.25θ	3870-89250
0	0	-6.75θ	0	3.15-3.5θ	0	.025+.75θ	.25+5.25θ	38,780-20475θ
0	0	0	0	3.15-3.5θ	0	.025+.75θ	.25-1.5θ	38,780-44,100θ

Table 1-2

				SLACKS				
A	B	C	LABOR	MACHINE	A	B	C	RHS
0	0	0	0	3.15-3.5θ	0	.025+.75θ	.25-1.5θ	38,780-44,100θ
0	0	0	1	-2	0	.5	-1	600
1	0	0	0	1	0	-1	-1.5	2,550
0	0	0	0	-1	1	1	1.5	2,450
0	1	0	0	0	0	1	0	4,200
0	0	1	0	0	0	0	1.	3,500

Table 1-3

			SLACKS					
A	B	C	LABOR	MACHINE	A	B	C	RHS
0	0	0	0	3.3167-4.5θ	θ-1/6	1.75θ-.1416θ	0	38371.66-41650θ
0	0	0	1	-16/6	4/6	7/6	0	6700/3
1	0	0	0	0	1	0	0	5000
0	0	0	0	-2/3	2/3	2/3	1	4900/3
0	1	0	0	0	0	1	0	4200
0	0	1	0	2/3	-2/3	-2/3	0	5600/3

Table 1-4

			SLACKS					
A	B	C	LABOR	MACHINE	A	B	C	RHS
0	0	0	0	0	3.15-3.5θ	3.175-2.75θ	0	29085-290500θ
0	0	4	1	0	-2	9/16	0	9700
1	0	0	0	0	1	0	0	5000
0	0	1	0	0	0	0	1	3500
0	1	0	0	0	0	1	0	4200
0	0	3/2	0	1	-1	-1	0	2800

Table 1-5

			SLACKS					
A	B	C	LABOR	MACHINE	A	B	C	RHS
3.50-3.15θ	0	0	0	0	0	3.175-2.75θ	0	13335-115500
2	0	4	1	0	0	9/16	0	19700
1	0	0	0	0	1	0	0	5000
0	0	1	0	0	0	0	1	3500
0	1	0	0	0	0	1	0	4200
1	0	3/2	0	1	0	-1	0	7800

Table 1-6

A	B	C	LABOR	MACHINE	SLACKS A	SLACKS B	SLACKS C	RHS
0	0	0	1.5θ-.05	3.05+.5	0	0	.2	38750-45000θ
0	0	0	2	-4	0	1	-2	1200
1	0	0	2	-3	0	0	-2.5	3750
0	0	0	-2	3	1	0	.5	1250
0	0	1	-2	4	0	0	1	3000
0	0	1	0	0	0	0	1	3500

for $\theta < 0$

A Planning Decision Problem:

A company's short run objective is to maximize her total contribution margin by deciding on the production quantities of various products subject to the production costs, productive capacities and the market demand constraints.

Assume the following facts

Products	Price	Material	Labor*	Machine Hours	Demand
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C	$45	$14.25	4	1.5	3500
TOTAL AVAILABLE			26,000	12,000	

*Variable Overhead = [(Machine Hours + Labor Hours)/2] x Labor Rate
Direct Labor cost per standard hour is assumed to be $5.30 per hour.

Then, the short run decision can be modeled in the Linear Programming framework.

$$
\begin{array}{llllll}
\text{Max} & 3.15A & + 3.175B & + 4.975C & & \\
\text{S.T.} & 2A & + 1.5B & + 4C & \leq & 26,000 \\
 & 1A & + 1B & + 1.5C & \leq & 12,000 \\
 & A & & & \leq & 5,000 \\
 & & B & & \leq & 4,200 \\
 & & & C & \leq & 3,500 \\
 & A, & B, & C & \geq & 0
\end{array}
$$

Approach	Inference	Decision
Variable of Interest	x	a
Measure of error	$g(x-\tilde{x})$	$L(x,\hat{x}) = O[(a^*\|x),x]-O[(a^*\|\hat{x}),x]$
Estimate	1) $\underset{\tilde{x}\in X}{\text{Min}}\ g(x-\tilde{x})f(x)$	1) $\underset{\hat{x}\in X}{\text{Min}}\ L(x,\hat{x})f(x)$
	2) $\underset{a^*\in A}{\text{Max}}\ O(a^*\|\tilde{x})f(x)$	$= \underset{\hat{x}\in X}{\text{Min}}\{[O(a^*\|x),x]-O[(a^*\|x),\hat{x}]\}f(x)$

The key concern was whether the decision based on inference approach is the same as the one based on decision approach. And if they are not, which is the proper one? Given a managerial decision context, the decision theory approach provided in this paper considers the information system and production planning issues as one integrated problem. The conventional approach separates this problem into data generating phase and alternative choosing phase, and this paper has shown that the two phases are not separable in general.