SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

PART II.

INVESTIGATION OF THE SOLAR DIURNAL VARIATION IN THE MAGNETIC DECLINATION AND ITS ANNUAL INEQUALITY.

BY

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INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, AND ITS ANNUAL INEQUALITY.

Having discussed, in Part I, the eleven-year period in the amplitude of the solar-diurnal variation, as well as in the disturbances of the magnetic declination, I now proceed to the analysis of the annual inequality of the solar-diurnal variation.

To obviate the difficulty which would occur in cases of months of unusal disturbance, if the crude observations were used, the normals or means freed from the disturbances have been employed in the discussion. This mode of proceeding not only obviates the necessity for rejecting the observations of particular months, but brings out the most consistent results which the observations can furnish, for both diurnal and annual variation. It is the course adopted by General Sabine in the third volume of his discussion of the Toronto observations.[1]

Returning, then, to the hourly normals, they are rearranged in the tables which follow, according to the different months of the year. The normals for 1840 are corrected for the index error by the addition of 93.3 scale divisions. All corrections for referring the partial monthly readings to the annual mean are, of course, omitted.

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[1] Table LXVI, of this volume, exhibits the solar-diurnal variation of the declination after the separation and omission of the larger disturbances; whereas Table VII, of the preceding volume, similar in form, differs from the latter, being derived from all the observations including the disturbances.

Hourly Declination. Normals for January.[1]

Observations $19\frac{1}{2}$ minutes later than indicated. Value of one scale division = 0′.453.
Increase of scale readings corresponds to a decrease of westerly declination.

Year.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	579.3	...	577.0	...	578.6	...	576.9	...	580.7	...	581.9	...
1842	564.3	...	563.8	...	565.3	...	565.9	...	570.9	...	566.4	...
1843	...	...	...	...	...	...	...	...	...	...	...	...
1844	558.6	558.2	558.4	559.2	558.9	558.8	559.7	561.2	562.9	563.3	559.1	555.9
1845	530.9	531.3	531.1	531.5	533.0	531.6	532.9	535.2	535.8	533.8	530.2	526.7
Mean[2]	558.28	...	557.57	...	558.95	...	558.85	...	562.57	...	559.40	...
Same refer'd to its mean epoch[3]	565.25	564.80	564.35	565.62	565.70	564.66	565.47	567.74	569.27	569.51	566.65	561.88

Year.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	570.0	...	568.8	...	570.3	...	574.2	...	578.0	...	580.1	...
1842	556.7	...	556.0	...	562.9	...	563.2	...	566.1	...	567.8	...
1843	...	...	555.4	...	...	...	...	...	...	...	...	...
1844	552.9	552.4	553.2	554.1	556.3	556.9	557.8	559.2	559.5	560.9	560.8	559.6
1845	524.2	525.2	526.2	528.0	530.1	531.8	532.7	532.8	533.3	533.0	532.4	532.0
Mean[2]	550.95	...	551.92	...	554.90	...	556.97	...	559.22	...	560.27	...
Same refer'd to its mean epoch[3]	557.72	557.31	557.55	558.97	561.20	562.41	563.38	564.82	565.90	567.00	567.20	566.35

[1] The hours refer to mean local time, reckoned from midnight to 24 hours.

[2] The mean given is the simple mean of the four readings, and at 14h. of five readings, and is here inserted for comparison with the corrected mean in the line below, which would have been obtained if there had been no omissions in the observations.

[3] To obtain the normals referring to January of the mean year, the readings for the defective years 1840 and 1843 have been interpolated in the following manner: 1. *For the even hours.*—The normals for any two consecutive years differ simply by the annual effect of the secular change, which may be regarded as uniform when the same hours and months are compared, as in the present case. The values derived from the comparison of the several months of any two years differ, however, by the accidental errors of the observations; thus, taking the difference of the normals for 1840 and 1841, we obtain for the several months the values—

June	+15d.7	September	+21d.9	December	+20d.0
July	20.5	October	12.7		
August	18.5	November	17.5	Mean	16.86

Which mean corresponds exactly to the difference of the constant terms in Part I, for 1840 and 1841. By adding, therefore, 16.9 scale divisions to the normals for 1841, we obtain interpolated values for 1840. The values from January to May, 1840, were thus supplied. The normals for 1843 were supplied in a different manner, by making use of the readings at 2 P. M., which were taken for the purpose of keeping up the continuity of the series. Subtracting 0.6 scale division from the hourly readings of 1842, we obtain those for 1843—this being the difference at 14h.; in like manner, adding 2.2 scale divisions to the readings of 1844, we obtain a second value for the normals of 1843. The mean of these two independent determinations has been used in supplying the readings for 1843. The normals for 1840 and 1843 being thus supplied, the figures in the last line of the preceding table are obtained by simply taking the mean of the six readings at each even hour. 2. *For the odd hours.*—The difference in the mean readings for any given odd hour, in 1844 and 1845, from the two adjacent even hours, was applied to the normals of these hours, and the mean taken as the normal of the intermediate odd hour. Thus, the mean reading at noon of 1844 and 1845 is 538.55, at 13h, 538.80, difference +0.25; which, added to the noon normal 557.72, gives 557.97; and, in like manner, by comparison with 14h., the correction to its normal is —0.90, and the normal for 13h. becomes 556.65. The mean of the two results, 557.31, is the resulting normal for this hour as given in the table.

The same principle of interpolation was applied throughout the tables. Due attention must be paid, in the deductions, for the unequal weight of the normals for the even and odd hours; these weights being generally as 5 : 2, or proportional to the number of separate readings. The application of a nearly constant quantity to refer means from a defective number of years to the mean epoch of all the years, is not of much consequence in regard to the diurnal and annual inequalities, which depend mainly on differences of readings, but it is essential that no changes should have occurred in the zero of the scale during any interval under discussion.

Hourly Declination. Normals for February.

Observations 19½ minutes later than indicated. One division of scale = 0.′453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	575.0	...	573.2	...	575.6	...	577.8	...	582.1	...	579.5	...
1842	564.5	...	564.3	...	563.8	...	565.2	...	567.8	...	565.5	...
1843	...	...	...	...	...	...	...	...	...	...	...	...
1844	559.1	558.5	559.1	559.2	559.9	561.1	560.8	562.1	562.2	560.7	557.3	554.5
1845	531.6	531.1	531.0	532.4	532.3	533.1	534.7	535.9	535.7	535.4	533.0	528.6
Mean	557.55	...	556.90	...	557.90	...	559.62	...	561.95	...	558.82	...
Same refer'd to its mean epoch	563 88	563.10	563.13	563.90	564.23	565.25	565.93	567.88	568.53	567.97	565.42	561.47

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	569.5	...	566.0	...	569.5	...	572.4	...	574.4	...	575.8	...
1842	558.2	...	559.9	...	558.0	...	561.9	...	565.3	...	565.5	...
1843	...	...	555.9	...	...	...	...	...	...	...	...	...
1844	551.1	551.1	553.0	554.7	556.4	556.6	557.6	558.4	559.9	559.4	560.1	559.0
1845	524.4	523.0	525.3	527.5	529.7	530.4	532.4	531.3	533.6	534.4	532.3	531.9
Mean	550.80	...	552.02	...	553.40	...	556.07	...	558.30	...	558.42	...
Same refer'd to its mean epoch	557 33	555.85	557.17	558.30	559.43	560.25	562.13	562.25	564.42	565.02	564.77	564.00

Hourly Declination. Normals for March.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	577.1	...	577.6	...	580.9	...	582.9	...	586.8	...	578.9	...
1842	564.8	...	564.1	...	565.4	...	566.1	...	571.8	...	565.9	...
1843	...	...	...	...	...	...	...	...	...	...	...	...
1844	558.0	559.0	559.2	557.9	559.8	560.2	561.3	563.6	564.8	564.1	560.3	554.9
1845	532.9	532.7	533.7	533.6	535.0	533.9	536.0	538.8	539.4	538.6	534.5	529.4
Mean	558.20	...	558.65	...	560.27	...	561.58	...	565.70	...	559.90	...
Same refer'd to its mean epoch	565.60	565.72	566.03	565.75	567.82	567.53	569.20	572.11	573.37	571.95	567.32	562.02

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	569.4	...	567.7	...	571.8	...	576.4	...	577.4	...	577.7	...
1842	555.6	...	553.9	...	556.4	...	560.3	...	564.5	...	564.9	...
1843	...	...	557.2	...	...	...	...	...	...	...	...	...
1844	550.6	549.4	549.6	551.7	553.0	555.2	556.6	558.0	558.4	558.2	558.6	559.7
1845	524.8	522.5	522.8	524.8	527.8	529.7	531.6	533.0	533.0	533.8	533.5	534.0
Mean	550.10	...	550.24	...	552.25	...	556.22	...	558.32	...	558.67	...
Same refer'd to its mean epoch	557.52	555.75	555.97	557.75	559.63	561.85	563.68	565.31	565.75	566.04	566.08	566.94

Hourly Declination. Normals for April.

Observations $19\frac{1}{2}$ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	580.0	...	581.9	...	582.9	...	585.6	...	587.6	...	579.4	...
1842	563.3	...	565.4	...	566.1	...	568.5	...	569.7	...	563.6	...
1843	569.7	...	570.0	...	571.0	...	574.7	...	576.2	...	566.2	...
1844	556.6	557.0	557.2	556.9	557.5	558.4	561.7	558.5	564.4	561.8	557.1	552.0
1845	529.1	528.8	529.0	529.2	529.8	531.7	534.0	535.6	537.5	535.4	528.5	522.5
Mean	559.74	...	560.70	...	561.46	...	564.90	...	567.08	...	558.96	...
Same refer'd to its mean epoch	565 93	566.42	567.05	567.12	567.85	568.31	571.17	569.90	373.32	570.98	565.18	559.76

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	568.8	...	566.1	...	571.7	...	576.9	...	578.0	...	579.1	...
1842	554.0	...	552.5	...	555.1	...	560.6	...	561.3	...	563.0	...
1843	557.8	...	555.7	...	562.6	...	564.8	...	568.5	...	568.7	...
1844	547.4	545.7	546.2	547.6	549.6	553.4	553.4	553.8	556.2	555.1	555.7	559.3
1845	517.8	513.9	514.0	517.2	521.5	525.8	527.8	527.9	528.1	528.5	528.0	529.4
Mean	549.16	...	546.90	...	552.10	...	556.70	...	558.42	...	558.90	...
Same refer'd to its mean epoch	555 25	552.54	552.92	555.13	558.18	562.05	562.88	563.16	564.50	564.59	565.08	567.50

Hourly Declination. Normals for May.

Observations $19\frac{1}{2}$ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	579.1	...	579.8	...	581.9	...	587.4	...	589.1	...	578.6	...
1842	563.3	...	564.3	...	566.0	...	571.2	...	569.5	...	560.0	...
1843	567.0	...	567.3	...	569.6	...	574.6	...	575.6	...	565.7	...
1844	548.4	548.7	547.8	547.0	549.3	552.5	555.8	556.8	555.1	552.3	546.7	542.2
1845	529.9	531.3	529.7	531.7	533.2	536.3	539.3	541.9	540.7	536.0	528.0	522.6
Mean	557.54	...	557.78	...	560.00	...	565.66	...	566.00	...	555.80	...
Samer fer'd to its mean epoch	563 95	565.16	564.27	564.72	566.47	569.28	572.10	574.01	572.67	569.07	562.42	557.72

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	569.4	...	567.9	...	573.6	...	577.4	...	578.5	...	580.1	...
1842	552.6	...	552.3	...	557.7	...	560.8	...	561.8	...	562.3	...
1843	556.0	...	556.2	...	562.2	...	566.4	...	566.9	...	567.3	...
1844	538.3	535.8	536.5	538.9	542.1	545.1	545.2	546.5	546.3	547.3	547.3	547.8
1845	517.1	516.8	518.9	522.1	526.7	529.3	529.6	530.4	529.7	530.3	530.5	530.3
Mean	546.68	...	546.36	...	552.46	...	555.88	...	556.64	...	557.50	...
Same refer'd to its mean epoch	553.28	551.62	552.77	555.23	558.80	561.94	562.28	563.44	563.10	563.94	564.09	564.04

Hourly Declination. Normals for June.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	587.7	...	588.3	...	590.8	...	597.3	...	596.0	...	587.1	...
1841	571.7	...	572.2	...	574.7	...	583.3	...	582.6	...	571.1	...
1842	564.6	...	563.7	...	567.2	...	573.7	...	573.0	...	565.2	...
1843	566.0	...	565.6	...	568.4	...	574.1	...	573.9	...	564.8	...
1844	548.7	549.0	549.3	549.1	551.6	553.9	557.6	559.1	558.2	554.3	547.9	541.8
1845	531.5	531.7	531.6	532.0	534.8	537.9	541.9	543.5	542.5	538.6	532.2	524.9
Mean	561.70	...	561.78	...	564.58	...	571.32	...	571.03	...	561.38	...
Same refer'd to its mean epoch	...	561.81	...	561.91	...	567.38	...	572.42	...	567.46	...	555.22

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	578.8	...	576.7	...	581.2	...	586.1	...	585.8	...	586.9	...
1841	561.6	...	560.3	...	565.0	...	570.1	...	570.9	...	570.8	...
1842	555.1	...	552.5	...	558.3	...	561.8	...	563.7	...	564.1	...
1843	556.4	...	556 0	...	561.1	...	564.3	...	564.0	...	565.6	...
1844	537.4	535.0	537.3	540.0	542.4	545.2	545.6	546.2	546.5	546.8	548.0	548.5
1845	521.3	519.6	520.0	522.1	525.4	528.9	530.3	530.7	530.1	530.7	530.3	531.4
Mean	551.77	...	550.47	...	555.57	...	559.70	...	560.17	...	560.95	...
Same refer'd to its mean epoch	...	549.42	...	552.80	...	558.76	...	560.26	...	560.58	...	561.65

Hourly Declination. Normals for July.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	590.6	...	590.5	...	592.2	...	598.0	...	598.8	...	588.7	...
1841	569.9	...	568.5	...	571.6	...	578.4	...	581.2	...	571.8	...
1842	566.0	...	566.0	...	568.4	...	576.6	...	576.4	...	565.8	...
1843	566.9	...	565.9	...	568.2	...	574.2	...	574.6	...	564.5	...
1844	549.0	550.5	548.4	549.4	551.0	554.3	556.9	559.8	558.6	554.8	548.0	540.8
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	568.48	...	567.86	...	570.28	...	576.82	....	577.92	...	567.76	...
Same refer'd to its mean epoch	561.77	563.26	561.15	562.07	563.60	567.16	570.02	572.67	571.23	567.61	561.00	535.47

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	577.8	...	577.3	...	582.0	...	586.6	...	588.8	...	589.6	...
1841	558.9	...	557.3	...	562.3	...	567.2	...	568.8	...	568.6	...
1842	556.3	...	553.8	...	558.5	...	562.4	...	564.2	...	567.1	...
1843	555.1	...	554.1	...	559.5	...	563.6	...	563.8	...	565.6	...
1841	538.3	535.5	536.3	538.8	541.9	544.5	545.8	546.2	546.6	547.4	548.8	549.3
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	557.28	...	555.76	...	560.84	...	565.12	...	566.44	...	567.94	...
Same refer'd to its mean epoch	550 65	548.05	549.05	551.33	554.22	556.98	558.43	559.05	559.67	560.18	561.28	561.97

Hourly Declination. Normals for August.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	588.6	...	589.0	...	592.1	...	599.7	...	602.4	...	582.7	...
1841	568.4	...	570.3	...	571.6	...	580.1	...	583.9	...	568.9	...
1842	564.8	...	566.0	...	568.5	...	573.7	...	575.0	...	560.0	...
1843	564.2	...	564.5	...	267.2	...	573.5	...	572.7	...	560.5	...
1844	548.6	547.8	547.3	547.4	550.9	552.4	557.5	560.3	558.2	551.8	543.3	536.4
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	566.92	...	567.42	...	570.06	...	576.90	...	578.44	...	563.08	...
Same refer'd to its mean epoch	560.40	559.85	560.60	560.80	563.40	565.00	570.20	573.35	571.60	565.01	556.32	549.14

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	573.8	...	575.2	...	581.5	...	586.5	...	588.2	....	589.4	...
1841	558.3	...	556.9	...	564.0	...	566.8	...	568.6	...	568.9	...
1842	552.3	...	553.7	...	561.5	...	562.2	...	564.1	...	564.5	...
1843	555.1	...	554.6	...	561.2	...	563.6	...	562.3	...	564.2	...
1844	531.8	532.0	534.3	538.7	542.1	544.3	546.0	546.5	546.7	546.6	547.8	547.7
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	554.26	...	554.94	...	562.06	...	565.02	...	565.98	...	566.96	...
Same refer'd to its mean epoch	547 05	546.49	548.03	552.15	555.27	557.12	558.38	558.99	559.30	559.15	560.30	559.85

Hourly Declination Normals for September.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	585.8	...	588.5	...	590.2	...	596.5	...	595.8	...	584.1	...
1841	565.1	...	564.5	...	565.5	...	569.4	...	571.1	...	564.1	...
1842	567.4	...	567.8	...	570.0	...	576.8	...	574.9	...	561.2	...
1843	560.4	...	560.4	...	560.3	...	565.7	...	566.6	...	554.6	...
1844	543.3	543.1	544.1	546.0	546.5	547.1	550.0	552.9	552.4	545.8	538.3	532.5
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	564.40	...	565.06	...	566.50	...	571.68	...	572.16	...	560.46	...
Same refer'd to its mean epoch	557 42	557.16	558.10	559.60	559.70	561.00	564.60	566.70	565.40	559.80	553.30	547.47

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	570.6	...	572.8	...	581.7	...	583.2	...	586.6	...	585.9	...
1841	553.6	...	554.5	...	559.5	...	562.9	...	563.8	...	564.0	...
1842	556.0	...	555.4	...	562.0	...	565.7	...	566.7	...	566.6	...
1843	547.5	...	550.5	...	556.8	...	558.0	...	560.0	...	558.7	...
1844	529.3	530.0	534.1	538.3	539.4	541.9	542.4	541.9	543.0	544.6	543.7	543.3
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	551.40	...	553.46	...	559.88	...	562.44	...	564.02	...	563.78	...
Same refer'd to its mean epoch	544.25	543.81	546.77	551.44	553.00	555.31	555.63	556.04	557.05	558.26	556.97	557.00

Hourly Declination. Normals for October.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	585.8	...	583.7	...	584.4	...	582.4	...	582.5	...	577.4	...
1841	566.8	...	566.3	...	565.5	...	567.6	...	569.4	...	568.2	...
1842	563.1	...	563.1	...	564.4	...	566.0	...	568.8	...	564.0	...
1843	559.6	560.2	559.6	559.1	559.9	560.6	562.1	565.1	566.0	565.0	560.8	556.5
1844	545.1	545.3	544.2	546.1	545.8	544.4	548.6	550.9	551.5	548.7	545.3	540.8
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	564.08	...	563.38	...	564.00	...	565.34	...	567.64	...	563.14	...
Same refer'd to its mean epoch	557 45	557.71	556.72	557.33	557.50	556.67	559.08	561.23	561.48	560.04	556.70	552.36

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	571.7	...	570.6	...	575.2	...	579.6	...	579.0	...	586.4	...
1841	564.0	...	562.3	...	564.7	...	573.5	...	568.6	...	569.3	...
1842	556.0	...	555.0	...	558.2	...	564.3	...	565.0	...	565.3	...
1843	553.6	552.6	552.7	554.2	556.2	557.0	558.2	559.7	560.1	561.1	559.7	560.7
1844	541.1	539.5	541.4	544.0	545.7	545.4	545.6	545.0	544.9	544.6	544.5	544.6
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	557.28	...	556.40	...	560.00	...	564.24	...	563.56	...	565.04	...
Same refer'd to its mean epoch	551.12	549.62	550.43	552.39	554.15	555.68	557.67	557.47	556.98	558.12	558.15	558.22

Hourly Declination. Normals for November.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	574.4	...	573.9	...	576.2	...	577.0	...	579.7	...	575.0	...
1841	557.2	...	558.5	...	558.5	...	557.6	...	561.7	...	557.1	...
1842	564.2	...	563.8	...	565.6	...	566.9	...	569.2	...	563.3	...
1843	556.3	556.7	556.6	556.6	557.4	557.4	559.1	561.8	561.3	560.1	556.2	552.6
1844	546.8	546.8	548.3	548.6	547.4	548.5	551.5	549.2	548.4	547.9	546.2	542.8
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	559.78	...	560.22	...	561.02	...	562.42	...	564.06	...	559.56	...
Same refer'd to its mean epoch	554.15	554.21	554.77	555.20	555.28	555.30	557.13	557.98	557.98	556.90	553.87	550.00

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	567.5	...	565.8	...	570.8	...	574.1	...	576.9	...	576.0	...
1841	551.8	...	549.9	...	553.4	...	554.9	...	558.0	...	558.6	...
1842	556.6	...	557.3	...	561.2	...	564.0	...	565.5	...	565.0	...
1843	550.4	550.0	551.1	552.6	553.8	554.9	556.3	557.5	557.5	557.7	557.3	557.4
1844	542.8	541.7	544.5	546.1	545.6	547.9	548.8	548.2	548.3	549.6	548.0	548.0
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	553.82	...	553.72	...	556.96	...	559.62	...	561.24	...	560.98	...
Same refer'd to its mean epoch	548 52	547.32	548.72	550.76	551.60	553.25	554.35	555.26	555.62	556.36	555.35	555.35

HOURLY DECLINATION. NORMALS FOR DECEMBER.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	571.2	...	568.5	...	573.1	...	572.8	...	573.8	...	573.9	...
1841	560.1	...	559.3	...	560.5	...	559.6	...	560.1	...	558.1	...
1842	561.7	...	560.7	...	562.1	...	562.7	...	565.5	...	564.2	...
1843	559.0	558.1	557.4	558.2	557.8	558.8	560.0	560.8	561.2	561.9	559.9	556.7
1844	536.1	535.8	535.4	535.9	536.8	537.3	537.2	536.8	537.9	539.3	536.1	532.9
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	557.62	...	556.26	...	558.06	...	558.46	...	559.70	...	558.44	...
Same refer'd to its mean epoch	550 57	549.92	549.32	550.38	551.05	551.35	551.45	551.75	552.60	553.75	551.25	547.78

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	564.0	...	564.9	...	566.0	...	571.8	...	572.3	...	574.5	...
1841	552.9	...	551.7	...	555.8	...	559.6	...	563.3	...	561.6	...
1842	556.6	...	556.2	...	560.1	...	562.0	...	563.5	...	563.8	...
1843	552.9	551.4	550.9	553.1	554.6	557.5	558.2	558.9	559.6	560.0	559.9	559.5
1844	530.6	529.3	529.4	532.1	533.2	534.8	535.9	537.0	536.8	537.4	537.8	537.1
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	551.40	...	550.62	...	553.94	...	557.50	...	559.10	...	559.52	...
Same refer'd to its mean epoch	544.47	543.45	543.62	546.35	547.02	549.40	550.43	551.50	551.92	552.35	552.43	551.60

The following table contains the recapitulation of the monthly normals for each hour of the day, and for the mean epoch 1842 to 1843, and forms the basis for the discussion of the diurnal variation and its annual inequality. The table exhibits at one view the mean hourly readings for each month, unaffected by the larger disturbances.

RECAPITULATION.—MONTHLY DECLINATION-NORMALS FOR EACH HOUR OF THE DAY, AND FOR THE MEAN EPOCH 1842–43.

Increasing scale divisions denote an easterly movement of the north end of the magnet. The readings belong to an hour 19½ minutes later than indicated by the figures at the head of the columns. Value of a scale division = 0'.453. Readings derived from five years of observation between 1840 and 1845.

PHILADELPHIA MEAN TIME.

Mean Epoch 1842–43.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	565.25	564.80	564.35	565.62	565.70	564.66	565.47	567.74	569.27	569.51	566.65	561.88
February	563.88	563.10	563.13	563.90	564.23	565.25	565.93	567.88	568.53	567.97	565.42	561.47
March	565.60	565.72	566.03	565.75	567.82	567.53	569.20	572.11	573.37	571.95	567.32	562.02
April	565.93	566.42	567.05	567.12	567.85	568.31	571.17	569.90	573.32	570.98	565.18	559.76
May	563.95	565.16	564.27	564.72	566.47	569.28	572.10	574.01	572.67	569.07	562.42	557.72
June	561.70	561.81	561.78	561.91	564.58	567.38	571.32	572.42	571.03	567.46	561.38	555.22
July	561.77	563.26	561.15	562.07	563.60	567.16	570.02	572.67	571.23	567.61	561.00	553.47
August	560.40	559.85	560.60	560.80	563.40	565.00	570.20	573.35	571.60	565.01	556.32	549.14
September	557.42	557.16	558.10	559.60	559.70	561.00	564.60	566.70	565.40	559.80	553.30	547.47
October	557.45	557.71	556.72	557.33	557.50	556.67	559.08	561.23	561.48	560.04	556.70	552.36
November	554.15	554.21	454.77	555.20	555.28	555.30	557.13	557.98	557.98	556.90	553.87	550.00
December	550.57	549.92	549.32	550.38	551.05	551.35	551.45	551.75	552.60	553.75	551.25	547.78

Mean Epoch 1842–43.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mean.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	557.72	557.31	557.55	558.97	561.20	562.41	563.38	564.82	565.90	567.00	567.20	566.35	564.20
February	557.33	555.85	557.17	558.30	559.43	560.25	562.13	562.25	564.42	565.02	564.77	564.00	562.98
March	557.52	555.75	555.97	557.75	559.63	561.85	563.68	565.31	565.75	566.04	566.08	566.94	564.86
April	555.25	552.54	552.92	555.13	558.18	562.05	562.88	563.16	564.50	564.59	565.08	567.50	564.03
May	553.28	551.62	552.77	555.23	558.80	561.94	562.28	563.44	563.10	563.94	564.09	564.04	563.18
June	551.77	549.42	550.47	552.80	555.57	558.76	559.70	560.26	560.17	560.58	560.95	561.65	560.84
July	550.65	548.05	549.05	551.33	554.22	556.98	558.43	559.05	559.67	560.18	561.28	561.97	560.24
August	547.05	546.49	548.03	552.15	555.27	557.12	558.38	558.99	559.30	559.15	560.30	559.85	559.07
September	544.25	543.81	546.77	551.44	553.00	555.31	555.63	556.04	557.05	558.26	556.97	557.00	556.07
October	551.12	549.62	550.43	552.39	554.15	555.68	557.67	557.47	556.98	558.12	558.15	558.22	556.43
November	548.52	547.32	548.72	550.76	551.60	553.25	554.35	555.26	555.62	556.36	555.35	555.35	553.97
December	544.47	543.45	543.62	546.35	547.02	549.40	550.43	551.50	551.92	552.35	552.43	551.60	549.82
Mean	...	...	...	...	...	...	...	...	...	...	...	...	559.64

This table shows plainly the relation of the mean hourly position of the magnet of each month to its general mean position, after the separation of the larger disturbances, and also, by running the eye along any horizontal line, the solar-diurnal variation for each month. It does not, however, show distinctly the annual inequality, on account of the changes in the numbers by the secular change. To eliminate the effect of this change, each hourly normal has been compared, in the following table, with the corresponding mean monthly value, as given in the last right-hand column; the sign + indicating a westerly direction, and — an easterly direction,[1] of the north end of the magnet from the mean monthly position. The scale divisions have been converted into minutes of arc.

[1] The sign + being generally taken to signify west declination, it has been retained to indicate a movement of the north end of the magnet to the west.

2

TABLE OF THE SOLAR DIURNAL VARIATION OF THE MAGNETIC DECLINATION FOR EACH MONTH OF THE YEAR, SHOWING THE ANNUAL INEQUALITY.

Observations 19½ minutes later than indicated in the headings.

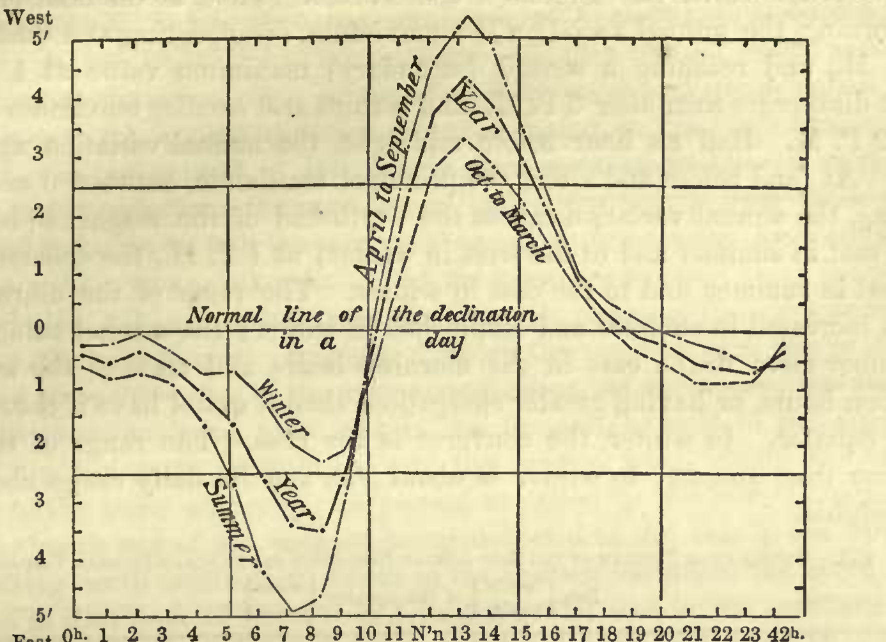
PHILADELPHIA MEAN TIME.

MEAN EPOCH 1842–43.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
January	—0′.47	—0′.27	—0′.07	—0′.64	—0′.68	—0′.21	—0′.57	—1′.61	—2′.29	—2′.40	—1′.11	+1′.06
February	—0.41	—0.06	—0.07	—0.42	—0.56	—1.03	—1.34	—2.22	—2.51	—2.26	—1.11	+0.68
March	—0.34	—0.39	—0.53	—0.40	—1.35	—1.21	—1.97	—3.28	—3.85	—3.21	—1.12	+1.29
April	—0.86	—1.09	—1.37	—1.40	—1.73	—1.94	—3.24	—2.65	—4.21	—3.15	—0.50	+1.93
May	—0.35	—0.90	—0.49	—0.70	—1.49	—2.77	—4.04	—4.90	—4.30	—2.66	+0.35	+2.47
June	—0.39	—0.44	—0.43	—0.48	—1.70	—2.97	—4.75	—5.25	—4.62	—3.00	—0.25	+2.54
July	—0.68	—1.37	—0.41	—0.82	—1.53	—3.18	—4.44	—5.63	—4.98	—3.34	—0.35	+3.07
August	—0.60	—0.36	—0.69	—0.78	—1.96	—2.68	—5.03	—6.47	—5.68	—2.69	+1.25	+4.50
September	—0.61	—0.49	—0.92	—1.60	—1.64	—2.23	—3.86	—4.81	—4.23	—1.69	+1.26	+3.89
October	—0.46	—0.58	—0.13	—0.41	—0.48	—0.10	—1.20	—2.17	—2.28	—1.63	—0.12	+1.84
November	—0.09	—0.11	—0.36	—0.55	—0.59	—0.60	—1.44	—1.81	—1.81	—1.33	+0.05	+1.80
December	—0.34	—0.05	+0.23	—0.26	—0.55	—0.69	—0.73	—0.87	—1.27	—1.78	—0.64	+0.93
Summer	—0.58	—0.78	—0.72	—0.96	—1.68	—2.63	—4.23	—4.95	—4.67	—2.76	+0.29	+3.07
Winter	—0.35	—0.24	—0.16	—0.45	—0.70	—0.64	—1.22	—1.99	—2.33	—2.10	—0.67	+1.27
Year	—0.47	—0.51	—0.44	—0.71	—1.19	—1.64	—2.72	—3.47	—3.50	—2.43	—0.19	+2.17

MEAN EPOCH 1842–43.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
January	+2′.94	+3′.12	+3′.01	+2′.36	+1′.36	+0′.81	+0′.37	—0′.28	—0′.77	—1′.27	—1′.36	—0′.98
February	+2.55	+3.23	+2.62	+2.12	+1.61	+1.24	+0.38	+0.33	—0.65	—0.93	—0.81	—0.46
March	+3.33	+4.13	+3.02	+3.22	+2.36	+1.36	+0.53	—0.20	—0.40	—0.54	—0.55	—0.95
April	+3.98	+5.20	+5.02	+4.03	+2.64	+0.90	+0.52	+0.39	—0.21	—0.26	—0.47	—1.57
May	+4.49	+5.24	+4.71	+3.60	+1.99	+0.56	+0.41	—0.12	+0.04	—0.35	—0.41	—0.39
June	+4.11	+5.16	+4.70	+3.64	+2.38	+0.95	+0.51	+0.27	+0.30	+0.12	—0.05	—0.36
July	+4.35	+5.53	+5.07	+4.03	+2.73	+1.47	+0.81	+0.53	+0.26	+0.03	—0.47	—0.78
August	+5.45	+5.71	+5.00	+3.14	+1.72	+0.88	+0.32	+0.04	—0.10	—0.04	—0.56	—0.36
September	+5.35	+5.56	+4.17	+2.09	+1.39	+0.35	+0.20	+0.01	—0.45	—1.00	—0.41	—0.42
October	+2.40	+3.08	+2.72	+1.83	+1.04	+0.35	—0.56	—0.47	—0.25	—0.76	—0.78	—0.81
November	+2.46	+3.01	+2.37	+1.45	+1.08	+0.33	—0.18	—0.59	—0.74	—1.09	—0.63	—0.63
December	+2.42	+2.89	+2.81	+1.57	+1.27	+0.19	—0.27	—0.76	—0.96	—1.15	—1.18	—0.81
Summer	+4.62	+5.40	+4.78	+3.42	+2.14	+0.85	+0.46	+0.19	—0.03	—0.25	—0.40	—0.65
Winter	+2.68	+3.24	+2.76	+2.09	+1.46	+0.71	+0.05	—0.33	—0.63	—0.95	—0.88	—0.77
Year	+3.65	+4.32	+3.77	+2.76	+1.80	+0.78	+0.25	—0.07	—0.33	—0.60	—0.64	—0.71

The distinctive features of the above table are next to be considered analytically as well as graphically. The inequality in the diurnal variation is most conspicuous when the tabular numbers in the horizontal lines for the months of February and August are compared. The annual variation appears plainest by carrying the eye over the vertical column at the hours 6 or 7 A. M. The annual variation depends on the earth's position in its orbit; the diurnal variation being subject to an inequality depending on the sun's declination. The diurnal range is greater when the sun has north declination, and smaller when south declination; the phenomenon passing from one state to the other about the time of the equinoxes. To show the diurnal variation at these periods, the summer and winter means, as well as the annual means, were tabulated. The months from April to September (inclusive) comprise the summer period, and from October to March (inclusive) the winter period. The first diagram (A) shows this variation, and contains the type curves for these half yearly periods. We find for the summer months a diurnal range of nearly 10½ minutes, and for the winter months of but 5½ minutes. These and other curves will be further analyzed hereafter.

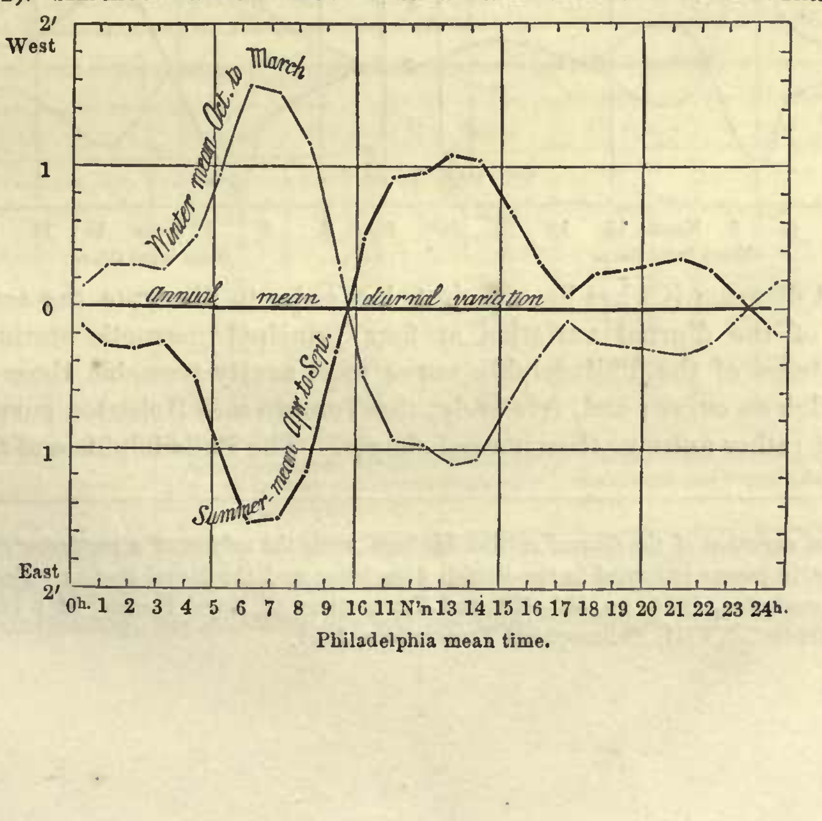
(A).—Mean Solar-Diurnal Variation of the Declination for Summer, Winter, and the whole Year.



Philadelphia mean time.

The second diagram (B) exhibits the same phenomenon in a different way; the yearly curve of the first diagram being straightened out and forming the axis of the second diagram, which thus shows the deviations from the annual mean value for the two seasons when the sun has north and south declination. The ordinates are obtained by subtracting the annual mean from either the summer or winter mean in the preceding table. This diagram exhibits, in quite a characteristic manner, the course of the annual variation at the different hours of the day, at

(B).—Semi-Annual Irregularities of the Solar-Diurnal Variation of the Declination.

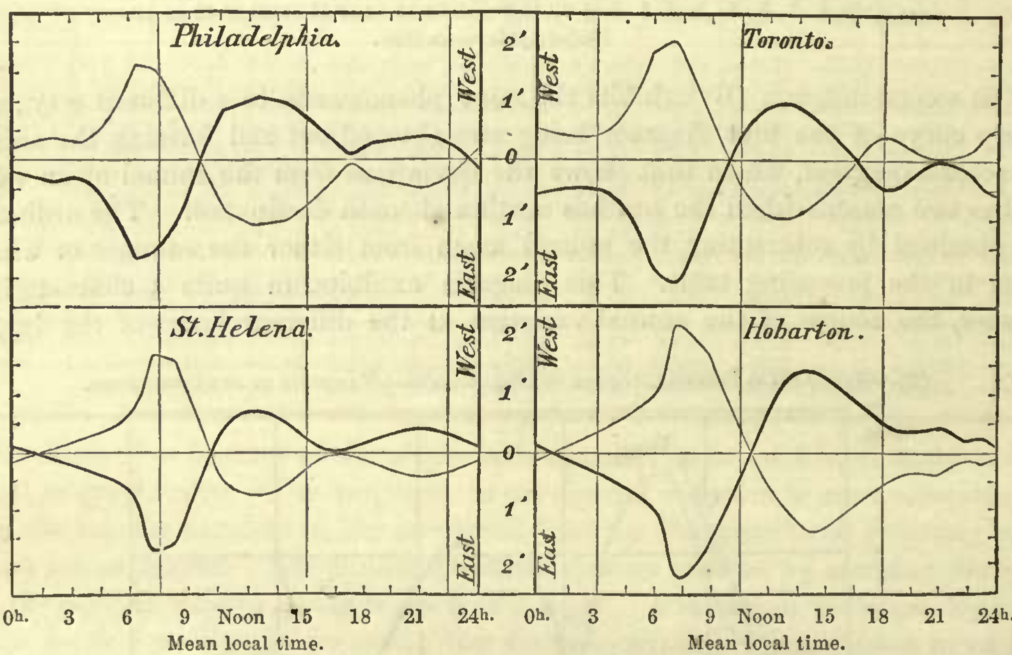


Philadelphia mean time.

the season for which the diagram is constructed. Thus, at the hour of 6 or 7 in the morning, the annual variation is a maximum, disappearing at a quarter before 10 A. M., and reaching a second (secondary) maximum value at 1 P. M. It almost disappears soon after 5 P. M., and a third still smaller maximum is reached after 9 P. M. Half an hour before midnight, the annual variation again disappears. At (and before and after) the principal maximum, between 6 and 7 in the morning, the annual variation causes the north end of the magnet to be deflected to the east in summer and to the west in winter; at 1 P. M., the deflections are to the west in summer and to the east in winter. The range of the diurnal motion is thus increased in summer and diminished in winter; the magnet being deflected in summer more to the east in the morning hours, and more to the west in the afternoon hours, or having greater elongations than it would have if the sun moved in the equator. In winter, the converse is the case. The range of the annual variation from summer to winter is about 3′.0, and its daily range about 2′.6 at Philadelphia.

(C).—Comparative Diagram of the Semi-Annual Deflection of the Solar-Diurnal Variation.

Sun {north / south} of the equator {—— / ——}



The next diagram (C) has been projected in order to illustrate the semi-annual inequality of the diurnal variation at four principal magnetic stations.[1] The general features of the Philadelphia curve most nearly resemble those exhibited in the St. Helena curve; and, relatively, the Toronto and Hobarton curves appear to represent rather extreme than normal shapes. The Philadelphia and St. Helena

[1] The annual variation of the diurnal motion has been made the subject of a particular discussion by General Sabine, in papers presented to the British Association and the Royal Society. See Reports of the British Association, 1854, pp. 355–368, and Transactions of Royal Society, May 18, 1854, pp. 67–82; also, article XXVIII, Philosophical Transactions, 1851.

curves have another feature in common: the amplitude at its maximum value, shortly after 6 A. M., is less than the amplitude at Toronto and Hobarton; and, upon the whole, the Philadelphia type confirms the idea that all forms partake of the same general character, more or less affected by incidental irregularities.

In reference to the annual variation, General Sabine, in the "rectifications and additions" to the last volume of Humboldt's *Cosmos*, expresses himself as follows: "Thus, in each hemisphere, the semi-annual deflections concur with those of the mean annual variation for half the year, and consequently augment them, and oppose and diminish them in the other half. At the magnetic equator, there is no mean diurnal variation, but in each half year the alternate phases of the sun's annual inequality constitutes a diurnal variation, of which the range in each day is about 3′ or 4′, taking place every day in the year except about the equinoxes; the march of this diurnal variation being from east in the forenoon to west in the afternoon, when the sun has north declination, and the reverse when south declination." According to the same authority, the *annual* variation is the *same* in both hemispheres, the north end of the magnet being deflected to the east in the forenoon, the sun having north declination; when in the *diurnal* variation, the north end of the magnet at that time of the day is deflected to the east in the northern hemisphere and to the west in the southern hemisphere. In other words, in regard to direction, the law of the annual variation is the same, and that of the diurnal variation the opposite, in passing from the northern to the southern magnetic hemisphere.

I next proceed to consider more in detail the annual variation at the hours of 6 and 7 in the morning and of 1 and 2 in the afternoon, these being the hours of the principal and secondary maxima respectively. By subtracting the annual mean from each monthly value at the respective hours, we obtain from the preceding general table the following columns:—

Annual Variation at the Hours of the Principal and Secondary Maxima of Range.

$\left.\begin{matrix}+\\-\end{matrix}\right\}$ indicates $\left\{\begin{matrix}\text{west}\\\text{east}\end{matrix}\right\}$ deflection from the mean annual position.

	6h. A. M.	7h. A. M.	Mean.	1h. P. M.	2h. P. M.	Mean.
January	+2′.15	+1′.86	+2′.01	—1′.20	—0′.76	—0′.98
February	+1.38	+1.25	+1.31	—1.09	—1.15	—1.12
March	+0.75	+0.19	+0.47	—0.19	—0.75	—0.47
April	—0.52	+0.82	+0.15	+0.88	+1.25	+1.06
May	—1.32	—1.43	—1.38	+0.92	+0.94	+0.93
June	—2.03	—1.78	—1.90	+0.84	+0.93	+0.89
July	—1.72	—2.16	—1.94	+1.21	+1.30	+1.25
August	—2.31	—3.00	—2.66	+1.39	+1.23	+1.31
September	—1.14	—1.34	—1.24	+1.24	+0.40	+0.82
October	+1.52	+1.30	+1.41	—1.24	—1.05	—1.14
November	+1.28	+1.66	+1.47	—1.31	—1.40	—1.35
December	+1.99	+2.60	+2.30	—1.43	—0.96	—1.20
	Maximum range at the above hours, 5′.0; the easterly deflection being greater by 0′.4 than the westerly.			Range at the hours 1 and 2 P. M., 2′.7; the eastern and western deflections being equal.		

A general inspection of the above columns containing the mean values shows that, approximately, the solstices are the turning epochs of this annual variation, the signs changing at the time of the equinoxes. To ascertain how nearly this is true, and in order to obtain a more precise expression, the means of the two columns (after changing the signs in the second) for each month respectively, were put into an analytical form, using Bessel's well-known formula for periodic functions—

$$\Delta_a = +1'.78 \, sin \, (\theta + 90°) + 0'.32 \, sin \, (2\theta + 180°);$$

$$\text{or, } \Delta_a = +1'.78 \, cos \, \theta - 0'.32 \, sin \, 2\theta;$$

the angle θ counting from January 1st.

The maximum values will occur on the first of January and the first of July; and the transition from a positive to a negative value, and the reverse, will take place on the first of April and the first of October, the equation $1.78 \, cos \, \theta = 0.32 \, sin \, 2\,\theta$, being only satisfied for $\theta = 90°$ and $270°$. That the angles C_1 and C_2 should be exactly 90° and 180° is remarkable. The monthly values are satisfied as follows:—

Middle of	By observation.	By calculation.
January	$+1'.50$	$+1'.56$
February	$+1.22$	$+0.94$
March	$+0.47$	$+0.30$
April	-0.46	-0.30
May	-1.16	-0.94
June	-1.40	-1.56
July	-1.59	-1.56
August	-2.00	-0.94
September	-1.03	-0.30
October	$+1.28$	$+0.30$
November	$+1.41$	$+0.94$
December	$+1.76$	$+1.56$

The regular progression of the monthly values is a feature of the annual variation deserving particular notice. There is no sudden transition from the positive to the negative side, or *vice versâ,* at or near the time of the equinoxes (certainly not at the vernal equinox); on the contrary, the annual variation seems to be regular in its progressive changes. The method here pursued is entirely different from that employed by General Sabine for the same end, but the results are, nevertheless, in close accordance. He remarks (in the British Association report above cited): "When a mean is taken corresponding to the 10th or 11th day after the equinox, the transition from the character of the preceding six months has already commenced and advanced very far towards its completion, and, by the middle of October, is quite complete; apparently, the progress of the change is somewhat more tardy in the March than in the September equinox." From the above analysis, we have found that the transition took place *ten* days after either equinox, and also that the turning points occur ten days after the solstices.

For the more precise determination of the law of the phenomenon, and in order to render the results of similar investigations comparable with one another, the *regular* solar-diurnal variation is now to be expressed as a function of the time. The preceding tabular values, given in minutes of arc, when treated as required by Bessel's[1] periodic function, furnish the following expressions for each month of the year:—

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[1] For another development of the formula, see Rev. Dr. H. Lloyd, "On the Mean Results of Observations," Transactions Royal Irish Academy, 1848, Vol. XXII, Part I. Dublin, 1849.

For January, $\Delta_d = +1'.423 \sin(15n + 225° 09') + 1'.491 \sin(30n + 16° 38')$
$+0'.579 \sin(45n + 220° 23') + 0'.548 \sin(60n + 53° \ldots)$
For February, $\Delta_d = +1'.469 \sin(15n + 211° 09') + 1'.456 \sin(30n + 20° 50')$
$+0'.472 \sin(45n + 231° 59') + 0'.352 \sin(60n + 60° \ldots)$
For March, $\Delta_d = +2'.098 \sin(15n + 206° 46') + 1'.827 \sin(30n + 26° 34')$
$+0'.693 \sin(45n + 230° 10') + 0'.413 \sin(60n + 84° \ldots)$
For April, $\Delta_d = +2'.906 \sin(15n + 213° 21') + 2'.001 \sin(30n + 34° 01')$
$+0'.926 \sin(45n + 223° 29') + 0'.245 \sin(60n + 80° \ldots)$
For May, $\Delta_d = +2'.746 \sin(15n + 210° 38') + 2'.377 \sin(30n + 45° 50')$
$+0'.970 \sin(45n + 251° 57') + 0'.100 \sin(60n + 161° \ldots)$
For June, $\Delta_d = +2'.883 \sin(15n + 204° 09') + 2'.438 \sin(30n + 44° 15')$
$+0'.941 \sin(45n + 254° 03') + 0'.216 \sin(60n + 114° \ldots)$
For July, $\Delta_d = +3'.310 \sin(15n + 204° 19') + 2'.465 \sin(30n + 38° 48')$
$+1'.047 \sin(45n + 251° 38') + 0'.092 \sin(60n + 176° \ldots)$
For August, $\Delta_d = +3'.161 \sin(15n + 211° 37') + 2'.849 \sin(30n + 52° 16')$
$+1'.375 \sin(45n + 265° 49') + 0'.201 \sin(60n + 51° \ldots)$
For September, $\Delta_d = +2'.706 \sin(15n + 220° 05') + 2'.372 \sin(30n + 55° 54')$
$+1'.126 \sin(45n + 261° 14') + 0'.414 \sin(60n + 115° \ldots)$
For October, $\Delta_d = +1'.271 \sin(15n + 226° 29') + 1'.325 \sin(30n + 33° 12')$
$+0'.727 \sin(45n + 230° 52') + 0'.150 \sin(60n + 47° \ldots)$
For November, $\Delta_d = +1'.259 \sin(15n + 229° 06') + 1'.257 \sin(30n + 39° 15')$
$+0'.390 \sin(45n + 236° 30') + 0'.242 \sin(60n + 87° \ldots)$
For December, $\Delta_d = +1'.212 \sin(15n + 231° 46') + 1'.321 \sin(30n + 23° 34')$
$+0'.367 \sin(45n + 205° 46') + 0'.418 \sin(60n + 32° \ldots)$

In like manner, we obtain for the summer half-year (from April to September inclusive), for the winter half-year (from October to March inclusive), and for the whole year, the following expressions for the diurnal variation:—

For summer half-year, $\Delta_d = +2'.936 \sin(15n + 210° 36') + 2'.404 \sin(30n + 46° 07')$
$+1'.031 \sin(45n + 253° 37') + 0'.178 \sin(60n + 132° 20')$
For winter half-year, $\Delta_d = +1'.420 \sin(15n + 220° 41') + 1'.399 \sin(30n + 26° 39')$
$+0'.520 \sin(45n + 227° 26') + 0'.310 \sin(60n + 61° 17')$
For the whole year,[1] $\Delta_d = +2'.167 \sin(15n + 213° 55') + 1'.875 \sin(30n + 38° 52')$
$+0'.759 \sin(45n + 244° 40') + 0'.198 \sin(60n + 83° 05')$

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[1] For the purpose of showing the correspondence when the above equation is deduced *independently*, from the observations at the even and odd hours, I add here the values for the two cases:—

From even hours, $\Delta_d = +2'.170 \sin(15n + 213° 27') + 1'.888 \sin(30n + 38° 59')$
$+0'.729 \sin(45n + 244° 57') + 0'.183 \sin(60n + 83° 26')$
From odd hours, $\Delta_d = +2'.159 \sin(15n + 215° 19') + 1'.835 \sin(30n + 38° 31')$
$+0'.848 \sin(45n + 243° 49') + 0'.242 \sin(60n + 82° 01')$

The relative weights of the results by the even hours and the odd hours are as 3 : 1.

If, for the purpose of comparison with the previous results in Part I of this discussion, and with other similar expressions, we change the angles C_1, C_2, C_3, C_4, by 180°, which is equivalent to an easterly deviation from the mean for positive results and to a westerly deviation for negative results, we find—

For Philadelphia, $\Delta_d = +2'.167 \sin(\theta + 33° 55') + 1'.875 \sin(2\theta + 218° 52')$
$+0'.759 \sin(3\theta + 64° 40') + 0'.198 \sin(4\theta + 263° 05')$
For Dublin, $\Delta_d = +3'.519 \sin(\theta + 64° 18') + 2'.127 \sin(2\theta + 225° 22')$
$+0'.688 \sin(3\theta + 70° 40') + 0'.322 \sin(4\theta + 242° 27')$

This latter expression is copied from the Rev. H. Lloyd's discussion of the Dublin observations in 1840–'43.

For a comparison of the monthly equations, the reader may also consult similar expressions obtained

In determining the least square coefficients in these equations, allowance has been made for the different weights due to the readings at the even and odd hours. θ is reckoned from midnight at the rate of 15° an hour. To compare the numerical quantities of the angles C_1, C_2, C_3, C_4, in the general expression—

$$\Delta_d = B_1 \sin(\theta + C_1) + B_2 \sin(2\theta + C_2) + B_3 \sin(3\theta + C_3) + B_4 \sin(4\theta + C_4),$$

with the same quantities in the formula of the diurnal variation (pp. 8 and 9 of Part I), 180° must first be added or subtracted from each angle given there; since, in the discussion of Part I, *increasing* numbers correspond to a *decrease* of western declination, the scale being thus graduated, whereas, in the *present* case, *increasing* positive numbers correspond to an *increase* of western declination, as stated above.

The following table exhibits the close correspondence of the computed and observed mean annual value of the regular solar-diurnal variation :—

Philadelphia mean time.	Diurnal Variation.		C—O.	Philadelphia mean time.	Diurnal Variation.		C—O.
	Computed.	Observed.			Computed.	Observed.	
0h. 19½m.	−0′.49	−0′.47	−0′.02	Noon 19½m.	+3′.69	+3′.65	+0′.04
1 19½	−0.48	−0.51	+0.03	13h. 19½	+4.28	+4.32	−0.04
2 19½	−0.51	−0.44	−0.07	14 19½	+3.81	+3.77	+0.04
3 19½	−0.67	−0.71	+0.04	15 19½	+2.77	+2.76	+0.01
4 19½	−1.09	−1.19	+0.10	16 19½	+1.71	+1.80	−0.09
5 19½	−1.82	−1.64	−0.18	17 19½	+0.88	+0.78	+0.10
6 19½	−2.77	−2.72	−0.05	18 19½	+0.33	+0.25	+0.08
7 19½	−3.49	−3.47	−0.02	19 19½	−0.07	−0.07	0.00
8 19½	−3.44	−3.50	+0.06	20 19½	−0.38	−0.33	−0.05
9 19½	−2.29	−2.43	+0.14	21 19½	−0.57	−0.60	+0.03
10 19½	−0.24	−0.19	−0.05	22 19½	−0.62	−0.64	+0.02
11 19½	+2.03	+2.17	−0.14	23 19½	−0.57	−0.71	+0.14

The maximum difference at any one hour is less than 11″, and the probable error of any single hourly result is ±0′.05. The probable error of any single computed value from a monthly expression is ±0′.19.

By means of the preceding equations, the hourly values of the diurnal variation for each month of the year have been computed; and the results, projected in curves, are given in Diagrams D and E. The first contains the curves for the six months of the summer half-year, and the second those of the winter half-year. Positive ordinates correspond to a westerly movement, and negative ordinates to an easterly movement, of the north end of the magnet. The diagram following (F) contains the type curves for summer, winter, and the whole year, all being upon the same scale.

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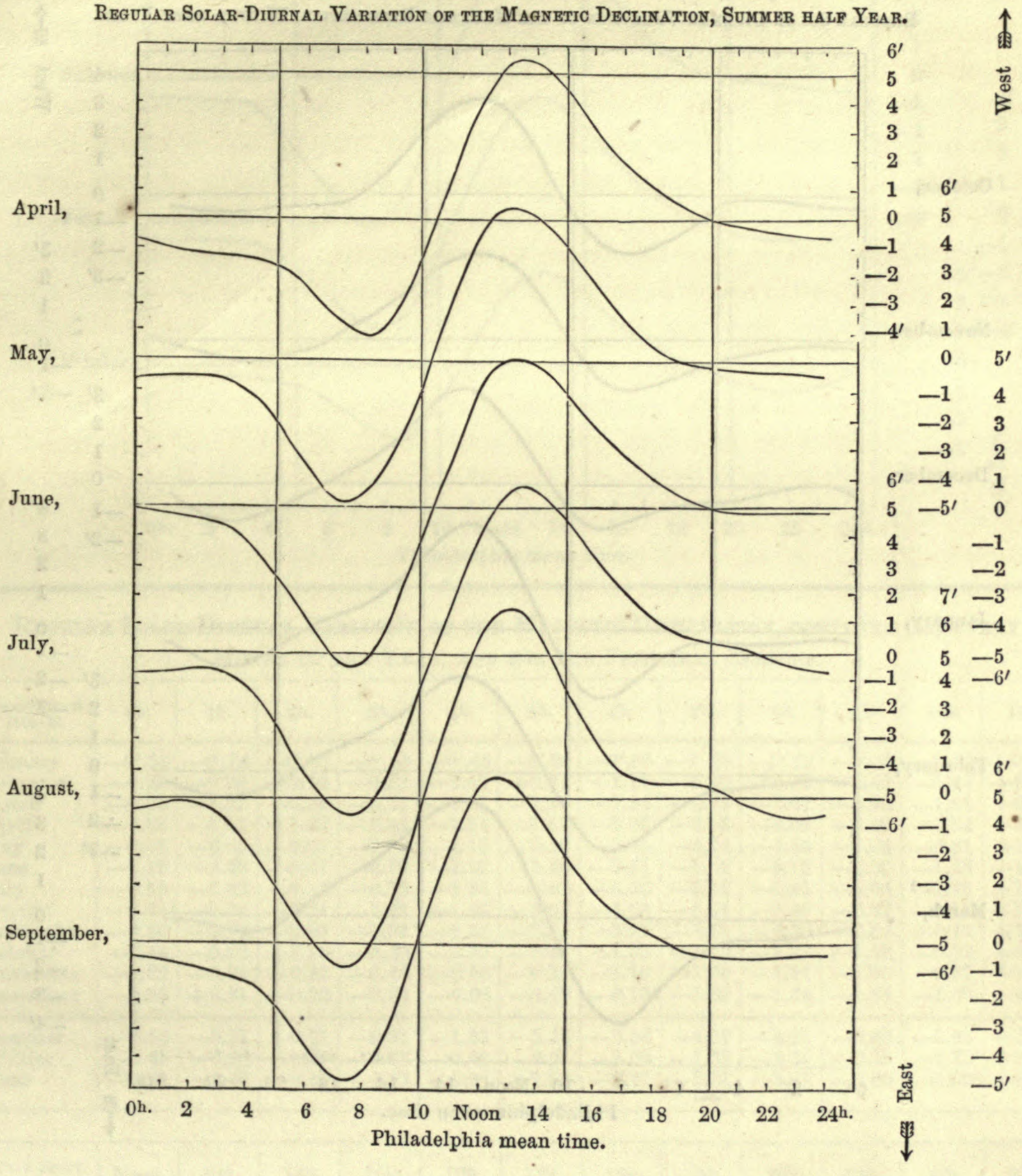
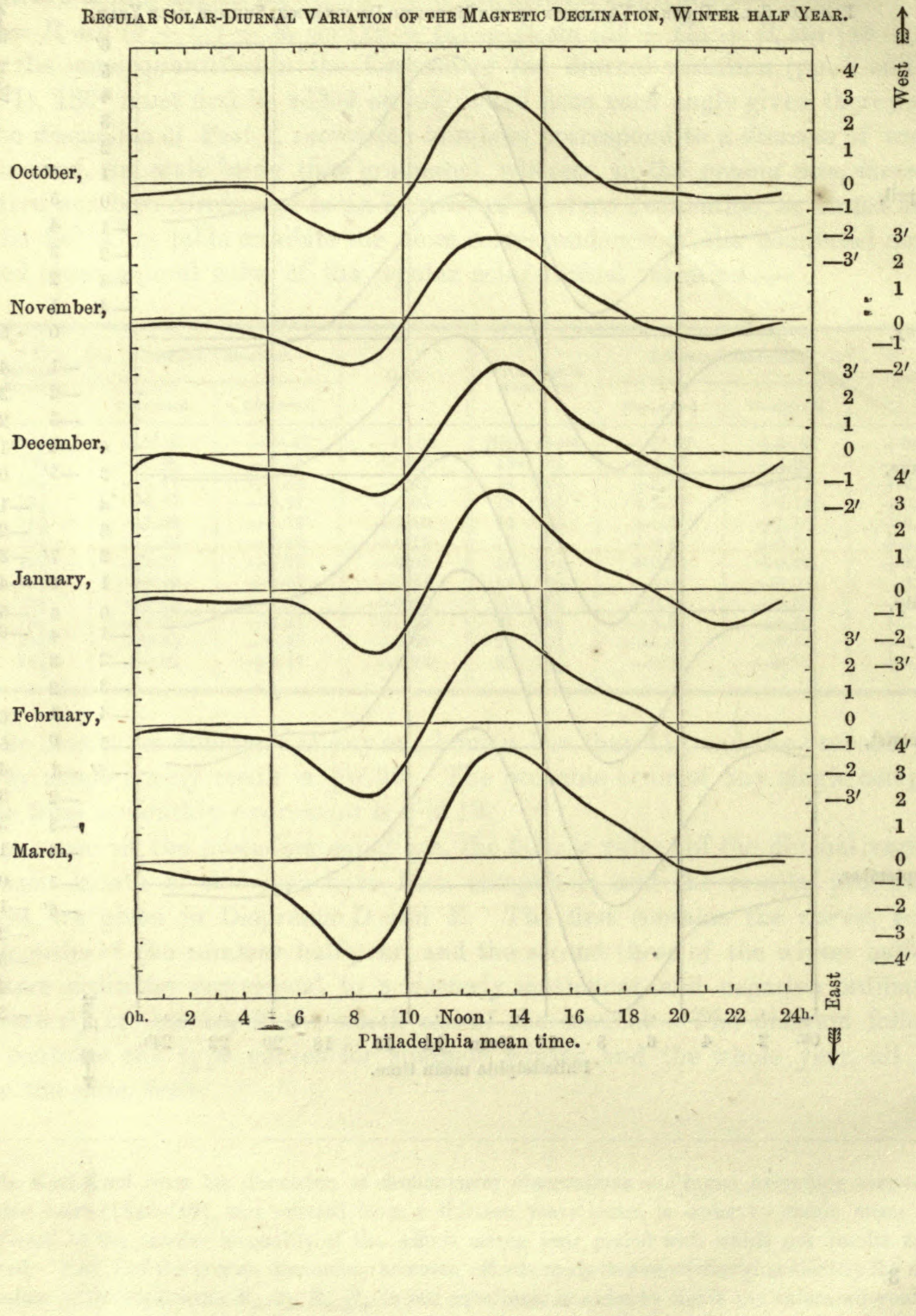
by Mr. Karl Kreil from his discussion of declinometer observations at Prague, extending over ten consecutive years (1840–'49), and selected from a thirteen years' series, in order to obtain mean results *unaffected* by the smaller inequality of the ten or eleven year period with which our results are still affected. Part I of the present discussion, however, affords ready means of changing slightly the numerical values of the coefficients B_1, B_2, B_3, B_4, in our equations, in order to obtain the values we would have obtained, had we discussed a consecutive eleven year series of observations or one extending over a series of years corresponding to the actual length of the solar period then observed. Mr. Kreil's discussion will be found in Vol. VIII of the proceedings of the mathematical and physical section of the Imperial Academy of Sciences at Vienna (1854).

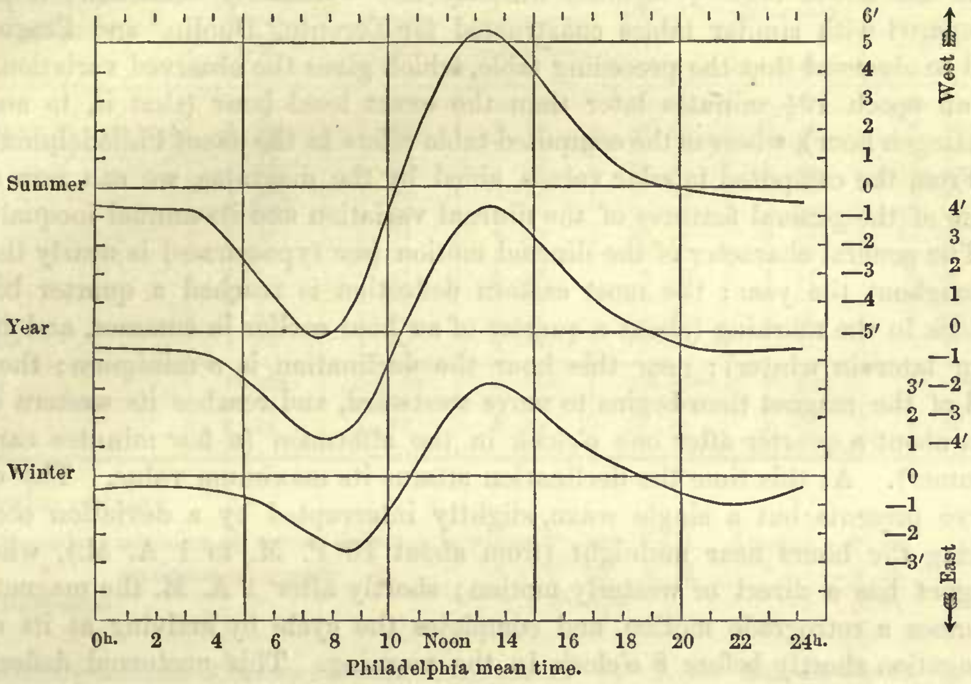
REGULAR SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, SUMMER HALF YEAR.



REGULAR SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, WINTER HALF YEAR.



TYPE-CURVES OF THE REGULAR SOLAR-DIURNAL VARIATION OF THE DECLINATION.



REGULAR SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, COMPUTED FOR EVERY MONTH OF THE YEAR, AND FOR THE PRINCIPAL SEASONS.

MEAN EPOCH 1842–43.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
January	—0′.52	—0′.24	—0′.30	—0′.48	—0′.48	—0′.38	—0′.55	—1′.24	—2′.12	—2′.45	—1′.59	+0′.26
February	—0.30	—0.09	—0.14	—0.32	—0.49	—0.71	—1.18	—1.90	—2.52	—2.49	—1.49	+0.24
March	—0.25	—0.32	—0.55	—0.71	—0.81	—1.09	—1.84	—2.93	—3.67	—3.40	—1.81	+0.55
April	—0.88	—1.14	—1.34	—1.44	—1.54	—1.88	—2.64	—3.55	—4.02	—3.42	—1.54	+1.11
May	—0.58	—0.59	—0.50	—0.59	—1.18	—2.32	—3.75	—4.74	—4.66	—3.24	—0.81	+1.93
June	—0.19	—0.28	—0.41	—0.69	—1.32	—2.45	—3.87	—5.02	—5.13	—3.80	—1.23	+1.69
July	—0.80	—0.82	—0.67	—0.72	—1.34	—2.62	—4.22	—5.42	—5.45	—4.04	—1.47	+1.47
August	—0.62	—0.33	—0.21	—0.59	—1.60	—3.09	—4.68	—5.71	—5.48	—3.67	—0.58	+2.87
September	—0.52	—0.72	—0.90	—1.06	—1.42	—2.23	—3.51	—4.55	—4.50	—2.84	+0.11	+3.18
October	—0.44	—0.52	—0.30	—0.20	—0.16	—0.45	—1.33	—1.72	—2.19	—1.92	—0.79	+0.77
November	—0.23	—0.21	—0.32	—0.44	—0.56	—0.75	—1.18	—1.68	—1.91	—1.50	—0.37	+1.11
December	—0.36	+0.01	—0.02	—0.34	—0.60	—0.67	—0.73	—1.00	—1.44	—1.64	—1.09	+0.26
Summer	—0.63	—0.71	—0.71	—0.81	—1.33	—2.43	—3.84	—4.92	—4.91	—3.43	—0.83	+2.12
Winter	—0.41	—0.26	—0.29	—0.42	—0.50	—0.68	—1.09	—1.73	—2.25	—2.15	—1.12	+0.56
Year	—0.49	—0.48	—0.55	—0.62	—0.94	—1.54	—2.41	—3.31	—3.54	—2.80	—1.02	+1.32

MEAN EPOCH 1842–43.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
January	+2′.26	+3′.40	+3′.34	+2′.46	+1′.52	+0′.92	+0′.57	+0′.08	—0′.64	—1′.29	—1′.45	—1′.08
February	+1.96	+2.97	+3.02	+2.42	+1.71	+1.17	+0.76	+0.26	—0.36	—0.85	—0.97	—0.70
March	+2.71	+3.86	+3.85	+3.17	+2.33	+1.65	+1.02	+0.35	—0.31	—0.70	—0.67	—0.43
April	+3.60	+5.06	+5.18	+4.28	+2.98	+1.76	+0.88	+0.27	—0.14	—0.38	—0.54	—0.67
May	+4.06	+5.07	+4.88	+3.85	+2.48	+1.22	+0.39	—0.02	—0.12	—0.14	—0.21	—0.43
June	+3.99	+5.00	+4.79	+3.79	+2.60	+1.59	+0.87	+0.38	+0.07	—0.10	—0.13	—0.15
July	+3.90	+5.26	+5.37	+4.54	+3.28	+2.04	+1.16	+0.66	+0.39	+0.18	—0.15	—0.53
August	+5.44	+6.35	+5.55	+3.75	+1.98	+0.87	+0.50	+0.45	+0.26	—0.13	—0.56	—0.77
September	+5.18	+5.54	+4.48	+2.99	+1.68	+0.85	+0.33	—0.11	—0.44	—0.56	—0.55	—0.44
October	+2.60	+3.17	+3.00	+2.20	+1.08	+0.25	—0.39	—0.36	—0.39	—0.52	—0.69	—0.69
November	+2.31	+2.81	+2.58	+1.90	+1.18	+0.57	+0.06	—0.40	—0.59	—0.92	—0.77	—0.49
December	+1.86	+2.95	+3.04	+2.32	+1.34	+0.57	+0.11	—0.28	—0.78	—1.22	—1.33	—0.96
Summer	+4.35	+5.29	+4.99	+3.89	+2.57	+1.43	+0.64	+0.18	—0.05	—0.17	—0.33	—0.44
Winter	+2.21	+3.12	+3.09	+2.38	+1.52	+0.84	+0.37	—0.09	—0.55	—0.89	—0.94	—0.72
Year	+3.25	+4.22	+4.09	+3.14	+2.06	+1.12	+0.45	+0.05	—0.32	—0.52	—0.58	—0.58

In the above table + signifies westerly and — easterly deflection; it may be compared with similar tables constructed for Toronto,[1] Dublin,[2] and Prague.[3] It will be observed that the preceding table, which gives the observed variation, refers to an epoch 19½ minutes later than the exact local hour (that is, to an exact Göttingen hour), whereas the computed table refers to the exact Philadelphia hours.

From the computed tabular values, aided by the diagrams, we can now deduce some of the general features of the diurnal variation and its annual inequality.

The general character of the diurnal motion (see type-curves) is nearly the same throughout the year; the most eastern deflection is reached a quarter before 8 o'clock in the morning (about a quarter of an hour earlier in summer, and half an hour later in winter); near this hour the declination is a minimum; the north end of the magnet then begins to move westward, and reaches its western elongation about a quarter after one o'clock in the afternoon (a few minutes earlier in summer). At this time the declination attains its maximum value. The diurnal curve presents but a single wave, slightly interrupted by a deviation occurring during the hours near midnight (from about 10 P. M. to 1 A. M.), when the magnet has a direct or westerly motion; shortly after 1 A. M. the magnet again assumes a retrograde motion, and completes the cycle by arriving at its eastern elongation shortly before 8 o'clock in the morning. This nocturnal deflection is well-marked in winter, vanishes in the summer months, and is hardly perceptible in the annual curve. According to the investigations of General Sabine, it is probable that, if we had the means of entirely obliterating the effect of disturbances, this small oscillation would almost disappear. In summer, when it has no existence, the magnet remains nearly stationary between the hours of 8 P. M. and 3 A. M., a feature which is also shown by the annual type-curve.

The two preceding plates show a close general resemblance in the diurnal curves for the six months when the sun has north declination, and a similar resemblance in the other six months when it has south declination.

The analytical expressions give the epoch and amount of variation with greater precision. The hours of minimum and maximum deflection are obtained from the equation $\frac{d\,\Delta_d}{d\,n} = o$; and the hours of the mean declination, when the curves cross the axis of abscissæ, from the condition $\Delta_d = o$. The following table contains these results for each month and the two principal seasons of the year, also the critical interval between the two adjacent hours of the mean position.

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[1] Vol. III., Table LXVI; compare also with Table VII. of Vol. II.

[2] Trans. Royal Irish Academy, Vol. XXII., Part I., Table III.

[3] Academy of Sciences at Vienna, Vol. VIII. of Math. Section, Table II.

Month.	Eastern elongation A. M.	Western elongation P. M.	Critical interval from minimum to maximum.	Epoch of Mean Declination. A. M.	Epoch of Mean Declination. P. M.	Critical interval.
January	8h. 58m.	1h. 27m.	4h. 29m.	10h. 52m.	7h. 08m.	8h. 16m.
February . . .	8 34	1 32	4 58	10 52	7 26	8 34
March	8 07	1 34	5 27	10 46	7 32	8 46
April	8 12	1 27	5 15	10 34	7 40	8 56
May	7 29	1 21	5 52	10 19	6 57	8 38
June	7 33	1 20	5 47	10 25	8 26	10 01
July	7 36	1 28	5 52	10 30	9 32	11 02
August	7 18	1 05	5 47	10 10	8 40	10 30
September . . .	7 30	0 45	5 15	9 58	6 45	8 47
October	8 00	1 17	5 17	10 30	5 23	6 53
November . . .	7 54	1 08	5 14	10 16	6 08	7 52
December . . .	8 54	1 40	4 46	10 50	6 17	7 27
Summer	7h. 33m.	1h. 8m.	5h. 35m.	10h. 17m.	7h. 43m.	9h. 26m.
Winter	8 24	1 25	5 01	10 40	6 49	8 09
Year	7 48	1 16	5 28	10 26	7 08	8 42

We likewise obtain:

Secondary minimum of eastern deflection in winter	$9^{h.}\ 42^{m.}$ P. M.	Amount	—0′.97
" maximum of western " " "	1 15 A. M.	"	—0 .26
Differences:	$3^{h.}\ 33^{m.}$		0′.71

and

Secondary minimum of eastern deflection for the year	$10^{h.}\ 11^{m.}$ P. M.	Amount	—0′.62
" maximum of western " " "	1 13 A. M.	"	—0 .47
Differences:	$3^{h.}\ 02^{m.}$		0′.15

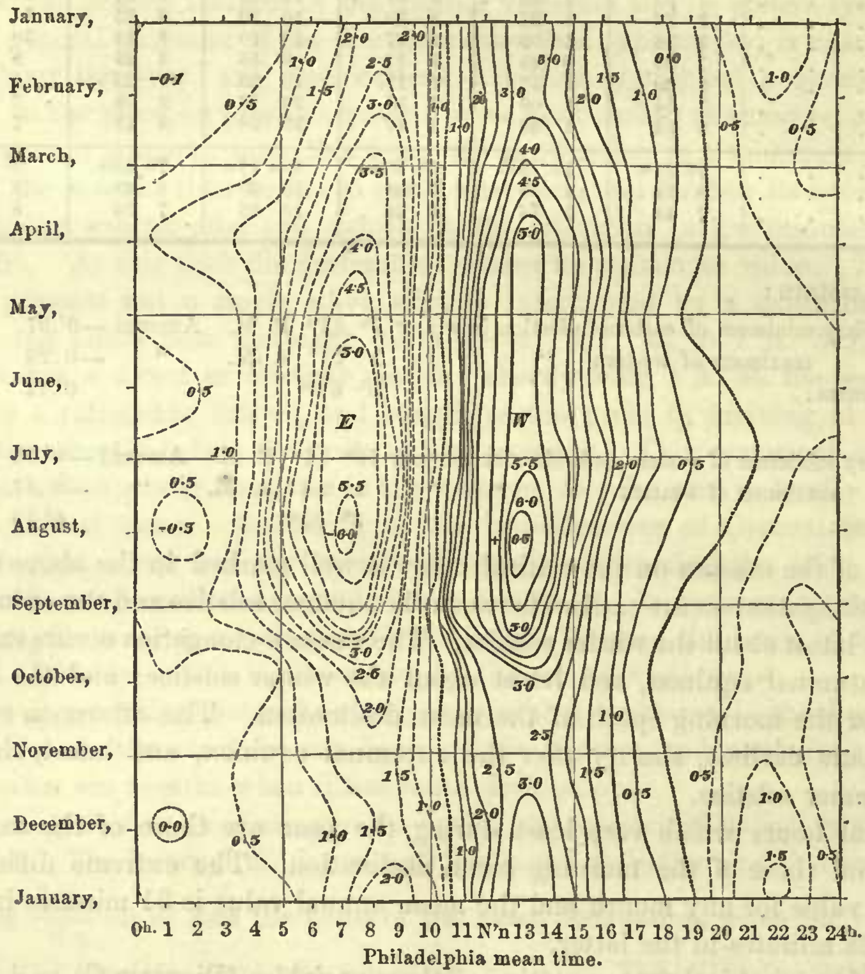
The effect of the seasons on the critical hours is well marked in the above table. The eastern elongation occurs earliest between the summer solstice and the autumnal equinox, and latest about the winter solstice. The western elongation occurs earliest about the autumnal equinox, and latest about the winter solstice; and the same holds good for the morning epoch of the mean declination. The afternoon epoch, however, occurs earliest, shortly after the autumnal equinox, and latest, shortly after the summer solstice.

The critical hours which vary least during the year are those of the western elongation and those of the morning mean declination. The extreme difference between the value for any month and the mean annual value is 31 minutes in the former and 28 minutes in the latter.

The following graphical representation of three variables (Diagram G) will serve to show at a glance the various features of the diurnal variation and its annual inequality: The magnetic surface is formed by contour lines, 0.′5 apart; the dotted curves are lines of mean position, the curves represented by dashes correspond to eastern, and the full curves to western deflection from the normal position. This diagram, as well as the computed tabular values from which it has been constructed, serve equally to furnish the correction necessary to reduce any single observation taken at any hour of the day and month to its mean value. It also enables us in a measure to dispense with developing the annual variability of the coefficients B_1 B_2 B_3 ... and C_1 C_2 C_3 .... (or rather the equivalents a_1 b_1 a_2 b_2 a_3 b_3 .... from which they are derived) in the general expression $A + B_1 \sin (\theta + C_1) +$ etc. In most cases either a tabular or graphical interpolation between the two adjacent monthly values will fully answer the purpose. The diagram also distinctly exhibits the diurnal minima and maxima, the former represented by a valley, the latter by a ridge in the magnetic surface.

The magnitude of the diurnal range is next to be considered.

Diagram showing the Deflection (in minutes of arc) of the North end of the Magnet from its Monthly Normal Position for every hour of the day and month of the year, derived from the Declinometer Observations at Philadelphia between 1840 and 1845.



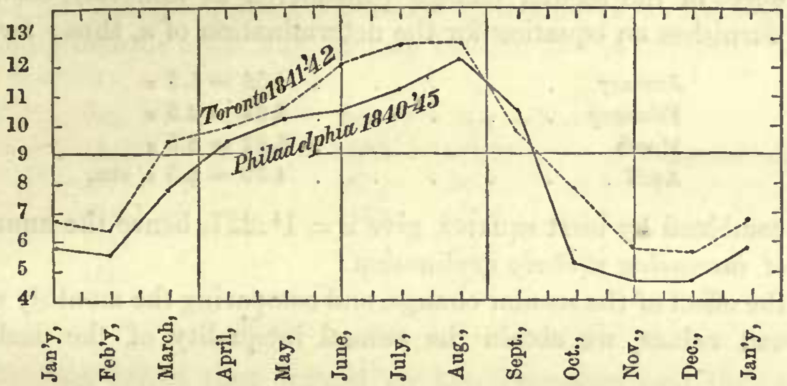
The following table contains the amount of the deflection at the eastern and western elongations and the diurnal amplitude of the declination for each month of the year, derived from the preceding equations:—

	Deflection at		Diurnal range.		Deflection at		Diurnal range.
	E. Elong.	W. Elong.			E. Elong.	W. Elong.	
January	—2′.46	+3′.52	5′.98	July	—5′.58	+5′.46	11′.04
February . . .	—2.64	+3.11	5.75	August . . .	—5.79	+6.36	12.15
March	—3.73	+4.03	7.76	September . . .	—4.71	+5.60	10.31
April	—4.02	+5.28	9.30	October . . .	—2.18	+3.23	5.41
May	—4.89	+5.16	10.05	November . . .	—1.92	+2.85	4.77
June	—5.26	+5.06	10.32	December . . .	—1.65	+3.14	4.79

The diurnal range for the summer months is 10′.45, for the winter months 5′.56, and for the whole year 7'.89; all corresponding to an epoch removed about one year and a half from the epoch of a minimum of the solar period.

The numbers expressing the diurnal range exhibit three remarkable features, viz., the maximum value in the month of August, the sudden falling off in the months of September and October (see the graphical representation), and the

Diurnal Range of the Declination.



minimum value in November or December. Otherwise the progression is regular; the curve is *single-crested*, a feature equally true for the *eastern* as well as for the *western* deflection when viewed separately. This latter circumstance is of special importance, since it is probable that it is mostly by the interference of these two separate curves that we observe at other stations the curve of the diurnal range at some stations apparently to be a double-crested one. The curves for Milan, Munich, Göttingen, Brussels, Greenwich, Dublin, etc., for instance, exhibit two maxima, one after the vernal equinox, and a second, generally the smaller one, about the summer solstice, with more or less regularity. The system to which Philadelphia belongs is exemplified by the annual curve of the diurnal range at Prague and at some Russian stations, especially at Nertschinsk, but principally at Toronto, for which last station the curve is shown in the diagram. Neither station appears to have a tendency to a secondary maximum about the month of April, leaving the maximum about a month and a half after the summer solstice, a well-marked North American feature.

Annual Variation of the Declination.—In connection with the preceding discussion the annual inequality in the magnetic declination next claims attention.

This subject presents greater difficulty, inherent in the observations, than the diurnal inequality; not so much on account of the length of the period as on account of the difficulty of keeping the instrument in precisely the same condition of adjustment throughout the year. In the first part of this discussion I have already had occasion to refer to this circumstance while investigating the annual effect of the secular change, and it was there shown that the Philadelphia observations share in this respect the difficulties of those of other stations,[1] in consequence of which the results must be received with caution.

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[1] It may be proper to give here, in full, Dr. Lloyd's instructive note on this subject, in his discussion of the Dublin observations: "The determination of the annual variation is much more difficult than that

Returning to the last vertical column in the table, headed "mean," we have there the monthly values of the declinometer readings (in scale divisions), and in their differences when compared month for month, the joint effect of the secular change, and of the annual inequality. To eliminate the effect of the secular change, we determine its annual amount as follows: Subtracting the mean annual reading 559.64, corresponding to July 1, from each monthly mean, and putting $x =$ monthly effect of the secular change (considered as uniform), each monthly mean reading furnishes an equation for the determination of x, thus: for

January	$4.56 = 5.5\,x$
February	$3.34 = 4.5\,x$
March	$5.22 = 3.5\,x$
April	$4.39 = 2.5\,x$, etc.,

which, when combined by least squares, give $x = 1^{d.}.227$, hence the annual change $14^{d.}.7$ or $6'.7$ of *increasing westerly* declination.[1]

Deducting the effect of the secular change, and comparing the monthly remainders with their mean values, we obtain the annual inequality of the declination as follows:—

MONTH.	Mean reading.	Reduction for sec. change.	Reduced reading.	Annual inequality.	MONTH.	Mean reading.	Reduction for sec. change.	Reduced reading.	Annual inequality.
	d.	d.	d.	d.		d.	d.	d.	d.
January . . .	564.20	—6.75	557.45	+2.2	July	560.24	+0.61	560.85	—1.2
February . .	562.98	—5.52	557.46	+2.2	August . . .	559.07	+1.84	560.91	—1.3
March . . .	564.86	—4.29	560.57	—0.9	September . .	556.07	+3.07	559.14	+0.5
April	564.03	—3.07	560.96	—1.3	October . . .	556.43	+4.29	560.72	—1.1
May	563.18	—1.84	561.34	—1.7	November . .	553.97	+5.52	559.49	+0.2
June	560.84	—0.61	560.23	—0.6	December . .	549.82	+6.75	556.57	+3.1

The sign { — / + } in the annual inequality indicates an { *easterly* / *westerly* } deflection.

According to these results the magnet (north end) is deflected to the east of its mean annual position in summer, and to the west in winter. It is, however, desirable to test the result by submitting the first and the second $2\frac{1}{2}$ years of observations separately to the same process of investigation. The first 31 months in the years 1840, '41, and '42, give a result almost identical with that just deduced;

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of the diurnal, both on account of the much smaller frequency of the period, and the difficulty of preserving the instrument in the same unchanged condition during the much longer time, or of determining and allowing for its changes when they do occur. Accordingly, although the annual period may be traced in the observations of Gilpin and is decidedly displayed in those of Bowditch, it has evaded the researches of recent observers. There is but a faint indication of its existence in the Göttingen observations, which were made at the hours of 8 A. M. and 1 P. M., and Professor Gauss and Dr. Goldschmidt find, in their analysis of these observations, no important fluctuation dependent on season. A similar negative result is deduced by Dr. Lamont from the Munich observations, which were made twelve times in the day."

[1] This value (+6'.7), as resulting from a different combination of observed and partly interpolated values, may not be preferable to that (+4'.5) deduced in Part I. of this discussion, but must necessarily be employed in the present investigation. The most reliable value, +5'.0, was deduced from independent observations, as already remarked, and lies between the two.

the remaining 27 months in the years 1843, '44, and '45, when discussed in the same manner, give a rather different result.

Some improvements, however, can be made in the preceding investigation by omitting the December mean of 1844, which is obviously about 12 scale divisions too small; the observed value is $535^{d.}.2$, and the interpolated value $547^{d.}.0$. An examination of the first series shows a defect in the monthly means of 1841, between May and June, requiring a constant correction of + 8.0 scale divisions for the remaining months after May, as may be seen by the following table:—

Year.	May.	June.	Diff. May–June.	
1841 . . .	578.5	571.2?	. . . .	Computed value for June, 579.2
1842 . . .	561.8	563.6	—1.8	
1843 . . .	566.2	565.0	+1.2	
1844 . . .	546.5	547.5	—1.0	
1845 . . .	529.7	531.0	—1.3	
Mean			—0.7	

The following values then remain for the discussion, and they should be considered as forming the basis from which the legitimate results are to be deduced. The numbers marked with an asterisk have been increased by $8^{d.}.0$. Interpolated values are between brackets, and were obtained by comparing the means of the remaining months of the year with the corresponding means of every other year; by this process several values are obtained for each interpolated number; the resulting mean is given in the table. The high value of 1841, and the low value of 1844, for the month of May, in some measure compensate.

Year.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	(586.9)	(585.9)	(586.3)	(586.4)	(584.1)	586.9	588.4	587.4	585.2	579.9	573.9	570.5
1841	576.3	574.3	577.0	578.1	578.5	*579.2	*576.7	*576.9	*571.1	*575.2	*564.4	*567.4
1842	564.1	563.3	562.0	561.9	561.8	563.6	565.2	563.8	566.7	562.7	563.5	561.6
1843	(566.5)	(565.6)	(565.9)	567.2	566.2	565.0	564.7	563.6	558.3	559.0	556.1	557.6
1844	558.0	558.0	557.6	555.2	546.5	547.5	547.5	546.2	542.2	545.3	547.2	(547.0)
1845	531.0	531.3	532.0	527.1	529.7	531.0	(529.9)	(529.0)	(527.0)	(526.2)	(522.7)	(522.1)
Means	563.8	563.1	563.5	562.6	561.1	562.2	562.1	561.2	558.4	558.1	554.6	554.4
	560.4											
Correct'n for sec. changes	—4.4	—3.6	—2.8	—2.0	—1.2	—0.4	+0.4	+1.2	+2.0	+2.8	+3.6	+4.4
Corrected means	559.4	559.5	560.7	560.6	559.9	561.8	562.5	562.4	560.4	560.9	558.2	558.8
Annual variation	+1.0	+0.9	—0.3	—0.2	+0.5	—1.4	—2.1	—2.0	—0.0	—0.5	+2.2	+1.6
(in arc)	+0'.5	+0'.4	—0'.1	—0'.1	+0'.2	—0'.6	—1'.0	—0'.9	—0'.0	—0'.2	+0'.9	+0'.7

This last result accords in general with that before deduced, but is much to be preferred.

From June to October the north end of the magnet is accordingly to the eastward of the mean annual position (after the elimination of the secular change), and in the remaining months of the year it is to the westward of this position. From the vernal equinox till after the summer solstice the motion is to the eastward or retrograde in regard to the advance of the secular change (to the westward); this is in conformity with the law as given by Dr. Lloyd in the Dublin discussion, where the motion of the magnet is to the westward at this period of the year, or the reverse of the Philadelphia deflection, but the secular change is likewise reversed, the west declination diminishing at Dublin (at the same time or more accurately between 1840 and '43).

For further comparison I give here the results deduced from seven years' observation at Toronto between the years 1845 and '51, a previous working up of a three years' series (middle year 1846) not being deemed sufficiently distinctive in its results. The secular change is here 2'.0 per annum, increasing westerly declination, whereas it was 4'.4 per annum at Philadelphia in 1843; as in the above result + indicates west, — east deflection.

Annual Variation at Toronto between 1845 and 1851.

January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
+0'.1	—0'.5	—0'.2	0'.0	—0'.1	—0'.5	—0'.8	—0'.2	+0'.7	+1'.0	+0'.3	+0'.3

In regard to the amount of the inequality, the two stations agree remarkably well, the range remaining slightly below 2' of arc. It has been supposed that this range at the same station is increasing or diminishing as the secular change increases or diminishes.

It may further be remarked that the general mean resulting from the above discussion at Philadelphia, viz., 560.4, is identical with the value given in Part I. of the discussion, there deduced by an entirely different combination. The annual effect of the secular change, + 4'.4, is likewise in very close conformity with the value given in Part I., as found by a very different process.

The monthly values of the annual variation may serve to give the corrections to observed declinations in any month of the year needed to refer the same to the mean declination of the year, and may also be used in the more refined discussion of the secular change, in both cases, only, when the greatest accuracy is required.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

PART III.

INVESTIGATION OF THE INFLUENCE OF THE MOON ON THE MAGNETIC DECLINATION.

BY

A. D. BACHE, LL. D.

[ACCEPTED FOR PUBLICATION, SEPTEMBER, 1860.]

INVESTIGATION

OF THE

INFLUENCE OF THE MOON ON THE MAGNETIC DECLINATION.

THE existence of a sensible lunar effect on the magnetic declination has already been established by the labors of Broun, Kreil, Sabine, and others. It is nevertheless important to add the weight of new numerical results to those already obtained.

In the discussions of the Philadelphia observations of magnetic declination, already presented to the Association, I have shown how the influence of magnetic disturbances, of the eleven year period of the solar diurnal variation and its annual inequality, of the secular change, and of the annual variation may be severally eliminated, leaving residuals from which the lunar influence is to be studied. Each observation was marked with its corresponding lunar hour and the hourly normals used for comparison.

This method of treatment of the subject is that followed by General Sabine in his discussion of the results of the British observations.[1]

The details of the method will be better understood by an example.

The time of the moon's passage over the meridian of Philadelphia (upper transit) was obtained from the American Almanac, the small correction for the difference

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[1] In reference to methods and results, in general, on this subject, the following papers may be consulted: Observations in Magnetism and Meteorology made at Makerstown, in Scotland, in the observatory of General Sir Thomas M. Brisbane, Bart., in 1845 and 1846, forming vol. xix., part i. of the Trans. Royal Society of Edinburgh. By John Allan Broun. Edinburgh, 1849; also vol. xix. part ii., containing the general results (1850).

Einfluss des Mondes auf die magnetische Declination by Carl Kreil. Vol. iii. of the Proceedings of the Mathematical and Physical Section of the Imperial Academy of Sciences of Vienna, 1852; also, vol. v., ibid., 1853.

Philosophical Trans. Royal Society, art. xix., 1853: On the Influence of the Moon on the Magnetic Declination at Toronto, St. Helena, and Hobarton. By Col. E. Sabine.

Phil. Trans. Royal Society, art. xxii., 1856: On the Lunar-diurnal Magnetic Variation at Toronto. By Major General E. Sabine. And—

Phil. Trans. Royal Society, art. i., 1857: On the evidence of the Existence of the Decennial Inequality in the Solar-diurnal Magnetic Variations and its Non-existence in the Lunar-diurnal Variation, of the Declination at Hobarton. By Major General E. Sabine.

of longitude being neglected. The observation nearest to the local mean solar time of the moon's transit was marked with a zero, signifying $0^{h.}$ of lunar time. The time of the inferior transit was next obtained; and the observation nearest to it in time was marked 12^{h}. The greatest difference in interval between the moon's transit and the time of observation could in no instance exceed half an hour. In the bi-hourly series, the observations nearest the moon's transit, or to either hour angle, one hour before or one hour after the transit was marked. The mean of a number of differences for the same hours thus gave a result corresponding nearly enough with the hour. The number of observations intermediate between those marked $0^{h.}$ and $12^{h.}$ were marked with the corresponding hour angle by interpolation, care being taken to note the nearest full hour against each observation in the bi-hourly series. The hourly series begins with October, 1843. In the case of thirteen observations within twelve lunar hours, the one nearest midway between the two consecutive lunar hours was omitted.

In the month of March, 1842, which is selected as an example of the details of working the bi-hourly series, the number of observations available is 298, of which 148 correspond to western and 150 to eastern hour angles. In the abstract which follows + indicates a deviation of the north end of the magnet to the west, and — a deviation to the east of the respective normal position for the hour. The hourly normals are given in the first part of the discussion. No difference exceeds eight divisions, this being the limit in number indicated by the criterion.

LUNAR-DIURNAL VARIATION FROM OBSERVATIONS AT PHILADELPHIA IN MARCH, 1842.
Differences from the hourly normals.

☽'s Upper transit. Western hour-angles.

	0^h.	1^h.	2^h.	3^h.	4^h.	5^h.	6^h.	7^h.	8^h.	9^h.	10^h.	11^h.
	+4.1	+6.1	−0.1	−5.0	+1.3	−0.9	−0.1	+1.0	+2.5	+3.1	+0.9	−4.1
	−2.5	−4.2	+2.0	+1.0	+7.2	+0.2	+7.8	0.0	+1.2	−2.0	+2.1	−1.8
	+3.1	−1.4	+4.6	+6.1	+1.4	+0.9	−0.6	−0.5	+4.2	−2.7	+1.6	−6.8
	−4.3	+4.3	−0.7	+1.8	+0.3	+1.2	+2.3	−3.0	+1.0	+5.2	−0.4	−2.0
	−1.9	−0.5	+3.9	−1.3	−0.6	−2.9	−0.5	+1.4	+0.7	−1.8	−1.1	−5.2
	+6.4	+1.0	+3.6	+1.9	−4.8	+1.3	−0.1	+2.1	−3.3	+7.0	+1.7	+5.6
	+1.7	+3.3	+1.6	−0.9	+3.1	−2.3	−1.6	−6.2	+2.3	−1.1	+6.5	+6.6
	−0.7	+0.2	−3.2	−0.9	+2.9	−3.8	−3.8	−3.1	−4.5	−5.5	−1.9	−1.8
	+0.8	−5.1	+3.0	0.0		−7.2	−6.3	−5.9	−2.9	−3.0	+2.6	+1.1
	+2.9	−2.7	+3.1	+1.8		−3.7	−2.4	−4.6	−0.8	+4.4	−1.7	+0.7
	−1.7	+2.8		−3.0		+0.5	+0.5	−5.1	−0.9	−2.2	−0.9	−1.6
		−1.3		−1.9		+2.9	−0.2	−4.6		−1.6	−0.5	
		+2.2		−1.4			−5.9	−4.2		−1.9	−2.9	
				+2.9			+3.5	−1.6		−1.9		
				+2.1				+2.0				
				+1.6								
Means	$+0^d.72$	+0.38	+1.78	+0.30	+1.35	−1.15	−0.53	−2.15	−0.05	−0.29	+0.46	$-0^d.84$

☽'s Lower transit. Eastern hour-angles.

	12^h.	13^h.	14^h.	15^h.	16^h.	17^h.	18^h.	19^h.	20^h.	21^h.	22^h.	23^h.
	−2.7	−0.5	−2.9	+0.5	+5.6	+0.5	+0.6	−4.1	+1.6	−4.9	+4.4	−4.2
	−0.2	−2.4	−1.9	−0.4	−3.4	−1.7	−2.8	+1.3	+6.5	−5.4	−0.3	+0.6
	−1.7	+3.0	+2.9	−0.6	−2.0	−0.6	−1.6	+2.5	−5.1	−0.4	+6.9	−3.1
	−1.0	−0.6	+0.5	−7.2	−1.4	−3.2	−1.0	−0.6	−0.6	+0 4	−0.9	−0.1
	+3.2	−0.2	−5.2	−0.5	−2.3	−0.4	−6.0	+0.3	+0.6	+0.7	+1.6	+1.2
	+0.9	+2.7	+3.4	−1.1	+1.4	+2.2	−3.3	−4.2	+0.2	+0.1	−1.1	+1.7
	+2.4	+2.6	+4.5	+3.4	+1.0	+1.4	−3.8	−3.3	−0.7	−0.4	−2.8	+1.6
	+1.9	+4.9	+5.4	+4.7	−1.0	+2.3	+2.9	−0.3	−3.6	−2.2	−1.5	−3.5
	+7.4	−0.7	−1.1	+7.6	+5.8	−0.1	−0.7	−2.0	+5.9	+1.5	+1.4	−4.0
	+0.3	−4.6	+6.4	+3.4	+3.0	−0.5	−3.0	+3.6	−1.0	+1.2	+3.0	−3.3
	+5.3	−3.5	+1.6	−0.1	−0.3	−3.0	+2.5	−6.6	−1.6	−0.8	−0.4	+0.6
	−1.2	−0.3	+1.5		−1.1	−1.0	−0.7	−3.7	+3.4	+5.6	−3.4	+5.1
					+3.7		+4.9				−6.5	+2.8
					−1.5							
					+0.6							
					−1.1							
Means	$+1^d.22$	+0.20	+1.25	+0.88	+0.44	−0.34	−0.92	−1.43	+0.47	−0.39	+0.03	$-0^d.35$

Number of observations or differences at western hour-angles . . 148
" " " " eastern " . . 150

Total 298

The following table contains the number of observations used in the discussion of the lunar-diurnal variation:—

	1840.	1841.	1842.	1843.	1844.	1845.
January	...	168	265	...	577	591
February	...	263	257	...	571	535
March	...	293	298	...	551	575
April	...	283	278	276	522	561
May	...	276	285	309	596	603
June	300	276	280	300	566	542
July	272	292	267	290	593	...
August	269	262	254	244	541	...
September	253	250	247	283	522	...
October	223	214	221	571	517	...
November	271	230	289	590	517	...
December	237	268	316	595	549	...
Sum	1825	3075	3257	3458	6622	3407
Total sum 21644						

If divided into western and eastern hour-angles, the annual numbers stand as follows:—

	Western hour-angles.	Eastern hour-angles.
1840	916	909
1841	1523	1552
1842	1618	1639
1843	1724	1734
1844	3288	3334
1845	1700	1707
Sum	10769	10875

The preceding mean results will be found inserted in their proper place in the following abstract of the mean monthly values for each observing month between 1840 and 1845.

Proceeding in this way the following results are obtained for the different months discussed.

☽'s Upper transit. Moon's hour-angle.

1840.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
June[1]	−0.23	−0.25	−1.28	+0.95	−1.09	+0.11	−0.21	+0.30	−1.12	+1.60	−0.02	+0.55
July[2]	+0.52	+1.87	−0.56	+2.04	−1.98	+1.60	−1.34	+0.40	−0.21	+0.47	+0.11	+0.75
August	−0.71	−0.10	+1.41	+0.73	+1.05	+1.20	−0.50	−0.44	+0.10	+0.86	+0.20	+0.75
September[3]	+1.74	−0.52	+1.05	−0.87	−0.40	−2.05	−0.67	−1.18	+0.49	+0.28	+0.52	+1.53
October	+0.77	−1.13	+0.37	+0.98	+0.25	+1.23	−0.01	+0.71	−0.78	−0.63	−0.68	−3.61
November[4]	+1.11	+1.04	+1.21	+0.77	+1.07	+1.44	−0.39	−0.53	−1.44	−2.03	−0.08	−1.61
December	−1.43	+1.14	+0.37	+0.37	+0.16	−0.90	−0.73	−1.44	−1.03	+1.01	−0.81	+1.24

☽'s Lower transit.

1840.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
June[1]	+0.50	+0.75	+0.38	+0.86	+0.19	+1.65	−0.72	+0.68	−1.35	+0.69	−2.30	+0.98
July[2]	+1.15	−1.08	−0.41	+0.32	−1.71	+1.03	+0.15	−0.18	−0.37	+1.00	−1.38	−0.03
August	+0.18	−1.56	−0.91	−0.65	−1.15	−0.03	+0.06	−2.61	+1.50	−1.30	−1.27	−0.50
September[3]	+0.64	+0.38	+0.63	+2.25	+0.84	+1.26	−0.61	−0.01	−1.05	−0.61	−0.23	+0.20
October	+0.53	−0.59	+0.30	+1.18	−1.19	+0.63	−0.31	−0.99	−0.40	−0.40	+1.51	+1.05
November[4]	+0.75	−0.62	+0.02	−0.82	−0.49	+0.01	−0.02	+1.09	+0.88	+0.57	+0.14	+0.18
December	+0.91	−0.78	−0.67	−1.82	−0.06	−0.70	−2.57	+1.21	+0.63	+0.86	+0.64	+1.48

[1] The tabular values for this month are expressed in parts of the new or observatory scale, the quantities having been converted from parts of the old or college scale into parts of the new scale.

[2] The tabular numbers refer to the new scale, the values for the first eighteen days of the month having been converted as above.

[3] Attention was paid to the half-monthly normals for the hour $8^{h.}\ 19\frac{1}{2}^{m.}$ (mean observatory time).

[4] The index correction, on and after the twenty-third day of the month, was applied before the differences were taken.

☽'s Upper transit. Moon's hour-angle.

1841.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	+0.86	−1.07	+0.54	+1.39	+0.50	−2.01	+0.89	−0.11	−1.52	+0.48	−0.12	−1.10
February	+1.48	−2.17	+1.12	+0.49	+0.49	+0.10	−0.10	−0.57	−0.38	+0.32	+0.92	+1.40
March	+1.67	+0.82	+0.64	+1.00	+0.61	+0.40	−0.39	−1.07	−1.21	+0.69	−0.65	−0.91
April	+1.57	+1.01	+0.45	+0.97	+0.20	+0.12	+0.39	+1.40	−0.27	−1.52	+0.48	−1.43
May	+0.19	+2.11	+0.69	+1.94	−0.05	+0.92	−0.39	−0.60	−0.73	−0.20	−0.94	+1.21
June	−0.56	+1.77	+0.07	+0.45	+2.18	+1.25	−1.15	−0.59	−2.40	−1.13	−0.42	−1.24
July	+0.84	+1.86	+0.46	−1.06	−0.62	−1.52	−0.80	−0.55	−0.88	−1.71	−0.24	+1.63
August	+1.95	+1.31	+1.73	+1.42	−1.17	−1.46	−1.48	−1.39	−2.06	−2.24	−1.72	+0.60
September	+1.05	+0.10	−0.45	−0.17	−3.50	−0.54	−0.55	−0.83	−1.47	+0.86	+1.29	+0.03
October[1]	−1.15	+0.26	−0.77	−0.06	−1.31	−0.82	−0.66	−0.61	−1.73	+1.73	+0.22	+1.09
November	+0.01	−0.08	+0.02	+0.54	+0.23	−1.08	+1.54	+0.52	+1.39	+0.02	−0.24	−0.06
December	−0.41	+0.10	+0.45	−0.71	−0.94	+0.55	−0.51	+1.09	+0.62	−0.47	+0.48	+0.08

☽'s Lower transit.

1841.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	+1.33	+0.57	−0.04	−0.51	−0.50	+0.21	+0.25	−2.10	−0.21	−1.32	−0.07	0.00
February	−0.03	−1.30	−0.78	−0.30	−1.23	−2.01	−1.12	−1.08	+0.60	+1.30	+0.56	+1.07
March	+0.15	+0.18	+1.05	+0.23	−0.15	−0.59	−0.23	−0.93	−0.47	−0.98	+1.89	+0.35
April	+1.35	−1.05	−0.09	+0.90	−0.02	−1.13	−0.32	−1.67	−0.89	−0.13	−0.63	+0.02
May	+0.42	+1.44	+0.56	+0.24	−1.21	−0.89	−2.64	−0.85	−2.20	−1.09	+0.96	+0.90
June	+0.11	−1.42	−0.13	+0.67	+1.18	−0.53	+0.62	−1.14	+1.79	+0.01	−0.22	+0.80
July	+1.26	+1.50	+1.09	+1.76	+0.32	+0.45	−0.80	+0.01	−0.95	+0.27	−0.87	+0.44
August	+2.28	+0.51	+1.97	+1.19	+0.62	−1.81	−0.50	−1.07	−0.59	+1.66	+0.06	+1.20
September	+0.37	+0.41	+1.21	+0.95	−1.66	−0.44	−0.25	−0.45	+0.45	+0.19	+0.85	+0.44
October[1]	−1.73	+1.04	+0.76	+0.34	+0.18	+1.60	+0.97	+3.14	+1.30	+3.10	−0.61	−1.54
November	+1.01	+0.03	−1.20	−0.30	−1.89	−1.33	−0.72	−0.49	+0.50	−1.89	+0.79	−0.27
December	+0.73	−0.59	+0.80	−0.49	+0.71	−0.92	−0.67	−1.27	+0.12	+1.21	+1.76	+0.83

☽'s Upper transit. Moon's hour-angle.

1842.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	−0.30	+0.64	−0.53	+0.02	+0.66	−0.61	+0.14	−1.48	+0.44	−1.20	+0.26	−1.84
February	−0.73	+0.88	+0.36	−0.13	−0.83	+0.67	+0.18	−1.80	−0.92	−0.73	−0.27	+0.04
March	+0.72	+0.38	+1.78	+0.30	+1.35	−1.15	−0.53	−2.15	−0.05	−0.29	+0.46	−0.84
April	−0.77	+0.92	+0.53	+0.37	−0.07	−0.39	−0.20	−1.65	+0.27	−0.42	+1.21	+0.10
May	−0.57	+1.78	+0.01	−0.16	+0.18	−1.01	−1.41	−0.97	−0.92	+0.08	−0.43	+0.42
June	+0.38	+0.69	−0.95	+1.64	−0.18	+0.77	−0.25	−0.32	+0.76	+1.18	+0.38	−0.74
July	+0.78	+0.16	+0.69	−0.07	+0.60	−0.76	−2.08	+0.08	−1.65	+0.87	−1.04	+3.03
August	+0.88	+0.82	−0.08	−1.03	+1.17	−0.91	−0.95	+0.67	+0.72	−1.24	−0.17	+1.65
September	+0.71	−0.52	−0.13	−0.95	+0.67	+0.96	−0.82	+0.34	+0.82	+0.35	+0.62	+1.36
October	+3.46	+0.38	+0.77	−0.29	+0.06	+0.02	−0.25	−2.21	−0.98	−1.39	+0.52	−1.09
November	−0.05	+0.38	−1.07	−0.48	−0.36	−1.10	−0.53	+0.43	−0.95	+0.54	+0.14	+0.29
December	−0.59	−0.36	−0.34	−1.15	−0.75	+0.26	−0.57	+0.24	+0.39	+0.64	+0.87	+0.16

☽'s Lower transit.

1842.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	−0.18	−1.19	−0.11	−2.13	+0.17	−0.61	+0.45	+1.05	+0.72	+0.66	−0.12	+0.62
February	−0.91	+0.71	+0.52	+0.40	−0.97	−0.86	−1.11	+0.44	−0.12	+0.14	+0.08	+0.84
March	+1.22	+0.20	+1.25	+0.88	+0.44	−0.34	−0.92	−1.43	+0.47	−0.39	+0.03	−0.35
April	+3.28	−0.86	+0.13	−0.12	−1.05	−1.34	−1.36	+0.15	−1.22	+0.19	−0.94	+1.11
May	+1.13	+1.78	+1.59	+1.10	−0.59	−0.52	−0.68	−1.47	−1.05	+0.15	−0.70	+1.01
June	+0.20	−0.82	+1.45	+0.33	+1.73	−1.19	+0.05	−1.36	−1.04	−1.43	−1.35	−1.37
July	−0.32	+1.84	−0.86	−0.72	+0.59	−0.95	−0.27	+0.03	−1.22	+0.09	−0.58	+0.68
August	−0.68	+2.50	−1.34	+0.59	−1.41	−0.67	−0.79	−0.58	−0.96	−0.26	+1.68	+0.81
September	+0.46	+1.11	−1.94	+0.25	+0.99	−0.45	−1.64	+0.10	−1.70	+2.14	+1.50	+0.96
October	+1.31	+1.68	−0.62	+0.74	−1.87	−0.14	+1.08	+0.43	−0.16	−0.25	+0.71	−0.56
November	+0.47	+0.40	+0.91	−1.13	+0.02	+0.11	−0.22	−1.46	+0.05	+0.68	+0.94	+1.58
December	+0.53	+0.35	+0.12	−0.45	−1.12	+0.15	+0.35	+0.54	+0.40	+0.57	+0.21	+0.07

[1] At $14^{h.}\ 19\frac{1}{2}^{m.}$ (observatory time) the difference from the half-monthly normals was used.

☽'s Upper transit. Moon's hour-angle.

1843.[1]	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
April	+0.87	+1.47	+1.66	+0.39	−0.42	−1.30	−2.64	−1.72	−1.99	−0.12	−1.63	−0.48
May	+0.94	+0.89	+1.54	+0.45	+0.27	+0.38	+0.23	−1.02	−0.79	−1.01	+0.47	+1.08
June	−0.13	−1.58	+0.18	−0.81	+0.67	+1.21	−0.31	+0.83	+0.16	+0.61	−0.10	+1.30
July	+2.10	+0.91	−0.71	+0.65	+0.69	+0.54	−0.62	+0.56	−0.39	−2.29	+1.05	−0.16
August[2]	−1.56	−0.81	−2.28	+1.17	−0.05	−1.12	+0.32	−1.24	+0.26	−0.22	−0.69	+0.46
September	−0.71	+0.26	−0.58	−0.85	−1.08	−0.23	−0.30	+1.74	−0.74	+0.37	−0.42	+0.58
October[3]	+1.05	+0.14	+0.28	+0.17	−0.03	−0.93	+0.19	−0.52	−1.16	+0.27	+0.33	+0.33
November	+0.52	+0.16	−0.72	−0.47	−0.80	−0.84	−0.57	−0.72	−0.02	+0.23	−0.17	+0.72
December	−0.41	−0.24	−0.64	−1.15	−0.88	−0.41	+0.07	+0.08	+0.39	+0.99	+1.09	+1.28

☽'s Lower transit.

1843.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
April	+0.79	+1.92	+0.72	−0.06	+0.53	+0.05	−1.10	−1.05	−0.22	−1.06	−0.56	+1.58
May	+0.67	+0.74	+1.01	−0.58	−1.01	−1.03	−1.43	−0.27	−0.52	−0.49	+0.70	+0.08
June	+0.94	+1.46	−0.55	+0.29	−0.99	−0.05	−0.63	+0.07	−0.38	−0.22	+0.74	−0.20
July	−0.25	+0.61	+0.66	+0.66	−0.43	−1.10	−2.00	−1.05	−0.20	−0.06	−0.54	+1.73
August[2]	+0.91	−0.59	−0.77	+0.59	−1.85	+0.01	−1.00	+1.37	−0.92	+0.74	+0.49	+0.06
September	+1.63	+1.85	+0.78	+2.32	+1.15	−0.29	−0.86	+1.08	+0.65	−0.37	−0.90	−0.78
October[3]	+0.76	+1.50	+1.30	+0.53	−0.71	−0.92	−1.76	−0.70	−0.08	+0.50	−0.37	+0.78
November	+0.67	+0.45	−0.33	−0.25	−0.54	+0.04	−0.24	+0.17	+1.06	+1.00	+0.27	+0.50
December	+0.83	+0.51	+0.60	+0.62	+0.28	−1.14	−0.59	−0.74	−0.46	+0.46	+0.24	−0.42

☽'s Upper transit. Moon's hour-angle.

1844.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January[4]	−0.79	−0.18	−0.26	+0.07	+0.20	+0.94	+0.58	+0.19	+0.22	+0.37	−0.46	+0.43
February	+1.43	+0.87	+0.67	−0.52	−0.69	−0.82	−0.56	−0.74	−0.29	+0.77	+1.03	+0.96
March	+1.10	+1.06	+0.42	+0.04	−0.72	−0.55	−0.69	−0.16	+1.18	+0.05	+0.93	−0.02
April	−0.52	+0.08	+0.23	+0.54	+0.09	+0.35	−0.49	−0.12	−0.55	−0.41	+0.16	−0.04
May	+0.76	+1.17	+0.88	+0.27	+0.02	−0.49	−0.18	−0.60	−0.35	−0.10	+0.14	+0.27
June	+1.11	+0.68	+1.07	+0.44	+0.09	−0.64	−0.24	−1.33	−1.58	−1.47	−0.40	+0.22
July	+1.09	+1.27	+0.78	+0.97	+0.18	−0.73	−1.05	−1.77	−0.17	−0.13	+0.68	+0.37
August	+2.30	+0.93	+0.19	−0.14	−0.16	−1.55	−0.78	−0.69	−0.38	−0.66	+0.45	+0.45
September	+1.13	+1.47	−0.21	−0.05	−0.61	−1.15	−0.81	+1.05	+1.10	−0.18	+0.12	−0.34
October	−0.22	+0.42	−0.02	+0.22	−0.41	−0.59	−0.78	+0.38	−0.02	+1.04	+1.10	+1.01
November	−0.91	−1.12	−0.71	−0.57	−0.76	+0.03	−0.01	+0.45	−0.77	+0.06	+0.62	+2.57
December[5]	−0.26	−0.74	−0.21	−0.44	−1.14	−0.33	−0.41	−0.18	+0.14	+0.33	+0.36	+0.60

☽'s Lower transit.

1844.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January[4]	+0.32	+0.10	+0.31	−0.09	−0.61	−0.17	+0.84	+0.95	+0.32	−0.10	−0.80	−0.48
February	+0.44	0.00	+0.54	−0.26	−1.10	−0.49	−1.13	−0.39	−0.02	−0.05	−0.02	+0.84
March	+1.33	+0.52	−0.50	−0.21	−0.21	−0.68	−1.60	−0.50	−0.31	−0.48	+0.35	−0.10
April	+0.87	+0.70	+0.37	+0.64	−0.22	−0.54	−0.50	+0.05	−0.66	−0.42	−0.02	−1.63
May	+0.46	+0.09	+0.74	+0.43	+0.62	+0.06	−0.19	−1.10	−0.85	−1.17	+0.10	+0.06
June	+0.19	+0.48	−0.30	−0.36	−0.01	+0.35	+0.31	+0.29	+0.25	+0.20	+0.11	0.00
July	+1.27	+0.36	+0.46	−0.70	−0.51	−0.70	−1.03	−0.13	−0.31	−0.57	−0.12	+0.78
August	+0.50	+0.22	+0.84	−0.30	−0.19	−0.77	−1.06	−0.75	−0.34	−0.14	−0.43	+0.76
September	+0.25	+0.04	+0.73	−0.20	−0.03	−1.20	−1.89	−1.27	−1.33	−0.62	+0.13	+1.15
October	+0.56	+1.19	+0.78	−0.10	−0.36	−0.32	−0.03	−0.83	−0.58	−0.55	+0.06	+0.06
November	+0.36	+1.09	+0.67	+0.43	+0.05	+0.40	+0.07	−0.77	−0.68	+0.41	+0.15	−0.11
December[5]	+0.48	+0.64	+1.06	−0.12	−0.12	−0.64	−0.22	+0.23	+0.26	+0.68	+0.17	+0.42

[1] There are no observations in January, February, and March, of this year.

[2] Attention was paid to the shifting of the zero of the scale between the 9th and 10th.

[3] Commencement of the hourly series of observations.

[4] Proper attention was paid to the change in the zero of divisions after the 10th.

[5] The half-monthly normals were used.

☽'s Upper transit. Moon's hour-angle.

1845.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	−0.46	−1.65	−1.52	−1.65	−1.63	−0.24	+0.11	+1.41	+0.96	+1.83	+0.91	+1.11
February	−0.13	+0.48	−0.26	−1.15	−0.56	−0.81	−0.39	−0.28	+0.18	+1.03	+0.98	+1.28
March	−0.42	−0.47	−0.26	−0.48	−0.25	−0.75	−0.81	−0.25	+0.20	+0.39	+0.79	+0.91
April	+0.45	+0.54	+0.07	+0.52	−0.21	−0.47	−0.27	−0.07	−0.25	−0.03	+0.27	+1.08
May	+0.53	+0.49	+0.01	+0.16	−0.21	−0.22	−0.66	−0.25	−0.88	+0.04	+0.92	+0.43
June	+1.77	+1.63	+0.90	+1.24	+0.86	+0.54	−0.66	−1.09	−0.75	−0.93	−0.83	−0.31

☽'s Lower transit.

1845.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	+0.02	−0.28	−1.07	−0.60	−0.30	+0.14	+1.09	+0.29	+0.86	+0.34	+0.39	+0.38
February	+1.70	+0.67	−0.13	+0.40	+0.03	−0.76	−0.92	−1.26	−0.46	+0.17	−0.05	−0.09
March	+1.15	+0.95	+1.79	+0.35	+0.86	−0.08	−0.83	−1.27	−0.56	+0.37	−0.39	−0.73
April	+0.54	+0.56	0.00	+0.76	+1.01	−0.30	−1.00	−1.67	−1.62	−0.97	+0.37	−0.78
May	+0.53	+0.03	−0.63	−0.01	−0.24	−0.48	−0.70	−0.30	−0.40	−0.53	+1.16	+0.63
June	+0.01	+0.86	+0.30	+0.18	−0.33	−1.27	−0.82	−0.59	−0.92	+0.05	+0.74	+0.64

Value of a scale division 0′.453.

One of the first questions to determine is how many of these residuals must be used to give a definite result, and another one is whether numbers deduced from different parts of the series would give harmonious results. To test both of these the observations were formed into three groups—one containing 4,900 in 19 months of 1840, '41; another, 6,715 results in 21 months of 1842, '43; and a third, 10,029 results in 18 months of 1844, '45. In all, 21,644 results.

The following table contains the result for each group. Group II includes three months of the hourly series of observations treated as if only equal in weight to the bi-hourly series.

The sign Σ indicates the algebraic sum of the values in the preceding tables for the months comprised in each group, and for every hour-angle of the moon. The lines headed I, II, III, contain the preceding values divided by their respective number of months and expressed in minutes of arc, or the lunar diurnal variation.

☽'s Upper transit. Moon's hour-angle.

	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Σ of group I	+10.27	+8.07	+7.52	+11.17	−4.32	−1.46	− 7.06	− 5.49	−14.63	−1.61	−1.70	+ 0.30
Σ " II	+ 6.59	+7.35	−0.23	− 2.38	+0.87	−5.95	−10.90	−10.83	− 6.35	−2.78	+2.48	+ 7.65
Σ " III	+ 7.96	+6.93	+1.77	− 0.53	−5.91	−7.48	− 7.60	− 4.05	− 2.01	+2.00	+7.77	+10.98
I	+0′.24	+0′.19	+0′.18	+0′.27	−0′.10	−0′.04	−0′.17	−0′.13	−0′.35	−0′.04	−0′.04	+0′.01
II	+0.14	+0.16	+0.00	−0.05	+0 02	−0.13	−0.24	−0.23	−0.14	−0.06	+0.05	+0.16
III	+0.20	+0.17	+0.05	−0.02	−0.15	−0.19	−0.20	−0.10	−0.05	+0.05	+0.20	+0.28

☽'s Lower transit. Moon's hour-angle.

	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Σ of group I	+11.91	− 2.18	+4.54	+6.00	−7.22	− 3.54	− 9.43	−8.71	−0.71	+3.14	+1.58	+7.60
Σ " II	+13.46	+16.15	+4.52	+3.86	−6.64	−11.24	−14.67	−4.68	−6.90	+2.79	+1.53	+8.73
Σ " III	+10.98	+ 8.22	+5.96	+0.24	−1.66	− 7.45	− 9.61	−9.02	−7.35	−3.38	+1.90	+1.80
I	+0′.29	−0′.05	+0′.11	+0′.14	−0′.17	−0′.09	−0′.23	−0′.21	−0′.04	+0′.08	+0.04	+0′.18
II	+0.29	+0.35	+0.10	+0.08	−0.14	−0.24	−0.32	−0.10	−0.14	+0.06	+0.03	+0.19
III	+0.28	+0.21	+0.15	0.00	−0.04	−0.19	−0.24	−0.23	−0.19	−0.08	+0.05	+0.05

+ indicates west, — east, deflection from the normal position.

These results, I, II, III, when expressed analytically by means of Bessel's form of periodic functions, and when treated by the method of least squares, are represented by the following equations, in which the moon's hour-angle θ is reckoned from the upper transit westwards at the rate of 15° to each hour. $\Delta_{☾}$ represents the lunar diurnal variation.

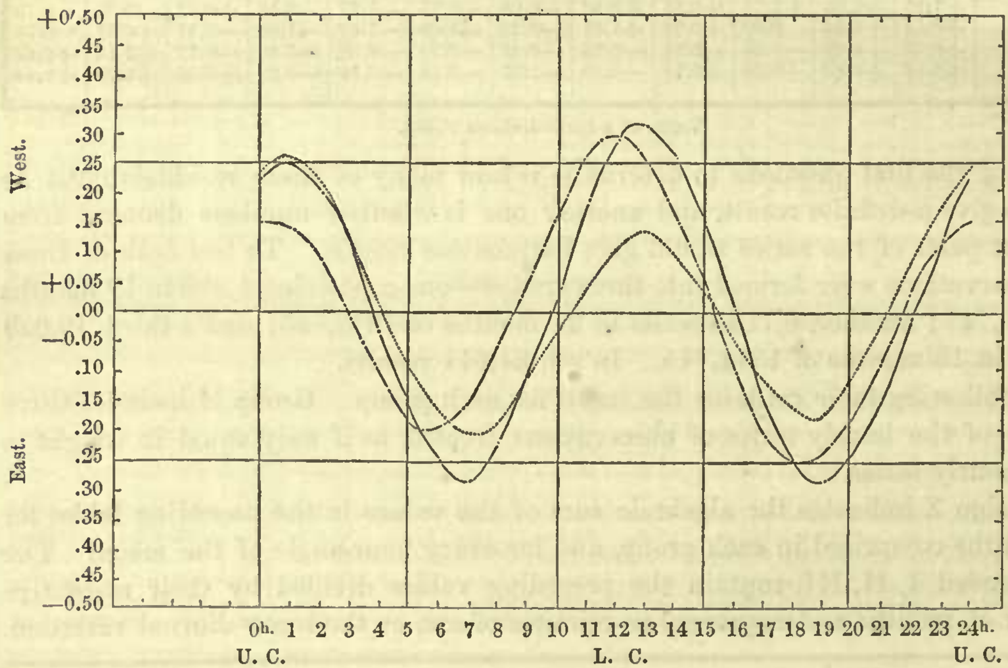
Group I, 1840–'41. $\Delta_{☾} = +0'.003 + 0'.063 \sin. (\theta + 92°) + 0'.189 \sin. (2\theta + 67°)$

" II, 1842–'43. $\Delta_{☾} = -0'.006 + 0'.030 \sin. (\theta + 263°) + 0'.282 \sin. (2\theta + 63°)$

" III, 1844–'45. $\Delta_{☾} = 0'.000 + 0'.075 \sin. (\theta + 292°) + 0'.219 \sin. (2\theta + 88°)$

The numerical results from these equations are presented graphically on the following diagram.

LUNAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION.



.... from 4,900 observations in 1840, '41.
———— from 6,715 observations in 1842, '43.
-.-.-.-.-.- from 10,029 observations in 1844, '45.

The curves all agree in their distinctive characters, and show two east and two west deflections in a lunar day, the maxima W. and E. occurring about the upper and lower culminations, and the minima at the intermediate six hours. The total range hardly reaches 0'.5. These results agree generally with those obtained for Toronto and Prague.

From 8,000 to 10,000 observations seem to be required to bring out the results satisfactorily, and the best results are derived from the use of all the groups.

The following table contains annual sums of deflections for each hour, and the resulting lunar-diurnal variation from the 21,644 observations available for the purpose:—

Upper curve. Westerly hour-angles.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Σ for 1840	+1.77	+2.05	+2.57	+4.97	—0.94	+2.63	—3.85	—2.18	—3.99	+1.56	—0.76	—0.40
" 1841	+8.50	+6.02	+4.95	+6.20	—3.38	—4.09	—3.21	—3.31	—10.64	—3.17	—0.94	+0.70
" 1842	+3.92	+6.15	+1.04	—1.93	+2.50	—3.25	—7.27	—8.82	—2.07	—1.61	+2.55	+2.54
" 1843 (*a*)	+1.51	+1.14	—0.19	+1.00	+0.08	—0.52	—3.32	—0.85	—3.49	—2.66	—1.32	+2.78
" 1843 (*b*)	+1.16	+0.06	—1.08	—1.45	—1.71	—2.18	—0.31	—1.16	—0.79	+1.49	+1.25	+2.33
" 1844	+6.22	+5.91	+2.83	+0.83	—3.91	—5.53	—4.92	—3.52	—1.47	—0.33	+4.73	+6.48
" 1845	+1.74	+1.02	—1.06	—1.36	—2.00	—1.95	—2.68	—0.53	—0.54	+2.33	+3.04	+4.50
Mean $\frac{\Sigma}{79}$	+0.43	+0.37	+0.12	+0.08	—0.21	—0.31	—0.42	—0.32	—0.33	+0.01	+0.23	+0.41
Same in arc	+0′.19	+0.17	+0.05	+0.04	—0.10	—0.14	—0.19	—0.14	—0.15	+0.01	+0.10	+0′.19

Lower curve. Easterly hour-angles.

YEAR.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mos.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	
Σ for 1840	+4.66	—3.50	—0.66	+1.32	—3.57	+3.85	—4.02	—0.81	—0.16	+0.81	—2.89	+3.36	7
" 1841	+7.25	+1.32	+5.20	+4.68	—3.65	—7.39	—5.41	—7.90	—0.55	+2.33	+4.47	+4.24	12
" 1842	+6.51	+7.70	+1.10	—0.26	—3.07	—6.81	—5.06	—3.56	—5.83	+2.29	+1.46	+5.40	12
" 1843 (*a*)	+4.69	+5.99	+1.85	+3.22	—2.60	—2.41	—7.02	+0.15	—1.59	—1.46	—0.07	+2.47	6
" 1843 (*b*)	+2.26	+2.46	+1.57	+0.90	—0.97	—2.02	—2.59	—1.27	+0.52	+1.96	+0.14	+0.86	3
" 1844	+7.03	+5.43	+5.70	—0.84	—2.69	—4.70	—6.43	—4.22	—4.25	—2.81	—0.32	+1.75	12
" 1845	+3.95	+2.79	+0.26	+1.08	+1.03	—2.75	—3.18	—4.80	—3.10	—0.57	+2.22	+0.05	6
Mean $\frac{\Sigma}{79}$	+0.63	+0.42	+0.29	+0.14	—0.23	—0.40	—0.58	—0.42	—0.27	+0.01	+0.09	+0.26	37 21
Same in arc	+0′.29	+0.19	+0.13	+0.06	—0.10	—0.18	—0.26	—0.19	—0.12	+0.01	+0.04	+0′.12	21
													79

The two values for 1843, marked (*a*) and (*b*), exhibit the separate sums for the bi-hourly and the hourly observations, and were required to give proper weights to each. There are 37 months of bi-hourly, and 21 months of hourly observations —the latter having double weight, as found from a consideration of the probable errors derived respectively from all the results of the years 1842 and 1844. The probable error of any single monthly mean for any hour in the year 1842 was found $= \pm\, 0^{d}.60$, and the same for the year 1844 was $= \pm\, 0^{d}.40$. Hence the weights for a resulting value in the bi-hourly series is to the weight for a value in the hourly series nearly as 1 : 2, or the weights are nearly proportional to the number of observations—a result which indicates that no constant errors influence the result. The accordance among themselves of the values for the easterly hour-angles is somewhat better than the corresponding values for the westerly hour-angles—a circumstance which seems to connect itself with another phenomenon to be mentioned presently. Giving, therefore, double weight to months of the hourly series, the lunar-diurnal variation resulted as given above. When expressed analytically, it takes the form

$$\triangle_{☾} = +0'.001 + 0'.029 \sin(\theta + 295°) + 0'.207 \sin(2\theta + 85°)$$

which may also be written

$$\triangle_{☾} = 0''.0 + 1''.7 \sin(15n + 295°) + 12''.4 \sin(30n + 85°)$$

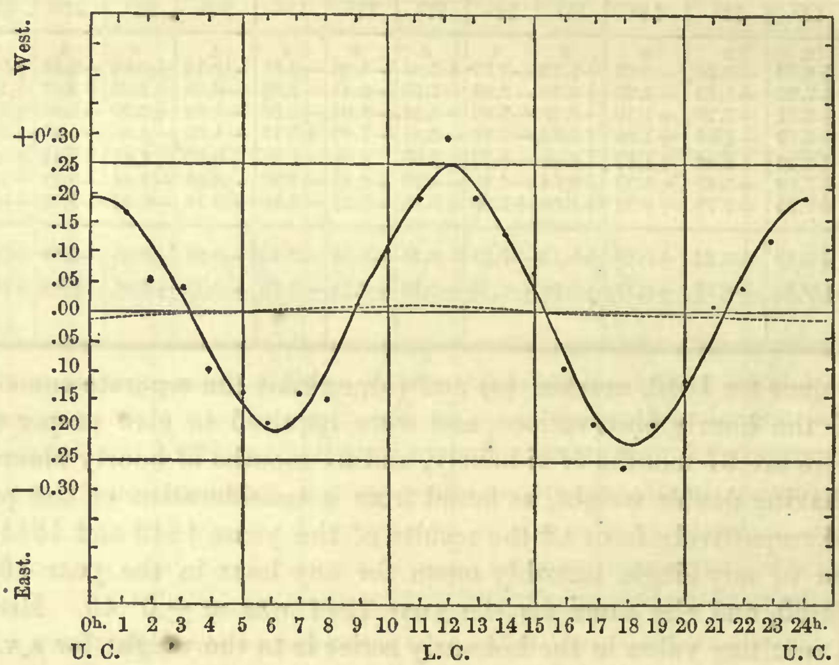
where θ represents the moon's hour-angle, reckoned from the upper culmination, or n the number of hours after the same epoch: + indicates west, and — east deflection.

The constant in Bessel's formula comes out zero, and hence it is inferred that the moon has no specific action in deflecting the magnet by a constant quantity. The coefficient of the first term of the formula is small, and it is from the second term that the distinctive features of the double-crested curve result. These results are all represented by curves.

Both the east and west deflections are well marked, those occurring when the moon is east of the meridian being greater than those when west.

It is not at all necessary to take in the third or higher terms. The progression of the hourly values is systematic, and the agreement between the computed and observed values is deemed satisfactory. The following diagram represents the curve resulting from the above equation, the observed values being indicated by dots.

Lunar-Diurnal Variation.



From 21,644 observations at Philadelphia, from 1840 to 1845.

The principal western maximum occurs 6 minutes after the lower culmination of the moon, and amounts to 0′.23. The secondary maximum occurs 14 minutes after the upper culmination, and amounts to 0′.18. The principal minimum occurs at $6^{h.}$ $17^{m.}$ after the lower culmination, the easterly deflection being 0′.22. The secondary minimum at $6^{d.}$ $03^{m.}$ after the upper culmination, with a deflection of 0′.19. The greatest range is 27″, and the secondary 22″. The epochs of the maxima and minima are found from the formula to be at a mean 10 minutes after culmination. The probable error of a single computed value of the lunar declination is $\pm$ 1″.32. The Toronto observations gave $\pm$ 1″.37 from more than twice the number of observations, so that the Philadelphia results are worthy of every confidence.

At Toronto, from the second investigation, embracing about 44,000 observations, the western and eastern deflections balanced, giving for the range 38″.3. The

Prague observations also confirm the nearly equal deflections (mean) to the west and east. The epochs of the maxima and minima were found from the four roots of the equation $0 = 0.029 \cos(\theta + 295°) + 0.414 \cos(2\theta + 85°)$, which gave 10 minutes as the mean time elapsed between the moon's passing the meridian, and the time of maxima deflections. If we take the four phases into account, the lunar action seems to be retarded 10 minutes, which quantity may be termed the *lunar-magnetic interval* for the Philadelphia station. At Toronto the intervals are not so regular.

The secondary range exists at Toronto, and is a marked feature in the Prague result.

The following table contains the observed and computed values and their differences:—

	Upper Curve.				Lower Curve.		
	Observed.	Computed.	Difference.		Observed.	Computed.	Difference.
0h.	+0′.19	+0′.18	+0′.01	12h.	+0′.29	+0′.23	+0′.06
1	+0.17	+0.17	0.00	13	+0.19	+0.21	—0.02
2	+0.05	+0.10	—0.05	14	+0.13	+0.13	0.00
3	+0.04	+0.01	+0.03	15	+0.06	+0.03	+0.03
4	—0.10	—0.09	—0.01	16	—0.10	—0.08	—0.02
5	—0.14	—0.16	+0.02	17	—0.18	—0.18	0.00
6	—0.19	—0.19	0.00	18	—0.26	—0.22	—0.04
7	—0.14	—0.17	+0.03	19	—0.19	—0.21	+0.02
8	—0.15	—0.09	—0.06	20	—0.12	—0.14	+0.02
9	+0.01	+0.01	0.00	21	+0.01	—0.05	+0.06
10	+0.10	+0.12	—0.02	22	+0.04	+0.06	—0.02
11	+0.19	+0.20	—0.01	23	+0.12	+0.14	—0.02

The formula or curve enables us to divide the observed curve so as to show the diurnal and semi-diurnal part of the observed variations. The decomposition of the curve is made on the diagram where the resulting curve for the diurnal period is given.

The lunar-diurnal variation seems to be subject to an inequality depending on the solar year, for the investigation of which the preceding results were rearranged in two groups, one containing the hourly values for the summer months (April to September), the other the values for the winter months (October to March). For the summer season we have 11,087 observations, and for the winter 10,557.

HOURLY SUMS OF THE LUNAR VARIATION FOR THE SUMMER SEASON.

Moon's hour-angle.

	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.	
Σ 1840–3	+9.29	+14.15	+3.45	+7.20	—2.93	—2.23	—15.73	—6.18	—12.04	—4.57	—1.49	+12.38	
Σ 1844–5	+8.62	+8.26	+3.92	+3.95	+0.05	—4.36	—4.64	—4.87	—3.81	—3.87	+1.51	+2.13	
$\frac{\Sigma}{40}$	+0.66	+0.77	+0.28	+0.38	—0.07	—0.27	—0.63	—0.40	—0.49	—0.31	+0.04	+0.42	
Same in arc	+0′.30	+0.35	+0.13	+0.17	—0.03	—0.13	—0.28	—0.18	—0.22	—0.14	+0.02	+0.19	
	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mo's.
Σ 1840–3	+17.02	+11.44	+5.18	+13.14	—4.94	—7.97	—16.72	—10.27	—12.44	+0.11	—5.49	+10.12	22
Σ 1844–5	+4.62	+3.32	+2.51	+0.44	+0.10	—4.85	—6.88	—5.47	—6.18	—4.17	+2.04	+1.61	9
$\frac{\Sigma}{40}$	+0.66	+0.45	+0.26	+0.35	—0.12	—0.44	—0.76	—0.53	—0.62	—0.21	—0.04	+0.33	
Same in arc	+0′.30	+0.20	+0.12	+0.16	—0.05	—0.20	—0.34	—0.24	—0.28	—0.09	—0.02	+0.15	

Hourly Sums of the Lunar Variation for the Winter Season. Moon's hour-angle.													
	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.	
Σ 1840–2	+6.42	+1.21	+4.92	+3.04	+1.19	—3.00	—1.92	—8.98	—8.15	—1.31	+1.02	—6.76	
Σ 1843–5	+0.50	—1.27	—3.23	—5.93	—7.67	—5.30	—3.27	—0.34	+1.02	+7.36	+7.51	+11.18	
$\frac{\Sigma}{39}$	+0.19	—0.04	—0.04	—0.23	—0.36	—0.35	—0.22	—0.25	—0.16	+0.35	+0.41	+0.40	
Same in arc	+0′.09	—0.02	—0.02	—0.10	—0.16	—0.16	—0.10	—0.11	—0.07	+0.16	+0.18	+0.18	
	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mo's.
Σ 1840–2	+6.09	+0.09	+2.31	—4.18	—7.95	—4.79	—4.79	—1.85	+4.31	+3.86	+8.46	+5.35	15
Σ 1843–5	+8.62	+7.34	+5.02	+0.70	—2.73	—4.62	—5.32	—4.82	—0.65	+2.75	0.00	+1.05	12
$\frac{\Sigma}{39}$	+0.60	+0.38	+0.32	—0.07	—0.35	—0.37	—0.40	—0.29	+0.08	+0.25	+0.22	+0.19	
Same in arc	+0′.27	+0.17	+0.14	—0.03	—0.16	—0.17	—0.18	—0.13	+0.04	+0.11	+0.10	+0.09	

Expressed analytically, the lunar-diurnal variation in the two seasons is as follows:—

In summer,

$$\Delta_{☾} = -0'.006 + 0'.028 \, sin \, (\theta + 18°) + 0'.278 \, (2\theta + 67°)$$

In winter,

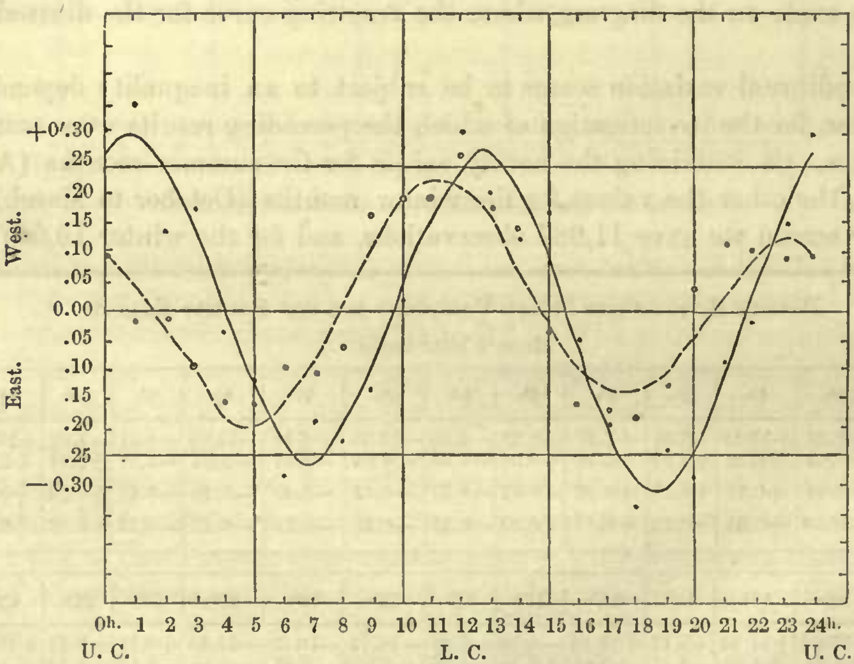
$$\Delta_{☾} = +0'.005 + 0'.058 \, sin \, (\theta + 264°) + 0'.173 \, (2\theta + 115°)$$

The characteristic feature of the annual inequality in the lunar-diurnal variation is, therefore, a much smaller amplitude in winter than in summer. Kreil, indeed,

Lunar-Diurnal Variation.

In summer ————

In winter — — — —



Summer curve from 11,087 observations (.)
Winter " " 10,557 " (°)

inferred from the ten-year series of the Prague observations, that in winter the lunar-diurnal variation either disappears, or is entirely concealed by irregular fluctuations, requiring a long series for their diminution. The method of reduction which he employed was, however, less perfect than that now used. The second characteristic of the inequality consists in the earlier occurrence of the maxima and minima in winter than in summer. The winter curve precedes the summer curve by about one and three-quarter hours. Both these features are well expressed in the above diagram. At Toronto, the same shifting in the maxima and minima epochs was noticed, but the other inequality in the amount of deflection is not exhibited. It seems probable that the Philadelphia results are more typical in form than those either of Prague or Toronto. It is also apparent that the smaller deflection at the upper culmination in the annual mean, when compared with the deflection at the lower culmination, is entirely produced by the feeble lunar action in winter. The maximum west deflection in summer occurs actually near the upper culmination. At the same season the maximum east deflection is still retained (as in the annual curve) about six hours after the lower culmination. In the winter season this last mentioned maximum east deflection is actually the smaller of the two. We have—

Maximum summer range	35″.4,	Secondary,	31″.8
" winter "	25 .2,	"	15 .6
Difference	10 .2,		16 .2

At Prague the maximum summer range was 44″.

Next I proceed to examine whether the phases of the moon, the declination, or parallax, have any sensible effect upon the magnetic declination. Mr. Kreil found, from a ten years' series of observations at Prague, that there was no specific change in the position of the magnet depending upon the moon's phases and parallax, but that the declination was 6″.8 greater when the moon was at the greatest northern declination than when at the greatest southern declination. On the contrary, Mr. Broun, from the Makerstoun observations, a much shorter series than the one at Prague, inferred that there was a maximum of declination two days after the full moon. He also found a maximum corresponding to the greatest northern declination of the moon, but does not appear to have investigated the effect of distance. The residuals which we have been treating enable us at once to examine these several points.

Beginning with the lunar phases, the daily means for the day of full and new moon, and for two succeeding days, were compared with the monthly mean declination. In case any of the hours were disturbed, the monthly normal for the hour was substituted for the disturbed observation before the mean was taken. If one-half or more of the hourly readings were disturbed, the daily mean was altogether omitted. Accidental omissions of hourly observations were supplied by the hourly normal. The half-monthly normals were then compared with the half-monthly means. In the table of differences thus formed, equal weight is given to the bi-hourly and hourly observations. The daily mean having been subtracted from the monthly mean, the positive sign indicates a western deflection, and the negative sign an eastern one, as compared with the normal position. The following table contains the result:—

	Sum of deflections.	Number.	Deflection.		
Full moon ☉	+11.6	52	$+0^{d}.22$	$+0'.10$	$+0'.07$
1st day after	—7.1	51	—0.14	—0.06	—
2d day after	—9.3	48	—0.19	—0.08	
New moon ●	—11.5	43	—0.27	—0.12	$+0'.09$
1st day after	+1.5	47	+0.03	+0.01	—
2d day after	+4.4	49	+0.09	+0.04	

The effect is very small, scarcely much beyond the probable error, but the table indicates that the north end of the magnet is deflected to the westward $0'.1$ at the full, and as much to the eastward at the change day, the range between full and new moon being $0'.2$. A more definite result could hardly be expected from a series of observations extending over but five years.

Treating the subject of the effect of the moon's variation in declination in precisely the same manner, we obtain the following result:—

	Mean deflection.				
One day before	$-0'.20$	from 54	days of observation.		
At moon's max. declination . .	—0.10	" 53	"	"	
One day after	—0.09	" 55	"	"	
Mean . . .	$-0'.13$	" 162	"	"	
One day before	$-0'.04$	" 54	"	"	
At moon's min. declination . .	—0.07	" 52	"	"	
One day after	+0.14	" 52	"	"	
Mean . . .	$+0'.01$	" 158	"	"	

These results do not positively fix a deflection of the magnet as depending on the moon's greatest north and south declination, the amount resulting from the comparisons being of nearly the same magnitude as its probable error.

A similar investigation, with respect to the moon's distance from the earth, gives the following results:—

	Mean deflection.				
One day before	$-0'.18$	from 50	days of observation.		
At moon's perigee . . .	—0.18	" 41	"	"	
One day after	0.00	" 59	"	"	
Mean . . .	$-0'.12$	" 150	"	"	
One day before	$-0'.02$	" 55	"	"	
At moon's apogee	—0.20	" 53	"	"	
One day after	—0.13	" 47	"	"	
Mean . . .	$-0'.12$	" 155	"	"	

The differences being of the same order of magnitude as the probable errors, no conclusion as to the effect of distance can be drawn from them.

I propose hereafter to extend the discussion of the moon's effect on the declination to the effect on the earth's magnetic force.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

SECOND SECTION,

COMPRISING PARTS IV, V, AND VI. HORIZONTAL FORCE.

INVESTIGATION OF THE ELEVEN (OR TEN) YEAR PERIOD AND OF THE DISTURBANCES OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE, WITH AN INVESTIGATION OF THE SOLAR DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE HORIZONTAL FORCE; AND OF THE LUNAR EFFECT ON THE SAME.

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PART IV.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD AND OF THE DISTURBANCES OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD, AND OF THE DISTURBANCES OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

Volume XI of the Smithsonian Contributions to Knowledge contained a discussion, in three parts, of the observations for magnetic declination. The first part referring to the eleven (or ten) year period in the amplitude of the solar diurnal variation, and of the disturbances of the magnetic declination; the second, to the annual inequality of the solar diurnal variation, and the third, to the influence of the moon on the magnetic declination. The present discussion refers to the changes of horizontal force, and will be carried on in the same order as the former, so as to dispense with explanations in the mode of treatment, unless in those portions involving the peculiarities of the horizontal force instrument and record. Charles A. Schott, Esq., has rendered me the same assistance in this work, stated in the introduction to Part I.

The horizontal force instrument was one of Gauss's large bifilar magnetometers, made by Meyerstein, of Göttingen, the weight of the magnetic bar being about twenty-five pounds, and its length being thirty-six inches and five-eighths. The suspension wires were slightly inclined, the smaller distances between them being above the larger. The value of one division of the scale in parts of the horizontal force was determined to be:—

in May, 1840, ...	0.000035
in June, 1841, ...	0.000038

The mean, or 0.0000365 is the value used throughout the series. The sensibility of the instrument was thus very considerable. The instrument having been properly adjusted with the bar at right angles to the mean magnetic meridian, the torsion angle Z was found to be 71° 43′. The relation $k = a\ cotan.\ Z$ expresses the value of one scale division k in parts of the horizonal force, a being the value of a scale division in parts of the radius, or $0.00011 = 0'.38$, and Z the angle of torsion. Increase of readings on the scale corresponded to decrease of horizontal force.

The instruments were placed in position by the equations deduced by Professor Lloyd, for the case of the declinometer in equilibrium with the horizontal and vertical force magnetometers, the position of instable equilibrium being taken necessarily from the form and position of the observatory. The effect of the small vertical force bar at first used, upon the bifilar was quite insensible, and that of the declinometer bar affected the value of the scale but slightly, the effect of both instruments changing the value of the scale divisions only in the ratio of 1 to 0.9956.

A thermometer, by Francis, of Philadelphia, divided to half degrees of Fahrenheit's scale, and easily read to tenths, was placed in the box of the horizontal force magnetometer and as near as practicable to the bar.

After the bifilar was set up, a motion commenced in the direction indicating decrease of force; it was progressive though not steadily so. After a time an extra scale was required on occasions of auroral, or other disturbances, and finally the ordinary readings were upon this extra scale. On the occasion of the change of the vertical force magnetometer, in January, 1841, by the substitution of Saxton's balance magnetometer for Lloyd's, the magnetism of the horizontal force bar was examined and found to have sensibly decreased; its force amounted to 0.9601 of its original force, in May, 1840. The experiments were made by means of deflections with a subsidiary declinometer bar, the only means then available. A further experiment of the loss of force was made in June, 1841, when the instrument was accidentally disturbed by one of the observers. The loss of magnetism then found, by means of a new determination of the angle Z, was 0.0314 of its amount in January, 1841. To ascertain the change of magnetism of the bars of the magnetometers, vibrations were also made use of, but they led to no satisfactory result. The progressive change of the scale readings from the change of the horizontal force and loss of magnetism of the bar, will be investigated further on.

The observations, between June, 1840, and September, 1843, were made bi-hourly, and from October, 1843, to the close of the series, hourly. The series extending over five years is not quite continuous; no observations were made on eleven days in January, 1841, on the occasion of the introduction of a new vertical force magnetometer, and the consequent necessity of readjusting the instruments; in January, February, and March, 1843, the work was reduced to but a single reading a day, by circumstances elsewhere stated; there are also some minor disturbances at other times when the difference in the readings, however, were ascertained and allowed for. Full statements bearing on the continuity of the series will be given in subsequent pages.

The reduction proper, necessarily commences with the operation of bringing all the readings to the same standard temperature, to render them comparable among themselves.

Correction of the Readings of the Bifilar Magnetometer for Changes of Temperature.

The care bestowed on the experiments to ascertain the effect of the temperature on the instrument, and the perseverance with which they were carried out were not rewarded with a corresponding degree of agreement in the results obtained, by the various processes employed. This it will be recollected was also the case at other observatories. The subject of the co-efficient of temperature for the bifilar magnet is fully treated in the preface to the three volumes containing the record, and it will, therefore, in this place only be necessary to recapitulate in general the results and to state the nature of the experiments there described.

The first observations for the temperature co-efficient were made on July 16, 1840. Oscillations were observed alternately at the ordinary temperature and near the freezing point, obtained by surrounding the box containing the magnet with ice; at the same time comparative oscillations of a bar in another building were observed to furnish the necessary data to correct the bifilar results for any change in the horizontal force during the progress of the experiments. The value deduced was 2.8 scale divisions for a change of 1° Fahrenheit. No reliance was placed on this result on account of the comparatively rude indications of the subsidiary instrument, and also on account of an irregularity at a certain point in the curve representing the connection of change of force with change of temperature.

The method of deflections was tried, and abandoned on account of the small amount of deflection at a distance sufficiently great to prevent the chance of permanent changes from the mutual action of the bars.

On the 22d of February, 1841, comparisons by vibrations were again resorted to, but with no better success, the correction for change of force during the interval being unsatisfactory. The result deduced was 3.0 scale divisions for 1° Fahr.

Applying the results to the readings of the bar when mounted on the bifilar suspension wires in the observatory, they were so little satisfactory that it was determined to get the change of intensity of the bar by heating and cooling the observatory while the bar remained in situ.

In January and February, 1842, a continuous series of observations was made by allowing the observatory to attain the winter temperature on one day, and obtaining thus a result by comparison with the preceding and succeeding days, when the room was artificially warmed. The value found was 1.55 scale divisions for 1° Fahr. At this time the observatory was warmed by a soap-stone stove with copper fixtures.

About the close of the year 1842 an efficient set of subsidiary instruments was mounted in one of the College buildings, the bifilar magnet being about nine inches in length. After the relative value of the scales of the instruments had been ascertained, comparative observations were made, six each day, in the morning and afternoon. These observations and results are given in a table extending over eleven months, in 1843, and over eleven months, in 1844. The results were fluctuating, and the discrepancies proved conclusively, that other causes were at work which would not be accounted for. The changes in the force were generally small. In the course of these experiments I found, beyond a doubt, that instruments of the same dimensions were required to give comparative results. During an aurora the small instrument in the College gave by no means the same results as the large instrument in the observatory; there were numerous comparisons determining this. I had reason also to believe that the large bar had its induced magnetism easily disturbed, and not regularly renewing itself, so that the correction for temperature may be supposed compound, one part permanent and one part temporary. The following results were obtained:—

Observations between	February and June, 1843,	2.50 scale divisions
" "	July and December, 1843,	2.28 " "
" "	January and June, 1844,	1.94 " "
" "	July and December, 1844,	2.00 " "

for 1° Fahr. It may also be stated that no reasonable supposition in regard to differences of temperature between the indications of the thermometer and magnetic bar, or to changes in the co-efficient varying with the temperature, will explain all the cases of discrepancies. In these comparisons, always near each other in time, small differences in intensity, as shown by the subsidiary instrument, were allowed for, but the corrections for temperature of this latter instrument were neglected, as the changes of temperature in the building where it was placed were small.

Another method, not quite so unobjectionable as the preceding one, was tried; it consisted in taking the results corresponding to the highest temperatures during each winter, and comparing them with those corresponding to the lowest temperatures, a correction being made to reduce the changes of force by means of the secondary instrument. These comparisons were liable to be affected by the unequal distribution of the results used over the different parts of the month. The result was: for combinations and comparisons, from

January, 1844, to June, 1844,	2.03
July, 1844, to December, 1844,	2.29

scale divisions for each degree of Fahrenheit's scale.

The mean value of all the results obtained by the various processes explained, is 2.6 scale divisions, and as a preliminary measure, it was supposed that the co-efficient was changeable, and hence a correction for change of temperature was applied, varying from 3.2 scale divisions, in 1840, to 2.0 scale divisions, in 1844.

On resuming the discussion it was thought desirable to deduce a value for this co-efficient directly from the entire mass of observations, as this could not fail to satisfy the whole series. For this purpose it was indispensable to make the series of observations continuous, or, in other words, to refer the readings, extending over five consecutive years, to the same initial division of the scale. This is, therefore, a proper place for stating all cases when the instrument suffered any disturbance and the amount of scale correction required. All necessary explanations are given in the record.

The first break in the series occurred August 27, 1840, at $12^h\ 22^m$ (Philadelphia time), when the mirror was accidentally deranged. The observed numbers from this date to September 22, at $12^h\ 22^m$ have been brought to comparison with former numbers by the mean position of the bar for six previous days (in some cases seven) and by the hours, from $0^h\ 22^m$ to $22^h\ 22^m$ inclusive. This correction is already applied in the record, its probable error is given as 3.3 scale divisions.

On September 22, 1840, the instrument was readjusted.

An interruption of eleven days occurred, in January, 1841, owing to the introduction of a reflecting vertical force magnetometer, and requiring a new arrangement of the instruments. The horizontal force magnetometer was left in its place. The mean values for January, viz: 944.6 divisions for the bifilar, and 36°.5 for the corresponding temperature, as given in volume I of the record, may be reduced to the true mean by the interpolation of values, between December 31 and January 12. The daily mean (at 32°), on December 31, was 842.3, and on January 12, 913.0, hence, omitting the readings for January 3d, and 10th, as Sundays, the complete monthly mean should be 18.6 divisions less or equal 926.0.

The observations were resumed on the 12th, and continued to February 8th at $22^h\ 49\frac{1}{2}^m$, when the wires were found to have been slightly deranged, two days previously, February 6, $18^h\ 22^m$ (Philadelphia time), a great change in the position was noticed; on re-arranging the instrument it did not return to its former readings. A correction of + 116 has been applied (in the record) to the previous *mean* readings only in this month, and in consequence + 116 divisions should be added to each individual reading from the commencement of the series; but on account of another disturbance of the instrument, on the 22d, at $16^h\ 22^m$ (Philadelphia time), a further correction of + 92.8 scale divisions should be applied. The total correction is therefore + 208.8. Besides these corrections the readings on the 22d from $0^h\ 22^m$ (Philadelphia time) to $10^h\ 22^m$ (Philadelphia time), inclusive, should be increased by + 25.1 divisions, the alhidade of the instrument having been disturbed.[1]

On the 2d of June, 1841, the suspension wires were struck accidentally, deranging the instrument; the readings were then near the end of the subsidiary scale, and in rearranging the instrument the new readings were brought near the middle of the scale. The total difference between the old and new scale readings, the latter commencing with the first of the month, is 900 scale divisions. The means between June 1st and 5th are already corrected in the record, but the individual bi-hourly readings require a correction of + 213[2] scale divisions to produce these means. It was thought best not to apply this correction of — 900 divisions to the observations between June, 1840, and June, 1841, but simply to state the quantity since it can be applied easily to any result hereafter.

At the close of 1842 the regular observations were discontinued for three months, during January, February, and March, 1843; a daily reading was taken at $14^h\ 22^m$ (Philadelphia time), in order to keep up a continuity in the series. By means of the reduced readings in the same months in the other years, it was found that a correction of — $3^d.4$ — $3^d.7$ and + $1^d.5$ for January, February, and March, respectively, was required to refer the mean at $14^h\ 22^m$ to the mean of a complete bi-hourly daily series. Applying these corrections, the corrected monthly means become:—

---

[1] The corrected daily means for the month of February, 1841, should, therefore, read as follows:—

1st	1163.5	10th	1131.1	19th	1127.9
2d	1144.8	11th	1103.8	20th	1130.0
3d	1141.9	12th	1082.5	22d	1182.9
4th	1133.0	13th	1083.5	23d	1182.6
5th	1138.1	15th	1100.0	24th	1128.0
6th	1138.6	16th	1122.1	25th	1107.7
8th	1181.2	17th	1139.7	26th	1144.6
9th	1150.6	18th	1137.0	27th	1162.3
Mean					1135.7

[2] For the first day only + 142, according to the mean in the record.

For January, 1843, ...	$803^d.7$ at $59°.2$
For February, 1843, ...	$798^d.9$ at $51°.9$
For March, 1843, ...	$815^d.1$ at $48°.7$

On the 15th of April, 1843, the instrument was carefully examined and found in adjustment.

At 6^h 50^m on May 4, 1843, the bifilar was disturbed, but readjusted on May 5, before the regular observation at 2^h 21^m P. M. A correction of — 16 divisions during the interval is to be applied to the readings. After this date the instrument remained undisturbed.

We have, therefore, for discussion the following continuous series of monthly means of the readings of the bifilar magnetometer with its corresponding mean temperature. The series extends over five years and one month. To obtain a better view of the series, the correction of — 900 divisions for the first twelve months has been applied, it gives a negative value to the June mean of 1840.

TABLE I.—RECAPITULATION OF MONTHLY MEAN READINGS OF THE BIFILAR MAGNETOMETER, CORRECTED SO AS TO PRESENT A CONTINUOUS SERIES.

	1840-41.	1841-42.	1842-43.	1843-44.	1844-45.
June	— 85.4	+432.3	+663.5	+901.0	+1092.0
July	+ 90.1	463.9	710.2	946.5	1126.6
August . . .	146.2	511.6	718.1	956.3	1149.5
September . .	162.1	537.9	740.3	985.4	1124.8
October . . .	149.4	515.6	768.8	988.6	1140.7
November . .	136.8	503.1	777.8	983.7	1135.1
December . .	156.0	535.4	775.9	986.1	1191.3
January . . .	234.8	561.0	803.7	988.3	1227.2
February . . .	235.7	576.4	798.9	1018.1	1221.6
March	248.9	572.1	815.1	1052.1	1235.3
April	266.5	606.7	869.5	1067.6	1257.3
May	307.8	625.1	873.6	1072.4	1250.8
June	...	...	...	...	1291.7
Temperature of the bifilar magnet.					
June	+72°.1	+74°.1	+71°.3	+75.1	+72.9
July	75.6	77.3	76.8	76.8	77.8
August . . .	75.5	75.4	74.7	77.2	75.8
September . .	65.0	70.6	72.5	73.1	71.5
October . . .	58.7	53.7	67.9	66.3	68.8
November . .	47.4	47.1	61.8	60.5	61.5
December . .	35.7	55.4	57.3	57.7	57.4
January . . .	36.5	61.5	59.2	51.7	58.8
February . . .	34.7	60.5	51.9	54.6	53.6
March	43.5	64.1	48.7	62.8	58.2
April	50.5	65.5	67.4	63.8	64.1
May	60.3	68.3	68.4	68.9	64.3
June	...	...	...	...	74.8

Under the supposition of a uniform progression in the change of the mean monthly readings (due to change in the horizontal force and loss of magnetism of the bar) the bifilar readings for a given period may be represented by the form:—

$$B = B_m + \triangle e\, x + \triangle t\, y$$

where B_m a mean bifilar reading for the period.

x the change during a period.

y the change in the reading due to a change of 1° Fahr.

$\triangle e$ = difference between any single period and the mean epoch.

$\triangle t$ = " " any temperature and the mean temperature.

The formula was first applied to the monthly means resulting from five years of observation; it gave $y = +$ 1.0 scale division; but the remaining differences showed that the irregular changes between June and July, and December and January, of the years 1840–41, had an undue effect on the result, the first year's observations were, therefore, omitted, and the process repeated for the remaining four years. The twelve conditional equations gave the normal equations:—

$$+2143.15 = +143x - 200.4\,y.$$
$$-2549.73 = -200.4x + 711.1\,y.$$

whence x= monthly effect of the progression = + 16.5 scale divisions.

y= temperature correction for 1° Fahr. = + 1.8 " "

An examination of the observed and computed values showed that the introduction of a term $\triangle e^2 z$ would improve the agreement, solving the three normal equations we found

$$x = +17.6$$
$$y = +\ 1.62$$
$$z = -\ 0.31$$

The following table shows the comparison of the observed and computed monthly mean readings of the bifilar:—

1841–1845.	Mean temperature.	Mean observed bifilar reading.	Mean computed.	Difference $c. - o.$	$C. - o. + 3.5.$
June	73.3	772.2	779.2	+ 7.0	+10.5
July	77.2	811.8	806.2	— 5.6	— 2.1
August . . .	76.5	833.9	824.7	— 9.2	— 5.7
September . .	71.9	847.1	837.0	—10.1	— 6.6
October . . .	64.2	853.4	843.3	—10.1	— 6.6
November . .	57.7	849.9	851.4	+ 1.5	+ 5.0
December. . .	57.0	872.2	867.8	— 4.4	— 0.9
January . . .	57.8	895.0	886.0	— 9.0	— 5.5
February . . .	55.2	903.8	897.9	— 5.9	— 2.4
March	58.5	918.6	919.3	+ 0.7	+ 4.2
April	65.2	950.3	945.4	— 4.9	— 1.4
May	67.5	955.5	963.5	+ 8.0	+11.5
Mean	65.17	872.0			

Adding + 3.5 scale divisions to the mean value of B_m the above differences will balance. According to the above results, the annual progressive change is + 17.6 × 12 = 211.2 scale divisions, and the change in magnetic moment of the bar for a change of 1° Fahr. in the temperature, or $q = +1.62 \times 0.0000365 = 0.0000591$. This agrees with the best direct determination, being the one in which the observatory was alternately heated and cooled.

To test these results, a combination of the six warmest months with the six coldest months, by alternate means furnished several values for q depending merely on the assumption of a gradual regular progressive change during each year and a half, for which separate results were deduced; this series commences with May, 1841, and ends with April, 1845, and contains, therefore, the same number of months as the first combination, excluding at the same time the two defective portions noticed above. This combination also possesses the advantage of showing the variations in the values of q.

COMBINATION BY ALTERNATE MEANS OF THE WARMER MONTHS, FROM MAY TO OCTOBER INCLUSIVE, WITH THE COLDER MONTHS, FROM NOVEMBER TO APRIL INCLUSIVE.

	Bifilar.	Temperature.	Alternate Means.		Δd	Δt	q in scale divisions.
May, 1841 to Oct., 1841	461.5	68.57					
Nov., 1841 to April, 1842	559.1	59.05	582.9	70.25	23.8	11.20	+2.1
May, 1842 to Oct., 1842	704.3	71.92	683.0	58.38	21.3	13.54	+1.6
Nov., 1842 to April, 1843	806.8	57.72	823.1	72.37	16.3	14.65	+1.1
May, 1843 to Oct., 1843	941.9	72.82	911.4	58.12	30.5	14.70	+2.1
Nov., 1843 to April, 1844	1016.0	58.52	1029.8	72.72	13.8	14.20	+1.0
May, 1844 to Oct., 1844	1117.7	72.62	1113.6	58.72	4.1	13.90	+0.3
Nov., 1844 to April, 1845	1211.3	58.93					
				Sum	109.8	82.19	+1.3

The result from this combination + 1.3 confirms the preceding value, the result, according to weight or + 1.5 scale divisions or q=0.0000548 in parts of the horizontal force has, therefore, been adopted in the reduction of the bifilar readings to a standard temperature, for which + 63°.0 Fahr. has been determined upon as the mean temperature of the magnetic bar during the five years series of observations.

The difference in the resulting value for q, when obtained from deflections or vibrations, and from combinations of the bifilar readings themselves, has been remarked before, and no satisfactory explanation has as yet been given of it. Thus, for instance, at Toronto, the two respective values were 2.69 and 1.63 scale divisions, as shown in General Sabine's remarks (Vol. III.) The existence of a similar discrepancy in the case of the Makerstoun bifilar has been detected by Mr. Broun. Whatever may be the cause of the difference, there can be no hesitation in saying that the result derived from the bifilar observations themselves is the one to be preferred. At St. Helena (Vol. II., London, 1860), the two values were 1.45 and 0.98, the half yearly comparisons at this station even show a less value, viz., 0.88 scale divisions; 0.98 (for convenience 1.0) was adopted in the reduction. Dr. Lamont, in his Handbook of Terrestrial Magnetism (p. 206, edition of 1849), says: "It deserves to be remarked that the value obtained by comparing monthly mean readings of the bifilar at high and low temperatures is smaller than that obtained by direct observation."

In the present discussion the value $\frac{q}{k} = \frac{0.0000548}{0.0000365} = 1.5$ has been adopted. At Toronto this value was $\frac{q}{k} = \frac{0.000142}{0.000087} = 1.63$, and at St. Helena $\frac{q}{k} = \frac{0.00019}{0.00019} = 1.0$.

It will be seen from these values that the Philadelphia bifilar magnetometer was very sensitive; its scale value in parts of the horizontal force is but four-tenths of the Toronto value, and only two-tenths of that of the St. Helena instrument.

In the computations which follow the tenths of scale readings have been omitted (keeping only the nearest unit) as contributing nothing to the accuracy of the results, and merely increasing the labor of reduction. The uncertainty in the readings arising from the uncertainty in the value of q probably affects the units, and the same may be said of the declination changes, so that in extreme (individual) cases the next higher figure may be affected.

The next step of the reduction consisted in transcribing the whole body of the observations after correcting them individually for differences of temperature; the adopted standard temperature being 63° Fahr.

The following table contains the monthly means of the bifilar readings reduced to the standard temperature; the series has been made continuous by the application of certain corrections explained before.

The readings are in scale divisions of 0.0000365 parts of the horizontal force; increasing numbers denote decrease of force. The time is Observatory mean time, counted to twenty-four hours for convenience sake.

Table II.—Monthly Means of the Bifilar Readings taken at intervals of two hours and reduced to the standard Temperature 63° Fahrenheit.

Philadelphia time (A. M.)						(P. M.)						
	$0^h\ 22^m$	$2^h\ 22^m$	$4^h\ 22^m$	$6^h\ 22^m$	$8^h\ 22^m$	$10^h\ 22^m$	$12^h\ 22^m$	$14^h\ 22^m$	$16^h\ 22^m$	$18^h\ 22^m$	$20^h\ 22^m$	$22^h\ 22^m$
1840.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
June	—96	—98	—101	—113	—102	— 79	— 94	—115	—117	— 96	— 88	— 90
July	+74	+67	+ 63	+ 60	+ 86	+100	+ 81	+ 52	+ 41	+ 74	+ 79	+ 79
August	129	117	117	113	146	157	137	113	110	129	126	133
Sept.	158	147	143	138	169	201	183	157	152	153	157	153
October	155	149	137	140	153	179	177	161	155	157	158	152
Nov.	160	157	149	141	153	171	179	165	167	159	160	164
Dec.	203	192	184	178	184	210	218	206	192	196	202	202
1841.	$0^h\ 22^m$	$2^h\ 22^m$	$4^h\ 22^m$	$6^h\ 22^m$	$8^h\ 22^m$	$10^h\ 22^m$	$12^h\ 22^m$	$14^h\ 22^m$	$16^h\ 22^m$	$18^h\ 22^m$	$20^h\ 22^m$	$22^h\ 22^m$
*January	296	287	286	276	272	294	322	306	289	298	294	298
Feb.	279	270	265	256	261	286	303	295	276	283	289	275
March	276	273	267	260	272	298	299	272	279	281	282	280
April	285	278	268	265	287	312	314	282	273	280	289	286
May	311	312	311	303	318	335	323	304	298	307	312	315
June	420	417	414	405	418	427	406	402	408	416	426	427
July	444	440	435	436	447	457	449	429	430	442	453	448
August	490	490	485	481	499	515	500	479	481	496	501	497
Sept.	517	520	517	514	534	561	538	522	521	528	523	524
October	528	520	517	518	532	540	545	535	529	530	531	530
Nov.	528	529	522	515	525	535	539	525	523	525	528	529
Dec.	545	541	537	534	539	551	562	550	547	553	553	551
1842.	$0^h\ 21\frac{1}{2}^m$	$2^h\ 21\frac{1}{2}^m$	$4^h\ 21\frac{1}{2}^m$	$6^h\ 21\frac{1}{2}^m$	$8^h\ 21\frac{1}{2}^m$	$10^h\ 21\frac{1}{2}^m$	$12^h\ 21\frac{1}{2}^m$	$14^h\ 21\frac{1}{2}^m$	$16^h\ 21\frac{1}{2}^m$	$18^h\ 21\frac{1}{2}^m$	$20^h\ 21\frac{1}{2}^m$	$22^h\ 21\frac{1}{2}^m$
January	560	558	557	554	553	575	579	564	559	568	565	565
Feb.	582	576	574	568	570	580	593	582	578	583	589	582
March	573	564	561	561	567	580	577	567	568	574	576	577
April	605	599	598	593	601	618	612	596	592	605	607	607
May	618	614	609	609	624	632	622	607	609	618	620	622
June	652	655	649	641	652	664	654	642	639	652	655	656
July	684	689	682	683	695	710	698	681	674	687	693	697
August	702	695	695	693	712	722	703	689	690	700	704	703
Sept.	721	723	719	712	732	746	734	722	718	729	730	727
October	757	750	747	747	755	774	778	772	766	764	765	762
Nov.	780	774	772	769	778	791	786	782	778	778	781	785
Dec.	783	780	778	776	779	793	800	791	780	781	784	785
1843.	$0^h\ 21\frac{1}{2}^m$	$2^h\ 21\frac{1}{2}^m$	$4^h\ 21\frac{1}{2}^m$	$6^h\ 21\frac{1}{2}^m$	$8^h\ 21\frac{1}{2}^m$	$10^h\ 21\frac{1}{2}^m$	$12^h\ 21\frac{1}{2}^m$	$14^h\ 21\frac{1}{2}^m$	$16^h\ 21\frac{1}{2}^m$	$18^h\ 21\frac{1}{2}^m$	$20^h\ 21\frac{1}{2}^m$	$22^h\ 21\frac{1}{2}^m$
January								813				
Feb.								819				
March								835				
April	860	859	853	853	867	880	875	860	859	863	866	859
May	866	864	862	860	875	877	862	855	856	863	873	870
June	884	883	879	876	886	895	887	873	873	884	887	888
July	924	921	921	920	933	940	932	920	916	921	931	929
August	932	931	931	928	950	957	944	924	925	930	936	935
Sept.	968	967	962	957	977	990	981	966	968	970	970	966

Hourly Series.

	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
1843.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
October	983	978	980	978	976	978	977	980	984	987	991	992
Nov.	988	987	986	984	983	981	981	984	988	992	994	994
Dec.	996	994	993	992	990	988	988	988	992	992	993	998
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
October	991	989	985	983	983	983	985	985	985	984	983	984
Nov.	992	990	988	987	985	986	987	987	988	988	989	989
Dec.	1000	999	997	994	992	991	993	996	996	996	998	999
1844.	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
January	1009	1007	1006	1004	1002	1002	1001	1001	1004	1007	1010	1013
Feb.	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1036
March	1050	1048	1047	1046	1045	1044	1045	1046	1051	1058	1060	1062
April	1067	1066	1065	1062	1059	1059	1062	1062	1067	1075	1079	1079
May	1066	1066	1064	1063	1063	1062	1062	1065	1069	1075	1076	1071
June	1080	1079	1078	1079	1079	1077	1075	1079	1082	1084	1086	1083
July	1103	1104	1106	1107	1107	1106	1105	1105	1110	1117	1119	1115
August	1129	1130	1130	1130	1129	1127	1126	1131	1139	1148	1149	1143
Sept.	1108	1108	1108	1109	1105	1107	1106	1113	1123	1129	1133	1129
October	1132	1128	1127	1123	1122	1124	1125	1130	1137	1143	1146	1141
Nov.	1136	1135	1133	1132	1131	1127	1128	1129	1134	1141	1147	1149
Dec.	1203	1201	1198	1196	1194	1192	1188	1191	1192	1196	1207	1215
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
January	1011	1008	1005	1001	1000	1002	1004	1005	1005	1006	1007	1009
Feb.	1035	1032	1028	1028	1032	1031	1032	1033	1034	1033	1034	1033
March	1067	1063	1056	1049	1052	1054	1054	1053	1051	1052	1052	1051
April	1074	1069	1063	1059	1061	1059	1065	1067	1068	1069	1066	1069
May	1065	1058	1054	1054	1052	1055	1060	1064	1065	1065	1064	1064
June	1079	1074	1069	1067	1067	1069	1073	1075	1077	1079	1079	1080
July	1107	1101	1097	1094	1093	1094	1097	1100	1102	1103	1104	1105
August	1134	1125	1117	1115	1117	1123	1130	1131	1132	1131	1132	1131
Sept.	1119	1108	1102	1100	1101	1105	1108	1110	1111	1111	1112	1116
October	1139	1134	1128	1129	1128	1132	1133	1133	1135	1132	1133	1130
Nov.	1146	1145	1139	1137	1138	1138	1138	1143	1141	1138	1135	1139
Dec.	1215	1210	1205	1200	1195	1196	1197	1197	1197	1201	1201	1201
1845.	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
January	1233	1230	1231	1229	1227	1225	1224	1226	1230	1238	1244	1248
Feb.	1232	1234	1232	1230	1230	1227	1224	1228	1234	1238	1246	1249
March	1237	1237	1235	1236	1235	1235	1231	1234	1242	1250	1256	1262
April	1253	1250	1249	1247	1245	1243	1241	1247	1255	1270	1280	1279
May	1249	1248	1246	1245	1241	1238	1235	1242	1254	1264	1265	1263
June	1274	1274	1274	1273	1268	1267	1262	1266	1273	1284	1290	1289
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
January	1245	1241	1238	1235	1233	1236	1237	1233	1232	1231	1231	1229
Feb.	1251	1247	1240	1236	1235	1233	1234	1236	1236	1232	1232	1233
March	1261	1254	1246	1240	1241	1243	1245	1242	1241	1238	1241	1240
April	1271	1267	1255	1253	1249	1251	1254	1257	1257	1254	1251	1252
May	1256	1248	1242	1242	1242	1246	1251	1251	1251	1253	1251	1245
June	1282	1278	1269	1267	1266	1269	1274	1278	1277	1276	1275	1275

The monthly means are contained in the following table:—

TABLE III.—MONTHLY MEANS OF THE PRECEDING BIFILAR READINGS REDUCED TO THE STANDARD TEMPERATURE 63° FAHRENHEIT.

	1840–41.	1841–42.	1842–43.	1843–44.	1844–45.
	Div's.	Div's.	Div's.	Div's.	Div's.
June	— 99				
July	+ 71	443	689	926	1104
August . . .	127	493	701	935	1130
September . .	159	527	726	970	1112
October . . .	156	530	761	984	1132
November . .	160	527	780	987	1138
December . .	197	547	784	994	1199
January . . .	[1]274	563	[3]808	1005	1233
February . . .	278	580	[3]814	1031	1235
March	278	570	[3]835	1052	1243
April	285	603	863	10 6	1255
May	312	617	865	10 4	1249
June	[2]415	651	883	10.7	1274

Correction for progressive change in the readings.—The observations having been referred to a uniform temperature, still require a correction for the effect of the progressive change during each month before Peirce's criterion can be applied for the purpose of separating the disturbances. We have seen that the mean monthly value of this change due to loss of magnetism of the bar and to change in the horizontal force itself, was 17.6 scale divisions; on the average, therefore, a correction must be applied to the observations on the first and last day of each month of + 8.8 and — 8.8 scale divisions, and in proportion for the intermediate days. At Toronto, also, the progressive change in some months was so great as to present a practical difficulty by its interference with the proper comparability of the observations, and in these cases new means at shorter intervals than a month were taken.

[1] The actual mean of 17 days was 293; to reduce this to the mean of 27 days, 19 scale divisions were subtracted, resulting from an interpolation between January 1st and January 12th; the mean of 7 days preceding and following the gap was made use of.

[2] Owing to causes already explained, the means of May and June differ so much as to affect the continuity of the series; the same is to be said of the differences between June and July, 1840, and between December, 1840, and January, 1841; the corresponding differences between the same months in the other four years furnish us with the means of correcting the series for the first year, as will be seen hereafter; it also appeared advisable to omit the readings in June, 1840, altogether, the instrument not having then been in stable adjustment.

[3] The numbers in table II have been slightly changed, to refer the mean of the hour of observation to the mean resulting from observation of 12 hours a day. Comparing the mean at $14^h\ 22^m$ in each month with the respective monthly means in the other four years, the above corrections became —5, —5 and 0 for January, February, and March.

The bar between September and October, 1843, separates the means from the bi-hourly and the hourly series.

In the application of the reduction for temperature no attempt whatever has been made at interpolation in the magnetic series, but whenever a temperature reading was accidentally omitted, it has been supplied by comparison with the observed temperature immediately preceding and following. No magnetic reading can be supplied by interpolation, however short the interval, as long as the law of the occurrence of the disturbances remains unknown.

At Philadelphia the progressive change is so large as to require a systematic correction throughout the series. In the manuscript tables used for the preparation of the monthly normals and containing the observations reduced to 63° Fahr., the readings corrected for progressive change were written in blue ink underneath each observation. If the monthly differences are taken from Table No. III., it is apparent that the change is irregular, and in three cases at least it is certain that other causes were in operation, which produced larger monthly differences than could be attributed to the gradual loss of magnetism. These cases are the following (already noticed in the preceding temperature discussion): between June and July, 1840, a difference of 170 divisions; between December and January, 1840–41, a difference of 77; and between May and June, 1841, a difference of 103 divisions. They require separate treatment, as will be presently explained. For the correction of the progressive change the mean reading from *one* month's series was made out for the first, middle, and last of each month. By this process of taking the mean from 14 days preceding and 14 days following each of the epochs the lunar effect on the solar variation is practically eliminated from the resulting mean value.[1] These means corresponding in time to the beginning, the middle, and the end of each month, furnish the *rate* of change for the first and second half of the month, and by simple interpolation give the correction for progressive change for each day. If the rates for the first and second half of the month are different, the monthly means of each hour (from the blue figures) will differ by a small but *constant* quantity from the former monthly means. Thus, for instance, for the month of June, 1842, the monthly mean is 651 divisions, corresponding in time to the middle of the month, the mean of the readings (at 63°) for the second half of May and the first half of June is 641, corresponding in time to the first of June, and the mean of the readings (at 63°) of the second half of June and the first half of July is 673, corresponding in time to the last of June; the correction applied to the bi-hourly readings (at 63°) on June 1st was + 10, and to the readings on June 30th was — 22 divisions. At the middle of the month the correction is zero, and for the intermediate days it is in proportion to their respective distances from the middle. The algebraic sum of the daily corrections divided by the number of days of observation is — 3, which gives the new monthly mean 648, as corrected for irregularity in the progressive change. In the exceptional case of a break, or beginning and termination, the required rate of change for half the month was found by a similar process, using half monthly and quarterly means.

The following table, No. IV., contains the monthly means of the bi-hourly and hourly readings of the bifilar magnetometer referred to a uniform temperature (63° Fahr.), and corrected for irregularity in the progressive change. It is here inserted for the purpose of comparing it with the monthly normals, showing the change produced by the exclusion of the disturbances. The means in the month of June, 1840, are suppressed, and the readings between June 1 and June 5, 1841, were not used.

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[1] In connection with this subject, the first part of an interesting paper by Mr. Bronn may be consulted, viz.: "On the lunar diurnal variation of the magnetic declination at the magnetic equator." —*Proceedings Royal Society*, vol. X., No. 39, 1860.

TABLE IV.—MONTHLY MEANS OF THE BI-HOURLY AND HOURLY READINGS OF THE BIFILAR MAGNETOMETER, REDUCED TO A UNIFORM TEMPERATURE AND CORRECTED FOR IRREGULARITY IN THE PROGRESSIVE CHANGE.

Philadelphia time (A. M.)						(P. M.)						
	$0^h\,22^m$	$2^h\,22^m$	$4^h\,22^m$	$6^h\,22^m$	$8^h\,22^m$	$10^h\,22^m$	$12^h\,22^m$	$14^h\,22^m$	$16^h\,22^m$	$18^h\,22^m$	$20^h\,22^m$	$22^h\,22^m$
1840.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
July	90	83	79	76	102	116	97	68	57	90	95	95
August	130	118	118	114	147	158	139	115	112	130	127	134
Sept.	161	150	146	141	172	204	186	160	155	156	160	156
October	153	147	135	138	151	177	175	159	153	155	156	150
Nov.	155	152	144	136	148	166	174	160	162	154	155	159
Dec.	202	191	183	177	183	209	217	205	191	195	201	201
1841.	$0^h\,22^m$	$2^h\,22^m$	$4^h\,22^m$	$6^h\,22^m$	$8^h\,22^m$	$10^h\,22^m$	$12^h\,22^m$	$14^h\,22^m$	$16^h\,22^m$	$18^h\,22^m$	$20^h\,22^m$	$22^h\,22^m$
[1]January	300	291	290	280	276	298	326	310	293	302	298	302
Feb.	279	270	265	256	261	286	303	295	276	283	289	275
March	276	273	267	260	272	298	299	272	279	281	282	280
April	283	275	265	262	284	309	311	279	270	277	286	283
May	307	308	307	299	314	331	319	300	294	303	308	312
[2]June	392	390	389	383	400	406	390	380	386	392	402	400
July	445	441	436	437	448	458	450	430	431	443	454	449
August	492	492	487	483	501	517	502	481	483	498	503	499
Sept.	519	522	519	516	536	562	540	524	523	530	525	526
October	527	519	516	517	531	539	544	534	528	529	530	529
Nov.	525	526	519	512	522	532	536	522	520	522	525	526
Dec.	546	542	538	535	540	552	563	551	548	554	554	552
1842.	$0^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$12^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$
January	558	556	555	552	551	573	577	562	557	566	563	563
Feb.	585	579	577	571	573	583	596	585	581	586	592	585
March	569	560	557	557	563	576	573	563	564	570	572	573
April	610	604	603	598	606	623	617	601	597	610	612	612
May	614	610	606	605	621	629	618	604	606	615	617	619
June	649	652	645	638	649	661	651	639	636	649	652	653
July	687	692	685	686	698	713	700	684	677	690	696	700
August	701	694	695	692	711	721	702	688	689	699	703	702
Sept.	723	725	720	713	734	748	736	724	720	731	732	729
October	761	754	751	751	759	778	782	776	770	768	769	766
Nov.	779	773	771	768	777	790	785	781	777	777	780	784
Dec.	780	777	775	773	776	790	797	788	777	778	781	782
1843.	$0^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$12^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$
January								818				
Feb.								819				
March								831				
April	863	862	856	856	870	883	878	863	862	866	869	862
May	865	863	861	859	874	876	861	854	855	862	872	869
June	881	880	876	873	883	892	884	870	870	881	884	885
July	927	924	924	923	936	943	935	923	919	924	934	932
August	931	930	930	927	949	956	943	923	924	929	935	934
Sept.	971	970	965	960	980	993	984	969	971	973	973	969

[1] The mean of 17 days is given; to refer it to a complete month subtract 19 divisions.
[2] The mean of 19 days is given; to refer it to a complete month add 8 divisions.

Hourly Series.

	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
1843.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
October	983	978	980	978	976	978	977	980	984	987	991	992
Nov.	987	986	985	983	982	980	980	983	987	991	993	993
Dec.	995	993	992	991	989	987	987	987	991	991	992	997
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
October	991	989	985	983	983	983	985	985	985	984	983	984
Nov.	991	989	987	986	984	985	986	986	987	987	988	988
Dec.	999	998	996	993	991	990	992	995	995	995	997	998
1844.	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
January	1007	1005	1004	1002	1000	1000	999	999	1002	1005	1008	1011
Feb.	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1036
March	1051	1049	1048	1047	1046	1045	1046	1047	1052	1059	1061	1063
April	1070	1069	1068	1065	1062	1062	1065	1065	1070	1078	1082	1082
May	1065	1065	1063	1062	1062	1061	1061	1064	1068	1074	1075	1070
June	1078	1077	1076	1077	1077	1075	1073	1077	1080	1082	1084	1081
July	1102	1103	1105	1106	1106	1105	1104	1104	1109	1116	1118	1114
August	1133	1134	1134	1134	1133	1131	1130	1135	1143	1152	1153	1147
Sept.	1102	1102	1102	1103	1099	1101	1100	1107	1117	1123	1127	1123
October	1136	1132	1131	1127	1126	1128	1129	1134	1141	1147	1150	1145
Nov.	1132	1131	1129	1128	1127	1123	1124	1125	1130	1137	1143	1145
Dec.	1205	1203	1200	1198	1196	1194	1190	1193	1194	1198	1209	1217
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
January	1009	1006	1003	999	998	1000	1002	1003	1003	1004	1005	1007
Feb.	1035	1032	1028	1028	1032	1031	1032	1033	1034	1033	1034	1033
March	1068	1064	1057	1050	1053	1055	1055	1054	1052	1053	1053	1052
April	1077	1072	1066	1062	1064	1062	1068	1070	1071	1072	1069	1072
May	1064	1057	1053	1053	1051	1054	1059	1063	1064	1064	1063	1063
June	1077	1072	1067	1065	1065	1067	1071	1073	1075	1077	1077	1078
July	1106	1100	1096	1093	1092	1093	1096	1099	1101	1102	1103	1104
August	1138	1129	1121	1119	1121	1127	1134	1135	1136	1135	1136	1135
Sept.	1113	1102	1096	1094	1095	1099	1102	1104	1105	1105	1106	1110
October	1143	1138	1132	1133	1132	1136	1137	1137	1139	1136	1137	1134
Nov.	1142	1141	1135	1133	1134	1134	1134	1139	1137	1134	1131	1135
Dec.	1217	1212	1207	1202	1197	1198	1199	1199	1199	1203	1203	1203
1845.	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
January	1234	1231	1232	1230	1228	1226	1225	1227	1231	1239	1245	1249
Feb.	1231	1233	1231	1229	1229	1226	1223	1227	1233	1237	1245	1248
March	1236	1236	1234	1235	1234	1234	1230	1233	1241	1249	1255	1261
April	1255	1252	1251	1249	1247	1245	1243	1249	1257	1272	1282	1281
May	1244	1243	1241	1240	1236	1233	1230	1237	1249	1259	1260	1258
June	1281	1281	1281	1280	1275	1274	1269	1273	1280	1291	1297	1296
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
January	1246	1242	1239	1236	1234	1237	1238	1234	1233	1232	1232	1230
Feb.	1250	1246	1239	1235	1234	1232	1233	1235	1235	1231	1231	1232
March	1260	1253	1245	1239	1240	1242	1244	1241	1240	1237	1240	1239
April	1273	1269	1257	1255	1251	1253	1256	1259	1259	1256	1253	1254
May	1251	1243	1237	1237	1237	1241	1246	1246	1246	1248	1246	1240
June	1289	1285	1276	1274	1273	1276	1281	1285	1284	1283	1282	1282

Table V.—Monthly Means of the preceding Bifilar Readings referred to a Uniform Temperature and Corrected for Irregularity in the Progressive Change.

The column 1840–41 contains a double set of figures, the first are the monthly means directly obtained from Table IV, the second contains the means when the series is made continuous for the two breaks already noticed. The mean difference between May and June (from four years) is 25 scale divisions, and between December and January it is 22 scale divisions; these corrections were applied in the second set of figures.

	1840–1841.		1841–1842.	1842–1843.	1843–1844.	1844–1845.	Monthly Means of Series.
	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	
July	87	215	444	692	929	1103	677
August	128	256	495	700	934	1134	704
September	162	290	529	728	973	1106	725
October	154	282	529	765	984	1136	739
November	155	283	524	779	986	1134	741
December	196	324	548	781	993	1201	769
January	297—19	346	561	813	1003	1234	791
February	278	346	583	814	1031	1234	802
March	278	346	566	831	1053	1242	808
April	282	350	608	866	1069	1257	830
May	308	376	614	864	1063	1244	832
June	393+8	401	648	880	1075	1281	857
Annual Means		·318	554	793	1008	1192	773

The differences in the successive annual means indicate that the progressive change may be assumed to have been uniform from year to year, and applying the usual method we find an annual progressive change of 220 scale divisions.

Introduction of the Horizontal Intensity in absolute measure and separation of the effect of the loss of Magnetism of the Bifilar bar from the effect due to the secular change of the Horizontal Intensity.—Although some experiments were made to determine the gradual loss of magnetism of the bar, as, for instance, in January, 1841, when the amount was found to be 0.9601 of the force in May, 1840, and again in June, 1841, when the amount was 0.9686 of its amount in January, 1841, yet the experiments do not extend over the whole period of observation, and consequently we are obliged to deduce the effect of the secular change of the horizontal intensity from other independent means, and, after converting it into scale divisions, we can assign the proper proportion of what is due to secular change and to loss of magnetism, in the whole progressive change of 220 scale divisions in a year.

In connection with the operations of the U. S. Coast Survey, Assistant Schott has investigated[1] the secular change of the horizontal intensity at a number of stations on the Atlantic and Pacific coasts. At several stations the results were subsequently improved by a discussion of my observations for intensity, made in part in connection with a magnetic survey of Pennsylvania, and also extending into adjoining States, and, in one of the journeys, into Canada. From the complete material the values in the following table of observed horizontal and total intensities have been collected. The horizontal intensity X and the total intensity ϕ are expressed in absolute measure (grains and feet).

[1] Report to Superintendent, dated January 19, 1861.

No.	Year.	Observer.	Reference from which the values were derived or taken.	X.	φ.
1	1835.0	Bache and Courtenay.	Trans. Amer. Phil. Soc., Vol. V, 1837.	4.195	13.58
2	1836.7	Bache.		4.159	13.46
3	1839.5	Loomis.	Trans. Amer. Phil. Soc., Vol. VIII.	4.149	13.41
4	1840.9	Bache.		....	13.41
5	1841.5	Locke.	Phil. Trans. Roy. Soc., 1846.	4.172	13.51
6	1841.8	Bache.		....	13.46
7	1842.5	Locke.	Phil. Trans. Roy. Soc., 1846.	4.174	13.52
8	1842.8	Lefroy.	" " " " "	4.176	13.50
9	1843.6	Bache.		4.172	13.46
10	1844.5	Locke.	Phil. Trans. Roy. Soc., 1846.	4.162	13.47
11	1846.4	Locke.	U. S. Coast Survey Records.	4.143	13.42
12	1855.7	Schott.	" " " "	4.226	13.89
[1]13	1862.6	Schott.	" " " "	4.088	13.30

The first three observations were not made at the Girard College grounds; and it appears from Prof. Loomis' observation when compared with Dr. Locke's, that a correction of 0.023 in the value of X should be added to these; to the twelfth observation I have assigned only half weight; it was probably made during a disturbance. From the general discussion an annual diminution in the horizontal force of 0.0011 parts was deduced for a number of stations on the Atlantic coast. At Toronto (vol. III of General Sabine's Discussion) the annual decrease was found 0.0010 in parts of the horizontal force. Being somewhat guided by these results, after several trials, the following combination of the results in the table has been adopted, as perhaps best representing the values for the time during which the Girard College observations were made, these latter being merely of a differential character:—

Combination.	Mean epoch.	Mean horiz'l int. X.
1, 2, 3	1837.1	4.191
5, 7, 8, 9	1842.6	4.174
10, 11, 12, 13	1852.3	4.145

The annual diminution of X is 0.0030, or, when expressed in parts of the horizontal force, = 0.0007; its equivalent in scale divisions is 19.2. The total annual change was found to be 220 scale divisions; hence, 200.8 scale divisions of annual change is due to loss of magnetism of the bar.

The mean epoch is 1844.0, and the corresponding mean $X = 4.170$; the mean epoch of the observation taken at the Girard College, is January, 1843, for which, therefore, the mean value of $X = 4.173$. This value has been adopted whenever it was desirable to introduce the horizontal force in absolute measure.

Separation of the Larger Disturbances.—The observations having been referred to a uniform temperature, and corrected for progressive change, Peirce's criterion was applied separately to each month. For this purpose, a systematic application was made extending over the whole series of observations, commencing with the hour 0 and the month of July, next with the hour 2 and August, followed by hour 4 and September, and so on in regular progression. This process eliminates from the result the diurnal variation and the annual variation of the disturbances themselves. The value for 0^h in July, 1840, was omitted as affected by two very large disturbances. The following table shows the limiting value of difference from the

[1] Added while this paper is passing through the press.

mean (the monthly mean for the respective hour), also the number of observations in each year subjected to the process:—

LIMITS OF REJECTION BY PEIRCE'S CRITERION.

		Div's.	
1840–41	$ex =$	53	$n =$ 241
1841–42	"	44	312
1842–43	"	37	309
1843–44	"	28	313
1844–45	"	33	313
	Mean value	39	Sum 1488

The limiting value derived from nearly 1,500 observations is 39 scale divisions, and the separate annual values show plainly the effect of the eleven (ten ?) year period, the year 1843–4 being a minimum year. Certain limits in the adoption of a separating value are allowable, and upon trial as to the actual number of disturbances separated, the value 33 scale divisions was finally adopted. Any observation differing 33 divisions or more from its respective monthly mean, was therefore marked and excluded from the mean. 33 divisions equal 0.0012 parts of the horizontal force, and in the value of the absolute scale it amounts to 0.005. At Toronto the limiting value was 14 divisions, = 0.0012 parts of the horizontal force, equal to 0.004 in the absolute scale. (Vol. III of the Toronto Obser's.)

TABLE VI.—SHOWS THE NUMBER OF OBSERVATIONS AND THE NUMBER OF THE LARGER DISTURBANCES SEPARATED BY THE VALUE 33, AS THE LIMIT, FOR EACH MONTH, YEAR, AND THE WHOLE PERIOD.

MONTH.	1840—1841.		1841—1842.		1842—1843.		1843—1844.		1844—1845.	
	Obser's.	Dist's.	Obser's.	Dist's.	Obser's.	Dist's.	Obser's.	Dist's.	Obser's.	Dist's.
July	323	165	323	26	308	24	312	15	648	0
August . . .	308	73	312	17	321	3	324	11	648	4
September . .	312	54	310	41	308	44	312	16	600	27
October . . .	323	68	308	28	310	53	624	3	648	32
November . .	293	49	312	32	312	15	624	1	624	42
December . .	321	120	323	26	323	5	624	0	624	46
January . . .	201[1]	23	311	14	26[3]	0	646	3	648	27
February . .	288	50	287	37	24[4]	1	600	5	576	18
March . . .	320	62	323	26	27[5]	1	624	29	624	3
April	309	48	309	38	300	14	624	16	624	33
May	310	46	300	29	324	25	648	3	648	19
June	225[2]	13	311	16	312	4	600	0	600	56
Sums	3533	770	3729	330	2895	189	6562	102	7512	307
Ratio	1 dist. in 4.6 ob's.		1 dist. in 11.3 ob's.		1 dist. in 15.3 ob's.		1 dist. in 64.3 ob's.		1 dist. in 24.4 ob's.	

Total number of observations 24,231
Total number of disturbances 1,698

The limiting value separated, therefore, one in every 14.3 observations. At Toronto one in every 12.5 was marked as a disturbance.

[1] In 17 days.
[2] In 19 days.
[3] One observation a day.
[4] One observation a day.
[5] One observation a day.

The larger disturbances having been excluded, new monthly means were taken, and the process was repeated several times, when required, until all readings differing 33 scale divisions or more had been excluded; the final means constitute the normals as given in the following table:—

TABLE VII.—MONTHLY NORMAL OF THE BI-HOURLY AND HOURLY READINGS OF THE BIFILAR MAGNETOMETER REDUCED TO A NORMAL TEMPERATURE AND CORRECTED FOR IRREGULARITY IN THE PROGRESSIVE CHANGE.

Philadelphia time (A. M.)						(P. M.)						
	$0^{h}\,22^{m}$	$2^{h}\,22^{m}$	$4^{h}\,22^{m}$	$6^{h}\,22^{m}$	$8^{h}\,22^{m}$	$10^{h}\,22^{m}$	$12^{h}\,22^{m}$	$14^{h}\,22^{m}$	$16^{h}\,22^{m}$	$18^{h}\,22^{m}$	$20^{h}\,22^{m}$	$22^{h}\,22^{m}$
1840.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
July	113	97	89	50	112	116	94	59	52	92	93	108
August	108	112	117	106	138	153	134	103	111	114	121	126
September	155	147	139	141	180	202	177	155	153	158	157	150
October	142	137	122	138	153	166	159	158	148	146	148	151
November	155	150	144	133	144	154	175	157	151	148	144	160
December	196	188	176	166	178	208	217	193	182	185	200	194
1841.	$0^{h}\,22^{m}$	$2^{h}\,22^{m}$	$4^{h}\,22^{m}$	$6^{h}\,22^{m}$	$8^{h}\,22^{m}$	$10^{h}\,22^{m}$	$12^{h}\,22^{m}$	$14^{h}\,22^{m}$	$16^{h}\,22^{m}$	$18^{h}\,22^{m}$	$20^{h}\,22^{m}$	$22^{h}\,22^{m}$
January[1]	298	300	294	284	281	302	326	311	289	296	301	302
February	269	261	264	257	265	288	297	289	275	274	275	272
March	268	272	267	257	271	294	286	267	266	282	264	272
April	273	271	262	262	283	317	315	279	268	271	283	280
May	311	305	306	297	306	323	313	301	294	306	309	313
June[2]	392	390	392	386	400	401	395	382	385	392	402	392
July	442	442	435	435	447	458	449	428	430	444	448	439
August	490	494	487	482	501	518	502	483	483	497	500	495
September	510	514	515	508	531	542	537	516	519	520	515	515
October	521	517	518	514	526	537	547	530	525	527	529	528
November	519	517	515	509	518	529	531	514	518	513	516	518
December	546	541	538	535	537	548	562	549	545	547	550	552
1842.	$0^{h}\,21\frac{1}{2}^{m}$	$2^{h}\,22\frac{1}{2}^{m}$	$4^{h}\,21\frac{1}{2}^{m}$	$6^{h}\,21\frac{1}{2}^{m}$	$8^{h}\,21\frac{1}{2}^{m}$	$10^{h}\,21\frac{1}{2}^{m}$	$12^{h}\,21\frac{1}{2}^{m}$	$14^{h}\,21\frac{1}{2}^{m}$	$16^{h}\,21\frac{1}{2}^{m}$	$18^{h}\,21\frac{1}{2}^{m}$	$20^{h}\,21\frac{1}{2}^{m}$	$22^{h}\,21\frac{1}{2}^{m}$
January	561	556	555	558	553	573	577	559	554	564	563	564
February	580	573	572	567	568	582	589	578	578	580	590	578
March	565	559	557	554	563	574	575	561	565	571	567	566
April	595	598	597	594	604	620	618	603	598	607	608	611
May	614	610	611	605	621	630	622	606	607	615	618	619
June	649	652	646	638	649	659	650	639	638	649	648	650
July	692	686	682	678	695	708	700	680	677	690	694	700
August	699	694	695	692	711	721	700	688	689	701	703	702
September	726	733	722	717	739	750	737	730	727	737	737	734
October	764	759	757	757	764	781	783	776	776	768	769	764
November	774	770	771	768	777	789	787	781	778	775	776	776
December	780	777	775	773	776	790	795	786	773	776	781	782
1843.	$0^{h}\,21\frac{1}{2}^{m}$	$2^{h}\,21\frac{1}{2}^{m}$	$4^{h}\,21\frac{1}{2}^{m}$	$6^{h}\,21\frac{1}{2}^{m}$	$8^{h}\,21\frac{1}{2}^{m}$	$10^{h}\,21\frac{1}{2}^{m}$	$12^{h}\,21\frac{1}{2}^{m}$	$14^{h}\,21\frac{1}{2}^{m}$	$16^{h}\,21\frac{1}{2}^{m}$	$18^{h}\,21\frac{1}{2}^{m}$	$20^{h}\,21\frac{1}{2}^{m}$	$22^{h}\,21\frac{1}{2}^{m}$
January								818				
February								817				
March								829				
April	861	861	854	854	868	883	878	863	860	861	865	859
May	864	862	858	857	875	872	864	855	856	862	867	863
June	881	879	876	873	883	894	884	870	870	881	881	885
July	927	924	924	923	935	941	934	923	916	221	928	931
August	931	930	931	927	947	954	938	921	924	929	932	933
September	974	967	965	960	980	992	985	972	972	975	974	973

[1] The mean of 17 days. [2] The mean of 19 days.

Hourly Series.

	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
1843.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
October	983	978	983	978	976	978	977	980	983	987	991	992
November	987	986	985	983	982	980	980	983	987	991	992	993
December	995	993	992	991	989	987	987	987	991	991	992	997
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
October	991	989	985	983	983	983	985	985	985	986	983	984
November	991	989	987	986	984	985	986	986	987	987	988	988
December	999	998	996	993	991	990	992	995	995	995	997	998
1844.	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
January	1006	1005	1004	1002	1000	1000	999	999	1002	1005	1008	1011
February	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1034
March	1048	1047	1046	1046	1046	1043	1043	1047	1050	1054	1057	1063
April	1070	1069	1068	1065	1063	1062	1064	1061	1067	1074	1078	1078
May	1065	1065	1063	1062	1062	1061	1061	1064	1068	1076	1077	1070
June	1078	1077	1076	1077	1077	1075	1073	1077	1080	1082	1084	1081
July	1102	1103	1105	1106	1106	1105	1104	1104	1109	1116	1118	1114
August	1133	1134	1134	1133	1133	1131	1130	1135	1143	1152	1153	1147
September	1106	1104	1107	1105	1101	1101	1100	1107	1117	1125	1128	1125
October	1133	1132	1131	1127	1124	1125	1129	1134	1141	1149	1152	1145
November	1131	1130	1127	1126	1125	1123	1122	1125	1130	1135	1138	1142
December	1213	1202	1200	1198	1196	1194	1190	1193	1194	1197	1209	1217
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
January	1009	1006	1003	999	998	1000	1002	1003	1003	1004	1004	1005
February	1035	1032	1028	1028	1030	1031	1032	1033	1034	1033	1032	1030
March	1063	1061	1057	1050	1051	1050	1050	1052	1050	1048	1050	1048
April	1077	1071	1066	1062	1064	1062	1068	1068	1071	1071	1068	1069
May	1064	1057	1053	1053	1051	1054	1059	1063	1064	1064	1065	1063
June	1077	1072	1067	1065	1065	1067	1071	1073	1075	1077	1077	1078
July	1106	1100	1096	1093	1092	1093	1096	1099	1101	1102	1103	1104
August	1138	1129	1121	1119	1121	1127	1134	1135	1135	1134	1135	1135
September	1115	1104	1097	1095	1095	1100	1102	1104	1104	1108	1107	1108
October	1145	1137	1134	1130	1132	1134	1135	1137	1138	1134	1137	1135
November	1136	1133	1129	1127	1124	1131	1128	1129	1130	1130	1131	1131
December	1220	1212	1209	1202	1201	1201	1203	1198	1200	1204	1206	1206
1845.	$0^h\,21\frac{1}{2}^m$	$1^h\,21\frac{1}{2}^m$	$2^h\,21\frac{1}{2}^m$	$3^h\,21\frac{1}{2}^m$	$4^h\,21\frac{1}{2}^m$	$5^h\,21\frac{1}{2}^m$	$6^h\,21\frac{1}{2}^m$	$7^h\,21\frac{1}{2}^m$	$8^h\,21\frac{1}{2}^m$	$9^h\,21\frac{1}{2}^m$	$10^h\,21\frac{1}{2}^m$	$11^h\,21\frac{1}{2}^m$
January	1233	1228	1231	1230	1228	1226	1225	1226	1231	1241	1248	1252
February	1230	1230	1231	1229	1229	1226	1223	1227	1231	1236	1243	1244
March	1236	1236	1234	1235	1234	1234	1231	1233	1241	1249	1255	1261
April	1252	1250	1249	1247	1245	1243	1241	1244	1253	1268	1278	1281
May	1244	1243	1241	1239	1236	1233	1229	1236	1251	1261	1262	1258
June	1280	1281	1281	1281	1275	1271	1266	1273	1282	1293	1295	1292
	$12^h\,21\frac{1}{2}^m$	$13^h\,21\frac{1}{2}^m$	$14^h\,21\frac{1}{2}^m$	$15^h\,21\frac{1}{2}^m$	$16^h\,21\frac{1}{2}^m$	$17^h\,21\frac{1}{2}^m$	$18^h\,21\frac{1}{2}^m$	$19^h\,21\frac{1}{2}^m$	$20^h\,21\frac{1}{2}^m$	$21^h\,21\frac{1}{2}^m$	$22^h\,21\frac{1}{2}^m$	$23^h\,21\frac{1}{2}^m$
January	1249	1242	1239	1233	1229	1230	1233	1231	1230	1230	1229	1229
February	1250	1242	1238	1231	1233	1229	1231	1233	1235	1231	1231	1232
March	1260	1253	1245	1239	1240	1242	1244	1240	1239	1237	1240	1239
April	1268	1267	1255	1252	1248	1253	1256	1254	1254	1253	1250	1254
May	1253	1244	1238	1236	1237	1239	1245	1246	1246	1248	1247	1242
June	1286	1280	1272	1269	1269	1273	1278	1281	1280	1277	1279	1280

Increase of scale readings corresponds to decrease of force. Value of one division of the scale = 0.0000365 parts of the horizontal force, or in the absolute scale equal to 0.0001523.

Investigation of the Eleven Year (also called Ten Year) Period, as shown in the Changes of the Amplitude of the Solar Diurnal Variation of the Horizontal Force.— The variation in the amplitude of the diurnal motion of the horizontal force is subject to the same inequality of about eleven years as the declination, and the means of investigation will be analogous to those used in Part I of this discussion. For greater convenience, the preceding monthly normals were united into annual means and the results put into an analytical form, using Bessel's function applicable to periodical phenomena, and determining the numerical quantity by the application of the method of least squares.

In the following table of the regular solar diurnal variation of the horizontal force the means for 1842–43 depend only on nine months of observation; the correction given to refer them to twelve months of observation depends on the mean difference between the results of the same nine months and twelve months of the preceding and following year; this correction is nearly constant and the same within one scale division for the adjacent years. In the second corrected column for 1842–43 the effect of the annual inequality is thus eliminated. In the year 1843–44 the results from nine months of observation at the odd hours were reduced to twelve months by means of corresponding differences in the series of even hours; thus (omitting the minutes) at hour 2, mean of 12 months = 1006, mean of 9 months = 1028; at hour 3 for the same 9 months, mean = 1026, or 2 divisions less; at hour 3 for 12 months the mean is therefore 1004, and the same result is found by comparing with the following hour 4; the mean is given in case of a difference in the two results.

TABLE VIII.—REGULAR SOLAR DIURNAL VARIATION OF THE HORIZONTAL FORCE FOR EACH YEAR OF OBSERVATION EXPRESSED IN SCALE DIVISIONS.

Increased numbers indicate decrease of force. The minutes at the head of each column are to be added to the hours given in the first vertical column. Each year commences with the month of July.

Hour of the day.	1840–41. 22^m	1841–42. $21\frac{3}{4}^m$	1842–43 (9 m'ths). $21\frac{1}{2}^m$	Correction.	1842–43. $21\frac{1}{2}^m$	1843–44. $21\frac{1}{2}^m$	1844–45. $21\frac{1}{2}^m$
	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
0 (A. M.)	223	549	782	+6	788	1008	1191
1						1007	1189
2	219	548	780	+6	786	1006	1189
3						1004	1188
4	214	545	777	+6	783	1003	1186
5						1002	1184
6	206	542	774	+6	780	1002	1182
7						1005	1186
8	226	552	788	+5	793	1010	1194
9						1013	1202
10	244	564	799	+5	804	1017	1206
11						1016	1207
12 (P. M.)	241	563	792	+6	798	1014	1202
13						1010	1195
14	221	547	781	+7	788	1005	1189
15						1002	1186
16	215	547	778	+7	785	1002	1185
17						1004	1188
18	222	553	783	+7	790	1006	1190
19						1008	1191
20	225	554	786	+7	793	1008	1191
21						1009	1191
22	227	553	785	+6	791	1008	1191
23						1008	1191
Mean.	223.5	551.5			789.9	1007.4	1191.4

(Philadelphia local time, counted from midnight to midnight, 24 hours.)

The preceding mean diurnal variations were put in the following analytical form, in which the angle θ counts from midnight at the rate of 15° an hour.

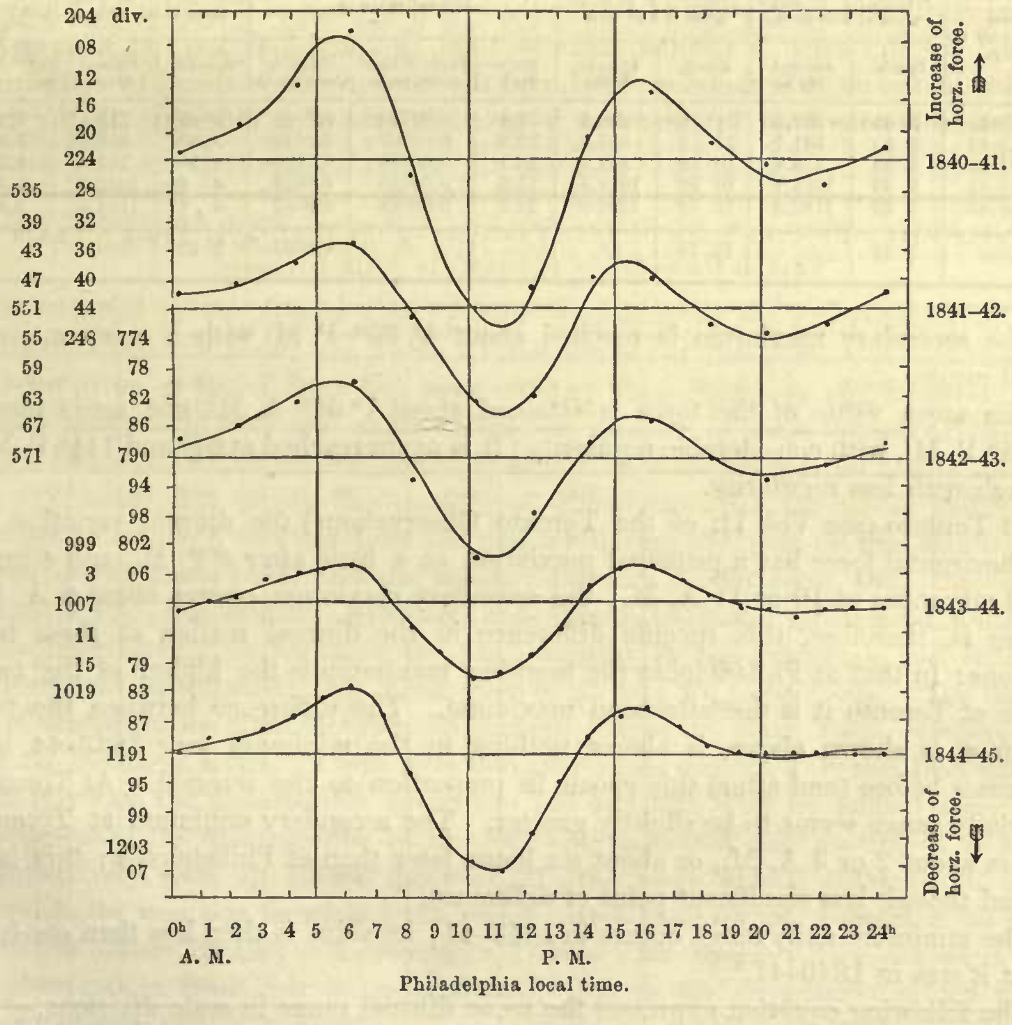
$$\text{Year 1840–41} \quad H = 223^d.5 + 5^d.98 \sin(\theta + 252°\ 14') + 11^d.68 \sin(2\theta + 121°\ 16') + 5^d.89 \sin(3\theta + 314°\ 42')$$
$$\text{“ 1841–42} \quad H = 551.5 + 4.03 \sin(\theta + 244\ 07) + 6.58 \sin(2\theta + 131\ 32) + 4.48 \sin(3\theta + 312\ 19)$$
$$\text{“ 1842–43} \quad H = 789.9 + 4.14 \sin(\theta + 250\ 06) + 7.07 \sin(2\theta + 132\ 24) + 3.74 \sin(3\theta + 323\ 06)$$
$$\text{“ 1843–44} \quad H = 1007.4 + 2.14 \sin(\theta + 273\ 55) + 5.09 \sin(2\theta + 128\ 58) + 2.35 \sin(3\theta + 317\ 58)$$
$$\text{“ 1844–45} \quad H = 1191.4 + 4.40 \sin(\theta + 271\ 13) + 6.86 \sin(2\theta + 123\ 25) + 4.11 \sin(3\theta + 321\ 26)$$

To show the degree of correspondence in the formulæ when deduced from the observations of the even and odd hours separately, the results for the last year have been added, viz:—

$$\text{Even hours} \quad H = 1191^d.3 + 4^d.20 \sin(\theta + 271°\ 28') + 6^d.98 \sin(2\theta + 122°\ 36') + 4^d.11 \sin(3\theta + 322°\ 35')$$
$$\text{Odd hours} \quad H = 1191.5 + 4.60 \sin(\theta + 270\ 59) + 6.73 \sin(2\theta + 124\ 13) + 4.12 \sin(3\theta + 320\ 17)$$

The close agreement between the observed and computed values is shown generally in the annexed diagram.

(A).—Inequality in the Diurnal Variation of the Horizontal Intensity.



The following table exhibits the differences for the year 1842–43, as an example of the numerical correspondence.

A. M.	Computed.	Observed.	C—O.	P. M.	Computed.	Observed.	C—O.
$0^m\ 21\frac{1}{2}^h$	788.7	788	+0.7	$12^m\ 21\frac{1}{2}^h$	799.5	798	+1.5
2 "	786.6	786	+0.6	14 "	787.6	788	—0.4
4 "	781.3	783	—1.7	16 "	784.5	785	—0.5
6 "	781.2	780	+1.2	18 "	790.2	790	+0.2
8 "	792.5	793	—0.5	20 "	792.9	793	—0.1
10 "	803.3	804	—0.7	22 "	720.5	791	—0 5

The differences, using three terms in the equations, are within the uncertainty of the observed values. The probable error of a single representation is $\pm$ 0.6 scale divisions, or $\pm$ 0.00009 in the absolute scale.

The curves show a double progression in the daily motion, with a principal maximum of horizontal force in the morning, a principal minimum before noon, and a secondary maximum in the afternoon; the precise epochs (to the nearest five minutes) and extreme values were computed by means of the preceding formulæ.

Year. From July to July.	Principal A. M. Maximum of hor. force.		Principal A. M. minimum of hor. force.		Diurnal range in			Secondary P. M. maximum of hor. force.		Less than A. M. max. by div's.
	Epoch.	Amount. Div's.	Epoch.	Amount. Div's.	Scale div's.	Parts of horizontal force.	Value in absol. scale.	Epoche.	Amount. Div's.	
1840–41	$5^h\ 45^m$	207.3	$11^h\ 0^m$	246.1	38.8	0.00142	0.0059	$4^h\ 05^m$	213.5	6.2
1841–42	5 50	541.7	11 5	565.5	23.8	0.00087	0.0036	3 50	545.1	3.4
1842–43	5 30	779.8	10 55	803.9	24.1	0.00088	0.0037	3 50	784.0	4.2
1843–44	5 40	1001.7	10 50	1016.9	15.2	0.00055	0.0023	4 0	1002.0	0.3
1844–45	5 40	1182.4	10 50	1206.6	24.2	0.00088	0.0037	4 0	1184.8	2.4
Mean	5 41		10 56				0.0038	3 57		

The secondary maximum is reached about $8^h\ 30^m$ P. M. with a comparatively small range.

The mean value of the force is attained about $7^h\ 55^m$ A. M., and again about $1^h\ 55^m$ P. M., with considerable regularity; it is again reached at $6\frac{3}{4}^h$ and $11\frac{1}{2}^h$ P. M., though with less regularity.

At Toronto (see Vol. II. of the Toronto Observations) the diurnal variation of the horizontal force has a principal maximum at a little after 4 P. M., and a principal minimum at 10 or 11 A. M.; the secondary maximum occurs about 6 A. M. There is, therefore, this specific difference in the diurnal motion at these two stations: in that at Philadelphia the morning maximum is the higher of the two, while at Toronto it is the afternoon maximum. The difference between the two maxima, as shown above, is almost nothing in the minimum year 1843–44, but increases before (and after) this epoch in proportion to the interval. At Toronto the daily range seems to be slightly greater. The secondary minimum at Toronto occurs about 2 or 3 A. M., or about six hours later than at Philadelphia; this is a second though less significant point of difference.

The minimum daily range occurs in 1843–44; its value is then less than one-half what it was in 1840–41.

The following equation expresses the mean diurnal range in scale divisions:—

$$R = + 19.68 - 3.78\,(t - 1843) + 2.77\,(t - 1843)^2.$$

It represents the observed values as follows:—

	Observed range.	Computed range.
January, 1841	38.8	38.3
" 1842	23.8	26.2
" 1843	24.1	19.7
" 1844	15.2	18.7
" 1845	24.2	23.2

The minimum range as given by the formula is in September, 1843. In Part I. of the discussion we found the minimum range of the declination in May, 1843, and the minimum from the disturbances of the declination in August, 1843.

Before proceeding to the discussion of the disturbances in the horizontal force, the formulæ given for the diurnal variation require to be put in a different form for future use and for convenience of comparison with other places.

The scale divisions were multiplied by the value of one division of the scale (0.0000365), and again by the value of X found for the year; the numerical constant was replaced by X and the angular quantities were changed by 180° so as to make increasing numbers correspond to increase of force; we then obtain in absolute measure the following expressions for the regular solar-diurnal variation of the horizontal force at the Girard College:—

Year 1840–41 $H = 4.178 + 0.00091 \sin(\theta + 72°\ 14') + 0.00178 \sin(2\theta + 301°\ 16') + 0.00090 \sin(3\theta + 134°\ 42')$
" 1841–42 $H = 4.175 + 0.00061 \sin(\theta + 64\ \ 07) + 0.00100 \sin(2\theta + 311\ \ 32) + 0.00069 \sin(3\theta + 132\ \ 19)$
" 1842–43 $H = 4.173 + 0.00063 \sin(\theta + 70\ \ 06) + 0.00108 \sin(2\theta + 312\ \ 24) + 0.00057 \sin(3\theta + 143\ \ 06)$
" 1843–44 $H = 4.170 + 0.00033 \sin(\theta + 93\ \ 55) + 0.00078 \sin(2\theta + 308\ \ 58) + 0.00036 \sin(3\theta + 137\ \ 58)$
" 1844–45 $H = 4.168 + 0.00067 \sin(\theta + 91\ \ 13) + 0.00104 \sin(2\theta + 303\ \ 25) + 0.00063 \sin(3\theta + 141\ \ 26)$

The angle θ counts from midnight; the middle epoch to which each equation refers is January.

Investigation of the Eleven (Ten?) Year Inequality in the Disturbances of the Horizontal Magnetic Force.—In Table VI. the number of disturbances in each month has been given as found from the observations; these numbers are, however, not directly comparable with one another, first, on account of some omissions in the record, and secondly, on account of the change from a bi-hourly to an hourly series. For any incomplete month the number of disturbances for the whole month is obtained by simple proportion from the number during the part of the month recorded; for January, 1841, the total number becomes 35, for June, 1841 the total number is 18. For January, February, and March, 1843, the mean total number of the disturbances, as found in the same months in the preceding and following year, was substituted; this mean gave 8, 20, and 20, respectively. The number of disturbances after October, 1843, were halved to make them comparable with the bi-hourly series. There were two anomalous months, July and December, 1840, in which the disturbances amount to 165 and 120, with an annual mean of 64, whereas in the same months in the following year they only amount to 26 and 26 respectively, with an annual mean of 27; the mean annual difference 37 was applied to the numbers found in 1841, which give 63 and 63 as a substitute for the anomalous values in July and December, 1840. This anomaly does not exist in the phenomenon itself, but is unquestionably due to the irregularity in the progressive change.

Table IX. contains the number of disturbances as distributed over the several years and months, all referred to a uniform series of bi-hourly observations. To this table the monthly means and their ratio, when compared with the annual mean, have been added; also, for comparison, the corresponding ratios found in Part I. of the discussion of the disturbances of the declination.

MONTH.	1840-41.	1841-42.	1842-43.	1843-44.	1844-45.	Mean.	Hor. force. Ratio.	Declination. Ratio.
July	(63)	26	24	15	0	26	1.09	0.86
August	73	17	3	11	2	21	0.89	1.59
September	54	41	44	16	13	34	1.43*	1.36
October	68	28	53	2	16	33	1.39	2.12*
November	49	32	15	0	21	24	1.00	1.08
December	(63)	26	5	0	23	23	0.97	1.00
January	35	14	8	1	13	14	0.59	0.77
February	50	37	20	3	9	24	1.00	0.52
March	61	25	20	14	2	25	1.06	0.68
April	48	38	14	8	16	25	1.06	0.91
May	46	30	25	2	10	23	0.97	0.58
June	18	16	4	0	28	13	0.55*	0.53*
Sums	628	330	235	72	153	285	12.00	12.00
Mean	52	28	20	6	13	24		

In the columns of ratios the principal maxima and minima are indicated by an asterisk.

The annual means exhibit plainly the eleven year inequality; they have been represented by the formula:—

$$N = + 14.4 - 10.2 (t - 1843) + 4.8 (t - 1843)^2.$$

	Observed N.	Computed N.
January, 1841	52	54
" 1842	28	29
" 1843	20	14
" 1844	6	9
" 1845	13	13

According to the formula, the minimum occurs in January, 1844.

We have next to consider the eleven year inequality in the magnitude of the disturbances of the horizontal force. Table X. contains the aggregate amount of the disturbances expressed in scale divisions, and also their mean amount obtained by application of the number of disturbances already given in Table VI.

For reasons already explained, the amount of disturbances in July, 1840, equal to 10761 scale divisions, has been diminished in the ratio of 165 : 63. The ratio of each monthly mean to the mean amount of the year is also given, together with a column of corresponding ratios derived from the disturbances of the declination, as made out in Part I. of the discussion.

TABLE X.—AGGREGATE AND MEAN AMOUNT OF THE DISTURBANCES OF THE HORIZONTAL FORCE. EXPRESSED IN SCALE DIVISIONS.

MONTH.	1840–41.	1841–42.	1842–43.	1843–44.	1844–45.	Mean Amount.	Hor. force. Ratio.	Declination. Ratio.
July	(4089)	1157	1295	659	0	56	1.10	0.87
August	4084	755	131	471	142	52	1.03	1.61
September	3092	3075	2099	660	1228	56	1.11*	1.56
October	3720	1284	2399	169	1412	49	0.97	2.06*
November	2390	1991	915	34	2173	54	1.06	1.06
December	6515	1225	239	0	2283	52	1.03	1.00
January	1186	601	0	111	1402	49	0.97	0.72
February	2664	1822	44	200	806	50	0.99	0.54
March	3112	1176	39	1412	127	49	0.97	0.66
April	2138	2075	676	861	1604	49	0.97	0.94
May	2456	1211	1187	131	789	47	0.93	0.56
June	560	794	164	0	2390	44	0.87*	0.42*
Mean amount	53.9	52.0	48.6	46.3	46.8	50.6	1.00	1.00

Maxima and minima in the columns of ratios are marked with au asterisk.

The inequality in the mean amount of the horizontal force disturbances in each year, indicates the year 1843-44 as the minimum year.

From the preceding results, we may assume the month of November, 1843, as the epoch for the minimum of the eleven (ten?) year inequality, as far as indicated by the differential observations of the horizontal force.

Further Analysis of the Disturbances of the Horizontal Force.—The distribution of the disturbances in number and mean amount over the several months of the year has been given in Tables IX. and X. From Table IX. we learn that the disturbances are greatest in number in September and March or April, or about the time of the equinoxes, and least in number about January and June, or about the time of the solstices. At the autumnal equinox the numbers exceed those of the vernal equinox; the same law was found at Toronto; also the numbers are smaller at the summer solstice than at the winter solstice, in perfect accordance with the result found at Toronto. These results are shown graphically on the annexed diagram, which contains also the ratio of the disturbances for the declination in which the same law is apparent.

(B).—DISTRIBUTION OF THE NUMBER OF DISTURBANCES IN THE SEVERAL MONTHS OF THE YEAR.

Full line for horizontal force. Dotted line for declination.

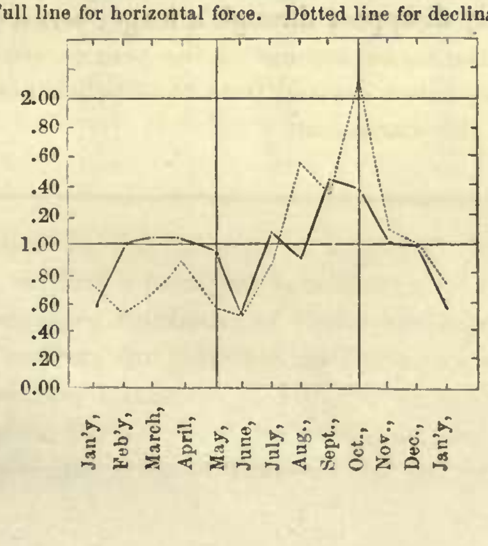


Table X. shows that, in reference to the average magnitude of the disturbances, the same law holds good, viz: the greatest relative magnitude occurring about the time of the equinoxes; the greatest amount corresponding to the autumnal equinox, and the least to about the time of the solstices, the smaller amount occurring near the summer solstice. The average magnitude of the disturbances of the declination was found subject to the same law.

If we separate the disturbances which increase the force from those which decrease it, we may form the two following tables of the distribution of the disturbances in number and average amount over the several months of the years.

Table XI.—Annual Inequality in the Number of Disturbances, Increasing and Decreasing the Horizontal Force.

	1840–41.		1841–42.		1842–43.		1843–44.		1844–45.		Sum.		Ratios.	
	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.
July	(38)	(25)	6	20	5	19	1	14	0	0	50	78	1.2	1.0
August	18	55	6	11	1	2	2	9	0	2	27	79	0.7	1.0
September	25	29	5	36	38	6	11	5	9	4	88	80	2.1*	1.1
October	18	50	11	17	37	16	1	1	8	8	75	92	1.8	1.2
November	13	36	1	31	4	11	0	0	0	21	18	99	0.4	1.3*
December	(25)	(38)	8	18	0	5	0	0	15	8	48	69	1.1	0.9
January	19	16	6	8	3	5	0	1	3	10	31	40	0.8	0.6
February	15	35	4	33	2	18	0	3	0	9	21	98	0.5	1.2
March	17	44	10	16	3	17	0	14	1	1	31	92	0.8	1.2
April	18	30	14	24	1	13	1	7	0	16	34	90	0.8	1.2
May	24	22	16	13	10	15	1	1	5	5	56	56	1.3	0.7
June	9	9	6	10	1	3	0	0	7	21	23	43	0.5*	0.6*
Sum	239	389	93	237	105	130	17	55	48	105	502	916	12.0	12.0

In each year the number of disturbances increasing the force is less than the number which decreases it; the numbers of increase are to the numbers of decrease as 1 : 1.8. The numbers of the monthly ratio for the increasing disturbances exhibit the same law as found in Table IX.: with respect to the numbers for the decreasing force the law is apparently less distinctly marked; the maximum seems to occur about two months later (before the winter solstice), at a time when the number for increasing force is apparently at its minimum. This indistinctness in the law may possibly be due to an irregular distribution in reference to the hours of the day, and could only disappear through a longer series of observations.

TABLE XII.—ANNUAL INEQUALITY IN THE MEAN AMOUNT OF THE DISTURBANCES OF THE HORIZONTAL FORCE. AGGREGATE AMOUNT FOR INCREASING AND DECREASING DISTURBANCES, EXPRESSED IN SCALE DIVISIONS.

Month.	1840–41.		1841–42.		1842–43.		1843–44.		1844–45.		1840–45.		Aver. am't.		Ratios.	
	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec
July	(2202)	(1887)	214	943	292	1003	41	628	0	0	2749	4461	55[d]	57[d]	1.2	1.1
August	794	3290	261	494	51	80	69	402	0	142	1175	4408	44	54	1.0	1.0
Sept.	1082	2010	186	2889	1857	242	452	208	873	355	4450	5704	45	56	1.0	1.1
October	726	2994	421	863	1685	714	128	41	691	721	3651	5333	44	53	1.0	1.0
Nov.	520	1870	35	1956	185	730	0	34	0	2173	740	6763	41	56	0.9	1.1
Dec.	2204	4311	289	936	0	239	0	0	1483	800	3976	6286	47	56	1.0	1.1
January	723	463	231	370	0	0	0	111	302	1100	1256	2044	48	50	1.1	0.8
Feb.	649	2015	140	1682	0	44	0	200	0	806	789	4747	42	52	1.0	1.0
March	643	2469	415	761	0	39	0	1412	37	90	1095	4771	39	52	0.9	1.0
April	732	1406	550	1525	54	622	75	786	41	1563	1452	5902	40	52	0.9	1.0
May	1000	1456	696	515	412	775	83	48	398	391	2589	3185	42	52	1.0	1.0
June	307	253	284	510	50	114	0	0	604	1786	1245	2663	44	44	1.0	0.8
Sum	11582	24424	3722	13444	4586	4602	848	3870	4429	9927	25167	56267			12.0	12.0
Number	254	414	93	237	97	92	20	82	96	211	560	1036				
Mean	46	59	40	57	47	50	42	47	46	47	45	54				

The average amount of a disturbance increasing the horizontal force is 45 scale divisions, or 0.0069 in absolute measure; the average amount of a disturbance decreasing the same is 54 scale divisions, or 0.0082 in absolute value. The ratio of these numbers is as 1 : 1.2, whereas at Toronto the ratio is 1 : 6.4.

The law of the monthly inequality for amount of increasing or decreasing disturbances is, as in the preceding case, very indistinct and further obscured by the small absolute amount of variation.

In the following Table, XIII., the larger disturbances have been distributed over the different hours of their occurrence; in this combination the bi-hourly series (of the even hours) of observation has been used throughout.

Hour.	Aggregate amount in sc. div.	Number of occurrence.	Average amount.	Ratio of numbers.
0 (Midnight)	8116	142	57	1.12
2	5967	109	55	0.86
4	4961	93	53	0.73*
6	4751*	94	51	0.74
8	5562	104	53	0.83
10	7721*	146	53	1.15
12 (Noon)	6825	161	42	1.27*
14	6636	127	52	1.00
16	6634	135	49	1.07
18	6894	132	52	1.05
20	7574	139	55	1.09
22	7358	139	53	1.09

Directing our attention to the columns of aggregate amount and of ratios of number of occurrence, we find a principal maximum about 11 A. M., which seems to correspond to the *secondary* maximum of corresponding ratios at Toronto occurring about three hours earlier; the principal minimum occurs about 5 A. M., which corresponds to the *secondary* minimum at Toronto occurring between 5 and 6 A. M.; again, at Philadelphia, the secondary maximum at midnight is about two hours earlier than the *principal* maximum at Toronto, and the secondary minimum about

4 P. M. corresponds in time to the *principal* minimum at Toronto occurring between 2 and 6 P. M. Thus, the curves at the two stations, representing the diurnal variation of the disturbances (irrespective of increase or decrease) of the horizontal force, is double crested with an exchange of the principal and secondary maximum and also of the principal and secondary minimum.

In the next Table, XIV., the diurnal variation of the disturbances is exhibited separately for disturbances increasing and disturbances decreasing the horizontal force.

Hour.	Disturbances Increasing Horizontal Force.			Disturbances Decreasing Horizontal Force.			Excess of aggregate decrease over aggregate increase.
	Number of occurrences.	Aggregate amount.	Ratio.	Number of occurrences.	Aggregate amount.	Ratio.	
0 (Midn't)	57	2878	1.28	85	5238	1.21	2360
2	44	2173	0.97	65	3794	0.87	1621
4	42	1998	0.89	51	2963*	0.68	965
6	28	1213*	0.54	66	3538	0.81	2325
8	48	2345	1.04	56	3217	0.74	872
10	61	2732	1.22	85	4989	1.15	2257
12 (Noon)	74	3134*	1.39	87	3691	0.85	557
14	48	2239	1.00	79	4397	1.01	2158
16	49	2200	0.98	86	4434	1.03	2234
18	45	2005	0.89	87	4889	1.13	2884
20	39	1758	0.78	100	5816*	1.34	4058
22	50	2296	1.02	89	5062	1.18	2766
Sums.	585	26971	12.00	936	52028	12.00	25057

The disturbances increasing and those decreasing the horizontal force evidently follow different laws; at Toronto they were found completely opposed; they are less so at Philadelphia. The principal maximum of increasing disturbances (at noon) seem to be contemporaneous with a secondary minimum of the decreasing disturbances; again the principal maximum of the decreasing disturbances (at 8 P. M.) corresponds to a secondary minimum of the increasing disturbances. In reference to the main feature, the maximum disturbance of those increasing the force and of those decreasing the force, the Philadelphia ratios show even a greater resemblance to the results at St. Helena and the Cape of Good Hope than to those at Toronto. At the two southern stations the maximum in the disturbances which increase occurs at 11 A. M. and the maximum in the disturbances which decrease occurs about 6 or 7 P. M. (See Vol. II. of the St. Helena Observations.)

Table XIV. contains also the hourly excess of the aggregate amount of the disturbances which decrease the horizontal force over those which increase the same. If we divide the numbers by the whole number of days of observation (nearly 1500) we obtain the diurnal disturbance variation expressed in scale divisions.

Table XV.—Diurnal Disturbance Variation.

Hour.	S. D.	In absolute measure.	Hour.	S. D.	In absolute measure.
0 (Midn't)	1.6	0.00024	12 (Noon)	0.4	0.00006
2	1.1	17	14	1.4	21
4	0.7	11	16	1.5	23
6	1.6	24	18	2.0	30
8	0.6	09	20	2.8	43
10	1.5	23	22	1.9	29

The average amount by which the disturbances tend to decrease the diurnal variation of the horizontal force is 1.4 scale divisions or 0.00021 in the absolute scale. The maximum effect takes place at 8 P. M., at exactly the same hour when the declination disturbances reach their greatest effect.

In the preceding Tables, XIII., XIV., and XV., to the hours indicated $21\frac{1}{2}$ minutes should be added, the observations being made so much later than the even hours.

The preceding discussion shows that for two stations, even at a comparatively short distance, as for Philadelphia and Toronto, there are, generally speaking, some close coincidences in the laws derived from independent observations; but there are also certain differences in other results; yet it must not be forgotten that for a strict comparability we require, if not simultaneous observations, at least observations extending over similar parts or the whole of an eleven year period. The Philadelphia series includes a minimum year of that inequality, with the greater extent of observations before that epoch, whereas at Toronto the series begins after the minimum epoch and barely extends to a maximum year.

For the purpose of obtaining a better view of the absolute amount of the disturbances and their frequency of occurrence,[1] they were classified in nine groups of equal differences of 20 scale divisions; the number of disturbances in each was found as follows:—

Limits Adopted.			Number of disturbances.
In scale divisions.	In parts of horizontal force.	In the absolute scale.	
33 to 53	0.0012 to 0.0019	0.005 to 0.008	1159
53 " 73	19 " 27	08 " 11	348
73 " 93	27 " 34	11 " 14	93
93 " 113	34 " 41	14 " 17	45
113 " 133	41 " 48	17 " 20	27
133 " 153	48 " 55	20 " 23	14
153 " 173	55 " 62	23 " 26	4
173 " 193	62 " 70	26 " 29	6
193 " 213	0.0070 " 0.0077	0.029 " 0.032	2
Beyond.	....	....	0

The numbers in the last column cannot be considered as entirely independent of the eleven year period, and in attempting to apply the theory of probabilities in

[1] A table analogous to that given above, showing the distribution of the disturbances in *declination*, is here added for comparison:—

Limits Adopted.		Number of disturbances.
In scale divisions.	In minutes of arc.	
8 to 16	3′.6 to 7′.2	1856
16 " 24	7.2 " 10.8	333
24 " 32	10.8 " 14.4	105
32 " 40	14.4 " 18.1	42
40 " 48	18.1 " 21.7	16
48 " 56	21.7 " 25.3	2
56 " 64	25.3 " 29.0	2
64 " 72	29.0 " 32.6	1
Beyond	....	0

reference to the number of disturbances which ought to occur between the assigned limits, it became apparent that the larger disturbances greatly preponderate, a fact no doubt intimately connected with the difficulty in correctly allowing for the progressive change during the first year of observation.

PART V.

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION AND OF THE ANNUAL INEQUALITY OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

THE discussion of the diurnal and annual variations of the horizontal force is based on the resulting monthly normal values for each observation hour as given in the preceding part (IV.), in which the horizontal force has been discussed in relation to the ten or eleven year period, and which also contains the investigation of the disturbances; in the same part all necessary statements are given relating to the instrumental data and the absolute values of the horizontal force.

The normals, as has been shown, are referred to a uniform standard temperature; they are corrected for irregularity in the progressive change, and are necessarily freed from all the larger disturbances. The use of the normals instead of the simple means of the readings (corrected for difference of temperature) will insure greater regularity in the variations of the horizontal force, now under consideration.

The diurnal variation requires an arrangement of the five year series of monthly normals according to the months of the year and hours of the day; in general, the method of interpolation for an occasional omission in either a month or hour, is the same as that used in Part II. of the discussion of the Girard College observations; there is, however, this difference in the tabulation of the monthly values, that in the present case the results are consolidated in a five years' arrangement, and in consequence the year commences with the month of July. This arrangement was preferred, particularly since it was found desirable to make no use of the observations in the first month of the series.

Tabulation of monthly normals for each observing hour and each observing year, beginning and ending with July. The individual values are taken from Table VII. of the preceding Part IV.

After applying the corrections of — 19 scale divisions to the normals for January, 1841, and of +8 scale divisions to those of June, 1841, to allow for defective number of observations in these months, a further correction of + 68 scale divisions was applied to all values between July, 1840, and May, 1841, inclusive, and of + 60 to all values between July, 1840, and December, 1840, inclusive, to allow for defects in the regularity of the progressive change, thus making the total correction for the latter months = 128 scale divisions. The above corrections, when divided by 5, in order to give the correction to the means derived from five years, become, therefore: for months between July and December inclusive, + 26; for January + 10; for February, March, April, and May + 14; for June + 2. These corrections are constant for each hour of the day in any one month, and consequently do not affect the diurnal variation; but they have nevertheless been applied at once to facilitate subsequent deductions. Their origin has also been explained in the remarks accompanying Table V. of the preceding part.

The following example of the process of interpolation for the odd hour values will suffice for all similar cases: Required the mean normal from the 5 year series for $5^h\ 21\frac{1}{2}^m$ A. M. in June (see tabular values and results below). The mean normals for the two last years at $4^h\ 21\frac{1}{2}^m$, $5^h\ 21\frac{1}{2}^m$, and $6^h\ 21\frac{1}{2}^m$, are 1176, 1173, and 1169 respectively; the mean at $5^h\ 21\frac{1}{2}^m$ is therefore 3 divisions less than the mean at $4^h\ 21\frac{1}{2}^m$, and since the mean of the 5 year series at $4^h\ 21\frac{1}{2}^m$ is 853, the result for $5^h\ 21\frac{1}{2}^m$ becomes 849; again, adding 4 divisions to 847, the mean at $6^h\ 21\frac{1}{2}^m$, we find 851; the mean of the two values, or 850, is that given in the table, to which + 2 has been added, making the final result 852. The means of the odd hours, thus found from the adjacent even hours, in general, do not differ by as much as a scale division.

The time given in the tables of the normals is mean local time, counting from midnight to midnight to twenty-four hours. The observations were taken (on the average) $21\frac{1}{2}$ minutes after the full hours, as indicated in the tables. Increase of scale readings indicates decrease of horizontal force; the value of a scale division equals 0.0000365 parts of the horizontal force, or 0.0001523 in absolute measure, the mean horizontal force being 4.173 (in absolute measure). Proper weights have been given to the normals of the even and odd hours, in proportion to the number of observations, as will be seen hereafter. Other special remarks will be found at the end of the month to which they refer.

Tabulation of the hourly normals for each month and the mean of the five year series, expressed in scale division readings and reduced to the standard temperature of 63° (Fahrenheit's scale), also corrected for all irregularities in the progressive change. The regular progressive and secular change, therefore, remains in the tabular quantities.

Normals of the Horizontal Force for July.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1840	113		97		89		50		112		116		
1841	442		442		435		435		447		458		
1842	692		686		682		678		695		708		
1843	927		924		924		923		935		941		
1844	1102	1103	1105	1106	1106	1105	1104	1104	1109	1116	1118	1114	
Mean	605		651		647		638		660		668		
Referred mean		653		649		642		647		666		664	
Constant correction + 26													
Normals	681	679	677	675	673	668	664	673	686	692	694	690	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1840	94		59		52		92		93		108		
1841	449		428		430		444		448		439		
1842	700		680		677		690		694		700		
1843	934		923		916		921		928		931		
1844	1106	1100	1096	1093	1092	1093	1096	1099	1101	1102	1103	1104	
Mean	657		637		633		649		653		656		
Referred mean		646		634		640		651		655		657	
Constant correction + 26													
Normals	683	672	663	660	659	666	675	677	679	681	682	683	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 676.3, weight 5.
" " " odd " " 676.3, " 1.

Normals of the Horizontal Force for August.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1840	108		112		117		106		138		153		
1841	490		494		487		482		501		518		
1842	699		694		695		692		711		721		
1843	931		930		931		927		947		954		
1844	1133	1134	1134	1133	1133	1131	1130	1135	1143	1152	1153	1147	
Mean	672		673		673		667		688		700		
Referred mean		673		672		669		676		698		692	
Constant correction + 26													
Normals	698	699	699	698	699	695	693	702	714	724	726	718	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1840	134		103		111		114		121		126		
1841	502		483		483		497		500		495		
1842	700		688		689		701		703		702		
1843	938		921		924		929		932		933		
1844	1138	1129	1121	1119	1121	1127	1134	1135	1135	1134	1135	1135	
Mean	682		663		666		675		678		678		
Referred mean		672		662		670		677		677		676	
Constant correction + 26													
Normals	708	698	689	688	692	696	701	703	704	703	704	702	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 702.2, weight 5.
" " " odd " " 702.2, " 1.

Normals of the Horizontal Force for September.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1840	155		147		139		141		180		202		
1841	510		514		515		508		531		542		
1842	726		733		722		717		739		750		
1843	974		967		965		960		980		992		
1844	1106	1104	1107	1105	1101	1101	1100	1107	1117	1125	1128	1125	
Mean	694		694		688		685		709		723		
Referred mean		692		692		687		695		718		720	
Constant correction + 26													
Normals	720	718	720	718	714	713	711	721	735	744	749	746	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1840	177		155		153		158		157		150		
1841	537		516		519		520		515		515		
1842	737		730		727		737		737		734		
1843	985		972		972		975		974		973		
1844	1115	1104	1097	1095	1095	1100	1102	1104	1104	1108	1107	1108	
Mean	710		694		693		698		697		696		
Referred mean		700		692		697		699		699		696	
Constant correction + 26													
Normals	736	726	720	718	719	723	724	725	723	725	722	722	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 724.4, weight 5.
" " " odd " " 724.9, " 1.

Normals of the Horizontal Force for October.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1840	142		137		122		138		153		166		
1841	521		517		518		514		526		537		
1842	764		759		757		757		764		781		
1843	983	978	983	978	976	978	977	980	983	787	991	992	
1844	1133	1132	1131	1127	1124	1125	1129	1134	1141	1149	1152	1145	
Mean	709		705		699		703		713		725		
Referred mean		705		701		702		708		720		724	
Constant correction + 26													
Normals	735	731	731	727	725	728	729	734	739	746	751	750	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1840	159		158		148		146		148		151		
1841	547		530		525		527		529		528		
1842	783		776		776		768		769		764		
1843	991	989	985	983	983	983	985	985	985	986	983	984	
1844	1145	1137	1134	1130	1132	1134	1135	1137	1138	1134	1137	1135	
Mean	725		717		713		712		714		713		
Referred mean		721		714		713		713		713		712	
Constant correction + 26													
Normals	751	747	743	740	739	739	738	739	740	739	739	738	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 738.3, weight 5.
" " " odd " " 738.2, " 2.

Normals of the Horizontal Force for November.

Year.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+ 21½m
1840	155		150		144		133		144		154		
1841	519		517		515		509		518		529		
1842	774		770		771		768		777		789		
1843	987	986	985	983	982	980	980	983	987	991	992	993	
1844	1131	1130	1127	1126	1125	1123	1122	1125	1130	1135	1138	1142	
Mean	713		710		707		702		711		720		
Referred mean		712		708		704		706		717		725	
Constant correction + 26													
Normals	739	738	736	734	733	730	728	732	737	743	746	751	

Year.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+ 21½m
1840	175		157		151		148		144		160		
1841	531		514		518		513		516		518		
1842	787		781		778		775		776		776		
1843	991	989	987	986	984	985	986	986	987	987	988	988	
1844	1136	1133	1129	1127	1124	1131	1128	1129	1130	1130	1131	1131	
Mean	724		714		711		710		711		715		
Referred mean		720		712		713		710		713		714	
Constant correction + 26													
Normals	750	746	740	738	737	739	736	736	737	739	741	740	

Monthly mean normal from the even hours (+ 21½m) 738.3, weight 5.
" " " odd " " 738.8 " 2.

Normals of the Horizontal Force for December.

Year.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+ 21½m
1840	196		188		176		166		178		208		
1841	546		541		538		535		537		548		
1842	780		777		775		773		776		790		
1843	995	993	992	991	989	987	987	987	991	991	992	997	
1844	1213	1202	1200	1198	1196	1194	1190	1193	1194	1197	1209	1217	
Mean	746		740		735		730		735		749		
Referred mean		741		738		733		732		740		757	
Constant correction + 26													
Normals	772	767	766	764	761	759	756	758	761	766	775	783	

Year.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+ 21½m
1840	217		193		182		185		200		194		
1841	562		549		545		547		550		552		
1842	795		786		773		776		781		782		
1843	999	998	996	993	991	990	992	995	995	995	997	998	
1844	1220	1212	1209	1202	1201	1201	1203	1198	1200	1204	1206	1206	
Mean	759		747		738		741		745		746		
Referred mean		752		742		739		742		746		745	
Constant correction + 26													
Normals	785	778	773	768	764	765	767	768	771	772	772	771	

Monthly mean normal from the even hours (+ 21½m) 768.6, weight 5.
" " " odd " " 768.2, " 2.

NORMALS OF THE HORIZONTAL FORCE FOR JANUARY.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1841	298		300		294		284		281		302		
1842	561		556		555		558		553		573		
1843	(820)		(817)		(814)		(815)		(814)		(827)		
1844	1006	1005	1004	1002	1000	1000	999	999	1002	1005	1008	1011	
1845	1233	1228	1231	1230	1228	1226	1225	1226	1231	1241	1248	1252	
Mean	784		782		778		776		776		792		
Referred mean		782		780		777		774		785		798	
Constant correction + 10 Normals	794	792	792	790	788	787	786	784	786	795	802	808	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1841	326		311		289		296		301		302		
1842	577		559		554		564		563		564		
1843	(830)		818		(813)		(820)		(820)		(821)		
1844	1009	1006	1003	999	998	1000	1002	1003	1003	1004	1004	1005	
1845	1249	1242	1239	1233	1229	1230	1233	1231	1230	1230	1229	1229	
Mean	798		786		777		783		783		784		
Referred mean		791		780		780		784		784		786	
Constant correction + 10 Normals	808	801	796	790	787	790	793	794	793	794	794	796	

Monthly mean normal from the even hours 793.3, weight 4.
" " " odd " 793.4, " 2.

The values for 1843 within brackets are interpolated by means of the continued readings at $14^h\ 21\frac{1}{2}^m$; at this hour the difference of reading from the preceding year is 259, which added to the values of 1842 gave resulting normals for 1843; in the same manner the reading in 1843 at $14^h\ 21\frac{1}{2}^m$ when compared with the reading in the following year (1844) leaves the difference 185, which quantity when subtracted from each hourly value in 1844 gives a second determination for the year 1843; the mean of the two determinations for each hour has been inserted above.

NORMALS OF THE HORIZONTAL FORCE FOR FEBRUARY.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1841	269		261		264		257		265		288		
1842	580		573		572		567		568		582		
1843	(820)		(816)		(813)		(810)		(812)		(822)		
1844	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1034	
1845	1230	1230	1231	1229	1229	1226	1223	1227	1231	1236	1243	1244	
Mean	786		782		781		777		781		794		
Referred mean		784		782		779		779		786		796	
Constant correction + 14													
Normals	800	798	796	796	795	793	791	793	795	800	808	810	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1841	297		289		275		274		275		272		
1842	589		578		578		580		590		578		
1843	(826)		817		(818)		(820)		(826)		(819)		
1844	1035	1032	1028	1028	1030	1031	1032	1033	1034	1033	1032	1030	
1845	1250	1242	1238	1231	1233	1229	1231	1233	1235	1231	1231	1232	
Mean	799		790		787		787		792		786		
Referred mean		794		786		786		790		788		786	
Constant correction + 14													
Normals	813	808	804	800	801	800	801	804	806	802	800	800	

Monthly mean normal from even hours ($+ 21\frac{1}{2}^m$) 800.8, weight 4.
" " " " odd " " 800.4, " 2.

The values of 1843 inclosed in brackets are derived from the reading at $14^h\ 21\frac{1}{2}^m$ in the same manner as explained in the preceding month.

Normals of the Horizontal Force for March.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1841	268		272		267		257		271		294		
1842	565		559		557		554		563		574		
1843	(827)		(823)		(822)		(819)		(827)		(836)		
1844	1048	1047	1046	1046	1046	1043	1043	1047	1050	1054	1057	1063	
1845	1236	1236	1234	1235	1234	1234	1231	1233	1241	1249	1255	1261	
Mean	789		787		785		781		790		803		
Referred mean		788		786		783		785		798		808	
Constant correction + 14													
Normals	803	802	801	800	799	797	795	799	804	812	817	822	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1841	286		267		266		282		264		272		
1842	575		561		565		571		567		566		
1843	(839)		829		(828)		(831)		(828)		(828)		
1844	1063	1061	1057	1050	1051	1050	1050	1052	1050	1048	1050	1048	
1845	1260	1253	1245	1239	1240	1242	1244	1240	1239	1237	1240	1239	
Mean	805		792		790		796		790		791		
Referred mean		800		787		793		794		789		790	
Constant correction + 14													
Normals	819	814	806	801	804	807	810	808	804	803	805	804	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 805.6, weight 4.
" " " " " odd " " 805.8, " 2.

The values for 1843 are interpolated as in the preceding two months

Normals of the Horizontal Force for April.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1841	273		271		262		262		283		317		
1842	595		598		597		594		604		620		
1843	861		861		854		854		868		883		
1844	1070	1069	1068	1065	1063	1062	1064	1061	1067	1074	1078	1078	
1845	1252	1250	1249	1247	1245	1243	1241	1244	1253	1268	1278	1281	
Mean	810		809		804		803		815		835		
Referred mean		809		806		803		806		827		837	
Constant correction + 14													
Normals	824	823	823	820	818	817	817	820	829	841	849	851	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1841	315		279		268		271		283		280		
1842	618		603		598		607		608		611		
1843	878		863		860		861		865		859		
1844	1077	1071	1066	1062	1064	1062	1068	1068	1071	1071	1068	1069	
1845	1268	1267	1255	1252	1248	1253	1256	1254	1254	1253	1250	1254	
Mean	831		813		808		813		816		814		
Referred mean		825		810		808		813		816		813	
Constant correction + 14													
Normals	845	839	827	824	822	822	827	827	830	830	828	827	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 828.2, weight 5.
" " " " " odd " " 828.4, " 2.

Normals of the Horizontal Force for May.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1841	311		305		306		297		306		323		
1842	614		610		611		605		621		630		
1843	864		862		858		857		875		872		
1844	1065	1065	1063	1062	1062	1061	1061	1064	1068	1076	1077	1070	
1845	1244	1243	1241	1239	1236	1233	1229	1236	1251	1261	1262	1258	
Mean	820		816		815		810		824		833		
Referred mean		819		815		812		815		832		829	
Constant correction + 14													
Normals	834	833	830	829	829	826	824	829	838	846	847	843	

Year	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1841	313		301		294		306		309		313		
1842	622		606		607		615		618		619		
1843	864		855		856		862		867		863		
1844	1064	1057	1053	1053	1051	1054	1059	1063	1064	1064	1065	1063	
1845	1253	1244	1238	1236	1237	1239	1245	1246	1246	1248	1247	1242	
Mean	823		811		809		817		821		821		
Referred mean		816		810		811		818		822		818	
Constant correction + 14													
Normals	837	830	825	824	823	825	831	832	835	836	835	832	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 832.3, weight 5.
" " " " " odd " " 832.1, " 2.

Normals of the Horizontal Force for June.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1841	392		390		392		386		400		401		
1842	649		652		646		638		649		659		
1843	881		879		876		873		883		894		
1844	1078	1077	1076	1077	1077	1075	1073	1077	1080	1082	1084	1081	
1845	1280	1281	1281	1281	1275	1271	1266	1273	1282	1293	1295	1292	
Mean	856		856		853		847		859		867		
Referred mean		856		856		850		853		865		864	
Constant correction + 2													
Normals	858	858	858	858	855	852	849	855	861	867	869	866	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1841	395		382		385		392		402		392		
1842	650		639		638		649		648		650		
1843	884		870		870		881		881		885		
1844	1077	1072	1067	1065	1065	1067	1071	1073	1075	1077	1077	1078	
1845	1286	1280	1272	1269	1269	1273	1278	1281	1280	1277	1279	1280	
Mean	858		846		845		854		857		857		
Referred mean		853		845		849		856		856		857	
Constant correction + 2													
Normals	860	855	848	847	847	851	856	858	859	858	859	859	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 856.6, weight 5.
" " " odd " " 857.0, " 2.

Table I.—Recapitulation of the Hourly Normals of the Horizontal Force (expressed in Scale Divisions) for each Month of the Year.

Increase of scale readings denotes decrease of force.

1840–1845.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
July . . .	681	679	677	675	673	668	664	673	686	692	694	690	
August . . .	698	699	699	698	699	695	693	702	714	724	726	718	
September . .	720	718	720	718	714	713	711	721	735	744	749	746	
October . .	735	731	731	727	725	728	729	734	739	746	751	750	
November . .	739	738	736	734	733	730	728	732	737	743	746	751	
December . .	772	767	766	764	761	759	756	758	761	766	775	783	
January . .	794	792	792	790	788	787	786	784	786	795	802	808	
February . .	800	798	796	796	795	793	791	793	795	800	808	810	
March . . .	803	802	801	800	799	797	795	799	804	812	817	822	
April . . .	824	823	823	820	818	817	817	820	829	841	849	851	
May	834	833	830	829	829	826	824	829	838	846	847	843	
June . . .	858	858	858	858	855	852	849	855	861	867	869	866	
Year . . .	771.5	769.8	769.1	767.4	765.7	763.7	761.9	766.7	773.7	781.3	786.1	786.5	
Summer . .	769.2	768.3	767.8	766.3	764.7	761.8	759.7	766.7	777.2	785.7	789.0	785.7	
Winter . . .	773.8	771.3	770.3	768.5	766.8	765.7	764.2	766.7	770.3	777.0	783.2	787.3	

1840–1845.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
July . . .	683	672	663	660	659	666	675	677	679	681	682	683	
August . . .	708	698	689	688	692	696	701	703	704	703	704	702	
September .	736	726	720	718	719	723	724	725	723	725	722	722	
October . .	751	747	743	740	739	739	738	739	740	739	739	738	
November . .	750	746	740	738	737	739	736	736	737	739	741	740	
December . .	785	778	773	768	764	765	767	768	771	772	772	771	
January . .	808	801	796	790	787	790	793	794	793	794	794	796	
February . .	813	808	804	800	801	800	801	804	806	802	800	800	
March . . .	819	814	806	801	804	807	810	808	804	803	805	804	
April . . .	845	839	827	824	822	822	827	827	830	830	828	827	
May	837	830	825	824	823	825	831	832	835	836	835	832	
June . . .	860	855	848	847	847	851	856	858	859	858	859	859	
Year . . .	782.9	776.2	769.5	766.5	766.2	768.6	771.6	772.6	773.4	773.5	773.4	772.8	
Summer . .	778.2	770.0	762.0	760.2	760.3	763.8	769.0	770.3	771.7	772.2	771.7	770.8	
Winter . . .	787.7	782.3	777.0	772.8	772.0	773.3	774.2	774.8	775.2	774.8	775.2	774.8	

In the preceding table the normals for the summer half year comprise the months between April and September inclusive; those for the winter half year comprise the months between October and March inclusive.

The following table contains the mean values of the normals for each month and season.

Table II.

1840–1844.	Normal.	1841–1845.	Normal.	1840–1845.	Normal.
July	676.3	January . . .	793.3	Year	772.1
August . . .	702.2	February . . .	800.6	Summer	770.1
September . .	724.6	March	805.7	Winter	774.1
October . . .	738.2	April	828.3		
November . .	738.5	May	832 2		
December. . .	768.4	June	856.8		

Regular Solar-Diurnal Variation of the Horizontal Force.—If we subtract the hourly normals of Table I. from their respective monthly mean value as given in Table II., the difference (in scale divisions) will represent the regular solar-diurnal variation for each month in the year. In like manner we obtain the diurnal variation of the horizontal force—free of the larger disturbances—for the summer and winter half, and for the whole year. Table III. will exhibit these differences after their conversion from scale divisions into parts of the horizontal force (one scale division equalling 0.0000365 parts of the horizontal force). The tabular numbers are expressed in units of the sixth place of decimals. A plus sign indicates a greater force, a minus sign a less force than the mean value. Casting the eye over the vertical columns, we obtain also a view of the annual inequality of the diurnal variation, which will be examined further on.

Table III.—Regular Solar-Diurnal Variation of the Horizontal Component of the Magnetic Force expressed in parts of the Horizontal Force.

A plus sign indicates greater force than the mean. For convenience sake, the first three decimals (0.000) have been placed on the side of the table.

	1840–1845.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+ 21½m
	July	—171	—098	—025	+047	+120	+303	+449	+120	—353	—572	—646	—499	
	August	+153	+116	+116	+153	+116	+262	+335	+007	—430	—795	—868	—576	
	September	+168	+241	+168	+241	+387	+423	+481	+131	—380	—708	—891	—781	
	October	+116	+262	+262	+408	+481	+372	+335	+153	—029	—284	—467	—430	
	November	—018	+018	+091	+164	+200	+310	+383	+237	+054	—164	—273	—456	
	December	—131	+051	+088	+161	+270	+343	+453	+380	+270	+088	—241	—533	
0.000	January	—025	+047	+047	+120	+193	+230	+266	+339	+266	—061	—318	—536	
	February	+022	+095	+146	+168	+204	+277	+350	+277	+204	+022	—270	—343	
	March	+098	+134	+171	+207	+244	+317	+390	+244	+061	—230	—412	—595	
	April	+157	+193	+193	+303	+376	+412	+412	+303	—025	—463	—755	—828	
	May	—065	—029	+080	+116	+116	+226	+299	+116	—211	—503	—540	—394	
	June	—043	—043	—043	—043	+065	+175	+284	+065	—153	—372	—445	—335	
	Year	+022	+082	+108	+170	+231	+304	+370	+198	—060	—337	—511	—526	
	Summer	+033	+063	+082	+136	+197	+300	+377	+127	—259	—570	—691	—569	
	Winter	+010	+101	+134	+205	+265	+308	+363	+272	+138	—105	—330	—482	

	1840–1845.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+ 21½m
	July	—244	+157	+485	+595	+631	+376	+047	—025	—098	—171	—244	—244	
	August	—211	+153	+481	+518	+372	+226	+043	—029	—065	—029	—065	+007	
	September	—416	—051	+168	+241	+204	+058	+022	—015	+058	—015	+095	+095	
	October	—467	—321	—175	—065	—029	—029	+007	—029	—065	—029	—029	+007	
	November	—419	—273	—054	+018	+054	—018	+091	+091	+054	—018	—091	—054	
	December	—606	—350	—168	+015	+161	+124	+051	+015	—095	—131	—131	—095	
0.000	January	—536	—317	—098	+120	+230	+120	+011	—025	+011	—025	—025	—098	
	February	—453	—270	—124	+022	—015	+022	—015	—124	—197	—051	+022	+022	
	March	—485	—303	—011	+171	+061	—047	—157	—088	+061	+098	+025	+061	
	April	—609	—390	+047	+157	+230	+230	+047	+047	—061	—061	+011	+047	
	May	—175	+080	+262	+299	+335	+262	+043	+007	—102	—138	—102	+007	
	June	—116	+065	+321	+357	+357	+211	+029	—043	—080	—043	—080	—080	
	Year	—395	—152	+095	+204	+216	+128	+018	—019	—049	—051	—051	—027	
	Summer	—295	+002	+294	+361	+355	+227	+037	—009	—058	—076	—064	—028	
	Winter	—494	—306	—105	+047	+077	+029	—002	—027	—039	—026	—038	—026	

Table IV.

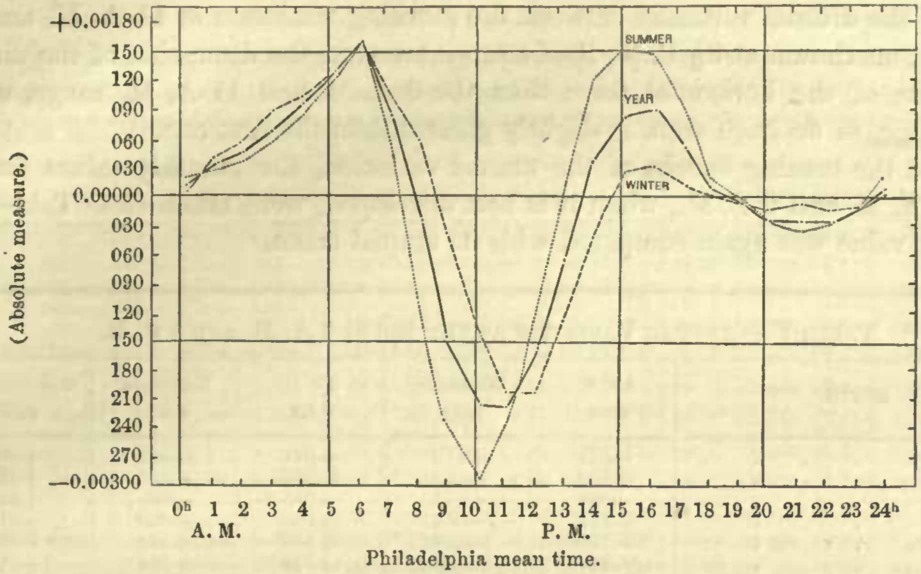
Table IV. is derived from Table III. by multiplication with the absolute value of the horizontal force (4.173); it contains, therefore, the regular solar-diurnal variation of the horizontal force in absolute measure. A plus sign indicates greater force than the mean. Two places of decimals have been placed on the side of the table.

	1840–1845.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+21½m
	July	—071	—041	—010	+020	+050	+127	+188	+050	—147	—239	—270	—208	
	August	+064	+048	+048	+064	+048	+109	+140	+003	—180	—332	—362	—241	
	September	+070	+101	+070	+101	+162	+177	+201	+055	—159	—296	—372	—326	
	October	+048	+109	+109	+170	+201	+155	+140	+064	—012	—119	—195	—180	
	November	—008	+008	+038	+068	+083	+129	+160	+099	+022	—068	—114	—190	
	December	—055	+021	+037	+067	+113	+143	+189	+159	+113	+037	—101	—223	
0.00	January	—010	+020	+020	+050	+081	+096	+111	+141	+111	—025	—132	—224	
	February	+009	+040	+061	+070	+085	+116	+146	+116	+085	+009	—113	—143	
	March	+041	+056	+071	+086	+102	+132	+163	+102	+025	—096	—172	—248	
	April	+066	+081	+081	+127	+157	+172	+172	+127	—010	—193	—315	—346	
	May	—027	—012	+033	+048	+048	+094	+125	+048	—088	—210	—226	—165	
	June	—018	—018	—018	—018	+027	+073	+119	+027	—064	—155	—186	—140	
	Year	+009	+034	+045	+071	+096	+127	+155	+083	—025	—141	—213	—220	
	Summer	+014	+026	+034	+057	+082	+125	+157	+053	—108	—238	—289	—238	
	Winter	+004	+042	+056	+086	+111	+129	+152	+114	+058	—044	—138	—201	

	1840–1845.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+21½m
	July	—102	+065	+203	+248	+263	+157	+020	—010	—041	—071	—102	—102	
	August	—088	+064	+201	+216	+155	+094	+018	—012	—027	—012	—027	+003	
	September	—174	—021	+070	+101	+085	+024	+009	—006	+024	—006	+040	+040	
	October	—195	—134	—073	—027	—012	—012	+003	—012	—027	—012	—012	+003	
	November	—175	—114	—022	+008	+022	—008	+038	+038	+022	—008	—038	—022	
	December	—253	—146	—070	+006	+067	+052	+021	+006	—040	—055	—055	—040	
0.00	January	—224	—132	—041	+050	+096	+050	+005	—010	+005	—010	—010	—041	
	February	—189	—113	—052	+009	—006	+009	—006	—052	—082	—021	+009	+009	
	March	—203	—127	—005	+071	+025	—020	—065	—037	+025	+041	+010	+025	
	April	—254	—163	+020	+065	+096	+096	+020	+020	—025	—025	+005	+020	
	May	—073	+033	+109	+125	+140	+109	+018	+003	—043	—058	—043	+003	
	June	—048	+027	+134	+149	+149	+088	+012	—018	—033	—018	—033	—033	
	Year	—165	—063	+040	+085	+090	+053	+008	—008	—020	—021	—021	—011	
	Summer	—123	+001	+123	+151	+148	+095	+015	—004	—024	—032	—027	—012	
	Winter	—206	—128	—044	+020	+032	+012	—000	—011	—016	—011	—016	—010	

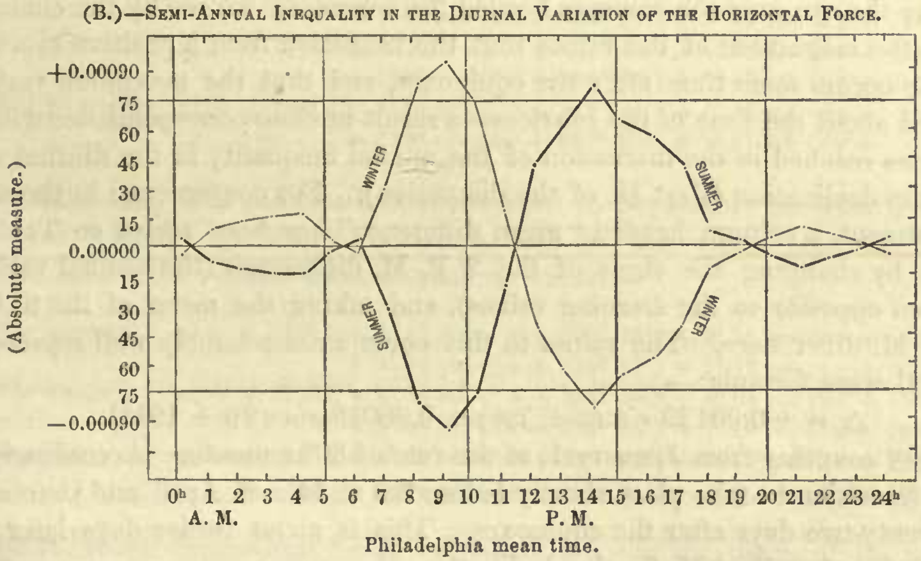
Annual Inequality in the Diurnal Variation of the Horizontal Force.—The distinctive feature of the diurnal variation is shown in the annexed diagram (A), constructed from the mean annual and half-yearly values given in the preceding table, IV. It exhibits in the annual mean, as its characteristic type, a maximum value about 6 A. M., a minimum value about 11 A. M., a secondary maximum value about 3½ P. M., and a secondary minimum about 9 P. M. For the half year when the sun has north declination, the morning minimum becomes smaller and the afternoon maximum larger, thus increasing the diurnal range; the converse takes place in the other half of the year, when the sun has south declination. The 6 A. M. maximum remains nearly unchanged throughout the year. The average summer range (April to September inclusive) is 0.0046, and the average winter range (October to March inclusive) is 0.0025, both expressed in absolute measure. The range between the morning maximum and the morning minimum is 0.0045 in summer and 0.0036 in winter, as will be explained further on.

(A.)—Diurnal Variation of the Horizontal Force in Summer, Winter, and for the whole Year.



This semi-annual change in the diurnal amplitude is more conspicuously represented in the annexed diagram (B), derived from diagram (A) by straightening out the annual curve and using it as an axis of abscissæ for laying off the differences between the annual values and the summer and winter values at the same respective hours of the day.

(B.)—Semi-Annual Inequality in the Diurnal Variation of the Horizontal Force.



This diagram (B) may, with advantage, be compared with the analogous one representing the annual change of the diurnal variation of the declination as given in Part II. of this discussion. The construction is the same in either case.

At 6 A. M. there is hardly any change throughout the year. The maximum variation, in the course of a year, takes place at 9 A. M. (range 0.00194 in absolute measure); about $11\frac{1}{2}$ A. M. there is an epoch of no variation; at 2 P. M. a second maximum is reached (range 0.00167); again at $7\frac{1}{2}$ and 11 P. M. points of no variation are reached. Owing to the prominent annual variation near 2 P. M., the range of the diurnal variation between the morning minimum at 11 A. M. and the afternoon maximum at $3\frac{1}{2}$ P. M. is of more interest in the discussion of the diurnal fluctuation of the horizontal force than the 6 A. M. and 11 A. M. range, which latter range, as we have seen, is slightly greater than the first one.

To find the turning epochs of the annual variation, the monthly values for the hours 9 A. M. and 2 P. M., when it is best developed, were taken from Table IV., and each value was again compared with its annual mean.

TABLE V.—ANNUAL VARIATION AT THE HOURS 9 A. M. AND 2 P. M.

MONTH.	9 A. M. 0.00	Differences. 0.00	2 P. M. 0.00	Differences. 0.00	Mean difference. 0.00
January	—025	+116	—041	—081	+099
February	+009	+150	—052	—092	+121
March	—096	+045	—005	—045	+045
April	—193	—052	+020	—020	—016
May	—210	—069	+109	+069	—069
June	—155	—014	+134	+094	—054
July	—239	—098	+203	+163	—130
August	—332	—191	+201	+161	—176
September	—296	—155	+070	+030	—092
October	—119	+022	—073	—113	+046
November	—068	+073	—022	—062	+068
December	+037	+178	—070	—110	+144
Mean	—141		+040		

Casting the eye over the columns headed "differences," we see by the change of sign and the magnitude of the values that the transition from a positive to a negative value occurs some time after the equinoxes, and that the maximum variation is reached about the time of the solstices—a result in close correspondence with the conclusions reached in the discussion of the annual inequality in the diurnal variation of the declination (Part II. of the discussion). For convenience in the analytical treatment, a column headed "mean difference" has been added to Table V., obtained by changing the signs of the 2 P. M. differences (the annual variation being then opposite to the morning values), and taking the mean of the 9 A. M. and 2 P. M. differences. The values in this column are tolerably well represented by the following formula:—

$$\Delta_a = +0.00129 \, sin\,(\theta + 79°) + 0.00018 \, sin\,(2\theta + 191°),$$

the angle θ counting from January 1, at the rate of 30° a month. Accordingly, we find the transition to take place shortly before the middle of April and October, or about twenty-two days after the equinoxes. This is about twelve days later than the epoch found in Part II. for the declination.

Analysis of the Solar-Diurnal Variation of the Horizontal Force.—For convenience of investigation and proper comparison with similar results at other localities, the values given in Table I. have been put in an analytical form, and are represented by the following expressions. It will be seen that the difference between any monthly normal mean and the corresponding mean in Table V. of Part IV., which latter mean is affected with the disturbances, does not exceed $2\frac{1}{2}$ scale divisions. This small difference includes also a small effect due to the necessity of different methods of interpolation in the construction of the two tables. In the determination of the numerical quantities (by application of the method of least squares) in the monthly equations, due attention was paid to the relative weights of the values for the even and odd hours. The coefficients are expressed in scale divisions (increasing numbers denoting decrease of force), and the angle θ counts from midnight at the rate of 15° an hour.

For January, $\Delta_h = +\,793^d.3 + 3^d.77 \sin(\theta + 236^\circ 52') + 6^d.56 \sin(2\theta + 96^\circ 52')$
$\quad + 3^d.99 \sin(3\theta + 282^\circ 13') + 2^d.00 \sin(4\theta + 97^\circ)$

For February, $\Delta_h = +\,800^d.6 + 5^d.50 \sin(\theta + 218^\circ 26') + 4^d.57 \sin(2\theta + 102^\circ 29')$
$\quad + 3^d.27 \sin(3\theta + 282^\circ 40') + 1^d.66 \sin(4\theta + 121^\circ)$

For March, $\Delta_h = +\,805^d.7 + 6^d.56 \sin(\theta + 243^\circ 31') + 5^d.35 \sin(2\theta + 114^\circ 14')$
$\quad + 4^d.23 \sin(3\theta + 316^\circ 04') + 1^d.91 \sin(4\theta + 113^\circ)$

For April, $\Delta_h = +\,828^d.3 + 7^d.65 \sin(\theta + 257^\circ 37') + 9^d.55 \sin(2\theta + 123^\circ 06')$
$\quad + 5^d.15 \sin(3\theta + 306^\circ 44') + 1^d.18 \sin(4\theta + 163^\circ)$

For May, $\Delta_h = +\,832^d.2 + 2^d.24 \sin(\theta + 314^\circ 31') + 7^d.81 \sin(2\theta + 140^\circ 53')$
$\quad + 4^d.40 \sin(3\theta + 330^\circ 05') + 1^d.34 \sin(4\theta + 214^\circ)$

For June, $\Delta_h = +\,856^d.8 + 2^d.12 \sin(\theta + 356^\circ 03') + 6^d.40 \sin(2\theta + 140^\circ 32')$
$\quad + 4^d.48 \sin(3\theta + 327^\circ 14') + 0^d.92 \sin(4\theta + 216^\circ)$

For July, $\Delta_h = +\,676^d.3 + 3^d.42 \sin(\theta + 4^\circ 11') + 11^d.50 \sin(2\theta + 139^\circ 14')$
$\quad + 6^d.14 \sin(3\theta + 330^\circ 15') + 0^d.78 \sin(4\theta + 210^\circ)$

For August, $\Delta_h = +\,702^d.2 + 5^d.32 \sin(\theta + 310^\circ 58') + 10^d.37 \sin(2\theta + 153^\circ 46')$
$\quad + 6^d.79 \sin(3\theta + 335^\circ 55') + 2^d.88 \sin(4\theta + 203^\circ)$

For September, $\Delta_h = +\,724^d.6 + 8^d.02 \sin(\theta + 271^\circ 57') + 9^d.59 \sin(2\theta + 137^\circ 25')$
$\quad + 7^d.08 \sin(3\theta + 345^\circ 17') + 1^d.99 \sin(4\theta + 215^\circ)$

For October, $\Delta_h = +\,738^d.2 + 8^d.06 \sin(\theta + 237^\circ 57') + 6^d.40 \sin(2\theta + 123^\circ 37')$
$\quad + 1^d.34 \sin(3\theta + 325^\circ 20') + 0^d.29 \sin(4\theta + 174^\circ)$

For November, $\Delta_h = +\,738^d.5 + 4^d.13 \sin(\theta + 237^\circ 36') + 6^d.08 \sin(2\theta + 100^\circ 01')$
$\quad + 1^d.93 \sin(3\theta + 310^\circ 45') + 0^d.46 \sin(4\theta + 211^\circ)$

For December, $\Delta_h = +\,768^d.4 + 5^d.03 \sin(\theta + 212^\circ 48') + 8^d.07 \sin(2\theta + 94^\circ 14')$
$\quad + 3^d.98 \sin(3\theta + 269^\circ 17') + 1^d.31 \sin(4\theta + 88^\circ)$

We have also: For summer half year (April to September inclusive), for winter half year (October to March inclusive), and for the whole year, the following expressions for the regular solar diurnal variations:—

For summer, $\Delta_h = +\,770^d.1 + 3^d.79 \sin(\theta + 293^\circ 49') + 9^d.11 \sin(2\theta + 139^\circ 10')$
$\quad + 5^d.36 \sin(3\theta + 329^\circ 17') + 1^d.42 \sin(4\theta + 202^\circ)$

For winter, $\Delta_h = +\,774^d.1 + 5^d.36 \sin(\theta + 231^\circ 36') + 6^d.04 \sin(2\theta + 104^\circ 46')$
$\quad + 2^d.88 \sin(3\theta + 293^\circ 54') + 1^d.11 \sin(4\theta + 108^\circ)$

For year, $\Delta_h = +\,772^d.1 + 3^d.95 \sin(\theta + 256^\circ 19') + 7^d.25 \sin(2\theta + 125^\circ 05')$
$\quad + 3^d.96 \sin(3\theta + 317^\circ 31') + 0^d.86 \sin(4\theta + 165^\circ)$

The following expressions for January may serve as specimens of the agreement of the result derived from the even and odd hours independently:—

From even hours, $\Delta_h = 793^d.3 + 3^d.81 \sin(\theta + 238^\circ 01') + 6^d.56 \sin(2\theta + 94^\circ 32')$
$\quad + 4^d.10 \sin(3\theta + 280^\circ 19') + 2^d.08 \sin(4\theta + 86^\circ)$

From odd hours, $\Delta_h = 793^d.4 + 3^d.71 \sin(\theta + 234^\circ 35') + 6^d.56 \sin(2\theta + 101^\circ 32')$
$\quad + 3^d.76 \sin(3\theta + 286^\circ 00') + 1^d.85 \sin(4\theta + 119^\circ)$

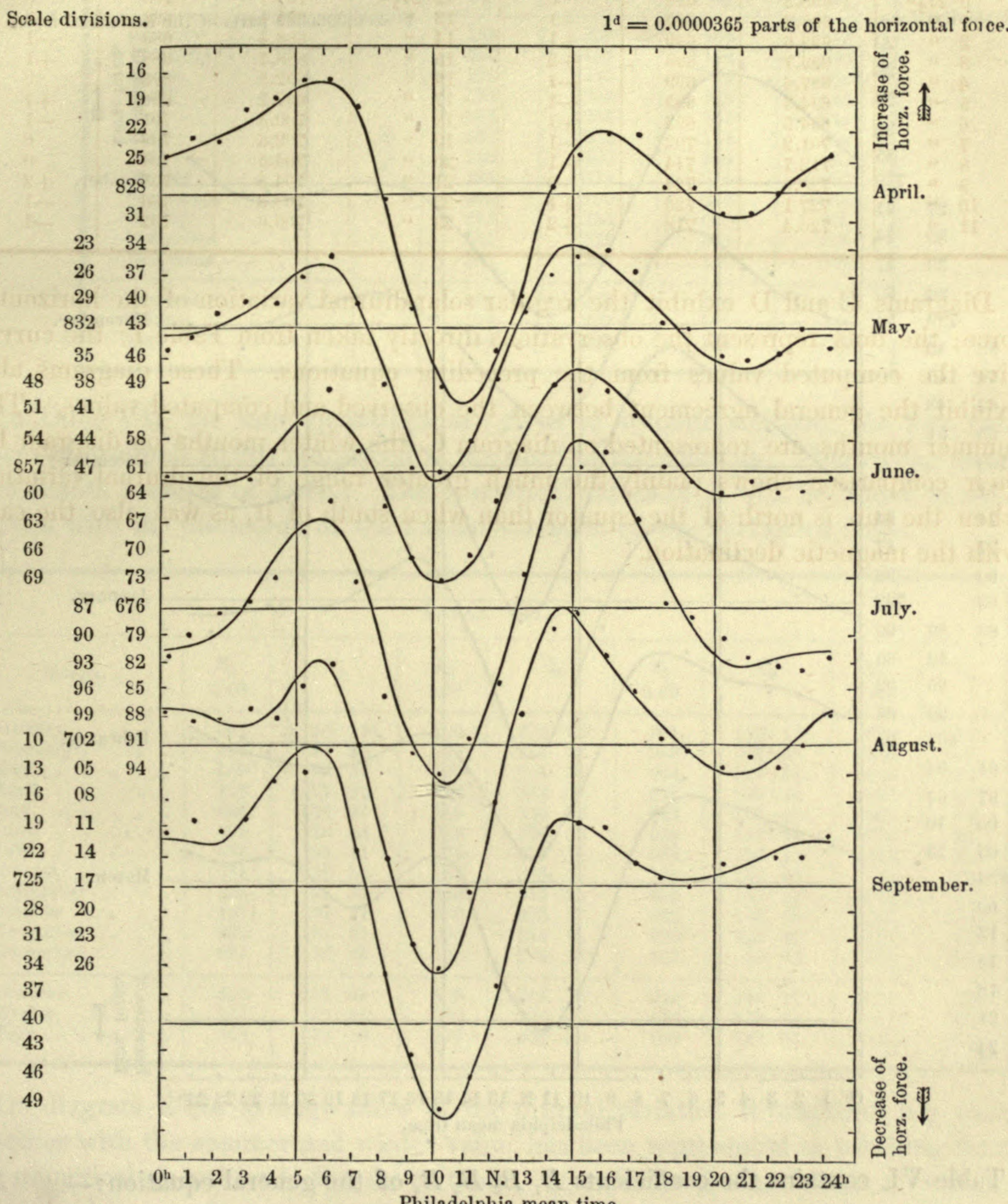
giving to the first equation the weight 2 and to the second the weight 1, we obtain the equation as given above.

The following comparison will show the agreement of the observed and computed values we have for August:—

(A. M.)	Computed.	Observed.	Δ	(P. M.)	Computed.	Observed.	Δ
$0^h\ 21\frac{1}{2}^m$	698.3	698	0	$12^h\ 21\frac{1}{2}^m$	707.7	708	0
1 "	698.3	699	—1	13 "	695.1	698	—3
2 "	699.6	699	+1	14 "	688.4	689	—1
3 "	699.7	698	+2	15 "	888.7	688	+1
4 "	697.6	699	—1	16 "	692.5	692	0
5 "	694.3	695	—1	17 "	697.1	696	+1
6 "	694.5	693	+1	18 "	700.3	701	—1
7 "	701.2	702	—1	19 "	702.6	703	0
8 "	712.7	714	—1	20 "	704.5	704	0
9 "	723.6	724	0	21 "	704.8	703	+2
10 "	727 1	726	+1	22 "	703.3	704	—1
11 "	720.4	718	+2	23 "	700.6	702	—1

Diagrams C and D exhibit the regular solar-diurnal variation of the horizontal force; the dots represent the observations directly taken from Table 1; the curves give the computed values from the preceding equations. These diagrams also exhibit the general agreement between the observed and computed values. The summer months are represented on diagram C, the winter months on diagram D; their comparison shows plainly the much greater range of the diurnal variation when the sun is north of the equator than when south of it, as was also the case with the magnetic declination.

(C.)—Solar-diurnal Variation of the Horizontal Force; April to September, 1840 to 1845.



(D.)—SOLAR-DIURNAL VARIATION OF THE HORIZONTAL FORCE; OCTOBER TO MARCH, 1840 TO 1845.

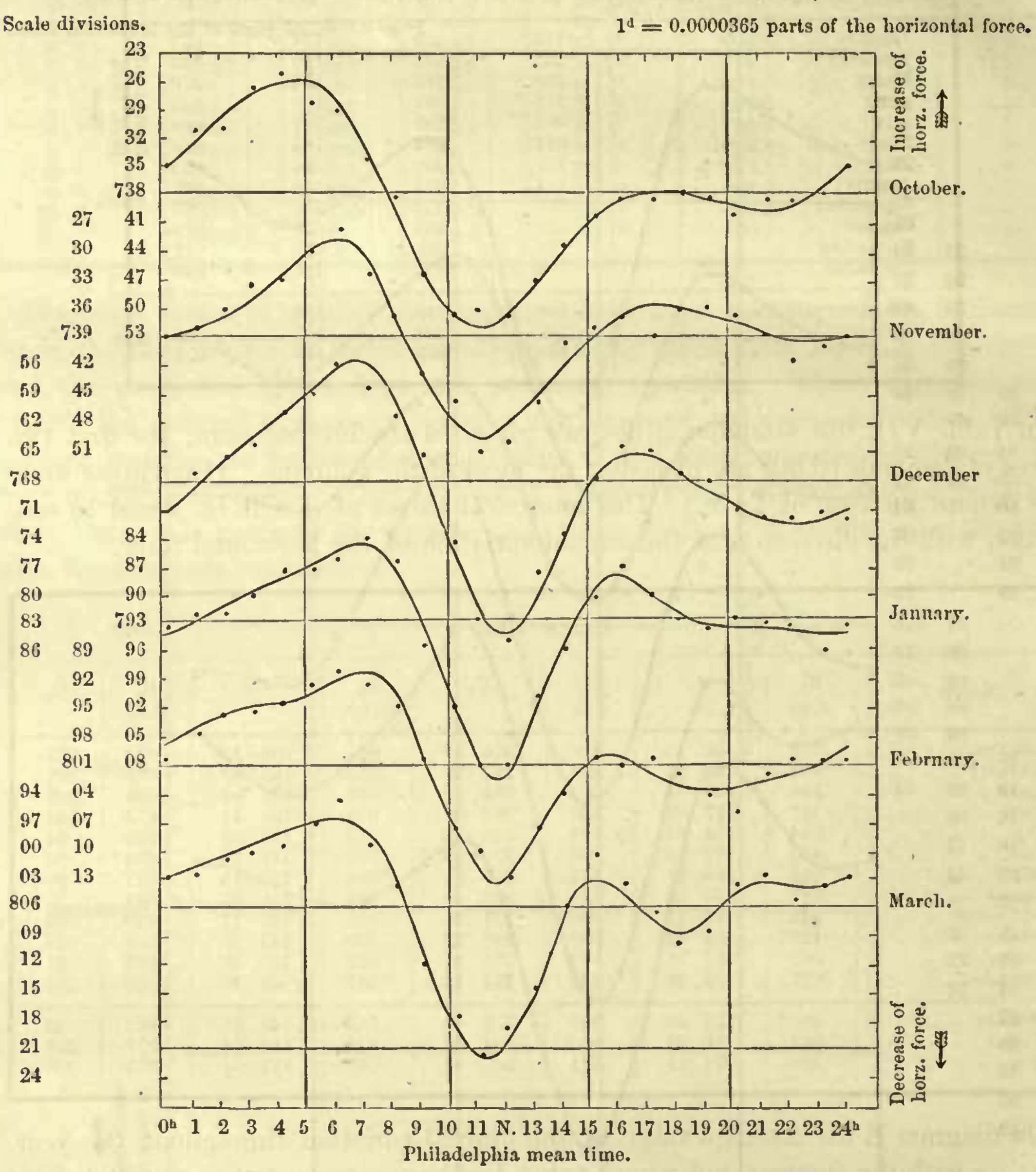


Table VI. contains the coefficients B_1 B_2 B_3 B_4 of the general equation:—

$$\Delta_h = A + B_1 \sin(\theta + C_1) + B_2 \sin(2\theta + C_2) + B_3 \sin(3\theta + C_3) + B_4 \sin(4\theta + C_4)$$

expressed in parts of the horizontal force, by multiplying the corresponding quantities in the preceding equations with the value of a scale division. The angles C_1 C_2 C_3 C_4 will be found in Table VII.; they are the same as given before, increased by 180°, so as to make a corresponding change in the direction of the scale readings; increasing numbers will now indicate increasing force.

The first three decimals (0.000) have been placed in front of the table.

TABLE VI.

MONTH.		B_1	B_2	B_3	B_4
January		138	239	146	073
February		202	167	119	060
March		239	195	154	070
April		279	349	188	043
May		082	285	161	049
June	0.000	077	234	164	034
July		125	420	224	029
August		194	379	248	105
September		295	350	258	073
October		294	234	048	011
November		151	222	071	017
December		184	295	145	048
Summer		138	333	196	052
Winter		196	220	105	040
Year		144	265	145	031

In Table VII. the same quantities are given in absolute measure; the first two places of decimals (0.00) are placed at the head of the columns. (Increasing numbers denote increase of force.) The numerical values of A will be found in connection with the discussion of the annual variation of the horizontal force.

TABLE VII.

MONTH.	B_1 0.00	C_1	B_2 0.00	C_2	B_3 0.00	C_3	B_4 0.00	C_4
January	057	56° 52′	100	276° 52′	061	102° 13′	030	277°
February	084	38 26	070	282 29	050	102 40	025	301
March	100	63 31	082	294 14	064	136 04	029	293
April	117	77 37	146	303 06	079	126 44	018	343
May	034	134 31	119	320 53	067	150 05	020	34
June	032	176 03	098	320 32	068	147 14	014	36
July	052	184 11	175	319 14	094	150 15	012	30
August	081	130 58	158	333 46	104	155 55	044	23
September . . .	122	91 57	146	317 25	108	165 17	030	35
October	123	57 57	098	303 37	020	145 20	005	354
November . . .	063	57 36	093	280 01	029	130 45	007	31
December . . .	077	32 48	123	274 14	061	89 17	020	268
Summer	058	113 49	139	319 10	082	149 17	022	22
Winter	082	51 36	092	284 46	044	113 54	017	288
Year	060	76 19	111	305 05	060	137 31	013	345

On diagram E the average value of the diurnal variation throughout the year, together with the summer and winter value, has been represented as resulting from the numerical quantities in the above table. It exhibits the noticeable feature in the annual curve of a greater morning maximum (about 6 A. M.) than afternoon maximum (about 3½ P. M.), whereas in the summer curve it is the afternoon maximum which is the greater of the two.[1] In the winter season the contrast is more

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[1] The same is the case at Prague; in May, June, and July, the afternoon maximum was the greater of the two. Karl Kreil, in vol. VIII. Proceedings of the Academy of Sciences of Vienna, 1855: "Resultate aus den magnetischen Beobachtungen zu Prag."

marked, the morning maximum being considerably greater. These curves also show the gradual shifting of the maxima and minimum to a later hour in winter than in summer, a phenomenon also well exhibited in the preceding diagrams C and D. The numerical values of this change of hours will be given in tabular form further on. The small afternoon minimum about 9 P. M. is less distinctly marked than any other feature of the diurnal curve.

(E.)—Regular Solar-Diurnal Variation of the Horizontal Force for Summer, Winter, and whole Year. (In absolute measure.)

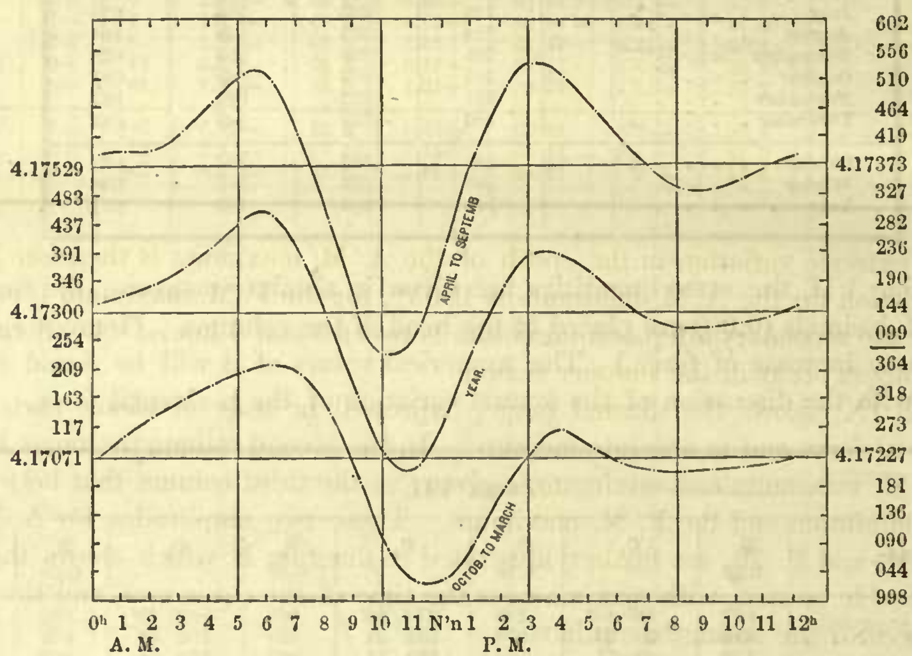


Table VIII. contains the computed values of the time and amount of the morning maximum and minimum, and of the afternoon maximum. The values for the secondary afternoon minimum are taken from the diagrams. The time of the A. M. maximum and minimum is within the nearest eighth minute; that of the P. M. maximum within the nearest tenth minute. The time for the P. M. secondary minimum is within the nearest hour. The amount of change of horizontal force is expressed in scale divisions.

TABLE VIII.

MONTH.	Morning maximum.		Morning minimum.		Afternoon maximum.		Secondary afternoon minimum.		Interval A. M. min. to P. M. max.
January . . .	$7^h 10^m$	— $9^d.2$	$11^h 50^m$	+$15^d.7$	$4^h 10^m$	— $5^d.3$	11^h	+2^d	$4^h 20^m$
February . . .	7 15	— 9.6	11 40	+12.7	4 00	— 0.9	7	+2	4 20
March	6 15	— 9.2	11 30	+16.4	3 20	— 2.3	6	+3	3 50
April	6 00	—12.3	11 20	+22.5	3 55	— 6.6	9	+3	4 35
May	5 50	— 7.9	10 25	+15.5	3 10	— 9.8	9	+4	4 45
June	5 50	— 6.3	10 30	+12.5	3 20	—10.4	8	+3	4 50
July	5 35	— 9.9	10 30	+19.3	3 25	—17.5	9	+6	4 55
August	5 55	— 8.5	10 10	+24.8	2 45	—14.2	9	+3	4 35
September . . .	5 35	—14.9	10 20	+25.9	3 05	— 6.7	7	—1	4 45
October . . .	5 00	—12.6	11 15	+13.7	5 10	— 0.1	9	+2	5 55
November . . .	6 00	— 9.8	11 25	+11.0	5 15	— 3.0	11	+0	5 50
December . . .	7 05	—12.1	12 05	+16.1	4 35	— 5.1	10	+4	4 30
Summer . . .	5 50	— 9.8	10 30	+19.6	3 25	—10.5	20½	+3	4 55
Winter	6 15	— 9.4	11 45	+13.9	4 10	— 2.2	21	+2	4 25
Year.	5 55	— 9.6	11 00	+15.6	3 35	— 6.0	20¾	+2.5	4 35

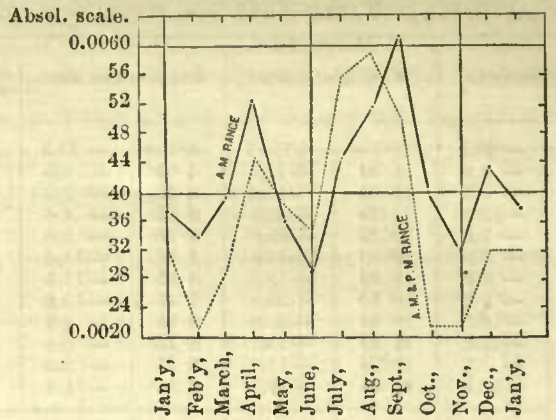
The extreme variation in the epoch of the A. M. maximum is therefore $2^h 15^m$; the variation for the A. M. minimum is $1^h 55^m$; for the P. M. maximum it is $2^h 30^m$, and for the secondary afternoon minimum between 3 and 4 hours. In all cases, the earlier hours occur in the summer season.

Table IX. shows the diurnal range, expressed in scale divisions, parts of the horizontal force and in absolute measure. In the second column the range between the A. M. maximum and minimum is given; in the third column that between the A. M. minimum and the P. M. maximum. These two amplitudes for A. M., and for A. M. and P. M., are further illustrated in diagram F, which shows the curve to be double crested, with maxima near the time of the equinoxes, and the greater of these near the autumnal equinox.

TABLE IX.—AMPLITUDE OF THE DIURNAL VARIATION OF THE HORIZONTAL FORCE.

MONTH.	For A. M.	For A. M. and P. M.	For A. M.	For A. M. and P. M.	For A. M.	For A. M. and P. M.
January	$24^d.9$	$21^d.0$	0.00091	0.00077	0.0038	0.0032
February	22.3	13.6	081	050	34	21
March	25.6	18.7	093	068	39	29
April	34.8	29.1	127	106	53	45
May	23.4	25.3	085	092	36	38
June	18.8	22.9	069	084	29	35
July	29.2	36.8	106	134	45	56
August	33.3	39.0	122	142	51	59
September	40.8	32.6	149	119	62	50
October	26.3	13.6	096	050	40	21
November.	20.8	14.0	076	051	32	21
December	28.2	21.2	0.00103	077	0.0043	0.0032
Summer	29.4	30.1	0.00107	0.00110	0.0045	0.0046
Winter	23.3	16.1	0.00085	0.00059	0.0036	0.0025
Year	25.2	21.6	0.00092	0.00079	0.0038	0.0033
	In scale divisions.		In parts of the horizontal force.		In absolute measure.	

(F.)—SOLAR-DIURNAL RANGE OF THE HORIZONTAL FORCE FOR EACH MONTH OF THE YEAR.



The next table contains the epochs when the mean horizontal force is reached in each day, as computed by the preceding formulæ. The diurnal curves intersect the axis of abscissæ four times, of which the table contains only the A. M. and first P. M. intersection: those later in the afternoon and near midnight occur in summer, winter, and whole year at 7 P. M., $5\frac{3}{4}$ P. M., and $6\frac{1}{2}$ P. M. respectively, and at $11\frac{1}{4}$ P. M., 12 P. M., and $11\frac{3}{4}$ P. M. respectively.

TABLE X.—PRINCIPAL EPOCHS OF MEAN HORIZONTAL FORCE.

MONTH.	A. M.	P. M.
January	$9^h 20^m$	$2^h 36^m$
February	9 23	2 58
March	8 42	2 28
April	8 14	2 19
May	7 44	0 59
June	7 47	0 48
July	7 57	0 53
August	7 28	0 44
September	7 42	1 29
October	8 08	5 00
November	8 40	3 28
December	9 34	3 03
Summer	$7^h 45^m$	$1^h 12^m$
Winter	9 00	3 07
Year	8 14	1 54

The above times are generally correct within two minutes (according to the formulæ). The morning hour of average daily horizontal force is less variable in the course of a year than the afternoon hour.

The following table contains the computed diurnal variation of the horizontal force. The values have been expressed in absolute measure. It compares directly with Table IV., which contains the observed values. It will be useful for the interpolation of observations, or for their reduction to the mean value of the day from observations taken at irregular hours. The table also forms the basis for the construction of diagram G.

Table XI.—Computed Solar-Diurnal Variation of the Horizontal Force in Absolute Measure.

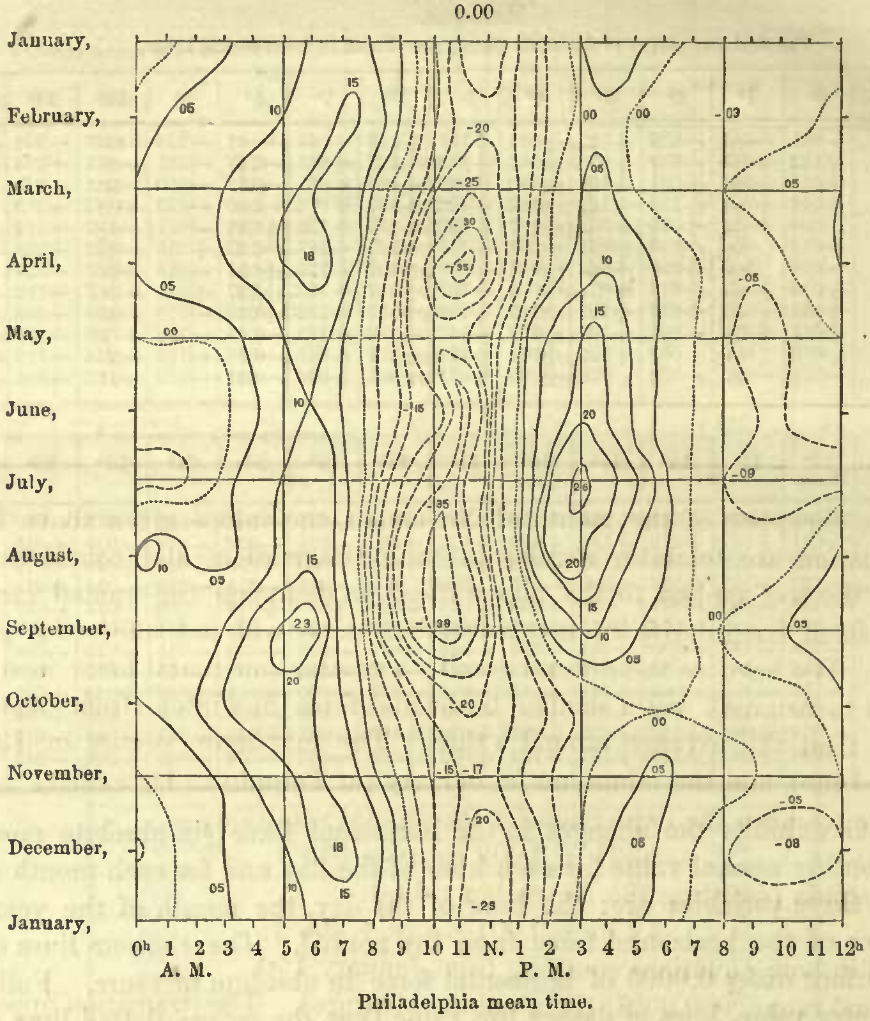
The first two places of decimals (0.00) are placed in front of the table.

	1840–1845.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	$+21\frac{1}{2}^m$
	July	—061	—061	—030	+015	+091	+137	+137	+046	—107	—244	—290	—244	
	August	+122	+061	+030	+030	+061	+122	+122	+015	—167	—335	—381	—274	
	September	+061	+061	+061	+107	+182	+229	+198	+061	—152	—320	—381	—320	
	October	+046	+091	+122	+167	+182	+182	+137	+076	—030	—122	—182	—213	
	November	000	+015	+030	+061	+107	+152	+152	+122	+030	—061	—137	—167	
0.00	December	—046	000	+030	+061	+091	+137	+167	+182	+122	+015	—122	—229	
	January	—030	000	+030	+046	+061	+091	+107	+122	+091	—015	—137	—229	
	February	+030	+061	+076	+076	+091	+107	+137	+152	+137	+015	—107	—182	
	March	+046	+061	+076	+107	+107	+137	+137	+122	+030	—076	—198	—244	
	April	+061	+076	+091	+107	+137	+167	+182	+122	—015	—198	—320	—351	
	May	000	000	000	+030	+061	+107	+107	+046	—076	—198	—244	—182	
	June	—015	—030	—030	000	+046	+091	+091	+046	—061	—152	—182	—305	

	1840–1845.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	$+21\frac{1}{2}^m$
	July	—091	+076	+213	+259	+229	+152	+046	—015	—061	—091	—076	—076	
	August	—091	+107	+213	+198	+152	+076	+030	—015	—030	—046	—015	+015	
	September	—152	—015	+091	+107	+076	+046	+015	000	+030	+030	+046	+076	
	October	—182	—137	—076	—030	—015	000	000	—015	—015	—030	—015	+015	
	November	—137	—091	—046	000	+030	+046	+046	+030	+015	000	—015	—015	
0.00	December	—244	—182	—076	+015	+061	+061	+015	—015	—046	—061	—076	—061	
	January	—229	—137	—030	+061	+076	+046	+015	—015	—030	—015	—015	—030	
	February	—182	—107	—030	+015	+015	—015	—030	—030	—030	—015	—015	000	
	March	—198	—107	000	+046	+015	—015	—046	—030	+015	+046	+046	+046	
	April	—274	—137	000	+076	+091	+061	+030	000	—030	—030	000	+030	
	May	—076	+046	+122	+152	+122	+076	+030	—015	—046	—061	—046	—015	
	June	—046	+061	+137	+167	+137	+076	+030	—015	—030	—046	—030	—015	

Diagram G exhibits the changes in the horizontal force (in absolute measure) from the monthly normal value for each hour of the day and for each month of the year. The three variables are: the hour of the day, the month of the year, and the difference of the horizontal force from the normal. The contour lines of the magnetic surface differ 0.0005 of horizontal force in absolute measure. Full lines indicate greater value, lines of dashes less value than the mean; dotted lines represent the normal value.

(G.)—Changes of the Horizontal Force from its Normal Value, for each Hour of the Day and Month of the Year. Expressed in absolute measure.



Annual Variation of the Horizontal Force.—For the discussion of the annual variation we make use of the monthly normal readings of the horizontal force as given in Table II. If m equals the monthly effect of the total progressive change, we obtain from the twelve equations by the usual method the value $m = +\ 15.49$, and the correction for progressive change for July and June, for instance, becomes $+\ 5.5\ m$ and $-\ 5.5\ m$ respectively. The following table contains the monthly normals uncorrected and corrected for progressive change; also the differences from the mean for each month, constituting the annual variation.

TABLE XII.

MONTH.	Normals.	Corrected for progressive change.	Corrected normals.	Differences, or annual variation.		
					0.000	0.00
July	676.3	+85.2	761.5	+10.6	+39	+16
August	702.2	+69.7	771.9	+ 0.2	+01	+00
September . . .	724 6	+54.2	778.8	— 6.7	—24	—10
October . . .	738.2	+38.7	776.9	— 4.8	—17	—07
November . . .	738.5	+23.2	761.7	(+10.4)	(+38)	(+16)
December . . .	768.4	+ 7.7	776.1	— 4.0	—15	—06
January . . .	793.3	— 7.7	785.6	—13.5	—49	—20
February . . .	800.6	—23.2	777.4	— 5.3	—19	—08
March	805.7	—38.7	767.0	+ 5.1	+19	+08
April	828.3	—54.2	774.1	— 2.0	— 7	—03
May	832 2	—69.7	762.5	+ 9.6	+35	+15
June	856.8	—85.2	771.6	+ 0.5	+02	+01
Mean	772.1	0.0	772.1	In scale divisions.	In parts of the horizontal force.	In absolute measure.

With the exception of the month of November, the values given above for the annual variation are tolerably regular in their progression, and considering the delicacy of the test applied to the observations in deducing the annual variation, this exceptional irregularity in the November value will not affect the general conclusion. We have as the general result: a greater horizontal force in summer (from April to August), and a smaller horizontal force in winter (from September to March) than the average annual value. The maximum occurs in July (at Toronto in June), and the minimum in January (at Toronto in December).

For Toronto we have the expression for the annual variation:—

$$3.531 + 0.002 \, sin \, (\theta + 312°).$$

For Philadelphia (omitting the November value):

$$4.176 + 0.001 \, sin \, (\theta + 306°);$$

the angle θ in both equations counting from January 15th.

The annual range is 0.0021 (in absolute measure). The transition appears to take place about the time of the equinoxes or a short time before.

Table XIII. contains the monthly normal values of the horizontal force in absolute measure, obtained by adding (algebraically) 4.1730 to the values in the last column of Table XII. These numbers, it will be observed, are corrected for secular change; if we apply the same we obtain the resulting monthly mean values of the horizontal force answering to the epoch January, 1843. The quantity A, mentioned in the explanatory remarks to Table VII., is given in the last column of Table XIII.

TABLE XIII.

MONTH.	Normals corrected for secular change.	Monthly means (affected with secular change).
July	4.1746	4.1759
August	4.1730	4.1740
September	4.1720	4.1727
October	4.1723	4.1728
November	4.1746	4.1749
December	4.1724	4.1725
January	4.1710	4.1709
February	4.1722	4.1719
March	4.1738	4.1733
April	4.1727	4.1720
May	4.1745	4.1735
June	4.1731	4.1718
Mean	4.1730	4.1730

PART VI.

INVESTIGATION

OF THE

LUNAR INFLUENCE ON THE MAGNETIC HORIZONTAL FORCE.

INVESTIGATION

OF THE

INFLUENCE OF THE MOON ON THE MAGNETIC HORIZONTAL FORCE.

THE method pursued in the investigation of the lunar effect on the horizontal force is, in general, the same as that explained in Part III. of the discussions of the Girard College observations. The process may be briefly recapitulated as follows: Each horizontal force observation, after it had been corrected for the effect of difference from the standard temperature and for progressive change, the disturbed readings being omitted (as fully explained in Part IV.), was marked with its corresponding lunar hour; the observation nearest to the time of the moon's upper transit over the true meridian of the observatory was marked 0^h, that nearest to the lower transit was marked 12^h, and the observations between, for western and eastern hour angles of the moon, were marked with the proper lunar hour by interpolation. In the hourly series where thirteen observations are recorded in twelve lunar hours, that observation which is nearest midway between any two consecutive lunar hours was omitted. Each observation and reduced reading thus marked with its corresponding lunar hour was subtracted from the monthly normal belonging to its respective hour, and these differences were set down in tabular form, arranged according to lunar hours and keeping each monthly result separate for future combination. Let n = any normal belonging to any reduced reading r, the following tables contain the mean monthly values of the differences $n - r$; a positive sign, therefore, indicates greater force, a negative sign less force than the normal. It need hardly be repeated that in the original record of the horizontal force increasing numbers denote a decrease of the force. The greatest possible difference is 33, the number of scale divisions, which, according to the criterion, separates a disturbed from an undisturbed observation. For the formation of these differences which amount to more than 22,000, the manuscript tables of the reduced record were used: these tables have already been referred to in the preceding Part IV.

The units in which the differences $n - r$ are expressed are scale divisions, one division being equal 0.0000365 parts of the horizontal force, or equal to 0.000152 in absolute measure, the mean X being = 4.173 (in units of grains and feet).

The lunar effect on terrestrial magnetism being exceedingly minute, the process required for its elucidation is proportionally delicate; all the regular and irregular deviations arising from other sources must first be eliminated. In the method, as indicated above, the magnetic disturbances (as far as they could be recognized as such), the diurnal and annual solar variation, as well as the eleven (or ten) year inequality and secular change, are all eliminated, leaving numbers fitted for the lunar research.

The readings taken in the month of June, 1840, have not been used in this discussion (these had likewise been rejected in the two preceding parts), on account of the imperfect manner in which the allowance for the progressive change could only be made at that time. For the lunar hour 21 in July, 1840, the number of differences is so small that the mean had necessarily to be reduced; one-fourth of its amount was set down in the table. In January, February, and March, 1843, the observations were discontinued, excepting a single daily reading. These months, therefore, do not occur in the lunar discussion.

The number of observations used are distributed over the several months and years, as shown in the following table.

Table I.—Number of Observations for Lunar Discussion.

Month.	1840–1841.	1841–1842.	1842–1843.	1843–1844.	1844–1845.	Sum.
July	157	297	284	294	627	1659
August	235	295	318	313	622	1783
September	258	269	265	296	556	1644
October	255	281	257	*602	597	1992
November	245	279	297	603	564	1988
December	199	297	318	603	559	1976
January	179	298	- - - -	621	601	1699
February	238	250	- - - -	575	541	1604
March	260	297	- - - -	576	601	1734
April	262	271	286	586	575	1980
May	264	271	299	623	612	2069
June	212	295	309	579	522	1917
Sum	2764	3400	2633	6271	6977	22045

Table II.—Distribution of the Number of Observations according to Western and Eastern Hour Angles of the Moon.

Year.	Western hour angles.	Eastern hour angles.
1840–41	1371	1393
1841–42	1688	1712
1842–43	1320	1313
1843–44	3138	3133
1844–45	3499	3478
Sum	11016	11029

Tables III., IV., V., VI. and VII. contain the monthly and annual means of the lunar diurnal variation for the years 1840 to 1845. The numbers are expressed in scale divisions.

* Commencement of the hourly series.

TABLE III.—DIFFERENCES FROM THE MONTHLY NORMALS, 1840–41, WESTERN HOUR ANGLES OF THE MOON.

1840–41.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
July	+2	+1	+3	−9	+3	+7	−1	−17	+12	−4	−3	− 8
August	0	−4	+3	−5	+6	+2	−4	− 4	− 2	+2	+5	− 1
September	−2	0	+8	+1	0	−4	−5	+ 2	− 4	−7	+8	+ 4
October	−3	−1	−1	+1	0	−4	−4	+ 1	− 7	+7	+4	+10
November	−6	+3	0	−4	+1	−5	+4	− 4	− 4	+1	+6	− 1
December	−4	−3	+3	+4	+2	−7	−5	− 3	0	+3	−8	+ 9
January	+1	+3	−1	+8	−7	+3	−8	+ 1	− 9	−4	0	+ 2
February	+7	+5	0	+6	−3	+4	−2	0	− 4	0	−2	0
March	−4	+4	+1	0	+3	+3	−7	− 1	+ 8	+1	+3	− 2
April	0	+6	+1	−2	−2	−1	−1	+ 3	− 1	−2	+1	− 1
May	+3	−3	+1	−1	−8	+2	−1	− 1	− 6	0	+2	− 5
June	−1	−5	0	−3	+4	+4	+5	+ 8	− 1	+1	−1	− 8
Mean	−0.4	+0.5	+1.5	−0.3	−0.1	+0.3	−2.4	− 1.3	− 1.5	−0.2	+1.3	− 0.1

1840–41.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
July	+11	− 9	−5	+ 2	+6	0	− 2	− 5	+6	− 5	−4	− 2
August	+ 7	+ 6	+9	+ 1	+5	+2	+ 5	− 3	+5	−11	0	− 2
September	− 2	− 1	+2	+ 6	+5	+4	− 4	+ 1	−2	− 3	−1	− 2
October	−16	+14	−9	+ 4	−7	+3	−10	− 2	−1	+ 6	−3	+ 5
November	− 2	+ 1	−1	+ 4	−6	0	+ 1	− 1	+4	+ 6	+1	+ 5
December	+ 6	+ 9	+2	+10	−3	+2	− 6	−12	−3	− 6	+3	+ 5
January	− 2	− 4	+3	− 1	+1	−1	+ 4	− 2	−2	+ 1	+3	+ 7
February	− 5	+ 4	−4	− 7	−6	+5	+ 1	+ 2	+1	− 5	+3	+ 4
March	− 4	0	−5	+ 2	−1	+4	−10	+ 2	−2	− 2	+2	+ 2
April	− 1	− 3	+3	− 8	−3	−4	0	+ 3	−2	+ 2	+4	+ 2
May	+ 8	− 3	0	− 3	0	0	− 2	+ 8	+3	− 2	−2	+ 2
June	+ 8	− 4	+6	− 5	+7	−8	− 5	− 7	0	− 7	+1	−11
Mean	+ 1.0	+0.8	+0.1	+ 0.4	−0.2	+0.6	− 2.3	− 1.3	+0.6	− 2.1	+0.6	+ 1.2

Table IV.—Differences from the Monthly Normals, 1841–42, Western Hour Angles of the Moon.

1841–42.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
July	+4	+5	0	+8	−4	0	0	−2	0	−8	+2	0
August	−1	0	− 2	+2	+2	+3	+3	0	−5	0	+1	+ 5
September	−3	+8	+ 2	−1	0	−1	−5	−3	+1	0	−3	+10
October	+7	−1	+ 4	+4	0	+1	−1	+4	+1	−2	0	+ 1
November	0	+6	− 3	+1	−7	0	−3	−1	+6	+3	+1	+ 3
December	+8	−4	+12	−2	−1	−3	+2	−3	−2	−1	−3	0
January	−2	+8	+ 2	+2	−1	+7	0	+3	−2	+1	+5	0
February	−5	+1	− 1	−3	+4	+2	+4	−7	−5	+5	0	+ 2
March	+4	+3	+ 2	−1	+2	−1	+2	0	−1	−1	−2	− 3
April	0	0	+ 1	0	0	+4	+1	+3	+2	−1	0	+ 1
May	0	−2	+10	+1	+5	+4	+6	−4	+5	−7	−3	− 4
June	+1	0	+ 3	0	+4	−3	−1	−3	−5	−5	0	− 3
Mean	+1.1	+2.0	+ 2.5	+0.9	+0.3	+1.1	+0.7	−1.1	−0.4	−1.3	−0.2	+ 1.0

1841–42.	12h Low. cul.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
July	+3	−5	+3	+5	+1	−1	−8	−4	+1	−1	−1	+3
August	+1	+3	0	+2	+1	−5	−1	+3	−4	−3	−5	−1
September	+3	+2	+2	−6	+5	−1	−5	−2	+1	+4	−5	+6
October	+3	−1	−3	−5	−4	−3	+7	−3	−1	−3	−1	+1
November	−1	+4	+3	−6	−1	−5	+1	−2	0	−3	−4	−3
December	−1	0	−1	+2	−4	−3	−1	+1	+1	+1	+6	+1
January	+4	−2	−2	−4	−1	−5	−3	−3	+5	0	−3	+2
February	+7	+1	+1	−2	−3	0	−8	+6	−7	+1	+3	+2
March	−2	+3	−2	0	−6	−1	0	−2	−1	+2	−6	−1
April	+1	+1	+3	−3	+1	−1	−3	+2	−5	−3	−2	+1
May	0	−5	−3	−3	+4	−6	+6	−4	0	+3	+4	+2
June	−4	−2	−4	+2	0	+2	+2	+6	+6	−2	+2	+4
Mean	+1.2	−0.1	−0.2	−1.5	−0.6	−2.4	−1.1	−0.2	−0.3	−0.3	−1.0	+1.4

TABLE V.—DIFFERENCES FROM THE MONTHLY NORMALS, 1842–43, WESTERN HOUR ANGLES OF THE MOON.

1842–43.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
July	+3	—3	+2	— 1	+1	0	+7	+2	0	—4	+1	—8
August	+3	+1	—3	0	—1	—4	—3	+4	+2	+1	+1	+3
September	+3	—6	—1	+ 9	+4	—1	+7	+1	0	+2	—3	0
October	+2	—7	0	+ 1	—6	+3	—1	+3	—3	+5	—3	+2
November	+1	+3	—1	+ 2	+1	+1	0	—2	—2	—1	—4	+1
December	—2	—3	—6	+ 1	—5	0	—2	0	+1	—1	+5	0
January	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
February	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
March	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
April	—1	+2	0	+10	+4	+9	—1	+1	—3	—1	+1	—4
May	+3	—2	+2	+ 5	+3	+4	—1	+9	—1	+1	—6	+3
June	—6	+7	—4	— 1	0	+2	0	—1	—5	+4	—1	+3
Mean	+0.7	—0.9	—1.0	+ 2.9	+0.1	+1.6	+0.7	+1.9	—1.2	+0.7	—1.0	0.0

1842–43.	12h Low. cul.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
July	+1	+1	+4	—2	+4	—3	—4	—1	0	—2	+ 3	—1
August	—2	+1	+1	+2	+2	—4	+3	—5	—1	+2	+ 2	—2
September	+6	—1	—1	—8	—3	—1	—4	—1	—2	—7	+ 1	—6
October	—7	—3	+2	—1	+1	0	—1	+4	+4	—3	+11	+3
November	—2	—2	—1	—3	+1	+4	—1	+6	0	+1	+ 1	+2
December	+3	+3	+2	+1	+3	0	+1	+2	—2	+4	— 3	+3
January	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
February	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
March	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
April	+2	—2	+3	+2	0	—2	—5	0	—3	—2	+1	—1
May	0	+1	+4	0	+1	—4	—3	—1	—1	—1	—1	—2
June	0	+3	—1	+4	—2	+2	—3	—3	+2	+1	—7	+4
Mean	+0.1	+0.1	+1.4	—0.6	+0.8	—0.9	—1.9	+0.1	—0.3	—0.8	+0.9	0.0

TABLE VI.—DIFFERENCES FROM THE MONTHLY NORMALS, 1843–44, WESTERN HOUR ANGLES OF THE MOON.

1843–44.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
July	+6	+4	+2	+4	+5	—4	+3	+1	+1	—2	—5	—2
August	+2	+2	0	—1	+2	+1	—3	+4	0	—1	—2	—2
September	+1	—1	—3	+6	0	—2	—1	—4	—1	0	—2	0
October	—1	+4	+3	+5	+2	+3	+2	+1	0	—2	—1	—3
November	+1	+1	0	0	0	0	0	—2	0	0	+1	+1
December	+2	+1	+2	0	0	—2	—1	—1	—1	—2	+1	—1
January	+1	0	0	0	—1	—1	+1	+1	+1	0	—1	0
February	—1	—1	+1	+2	+1	—1	0	0	+3	0	—1	+2
March	—1	—3	+1	+1	+1	+1	+2	+2	0	+2	+1	+1
April	+2	+2	+3	+2	0	0	+1	+1	—1	—2	—3	—2
May	—2	—2	0	—1	—2	0	—1	—1	—2	—1	0	—1
June	0	—2	0	0	+2	+2	+2	+2	+1	—1	—2	0
Mean	+0.9	+0.4	+0.8	+1.5	+0.9	—0.3	+0.4	+0.3	+0.1	—0.8	—1.2	—0.6

1843–44.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
July	—2	—7	—2	—3	+3	—1	+4	—2	+1	—1	+2	+2
August	+4	0	+2	—1	—2	0	+2	+1	+1	—2	+4	0
September	+3	0	+3	+3	+8	+2	—6	—1	—6	—3	—2	0
October	—3	—4	—2	—1	—2	0	0	—1	—1	+1	—2	—1
November	+1	+2	+2	+2	+2	0	—1	—2	—1	—1	0	—1
December	0	+1	+1	0	+1	—1	—3	—4	—4	—3	—2	0
January	+1	+2	—1	—1	—1	—2	0	—1	+1	+2	+1	+2
February	+2	+1	+1	+2	+2	—3	—2	+1	—2	—1	—1	—2
March	+1	0	—1	0	0	0	—1	—1	—1	—1	+1	—3
April	—4	0	0	0	+2	0	0	+1	0	0	—1	0
May	0	0	—2	0	0	—2	—3	+1	+1	+1	+2	+1
June	0	+2	+1	+3	+2	0	—1	0	0	—1	—2	0
Mean	+0.3	—0.2	+0.2	+0.3	+1.3	—0.6	—0.9	—0.7	—0.9	—0.8	0.0	—0.2

Equal weight has been given to each monthly result in the formation of the annual mean.

TABLE VII.—DIFFERENCES FROM THE MONTHLY NORMALS, 1844–45, WESTERN HOUR ANGLES OF THE MOON.

1844–45.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
July	0	+1	+1	+1	0	+1	+2	+2	0	+1	0	−2
August	−3	+1	−1	0	+2	0	+1	+3	+1	−3	−2	+1
September	−2	0	−1	0	−2	+2	0	+3	+2	+2	+2	+4
October	0	+4	+5	+2	+3	+4	+2	0	0	+1	−3	0
November	−1	+3	+1	+2	+1	+3	+3	+3	+3	+3	+2	−1
December	−1	0	−1	0	−2	−3	−3	−2	−1	−1	+1	−1
January	+1	+2	+4	−2	−3	−4	−1	−3	0	−1	+4	+2
February	+1	+1	0	0	+1	+1	+1	+2	−1	−2	0	+4
March	+1	−3	−3	−3	0	0	+1	+1	0	+1	+1	0
April	−4	+2	+2	+2	0	+2	0	+2	−2	−2	−1	−1
May	+2	0	+2	+2	0	−2	0	−1	−2	−2	+1	0
June	−5	−4	−3	−1	0	+3	+1	+1	+1	0	−5	−4
Mean	−0.9	+0.6	+0.5	+0.3	0.0	+0.6	+0.6	+0.9	+0.1	−0.3	0.0	+0.2

1844–45.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
July	0	0	+1	0	−1	0	0	0	−2	−2	−2	−3
August	+3	+2	+3	−1	0	−2	0	0	−3	−3	0	−2
September	+2	+3	+1	+1	−1	−2	−3	−3	−4	−3	−4	−4
October	+1	+2	+1	+2	0	−2	−2	−4	−4	−5	0	−2
November	−1	−4	0	−2	0	0	−3	−2	−1	+1	−4	−3
December	+1	0	−1	+2	0	+2	+2	+1	0	+3	+2	0
January	+4	0	+2	−1	−5	−4	−4	−4	0	+2	+1	+2
February	+1	+2	+1	−1	−3	−1	−2	−1	−5	−1	−1	−1
March	+1	+2	−1	0	+4	+3	+2	+1	−3	−2	−1	−3
April	−2	+1	0	0	+1	+1	+3	−1	−3	−4	−3	−4
May	+1	−2	−2	−2	−2	0	−1	0	+1	−3	0	+2
June	+1	−1	+1	+2	−1	+4	+4	+3	+2	+2	0	−1
Mean	+1.0	+0.4	+0.5	0.0	−0.7	−0.1	−0.3	−0.8	−1.8	−1.2	−1.0	−1.6

TABLE VIII.—RECAPITULATION OF THE ANNUAL MEANS EXHIBITING THE LUNAR-DIURNAL VARIATION, FROM 22,045 OBSERVATIONS BETWEEN 1840 AND 1845, EXPRESSED IN SCALE DIVISIONS.

July to July.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
1840–41	−0.4	+0.5	+1.5	−0.3	−0.1	+0.3	−2.4	−1.3	−1.5	−0.2	+1.3	−0.1
1841–42	+1.1	+2.0	+2.5	+0.9	+0.3	+1.1	+0.7	−1.1	−0.4	−1.3	−0.2	+1.0
1842–43	+0.7	−0.9	−1.0	+2.9	+0.1	+1.6	+0.7	+1.9	−1.2	+0.7	−1.0	0.0
1843–44	+0.9	+0.4	+0.8	+1.5	+0.9	−0.3	+0.4	+0.3	+0.1	−0.8	−1.2	−0.6
1844–45	−0.9	+0.6	+0.5	+0.3	0.0	+0.6	+0.6	+0.9	+0.1	−0.3	0.0	+0.2
Mean	+0.3	+0.5	+0.9	+1.1	+0.2	+0.7	0.0	+0.1	−0.6	−0.4	−0.2	+0.1

July to July.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
1840–41	+1.0	+0.8	+0.1	+0.4	−0.2	+0.6	−2.3	−1.3	+0.6	−2.1	+0.6	+1.2
1841–42	+1.2	−0.1	−0.2	−1.5	−0.6	−2.4	−1.1	−0.2	−0.3	−0.3	−1.0	+1.4
1842–43	+0.1	+0.1	+1.4	−0.6	+0.8	−0.9	−1.9	+0.1	−0.3	−0.8	+0.9	0 0
1843–44	+0.3	−0.2	+0.2	+0.3	+1.3	−0.6	−0.9	−0.7	−0.9	−0.8	0.0	−0.2
1844–45	+1.0	+0.4	+0.5	0.0	−0.7	−0.1	−0.3	−0.8	−1.8	−1.2	−1.0	−1.6
Mean	+0.7	+0.2	+0.4	−0.3	+0.1	−0.7	−1.3	−0.6	−0.5	−1.0	−0.1	+0.2

If we give weight to the annual means according to the number of observations, they would be; one for the first and second year, three-fourths for the third year, one and three-fourths for the next year, and two for the last year: a general examination, however, shows that, owing to the disturbing effect of the progressive change, the monthly means are very nearly of equal value, derived either from the bi-hourly or the hourly series. It will also be shown in the sequel that the lunar diurnal variation is nearly the same in the summer and winter seasons; the means of Table V. and the final means of Table VIII. have therefore been adopted without reference to combinations or weights.

A comparison of the values of Table VIII. among themselves shows them to be very irregular, although derived from many thousand observations; a five year series of observations seems barely sufficient to exhibit a tolerably regular progression. In the following table two groups have been formed, one of results from three years, 1840 to 1843, comprising 8,797 observations, the other from the remaining two years comprising 13,248 observations. From these it appears that the lunar diurnal variation during these two periods exhibits the same general character.

LUNAR-DIURNAL VARIATION DURING THE PERIODS 1840–43 AND 1843–45.

Groups.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
1840–43	+0.5	+0.5	+1.0	+1.2	+0.1	+1.0	—0.3	—0.2	—1.0	—0.3	0.0	+0.3
1843–45	0.0	+0.5	+0.7	+0.9	+0.4	+0.3	+0.5	+0.6	+0.1	—0.6	—0.6	—0.2

Groups.	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
1840–43	+0.8	+0.3	+0.4	—0.6	0.0	—0.9	—1.8	—0.5	0.0	—1.2	+0.2	+0.9
1843–45	+0.7	+0.1	+0.4	+0.2	+0.3	—0.4	—0.6	—0.7	—1.3	—1.0	—0.5	—0.9

Before proceeding to the analysis of the final result of Table VIII. the separate results have been combined into summer and winter groups; the first group comprising the months from April to September, the second group the months from October to March.

Table IX. exhibits the lunar diurnal variation of the horizontal force during the summer and winter seasons.

TABLE IX.—LUNAR-DIURNAL VARIATION IN SUMMER.

(In scale divisions.)

Apr. to Sept.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
1840–41	+0.7	—0.9	+2.7	—3.2	+0.5	+1.7	—1.1	—1.5	—0.3	—1.7	+2.0	—3.2
1841–42	+0.2	+1.8	+2.3	+1.7	+1.2	+1.2	+0.7	—1.5	—0.3	—3.5	—0.5	+1.5
1842–43	+0.8	—0.2	—0.7	+3.6	+1.8	+1.7	+1.5	+2.7	—1.2	+0.5	—1.2	—0.5
1843–44	+1.5	+0.5	+0.3	+1.7	+1.2	—0.5	+0.2	+0.5	—0.3	—1.2	—2.3	—1.2
1844–45	—2.0	0.0	0.0	+0.7	0.0	+1.0	+0.7	+1.7	0.0	—0.7	—0.8	—0.3
Mean	+0.2	+0.2	+0.9	+0.9	+0.9	+1.0	+0.4	+0.4	—0.4	—1.3	—0.6	—0.7

	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
1840–41	+5.8	—2.3	+2.5	—1.2	+3.2	—1.0	—1.3	—0.5	+1.7	—4.3	—0.3	—2.2
1841–42	+0.7	—1.0	+0.2	—0.5	+2.0	—2.0	—1.5	+0.2	—0.2	—0.3	—1.2	+2.5
1842–43	+1.2	+0.5	+1.7	—0.3	+0.3	—2.0	—2.7	—1.8	—0.8	—1.5	—0.2	—1.3
1843–44	+0.2	—0.8	+0.3	+0.3	+2.2	—0.2	—0.7	0.0	—0.5	—1.0	+0.5	+0.5
1844–45	+0.8	+0.5	+0.7	0.0	—0.7	+0.2	+0.5	—0.2	—1.5	—2.2	—1.5	—2.0
Mean	+1.7	—0.6	+1.1	—0.3	+1.4	—1.0	—1.1	—0.5	—0.3	—1.9	—0.5	—0.5

LUNAR-DIURNAL VARIATION IN WINTER.

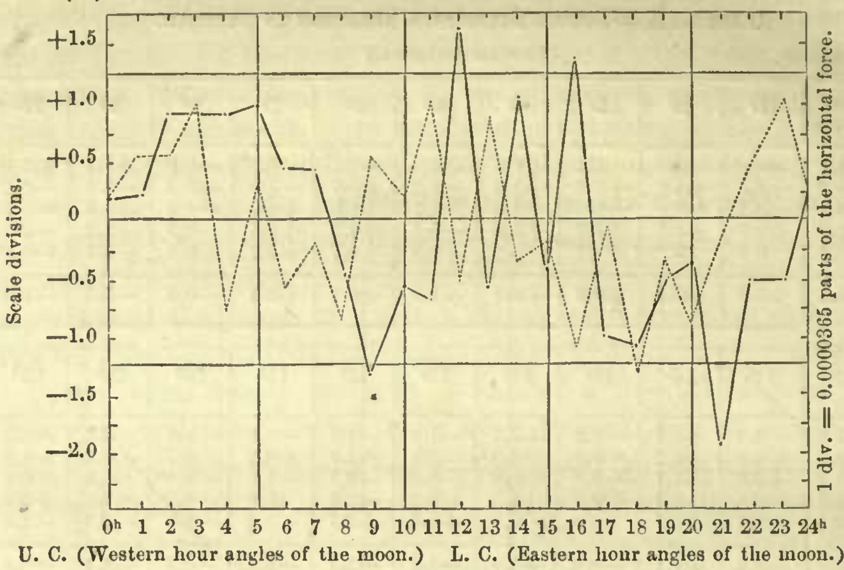
(In scale divisions.)

Oct. to Mar.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
1840–41	—1.5	+1.8	+0.3	+2.5	—0.7	—1.0	—3.7	—1.0	—2.7	+1.3	+0.5	+3.0
1841–42	+2.0	+2.2	+2.7	+0.2	—0.5	+1.0	+0.7	—0.7	—0.5	+0.8	+0.2	+0.5
1842–43	+0.3	—2.3	—2.3	+1.3	—3.3	1.3	—1.0	+0.3	—1.3	+1.0	—0.7	+1.0
1843–44	+0.2	+0.3	+1.2	+1.3	+0.5	+0.0	+0.7	+0.2	+0.5	—0.3	0.0	0.0
1844–45	+0.2	+1.2	+1.0	—0.2	0.0	+0.2	+0.5	+0.2	+0.2	+0.2	+0.8	+0.7
Mean	+0.2	+0.6	+0.6	+1.0	—0.8	+0.3	—0.6	—0.2	—0.8	+0.6	+0.2	+1.0

	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
1840–41	—3.8	+4.0	—2.3	+2.0	—3.7	+2.2	—3.3	—2.2	—0.5	0.0	+1.5	+4.7
1841–42	+1.7	+0.8	—0.7	—2.5	—3.2	—2.8	—0.7	—0.5	—0.5	—0.3	—0.8	+0.3
1842–43	—2.0	—0.7	+1.0	—1.0	+1.7	+1.3	—0.3	+4.0	+0.7	+0.7	+3.0	+2.3
1843–44	+0.3	+0.3	0.0	+0.3	+0.3	—1.0	—1.2	—1.3	—1.3	—0.5	—0.5	—0.8
1844–45	+1.2	+0.3	+0.3	0.0	—0.7	—0.3	—1.2	—1.5	—2.2	—0.3	—0.7	—1.2
Mean	—0.5	+0.9	—0.3	—0.2	—1.1	—0.1	—1.3	—0.3	—0.8	—0.1	+0.5	+1.1

The results are exhibited in the annexed diagram. The number of observations (about 11,000 for each group) is evidently too small to eliminate the greater irregularities.

(A.)—Lunar-Diurnal Variation of the Horizontal Force in Summer and Winter.



——— summer deflection.
------- winter deflection.

If there is any marked difference in the lunar diurnal variation in the summer and winter season, the summer range is slightly greater than the winter range; as to the epoch, there is no doubt that in winter the lunar maxima and minima are earlier than in summer. It is a remarkable fact that we have found the same features in the lunar effect on the declination, viz., a greater amplitude in summer and an earlier occurrence of the maxima and minima in winter; the amount of the shifting of the two curves appears to be nearly the same. From the ten year series of observations at Prague (1840–49) Mr. Karl Kreil found a larger lunar effect in the summer months than in the winter months.

Recurring to the final values of the lunar-diurnal variation of the horizontal force, as given in Table VIII., they can be represented by the usual Besselian form of periodic functions.

The angle θ counts from the moon's upper culmination westward at the rate of 15° to an hour; a + sign indicated greater, a — sign, less force than the average normal. The observed values are represented by the following expression:—

$$H_{☾} = -0.01 + 0.40 \sin(\theta + 13° 29') + 0.60 \sin(2\theta + 38° 43') + 0.155 \sin(3\theta + 244° 31').$$

The three coefficients are expressed in scale divisions; if expressed in parts of the horizontal force the equation may be written as follows: (M signifies millionth parts of the force.)

$$H_{☾} = -\overset{M}{0.36} + \overset{M}{14.60} \sin(\theta + 13°.5) + \overset{M}{21.90} \sin(2\theta + 38°.7) + \overset{M}{5.64} \sin(3\theta + 244°.5.)$$

If expressed in absolute measure and if n = number of hours after the upper culmination, it may be written

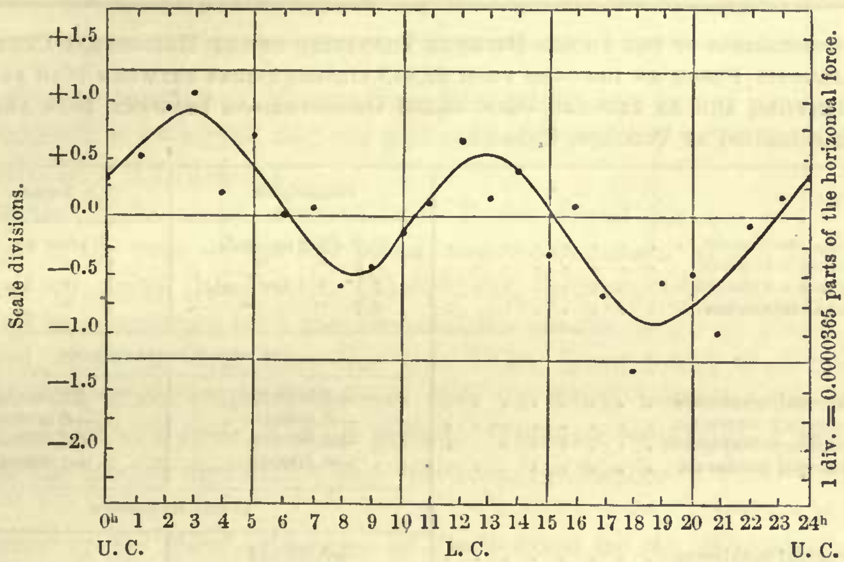
$$H_{☾} = -\overset{M}{1.5} + \overset{M}{61.0} \sin(15n + 13°.5) + \overset{M}{91.5} \sin(30n + 39°) + \overset{M}{23.6} \sin(45n + 244°.5.)$$

The curve is double-crested and is exhibited, together with the observed values, in the annexed diagram. It presents two maxima and two minima, which are found from the equation

$$\frac{d\,H}{d\,\theta} = 0 = +\,0.40 \cos(\theta + 13^\circ) + 1.20 \cos(2\,\theta + 39^\circ) + 0.45 \cos(3\,\theta + 245^\circ).$$

The lunar effect on the declination we have found likewise to present two maxima and two minima. (See Part III. of the discussion.)

(B.)—Lunar-Diurnal Variation of the Horizontal Force Observed and Computed.



We find: Principal maximum	$2^h\ 52^m$	after Upper Culmination;	$+ 0.87$	scale divisions.		
Secondary "	1 7	" Lower "	$+ 0.51$	"	"	
Principal minimum	6 41	" " "	$- 0.87$	"	"	
Secondary "	8 19	" Upper "	$- 0.45$	"	"	

The epoch of the horizontal force tide for the high values is nearly 2 hours after the culminations, and for the low values it is $7\frac{1}{2}$ hours after the same phases.

For Makerstoun, in Scotland, at General Sir Thomas M. Brisbane's observatory, in 1843–46, Mr. J. A. Broun found (Trans. Royal Society of Edinburgh, Vol. XIX. p. 11, 1849) the smaller maximum of the horizontal force 2 hours after upper culmination, the greater maximum $1\frac{1}{4}$ hours after the lower culmination, the smaller minimum 8 hours after the upper culmination, and the greater minimum 9 hours after the lower culmination.

At Prague all extremes appear from 2 to 3 hours later. Mr. Karl Kreil (Denkschriften of the Imperial Academy of Sciences, at Vienna, Vol. V. 1853), found from the ten year series at Prague (1840—49) maxima of horizontal force between four and five hours after the upper and lower culminations, the latter being the greater of the two; and minima between ten and eleven hours after the same epoch, that after the upper culmination being the greater of the two.

From the Toronto observations, continued for five years, Major-General Sabine deduced the formula (see Vol. III. of the Toronto Magnetical and Meteorological Observations, London, 1857).

$$\Delta_x = +\,0.05 + 0.215 \sin(a + 353^\circ.6) + 0.3324 \sin(2\,a + 13^\circ.5).$$

The coefficients are in decimals of scale divisions (1 div. = 0.000087) parts of the horizontal force); the angle a counts from the superior culmination, giving a curve of which the general features are in exact accordance with those deduced from the Philadelphia observations, viz: a principal maximum after Upper Culmination, followed by the secondary minimum; the secondary maximum after the Lower Culmination, followed by a principal minimum. The times and amount of these values are compared in the following Table X.

Table X.—Comparison of the Lunar-Diurnal Variation of the Horizontal Component of the Magnetic Force as deduced from 22,045 Observations between 1840 and 1845 at Philadelphia, and as deduced from 34,303 Observations between 1844 and 1848 (a five year series) at Toronto, Canada.

	Philadelphia.	Toronto.
Time of principal maximum	$2^h.9$ after up. cul.	3^h after up. cul.
" " secondary minimum	8.3 " "	9 " "
" " secondary maximum	1.1 " low. cul.	2 " low. cul.
" " principal minimum	6.7 " "	8 " "
	In parts of horizontal force.	
Amount of principal maximum	+0.000032	+0.000046
" " secondary minimum	—0.000016	—0.000010
" " secondary maximum	+0.000019	+0.000024
" " principal minimum	—0.000032	—0.000041
	In absolute measure.	
Amount of principal maximum	+0.000133	
" " secondary minimum	—0.000068	
" " secondary maximum	+0.000078	
" " principal minimum	—0.000133	

Probable error of any single representation of the Philadelphia values = $\pm$ $0^d.25$ = $\pm$ 0.000009 parts of the horizontal force = $\pm$ 0.000038 in absolute measure.

Investigation of the Horizontal Force in Reference to the Lunar Phases.—The following process of reduction has been adopted: After marking the days of the full and new moon, and also the days preceding and following, the daily means of the horizontal force readings were taken (already corrected for difference of temperature and progressive change.) In the place of any disturbed observation, the monthly normal, belonging to the respective hour, was substituted before taking the daily mean. All accidental omissions in the record of the hourly or bi-hourly series were supplied by the hourly normal of the month. The means thus obtained are independent of the solar diurnal variation. The monthly normal was next compared with each daily mean and the differences (normal minus mean) were tabulated.

A positive sign signifies a greater; a negative sign, a less force than the normal value. As the results deduced from a single year are yet too much affected by the incidental irregularities of the observations, the collective results from the five year series (1840–45) are herewith presented.

TABLE XI.—INFLUENCE OF THE LUNAR PHASES ON THE HORIZONTAL FORCE.

	Scale divisions.	Parts of the hor. force.	In absolute measure.
One day before full moon	—1.0	—0.000036	—0.00015
On the day of full moon	—1.5	—0.000055	—0.00023
One day after full moon	—0.2	—0.000007	—0.00003
One day before new moon	+0.0	+0.000000	+0.00000
On the day of new moon	+2.4	+0.000091	+0.00038
One day after new moon	+0.9	+0.000033	+0.00014
Difference for new-full moon	3.9	0.000146	0.00061

The average number of observations from which any one of the above six means were deduced, is over 800, and the probable error, in scale divisions, of any one of the results is $\pm$ 0.7 (nearly).

From the Makerstoun observations, Broun found for the years 1843–46, a minimum at the time of the full moon, and a maximum at the time of the new moon; Kreil, from the Prague observations, between 1843–46, found the same result, all in accordance with the Philadelphia results, as given above. It must be remarked, however, that after the year 1848, Kreil found that the signs were reversed and consequently it appears that the lunar influence on the horizontal force is subject to a cycle of short period. This last remark does not apply to the effect of the moon's declination and variations in distance.

Influence of the Moon's Changes of Declination on the Horizontal Force.—The method of investigation is precisely the same as that adopted for the phases. We find:—

TABLE XII.

	Scale divisions.	
One day before the greatest north declination	+0.8	Mean +1.1.
On the day of " " " "	+0.6	
One day after " " " "	+2.2	
Two days after " " " "	+0.9	
On the day of the moon's crossing the equator	—1.2	Probable error of any one result $\pm$0.9.
One day before the greatest south declination	—3.4	Mean —0.6.
On the day of " " " "	—0.9	
One day after " " " "	+0.9	
Two days after " " " "	+1.0	

It seems probable that the greatest effect takes place rather a day after than on the day of the moon's greatest declination. Taking means, as indicated in the above table, we find about the time of the maximum north declination an increase of horizontal force of 1.1 scale divisions (or 0.000040 parts of the horizontal force); at the time of the moon's crossing the equator the force is decreased 1.2 scale divisions (or 0.000044 parts of the horizontal force); the horizontal force also appears decreased about the time of the moon's greatest south declination; the amount is about half that of the other two cases, and is somewhat doubtful, from an apparently excessive value on the preceding day.

According to Broun, there is at Makerstoun a maximum horizontal force at the time of the moon's greatest north and south declination, with a minimum force at the time of her crossing the equator; in two cases, therefore, viz: for north declination and no declination, the Makerstoun and Philadelphia results agree; while in the third case they disagree or remain doubtful. Kreil's results, from the Prague observations, do not appear to me sufficiently decisive and regular to admit of comparison.

Influence of the Moon's Variation in Distance on the Horizontal Force.—By a process of reduction similar to that followed in the preceding investigation we find:—

TABLE XIII.

	Scale divisions.	s. d.
One day before perigee	—1.5	
On the day of "	—1.9	Mean —1.8.
One day after "	—2.0	
One day before apogee	+2.3	
On the day of "	+2.3	Mean —2.4.
One day after "	+2.7	

The probable error of any one result is about the same as in the preceding results (Tables XI. and XII.). The results for variation in the moon's distance are more consistent and satisfactory than those depending on the phases and declination changes. The lunar effect is to diminish the horizontal force by its 0.000066 part in perigee, and to increase it by its 0.000088 part when she is in apogee.

The Prague results are the same, viz: a greater horizontal force at and after the moon's apogee than at and after her perigee; a three years' series of observations at Milan, however, do not agree therewith.

In no branch of magnetic research would additional results from independent observations, particularly at stations widely apart, be more acceptable and valuable than in the study of the lunar effect in its various manifestations.

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