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Development Policies in LDC's with Several Ethnic Groups — A Theoretical Analysis

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This is an extensively revised version of *Johns Hopkins Working Paper No. 45* and represents research originally begun by the first author while he was visiting the Department of Economics and the Math Center at Northwestern University in 1978. A preliminary version was presented at a seminar at the Development Research Center, IBRD in March 1979, and some of the results of Section 2 were announced, without proof, in *Economic Letters*, 2 (1979), 369-75.

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The presence of economically and socially disadvantaged groups is a common feature of most less-developed countries (LDC's). These "backward classes" share a common religion, or belong to the same ethnic or tribal group or originate from a particular geographic region. In each instance they form an easily-identifiable minority. The "untouchables" or "scheduled castes" in India readily come to mind but even a casual observer of other South Asian, or indeed African and Latin American economies can readily furnish his own examples.

Economic planners in LDC's have long been concerned with the economic and social advancement of such groups[1] and various national plan documents are replete with a variety of policy measures introduced to help such communities. Such measures have typically taken the form of specific employment quotas, minimum wage legislation, regional subsidies and specially targeted development expenditures.[2] A natural question arises as to whether such policies accomplish what they are intended to do and the impact they have on the welfare of other groups and on the country as a whole. An analysis of these issues has been lacking in the development literature. This literature, by and large, confines itself to a two-sector setting with a homogeneous labor force whereas the very nature of the problem calls for a general equilibrium analysis in a multi-sectoral setting. Such an analysis is presented in this paper.

The plan of the paper is as follows. Section 1 presents the model and Section 2 is devoted to checking out the viability of our equilibrium concept; namely whether it exists, is locally unique and robust, and is stable in terms of a reasonable adjustment process. Section 3 moves on to a consideration of policy issues and Section 4 indicates several other questions which our model can easily answer. Section 5 concludes the paper and three appendices establish the results presented in the paper.

1. THE MODEL AND THE EQUILIBRIUM CONCEPT

We study a small, open economy with one urban city-center and n rural sectors, some or all of which typically may be regarded as backward. There are n ethnic groups, one associated with each of the n-rural sectors whose members come to the urban center to find employment. The volume of migration is controlled by expected wage equalization, a seminal idea introduced into the development literature by Harris-Todaro [10]. However, our multisectoral setting allows us to extend the Harris-todaro equilibrium condition by postulating that each migrant calculates the probability of finding a city-job on the basis of the unemployment rate specific to his own tribe rather than the aggregate unemployment rate. This allows us to capture the fact that information about employment possibilities in the city flows to a village largely, or even solely, through those of its members who are already in the city. Further, our extension also emphasizes the fact that during the period when he is unemployed and looking for a city-job, a migrant has to fall back for support on the employed members of his tribe or his region.

Our second departure from the original treatment of Harris-Todaro lies in assuming, along with several recent studies, that urban wages are not necessarily rigidly set but may be a general function of the rural wage and the unemployment rate. A basis for such a function has been succinctly set out in Sen [17, p. 55], and more thoroughly discussed in a two sector, mobile capital context by Khan [13]. The only point worth underscoring here is that the urban wage function may differ from tribe to tribe reflecting different supply prices to the urban employer or more generally, different institutional arrangements for each tribe.

In summary, then, the basic ingredients of our model can be simply stated. There are $n+1$ commodities and $2n+1$ factors of production, $n+1$ of which are commodity specific and non-shiftable and the remaining n, i.e., the different types of laborers, flow freely between the urban center and the rural sectors they come from. For any tribe, laborers are allocated between the urban center and the rural sector on the basis of the Harris-Todaro equality of expected returns. All factor returns are endogenously determined and it is assumed that the supplies of each of the $2n+1$ factors and the prices of $n+1$ commodities are exogenously given. We shall devote the remainder of this section to a more formal presentation of the model and our equilibrium concept.

Let each of the n rural regions be indexed by i, and let $\mathcal{L}_i$ denote the population coming from region i. Let L_{ir} denote the rural employment, L_{iu} the urban employment and λ_i the ratio of urban unemployed to urban employed,[3] all these variables pertaining to region i. We can thus write

$$L_{ir} + (1+\lambda_i)L_{iu} = \mathcal{L}_i \qquad i=1,2,\ldots,n \qquad (1.1)$$

The technology available to the economy is given by

$$X_{ir} = F_{ir}(L_{ir},K_{ir}) \qquad i=1,2,\ldots,n \qquad (1.2a)$$

$$X_u = F_u(L_{1u},L_{2n},\ldots,L_{nu},K_u) \qquad (1.2b)$$

where K_{ir} and K_u are the community-specific, non-shiftable factors of production.

The endogenous urban wage of the i^{th} tribe is given by w_{iu} and its endogeneity is brought out by

$$w_{iu} = \Omega_i(w_{ir}, \lambda_i, \mathcal{T}_i) \qquad (i = 1,2,\ldots,n) \qquad (1.3)$$

where $\mathcal{T}_i$ is a shift parameter. For details about the microeconomics underlying the $\Omega(\cdot)$ functions, see Khan[4] [13].

Labor migrates until expected returns are equalized and this can be expressed as

$$\Omega_i(w_{ir}, \lambda_i, \mathcal{T}_i) = w_{iu} = w_{ir}(1 + \lambda_i) \qquad (i = 1,\ldots,n) \qquad (1.4)$$

with the probability of finding a job in the urban sector given by $(1/1 + \lambda_i)$.

Finally, we shall assume that there is marginal productivity pricing of labor in each sector. Thus

$$\partial F_{ir}/\partial L_{ir} = w_{ir}/p_{ir} \qquad (i = 1,\ldots,n) \qquad (1.5a)$$

$$\partial F_u/\partial L_{iu} = w_{iu}/p_u \qquad (i = 1,\ldots,n) \qquad (1.5b)$$

where p_{ir}, p_u are the exogenously given prices for the $n+1$ commodities.

Putting all these facets together yields us our basic equilibrium concept.

D.1: <u>A Harris-Todaro Equilibrium</u> is a n-tuple of the quadruple $(w^*_{ir}, \lambda^*_i, L^*_{ir}, L^*_{iu}) >> 0$ such that for all i

(i) L^*_{ir} maximizes $p_{ir}F_{ir}(L_{ir}, K_{ir}) - w^*_{ir}L_{ir}$

(ii) L^*_{iu} maximizes $p_u F_u(L_{1r},\ldots,L_{nr},K_u) - \sum_{i}^{n} \Omega_i(\cdot) L_{iu}$

(iii) $L^*_{ir} + (1+\lambda^*_i)L^*_{iu} = \mathscr{L}_i$

(iv) $w^*_{ir}(1+\lambda^*_i) = \Omega_i(w^*_{ir},\lambda^*_i,\mathscr{T}_i) \equiv w^*_{iu}$

2. EXISTENCE, LOCAL UNIQUENESS AND DYNAMIC STABILITY OF HARRIS-TODARO EQUILIBRIA

In this section we present sufficient conditions under which Harris-Todaro equilibria exist, are locally unique and stable in terms of an "intuitively reasonable" adjustment process. These results are presented as preliminary consistency tests of the viability of the model and as a prelude to the more substantive, comparative static investigation presented in Section 3. The fact that such results are a prerequisite for comparative static investigations is evident in the works of Bhagwati, Srinivasan, Calvo, Neary, Khan and others.

2.1 Existence of Equilibrium

The result of this section can be viewed as a generalization to a multisectoral setting of the existence results in Srinivasan-Bhagwati [18] and Calvo [6].

Let w_r denote a vector of rural wages $(w_{1r},\ldots,w_{nr})$ and $w_r(i)$ denote w_r with the i^{th} component deleted. Let the excess demand for labor of the i^{th} tribe be denoted by $\phi_i(w_r)$ where

$$\phi_i(w_r) = L_{ir}(w_{ir},p_{ir}) + (1+\lambda_i)L_{iu}(w_{1u},\ldots,w_{nu},p_u) - \mathscr{L}_i \quad (2.1.1)$$

If we focus on situations where the equality of expected wages is maintained, (1.4) allows us to write λ_i as a function of w_{ir}, say $\ell_i(w_{ir})$. Using (1.3) we can rewrite (2.1.1) as

$$\phi_i(w_r) = L_{ir}(w_{ir},p_{ir}) + (1 + \ell_i(w_{ir}))L_{iu}(w_{1r},\ldots,w_{nr},p_u) - \mathcal{L}_i \quad (2.1.2)$$

We shall now assume the following

Assumption 2.1.1: For all i, and for all $w_{ir} > 0$, $\Omega_i(\cdot) > w_{ir}$.

Assumption 2.1.2: For all i, $\phi_i(\cdot)$ is a continuous, single valued function decreasing in w_{ir}.

Assumption 2.1.3: $\underset{w_{ir}\to 0}{\mathrm{Lt}}\ \phi_i(w_{ir},w_r(i)) = \infty$ and $\underset{w_{ir}\to\infty}{\mathrm{Lt}}\ \phi_i(w_{ir},w_r(i)) = -\mathcal{L}_i$ (2.1.

Assumptions 2.1.2 and 2.1.3 formalize the fact that the excess demand in each market is a single-valued function of rural wages and that it is a decreasing function of its own rural wage, unbounded from above and bounded below by a negative number. It is clear that there exist inocuous sufficient conditions[5] on the technologies and the $\Omega(\cdot)$ functions which imply Assumptions 2.1.2 and 2.1.3. Assumption 2.1.1 is adduced to insure that in equilibrium the unemployment rate for each tribe is positive, or to put it another way, there does not exist any tribe for which the urban wage is lower than the rural wage. However, the reader should be clear in the original Harris-Todaro setting of a rigid wage, Assumption 2.1.1 leads us into difficulties.[6] In such a context, a direct translation of this assumption yields

Assumption 2.1.1': For all i, for all $w_{ir} > 0$, $w_{ir} < \Omega(\cdot) = T_i$.

This assumption makes little sense since it keeps open the possibility that each of the rigid wages T_i be unbounded numbers, there being no way of telling *á priori* what the equilibrium rural wage would be. Corollary 2.1 below presents a version of our existence result which takes this difficulty into account.

We can now present

Theorem 2.1.1: *Under Assumptions 2.1.1 to 2.1.3, there exists at least one Harris-Todaro equilibrium.*

In the special case of exogenously-given urban wages, i.e., $\Omega(\cdot) = T_i$ for all i, Theorem 2.1 can be specialized and stated in a way that brings out its dependence solely on

Assumption 2.1.4: F_u and for all i, F_{ir} give rise to continuously differentiable, strictly decreasing demand functions for labor. The production functions F_{ir} also satisfy Inada conditions, i.e., for all i.

$$\partial F_{ir}(0,K_{ir})/\partial L_{ir} = \infty; \quad \partial F_{ir}(\infty,K_{ir})/\partial L_{ir} = 0 \qquad (2.1.4)$$

We can now state[7]

Corollary 2.1.2: *Under Assumption 2.1.4, for all i, there exist T_i^* such that for all $T_l > T_i^*$, there exists at least one Harris-Todaro equilibrium with exogenous wages.*

Corolarry 2.1.2 avoids the difficulty inherent in Assumption 2.1.1'. If for all i, $T_i = T_i^*$, one obtains competitive equilibria with no unemployment; on the other hand, $T_i < T_i^*$ for even one i would generate a negative value of λ_i and hence negate the existence of Harris-Todaro equilibria.[8]

2.2 Local Uniqueness of Equilibrium

In this section we show that for "almost all" values of $\mathcal{L} = (\mathcal{L}_1, \ldots, \mathcal{L}_n)$, Harris-Todaro equilibria are locally unique and continuous in $\mathcal{L}$. Such a result is a minimal requirement for the validity of comparative-statics exercises once we are guaranteed that equilibria exist. We shall need

Assumption 2.2.1: For all i, $\phi_i(\cdot)$ is a continuously differentiable function of w_r.

Theorem 2.2.1: *Under Assumption 2.2.1, the set of $\mathcal{L}$ for which Harris-Todaro equilibria are not locally unique and continuous in $\mathcal{L}$ is closed and of Lebesgue-measure zero.*

Remark 2.2.2: Under the hypothesis of Theorem 2.2.1, the set of Harris-Todaro equilibria is finite for all $\mathcal{L}$ in a closed set of Lebesgue measure zero.

2.3 Stability of Equilibrium

In this section we ask whether a Harris-Todaro equilibrium, if disturbed, will have tendencies to establish itself. We put forward an adjustment process and investigate conditions under which Harris-Todaro equilibria are stable in terms of this process.[9] These conditions revolve around gross-substitutability and unfortunately, turn out to be fairly strong. Nevertheless, they are worth having if only to provide a benchmark against which the assumptions we make for comparative statics results are to be judged. As Neary [15] has recently noted, a number of paradoxes in trade theory have arisen as a result of the equilibria being unstable in terms of natural adjustment processes.

The dynamic process $\mathcal{P}$ that we study is given by

$$Dw_{ir} = \phi_i\{((L_{ir} + (1+\lambda_i)L_{iu})/\mathcal{L}_i) - 1\},\ \phi_i'(0) > 0,\ \phi_i(0) = 0 \quad (2.3.1a)$$

$$D\lambda_i = \psi\{(\Omega_i(\cdot)/(1+\lambda_i)w_{ir}) - 1\},\ \psi_i'(0) > 0,\ \psi_i(0) = 0 \quad (2.3.1b)$$

where i runs from 1 to n and D denotes the differential operator.

Equation (2.3.1a) states that rural wages for the i^{th} tribe go up if there is positive excess demand in the i^{th} labor market and go down if the excess demand is negative. $\phi_i(\cdot)$ specifies the speed of adjustment of the rural wage. Equation (2.3.1b) relates the rate of change of unemployment of the i^{th} tribe to the discrepancy between expected wages. If the expected urban wage of the i^{th} tribe is greater than the rural wage, i.e., $\Omega_i(\cdot)/(1+\lambda_1) > w_{ir}$, there is increased migration to the city and the unemployment rate rises. Depending on the context, it may be useful to regard λ_i as a flow and, in such a case, $D\lambda_i$ can be interpreted as the rate of change of this flow, i.e., migrants come to the city or leave it at an increased rate.

Before presenting our results, we shall need some further notation. Let

$$e_w^i = \frac{\partial \log \Omega(\cdot)}{\partial \log w_{ir}} \text{ and } e_\lambda^i = (1+\lambda)\frac{\partial \log \Omega(\cdot)}{\partial\lambda}$$

Thus the e's refer to elasticities which pertain to the urban wage.

We shall need the following assumptions

<u>Assumption 2.3.1</u>: For all i, and for all w_{ir}, λ_i, $1 \geq e_w^i \geq 0$ and $e_\lambda^i \leq 0$.

<u>Assumption 2.3.2</u>: The Hessian of F_u given by $H = [\partial^2 F_u/\partial L_{iu}\partial L_{ju}]$ is

symmetric and negative definite. In addition $\partial^2 F_{ir}/\partial L_{ir}^2 < 0$ for all i.

Assumption 2.3.3: For all i and j, labor of the i^{th} tribe is a *gross substitute* for labor of the j^{th} tribe, i.e., $\eta_{iu}^{j} > 0$ for all $i \neq j$ where $\eta_{iu}^{j} = \partial \log L_{iu}/\partial \log w_{ju}$.

A justification of Assumption 2.3.1 hinges on the microfoundation of $\Omega_i(\cdot)$ for which the reader is again[10] referred to Khan [13]. Assumption 2.3.2 is innocuous. In the sequel, reference to stability of equilibrium is to be understood to mean stability in terms of the adjustment process $\mathcal{P}$ given by (2.3.1). We can now state

Theorem 2.3.1: *Under Assumption 2.3.1 to 2.3.3, locally unique Harris-Todaro equilibria are partially locally asymptotically stable*[11] *in the sense that $w_r(t)$ locally converges to w^* if $\lambda(t)$ is held fixed at λ^* and vice versa.*[12]

Theorem 2.3.1 is, unfortunately, only a partial answer to the stability question. A motivation for a more complete answer can be had by examining the matrix Δ given in Figure 3. Δ represents the matrix of partial derivatives of equations 2.3.1 if we disregard the functions $\phi_i(\cdot)$ and $\psi_i(\cdot)$. Assumptions 2.3.1 to 2.3.3 guarantee, as well as become clear in the proofs that the diagonal blocks have eigenvalues with negative real parts. Thus, for complete local asymptotic stability, we have to ensure that when the off-diagonal blocks are brought into the picture, they do not upset anything. In other words, the diagonal blocks *dominate* the off-diagonal blocks. This is precisely the force of the next assumption. Let ℓ_{ir} denote $L_{ir}/\mathscr{L}_i$ and ℓ_{iu} denote $L_{iu}(1+\lambda_i)/\mathscr{L}_i$.

Assumption 2.3.4: There exist positive numbers d_1 and d_2 such that

$$\text{(i)}\quad d_1 \underset{i}{\text{Max}}\{\left|(\frac{\ell_{ir}}{\ell_{iu}})n_{ir} + e_w^i n_{iu}^i\right| + \sum_{j \neq i} e_w^j n_{iu}^j\} > d_2 \underset{i}{\text{Max}}\{(1 + n_{iu}^i e_\lambda^i) + \sum_{j \neq i} |n_{iu}^j e_\lambda^j|\}$$

$$\text{(ii)}\quad d_2 \underset{i}{\text{Max}}\{|e_\lambda^i - 1|\} > d_1 \underset{i}{\text{Max}}\{|e_w^i - 1|\}$$

We can now state

Theorem 2.3.2: *Under Assumption 2.3.1 to 2.3.4, Harris-Todaro equilibria are locally unique and locally asymptotically stable.*

Under Assumption 2.3.1, Assumption 2.3.4(ii) is automatically fulfilled with $d_1 = d_2 = 1$. It is easy to check that given Assumptions 2.3.1 to 2.3.3, a sufficient condition for the validity of Assumption 2.3.4 is simply

$$|n_{ir}| > (\ell_{iu}/\ell_{ir})(1 + \sum_{j=1}^{n} |n_{iu}^j e_\lambda^j|) \text{ for all } i.$$

It is also easy to check that in a setting with one rural sector, Assumptions 2.3.1 to 2.3.3 insure that Assumption 2.3.4 is redundant.

We conclude this subsection with the remark that Theorem 2.3.2 strengthens the local uniqueness result given as Theorem 2.2.

3. DEVELOPMENT POLICIES

In this section we analyze four policy issues; namely, immigration policy, sector-specific capital inflows, minimum wage legislation and employment quotas. In keeping with the basic thrust of our paper we shall be primarily concerned with tracing out the differing welfare-effects of a particular policy on the variety of groups in our stylized economy.

The model can be usefully thought of in terms of Figure 2 which brings out the fact that the only linkage between any two tribes occurs through the city. This rather straightforward observation is of some consequence for the analysis to follow because it emphasizes the essential block-diagonal structure of the model. If we ignore this and follow the route we have taken so far by working with the independent variables $(w_{ir}, \lambda_i)_{i=1}^{n}$, we end up with the Δ matrix[13] we used in our stability analysis, the structure of whose inverse is far from evident. With such variables the model is reduced to an extent that is useful for the purposes of Section 2 but which obscures its essential simplicty in the context of comparative static analysis.

In this section we shall work with the independent variables $(L_{ir}, L_{iu}, \lambda_i)_{i=1}^{n}$ and reduce the model to its primal form:

$$L_{ir} + (1+\lambda_i)L_{iu} = \mathcal{L}_i \qquad i=1,\dots,n \qquad (3.1a)$$

$$p_{ir}(\partial F_{ir}/\partial L_{ir})(1+\lambda_i) = p_u(\partial F_u/\partial L_{iu}) \qquad i=1,\dots,n \qquad (3.1b)$$

$$p_u(\partial F_u/\partial L_{iu}) = \Omega_i(p_{ir}(\partial F_{ir}/\partial L_{ir}), \lambda_i, \mathcal{T}_i) \qquad i=1,\dots,n \qquad (3.1c)$$

Total differentiation of the above system of equations yields

$$\begin{array}{c} 2n \\ \\ n \end{array}\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]\left[\begin{array}{c} \hat{x} \\ \hline \hat{L}_u \end{array}\right] = \left[\begin{array}{ccc|ccc} A_1 & & & b_1 & & \\ & \ddots & & & \ddots & \\ & & A_n & & & b_n \\ \hline C & \cdots & C_n & & D & \end{array}\right]\left[\begin{array}{c} \hat{x}_1 \\ \vdots \\ \hat{x}_n \\ \hline \hat{L}_{iu} \end{array}\right] = \left[\begin{array}{c} \hat{u}_1 \\ \vdots \\ \hat{u}_n \\ \hline \hat{v} \end{array}\right] \qquad (3.2)$$

where all submatrices except D are block-diagonal such that

$$A_i = \begin{bmatrix} \ell_{ir} & \ell_{iu} \\ (1 - e^i_w)/\eta_{ir} & 1 - e^i_\lambda \end{bmatrix}, \; b_i = \begin{bmatrix} \ell_{iu} \\ 0 \end{bmatrix}, \; c_i = (\frac{-e^i_w}{\eta_{ir}}, -e^i_\lambda), \; d_{ij} = \frac{1}{\eta^i_{ju}} \qquad (3.3)$$

and where for any variable z,[14] $\hat{z}$ denotes the proportional change dz/z, T denotes transpose, and

$$\hat{x}_i = (\hat{L}_{ir}, \hat{\lambda}_i)^T \qquad (3.4a)$$

$$\hat{u}_i = (\hat{\mathcal{L}}_i,\ e^i_{\mathcal{T}}\hat{\mathcal{T}}_i - (1 - e^i_w)\hat{p}_{ir} - (1 - e^i_w)(\hat{K}_{ir}/\rho_{ir}))^T \qquad (3.4b)$$

$$\hat{v}_i = (e^i_w\hat{p}_{ir} + e^i_{\mathcal{T}}\hat{\mathcal{T}}_i + e^i_w\hat{K}_{ir} - \hat{p}_u - (\hat{K}_u/\rho_{iu})) \qquad (3.4c)$$

with $\rho_{ir} = \partial \log K_{ir}/\partial \log w_{ir}$ and $\rho_{iu} = \partial \log K_u/\partial \log w_{iu}$ (3.4d)

The equation system (3.2) should be seen as a formalization of Figure 2 with the central linking role of the urban center being parametrized by the elements of D. Rewriting (3.2) in terms of the inverse, we obtain

$$\begin{bmatrix} \hat{x} \\ \hat{L}_u \end{bmatrix} = \begin{bmatrix} \alpha(2n \times 2n) & \beta(2n \times n) \\ \gamma(n \times 2n) & \delta(n \times n) \end{bmatrix} \begin{bmatrix} \hat{u} \\ \hat{v} \end{bmatrix} \qquad (3.5)$$

with the order of each submatrix as indicated in the brackets. The qualitative information on the entries in δ is crucial for signing the entries in α, β and γ and such information is furnished in the following Lemma.

Lemma 3.0: *Under Assumptions 2.3.1 to 2.3.3, $\delta_{ij} \leq 0$ for all i,j.*

We shall supply a proof of this lemma in Appendix III. However, the reader should note that $\delta = (D - CA^{-1}B)^{-1}$ and that $CA^{-1}B$ is a diagonal matrix with a typical entry

$$\ell_{iu}(e^i_\lambda - e^i_w)/\Delta_i n_{ir};\ \Delta_i = |A_i| = \ell_{ir}(1 - e^i_\lambda) - \ell_{iu}(1 - e^i_w)/n_{ir}$$

Once we have δ, the remaining matrices are easy. By straightforward calculations, the reader can derive the following.

$$-\gamma = \begin{bmatrix} \\ \delta \\ \\ \end{bmatrix} \begin{bmatrix} c_1A_1^{-1} & & \\ & \ddots & \\ & & c_nA_n^{-1} \end{bmatrix} = \begin{bmatrix} \delta_{11}c_1A_1^{-1} & \cdots & \delta_{1n}c_nA_n^{-1} \\ \vdots & & \vdots \\ \delta_{n1}c_1A_1^{-1} & \cdots & \delta_{nn}c_nA_n^{-1} \end{bmatrix} \tag{3.6a}$$

$$\text{where } c_iA_i^{-1} = ((e^i_\lambda - e^i_w),\ \ e^i_w\ell_{iu} - e^i_\lambda\ell_{ir}n_{ir})/\Delta_i n_{ir} \tag{3.6b}$$

$$-\beta = \begin{bmatrix} b_1A_1^{-1} & & \\ & \ddots & \\ & & b_nA_n^{-1} \end{bmatrix} \begin{bmatrix} \delta \end{bmatrix} = \begin{bmatrix} \delta_{11}b_1A_1^{-1} & \cdots & \delta_{1n}b_1A_1^{-1} \\ \vdots & & \vdots \\ \delta_{n1}b_nA_n^{-1} & \cdots & \delta_{nn}b_nA_n^{-1} \end{bmatrix} \tag{3.7a}$$

$$\text{where } b_iA_i^{-1} = (\ell_{iu}(1 - e^i_\lambda),\ -\ell_{iu}(1 - e^i_w)/n_{ir})^T/\Delta_i \tag{3.7b}$$

$$\alpha = A^{-1}(I - B\gamma) = \begin{bmatrix} \alpha_{ij} \end{bmatrix} \tag{3.8a}$$

$$= \begin{bmatrix} A_1^{-1}(I + b_1\delta_{11}c_1A_1^{-1}) & \cdots\cdots & A_1^{-1}b_1\delta_{11}c_nA_n^{-1} \\ \vdots & & \vdots \\ A_n^{-1}b_n\delta_{n1}c_1A_1^{-1} & \cdots\cdots & A_n^{-1}(I + b_n\delta_{nn}c_nA_n^{-1}) \end{bmatrix} \tag{3.8b}$$

where

$$\alpha_{ij} = \frac{\delta_{ij}}{\Delta_i\Delta_j n_{jr}} \begin{bmatrix} \ell_{iu}(1 - e_\lambda^i)(e_\lambda^j - e_w^j) & \ell_{iu}(1 - e_\lambda^i)(e_w^j\ell_{ju} - e_\lambda^j\ell_{jr}n_{jr}) \\ -\ell_{iu}(1 - e_w^i)(e_\lambda^j - e_w^j)/n_{ir} & -\ell_{iu}(1 - e_w^i)(e_w^j\ell_{ju} - e_\lambda^j\ell_{jr}n_{jr})/n_{ir} \end{bmatrix} \tag{3.8c}$$

$$\alpha_{ii} = \frac{1}{\Delta_i} \begin{bmatrix} (1 - e_\lambda^i)\{1 + \frac{\delta_{ii}}{\Delta_i n_{ir}}\ell_{iu}(e_\lambda^i - e_w^i)\} & -\ell_{iu}\{1 - \frac{\delta_{ii}}{\Delta_i n_{ir}}(1 - e_\lambda^i)(e_w^i\ell_{iu} - e_\lambda^i\ell_{ir}n_{ir})\} \\ \frac{-(1 - e_w^i)}{n_{ir}}\{1 + \frac{\delta_{ii}}{\Delta_i n_{ir}}\ell_{iu}(e_\lambda^i - e_w^i)\} & \ell_{ir}\ \frac{-\delta_{ii}\ell_{iu}}{\Delta_i n_{ir}}(1 - e_w^i)(e_w^i\ell_{iu} - e_\lambda^i\ell_{ir}n_{ir}) \end{bmatrix}$$

(3.8d)

In the sequel we shall denote the ij^{th} element of a particular matrix α_{ij} by $\alpha_{ij}(ij)$. We shall also use the hypothesis of Lemma 3.0 as a Standing Hypothesis for the discussion to follow.

3.1 Population Growth

In this subsection we discuss the positive and normative effects of the growth in the numbers of a particular tribe. This is a problem of some importance for development theory. Firstly, attempts at population control in the LDC's meet the greatest resistance when they are perceived to be principally directed at a particular ethnic group. It is interesting to see, albeit in our simple stylized setting, whether there is in fact an economic rationale for this resistance. Secondly, there is substantial emigration especially from South Asian economies to the Middle Eastern Gulf States. This emigration has tended to be region-specific partly as a result of information flows and partly as a consequence of the costs of migration becoming lower for the group whose members are already there. It is of interest to see what such emigration implies for the other groups in the economy.

The algebraic groundwork for the relevant comparative static exercise has already been laid out above. We just have to read off the various entries. Thus $\hat{L}_{ir}/\hat{\mathcal{L}}_j = \alpha_{ij}(11)$, $\hat{\lambda}_i/\hat{\mathcal{L}}_j = \alpha_{ij}(21)$ and $\hat{L}_{iu}/\hat{\mathcal{L}}_j = \gamma_{ij}(1)$. After again reminding the reader that we are assuming the hypothesis of Lemma 3.0, we can collect these results as

Proposition 3.1.1: *An increase in the population of a particular tribe leads to an increase in the urban employment of all tribes. It leads to a decrease in rural employment as well as the unemployment rates of all other tribes. Finally, it leads to an increase in rural employment and unemployment rate of its own tribe if and only if*

$$1 + \frac{\ell_{iu}\delta_{ii}(e^i_\lambda - e^i_w)}{\Delta_i\eta_{ir}} > 0 \qquad (3.1.1)$$

Thus, an increase in the population of the i^{th} tribe leads to increased urban employment of all groups and to a decrease in the rural work force of all other groups. Put differently, and somewhat boldly, Punjabi emigration to the Middle East leads to an increase in Pathan or Baluchi urban unemployment rates.[15]

These results are interesting and at first seem counter-intuitive. As population of a particular group increases, one would expect labor of that group to become cheaper and to be more intensively employed in the city. Given the overriding assumption of labor substitutability one would expect labor of all other groups to be substituted against, leading to their increased urban unemployment and to increased rural employment rates. This is contrary to what the results suggest. The reason is that these are only the first round effects and equilibrium is not attained at this stage. As the labor of all other groups is let off from the city, it too becomes cheaper. Since the urban wages are determined from rural wages and urban unemployment rates, they fall to an extent that nullifies the first round effects leading to Proposition 3.1.1.

Some special cases of Proposition 3.1.1 may be worth pointing out briefly. If the group whose population increases has a fixed rigid wage in the urban center, i.e., $e^i_\lambda = e^i_w = 0$, (3.1.1) is automatically fulfilled and thus the elasticities of substitution for all other sectors are strictly irrelevant. In this case, the urban and rural employment rates of all other tribes are unaffected. It is worth reminding the reader that it is precisely this case that was studied by Harris-Todaro in a two-sector context.

Another special case is of interest. This is a situation when there is an exogenously given proportional wage differential between the urban and rural wages of the tribe whose numbers increase. In this case $e^i_w = 1$ and (3.1.1) applies only to rural employment since the own unemployment rate is independent of population growth.

We can now use the results of Proposition 3.1.1 to discuss the normative effects of population growth of a particular tribe. As any reader of Bhagwati [3] knows, population growth may well be immiserizing in terms of aggregate welfare, given the variety of distortions in our model. It is of some interest to see how the above result could be sharpened and also generalized to the welfare of the variety of groups in our model.

Consider first aggregate welfare given by

$$W_A = (\sum_{i=1}^{n} p_{ir}X_{ir}) + p_u X_u \tag{3.1.2}$$

It is easy to check that

$$\partial W_A/\partial \mathcal{L}_i = -\sum_{j=1}^{n} w_{jr}L_{ju}(\partial\lambda_j/\partial\alpha_i) \tag{3.1.3}$$

This is intuitively appealing as it says that aggregate welfare will go up if the size of urban unemployment of each group, weighted by its shadow wage, goes down. We have seen from Proposition 3.1.1 that this is indeed so for all $j \neq i$. Thus the reader can write for himself necessary and sufficient conditions for population growth to be immiserizing in terms of aggregate welfare. It should be noted that for the special case $e_w^i = 1$, population growth is *never* immizerizing.

The situation as regards regional welfare is much more clearcut, when we consider the value of regional output $W_R^j = p_{jr}X_{jr}$ as an index of welfare. Then $\frac{\partial W_R^j}{\delta \mathcal{L}_i} = w_{jr}\frac{\partial L_{jr}}{\delta \mathcal{L}_i}$ shows clearly that with population growth in the i^{th} tribe, j^{th} regional welfare falls.[16] Further if the necessary and sufficient condition in Proposition 3.1.1 is not satisfied, then the welfare of all the rural regions taken together falls. Thus inflow of refugees in a particular region can reduce aggregate regional welfare.

It is interesting to note that if the necessary and sufficient condition of Proposition 3.1.1 does not hold, then the welfare of the working class as a whole given by $\sum_j w_{jr}L_{jr} + \sum_j w_{ju}L_{ju}$ improves and that of the rural landlords given by $\sum_j p_{jr}X_j - \sum_j w_{jr}L_{jr}$ worsens.

The reader can now provide for himself necessary and sufficient conditions under which tribal welfare $W_T^j(= p_{jr}X_{jr} + w_{ju}L_{ju})$ and the urban capitalists' welfare $W_u(= p_uX_u - \sum_{j=1}^{n} w_{ju}L_{ju})$ improves or worsens.

3.2 Region-Specific Capital Inflows

The importance of the study of region-specific capital inflows needs little justification. In the context of South-East Asian economies, the problem has attained even more significance once one views the introduction of the Green Revolution technology with its attendant changes in irrigation methods and use of fertilizers as a capital inflow. Huge capital investments were made in order to increase the effective area under cultivation and its fertility. The socio-economic implications of this technology and what it has achieved have been extensively discussed elsewhere but the general equilibrium repercussions of inflow of capital has been seldom modelled. The following proposition brings out the nature of such repercussions in the context of our model and that is without even introducing essential complications like inequality of land holding and indebtedness of the rural peasantry.

Proposition 3.2.1: *An increase in the inflow of capital in the i^{th} region will increase rural employment and decrease urban employment for members of the i^{th} region if*

$$\xi_i = \frac{e^i_w \ell_{iu}}{\eta_{ir}} - e^i_\lambda \ell_{ir} \geqq 0 \tag{3.2.1}$$

Satisfaction of this condition also ensures an increase in rural employment of all other regions, a decrease in the urban employment of their members and a rise in their unemployment rates.[17]

So under the above mentioned condition total rural employment in the economy increases and total urban employment falls. The decrease in urban employment of other regions is because of second round effects dominating the first round ones as explained in the context of population growth in Section 3.1. It is worth pointing out that if the members of the i^{th} region face rigid urban wages, i.e., $e^i_W = e^i_\lambda = 0$, then the above proposition is automatically true.[18]

We observe readily that under (3.2.1) the welfare of the working class as a whole deteriorates, that of the landlords improves and that the regional welfare of *all* the regions improves. Moreover, following Calvo [6], if we assume that the members of the i^{th} tribe only[19] are unionized in the urban sector[20] then their unemployment rates remain unchanged and combining this with (3.2.1) and (3.1.3) we get the result that the overall welfare of the economy deteriorates as a result of such capital inflow. The reader can provide conditions for himself for the more general case.

Since (3.2.1) is a sufficient condition only, it is worthwhile to look at conditions under which capital flows to any rural region will lead to a decrease in employment there. It is important, because, in the context of labor surplus LDC's, such capital inflows are thought of as undesirable since they tend to be labor replacing.

Proposition 3.2.2: *An increase in the inflow of capital in the i^{th} region will reduce rural employment there if*

$$\xi_i < 0 \quad \text{and} \tag{3.2.2}$$

$$(1 - e_w^i)(1 - e_\lambda^i)\frac{\delta_{ii}}{\Delta_i}\xi_i + (1 - e_\lambda^i)\delta_{ii}e_w^i - (1 - e_w^i) > 0 \tag{3.2.3}$$

We conclude this subsection with the observation that a change in p_{ir}, the price of the i^{th} rural commodity, has identical qualitative effects as a change in K_{ir}, the capital stock of the i^{th} region. This can be seen on inspection of (3.4b and c).

3.3 Minimum Wage Legislation

The policy issue we consider now is a change or a repeal of the minimum wage laws. It is worth reminding the reader that we consider a situation where the urban, minimum wage of the i^{th} tribe is greater than the market-determined wage.

Let $w_{iu} = \mathcal{J}_i$ be the institutionally fixed minimum wage for the i^{th} tribe and note that in this case $e_w^i = e_\lambda^i = 0$ and $e_{\mathcal{J}}^i = 1$.

Proposition 3.3.1:

a) *An increase in the urban minimum wages of the i^{th} tribe will* *lead to a decrease in their urban employment and an increase in their unemployment rate. It will increase rural employment if and only if*

$$|\delta_{ii}| > 1 \tag{3.3.1}$$

b) It necessarily leads to a decrease in urban employment of all other tribes and an increase in their rural employment and urban unemployment rates.

The above result is interesting in that it gives conditions under which an increase in the urban wage rate of a particular ethnic group will *increase* the overall unemployment rate $\sum_{i=1}^{n} \lambda_i$. Further, if (3.3.1) is not satisfied, then the level of unemployment of the i^{th} tribe will increase. This condition in a two sector setting reduces to the elasticity of labor demand in the urban sector being greater than unity which is precisely the Corden-Findlay [7] result. But since theirs is a two-sector setting they do not have any proposition like 3.3.1b.

Given Proposition 3.3.1 and the various welfare functions discussed in Section 3.1 the reader can supply for himself as detailed a welfare analysis as he desires; the results do not merit a case by case treatment here.

3.4 Employment Quotas

The last policy issue we consider is in connection with the government fixing the quota of urban employment of a particular tribe. In a country like India where the labor force is heterogeneous, one of the ways the government has been trying to bring the backward and socially handicapped people classified under "scheduled castes and tribes" to the forefront is by directly increasing their opportunities for employment in the urban centers.[21] Of course, employment quotas for socially backward groups are not only found in India but in many other LDC's.

Recall that the size of the urban unemployed of the i^{th} tribe is given by $\lambda_i L_{iu}$ and let the i^{th} region be the one whose members enjoy an urban employment quota. Let $k_i = \dfrac{\lambda_i L_{iu}}{\mathcal{L}_i}$ be the unemployment rate specified initially by the government. Such a quota leads to two modifications in the equation system (3.2) namely (i) the submatrix A_i has zero in place of ℓ_{iu} leading to $\Delta_i = |A_i| = \ell_{ir}(1 - e^i_\lambda)$ and (ii) $((1 - k_i)\hat{\mathcal{L}}_i - k_i\hat{k}_i)$ replaces $\hat{\mathcal{L}}_i$ in $\hat{u}_i$.

Proposition 3.4: *An increase in the employment quota of the i^{th} tribe will increase urban employment and decrease both rural employment and urban unemployment rate of all tribes.*

Since, with a rise in employment quota the unemployment rate falls for the i^{th} tribe, there is an increased tendency for them to migrate which reduces their rural employment. Intuition suggests that since the urban employers have to employ a higher number of i^{th} tribal people, they will substitute against the members of the other tribes and their employment levels will fall. But this fall in employment causes an excess supply of rural labor leading to a fall in their rural wage rates and hence urban wage rates and eventual increased employment. Since their unemployment rates drop too, increased migration takes place causing a fall in their rural employment. The reader can readily see that given the welfare functions in Section 3.1, the overall workers' welfare will rise and that of landlord's fall. Regional value of production and hence regional welfare will fall too. Again, conditions can be derived to sign overall welfare, urban capitalists' welfare and tribal welfare.

4. OTHER POLICY ISSUES

In the introduction we pointed out that the analytical structure of this paper could handle a wide range of policy issues other than the ones we have already discussed. Our purpose in this section is to briefly point out the different situations in which this framework can be useful.

4.1 Manpower Planning

One of the assumptions we have made throughout the paper is that the cost per man to the urban employer is identical to the wage that is paid out. Suppose this is not so and the urban employer also bears some additional training costs ψ that are not passed on to the employee.[22] In this case, (1.5b) is replaced by $p_u \frac{\partial F_u}{\partial L_{iu}} = w_{iu} + \psi_i(\cdot)$, where $\psi(\cdot)$ typically depends on $\mathcal{T}_i$ and the quit rate q_{iu} which would itself depend on λ_i. It is easy to check that our methods can easily handle this case.

4.2 Urbanization

An extensive analysis of the effects of capital inflow in the rural sector has been made in Section 3.2. The positive and normative effects of capital inflow in the urban sector can also be arrived at along the same lines. This is important since LDC's have gone through extensive urbanization in the recent past. Equally important is an analysis of excessive pressure on the urban center as a result of migration. This has been of some concern to urban planners because of the resource waste associated with it, e.g., time waste due to congestion, high maintenance costs of rapidly deteriorating sanitage and sewerage systems, etc. Our framework can accommodate analysis of such issues.

At any point in time the total number of people in the urban center is $\sum_{i=1}^{n} L_{iu}(1+\lambda_i)$. Let the cost associated with the population pressure be given by a function

$$C = \phi\left[\sum_{i=1}^{n} L_{iu}(1+\lambda_i)\right] \quad \text{where } \phi' > 0,\ \phi'' > 0 \tag{4.2.1}$$

Total welfare is now given by

$$W_A = p_n X_n + \sum_{i=1}^{n} p_{ir} X_{ir} - \phi\left[\sum_{i=1}^{n} L_{iu}(1+\lambda_i)\right] \tag{4.2.2}$$

The policy measures we have talked about can now be evaluated in the light of this modified welfare function and the costs associated with urbanization will play a crucial role in dictating the choice of policies.

4.3 Wage Subsidies

Wage subsidies have been discussed at length in Khan [13] in the context of a two sector model with intersectorally mobile capital. We leave it to the reader to discuss the consequence of such subsidies in the context of our model here. It is worth pointing out, however, that in the case of $w_{iu} = \mathcal{T}_i$, the effects of wage subsidies are identical to changes in $\mathcal{T}_i$, a problem already discussed above in section 3.3.

4.4 Sub-Optimal Tariff Policy

Finally, it is worth pointing out the relevance of our model to questions dealing with the positive and normative effects of tariffs in a multisectoral economy riddled with distortions. In particular, one can study this problem under a variety of assumptions pertaining to the disbursement of tariff revenue.

5. CONCLUDING REMARKS

In this paper we have presented a model of a small, open economy which can be used to study the effects of various policy changes on the distribution of income between landlords, laborers, capitalists and on different regional groups. We obtain worthwhile comparative-static results in a setting for which the existence, uniqueness and local stability of equilibria cannot be deduced routinely from corresponding results pertaining to the general competitive models as set out in Debreu [9]. It is thus satisfying that the hypothesis $0 \leq e^i_w \leq 1$, $e^i{}_\lambda \leq 0$, for example, has a role to play in the existence, uniqueness, stability and comparative static results.

Our basic model can be seen as a multisectoral generalization of the Ricardo-Viner model, one that is somewhat different from the generalization presented by Jones [12]. If we let $X_u = F_u(\sum_{i=1}^{n} L_{iu}, K_u)$, and $\Omega_i(w_{ir}, \lambda_i, \mathcal{T}_i) =$

$w_{ir} \equiv w_r$ in equations (2.2b) and (2.3a) respectively, we obtain Jones' model in [12]. As such we have presented a multicommodity trade model that may also be well-suited for answering trade-theoretic questions particularly for LDC's.

APPENDIX I: PROOF OF THEOREM 2.1.1

We introduce the formal proof by sketching the underlying ideas; focus on the i^{th} market and assume that $w_r(i)$ is fixed. Given Assumptions 2.1.2 and 2.1.3, we can find the value of w_{ir} tht will clear that i^{th} market, i.e., equate $\phi_i(w^o_{ir}, w_r(i))$ to zero. Figure 1 illustrates the procedure for doing this. Given the boundary conditions, we can find $\underline{m}$, $\bar{m}$ at which $\phi_i(\cdot, w_r(i))$ is respectively positive and negative and an application of Bolzano's Theorem (see [1, p. 73]) yields the result. We thus get a mapping ψ from $w_r(i)$ to w_{ir}. Doing this for all the markets, we obtain a mapping $\psi(\cdot) = (\psi_1(\cdot), \ldots, \psi_n(\cdot))$ which takes the set of rural wages into itself. A fixed point of such a mapping, if it existed, would give us a Harris-Todaro equilibrium. However, the problem is that we cannot prove the existence of such a fixed point through any of the usual theorems since we cannot limit $\psi(\cdot)$ to a compact set. This is because the bounds $\underline{m}$, $\bar{m}$ vary from market to market and depend on $w_r(i)$. To overcome this difficulty, one has to choose bounds $1/m$ and m for all markets right from the start and restrict each $\phi_i(\cdot)$ to the closed interval $[1/m, m]$. If, in this interval, we do not find a wage that clears that market, we choose one that minimizes absolute value of excess demand, i.e., in Figure 1, A' when the interval is AA' or B when it is BB'. We then proceed as above, and find a fixed point of the mapping ψ. As m is allowed to get larger and larger, we can generate a sequence of fixed points. We now bring in the boundary conditions embodied in Assumption 2.1.3 to show that there exists a Harris-Todaro equilibrium for large enough m. Assumption 2.1.1 is used to generate non-negative equilibrium values of λ_i. In conclusion, it is worth mentioning that it is the absence of a direct analogue of Walras' Law for Harris-Todaro equilibria that necessitates the use of an argument different from the one conventionally used to prove the existence of competitive equilibria.

We can now present

Proof of Theorem 2.1.1: We shall need the following notation, additional to that provided in the text.

$$M = \{x \in R \mid (1/m) \leq x \leq m\}, \text{ m a positive integer;}$$

$$M^j = \prod_{k=1}^{j} M; \; C_i : M^n \to M; \; \psi_i : M^n \to M \text{ and } \psi : M^n \to M^n$$

$$C_i(w_r) = \{z \in M \mid \phi_i(z, w_r(i)) \geq 0\} \cup \{1/m\}$$

$$\psi_i(w_r) = \{w_{ir} \in C_i(w_r) \mid w_{ir} \text{ minimizes } \phi_i(w_r) \text{ over } C_i(w_r)\}$$

$$\psi = (\psi_1, \psi_2, \ldots, \psi_n)$$

$C_i(w_r)$ is nonempty and bounded for all $w_r \in M^n$ and for all i. Given continuity of $\phi_i(\cdot)$ in w_r, it is also closed.

$C_i(\cdot)$ is a continuous correspondence over M^n. To show this, we need only consider the restriction of $C_i(\cdot)$ to M^{n-1}. We use $C_i(\cdot)$ or $C_i(w_r(i))$ to denote this restriction. The upper-semi-continuity of $C_i(\cdot)$ is straightforward. For lower semi-continuity, let $w_r^\nu(i) \to w_{ir}^o \in C(w_r^o(i))$. We have to construct a sequence $\{w_{ir}^\nu\}_\nu$ such that $w_{ir}^\nu \to w_{ir}^o$ and $w_{ir}^\nu(i)$ for all ν. The argument revolves around three cases.

Let $\phi_i(w_r^o) > 0$ and let

$$w_{ir}^{\nu} = w_{ir}^{o} - (1/\nu) \text{ if } w_{ir}^{o} = m$$

$$= w_{ir}^{o} + (1/\nu) \text{ otherwise}$$

There exists $\bar{\nu}$ such that $\nu \geq \bar{\nu}$ implies $\phi_i(w_{ir}^{\nu}, w_r^{\nu}(i)) > 0$. Suppose not, i.e., for all $\bar{\nu}^j$, there exists $\nu^j \geq \bar{\nu}^j$ such that $\phi_i(w_{ir}^{\nu_j}, w_r^{\nu_j}(i)) \leq 0$. $\underset{j\to\infty}{\text{Lt}}\ \phi_i(w_{ir}^{\nu_j}, w_r^{\nu_j}(i)) = \phi_i(w_r^o) \leq 0$, a contradiction. We can now easily construct the required sequence.

Let $\phi_i(w_r^o) = 0$ and $w_{ir}^o > (1/m)$. For each ν consider the sign of $\phi_i(w_{ir}^o, w_r^{\nu}(i))$. Since $\phi_i(\cdot)$ is a single-valued function, it cannot be zero. If it is positive, let $w_{ir}^{\nu} = w_{ir}^{o}$. If it is negative, let $w_{ir}^{\nu} = x^{\nu}$ where $\phi_i(x^{\nu}, w_r^{\nu}(i)) = 0$. Given that $\phi_i(\cdot)$ is a continuous function and that $\phi_i(0, w_r^{\nu}(i)) = \infty$, we are guaranteed by Bolzano's Theorem (see [1, p. 73]), that x^{ν} exists. The only question is whether $x^{\nu} \in M$. We can assert that there exists $\bar{\nu}$ such that $\nu \geq \bar{\nu}$ implies $x^{\nu} \in M$. Suppose not, i.e., for all $\bar{\nu}^j$, there exists $\nu^j \geq \bar{\nu}^j$ such that $x^{\nu_j} \notin M$. Certainly $x^{\nu_j} \leq w_{ir}^o$ for all j. If not, $\phi_i(x^{\nu_j}, w_r^{\nu_j}(i)) = 0$ implies $\phi_i(w_{ir}^o, w_r^{\nu_j}(i)) > 0$, given that $\phi_i(\cdot)$ is a decreasing function of w_{ir}^o. This case has been disposed of earlier. Thus x^{ν_j} is a bounded sequence and by the Bolzano-Weierstrass Theorem, (see [1, p. 43]), it must have a convergent subsequence $x^{\nu_{jk}}$. Let $\underset{k\to\infty}{\text{Lt}}\ x^{\nu_{jk}}$ be $\bar{x}$. Then $\underset{k\to\infty}{\text{Lt}}\ \phi_i(x^{\nu_{jk}}, w^{\nu_{jk}}(i)) = \phi_i(\bar{x}, w_r^o(i))$. Since ϕ_i is a single valued function, $\bar{x} = w_{ir}^o$, and since $w_{ir}^{o} > 1/m$, for all large enough k, $x^{\nu_{jk}} \varepsilon M$. We have our required contradiction. We also have a sequence to complete the argument for lower semi-continuity in this case.

The only case remaining is when $\phi_i(w_r^o) < 0$ or $\phi_i(w_r^o) = 0$ and $w_{ir}^o = 1/m$. In this case, let $w_{ir}^{\nu} = 1/m$ for all ν.

Given that $C(\cdot)$ is a continuous correspondence and that $\phi_i(\cdot)$ is a continuous function, we can appeal to a Theorem given in Debreu, (see [8], p. 19]), to assert that $\psi_i(\cdot)$ is an upper semicontinuous correspondence. Given that $\phi_i(\cdot)$ is a single-valued function, upper semi-continuity reduces to continuity of $\psi_i(\cdot)$.

ψ is thus a continuous mapping of M^n into itself. Since M^n is easily seen to be a nonempty, convex, compact subset of R^n, we can apply Brouwer's Fixed Point Theorem (see [8, p. 26]), to assert the existence of $w_r^m \varepsilon M^n$. Let $\lambda_i^m = \ell_i(w_{ir}^m)$ and $w_{iu}^m = (1+\lambda_i^m)w_{ir}^m$. Under Assumption 2.1, $\lambda_i^m > 0$.

If $\phi^i(w_r^m) = 0$ for all i, the proof is finished, but this will, of course, not be the case for arbitrary m. We will now show that there exists m^* such that for all $m \geq m^*$,

$$1/m^* < w_{ir}^m < m^* . \tag{A.1.1}$$

We first provide an argument for the upper bound. Suppose not, i.e., for all m_j^*, there exists $m_j \geq m_j^*$ and an i such that $w_{ir}^{m_j} \geq m_j$. Since i is between 1 and n, there at least one i_o for which $w_{i_or}^{m_j}$ is becoming arbitrarily large. By construction $\phi_{i_o}(w_{i_or}^{m_j}, w_r^{m_j}(i_o)) \geq 0$. But we know from Assumption 2.1.3 that there exists $\bar{j}$ such for all $j > \bar{j}, \phi_{i_o}(w_{i_or}^{m_j}, w_r^{m_j}(i_o)) < 0$, a contradiction. The argument for the lower bound is similar. Suppose for all m_j^*, there exists $m_j \geq m_j^*$ such that for some i_o $w_{i_o r}^{m_j} = (1/m_j)$. By construction, this implies that $\phi_{i_o}(w_{i_o r}^{m_j}, w_r^{m_j}(i_o)) \leq 0$. But, again we know from Assumption 2.1.3 that there exists $\bar{j}$ such that for all $j > \bar{j}, \phi_{i_o}(w_{i_o r}^{m_j}, w_r^{mj}(i)) > 0$, a contradiction.

Now let $w_r^* = w_r^{m^*}$, $\lambda_i^* = \lambda_i^{m^*}$, $w_{iu}^* = w_{iu}^{m^*}$, $L_{iu}^* = L_{iu}(w_u^*)$ and $L_{ir}^* = L_{ir}(w_{ir}^*)$, giving us our Harris-Todaro equilibrium. Certainly for all i, $\phi_i(w_r^*) = 0$. If not, we contradict (A.1.1) Q.E.D.

<u>Remark A1:</u> If we do not assume that $\phi(\cdot)$ is a decreasing function of w_{ir}, the proof fails on several counts, most important of which is probably the failure of lower semi-continuity of the correspondence $C_i(w_r(i))$. Figure 4 illustrates this. Of course, if there is only one market, i.e., only one equation 2.1.1, we need rely only on Balzano's Theorem and the simple argument given in the Idea of Proof will suffice. In this case, we do not need to assume that $\phi(\cdot)$ is a decreasing function of w_{ir} if we do not insist on a unique equilibrium.

<u>Remark A2:</u> It is well to remind the reader that Assumption 2.1.2 does not guarantee a unique Harris-Todaro equilibria.

APPENDIX II: PROOF OF THEOREM 2.2.1

The proof of Theorem 2.2.1 is a consequence of Sard's Theorem [9], which in a informal version states that the set of values taken by a differentiable function at points at which its derivatives are zero, is "small," i.e., of Lebesgue measure zero. We can now present

Proof of Theorem 2.2.2: Define a mapping $\hat{\phi}$ from the strictly positive orthant of R^n, R^n_{++}, into R^n, where

$$\hat{\phi}_i(w_r) = L_{ir}(w_{ir}/p_{ir}) + (1+\ell_i(w_{ir}))L_{iu}(w_r) \qquad (i=1,\ldots,n).$$

Then a Harris-Todaro equilibrium can be characterized by $\hat{\phi}^{-1}(\mathcal{L})$. Given Assumption 2.2.1 a routine application of Sard's Theorem, [9 , p. 388], allows us to deduce that the set C of $\mathcal{L}$ in R^n for which the Jacobian of $\hat{\phi}$ has rank smaller than n is of Lebesgue measure zero. Since the Jacobian of $\hat{\phi}$ is a continuous function of w_r, $\phi^{-1}(C)$ is a closed set. Hence C is a closed set. For any $\mathcal{L} \notin C$, the inverse function theorem, [1, p. 144], applies and the corresponding Harris-Todaro equilibria are locally unique, and continuous in $\mathcal{L}$. Q.E.D.

Proof of Remark 2.2.2: As in Debreu [9], it can be shown that for a compact subset K of R^n, $\phi^{-1}(K)$ is compact. This allows us to denote that the set of Harris-Todaro equilibria is finite for all $\mathcal{L}$ in a closed set of Lebesgue measure zero.

APPENDIX III

In this section, we provide the proofs of the results in Section 2.3. We begin with a

Lemma A.1: Under Assumptions 2.3.1 to 2.3.3, the matrices $A,(-B)$ in Δ, (see Figure 3 are stable Metzler matrices, such a matrix being one with diagonal elements negative, off-diagonal elements non-negative and the real parts of whose eigenvalues are negative.

Proof: The necessary conditions for profit maximization in the urban sector give us $(\partial F_u/\partial L_{iu}) = w_{iu}$, $(i=1,\ldots,n)$. Differentiating these equations with respect to w_{ju}, we obtain

$$[\partial^2 F_u/\partial L_{iu}\partial L_{ju}][\partial L_{ju}/\partial w_{iu}] \equiv H[\partial L_{ju}/\partial w_{iu}] = I$$

where I is an identity matrix of order n. Under Assumption 2.3.2, H is symmetric negative definite. Hence H^{-1} is symmetric and negative definite. Thus by Assumption 2.3.2 $[\partial L_{ju}/\partial w_{iu}]$ is a stable Metzler matrix. Now by a Theorem in Arrow, [2, p. 8], the class of stable Metzler matrices is closed under pre or post multiplication by a non-null, non-negative diagonal matrix. It can be easily checked that the class of symmetric, stable matrices is closed under addition of a negative, diagonal matrix. These facts along with Assumption 2.3.3, allow us to complete the proof. Q.E.D.

Proof of Theorem 2.3.1: Linearizing the differential equations (2.3.1) around a locally unique Harris-Todaro equilibrium, we obtain

$$Dw_r = hA(w_r - w_r^*) + hB(\lambda - \lambda^*)$$

$$D\lambda = gC(w_r - w_r^*) + gD(\lambda - \lambda^*)$$

where h and g are diagonal, matrices with typical entries $H_i'(0)$ and $G_i'(0)$ and A, B, C, D are square matrices constituting Δ, (see Figure 3). Using again, the theorem in Arrow quoted in the proof of lemma B.1 above, we can show that all the eigenvalues of hA and gD have negative real parts. Application of the Theorem in [11, p. 181], completes the argument. Q.E.D.

Proof of Theorem 2.3.2: The theorem is an easy consequence of two theorems in Okuguchi, [16], on matrices with dominant diagonal blocks. Adopt the Minkowski norm for a matrix with the absolute value norm for vectors. Then local uniqueness of Harris-Todaro Equilibria follows from Theorem 1 in [16] and local asymptotic stability from Theorem 3 in [16], and the Theorem in [11, p. 181]. Note that A and D are stable, Metzler matrices, the former by virtue of Lemma B.1, as is required for Theorem 3 in [16]. Q.E.D.

Proof of Lemma 3.0: Using the arguments in the proof of Lemma A1, it is easy to show that D is a stable Metzler matrix. Sinc $CA^{-1}B$ is a diagonal matrix, δ is stable Metzler matrix. Now by a theorem in Arrow [2, p. 7], the inverse of a stable, Metzler matrix has all non-positive entries. Q.E.D.

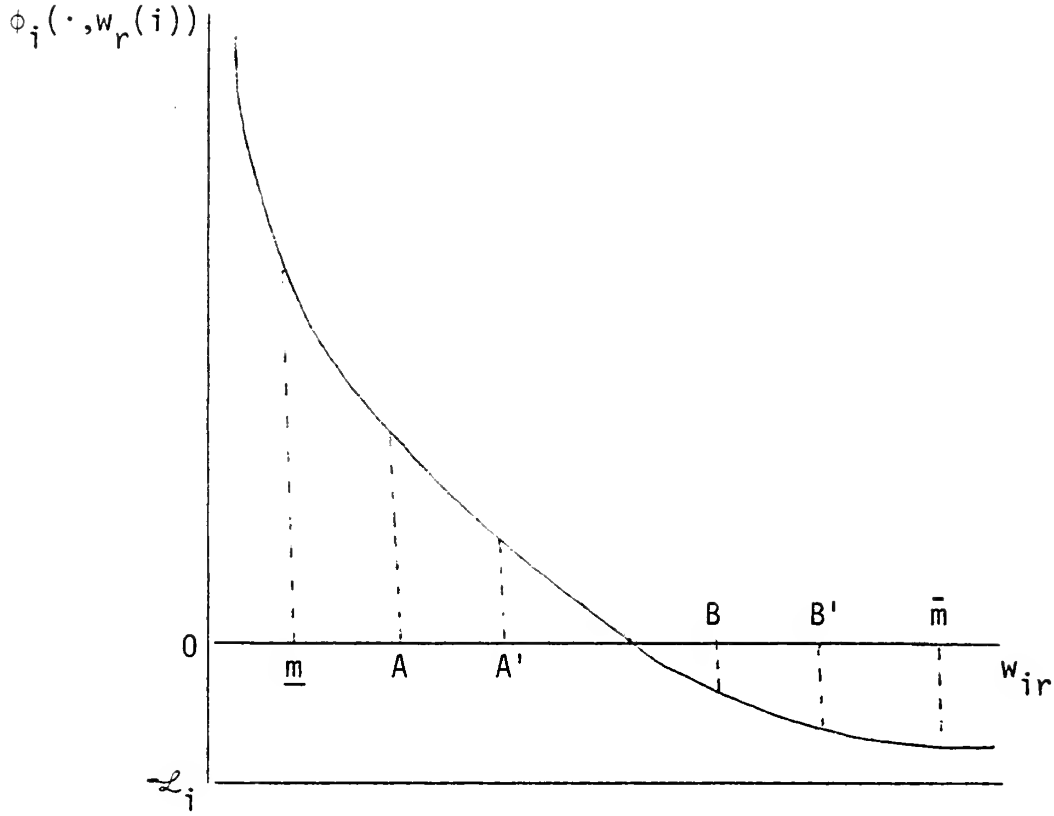

FIGURE 1

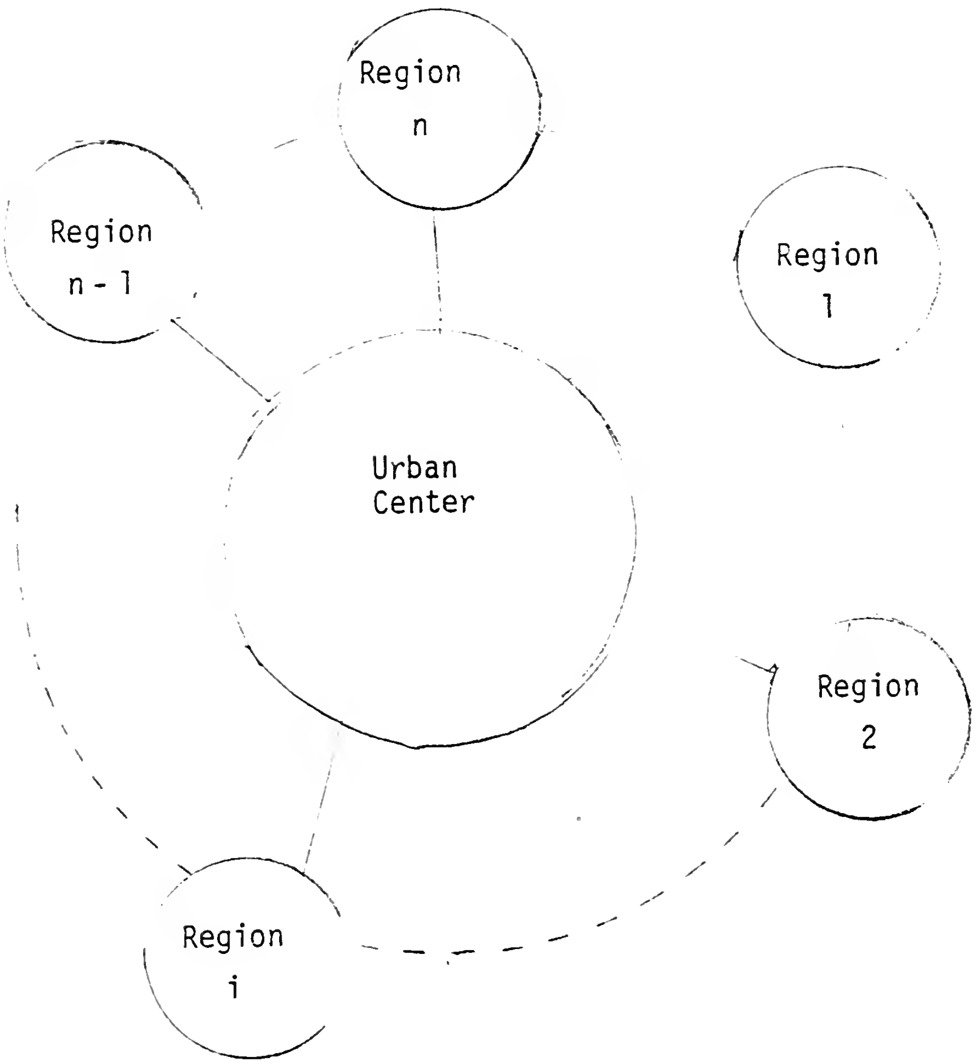

FIGURE 2

$$
\Delta = \left[\begin{array}{ccc|ccc}
(\ell_{1r}\eta_{1r} + \ell_{1u}e_w^1\eta_{1u}^1) & \cdots & \ell_{1u}e_w^n\eta_{1u}^n & \ell_{1u}(1+\eta_{1u}^1 e_\lambda^1) & \cdots & \ell_{1u}\eta_{1u}^n e_\lambda^n \\
\vdots & & \vdots & \vdots & & \vdots \\
\ell_{nu}e_w^1\eta_{nu}^1 & \cdots & (\ell_{nr}\eta_{nr} + \ell_{nu}e_w^n\eta_{nu}^n) & \ell_{nu}\eta_{nu}^1 e_\lambda^1 & \cdots & \ell_{nu}(1+\eta_{nu}^n e_\lambda^n) \\
\hline
(e_w^1 - 1) & & & (e_\lambda^1 - 1) & & \\
 & \ddots & & & \ddots & \\
 & & (e_w^n - 1) & & & (e_\lambda^n - 1)
\end{array}\right]
$$

FIGURE 3

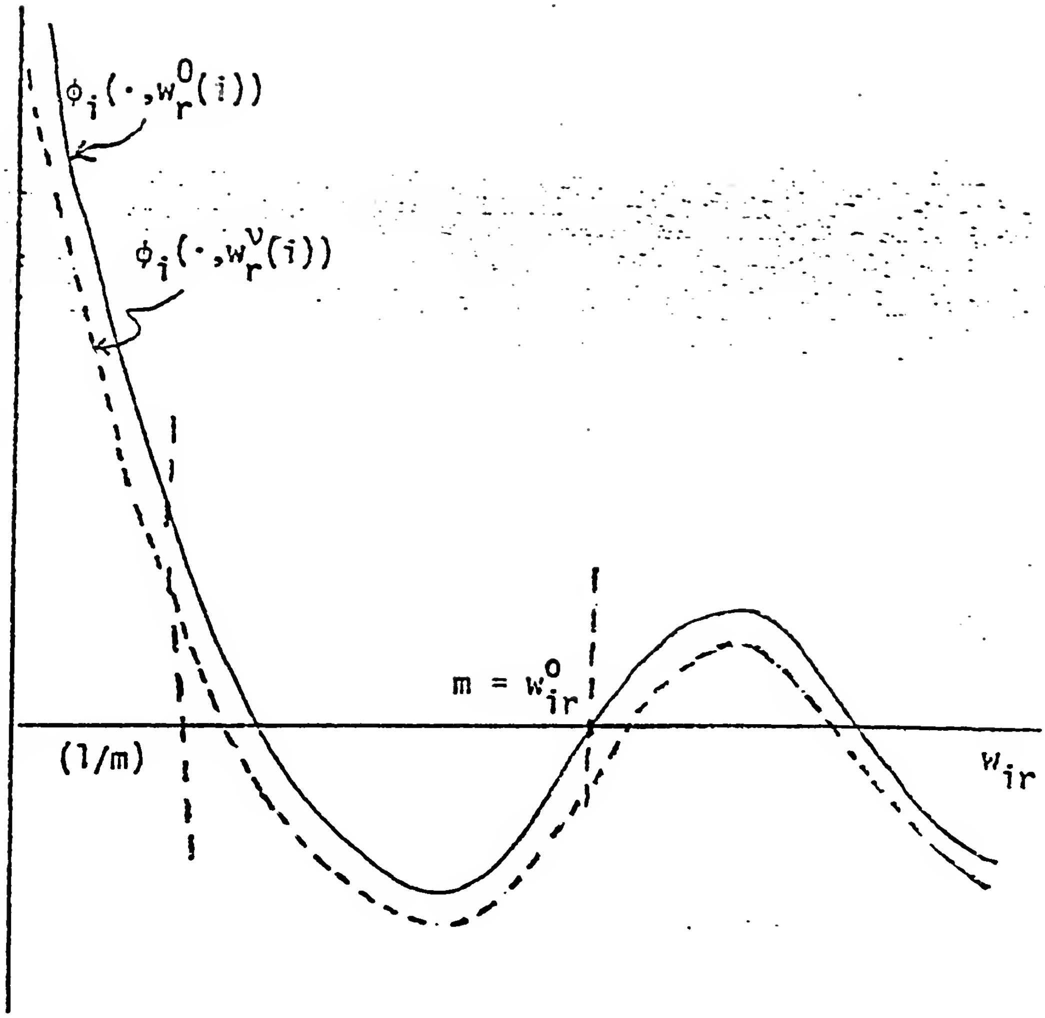

FIGURE 4

FOOTNOTES

[1]For example see the Five Year Plan documents of the Government of India. Indeed, in terms of general objectives of the Indian Five Year Plans, the following excerpt from Chapter XXXIV of the Third Five Year Plan is illuminating. "... besides ensuring rapid and sustained growth for the economy as a whole, at least during the next two or three Plans, measures for advancing the economic and social interests of scheduled tribes, scheduled castes and other weaker sections of the community should be so intensified, that they do, in fact, reach a level of well-being comparable with that of other sections of the population."

[2]For example, see the reports of the Committee on the Welfare of Scheduled Castes and Scheduled Tribes published yearly by the Lok Shabha Secretariat, Government of India. Together with general developmental expenditures, there exists regulations for the reservation of posts for the members of scheduled castes and tribes in the public sector. The Committee notes "... that the present policy of the Government in reserving a percentage of vacancies occurring every year for Scheduled Castes and Scheduled Tribes is not only equitable but is also in the overall interest of the Administration."

[3]Note that this is a somewhat unconventional way of defining the unemployment rate. In [6] and [19], for example, this rate is defined as the ratio of the unemployed to the total urban labor force. This translates to $\lambda_i/(1+\lambda_i)$ in our notation.

[4]Also Table 1 in [14].

[5] One such set of sufficient conditions are the Inada conditions (see 2.1.4 below) and Assumptions 2.3.1 and 2.3.2 below.

[6] This was first pointed out to the authors by T. N. Srinivasan.

[7] The reader can usefully compare Corollary 2.1.2 with the existence theorems given in Srinivasan-Bhagwati [18] and Calvo [6].

[8] These facts are exploited in a two-sector, mobile capital setting to present an example of non-existence of a Harris-Todaro equilibrium in Khan [15].

[9] It must be emphasized that the results presented in this section are subject to all the blemishes of the *tâtonnement* stability theory of competitive equilibria. We have in mind particularly an adjustment process not based on maximizing behavior of individual agents, presence of an auctioneer and no trading out of equilibrium.

[10] Also Table 1 in [16].

[11] For a precise definition of local asymptotic stability of equilibria, see, for example, [11, pp. 185-86].

[12] Theorem 2.3.1 does not need $\dot{e}_w^i \leq 1$ for its validity.

[13] See Figure 3.

[14] The only exception to this is $\hat{\lambda}_i$ which is given by $d\lambda_i | (1 + \lambda_i)$ to allow for situations when λ_i may be zero.

[15] Pathans, Baluchis and Punjabis are all ethnic groups of Pakistan.

[16] Note that K_{jr} is a parameter.

[17] Unlike subsection 3.1, no necessary and sufficient condition can be provided.

[18] If we follow Stiglitz [19] and assume $e_w^i = 0$, then also the above

proposition is automatically true.

[19]If other tribes also are unionized, then there might exist linkages between trade unions, an analysis of which is outside the scope of our study here.

[20]Following Calvo [6], this implies $e_w^i = 1$ and $e_\lambda^i = 0$.

[21]See footnote 2.

[22]This has been considered in a two-sector setting in Stiglitz [19].

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