# THE DESIGN AND CONSTRUCTION OF SHIPS.

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AND

# CONSTRUCTION OF SHIPS.

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VOL. I.—CALCULATIONS AND STRENGTH,

COMPLETE IN ITSELF,

With 36 Folding Plates and 245 other Illustrations.

LONDON:
CHARLES GRIFFIN AND COMPANY, LIMITED.
PHILADELPHIA: J. B. LIPPINCOTT COMPANY.
1908.

# PREFACE.

This book is the written word of many years of lecturing in Glasgow University. As these lectures have never been written as they were delivered, and as new matter was necessarily added to the lectures as it became available, the chapters of this book do not represent any one systematic series of lectures, though there is little in the book but what has been given at some time to the students. The process of writing such a book is very dissimilar to that of writing most books. Much of it at first was collected from lecture notes by one or other of my assistants. These collections were elaborated, modified, rewritten in parts, and finally corrected in the form in which they now appear. In the collecting and correcting process I was continually helped by my assistants and students, but the bulk of the assistance was given me by Mr J. G. Johnstone, B.Sc., to whom I am indebted for saving me a great deal of detail work in the collection of what I have given from time to time in my lectures. Some of the calculations were made by students of the Naval Architecture Class.

This book cannot possibly lay any large claim to originality. The greater part is necessarily the work of others who went before or who are now working for the advancement of the science of Naval Architecture. Very little attempt has been made to acknowledge the sources of such help. The subject is too wide to attempt to acknowledge all the sources, even if one were qualified to do so. But there are parts which are original work given for the first time in the lecture-room, though some has been already published, mostly in the *Transactions of the Institution of Naval Architects.*

The subject is developing year by year, and often so rapidly as to make it very difficult, if not almost impossible, to read all that is published on the subject. There are further stores of knowledge which are never published, and are only accessible to individuals associated with the shipbuilding establishments, in which much expense is incurred in investigations and experiments for the advancement of knowledge of the subject. To many such I have been indebted from time to time for the ready reply to requests for information. It would be difficult to name all these individuals, but from circumstances of close proximity and old friendships, as well as their ability to give, the firms of Messrs John Brown & Co. and Messrs Wm. Denny Brothers have been most severely taxed by demands of all kinds, to which they have never failed to accede. My thanks to these and all others who have assisted me are gratefully offered.

This book is primarily intended for young students, but it is hoped that many who have been students, and some who in their daily work are interested in the problems dealt with, may find some assistance. There must be some errors and shortcomings in such an attempt; but it is sincerely hoped that, in spite of these, the work may serve the purpose of assisting some to a better knowledge of the science of Naval Architecture.

JOHN HARVARD BILES.

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# THE DESIGN AND CONSTRUCTION OF SHIPS.

## INTRODUCTION.

### GENERAL CONSIDERATIONS.

Usually the problem of making a ship is one of doing something very like what has been done before. To understand fully how to make a new ship, it is necessary to study the qualities and history of creation of existing ships.

The finished ship is generally first seen floating at rest in smooth water. All the forces acting upon the ship are in equilibrium. The weight of the whole structure and all the ship contains must be balanced by the supporting forces of the water. The pressures of the water on the sides and bottom of the vessel are balanced by the resistances of the material to change of form. The vessel, when floating in absolutely smooth water, is in such a condition that a small inclination from its position of rest is not followed by an increased departure from this position, but by a return to it. This tendency is known as the stability of the vessel.

If we consider the duty which a ship has to perform by moving in smooth water, we see that force from some source has to be developed equal and opposite to the resistance to be overcome. If the circumstances are still further complicated by the water not being smooth, the ship will have movements other than in the direction of intended locomotion, and the movements impressed upon one part of the vessel will develop resistances to change of form in the material of the structure. These resistances are known as the strength of the ship.

When the vessel is in disturbed water, her movements other than directly forward will be oscillatory about the position of rest in smooth water.

The result of this general survey is that we must consider the vessel from the following points of view:—

| | |
|---|---|
| Support or buoyancy. | Resistance. |
| Strength. | Oscillations. |
| Stability. | |

The book has therefore been divided into Parts which treat the subject as follows:—

| | | | |
|---|---|---|---|
| Volume I. | PART | I. | Areas, Volumes, and Centres of Gravity. |
| | PART | II. | Ship Calculations. |
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# PART I.

## AREAS, VOLUMES, AND CENTRES OF GRAVITY.

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### CHAPTER I.

### GENERAL CONSIDERATIONS OF THE EQUILIBRIUM OF A FLOATING BODY.

**Buoyancy.**—The intensity of pressure in still water varies as the depth and as the weight of a cubic unit of the water. A cubic foot of fresh water at 62° F. weighs 62½ lbs., and salt water 64 lbs. Water is, for all questions connected with ship design, practically incompressible, so that the effect of variation of atmospheric pressure may be neglected. At a depth of 10 feet the pressure upon a square foot of area in salt-water is $10 \times 64 = 640$ lbs., at 62° F.

Consider the case of a box-shaped vessel floating in undisturbed water.[1] The pressures on the sides and ends are horizontal, and on the bottom vertical. The horizontal pressures will balance each other, and the vertical pressures will balance the weight. The total pressure on any small area of the bottom is the area in square feet multiplied by the depth in feet of the centre of the area, and by the weight of a cubic foot of water. The vertical pressure is the projection of this area on a horizontal plane multiplied by the above depth and unit weight.

The projected area multiplied by the depth is the volume of the submerged part of the box, so that the vertical pressures are equal to the weight of a volume of water equal to the submerged volume of the vessel. This volume is called the displacement: and in still water the weight of the displacement, which is equal to the sum of the vertical pressures, is equal to the weight of the floating body. Hence the weight of the body is frequently spoken of as the displacement.

Consider the case of a body of any form. The pressure upon all the submerged points in the surface of the body may be resolved into vertical and horizontal components, and the latter must balance among themselves or else motion of the body relatively to the water would be produced. The sum of

[1] The sides and ends are assumed to be floating vertically.

the vertical parts of the pressure will, as before, be balanced by the weight of the body.

Let fig. 1 represent the part of a floating body intercepted between two parallel vertical planes a unit distance apart.

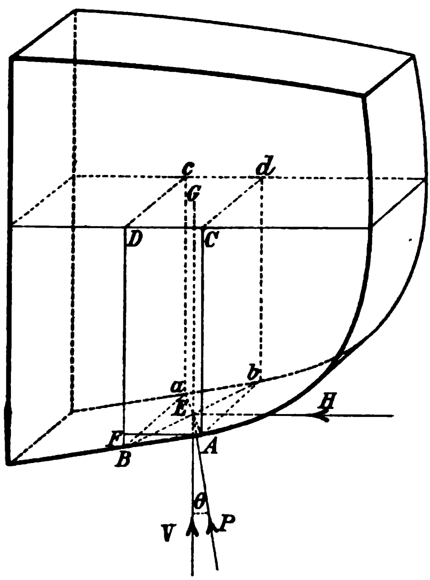



Fig. 1.

Consider a small part A B *a b* of the submerged area formed by the intersection of the parallel planes and with two other vertical planes perpendicular to them.

Let C D *c d* be its projection on the waterplane or plane of flotation.

Let E be the C.G. of A B *a b*, and G the projection of E on C D *c d*. The resultant pressure P on A B *a b* will act normally to the part A B *a b*, and will be equal to its area multiplied by G E and by *w*, the weight of a unit volume of water. Let $\theta$ be the angle of inclination of P to the vertical.

Then the vertical component

$$= P \cdot \cos\theta \equiv V,$$

and the horizontal component

$$= P \cdot \sin\theta \equiv H,$$

which latter can be resolved into two others respectively, parallel and perpendicular to the original vertical planes.

From A draw A F perpendicular to D B.

Let $a$ be the area of A B *a b*, then $a \cdot \cos\theta$ = the projection of A B *a b* on a horizontal plane = C D *c d*.

The vertical pressure $= P \cdot \cos\theta$,

$= w \cdot GE \times a \cos\theta$,

$= w \cdot GE \times CDcd$,

$= w$ multiplied by the elemental volume D *b*, which, if *c* D be of unit thickness, is represented by A B D C.

We therefore see that the outline of a vertical section of a body is a curve of still water pressures referred to the water-line as base, and the area of the submerged part of the section represents the buoyancy per unit of length of the body at that section. Hence the weight of a volume of water equal to the volume of the part of the body immediately over A B *a b* is equal to the vertical pressure on A B *a b*. The same applies to every portion of the surface similar to A B *a b*, so that the total vertical pressure on the body is equal to the weight of a volume of water equal to the volume of the body below the water surface. This holds for all bodies floating in still water.

Hence to find the weight of a floating body we have only to find the volume submerged (called the displacement), and find the weight of an equal volume of the water in which the body is floating. Or, conversely, to provide buoyancy or support for a body of known weight, we must provide a submerged volume (or displacement of water) equal in weight to that of the body.

Another proof of this proposition may be obtained by imagining the body to be removed from the water and the space left vacant to be then filled with some of the surrounding fluid. Equilibrium will be established when the void is filled, and evidently the vertical forces supporting the water which has filled the void must equal the weight of the water. These vertical forces which are supplied by the pressure of the water equalled the weight of the vessel before it was removed, because they are still solely dependent on the depth and area, so that the weight of the vessel must be equal to the weight of water which would fill the space occupied by the submerged portion of the volume of the vessel.

**Centre of Gravity and Centre of Buoyancy.**—It is also evident that, for equilibrium, the centre of gravity of the volume displaced must be in the line of action of the resultant of the vertical upward forces due to water pressure. The centre of gravity of the displaced volume is in consequence usually called the centre of buoyancy. The line of action of the resultant of all the vertical downward forces acting upon the vessel must for equilibrium coincide with the vertical through the centre of buoyancy. Hence if the only vertical forces acting on the body be its weight forces, to find the vertical line through its centre of gravity we have to find the centre of buoyancy or centre of gravity of the displaced volume; and, conversely, to ensure equilibrium in a vessel of given form, the disposition of the weights of the parts of a vessel must be such that the resultant centre of gravity is in the vertical line through the centre of buoyancy.

From these considerations we see that in order to examine the conditions of equilibrium for any assumed position of flotation, it is necessary to determine the submerged volume and the position of the centre of gravity of that volume.

# CHAPTER II.

## METHODS OF DETERMINATION OF THE VOLUME AND CENTRE OF GRAVITY OF A KNOWN SOLID.

THE volume of any solid may be found by supposing the solid to be cut by a series of parallel planes at equal small distances apart. If the area of each of these planes be found, and be multiplied by the small distance or interval between the planes, we shall get a series of volumes, each volume being approximately equal to the corresponding volume between consecutive planes. The smaller the distance between the planes the more nearly will the sum of the volumes so found approximate to the actual volume of the solid. Hence the determination of volume resolves itself into the determination of areas of cross-sections.

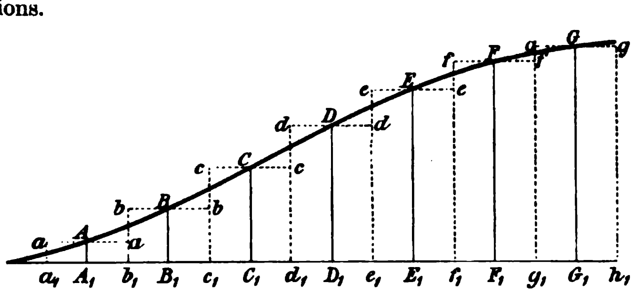



FIG. 2.

If we find a series of areas of such cross-sections and set them up as in fig. 2 as ordinates at $A A_1$, $B B_1$, $C C_1$, etc., we can draw a curve through the spots A B C D, etc., so obtained. This will be a curve of sectional areas, and the area between two ordinates will give the volume of the solid between the parallel planes whose positions the ordinates represent.

Set off on each side from $A A_1$, $B B_1$, etc., the dotted lines $a a_1$, $b b_1$, $c c_1$, etc., at one-half the distance between consecutive planes. Then the areas of the rectangles between dotted lines such as $a_1 a a b_1$, $b_1 b b c_1$, etc., will represent the area of the cross-section multiplied by the distance between consecutive planes; that is, the volume between consecutive planes. The ordinate of the curve A B C D will represent at any point the area of the cross-section at that point, and the smaller the distance between the planes the nearer will the sum of the rectangles be to the area of the curve A B C D. As the sum of the areas of these rectangles approximately represents the total volume of the solid, it is evident that the area of the curve A B C D represents the volume of the solid. Hence the determination of any volume resolves itself into a determination, first, of the areas of a series of cross-sections, and second, of the area of a curve representing these areas of cross-sections.

In practice it is sufficient to find the areas of a few cross-sections and to set up these areas as ordinates $a_1a$, $b_1b$, etc., along a base O X, fig. 3, at the corresponding positions of the planes, and run a curve through the extremities of the ordinates. The area of the curve *a b c d*, etc., between two ordinates gives the volume of the solid between the parallel cross-sections whose positions the ordinates represent.

There are several methods of finding areas of plane curves. For many plane curves of known form definite rules can be discovered and made use of. These rules will be described in Chapter III. But in naval architecture the curves are generally not of known form, but are created by the naval architect to suit each particular case. Hence it is necessary to have rules for the determination of areas which shall be applicable to any curve.

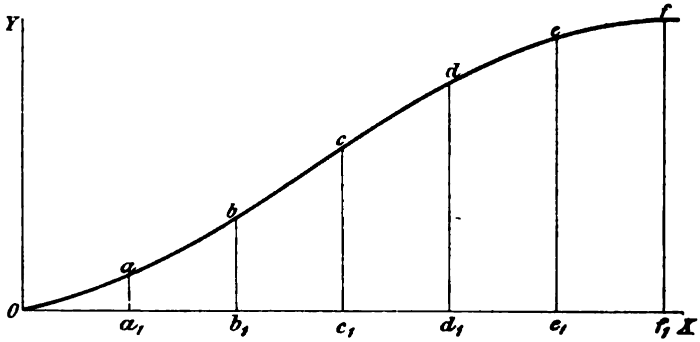



FIG. 3.

In mathematical treatises the subject of curves is fully treated. For the purpose of the ordinary student of naval architecture it is sufficient to understand one or two elementary principles.

Suppose O X and O Y (fig. 4) to be two straight lines at right angles to each other in the vicinity of a curve A B, of which any point P is distant P N and P M from O Y and O X respectively. O X and O Y are called the rectangular axes of X and Y respectively, and P N, P M (called $x$ and $y$) are said to be the co-ordinates of P with reference to the axes O Y, O X. Suppose that A B happens to be a curve such that the product of P M and P N is always the same, wherever P may be on the curve; the relation between P M and P N or $y$ and $x$ may be expressed by an equality or equation

$$x \, . \, y = C,$$

where C is a constant. This expression is said to be the equation to the curve A B.

It is evident that there may be many equations each expressing a relation between $x$ and $y$, and each equation may be graphically expressed by a curve In this case the curve takes the form as shown at A B in fig. 4.

As drawn, the values of $x$ and $y$ are usually considered positive. If $x$ is measured to the left of O Y it is negative. If $y$ is measured below O X it is negative.

If we can express the equation in the form $y$ equals some expression having only $x$ and constants in it, we shall be able to construct the curve appertaining to that equation. To find its area it is only necessary to divide the curve into a series of parallel strips, each having a small width, and sum up the area of these strips.

Suppose these strips to be of equal width $dx$, then $y.dx$ will be the area of a strip.

It is necessary to choose limits. Suppose we want to find the area of the curve enclosed by $a\,b\,c\,d$, then the limits of $x$ are $Oa = h$ and $Ob = H$ (see fig. 5).

The area is the sum of all the elemental strips parallel to O Y. Suppose

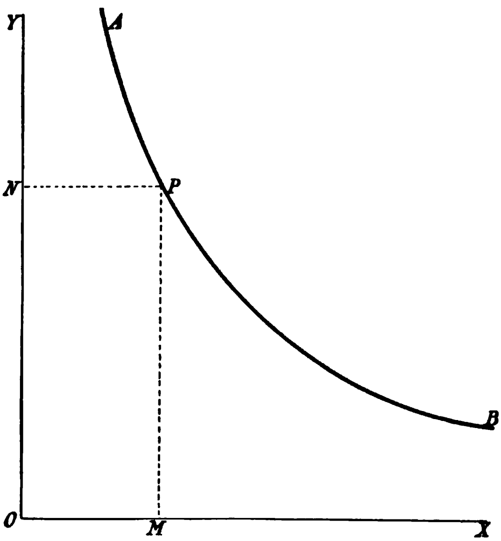



FIG. 4.

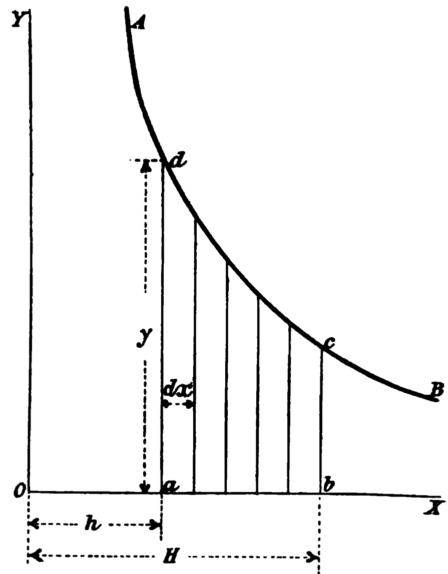



FIG. 5.

we express the sum of all values of $y.dx$ by $\int y.dx$, and the sum between definite limiting values of $x$, say H and $h$, by $\int_h^H y.dx$, then we may say that the area of a curve between the limits of H and $h$ is $\int_h^H y.dx$.

The summation of the expression $\int y.dx$ can in some cases be effected by the methods of the integral calculus. The only thing in this branch of mathematics that is necessary to be known in order to arrive at practical results in the determination of volume and C.G. of a ship's displacement is that the summation or integration of $x^n$ is $\frac{x^{n+1}}{n+1} + C$, where $n$ is the power to which $x$ is raised and C is a constant.[1]

[1] If $n$ is $-1$, the integral of $x^n$ is $\log x + C$.

If the equation between $y$ and $x$ takes the form $y = x^n$ then

$$\int y.dx = \int x^n dx = \frac{x^{n+1}}{n+1} + C.$$

Hence if we know $y$ in terms of $x$ raised to some power $n$, we can find $\int_h^H y.dx$ by applying the above rule for the integration of $x$ and substituting H and $h$ for $x$ in succession in the resulting expression, and then taking the difference of these results. For instance, if we have

$$y = ax^2 + bx + c \quad . \quad . \quad . \quad . \quad . \quad (1)$$

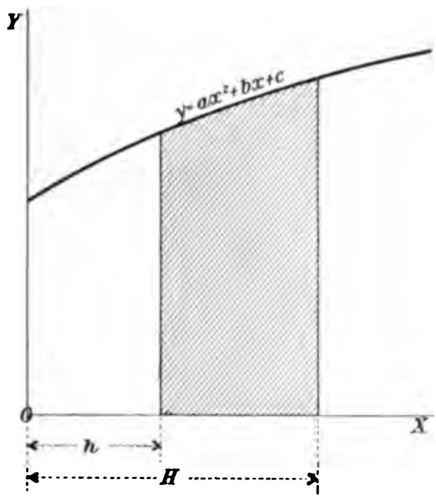



FIG. 6.

as the equation to a curve, then the area of that curve between the limits of H and $h$ (see fig. 6) will be

$$= \int_h^H y.dx,$$

$$= \int_h^H (ax^2 + bx + c)dx,$$

$$= \left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx + C\right]_h^H,$$

$$= \frac{a}{3}H^3 + \frac{b}{2}H^2 + cH + C - \left(\frac{a}{3}h^3 + \frac{b}{2}h^2 + ch + C\right)$$

$$= \frac{a}{3}(H^3 - h^3) + \frac{b}{2}(H^2 - h^2) + c(H - h) \quad . \quad . \quad . \quad . \quad (2)$$

The equation (1) is that of a parabola of the second order, and its area between ordinates whose abscissæ are $h$ and H respectively is expressed by (2).

In general, if we have any curve $y = f(x)$ where $f$ denotes "function of," the area of the curve between the limits of $x = \mathrm{H}$ and $x = h$ is given by $\int_h^{\mathrm{H}} y.dx$, or is obtained by integrating the curve between the limits. Thus we see that the process of finding an area can be performed by any of the rules for integration which are applicable, if the given curve can be represented mathematically by an equation.

**Centre of Gravity of an Area.**—The moment of an area about an axis is obtained by dividing the area into a number of small parts and multiplying the area of each part by its perpendicular distance from the axis. The moment is the limit of the sum of these products, as the parts are taken smaller and smaller.

Let the O Y axis (fig. 7) be the axis about which the moments of the area are to be taken.

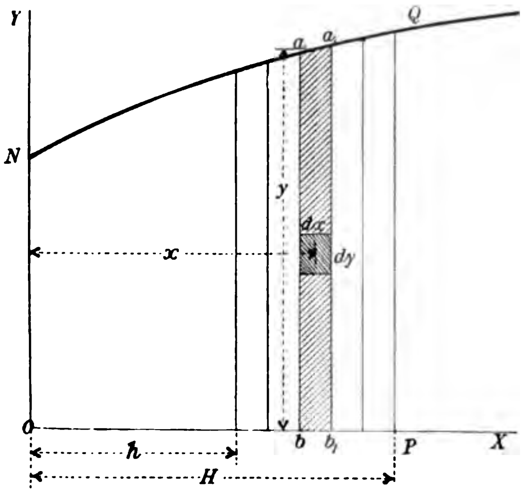



FIG. 7.

Elemental area $= dx.dy$.

Moment of elemental area about O Y $= x.dx.dy$.

$\therefore$ Moment of area $a\,b\,b_1\,a_1$ about O Y $= \int_o^{ab} x.dx.dy$.

and $\therefore$ the moment of area O N $a\,b$ about O Y $= \int_o^{ob}\int_o^{ab} x.dx.dy$.

The limits for integrating $x.dy$ are zero and $ab$ or $y$ the coordinate of the corresponding point in the curve. $x$ is a constant for this integration.

$$\therefore \qquad \iint x\,dx\,dy = \int x.y.dx.$$

We may choose any limits we like for $x$. Therefore we can say—if we have a curve $y = f(x)$—the moment of the area of this curve O P Q N about the axis of O Y is $= \int_h^{\mathrm{H}} x.y.dx$ where $h$ and H are the limits of $x$. The area of the curve between these limits is, as we have seen, $\int_h^{\mathrm{H}} y.dx$.

Call $$M_x = \int_h^H xy.dx.$$

$$A = \int_h^H y.dx.$$

The distance $\frac{M_x}{A}$ is called the leverage of the area about the O Y axis.

Calling this leverage $\bar{x}$

$$\bar{x} = \frac{\int_h^H xy.dx}{\int_h^H y.dx}$$

$$\therefore \quad \bar{x} \times \int_h^H y.dx = \int_h^H xy.dx.$$

The leverage is also defined as the distance of the centre gravity of the area from the axis. Therefore we see that when $\bar{x}$ is zero, or when the axis about which the moment of the area is taken passes through the centre of gravity, the moment is zero, *i.e.* $\int xy.dx = 0$.

There will therefore be a point in the area through which if we draw any axis the moment of the area about that axis will be zero. This point is the centre of gravity of the area.

The moment of the area about the axis O X can be determined as follows: Consider one of the parallel strips $a\,b\,b_1\,a_1$ in fig. 7. The distance of its centre of gravity from the O X axis is $\frac{1}{2}\,y$. Therefore the moment of the strip is $y.dx \times \frac{y}{2} = \frac{1}{2}y^2.dx$, so that the total moment of area about $OX = \frac{1}{2}\int_h^H y^2.dx$, between the limits $x = H$ and $x = h$. Call this moment $M_y$.

$\therefore$ the distance of the centre of gravity from O X, or leverage about O X, is called $\bar{y}$

and $$\bar{y} = \frac{M_y}{A} = \frac{\int_h^H y^2.dx}{2\int_h^H y.dx}.$$

The centre of gravity of the area is now completely determined: its X coordinate is

$$\frac{\int_h^H xy.dx}{\int_h^H y.dx},$$

and its Y coordinate is

$$\frac{\int_h^H y^2.dx}{2\int_h^H y.dx}.$$

Applying the above rule to the curve whose equation is $y = ax^2 + bx + c$ we proceed as follows:—

Moment of area about O Y

$$= \int_h^H xy.dx,$$

$$= \int_h^H x(ax^2 + bx + c)dx,$$

$$= \left[\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + C\right]_h^H,$$

$$= \frac{aH^4}{4} + \frac{bH^3}{3} + \frac{cH^2}{2} + C - \left(\frac{ah^4}{4} + \frac{bh^3}{3} + \frac{ch^2}{2} + C\right),$$

$$= \frac{a}{4}(H^4 - h^4) + \frac{b}{3}(H^3 - h^3) + \frac{c}{2}(H^2 - h^2) \quad . \quad . \quad . \quad . \quad (3)$$

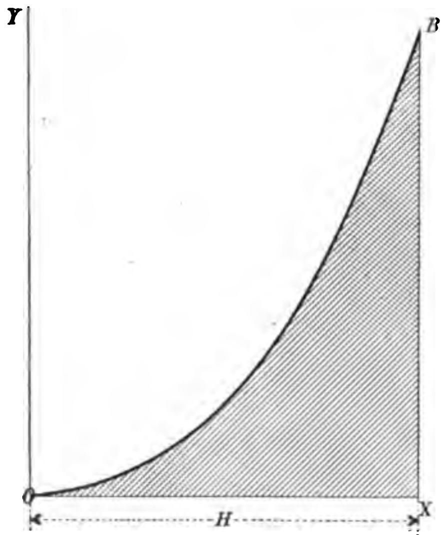



FIG. 8.

The area is $= \int_h^H y.dx,$

which, from (2), p. 9,

$$= \frac{a}{3}(H^3 - h^3) + \frac{b}{2}(H^2 - h^2) + c(H - h).$$

$$\therefore \quad \bar{x} = \frac{\frac{a}{4}(H^4 - h^4) + \frac{b}{3}(H^3 - h^3) + \frac{c}{2}(H^2 - h^2)}{\frac{a}{3}(H^3 - h^3) + \frac{b}{2}(H^2 - h^2) + c(H - h)}.$$

Taking a simpler case of a parabola with vertex at O (fig. 8) and O Y the axis of parabola, equation becomes

$$y = ax^2$$

and if $h = 0$

$$\text{area} \quad = \frac{a\text{H}^3}{3},$$

$$\text{and} \quad \bar{x} = \frac{\frac{a}{4}\text{H}^4}{\frac{a}{3}\text{H}^3} = \frac{3}{4}\text{H}.$$

The moment about $\text{O X} = \frac{1}{2}\int y^2.dx = \frac{1}{2}\int_0^{\text{H}} a^2x^4.dx,$

$$= \frac{a^2}{2}\frac{\text{H}^5}{5}.$$

$$\therefore \quad \bar{y} = \frac{\frac{a^2\text{H}^5}{10}}{\frac{a\text{H}^3}{3}} = \frac{3}{10}a\text{H}^2 = \frac{3}{10}\text{B X}.$$

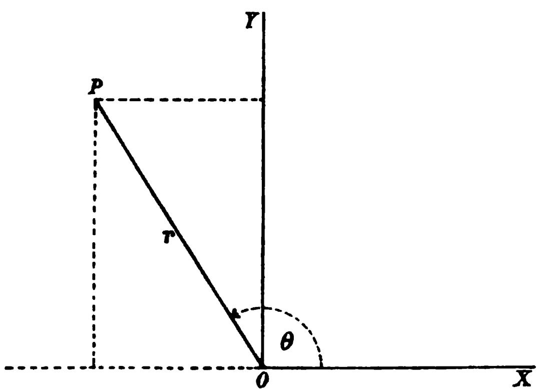



FIG. 9.

**Polar Integration.**—In the rectangular system of coordinates a point is fixed when we know the values of $x$ and $y$. In some cases it is more convenient to know the polar coordinates.

Let O X (fig. 9) be a fixed straight line. Let P be any point. Join P to O. Then if we specify the angle X O P and the distance O P we fix the position of the point P. Hence the polar coordinates of the point P are $r$ and $\theta$ where $r$ is the distance O P and $\theta$ is the angle X O P.

In this system, as in the rectangular system, it is necessary to consider the sign of the coordinate. The $r$ coordinate is called the polar distance of the point P. The ordinary method of measuring the angle $\theta$ is to consider O P as a radial arm which has revolved from O X to O P in a counter-clockwise direction. This direction is positive, so that if the angle be measured in the clockwise direction the sign of $\theta$ is negative. After the position of the line O P is fixed the distance O P equals can be set off as shown. If $r$ is negative the distance $r$ will be set off on the other side of O from that shown in fig. 9.

P is the point $r\,\theta$ in polar coordinates. Draw O Y perpendicular to O X. Draw the perpendiculars from P on O X and O Y respectively. The intersections of these perpendiculars with O X and O Y give the values of the rectangular coordinates $x$ and $y$ of the point P. We therefore have the following relations:

$$x = r \cos \theta$$
$$y = r \sin \theta$$

$$\therefore \quad r = \pm\sqrt{x^2 + y^2} \quad \text{and} \quad \tan \theta = \frac{y}{x}.$$

Therefore if we have any equation to a curve in rectangular coordinates we can at once get the polar equation by substituting for $x$ and $y$ the above values, and *vice versa*.

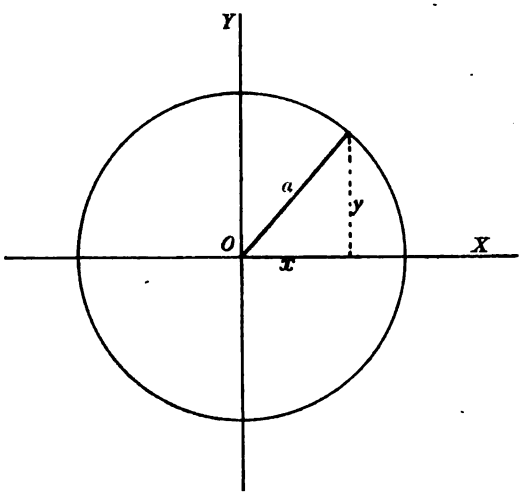



FIG. 10.

A short way of writing the equation to a curve referred to rectangular axes is $y = f(x)$.

This represents a curve in its most general form, and $f(x)$ is a rational function of $x$. In the same way we may also write the general form of the polar equation to a curve $r = f(\theta)$, where $f(\theta)$ has a similar meaning for $\theta$ as $f(x)$ for $x$.

Consider a simple example. The equation to a circle of radius $a$ with its centre at the origin is

$$x^2 + y^2 = a^2.$$

Since $x = r \cos \theta$ and $y = r \sin \theta$, this equation becomes

$$r^2 \cos {}^2\theta + r^2 \sin {}^2\theta = a^2$$
$$\text{or} \qquad r^2 = \text{constant}$$

is the equation to a circle in polar coordinates.

For an ellipse whose equation is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

the polar equation is

$$\frac{r^2\cos^2\theta}{a^2}+\frac{r^2\sin^2\theta}{b^2}=1$$

or

$$\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}=\frac{1}{r^2}.$$

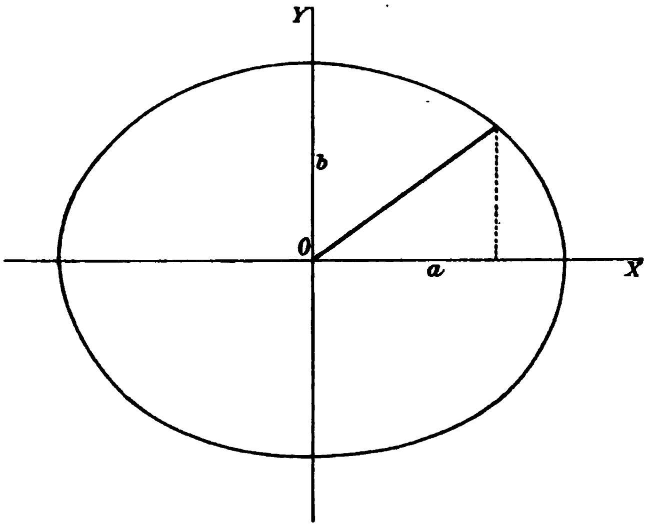



Fig. 11.

# CHAPTER III.

## ARITHMETICAL RULES FOR INTEGRATION.

WHAT has been explained in Chapter II. has reference to the integration of an area, and we have seen that this can be done by means of formulæ, or rules, if we know the equation to the curve. In solids of shipshape form it is generally not possible to express a curve of section by a simple equation. If, however, we divide a curve such as $A_1 A_2 A_3$ in fig. 12 into a sufficient number of parts by parallel ordinates like $A_1 B_1$, $A_2 B_2$, etc., the parts $A_1 A_3$, $A_3 A_5$, etc., may be sufficiently like some known curve, such as a parabola,

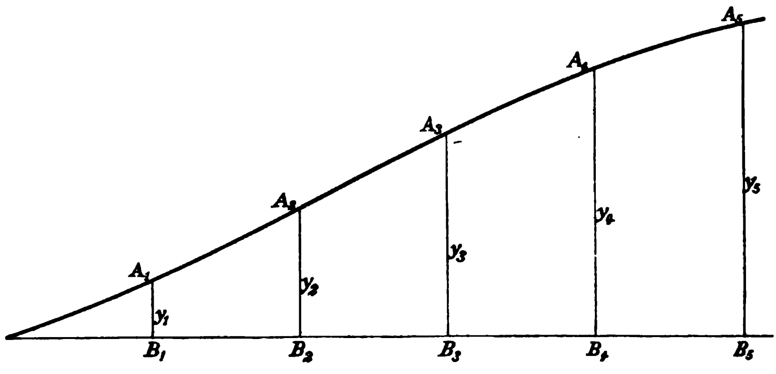



FIG. 12.

as to involve no appreciable error in assuming that the parts are of a known form. On this assumption the rules applicable to known forms may be applied to the ship form under consideration.

*Trapezoidal Rule.*—The assumption that is made for this rule is that the parts $A_1 A_2$, $A_2 A_3$, etc., are straight lines (see fig. 13). $k$ is the interval between the ordinates, which is assumed to be the same for all the ordinates, and the smaller the value of $k$ the more accurate will be the result.

The area of the part $A_1 B_1 B_2 A_2 = k\dfrac{y_1 + y_2}{2}$,

" " $A_2 B_2 B_3 A_3 = k\dfrac{y_2 + y_3}{2}$.

$\therefore$ Area of $A_1B_1B_3A_3 = \dfrac{k}{2}(y_1 + 2y_2 + y_3)$.

Extending this to the curve in fig. 12, we have the general formula for the Trapezoidal Rule:—

$$\text{Area } A_1B_1B_nA_n = \frac{k}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 \ldots\ldots + y_n),$$

$$= \frac{k}{2}\cdot(y_1 + y_n) + k(y_2 + y_3 + \ldots\ldots y_{n-1}).$$

*Simpson's First Rule.*—The assumption in this rule is that the parts like $A_1 A_2 A_3$, etc., are parabolic. Suppose $A_1 A_2 A_3$, fig. 13, are points in a parabola whose equation is $y = ax^2 + bx + c$.

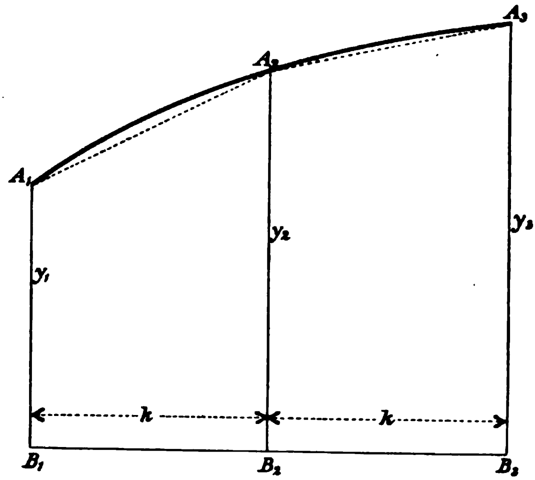



FIG. 13.

$$\text{The area of } A_1 B_1 B_3 A_3 = \int_0^{2k} y.dx,$$

$$= \int_0^{2k} (ax^2 + bx + c)dx,$$

$$= \frac{8ak^3}{3} + \frac{4bk^2}{2} + 2ck,$$

$$= \frac{k}{3}(8ak^2 + 6bk + 6c).$$

We can find the values of $y_1\, y_2\, y_3$ by substituting the corresponding values of $x$ in the equation.

Thus when $x = 0 \qquad y_1 = c,$

" " $x = k \qquad y_2 = ak^2 + bk + c,$

" " $x = 2k \qquad y_3 = 4ak^2 + 2bk + c.$

By inspection $\qquad y_1 + 4y_2 + y_3 = 8ak^2 + 6bk + 6c,$

$$\therefore \quad \int_0^{2k} y.dx = \frac{k}{3}(y_1 + 4y_2 + y_3).$$

$$\therefore \quad \text{Area } A_1 B_1 B_3 A_3 = \frac{k}{3}(y_1 + 4y_2 + y_3).$$

Applying this to the curve in fig. 12,

$$\text{Area } A_1\,B_1\,B_n\,A_n = \frac{k}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 \,.\,.\,.\,.\,.\,.\, y_n),$$

where $n$ is an odd number.

This is Simpson's First Rule.

It will be seen in this case that every third ordinate forms a stop-point in the integration, and that $n$ must be an odd number.

*Simpson's Second Rule.*—If we assume that the points $A_1\,A_2\,A_3\,A_4$ be on a parabola whose equation is $y = ax^2 + bx + c$.

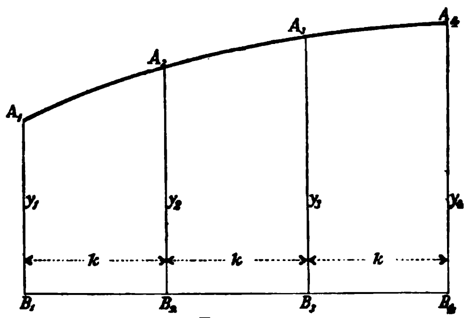



FIG. 14.

$$\text{Area } A_1\,B_1\,B_4\,A_4 = \int_0^{3k} y.dx,$$

$$= \int_0^{3k} (ax^2 + bx + c)dx,$$

$$= 9ak^3 + \frac{9}{2}bk^2 + 3ck,$$

$$= \frac{3}{8}k(24ak^2 + 12bk + 8c).$$

| | | |
|---|---|---|
| Now when | $x = 0$ | $y_1 = c,$ |
| ,, | $x = k$ | $y_2 = ak^2 + bk + c,$ |
| ,, | $x = 2k$ | $y_3 = 4ak^2 + 2bk + c,$ |
| ,, | $x = 3k$ | $y_4 = 9ak^2 + 3bk + c.$ |

By inspection $y_1 + 3y_2 + 3y_3 + y_4 = 24ak^2 + 12bk + 8c,$

$$\therefore \quad \frac{3}{8}k(y_1 + 3y_2 + 3y_3 + y_4) = \int_0^{3k} y.dx.$$

$$\therefore \quad \frac{3}{8}k(y_1 + 3y_2 + 3y_3 + y_4) = \text{Area } A_1\,B_1\,B_4\,A_4.$$

The general formula is

$$\text{Area of curve} = \frac{3}{8}k(y_1 + 3y_2 + 3y_3 + 2y_4 + 3y_5 + 3y_6 \,.\,.\,.\,.\,.\,.\, y_n),$$

where $n = 3p + 1$ where $p$ is an integer.

If we assume that the points $A_1\,A_2\,A_3\,A_4$ lie on a cubic parabola whose equation is $y = ax^3 + bx^2 + cx + d$, it can be shown that the area $A_1\,B_1\,B_n\,A_n$ can be obtained by the same general formula.

*Five-Eight Rule.*—This rule expresses the area between consecutive ordinates, like the area of the part $A_1 A_2 B_2 B_1$.

$$\text{Area of part } A_1 A_2 B_2 B_1 = \int_0^k y.dx$$

$$= \frac{ak^3}{3} + \frac{bk^2}{2} + ck$$

$$= \frac{k}{12}(4ak^2 + 6bk + 12c).$$

Now

$$y_1 = c$$
$$y_2 = ak^2 + bk + c$$
$$y_3 = 4ak^2 + 2bk + c.$$

By inspection $5y_1 + 8y_2 - y_3 = 4ak^2 + 6bk + 12c.$

$\therefore$ Area part $A_1 A_2 B_2 B_1 = \frac{k}{12}(5y_1 + 8y_2 - y_3).$

If the curve is fairly uniform in character, we may apply this rule to a succession of ordinates thus:—

$$k\left(\frac{5}{12}y_1 + \frac{13}{12}y_2 + y_3 + y_4 + \ldots\ldots + y_{n-2} + \frac{13}{12}y_{n-1} - \frac{y_n}{12}\right)$$

$$= \frac{k}{12}(5y_1 + 13y_2 + 13y_{n-1} - y_n) + k(y_3 + y_4 + \ldots + y_{n-2}).$$

*Tchebycheff's Rule.*—In the foregoing cases that have been considered for measuring area the ordinates are spaced equidistantly, though for purposes of greater accuracy intermediate ordinates can be introduced, but their spacing bears a direct proportion to the common interval.

In Tchebycheff's Rule the ordinates are unequally spaced, and always bear a definite proportion to the length of the base of the curve to be integrated. The area of a curve may be found by Tchebycheff's method with 2, 3, 4, 5, 6, 7, 8, 9, or 10 ordinates, or with any multiples or combinations of these.

Table I. gives the spacing of these ordinates in proportion to the half length of the base of the curve.

TABLE I.

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| ... | ... | ... | ... | ... | ... | ... | ... | ·9162 |
| ... | ... | ... | ... | ... | ... | ·8974 | ·91159 | ·6873 |
| ... | ... | ... | ... | ·8662 | ·88386 | ·5938 | ·60102 | ·5000 |
| ... | ... | ·794654 | ·8825 | ·4225 | ·52966 | ·4062 | ·52876 | ·3127 |
| ·577350 | ·707107 | ·187592 | ·3745 | ·2666 | ·32391 | ·1026 | ·16791 | ·0838 |
| | ·000000 | | ·0000 | | ·00000 | | ·00000 | |
| ·577350 | ·707107 | ·187592 | ·3745 | ·2666 | ·32391 | ·1026 | ·16791 | ·0838 |
| ... | ... | ·794654 | ·8325 | ·4225 | ·52966 | ·4062 | ·52876 | ·3127 |
| ... | ... | ... | ... | ·8662 | ·88386 | ·5938 | ·60102 | ·5000 |
| ... | ... | ... | ... | ... | ... | ·8974 | ·91159 | ·6873 |
| ... | ... | ... | ... | ... | ... | ... | ... | ·9162 |

When the ordinates have been drawn in their proper places they are measured off and added together, the total sum being multiplied by the whole length of the curve and divided by the number of ordinates.

The simplicity of this rule is obvious.

With 9 as the number of ordinates, the work is no more than with 9 ordinates in the case of the Trapezoidal Rule.

Consider the curve to be a portion of the parabola of the $n$th order.

Let the origin be at the central ordinate and let the base be the axis O X.

In a parabola of $n$th order $y = a_0 + a_1x + a_2x^2 \ldots\ldots a_nx^n$.

Let length of base of curve be $2l$.

$$\text{Area of curve} = \int_{-l}^{+l} y.dx.$$

If $n$ be odd

$$= 2l\left\{a_0 + a_2\frac{l^2}{3} + a_4\frac{l^4}{5} + \ldots\ldots + a_{n-1}\frac{l^{n-1}}{n}(n \text{ odd})\right\} \quad . \quad . \quad (1)$$

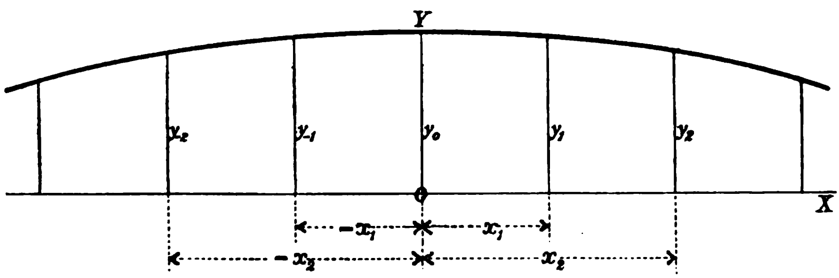



FIG. 15.

Let area of curve = c times the sum of $n$ ordinates

$$= c\left\{y_0 + (y_1 + y_{-1}) + (y_2 + y_{-2}) + \ldots\ldots + \left(y_{\frac{n-1}{2}} + y_{-\frac{n-1}{2}}\right)\right\} \quad . \quad (2)$$

$$= 2c\left\{\frac{n}{2}a_0 + a_2\left(x_1^2 + x_2^2 + x_3^2 + \ldots\ldots + x^2_{\frac{n-1}{2}}\right) + \ldots\ldots\right.$$

$$\left. \ldots + \underset{(n \text{ odd})}{a_{n-1}}\left(x_1^{n-1} + x_2^{n-1} + \ldots\ldots + x^{n-1}_{\frac{n-1}{2}}\right)\right\} \quad . \quad . \quad (3)$$

Equating the coefficients of $a_0, a_2, a_4, \ldots\ldots a_{n-1}$ in (1) and (3) we get

$$c = \frac{2l}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$x_1^2 + x_2^2 + \ldots\ldots + x^2_{\frac{n-1}{2}} = \frac{l^2n}{6}$$

$$x_1^4 + x_2^4 + \ldots\ldots + x^4_{\frac{n-1}{2}} = \frac{l^4n}{10}$$

. . . . . . . . . . . . .

$$x_1^{n-1} + x_2^{n-1} + \ldots\ldots + x^{n-1}_{\frac{n-1}{2}} = \frac{l^{n-1}n}{2n}.$$

A similar proof will apply if $n$ is even.

It is from these equations that we can get the values given in the table for the abscissæ by substituting the value for $n$.

*e.g.* if we take $n=3$

$$x_1^2=\frac{l^2}{2} \quad \therefore \quad x_1=\frac{l}{\sqrt{2}}.$$

By substituting in (2) the value of $c$ given by (4) it is seen that the area required = arithmetic mean of the ordinates multiplied by the whole length of the base.

The question of the mathematical accuracy of this rule has been discussed very fully by Professor Kriloff, and he has given some examples showing the extent of the inaccuracy with a varying number of ordinates.

A summary of his results, all reduced to coefficients of area or ratio of the actual area of the curve to the circumscribing rectangle, is given in Table II. The curves are in each case of definite mathematical form, so that the areas can be exactly calculated. The comparison is made for two sections, one amidships and one at about one-third of the length of the ship from aft. Results are also given for the load water line.

TABLE II.

| | Midship Section. | After Section. | Load Water Line. |
|---|---|---|---|
| Exact value . . . . . . . . | ·8351 | ·5903 | ·8066 |
| Tchebycheff's 7 ords. . . . . . . | ·8354 | ·5904 | ·8076 |
| ,, 9 ,, . . . . . . | ·8348 | ... | ·8064 |
| ,, 14 ,, (2 series 7 each) . . | ... | ... | ·8066 |
| Simpson's 8 ords. . . . . . . . | ·8341 | ·5942 | ... |
| ,, 10 ,, . . . . . . . . | .. | ·5914 | ... |
| ,, 21 ,, . . . . . . . . | ... | ... | ·8064 |
| Trapezoidal 21 ords. . . . . . . | ... | ... | ·8033 |
| ,, 21 ,, and three half ords. at each end . . . . . . . . | ... | ... | ·8066 |

The above results also show a comparison with Simpson's and the Trapezoidal rules. The following results give a comparison for 2, 3, 4, 5, and 6 ordinates.

The case taken is the midship section.

| | Midship Section. |
|---|---|
| Exact value . . . . . . . | ·8351 |
| Tchebycheff's 2 ords. . . . . . | ·8540 |
| 3 ,, . . . . . | ·8468 |
| 4 ,, . . . . . | ·8896 |
| 5 ,, . . . . . | ·8378 |
| 6 ,, . . . . . | ·8352 |

The following example illustrates the method of finding an area by Tchebycheff's Rule: Let fig. 16 be a curve, say the water-line of a vessel 350 feet long, and let the number of ordinates be seven. Then from the table we find

$$
\begin{aligned}
x_0 &= 0 &&= 0 \\
x_1 &= \cdot 32391 \times 175 &&= 56\cdot 684 \\
x_2 &= \cdot 52966 \times 175 &&= 92\cdot 691 \\
x_3 &= \cdot 88386 \times 175 &&= 154\cdot 676.
\end{aligned}
$$

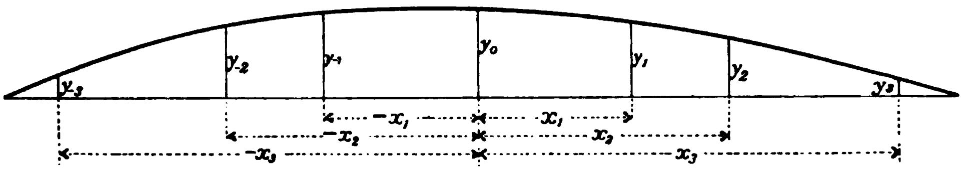



FIG. 16.

Setting up ordinates at these positions we get

$$
\begin{aligned}
& y_0 = 23\cdot 8 \\
& \begin{cases} y_1 = 21\cdot 2 \\ y_{-1} = 23\cdot 1 \end{cases} \\
& \begin{cases} y_2 = 15\cdot 9 \\ y_{-2} = 20\cdot 6 \end{cases} \\
& \begin{cases} y_3 = 2\cdot 8 \\ y_{-3} = 5\cdot 5 \end{cases}
\end{aligned}
$$

$\therefore$ Sum of ordinates $= \Sigma y = 112\cdot 9.$

$\therefore$ Area $= 112\cdot 9 \times \frac{350}{7}$

$= 5645$ ft.$^2$

# CHAPTER IV.

## APPLICATION OF ARITHMETICAL RULES TO THE DETERMINATION OF VOLUMES, CENTRES OF GRAVITY, AND MOMENTS OF INERTIA.

HAVING obtained rules for finding the area of any curve, it is easy to find volumes and centres of gravity of volumes. As already explained, the curve whose ordinates represent the area of a cross-section is such that its area gives the volume of the solid.

The curve $A_1 A_2 A_3$ represents such a curve, fig. 17. If the area of each cross-section be multiplied by its thickness, we shall have an element of the volume. If, now, we take account not only of the volume, but its position in the solid, we shall be able to obtain the centre of gravity of the solid. If, in other words, we take account of each piece of volume about some fixed plane parallel to the planes of section, we can get what is known as the moment of the whole solid.

Taking an element of volume $a\,b\,b_1\,a_1$, which we have already called $y.dx$, and multiplying it by its distance $x$ from the plane through O Y, then we have the moment $x.y.dx$ of the volume $a\,b\,b_1\,a_1$ about O Y. Summing all the values of such moments we have

$$\text{Total moment of solid} = \int_h^H x.y\,dx.$$

Taking as before $A_1 B_1$, $A_2 B_2$, $A_3 B_3$ as equidistant ordinates $k$ apart, we can obtain a new curve by multiplying $A_1 B_1$ by $O B_1$, $A_2 B_2$ by $O B_2$, and $A_3 B_3$ by $O B_3$, etc., each ordinate of which $B_1 M_1$, $B_2 M_2$, etc., will represent the moment of the elementary volumes such as $a b b_1 a_1$.

$$\text{Hence} \qquad \int_h^H \text{BM}.dx = \text{total moment}.$$

From this we see that the finding of the total moment about O Y resolves itself into finding the area of a curve, each ordinate of which represents the product $x.y$.

To find the moment about O X we notice that the centre of gravity of the element $a\,b\,b_1\,a_1$ is at C, distant $\frac{y}{2}$ from O X.

The moment of the element will be

$$y.dx \times \frac{y}{2} = \frac{1}{2}y^2dx,$$

and summing the moments of all the elements we have

$$\int_h^H \frac{y^2}{2}.dx = \text{Total moment of solid about O X.}$$

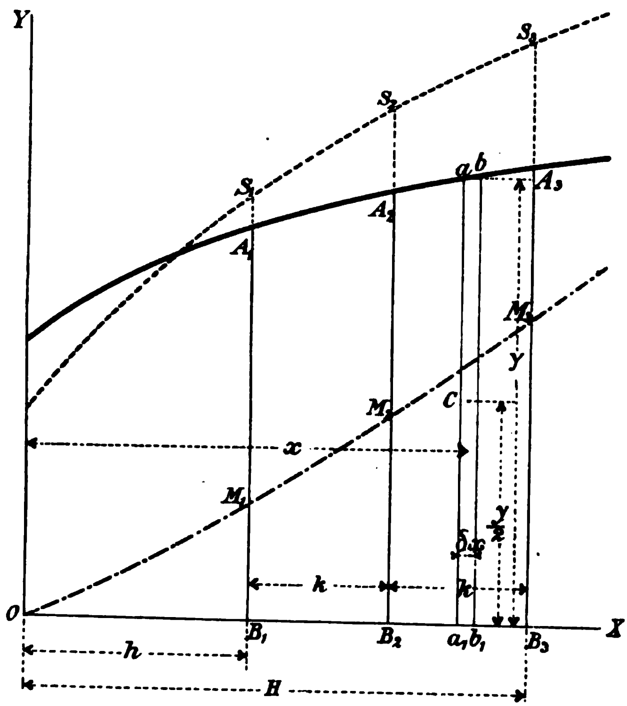



Fig. 17.

This is equivalent to finding the area of a new curve $S_1 S_2 S_3$, each ordinate of which is one-half the square of the ordinate of the curve $A_1 A_2 A_3$. The rules already determined for finding the area of a curve have now to be applied to these special cases.

Using Simpson's First Rule, we have

$$\text{Moment of } A_1 B_1 B_3 A_3 \text{ about O Y} = \int_h^H BM.dx$$

$$= \frac{k}{3}(x_1y_1 + 4x_2y_2 + x_3y_3),$$

where $x_1 = OB_1$, $x_2 = OB_2$, $x_3 = OB_3$;

and $y_1 = B_1A_1$, $y_2 = B_2A_2$, $y_3 = B_3A_3$.

General formula: the moment of an area about O Y

$$= \frac{k}{3}(x_1y_1 + 4x_2y_2 + 2x_3y_3 + 4x_4y_4 + \ldots + x_ny_n),$$

where $n$ is an odd integer.

Distance of centre of gravity from O Y

$$= \frac{x_1y_1 + 4x_2y_2 + 2x_3y_3 + 4x_4y_4 + \ldots + x_ny_n}{y_1 + 4y_2 + 2y_3 + 4y_4 + \ldots + y_n}.$$

It will be seen from this formula that it does not matter where the axis for moments is taken. Altering the position of the axis only alters the abscissæ $x_1$, $x_2$, $x_3$, etc. This formula can be put in a practicable form as in the following table:—

TABLE III.

| Number of Ordinate. | Length of Ordinate. | Multipliers for Simpson's First Rule. | Functions for Area. | Distances from O Y axis. | Functions for Moments. |
|---|---|---|---|---|---|
| 1 | $y_1$ | 1 | $y_1$ | $x_1$ | $x_1 y_1$ |
| 2 | $y_2$ | 4 | $4y_2$ | $x_2$ | $4x_2 y_2$ |
| 3 | $y_3$ | 2 | $2y_3$ | $x_3$ | $2x_3 y_3$ |
| 4 | $y_4$ | 4 | $4y_4$ | $x_4$ | $4x_4 y_4$ |
| 5 | $y_5$ | 2 | $2y_5$ | $x_5$ | $2x_5 y_5$ |
| ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... |
| $n$ | $y_n$ | 1 | $y_n$ | $x_n$ | $x_n y_n$ |
| | | | $\Sigma A$ | | $\Sigma M$ |

Distance of centre of gravity from O Y axis $= \frac{\Sigma M}{\Sigma A}$.

Area of curve $= \Sigma A \times \frac{k}{3}$.

Moment of curve about O Y axis $= \Sigma M \times \frac{k}{3}$.

Usually the abscissæ $x_1, x_2, x_3$ are expressed as multiples of $k$ if the axis about which the moments are taken coincides with one of the ordinates. In this case, instead of multiplying by $x_1, x_2, x_3$, etc., we multiply in the column by 0, 1, 2, 3, etc., or whatever the multiple may be, and then multiply the $\Sigma M$ by $k$ and $\frac{k}{3}$ finally for the true moment.

If the axis O Y be at number 1 ordinate, then the leverages $x_1$, $x_2$, $x_3$, etc. become 0, $k$, $2k$, $3k$ respectively, and therefore the moment M

$$= \frac{k}{3}(0 + 4y_2.k + 2y_3.2k + 4y_4.3k + \ldots \text{ etc.})$$

$$= \frac{k^2}{3}(4y_2 + 4y_3 + 12y_4 + \ldots \text{ etc.})$$

$\therefore$ Distance of C.G. from O Y $= \dfrac{k(4y_2 + 4y_3 + 12y_4 + \ldots + (n-1)y_n}{y_1 + 4y_2 + 2y_3 + 4y_4 + \ldots + y_n}.$

The moment of $A_1 B_1 B_3 A_3$ about O X $= \int_h^H BS.dx$

$$= \frac{k}{3}\left(\frac{y_1^2}{2} + \frac{4y_2^2}{2} + \frac{y_3^2}{2}\right).$$

$\therefore$ General formula for moment of area about O X

$$= \frac{k}{6}(y_1^2 + 4y_2^2 + 2y_3^2 + 4y_4^2 + \ldots + y_n^2).$$

General form for distance of C.G. from O X

$$= \frac{1}{2}\left(\frac{y_1^2 + 4y_2^2 + 2y_3^2 + 4y_4^2 + \ldots + y_n^2}{y_1 + 4y_2 + 2y_3 + 4y_4 + \ldots + y_n}\right).$$

This can be tabled in the form :

TABLE IV.

| Number of Ordinate. | Length of Ordinate. | Multipliers for Simpson's First Rule. | Functions for Area. | Squares of Ordinates. | Functions of Squares. |
|---|---|---|---|---|---|
| 1 | $y_1$ | 1 | $y_1$ | $y_1^2$ | $y_1^2$ |
| 2 | $y_2$ | 4 | $4y_2$ | $y_2^2$ | $4y_2^2$ |
| 3 | $y_3$ | 2 | $2y_3$ | $y_3^2$ | $2y_3^2$ |
| 4 | $y_4$ | 4 | $4y_4$ | $y_4^2$ | $4y_4^2$ |
| 5 | $y_5$ | 2 | $2y_5$ | $y_5^2$ | $2y_5^2$ |
| ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... |
| $n$ | $y_n$ | 1 | $y_n$ | $y_n^2$ | $y_n^2$ |
| | | | $\Sigma A$ | | $\Sigma M$ |

Distance of C.G. from O X axis $= \dfrac{\Sigma M}{2\Sigma A}$.

Area of curve $= \Sigma A \times \dfrac{k}{3}$.

Moment of curve $= \Sigma M \times \dfrac{k}{6}$.

**Moment of Inertia.**—The moment of inertia of an area about a given axis is the limit of the sum of the product of each element of the area, and the square of its distance from the given axis, as the element is taken smaller and smaller in size.

The moment of inertia of a volume or of a mass is obtained by taking each element of the volume or mass, multiplying it by the square of its distance from the axis, finding the expression which gives the sum of these products, and then taking the limit of this sum as the size of the element gets smaller and smaller.

Let the equation to the curve in fig. 18 be $y = f(x)$.

Consider the area $A_1 B_1 B_3 A_3$ between ordinates whose abscissæ are $h$ and H as before.

Area of elemental strip $a\,b\,b_1\,a_1 = y.dx$.
Moment of „ „ about O Y $= x.y.dx$.
„ „ „ „ O X $= \frac{y^2}{2}.dx$.

Moment of inertia of elemental strip about O Y $= \int_0^y x^2.dx.dy = x^2.y.dx$.

Moment of inertia of elemental strip about O X $= \int_0^y y^2.dx.dy = \frac{y^3}{3}.dx$.

$\therefore$ M.I. of area $A_1 B_1 B_3 A_3$ about O Y $= \int_h^H x^2.y.dx$.

and „ „ O X $= \frac{1}{3}\int_h^H y^3.dx$.

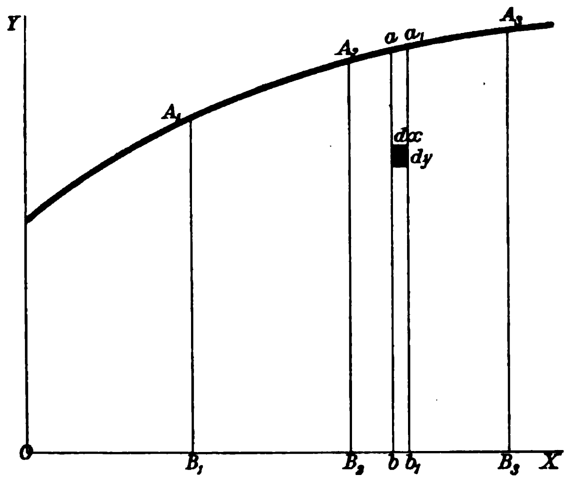



FIG. 18.

In the same way as for finding the moments of an area we can apply the arithmetical rules to find the moment of inertia.

The general formulæ are, using Simpson's First Rule :—

$$\text{M.I. about O Y} = \frac{k}{3}(x_1^2y_1 + 4x_2^2y_2 + 2x_3^2y_3 + 4x_4^2y_4 + \ldots + x_n^2y_n)$$

$$\text{,, O X} = \frac{1}{3}\cdot\frac{k}{3}(y_1^3 + 4y_2^3 + 2y_3^3 + 4y_4^3 + \ldots + y^3_n).$$

This can be tabled in a similar way to that for the moments.

**Moment of Inertia about an axis through C.G.**—The moment of inertia is least when the axis passes through the centre of gravity (for axes fixed in direction).

This we can see by considering two parallel axes, one of them being through the C.G. and the other parallel to it, but at a distance $h$ as represented by fig. 19. Let I represent the moment of inertia of the area (A) about O Y. Let O be the position of C.G. If we want to find $I_1$, the moment of inertia about the new axis, all the $x$ coordinates are changed by an amount equal to $h$.

$$I = \int x^2.y.dx.$$

$$\therefore \quad I_1 = \int (x+h)^2 y.dx.$$

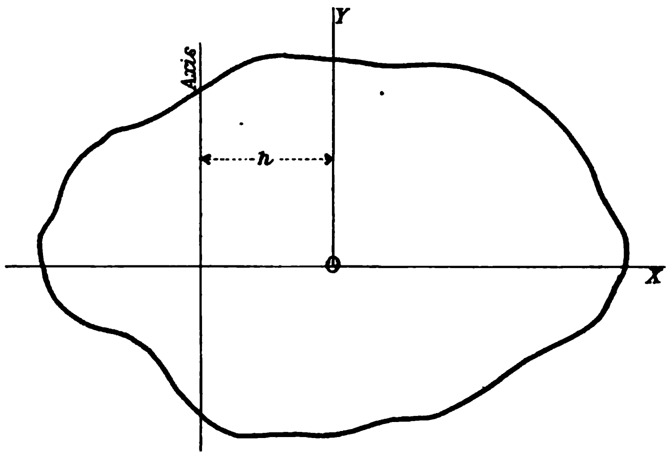



FIG. 19.

The $y$ coordinates remain unchanged.

$$\therefore \quad I_1 = \int (x^2 + 2hx + h^2) y.dx.$$

$$\therefore \quad I_1 = \int x^2.y.dx + 2h \int x.y.dx + h^2 \int y.dx.$$

$$\text{Now} \quad \int x^2.y.dx = I$$

and $\int x.y.dx$ the moment of area = zero, since O Y passes through the C.G.

$$\text{and} \quad h^2 \int y.dx = h^2 \text{ (A)}$$

$$\therefore \quad I_1 = I + h^2(A) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Since $h^2$ is always positive, I will be less than any value $I_1$, and the moment of inertia is therefore least when $h = 0$, that is, when the axis passes through the C.G.

The above formula enables us to determine the moment of inertia about any axis parallel to a given axis through the C.G. when the value of the moment of inertia about the given axis through the C.G. is known.

The following table is drawn to show the method of finding the moment of inertia of a given area about an axis through its C.G., using Simpson's First Rule.

The position of the C.G. of the given area is unknown, and this has to be first determined in the table by taking moments about an assumed axis. By this we can get the correct position of the C.G. Ordinate No. 1 is the assumed axis. Let the area be divided by seven ordinates as shown in fig. 20. Common interval $= k$.

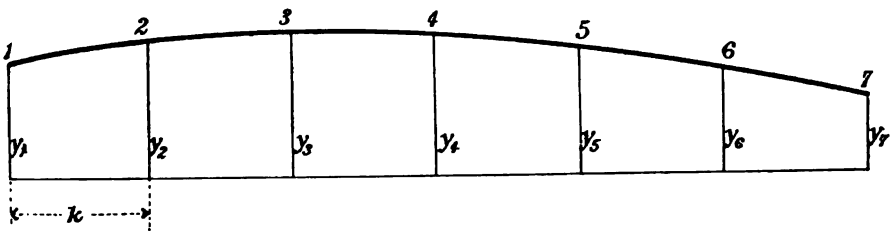



FIG. 20.

TABLE V.

| Number of Ordinate. | Length of Ordinate. | Simpson's Multipliers. | Functions for Areas. | Leverages about assumed Axis. | Function for Moments. | Function for Moment of Inertia. |
|---|---|---|---|---|---|---|
| | $y$ | | $y.dx$ | multiples of $k$ $x$ | $x.y.dx$ | $x^2y.dx$ |
| 1 | $y_1$ | 1 | $y_1$ | 0 | 0 | 0 |
| 2 | $y_2$ | 4 | $4y_2$ | 1 | $4y_2$ | $4y_2$ |
| 3 | $y_3$ | 2 | $2y_3$ | 2 | $4y_3$ | $8y_3$ |
| 4 | $y_4$ | 4 | $4y_4$ | 3 | $12y_4$ | $36y_4$ |
| 5 | $y_5$ | 2 | $2y_5$ | 4 | $8y_5$ | $32y_5$ |
| 6 | $y_6$ | 4 | $4y_6$ | 5 | $20y_6$ | $100y_6$ |
| 7 | $y_7$ | 1 | $y_7$ | 6 | $6y_7$ | $36y_7$ |
| | | | $\Sigma A$ | | $\Sigma M$ | $\Sigma I$ |

$$\text{Position of C.G.} = \frac{\Sigma M}{\Sigma A}.k = \frac{k(4y_2 + 4y_3 + 12y_4 + 8y_5 + 20y_6 + 6y_7)}{y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7} = h.$$

$\therefore$ $h$ = distance of assumed axis (No. 1 ord.) from correct position of C.G.

And the area $= \Sigma A.\frac{k}{3} = \frac{k}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7)$.

$\therefore$ $Ah^2$ can be found.

$\therefore$ True moment of inertia about axis through the C.G. $= \left(\Sigma I \times \frac{k^3}{3}\right) - Ah^2$.

We have now considered the general formulæ for areas, moments, and moments of inertia.

It is necessary to be thoroughly acquainted with these formulæ before dealing with the actual determination of displacement, position of centre of buoyancy, and other ship calculations.

The formulæ are appended below.

We have seen that volume is determined by finding the area of a curve of sectional areas. All sections are taken in the plane of X Y.

The direction of Z is the direction for integrating the areas of the sections.

### List of Rectangular Integrals.

| | |
|---|---|
| Area of section . . . . . . . . . . | $= \int y.dx.$ |
| Moment of area about axis O X . . . . . | $= \frac{1}{2}\int y^2.dx.$ |
| " " O Y . . . . . . | $= \int x.y.dx.$ |
| Volume of displacement . . . . . . . . | $= \int\int y.dx.dz.$ |
| Moment of inertia of area about axis O Y . . . . | $= \int\int x^2.dx.dy.$ |
| | $= \int x^2.y.dx.$ |
| " " " " O X | $= \int\int y^2.dx.dy.$ |
| | $= \frac{1}{3}\int y^3.dx.$ |

**Polar Integrals.**—Let A B be a curve for which we have the polar equation $r = f(\theta)$.

Consider the area between the limits for $\theta$, $a_1$ and $a_2$.

Draw O B where X O B $= a_1$.

" O A " X O A $= a_2$.

Consider an elemental sector between the radii $op_1$ and $op_2$.

Let $p_1$ be the point $r$, $\theta$.

Then $p_2$ is the point $(r + dr)(\theta + d\theta)$.

The area of $op_1 p_2 = \frac{1}{2}r.r.d\theta$.

$$\therefore \quad \text{The total area of O B A} = \frac{1}{2}\int_{a_1}^{a_2} r^2.d\theta.$$

The moment of the elemental sector about O X

$$= \tfrac{1}{2}r^2.d\theta \times \tfrac{2}{3}r.\sin\theta$$
$$= \tfrac{1}{3}r^3 \sin\theta.d\theta.$$

$$\therefore \quad \text{The total moment of O B A about O X} = \frac{1}{3}\int_{a_1}^{a_2} r^3 \sin\theta.d\theta.$$

The elemental moment about the axis O Y

$$= \tfrac{1}{2} r^2 . d\theta \times \tfrac{2}{3} r \cos\theta$$
$$= \tfrac{1}{3} r^3 \cos\theta . d\theta.$$

$\therefore$ The total moment of O B A about O Y $= \tfrac{1}{3}\int_{\alpha_1}^{\alpha_2} r^3 \cos\theta . d\theta.$

The volume of a solid integrated polarly is obtained by integrating the areas of sections in the $z$ direction and is therefore $= \tfrac{1}{2}\int\int r^2 . d\theta . dz.$

The moments of the solid can be obtained in the same way.

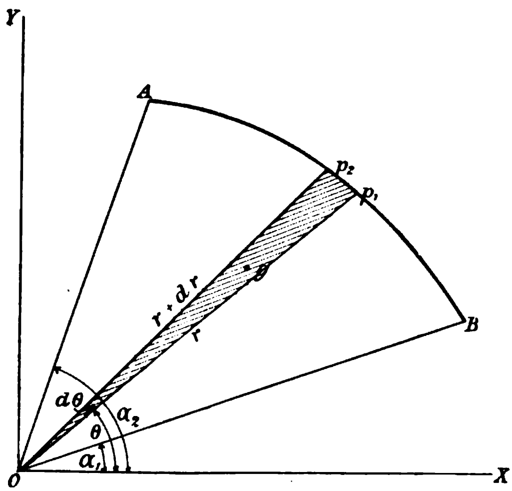



Fig. 21.

The following list gives the polar integrals.

List of Polar Integrals.

| | |
|---|---|
| Area . . . . . . . . . . | $= \tfrac{1}{2} \int r^2 . d\theta.$ |
| Moment of area about axis O X . . . . . | $= \tfrac{1}{3} \int r^3 . \sin\theta . d\theta.$ |
| ,, ,, ,, O Y . . . . . | $= \tfrac{1}{3} \int r^3 . \cos\theta . d\theta$ |
| Volume . . . . . . . . | $= \tfrac{1}{2}\int\int r^2 . d\theta . dz.$ |
| Moment of wedge about the plane Y Z . . . . | $= \tfrac{1}{3}\int\int r^3 . \cos\theta . dz\, d\theta.$ |
| ,, ,, ,, X Z . . . . | $= \tfrac{1}{3}\int\int r^3 . \sin\theta . dz . d\theta$ |

Moment of wedge about the axis O Z . . . $= \frac{1}{3}\int\int r^3.d\theta.dz.$

Moment of inertia of wedge about plane Y Z . . $= \frac{1}{4}\int\int r^4.\sin^2\theta.d\theta.dz.$

" " " " X Z . . $= \frac{1}{4}\int\int r^4.\cos^2\theta.d\theta.dz.$

Moment of inertia about axis O Z . . . . $= \frac{1}{4}\int\int r^4.d\theta.dz.$

TABLE VI.—AREAS AND CENTRES OF GRAVITY OF FIGURES.

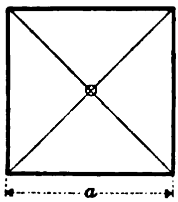
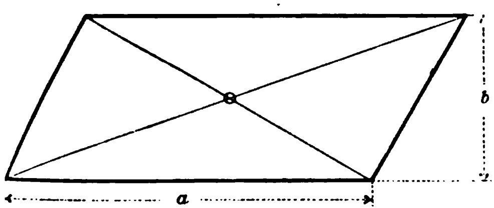
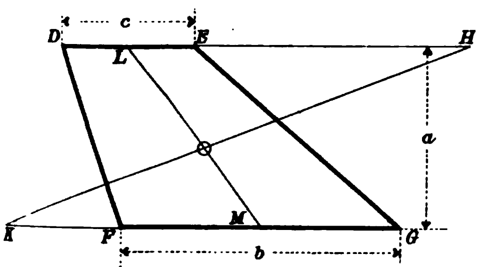
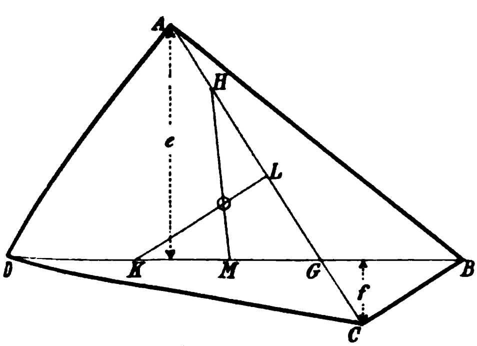
| Form. | Area. | Centre of Gravity. |
|---|---|---|
|   Square. | $A = a^2$. | Centre of gravity at intersection of diagonals. |
|   Parallelogram. | $A = a \times b$. | Centre of gravity at intersection of diagonals. |
|   Trapezoid. | $A = \frac{b + c}{2} \times a$. | Construction:<br>Bisect D E in L,<br>,, F G in M.<br>Join L M.<br>Produce D E to H, making E H = F G.<br>Produce G F to K, making F K = D E.<br>Join H K.<br>C.G. is at intersection of of L M and H K. |
|   Trapezium. | $A = \frac{e + f}{2} \times DB$. | Construction:<br>Make A H = C G,<br>,, D K = B G.<br>Bisect A C in L,<br>,, D B in M.<br>Join K L and H M.<br>C.G. is at the intersection of H M and K L. |

TABLE VI.—AREAS AND CENTRES OF GRAVITY OF FIGURES—*continued.*

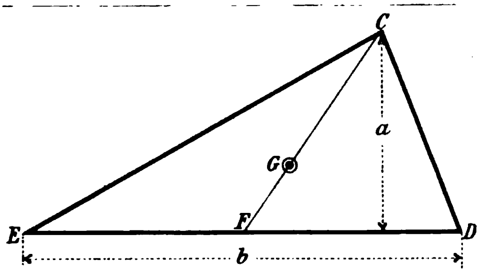
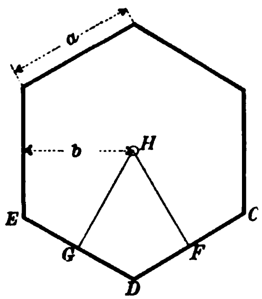
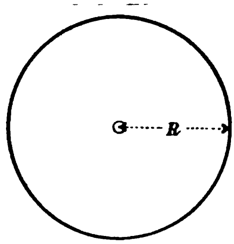
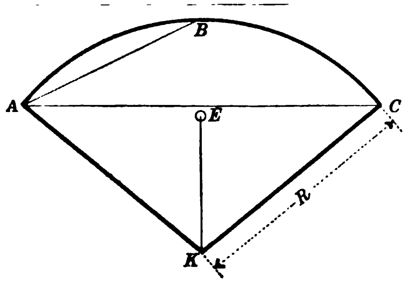
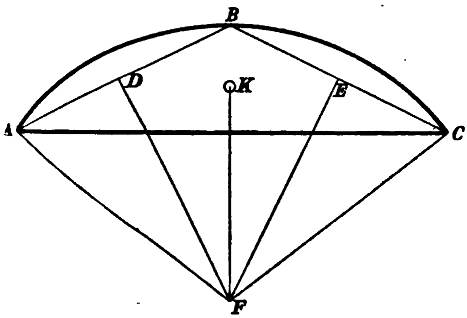
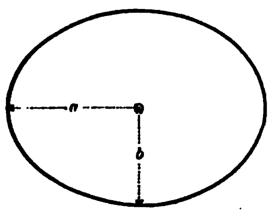
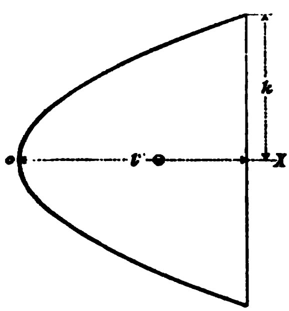
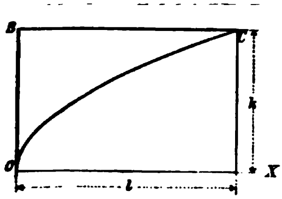
| Form. | Area. | Centre of Gravity. |
|---|---|---|
| <br>Triangle. | $A = \frac{a \times b}{2}$. | Construction :<br>Bisect E D in F.<br>Join F C.<br>Divide F C in G such that F G : G C = 1 : 2.<br>C.G. is at G. |
| <br>Regular Polygon. | $A = \frac{nab}{2}$. | Construction :<br>Bisect C D in F,<br>,, D E in G.<br>Draw F H perpendicular to C D,<br>Draw G H perpendicular to D E.<br>C.G. is at intersection of F H and G H. |
| <br>Circle. | $A = \pi R^2$. | C.G. at centre. |
| <br>Sector of Circle. | Construction :<br>Bisect the arc A C in B.<br>$A = \frac{R}{6}(8\ AB - AC)$. | C.G. at E, where<br>$KE = \frac{2\ R\ .\ AC}{8\ .\ AB - AC}$. |

TABLE VI.—AREAS AND CENTRES OF GRAVITY OF FIGURES—*continued.*

| Form. | Area. | Centre of Gravity. |
|---|---|---|
|   Segment of Circle. | Construction: Bisect the arc A C in B, ,, A B in D, ,, B C in E. Draw D F perpendicular to A B, and E F perpendicular to B C, to intersect in F. A = (area of sector A B C F) − (area of triangle A C F). | C.G. at K, where $F K = \frac{A C^3}{12 \text{(area of segment)}}$ |
|   Ellipse. | Note: $a = \frac{1}{2}$ major axis, $b = \frac{1}{2}$ minor axis. $A = \pi . a . b.$ | C.G. at intersection of major and minor axes. |
|   Parabola. | $A = \frac{4}{3} k . l.$ | C.G. $\frac{3}{5} l$ from vertex O. Half parabola has C.G. $\frac{3}{8} k$ from O X. |
|   Parabolic Spandrel. | $A = \frac{1}{3} k . l.$ | C.G. at $\frac{4}{3} k$ from O X, and $\frac{3}{10} l$ from O B. |

## TABLE VI.—*continued.*

### VOLUMES AND CENTRES OF GRAVITY OF SOLIDS.

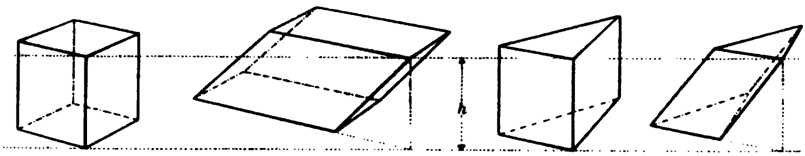
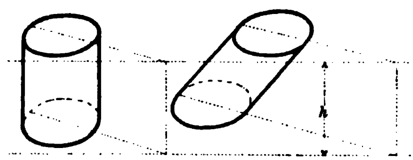
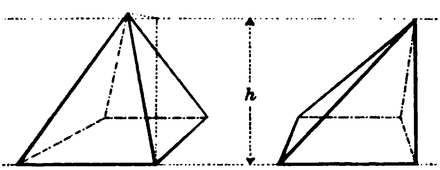
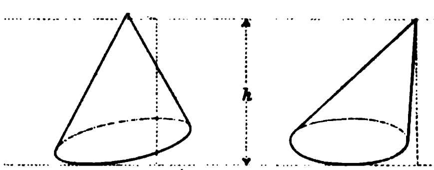
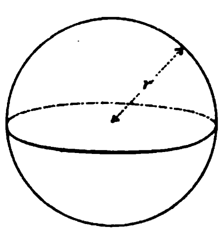
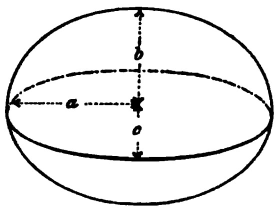
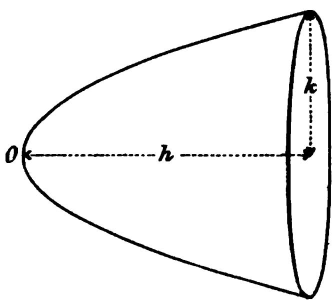
| Form. | Volume. | Centre of Gravity. |
|---|---|---|
|   Parallelopipedon Prism or Cylinder. | V = (area of base) × (perpendicular height). | C.G. at the middle of the line joining the centres of base and top. |
|   Pyramid or Cone. | V = (area of base) × $\frac{1}{3}$ (perpendicular height). | C.G. on the line joining the vertex to the C.G. of base, and ¼th of the line measured from the base end. |
|  Sphere. | $V = \frac{4}{3}\pi r^3$. | C.G. at centre. Hemisphere : C.G. at $\frac{3}{8}r$ from centre. |

TABLE VI.—VOLUMES AND CENTRES OF GRAVITY OF SOLIDS—*continued.*

| Form. | Volume. | Centre of Gravity. |
|---|---|---|
|   Ellipsoid. | $V=\frac{4}{3}\pi.a.b.c.$ | C.G. at intersection of axes. Semi-ellipsoid: C.G. at $\frac{4}{3}\frac{b}{\pi}$ along $b$ from base. |
|   Paraboloid. | $V=\frac{\pi.k^2.h}{2}$. | C.G. at $\frac{2}{3}h$ from vertex O. |

# CHAPTER V.

## DELINEATION AND DESCRIPTIVE GEOMETRY OF A SHIP'S FORM.

Before considering more fully how to make calculations relating to a ship's form, it is desirable to consider the geometry of a ship's form, or its delineation.

Suppose O X and O Y (fig. 22) to be two lines at right angles to each other in the plane of the paper, and a third line O Z at right angles to the plane of the paper. The points O, X, and Y lie in the plane of the paper; O, Y, and Z in a plane perpendicular to the paper which cuts the plane of it in O Y. Similarly the points O, Z, and X are in a plane perpendicular to the paper, which plane intersects it in O X. Hence the planes represented by X Y, Y Z, and Z X are at right angles to each other.

The intersections of any plane with the three planes of reference are called the traces of that plane.

Traces of planes parallel to the planes of reference:—

In fig. (i) P $a$ and P $c$ are the traces of a plane parallel to Y Z in the planes of X Y and Z X respectively. P $a$ is parallel to O Y and P $c$ is parallel to O Z.

In fig. (ii) the plane is parallel to X Z and the traces are Q $a$ and Q $b$ in the planes of X Y and Y Z respectively. Q $a$ is parallel to O X and Q $b$ is parallel to O Z.

Similarly in fig. (iii) R $b$ and R $c$ are the traces of a plane parallel to X Y.

Traces of a single cant plane:—If a plane is perpendicular to one of the planes of reference and inclined to the other two, then it is called a single cant plane, and it has a trace in each of the three planes of reference.

Suppose the plane $c$ P $a$ is revolved about the trace P $c$, then the trace P $a$ revolves round P to some other position, say P $a_{,}$. P $a_{,}$ and P $c$ are therefore the traces in the planes X Y and Z X respectively of a single cant plane perpendicular to X Y.

To find the trace in the plane Y Z we have to produce $a_{,}$ P to meet O Y produced in $c''$. The trace required will lie in Y Z, pass through $c''$ and be perpendicular to X Y. Hence it will be parallel to O Z.

Similarly in fig. (ii) Q $a_{,}$ and Q $b$ are the traces of a single cant plane perpendicular to X Y, but inclined to X Z and Y Z.

In fig. (iii) R $b$ and R $c_{,}$ are the traces of a single cant plane perpendicular to X Z, but inclined to X Y and Y Z.

A double cant plane is a plane inclined to all the three planes of reference.

If the single cant plane $c \, P \, a_{,}$ in fig. (i) be revolved through an angle less

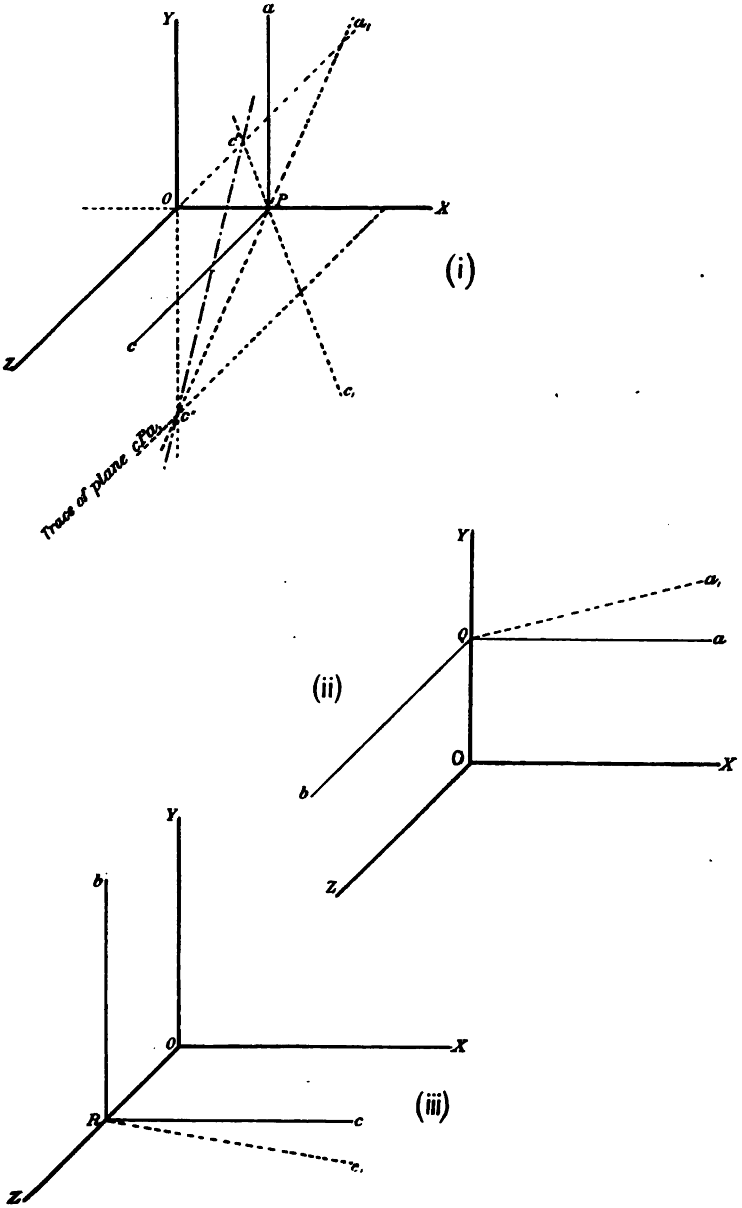



FIG. 22.

than a right angle about the trace $P \, a_{,}$, then it will be inclined to the plane X Y; so that it will now be inclined to all three—X Y, Y Z, and Z X. The trace in the plane X Z may be like that shown P $c_,$. P $a_,$ and P $c_,$ are therefore the traces of a double cant plane in the planes X Y and Z X respectively. To find the trace in the plane Y Z, produce $c_,$ P in the plane X Z to meet the axis O Z in $c'$. Produce $a_,$ P to meet the axis O Y in $c''$. Then $c'$ and $c''$ are two points in the plane $a_,$ P $c_,$ and also in the plane Y Z. Therefore the line $c'$ $c''$ is the intersection of the planes $a_,$ P $c_,$ and Y Z, and therefore $c'$ $c''$ is the trace of the double cant plane in the plane Y Z.

The following are some of the properties of the traces of a plane.

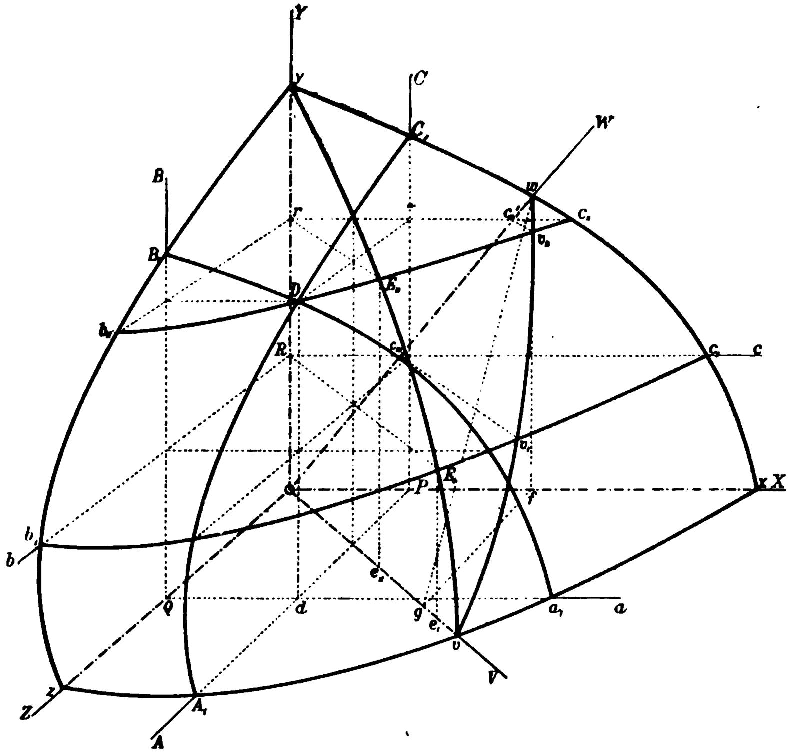



Fig. 23.

*Plane parallel to one of the planes of reference.*—The traces are parallel to the axes of reference.

*Single cant plane.*—One of the traces is parallel to one of the axes of reference, and the other two traces are inclined.

*Double cant plane.*—All three traces are inclined to the axes of reference.

Referring now to fig. 23, a plane parallel to Y Z at a distance O P from it will intersect X Y and X Z in the lines P C and P A respectively, and the new plane will be represented by A C. Similarly, planes parallel to Y X and X Z at distances O Q and O R respectively will be represented by B *a* and *c b* respectively. The lines of intersection of the plane parallel to Y X with the planes of reference are Q *a* and Q B, and the lines of intersection of the plane parallel to X Z with the planes of reference are R *c* and R *b*.

Suppose we have, as in the figure, a surface, part of which is in the angle formed by the three planes of reference X Y, Y Z, and Z X, *i.e.* in the first dihedral angle. The sections of the surface by these planes will be curves passing through the points *x* and *y*, *y* and *z*, *z* and *x*, respectively; *x*, *y*, and *z* being points on the surface and in the axes of reference O X, O Y, and O Z. The plane *b c* parallel to X Z will also cut the surface in some curve. Evidently *c* R will cut the curve *y x* in some point $c_{,}$ distant O R from the axis O X. Similarly, *b* R will cut the curve *y z* at some point $b_{,}$, distant O R from the axis O Z. We have now two points on the curve in which the plane *b c* cuts the surface.

In the same way we can get two points $C_1$ and $A_1$ in the curve in which the plane A C cuts the surface, and two points $B_1$ and $a_1$ in the curve in which B *a* cuts the surface.

Hence, in each of the curves *x y*, *y z*, and *z x* we have four points formed by the intersection of planes of reference and other known planes.

If we have the measurements or coordinates of the points $x\, a_1\, A_1\, z$ such that we can set off the positions of each of the four points in the curves, and if we have in addition the condition that the curve $x\, a_{,}\, A_{,}\, z$ must be a fair curve, we shall be able to set off the curve completely.

The form of a ship is generally a fair surface to its upper deck line. If we have a model of the surface and cut it by planes, we shall find the curves of section to be fair curves. In obtaining the form upon paper which we wish the ship to have, it is necessary to create the curves of section in the three planes of reference, and in doing this it is necessary to ensure that the curves are fair and consistent. The former is usually determined by the eye, the latter must be determined by measurement.

For instance, the curves $A_{,} C_{,}$ and $B_{,} a_{,}$ may each be quite fair, but unless the point D where they intersect is vertically over the intersection *d* of $A_{,}$ P and Q $a_{,}$ these curves will not be consistent with a fair surface.

The operation of delineating the curves of section of a ship's surface is known as laying-off. The two conditions of fairness and consistency have to be fulfilled, and almost the whole difficulty of laying-off a ship consists in, at the same time, fulfilling these two conditions. To make the curves consistent they may become unfair, and in making them fair they may become inconsistent. Only by practice and judgment can proficiency be gained in this subject.

There are a few problems in the geometry of surfaces which are of interest and value in so far as they give facility in appreciating and representing surfaces by means of plane curves on planes of reference.

We may consider a few here.

Suppose that we have three curves of section made by horizontal planes. If we put these curves upon one plane with O X and O Z as axes, we shall have them as in the figure 24. These curves now occupy the position on the plane X Z which they would have, had they been dropped vertically from their actual position on this plane. This operation is known as projecting the curves upon the plane X Z.

Referring to fig. 23, suppose we cut the surface by a vertical plane through O Y and O V. This plane will cut the surface in a curve $y\, E_{,,}\, E_{,}\, v$. The form of this curve may be obtained by drawing (fig. 24) a line O V occupying the same actual position in this figure that it does in fig. 23, the axes O X and O Z being drawn at right angles as they actually are. Any one of the horizontal planes such as $b_{,,}\, c_{,,}$ will cut the plane Y V in a line $r\, E_{,,}$ parallel to O V, and the distance that the point of intersection of this line and the surface is from $r$ fig. 23 is equal to the distance $O e_{\prime\prime}$ in fig. 24. If we set off in fig. 24 $O e_{\prime\prime} = r E_{\prime\prime}$ in fig. 23 we shall get $e_{\prime\prime}$ a point on the curve of section of the plane Y V and the surface. Similarly, the points $e_{\prime}$ and $v$ can be found, and the exact position of the curve of section $y v$ in fig. 23 can be

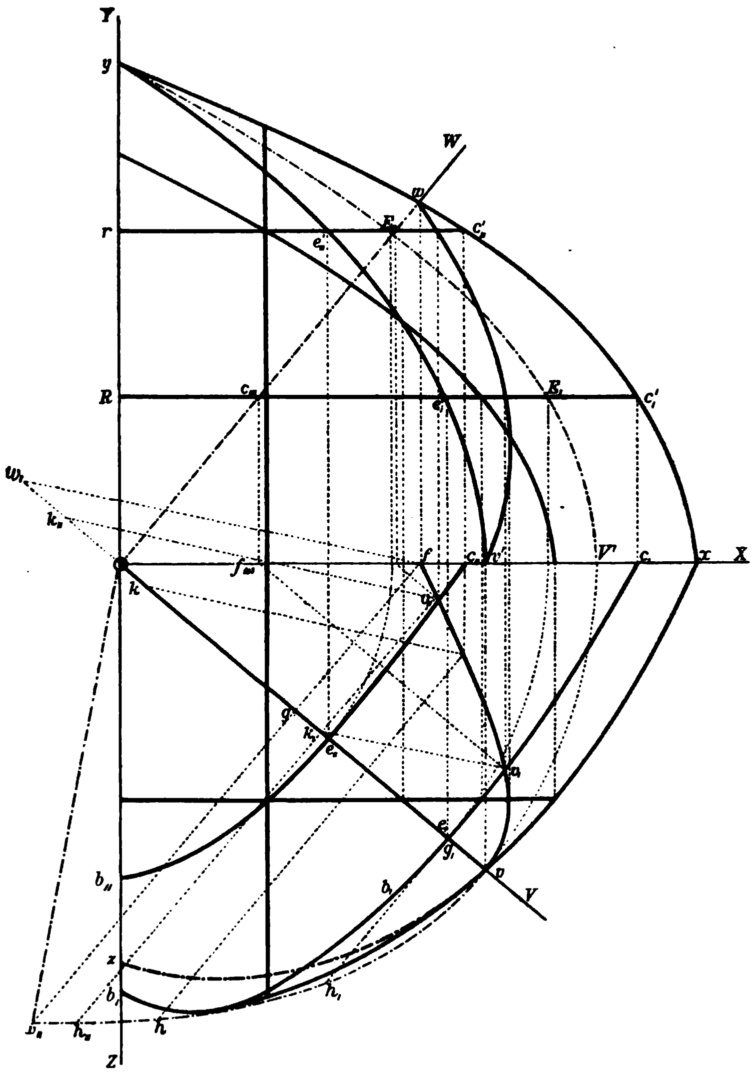



FIG. 24.

determined in fig. 24. To find the true form of this curve, set up in fig. 24 O Y perpendicular to O X and set off O R, O $r$, and O $y$ the same lengths as in fig. 23. Draw through R and $r$ horizontal lines, and from R and $r$ respectively set off $R E_{\prime}$ and $r E_{\prime\prime}$ equal to $o e_{\prime}$ and $o e_{\prime\prime}$, also set off O V′ equal to

O *v*. Then $y E_{\prime\prime} E_{\prime} V'$ will be the true curve of section. In shipbuilding operations this is called a single cant section. The curve $y e'_{\prime\prime} e'_{\prime} v'$ is its projection in the plane Y X.

A double cant section with the surface is one such as would be made by the plane passing through O V and O W, fig. 23. A point on the surface can be determined for each horizontal plane as follows. At the point $c_{\prime\prime\prime}$ fig. 23, where O W cuts R *c* draw $c_{\prime\prime\prime} v_{\prime}$ parallel to O *v* cutting the horizontal curve of section $b_{\prime} E_{\prime} v_{\prime} c_{\prime}$ in $v_{\prime}$, then $v_{\prime}$ is a point on the curve of section of the plane V W with the surface. *v* and *w* are two other points, and the whole curve of section is $w v_{\prime} v$. To get this curve accurately, more horizontal plane sections should have been drawn, but to avoid making the figure more complicated the method of obtaining only one point has been shown.

To find the true form of the curve of section made by a double cant plane with the surface, it is necessary to turn the plane V W, with all that is represented in it, about either O V or O W, say O V, fig. 23. Every point in this plane will move perpendicularly to O V. For instance, the point *w* fig. 23 is in the triangle *w f g* and is distant *g w* from *g*. If in fig. 24 we draw *w f* perpendicular to O X and *f g* perpendicular to O V, when we turn the plane V W about O V the point *w* will be somewhere in the line *f g* produced at a distance equal to the true length of the line *g w*. To find this true length it is necessary to construct the triangle *w f g* in its true form. The true lengths of two sides are given in fig. 24. They are *f g* and *f w*, and as these two are at right angles to each other in their actual position we can construct *g w*, which will be the hypotenuse of the triangle of which *f g* and *f w* are the other two sides. To do this, set off $g w_{\prime}$, equal to *w f* (fig. 24) and join $w_{\prime} f$. Then $w_{\prime} f$ is the true length of *g w* fig. 23. Set off $g w_{\prime\prime}$ equal to $f w_{\prime}$ and we have a point on the true curve of double cant section.

To find the other points on the curve a sufficient number of horizontal sections must be obtained. Take the plane *b c* fig. 23 as an instance: $c_{\prime\prime\prime}$ is the point in R *c* and O W, and is therefore in the line in which these two planes intersect. The line $c_{\prime\prime\prime} v_{\prime}$ is parallel to O V. Through $c_{\prime\prime\prime}$ (fig. 24) draw $c_{\prime\prime\prime} f_{\prime\prime\prime}$ perpendicular to O X and $f_{\prime\prime\prime} v_{\prime}$ parallel to O V cutting the curve $b_{\prime} c_{\prime}$ in $v_{\prime}$. $v_{\prime}$ is the projection in the plane Z O X of a point on the curve of the section required. To get its position in the true curve, draw through $v_{\prime}$, $v h_{\prime}$ perpendicular to O V cutting it in $g_{\prime}$. Set off $g_{\prime} k_{\prime}$ equal to O R and join $k_{\prime} v_{\prime}$. Set off $g_{\prime} h_{\prime}$ equal to $v_{\prime} k_{\prime}$. $h_{\prime}$ is the position in the true curve of the point in the double cant section which is in the plane *b c*. Finding other points similarly we get $w_{\prime\prime} h_{\prime\prime} h_{\prime} v$, the true form of the double cant section.

If the reader will follow these two diagrams carefully through he will be able to determine the form of section made by any plane which can cut a ship. We commenced with planes parallel to planes of reference. These are similar to waterlines, transverse sections, and bow or buttock planes, which will be described in the following chapter. We next considered a plane cutting one plane of reference at right angles, but oblique to the two others. This is the single cant plane. We next considered a plane oblique to all three planes of reference. This is a double cant plane.

These three cases include all possible planes of section.

# CHAPTER VI.

## DESCRIPTION AND INSTANCES OF SHIPS' FORMS.

Having devoted the foregoing chapters to considerations of buoyancy, centres of gravity and buoyancy, and moments of inertia of floating bodies generally, it is necessary before going further to describe and give instances of ship forms.

It is usual to define a form by giving three series of sections of the form made by planes parallel respectively to three planes at right angles to each other, called planes of reference. In a ship form as delineated in practice, the three planes are (1) the vertical transverse plane at the middle of the length of the ship; (2) the vertical longitudinal middle line plane; (3) the horizontal plane through the top of the keel amidships.

As the form of the ship is symmetrical about the vertical longitudinal plane, it is fully delineated for most purposes when we know the form of one side, and it is usual to deal only with curves showing one side. These curves of form are shown on three separate diagrams similar to those shown in Plate I. The body plan gives the sections with the ship's form made by planes referred to above under heading (1); the elevation, or sheer plan, those by planes referred to under heading (2); and the half-breadth plan those by planes under heading (3).

The horizontal plane through the top of keel amidships is taken parallel to the water surface when the ship is floating at her deepest draught, or sometimes her designed draught and trim. The plane of the water surface when the vessel is at her deepest draught is called the load water plane. Generally in mercantile vessels the top of the keel is parallel to the load water plane. In warships and yachts this is not generally so.

Fig. 25 shows a skeleton of the drawing in Plate I. The full curved lines represent the largest sections in each individual plan. The *dotted* lines represent one section in each plan, in each body. It is usual to speak of the two portions of the ship divided by the largest transverse section as the fore body and after body respectively. A B, A′ B′, and A″ B″ are the lines in the body, sheer, and half-breadth plans respectively corresponding to the section made by a transverse vertical plane in the fore body; *a b*, *a′ b′*, and *a″ b″* are similar lines in the after body. C D, C′ D′, and C″ D″ are the lines in the three plans in the fore body made by a longitudinal vertical plane parallel to the middle line plane; *c d*, *c′ d′*, and *c″ d″* are similar lines in the after body. E F, E′ F′, and E″ F″ are lines in the three plans in the fore body made by a horizontal plane; *e f*, *e′ f′*, and *e″ f″* are similar lines in the after body.

It will be observed that the lines made by the intersection of these planes are straight in two plans and curved only in one.[1] The form of the side of the ship at its highest part is not a plane curve, and is therefore represented by curves in each plane. These curves are the projections in the respective planes of the actual curve.

For instance, the transverse vertical planes make curved lines A B and *a b* in the body plan only. These are usually called sections. The longitudinal vertical plane makes curved lines *c′ d′*, D′ C′ in the sheer plan only. These are called buttock lines in the after body, and bow lines in the fore body. The horizontal plane makes curved lines *e″ f‴* F″ E″ only in the half-breadth plan. These are called water-lines. It is a series of such lines which makes up the full delineation shown in Plate I. This representation, by showing the curves in three plans, gives us sufficient knowledge of the form of the ship.

In Chapter V. it was shown how the intersection of a double cant plane with any curved surface could be drawn in plan and elevation,

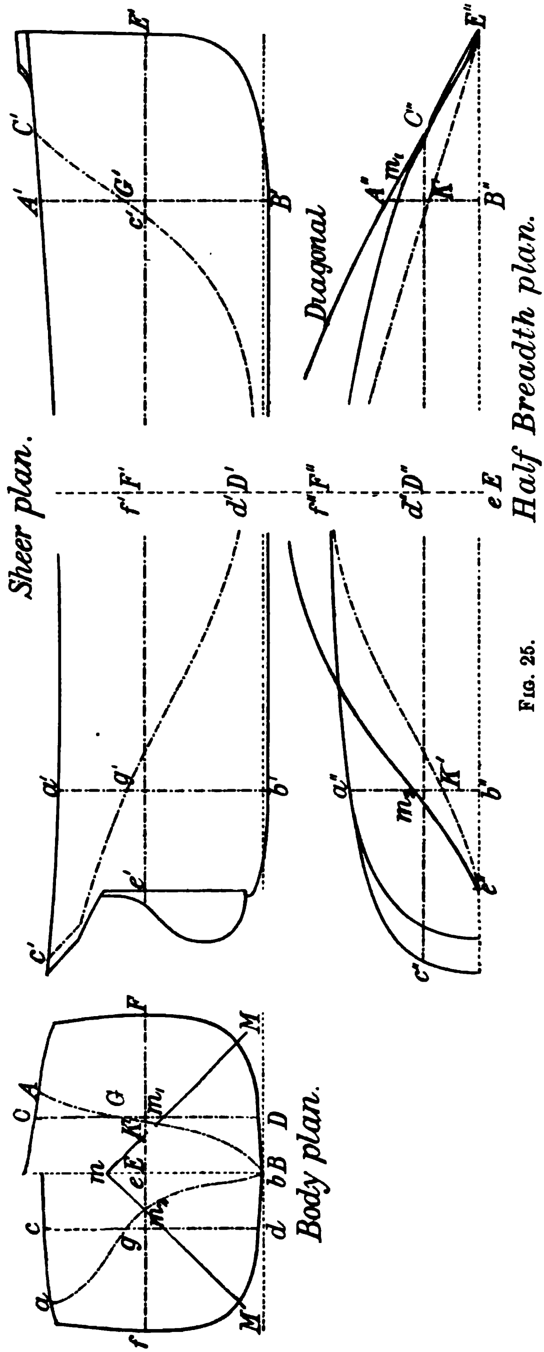



Fig. 25.

[1] Each of these measurements can be shown in two of the three planes. For instance, B′ G′ and D G are the same, B″ K′ and E K are the same.

also how its true shape could be found. We will now proceed to find the above curves for a ship form.

A single cant plane is one which is inclined to two planes of reference and perpendicular to the third. A plane such as is shown by M *m* in fig. 25, inclined to the plane of the waterlines and to the vertical middle line plane, but perpendicular to the plane of the sections, is a single cant plane and is called a diagonal.

In the body plan a diagonal can be represented by a straight line, such as M *m*. Generally the true form of the diagonal is drawn on the half-breadth plan, being obtained by rabatting the plane containing the diagonal about the line through *m* into the horizontal plane. It will be seen, if reference is made to a body plan, that a diagonal can be drawn which will be nearly square to most of the sections. For this reason the diagonal is useful in fairing up the lines. The method of drawing the diagonal is to measure the intersections such as $m m_1$, *m* M, $m m_2$, on the body plan and lay those distances off from the middle line on the half-breadth plan.

The intersection of a double cant plane with a ship's form is sometimes required. We can imagine a double cant plane by rabatting a transverse plane about the vertical middle line and then rabatting it about a horizontal line.

Fig. 26 shows in elevation and plan the intersection of a double cant plane with the surface of the vessel. The data here are the waterlines K, L, M, etc., shown in elevation, and their intersections *k*, *l*, *m*, etc., with the ship form shown on the plan; also the vertical trace O A and the horizontal trace O B of the double cant plane A B.

To get the points E and *e* in elevation and plan respectively where the plane A B cuts the waterline K *k* we proceed as follows:—

The water plane K cuts the trace O A in C on the elevation, and, therefore, at *c* on the plan. K will cut the plane A B in a line parallel to its horizontal trace O B, therefore we draw *c e* parallel to O B, and its intersection with *k* will give the point *e*, which is one of the points required. The other point E is got by drawing *e* E vertically to cut K in E.

The points F, G, H, etc., and *f*, *g*, *h*, etc., are got in the same manner, and thus the curves O E and O *e* can be drawn.

To get the true shape of the curve we proceed as follows:—

From E draw E N perpendicular to O A. Draw E P perpendicular to E N and equal in length to *e p* in plan. Then N P will be the perpendicular distance from E to O A. If now we swing the plane A B about O A into the vertical plane and draw N J at right angles to O A and of length equal to N P, we get at J the relation which the point E actually bears to the line O A and to the point O.

In a similar manner the points G, R, S, etc. can be got and the curve J O drawn. This curve is the true shape of the intersection of the double cant frame with the ship form.

A shorter method of construction can be used to find such points as J, G, R, S, etc.

We want to find the true length of N E, the side of the triangle which in elevation is E N C. Therefore with C as centre set off C J = *e c* so as to cut E N produced in J. J is the point on the line giving the true form.

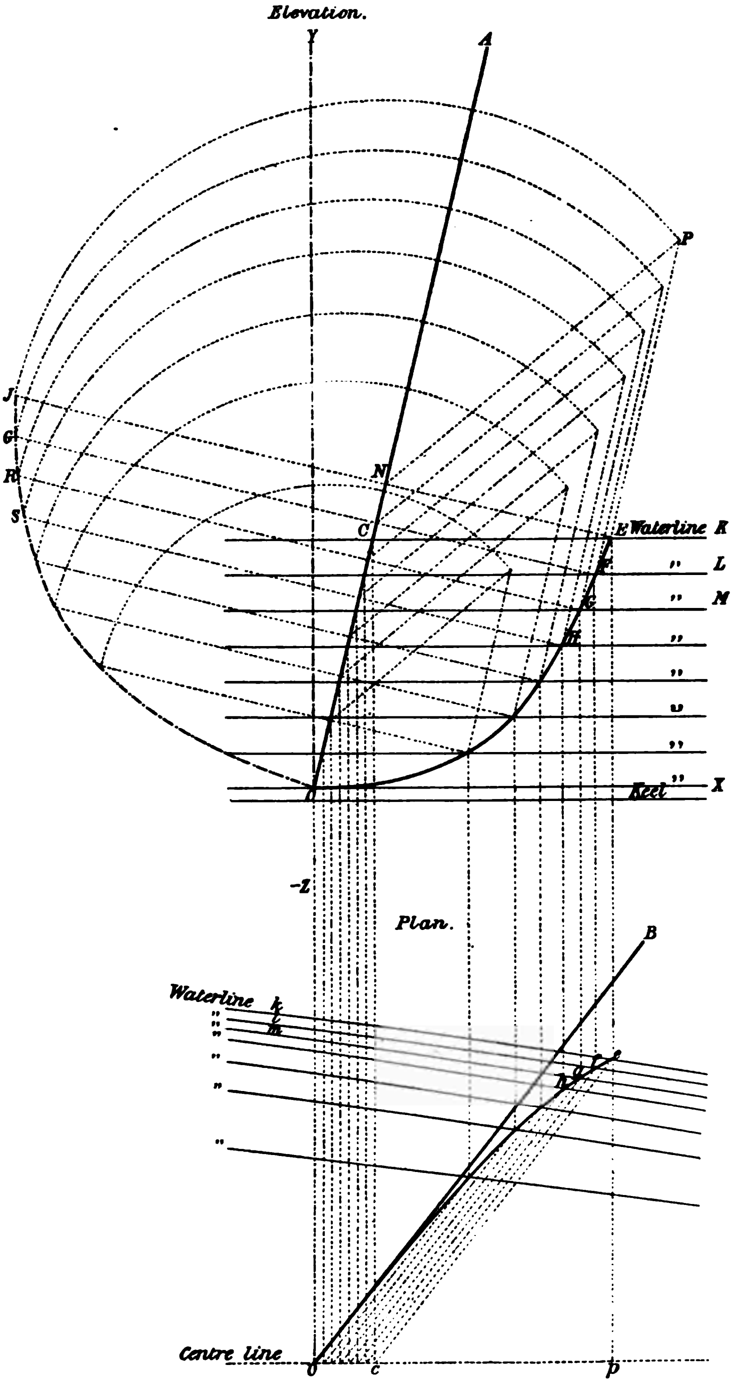

Elevation.
Y
A
P
J
G
R
S
N
C
E Waterline K
F ,, L
G ,, M
H ,,
,,
,,
,,
Keel ,, X
O
-Z
Plan.
B
Waterline k
l
m
e
f
g
h
Centre line
o
c
p

FIG. 26.

## DIMENSIONS.

In taking account of dimensions the following measurements are taken, viz. lengths, breadths, and depths, or heights.

*Lengths* are those measurements which are taken parallel to the middle longitudinal plane and to the horizontal planes.

*Breadths* are those measurements which are taken parallel to the transverse and to the horizontal water planes.

*Depths* or *Heights* are those measurements which are taken parallel to the transverse and to the middle longitudinal planes.

*The draught* of a point is the vertical depth of that point below the water surface at which the ship is floating.

It is in accordance with these directions of measurement that the dimensions of a vessel are fixed. There are three sets of dimensions:—

(1) Moulded dimensions.
(2) Overall dimensions of the whole body.
(3) Dimensions for classification rules.

(1) **Moulded dimensions.**—In all calculations dealing with the under-water portion of the body, usually taken as that part of the body below the load water plane, the moulded dimensions have to be used.

The form of the vessel is the outside surface which lies fair over the outside of the frames, *i.e.* the surface to which the frames are moulded.

*Length.*—The length for displacement (or other calculations) is taken between two perpendiculars, and is called the length between perpendiculars, or the L.B.P. The perpendiculars are chosen thus: The after perpendicular is taken at the after side of the after sternpost in single-screw steamers having an aperture for the screw, and the after side of sternpost if it is vertical in a twin-screw steamer with a small or no aperture. The forward perpendicular is taken where the fore side of stem cuts the load waterline in steamers with a straight stem.

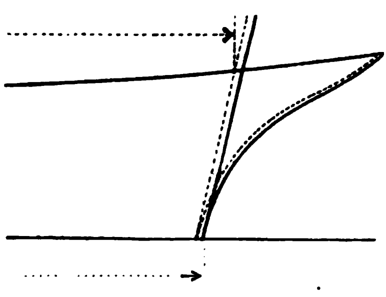


Fig. 27.

*Breadth.*—The moulded breadth is the greatest breadth of the midship section of the form which is stated above as being the outside surface over the frames.

*Draught.*—The moulded draught is the draught of the top of keel amidships. Most vessels have their load line parallel to the keel, so that the draught of any point on the top of keel is the same; but in vessels with a droop keel this is not so, in which case the draught dimension is fixed from a base line at a point on the top of keel amidships.

*Depth.*—The moulded depth is the depth from the uppermost complete deck taken from the top of beam at side at the middle of the length between perpendiculars, to the top of keel.

(2) **Overall dimensions.**—Overall length is the length between the extreme points at the bow and stern.

Overall breadth is the breadth at the widest part of the widest section in the ship. This is usually spoken of as extreme breadth. The term overall depth is seldom, if ever, used.

Extreme draught is the draught of the lowest point of the keel.

The overall dimensions are not of much use in giving correctly the size of the ship; for instance, consider the overall lengths of two vessels of the same breadth and depth, one of which has a very large overhang at each end, and the other has a vertical stem and only a small overhang at the stern. These vessels might have equal displacements and yet their overall lengths might be very different. So with the other overall dimensions.

**(3) Dimensions for classification rules.**—When fixing numerals for classification purposes, other dimensions are taken which afford a measurement for the body, and consequently for the strength of the different parts. The subject of classification will be treated later, but just now we are concerned with the "dimensions." In Lloyd's book of Rules for Scantlings they are defined thus:—

*Length* to be measured from the after part of the stem to the fore part of the sternpost on the range of the upper deck beams in one, two, and three-decked and spar-decked vessels, but on the range of main-deck beams in awning-decked vessels.

In vessels[1] where the stem forms a cutwater, the length to be measured from the place where the upper deck beam line would intersect the after edge of stem if it were produced in the same direction as the part below the cutwater. See fig. 27.

*Breadth.*—The breadth is in all cases to be the greatest moulded breadth of vessel.

*Depth.*—The depth in one- and two-decked vessels is to be taken from the upper part of the keel to the top of the upper-deck beam at the middle of the length, assuming a normal round-up of beam of one quarter of an inch to a foot of breadth. In spar-decked and awning-decked vessels the depth is to be taken from the upper part of the keel to the top of the main-deck beam at the middle of the length, with the above normal round-up of beam. Dimensions for tonnage and the ship's registry are described in Chapter XV.

**Spacing of waterlines and sections.**—It is usual to space the waterline planes a uniform distance apart, except in the region near the keel, where the spacing is sometimes halved.

This close spacing is required for accuracy in using Simpson's Rules wherever the form changes rapidly, as it does frequently near the keel. The sections are also uniformly spaced, except near the ends, where the form changes more rapidly than elsewhere, and in this case it is usual to halve the spacing also.

The advantage of halving the spacing is apparent when we consider the calculation of volume by Simpson's Rules. If we have a series of equally-spaced ordinates of a curve, the multipliers will be 1, 4, 2, 4, 2, - - 4, 1, the whole being summed up and multiplied by one-third of the interval. If the interval be halved, we can obtain the area by using the same multipliers and, after summation, multiplying by one-third of half the former interval. If, instead, we use the multipliers divided by two, such as $\frac{1}{2}$, $\frac{4}{2}$, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{2}{2}$, - - $\frac{4}{2}$, $\frac{1}{2}$, or $\frac{1}{2}$, 2, 1, 2, 1, - - 2, $\frac{1}{2}$, we can, after summation, multiply by two-thirds of the original interval. Hence, if we have a series of ordinates, some

[1] These different classes of vessels are described in a later chapter.

4

of which are spaced a whole interval and some a half interval apart, we may add the products of the multipliers by either 1, 4, 2, 4, - - - 1, or by $\frac{1}{2}$, 2, 1, 2, - - $\frac{1}{2}$, according as the ordinates are half spacing or whole spacing, and multiply the sum by one-third of the interval.

Suppose fig. 28 represents such a curve. From A to B the ordinates are spaced $k$ apart.

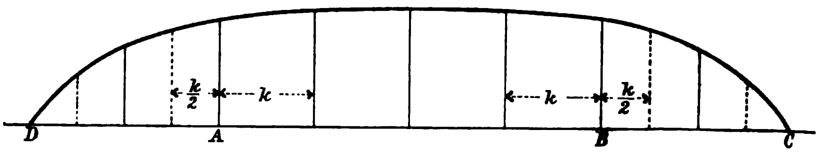



FIG. 28.

From D to A and from B to C they are spaced $\frac{k}{2}$ apart. The multipliers will be

$$\begin{array}{l} \frac{1}{2},\ 2,\ 1,\ 2,\ \frac{1}{2} \qquad\qquad\qquad \frac{1}{2},\ 2,\ 1,\ 2,\ \frac{1}{2} \\ \qquad\qquad\quad 1,\ 4,\ 2,\ 4,\ 1, \\ \hline \frac{1}{2},\ 2,\ 1,\ 2,\ 1\frac{1}{2},\ 4,\ 2,\ 4,\ 1\frac{1}{2},\ 2,\ 1,\ 2,\ \frac{1}{2}. \end{array}$$

The products of these multipliers and the ordinates are called the functions of the ordinates, and the sum of the function multiplied by $\frac{k}{3}$ gives the area.

Similarly, the multipliers for such a curve as in fig. 29 would be

$$\begin{array}{lccccccc} & 1 & 4 & 2 & 4 & 1 & & \\ \text{and} & & & & & \frac{1}{2} & 2 & \frac{1}{2} \\ \hline \text{or} & 1 & 4 & 2 & 4 & 1\frac{1}{2} & 2 & \frac{1}{2}. \end{array}$$

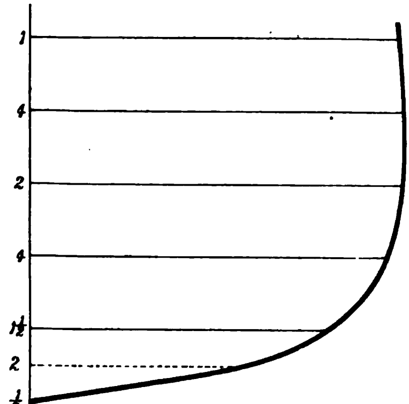



FIG. 29.

Referring again to fig. 25, the buttock lines or the lines such as $cd$ and CD are usually spaced uniformly, the spacing being a proportion of the extreme half breadth of the vessel, but as no use is made of these for calculation, no half-spaced buttocks or ordinates are necessary.

# CHAPTER VII.

## DESCRIPTION OF TYPES OF SHIPS.

In the foregoing chapters the questions of buoyancy, displacement, and form have been treated.

These questions we have seen are connected with the finished ship, and have been treated in such a way that we can now enter upon a fuller and more detailed examination of the various problems that arise in the process of designing and building a ship; but in order to understand how these problems affect the designer and the builder, it will be better at this stage to first describe the different types of ships in existence, and to review briefly the main points to be considered before the ship can be considered to be ready for its first trial or sea-voyage.

A discussion of the main features found in existing types of ships will be sufficient to enable them to be classified and compared so that the reader may have a general idea of the work with which the naval architect has to deal. Ships whose types are dying out and of which no new ones are being built need not be considered, as they have probably become obsolete. At present ships may be grouped in the following manner according to the purposes for which they are built, but each of these groups, as we shall see, contains a variety of types.

1. Purposes of war.
2. Passenger and cargo-carrying.
3. Miscellaneous.

The last class embraces a variety of ships built for special purposes, such as fishing, dredging, life-saving, towing, oil-carrying, etc., ships for river traffic, etc. For each of these purposes the vessel is specially built, and each design embodies its own special purpose, but must also embody the general principles connected with the design and building of every vessel. We shall therefore examine generally only the first two classes.

## (1) SHIPS FOR PURPOSES OF WAR.

These ships form the navies of the countries, and are built to the orders of the government. The object of a navy is to be able to capture or destroy the ships of an enemy, and, incidentally, to be capable of protecting the commerce of the country, and also of protecting the country from invasion by an armed force carried over sea. At the present time we find the following classes in countries which carry on an extensive commerce:—

i. Battleships.
ii. Cruisers and Gunboats.
iii. Torpedo Boats and Torpedo-boat Destroyers.
iv. Submarines.

These classes have different functions in time of war. The **battleship** (fig. 30) is essentially a ship which must be able to fight in a line of battle; it must also be able to keep the seas in all weathers; it must be constructed

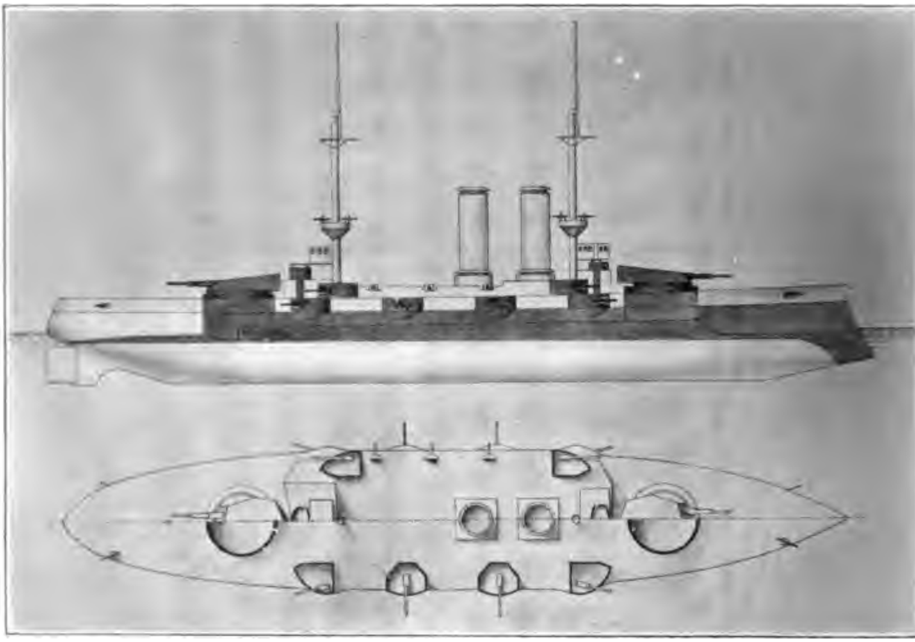


Fig. 30.—Battleship.

so as to resist shot and shell attack, and to localise as much as possible any damage that may happen to be caused by them, and it must have good speed and manœuvring powers. In order to fulfil these functions a battleship must be armed with powerful guns so as to inflict damage on the enemy's ships; she must be well armoured to resist penetration from the enemy's shot and shell; the hull must be well subdivided, so as to localise damage and, if possible, prevent her sinking; the vessel must be so designed that the battleship can keep the seas in all weathers, and that the quarters for the men are habitable; and she must carry sufficient coal and stores to remain at sea for a certain length of time. Battleships are, in consequence, the heaviest ships in a navy, the displacement of the largest being about 18,000 tons.

In some navies there are also vessels of much smaller displacement, called monitors (fig. 31), which are classed as battleships. Their displacement is from 2000 to 4000 tons. They are built for coast defence only, and the features in which they chiefly differ from battleships are their slow speed and low freeboard. Their small displacement is a consequence of this low freeboard, and also of their low speed and small coal and store supply. These detract from their habitability and sea-keeping qualities, and in consequence they are not able to act far away from a base. They possess heavy guns and are powerfully armoured, but in the event of penetration, subdivision of the hull does not prolong their life to any great extent, as their freeboard is so small. Their best chance lies in the small mark they present.

In considering the qualities of a sea-going battleship, we may first consider the protection of the hull. In all modern battleships the vitals (engines, boilers, magazines, etc.) are under a protective deck of steel. This

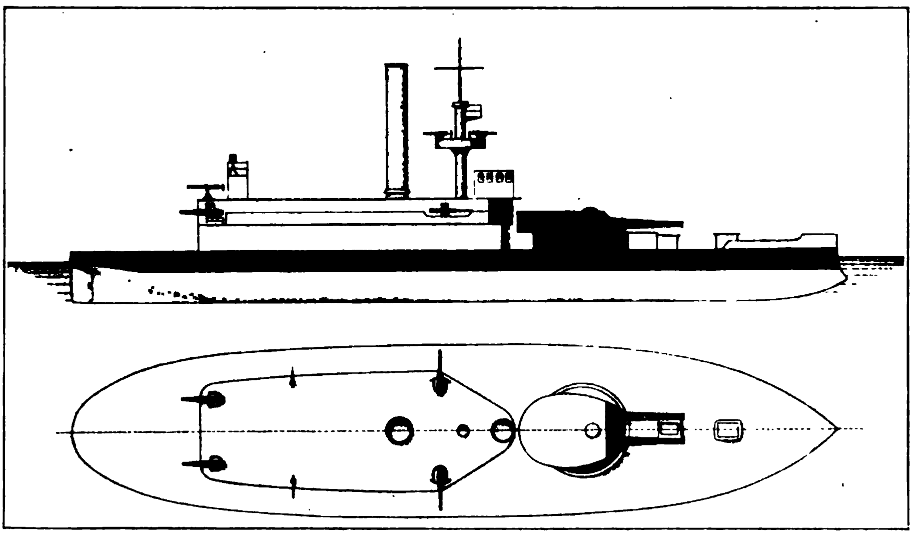


FIG. 31.—Monitor.

deck is about 3 feet higher than the load water plane at the middle line, but sometimes slopes at the fore and after ends to from 5 to 10 feet below the waterline. This deck also slopes to the ship's side at an angle of about 40° to the horizontal, meeting the ship's side at about 5 to 6 feet below the load water plane. The protective deck is formed of two or three thicknesses of steel, amounting to 2 in. or 3 in. in all on the flat part, and the sloped parts at the side are from 2 in. to 4 in. thick. Above the protective deck is the main deck, which in recent ships is made of plating 1½ in. thick. In these two decks all the openings, ventilators, uptakes, ladderways to engines and boilers are covered with armour hatches or gratings, and all the hatchway covers which can be made solid are made of the same thickness of plates as the decks, so that there is no part in these decks where a shot or a splinter can enter the vitals unless it passes through the thick plating of these decks.

The main protection to the hull, however, is given by a belt of armour.

This belt runs along the ship's side for a breadth of about 15 feet in the vicinity of the waterline, either for the greater part or for the whole of the vessel's length. The maximum thickness varies in battleships from 7 in. to 12 in., and is maintained over the length of the engine and boiler space. Towards the end it tapers off to 2 in. or 4 in. In some battleships this belt is the same thickness, 7 in., from the protective deck, where it slopes to the lower edge of the armour, to the main deck, but in some cases the maximum thickness of 12 in. is in the vicinity of the waterline. For protection against ahead or astern fire, transverse or oblique armour bulkheads, usually thinner than the side-belt, are fitted between the protective and main decks, one forward of the boiler space and one at the after end of the engine space.

Battleships are all constructed on the double-bottom principle for about two-thirds of the length of the ship. The combination of longitudinal and transverse framing is utilised to divide the space between the outer and inner bottom plating into a number of watertight compartments. All the flats of storerooms and magazines are made watertight, so also are all the bulkheads. The doors in these bulkheads are also watertight, and can be closed by gearing leading from the main or upper deck. Further, the bulkheads to each compartment are made strong enough to withstand the pressure of water in the event of the compartment filling. The coal bunkers are ranged alongside the engines and boilers below the protective deck and abreast the boiler and machinery casings, between the main and protective decks. This arrangement of bunkers and subdivision of hull gives an excellent protection, as in the event of any damage under water only the damaged compartment is filled, so that before the ship can be sunk the damage inflicted must be such as to flood as many compartments as would either bring the openings in the upper works under water, or would cause the vessel to capsize.

The question of guns and gun protection is of the next importance.

The guns of a warship are divided into two batteries, the main and secondary. The main battery includes all the heavy guns, usually 12-inch to 9-inch. These guns are protected by armour and worked by machinery. The secondary battery comprises all the lighter guns which are distributed over the upper works and have less or even no protection; these guns and their ammunition can be worked by hand.

*Main Battery.*—Modern battleships carry four large guns, 10-inch to 12-inch B.L.R.[1] The 10-inch to 12-inch guns are mounted in pairs in two revolving turrets, one on the forecastle deck and one on the quarter-deck. These turrets have their bases surrounded by armoured barbettes, which protect the men and mechanism for loading the gun and working the ammunition. The turrets can swing through an angle of 120° on each side of the middle line of the ship, and in this range the decks are clear of erections. The armour of the turret is from 10 in. to 15 in thick at the sides, and the top covering plate is 3 in. or 4 in. The barbette is armoured from the protective deck to the gun turret, in some ships, with 10-inch to 15-inch armour. In others the barbette armour only extends from the gun platform to the turret. The position of the turrets is usually near the transverse armour bulkheads, so that, in some cases, this armour is worked diagonally across the ship to meet the barbette. When the barbette armour extends only from the main deck to the turret an armour tube extends from the centre of gun platform to the protective deck to afford

[1] Breech-loading rifled.

protection to the gear for working the guns and hoisting the ammunition. If the battleship is fitted with other guns heavier than 6-inch, *e.g.* 8-inch to 10-inch guns, these are also placed in turrets worked on the same principle as the large guns, sometimes with only one gun in each turret. One method of arranging these guns is to place a pair in a turret directly above the 12-inch turret, thus having two 12-inch and two 8-inch over one barbette.[1] Otherwise the 8-inch guns are placed in pairs in turrets firing broadside and ahead or astern. The modern 6-inch or 7·5-inch guns are quick-firing, and are commonly arranged in broadside on the main decks in what is called a citadel, and a few on the upper deck. There are two distinct methods adopted to protect these guns—the casemate system and the side-belt protection. The casemate system was largely adopted in the British Navy. Each gun is enclosed in a casemate which at the ship's side forms a sponson to give the gun a large angle of train, about 140°. Usually four casemates and, consequently, four 6-inch guns can be ranged on each side on the main deck. The guns in the forward and after casemates can fire directly ahead and directly astern respectively. On the upper deck there are usually two or three casemates on each side. Consequently a battleship with the 6-inch guns in casemates has seldom more than fourteen of these guns. The armour of the casemates is 6 in. thick on the outside and 2 in. on the rear walls. This protection is considered sufficient against the fire of 6-inch quick-firing guns.

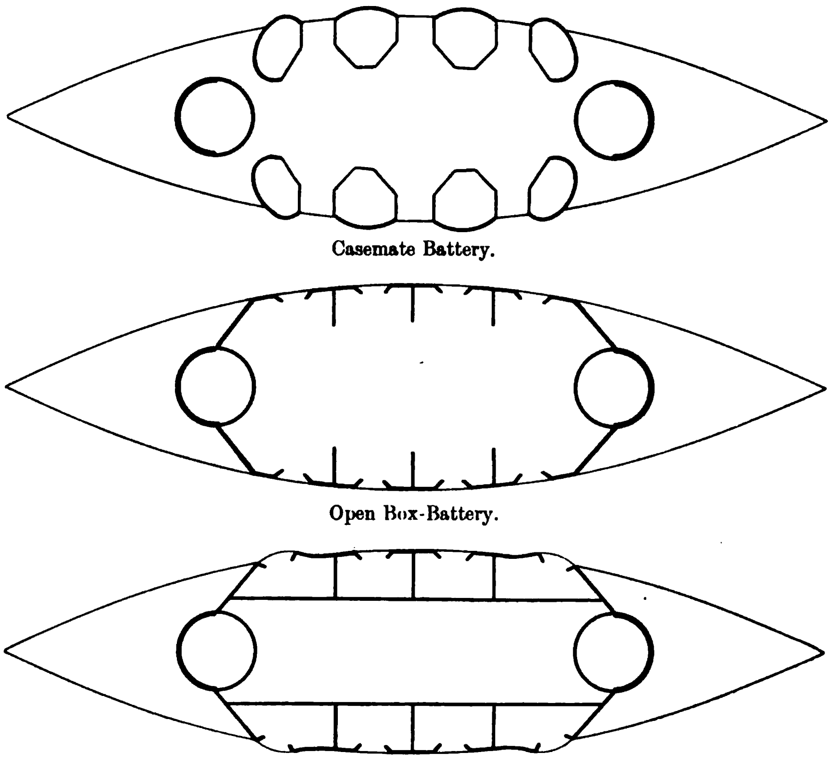
The other method of protecting the 6-inch guns is by a continuous side-belt of armour, 5 in. to 7 in. in thickness, on the side; screen bulkheads of 2 in. being erected between each gun, and at the ends of the citadel transverse armoured bulkheads are fitted between the main and upper decks. This is called the box-battery arrangement. As many as seven guns on each broadside have been mounted by this method. The main disadvantage in this system is that the guns have not such a large angle of train as the guns in casemates, 110° as against 140°, but the gun mechanism and gun crew are well protected from damage from gun-fire in the rear, and they have the advantage that the guns and gun crews on one side are all under the supervision of the gunnery officers.

A system which still further reduces the chance of damage in rear has been adopted on some battleships. This is the box-battery arrangement, in which a screen bulkhead of 1 in. to 2 in. in thickness is worked fore and aft in rear of the guns, and this, with the screen bulkheads between each gun, completely isolates each gun and gun crew. The tactical advantage of control which the open box-battery gives is, however, sacrificed in this arrangement. These three systems are shown in diagrammatic form in fig. 32.

The ammunition for the 6-inch guns is brought up through armoured tubes by hoists, but can be placed into the guns by the crew; whereas in the large guns, 12-inch to 8-inch, the gun is loaded and the hoists are worked by special machinery. A point in this connection in the design of modern battleships and also of cruisers is the ammunition passage, which is constructed immediately under the protective deck. This passage is on each side of the ship, and connects two transverse ammunition lobbies at each end of the engine and boiler space. From these lobbies lead the hatchways to all the magazines (except for the large guns and the small arms), from which the ammunition can be brought up and conveyed along the passages, where it is laid in racks ready to be hoisted up the armoured tubes to the 6-inch or other secondary guns.

[1] Usually called a "superimposed turret."

The other important considerations in a battleship are the speed and the coal supply. A speed of 21 knots has been given to our latest battleships. The maximum coal supply seldom exceeds 2000 tons, and this is considered sufficient supply for a steaming distance of about 7000 nautical miles at the rate of 10 knots. The coal supply at the normal or designed load draft is usually nominally 900 tons, though in most British ships it actually exceeds this amount by 200 tons or more.



Casemate Battery.

Open Box-Battery.

Subdivided Box-Battery.

FIG. 32.

**Cruisers.**—Ships of the cruiser class are built to fulfil two functions:—(1) scouting; (2) protection of the routes of commerce. When they are associated with a fleet of battleships their duty is to act as scouts and give intelligence of the approach of the enemy's ships, or prevent the enemy's patrols from obtaining information of our own fleet's position and strength. In protecting the routes of commerce, a cruiser may have to catch and fight other cruisers. Accordingly, to fulfil all these functions, a cruiser requires a greater speed than the battleship, which can only be obtained by sacrificing other qualities, but she must also have good fighting and good sea-going qualities. Some cruisers are armoured, and these will be considered first as they are the most important vessels in the cruiser class.

**Armoured Cruisers** (fig. 33).—The addition of the protection of an armour belt to the hull constitutes the difference between an armoured and what is known as a protected cruiser of the first class. Some armoured cruisers are practically battleships of two or three knots more speed, but with the heaviest guns reduced in number, calibre, and protection. An armoured cruiser, therefore, possesses to a certain extent the same qualities as a battleship, and might rather be termed a "swift battleship," because the same features as the battleship exist in her, a compromise being made to obtain more speed by decreasing the weight of armour and guns. In an armoured cruiser the same principles as exist in the battleship regarding protection of the hull govern the design. There is the double bottom, protective deck, subdivision of the hull, and arrangement of coal bunkers at the side. The largest

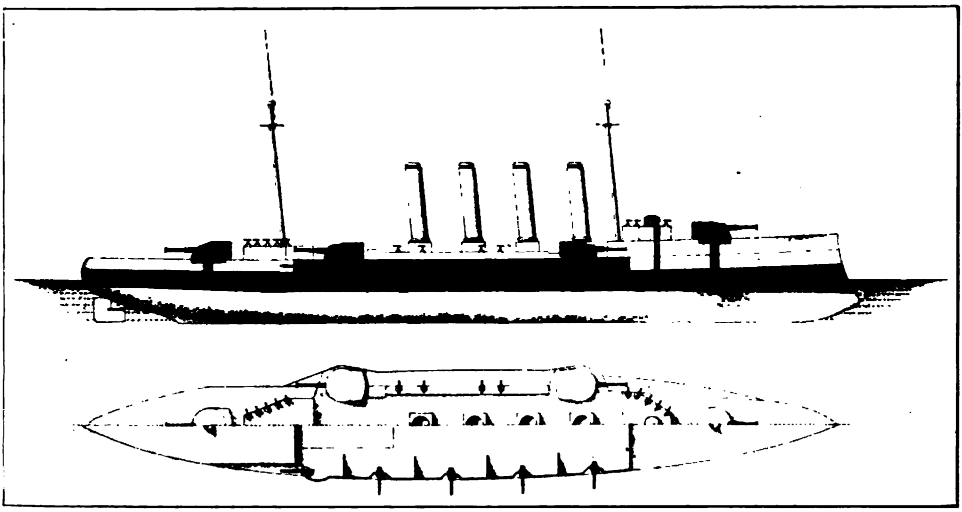


FIG. 33.—Armoured Cruiser ("Duke of Edinburgh").

vessels of this class have a length of 500 feet (about 75 feet longer than the larger battleships) and a displacement of 14,500 tons, though larger armoured cruisers are being built. The disposition of the armour belt is very similar to the battleships, but the maximum thickness is seldom greater than 6 in. If the belt is only over a portion of the length, thick athwartships armour bulkheads are fitted between the protective and main decks; but if the belt is carried to the stem or stern, the armour bulkhead at this part is sometimes not fitted. If carried to the extreme ends of the vessel the thickness of the belt is generally 2 in. to 3 in. The guns are generally lighter than in a battleship; in place of the pair of 12-inch guns at each end of the ship there is usually one 9·2-inch, or two 8-inch. These are in turrets similar to the battleships, but the armour for the barbette usually extends only from gun platform to the turret. The maximum thickness of turret and barbette armour is about 10 in., but in some cruisers the armour is only 6 in. The next size of gun generally fitted is the 6-inch, of which several are placed on broadside in the citadel. This battery is very similar to the battery of 6-inch guns in the battleship, and either of the two different systems of protection by casemates or by a side belt of armour can be adopted. In the case of some of the large cruisers, four casemates are fitted on the upper deck on each side directly over the casemates on the main deck, in which are additional casemates and guns, thus giving on the broadsides in that battery ten or more 6-inch guns. The lighter guns are placed in convenient positions on the upper deck and bridges, and have no other protection than light shields fitted forward of the breach mechanism. As regards speed, 21 to 23 knots is aimed at in the latest types, and for this speed in the largest cruisers 30,000 H.P. is

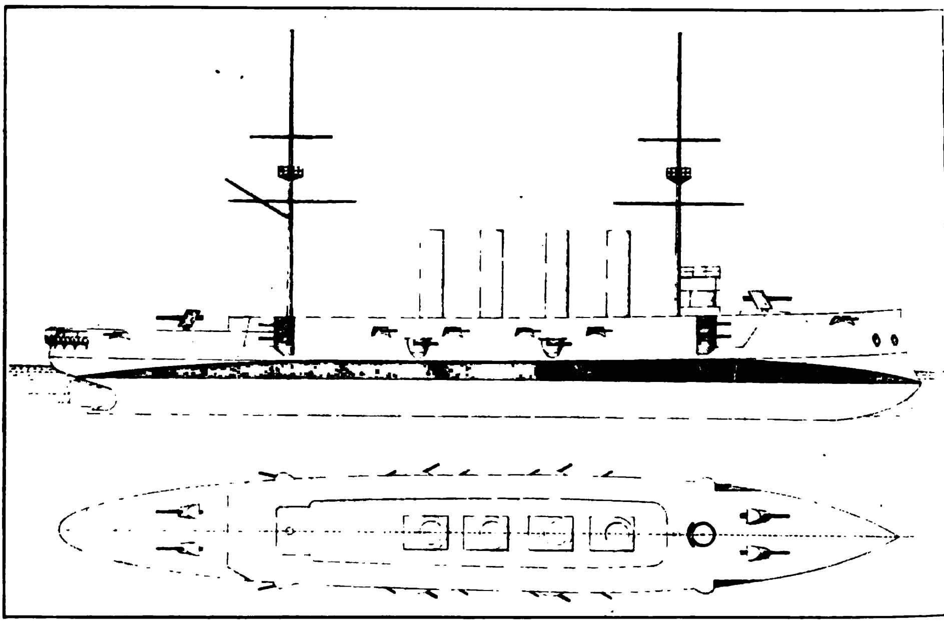


FIG. 34.—Protected Cruiser ("Andromeda").

necessary. This means that the engines and boilers take up a very much larger space than in the battleship which has engines of about 19,000 H.P. and a speed of 19 knots. Provision is made for a coal supply of 2000 tons, so that the steaming radius at 10 knots will be about the same as the battleships.

**Protected Cruisers** (fig. 34).—Many cruisers have been designed and built with the intention of being commissioned for foreign stations; and as the duties attached to these commissions vary greatly in importance, cruisers are of varying size and cost.

There are three classes—

(1) First class.
(2) Second class.
(3) Third class.

These classes differ in the main elements of size and cost, so that the first class includes the largest and costliest. An armoured cruiser may therefore be reckoned a first class cruiser when cruisers are considered generally. A *first class cruiser* is practically of the same design, and has the same armament as the armoured cruiser we have just described, but its hull is protected mainly by means of a protective deck, similar to that of battleships. It is necessarily of less displacement, about 11,000 tons, than an armoured cruiser of the same speed and gun-power, on account of the absence of the armoured belt. This class is not being built at present.

*Second class cruiser.*—This class is a stage below the first class in size and in cost. In speed and in armament vessels of this class are also inferior. The same principles of subdivision and other arrangements described for the previous vessels are carried out in the construction of the hull. The length of vessels of this class is about 300 to 350 feet, and the displacement 3500 to 6000 tons. The speed is about 20 knots, and the coal supply 600 to 900 tons. New vessels of this class will probably be more powerful and will displace protected ships of the first class.

Usually one 6-inch quick-firer is placed on the forecastle and two 6-inch on the quarter-deck, protected only by shields or hoods of 6-inch armour which are fitted over the breach mechanism and revolve with the gun. The broadside battery is usually one 6-inch quick-firer and three 4·7-inch quick-firers on each side, but in some of the latest vessels four or more 6-inch quick-firer guns are mounted on each side. These guns are also protected by light shields or, as in some larger cruisers, by casemates. It is a very general practice to sheath the vessels of this class with teak and copper, as they are often commissioned for distant stations where there are no docking facilities, and the sheathing prevents excessive fouling and corrosion of the skin.

*Third class cruisers* are smaller and of less cost than the second class cruisers and have less speed, protection and armament. The general features in design are simpler than in the second class. The average displacement of ships of this class is about 2000 tons, with a speed of 19 knots. Their main armament consists of twelve 4-inch guns, and their coal supply is about 500 tons. New vessels of this class are faster and of more displacement.

A few general features are common to all the classes just considered. The protective deck, methods of subdivision, double bottom, arrangement of coal, bunkers, and the ammunition passage have been described.

Another method of protection which has been adopted extensively by some navies is the fitting behind the armour belt of a cofferdam, packed in some cases with corn pith cellulose. In the event of a shot penetrating the side the cellulose is expected to swell up on contact with the water and thus close up the hole and prevent the inrush of the sea. This cofferdam is usually 1 to 2 feet broad. This system is not approved in the British Navy, as the cellulose appears to be of little use for this purpose after it has been some time on a ship.

The practice of sheathing the hulls of second class cruisers has been noted; this also is done with a certain proportion of battleships and cruisers that are intended for distant stations where dock accommodation is limited.

With the third class cruisers may be considered the smaller gunboats and sloops intended for special service, such as on shallow rivers, but the main points in their design are based on the special circumstances to be met with in each case.

The following table shows the main features of the classes just considered, and affords a means of comparison :—

TABLE VII.

| Particulars. | Battleship. | Large Armoured Cruiser. | Small Armoured Cruiser. | Protected. 1st Class Cruiser. | Protected. 2nd Class Cruiser. | Protected. 3rd Class Cruiser. | Scout. |
|---|---|---|---|---|---|---|---|
| When ordered | 1902 | 1903 | 1900 | 1895 | 1894 | 1902 | 1903 |
| Cost | £1,500,000 | £1,150,000 | £800,000 | £580,000 | £270,000 | £230,000 | £280,000 |
| *Dimensions:* | | | | | | | |
| Length in feet | 425 | 480 | 440 | 435 | 350 | 360 | 360 |
| Breadth in feet | 78 | 73½ | 66 | 69 | 54 | 40 | 39 |
| Draught in feet | 26¾ | 27 | 24½ | 26 | 21 | 14½ | 14 |
| Displacement in tons | 16,350 | 13,550 | 9,800 | 11,000 | 5,600 | 3,000 | 2,945 |
| Coal (normal) | 950 | 1,000 | 800 | 1,000 | 550 | 300 | 380 |
| Indicated Horse-Power | 18,000 | 23,500 | 22,000 | 16,500 | 9,600 | 9,800 | 16,500 |
| Speed in knots (designed) | 18·5 | 22·0 | 23·0 | 20·5 | 19·5 | 21·75 | 25 |
| Complement | 800 | 700 | 500 | 680 | 470 | 280 | 270 |
| *Armour:* | | | | | | | |
| Belt in inches | 9 K.S. | 6 K.S. | 4–2 K.S. | .. | .. | .. | .. |
| Deck in inches | 2–1 | 2–1 | 2–¾ | 4–2½ | 2½ | .. | 1½–⅝ in. |
| Side above belt in inches | 8 K.S. | 6 | 4 K.S. | .. | .. | .. | .. |
| Bulkhead in inches | 12 K.S. | 6 | 5 K.S. | .. | .. | .. | .. |
| *Gun position:* | | | | | | | |
| Heavy guns in inches | 12–6 N.S. | 6 | 5–4 N.S. | .. | .. | .. | .. |
| Secondary guns in inches | .. | 6 | 4 K.S. | 4½–2 | 3 | .. | .. |
| Guns in inches of calibre | 10 6-in. | 10 6-in. | 14 6-in. | 16 6-in. | 11 6-in. | .. | .. |
| | 4 12-in. | .. | 8 12-pr. | 14 12-pr. | 9 12-pr. | 12 4-in. | 10–12-pr. |
| | 4 9·2-in. | 6 9·2-in. | 3 3-pr. | 4 3-pr. | 7 3-pr. | 8 3-pr. | 8–3-pr. |
| | 24 small | 30 small | 8*m* 2*l* | 8*m* | 5*m* 1*l* | .. | .. |
| Torpedo tubes | 2 | 2 | .. | .. | 3 | .. | 2 |

K.S., Krupp steel. *l*, Light guns under 15 cwt., including boats' guns.
N.S., Nickel steel. *m*, Machine guns.

As vessels get older, they are sometimes placed in a lower class than that in which they are built.

In 1903 four vessels forming a new class, the scout class, distinct from the cruiser class, were ordered by the British Government. These vessels were of the same length as some of the second class cruisers, but they were designed to obtain a speed of at least 25 knots. Their particulars are given in the table. The function of these vessels in war time is scouting for a fleet of battleships, and in that duty they are much more able than the fast destroyer, being much larger, and therefore more able to maintain speed in a seaway.

**Modern large Battleships and Cruisers.**—It will be seen from any survey of tables giving the particulars of warships, such as Brassey's *Naval Annual*, that there has been a tendency in recent years to increase the size of battleships and armoured cruisers. The most important departure in this respect was made in 1904 by the British Admiralty. In the programme of new construction for that year it was announced that one large battleship and three large armoured cruisers would be laid down.

The battleship "Dreadnought" provided for in the 1904 programme was laid down and constructed with great despatch, and in 1906 she ran her trials successfully. Previous to her class the largest British battleship was the "Lord Nelson" class, which was 450 feet long and 16,350 tons displacement.

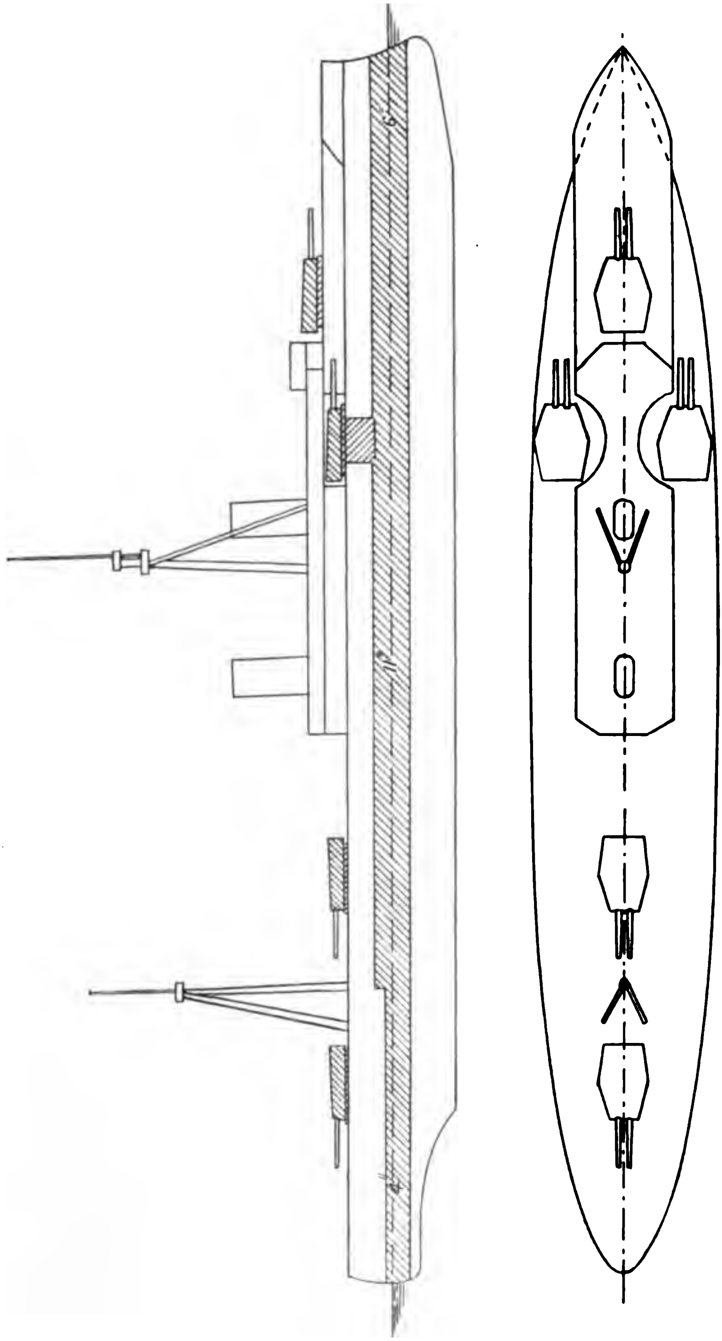


Fig. 35.

The following are the published leading particulars of the "Dreadnought":—

| | |
|---|---|
| Length . . . . . | 490 ft. |
| Breadth . . . . . | 82 ft. |
| Draught . . . . . | 26 ft. 6 in. |
| Displacement . . . . | 17,900 tons. |
| Armament . . . . | Ten 12-in. guns in five barbettes. |
| ,, . . . . . | About 30 small quickfires. |
| Armour . . . . . | Main side armour belt 11 in. |
| Engines . . . . . | Turbines, 23,000 H.P. |
| Speed . . . . . | 21 knots. |

The diagram No. 35 shows the chief features in the arrangement of the "Dreadnought."

Since the time of the announcement of the 1904 programme, the policy of building similar battleships has been adopted in all the first-class navies.

Regarding the new armoured cruisers, a reference to the Table VII. will show the tendency of increasing the size of the vessels of this class.

From statements that have been recently made regarding the designs of the new armoured cruisers, the following particulars have been taken :—

| | |
|---|---|
| Length . . . . . | 530 ft. |
| Breadth . . . . . | 78 ft. 6 in. |
| Draught . . . . . | 26 ft. |
| Displacement . . . . | 17,250 tons |
| Engines . . . . . | Turbines of 40,000 H.P. |
| Speed . . . . . | 25 knots. |
| Coal capacity . . . . | 1000 tons. |
| Armament . . . . | Eight 12-in. guns. |
| Armour belt . . . . | 7 in. |

The diagram No. 36 shows the chief features of this class of vessel—the "Inflexible" class.

The adoption of the one calibre gun for the heavy armament of the big battleship or cruiser has simplified greatly the internal arrangements of magazines, etc.

In the cruisers of the "Inflexible" class two of the barbettes are to be placed fore and aft on the middle line, and, as will be seen from the plan view, each pair of guns has a large arc of training on either side. The other two barbettes are placed *en échelon* towards the middle of the length and between two boiler-rooms. The guns in the midship barbettes are available on either broadside, and are capable also of firing parallel to the line of keel.

The "Dreadnought" and the subsequent large vessels for the British Navy are each fitted with two rudders abreast.

**Torpedo Boats.**—The torpedo boats are boats essentially for carrying and discharging torpedoes. The most important requisites in these boats are speed and invisibility. The former is necessary to enable them to get quickly in and out of range for torpedo firing. The latter is secured by their small size, which also affords a small mark to gun-fire. If made too small, however, they cannot have good sea-going qualities. In the earlier boats a length of about 60 feet with a speed of 15 knots was adopted, but their sea-going qualities were limited. For this reason a class of boat was built about 125 feet in length and with a speed of about 20 knots.

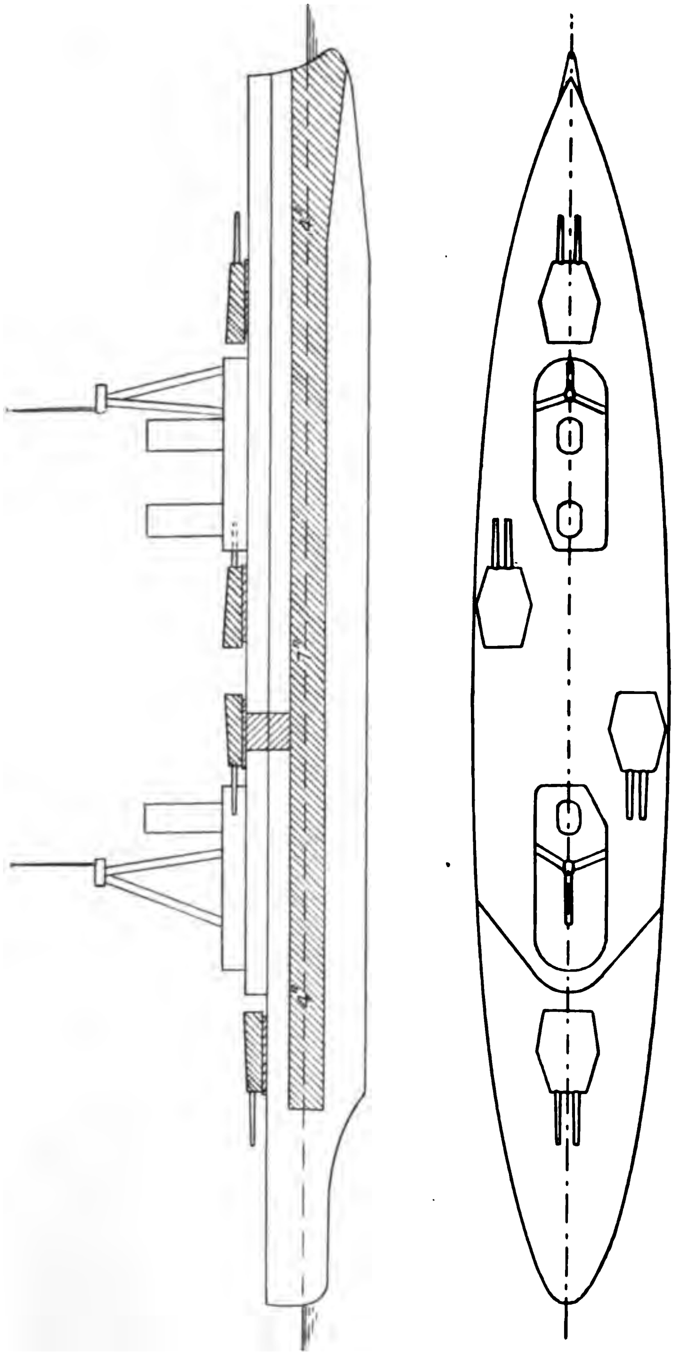


FIG. 36.

As the speeds of ships increased, however, the speed of torpedo boats had to be increased, until torpedo boats of 150 feet long were produced, capable of steaming 23 to 24 knots. To deal with this class of vessel a class called the "torpedo-boat destroyer" was produced. These were at first about 180 feet long and 27 knots speed, but later were 210 feet long and 30 knots speed.

Fig. 37 shows this type of torpedo boat. Fig. 38 shows a torpedo-boat destroyer.

The object of the destroyer is to catch, or at least put out of action, the torpedo boats of the enemy; but, at the same time, they possess means of firing torpedoes, so that they have therefore the same functions as a torpedo boat as regards attacking the enemy's ships. The main point in the design of a destroyer is to obtain as much speed as possible, for which end every effort is made to cut down weight in the hull and fittings, and to get as much horse-power as possible out of a given weight of machinery. A speed of 32 knots has been reached by the later vessels of this class, but some with turbine machinery obtained 36 knots.

On account of their high speed, these vessels are very useful for duties in and around harbours, and also when attached to a fleet. They can act as scouts like the cruisers, and they make very good vessels for carrying despatches, but their speed at sea is very soon reduced as the sea rises.

The types of destroyers built before 1902 have a length of 200 to 215 feet, with a displacement of 450 tons on a draught of 5 to 7 feet. The coal capacity is usually 80 to 90 tons, but a few have been designed to carry about twice that amount (having a much larger displacement), and are consequently able to keep the sea longer. The armament is generally one 12-pounder, five 6-pounders, and two torpedo tubes. As regards the construction of the hull, a destroyer is merely a thin shell with a continuous deck running from the forecastle, which is sunk and protected by a turtle-backed deck, to right aft. On the top of the forecastle is mounted the 12-pounder gun.

The crew accommodation is forward, and the officers' quarters and a few store-rooms aft. The engines and boilers take up about two-thirds of the volume of the hull. The torpedoes are fired from tubes which are pivoted on the deck near amidships.

The speed of 30 knots which has been mentioned as the speed of the foregoing torpedo-boat destroyer is the speed which the builders guaranteed under the trial trip conditions. In most cases the load on board was only 40 tons. The trial trip results gave no guarantee that this speed could be maintained in rough weather. It was considered that this type of destroyer had insufficient strength for general sea-going purposes, so that a new type was evolved with greater strength. The designed speed was only 26 knots, but it had to be made on trial with the full load on board. The quarters for the men were more habitable, the chief feature being a full forecastle for the accommodation of the crew. This full forecastle gave a much higher freeboard forward and made a drier ship in rough weather. The dimensions are -- length 222 ft. and breadth 23½ ft. The draught is 9 ft. 6 in., with a displacement of about 600 tons, and the armament is the same as in the previous type.

*Ocean Destroyers.*—Since this type has been tried a new type called ocean destroyers, of 800 tons displacement, has been created. Their speed is 33 knots, with sufficient fuel on board to steam 1500 nautical miles at 14 knots. They have turbines and water tube boilers fired

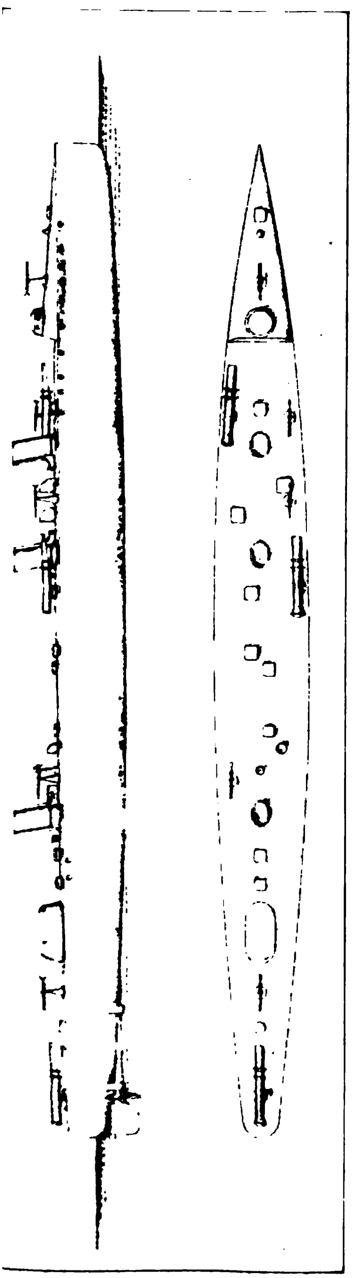


FIG. 37.—Torpedo Boat.

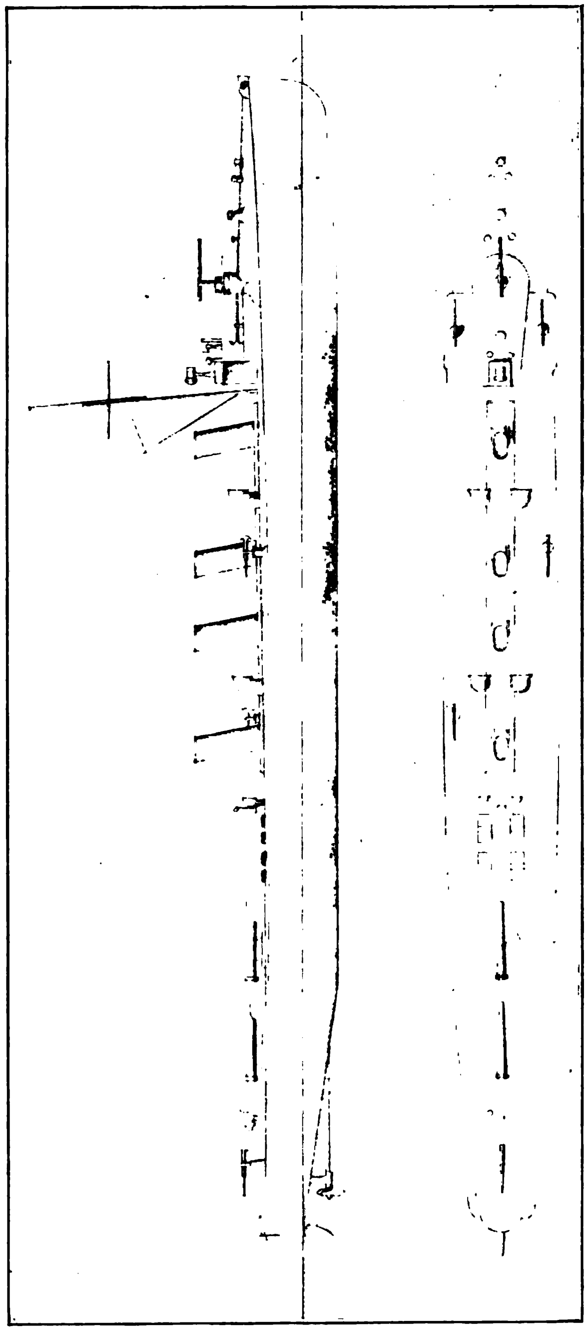


FIG. 38.—Torpedo-boat Destroyer.

with oil. They carry two 4-inch guns and two torpedo tubes. Their dimensions are about 260 ft. by 27 ft. by 17 ft.

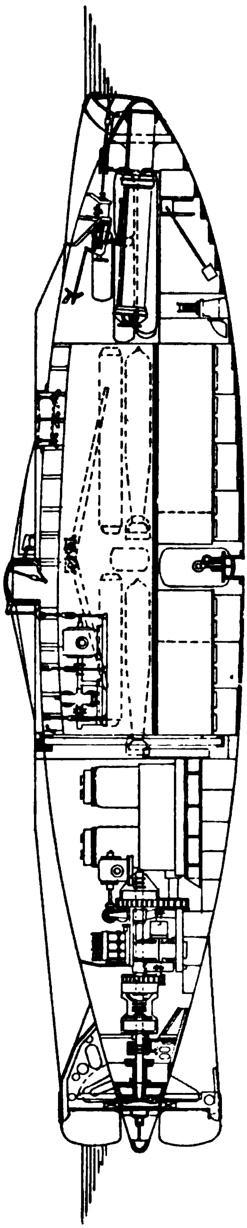


FIG. 39.—Submarine Vessel.

Torpedo boats as built now are 160 ft. to 180 ft. in length. The speed of those of this size is about 26 knots, the displacement 250 tons, and the fuel capacity 40 tons. They are each armed with two guns, besides provision for firing three torpedoes. They can steam 1500 nautical miles at 12 knots.

*Submarines.*—In recent years an entirely new type of fighting vessel, the submarine, has been constructed by several navies. At present France possesses a larger number of these vessels than any other navy. Most of the submarines now built are developments of the Holland type, which is illustrated in fig. 39.

Many of the submarines are provided with petrol engines for propelling the vessel while open on the surface, and electric power for propulsion when closed and submerged. Trim is altered by filling tanks with water. The vessel is prepared for diving by filling these tanks with water so that the weight of the vessel is increased and she can be entirely submerged. The buoyancy is regained by expelling the water from the tanks by means of compressed air. There are two sets of rudders, one set for horizontal and the other for vertical change of direction. Provision is made at the bow for carrying one or two ordinary torpedo tubes, and the vessel is able to carry two or more torpedoes, which she can fire in the submerged condition. The lengths of the submarines are from 100 to 150 ft., and the total displacement of the largest size is about 300 tons. Some of 600 tons have been designed.

## (2) SHIPS FOR PASSENGER AND CARGO CARRYING.

The next class of vessels we have to consider is the class for carrying passengers and cargo. Of this class there is a large number of different types, which to describe in detail would be too laborious a task.

Some merchant vessels are driven by the wind pressure on the sails. Even at the present time, when the method of driving by steam is in such a state of perfection, some sailing-vessels can still be run with profit, and new ones are still occasionally ordered.

Practically all sailing-vessels are used only for cargo-carrying purposes. There is no special point requiring notice in the construction of the hull: it is merely a shell, with one or two continuous decks.

Only one bulkhead is necessary, the forward or "collision" bulkhead. Nearly all the space in the hull is available for carrying cargo, and there is the advantage that the holds are free and open, except for the pillars and masts, thus allowing the stowage of long timbers, rails, or beams. The only erections are usually a poop and a forecastle for accommodation and ship's stores. Sometimes there is a house amidships in which the crew is housed. The masts are carried through the decks and stepped on the floors above the keel.

An important point in a sailing-vessel is the rig. There are different types of rigs.

A schooner is rigged fore and aft on each mast (fig. 40), and may have as many as seven masts. Some schooners have one or two square sails on the foremast (fig. 41), and are called topsail schooners.

FIG. 40.—Schooner.

A brigantine is a vessel square rigged on the foremast and fore-and-aft rigged on the main (fig. 42).

A brig is a vessel square rigged on both masts (fig. 43).

A barque has two masts square rigged and one fore-and-aft rigged (fig. 44).

A barquentine has one mast square rigged and two masts fore-and-aft rigged (fig. 45).

A ship has three masts square rigged. Some ships have four masts, sometimes three only being ship rigged and the fourth fore-and-aft rigged. The three masts of a ship are called fore, main, and mizzen, which are all square rigged.

In the four-masted ship the after mast is called the jigger (fig. 46).

Merchant steamers form by far the largest proportion of ships afloat. They carry on the commerce between the countries. Most of them, especially those carrying passengers, keep up a regular trade, and ply along fixed routes or lines and are called liners. They are mostly owned by shipping companies

FIG. 41.—Topsail Schooner.

FIG. 42.—Brigantine.

or companies engaged in a certain trade, and some of the passenger vessels are paid by the Government to carry the mails. Many steamers, however, engage in trade of an irregular nature, sailing from any port to any other port with any cargo that has to be shipped. These vessels are commonly called "tramps." If built to act only as tramps their speed is small, and the general fittings inexpensive.

FIG. 43.—Brig.

FIG. 44.—Barque.

Many ships that are intended for passengers or cargo are built to a certain class of one of the Societies of Registration, among which may be mentioned "Lloyd's Register," "Germanischer Lloyd's," "Bureau Veritas," "British Corporation," and the "Record of American and Foreign Shipping."

The class is determined by the scantlings and the equipment of the vessel, and is adopted for purposes of insurance in some cases.

FIG. 45.—Barquentine.

FIG. 46.—Four-masted Ship.

In building to a class, the scantlings of all the structural parts of the ship are fixed by the tables or rules of the classification society, thus ensuring uniformity of weight of structure. This is considered by some as giving a guarantee of sufficient strength. To suit these conditions, and for carrying certain specific kinds of cargo, various types of merchant steamers are constructed, but in the general description of the hull they are practically the same.

Merchant steamers, except in special cases, are generally built on the double-bottom principle.

Fig. 47 shows a section through the double bottom of a vessel. The double bottom consists of an inner and an outer skin extending almost the whole length of the vessel, each being made watertight. In the centre line is built a vertical girder, to which are connected the transverse brackets or floors. This arrangement, besides giving safety in the event of damage to the outer bottom, affords stiffening to the ship at every frame space. The double bottom is further stiffened by fore-and-aft longitudinals or intercostals at the sides of the vertical keel, and a strong margin plate at the outside. Beyond each end of the double bottom rises the stem and sternpost, and from the margin plate to the top deck or deck erections runs the transverse framing. The framing is stiffened by stringers and web frames at intervals. The system of fitting web frames to stiffen the framing is, however, being superseded by making all the frames of a uniform depth, but deeper than the

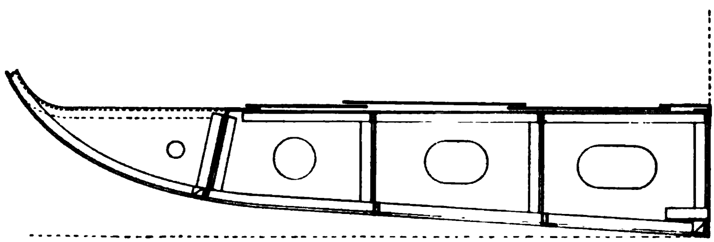


FIG. 47.

requirements for frames when the web frames are fitted. Stiffening to the framing and the shell is given by the deck beams and plating and the transverse bulkheads. The decks themselves are also stiffened by tiers of pillars attached under the beams to the deck below.

As regards arrangement, in most sea-going vessels the engines and boilers are in compartments amidships. Hatches for air, light, and ventilation are led up from the engines and boilers. The cargo-holds are forward and aft of the engine and boiler compartments, and hatches or openings for the handling of the cargo are made in the decks at suitable places. In the extreme forward end are usually the cable lockers, ship's storerooms, and trimming tanks. Storerooms and trimming tanks are sometimes also arranged at the after end. Accommodation is usually provided in the deck erections, poop, bridge, and forecastle.

The crew's quarters are generally in the forecastle, and the passengers and officers in the bridgehouse or in the poop. If the vessel carries a large number of passengers, sometimes accommodation is arranged on the main and lower decks, first-class passengers being generally amidships, where the motion of the vessel is least. Accommodation is also provided on the upper deck, and in the large types of passenger vessels a promenade deck is erected above the upper deck, fitted with saloons, etc. The boat deck is usually over the promenade deck. In recent large passenger vessels the number of decks has outrun the usual nomenclature, and the decks are given names like streets, or are lettered. Fig. 57 is a longitudinal elevation of a large passenger Atlantic liner having seven decks.

For working the cargo through the hatches or for coaling, derricks on the masts are arranged over the hatches.

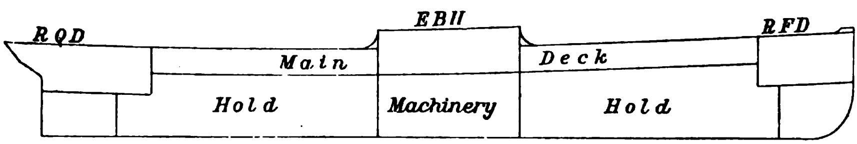



FIG. 48.—One-deck Steamer with short raised quarter and fore decks and enclosed bridgehouses.

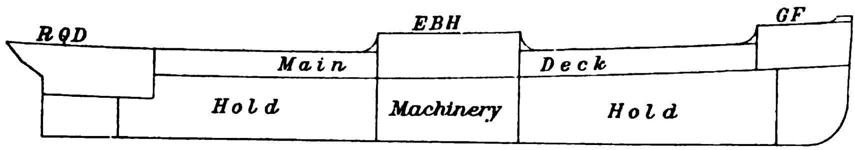



FIG. 49.—One-deck Steamer with short raised quarter-deck, enclosed bridgehouses, and topgallant forecastle.

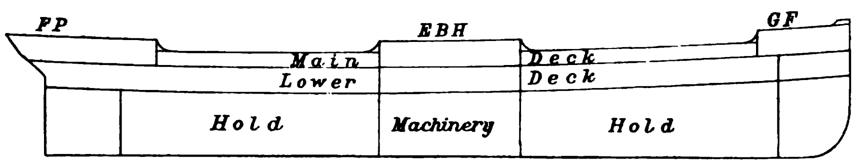



FIG. 50.—Two-deck Steamer with full poop, enclosed bridgehouses, and forecastle.

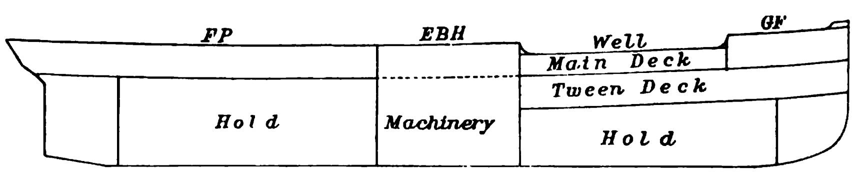



FIG. 51.—Steamer with long full poop, enclosed bridgehouses, and topgallant forecastle. Known as a "well-decker."

The different types may be classed as follows :—

| | |
|---|---|
| One decker. | Awning decker. |
| Two ,, | Shelter ,, |
| Three ,, | Well ,, |
| Spar ,, | Turret or Trunk. |

The existence of these different types is more fully understood when we consider the rules for freeboard measurement (which will be treated in a later

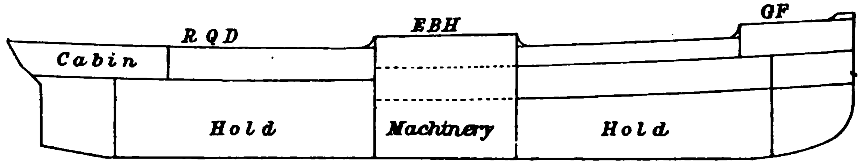



FIG. 52.—Steamer with long raised quarter-deck, enclosed bridgehouses, and topgallant forecastle. Also known as a "well-decker."

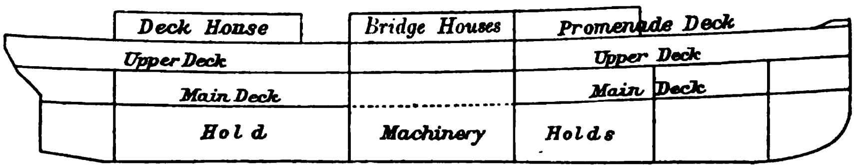



FIG. 53.—Steamer with promenade deck and long bridgehouses.

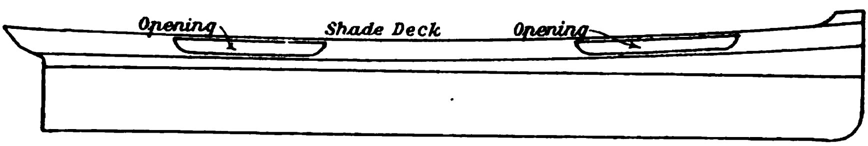



FIG. 54.—"Shade-deck Vessel." This type of vessel has a continuous upper deck of light construction with openings in the sides.

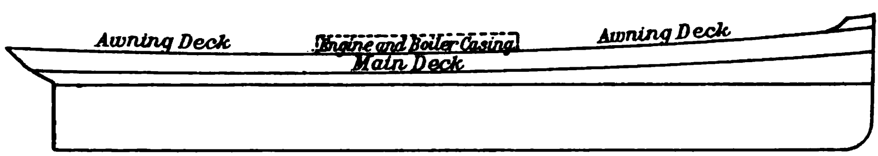



FIG. 55.—"Awning-deck Vessel." This type of vessel has a continuous upper deck of light construction and the sides completely enclosed above the main deck.

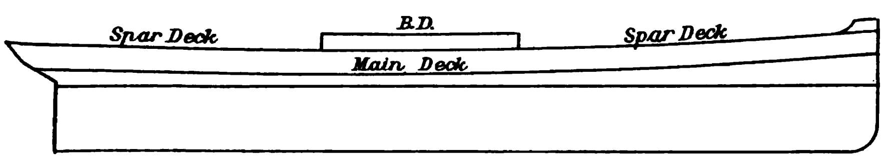



FIG. 56.—"Spar-decked Vessel." This type of vessel is constructed with the scantlings above the main deck heavier than an "awning-decked vessel," but not so heavy as in a "three-decked vessel."

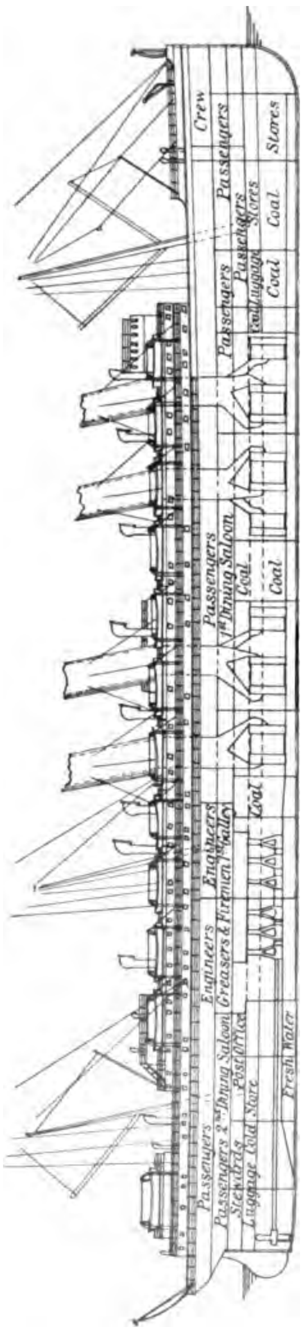



FIG. 57.

chapter); but as there are certain points of difference in the construction, these can be now noted.

In the one, two, and three deck classes the strength is carried up to the top deck in all cases, and the sides are watertight up to that deck. There are deck erections or houses, such as poop, bridgehouse, and forecastle, on the top deck, of light construction. The three-decker is a very common type of passenger vessel, and the deep vessels of this class have usually an extra deck below the lower deck, called the orlop deck, which is sometimes structurally only a tier of beams in the holds for stiffening purposes, upon which planks may be laid for carrying cargo. When carrying cargo in bulk, these planks may be removed.

The spar-decker is one in which the strength is carried up to the main deck, the part above being of lighter construction than that required in a three-decker. The deck corresponding to the upper deck in a three-decker is called the spar deck.

The awning-decker resembles the spar-deck type, but the sides and deck above the main deck, called the awning deck, are of lighter construction than for the spar-deck class.

A shade deck or shelter deck is a light deck above the upper deck, and was only for shelter of passenger or cargo. In large modern vessels the shelter deck is made stronger than formerly, and frequently is the strongest deck of the ship. The ship's side is sometimes completely carried up above the upper deck. In large passenger steamers this deck is called the promenade deck.

An arrangement called the raised quarter-deck is sometimes made by stopping the upper deck abaft amidships, and continuing it right aft at a greater height, about one-half of an ordinary 'tween-deck space above the forward part of it.

When this arrangement is adopted, and also a bridgehouse and forecastle are added to a two-decker, we have what is called the well-decker.

Specimens of these types are shown in figs. 48–56.

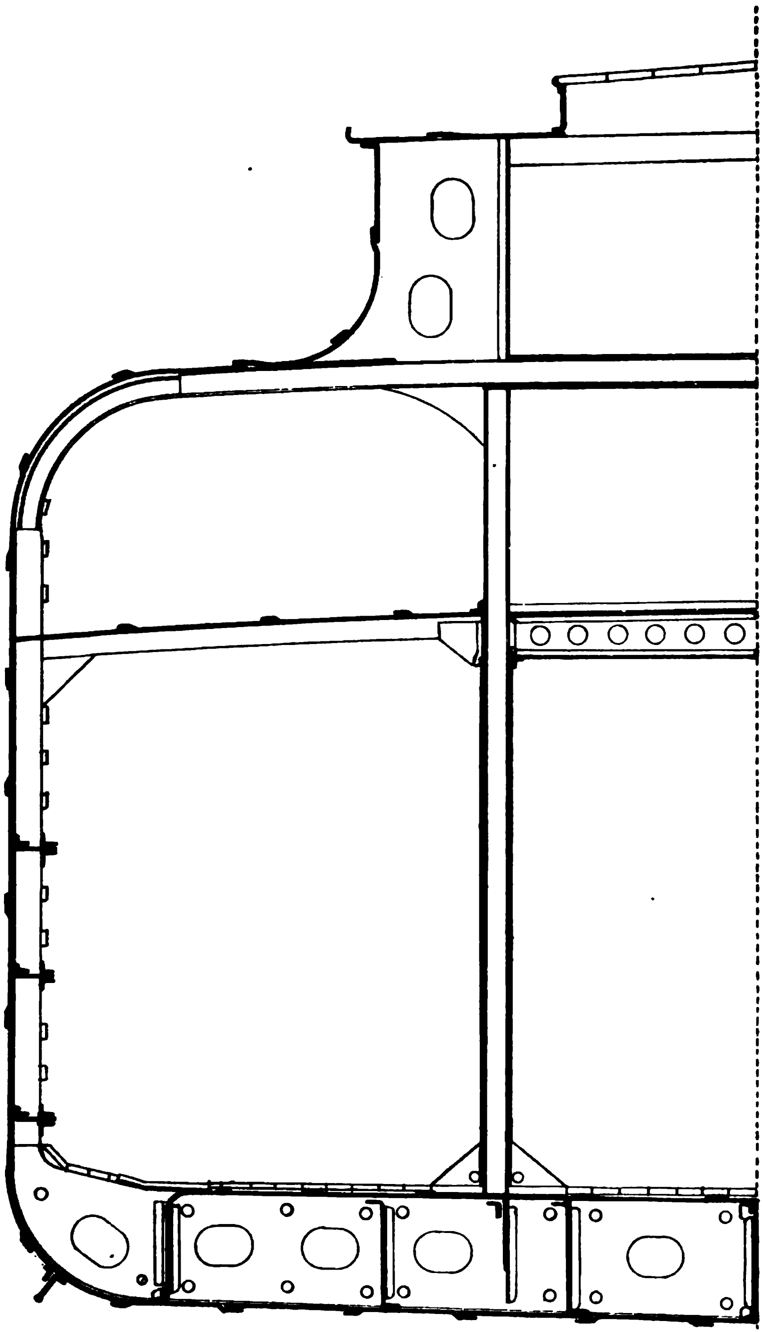


FIG. 58.

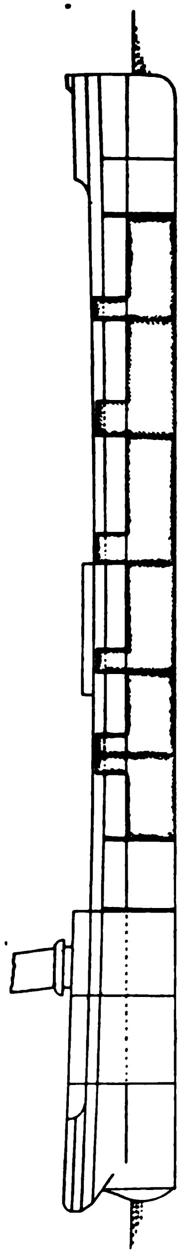


Fig. 59.—Oil-carrying Steamer.

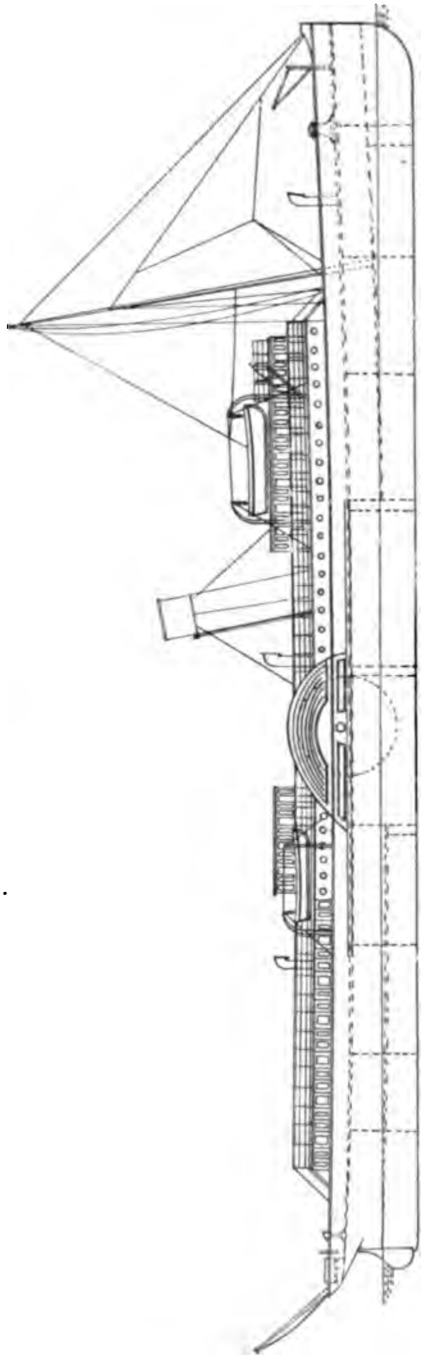


Fig. 60.—Paddle Steamer.

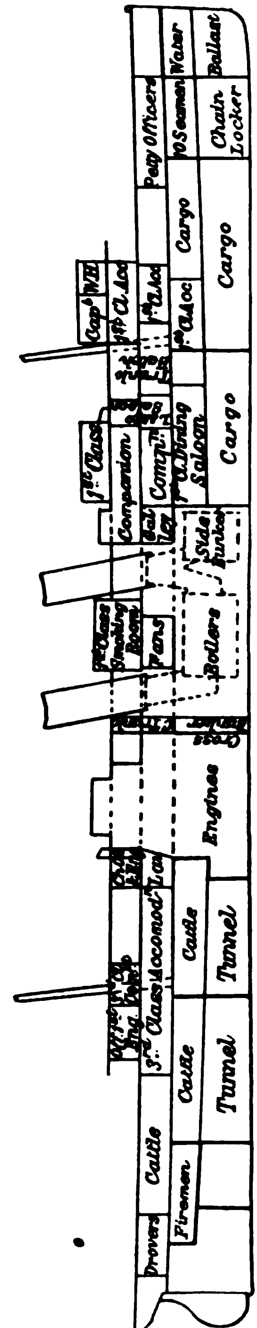



Fig. 61.—Fast Channel Steamer.

The turret-deck type of steamers has a section of the form shown in fig. 58, and general arrangement as shown in fig. 64. A trunk-deck steamer is similar to a turret, but there are no rounded corners at the sheerstrake or deck erection. The general arrangement of a single deck type of steamer that is being much used at present is given in Plate XIII.,[1] and a section in Plate XIV.[1] For ships in ballast it is found desirable to have arrangements of the structure made so that water ballast may be carried in parts other than in the bottom.

3. **Miscellaneous class.**—This class is so varied that no general description can be made. Each type of vessel in this class has its own special duties. Illustrations have been made of some of the more common types.

Fig. 59 is a longitudinal sectional elevation of an oil-carrying steamer. It

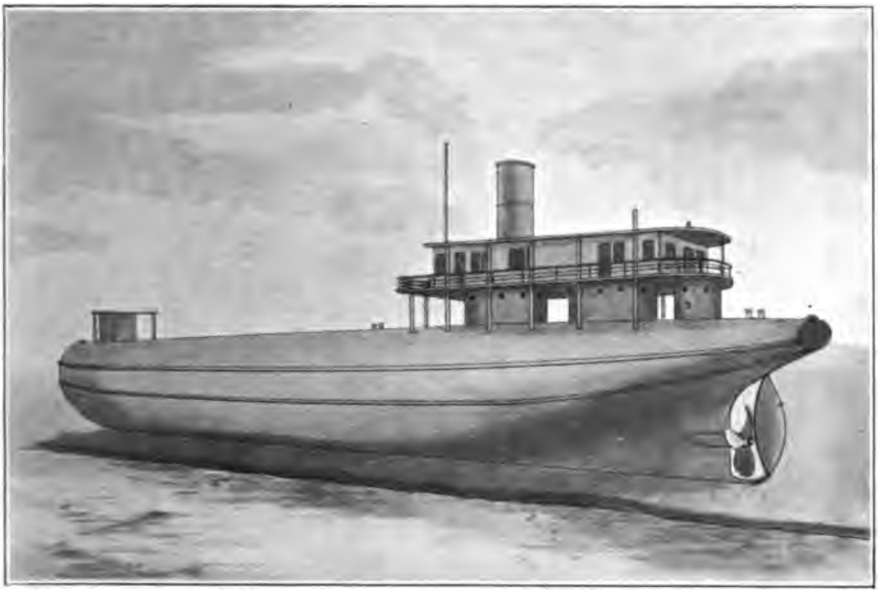


**Fig. 62.—Whale-back Steamer.**

shows the division of the hull by strong watertight bulkheads. Sometimes the trunks shown in the 'tween decks, which are fitted to allow for the expansion of the liquid, extend through the whole length of the oil compartments. Fig. 62 is an illustration of the type of steamer known as the whale-back. This type only exists on the American coast and the Great Lakes. Fig. 63 shows a large type of bucket dredger. Fig. 60 is an illustration of a paddle steamer. This is the usual type of steamer for the passenger traffic in rivers, and generally where the water is shallow. Fig. 61 is an illustration of a fast Channel steamer. These steamers have generally a high speed, and are designed mostly for the passenger traffic across the channels of the North Sea, south coast of England, and the Irish Sea. Figs. 65 and 66 are pictures of a steam yacht and a sailing yacht respectively. Fig. 67 is an illustration of a new type of pleasure craft —the motor boat. These vessels can be made to attain a very high speed owing to the lightness of their engines (generally petrol motors) and the light construction of the hull.

[1] Facing page 224.

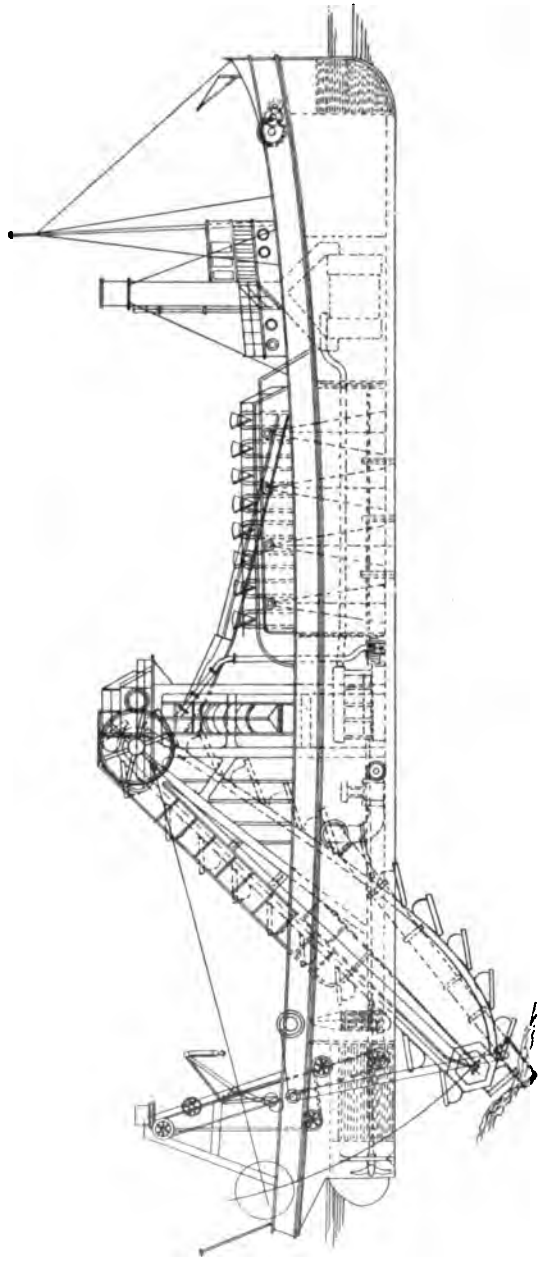


Fig. 63.—Bucket Dredger.

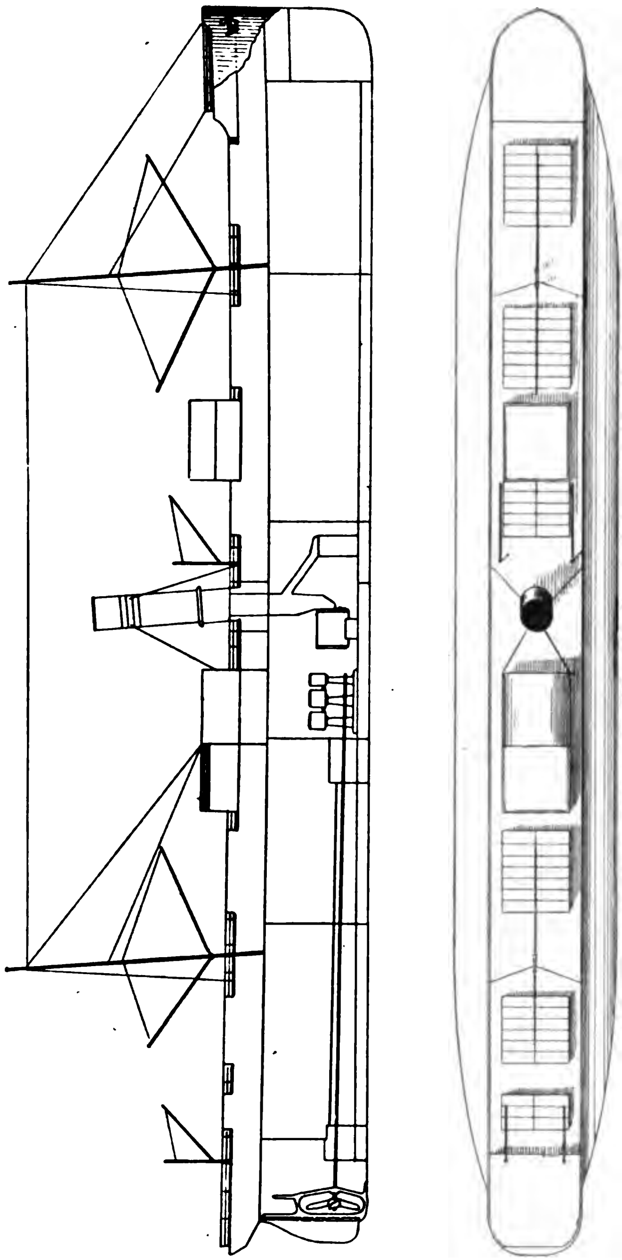


FIG. 64.

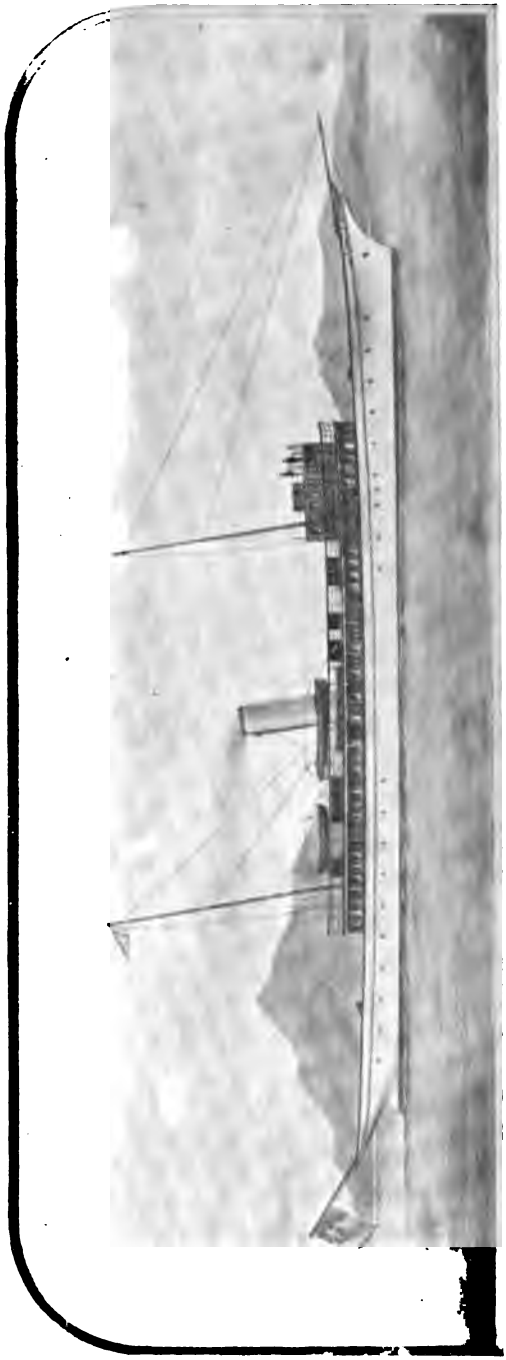


FIG. 65.—Steam Yacht.

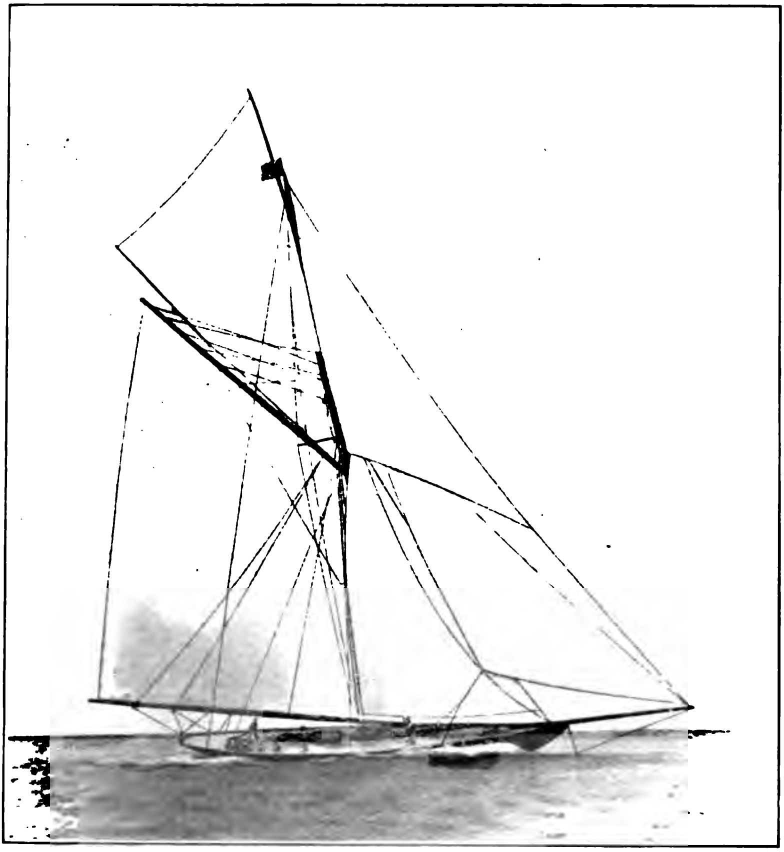


FIG. 66.—Sailing Yacht.

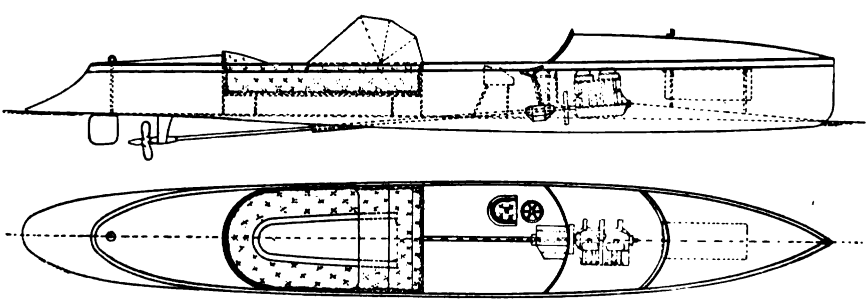


FIG. 67.—Motor Boat.

# PART II.

## SHIP CALCULATIONS.

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## CHAPTER VIII.

### CALCULATIONS OF DISPLACEMENT, CENTRE OF BUOYANCY, AND AREAS.

In the process of designing a ship's form, calculations have to be mad relating to the displacement and the trim.

The designer of a new form, however, is more or less guided by coefficient which have been accurately calculated from existing types.

In this Part it is proposed to deal with the calculations that are usuall made in the drawing-office after the form has been fixed and faired.

As already indicated, these calculations are very useful to a designer i fixing the form of a new vessel of a similar type, but they are of further us when the ship is in service. For instance, if the results are plotted in tabula form or in curves, the master of the ship is able to determine easily what th displacement is after the draughts have been noted. He is able to judge b these curves how to load his ship so as to bring the condition into one suitabl for going to sea.

The calculations that will be dealt with may be enumerated as follows:—

| | |
|---|---|
| Displacement. | Metacentres. |
| Centres of buoyancy. | Trim. |
| Areas of waterplanes. | Coefficients. |
| Areas of sections. | |

These calculations, if made for the finished ship, will be useful to tho handling the ship while in service. They are usually made on what are calle "displacement sheets,"—see Plates II. and III.

The first sheet is ruled so as to give the working and the results of th displacement and the centres of buoyancy calculations. The second shee contains the calculations for the metacentric heights.

Detailing the calculations that can be made on the displacement sheet we have—

(1) Displacement.
(2) Vertical centre of buoyancy.
(3) Longitudinal " "

I.

...acement
...0′ abaft
...gth.

| ...ent ...ions. | No. 4. 6 FEET. | 1 | ...EVERS about No. 5 ...ection. |
|---|---|---|---|
| . | ... | ... | ... |
| . | ... | ... | 5 |
| | 0·75 | 1·5 | ... |
| ·49 | 0·75 | ... | 4½ |
| | 2·52 | 3·7 | ... |
| ·92 | 2·52 | ... | 4 |
| | 7·76 | 31·0 | ... |
| ·86 | 7·76 | ... | 3 |
| | 14·58 | 39·1 | ... |
| ·36 | 14·58 | ... | 2 |
| | 19·42 | 77·6 | ... |
| ·72 | 19·42 | ... | 1 |
| | 20·86 | 41·7 | ... |
| ·85 | 20·86 | ... | ... |
| | 18·71 | 74·8 | ... |
| ·24 | 18·71 | ... | 1 |
| | 18·78 | 27·5 | ... |
| ·72 | 18·78 | ... | 2 |
| | 8·42 | 33·6 | ... |
| ·64 | 8·42 | ... | 3 |
| | 3·76 | 5·64 | ... |
| ·52 | 3·76 | ... | 4 |
| | 1·75 | 3·56 | ... |
| ·51 | 1·75 | ... | 4½ |
| . | ... | ... | ... |
| . | ... | ... | 5 |
| ·63 | ... | 330·10 | |
| ·85 | ... | 1 | |
| ·78 | ... | 330·10 | |
| ·729 | ... | 3 | |
| . | ... | 990·30 | |
| 2′ | | | |
| ...o. 5 | | | |

... page 82.

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 5 | ... | ... | 5 | ... | ... | ... | 5 | ... | ... | ... | 5 | ... | ... | ... | 5 | ... |
| ½ | 4½ | ·64 | 2·88 | 4½ | 12·96 | 1·50 | 6·75 | 4½ | 30·37 | 3·74 | 16·83 | 4½ | 75·73 | 5·10 | 22·95 | 4½ | 103·23 |
| 1 | 4 | 1·92 | 7·68 | 4 | 30·72 | 3·78 | 15·12 | 4 | 60·48 | 6·24 | 24·96 | 4 | 99·84 | 10·56 | 42·24 | 4 | 168·96 |
| 2 | 3 | 14·88 | 44·64 | 3 | 133·92 | 31·04 | 93·12 | 3 | 279·36 | 46·04 | 138·12 | 3 | 414·36 | 60·40 | 181·20 | 3 | 543·60 |
| 3 | 2 | 16·76 | 33·52 | 2 | 67·04 | 29·16 | 58·32 | 2 | 116·64 | 35·38 | 70·76 | 2 | 141·52 | 38·58 | 77·16 | 2 | 154·32 |
| 4 | 1 | 59·84 | 59·84 | 1 | 59·84 | 77·68 | 77·68 | 1 | 77·68 | 82·08 | 82·08 | 1 | 82·08 | 83·12 | 83·12 | 1 | 83·12 |
| 5 | 0 | 36·28 | 148·56 | 0 | ... | 41·72 | 250·99 | 0 | ... | 42·00 | 332·75 | 0 | ... | 42·00 | 406·67 | 0 | ... |
| 6 | 1 | 64·00 | 64·00 | 1 | 64·00 | 74·84 | 74·84 | 1 | 74·84 | 78·24 | 78·24 | 1 | 78·24 | 79·88 | 79·88 | 1 | 79·88 |
| 7 | 2 | 21·24 | 42·48 | 2 | 84·96 | 27·56 | 55·12 | 2 | 110·24 | 30·64 | 61·28 | 2 | 122·56 | 33·00 | 66·00 | 2 | 132·00 |
| 8 | 3 | 22·32 | 66·96 | 3 | 200·88 | 33·68 | 101·04 | 3 | 303·12 | 40·20 | 120·60 | 3 | 361·80 | 45·96 | 137·88 | 3 | 413·64 |
| 9 | 4 | 3·18 | 12·72 | 4 | 50.88 | 5·64 | 22·56 | 4 | 90·24 | 7·38 | 29·52 | 4 | 118·08 | 8·94 | 35·76 | 4 | 143·04 |
| 9½ | 4½ | 1·60 | 7·20 | 4½ | 32·40 | 3·50 | 15·75 | 4½ | 70·87 | 4·90 | 22·05 | 4½ | 99·22 | 6·12 | 27·54 | 4½ | 123·93 |
| 10 | 5 | ... | ... | 5 | ... | ... | ... | 5 | ... | ... | ... | 5 | ... | ... | ... | 5 | ... |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (A) | 242·66 | 193·36 | 737·60 | 330·10 | 269·31 | 1213·84 | 376·84 | 311·69 | 1593·43 | 413·66 | 347·06 | 1945·72 |
| Met're Multiplier = † $\frac{\frac{33·729 \times 2}{3}}{3}$ = 25580 | | 148·56 | Correction 8·27 | | 250·99 | 1·02 | | 332·75 | 1·18 | | 406·67 | 8·59 |
| | (B) + 44·8 | 33·729 | 729·33<br>25580 | | + 18·32<br>33·729 | 1212·82<br>25580 | | − 21·06<br>33·729 | 1592·25<br>25580 | | − 59·61<br>33·729 | 1937·13<br>25580 |
| | 242·66 | | 7501 | 330·1 | | 33882 | 376·84 | | 65800 | 413·66 | | 101380 |
| C.G. Before No. 5, | | 6·23′ | BM = 2487·3 | Before No. 5, 1·87′ | | 915·6 | Abaft No. 5, 1·88′ | | 619·0 | Abaft No. 5, 4·86′ | | 488·8 |

Correction is $\frac{(B)^2}{A}$.

[To face page 82.

(4) Areas of midship section.
(5) Areas of waterplanes or tons per inch.
(6) Transverse metacentre.
(7) Longitudinal metacentre.
(8) C.G.'s of waterplanes.
(9) Moment to trim one inch.
(10) Block coefficient.
(11) Prismatic „
(12) Midship section coefficient.
(13) Waterplane areas „

We will treat each of these calculations in the above order.

**Displacement.**—This has already been defined. We have seen that displacement is merely a measurement of the volume of water displaced by the ship. The unit that was taken was a volume of 35 cubic ft. This volume of sea-water weighs one ton. Therefore the displacement in cubic feet of a ship when floating in sea-water divided by 35 gives the weight of the ship in tons. When the ship is floating in fresh-water, in order to get the weight of the ship we have to divide the displacement in cubic feet by the factor $\frac{35 \times 64}{62 \cdot 5} = 35 \cdot 84$.

This figure is arrived at from the fact that 1 cubic ft. of salt-water weighs 64 lbs. and 1 cubic ft. of fresh-water weighs 62·5 lbs. at a temperature of 62° F.

A stated displacement for a cargo-carrying ship must correspond to some draught. The limits of the draught of a vessel when in ordinary service are the light and the load draughts. The "light draught" is the draught of the ship in the "light condition," *i.e.* all cargo, coal, and consumable stores, and men out, all water out of ballast, fresh-water, and reserve feed water tanks. In fact, nothing in the ship but such as is absolutely necessary to have on board to complete the hull, machinery and equipment, including spare gear usually carried in the ship. The "load draught" is the draught fixed by statute in the condition when the ship has sufficient cargo, coal, stores, and ballast to sink her to the draught beyond which it is not generally prudent to load her. For all draughts and trims between these limits it is necessary to know the displacement. In many ships the difference between load and light displacement is greater than one-half, and sometimes is as much as nearly three-fourths of the load displacement.

The best way to represent the different displacements is, therefore, by means of a curve, and for this purpose the displacement is calculated to four or more different draughts in the displacement sheet calculation.

In the first displacement sheet the calculations for the positions of the centre of buoyancy are also made. These are done at each stage of draught for which the displacement is calculated. We have thus the means of setting off curves giving the positions of centre of buoyancy corresponding to any draught.

In order to illustrate clearly the methods of the first displacement sheet it will be better to consider the simple case of the solid shown in fig. 68.

The surface $A_1 A_3 C_3 C_1$ we may consider as part of a ship's form, and consequently that it is fair. Let $a_1 a_3 c_3 c_1$ be the projection of it on a plane of reference $xz$. Let $a_1 a_3 = 2x$, $a_1 c_1 = 2z$. To find the volume $a_1 C_3$ we can apply the rule of the previous chapter, viz. divide the volume by parallel sections, calculate the area of each section, and integrate these areas. Suppose, in this case, we divide the solid by a plane parallel to $A_1 a_1 a_3 A_3$ and midway between $a_1$ and $c_1$, we get a new section $B_1 b_1 b_3 B_3$ as shown. There are thus three sections $z$ apart. Let A, B, C represent their areas. Then $(A + 4B + C)\frac{z}{3}$ is the volume of solid, using Simpson's First Rule.

Again, if we divide the solid by a plane parallel to $A_1 a_1 c_1 C_1$ and midway between $a_1$ and $a_3$, we get a new section $A_2 a_2 c_2 C_2$ which intersects the first sectional plane in $B_2 b_2$.

Let the areas of the sections formed in this way be A′ A″ A‴, then the volume equals $(A' + 4A'' + A''')\frac{x}{3}$. This must be equal to $(A + 4B + C)\frac{z}{3}$.

This affords a useful means of cross-checking the result.

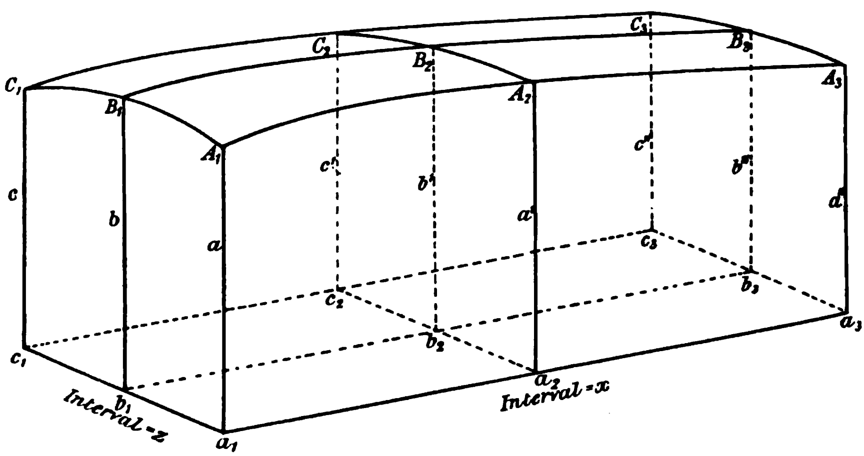



FIG. 68.

Let the ordinates of A be $a$, $a'$, $a''$,
" " B be $b$, $b'$, $b''$,
" " C be $c$, $c'$, $c''$;

then
$$A = (a + 4a' + a'')\frac{x}{3},$$
$$B = (b + 4b' + b'')\frac{x}{3},$$
$$C = (c + 4c' + c'')\frac{x}{3}.$$

$$\text{Volume of solid} = (A + 4B + C)\frac{z}{3}.$$

$$\therefore \quad " \quad " = \left\{a + 4b + c + 4(a' + 4b' + c') + a'' + 4b'' + c''\right\}\frac{xz}{9}.$$

Taking it the other way: integrating areas parallel to $A_1 a_1 c_1 C_1$,

$$A' = (a + 4b + c)\frac{z}{3},$$

$$A'' = (a' + 4b' + c')\frac{z}{3},$$

$$A''' = (a'' + 4b'' + c'')\frac{z}{3}.$$

$$\text{Volume of solid} = (A' + 4A'' + A''')\frac{x}{3}.$$

$$\therefore \quad " \quad " = \left\{ a + 4b + c + 4(a' + 4b' + c') + a'' + 4b'' + c'' \right\}\frac{xz}{9},$$

which is the same expression as above.

The quantity within the bracket is called the "function" of the displacement, and $\frac{xz}{9}$ the multiplier for displacement.

The displacement table columns are ruled so that this cross-check is automatically performed. A displacement calculation is a laborious operation, and accuracy is most probably assured when the result is checked by the above method. It is, however, possible that an error, the same in amount, has crept into both processes.

This method has the further advantage in so far as it is a convenient one to apply in finding both the longitudinal and vertical positions of the centre of buoyancy by the same operation.

Applying this to the simple case; if we integrate the moments of the areas in the $z$ direction we can find the $z$ coordinate of the centre of gravity by dividing by the volume, and similarly by integrating the moments of areas in the $x$ direction we can get the $x$ coordinate.

For simplicity let the actual figures be assumed,

$$a = 1\cdot5 \qquad b = 1\cdot6 \qquad c = 1\cdot4$$
$$a' = 1\cdot8 \qquad b' = 1\cdot9 \qquad c' = 1\cdot7$$
$$a'' = 1\cdot6 \qquad b'' = 1\cdot7 \qquad c'' = 1\cdot5$$
$$x = 3' \qquad z = 3'.$$

$$\text{Then displacement or volume} = 63\cdot6 \times \frac{xz}{9} = 63\cdot6 \text{ cu. ft.}$$

$$\text{Distance of C.B. in } z \text{ direction} = \frac{63\cdot0}{63\cdot6} \times z = 2\cdot97 \text{ ft.}$$

$$" \quad " \quad x \quad " \quad = \frac{64\cdot2}{63\cdot6} \times x = 3\cdot03 \text{ ft.}$$

This method is carried out in the displacement sheet: see Table VIII.

*Preparation of Offsets, etc.*—At the head of the sheet (Plate II.) are a few of the important particulars connected with the ship:—

Name.
Length for displacement.
Moulded dimensions.
" draught.
Particulars of depth of keel.
Horizontal interval.
Vertical interval.
Positions of perpendiculars and C.B.P.

### Table VIII.

| Simpson's Multipliers. | Number of Sections. | First Plane x Direction. | | Second Plane x Direction. | | Third Plane x Direction. | | Sum of Functions (Areas). | Functions for Displacement. | Leverages. | Functions for Moments. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 4 | | 1 | | | | | |
| 1 | 1 | 1·5 | 1·5 | 1·8 | 1·8 | 1·6 | 1·6 | | 10·3 | 0 | ... |
| | | 1·5 | ... | 7·2 | ... | 1·6 | ... | 10·3 | | | |
| 4 | 2 | 1·6 | 6·4 | 1·9 | 7·6 | 1·7 | 6·8 | | 43·6 | 1 | 43·6 |
| | | 1·6 | ... | 7·6 | ... | 1·7 | ... | 10·9 | | | |
| 1 | 3 | 1·4 | 1·4 | 1·7 | 1·7 | 1·5 | 1·5 | | 9·7 | 2 | 19·4 |
| | | 1·4 | ... | 6·8 | ... | 1·5 | ... | 9·7 | | | |
| Sum of Functions of Area. | | ... | 9·3 | ... | 11·1 | ... | 9·9 | ... | 63·6 | ... | 63·0 |
| Functions for Displacement. | | 9·3 | | 44·4 | | 9·9 | | 63·6 | | | |
| Leverages. | | ... | | 1 | | 2 | | | | | |
| Functions for Moments. | | ... | | 44·4 | | 19·8 | | 64·2 | | | |

Suppose we have a set of lines which are fair, the first thing to arrange is the position of the sections. The directions of integration are vertical and longitudinal, therefore in each of these directions the sections must be spaced to suit Simpson's Rule, or whatever rule is being used. It is suitable in fairing up the form to use sections spaced according to Simpson's First Rule, and generally it is the practice to have the number of longitudinal intervals 10, 12, 14, or 16, according to the length and form of the ship.

The perpendiculars for the displacement length, as it is called, can be chosen somewhere near the endings of the waterplanes (see definition of perpendiculars for length in Chapter VI.). This length is divided up into the required number of intervals, and generally the interval at each end is halved by the interposition of another section. This is necessary in full vessels, as the form changes most rapidly at the ends in this class.

These longitudinal intervals give the positions of the transverse sections; these may be transferred to a body plan.

The vertical interval is determined by dividing the distance between the load draught and the keel into eight to twelve spaces. The form can be faired up originally to waterlines spaced according to this rule.

When vessels such as yachts and warships have a keel which is not parallel to the load waterline, the spacing is taken from the L.W.L. to the line which makes the lowest completed waterline. Below this the volume and C.G. are independently calculated. This part is called an "appendage."

The forms of nearly all ships change rapidly at the draughts below the turn of bilge, so it is convenient at this part to subdivide the interval by the addition of one or more waterplanes. All these waterlines are drawn on the body plan, and the half ordinates can be read off either there or in the half-breadth plan.

In a displacement sheet the columns are generally headed :—

| | | | |
|---|---|---|---|
| No. 1 | Waterline : | keel, | |
| ,, 2 | ,, | $a$ | feet, |
| ,, 3 | ,, | $b$ | ,, |
| ,, 4 | ,, | $c$ | ,, |
| ,, 5 | ,, | etc. etc. | |

—where $a$, $b$, $c$, etc. are the draughts of the waterline amidships.

The half ordinates are read off in feet and decimals of a foot, and they are transferred to the columns under the proper headings, and in the same horizontal line as the number of the section given in the vertical column at the extreme left of the table. The half ordinates or offsets in Plate II. are in heavy-faced type.

For the purpose of obtaining the displacement at any draught a curve is required, and in order to obtain a displacement curve at least four spots are necessary; hence the integration is performed in four or more stages. Referring to Plate II. there are four stages :—

| | | | | |
|---|---|---|---|---|
| 1st | to waterplane | No. 2 at | 2 | feet draught, |
| 2nd | ,, | No. 4 ,, | 6 | ,, |
| 3rd | ,, | No. 6 ,, | 10 | ,, |
| 4th | ,, | No. 8 ,, | 14 | ,, |

Considering only the first stage, we have the multipliers $\frac{1}{2}$, 2, and $\frac{1}{2}$ for vertical integration, and we have the multipliers $\frac{1}{2}$, 2, $1\frac{1}{2}$, 4, 2, 4, 2, 4, 2, 4, $1\frac{1}{2}$, 2, and $\frac{1}{2}$ for longitudinal integration.

These give the function of displacement = 500·29.

This has to be multiplied by $\frac{x}{3}$ which is 11·243 feet,

and ,, $\frac{y}{3}$ ,, $\frac{2}{3}$ ,,

and ,, 2 for both sides,

and divided ,, 35 for tons.

The multiplier for displacement is therefore $\frac{11 \cdot 243 \times 2 \times 2}{3 \times 35} = \cdot 4283$.

∴ the displacement up to waterplane No. 2 = 214 3 tons.

Taking the column headed "Multiples of Areas," if these be multiplied by the levers about No. 5 section we get the next column, "Moment Functions"; to each of these must be affixed an algebraic sign, and the algebraic sum of the "Moment Functions" gives the function of moment about No. 5 section. Leverages forward of No. 5 may be called positive, and those aft will then be negative.

The multiplier in this case to give the true moment in foot tons is $\frac{x}{3} \cdot \frac{y}{3} \cdot \frac{2}{35} \cdot x$.

Function of moment $= 393 \cdot 84 - 280 \cdot 98 = 112 \cdot 86$.

„ displacement $= 500 \cdot 29$.

Therefore the distance of centre of buoyancy from No. 5 section is equal to $\frac{112 \cdot 86 \times 33 \cdot 729}{500 \cdot 29} = 7 \cdot 61$ ft.; and as the greater moment was on the forward side of No. 5, the longitudinal centre of buoyancy is 7·61 ft. forward of that section.

The vertical position of the centre of buoyancy is obtained by taking moments of the multiples of waterplane areas about the base line or top of keel.

In yachts and warships it is usually taken about the L.W.L.

In Plate II. the lever multipliers about the base line are 0, ½, 1, 2, 3, etc. for waterplanes respectively; this gives function of moment up to No. 2 waterplane, equal to 311·64 and height of centre of buoyancy $= \frac{311 \cdot 64 \times 2}{500 \cdot 29} = 1 \cdot 246$ ft. above the top of keel.

The next stage is from waterplane No. 2 to No. 4.

Here it is sometimes the practice to first calculate the displacement between these waterlines, but it is more convenient to add the functions obtained in the preceding stage to the functions obtained in the last stage, so that the displacement, L.C.B., and V.C.B., can be obtained for the whole volume up to the last waterline.

If the displacement is calculated in separate stages the position of centre of buoyancy of the total volume is found as follows:—

Let $v$ = volume up to draught $d$,
„ $h$ = distance of centre of buoyancy of $v$ from axis,
„ $v_1$ = volume between draughts $d$ and D,
„ $h_1$ = distance of centre of buoyancy of $v_1$ from axis;

then total volume $= v + v_1$.

Centre of buoyancy of $v + v_1$ from axis $= \frac{vh + v_1 h_1}{v + v_1}$.

*Multipliers.*—In the displacement table, after the column headed "Multiples of Areas" has been added, we get the function of displacement. Summing the vertical column headed "Moment Functions" we get the function of moment of the displacement longitudinally, and the horizontal column of moment functions gives the function of moment of the displacement vertically.

The multipliers for the functions are as follows:—

For displacement we have $\frac{x}{3}$ for the horizontal integration.

" " $\frac{y}{3}$ " vertical "

" " 2 for both sides.

" " $\frac{1}{35}$ for cubic feet to tons.

In addition we have the factor $x$ for horizontal moments,
and " " " $y$ for vertical "

These give—

Multiplier for tons displacement . . $= \frac{2.x.y}{9 \times 35} = \cdot 4283$ ft.

" " foot tons moment vertically $= \frac{2.x.y^2}{9 \times 35}, = \cdot 8566$ ft.

" " " " horizontally $= \frac{2.x.^2y}{9 \times 35} = 14 \cdot 44$ ft.

This also gives: Let M = vertical moment function.
$M_1$ = horizontal " "
F = displacement function.

Position of centre of buoyancy vertically $= \dfrac{M \frac{2xy^2}{9.35}}{F \frac{2xy}{9.35}} = \frac{M}{F} y.$

Position of centre of buoyancy longitudinally $= \dfrac{M_1 \frac{2x^2y}{9.35}}{F \frac{2xy}{9.35}} = \frac{M_1}{F} x.$

The rule, therefore, for position of centre of buoyancy is to divide the moment function by the displacement function and multiply the result by the interval used in the leverage factors.

**Curves of Displacement and Centres of Buoyancy.**—The displacement and position of C.B. are set off in terms of draught, which is generally set off vertically.

Fig. 69 shows the method of setting up these curves. O A is the vertical line on which is marked a scale of draughts amidships.

At the draughts in the displacement sheet, for which the displacement has been calculated, horizontal lines can be drawn, and the corresponding displacements set off to a convenient scale.

The range we can have is from zero up to the load displacement. It is usual to draw this curve to a draught slightly greater than the load draught.

The V.C.B. heights are set off in a manner similar to setting off the displacement.

The scale of each curve is marked on the top so that the lengths of any horizontal ordinate can be easily measured.

The L.C.B. has to be set off with reference to a line which represents the C.B.P. or any axis which has been chosen for reference. The distances can be measured to one side or other of this line according as the L.C B. is before or abaft the C.B.P.

*Intermediate Spots on the Displacement.*—In Plate II. the table is arranged to calculate the displacement and positions of centre of buoyancy at four stages of draught, viz. at waterplanes Nos. 2, 4, 6, and 8.

Intermediate spots can be calculated by using the $\frac{5}{8}$ rule. For instance, taking waterlines Nos. 2, 3, and 4 we can get the displacement between

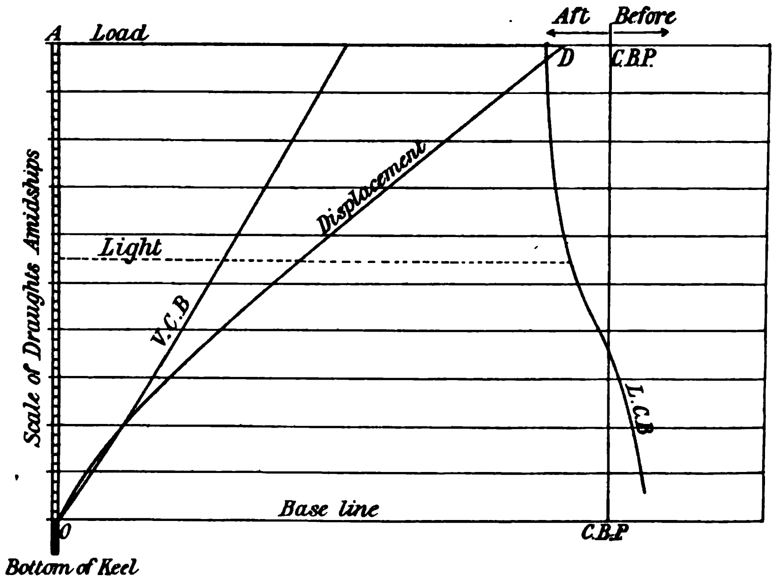



Fig. 69.

waterlines Nos. 2 and 3, *i.e.* from the waterline at 2 ft. draught to the waterline at 4 ft. draught. See Table IX.

TABLE IX.

| | Functions. | Multipliers. | Functions. |
|---|---|---|---|
| No. 2 w.l. | 242·66 | 5 | 1213·3 |
| No. 3 w.l. | 296·77 | 8 | 2374·16 |
| | | | 3587·46 |
| No. 4 w.l. | 330·10 | -1 | -330·10 |
| | | | 3257·36 |

The multiplier for the vertical integration is $\frac{x}{12}$. This gives a coefficient of ·107.

∴ Displacement from No. 2 w.l. to No. 3 w.l.

$$= 3257{\cdot}36 \times {\cdot}107 = 348{\cdot}5 \text{ tons},$$

but Displacement to w.l. No. 2 = 214·3 tons.

∴ Total to w.l. No. 3 = 562·8 tons.

*Appendages Correction.*—The form that is represented by the half ordinates in the displacement sheet is the form whose sections are given by the finished lines, *i.e.* the outside surface of the frames.

Additional buoyancy is given by the appendages, which include in an ordinary vessel—

(1) The shell or skin.
(2) The keel or keel doubling.
(3) The rudder.
(4) The bossing of the shafting and the propellers.

The volume and the position of the centre of buoyancy of these appendages is calculated, and a correction is made on the results of the displacement sheet calculation.

(1) *Skin Correction.*—To make this correction, the area of wetted surface can be estimated at a certain draught. One way of estimating this area is to find the mean girth and multiply by the mean length of waterline, but a convenient and sufficiently correct method between the light and load draughts is to use the following formula:—

$$A = 1.7L\delta + \frac{V}{\delta},$$

where $L$ = displacement length.
$\delta$ = draught.
$V$ = volume of displacement at that draught.

Having the area of wetted surface, the mean thickness of shell plating can be easily obtained, and hence the displacement of the shell plating obtained.

Let $t$ = thickness of shell in ft., then $\frac{At}{35}$ = skin correction.

At each stage of draught this can be done, and the result added to the results for the displacement of the form.

(2) *Keel* or *Keel Doubling.*—The displacement of the keel or doubling can be very easily calculated, and it remains a constant correction throughout the different draughts.

(3) *Rudder.*—This can also be separately calculated by taking approximate dimensions for it at each draught. It is hardly necessary to make this correction in ships with single plate rudders.

(4) *Bossing* or *Shafting and Propellers.*—This forms the largest correction, especially in twin-screw vessels with bossing round the shafting.

If the vessel has no bossing, then the volume of the shafting and of the brackets and propellers has to be estimated.

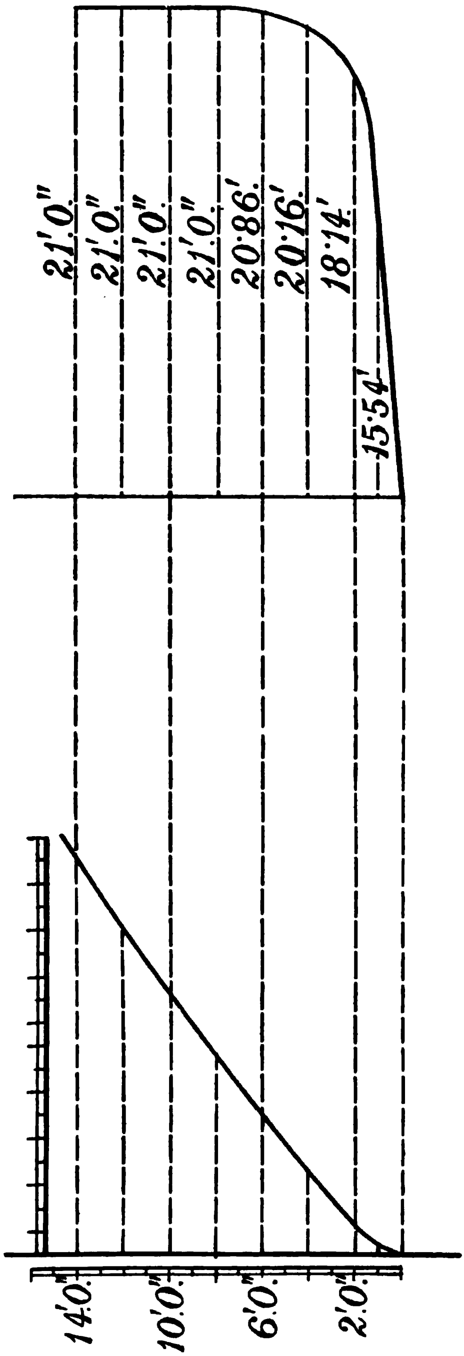



Fig. 70.

If the vessel's hull is bossed to take the shaft, it is necessary to calculate the volume of the bossing by taking the areas of sections and integrating these areas by some rule.

It is usually unnecessary to correct the positions of L.C.B. and V.C.B. for any of the appendages 1, 2, or 3, but the bossing being considerable in screw ships, and also being at the stern, account has to be taken of the change it makes in the position of the centre of buoyancy. Therefore it is necessary to make a calculation according to the formula on page 88 in order to get the position of the centre of buoyancy of the vessel when taking bossing into account, and for this purpose the position of the centre of buoyancy of the bossing has to be estimated. In yachts and warships the appendage below the lowest waterline must be calculated by integrating the areas and moments of the sections.

*Curve of Displacement in Fresh-water.*—Having a certain volume of displacement, say V cubic ft., the weight of sea-water displaced is $\frac{V}{35}$, but if the vessel be floating at this displacement in fresh-water the weight is now $\frac{V}{x}$, where $x$ represents the number of cubic ft. of the water in a ton. In absolutely fresh-water at a temperature of 62°, $x$ is 35·84.

If the ordinates of the displacement curve be altered in this proportion $\frac{35}{35 \cdot 84}$ we get a new curve which will be the curve of displacement for the ship in fresh-water. In some rivers or harbours the water is brackish, and in order to get the correct weight of a ship the density of the water should be taken by a hydrometer.

**Areas of Midship Section.**—The midship section is usually the section at the centre between perpendiculars. In the calculation of Plate II. it is 3·9 ft. forward of section No. 5, but is of the same form as that section.

At the right hand of fig. 70 the midship section has been drawn. From the displacement sheet we have the functions of the area of section No. 5 up to the different waterlines Nos. 2, 4, 6, and 8, as in the following table :—

TABLE X.

| No. of Waterline. | Functions of Area. | Multiplier $\frac{2y}{3}$ | Areas of Midship Section. |
|---|---|---|---|
| 2 | 39·12 | 1·333 | 52·2 |
| 4 | 158·76 | ... | 211·1 |
| 6 | 284·62 | ... | 379·5 |
| 8 | 410·62 | ... | 546·9 |

These areas can be plotted in terms of draught, as to the left of fig. 70, and a curve obtained, in a similar manner to the displacement curve.

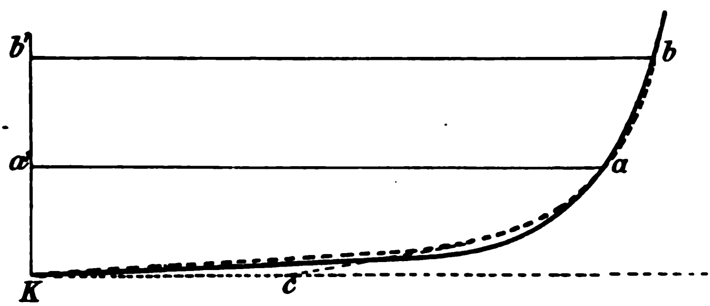



FIG. 71.

*Inaccuracy of area at the lowest part of sections.*—It will be noticed that half ordinates of considerable length have been assumed at waterplane No. 1, which is at the top of keel.

In the actual form the ordinates at this place are zero. It has been found, however, that for the shape of a full section divided up by lines as shown in fig. 71 at $a^1$ $a$, $b^1$ $b$, Simpson's Rule does not give this area correctly.

The actual section is given by the full line, while the dotted line represents what the area is like that would be more correctly measured by the ordinates $a^1 a$, and $b^1 b$. To correct for this an ordinate K C is assumed at the base line. This is fixed by experience generally, but can be determined, if necessary, by the planimeter.

The following table gives the results of tests of areas of curves and the value of K C necessary to ensure accuracy. Fig. 72 shows the different curves for which K C has been calculated.

TABLE XI.

| | | Area by Integrator (Square Inches). | Area by Simpson's First Rule (Square Inches). | Value required for K C in inches. |
|---|---|---|---|---|
| A | Section 1 | 14·64 | 15·22 | −1·74 |
| | ,, 2 | 12·46 | 12·96 | −1·5 |
| | ,, 3 | 10·81 | 11·12 | − ·93 |
| B | Section 1 | 14·42 | 12·5 | 7·68 |
| | ,, 2 | 12·16 | 11·25 | 3·64 |
| | ,, 3 | 10·23 | 10·0 | ·92 |
| C | Section 1 | 8·09 | 8·12 | − ·18 |
| | ,, 2 | 6·66 | 7·10 | −2·64 |
| | ,, 3 | 5·57 | 5·9 | −1·98 |

The figure K C $a$ $b$ now gives an area which can be more correctly calculated by the 1, 4, 1 rule, and will approximate more closely to the actual area of the section. The values of the K C ordinates for each section are inserted in the column headed "Waterline No. 1" in Plate II., and the calculation for displacement is made thereby more correct. Sometimes there is a part below waterplane No. 1 considered as an appendage.

**Areas of Waterplanes, or Tons per Inch.**—The areas of waterplanes can be easily obtained from the sums of the vertical columns. Set these out in a table of the following form and multiply by $\frac{2x}{3} = 22{\cdot}486$ ft. and we get the actual areas in square feet of the whole waterplanes.

TABLE XII.

| No. of Waterplane. | Function of Areas of Waterplanes. | Areas in Square Ft. of Waterplanes. | Tons per Foot. | Tons per Inch. |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1½ | 190·31 | 4280·0 | 122·3 | 10·2 |
| 2 | 242·66 | 5456·0 | 155·9 | 12·97 |
| 3 | 296·77 | 6672·0 | 190·7 | 15·90 |
| 4 | 330·10 | 7423·0 | 212·1 | 17·67 |
| 5 | 355·55 | 7995·0 | 228·4 | 19·03 |
| 6 | 376·84 | 8473·0 | 242·1 | 20·17 |
| 7 | 395·83 | 8900·0 | 254·3 | 21·19 |
| 8 | 413·66 | 9301·0 | 265·8 | 22·15 |

A curve can be set up in terms of draught for areas of waterplanes, as shown in fig. 73.

*Tons per Inch.*—The term "tons per inch" means the number of tons of weight which require to be added to the ship in order to increase the draught one inch in salt water, supposing the added weight so placed that the new waterline is parallel to the old. Sometimes "tons per foot" is spoken of and used in the displacement sheet calculations. "Tons per foot" is the number of tons that would sink the ship bodily one foot.

Suppose we consider a ship to have been sunk bodily through one inch change of draught; then the change in buoyancy equals $A \times \frac{1}{12}$ cubic feet, where $A$ = area of waterplane. That is, we suppose the waterplane area to be constant throughout the change of draught of one inch or $\frac{1}{12}$ of a foot, so

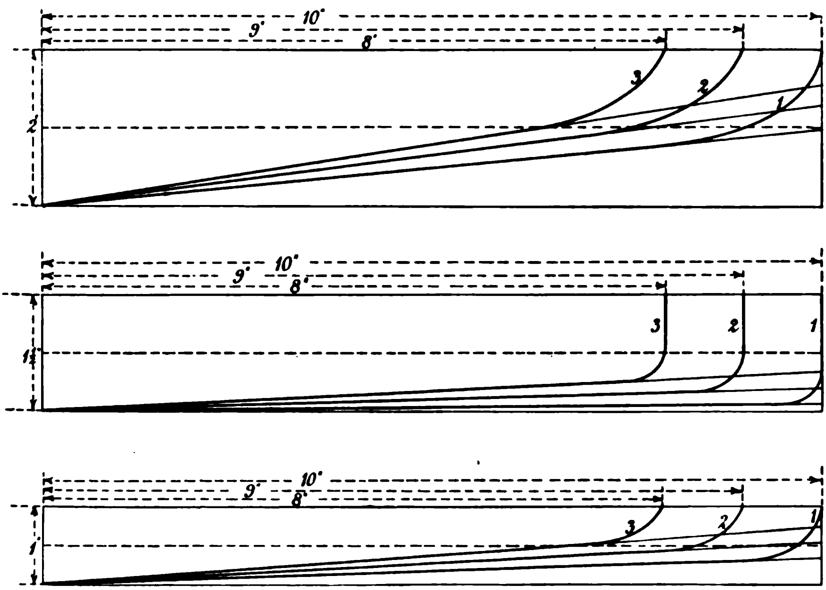



FIG. 72.

that the added layer $= A \times \frac{1}{12}$. This in tons is $\frac{A}{12 \times 35} = \frac{A}{420}$. Therefore to get "tons per inch" we divide the area of the waterplane in square feet by 420.

It is to be noted that this formula only gives the tons to sink the ship one inch correctly for a wall-sided ship, or where $A$ is unaltered throughout the change of draught, but it is quite correct to speak of $\frac{A}{420}$ as the tons *per* inch at the waterline whose area is $A$.

Suppose the ship to have sunk one foot, and the area of the original waterplane $= A_1$, and the area of the new waterplane at one foot deeper draught $= A_2$. Then $\frac{A_1 + A_2}{2}$ will be, very nearly, the mean waterplane area.

$\therefore \frac{A_1 + A_2}{70}$ would be more approximately the tons to sink the ship one foot than the value $\frac{A_1}{35}$. But at the original waterline the tons *per* foot will be $\frac{A_1}{35}$.

Since "tons per inch" is obtained from the waterplane areas by dividing

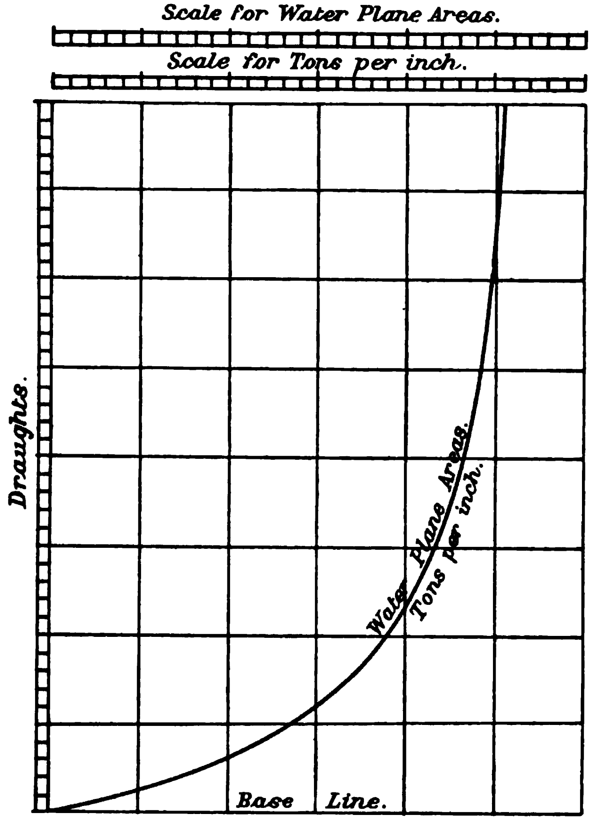



FIG. 73.

by a constant figure, viz. 420, the curve of tons per inch can only differ from the waterplane areas' curve in scale. It is usual to make one curve do for both and to have two scales, one 420 times the other, for measurement of the ordinates.

The tons per inch curve is convenient for calculating the new draught after a known weight is put aboard; or, when the new draught is known, the change in weight can be estimated.

# CHAPTER IX.

## METACENTRES.

The term "metacentre" will be fully treated in that part that deals with stability. Here we are only concerned with the part it plays in the initial stability of the vessel.

If any body floating in equilibrium in still water be slightly inclined by any force which leaves its displacement unchanged, so that all particles in the body move parallel to a fixed vertical plane, the plane parallel to this vertical plane passing through the centre of buoyancy in the original position may, or may not, contain the centre of buoyancy in the inclined position. If the plane through the original centre of buoyancy contains the centre of buoyancy in the inclined position, then the vertical straight line through the centre of buoyancy in the original position, and the vertical through the centre of buoyancy in the inclined position, will intersect. When the angle of inclination becomes indefinitely small, this point of intersection is called the metacentre for that direction of inclination.

When the centre of buoyancy in the inclined position does not lie in the plane of inclination through the original centre of buoyancy, the verticals do not intersect. As the angle of inclination becomes indefinitely small these verticals may, or may not, intersect. If they do not intersect, the intersection of the projection on to the plane of inclination of the vertical through the centre of buoyancy in the inclined position, with the vertical through the original centre of buoyancy, is called the metacentre for that direction of inclination.

In representing this by fig. 74 the plane of the paper is the plane of inclination. The centre of buoyancy in the inclined position, and the vertical through it, are not in the plane of the paper, but only their projections are drawn, so that on the figure we have the projections of the verticals, and these projections must intersect in the point we have called the metacentre.

When the verticals do not intersect there will be a couple tending to incline the body in a plane of inclination at right angles to the plane of the paper, which is the one we are considering, and this couple must be balanced by an extraneous couple in order that the body may be constrained to move only in the chosen plane of inclination.

The metacentre M is important, as its position relatively to the centre of gravity determines whether the vessel is in stable, neutral, or unstable equilibrium.

If M is above G the vessel is in stable equilibrium.
" M coincides with G " " neutral "
" M is below G " " unstable "

When a vessel in stable equilibrium is inclined slightly by an extraneous force, it will return to its original position when the force ceases to act.

In neutral equilibrium it will neither return nor upset, and in unstable equilibrium it will continue to heel over further.

**Transverse Metacentre.**—Let fig. 74 represent a transverse vertical section of the ship through the centre of buoyancy B. W L is the waterline when the ship is floating upright and in equilibrium. Let it receive a small inclination $\theta$ in the plane of the paper, *i.e.* a vertical plane through B. Let G be the position of the centre of gravity of the ship, which must also be in plane of inclination, and $B_1$ the projection of the position of the new centre of buoyancy on the plane of the paper when the vessel is inclined through an angle $\theta$ and has the same displacement as in the original position. Let $W_1L_1$ be the waterline when the vessel is thus inclined.

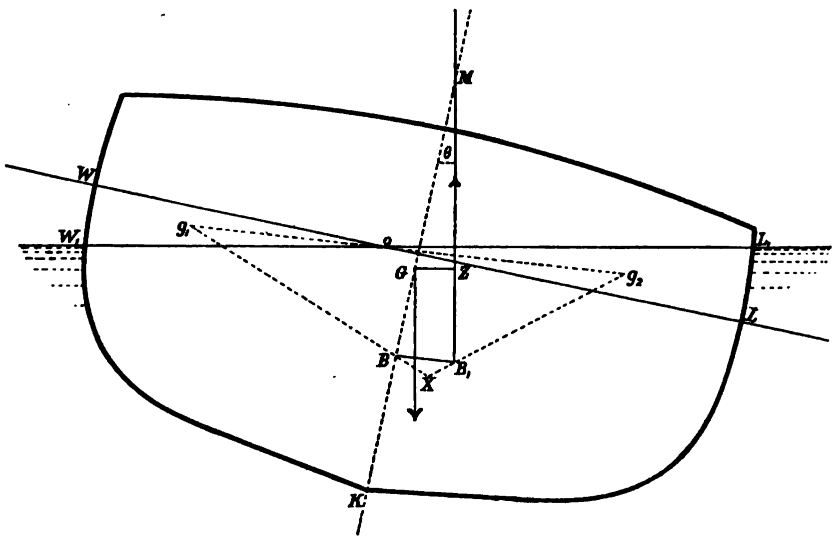



FIG. 74.

Let W L intersect $W_1 L_1$ in O.

In inclining the ship to this angle a volume $W_1$ O W of wedge shape has been transferred to L O $L_1$ along a line joining the centres of gravity of the wedges.

*Formula for the Distance B M.*—The metacentric height is the distance G M. M in the figure is the metacentre for the transverse vertical plane of inclination and is called the transverse metacentre. If the inclination had been made in a fore-and-aft direction the metacentre would have been called the longitudinal metacentre.

The method of determining the value of B M will, however, apply to any plane of inclination.

By hypothesis, the displacement remains the same.

$\therefore$ the volume of wedge W O $W_1$ = volume of wedge L O $L_1$.

Let $g_1$ and $g_2$ be the projections of the centre of gravity of W O $W_1$ and L O $L_1$ respectively on the transverse plane of inclination.

Join $g_1$ B and $g_2$ $B_1$; produce them to meet in X.
Then centre of gravity of $W_1$ K L must lie in line $g_1$ B,
" " " " $g_2 B_1$.
Therefore centre of gravity of $W_1$ K L is at X,
$v$ = volume of wedge,
and V = total volume of displacement.
Taking moments about X we have,

$$v.g_1 X = V.BX,$$

and $$v.g_2 X = V \cdot B_1 X.$$

$$\therefore \quad \frac{g_1 X}{g_2 X} = \frac{BX}{B_1 X}.$$

$\therefore$ $BB_1$ is parallel to $g_1 g_2$, and $\frac{g_1 g_2}{BB_1} = \frac{V}{v}$.

$$\therefore \quad v.g_1 g_2 = V.BB_1,$$
$$= V.BM.\theta \quad \text{(as } \theta \text{ is very small)}.$$

Now $v = \int \frac{1}{2} y^2 . \theta dx$, where $x$ is the direction of length,

and $y$ = distance O L or O W.

$og_2 = \frac{2}{3} y$, for a wedge of length $dx$.

If we consider all the sections of the wedges as triangles the moment of the wedges will be $v.g_1 g_2 = \int \frac{1}{2} y^2 \theta \times \frac{2}{3} y . dx$,

$$\therefore \quad V.BM.\theta = \frac{1}{3}.\theta \int y^3 . dx.$$

$$\therefore \quad BM = \frac{1}{3} \int \frac{y^3 . dx}{V}.$$

The integration must be made between the extreme values of $x$ for all values of $y$ on both sides of O.

Where O L is not equal to O W,

the formula becomes $BM = \frac{1}{3} \int \frac{(y_1{}^3 + y_2{}^3) dx}{V}$.

If the waterline is symmetrical about the middle line the values of $y$ will be the same for each value of $x$ and the formula becomes $BM = \frac{2}{3} \int \frac{y^3 dx}{V}$.

It has been already shown in Chapter IV. that the expression $\frac{1}{3} \int y^3 . dx$ is the moment of inertia of the waterplane W L about an axis through O, so that $BM = \frac{I}{V}$, where I is the moment of inertia of the waterplane about the line forming the intersection of consecutive waterplanes. Waterplanes are consecutive when they are inclined to each other at a very small angle and cut off the same displacement. If, as in the case of a ship, the floating body is symmetrical about a vertical plane, O will, in the limiting value of $\theta$, lie in the plane of symmetry for consecutive waterplanes.

We have the following theorem relating to consecutive waterplanes what ever the form of the waterline may be.

*The limit of the intersection of consecutive waterplanes is a straight line which passes through the centre of gravity of the waterplane, and is perpendicular to the plane of inclination.*

In fig. 75, *a* and *b* are views of small transverse and longitudinal inclinations respectively.

If the displacement remains the same, the volume of the wedge $WOW_1$ must be equal to the volume of the wedge $LOL_1$, *i.e.* $\frac{1}{2}\theta\int y_1^2.dx=\frac{1}{2}\theta\int y_2^2dx$ (fig. *a*) where $y_1$ and $y_2$ are the ordinates of the waterline on the port and starboard sides of the body respectively.

$\therefore\ \frac{1}{2}\int y_1^2dx=\frac{1}{2}\int y_2^2dx$, $\theta$ being a constant. But in Chapter IV. it has been shown that these are the expressions for the moments of the areas of the emerged and submerged portions of the waterplane WOL about a straight line perpendicular to $y$, and these moments of two parts of an area about an

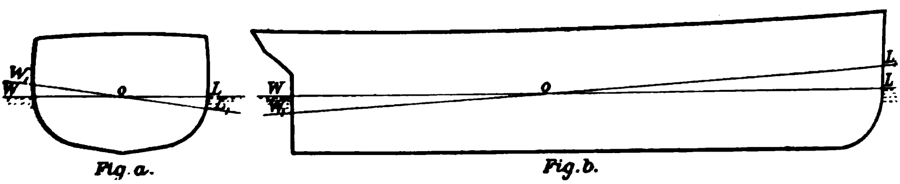



FIG. 75.

axis can only be equal when the axis passes through the centre of gravity of that area. Hence the straight line forming the intersection of the two wedges of equal volume passes through the C.G. of the waterplane. This line is perpendicular to the plane of inclination because it is the intersection of two planes which by hypothesis are each perpendicular to a vertical plane, and their intersection must be a straight line perpendicular to the plane of inclination.

The formula is approximately true when $\theta$ is very small, and the theorem is only true in the limit, *i.e.* when $\theta=0$, so that the centre of gravity through which the straight line must pass, must be the centre of gravity of the original waterplane.

For a longitudinal inclination:

Equating volumes of wedges,

$$\frac{1}{2}.\theta\int x_2^2.dy=\frac{1}{2}.\theta\int x_1^2.dy.$$

$$\therefore\quad \frac{1}{2}\int x_2^2dy=\frac{1}{2}\int x_1^2.dy,$$

and the proof holds good as in the former case.

Going back to the formula for the transverse metacentre we have the expression $BM=\frac{I}{V}$.

I is the moment of inertia of the waterplane about an axis through the centre of gravity of the waterplane, and O represents this point in fig. 75. This is true for any direction of inclination, and if applied to the longitudinal direction the value of I will be $\frac{1}{3}\int(x_2^3 + x_1^3)dx$.

There is given in Chapter IV. an alternative value for I, viz. $\int x^2.y.dx$, which is made use of in practically determining BM for a longitudinal inclination.

In a ship, O will be at the vertical middle line, and in the expression $y_1 = y_2$.

$$\text{Transverse I} = \frac{2}{3}\int y^3.dx.$$

$$\text{Longitudinal I} = \int x^2 y.dx.$$

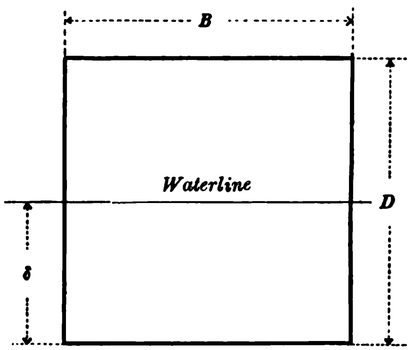



FIG. 76.

The formula is generally written

$$BM = \frac{I}{V}.$$

*Application of the Metacentric Formula to particular cases.*—For a rectangular vessel floating at a draught $\delta$ and dimensions L.B.D:—

$$\text{Transverse} \qquad I = \frac{1}{12} L.B^3.$$

$$V = L.B.\delta$$

$$\therefore \qquad BM = \frac{L.B^3.}{12L.B.\delta} = \frac{B^2}{12.\delta}.$$

$$\text{Longitutinal} \qquad BM = \frac{L^2}{12\delta}.$$

$$\text{The height of B is} \qquad \frac{\delta}{2}.$$

Therefore, if G is at a height $\frac{\delta}{2}+\frac{B^2}{12\delta}$, the vessel will be in neutral equilibrium transversely.

It will be unstable if G is at a height greater than $\frac{\delta}{2}+\frac{B^2}{12\delta}$, and stable if at a less height.

If the vessel be homogeneous the height of $G=\frac{D}{2}$.

The stability of a vessel as above therefore depends on the relation of the draught $\delta$ to D as well as on the position of G.

The vessel is stable, neutral or unstable, according as $\left(\frac{D}{2}-\frac{\delta}{2}-\frac{B^2}{12\delta}\right)$ is $\gtreqless 0$.

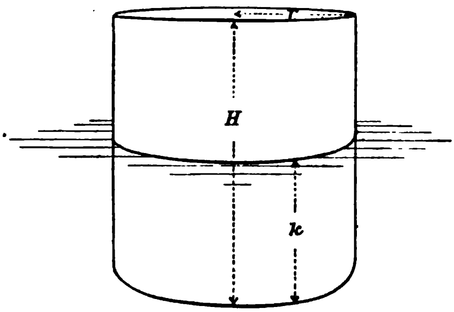


FIG. 77.

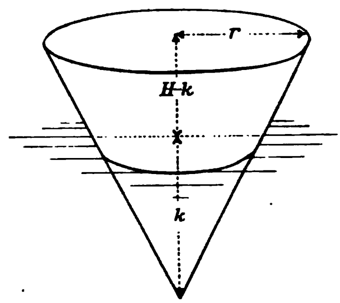


FIG. 78.

Case of a cylindrical vessel floating with its axis upright:—

Radius $=r$.
Height $=H$.
Draught $=k$.

$$\text{Then BM} =\frac{I}{V}=\frac{\pi}{4}r^4 \div \pi r^2 k,$$

$$=\frac{r^2}{4k}.$$

$$\text{Height of B above base} =\frac{k}{2}.$$

$$\therefore \quad \text{,, M ,,} =\frac{k}{2}+\frac{r^2}{4k}$$

Case of a cone floating apex down:—

Radius of base $=r$.
Height $=H$.
Draught $=k$.
Radius at waterline $=\frac{r.k}{H}$.

Then $\quad BM = \frac{I}{V} = \frac{\pi}{4}\left(\frac{r.k}{H}\right)^4 \div \frac{\pi k}{3}\left(\frac{r.k}{H}\right)^2,$

$$= \frac{3}{4}\frac{r^2.k}{H^2}.$$

Height of B above apex $= \frac{3}{4}k,$

" M " $= \frac{3}{4}k + \frac{3}{4}\frac{r^2.k}{H^2} = \frac{3}{4}k\left(1 + \frac{r^2}{H^2}\right).$

In these simple instances the volume and the value of I can be easily calculated from formulæ. In the case of a ship the calculation is more difficult, as we have to find the value of I, which cannot be done by simple formulæ.

Let us first consider the finding of the transverse BM. This is done on Plate III. of the displacement sheets. The formula for the transverse moment of inertia is $\frac{2}{3}\int y^3 dx$. The method for performing this calculation is shown in Table XIII.

TABLE XIII.

Waterplane No. 2. Longitudinal Interval = 33·729.
Vertical " = 2′ 0″.

| Number of Section. | Ordinates. | Cubes of Ordinates. | Multiplier. | Functions of Cubes. |
|---|---|---|---|---|
| 0 | ... | ... | ½ | ... |
| ½ | 0·32 | ... | 2 | ... |
| 1 | 1·28 | 2 | 1½ | 3 |
| 2 | 3·72 | 51 | 4 | 204 |
| 3 | 8·38 | 588 | 2 | 1176 |
| 4 | 14·96 | 3348 | 4 | 13392 |
| 5 | 18·14 | 5969 | 2 | 11938 |
| 6 | 16·00 | 4096 | 4 | 16384 |
| 7 | 10·62 | 1198 | 2 | 2396 |
| 8 | 5·58 | 174 | 4 | 696 |
| 9 | 2·12 | 10 | 1½ | 15 |
| 9½ | 0·80 | 1 | 2 | 2 |
| 10 | ... | ... | ½ | ... |
| | | | | 46206 |

$$\text{Multiplier} = \frac{2x}{9} = \underline{7 \cdot 4953}$$

Value of I $= 346300$ ft.$^4$

$$BM = \frac{46206}{500 \cdot 29 \times 2} = \underline{46 \cdot 18}$$

C.B. above top of keel $= \underline{1 \cdot 25}$ ft.

M above top of keel $= \underline{47 \cdot 43}$ ft.

The first column gives the numbers of the sections,
the second „ „ values of $y$,
the third „ „ „ $y^3$,
the fourth „ „ Simpson's multipliers,
and the last gives the functions.

The sum of the functions of the cubes is therefore a function of the moment of inertia.

The multiplier has the following factors:—

2 for both sides,

$\frac{x}{3}$ for the integration of the cubes,

$\frac{1}{3}$ for the formula $\frac{1}{3}\int y^3 dx$.

The multiplier is therefore $= \frac{2x}{9}$.

The displacement volume multiplier is $\frac{2xy}{9}$.

The height of the metacentre or distance BM $= \frac{I}{V}$,

$$= \frac{\text{function I} \times \frac{2.x}{9}}{\text{function V} \times \frac{2.x.y}{9}},$$

$$= \frac{\text{function I}}{(\text{function V})\, y}.$$

We can thus find a series of values of B M by calculating a series of values of I, transverse moment of inertia of waterplanes, and dividing by the corresponding volume of displacement.

**Metacentric Diagrams.**—The metacentric diagram is always associated with the vertical centre of buoyancy curve. At each stage of draught we have, from the displacement sheet, the height of the centre of buoyancy, and the height of the metacentre above the centre of buoyancy. Setting up the values of B M in terms of the draught does not afford any useful information; it is better to represent the height of M in relation to B and also to K the base line.

A diagram of this nature is shown below in fig. 79.

As in the other diagrams, let O A be a vertical line which gives the scale of draughts. At the required draughts set off, in the horizontal direction, the values of K B, B M, which give K M.

It is generally convenient to set these off to the same scale, but for convenience they are set off as O B and C D in the figure.

At any draught we can therefore, by drawing a horizontal line, find K B and B M, and, if the displacement curve is on the diagram, we can easily find the corresponding displacement.

The advantage of this diagram is that one can easily find the initial stability of the vessel in all conditions.

Sometimes this diagram is drawn in the form as shown in fig. 80.

O A, as before, is a vertical draught scale. Through O a line O O¹ is drawn at 45°. Horizontal lines are drawn at the draughts from the displacement sheet, and where these intersect O O¹ perpendiculars are drawn, and the distance down of the corresponding centre of buoyancy is marked, such as $C_1 B_1$, $C_2 B_2$, $C_3 B_3$, and $C_4 B_4$. The curve $B_1 B_2 B_3 B_4$ is therefore a vertical centre of buoyancy curve. The heights $B_1 M_1$, $B_2 M_2$, $B_3 M_3$, and $B_4 M_4$ are then set up on these vertical lines. The same scale as that of the draughts is used.

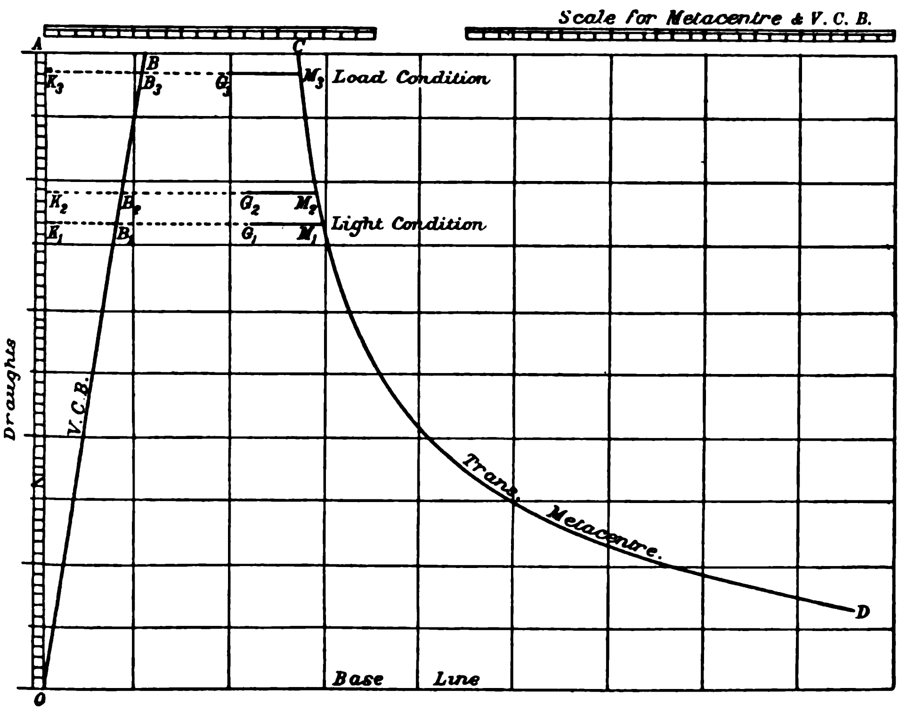



FIG. 79.

We thus have a new curve $M_1 M_2 M_3$ and $M_4$ which is the transverse metacentric curve.

These curves fulfil the same functions as those on fig. 79, although they are represented in a different way.

It is better to have on these diagrams the displacement curve, so as to be able to obtain values of displacement corresponding to the metacentric heights.

"Metacentric height" sometimes means the height B M, but in referring to the metacentric height of a vessel it is oftener the distance of M from the centre of gravity that is meant. This metacentric height determines the initial stability of the vessel and is called G M.

It will be seen that the height of M can be easily found from the displacement sheet calculation. The height of G, however, is not altogether dependent on the form, and depends upon the vertical distribution of the weight of or in the ship. The height of G varies with different conditions of loading. Therefore the G M varies with different conditions of loading.

It is a very laborious task to calculate the vertical height of the centre of

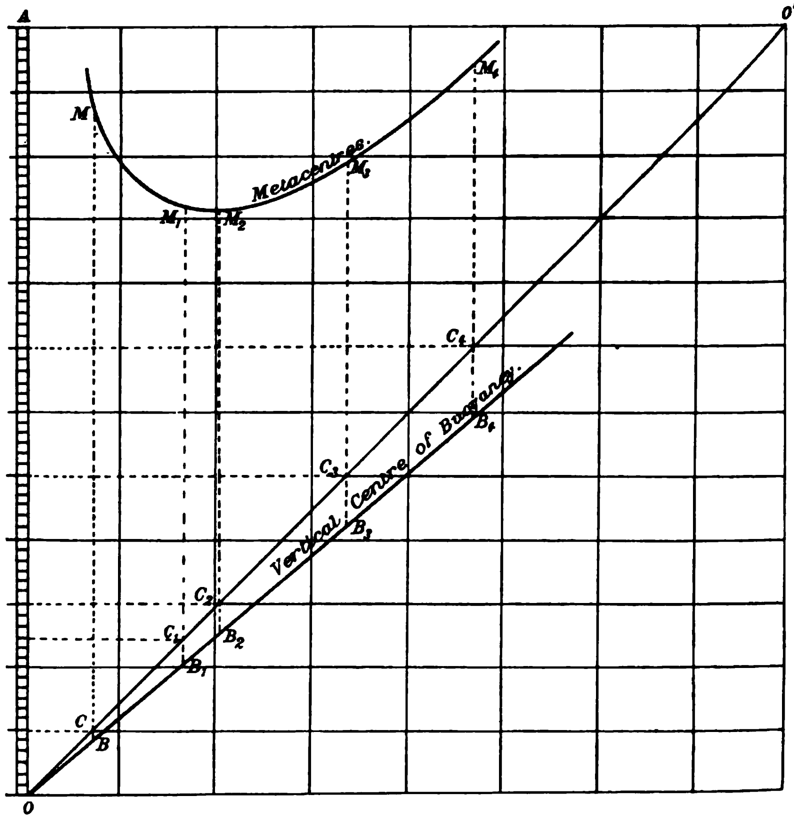



FIG. 80.

gravity of a vessel. In the process of designing, the height is approximately estimated by coefficients from previous types, these coefficients generally being a ratio of height of centre of gravity to moulded depth. For many purposes in design this is sufficient, but frequently cases arise where it is necessary to make a detailed calculation.

But after the vessel has been constructed and is approaching completion the G M can be accurately determined by experiment, and as the height of M is known from the calculations, the actual height of G in the experimental condition can be found.

This experiment is carried out on all vessels of importance, unless there have been sister vessels, or vessels of practically the same build, constructed and experimented on beforehand.

The experiment is called the Inclining Experiment. This experiment will be described after we have dealt with the subject of trimming the ship.

**Longitudinal Metacentre.**—We have seen that the formula $BM = \frac{I}{V}$ is equally applicable to transverse and longitudinal inclinations. I in each case must be taken about an axis perpendicular to the plane of inclination, and through the centre of gravity of the waterplane. In longitudinal inclination, or in any direction of inclination between the longitudinal and transverse, I is the moment of inertia of the waterplane about a line through the centre of gravity of the waterline, and perpendicular to the direction of inclination of the ship. As the ends of a ship are not generally alike, the centre of gravity of the waterplane is not generally at the middle of the length. This affects the calculation of the longitudinal moment of inertia.

Referring to Plate III. we see the form of table used in making the calculation for longitudinal metacentre.

The formulæ in Chapter IV. for moment of inertia have been used.

Let $y$ represent the transverse ordinates from middle line, and let $x$ represent the longitudinal ordinates from a chosen axis.

The moment of inertia of waterplane about the axis O Y is

$$\iint x^2.dy.dx,$$

$$= \int x^2.y.dx.$$

This value $\int x^2.y.dx$ only gives the longitudinal moment of inertia about the chosen axis of reference O Y. What is really wanted is the moment of inertia about the axis through the centre of gravity which is $\left(\int x^2.y.dx - Ah^2\right)$, where $h$ = distance of the centre of gravity from O Y, and A = area of waterplane. The latter has been already calculated in the displacement table. Provision is made in the longitudinal metacentre table for finding $h$.

The moment is given by $\int x.y.dx$, and the moment of inertia is given by $\int x^2.y.dx$.

The functions of the ordinates of the waterline are copied from the displacement sheet and multiplied by the multiple of leverage about O Y. This gives $\int x.y.dx$.

Multiplying again by the multiple of leverages we get a column which gives us $\int x^2.y.dx$.

Thus the table is of the following form :—

TABLE XIV.

Waterplane No. 2. Long. interval = 33·73.
No. 5 ordinate chosen as axis of O Y, Vert. „ = 2·0′.

| No. of Ordinate. | Length of Ordinate. | Simpson's Multipliers. | Functions of Ordinates. | Leverages about O Y. | Functions for Moments. | Leverage about O Y. | Functions for M.I. |
|---|---|---|---|---|---|---|---|
| $n$ | $y$ | | $y.dx$ | $x$ | $xy.dx$ | $x$ | $x^2.y.dx$ |
| 0 | ... | ½ | ... | 5 | ... | 5 | ... |
| ½ | ·32 | 2 | ·64 | 4½ | 2·88 | 4½ | 12·96 |
| 1 | 1·28 | 1½ | 1·92 | 4 | 7·68 | 4 | 30·76 |
| 2 | 3·72 | 4 | 14·88 | 3 | 44·64 | 3 | 133·92 |
| 3 | 8·38 | 2 | 16·76 | 2 | 33·52 | 2 | 67·04 |
| 4 | 14·96 | 4 | 59·84 | 1 | 59·84 | 1 | 59·84 |
| 5 | 18·14 | 2 | 36·28 | 0 | 148·56 | | |
| 6 | 16·00 | 4 | 64·00 | 1 | 64·00 | 1 | 64·00 |
| 7 | 10·62 | 2 | 21·24 | 2 | 42·48 | 2 | 84·96 |
| 8 | 5·58 | 4 | 22·32 | 3 | 66·96 | 3 | 200·88 |
| 9 | 2·12 | 1½ | 3·18 | 4½ | 12·72 | 4 | 50·88 |
| 9½ | ·8 | 2 | 1·60 | 4½ | 7·20 | 4½ | 32·40 |
| 10 | ... | ½ | ... | 5 | ... | 5 | 0 |
| | | | 242·66 | | 193·36 | | 737·6 |
| | | | | | 148·56 | Correction = | 8·27 |
| | | | $(h) = \frac{44·8}{242·66}$ | | 44·8 | | 729·33 |

Function Correction $= (Ah^2) = \frac{242·66 \times 44·8^2}{242·66^2} = 8·27$

$\frac{2x^3}{3}$ . . . . . 25580

I = 18658000

Correct value of I $= (737·6 - 8·27)\frac{2x^3}{3}$.

The multiplier for M.I. contains :—

$\frac{x}{3}$ for area of waterplane,
$x$ for moment levers,
$x$ for second moment levers,
2 for both sides,

and is $\therefore = \frac{2}{3}(x)^3$.

In obtaining the true value of the correction the multiplier is $\frac{2x^2}{3}$ for moments.

This is divided by the area multiplier which is $\frac{2x}{3}$, and the whole is squared and multiplied by the area.

$$\therefore \quad \text{Multiplier for the correction} = \left(\frac{\frac{2x^2}{3}}{\frac{2x}{3}}\right)^2 \times \frac{2x}{3}$$

$$= \frac{2x^3}{3} \text{ which is the same multiplier as for M.I.}$$

Therefore the correction can be deducted from the function M.I. The correction is obtained therefore by dividing the (*function of moments*)$^2$ by the *function of areas*, and subtracting this from the function M.I. The difference

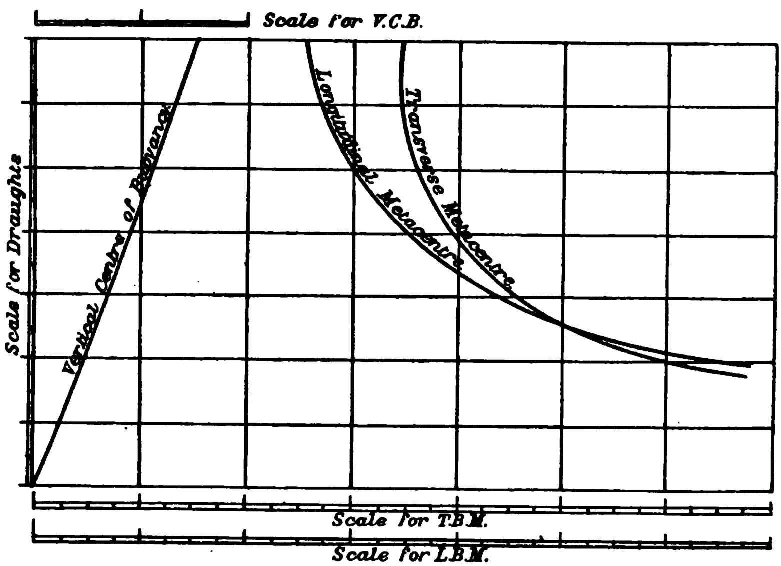



FIG. 81.

multiplied by $\frac{2x^3}{3}$ gives the I about the axis through the C.G. of the waterplane. It is usually more convenient to find the value of $h$ and deduct $Ah^2$ from the actual value of I about O Y. This value of $h$ gives the ordinate of the C.G. of waterplane.

The series of values for the longitudinal B M at different waterplanes can be set off in diagrammatic form in terms of the draught, but as the longitudinal B M is very large in comparison with the transverse B M, the scale has to be much reduced. Hence it is better to set off merely a curve of heights of longitudinal metacentre above B as a curve measuring the ordinates from the vertical line of draughts.

A specimen diagram is shown in fig. 81.

The longitudinal metacentric height is of use in the determination of the change of trim.

# CHAPTER X.

## TRIM.

**Trim** is the expression used to designate the difference in the draughts fore and aft at which the vessel is floating. For instance, if the draught forward is 6 ft. 6 in. and the draught aft is 7 ft., we say that the vessel has a trim of 6 in. by the stern.

Trimmed by the head would mean that the draught forward was greater than the draught aft. If the draught forward is equal to the draught aft, then the ship is said to be floating on an "even keel."

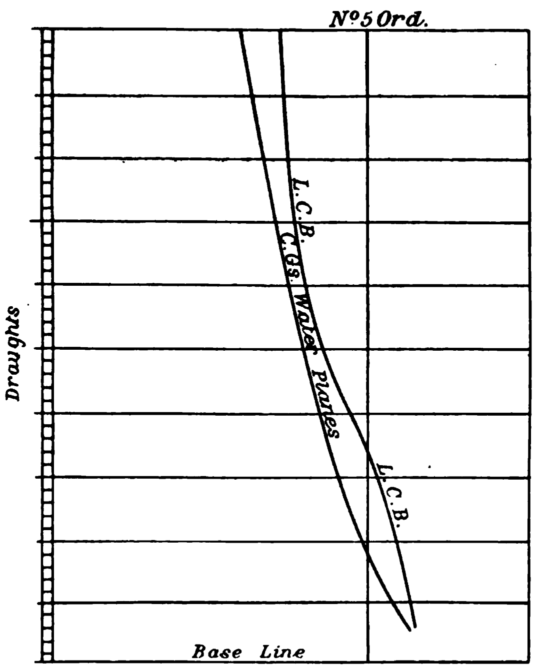



Fig. 82.

If a weight be added near the ends of a vessel there will be two effects:—

(1) The ship will sink bodily.

(2) The ship will trim by the head or the stern, according as the added weight is nearer the stem or stern.

There must be a certain place of the ship, however, where an added weight causes no change of trim. This can be easily seen by considering that a weight forward trims it by the head, and if moved aft the change of trim gets less and less until it is zero at a certain place.

If no change of trim occurs the additional buoyancy will lie between two parallel planes, and the centre of gravity of the added weight must be vertically over the centre of buoyancy of this added layer. This follows from the condition of equilibrium of the total buoyancy and total weight forces.

At this place, then, if a weight be added, the ship will sink bodily. If the weight be small the thickness of the layer will be small, and the centre of buoyancy of the layer can for all practical purposes be considered to be vertically over the centre of gravity of the waterplane.

**Centre of Gravity of Waterplanes.**—Hence, in all questions of trim it is necessary to know the centres of gravity of the waterplanes. We have also seen that if a ship receive a small inclination, the new waterplane will intersect the original waterplane in its centre of gravity.

The centres of gravity of the waterplanes have been calculated in Plate III. in the longitudinal metacentric forms. At each waterplane we have calculated $h$, the distance of the centre of gravity from the axis, which in this case has been No. 5 ordinate. See fig. 82.

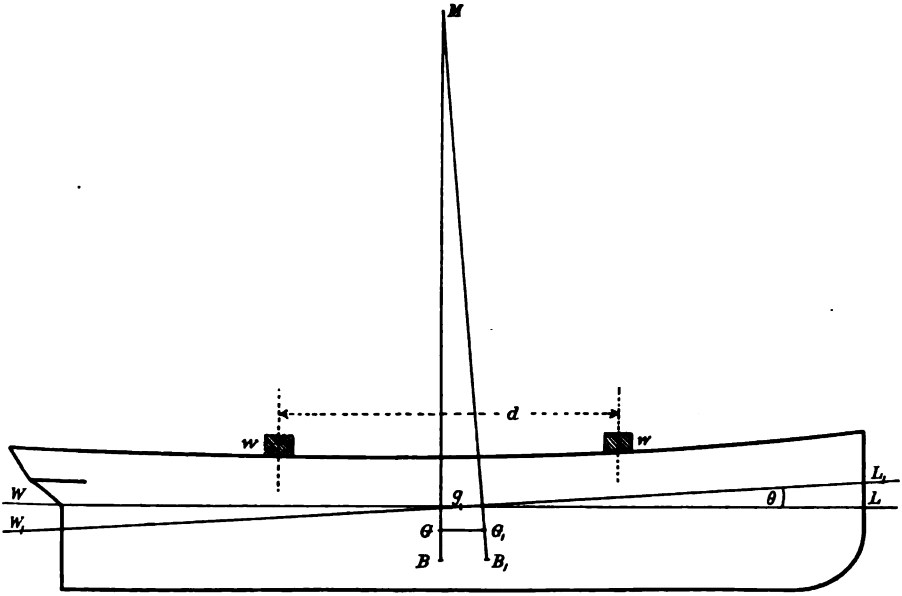



FIG. 83.

A curve of centres of gravity of waterplanes can therefore be constructed. This is done in a similar manner to that for the longitudinal centre of buoyancy.

The scale of draughts is vertical. A vertical line is chosen to represent No. 5 ordinate, and at each stage of draught the distance, fore or aft as the case may be, of the centre of gravity of the corresponding waterplane is set off. The characteristics of this curve are similar to those of the L.C.B. curve. These two curves can generally be plotted to the same scales. These curves should not extend below the lowest drawn waterline, as the form near the keel is very difficult to determine and the operation is quite valueless.

**Moment to change Trim.**—In the figure 83 let $w$ be a small weight on board when the waterline is W L. It is shifted forward a longitudinal distance $d$ and alters the waterline from W L to $W_1 L_1$. $w$ being small, the angle $\theta$ between W L and $W_1 L_1$ is small, and therefore the intersection of W L and $W_1 L_1$ may be taken as $g$, the centre of gravity of W L

Let $\Delta$ be the whole displacement in tons. The effect of shifting $w$ a distance $d$ will be to shift G, the centre of gravity of the whole, to $G_1$, such that

$$\Delta \times GG_1 = w.d.$$

This formula, therefore, gives us the horizontal shift of G, *i.e.* $GG_1 = \frac{w}{\Delta}.d.$

If we draw through G and $G_1$ the perpendiculars to the respective water-planes W L and $W_1$ $L_1$ we know that they contain B and $B_1$ respectively.

The intersection of these perpendiculars will be the longitudinal meta-centre.

From the figure approximately,

$$GG_1 = GM. \tan \theta;$$

$$\therefore \quad GM = \frac{w.d}{\Delta. \tan \theta}$$

$$\text{or} \quad w.d = GM.\Delta. \tan \theta.$$

$w.d$ is called the "trimming moment."

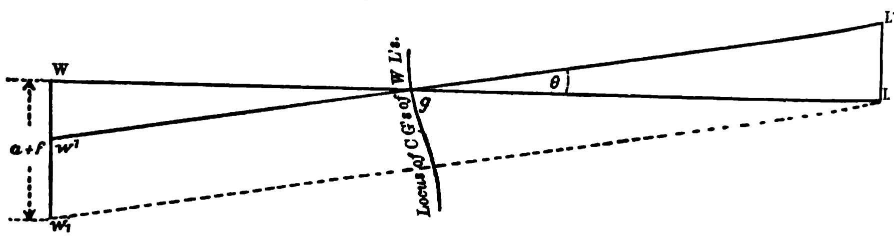



FIG. 84.

In fig. 84 let

$$WL = L,$$

$$WW^1 = a,$$

$$LL_1 = f.$$

Then $\frac{a+f}{L} = \tan \theta. = \frac{w.d}{\Delta.GM}$ very nearly if $\theta$ is small.

$\therefore$ the trimming moment, which is $w.d, = \Delta.GM \frac{a+f}{L}$. $(a+f)$ is the change of trim.

If $(a+f) = 1$ inch, we call this a change of trim of 1 inch.

$\therefore$ in the formula, putting $(a+f) = 1''$ or $\frac{1'}{12}$, the moment $w.d$ that would trim the vessel $1'' = \Delta.GM \frac{1}{12.L}, = \frac{\text{displacement} \times (\text{longitudinal GM})}{12 \times \text{length of ship}}$.

This is an important formula and should be remembered, as it is referred to in all questions relating to trim.

The curve of moment to trim 1", the units of which are foot tons, can be set off in relation to the draught.

In order to get the true displacement when the ship is floating at known draughts forward and aft, set up on the sheer plan the actual positions of the draught marks at the bow and stern, and at these positions set off the respective known draughts. Set along on each of the waterlines used for determining displacement the true position of its centre of gravity, and pass a curve through these spots. Through the known draughts as set off, draw the waterline W L at which the ship is floating (fig. 85). Then through the intersecting of this line with the curve of the centres of gravity of water-planes draw a line $wl$ parallel to the displacement waterlines. The displacement to the waterline $wl$ will be the displacement of the ship when floating at the observed waterline W L. The actual position of the draught marks should be measured at the ship before she is launched, and be placed on the sheer drawing.

To find the draughts fore and aft after a known weight has been shifted through a fore-and-aft distance $d$,

Draw the original waterline W L, fig. 84.

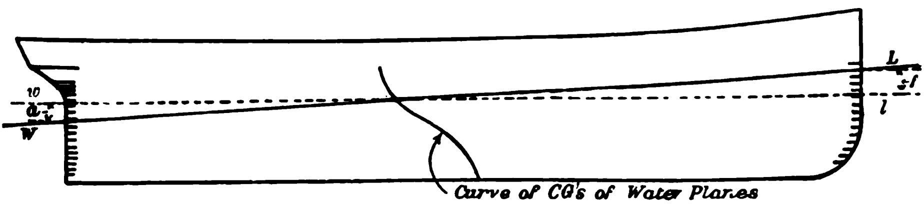



FIG. 85.

The trimming moment $= w.d$ foot-tons.

The moment to trim ship 1″ $= \dfrac{\Delta.\text{GM}}{12.\text{L}}$.

$\therefore$ number of inches change of trim $= \dfrac{w.d.12.\text{L}}{\Delta.\text{GM}}$.

$$\therefore \quad (a+f)'' = \frac{w.d.12.\text{L}}{\Delta.\text{GM}}.$$

At either of the perpendiculars, say the after one, set off $\text{W W}_1 = a + f$. Join $\text{W}_1$ L.

Through $g$, the intersection of W L with centres of gravity curve, draw a line $\text{W}^1 \text{L}^1$ parallel to $\text{W}_1$ L.

Then $\text{W W}^1 = a$ and $\text{L L}^1 = f$, so that $\text{W}^1 \text{L}^1$ is the required new waterplane, and hence the draughts forward and aft can be determined.

**Effect of putting a weight on board.**—Let a weight be put over the centre line of a deck, there will be a change of draught due to the increased displacement, and probably a change of trim. If it produces no change of trim then it is vertically over the centre of gravity of the added buoyancy, which, for all practical purposes, when the weight added is small, may be regarded as the centre of gravity of the waterplane.

Imagine the weight to be placed at some distance $d$ forward or aft from this position, then we are able to consider the effect on the sinkage and trim separately. This we can do by first supposing the weight $w$ to be placed vertically over the centre of gravity of the waterplane. The sinkage can be calculated from the tons per inch immersion. The change of trim due to shifting the weight a distance $d$ along the deck can be found from the moment to trim one inch.

8

In fig. 86, if $\Delta$ = original displacement,

$w$ = weight,

$\therefore \quad \Delta + w$ = new displacement.

If $t$ = tons per inch at the waterline,

then $\frac{w}{t}$ = distance vessel sinks bodily in inches.

Consider the vessel at this new waterline of an increased draught $\frac{w}{t}$ inches parallel to the original waterline.

Moment to trim by shifting weight a distance $d$ along the deck = $w.d$ foot-tons.

Let T = moment to trim 1″ at the increased draught.

Then $(a+f)$ inches $= \frac{w.d}{T}$.

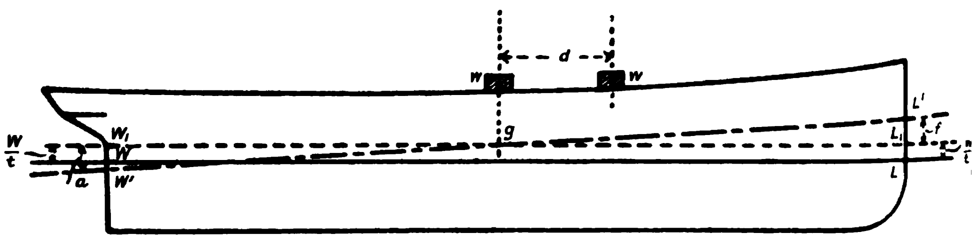



FIG. 86.

The original waterline is W L
„ parallel „ „ $W_1$ $L_1$
„ trimmed „ „ $W^1$ $L^1$

If $g$ is the position of centre of gravity of $W_1$ $L_1$,

$$\text{then} \quad \frac{a}{f} = \frac{W_1g}{L_1g},$$

$$\text{and} \quad \frac{a}{a+f} = \frac{W_1g}{W_1L_1} = \frac{W_1g}{L},$$

$$\text{also} \quad \frac{f}{a+f} = \frac{L_1g}{L}.$$

$$\text{We saw that} \quad (a+f) = \frac{w.d}{T},$$

$$\therefore \quad a = \frac{w.d.W_1g}{T.L},$$

$$\text{and} \quad f = \frac{w.d.L_1g}{T.L}.$$

The change of draught forward in inches $= f + \frac{w}{t} = \frac{w.d.L_1g}{T.L} + \frac{w}{t}$.

" " aft " $= a - \frac{w}{t} = \frac{w.d.W_1g}{T.L} - \frac{w}{t}$.

If the total change of draught aft = 0

$$\text{then} \quad \frac{w}{t} = \frac{w.d.W_1g}{T.L},$$

$$\therefore \quad d = \frac{T.L}{t.W_1g}.$$

$$\text{If} \quad W_1g = \frac{L}{2}$$

$$\text{then} \quad d = 2\frac{T}{t}$$

$$= \frac{\text{Twice moment to trim ship } 1''}{\text{Tons per inch immersion}}.$$

This, therefore, in the case where the C.G. of waterplanes is about amidships, roughly gives the longitudinal position where a weight should be added so that no change of draught aft should take place.

It will be noted that this position is independent of the amount of the weight *added*, and is true if the weight be *taken away* from this position. But $w$ must not be large or $t$ and T will not remain constant.

**Transverse Change of Trim.**—If a weight be moved transversely we shall have a change of C.G. and C.B. similar to that in the foregoing cases, but it is not usual to calculate transverse moment to change trim 1 inch, nor are draught marks usually put on the sides of the ship amidships. The change of trim transversely is measured by the angle of heel of the ship.

In transverse inclinations the axis of the waterplane is the longitudinal middle line plane. Since the centre of gravity lies in that plane, the change of trim for a given moment is the same on either side if it is small.

Let fig. 87 represent the transverse section of a ship which has been inclined from W L to $W_1 L_1$ by moving a weight $w$ a transverse distance $d$.

As in the case we have already considered for longitudinal inclination,

$$w.d = \Delta \times GG_1$$

$$\therefore \quad GG_1 = \frac{w.d}{\Delta}.$$

But $GG_1 = GM. \tan\theta$, and $\tan\theta = \frac{GG_1}{GM} = \frac{w.d}{\Delta.GM}$.

If the positions of G and M are both known we can find the value of $\theta$ for a given value of $w.d.$

$$\text{Also} \quad GM = \frac{w.d}{\Delta.\,\theta \tan}$$

so that if $\theta$ is measured and M is known we can find G M, and, therefore, the position of G.

**Inclining Experiment.**—We have seen that it is necessary to have approximate information about the position of the centre of gravity of a ship while she is yet in the process of design. Hence it is important, for purposes of future reference, to make an accurate determination of the centre of gravity when the ship is completed, and this is best done experimentally by observing $\theta$ for a known value of $w\,d$.

The inclining experiment consists in inclining the vessel transversely by moving a weight across the deck, noting the angle of inclination, and finding the G M by the above formula. The weights moved are generally in the form of pigs of iron which are called "inclining ballast."

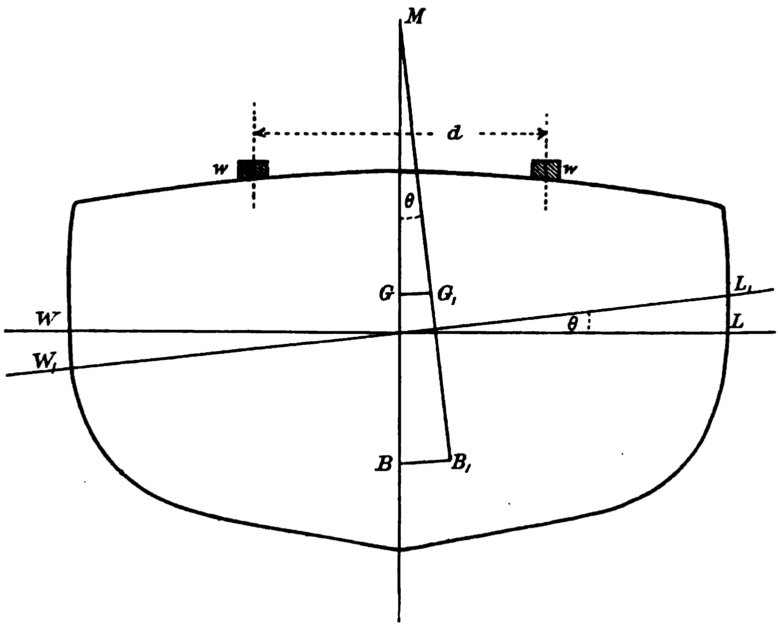



FIG. 87.

In order to get G M accurately, certain precautions have to be observed beforehand, but they all resolve themselves to avoidance upon the action of the vessel of forces which cannot be determined.

Precautions :—

(1) The ship should be inclined in a basin or dock where there are no strong currents.
(2) If there be any wind the ship should be laid head to wind.
(3) The ship should have perfect freedom to incline transversely.
(4) There should be no weights on the ship which are free to move in an undeterminable manner.
(5) No loose water should be aboard.

The observations to be taken consist of—

(1) Draught of vessel fore and aft.
(2) Condition of vessel fully detailed.
(3) Observations of the angle of inclination.
(4) Density of water—by a hydrometer.

(1) The draught of the vessel should be accurately taken, so that the correct displacement may be determined by the method already explained in this chapter; B M and the position of B must also be accurately obtained by the methods described in this Chapter.

(2) The condition of the vessel should be fully noted. In many inclining experiments the ship is just approaching completion, so that careful notes should be made of the extent to which the hull and the fittings are incomplete.

All weights liable to shift should be secured. The hull should be entirely free from any floating stages, etc. in the water outside. Only the observers and men for shifting the inclining ballast should be aboard, or, if this is unavoidable, the men should be retained in definite fixed positions.

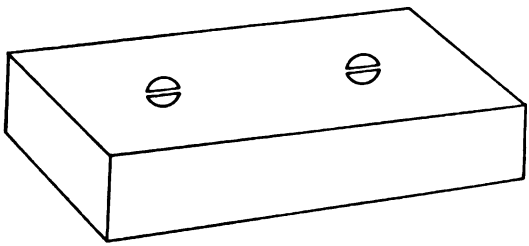


FIG. 88.

Positions on the deck should be allotted to the men who move the ballast.

The movable weight or inclining ballast is usually made up of a number of iron pigs which can be closely packed together so that the centre of gravity of the ballast can easily be ensured to be at a certain place.

These pigs are formed as shown in fig. 88, and usually weigh about 70 or 80 lbs. each, so that 32 or 28 of them are in one ton respectively.

A rough estimate is made beforehand of how much weight will be necessary to incline the vessel 4° or 5°. This weight and the distance it should be moved are fixed, and the position on the deck is selected.

When the ballast is placed with its centre of gravity over the centre line of the deck, *i.e.* when an equal weight is at each side of the deck at the same distance from the centre line, the ship should float upright. Positions are allotted to the men who shift the pigs, and the ship is ready for the observations to be made.

(3) The inclination is best observed by means of a plumb line and bob swinging over a scale as in fig. 89. Sometimes more than one of these arrangements are suspended throughout the length of the vessel. The length of the plumb line should be as long as possible.

In cargo vessels a convenient place for hanging the plumb line is in one of the hatchways. A small scale is fixed horizontally near the bob. The scale should be graduated to decimals of an inch. A small mark is made on the scale to show the position of the plumb line before the vessel is inclined, and when she is supposed to be floating freely and at rest. It can be easily determined whether the ship is actually upright or not. To avoid making small swings the bob is usually placed so as to hang in a pail of water. The length of the plumb line is the length from the scale to the point of suspension.

The ballast is usually laid in two or more equal heaps on each side of the deck. The men for shifting the weights are stationed about the middle of the ship in a position to which they always return while observations are being taken. The position of the plumb line on the scale is noted. They then shift all the weights in one heap from one side to the other and return to their stations. The positions of the plumb lines on the scales are again observed. The men, as before, then shift the remainder of the weight to the other side, and after resuming their stations observations are again taken.

It is advisable to go through this operation at least four times, and to leave the weights finally in the same position in which they were originally.

The position of the plumb lines in this condition forms a check on the first observation. If the lines are not hanging at the same marks as before, then some weight, other than the ballast weight, has shifted during the interval, or there has been some change in the forces acting on the vessel. If this is so, the matter must be elucidated before the experiment can be considered satisfactory.

For convenience in shifting the weights for inclining a large vessel, the pigs are sometimes loaded on to small trucks which are put on temporary guide rails laid across the deck. Fewer men are required by this arrangement.

Sometimes in inclining war vessels the large guns are moved to one side, and as the weight of these is known, the moment due to the transference of the weight can be found. As war vessels are very broad and have a fairly large metacentric height, the moving of the heavy guns to port or starboard lessens the necessity of having a large weight of inclining ballast for producing the inclination.

Another method that is sometimes adopted is to incline the vessel by partly filling the lifeboats with water.

The mean of the observations is taken to be the true reading.

Let fig. 89 represent the pendulum arrangement,

$PP' = h$ the horizontal shift in inches.

$CP = L$ the length of pendulum in inches.

$$\frac{PP'}{CP} = \frac{h}{L} = \tan\theta.$$

$$\therefore \qquad GM = \frac{w.d}{\Delta}.\frac{L}{h}.$$

Take the case where $w = 15$ tons

$d = 40$ feet

$\Delta = 3600$ tons

$h = 12$ inches

$L = 216$ „

$$GM = \frac{15 \times 40}{3600} \times \frac{216}{12}$$

$$= 3 \text{ feet.}$$

An important precaution, that has been already stated, is to see that there is no loose water in the ship. In some ships it may be impossible to do away with the loose water. The presence of loose water may be detected by the jerky oscillation of the ship, or by the time the ship takes to come back to the upright or original position.

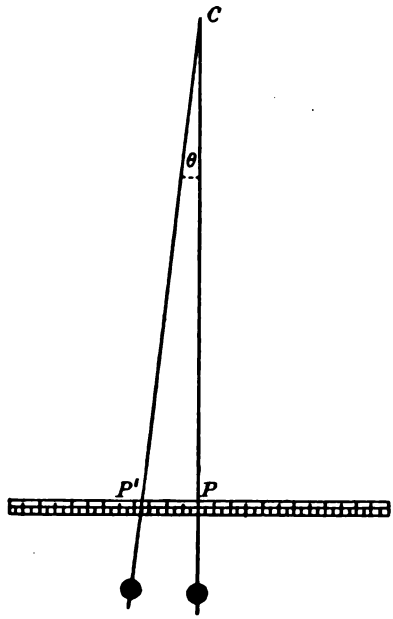



FIG. 89.

*To determine the metacentric height for any condition from the data of the inclining condition :—*

It is necessary to make a list of all the weights to "go in" or "come out" of the ship in order to bring the ship into the required condition.

A note is also made of the position of the centre of gravity of these weights separately, relatively to an axis which, for convenience, may be taken at amidships or at the centre-between-perpendiculars. The heights of the centres of gravity of the weights are also noted.

Generally in a cargo vessel this calculation is first made for the light condition.

The following tables give results for various conditions of loading.

[TABLES.

TABLE XV.—FAST PASSENGER STEAMER, 430 FEET LONG.

| No. | Condition. | Mean Draught. | Dispt. in Tons. | C.G. above Keel. | Meta-centric Height. | GM. | Angle of Maximum Stability. | Angle of Vanishing Stability. |
|---|---|---|---|---|---|---|---|---|
| 1st Condition. | Cargo stowed homogeneously to lower deck (1651 tons) at 40 cubic feet per ton. Coal bunkers full (1637 tons) at 42 cubic feet per ton . . . . . . . . . . . . | 25' 3¾" | 9207·1 | 19·456 | 22·44 | 2·984 | 57° 15' | Beyond 90° |
| 2nd ,, | Same as No. 1, but with coal out . . . . . . . | 21' 7½" | 7558 | 20·667 | 21·66 | 0·993 | 55° 2' | Beyond 90° |
| 3rd ,, | With cargo stowed homogeneously to main deck sufficient to load her to 25' 6" full draught. Coal in cargo (1744 tons) at 94·5 cubic feet per ton . . . . . . . . . . . | 25' 6" | 9300 | 20·872 | 22·5 | 1·628 | 53° 25' | Beyond 95° |
| 4th ,, | Same as No. 3, but with coal out . . . . . . . | 21' 10" | 7643 | 22·372 | 21·7 | 0·674 | 50° 31' | 85° 25' |
| 5th ,, | With cargo space only stowed to lower deck with cargo of a density of 48 cubic feet per ton, assuming that there are about 200 tons of stores stowed homogeneously in store spaces and bunkers full | 25' 1½" | 9131 | 19·648 | 22·4 | 2·752 | 56° 30' | Beyond 90° |
| 6th ,, | Same as in No. 5, but with coal out . . . . . . . | 21' 5½" | 7474 | 20·914 | 21·625 | 0·711 | 54° 40' | Beyond 90° |
| 7th ,, | As in No. 5, but with 'tween-decks filled with a cargo of cotton sufficient to load her down to a draught of 26' . . . | 26' 0" | 9525 | 19·906 | 22·625 | 2·719 | 56° 30' | Beyond 90° |
| 8th ,, | Same as No. 7, but with coal out . . . . . . . | 22' 4" | 7866 | 21·162 | 21·79 | 0·628 | 52° 40' | Beyond 90° |
| 9th ,, | Same as in No. 2, with 150 tons of coal placed in mid-bunkers, and 1051 tons of cargo taken out (600 tons) cargo and 150 tons of coal | 23' 4" | 8800 | 19·644 | 21·96 | 2·336 | 57° 15' | Beyond 90° |
| 10th ,, | As in No. 7, with sufficient cargo taken out of 'tween-decks to bring her to 25' full draught . . . . . . . | 25' 0" | 9063 | 19·623 | 22·375 | 2·752 | 55° 45' | Beyond 90° |
| 11th ,, | Light . . . . . . . . . . . . . | 17' 8½" | 5870 | 22·497 | 21·41 | 1·087 | 55° 47' | 88° 45 |
| 3A ,, | With sufficient cargo stowed homogeneously to load her to 25' 6" full draught coal in cargo = 1744·6 tons at 60 cubic feet per ton | 25' 6" | 9300 | 20·133 | 22·5 | 2·367 | 55° 15' | Beyond 90° |
| 4A ,, | Same as in No. 3A, but with coal out . . . . . . | 21' 10" | 7643 | 21·476 | 21·7 | 0·224 | 53° 20' | Beyond 90° |

TABLE XVI.—CARGO AND PASSENGER VESSEL, 425 FEET LONG.

| No. | Condition. | Mean Draught. | Displacement. | C.G. above Keel. | GM. |
|---|---|---|---|---|---|
| 1 | 3900 tons cargo to main deck and with 1000 tons coal on board | 24′ 0″ | 10350 | 21·64 | 0·24 |
| 2 | 3900 tons cargo to main deck (coal burnt out) | 22′ 1″ | 9350 | 21·97 | − 0·32 |
| 3 | As in No. 2 condition, but with 850 tons water ballast | 23′ 8″ | 10200 | 20·37 | 1·45 |
| 4 | 3900 tons cargo to upper deck and with 1000 tons coal | 24′ 0″ | 10350 | 22·9 | − 1·0 |
| 5 | 3900 tons cargo to upper deck (coal burnt out) | 22′ 1″ | 9350 | 23·4 | − 1·77 |
| 6 | As in No. 5 condition but with 850 tons water ballast | 23′ 8″ | 10200 | 21·7 | 0·12 |
| 7 | Light condition | 13′ 7″ | 5250 | 24·0 | − 0·8 |
| 8 | Light condition (850 tons water ballast) | 15′ 4″ | 6100 | 21·0 | 1·3 |
| 9 | Bunkers full (no cargo on board) | 16′ 6″ | 6635 | 23·1 | − 1·18 |
| 10 | Launching condition—95 tons water ballast on board | 10′ 9″ | 3830 | 23·0 | 2·65 |

TABLE XVII.—ATLANTIC LINER, 525 FEET LONG.

| Condition. | No. | Weight of Cargo. | Weight of Coal. | Stores, Fresh-water, Passengers and Baggage. | Weight of Water Ballast. | Displacement. | Mean Draught. | GM. | Remarks. |
|---|---|---|---|---|---|---|---|---|---|
| Light. | 1 | ... | ... | ... | ... | 10295 | 19′ 4″ | 0·4 | This includes 100 tons refrigerating fittings and 46 tons steward's furnishings. |
| Boilers and condensers empty, donkey boilers full. | 2 | ... | ... | ... | ... | 9920 | 18′ 9″ | 0·05 | Same as No. 1. |
| Docking. | 3 | ... | ... | ... | 510 | 10430 | 19′ 7½″ | 0·75 | Ship on even keel. |
| Load with cargo to lower deck. | 4 | 900 at 150 c. ft. per ton. | 2650 | 420 | ... | 14220 | 25′ 3″ | 0·7 | |
| Crossing the bar. | 5 | 900 | 100 | 285 | 506 | 12038 | 21′ 11¾″ | 1·46 | Draught ext. 22′ 0¼. |
| Load coal out. | 6 | 900 | ... | 285 | ... | 11432 | 21 1′ | 0·7 | |
| Deep load with cargo to upper deck. | 7 | 1798 | 2650 | 421 | ... | 15118 | 26 7′ | 0·52 | |
| Crossing the bar. | 8 | 1798 | 100 | 283 | 766 | 13196 | 23 | 1·39 | Draught ext. 23′ 10½″. |
| Deep load, coal out. | 9 | 1798 | ... | 283 | ... | 12330 | 22 6′ | 0·16 | |
| | 9A | 1798 | ... | 283 | 190 | 12520 | 22′ 9½ | 0·3 | |

### TABLE XVIII.—INCLINING EXPERIMENT—CRUISER.

**Length = 440 feet.**

**Draught—Forward 26′ 4″. Aft. 26′ 0″. Mean, 26′ 2″.**

**Ballast—100 tons used. Shifted 50 feet.**

**Pendulums—Each 15 feet long.**

**Commencement—Ballast, 50 tons each side.**

**Readings during Experiment.**

| Number of Experiment. | Amount Shifted. | Reading. Forward Pendulum. | Reading. After Pendulum. | Mean Deflection for 25 tons. |
|---|---|---|---|---|
| I. | 25 tons port to starboard . . | $5\frac{15}{16}$″ | $5\frac{15}{16}$″ | $5\frac{15}{16}$″ |
| II. | 50 ,, ,, . . | 12″ | 12″ | 6″ |
| III. | Ballast replaced . . . . | ... | ... | 6″ |
| IV. | 25 tons starboard to port . . | $5\frac{15}{16}$″ | $5\frac{15}{16}$″ | $5\frac{15}{16}$″ |
| V. | 50 ,, ,, . . | 12″ | 12″ | 6″ |
| | | | 5 | $29\frac{7}{8}$ |

Mean deflection for 25 tons for 5 shifts—$5\frac{39}{40}$″.

**Calculation of GM.**

$$GM = \frac{w.d}{\Delta \times \frac{h}{l}}$$ where $w = 25$ tons.

$d = 50$ feet.

$\Delta = 11950$ at 26′ 2″ mean draught.

$h = 5\frac{39}{40}$″.

$l = 180$″.

$$= \frac{25 \times 50}{11950 \times \frac{5\frac{39}{40}}{180}}$$

$$= \underline{3 \cdot 15 \text{ feet.}}$$

TABLE XVIII.—*continued.*—" DEEP LOAD CONDITION."

| Items. | Weight in Tons. | C.G. Above Keel (Feet). | Moment. | C.G. Forward Section II. | Moment For'd. | C.G. Aft Section II. | Moment Aft. |
|---|---|---|---|---|---|---|---|
| **Weights to go in to Complete " Deep Load Condition."** | | | | | | | |
| Stores, crew and effects | 197·5 | ... | 6305 | ... | ... | ... | 1429 |
| Shells and ammunition | 168·5 | ... | 2715 | ... | ... | ... | 2140 |
| Water | 260·0 | ... | 3850 | ... | ... | ... | 5004 |
| Coal | 75·0 | ... | 1413 | ... | ... | ... | 1790 |
| Boats and rigging | 56·5 | ... | 2686 | ... | ... | ... | 3421 |
| Torpedoes and stores | 7·5 | ... | 127 | ... | 917 | ... | – 935 |
| Gear for gun mountings | 2·0 | ... | 80 | ... | 16 | ... | ... |
| Total weights to " go in " | 767·0 | ... | +17176 | ... | ... | ... | 12851 |
| **Weights to be " Shifted " to Complete " Deep Load Condition."** | | | | | | | |
| Shells to be " shifted " from shell-rooms to gun-stations | (33·9) | ... | + 537 | ... | 72 | ... | ... |
| **Weights to " Come Out " to Reduce to " Deep Load Condition."** | | | | | | | |
| Inclining ballast | 100·0 | – 42·2 | – 4220 | ... | ... | 9·4 | – 940 |
| Plant and people | 23·5 | – 35·0 | – 822 | ... | ... | 15·2 | – 357 |
| Total weights to " come out " | 123·5 | ... | – 5022 | ... | ... | ... | – 1297 |
| Weights to go in | 767 | ... | +17176 | ... | ... | ... | 12851 |
| ,, be shifted | ... | ... | + 537 | ... | +72 | ... | ... |
| ,, come out | 123·5 | ... | – 5022 | ... | ... | ... | – 1297 |
| Total change | 643·5 | 19·7 | 12691 | ... | ... | 17·84 | 11482 |

Centre of Flotation Abaft Section = 15·98 Feet.
∴ C.G. of Weights go on Board Abaft C.F. = 1·86 Feet.

TABLE XVIII.—*continued.*

To Reduce from "Deep Load" to "Ordinary Load" Condition.

| Items to come out. | Weight in Tons. | C.G. Above Keel (Feet). | Moment. | C.G. Forward Section II. | Moment. Ford. | C.G. Abaft Section II. | Moment Aft. |
|---|---|---|---|---|---|---|---|
| Coal . . . . . . | 727 | ... | 20510 | ... | ... | ... | 1350 |
| Water . . . . . . | 61 | ... | 1865 | ... | 2443 | ... | ... |
| Reserve Feed . . . | 98 | ... | 294 | 16·3 | 1598 | ... | ... |
| | 886 | ... | 22·669 | 3·04 | 2691 | ... | ... |

∴ C.G. of Weights to "Come Out" Before C.F. = 19·04.

To Reduce from "Deep Load" to "Light Condition."

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *To come out:—* | | | | | | | |
| F. water . . . . . | 131·4 | 29·75 | 3895·0 | ... | 2653 | ... | ... |
| Provisions . . . . . | 45·0 | 23·15 | 1047·7 | ... | 1639 | ... | ... |
| Half Gunner's stores . . | 24·6 | 23·8 | 585·0 | ... | 664 | ... | ... |
| Half W. O.'s stores . . | 35·8 | 31·9 | 1142·0 | ... | ... | ... | 3065 |
| Half Engineer's stores . . | 51·0 | 22·4 | 1142·0 | ... | 3790 | ... | ... |
| R. F. water . . . . . | 98·0 | 3·0 | 294·0 | ... | 1598 | ... | ... |
| Coals . . . . . . | 1527·0 | 23·3 | 35550·0 | ... | 11450 | ... | ... |
| Officers' stores and slops . | 35·0 | 31·75 | 1110·4 | ... | ... | ... | 5202 |
| | 1947·8 | ... | 44750·1 | ... | 13524 | ... | ... |

∴ C.G. of Weights "to Come Out" Before C.F. 22·94.

TABLE XVIII.—*continued.*

**Calculation for Position of G.**

| | Weight. | Above Keel. "G." | Above Keel. Moment. | Before C.F. G. | Before C.F. Moment. | Abaft C.F. G. | Abaft C.F. Moment. |
|---|---|---|---|---|---|---|---|
| Inclining condition | 11950 | 27·00 | 322650 | ... | ... | ... | ... |
| To go in for deep condition | 645 | ... | 12691 | ... | ... | 1·86 | 1199 |
| Deep load condition | 12595 | 26·62 | 335341 | ... | ... | 0·08 | 1199 |

Increase in draught $= \frac{645}{51 \cdot 5} = 12\frac{1}{2}''$.

Change of trim $= \frac{5}{8}''$.

Draft:—Forward $(26'\ 4'') + (1'\ 0\frac{1}{2}'') - \frac{5}{8}'' = 27'\ 3\frac{7}{8}''$
Aft $(26'\ 0'') + (1'\ 0\frac{1}{2}'') + \frac{1}{4}'' = 27'\ 0\frac{3}{4}''$ } $27'\ 2\frac{3}{4}''$.

M above keel $= 30'\ 0''$.

GM $= 3 \cdot 38''$.

| | Weight. | Above Keel. "G." | Above Keel. Moment. | Before C.F. G. | Before C.F. Moment. | Abaft C.F. G. | Abaft C.F. Moment. |
|---|---|---|---|---|---|---|---|
| Deep load condition | 12595 | 26·62 | 335341 | ... | ... | ... | ... |
| To come out for ordinary load | 886 | ... | 22669 | 19·02 | 16850 | ... | ... |
| Ordinary load condition | 11709 | 26·70 | 312672 | ... | 16850 | ... | ... |

Decrease in draught $= \frac{886}{51 \cdot 5} = 17\frac{1}{4}''$, Change of trim $= 10\frac{3}{4}''$.

Draft:—Forward $(27'\ 3\frac{7}{8}'') - (1'\ 5\frac{1}{4}'') - 5\frac{7}{8}'' = 25'\ 4\frac{3}{4}''$
Aft $(27'\ 0\frac{3}{4}'') - (1'\ 5\frac{1}{4}'') + 4\frac{1}{2}'' = 26'\ 0\frac{3}{8}''$ } $25'\ 8\frac{3}{4}''$.

M above keel $= 30 \cdot 15$.

GM $= 3 \cdot 45$.

| | Weight. | Above Keel. "G." | Above Keel. Moment. | Before C.F. G. | Before C.F. Moment. | Abaft C.F. G. | Abaft C.F. Moment. |
|---|---|---|---|---|---|---|---|
| Deep condition | 12595 | 26·62 | 335341 | ... | ... | 0.08 | 1199 |
| To come out for light condition | 1948 | ... | 44750 | 22·94 | 44660 | ... | ... |
| | 10647 | 27·3 | 290591 | ... | ... | ... | ... |

Decrease in draught $= \frac{1948}{51} = 38\frac{1}{4}''$, Change of trim $= 28\frac{5}{8}''$ or $\begin{cases} 15\frac{3}{4} \text{ F.} \\ 12\frac{7}{8} \text{ A.} \end{cases}$

Draft:—Forward $= (27'\ 3\frac{7}{8}'') - (3'\ 2\frac{1}{4}'') - (1'\ 3\frac{3}{4}'') = 22'\ 9\frac{7}{8}''$
Aft $= (27'\ 0\frac{3}{4}'') - (3'\ 2\frac{1}{4}'') + (1'\ 0\frac{7}{8}'') = 54'\ 11\frac{3}{8}''$ } $23'\ 10\frac{3}{4}''$.

M above keel $= 30 \cdot 5$

GM $= 3 \cdot 2$.

TABLE XIX.—INCLINING EXPERIMENT—CHANNEL STEAMER.

Dimensions :—

Draught—Forward 9′ 6″. Aft 11′ 10″. Mean 10′ 8″.

Ballast—10 tons used. Shifted 35 feet.

Pendulums—Two used. 6′ 0″ long.

Density—35·11. Displacement at 10′ 8″ in dock = 2017.

Readings during Experiment.

| Number of Experiment. | Amount Shifted. | Reading. | | Mean Deflection for $2\frac{1}{2}$ tons. |
|---|---|---|---|---|
| | | Forward Pendulum. | After Pendulum. | |
| ... | 5 tons on each side | Vessel practi | cally upright | ... |
| 1 | $2\frac{1}{2}$ tons moved starboard to port | 1·04 | 1·08 | 1·06 |
| 2 | $2\frac{1}{2}$ ,, ,, ,, | 1·04 | 1·02 | 1·03 |
| 3 | 5 tons moved port to starboard | 2·08 | 2·10 | 1·045 |
| 4 | $2\frac{1}{2}$ ,, ,, ,, | 1·10 | 1·10 | 1·10 |
| 5 | $2\frac{1}{2}$ ,, ,, ,, | 1·10 | 1·07 | 1·085 |
| | | | 5 | 5·32 |

Mean deflection for $2\frac{1}{2}$ tons for 5 shifts = 1·064.

Calculation of GM.

$$GM = \frac{w.d}{\Delta \times \frac{h}{l}}$$ where $w = 2{\cdot}5$ tons.

$d = 35$ feet.

$\Delta = 2017$ tons at 10′ 8″ mean draught in dock.

$l = 72''$.

$h = 1{\cdot}064$.

$$= \frac{2{\cdot}5 \times 35}{2017 \times \frac{1{\cdot}06}{72}}$$

$= 2{\cdot}94$ feet.

Height of transverse M above base = 18·79′.
,, G ,, = 15·85′.

TABLE XIX.—*continued.*

| Items. | Weight in Tons. | C.G. Above Keel (in Feet). | Moment. | C.G. Forward Midships. | Moment Forward. | C.G. Aft Midships. | Moment Aft. |
|---|---|---|---|---|---|---|---|
| **Weights to "go in."** | | | | | | | |
| Indiarubber tiling . . | 1·6 | 26·5 | 42·4 | 1·0 | 1·6 | ... | ... |
| Spare tail shaft and propeller | 1·16 | 3·0 | 3·48 | ... | ... | 78·0 | 90·48 |
| Total to "go in". . . | 2·76 | ... | 45·88 | ... | 1·6 | ... | 90·48 |
| **Weights to "come out."** | | | | | | | |
| Inclining ballast . . . . | 10·0 | 27·2 | 272·0 | ... | ... | 60·5 | 605·0 |
| Tool-boxes, etc. . . . . | 4·42 | 24·0 | 106·1 | ... | ... | 10·0 | 44·2 |
| Coal . . . . . . . | 25·0 | 3·5 | 87·5 | 7·8 | 195·0 | ... | ... |
| Excess water in boilers. . | 14·0 | 14·5 | 208·8 | 9·5 | 133 0 | ... | ... |
| Men . . . . . . . | 1·5 | 29·0 | 43·5 | ... | ... | 60·5 | 90·7 |
| Fresh-water . . . . | 7·2 | 37·5 | 270·0 | ... | ... | 56·5 | 406·8 |
| Total to "come out" . . | 62·12 | ... | 987·9 | ... | 328·0 | ... | 1146·7 |
| **Equipped condition.** | | | | | | | |
| Vessel when inclined . . | 2017·0 | 15·85 | 31955·0 | ... | ... | 7·35 | 14820·9 |
| Weights to go ashore . . | 62·2 | ... | 987·9 | ... | 328·0 | ... | 1146·7 |
| | 1954·9 | ... | 30967·1 | ... | – 328·0 | ... | 13674·2 |
| Weights to go on board . | 2·76 | ... | 45·9 | ... | 1·6 | ... | 90·5 |
| | 1957·6 | 15·85 | 31013·0 | ... | ... | ... | 14091·0 |

Forward draught . . . . 9′ 3¼″
Aft ,, . . . . 11′ 8″
Mean ,, . . . . 10′ 5⅝″

TABLE XIX.—*continued.*

| | | |
|---|---|---|
| Transverse metacentre above base | = | 18·79′ |
| C.G. above base | = | 15·85′ |
| Trim by stern (11′ 10″) − (9′ 6″) | = | 2·33′ |
| L.C.B. at 10′ 8″ draught = 2·9′ aft of midship section. | | |
| L. metacentre above base = 608 feet. | | |
| ,, ,, ,, C.G. = 592·16 feet. | | |
| L.C.G. abaft L.C.B. $=\frac{592\cdot16 \times 2\cdot33}{330}$ | = | 4·181′ |
| L.C.B. abaft midship section at time of experiment | = | 2·9′ |
| L.C.G. ,, L.C.B. ,, ,, | = | 4·18′ |
| L.C.G. ,, midship section | = | 7·08′ |

Equipped Condition :—

L.C.G. abaft L.C.B. = 7·19 − 2·8 = 4·39

Trim by stern $=\frac{330 \times 4\cdot39}{604\cdot16} = 2\cdot39$

| | | | |
|---|---|---|---|
| KM transverse | = 18·92 | KM longitudinal | = 620 |
| KG | = 15·85 | KG | = 15·8 |
| Transverse GM | = 3·07 | L. meta. above C.G. | = 604·2 |

**Effect of loose water on the initial stability.**—We may take it that, in the upright condition, the stability for a given displacement depends upon the metacentric height G M.

If a weight be added to the ship the weight of the ship is altered, and the position of its centre of gravity may be changed. If, further, this weight be fixed on board and the vessel receive a small inclination there will be no change in the position of the centre of gravity of the added weight $w$.

Loose water in a vessel will not be like this, but will act similarly to a weight which is free to move as the ship is being inclined. As the ship is inclined the surface of the water tends to remain horizontal, and therefore the position of the centre of gravity of the volume of water changes relatively to the ship during the inclination. The centres of gravity of the ship and of the water are constantly changing during the inclination.

Let the figure 90 represent the transverse section of a ship floating with a certain volume $v$ of loose water in the hold.

Let W L be the waterline at which the ship is floating, and $w\,l$ the surface of the loose water in the upright condition.

If the ship be inclined by a weight $w$ moved across the ship a distance $d$ through a small angle $\theta$ the new waterlines will be as shown at $W_1\,L_1$ and $w_1\,l_1$ respectively.

Let B be the centre of buoyancy of the ship in the upright condition and $b$ the centre of gravity of the loose water.

Let these points B and $b$ be $B_1$ and $b_1$ respectively in the inclined condition.

Draw the new verticals through $b_1$ and $B_1$. These verticals will meet the original vertical in the points $m$ and M respectively.

The total moment inclining the vessel through the angle $\theta$ is $V.\,G\,G_1$; where V is the volume of displacement to W L, and G is the centre of gravity of the ship in the upright condition and $G_1$ the centre of gravity in the inclined. $G_1$ must lie on B M.

This moment $V.GG_1$ is made up partly by $wd$ the moment due to the transfer of the weight $w$ across the deck through a distance $d$, the remainder being due to the loose water. The value of this remaining moment is $v.bb_1$ and it is obtained by the same consideration from which we should obtain it if the vessel were floating at the line $wl$ and were inclined through $\theta$, so that the new waterline was $w_1 l_1$; the moment of the transfer of volume in the two cases being the same, viz. $v.bb_1$.

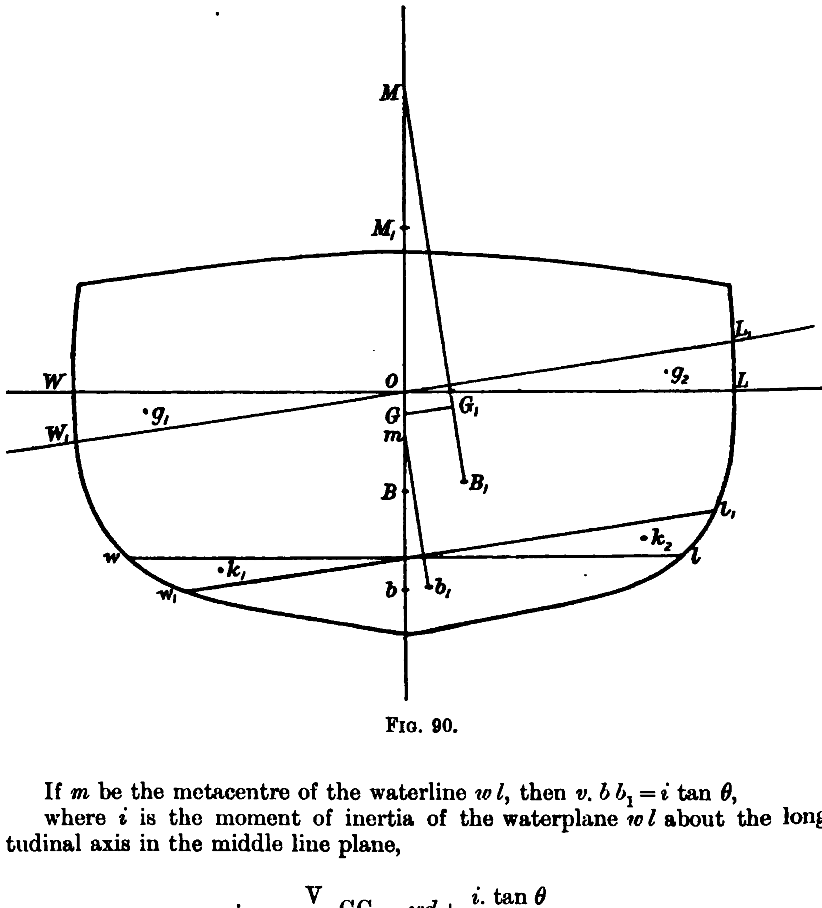



FIG. 90.

If $m$ be the metacentre of the waterline $wl$, then $v.bb_1 = i \tan\theta$, where $i$ is the moment of inertia of the waterplane $wl$ about the longitudinal axis in the middle line plane,

$$\therefore \quad \frac{V}{35} GG_1 = wd + \frac{i. \tan\theta}{35}$$

$$GM = \frac{35\,wd}{V \tan\theta} + \frac{i}{V}.$$

If we take $GM_1 = \frac{35\,wd}{V \tan\theta}$ which would be the distance between the $G$ and the metacentre if $wd$ alone heeled the ship through $\theta$, we get $M_1$ a virtual metacentre for the ship with water loose in the bottom,

$$GM_1 = GM - \frac{i}{V}.$$

Thus when the centre of gravity of the ship and all it contains remains constant the whole change during the inclination is measured by the moment of the buoyancy from one side to the other, but if, in addition, we have a transfer of a weight of loose water inside the ship, the total change is represented by the difference of the moments of the transfer of buoyancy and of the transfer of weight of water.

The metacentric height is therefore virtually reduced by an amount which is proportionate to the moment of the transfer of weight. This was equal to $v_0 . k_1 k_2$.

Thus the lowering of M due to loose water in the hold depends on the area of the free water surface (being equal to the moment of inertia of that area divided by the whole volume of displacement), and not on the volume of flooded water. If the ballast tanks be half full of water the value of $i$ is considerable, and the value $\frac{i}{V}$ may make a dangerous reduction in the metacentric height.

When the ballast tanks are full there is no free surface ($i = 0$); the water can be considered merely as a solid weight.

If the vessel has a large flat bottom and there is very little loose water, yet the value of $i$ may be considerable. It may be sufficient to make the vessel unstable, and she will heel over to such an angle until there is positive stability. In these circumstances, as soon as there is a slight inclination the area of the free water surface decreases very rapidly, so that the reduction $\frac{i}{V}$ becomes rapidly smaller.

The obvious way to obtain the value of G M is to find the moment of inertia of the waterline W L and deduct the moment of inertia of the waterline $wl$. The difference divided by V gives the value $BM_1$. The position of G must be determined by taking into account the weight of the loose water. The lowering of virtual M due to the free water surface will be the same for the same flooded area whether the water surface be in the hold or above a deck or anywhere else. But the position of G will depend on the position of the water.

**A Method of finding the Vertical Movement of the Centre of Buoyancy due to Change of Trim.**—Ships are usually designed to a fixed trim, and the calculations relating to the form are made for waterplanes parallel to the one having this trim. Frequently the trim is taken as on even keel; this is usual in designing merchant steamers, and there the calculations may be made from the base line as datum. In warships and in yachts it is customary to have trim by the stern as designed, and there the load line is taken as datum.

In both cases, however, the ships have to float at trims other than those for which they are designed, and it is desirable to see what effect this change of trim will produce on the results of the calculations as made in the designed conditions.

This is particularly so in considering questions of stability, as trim introduces changes into two of the important factors bearing on the case. These are: (1) the vertical position of the centre of buoyancy and the consequent change in height of the metacentre; and (2) change in the height of the metacentre directly due to the alteration in the moment of inertia of the waterplane area. As deviations in trim in ships are seldom very large in comparison with the length of a ship, there is not a great change in the position of M due to the trim, but it is well to bear in mind that the transverse metacentric height is also of small dimensions as compared with her size, so that any factors tending to alter the G M may be important.

Let fig. 91 represent the underwater portion of a ship with W L the designed waterplane, and $W_1 L_1$ the waterplane when she is trimmed by the stern. A C is the intersection of these two planes, and contains the common

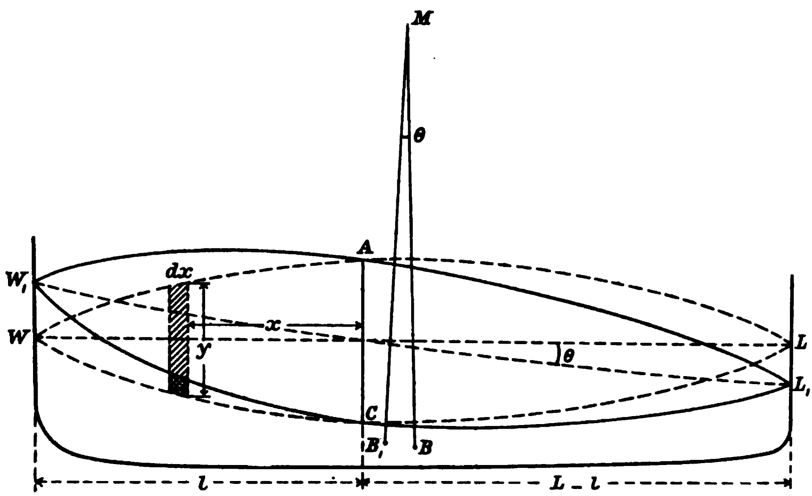



FIG. 91.

centre of areas of the waterplanes. B is the centre of buoyancy at W L, and $B_1$ the centre of buoyancy consequent on trimming to $W_1 L_1$; $y$ is an ordinate of the original waterplane W L at a distance $x$ from A C. A C is distant $l$ feet from the after draught marks and $(L - l)$ from the forward draught marks, L being the length of the ship.

If M be the moment trimming the ship $t$ inches, and $m$ the moment to trim ship one inch, $M = m t$, $t$ being $W W_1 + L L_1$,

Let W be the displacement of the ship in tons, V in cubic feet, and I the moment of inertia of the original waterplane W L. If the angle of trim $\theta$ be small, the volume of the submerged or after wedge will be $\int_0^l x\theta y dx$, and of the emerged or forward wedge $\int_0^{(L-l)} x\theta y dx$.

The vertical moment in foot tons of the submerged wedge about the original waterplane W L is

$\frac{1}{35}\int_0^l \frac{x\theta}{2} x\theta y dx$, and of the emerged wedge

$\frac{1}{35}\int_0^{(L-l)} \frac{x\theta}{2} x\theta y dx$, which can be simplified to

$\frac{\theta^2}{70}\int_0^l x^2 y dx$, and $\frac{\theta^2}{70}\int_0^{(L-l)} xydx$, and are equal respectively to

$\frac{\theta^2}{70}$(I of the after part), and $\frac{\theta^2}{70}$(I of the forward part).

The algebraic summation of these is $\frac{\theta^2}{70}$(I of the whole waterplane).

$$\begin{aligned}\frac{\theta^2}{70}\mathrm{I} &= \frac{\theta}{2}\left(\frac{\theta \mathrm{I}}{35}\right)\\ &= \frac{\theta}{2}\mathrm{W}\left(\frac{\mathrm{I}}{\mathrm{V}}\right)\theta\\ &= \frac{\theta}{2}\mathrm{W.BM}\theta\\ &= \frac{\theta}{2}\mathrm{W.BB_1}\\ &= \frac{\theta}{2}\mathrm{M}\\ &= \frac{t}{12\mathrm{L}}\,\frac{\mathrm{M}}{2}\\ &= \frac{tmt}{24\mathrm{L}}\\ &= \frac{t^2m}{24\mathrm{L}}.\end{aligned}$$

The total vertical moment of the wedges in foot tons about the original waterplane is given by the expression $\frac{t^2m}{24L}$, and the vertical movement of B to $B_1$ in relation to the original horizontal plane W L can be obtained from the formula $\frac{t^2m}{24L.W}$ in foot units. As the foregoing is only meant to apply where $\theta$ is of small magnitude, this vertical shift can be taken as approximately true with regard to the plane $W_1 L_1$.

The change in B M can only be determined by finding the value of I for the waterline $W_1 L_1$, which can be readily done from the drawings.

## CHAPTER XI.

# COEFFICIENTS AND STANDARDISING RESULTS OF SHIP CALCULATIONS.

**Block Coefficient.**—The value of coefficients in the preparation of a new design has already been mentioned. We describe a ship's underwater form as being full or fine according as the shape approaches to that of a block or rectangular vessel of the same dimensions as the ship. For instance, a barge is a full vessel, as it is almost rectangular in shape; a sailing yacht and a torpedo boat are examples of types of vessels with fine forms.

The block coefficient expresses the fulness or fineness of the form by the ratio of the displacement of the vessel to the displacement of the surrounding block (fig. 92).

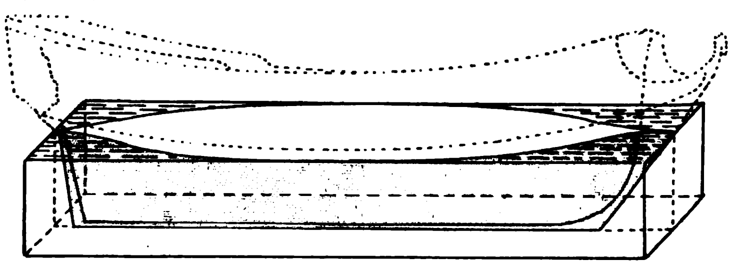


FIG. 92.

| | | | | | | |
|---|---|---|---|---|---|---|
| The | length | of the block is the | | length | of the ship | = L. |
| ,, | breadth | ,, | ,, | breadth | ,, | = B. |
| ,, | draught | ,, | ,, | draught | ,, | = $d$. |

If V = volume of displacement of a ship L, B, D, at draught $d$, call the block coefficient $C_B$; then

$$C_B = \frac{V}{L.B.d}.$$

If $$V = 35.\Delta$$

$$C_B = \frac{\Delta \times 35}{L.B.d}$$

$$\Delta = \frac{C_B \times L.B.d}{35}.$$

The $C_B$ can therefore be set off in terms of draught. Its value at certain stages of draught can be calculated on the displacement sheets. It can also be obtained direct from the displacement curve.

In fig. 93 let O D represent the displacement curve; at any draught $d$ draw a horizontal line $kd$.

Then $kd$ represents the displacement at draught $d$.

Let $kd = \Delta$.

$C_B = \frac{\Delta \times 35}{L.B.d} = ke$, which can be set off along the same horizontal line.

TABLE XX.—BLOCK COEFFICIENT—VALUES APPROXIMATE.

| | |
|---|---|
| Very full vessels, like barges | ·85 to ·9 |
| Very full cargo vessels (up to 8 knots) | ·8 to ·85 |
| Full cargo vessels (up to 12 knots) | ·76 to ·82 |
| Large cargo vessels (12 to 14 knots) | ·7 to ·76 |
| Intermediate—faster cargo vessels and coasters | ·65 to ·7 |
| Fast Atlantic liner | ·60 to ·65 |
| Channel passenger steamers | ·5 to ·6 |
| Paddle steamers | ·46 to ·57 |
| Steam trawlers, etc. | ·56 to ·6 |
| War vessels—battleships | ·6 to ·65 |
| „ „ cruisers | ·48 to ·55 |
| „ „ torpedo boats and destroyers | ·4 to ·48 |
| Merchant sailing ships | ·6 to ·72 |
| Steam yachts | ·45 to ·60 |
| Sailing yachts | ·15 to ·42 |

**Prismatic Coefficient.**—This coefficient is the ratio between the volume of displacement and the volume of the surrounding prism or cylinder, this prism being of the same length as the ship and of the same cross-sectional area as the immersed midship section. The prismatic coefficient, therefore, is used in comparing the fineness of forms with different midship sections.

Let $\otimes$ represent the area of the midship section up to the draught $d$.

Then $L.\otimes$ = volume of surrounding prism.

$V$ = volume of displacement up to draught $d$.

Then $C_p$, the prismatic coefficient,

$$= \frac{V}{L.\otimes}.$$

The prismatic coefficient curve can be easily obtained from the curves of the midship areas and displacement.

**Midship Areas Coefficient.**—The midship areas coefficient is the ratio of the area of the midship section submerged to the area of the surrounding rectangle $(B \times d)$.

$$\text{Midship area coefficient } C_{\otimes} = \frac{\otimes}{B.d}.$$

$$\therefore \quad \otimes = C_{\otimes} B.d.$$

$$\therefore \quad C_p = \frac{V}{L.B.d.C_{\boxtimes}} = \frac{L.B.d.C_B}{L.B.d.C_{\boxtimes}} = \frac{C_B}{C_{\boxtimes}}.$$

$$\therefore \quad \text{Prismatic coefficient} = \frac{\text{Block coefficient}}{\text{Midship areas coefficient}}.$$

**Coefficient of Waterplanes.**—The coefficient of waterplane is the ratio of the area of the waterplane to the area of the rectangle (B × L) which is constant.

$$C_w = \frac{\text{Area waterplane}}{L.B}.$$

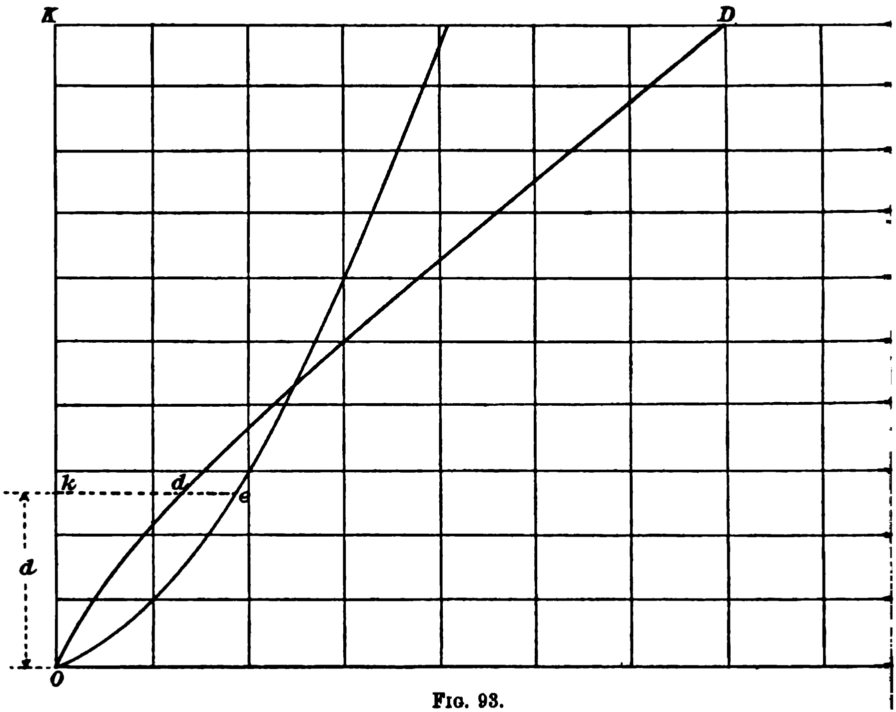


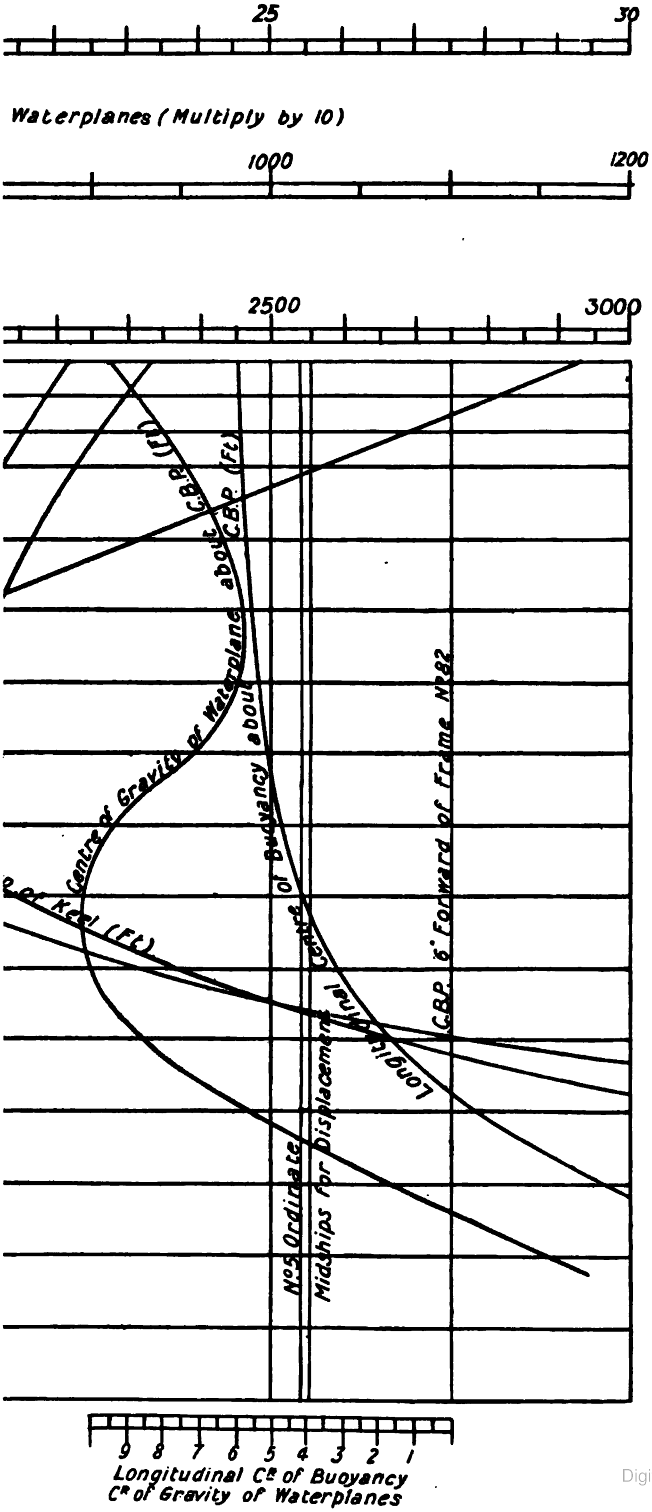

FIG. 93.

Therefore the curve of coefficients of waterplane areas is a similar one to the areas of waterplanes curve, and therefore similar to the tons per inch curve.

These three curves can be represented by one curve. The scales will be different and can be marked on the top of the diagram.

**Relation between Area of Waterplanes Curve and Displacement Curve.**—We have seen that by integrating the areas of waterplanes we get the displacement. Therefore by integrating the area of waterplanes curve we get the ordinates of the displacement curve.

**Standardising the Results of Ship Calculations.**—The desirability of directly comparing the curves of displacement, centre of buoyancy, and all those curves that we have considered in the previous chapters of this part, has led to a system of standardisation. This system was explained in a paper by the author to the Institution of Naval Architects in 1901.




[To face page 187.

The scales that are usually adopted for setting off the curves are chosen merely with a view to convenience in the size of the whole diagram.

Let us take first the displacement paper. The usual form is shown in Plates II. and III., which was devised for obtaining by Simpson's Rules the displacement of a set of lines at a series of draughts. This form is given as showing one method of arranging such work, and to illustrate the general question under consideration. The table enables the calculator to obtain displacement, vertical and horizontal position of centres of buoyancy, areas of waterlines, and mid-sections, for a series of draughts. If these results be set off in terms of draught, we shall have for the particular form we are dealing with curves of displacement, loci of vertical and longitudinal centres of buoyancy, waterline, and mid-section areas. Such a series of results is shown in Plate IV.

It is usual to set off these results upon some arbitrarily chosen scale, such as 1 in. = 1000 tons for displacement, or $\frac{1}{2}$ in. = 1 ft. for vertical and longitudinal measurements of centres of buoyancy. These curves convey nothing to the eye, except the variation of result in terms of draught. To make any use of them, the number of inches of their ordinates must be measured, and the result in inches must be multiplied by the scale upon which they have been set off. When this is done, we know the value of the particular ordinate we have been dealing with, and what it means in relation to the particular set of lines from which it was taken. Its usefulness stops at this point. If we have similar results for other sets of lines their usefulness is similarly limited. Each set of results is isolated when it is set off upon such scales as are usual.

Suppose, however, we adopt a method of setting off results, so that we can remove this isolation, and make each ship directly comparable in all these respects with all other ships. We shall then be able to compare all our own ships with each other, and, if other naval architects are willing, with all theirs also.

The results we have been discussing are due to form and dimensions. The system we shall now consider is one in which form only is taken into account, dimensions being altogether relegated to the domain of "practical application."

Suppose we have a vessel whose dimensions are L, B, $d$ (length, breadth, draught), $\delta$ being the load draught. The product $\frac{LB\delta}{35}$ will give us the number of tons of salt-water which the circumscribing parallelopipedon, or, as it is perhaps more commonly known, "the block," would displace. Suppose that for all ships, whatever their forms or dimensions, we let the displacement of this block be represented by a fixed length, say 10 in. (or, in those countries which are blessed with a metric system, say $\frac{1}{4}$ of a metre = 9·84 in.). Whatever the displacement at any draught in any *form* may be, it will have a definite relation to the volume of the circumscribing block, and this relation will remain absolutely unaltered, however we may alter the individual values of L, B, and $d$, provided that we alter all measurements in the directions in which L, B, and $d$ respectively are measured, in exactly the same ratio in which L, B, and $d$ have been altered. In other words, one of the qualities which remains unchanged for all variations of L, B, and $d$, when form characteristics are maintained, is the block coefficient. A block has a coefficient of unity, and this may be taken as our unit of value for displacement. We always consciously or unconsciously fall back upon this value as a unit in measuring fulness; why should we not begin with it?

If we agree to let 10 in. $= \frac{LB\delta}{35}$ (fig. 94), and at the same time let unity represent the volume of the block, we shall have 10 in. = (say, A B) = 1 of block coefficient, and 1 in. = .1 of block coefficient. But 10 in. = $\frac{LB\delta}{35}$ tons, and 1 in. = $\frac{LB\delta}{350}$ tons. If we set off the actual displacement at the load waterline of the ship we are considering on the scale of 1 in. $= \frac{LB\delta}{350}$ tons (say, A B), then the ratio of the number of inches which will represent the displacement to 10 in. will be the block coefficient of the set of lines under consideration.

Suppose we set down $\delta = AK = 10$ in. Then the draught at any intermediate point can be set up from K on a scale 1 in. $= \frac{\delta}{10}$ ft. of draught. If the values of the displacement at any other draught $d$ be set off on the scale,

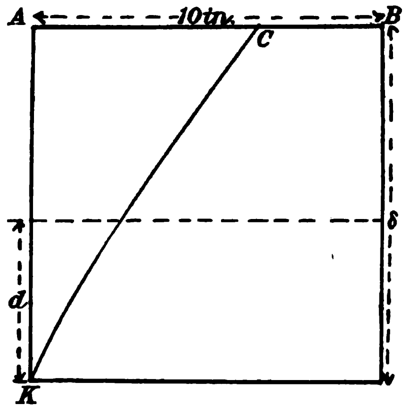



FIG. 94.

1 in. $= \frac{LB\delta}{350}$ tons at the corresponding draughts (which will be on a scale 1 in. $= \frac{\delta}{10}$ ft.), we can get a curve K C which will show the variation of displacement in terms of draught, but it will also show the displacement at any draught for any ship of similar form whose dimensions are known, upon a scale whose absolute value can be determined by substituting the particular values of L, B, and $\delta$ in the equation 1 in. $= \frac{LB\delta}{350}$ tons. Thus, by setting off the result of *one* ship's calculation in this suggested manner, we have removed the isolation in which it stood, and have made its results immediately applicable to an indefinite number of cases of differing dimensions. The only quality which we have retained constant is the character of the form. If we work out the results for another form and set them off in the same way, the comparison of the results due to difference of form will be at once apparent to the eye. All question of difference of dimension will be absolutely eliminated, and we shall have a comparison of the results of form only. In this way the result has been standardised.

Displacement has been chosen merely to illustrate the principle of standardisation. The unit length was chosen at 10 in. as a matter of convenience. Whatever length is chosen should be adopted by all who use this system, partly for purposes of direct comparison by placing tracings of the

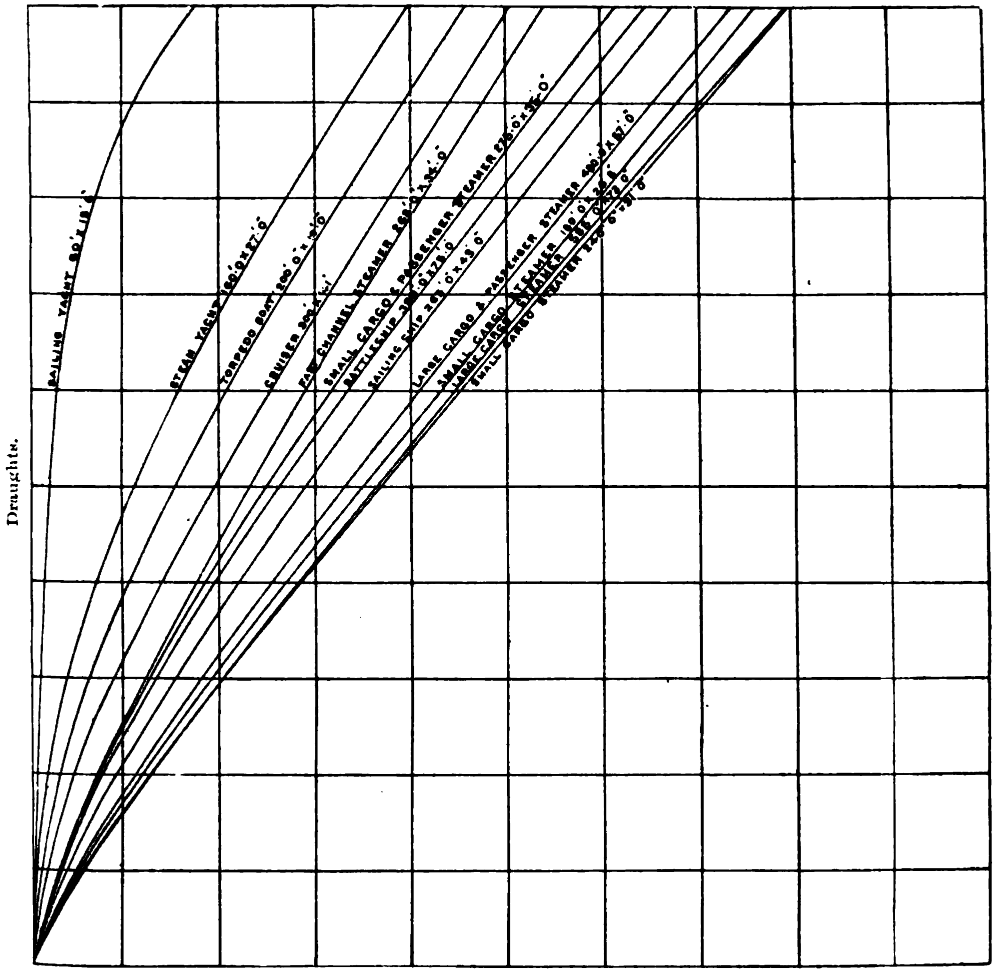



FIG. 95.—Displacements.

results of different forms above each other, and partly because the eye can judge immediately of the qualities if the absolute scale is always the same.

Fig. 95 shows a considerable number of forms, the displacement curves of which have been standardised. The dimensions of the actual ships from which they have been deduced are given, and also the types of vessels are stated. It will be seen that the comparison of form characteristics, as far as displacement is concerned, is shown for a very wide range of types.

In a similar manner the results of calculations of waterline areas, midship section areas, longitudinal and vertical positions of centres of buoyancy, heights of metacentre above centre of buoyancy, may all be set off. Such results are shown in figs. 96–100 respectively.

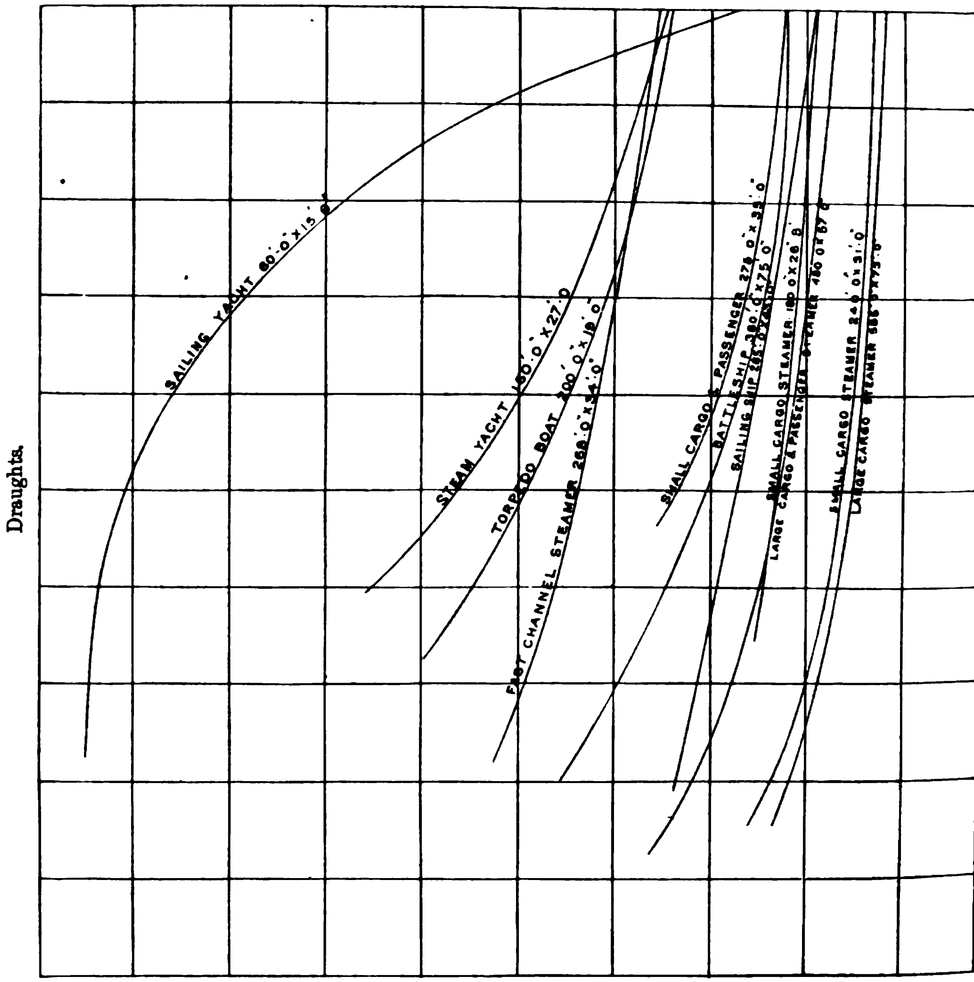



**Fig. 96.—Areas, and Coefficients of Waterplanes. Also Curves of Tons per Inch.**

The scales upon which the actual results have been set off are as follow :—

Fig. 96.—Waterline areas . . 10 in. = L.B sq. ft.

$$1 \text{ in.} = \frac{\text{L.B}}{10} \quad ,,$$

Fig. 97.—Midship section areas $10 \text{ in.} = B.\delta$ sq. ft.

$$1 \text{ in.} = \frac{B.\delta}{10} \quad ,,$$

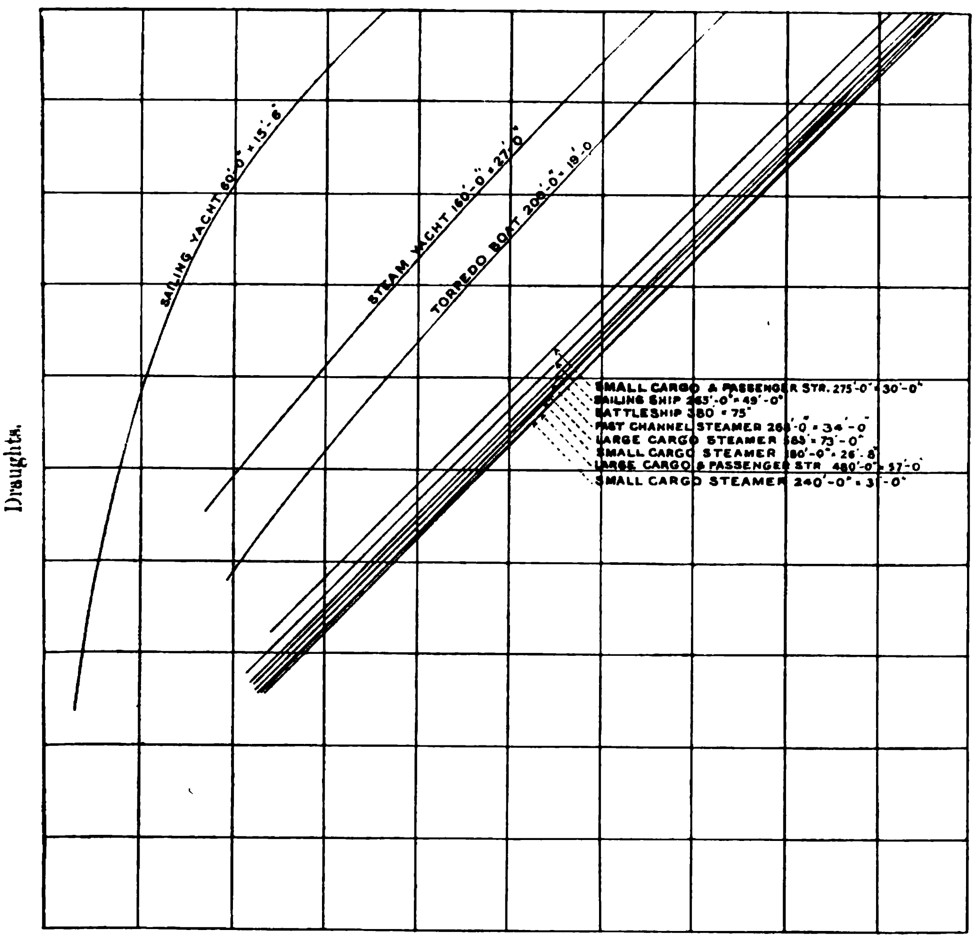



**Fig. 97.—Areas of Midship Sections.**

Fig. 98.—Distance of centre of buoyancy from middle ordinate $= l$.

If $100 \text{ in.} = L$, and we set off $l$ on the same scale, it will be on scale of $1 \text{ in.} = \frac{L}{100}$ ft., and is plotted from the centre line of the diagram, the after side being to the left.

Fig. 99.—Distance of centre of buoyancy above keel $= K B = b$.

If $10 \text{ in.} = \delta$ and we set off $b$ in this scale, it will be on scale $1 \text{ in.} = \frac{\delta}{10}$ ft.

Fig. 100.—Height of metacentre above centre of buoyancy = B M.
The transverse B M of a block is

$$= \frac{I}{V} = \frac{LB^3}{12LBd} = \frac{B^2}{12d}.$$

$$\text{If} \quad 5 \text{ in.} = \frac{B^2}{12\delta}, \; 1 \text{ in.} = \frac{B^2}{60\delta} \text{ ft.}$$

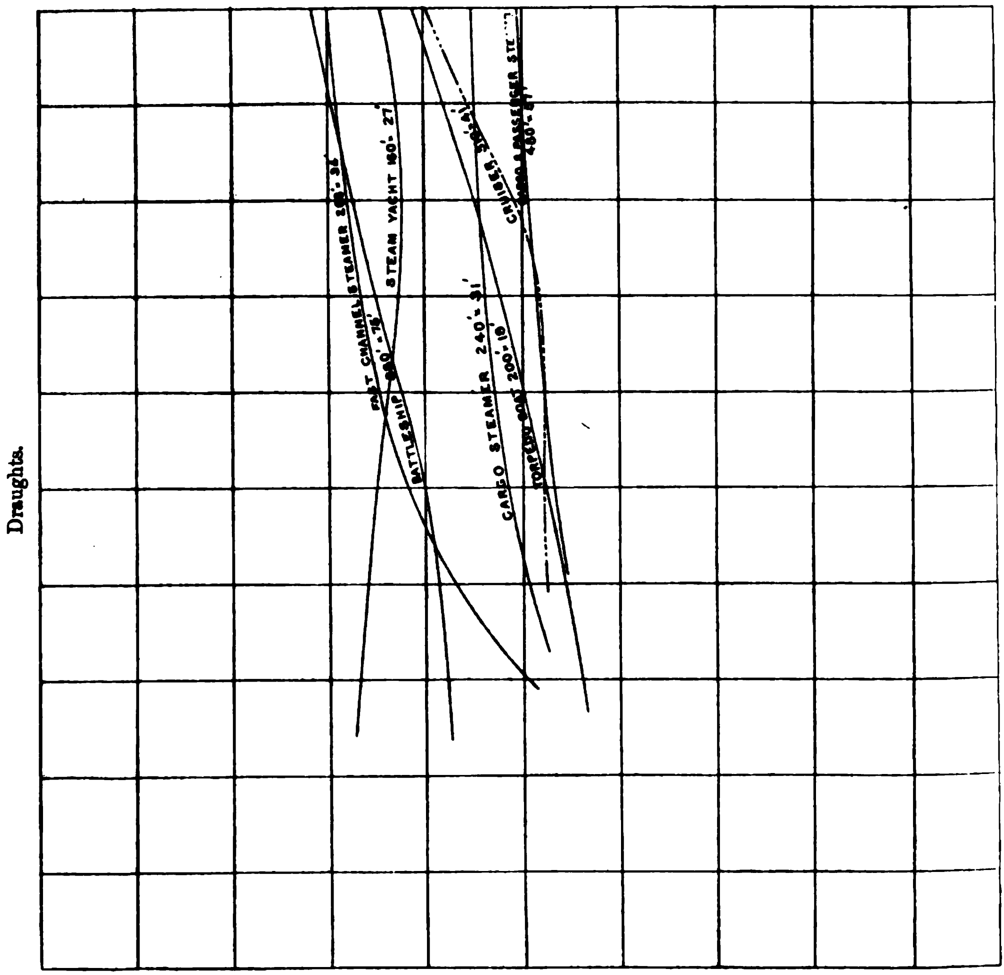



FIG. 98.—Longitudinal Centres of Buoyancy.

If we set off actual B M's on this scale at the corresponding draughts (on a scale of 1 in. = $\frac{\delta}{10}$ ft.) we shall get standardised B M curves, such as are shown in fig. 100. Longitudinal B M curves may be set off in a similar manner.

These results are perhaps not so valuable as those which give the actual vertical positions of the metacentre in the ship, because it is generally its relation to the centre of gravity which is of most interest.

To find this, it is necessary, as usual, to set up M in two operations, first by setting off K B, and then setting B M above it. If both K B and B M be

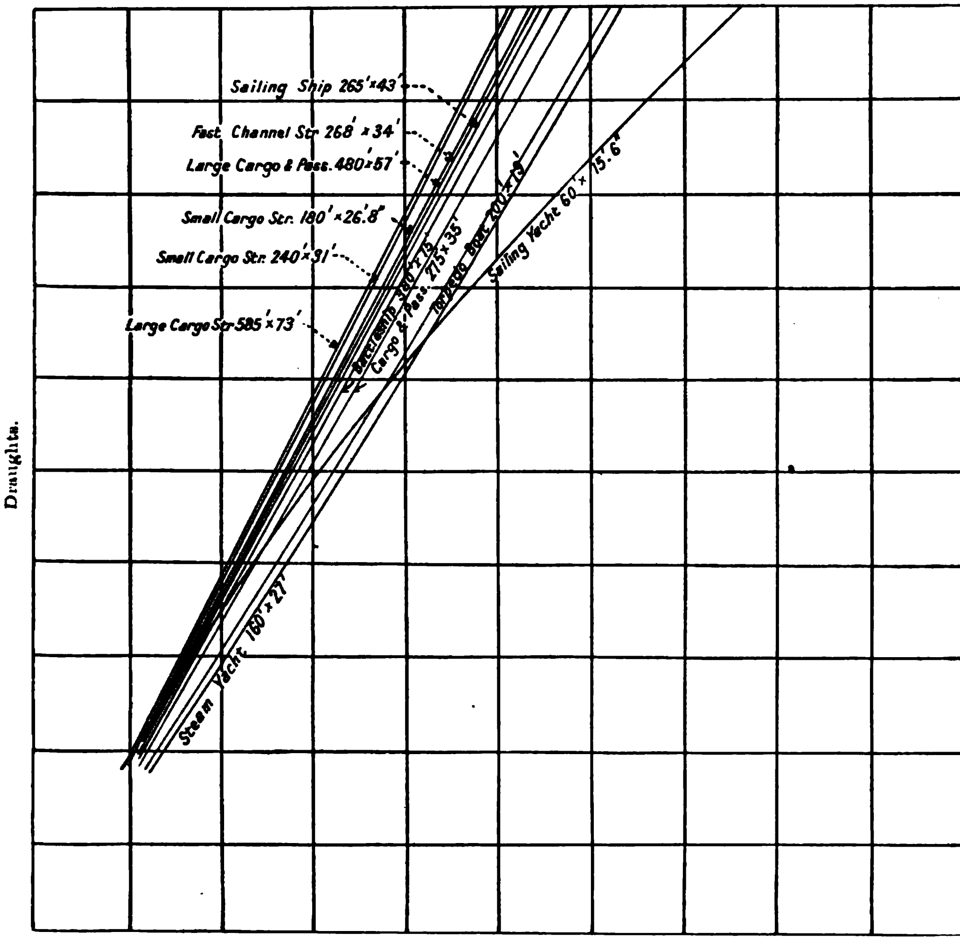



FIG. 99.—Centres of Buoyancy.

set off on the scales already used, viz. 1 in. $= \frac{\delta}{10}$ ft. and 1 in. $= \frac{B^2}{60\delta}$ ft., the resulting curve of K M will, especially in the lighter draught parts of the curve, be outside the scope of the 10-in. diagram. The K M of a block is $\frac{\delta}{2} + \frac{B^2}{12\delta}$, and let 5 in. $= \frac{\delta}{2} + \frac{B^2}{12\delta}$, then K M's can be set off on the scale 1 in. $= \left(\frac{\delta}{10} + \frac{B^2}{60\delta}\right)$ ft., and all will be directly comparable with the K M of the block.

The resulting curves of K M show the position of metacentre for each form in terms of the K M of a block. These curves are shown in fig. 101.

Ratio curves can also be set off by letting 10 in. represent a coefficient of unity, so that 1 in. = ·1 of coefficient. Curves of block, prismatic, midship

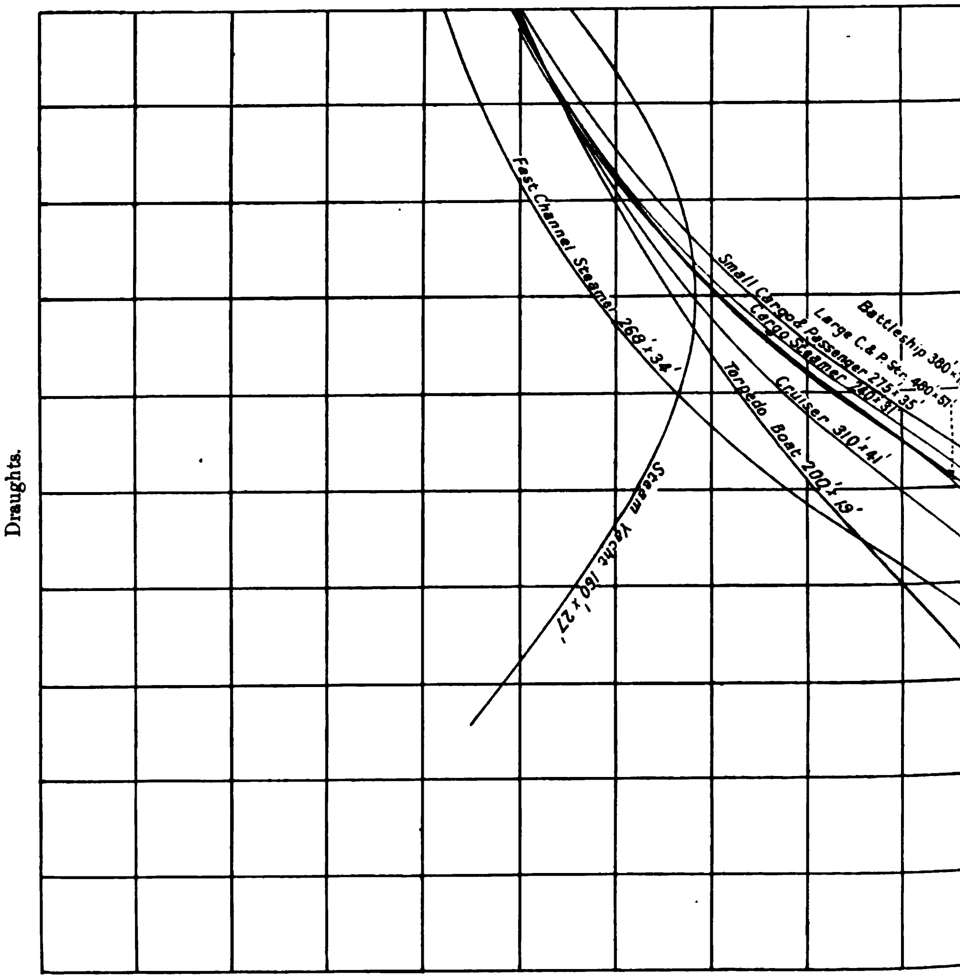



FIG. 100.—Transverse Metacentres, or B M's.

area and waterline area coefficients, in terms of draught, are shown in figs. 102, 103, 104, and 96 respectively. The reason why the waterline areas coefficient coincides with the waterline areas is that the draught is not involved in the calculation of the coefficient, and B remains constant.

The mid-section areas could easily be determined from the coefficient of mid-section areas by measuring the ordinate (at draught $d$, say) to the scale

1 in. $= \frac{B.d}{10}$ sq. ft. The corresponding ordinate at draught $d$ on the mid-section area curve is set up to scale 1 in. $= \frac{B.\delta}{10}$ sq. ft. ($\delta$ being load draught).

The scales corresponding to the different diagrams are given in the following table:—

TABLE XXI.—CURVES ON 10-INCH DIAGRAMS.

| No. of Fig. | | Vertical Scale. | Horizontal Scale. |
|---|---|---|---|
| 95 | Displacements . . . . | $1'' = \frac{\delta}{10}$ (draught scale) | $1'' = \frac{LB\delta}{350}$. |
| 96 | Areas waterline planes . . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \frac{LB}{10}$. |
| 96 | Coefficient waterline planes . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \cdot 1$ of coefft. *i.e.* $\frac{LB}{10}$. |
| 97 | Areas mid-sections . . . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \frac{B\delta}{10}$. |
| 98 | Longitudinal centres of buoyancy | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \frac{L}{100}$. |
| 99 | Centres of buoyancy, K B's . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \frac{\delta}{10}$ (draught scale). |
| 100 | Transverse metacentres, B M's . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \frac{B^2}{60\delta}$ |
| 101 | Transverse metacentres, K M's . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \frac{\delta}{10} + \frac{B^2}{60\delta}$ |
| 102 | Block coefficient . . . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \cdot 1$ of coefft. *i.e.* $\frac{LBd}{350}$. |
| 103 | Prismatic coefficient . . . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \cdot 1$ of coefft. *i.e.* $\frac{\text{mid-section area} \times L}{350}$ |
| 104 | Coefficient of mid-section . . | $1'' = \frac{\delta}{10}$ ,, ,, | $1'' = \cdot 1$ of coefft. *i.e.* $\frac{Bd}{10}$. |

*Note.*—$\delta$ = load draught. $d$ = general draught. L = length between perpendiculars. B = breadth moulded.

It is evident that we may set off the lines of a ship, that is, the form itself, in a similar manner. The breadth and draught ordinate may each be set off respectively on scales such that 10 in. $= \frac{1}{2}$ B, or 1 in. $= \frac{B}{20}$ ft., and 10 in. $= \delta$, or 1 in. $= \frac{\delta}{10}$ ft.

We may also set off length ordinates such that, say, 50 in. = L ft., or 1 in. $= \frac{L}{50}$ ft. All forms would by this means be brought to a directly comparable scale.

Other methods of comparing lines are given which have other advantages than this method. For instance, in fig. 1 of Plate V. the length is on a scale of 1 in. $= \frac{L}{40}$ ft., and the transverse sectional areas are plotted to a scale of

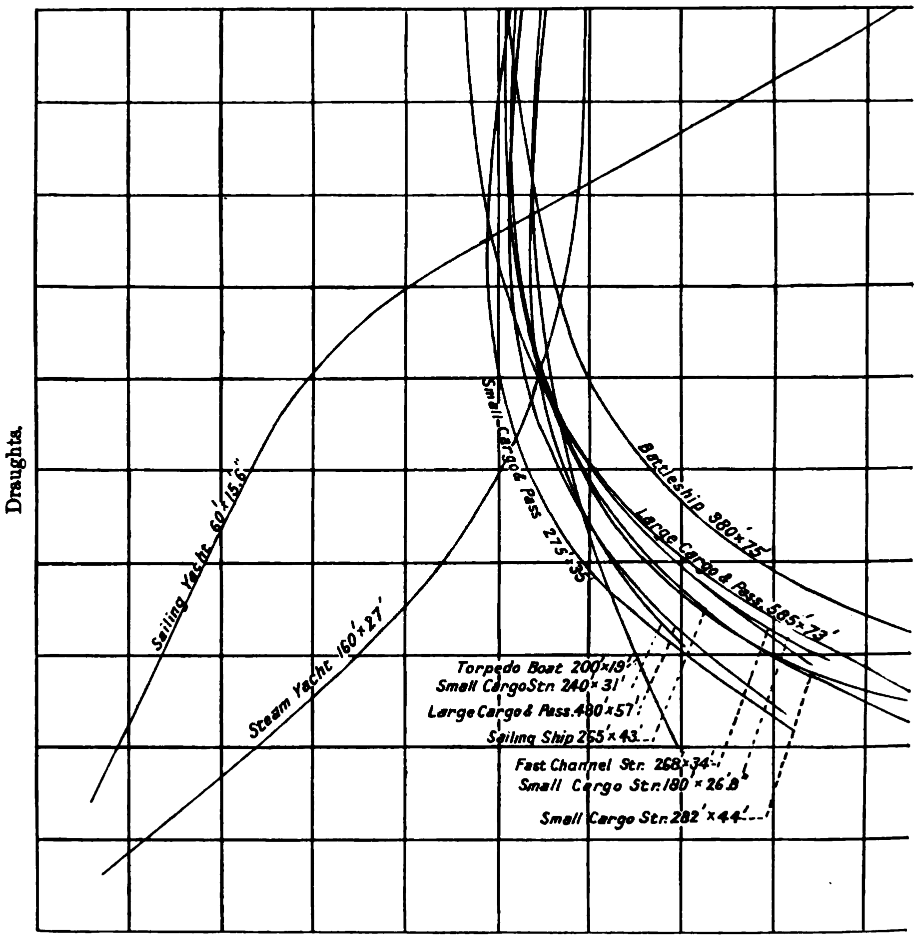



FIG. 101.—Transverse Metacentres, or K M's.

5 in. = B.$\delta$ sq. ft., or 1 in. $= \frac{B.\delta}{5}$ sq. ft. These curves show the longitudinal distribution of displacement.

This method does not measure the fineness quite so well as that shown in fig. 2 of the same plate, in which the length is set off on a scale of 1 in. $= \frac{L}{40}$ ft., and the ordinates are the $\frac{\text{area of half cross-section}}{\text{draught}}$ set off on the same scale as the length. This ordinate is really the mean half breadth

of the section, and, as the draught is assumed constant for each line, it is really a cross-sectional area curve, the same as in fig. 1, but to another scale, and one which is directly comparable by the eye with the length. The angles of the lines at the ends are better measures of the fineness than in fig. 1.

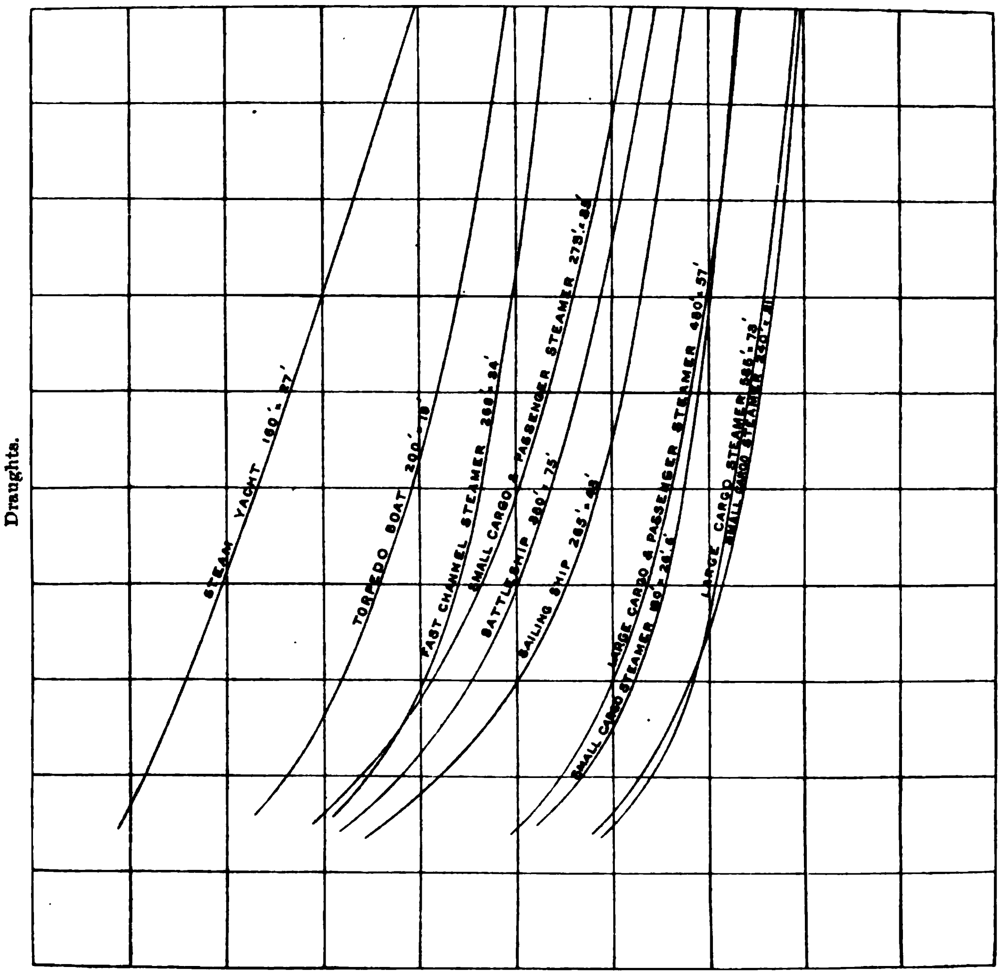



FIG. 102.—Block Coefficients.

For a more complete comparison of lines it is well to set off to the same scale—

1. The load waterplane.
2. A waterplane at $\frac{4}{10}$ths of the load draught.

These should be recorded on separate bases so that each characteristic of the lines of different forms can be compared with each other. For instance the load waterlines should be kept together.

It will be interesting to note that the $\frac{4}{10}$th waterline is very like the mean half-breadth line. With a few actual forms before one it is quite easy, when the load waterline and the mean half-breadth line are fixed, to draw the $\frac{4}{10}$ths waterline. With these and the midship section it is comparatively easy to draw a complete set of lines.

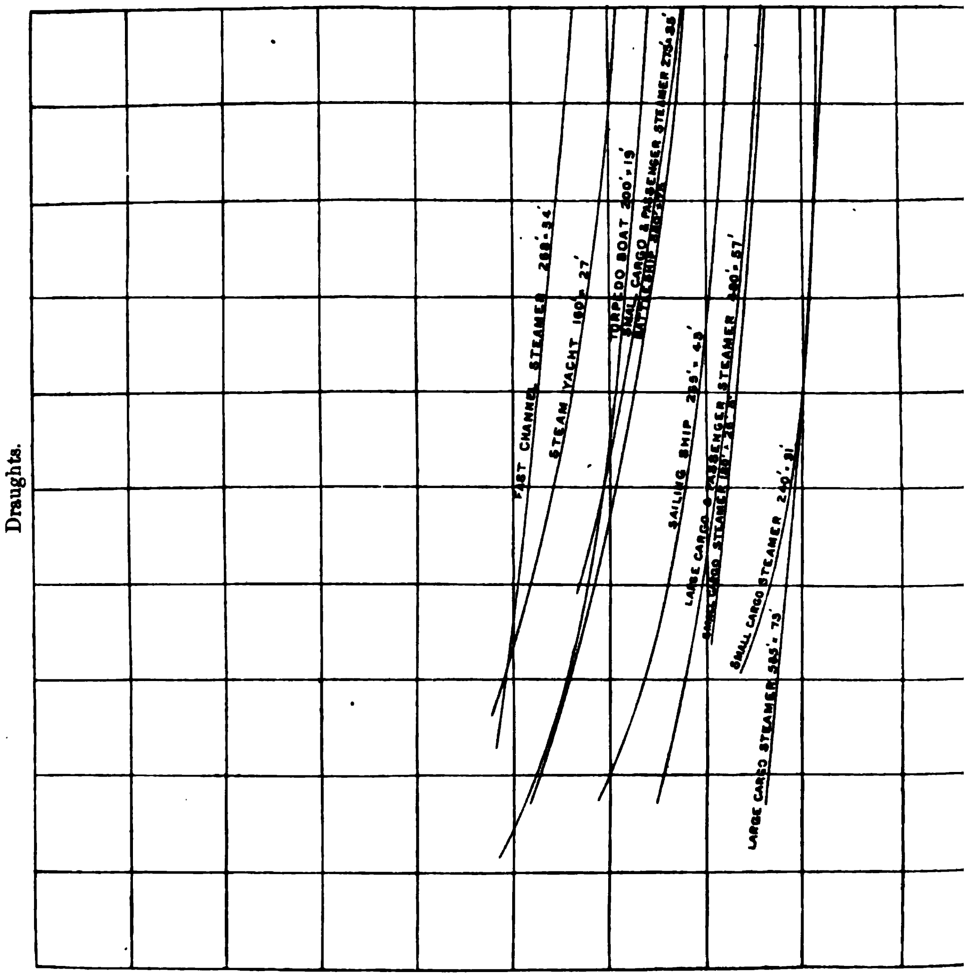
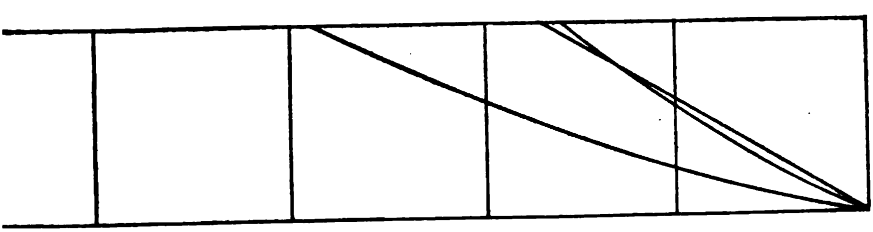
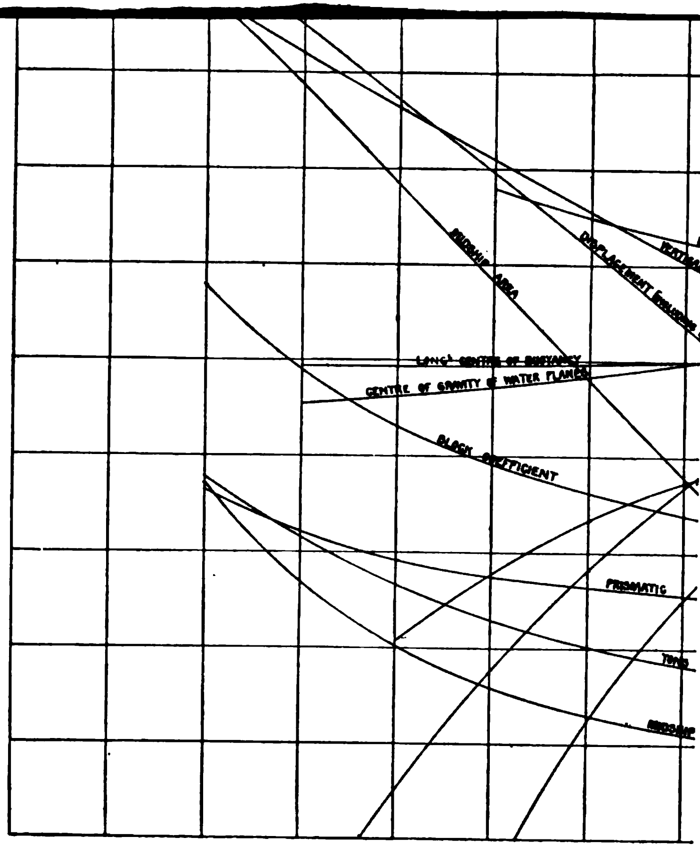



FIG. 103.—Prismatic Coefficients.

The curves of the form, for which the calculations have been made in Plates II. and III. and have been drawn in Plate IV., are shown standardised in Plate VI.

**Form of Displacement Sheet to suit Tchebycheff's Rules.**—See Plates VII. and VIII., which are arranged to suit Tchebycheff's three-ordinate Rule for the longitudinal integration and Simpson's Rules for the




MIDSHIP AREA
CENTRE OF GRAVITY OF WATER PLANES
PRISMATIC

vertical integration. Tchebycheff's Rules, and the methods of using them to determine areas and volumes, have been described in Chapter III. of Part I. If the sections and the waterplanes are both spaced to suit Simpson's Rules there is more labour in making the longitudinal integrations than the vertical

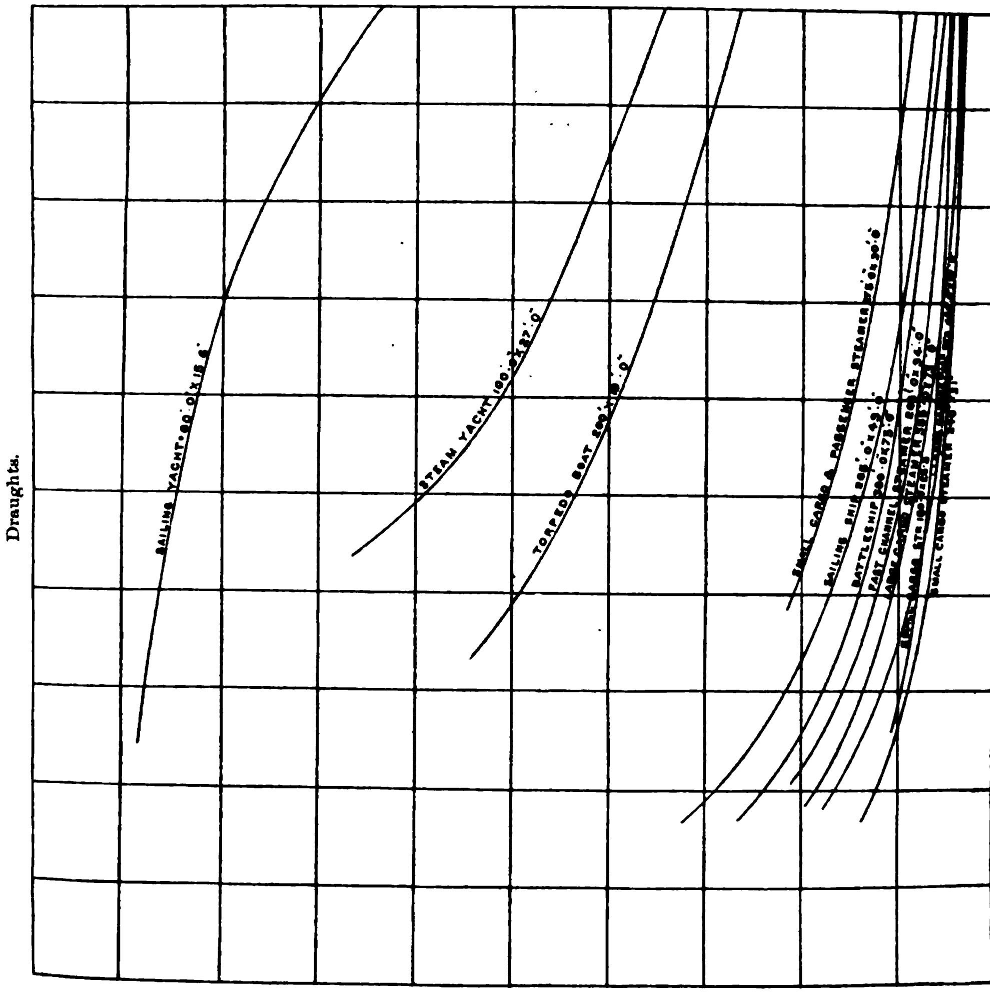



FIG. 104.—Coefficients of Mid-sections.

integrations, owing to the necessarily larger number of ordinates of the former.

The number of waterplanes drawn in the finished lines seldom exceed 9 or 10, and it makes very little difference in calculating displacement whether these are spaced to Simpson's or to any of Tchebycheff's Rules. It is desirable, for fairing purposes, to have the waterplanes in the finished lines spaced equidistantly, and it saves trouble to use these lines instead of making another set. Hence the equidistant spacing for the displacement sheet calculation is better for vertical integration, and consequently Simpson's Rules are applicable.

The Tchebycheff spacing can be drawn on the half-breadth plan and the ordinates measured directly therefrom and put in the displacement sheet, in this case in the columns headed A. This saves the necessity of drawing new sections. The Tables in Plates VII. and VIII. are very compact, and in comparison with the tables of Plates II. and III. involve much less arithmetical work. The form of Plates VII. and VIII. was designed by Mr W. J. Luke, the Naval Architect at Clydebank. The Tchebycheff Rule that is here adopted for the longitudinal integration is for three ordinates. It is applied to each fifth part of the length, thus giving fifteen sections in all. The rule for the spacing of the ordinates is clearly shown at the top left-hand corner of the sheet. The ordinates $a, b, c$ are spaced over the first fifth-part of the length. $b$, the central ordinate, coincides with the first tenth division of the length, and the distance between the ordinates $b$ and $c$ or $b$ and $a$ is equal to $\frac{L\sqrt{2}}{20}$ or $\cdot 7071\frac{L}{10}$. The divisions of the length into tenths are marked on the explanatory figure, and it will be seen that the Tchebycheff ordinates $b, e, h, m$, and $p$ coincide with the ordinates of the divisions 1, 3, 5, 7, and 9 respectively.

It is not generally necessary to draw the Tchebycheff sections in the body-plan, but if for any reason it should be, it is only those corresponding to $a\,c\,d\,f\,g\,k\,l\,n\,o$ and $q$ which have to be drawn, as the other sections have generally been drawn at intervals of one-tenth of the length.

In finding areas of a waterplane the ordinates corresponding to the sections $a, b, c$, etc. are placed in the corresponding vertical column under the heading A. The ordinates are then added, and the sum multiplied by $\frac{L}{15}$ gives the area of half of the waterplane.

The rules for multiplying the functions and obtaining all the results are given in a compact form at the top right-hand corner of the sheet. It will be seen that in the method of obtaining the longitudinal moment of the waterplanes the multiples of the leverages are integers. The actual distances of the ordinates from the axis do not correspond exactly to these multiples, but they are sufficiently close for all practical purposes. The following table (XXII.) shows the real and the assumed distances that have been taken to represent the leverages of the ordinate from $h$, the midship ordinate.

The levers shown at the left-hand side of the tables are assumed to be simple multiples of $\frac{L}{15}$. The errors in the moments of the ordinates tend to neutralise each other in the algebraic sum of all the moments, because they are of opposite signs in the opposite ends of the vessel. The error in individual ordinates varies from $-5\cdot 6$ per cent. to $+3\cdot 1$ per cent., and if all the ordinates were of equal length there would only be a mean error of minus one-third per cent. As the ordinates at the ends are less than toward the middle of the length the mean error will probably be less. But taking the algebraic sum of the errors will make the mean error negligible. The columns B are for the moments. The columns C are for the cubes of the ordinates, and these have simply to be added to obtain the function of the transverse moment of inertia. The columns D are for the moments multiplied by the levers, giving the functions for the longitudinal moment of inertia.

TABLE XXII.

| Ordinate. | Actual Distance from L. | | Assumed Distance. | % error. |
|---|---|---|---|---|
| *h* | 0 | = | 0 | 0·0 |
| *g* or *k* | $\frac{L\sqrt{2}}{20} = \cdot707L$ | > | $1.\frac{L}{15} = \cdot667L$ | −5·65 |
| *f* ,, *l* | $\frac{L}{5} - \frac{L\sqrt{2}}{20} = 1\cdot293L$ | < | $2.\frac{L}{15} = 1\cdot333L$ | 3·08 |
| *e* ,, *m* | $\frac{L}{5} = 2\cdot0L$ | = | $3.\frac{L}{15} = 2\cdot000L$ | 0·0 |
| *d* ,, *n* | $\frac{L}{5} + \frac{L\sqrt{2}}{20} = 2\cdot707L$ | > | $4.\frac{L}{15} = 2\cdot667L$ | −1·11 |
| *c* ,, *o* | $\frac{2L}{5} - \frac{L\sqrt{2}}{20} = 3\cdot293L$ | < | $5.\frac{L}{15} = 3\cdot333L$ | +1·21 |
| *b* ,, *p* | $\frac{2L}{5} = 4L$ | = | $6.\frac{L}{15} = 4\cdot00L$ | 0·0 |
| *a* ,, *q* | $\frac{2L}{5} + \frac{L\sqrt{2}}{20} = 4\cdot707L$ | < | $7.\frac{L}{15} = 4\cdot666L$ | − ·06 |
| | | | | −6·82<br>+4·29 |
| | | | 7 | −2·53 |
| | | | Average over | − ·36% |

It is only necessary to fill in column D for three or four waterplanes.

In the sheet there are columns for nine waterplanes. The part below No. 1 waterplane is calculated separately as an appendage at the right-hand side, Thomson's Rule[1] being used.

The lower half of the sheet has columns for combining the result obtained from the upper half.

For instance, the functions of the waterplanes got by adding up columns A are integrated by Simpson's 1st Rule and the displacement is obtained. By integrating the moments of the waterplane functions we can get the vertical C.B. and so on. Spaces are left for adding in the results of the calculations for appendages, etc. The table at the foot of the sheet shows the results at a glance. A space is left alongside this table for plotting the results in curves as is done in Plate IV.

Plate VII. is for the same form as is used for Plates II. and III. The calculation in Plate VIII. is for a large Atlantic liner, 580 feet, and the standardised curves are drawn on the same table.

[1] Thomson's Rule for finding the area of a plane curve:—Use any odd number of ordinates equally spaced. Erect "half" ordinates at the middle of each end space. Then the area equals the common interval multiplied by the combined sum of $\frac{1}{3}$rd the end ordinates, $\frac{4}{3}$th the half ordinates, $\frac{11}{12}$ths the ordinates next but one to each end, and all the remaining ordinates.

# CHAPTER XII.

## INSTRUMENTS USED TO DETERMINE AREAS, MOMENTS, AND MOMENTS OF INERTIA OF PLANE CURVES.

THE methods of determining areas, etc. of curves by the use of multipliers, though generally applicable to ship work, are laborious. Sometimes it is impracticable to use such rules. For both reasons, and to save labour and to increase accuracy, instruments known as planimeters, integrators, and integraphs are used.

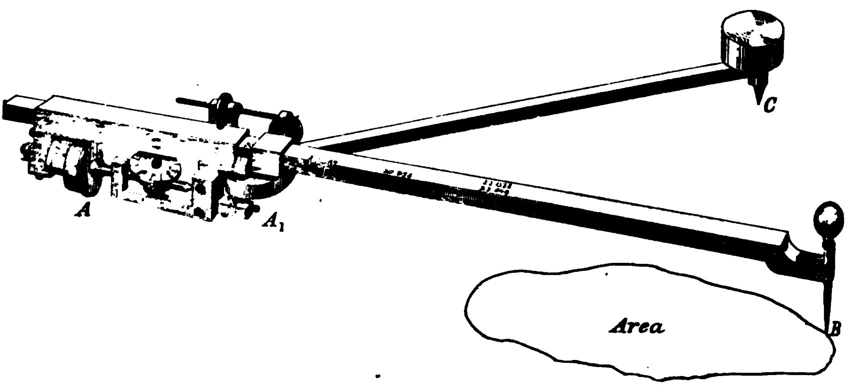



FIG. 105.—Planimeter.

The simplest form of these instruments is the planimeter, fig. 105, which records areas only. The instrument works by recording the motion of a tracing point which is guided over the boundary of a closed curve, generally beginning and ending at the same point.

**The Planimeter.**—It consists of two bars A B and A C jointed at $A_1$, as in fig. 105. The end C of one bar, called the radial bar, is fixed by a needle-point in the plane of the paper at a convenient distance from the area to be measured. The end B of the other bar, called the sliding bar, carries a pointer, which is guided by hand over the line bounding the given area. On the bar A B is a wheel A, which is free to revolve on an axis parallel to A B. The instrument rests on the three points, C, B, and the circumference of the wheel A. As B is moved over the line, the wheel A revolves and also slides, and the amount of revolution can be easily read off on a scale round its circumference. The principle of the planimeter is that the amount of revolution of this wheel is a function of the area swept out by the bar A B, and when the path of B is a closed curve the amount of revolution is proportional to the area enclosed by the path.

Fig. 106 shows the arrangement of the wheel and vernier for taking the readings; $a\,b$ is the axis of the wheel. The number of revolutions that the wheel makes is recorded on a horizontal dial by a worm-gearing arrangement. The fractions of a revolution can be read off on the wheel by means of a scale and a vernier.

The scale of the reading is altered by altering the length $A_1$ B, and for this purpose the bar is graduated and the joint $A_1$ is a sliding one. It can be temporarily secured to any point on the scale of the sliding bar. On the sliding bar marks are put at 5, 10, 15 and 20 square inches, etc. These

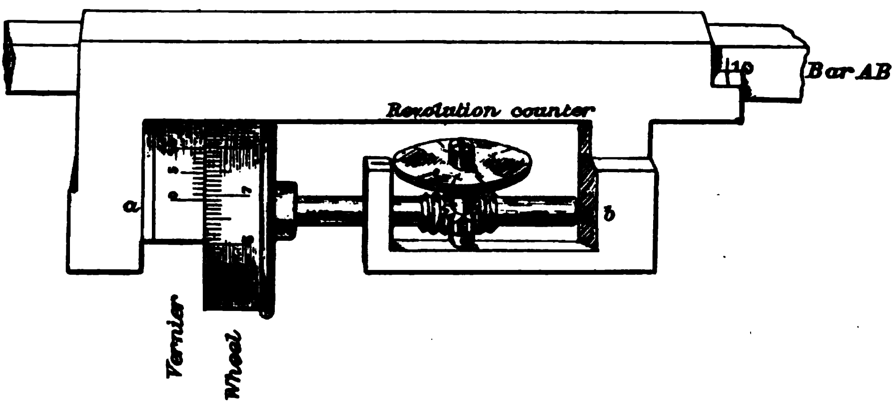



FIG. 106.—Planimeter. Vernier.

figures represent respectively the area in square inches corresponding to one revolution of the wheel A. Suppose we have, when the planimeter is fixed at 10 square inches, a reading $x$ when a certain area is traced over, then the actual area traced over $= 10 \times x$ square inches.

*Proposition.*—The area of the curve is a function of the length of the bar $A_1$ B and the revolutions of the wheel:—

Consider the area traced out by a line A B, fig. 107, the ends A and B of which always lie on fixed paths, A A′ and B B′ respectively. A and B correspond to the same lettered points in the planimeter.

Let A B and A′ B′ be two successive positions of the line, very close together.

Neglecting the second order of infinitesimals, the area of A B B′ A′ may be considered to be equal to the area of the parallelogram A B $B_1$ A′ plus the area of the triangle $B_1$ A′ B′.

The movement of A B to A′ B′ can be considered as made up of two movements,—

(1) A translation of A B to A′ $B_1$.

(2) A rotation about A′ of A′ $B_1$ to A′ B′.

Let $h$ be the perpendicular distance between A B and $A' B_1$, *i.e.* $h$ = normal displacement of A B.

Let $da$ be the angle of rotation $B_1 A' B'$, *i.e.* $da$ = rotational or angular displacement.

Suppose, now, we have a wheel fixed at P, any point in A B, A B being the axis.

$$\text{Let} \quad AP = p$$
$$\text{and} \quad AB = l.$$

As the line A B is moved to $A' B_1$ the wheel will partly rotate and partly slide.

The reading of the wheel, *i.e.* the travel of its circumference at $P_1$, will be equal to $h$. As the line is rotated from $A' B_1$ to $A' B'$ the wheel will only

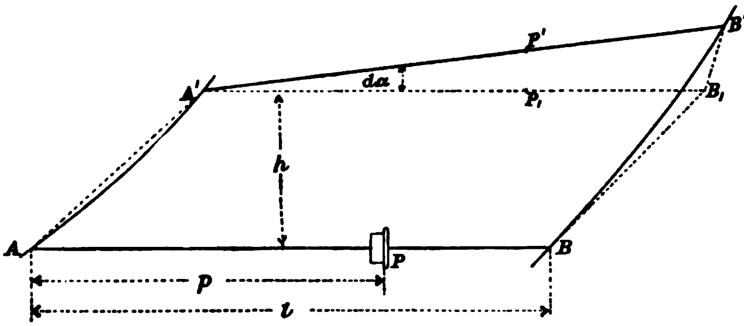



FIG. 107.

have a rotary motion, and its reading at P′ will be equal to $(h + p.da)$, which we will call $ds$.

$$ds = h + p.da$$
$$h = ds - p.da \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$\text{Let} \quad dz = \text{area } ABB'A'$$
$$= AB.h + \tfrac{1}{2}.AB^2.da$$
$$= lh + \tfrac{1}{2}.l^2 da \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting the value of $h$ from (1) in (2) we get

$$dz = l.ds - l.p.da + \tfrac{1}{2}l^2 da$$
$$\therefore \quad dz = l.ds + (\tfrac{1}{2}l^2 - lp)da.$$

If we integrate both sides of the equation we shall get:—

Total area = $l$ times the total reading $+ \frac{1}{2}(l^2 - lp)$ times the total angle which the sliding bar moves through.

If the sliding bar returns to its original position, which it will in a closed curve, the integrated or summed-up value of $a$ will be zero, and therefore

$$z = l.s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

*i.e.* the area enclosed by the curve, which is then the area traced out by the moving bar, is equal to the product of the length of the bar and the reading.

We therefore see that—

(1) The equation (3) is true, no matter what may be the path of A.
(2) The reading is independent of the position of the wheel in A B.
(3) The reading is inversely proportional to the length A B, so that A B determines the scale of the reading.

In the planimeter the path of A is a circle.

The scale of the planimeter is therefore dependent upon the radius of the wheel and the length A B.

If the radius of wheel be $r$, then the circumference $= 2\pi r$

and length of $AB = l$

$\therefore$ Each revolution $= (2\pi r \times l)$ square inches.

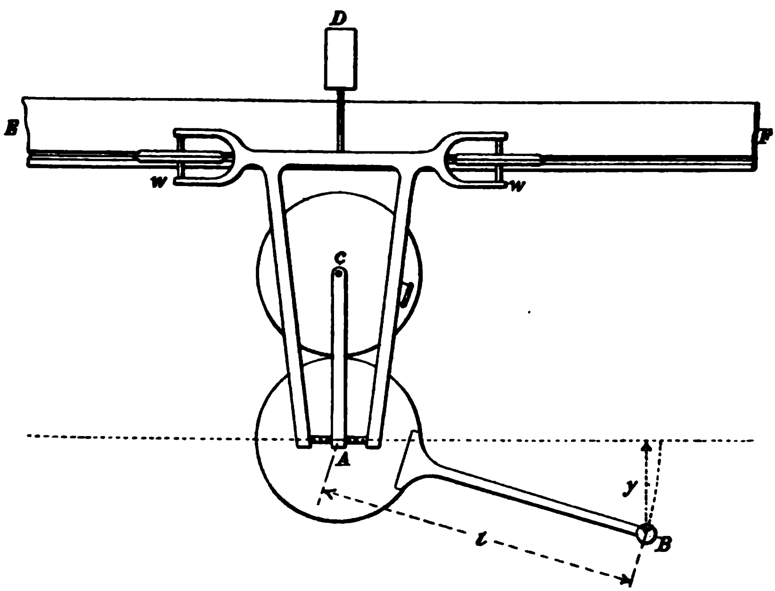



FIG. 108.—Planimeter (second form).

**Another Form of Planimeter.**—In this second type of planimeter (fig. 108) the path of A is a straight line. The bar A B, as shown in the figure, corresponds to the sliding bar of the ordinary type. The motion of A B is governed by two small wheels $w$ and $w$ as shown, which run in the groove of a steel set bar E F. The travel of A is therefore parallel to E F. This motion is equivalent to having a radial bar of infinite length in the previous type. A B is the rod carrying the tracing point B.

On A B and jointed at A is fixed a toothed disc which gears with another disc of equal diameter jointed at C as shown.

Any rotation of A B causes a revolution of the disc C. The movement of this combination can also be controlled by non-slipping wheels. The wheel for taking readings is fixed in the disc C, and is on the radius parallel to E F, when A B is parallel to E F.

The scale is, as before—1 revolution $= 2\pi rl$ square inches.

**Integrator.**—The integrator is an extension of the last form of planimeter.

The integrator is an instrument which gives the integrals

$$\int y.dx \text{ – Area function,}$$

$$\int y^2.dx \text{ – Moment function,}$$

$$\int y^3.dx \text{ – Second moment or moment of inertia function,}$$

$$\int y^4.dx \text{ – Third moment function,}$$

of a closed area about an axis O X, the axis of O X being parallel to the steel set bar E F of the machine.

Fig. 109 is an illustration of the most commonly used type of integrator. This type has wheels to give area, moment, and moment of inertia readings, but not the third moment.

In the figure 109, E F is a separate steel set bar which is first set parallel to, and at a fixed distance from, the axis O X about which moments are being taken. There is a straight groove in E F, and when the integrator is placed so that its wheels $w$ and $w$ run in this groove the centre A runs along the axis of O X. In addition, the integrator rests on the recording wheels P, Q, and R, and on the tracing point B, so that each connection carrying the wheels and the point B has to be independent of the support of the other points. The tracing point B is at the end of a bar A B which revolves about A. On this bar A B there is fixed the wheel P for area readings similar to the arrangement described for the planimeter reading wheel. A is the centre of a fixed disc which has a toothed circumference. On to this disc are geared smaller discs which carry the wheels for the moment, moment of inertia, and in some machines third moment readings.

It will be easily seen that the area wheel motion is simply that already described of the planimeter, with an infinitely long radial bar, as in fig. 108.

*Motion of Moment Wheel.*—Consider fig. 110, which has been drawn in simple diagrammatic form for clearness.

Let A B and C D be two discs geared as in the integrator. P is the tracing point at the end of the arm O P, O being the centre of disc A B.

Let the path of O be O O′ as in the dotted line.

Let $c$ be the centre of other disc C D, then $c c'$ is the path of $c$ and is parallel to O O′.

Consider the initial position of O P to be on line O O′, and let the radius, which in C D is perpendicular to O P when in that initial position, be $c d$.

If O P revolves through an angle $a$ to O $P_1$, then $c d$ will revolve through some angle $\phi$ to $c e$. This angle will be given by

$$\phi = n(-a)$$

where $n$ is the ratio of the radii of the discs, *i.e.* $\dfrac{\text{radius of AB}}{\text{radius of CD}} = n.$

Since the two discs A B and C D move in opposite angular directions, the two angles $a$ and $\phi$ will always be of opposite sign.

Now consider the machine in the position when P is at $P_1$, then $d$ is at $e$.

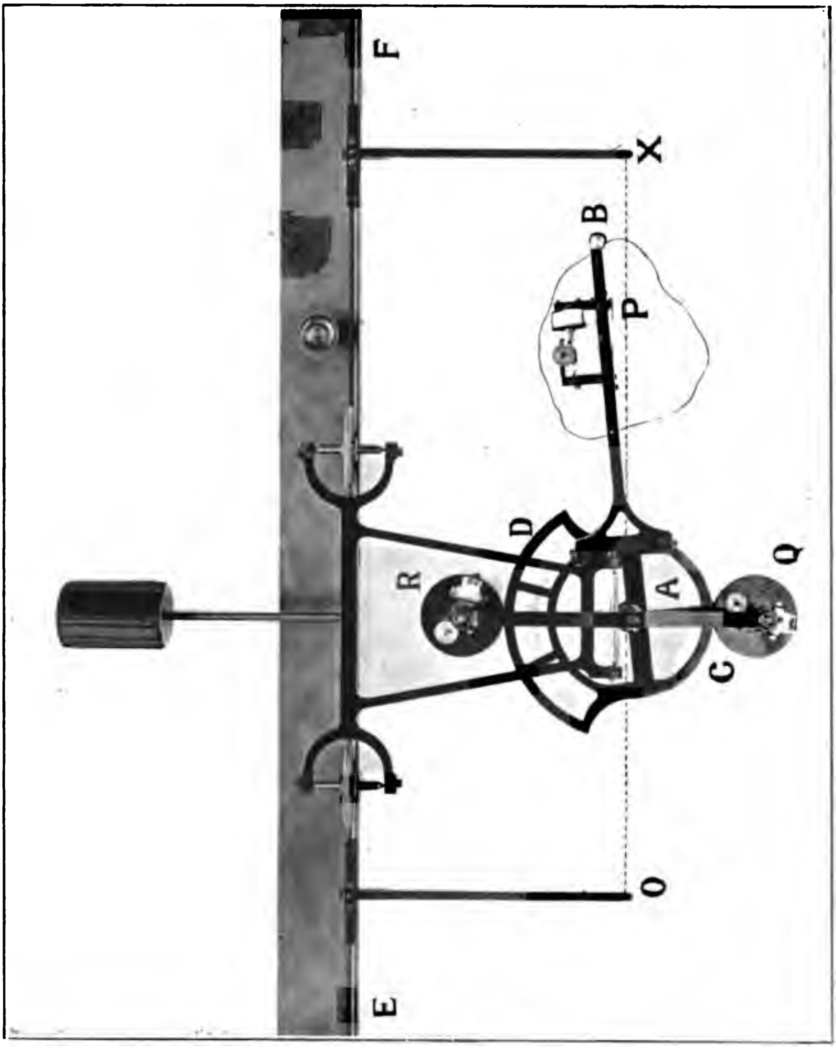



Fig. 109.—Integrator.

If $P_1$ moves a distance $dx$ to the right parallel to axis O O′, then the wheel in C D slides (without rotation) a distance $dx. \sin \phi$, and records (by its rotation) a distance $dx. \cos \phi$, which is equal to $dx. \cos(-na)$, or $dx. \cos na$.

Now revolve O $P_1$ back so as to coincide with O O′, then $c\,d$ will revolve through an angle $-\phi$.

Finally, move the tracing point a distance $dx$ towards the left, so as to arrive at the initial position. This movement $(= -dx)$ will be recorded by the wheel in C D.

Thus after the pointer P has traced completely round the element of area, the total angular movement of the disc C D is zero, and the wheel in C D has recorded a distance $dx.\cos na - dx = -(1-\cos na)dx$.

Therefore the reading of an area enclosed by a curve will be given by

$$-\int(1-\cos na)dx.$$

If $n=2$, this expression becomes $= -\int(1-\cos 2a)dx$

$$= -\int\{1-(1-2\sin^2 a)\}\,dx = -\int 2\sin^2 a.\,dx = -\int\frac{2y^2}{l^2}\,dx,$$

where $l$ = length of bar O P, and $y$ = displacement of P from O O′.

∴ Expression (numerically) $= \frac{2}{l^2}\int y^2.dx.$

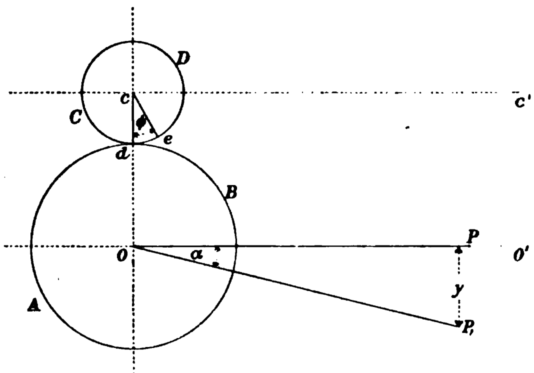



FIG. 110.—Integrator (diagrammatic view).

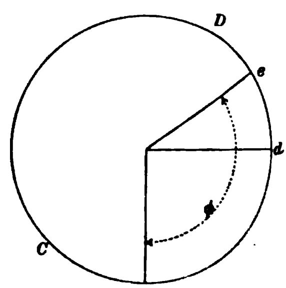



FIG. 111.

But we have seen in a previous chapter that the moment of the area about $OX = \frac{1}{2}\int y^2.dx$.

∴ The reading of the wheel $= \frac{4}{l^2}$ (moment of area about the axis O O′).

It will be here noticed that there are two conditions for the measurement of the moment :—

(1) The wheel must be on an axis perpendicular to O O′ when O P lies in O O′ (see fig. 110).

(2) Radius of disc A B must be twice that of disc C D.

*Motion of Moment of Inertia Wheel.*—Suppose the recording wheel to be placed in the disc C D along the radius $d$, fig. 111, so that its axis is parallel to O O′ when O P lies in O O′. This is equivalent to turning the disc C D through a right angle; then we have

$$\phi = na + \frac{\pi}{2}.$$

In this case the reading of the wheel $=\int \cos.\phi.dx$ for a movement of P around an enclosed area.

This expression $=\int \cos\left(-na+\frac{\pi}{2}\right)dx.$

If $n=3$,

this expression becomes
$$\int \cos\left(-3a+\frac{\pi}{2}\right)dx$$
$$=\int \sin 3a.dx$$
$$=\int 3\sin a.dx-\int 4\sin^3 a.dx$$
$$=\frac{3}{l}\int y.dx-\frac{4}{l^3}\int y^3.dx.$$

But $\int y.dx=$ Area,

and $\frac{1}{3}\int y^3.dx=$ Moment of inertia of area about O O′.

$\therefore$ Reading of wheel $=\frac{3}{l}$ area $-\frac{4\times 3}{l^3}$ (Moment of inertia of area about O O′).

The two conditions for the measurement of the moment of inertia are therefore—

(1) Axis of wheel must be parallel to O O′ in the initial position.
(2) Radius of disc A B must be three times that of disc C D.

Referring again to fig. 109, we see that this arrangement must hold for the radii and position of wheels in the disc. A convenient arrangement is shown in this figure. One part of the circumference of the disc C has a certain radius $r$, say, and the other part D has a radius $\frac{3}{2}r$. On the circumference with the larger radius, the moment of inertia disc R is fixed, and on the circumference with the smaller radius the moment disc Q is fixed. The radii of these two discs are each equal to $\frac{1}{2}r$. The placing of the reading wheels in these discs is in accordance with the conditions which we have seen must necessarily exist.

In some integrators the point P in the bar can be altered, thus altering $l$. This changes the scale. This is done by a lengthening bar in A B so that large areas can be measured in one operation.

This type of integrator has scales

20 for area,
40 for moment,
$(320\,a-100i)$ for moment of inertia,

where "$a$" is the reading of the area wheel P, and "$i$" is the reading of the moment of inertia wheel R. These numbers, when multiplied by the reading, give the area in square inches, the moment in inch.inch$^2$, and the moment of inertia in inch$^2$.inch$^2$.

It will be seen that the integrator gives the area and moment, etc. only for the whole curve. It is possible by dividing the curve into parts to find in succession the area of each part. This, however, can be done more conveniently by an instrument called the integraph, which continuously records the integral from point to point as the tracing point is guided over the curve.

**Description of the Integraph.**[1]—Fig. 112 represents the latest type of this machine. The figure has, for simplicity, been drawn in diagrammatic form.

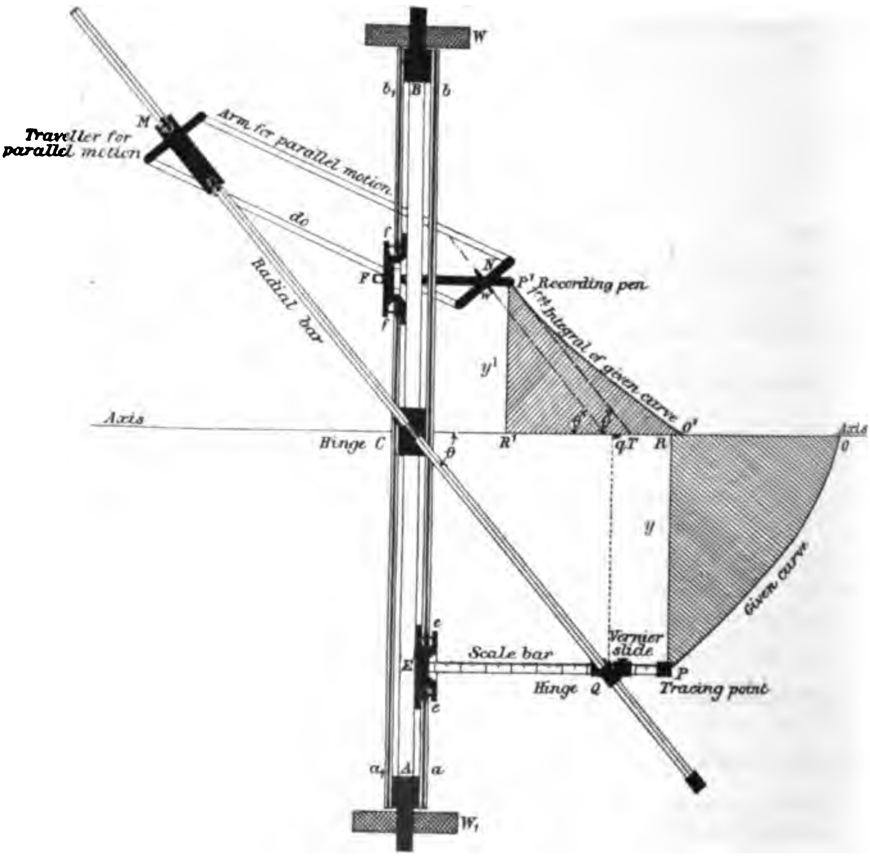



FIG. 112.—Diagram of the Integraph in a working position.

The motion of the machine is governed by the two non-slipping wheels W and $W_1$, which are fixed about 22 inches apart on the spindle A B. On this arrangement are suspended two grooved bars *a b* and $a_1 b_1$, each of which is grooved to carry an arrangement of travelling wheels. These travellers are shown at *e e* and *f f*. The bars *a b* and $a_1 b_1$ are fixed together at their ends and at the centre. In the centre piece there is a pivoted block C, through which a long radial bar M Q can slide and revolve C. A scale bar E P, on which is measured inches, is supported at right angles to A B, and is fixed to

[1] For a more detailed description of the integraph see a pamphlet, *L'Intégraphe Abdank-Abakanowicz*, by Henry Lossier, published by G. Coradi, Zurich, Suisse.

the traveller *e e*. At the end P of this scale bar there is a tracing point. The zero of the scale bar is at the middle between $a\,b$ and $a_1\,b_1$.

A vernier arrangement, with a pivot joint attachment Q, slides along E P, and can be fixed at any point on the bar along the scale. The radial bar M Q passes through the pivot Q, and can be fixed at Q so as to prevent it from sliding along E P, but permits it to revolve about Q. The recording pen $P^1$ is fitted into a bar F $P^1$ supported at right angles to the bar $a_1\,b_1$ from the traveller *f f* on $a_1\,b_1$.

$P^1$ records the movements of a small wheel *w* which has a sharp edge. This wheel can be lowered on to the paper when the machine is to integrate, and it is so fitted that it can revolve about a vertical axis through its centre. The direction of this wheel is governed by the parallel motion M N, the end M being free to travel on the radial bar M Q as shown. It will be seen that the wheel *w* is thus always kept parallel to the radial bar.

*Setting the Machine.*—When the machine is stationary the tracing point can be moved parallel to the bar $a\,b$. Before integrating a given curve, the machine must be placed so that it runs parallel to the axis of the curve. When the inclination of the radial bar is zero, *i.e.* when it is perpendicular to A B, the tracing point P should trace out the axis of the given curve, and should coincide with or be parallel to the axis of the integral curve.

A small adjustment in the machine provides for bringing the radial bar in the set position so as to test the direction of the motion of the machine when moved along. The scale is fixed by adjusting the vernier slide arrangement to the required number of inches along E P.

*Principle of the Machine.*—The operation of integrating a given curve, such as O P, consists in starting at O and tracing out O P with the tracing point P. The pen can be set at first to any axis parallel to the axis of the curve. In fig. 112 the pen is at O′ when the point P is at O, so that the axis of the integral curve coincides with the axis of the given curve.

When the radial bar is perpendicular to A B, as in the set position, so also is the plane of the wheel *w*, no matter what the position of *f f* on $a_1\,b_1$ may be, so that if the machine be moved, keeping the radial bar in this position, the pen will trace out a line parallel to the axis of the curve or to the radial bar. This is the method of drawing the axis of the integral curve.

Suppose the tracing point P to be kept fixed in the position in the figure and the machine be moved along by rolling on the wheels W $W_1$, the radial bar will keep its position relatively to A B, and the direction of the wheel *w* will not change. Therefore the wheel *w* will follow a line in the plane of its circumference or parallel to the radial bar. If P be moved up or down during the time the machine is moved, the radial bar will rotate about C, and the wheel *w* at any instant will be tangential to the curve traced out.

In fig. 112 let O′ P′ be the curve traced out by the pen as the tracing point P is moved from O to P. Then the wheel *w* is parallel to the tangent P′ T to the curve O′ P′ at P′. Let $\theta$ be the angle of inclination of this tangent. Then $\tan\theta$ is the $\frac{dy}{dx}$ of the curve O′ P′ at P′.

$$\text{But} \qquad \tan\theta = \text{tangent of angle } QCq$$

$$= \frac{Qq}{Cq} = \frac{PR}{EQ}.$$

$$\therefore \qquad \frac{dy}{dx} = \frac{PR}{EQ}.$$

Now E Q is the scale, say $n$, and is constant.

$$\therefore \qquad PR = n\frac{dy}{dx},$$

*i.e.* the ordinate of the given curve is a measure of the first differential coefficient of the curve traced out, and the integral of the given curve is therefore a measure of the ordinate of the curve traced out. Therefore, the ordinate of the curve O′ P′ measures the integral of the curve O P.

*Properties of the first three integral curves.*—We have seen that any ordinate of the given curve is a measure of the differential coefficient of the integral curve at a corresponding point, *i.e.* at a point whose abscissa with reference to the origin of the integral curve is equal to the abscissa of the ordinate of the given curve.

In fig. 112, P R and $P^1 R^1$ are corresponding ordinates of the given and integral curves respectively.

$$\text{Let} \qquad y = PR \text{ and } y^1 = P^1R^1, \ \frac{dy^1}{dx} = \frac{PR}{EQ} = \frac{y}{n}.$$

$$\frac{dy'}{dx} = \frac{y}{n}.$$

$$\therefore \qquad y^1 = \frac{1}{n}\int y.dx \text{ or } ny^1 = \int y.dx.$$

∴ (1) *The ordinate of the integral curve measured in inches and multiplied in inches by the scale n gives the number of square inches in the area of the integrated part of the given curve.*

Referring to fig. 113, let A A be the given curve $y = f(x)$ with reference to the axes O X and O Y.

Integrating A A along the axis of $x$, the first integral curve O $A_1$ or $ny^1 = \int y.dx$ is obtained; integrating O $A_1$ in the same way, the second integral curve O $A_2$ or $ny^{11} = \int y^1.dx$ is obtained; O $A_3$ is the third integral curve.

Take an elemental strip of A A between the parallel ordinates $b$ B and $b^1$ $B^1$ as shown. Then the area of elemental strip = $y.dx$. This area is represented by the horizontal distance between C and $C^1$, the intersections of the ordinates on the curve O $A_1$. Call $OX = l$.

Then the moment of elemental strip about X $A_1$ is equal to $y.dx.(l - x)$.

This is equal to $n$ times the area of strip C $C^1$ $c^1$ $c$. Therefore the area of the first integral curve O $A_1$ represents the moment of the area of the given curve O A A X about the axis of X $A_1$.

This property may therefore be expressed generally.

(2) *Any ordinate of the second integral curve represents the moment of the corresponding area of the given curve,* i.e. *the area between O Y and that ordinate about that ordinate as axis.*

The second integral curve may therefore be called a moment curve with reference to the given curve.

Then from the above property the ordinate $b$ D represents the moment of the area O A B $b$ about $b$ B as axis, and the ordinate $b^1$ $D^1$ represents the moment of the area O A $B^1$ $b^1$ about $b^1$ $B^1$ as axis. It can be easily shown that if the tangents D $t$ and $D^1$ $t^1$ are drawn to meet the line X $A_1$ in $t$ and $t^1$ the ordinate X $t$ represents the moment of the area O A B $b$ about X $A_1$ as axis, and the ordinate X $t^1$ represents the moment of O A $B^1$ $b^1$ about X $A_1$ as axis. Therefore $t\,t^1$ represents the moment of the elemental area strip about X $A_1$ as axis, *i.e.*, $tt^1 = (l - x).y.dx$.

Now the area enclosed between the tangents D $t$ and $D^1$ $t^1$ and the line X $A_1$ is equal to $\frac{1}{2}tt^1\,(l-x)$ which is therefore $= \frac{1}{2}(l-x)^2y.dx = \frac{1}{2}$ moment of inertia of area strip about X $A_1$.

Therefore the area of the triangular element D $tt^1$ of the curve O $A_2$ is equal to half the moment of inertia of area strip about the axis X $A_1$.

So that the whole area of the curve O X $A_2$ represents half the moment of inertia of the area of the given curve about X $A_1$ as axis. Therefore,

(3) *Any ordinate of the third integral curve represents half the moment of inertia of the corresponding area of the given curve about that ordinate as axis.*

The dotted curve O $A_3$ is the third integral curve.

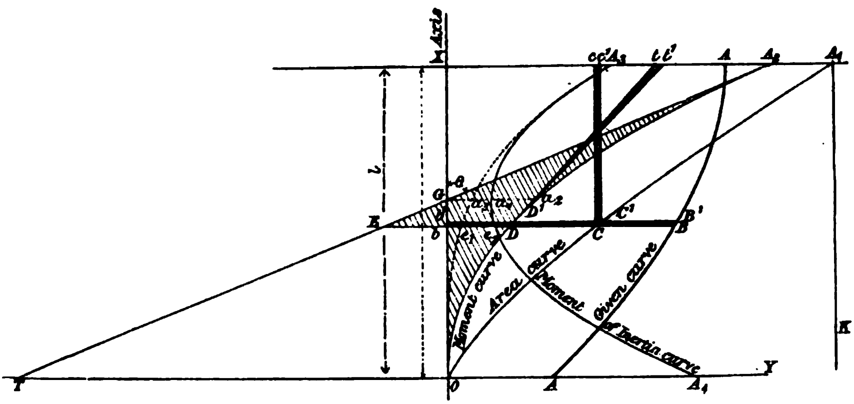



FIG. 118.—First three Integral Curves.

Now, at $A_2$ if the tangent $A_2$ G T to the curve O $A_2$ be drawn to meet the axis O X in G,

$$\text{Then}\quad XG = \frac{XA_2}{\tan\theta} = \frac{XA_2}{XA_1}$$

$$= \frac{\text{Moment of area OAAX about } XA_1}{\text{Area OAAX}}$$

= Distance of C.G. of area of given curve from X $A_1$. Therefore,

(4) *If a tangent be drawn at any point in the moment curve it meets the axis in a point which gives the coordinates of the C.G. of the corresponding area of the given curve.*

*Note.*—[This tangent can be accurately drawn by the machine, as for instance $A_2$ G T is traced out by the pen, when the pointer is made to return along the line $A_1$ K parallel to the axis O X.]

Again, the area of O X $A_2$ represents half the amount of inertia of the area O A A X about X $A_1$ as axis. If the moment of inertia about any other parallel axis is required, then a deduction must be made of $(A)(h^2 - h_1{}^2)$, where $(A) =$ area O A A X and $h_1$ = distance of new axis from G; and $h = GX$.

Let $h_1$ in this case be G $b$.
Then $(h^2 - h_1^2) = GX^2 - bG^2$.
Now $XA_2 = (A)GX$.
Triangle $GXA_2 = \frac{1}{2}(A)GX^2$, and triangle $GbE = \frac{1}{2}(A)bG^2$.

$\therefore$ The correction $(A)(h^2 - h_1^2)$ is twice the area of triangle $GXA_2$ - twice the area of triangle $GbE$. So that the area shown shaded represents half the moment of inertia of the area O A A X about the axis $b$ B.

This shaded area can be represented by the ordinate of the curve $A_3A_4$ which is obtained by tracing over the line $A_2$ T after having traced over $OA_2$.

Thus in the figure $be_1$ = area $ob$D.

$$e_1e_2 = DA_2ED.$$

$\therefore$ Area $be_2 = \frac{1}{2}$ moment of inertia of OAAX about $b$D as axis.

Therefore,

(5) *$A_3$ $A_4$ is a moment of inertia curve, any ordinate of which represents one-half the moment of inertia of the whole of the given area about that ordinate as axis.*

These propositions should be mastered and remembered by the student in order to familiarise himself with the use of the integraph and to simplify the resulting work.

## CHAPTER XIII.

## CARGO CAPACITIES.

BEFORE the centre of gravity of a ship in the loaded condition can be determined, when the centre of gravity in the light condition is known, it is necessary to find the centre of gravity of what has to be added to the ship to complete her. In a merchant ship the principal weight to be added is cargo. In warships the weights are usually well defined and constant, so that an exact estimate is not difficult to make, but cargoes in merchant ships vary very much. A vessel may be filled with a very light cargo, or only partially filled with a heavy one. The draught and centre of gravity of ship will be very different in these two cases. It is therefore necessary to consider cargoes in some detail.

Cargo is sometimes measured by its cubical capacity, and sometimes by its actual weight. Cargoes that occupy a small space in relation to their weight are measured by their actual weight in tons. Cargoes that generally fill a ship without fully loading her to her L.W.L. are sometimes measured by their cubical capacity. The density of a kind of cargo may be defined as its weight per unit of volume, so that the actual weight of cargo is equal to the cubical capacity multiplied by its density. The unit of volume most frequently used is the "measurement ton" of 40 cubic feet, and the unit of weight, the ton of 2240 lbs. The density is, however, often measured in terms of the number of cubic feet of the cargo per ton weight.

Table XXIII. gives an idea of the densities of the various kinds of cargo, that having 40 cubic feet to the ton being called unity. The weight in each 100 cubic feet is also given. It will be seen later that 100 cubic feet is the unit of register tonnage measurement.

A measurement cargo is one in which the cargo is usually sold by measurement and not by weight. Sometimes a cargo is spoken of as a measurement cargo when it has a density less than unity, even though it is sold by weight.

The first column of figures in the previous table gives the number of cubic feet a ton of the cargo would occupy, while the second column gives the number of tons each 40 cubic feet of volume will weigh, and the third column the number of tons each 100 cubic feet will weigh. In the ordinary working of ships the earning power may be spoken of in tons weight that the ship carries, or in tons measurement for which her cargo holds have capacity. The former is spoken of as a "dead weight ton," the latter as a "measurement ton." In the case of a cargo such as pig iron, which is so dense that no ship can be filled with it, the amount of space unoccupied by cargo is of no marketable value, and it will be of no use to talk of the number of measurement tons in a ship which carries nothing but pig iron. In other words, the ship is loaded to her load waterline long before she is full.

TABLE XXIII.

| Cargo. | Cubic feet per ton of 2240 lbs. | Density. | |
|---|---|---|---|
| | | Weight per 40 cubic ft. in tons of 2240 lbs. | Weight per 100 cubic ft. in tons of 2240 lbs. |
| Scotch coal | 44 [1] | ·91 | 2·27 |
| Newcastle coal | 44 | ·91 | 2·27 |
| Welsh coal | 40 | 1·0 | 2·50 |
| Pig iron | 9 | 4·44 | 11·10 |
| Alkali casks | 47 | ·85 | 2·13 |
| Wheat | 46–52 | ·87–·77 | 2·17–1·92 |
| Flour | 45 | ·89 | 2·22 |
| Maize | 46 | ·87 | 2·17 |
| Barley | 58 | ·69 | 1·72 |
| Oats | 72 | ·56 | 1·40 |
| Rice in bags | 50 | ·80 | 2·00 |
| Tea | 100 | ·40 | 1·00 |
| Raw sugar in baskets | 50 | ·80 | 2·00 |
| American cotton | 120 | ·33 | ·82 |
| Machine pressed Indian cotton | 60 | ·67 | 1·67 |
| Egyptian cotton | 70–220 | ·57–·18 | 1·42–·45 |
| Jute | 49–77 | ·82–·52 | 2·05–1·3 |
| Undumped wool | 235 | ·17 | ·42 |
| Washed and undumped wool | 84 | ·48 | 1·2 |
| Greasy dumped wool | 100 | ·40 | 1·0 |
| Potatoes | 50 | ·80 | 2·0 |
| Beans and peas | 43–53 | ·98–·76 | 2·45–1·9 |
| Beef, frozen and hung | 120 | ·33 | ·82 |
| Beef, including insulation | 150 | ·27 | ·67 |
| Bacon cases and ham | 64 | ·63 | 1·57 |
| Manchester bales | 50–160 | ·80–·25 | 2·0 –·62 |

[1] Cubic feet per ton of coal. This figure varies with the size and arrangement of bunkers. A number of small bunkers give more broken space than a few larger bunkers.

The Admiralty practice is 42 for Welsh coal; 45 is the usual figure for the softer kinds of coal.

Coal is usually stowed right out to the shell, up to the deck between the beams, and out to the bulkhead plating between the stiffeners. The number of cubic feet per ton takes account of the space that cannot be occupied. The smaller the bunker the larger should be the factor for stowage. (See *Trans. I.N.A.*, paper by Professor Purvis on Stowage.)

In the case of a vessel carrying a cargo such as undumped wool, she cannot generally be loaded to her L.W.L., and in consequence the space available for the stowage of the wool is the only marketable thing. It is of no use to say that she can carry so much more dead weight if there is no room in which to put the cargo.

In all the following calculations regarding cargo, it is assumed that the space occupied is filled with a homogeneous cargo. The centre of gravity of the space occupied is then the centre of gravity of the cargo, and can therefore be determined by the ordinary rules.

If the centre of gravity of the cargo space and the weight of the cargo are known, the centre of gravity of the ship and cargo can be found. We may have a cargo which fills the cargo holds, but, as only a certain weight of cargo can be carried, there is a limit to the density of cargo that completely fills the holds. Denser cargoes can only fill part of the holds. In the case of cargoes of varying densities, a detailed calculation must be made, taking account of the quantity and density of each kind of cargo.

The total dead weight which can be carried is limited by the maximum draught of water to which the ship can load. This is fixed by law by the Board of Trade. The Board of Trade generally accepts the load line assigned

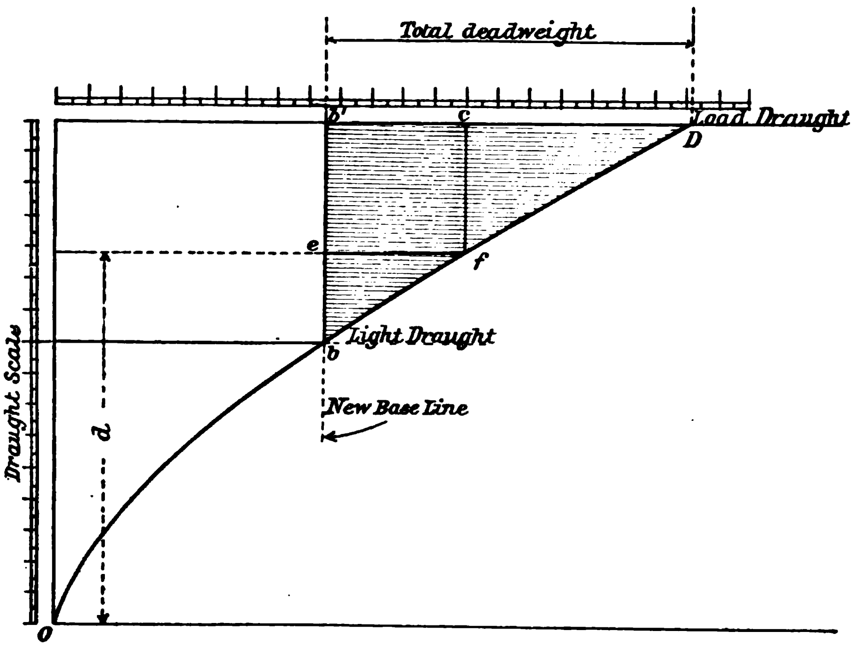



FIG. 114.—Displacement Curve.

by Registration Societies such as the British Corporation, Lloyds, or Bureau Veritas. In some cases there are other limitations to the amount of dead weight which can be carried, such as a limit of draught of water imposed by other considerations than the statutory load line.

Docks and harbour entrances may be such that a vessel may not be able to pass out or in when loaded to the statutory load line. In other words, the vessel may be quite safe to load to this line, but it may not be expedient to load her so deeply. Whatever the actual line may be to which the ship is to be loaded, the total dead weight carrying capacity is got by taking the difference of the load and the light displacements. The carrying capacity at any draught is best represented by the displacement curve, fig. 114.

A vertical line $b\,b'$ is drawn through the intersection of the curve with the light draught line. The ordinate at any draught of the remaining part of the curve $b$ D measured from this vertical line gives the dead weight carrying capacity at that draught.

In a steamer, in addition to the cargo, the dead weight is made up of coals, passengers, crew, baggage, fresh-water, and consumable stores.

Suppose $ef$ to be these weights, then $c$ D will be the weight of cargo, and the condition at this draught $d$ may be called the "cargo out" condition.

Let $w$ be the total weight of cargo in tons that can be carried at any draught.

Let V be the total volume of space available for cargo carrying.

Then $\frac{V}{w}$ = the number of cubic feet per ton carried when the cargo holds are full. This value will be a minimum for the value of $w$ at load draught. Call this value C, so that $\frac{V}{w} = C$.

When the cargo is of so great a density that it will overload the ship if the holds are full, these holds cannot be filled.

If $v$ = the volume actually occupied,

and W = the value of $w$ at load draught,

then $\frac{v}{W} = c$ = number of cubic feet per ton of cargo when holds are partly filled.

It may be noticed that in the equation $C = \frac{V}{w}$, as C increases $w$ decreases. If the curve $f$ D, fig. 114, is a straight line, the value of C can be represented by a hyperbola.

In the equation $\frac{v}{W} = c$, the value of $c$ will be represented by a curve of the same form as $v$.

In order to represent the conditions showing the variation of metacentric height in terms of density of cargo (assumed homogeneous), a diagram as shown in fig. 115 is generally made.

This diagram shows the two conditions when—

(1) The cargo completely fills the holds. It varies in density from $C = \infty$ down to $C = \frac{V}{w}$.

(2) The dead weight of cargo is constant and equal to W. The density varies from $c = \frac{v}{W}$ to $c = 0$. The space occupied by the cargo varies from V to zero.

The curves for condition (1) are put on the left hand and for condition (2) on the right hand of the line X Y in fig. 115.

For the preparation of this diagram, it is necessary to make a curve of cargo capacities and a curve giving the locus of the centre of gravity of the cargo space. The following plans and information are necessary:—

Arrangement of cargo holds, tunnels, bunkers, etc.

Lines, or the vessel's body plan.

Particulars as to height of tanks, depth of framing, beams, etc., in way of the holds.

From this information is made a body plan of sections of the internal form of the vessel throughout the cargo space.

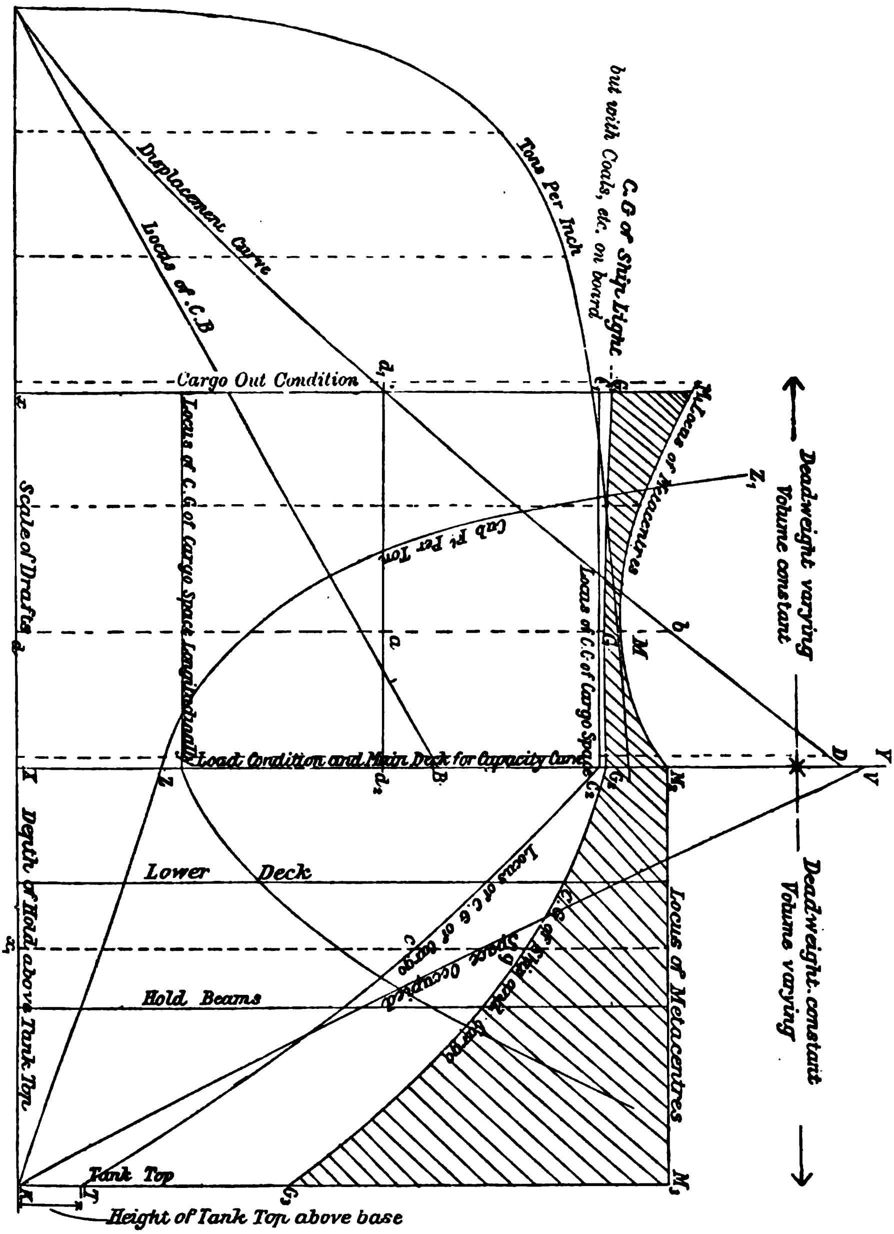



Fig. 116.—Diagram showing variation of Metacentric Height.

Lines are drawn to represent the outline of the cargo space. This is taken to the top of ceiling, or to the top of the tank if there is no ceiling on the tank, to the inside of sparring on frames, and to the under side of beams if fitted on every frame.

In cases where beams are on alternate frames there is a considerable amount of space available between the beams, so that it is not far wrong to take the outline at a half of the beam depth.

A body plan of this nature is shown in fig. 116.

There may be a number of items, such as boxing for pipes, stanchions, ventilators, keelsons, stringers, pillars, etc., which occupy a considerable volume, and it is therefore necessary to make a deduction for these from the volume of the cargo space.

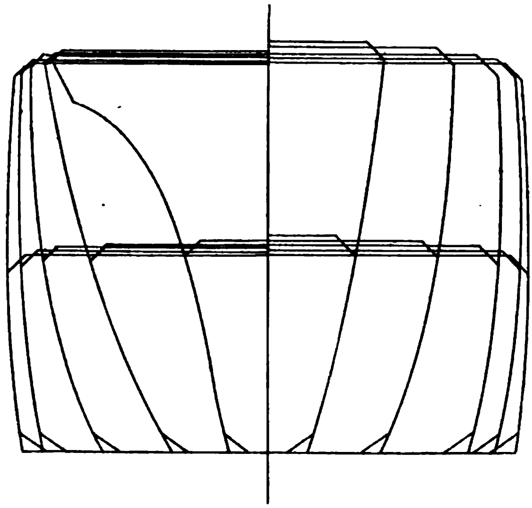



Fig. 116.[1]

Before deductions are made, the best way is to make curves from the above plan showing separately the capacity in holds and each 'tween decks. A base line is drawn to represent the length, the sections are spaced along it as ordinates, and at each section the area up to the several decks is determined from the body plan and set up. We thus obtain a series of curves like fig. 117. These are area curves, and, at the corresponding points of the length, the bulkheads are drawn which divide the holds from the other spaces in the ship. We have thus a capacity curve for each deck. If the holds be very deep it may be necessary to draw a capacity curve for an imaginary deck at a half the depth of hold, in order to get a spot on the capacity curve between the top of tank and the lowest deck.

The deductions can now be made on this plan. For instance, in the after hold the capacity of the tunnels can be represented, and also the capacity of those other parts mentioned above can be estimated and drawn in.

The cargo space is shown shaded.

The area of the shaded part between each deck is then determined by the planimeter or otherwise, and a curve of hold capacity can be

[1] To maintain clearness of lines in this diagram, the top of each deck is not shown.

set up in terms of depth of hold. A curve of this nature is shown in fig. 118 alongside the midship section.

It will be seen that this curve is zero at a point in the depth corresponding to the tank top or top of ceiling.

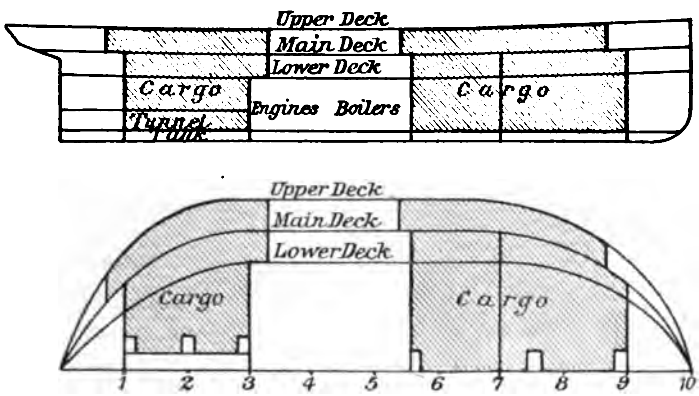



Fig. 117.

The horizontal ordinates of the curve in fig. 118 represent the volume of the cargo holds taken to the height at which the ordinate is measured. It is analogous to the displacement curve for the volume of the submerged portion

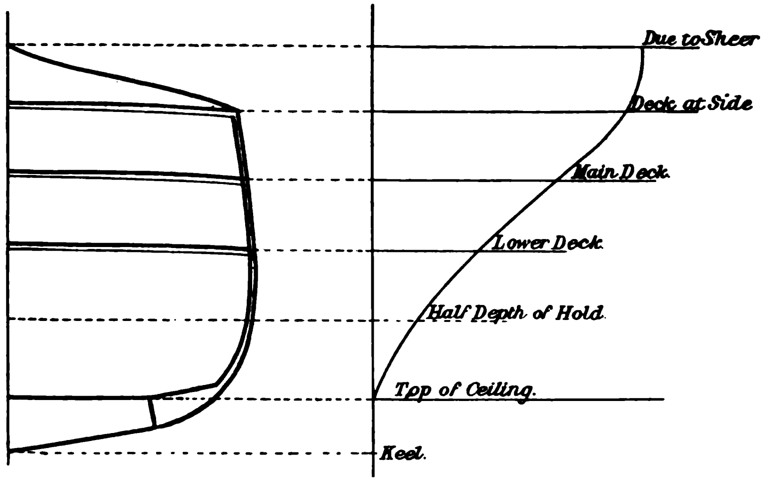



Fig. 118.

of the ship. In the same way as the centre of buoyancy of the displaced volume is determined from the displacement curve, so can the centre of gravity of the cargo space be determined from the capacity curve.

The rule, which has been already proved, is :—

Let O D (fig. 119) be the height of the volume of cargo space for which the centre of gravity is required. Draw D $d$, the ordinate of the volume; find the area of the space O K $d$ and divide by D $d$.

Thus :—

$$\frac{\text{Area of O K } d \text{ in square inches}}{\text{Length of D } d \text{ in inches}} = \text{Distance in inches of centre of gravity of}$$

curve O D $d$ above O.

The actual distance is obtained by applying the scale of the drawing.

Following this rule, we can thus get a curve giving the height of centre of gravity above O. It is, however, more convenient to set off the curve giving the height of the centre of gravity above the keel, and for this purpose a constant equal to the depth of the tank has to be included.

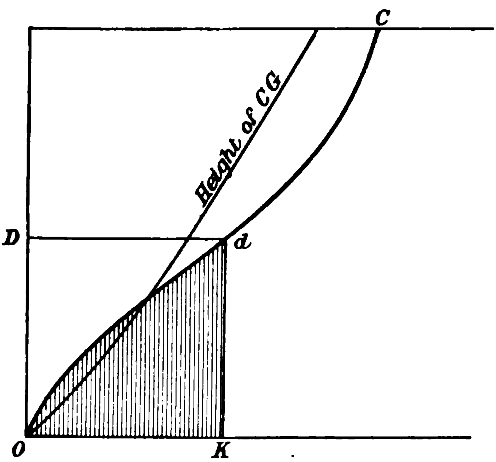



Fig. 119.

The maximum ordinate of the capacity curve represents the total volume of cargo holds, and the maximum ordinate of the centre of gravity curve represents the height of the centre of gravity of the total volume of cargo holds.

*Preparation of Curves to left-hand side of Diagram.*—As will be seen by reference to fig. 115, the base line on the left-hand side of X Y represents the draught, the range being from 0 to load draught.

Vertical lines are set up at the cargo-out and the load draughts.

From the displacement sheet we are able to plot the curves of displacement, vertical centre of buoyancy, and transverse metacentres. It is necessary to plot the last two on the same scale, so that the length of any vertical ordinate measured from the base line to the metacentric curve gives the height of the metacentre above the keel. This scale should be the same as that used in setting up the height above keel of centre of gravity of cargo space.

As has been already stated, the condition for the curves to the left-hand side is that the cargo space is always full, but the weight varies.

The centre of gravity of the cargo space therefore is constant for any variation in the density of the cargo. This can be represented by a horizontal line $C_1 C_2$ drawn at a vertical distance from the base line to represent the height of the centre of gravity of cargo volume. The centre of gravity of the ship with cargo out is represented by the point $G_1$.

From these curves we can now determine the centre of gravity of the ship and cargo as the density of the cargo varies.

The condition at any draught between the "cargo out" and the load condition is the displacement in "cargo out" condition *plus* a varying amount of cargo. The weight $w$ of the added cargo is obtained from the displacement curve.

Where the "cargo out" line cuts the displacement curve in $d_1$ a horizontal line $d_1 d_2$ is drawn. The ordinate of the displacement curve measured from this new base line $d_1 d_2$ gives the weight of cargo carried at that draught.

The density curve may then be plotted as at Z $Z_1$, which will be asymptotic to the "cargo out" line, because there $w=0$, and $C=\frac{V}{0}=\infty$.

At the load draught the ordinate of this curve will be $\frac{V}{W}=\frac{V}{d_2D}$ in the diagram.

The next curve to plot is the centre of gravity of ship and cargo. This we can do by first measuring the ordinates of the curves in the following fashion:—

At any draught $d$ draw a vertical line $dab$ cutting the displacement curve in $b$.

Weight of cargo $=ab$.
Height of C.G. of cargo $=XC_2$.
" " of ship $=xG_1$.
Weight of ship, "cargo out" $=xd_1=ad$.

Height of C.G. of ship and cargo

$$=\frac{\text{Moment of cargo about keel} + \text{moment of ship with cargo out about keel}}{\text{Weight of ship and cargo}}$$

$$\therefore \quad =\frac{ab\times xC_2+xd_1\times xG_1}{ab+xd_1}.$$

This enables us to find the ordinates of the curve $G_1$ $G_2$, which gives the height of the centre of gravity of ship and cargo subject to the conditions stated for the left-hand side of X Y.

It will be seen that this curve passes through $G_1$; at the load draught its value X $G_2$ is equal to

$$\frac{d_2D\times XC_2+xd_1\times xG_1}{\text{Load }\Delta}.$$

At any draught, therefore, by measuring the distance G M we get the value of the metacentric height in the condition corresponding to that draught.

Turning to the curves on the right-hand side of the line X Y we have to deal with the load condition, but the volume of the cargo, and consequently the density, varies. When the volume is greatest, *i.e.* when the cargo holds are completely full, we have exactly the same condition as on the left-hand side at the load draught. The cargo holds are now supposed to be loaded by denser cargo, and therefore to a less height. The base line for the right-hand side represents height of cargo in the hold, which is supposed to be trimmed level throughout uniformly to this height. The scale is the same as the draught scale, and the range is from zero, which represents the top of tank or of ceiling, to the depth of hold from the highest deck.

The density curve is determined from the same formula, viz. cubic feet per ton, $c=\frac{v}{W}$, only in this case W is a constant, and $v$ is the volume of that part of the cargo space which is filled with cargo. The density curve is therefore obtained by setting up the capacity curve to a different scale. In the figure, K V represents the capacity curve, and K Z the density curve. The centre of gravity of cargo above keel is also plotted in terms of the depth of hold: it is shown by the curve T $C_2$.

Since the draught for the right-hand condition is constant and is equal to the load draught, the locus of transverse metacentres is a straight line $M_2$ $M_3$ parallel to the base passing through $M_2$ for the left-hand side.

To determine the locus of the centre of gravity of ship and cargo,

C.G. of ship and cargo above keel

$$= \frac{\text{Moment of ship with cargo out about keel} + \text{moment of cargo about keel}}{\text{Load } \Delta}$$

$$= \frac{xd_1 \times xG_1 + W \times x_1c}{\text{Load } \Delta}.$$

Here the only variable is the distance $x_1 c$, the height of the centre of gravity of cargo space above the base line, when the cargo fills the hold to a height $K x_1$ above the tank top. The above formula determines $x_1 g$, and hence we are able to plot the curve of $G_2 G_3$, the locus of the centre of gravity of ship and cargo.

The distances from this curve to the curve $M_2 M_3$ give the metacentric heights.

The diagram which has been described in the foregoing pages can be altered to suit different conditions of loading.

It can also be extended to include the changes of trim due to varying characters of cargo and lading. The work is similar in character to that already described.

It is necessary to have the centre of gravity of the ship accurately in the light condition. From the light condition, and knowing the distribution of the added weights, we can find the centre of gravity and trim of any new condition. Usually a table is made out, of the following form, similar to that used in the calculations from the results of the inclining experiments.

TABLE XXIV.

| Item. | Weight. | C.G. above Keel. | Vertical Moments. | C.G. from C.B.P. | | Horizontal Moments. | |
|---|---|---|---|---|---|---|---|
| | | | | For'd. | Aft. | For'd. | Aft. |
| Ship Light . . | | | | | | | |
| ,, Cargo . . | | | | | | | |
| ,, Baggage . | | | | | | | |
| ,, Passengers | | | | | | | |
| ,, Fixed stores | | | | | | | |
| As arriving in port. | $\Sigma w$ Displacement. | | $\Sigma m$ Ht. of C.G. | | | $\Sigma Mg$. | $\Sigma Ma$ C.G. fore and aft. |
| Coal . . | | | | | | | |
| Consumable stores | | | | | | | |
| | Load Dispt. | | Load C.G. Vertical. | | | | Load C.G. fore and aft. |

Setting up on the diagram the heights of C.G. thus calculated, we can get at once the corresponding G M. In the above table the C.G. can be calculated for port-leaving and port-arriving conditions. From this calculation we can

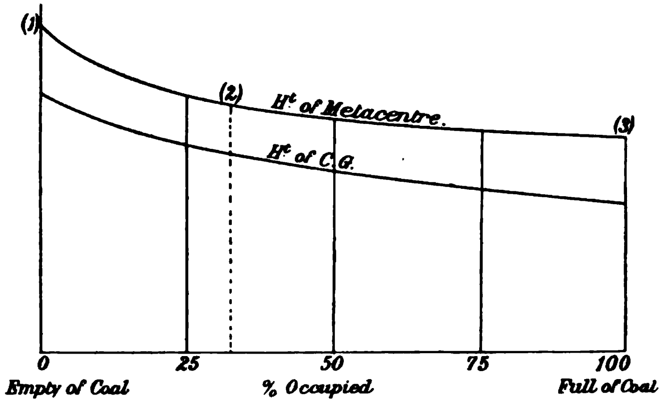



FIG. 120.

get the effect of burning out the coal. To find the variation in the G M during a voyage, the problem is treated in the same way as the varying cargo space on the right-hand side of the diagram. Assuming that the coal works down from the top, we can find the effect on the centre of gravity at certain

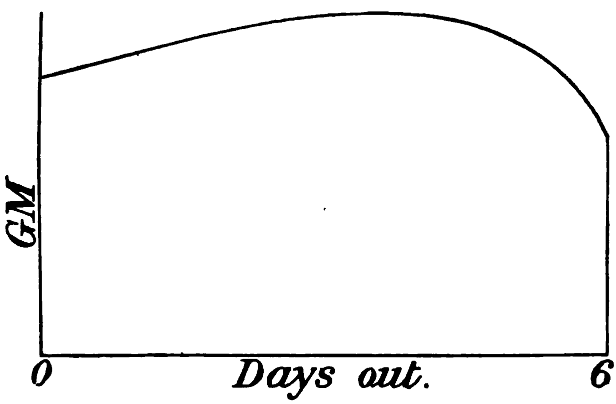
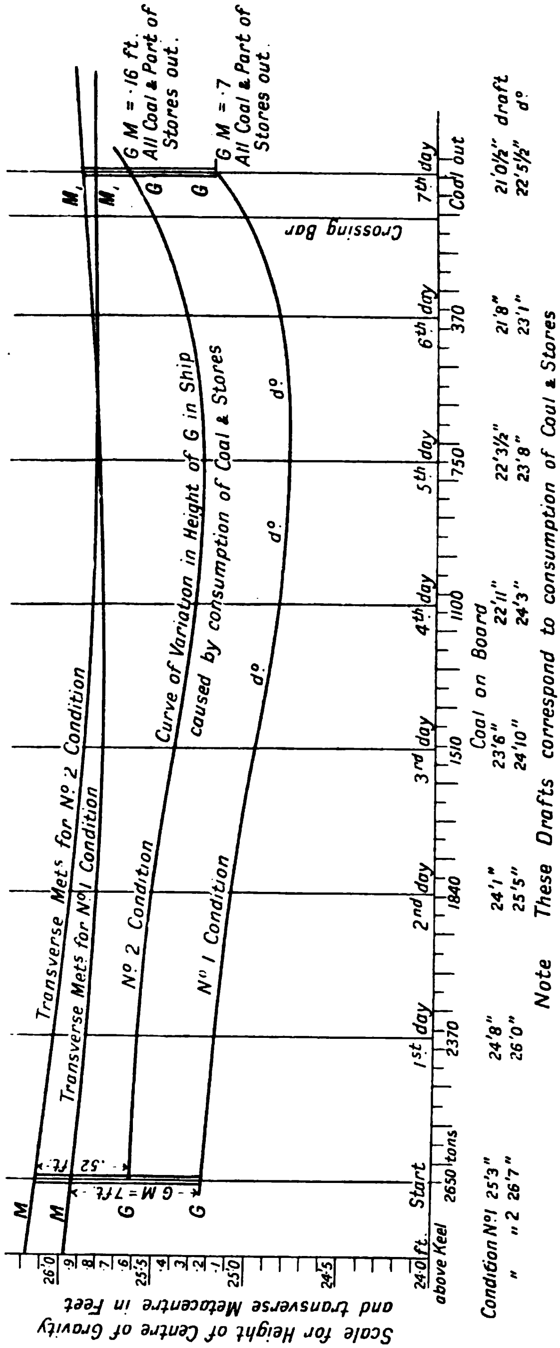



FIG. 121.

stages, say 20 per cent., 40 per cent., etc. of coal burnt. This will give a curve for the height of C.G. of the following nature (fig. 120),—

(1) Light condition.
(2) Port arrival.
(3) Port departure.

The G M can be traced all through the voyage as the coal is being burned out.



Scale for Height of Centre of Gravity and Transverse Metacentre in Feet
24·0 ft. above Keel
24·5
25·0
25·5
26·0
M
G
Start
2650 tons
·52 ft.
·7 ft.
G M = ·7 ft.
1st day
2370
2nd day
1840
3rd day
1510
4th day
1100
5th day
750
6th day
370
7th day
Coal out
Crossing Bar
Transverse Mets for No. 2 Condition
Transverse Mets for No. 1 Condition
No. 2 Condition
No. 1 Condition
Curve of Variation in Height of G in Ship
caused by consumption of Coal & Stores
do.
G M = ·16 ft. All Coal & Part of Stores out.
G M = ·7 All Coal & Part of Stores out.
M₁
Coal on Board
Condition No. 1 25'3" 24'8" 24'1" 23'6" 22'11" 22'3½" 21'8" 21'0½" draft
" " 2 26'7" 26'0" 25'5" 24'10" 24'3" 23'8" 23'1" 22'5½" do.
Note These Drafts correspond to consumption of Coal & Stores


FIG. 122.

In the case of the "New York" the bunkers are carried to the upper deck, hence this ship gains in stability as the voyage goes on until near the end of the voyage. The value of G M in terms of time is graphically represented in fig. 121.

The centre of gravity of the coal is for a certain stage above the centre of gravity of the ship. The centre of gravity, however, is lowering as the coal is burning out until this stage is completed, after which the coal is taken out below the C.G. of the ship and it begins to rise. When about 70 per cent. of the coal is burnt out in the "New York," the G M is a maximum; part of this is due to the rise of metacentre.

In some ships the G M becomes negative as the coal burns out, and water ballast must be added.

Suppose G G, fig. 122, to be the locus of centre of gravity of ship, coal, and stores, in terms of draught, or days on voyage, and $M M_1$ is the locus of metacentre. If we assume that a distance $g$ M is the least metacentric height the ship ought to have, then the height of G should never rise above the line $g\,m$ drawn parallel to $M M_1$ at a distance $g$ M below. Where this line crosses G G, find the effect of filling up one ballast tank. G will be lowered, and the locus of C.G. will be shifted. In this way we get the effect of filling tanks, and also an indication of the period during the voyage when this is necessary.

## CHAPTER XIV.

## EFFECTS ON DRAUGHT, TRIM, AND INITIAL STABILITY DUE TO FLOODING COMPARTMENTS.

One of the risks that a sea-going vessel runs is that of collision with another vessel or rocks. The importance of subdividing the hull in order to secure immunity from total disaster in case of damage resulting from collision or running on rocks is obvious. It is evident that with internal bulkheads strong enough to withstand the water pressure which may come upon them when the adjacent compartment is open to the sea, the amount of water which enters a ship may be limited to the size of the compartment when the damage by collision is confined to a space between the limiting bulkheads of this compartment.

If, however, one of these bulkheads is struck, the amount flooded may extend to two adjacent compartments, but will be limited to these. Therefore, the safety of a vessel, in the event of serious damage of the above nature, depends on her ability to keep afloat with any two adjacent compartments flooded.

When a vessel is flooded by going on the rocks she may have more than two compartments flooded, but she is not likely to sink on account of this flooding unless she is taken off the rocks by some means. Further, a double bottom is a probable protection from such damage.

In warships, which, in addition to the foregoing, are liable to flooding in a naval battle by having more than one compartment in communication with the sea, a much more minute system of subdivision is necessary than in ships where flooding is only likely to take place by damage at one place in the structure.

The more minute subdivision makes the vessel less liable to change of trim, draught, or heel than the less minute; but inasmuch as the principles for determining these changes are the same in all cases, and the changes are greater in the less minutely subdivided ship, it is only necessary to consider the cases of the smaller number of divisions, such as exist in mercantile ships.

In a warship the space between the outer and inner watertight bottoms is divided into a large number of watertight compartments by transverse and longitudinal girders. This space in some cases extends for about two-thirds of the length of the ship, in way of the bunkers, boilers, engines, and magazines. The bunker bulkheads are watertight, and these spaces are subdivided as much as is practicable for the satisfactory handling of coal. There is a strong watertight bulkhead between each boiler-room, and, generally, there is a longitudinal bulkhead dividing the engine-room into two watertight compartments. This latter method of subdivision can only be done in ships having at least two screws. All the compartments forward and aft of the machinery space and under the protective deck are made watertight. The bulkheads between the protective and main decks are also watertight, and watertight doors and scuttles are fitted at every opening where a passage is necessary from one compartment to another.

Not only does this extensive system of subdivision tend to localise the extent of the flooding that may be caused by shell or by collision, but it also serves the purpose of enabling the vessel to be brought to the upright by deliberately flooding compartments on the opposite side of the ship to that where damage has happened. Also, if the damage has taken place near the ends, the vessel can be trimmed by flooding the compartments at the other end of the ship. Thus the danger of an excessive list or trim may be reduced, and at the same time the ship may be brought into a more favourable position for working and fighting.

In an ordinary merchant vessel, a watertight bulkhead must be placed near the bow for localising the flooding that may result from running into another vessel. A similar watertight bulkhead is also placed near the stern in order to localise flooding due to breaking the stern tube or damaging the stern frame. The bulkheads separating the machinery from the cargo holds are also made watertight. Lloyd's Rules require—

> "screw steamers to have a watertight bulkhead at each end of the "engine and boiler space. In addition, a watertight collision bulk-"head is to be fitted at not less than one-twentieth of the vessel's "length abaft the stem, measured at the lower deck, and a watertight "bulkhead is also to be fitted at a reasonable distance from the after "end of the vessel. In all cases the foremost or collision bulkhead is "to extend from the floor plate to the upper, spar, or awning deck, "and its watertightness is to be tested by filling the peak with water "to the height of the load line. Where the machinery is fitted aft in "vessels 220 ft. and under 280 ft. long, a watertight bulkhead is to "be fitted about midway between the collision bulkhead and the "bulkhead at the fore end of the engine and boiler space, making "four bulkheads in all. In steamers 280 ft. and under 330 ft. in "length, in addition to the collision, machinery space, and after bulk-"heads, an extra watertight bulkhead is to be fitted in the main hold "about midway between the collision and boiler-room bulkheads, "making five in all.
>
> "In steamers 330 ft. and under 400 ft. in length an additional "watertight bulkhead is to be fitted in the after hold, making six in "all; in steamers 400 ft. and under 470 ft. in length seven watertight "bulkheads are to be fitted; in steamers 470 ft. and under 540 ft. in "length eight watertight bulkheads are to be fitted; and in steamers "540 ft. and under 600 ft. in length nine watertight bulkheads are to "be fitted. These results are given in tabular form as follows:—

| Length in Feet. | No. of Bulkheads. |
|---|---|
| 220 to 280 | 4 |
| 280 „ 330 | 5 |
| 330 „ 400 | 6 |
| 400 „ 470 | 7 |
| 470 „ 540 | 8 |
| 540 „ 600 | 9 |

" These bulkheads are to extend to the height of the upper deck in " vessels with one, two, or three decks, to the spar deck in spar-deck " vessels, and to the main deck in awning- or shelter-deck vessels.

" In sailing vessels the foremost or collision bulkhead only will be " required."

It is evident that for equal safety in ships there should be equal division; in fact, smaller ships should have rather more subdivision than large ones, as the damage caused by collision is likely to extend to more compartments in the former. Hence length as a determining factor in the number of bulkheads cannot be chosen for purposes of safety. The number of bulkheads in a merchant ship is determined generally by practical considerations as to shortness of holds permissible for carrying cargo. The larger ships are much safer than the smaller, which is what they should be when their value is taken account of.

In large passenger merchant steamers it is usual to further subdivide the hull. The greater the size of the vessel, the greater the number the hatches must be in order that the cargo may be discharged or loaded at a satisfactory rate. This permits of correspondingly increased subdivision. In some cases the engine-room of twin-screw vessels is divided by a longitudinal watertight bulkhead. In high-powered ships additional transverse bulkheads are fitted in the boiler spaces.

In order to determine the number of watertight bulkheads that a vessel (for safety) should have, the effect on the draught, stability, and trim due to flooding the compartments has to be calculated.

One condition, for instance, may be specified, viz. that the vessel has to be subdivided so that she is able to keep afloat with any two adjacent compartments flooded. An arrangement of bulkheads can be approximately decided upon, and the position of the waterplane for each separate flooded condition can be calculated. It will then be seen if any of these conditions are dangerous, and the compartments may then be changed until the spacing of the bulkheads is satisfactory. This, however, is rather a laborious method of arriving at the required result. It is therefore necessary to make a more general solution of the question.

It is evident that, with any one length of compartment (*i.e.* any one spacing of transverse watertight bulkheads), for a given condition of ship there will be one definite change of draught, trim, and transverse stability, due to the flooding of this one compartment, for one position of the bulkheads.

A variation in the longitudinal position of the compartment will cause a variation in the above changes of draught, trim, and transverse stability. A variation in the length of the compartment will also cause a variation in these items. Hence we may find the value of the changes, first, for constant length of compartment and varying longitudinal position. We may then find the changes due to another, but different, constant length and varying longitudinal position, and so on, so that we may cover the whole of the possible ground of variation of length and position of compartment. When we fully know the effect of all these variations, we may then determine which of them causes the changes of trim, draught, or stability to be of a dangerous character. Such a general flooding calculation is usually made for the vessel, and the results are given in curves which show, in terms of the percentage of length of the ship assumed to be flooded, the actual changes of trim, draught, and stability. If upon these curves the permissible limiting values of these changes be drawn, the suitable positions of the transverse watertight bulkheads can be easily determined.

It is necessary, before making this general calculation, to fix these permissible limits of change of draught and trim; in other words, to determine a margin of safety line. The margin of safety line for change of trim and draught will give the limiting positions of the waterplane in the various flooded conditions, beyond which, if any length of compartment causes the waterline to pass, that length of compartment is inadmissible. The permissible limit of transverse stability may be considered separately, but will generally be a minimum G M of a small positive amount.

In a merchant vessel the transverse watertight bulkheads are generally carried up to the upper deck or weather deck. The margin of safety line is a line in the profile which is fixed at distances from the upper deck line at side as shown in fig. 131. These percentages are proportions of the moulded depth.

With respect to stability, it is possible to determine the change in metacentric height due to flooding, and to plot these changes in terms of length and position of compartments. If a value of the least permissible G M be

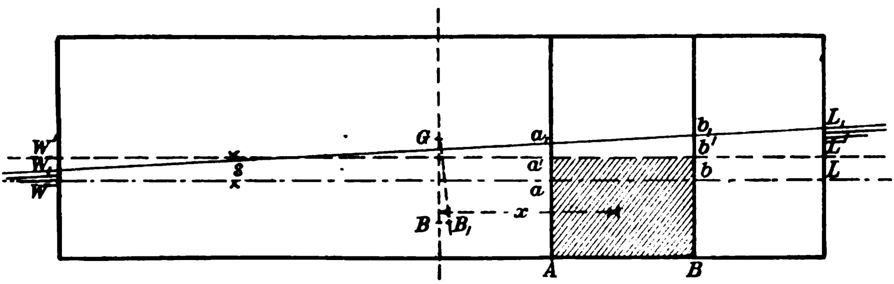



FIG. 123.

fixed, it will be easy to fix the maximum length of compartment at any position in the ship which will give this limiting value of G M. Generally, this method of solution is satisfactory, but there may be cases in which it may be desirable to consider stability not only in the upright but at finite angles. These calculations are generally laborious, and would only be undertaken in special cases where it appeared from the results of the G M calculation to be desirable to investigate further.

We will first consider the method of finding the change of trim and draught due to flooding definite lengths of compartments.

*Calculation to find the effect on Draught and Trim due to flooding a known length of a vessel.*—Let us consider first the simple case of an empty rectangular vessel of length $l$ and breadth $b$, floating at a draught $d$ with plenty of freeboard (see fig. 123).

Let the centre of gravity be amidships at G, so that the vessel is floating on an even keel. The centre of buoyancy is at B vertically below G, and at a height $\frac{d}{2}$ above the bottom.

Let A and B be the position of the watertight bulkheads forming the compartment that is to be flooded. When this compartment is opened to the sea a bodily sinkage of the vessel will take place, and at the same time the vessel will change trim, so as to occupy a new position, as shown by waterline $W_1 L_1$. The vessel will trim by the head or the stern according as the centre of gravity of the compartment is forward or aft of the centre of gravity of the original waterplane.

The original waterplane is W L, the waterplane W′ L′ is the line to which the vessel would sink if there were no change of trim, while $W_1 L_1$ is the actual line at which she will float with the compartment between A and B flooded.

In the flooded condition water has risen in the compartment to $a_1 b_1$. For simplicity we may consider the buoyancy in the region of A B as lost, though there will be a negligible amount due to the buoyancy of the material of the structure; and since the flooded condition is a statical one, we have the following:—

(1) Volume $W_1$ A + Volume $L_1$ B = Original volume of displacement, or $l.b.d.$

(2) Centre of buoyancy $B_1$ of the combined volumes of $W_1$ A and $L_1$ B is in the new vertical through G.

The first condition is satisfied by the vessel sinking bodily, so that she gains as much buoyancy as she has lost by flooding the compartment. She cannot gain buoyancy in the flooded compartment, so that it is only in the intact part of the waterplane that support can be obtained. The second condition is satisfied by the vessel trimming until $B_1$ is in the same vertical as G.

These two effects can be calculated separately.

*Calculation of sinkage:—*

Let A B $= \dfrac{l}{n}$.

The volume of displacement $= l.b.d.$

The area of whole waterplane $= l.b.$

The area of flooded part of waterplane $= \dfrac{l.b}{n}$.

The area of intact " " $= l.b\left(1 - \dfrac{1}{n}\right) = l.b\left(\dfrac{n-1}{n}\right)$.

Let the centre of gravity of the volume of the compartment be at a distance $x$ from the middle of the vessel. This distance $x$ is also the distance from the middle of the vessel of the centre of gravity of the flooded part of the waterplane.

Let S = the mean sinkage.

The volume of the flooded compartment up to $a\,b = \dfrac{l.b.d}{n}$.

From condition (1) we have

$$S \times l.b\left(\frac{n-1}{n}\right) = \frac{l.b.d}{n}.$$

$$\therefore \quad S = \frac{d}{n-1}.$$

This is the same as finding the increase in draught caused by flooding a compartment of the same size as A B with its centre of gravity in the same vertical line as the centre of gravity of the intact waterplane.

If the centre of gravity of the volume of the flooded compartment is vertically under the centre of gravity of the waterplane, then the vessel will sink to a new waterplane which will be parallel to the original waterplane. In this case the centre of gravity of the waterplane is at the middle of the length, and the compartment would therefore have to be amidships in order that no change of trim should take place. At any other position the centre of gravity of the compartment and the centre of gravity of the waterplane cannot coincide, and there must therefore be a change of trim.

*Calculation of change of Trim.*—First, imagine the water in the compartment to be a solid weight put on board with its centre of gravity in the same vertical line as that of the centre of gravity of the whole waterplane. The vessel will sink to the waterplane W′ L′.

The weight of added water is

$$\frac{l.b.d}{35\,n}+\frac{l.b}{35n}\,\frac{d}{(n-1)}=\frac{l.b.d}{35\,(n-1)}.$$

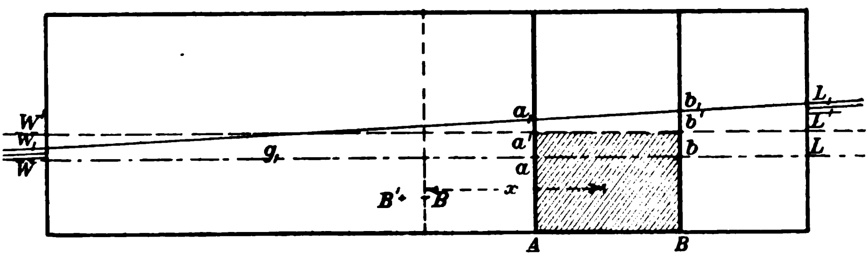



FIG. 124.

This weight is then transferred a distance $x$, and therefore the moment causing the change of trim is

$$=\frac{l.b.d.x}{35(n-1)}.$$

This moment to trim can be considered in another way.

B is the centre of buoyancy of the original displacement to W L. If we consider that the buoyancy up to $a\,b$ is lost, this can be regained by the vessel sinking to W′ L′ through a distance $\frac{d}{n-1}$.

In fig. 124 let B′ be the position of the centre of buoyancy of the combined volumes W′ A and L′ B.

Then, in order that the vessel may be at rest to satisfy the condition of trim, the moment to change trim is $\frac{\text{BB}'\times l.b.d}{35}$.

$$\text{But}\qquad \text{BB}'=\frac{\dfrac{l.b.d.x}{n}}{\dfrac{l.b.d(n-1)}{n}}=\frac{x}{n-1}.$$

∴ The moment to trim $=\frac{l.b.d.x}{(n-1)35}$, which is the same as that already obtained.

If we divide this moment, causing the change of trim by the moment to trim one inch, we get the total change of trim in inches.

The moment to trim one inch has to be taken in the flooded condition. It is given by

$$M_t = \frac{\Delta \times GM}{12 \times l}, \text{ where } \Delta = \text{total displacement,}$$

and GM = the longitudinal metacentric height in the flooded condition.

$$\therefore \quad M_t = \frac{\Delta \times (BM - BG)}{12 \times l} = \frac{\Delta.BM}{12 \times l} - \frac{\Delta.BG}{12.l}.$$

This latter expression, $\frac{\Delta.BG}{12.l}$, is a constant, and in most cases is so small that it can be neglected. The error in the final result will be in the proportion that BG is to BM.

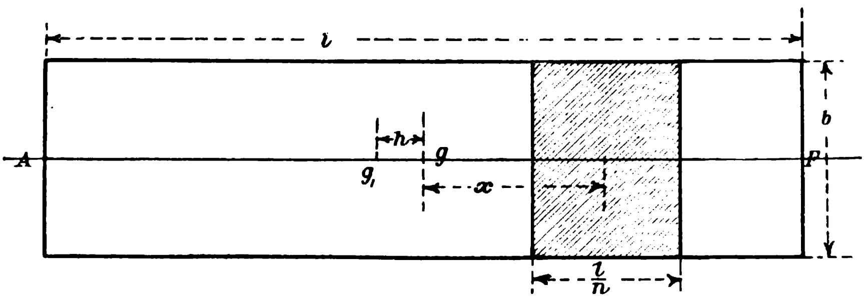



FIG. 125.

$$BM = \frac{I_i}{V}, \text{ where } I_i = \text{moment of inertia of intact portion of the}$$

waterplane,

V = total displacement, *l.b.d.*

$$\therefore \quad M_t = \frac{I_i}{420 \cdot l}.$$

We have to find $I_i$.

$I_i$ has to be taken about an axis through the centre of gravity of the intact portion of the waterplane. Fig. 125 is a plan of the waterplane W′L′. $g_1$ is the centre of gravity of the intact portion.

$g$ is the centre of gravity of the whole waterplane, and is therefore at the middle of the length.

$$g_1 \text{ is at a distance } h \text{ from } g = \frac{x}{n-1}.$$

$$\therefore \quad I_i = \frac{l^3.b}{12} + l.b\left(\frac{x}{n-1}\right)^2 - \frac{l.^3b}{12.n^3} - \frac{l.b}{n}\left(\frac{xn}{n-1}\right)^2,$$

$$= \frac{l^3.b}{12}\left(1 - \frac{1}{n^3}\right) - \frac{l.b.x^2}{n-1}.$$

$$\therefore \quad \text{Moment to trim one inch} = \frac{\frac{l^2 b}{12}\left(1 - \frac{1}{n^3}\right) - \frac{b.x^2}{n-1}}{420}.$$

The moment causing change of trim we have seen to be

$$= \frac{l.b.d.x}{35(n-1)}.$$

$$\therefore \quad \text{The total change of trim} = \frac{l.b.d.x}{35(n-1)} \times \frac{420}{\left\{\frac{l^2.b}{12}\left(1 - \frac{1}{n^3}\right) - \frac{bx^2}{n-1}\right\}},$$

$$= \frac{144l.d.x}{l^2\left(1 - \frac{1}{n^3}\right)(n-1) - 12x^2}.$$

In order to find the change of trim at either end, the total change of trim has to be divided in the proportion of $W'g_1$ to $L'g_1$, for, approximately, the consecutive waterplanes intersect in a line through their centre of gravity. In this case the consecutive waterplanes are $W'L'$ and $W_1 L_1$ with the flooded part taken away from the area of both. The centre of gravity of $W'L'$ is $g_1$.

$$W'g_1 = \frac{l}{2} - \frac{x}{n-1},$$

$$L'g_1 = \frac{l}{2} + \frac{x}{n-1};$$

$$\therefore \quad f, \text{ the change of trim forward,} = \frac{\left(\frac{l}{2} + \frac{x}{n-1}\right)(144d.x)}{l^2\left(1 - \frac{1}{n^3}\right)(n-1) - 12x^2},$$

$$\text{and} \quad a, \quad " \quad " \quad \text{aft,} \quad = \frac{\left(\frac{l}{2} - \frac{x}{n-1}\right)(144d.x)}{l^2\left(1 - \frac{1}{n^3}\right)(n-1) - 12x^2}.$$

Curves representing the values of $f$ and $a$ as deduced from these expressions are given in fig. 126 for varying values of $x$ and $n$. The curves A M F are cross curves of values of $n$ for values of $x$ at which the change of draught is not more than sufficient to immerse the vessel below the chosen safety line.

The foregoing calculation assumes that the space in the compartment is entirely empty. This space may be partly filled with cargo, and, consequently, the amount of water that flows into the compartment depends upon the space occupied by the cargo.

The most dangerous case for a given draught of water happens when the hold is empty, but as this draught will generally be a light one, it may not be so dangerous as one in which the holds contain cargo and the draught is much deeper. It is necessary to calculate the effect of flooding with the compartment partly full. The nature of the cargo must be taken into account in order that its buoyancy or water-excluding capability may be calculated.

First, assume the cargo to be of a homogeneous character and such as will fill the holds, though still being capable of admitting some water between the spaces actually occupied by the cargo. It is necessary to assume a proportion of the hold which admits the water, say $\frac{1}{m}$th of the volume of the compartment. This is equivalent to flooding an empty compartment between

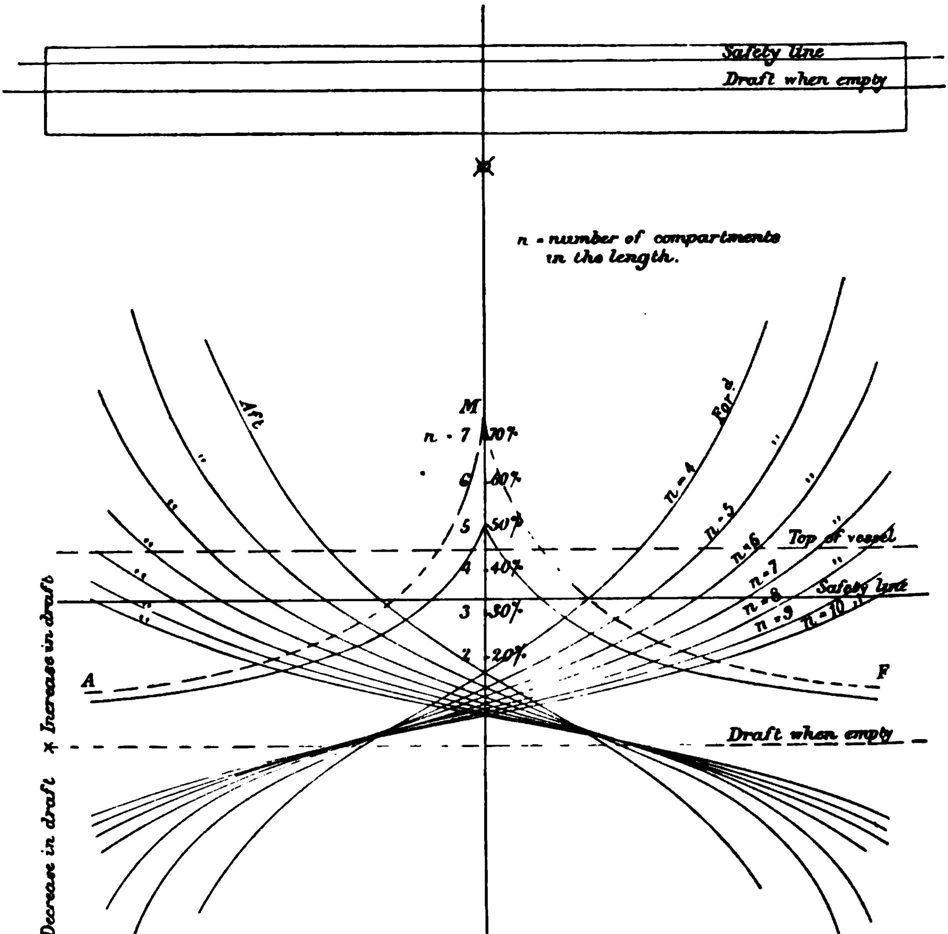



FIG. 126.

bulkheads a distance $\frac{l}{m}$ apart. Then the amount of water which can enter the compartment will be $\frac{1}{m}$th times the amount which enters when the compartment is empty. The volume of water then becomes $\frac{l.b.d}{(m.n-1)}$.

This is obtained by substituting $m.n$ instead of $n$ in the formulæ for the previous case when the bulkheads were $\frac{l}{n}$ apart. The sinkage, change of trim, and metacentric height can be obtained in the same way as before.

If the cargo is of such a nature that it only partially fills the holds, it may be that in some conditions of flooding the loss of buoyancy at the waterline is the same as if the compartment were empty, in which case the alteration of waterline area for calculation of sinkage, change of trim, and metacentric height by changing to $m.n$ will not be correct, but the work must be done in detail in accordance with the circumstances.

The same principles are used in the calculation for sinkage and change of trim in an ordinary shipshape vessel.

Fig. 127 represents a vessel which has a compartment A B flooded so that the waterplane W L becomes $W_1 L_1$. In the plan view which shows the waterplane W L, $g$ is the centre of gravity of the whole waterplane. For a

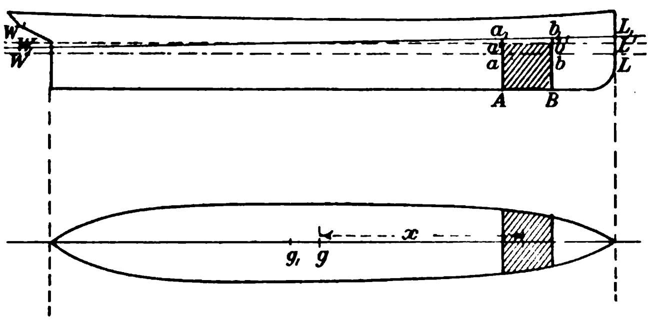



FIG. 127.

small compartment the shape of the waterplane $W_1 L_1$ may be considered the same as W L.

Let the centre of gravity of the compartment A B be at a distance $x$ from $g$. It is usual to consider the centre of gravity of the compartment midway between the bulkheads A and B. $x$ can also be considered as the distance of the centre of gravity of the damaged area of the waterplane from $g$.

To obtain the sinkage :—

First calculate the volume $v$ of the compartment up to the original waterplane W L.

Let A = area of whole waterplane.

„ $a$ = area of damaged part.

Then the sinkage $= \frac{v}{A - a}$.

The distance of the centre of gravity of the intact waterplane from the centre of gravity of the original waterplane $= gg_1 = \frac{a.x}{A - a}$.

The moment to trim is equal to $\{v\,(x + gg_1)\} \div 35$,

$$= \frac{A.v.x}{35(A - a)}.$$

The same result can be obtained, as it was in the rectangular vessel, by considering the transference of the weight of water in the compartment.

The volume of water in the compartment up to $a'\,b'$ is equal to $v+a.\text{S}$, where $\text{S}=$ the sinkage.

$$\therefore\quad \text{Weight of water} = \frac{v+\dfrac{a.v}{\text{A}-a}}{35}.$$

$$\text{Transference} = x.$$

$$\therefore\quad \text{Moment causing change of trim} = \frac{x\left(v+\dfrac{a.v}{\text{A}-a}\right)}{35} = \frac{\text{A}.v.x}{35(\text{A}-a)}.$$

The moment to change trim one inch is $= \dfrac{\text{I}_i}{420.l}$,

neglecting the constant $\dfrac{\Delta \times \text{BG}}{12 \times l}$.

$\text{I}_i =$ moment of inertia of the intact waterplane about its centre of gravity $g_1$.

Let $\text{I}$ = moment of inertia of the whole waterplane W L about $g$ (this can be obtained from the displacement form).

,, $i$ = moment of inertia of the area $a$ about its own centre of gravity.

$$\text{Then}\qquad \text{I}_i = \text{I} + \text{A}.gg_1{}^2 - i - a(x+gg_1)^2,$$

$$= \text{I} - i - \frac{\text{A}.a.x^2}{\text{A}-a}.$$

$$\therefore\quad \text{Moment to trim one inch} = \frac{\text{I}-i-\dfrac{\text{A}.a.x^2}{\text{A}-a}}{420.l}.$$

$$\therefore\quad \text{Total change of trim} = \frac{\dfrac{\text{A}.v.x}{35(\text{A}-a)}}{\dfrac{\text{I}-i-\dfrac{\text{A}.a.x^2}{\text{A}-a}}{420.l}},$$

$$= \frac{12\text{A}.v.x.l}{(\text{I}-i)(\text{A}-a)-\text{A}.a.x^2}.$$

$$\text{Change of trim forward} = \frac{\text{L}g_1}{l}\left\{\frac{12\text{A}.v.x.l}{(\text{I}-i)(\text{A}-a)-\text{A}.a.x^2}\right\},$$

$$\text{,,}\quad \text{,,}\quad \text{aft} = \frac{\text{W}g_1}{l}\left\{\frac{12\text{A}.v.x.l}{(\text{I}-i)(\text{A}-a)-\text{A}.a.x^2}\right\}.$$

$$\text{Change of draught forward} = \frac{v}{\text{A}-a} + \frac{\text{L}g_1}{l}\left\{\frac{12\text{A}.v.x.l}{(\text{I}-i)(\text{A}-a)-\text{A}.a.x^2}\right\},$$

$$\text{,,}\quad \text{,,}\quad \text{aft} = \frac{v}{\text{A}-a} - \frac{\text{W}g_1}{l}\left\{\frac{12\text{A}.v.x.l}{(\text{I}-i)(\text{A}-a)-\text{A}.a.x^2}\right\}.$$

In the above calculation, the application can be made general by assuming $x$ to vary for a constant length of compartment.

By varying $x$ we get two curves like fig. 128, one of which represents $a$, the change of draught aft, the other $f$, the change of draught forward. An increase in the draught is plotted above the line, and a decrease below. The base line represents the length. In this way we get curves which give, at

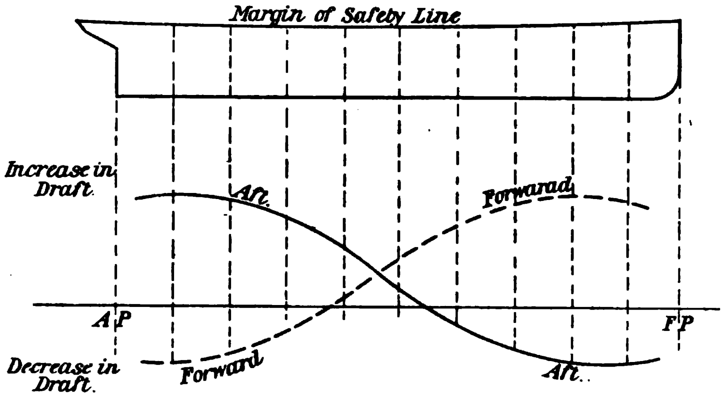



FIG. 128.

any ordinate, the particular values of $f$ and $a$ for a certain length of compartment flooded in that longitudinal position in the ship. The ordinate is drawn at the middle of the compartment, at which point we have seen it is convenient to assume the centre of gravity of the compartment to be. These curves end at the centre of gravity of the end compartments.

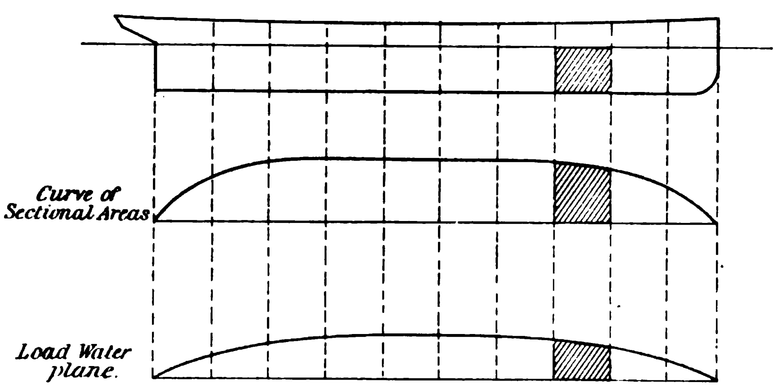



FIG. 129.

A convenient way of making this calculation is to draw a sectional area curve immediately below an outline of the profile, and below this the load waterplane is drawn (fig. 129).

The length of the vessel is then divided, each space being the length of compartment chosen, say 10 per cent. of length. This would give ten compartments. From the curves the volume of the compartment and the area of the flooded part of the W.L. can be calculated.

A is a constant equal to the area of the load waterplane.

I " " " moment of inertia "

$v$ and $a$ can be at once determined from the sectional area and load wate plane curves respectively. The position of $g_1$ can be calculated for eac compartment.

$g$ is constant, and $gg_1 = \frac{a.x}{A - a}$.

$x$ is the distance of the centre of the compartment from $g$.

$i$ has to be calculated for each flooded part of the waterplane. It is a ver close approximation to find the mean half ordinate $y$ of the area, and then

$$i = \frac{2}{12} \cdot \frac{l^3}{n^3} \cdot y = \frac{l^3 . y}{6n^3}.$$

All that have to be calculated for each value of $x$ are the following :—

$i$ = moment of inertia of area $a$,
$v$ = volume of compartment up to W L,
$a$ = area of flooded waterplane,
and the position of $g_1$.

A table of the following form is convenient for working the results.

The first table is made for each 5 per cent. or 10 per cent. of the length of the ship. The second, for 20 per cent., can be largely deduced by combining the results of the 10 per cent. table; similarly for a 30 per cent. table. The values of $f$ and $a$ for each percentage are plotted on a base line of length at the positions of the centre of gravity of the compartment. A series of curves for percentages of 5, 10, 15, 20, and 25 is shown in Plate IX.

The corresponding values of $f$ and $a$ should be set up and joined by straight lines on the sheer drawing on which the margin of safety line has been drawn. Some of these straight lines will in part pass above the margin and others below, and by inspection it will be easy to fix the percentage which will just reach the margin. This percentage should be set up at its corresponding longitudinal position, and a curve passed through the spots so obtained.

In practice it is sufficiently accurate to draw the margin of safety line on the side towards the increase in draught. The intersection of this line with the lines of $f$ and $a$ gives at any ordinate the percentage of length that can be flooded so that the vessel shall not sink below, but shall just sink to, the margin of safety line. This curve is shown in Plate IX.

The curves in this plate have been calculated for an actual vessel, the particulars of which are given.

It will be seen that a much greater percentage can be flooded amidships than at the ends, though at the extreme ends, due to their small volume, the percentage tends to increase.

Curves like the above can be made for different conditions of loading of the ship. Generally it is sufficient to make this curve for the load condition only.

From the above curve the maximum length of compartment, and consequently the limits of position of bulkheads, can be easily determined. If it is required that there are enough bulkheads to make the ship safe with any two compartments flooded, then the maximum length given by the curve must be that of two compartments.[1]

[1] See *Transactions I.N.A.*, 1907, for a description of a method devised by Mr J. G. Johnstone, B.Sc., of obtaining these curves by the integraph.

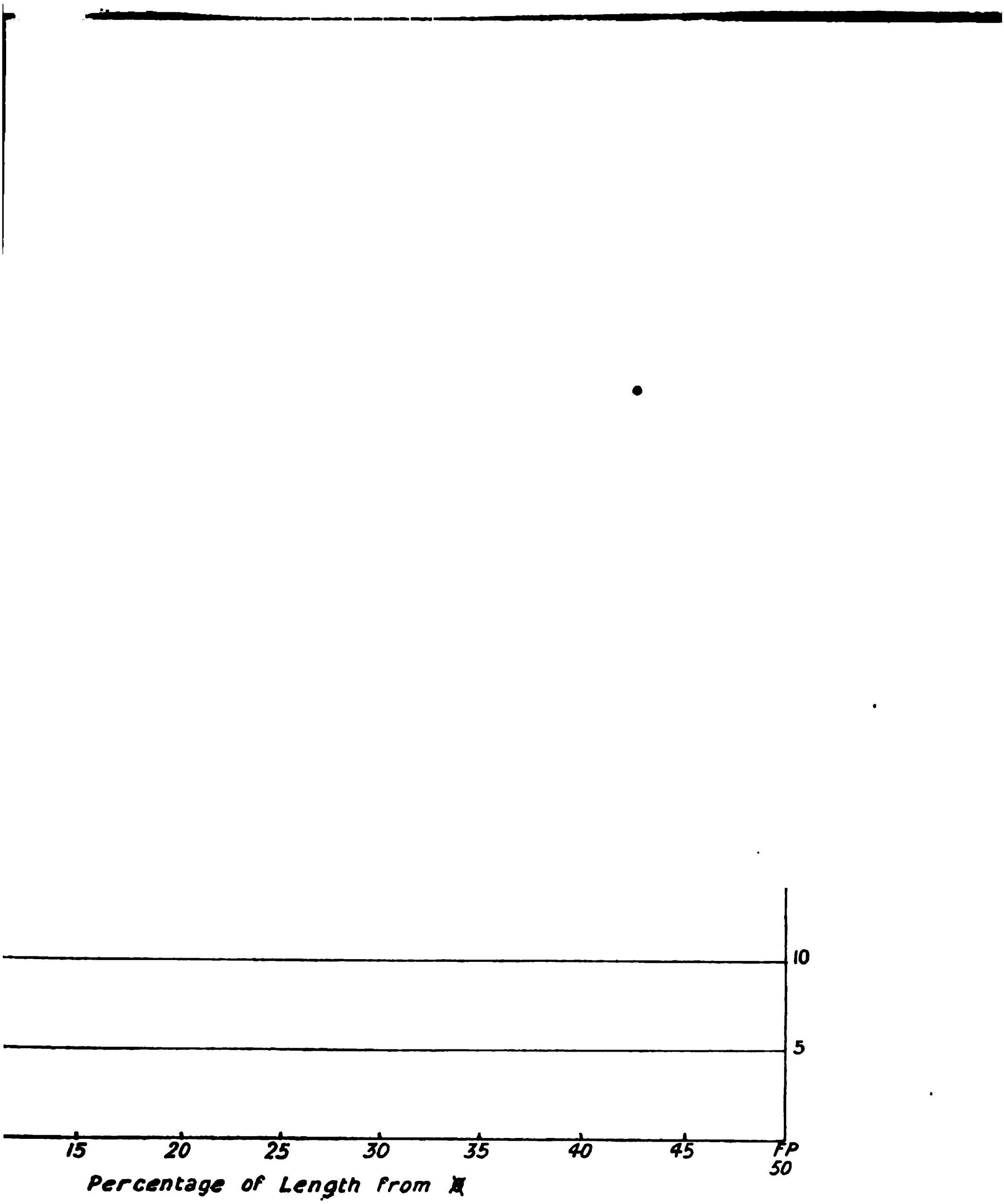



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TABLE XXV.

Particulars of Vessel L = 200′ B D $d$
A =
I =
W$g$ =

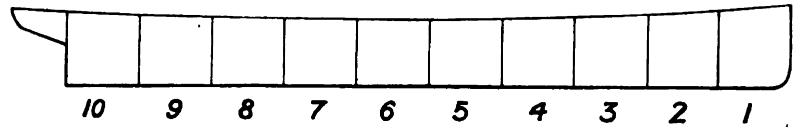



FIG. 130.

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of Compts. | Distance $cg$ | Area L. W. L. | Area Flooded Part. | Momt. Inertia L. W. L. | Momt. Inertia Flooded Part. | Volume of Compt. | | | | | | | | | | | |
| $n=10$ | $x$ | A | $a$ | I | $i$ | $v$ | $12Avxl$ | $I-i$ | $A-a$ | $Aax^2$ | $(I-i)(A-a) - Aax^2$ | $f+a$ | $A-a$ | $\frac{Wg_1}{l}$ | $\frac{Lg_1}{l}$ | $a$ | $f$ |
| 1 | 90 | | | | | | | | | | | | | | | | |
| 2 | 70 | | | | | | | | | | | | | | | | |
| 3 | 50 | | | | | | | | | | | | | | | | |
| 4 | 30 | | | | | | | | | | | | | | | | |
| 5 | 10 | | | | | | | | | | | | | | | | |
| 6 | 10 | | | | | | | | | | | | | | | | |
| 7 | 30 | | | | | | | | | | | | | | | | |
| 8 | 50 | | | | | | | | | | | | | | | | |
| 9 | 70 | | | | | | | | | | | | | | | | |
| 10 | 90 | | | | | | | | | | | | | | | | |
| $n=5$ | | | | | | | | | | | | | | | | | |
| 1 + 2 | 80 | | | | | | | | | | | | | | | | |
| 2 + 3 | 60 | | | | | | | | | | | | | | | | |
| 3 + 4 | 40 | | | | | | | | | | | | | | | | |
| 4 + 5 | 20 | | | | | | | | | | | | | | | | |
| 5 + 6 | 0 | | | | | | | | | | | | | | | | |
| 6 + 7 | 20 | | | | | | | | | | | | | | | | |
| 7 + 8 | 40 | | | | | | | | | | | | | | | | |
| 8 + 9 | 60 | | | | | | | | | | | | | | | | |
| 9 + 10 | 80 | | | | | | | | | | | | | | | | |
| $n$ per cent. = 30. | | | | | | | | | | | | | | | | | |

To determine the G M in the flooded condition involves a great deal of work, and it is desirable to consider the G M in the worst conditions only. These can be selected when the bulkheads are fixed to satisfy the longitudinal condition of safety. It is evident that the loss of G M will be greatest when the midship compartments are flooded, because these are likely to be so much longer.

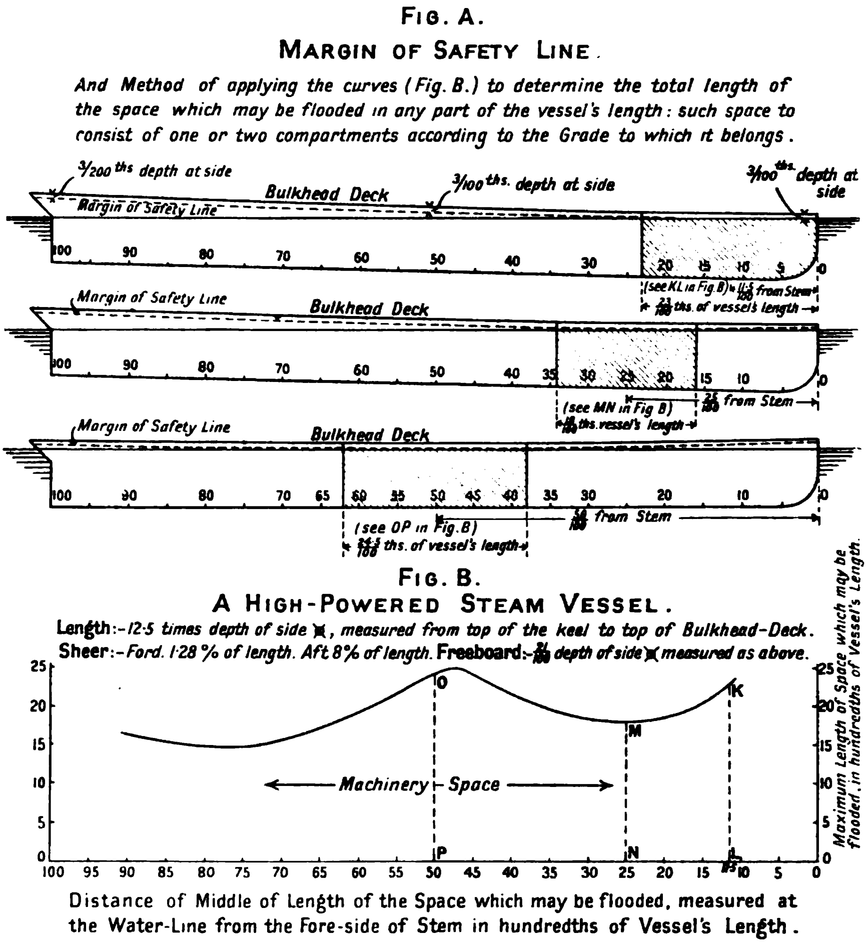



FIG. 131.

*Notes on the Report of the Committee appointed to consider the Strength and Spacing of Watertight Bulkheads.*—The Committee was appointed by the President of the Board of Trade, and the report of the work was published in 1891.

A vessel is considered to be safe, even in the event of serious damage, if she is able to keep afloat with two adjoining compartments in free communication with the sea. The vessel must therefore have efficient transverse watertight bulkheads so spaced that when any two adjoining compartments are open to the sea, the uppermost watertight deck to which all the bulkheads extend is not brought nearer to the surface of the water than a certain prescribed margin.

The watertight deck referred to is called the "bulkhead" deck. The line past which the vessel may not sink is called the Margin of Safety Line.

The Margin of Safety Line, as defined in the above Report, is a line drawn round the side at a distance amidships of $\frac{3}{100}$ths of the depth at side at that place below the Bulkhead Deck, and gradually approaching it towards the aft end, where it may be $\frac{3}{200}$ths of the same depth below it.

The Report contains a number of tables and diagrams, which have been made on the supposition that the Bulkhead Deck is continuous, and that the water surface when the compartment in question is flooded is not nearer the top of the Bulkhead Deck than the Margin of Safety Line. Such a condition is illustrated in fig. 131, in which the length and the position of the centre of the flooded compartment have been determined from the curve at the bottom.

The curve may be plotted from the tables which are published in the Report. These tables give the maximum length of the space which may be flooded with water, and the vessel still float in moderate weather. Table XXVI. is an abridged reproduction of the table in the Report. Fig. 131 is the curve showing the upper row of figures in this table.

*Example.*—If the "bulkhead-freeboard" be $\frac{21}{100}$ths of the depth at side, and the middle of the length of space which can be flooded be fixed at a distance of $\frac{25}{100}$ths of the length from the stem, the maximum length of such a space at that portion must not be greater than $\frac{18}{100}$ths of the vessel's length. This corresponds to the ordinate M N in fig. 131. The freeboard required is not necessarily that stated in the Freeboard tables under the Merchant Shipping Act (see Chap. XVI.), but may be any freeboard, which on the application of the shipowner may be registered by the B.O.T. Such Freeboard is called the Bulkhead Freeboard, and corresponds to the Bulkhead Load Line. It is not intended that the curves and tables in the Report are suitable for all types of ships. Many other considerations affect the result, such as—

Weight of hull and machinery.
Position and extent of the machinery.
Nature of cargo and its distribution.
Trim, sheer, etc.

The Report contains diagrams illustrating the modification produced in the spacing of bulkheads by a variation in the above condition.

Fig. 132 shows the effect of a variation in the length of machinery space. Curves are given for lengths of machinery space 15 per cent., 22 per cent., 29 per cent., 36 per cent., and 43 per cent. of the vessel's length, first for a freeboard of 29 per cent., and second for a freeboard of 21 per cent.

Fig. 133 shows the effect of varying the nature of the cargo. Curves are given for three different kinds of cargo, viz. coal, salt, and iron.

Fig. 134 shows the effect of varying the freeboard and varying the sheer.

Curves A and *a* are for the same freeboard, but a different sheer. So also are curves B and *b*.

The curves A and B are for the same sheer, but different freeboard; so also are curves *a* and *b*.

In vessels where the bulkhead deck is discontinuous or stepped, the margin of safety line may be drawn round the side to follow the bulkhead deck in its several steps.

TABLE XXVI.

CARGO—COAL.

| Minimum Freeboard to Top of Bulkhead Deck in Hundredths of the depth at side )( measured from top of keel to top of deck at side. | Aft. Distance, measured at the waterline from the fore side of the stem to the middle of the Length of space which may be flooded in hundredths of the vessel's length. For'd. | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 85 | 80 | 75 | 70 | 65 | 60 | 55 | 50 | 45 | 40 | 35 | 30 | 25 | 20 | 15 | 10 |
| | Maximum Length of the space which may be flooded in $\frac{1}{100}$ths of the length. | | | | | | | | | | | | | | | |
| 21 | 15·1 | 15 | 15·2 | 15·8 | 17·0 | 19·4 | 22·0 | 24·5 | 24·5 | 21·1 | 19·7 | 18·4 | 18 | 18·4 | 20·1 | 24· |
| 23 | 15·2 | 15 | 15·3 | 16·1 | 18·0 | 20·5 | 24·3 | 26·8 | 26·7 | 24·0 | 21·3 | 19·3 | 18·3 | 18·5 | 19·9 | |
| 25 | 15·3 | 14·9 | 15·3 | 16·5 | 18·9 | 22·6 | 26·5 | 29·1 | 29·0 | 25·9 | 22·9 | 20·4 | 18·7 | 18·4 | 19·7 | |
| 27 | 15·5 | 14·9 | 15·4 | 16·8 | 19·9 | 24·7 | 28·8 | 31·4 | 31·2 | 28·0 | 24·5 | 21·4 | 19·2 | 18·4 | 19·5 | |
| 29 | 15·6 | 14·9 | 15·5 | 17·3 | 20·9 | 25·8 | 31·1 | 33·9 | 33·5 | 29·7 | 26·1 | 22·3 | 19·6 | 18·4 | 19·3 | |

Curves for Steam Vessels with various lengths of machinery space.

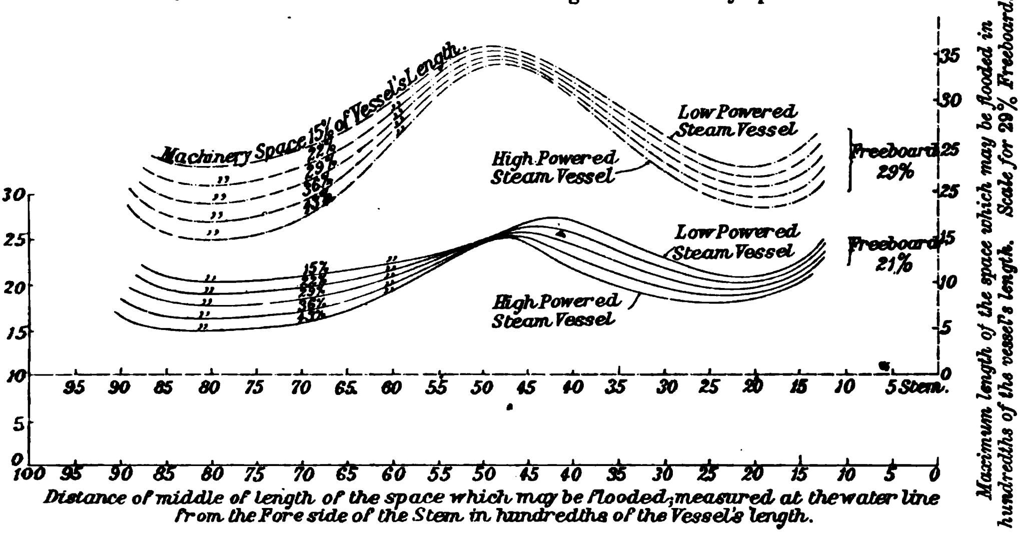



FIG. 132.

SAILING VESSEL.

Length 12·5 times depth of side amidships, measured from top of Keel to top of Bulkhead Deck.
Freeboard $\frac{21}{100}$ „ „ „ „ „
Sheer, forward 1·28 and aft ·8 per cent. of vessel's length.

Diagram showing the effect of Cargo.

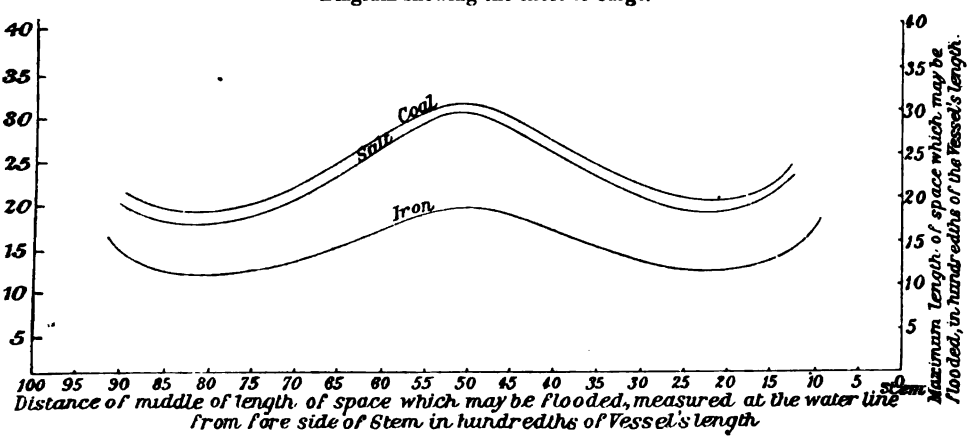
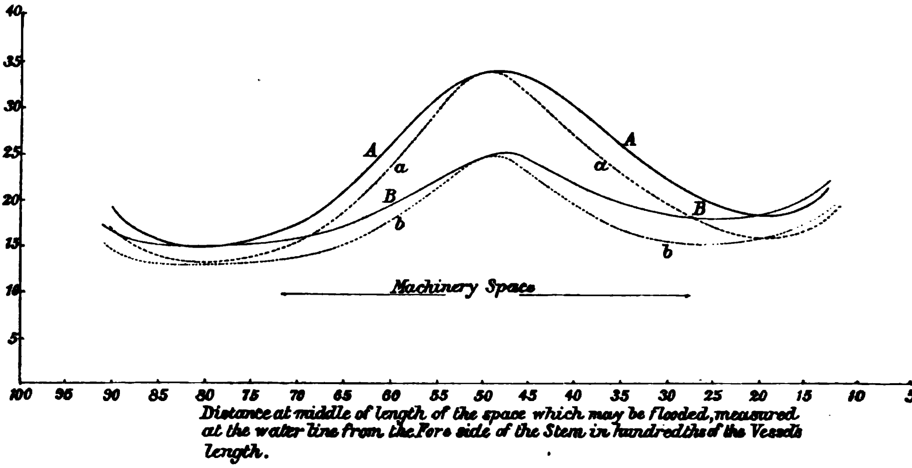



FIG. 133.

In the well decked vessels, having wells "so called" not exceeding $\frac{1}{10}$th o the vessel's length, the length of the compartment or compartments immedi ately below the lowest step may be calculated on the assumption that th margin of safety line at that point coincides with the level of such a step o deck, providing that the compartments at the fore and after sides of the we be made watertight and the bulkheads sufficiently strong. Whether th




HIGH-POWERED STEAM VESSEL.

Length 12·5 times depth amidships, measured from top of Keel to top of Bulkhead Deck.

*Cargo Coal.*

| | | | | | | |
|---|---|---|---|---|---|---|
| Curve A for Freeboard of $\frac{20}{100}$ of depth of side amidships, measured from top of Keel to top of Bulkhead Deck, | Sheer forward | 1·28 | and aft | ·8 | per cent. | of ve |
| Curve *a* for Freeboard of $\frac{20}{100}$ of depth of side amidships, measured from top of Keel to top of Bulkhead Deck, | ,, | ·64 | ,, | ·4 | ,, | |
| Curve B for Freeboard of $\frac{21}{100}$ of depth of side amidships, measured from top of Keel to top of Bulkhead Deck, | ,, | 1·28 | ,, | ·8 | ,, | |
| Curve *b* for Freeboard of $\frac{21}{100}$ of depth of side amidships, measured from top of Keel to top of Bulkhead Deck, | ,, | ·64 | ,, | ·4 | ,, | |

FIG. 134.

flooded portion consist of one or of two adjacent compartments depends o the "grade" of the vessel.

The Committee classified the vessels in their Report into six "grades, ranging from Grade I.—sea-going steam vessels, whether screw or paddl which have passenger certificates under the Merchant Shipping Acts an which are not less than 425 feet in length—to Grade VI., which include "sea-going sailing vessels between 275 feet and 225 feet in length, and sea-goin steam vessels between 300 feet and 260 feet in length."

Names, Residence and Description of the Owners, and Number of Sixty-fourth Shares held by each ... } viz.;

Volana Shipping Company Limited
having its principal place of business at
17 Water Street, Liverpool — Sixty-four.

Mr. Ernest Cook of 17 Water Street, Liverpool designated the person to whom the management of the vessel is entrusted by and on behalf of the owners. Advice received the 16 day of August 1898 under the seal of the Volana Shipping Company Limited.

Dated 16 August 1898.

Registrar J. Bradford

[*To face page* 197.

# CHAPTER XV.

## TONNAGE.

In ship calculations the ton weight, 2240 lbs., is most commonly used. In the United States the short ton, 2000 lbs., is sometimes used. With ship-owners and those who work ships the "ton" is of a varying nature. A ship is spoken of as having so many gross tons of measurement and nett tons of measurement, and the cargo she can carry is spoken of as so many tons of freight. These units are units of measurement, and are not units of weight.

The statutory measurement unit, as by law established, is called the "register ton measurement," and is 100 cubic feet. It is the unit of the tonnage placed upon the ship's register. The form of the register is in accordance with the Merchant Shipping Act of 1894. A specimen copy of a ship's register is given in Plate X.

The register of the ship is a record of her ownership and measurement tonnage, and is kept in the Custom House of the port in which she is registered. The figures of dimension and tonnage are supplied by a Board of Trade surveyor, who makes all the necessary calculations for the tonnage or tons measurement.

Instructions to the measuring surveyors are issued in a printed form, and should be studied if a full knowledge of the measurement of tonnage is required. The pamphlet is issued and prepared by the Board of Trade, and is called "Instructions relating to the Measurement of Ships."

Before we describe the measurement of register tonnage, it may be well to review the other rules of measurement.

We have seen in the chapter on Cargo Capacities that one ton of 40 cubic feet is sometimes taken as a unit in measuring cargo. This unit is called the freight ton. This unit is occasionally used, and then only in reference to the capacity of cargo holds and to certain kinds of cargo.

The first English tonnage law was passed in 1422, and was applied to coaling vessels entering Newcastle. Later, this Act was extended so that it applied to vessels in other coaling ports. An Act passed in 1694 was intended to express the maximum dead weight of cargoes which could be carried, in terms of the principal dimensions; but it was soon repealed.

There used to be a system of measurement of vessels called the Builders' Old Measurement. It was passed in 1773, and existed until 1835. The unit was referred to as the B.O.M. The B.O.M. was obtained by the following formula:—

$$\text{B.O.M} = \frac{(L - \frac{3}{5}B)B \times \frac{B}{2}}{94},$$

where L = distance along the deck between the fore side of the stem continued to the upper deck and the after side of the sternpost continued to the upper deck, and where B = the maximum breadth of ship.

In the formula $(L - \frac{3}{5}B)$ was called the length for tonnage. $\frac{B}{2}$ was intended to represent the depth.

$(L - \frac{3}{5}B)$, B was supposed to be an approximation to the area of the mean horizontal area of the ship's form.

During the period in which this formula was used the tonnage was measured by size, and not weight, just as in the system of gross measurement at the present day. Ships of that time had a very full form. The tonnage increased with the square of the breadth. The breadth was severely penalised, and in consequence led to the adoption of long narrow deep vessels, as the depth was not measured. The vessel tended to become unsafe, and the law was repealed in 1835.

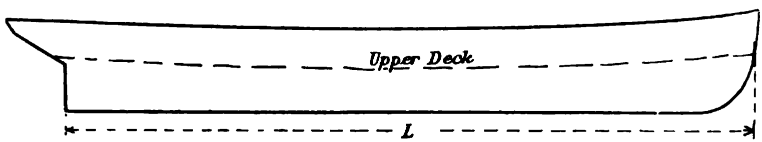



FIG. 135.

The Thames system of measurement still exists, but it is only used for yachts. It differs from the B.O.M. in that the length for tonnage is taken as (L – B) and not $(L - \frac{3}{5}B)$, so that the formula becomes

$$\text{Thames measurement} = \frac{(L - B)B \times \frac{B}{2}}{94},$$

where L is the length along the deck, as before.

An older rule that was applied to yacht measurement was

$$\frac{L \times B \times \text{depth of hold}}{94},$$

where L = length on the keel,
B = inside breadth amidships,
and D = depth from top of ceiling to under side of upper-deck planking.

In 1821 an Admiralty Committee was appointed to inquire into the system of tonnage measurement. Nothing came of this investigation, however. Another Commission appointed in 1833 reported on the internal capacity as being the fairest standard of measurement, and in 1835 a new tonnage law was enacted on a basis of measurement known as the New Measurement, in accordance with the recommendation of the Commission. The New Measurement rules existed until 1854. The rules embodied in this Act provided a system for measuring the capacity of all the cargo-carrying spaces on the basis of the smallest number of measurements which would yield satisfactory results. The total capacity was called the Gross Tonnage, and a deduction was allowed for the capacity of the engine and boiler spaces. It was found, however, that advantage could be taken of this system, and it was revised thoroughly, and a better system of measurement was defined in the Merchant Shipping Act of 1854.

The system of measurement embodied in this Act was known as Moorsom's, and was intended to give a fairly accurate estimate of internal capacity of the vessel under the upper deck, and of the permanently closed-in spaces above the upper deck, available for carrying cargo, stores, or for accommodation. This Act was slightly amended in 1867, 1876, and 1889.

The cubic capacity divided by 100 gave the tonnage. This number was chosen for convenience, and because it gave a close approximation to the tonnage of existing ships. Further amendments were required to be made in the Act as due to the construction of vessels with double bottoms or with tanks for water-ballast only.

The modern system of tonnage measurement is based upon the Merchant Shipping Act of 1854. The register ton is defined to be the volume of 100 cubic feet. The Merchant Shipping Act of 1854 was revised, and a new Merchant Shipping Act was passed in 1894, and the rules of measurement therein contained are now universally adopted throughout civilised countries.

The tonnage is based upon an attempt to determine the carrying capacity of the vessel. What the vessel carries in the capacity available has no effect on the amount of tonnage.

There are two views of the basis of tonnage upon which payment should be made by shipowners to dock-owners and others. They are—

(1) According to the ship's money-earning capability.

(2) According to the service rendered to the ship.

The present system of tonnage measurement is supposed to be based upon (1)—that is, on the principle that taxation should be based upon ability to pay.

There are, however, objections to this mode of measurement from the point of view of dock-owners and others, who consider that the second principle is the one which holds in all commercial transactions, and should be applied to shipowners, whose gross earnings are usually based upon the principle of service rendered.

In the ship's register we see that there are two kinds of tonnage—gross and nett.

The gross tonnage is a measurement in register tons of the internal capacity of the whole ship. This capacity is obtained according to the rules for measurement of tonnage; and the nett register tonnage, which is the tonnage upon which payment is made, is intended to be the volume of the spaces available for cargo-carrying (and other money-earning purposes, such as passenger space). There is an obvious fallacy in this, even from the point of capability of earning money, because, while it takes account of the quantity of the space, it takes no account of the quality. In fact, it assumes that the money-earning capacity is the same whether the space is devoted to carrying cargo or carrying passengers. It is well known that, per cubic foot, passengers pay much better than cargo.

These measurements are classified so that they can be put in the different headings under the gross tonnage side in the register certificate. Thus we have—

Space under tonnage deck.
Closed-in spaces above the tonnage deck (if any).
Space or spaces between decks.
Poop or raised quarter-deck.
Forecastle.
Roundhouses.

Other closed-in spaces as follows :—

Spaces for machinery, light, and air.

It will be seen here that the capacity is first calculated to the tonnage deck, which is defined thus :—

Tonnage deck shall be taken to be the upper deck in ships which have less than three complete decks, and to be the second complete deck from below in all other ships.

The gross tonnage is the sum of the capacity in register tons of all the items mentioned.

The nett register tonnage is the gross tonnage, with certain deductions from it which are allowed by the Act to be made.

The deductions allowed are tabulated on the right hand of the register certificate.

Deductions are allowed from the gross tonnage, as the spaces so deducted are assumed to be non-money-earning. The deductions allowed are—

On account of space required for propelling power.

On account of spaces occupied by seamen or apprentices, and appropriated to their use, and kept free from goods or stores of every kind, not being the personal property of the crew.

These spaces are the deductions allowed under section 79 of the Merchant Shipping Act of 1894.

The total deductions are then subtracted from the gross tonnage, and the remainder gives the nett register tonnage.

The following rules give directions for measuring the capacity of the spaces.

The length taken in tonnage measurement is the length on the tonnage deck.

The measurement of the capacity of the under-deck tonnage is made by dividing up the length so defined into a sufficient number of parts for integration by Simpson's Rules. At these divisions the depths of the sectional areas are subdivided vertically into a sufficient number of intervals for Simpson's Rules, and the half-breadths at these points of subdivision are measured in feet.

In vessels of 225 ft. in length and above, twelve divisions are made in the length, which give thirteen ordinates. If the depth is under 16 ft., only four divisions are necessary for the half-breadth ordinates, and if the depth is above 16 ft., six divisions are necessary.

Table XXVII. gives the number of divisions to be taken.

TABLE XXVII.

| Class. | Length of Tonnage Deck. | Number of Divisions. |
|---|---|---|
| I. | 50 ft. or under . . . . | 4 |
| II. | Above 50 ft. not exceeding 120 ft. | 6 |
| III. | ,, 120 ,, ,, 180 ,, | 8 |
| IV. | ,, 180 ,, ,, 225 ,, | 10 |
| V. | ,, 225 ft. . . . . | 12 |

The depth referred to above is the depth at the section, and is taken from the under side of the tonnage deck to the upper side of the floor timber at the inside of the limber strake. From this is deducted one-third of the round of beam, and also the average thickness of the ceiling on the floor timber.

*Measurement of Tonnage under Tonnage Deck when vessel has a double bottom for water ballast.*—In the case of a break or breaks in a double bottom for water ballast, the length of the vessel is to be taken in parts according to the number of breaks, and each part must be divided into a number of equal parts according to the class in the above table to which such length belongs. The depth at each point of division is measured from a point at a distance of one-third the round of the beam below the tonnage deck (or, in the case of a break, to a line stretched in continuation thereof) to the upper side of the floor timber (upper side of the inner plating of the double bottom) at the inside of the limber strake, after deducting the average thickness of the ceiling which is between the bilge planks and limber strake.

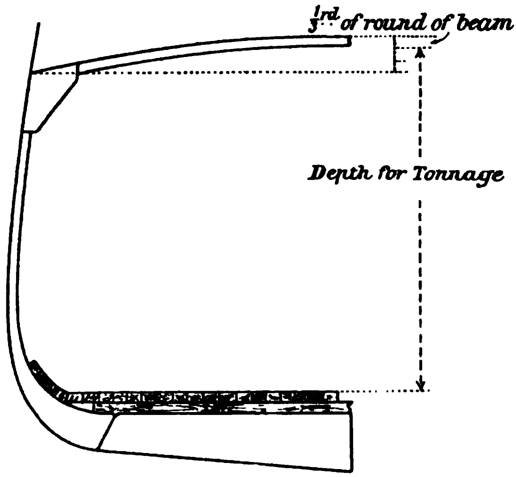



FIG. 136.

The area of each section is calculated in the following way:—If the depth at the midship section do not exceed 16 feet, divide it into four equal parts. Calculate the area of the four parts, beginning at the top, by Simpson's Rules. Then find the area of the remaining part by dividing the depth of the lowest part into four equal parts, and calculate the area by Simpson's Rules. If the midship depth exceed 16 feet, divide each depth into six equal parts, and measure as before.

In double-bottom ships with ceiling placed over grounds, only the thickness of ceiling itself is allowed to be deducted for tonnage.

The under-deck tonnage is then obtained from the half-breadths by multiplying them by the Simpson's multipliers, first in the vertical direction for areas, and second in the lengthwise direction for volume. After the total volume in cubic feet is obtained, the result is divided by 100 to give the under-deck tonnage.

The above-deck tonnage is the capacity in tons measurement of all the spaces mentioned in the previous list for gross tonnage excepting the under-deck tonnage. The 'tween-deck space, if any, is measured in a similar way to that described for under-deck tonnage, by taking sections at suitable intervals along the length of the 'tween-deck space. The depth is taken from the top of wood deck to the top of beam above.

The remainder of the above-deck tonnage includes all the other closed-in spaces, such as poop or raised quarter-deck, bridge, forecastle, etc.

If the poop has doors, the total volume from the upper side of deck to upper side of poop beams is calculated. If there are no doors in the poop, the volumes of the enclosed houses under the poop deck are calculated separately. The same applies to the bridge deck and forecastle. Enclosed deckhouses are calculated separately.

The following deck erections are not taken account of :—

Wheelhouses (including all houses containing steering gear of any kind).

Deck W.C.'s and urinals, whether for passengers or crew.

Donkey boiler space when above upper deck.

All companion houses and stairways above upper deck.

Galleys, whether above or below upper deck, are ignored in the calculations.

The remaining deck erections are to be calculated and put under three headings, as in Table XXVIII.

TABLE XXVIII.

| | A | B | C |
|---|---|---|---|
| Items. | Machinery Openings or Erections. | Officers' and Crew's Houses and Rooms. | Other Erections. |
| | | | |

The machinery openings are calculated and kept separate from the other items, and put in the column headed A. These are part of the light and air spaces, the remainder being these spaces below the tonnage deck.

The gross tonnage, or G.R.T., is generally obtained by adding the underdeck tonnage, the 'tween-deck tonnage, and the volume of $(A + B + C)$.

*Nett Register Tonnage*—N.R.T.—The N.R.T. is the G.R.T. less—

(1) Deduction for crew space.
(2) ,, propelling space.

(1) is found by calculating the volume of the space occupied by the crew, firemen, officers, and stewards, etc. below the upper deck, and adding to this the item B.

*Note.*—Under the Act of 1889, under the title of "Navigation Spaces (N.S.) which can be claimed at the option of the owners," the captain's room, chartroom, boatswain's storeroom, and sailroom can be deducted from the G.R.T. The limits as to the amounts allowed for each of these spaces lie in the hands of the local surveyor of the Board of Trade, but generally are—

6 tons for the captain's room.
6 tons for the chartroom.
2½ per cent. of the G.R.T. for the boatswain's storeroom and sailroom respectively.

(2) is found by calculating the total volume of the space occupied by machinery, including shaft tunnel, engineroom (but not coal bunkers in it), passages between engine-. and boiler-rooms, and between boiler-rooms; boiler-rooms (but not coal bunkers); all casings between decks which contain either

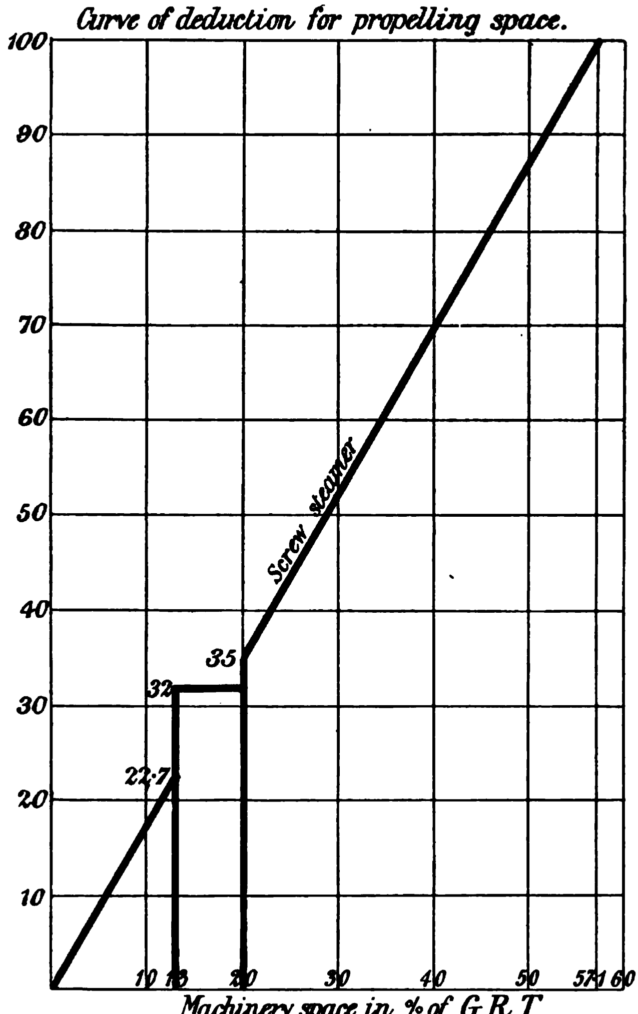



FIG. 137.

light and air for machinery spaces or part of machinery or boilers. To this volume add the item A, and find what percentage this total is of the gross tonnage. If this percentage is between 13 and 20 in screw vessels, then the deduction for propelling space is 32 per cent. of the G.R.T.

In paddle steamers, if this percentage is between 20 and 30, the deduction for propelling space is 37 per cent. of the G.R.T. When the percentage is under or above these ranges, the deduction is $1\frac{3}{4}$ times the volume of the spaces in the case of screw vessels, and $1\frac{1}{2}$ times in the case of paddle vessels.

*Note.*—Under the Merchant Shipping Act of 1889 the deduction for light and air spaces above the upper deck must be the subject of special application by the owner, and the spaces must be constructed to admit both *light and air*, otherwise the deduction will not be allowed. If no application is made for this deduction, the volume A, that of light and air, above the decks need not be added to the gross. No deduction can be made for spaces not previously included in the G.R.T.

In order to show the amount of deduction allowed in screw-propelled vessels, a diagram (fig. 137) has been prepared, which gives the amount of deduction in terms of the percentage that the machinery space is of the G.R.T. It will be seen that the curve must be a line parallel to the base at the region where the machinery space is between 13 per cent. and 20 per cent. of the G.R.T. Elsewhere the curve is a straight line, whose ordinates are $1\frac{3}{4}$ times the abscissæ, if the percentages are set up to the same scale.

It will be seen from this diagram that the deduction is 100 per cent. of the G.R.T. if the propelling space were 57·1 per cent. of the G.R.T.

In the case of a steamer whose propelling space is 50 per cent. of the G.R.T. the deduction for machinery space would be 87·5; and if the other spaces amounted to 12·5 per cent. of the G.R.T., the total reduction would be $87{\cdot}5 + 12{\cdot}5 = 100$, and the N.R.T. would consequently be zero.

It is obvious that if N.R.T. is based on money-earning capacity, a ship with a zero of N.R.T. would earn nothing, and ought not to be built. If it were built, it would be because its money-earning depended on something which N.R.T. would not measure.

Tables XXIX., XXX., and XXXI. give particulars of the tonnage measurement of a number of ships which are representative of the types indicated in the heading. The particulars have been taken from the Register. In each case the percentage of nett to gross has been calculated.

In nearly all of the large vessels with comparatively small power, the machinery space is just over the 13 per cent. of G.R.T., so that the deduction is 32 per cent. This is the most favourable point below 20 per cent. for obtaining the advantage of the reduction. The great bulk of the tonnage of the world has therefore the relation of nett to gross, got from taking advantage of this proportion.

A similar curve, fig. 138, can be drawn for the deduction allowed in a paddle steamer. It will thus be seen that the larger the machinery space in a vessel, the less the proportion of N.R.T. to G.R.T. High-powered vessels have a small proportion of N.R.T. to G.R.T. Cross-Channel steamers and fast Atlantic liners are high-powered in relation to their size. The size of a vessel is roughly in proportion to the G.R.T., but dock-owners are paid on the N.R.T. The dues paid upon the basis of N.R.T. are not in proportion to service rendered to the vessel in dock in so far as they are not in proportion to the size of the vessel. In over 90 per cent. of the tonnage of the country the percentage of nett to gross is about 63 per cent., because the deduction for machinery is 32 per cent. and for crew and other places 5 per cent., so that it is practically a constant proportion, and gross measures dock dues as much as nett does. In the remaining vessels the plea is that the payment of dues should not be on a fixed proportion of the G.R.T. (or size of the vessel),

TABLE XXIX.

| | Paddle Steamers. | | | Single Screw Small Coasters. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | "Empress Queen." | "Iverna." | "Adder." | "Ape." | "Jabiru." | "Burnbrae." | "Fluor." | "Donegal Castle." | "Joseph Fisher." | "Fern." | "Dauntless." |
| Length for tonnage | 360 | 255 | 280 | 175 | 260 | 105 | 200 | 185 | 140 | 180 | 256 |
| Breadth | 42 | 30 | 33 | 28 | 34 | 20 | 31 | 22 | 21 | 29 | 32 |
| Depth in hold, tonnage deck | 17 | 15 | 13 | 13 | 18 | 9 | 11 | 10 | 9 | 12 | 15 |
| Length, engineroom | 114 | 64 | 97 | 38 | 40 | 28 | 44 | 29 | 33 | 40 | 44 |
| Displacement to ¾ depth from weather deck | 3182 | 1728 | 1710 | 875 | 2655 | 295 | 1400 | 420 | 446 | 910 | 2170 |
| Tons per inch at same depth | 25·5 | 14·3 | 15·3 | 9 | 17·4 | 3·4 | 13 | 5·59 | 6·1 | 10 | 15·5 |
| I.H.P. | 10500 | 2000 | 4250 | 600 | 1030 | 190 | 650 | 350 | 400 | 720 | 1400 |
| Speed | 21·5 | ... | 19 | 10 | 10¾ | ... | 11½ | 8½ | 9½ | 12 | 12 |
| *Gross Tonnage:—* | | | | | | | | | | | |
| Under deck | 1597 | 820 | 771 | 426 | 1200 | 130 | 641 | 206 | 201 | 401 | 955 |
| 'Tween-decks | ... | ... | ... | .. | ... | ... | ... | 14 | 23 | ... | ... |
| Forecastle and poop | 48 | 103 | 26 | 6 | 33 | 39 | 144 | 36 | 32 | 4 | 72 |
| Other closed-in spaces | 196 | 19 | 37 | 8 | 89 | 15 | 46 | 14 | 11 | 38 | 74 |
| Spaces for L. and A. | 299 | 53 | 116 | 25 | 78 | 16 | 52 | 20 | 25 | 60 | 92 |
| Gross tonnage | 2140 | 995 | 950 | 465 | 1400 | 200 | 883 | 290 | 292 | 503 | 1233 |
| Deductions as under | 1669 | 612 | 804 | 272 | 703 | 163 | 588 | 218 | 222 | 411 | 562 |
| Nett register tonnage | 471 | 383 | 146 | 193 | 697 | 37 | 295 | 72 | 70 | 92 | 671 |
| Percentage nett/gross | 22·0 | 38·5 | 15·4 | 41·5 | 49·8 | 18·5 | 33·4 | 25 | 24 | 18·3 | 54·4 |
| Deductions: Propelling space | 1559 | 542 | 722 | 237 | 638 | 136 | 508 | 174 | 177 | 356 | 495 |
| Deductions: Other spaces | 110 | 70 | 82 | 35 | 65 | 27 | 80 | 44 | 45 | 55 | 67 |
| Total deductions | 1669 | 612 | 804 | 272 | 703 | 163 | 588 | 218 | 222 | 411 | 562 |

TABLE XXX.

| | Intermediate Atlantic Liners. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | "Sachem." | "Ultonia." | "Ivernis." | "Haverford." | "Roanmore." | "Celtic." | "Boric." | "Cymric." | "Romanic." | "Saxonia." |
| Length for tonnage | 445 | 500 | 582 | 531 | 521 | 680 | 470 | 585 | 550 | 580 |
| Breadth | 46 | 57 | 64 | 59 | 59 | 75 | 53 | 64 | 59 | 64 |
| Depth in hold, tonnage deck | 23 | 33 | 21 | 27 | 29 | 28 | 23 | 30 | 28 | 30 |
| Length, engineroom | 74 | 70 | 107 | 84 | 75 | 142 | 73 | 92 | 101 | 105 |
| Displacement to ¼ depth from weather deck | 10760 | 17266 | 24400 | 18900 | 20100 | 38200 | 13960 | 24700 | 20130 | 24900 |
| Tons per inch at same depth | 39·6 | 58·2 | 73·6 | 63·5 | 60 | 99 | 50·2 | 75 | 64·2 | 74·3 |
| I.H.P. | 3000 | 5500 | 10000 | 5400 | 5000 | 13000 | 3350 | 6700 | 7800 | 9500 |
| Speed | 12 | 14½ | 16 | 14 | 14½ | 16 | 13 | 14 | 15½ | 15 |
| *Gross Tonnage:—* | | | | | | | | | | |
| Under deck | 3596 | 7725 | 6132 | 6558 | 6987 | 10663 | 4661 | 8920 | 7103 | 8813 |
| 'Tween-decks | 1268 | 2023 | 4925 | 4213 | 2009 | 6452 | 1609 | ... | 2007 | 2463 |
| Forecastle and poop | 97 | ... | 1392 | 506 | 80 | 299 | 75 | 2333 | 420 | 206 |
| Other closed-in spaces | 134 | 444 | 1507 | 165 | 321 | 3490 | 237 | 1843 | 1863 | 2797 |
| Spaces for L. and A. | 109 | 211 | 101 | 193 | 59 | ... | ... | ... | ... | ... |
| Gross tonnage | 5204 | 10402 | 14057 | 11635 | 9456 | 20904 | 6583 | 13096 | 11394 | 14280 |
| Deductions as under | 1866 | 3808 | 5006 | 4142 | 3298 | 7455 | 2353 | 4588 | 3977 | 5180 |
| Nett register tonnage | 3338 | 6594 | 9051 | 7493 | 6158 | 13449 | 4230 | 8508 | 7417 | 9100 |
| Percentage $\frac{\text{nett}}{\text{gross}}$ | 64·1 | 63·4 | 64·4 | 64·4 | 65·1 | 64·3 | 64·3 | 65·0 | 65·1 | 63·7 |
| Deductions { Propelling space | 1665 | 3328 | 4498 | 3723 | 3026 | 6689 | 2107 | 4191 | 3646 | 4570 |
| Deductions { Other spaces | 201 | 480 | 508 | 419 | 272 | 766 | 246 | 397 | 331 | 610 |
| Total deductions | 1866 | 3808 | 5006 | 4142 | 3298 | 7455 | 2353 | 4588 | 3977 | 5180 |

TABLE XXXI.

| | Fast Liners. | | | Channel Steamers. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | "Campania." | "Majestic." | "Oceanic." | "Spaniel." | "Dundalk." | "Antrim." | "Connaught." | "Magic." | "Mystic." | "Alberta." | "Inniscarra." |
| Length for tonnage . . | 601 | 565 | 685 | 250 | 236 | 330 | 360 | 311 | 220 | 270 | 280 |
| Breadth . . . . . | 65 | 57 | 68 | 33 | 32 | 42 | 41 | 38 | 29 | 35 | 38 |
| Depth in hold, tonnage deck | 28 | 22 | 26 | 15 | 15 | 17 | 18 | 15 | 14 | 14 | 17 |
| Length, engineroom . . | 183 | 159 | 307 | 48 | 56 | 38 | 40 | 81 | 53 | 68 | 76 |
| Displacement to ½ depth from weather deck . | 23352 | 19425 | 33500 | 1922 | 1583 | ... | 812 | 2515 | 1405 | 1620 | 2213 |
| Tons per inch at same depth | 68·5 | 59 | 85 | 15·3 | 13·7 | ... | 16 | 20·4 | 11·8 | 15·1 | 17·9 |
| I.H.P. . . . . . | 26500 | 16000 | 27000 | 1800 | 2300 | 6100 | 8500 | 3500 | 1100 | 5000 | 3600 |
| Speed . . . . . | 21 | 20 | 21 | 13¾ | 14 | 20 | 23 | 17 | 13¼ | 19½ | 16 |
| *Gross Tonnage* :— | | | | | | | | | | | |
| Under deck . . . | 7592 | 4749 | 8484 | 873·1 | 703 | 1383 | 1516 | 1125 | 602 | 786 | 1042 |
| 'Tween-decks . . | 2674 | 4079 | { 2892<br>{ 3356 | } ... | 15 | 317 | 943 | 188 | ... | ... | ... |
| Forecastle and poop . | 77 | 31 | | 154 | 69 | 51 | 138 | 30 | 17 | 73 | 53 |
| Other closed-in spaces . | 2540 | 1081 | 1893 | 70 | 3 | 203 | 14 | 161 | 59 | 251 | 317 |
| Spaces for L. and A. . | 66 | 206 | 649 | 76 | 74 | 145 | 30 | 135 | 48 | 216 | ... |
| Gross tonnage . . . | 12950 | 10146 | 17274 | 1173 | 863 | 2099 | 2641 | 1639 | 726 | 1236 | 1412 |
| Deductions as under . | 7976 | 5704 | 10357 | 683 | 788 | 1496 | 1893 | 1161 | 556 | 911 | 847 |
| Nett register tonnage. . | 4974 | 4442 | 6917 | 490 | 75 | 603 | 747 | 478 | 170 | 325 | 565 |
| Percentage $\frac{\text{nett}}{\text{gross}}$ . . | 38·4 | 43·8 | 40·0 | 41·8 | 8·7 | 28·7 | 28·3 | 29·2 | 23·4 | 26·3 | 40 |
| Deductions { Propelling space | 7373 | 5117 | 9378 | 607 | 742 | 1360 | 1693 | 1070 | 512 | 825 | 769 |
| Deductions { Other spaces . | 603 | 587 | 979 | 75 | 46 | 163 | 200 | 91 | 44 | 85 | 78 |
| Total deductions . . | 7976 | 5704 | 10357 | 683 | 788 | 1496 | 1893 | 1161 | 556 | 910 | 847 |

because the ability to pay in these cases is reduced by the increase of machinery space. This plea has never been justified by balance sheets, nor does it seem likely that the higher-powered vessels, which cost more per gross ton than the low-powered, will in the long-run pay a lower rate of

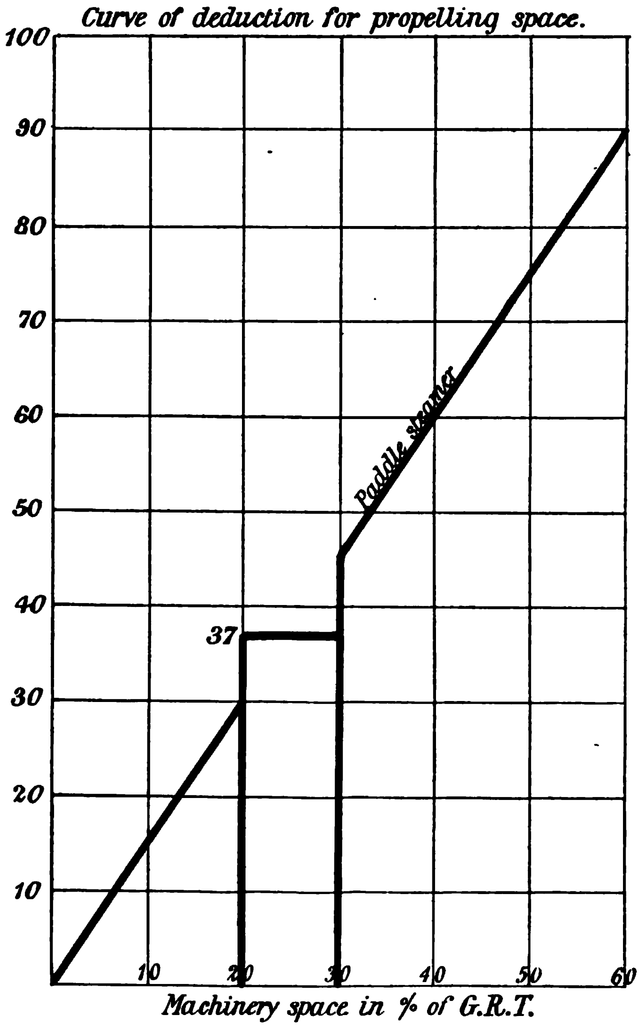



FIG. 138.

interest on money invested, or they would cease to be built. If they pay the same rate, their ability to earn per ton of G.R.T. must be greater in proportion to the greater first cost. Hence it seems not to be an unreasonable claim on the part of the dock-owners that all vessels should pay on an approximately uniform percentage of their gross tonnage.

An attempt was made by the Mersey Dock Board to get Parliament to sanction the levying of dock dues on *not less* than 50 per cent. of the G.R.T. In the cases of several small ports Parliament has sanctioned a limit of 40 per cent., and in some cases of 50 per cent., but as Liverpool has such a large amount of tonnage entering the port, the Government considered that to make a rule in the case of Liverpool would practically be so large a change in the effect of the tonnage law that it was desirable to deal with the matter by general rather than special legislation, so they appointed a Committee to examine and report upon the basis of deductions for machinery space, with a view to dealing with the question as a whole if such should be considered necessary.

As a result of this Committee's Report, an Act was passed in 1907 limiting the amount of the deduction for propelling spaces. The basis of the limit is that no deduction shall exceed 55 per cent. of the G.R.T. minus the deductions for crew spaces, etc. In the case of a ship having a crew space amounting to 10 per cent. of the G.R.T., the maximum deduction for propelling spaces is not to exceed $55 \times (100 - 10) = 49{\cdot}5$ per cent. of the G.R.T.

The object of basing the limit of deduction upon the G.R.T. less the crew space is to give the shipowner an inducement in vessels having machinery spaces large in proportion to G.R.T. to have large crew spaces. In effect it gives the dock-owner smaller dock dues in order to prevent the shipowner from giving the crew insufficient living spaces. The dock-owner pays the shipowner to do his duty to the crew.

It will, however, it is hoped, prevent the dock-owner from having to receive ridiculously small dues from vessels which have been specially designed to obtain an unreasonable deduction for propelling spaces.

# CHAPTER XVI.

## FREEBOARD.

**Freeboard** is defined to be the height of the side of a ship above the waterline at the middle of her length, measured from the top of the deck at the side, or in cases where a waterway is fitted, from the curved line of the top of the deck continued through to the side.

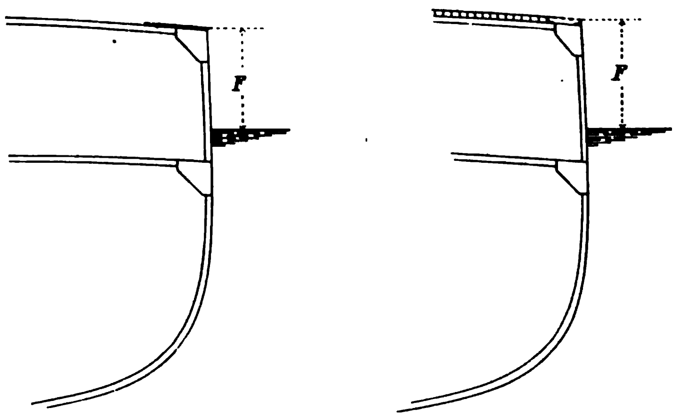



Fig. 139.

The deeper a vessel can load the less freeboard she has, and the more she can carry; but this can be extended too far. The freeboard may be so reduced that the vessel may sink. It is therefore necessary to determine the minimum permissible freeboard which may be assigned to a vessel to give a reasonable margin of safety in the work she has to do.

The limit to which vessels can load is fixed by the regulations given in the freeboard tables, incorporated in the Load Line Act of 1885, and issued as Instructions to Surveyors, etc. Further modifications since made are incorporated in these instructions.

The determination of freeboard for safety depends largely upon purely practical considerations, and therefore most of the elements which must be secured in order to have safety can only be determined by experience. A vessel at rest which is perfectly watertight, like a sealed empty bottle, needs practically no reserve buoyancy to prevent her sinking. If there be openings for the admission of air or the outlet of gases, like a funnel, then as long as no water enters by these openings the vessel requires practically no reserve buoyancy.

Evidently, in a sea where waves are high and spray is breaking freely, the question of complete exclusion of water for a given reserve buoyancy and height of opening is one of experience alone. Water will, however, accumulate in a ship from condensation alone, if from none of the other many causes. Water can, of course, be expelled even though it enters, but the rate at which it enters can only be determined by experience. Hence for such a simple case as a vessel at rest, the amount of reserve buoyancy necessary cannot be determined by *à priori* considerations. With a vessel in motion waves break over the vessel, and the pitching and rolling motions place the vessel in a condition of altered and generally reduced longitudinal and transverse stability, so that, unless there is sufficient margin of buoyancy or side, or something to give stability in every direction, the vessel may capsize, either head first or broadside on. The former points to the desirability of increased freeboard at the ends as compared with amidships, but how much, experience only can determine. This *increased* freeboard is called sheer, and a standard quantity of it is assumed in fixing the freeboard amidships. There is a further necessity for freeboard for convenience in navigating the ship at sea. In the case of ships of very low freeboard, it is impracticable for the crew to get along the deck in bad weather, and navigation is prevented. There must, therefore, be a "height of platform" above water sufficient to ensure that communication can be obtained from the crew's quarters to the ends of the ship. This height can only be determined by experience. This platform is sometimes provided in the form of a gangway, and not in the form of increased freeboard. Hence the determination of freeboard is based upon the experience of successful and unsuccessful loading of vessels. Those which have been lost from overloading give data for fixing load lines.

There is the further question of sinking by flooding compartments.

From a consideration of Chapter XIV. it will be seen that a vessel's safety, independently of her stability, in the event of any of the compartments becoming flooded, depends upon the amount of reserve buoyancy that she has. But having fixed, from experience, a freeboard for the vessel in the intact condition, we can, from *à priori* considerations, fix the extra freeboard, if any, necessary to ensure safety when flooded. The freeboard is roughly proportional to the amount of reserve buoyancy, so that the assignment of a certain amount of freeboard to a vessel gives her a certain amount of reserve buoyancy, and therefore a definite margin of safety at sea.

The amount of freeboard to be assigned to each vessel was fully considered by a Committee appointed in 1885, and another appointed in 1898. The basis upon which the amount of freeboard was fixed was percentage of reserve buoyancy of the total buoyancy of the ship. By collecting from all sources the data of draughts and freeboards, it was possible to determine the actual percentage of reserve buoyancy of all classes of ships. In classes which had been found to have an abnormal percentage of losses due to overloading, it was found possible to fix the percentage of reserve buoyancy at which the danger appeared to begin. From this a limit of safety was fixed. Other considerations, such as strength of ship, strength and extent of deck erections, had a qualifying effect upon the determination of this limit, but these were given effect to largely upon the basis of experience.

The reports of the Committee are embodied in the Instructions to Surveyors published by the Board of Trade, which contain the tables used in fixing the actual amount of freeboard for any given vessel.

The amount of freeboard assigned in the tables to any vessel is dependent upon its size, type, and structural strength, and upon an assumed standard sheer and round of beam. Modifications have to be made for deviation from these standards, amount of sheer and camber or round of beam, and also for the extent and nature of houses built on deck, usually called deck erections.

The Instructions to Surveyors contain tables of freeboard, which class the vessels to be assigned under one of the following :—

A. Cargo-carrying steam vessels not having spar or awning decks.
B. " " having spar decks.
C. " " " awning decks.
D. " sailing vessels.

There are 36 tables in all,—

13 A tables
6 B "
7 C "
10 D "

It will be seen that Tables A and D are for full scantling vessels. Any vessel of a type not described by either of the above is classed in Table A, and corrections are made by the surveyor for the deviations from A in the construction and strength of this vessel. In some cases the rules for the allowances to be made are given. Well-deckers, turret vessels, and shelter-deck vessels are cases in point.

In the explanation of the tables, it is stated that the exact freeboard for a given ship belonging to either of the classes A or D may be calculated by constructing a displacement scale to the height of the upper deck, to which the freeboard is measured, so as to give the whole external volume up to the upper surface of that deck.

The percentage of the whole volume which is given in the tables as the reserve buoyancy percentage gives the amount of volume that must be left out of water. It is given in terms of depth moulded for a vessel whose length is twelve times its depth. If a waterline be drawn upon the displacement scale to cut off the given percentage of total volume, this line will give the freeboard required.

The amount of freeboard thus obtained is subject to slight modification if the vessel is outside the proportions stated.

The freeboards necessary for spar- and awning-deck steamers are given in the Tables B and C respectively, and are determined by considerations of the structural strength. The freeboards and percentages of reserve buoyancy thus obtained being in excess of what would otherwise be required, the amounts of such percentages are not given in the Tables B and C.

The tables in the Rules are arranged so as to obviate the necessity for making the displacement curve up to the top deck. It may be mentioned here that if the lines of the vessel are at hand, the full displacement curve can easily be obtained by using the integraph.

The tables are therefore intended to save the labour of making a complete displacement scale for the whole external volume of the ship, and are convenient in the respect that in cases where sufficient data are not available for the construction of this displacement scale, the freeboard can nevertheless be easily obtained.

The coefficients of fineness of the external and internal volumes of the ship bear a fairly fixed relation to each other. The latter can be easily obtained from the measurement of tonnage given in the ship's register.

**Coefficient of Fineness.**—The coefficient of fineness in one-, two-, and three-deck and spar-deck vessels of standard construction is found by dividing 100 times the registered tonnage of the vessel below the upper deck by the product of the length, breadth, and depth of hold. In awning-deck vessels the registered depth and tonnage are taken below the main deck.

Let T be the under-deck tonnage as obtained above.

$$\text{Then the coefficient of fineness} = \frac{T \times 100}{L \times B \times D}.$$

The values of T, L, B, and D are those for a ship of standard construction. The standard vessel is one built on the ordinary system of flooring, with floors of the usual depth. If the vessel is built on the double-bottom principle, then two corrections may have to be made.

(a) *Corrections in the value of T.*—If the vessel has a double bottom for the whole or part of her length, a calculation of the change of volume caused by the departure from the ordinary system of flooring has to be made, and the value of T modified accordingly.

(b) *Correction in the value of D.*—This correction only needs to be made if the double bottom extends in the region of midships. The depth of hold measured for the tonnage would in this case be taken to the top of the ceiling in the inner bottom at amidships, so that a correction has to be made in D to give the depth to the top of the ordinary floors if such were fitted.

In order to allow for differences in fulness of form, the freeboards are given in terms of a coefficient of fineness.[1] The first vertical column gives the coefficient of fineness, which ranges from ·68 to ·82, giving eight different values.

The first horizontal column is percentage reserve buoyancy.

The second horizontal column is the moulded depth, and the third horizontal column the moulded length.

At the foot there are columns containing corrections for changes in the length, etc.

In the tables the dimensions of the vessel are arranged horizontally.

*Length.*—The length in the table is measured on the load waterline from the fore side of the stem to the aft side of the sternpost in sailing vessels, and to the aft side of the afterpost in steamers.

*Depth*—is the moulded depth. In an iron or steel vessel this is the perpendicular depth taken from the top of the upper-deck beam at side, at the middle of the length of the vessel, to the top of the keel and the bottom of the frame at the middle line, except in spar- and awning-deck vessels, in which the depth is measured from the top of the main-deck beams at side.

*Freeboard.*—The moulded depth as described above is that used in the tables for ascertaining the amount of reserve buoyancy and corresponding freeboard in vessels having a wood deck, and the freeboard is measured

[1] See Board of Trade Instructions and Surveyors' Tables of Freeboard.

from the top of the wood deck at side, at the middle of the length of the vessel.

*Freeboard of flush-deck vessels with upper deck uncovered with wood.*—On the same principle, in flush-deck vessels other than spar- or awning-decked, and in vessels fitted with short poop and forecastle having an iron upper deck not covered with wood, the usual thickness of a wood deck should be deducted from the moulded depth of the vessel measured as above, and the amount of reserve buoyancy and corresponding freeboard taken from the column in the tables corresponding to this diminished depth.

*Example.*—Steamer with an iron upper deck not covered with wood and having a moulded depth of 19 ft. 10 in.; 4 in., or the usual thickness of a wood deck, must be deducted from this, leaving a depth of 19 ft. 6 in. The freeboard of such a vessel with a coefficient of fineness of 0·76 taken from the column under 19 ft. 6 in. is 3 ft. 8½ in., which should be measured from the top of the iron deck at side.

*Freeboard of spar- and awning-decked vessels. Correction for part of deck covered with erections.*—In spar- and in awning-deck vessels having iron main decks the freeboard required by the tables should be measured as if those decks were wood-covered. Also in vessels where $\frac{7}{10}$ths or more of the main deck is covered by substantial erections the freeboard found from the tables should be measured amidships from a wood deck, whether the deck be of wood or iron.

In applying this principle to vessels having shorter lengths of substantial enclosed erections, the reduction in freeboard, in consideration of its being measured from the iron deck, is to be regulated in proportion to the length of the deck covered by such erections. Thus in a vessel having erections covering $\frac{6}{10}$ths of the length, the reduction is $\frac{6}{10}$ths of 3½ in., *i.e.* 2 in.

"*Freeboard correction.*—In spar-decked vessels and awning-decked vessels having iron main deck, freeboards are calculated as if those decks were wood-covered, *i.e.* the ordinary thickness of a wood deck, less the thickness of the stringer plate, should be deducted from the freeboard; also, in vessels where $\frac{7}{10}$ths or more of the main deck is covered by substantial enclosed erections, the freeboard found from the tables should be measured amidships from a wood deck, or if the deck is of iron, it should be measured from the iron deck, and the ordinary thickness of a wood deck required for that size of ship, less the thickness of the stringer plate, should in that case be deducted from the freeboard. In vessels which have $\frac{6}{10}$ths of the deck covered, $\frac{6}{10}$ths the thickness of a wood deck, less the thickness of the stringer plate, is to be deducted from the freeboard. Between $\frac{6}{10}$ths and $\frac{7}{10}$ths a proportionate quantity."

*Freeboard of vessels which trim by the stern.*—In cases where the trim by the stern is such as to cause an error in the percentage reserve buoyancy, if the usual freeboard as given by the tables were applied, it is necessary to have full information regarding the trim of the vessel in the loaded condition.

*Freeboard of vessels of extreme proportions.*—For vessels whose length is greater or less than that of the vessel of the same moulded depth for which the tables are framed, viz. $\frac{L}{D}=12$, the freeboard should be increased or diminished as specified in the footnote to the tables.

*Correction to be made in freeboard for erections on deck.*—For steam vessels with topgallant forecastles having long poops, or raised quarter-decks connected with bridgehouses, covering in the engine and boiler openings, the latter being entered from the top, and having an efficiently constructed iron bulkhead at the fore end, a deduction may be made from the reserve buoyancy given in the tables, according to the following scale :—

(*a*) When the combined length of the poop or raised quarter-deck, bridge-house, and topgallant forecastle is $\frac{9}{10}$ths of the length of the vessel, deduct 85 per cent. of the reduction in the reserve buoyancy allowed for a complete awning deck, or $\frac{85}{100}$ths of the difference between freeboards in Table A (after correction for sheer) and Table C; for $\frac{8}{10}$ths of the length of the vessel deduct 75 per cent., and so on.

The above outline description of the method of obtaining freeboard is given to enable the reader to get an idea of how freeboard is determined. In any particular case the freeboard can only be obtained by reference to the tables given in the Board of Trade Instructions and Surveyors' Tables of Freeboard, which can be obtained for a few pence. The actual assigning of a statutory freeboard is done by the Board of Trade, or one of the Registration Societies to whom these powers have been delegated.

# CHAPTER XVII.

## LAUNCHING.

THE launching operation of a vessel is attended, in addition to other risks, with considerable risk to the structure; and in order to lessen the danger of straining the structure, it is important to take proper precautions beforehand in laying out the building berth, and in making the launching arrangements. It is also necessary to make a calculation in order to determine the relation of the forces and moments acting on a vessel, and the intensity of the straining forces on the vessel as a whole as she is passing from the ways into the water.

It is a disadvantage if the expanse of water fronting the shipyard is narrow. In some shipyards, especially those situated on rivers, the breadth of the channel is too narrow for launching the vessel at right angles to the water's edge. Hence it is necessary to lay the building berth at an angle to the water's edge. This angle of slant will depend on the length of the vessel and the breadth of the channel, and it should be such as to give enough clearance from the opposite shore after the vessel has been launched and stopped. The vessel is usually stopped by heavy drags in the yard. These drags are attached by strong cables to each side of the hull, and they are arranged to offer resistance when the vessel is clear of the ways, though in some cases this resistance begins to operate before that point. Sometimes, when the breadth of the river is insufficient to allow the vessel to be launched in a continuous straight line, a drag begins to act on one side before the other, so as to pull the vessel round, away from the opposite bank.

When there is sufficient water in front of the building berths no such arrangement is necessary to stop the vessel's motion. The vessel is allowed to go freely into the water, and one or more anchors are dropped from the vessel some time after she is clear.

The berth foundation must be firm, and able to withstand the weight of the ship (usually called the launching weight) without undue sinkage. If the launching weight is great and the berth is laid on soft yielding ground, the precaution has to be taken of making it firm by driving in deep piles of timber.

The declivity of the keel blocks varies slightly with the size of the vessel. The larger the vessel, the less the declivity. The height of the keel blocks above the ground should be not less than enough to allow room for working underneath the vessel's keel. A height of 4 feet is a minimum headroom for ordinary working. A vessel is usually launched upon a wood surface erected on each side of the keel. These surfaces are called the launching ways. They are laid after the vessel's structure has been completed upon blocks similar to the keel blocks, and at a height which is considerably less than the latter. The distance apart of the launching ways is generally equal to one-third of the breadth of the vessel. The inclination of the ways is generally a little more than that of the keel blocks unless the vessel's keel is not parallel to her waterline.

In places where there is a considerable rise and fall of the tide the launching ways extend usually to the level of the water at low tide, but in cases where the tidal rise is small it may be necessary to carry them further out than this. The bank at the end of the ways should slope suddenly into deep water, and there should be plenty of clearance to prevent the vessel's forefoot touching the ground when she slips off the end of the ways. The launching ways are made of strong oak or pitch pine square timbers, well fixed together. The breadth depends upon the launching weight of the ship. It is usual, if the berth is upon ordinary ground, to give the ways some camber or longitudinal curvature. The camber is the rise of the curve of the top of the ways above a straight line joining the ends of the ways. The amount of camber given depends upon the probable sinkage of the ways that may take place as the vessel is passing over them. If the declivity of the ways is small and no camber is given, the sinkage may be as great as to reduce the vessel's speed to a dangerous extent and probably cause her to stop. Twelve-inch camber in a length of 500 feet is not unusual.

If the berth is on rock or made of masonry or concrete, as in the Royal Dockyards, practically no sinkage can take place and camber is not necessary.

It often happens however that, on account of the formation of the ground and the extent of the water, it is desirable to have much more declivity at the lower than at the upper end of the ways; this is got by giving camber.

The following tables give particulars of the above nature for several vessels which have been successfully launched.

With a large amount of water fronting the shipyard the slope can be greater than that given by the above figures, as a vessel may be allowed to have a good velocity after launching.

For an ordinary merchant steamer above 400 feet in length, $\frac{10}{16}$ in. to the foot may be considered an average figure for the declivity of keel and launching ways. This is a declivity of 1 in 19·2. A declivity of 1 in 18·2 was used in launching the "Kaiser Wilhelm II."

In the Royal Dockyards the declivity generally used in launching battleships is $\frac{6}{8}$ in., or 1 in 16; the keel blocks being usually laid to a declivity of $\frac{5}{8}$ in., or 1 in 19·2. The launching weight of a large battleship is sometimes as much as 9000 tons.

In private yards the following figures represent the general practice for building and launching warships:—

| | Keel. | Ways. |
|---|---|---|
| Torpedo Boats, . . . . . | $\frac{15}{16}$ in. | $\frac{12}{16}$ to $\frac{15}{16}$ in. |
| Cruisers, . . . . . . | $\frac{9}{16}$ ,, | $\frac{10}{16}$ to $\frac{12}{16}$ ,, |
| Battleships, . . . . . | $\frac{1}{2}$ ,, | $\frac{9}{16}$ to $\frac{10}{16}$ ,, |

The launching cradle carries the vessel and is made to slide over the launching ways. It is built underneath the vessel after the launching ways have been laid, and extends for about ·8 of its length. It supports the vessel forward and aft by means of vertical shores called poppets, and amid-

TABLE XXXII.

| | Twin-screw War Vessel. | Screw Steamer. | Screw Steamer. | Screw Steamer. | Screw Steamer. |
|---|---|---|---|---|---|
| Moulded dimensions of vessel in ft. | 300 × 56 × 37 | 360 × 36 × 28 | 400 × 42 × 29·5 | 360 × 42·5 × 29 | 330 × 43·5 × 30·5 |
| Declivity of ways (inches/ft.) . | $\frac{5}{16}$ to $\frac{13}{16}$ | $\frac{5}{16}$ to $\frac{13}{16}$ | $\frac{5}{16}$ to $\frac{11}{16}$ | $\frac{5}{16}$ to $\frac{11}{16}$ | $\frac{5}{16}$ to $\frac{11}{16}$ |
| Camber of ways (inches) . . | 27 | 12 | 14 | 12 | 23 |
| Inclination of ship on blocks . | $\frac{9}{16}$″ | $\frac{5}{16}$″ | $\frac{5}{16}$″ | $\frac{5}{16}$″ | $\frac{9}{16}$″ |
| Water on end of ways at launch . | 8′ 7″ | 6′ 0″ | 4′ 4″ | 2′ 6″ | 3′ 9″ |
| Draught of ship forward . . | 11′ 2″ | 11′ 6″ | 7′ 0″ | 8′ 0¾″ | 6′ 6½″ |
| ,, ,, aft . . . | 16′ 6″ | 14′ 0″ | 10′ 10½″ | 10′ 5″ | 9′ 5½″ |
| ,, ,, mean . . . | 13′ 10″ | 12′ 9″ | 9′ 0¾″ | 9′ 2¾″ | 8′ 0″ |
| Displacement in tons launching . | 2850 | 2500 | 2157 | 2240 | 1660 |
| Length of standing ways . . | 345′ | 367′ | 395′ | 370′ | 348′ |
| ,, ,, sliding ways . . . | 240′ | 284′ | 330′ | 305′ | 240′ |
| Breadth of ,, ,, . . . | 44″ | 1′ 9″ | 1′ 9″ | 1′ 9″ | 1′ 10″ |
| Area ,, ,, ,, (sq. ft.) . | 1430 | 994 | 1155 | 1067 | 880 |
| $\frac{\text{Tons disp.}}{\text{Sq. ft.}}$ ,, ,, . . | 2·0 | 2·51 | 1·90 | 2·09 | 1·89 |
| Calculated tipping moment . . | 10,500 | 33,250 | 80,000 | 35,300 | 53,500 |
| $K = \frac{\text{Calculated tipping moment}}{\Delta}$ . | 3·68 | 13·3 | 37·08 | 15·77 | 32·23 |
| Length of Ship ÷ K . . . | 81·5 | 27·0 | 10·8 | 22·8 | 10·2 |

| | Screw Steamer. | Screw Steamer. Fine Lines. | Screw Steamer. | Sailing Vessel. | Paddle Steamer. |
|---|---|---|---|---|---|
| Moulded dimensions of vessel in ft. | 280 × 36 × 24 | 270 × 34 × 19 | 234 × 33 × 18 | 220 × 35 × 22 | 190 × 22 × 9 |
| Declivity of ways (inches/ft.) | $\frac{5}{16}$ to $\frac{13}{16}$ | $\frac{5}{16}$ to $\frac{13}{16}$ | $\frac{5}{16}$ to $\frac{13}{16}$ | $\frac{5}{16}$ to $\frac{13}{16}$ | $\frac{12}{16}$ to $\frac{14}{16}$ |
| Camber of ways (inches) . . | 22 | 10 | 12 | 10 | 8 |
| Inclination of ship on blocks . | $\frac{5}{16}$″ | $\frac{5}{16}$″ | $\frac{9}{16}$″ | $\frac{5}{16}$″ | $\frac{9}{16}$″ |
| Water on end of ways at launch . | 3′ 10″ | 3′ 7″ | 2′ 8″ | 4′ 5″ | 2′ 9″ |
| Draught of ship forward . . | 6′ 0″ | 5′ 7″ | 5′ 9″ | 8′ 7″ | 4′ 0″ |
| ,, ,, aft . . . | 8′ 2″ | 10′ 8″ | 9′ 0″ | 7′ 1″ | 3′ 10″ |
| ,, ,, mean . . . | 7′ 1″ | 8′ 1½″ | 7′ 4½″ | 7′ 10″ | 3′ 11″ |
| Displacement in tons launching . | 1100 | 1000 | 865 | 700 | 215 |
| Length of standing ways . . | 308′ | 300′ | 267′ | 250′ | 195′ |
| ,, ,, sliding ways . . . | 200′ | 200′ | 180′ | 170′ | 150′ |
| Breadth of ,, ,, . . . | 1′ 8″ | 1′ 9″ | 1′ 9″ | 1′ 9″ | 1′ 3″ |
| Area ,, ,, ,, (sq. ft.) . | 666 | 700 | 630 | 595 | 375 |
| $\frac{\text{Tons disp.}}{\text{Sq. ft.}}$ ,, ,, . . | 1·65 | 1·40 | 1·37 | 1·16 | ·57 |
| Calculated tipping moment . . | 39,000 | 5400 | 9700 | 12,300 | 5500 |
| $K = \frac{\text{Calculated tipping moment}}{\Delta}$ . | 35·45 | 5·4 | 11·21 | 17·57 | 25·58 |
| Length of ship ÷ K . . . . | 7·8 | 50 | 20·9 | 12·5 | 7·42 |

TABLE XXXIII.

| | Fast Liner. | Cruiser. | Cruiser. | Cruiser. | Fast Liner. |
|---|---|---|---|---|---|
| Length of vessel | 480 | 220 | 225 | 330 | 525 |
| Declivity of ways (inches) | $\frac{8}{16}$ | $\frac{9}{16}$ | $\frac{9}{16}$ | $\frac{9}{16}$ | ·47 |
| Inclination of ship on blocks | $\frac{8}{16}$ | $\frac{8}{16}$ | $\frac{8}{16}$ | $\frac{9}{16}$ | ·47 |
| Water on end of ways at launch | 4′ 3″ | 4′ 8″ | 4′ 9″ | 5′ 6″ | 3′ 9″ |
| Draught of ship forward | 11′ 2″ | .. | 7′ 0″ | 8′ 4″ | 10′ 0″ |
| ,, ,, aft | 15′ 6½″ | .. | 11′ 10″ | 14′ 6″ | 17′ 0″ |
| ,, ,, mean | 13′ 4¼″ | .. | 9′ 5″ | 11′ 5″ | 13′6″ |
| Displacement in tons launching | 3982 | 1050 | E. & B. aboard 1000 | 2280 | 6420 |
| Length of standing ways | 495′ | 314′ 6″ | 295′ 6″ | 417′ 6″ | 570′ |
| ,, ,, sliding ways | 361′ | 194′ 6″ | 193′ 9″ | 252′ 0″ | 423′ |
| Breadth of ,, ,, | 24″ | 20″ | 20″ | 36″ | 36″ |
| Area ,, ,, ,, in sq. ft. | 1440 | 649 | 660 | 1512 | 2538 |
| $\frac{\text{Tons disp.}}{\text{Sq. ft.}}$ ,, ,, | 2·76 | 1·62 | 1·5 | 1·51 | 2·53 |
| Time of leaving ways (secs.) | .. | 39 | 51 | 110 | 68 |
| Maximum velocity in ft. per sec. | .. | 13·5 | about 10 to 12 | .. | 19·7 |
| Velocity on leaving standing ways | .. | 12·0 | .. | 18·3 | 7 3 |
| Calculated tipping moment | 50,000 | 20,000 | 12,000 | 37,000 | 194,000 |
| $K = \frac{\text{Calculated tipping moment}}{\Delta}$ | 12·5 | 19·6 | 12 | 16·2 | 30·2 |
| Length of ship ÷ K | 37·4 | 11·2 | 18·75 | 18·2 | 17·4 |

TABLE XXXIV.

| | Battleships. | | | | | | |
|---|---|---|---|---|---|---|---|
| Length of vessel | 330 | 330 | 330 | 360 | 430 | 435 | 435 |
| Declivity of ways (inches) | $\frac{13}{16}$ | $\frac{13}{16}$ | $\frac{43}{64}$ | $\frac{13}{16}$ | $\frac{11}{16}$ | $\frac{11}{16}$ | $\frac{3}{4}$ |
| Inclination of ship on blocks | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{15}{16}$″ | $\frac{11}{16}$″ | $\frac{5}{8}$″ |
| Water on end of ways at launch | 16½″ | 16½″ | 17″ | 17″ | 9′ 10″ | 8′ | 6′ 6″ |
| Draught of ship forward | 14′ 10″ | 14′ 10″ | 14′ 4½″ | 15′ 3″ | 10′ 2½″ | 11′ 0″ | 8′ 6″ |
| ,, ,, aft | 18′ 3″ | 18′ 3″ | 18′ 5½″ | 16′ 9″ | 12′ 0½″ | 10′ 9″ | 13′ 5″ |
| ,, ,, mean | 16′ 6½″ | 16′ 6½″ | 16′ 5″ | 16′ 0″ | 11′ 1½″ | 10′ 10½″ | 10′ 11½″ |
| Displacement in tons launching | 4926 | 4926 | 6100 | 7500 | 6540 | 7344 | 5879 |
| Length of standing ways | 430′ | 430′ | 445′ | 470′ | 485′ 6″ | 562′ 3″ | 488′ 4″ |
| ,, ,, sliding ways | 270′ | 270′ | 278½′ | 307′ | 380′ | 364′ | 402′ |
| Breadth of ,, ,, | 3¾′ | 3¾′ | 3′ 11″ | 4·25′ | 4′ 2″ | 3′ 6″ | 3′ 6″ |
| Area ,, ,, ,, in sq. ft. | 2025 | 2025 | 2180 | 2609·5 | 3000 | 2700 | 2556 |
| $\frac{\text{Tons disp.}}{\text{Sq. ft.}}$ ,, ,, | 2·43 | 2·43 | 2·8 | 2·86 | 2·18 | 2·72 | 2·30 |

ships by blocks which are fitted vertically between the shell of the vessel and the sliding ways (fig. 140). At the fore end of the sliding ways in large vessels a forging with deep flanges is fitted to the shell at each side to take the fore poppets, and in large single-screw vessels a similar arrangement is fitted to the shell for the after poppets. In twin-screw vessels the bossing for the propeller shafting affords a convenient place for fitting the after poppets. The top ends of the poppets are fitted closely to the shell and to the forging, and the bottom ends are wedged up on the sliding ways.

In very heavy ships the poppets towards the ends of the ship are stiffened by longitudinal angle bars, and they are further tied from side to side by angle bars and chains which extend underneath the vessel.

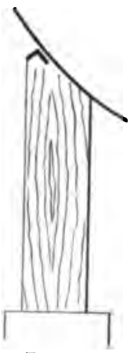


Fig. 140.

On the day of the launch most of the keel blocks are removed from underneath the vessel. The weight of the vessel then rests on the cradle which is borne by the launching ways. The vessel is prevented from moving by bilge blocks, wedged-up shores, and dog shores, which latter keep the sliding and launching ways together. These dog shores are really triggers, and just before the launch the bilge blocks and shores are knocked away so that the vessel is kept from moving only by the triggers, which are finally released simultaneously by some mechanical arrangement, and the vessel slides down the ways into the water. A few remaining keel blocks forward are tripped by the ship in going off.

Plate XI. is a diagram of the launching arrangements for a large battleship. The forging attached to the shell plating at the top of the forward poppets is clearly shown.

This arrangement is also shown in the sectional view. There are two sets of poppets forward—the set furthest forward being inclined towards the ship's side. This is sometimes necessary in fine-ended ships. It will be seen that the forging is horizontal at this stage. The diagram also shows the fore and aft ties and the transverse ties for the poppet.

The photographs give a good idea of the launching arrangement. Figs. 141 and 142 are illustrations taken from the launching arrangement of the Cunard liner, "Carmania." Fig. 143 shows the arrangement for releasing the dog shores for another vessel.

The most critical times during the launch, so far as the straining of the structure is concerned, may be—

(1) After the centre of gravity of the vessel has passed over the ends of the ways, because there is little support aft, and the tendency of the ship may be to turn about the after end of the ways and so concentrate the weight at that point.

(2) When the buoyancy aft is sufficient to lift the vessel and cause her to turn about the fore end of the cradle, there is then a long length of structure completely unsupported, and a great pressure is exerted over a short length at the fore end of the cradle and launching ways.

Let us consider the forces acting on the vessel at any instant after the stage of motion stated in (1) has been reached, and before she is entirely waterborne (see fig. 144).

The statical forces may be compounded into three resultant forces,—

(1) An upward force near the stern of the vessel, due to the buoyancy of the part of the vessel and cradle immersed.

(2) A downward force equal to the weight of the vessel and cradle acting through the centre of gravity of the vessel and cradle.

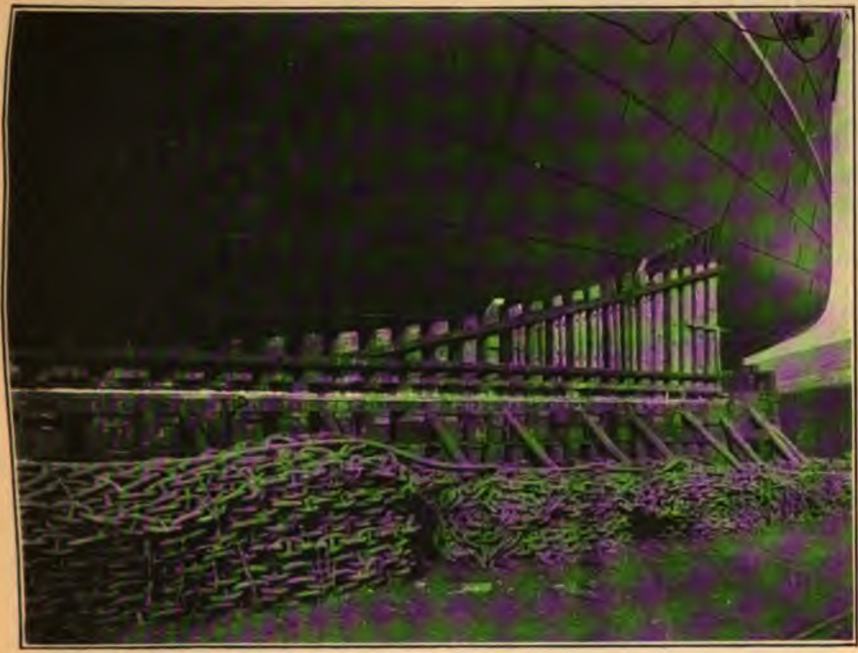


FIG. 141.

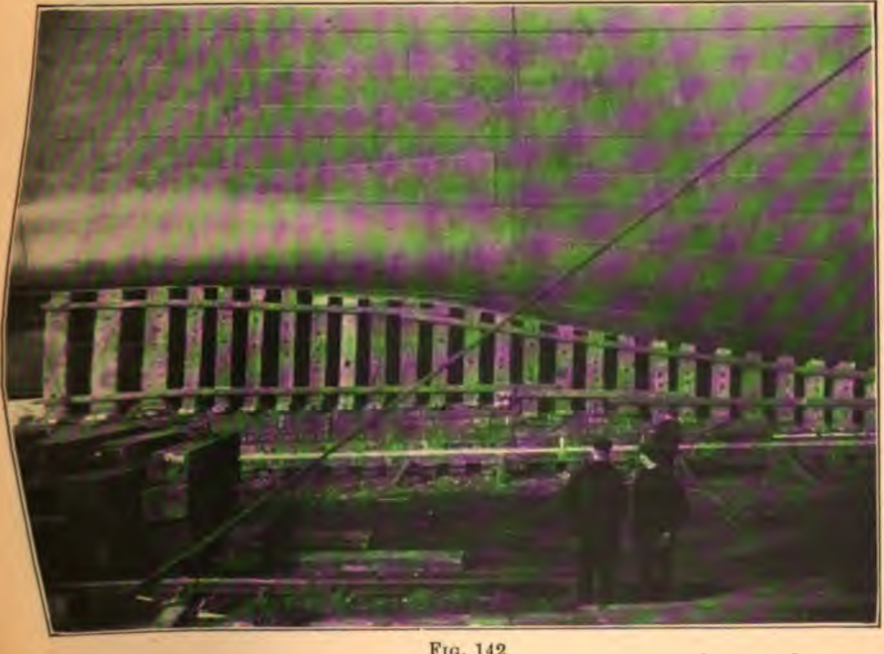


FIG. 142.

(3) An upward force, representing the support which the launching ways give to the ship.

The resultant of these three forces will be a zero force and a couple. If we take moments of these forces about the after ends of the ways we shall have a couple which, if it tends to turn the ship by lowering the after end and raising the fore end, is called a tipping moment. In other words, if the moment of the weight force is greater than the moment of buoyancy force about the after end of the ways, then the vessel tends to turn about the end of the ways so that the bow tends to tip up. The third force will be at the after end of the ways and its

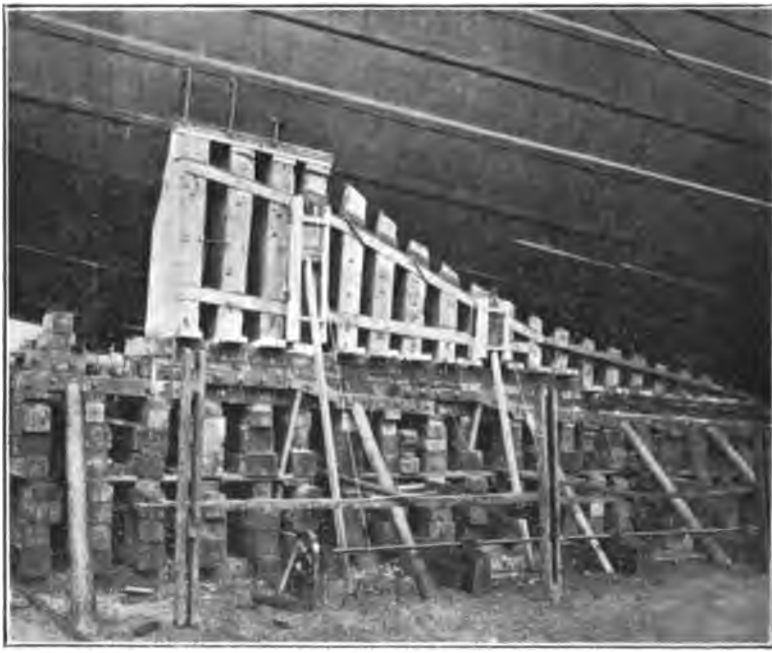


FIG. 143.

moment will be zero. If, however, the moment tends to turn the vessel in the other direction there will be no tendency to tip, and the force (3) will not be at the after end of the ways. The vessel may turn about the fore end of the ways, and will do so if the moment of the buoyancy is greater than the moment of the weight of the vessel about this point. This moment is called the lifting moment. The force (3) will act through the fore end of the ways and its moment will be zero.

In a launching calculation these moments are calculated at definite stages of the vessel's motion down the ways. The following curves are plotted on a base line representing distances travelled down the ways:—

(1) Buoyancy of the part of the vessel immersed.
(2) Position of the C.B. corresponding to the above.

(3) Moment of the buoyancy about the fore end of sliding ways.
(4) Moment of the weight about the fore end of sliding ways.
(5) Moment of the buoyancy about the after end of the fixed ways.
(6) Moment of the weight about the after end of the fixed ways.

The curves so plotted, shown in Plate XII., enable one to determine easily the values of the tipping or lifting moments at any stage of the vessel's progress. If a tipping moment exists at any stage, the ordinate of the curve No. 5 will be less than the ordinate of the curve No. 6 at that stage; and if a lifting moment exists, the ordinate of curve No. 3 will be greater than the

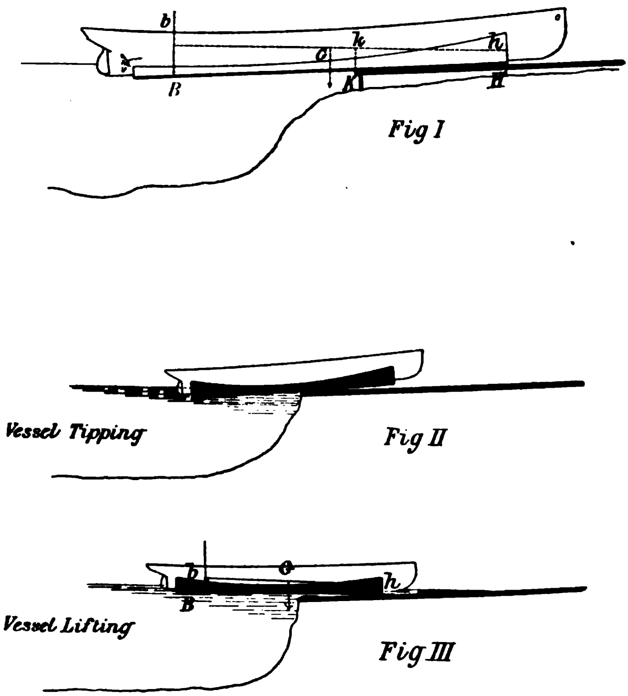



FIG. 144.

corresponding ordinate of curve No. 4. Curves showing these differences are given in the plate. No account is taken in these curves of the dynamical effect of either the motion down the ship or the angular motion about the ends of the ways.

It is desirable that the tipping moment should not be excessive. A lifting moment must occur at some stage before the vessel is entirely clear of the ways, but it is desirable that it should not occur at too early a stage, or the pressure on the fore end of the ways will be too great.

The means of judging whether either of these moments is excessive or not is afforded by the launching diagram.

Fig. 144 I represents a vessel being launched, and at a stage soon after the centre of gravity G has passed the ends of the ways K.

Let B be the position of the centre of buoyancy of the part immersed.

H is the fore end of the sliding ways.

Draw verticals at B, K, G, and H.

Let *b*G, G*k*, and *kh* be the horizontal distances between these perpendiculars.

Let W = weight of vessel.

„ Δ = buoyancy of the part immersed.

Then at this stage of the vessel's progress we have—

(1) Buoyancy = Δ.
(2) Position of B = *bk* from after end of ways.
(3) Moment of buoyancy about H = Δ.*bh*.
(4) „ weight of vessel about H = W.G*h*.
(5) „ buoyancy „ K = Δ.*bk*.
(6) „ weight „ „ = W.G*k*.

The resultant moment about K is Δ*bk* − W.G*k*; and if W.G*k* is greater than Δ*bk*, then tipping tends to take place, and the above expression gives the tipping moment. Fig. 144 II illustrates tipping.

The resultant moment about H is Δ.*bh* − W.G*h*; and if Δ.*bh* is greater than W.G*h* lifting tends to take place, and (Δ.*bh* − W.G*h*) gives the value of the lifting moment. This is illustrated by fig. 144 III.

A tipping or a lifting moment may exist, and yet little or no movement may take place owing to the momentum of the vessel down the ways, and to the fact that before the unbalanced moment has time to turn the vessel appreciably it has ceased to be a tipping or a lifting moment.

As has already been noted, it is desirable to avoid an excessive tipping moment. This can be done by altering the fore and aft position of G. If the ballast tanks be filled forward the C.G. can be brought forward, and therefore the leverage of G about the ends of ways at any stage will be less. Another method that can be employed to lessen tipping is to box in the stern so as to increase the buoyancy aft. This can be conveniently done on a twin-screw vessel, as the boxing can be built round the bossing. It is only steamers with fine after bodies that are likely to have tipping moments. If the lifting moment is excessive, this may be reduced by filling the after tanks.

Referring to the launching diagram, Plate XII., it will be seen that curve No. 6 is below the line which represents No. 5. The difference of the ordinates at this stage (such as T T) gives the moment against tipping; and if the curve (5) crossed the line (6), the maximum tipping moment can be found.

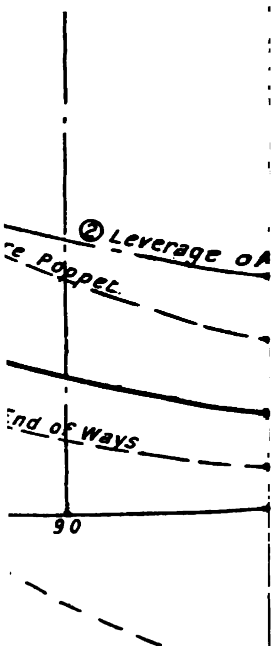
The maximum tipping moment is often related to the product W.L by the following factor C, called the tipping factor.

$$C = \frac{\text{W.L}}{\text{Maximum tipping moment}}.$$

The tipping factor equals Length of Ship ÷ K, values of which are given in Tables XXXII. and XXXIII.

Vessels have been successfully launched with tipping factors ranging from 18 to 7,—*i.e.* the tipping moment has been $\frac{1}{18}$th to $\frac{1}{7}$th of the weight multiplied by the length of the ship.

**Construction of a Launching Diagram** (see Plate XII.).—The base line of the diagram is made to represent the distance travelled by the vessel


② Leverage o
Poppet.
of Ways
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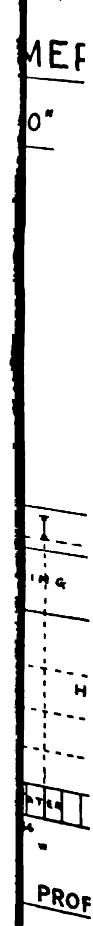

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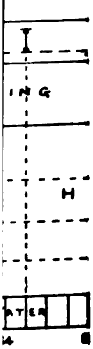


PROFI

down the ways. We may take as their zero or starting-point the stage when the after perpendicular of the vessel is at the end of the way.

A diagram is made showing a profile view of the vessel and the ways. The stages for which the calculations are to be made are decided upon, say, every 20 ft. down. At the distances 20 ft., 40 ft., 60 ft., 80 ft., etc. down the ways, we get from the above diagram the corresponding position of the

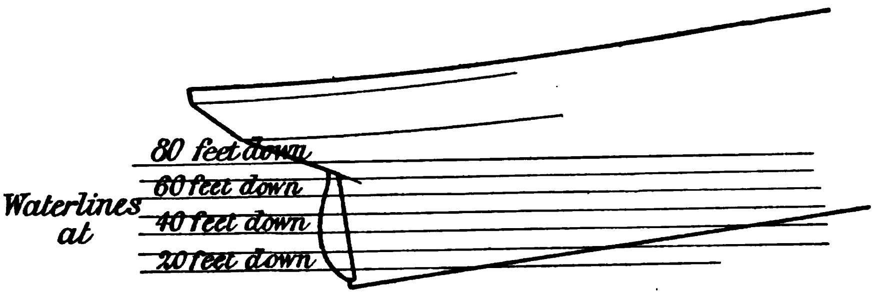


FIG. 145.

water-level (see fig. 145). From these we get a series of curves, each showing the cross-sectional area of the part of the ship that is immersed. There will be one of these curves for each distance down. The waterlines can be marked on the profile as shown.

By taking the immersed area of each cross-section for any one distance down, we get the curve of buoyancy or support for that particular distance.

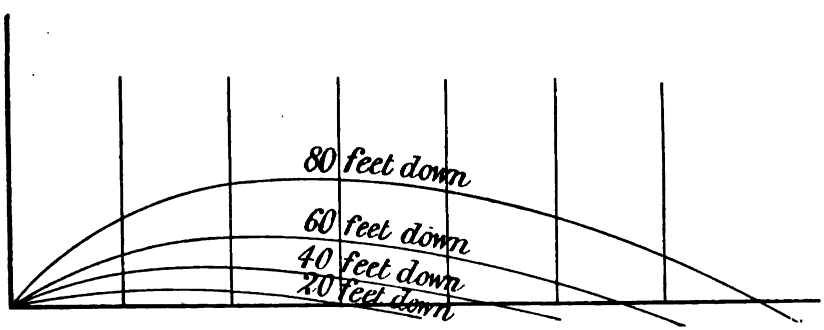


FIG. 146.

A series of curves are obtained in this manner, as shown by fig. 146.

From these curves we calculate—

(1) The total area of each curve which gives values of Δ, the buoyancy or support.

(2) The position of the C.G. of each curve, which gives the position of C.B. of the volume immersed.

The positions of B so found are marked on the base line.

A list is made of the values of—

(1) $\Delta$.
(2) Position of B or $bk$.
(3) $\Delta.bh$.
(4) $W.Gh$.
(5) $\Delta.bk$.
(6) $W.Gk$.

The corresponding values for each position of vessel down the ways are set up as ordinates, and curves of support and moments about fore and aft ends of the curves are obtained as in Plate XII.

It is convenient to tabulate the results as follows:—

TABLE XXXV.

| Distance that Vessel has travelled down ways, in feet. | Buoyancy. | Leverage about | | Moment of Buoyancy about | | Moment of Weight about | | Tipping Moment. | Lifting Moment. |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | H | K | H | K | H | K | | |
| 20 | $\Delta_1$ | $l_1$ | $m_1$ | $\Delta_1 l_1$ | $\Delta_1 m_1$ | $Wp$ | ... | ... | ... |
| 40 | $\Delta_2$ | $l_2$ | $m_2$ | $\Delta_2 l_2$ | $\Delta_2 m_2$ | ,, | ... | ... | ... |
| 60 | $\Delta_3$ | $l_3$ | $m_3$ | $\Delta_3 l_3$ | $\Delta_3 m_3$ | ,, | ... | ... | ... |
| 80 | $\Delta_4$ | $l_4$ | $m_4$ | $\Delta_4 l_4$ | $\Delta_4 m_4$ | ,, | ... | ... | ... |
| 100 | $\Delta_5$ | $l_5$ | $m_5$ | $\Delta_5 l_5$ | $\Delta_5 m_5$ | ,, | $Wk_1$ | ... | ... |
| 120 | $\Delta_6$ | $l_6$ | $m_6$ | $\Delta_6 l_6$ | $\Delta_6 m_6$ | ,, | $Wk_2$ | ... | ... |
| 140 | $\Delta_7$ | $l_7$ | $m_7$ | $\Delta_7 l_7$ | $\Delta_7 m_7$ | ,, | $Wk_3$ | ... | ... |
| 160 | $\Delta_8$ | $l_8$ | $m_8$ | $\Delta_8 l_8$ | $\Delta_8 m_8$ | ,, | $Wk_4$ | ... | ... |
| Vessel waterborne | W | ... | ... | ... | ... | ... | ... | ... | ... |

*To determine the breadth of ways:*—

Let $W$ = probable launching weight.
$l$ = length of cradle or of sliding ways.
$b$ = required breadth of each way.

Area of sliding ways $= 2b.l$.

$$\therefore \text{ Average pressure per square foot on ways} = \frac{W}{2.b.l}.$$

The area of ways provided should be such that the pressure per square foot should not be more than about 2·5 tons.

$$\text{Hence} \qquad 2{\cdot}5 = \frac{W}{2.b.l}.$$

$$\therefore \qquad b = \frac{W}{5l}.$$

If $W = 7000$ and $l = 350$ for a large battleship, then $b = \dfrac{7000}{5 \times 350} = 4$ feet.

Sometimes a curve giving the average pressure per square foot on the ways is drawn.

Suppose the vessel is at such a stage that the length of sliding ways bearing on the standing ways is $l$.

Then pressure per square foot $= \frac{W - \Delta}{2.b.l}$.

Hence for each distance down the ways we may find the average pressure on the ways. These can be shown in a curve.

For small ships the value of $b$ given by the foregoing formula would be inconveniently small, but there is no practical objection to the use of a greater breadth. Ways are usually made up of sawn logs of about 1 foot wide, and consequently any variation in breadth of ways must be made by increasing or decreasing the number of logs. The breadth can thus generally only vary by about 1 foot.

The average pressure as given by the above formula may not form an indication as to the maximum pressure that may come upon any part of the bearing structure. In order to determine the variation of pressure along the ways at any instant during the passage of the vessel down the ways, assumptions have to be made regarding the elasticity of the ways. As the questions connected with the variation of pressure of a vessel resting on blocks are of a similar nature, fuller consideration will be made of this problem in Chapter XXVII. (Part III.).

# CHAPTER XVIII.

## APPLICATION OF THE INTEGRAPH TO SHIP CALCULATIONS.*

THE integraph has been in existence for about twenty years.

The majority of ship calculations are in the nature of an integration. The integration is ordinarily performed by the aid of rules such as Simpson's, or such instruments as the integrator and the planimeter. These instruments, however, only give a definite integral for one complete operation. For instance, the planimeter or the integrator, after having been towed round the boundary of a given area, records only one result—the area, moment, or moment of inertia of the whole curve which has been traced over. The integraph, on the other hand, traces out graphically the integral of the curve, point by point, from the beginning to the end of the operation. This graphic integral curve can only be obtained by a series of operations of the integrator or the planimeter, and by setting off the readings obtained at the end of each operation as ordinates to form the curve.

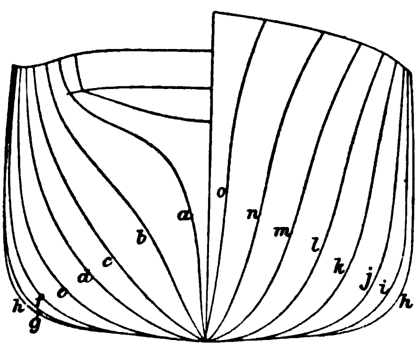

FIG. 147.

The machine, in its latest type, is illustrated in simple diagrammatic form in fig. 112.

This type will integrate in one operation a curve whose maximum ordinate on either side of the axis does not exceed 10 inches, and it will integrate an area not exceeding 120 square inches.

**Application to Ship Calculations.**—The ordinary ship calculations of displacement and position of centre of buoyancy are much simplified by making use of Tchebycheff's rules for the spacing of ordinates. In the examples which have been worked out here, fig. 147 shows a body plan with sections spaced to the Tchebycheff Rule for three ordinates; and in the body plan (fig. 148) the sections are spaced to the rule for two ordinates.

**Curves of Integrated Sections.**—The machine is set so that it runs along the vertical middle line of the body plan as axis. The sections are then

* See *Trans. I.N.A.*, 1907, for a very full development of this subject by Mr J. G. Johnstone, B.Sc.

separately traced over with the pointer, and corresponding integrated sections are thereby quickly and conveniently obtained (fig. 149).

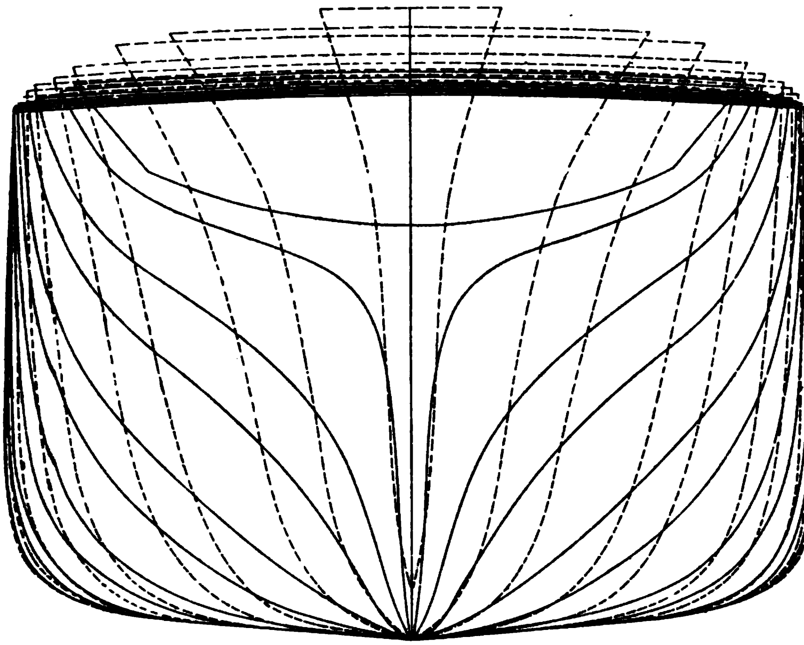

FIG. 148.—Body Plan.

Comparing the body plans of ordinary and integrated sections, corresponding points are at the same height above the base line through the keel. Any

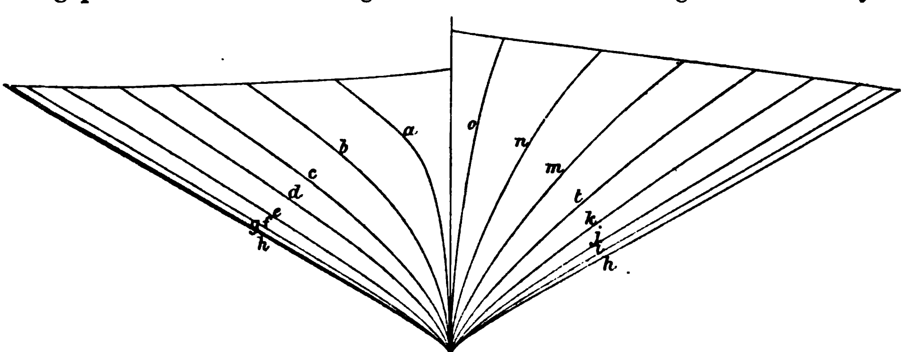


FIG. 149.

ordinate of an integrated section gives the area of the corresponding section up to that ordinate. Therefore, if the ordinates of the integrated sections at any waterline be added, the sum is a function of the displacement. This enables a displacement curve to be set off.

**Displacement Curve.**—There is a quicker method than that described in the preceding paragraph for obtaining a displacement curve. If the ordinates at any waterline of the sections in the body plan, fig. 148, are added, the sum measures the area of the waterplane. It is easy to set off a curve of "areas of waterplanes," and this, when integrated, gives the displacement curve.

In fig. 150, K A is a curve of waterplane areas, and K D is the integral curve which is the displacement curve.

**Centre of Buoyancy Curve.**—The displacement curve, when integrated, gives a moment of displacement curve. (See Chap. XII.)

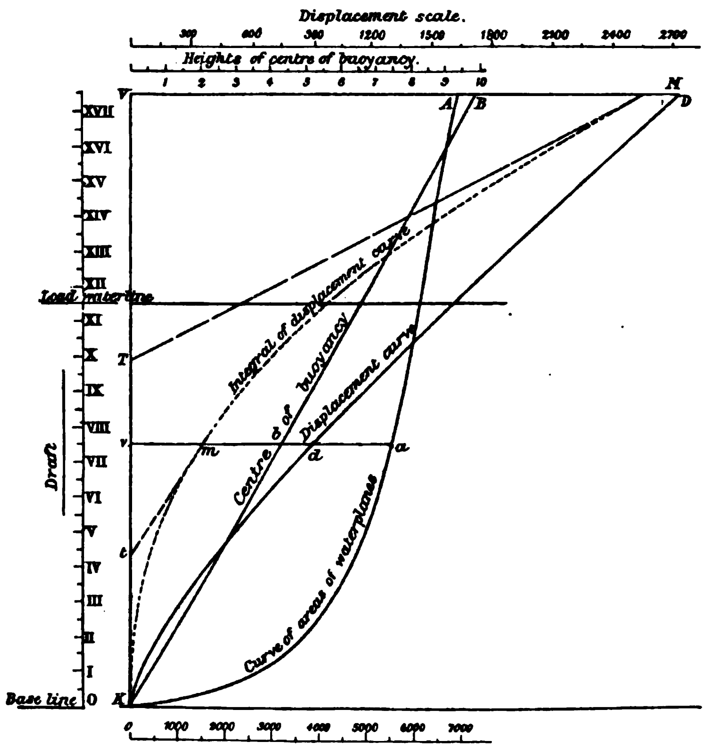


FIG. 150.

The ratio of the corresponding ordinates of these two curves fixes the vertical position of the centre of buoyancy. Thus, in fig. 150, K M is the moment of displacement curve, the displacement curve being K D. The corresponding ordinates at the 7 ft. 6 in. waterline are $vm$ and $vd$ respectively. The distance of the centre of buoyancy $t$ for the 7 ft. 6 in. waterline is given by $vt$ in. $= \frac{vm}{vd} \times n$ in. for scale. Similarly, at draught 17 ft. 6 in. the centre of buoyancy T is given by V T in. $= \frac{\text{VM}}{\text{VD}} \times n$ in. (In this case $n = 6$.)

If the heights K $t$ and K T are plotted in terms of the draughts, a centre of buoyancy curve is the result, which, in this case, is shown by K $b$ B.

Many calculations can be simplified by using the body plan of integrated sections. For instance, the displacement at any given trim can be easily obtained. The ordinates at any waterline of the integrated sections, set off in terms of the length, give a curve of sectional areas. These curves, when integrated twice along the length, will give the longitudinal positions of the centres of buoyancy in a manner similar to that described for obtaining the vertical centre of buoyancy. Curves of waterplanes, when integrated twice along the length, give the positions of the centres of gravity of waterplanes.

The integrated midship section is a curve of midship areas, and from this curve and the displacement curve the prismatic coefficient curve can be obtained.

The longitudinal BM $= \frac{I}{V}$, where I is the moment of inertia of the waterplane about a transverse axis through its C.G. This I can be found by the machine by performing the third integration of the waterplane along its length.

The transverse BM $= \frac{I}{V}$. The value of I, the transverse moment of inertia of the waterplane, can be obtained by the machine in three operations, but the waterplane first requires to be plotted to a convenient scale. It would seem that the ordinary arithmetical method of calculating I is slightly quicker in this special case.

The following curves are usually obtained by the method framed in the displacement sheets. They can be readily obtained by using the integraph as already described:—

Displacement curve.
Block coefficient curve.
Waterline areas or tons per inch.
Vertical centres of buoyancy.
Longitudinal centres of buoyancy.
Locus of C.G.'s of waterplanes.
Moment to change trim 1 inch.
Midship areas and coefficients.
Prismatic coefficient curve.
Longitudinal metacentres.

And for transverse metacentres the calculation is simplified.

The scales used in the integrations for obtaining the above curves are fixed by the scope of the machine. It is found that the curves can be worked very conveniently to standardised scales, so that the results can be traced on to the 10-in. standardised diagram, Plate VI.

# PART III.

## STRENGTH OF SHIPS.

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## CHAPTER XIX.

### STRAINING ACTIONS DUE TO UNEQUAL LONGITUDINAL DISTRIBUTION OF WEIGHT AND BUOYANCY.

When a ship is floating at rest in still water the weight of the ship is equal to the weight of the volume of water which it displaces, and the centre of gravity of the ship is in the same vertical line as the centre of gravity of the volume of water displaced.

This is the condition for equilibrium of any floating body; but when we apply this condition to the component parts of an actual ship, we shall generally find that the weight of the volume displaced is not equal to the weight of the part of the ship vertically over the boundary of the displaced volume. The aggregate weight of the ship is equal to the aggregate weight of the volume displaced, but parts of the ship are heavier than the weight of the volume displaced within its limits, and if the component parts of the ship were not united by some material capable of developing resistance to change of form, the equality of weight of ship and weight of water displaced would be established by a rise or fall of that part of the ship. It is this resistance to the rise or fall of the component parts of the structure which is called the strength of a ship. This resistance or strength may be determined by first considering the forces which develop it.

First, let us consider the support or buoyancy which in the aggregate equals the weight of the displaced water, and is also equal to the weight of the ship. If we have a section of a ship between two transverse vertical planes very close together, the volume of the displaced water is equal to the area of the section midway between the two planes multiplied by the distance between the planes.

Suppose fig. 151 to represent two sections close together, and A B C D to be an element of the surface upon which the water exerts a total pressure P. If this pressure be resolved in the vertical and in two horizontal directions perpendicular and parallel to the vertical longitudinal plane of the ship, these forces may be called V, H, and F respectively. All the forces H and F throughout the ship will balance each other, and the forces V will balance the weight of the vessel. The H forces will tend to compress the section transversely, while the F forces will compress it longitudinally. If the V forces in the section under consideration do not balance the weights, there will be a tendency to move the section, which will be resisted by the strength of the material of the ship.

It will simplify the consideration of this question to take a section of indefinitely small thickness, so that H is the resolved part of P in a transverse plane (fig. 152); V will then be P resolved in a vertical line, and will be $= w.h. \cos\theta. \cos\alpha$, where $\theta$ is the inclination to the vertical of the normal in which P acts, and $\alpha$ is its inclination to the transverse plane; $w$ is the weight of a cubic unit of the liquid and $h$ is the depth of C below the surface. The element of area in which this pressure acts when multiplied by $\cos\theta$.

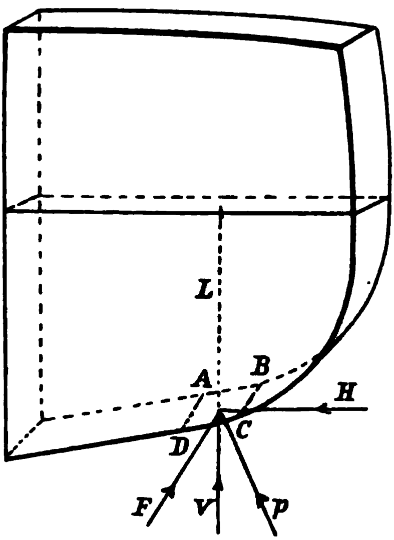



FIG. 151.

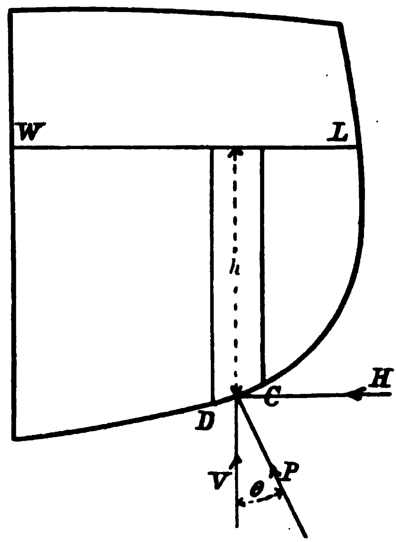



FIG. 152.

$\cos\alpha$ is the projection on the waterplane of that element. Hence $h \cos\theta. \cos\alpha$ is the volume of the layer between the waterplane and the outer bottom. Integrating this over the whole section we get the total vertical force for that section.

Hence the curve of the form of the section is a curve of vertical pressures with the waterline as base, and the aggregate vertical force or the buoyancy is represented by the area of the section. The buoyancy per foot of length, therefore, of the ship is equal to the area of the section multiplied by 1 foot, and by the weight of a cubic unit of the water; hence a curve of cross-sectional areas is a curve of supporting forces per foot of length. An adjustment for the scale is necessary which will turn area of section into weight of volume of water per foot of length of ship.

If we can graphically represent the weight per foot of length upon the same scale, we shall have, for every point in the ship's length, a representation of the longitudinal distribution of the weight and buoyancy. This is shown in fig. 153. Where the weight exceeds the buoyancy there is an unbalanced downward force, and *vice versâ*.

This unbalanced force develops in the structure a resistance to separation equal and opposite to it.

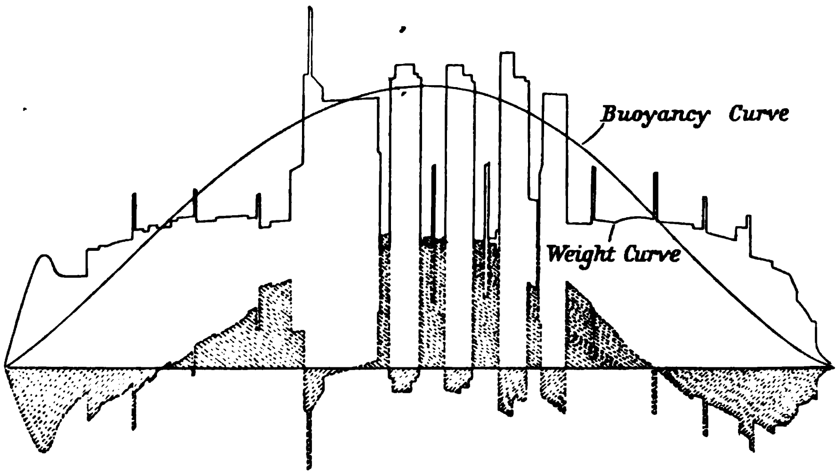



FIG. 153.

This force is called the load upon the section; and if we make a new curve, as in fig. 154, whose ordinate is the difference between the weight and buoyancy curve, we shall get a curve of loads. The vessel may now be considered as acted on by forces directly proportioned to the ordinates of this curve, and so the forces are determined which tend to cause separation of

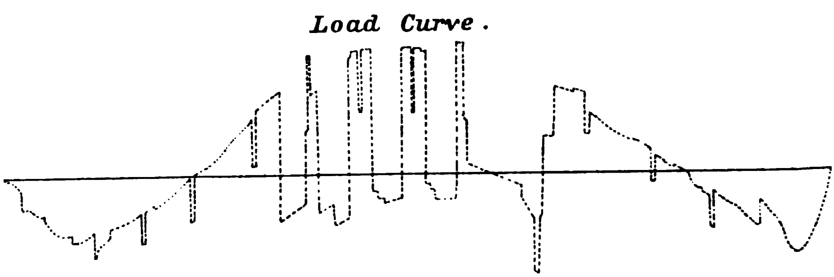



FIG. 154.

contiguous parts, and also the consequent stresses brought upon the structure by these forces may be determined.

The otherwise unbalanced forces are the deforming forces acting on the structure, and the resistances set up by these forces are called the stresses developed in the structure.

Greater stresses are generally experienced by a ship among waves than in still water; and in order to determine these, we must be acquainted with the laws relating to the formation of waves, and to the variation of pressure throughout the wave structure. These pressures are called wave pressures. The stresses will also vary on account of the motion of the vessel as it is passing through a series of waves. In the case of the ship floating in still water the calculation of the forces is simple and exact. When the ship is among waves, however, we have to assume certain conditions for equilibrium, or make our calculation upon what we suppose to be the worst condition in which the ship can be placed.

We cannot arrive at an exact determination of the stresses due to pitching or heavy rolling, but by considering these motions upon certain limited assumptions we are able to state generally the effect of each of these motions on the stresses.

When the vessel tends to bend longitudinally, that is, in planes parallel to the vertical longitudinal middle-line plane, stresses are produced in transverse sections, which we call longitudinal stresses. This tendency to bend is greatest in the longitudinal direction, because the resistance to it can only come from the breadth and depth of the ship, while the forces causing the bending depend on the length of the ship and, therefore, have greater leverages than in any other directions. Consequently the longitudinal stresses are generally the greatest.

Transverse stresses are those which are developed in the structure by a tendency to change form transversely. The same considerations of inequality

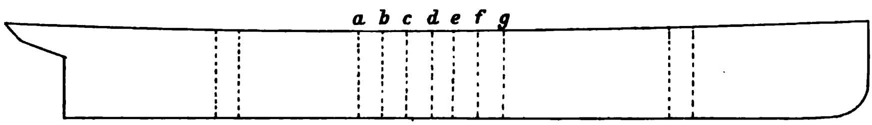



Fig. 155.

of weight and buoyancy for a section between two parallel transverse sections will lead to a transverse curve of loads similar to the longitudinal one. The stresses so produced are generally small compared to longitudinal stresses, though large ones may be caused by rolling or by the loading of the ship. Other transverse stresses may also be caused by the tendency of the longitudinal stresses to alter the form in a transverse direction.

We shall consider first the longitudinal stresses, and later shall deal with the transverse stresses.

Let us briefly examine the distribution of the forces that are likely to act on a ship's structure. First, when she is at rest.

The figure 155 represents the profile of a ship.

Consider the ship divided by vertical watertight sectional planes into layers, as shown at *abcde*. One of these layers may be full of coal and have a bulkhead or other heavy part of the structure in it, while the adjacent layer may be through an open space with very little in it, such as would be the case in a transverse stokehold. The buoyancy forces of these two layers are nearly equal, but their weights are very different. In the case of the layer through the coal bunker the weight will probably be in excess of the buoyancy, and the layer will sink until the buoyancy becomes equal to the weight. The weight of the other layer will be less than the buoyancy force, and it will rise until the equality of weight and buoyancy be established. In an actual ship this change of relative position of the sections is prevented by the resistance of the material of the structure to relative movement. The force causing this tendency to change of position at any section is the difference between weight and buoyancy in adjacent sections. Hence it is necessary to know the weight per section throughout the ship, and also the buoyancy.

We must first prepare the two curves of weight and buoyancy. For these we must know the construction and the shape of the ship, all about her weights, and the disposition of those weights. In order to draw the curve of weights exactly, the weight per frame space is calculated at intervals of the structure; these intervals are chosen where plans of the sections showing structural parts are obtainable, and if not obtainable they must be made. A base line is drawn representing the length of the ship to scale, and at the corresponding points the weights per foot of structure are set off as ordinates. In calculating the weight per frame space (which need only be done for the half section) the weight of one complete frame and all its component parts and attachments is calculated, and also the weight of one beam with its transverse connections to frame, the weight of deck plating, shell plating, casings, stringer angles and bulbs, etc., wood deck, ceiling, etc., for a length of one frame space. The total weight divided by the distance between frames gives the weight per foot, which is then set up as an ordinate at the centre of the frame space. Discontinuous parts of the structure, such as web frames, bulkheads, heavy castings or forgings of the stem and stern post, propellers, and brackets, are added at their proper places in the curve. It should be remembered that all continuous weights should be first plotted, including such items as cargo when of uniform density, before adding the discontinuous items of weight. A list of weights in the ship for the condition under consideration should be made out. This usually includes such items as—engines; shafting; coal; stores; armament and armour; boilers; propellers; cargo; water ballast; and passengers. The disposition of these weights longitudinally is taken from the longitudinal sectional drawing of the ship. These weights are added to the curve at their proper places. Cargo and coal are assumed to be stored in proportion to the volume of the occupied holds or bunkers.

Fig. 156 shows a weight curve made up in detail for the vessel shown in fig. 155. A simpler method of setting up the hull part of the weight curve is sometimes followed when the strength of a proposed vessel is required in the early stages of design, and sometimes in completed ships when rapid comparative results only are required. It will be described later.

The buoyancy forces can be represented by a curve. It has been shown that the cross sections of a ship are curves of pressure on the bottom of the ship, and that the areas of the cross sections up to the waterline represent the supporting forces on the ship per foot of length. In finding, therefore, the buoyancy per foot of length at any section, we calculate the area of the section (in square feet), divide by 35, and so get the buoyancy in tons per foot of length in salt water. Plotting successive values of the above along the base line we get the curve of buoyancy, which is a fair curve.

The area of the weight curve, since the ordinate gives weight per foot of length, represents the total weight of the ship. The area of the buoyancy curve will likewise give the total buoyancy. But the total weight equals the total buoyancy or displacement of the ship; therefore the areas of the two curves are equal.

Another condition of equilibrium is that, as the centre of gravity of the weight and the centre of gravity of the volume of water displaced are in the

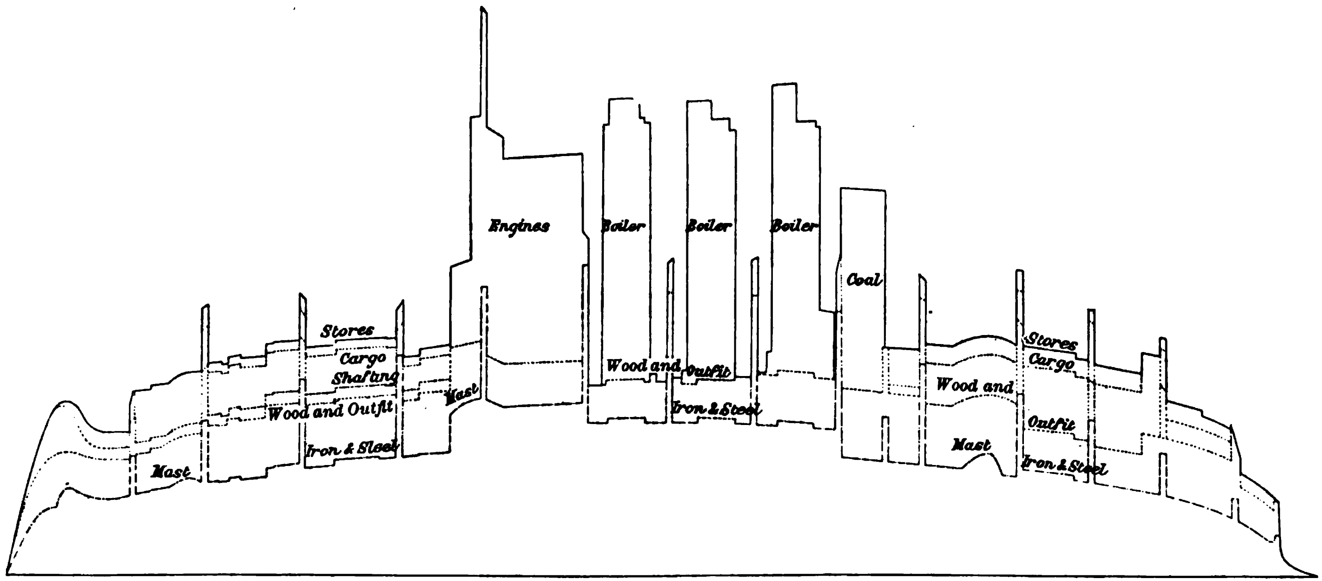



FIG. 156.

*Weights.*

| | | |
|---|---|---|
| Iron and steel . . . | 5,466 | tons. |
| Wood and outfit . . . | 2,200 | ,, |
| Machinery . . . . | 2,600 | ,, |
| Cargo . . . . . | 1,144 | ,, |
| Stores . . . . . | 220 | ,, |
| Coal . . . . . | 320 | ,, |
| | 11,950 | ,, |

| | | |
|---|---|---|
| Coal consumed | = 2330 | tons. |
| Stores ,, | = 260 | ,, |
| Water ,, | = 191 | ,, |
| | 2781 | ,, |

same vertical line, therefore the centre of gravity of the area of the weight curve must be in the same longitudinal position as the centre of gravity of the area of the buoyancy curve, since each curve represents graphically the distribution of the weight and the buoyancy forces.

Fig. 153 shows the weight and buoyancy curves of a vessel about 500 feet long. The irregularity of the weight curve compared with the buoyancy curve is its chief characteristic. In some places the ordinate of the weight curve is much greater than the ordinate of the buoyancy curve, and in other places it is less. Since the areas of the two curves are equal, the excess of weight curve over buoyancy curve will equal the excess of buoyancy curve over weight curve. At any point we can determine the difference between the ordinates representing the buoyancy and weight per foot at that point. This difference is the unbalanced force which is the ordinate of the load curve previously referred to. It may be clearer to set up a curve on the base line where the ordinates give this difference. Doing so, we get a curve of loads. Some parts of this curve will be below the base line and others will be above, but, from what we have previously seen, the area of the parts above the base line must equal the area of the parts below. The curve of loads, therefore, gives the distribution of unbalanced forces acting on the ship, and in this case is the system of forces acting upon the structure when the ship is floating at rest. In practice it is not generally necessary to construct this curve, as its area is equal to the area of excess or defect of the weight curve with regard to the buoyancy.

Having the load curve, the ship may now be assumed to be out of the water, and may be considered in the same way that any other structure would be if the applied forces were as represented by the curve of loads.

We have now to deduce the magnitudes of the resistances of stresses that are likely to be developed in the structure by these inequalities.

It will be necessary, first, to examine the nature of these stresses by considering what happens when a bar of uniform section and material is subjected to the action of a straining force.

If the straining force is a direct pull, the stress in the bar is a tensile stress. If straining is a direct push, the stress in the bar is a compressive stress. If the straining tends to shear the bar at any section, or to produce angular distortion, the stress in the bar is a shearing stress.

Take the case of a weightless beam under the action of a system of parallel forces (fig. 157).

The beam is in equilibrium. On every section of the beam there must be a force or forces acting.

Consider any section A B. Let the forces be $W_1$, $W_2$, $W_3$, $W_4$, and $W_5$, at distances $X_1$, $X_2$, $X_3$, $X_4$, and $X_5$ from A B respectively. Consider force $W_5$ at distance $X_5$. If we apply two equal and opposite forces $= W_5$ on A B, the equilibrium will not be disturbed. We then have, considering $W_5$ and the equal and opposite forces applied at A B, a couple $W_5 \times X_5$ and a force $W_5$ acting over the section A B in the same direction in which $W_5$ acts at Q. This force over the section is the shearing force due to $W_5$, and tends to make one part slide relatively to another. The couple $W_5.X_5$ due to the force $W_5$ is the tendency to bend the bar at A B, and is called the bending moment due to the force $W_5$. Similarly, the shearing force on A B due to $W_4$ is $= W_4$, but it acts in the opposite direction to $W_5$. Treating each of the forces in the same way, we have total shearing force over right-hand side of $A B = W_5 + W_3 - W_4 - W_2$. As all the forces acting upon the bar are in equilibrium,

$W_5 + W_3 + W_1 - W_4 - W_2 = 0$, and therefore $W_5 + W_3 - W_4 - W_2 = -W_1$. The only force acting on the left of A B is $+W_1$. The force $-W_1$ on the right of A B tends to push the bar down, while the force $+W_1$ on the left tends to push it up. These two constitute a shearing force which the resistance to shearing of the material balances. Hence the total shearing force over any section of a beam equals the algebraic sum of all the forces acting on the beam on one side of the section.

The bending moment due to force $W_4 = W_4 \times X_4$, but it acts in the opposite direction to $W_5$, and so on for the other forces, so that the

Total bending moment on the right of A B $= W_5.X_5 + W_3.X_3 - W_4.X_4 - W_2.X_2$.

As the bar is in equilibrium,

$$W_5.X_5 + W_3.X_3 - W_4.X_4 - W_2.X_2 - W_1.X_1 = 0,$$

and therefore

$$W_5.X_5 + W_3.X_3 - W_4.X_4 - W_2.X_2 = W_1.X_1.$$

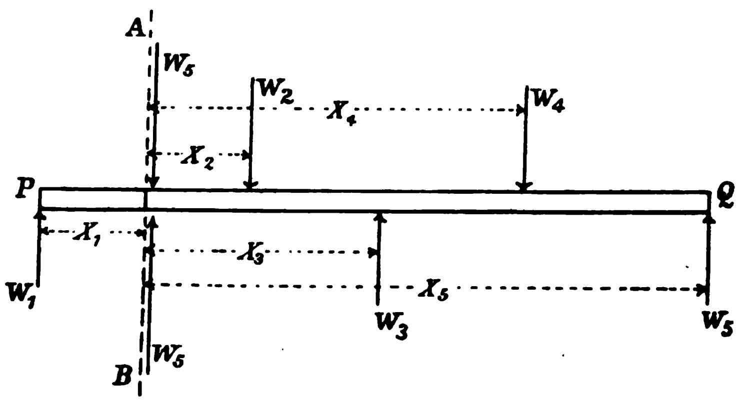



FIG. 157.

The bending moment on the left of A B is $-W_1.X_1$. The section at A B thus has a bending moment $-W_1.X_1$, tending to cause the part of the bar to the left of it to move clockwise, while the part to the right tends to move anti-clockwise under the action of the bending moment $W_1.X_1$. The material at A B offers resistance sufficient to prevent this motion, and the moment of this resistance must balance the moment $W_1.X_1$. Hence the total bending moment on any section of a bar is equal to the algebraic sum of the moments of all the forces acting on the bar on one side of the section.

If we look at fig. 158 we see that in the two ends of the ship the resultant weight acts further away from the midship section than the resultant buoyancy acts. The difference of moment of $B_1$ and $W_1$ about the centre of gravity of ship is equal and opposite to the difference of moment of B and W about the same point. These moments are prevented from turning the two halves of the ship round by the resistance to change of form of the material of the structure. Either of these may be called the bending moment.

$$B + B_1 = W + W_1.$$

$$W \times X - B \times x = -[W_1 \times X_1 - B_1 \times x_1] = \text{bending moment.}$$

Consider two parts of a beam, fig. 159, A and B, held together at the faces of a section by two parallel rods $x$ and $y$ pin-jointed as shown at P, Q, $q$, and $p$, fig. 159.

Let A and B be under the action of a system of parallel forces which would cause the beam A B to be in equilibrium.

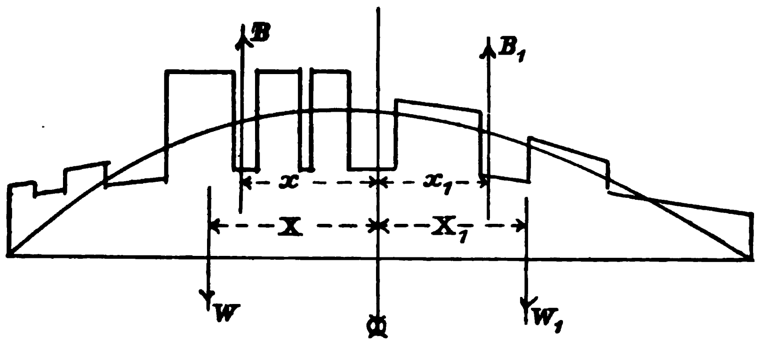



FIG. 158.

If there is no shearing force between P$p$ and Q$q$ there will be no tendency for A to move relatively to B, therefore the parts A and B will be in equilibrium by themselves. If there is a bending moment between P$p$ and Q$q$ the tendency will be for B to turn about A. There will,

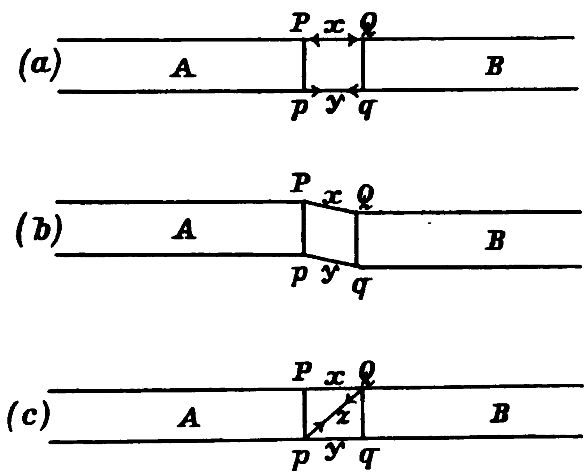



FIG. 159.

therefore, be a pull in one of the rods, say $x$, and a thrust in the other, $y$. These forces will be equal, and their magnitude multiplied by the distance between $x$ and $y$ will give a moment which, when the bar is in equilibrium, will equal the bending moment. Suppose that the bending at the section P$p$ is not the same as at Q$q$, and in consequence there is a shearing force between the parts P$p$ and Q$q$, so that B tends to move downwards relatively to A, as shown in (*b*). If we now introduce another rod *z* as shown in (*c*) the relative motion will be prevented, and a thrust will be set up in the rod *z*. The condition will therefore be the same as if A B were a continuous bar. Imagine now the faces of the sections to be very close together. As they approach, keeping the same condition, the thrust in the rod *z* will become more and more nearly equal to the shearing force over the section, and the moment of the force acting in the horizontal rods will more nearly equal the bending moment at either of the sections P*p* or Q*q*.

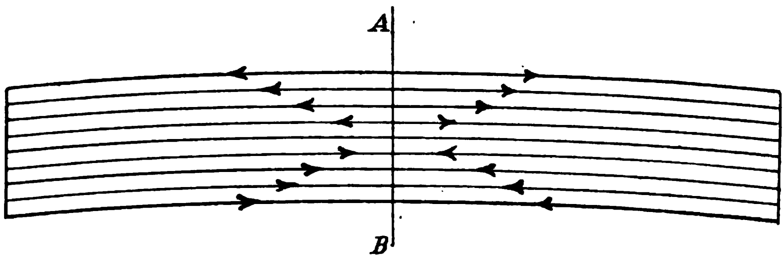



FIG. 160.

In the limit when the sections touch each other or are united, the forces that act over the section can be replaced by a force equal to the shearing force across the section, and a couple equal to the bending moment. These must be balanced by the resistance due to change of form of the material, which will be compression and tension of the fibres of the bar, and shearing stress of the material.

When a girder is bent as in fig. 160, the upper part lengthens and the lower part compresses.

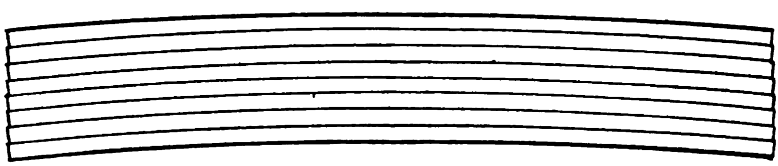


FIG. 161.

If the girder were merely a series of layers perfectly disconnected from each other, this would not happen, but the layers would bend individually, and the girder would take the shape shown in fig. 161.

If, however, the touching surfaces of the layers be connected together they will not slide by each other as in fig. 161, but will maintain their positions as in fig. 160. The resistance to this sliding tendency produces a stress in the layers which is a shearing stress. Therefore, at any cross section (fig. 162) there will be on adjacent surfaces a series of shearing stresses balancing each other.

These forces will act on the two opposite sides of a particle in a layer, and will produce in it a tendency to rotate, which tendency will be balanced by a resistance to shearing in the vertical direction.

Suppose A B C D to be a particle having a shearing stress $q$ in A B which balances a shearing stress $q$ in C D. This causes a tendency to rotate the particle, which is balanced by similar stresses $q$ in A D and C B. Hence the shearing force acting across the section produces the same intensity of shearing stress in a horizontal and a vertical layer.

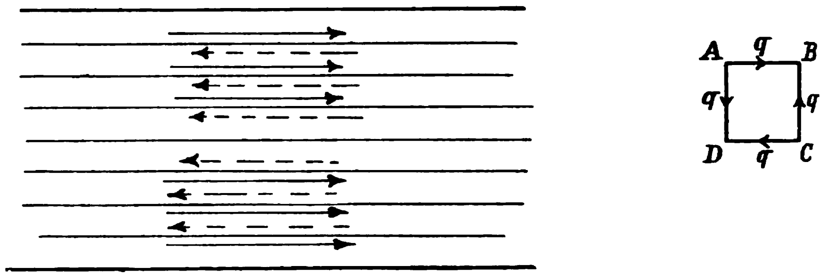



FIG. 162.

We have seen how to construct a curve of loads for a ship floating at rest. Let fig. 163 represent a curve of loads for a ship or for any other structure in equilibrium under a system of forces. We want to find the shearing force, called F, and the bending moment, called M, at any section A B.

Choose a point P in the base line and consider an element of load at P which will be $l.du$, where $l$ equals the length of ordinate or load per foot of

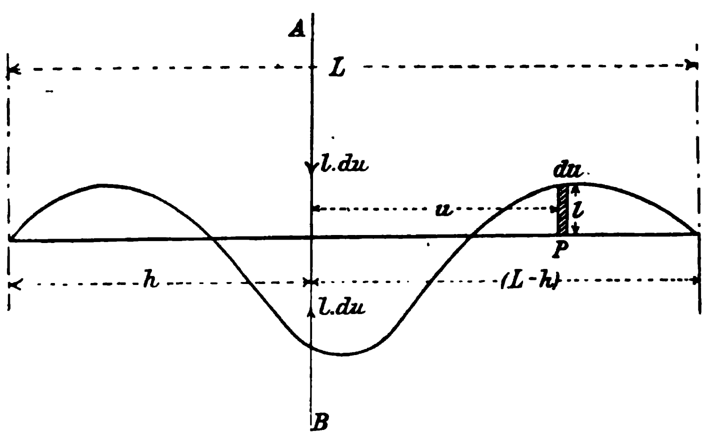



FIG. 163.

length, and $du$ is the breadth of strip measured parallel to base line. Let the distance of P from the section A B be $u$. On applying two equal and opposite forces at A B each equal to $l.du$ the condition of equilibrium is unaffected; hence the force $l.du$ at $p$ is equivalent to a single force $l.du$ at A B and a bending couple $u.l.du$ at A B.

Therefore the shearing force at A B due to $l.du$ at P $= l.du$, . . (a)

and the bending moment ,, ,, P $= u.l.du$ . (b)

We may call (*a*) the element of the total shearing force at A B $= dF$,

and (*b*) „ „ bending moment at A B $= dM$.

So that we have $dF = l.du$,

and $dM = u.l.du$.

Integrating these equations between the limits $u=0$ and $u=h$, we get

$$F_L = \int_0^h l.du, \quad M_L = \int_0^h u.l.du,$$

and between limits $u=h$ and $u=L$ we get

$$F_R = \int_h^L l.du = \int_0^L l.du - \int_0^h l.du \quad . \quad . \quad . \quad (c)$$

$$M_R = \int_h^L l.u.du = \int_0^L l.u.du - \int_0^h l.u.du. \quad . \quad . \quad . \quad (d)$$

The first expression in these equations (*c*) and (*d*) is the algebraic sum of all the forces and moments acting on the girder which is in equilibrium.

Hence $F_R + F_L = 0$

$M_R + M_L = 0$.

Hence at the section A B we have to the right of it a force equal and opposite to that at the left of the section, and these two are prevented from causing the two parts of the bar to separate by the resistance of the material to shearing. Hence $F_R = -F_L$ is called the shearing force. Similarly, $M_R$ and $M_L$ are resisted by the material from causing the two parts of the bar to turn relatively to each other, and are called the bending moment.

So that the total shearing force at A B $= \int_0^h l.du$

and the total bending moment at A B $= \int_0^h u.l.du$.

In the equation $M = \int_0^h u.l.du = -\int_h^L u.l.du$ we see that this is the second integral of $l.du$.

We may therefore write $M = \int_0^h \int_0^h l.du.du$

which may be written as $M = \int_0^h F.du$.

If we find the value of F for every point along the base we can set up a curve of F's which will represent the shearing force throughout the length of the bar.

Suppose F F to be such a curve (fig. 164); then the total summation of the values of $F.du$ from one end to A B gives the total value of M at A B; hence the area of the curve of F's on one side of A B gives the value of M at that section. A similar proof to that given above for F will show that the area of the F curve on the right of any section is the same as the area on the left with the sign reversed. Hence the shearing-force curve is the integral of the load curve, and the bending-moment curve is the integral of the shearing-force curve. These relations between the load, shearing-force, and bending-moment curves give the following characteristics:—

The load per foot at any section is equal to $\frac{dF}{du}$, and therefore where the load curve crosses the base line the shearing-force curve is a maximum or minimum since $\frac{dF}{du}=0$. Where the shearing-force curve crosses the base line the bending-moment curve is a maximum or minimum since $\frac{dM}{du}=F$, which is zero at the point where the F curve crosses the base line.

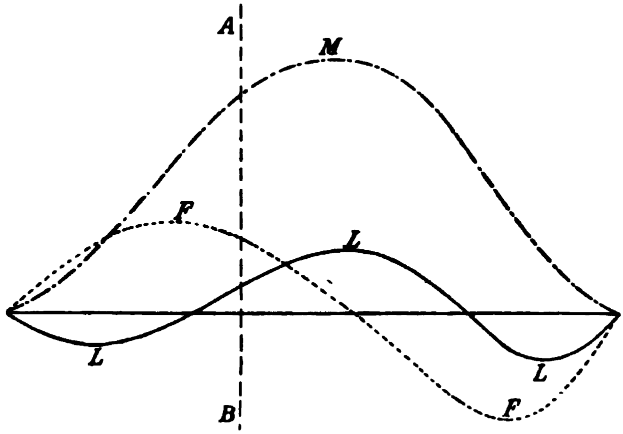



FIG. 164.

Where the shearing force is a maximum or minimum, the curvature of bending-moment curve changes sign since $l=\frac{dF}{du}=\frac{d^2M}{du^2}$. Where $\frac{dF}{du}=0$, $\frac{d^2M}{du^2}$ is $=0$, and the curvature changes sign.

*Problem.*—To find the maximum shearing force and bending moment on a homogeneous log floating in salt water and loaded as follows:—Weight of log $=8$ tons. Dimensions $48' \times 4' \times 3'$. It is loaded uniformly for 20 feet of its length amidships by a weight of 4 tons (fig. 165).

Total weight 12 tons.

Buoyancy per foot of length is uniform and $=\frac{12}{48}=\frac{1}{4}$ ton.

Weight of log per foot of length $=\frac{8}{48}=\frac{1}{6}$ ton.

Weight per foot of length for 14′ at each end $=\frac{1}{6}$ ton.

" " of load for 20′ amidships $=\frac{4}{20}=\frac{1}{5}$ ton.

" " 20′ amidships $=\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$ ton.

From these figures the load curve can be constructed as shown in fig. 165. The maximum shearing force is where the load curve crosses the base line, and the maximum shearing force $=$ area of load curve to that point $=\frac{1}{12}\times 14=\frac{7}{6}$ tons.

The maximum bending moment may, from symmetry, be seen to be at the middle of the length, and it is equal to the area of the shearing-force curve to this point:

$$= \{(\tfrac{1}{2} \times \tfrac{7}{6} \times 14) + (\tfrac{1}{2} \times \tfrac{7}{6} \times 10)\} \text{ foot-tons,}$$

$$= 14 \text{ foot-tons.}$$

The curve of bending moments is made up of four parabolas.

An outline of the method usually adopted to obtain the curves of shearing force and bending moment for a ship floating at rest is as follows:—

**Shearing-Force Curve.**—Having obtained the curves of weight and buoyancy, the load curve can be constructed by simply setting up the

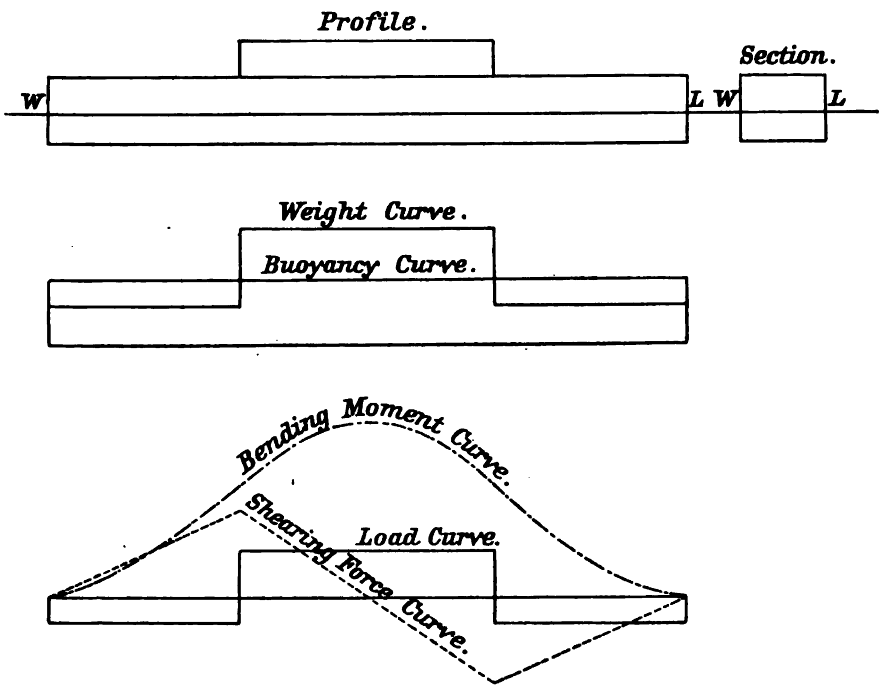



Fig. 165.

difference of the ordinates of the weight and buoyancy curves as the ordinates of a new curve, but it is generally not necessary to do this. The area of the excess or defect of the weight curve and the buoyancy curve is exactly the same in magnitude and longitudinal distribution as the area of the load curve, so that we only need to integrate this area to get the shearing force. Starting at one end of the base line we obtain the shearing-force ordinate at any point P (see fig. 166) by finding the area of the load curve up to the line $PP_1$. This area is shown shaded. Where the weight is in defect of the buoyancy the supporting force is positive, and where in excess it is negative. Adopting this nomenclature, we call the area of excess of weight negative, and area of defect of weight positive. As shown in the figure, the area (*a*) is negative and (*b*) is positive. In practice this can be directly calculated by the planimeter by first starting the instrument at A and moving it along the buoyancy curve from A to a series of positions in succession, returning from each to A. (The buoyancy and weight curves must necessarily be set up on the same scale.) If the base line scale is 1″ = $m$ feet, and the buoyancy and weight curve ordinate scales are 1″ = $n$ tons per foot, then the scale for the area of the load curve is 1 square inch = $m\,n$ tons of shearing force. Hence the number of square inches in each portion of the load curve selected for integration has to be multiplied by $m\,n$ to give the number of tons of shearing force which it represents. When the value of F at a series of points is found, the shearing force can be set off to any convenient scale, and the ordinate placed above or below the base line according as the area

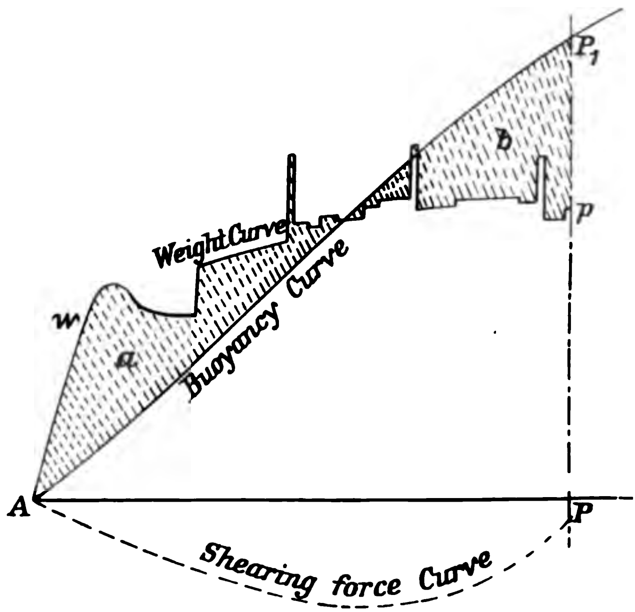



FIG. 166.

of the load curve, *i.e.* the shearing force, is positive or negative. Since the area of excess of weight over buoyancy equals the area of excess of buoyancy over weight, the total area of the load curve must be zero. That is to say, that at the finishing end of the base line, when we have integrated the whole of the load curve, the shearing-force ordinate will be zero.

**Bending-Moment Curve.**—The bending-moment curve is obtained by integrating the shearing-force curve. Starting from one end of the base line in the same way as integrating the load curve, the area of the shearing-force curve is taken up to the ordinate where we want to determine the bending moment. This area of the shearing-force curve gives foot-tons, since the ordinate of the shearing-force curve gives tons and the length along the base gives feet. If the shearing-force curve is set up to the scale 1″ = $p$ tons, and the length scale is 1″ = $m$ feet, then 1 square inch of shearing force curve = $pm$ foot-tons of bending moment. When the value of a sufficient number of M's is found the bending moment can be set off to any convenient scale.

**Short Method of constructing the Weight Curve.**—A detailed way has been described for setting off the curve of weights of the hull. It is found to be very convenient, however, to adopt a short method for arriving at an approximation to the curve of weights of hull, especially in the preliminary stages of design, and when only the form and the principal weights are known. It is found that by the method about to be described the maximum bending moment is, for approximate purposes, the same as that obtained from a more detailed curve of weights. Also the maximum shearing force is sufficiently close to the correct value in all conditions and in different types of vessels to enable the method to be adopted for the purpose intended. It is usual to divide the length up into three equal parts (see fig. 167). At the ends and points of division the ordinates are set up as shown, being proportions of the mean ordinate of the weight of structure. The total weight of hull is known, say H tons; then the mean ordinate $=\frac{H}{L}$, where L = length of ship.

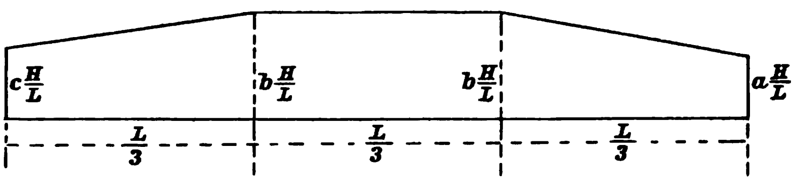



Fig. 167.

For an ordinary passenger or cargo vessel, the following coefficients are sufficiently correct for all practical purposes.

The forward ordinate is taken $\cdot 566\,\frac{H}{L}=a\frac{H}{L}$.

The middle ordinates are taken $1\cdot 195\,\frac{H}{L}=b\frac{H}{L}$.

The aft ordinate is taken $\cdot 653\,\frac{H}{L}=c\frac{H}{L}$.

$$\text{Then}\qquad H=\frac{H}{L}\left\{\left(\frac{c+b}{2}\times\frac{L}{3}\right)+\frac{bL}{3}+\left(\frac{b+a}{2}\times\frac{L}{3}\right)\right\}.$$

$$\therefore\qquad \frac{c+b}{6}+\frac{b}{3}+\frac{b+a}{6}=1.$$

$$\therefore\qquad \frac{a+c}{6}+\frac{2b}{3}=1.$$

Substituting the actual values we get

$$\frac{\cdot 566+\cdot 653}{6}+\frac{2}{3}\times 1\cdot 195$$
$$\fallingdotseq 1\cdot 000.$$

The above ratios therefore give the required area for the structural curve. The figures are consistent in themselves, and apply mostly to merchant ships.

Having the distribution and weight of hull, the other weights which are to be added to give the displacement at the given condition are built up or laid on the top of this geometrical figure. These other weights include the machinery, which is usually divided up into—

(1) Boilers. Everything connected with the boilers, and all weights above them, including funnels, mountings, gratings, pumps in boiler-room, stop and safety valves, etc., to be uniformly distributed over the length of the boilers.

(2) Engines. Everything connected with the engine-room is assumed to be uniformly distributed over the length of the engine bedplates, *i.e.* from fore end of engine bedplate to forward coupling of thrust shaft.

(3) Shafting. The weights are taken from the forward end of the thrust shaft to the after end of the propeller shaft, and assumed to be uniformly distributed over that length. This weight includes the weight of the plummer blocks, coupling bolts, and any spare gear stowed in the tunnel.

(4) Propellers. The weight of propellers is assumed to be uniformly distributed over the length of the propeller boss.

**Cargo.**—If the cargo is uniformly distributed, it is better to add it on to the hull-weight curve before adding the machinery weights. The distribution of cargo stowed in the holds is easily got from the capacity curves of the holds. The same remarks apply to coal.

# CHAPTER XX.

## CONSIDERATION OF THE STRESSES IN A GIRDER.

We have seen how to determine the shearing force and the bending moment at any transverse vertical section of a ship's structure, having been given the curves of weight and buoyancy.

If we assume the ship's structure to be a girder, the strength and stiffness of which is equal to that of the longitudinal parts of the ship, we can, by means of the formulæ in this chapter, determine the stresses that come upon any of the longitudinal parts.

Suppose we have a girder which has no discontinuity in its form, and which is made up of homogeneous and perfectly elastic material, and is in equilibrium under the action of a bending moment, either uniform or varying, acting in a longitudinal vertical plane. Since the girder is in equilibrium, the following propositions hold true for it as a whole:—

1. The algebraic sum of the horizontal or of the vertical components of the external forces acting on the girder, resolved into any plane, is zero.

2. The algebraic sum of the moments of the external forces, resolved parallel to any plane, about any line perpendicular to the plane, is zero.

Let us examine the stresses brought to bear on a cross section of this girder under the action of such a system of external forces. If we consider a vertical section of the girder perpendicular to the longitudinal vertical middle-line plane, we must have the following conditions fulfilled regarding the stresses at the section, in order that the above propositions may hold:—

(1) The sum of the vertical components of the stresses acting at the section must be equal to the sum of the vertical components of the external forces acting upon the body on either side of that section.

(2) The sum of the horizontal components of the stresses acting on the section must be equal to the sum of the horizontal components of the external forces acting upon the body on either side of that section.

(3) The sum of the moments about a horizontal line in the section of the stresses acting on that section must be equal to the sum of the moments of the external forces about that line acting on the body on either side of that section.

In the case of a ship, the external forces can always be resolved into vertical and horizontal forces and moments. The above propositions therefore become—

(1) The shearing force or sum of all the shearing forces over any section is equal to the sum of all the external forces on either side of that section.

(2) The algebraic sum of the horizontal stresses acting on the section is equal to zero.

(3) The algebraic sum of the moments of the stresses acting on that section about any horizontal line in the section is equal to the sum of the moments of the external forces about that line.

The effect of a bending moment acting on a girder is to bend or deflect it, thus lengthening the fibres on the convex side and shortening them on the concave side. If the material be perfectly elastic, particles which were originally in one plane before bending may remain in one plane relatively to each other after bending. This could only be true if the strain were constant, or if it varied uniformly as the distance from some fixed axis. Assume that particles originally in one transverse vertical plane remain in a plane after bending.

At the convex side the fibres are in tension, and at the concave side the fibres are in compression. Therefore at some lamina between the extreme fibres there is neither tension nor compression. This is called the neutral lamina, and its intersection with the cross section is called the neutral axis. The deformation of the fibres takes place most at the extreme edges, and to satisfy the geometrical condition that particles should remain in the same planes relatively to each other before and after bending, the stretch or strain must vary as the distance from the neutral axis.* Therefore, if the material is perfectly elastic, the stresses will vary as the distance from the neutral axis.

The relation between the stress at any point in the cross section of a girder and the bending moment acting on the cross section can be deduced from the above propositions.

To simplify the consideration, let us suppose the girder to be symmetrical about the vertical middle line, and let the middle-line longitudinal vertical plane be the plane of the bending couple. This supposition makes a similarity to the conditions of a ship's structure when upright and under the action of a bending moment in the middle-line vertical plane.

Let fig. 168 represent the girder, and let A B (fig. 169) represent the cross section to be considered (symmetrical about A B).†

The relation between the stress at any point, and the bending moment over the section when the cross section is not symmetrical about any line, will be treated later on.

Let N N′ be the neutral axis.

Consider any point P distant $y$ from N N′. Then the stress on P is assumed to be in direct proportion to the distance $y$.

Let $p$ be the stress per unit of area of material at P.

Then $\frac{p}{y} = \text{constant} = a$, say.

$\therefore\ p = ay.$

Area of an elemental horizontal strip of cross section at $p = b.dy$, where $b$ is the breadth at P.

The stress on the elemental area $= p.b.dy$.

Let $y_1$ and $y_2$ be the distances of extreme top and bottom fibres respectively.

* The conditions of equilibrium may be fulfilled by other arrangements of strain than the above.

† Since A B is an axis of symmetry, it is one of the principal axes of moment of inertia, so that the neutral axis is perpendicular to A B. This fact simplifies the consideration, and will be better understood later.

Then total stress on the area above NN $= \int_0^{y_1} p.b.dy$, and will act, say, from right to left,

and ,, ,, below ,, $= \int_{-y_2}^{0} p.b.dy$, and will act, say, from left to right.

But since the algebraic sum of the horizontal stresses is zero (prop. 2),

$$\therefore \quad \int_0^{y_1} p.b.dy + \int_{-y_2}^{0} p.b.dy = 0 \quad . \quad . \quad . \quad . \quad (1)$$

∴ *The total stress on the area above N N must be equal to the total stress on the area below N N.*

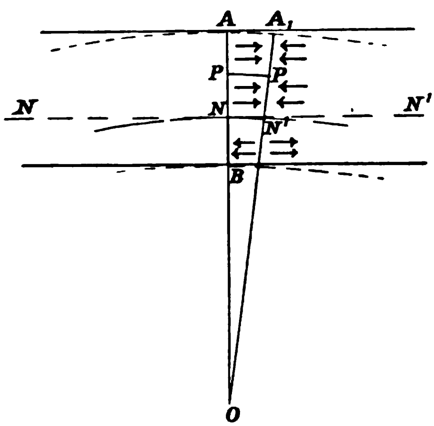



Fig. 168.

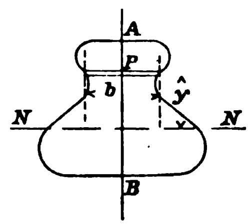



Fig. 169.

Also the moment about N N of the stress on the elemental strip $= y.p.b.dy$.

∴ Moment of total stress above N N $= \int_0^{y_1} y.p.b.dy$, acting anti-clockwise, say,

and ,, ,, below ,, $= \int_{-y_2}^{0} y.p.b.dy$ ,, ,,

But since the algebraic sum of the moments of all the stress forces about N N equals the bending moment acting on the cross section (prop. 3),

$$\therefore \quad \int_0^{y_1} y.p.b.dy + \int_{-y_2}^{0} y.p.b.dy = \text{M},$$

where M is the total bending moment acting over the cross section.

Hence $\int_{-y_2}^{+y_1} y.p.b.dy = \text{M}.$

Now $p = ay,$

$$\therefore \quad \text{M} = a\int_{-y_2}^{+y_1} y^2.b.dy \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equation (1) we have

$$\int_0^{y_1} p.b.dy + \int_{-y_2}^{0} p.b.dy = 0.$$

$$\therefore \quad \int_{-y_2}^{+y_1} p.b.dy = 0.$$

$$\therefore \quad a\int_{-y_2}^{+y_1} y.b.dy = 0.$$

$$\therefore \quad \int_{-y_2}^{+y_1} y.b.dy = 0, \text{ since } a \text{ is constant,}$$

*i.e.* the moment of the area of the cross section about N N is zero.

$\therefore$ *N N passes through the centre of gravity of the area of cross section.*

Now $\int_{-y_2}^{+y_1} y^2.b.dy$ is the expression for the moment of inertia of the area of the cross section about the neutral axis.

Calling this I we have

$$M = aI.$$

$$\text{But} \quad a = \frac{p}{y}.$$

$$\therefore \quad \frac{M}{I} = \frac{p}{y},$$

which expresses the relation between $p$ and M in terms of the geometrical quantities $y$ and I.

When the material is perfectly elastic, the formula $p = Ea$, which is known as Hooke's Law, will hold, where E is a constant, called the modulus of elasticity, and $a$ is the ratio of the alteration in dimension to the original dimension. In fig. 168, if O be the centre and R be the radius of curvature of the form into which the beam bends, $P P_1$, which was originally the same length as $N N_1$, will have increased in ratio $\frac{OP}{ON}$ and $\frac{NP}{ON}$ will be $a$.

$$\text{Hence} \quad a = \frac{y}{R} = \frac{p}{E}$$

$$\text{and} \quad \frac{p}{y} = \frac{E}{R}.$$

$$\text{Hence} \quad \frac{M}{I} = \frac{p}{y} = \frac{E}{R}.$$

In the above formula, if $y_1$ or $y_2$ be substituted for $y$ we get the stress at the extreme top or bottom fibre respectively due to the given bending moment M,

*i.e.* $p$ for the extreme top fibre $= \frac{y_1 M}{I}$,

and $p$ ,, ,, bottom ,, $= \frac{y_2 M}{I}$.

It must be understood that this formula is only true within the limits of elasticity of the material, and in no case is it true at the stage where breaking takes place.

A few simple cases may be examined. Assume the bending moment M to be acting on the cross section in each case.

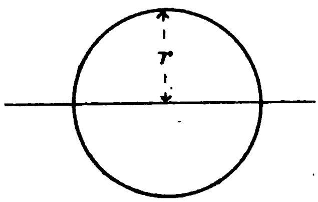



FIG. 170.

(1) Uniform bar of circular section having radius $r$, fig. 170.

$$I = \frac{\pi r^4}{4},$$

$$\therefore \quad \frac{p}{y} = \frac{4M}{\pi r^4};$$

so that $p$ on extreme fibre top or bottom $= \frac{4}{\pi r^3}M$.

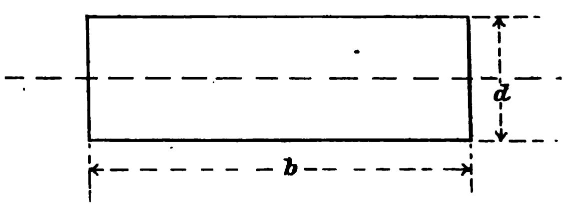



FIG. 171.

(2) Uniform rectangular bar:

breadth $b$, depth $d$,

$$I = \frac{b.d^3}{12},$$

$$\therefore \quad \frac{p}{y} = \frac{12}{b.d^3} \times M;$$

and $p$ on extreme top or bottom fibre $= \frac{6}{bd^2}M$.

(3) Girder of I section:

Thickness of web and flanges $= t$,
Breadth of each flange $= b$,
Depth of web $= d$.

Centre of gravity of section is midway between top and bottom.

$$I = \frac{bd^3}{12} - \frac{(b-t)(d-2t)^3}{12},$$

$$\therefore \quad \frac{p}{y} = \frac{12\,M}{bd^3 - (b-t)(d-2t)^3};$$

$$\therefore \quad p \text{ at extreme top or bottom fibre} = \frac{6.d.M}{b.d^3 - (b-t)(d-2t)^3}.$$

It will be seen from the formula $p = \frac{My}{I}$, where $y$ is the distance of extreme fibre, that when $y$ and the area of cross section are both constant, if I be increased, $p$ will be decreased. I can be increased by shifting the area from the region of the neutral axis to the region of the extreme fibres. This gives a stronger form of beam to resist bending.

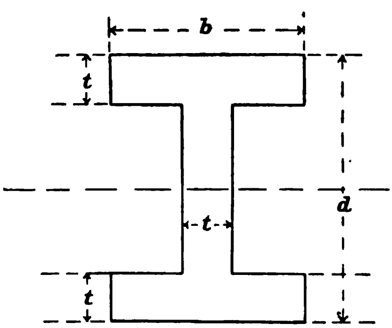



FIG. 172.

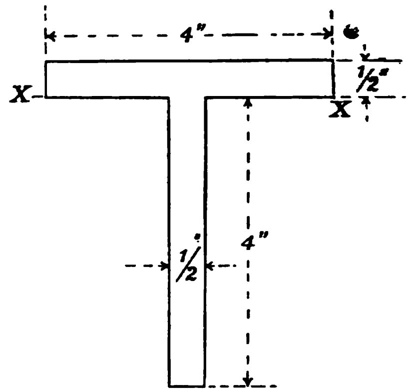



FIG. 173.

(4) Tee bar $4'' \times 4\frac{1}{2}'' \times \frac{1}{2}''$.

Maximum bending moment = 12 inch-tons hogging.*

Find the maximum stresses:

First find M.I. about X X:

| | Area in sq. in. | Leverage of Area about X X. | Moment of Area about X X. | Moment of Inertia of Area about X X. |
|---|---|---|---|---|
| | | | | $A.l^2 + \frac{1}{12}AD^2$ |
| Flange $4'' \times \frac{1}{2}''$ | 2 | $\frac{1}{4}$ | − ·5 | ·125 + ·042 |
| Web $4'' \times \frac{1}{2}''$ | 2 | 2 | + 4·0 | 8·000 + 2·666 |
| | 4 | | + 3·5 or $m$ | 8·125 + 2·708 |
| | | | | 2·708 |
| | | | | 10·833 |

* For definition of Hogging Bending Moment, see Chap. XXI.

Distance of centre of gravity from X X $= \frac{3 \cdot 5}{4} = h.$

Correction for moment of inertia $= Ah^2 = m.h.$

$$= \frac{3 \cdot 5^2}{4} = 3 \cdot 062,$$

∴ Moment of inertia about axis through centre of gravity

$$= 10 \cdot 833 - 3 \cdot 062.$$

$$= 7 \cdot 771 \text{ in.}^4$$

∴ $p$, bottom fibre, $= \frac{M \times (4 - \cdot 875)}{7 \cdot 771} = \frac{M \times 3 \cdot 125}{7 \cdot 771},$

$$= \frac{12 \times 3 \cdot 125}{7 \cdot 771} = 4 \cdot 82 \text{ tons, compression.}$$

∴ $p$, top fibre, $= \frac{12 \times 1 \cdot 375}{7 \cdot 771} = 2 \cdot 12$ tons, tension.

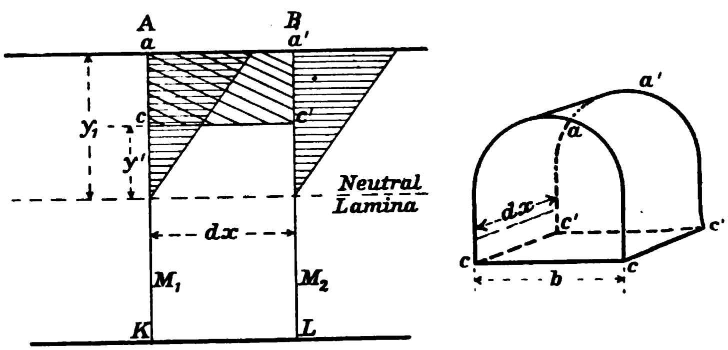



FIG. 174.

**Distribution of Shearing Stress over the Cross Section of a Girder.**—We have seen that the total shearing force, which equals the sum of the shearing stresses over the cross section, equals the algebraic sum of the external forces to either side of that section. The relation that exists between the curves of shearing force and bending moment is expressed by the equation, $M = \int F.\frac{dx}{du},$

That is to say, that the total shearing force on any transverse section of a girder is equal to the first differential coefficient of the bending moment.

It follows from this that—

Where the bending moment is constant, the shearing force is zero.

Where the shearing force is zero, the bending moment is a maximum or minimum.

Consider a girder, as in fig. 174, under the action of a varying bending moment.

Let A K and B L be two transverse sections a small distance $dx$ apart, and let the bending moments at these sections be $M_1$ and $M_2$ respectively.

In the perspective view the sections are $a\,c\,c$ and $a'\,c'\,c'$. Consider the layer $c\,c'$ parallel to the neutral lamina and distant $y'$ from it. Consider the equilibrium of the part $a\,a'\,c\,c'$.

There is a tension or pull on the particles of the section $a\,c$, which can be determined point by point from $\frac{p}{y} = \frac{M}{I}$, and since the bending moment is varying, there will be a different pull on $a'c'$. The difference in these pulls will be balanced by the resistance to shearing in the material of $a\,a'\,c'\,c$, and the net resultant of these resistances will be a shearing force over the layer $c\,c'$. The intensity of shearing stress over this layer will be the total shearing force divided by the area. Let $b$ be the breadth of the layer $c\,c'$, then the area of the layer $= b.dx$.

Now the total force over this area $b.dx$ is equal to the difference between the total tensional forces on $a\,c$ and $a'\,c'$.

But the force on $a\,c = \int_{y^1}^{y_1} p_1 b_1 \,.\, dy = \int_{y^1}^{y_1} \frac{b_1 M_1 y}{I_1} dy$,

and similarly on $a'\,c'$ $= \int_{y^1}^{y_1} p_2 b_2 \,.\, dy = \int_{y^1}^{y_1} \frac{b_2 M_2 y}{I_2} .dy.$

In a girder of uniform section

$$b_1 = b_2 \text{ and } I_1 = I_2.$$

$\therefore$ Difference of total forces on $a\,c$ and $a'\,c'$

$$= \frac{M_1 \backsim M_2}{I} \int_{y^1}^{y_1} by.dy.$$

But $M_1 \backsim M_2 = dM$, the small difference between consecutive bending moments, and $dM = F.dx$,

so that the total force on layer $c\,c' = \frac{F dx}{I} \int_{y^1}^{y_1} by.dy.$

Let $q$ be the unital stress on $c\,c'$,
then the total stress on plane $c\,c' = q\,b\,dx$.

$$\therefore \quad qbdx = \frac{F dx}{I} \int_{y^1}^{y_1} by.dy.$$

$$\text{or} \quad q = \frac{F}{bI} \int_{y^1}^{y_1} by.dy.$$

Now $\int_{y^1}^{y_1} by.dy$ is the expression for the statical moment about the neutral axis of the area of cross section above $cc'$. Calling this area A and $\bar{y}$ the distance of its centre of gravity from the neutral axis we have

$$A\bar{y} = \int_{y'}^{y_1} b.y.dy.$$

$$\therefore \quad q = \frac{FA\bar{y}}{bI}.$$

That is to say, the intensity of the shearing stress at any point in the cross section of a beam of uniform section is equal to the total shearing force on that cross section, multiplied by the statical moment of the area of that portion which is above the horizontal plane of shear in question (the moment being taken about the axis in the neutral plane), divided by the product of the moment of inertia of the area of the whole cross section, and of its breadth at the point considered.

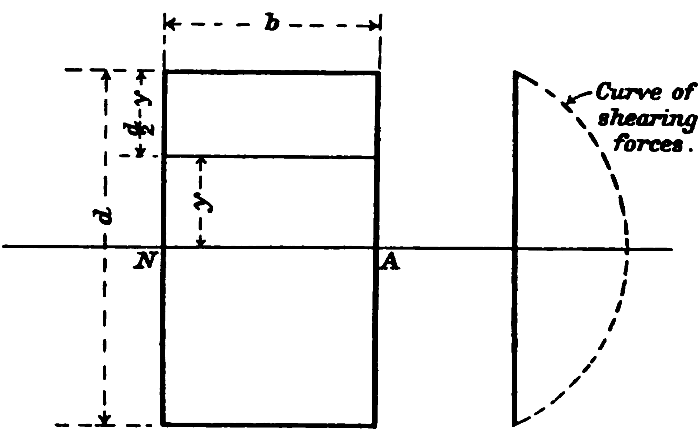



FIG. 175.

Apply this formula to the simple case of a beam of breadth $b$ and depth $d$ under a varying bending moment. Let the shearing force at the section considered be F. I for the section $= \frac{bd^3}{12}$. At any point in the section the breadth $= b$. Consider the shearing stress at a point $p$ distant $y$ from the neutral axis.

There $\quad A = \left(\frac{d}{2} - y\right)b$

and $\quad \bar{y} = \frac{d}{4} + \frac{y}{2}.$

$$\therefore \quad q = \frac{6F\left(\frac{d^2}{4} - y^2\right)}{bd^3}$$

which is the equation to a parabola.

When $y = \frac{d}{2}$, $q = 0$,

and when $y = 0$, $q = \frac{3F}{2bd}$ and is a maximum.

But $\frac{F}{bd}$ = mean shearing stress.

∴ At the neutral axis in a rectangular section, the shearing stress equals one and a half times the mean shearing stress.

To find the maximum shearing stress in a girder of I section: Let the shearing force over the section be 40 tons. Flanges 6 in. by ½ in.: Web thickness, ½ in. Depth of girder, 12 in.

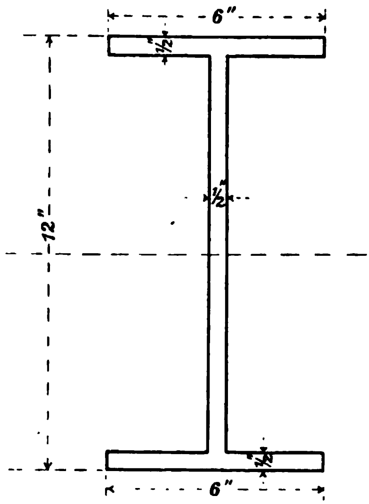



FIG. 176.

We have $q = \frac{FA\bar{y}}{bI}$.

Moment of inertia of girder = 254 inches$^4$.

$A\bar{y}$ = 24·8 inches$^3$.

∴ At the neutral axis $q = \frac{40 \times 24 \cdot 8}{254 \times 0 \cdot 5} = 7 \cdot 8$ tons per square inch.

$q$, where web joins flange, $= \frac{40 \times (17 \cdot 25)}{254 \times 0 \cdot 5}$

$= 5 \cdot 4$ tons per square inch.

# CHAPTER XXI.

## APPLICATION OF THE FORMULÆ $\frac{p}{y}=\frac{M}{I}$ FOR TENSILE AND COMPRESSIVE STRESSES AND $q=\frac{FAy}{bI}$ FOR SHEARING STRESSES TO THE SECTION OF A SHIP.

The conditions which produce severe shearing forces and bending moments in the structure of a ship will be fully dealt with later. We shall only now consider what stresses come upon the material of the structure in resisting the force F and the moment M. Suppose that we know for one condition of the ship the values of F and M at all points of the length,—*i.e.* suppose we know the shearing-force and bending-moment curves for bending in a longitudinal vertical plane.

From the nature and direction of the stresses resisting the bending moment and shearing force, it is only the continuous longitudinal parts that need be considered. We are not concerned at this stage with any transverse or any local stresses that may be indirectly set up in other parts of the structure by F and M.

In assuming the ship's structure to be equivalent to that of a girder, we have only to take into account the longitudinal parts.

When a ship is under the action of a bending moment that tends to cause the ends of the ship to droop relatively to the middle, we say the moment is hogging. The top of the ship's structure is therefore in tension and the bottom in compression. Hogging stresses resulting from a hogging bending moment are therefore—top parts in tension, bottom parts in compression.

We say the moment is sagging when the tendency is to cause the ends of the vessel to rise relatively to the middle,—the effect being the opposite to the effect produced by hogging. Sagging stresses resulting from a sagging bending moment are therefore—top parts in compression and bottom parts in tension.

From the nature of the distribution of the bending moment that comes on a ship's structure in most conditions, the maximum bending moment is usually near amidships, and the bending moment does not vary much for some distance on each side of the maximum. This fact confines the consideration of the question of the maximum stresses that are likely to come upon a ship to that part of the structure in the region of midships.

In the region of midships, therefore, the structure can be examined by taking sections at certain intervals, these sections showing only the longitudinal parts intended to resist the bending moment and shearing force, and applying the formulæ for the stresses to each section. The section which gives the largest maximum stress for a given condition of hogging or sagging is called the unavoidably weakest section of the ship near amidships.

A section for strength through the ship's structure shows generally the following parts:—

Shell plating.
Deck plating and wood deck.
Inner bottom plating.
Margin plate.
Double bottom longitudinals.
Longitudinal bulkheads.
Side stringers.
Deck fore and afters.
All longitudinal angles.
Bilge keels.
Keel bars, etc.

These items may be modified to a certain extent according to the type of vessel. For instance, a vessel with ordinary floors will have, instead of an inner bottom and margin plate, strong keelsons. Other modifications also will have to be made, such as, for instance, in a war vessel, especially if armour is fitted. The principle is, however, the same in all cases.

A section prepared for the determination of the stresses in a cargo vessel with a double bottom is shown, fig. 177. If we consider the ship as a girder, we have a section symmetrical about a vertical middle line, but very irregular in breadth. An equivalent girder can be constructed, the sectional area between two adjacent horizontal lines at a point at any height being equal to the sectional area of all the material in the section of the ship at the same height. The area of the equivalent girder equals the area of the section, and its strength is the same. If both sides of the ship are drawn similarly to fig. 177, it gives the true girder section.

An equivalent girder need not be constructed. It is only of interest geometrically as showing the distribution of the area of the material in the cross sections of various types of ships.

For instance, the rise of floor has a considerable effect on the shape of the girder. In the case of an armoured vessel, the breadth of this girder at the region of the armour would be very great. A girder that approaches ideal conditions for resisting bending-moment stresses would be one in which as little material as possible were put in the webs, and as much as possible of the material distributed equally at the top and bottom; but in such a girder the shearing stress at the neutral axis might be large, and, unless the web of the girder were well stiffened, it might collapse if too thin. It is important to note that the area of the longitudinal parts above the strength deck, not intended to resist longitudinal stresses, should be left out of the calculation. Precautions should be taken in designing the structure that these parts are not capable of having the main structural stresses transmitted to them. This will be discussed more fully later.

Having a section or sections prepared as fig. 177, and knowing the value of F and M, we can deduce the maximum tensile and compressive stresses from the formula $\frac{p}{y} = \frac{M}{I}$.

T.S.S. 530′ 0″ × 65′ 0″ × 45′ 0″.

Section for Moment of Inertia Calculation.

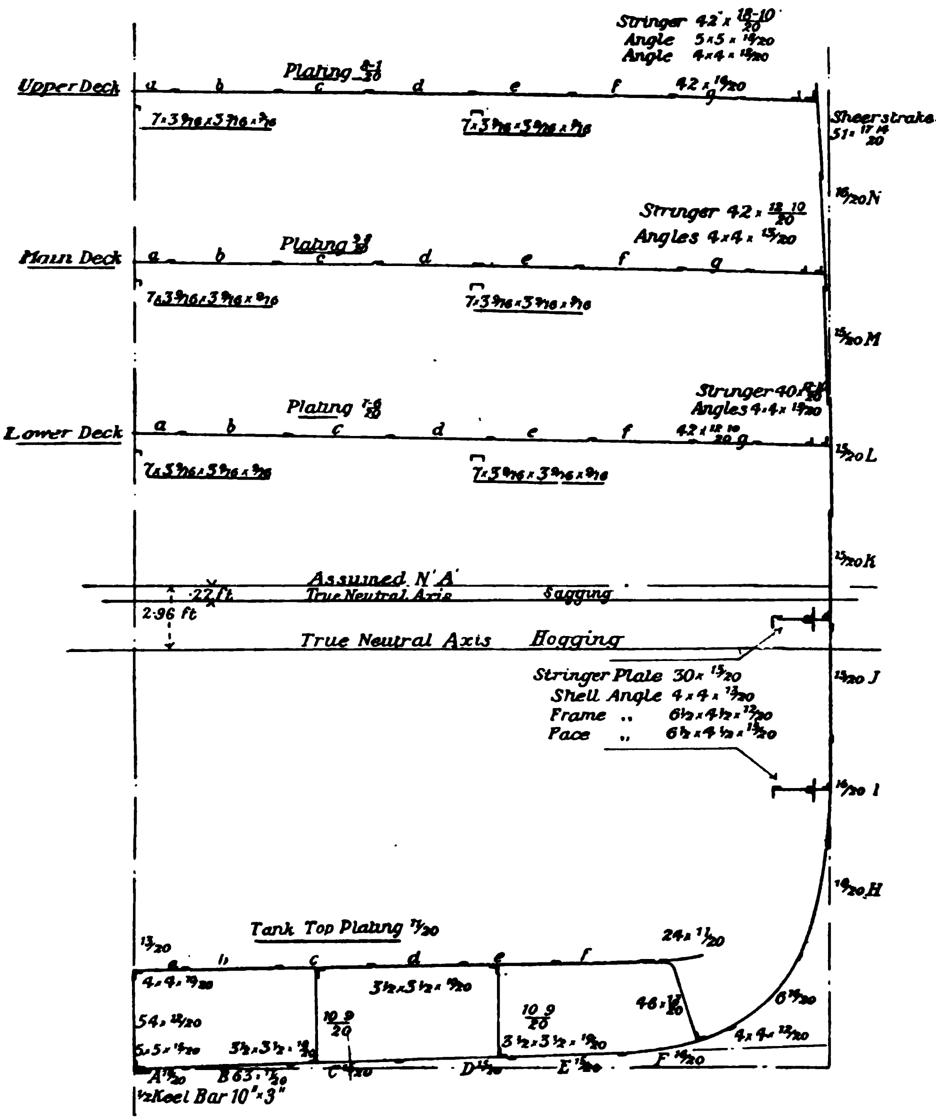



FIG. 177.

It only remains to calculate I for the section. This calculation is made as shown in the accompanying tables, which show in detail the calculation for the moment of inertia of a large 530-ft. cargo and passenger vessel. The tables are:—

Table XXXVI. Preliminary calculation.
„ XXXVII. Calculation for position of true neutral axis.
„ XXXVIII. „ „ true moment of inertia.
„ XXXIX. Results.

First an arbitrary position $N^1 A^1$ is chosen for the neutral axis. Generally this position is at about a height of half the depth. The various items are then tabulated in Table XXXVI., and a separation is made between the items above the neutral axis and the items below.

In Table XXXVI. the column headed scantlings is filled in. This column enables the area of each item to be calculated. This area is placed alongside in the area column. It is convenient to calculate the area in square inches.

In vessels such as warships, whose material is specified in lbs. per square foot for plates, and lbs. per foot of length for angles, the calculation is made with these units as a basis.

The lbs. per foot of length of a part of the structure is merely a function of the area, and this function being determined for the material, it remains a constant factor throughout the calculation; and therefore, if the calculation is worked on the basis of lbs. per foot of length instead of area, the final results have only to be multiplied by the factor giving the function. These final results can easily be transformed to square inches by remembering that a steel plate or bar 1 foot long and 12 square inches of section weighs 41 lbs.

The distance of the centre of gravity of any item from the $N^1 A^1$ gives the figure for the leverage column. The leverage scale is generally in feet, but in vessels as small as torpedo-boat destroyers it is more convenient to use inches.

Multiplying areas by leverages, we get the moments of each item about the $N^1 A^1$, and multiplying the moments again by leverages we get the moment of inertia of items about the $N^1 A^1$. For some of the items it is necessary to add to each the moment of inertia of the item about its own axis. These are put in separate columns, the last columns being for the moment of inertia of each item about its own axis. In this last column it is only necessary to calculate those items which have considerable depth, like the sheerstrake and side shell plates, longitudinals, etc. Items such as deck plates, inner bottom plates, need not come in this column, as their moment of inertia about their own horizontal axis is so small.

It is preferable to keep items of wood separate from those of metal.

Table XXXVI. is worked out and the total of the metal items added for the area, moment, and moment of inertia columns. The same can be done separately for wood.

At this stage it is necessary to take note of the different considerations when the bending moment is a hogging moment and when it is a sagging moment.

The section is assumed to be at a line of rivet holes in the way of a frame and beam. The longitudinal parts are generally all connected to some kind of transverse stiffening at a frame line. The longitudinal parts are therefore weakened by the rivet holes when under tension. As the spacing of rivets in a frame line is about 7 to 8 diameters, a deduction of $\frac{1}{8}$th of the area is made for the parts in tension. It is assumed that the rivets in the longitudinal parts in compression completely fill the holes, and so take up the proportion of stress that would have been taken up by the material punched out to form the holes, so that no deduction is made for parts in compression.

Very often the decks are either covered with wood or built of wood and steel ties. In these cases the wood is capable of resisting compression, and if properly connected to the plating or beams, may be able also to resist tension to a certain extent.

The strength of wood is inferior to that of iron or steel, its modulus of elasticity being about $\frac{1}{18}$th that of steel. It is assumed in the calculations that the wood in compression takes up as much stress as $\frac{1}{18}$th of its area in metal, but in tension, as about $\frac{1}{4}$th of the deck planking is weakened by butts and the remaining $\frac{3}{4}$ths is wounded by bolt holes, it is only taken as equivalent to $\frac{1}{25}$th of its area in metal.

This allowance of $\frac{1}{8}$th for rivet holes is the usual practice, and is adopted in order that the results may be directly comparable with previous results. The deduction of area is, however, only strictly applicable to ultimate stresses, but within the limits of elasticity it would probably be more nearly accurate to assume the longitudinal parts to be solid in all cases, *i.e.* to make no deduction for rivet holes. Vessels are not generally designed with little enough material to cause the stresses upon them to reach the elastic limit, so that the taking account of rivet holes is generally inaccurate. This matter will be discussed more fully later.

These allowances are made in the next part of the calculation, Table XXXVII. In this table the calculation for the position of the true neutral axis is made.

The neutral axis for hogging is generally below that for sagging, so that two independent calculations have to be made. In hogging, the top parts are in tension, so that the area above the assumed $N^1 A^1$ is $\frac{7}{8}$ths of that got by adding up the area columns in Table XXXVI. To this also has to be added $\frac{1}{25}$th of the area of wood above the $N^1 A^1$.

This deduction also has to be made in the moment of the area, and also in the moment of inertia of the area about $N^1 A^1$.

These deductions are made for the areas and moments below the $N^1 A^1$ in the calculation for sagging. The true neutral axis passes through the centre of gravity of the reduced area of the section, and therefore the distance of the true neutral axis from the chosen $N^1 A^1$ is equal, by the principle of moments, *to the difference of the moments above and below the $N^1 A^1$ divided by the sum of the areas above and below.* Strictly speaking, there should be a further correction made for the material between $N^1 A^1$ and N A, which changes from compression to extension, or *vice versâ*, by the change in the position of the axis of moments. This, however, is not usually given effect to, as it is generally very small.

Let $A$ = area above $N^1 A^1$ from Table XXXVI.
$M$ = moment of " " "
$A_1$ = area below $N^1 A^1$ " "
$M_1$ = moment of " " "

Then area in tension = $\frac{7}{8}A$. Area in compression = $A_1$.
" moment of area in tension = $\frac{7}{8}M$. Moment of area in compression = $M_1$.

$$\therefore \text{ Distance of neutral axis from } N^1 A^1 = \frac{M_1 \backsim \frac{7}{8}M}{A_1 + \frac{7}{8}A}.$$

This operation for both hogging and sagging is shown in Table XXXVII.

Table XXXVIII. shows how to obtain the true moment of inertia. In this table a correction has to be made to obtain the moment of inertia about the true neutral axis N A. The method has been already explained for plane curves.

The moment of inertia about any axis is $(I + Ak^2)$, where I is about a parallel axis (through the C.G.) distant $k$ from the chosen axis, and A is the area of the material.

In the tables we have the moment of inertia about the assumed $N^1 A^1$.

In the case of hogging let $i$ = M.I. of area above $N^1 A^1$
,, $i_1$ = ,, ,, below $N^1 A^1$.

Then, the top being in tension, the total moment of inertia about $N^1 A^1 = i_1 + \frac{7}{8}i$.

We have also the distance between N A and $N^1 A^1$ corresponding to $k$ in the above equation.

∴ I, the true moment of inertia about N A,

$$= i_1 + \tfrac{7}{8}i - (A_1 + \tfrac{7}{8}A)\left(\frac{M_1 \backsim \frac{7}{8}M}{A_1 + \frac{7}{8}A}\right)^2.$$

Similarly, we can obtain the value of I for sagging.

The results of these calculations can be put in Table XXXIX., which gives a convenient method of tabulating them, and also tabulates certain ratios which may be interesting as a basis of comparison between different types of vessels.

We now have the values of I for hogging and for sagging. The maximum stresses occur at the extreme parts of the section distant from the neutral axis. In hogging, let these distances of the extreme top and bottom parts be respectively $y_1$ and $y_2$. Then if M is the hogging bending moment,

$p_1$ the maximum tensile stress $= \frac{M}{I} y_1$

and $p_2$ ,, compressive ,, $= \frac{M}{I} y_2$.

The maximum stresses, if M is sagging, are obtained in a similar way. These calculations are fully shown in Table XXXIX.

The desirable maximum value of $p$ can only be arrived at from experience. We may by the above method find $p$ for actual ships which have been successful, but all we can do for a proposed ship is to make the section sufficiently strong, so that the maximum value of $p$ is not greater in the new ship than the value deduced from the calculations made for actual cases of successful ships.

To arrive at an exact determination of the necessary amount of material in the structure which would just be sufficient to withstand breaking is impossible. We can, by comparison and judicious use of the results of calculations of successful ships, so arrange the material that it shall not be subjected to a greater stress than the prototype, and also that it shall not approach by a reasonable amount the stress which is sufficient to cause permanent distortion of any kind. The considerations governing the conditions at sea that impose severe stresses on the structure are dealt with later.

**Stress due to Shearing Force.**—The shearing force varies over the length of a ship in any condition; and in a condition producing a severe bending moment there are generally two sections where it is a maximum. It is therefore only these sections that need be considered for any one condition of distribution of weight and support. The section is made in the same way as the section for the previous moment of inertia calculation, and its moment of inertia is calculated according to the method of the tables, but there is no reason to make a deduction in the area for rivet holes, as it is supposed that the rivets connecting the various parts transmit fully the shearing stress. The value of I, then, for a section under a shearing force F is the moment of inertia of the whole area of the longitudinal parts. The position of the neutral axis of the section is also determined without deductions.

We have seen, the shearing stress $q$ at any point in the section is equal to $\frac{FA\bar{y}}{bI}$. It only remains to find $A\bar{y}$ and $b$. $b$ is easily determined for any point, as it is the breadth of the material at that point. At any point in the section the value of $A\bar{y}$ is the moment of the area of the material on the other side from the neutral axis of a horizontal line through the point under consideration. This value $A\bar{y}$ can be obtained from the Table XXXVI. in the moment of inertia calculation. Thus the shearing stress at any point in the section can be determined. The value of the intensity at any point depends on two things: (1) the moment of the area to one side of the point, and (2) the breadth of the material at the point. (1) is a maximum at the neutral axis, and therefore if (2) is constant, $q$ is a maximum at the neutral axis.

Let the fig. 178 represent the section of the longitudinal parts of a ship where there is a maximum shearing force F. From the formula it will be seen that the stress is zero at the centre of the top deck and at the bottom of keel. If the parts were perfectly fixed together so that the section as shown be perfectly homogeneous, it would be correct to take $b$ as the horizontal breadth for all points; but when we consider the horizontal members, such as decks or flat stringers, in fig. 178, it will be seen that the shearing stress upon the section $a\,b$ which the total shearing force compels will be obtained by dividing $\frac{FA\bar{y}}{I}$ by the thickness of the part at $a\,b$. But practically this same force will act at $c\,d$, and the intensity of the stress along $c\,d$ will be much less than at $a\,b$. It should, however, be noticed that if the material sheared at $c\,d$, then the part to the left of $c\,d$ would not transmit shearing to its adjacent parts, and, for all strength purposes, it would cease to be of value. Hence the shearing stress through $a\,b$ will be the same as through $c\,d$ if the thickness is the same. There must be about as much shearing stress at the edge of the stringer plate, where the horizontal breadth is large, as in the material adjacent to the plate, where the horizontal breadth is small.

In the section the shearing stress has been set off perpendicularly to the plate for each shell plate. In this calculation also the breadth $b$ for any part has been taken to be the perpendicular or least dimension of the part.

## Table XXXVI.

*Preliminary Calculation.* Items *below* chosen N[1] A[1].

**Section at midships.**
**Neutral axis chosen 22·5 feet above base line.**

| Items. | Scantlings Inches. | Area Sq. Inches. | Leverages Feet. | Moment In² Feet. | Moment of Inertia. In² Feet². $Ah^2$ | Moment of Inertia. In² Feet². $i = \frac{AD^2}{12}$ |
|---|---|---|---|---|---|---|
| Keel | $10 \times 3$ | 30 | 22·6 | 678 | 15320 | ... |
| Shell Plating. A Strake | $30 \times \frac{18}{20}$ | 27 | 22·45 | 606 | 13620 | ... |
| B ,, | $63 \times \frac{17}{20}$ | 52·55 | 22·35 | 1182 | 26420 | ... |
| C ,, | $63 \times \frac{15}{20}$ | 47·25 | 22·15 | 1053 | 23350 | ... |
| D ,, | $66 \times \frac{15}{20}$ | 49·5 | 22·0 | 1089 | 23950 | ... |
| E ,, | $63 \times \frac{15}{20}$ | 47·25 | 21·85 | 1035 | 22650 | ... |
| F ,, | $66 \times \frac{16}{20}$ | 52·8 | 21·45 | 1132 | 24300 | ... |
| G ,, | $63 \times \frac{16}{20}$ | 50·4 | 19·0 | 958 | 18400 | 51 |
| H ,, | $66 \times \frac{16}{20}$ | 52·8 | 14·7 | 777 | 11820 | 133 |
| I ,, | $60 \times \frac{16}{20}$ | 48·0 | 9·5 | 356 | 3380 | 100 |
| J ,, | $66 \times \frac{15}{20}$ | 49·5 | 4·5 | 222 | 999 | 125 |
| K ,, Part | $24 \times \frac{15}{20}$ | 18·0 | 1·0 | 18 | 18 | 72 |
| Tank Top Plating. *a* ,, | $30 \times \frac{13}{20}$ | 19·5 | 17·9 | 349 | 6260 | ... |
| *b* ,, | $57 \times \frac{11}{20}$ | 31·35 | 17·85 | 560 | 10000 | ... |
| *c* ,, | $54 \times \frac{11}{20}$ | 29·7 | 17·75 | 527 | 9360 | ... |
| *d* ,, | $57 \times \frac{11}{20}$ | 31·35 | 17·65 | 553 | 9760 | ... |
| *e* ,, | $54 \times \frac{11}{20}$ | 29·7 | 17·55 | 522 | 9170 | ... |
| *f* ,, | $57 \times \frac{11}{20}$ | 31·35 | 17·45 | 547 | 9560 | ... |
| Gusset Plate | $24 \times \frac{11}{20}$ | 13·2 | 17·4 | 230 | 4000 | . |
| ½ Centre Girder | $54 \times \frac{6}{20}$ | 16·2 | 20·2 | 327 | 6610 | 24 |
| ,, Angles Top | $7\frac{1}{2} \times \frac{13}{20}$ | 4·87 | 18·0 | 88 | 1565 | ... |
| ,, ,, Bottom | $9\frac{1}{2} \times \frac{15}{20}$ | 7·37 | 22·4 | 165 | 3700 | . |
| Margin Plate | $63 \times \frac{13}{20}$ | 40·95 | 18·9 | 774 | 14620 | 48 |
| ,, Angle | $7\frac{1}{2} \times \frac{12}{20}$ | 4·5 | 21·1 | 95 | 2100 | ... |
| 1st Side Girder | $54 \times \frac{10}{20}$ | 27·0 | 19·8 | 535 | 10600 | 46 |
| 2nd ,, ,, | $51 \times \frac{10}{20}$ | 25·5 | 19·7 | 503 | 9920 | 38 |
| Top Angles Side Girder (2) | $6\frac{1}{2} \times \frac{10}{20} \times 2$ | 6·5 | 17·75 | 115 | 2040 | ... |
| Bottom ,, ,, ,, (2) | $6\frac{1}{2} \times \frac{10}{20} \times 2$ | 6·5 | 22·0 | 143 | 3146 | ... |
| Lower Stringer Plate | $30 \times \frac{15}{20}$ | 22·5 | 9·5 | 214 | 2030 | ... |
| Face Angle | $10\frac{1}{4} \times \frac{15}{20}$ | 7·69 | 9·5 | 73 | 694 | ... |
| Shell ,, | $7\frac{1}{2} \times \frac{13}{20}$ | 4·87 | 9·45 | 46 | 434 | .. |
| Frame ,, (2) | $10\frac{1}{2} \times \frac{12}{20} \times 2$ | 12·6 | 9·5 | 120 | 1140 | ... |
| Upper Stringer Plate | $30 \times \frac{15}{20}$ | 22·5 | 1·6 | 38 | 61 | ... |
| Face Angle | $10\frac{1}{4} \times \frac{15}{20}$ | 7·69 | 1·6 | 12 | 19 | ... |
| Shell ,, | $7\frac{1}{2} \times \frac{13}{20}$ | 4·87 | 1·55 | 8 | 12 | .. |
| Frame ,, (2) | $10\frac{1}{2} \times \frac{12}{20} \times 2$ | 12·6 | 1·6 | 20 | 32 | ... |
| Total Hogging | ... | 945·81 | ... | 15670 | 301060 | 637 |
| 1st Side Girder | ... | 27·0 | ... | 535 | 10600 | 46 |
| 2nd ,, ,, | ... | 25·5 | ... | 503 | 9920 | 38 |
| Top Angles Side Girder (2) | ... | 6·5 | ... | 115 | 2040 | ... |
| Bottom ,, ,, ,, (2) | ... | 6·5 | ... | 143 | 3146 | ... |
| Lr. Stringer Shell Angle | ... | 4·87 | ... | 46 | 434 | ... |
| Up. ,, ,, ,, | ... | 4·87 | ... | 8 | 12 | ... |
| | ... | 75·24 | ... | 1350 | 26152 | 84 |
| Total Sagging | ... | 870·57 | ... | 14320 | 274908 | 753 |

TABLE XXXVI.—*continued.*

*Preliminary Calculation.* Items *above* chosen N[1] A[1].

**Section at midships.**
**Neutral axis chosen 22·5 feet above base line.**

| Items. | | Scantlings Inches. | Area Sq. Inches. | Lever-ages Feet. | Moment In² Feet. | Moment of Inertia. In² Feet². Ah² | $i = \frac{AD^2}{12}$ |
|---|---|---|---|---|---|---|---|
| Shell Plating. | K Strake | $42 \times \frac{15}{20}$ | 31·5 | 1·75 | 55 | 86 | 32 |
| | L ,, | $66 \times \frac{15}{20}$ | 49·5 | 5·75 | 284 | 1632 | 125 |
| | M ,, | $66 \times \frac{15}{20}$ | 49·5 | 10·15 | 502 | 5100 | 125 |
| | N ,, | $66 \times \frac{16}{20}$ | 52·8 | 16·50 | 872 | 14380 | 132 |
| | Sheerstrake | $51 \times \frac{17}{20}$ | 43·25 | 21·25 | 921 | 19550 | 64 |
| Lr. Dk. Stringer | | $40 \times \frac{12}{20}$ | 24·0 | 6·5 | 156 | 1014 | ... |
| ,, ,, Angles (2) | | $7\frac{1}{2} \times \frac{13}{20} \times 2$ | 9·75 | 6·5 | 63 | 412 | ... |
| ,, *g* Strake | | $40 \times \frac{11}{20}$ | 22 | 6·6 | 145 | 957 | ... |
| ,, *f* ,, | | $60 \times \frac{7}{20}$ | 21 | 6·7 | 141 | 946 | ... |
| ,, *e* ,, | | $63 \times \frac{7}{20}$ | 22·05 | 6·8 | 150 | 1020 | ... |
| ,, *d* ,, | | $60 \times \frac{7}{20}$ | 21 | 6·9 | 145 | 1000 | .. |
| ,, *c* ,, | | $60 \times \frac{7}{20}$ | 21 | 7·0 | 147 | 1029 | ... |
| ,, *b* ,, | | $63 \times \frac{7}{20}$ | 22·05 | 7·05 | 156 | 1100 | ... |
| ,, *a* ,, | | $30 \times \frac{7}{20}$ | 10·5 | 7·1 | 75 | 532 | ... |
| ½ Cr. Fore and After L.D. | | $6\frac{1}{2} \times \frac{9}{16}$ | 3·66 | 6·2 | 23 | 142 | ... |
| Side ,, ,, L.D. | | $13 \times \frac{9}{16}$ | 7·3 | 6·0 | 44 | 264 | ... |
| Main. Dk. Stringer | | $42 \times \frac{12}{20}$ | 25·2 | 14·6 | 368 | 5370 | ... |
| ,, ,, Angles (2) | | $7\frac{1}{2} \times \frac{13}{20} \times 2$ | 9·75 | 14·7 | 146 | 2140 | ... |
| ,, *g* Strake | | $42 \times \frac{12}{20}$ | 25·3 | 14·7 | 363 | 5340 | ... |
| ,, *f* ,, | | $60 \times \frac{9}{20}$ | 27·0 | 14·8 | 400 | 5920 | ... |
| ,, *e* ,, | | $63 \times \frac{9}{20}$ | 28·35 | 14·9 | 423 | 6260 | ... |
| ,, *d* ,, | | $60 \times \frac{9}{20}$ | 27·0 | 14·95 | 404 | 6050 | ... |
| ,, *c* ,, | | $63 \times \frac{9}{20}$ | 28·35 | 15·0 | 426 | 6390 | .. |
| ,, *b* ,, | | $63 \times \frac{9}{20}$ | 28·35 | 15·05 | 427 | 6430 | ... |
| ,, *a* ,, | | $24 \times \frac{9}{20}$ | 10·8 | 15·1 | 163 | 2460 | ... |
| ½ Cr. Fore and After M.D. | | $6\frac{1}{2} \times \frac{9}{16}$ | 3·66 | 14·2 | 52 | 739 | ... |
| Side ,, ,, M.D. | | $13 \times \frac{9}{16}$ | 7·3 | 14·0 | 102 | 1428 | .. |
| U. Dk. Stringer | | $42 \times \frac{16}{20}$ | 33·8 | 22·6 | 765 | 17270 | ... |
| ,, ,, Angles | | $9 \times \frac{18}{20}$ | 8·1 | 22·7 | 187 | 4245 | ... |
| ,, ,, ,, | | $7\frac{1}{2} \times \frac{13}{20}$ | 4·87 | 22·7 | 110 | 2497 | ... |
| ,, *g* Strake | | $42 \times \frac{16}{20}$ | 33 6 | 22·7 | 764 | 17340 | ... |
| ,, *f* ,, | | $60 \times \frac{8}{20}$ | 24·0 | 22·8 | 547 | 12480 | ... |
| ,, *e* ,, | | $60 \times \frac{8}{20}$ | 24·0 | 22·9 | 550 | 12580 | ... |
| ,, *d* ,, | | $60 \times \frac{8}{20}$ | 24·0 | 23·0 | 552 | 12670 | ... |
| ,, *c* ,, | | $54 \times \frac{8}{20}$ | 21·6 | 23·05 | 498 | 11480 | ... |
| ,, *b* ,, | | $60 \times \frac{8}{20}$ | 24·0 | 23·1 | 555 | 12820 | ... |
| ,, *a* ,, | | $27 \times \frac{8}{20}$ | 10·8 | 23·15 | 250 | 12740 | ... |
| ½ Cr. Dk Fore and After | | $6\frac{1}{2} \times \frac{9}{16}$ | 3·66 | 22·3 | 82 | 1830 | ... |
| Side ,, ,, ,, | | $13 \times \frac{9}{16}$ | 7·3 | 22·15 | 162 | 3590 | ... |
| Totals—Hogging or Sagging | | | 851·65 | ... | 12175 | 219233 | 478 |

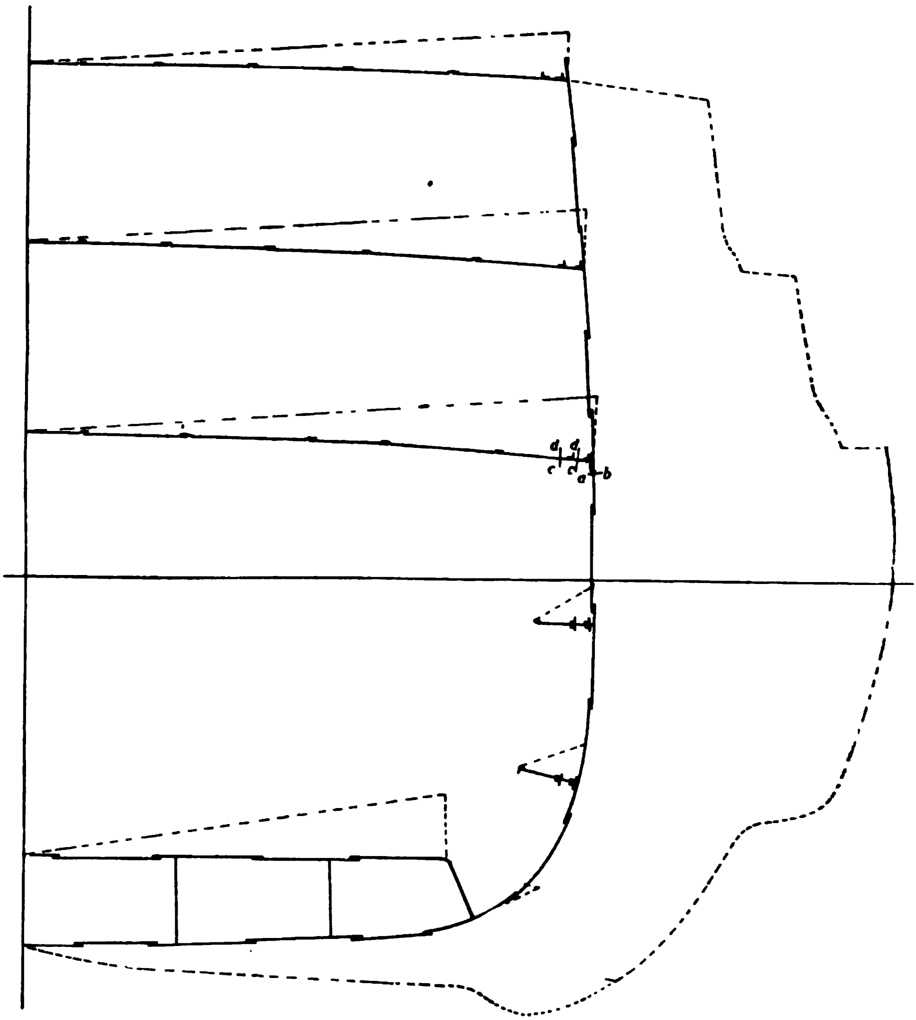


Fig. 178.

Dimensions, 420′ × 48′ × 36′.

Total—Area of section = 2100 sq. in.
Total shearing force = 1200 tons.
Mean shearing force = $\frac{1200}{2100}$ = ·55 tons/sq. in.
Shearing stress set off perpendicular to plate.
Shearing stress at neutral axis $= q = \frac{FA\bar{y}}{bI} = \frac{1200 \times 17100}{2 \times \frac{11}{20} \times 520000 \times 12}$ = 3 tons/sq. in.
Value of $b$ at N A is $2 \times \frac{11}{20}$″.
I is 520000 in.$^2$ ft.$^2$ $A\bar{y}$ is 17100 in.$^2$ ft.$^2$

## TABLE XXXVII.

### CALCULATION FOR POSITION OF NEUTRAL AXIS.

*Moment hogging:*

| | | | | | |
|---|---|---|---|---|---|
| $\frac{7}{8}$ Area metal above $N^1 A^1$ = | | 745·2 | $\frac{7}{8}$ Moment metal above $N^1 A^1$ = | | 10653 |
| $\frac{1}{25}$ ,, wood ,, ,, = | | | $\frac{1}{25}$ ,, wood ,, ,, = | | |
| | (a) | 745·2 | | (b) | 10653 |
| Area metal below $N^1 A^1$ = | | 945·8 | Moment metal below $N^1 A^1$ = | | 15670 |
| $\frac{1}{16}$ ,, wood ,, ,, = | | | $\frac{1}{16}$ ,, wood ,, ,, = | | |
| | (c) | 945·8 | | (d) | 15670 |

Distance of true N A from assumed $N^1 A^1 = \frac{b \backsim d}{a+c} = (\epsilon) = \frac{5017}{1691} = 2{\cdot}96$ ft.

N A must be taken ($\epsilon$) towards the side of greater moment.

*Moment sagging:*

| | | | | | |
|---|---|---|---|---|---|
| Area metal above $N^1 A^1$ = | | 851·6 | Moment metal above $N^1 A^1$ = | | 12175 |
| $\frac{1}{16}$ ,, wood ,, ,, = | | | $\frac{1}{16}$ ,, wood ,, ,, = | | |
| | $(a^1)$ | | | $(b^1)$ | |
| $\frac{7}{8}$ Area metal below $N^1 A^1$ = | | 760·9 | $\frac{7}{8}$ Moment metal below $N^1 A^1$ = | | 12530 |
| $\frac{1}{25}$ ,, wood ,, ,, = | | | $\frac{1}{25}$ ,, wood ,, ,, = | | |
| | $(c^1)$ | 1612·5 | | $(d^1)$ | 355 |

Distance of true N A from assumed $N^1 A^1 = \frac{b^1 \backsim d^1}{a^1+c^1} = (\epsilon^1) = \frac{355}{1612{\cdot}5} = {\cdot}22$ ft.

N A must be taken ($\epsilon^1$) towards side of greater moment.

## TABLE XXXVIII.

### CALCULATION FOR TRUE MOMENT OF INERTIA.

*Hogging moment:*

| | | |
|---|---|---|
| $\frac{7}{8}$ Moment of inertia of area of metal above $N^1 A^1$ = | 192247. | |
| $\frac{1}{25}$ ,, ,, ,, wood ,, ,, = | | |
| ∴ Total moment of inertia above $N^1 A^1$ = | 192247 | $=(g)$. |
| Moment of inertia of area of metal below $N^1 A^1$ = | 301697 | |
| $\frac{1}{16}$ ,, ,, ,, wood ,, ,, = | | |
| ∴ Total moment of inertia below $N^1 A^1$ = | 301697 | $=(h)$. |
| ∴ Total moment of inertia about $N^1 A^1$ = | 495144 | $=(g+h)$. |
| Correction for above subtract $\{\epsilon^2(a+c)\}$ = | 14870 | $=(k)$. |
| ∴ True moment of inertia for hogging = | 479074 | $=(g+h-k)$. |
| Both sides ,, ,, ,, = | 958148 inch² feet². | |

*Sagging moment:*

| | | |
|---|---|---|
| Moment of inertia of area of metal above $N^1 A^1$ = | 219711 | |
| $\frac{1}{16}$ ,, ,, ,, wood ,, ,, = | | |
| ∴ Total moment of inertia above $N^1 A^1$ = | 219711 | $=(g^1)$. |
| $\frac{7}{8}$ Moment of inertia of area of metal below $N^1 A^1$ = | 241203 | |
| $\frac{1}{25}$ ,, ,, ,, wood ,, ,, = | | |
| ∴ Total moment of inertia below $N^1 A^1$ = | 241203 | $=(h^1)$. |
| ∴ Total moment of inertia about $N^1 A^1$ = | 460914 | $=(g^1+h^1)$. |
| Correction for above subtract $\epsilon^2(a^1+c^1)$ = | 780 | $=(k^1)$. |
| ∴ True moment of inertia for sagging = | 460134 | $=(g^1+h^1-k^1)$. |
| ∴ ,, ,, ,, ,, both sides = | 920268 inch² feet². | |

## Table XXXIX.

*Results. Hogging.*

Dimensions of ship = 530 × 65 × 45 Mld. to Upper Deck.
Displacement = 18900 tons.
Draft = 26 ft. 7 in. (mean) in still water.
Cargo in. Coal out.
Vessel on crest of wave. Length = 530, Height = 26·5.
Maximum bending moment = 433700 foot tons.

*Stresses:*

Top in tension. Upper Deck $p=\frac{M}{I}y_1=\frac{433700}{958148}\times 26=11\cdot 76$ tons sq. in.

Bottom in compression. Keel $p=\frac{M}{I}y_2=\frac{433700}{958148}\times 19\cdot 5=8\cdot 82$.

*Results. Sagging.*

Dimensions of ship = 530 × 65 × 45 Mld. to Upper Deck.
Displacement = 11000 tons.
Cargo out. Coal in.
Vessel in wave hollow. Length = 530, Height = 26·5.
Maximum bending moment = 320000.

*Stresses:*

Top in compression. Upper Deck $p=\frac{M}{I}y_1=\frac{320000}{920000}\times 23\cdot 4=8\cdot 13$ tons.

Botton in tension. Keel $p=\frac{M}{I}y_2=\frac{320000}{920000}\times 22\cdot 1=7\cdot 68$ tons sq. in.

*Results for comparison with other ships.*

Name of ship:—
Description:—
Material—iron or steel.
Dimension:—Length 530 ft., breadth 65 ft. 0 in., depth 45 ft. 0 in.
Draught:—Forward 16 ft. 7 in., aft 16 ft. 7 in.
Condition of loading:—Ship complete. Cargo in. Coal out.
Position of section:—Amidships.
If *hogging*, top in tension—bottom in compression.
If *sagging*, top in compression—bottom in tension.

| | | | | |
|---|---|---|---|---|
| 1. | Area material above neutral axis | =1703·2 | sq. ins. | |
| 2. | ,, ,, below ,, | =1891·6 | ,, | |
| 3. | Total area of section of material | =3594·8 | ,, | |
| 4. | Moment of area above or below neutral axis | =27800 | ,, | × feet. |
| 5. | Moment of inertia above neutral axis | =424000 | ,, | × sq. ft. |
| 6. | ,, ,, below ,, | =534150 | ,, | ,, |
| 7. | Total moment of inertia about neutral axis | =958150 | ,, | ,, |
| 8. | Height of neutral axis above keel | =19·54 | feet | |
| 9. | Ratio of (8) to moulded depth | =·434 | ratio | |
| 10. | Greatest distance of any particle above N A | =26·5 | feet | |
| 11. | ,, ,, ,, below ,, | =19·6 | ,, | |

## TABLE XXXIX.—*continued.*

***Shearing Stress for comparison with other ships.*** **See fig. 178.**

| | | |
|---|---|---|
| (*a*) | Maximum shearing force forward | 1200 = tons. |
| (*b*) | ,, ,, ,, aft | 1130 ,, |
| (*c*) | $\frac{(a)}{\text{Dispt.}}$ | ·133 = ratio. |
| (*d*) | $\frac{(b)}{\text{Dispt.}}$ | ·125 = ,, |
| (*e*) | Section (*a*) from fore perpendicular | 23 = per cent. |
| (*f*) | Section (*b*) ,, ,, | 20·5 = ,, |
| (*g*) | Mean shearing stress caused by (*a*) | ·55 = tons/sq. inch. |
| (*h*) | ,, ,, ,, ,, (*b*) | ·54 = ,, |
| (*i*) | Maximum ,, ,, (*a*) | 3·0 = ,, |
| (*j*) | ,, ,, ,, (*b*) | 2·9 = ,, |
| (*k*) | Thickness of material in way of (*i*) | $\frac{11}{16}''$ = inches. |
| (*l*) | ,, ,, ,, (*j*) | $\frac{11}{16}''$ = ,, |
| (*o*) | $\frac{i}{g} = \frac{\text{Max. shearing stress at } (a)}{\text{Mean shearing stress at } (a)}$ | = 5·55 ratio. |
| (*p*) | $\frac{j}{h} = \frac{\text{Max. shearing stress at } (b)}{\text{Mean shearing stress at } (b)}$ | = 5·37 ,, |

# CHAPTER XXII.

## ABILITY OF STRUCTURES TO RESIST STRESSES.

Many considerations affect the strength of a ship's structure. The section that is obviously weakest for ultimate strength is usually through a frame line in the region where the maximum bending moment occurs. This weakness is due to the rivet holes necessary for the rivets which unite the transverse framing at this section to the longitudinal parts, and we have seen that the assumption generally made as to the necessary deduction is *one-eighth for parts in tension.* It may, however, be the case that a section between two frames is weaker than a frame-line section on account of openings in the deck or shell plating where these openings are not fully compensated for. Many questions also affect the consideration of the ability of several of the parts to take up their proportion of the stress.

In tension the *ultimate* strength of any of the longitudinal parts, such as deck plates, girders, or any other continuous part of the structure, is equal to the *ultimate* strength of the riveted joint.

The length of these continuous parts is limited by practical considerations as to the length which can be worked either in the steel mill or the shipyard. Therefore, in the region of any transverse section, one or more joints of the main longitudinal parts may occur. It is the aim, however, to arrange a shift of joints or butts, so that as few joints come together or near any transverse section as possible.

In compression, longitudinal parts depend for strength on their individual stiffness and also on the stiffness of intercostal parts. It may be taken that in tension, intercostals, which are generally in the form of plates fitted to the shell or deck between frames, floors, or beams, only resist longitudinal stresses to the extent of their rivet connections. In large double-bottom steamers the inner and outer bottoms are usually connected by intercostal longitudinals which are riveted to the floors, the connection either being made by a flange on the plate or by an angle bar. When the bottom parts are in tension these longitudinals are able only to contribute to the total strength of the ship the strength of their rivet connections to the floors.

It is to be expected that this is not a very large proportionate amount of the full strength of the plate forming the intercostal, and also that the strength of the rivet connection almost entirely depends on the workmanship of the riveting. In a moment of inertia calculation for sagging, therefore, it is more nearly correct to leave these parts out than to include them.

Again, in these same parts which, when under hogging, are in compression, it is more correct to include them, because if closely fitted to the floors they are able to take up directly a large portion of the compressive force.

The most important use of intercostals, however, is to stiffen the shell plating or connect the shell plating with stiff longitudinals. It is well known that thin plating unstiffened withstands very little compression, but that thin plating stiffened by a web or girder, which itself may not be receiving the stress directly (such as an intercostal in a ship's structure), withstands a good deal of compression.

The same is true of any other intercostals, such as are fitted to connect fore and afters with the deck plating, only it has to be borne in mind that when the neighbouring parts are in tension the intercostal takes up very

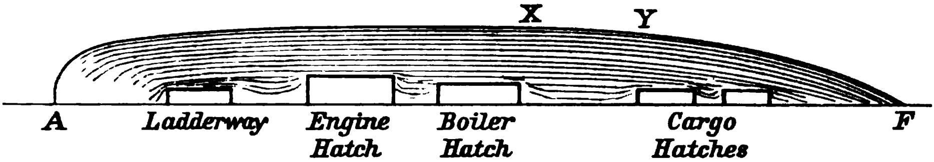



FIG. 179.

little of the stress; consequently a deck intercostal plate to a fore and after should only be included in a sagging calculation, and even in such cases the fit of the intercostal at its ends can seldom be good enough to justify more than the assumption that the intercostal does more than to enable the thin plating to efficiently resist compression in proportion to its sectional area. The effect of an opening in the deck in way of the section is generally to reduce the strength, not so much because the area of deck plating is reduced, but because there is an irregularity caused in the distribution of that area. The area of the deck plating cut away is generally compensated for in some form or other; for instance, at a cargo hatch, a stiff coaming is fitted round the edge of the opening. It would not be quite correct to assume, however, that this coaming takes up the stress in proportion to its area. If we

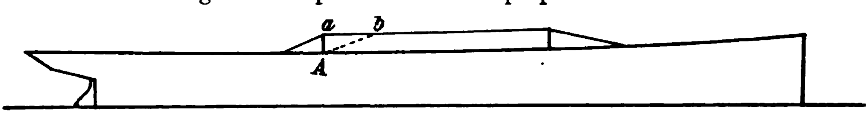



FIG. 180.

examine the distribution of the tensile forces coming on deck, we see that if the deck were uniform in cross section the stress lines would be parallel, as in the part from X to Y in fig. 179.

Where an opening occurs, the stress lines diverge before they come to the opening and follow lines round the corners of the opening, as shown. Therefore the stress is obviously increased at the corners. If the thickness of the plating remains the same, the unital stress is therefore increased. To compensate for this it is usual to fit thick patches or doublings at the corners of openings so that the area of plating is increased at these places. If the opening is rounded at the corners, the serious danger of having a sharp discontinuity, and its consequent increased stress, is very much lessened.

In the shell plating there is generally a butt to every three adjacent unbutted strakes, and this may also weaken the section appreciably.

Discontinuities in some cases exist in the top sides of vessels. It is sometimes the practice to carry the strength of the upper deck amidships up to the bridge deck. The efficiency of this arrangement depends upon the length of the bridge deck, and also upon the method adopted for strengthening at the break.

The figure 180 shows a case of this kind. The directions of the stress lines can be easily imagined. If the bridge structure were abruptly ended at A like A *a*, then the part A *a b* would be of little use for strength, and at A the unital stress, unless the area of material was much increased, would be great. If, however, it is carried on as at *a c*, the part A *a b* would be of use, and the stress at C would not be great.

The strength of the bridge should therefore be as continuous as possible at its ends. This consideration applies to similar features in those types of ships with raised quarter-decks ending at midships, etc., or at breaks for gangways in the sheerstrakes.

**Riveted Joints.**—In determining the weakness caused by the presence of a butt in shell or deck plating, it is necessary first to examine the strengths of riveted joints.

Mild steel is the material most commonly used in ship construction. It is only for special purposes in some ships that wrought-iron is used. Steel is of many qualities, but mild steel contains a relatively small proportion of carbon. The percentage of carbon in steel varies from about ·1 to at most 2 per cent.

Ordinary steel has the following characteristics:—

TABLE XL.

| | Per cent. Carbon. | Modulus of Elasticity lbs./□″. | Ultimate Tensile Strength lbs./□″ T. | Per cent. Elongation in 8″. | Ultimate Comp. Strength lbs./□″ C. | Ultimate Shearing Strength lbs./□″ S. |
|---|---|---|---|---|---|---|
| Mild Steel . . | ·14 | 32000000 | 63000 | 19 to 24 | 68000 | 48500 |
| | ·19 | 31000000 | 68000 | 18 ,, 22 | 76000 | 52800 |
| High Tension . | ·46 | 32000000 | 80000 | 17 ,, 18 | 90000 | 51800 |
| | ·51 | 31000000 | 79000 | 12 ,, 16 | 99600 | 57000 |
| Carbon Steel . . | ·54 | 30600000 | 79000 | 17 ,, 18 | 86900 | 55900 |

In the latest high-speed vessels, especially torpedo-boat destroyers, where everything is done to get as much strength as possible for a given weight of hull, material can be used which possesses a higher tensile strength than mild steel. Nickel steel (steel with a small percentage, about 5, of nickel in its composition) is also used when a high tensile strength is required.

In conjunction with steel plates, iron or steel rivets are used.

The principal kinds of joints are shown in fig. 181. These figures serve to show the varieties of joints in the deck or shell plating of a ship. Treble riveted laps and treble and quadruple riveted butts are necessary in large ships with heavy plates.

In the following calculations let—

$p$ = pitch of rivets, *i.e.* distance apart centre to centre.
$d$ = diameter of rivet.
T = ultimate tensile stress per sq. in. = 65,000 lb.
C = ultimate crushing „ „ = 70,000 „
S = ultimate shearing „ „ = 50,000 „

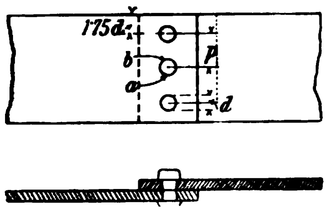



**Fig. 181a.**—Single riveted Lap.

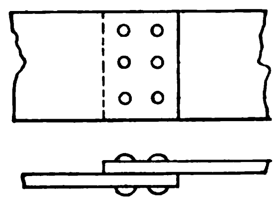


**Fig. 181b.**—Double riveted Lap.

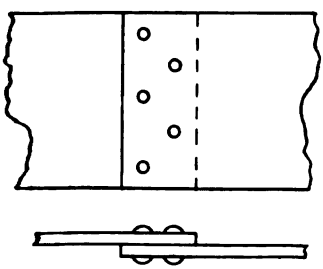


**Fig. 181c.**—Zigzag riveted Lap.

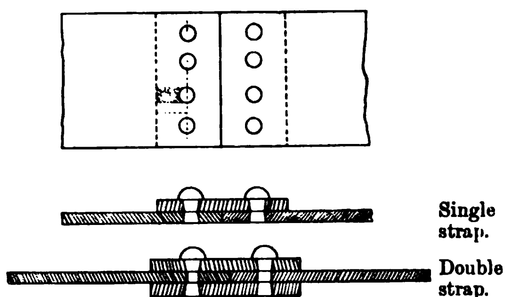



**Fig. 181d.**—Single riveted Butt.

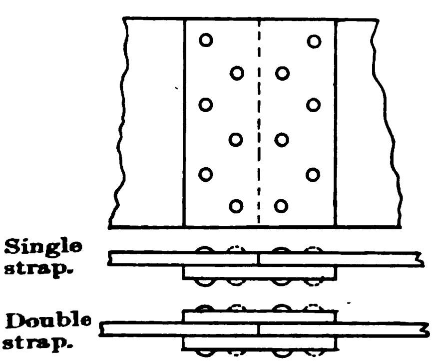



**Fig. 181e.**—Zigzag riveted Butt.

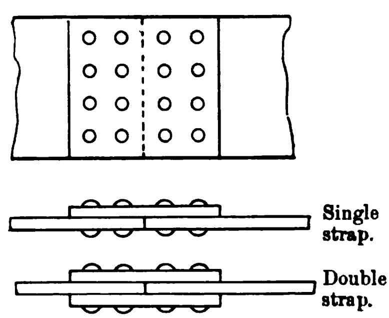



**Fig. 181f.**

**Cases of a Single riveted Lap Joint** (see fig. 181a).—In this case the **rivets** are in single shear. It will be seen that only an approximate determination of the distribution of stress in a riveted joint can be made.

For, in order that the plates and rivets, straps, etc., may take up a stress in proportion to the amount of the material, an absolutely accurate adjustment of these parts must be attained, but all workmanship

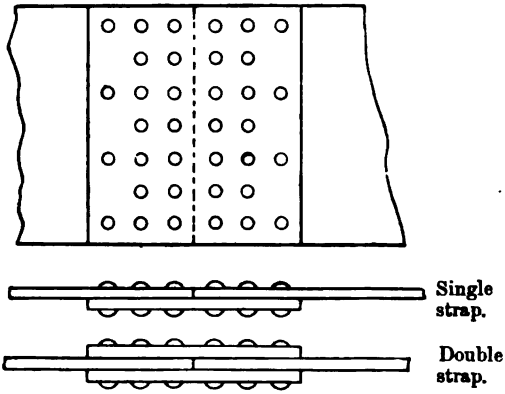



FIG. 181G.—Treble riveted Butt.

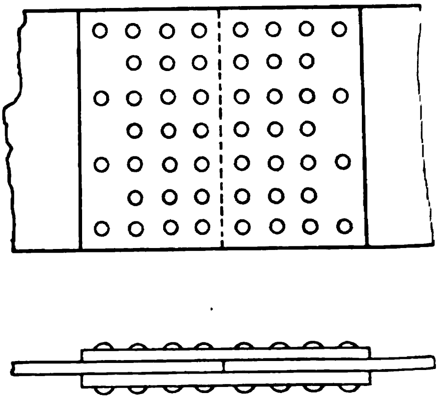


FIG. 181H.—Quadruple riveted Butt.

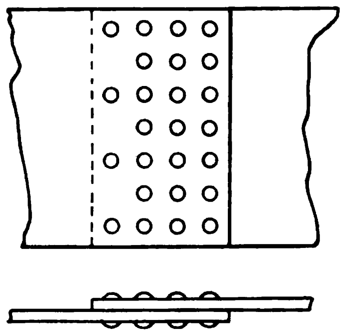


FIG. 181K.—Quadruple riveted Lap.

is more or less imperfect, so that this condition cannot always be realised. Also, when the joint is under the action of a severe pull, there are many features that tend to disturb the ideal conditions in a joint. The plates

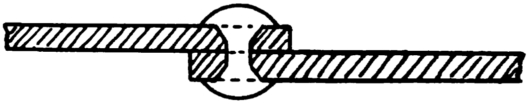


FIG. 182.

of a single riveted lap joint would tend to bend, as shown in the figure 184.

The rivets in this case would not be under the action of a pure shear. The material of the plate at the point where the rivet bears hardest, say $a$, is compressed, while the part at $b$ is in tension, so that at some point between $a$ and $b$ there will be no stress.

Assume, however, that the rivets be under the action of a shearing stress only.

Breadth of unpunched plate $= b$.
Thickness of unpunched plate $= t$.
$\therefore$ Area of unpunched plate $= bt$.
$\therefore$ Strength of unpunched plate $= bt\text{T}$.

Consider a section of the plate through the line of rivet holes and let there be $n$ rivets.

Breadth of punched plate $= b - nd$.
Thickness of punched plate $= t$.
$\therefore$ Area of punched plate $= (b - nd)t$.
$\therefore$ Strength of punched plate $= (b - nd)t\text{T}$.

In order to make the most efficient joint of this type, the tensile strength of the punched plate should be equal to the shearing strength of all the rivets, since these are the two most likely ways in which the joint may break.

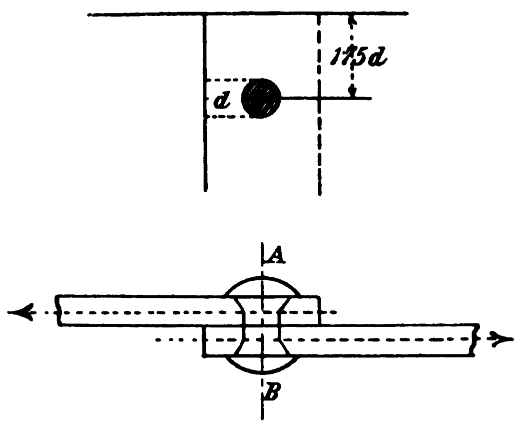



FIG. 183.

Shearing stress of $n$ rivets $= \frac{\pi d^2}{4} \times n \times \text{S}$.

$\therefore$ For the most efficient joint

$$\frac{n\pi d^2 \text{S}}{4} = (b - nd)t\text{T}.$$

It is a common practice to make the breadth from the centre of rivet to the edge equal to $1{\cdot}75d$.

$$\therefore \quad b = (n - 1)p + 3{\cdot}5d.$$

In these two equations the unknowns are $n$, $d$, $p$, so that if any one of these quantities be previously decided, the other two can be obtained from the equations.

**Bending of Lapped Joint under Tensile Stress.**—It will be noticed that we have assumed the stress to be uniform across the section of each plate. This is far from being the case. Take the same example of joint, and suppose the plates to be under a pull stress as shown in the figure 183. Let the mean intensity of stress on each unpunched plate be T. Then the pull $= T \times t \times p$. Suppose the breadth of strap to equal $p$. There is a couple acting on each plate and tending to bend it; if we assume the line of resultant stress for each to be midway in each plate, the couple $= Tpt \times t = Tpt^2$.

Mean intensity of stress at $AB = T_1$.

$$T_1 = \frac{Tpt}{(p-d)t} = \frac{Tp}{p-d}.$$

Let the tensile stress due to the couple $Tpt$ be K,

$$K = \frac{M}{I} \times \frac{t}{2} = \frac{Tpt^3}{2I} = \frac{6Tp}{(p-d)} = 6T_1$$

∴ Greatest stress on the material at A B due to pull of $Ttp$,

$$= T_1 + K = 7T_1.$$

In the testing machine, however, the resultant line of stress in each plate might pass nearer to the centre of the joint. In this case the tensile stress due to bending (deduced in the same manner as on the first assumption) would be given by

$$\frac{Tp}{(p-d)} + \frac{6Tp}{(p-d)x}$$

where $x$ is the proportion of $\frac{t}{2}$ that the line of pull in each plate is nearer the centre of joint.

These investigations are based upon the virtual assumption that the plates remain straight under the action of a pull. This condition of things does not

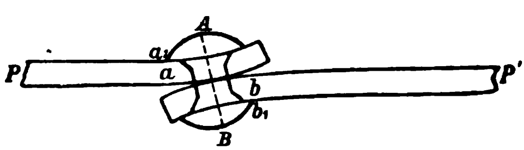



FIG. 184.

exist. It will approximately exist when the stresses are much below the elastic limit, but when the stresses become greater the tendency of the plates is to bend as shown in fig. 184. The greatest stress, which is the mean stress due to the pull, with the additional tensile stress due to bending, occurs at $a$ and $b$. So that at these points the material will stretch more than at $a_1$

and $b_1$ where the action of the pulling couple is to cause a compression. The result is to bring the centre lines of the plates P and P′ more nearly into the same line, thus lessening the bending couple. The stress in the section A B will then ultimately become more uniformly distributed. If the plates are thin they will bend more easily, and the joint may yield to the action of the bending couple before the stress reaches the elastic limit of the material at A. This is not likely to be the case in thick plates, where the stresses might easily be beyond the elastic limit at A before P and P′ are in line. Hence the mean intensity of stress in a thick plate, other things being equal, at the instant of breaking, will be considerably less than that in a thin one. This points to the conclusion that a lap joint with thin plates is relatively stronger than one with thicker plate. This conclusion is verified by experiment.

**Effect of Punching.**—The existence of a round hole in a specimen generally tends to increase slightly the ultimate strength of the material in way of the hole. This effect, however, is modified by the injury to the material round the hole if punched. Again, if the hole is in a joint, there is a bearing pressure between the rivet and plate which affects the distribution of stress when the joint is being pulled. The rivet or rivets for one plate may be considered as a form of grip. It therefore follows that the stress will be greater in the material of the plate immediately in the vicinity of the holes, than that in the centre of the material between the holes, and this unequal distribution will be increased by the bearing pressure of the rivet. These two influences tend to reduce the ultimate strength of the material of the joint.

In order to avoid decrease in the ultimate strength, the punched hole should be reamed or the material annealed after the punching operation. It has been found that after reaming the hole $\frac{1}{8}$ in., *i.e.* widening the diameter $\frac{1}{8}$ in., the injurious effect of the die has been done away with. With regard to drilling or reaming holes after they are punched, it is good practice to remove the sharp corners. In rivet holes in ship plates a fair amount of countersink is usually given to the holes. Lloyd's practice can be seen from their tables of rules for rivet spacing. The removal of sharp corners greatly lessens the liability to a sharp shearing action on the rivet. This consideration is important in the material at the neck of the rivet, where a sharp corner might easily cause a breaking off of the rivet.

It has been found experimentally that reaming the holes in steel plates certainly increases the ultimate strength of the joint, as compared with that of a joint where the holes have not undergone that operation.

The difference in the loss of strength due to punching is dependent also to a slight extent on the pitch of the rivets, and also on the thickness of the plate.

The increase in ultimate strength of a plate with holes drilled in it varies from 20 per cent. downwards, according to the pitch of the holes and thickness of plate.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| In a $\frac{3}{8}$ in. | steel plate | holes | 1·9 | diam. | apart increase | = 20 | per cent. |
| ,, $\frac{3}{8}$ in. | ,, | ,, | 3·9 | ,, | ,, | = 6 | ,, ,, |
| ,, $\frac{3}{4}$ in. | ,, | ,, | 1·9 | ,, | ,, | = 20 | ,, ,, |
| ,, $\frac{3}{4}$ in. | ,, | ,, | 2·8 | ,, | ,, | = 1·8 | ,, ,, |

Loss or otherwise due to punching, plating of steel of 30 tons ultimate strength,

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{1}{4}$ | in. | plate | increased to | 32 | tons | per sq. in. |
| $\frac{3}{8}$ | ,, | ,, | ,, | 31 | ,, | ,, |
| $\frac{7}{16}$ | ,, | ,, | remained at | 30 | ,, | ,, |
| $\frac{1}{2}$ | ,, | ,, | fell to | 29 | ,, | ,, |
| $\frac{9}{16}$ | ,, | ,, | ,, | 28 | ,, | ,, |
| $\frac{5}{8}$ | ,, | ,, | ,, | 27 | ,, | ,, |

The above figures are for comparatively closely pitched holes.

**Strengths of Joints generally.**—Consider the different types of joints according to the first assumption, namely, that the stress on each plate is uniformly distributed, and that the plates remain in the same line after being pulled.

*For a single-riveted butt with single strap* the consideration is similar to that of a single-riveted lap.

*In the case of a single-riveted butt with double straps*, however, the rivets are in double shear. The ultimate tensile strength of the plate is the same for the same arrangement and size of rivets as for a lap, except that there is no stress due to the bending of the plate. If possible we want to make the tensile strength equal to the shearing strength of the rivets.

$$\text{Tensile strength} = (b - nd)tT.$$

$$\text{Strength of rivet in single shear} = \frac{\pi d^2}{4}S.$$

From experimental results the strength of a rivet in *double* shear is equal to 1·75 that of rivet in *single* shear.

$$\therefore \quad \text{Strength of a rivet in double shear} = \frac{1{\cdot}75\pi d^2 S}{4}.$$

$$\therefore \quad \text{,,} \quad n \text{ rivets} \quad \text{,,} \quad \text{,,} = \frac{1{\cdot}75 \times nd^2\pi S}{4}.$$

$$\therefore \quad (b - nd)tT = \frac{1{\cdot}75 n\pi d^2 S}{4}$$

$$\text{and as before} \quad b = (n-1)p + 3{\cdot}5d.$$

These two equations give the relation of $d$ and $p$ in a single-riveted butt with double straps.

*In a double-riveted or zigzag-riveted lap* there is twice the area of rivets to shear, and the same area to resist tension as in the case of single-riveted lap with same size and spacing of rivets.

The diameter of the rivets can therefore be smaller in a double-riveted lap than in a single-riveted lap, thus giving a greater ultimate tensile strength. The formula for equality of shearing and tensile strengths is therefore

$$(b - nd)tT = \frac{2n\pi d^2 S}{4} \quad \text{where } n = \text{number of rivets in one row}$$

$$= \frac{n\pi d^2 S}{2}.$$

*In a double-riveted or zigzag-riveted butt with double straps* the formula becomes

$$(b - nd)t\mathrm{T} = \frac{1{\cdot}75 n\pi d^2 \mathrm{S}}{2}$$

In *treble riveting* the formula becomes

*Lap or single butt* $(b - nd)t\mathrm{T} = \frac{3n\pi d^2\mathrm{S}}{4}.$

*Butt with double straps* $(b - nd)t\mathrm{T} = \frac{1{\cdot}75 \times 3n\pi d^2\mathrm{S}}{4}.$

**Breadth of Lap.**—The amount of lap is fixed with regard to the consideration of crushing of the material behind the rivet. From the figure the method of breaking shown is a likely one in a single-riveted joint. This could easily be prevented by giving sufficient lap. It will be seen that at least the distance between the rivet and edge of plate should be equal to a half the space of material between the rivet holes, otherwise the material will shear along the lines *a b*. The allowance usually made in practice as regards the space between the centre of rivet and the edge of lap is :—$1{\cdot}75\ d$ = space of centre of outside rivet from landing edge of plate. (See Lloyd's Spacing Rules.)

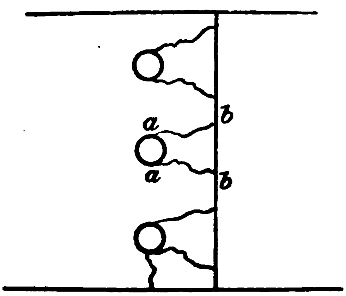


FIG 185.

**Pitch and Diameter of Rivets.**—If we want to make the crushing strength of the material behind the rivet equal to the breaking stress between the rivets, the following is the equation :—

$$(p - d)t\mathrm{T} = dt\mathrm{C}$$

or $$(p - d)\ \mathrm{T} = d\mathrm{C},\ p = d\left(1 + \frac{\mathrm{C}}{\mathrm{T}}\right)$$

T = 65000 average result lbs. per square inch.

C = 70000 " " "

$$\therefore \quad p = \frac{27}{13}d.$$

This gives a value for $p$ which is too small, as it would make $n$ too large and the value of $(b - nd)t\mathrm{T}$ (the strength of the plate) too small. Making the shearing strength of one rivet equal to the crushing strength of the material behind the rivet, the equation is

$$\frac{\pi d^2\mathrm{S}}{4} = dt\mathrm{C}.$$

$$d = \frac{4\mathrm{C}t}{\pi\mathrm{S}}$$

$$= \frac{9}{5}t.$$

Or $d$ should not be greater than $1\frac{1}{8}$ times $t$.

Let $n$ = number of rivets in a row of single riveting.

$$b = p(n-1) + 3\cdot5d \qquad \fallingdotseq 2d(n + \tfrac{3}{4}) \text{ if } p = 2d.$$

$$b = d(n-1)\left(1 + \frac{C}{T}\right) + 3\cdot5d \fallingdotseq 2\cdot07dn + 1\cdot43d \text{ if } p = d\left(1 + \frac{C}{T}\right)\frac{27d}{13}$$

$$n = 1 + \frac{b - 3\cdot5d}{d\left(1 + \frac{C}{T}\right)} = \frac{b - 1\cdot43d}{2\cdot07d} = \frac{b}{2\cdot07d} - \cdot7,$$

which may be written approximately $= \dfrac{b}{2d}$.

$$\therefore \quad \text{Efficiency of combination} \quad = 1 - \frac{nd}{b} = 1 - \left(\frac{d}{b} + \frac{1 - \frac{3\cdot5d}{b}}{1 + \frac{C}{T}}\right)$$

$$= \frac{1}{2} + \frac{\cdot7d}{b} \text{ approximately.}$$

The question of the material of the rivet in relation to the material of plate is of importance. With nickel steel or high tensile steel, nickel steel or crucible rivets have to be used.

Crucible steel has a reputed elastic limit of 38 tons per square inch, and breaks at 50 tons per square inch.

Lloyd's give as value of $\frac{S}{T}$ for rivets in boiler work ·85, and the Board of Trade, ·825.

In steel of higher tensile strength than 30 tons per square inch the shearing strength does not increase so rapidly as the tensile strength.

| Tensile Strength. | Shearing Strength ratio $\frac{S}{T}$. |
|---|---|
| 30 | ·8 |
| 36 | ·72 |
| 52 | ·63 |

This constitutes a difficulty in the adoption of high tensile steel, as the percentage strength of the riveted joints is not likely to be so great.

**True Area of Rivets and Plates for Strength.**—The deduction that has been made in the foregoing work for rivet holes is not quite correct. The area assumed for a rivet hole has been $dt$. A more correct basis would be to calculate the loss of area due to the countersinking as well.

It is usual to take the working stresses as being those on the area of the plate left when the area for rivet holes is deducted as if the holes had been bored through parallel. If the strength of the plate is a little less than the strength of the rivets we have a fairly satisfactory result.

Generally it is found that work fails through the rivets slacking. This may readily cause fracture of the plate. It is therefore important to err on the side of having too much rivet area rather than too little. Rivet strength for tensile and compression has therefore to be greater than the strength of plate through the weakest section. Apart from this increase of rivet area in part under tension, the tendency is to increase the number of rivets in edge laps and to increase the number of frame rivets, even at the expense of the frames.

**Strength of a Butt or Lap Connection.**—We can now examine the nature and extent of the weakening of a section through a frame line caused by the presence of a butt in the shell or deck plating. If the butt causes a weakness to the section, then in the region of that section the line of fracture would extend to the butt. The estimation of the strength of the butt is done by calculating the resistance to breaking of the combination of a butt joint and a certain number of adjacent whole strakes (see fig. 186). This number of whole strakes is the number of passing strakes on either side of the butt, and is equal to two or three (the practice most commonly adopted). It is usual to take one strake on each side of the butt as contributing to the strength of the butt.

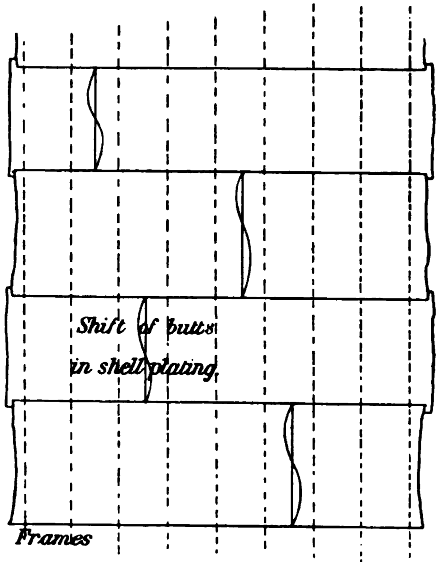



FIG. 186.

Consider the example given in fig. 187 as one of the butts in fig. 186.

Frame spacing = 28 in.
Butt strap = 19 „
Edge lap = 6 „
Breadth strake = 60 „
Rivet diameter = 1 „
Thickness = $\frac{3}{4}$ „
Spacing rivets in butt $3\frac{1}{2}$ in.
„ „ in edges $4\frac{1}{2}$ „
„ „ in frames 7 „

The line of fracture, if the ship were breaking, would generally be in frame line A B for the whole plates. In the region of the butt it would depend on whether there was a weaker way in which this strake could break.

(1) Consider a section through the frame line of rivets. Spacing the rivets, we find there will be eight in frame A B for this strake *plus* two in each landing, making twelve in all. Sometimes one only in each landing is fitted.

$\therefore$ Sectional area at frame line $= 60 \times \frac{3}{4} - 12 \times 1'' \times \frac{3}{4}$
$= 36$ sq. ins.

If strength of material = 30 tons per sq. in.
Then strength at this section $= 36 \times 30$
$= 1080$ tons.

The standard of strength is the standard of three strakes, two without and one with a butt, *i.e.* strength of three strakes through frame line = 3240 tons.

(2) Suppose this frame line of the butted plate to remain intact, but all the rivets between the butt and the frame line to shear:

Number of rivets in a row = 14 + 4 in the landings = 18
∴ " " 3 " = 54
In addition there are four rivets in landing to shear.

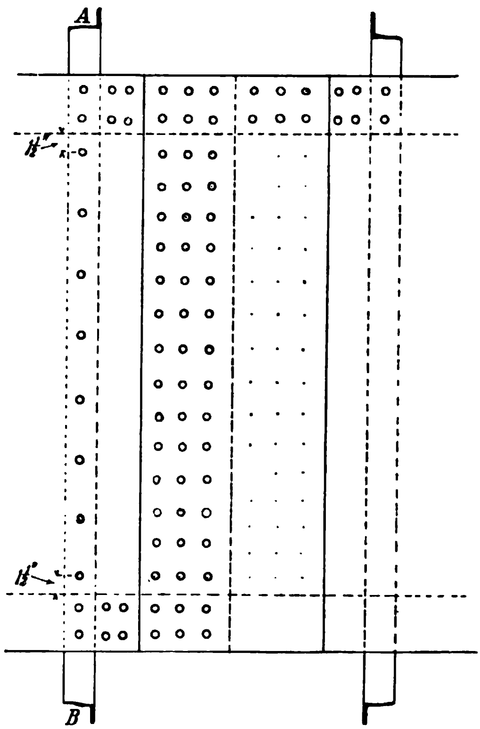



FIG. 187.

∴ Total number of rivets to shear = 58
Shearing strength of one rivet = ·7854 × 22
= 17·28 tons.
∴ Shearing strength of all the rivets = 17·28 × 58
= 1002·2 tons.

Being less than the strength 1080 tons required to tear through the frame line of rivets, the break would occur more readily by shearing the rivets.

(3) Suppose the rivets in the landing to shear and the plate to tear along the outer row of rivets in the strap,

Number of rivets to shear = 8
Strength required = 8 × 17·28
= 138·2 tons.

Area resisting tearing (number of rivets in the row = 18),

$$= 60 \times \tfrac{3}{4} - 18 \times \tfrac{3}{4}$$
$$= \tfrac{3}{4}\,(42) = 31 \cdot 5 \text{ sq. in.}$$

∴ Strength to resist tearing – 31·5 × 30
= 945 tons.

∴ Total strength to resist this method of breaking

= 945 + 138
= 1083 tons.

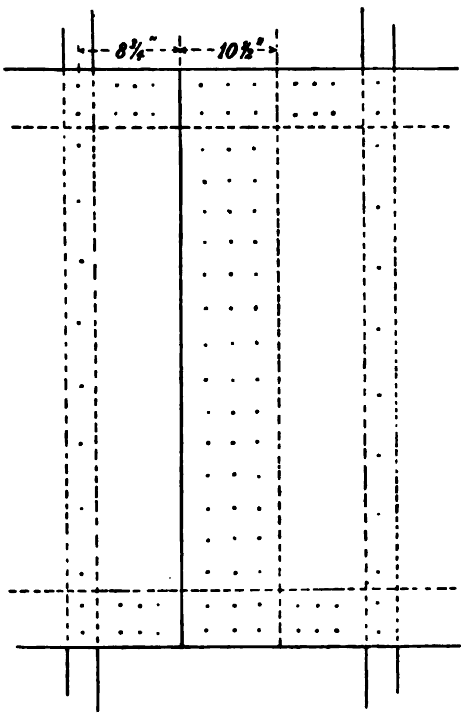



FIG. 188.

This method is stronger than (2) and (1).

If the joint had been lapped this would have left room for more rivets in the landings. See fig. 188.

Case (1) remains the same, viz. 1080 tons.

„ (2) There are two extra rivets in each landing to shear four in all.

In butt strap there were 58.

In this case „ 62.

∴ Strength to resist shearing = 62 × 17·28 = 1071·4 tons, which is only $8\frac{1}{2}$ tons less than (1), and considerably more than case (2) for the butt strap.

Case (3) has the same extra strength over former case (3) of the butt strap, *i.e.* shearing of four rivets $= 4 \times 17{\cdot}28$
$= 69{\cdot}1$ tons.

$\therefore$ Total strength $= 1083 + 69$
$= 1152$ tons.

Had the joint been made up with double-butt straps, then the shearing strength of each rivet in the strap would have been 1·75 times the single shear, and case (2) would have been much stronger than case (1). Cases (1) and (3) would remain the same.

Results :—

| | Single-butt Strap. | Lap. | Double butt Strap. |
|---|---|---|---|
| Case (1) | 1080 | 1080 | 1080 |
| „ (2) | 1002 | 1071 | Plenty of strength |
| „ (3) | 1083 | 1152 | 1083 |

A lapped joint is generally a very good joint, though in some places it is not desirable. One additional reason for its adoption in preference to a butt with a single strap is that the latter joint, due to bending, tends to open out at the caulked seam. Sometimes a fore and aft stiffening angle is fitted to prevent the strap bending. The lapped joint also has the economical advantage of having only half the number of rivets and less plate than a strapped butt has. The double-butt strap is the best joint, and is a good joint for caulking. It requires, however, about twice the amount of caulking and riveting that a lapped joint does. It is a better-looking job, and is used in the sheerstrakes and upper-deck stringers of large steamers.

**Tests on Riveted Joints of thin Plating.**—The appended Tables XLI. to XLVIII. give results of an extensive series of tests that were carried out at Glasgow University in 1902. The specimens were made from thin plating, such as is used in the construction of torpedo-boat destroyers. From mild steel plates of 5 lb. and 6 lb. per sq. ft. (equal to about $\frac{1}{8}$ of an inch in thickness) two sets of specimens were made, each set consisting of five groups.

Group A, Plain strips – all lengthwise and across.
„ B, „ „ – hammered locally, similarly to work done in straightening plates.
„ C, „ „ – holes punched in them.
„ D, Riveted joints to test strength of joint.
„ E, „ „ „ shearing strength of rivets.

A few tests were also carried out on specimens cut from a 6 lb. high tensile steel plate, and on specimens cut from plates taken from destroyers undergoing repairs.

TABLE XLI.

Specimens all cut from a 5 *lb. Mild Steel Plate.* (All 20″ long.)

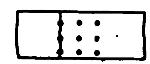
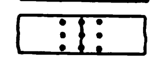
| Group and Number. | Description and Size. | Sectional Area (sq. ins.) | Total Breaking Load in Tons | Ultimate Tensile Strength. Tons/sq. in. | Percentage Extension on 10″. | Remarks. |
|---|---|---|---|---|---|---|
| A. 1 | 1″ broad; cut lengthwise | 0·123 | 3·84 | 31·21 | 15·20 | Uneven fracture. Central. |
| A. 2 | $1\frac{5}{16}$″ broad; cut lengthwise | 0·146 | 4·28 | 29·31 | 15·85 | Oblique ,, 5″ from end. |
| A. 3 | $2\frac{1}{2}$″ ,, ,, ,, | 0·307 | 9·20 | 29·96 | 18·92 | ,, ,, $5\frac{1}{4}$″ ,, |
| A. 4 | $3\frac{15}{16}$″ ,, ,, ,, | 0·484 | 14·20 | 29·34 | 21·15 | Uneven ,, $5\frac{1}{4}$″ ,, |
| A. 5 | $2\frac{1}{2}$″ ,, ,, across | 0·307 | 8·90 | 29·00 | 15·28 | ,, ,, Central. |
| A. 6 | $3\frac{15}{16}$″ ,, ,, ,, | 0·484 | 13·24 | 27·35 | 22·50 | Oblique ,, ,, |
| | Averages . . . . | ... | ... | 29·36 | 18·15 | |
| B. 1 | Same as A. 1; hammered locally | 0·123 | 3·46 | 28·13 | 16·93 | Oblique fracture; $6\frac{1}{2}$″ from end, clear of hammered part. |
| B. 2 | ,, A. 2; ,, ,, | 0·161 | 4·41 | 27·40 | 16·25 | Oblique fracture; 8″ from end, through hammered part. |
| B. 3 | ,, A. 3; ,, ,, | 0·307 | 8·56 | 27·88 | 20·00 | Oblique fracture; $6\frac{3}{4}$″ from end, clear of hammered part. |
| B. 4 | ,, A. 4; ,, ,, | 0·484 | 14·03 | 29·00 | 20·00 | Oblique fracture; $6\frac{1}{2}$″ from end, clear of hammered part. |
| B. 5 | ,, A. 5; ,, ,, | 0·307 | 8·46 | 27·55 | 22·50 | Clean central fracture; clear of hammered part. |
| B. 6 | ,, A. 6; ,, ,, | 0·484 | 13·87 | 28·65 | 21·25 | ,, ,, ,, ,, |
| | Averages . . . . | ... | ... | 28·07 | 19·5 | |
| C. 1 | ,, A. 3; 4 $\frac{5}{16}$″ holes punched | 0·215 | 5·83 | 27·11 | very small and local |  |
| C. 2 | ,, A. 4; 9 $\frac{5}{16}$″ ,, ,, | 0·346 | 8·94 | 25·84 | ,, ,, |  |
| C. 3 | ,, A. 5; 4 $\frac{5}{16}$″ ,, ,, | 0·215 | (not tested properly 1·47) | (6·84) | ,, ,, |  |
| C. 4 | ,, A. 6; 9 $\frac{5}{16}$″ ,, ,, | 0·346 | 9·37 | 27·08 | ,, ,, |  |

TABLE XLII.

Riveted Joints. Specimens all cut from a 5 *lb. Mild Steel Plate.* (Rivets $\frac{5}{16}$" Diameter Galvd. Panheads.)

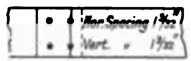
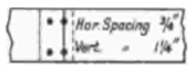
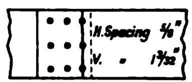
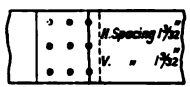
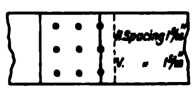
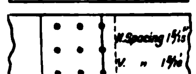
| Group and Number. | Description and Size. | Sketch. | Sectional Area (sq. in.). | Breaking Load in Tons. | Tensile Strength. Tons/sq. in. | Remarks. |
|---|---|---|---|---|---|---|
| D. 3 | $2\frac{3}{16}$" broad ; lapped $1\frac{1}{2}$" |  | ·165 | 4·97 | 30·12 | Broke through line of rivets on counter-sunk side. |
| D. 4 | $2\frac{3}{16}$" ,, ,, $2\frac{3}{16}$" |  | ·165 | 5·00 | 30·3 | ,, ,, ,, |
| D. 5 | $2\frac{1}{2}$" ,, ,, $1\frac{1}{2}$" |  | ·211 | 5·73 | 27·1 | ,, ,, ,, |
| D. 6 | $2\frac{1}{2}$" ,, ,, $2\frac{1}{2}$" |  | ·204 | 5·78 | 28·33 | ,, ,, ,, |
| D. 7 | $3\frac{9}{32}$" ,, ,, $2\frac{1}{2}$" |  | ·244 | 7·44 | 30·5 | ,, ,, ,, |
| D. 8 | $3\frac{9}{32}$" ,, ,, $3\frac{9}{32}$" |  | ·259 | 7·24 | 28·04 | ,, ,, ,, |
| D. 9 | $3\frac{5}{16}$" ,, ,, $3\frac{5}{16}$" |  | ·328 | 8·96 | 27·31 | ,, ,, ,, |
| D. 10 | $3\frac{5}{16}$" ,, rivets [illegible] ,, $3\frac{5}{16}$" |  | ·305 | 8·60 | 28·22 | ,, ,, ,, |

TABLE XLII.—*continued.*

Riveted Joints—*continued.* Specimens cut from a 5 *lb. Mild Steel Plate.* (Rivets $\frac{5}{16}$″ Diameter Galvd.)

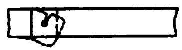
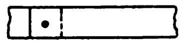
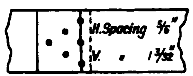
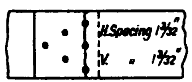
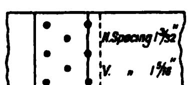
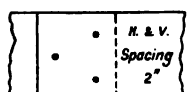
| Group and Number. | Description and Size. | Sketch. | Sectional Area (sq. in.) | Breaking Load in Tons. | Tensile Strength. Tons/sq. in. | Remarks. |
|---|---|---|---|---|---|---|
| D. 11 | 1″ broad ; lapped 1″ |  | ·071 | 1·83 | 25·77 | Broke through rivet line on countersunk side. |
| D. 12 | $1\frac{5}{16}$″ ,, ,, $1\frac{5}{16}$″ |  | ·110 | Shearing Force 1·80 | Shearing Force 16·43 | Rivet sheared. |
| D. 13 | $2\frac{3}{16}$″ ,, ,, $1\frac{1}{2}$″ |  | ... | 4·56 | ... | Broke through rivet line on countersunk side. |
| D. 14 | $2\frac{1}{2}$″ ,, ,, $1\frac{1}{2}$″ |  | ... | 4·91 | ... | Two rivets tore through and plate crushed at other rivet. |
| D. 15 | $3\frac{9}{32}$″ ,, ,, $2\frac{1}{2}$′ |  | ·253 | 7·12 | 28·21 | Broke through line of rivets on countersunk side. |
| D. 16 | $3\frac{9}{32}$″ ,, ,, $3\frac{9}{32}$″ |  | ·261 | 8·32 | 31·88 | Broke through line of rivets on pan-head side. |
| D. 17 | $3\frac{15}{16}$″ ,, ,, $3\frac{9}{32}$″ |  | ·321 | 8·47 | 26·40 | Broke through line of rivets on countersunk side. |
| D. 18 | 4″ ,, ,, 4″ (To test shearing strength) |  | ·331 | 6·16 | Shearing Force 18·60 | All rivets sheared. |

TABLE XLIII.

Rivet Shearing Tests. Specimens of Rivets from various Firms. ($\frac{5}{16}$" Rivets.)

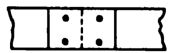
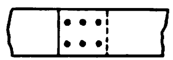
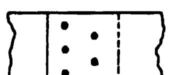
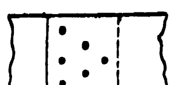
| Group and Number. | Description. | Sketch. | Sectional Area (sq. in.) | Breaking Load in Tons. | Shearing Stress. One Rivet. | Shearing Stress. Tons/sq. in. | Remarks. |
|---|---|---|---|---|---|---|---|
| D. 1 | $2\frac{3}{4}$" broad; 2 $\frac{5}{16}$" rivets, lap joint |  | ·220 | 4·21 | 2·105 | 19·06 | Sufficient strength of plating to shear all the rivets. |
| D. 2 | $2\frac{3}{4}$" broad; 2 $\frac{5}{16}$" rivets, butt |  | ·220 | 3·35 | 1·675 | 15·17 | ,, ,, ,, |
| E. 1 | 6 $\frac{5}{16}$" rivets (chain), snap-headed |  | ·6624 | 12·35 | 2·058 | 18·64 | ,, ,, ,, |
| E. 2 | 3 $\frac{5}{16}$" rivets (zigzag), snap-headed |  | ·3312 | 6·10 | 2·033 | 18·41 | ,, ,, ,, |
| E. 3 | 2 $\frac{5}{16}$" rivets, countersunk heads |  | ·2208 | 4·26 | 2·130 | 19·29 | ,, ,, ,, |
| E. 4 | 2 $\frac{5}{16}$" rivets, countersunk heads |  | ·2208 | 3·23 | 1·615 | 14·63 | ,, ,, ,, |
| E. 5 | 2 $\frac{5}{16}$" rivets, double butt-strap, panheaded |  | ·2208 | 6·36 | ... | ... | Plate crushed at one rivet. The other double-sheared. |
| E. 6 | 5 $\frac{5}{16}$" rivets (zigzag), countersunk heads |  | ·552 | 11·50 | 2·30 | 20·83 | Sufficient strength of plating to shear all the rivets. |
| E. 7 | 6 $\frac{5}{16}$" rivets (zigzag), pan-headed |  | ·6624 | 12·11 | 2·018 | 18·28 | ,, ,, ,, |
| | | | | | Average . | 18·04 | |

TABLE XLIV.

Specimens all cut 20″ long from a 6 *lb. Mild Steel Plate.*

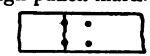
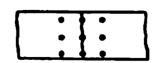
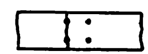
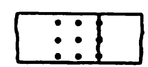
| Group and Number. | Description and Size. | Sectional Area (sq. in.). | Total Breaking Load in Tons. | Ultimate Tensile Strength. Ton/sq. in. | Percentage Ext. on 10″. | Remarks. |
|---|---|---|---|---|---|---|
| A. 1 | 1″ broad; cut lengthwise | 0·150 | 4·60 | 30·66 | 15·29 | Clean fracture; central. |
| A. 2 | 1 5/16″ ,, ,, ,, | 0·197 | 5·80 | 29·44 | 15·62 | ,, ,, 9″ from end. |
| A. 3 | 2½″ ,, ,, ,, | 0·375 | 9·80 | 26·18 | 19·00 | Oblique ,, 8″ ,, |
| A. 4 | 3 15/16″ ,, ,, ,, | 0·591 | 15·32 | 25·92 | 12·50 | ,, ,, 6¾″ ,, |
| A. 5 | 2½″ ,, ,, across | 0·375 | 11·35 | 30·26 | 16·40 | Uneven ,, 3½″ ,, |
| A. 6 | 3 15/16″ ,, ,, ,, | 0·590 | 17·80 | 30·17 | 17·80 | ,, ,, 4½″ ,, |
| | Averages . . | ... | ... | 28·76 | 16·10 | |
| B. 1 | Same as A. 1; hammered locally | 0·150 | 4·22 | 28·12 | 15·62 | Clean fracture; 7½″ from end, clear of hammered part. |
| B. 2 | ,, A. 2; ,, ,, | 0·197 | 4·80 | 24·36 | 13·25 | Clean fracture; 5½″ from end, clear of hammered part. |
| B. 3 | ,, A. 3; ,, ,, | 0·375 | 9·90 | 26·40 | 9·78 | ,, ,, ,, ,, |
| B. 4 | ,, A. 4; ,, ,, | 0·591 | 16·42 | 27·78 | 22·50 | Oblique fracture; 8½″ from end, through hammered part. |
| B. 5 | ,, A. 5; ,, ,, | 0·375 | 10·97 | 29·25 | 17·50 | Oblique fracture; 6″ from end, clear of hammered part. |
| B. 6 | ,, A. 6; ,, ,, | 0·591 | 17·00 | 28·81 | 17·78 | Clean fracture; 6″ from end, clear of hammered part, and through punch mark. |
| | Averages . . | ... | ... | 27·45 | 16·07 | |
| C. 1 | ,, A. 3; 4 ⅜″ holes punched | 0·243 | 4·00 | 16·41 | very small and local |  |
| C. 2 | ,, A. 4; 9 ⅜″ ,, ,, | 0·393 | 11·82 | 30·07 | ,, ,, |  |
| C. 3 | ,, A. 5; 4 ⅜″ ,, ,, | 0·243 | 7·58 | 31·10 | ,, ,, |  |
| C. 4 | ,, A. 6; 9 ⅜″ ,, ,, | 0·393 | 11·34 | 28·88 | ,, ,, |  |

TABLE XLV.

Riveted Joints. Specimens all cut from a 6 *lb. Mild Steel Plate.* (Rivets ⅜" Diameter Galvd. Panheads.)

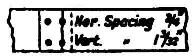
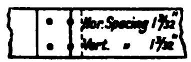
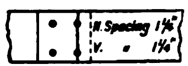
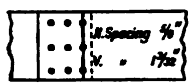
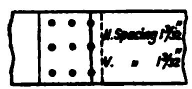
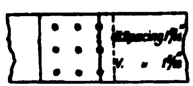
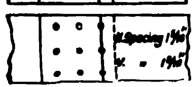
| Group and Number. | Description and Size. | Sketch. | Sectional Area (sq. in.) | Breaking Load in Tons. | Tensile Strength. Tons/sq. in. | Remarks. |
|---|---|---|---|---|---|---|
| D. 3 | $2\frac{3}{16}$" broad ; lapped $1\frac{1}{2}$" |  | ·187 | 5·35 | 28·53 | Broke through rivet line on countersunk side. |
| D. 4 | $2\frac{3}{16}$" ,, ,, $2\frac{3}{16}$" |  | ·1687 | 4·70 | 27·85 | ,, ,, ,, |
| D. 5 | $2\frac{1}{2}$" ,, ,, $1\frac{1}{2}$" |  | ·234 | 5·99 | 25·60 | ,, ,, ,, |
| D. 6 | $2\frac{1}{2}$" ,, ,, $2\frac{1}{2}$" |  | ·229 | 6·30 | 27·50 | ,, ,, ,, |
| D. 7 | $3\frac{9}{32}$" ,, ,, $2\frac{1}{2}$" |  | ·274 | 7·87 | 28·70 | ,, ,, ,, |
| D. 8 | $3\frac{9}{32}$" ,, ,, $3\frac{9}{32}$" |  | ·274 | 7·40 | 27·04 | ,, ,, ,, |
| D. 9 | $3\frac{1}{8}$" ,, ,, $3\frac{1}{8}$" |  | ·372 | 8·44 | 22·68 | ,, ,, ,, |
| D. 10 | $3\frac{1}{8}$" ,, ,, $3\frac{1}{8}$" |  | ·372 | 10·21 | 27·50 | ,, ,, ,, |
| | | | | Average . | 26·92 | |

TABLE XLV.—*continued.*

Riveted Joints—*continued.* Specimens cut from a 6 *lb. Mild Steel Plate.* (Rivets ⅜" Diameter Galvd. Panheads.)

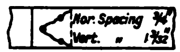
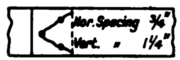
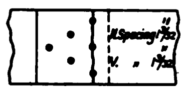
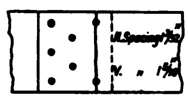
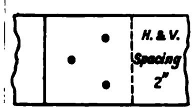
| Group and Number. | Description and Size. | Sketch. | Sectional Area (sq. in.) | Breaking Load in Tons. | Tensile Strength. Tons/sq. in. | Remarks. |
|---|---|---|---|---|---|---|
| D. 11 | 1" broad ; lapped 1" |  | ·077 | 2·28 | 29·61 | Broke through rivet line on countersunk side. |
| D. 12 | 1 5/16" „ „ 1 5/16" |  | ·150 | 2·46 | Shearing Force 16·36 | Rivet sheared. |
| D. 13 | 2 3/16" „ „ 1½" |   | ... | 5·69 | ... | Broke through rivet line obliquely on countersunk side. |
| D. 14 | 2½" „ „ 1½" |   | ... | 5·73 | ... | „ „ „ |
| D. 15 | 3 9/32" „ „ 2½" |   | ... | 9·30 | ... | Broke obliquely through six rivets on countersunk side. |
| D. 16 | 3 9/32" „ „ 3 9/32" |  | ·295 | 9·22 | 31·25 | Broke through line of rivets on panhead side. |
| D. 17 | 3 15/16" „ „ 3 9/32" |  | ·445 | 10·98 | 24·67 | Broke through rivet line on countersunk side. |
| D. 18 | 4" „ „ 4" |   | ·450 | 8·1 | Shearing Force 17·96 | All rivets sheared. |

TABLE XLVI.

Rivet Shearing Tests. Specimens of Rivets from various Firms. ($\frac{5}{8}$" and $\frac{7}{16}$" Rivets.)

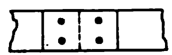
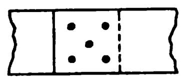
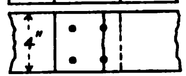
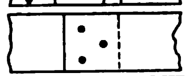
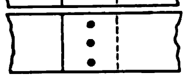
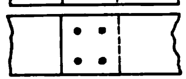
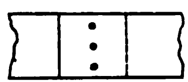
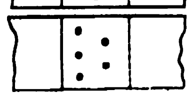
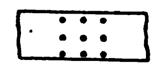
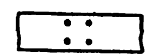
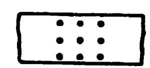
| Group and Number. | Description. | Sketch. | Sectional Area (sq. in.) | Breaking Load in Tons. | Shearing. One Rivet. | Shearing Stress. Tons/sq. in. | Remarks. |
|---|---|---|---|---|---|---|---|
| D. 1 | $2\frac{3}{4}$" broad; 2 $\frac{3}{8}$" rivets, lap joint |  | ·300 | 5·47 | 2·73 | 18·19 | Sufficient strength of plating to shear all the rivets. |
| D. 2 | $2\frac{3}{4}$" broad; 2 $\frac{3}{8}$" rivets, butt |  | ·300 | 5·67 | 2·83 | 18·86 | ,, ,, ,, |
| E. 8 | 5 $\frac{5}{8}$" panheaded rivets, zigzag |  | ·7815 | 14·08 | 2·81 | 18·73 | ,, ,, ,, |
| E. 9 | 4 $\frac{5}{8}$" countersunk rivets, chain |  | of plate ·4756 | 12·98 | ... | Tensile Stress 27·29 | Plate broke across line of rivets. |
| E. 10 | 3 $\frac{5}{8}$" panheaded rivets, zigzag |  | ·4509 | 7·79 | 1·94 | 17·27 | Sufficient strength of plating to shear all the rivets. |
| E. 11 | 3 $\frac{5}{8}$" countersunk rivets |  | ·4509 | 8·55 | 2·85 | 19·00 | ,, ,, ,, |
| E. 12 | 4 $\frac{5}{8}$" countersunk rivets, chain |  | ·6012 | 8·94 | 2·23 | 14·87 | ,, ,, ,, |
| E. 13 | 2 $\frac{5}{8}$" countersunk rivets |  | ... | 4·50 | ... | ... | Rivets tore through the rivet holes. |
| E. 14 | 3 $\frac{7}{16}$" panheaded rivets |  | ·5889 | 9·08 | 3·026 | 15·41 | Sufficient strength of plating to shear all the rivets. |
| E. 15 | 5 $\frac{7}{16}$" panheaded rivets |  | ·9815 | 14·86 | 2·972 | 15·14 | ,, ,, ,, |
| | | | | Average Shear Stress . | | 17·18 | |

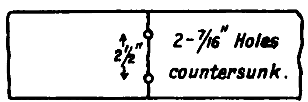
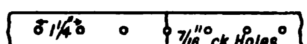
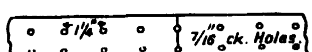
TABLE XLVII.

Specimens all cut 20″ long from a 6 *lb. High Tensile Steel Plate.*

| Group and Number. | Description and Size. | Sectional Area (sq. in.) | Total Breaking Load in Tons. | Ultimate Tensile Strength. Tons/sq. in. | Percentage Extension on 10″. | Remarks. |
|---|---|---|---|---|---|---|
| A. 1 | 1″ broad ; cut lengthwise | 0·150 | 5·89 | 39·26 | 12·5 | |
| A. 2 | 1 5/16″ ,, ,, ,, | 0·197 | 7·63 | 38·77 | (13·5) | Broke through punch mark. |
| A. 3 | 2½″ ,, ,, ,, | 0·375 | 14·82 | 39·52 | 12·47 | |
| A. 4 | 3 15/16″ ,, ,, ,, | 0·591 | 23·59 | 39·91 | 16·64 | |
| A. 5 | 2½″ ,, ,, across | 0·375 | 15·30 | 40·80 | 11·51 | |
| A. 6 | 3 15/16″ ,, ,, ,, | 0·591 | 23·49 | 39·75 | 11·64 | |
| | Averages . . | ... | ... | 39·67 | 13·04 | |
| B. 1 | Same as A. 1; hammered locally | 0·150 | 6·35 | 42·33 | 11·5 | |
| B. 2 | ,, A. 2 ; ,, ,, | 0·197 | 7·85 | 39·86 | (11·0) | Broke through punch mark. |
| B. 3 | ,, A. 3 ; ,, ,, | 0·375 | 15·45 | 41·15 | 11·8 | |
| B. 4 | ,, A. 4 ; ,, ,, | 0·591 | 24·06 | 40·70 | 16·5 | |
| B. 5 | ,, A. 5 ; ,, ,, | 0·375 | 15·03 | 40·08 | (12·5) | ,, outside ,, |
| B. 6 | ,, A. 6 ; ,, ,, | 0·591 | 23·43 | 39·65 | (12·5) | ,, through ,, |
| | Averages . . | ... | ... | 40·63 | 12·63 | |
| C. 1 | ,, A. 3 ; 4 ⅜″ holes punched | 0·243 | 10·54 | 43·37 | very small, local |  |
| C. 2 | ,, A. 4 ; 9 ⅜″ ,, ,, | 0·393 | 16·36 | 41·62 | ,, ,, |  |
| C. 3 | ,, A. 5 ; 4 ⅜″ ,, ,, | 0·243 | 9·55 | 39·30 | ,, ,, |  |
| C. 4 | ,, A. 6 ; 9 ⅜″ ,, ,, | 0·393 | 15·95 | 40·58 | ,, ,, |  |

# Table XLVIII.

(F″) Miscellaneous Specimens from Plates taken from Destroyers undergoing Repairs.

| Group and Number. | Description and Size. | Sketch. | Sectional Area (sq. in.) | Breaking Load in Tons. | Ultimate Tensile Strength. Tons/sq. in. | Percentage Extension on 10″. | Remarks. |
|---|---|---|---|---|---|---|---|
| F. 1 | Cut lengthwise 20″ × 1″ from deck plate of 6 lb. hard steel | ... | 0·15 | 5·22 | 34·8 | ... | Fracture 1½″ from end. |
| F. 2 | ,, ,, 20″ × $1\frac{5}{16}$″ | ... | 0·197 | 7·04 | 35·74 | ... | ,, ½″ ,, |
| F. 3 | ,, ,, 20″ × 2½″ | ... | 0·375 | 13·4 | 35·73 | 17·2 | ,, 7½″ ,, |
| F. 4 | ,, ,, 20″ × $3\frac{1}{16}$″ | ... | 0·591 | 21·03 | 35·58 | 23·5 | ,, 9¼″ ,, |
| F. 5 | ,, ,, 20″ × 2½″ | ... | 0·375 | 13·55 | 36·13 | 15·0 | ,, 6¼″ ,, |
| F. 6 | ,, ,, 20″ × $3\frac{1}{16}$″ | ... | 0·591 | 19·98 | 33·81 | ... | ,, 8″ ,, |
| | Averages . . | ... | ... | ... | 35·3 | | |
| F. 7 | Cut lengthwise 24″ × 2″ from sheerstrake of 10 lb. mild steel | ... | 0·530 | 14·20 | 26·79 | ... | ,, 5¼″ ,, |
| F. 8 | ,, ,, 23″ × 2½″ | ... | 0·662 | 17·13 | 25·88 | ... | ,, 2¼″ ,, |
| F. 9 | Cut lengthwise 20″ × 5″ from deck plate of 6 lb. high tensile steel |   | 0·619 | 17·41 | 28·13 | ... | ,, through holes. |
| F. 10 | ,, ,, 20″ × 1⅞″ |   | 0·141 | 4·24 | 30·07 | ... | ,, through fourth hole from end. |
| F. 11 | Cut across 20″ × 2½″ from deck plate of 6 lb. high tensile steel |   | 0·244 | 7·45 | 30·53 | ... | ,, through fourth row of holes from end. |

## CHAPTER XXIII.

# SHEARING FORCES AND BENDING MOMENTS IN A SHIP AMONGST WAVES.

We have dealt with the method of determining the shearing forces and bending moments which come upon a ship when under a known distribution of weight and buoyancy. We know also what stresses these shearing forces and bending moments produce in the structure, and how the ship resists these stresses. It is now necessary to consider what shearing forces and bending moments are likely to come upon a ship, and the effects they produce.

In dealing with the relation that exists between the curves of bending moment, shearing force, weight, and buoyancy, we considered the ship to be floating at rest in still water. When we pass from the case of a ship in still water to one in disturbed water, we are at once face to face with the fact that the ship will probably be in motion, and we must take into account the forces necessary to move the various parts of the ship. The mass of the ship will remain unchanged, except in such a case as taking water over the bow and holding it on board a sufficient time to seriously affect the total mass to be moved. The supporting force will be continually varying as the vessel passes through the various combinations of crests and hollows which form the surface of the sea.

It is obvious that an infinite variety of cases may be taken upon which calculations may be made. The variation in the distribution of buoyancy as a ship meets a series of waves causes a varying bending moment on the ship's structure.

The passage of a series of waves also causes the ship to oscillate and to heave, these motions generally accentuating the features in the distribution of buoyancy that tend to produce severe bending moments. For purposes of comparing the strengths of vessels of various types, it serves the purpose of the designer sufficiently well to find out the stresses in the worst probable combination of sea. The stresses are deduced by means of what is called an ordinary strength calculation. In making an ordinary strength calculation no attempt is made to determine the actual bending moment that comes upon a ship's structure in all conditions in a seaway, but only the bending moment that comes upon the ship under certain assumptions as to the conditions. It will be shown later that under the assumed conditions greater stresses will come on a vessel than those actually sustained.

The following assumptions are made for an ordinary strength calculation :—
Regarding the size and form of wave—

(*a*) In all cases the wave or waves are trochoidal in profile, the trochoid being traced by a point in the circumference of a circle which rolls on the underside of a straight line.

(*b*) The wave of a given height that produces the most severe stresses on a ship is a wave equal in length to the length of the ship.

(*c*) The ship meets the wave at right angles, and therefore remains upright as the wave passes.

(*d*) The proportion of height to length of wave is $\frac{1}{20}$th, and the length of wave equals the length of the ship. (From observations made of large deep-sea waves about 200 feet long and above, the greatest proportion of height to length is $\frac{1}{20}$th.) In shallow water, waves have generally a greater proportion of height to length than 1 to 20, but they are not so long as deep-sea waves. The dimensions of waves that are sometimes taken for a calculation for small vessels such as Channel or river steamers, say about 200 ft., that may go into shallow water, are—length equal to $\frac{2}{3}$rds of the length of the vessel, and a proportion of height to length of wave of 1 to 10.

(*e*) It is generally assumed that the pressure at any point in the bottom of a vessel in a wave is proportional to the depth of that point below the trochoidal surface.

(*f*) No account is taken of the change of buoyancy due to the deflection of the vessel.

(*g*) It is assumed that the ship is instantaneously at rest upon the wave: this is equivalent to the condition of a wave passing a ship infinitely slowly, so that no account is taken of the dynamical effects of heaving and pitching.

(*h*) Two positions of the wave relatively to the ship are chosen: (1) crest of the wave amidships; (2) hollow of the wave amidships.

When a vessel, which in smooth water floats at a certain waterline, meets with a wave, the distribution of buoyancy is changed. When the crest of the wave is amidships, the midship portion has gained and the ends have lost buoyancy. The general effect is that a hogging bending moment is produced on the ship's structure. The nature of a hogging bending moment we have already examined—its tendency is to cause the ends of the vessel to droop relatively to the middle.

The condition that produces a hogging bending moment is called a hogging condition, and the effect is called, for short, "hogging."

In the case when the hollow of the wave is amidships, the buoyancy is decreased there and increased at the ends. The effect of this change on the structure is opposite to that already stated, and is called "sagging." The moment being a sagging moment, it tends to cause the ends of the vessel to rise relatively to the middle.

The ship on the crest, the crest being amidships, gives the maximum hogging moment, and the ship in the hollow, the hollow being amidships, gives the maximum sagging moment; but for intermediate positions there is a smaller maximum hogging or maximum sagging moment, assuming that, as the wave passes, there is an equality between the resultant forces of buoyancy and weight.

**Loading of the Vessel.**—The distribution of the internal heavy weights in a vessel considerably affects the bending moment. In most vessels the engine and boiler spaces are amidships, and the holds for cargo are towards the ends.

The worst condition of loading for "sagging" is, therefore, when the ends are as light and the amidship portion as heavy as possible, and *vice versâ* for "hogging."

For the worst possible combination at sea, the following conditions are assumed and are taken as being standard conditions. They are based on the foregoing assumptions:—

### Standard Hogging Condition:—

Loading: Coal, if usually carried amidships, to be out of the ship.
Cargo, stores, etc., or whatever is carried in the ends of the ship, is to be on board.
If water ballast is necessary to bring the ship to a safer sea-condition, the tanks at the ends of the ship are to be assumed full.

Wave: Trochoidal.
Hydrostatic law of pressures.
Crest amidships.
Length equals length of ship.
Height equals $\frac{1}{20}$th of length.

### Standard Sagging Condition:—

Loading: Coal bunkers full.
All cargo, stores, etc. on board.
Water ballast, if any, amidships, and none at ends.

Wave: Hollow amidships,
and in every other respect same as for hogging.

A strength diagram for a standard hogging or sagging condition is prepared in the following manner:—

**Weight Curve.**—A full description of the method of setting up a weight curve has already been given. A note is made as to the condition of loading to which the weight curve corresponds.

Its area represents to scale the displacement of the ship; and its longitudinal C.G. gives the position of the C.G. of the ship in the given condition of loading.

**Buoyancy Curve.**—The conditions to be satisfied, deduced from the assumption, are—

(1) Total buoyancy equals the total weight of ship.
(2) The resultant forces of buoyancy and weight act in the same vertical line.

### Graphic Representation of the Trochoidal Wave Surface.—

Length of wave = L = length of ship.

Height of wave $= \frac{L}{20}$.

Let radius of rolling circle = R.

" " tracing " $= r$.

Then $L = 2\pi R \quad \therefore \; R = \frac{L}{2\pi}$,

and $2r = \frac{L}{20} \quad \therefore \; r = \frac{L}{40}$.

In drawing the form of wave (see fig. 189), only the path of the centre of the tracing or rolling circle need be drawn. This path is a straight horizontal line midway between the hollow and crest. This line is A B in the figure. A B is drawn to scale to represent the length of the wave. Divide A B into a convenient number of intervals, say eight, shown at $a_1 a_2 a_3$, etc.

At each of these points $a_1$ $a_2$, etc., describe a circle with radius equal to $r$,—*i.e.* $\frac{L}{40}$ to the same scale as the length A B. As the centre of the circle moves along we can trace the path of any point on its circumference. The circle makes one complete revolution as it travels from A to B. It makes one-eighth in travelling from A to $a_1$, from $a_1$ to $a_2$, etc. The direction of rolling is indicated by the arrow at each circle.

Suppose P is the position of the point when the tracing circle is at A. Then the radius A P is in the position $a_1 p_1$ when A has reached $a_1$, and so on for the other positions. The trochoid is got by drawing a fair curve through the points P $p_1$ $p_2$, etc.

Having obtained the curve showing the wave surface, the next step is to make a profile of the ship with the sections marked on it, on tracing paper. The profile of the ship is then placed, in regard to the wave curve, as near as possible to what is thought to be the correct position. The correct position is the position that satisfies the two conditions already noted.

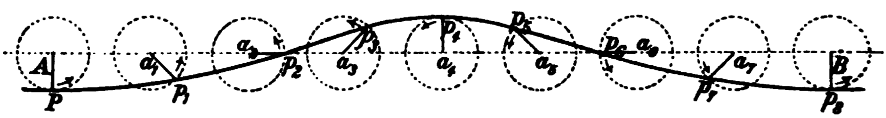



FIG. 189.

Since the pressure at any point is assumed to be proportional to the depth of the point, the immersed part of the sections of the ship give the curves of water pressure, and the areas of the sections give the buoyancy per foot of length (see p. 233). If the heights of the intersections of the wave curve on the profile be then transferred to the body plan, we can easily get the buoyancy per foot of length at each section.

Integrating these areas of sections, we can get the total buoyancy and longitudinal position of the C.B. for the arbitrarily chosen position.

Let $\Delta_1$ = displacement of the buoyancy.
" $\Delta$ = weight of ship.
Then $\Delta \sim \Delta_1$ = error in displacement.

From the body plan we can now read off the half ordinates of the wave waterline. This will give the tons per inch at that waterline.

Let tons per inch of wave waterline = $t$.

∴ Amount vessel requires to be raised or lowered with regard to the wave is equal to $\frac{\Delta \sim \Delta_1}{t}$ inches. This operation ensures the equality of weight and buoyancy.

The adjustment, in order to get the C.B. vertically in line with the C.G is made in the following manner:—

Let the horizontal distance between the C.B. in the chosen position and the C.G. of ship be $x$. Then the ship requires to be trimmed so that C.B. is brought to the required position.

The necessary moment to trim the ship to effect this change in C.B. is $(\Delta \times x)$ foot tons. The moment to trim ship 1 inch is calculated for the wave waterplane by the formula $M_t = \frac{\Delta \times GM}{12 \times L}$. The only thing that requires to be calculated in this expression is G.M., which is taken to be B.M. This calculation for B.M. is done in the ordinary manner, using the half ordinate of the wave waterplane, and the horizontal intervals between sections.

The number of inches, therefore, that the ship requires to be trimmed

$$= \frac{\Delta \times x}{\frac{\Delta \times GM}{12L}}$$

$$= \frac{12 \times x \times L}{GM} = (f + a)''.$$

The ship trims about the C.G. of the waterplane, so that the trim fore and aft is determined. Fig. 190 shows the method of determining $f$ and $a$, the trim forward and aft respectively: $(f + a)$ is got by the formula.

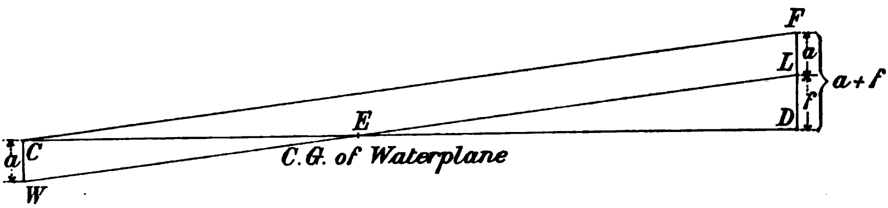



FIG. 190.

Let C D be length of ship, and E the position of C.G. of waterplane. Set off D F $= (f + a)$ at the fore end of the ship.

Join C F, and draw through E a line parallel to C F, meeting the perpendiculars at C and D in W and L respectively; then D L $= f$ and C W $= a$.

If the ship be trimmed this amount fore and aft, and at the same time adjusted for buoyancy, the correct buoyancy curve can then be obtained.

The two curves of weight and buoyancy being obtained, the shearing-force and bending-moment curves can then be obtained by integrating the difference of the weight and buoyancy curves, in the same manner as described for the ship floating in still water.

Strength diagrams for standard conditions can be very suitably standardised in a manner similar to that already explained in Chapter XI. for other ship calculations, so that the ordinates of the bending-moment curve always represent the intensity of the bending moment independently of the absolute dimensions of the particular ship chosen.

The rules for standardising a strength calculation are:—

1. Length of ship is represented by 20 in.
2. Mean ordinate of weight or buoyancy curves shall be 3 in.
3. 1 in. of ordinate of shearing force shall equal 2 square inches of area of load curve.

4. 1 in. of ordinate of bending-moment curve shall equal 3 square inches of area of shearing-force curve.

These rules are adopted to give a diagram of convenient and uniform size.

## STANDARDISED SCALES.

### *Scale for Length.*

Since the mean ordinate of weight curve = 3 in. and length = 20 in.:

∴ Area of weight curve = 60 square inches.

$$L = 20 \text{ in.} \quad \therefore \text{ Scale for length 1 in.} = \frac{L}{20} \text{ ft.}$$

### *Scale for Ordinates of Weight and Buoyancy Curves.*

60 square inches = Δ, and 20 in. = L.

$$\text{Mean ordinate} = \frac{60}{20} = 3 \text{ in.} = \frac{\Delta}{L} \text{ tons per foot of length of ship.}$$

$$1 \text{ in.} = \frac{\Delta}{3L} \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

### *Scale for Shearing Force Curve.*

Now $1 \text{ in.} = \frac{L}{20}$ ft. in a longitudinal direction.

$$1 \text{ in. ordinate of load curve} = \frac{\Delta}{3L} \text{ tons per foot.}$$

$$\therefore \quad 1 \text{ square inch of load curve} = \frac{\Delta}{3L} \times \frac{L}{20} \text{ tons} = \frac{\Delta}{60} \text{ tons.}$$

$$\therefore \quad 2 \text{ square inches of load curve} = \frac{\Delta}{30} \text{ tons.}$$

$$\therefore \quad 1 \text{ in. ordinate of shearing-force curve} = \frac{\Delta}{30} \text{ tons.}$$

### *Scale for B.M. Curve.*

$$1 \text{ in. length} = \frac{L}{20} \text{ ft.}$$

$$\therefore \quad 1 \text{ square inch of shearing-force curve} = \frac{L}{20} \times \frac{\Delta}{30} = \frac{\Delta \times L}{600} \text{ ft. tons.}$$

$$\therefore \quad 3 \text{ square inches of shearing-force curve} = \frac{\Delta \times L}{200} \text{ ft. tons.}$$

$$\therefore \quad 1 \text{ in. ordinate of bending-moment curve} = \frac{\Delta \times L}{200} \text{ ft. tons.}$$

Summarised, these scales are—

Length $\quad 1 \text{ in.} = \frac{L}{20}$ feet.

Ordinates of weight or buoyancy 1 in. $= \frac{\Delta}{3L}$ tons / ft. 1 sq. in. area $= \frac{\Delta}{60}$ tons.

Ordinates of shearing-force curve 1 in. $= \frac{\Delta}{30}$ tons. 1 sq. in. area $= \frac{\Delta \times L}{600}$ ft. tons.

Ordinates of bending-moment curve 1 in. $= \frac{\Delta \times L}{200}$ ft. tons.

The maximum bending is usually expressed as a factor of the product of the displacement and the length of the vessel. For similar ships and the same conditions, the bending moment which is of the fourth (foot-tons) order must vary as the fourth power of the lineal dimensions or displacement multiplied by length. If M = bending moment, then $M = \frac{\Delta \times L}{f}$ where $f$ is a factor. The reciprocal of $f$ is a measure of the intensity of the bending moment.

Let $x$ = number of inches of ordinate in the maximum bending moment of a standardised strength calculation.

$$\text{Then} \qquad M = x \times \frac{\Delta \times L}{200}.$$

$$\therefore \qquad \frac{x}{200} = \frac{1}{f}.$$

So that $x$ is a measure of the intensity of the bending moment, and $\frac{200}{x}$ gives the factor $f$.

**Results of Strength Calculations made for some Large-sized Vessels.**—Figs. 191, 192, 193, and 194 show diagrams worked out in the manner just described for definite sizes, but which may be considered as suitable to the types of about

| | | | | |
|---|---|---|---|---|
| 525 ft. long | which | we will | call | A |
| 560 | ,, | ,, | ,, | B |
| 610 | ,, | ,, | ,, | C |
| 680 | ,, | ,, | ,, | D |

In all these cases the assumptions made are as for the standard hogging condition.

Fig. 191 shows the curves of distribution of weight and buoyancy for the type A. The vessel is 63 ft. beam and 42 ft. to the upper deck at side. These dimensions are similar to those of existing Atlantic ships. Above the upper deck is a strong deckhouse about 8 ft. from the side. The top of this deckhouse, when extended to the ship's side, forms the promenade deck, as is usual in such ships.

Calculations have been made in this case showing the stress at the upper deck :—

(1) Supposing the promenade deck not to contribute in any way to the longitudinal strength of the vessel.

(2) Supposing the promenade deck to be plated, and to contribute its utmost to the strength.

The bending moment is usually shown at every point in the length of the ship, but, as already stated, the midship section is the only one whose strength is usually calculated. In this case, however, the values of I and $y$ have been calculated at various points in the length of the ship, and the stresses determined in the usual way. The thickness of promenade-deck plating is assumed to be $\frac{6}{20}$ths of an inch. It should be noted that in the expression $p = \frac{yM}{I}$ the greater $y$ is, the greater is the stress, so that bringing in the promenade-deck plating increases the maximum stress on the section unless the increase of I is proportionally equal to or greater than that of $y$.

Table XLIX. shows the percentage increase of $y$ and I throughout the ship in passing from the condition of excluding the promenade deck to that of including it.

T.S.S. 525′ × 63′ × 43′.
Standard Hogging Condition.
Displacement = 11950 tons.

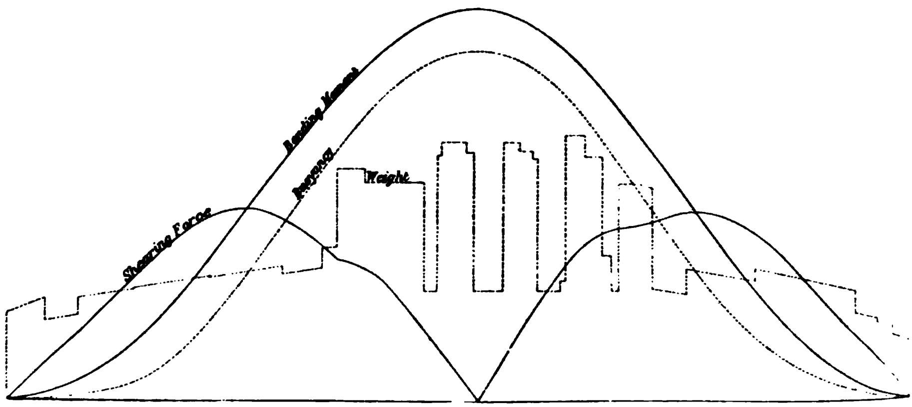



FIG. 191.

CASE A.

*Standardised Scales.*

| | |
|---|---|
| Length | 1″ = 26·25 ft. |
| W. and B. | 1″ = 7·588 tons per ft. |
| S.F. | 1″ = 398·3 tons |
| B.M. | 1″ = 31370 ft. tons. |

*Results.*

| | | |
|---|---|---|
| Maximum shearing force aft | = 4·04″ | = 1610 tons. |
| ,, ,, ,, forward | = 3·98″ | = 1585 tons. |
| ,, bending moment | = 8·225″ | = 258000 ft. tons. |

It will be seen that the reduction in stress due to properly plating this deck is 12½ per cent. amidships.

In order to get the full value of this plating, the side of the deckhouse between the upper and promenade decks must be thoroughly well stiffened inside the house by web frames or some equivalent stiffening. The sides of the deck must be well supported by stanchions having good connections to the upper-deck sheerstrake, the promenade-deck beams and the washplate. If the plating of the promenade deck be of such a thickness that the value of I is not increased as much as $y$, then the stress upon the promenade-deck plating will be greater than the stress would have been had the promenade deck not been plated.

TABLE XLIX.

| Position of Section. | Percentage Increase. | | Percentage Reduction on Stress. |
|---|---|---|---|
| | I | $y$ | |
| ⅗ length aft | 47·3 | 22·3 | 13·8 |
| ½ ,, ,, | 43·0 | 23·5 | 13·6 |
| Midships | 38·7 | 21·4 | 12·4 |
| ½ length forward | 38·0 | 22·0 | 12·0 |
| ⅗ ,, ,, | 35·0 | 22·8 | 8·7 |

T.S.S. 560′ × 57′ 6″ × 42′.
Standard Hogging Condition.
Displacement = 13000 tons.

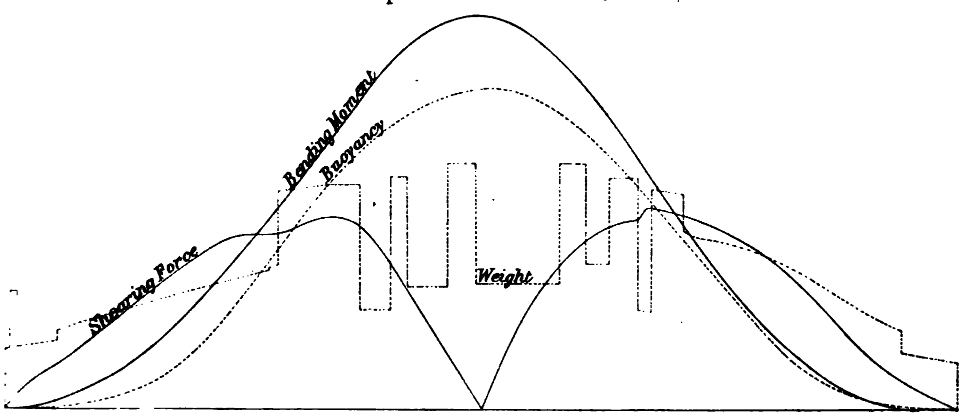



FIG. 192.

CASE B.

*Standardised Scales.*

Length 1″ = 28 ft.
W. and B. 1″ = 7·739 tons per ft.
S.F. 1″ = 433·3 tons.
B.M. 1″ = 36400 ft. tons.

*Results.*

Maximum shearing force aft = 4·08″ = 1768 tons.
,, ,, ,, forward = 4·28″ = 1855 tons.
,, bending moment = 8·38″ = 305000 ft. tons.

Fig. 192 shows the shearing-force and bending-moment curves for the type B. The depth is the same as in the shorter vessel A, and the beam has been reduced in the same proportion as the length has been increased. This alteration of beam has necessitated changes in the internal arrangements of the vessel, and the general character of existing vessels of about these dimensions has been assumed in making these calculations. The increase in the proportion of length to breadth is accompanied by an increase in the bending moment of 18·7 per cent., but the moment of inertia of the section is 11 per cent. less. Hence the stress, neglecting the promenade deck, has been increased amidships at least 33 per cent.

If we add a promenade deck $\frac{5}{20}$th in. thick for the breadth of the deck-house, the stress on this deck will be 8 per cent. more than it would be at the upper deck if the promenade deck had not been plated.

Fig. 193 shows the shearing forces and bending moments of type C. The depth is about the same as in the other two cases, A and B. The beam has been increased to $65\frac{1}{4}$ feet. The general arrangement of existing vessels of about this size has been followed.

The bending moment in C is 67 per cent. more than in A, and 40 per cent. more than in B. The moment of inertia of the section, neglecting the promenade deck, is 4 per cent. more than in A, and 17 per cent. more than in B, the maximum stress being 68 per cent. and 14 per cent. in excess of A and B respectively.

T.S.S. 610′ × 65′ 3″ × 41′ 6″.
Standard Hogging Condition.
Displacement = 15950 tons.

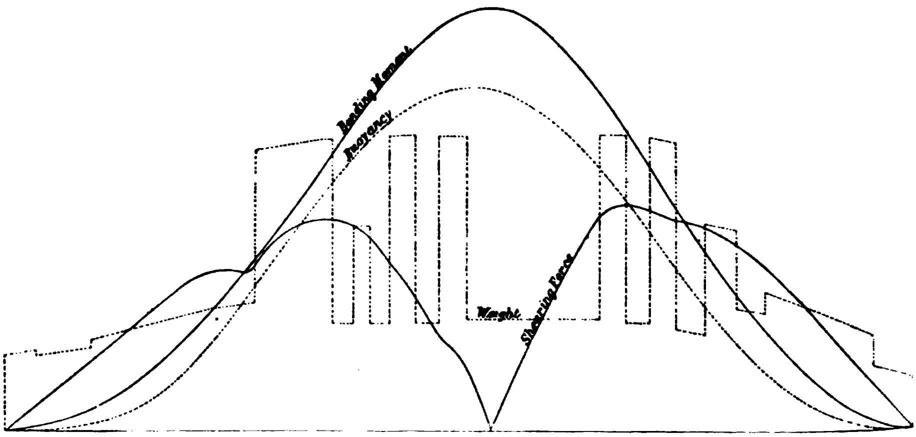



FIG. 193.

CASE C.

*Standardised Scales.*

Length 1″ = 30·5 ft.
W. and B. 1″ = 8·716 tons per ft.
S.F. 1″ = 531·6 tons.
B.M. 1″ = 48647 ft. tons.

*Results.*

Maximum shearing force aft = 4·43″ = 2355 tons.
,, ,, ,, forward = 4·73″ = 2515 tons.
,, bending moment = 8·87″ = 431500 ft. tons.

If the promenade deck be assumed to be $\frac{6}{20}$ths, the same thickness as in case A, the stress upon this deck will be $3\frac{1}{2}$ per cent. more than upon the upper deck when the effect of the promenade deck is neglected.

Hence it is seen that a thickness sufficient in case A is not sufficient in case C to reduce the maximum stress, as the value of I up to the upper deck is such that a greater thickness of promenade-deck plating becomes necessary to increase I proportionally as much as $y$ is increased.

It will be seen, then, that as the length of the ship is increased, the moment of inertia must also be increased if the standard of strength of the case A is to be maintained. It is seen that the strengthening given by the promenade-deck plating in A is not sufficient in the cases of B and C, and cannot be relied on to reduce the stress unless its thickness is increased.

In considering the question of the best dimensions for fast Atlantic ships, the most importance has to be given to the strength for successfully running vessels of increased length. This is treated in the following manner in the designs C and D.

In the types A, B, and C the importance of the promenade deck in the strength was discussed. The assumption is made in the type now to be considered that the promenade deck is the strongest, and that the promenade-deck sheerstrake will consequently be the main sheerstrake of the ship. What corresponded to the upper or saloon deckhouse in the previous types

T.S.S. 680′ × 67′ × 52·5′.
Standard Hogging Condition.
Displacement = 16950 tons.

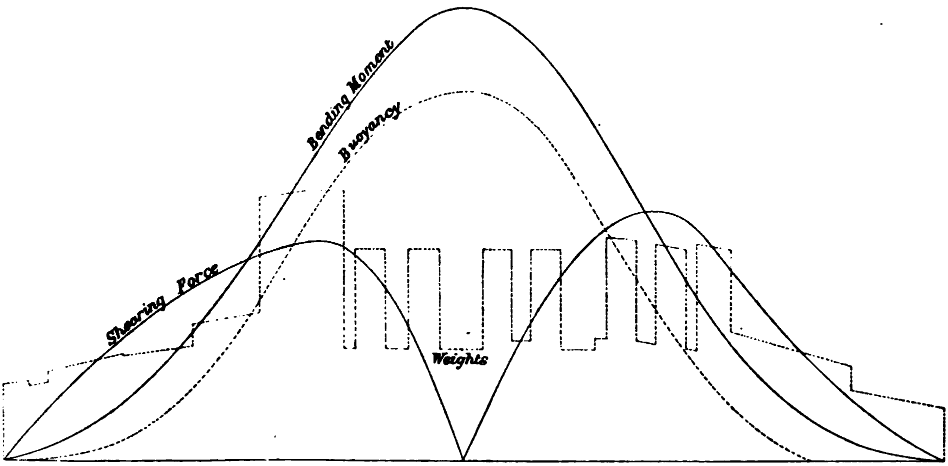



FIG. 194.

CASE D.

*Standardised Scales.*

| | |
|---|---|
| Length | 1″ = 34 ft. |
| W. and B. | 1″ = 8·309 tons per ft. |
| S.F. | 1″ = 565 tons. |
| B.M. | 1″ = 57630 ft. tons. |

*Results.*

| | | |
|---|---|---|
| Maximum shearing force aft | = 4·64″ | = 2622 tons. |
| ,, ,, ,, forward | = 5·28″ | = 2983 tons. |
| ,, bending moment | = 9·58″ | = 552000 ft. tons. |

is now carried out to the ship's sides. The depth considered, in view of the above condition, is 52 ft. 6 in. to the promenade deck, as against 42 ft. to the upper deck in types A and B. The effect of this increased depth is to necessitate a beam of 67 feet for stability. In a vessel 610 ft. long of this depth and breadth, and with internal rearrangements which necessarily followed, the moment of inertia was increased 73 per cent. above that in the case C which is 42 ft. deep. The resulting stress was 20 per cent. less than in C.

With a view to determining the effect of a further increase in depth, the dimensions assumed for another design were 610 ft. length, 70 ft. beam, 56½ ft. deep. The I was increased 109 per cent. and the stress reduced 30 per cent. from that in C.

In considering the question of the effect of a considerable increase of length upon the strength of such a section, calculations were made for a vessel type D, 680 ft. long, but with the same structural midship section. The result was that the stress was found to be about the same as in C.

All these calculations have been based upon the same assumption as to proportion of height to length of wave, so that it may be seen that it is quite practicable to produce a 680 ft. ship as strong as a 610 ft. ship.

The conditions of weight-carrying necessary to produce a practicable working vessel have been kept in view in preparing the outline designs upon which the necessary calculations were based.

It may be interesting at this stage to consider the variation of stress throughout the length.

Fig. 202 shows the variation of stress throughout the length of the ship of type A. Taking the stress amidships as 100, we see that at half length it is reduced—forward to 45 and aft to 58. The stress is less than three-fourths of the maximum beyond one-third of the length amidships.

The shearing force is a maximum at about one-half length aft. The mean shearing stress on this section is only about one-fourth of the maximum tensile stress. The maximum shearing stress is about 50 per cent. more than the mean, or about 39 per cent. of the maximum tensional stress on this section.

In other words, the maximum shearing is in no case more than 23 per cent. of the maximum tension in the ship. The shearing stress in the other cases B, C, D will not be a greater percentage of the maximum tensional strains. The maximum shearing stress does not occur at the same section of the ship as the maximum tension, so that the two stresses together are not likely to be greater than the maximum tensile stress.

The maximum compressive stress in these cases is less amidships than the tensile, though it is slightly greater in the ends; but it falls off almost as rapidly in the ends as the tensile. The maximum tensile stress has been taken as the stress at the gunwale amidships, and the maximum compressive stress at the bottom of keel.

**Results of Standard Strength Calculations for various Types of Vessels.**—Tables L. and LI. give the results of strength calculations for different types of vessels in the standard hogging and sagging conditions respectively. The tables show at a glance the comparative maximum tensile and compressive stresses on the structure of each vessel. The dimensions of each type are given, the depth noted being the moulded depth to the strength deck. The calculation for moment of inertia has been made according to the method described in a previous chapter, and no allowances have been made for openings in the deck. The load displacement and the displacement in the condition for the calculation are given. In Table L. the difference between these displacements represents the amount of coal and consumable stores near midships which has been excluded from the total weight. We have seen that the loading of the vessel in the standard hogging condition represents the worst condition the vessel can be in at sea, for hogging, so that in Table LII. coal and stores amidships are out.

In the hogging condition the maximum tensile stress occurs at the top of the sheerstrake or at the top of strength deck at centre, and the maximum compressive stress occurs at the bottom of the keel plate or bar.

The results of the Table L. show that in the torpedo-boat destroyer

DARD HOGGING CONDITION.

| | | Type A. | Type A lengthened 75′. | Type B. | Type C. | Type D. | |
|---|---|---|---|---|---|---|---|
| | Large Cargo and Passenger. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. |
| Length . . | 530′ 0″ | 525′ 0″ | 600′ 0″ | 560′ 0″ | 610′ 0″ | 680′ 0″ | 720′ 0″ |
| Breadth . . | 65′ 0″ | 63′ 0″ | 63′ 0″ | 57′ 6″ | 65′ 3″ | 67′ 0″ | 76′ 0″ |
| Depth . . | 45′ 0″ | 42′ 0″ | 42′ 0″ | 42′ 0″ | 41′ 6″ | 52′ 6″ | 58′ 0″ |
| Load draught (mld.) | 29′ 0″ | 26′ 0″ | 26′ 0″ | 27′ 0″ | 28′ 6″ | 28′ 6″ | 31′ 0″ |
| Load displacement | 20000 | 13700 | 18000 | 15000 | 18500 | 20000 | 29000 |
| Displacement for B. | 18900 | 11950 | 16200 | 13000 | 15950 | 16950 | 24800 |
| Maximum B.M. | 433700 | 258000 | 319900 | 305000 | 431500 | 552000 | 714200 |
| Moment of inertia | 958000 | 941400 | 941400 | 834500 | 959000 | 1514000 | 1955000 |
| $y$ for top . . | 26·0 | 23·9 | 23·9 | 24·2 | 23·3 | 29·0 | 29·0 |
| $y$ for bottom . | 19·5 | 20·0 | 20·0 | 18·6 | 19·0 | 24·6 | 25·6 |
| Maximum tension t | 11·76 | 6·52 | 8·12 | 8·85 | 10·50 | 10·60 | 10·60 |
| Maximum compress | 8·82 | 5·5 | 6·80 | 6·80 | 8·55 | 8·97 | 9·35 |
| Factor $\frac{\Delta \times L}{M}$ . | 23·1 | 24·3 | 30·38 | 23·87 | 22·55 | 20·90 | 25·0 |
| Standardised inche | 8·7 | 8·26 | 6·6 | 8·4 | 8·83 | 9·6 | 8·0 |
| | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

NDARD SAGGING CONDITION.

| | | Type A. | Type A lengthened 75′. | Type B. | Type C. | Type D. | |
|---|---|---|---|---|---|---|---|
| | Large Cargo and Passenger. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. | Fast Atlantic Liner. |
| Length . . | 530′ 0″ | 525′ 0″ | 600′ 0″ | 560′ 0″ | 610′ 0″ | 680′ 0″ | 720′ 0″ |
| Breadth . . | 65′ 0″ | 63′ 0″ | 63′ 0″ | 57′ 6″ | 65′ 3″ | 67′ 0″ | 76′ 0″ |
| Depth . . | 45′ 0″ | 42′ 0″ | 42′ 0″ | 42′ 0″ | 41′ 6″ | 52′ 6″ | 58′ 0″ |
| Load draught (mld.) | 28′ 0″ | 26′ 0″ | 26′ 0″ | 27′ 0″ | 28′ 6″ | 28′ 6″ | 31′ 0″ |
| Load displacement | 20000 | 14700 | 18000 | 15000 | 18500 | 20000 | 29000 |
| Displacement for B. | 11000 | 13400 | 16500 | 9500 | 11800 | 12100 | 22200 |
| Maximum B.M. | 320000 | 207000 | 330000 | .. | .. | .. | .. |
| Moment of inertia | 920000 | 922000 | 922000 | 823000 | 944000 | 1492000 | 1980000 |
| $y$ for top . . | 23·4 | 21·4 | 21·4 | 22·9 | 20·8 | 26·1 | 25·8 |
| $y$ for bottom . | 22·2 | 22·5 | 22·5 | 19·9 | 21·5 | 27·5 | 28·8 |
| Maximum compress | 8·13 | 4·8 | 7·67 | .. | .. | .. | .. |
| Maximum tension bo | 7·68 | 5·05 | 8·04 | .. | .. | .. | .. |
| Factor $\frac{\Delta \times L}{M}$ . | 18·3 | 33·6 | 30·0 | .. | .. | .. | .. |
| Standardised inches | 10·92 | 5·95 | 6·66 | .. | .. | .. | .. |
| | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

and cruiser classes the stresses due to hogging are small, ranging from 4·5 tons to 7·8 tons tension and 4·5 to 6·5 tons compression. A full consideration of the strength of a vessel of the destroyer type is made in Chapter XXXI.

In the moment of inertia of cross section of vessels like armoured cruisers or battleships the side armour is not included. The side-armour plates are not long and cannot very effectively add to the longitudinal strength. The side armour may contribute to the longitudinal strength by the bolts through the backing connecting the armour to the side plating, but it is considered that these bolts are also likely to set up local stresses in the plating, and for this reason it is better to neglect the area of the side armour in the moment of inertia calculation.

Cruisers and battleships are built on the double-bottom and longitudinal-framing principle up to the protective deck. This system affords great longitudinal stiffness. Sometimes the main deck is made of the same thickness as the protective deck, and this adds greatly to the moment of inertia. The longitudinal casings and bulkheads should also be included in the calculation. The structure in the cruiser and battleship classes has to be made strong enough to resist great local stresses. For instance, the decks have to be strong enough to support the casemates and guns, etc. The side plating has to be strong enough to support the side armour. The inner bottom has to be strong enough to take the weights of boilers and machinery, to carry water in the compartments, and the outer bottom has to be strong enough to resist the water pressure from the outside. It may be generally said that, if the main longitudinal parts of the structure of these vessels are strong enough to withstand the local stresses, the longitudinal strength is sufficient to resist the stresses due to hogging or sagging moments at sea.

In the Table LI. for sagging, it will be seen that the stresses in the destroyer class are 7·5 to 8·5 tons tension, and 6 to 7·2 tons compression. It will be seen later that this figure, 7·2 tons compression, is rather a large figure for a small vessel which is likely to be at sea in heavy weather. Generally merchant vessels of this size have sufficient thickness of shell plating and decks to resist compression, on account of its being necessary to have a definite thickness of plating to provide against the weakening due to corrosion. In small vessels this added thickness is sometimes as much as the thickness necessary to give the hull sufficient longitudinal strength, viz. $\frac{1}{8}$ inch. In destroyers no such margin for corrosion is provided, as it is necessary to cut down the weight of hull as much as possible in order to obtain a high speed. Consequently the stresses in vessels of this type are much larger than in merchant vessels of the same length. The sagging bending moment in vessels of the destroyer class is also severe on account of the large proportion of length to the transverse dimensions, and on account of the disposition of weight. A swift destroyer has a relatively large weight of machinery distributed over the midships for nearly two-thirds of the length, and the effect of this is to make the sagging moment more severe than the hogging moment. Also the large proportion of length to depth in a destroyer makes the stresses usually more severe than that in a merchant vessel of the same length.

Passing to the consideration of vessels of the merchant class, it may be noted that, with few exceptions, the hogging bending moment of an ordinary merchant vessel is more severe than the sagging bending moment. Only in exceptional circumstances of loading is this not the case. The maximum tensile stress in the hogging condition varies from 6·5 to 11·7 tons in the cases given, and the maximum compressive stress from 5·5 to 9·4 tons. The average maximum tensile stress for the merchant vessels in Table LL 9·4 tons per square inch, and the average compressive stress is 7·4 tons square inch. All the vessels in this table have double bottoms except N 6, 7, and 9, which are vessels constructed with floors on the ordinary sin bottom principle. In these latter vessels it will be seen that the height the neutral axis is relatively higher in proportion to the depth than in oth

These vessels Nos. 6, 7, and 9 are also constructed with bar keels, so t the height of neutral axis or the *y* for bottom part is relatively greater t for the other vessels which have flat plate keels or rubbing plates. No w has been taken into account in any of the moment of inertia calculations.

With regard to what stress, as deduced in the manner of those in Ta LII., may be considered a safe working stress for a merchant vessel, it has be borne in mind that these results are only comparative, and only by exp ence of cases that have shown weakness in longitudinal strength can we h an estimate of a safe working stress. A fuller discussion will be made on question in the chapter on the "Wolf" experiments, but some further res have been given in this chapter which may be some guide.

The stresses of Nos. 13 and 14 appear to be high. Two vessels of a t somewhat similar to these vessels showed a weakness at the ends of the bri on the upper deck. The ends of the bridge formed a point of discontinu in the strength of the upper deck, and hence were the most likely parts the weakness, if there was any, to show itself.

The large vessels, say above 500 feet in length, are very unlikely to be the standard condition, *i.e.* they are unlikely to experience waves of their o length. Hence the factor of safety adopted for very large vessels may be than that for smaller vessels. It will be seen from Table L. that the stre are greater as the length is greater, and the scantlings may therefore be c sidered to be in the proper proportion for strength.

The Channel steamer No. 6 is of the light scantling type, and is intended to be strong enough to withstand heavy Atlantic weather, but stresses are low. These vessels go to sea in all weathers and show no si of weakness.

The coasting steamer No. 7 is designed for the American coasting tra where severe Atlantic storms are likely to occur. In shoal water the prop tion of height to length of wave increases, therefore it is possible that a coast steamer on the Atlantic border may experience waves which give more sev bending moments than the standard waves. Consequently vessels of t type have to be made strong enough to stand heavy weather.

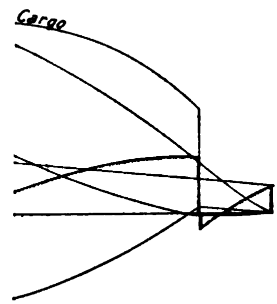
In considering the condition that gives the most severe sagging stres the effect of any change in the distribution of the weight on the bendi moment can be roughly judged from the weight and buoyancy curves for one sagging condition. The standard sagging condition should be when cargo is out and the coal and weights are in amidships. In some vess however, other dispositions of loading can be made which give larger bendi moments than that due to the above standard condition. It is necessary calculate the bending moment for the worst possible condition of loading t is likely to occur at sea. No very reliable standard of loading can be ma for vessels in the sagging condition.

**Results of Strength Calculations made on a Vessel in differe Conditions of Loading.**—To illustrate the variety of calculations that c be made, Table LII. and Plate XV., diagrams A to F, are given for a ves 530 ft. × 59 ft. × 39 ft.

39.
Wat
< 26·

*Weights.*

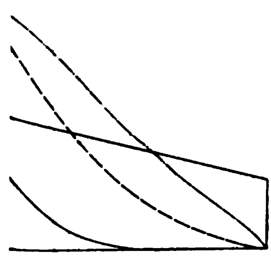
| | |
|---|---|
| . . . . | 6000 tons |
| nes . . . . . | 360 ,, |
| rs . . . . . | 560 ,, |
| ing . . . . . | 110 ,, |
| ellers . . . | 20 ,, |
| . . . . . . | 950 ,, |
| o . . . . . | 10000 ,, |
| Total Displacement | 18000 ,, |




).
l W
6·5
f ta
essu

*Weights.*

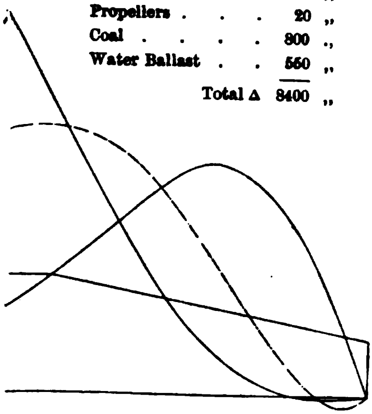
| | |
|---|---|
| Hull . . | 6000 tons |
| Engines . | 360 ,, |
| Boilers . . | 560 ,, |
| Shafting . | 110 ,, |
| Propellers . | 20 ,, |
| Total Δ . | 7050 ,, |



r Ballast in.
5

*Weights.*

| | |
|---|---|
| Hull . . . . . | 6000 tons |
| Engines . . . . | 360 ,, |
| Boilers . . . . | 560 ,, |
| Shafting . . . . | 110 ,, |
| Propellers . . . . | 20 ,, |
| Coal . . . . . | 800 ,, |
| Water Ballast . . . | 550 ,, |
| Total Δ | 8400 ,, |

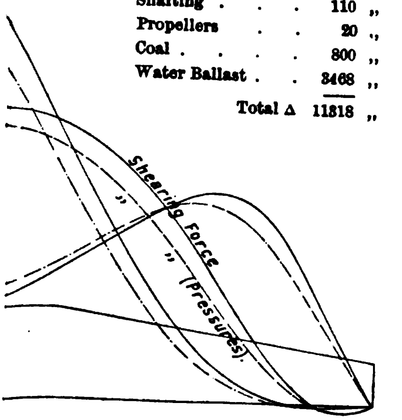


Vater Ballast in.

king into
ıre.

*Weights.*

| | |
|---|---|
| Hull . . . . . | 6000 tons |
| Engines . . . . | 360 ,, |
| Boilers . . . . | 560 ,, |
| Shafting . . . . | 110 ,, |
| Propellers . . . | 20 ,, |
| Coal . . . . . | 800 ,, |
| Water Ballast . . . | 3468 ,, |
| Total Δ | 11318 ,, |




[To face page 311.

## TABLE LII.

### T.S.S. CARGO AND PASSENGER STEAMER.

**Legend of Weights**—Hull . . . 6000 tons
Engines . . 360 ,,
Boilers . . 560 ,,
Shafting . . 110 ,,
Propellers . 20 ,,
Coal . . . 800 ,,
Cargo . . 10000 ,,
Water Ballast 150 ,,
Total . 18000 tons

**Dimensions**—Length . 530′
Breadth 59′
Depth . 39′
Wave Length . . 530′
Wave Height . . 26·5′

Load Dispt. = 18000 tons.
Load Draft = 28′ 0″

| Position on Wave. | Condition of Loading. | Displacement. | Maximum Shearing Force. Tons. | | Maximum Bending Moment. | Maximum Stresses. Tons per □ in. | | x = inches max. B.M. in Standardised Curve. | Δ × L / M | No. of Diagram on Plate. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Aft. | For'd. | | Top. | Bottom. | | | |
| | | Tons. | | | Ft. tons. | | | | | |
| 1. Still water | Fully loaded . . | 18000 | 1260 | 1150 | 165000 | 5·64 T | 4·64 C | 3·46 | 57·8 | A |
| 2. Crest | Light . . . | 7050 | 1140 | 1220 | 179500 | 6·13 T | 5·04 C | 9·61 | 20·8 | B |
| 3. Crest | Load condition, but with coal out and 150 tons W.B. in | 17200 | 2184 | 2030 | 329000 | 11·24 T | 9·25 C | 7·22 | 27·7 | C |
| 4. Crest | Same as above (corrected for wave pressures) | 17200 | 1900 | 1730 | 289000 | 10·34 T | 8·12 C | 6·35 | 31·5 | C |
| 5. Hollow | Load condition, coal out and 150 tons W.B. in at ends | 17200 | 653 | 825 | 107000 | 3·7 C | 3·2 T | 2·35 | 85·2 | D |
| 6. Hollow | Light condition, 800 tons coal in, 550 tons W.B. in | 8400 | 1565 | 1797 | 242200 | 8·36 C | 7·23 T | 10·87 | 18·4 | E |
| 7. Hollow | Light condition, coal in all W.B. tanks ✠ full | 11318 | 2565 | 2263 | 310000 | 10·7C | 9·26 T | 10·36 | 19·3 | F |
| 8. Hollow | Same as above (corrected for wave pressures) | 11318 | 2354 | 2128 | 277500 | 9·58 C | 8·28 T | 9·26 | 21·6 | F |

Moment of Inertia (Hogging) = 644000 in.$^2$ ft.$^2$ $y$. top = 22·0, $y$. bottom = 18·1.
,, ,, (Sagging) = 623000 in.$^2$ ft.$^2$ $y$. ,, = 21·5, $y$. ,, = 18·6.

It will be seen that No. 3 is the standard hogging condition and that No. 6 is the standard sagging condition. No. 7 condition gives a larger sagging moment than No. 6, but this condition of loading could be avoided at sea by filling the end ballast tanks instead of the ballast tanks amidships.

It will be seen from the diagrams for Table LII. that a condition which is bad for hogging is good for sagging, and *vice versâ*.

The coefficients at the bottom of the Tables L. and LI. represent the intensity of the bending moment in terms of displacement and length.

In Table L. the intensity coefficient $x$ varies from 6·0 to 9·6 for hogging, but in Table LI. this coefficient has a much larger range for sagging, viz. 4·36 to 11·22, while in Table LII. it ranges from 2·35 to 10·87. It will be seen that many things tend to produce a variation in the $x$ coefficient for sagging. The proportion of length to draught is involved, and the displacement of the vessel greatly affects this coefficient. In a cargo vessel the cargo is generally all out in the worst sagging condition, and the displacement for the calculation small in relation to the load displacement. The coefficient may therefore be very large, because the bending moment may be large even with a small displacement.

TABLE LIII.

Dimensions—L = 300′ 0″
B =
D = 24′ 6″
Draught—F = 1′ 10½″
,, A = 12′ 2″
Displacement = 1820 tons.

Moment of Inertia. Hogging = 66600 in.$^2$ ft.$^2$
Height of N.A. ,, = 6·9 ft.

Moment of Inertia. Sagging = 69000 in.$^2$ ft.$^2$
Height of N.A. ,, = 8·0 ft.

| Dimensions of Wave. | Maximum Bending Moment. | Maximum Shearing Force. Tons. | | Percentage of Length of Ship from C. B. P. | | Percentage of Length of Max. B.M. from C.B.P. | $\frac{\Delta \times L}{M}$ | Maximum Stresses. Tons per □ in. | |
|---|---|---|---|---|---|---|---|---|---|
| | | Aft. | For'd. | Aft. | For'd. | | | Top in Tension. | Bottom in Compression. |
| | Ft tons. | | | | | | | | |
| Still water | 16200 | 154 | 151 | 27 | 25 | 7·5 A | 33·6 | 4·34 | 1·70 |
| Crest 400 × 15 | 32800 | 368 | 322 | ... | ... | ... | 16·6 | 8·79 | 3·45 |
| ,, 400 × 30 | 37100 | 432 | 392 | ... | ... | ... | 14·7 | 10·0 | 3·9 |
| ,, 400 × 45 | 41200 | 491 | 454 | ... | ... | ... | 13·2 | 11·0 | 4·33 |
| ,, 300 × 15 | 35500 | 394 | 371 | 25 | 18 | 16 A | 15·4 | 9·5 | 3·7 |
| ,, 300 × 30 | 45100 | 564 | 482 | 24 | 10 | 18 A | 12·1 | 12·04 | 4·7 |
| ,, 300 × 45 | 49100 | 643 | 528 | 24 | 6 | 19 A | 11·1 | 13·1 | 5·16 |
| ,, 200 × 15 | 37700 | 428 | 403 | 25 | 14 | 15 A | 14·5 | 10·07 | 3·9 |
| ,, 200 × 30 | 46400 | 607 | 500 | 24 | 9 | 19 A | 11·7 | 12·4 | 4·88 |
| ,, 200 × 45 | 50800 | 667 | 545 | 22 | 6 | 7 A | 10·7 | 13·6 | 5·3 |
| | | | | | | | | Bottom in Tension. | Top in Compression. |
| Hollow 200 × 15 | 8870 | 218 | 194 | 19 | 19 | ⊕ | 61·5 | 1·05 | 2·1 |
| ,, 200 × 30 | 14200 | 337 | 321 | 22 | 23 | 7 A | 38·5 | 1·69 | 3·4 |
| ,, 200 × 45 | 32200 | 506 | 397 | 28 | 30 | 7 A | 16·9 | 3·82 | 7·78 |
| ,, 300 × 15 | 14300 | 190 | 188 | 25 | 23 | ⊕ | 38·1 | 1·7 | 3·46 |
| ,, 300 × 30 | 31100 | 405 | 363 | 30 | 27 | 10 A | 17·5 | 3·7 | 7·8 |
| ,, 300 × 45 | 35200 | 484 | 417 | 30 | 30 | 9 A | 15·5 | 4·2 | 8·5 |
| ,, 400 × 15 | 3550 | 109 | 31 | 47 | 40 | ⊕ | 15·4 | ·41 | ·83 |
| ,, 400 × 30 | 14500 | 197 | 163 | 25 | 26 | 6 F | 37·6 | 1·70 | 3·5 |
| ,, 400 × 45 | 27300 | 346 | 320 | 32 | 24 | 12 A | 19·9 | 3·2 | 6·6 |

In Table LII. the difference between the light and load conditions represents cargo and the weights that can be taken out at the ends, so that it will be seen what cases give a small coefficient. Also the relative amount and position of the machinery affects the sagging moment. This will be seen in the Channel-steamer type, which has heavy machinery amidships and little or no cargo. This resembles the features in the weight curve of a vessel of the destroyer type. The necessity for following some standard in making strength calculations, either hogging or sagging, will be seen from a study of the results in these tables.

**Special Strength Calculations made on certain Vessels.**—Table LIII. gives a series of results that were calculated in connection with investigations made of the strength of an ore-carrying steamer which broke and sank in heavy weather during a voyage across one of the great lakes of America. She was at the time of the disaster in a light condition, but had about 270 tons of water in ballast tanks aft and had 110 tons of coal on board. The machinery was situated aft. Her draught was 1 ft. 10½ in. forward and 12 ft. 2 in. aft, and her displacement in this condition was 1820 tons. As there was no reliable evidence as to the size of the waves she was meeting, calculations were made for different sizes of waves. The length of the vessel was 300 ft. The size of a standard wave is therefore 300 ft. long and 15 ft. high. Three different lengths of waves were chosen, 200 ft., 300 ft., and 400 ft. long respectively, and for each length three different heights were assumed, viz. 15, 30, and 45 ft. A sagging and a hogging calculation for each wave and also the strength calculation for the still-water condition were made, giving twenty calculations in all. The results are given in Table LIII. On the standard wave, 300 ft. × 15 ft., hogging, the stresses are 8·5 tons tension and 3·7 tons compression. These figures are small if we compare them with the results in Table L. The vessel was therefore comparatively strong enough, so that she must have met with very exceptional conditions while being driven into the sea, to cause the disaster. The severest condition in the table is when the wave is 300 ft. × 45 ft. hogging, but it is unlikely that the waves reached this proportion of height to length. It may be seen that a wave 30 ft. high causes a stress of 12 tons. No details of the connections of the structure were available, but it is quite conceivable that some material at the gunwale may have had a very low efficiency in the connection, so that an apparent stress of 12 tons may have become a very much higher one at a weak point.

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A similar series of calculations was made for a comparison with those just described on a vessel of somewhat the same type 405 ft. long. The lengths of waves chosen for this vessel were 305 and 405 ft., and the heights were 20¼, 30, and 40 ft. respectively. The standard wave is 405 ft. by 20¼ ft. high. The results are given in Table LIV. The standard hogging wave gives stresses of 7·58 tons tension and 5·52 tons compression, as against 9·5 and 3·7 for the former case respectively. Comparing the results in the two tables, it will be seen that for the same length of waves the stresses in the latter are more severe than those in the former case. As far as these tables show, we can say that the latter vessel is not so strong as the former, yet the latter has successfully withstood bad weather while the other broke in two. With regard to the coefficient $\frac{\Delta \times L}{M}$ it will be seen that the intensity of bending moment is much greater in the case of the former. In the standard hogging condition the coefficient is 15·4 in the former as against the 30·3 for the latter. This shows that the former was very unfavourably loaded in relation to her displacement and length. The draught of the vessel in the condition in which she met with the disaster also shows this.

### Table LIV.

Dimensions—L = 405′
B = 47·5′
D = 30·8′
Draught F = 7′ 7½″
A = 17′ 2½″
Displacement = 4660 tons

*Condition*—Light, with 200 tons coal and 920 tons water ballast.
Moment of Inertia. Hogging = 146200 in.² ft.² Ht. of N.A. = 13
" " Sagging = 153200 " " " = 14

| Dimensions of Wave. | Maximum Bending Moment. | Maximum Shearing Force. Tons. | | Percentage of Length from C.B.P. | | Percentage of Length from C.B.P. of Max. B.M. | $\frac{\Delta \times L}{M}$ | Maximum Stresses. Tons per □ in. | |
|---|---|---|---|---|---|---|---|---|---|
| | | Aft. | For'd. | Aft. | For'd. | | | Top in Tension. | Bottom in Compressio[n] |
| | Ft. tons. | | | | | | | | |
| Still water | 14130 | 264 | 302 | 18 | 3 | 71 F | 133·6 | 1·72 | 1·24 |
| Crest 305′ × 20¼′ | 71590 | 527 | 698 | 10 | 22 | 12 F | 26·4 | 8·71 | 6·35 |
| " 305 × 30 | 94670 | 721 | 863 | 12 | 27 | 12 F | 19·9 | 11·5 | 8·4 |
| " 305 × 40 | 107400 | 855 | 976 | 13 | 17 | 12 F | 17·5 | 13·1 | 9·53 |
| " 405 × 20¼ | 62200 | 379 | 581 | 29 | 26 | 28 F | 30·3 | 7·58 | 5·52 |
| " 405 × 30 | 79000 | 612 | 738 | 27 | 23 | 14 F | 24·0 | 9·64 | 7·01 |
| " 405 × 40 | 93260 | 759 | 806 | 25 | 20 | 5 F | 20·2 | 11·37 | 8·27 |
| | | | | | | | | Bottom in Tension. | Top in Compressio[n] |
| Hollow 405 × 20¼ | 50000 | 705 | 434 | 17 | 12 | 24 A | 37·8 | 4·9 | 5·2 |
| " 405 × 30 | 122400 | 1201 | 992 | 20 | 15 | 12 A | 15·4 | 11·77 | 12·7 |
| " 405 × 40 | 143200 | 1255 | 1120 | 23 | 20 | 21 A | 13·2 | 13·77 | 14·9 |

The diagrams in connection with the results of the calculations in Table LIV. are given in Plate XVI.

Fig. A gives the weight curve and the strength curves for the still water condition.

Fig. B shows the buoyancy curves for the waves 305 ft. long.

Fig. C shows the shearing-force and the bending-moment curves for the waves 305 ft. long.

Figs. D and E give similar curves for the waves 405 ft. long—crests.

Figs. F and G give similar curves for the waves 405 ft. long—hollows.

It will be seen that the crests of the steepest wave overtop the deck edge so that the buoyancy of the wave crest which comes above the deck is lost. This feature is shown in diagram, fig. B.

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Table LV. gives the results of calculations for bending moments and stresses on a light-draught river paddle steamer 215 ft. long. A vessel of this class is built of very light scantlings, and is only intended for trade in smooth

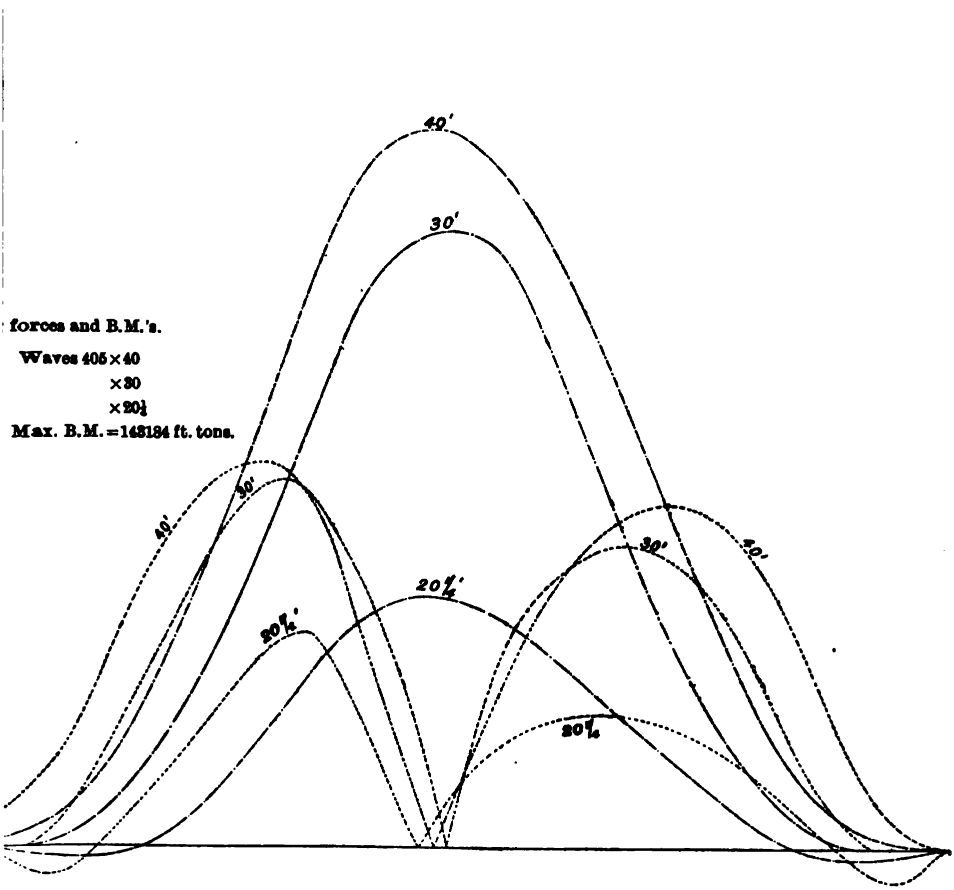


Fig. G.

[*To face page* 314.

P.S. 215 × 23 × 7·5. Δ = 337 tons.
Draft = 3′ 10″.
Hollow 215 × 5′.

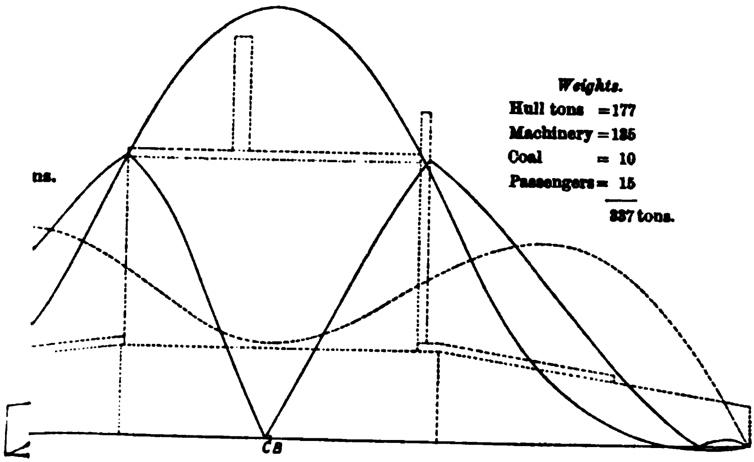


FIG. C.

P.S. = 215 × 23 × 7·5.
Wave hollow 215 × 10′.

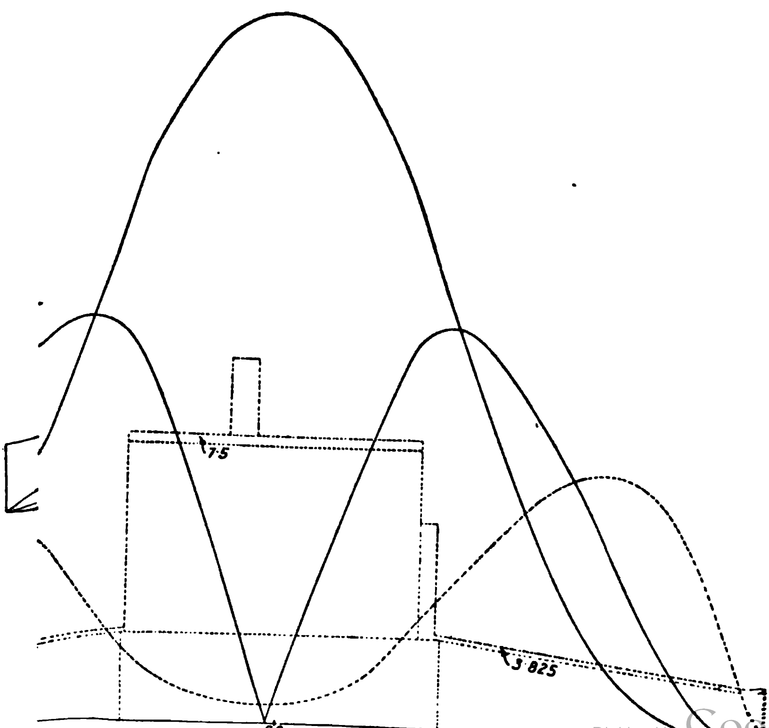


FIG. D.



[To face page 315.

water, like a river, channel, or mouth. The standard conditions are, therefore, never met with. In the table the results are given for four calculations—

(1) Still-water condition.
(2) Vessel on crest of a wave 215 ft. long and 5 ft. high.
(3) Vessel in hollow of a wave 215 ft. long and 5 ft. high.
(4) Vessel in hollow of a wave 215 ft. long and 10 ft. high.

### Table LV.

Strength Calculations—Light Draught Paddle Steamer.

| | |
|---|---|
| Length = 215′ 0″ | Load draught = 3′ 10″ |
| Breadth = 23′ 0″ | Displacement = 337 tons |
| Depth = 7′ 6″ | Condition for calculations, vessel fully loaded. |

| | Still water. (1) | Wave. (2) | Wave. (3) | Wave. (4) |
|---|---|---|---|---|
| Wave length | ... | 215′ | 215′ | 215′ |
| ,, height | ... | 5′ | 5′ | 10′ |
| Position on wave | ... | Crest | Hollow | Hollow |
| Max. shearing force aft | 28·4 | 27·4 | 66·0 | 94·5 |
| ,, ,, ,, forward | 29·2 | 26·0 | 66·0 | 94·0 |
| Max. bending moment | 761 sagging | 1222 | 3280 | 5420 |
| Moment of inertia | 2410 | 2450 | 2450 | 2450 |
| $y$ for top | 5·4 | 5·8 | 5·8 | 5·8 |
| $y$ for bottom | 3·8 | 3·4 | 3·8 | 3·8 |
| Max. stress top | 1·70 C | 2·9 T | 7·35 C | 12·15 C |
| ,, ,, bottom | 1·20 T | 1·7 C | 5·17 T | 8·55 T |
| $\frac{\Delta \times L}{M}$ | 95·1 | 59·2 | 22·08 | 13·3 |
| Standardised inches | 2·10″ | 3·37″ | 9·06″ | 14·95 |

The still-water condition gives a sagging bending moment owing to the distribution of weight, which will be seen from the weight curve in diagram, fig. 295. In the wave hollow 215 ft. by 5 ft. the sagging bending moment is severe, and the stresses are 7·35 tons compression and 5·17 tons tension. The height of this wave is about half the height of the standard wave. In order to show a comparison of strength of this vessel with those in Tables L. and LI.,. calculation (4) was made. This is the standard sagging condition. The stresses due to this condition are 12·15 tons compression and 8·55 tons tension. The maximum compression in any of the other cases was about 9 tons, so that it would seem that the river paddle steamer is very much weaker than the ordinary sea-going merchant vessel.

The diagrams for each of the conditions in Table LV. are given (see Plate XVII.).

## CHAPTER XXIV.

## EFFECT OF TAKING ACCOUNT OF THE ORBITAL MOTION OF THE PARTICLES IN DETERMINING THE WAVE PRESSURES AT ANY POINT.

It has been assumed in the foregoing considerations that the pressure at any point in the wave was proportional to the depth of the point from the surface. This is not quite the case. The true law of wave pressures is determined by the trochoidal wave theory, which may be taken as true, for all practical purposes, for large deep-sea waves. If we assume that the pressure is the same in a wave when the ship is in it as when the ship is not there, we can determine the supporting force, though not without considerable labour. Taking accurate account of the pressures of the particles in the wave was shown by Mr W. E. Smith, of the Admiralty, to have the effect of reducing the supporting force in the crest, and of increasing it in the hollow. Hence the bending moment due to the excessive support in the centre of the ship, and deficient support in the ends, when the ship is on the crest of a wave, is not so great as would be the case if the hydrostatic law of pressures was assumed, providing that the pressures in a wave are the same when the ship is in it as when the wave is undisturbed by the passage of the ship.

The question whether the pressures in a wave are much altered in their passage by the ship cannot be decided without noticing in which direction the ship is going in relation to the waves. A wave 600 ft. long would travel at a speed of about 33 knots. If a vessel of this length were travelling at 21 knots per hour in the same direction as the wave, the crest would pass her from aft to forward at a speed of 12 knots, and would take about 30 seconds to pass her. If, on the contrary, the wave be travelling in the opposite direction to the ship, it would pass her from forward to aft at 54 knots, or in about 6¼ seconds. Hence the disturbance in the wave is likely to be much less in the former case than in the latter, though the action of the propeller or propellers in a steamship would cause some disturbance in the former case, but the effect of this might possibly have become reduced by the time the crest reached amidships.

The effect of pitching, which is the oscillatory motion imparted to the ship, upon the pressures in a wave is not easy to determine, but it is evident that pitching will be much greater when the wave crest is passing the vessel once in 6¼ seconds than when passing only once in 30 seconds, more particularly if it happens that the pitching period of the ship is about 6¼ seconds, which is not unlikely. In this case synchronism of the waves with the ship's pitching will inevitably cause large angles of pitching motion, which cannot fail to disturb the orbital motion of the particles of the wave.

We get a better idea of what is likely to happen by considering the wave form to be at rest, and the whole ocean to be moving past it at 33 knots. The water will flow through trochoidal channels, as shown in fig. 195.

The passage of the vessel through the water will be 21 knots as before, but she will be moving in one case at 54 knots ahead, and in the other at 12

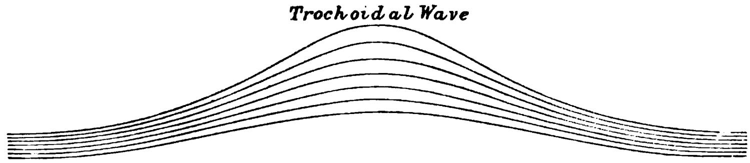


FIG. 195.

knots astern, the water moving in trochoidal channels ahead in the former case, and astern in the latter (see fig. 196). As the ship in the former case passes through the wave crest at 54 knots ahead, and in the latter at 12 knots astern, the water in the trochoidal channels will have less time to adapt itself to the form of the ship in the former case than in the latter, and there is likely to be more disturbance of the orbital motion. Hence it seems probable that with the wave moving in an opposite direction to the ship, the

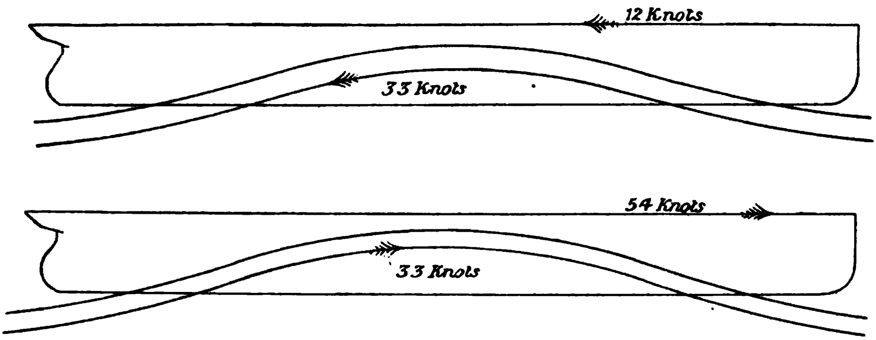


FIG. 196.

pressures more nearly approximate to those due to the depth below the surface than they will when the wave is going in the same direction. In the latter case the stress will be less, but it will be of longer duration. In the former it will be nearly if not quite equal to what is got by neglecting the orbital motion, and will be much more frequently repeated. The stresses due to pitching will be greater and much more frequent in the former than in the latter condition, so that it seems probable that a vessel will experience stresses at least equal to those deduced by assuming her to be on a trochoidal surface instantaneously at rest throughout the whole of its depth, and that the maximum stresses will be experienced when going head to sea.

The above remarks are based on the assumption that the vessel falls in with a regular series of waves of her own length. The frequency of such an event must be taken into account, but no very reliable information is at present available. From the results obtained by Lieutenant Paris, the average periods and lengths of waves in various seas can be obtained, and it may be seen that the average lengths vary from 180 to 450 feet in different localities. Lieutenant Paris gives 348 feet long by 16½ feet high as the average dimensions of a wave corresponding to what sailors call a high sea, and 485 feet by 25½ feet a very high sea.

The following table gives the Heights of Waves noted by three different observers:—

TABLE LVI.

| | Desbois. | Paris. | Wilson-Barker. | Mean. |
|---|---|---|---|---|
| Hurricane | 28·5 | 25·5 | 28 | 27·3 |
| Strong Gale | 20·6 | 16·5 | 23 | 20·0 |
| Gale | 15·4 | ... | 14 | 14·7 |
| Strong Breeze | 10·8 | ... | 8 | 9·4 |

We may therefore infer that regular series of waves of 500 feet and upwards are not very frequently met by ships, but for purposes of comparison we may assume that the stresses which would be brought upon a vessel when poised on a wave of her own length, and of height one-twentieth of the length, ought to be provided for in the disposition of material in the structure.

The character of the loading, and the effect of such on the hogging and sagging moments, have been already discussed.

At the beginning of a voyage the ship is generally full of cargo and coal, and at the end of the voyage coals and stores are nearly, if not quite, consumed. From an examination of sea-going vessels, strength diagrams of several types of which are given, it is seen that the worst condition is met with at the end of the voyage. The coal is generally stowed in a part of the ship where buoyancy exceeds weight, and consequently when the bunkers are full this local inequality is greatly diminished. It is also to be noted that many large ships have ballast tanks, the weight of water in which, in the parts where buoyancy exceeds weight, tends considerably to reduce the maximum bending moment. If the burning out of coal so modifies the stability as to make it desirable to fill the ballast tank, it is desirable for strength purposes to fill up those amidships. But it may be necessary to fill some aft in order to ensure proper immersion to the propellers, especially if the vessel is steaming in a sea which causes her to pitch and race. In this case the stresses will generally be greater for a hogging moment than if the ballast tanks had been empty.

In constructing a strength diagram for a ship in a wave, taking into account the true wave pressures, it is only the method of obtaining the buoyancy curve that need be described, as all the other work is the same as that for an ordinary strength diagram, which has been dealt with in the preceding pages.

It is first necessary to draw the wave and also a series of subsurfaces at convenient intervals of depth.

**Trochoidal Wave Theory.**—The basis of the trochoidal wave theory is as follows:—

The particles of water in a trochoidal wave move in vertical planes in the direction of the advance of the wave in circular orbits with uniform angular velocity.

The radii of the orbits of particles are equal for those particles which lie in the same horizontal layer in still water.

The particles in any originally horizontal layer take up the orbital motion in uniform succession.

The particles in a vertical plane perpendicular to the plane of orbital motion take up the motion simultaneously.

From these can be deduced certain geometrical properties and formulæ by which the variation of the water pressure in the whole wave can be determined. These are proved in the discussion on waves in the chapter on Waves, Vol. II. An abstract need only be given here.

In order to imagine the nature of the wave, it is well to consider that layers which are horizontal in still water take the form of trochoidal surfaces when the water is in wave motion.

The crests and hollows of each subsurface are in the same vertical lines respectively.

The formula giving the means by which the subsurfaces can be constructed is

$$r_1 = r_0 e^{-\frac{Y}{R}}$$

where $r_0$ is the radius of the tracing circle of the surface trochoid,

and $r_1$ is the radius ,, ,, ,, subsurface trochoid,

and Y is the vertical distance between centres of the rolling or tracing circles of the subsurface and surface trochoids,

and R is the radius of the rolling circle.

The values of $r_0$ and R are fixed from the dimensions of the wave. If the wave is a standard wave

we have $2\pi R = L = \text{length of ship}$

and $2r_0 = \frac{L}{20}$

so that $r_0 = \frac{L}{40}$

$$\frac{r_0}{R} = \frac{2\pi}{40}.$$

It is usual to apply the formula to a series of layers which in still water are horizontal and 1 foot apart, so that the distance between the rolling-circle centres is the same at 25 feet; hence Y = 1.

For the radius of 1st subsurface we have $r_1 = r_0 e^{-\frac{1}{R}}$

" " 2nd " " $r_2 = r_0 e^{-\frac{2}{R}}$

" " 3rd " " $r_3 = r_0 e^{-\frac{3}{R}}$

and so on.

These formulæ are worked in logarithmic form.

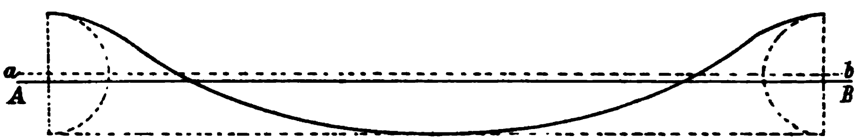


FIG. 197.

Taking logs of both sides we have—

$$\text{Log } r_1 = \text{Log } r_0 - \frac{1}{R}$$

$$= \text{Log } \frac{L}{40} - \frac{2\pi}{L}$$

$$\text{Log } r_2 = \text{Log } \frac{L}{40} - \frac{4\pi}{L}$$

$$\text{Log } r_3 = \text{Log } \frac{L}{40} - \frac{6\pi}{L}$$

etc.

We can from these get the radii of the tracing circles of the subsurface and surface trochoids, $r_0\, r_1\, r_2\, r_3$, etc. It will be noticed that these decrease as the depth increases.

The next thing to determine is the distance between the rolling lines of the circle which trace out the trochoids. Suppose A B, fig. 197, to represent one of the horizontal still-water layers. Then the position of $a\,b$, the line along which the centre of the tracing circle moves, is given by

$$h = \frac{r^2}{2R}$$

where $h$ = vertical distance between $a\,b$ and A B
and $r$ = radius of tracing circle.

Therefore the distances of the paths of the tracing circles of the wave surface and subsurfaces above their still-water positions are

$$h_0 = \frac{r_0^2}{2R}$$

$$h_1 = \frac{r_1^2}{2R}$$

$$h_2 = \frac{r_2^2}{2R}$$

$$h_3 = \frac{r_3^2}{2R}$$

etc.

Each trochoid is then drawn as described for a standard wave (fig. 199). Having obtained the wave as shown in outline by its surface and subsurface, we

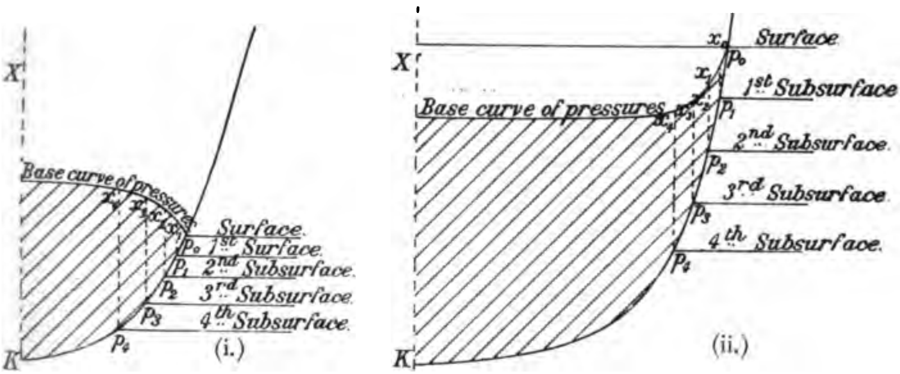


FIG. 198.

proceed in the usual way by arbitrarily choosing a position of the vessel near to what is thought to be the correct position.

The law of pressures in a trochoidal wave is: The pressure at any particle in a wave is equal to the pressure on the same particle in still water.

What we have in the wave is a series of subsurfaces which we know correspond to a series of horizontal planes in still water 1 foot apart. Therefore the pressure at the surface trochoid, which corresponds to the surface in still water, is zero. The pressure at the first subsurface trochoid which corresponds to the horizontal plane in still water at 1 foot of depth is equal to the pressure of 1 foot of water. The pressure at the second subsurface trochoid is equal to the pressure of 2 feet of water, and so on for the other subsurfaces. The buoyancy per foot of length will not be measured now by the submerged area of cross section, as in cases in which pressure varies as depth below the free surface, so that new curves of pressure require to be drawn for each immersed section.

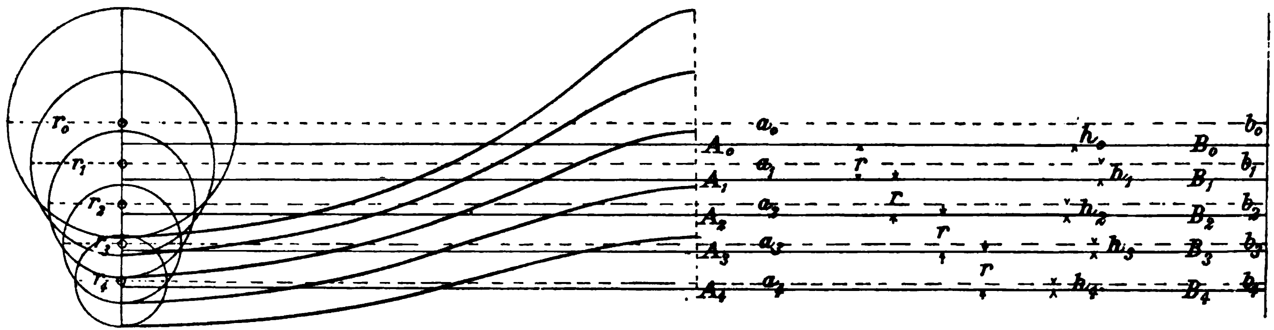


FIG. 199.

*Formulæ* :—

$L$ = length of ship

$R = \frac{L}{2\pi}$ radius of rolling circle

$r_0 = \frac{L}{40}$ ,, ,, surface tracing circle $h_0 = \frac{\pi}{L} r_0^2$

$\text{Log}\, r_1 = \log \frac{L}{40} - \frac{2\pi}{L}$ $\qquad h_1 = \frac{\pi}{L} r_1^2$

$\text{Log}\, r_2 = \log \frac{L}{40} - \frac{4\pi}{L}$ $\qquad h_2 = \frac{\pi}{L} r_2^2$

$\text{Log}\, r_3 = \log \frac{L}{40} - \frac{6\pi}{L}$ $\qquad h_3 = \frac{\pi}{L} r_3^2$

$\text{Log}\, r_4 = \log \frac{L}{40} - \frac{8\pi}{L}$ $\qquad h_4 = \frac{\pi}{L} r_4^2$

$A_0 B_0$ = surface still water
$A_1 B_1$ = horizontal layer still water at 1 ft. down
$A_2 B_2$ = ,, ,, ,, 2 ,,
$A_3 B_3$ = ,, ,, ,, 3 ,,
$A_4 B_4$ = ,, ,, ,, 4 ,,

$a_0 b_0$ = path tracing circle surface trochoid
$a_1 b_1$ = ,, ,, 1st subsurface
$a_2 b_2$ = ,, ,, 2nd ,,
$a_3 b_3$ = ,, ,, 3rd ,,
$a_4 b_4$ = ,, ,, 4th ,,

Let K X be a section (fig. 198). In the figures the intersection of the trochoids are shown at $p_0\, p_1\, p_2\, p_3\, p_4$; the section K X is in the region of the hollow of the wave in (i.) and in the region of the crest of the wave in (ii.) of fig. 198.

From the law of pressures :—

Pressure at $p_0 = 0$
„ at $p_1 = 1$ foot of water
„ at $p_2 = 2$ feet „
„ at $p_3 = 3$ „ „
„ at $p_4 = 4$ „ „

These pressures are then set up vertically to the proper scale of feet $p_0$, $p_1\,x_1 =$

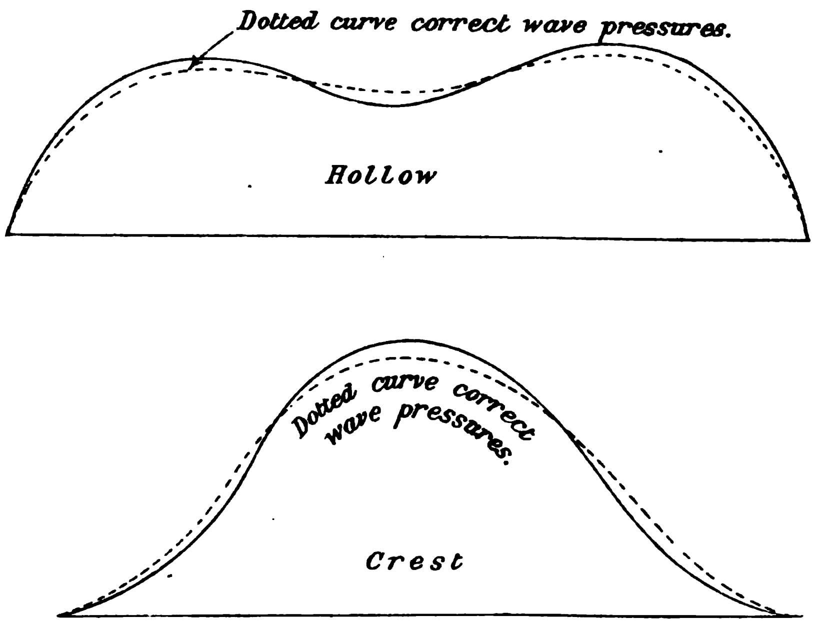


FIG. 200.

1 ft., $p_2\,x_2 = 2$ ft., $p_3\,x_3 = 3$ ft, $p_4\,x_4 = 4$ ft. A curve is drawn through the points $x_1\,x_2\,x_3\,x_4$ so obtained. Had the pressures been proportional to the depth below the surface, the points $x_1\,x_2\,x_3\,x_4$ would have been on a straight horizontal line through $p_0$, and the area of section so formed would have given the buoyancy per foot of length. The wave-pressure curve ordinates are now the vertical distances between the curves through the points $x_1$–$x_4$ and $p_1$–$p_4$; so that the buoyancy per foot of length at the section K X is represented by the area of the section between these two curves. This area is shown shaded in the figures. It will be noticed that in the region of the hollows the pressure

is greater than the ordinary hydrostatical pressure, and therefore the curve $x_1$ to $x_4$ is above the horizontal line through $p_0$. In the region of the crest this curve is below the horizontal line through $p_0$. Therefore the buoyancy per foot of length is less in the hollow to more in the crest on the true assumption as to wave pressures than if the ordinary hydrostatical law were assumed.

Having placed the ship in an arbitrarily chosen position in the wave, the intersections of the vertical line of section and the trochoidal subsurfaces are transferred to the body plan, and the pressure areas drawn out.

The adjustments, in order to satisfy the two conditions of equilibrium, are similar to those already described in obtaining the buoyancy curve for the standard conditions. The general features of the buoyancy curves, one

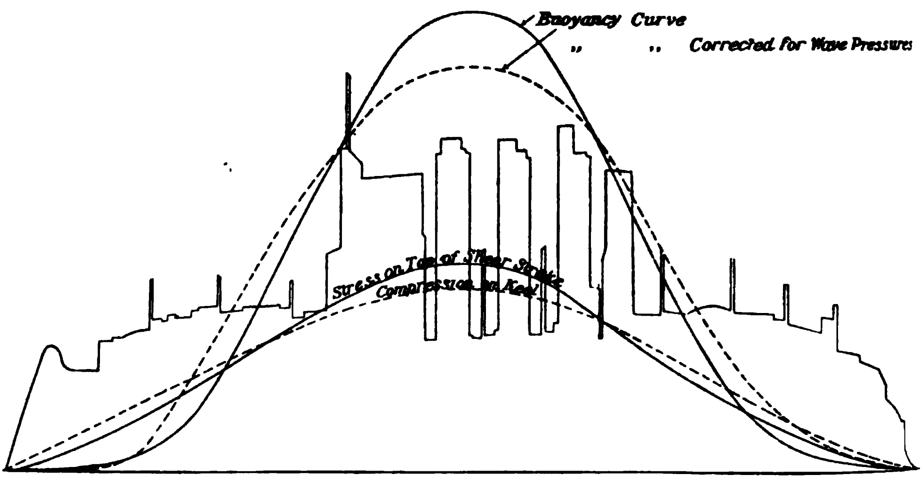


FIG. 201.

corrected for wave pressures and the other assuming the hydrostatical pressures, are shown in fig. 200.

In the case of a vessel in the hollow, the ordinates of the buoyancy curve are larger amidships, and less at the ends in the curve corrected for wave pressures. This result leads to smaller shearing-forces ordinates, and hence to a smaller sagging moment.

When the vessel is on the crest the ordinates are smaller amidships and greater at the ends, thus giving smaller shearing forces, and hence a smaller hogging moment.

The vessel for fig. 201 will sink bodily into the wave 22 inches from the position she has when no correction is made for the true wave pressures. The variation in supporting force is shown by the dotted line, fig. 201. The alteration due to this change is shown in the bending-moment curve in fig. 202 by a dotted line. The maximum bending moment is reduced 13 per cent., and the stress the same amount.

The results of correcting for wave pressures are given in Table LVII. Calculations were made on four different types of vessels:—

Torpedo-boat destroyer.
Passenger steamer, 420 ft. long.
Large cargo steamer, 530 ft long.
Fast Atlantic liner.

In the table the maximum shearing force, forward and aft, and the maximum bending moment, are given for the two conditions—

(*a*) When the pressures are taken proportional to the depth below the wave surface (this is referred to as the ordinary method), and

(*b*) When the pressures are taken according to the formula for wave pressures in a trochoidal wave. This is the correct method.

At the foot of the table the figures giving the percentage reduction due to taking true account of the wave pressures are tabulated.

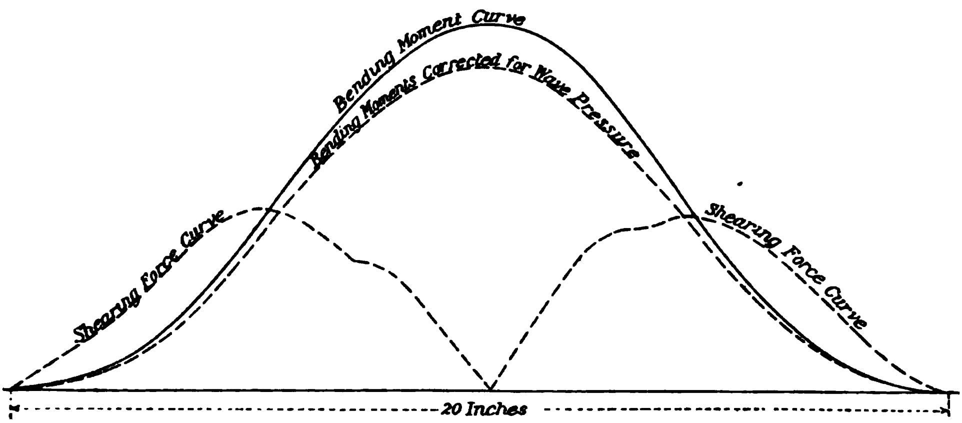


FIG. 202.

Seven conditions for the destroyer were calculated. Nos. 1 and 3 are the standard hogging and sagging conditions respectively. They show a reduction in bending moment of 9 and 9·9 per cent. respectively.

Comparing No. 1 with No. 2, the conditions are similar, except that No. 2 has a larger displacement. The percentage reduction is 15·7 in 2, as against 9 in 1. This same comparison can be made with the crest conditions (5) and (6). No. 6 has the larger displacement, and the reduction in (6) is 14 as against 10 in (5). Condition (4) is for the vessel inclined 35° to the upright in the hollow of a standard wave, the reduction being slightly greater, 10·3 as against 9·8 for the upright standard sagging condition No. 3.

In conditions (5), (6), and (7) the wave-length is two-thirds the length of the vessel, but is proportionally steeper than the standard waves, having a proportion of height to length of $\frac{1}{10}$th. This makes their size 150 ft. by 15 ft. high. Conditions (5), (6), and (7) may be contrasted with Nos. 1, 2, and 3 respectively. Regarding the other types, Nos. 8, 9, and 11 are for standard conditions. The figures for percentage reduction of bending moment due to taking account of wave pressures vary from 11·5 to 12·2.

# TABLE LVII.

## COMPARISON OF RESULTS OF BENDING-MOMENT CALCULATIONS.

I. Pressures in wave taken proportional to depth (ordinary method).
II. Pressures in wave taken according to formula for wave pressures in a trochoidal wave. (Correct method.)

| | | Torpedo-boat Destroyer, 225′ × 21′ 6″ × 13′ 6″. | | | | | | | Passenger Steamer, 420′ × 51′ × 36′. | Large Cargo and Passenger Vessel, 530′ × 59′ × 39. | | Fast Atlantic Liner, 525′ × 63′ × 43′. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Displacement | 390 | 490 | 490 | 490 | 390 | 490 | 490 | 7550 | 17200 | 11818 | 11950 | 11950 |
| | Wave length | 225′ | 225′ | 225′ | 225′ | 150′ | 150′ | 150′ | 420′ | 530′ | 530′ | 525′ | 525′ |
| | Wave height | 11·2′ | 11·2′ | 11·2′ | 11·2′ | 15′ | 15′ | 15′ | 21′ | 26·5′ | 26·5′ | 26·2′ | 26·2′ |
| | Position on wave | crest | crest | hollow | hollow inclined 35° | crest | crest | hollow | crest | crest | hollow | crest | crest inclined 20° to upright; ,, 20° to wave |
| Ordinary Method. | Maximum S.F. aft | 53 | 44 | 100 | 103 | 92 | 72 | 137 | 1208 | 2184 | 2565 | 1610 | 1630 |
| | ,, S.F. forward | 70 | 50 | 103 | 107 | 93 | 77 | 139 | 1340 | 2035 | 2263 | 1585 | 1560 |
| | ,, bending moment | 3810 | 2908 | 6240 | 6313 | 4787 | 4000 | 7300 | 146600 | 329200 | 310000 | 258000 | 256300 |
| Correct Wave Pressures. | Maximum S.F. aft | *51* | *36* | *92* | *94* | *78* | *57* | *118* | *1070* | *1900* | *2354* | *1364* | *1440* |
| | ,, S.F. forward | *61* | *38* | *98* | *99* | *84* | *66* | *123* | *1170* | *1730* | *2128* | *1367* | *1370* |
| | ,, bending moment | *3464* | *2450* | *5634* | *5662* | *4310* | *3445* | *6320* | *129500* | *289000* | *277500* | *228400* | *225900* |
| | % reduction, S.F. aft | 13·3 | 18·1 | 8·0 | 8·7 | 17·9 | 20·8 | 13·8 | 11·4 | 17·5 | 8·2 | 15·3 | 11·6 |
| | % ,, S.F. forward | 12·9 | 24·0 | 4·8 | 7·4 | 9·6 | 14·3 | 11·5 | 12·6 | 14·9 | 5·9 | 13·8 | 12·1 |
| | % ,, bending moment | 9·08 | 15·75 | 9·87 | 10·31 | 9·96 | 13·88 | 13·42 | 11·66 | 12·21 | 10·5 | 11·46 | 11·86 |
| | Number | 1 | 2 | 3 | 4 | | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| | | | | | | | | | See dia. A, Plate XVIII. | See dia. C, Plate XV. | See dia. F, Plate XV. | | See diagram B, Plate XVIII. |

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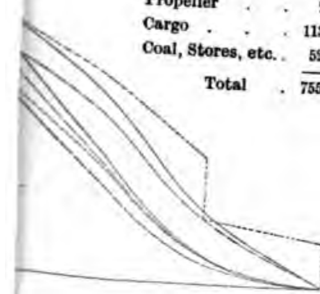
*Weights.*

| | Tons. |
|---|---|
| Hull . . . . | 4320 |
| Engine . . . | 559 |
| Boiler A. . . | 439 |
| " F. . . . | 439 |
| Donkey Engine . | 27 |
| Shaft . . . . | 88 |
| Propeller . . | 25 |
| Cargo . . . . | 1131 |
| Coal, Stores, etc. . | 522 |
| Total . | 7550 |



essel on crest.
nclined 20° to upright.
" 20° to wave.
Vave=525. Cor. 20°.

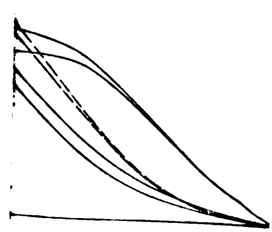


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TABLE LVIII.

| | Length of Vessel. | Block Coefficient. | Percentage Reduction. |
|---|---|---|---|
| Cargo steamer . . | 530′ | ·68 | 12·2 |
| Passenger steamer . | 420′ | ·56 | 11·7 |
| Atlantic liner . . | 525′ | ·51 | 11·5 |
| Torpedo-boat destroyer | 220′ | ·48 | 9·1 |

Table LVIII. shows the percentage reductions in the different types of ships due to taking account of wave pressures.

All the conditions are standard hogging conditions. It should be noted that in the shallower draught vessels the difference is necessarily not so much as in the deep draught vessels.

Diag. A Plate XVIII. is for condition 8.
" C " XV. " " 9.
" F " XV. " " 10.
" B " XVIII. " " 12.

The nature of the changes made in the buoyancy curve, by taking true account of the wave pressures, can be seen from these diagrams on which both curves have been drawn.

# CHAPTER XXV.

## CONSIDERATION OF THE STRESSES DUE TO HEAVING AND PITCHING.

THE motion of a ship among waves moving at right angles to her length is of two kinds—pitching and heaving. The pitching is an oscillatory rotational motion about a transverse axis through the centre of gravity. This motion is independent of the heaving motion, which is the vertical oscillatory movement of the centre of gravity as the vessel passes through the waves. These motions depend upon the period of the disturbing forces, *i.e.* the waves, relatively to the ship.

The pitching period is generally smaller than the rolling period. When the vessel is oblique to the waves the pitching is not so excessive, but it is accompanied by a certain amount of rolling. These two rotational motions give a resultant corkscrew-like motion. It is the intention in this chapter to consider the effect of the heaving and pitching motion only on a ship steaming at right angles to the waves.

The mathematical solution of the motion of a ship among waves is a very complicated problem. It is necessary to limit the conditions by certain assumptions. In a paper by the late Mr Read to the Institution of Naval Architects, the author gave a solution to the heaving motion on the following assumptions:—

(1) That the ship is steaming at right angles to the crests of the waves.
(2) That the waves are regular.
(3) That the pitching motion may be neglected.
(4) That the positions of the ship in the wave in which equilibrium of weight and support exist are such that the locus of the centre of gravity is a curve of sines.

The first assumption is made as it gives the position of ship when the effect of the heaving motion is greatest. The second assumption has to be made in order to arrive at a mathematical solution.

The third assumption is made for the following reason:—

If the bending moment amidships is M when the vessel is in any position relatively to the wave, and the vessel be supposed to change trim slightly, then, considering the ship in two parts divided by a section through the centre of gravity, there will be a force in the fore body equal to the change in buoyancy of the fore end. This force will act either upwards or downwards according as the vessel trims by the head or by the stern. Let this force be

$f$, fig. 203. Then, at the section through the centre of gravity, we can apply two equal and opposite forces, each of magnitude $f$. There will thus be a couple acting on the fore end of magnitude $m$, say, and a force $f$ at the section through the centre of gravity. If the ends of the vessel are similar in form, the force corresponding to $f$ at the after end will be equal in magnitude, but of opposite sign. The couple acting at the after end will therefore be $-m$, and the force amidships will be $-f$, which will balance the force $+f$ due to the excess or defect of buoyancy at the fore end.

These unbalanced couples will tend to turn the two ends in the same direction. If the moments of inertia of the two ends about an axis through the section be the same, the velocities induced will be identical, and there will be no tendency to separate at this section. In other words, there will be no stress at the midship section due to the forces causing pitching. If the moments of inertia of the ends are unequal, the end which is less will tend to move more quickly than the other. To avoid this, stresses must be exerted in the material at the midship section whose moments about the neutral axis are sufficient to cause the velocity of the two ends to be the same.

It will be shown later that, while there may be no change of stress amid-

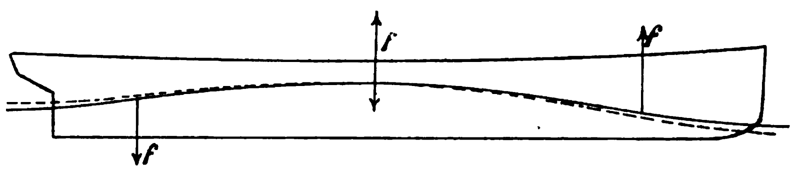



FIG. 203.

ships due to pitching, yet at some distance forward and aft there will be a change. The stresses due to bending are greatest amidships, and the vessel is less able to withstand an increase of stress there than at some distance forward and aft, where the bending stresses are much less. We shall only, therefore, deal with the vertical motion of the centre of gravity due to the inequality of the resultant weight and buoyancy forces as a wave passes along the vessel, and at present only attempt to find the change of stresses due to heaving.

**Heaving Calculations.**—Consider the simple case of a box-shaped vessel, and imagine a wave of the same length to be passing along the side: for any two successive positions the total buoyancy is the same while the vessel remains at the same height relatively to the wave. There will therefore be no heaving motion. Now consider this vessel tapered towards the ends. When the hollow of the wave is amidships, the vessel has a certain position relatively to the axis of the wave. When the crest of the wave is amidships, the proportion of buoyancy is very much greater there than at the ends, so that the vessel will have to rise bodily a certain amount in order to adjust the weight and buoyancy.

Supposing a wave to pass slowly along a ship's side so as to allow sufficient time for the adjustment of the weight and buoyancy forces, and that at successive positions the height of the centre of gravity of the vessel above the axis of the wave is found. Plotting these heights in terms of the position of the crest of the wave relatively to the ship, we obtain a curve giving the locus of the centre of gravity of the ship under the above conditions.

The author of the paper referred to above found that this curve for a number of ships and a trochoidal wave of length equal to that of the vessel closely approximated to a curve whose equation is given by

$$h = a \sin \frac{\pi t}{T},$$

where $h$ is the height of any point on the curve relatively to an axis,

$2a$ is the vertical movement of the centre of gravity from the hollow to the crest of the wave,

T is the half period of the wave relatively to the ship,

and $t$ is the elapsed time.

Let A B in fig. 204 represent this curve with reference to the axes O X and O Y.

Then drawing B C and A C parallel to O Y and O X respectively, we have $BF = FC = a$ and $AG = GC$.

In order to find "$a$," the positions of equilibrium, taking into account the

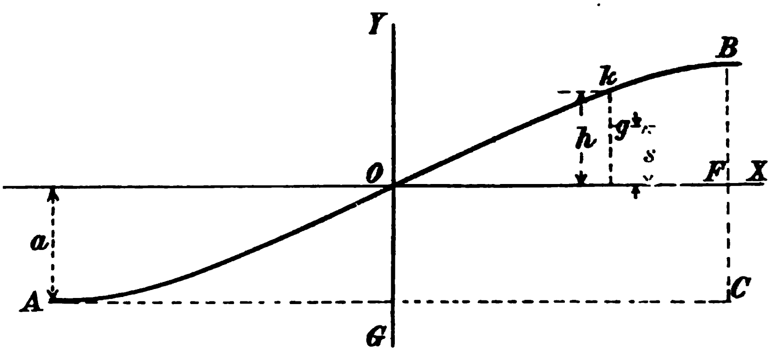



FIG. 204.

wave pressures, must be determined. The change in the vertical position of the centres of gravity of the ship in these two positions is $2a$.

This value will be the "heave" as the wave passes sufficiently slowly, but the rapid passage of a wave does not give time for the adjustment of weight and support, consequently the centre of gravity of the ship is generally either above or below the position it would take up if this equality were established.

Suppose, at any instant, the centre of gravity to be at $g$ instead of at $k$, where it would be if the weight and support were equalised. The vessel will be immersed a distance $kg$ below its position of equilibrium, and the excess of buoyancy over weight will equal $p \cdot kg$ (where $p$ is the tons per foot of immersion of the ship). This force of $p \cdot kg$ tons will tend to lift the vessel, and thereby give vertical motion to it. The centre of gravity rises under the action of this force until it reaches $k$, when the momentum of the vessel will carry it past this position, and there will then be an excess of weight over buoyancy, which will in time bring the vessel to rest, and subsequently cause her to accumulate a downward motion. This downward motion will continue until the buoyancy again exceeds the weight, when in time the vessel's motion will be stopped and reversed, thus causing an oscillatory vertical movement or "heave."

The following is the method of solution in the determination of the maximum heave:—

Let $g$ be at a time $t$ the position of the centre of gravity distant $s$ from the axis O X. Then the upward force acting on the vessel is equal to $p(h-s)$, where $h$ is the height of $k$ above the datum line O X.

The equation of motion is therefore

$$M\frac{d^2s}{dt^2}=p(h-s), \text{ where } M=\text{mass of vessel.}$$

$$\therefore \quad \frac{d^2s}{dt^2}=\frac{p}{M}\left(a\sin\frac{\pi t}{T}-s\right) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The solution of this equation is of the form

$$s=A\sin\frac{\pi t}{T}+B\sin at+C\cos at+D \quad . \quad . \quad . \quad (2)$$

$$\therefore \quad \frac{ds}{dt}=A\frac{\pi}{T}\cos\frac{\pi t}{T}+Ba\cos at-Ca\sin at \quad . \quad . \quad . \quad (3)$$

$$\therefore \quad \frac{d^2s}{dt^2}=-A\frac{\pi^2}{T^2}\sin\frac{\pi t}{T}-Ba^2\sin at-Ca^2\cos at \quad . \quad . \quad (4)$$

Assuming the vessel to start from a position when the centre of gravity was in the still-water position at 0 when $s=0$ and $V=0$ and $t=0$.

Then from (2) we have $C+D=0$ . . . . . . . . . (5)

and from (3) we have $\frac{A\pi}{T}+Ba=0$ . . . . . . . . . (6)

Substituting from equations (2) and (4) in (1) we have

$$-A\frac{\pi^2}{T^2}\sin\frac{\pi t}{T}-Ba^2\sin at-Ca^2\cos at,$$

$$=\frac{p.a}{M}\sin\frac{\pi t}{T}-\frac{p.A}{M}\sin\frac{\pi t}{T}-\frac{p.B}{M}\sin at-\frac{p.C}{M}\cos at-\frac{pD}{M} \quad . \quad . \quad (7)$$

Substituting from equations (5) and (6) in (7) we have

$$-\frac{A\pi}{T}\left(\frac{\pi}{T}\sin\frac{\pi t}{T}-a\sin at+Ca^2=\frac{pa}{M}\sin\frac{\pi t}{T}-\frac{pA}{M}\sin\frac{\pi t}{T}+\frac{pA\pi}{MaT}\sin at\right.$$

$$-\frac{pc}{M}\cos at+\frac{pc}{M}.$$

This is true for all values of $t$ if coefficients of $\sin\frac{\pi t}{T}$, $\sin at$, and $\cos at$, are equal, and $c=0$.

Hence $$-A\frac{\pi^2}{T^2}=\frac{p.a}{M}-\frac{p.A}{M}, \text{ coefficient of } \sin\frac{\pi t}{T},$$

whence $$A=\frac{\frac{p}{M}.a}{\frac{p}{M}-\frac{\pi^2}{T^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Again $$\frac{A\pi a}{T}=-\frac{pA\pi}{MaT}, \text{ coefficients of } \sin at,$$

whence $$a=\sqrt{\frac{p}{M}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The solution to (2) is obtained by substituting these values for A, B, C, D, and $a$.

Therefore the solution becomes

$$s=\frac{\frac{p.a}{M}}{\frac{p}{M}-\frac{\pi^2}{T^2}}\left\{\sin\frac{\pi t}{T}-\frac{\frac{\pi}{T}}{\sqrt{\frac{p}{M}}}\sin\sqrt{\frac{p}{M}}.t\right\} \quad . \quad . \quad (10)$$

When the vessel is on the crest $\sin\frac{\pi t}{T}=1$ (since $t=\frac{T}{2}+nT$), and as the factor outside the brackets is always the same sign, the maximum value of $s$ will necessarily occur when

$$\sin\sqrt{\frac{p}{M}}.t=-1.$$

$\therefore$ $s$ is a maximum, and is then $$\frac{\frac{pa}{M}}{\frac{p}{M}-\frac{\pi^2}{T^2}}\left(1+\frac{\frac{\pi}{T}}{\sqrt{\frac{p}{M}}}\right)=\frac{a\sqrt{\frac{p}{M}}}{\sqrt{\frac{p}{M}}-\frac{\pi}{T}}.$$

The lowest position on crest is when $\sin\sqrt{\frac{p}{M}}.t=+1$, because $\sin\frac{\pi t}{T}=1$.

Then $$s=\frac{a\sqrt{\frac{p}{M}}}{\sqrt{\frac{p}{M}}+\frac{\pi}{T}}.$$

When the vessel is in the hollow, $\sin\frac{\pi t}{T}=-1$, and the maximum value of $s$ will occur when

$$\sin\sqrt{\frac{p}{M}}.t=1,$$

so that $$s=\frac{a\sqrt{\frac{p}{M}}}{\sqrt{\frac{p}{M}}-\frac{\pi}{T}}.$$

*Example of a Calculation based on the foregoing Method.*—The vessel is a torpedo-boat destroyer of 500 tons displacement.

Length of vessel = 220 feet.
,, wave = 220 feet.
Height of wave = 11 feet.
Speed of a wave of this size = 20 knots, or 33 feet per second.

The speed of the vessel against the waves has been taken to be 10 knots, say 17 feet per second, this speed being about the greatest safe speed at which a vessel of this kind could steam against the waves.

The distance $2a$ obtained by placing the body statically at rest in the hollow and on the crest and measuring the vertical distance between the two positions of the centre of gravity, was found to be 2·32′, so that

$$a = 1{\cdot}16'.$$

$p$, the tons per foot immersion = 101 tons.

M, the mass of the vessel $= \frac{\Delta}{g} = \frac{500}{32}$ absolute units.

We have, therefore, $\sqrt{\frac{p}{M}} = 2{\cdot}55.$

T = half period of wave relatively to the ship.
Relative speed of wave to vessel = 50 feet per second.

$$\therefore \quad 2\,T = \frac{220}{50} = 4{\cdot}4 \text{ sec.,}$$

$$\therefore \quad T = 2{\cdot}2,$$

$$\therefore \quad \frac{\pi}{T} = 1{\cdot}4.$$

In this type of vessel the hogging moment when on the crest is much less than the sagging moment when in the hollow. The calculation has been made only for the case of the increase on the maximum moment, namely, the sagging, this increase being due to the extra immersion into the hollow at the time of the maximum heave. The maximum heave is given by

$$s_{\text{max.}} = \frac{\sqrt{\frac{p}{M}} \cdot a}{\sqrt{\frac{p}{M}} - \frac{\pi}{T}} = 2{\cdot}6.$$

$s$ is measured from the axis O X in fig. 204, therefore the extra immersion or the heave into the hollow from the ordinary statical position is equal to $2{\cdot}6 - 1{\cdot}16 = 1{\cdot}46'$.

Diagrams, figs. 205 and 206, show the method of determining the increased bending moment due to this extra immersion in the hollow.

We have found that during the passage of the wave and at the maximum heave the rising out of the crest or the immersion into the hollow from the statical position is 1·46 ft. The buoyancy curve for the whole length of the ship having been constructed for the statical position in the hollow, the area and the positions of the centres of buoyancy of parts of the curves on each side of the maximum bending moment can be found.

So also can be found the centre of gravity and area of each part of the weight curve on each side of the position of maximum bending moment. The forces which these areas measure are represented in fig. 205 at $g$ and $b$, and the couple they represent is equal to the maximum bending moment.

The buoyancy curve for the vessel heaved into the wave is next drawn (fig. 206). The centres of gravity of the weights of the fore and after ends and the centres of buoyancy of the excess or increase in buoyancy at each end, as shown shaded, are then determined for the parts to each side of the position of maximum bending moment.

Call $B_1$, $B_2$ the amount of the buoyancy at each end, $W_1$, $W_2$ the weights, and $b_1$, $b_2$ the added buoyancy.

The resultant couple at one end, in the ordinary statical position where

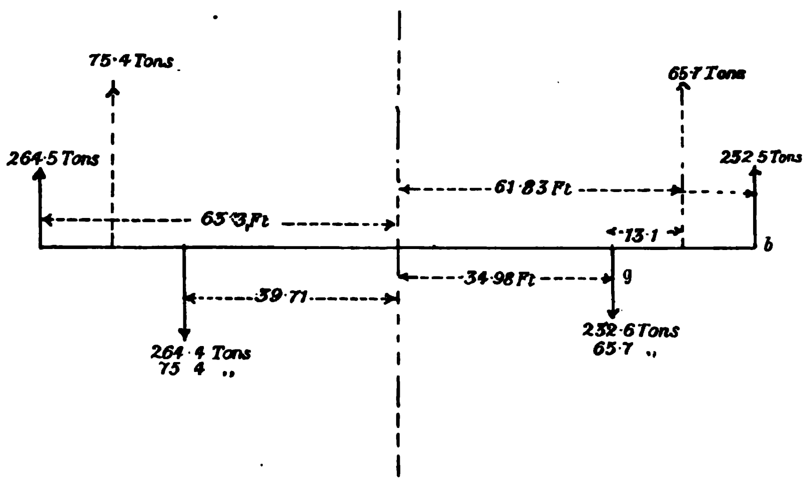



**Fig. 205.—Diagram of Forces for Heaving Condition.**

Statical Condition, BM = 232·5 × 61·83 − 232·6 × 34·98 = 6240 ft. tons.
Heaving Condition, BM = 6240 + 65·7 × 13·1 = 7101 ft. tons.

weight equals total buoyancy, is balanced by the resultant couple at the other end, and is equal to the maximum bending moment. If we introduce two equal and opposite forces, $b_1$ and $b_2$ at the lines of action of $W_1$ and $W_2$ respectively, we see that it is equivalent to an additional couple $m$, say, and two unbalanced forces, $b_1$ and $b_2$, acting upwards as shown. In this example (see fig. 205) the extra buoyancy at the fore side of position of maximum bending moment = 65·7 tons acting at an arm of 13·1 ft., having a moment $m$ of 861 tons and an unbalanced force of 65·7 tons.

| | Tons. | Leverage. |
|---|---|---|
| The buoyancy of fore end | = 232·5 | 61·83 ft. |
| The weight ,, ,, | = 232·6 | 34·98 ft. |

∴ Ordinary statical moment = 232·5 × 61·83 − 232·6 × 34·98
= 6240 ft. tons.

To this has to be added $m$, the moment due to the heave, which is 65·7 tons × 13·1 ft. = 861 ft. tons. The same result will be obtained by taking the after forces.

∴ Total bending moment at section of maximum bending moment = 7101 ft. tons, an addition of 13·8 per cent.

**Pitching.**—Fig. 207, A and B, represents a vessel on the hollow and crest respectively of a wave. The full line indicates the position of the wave contour on the vessel when equality of weight and buoyancy is established, and the centres of gravity and buoyancy lie on the same vertical.

Suppose the vessel trims slightly by the head, the new wave-profile will be as indicated by the dotted line. Let X Y be any transverse section dividing the vessel into two parts. Let $f_1$ be the defect of buoyancy of the after end acting at a distance $h_1$ from X Y and $f_2$ the excess of buoyancy at the

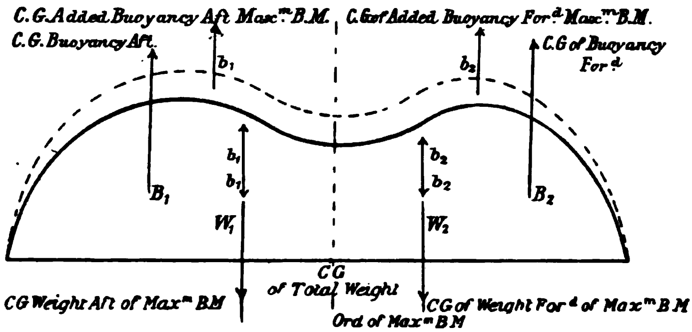



FIG. 206.

Dotted Curve, Buoyancy—Heaving Condition.
Drawn ,, ,, Statical ,,

fore end acting at the arm $h_2$. Since the motion is assumed to be purely pitching the displacement will remain constant, and consequently the wedge of emersion will equal the wedge of submersion. If $v$ is the volume of the part of the submerged wedge between $g$ and X Y,

$$f_1 + v = f_2 - (-v) = f_2 + v.$$

Therefore $$f_1 = f_2.$$

Without affecting the system of forces, two forces each equal and parallel to $f_1$ can be considered to act on the section X Y, but in opposite directions. The result of this will be that instead of a single force $f_1$ acting at a distance $h$ from X Y there will be a couple $f_1 h_1$ tending to turn the after body in a counter-clockwise direction and a force $f_1$ acting downwards at the section X Y. Similarly, on applying two opposite forces $f_2$ at X Y we get a couple $f_2 h_2$ tending to trim the fore body also in a counter-clockwise direction, and a force $f_2$ acting upwards at X Y. The forces $f_1$ and $f_2$ will create a shearing resistance $f_1 = f_2$ in the section.

In order that the angular acceleration of the two ends may be the same, there must pass through the section X Y a moment $m$, say, whose value is such that

$$\frac{W_1 K_1^2}{g} \cdot \frac{d^2\theta}{dt^2} + f_1 h_1 + m = 0 \quad . \quad . \quad . \quad . \quad (1)$$

for the part aft of X Y,
where $W_1$ is the weight of the after part and $K_1$ is the radius of gyration of the after part about the axis of rotation.

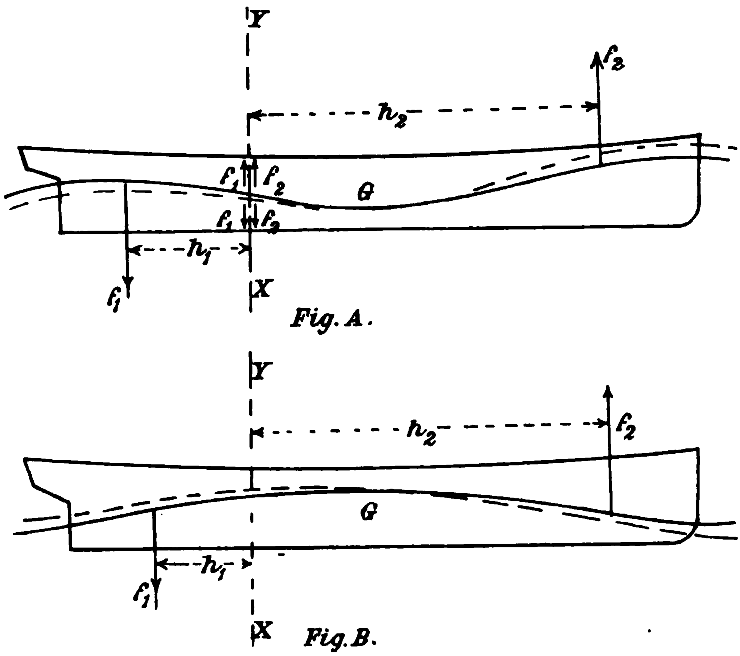



Fig. 207.

For the part forward of X Y we will have $m$ of opposite sign,

$$\frac{W_2 K_2^2}{g} \cdot \frac{d^2\theta}{dt^2} + f_2 h_2 - m = 0 \quad . \quad . \quad . \quad . \quad (2)$$

where $W_2$ and $K_2$ for the fore body are similar quantities to $W_1$ and $K_1$ for the after body.

The axis of rotation will be through G the centre of gravity.

Since the angular acceleration of the two ends is the same,

$$\left(\frac{W_1 K_1^2}{g} + \frac{W_2 K_2^2}{g}\right)\frac{d^2\theta}{dt^2} + f_1 h_1 + f_2 h_2 = 0$$

$$i.e. \quad \frac{W K^2}{g} \cdot \frac{d^2\theta}{dt^2} + fh = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where W = weight of whole body.

K = radius of gyration of whole body about G.

$(f_1h_1 + f_2h_2)$ is equal to the turning couple, which we can call $fh$.

Equation (3) is the equation of the pitching motion, from which we can determine the period of pitching and the acceleration at any instant.

From the equations (1) and (2) we can get the bending moment at any section X Y due to the pitching.

In making a calculation to determine the pitching moment at any section of a vessel which has trimmed through an angle from a known position of equilibrium, there are two things to be done:—

The moment of the change in buoyancy to one side of the section about that section has to be calculated.

The moment of inertia of the part of the vessel to one side of the section about the centre of gravity of the vessel has to be determined.

Having obtained these two expressions, we can find the total moment of the change of buoyancy, and the total moment of inertia of the weight of the vessel about its centre of gravity. These we substitute in equation (3) and so determine $\frac{d^2\theta}{dt^2}$, the acceleration. We then substitute the expressions calculated for the moment of the buoyancy about the section and the moment of inertia of the weight to one side in equation (1) or (2), and so we can get $m$.

An example of this calculation has been made,—see diagrams, figs. 208 and 209. The vessel is a torpedo-boat destroyer of 210 ft. length and 420 tons displacement. At this displacement the waterline is W L. Suppose the vessel in smooth water to be trimmed through an angle $\theta$ to $W_1 L_1$, still keeping the displacement the same, the trimming to take place by the action of some outside force. Let A B F be the buoyancy curve corresponding to W L, and let $A B_1 F$ be the buoyancy curve for $W_1 L_1$. Then a curve as at A $b$ $b$ F can be drawn which gives the change in distribution of buoyancy. This curve will represent the distribution of the force acting on the vessel causing pitching if the trimming force were now suddenly released after the vessel reached the position $W_1 L_1$. This curve can be easily integrated twice by the Integraph. The first integral is $A b_1 F$ and the second integral is the curve F $m$ C. This curve F $m$ C is the moment curve, and any ordinate gives the moment of the area of the disturbing force curve to the right of the ordinate about that ordinate as axis. This moment is therefore the moment that we have to determine. The final ordinate A C is therefore the total moment of the disturbing force. The proof of this proposition is similar to that for the relation of the load, shearing force, and bending moment given on page 141.

In diagram, fig. 209, the weight curve of this vessel is shown. The most convenient way to get the moment of inertia of any part about the centre of gravity as axis is to use a large type of Integrator and set it to the axis through the centre of gravity. The Integrator arm should be of such a length to span the whole weight curve. The weight curve can then be divided by a series of ordinates as shown, and the moment of inertia of the weight about the centre of gravity, say to the forward side of each ordinate, can be measured by tracing over the outline of the required part of the curve with the pointer and noting the readings. The moment of inertia for each part can then be calculated from the readings.

A convenient method can now be adopted here. We have seen that the acceleration $\frac{d^2\theta}{dt^2}$ is a constant for this position in the equation for determining

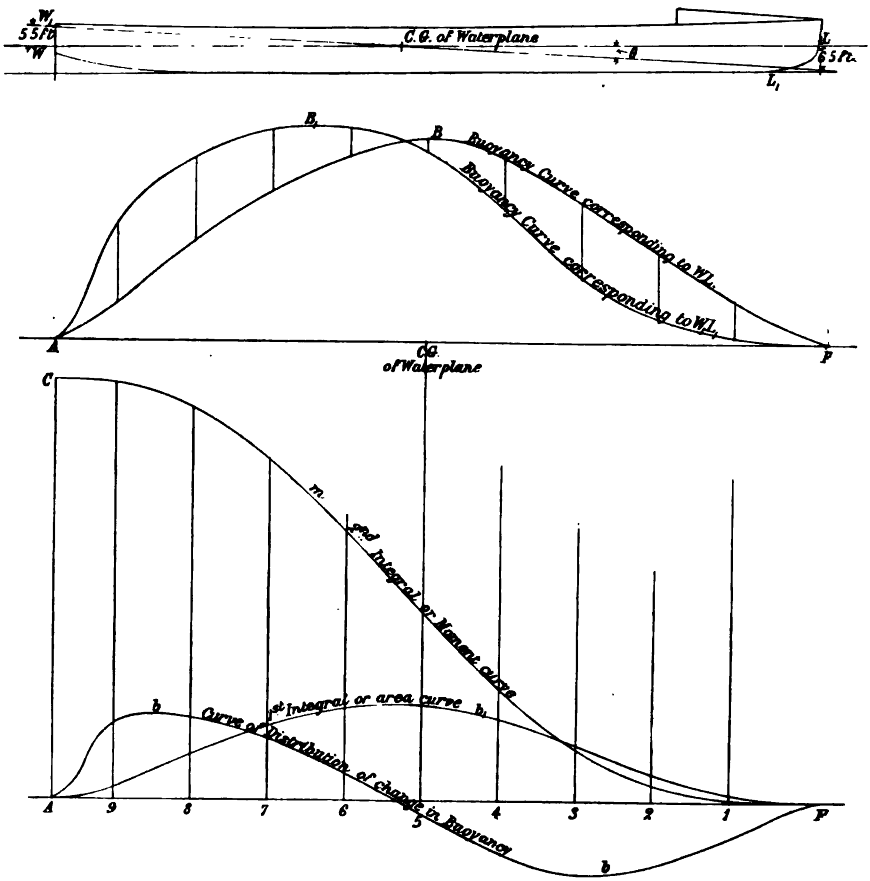



FIG. 208.

*Scales.*

| | |
|---|---|
| Length . . . . . | 1″ = 10·55 ft. |
| Buoyancy . . . . . | 1″ = ·668 tons per ft. |
| 1st Integral Buoyancy . | 1″ = 35·25 tons. |
| 2nd ,, ,, . | 1″ = 892·5 ft. tons. |

Standardised Scale for B M is $1'' = \frac{\Delta \times L}{200} = 446 \cdot 2$ ft. tons.

*m*. It is first determined from equation (3). A C is the total moment of the disturbing couple, so that A C represents $\frac{WK^2}{g} \cdot \frac{d^2\theta}{dt^2}$, but we have W K² from the readings and $\frac{d^2\theta}{dt^2}$ is a constant. Therefore all that is necessary is to set up the readings for moment of inertia to such a scale as will make the final ordinates equal. This has been done in diagram, fig. 209. The curve for moment of inertia is F I C. It can be easily proved that the difference between the ordinates of the curves F I C and F M C (fig. 209) at any section gives *m*. This difference has been plotted on a straight base to a standardised scale.

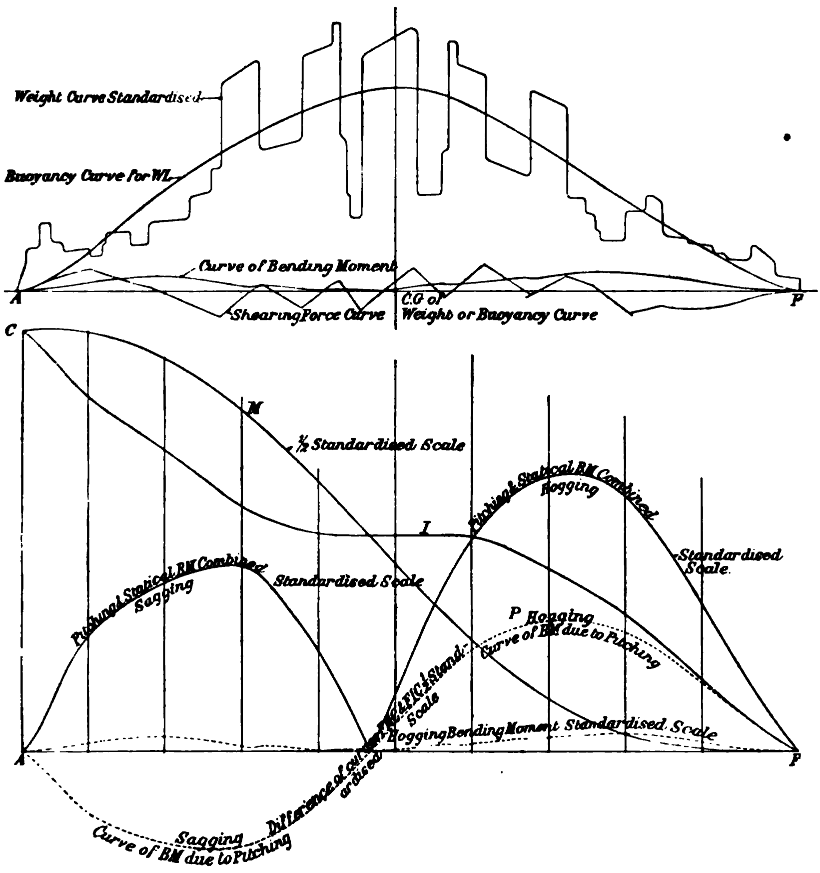



FIG. 209.

*Results.*

| | | |
|---|---|---|
| Maximum Hogging Bending Moment | . . | =3230 ft. tons. |
| ,, ,, ,, ,, | from F.P. | =29 per cent. of L. |
| ,, Sagging ,, ,, | . . | =2165 ft. tons. |
| ,, ,, ,, ,, | from A.P. | =27 per cent. of L. |

The curve F P P A gives the bending moment due to pitching. The forward part of this curve is a hogging moment and the after part is sagging. This moment due to pitching at any section has now to be added or subtracted to the statical moment at the section in the still-water condition W L.

The strength curves for the still-water condition have been drawn on the same diagram, and the statical bending-moment curve has been plotted on the same base as the pitching bending-moment curve. It will be seen that the pitching action causes a bending-moment which is zero at both ends and somewhere near amidships, and is a maximum at two places; in this case the maximum hogging is at about one-fourth the length from F and the maximum sagging at about one-fourth the length from A.

The following are the results of the calculation for which the diagrams are given. The vessel is one of the destroyer type and is 211 ft. long, with a displacement of 420 tons at the waterline W L. In the still-water condition the bending moment is inconsiderable. The vessel was then supposed to be trimmed by the stem 12 ft., *i.e.* 6·5 ft. forward and 5·5 ft. aft. In this trimmed condition the displacement remained the same. The result of the calculation for bending moment due to pitching shows that the maximum bending moments on the structure are 3230 ft. tons hogging acting at a section 29 per cent. of length from forward perpendicular, and 2165 ft. tons sagging acting at a section 27 per cent. of length from after perpendicular. These bending moments are considerable, and show that it may be necessary to make a calculation similar to this in vessels which are likely to experience considerable pitching. In the above vessel the standard sagging bending moment was 4100 ft. tons, and the standard hogging bending moment 2930 ft. tons, so that in this case pitching by the amount of 12 ft. causes a larger hogging moment than the standard hogging moment.

Other conditions can be calculated, and, in order to represent the condition at sea, the calculation should be made for a position which has been obtained by trimming the vessel from a position of equilibrium on a wave. The standard hogging condition is a position of equilibrium; and if the vessel be assumed to be trimmed through a definite angle, the pitching-moment curve can be found in exactly the same way as just described, and it can be combined with the statical standard hogging bending-moment curve. A mathematical treatment of the subject will be found in two papers to the Institution of Naval Architects by Captain Kriloff. These papers give the method of determining the equations to the pitching and heaving motion, but being mathematical, the method involves the use of a large number of tables and equations which make the calculation very complicated. We think that the subject is treated in a simpler and clearer fashion in this chapter as far as the determination of the pitching bending moment is concerned, and it will be better for the student who is interested in this subject to master its contents before attempting the more elaborate methods in the paper just mentioned.

# CHAPTER XXVI.

## TRANSVERSE STRENGTH.

The fact that the longitudinal bending moments that come upon a vessel's structure are so much greater than any other straining forces that the vessel can meet with in ordinary service, makes the consideration of the longitudinal stresses of the first importance in the question of the vessel's strength. Nevertheless, it is important to examine the other causes that produce stresses in the vessel's structure.

In the general question of strength, local stresses, such as those due to heavy parts of the structure, or, in a warship, to the weight of guns, barbettes, or armour, to the firing of the guns, to the strains in the deck in way of the mast, are not included.

The longitudinal bending moments change the form of the vessel in a fore and aft vertical plane, and also deform the vessel transversely. Consequently, the transverse members do a certain amount of work in keeping the longitudinal members in place. Stresses are thus indirectly set up in the transverse members. The transverse members may also experience stresses directly due to the action of the sea or the loading of the vessel, etc., or by the inequality of the transverse distribution of the weight and support.

The method of determining the longitudinal stresses is simple.

We have seen that the longitudinal continuous parts may be considered as a rigid girder symmetrical about a middle line, so that the formula $\frac{p}{y}=\frac{M}{I}$ may be applied. The method of determining the stresses in the transverse framing is more complex, and can only be done by making certain assumptions. The method about to be described was developed by Dr Bruhn, and is contained in two papers given to the Institution of Naval Architects, 1901 and 1904.

It is easy to imagine a transverse ring of one frame space in length containing only the members that contribute directly to the transverse strength. Such parts would include—

Frame,
Reverse frame,
Floor plate and bracket,
Knees,
Beam,
Deck and shell plating,
Pillars, and any transverse stiffening.

Knowing the system of loads on this arrangement, the stresses are determinable. The difficulty of the problem lies in determining the forces acting on the transverse framing.

The loads or straining forces may be mentioned :—

(1) Weight of structure itself.
(2) Weights carried by the structure.
(3) Water pressure or pressure of supports.
(4) Indirect loads due to the longitudinal bending moments.
(5) Shearing forces of the neighbouring sections.

The first three items can be easily determined. It is, however, difficult even to approximately determine (4). Dr Bruhn, in his most recent paper, gives a rough method, but for the present it may be left out of account. The calculations will therefore afford only a means of comparison of the transverse strength of one ship with that of another, and will not give absolute stresses.

In view of this, the method should be standardised, and a standard section or transverse ring should be taken.

In way of the boiler and engine room, there is usually extra transverse stiffening at some of the sections. It might be interesting to examine how the transverse strength is affected by these parts, but, for purposes of comparison between, say, similar ships of different size, and consequently different scantlings, only the standard section as indicated need be compared. The parts composing the section have been enumerated.

The principle of least work can be successfully applied to the determination of the stresses in a structure of this kind. The transverse members in a vessel with two or more decks are redundant, and the whole transverse structure is rigid. The principle of least work is used extensively on the Continent by bridge and mechanical engineers, but it is less known in this country.

On the Continent it has been developed, so that it can be systematically applied to many practical cases. Chief among those who developed it was Alberto Castigliano. The principle is sometimes referred to as the principle of Alberto Castigliano.

**The Principle of Least Work.**—Let us consider how it can be applied to a girder made up like the transverse framing of a vessel.

Let the figure 210 represent a rigid girder like that made up of a beam, frame and reverse frame, and floor.

This girder is symmetrical about the middle line K L, so that we need only consider one side, K A L.

At any section, N say, we may have three different straining actions :—

(1) A direct force, P, perpendicular to the plane of the section.
(2) A shearing force, Q, in the plane of the section.
(3) A bending moment, M, in a plane perpendicular to the plane of the section.

These values of P, Q, and M may vary along the section. If we know the values of P, Q, and M for any particular section, we can find them for any other section, say at K, where they will be, say, $P_0$, $Q_0$, $M_0$. Hence it only remains for us to find the values of P, Q, and M at any particular section.

Let us assume these values at the section K to be $P_0$, $Q_0$, $M_0$ respectively.

Let $a$ be the angle between the longitudinal vertical plane L K and the plane of section through N.

*The System of Loading.*—Suppose the vessel to be floating in water and to be laden with cargo in the hold. Then we have to consider the values of the P, M, and Q caused by the changes of the loads on the forces $P_0$, $Q_0$, and $M_0$. We are given the dimensions of the girder, and by taking into account the intermediate forces acting on the girder K N we can deterime P, Q, and M.

Let $v$ be the component vertically of the total water pressure on the girder K N.
,, $h$ be the component horizontally of the total water pressure on the girder K N.
,, $w$ be the weight of the structure between K and N.
,, $c$ ,, ,, cargo ,, ,, *i.e.* the dead load acting on the part K N, suppose it to have no horizontal component.

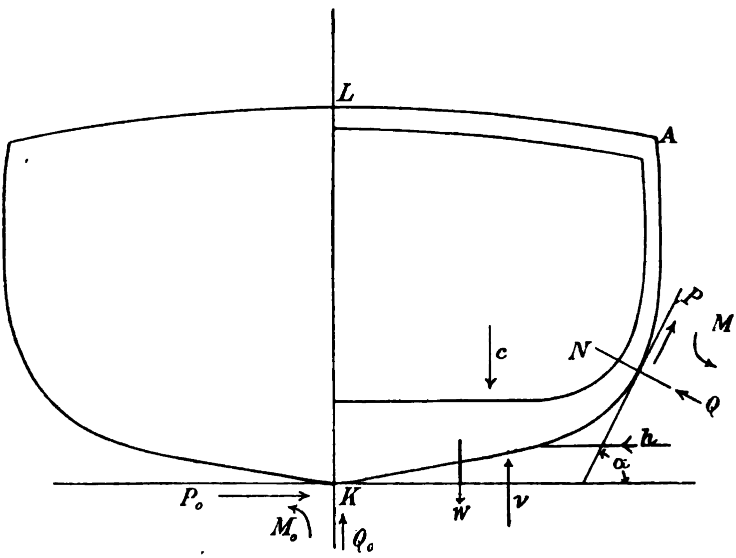



FIG. 210.

Let us assume also that the resultant loads are balanced, *i.e.* that the girder is in equilibrium under the above loads $v$, $h$, $w$, $c$, the forces $P_0$, $Q_0$, and the bending moment $M_0$ at K.

The equations of equilibrium are therefore,

$$P = P_0 \cos a + Q_0 \sin a + v \sin a - h \cos a - w \sin a - c \sin a.$$
$$Q = -P_0 \sin a + Q_0 \cos a + v \cos a - h \sin a - w \cos a - c \cos a.$$

If we call V, H, W, and C the moments of $v$, $h$, $w$, and $c$ about a point in the section N, and let the coordinates of this point with reference to the axis through K be $x$ and $y$,

Then $$M = M_0 + P_0 y - Q_0 x - V - H + W + C.$$

These three equations determine P, Q, and M for any section taken at a point $x\,y$ in the girder.

We can now apply the principle of least work to the determination of the values of $P_0$, $Q_0$, and $M_0$ at the section K.

Expressions for the work done by the straining forces:—The general expression for the work done in straining a structure is

$$\iiint \frac{p^2}{2E}\,dy,\,dx,\,dz,$$

where $p$ is the stress at any point and E the modulus of elasticity of the material.

For a direct push or pull stress P this expression becomes $\frac{1}{2E}\int\frac{P^2}{A}\,.\,dl$, where A is the sectional area and $l$ is the length of the bar.

For a shearing stress Q the expression becomes $\frac{1}{2G}\int\frac{Q^2}{A}\,.\,dl$, where G is the modulus for shearing. This also assumes that the shearing stress is uniform over the section.

Actually this expression should be $\frac{1}{2G}\int\frac{\mu Q^2}{A}\,dl$, where $\mu$ is a coefficient $>1$,

and $=\frac{\iint q^2 dy, dx}{Q_1{}^2 A}$, where $Q_1$ is the total shearing force over the section and $q$ is the shearing force at the point $x\,y$ in the section. A = total area of section.

For the work done by bending moments the expression becomes

$$\frac{1}{2E}\int\frac{M^2}{I}.dl.$$

Where I = moment of inertia of the cross section, the total work done over the whole of the girder K N is therefore

$$U = \frac{1}{2E}\int\frac{P^2}{A}dl + \frac{1}{2G}\int\frac{\mu Q^2}{A}dl + \frac{1}{2E}\int\frac{M^2}{I}dl.$$

We can apply the principle of least work here; U must be a minimum with regard to the applied forces,

$$i.e.\quad \frac{dU}{dP_0},\frac{dU}{dQ_0}\quad\text{and}\quad\frac{dU}{dM_0}\quad\text{must each be zero.}$$

$$i.e.\quad \frac{dU}{dP_0} = \frac{1}{E}\int\frac{P}{A}.\frac{dP}{dP_0}.dl + \frac{1}{G}\int\frac{\mu Q}{A}.\frac{dQ}{dP_0}dl + \frac{1}{E}\int\frac{M}{I}.\frac{dM}{dP_0}dl$$

$$\text{and}\quad \frac{dU}{dQ_0} = \frac{1}{E}\int\frac{P}{A}\frac{dP}{dQ_0}dl + \frac{1}{G}\int\frac{\mu Q}{A}.\frac{dQ}{dQ_0}dl + \frac{1}{E}\int\frac{M}{I}.\frac{dM}{dQ_0}dl$$

$$\text{and}\quad \frac{dU}{dM_0} = \frac{1}{E}\int\frac{P}{A}\frac{dP}{dM_0}.dl + \frac{1}{G}\int\frac{\mu Q}{A}.\frac{DQ}{DM_0}dl + \frac{1}{E}\int\frac{M}{I}\frac{DM}{DM_0}dl.$$

These three equations determine $P_0$, $Q_0$, and $M_0$; and P, Q, and M for any section are determined from

$$P = P_0 \cos a + Q_0 \sin a + v \sin a - h \cos a - (w + c) \sin a$$
$$Q = -P_0 \sin a + Q_0 \cos a + v \cos a - h \sin a - (w + c) \cos a$$
$$M = M_0 + P_0 y - Q_0 x - V - H + W + C.$$

For all practical purposes the first two terms in the equations for the work done may be neglected. The first term gives the work done by the direct straining forces, and in the case of vessels the direct pull at any part of any of the transverse members is so small that the term is of no practical importance. The second term, namely, the work done by the shearing forces, is very small compared with the work done by the bending couples, and its exclusion makes no appreciable error. We can now substitute the expressions for P, Q, and M in the equations for the work done and we have

$$\int \frac{M}{I} \cdot \frac{dM}{dP_0} dl = \int \frac{(M_0 + P_0 y - Q_0 x - V - H + W + C)}{I} y.d$$
$$\int \frac{M}{I} \cdot \frac{dM}{dQ_0} \cdot dl = \int \frac{(M_0 + P_0 y - Q_0 x - V - H + W + C)}{I} x.dl$$
$$\int \frac{M}{I} \cdot \frac{dM}{dM_0} \cdot dl = \int \frac{M_0 + P_0 y - Q_0 x - V - H + W + C}{I} .dl.$$

We see, therefore, that all we have to do is to integrate the following expressions along the length of the girder

$$(M_0 + P_0 y - Q_0 x - V - H + W + C) \quad . \quad . \quad . \quad . \quad (1)$$
$$(M_0 + P_0 y - Q_0 x - V - H + W + C)x \quad . \quad . \quad . \quad (2)$$
$$(M_0 + P_0 y - Q_0 x - V - H + W + C)y \quad . \quad . \quad . \quad (3)$$

In the case of a girder of uniform cross section the value of I is constant, and therefore we can equate the above expressions to zero.

When the girder is not uniform, the value of I has to be calculated from point to point along the length.

The integration of the above expressions can be performed either by Simpson's or by Tchebycheff's Rules.

The method of the calculation may be easily followed if the results are put in tables.

The deck edge should be taken as a stop-point in the integration. The girder is divided into a convenient number of intervals to suit the integration, first from the keel to the deck edge, and secondly from the deck edge to the deck at centre along the beam. See figs. 212 and 214.

At each section the value of I is first calculated, and also the position of the neutral axis.

The neutral lamina of the girder contains the points at which $x$ and $y$ for each section are determined.

In starting with the unknowns $P_0$, $Q_0$, and $M_0$ we can assume them to be each = unity.

*First Table* (LIX.).

First make $P_0$, $Q_0$, and $M_0$ each equal to unity.

For section (1) the expression

$$M_0 + P_0 y - Q_0 x - V - H + W + C$$

becomes when $\begin{matrix} x=0 \\ y=0 \end{matrix} \left\{ 1+0-0-0-0+0+0 \right\}$

For section (2), say, where $x=x_1$, $y=y_1$, $V=V_1$, $H=H_1$, $W=W_1$, $C=C_1$ the expression becomes

$$1 + P_0 y_1 - Q_0 x_1 - V_1 - H_1 + W_1 + C_1$$

and so on.

These are best put in a table of the following form. The table is separated at the interval for the deck edge, because the interval for the frame part is chosen different from the interval for the beam part.

*First Table* (LIX.).

| | Number of Section. | $M_0$. | $P_0 y$. | $Qx$. | $v$. | $h$. | $w$. | $c$. |
|---|---|---|---|---|---|---|---|---|
| | 0 | +1 | | | | | | |
| | 1 | +1 | | | | | | |
| | 2 | +1 | | | | | | |
| | 3 | +1 | | | | | | |
| | 4 | +1 | | | | | | |
| | 5 | +1 | | | | | | |
| | 6 | +1 | | | | | | |
| | 7 | +1 | | | | | | |
| Deck Edge. | 8 | +1 | | | | | | |
| | 8 | +1 | | | | | | |
| | 9 | +1 | | | | | | |
| | 10 | +1 | | | | | | |
| | 11 | +1 | | | | | | |
| Deck at Centre. | 12 | +1 | | | | | | |

*Second Table* (LX.).

Each figure of the first table is divided by the moment of inertia of the corresponding section.

| Number of Section. | Moment of Inertia of Section. | $\frac{M}{I}$. | $\frac{P_0y}{I}$. | $\frac{Q_0y}{I}$. | $\frac{v+h}{I}$. | $\frac{w}{I}$. | $\frac{c}{I}$. |
|---|---|---|---|---|---|---|---|
| 1 | I | | | | | | |
| etc. | | | | | | | |

*Third Table* (LXI.).

Each figure of the second table is multiplied by the corresponding multiplier for Simpson's Rules.

| Number of Section. | Simpson's Multipliers. | $\int\frac{M_0}{I}dl.$ | $\int\frac{P_0y}{I}.dl.$ | $\int\frac{Q_0x}{I}.dl.$ | $\int\frac{v+h}{I}.dl.$ | $\int\frac{w}{I}.dl.$ | $\int\frac{c}{I}.dl.$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | | |
| 2 | 4 | | | | | | |
| 3 | 2 | | | | | | |
| etc. | etc. | | | | | | |

The sums of the columns in this table give

$$\int\frac{M}{I}\cdot\frac{dM}{dM_0}dl,$$

or
$$\int\frac{M_0+P_0y-Q_0x-V-H+W+C}{I}dl,$$

which is equal to zero.

*Fourth Table* is the third table multiplied by the corresponding value of $y$ for each section, and gives

$$\int\frac{M_0y}{I}dl+\int\frac{P_0y^2}{I}dl-\int\frac{Q_0xy}{I}dl-\int\frac{(V+H)y}{I}.dl+\int\frac{Wy}{I}dl+\int\frac{Cy}{I}dl=0.$$

*Fifth Table* is the third table multiplied by the corresponding value of $x$ for each section, and gives

$$\int \frac{M_0 x}{I} dl + \int \frac{P_0 xy}{I} dl - \int \frac{Q_0 x^2}{I} . dl - \int \frac{(V+H)x}{I} . dl + \int \frac{Wx}{I} dl + \int \frac{Cx}{I} dl = 0.$$

The sums of the columns of the last three tables give equations containing only the unknown $P_0$, $Q_0$, and $M_0$, which can now be determined.

The values of $P_0$, $Q_0$, and $M_0$ may then be substituted in the first table, and the actual values of M at each section may thus be obtained.

The values of P and Q can be obtained from the equations for P and Q.

The foregoing method of investigation deals with a standardised transverse frame made up of beams, frames, reverse frame, floors, brackets, and a frame-space length of deck and shell plating. It is necessary to consider more fully the forces acting on the transverse members in a vessel. These have been already enumerated, but we can consider them in the following order:—

(1) Supporting forces.
(2) Dead loads.
(3) Weight of the structure.
(4) Shearing forces.
(5) Indirect loads due to longitudinal bending.

First of all should be considered the share of the total direct load that is taken up by the standard transverse ring.

When the vessel is in water the supporting forces may be conveniently resolved into horizontal and vertical components. For a standard transverse ring, we shall be very nearly correct to take the pressure over the frame-space length which is submerged. The accuracy of this assumption depends upon the proportion of this load that is taken up by the longitudinal parts, but it may be safely taken that this proportion will be the same in vessels of a similar type. The proportion is also affected by the extra transverse stiffening, like web frames, boiler seats, engine seats, and bulkheads. In a very long hold the transverse ring midway between the bulkheads would have to support almost the total load. A similar reasoning holds with other loads, such as the weight of cargo, etc.

(1) Considering the *method of support*.—The pressure of the water is resolved into horizontal and vertical components, and the pressure calculated according to the hydrostatic law of pressure, the surface over which it acts being the submerged surface of the transverse ring.

A condition that is important to examine is when the vessel is supported upon blocks, as in dry dock. In this case the resultant of the vertical loads is balanced by the equal and opposite pressure of the blocks. The calculation is made simpler in this way.

(2) *Dead Loads.*—In a hold which is full of solid cargo, the longitudinal distribution of the load may be supposed to be uniform. The weight of the cargo per frame space should therefore be taken as the dead load on the transverse ring, and it should act over the top of floors, and be distributed transversely according to the depth of the hold. Concentrated loads, such as engines and boilers, may be considered in the same way as applied forces

acting downwards through their centre of gravity on the top of the floors. If the load is spread over, say, $n$ frame spaces, then $\frac{1}{n}$th part of the load should be the applied force on the transverse ring.

(3) The *weight of the structure* is easily calculated from the scantlings of the material.

(4) *Shearing forces.*—When the vessel is in water there may be considerable shearing forces on each end of the transverse ring. The difference of these shearing forces gives the unbalanced vertical load which may be assumed to act over one section. For all practical purposes, it will be correct enough to assume this force distributed uniformly over the vertical side plating, and it can be included in the calculation in the same way as the loads.

(5) *Indirect loads.*—It is impossible to determine the effect of longitudinal bending moments on the transverse members of a vessel. The structure of a vessel, considering it as a continuous girder stiffened by a transverse system of framing, is too complex for the application of any general rule as to the work done in straining each member.

Dr Bruhn gives the rule which would apply to a plate which is subjected to a uniform normal pressure, and is stiffened by lateral and longitudinal girders, viz. that the work done by the girders at right angles to each other varies directly as the aggregate moment of inertia of the section of the girders, and inversely as the cube of the lengths of the girders. The load is then split up in this ratio, and each set of girders dealt with as if only loaded with the part apportioned to it.

Considering transverse strength merely, the effect of longitudinal bending moments may be left out of account. Properly speaking, the strength of the ship ought to refer to the stresses produced by both the longitudinal and transverse distribution of the loads, because, in reality, these two questions cannot be separated. Treating the problem in a narrower way, we can divide the general question of a ship's strength into two, viz. the longitudinal strength and the transverse strength; and in order to simplify the solutions, we only consider the longitudinal distribution of loads in the former question and the transverse distribution in the latter.

The longitudinal question is the simpler problem, and it is possible to obtain an idea of how the stresses vary when the vessel is at sea. The results of such calculations have been proved by experiment to give a true indication of what actually is experienced by a vessel in a sea-way.

Regarding "transverse strength," the results, although they cannot be said to be so trustworthy, are also of use in affording comparisons of one type with another, and are certainly very useful in estimating the effect of changing the transverse structural arrangement.

Two examples of transverse strength calculations have been appended. Both are for Channel steamers in which the loads in proportion to the scantlings are large. One of the calculations has been given in detail. See Tables LXII. to LXVII., and figs. 211 and 212.

The results given in these calculations in the form of stresses must not be taken as absolute, but only as comparative. The case in the calculation gives a stress of 21·4 tons, but the vessel has been docked successfully several times in the conditions assumed without signs of straining. The factor for similar structures may be used to determine whether the scantlings are sufficient in a new structure.

**Transverse Strength Calculation.**

**Channel Steamer resting on keel blocks in dry dock.**

**Dimensions 270′ × 34′ × 14′ 6″. Vessel has four S.E. boilers 13′ 5″ × 12′ 1″.**

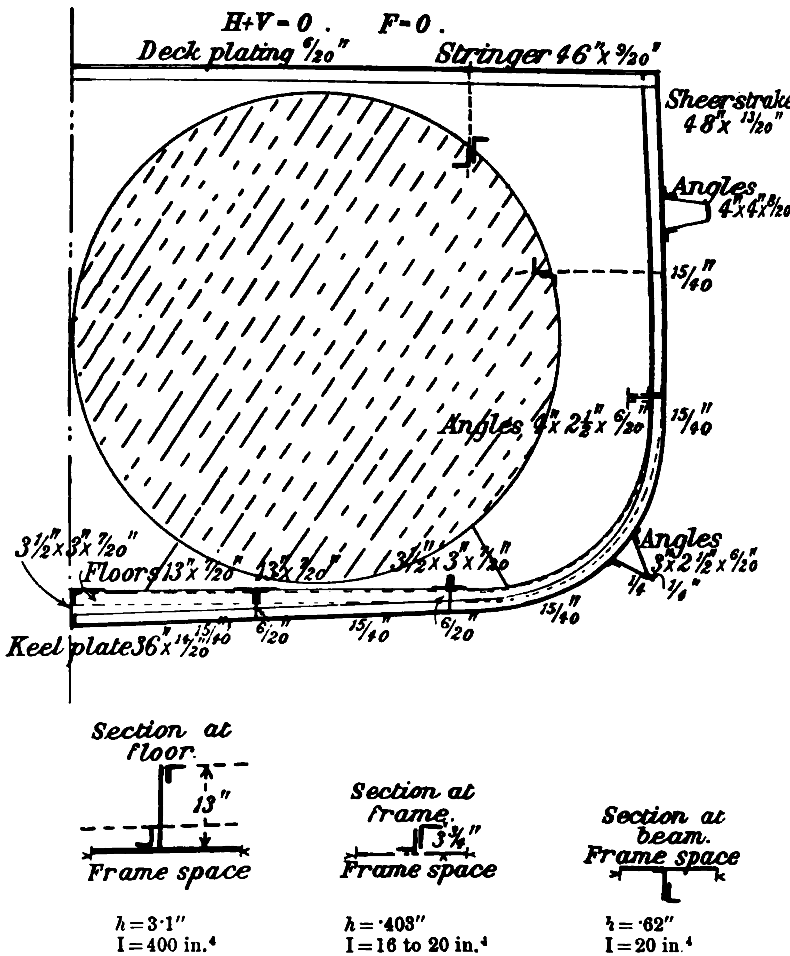
**Sketch of Part of Structure taken into account in the Transverse Ring.**

**S = Weight of Structure per frame space.**

**B = ,, Boiler ,, ,,**

**Frame spacing = 22″.**

**Boiler weight in this frame space = 10 tons.**




Figs. for the calculation of position of N A and value of I for section of the framing and floors.

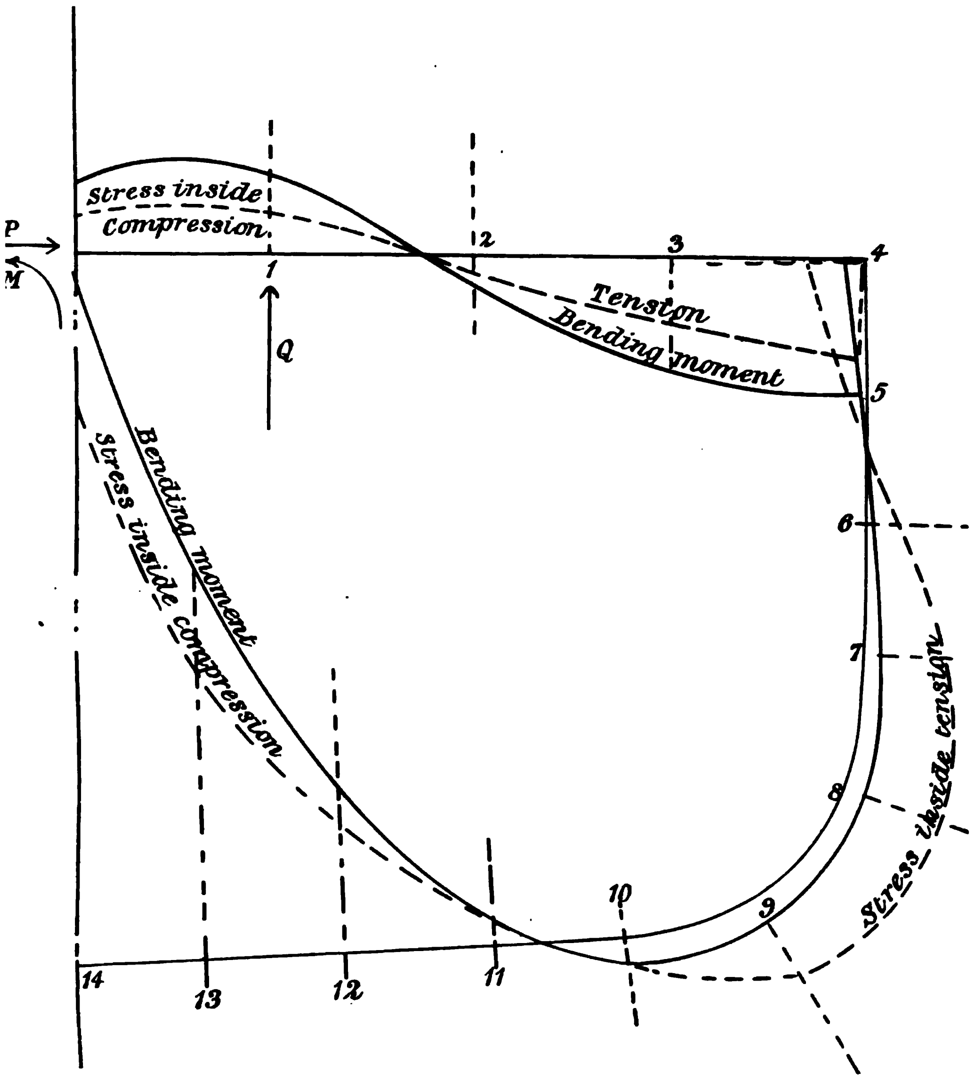
Fig. 211.

Sketch giving line of Neutral Axis. Scale $\frac{1}{4}'' = 1$ ft.
Stresses and bending moments plotted from Table LXVII.

Interval for beam $= \frac{16 \cdot 5}{4} = 4 \cdot 13'$.

,, ,, girth $= \frac{28 \cdot 4}{10} = 2 \cdot 84'$.

Bending-moment scale $\frac{1}{2}'' = 10$ ft. tons.
Stress ,, $\frac{1}{8}'' = 1$ ton per sq. in.




*Note.*—Q, the vertical force, is in line of pillar support in boiler-room.

Fig. 212.

## Table LXII.

| No. | Moments. | | | | | | | I. |
|---|---|---|---|---|---|---|---|---|
| | M | Py | Qx | S | B | H + V | F | |
| 0 | +1 | −·0 | − 0 | + 0 | ... | ... | ... | 20 |
| 1 | 1 | ·05 | 0 | ·12 | ... | ... | ... | 20 |
| 2 | 1 | ·2 | 4·13 | ·50 | ... | ... | ... | 20 |
| 3 | 1 | ·4 | 8·26 | 1·1 | ... | ... | ... | 20 |
| 4 | 1 | ·7 | 12·39 | 2·0 | ... | ... | ... | 20 |
| | | | | | | | | |
| 4 | 1 | ·7 | 12·39 | 2·1 | ... | ... | ... | 25 |
| 5 | 1 | 3·6 | 12·55 | 2·1 | ... | ... | ... | 25 |
| 6 | 1 | 6·4 | 12·6 | 2·1 | ... | ... | ... | 20 |
| 7 | 1 | 9·25 | 12·5 | 2·1 | ... | ... | ... | 20 |
| 8 | 1 | 12·0 | 11·9 | 2·0 | ... | ... | ... | 40 |
| 9 | 1 | 14·0 | 10·0 | 1·0 | ... | ... | ... | 20 |
| 10 | 1 | 14·65 | 7·3 | − ·5 | − ·6 | ... | ... | 220 |
| 11 | 1 | 14·7 | 4·4 | − 3·0 | − 5·3 | ... | ... | 300 |
| 12 | 1 | 14·8 | 1·6 | − 5·0 | −18·3 | ... | ... | 350 |
| 13 | 1 | 14·85 | 0 | − 7·2 | −40·05 | ... | ... | 400 |
| 14 | 1 | 14·9 | 0 | − 10·0 | −69·0 | ... | ... | 400 |

## Table LXIII.

| No. | Quotients. | | | | | | |
|---|---|---|---|---|---|---|---|
| | M | Py | Qx | S | B | H + V | F |
| 0 | + ·05 | − ·0 | − ·0 | + ·0 | ... | ... | ... |
| 1 | ·05 | ·0025 | ·0 | ·006 | ... | ... | ... |
| 2 | ·05 | ·01 | ·206 | ·025 | ... | ... | ... |
| 3 | ·05 | ·02 | ·413 | ·055 | ... | ... | ... |
| 4 | ·05 | ·035 | ·619 | ·100 | ... | ... | ... |
| | | | | | | | |
| 4 | ·04 | ·035 | ·495 | ·084 | ... | ... | ... |
| 5 | ·04 | ·145 | ·500 | ·084 | ... | ... | ... |
| 6 | ·05 | ·32 | ·63 | ·105 | ... | ... | ... |
| 7 | ·05 | ·462 | ·625 | ·105 | ... | | ... |
| 8 | ·05 | ·60 | ·595 | ·100 | ... | ... | ... |
| 9 | ·025 | ·35 | ·25 | − ·025 | ... | ... | ... |
| 10 | ·005 | ·062 | ·033 | − ·0002 | − ·002 | ... | ... |
| 11 | ·0035 | ·049 | ·014 | − ·01 | − ·017 | ... | ... |
| 12 | ·0031 | ·042 | ·005 | − ·0143 | − ·052 | ... | ... |
| 13 | ·0002 | ·037 | ·0 | − ·018 | − ·100 | ... | ... |
| 14 | ·0002 | ·037 | ·0 | − ·025 | − ·172 | ... | ... |

TABLE LXIV.

| No. | S.M. | Products. | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M | Py | Qx | S | B | H+V | F |
| 0 | 1 | + ·05 | − ·0 | − ·0 | + ·0 | ... | ... | ... |
| 1 | 4 | ·2 | ·01 | ·0 | ·024 | ... | ... | ... |
| 2 | 2 | ·1 | ·02 | ·412 | ·05 | ... | ... | ... |
| 3 | 4 | ·2 | ·08 | 1·652 | ·22 | ... | ... | ... |
| 4 | 1 | ·05 | ·035 | ·619 | ·1 | ... | ... | ... |
| ... | ... | + ·6 | − ·145 | − 2·683 | + ·394 | ... | ... | ... |
| × 4·13/2·84 | =1·454 | + ·8724 | − ·211 | − 3·901 | + ·5728 | ... | ... | ... |
| 4 | 1 | ·04 | ·03 | ·495 | ·084 | ... | ... | ... |
| 5 | 4 | ·16 | ·58 | 2·0 | ·336 | ... | ... | ... |
| 6 | 2 | ·1 | ·64 | 1·26 | ·21 | ... | ... | ... |
| 7 | 4 | ·2 | 1·848 | 2·5 | ·42 | ... | ... | ... |
| 8 | 2 | ·1 | 1·2 | 1·19 | ·20 | ... | ... | ... |
| 9 | 4 | ·1 | 1·4 | 1·0 | ·01 | ... | ... | ... |
| 10 | 2 | ·01 | ·124 | ·066 | − ·0004 | −.004 | ... | ... |
| 11 | 4 | ·014 | ·196 | ·056 | − ·04 | − ·068 | ... | ... |
| 12 | 2 | ·006 | ·084 | ·01 | − ·0286 | − ·104 | ... | ... |
| 13 | 4 | ·0008 | ·148 | ·0 | − ·072 | − ·400 | ... | ... |
| 14 | 1 | ·0002 | ·037 | ·0 | − ·025 | − ·172 | ... | ... |
| | | + ·731 | − 6·287 | − 8·577 | + 1·094 | − ·748 | | |
| | | + ·872 | − ·211 | − 3·901 | + ·5728 | | | |
| | Totals | + 1·603 | − 6·498 | − 12·478 | + 1·6668 | − ·748 | S+B = + ·9188 | |

TABLE LXV.

| No. | y | Products (P). | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M | Py | Qx | S | B | H+V | F |
| 0 | 0 | + 0 | − 0 | − 0 | + 0 | ... | ... | ... |
| 1 | ·05 | ·01 | ·0005 | ·0 | ·0012 | ... | ... | ... |
| 2 | ·2 | ·02 | ·004 | ·0083 | ·01 | ... | ... | ... |
| 3 | ·4 | ·08 | ·032 | ·6608 | ·088 | ... | ... | ... |
| 4 | ·7 | ·035 | ·0245 | ·4333 | ·07 | ... | ... | ... |
| ... | ... | + ·145 | − ·061 | − 1·1024 | + ·1692 | ... | ... | ... |
| × 4·13/2·84 | ... | + ·211 | − ·0887 | − 1·6029 | + ·246 | ... | ... | ... |
| 4 | 7 | ·028 | ·021 | ·3465 | ·0588 | ... | ... | ... |
| 5 | 3·6 | ·576 | 2·088 | 7·20 | 1·21 | ... | ... | ... |
| 6 | 6·4 | ·64 | ·409 | 8·06 | 1·34 | ... | ... | ... |
| 7 | 9·25 | 1·85 | 17·1 | 23·13 | 3·885 | ... | ... | ... |
| 8 | 12·0 | 1·2 | 14·4 | 14·28 | 2·4 | ... | ... | ... |
| 9 | 14·0 | 1·4 | 19·6 | 14·0 | ·14 | ... | ... | ... |
| 10 | 14·65 | ·146 | 1·817 | ·967 | − ·0058 | − ·0586 | ... | ... |
| 11 | 14·7 | ·206 | 2·881 | ·823 | − ·588 | − 1·00 | ... | ... |
| 12 | 14·8 | ·0888 | 1·243 | ·148 | − ·423 | − 1·54 | ... | ... |
| 13 | 14·85 | ·0119 | 2·198 | ·0 | − ·107 | − 5·94 | ... | ... |
| 14 | 14·9 | ·003 | ·5513 | ·0 | − ·37 | − 2·563 | ... | ... |
| | | 6·1496 | − 62·308 | − 68·9595 | + 7·54 | − 11·1016 | | |
| | | ·211 | − ·0887 | − 1·6029 | + ·246 | ... | | |
| | Totals | + 6·3606 | − 62·397 | − 70·557 | + 7·786 | − 11·1016 | | |

(S + B) = − 3·3156.

TABLE LXVI.

| No. | x | Products (Q). | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | M | Py | Qx | S | B | (H+V) | F |
| 0 | 0 | + 0 | − 0 | − 0 | + 0 | ... | ... | ... |
| 1 | 0 | 0 | 0 | 0 | 0 | ... | ... | ... |
| 2 | 4·13 | ·413 | ·0826 | 1·701 | ·206 | ... | ... | ... |
| 3 | 8·26 | 1·652 | ·6608 | 13·645 | 1·817 | ... | ... | ... |
| 4 | 12·39 | ·6195 | ·4337 | 7·669 | 1·239 | ... | ... | ... |
| ... | ... | 2·6845 | 1·1771 | 23·015 | 3·262 | ... | ... | ... |
| $\times \frac{4\cdot13}{2\cdot84}$ | ... | 3·9032 | 1·7115 | 33·464 | 4·743 | ... | ... | ... |
| 4 | 12·39 | ·4956 | ·3717 | 6·133 | 1·041 | ... | ... | ... |
| 5 | 12·55 | 2·008 | 7·279 | 25·10 | 4·217 | ... | ... | ... |
| 6 | 12·6 | 1·26 | 8·064 | 15·87 | 2·646 | ... | ... | ... |
| 7 | 12·5 | 2·50 | 23·098 | 31·25 | 5·25 | ... | ... | ... |
| 8 | 11·9 | 1·19 | 1·428 | 14·161 | 2·38 | ... | ... | ... |
| 9 | 10·0 | 1·10 | 14·00 | 10·00 | ·1 | ... | ... | ... |
| 10 | 7·3 | ·073 | ·9052 | ·482 | − ·0029 | − ·0292 | ... | ... |
| 11 | 4·4 | ·0616 | ·8624 | ·246 | − ·176 | − ·2992 | ... | ... |
| 12 | 1·6 | ·0096 | ·1344 | ·016 | − ·0457 | − ·1664 | ... | ... |
| 13 | ·0 | ·0 | ·0 | ·0 | 0 | − 0 | ... | ... |
| 14 | 0 | ·0 | ·0 | ·0 | 0 | 0 | ... | ... |
| | | 8·5978 | 56·1427 | 103·258 | 15·4094 | − ·4948 | | |
| | | 3·9032 | 1·7115 | 33·464 | 4·743 | | | |
| Totals | | +12·501 | −57·854 | −136·722 | +20·1524 | − ·4948 | S+B=+19·6576 | |

EQUATIONS FOR P, Q, AND M.

(1) $1{\cdot}6034\,M - 6{\cdot}498\,P - 12{\cdot}378\,Q + {\cdot}9188 = 0$ . . . . (1)

(2) $6{\cdot}3606\,M - 62{\cdot}397\,P - 70{\cdot}557\,Q - 3{\cdot}3156 = 0$ . . . . (2)

(3) $12{\cdot}501\,M - 57{\cdot}854\,P - 136{\cdot}721\,Q + 19{\cdot}657 = 0$ . . . . (3)

(1) × 3·766 $\quad 6{\cdot}3606\,M - 25{\cdot}775\,P - 49{\cdot}091\,Q + 3{\cdot}654 = 0$

$-6{\cdot}3606\,M + 62{\cdot}397\,P + 70{\cdot}557\,Q + 3{\cdot}3156 = 0$

$36{\cdot}622\,P + 21{\cdot}466\,Q + 6{\cdot}969 = 0$ . . . . (4)

(2) × 1·965 $\quad 12{\cdot}501\,M - 122{\cdot}610\,P - 138{\cdot}644\,Q - 6{\cdot}5151 = 0$

$-12{\cdot}501\,M + 57{\cdot}854\,P + 136{\cdot}721\,Q - 19{\cdot}657 = 0$

$-64{\cdot}756\,P - 1{\cdot}923\,Q - 26{\cdot}273 = 0$ . . . . (5)

(4) × 1·768 $\quad +64{\cdot}756\,P + 37{\cdot}952\,Q + 12{\cdot}321 = 0$

$36{\cdot}029\,Q - 13{\cdot}952 = 0$

$\therefore Q = {\cdot}3872$

$\therefore P = -{\cdot}4172$

$\therefore M = {\cdot}7252$

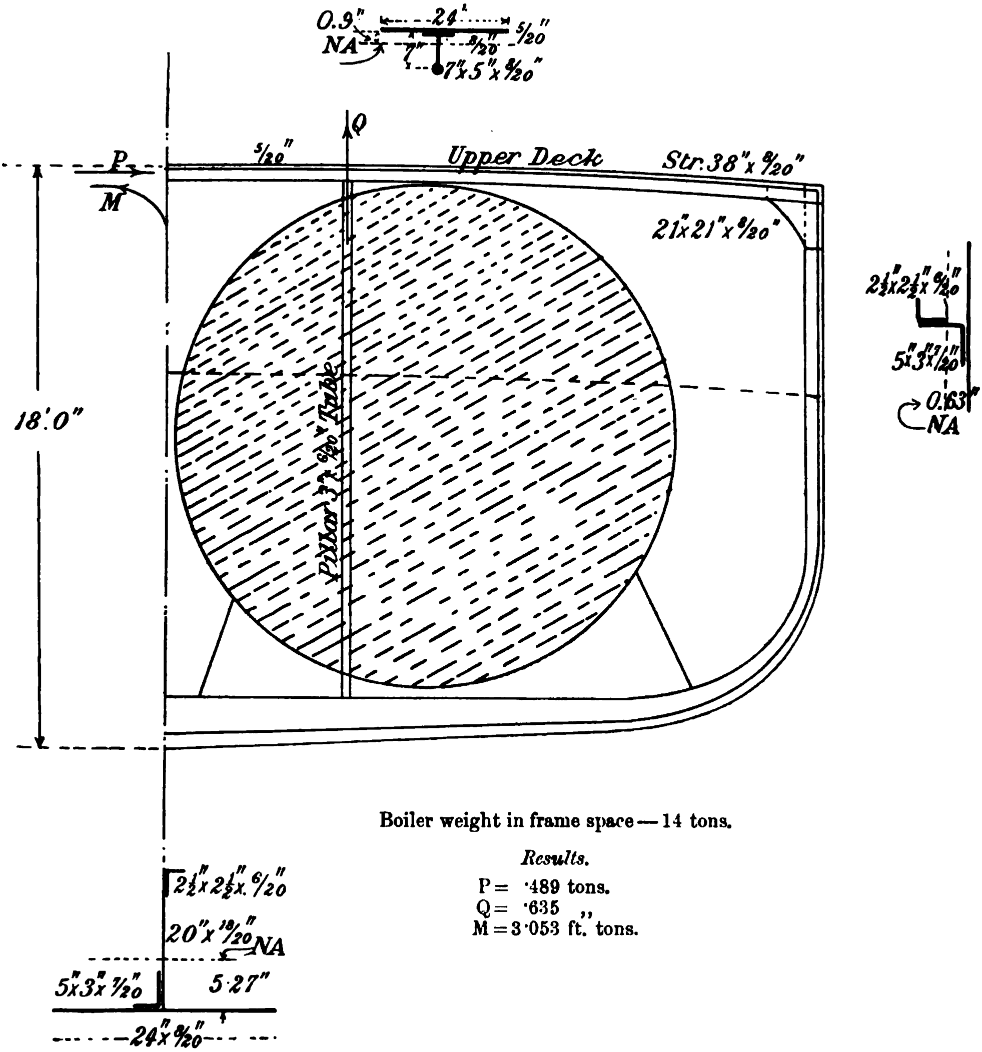
TRANSVERSE STRENGTH CALCULATION.

Channel Steamer resting on keel blocks in dry dock.

For simplicity, only one deck—the upper deck—has been included.

Half beam = 20·7 ft.

,, girth = 34·8 ft.




Boiler weight in frame space — 14 tons.

*Results.*

P = ·489 tons.
Q = ·635 ,,
M = 3·053 ft. tons.

FIG. 213.

TABLE LXVII.

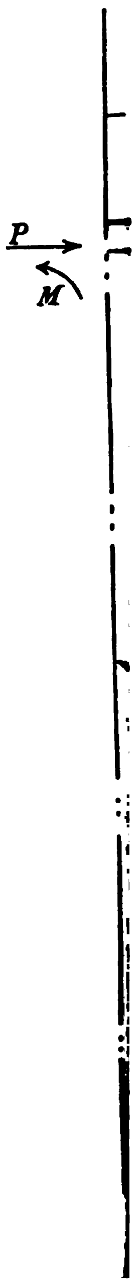
| No. | I | y | $\frac{I}{y}$ | Bending Moment. | Stress Tons/sq. in. |
|---|---|---|---|---|---|
| 0 | 20 | 3·61 | 5·54 | + ·725 | + 1·57 |
| 1 | 20 | 3·61 | 5·54 | + ·866 | + 1·87 |
| 2 | 20 | 3·61 | 5·54 | − ·29 | − ·63 |
| 3 | 20 | 3·61 | 5·54 | − 1·208 | − 2·62 |
| 4 | 20 | 3·61 | 5·54 | − 1·781 | − 3·86 |
| 4 | 25 | 3·35 | 7·46 | − 1·671 | − 2·24 |
| 5 | 25 | 3·35 | 7·46 | − ·533 | − ·85 |
| 6 | 20 | 3·35 | 5·97 | + ·515 | + 1·03 |
| 7 | 20 | 3·35 | 5·97 | + 1·844 | + 3·7 |
| 8 | 20 | 3·35 | 5·97 | + 2·124 | + 4·07 |
| 9 | 40 | 3·4 | 11·76 | + 3·693 | + 3·76 |
| 10 | 220 | 6·1 | 36·06 | + 2·909 | + ·96 |
| 11 | 300 | 8·3 | 36·14 | − 3·146 | − 1·04 |
| 12 | 350 | 9·5 | 36·84 | −17·02 | − 5·54 |
| 13 | 400 | 9·8 | 40·82 | −40·33 | −11·85 |
| 14 | 400 | 9·9 | 40·4 | −72·06 | −21·4 |

The next case (see figs. 213 and 214) is for a vessel of a similar type to the previous ones. The curves in fig. 212 are drawn to the same scales as those for fig. 211, so that the results can be compared.

The first case is one in which it was expected that the transverse stresses in the top of the floors would be severe in the dry-dock condition. The vessel is a high-speed vessel with a comparatively light structure and with two heavy boilers abreast of each other. The worst condition of support for a vessel of this type would therefore be experienced when she was in dry dock, supported by a line of keel blocks.

The second case is one of a somewhat similar vessel which had been many times successfully docked. A comparison of the stresses showed that while both cases, under the same assumptions, showed high stresses, the results were not very unlike. The first vessel has been successfully docked several times.

The above cases are undoubtedly extreme. They represent the combination of very heavy boilers away from the keel with very light scantlings of the hull. In order that the above calculations should have an absolute rather than a relative application, the circumstances of the structure beyond the limits of the boilers in a fore and aft direction should be considered. At about eight feet from the boiler ends a complete transverse bulkhead is fitted. These are, compared with the ordinary frames, practically rigid. Connected to these and extending beyond them is the practically vertical side-plating to which the frames are attached, and this in turn is practically rigid, so that the stresses brought upon the frames carrying the boilers can be distributed to adjoining frames and the transverse bulkheads. It would enormously add to the complexity of the calculation to take this support into account, and would in similarly constructed ships lead only to similar comparative results to those already obtained. It may be stated generally that the transverse stresses which come upon ordinary vessels are well within the limits of safety of the material, but the examples given show how in any special cases comparative values of these stresses may be obtained.

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## CHAPTER XXVII.

# DISTRIBUTION OF PRESSURE ON THE KEEL BLOCKS SUPPORTING A VESSEL IN DRY DOCK.

THE special case discussed in Chapter XXVI. leads to a more detailed consideration of the forces supporting a ship in dry dock.

In the description of the launching arrangements of a vessel we saw that the vessel may rest partly, or wholly, on blocks.

Vessels have to be dry-docked at regular periods for purposes of painting and repair of the outer bottom. When in dry dock the total weight of the vessel is generally supported by a line of keel blocks. The longitudinal spacing of the keel blocks is generally 3 to 4 ft., but often has to be modified according to the size or weight per foot run of the vessel, and at parts of the length in the region of heavy weights aboard. Some vessels, especially heavy battleships, require additional shoring under the bottom at the bilges. The vessel is kept upright by side shores fitted horizontally from the walls of the dock, and often diagonal shores are fitted.

Several accidents have happened to vessels in dry dock due to the crushing of the keel blocks, and these have led to investigations of the question of the pressure caused by the vessel's weight upon the blocks. In a vessel with a considerable part of the keel not in a straight line with the rest, care has to be taken to fit the blocks at the end of the straight part of the keel closer together, or to have blocks arranged to take the weight of the overhanging part. For this latter purpose the end blocks must be prepared and their height determined from a profile of the vessel, but the difficulty of bringing the vessel to the exact longitudinal position necessary to cause the part of the keel which is not in the keel line to rest exactly on these inclined blocks makes it necessary to provide against there being no pressure on them until the water has fallen low enough to allow of their being wedged up to their proper position.

The problem of arriving at an accurate determination of the pressure likely to come upon each block is practically insoluble; but if we make assumptions regarding the elasticity of the vessel and of the blocks, we can arrive at a sufficiently close approximation.

We shall consider, first, the vessel rigid and the blocks perfectly elastic. This method of consideration involves the assumption that the line of the tops of the blocks is a straight line to begin with. For all practical purposes, the blocks may be considered perfectly elastic. The cap of the block is usually made of soft wood. The base blocks are of some kind of heavy wood or of cast-iron. Two of them near the top are wedge-shaped, so that the height of the cap may be adjusted to the keel line. It will simplify the consideration of the problem to first take a simple case.

Let A F, fig. 215, be a heavy beam of uniform weight resting on blocks *b b*, etc. for a part *l* of its length. Let the overhang at the ends A and F be "*a*" and "*f*" respectively.

First assume that the beam is perfectly rigid. Let G be the centre of gravity which will be at the centre of the beam. Let W = weight of beam. G must lie between $b_a$ and $b_f$ the aftermost and foremost blocks respectively.

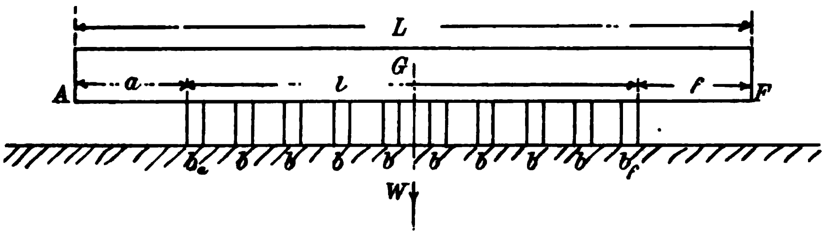


FIG. 215.

Since there is equilibrium, the total weight must be equal to the total support, and their resultants must act in the same vertical line. Hence the resultant support acts vertically upwards through G and is equal to W, and therefore the sum of the supports of the blocks must be equal to W. Let us now consider the case of a ship. If, instead of considering the support of each block, we convert this into support per foot of length of the part of the keel supported by blocks, the distribution of support can then be represented by a curve, and the area of this curve of supporting forces must be equal to the area of the weight curve of the ship, and, further, the centres of gravity of these two curves must be in the same longitudinal position. If the

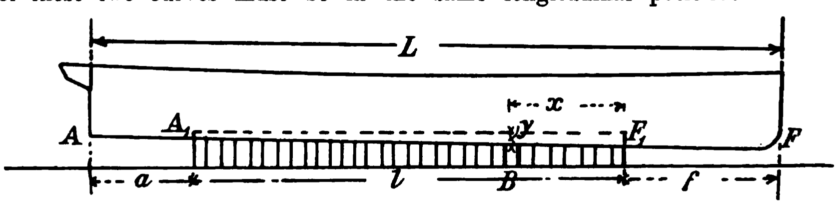


FIG. 216.

blocks as well as the vessel are perfectly rigid, it is evident that no variation of pressure throughout the length of the blocks or the ship will cause any change of form, and therefore there cannot be developed any pressure due to the resistance of the material to change of form. We may therefore have any number of curves representing the distribution of support, each of them fulfilling the conditions of having its area equal to and its centre of gravity in the same longitudinal position as that of the weight curve. The problem of finding the pressure at any point in the supported length of a rigid ship resting on rigid blocks is in consequence indeterminable. If, however, we assume the blocks to be perfectly elastic within the limits to which they are likely to be compressed, and the ship to remain perfectly rigid, we can determine the supporting force upon each block, or the support per ft. of length, so as to fulfil the necessary conditions of equilibrium.

Suppose the vessel to sink into the blocks, as shown by the dotted line in fig. 216. The distance that the keel sinks into any block is a direct measure of the pressure upon that block, and we can thus draw a curve of pressures if we know the amount of this sinkage or compression.

Let $y$ be the amount of sinkage at any block B. Then $P = Ey$ gives the pressure on that block, where E is the modulus of elasticity of the material of the cap block under compression. Let $k$ be the spacing of the blocks, centre to centre. Then $p$ the pressure per foot of length is equal to $\frac{P}{k}$.

$$\therefore \quad p = \frac{Ey}{k}.$$

If $x$ be the distance of the block B from F, the foremost block, then the curve of the keel will be the curve of $y$ the sinkage, and it must be a straight line for a rigid ship with a straight keel. Its equation will therefore be

$$y = mx + c$$

where $m$ is the tangent of the angle of inclination of the keel with the

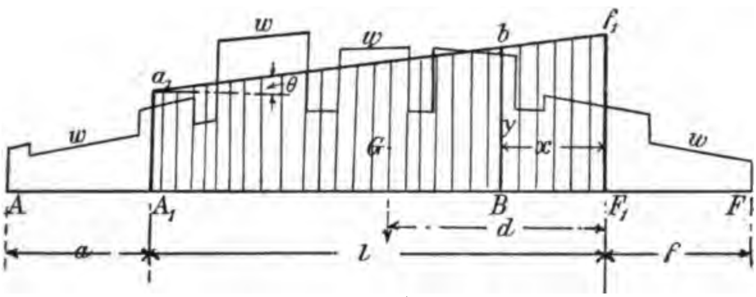


FIG. 217.

original line at the top of the caps, and $c$ is the distance the foremost block has sunk.

From these two equations we have

$$p = \frac{Ey}{k} = \frac{(mx + c)E}{k}.$$

Let this curve be drawn (fig. 217).

$a_1 f_1$ is the curve of supporting forces per foot of length.

$A_1 a_1$ is the support at the aftermost block.

$F_1 a_1$ „ „ foremost „

$Bb$ „ „ block B distant $x$ from F.

Let A $w$ $w$ F be the weight curve of the vessel, and let G be the position of its centre of gravity. Then the area of weight curve = area of support curve.

$$\therefore \quad W = \int_0^l p\,dx$$

$$= \frac{E}{k}\int_0^l (mx + c)dx$$

$$= \frac{E}{k}\left(\frac{ml^2}{2} + cl\right) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also the centres of gravity must be in the same vertical line.

∴ Taking moments about $F_1$,

$$
\begin{aligned}
Wd &= \int_0^l p\,x\,dx \\
&= \frac{E}{k}\int_0^l x\,(mx+c)dx \\
&= \frac{E}{k}\left(m\frac{l^3}{3}+c\frac{l^2}{2}\right) \qquad . \quad . \quad . \quad . \quad . \quad (2)
\end{aligned}
$$

Equations (1) and (2) determine the unknowns, $m$ and $c$,

$$\text{whence} \qquad c=\frac{kW}{El^2}(4l-6d)$$

$$\text{and} \qquad m=\frac{kW}{El^3}(12d-6l).$$

Therefore the sinkage $c$ at the foremost block

$$=F_1f_1=\frac{kW}{El}(4l-6d)$$

$$\text{and} \qquad \tan\theta=\frac{kW}{El^3}(12d-6l),$$

where $\theta$ is the inclination of the keel to the ground.

Substituting for $m$ and $c$ in the original equation for $p$ we have

$$
\begin{aligned}
p &= \frac{E}{k}(mx+c) \\
&= \frac{W}{l^2}\left(\frac{12dx}{l}-6x+4l-6d\right).
\end{aligned}
$$

At aftermost block $x=l$.

$$\therefore \quad \text{Pressure per foot of length}=\frac{2W}{l^2}(3d-l) \qquad . \quad . \quad (3)$$

At foremost block $x=0$.

$$\therefore \quad \text{Pressure per foot of length}=\frac{W}{l^2}(4l-6d) \qquad . \quad . \quad (4)$$

From these equations we see (1) that if $d$ is greater than $\frac{2}{3}l$ the pressure at the foremost block becomes negative, and (2) that if $d$ is less than $\frac{1}{3}l$ the pressure on the after block becomes negative. When the pressure according to the above expressions on the after block changes sign the vessel will touch that block but will not sink into it; consequently the pressure cannot be negative, but the vessel lifts at the after block and her weight will be borne by a fewer number of blocks forward.

Fig. 217 shows the curve when all the blocks are giving support. When the centre of gravity of the vessel is at a distance $d$ from F such that $d$ is less than $\frac{l}{3}$ the curve will be like that shown in the fig. 218.

When $d=\frac{l}{3}$ the curve will be like $a_2 f_2$ and the pressure at A will be zero. When $d$ is smaller than $\frac{l}{3}$ the curve will be like $a_3 f_3$, *i.e.* at some point $a_3$ the pressure will be zero.

Let the distance of the *c.g.* from F, in this case, be $d_1$.

Then $a_3 f_1 = 3d_1$

and $W = \text{area } a_3 f_3 F_1 = \frac{3d_1 \times F_1 f_3}{2}.$

$$\therefore \quad F_1 f_3 = \frac{2W}{3d_1}.$$

$\therefore$ The support at foremost block per foot of length $= \frac{2}{3}\frac{W}{d_1}$

when $d = \frac{l}{3}$.

Support at foremost block per foot of length $= \frac{2W}{l}$.

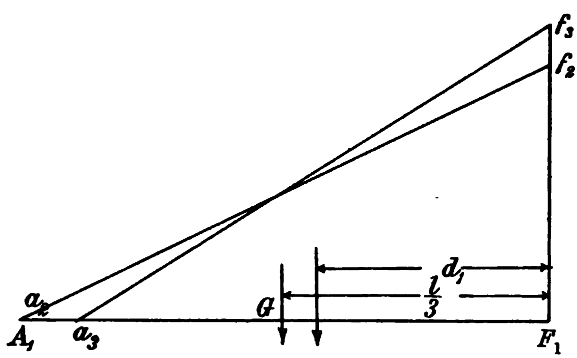


FIG. 218.

*Pressure on blocks assuming ship to be elastic.*—It has been shown in the chapter on deflection of ships how to obtain the form in which a ship will bend when subjected to a bending action due to bending moments which are completely known throughout the length of the vessel. If the known bending moment curve be twice integrated and the extremities of the second integral curve be joined by a straight line, the ordinates of the curve intercepted between it and the straight line will give us the deflection of the ship at the position of the ordinate.

If we can determine the value of M throughout the length of the ship when resting on elastic blocks for only part of her length, we can determine the deflection of the ship at every point. But the curve of pressure upon the blocks can only be determined when the exact form of the deflection curve is known, and consequently we cannot find the M throughout until we know the deflection due to M. A process of trial and error may, however, be adopted which will lead to a practical solution of the problem.

Let A F, fig. 219, represent the length of a ship whose weight curve is represented by A *w w* F. Let the support of the blocks for a length $A_1 F_1$ be represented by a curve *p p p*, whose form is not known exactly, but whose general character is as shown. To obtain the bending moment curve,

(1) integrate twice the curve A *w w* F
(2) " " " $A_1 p p F_1$.

The difference of ordinates of these two integral curves will give the M curve required.

The first curve can be obtained at once by the integraph, as A *w w* F is known. Let this be the curve A M M, the maximum ordinate being F M. To

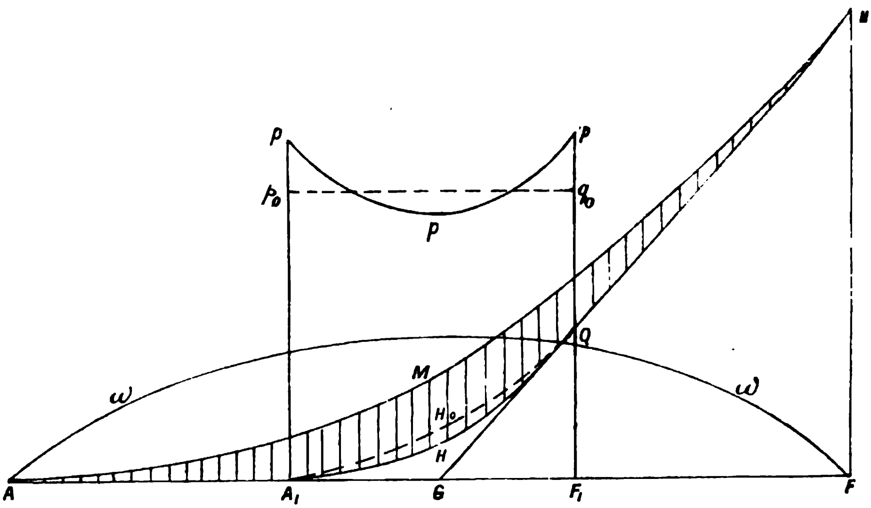


FIG. 219.

obtain the second, we must first consider how any curve *p p p* can be obtained which will fulfil the necessary conditions of support. As before, its area and position of centre of buoyancy longitudinally, say G, must coincide with those of the curve $A_1 w w F_1$. Join G M cutting $p F_1$ in Q. $F_1$ Q must be the maximum ordinate of the doubly integrated curve $A_1 p p F_1$.

Let us first suppose the curve of pressures on the blocks to be uniform throughout, but that the ship is free to deflect. This would be so if we were to support her on a series of uniform hydraulic rams in the place of the blocks, each under the same pressure. The curve of pressures would then be represented by the horizontal line $p_0 q_0$, and there would be equilibrium if G were at the middle of $A_1 F_1$. If, as would generally be the case, this were not so, we could assume a straight line pressure curve and determine the two end ordinates from (3) and (4), page 360. The line $p_0 q_0$ will, however, serve the purpose of illustration as well. The curve of M's can be at once determined. It is a common parabola $A_1 H_0 Q$ passing through $A_1$ and Q; and the curve of M, from whose double integration the deflection is to be determined, will be as shown in the shaded curve. We may then proceed as follows:—

First. Double integrate M curve as crossed to curve $A_1 \dot{H}_0 Q$. This will give the deflection supposing each block pumped up by hydraulic ram to the same pressure.

Second. Assume line of pressure to be that which would exist if the ship, after being deflected as in the previous case, were to become rigid and were to be supported by elastic blocks. It is to be noticed that total area of pressure curve is constant, and its centre of gravity must be at G. From this new line of pressure curve find new M curve as $A_1$ H Q. Double integrate this and find new curve of deflection.

Third. Assume third line of pressure to be due to the deflection curve of the second case, and repeat the double integration process. It will then be possible to spot the line of pressure curve which will just fit the deflection, and so get the pressure at extreme blocks. If it should happen that the deflection is such that there is no pressure on some of the blocks on account of the form of the curve passing below the base, then the part below the base

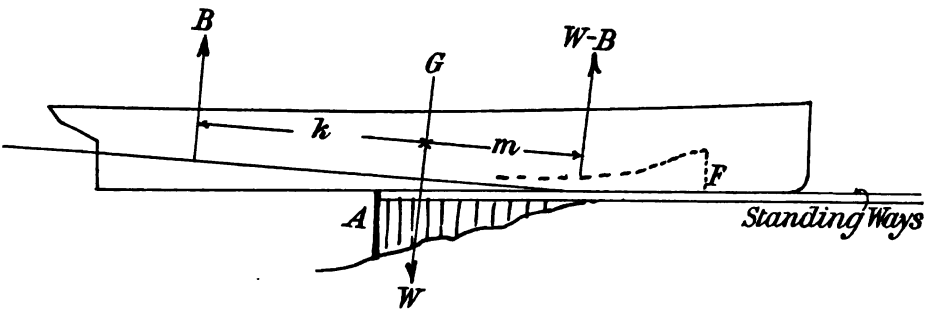


FIG. 220.

must be ignored, and only the part above which shows the positive pressures be taken into account.

The same consideration may be applied to the pressure at the end of ways while a vessel is being launched. In the chapter on launching in Part II. we dealt with the moment acting on the vessel during the time she takes in travelling from the ways to the water. When a tipping moment exists there is likely to be a severe pressure on the after end of ways (see fig. 220).

Making the same assumptions that we made in the case of the vessel resting on blocks, viz. that the ship is rigid and the ways elastic, we can find the curve of pressures in the same way.

Let W = launching weight of ship.
Let B = buoyancy of water.
Let G be position of centre of gravity of ship.

Then the land-borne weight = W − B and the moment $(W-B) \times m$ = the moment $B \times k$ as shown in figure. The resultant support of the standing ways is therefore (W − B), and it acts at a distance $m$ forward of G such that

$$m = \frac{Bk}{(W-B)} = \frac{\text{Moment of buoyancy about G}}{\text{Land-borne weight}}.$$

Let W – B act through the point P. Then, in the same way as before, we can find the curve of pressures over A F (fig. 221).

Let *a f* be this curve. *a f* will be a straight line if the keel remains straight.
A *a* represents pressure at end of ways.
F *f* „ „ at fore poppet.
Area A *a f* F = W – B.
Centre of gravity of A *a f* F is at P.
Let P F = *d* and A F = *l*.

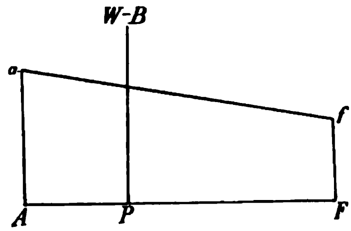


FIG. 221.

Then $Aa = \frac{2(W - B)}{l^2}(3d - l)$ from equation (3), p. 360.

and $Ff = \frac{2(W - B)}{l^2}(2l - 3d)$. „ (4) „

When $d = \frac{2}{3}l$ pressure at F is zero.

„ $d = \frac{l}{3}$ „ A „

Curves for values of A *a* and F *f* can therefore be plotted in terms of the distance travelled down ways by the vessel, and from these curves it will be seen whether the pressure per square foot at A or F is excessive in the event of the vessel tipping or lifting.

# CHAPTER XXVIII.

## BENDING STRESSES UPON A SHIP WHEN INCLINED TO THE UPRIGHT.

**Distribution of Stress over an unsymmetrical section.**—When a vessel is upright in still water or steaming at right angles to a series of waves, the plane of the resultant longitudinal bending couple is the vertical middle longitudinal plane of the vessel. The plane of the bending couple in this case is a plane of symmetry with respect to the area of the parts resisting the bending couples.

The majority of problems connected with the resistance of structures to bending deals only with symmetrical structures. Such structures as bridges and cantilevers, and any arrangement of strong beams designed to withstand a bending action, are symmetrical about the plane of the resultant bending couple, or about a plane parallel to it.

When a vessel is rolling among waves, or when she is inclined to the horizontal, the plane of the resultant bending couple may be inclined to the middle longitudinal plane.

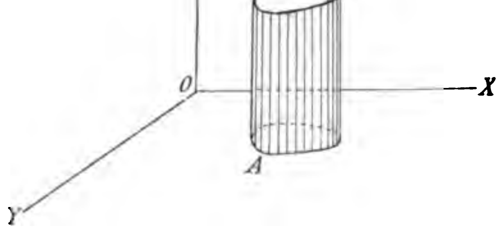


FIG. 222.

The structure is then unsymmetrical with regard to the plane of the resultant bending couple, and hence the formula $\frac{p}{y}=\frac{M}{I}$ cannot be directly applied. In deducing this formula it was assumed that the neutral axis is perpendicular to the plane of the bending couple.

In this chapter the general case of the bending of a beam of irregular section will be considered.

To find a formula giving the stress at any point in the section of an irregular beam, it will be necessary first to consider the relations governing the distribution of stresses acting obliquely on a plane surface of an elastic material.

(1) Let the area of the plane surface, fig. 222, be A, and let the stresses acting over A be uniform and parallel.

Let the intensity of the stress be $p$. Then the total stress P is equal to $p$ A. P is therefore the magnitude of the resultant, and it acts in the same direction as the stresses through the centre of gravity of the plane area.

(2) If the stress is not uniform, consider any element of the area $dx.dy$. Let $p$ be the intensity of stress at this element. Then the stress on the element $= p.dx.dy$.

$$\therefore \quad P = \iint p.dx.dy.$$

The area $A = \iint dx.dy$.

Therefore the mean intensity of stress $= \frac{P}{A} = \frac{\iint p.dx.dy}{\iint dx.dy}$.

Let the plane X Y, fig. 223, contain the area A, fig. 222, and let the direction of the stresses be perpendicular to the plane X Y.

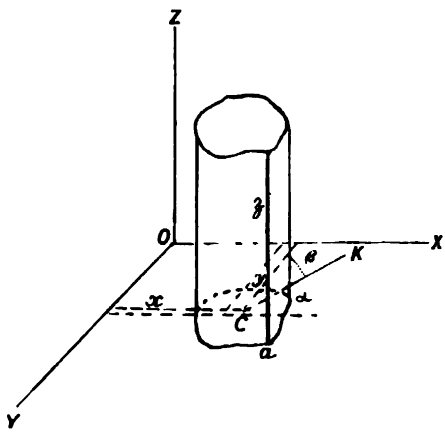


FIG. 223.

At any point in A draw the line which completely specifies the intensity of the stress at that point. The solid so formed is $\iint z.dx.dy$. This corresponds to the total stress $\iint p.dx.dy$, and therefore the mean stress corresponds to the mean height of the solid, which is

$$\frac{\iint z.dx.dy}{\iint dx.dy}.$$

The point $c$ on the area A at which the resultant stress acts is called the centre of stress.

In order to find the coordinates of this point, let A be the area as before in the plane X Y, fig. 223. Consider the stress on an element $dx.dy$.

Let C K the line of stress on $dx.dy$ make an angle $\alpha$ with O X and an angle $\beta$ with O Y.

Then the component of the stress parallel to Z O X is $p.dx.dy \sin \beta$,
and „ „ „ Z O Y is $p.dx.dy. \sin \alpha$.

∴ The component of the moment relatively to Z O X is $y.p.dx.dy. \sin \beta$, and the component of the moment relatively to Z O Y is $-x.p.dx.dy. \sin \alpha$.

Let $x_0$ and $y_0$ be the coordinates of the centre of stress.

Then $$y_0 P. \sin \beta = \iint y.p.dx.dy. \sin \beta.$$

$$\therefore \quad y_0 P. = \iint y.p.dx.dy.$$

Similarly $$x_0 P = \iint x.p.dx.dy.$$

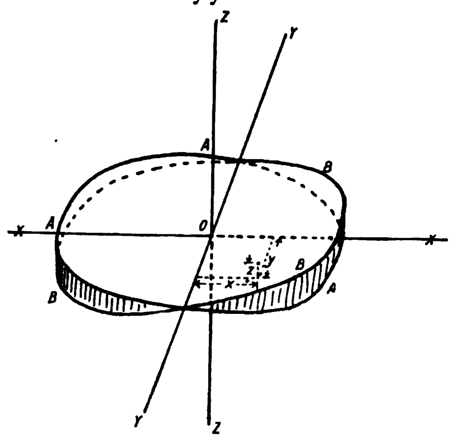


FIG. 224.

∴ The coordinates of the centre of stress are

$$x_0 = \frac{\iint x.p.dx.dy}{\iint p.dx.dy}.$$

$$y_0 = \frac{\iint y.p.dx.dy}{\iint p.dx.dy}.$$

If the stress is uniform, the coordinates $x_0$ and $y_0$ are those of the centre of gravity of the area A, viz.—

$$\frac{\iint x.dx.dy}{\iint dx.dy} \quad \text{and} \quad \frac{\iint y.dx.dy}{\iint dx.dy}.$$

(3) In a uniformly varying stress the intensity at any point is proportional to the distance of the point from a fixed straight line.

Let the area be A A A, fig. 224.

Let the fixed line be the axis of O Y, as in the figure 224.

The total stress $$P = \iint p.dx.dy.$$

We have this equation governing the distribution of the stress.

$$p = ax \quad \text{where } a \text{ is a constant.}$$

$$\therefore \quad P = a\iint x.dx.dy.$$

This expression is zero if O Y passes through the centre of gravity of A A A, and in that case O Y would be called the neutral axis of the section.

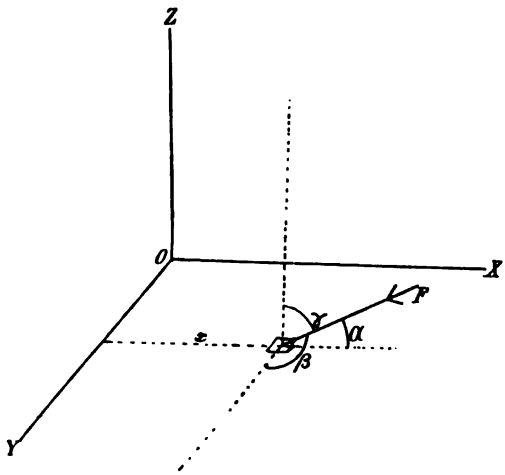


Fig. 225.

The stress forces can be represented as vertical ordinates of a cylindrical solid made up of two wedges, sides parallel to Z O Z, and between the planes A A and B B.

The forces being proportional to their distance from O Y, the vertical ordinates of this solid are proportional to the ordinates parallel to O X.

Consequently, the forces on opposite sides of O Y are opposite in sign, and the resultant of the whole stress is a couple whose moment and the position of its axis are found in the following manner.

Let $\alpha, \beta, \gamma$ be the angles which the direction of the stress makes with O X, O Y, and O Z respectively (fig. 225).

F = stress over the elemental area
$= p.dx.dy = ax.dx.dy.$

Then the

moment of stress round O X $= ax.dx.dy.y.\ \cos\gamma$
" " " " O Y $= -ax.dx.dy.x.\ \cos\gamma$
" " " " O Z $= ax.dx.dy(x\cos\beta - y\cos\alpha).$

Summing and integrating these moments,

The total moment round O X $= M_x = \quad a\cos\gamma\iint x.y.dx.dy.$

" " " O Y $= M_y = -a\cos\gamma\iint x^2.dx.dy.$

" " " O Z $= M_z = a\left\{\cos\beta\iint x^2dx.dy - \cos\alpha\iint xy.dx.dy\right\}$

$$\text{Now} \quad I = \iint x^2.dx.dy$$

$$\text{and} \quad K = \iint x.y.dx.dy$$

$$\therefore \quad M_x = aK\cos\gamma$$
$$M_y = -aI\cos\gamma$$
$$M_z = a(I\cos\beta - K\cos\alpha).$$

Moment of resultant couple $= M = \sqrt{M_x^2 + M_y^2 + M_z^2}$

$$= a\sqrt{(I^2 + K^2)\cos^2\gamma + I^2\cos^2\beta + K^2\cos^2\alpha - 2IK\cos\alpha\cos\beta}$$
$$= a\sqrt{I^2\sin^2\alpha + K^2\sin^2\beta - 2IK\cos\alpha\cos\beta}$$

since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

The angles $\lambda$, $\mu$, $\nu$ that the axis of this resultant couple makes with the axis of X, Y, and Z are given by

$$\cos\lambda = \frac{M_x}{M}$$
$$\cos\mu = \frac{M_y}{M}$$
$$\cos\nu = \frac{M_z}{M}.$$

Also $\cos\alpha\cos\lambda + \cos\beta\cos\mu + \cos\gamma\cos\nu = 0$.

This last equation indicates that the axis of the resultant couple is perpendicular to the direction of the stress.

From these considerations we can now pass to the question of the stresses resisting bending in a beam. The assumptions that must be made are—

(1) The beam is in equilibrium.

(2) The material is stretched or compressed only within the elastic limits, so that Hooke's Law holds.

(3) There is no discontinuity of form in the beam.

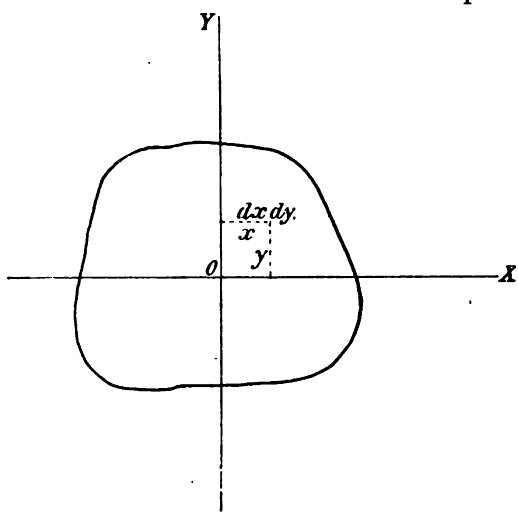


FIG. 226.

Let us consider the case of a section of the beam perpendicular to the plane of bending. Assume that the stress over this section varies uniformly. Since the beam is in equilibrium we have—

(1) The algebraic sum of all the stresses (which in this case are perpendicular to the section) is zero.

(2) The algebraic sum of all the moments of the stresses about the neutral axis is equal to the bending moment.

We have seen that in a stress varying uniformly as the distance that its point of application is from a straight line, the above two conditions will be satisfied if the stress varies directly as the distance from the neutral axis, and if the neutral axis passes through the centre of gravity of the cross section.

Applying the previous equation to the beam : Let the plane of the cross section be the plane of O X Y, and let O Y be the neutral axis as before. The figure 226 represents the section of a beam which is irregular.

We have then

$$\cos\ \alpha = 0$$
$$\cos\ \beta = 0$$
$$\cos\ \gamma = 1.$$

Then $M_z = 0$

and $\cos\ \nu = 0.$

So that the axis of the resultant couple is in the plane of the cross section.

The equations become

$$M_x = aK.$$
$$M_y = -aI$$
$$\therefore \quad M = a\sqrt{(I^2 + K^2)}$$
$$\cos\lambda = \sin\mu = \frac{K}{\sqrt{I^2 + K^2}}$$
$$\cos\mu = \sin\lambda = -\frac{I}{\sqrt{I^2 + K^2}}.$$
$$\therefore \quad \text{Tan}\ \mu = -\frac{K}{I}.$$

At an elemental area $dx\ dy$ let the stress be $p$,

$$p = ax.$$

$\therefore$ Stress on elemental area $= p.dx.dy$,

$$= ax.dx.dy.$$

Moment of elemental stress about $OX = -a\,x\,y\,dx\,dy$,

" " " " $OY = +a\,x^2\,dx\,dy$.

$\therefore$ $M_x$ the total amount about $OX = -a\iint x\,y\,dx\,dy$,

$M_y$ " " $OY = a\iint x^2\,dx\,dy$.

The resultant moment $M = \sqrt{M_x^2 + M_y^2}$

$$= a\sqrt{\left(\iint y\,x\,dx\,dy\right)^2 + \left(\iint x^2 \cdot dx \cdot dy\right)^2}.$$

The axis of M is inclined $\mu$ to $OY$ where

$$\text{Tan}\ \mu = \frac{M_x}{M_y} = -\frac{\iint xy.dx.dy}{\iint x^2.dx\,dy} = -\frac{K}{I}.$$

If the axis $OX$ or $OY$ divides the section symmetrically then $\iint x\,y\,dx\,dy$ would be zero.

$\therefore$ $\mu$ would be zero, and the axis of the couple would coincide with the neutral axis. Therefore in cross sections which are symmetrical about the plane of bending or about a plane perpendicular to the plane of bending the axis of the resultant couple coincides with the axis of the bending couple, *i.e.* it is perpendicular or parallel to the plane of symmetry.

In the case of a symmetrical section we also have

$$M_x = 0$$
$$M = M_y = -aI$$
$$\mu = 0$$

so that $\frac{p}{x} = \frac{M}{I}$ is the formula.

The general one being

$$\frac{p}{x} = \frac{M}{\sqrt{I^2 + K^2}}$$

and for the angle that the neutral axis or axis of O X makes with the axis of the bending couple we have $\tan \mu = -\frac{K}{I}$.

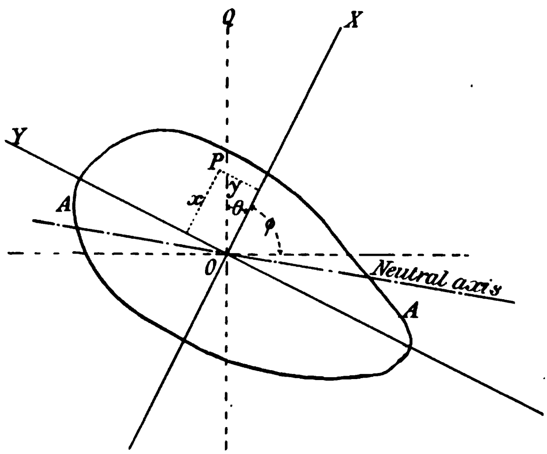


FIG. 227.

In any section there are two axes, called the principal axes, perpendicular to each other, about one of which the moment of inertia is a maximum and about the other a minimum.

The value of K or $\iint xy\,dx\,dy$ for the principal axis if chosen as the axis of coordinates is zero. This relation is proved in the ordinary moment of inertia formulæ. If, then, the neutral axis coincides with one of the principal axes, the axis of the bending couple must also coincide, and therefore the plane of bending contains the other principal axis. This last theorem enables us to determine the stress at any point in an irregular section when we know M and the plane of bending.

Let O X and O Y be the principal axes of the irregular cross section A A, fig. 227. Let O Q be the plane of the bending moment M acting over the section. Let $QOX = \theta$. Resolving the couple M in the planes of O X and O Y,

The component of M in the plane $OX = M \cos \theta$,

„ „ „ $OY = M \sin \theta$.

We can now apply the simple formula to each of the component moments. There will be a stress $p_1$ at P due to the moment M cos $\theta$ in plane of O X.

$$\therefore \quad p_1 = \frac{M \cos \theta x}{I_y} \text{ where } I_y = \iint x^2 dx.dy.$$

There will also be a stress $p_2$ at P due to the moment M sin $\theta$ in plane of O Y.

$$\therefore \quad p_2 = \frac{M \sin \theta y}{I_x} \text{ where } I_x = \iint y^2 dx.dy.$$

$\therefore$ The total stress $p$ at P $= p_1 + p_2$

$$= M\left(\frac{\cos \theta x}{I_y} + \frac{\sin \theta y}{I_x}\right).$$

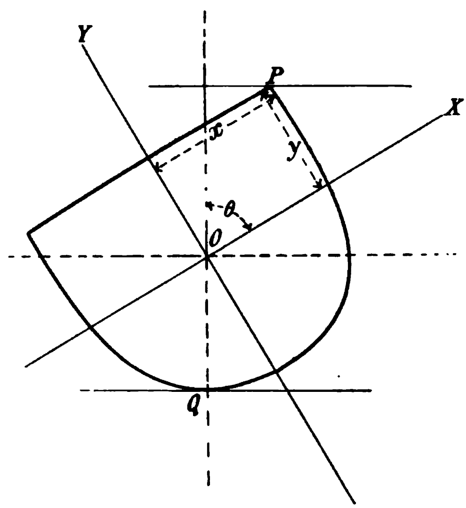


FIG. 228.

Knowing M and $\theta$, and after estimating $I_y$ and $I_x$, we can find the stress at any point $xy$ by the above formula.

The equation to the neutral axis is obtained by putting $p = 0$, since there is no stress at the neutral axis,

$$i.e. \quad \frac{\cos \theta x}{I_y} + \frac{\sin \theta y}{I_x} = 0.$$

$$\therefore \quad \frac{y}{x} = -\cot \theta . \frac{I_x}{I_y}, \text{ which is the equation to a straight line.}$$

Let the neutral axis make an angle $\phi$ with O X.

$$\text{Then} \qquad \tan \phi = -\cot \theta . \frac{I_x}{I_y}.$$

**Stresses over the cross section of a ship when inclined.**—In the case of a vessel the section is symmetrical about the vertical middle line plane, O Y, in fig. 228. The value of $I_x$ is calculated in the ordinary case for the vessel upright. It only remains for a calculation to be made for the moment of inertia $I_y$ about O Y. O X and O Y are principal axes.

The stresses at the highest and lowest points in the section are generally calculated. The highest point when the vessel has inclined to a considerable angle will be the deck edge, as at P, and the lowest point will be at the tangent to the bilge, as at Q. The coordinates of these points may be put in the equation for $p$ and the stress obtained. Generally it is found that the maximum stresses are slightly increased on account of the heeling. The diagram fig. 229 shows the inclination of the neutral axis as the angle of heel varies from 0° to 90°.

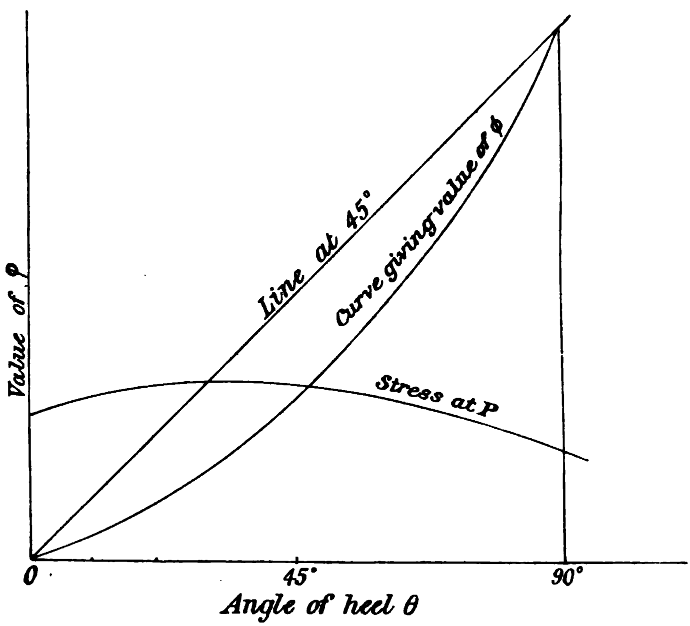


FIG. 229.

A line has been drawn at 45° to the base line, which latter indicates angle of heel. The vertical ordinates measure $\phi$. The curve below 45° line shows how much the neutral axis lags behind the axis of the bending couple where O X in the upright condition was the neutral axis.

Where $\theta = 90°$ as in upright, $\tan\phi = 0 \quad \therefore \quad \phi = 0,$

„ $\theta = 45°$ „ inclin. of 45° „ $= -\frac{I_x}{I_y} \quad \therefore \quad \phi = \tan^{-1} - \frac{I_x}{I_y},$

„ $\theta = 0°$ „ inclined 90° „ $= -\infty \quad \therefore \quad \phi = -90°.$

$$\text{Tan } \phi = -\cot\theta . \frac{I_x}{I_y}.$$

In this equation $\frac{I_x}{I_y}$ is a constant, and therefore cot $\theta$ is a maximum when tan $\phi$ is a maximum, *i.e.* when $\theta = 90°$ cot $\theta$ is infinity, and therefore

$\phi = -90°$. The axis of the bending couple is perpendicular to the plane of the bending moment. Therefore a line drawn at 45° in the diagram represents the inclination of the axis of the couple to the axis of O X as a vessel is heeled over.

$I_y$ is generally about equal to $2 I_x$ in vessels with two or more decks. Taking this value, the equation to the neutral axis becomes $2 \tan \phi = -\cot \theta$.

The Table LXVIII. contains results of calculations of vessels in the inclined condition. It will be seen that the maximum bending moment is reduced slightly in some cases and increased slightly in others when compared with the corresponding bending moment in the upright condition. The maximum tensile stresses are all reduced and the maximum compressive stresses are all increased.

**Determination of the Longitudinal Bending-Moment Curve when the Vessel is Heeling and Steaming Obliquely to the Waves.**—The results of some examples of the determination of the longitudinal bending moment on a vessel when inclined to the upright and to the line of advance of the waves are given in this chapter.

It has been stated that the worst size of wave that a vessel can meet is one her own length, assuming a given proportion of height to length of wave. This is clearly seen to be true for a vessel meeting the wave at right angles. The results of the examples referred to show that the above statement is true when the vessel is inclined to the upright, or steaming in a direction inclined to the waves. It will be seen that, in the case of a vessel steaming obliquely to the waves, the length of wave from crest to crest will be the projection of the length of vessel in the direction of motion of the waves if the bow and stern of the vessel are at any instant in consecutive hollows or crests of the wave.

When a vessel is steaming obliquely to the waves she will have a motion which may be resolved into two motions, viz. pitching and rolling. The support of buoyancy at any instant is no longer symmetrical about the middle line plane, and consequently twisting moments are produced. The pitching motion is produced by the periodic change of the longitudinal distribution of buoyancy. Rolling and the consequent racking stresses can therefore only be produced when the vessel is oblique to the waves. From the results of a number of calculations made on different types of vessels, it has been seen that the maximum longitudinal bending moment is only altered slightly when the vessel is inclined. Some results of calculations of this nature are given in Table LXVIII. From these results it will be seen that in some cases the bending moment is slightly increased and in others slightly decreased. For all practical purposes, therefore, it may be considered that the bending moment is not greatly altered for moderate angles of heel of the vessel, other conditions remaining the same.

The most general case of a vessel among waves is to assume the vessel inclined at an angle $\theta$ to the upright, and at an angle $a$ to the line of motion of the waves.

In fig. 230 let C and C be the crest lines of a wave of length A B.

If L is the length of the vessel, then if the wave is of the worst length

$$A B = L \cos a,$$

and the stem and stern will lie in consecutive crests as shown.

## TABLE LXVIII.

COMPARISON OF RESULTS OF CALCULATIONS FOR BENDING MOMENT AND STRESSES. VESSELS IN INCLINED CONDITION.

| | Torpedo-boat Destroyer. | | Torpedo-boat Destroyer. | | Torpedo-boat Destroyer. | | Torpedo-boat Destroyer. | | Torpedo-boat Destroyer. | | Channel Steamer. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Length | 225′ | | 225′ | | 218′ | | 211′ | | 220′ | | 330′ | |
| Breadth | 21′ 6″ | | 21′ 6″ | | 20′ | | 21′ 6″ | | 20′ 6″ | | 42′ | |
| Depth | 13′ 6″ | | 13′ 6″ | | 12′ 6″ | | 12′ 9″ | | 12′ 6″ | | 17′ 6″ | |
| Displacement | 490 | | 490 | | 390 | | 423 | | 404 | | 2385 | |
| Wave length | 225′ | | 150′ | | 218′ | | 211 | | 220′ | | 330′ | |
| ,, height | 11·25′ | | 15′ | | 10·9 | | 10·5′ | | 11′ | | 16·5′ | |
| Position on wave | hollow upright | hollow inclined 35° | hollow upright | hollow inclined 35° | hollow upright | hollow inclined 35° | hollow upright | hollow inclined 31° | hollow upright | hollow inclined 34½° | crest upright | crest inclined 20½° |
| Max. S.F. aft | 100 | 103 | 138 | 133 | 78 | 76 | 82 | 79 | 95 | 89 | 289 | 307 |
| ,, S.F. forward | 103 | 107 | 140 | 188 | 74 | 74 | 74 | 73 | 95 | 91 | 310 | 321 |
| ,, Bending moment | 6241 | 6313 | 7300 | 6960 | 4090 | 4027 | 4021 | 3841 | 5151 | 4868 | 26760 | 28970 |
| Moment of inertia upright | 5770 | | 5770 | | 3284 | | 4178 | | 5200 | | | |
| ,, ,, 90° | 10580 | | 10580 | | 6825 | | 3855 | | 8029 | | | |
| Stress, bottom of bilge | 8·5 keel | 8·12 | 9·94 | 8·95 | 9·11 | 7·52 | 7·3 | 5·45 | 8·17 | 7·32 | | |
| ,, gunwale angle | 7·17 deck | 9·1 | 8·39 | 10·05 | 7·60 | 8·73 | 5·67 | 6·33 | 4·83 | 6·48 | | |

As in the ordinary calculation we have to find the position of the vessel in the wave, that will satisfy the two conditions of equilibrium:—

(1) The displacement must be equal to the weight of the vessel.

(2) The centre of buoyancy must be in the same longitudinal position as the centre of gravity.

The transverse position of the centre of buoyancy may not be the same as that of the centre of gravity. This, however, only affects the heeling moments, and need not be taken into account in calculating the longitudinal bending moment. The proportion of height to length of wave is 1 to 20. The wave profile, and, if necessary, a series of subsurfaces, are first constructed. The most convenient way to obtain the buoyancy when the vessel is inclined to the upright and to the line of wave motion is to make a body plan of section of the form. These sections will be the intersections of the vertical planes inclined $a$ to the transverse planes of the vessel and $\theta°$ to the longi-

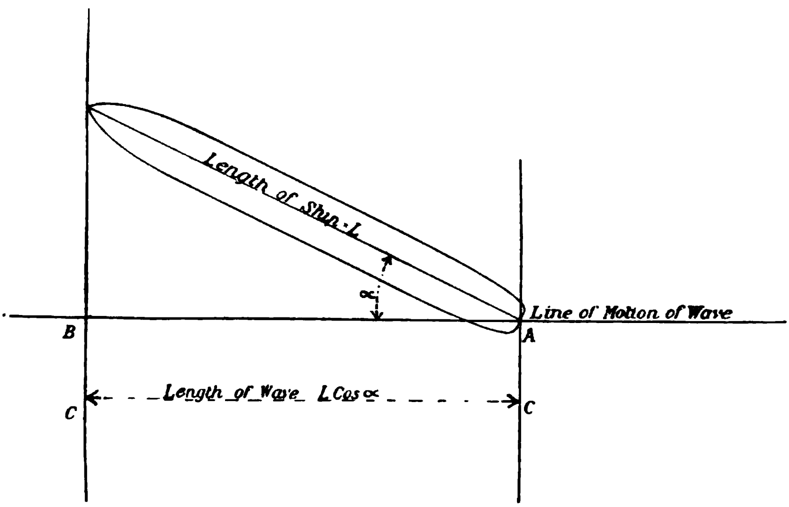


FIG. 230.

tudinal upright plane of the vessel. These sections are obtained in the following manner. A series of parallel waterlines at an inclination of $\theta$ to the horizontal are drawn on the body plan as shown in fig. 231. The true forms of the waterplanes are next drawn in their relative positions as viewed in a direction perpendicularly to them.

The waterplanes thus shown in plan are $wl_1$ $wl_2$ $wl_3$ $wl_4$ $wl_5$. The ordinary transverse sections of the vessel are numbered 0 to 10 consecutively. The lines of the crest and hollow will be parallel to A $a$ and F $f$ where $af$ is a line inclined $a$ the inclination of vessel to line of wave's advance. New sections can therefore be drawn parallel to F $f$ and A $a$, and their forms can be laid off to form a new body plan. The body plans thus obtained are shown in figs. 232 and 232A, and give sections of a vessel made by planes inclined 30° to the wave's advance and 30° to the upright. The sections are no longer symmetrical, so that they have to be drawn for both sides of the vessel.

A tracing of the elevation of these sections on a vertical plane through $af$ shows only straight lines. Draw the waterplanes in their relative positions

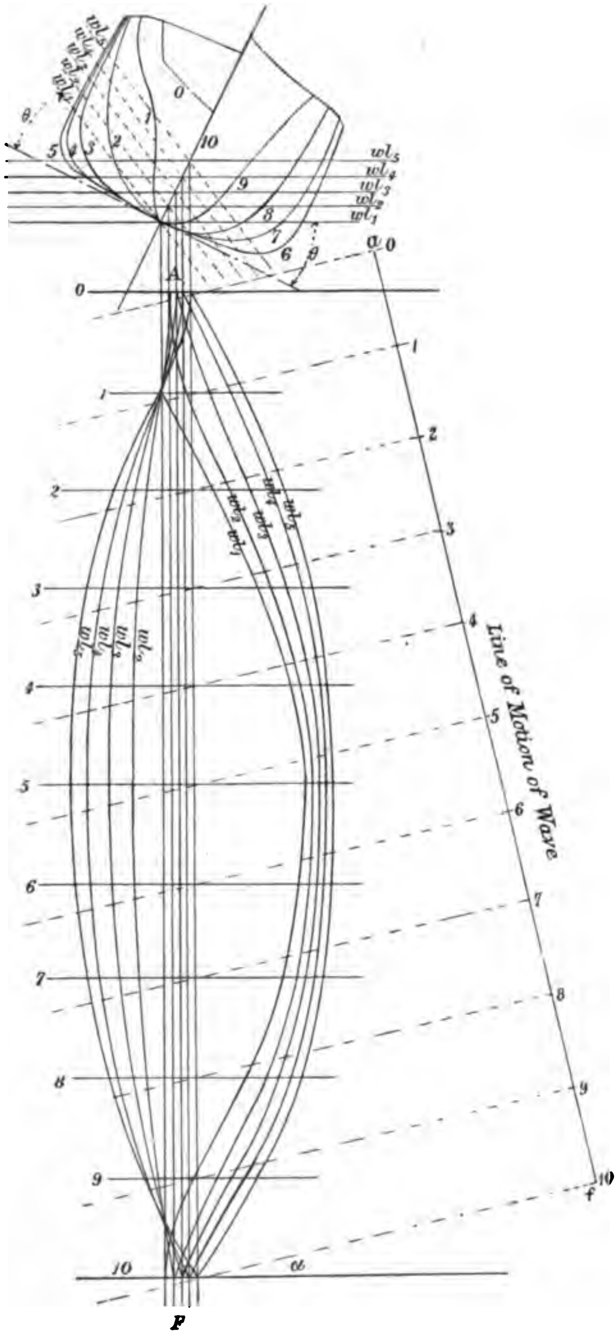


FIG. 231.

in this elevation. These will also be straight lines. This tracing is laid over the wave outline or the wave surface and subsurfaces in an arbitrarily chosen position, and the heights of the subsurfaces on the sections are transferred to the body plan. In this way the buoyancy per foot of wave-length at each of these sections can be calculated. It is necessary here to make the usual adjustments for displacement and longitudinal position of centre of buoyancy

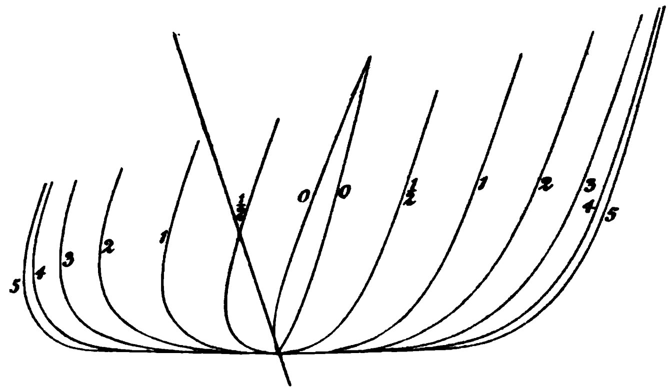


FIG. 232.—Fore Body Sections.

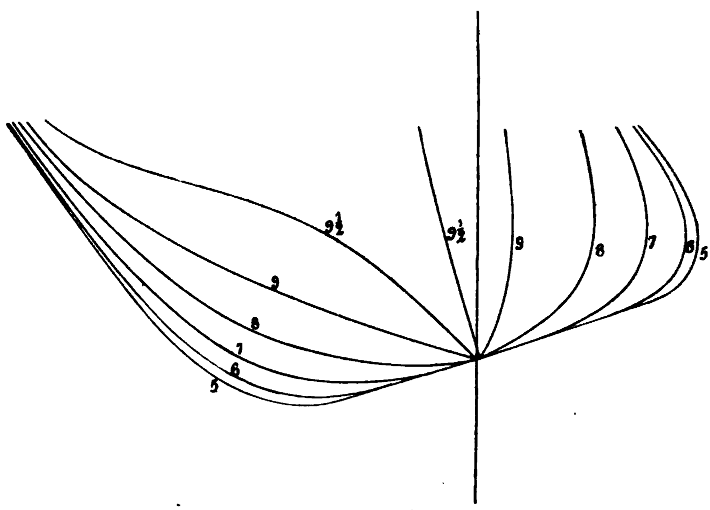


FIG. 232A.—After Body Sections.

to satisfy the conditions of equilibrium. The area of the immersed part of any section gives the buoyancy per foot of wave-length at that section. We want, however, to find the buoyancy at any transverse section in terms of the length of the ship. A close enough approximation can be made if we multiply the buoyancy per foot of wave-length by $\frac{1}{\cos a}$, where $a$ is the inclination of the vessel to the wave's advance. $a$ is also the inclination of a

T.S.S. 525 × 63 × 48.
Vessel. Coal out. Δ = 11950.

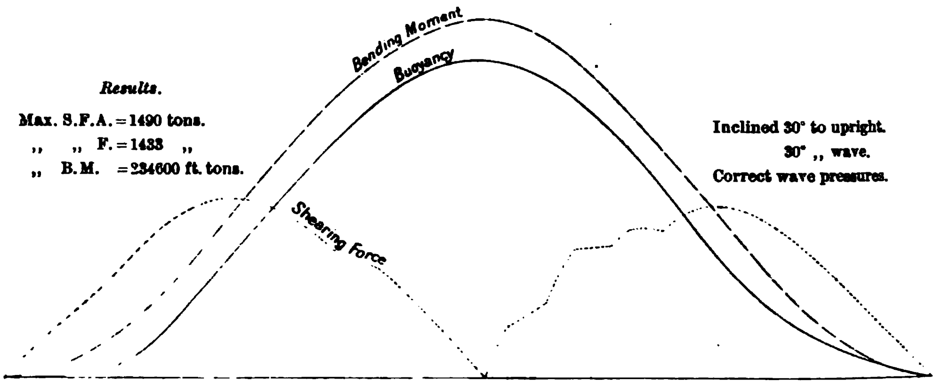


FIG. A.

T.S.S. 525 × 63 × 48.
Vessel full load. Coal out. Δ = 11950 tons.

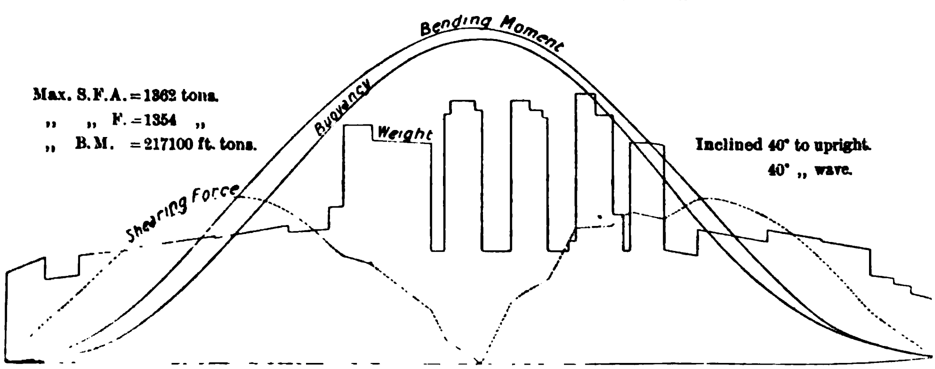


FIG. B.

[*To face page* 378.

transverse plane in the vessel to the plane of the sections at which we have measured the areas. Having obtained the buoyancy curve we proceed in the usual way to get the shearing-force and the bending-moment curves by using the weight curve for the condition which the vessel is in.

The following Table LXIX. gives the results of calculations that were made for inclined conditions of a large Atlantic liner. The strength diagrams corresponding to each case are also given.

TABLE LXIX.

Dimensions, 525′ × 63′ × 43′.
Displacement, 11,950 tons.

| Condition. | Shearing Force. | | Maximum Bending Moment. | Number of Fig. which shows the Diagram. |
|---|---|---|---|---|
| | Aft. | Forward. | | |
| Standard hogging . . . . . . . | 1610 | 1585 | 258000 | 201, 202 |
| Standard hogging (corrected for wave pressures) | 1364 | 1367 | 228400 | 201, 202 |
| Inclined 20° to vert., 20° to waves . . . . | 1630 | 1560 | 256300 | Diag. B, Pl. XVIII |
| Inclined 20° to vert., 20° to waves (corrected for wave pressures) . . . . . . | 1440 | 1370 | 225900 | Diag. B, Pl. XVIII |
| Inclined 30° to vert., 30° to waves (corrected for wave pressures) . . . . . . | 1490 | 1433 | 234600 | Diag. A, Pl. XVIIIA |
| Inclined 40° to vert., 40° to waves (corrected for wave pressures) . . . . . | 1362 | 1354 | 217100 | Diag. B, Pl. XVIIIA |

**Twisting Moment.**—In order to determine the twisting moment due to inequality of weight and buoyancy from point to point transversely in the wave, it would be necessary to draw transverse curves of weight and buoyancy. The former would be a very difficult operation for the structural part of the ship and her machinery, but the curve of buoyancy and the curve of distribution of a homogeneous cargo can be easily obtained by plotting the areas of a series of vertical sections of the ship perpendicular to the wave crest. The twisting moments that a vessel may experience among waves are in general much less than the bending moment. The formula for shearing stress due to twisting in a circular cylinder is $\frac{q}{r}=\frac{M}{I}$, where $q$ is the shearing stress at a radius of $r$ from the axis of zero stress, and M is the twisting moment.

I is the moment of inertia about the axis of zero stress. The formula is only strictly applicable to a girder of circular section, but for girders with sections approximately similar, comparisons may be made upon the basis of the formula.

# CHAPTER XXIX.

## DETERMINATION OF THE DEFLECTION OF A SHIP DUE TO A GIVEN CHANGE IN THE BENDING MOMENT.

A GIRDER when under the action of a bending moment changes form owing to the stretching of the fibres on one side and the compression of the fibres on the other side of the neutral axis.

A simple case is presented when the beam or girder under the bending moment is symmetrical about the plane containing the resultant bending couple.

In Chapter XIX. the formula $\frac{M}{I} = \frac{E}{R}$ was proved for the above case. This formula gives a relation between the curvature $\frac{I}{R}$ of the girder and the bending moment M producing it.

Taking the neutral axis in the direction of the length of the girder as the axis of $x$ we have the formula for the curvature when R is large (as it is in all ship work),

$$\frac{1}{R} = \frac{d^2y}{d^2x}$$

where $y$ is the deflection at any point from the axis O X of the girder, which is here a straight line tangential to the neutral axis.

Substituting this value in the above equation,

$$E\frac{d^2y}{dx^2} = \frac{M}{I}.$$

Integrating this equation twice,

$$y = \iint \frac{M}{EI}\,dx\,dx + ax + b$$

(where $a$ and $b$ are constants), which is an equation giving the deflection at any point from the axis O X due to the applied bending moment M. When M and I vary continuously this formula is true. If I is constant the formula becomes

$$EIy = \iint M\,dx\,dx.$$

Therefore in a uniform homogeneous girder the curve giving the values of $y$ may be obtained by integrating the curve of bending moments twice. The maximum value of $y$ in cases such as these can be calculated from the above formula.

From a consideration of the formula

$$y = \iint \frac{M}{EI}\, dx\, dx \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

it will be seen that if E is some known value, the values of $y$ can be obtained by plotting a curve of values $\frac{M}{EI}$ in terms of $x$, and integrating this curve twice. The value of E has to be taken from experience. For a homogeneous girder of solid rectangular or circular section E is the modulus of elasticity of the material. In a built-up structure this value is affected by the imperfections of the rivet connections between the individual members, so that E in the formula represents the modulus of the structure. This modulus will be less than the modulus of elasticity of the material by an amount depending upon the efficiency of the connections. The value of E for steel or iron is known, and for any material it can be experimentally obtained. The value of E for a structure or for any girder can be calculated from the results of bending tests. If we assume the truth of the formula $Ey = \iint \frac{M}{I}\, dx\, dx$ we can get a curve giving values of E$y$, and if we have the actual values for $y$ by observing the deflection during the experiment we can thereby obtain the value of E. This method is often used in the test laboratory to determine the value of E for the material. This method of determining E for a structure was first applied to the case of a ship by Messrs Read and Stanbury, who gave the results of their investigations in a paper to the Institution of Naval Architects.

The formula for E$y$, which has just been given, is applicable to the case of a ship if we are only considering the ordinary case of the bending moment acting in the vessel's plane of symmetry.

**Values of E obtained by Experiments on Bars.**—In the above paper the authors gave the results of some practical experiments on the deflection of girders made up of frames or bars commonly used in ship construction. The most convenient method of applying the bending moment, and the one that was adopted in the above experiments, is to rest the bar on two supports which are at a fixed distance apart, the ends of the girders being free to slide, and to apply a load midway between the supports. For a beam supported and loaded in this fashion the maximum deflection $y$ is at the centre and is given by

$$y = \frac{WL^3}{48EI}$$

where W is the applied load. (The weight of the girder is assumed to be negligible.) L is the length between the supports.

The results of Messrs Read and Stanbury's experiments are summarised in the following table. Structures Nos. 1 and 2 were each built up of two angles riveted together similarly to the frame and reversed frame of a ship.

The remaining girders were of ⅂ and [ section.

TABLE LXX.

| No. of Test. | Girder. | Scantlings. | Material. | Distance between Supports. | Load in Tons. | Actual Deflection. | Calculated Deflection. |
|---|---|---|---|---|---|---|---|
| 1 | ¬L | $5 \times 3 \times \frac{8}{20}$<br>$3 \times 3 \times \frac{15}{20}$ | Mild Steel. | 7′ 0″ | 10 | ·87″ | ·7″ |
| 2 | ¬L | $5 \times 3 \times \frac{8}{16}$<br>$3 \times 3 \times \frac{7}{16}$ | Iron. | 7′ 0″ | 10 | 1·1″ | ·64″ |
| 3 | ʅ | $5 \times 3 \times 3 \times \frac{7}{20}$ | Mild Steel. | 7′ 0″ | 8 | ·62″ | ·6″ |
| 4 | ʅ | $5 \times 3 \times 2\frac{1}{2} \times \frac{8}{20}$ | ,, | 7′ 0″ | 6 | ·5″ | ·51″ |
| 5 | [ | $6 \times 2\frac{1}{2} \times 2\frac{1}{2} \times \frac{8}{20}$ | ,, | 7′ 0″ | 6 | ·5″ | ·37″ |
| 6 | [ | $10 \times 3\frac{1}{2} \times 3\frac{1}{2}$ $\times \frac{10}{20}$ flanges, $\times \frac{11}{20}$ web | ,, | 12′ 0″ | 8·2 | ·43″ | ·46″ |
| 7 | [ | $10 \times 3\frac{1}{2} \times 3\frac{1}{2} \times \frac{11}{20}$ | ,, | 12′ 0″ | 8·2 | ·39″ | ·43″ |
| 8 | [ | $12 \times 3\frac{1}{2} \times 3\frac{1}{2} \times \frac{10}{20}$ | ,, | 12′ 0″ | 10·2 | ·43″ | ·37″ |

The calculated deflections have been all made on the assumption that E is 10,000 tons per square inch. In Nos. 1, 2, 3, 5, and 8 the actual deflections are greater than the corresponding calculated deflections, indicating that the value of E should have been less than 10,000, in order to give the actual deflection. The true value of E for the others, Nos. 4, 6, and 8, would seem to be a little greater than 10,000. Considering only Nos. 3 to 8, the average value for E is about 10,000. It will be seen later that the calculated deflections for Nos. 1, 2, 3, and 4 from the formula

$$y = \frac{WL^3}{48EI}$$

are not strictly correct unless lateral deflection has been prevented.

A very extensive series of experiments was carried out by the Committee of Lloyd's Register to ascertain the value of various methods of construction adopted or proposed in the arrangements of girders and attachments to the shell or deck plating. These results have been published. A full description of them may be obtained from a paper read on the subject to the Institution of Naval Architects, 1905, by Dr Bruhn.

The results of all the tests were given, and also the curves showing the amount of deflection produced in each case. It will be interesting in this chapter to consider the results of the observations on deflection in the above experiments. In an appendix to the paper, Dr Bruhn gives the method of calculating the deflection of a girder whose cross section is not symmetrical with respect to a straight line.

In Chapter XXVII. the formula was given for determining the stress at any point of a cross section of irregular shape, and we deduced the formula for the stress in a symmetrical section like that of a ship, but with the bending-moment couple not in the plane of symmetry. We shall briefly mention the formulæ that have to be applied in such cases for estimating the deflection. When the cross section is symmetrical and the bending moment M acts in the plane of symmetry we have the formula

$$\frac{p}{y} = \frac{M}{I} = \frac{E}{R}.$$

This formula is also true when the cross section is not symmetrical, and when M acts in a plane containing either of the principal axes.

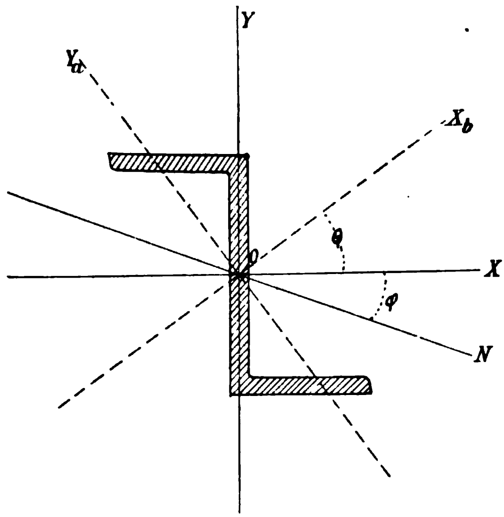


FIG. 233.

When the cross section is not symmetrical and the bending moment does not act in either of the planes containing the principal axes we have the formula

$$\frac{p}{y} = \frac{M}{\sqrt{I^2 + K^2}} \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

where I is the moment of inertia of the section about the neutral axis O X, and K is the rectangular moment of the section about the axes O X and O Y. It is therefore necessary in the latter case to first find the position of the principal axes. The bending moment M can be resolved into its component moments in the planes containing the principal axes, and the component deflection of the resultant deflection calculated by formula (i).

Let fig. 233 represent the cross section of an unsymmetrical girder.

Let O Y be the given plane of the bending moment M. Take the axis O X perpendicular to O Y. Then we first calculate

$I_y$ the moment of inertia of cross section about O X.
$I_x$ „ „ „ „ O Y.
K „ rectangular moment of cross section about the axes O X and O Y.

We next have to find the positions of the principal axes. Let O $X_b$ and O $Y_a$ be the principal axes for maximum and minimum moments of inertia respectively.

Let $\theta$ be the angle X O $X_b$.

Then the position of the axes is found from the equation

$$\text{Tan} \quad 2\theta = \frac{2K_{xy}}{I_y - I_x}$$

$$\text{and} \quad I_a = \frac{I_y + I_x}{2} + \frac{K_{xy}}{\sin 2\theta}$$

$$\text{„} \quad I_b = \frac{I_y + I_x}{2} - \frac{K_{xy}}{\sin 2\theta}.$$

We have also seen that the angle which is made by the neutral axis with the axis O X is given by

$$\text{Tan} \quad \phi = -\frac{I_b}{I_a} \tan \theta.$$

Having thus obtained the position of the neutral axis we can calculate the actual deflection $D_y$ due to the given applied couple in the plane O Y. This deflection $D_y$ will be perpendicular to the neutral axis O N. Let $D_a$ be the deflection perpendicular to axis O Y due to moment M sin $\theta$ in O X. Let $D_b$ be the deflection perpendicular to axis O $X_b$ due to moment M cos $\theta$ in O $Y_a$.

$$\text{Then} \quad D_a = \frac{\sin \theta}{EI_a} \iint M \, dx \, dx$$

$$\text{and} \quad D_b = \frac{\cos \theta}{EI_b} \iint M \, dx \, dx$$

$$\text{and} \quad D_y = \sqrt{D_a^2 + D_b^2}.$$

$D_y$ may thus be found.

If lateral deflection be prevented, then the formula can be directly applied, and the deflection $y$ will be

$$\frac{I}{EI_y} \iint M.dx.dx.$$

In the paper referred to above, the first series of experiments was made on frame girders, to determine the relative stiffness of the various forms of framing commonly used in ship construction.

The samples were made 9 ft. long and were supported freely on blocks leaving an unsupported span of 7 ft. They were then bent by weights hung at the centre of the span. Observations were made on the deflections horizontally and vertically for each increment of load, which was gradually applied until the frame failed. This series of tests was divided into two groups. The first group of specimens was made up of frame angle, frame and reverse frame angle, channel, bulb angle, and Z sections. The second group was made up of similar sections, but had in addition a horizontal plate 12 in. × ½ in. riveted to the top flange. The result in the latter case was to prevent

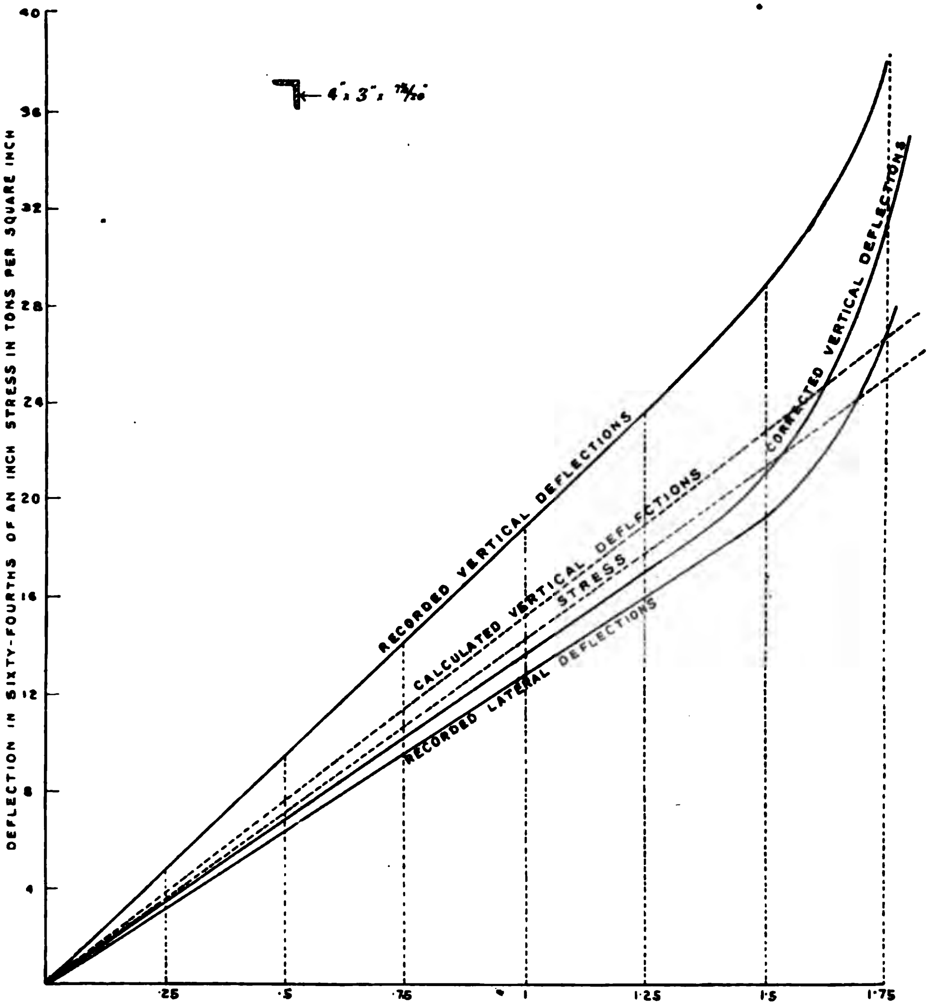


Fig. 234.

horizontal deflection almost entirely. Figs. 234 and 234A show the results, plotted in curves, of two of the tests. The corrected deflections are the deflections that would have taken place if the lateral deflection had been prevented.

**Application of Formula to the Case of a Vessel.**—In passing from the simple case of a uniform girder to a vessel we have to deal with a structure, the section of which varies along the length. It is necessary to construct a curve of moment of inertia. This can be done by calculating the value of I at definite intervals along the length, these intervals being chosen at each stage of the length where a change occurs in the scantlings, or where there is considerable change in the cross-sectional form. Such a curve is shown by *af* in fig. 235. This figure illustrates the method of calculating the deflection

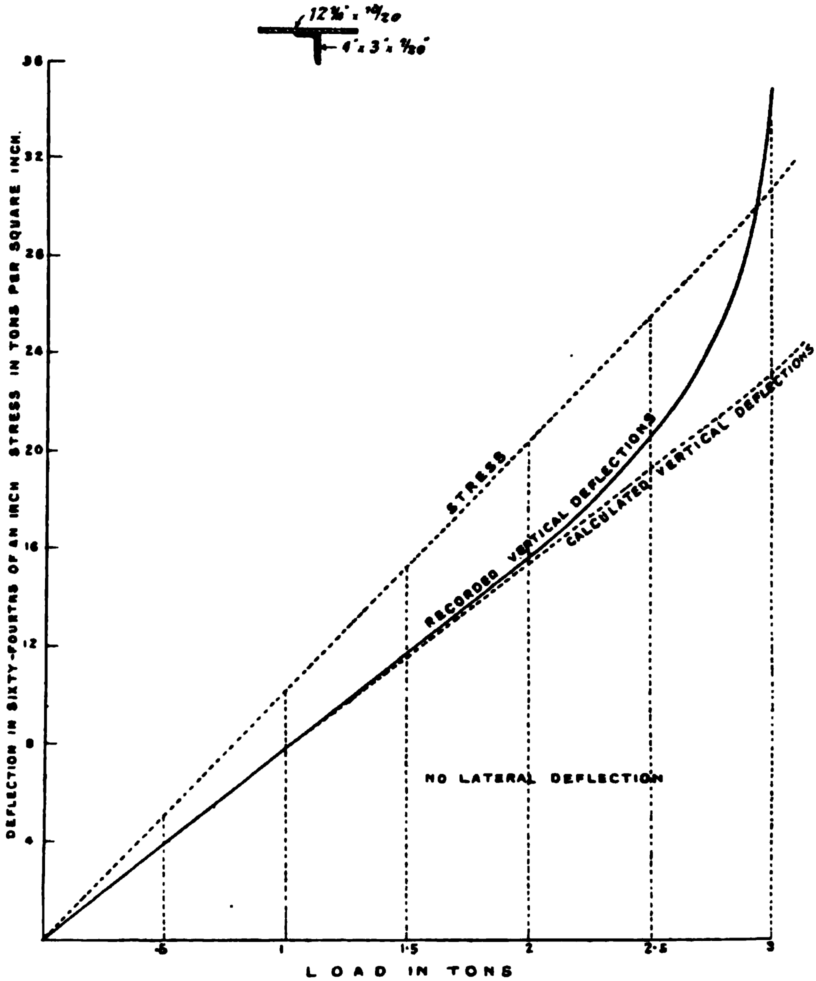


FIG. 234A.

of a vessel supposing it to meet the standard size of waves, *i.e.* waves of her own length. Suppose the vessel to pass from the condition of equilibrium on the wave crest to a similar condition in the wave hollow. The bending moment will change from hogging to sagging, and the moment causing deflection will be the change of bending moment between the two conditions. The deflection that we calculate is, therefore, the change in form that occurs from the hogging condition to the sagging. The bending-moment curves for these two conditions are drawn. The sum of the ordinates at any point gives

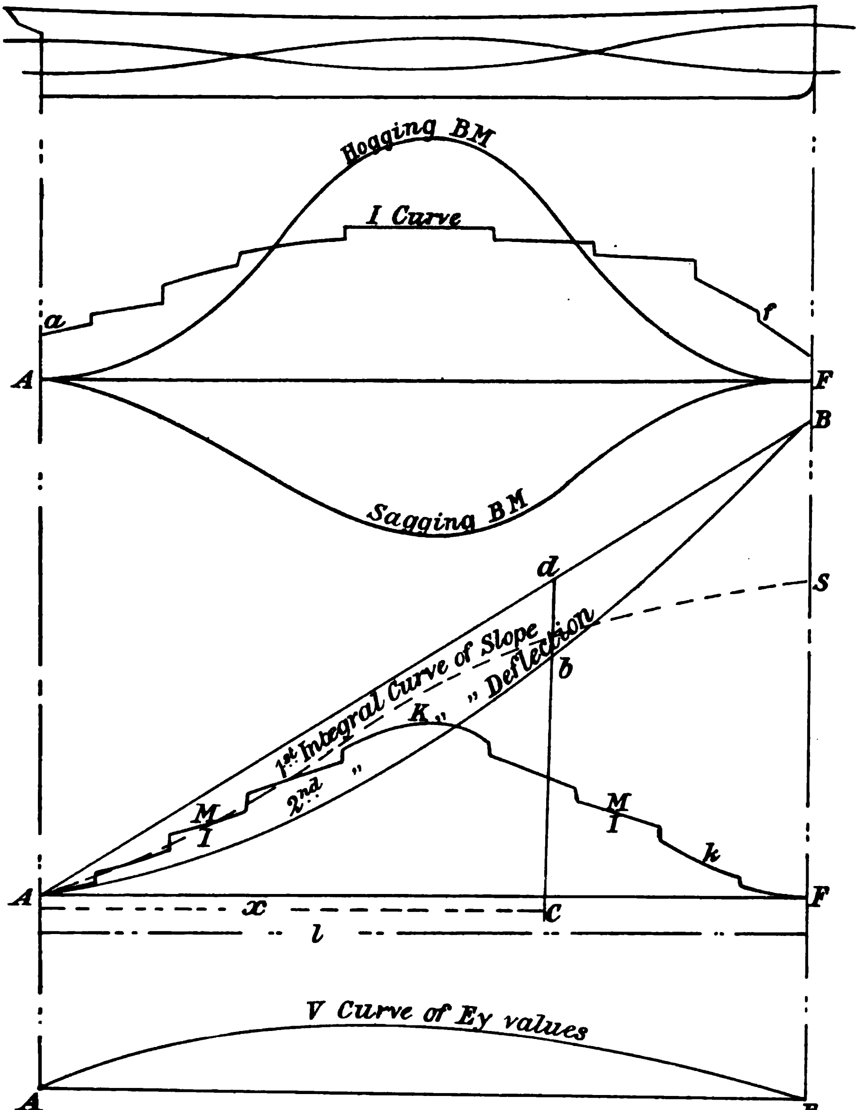


FIG. 235.

the change in bending moment at the corresponding section. We therefore divide this sum by the I of the section, and so get the ordinate for the curve of $\frac{M}{I}$. See the curve A $k$ F, fig. 235.

The equation to this curve A $k$ F is

$$\frac{M}{I} = E\frac{d^2y}{dx^2}.$$

On integrating this we get the curve of slope A S. The equation to A S is

$$E\frac{d^2y}{dx} = \int \frac{M}{I}dx + a$$

where $a$ is a constant.

The integration of O S gives the second integral of A K F. This is the curve A $b$ B, and the equation giving the deflection is

$$Ey = \int\int \frac{M}{I}dx.dx + ax + b.$$

If we consider the ends of the vessel to remain in the axis O X we can find the value of the constants $a$ and $b$.

When $x = 0$, $y = 0$ $\therefore$ $b = 0$.

When $x = l$, $y = 0$ $\therefore$ $a = -\dfrac{\int_0^l \int_0^l \frac{M}{I}dx.dx}{l}$

$\therefore$ The equation giving the deflection is

$$Ey = \int\int \frac{M}{I}dx.dx - \frac{x}{l}\int_0^l \int_0^l \frac{M}{I}dx.dx$$

$$= \int\int \frac{M}{I}dx.dx - \frac{x}{l}\cdot BF.$$

Any ordinate of A $d$ B, $cd = \frac{x}{l}\cdot BF.$

" " A $b$ B, $cb = \int\int \frac{M}{I}dx.dx.$

$\therefore$ $Ey = cb - cd = -bd.$

The deflection is, therefore, the vertical distance between the second integral curve and the straight line A B joining its ends. This curve can be better examined if it be set off a horizontal base. This has been done in fig. 235, A V F.

If we assume a value for E, say 10,000 tons, we can get the scale with which to measure the ordinates of the curve A V F in order to obtain the actual deflection in inches.

Actual values for E as determined by experiments on a ship are given in a later chapter, p. 413.

## CHAPTER XXX.

# CONSIDERATION OF THE STRENGTH OF THE DECK OF A VESSEL TO RESIST CRUSHING.

In the formula $\frac{p}{y}=\frac{M}{I}$, $p$ gives the compressive stress per unit of area at the point distant $y$ from the neutral axis. If we sum up all the stresses acting on the area of a deck or the flat of the outer bottom we shall get a large compressive force acting upon a considerable extent of thin plating. Unless this thin plating is capable of resisting this force without buckling, the assumed conditions upon which the formula is based do not hold and the formula cannot be applied. No such consideration applies in plating under tension. It becomes necessary therefore to consider the question of crushing by buckling, especially in the critical cases of thin decks under severe compressive stresses.

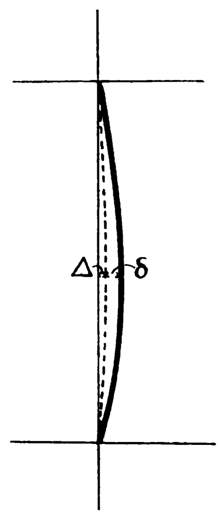


Fig. 236.

The crushing strength of a material is the compressive load per square inch of section that would fracture the material. When the specimen or strut is long so that the ratio of the length (the direction in which the load is applied) to the moment of inertia of the cross section is above a certain limit, the strut is liable to give way first by buckling. Tests to determine the crushing strength are therefore carried out on specimens whose length is small compared with the cross-sectional dimensions. The load that will buckle a strut will be less than the crushing strength of the material. We will examine a formula which gives the buckling load of a strut. It was derived by Professor T. C. Fidler.

In a long strut or column which has been deflected by an applied compressive load at the ends which are assumed to be rounded, when the load reaches a certain amount R a considerable increase of deflection will take place for practically no increase of load,

$$R=\frac{\pi^2 IE}{L^2} \text{ (Euler's formula)},$$

where R = force bending the column, called the resilient force,

L = length of column,

E = modulus of elasticity of the material,

and I = the minimum moment of inertia of the cross-sectional area about a diameter through the C.G.

Let A = cross-sectional area of column in square inches.

Then $\frac{R}{A} = \rho$ resilient force per square inch of areas.

$\therefore \quad \rho = \pi^2 E \left(\frac{r}{L}\right)^2$ where $r$ = minimum radius of gyration of cross section about a diameter.

It is practically impossible for the strut to be absolutely straight.

Let the strut have a small initial deflection $\Delta$. Let $\delta$ be the additional deflection produced by the load P, which is less than R, but which, on account of the strut having an initial deflection with no load, is sufficient to bring it into the resilient condition (fig. 236).

$$\text{Then} \qquad \delta = \Delta . \frac{P}{R - P}.$$

Let $E_1$ be the modulus of elasticity of the inner fibre which is under compression.

Let $E_2$ be the modulus of elasticity of the outer fibre which is under tension.

Then the eccentricity of the neutral axis is

$$e = \frac{E_2 - E_1}{E_2 + E_1} \times r.$$

$$\therefore \quad \delta = \frac{\pi r}{2} \cdot \frac{E_2 - E_1}{E_2 + E_1} \cdot \frac{P}{R - P}.$$

The resilient force bends the strut and produces a maximum bending moment $M_0$, which acts at the centre of the strut,

$$M_0 = P\delta.$$

$$\therefore \quad \text{The maximum stress} \qquad \pm f_1 = \frac{M_0 y}{I} = \frac{P.\delta.y}{Ar^2} = \frac{p.\delta.y.}{r^2}$$

$$\text{where} \qquad p = \frac{P}{A}.$$

$$\therefore \qquad \pm f_1 = \frac{py\pi}{2r} \cdot \frac{E_2 - E_1}{E_2 + E_1} \cdot \frac{P}{R - P}$$

$$= \frac{\pi}{2} \cdot \frac{y}{r} \cdot \frac{E_2 - E_1}{E_2 + E_1} \cdot \frac{p^2}{\rho - p}$$

$$= \frac{\phi p^2}{\rho - p} \quad \text{where} \quad \phi = \frac{\pi}{2} \cdot \frac{y}{r} \cdot \frac{E_2 - E_1}{E_2 + E_1}.$$

∴ Total compressive stress on concave side,

$$f = p + f_1$$
$$= p\left(1 + \frac{\phi p}{\rho - p}\right).$$

Solving this equation for $p$,

$$p = \frac{\rho + f - \sqrt{(\rho + f)^2 - 4f\rho(1 - \phi)}}{2(1 - \phi)} \quad . \quad . \quad . \quad . \quad (1)$$

From a large number of experiments the results seem to show that the value of $\frac{E_2 - E_1}{E_2 + E_1}$ may be taken as ·117.

$$\therefore \qquad \phi = \cdot 117 . \frac{\pi}{2} . \frac{y}{r}.$$

This formula, therefore, gives the load which would just produce the crushing stress $f$ on the extreme fibre under the deflection due to an initial eccentricity $e = \cdot 117 r$.

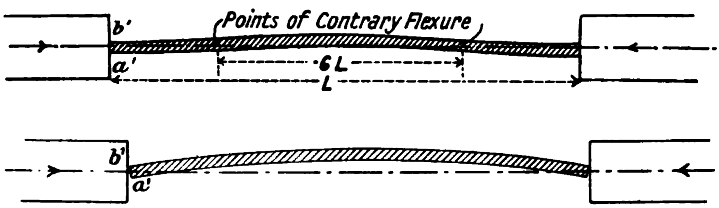


FIG. 237.

In the case of struts with fixed ends, or short struts in comparison with the value of $r$ with the ends flush, the above formula, No. 1, is modified by taking the free length for flexure in one direction at ·6L, so that $\rho$ becomes equal to $\pi^2 E\left(\frac{r}{\cdot 6L}\right)^2$. This is shown in the upper case in fig. 237.

Calling this value $\rho_1$, the formula for a strut with fixed ends is

$$p = \frac{\rho_1 + f - \sqrt{(\rho_1 + f)^2 - 4\rho_1 f(1 - \phi)}}{2(1 - \phi)} \quad . \quad . \quad . \quad (2)$$

If the material is weak in tensile strength, and only capable of bearing a small tensile stress $f_1$, it may give way on the convex side; for this case the formula is

$$p = \frac{\rho_1 - f_1 + \sqrt{(\rho_1 - f_1)^2 + 4\rho_1 f_1(1 + \phi)}}{2(1 + \phi)} \quad . \quad . \quad . \quad (3)$$

When tests are made with struts which are neither pointed at the ends nor fixed, but only bearing with squared ends, they may come under (3) equation. The behaviour of a strut like this would be as represented in the figures 237. The strut would at first bend as in the upper figure, similarly to a strut with fixed ends. At a certain point, as soon as the line of pressure, shown dotted, passes the edge so as to produce the slightest tensile stress at $a'$, it would suddenly spring open at $a'$, turning about the corner $b'$ (lower figure). At this point the tensile stress is zero. This kind of buckling will most readily take place in thin struts. Accordingly, putting $f_1=0$ in the last formula, No. 3, we have

$$p=\frac{\rho_1}{1+\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $$\rho_1=\pi^2 E\left(\frac{r}{\cdot 6L}\right)^2$$

and $$\phi=\cdot 117\frac{\pi}{2}\cdot\frac{y}{r}$$

These formulæ (1), (2), (3), and (4) have been applied to find theoretically the strength of several specimens for comparison with the actual results of the tests upon plates which have yielded by buckling under compression. Applying formula No. 4, the following are results of a series of tests on thin plating :—

TABLE LXXI.

| Test No. | Description. | Crushing Load (Tons). | |
|---|---|---|---|
| | | By Formula. | By Experiment. |
| 1 | 9 lb.[1] M.S. plate 21″ × 16″ wide covered with corticine . . . | 9·1 | 9·0 |
| 2 | Same as above, but 6″ hole cut out of centre . . . | 5·72 | 7·0 |
| 3 | 6 lb.[2] H.T. plate 21″ × 12″ wide. . . | 1·09 | 1·04 |
| 4 | 8 lb.[2] H.T. plate 21″ × 12″ wide. . . | 4·17 | 4·35 |
| 5 | 10 lb.[2] H.T. plate 21″ × 12″ wide . . | 5·4 | 5·15 |

[1] Mild steel. [2] High-tension steel.

E in all cases is taken as 29,000,000 lb.

Applying formula No. 1 for pointed ends, the results are very low in comparison with the experimental results, thus showing that the strength of thin plates is largely due to the support given by the squared ends.

Table LXXII. gives the results of tests made on corrugated plates. Theoretical results have been worked out by means of formulæ Nos. 1, 2, and 4 :—

No. 1 for pointed ends.
No. 2 ,, fixed ends.
No. 4 ,, squared ends.

Certain assumptions as to the shape of the corrugation have been made in working out the value of *r*, but as the shape of the corrugation affects the value of *r* very little, these need not be noted.

Formula No. 1 for pointed ends gives, on the average, the closest agreement with the actual results. This would seem to indicate that the corrugated plates do not depend much on the squareness of their ends for extra strength. It may therefore be deduced that the squareness of the ends adds largely to the strength of very thin plates or plates very little stiffened, but in the case of plates made stiffer by corrugation to the plate, the support of the squared ends does not seem to give much extra strength, at least not as much as is indicated by the formula, deduced on the assumption that the ends are pointed.

TABLE LXXII.

Crushing load in tons per square inch.

| | By Formula No. 1, Pointed Ends. | By Formula No. 2, Fixed Ends. | By Formula No. 4, Squared Ends. | Experiment. |
|---|---|---|---|---|
| 6 lb. H.T. plate 21″ × 12″ wide, corrugations 2″ apart and ¼″ high . . . | 3·16 | 9·1 | 6·58 | 3·24 |
| 6 lb. H.T. plate 21″ × 12″ wide, corrugations 2″ apart and ⅜″ high . . . | 5·59 | 14·2 | 12·61 | 9·93 |
| 6 lb. H.T. plate 21″ × 12″ wide, corrugations 9″ apart and ½″ high . . . | 5·33 | 14·0 | 10·55 | 7·25 |
| 8 lb. H.T. plate 21″ × 12″ wide, corrugations 2″ apart and ¼″ high . . . | 4·00 | 11·1 | 8·55 | 5·41 |
| 8 lb. H.T. plate 21″ × 12″ wide, corrugations 2″ apart and ⅜″ high . . . | 6·41 | 15·6 | 14·82 | 6·40 |
| 8 lb. H.T. plate 21″ × 21″ wide, corrugations 9″ apart and ½″ high . . . | 6·25 | 15·2 | 12·48 | 7·63 |

The next table, LXXIII., shows the comparison of another series of tests. The specimens in this case were each made up of a 9 lb. steel plate

12 in. wide, which was stiffened longitudinally by two H.T. steel angle bars $1\frac{3}{4}$ in. × $1\frac{3}{8}$ in. × $1\frac{1}{2}$ lb. Six different lengths of specimens were tested, and two similar sets of experiments were made, one set with the plate of mild steel and the other with the plate of H.T. steel.

TABLE LXXIII.

Crushing load in tons per square inch.

| Length. | By Formula No. 2, Fixed Ends. | By Formula No. 1, Pointed Ends. | Experiment Mild Steel. | Experiment H.T. Steel. |
|---|---|---|---|---|
| 6″ | 21·25 | 21·0 | 20·0 | 22·61 |
| 12″ | 20·75 | 19·2 | 18·8 | 17·33 |
| 18″ | 19·50 | 16·5 | 18·8 | 19·8 |
| 24″ | 18·25 | 14·0 | 13·9 | 17·44 |
| 30″ | 16·50 | 11·5 | 12·24 | 17·34 |
| 36″ | 15·00 | 9·5 | 11·5 | 15·37 |

The formula for fixed ends seems to give a result agreeing fairly closely with the results of experiments on H.T. steel. It is interesting to note that it is only in the long specimens, or specimens in which the proportion of 1 to $r$ is great, that the results obtained by formulæ Nos. 1 and 2 differ widely.

As the proportion of 1 to $r$ is made smaller, the value of the crushing load approaches more nearly to the maximum value of $f$ for pure crushing, viz. 21·5, and in the case of the specimen 6 in. long this maximum value is practically reached. The experiment shows in the high-tension steel 6-in. specimen a crushing strength of 22·61 tons per square inch. Probably if 23 had been used as the value of $f$ in the formulæ, the results would have been more comparable with the H.T. steel specimens.

Table LXXIV. gives the actual results of another series of tests made on specimens stiffened by beams riveted to the plates crosswise and by angles, and a deep girder riveted to the plates lengthwise. The results as worked out by formulæ give far too high a result. The results by the formulæ Nos. 1 and 2 range from 16 tons per square inch to 20 tons per square inch. The best of the results as obtained by experiment only average about 10 or 11 tons per square inch. This would seem to be due partly to the presence of the beams and partly to the thinness of the plate and girder together with the breadth of plate and depth of girder.

Table LXXIV. gives the values of $a$ and $r^2$, and also shows the actual results at a glance. Coefficients have been worked out for each test, giving the ratio of total crushing load to the product $ar^2$.

Generally, the boxing of the angles round the beams gives a slightly higher result, although there are a few cases where the result is lower.

TABLE LXXIV.

| Specimen. Width. | Specimen. Lb./Ft. | Calculated Value $r^2$ Sq. In. | Calculated Value $a$ Sq. In. | Value $ar^2$. | Crushing Loads. Unboxed Angles. Stiffening *Short*. | Crushing Loads. Boxed Angles. Stiffening *Short*. | Crushing Loads. Unboxed Angles. Stiffening Flush. | Crushing Loads. Boxed Angles. Stiffening Flush. | Ratio of $\frac{\text{Crushing Loads}}{ar^2}$. Unboxed Angles. Stiffening *Short*. | Ratio of $\frac{\text{Crushing Loads}}{ar^2}$. Boxed Angles. Stiffening *Short*. | Ratio of $\frac{\text{Crushing Loads}}{ar^2}$. Unboxed Angles. Stiffening Flush. | Ratio of $\frac{\text{Crushing Loads}}{ar^2}$. Boxed Angles. Stiffening Flush. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 18″ | 6 | 9·24 | 5·77 | 53·3 | 31·7 | 36·83 | 72·32 | 60·0 | ·595 | ·691 | 1·36 | 1·13 |
| 27″ | 6 | 8·23 | 7·09 | 58·3 | 23·66 | 37·05 | 71·07 | 69·78 | ·405 | ·635 | 1·22 | 1·20 |
| 36″ | 6 | 7·35 | 8·42 | 61·9 | 24·1 | 38·95 | ... | 56·25 | ·394 | ·636 | ... | ·919 |
| 18″ | 8 | 8·61 | 6·65 | 57·3 | 43·75 | 36·83 | 82·72 | 70·4 | ·764 | ·643 | 1·44 | 1·23 |
| 27″ | 8 | 7·40 | 8·42 | 62·3 | 41·07 | 43·53 | 84·6 | 78·8 | ·659 | ·699 | 1·36 | 1·26 |
| 36″ | 8 | 6·44 | 10·18 | 65·6 | 36·83 | 50·0 | 76·56 | 80·0 | ·562 | ·762 | 1·17 | 1·22 |
| 18″ | 10 | 7·84 | 7·79 | 61·1 | 56·25 | 65·0 | over 100 | 98·21 | ·921 | 1·06 | over 1·63 | 1·61 |
| 27″ | 10 | 6·58 | 10·00 | 65·8 | 60·94 | 59·17 | 93·3 | 100 | ·926 | ·899 | 1·42 | 1·52 |
| 36″ | 10 | 5·64 | 12·20 | 68·8 | 60·0 | 60·0 | 98·21 | 96·87 | ·872 | ·872 | 1·43 | 1·41 |

The effect of making the stiffening flush as compared with cutting the stiffening short of the plate is to give a much higher result. The strength of the specimens with the stiffening short is only about 50 per cent. in the 6 and 8 lb. plates, and 60 per cent. in the 10 lb. plate specimens, of the strength of the specimens with the stiffening flush. When the stiffening is cut short, its only use is to prevent buckling of the plate; when the stiffening is flush, it takes up directly a part of the stress and also prevents buckling of the plate. This is of importance in the consideration of the strength of thinly plated decks when they are cut up by large openings. The coamings and casings round these openings cannot take up directly the compressive stress, and are therefore only of use to prevent local buckling of the deck plating.

Diagram fig. 238 has been prepared, which gives the sectional views of the decks of two small vessels. The strength of a deck such as is shown in either of the figures can be estimated in the following way. From the bending moment and consequent stress calculated, it is possible to find the total thrust brought upon the deck. To find the capability to resist this total thrust, assume that the length taken is a frame space. This assumes that the beams and frames do not shift out of their plane relatively to each other. The deck or portion of the deck of this length can be considered like the stiffened specimens of Table LXXIV., the values of $a$ and $r^2$ can be found. The most important question in this connection is the amount of stiffening that is afforded by the sheerstrakes and by the bunker fore and aft bulkhead.

When these parts are not taken into account, the deck plating has only the stiffening of the girders and the round of beam to depend on for strength.

In making a comparison with the specimens of Table LXXIV., those under the heading of "unboxed angles and stiffening flush" correspond to the conditions of crushing in the deck under consideration. The angles connecting the stiffening of the deck are intercostal, and the parts below the beams are continuous, thus being able to take up their full proportion of the stress.

In the case of No. 1, fig. 238, the deck amidships is very much broken up by the large openings. Openings occur at this section for the scuttle to the stokehold. As will be seen from the sectional view, there is a portion of the deck between the funnel opening and scuttle which might easily give way independently of any movement in the bunker bulkhead or

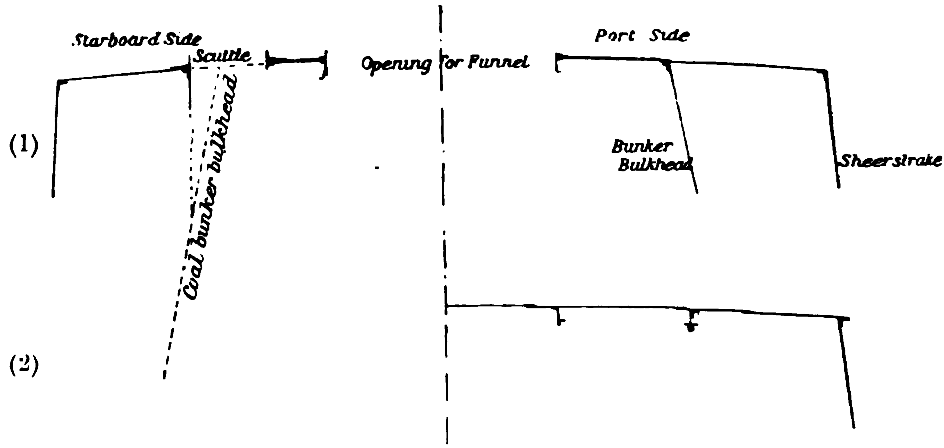


FIG. 238.

in the sheerstrake. The bunker bulkhead is worked round the scuttle on the starboard side, and being only of thin plating it is not likely that it can affect the strength of this intermediate portion. The remaining portion of the deck on this side consists of the stringer plate, which is stiffened on the outside by the gunwale angle and sheerstrake, and on the other by the bunker bulkhead and scuttle coaming.

Considering the strength of this section at the intermediate part,

$$\text{Area} = 9 \cdot 4 \text{ sq. in.}$$
$$r^2 = 3$$
$$ar^2 = 28 \cdot 2.$$

Assumed coefficient from table $= 1 \cdot 5$.
Crushing load in tons $= 42$.

This crushing load is deduced on the assumption that all the stiffening takes up the stress, but as the coaming only prevents buckling, we will take the crushing load for this part of the deck to be 40 *tons*,—*i.e.* this portion of the deck would give way at an average compressive stress of 4·3 tons per square inch, and if the compressive stress calculated from the bending moment is greater than 4·3 tons, this part will probably yield, and its area is then not available in the moment of inertia calculation.

The other portion of deck is the stringer plate, gunwale angle, and stiffening. The sheerstrake gives considerable support to the deck, but the formula used probably overestimates this support when a large depth of sheerstrake is taken into account, as, being a thin plate, it will probably buckle at some distance below the deck, and only a limited depth of it will thus be available. The values of $a$ and $r^2$ should be calculated for varying depths of sheerstrake, and the results plotted in the form of curves. Only about 4 in. of bunker bulkhead should be included, as all the stiffening on this side is only of use to prevent buckling. The value of the coefficient can be safely taken as 1·3 in this case.

If the amount of depth of sheerstrake which has to be taken into account in order to resist the total compressive force is a reasonable amount, the deck may be considered as strong enough. It is, however, evident that the deduction drawn is not very reliable, and it is therefore desirable to make quite sure that the girders stiffening the deck are sufficient to resist the total compressive force without relying upon a depth of sheerstrake much greater than the depth of the girders, say 1 foot.

Similar methods may be applied to larger ships, but until experiments upon thicker plates and angles in combination are made, it is difficult to draw reliable conclusions.

## CHAPTER XXXI.

# METHOD OF DETERMINING STRAINS IN A SHIP.

THE only known complete set of experiments to determine strains in a ship are here recorded.

A series of experiments was carried out on H.M.S. "Wolf" at Portsmouth Dockyard with the object of determining the following:—

(1) The strains which known longitudinal bending moments produced in the structure.

(2) The stresses corresponding to these strains.

This involved a determination of the modulus of elasticity of the structure.

(3) A calibration of the structure in relation to definite known bending moments, so that when the vessel at sea became subjected to an unknown bending moment which produced observable and measurable strains in the calibrated parts, the bending moments could be inferred from these indications.

To obtain the results, the vessel was subjected to—

1st. Hogging stresses, *i.e.* the stresses to which the vessel is subjected when supported near amidships.

2nd. Sagging stresses, *i.e.* the stresses to which the vessel is subjected when supported near the ends.

For the purpose of carrying out these experiments the vessel was placed in a dry dock, and the following method adopted for supporting her.

First, for hogging stresses: Two strongly constructed cradles of steel plates and angles supported by pillars, and resting on the floor of the dock, were built around the vessel amidships,—see figs. 239, 240, 241, and 242, Plates XIX. and XX. These cradles were each 6 ft. long and 26 ft. apart from centre to centre. Wood packing was fitted tightly between the cradle and the skin of the ship. This wood packing extended 2 ft. over the edge of the cradles at each end, and was gradually tapered off at the ends to make it slightly flexible, so that the support given did not end abruptly. The surface of the wood packing and the plating of the ship in way of it were thickly coated with grease to prevent the vessel from being held by the cradle when under deflection. A system of internal fortification, consisting of wood shores and packing, extending from the keel and outer or shell plating to the upper deck, was fitted in way of the cradles to give local support.

The observations in dock were made with the object of determining the following:—

(*a*) The actual strains in the structural material of the vessel when in hogging and sagging conditions.

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FRAME

(*b*) The actual deflection of the vessel in a longitudinal direction.

(*c*) The transverse change of form.

To obtain the first (*a*), Stromeyer's strain indicators were used, and from the observations of strain taken upon them the stresses were deduced by the method described later on.

Indications were taken at two cross sections of the vessel, viz.—

(1) In the engineroom about amidships; and (2) in the after stokehold, about station 84.

To obtain (*b*), a series of vertical battens was arranged, one at the bow, one at the stern, and nine on each side, spaced at about equal distances apart, and directly opposite each other along the ship's side. These battens were securely fastened and stayed in the dock, and extended well above the upper deck. They were quite free from the ship.

Horizontal battens supported on wood chocks securely fastened to the stringer plates were arranged directly against each vertical batten. These horizontal battens carried a sliding vernier arrangement, free to slide up or down over a graduated index on the vertical battens, so that the amount of deflection either upwards or downwards could be easily read off very accurately at each position.

To obtain (*c*), horizontal battens were placed across the ship at a sufficient height above the upper deck to allow the ship to float. Each one was connected to the pair of vertical battens on opposite sides of the ship. The distance of the deck from these horizontal battens was measured at intervals.

Means were also adopted so that any other changes of form could be easily observed. The vessel was kept in the proper longitudinal and transverse position by ropes, so that when the water was lowered in the dock sufficiently for her to rest in the cradles she came to the desired position, and then these cradles were the only means by which she was supported from the dock bottom.

Whilst floating, the draught marks forward, amidships, and aft, on both sides were observed, so that the actual displacement at any time of the experiment could be accurately calculated. The water was then gradually lowered in the dock, and by carefully observing the vernier readings on the vertical battens the vessel was brought to the same position as she occupied when supported on the ordinary docking blocks and shores before the water was let in. In this position she was a very little less than waterborne, and was at that stage in which any further lowering of the water in the dock would be the means of increasing materially the support from the cradles.

Whilst in this position initial readings were observed on the strain indicators and on the verniers, and the distances from the horizontal battens to the deck were measured. The water was then lowered 6 in. in the dock.

The readings of the strain indicators and verniers and the distances down from the horizontal battens to the deck were again noted. The water was then lowered another 6 in. and observations were again taken. This process was repeated until the water had completely left the ship and the vessel was wholly supported by the cradles. In this condition the vessel was allowed to remain for about two hours, during which period careful observations were taken of the effect of the continued stress on the structure of the vessel.

After this, the water-level was raised by steps of 6 in. and observations were made similar to those already described at the same water-levels as before. This operation of lowering and raising the water-levels was gone through three times for hogging and eight times for sagging.

In the series of experiments for hogging stresses the boilers and reserve feed tanks and coal bunkers were empty.

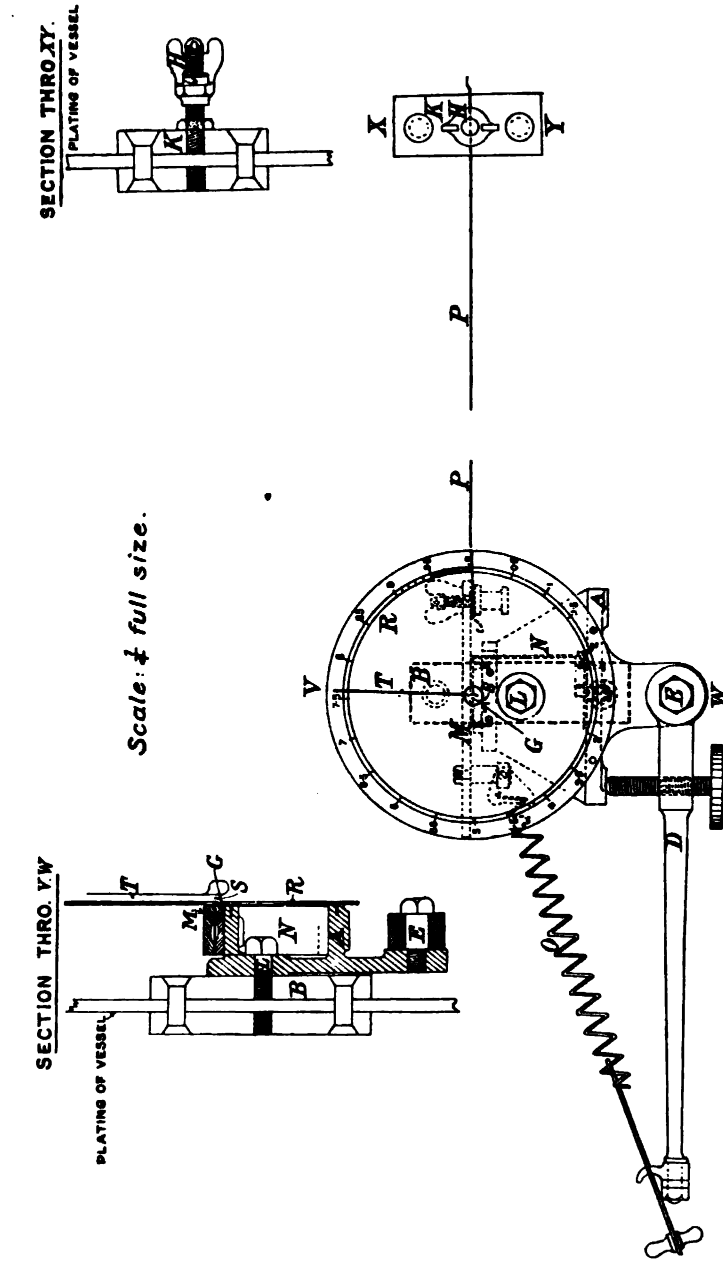



FIG. 243.—H.M. Torpedo-boat Destroyer "Wolf." Sketch showing Stromeyer's Strain Indicator and Method of Attachment to plating of vessel during experiments.

At the conclusion of the hogging series the vessel was floated, the caps of the ordinary docking blocks replaced, and the water pumped completely out of the dock. The vessel was then in a similar position to that which she occupied before the experiments were commenced, and it was found that she had returned to her original form.

The arrangements for the sagging series of experiments were precisely similar to those for the hogging series, with the exception that the cradles were built about 120 feet apart, one forward and the other aft, and the vessel was brought to correspond closely to her fully-laden condition by filling the bunkers with coal or equivalent weight. The experiments made and the observations taken were of a similar nature in every respect to those described for the hogging series.

Stromeyer's strain indicators were used during the experiments to ascertain the strains coming on the various parts of the structure. A sketch of one is given in fig. 243. It is an instrument which, when either tensile or compressive stresses are set up in the material of a structure to which it is attached, indicates by the readings on a dial what amount of strain takes place in the material.

The instrument consists of two plates, with accurate plane faces which slide over each other. Between the faces is placed a very small cylindrical steel rolling pin, to which a light straw indicator is attached. When the plates slide over each other they cause the rolling pin to rotate, and by this means the straw indicator is made to revolve round a graduated dial plate. One complete revolution of the rolling pin, causing the index pointer to make one complete revolution on the dial plate, corresponded to an extension or compression of about $\frac{1}{16}$th of an inch; as the dial is subdivided into $\frac{1}{1000}$th of its circumference, a high degree of sensitiveness exists.

Fig. 243 shows the instrument as fixed during the experiments. In the experiments made on the "Wolf," the distance over which extension or compression was measured was 20 in.—the spacing of the frames of the vessels.

It is evident that when a stress comes upon the material, causing it to elongate, the distance between the bolt L and thumb-screw H increases, thereby causing the plate M to slide on the plate G, by which means the rolling pin S is made to rotate, and by means of the strain indicator the exact amount of rotation can be read from the graduated dial-plate.

With these data it is possible to ascertain the actual strain in the material, while $p$, the stress, can be determined by calculation. These two quantities being known, from the formula $p = E\alpha$ the modulus E can be determined.

Table LXXV. shows the number of times indications were taken at the same positions during the experiments, also the number of resulting curves from which the mean stress curve for each position was obtained.

The following is a description of the method adopted to obtain actual stress from observations of the strain indicators.

Two indicators, one on each side of the plate, were generally used for each position.

The positions of the indicators during the experiments are shown in fig. 245. The observations that were taken were dial readings which were noted at every approximately 6-in. change of draught. The dial readings of each pair of indicators were set up as ordinates from a base line of draughts of water at the ship amidships, and curves were drawn through the points so obtained. By this method any obvious error in the readings was eliminated. The means of all the pairs of dial readings were set up, and a mean of means

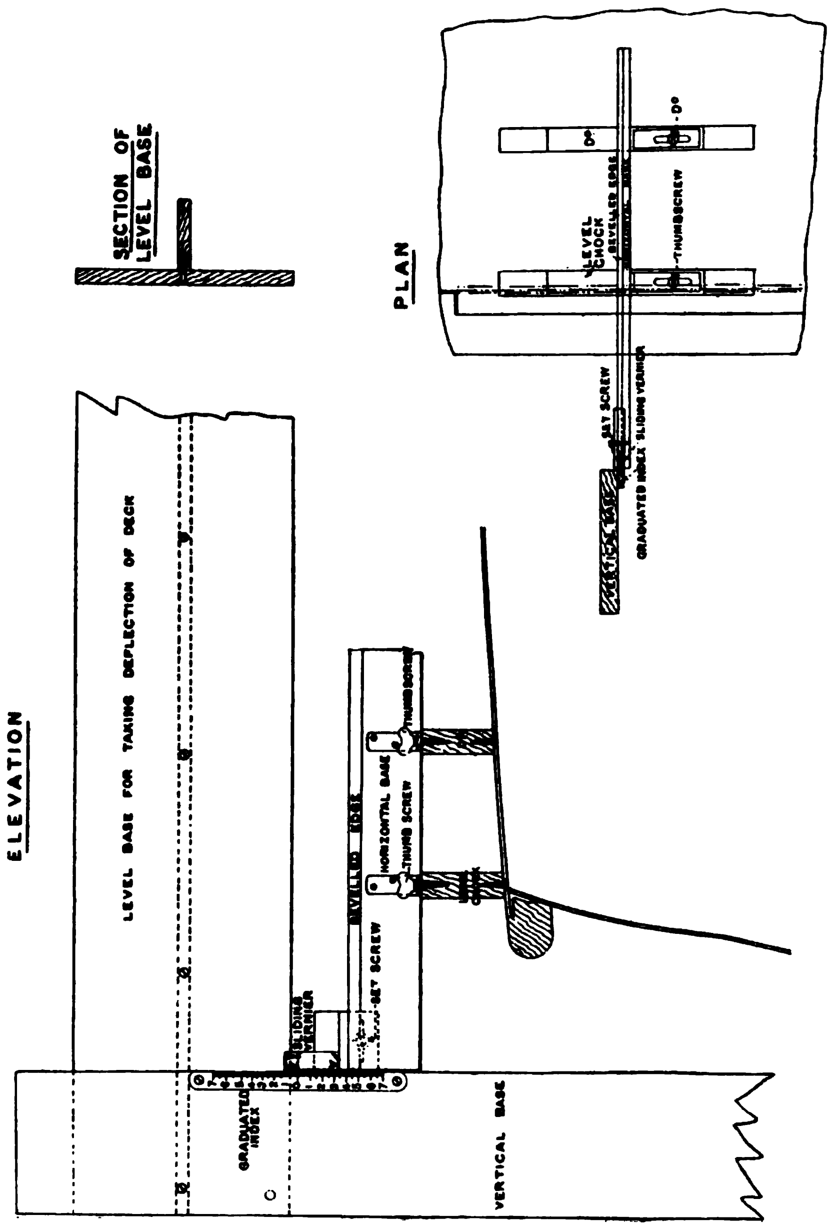



Fig. 244.—Vernier Arrangement for Measuring the Deflection at ship's side. Scale, $\frac{1}{6}$th full size.

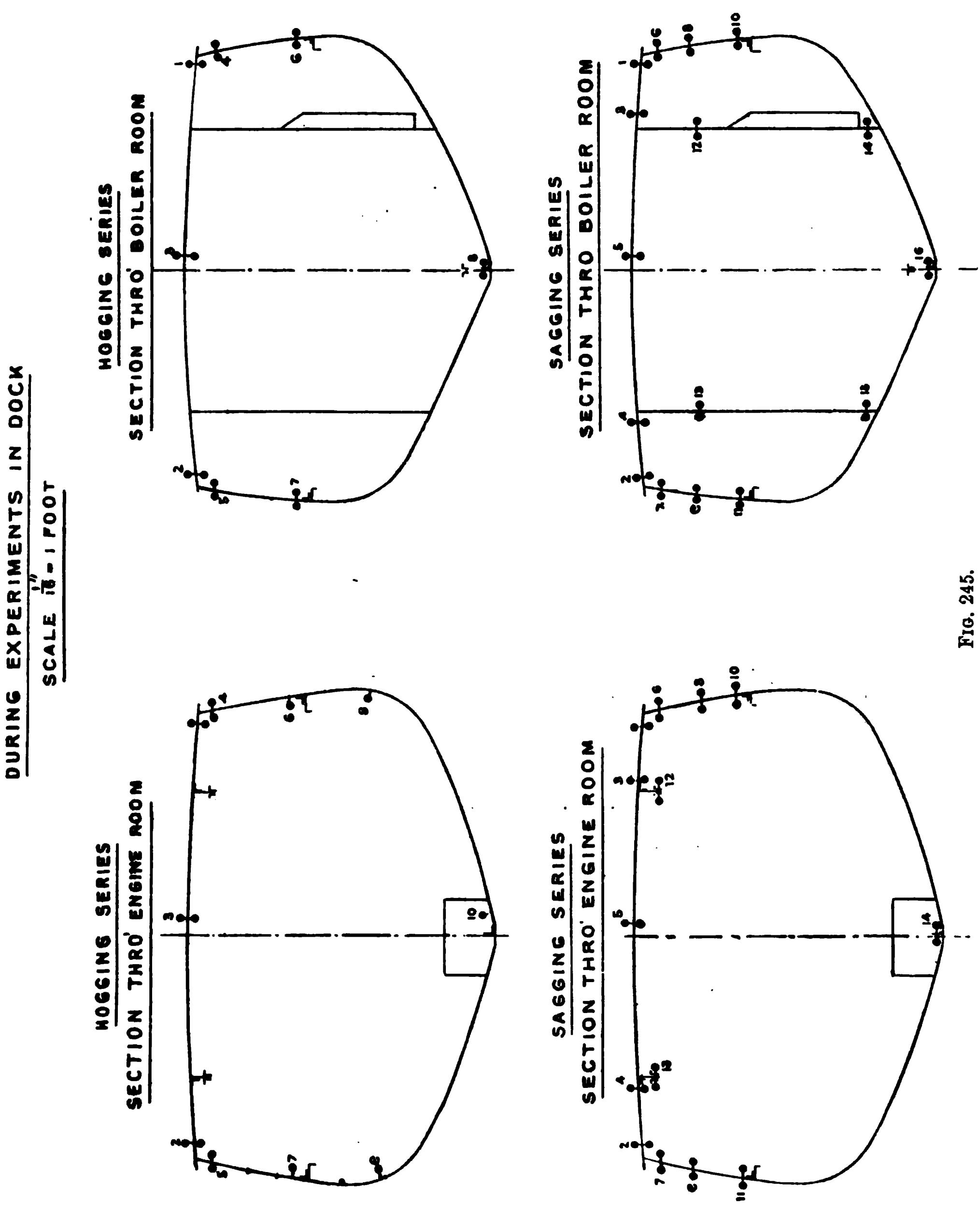

DURING EXPERIMENTS IN DOCK
SCALE 1/16" = 1 FOOT
HOGGING SERIES
SECTION THRO' ENGINE ROOM
HOGGING SERIES
SECTION THRO' BOILER ROOM
SAGGING SERIES
SECTION THRO' ENGINE ROOM
SAGGING SERIES
SECTION THRO' BOILER ROOM

FIG. 245.

determined from which the stresses were deduced. Specimens of these curves are given in figs. 262 to 268.

TABLE LXXV.

| Position. | | After Boiler-room. Number of Times Indications taken. | After Boiler-room. Number of Curves from which Mean Curve was obtained. | Engineroom. Number of Times Indications taken. | Engineroom. Number of Curves from which Mean Curve was obtained. |
|---|---|---|---|---|---|
| On deck, at middle line | | 5 | 5 | 2 | 2 |
| ,, near middle line (port) | | 5 | 4 | 2 | ... |
| ,, stringer (starboard) | | 5 | 3 | 3 | ... |
| ,, over bulkhead or girder | port | 4 | 4 | 2 | 2 |
| | starboard | 8 | 8 | 2 | 2 |
| On sheerstrake | port | 12 | 12 | 4 | 4 |
| | starboard | 12 | 12 | 4 | 2 |
| On side, at one sixth depth of ship down from upper deck | port | 4 | 4 | 2 | 2 |
| | starboard | 4 | 4 | 2 | 2 |
| On side, at one-third depth of ship down from upper deck | port | 4 | 3 | 2 | 2 |
| | starboard | 4 | 2 | 2 | 2 |
| Fore and aft girder under deck | port | ... | ... | 2 | 2 |
| | starboard | ... | ... | 4 | 4 |
| Top of bunker bulkhead | port | 3 | 4 | ... | ... |
| | starboard | 8 | 4 | ... | ... |
| Bottom of bunker bulkhead | port | 8 | 8 | ... | ... |
| | starboard | 8 | 2 | ... | ... |
| Vertical keel | | 12 | 12 | 5 | 5 |

*Note.*—For the hogging series, one set of indications only was taken for each position.

The mean of the strains of the two indicators for any one position gave the actual strain at that position for the corresponding draught of water.

It will be seen from Chapter XIX. that the following statements are assumed to be true of a homogeneous girder when subjected to a bending moment:—

(1) The neutral axis is a horizontal straight line passing through the C.G. of the area of the cross section of the material which is subjected to longitudinal stress.

(2) The stresses are proportional to the distance from the neutral axis, *i.e.* $\frac{p}{y}$ is constant over a transverse section.

(3) Considering the tensile and compressive stresses as respectively positive and negative forces acting on the cross section, the algebraic sum of the forces must be zero; and the sum of the moments of the stresses about the neutral axis must equal the bending moment upon the cross section if the girder remains at rest.

Underlying these assumed truths is another, viz. that the modulus of elasticity (the relation between stress and strain) is the same in all parts of the structure.

In the formula $\frac{p}{y}=\frac{M}{I}$ we see that if we know $y$, I, and M, $p$ can be determined.

The relation between stress and strain can be obtained from the formula $p=E\alpha$, where $\alpha$ is the ratio of the stressed length to the original length and E is the modulus of elasticity. If we can measure $\alpha$ at various points of a section of a structure we can plot a curve showing the ratio $\frac{p}{E}$. If E is constant and $p$ varies as the distance from the neutral axis, then this curve of $\alpha$ will be a straight line. Observations of $\alpha$, therefore, are necessary to obtain a direct check on the validity of the theory.

If, in addition to measuring the strain, we measure the deflection of the girder throughout, we can, by a simple graphic process similar to that explained in Chapter XXVII., determine the value of E from point to point throughout the length of the vessel. This value of E is the mean modulus of elasticity of a section. If E so obtained throughout the length of the structure is constant it will be a fair assumption that the modulus throughout the section is also constant. It is obvious that no correct conclusions as to the validity of the theory of strength of structures can be reached without a full knowledge of the value of E.

Two sets of experiments were carried out by supporting the vessel on two specially built transverse cradles. See figs. 239 to 242, Plates XIX. and XX. The spacing of the cradles was such as to produce (1) a hogging moment and (2) a sagging moment slightly greater (when the vessel was entirely supported by the cradles) than the moments obtained by the standard hogging wave calculation and the standard sagging wave calculation. See fig. 254.

When the vessel began to rest on the cradles, a gradual lowering of the water in the dock gradually increased the bending moment.

The bending-moment curves, not only for the ship entirely supported by the cradles but also for those conditions in which the vessel was partially supported by the water and partially by the cradles, were made at stages of support corresponding to each foot of depth of water pumped out of the dock, starting from the waterline at which the ship just floated.

From these bending-moment curves the maximum stresses on the section subjected to the maximum bending moment were obtained by applying the formula $\frac{p}{y}=\frac{M}{I}$ and were plotted in terms of draught of water.

The maximum shearing force was determined and the maximum shearing stress at the section where the maximum shearing force occurred was obtained from the formula $q=\frac{FA\bar{y}}{bI}$.

*Description of Method of obtaining the Bending-Moment Curves of the Vessel when on Cradles.*—Seven buoyancy curves were made for the sagging condition of loading, one for each foot of draught from 0 to 7 ft. (the load waterline).

The bending moment in the still-water floating condition was almost zero.

Most of the derived curves necessary for carrying out the work were made by the integraph.

The buoyancy curves were obtained from the body plan of integral sections. They are shown in fig.[1] 246 at every foot of draught above the keel. The weight curve of the vessel is also shown in the same figure.

Fig.[1] 247 is a series of load curves, constructed from the previous figure by plotting the differences between the ordinates of each buoyancy curve and the weight curve for the sagging experiments. As the water is lowered in the dock the aggregate net excess of weight over buoyancy will continually increase, and the load to be supported by the cradles will increase correspondingly. At each of the seven stages of the draught this was calculated, so that there are thus seven load curves in this figure. The load curve when the vessel is entirely clear of water is the weight curve with the supporting forces on the cradles drawn in negatively.

The algebraic sum of the areas of the load curves represents the total amount of support given by the cradles. Having the area, and also the longitudinal position of the C.G. of a certain load curve, the reactions of the cradles are inversely proportional to their distance from the C.G. so found, and their sum equals, or is represented by, the area of the curve. The dotted lines in fig.[1] 248 are the integrals of the load curves.

The final ordinate of any one of these curves represents the area of the corresponding load curve. To get the shearing-force curve it is necessary to incorporate the reactions of the cradles. Let the dotted line in fig. 249 represent the integral $a'\,a$ of a load curve on base line $a\,b$. Let $g$ be position of C.G. of this load curve.

Then the form of the shearing-force curve is obtained by dividing $a\,a'$ the final ordinate in the proportion of $\frac{ga}{gb}$ and bringing down the curve $a'\,b$ parallel to its base from the integrated curve as shown. To get the form of the shearing-force curve at the cradles, the line is drawn as shown in fig. 249 at $b\,b'$. The curves of shearing force are shown in full lines on the upper figure, and also in fig.[1] 248.

These curves of fig. 248 were then integrated by the integraph, and the resulting bending-moment curves were obtained; these are shown on fig.[2] 250.

Similar diagrams were prepared for the vessel in the hogging condition at the load draught of about 6 ft.

Fig.[2] 251 is the load curve similar to those of fig. 246.

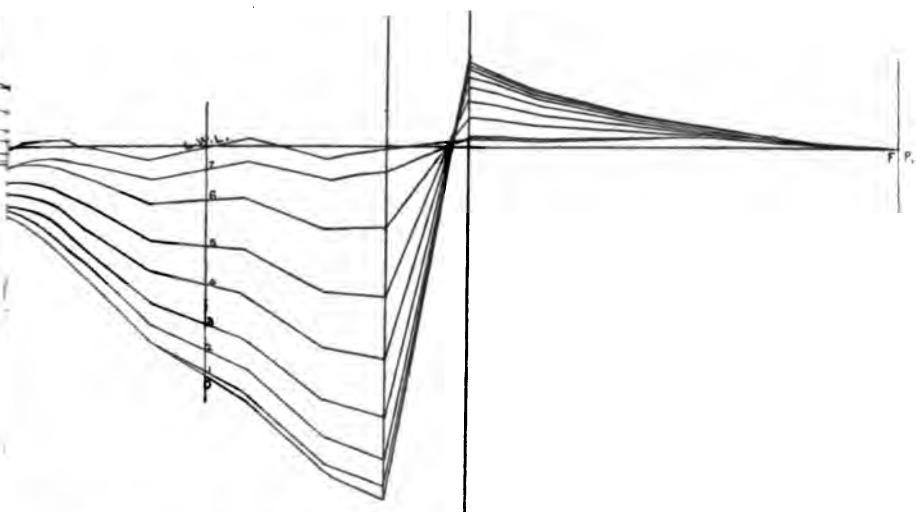
Fig.[2] 252 is the shearing forces, and fig.[2] 253 the curves of bending moments.

The spacing of cradles was chosen with regard to the practicability of stiffening the structure in way of the cradles by means of internal fortification, and from the results of these calculations it was seen to give a bending moment in both the hogging and sagging experiments more severe than that under standard conditions. The cradles were spaced 120 ft. apart for sagging and 26 ft. apart for hogging.

Fig. 254 shows the position of blocks as arranged for the actual carrying out of the experiments.

From the fig.[2] 250 a bending-moment curve at the weakest section amidships, in terms of draught of water, was constructed, and maximum stress curves were deduced. These stress curves gave the variation of the maximum stresses that would come upon the structure while undergoing the sagging experiments. From fig. 253 three cross curves of bending moment were made for each of the three positions—one in the region of each cradle and one at the weakest section amidships. For each of these bending-moment curves the corresponding stress curves were made (fig. 255, Plate XXIII.).

[1] Plate XXI. [2] Plate XXII.




Obtained by integrating the load curves in fig. **247.**

= ft.
= load = tons.

[*To face page* **406.**

The values of the maximum shearing forces and stresses are also given on fig. 255, Plate XXIII.

In both the cases of hogging and sagging, the shearing force is a maximum only at the outer edges of cradles. It was assumed that the worst possible

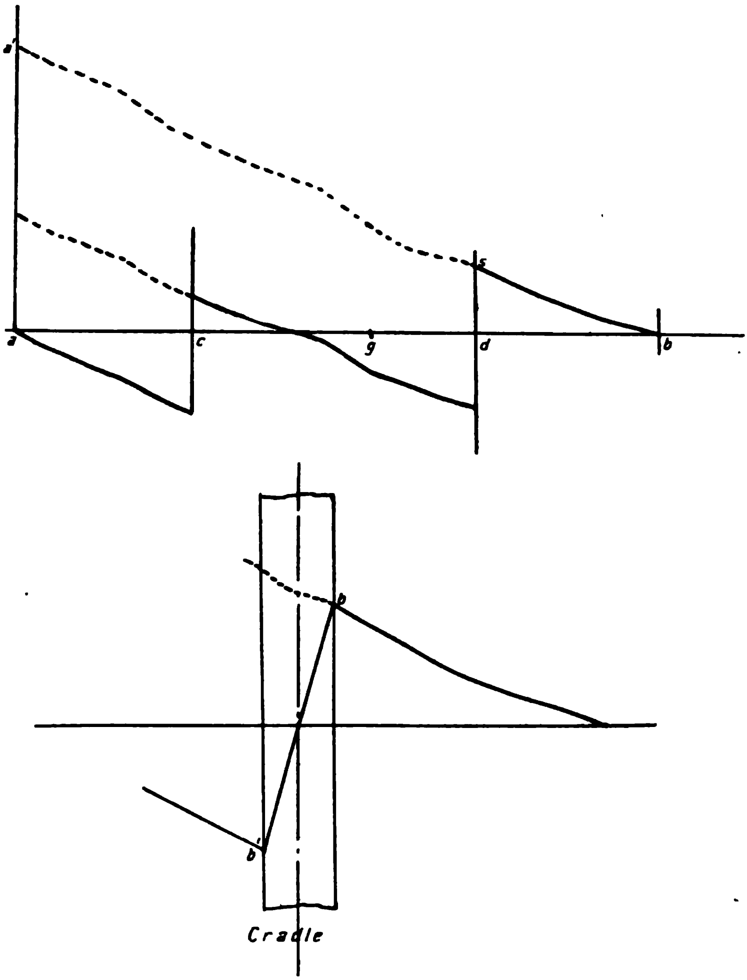



FIG. 249.

case was at the edge of a cradle, where the chocks end abruptly, and where there is no internal stiffening. Let F equal the total shearing force at that section, then $q$, the maximum shearing stress, is given by

$$q = \frac{FA\bar{y}}{bI}$$

which has been already referred to.

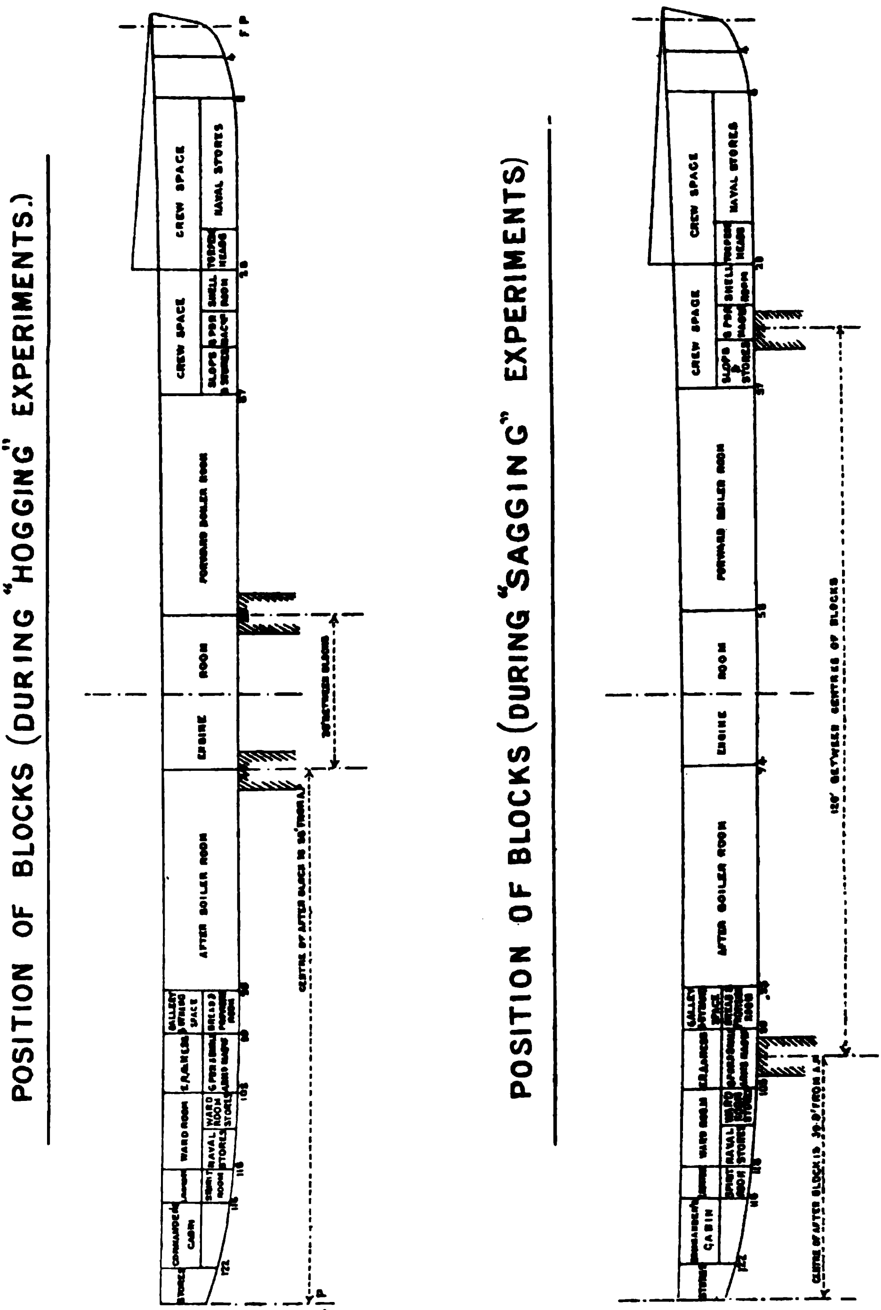
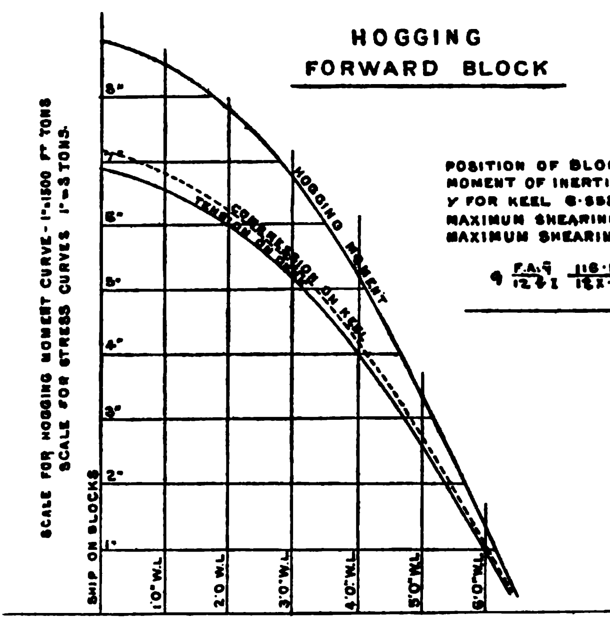

POSITION OF BLOCKS (DURING "HOGGING" EXPERIMENTS.)
CREW SPACE
NAVAL STORES
CREW SPACE
FORWARD BOILER ROOM
ENGINE ROOM
AFTER BOILER ROOM
WARD ROOM
CABIN
F P
A P
POSITION OF BLOCKS (DURING "SAGGING" EXPERIMENTS)
CREW SPACE
NAVAL STORES
CREW SPACE
FORWARD BOILER ROOM
ENGINE ROOM
AFTER BOILER ROOM
WARD ROOM
CABIN

Fig. 254.

## HOGGING MOMENTS ON FORWARD BLOCK ARE INCONSIDERABLE.

POSITION OF BLOCK – 159·0 FT FROM A.P
MAXIMUM SHEARING FORCE – 166·7 TONS
MAXIMUM SHEARING STRESS – 3·95 TONS

$$q = \frac{F.A.\bar{y}}{12\,t\,I} = \frac{166{\cdot}7 \times 488}{12 \times {\cdot}343 \times 5000} = 3{\cdot}95$$




POSITION OF BLOCK – 114' FT FROM A.P.
MOMENT OF INERTIA – 4277·8
Y FOR KEEL 6·5556'; Y FOR DECK – 6·6164
MAXIMUM SHEARING FORCE 116·1 TONS
MAXIMUM SHEARING STRESS 2·54 TONS

$$q \; \frac{F.A.\bar{y}}{12\,t\,I} \; \frac{116{\cdot}1 \times 698}{12 \times {\cdot}365 \times 7164} \; 2{\cdot}54.$$

[To face page 408.

MEASURED FROM A STRAIGHT LINE BASE THROUGH THE EXTREMETIES OF THE CURVES
THE CURVE $m_7$ GIVES THE DEFLECTION OF THE SHIP, UNDER THE BENDING MOMENT AT
THE FLOATING WATERLINE FOR THE HOGGING CONDITION.

$m_4 m_5 m_6$ ARE $\frac{m}{I}$ CURVES,
BENDING MOMENT CURVES (
PONDING SECTIONS OF THE
$m'_3 m'_4 m'_5 m'_6$ ARE THE FIR
$m''_6 m''_7 m''_2 m''_3 m''_4 m''_5 m''_6$ ARE T
E DEFLECTIONS OF THE S
E BASE THROUGH THE EXT
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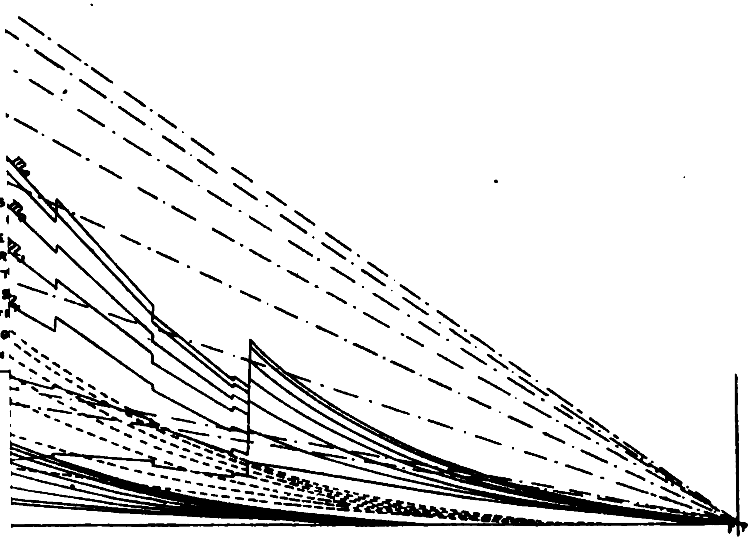
N THE FLOATING CONDITION



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In the weight curve, which is shown on fig.[1] 246, the dotted parts give the outline of the curve which represented the condition during the actual hogging experiments, viz. coal out; the full line giving the curve which represented the condition during the sagging experiments, viz. 95 tons coal aboard.

Bending-moment curves in terms of draught of water were constructed for the weakest section amidships, and for a section at 84 frame through the after boiler-room, where the strain indicators referred to above could be more readily placed and observed. Most of the observations by the strain indicators were made on the structural parts in the region of frame 84, about midway between the bulkheads of the after boiler-room.

Stress curves were deduced from the cross curves of bending moments for all the parts tested by the strain indicators, for comparison with the stress curves as obtained from the reading of the instruments. These are all described hereafter.

Having measured during the experiments the deflection of the vessel, a rough estimate was made of the change of buoyancy due to the deflection. In hogging, the ordinates of the buoyancy curves are increased at the ends slightly, while amidships they remain practically the same. The stage at which this makes the greatest change is at about one-half the load draught. The change is very slight, and can hardly be seen in the stress curves for the hogging series of experiments. In the sagging series the ordinate of the stress curve at about half the draught was decreased very slightly on account of the estimated change on the bending moment, and the curve was faired into the spots at zero waterline and load waterline.

**Determination of Deflection from the Bending Moment.**—The method of obtaining the deflection of a ship due to a given change in bending moment has been described in Chapter XXVIII. In order to get the curve of deflection, or the curve giving the value E$y$, at any point of the length, it is necessary to integrate twice the $\frac{M}{I}$ curve. The I curve was first constructed. Calculations were made to determine I at every part of the length where there was a change in the scantlings. On dividing the ordinates of any one of the bending-moment curves by the corresponding ordinates of the I curve, the $\frac{M}{I}$ curve was obtained. The $\frac{M}{I}$ curves for the sagging series of experiments are shown in fig.[2] 256. These curves were then integrated twice by the integraph, and are given on the same figure.

From the second integral curves the values of E$y$ were measured and plotted on a horizontal base line, fig.[3] 258.

Fig.[2] 257 gives the results of a similar calculation for the hogging series of experiments.

It will be noticed in these calculations that the deflection curves have been obtained on the assumption that there is zero of deflection at zero bending moment. In the dock experiments, at the initial draught of 7 ft., the bending moment was practically zero in the sagging condition. In the hogging condition there was, however, a considerable hogging moment at the initial draught of 6 ft. Deflection curves in this condition were obtained in the manner described above, and from each of these was subtracted the deflection produced on the vessel under the hogging moment in the initial condition. The net values of E$y$ due to change of M for the hogging series are shown in fig.[3] 259.

[1] Plate XXI. [2] Plate XXIV. [3] Plate XXV.

If, then, a certain value of E is assumed, the actual deflection in inches can be obtained, and conversely, if the actual deflection is known, the value of E can be determined.

In these calculations a comparison with the observed deflection was made. The arrangement used for measuring the actual deflection is shown in fig. 244. The curves giving the observed deflection of the vessel during the experiments are shown in the figs.[1] 260 and 261.

The ratios of corresponding ordinates of the calculated E*y* and the observed deflection give the values of E over the length of the ship, and at the definite stages of draught already described.

The values of E so obtained are figured on the diagrams, and later are compared with similar values obtained quite independently (see Table LXXVII.) from the following considerations.

By the ordinary law of elasticity, in which within the limits of elasticity the strain is directly proportional to the stress, it is possible to deduce the stress from the strain. To do so the proportionality, or, as it is usually termed, the modulus of elasticity, must be determined.

If the part of the vessel tested were perfectly elastic the amount of strain for a known stress would give this modulus. But many things on a structure like a ship prevent this ideal condition from existing. The structure is composed of many parts, each under different stress for the same condition of support. The parts are connected by riveted joints, which may be approximately perfectly elastic up to the point at which the friction of the joints is sufficient to prevent slipping, but at this point a sudden change takes place in the relative position of the parts, and sudden changes take place in the stresses of adjoining material. Further, the stresses in the vessel vary from point to point, both vertically and longitudinally, and the slipping point is reached in some joints and not in others. With a variation in load there would follow a further variation in stress throughout the structure, which would be accompanied by a further variation in the condition of the joints as to slipping. It is also to be remarked that the ship is not a solid structure, but is cellular in character, and in consequence the sides of the cells, such as the bulkheads, the deck, and the bottom, tend to change their relative position when under stress.

All these things tend to make the modulus of elasticity of a structure, as a whole, different from that of the test pieces of the material of which it is composed. It is, however, to be noted that the measurements by the strain indicator were made over a distance of 20 in. of solid plate, so that it may be expected that the modulus will be higher over this length than over the structure as a whole. It was at first assumed that a modulus of 14,000 tons would be a convenient one upon which to set off the numerous results of the experiments. On this assumption the stress in the vessel at this position, and for each draught of water at which observations were recorded, was obtained from the formula $p = E\alpha$.

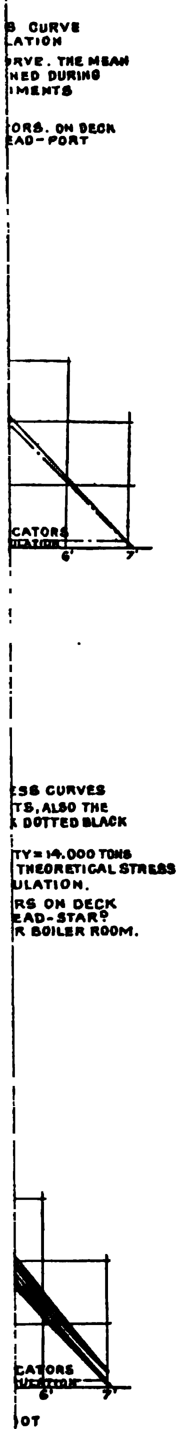
The stresses so obtained were then set up as ordinates at distances apart corresponding to the draughts of water where observed, and curves of stresses were drawn through the points so obtained. See figs.[2] 262 to 268.

There is also shown on each of these diagrams a theoretical stress curve for the positions under consideration.

It will be observed that these curves are set up from different base lines, and for the following reasons:—

The readings of the indicators, and the consequent deduced stresses at any

[1] Plate XXV. [2] Plate XXVI.



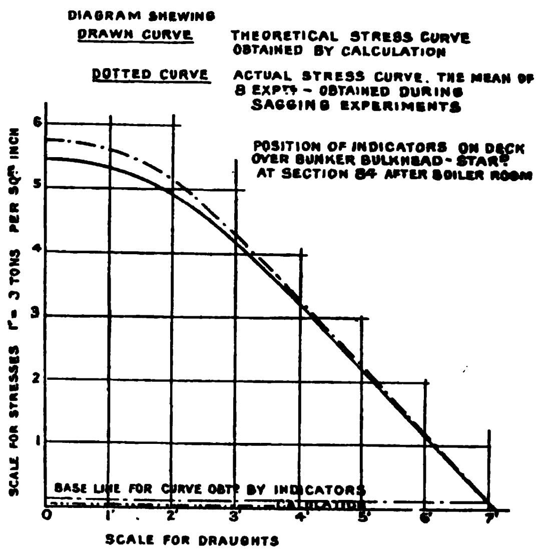




Fig. 268.



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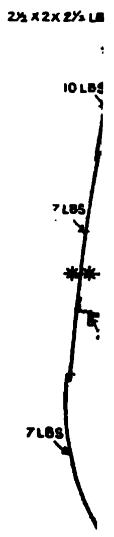


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draught, are only differences between the readings and stresses at the beginning of the observations, and at that draught; but the zero of stress as calculated for the theoretical stress curve is at a different draught of water from that at which observations began, and in order that the calculated curves and those deduced from observed strain-indicator dial readings may be readily comparable, a new base line had to be taken for the latter curves.

This base was obtained by drawing on the theoretical curve, at the draught of water at which observations commenced, a line parallel to the original base line.

The results of the deduced stresses obtained during the experiments were very reliable for a large number of positions. In the hogging series the experiment for each position was only conducted once, but in the sagging series the experiments were repeated several times with the indicators in the same positions, so that it was possible to compare the curves obtained during different experiments under exactly the same conditions. In the latter case it will be observed, on reference to figs.[1] 265, 266, 268, that during different experiments made under the same conditions of bending the dial readings and consequent deduced stresses were for most of the positions practically the same. This demonstrated that whenever the instruments are fixed in these positions, and the vessel is subjected to the same bending moment, the same dial readings are recorded.

The best results were obtained at the section through the after boiler-room, and the most reliable positions for the instruments in this section were, on deck at middle line, on deck over coal bunker bulkhead (starboard and port), sheer strake (starboard and port), and at vertical keel.

All the stress curves having been obtained in the manner previously described for each position during each experiment, it will be noticed that for the sagging series, curves were obtained for the same positions under the same conditions of bending, but during different experiments. The curves so obtained from any position were then taken and grouped together, and the mean curve of these plotted, with this difference, that the first series of stress curves was obtained on the assumption that the modulus of elasticity was 14,000 tons, but in the latter case the assumed modulus of elasticity was 10,000 tons.

But this means a series of diagrams was constructed for each of the positions in which strain indicators were fixed, showing—

(*a*) Theoretical stress curves obtained from the calculated bending moment and the formula

$$p = \frac{M}{I} y.$$

(*b*) Actual stress curves deduced from recorded indicator dial readings and the formula $p = E\alpha$ on the assumption that $E = 10,000$ tons.

Specimens of these curves are given on figs. 262 to 268.

The stress, on an assumed modulus of elasticity of 10,000 tons for each position of indicator at each draught of water at which observations were taken, having been obtained in the manner described, a structural transverse section was made of the vessel in the longitudinal position in which the instruments were fixed during the experiments. The positions of the indicators were set off on it, and the stress set off on horizontal ordinates at these positions. The middle line of the transverse section was taken as the base line, and a curve passed through the points so obtained (see figs.[2] 269 and 270).

[1] Plate XXVI.  [2] Plate XXVII.

The intersection of these curves with the middle line of section gives the position in the section where the stress is zero, which is the neutral axis of the section for the particular condition of vessel under consideration. The ordinates of this stress curve give the intensity of stress, and hence, by taking into account the sectional area of the material in the section and the stress indicated by the curve, the total forces acting on the section for the condition of bending under consideration can be determined. The section being in equilibrium, the resultant of all the stresses for the section must be zero. All the stresses above the neutral axis act in the opposite direction to all below, and hence the sum of the forces above the neutral axis must balance the sum of all those below.

Referring to figs.[1] 269 and 270, it may be seen that above the neutral axis there is a sufficient number of spots to determine the character of this stress curve, but that below the neutral axis there is only one, namely, that at the vertical keel. The difficulty of getting reliable indications for the underwater portion of the "Wolf" was considerable, and was never overcome. Having no observations below (except the vertical keel) made the process of getting the true form of the curve below the neutral axis a tentative one. The forms of these curves shown in the diagrams are those which fulfil the condition of equality of total force above and below the neutral axis, with the least rapid change of form.

TABLE LXXVI.

| Draught of Water. | Sagging. | | Hogging. | |
|---|---|---|---|---|
| | By Strain Indicator. | By Calculation.[2] | By Strain Indicator. | By Calculation.[2] |
| Feet. | Height above Keel—Feet. | Height above Keel—Feet. | Height above Keel—Feet. | Height above Keel—Feet. |
| 6 | 7·5 | 7·8 | ... | ... |
| 5 | 7·5 | 7·8 | 6·5 | 7·5 |
| 4 | 7·5 | 7·8 | 7·0 | 7·5 |
| 3 | 7·5 | 7·8 | 7·1 | 7·5 |
| 2 | 7·55 | 7·8 | 7·2 | 7·5 |
| 1 | 7·55 | 7·8 | 7·2 | 7·5 |
| Dry | 7·55 | 7·8 | 7·2 | 7·5 |

Table LXXVI. gives the positions of neutral axes obtained by this method, and also as obtained by calculation of the position of the centre of gravity of the area of section of material. *The comparison shows that the theoretical position is practically accurate.*

[1] Plate XXVII.

[2] In Table LXXVI. the height of the neutral axis has been calculated in the usual manner, *i.e.* making a deduction of one-eighth for rivet holes in material in tension. Taking account of all the material in the section, *i.e.* making no deduction for rivet holes, the height of the neutral axis in sagging = 7·67 feet, in hogging = 7·62 feet.

After obtaining the *forces* acting on the section in the manner described, the sum of the *moments of the forces* about the neutral axes was obtained for each experimental draught of water. As the section is in equilibrium the bending moment acting on it must be equal to the sum of the moments of the actual stress forces about the neutral axis.

If the modulus of elasticity of the material be accurately 10,000 tons, as assumed, the sum of the moments of the forces due to the stresses given in the curves in figs.[1] 269 and 270 will equal the corresponding bending moment. The extent to which they differ is a measure of the error in choosing the modulus, and the sum of the moments so obtained will bear the same relation to the actual bending moment which the assumed modulus of elasticity of 10,000 tons does to the actual mean modulus of elasticity.

The actual bending moment on the section was calculated exactly for each condition as previously described, and in the following manner the actual modulus of elasticity of the vessel for each condition was obtained.

Suppose E to be the actual modulus of elasticity, and $M_p$ to be the sum of the moments of the forces about the neutral axis based on an assumed modulus of elasticity of 10,000 tons.

If M is the actual bending moment on the section under consideration, then the actual modulus of elasticity

$$E = \frac{10000 \times M}{M_p}.$$

By this method the modulus of elasticity of the vessel was obtained at each draught of water at which observations for hogging and sagging were recorded.

Table LXXVII. gives the modulus of elasticity at the various draughts during the experiments, compared with those obtained by the deflection method.

TABLE LXXVII.

| Draught of Water. | Sagging. | | | Hogging. | | |
|---|---|---|---|---|---|---|
| | By Strain Indicator. | By Deflection Curves. | | By Strain Indicator. | By Deflection Curves. | |
| Feet. | | At 84 Frame. | Average over Length. | | At 84 Frame. | Average over Length. |
| 6 | 12102 | 11415 | 11813 | ... | ... | ... |
| 5 | 12125 | 11020 | 11950 | 15977 | 11750 | 11820 |
| 4 | 11419 | 10780 | 11390 | 15110 | 10510 | 10795 |
| 3 | 11431 | 10570 | 11500 | 12957 | 9935 | 10385 |
| 2 | 10774 | 10220 | 11110 | 12720 | 9412 | 9610 |
| 1 | 10725 | 9830 | 10550 | 11705 | 9350 | 9870 |
| Dry | 10247 | 9512 | 10340 | 11806 | 9300 | 9746 |

[1] Plate XXVII.

In obtaining the above figures the following were taken into consideration :—

All shell plating.
All deck plating and gunwale angles.
Vertical keel and angles.
Side girder and angles.

The modulus of elasticity decreases with increase of stress, which is not unreasonable in a riveted cellular structure, and is not uncommon in test specimens.

After the above-described experiments in dock had been completed the "Wolf" was sent to sea to find bad weather and obtain results on the strain indicators which would determine the actual bending moments which the sea produced. The vessel was under the command of Lieut. (now Commander) the Hon. F. Butler, R.N.

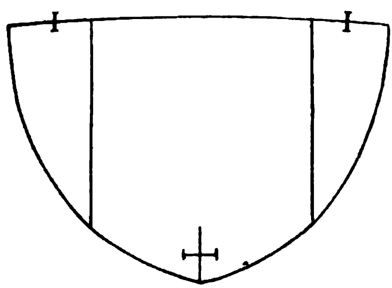


FIG. 271.

The section chosen in which to fix Stromeyer's strain indicators for experiments at sea was at about 84 frame, which was near the centre of the after stokehold, arrangements being made that this stokehold should be kept quite clear, and used for observation purposes only, Nos. 1 and 2 boilers in the fore stokehold being used for steaming purposes. Three positions were decided upon at which to fix the indicators in the section, as shown in the annexed diagram, viz.—

(a) At vertical keel.
(b) On deck in vicinity of coal bunker bulkhead (port).
(c) " " " " (starboard).

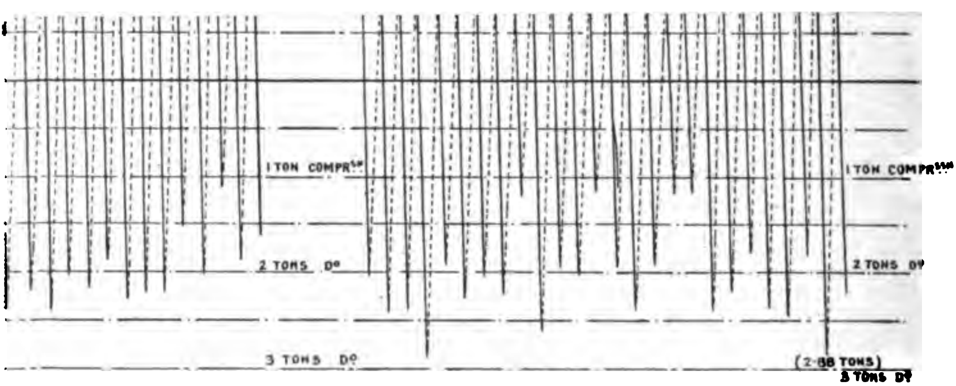
These positions had given the most uniform results in the dock experiments.

In the first set of observations at sea (see figs. 272, 273, 274, Plates XXVIII., XXIX., and XXX.) the maximum stresses obtained were—

TABLE LXXVIII.

| | Keel. | Deck—Port. | Deck—Starboard. |
|---|---|---|---|
| | Tons. | Tons. | Tons. |
| Tension, . . . . | 3·48 | 1·6 | 1·6 |
| Compression . . . | 2·56 | 2·18 | 1·88 |

The second set of observations were taken off the "Wolf" light, where a squall which attained the force of a moderate gale (force 7) with rough sea was encountered. The vessel was steamed head to sea at 12 knots. The deduced stresses are all shown in a diagram form. The maximum stresses obtained were—

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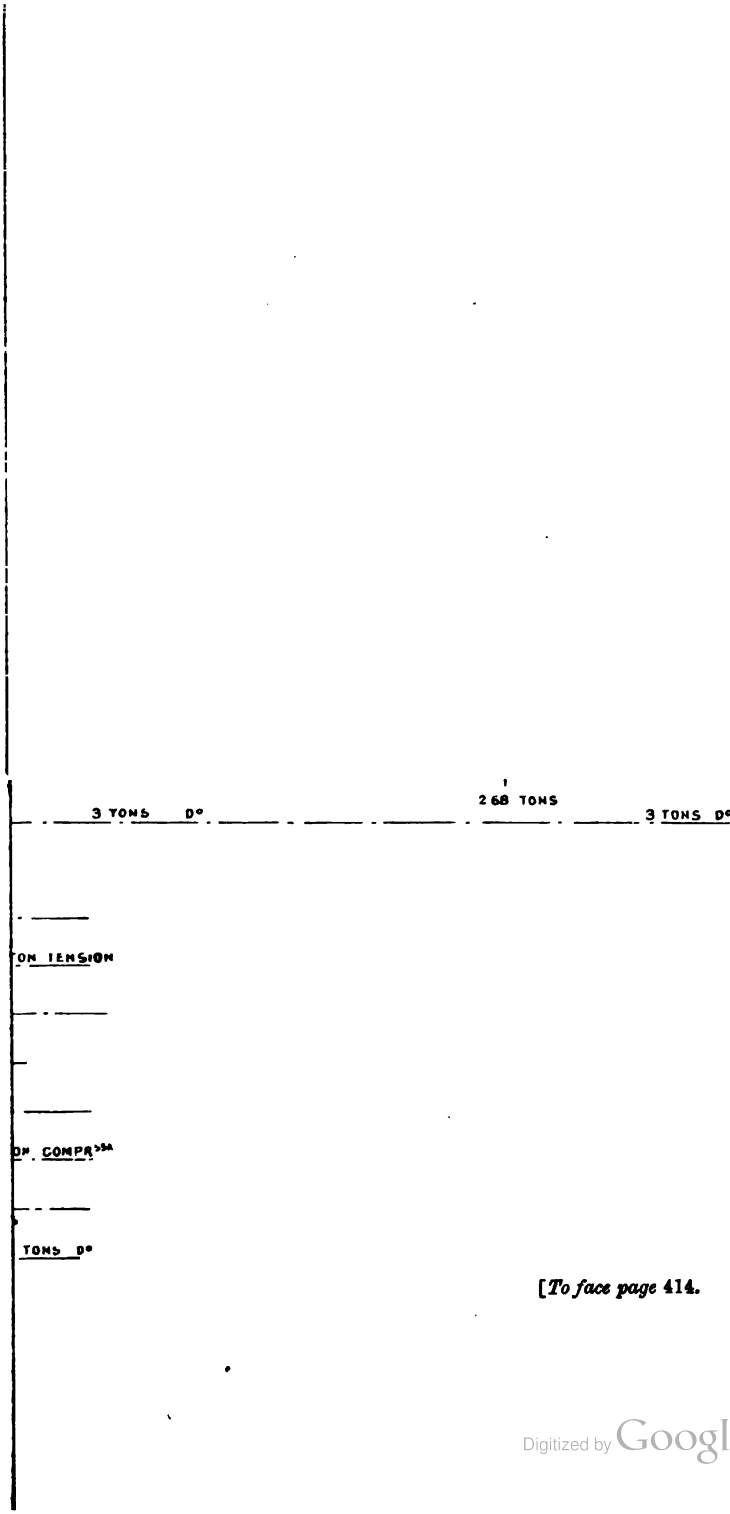


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Table LXXIX.

| | Keel. | Deck—Port. | Deck—Starboard. |
|---|---|---|---|
| | Tons. | Tons. | Tons. |
| Tension, . . . | 3·86 | 1 84 | 1·3 |
| Compression, . . . | 2·9 | 2·54 | 1·68 |

The third set of observations were taken whilst running through the race off the Lizard. The wind was strong from the south, and the sea rough and especially steep, with much force and vigour, and for this class of vessel a passage in it would be considered very bad. The vessel steamed through the race at 13·2 knots, the maximum stresses obtained being—

Table LXXX.

| | Keel. | Deck—Port. | Deck—Starboard. |
|---|---|---|---|
| | Tons. | Tons. | Needle in indicator out of order. |
| Tension, . . . | 5·38 | 1·7 | |
| Compression, . . . | 2·5 | 2·68 | |

The condition of lading of the boat during experiments was that of fully-equipped sea conditions. The draught of water ranged from 7 ft. 3 in. to 7 ft., even keel. From observations of the indicators under different conditions of weather and steaming it would appear that when the ship is rolling, even to such an extent as to submerge the gunwale and a portion of the deck, the stresses set up are very small, the needles in the indicators being scarcely disturbed. In the case of pitching and rolling combined, such as the vessel crossing the seas in an oblique direction, the stresses set up are very much less than those through pitching only, the needles in the instruments moving at the greatest from two to three dial divisions each way, equivalent to a stress of 0·95 tons to 1·45 tons, even in a rough sea.

The greatest observed effects on the indicators were when the vessel was steaming at fastest speed with head on to wind and sea, her bow lifting and riding on one sea, and then diving in the next advancing one, this latter striking her to such an extent as to cause quite a perceptible tremor to pass through her; the main portion of the hull being then in the trough of the sea, and the vessel being subjected to sagging stresses. Whilst steaming slowly through the sea the period of time in passing from maximum tension to maximum compression varied from 10 to 14 seconds. Whilst steaming at fastest, head to wind and sea, it was rather difficult to observe, but from best observations it appeared to be about from 6 to 8 seconds.

The strains observed in the vicinity of the after boiler-room were fully recorded, and from these records the following table, which gives a summary of the maximum stresses experienced at the section in the vicinity of the after boiler-room, has been made :—

TABLE LXXXI.

| Date of Sea Trial. | Sagging Stresses. | | | Hogging Stresses. | | |
|---|---|---|---|---|---|---|
| | Keel Tension. | Deck Port Compression. | Deck Starboard Compression. | Keel Tension. | Deck Port Compression. | Deck Starboard Compression. |
| April 22 | 2·2 | 1·88 | 1·72 | 1·68 | 1·62 | 1·48 |
| ,, 28 | 3·1 | 1·85 | 1·58 | 2·15 | 1·2 | 1·16 |
| ,, 29 | 3·39 | 2·63 | 1·72 | 2·5 | 1·8 | 1·54 |
| May 1 | 3·48 | 2·88* | ... | 2·82 | 1·78 | ... |
| ,, 1 | 2·55 | 1·82 | 1·72 | 2·34 | 1·75 | 1·68 |
| ,, 3 | 3·56 | 2·38 | 2·14* | 2·5 | 2·05* | 1·95* |
| ,, 8 | 3·48 | 2·18 | 1·88 | 2·56 | 1·6 | 1·6 |
| ,, 8 | 3·88 | 2·54 | 1·68 | 2·9* | 1·84 | 1·3 |
| ,, 8 | 5·38* | 2·68 | ... | 2·5 | 1·7 | ... |

* Maximum stresses.

The greatest recorded stresses were due to sagging, and are as follows :—

TABLE LXXXII.

| Keel. | Deck—Port. | Deck—Starboard. |
|---|---|---|
| 5·38 tension | 2·88 compression | 2·14 compression |

From calculations made it has been determined that the maximum theoretical stresses which come upon the "Wolf" in the vicinity of the after boiler-room when on a standard wave are—

TABLE LXXXIII.

| Keel. | Deck—Port. | Deck—Starboard. |
|---|---|---|
| 7·14 | 5·30 | 5·30 |

Hence the stresses actually experienced after prolonged seeking for high seas were much less than those obtained under standard conditions.

Table LXXXIV. shows the actual stresses which the "Wolf" successfully withstood in dock :—

TABLE LXXXIV.

| | After Boiler-room (84 frame). | |
|---|---|---|
| | Calculated Stresses. | Actual Stresses. |
| Deck, middle line | 5·4 | 6·98 |
| Stringer, port | 4 8 | 4·2 |
| ,, starboard | 4·8 | 4·58 |
| Deck above girder, port | 5·4 | 5·3 |
| ,, ,, ,, starboard | 5·4 | 5·72 |
| Sheerstrake, port | 4·1 | 4·0 |
| ,, starboard | 4·1 | 4·75 |
| Vertical keel | 7·2 | 6·4 |

It will be seen, therefore, that the vessel successfully stood in dock stresses much higher than those found at sea. It may be inferred that the bending moments which come upon a ship at sea are less than those assumed in the standard conditions.

The strains observed in dock compared closely with those determined by calculation upon a value for E which was reasonable. The girder theory of calculation was seen to be a reliable practical guide to obtaining the stresses due to known bending moments.

# CHAPTER XXXII.

## APPLICATION OF THE INTEGRAPH TO STRENGTH CALCULATIONS.

THE figs.[1] 275 and 277 illustrate an example of the ordinary strength calculation, the vessel in this case being a torpedo-boat destroyer, supported in the hollow of a wave.

In a strength calculation the first thing to do is to set up a weight curve from a given list of weights. It is necessary to determine accurately the centre of gravity of this curve, and, as the area of a weight curve is not usually within the scope of an integrator or a planimeter, this operation is rather long and troublesome. The centre of gravity can be determined by the integraph by integrating the curve twice, and drawing the tangent to the second integral curve at its final ordinate. The longitudinal position of the centre of gravity of the weight curve is where this tangent cuts the base line, as shown in fig.[1] 276. This tangent can be accurately drawn by the integraph. The final ordinate of the first integral of the weight curve represents the total weight. Suppose the buoyancy curve to be constructed, then it satisfies the following two conditions:—

(1) The area of buoyancy curve equals the area of the weight curve.
(2) The centre of gravity of the buoyancy curve is in the same longitudinal position as the centre of gravity of the weight curve.

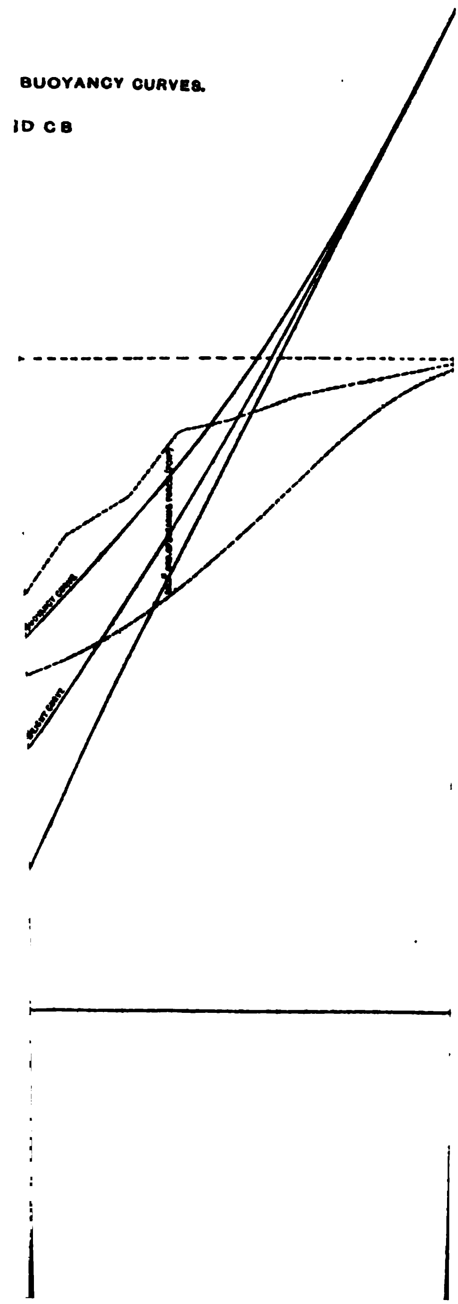
Therefore, on fig.[1] 276,

(1) The final ordinates of the first integrals of the weight and the buoyancy curves are equal.
(2) The final ordinates of the second integrals are equal and the tangents are coincident.

The quickest way to obtain the buoyancy curve is to draw the integrated sections in their corresponding position on a profile of the vessel. Then, placing the waterline over this, the buoyancy per foot of length at each section can be read off (fig.[1] 275). This method saves the necessity of transferring the vertical heights of the intersections of the waveline to the body plan, and then taking the areas of the submerged parts of each section. It is also easy by this method to get the correct position of the waveline relatively to the ship by a trial and error process.

Having obtained the weight and buoyancy curves, the load curve can be

[1] Plate XXXI.


BUOYANCY CURVES.
ID CB

## VES OF BENDING MOMENTS

:NTIRELY ON PAIR OF BLOCKS EACH 6′ 0″ LONG

:- BUNKERS FULL; DISPLACEMENT = 423 TONS.

ES { LONGᴸ $\frac{1}{3}''$ $\frac{L}{20}$ = 10·55 FEET

ES { VERTICAL $\frac{1}{3}''$ $\frac{AL}{50}$ = 1795·06 FOOT TONS

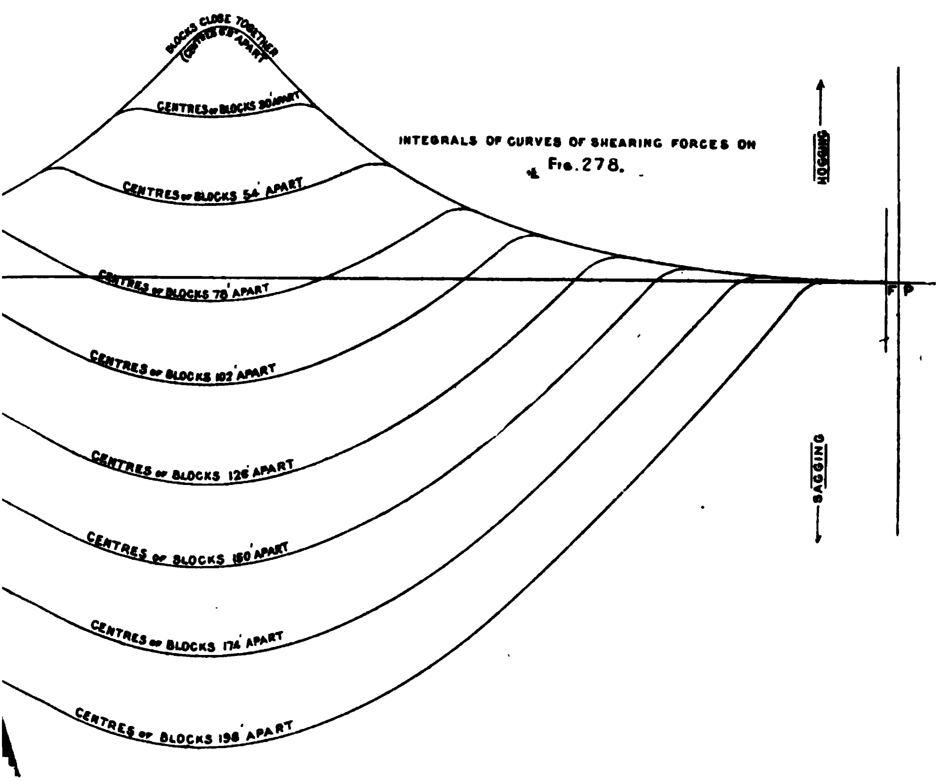



FIG. 279.

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# CURVES OF BENDING MOMENTS.

·ING ENTIRELY ON PAIR OF BLOCKS EACH 6'.0" LONG
SPACED EQUIDISTANT FROM C.G. OF SHIP.
·ITION:- BUNKERS EMPTY DISPLACEMENT = 328 TONS.

SCALES { LONG$^L$ $\frac{1}{8}$" = $\frac{1}{20}$ = 10·55 FEET
VERTICAL $\frac{1}{8}$" = $\frac{1}{50}$ = 1384·16 FOOT TONS.

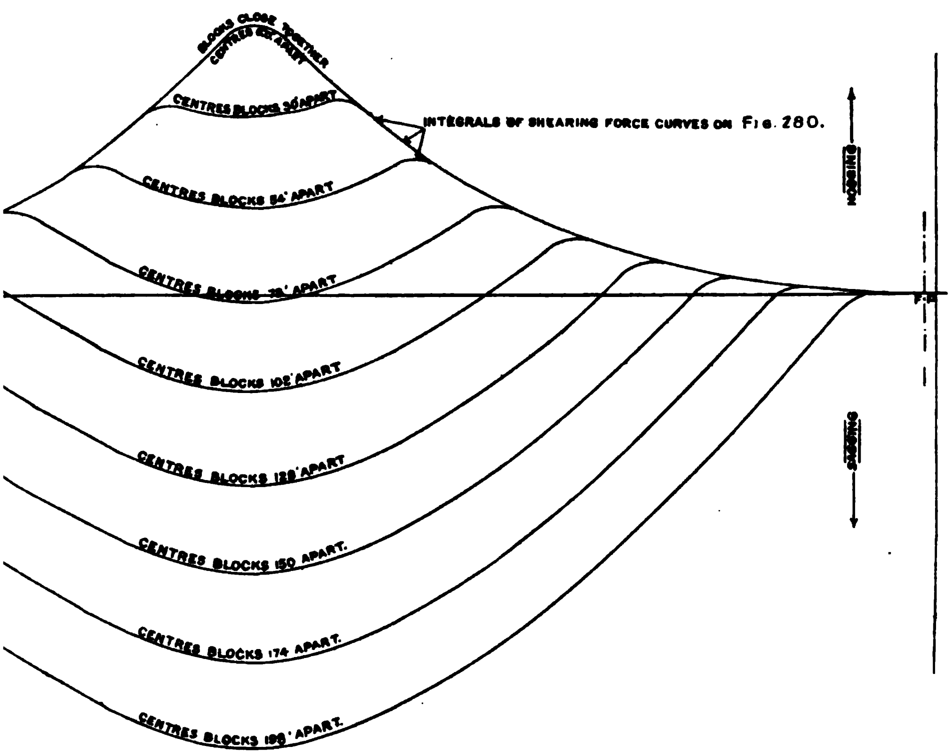


FIG. 281.

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next constructed, and this curve when integrated gives the curve of shearing forces for the first integral, and the bending-moment curve for the second integral (fig.[1] 275).

In fig.[1] 276 it has not been necessary to draw the load curve. In this figure the weight and the buoyancy curves have been integrated twice. The difference of the ordinates of the first integrals gives the shearing force, and the difference of the ordinates of the second integrals gives the bending moment. The scales for the shearing force and bending moment in the method illustrated by fig.[1] 276 are necessarily smaller than those for fig.[1] 275.

If the maximum bending moment only is required the method of fig.[1] 276 can be followed, and the area of the difference of the first integrals to one side of their intersection gives the maximum bending moment. If the buoyancy curve has been hurriedly constructed, and the position of the centre of buoyancy is, in consequence, only approximately correct, a sufficient approximation to the true maximum bending moment will be found by taking a mean of the areas of the difference of the integral curves on either side of their intersection.

*Moment of Inertia Calculation.*—The calculation for the moment of inertia of the cross section of a ship is generally done arithmetically by calculating the moment of inertia of the cross-sectional area of each item that contributes directly to the longitudinal strength. If, however, the "equivalent girder" is set up by the usual method and integrated three times, the result is a curve of moment of inertia about any axis; the correct position of the centre of gravity can also be found, and hence the neutral axis. If it can be said that the labour of constructing the "equivalent girder" is less than the labour of calculating arithmetically the moment of inertia, then this method is to be recommended. It has the further advantage that if it is desirable to plot a shearing-stress curve, the second integral curve gives the value of $Ay$ in the expression for shearing stress, namely $q = \frac{FAy}{bI}$.

Fig.[1] 277 illustrates an example of a moment of inertia calculation for a small one-deck vessel with light scantlings.

Deflection calculations can be quickly made by using the integraph to integrate the curve of $\frac{M}{I}$, the second integration of which gives the deflection. Examples of this calculation have been given in the previous chapter (see figs. 256 and 257).

In order to illustrate the special use of this machine, the diagrams figs.[2] 278 to 281 have been given. These diagrams were drawn for the experiments on the "Wolf." Diagram fig.[2] 278 gives the shearing forces on the "Wolf" resting entirely on the cradles; the cradles are in different positions, and a series of shearing-force curves have been drawn.

The method of obtaining the shearing forces has already been fully described.

These shearing forces when integrated give the bending-moment curves.

All these diagrams were easily and quickly obtained by using the Integraph.

[1] Plate XXXI.

[2] Plates XXXII. and XXXIII.

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