Design of a 3 Span Double Track Reinforced Concrete Railroad Arch Bridge

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The bridge, as designed, is a proposed structure for the C. & N. W. R. R. across the North Branch of the Chicago River, about four miles above the head of navigation. At the present time a wooden pile trestle carries the railroad across a rather shallow wooded valley. This valley of the upper waters of the North Branch has been under consideration, by various improvement associations, as a desirable link in an Outer Park System for the city of Chicago. With this fact in mind, the bridge was provided with two forty foot arches to accommodate future driveways.

METHOD OF DESIGN.

The method of design used in the analyses of the arches, was the graphical system developed by Mr. Burton R. Leffler, Bridge Engineer for the L. S. & M. S. R. R., in his treatise on "The Elastic Arch." In indicating the method of design, the calculations for one of the forty foot arches will be explained.

THE FORTY FOOT ARCH.

Two assumptions must be made to start with, (1) the thickness of the arch ring at the crown, (2) the curve of the intrados. The thickness of the ring at the crown was assumed to be 2 feet, the curve of the intrados was struck from 3 centers.

The theory is based on an equal number of horizontal divisions of the arch ring. The divisions were made on the dotted ordinates with an ordinate at the center. See Plates I, II and III. This determines $\frac{dl}{I}$ at the crown, where dl is the length of a division measured along the gravity axis, which is yet undeter-

mined. For a trial value, dl is measured along the intrados. I is taken at the center of each division of the gravity axis, assuming a width of arch ring of 1 foot. The remainder of the arch ring must be taken so that $\frac{dl}{I}$ is a constant.

$$\frac{dl}{I} = \frac{dl'}{I'}$$

Where dl' and I' are taken at the springing of the elastic arch.

$dl' = 2.33$ $dl = 2$ $I = \frac{bh^3}{12}$ $I' = \frac{bh'^3}{12}$

h' = depth of the ring at the springing.

$$h' = \frac{2.33 \times 12}{.666} = 2.1$$

The span of 40 feet is not the true span of the elastic arch. The span of the elastic arch lies between the points where the tangents to the arch are fixed in direction. These points may be located at that portion of the ring where a sudden enlargement of section takes place, or where $\frac{dl}{I}$ ceases to be a constant. In the arch under consideration, this length of span was taken as 32 feet.

LOADS.

The live load was taken as the equivalent uniform load for Cooper's E60 with 100% impact added. The load was considered to be distributed over a width of 12 feet. From Cooper's Specifications for Railroad Bridges, the equivalent uniform load for E40 with a span of 32 feet = 7120# per foot. For E60 this equals $\frac{6}{4} \times 7120$ = 10680#. With 100% impact added this equals 21360# per foot. In designing the arch, for convenience, a section 1 foot

wide was used. The load on a width of one foot = 1780#. 1800# per linear foot was the load used. The volume of the dead load was scaled from the drawing. The weight of the earth fill was taken as 100# per cubic foot, and the weight of the arch ring 150# per cubic foot. The volume of fill at section 0 = 1.5 x 2 = 3 cubic feet. The weight of earth = 3 x 100# = 300#. The weight of the arch ring = 2 x 2 x 150 = 600#. The total load at the crown = 900# + 3600# = 4500#. The length of a division was taken as 2ft. and therefore the live load per section = 3600#.

THE DETERMINATION OF THE TRUE VALUE OF "H."

In order to determine the maximum stresses at the various sections, at least three positions of the live load must be considered. The positions used were 1/2, 3/4 and full load. Taking the position of half load as an example, the load line was laid off vertically, each load being numbered corresponding to its position along the arch. Assuming H, the horizontal thrust as 40000#, a trial stress diagram and equilibrium polygon was constructed.

The next portion of the problem, was to locate a line [illegible], (See Plates) so that the sum of the ordinates called [illegible] ordinates, from this line to the trial equilibrium polygon, would be zero. All ordinates were measured to the same scale as that of the arch, 3/8 inch = 1 foot. The method of determining [illegible] can better be understood by referring to the table on Plate 1. Column 1 contains the number of the section. Columns 2 and 3 give the ordi-

notes to the trial equilibrium polygon on the right and left sides of the center; 4 gives the difference between the right and left ordinates of the same section; 5 gives the summation of these differences.

$$\Sigma(d_1 + 2d_2 + 3d_3 + ---- 7d_7)$$

This summation was used in the formula for determining v w

$$vw = \frac{12(d_1 + 2d_2 + 3d_3 + ---- 7d_7)}{(n+1)(n+2)}$$

Where **n** equals the number of equal spaces that the arch was divided into. To determine the direction from m m₁ v w was laid off above v and v₁ w drawn. R, the sum of the ordinates in columns 2 and 3 of the table = 48.95.

The formula $\frac{R}{n+1}$ gives the distance that [illegible]₁ is above v₁ w at the mid-ordinate of the polygon.

$$\frac{R}{n+1} = \frac{48.95}{17} = 2.88$$

Having drawn m m₁ parallel to v₁ w, the m ordinates were measured. These ordinates were recorded in columns [illegible] and 7.

The y ordinates are given in column [illegible] of the table. They were measured from the line joining the springing points o y, to the gravity axis of the arch ring. The line k k₁ was next drawn parallel to o y, and at a distance above it equal to $\frac{\Sigma y}{n+1}$. The line m m₁ cuts the polygon at E and E₁. Projecting these points upward to e and e₁, in the line k k₁ locates 2 points in the required pressure curve. The k ordinates were measured from k k₁ to the gravity axis of the arch. The summations Σmy and Σky recorded

in columns 12 to 1[illegible], and the sums of the products [illegible], [illegible].

To determine the true pole distance, the formula $\frac{\sum 1[illegible]}{\sum [illegible]}$ x Trial Pole Distance is necessary. If the true value of [illegible] was assumed in the first place, then $\sum$ [illegible] = $\sum$ [illegible].

True Pole Distance = $\frac{32.0[illegible]}{27.3[illegible]}$ x 40000 = 4[illegible]100[illegible].

To locate the true pole P, P_1 r was drawn parallel to [illegible],

P then lies on a horizontal line through r.

In the fundamental equation $S = \frac{T}{A} + \frac{Mc}{I}$ S = stress, T the thrust, A the area, M the moment, c the distance to any fibre and I the moment of inertia of the section. In this analysis of the arch the effect of $\frac{T}{A}$ has been omitted.

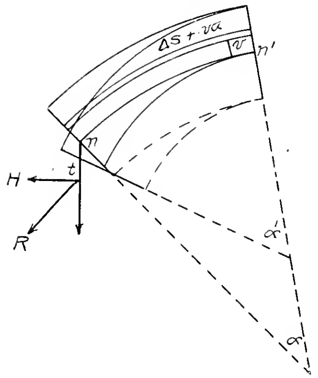
By experiment it has been determined that the effect of $\frac{T}{A}$ is to decrease the value of H. The experiments of Prof. Howe have shown that for an arch having a rise of $\frac{1}{4}$ the span, the true value of H is 93-1/2% of the approximate value; for a rise of 1/6, 86%; and for a rise of 1/10, 69%. In the arch designed, the rise was 3 feet 10 inches, and the span 32 feet, giving a ratio of $\frac{1}{8.15}$. By interpolating between the values already mentioned the correct percentage for H was determined.

True Pole Distance = 72.[illegible]% of the approximate value.

Having determined the true pole distance, a new stress diagram and equilibrium polygon was drawn. e and e_1 being points on the required curve, the new polygon must be drawn from one of these points, and as a check on the accuracy of the work, it should pass through the other point. The pressure curve having

been located, it only remains to determine the unit stresses.

UNIT STRESSES



The above figure is a side view of a portion of an arch ring contained between two planes perpendicular to the neutral surface n n' and making an angle α in circular measure, before strain, between them. A vertical plane midway between the faces of the arch intersects the neutral surface in the line n n' = Δs feet in length, which may be called the neutral line. The forces considered all act in this plane.

Let R be the resultant of all external forces acting upon the section passing through n. Consider applied at n two opposed forces +R and -R, each equal and parallel to R. The single force R is thus replaced by a couple Rh and a force +R acting at n. The latter may be resolved into components T and N, tangential and normal to n n' at n. The force T causes a uniform shortening in all the fibres. The force N is a shearing force and may be

neglected in determining the longitudinal stresses.

The couple RM is principally effected in changing the curvature of the arch and its moment is most conveniently found by multiplying its horizontal component H by the vertical distance from n to R, which we can call t feet. Then we have

$$M = Ht, \text{ in foot pounds } (1).$$

Under the action of this couple the angle α is changed to α' and the curvature is increased if R cuts the section below n, and decreased when R cuts the section above n.

Call $\alpha' - \alpha = \Delta\alpha$ and regard M as right handed.

Call distance of any fibre from n n' V, this being + above and − below. The length of the fibre before flexure $= \Delta S + V\alpha$ after flexure $= \Delta S + V\alpha'$ its change of length is $V(\alpha' - \alpha) = V\Delta\alpha$. Calling its cross section a in square feet and the unit stress due to M = f pounds per square foot, the stress on the fibre of concrete

$$fa = \frac{V\Delta\alpha}{\Delta S + V\alpha} aE_1 \qquad (2)$$

and of steel

$$fa = \frac{V\Delta\alpha}{\Delta S + V\alpha} anE_1 \qquad (3)$$

since $f = \frac{\text{elongation of fibre}}{\text{length of fibre}} \times E$.

In (3) $n = \frac{E_2}{E_1}$ where E_1 = modulus of elasticity of concrete and E_2 = modulus of elasticity of steel. $\Delta S + V\alpha$ can be replaced by ΔS without appreciable error. The same [illegible] all the

stresses due to flexure on the entire section at n for concrete

$$\Sigma(fa) = \frac{E_1 \Delta\alpha}{\Delta S} = \Sigma(va) \qquad (4)$$

or for steel

$$\Sigma(fa) = \frac{E_1 \Delta\alpha}{\Delta S} = \Sigma(vna) \qquad (5)$$

The moment of the stress (af) about n of any fibre is (afv)

$$M = \Sigma(afv) = E_1 \frac{\Delta\alpha}{\Delta S} \Sigma(v^2 a)$$

Let I_1 = moment of inertia of the concrete of area A_1 in feet, and I_2 = moment of inertia of the steel of area A_2 in feet.

$$\Sigma v^2 a = \Sigma(v^2 a) \text{ for concrete}$$

$$= \Sigma(v^2 na) \text{ for steel}$$

$$= I_1 + nI_2$$

$$\therefore \quad M = E_1 \frac{\Delta\alpha}{\Delta S}(I_1 + nI_2)$$

$$\therefore \quad \Delta\alpha = \frac{M\Delta S}{E_1(I_1 + nI_2)}$$

Let f_1 = stress per square inch on an extreme fibre of the concrete whose distance from the neutral axis is v_1 feet. Then from (2)

$$f_1 = v_1 E_1 \frac{\Delta\alpha}{\Delta S}$$

and eliminating $\frac{\Delta\alpha}{\Delta S}$ between this equation and ([illegible]) we get

$$f_1 = \frac{Mv_1}{I_1 + nI_2}$$

and for steel,

$$f_2 = \frac{Mv_2 n}{I_1 + nI_2}$$

The direct thrust of the arch must now be found. Let P = the uniform compression on concrete of area A_1 and let nP be the unit compression on steel of area A_2.

Then the total compression on a section $= P(A_1 + nA_2)$
This is equal and opposite to T.

$$\therefore \; T = P(A_1 + nA_2)$$

from which

$$P = \frac{T}{A_1 + nA_2} \qquad nP = \frac{nT}{A_1 + nA_2}$$

The total stress would now be the sum of the bending stresses and the direct thrust. Let s_1 and s_2 be the stress in pounds per square foot on the concrete and steel respectively at the upper and lower edges.

Then

$$S_1 = \frac{T}{A_1 + nA_2} \pm \frac{Mv_1}{I_1 + nI_2}$$

$$S_2 = \left(\frac{T}{A_1 + nA_2} \pm \frac{Mv_2}{I_1 + nI_2}\right) n$$

As an example take point 1 on the 80 foot arch.

Live load Thrust = 100,000#

Temperature Thrust = 15,600#
T = 115,600#

Live load Bending Moment = 133,[illegible] ft. pds.

Temperature " " = [illegible]0,000 " "
M = 1[illegible]3,000 " "

$A_1 = 3$ square feet

$A_2 = .0455$ square feet

$n = 15, \quad I_1 = 2.25, \qquad I_2 = .0455 \times 1.25^2$

$V_1 = 1.5, \quad V_2 = 1.25$

$$\frac{S_1}{144} = \frac{115{,}600}{3 + 15 \times .0455} \pm \frac{175{,}000 \times 1.5}{2.25 \times .0455 \times 1.25^2}$$

$\begin{matrix} -770 \\ +333 \end{matrix}$ pounds per square inch

$$\frac{S_2}{144} = \left(\frac{115{,}600}{3 + 15 \times .0455} \pm \frac{175{,}000 \times 1.25}{2.25 + 15 \times .0455 \times 1.25^2} \right) 15$$

$\begin{matrix} -10{,}150 \\ +5{,}600 \end{matrix}$ pounds per square inch.

TEMPERATURE STRESSES.

Let t_o = rise or fall of temperature in degrees Fahrenheit.

D = span in feet

f = coefficient of expansion of concrete.

The lengthening or shortening of the span is Dtf.

This is the horizontal movement along the X axis and is also given by the formula

$$\frac{H\,dl}{EI} \int ty$$

which is one of the equations leading up to the fundamental equations.

$$Dt_o f = \frac{H\,dl}{EI} \int ty$$

or $$H = \frac{I D t_o f E}{dl \int ty}$$

Since the end tangents are fixed in direction $\Sigma t = 0$ which means that H acts along the line $K K_1$.

$\int ty$ is the same as Σky which was found in computing the true pole distance.

The range of temperature was 28°. If the arch was built at 53° the range would be from 25° to 81°. The concrete is quite massive and is also covered with earth so that this range is undoubtedly sufficient.

As an example we will find H for the 80 foot arch.

$$I = \frac{9}{4} \qquad d_1 = 4 \qquad D = 64$$

$$t = 28° \qquad E = 144 \times 2,000,000$$

$$\int ty = 102.76, \qquad f = .0000055$$

$$H = \frac{\frac{9}{4} \times 64 \times 28 \times .0000055 \times 2000000 \times 144}{4 \times 102.76} = 15,500\#.$$

FOUNDATIONS.

The direct thrust of the 80 foot arch for full load is 125,000 pounds. The direct thrust of the 40 foot arch for no load is 46,000 pounds. The weight of the concrete and filling between the points considered in determining the above thrusts is 60,000 pounds. Combining these graphically and obtaining the resultant gave a thrust of 185,000 pounds acting diagonally through the center of the pier base. The vertical component of this is 172,000 pounds which must be taken by the foundation per

foot of width.

Considering the arch as 24 feet wide (since the load from each track was distributed over 12 feet) we get a total load on the foundation of 4,128,000 pounds or 2,064 tons. This load is taken by 96 piles giving a load of 21 tons on one pile. This may seem excessive, but the impact which was included in the thrusts never reaches the piles, but is absorbed by the inertia of the filling and concrete.

PLATE I

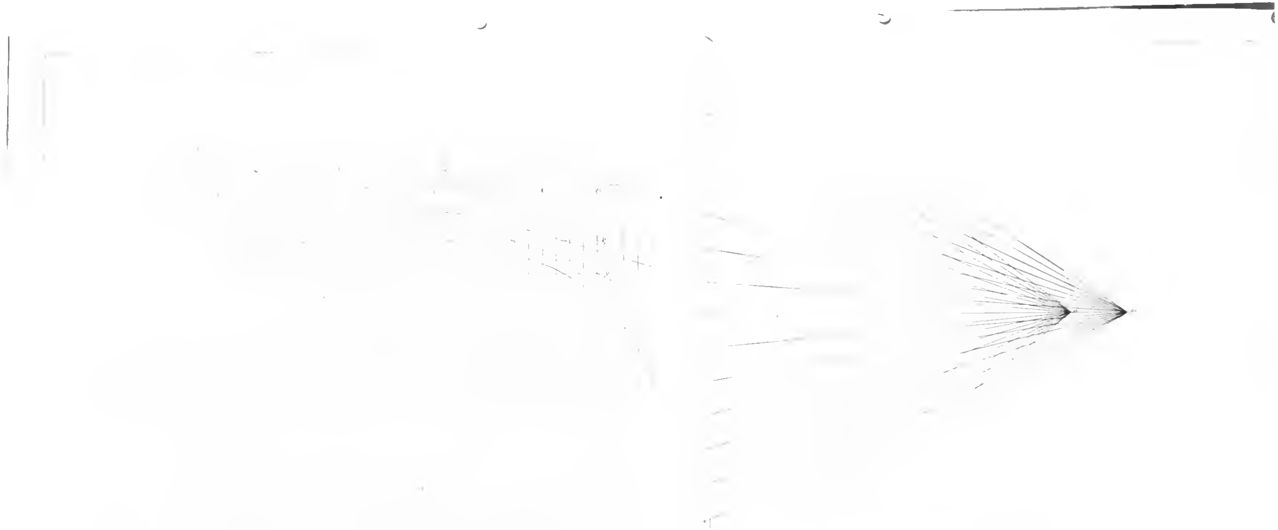
For scale of original drawing multiply by 3.97

	[illegible]		[illegible]	[illegible]	Moments		y	x	my		xy
	L	R			L	R			L	R	
0	+15	335	0	0	+129	+29	90	-68	[illegible]	+432	+760
[illegible]	+69	651	18	19	+250	[illegible]	370	+30	+14	+966	+496
2	+42	[illegible]	32	64	+235	[illegible]	360	+14	+80	+738	+428
3	593	546	51	53	+170	+42	330	+90	[illegible]	+468	+297
4	528	454	74	[illegible]	+98	+60	290	+49	+284	+74	+42
5	435	353	82	410	0	38	239	0	0	91	0
[illegible]	35	242	73	438	-125	47	75	-68	219	249	-19
[illegible]	68	29	39	273	-272	252	95	49	258	-240	-142
8	0	0	0	0	-442	378	0	242	0	0	0
	[illegible]			652	Σm = 0		4110	Σx = 0	Σmy = 4809		2725

True Pole Distance = (Σmy × Trial Pole Dist) / (Σxy) = 70600 [illegible]

STRESS SHEET
FOR
CONCRETE STEEL ARCH
PLATE II
SPAN - 40
¾ LOAD
THESIS OF
C. D. Willard
G. R. Haggander
Scale 1" = 50000#, 3/8" = 1'

PLATE II
For scale of original drawing multiply by 3.98



	2	3	4	5	6	7	8	9	0
[illegible]	238	238	0	[illegible]	110	110	90	68	[illegible]
	586	298	0	0	27	[illegible]	[illegible]	[illegible]	[illegible]
2	56	561	0	0	56	56	[illegible]	[illegible]	[illegible]
3	518	518	0	0	40	40	330	90	[illegible]
4	452	451	0	0	79	9	290	49	[illegible]
5	370	370	0	0	-06	06	239	0	[illegible]
6	270	210	0	0	06	06	5	60	
7	47	147	0	0	230	230	95	14	
8	0	0	0	0	-37	-31	0	24	
					Σm = 0		41.0	1.0	Σ

True Pole Distance = [illegible]

THESIS SHEET FOR CONCRETE-STEEL ARCH PLATE III

SPAN 40 ft.

THESIS OF [illegible]

PLATE III

For scale of original drawing, multiply by 4.09

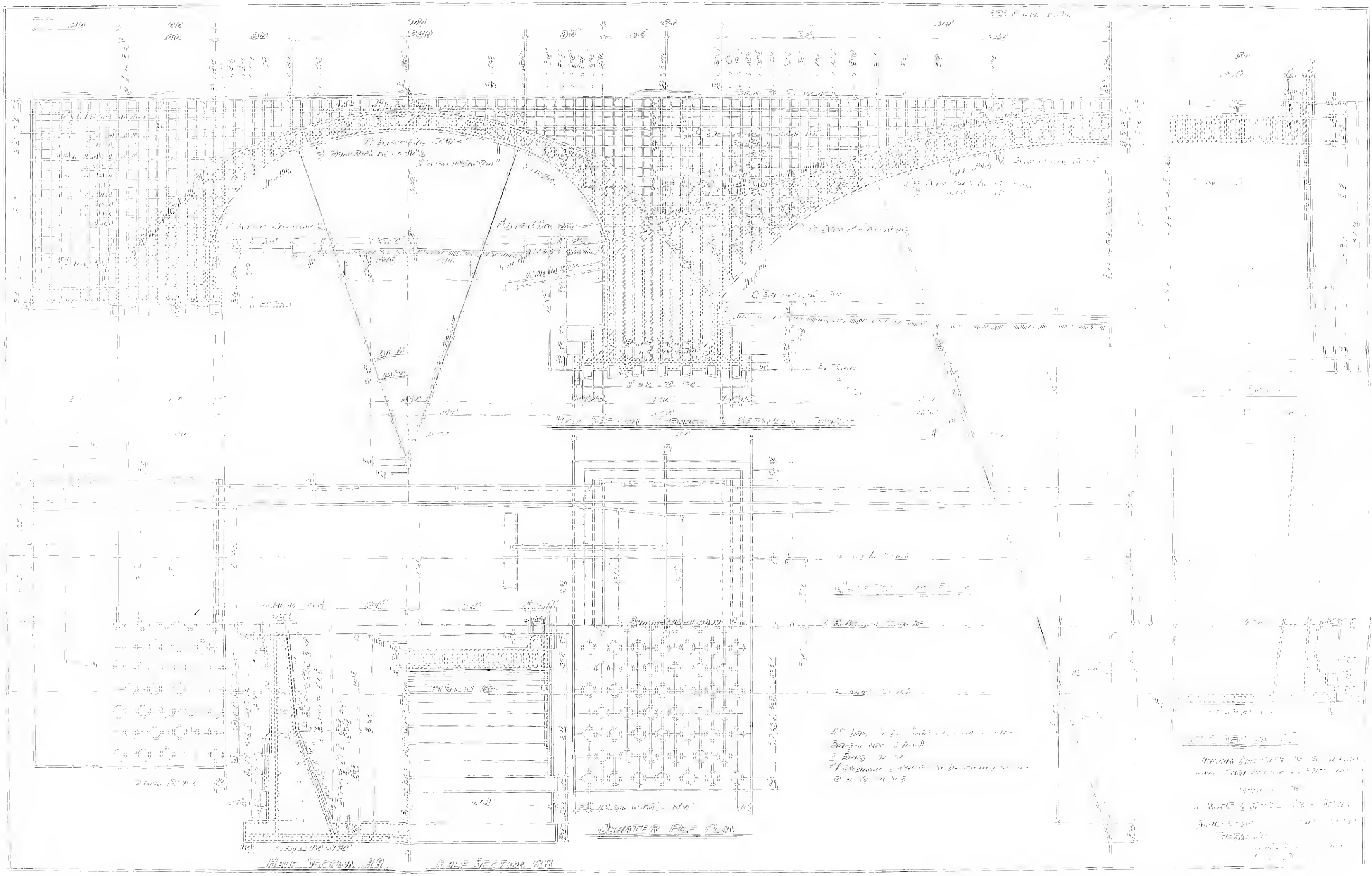
PLATE IV

For scale of original drawing multiply by 4.14

This part necessary for [illegible]							This part for [illegible]			This [illegible]			
	Ordinates		d_s	[illegible]	m Ordinates		[illegible] Ord	k Ord		m[illegible]		k[illegible]	
	L	R			L	R		L	R	L	R	L	R
1	2	3	4	5	6	7	8	9	[illegible]	[illegible]	12	13	14
0	625	625	000	000	+239	+239	744	+135	+135	+1778	1778	+004	[illegible]
1	1242	1214	028	028	466	450	730	258	258	3962	3285	883	1883
2	1200	1135	065	130	[illegible]	378	700	235	235	2912	2446	645	1645
3	1112	1024	088	264	325	246	644	[illegible]	171	2093	[illegible]	[illegible]	[illegible]
4	983	865	118	472	183	+112	566	+092	+092	[illegible]36	+[illegible]36	+52[illegible]	+52[illegible]
5	808	678	130	650	+016	-064	[illegible]	-012	-012	+076	-294	-055	-055
6	596	474	122	732	-2[illegible]6	-266	333	133	133	-720	-888	-443	443
7	32[illegible]	250	071	497	-500	-483	[illegible]	28[illegible]	28[illegible]	-905	-875	-5[illegible]	-5[illegible]
8	000	000	000	000	834	-725	[illegible]	-466	-466	000	000	000	000
	3[illegible]	[illegible]		2[illegible]3	Σm=0[illegible]		4358	Σk=[illegible]		Σm[illegible]	[illegible]89	[illegible]	[illegible]12

STRESS SHEET
FOR
CONCRETE-S[illegible]EL ARCH
PL[illegible]E
SPAN 80'0" 3/4 LOAD
THESIS OF
[illegible]
[illegible]

PLATE V
For scale of original drawing multiply by 4.03


QUARTER PILE PLAN
HALF SECTION AA
HALF SECTION BB


40'-0"