SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
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DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

THIRD SECTION,

COMPRISING PARTS VII, VIII, AND IX. VERTICAL FORCE.

INVESTIGATION OF THE ELEVEN (OR TEN) YEAR PERIOD AND OF THE DISTURBANCES OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE, AND APPENDIX ON THE MAGNETIC EFFECT OF THE AURORA BOREALIS; WITH AN INVESTIGATION OF THE SOLAR DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE VERTICAL FORCE; AND OF THE LUNAR EFFECT ON THE VERTICAL FORCE, THE INCLINATION, AND TOTAL FORCE.

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PART VII.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD, AND OF THE DISTURBANCES OF THE VERTICAL FORCE.

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ELEVEN (OR TEN) YEAR PERIOD, AND OF THE DISTURBANCES OF THE VERTICAL FORCE.

The observations of the vertical component of the magnetic force were commenced in June, 1840, and continued, with an exception in January, 1841, without interruption to the last of June, 1845. To keep up the continuity of the series, a daily reading was taken at $2^h\ 17^m$ P. M. during the months of January, February, and March, 1843. Up to October, 1843, the observations were bi-hourly, afterwards hourly.

Instruments.—From June, 1840, to the end of the year, the observations were made with a balance vertical force magnetometer of Lloyd's pattern. It was at first mounted in the eastern building of the College, but was removed to the observatory in the latter part of July. While in the College an increase of the readings corresponds to a decrease of vertical force; at the observatory increasing readings denote increasing force. The instrument was made by Robinson, of London; the magnet, the axis of which was mounted as nearly as possible transversely to the magnetic meridian, was 12 inches in length, having at its ends cross wires set in copper rings. For a full description see Dr. Lloyd's account of the Magnetical Observatory of Dublin, and the preface in volume I, of the record of the Philadelphia observations.

In January, 1841, the Lloyd instrument was replaced by a reflecting vertical force magnetometer, made at my suggestion by Mr. J. Saxton. The bar of this instrument was two feet and one inch in length, two inches wide in the middle, one and a half near the ends, tapering to nothing at the ends, and a quarter of an inch thick. The magnet was of steel and hardened as perfectly as the maker could effect. By means of a ball moving on a fine screw, its equilibrium could be changed. The mirror projected outside the box, and the motion of the bar was observed by means of a telescope. At the top of the box was a piece of plate glass through which a thermometer (of Francis' make) could be read. For further particulars see p. vii, of the preface to volume I of the record. For some time (between three and four months) after being put up, the bar lost considerably of its magnetic force, and after being in use four months, a movement of the adjusting ball upon the screw was required for placing the readings again near the middle of the scale. By this adjustment, the sensibility of the apparatus was not interfered with.

The value of a scale division of the Lloyd instrument, expressed in parts of the vertical force, was carefully determined and found to be =0.0000165, both in the

College building and at the observatory. This value being known, I considered that the value of the scale of the new reflecting magnetometer could best be ascertained by comparison with the former. The result of this, continued at intervals, was, that two divisions of the new scale were equivalent to one of the old, or that a change of one division of the reflecting instrument corresponded to a change of vertical force of 0.000033 parts. This was after the instrument had been finally adjusted.

The only disadvantage in the new instrument was the large effect of changes of temperature upon it; by direct observations it was found that a change of 1° (F.) of temperature produced a corresponding change of 13.5 ± 0.25 scale readings, whereas in the Lloyd instrument the corresponding change was but 3.12 scale divisions. We have accordingly for the Lloyd instrument q=0.0000515, and for the reflecting instrument q=0.000446. The values actually used in the reduction of the observed reading to a standard temperature will be seen further on.

The importance of ascertaining the most correct and suitable coefficients of temperature for the two series of observations, demands a more detailed statement and elaborate discussion of the observations themselves independently of the special trials. Experience has shown that the value for q deduced from the differential intensity observations themselves, with the magnet subject generally to gradual and small changes of temperature, is smaller by a considerable fraction than the value found by direct and special observation during which the temperature changes are necessarily more violent. There is no doubt that in the reduction to a standard temperature that value of q should be used which was obtained while the magnet was under its ordinary influences and condition. The same view is taken by General Sabine, and was also carried out in the discussion of the horizontal component of the magnetic force; for which see the preceding paper (Part IV).

Determination of the Effect of a Change of Temperature on the Readings of the Vertical Force.

(A.) Results of special observations made for determining the temperature coefficient. The correction for temperature of the Lloyd vertical force magnetometer was ascertained by the usual method of vibrating the bar when suspended horizontally, and when alternately heated and cooled artificially. The thermometer was placed with its ball near the axis of the magnet. The changes of the horizontal force magnetometer, while these experiments were going on, were noted and allowed for.

Date. Feb'y, 1841.	Time of 10 oscillations.	Temp. (F.)	Readings of Horz'l force.	Temp. (F.)
9th	87ˢ.950	37°.2	1128.8	25°.6
"	87.900	41.0	1079.3	36.5
"	88.117	94.6	1139.5	36.1
Result	87.990	39.1		
"	88.117	94.6		

hence $q = \frac{2}{t'-t} \cdot \frac{\tau'^2 - \tau^2}{\tau'^2 + \tau^2} = 0.0000520$

which is equivalent to 3.15 scale divisions; in the first reduction of the record 3.12 was used.

Before putting the reflecting vertical force magnetometer in its place in January, 1841, observations were made for its correction for temperature by means of deflections; the result, however, was not satisfactory, owing to the small difference in the deflections at high and low temperatures, and the necessity of keeping the bar at a proper distance from the declinometer to prevent the possibility of a permanent change of magnetism. The weight of the mirror and other fixtures of the bar rendered the method of horizontal oscillations impracticable without their removal, and it was finally decided to determine the value of q by means of a subsidiary instrument kept at a uniform temperature in a separate building, while the vertical force instrument at the observatory was subject to considerable fluctuations of temperature. The subsidiary instrument consisted of a small dipping needle mounted on a knife edge, and rendered horizontal by weighting it. The indications, however, did not prove very satisfactory; 14 scale divisions were indicated as the correction for 1° change in temperature. Subsequently an inclinometer, according to Prof. Lloyd's plan, was mounted as a subsidiary instrument, and observed twice a day with the vertical force instrument at the observatory. The mean values, expressed in scale divisions, thus found between February, 1843, and January, 1844, are as follows:—

13.3 14.3 14.4 12.3 12.2 13.1 and 15.4.

Average value 13.56 ± 0.25. In the first reduction the value 13.5 was used.

(B.) Investigation of the temperature coefficient from the regular series of observations. We will first examine the principal series observed between 1841 (February), and 1845 (June), with the reflecting magnetometer. In February, March, April, and May, 1841, the readings gradually increased and approached the end of the scale, requiring a readjustment of the instrument after May 22. It was supposed that —529 scale divisions would be an approximate correction for referring the observations to the indications of the scale subsequent to May 22, the uninterrupted series of observations commencing with June 1, 1841. The following table contains the *uncorrected* monthly means of the vertical force magnetometer together with the observed mean monthly temperature taken directly from the record. The tabular means for January, February, and March, 1843, when the instrument was read only once a day (at $2^h\ 17^m$ P. M.), were obtained as follows: The difference between the daily mean and the mean at $2^h\ 17^m$ P. M. was ascertained for each month, from the records of the preceding year (1842) and the following year (1844). The mean correction to the average reading at $2^h\ 17^m$ P. M. to refer the same to the mean of the day and month is +18.6, +14.4, and +11.2 scale divisions for the months of January, February, and March, respectively. These corrections have been applied.

TABLE I.—UNCORRECTED MONTHLY MEANS OF VERTICAL FORCE MAGNETOMETER AND CORRESPONDING MEAN MONTHLY TEMPERATURES.

1841.			July . . .	$792^{d}.9$	$77^{\circ}.5$	1844.		
February . .	$1059^{d}.0$	$35^{\circ}.58$	August . .	775.9	75.4	January . .	$636^{d}.1$	$57^{\circ}.0$
March . . .	1236.2	43.96	September .	775.0	74.3	February . .	657.4	59.2
April . . .	1357.4	50.8	October . .	763.8	71.4	March . . .	769.5	67.2
May . . .	1445.6	56.2	November .	725.0	66.8	April . . .	750.1	65.4
June . . .	722.6	74.5	December .	672.8	62.5	May . . .	798.4	69.9
July . . .	810.3	77.6	1843.			June . . .	845.1	73.6
August . .	776.1	75.85	January . .	681.9	64.0	July . . .	913.2	77.2
September .	702.5	71.1	February . .	615.8	57.6	August . .	877.2	75.4
October . .	448.7	54.5	March . . .	555.5	53.8	September .	860.7	71.7
November .	339.8	47.8	April . . .	762.4	71.3	October . .	834.7	71.2
December .	534.1	59.6	May . . .	734.1	70.4	November .	748.2	63.9
1842.			June . . .	784.6	75.9	December .	695.6	60.8
January . .	679.4	67.7	July . . .	800.5	77.4	1845.		
February . .	695.5	65.0	August . .	817.1	77.8	January . .	712.6	62.2
March . . .	668.9	66.8	September .	797.9	74.3	February . .	650.1	56.3
April . . .	671.2	67.4	October . .	749.0	69.6	March . . .	775.7	60.2
May . . .	693.9	69.7	November .	728.0	64.7	April . . .	721.1	65.7
June . . .	718.0	71.9	December .	705.9	62.5	May . . .	688.8	64.1
						June . . .	811.1	74.2

Combining the preceding values by months, we obtain a complete series extending over four years. The first five months are necessarily omitted; the break in the series occurred between the 4th and 5th month.

Uncorrected readings of the vertical force magnetometer. One division of scale =0.000033 parts of the vertical force.

Month.	1841–42.	1842–43.	1843–44.	1844–45.	1841–45.
July	810.3	792.9	800.5	913.2	829.2
August	776.1	775.9	817.1	877.2	811.6
September	702.5	775.0	797.9	860.7	784.0
October	448.7	763.8	749.0	834.7	699.1
November	339.8	725.0	728.0	748.2	635.2
December	534.1	672.8	705.9	695.6	652.1
January	679.4	681.9	636.1	712.6	677.5
February	695.5	615.8	657.4	650.1	654.7
March	668.9	555.5	769.5	675.7	667.4
April	671.2	762.4	750.1	721.1	726.2
May	693.9	734.1	798.4	688.8	728.8
June	718.0	784.6	845.1	811.1	789.6
			Mean		721.3

Corresponding readings of the thermometer (F.).

Month.	1841–42.	1842–43.	1843–44.	1844–45.	1841–45.
July	$77^{\circ}.6$	$77^{\circ}.5$	$77^{\circ}.4$	$77^{\circ}.2$	$77^{\circ}.42$
August	75.85	75.4	77.8	75.4	76.11
September	71.1	74.3	74.3	71.7	72.85
October	54.5	71.4	69.6	71.2	66.68
November	47.8	66.8	64.7	63.9	60.80
December	59.6	62.5	62.5	60.8	61.35
January	67.7	64.0	57.0	62.2	62.72
February	65.0	57.6	59.2	56.3	59.53
March	66.8	53.8	67.2	60.2	62.00
April	67.4	71.3	65.4	65.7	67.45
May	69.7	70.4	69.9	64.1	68.52
June	71.9	75.9	73.6	74.2	73.90
			Mean		67.45

The last column contains the mean readings. They may be represented by the equation:—

$$V = V_m + \Delta e x + \Delta t y$$

where $x =$ monthly amount of loss of magnetism and effect of secular change.
$y =$ change in magnetometer reading for a change of temperature of 1° F.
$\Delta e =$ epoch—middle epoch. The middle epoch is January 1st.
$\Delta t =$ temperature—mean temperature.
$V_m =$ mean reading of the vertical force magnetometer.
$V =$ any of the monthly means to be represented.

From the 12 conditional equations, we form the normal equations

$$-\ 828.90 = +\ 143.000\ x - 85.335\ y$$
$$+\ 4685.73 = -\ 85.335\ x + 443.120\ y$$

whence $x = +\ 0.577$, the monthly change, equal to nearly 7 scale divisions for each year.

And $y = +\ 10.68$ scale divisions, the correction for temperature for 1° F. This is not quite three-fourths of the value found by direct measure.

Second determination of the temperature coefficient by means of alternate combinations by seasons.

The mean values for each season have been directly formed from table No. 1. The value in June, 1845, is necessarily omitted.

				Alternate means.		Differences.		Temp. coefficient.
1841	June to November	633d.3	66°.89					
1841–2	December to May	657.2	66.03	695d.8	69°.88	+38d.6	+3°.85	10d.0
1842	June to November	758.4	72.88	663.8	64.65	—94.6	—8.23	11.5
1842–3	December to May	670.4	63.27	769.0	73.08	+98.6	+9.81	10.1
1843	June to November	779.5	73.28	695.0	63.40	—84.5	—9.88	8.6
1843–4	December to May	719.6	63.53	813.0	72.72	+93.4	+9.19	10.2
1844	June to November	846.5	72.17	705.1	62.54	—141.4	—9.63	14.7
1844–5	December to May	690.6	61.55					
					Mean			10.85
					By preceding method . .			10.68
					Mean, adopted			10.77

We have for the reflecting magnetometer $k = 0.000033$ $\frac{q}{k} = 10.77$, hence $q = 0.000355$. For comparison we have the corresponding values at Toronto $\frac{q}{k} = 1.80$ and $q = 0.000113$.

The scale value k at Toronto is 0.0000628, nearly twice as large as at Philadelphia. The comparatively large value for q at Philadelphia is most probably due to the large size of the bar which prevents a thorough hardening, a circumstance which undoubtedly also contributes to the difference exhibited by the resulting value of q as found by the direct and indirect methods.

The magnitude of the temperature coefficient requires that the standard temperature should be the mean temperature at all the readings. The average temperature between February, 1841, and June, 1845, is 66°.0, which has been adapted as the

standard temperatare to which all the vertical force readings, taken with the reflecting magnetometer, have been referred.

A close examination of the record of the Lloyd balance magnetometer, which was used in June and July, 1840, in the College, and afterwards at the observatory during five months, proved that in point of accuracy it would not compete with the reflecting magnetometer mounted in January, 1841, and continued in use for four years and a half. Owing to some imperfection in the first named instrument, its indications were very unsteady, and at times fitfully changeable; thus in September, October, and December, there are differences in the daily means (deduced from twelve readings and referred to 32° Fahrenheit) of adjacent days of more than 200 scale divisions, and in one instance (October 19–20) amounting even to 256 divisions. In August there is a change of 389 scale divisions in three consecutive days, and in October (17th to the 20th) one of 477 divisions in the means during the same interval. There is besides a large progressive change, showing that the instrument was in a very unstable equilibrium; this change amounted in the first month to over 300 scale divisions. An attempt was also made to deduce a temperature coefficient by comparing mean daily readings of short and specially selected periods of a few days each, with average high and low temperatures, but it failed for want of sufficient uniformity in the indications of the instrument. In such a series the disturbed indications could not be recognized and separated from the regular readings. It was finally concluded to make no use of the observations prior to January, 1841.

Reduction of the Observations, between February, 1841, *and June*, 1845, *to a uniform Temperature.*—A table has been constructed, with the observed temperature as the argument, giving the reduction for difference of temperature from the normal temperature (66° Fahr.); by means of this table each observation has been referred to its corresponding value as the standard temperature. Table No. 2 contains the monthly mean readings for each observing hour; the time is local time, and reckoned from midnight to midnight to 24 hours. The tenths in the record have been omitted, as of no special value, since an error in the recorded temperature of only 0°.1 affects the magnetometer reading by more than a scale division. An increase of scale readings corresponds to a decrease of vertical force, and one division equals 0.000033 parts of the force. Accidental irregularities in the record are especially referred to in foot notes.

The tabular values are directly taken from the manuscript tables containing the single reduced readings and their monthly means.

In the present state of our knowledge regarding the occurrence of the disturbances it is not safe to make any interpolations in the magnetometer record in case of an accidental omission; a rule which has been strictly adhered to.

TABLE II.—RECORD OF THE MONTHLY MEANS OF THE VERTICAL FORCE MAGNETOMETER READINGS FOR EACH OBSERVING HOUR, AND REDUCED TO UNIFORM TEMPERATURE OF 66° FAH.

1841.	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	+17m
February	861	861	860	857	851	846	845	849	847	878	872	867	
March	956	954	949	943	936	933	931	928	941	953	957	957	
April	1004	997	994	988	982	982	981	982	989	997	1006	1009	
May	1033	1031	1030	1021	1012	1009	1008	1008	1012	1022	1037	1043	
June	641	634	631	621	618	618	616	620	627	641	652	653	
July	704	698	686	675	669	666	667	700	684	695	708	707	
August	684	680	673	665	664	655	653	659	663	672	686	690	
September	665	660	657	651	642	633	632	632	637	646	656	662	
October	583	582	578	572	562	559	561	561	569	577	581	582	
November	540	543	540	544	531	528	526	527	532	535	538	540	
December	602	595	595	599	593	598	605	600	606	613	613	613	

Notes to the above table:—February 25th, 6h 17m, temperature interpolated, 28°. March 2d, 0h 17m, reading 32 minutes late, $t=46°.8$. March 11th, 22h 17m, reading 49m late, $t=41°.7$. March 27th, 20h 17m, reading 40m late, $t=64°.8$. March 29th, 22h 17m, reading 43m late, $t=50°.5$. April 9th, 16h 17m, reading 25m late, $r=920$, $t=57°.6$. April 30th, 0h 17m, reading 59m late, $r=805$, $t=49°.3$. May 22d, 14h 17m, observations discontinued. Between this date and June 1st the instrument was readjusted, the corrections required to make the readings of the first four months comparable with the continuous series following will be investigated further on. June 29th, 22h 17m, reading 38m late, temperature 81°.8, interpolated. July 22d, 16h 17m, temperature 84°.5 interpolated. August 23d, 24th, seven observations were omitted between 20h 17m and 8h 17m on account of the magnet being fixed by a spider's line which was found attached to the mirror. August 24th, 14h 17m, observation rejected, the sun shining on the box. October 4th, 16h 17m, sun shining on the needle; October 13th, 22h 17m, observation 7m late; October 28th, 22h 17m, observation 67m late. In December the variations of temperature are unusually large; they seem to demand a greater value of the temperature coefficient. December 8th, 16h 17m, observation 8m late; December 20th, 4h 17m, observation 9m late; December 30th, 18h 17m, temperature 69°.0, interpolated.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1842.	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	+17½m
January	651	651	663	659	657	651	661	648	660	679	674	674	
February	705	704	714	714	707	698	704	696	699	714	707	701	
March	661	656	662	664	663	662	655	649	655	667	673	666	
April	666	655	659	655	653	654	649	643	645	655	667	670	
May	662	665	657	649	646	647	652	644	647	654	664	665	
June	674	669	663	658	648	643	639	640	636	652	665	663	
July	690	685	675	667	663	656	652	651	656	669	681	684	
August	689	687	685	682	675	668	655	655	665	677	684	686	
September	698	692	696	689	686	677	671	671	673	679	690	698	
October	707	699	703	712	696	709	708	707	707	709	711	706	
November	718	710	723	725	713	715	713	713	716	711	718	718	
December	708	708	714	709	716	711	709	707	705	710	713	713	

Notes to above table:—February 3d, 14h 17½m, the temperature 73°.5 is interpolated. May 9th, 10h 17½m, the temperature 59°.6 is interpolated. June 6th, 0h 17½m, 2h 17½m, and 18h 17½m, the temperatures 70°.4, 71°.0, and 74°.4 respectively, were interpolated. August 3d, 12h 17½m; 5th, 22h 17½m; 6th, 10h 17½m, and 31st, 14h 17½m, the temperatures 69°.0, 73°.7, 76°.0, and 67°.6 respectively, were interpolated. September 1st, 22h 17½m, the temperature 77°.0 is interpolated. October 8th, 2h 17½m; 21st, 10h 17½m, and 28th, 6h 17½m, the temperatures 66°.1, 68°.1, and 70°.8 respectively, were interpolated. November 3d, 14h 17½m; and 16th, 6h 17½m, the observations are 6m and 7m late.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1843.	0^h	2	4	6	8	10	Noon.	14 $+17^m$	16	18	20	22^h	$+23\frac{1}{2}^m$
January								685					
February								692					
March								676					
April	715	713	713	712	708	702	705	687	697	696	706	709	
May	698	697	695	690	683	680	677	666	678	679	697	690	
June	696	691	693	690	677	669	664	660	659	671	681	690	
July	692	693	693	689	681	673	664	660	660	667	678	686	
August	702	703	707	705	695	682	670	672	672	682	694	698	
September	722	720	721	716	707	705	694	693	694	701	710	708	
October	714	708	714	717	704	703	702	703	704	714	719	714	
November	740	737	744	744	743	740	734	735	746	750	748	745	
December	749	737	740	740	739	728	724	738	755	759	762	754	

Additional odd hours observed.

1843.	1^h	3	5	7	9	11	13	15	17	19	21	23^h	$+23\frac{1}{2}^m$
October	710	709	718	712	704	700	701	702	709	717	715	717	
November	737	740	745	747	742	735	731	741	750	751	746	747	
December	744	738	742	743	738	720	726	747	755	763	756	757	

Notes to the above table:—January 4th, February 1st, and March 24th, observations 7^m, $7\frac{1}{2}^m$, and 20^m late, respectively. In April seven readings were supplied by the observers, also one in May and one in June. July 14th, 0^h $23\frac{1}{2}^m$, observation 6^m late. August 10th, 16^h $23\frac{1}{2}^m$, observation supplied by observer. August 29th, 0^h $23\frac{1}{2}^m$, observation 12^m late. September 20th, 0^h $23\frac{1}{2}^m$, temperature supplied by observer. November, six readings supplied by observers. December 1st, 4^h $23\frac{1}{2}^m$; December 9th, 1^h $23\frac{1}{2}^m$; December 12th, 21^h $23\frac{1}{2}^m$, and December 22d, 5^h $23\frac{1}{2}^m$, observations 5^m, 6^m, 15^m, and 8^m late, respectively. December 19th, 2^h $23\frac{1}{2}^m$, a printing error of 200 scale divisions was corrected. December 30th, 9^h $23\frac{1}{2}^m$, reading supplied by observer.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1844.	0^h	1	2	3	4	5	6	7	8	9	10	11^h	$+23\frac{1}{2}^m$
January	735	733	731	731	733	736	733	733	731	730	726	717	
February	736	731	729	730	733	732	733	734	727	725	727	720	
March	763	758	758	759	760	762	763	760	755	759	761	762	
April	766	765	763	765	766	765	763	759	752	751	746	740	
May	772	769	766	768	767	764	760	754	749	747	747	744	
June	778	776	772	772	771	768	765	760	752	750	749	747	
July	809	807	803	802	801	798	794	789	780	779	778	777	
August	794	792	788	787	785	783	780	774	768	765	761	759	
September	815	813	811	808	809	807	805	802	796	793	788	783	
October	779	776	773	776	778	780	781	782	775	774	775	771	
November	775	771	768	769	772	772	771	773	768	772	772	767	
December	756	752	753	754	756	757	756	760	752	752	754	749	

1844.	Noon.	13	14	15	16	17	18	19	20	21	22	23^h	$+23\frac{1}{2}^m$
January	717	713	717	724	732	740	743	745	747	746	744	745	
February	716	719	720	723	731	735	742	743	743	739	738	737	
March	758	751	752	755	752	751	751	749	750	751	759	762	
April	739	735	738	744	750	754	758	764	764	762	765	766	
May	744	740	740	744	746	748	753	762	765	766	768	772	
June	746	745	748	752	754	757	765	772	775	774	775	778	
July	776	775	772	778	780	787	795	801	804	804	806	809	
August	756	756	752	765	767	769	777	787	788	788	790	793	
September	779	777	766	791	790	793	804	810	809	810	812	815	
October	769	767	771	778	787	789	792	790	786	786	782	782	
November	766	763	761	758	774	779	777	778	774	770	769	772	
December	745	739	740	735	750	756	754	753	753	750	751	754	

Notes to preceding table:—January 2d, $10^h\ 23\frac{1}{2}^m$, temperature observation 30^m late. January 8th, $10^h\ 23\frac{1}{2}^m$, instrument disturbed. January 15th, $3^h\ 23\frac{1}{2}^m$, temperature 56°.3 interpolated. February 6th, $4^h\ 23\frac{1}{2}^m$, and 13th, $9^h\ 23\frac{1}{2}^m$, temperature observation 15^m and 20^m late, respectively. April 11th, $0^h\ 23\frac{1}{2}^m$ and $1^h\ 23\frac{1}{2}^m$, readings supplied by observer. July 13th, $12^h\ 23\frac{1}{2}^m$, observation 36^m late. August 26th and 27th, thirteen readings supplied by observers. October 1st, $22^h\ 23\frac{1}{2}^m$, observation 8^m late.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH

1845.	0^h	1	2	3	4	5	6	7	8	9	10	11^h	$+23\frac{1}{2}^m$
January	754	748	749	751	752	756	762	767	760	758	753	751	
February	761	756	753	757	759	760	763	764	756	756	752	749	
March	749	743	739	742	745	744	745	743	735	733	732	729	
April	732	727	726	728	729	727	725	724	719	718	718	717	
May	722	720	718	721	718	717	715	711	707	704	702	701	
June	733	731	729	729	729	727	726	722	717	715	715	711	
1845.	**Noon.**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23^h**	**$+23\frac{1}{2}^m$**
January	752	748	749	739	754	759	756	753	748	746	746	753	
February	747	741	740	739	753	759	761	761	758	753	752	756	
March	730	729	713	731	737	741	745	746	740	740	739	742	
April	716	713	715	724	726	723	732	737	735	729	726	728	
May	699	698	680	700	704	704	712	719	719	717	719	720	
June	711	711	711	714	713	715	722	730	733	731	731	732	

Notes to above table:—April 27th, $4^h\ 23\frac{1}{2}^m$, reading supplied by observer. April 14th, $2^h\ 23\frac{1}{2}^m$, observation 12^m late. April 22d, 23d, and 28th, $14^h\ 23\frac{1}{2}^m$; also April 22d, $15^h\ 23\frac{1}{2}^m$, readings supplied by observer. April 22d, $16^h\ 23\frac{1}{2}^m$, temperature supplied by observer. May 2d, $14^h\ 23\frac{1}{2}^m$, reading supplied by observer. May 12th, $4^h\ 23\frac{1}{2}^m$, 9^m late. June 6th, $5^h\ 23\frac{1}{2}^m$, and 28th, $1^h\ 23\frac{1}{2}^m$, observations 13^m and 9^m late, respectively. June 12th, $0^h\ 23\frac{1}{2}^m$ and $1^h\ 23\frac{1}{2}^m$, readings supplied by observer.

TABLE III.—MEAN MONTHLY READINGS OF THE VERTICAL FORCE REDUCED TO THE TEMPERATURE OF 66° FAH.

	1841.	1842.	1843.	1844.	1845.
January		661	699	733	752
February	858	705	702	731	754
March	945	661	684	757	738
April	993	656	705	756	725
May	1022	654	686	756	710
June	631	654	678	762	722
July	688	669	678	792	
August	670	676	690	776	
September	648	685	708	799	
October	572	706	710	779	
November	535	716	742	770	
December	603	710	744	751	
Mean		679	702	763	

The monthly mean for January, February, and March, 1843, was obtained by adding 14, 10, and 8 divisions to the readings at $14^h\ 7^m$ respectively; these corrections were found by comparisons in 1842 and 1844.

Corrections for progressive and irregular changes.—The difficulty of fully eliminating all effects of changes of temperature, and adjustment, particularly during the first year (1841), demanded the application of a secondary process analogous to that used in the reduction of the horizontal force for progressive change. The progressive change in the readings of the vertical force is less decided and more fluctuating than in the horizontal force. Half monthly means, and in special cases, means of even less periods of time, have been taken and were compared with the monthly mean, the differences were applied either progressively (increasing or diminishing) or as constants, as the case seemed to demand.

Seventeen months required no such correction, and in many months it was applied very sparingly.

The process leaves the diurnal variation, relatively, undisturbed, and prepares the series for the application of Peirce's Criterion for the recognition of the disturbances. The individual figures thus corrected were inserted in blue ink in the manuscript tables.

Recognition and separation of the larger disturbances.—Peirce's Criterion for the recognition of the disturbances was applied to the observations extending over four years, and commencing with July, 1841, in the following order: July 0^h, August 2^h, September 4^h, October 6^h, November 8^h, December 10^h, January (1842) 12^h, etc. The odd hours were selected from July, 1844, to the close of the series, thus July 1^h, August 3^h, September 5^h, etc. The following limits of separation, in scale divisions, have been found for each year:—

July,	1841 —	June,	1842,	limit,	52
"	1842 —	"	1843,	"	46
"	1843 —	"	1844,	"	40
"	1844 —	"	1845,	"	33
				Average limit,	43

As this limit would only separate 1 in every 34 observations, and would not furnish a sufficient number of disturbances to investigate their laws to advantage, it was necessary to contract the above limit, and 30 scale divisions were finally selected. There can be no doubt that the limiting number as found by the use of the criterion is too high, owing to the unavoidable presence of irregularities ascribable to imperfection in the corrections for temperature in some cases, and in others due to apparently fitful changes in the instrument. 30 scale divisions =0.00099 parts of the vertical force =0.0127[1] in absolute measure, adopted as limit of deviation of any observation from its corresponding mean monthly value for the same hour, will furnish an average value for the ratio of the number of disturbances to the whole number of observations. The ratio of a disturbance to the whole force is also nearly the same for the horizontal and vertical component.

All deviations over 30 divisions from the mean were marked, and a new mean was taken, the hourly observations were again compared with this new mean, and

[1] The vertical force, in absolute measure, is on the average, between 1841 and 1845, equal to 12.84 (English units), as stated in a subsequent number of this discussion.

the process was repeated, if necessary, until all deviations above 30 had been separated; the final hourly means for each month, thus found and known as the "normals," are given in the following tables.

TABLE IV.—BI-HOURLY NORMALS OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE IN 1841.

One division of the scale =0.000033 parts of the vertical force. Increasing numbers denote decrease of force. The observations are made 17^m after the full hours.

1841.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h
February	664	654	662	656	654	650	644	648	647	673	668	665
March	670	661	661	655	651	645	643	646	656	665	666	673
April	671	662	658	653	645	646	646	649	656	666	670	676
May	669	665	660	654	647	649	644	646	650	660	674	679
June	646	631	622	617	624	616	603	624	635	650	653	657
July	703	697	687	671	667	664	665	676	680	698	708	706
August	686	680	676	664	662	652	653	662	666	676	689	691
September	663	655	646	647	637	631	627	628	634	645	653	660
October	579	578	573	568	558	556	561	562	571	577	581	583
November	532	537	538	533	526	523	520	521	526	529	531	538
December	597	591	590	606	592	598	605	597	607	610	605	604

The normals for February, March, April, and May, have been diminished by 198, 278, 333, and 361 scale divisions respectively; the uncorrected monthly means are 856, 936, 991, and 1019, which can be exactly represented by the expression $r=966+54.4\ \Delta t-12.8\ \Delta t^2$, where $r=$ monthly reading and t, expressed in units of a month, counts from April 1 as the epoch. It shows that the monthly increase is uniformly retarded. The mean reading from the four succeeding months is 658, the corrections to February, March, April, and May, as applied, will produce the same mean.

The rapid change in the monthly means for some adjacent months makes a small correction necessary to the monthly means, viz: of plus one scale division to the February, March, and December means of the hours 0, 2, and 4, and to the September and October means at the hours 18, 20, and 22; of minus one scale division to the February, March, and December means at the hours 18, 20, and 22, and to the September and October means at the hours 0, 2, and 4. This small correction is included in the above normals.

TABLE IV.—*Continued.* BI-HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1842.

The observations are made $17\frac{1}{2}^m$ after the full hours.

1842.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h
January	(658)	642	656	649	656	653	664	650	663	675	670	663
February	706	701	713	721	699	698	709	692	698	712	723	710
March	655	[illegible]43	654	663	657	661	651	650	657	668	673	665
April	668	658	655	655	656	654	657	651	650	660	672	672
May	673	670	661	644	646	647	651	644	645	656	668	670
June	674	669	664	658	647	642	635	639	635	653	671	668
July	683	672	664	659	657	650	643	643	647	663	677	682
August	689	689	683	682	679	672	652	659	669	679	688	688
September	692	686	689	690	681	671	671	673	672	679	687	693
October	706	698	702	714	695	708	707	706	706	708	710	709
November	717	713	723	725	712	715	711	713	716	712	718	718
December	713	707	709	706	715	711	709	707	706	711	713	713

In January at 0^h the final mean is 637 which differs so much from the standard value at this hour that it was preferred to substitute the mean of the month (658) as a close approximation.

TABLE IV.—*Continued.* BI-HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1843.
The observations are made $23\frac{1}{2}^m$ after the full hours.

1843.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h
January								690				
February								697				
March								686				
April	715	712	717	716	708	702	709	700	696	696	710	709
May	698	699	695	690	682	680	677	668	677	685	691	695
June	698	691	693	687	677	668	663	658	659	669	681	689
July	691	692	692	686	679	672	662	658	659	666	677	685
August	703	703	708	706	698	683	669	671	672	682	695	699
September	721	719	721	716	707	706	693	692	692	703	714	710
October	714	707	712	717	706	703	702	703	704	714	719	714
November	742	745	745	744	742	737	735	731	746	749	749	746
December	752	733	740	740	740	729	727	743	758	767	764	754

Normals at additional odd hours.

1843.	1^h	3	5	7	9	11	13	15	17	19	21	23^h
October	710	713	714	714	704	701	700	701	709	717	715	717
November	740	743	744	748	739	729	731	738	749	751	747	748
December	744	742	742	743	740	720	729	749	760	763	757	757

TABLE IV.—*Continued.* HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1844.
The observations were made $23\frac{1}{2}^m$ after the full hours.

1844.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
January	733	739	730	728	732	733	732	732	730	725	720	713
February	734	729	725	726	729	728	729	730	723	724	722	717
March	768	761	763	760	764	762	765	762	758	761	762	764
April	776	773	771	765	766	765	759	755	749	749	744	740
May	772	769	766	768	767	764	760	754	749	747	747	744
June	776	772	767	768	767	764	760	755	747	745	744	744
July	816	811	804	806	805	802	798	793	784	783	782	780
August	794	790	786	781	783	777	776	769	763	760	756	754
September	816	815	813	812	811	810	809	805	798	795	790	785
October	775	771	769	773	776	779	780	780	774	773	773	770
November	775	772	768	769	772	771	770	773	768	772	772	767
December	754	753	754	755	757	756	755	758	749	751	750	746

1844.	Noon.	13	14	15	16	17	18	19	20	21	22	23^h
January	715	708	715	724	737	740	741	743	744	745	744	744
February	718	716	716	720	727	731	738	739	740	737	735	733
March	762	753	758	760	754	757	753	754	752	753	763	764
April	737	733	737	741	745	744	756	767	768	765	767	775
May	744	740	747	744	746	748	754	763	766	766	768	772
June	742	739	744	747	749	754	760	769	771	770	771	775
July	780	776	773	781	783	791	799	805	808	809	811	816
August	750	750	746	759	764	771	778	789	790	789	792	794
September	780	779	772	794	793	795	806	813	812	812	814	817
October	769	766	773	777	786	788	791	789	785	785	781	781
November	766	763	762	765	774	779	777	778	774	770	769	772
December	743	733	736	724	748	757	755	751	751	748	750	750

TABLE IV.—*Continued.* HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1845.

The observations are made $23\frac{1}{2}^m$ after the full hours.

1845.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
January	754	747	747	752	752	757	763	767	760	758	753	752
February	760	756	752	757	759	760	763	764	756	756	752	749
March	749	741	736	742	746	748	749	746	736	733	732	729
April	732	727	728	729	728	727	725	721	718	715	717	717
May	720	718	717	719	716	715	713	709	705	702	701	699
June	733	731	729	730	729	728	726	722	717	715	715	711
1845.	**Noon.**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23^h**
January	751	748	749	742	753	760	756	753	749	746	747	754
February	747	741	740	744	753	759	761	761	758	753	752	756
March	729	729	713	734	740	747	751	746	741	740	739	743
April	716	713	717	720	724	724	732	737	736	729	726	727
May	697	696	701	698	702	705	710	720	718	717	718	719
June	711	711	713	714	713	714	722	730	735	731	731	733

TABLE V.—NUMBER OF OBSERVATIONS AND LARGER DISTURBANCES IN EACH MONTH.

	1841.		1842.		1843.		1844.		1845.	
	Obs.	Dis.	Obs	Dis.	Obs.	Dis.	Obs.	Dis.	Obs.	Dis.
January . . .			300	76	26	5	646	81	648	17
February . . .	288	49	284	86	24	3	600	33	576	5
March	321	64	322	36	27	12	624	106	624	68
April	304	46	306	51	300	50	624	83	624	24
May	223	16	293	47	324	36	648	8	648	28
June	310	91	310	37	312	16	600	52	600	9
July	323	64	305	24	312	4	648	45		
August . . .	304	21	318	34	324	10	648	94		
September . .	307	40	303	57	312	20	600	14		
October . . .	308	28	310	12	624	25	648	20		
November . .	312	37	312	13	624	79	624	10		
December . .	323	84	319	15	624	65	624	47		
Sum . . .	3323	540	3682	488	3833	325	7534	593	3720	151
Ratio . . .	1 dis. in 6.2 obs.		1 dis. in 7.5 obs.		1 dis. in 11.8 obs.		1 dis. in 12.7 obs.		1 dis. in 24.6 obs.	

Total number of observations used, 22092
" " larger disturbances, 2097
Ratio of disturbances to observations, 1 to 10.5

Investigation of the eleven year (also called ten year) period in the inequality of the amplitude of the Diurnal Variation of the Vertical Force.—The preceding monthly means of the bi-hourly and hourly normals were rearranged in four groups of one year each, necessarily omitting the first five months; the annual means have for their mean epoch, January, as the monthly means were arranged from July to July.

The means for the year 1842–43 depend on nine months only, to refer them to the mean of twelve months, the differences for every observing hour, between the same nine months and twelve months for the preceding and following year, were made out and the mean correction, giving the weight two to the following year, as indicated by the readings taken at the hour 14, was applied to the values of 1842–43.

	From 9 months.	Correction.	Annual means.		From 9 months.	Correction.	Annual means.
0h	701	+3	704	Noon	682	+8	690
2	696	+3	699	14	681	+6	687
4	697	+5	702	16	683	+7	690
6	697	+6	703	18	689	+7	696
8	690	+6	696	20	697	+5	702
10	686	+7	693	22	700	+5	705

The normals for 1843–44 at the even hours are complete, at the odd hours they extend only over nine months. To refer the latter to twelve months, the difference between the means of the same nine months and the annual mean at the even hours was made out and applied as a correction to the means of the odd hours; the correction thus applied is the mean difference as deduced from the preceding and following even hour.

	Means of 9 months.	Correction.	Annual means.		Means of 9 months.	Correction.	Annual means.
1h	749	—11	738	13h	728	—14	714
3	746	—10	736	15	726	—16	720
5	746	—11	735	17	744	—16	728
7	744	—11	733	19	752	—15	737
9	737	—12	725	21	751	—14	737
11	730	—13	717	23	754	—13	741

TABLE VI.—ANNUAL MEANS OF THE BI-HOURLY AND HOURLY NORMAL VALUES OF THE REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE.

The numbers are expressed in scale divisions, increasing values indicate decrease of force. The minutes at the head of each column are to be added to the hour given in the first column. Each year commences with the month of July. The time is local Philadelphia time counted from midnight to midnight.

Hour.	1841–42. +17¼m	1842–43. +20½m	1843–44. +23½m	1844–45. +23½m
0 A. M.	650	704	740	765
1 "			738	761
2 "	643	699	735	759
3 "			736	760
4 "	643	702	737	761
5 "			735	761
6 "	640	703	734	761
7 "			733	759
8 "	634	696	727	752
9 "			725	751
10 "	632	693	722	749
11 "			717	747
12 P. M.	633	690	717	745
13 "			714	742
14 "	631	687	718	741
15 "			720	746
16 "	636	690	724	753
17 "			728	758
18 "	647	696	732	762
19 "			737	764
20 "	654	702	738	763
21 "			737	761
22 "	652	705	738	761
23 "			741	764
Means	641	697	730	756

The following formulæ of the mean diurnal variation of the vertical force were deduced from the above tabular values. The angle θ counts from midnight at the rate of 15° an hour.

1841-42 $V = 641^d + 10^d.4 \sin(\theta + 106^\circ 40') + 3^d.1 \sin(2\theta + 198^\circ 25') + 1^d.7 \sin(3\theta + 250^\circ)$
1842-43 $V = 697 + 7.6 \sin(\theta + 69\ 17) + 2.9 \sin(2\theta + 196\ 48) + 1.3 \sin(3\theta + 195)$
1843-44 $V = 730 + 11.0 \sin(\theta + 79\ 54) + 3.4 \sin(2\theta + 226\ 29) + 0.6 \sin(3\theta + 45)$
1844-45 $V = 756 + 9.2 \sin(\theta + 83\ 40) + 4.3 \sin(2\theta + 233\ 41) + 1.1 \sin(3\theta + 1)$

In the construction of the equation for 1843-44 weighted normals were used, those of the even hours have the weight 4, of the odd hours the weight 3.

To show the degree of accordance in the expressions when deduced from the even and odd hours separately, the resulting equations for the last year are added:—

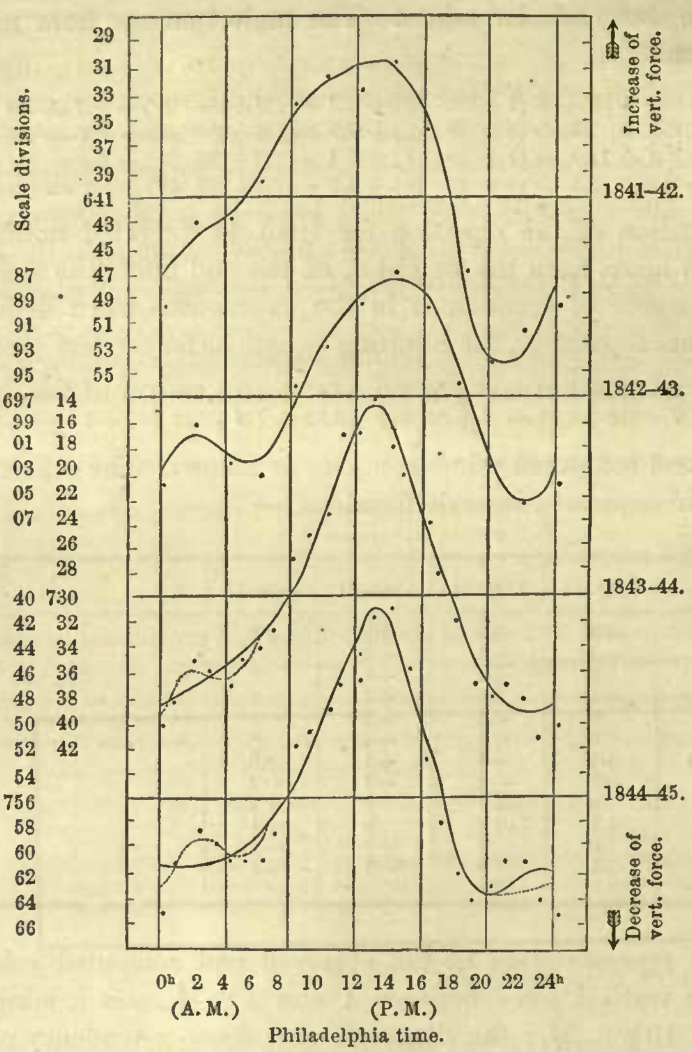
Even hours: $V = 756 + 9.32 \sin(\theta + 84^\circ 45') + 4.07 \sin(2\theta + 235^\circ 17') + 1.2 \sin(3\theta + 353^\circ)$
Odd " $V = 756 + 8.99 \sin(\theta + 82\ 36) + 4.52 \sin(2\theta + 232\ 05) + 1.0 \sin(3\theta + 10)$

The observed and computed values compare as follows. The differences, observed less computed, are expressed in scale divisions:—

Hour.	1841–42.	1842–43.	1843–44.	1844–45.	
0	+2	+2	+1	+3	17¼, 20½, 23¼, and 23½ minutes are to be added to the full hours for the four years respectively.
2	–1	–1	–2	–2	
4	0	+1	+1	0	
6	+1	+1	+1	+2	
8	0	–1	–2	–3	
10	0	+1	+1	0	
Noon	+1	+1	+1	+2	
14	–1	–1	+1	–2	
16	0	+1	0	+1	
18	+1	+1	0	+1	
20	0	0	+1	–1	
22	–1	0	–1	–1	

The graphical representation of the observed and computed values exhibits a maximum of the vertical force between 1 and 2 P. M., and a minimum of force between 8½ and 10½ A. M.; the diagrams also show a tendency of a secondary maximum about two hours after midnight followed by a secondary minimum about two hours later, with a range probably less than two scale divisions (0.000066 parts of the force, or 0.00085 in absolute measure). This small nocturnal inequality is only exhibited by one of the formulæ, in 1842–43, when it has its greatest value; in the preceding year there is but a faint trace of it, in the two succeeding years it is indicated in the diagram by dashes. The average diurnal range is nearly 22 scale divisions (0.00073 parts of the vertical force, or 0.00932 in absolute measure).

Inequality in the Diurnal Variation of the Vertical Force.



Epoch and Amount of the Principal Maximum and Minimum and Amplitude of the Diurnal Inequality.

Year.	Maximum.		Minimum.		Amplitude.		
	Reading.	Epoch.	Reading.	Epoch.	Scale div.	In parts of v. f.	In absol. meas.
1841–42	630d.9	13h 15m	654d.5	20h 50m	23d.6	0.00078	0.01000
1842–43	687.8	14 25	705.1	22 10	17.3	0.00057	0.00733
1843–44	715.1	13 00	739.3	23 00	24.2	0.00080	0.01025
1844–45	741.8	13 10	763.8	20 30	22.0	0.00073	0.00932
Mean		13½		21½	21.8		

The epochs are given to the nearest quarter of an hour.

If we compare the Philadelphia and Toronto curves, we find a general correspondence in their form, the early morning secondary in flection being well exhibited at Toronto; the epochs of the two curves, however, are shifted by nearly three hours, thus: at Toronto principal maximum at 5 P. M., at Philadelphia 1½ P. M.;

principal minimum at Toronto 10 A. M., at Philadelphia $9\frac{1}{2}$ A. M.; the epochs of the early morning inflection are also about $3\frac{1}{2}$ hours later at Toronto. The curves exhibit also a difference in the amplitude, at Toronto, Vol. III, the diurnal range is 0.00019 parts, whereas at Philadelphia we found it much larger.

The special study of the solar diurnal variation of the vertical force is reserved for Part VIII.

The minimum diurnal range occurred in 1842–43, on the average, therefore, we may assume May, 1843, as the epoch of the minimum range in the eleven (or ten) year period, resulting from the discussion of the declination, horizontal and vertical force observations.

To facilitate the comparison with similar expressions at other stations, the preceding equations of the diurnal variation are also presented, expressed in parts of the vertical force. The angles have been changed 180° to reverse the order of progression of the scale numbers.

1841–42 $V = +0.00034 \sin(\theta + 286^\circ 40') + 0.00010 \sin(2\theta + 18^\circ 25') + 0.00006 \sin(3\theta + 70^\circ)$
1842–43 $V = +0.00025 \sin(\theta + 249\ \ 17) + 0.00010 \sin(2\theta + 16\ \ 48) + 0.00004 \sin(3\theta + 15\ )$
1843–44 $V = +0.00036 \sin(\theta + 259\ \ 54) + 0.00011 \sin(2\theta + 46\ \ 29) + 0.00002 \sin(3\theta + 225)$
1844–45 $V = +0.00030 \sin(\theta + 263\ \ 40) + 0.00014 \sin(2\theta + 53\ \ 41) + 0.00004 \sin(3\theta + 181)$

The constant terms and numerical coefficients when expressed in absolute measure (English units) are as follows:—

		Term involving		
	Y	θ	2θ	3θ
1841–42	12.85	0.00441	0.00131	0.00072
1842–43	12.84	0.00322	0.00123	0.00055
1843–44	12.83	0.00468	0.00144	0.00025
1844–45	12.83	0.00388	0.00172	0.00047

The angle θ counts from midnight.

Investigation of the Eleven (or ten) Year Inequality in the Disturbances, and General Analysis of the Disturbances of the Vertical Force.—By means of Table V, a new table was formed of the number of disturbances in each month for the years 1841–42, 1842–43, 1843–44, 1844–45, commencing with July, and all referred to a uniform series of bi-hourly observations; the numbers for and after October, 1843, were halved. The number of disturbances for January, February, and March, 1843, are the means between the same months in the preceding and following year. The annual means of this Table (VII) are as follows:—

	Mean number of Disturbances.
In 1841–42	51
" 1842–43	34
" 1843–44	25
" 1844–45	16

This seems to indicate the end of the year 1844 as the epoch of the minimum number of disturbances in the eleven year period, taking the numbers collectively for declination, horizontal and vertical force, the minimum probably took place in the spring of 1844.

If we take the monthly aggregate amount of the disturbances, all referred to a uniform series of bi-hourly observations, and form a table of these values for each year (Table VIII), the mean aggregate amount for each year is as follows:—

	Mean amount of disturbances.
In 1841–42	2306 div.
" 1842–43	1521
" 1843–44	959
" 1844–45	636

This again points to the end of the year 1844 for the epoch of the minimum amount of disturbances, and considering the three elements, declination, horizontal and vertical force, the spring of 1844 might be assumed as the time of the minimum magnitude of the magnetic disturbances.

Altogether, the inequalities in the diurnal amplitude and in the number and magnitude of the disturbances of the magnetic elements, as observed at Philadelphia, fix the end of the year 1843, or the beginning of 1844, as the epoch of the minimum of the eleven (or ten) year inequality.

We now proceed with the analysis of the disturbances, their diurnal and annual inequality in number and amount, and for increasing and decreasing values.

Annual Inequality in the number of Disturbances.—The numbers for each month have been referred to a uniform series of bi-hourly observations as explained above. The ratios of the monthly means to the annual means is given, and also, for comparison, similar ratios found for the horizontal force and declination.

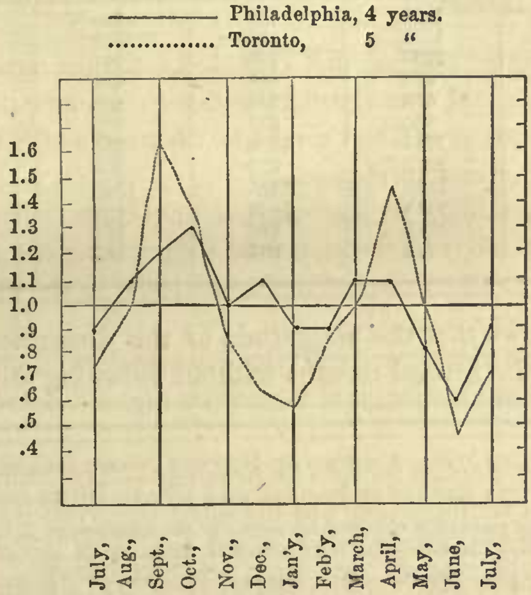
TABLE VII.—ANNUAL INEQUALITY IN THE NUMBER OF DISTURBANCES.

	1841–1842.	1842–1843.	1843–1844.	1844–1845.	Means from four years.	Vert. force ratio.	Hor. force ratio.	Declination ratio.	Mean ratio Vert. force hor. force and declination.	Mean ratio Hor. force and declination.
July	64	24	4	22	28	0.9	1.1	0.9	1.0	0.9
August	21	34	10	47	28	0.9	0.9	1.6	1.3	1.1
September	40	57	20	7	31	1.0	1.4	1.4	1.4	1.2
October	28	12	12	10	15	0.5	1.4	2.1	1.7	1.3
November	37	13	40	5	24	0.8	1.0	1.1	1.1	1.0
December	84	15	33	23	39	1.3	1.0	1.0	1.0	1.1
January	76	58	40	9	46	1.5	0.6	0.8	0.7	0.9
February	86	51	16	3	39	1.3	1.0	0.5	0.7	0.9
March	36	44	53	34	42	1.4	1.1	0.7	0.9	1.1
April	51	50	42	12	39	1.3	1.1	0.9	1.0	1.1
May	47	36	4	14	25	0.8	1.0	0.6	0.8	0.8
June	37	16	26	4	21	0.7	0.6	0.5	0.6	0.6
Mean	51	34	25	16	31					

The months of maximum disturbance are March and September (the high value in January and the low one in October appear anomalous, and would no doubt disappear in a longer series of observations). The minimum occurs in June; there is no well expressed second minimum. The horizontal force and declination ratios, as well as the ratios of the three elements at Toronto, give the maximum number of disturbances at the equinoxes, and the minimum number at the solstices, and as the winter solstice minimum only is wanting in the Philadelphia vertical force

ratios, it is probably due to the small number of observations, and the difficulty in keeping the instrument in adjustment and allowing for its irregularities. I have, therefore, given the mean ratio of the Philadelphia disturbances in the last column of Table VII, and compared the result, graphically, with those deduced by General Sabine for Toronto.[1]

Annual Inequality of Disturbances.



If we separate the disturbances into two parts, those increasing and those decreasing the force, we obtain the numbers of Table VIII. A positive sign indicates disturbances increasing, a negative sign those decreasing the vertical force. The law of the annual variation seems to be the same as shown by the ratios in the last two columns; this accords with the result at Toronto.

Table VIII.—Annual Inequality of Disturbances Increasing and Decreasing the Force.

	1841–42.		1842–43.		1843–44.		1844–45.		Ratios.	
	+	−	+	−	+	−	+	−	+	−
July	31	33	8	16	0	4	12	10	0.7	1.1
August	14	7	9	25	8	2	36	11	1.0	0.8
September	22	18	16	41	9	11	5	2	0.7	1.2
October	22	6	8	4	6	6	3	7	0.6	0.4
November	17	20	5	8	14	26	4	1	0.6	1.0
December	51	33	8	7	23	10	13	10	1.4	1.1
January	38	38	32	26	26	14	6	3	1.5	1.5
February	53	33	32	19	11	5	2	1	1.4	1.0
March	16	20	23	21	30	23	23	11	1.3	1.3
April	32	19	30	20	23	19	5	7	1.3	1.1
May	28	19	18	18	3	1	11	3	0.9	0.7
June	23	14	5	11	7	19	3	1	0.6	0.8

[1] Page lxx., Vol. III.

Table IX.—Aggregate and Mean Amount of Disturbances in each Month of the Year.

The numbers are expressed in scale divisions and referred to a uniform series of bi-hourly observations. The mean amount or average magnitude is found by dividing the number in the preceding column by 4 and by the number of disturbances found in Table VII.

	Aggregate amount.				Sum of 4 years.	Mean amount.
	1841–42.	1842–43.	1843–44.	1844–45.		
July . . .	2593	1255	149	784	4781	42
August . .	791	1323	622	2017	4753	43
September .	1612	2798	770	301	5481	44
October . .	1216	432	488	438	2574	42
November .	1564	503	1504	206	3777	40
December .	4187	560	831	872	6450	42
January . .	3899	2745	1592	314	8550	47
February . .	3900	2279	659	109	6947	45
March . . .	1672	1898	2125	1245	6940	42
April . . .	2324	2111	1642	468	6545	42
May . . .	2445	1625	186	704	4960	49
June . . .	1472	723	934	168	3297	40

The last column shows that the magnitude of the disturbances is rather irregularly distributed over the several months without following any apparent law.

Table X.—Aggregate and Mean Amount of Disturbances in each Month of the Year, separated into two Groups of Increasing (+) and Decreasing Force (—).

The mean amount is obtained by means of the numbers of Table VIII.

	1841–42.		1842–43.		1843–44.		1844–45.		Sum of 4 years.		Mean amount.	
	+	—	+	—	+	—	+	—	+	—	+	—
July	1130	1463	340	915	0	149	402	382	1872	2909	37	46
August	555	236	359	964	279	343	1568	449	2761	1992	41	45
September	835	777	775	2023	397	373	251	50	2258	3223	43	45
October	999	217	300	132	250	238	128	310	1677	897	43	39
November	653	911	223	280	504	1000	154	52	1534	2243	38	41
December	2745	1442	276	284	508	323	489	383	4018	2432	42	41
January	2128	1771	1589	1156	1050	542	208	106	4975	3575	48	44
February	2615	1285	1538	741	462	197	76	33	4691	2256	48	39
March	671	1001	910	988	1149	976	875	370	3605	3335	39	44
April	1471	853	1361	750	853	790	172	296	3856	2689	43	41
May	1535	910	928	697	170	16	598	106	3231	1729	54	42
June	999	473	246	477	246	688	133	35	1624	1673	43	37
Sums	16336	11339	8845	9407	5867	5635	5054	2572	36102	28953		
									Mean . . .		43	42

The magnitudes of the disturbances, as before, do not appear to follow any law. The disturbances which increase the force preponderate over those which decrease it; the ratio of the annual means is 1.3 to 1.0. At Toronto the reverse was found; the disturbances which decrease the force preponderate over those which increase in the ratio of 1.4 to 1.0.

Diurnal Inequality of the Disturbances.—In the bi-hourly combination of the disturbances we make use of the series of observations extending from February,

1841, to June, 1845, omitting only the single daily observation in January, February, and March, 1843. Strictly speaking the time is 21 minutes later than indicated in the table.

TABLE XI.—DIURNAL INEQUALITY IN THE NUMBER OF DISTURBANCES.
The ratios of the three elements have been collected to facilitate comparison.

	Number vertical force.	Ratios.		
		Vertical force.	Horizontal force.	Declination.
0^h	168	1.3	1.1	1.0
2	159	1.2	0.9	1.2
4	156	1.2	0.7	1.0
6	133	1.0	0.7	1.1
8	117	0.9	0.8	1.0
10	115	0.8	1.1	1.1
Noon	131	1.0	1.3	0.9
14	163	1.2	1.0	0.8
16	127	0.9	1.1	0.9
18	116	0.8	1.1	0.9
20	110	0.8	1.1	1.0
22	123	0.9	1.1	1.1
Mean	135			

The greatest number of disturbances occur about A. M. (at Toronto at 3 A. M.), with the least number at 10 A. M. (at Toronto at 11 A. M.); the secondary maximum and minimum occur about 2 P. M. and 7 P. M. (at Toronto the hours are 5 P. M. and 9 P. M.). On the average, therefore, the maxima and minima occur 1^h 40^m earlier at Philadelphia than at Toronto. At neither station do the three elements show the same law; they agree only in so far as to exhibit a systematic increase and decrease with the solar hours, and in having two maxima and two minima.

The diagram shows the law of the disturbances of the vertical force for Philadelphia and Toronto.

DIURNAL VARIATION OF DISTURBANCES.

———— Philadelphia, 4½ years.
.... Toronto, 5 "

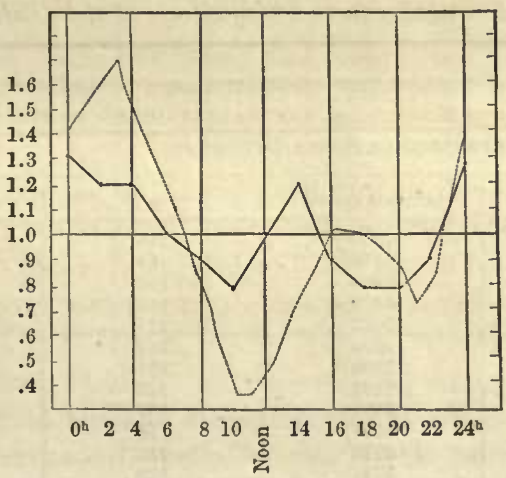


TABLE XII.—CONTAINS THE NUMBER OF DISTURBANCES DISTRIBUTED OVER THE HOURS OF THE DAY, SEPARATED INTO THOSE WHICH INCREASE (+) AND THOSE WHICH DECREASE (—) THE VERTICAL FORCE.

	Number of disturbances.		Ratios.	
	+	—	+	—
0^h	93	75	1.3	1.2
2	73	86	1.0	1.3
4	69	87	1.0	1.4
6	62	71	0.9	1.1
8	59	58	0.8	0.9
10	53	62	0.8	1.0
Noon	61	70	0.9	1.1
14	96	67	1.3	1.0
16	69	58	1.0	0.9
18	72	44	1.0	0.7
20	67	43	0.9	0.7
22	76	47	1.1	0.7
Mean	71	64		

The laws which regulate the diurnal occurrence of the nnmber of disturbances, increasing and decreasing the vertical force, are evidently not the same, yet they are by no means the converse of one another as has been found to be the case in the disturbances of the declination and the horizontal force. At Toronto also, where the horizontal force and declination curves were exactly opposed, that of the vertical force is not so, and at Philadelphia rather favors an agreement between the increasing and decreasing disturbances than an opposition.

Principal maximum of increasing disturbances 2 P.M., principal minimum 9 A.M. (at Toronto 5 P. M. and 5 A. M. respectively). Secondary maximum at midnight; this may possibly be the principal maximum; secondary minimum at 8 P. M.

Principal maximum of decreasing disturbances 4 A. M., principal minimum 8 P. M. (at Toronto 3 A. M. and 6 P. M. respectively). The secondary maximum at noon is less distinctly marked; secondary minimum at 8 A. M.

Thus the epochs of the curves for increasing and decreasing force seem to be 12 hours apart.

Diurnal Inequality in the Magnitude of the Disturbances.

TABLE XIII.—CONTAINS THE AGGREGATE AMOUNT OF DISTURBANCES AND THEIR AVERAGE MAGNITUDE, THE LATTER FOUND BY MEANS OF TABLE XI. ALL EXPRESSED IN SCALE DIVISIONS.

	Aggregate amount.	n	Mean amount.
0^h	7049	168	42
2	6876	159	43
4	6480	156	42
6	5418	133	41
8	5022	117	43
10	5096	115	44
Noon	5526	131	42
14	7101	163	44
16	5591	127	44
18	5571	116	48
20	4773	110	43
22	5246	123	43

Average magnitude 43 scale divisions, the disturbances appear to be nearly of the same size at all hours, there is a slight preponderance in magnitude between 10 A. M. and 10 P. M. over the other half of the day.

TABLE XIV.—AGGREGATE AMOUNT AND MEAN AMOUNT OF DISTURBANCES, SEPARATED INTO THOSE WHICH INCREASE AND THOSE WHICH DECREASE THE VERTICAL FORCE.

	Aggregate amount.		Mean amount.		Difference of aggregate amount.
	+	—	+	—	
0^h	3737	3312	40	44	+ 425
2	3221	3655	44	43	— 434
4	2866	3614	42	42	— 748
6	2577	2841	42	40	— 264
8	2524	2498	43	43	+ 26
10	2507	2589	47	42	— 82
Noon	2731	2795	45	40	— 64
14	4456	2645	46	40	+1811
16	2969	2622	43	45	+ 347
18	3456	2115	48	48	+1341
20	3003	1770	45	41	+1233
22	3258	1988	43	42	+1270
Mean			44.0	42.5	+4861

The magnitude of the disturbances, either increasing or decreasing the force, apparently does not vary with the hours of the day. The disturbances which increase the force preponderate between the hours 2 P. M. and 2 A. M.; those which decrease the same occur in the other half of the day, the average ratio of the preponderance of increase over decrease is as 4 to 1.

Dividing the numbers in the last column of the preceding table, or the excess of the sum of disturbances increasing the force over the sum of those decreasing the same, by the total number of days (1297) of observation, we find the diurnal disturbance variation as follows:—

	Scale divisions.	In parts of vertical force.	In absolute measure.
0^h	+0.3	+0.00001	+0.00013
2	—0.3	—0.00001	—0.00013
4	—0.6	—0.00002	—0.00025
6	—0.2	—0.00001	—0.00008
8	0.0	0.00000	0.00000
10	—0.1	0.00000	—0.00004
Noon	0.0	0.00000	0.00000
14	+1.4	+0.00004	+0.00059
16	+0.3	+0.00001	+0.00013
18	+1.0	+0.00003	+0.00042
20	+1.0	+0.00003	+0.00042
22	+1.0	+0.00003	+0.00042
Mean	+0.3	+0.00001	+0.00013

The value for the hour 14 is evidently anomalous, the mean of the hours 12 and 16 or + $0^d.2$ (0.00001 in parts of force, 0.00008 in absolute measure) should be substituted. The average daily effect of the larger disturbances is therefore to increase the vertical force between 1 P. M. and midnight, and to decrease it

between 1 A. M. and noon, with an amplitude of about 1.6 scale division (0.00005 parts of the force, 0.00067 in absolute measure). The maximum value takes place at 8 P. M., the same hour at which the horizontal force disturbance is greatest (decreasing that force).

The disturbance law at Toronto is nearly the same as at Philadelphia, the disturbances increase the force between noon and 9 P. M., and decrease it in the remaining hours of the day; the range at Toronto appears to be larger.

If we classify the disturbances according to their magnitude in eight groups, each differing 25 scale divisions from the preceding, we find the following scale numbers:—

Disturbances. In scale divisions.	Between limits in parts of the force.	Number.
30 and 55	0.00099 and 0.00181	1840
55 " 80	0.00181 " 0.00263	211
80 " 105	0.00263 " 0.00346	28
105 " 130	0.00346 " 0.00428	15
130 " 155	0.00428 " 0.00511	0
155 " 180	0.00511 " 0.00593	2
180 " 205	0.00593 " 0.00676	0
205 " 230	0.00676 " 0.00759	1
Beyond		None.

APPENDIX TO PART VII.

EFFECT OF THE AURORA BOREALIS ON THE MAGNETIC DECLINATION, AND THE HORIZONTAL AND VERTICAL FORCE AS OBSERVED AT THE GIRARD COLLEGE OBSERVATORY.

There were in all 22 auroras recorded; these, however, comprise only the brighter displays. Of those observed, 7 occurred between May 30, 1840, and July 1, 1841; 1 occurred between July, 1841, and July, 1842; 6 occurred between July, 1842, and July, 1843; and 7 between July, 1843, and July, 1844. One is recorded in the last year, ending June 30, 1845. They are distributed over the several months as follows:—

January	2	July	6
February	0	August	3
March	1	September	2
April	2	October	0
May	3	November	1
June	2	December	0

In the summer months there were 18, in the winter months 4. In reference to the hours of the night, the phenomenon was visible on the average between $9\frac{1}{2}$ P. M. and $11\frac{1}{4}$ P. M.

Individual examination of the magnetic record during auroral displays. The time is local time, counted for convenience' sake from midnight to midnight to 24 hours.

I. 1840, May 29th—30th. As the twilight faded an aurora became visible. In the course of the display there were moving pillars, flashes from a low segment of light in the north, and a beautiful arch nearly or quite at right angles to the magnetic meridian. Pillars of aurora from $21^h\ 18^m$ to $22^h\ 2^m$, varying in brightness and position; low segment of light to the north, continued throughout the appearances; at $22^h\ 5^m$ an arch forms from east to west; streams of light, varying in brightness, fading and reappearing from $22^h\ 20^m$ to about $23^h\ 10^m$; the brightest flash at $23^h\ 6^m$. From $18^h\ 54^m$ the declination magnet commenced to move eastward (declination decreasing), reaching an extreme position at $20^h\ 34^m$, difference from average position about 56 divisions or 19′; the movement then became westerly with smaller fluctuations till $22^h\ 39^m$, when it reached its westerly extreme of about 71 divisions or 24′ from the normal place; the magnet reached a second easterly extreme at $23^h\ 44^m$ of about 48 divisions or 17′, at $1^h\ 24^m$ (30th) again a westerly extreme of about 7′, and at $2^h\ 49^m$ an easterly deflection of about 14′; after this the needle returned gradually to its ordinary position. About the time of the brightest flash the change (easterly motion) was very rapid, no extreme value, however, was reached. When the arch formed, the position was nearly normal. The horizontal force decreased steadily until $22^h\ 42^m$, when the readings fell beyond the scale; a minimum was reached between that time and $22^h\ 52^m$ of at least 0.016 (parts of the force) below the normal force. At the time of the brightest flash the retrograde movement was in progress. The disturbance of the vertical force commenced before $17^h\ 52^m$, at which time the force was a maximum; it then decreased very rapidly, and finally moved off the scale after $22^h\ 2^m$. (The value of a division of the scale was not ascertained.)

II. 1840, July 4th. At 20^h auroral light in the N. N. W. about 10° above the horizon, at 22^h very faint aurora still visible in N W. The declination was not at all affected. The horizontal force

at these hours was 85 divisions (0.003 parts of the force) less than the normal amount. The vertical force is apparently undisturbed; it is slightly above the normal value.

III. 1840, July 6th. An aurora was noticed at 0^h 25^m and 2^h 25^m. The declination was disturbed at 0^h $19\frac{1}{2}^m$ and 2^h $19\frac{1}{2}^m$; it indicated 50 divisions and 34 divisions, or 17′ and 12′ of easterly deflection. It is likely that there were disturbances two hours preceding and two hours following the above times, as the scale could not be read. The horizontal force was disturbed from midnight till 2 P. M.; the force was less during this time, and reached its minimum value at 2^h 22^m of 130 divisions or 0.005 parts of the force; between 2 and 8 A. M. the diminution was about 0.004 parts. The vertical force was also less from midnight till after 2^h 17^m, the greatest diminution probably took place later as the observations failed at 4^h 17^m. Minimum value at 2^h 17^m, 0.004 parts of the force.

IV. 1840, July 29th. At 22^h 25^m a faint aurora. The declination was not disturbed. The horizontal force was very slightly affected. At 20^h 22^m it was 0.001 parts less than the normal, at 22^h 22^m it was nearly normal, and at 0^h 22^m (30th) it was greater by 0.002. There may have been ordinary disturbances not immediately connected with the aurora. At 22^h 17^m the vertical force was slightly affected, the force decreased 0.002 parts below the normal.

V. 1840, August 19th. At 20^h 25^m auroral light in N.; 22^h 25^m aurora continues in N. and N. W. The declination disturbance commenced at 22^h 20^m and continued to 2^h 20^m (20th), west deflection 48 divisions (22′), 10 divisions, and 10 divisions. The horizontal force was disturbed from 16^h 22^m to 22^h 22^m, force less 43 divisions, 49, 102, and 85 divisions (in minimo 0.004 parts of the force). The vertical force seems lower than usual, but hardly reached the limit of a recognized disturbance.

VI. 1840, August 28th and 29th. An aurora appeared at 20^h $39\frac{1}{2}^m$ in N. N. E., disappeared at 21^h $19\frac{1}{2}^m$, reviving at 21^h $59\frac{1}{2}^m$; at 22^h $9\frac{1}{2}^m$ streamers moving from E. to W.; light continued in N.; streamers again in N. E. at 22^h $59\frac{1}{2}^m$ and 1^h $14\frac{1}{2}^m$, after which time the aurora was not observed. An easterly movement of the needle commenced about 20^h 19^m with a maximum eastern deflection of 125 divisions (or 57′) at 21^h 0^m, the westerly motion continued till 21^h 55^m when the needle was yet 5′ east of its normal position; smaller fluctuations were observed till midnight, the deflection was then 19′ east; half an hour later it was 20′ east; the morning extreme was reached at 1^h 35^m, when the deflection was 25′ east; after this the disturbance gradually subsided. There was a disturbance of the horizontal force about 18^h 20^m; from about 21^h 52^m till 10 the next morning the horizontal force remained below its normal value. At 23^h 32^m it was 0.009 (parts) below, at 0^h 22^m it was 0.007, and at 1^h 22^m its minimum value of 0.010 (parts of the force) was reached. The disturbance in the vertical force appears to have commenced about 21^h 7^m, when the force gradually decreased till 21^h 57^m when it reached a minimum of about 0.003 (parts); after this it gradually increased.

VII. 1840, September 21st. At 20^h 25^m faint aurora, 22^h 25^m aurora disappeared. Disturbance of the declination commenced at 20^h 20^m and continued to 4^h 20^m (22d), deflections 40 divisions (18′) W., 10 divisions E., 14 divisions W., and 23 divisions E. The horizontal force disturbance commenced at 16^h 22^m and ceased at 4^h 22^m next day; force less 69 divisions, 47 divisions, 71 divisions, 42 divisions, 94 divisions, 124 divisions (0.005 parts of the force), and 93 divisions. The vertical orce between 16^h and 23^h was slightly above the average, but suddenly became much smaller than the normal between midnight and 3 A. M. Minimum about 0.002 parts of the force.

VIII. 1842, April 14th. At 22^h 40^m appearance of aurora, a bright light in the N.; at 0^h 20^m (15th) an arc of light was visible extending to about 15° above the north horizon. Declination disturbed from 22^h 20^m to 8^h 20^m (15th), deflections at the regular observing hours 23 divisions W., 39 E., 11, 37, 10, and 14 divisions W. Maximum west deflection at 22^h 56^m, 58 divisions (26′), maximum east deflection 39 divisions (18′), derived from the series of extra observations. The horizontal force disturbances commenced at 22^h 22^m and ceased at 4^h 22^m, force less 39 divisions, 149 divisions, 37 divisions, and 50 divisions, minimum 279 divisions (0.010 parts of the force) at 1^h 16^m (15th). But one of the 69 extra readings during this aurora shows an increase of force. The vertical force

disturbances commenced at $0^h\ 17\frac{1}{2}^m$ (15th) and continued to $6^h\ 17\frac{1}{2}^m$; force less 68 divisions, 69 divisions, 59 divisions, and 38 divisions. Minimum value 111 divisions or 0.0037 parts of the force at $1^h\ 24^m$ (15th).

IX. 1842, September 2d. At $2^h\ 22^m$ a bright light extending on each side of N. point about 15°, and to about 6° above the horizon; at $2^h\ 49^m$ light spreading and becoming more faint; at $3^h\ 12^m$ light faint and gradually subsiding. The declination was very slightly affected, maximum west deflection at $3^h\ 26^m$, 19 divisions (9′). The horizontal force was not disturbed. The vertical force was likewise undisturbed

X. 1842, November 21st and 22d. A well developed aurora and the best observed of the series. At $22^h\ 23^m$ a very luminous arc extending to about 15° above the horizon, and about 90° along it in the north; $22^h\ 38^m$ light slightly increasing; $22^h\ 53^m$ a slight decrease of light; $23^h\ 18^m$ light alternately appearing and disappearing; $23^h\ 33^m$ four streamers of unusual brightness reaching 30° above horizon; $23^h\ 36^m$ light particularly bright in N. W., whence a large streamer of 20° is shooting, also one due north of 15°; $23^h\ 40^m$ light subsided, no streamers; $23^h\ 43^m$ small streamers appearing; $23^h\ 46^m$ large streamers attended with great light in N. W.; $23^h\ 48^m$ the arc still remains about 15° above horizon, but has shortened its chord to 30°, no streamers; $23^h\ 51^m$ arc scarcely visible; $0^h\ 23^m$ (22d) two arcs visible; $0^h\ 28^m$ a large streamer of 20° in length; $0^h\ 36^m$ considerable light without the arc; $1^h\ 08^m$ light very faint; $1^h\ 23^m$ slight appearances of arc; $1^h\ 33^m$ faint streamer of 10°; $1^h\ 58^m$ faint streamer of 20°; $2^h\ 48^m$ a large but faint streamer due N. about 20° in length; light has nearly disappeared; $3^h\ 3^m$ light scarcely visible; $3^h\ 33^m$ no light visible, and readings of instrument ordinary. The declination disturbances commenced at $22^h\ 20^m$, and ceased at $10^h\ 20^m$ (22d); deflections 49 divisions W., 20 divisions W., 25, 20, 8, 25, and 16 divisions E. The maximum W. deflection (22′) occurred at the commencement, with the appearance of the luminous arc, the needle remained deflected to the westward until towards the end, when there was a smaller easterly deflection. No special effect of the streamers is noticed. The horizontal force disturbances commenced at $16^h\ 22^m$, and continued to $2^h\ 22^m$ (22d); horizontal force less 33 divisions, 68, 82, 183 divisions (0.007 parts of the force); this diminution was about the time of the appearance of the arc, 125 divisions and 73 divisions at the last two regular observing times. The streamers did not appear to have any special effect. The horizontal force always remained smaller than the normal value at the respective hours. The vertical component was not affected.

XI. 1843, May 6th and 7th. At $19^h\ 48^m$ a bright light; at $2^h\ 18^m$ (7th) light to N. about 23° high, but faint. The declination disturbances commenced at $16^h\ 20^m$, and continued to about 3^h (7th). The deflections at the regular hours were 28, 30, 16 divisions E., 15 divisions W. (20^m after midnight), maximum east deflection 18′, succeeding maximum west deflection 9′, next following maximum east deflection 33′, following maximum west deflection 15′. The horizontal force disturbances commenced at the same hour with the declination disturbances, and continued to the end of the series of observations. The change commenced with a violent increase of 113 divisions above the normal value, and increased to 330 divisions (0.012 parts of the force) at $18^h\ 04^m$, corresponding in time to the first maximum east deflection. The force then decreased, reaching 132 divisions below the normal value, and attaining shortly (16^m) after midnight the extraordinary low value 348 divisions (0.013 parts of the force); up to the end of the disturbance the force remained below the standard amount. The vertical force was suddenly disturbed, at $18^h\ 23\frac{1}{2}^m$ it was 161 divisions greater than the mean, and at $22^h\ 23\frac{1}{2}^m$ but 41 divisions above the normal. Maximum value 164 divisions (equal to 0.0054 parts of the force) at $18^h\ 12^m$.

XII. 1843, May 8th. At 0^h an aurora visible to north. The declination was but slightly affected; at $19^h\ 32^m$ there was an easterly deflection of 15 divisions or 7′; at $20^h\ 26^m$ it was west 4′; after this there was an easterly motion, changing again to west, which reached an extreme value at $23^h\ 44^m$ of 17 divisions or 8′ W. From the commencement of the horizontal force observations ($17^h\ 38^m$) the force was less than the normal; at $20^h\ 4^m$ the greatest depression was 0.003 (parts of the force). The disturbance continued till 6 A. M., the force being less than the standard value. From $17^h\ 38^m$, when

the vertical force was observed, it was found less than the normal, at 20^h 36^m depression 20 divisions, at 0^h 23^m it was 31 divisions or 0.001 parts of the force below the standard value.

XIII. 1843, June 27th. At 22^h a bright diffused light to north, particularly bright to N. W., whence streamers are shooting up; general light weakens as it rises at 20^h 45^m; at 21^h 15^m a brilliant light, dark cumulus spots in the bright light, and long streaks of dark clouds to N. Fades at 22^h 13^m, light to N. faint; dark, fuzzy, low cumuli form and disappear to N. Neither the declination nor the horizontal force was disturbed by this aurora. The vertical force was slightly affected, force less 38 divisions or 0.0012 parts of the force.

XIV. 1843, June 30th. At 23^h aurora visible to the N. N. E., flaming to about 10°. The declination disturbances commenced at 22^h 20^m (9 divisions W.), they reach a maximum at 0^h 02^m (July 1st) of 20 divisions (9′), and gradually disappear, the deflections having been west throughout. The horizontal force is smaller than the normal value, a first minimum is reached about 20^h 44^m (about 45 divisions), and the principal minimum about 0^h 10^m (July 1st) of nearly 50 divisions (0.0018 parts of the force). The vertical force remained undisturbed.

XV. 1843, July 7th. At 20^h 52^m very light in the N. N. E. and N. W. The declination at 18^h 20^m is deflected 15 divisions E., the motion then became westerly and reached 29 divisions (13′) W. at 23^h 33^m; at 2^h 20^m (8th) the deflection is again 16 divisions W. The horizontal force is less than usual, with a minimum value about 20^h 52^m of 55 divisions (0.002 parts). The force then increases, and about midnight reaches slightly above the normal. The vertical force was not disturbed.

XVI. 1843, July 24th and 25th. According to a letter (dated July 25th) from one of the observers, auroral disturbances commenced about 16^h (July 24th) and quieted down about 21^h. At 16^h 20^m the declinometer was deflected 15 divisions E., about 4^h (25th) the disturbances reappeared deflecting 10 divisions W., and changed to east deflection at 6^h 20^m, reaching a maximum east of 34 divisions (15′) at 13^h 20^m. At 16^h 22^m the horizontal force was about 46 divisions less, with disturbances reappearing about 8^h 22^m, reaching at 8^h 46^m 96 divisions (0.0015 parts of the force) below the normal, and quieting down about one hour after noon. The vertical force was not sensibly disturbed.

XVII. 1843, July 25th. At 21^h 30^m (25th) streamers to N., flaming to about 30°; at 22^h streamers very bright, reaching about 40°; At 22^h 15^m light very faint and gradually disappearing. The declination was disturbed (deflections west) between 20^h 20^m and 22^h 20^m reaching a maximum at 21^h 58^m of 36 divisions (16′). The horizontal force decreased between 18^h 22^m and 22^h 22^m, reaching at 21^h 34^m 91 divisions (0.0033 parts) below the normal value. The vertical force apparently undisturbed.

XVIII. 1843, August 22d. At 20^h 22^m there were streamers of 35° in length, bright light in N. Between 14^h 20^m and 18^h 20^m there was a small east deflection of the magnet reaching 28 divisions at the latter hour; at 19^h 56^m it changed to a west deflection of the same amount (13′). At the time of the appearance of the streamers the declination was normal. During the aurora the horizontal force diminished, reaching at 20^h 28^m 91 divisions (0.0033 parts) below the normal. The low value continued for about two hours after this time. The vertical force was not sensibly affected.

XIX. 1844, January 24th and 25th. Aurora visible to N. and N. N. E. at 0^h 22^m (25th), streamers running up 30°; 0^h 33^m streamers running up 15° and 20°. During this aurora the horizontal needle was deflected to the westward about 10 divisions, reaching a maximum at 6^h 58^m of 15 divisions (7′); at the time of the appearance of the shorter streamers the deflection was near 7′, the horizontal force was below the normal value, viz: decrease 36 divisions, 41 and 35 divisions at 22^h 22^m, 23^h 22^m, and 0^h 22^m (25th), minimum 47 divisions (0.0017 parts). At the time of the longer streamers there was an average decrease, and during the continuance of the shorter streamers the horizontal force was

normal. At $0^h\ 23\frac{1}{2}^m$ and $2^h\ 23\frac{1}{2}^m$ the vertical force was 36 and 32 divisions smaller than the normal. Difference 0.0011 parts of the force.

XX. 1844, March 29th. At $16^h\ 51^m$ cloudy, aurora visible. The declination magnet is deflected to the east and west several times in succession; between $16^h\ 20^m$ and $18^h\ 20^m$ about 14 divisions E., and 16 divisions E.; the following greatest west deflection of 61 divisions (27′) occurred at $20^h\ 10^m$; the next east deflection reached a maximum at $0^h\ 22^m$ (30th) of 41 divisions; a maximum west deflection was again reached at $1^h\ 14^m$ of 50 divisions (23′). The horizontal force is throughout smaller than the normal value, with differences varying on the average from 50 to 70 divisions. The greatest difference was reached at $20^h\ 2^m$ of nearly 100 divisions (0.0036 parts of the force); at $23^h\ 47^m$ another small value of 90 divisions was observed. The vertical force was disturbed from $21^h\ 23\frac{1}{2}^m$ to $4^h\ 23\frac{1}{2}^m$ (30th). Force less 49 divisions, 55, 44, 73, 49, 52, 55, and 31 divisions. Minimum value 0.0024 parts of the force.

XXI. 1844, April 17th. At $2^h\ 20^m$, although cloudy, it was very bright at the north; same remark at $22^h\ 20^m$. The declination disturbances extend nearly over the whole day. The deflection was at first west (between $0^h\ 20^m$ and $4^h\ 20^m$) with a maximum value of 48 divisions (22′) at $3^h\ 10^m$; it then changed to the east, at $6^h\ 04^m$ it reached 52 divisions (23′); up to $20^h\ 20^m$ the deflection was slightly to the east. The horizontal force was diminished early in the morning, attaining a first minimum at $2^h\ 40^m$ of 47 divisions; it increased for a short time, reaching at $4^h\ 14^m$ 52 divisions above the normal, the force again decreased and reached at $5^h\ 47^m$ the lowest value of 151 divisions (0.0055 parts); it remained below the normal value for several hours. At $19^h\ 53^m$ the diminution was 41 divisions. Vertical force disturbed from 3^h to 8^h ($+23\frac{1}{2}^m$), force less 52 divisions, 58, 61, 66, 53, and 35 divisions. Minimum value 0.0022 parts of the force.

XXII. 1845, January 9th. At $17^h\ 20^m$ an aurora visible. The declination magnet is deflected east and west alternately; first maximum east deflection at $16^h\ 32^m$ of 20 divisions; following maximum west at $17^h\ 02^m$ of 11 divisions; following east deflection about 20 divisions 12^m later; next west deflection at $17^h\ 22^m$ 21 divisions; at $19^h\ 56^m$ the deflection again east 32 divisions; at $21^h\ 38^m$ it is west 40 divisions (18′), at $22^h\ 20^m$ it is east 33 divisions. The horizontal force between $15^h\ 52^m$ and midnight is considerably smaller than the normal value, a minimum is reached at $17^h\ 16^m$ of 155 divisions (0.0056 parts of the force). The disturbances ceased between 2^h and 3^h on the morning of the 10th. The vertical force was disturbed at 17^h, 20, 22, and 23^h ($+23\frac{1}{2}^m$), force greater 44 divisions, 31, 35, and 33 divisions. Average increase 0.0012 parts of the force.

From the preceding detailed account of the condition of the declination and of the horizontal and vertical components of the magnetic force during auroral displays, we obtain the following general results: Each of the 22 auroras recorded was accompanied by a corresponding disturbance of the earth's magnetism, at least in one of the three elements; in one case the declination alone was affected, in another case only the horizontal force, and in a third only the vertical force. The latter force was less subject to disturbances than the other two elements.

In the following table, showing the condition of the magnetic components during auroras, the first column contains the number of the aurora, the second the amount of declination deflection, the third its direction or the successive large excursions of the north end eastward or westward, the fourth the amount of the horizontal force disturbance expressed in parts of that force (a minus sign indicates less force than the normal belonging to that time, a plus sign indicates the reverse), the last column contains the amount of disturbance in the vertical force expressed in parts of that force; the signs have the same signification as for the horizontal force.

Number.	Amount of deflection.	Direction of deflection and excursions.	Excess (+) or defect (—) of horizontal force.	Excess (+) or defect (—) of vertical force.
1	24′	E. W. E. W. E.	—0.016	— (?)
2			—0.003	
3	17	E.	—0.005	—0.004
4			∓0.002	—0.002
5	22	W.	—0.004	
6	57	E.	—0.010	—0.003
7	18	W. E. W. E.	—0.005	—0.002
8	26	W. E. W.	—0.010	—0.004
9	9	W.		
10	22	W. E.	—0.007	
11	33	E. W. E. W.	±0.013	+0.005
12	8	E. W. E. W.	—0.003	—0.001
13				—0.001
14	9	W.	—0.002	
15	13	E. W.	—0.002	
16	15	E. W. E.	—0.002	
17	16	W.	—0.003	
18	13	E. W.	—0.003	
19	7	W.	—0.002	—0.001
20	27	E. W. E. W.	—0.004	—0.002
21	23	W. E.	∓0.006	—0.002
22	18	E. W. E. W. E. W. E.	—0.006	+0.001

The action on the declination magnet appears to be that of alternate deflections either way from the normal position; in 5 cases the deflection was west, in 2 cases east, more frequently there were one or two successions of west and east deflections (or the reverse) in one instance even three; these alternate excursions appear to be a characteristic sign. In 5 cases the tendency of the deflection was easterly, in 6 cases westerly, and in the remaining 8 cases in both directions. The average amount of deflection is 17′. With but one exception the uniform effect upon the horizontal force was to decrease it. In the exceptional case a decrease followed the increase; in another case the reverse took place; during one aurora there was at first a fall in the force, then a rise, and again a fall. The average depression of the horizontal force below the normal value was 0.005 parts of the force (0.021 in absolute measure). The effect upon the vertical force is small; in 9 cases no disturbance occurred; in general the force is less than the normal; there are two exceptions to this in the 13 cases. The average depression of the vertical force below its normal was 0.0007 parts of the force (0.009 in absolute measure), or irrespective of sign 0.0013 parts of the force. If we wish to compare the tabular differences in declination and horizontal and vertical force with the magnitude of the recognized disturbances, the latter are 4′ and 0.001 (parts of the force) for either the horizontal or vertical component.

Of the auroras noted, that of May 29, 1840, was in many respects the most remarkable, and the best observed both as to its appearance and as to its magnetic effect; its study can be recommended to those who have occasion to test their theoretical views in reference to this phenomenon, and to terrestrial magnetism. Its appearance at New Haven, Conn., has been described by E. C. Herrick (Sill. Jour., 1840, Vol. XXXIX, p. 194). It was seen over a great portion of the United States, in Canada, and England. See, also, description in the volume of the Toronto Observations; also an extract from the proceedings of the British Association (Sill. Jour. Vol. XL, p. 337); also (p. 338 ibid.) note on the same by the Astronomer Royal.

The total number of auroras which occurred at Philadelphia was much greater than the number given above, as has already been stated. At Toronto the annual distribution of the phenomenon over the same period of time, and for three years beyond the close of the record at Philadelphia, is as follows:—

In 1840 (from March)	on 23 nights.
" 1841	36 "
" 1842	14 "
" 1843	16 "
" 1844	20 "
" 1845	19 "
" 1846	27 "
" 1847	29 "
" 1848	66 "

These figures seem to indicate the existence of a period of frequency, probably of eleven years as conjectured by Prof. Wolf, the least number probably occurred in 1843, if we make an allowance for invisibility of the phenomenon either by daylight or by cloudy weather.

Between June, 1840, and July, 1845 (incl.), there were seen, according to the Toronto record, 109 auroras. The disturbances at Philadelphia on the dates of their appearance have been classified as follows: The numbers give the relative proportion to the total number, which latter is expressed by 100; the average numbers are given resulting from the examination of the disturbances of the declination, the horizontal and the vertical force.

	Number of cases.
No record at Philadelphia	19
None of the elements disturbed	30
But very few disturbances	20
An ordinary number of disturbances	14
An unusual number of disturbances	17

The number of unusual disturbances is therefore less than one-fifth of the total amount, and in fully one-half of the cases the magnetic elements were either not at all or but very slightly affected.

PART VIII.

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION AND OF THE ANNUAL INEQUALITY OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE.

DISCUSSION

OF THE

SOLAR DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE AT PHILADELPHIA.

THE necessary data for this investigation are given in the preceding Part (VII), which contains the normals resulting from the reduction of the observations to the same temperature (66° Fah.), from the allowance for irregularity in the progressive change and the exclusion of all recognized disturbances.

Owing to the greater irregularity in the indications of the vertical force instrument, and the comparatively small number of observations at odd hours, the normals are given for the even hours only; the observations at odd hours, however, are used to improve those taken at the intermediate even hours by means of a suitable process of interpolation.

The tabular numbers are expressed in scale divisions, one division being equal to 0.000033 parts of the vertical force, or equal to 0.000423 in absolute measure. Increasing numbers denote decrease of force. The hours count from midnight to midnight to 24 hours; the number of minutes the observations are made later than the full hour are given in the last column for each month.

NORMALS OF THE VERTICAL FORCE FOR JULY.

Year.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
1841	703		697		687		671		667		664	
1842	683		672		664		659		657		650	
1843	691		692		692		686		679		672	
1844	816	811	804	806	805	802	798	793	784	783	782	780
Means[1]	722		717		712		703		698		692	

Noon	13	14	15	16	17	18	19	20	21	22	23^h	Min.
665		676		680		698		708		706		+17
643		643		647		663		677		682		+17½
662		658		659		666		677		685		+23½
780	776	773	781	783	791	799	805	808	809	811	816	+23½
687		688		693		706		717		721		+20.4

[1] Let reading for any even hour $= n$ for the year 1844, for the odd hours preceding and following $n\,p$ & $n\,f$, mean $\frac{np + nf}{2}$; hence mean for the even hour $\frac{1}{2}\left(n + \frac{np + nf}{2}\right)$ which was substituted before the general mean for the four years was taken.

NORMALS OF THE VERTICAL FORCE FOR AUGUST.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	686		680		676		664		662		652	
1842	689		689		683		682		679		672	
1843	703		703		708		706		698		683	
1844	794	790	786	781	783	777	776	769	763	760	756	754
Means	718		715		712		706		700		691	

Noon	13	14	15	16	17	18	19	20	21	22	23	Min.
653		662		666		676		689		691		+17
652		659		669		679		688		688		+17½
669		671		672		682		695		699		+23½
750	750	746	759	764	771	778	789	790	789	792	794	+23½
681		686		693		704		715		718		+20.4

NORMALS OF THE VERTICAL FORCE FOR SEPTEMBER.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	663		655		646		647		637		631	
1842	692		686		689		690		681		671	
1843	721		719		721		716		707		706	
1844	816	815	813	812	811	810	809	805	798	795	790	785
Means	723		718		717		715		706		699	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
627		628		634		645		653		660		+17
671		673		672		679		687		693		+17½
693		692		692		703		714		710		+23½
780	779	772	794	793	795	806	813	812	812	814	817	+23½
693		693		698		708		717		719		+20.4

Normals of the Vertical Force for October.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	579		578		573		568		558		556	
1842	706		698		702		714		695		708	
1843	714	710	707	713	712	714	717	714	706	704	703	701
1844	775	771	769	773	776	779	780	780	774	773	773	770
[1]Means.	694		689		691		694		684		685	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
561		562		571		577		581		583		+17
707		706		706		708		710		709		+17½
702	700	703	701	704	709	714	717	719	715	714	717	+23½
769	766	773	777	786	788	791	789	785	785	781	781	+23½
685		685		692		697		699		697		+20.4

Normals of the Vertical Force for November.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	532		537		538		533		526		523	
1842	717		713		723		725		712		715	
1843	742	740	745	743	745	744	744	748	742	739	737	729
1844	775	772	768	769	772	771	770	773	768	772	772	767
Means	691		691		694		693		688		686	

Noon.	13	14	15	16	17	18	19	20	21	22	23h	Min.
520		521		526		529		531		538		+17
711		713		716		712		718		718		+17½
735	731	731	738	746	749	749	751	749	747	746	748	+23½
766	763	762	765	774	779	777	778	774	770	769	772	+23½
682		683		690		692		693		693		+20.4

[1] Let m equal any reading at an even hour in 1843 or 1844, mp and mf the same for the odd hours preceding and following, then mean for the even hour $\frac{1}{2}\left(m + \frac{mp + mf}{2}\right)$ which was substituted for the above value at the even hour in 1843 and 1844 before the general mean of the four years was taken.

NORMALS OF THE VERTICAL FORCE FOR DECEMBER.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	597		591		590		606		592		598	
1842	713		707		709		706		715		711	
1843	752	744	733	742	740	742	740	743	740	740	729	720
1844	754	753	754	755	757	756	755	758	749	751	750	746
Means	703		697		699		703		700		697	·

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
605		597		607		610		605		604		+17
709		707		706		711		713		713		+17½
727	729	743	749	758	760	767	763	764	757	·754	757	+23½
743	733	736	724	748	757	755	751	751	748	750	750	+23½
695		695		704		710		707		705		+20.4

NORMALS OF THE VERTICAL FORCE FOR JANUARY.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841												
1842	658		642		656		649		656		653	
1843												
1844	733	739	730	728	732	733	732	732	730	725	720	713
1845	754	747	747	752	752	757	763	767	760	758	753	752
Means	716		707		713		715		715		709	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
664		650		663		675		670		663		+17½
		[1]690										+17
715	708	715	724	737	740	741	743	744	745	744	744	+23½
751	748	749	742	753	760	756	753	749	746	747	754	+23½
709		705		717		724		721		719		+21.5

[1] No use is made of this reading, nor of the analogous readings in the following two months.

Normals of the Vertical Force for February.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	664		664		662		656		654		650	
1842	706		701		713		721		699		698	
1843												
1844	734	729	725	726	729	728	729	730	723	724	722	717
1845	760	756	752	757	759	760	763	764	756	756	752	749
Means	718		711		716		717		709		705	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
644		648		647		673		668		665		+17
709		692		698		712		723		710		+17½
		(697)										+17
718	716	716	720	727	731	738	739	740	737	735	733	+23½
747	741	740	744	753	759	761	761	758	753	752	756	+23½
704		700		706		720		722		716		+20.4

Normals of the Vertical Force for March.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	670		661		661		655		651		645	
1842	655		643		654		663		657		661	
1843												
1844	768	761	763	760	764	762	765	762	758	761	762	764
1845	749	741	736	742	746	748	749	746	736	733	732	729
Means	709		701		705		707		701		700	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
643		646		656		665		666		673		+17
651		650		657		668		673		665		+17½
		(686)										+17
762	753	758	760	754	757	753	754	752	753	763	764	+23½
729	729	713	734	740	747	751	746	741	740	739	743	+23½
696		694		703		709		708		710		+20.4

Normals of the Vertical Force for April.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	671		662		658		653		645		646	
1842	668		658		655		655		656		654	
1843	715		712		717		716		708		702	
1844	776	773	771	765	766	765	759	755	749	749	744	740
1845	732	727	728	729	728	727	725	721	718	715	717	717
Means	712		706		705		702		696		693	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
646		649		656		666		670		676		+17
657		651		650		660		672		672		+17½
709		700		696		696		710		709		+23½
737	733	737	741	745	744	756	767	768	765	767	775	+23½
716	713	717	720	724	724	732	737	736	729	726	727	+23½
693		691		694		702		711		711		+21.0

Normals of the Vertical Force for May.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	669		665		660		654		647		649	
1842	673		670		661		644		646		647	
1843	698		699		695		690		682		680	
1844	772	769	766	768	767	764	760	754	749	747	747	744
1845	720	718	717	719	716	715	713	709	705	702	701	699
Means	706		704		700		692		686		685	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
644		646		650		660		674		679		+17
651		644		645		656		668		670		+17½
677		668		677		685		691		695		+23½
744	740	747	744	746	748	754	763	766	766	768	772	+23½
697	696	701	698	702	705	710	720	718	717	718	719	+23½
683		680		684		693		703		706		+21.0

Normals of the Vertical Force for June.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	646		631		622		617		624		616	
1842	674		669		664		658		647		642	
1843	698		691		693		687		677		668	
1844	776	772	767	768	767	764	760	755	747	745	744	744
1845	733	731	729	730	729	728	726	722	717	715	715	711
Means	705		698		695		690		683		677	

Year.	Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
1841	603		624		635		650		653		657		+17
1842	635		639		635		653		671		668		+17½
1843	663		658		659		669		681		689		+23½
1844	742	739	744	747	749	754	760	769	771	770	771	775	+23½
1845	711	711	713	714	713	714	722	730	735	731	731	733	+23½
Means	671		675		678		691		702		703		+21.0

Table I.—Recapitulation of the Bi-hourly Normals of the Vertical Force (expressed in Scale Divisions) for each Month of the Year.

Increase of scale readings denotes decrease of force.

1841–5.	0h	2	4	6	8	10	Noon	14	16	18	20	22h	(+20m.6)
January	716	707	713	715	715	709	709	705	717	724	721	719	
February	718	711	716	717	709	705	704	700	706	720	722	716	
March	709	701	705	707	701	700	696	694	703	709	708	710	
April	712	706	705	702	696	693	693	691	694	702	711	711	
May	706	704	700	692	686	685	683	680	684	693	703	706	
June	705	698	695	690	683	677	671	675	678	691	702	703	
July	722	717	712	703	698	692	687	688	693	706	717	721	
August	718	715	712	706	700	691	681	686	693	704	715	718	
September	723	718	717	715	706	699	693	693	698	708	717	719	
October	694	689	691	694	684	685	685	685	692	697	699	697	
November	691	691	694	693	688	686	682	683	690	692	693	693	
December	703	697	699	703	700	697	695	695	704	710	707	705	
Year . .	709.7	704.5	704.9	703.1	697.2	693.2	689.9	689.6	696.0	704.7	709.6	709.8	
Summer .	714.3	709.7	706.8	701.3	694.8	689.5	684.7	685.5	690.0	700.7	710.8	713.0	
Winter .	705.2	699.3	703.0	704.8	699.5	697.0	695.2	693.7	702.0	708.7	708.3	706.7	

The months from April to September inclusive comprise the summer half year, those from October to March inclusive the winter half year.

Table II.—Mean Values of the Normals for each Month and Season.

1841–45.		1841–44.		1841–45.	
January	714.2	July	704.7		
February	712.0	August	703.3	Year	701.0
March	703.6	September	708.8	Summer	700.0
April	701.3	October	691.0	Winter	702.0
May	693.5	November	689.7		
June	689.0	December	701.2		

Subtracting each value of Table I from its respective monthly mean as given in Table II, and converting the remainder into parts of the force, we find the regular solar-diurnal variation presented in the following table:—

TABLE III.—REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE EXPRESSED IN PARTS OF THE FORCE.

A plus sign indicates a greater, a minus sign a less value than the mean. The first three places of decimals 0.000 have been placed on the side.

	1841–5.	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	+20m.6
	January	—059	+238	+040	—026	—026	+172	+172	+304	—092	—323	—224	—158	
	February	—198	+033	—132	—165	+099	+231	+264	+396	+198	—264	—330	—132	
	March	—178	+086	—046	—112	+086	+119	+251	+317	+020	—178	—145	—211	
	April	—353	—155	—122	—023	+175	+274	+274	+340	+241	—023	—320	—320	
	May	—412	—346	—214	+049	+247	+280	+346	+445	+313	+016	—313	—412	
0.000	June	—528	—297	—198	—033	+198	+396	+594	+462	+363	—066	—429	—462	
	July	—571	—406	—241	+056	+221	+419	+584	+551	+386	—043	—406	—538	
	August	—485	—386	—287	—089	+109	+406	+736	+571	+340	—023	—386	—485	
	September	—469	—304	—271	—205	+092	+323	+521	+521	+356	+026	—271	—337	
	October	—099	+066	—000	—099	+231	+198	+198	+198	—033	—198	—264	—198	
	November	—043	—043	—142	—109	+086	+122	+254	+221	—010	—076	—109	—109	
	December	—059	+139	+040	—059	+040	+139	+205	+205	—092	—290	—191	—125	
	Year	—287	—115	—131	—068	+129	+257	+366	+377	+165	—121	—282	—290	
	Summer	—469	—317	—221	—040	+175	+350	+508	+482	+333	—020	—353	—426	
	Winter	—106	+086	—040	—096	+082	+162	+224	+274	—000	—221	—211	—155	

Multiplying the above numbers by Y=12.83, we obtain the solar-diurnal variation in absolute value.

TABLE IV.—REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE IN ABSOLUTE MEASURE.

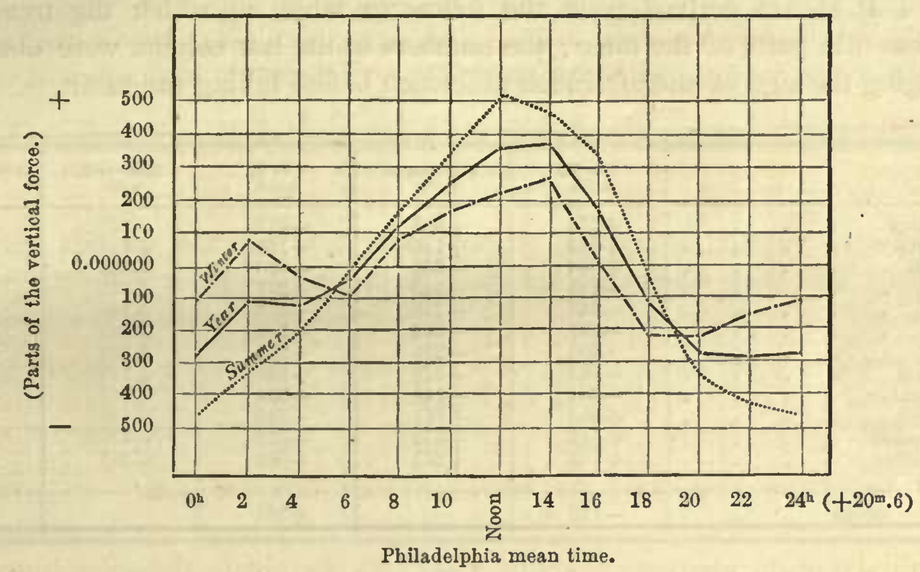
A greater force than the mean is indicated by a plus sign, a less force by a minus sign. The first two places of decimals 0.00 are placed on the side.

	1841–5.	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	+20m.6
	January	—076	+305	+051	—034	—034	+220	+220	+389	—118	—415	—289	—203	
	February	—254	+042	—170	—212	+127	+296	+338	+508	+254	—338	—423	—169	
	March	—229	+110	—059	—144	+110	+152	+321	+406	+025	—228	—186	—271	
	April	—452	—198	—157	—030	+224	+351	+351	+436	+309	—030	—410	—410	
	May	—529	—444	—275	+064	+317	+360	+444	+571	+402	+021	—402	—529	
0.00	June	—677	—380	—254	—042	+254	+508	+761	+592	+465	—085	—550	—592	
	July	—732	—520	—308	+072	+283	+537	+749	+706	+495	—055	—520	—690	
	August	—622	—495	—368	—114	+140	+520	+943	+732	+436	—030	—495	—622	
	September	—601	—389	—346	—263	+118	+415	+668	+668	+457	+034	—347	—431	
	October	—127	+085	—000	—127	+296	+254	+254	+254	—042	—254	—338	—254	
	November	—055	—055	—182	—140	+072	+157	+326	+283	—013	—098	—140	—140	
	December	—076	+178	+051	—076	+051	+178	+263	+263	—118	—372	—245	—161	
	Year	—368	—148	—168	—087	+165	+329	+470	+483	+212	—156	—364	—372	
	Summer	—601	—406	—284	—051	+224	+448	+651	+618	+426	—025	—453	—546	
	Winter	—136	+110	—051	—123	+106	+207	+288	+351	—000	—283	—271	—199	

Annual Inequality in the Diurnal Variation of the Vertical Force.—If we examine the average curve of the diurnal variation as observed throughout the year, and shown on diagram (A) by a full black line, we find the principal maximum value about 1 P. M., and the principal minimum value about 9½ P. M.; besides these characteristic values there is an indication of a secondary maximum about 2 A. M., and of a secondary minimum about 4 A. M. Dividing the year into a summer

and winter season, the diagram exhibits the diurnal variation in summer to be a curve of but one maximum and one minimum occurring about noon and midnight respectively, whereas in winter the double feature of the curve becomes very con-

(A.) Diurnal Variation of the Vertical Force in Summer, Winter, and for the whole Year.

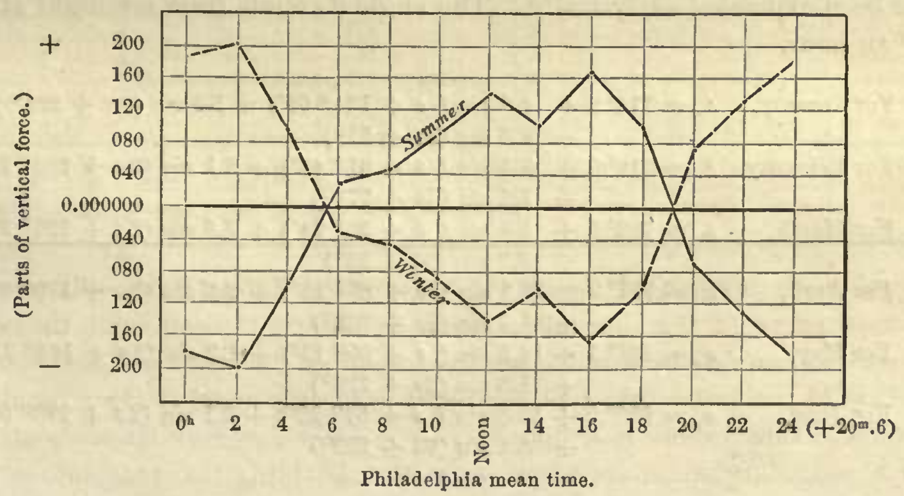


spicuous, the secondary maximum and minimum occurring about 2 and 6 A. M. respectively. The phenomenon, in the two seasons, changes therefore from a single to a double crested curve.

The semi-annual change of the diurnal variation is better shown in diagram (B), which contains the difference from the annual curve in summer and winter, viz. :—

	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	(+20m.6)
Summer	—182	—201	—90	+28	+46	+93	+142	+105	+168	+101	—71	—136	
Winter	+181	+202	+91	—28	—47	—95	—142	—103	—165	—100	+71	+135	

(B.) Semi-Annual Inequality in the Diurnal Variation of the Vertical Force.



At $5\frac{1}{2}$ A. M. and 7 P. M. there is no change in the diurnal variation throughout the year; at the hours 2 A. M. and 4 P. M. the change is a maximum, viz: range equal 0.000403 parts and 0.000333 parts of the force, or equal 0.00517 and 0.00427 when expressed in absolute measure.

The turning epochs of the annual inequality as found from the hours 2 A. M. and 4 P. M. are derived from the following table, in which the numbers are expressed in parts of the force; the numbers in the last column were obtained by changing the sign of the afternoon difference before taking the mean.

	2 A. M. 0.000.	Differences. 0.000.	4 P. M. 0.000.	Differences. 0.000.	Mean difference. 0.000.
January	+238	+353	—092	—257	+305
February	+033	+148	+198	+033	+058
March	+086	+201	+020	—145	+173
April	—155	—040	+241	+076	—058
May	—346	—231	+313	+148	—190
June	—297	—182	+363	+198	—190
July	—406	—291	+386	+221	—256
August	—386	—271	+340	+175	—223
September	—304	—189	+356	+191	—190
October	+066	+181	—033	—198	+190
November	—043	+072	—010	—177	+125
December	+139	+254	—092	—257	+256
Mean . . .	—115		+165		

The figures in the last column are represented by the equation

$$\Delta_a = + 0.000260 \, sin \, (\theta + 86°) + 0.000031 \, sin \, (2\,\theta + 180°)$$

the angle θ counting from January 1st at the rate of 30° a month. According to this expression the transition of the inequality from a positive to a negative value, and vice versa, takes place in the first quarter of April and October, or about 17 days after the equinoxes. The retardation of the phenomenon in the declination, horizontal and vertical force is, therefore, 10, 22, and 17 days respectively, or 16 days on the average.

Analysis of the Solar-Diurnal Variation of the Vertical Force.—For greater facility of the investigation, and for purposes of comparison, the numbers of Table I. have been expressed analytically. The angle θ counts from midnight at the rate of 15° an hour.

For January, $\Delta_v = 714^d.2 + 4.8 \, sin \, (\theta + 134° \, 09') + 5.5 \, sin \, (2\theta + 224° \, 22') + 0.8 \, sin \, (3\theta + 61°)$

For February, $\Delta_v = 712^d.0 + 7.5 \, sin \, (\theta + 91° \, 47') + 5.1 \, sin \, (2\theta + 226° \, 22') + 1.6 \, sin \, (3\theta + 273°)$

For March, $\Delta_v = 703^d.6 + 5.5 \, sin \, (\theta + 98° \, 24') + 3.6 \, sin \, (2\theta + 220° \, 22') + 0.7 \, sin \, (3\theta + 95°)$

For April, $\Delta_v = 701^d.3 + 10.5 \, sin \, (\theta + 89° \, 12') + 2.2 \, sin \, (2\theta + 175° \, 59') + 1.3 \, sin \, (3\theta + 232°)$

For May, $\Delta_v = 693^d.5 + 13.1 \, sin \, (\theta + 85° \, 17') + 1.9 \, sin \, (2\theta + 144° \, 31') + 1.8 \, sin \, (3\theta + 278°)$

For June, $\Delta_v = 689^d.0 + 15.8 \, sin \, (\theta + 87° \, 22') + 3.1 \, sin \, (2\theta + 193° \, 56') + 0.4 \, sin \, (3\theta + 210°)$

For July, $\Delta_v = 704^d.7 + 17.4 \sin(\theta + 86° 30') + 2.6 \sin(2\theta + 174° 16')$
$+ 0.7 \sin(3\theta + 300°)$

For August, $\Delta_v = 703^d.3 + 17.1 \sin(\theta + 81° 10') + 3.7 \sin(2\theta + 215° 50')$
$+ 0.5 \sin(3\theta + 75°)$

For September, $\Delta_v = 708^d.8 + 14.3 \sin(\theta + 73° 57') + 2.9 \sin(2\theta + 210° 24')$
$+ 0.3 \sin(3\theta + 165°)$

For October, $\Delta_v = 691^d.0 + 6.1 \sin(\theta + 119° 48') + 3.1 \sin(2\theta + 236° 28')$
$+ 1.1 \sin(3\theta + 210°)$

For November, $\Delta_v = 689^d.7 + 4.4 \sin(\theta + 83° 33') + 3.0 \sin(2\theta + 254° 00')$
$+ 0.0$

For December, $\Delta_v = 701^d.2 + 4.5 \sin(\theta + 133° 49') + 4.3 \sin(2\theta + 231° 57')$
$+ 1.0 \sin(3\theta + 63°)$

We have also for summer half year (April to September inclusive), for winter half year (October to March inclusive), and for the whole year the following expressions for the diurnal variation:—

For summer, $\Delta_v = 700^d.1 + 14.6 \sin(\theta + 83° 40') + 2.5 \sin(2\theta + 191° 01')$
$+ 0.5 \sin(3\theta + 255°)$

For winter, $\Delta_v = 702^d.0 + 5.1 \sin(\theta + 108° 54') + 4.0 \sin(2\theta + 229° 58')$
$+ 0.0$

For year, $\Delta_v = 701^d.0 + 9.7 \sin(\theta + 90° 17') + 3.0 \sin(2\theta + 216° 22')$
$+ 0.2 \sin(3\theta + 255°)$

The following comparison may serve to show the general representation of the observations by the analytical expressions:—

Comparison for August.

	Observed.	Computed.	Difference.
$0^h\ 20^m.6$	718	718.2	0
2 "	715	715.1	0
4 "	712	711.3	+1
6 "	706	707.0	—1
8 "	700	699.9	0
10 "	691	690.0	+1
12 "	681	683.0	—2
14 "	686	684.3	+2
16 "	693	693.5	0
18 "	704	705.0	—1
20 "	715	713.9	+1
22 "	718	718.4	0

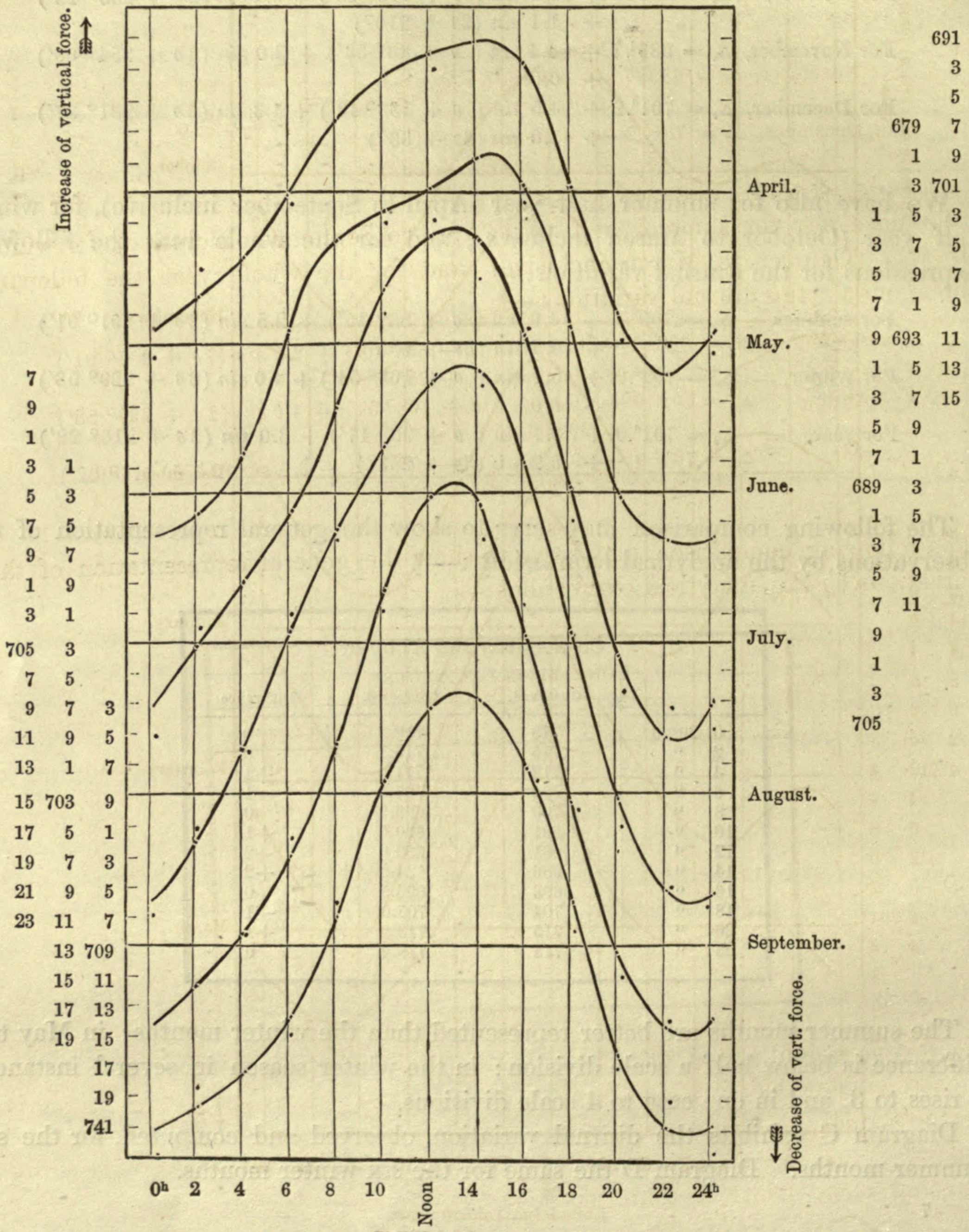
The summer months are better represented than the winter months; in May the difference is below half a scale division; in the winter season in several instances it rises to 3, and in one case to 4 scale divisions.

Diagram C exhibits the diurnal variation, observed and computed, for the six summer months. Diagram D the same for the six winter months.

(C). SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE, APRIL TO SEPTEMBER, 1841 TO 1845.

(Expressed in scale divisions.)

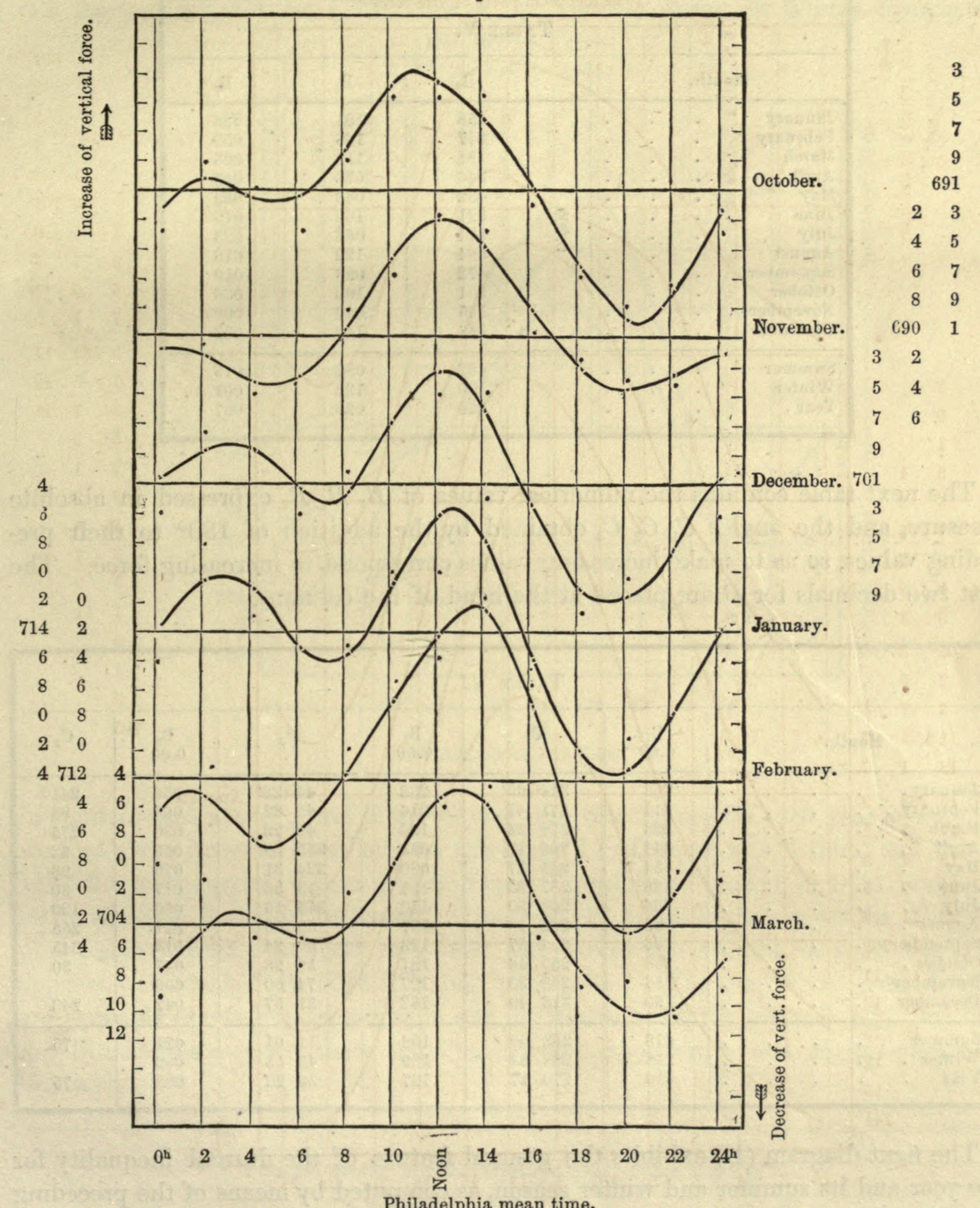
$1^d = 0.000033$ parts of the force.



(D). Solar-Diurnal Variation of the Vertical Force, October to March, 1841 to 1845.

(Expressed in scale divisions.)

$1^d = 0.000033$ parts of the force.



The numerical values of the coefficients B_1 B_2 B_3 in the general equation

$$\Delta_v = A + B_1 \sin(\theta + C_1) + B_2 \sin(2\theta + C_2) + B_3 \sin(3\theta + C_3)$$

expressed in parts of the horizontal force, are given in Table V. The first three decimals (0.000) have been placed in front of the table.

TABLE V.

Month.		B_1	B_2	B_3
January	0.000	158	181	026
February		247	168	053
March		181	119	023
April		346	073	043
May		432	063	059
June		521	102	013
July		574	086	023
August		564	122	016
September		472	096	010
October		201	102	036
November		145	099	000
December		148	142	033
Summer		482	082	016
Winter		170	132	001
Year		320	099	007

The next table contains the numerical values of B_1 B_2 B_3 expressed in absolute measure, and the angles C_1 C_2 C_3 obtained by the addition of 180° to their preceding values, so as to make increasing values correspond to increasing force. The first two decimals for B are placed at the head of the columns.

TABLE VI.

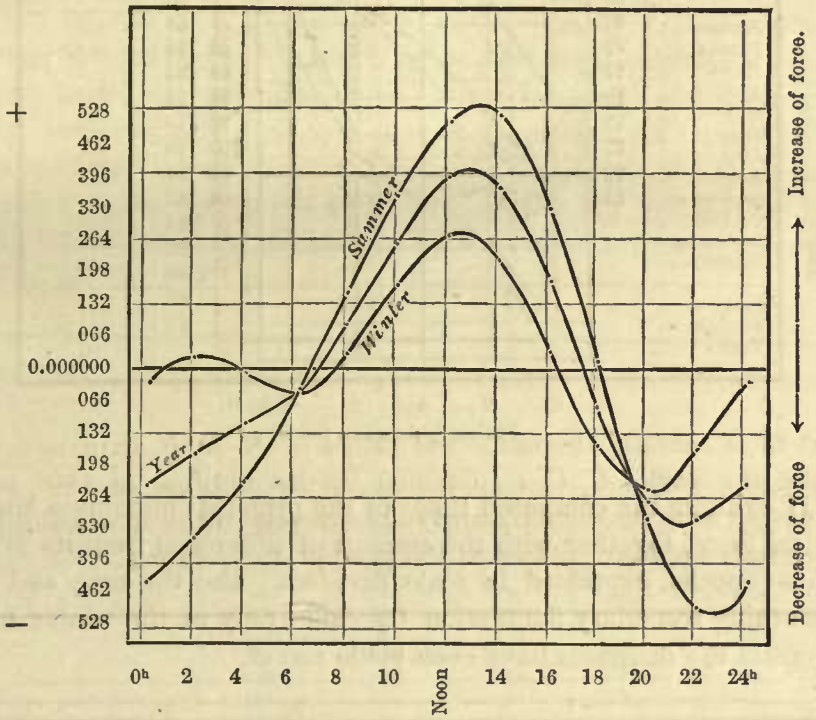
Month.	B_1 0.00	C_1	B_2 0.00	C_2	B_3 0.00	C_3
January	203	314° 09′	233	44° 22′	034	241°
February	317	271 47	216	46 22	068	93
March	233	278 24	152	40 22	030	275
April	444	269 12	093	355 59	055	52
May	554	265 17	080	324 31	076	98
June	668	267 22	131	13 56	017	30
July	736	266 30	110	354 16	030	120
August	723	261 10	157	35 50	021	255
September	606	253 57	123	30 24	013	345
October	258	299 48	131	56 28	047	30
November	186	263 33	127	74 00	000	
December	190	313 49	182	51 57	042	243
Summer	618	263 40	106	11 01	021	75
Winter	218	288 54	169	49 58	002	
Year	410	270 17	127	36 22	008	75

The next diagram (E) exhibits the general feature of the diurnal inequality for the year and its summer and winter season, as computed by means of the preceding formulæ. The greatest difference between the observed and computed values at any one hour is but $2\frac{1}{3}$ scale divisions at 2 A.M. in the winter season, and $1\frac{1}{2}$ divisions at the same hour in the annual curve. The absence of the secondary wave in the early morning hours during summer is as conspicuous as its presence

in the winter season; in the annual curve there is barely a trace of it left. We also recognize again the earlier occurrence of the maximum and minimum values in winter and their later appearance in summer. If we examine the resulting curves at Toronto[1] we find there the secondary morning fluctuation equally well marked

(E). Regular Solar-Diurnal Variation of the Vertical Force for Winter, Summer, and the whole Year.

(In parts of the force.)

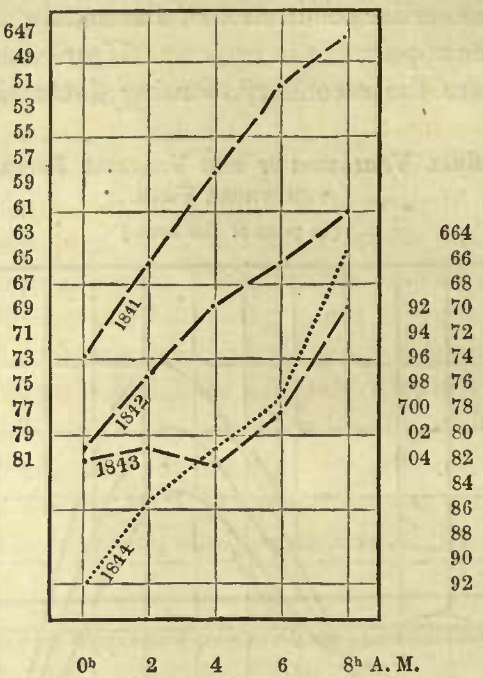


Philadelphia mean time.

in summer and winter; and if we inquire into this feature for each year separately, we find great irregularities between the hours 14 (Toronto astro'l time) and 22; in 1843 the secondary maximum and minimum is plainly developed, in 1844, and especially in 1845, it cannot be traced. Diagram (F) exhibits the curves for Philadelphia for each year. In 1841 and 1842 the curves are smooth, in 1843 the wave appears well marked, in 1844 it is just perceptible. These apparent irregularities are probably due to imperfections in our instruments; on the other hand, if we take the Philadelphia series, there may be a cyclic appearance and disappearance of this wave.

[1] Vol. III of the Toronto Observations, Table LXVIII.

DIAGRAM (F).



Philadelphia mean time.

Table VII exhibits the computed times of the principal maximum and minimum of the vertical force, together with the amount of difference from its average daily value at these epochs, expressed in scale divisions; also the time and amount of the early morning secondary fluctuation traceable only in the winter months; for these last values the diagrams have been made use of.

TABLE VII.

	Maximum at	Amount in scale divisions.	Minimum at	Amount in scale divisions.	Time elapsed between max. & min.	Secondary maximum at	Amount.	Secondary Minimum at	Am't.	Secondary range.
January	$13^h 07^m$	$-8^d.5$	$19^h 27^m$	$+9^d.4$	$6^h 20^m$	$2\frac{1}{2}^h$	-4	7^h	$+2$	6
February	13 41	-11.5	20 21	$+10.6$	6 40	$1\frac{1}{2}$	$+1$	$4\frac{1}{2}$	$+4$	3
March	12 46	-8.9	21 04	$+6.7$	8 18	3	-1	$6\frac{1}{2}$	$+2$	3
April	13 52	-10.2	22 04	$+11.8$	8 12	-----	----	-----	----	-----
May	14 34	-13.1	22 58	$+12.6$	8 24	-----	----	-----	----	-----
June	13 09	-17.2	22 35	$+16.2$	9 26	-----	----	-----	----	-----
July	13 33	-18.2	23 10	$+17.3$	9 37	-----	----	-----	----	-----
August	13 02	-20.8	23 06	$+15.5$	10 04	-----	----	-----	----	-----
September	13 23	-16.8	23 20	$+12.6$	9 57	-----	----	-----	----	-----
October	11 16	-7.6	20 34	$+8.7$	9 18	2	-1	5	$+1$	2
November	12 35	-7.4	19 59	$+3.9$	7 24	1	$+1$	$4\frac{1}{2}$	$+3$	2
December	12 31	-7.8	19 09	$+7.8$	6 38	3	-3	7	$+1$	4
Summer	13 29	-15.8	22 55	$+14.1$	9 26	-----	----	-----	----	-----
Winter	12 43	-8.2	20 08	$+7.6$	7 25	2	-1	6	$+2$	3
Year	13 02	-11.9	21 25	$+9.7$	8 23	-----	----	-----	----	-----

The extreme variation in the time of the maximum in the course of a year is 3^h 18^m, and of the minimum 4^h 11^m.

At Toronto the occurrence of the maxima and minima is later than at Phila-

delphia; from Table LXVIII, Vol. III of the Toronto Observations, we find the maximum at 5^h, a secondary minimum at 14^h, a secondary maximum at 18^h, and the minimum at 22^h; the maximum is therefore apparently delayed at Toronto 4^h, the minimum $4\frac{3}{4}^h$, the secondary wave is likewise retarded by about 4 hours. This epochal difference I take, most likely, to be a distinctive feature due to the localities; there is also a remarkable difference in the amount of the diurnal range as will presently appear. The degree of sensibility in the adjustment of the centre of gravity of the instrument affects most the latter difference, whereas the epochal difference may be supposed to depend, in a measure, upon the sensibility of the magnet in regard to changes of temperature and consequent changes of magnetism.

The change in the adopted value of the correction for 1° of change in the temperature (expressed in scale divisions) as used in present reduction (10.8), and as used in four volumes of record and reduction (13.5) gives us the means of a partial test of the effect on the epochs, we find from the plates in Vol. IV the time of the maximum $1\frac{1}{2}$ P. M. and of the minimum $11\frac{1}{2}$ P. M., which though somewhat nearer to the Toronto epochs, still leave a large discrepancy.

TABLE VIII.—AMPLITUDE OF THE DIURNAL VARIATION OF THE VERTICAL FORCE.

	Maximum 0.00	Minimum 0.00	Range 0.00	Maximum 0.00	Minimum 0.00	Range 0.0
January	028	031	059	359	398	0757
February	038	035	073	484	447	0931
March	029	022	051	377	283	0660
April	034	039	073	431	500	0931
May	043	042	085	555	535	1090
June	057	054	110	725	686	1411
July	060	057	117	769	733	1502
August	068	051	119	878	654	1532
September	055	041	097	712	532	1244
October	025	029	054	323	369	0692
November	024	013	037	313	166	0479
December	026	026	052	330	332	0662
Summer	052	046	098	667	597	1264
Winter	027	025	052	349	322	0671
Year	039	032	071	503	410	0913
	In parts of the force.			In absolute measure.		

The diurnal range at Toronto is very much less than at Philadelphia; in 1841–42 the range was but one-half of that observed at Philadelphia, and for later years (see Table LXVIII of Vol. III of the Toronto Observations) the ranges compare as follows: Toronto 0.00019, Philadelphia 0.00071.

In diagram G the diurnal range for each month is exhibited (expressed in parts of the force).

(G). Diurnal Range of the Vertical Force.

(In parts of the force.)

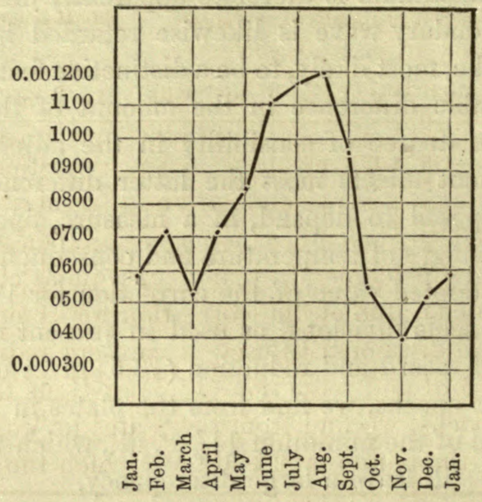


Table IX contains the times when the mean value of the vertical force is reached each day—arranged for monthly averages. In some months the average daily value is attained four times, but generally only twice. The table contains the two principal epochs (one A. M. the other P. M.); those produced by the secondary wave can easily be made out by means of the preceding diagrams.

Table IX.—Principal Epochs of Mean Vertical Force.

	A. M.	P. M.
January	8h 58m	3h 47m
February	7 42	5 16
March	8 27	4 53
April	6 15	6 13
May	5 55	6 21
June	6 36	6 04
July	6 08	6 10
August	7 30	6 04
September	7 51	6 28
October	6 34	4 19
November	8 11	4 49
December	8 27	3 39
Summer	6 43	6 13
Winter	7 52	4 29
Year	7 06	5 33

The next table contains the computed diurnal variation of the vertical force, expressed in absolute measure; compared with Table IV, it shows the differences between the observed and computed values.

Table X.—Computed Solar-Diurnal Variation of the Vertical Force, expressed in Absolute Measure.

The first two places of decimals 0.00 are placed ou the side; + indicates more, — less than the monthly average.

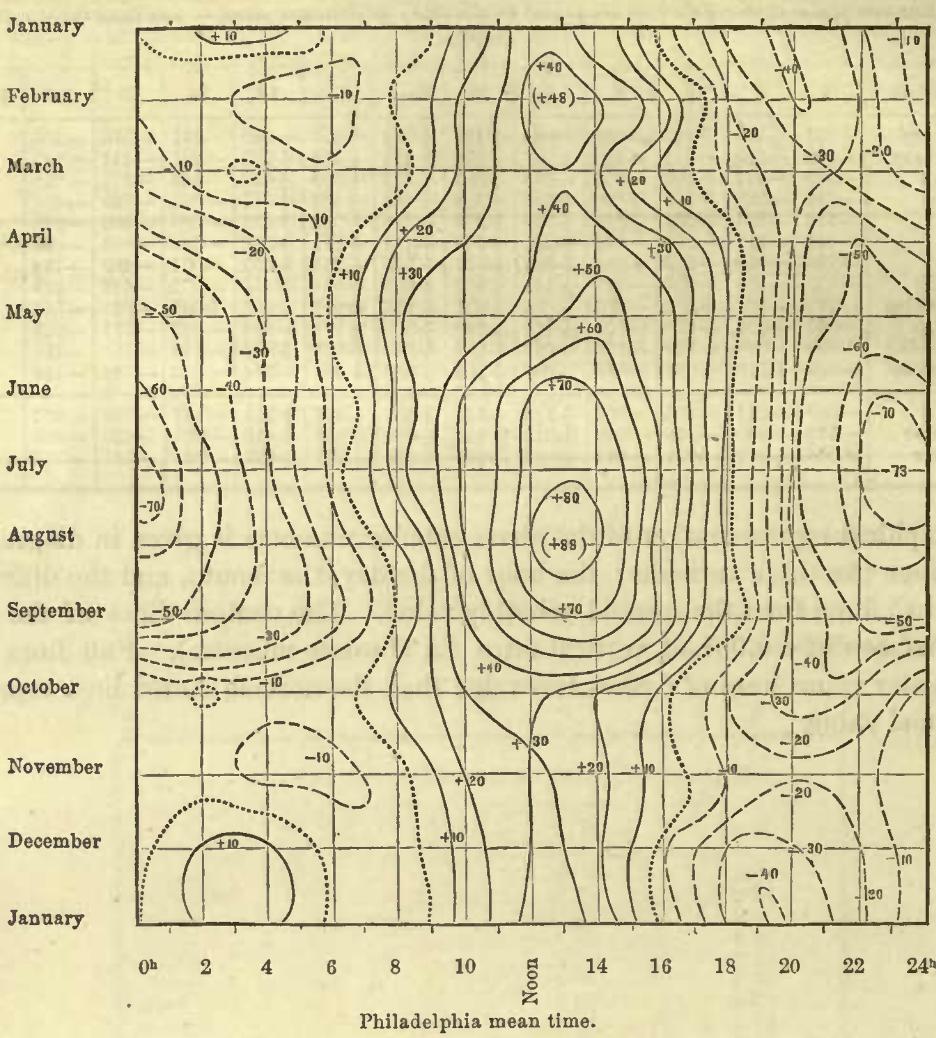
	1841-45.	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	+20m.6
	January	+025	+165	+123	—030	—046	+161	+355	+258	—080	—351	—376	—203	
	February	—072	—085	—178	—123	+063	+258	+436	+465	+203	—241	—444	—283	
	March	—135	—017	—008	—072	—013	+207	+372	+304	+059	—165	—275	—258	
	April	—385	—266	—165	+008	+216	+338	+402	+436	+317	—025	—385	—491	
	May	—516	—461	—262	+068	+309	+372	+448	+554	+423	—000	—402	—533	
0.00	June	—601	—419	—245	—042	+241	+533	+711	+681	+398	—068	—503	—685	
	July	—706	—550	—279	+034	+305	+537	+723	+744	+457	—051	—499	—714	
	August	—630	—499	—338	—157	+144	+563	+859	+804	+414	—072	—448	—639	
	September	—512	—431	—351	—207	+076	+431	+673	+677	+436	+047	—321	—516	
	October	—055	+030	—030	—008	+165	+309	+300	+182	—004	—237	—377	—275	
	November	—059	—089	—148	—131	+017	+207	+313	+233	+046	—123	—161	—106	
	December	—008	+110	+097	—008	—008	+182	+330	+203	—114	—313	—304	—165	
	Year	—309	—211	—152	—055	+123	+343	+495	+457	+211	—131	—368	—402	
	Summer	—558	—440	—275	—050	+212	+465	+634	+651	+410	—025	—423	—600	
	Winter	—046	+017	—021	—051	+034	+216	+338	+271	+021	—228	—321	—216	

A graphical representation of the above tabular numbers is given in diagram H, based upon the three variables: the hour of the day, the month, and the difference of vertical force from the normal monthly value. The contour lines of the magnetic surface differ 0.001 of vertical force (in absolute measure). Full lines indicate greater value, lines of dashes less value than the normal, dotted lines represent the normal value.

(H). DIFFERENCES FROM ITS NORMAL VALUE OF THE VERTICAL FORCE, FOR EACH HOUR AND MONTH.

(Expressed in absolute measure.)

0.00.



Annual Inequality of the Vertical Force.—The minor and irregular disturbances in the adjustment of the magnetometer, as well as the effect of the progressive and secular changes, tend to make the determination of the annual inequality in the vertical force a task of some delicacy, and the results deduced from our series of observations should be considered as approximate.

Taking the monthly normals of the years 1842, 1843, and 1844, the only years which could be made complete, and correcting the monthly means for 42 scale divisions of annual increase, the following table was formed:—

Monthly Normals.												
Year.	January.	Feb'u'ry.	March.	April.	May.	June.	July.	August.	Sept'ber.	October.	Nov'ber.	Dec'ber.
1842 . . .	658	707	658	659	656	655	662	677	682	706	716	710
1843 . . .	702	710	691	707	686	678	677	691	708	710	742	746
1844 . . .	731	728	760	756	756	758	796	773	801	778	770	749
Mean . . .	697	715	703	707	699	697	712	714	730	731	743	735
Corr'd . .	+19	+16	+12	+ 9	+ 5	+ 2	— 2	— 5	— 9	—12	—16	—19
Corr'd m. . .	716	731	715	716	704	699	710	709	721	719	727	716
Mean monthly val.	— 1	—16	0	— 1	+11	+16	+ 5	+ 6	— 6	— 4	—12	— 1

The vertical force appears, therefore, to be greater in May, June, July, and August, and less in the remaining months; the range is about 32 scale divisions $=0.00105$ parts of the force, or 0.0135 in absolute measure.

PART IX.

INVESTIGATION

OF THE

LUNAR INFLUENCE ON THE MAGNETIC VERTICAL FORCE, INCLINATION, AND TOTAL FORCE.

INVESTIGATION

OF THE

INFLUENCE OF THE MOON ON THE MAGNETIC VERTICAL FORCE.

THE method of discussion of the lunar effect on the vertical component of the magnetic force in no way differs from that employed for the horizontal component, which latter has been explained in Part VI.

The series of observations available for the lunar discussion extends from February, 1841, to June, 1845, inclusive. From February, 1841, to October, 1843, the observations are bi-hourly; from October, 1843, to the end of the series they are hourly. The record of May, 1841, is not quite complete, and in January, February, and March, 1843, but one observation a day is recorded. As increasing numbers denote a decrease of force, a positive sign of the tabulated differences between monthly normals and each individual undisturbed reading (at the normal temperature) indicates a greater force than the normal value, a negative sign indicates the reverse. 30 scale divisions being the limit beyond which difference an observation has been considered as belonging to the class of disturbances, all differences here recorded are below this limit. One scale division is 0.000033 parts of the force. The tabular numbers are expressed in scale divisions.

In tracing out the lunar effect upon the vertical force we have to contend with greater irregularities than was experienced in the case of the horizontal force. The vertical force magnetometer is more subject to changes, and the correction for temperature far exceeds that of the horizontal force.

The total number of observations and differences formed in the inquiry of the dependence of the force upon the moon's hour angle is 19513, which distribute themselves over the months and years as follows:—

TABLE I.—NUMBER OF OBSERVATIONS FOR LUNAR DISCUSSION.

Month.	1841.	1842.	1843.	1844.	1845.	Sum.
January	---	207	---	544	611	1362
February.	239	198	---	549	554	1540
March	257	286	---	502	539	1584
April	256	255	250	512	581	1854
May	207	246	288	617	602	1960
June	219	275	296	529	571	1890
July	258	281	308	581	---	1428
August	283	284	314	535	---	1416
September	267	246	292	568	---	1373
October	280	298	580	607	---	1765
November	275	299	528	596	---	1698
December	239	304	541	559	---	1643
Year	2780	3179	3397	6699	3458	19513

TABLE II.—DISTRIBUTION OF NUMBERS ACCORDING TO WESTERN AND EASTERN HOUR ANGLES OF THE MOON.

Year.	Western hour angles.	Eastern hour angles.
1841	1388	1392
1842	1588	1591
1843	1694	1703
1844	3321	3378
1845	1728	1730
Sum	9719	9794

TABLE III.—DIFFERENCES FROM THE MONTHLY NORMALS, 1841.

Western hour angles of the moon.

1841.	U. cul. 0^h	1	2	3	4	5	6	7	8	9	10	11^h
January	----	---	---	---	---	---	---	---	---	---	---	---
February	+3	+1	+3	−2	+8	−5	0	+1	−1	+4	−2	−3
March	−6	+5	−3	+6	+1	+9	−6	+2	−7	−1	−3	0
April	−1	+1	0	+2	−1	+3	−1	0	−1	+7	−7	+2
May	+5	−1	+2	+1	+4	+7	+3	+4	+4	+1	+5	−5
June	0	−6	−5	+2	−8	−2	+5	−3	−4	+8	+7	0
July	−4	+8	−6	+2	−7	+5	−3	+1	−1	−7	+5	−4
August	−1	−5	+1	−6	−5	−6	0	−1	0	+1	−3	−1
September	+3	+3	+1	+1	−6	+5	−2	+2	0	−4	−3	−3
October	−8	−1	−2	0	−2	−1	−4	0	+1	0	+6	+4
November	−3	+3	−1	+3	−3	−3	−6	−5	+1	−3	+4	−1
December	−5	−4	0	−1	−2	−5	−3	−3	+5	+2	−3	−2
Year	−1.5	+0.4	−0.9	+0.7	−2.0	+0.6	−1.5	−0.2	−0.3	+0.7	+0.5	−1.2

Eastern hour angles.

1841.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	---	---	---	---	---	---	---	---	---	---	---	---
February	−3	−2	+1	−3	+3	−2	+4	−3	+3	+2	+2	−1
March	0	+2	+3	+7	−1	+8	−7	+1	−8	0	−6	+1
April	0	−4	0	−1	0	−2	−1	−3	−1	−1	−1	+1
May	−2	−2	+1	−6	+1	−5	+1	−8	+3	−10	+3	−4
June	+1	+5	−2	+1	+5	−7	+5	+3	−1	−6	−2	−8
July	+1	−4	+6	0	−2	+5	−2	+5	−2	+2	−2	+7
August	+1	−1	+2	0	0	+1	+4	+5	+2	+4	+1	+3
September	−3	+2	−4	0	−1	+2	−1	−2	+1	0	+2	+8
October	+3	+4	+4	−1	+1	+1	+4	−1	+2	−2	−4	−4
November	+2	−3	+1	0	−1	+5	−3	+5	0	+8	−2	+2
December	+6	+6	+1	+6	+3	+2	+3	−10	0	−10	0	−7
Year	+0.5	+0.3	+1.2	+0.3	+0.7	+0.7	+0.6	−0.7	−0.1	−1.2	−0.8	−0.2

TABLE IV.—DIFFERENCES FROM THE MONTHLY NORMALS, 1842.

Western hour angles of the moon.

1842.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11ʰ
January	−6	0	−1	−7	−6	−5	+4	+8	−3	+9	+4	+7
February	−3	0	+5	−6	+4	−3	−9	−11	+6	+3	+7	+6
March	+5	+2	+3	0	+2	−4	−6	−6	+4	−1	−5	+5
April	+2	+2	0	0	−11	−2	−5	−3	−7	−1	+1	−5
May	−5	+3	−3	+2	−3	+4	0	0	+5	−2	+3	−7
June	+3	+5	+3	+2	−2	+1	+3	−3	+8	−7	+6	−7
July	−2	−4	−3	+2	−6	−2	0	+2	+4	+3	+7	+1
August	+2	−2	+1	−4	0	+1	−3	−2	+1	−1	+8	−3
September	+3	−5	−2	−7	−5	−3	+3	−1	−3	+6	−4	+5
October	0	0	−3	+4	+1	−5	+4	−1	+1	+4	−3	0
November	−3	+1	+1	−3	+2	+2	+3	0	+1	−4	+2	0
December	−1	−3	+1	+4	−4	+4	−1	+1	+4	−1	−3	+3
Year	−0.4	−0.1	+0.2	−1.1	−2.3	−1.0	−0.6	−1.3	+1.8	+0.7	+2.0	+0.4

Eastern hour angles.

1842.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11ʰ
January	+5	+4	−3	−1	−5	−3	−2	+3	−4	+4	0	−4
February	+6	+1	−2	+4	−3	+3	−1	−7	−1	+9	−5	+3
March	−4	+3	−4	−3	−4	−6	+2	−4	+2	+2	0	+4
April	−3	+4	−1	+6	+3	+10	+1	+4	+1	0	+6	+2
May	+5	+2	+6	−4	0	−2	−2	−1	−12	+8	−8	+3
June	0	−6	0	−8	+3	−5	−3	−9	+3	−5	+3	−5
July	+2	+1	+2	+1	−3	−1	+1	−2	−1	0	−3	−1
August	+5	−3	−1	0	+1	0	+2	0	+2	0	+7	−3
September	−6	−2	−1	+2	−1	−1	+2	+9	+2	+12	+4	−3
October	−9	+1	−4	−3	0	−3	+2	+2	+2	+2	0	+2
November	+3	−1	−6	−2	−2	0	+2	+2	+1	+3	−1	0
December	+5	−1	−2	−1	−1	0	0	0	−5	+1	−3	−3
Year	+0.8	+0.2	−1.3	−0.8	−1.0	−0.7	+0.3	−0.3	−0.8	+3.0	0.0	−0.4

TABLE V.—DIFFERENCES FROM THE MONTHLY NORMALS, 1843.

Western hour angles of the moon.

1843.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	---	---	---	---	---	---	---	---	---	---	---	---
February	---	---	---	---	---	---	---	---	---	---	---	---
March	---	---	---	---	---	---	---	---	---	---	---	---
April	+10	—3	+14	—5	+8	+1	0	+1	—5	+3	—2	—3
May	+6	—3	+4	+3	+3	—3	—3	—5	—1	—4	+1	—5
June	+1	—1	—4	+2	+2	+2	+1	+2	+1	+3	+4	+4
July	+7	+5	+4	+5	+4	+8	+3	+3	+3	+1	—1	—1
August	0	—2	+2	—1	+4	0	+5	—3	+3	—4	+2	—2
September	—3	+8	0	+5	—3	+4	+4	—7	+3	—3	+1	—3
October	+2	+2	+3	+3	+2	+2	—1	—4	—2	—3	—2	—3
November	+1	+2	+2	—3	+1	—2	—1	+1	0	+1	+4	0
December	+1	+2	+3	—2	0	+2	+2	+1	—1	0	+1	—2
Year	+2.4	+1.3	+3.0	+0.4	+2.0	+1.3	+0.8	—1.1	—0.2	—0.6	+0.9	—1.7

Eastern hour angles.

1843.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	---	---	---	---	---	---	---	---	---	---	---	---
February	---	---	---	---	---	---	---	---	---	---	---	---
March	---	---	---	---	---	---	---	---	---	---	---	---
April	—2	0	—1	—6	—5	—6	+5	+1	+4	0	—2	+1
May	—2	—6	—3	0	+2	—1	+1	+2	+4	+2	+6	+2
June	+2	0	+1	+2	—1	+2	—2	+1	—1	0	+2	—2
July	—5	—4	—5	—5	—3	—5	—6	—3	+2	+1	+3	+4
August	+2	—2	0	—1	—1	—1	+5	—2	+2	—4	+4	+2
September	—2	0	—3	—1	—6	—3	—2	+4	+3	+1	—2	+4
October	—5	—1	+1	+1	0	0	0	0	0	+2	+3	0
November	—1	0	—3	+1	+3	—1	—1	—3	—1	+2	—2	+1
December	—2	0	—2	—1	—3	—2	—3	—3	—3	—2	—3	0
Year	—1.9	—1.2	—1.6	—0.9	—1.2	—1.7	—0.6	—0.8	+0.5	0.0	+0.6	+1.1

In making up the annual means, the October, November, and December values have received double weight; they are derived from double the number of observations.

TABLE VI.—DIFFERENCES FROM THE MONTHLY NORMALS, 1844.

Western hour angles of the moon.

1844.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	+1	+5	0	0	—2	—1	—2	—1	—1	—1	—1	0
February	—1	—1	+1	+1	0	—1	+3	+3	0	+1	—1	—3
March	—4	—5	+2	—1	+3	—1	+1	—2	+1	—1	0	—3
April	+1	—1	+5	+2	0	+1	—4	+3	+3	0	—2	—3
May	0	+1	+1	+1	+1	+2	+3	+2	+2	+1	+2	0
June	+1	—2	—2	—2	—2	—1	—2	0	0	+3	0	+3
July	+2	+2	+2	+1	+1	+3	0	0	—2	0	0	0
August	+1	—2	—2	—2	—1	—2	—2	—6	—2	—4	—1	0
September	+1	+1	+1	—1	—1	—1	—2	0	—2	—2	—3	0
October	0	0	0	—2	—1	—3	—2	—3	—3	—2	0	+3
November	+2	+5	+5	+3	+2	0	+1	+3	+1	0	—2	—3
December	—1	0	+1	+3	0	—1	—1	—1	+2	+1	0	0
Year	+0.3	+0.3	+1.2	+0.3	0.0	—0.4	—0.6	—0.2	—0.1	—0.3	—0.7	—0.5

Eastern hour angles.

1844.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	—1	—4	—2	—3	—2	—2	—3	0	+1	—2	—1	+3
February	—3	—1	+2	+1	0	—3	—4	—2	+1	—1	—1	0
March	—6	+1	—4	—2	—1	+4	+2	+5	+2	—4	—4	—1
April	—1	+1	+1	0	—1	—1	—3	—1	0	—1	—1	+2
May	—1	—2	0	—2	—2	—1	—3	—2	—2	—1	0	0
June	+1	0	+1	+1	0	+1	+2	—2	—1	+2	0	—1
July	+1	0	0	+1	0	—1	+1	+2	+1	0	+1	+1
August	—2	—1	—1	0	+2	+6	+4	+4	+3	+2	+1	—1
September	—1	—2	—1	—1	0	0	+2	+3	+2	+1	+4	+4
October	+4	+1	+1	+1	0	+2	+1	+1	0	+3	+1	+3
November	—4	—4	—1	+1	+1	+1	+2	+1	0	+1	+3	+2
December	+2	+2	+1	+2	0	+1	—3	—3	—1	—1	+2	—1
Year	—1.0	—0.8	—0.2	—0.1	—0.3	+0.6	—0.2	+0.5	+0.5	—0.1	+0.4	+1.0

TABLE VII.—DIFFERENCES FROM THE MONTHLY NORMALS, 1845.
Western hour angles of the moon.

1845.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
January	+1	+2	+2	+2	+2	+4	+1	+1	—2	0	+3	+2
February	—2	0	—3	—2	—2	—2	0	0	+1	0	—1	+1
March	0	—1	—2	+1	+1	0	+2	+1	—2	0	+4	—1
April	—2	—4	—2	0	—1	—1	0	+2	+5	+4	+4	+2
May	—1	+1	+2	+1	+2	+3	—2	—1	0	+1	—1	0
June	+1	+1	+2	0	—2	0	—1	—1	—3	—4	—3	—2
Mean	—0.5	—0.2	—0.2	+0.3	0.0	+0.7	0.0	+0.3	—0.2	+0.2	+1.0	+0.3

Eastern hour angle.

1845.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
January	0	—1	—5	—3	0	—1	0	—2	—2	—4	—3	0
February	+2	+3	+3	+3	+2	0	0	—1	0	—1	—2	0
March	+4	+2	—1	+2	—2	+1	—1	+1	—2	—2	—3	—3
April	—1	+1	—1	—1	—2	—2	—1	—1	0	+1	+1	+1
May	—1	—1	—2	—3	—2	—1	—1	0	+1	+1	0	—1
June	—1	+1	0	+1	+2	+1	+2	+2	+1	+3	+2	+3
Mean	+0.5	+0.9	—1.0	—0.2	—0.3	—0.3	—0.2	—0.2	—0.3	—0.3	—0.9	0.0

Before we combine the above results of years and parts of years, it is desirable to inquire into the variability of the lunar effect in summer and winter. Considering the months from April to September (inclusive) as summer months, and those from October to March (inclusive) as winter months, and combining the differences from the monthly normals in each year according to the number of observations, we obtain the following results:—

TABLE VIII.—LUNAR-DIURNAL VARIATION IN SUMMER AND WINTER, 1841 TO 1845.
(Expressed in scale divisions.)
Western hour angles.

	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
Summer	+1.0	—0.1	+0.6	+0.2	—1.0	+0.8	—0.4	—0.4	+0.1	—0.1	+0.6	—1.0
Winter	—0.8	+0.8	+0.9	+0.1	+0.4	—0.6	—0.5	—0.5	0.0	+0.1	+0.4	0.0

Eastern hour angles.

	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
Summer	—0.5	—0.8	—0.3	—0.9	—0.4	—0.4	+0.4	+0.4	+0.6	+0.6	+1.1	+0.8
Winter	—0.2	+0.3	—0.9	+0.3	+0.4	+0.2	—0.5	—0.7	—0.5	0.0	—1.1	0.0

These numbers are sufficiently irregular to indicate that we cannot hope to deduce any separate results for the seasons, the number of observations (about 9800) being altogether insufficient. We can therefore in our general combination

of the annual means give equal weight to the results from the six months of hourly observations in 1845, and to the results from the twelve months of bi-hourly observations in 1842; compared with these results, those of 1844 have the weight two.

TABLE IX.—RECAPITULATION OF THE ANNUAL MEANS EXHIBITING THE LUNAR-DIURNAL VARIATION FROM OVER 19,500 OBSERVATIONS BETWEEN FEBRUARY, 1841, AND JUNE, 1845, INCLUSIVE.

Western hour angles.

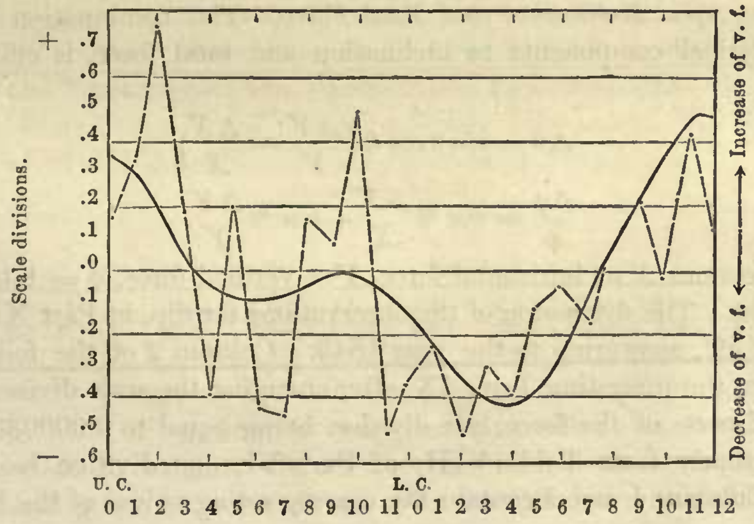
Weight.	Year.	U. cul. 0^h	1	2	3	4	5	6	7	8	9	10	11^h
1	1841	−1.5	+0.4	−0.9	+0.7	−2.0	+0.6	−1.5	−0.2	−0.3	+0.7	+0.5	−1.2
1	1842	−0.4	−0.1	+0.2	−1.1	−2.3	−1.0	−0.6	−1.3	+1.8	+0.7	+2.0	+0.4
1	1843	+2.4	+1.3	+3.0	+0.4	+2.0	+1.3	+0.8	−1.1	−0.2	−0.6	+0.9	−1.7
2	1844	+0.3	+0.3	+1.2	+0.3	0.0	−0.4	−0.6	−0.2	−0.1	−0.3	−0.7	−0.5
1	1845	−0.5	−0.2	−0.2	+0.3	0.0	+0.7	0.0	+0.3	−0.2	+0.2	+1.0	+0.3
	Mean	+0.10	+0.33	+0.75	+0.15	−0.38	+0.20	−0.42	−0.45	+0.15	+0.07	+0.50	−0.53

Eastern hour angles.

Weight.	Year.	L. cul. 0^h	1	2	3	4	5	6	7	8	9	10	11^h
1	1841	+0.5	+0.3	+1.2	+0.3	+0.7	+0.7	+0.6	−0.7	−0.1	−1.2	−0.8	−0.2
1	1842	+0.8	+0.2	−1.3	−0.8	−1.0	−0.7	+0.3	−0.3	−0.8	+3.0	0.0	−0.4
1	1843	−1.9	−1.2	−1.6	−0.9	−1.2	−1.7	−0.6	−0.8	+0.5	0.0	+0.6	+1.1
2	1844	−1.0	−0.8	−0.2	−0.1	−0.3	+0.6	−0.2	+0.5	+0.5	−0.1	+0.4	+1.0
1	1845	+0.5	+0.9	−1.0	−0.2	−0.3	−0.3	−0.2	−0.2	−0.3	−0.3	−0.9	0.0
	Mean	−0.35	−0.23	−0.52	−0.30	−0.40	−0.13	−0.05	−0.17	+0.05	+0.22	−0.05	+0.42

If we represent these values graphically, we find the general shape of the curve to be similar to that of the horizontal component, it is double crested with a principal maximum a little before the upper culmination, and a principal minimum about $3\frac{1}{2}$ hours after the lower culmination of the moon; the average epoch of the vertical force tide is, therefore, about one and a half hours apparently in advance of the culminations. The secondary wave is very feeble, its greatest value

(A.) LUNAR-DIURNAL VARIATION OF THE VERTICAL FORCE, 1841 TO 1845.



happens about 9^h, western hour angle, and its least value about three hours before, giving a range of nearly one-tenth part of the principal range. The observed values for the hours 8, 9, 10 (west) however, seem to indicate that the secondary wave is really larger, but in the present case apparently reduced by the accidentally low values at the hours 11 and 12.

The following expression has been deduced to express the lunar-diurnal variation of the vertical force:—

$$V_{☾} = -0.04 + 0.27 \sin(\theta + 72°) + 0.20 \sin(2\theta + 134°)$$

θ counts from the upper culmination, westward; $V_{☾}$ is expressed in scale divisions. The smooth, full curve in the diagram is computed by the formula; the differences between the observed and computed values are sufficiently well exhibited in the diagram. The probable error of any single hourly value is ± 0.20 scale divisions.

In the following expression M signifies millionth parts of the force:—

$$V_{☾} = \overset{M}{-1.3} + \overset{M}{8.9} \sin(\theta + 72°) + \overset{M}{6.6} \sin(2\theta + 134°).$$

Maximum value of $V_{☾}$, 28^m before the upper culmination, $= +.38$ scale divisions; minimum value at $15^h\ 30^m$, -0.43 scale divisions, hence lunar-diurnal range 0.81 scale divisions $= 0.000027$ parts of the force $= 0.00034$ in absolute measure. This range is so small that the correction for temperature due to a change of but 0°.08 would surpass it.

We have already seen that we cannot bring a sufficient number of observations to bear upon any *part* of the entire series, and are therefore not in a condition to pursue this subject of the lunar effect to any greater length.

At Toronto the curve is also double-crested with maxima three and a half hours after the moon's transits, but compared with Philadelphia the principal and secondary waves appear exchanged. The range at Toronto is 0.000012 parts of the force, nearly one-half of the Philadelphia range; we have already noticed a similar difference of range in the solar-diurnal variation, the Toronto range of which was also about one-half of that at Philadelphia. In connection with this it may be well to state that the dip at Toronto is 75° 15′, and at Philadelphia 71° 59.′

Lunar Effect upon Inclination and Total Force.—The combination of the horizontal and vertical components to inclination and total force, is effected by the formulæ:—

$$\Delta\theta = \sin\theta \cos\theta \left(\frac{\Delta Y}{Y} - \frac{\Delta X}{X}\right)$$

$$\frac{\Delta\phi}{\phi} = \cos^2\theta \frac{\Delta X}{X} + \sin^2\theta \frac{\Delta Y}{Y}$$

in which expressions X = horizontal force, Y = vertical force, ϕ = total force, and θ = inclination. The discussion of the observations for dip, in Part XII, gives the value $\theta = 71° 59'$, answering to the year 1843. Column 2 of the following table is derived from the preceding Table IX, after changing the scale divisions into their equivalents of parts of the force, one division being equal to 0.000033; column 3 is formed similarly from Table VIII, of Part VI, one division being equal to 0.0000365. Columns 4 and 5 contain the corresponding values of the lunar-diurnal

variation of the inclination and total force, the former expressed in seconds, the latter in parts of the total force. The letter M, heading columns 2, 3, and 5, signifies units of the sixth place of decimals or millionth parts of the force.

TABLE X.—LUNAR-DIURNAL VARIATION OF THE INCLINATION AND TOTAL FORCE.

☾'s hour angle.	$\frac{\Delta Y}{Y}$	$\frac{\Delta X}{X}$	$\Delta\theta$	$\frac{\Delta\phi}{\phi}$
	M.	M.	"	M.
U. C.	+ 3.3	+11.0	—0.5	+ 4.0
1	+10.9	+18.2	—0.4	+11.6
2	+24.7	+32.8	—0.5	+25.4
3	+ 5.0	+40.1	—2.1	+ 8.3
4	—12.5	+ 7.3	—1.2	—10.6
5	+ 6.6	+25.5	—1.1	+ 8.4
6	—13.9	0.0	—0.9	—12.6
7	—14.8	+ 3.6	—1.1	—13.0
8	+ 5.0	—21.9	+1.6	+ 2.4
9	+ 2.3	—14.6	+1.0	+ 0.7
10	+16.5	— 7.3	+1.4	+14.2
11	—17.5	+ 3.6	—1.3	—15.4
L. C.	—11.6	+25.5	—2.2	— 8.1
1	— 7.6	+ 7.3	—0.9	— 6.2
2	—17.2	+14.6	—1.9	—14.1
3	— 9.9	—11.0	+0.1	— 9.9
4	—13.2	+ 3.6	—1.0	—11.5
5	— 4.3	—25.5	+1.3	— 6.3
6	— 1.7	—47.4	+2.8	— 6.1
7	— 5.6	—21.9	+1.0	— 7.2
8	+ 1.6	—18.2	+1.2	— 0.3
9	+ 7.3	—36.5	+2.7	+ 3.1
10	— 1.7	— 3.6	+0.2	— 1.9
11	+13.9	+ 7.3	+0.4	+13.3

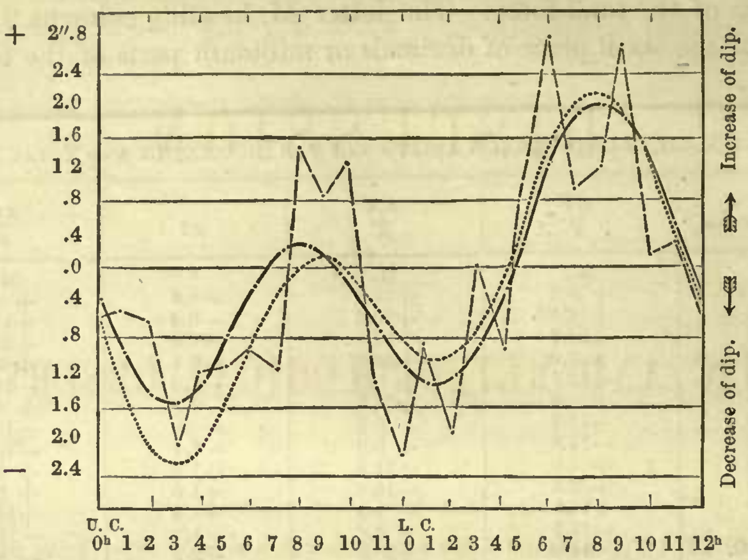
The numbers in column 2 are deduced from observations between 1841 and 1845, those in column 3 from observations between 1840 and 1845, the difference, however, is immaterial as far as it refers to the dip and total force, the lunar variations being nearly the same for a few adjacent years. The total number of observations employed in the combination is 41558.

The lunar-diurnal variation in the dip is well represented by the formula,

$$\theta_{☾} = -0''.06 + 0''.86 \sin(\theta + 156°) + 1''.30 \sin(2\theta + 206°)$$

the corresponding curve, as well as the observed values, are exhibited in the following diagram. The heavy smooth curve is the Philadelphia computed variation, the dotted curve the Toronto variation, inserted here for comparison. The correspondence between these curves is certainly remarkably close considering the minuteness of the lunar effect and the somewhat long process of deducing it.

(B.) LUNAR-DIURNAL VARIATION OF THE INCLINATION.



Maxima at 8^h and 20^h (principal), minima at 3^h (principal) and $13\frac{1}{2}^h$.

Total range 3″.6. Probable error of any single hourly representation ± 0″.7.

The lunar diurnal variation in the total force is represented by the equation:—

$$\overset{M}{\phi_{☾}} = -1.3 + \overset{M}{8.9}\, sin\, (\theta + 63°) + \overset{M}{6.3}\, sin\, (2\theta + 84°)$$

an expression not differing much from $V_{☾}$ owing to the large dip. The observed and computed values of $\frac{\Delta\phi}{\phi}$ are shown in the diagram, which nearly resembles that of the vertical force.

(C.) LUNAR-DIURNAL VARIATION OF THE TOTAL FORCE.

Maxima at $\frac{1}{2}^h$ (principal) and 11^h; minima at $7\frac{1}{2}^h$ and 17^h (principal).

Total range 0.000026. Probable error of any single hourly representation ± 0.000006.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

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DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

FOURTH SECTION,

COMPRISING PARTS X, XI, AND XII. DIP AND TOTAL FORCE.

ANALYSIS OF THE DISTURBANCES OF THE DIP AND TOTAL FORCE; DISCUSSION OF THE SOLAR DIURNAL VARIATION AND ANNUAL INEQUALITY OF THE DIP AND TOTAL FORCE; AND DISCUSSION OF THE ABSOLUTE DIP, WITH THE FINAL VALUES FOR DECLINATION, DIP AND FORCE BETWEEN 1841 AND 1845.

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PART X.

ANALYSIS

OF THE

DISTURBANCES OF THE DIP AND TOTAL FORCE.

ANALYSIS

OF THE

DISTURBANCES OF THE DIP AND TOTAL FORCE.

In the preceding discussion of the disturbances of the horizontal and vertical components of the magnetic force at the Girard College Observatory, the laws of their variations, as far as they have been recognizable from the series, were brought out and discussed, and this suffices, perhaps, in most cases, for any future application of theory, or for the purpose of testing hypotheses; but as it is also desirable for other comparisons to deduce the corresponding results for dip and total force from previous researches, it is proposed here to present the results of this combination numerically and in tabular form.

This combination is effected by the formulæ:—

$$\Delta\theta = \sin\theta\cos\theta\left(\frac{\Delta Y}{Y} - \frac{\Delta X}{X}\right) \text{ and } \frac{\Delta\phi}{\phi} = \sin^2\theta\frac{\Delta Y}{Y} + \cos^2\theta\frac{\Delta X}{X}$$

which expressions have already been used in the preceding part.

A strict treatment of the disturbances of either the dip or total force would require the formation of the difference of each observation of the vertical and horizontal force from its normal value (corresponding to the hour, month, and year), the conversion of these differences from units of scale value into parts of the respective force, and finally the numerical combination of the contemporaneous values of the two instruments by means of the above formulæ. To treat over 44,000 observations in this manner is impracticably laborious, and makes it desirable to substitute in its place another process less cumbrous, but, as regards results, equally effective. The method adopted avoids also the labor of forming normals, and especially that of separating the disturbances anew for each element. The method pursued in the discussion of the Toronto observations answers all purposes, and has also been adopted for the Philadelphia series; a more distinct idea, however, is here given for the limiting value beyond which disturbances are recognized. The method is as follows: Returning to the manuscript tables which contain the observations reduced to a uniform temperature and corrected for progressive change, as far as this was practicable, each observation marked there as a disturbance, that is, which differed as much or more than ± 30 scale divisions from the normal of the vertical force, and as much or more than ± 33 from the normal of the horizontal force, was transcribed and at once converted into its equivalent in parts of the force to which it respectively belonged. One scale division of the vertical force magnetometer equals 0.000033 parts, and of the horizontal force magnetometer 0.0000365 parts of the

respective force. These disturbances of the two components were tabulated in chronological order, and when, for any entry, but one of the constituent parts appeared disturbed, the corresponding difference from the normal of the contemporaneous second part, whatever amount that might be, was likewise entered in an adjoining column. The corresponding values of $\Delta\theta$ and $\frac{\Delta\phi}{\phi}$ were then easily computed for each disturbance, whether it occurred in both components or in one only.

Trustworthy contemporaneous readings of the two magnetometers commence with February, 1841, and continue to the close of the series in June, 1845; there is, however, an interval of time between the readings of the instruments which we are obliged to disregard; it amounts to but 5 minutes, the bifilar magnetometer having been read so much later.

As there is not generally a contemporaneous disturbance in the vertical and horizontal force, the total number of disturbed values obtained by the process explained above and employed, is necessarily much greater than it was for either of the components; it becomes therefore necessary to fix upon some limit of recognition for a disturbed value of the dip, and also of the total force. This is best done by the adoption of that value which will separate an equal proportion of disturbed values from the total number, as was done in the components; for the vertical component one in every 10.5 observations, for the horizontal component one in every 19.3 observations was separated as a disturbed value between February 1, 1841, and June 30, 1845; on the average, therefore, one in every 15 observations should be separated in the dip and total force series. During the time mentioned the number of observations of vertical force was 22,092, and of horizontal force 22,150, from which we should accordingly derive nearly 1470 disturbances. Now the number of computed values of $\Delta\theta$ and of $\frac{\Delta\phi}{\phi}$ is 2362, hence, marking in each set the 1470 highest values, the limit of $\pm$ 1.′1 is reached in the dip, and $\pm$ 0.00094 in the total force, which constitute the limiting values at and beyond which disturbances are recognized in each element. To render the series of disturbance results homogeneous, the disturbances at the odd hours after October, 1843, have been omitted. At Toronto the limit for the recognition of disturbances in the dip[1] was 1.′0, and in the total force 0.0004 parts of the force.

ANALYSIS OF THE DISTURBANCES OF THE INCLINATION.

The number of values of inclination differing $\pm$1.′1 or more from their normal amount, and which constitute the disturbance values is 1446, these are variously combined in the following tables, and, when necessary, are separated into two classes, those which increase and those which decrease the inclination; to the former the sign + is prefixed, to the latter the sign —. The aggregate and mean amount of disturbances are expressed in minutes of arc. The columns containing the number of disturbances are headed with the letter n. When ratios are given, they

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[1] Vol. III, p. xliii.

exhibit the proportion of the amount of disturbances during any given sub-period to the average amount of disturbances during the whole period. In the first and three subsequent tables the values for the first five months of 1841 are omitted, as there are no adequate means of extending the series beyond four full years.

TABLE I.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE INCLINATION IN EACH MONTH, DIVIDED INTO DISTURBANCES WHICH INCREASE AND WHICH DECREASE THE INCLINATION.

The values in brackets for January, February, and March, are interpolated, and are the average values of the corresponding months of the year preceding and the year following.

	1841 and 1842.				1842 and 1843.				1843 and 1844.				1844 and 1845.				Sums 1841 to 1845.				Ratio.	
	+	n	−	n	+	n	−	n	+	n	−	n	+	n	−	n	+	n	−	n	+	−
July . .	24.′5	16	15.′7	10	31.′5	17	17.′2	9	16.′9	10	4.′4	3	1.′1	1	12.′8	8	74.′0	44	50.′1	30	0.55	0.73
August .	34.5	16	14.0	9	13.6	8	33.5	22	20.8	13	0.0	0	42.4	29	3.9	3	111.3	66	51.4	34	0.83	0.75
Sept'ber .	104.7	40	14.4	10	20.7	9	114.8	64	8.4	4	7.2	6	25.4	14	10.9	6	159.2	67	147.3	86	1.19	2.15
October .	69.7	32	13.6	9	19.5	11	40.2	27	7.2	4	10.1	6	10.0	7	21.8	11	106.4	54	85.7	53	0.79	1.25
November	79.9	31	12.4	5	35.2	16	6.8	4	9.9	7	3.9	3	40.3	19	1.5	1	165.3	73	24.6	13	1.24	0.35
December	75.5	36	27.2	18	12.4	8	9.5	6	29.6	20	4.9	4	31.7	17	37.3	20	149.2	81	78.9	48	1.12	1.15
January .	72.3	33	40.5	24	(50.7)	(27)	(23.2)	(14)	29.1	21	5.9	4	23.7	10	7.7	5	175.8	91	77.3	47	1.31	1.12
February	101.6	45	40.3	26	(57.6)	(26)	(22.2)	(14)	13.7	8	4.2	3	5.8	3	0.0	0	178.7	82	66.7	43	1.34	0.97
March .	31.6	17	27.0	16	(37.5)	(20)	(18.6)	(12)	43.4	24	10.3	8	19.1	13	5.5	4	131.6	74	61.4	40	0.99	0.92
April . .	52.2	26	14 2	10	56.0	30	17.4	10	29.5	20	12.0	6	29.4	17	6.6	4	167.1	93	50.2	30	1.25	0.73
May . .	41.1	25	30.5	15	39.7	18	34.5	18	7.9	4	1.6	1	22.0	10	4.7	3	110.7	57	71.3	37	0.83	1.04
June . .	39.9	21	21.8	13	9.1	7	20.7	11	1.4	1	5.9	4	23.8	17	8.8	5	74.2	46	57.2	33	0.56	0.84

It would appear that during the colder season both sets of ratios present greater values; between September and February (inclusive) this ratio is, on the average, 1.16, and in the other months it is 0.84. Of the ratios decreasing the inclination the September and November values are somewhat anomalous, the first too high, the second too low.

The following table gives the annual inequality of the disturbances, irrespective of their sign.

TABLE II.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE INCLINATION IN EACH MONTH.

The series comprises the four years between July, 1841, and July, 1845. The ratio is that of the sums.

	Sum.	n	Ratio.
July	124.′1	74	0.62
August	162.7	100	0.80
September	306.5	153	1.51
October	192.1	107	0.95
November	189.9	86	0.94
December	228.1	129	1.13
January	253.1	138	1.25
February	245.4	125	1.22
March	193.0	114	0.96
April	217.3	123	1.07
May	182.0	94	0.90
June	131.4	79	0.65

The ratio near the autumnal equinox is the greatest of all, that about the vernal equinox is a little below the average value; the least value occurs probably near the summer solstice.

TABLE III.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE INCLINATION IN THE DIFFERENT YEARS OF OBSERVATION.

The ratios exhibit the variation due to the eleven year inequality.

Year.	Sum.	n	Ratio.
1841—1842	999.′1	503	1.65
1842—1843	742.1	408	1.22
1843—1844	288.2	184	0.47
1844—1845	396.2	227	0.66

The minimum amount of disturbances in the eleven year circle, therefore, occurred in the beginning of 1844.

TABLE IV.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES IN EACH YEAR ARRANGED FOR DISTURBANCES INCREASING THE INCLINATION AND FOR THOSE DECREASING IT; TOGETHER WITH THE RATIOS OF THE SUMS.

	Sum.		Sum.		Ratio.	
Year.	+	n	−	n	+	−
1841—1842	727.′5	338	271.′6	165	1.81	1.32
1842—1843	383.5	197	358.6	211	0.96	1.74
1843—1844	217.8	136	70.4	48	0.55	0.35
1844—1845	274.7	157	121.5	70	0.68	0.59

The minimum of the eleven year period is equally well marked in the disturbances increasing and in those decreasing the inclination. The sum of the positive values (1603.′5) is to the sum of the negative values (822.′1) as 1.95 to 1. At Toronto, between 1844 and 1848, this ratio was 5.6 to 1; the ratio, however, increased from 2.7 to 8.5 to 1 during this time.

In Tables V and VI, which exhibit the diurnal inequality of the disturbances, the whole series between February, 1841, and June, 1845, is employed. The sums, numbers, and ratios given do not in strictness apply to the even hours, but to an epoch 20 minutes later.

TABLE V.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE INCLINATION, DISTRIBUTED OVER THE EVEN HOURS OF THE DAY, AND RATIO SHOWING THE DIURNAL INEQUALITY OF THE SUM.

Hour.	Sum.	n	Ratio.
0	244.8	143	1.11
2	183.2	110	0.83
4	184.1	111	0.84
6	170.8	98	0.78
8	154.8	94	0.70
10	213.6	116	0.99
Noon	250.7	141	1.14
14	260.9	131	1.19
16	232.4	130	1.06
18	245.0	126	1.11
20	253.6	121	1.16
22	238.0	125	1.09

The hourly disturbances of the inclination exhibit a regular progression; between 1 A. M. and 11 A. M. the numbers fall short of the mean hourly value, and during the remaining afternoon and night hours they exceed this average value. The minimum occurs near 8 A. M. and the maximum near 8 P. M. There is, however, an indication of a superimposed smaller progression which, owing to the short series of observations, is not distinctly brought out. At Toronto we have a double progression, and the above ratios approximate to it. At Philadelphia a secondary maximum probably occurs about noon and a secondary minimum about 4 P. M.

Table VI shows the ratios at the different hours for disturbances increasing and disturbances decreasing the inclination.

TABLE VI.—AGGREGATE AMOUNT AND NUMBER OF HOURLY DISTURBANCES OF THE INCLINATION FOR INCREASING AND DECREASING VALUES; AND MEAN EFFECT OF DISTURBANCES.

Hour.	Sum. +	n	Sum. —	n	Ratio of sums. +	Ratio of sums. —	Excess of increasing disturbances.	Average diurnal effect of disturbances.
0	163.′9	94	80.′9	49	1.15	1.06	+83.′0	+0.′06
2	101.7	61	81.5	49	0.71	1.06	+20.2	+0.02
4	82.4	51	101.7	60	0.58	1.33	—19.3	—0.02
6	97.1	57	73.7	41	0.68	0.96	+23.4	+0.02
8	97.8	58	57.0	36	0.69	0.74	+40.8	+0.03
10	130.2	64	83.4	52	0.91	1.09	+46.8	+0.04
Noon . . .	139.3	75	111.4	66	0.98	1.45	+27.9	+0.02
14	193.2	90	67.7	41	1.35	0.89	+125.5	+0.10
16	148.5	79	83.9	51	1.04	1.09	+64.6	+0.05
18	178.7	86	66.3	40	1.25	0.87	+112.4	+0.09
20	198.3	87	55.3	34	1.39	0.72	+143.0	+0.11
22	181.5	91	56.5	34	1.27	0.74	+125.0	+0.10

The disturbances which increase the inclination show a very regular single progression (the value at 2 P. M. only being slightly anomalous); their minimum occurs at 4 A. M., and their maximum at 8 P. M. The disturbances decreasing the inclination are small in number at all hours, and show a tendency at double progression; principal maximum about noon, principal minimum about 8 P. M., secondary maximum about 4 A. M., and secondary minimum about 8 A. M. At Toronto the results appear different, but it is absolutely necessary for effective comparison to have results from contemporaneous series. As at Toronto, the disturbances increasing the inclination greatly preponderate over those decreasing it; the accumulated effect of this difference is shown in the column headed "Excess" (Table VI). At the hour 4 A. M. alone, we find the increasing disturbances inferior, at the hour 8 P. M. the difference has reached its maximum; at Toronto this maximum occurred an hour or two after midnight. The last column of Table VI exhibits the average diurnal effect of the disturbances (exceeding 1.′1 their normal value), the plus sign indicating a preponderance of increasing dip; the number of days is 1297.

The distribution of the disturbances according to their magnitude for an equal increase of 1′ is as follows:—

Between	Number of disturbances.
1.1 and 2.1 . . .	1096
2.1 " 3.1 . . .	247
3.1 " 4.1 . . .	65
4.1 " 5.1 . . .	27
5.1 " 6.1 . . .	5
6.1 " 7.1 . . .	3
7.1 " 8.1 . . .	3
beyond	none.

ANALYSIS OF THE DISTURBANCES OF THE TOTAL FORCE.

The number of values of total force differing 0.00094 parts of the force from its normal amount, and which constitute the disturbance values, is 1470, which have been combined in a manner similar to that of the disturbances of the dip; an increasing total force is indicated by a + sign, a decreasing one by a — sign. The aggregate amount and mean amount of disturbances are expressed in parts of the force, and the letter n indicates the number of disturbances. In the tables of the annual inequality the series commences with July, 1841, in those of the diurnal inequality it commences with February, 1841. The ratios given are those of the aggregate amount.

TABLE VII.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE TOTAL FORCE IN EACH MONTH, DIVIDED INTO DISTURBANCES WHICH INCREASE AND WHICH DECREASE THE FORCE.

The values in brackets for January, February, and March are interpolated, and are the average values of the corresponding months of the year preceding and the year following.

	1841 and 1842.				1842 and 1843.				1843 and 1844.			
	+	n	—	n	+	n	—	n	+	n	—	n
July	.02944	25	.04552	32	.01115	8	.02518	15	.00000	0	.00377	3
August . . .	.01239	10	.00438	4	.00758	5	.02658	24	.00830	8	.00200	2
September . . .	.01759	15	.02807	19	.02395	16	.06228	42	.01355	10	.00734	7
October . . .	.02658	20	.00697	6	.01148	10.	.00410	4	.00663	5	.00585	5
November . . .	.02357	17	.02413	20	.00781	6	.00960	9	.01563	13	.02485	21
December . . .	.07598	47	.05026	35	.00715	7	.00898	7	.03267	28	.00645	6
January . . .	.06321	38	.04939	36	(.04792)	(32)	(.03222)	(24)	.03264	27	.01506	13
February . . .	.07043	50	.03481	28	(.04151)	(30)	(.01984)	(16)	.01259	10	.00486	4
March . . .	.01895	15	.02730	19	(.02711)	(22)	(.02574)	(19)	.03527	29	.02419	19
April	.04122	28	.02820	19	.03523	24	.02084	18	.02181	19	.01940	14
May	.04586	27	.02443	16	.02697	17	.01736	14	.00977	6	.00000	0
June	.02777	21	.01191	11	.00561	3	.01038	7	.00342	3	.02193	20

	1844 and 1845.				Sums 1841 to 1845.				Ratios.	
	+	n	—	n	+	n	—	n	—	+
July	.00805	8	.01076	9	.04864	41	.08523	59	0.56	1.28
August . . .	.04661	35	.01348	10	.07488	58	.04644	40	0.87	0.70
September . . .	.00715	5	.00108	1	.06224	46	.09877	69	0.72	1.48
October . . .	.00331	3	.00809	5	.04800	38	.02501	20	0.56	0.38
November . . .	.00000	0	.00100	1	.04701	36	.05958	51	0.54	0.89
December . . .	.01365	12	.00416	4	.12945	94	.06985	52	1.50	1.05
January . . .	.00210	2	.00543	4	.14587	99	.10210	77	1.68	1.53
February . . .	.00000	0	.00216	2	.12453	90	.06167	50	1.44	0.92
March . . .	.02489	20	.01231	11	.10622	86	.08954	68	1.23	1.34
April	.00513	4	.00463	3	.10339	75	.07307	54	1.20	1.10
May	.02511	12	.00211	2	.10771	62	.04390	32	1.24	0.65
June	.00270	2	.00108	1	.03950	29	.04530	39	0.46	0.68

The ratios of the increasing disturbances of the force have a double progression, as have also those of the decreasing disturbances, though not so well marked. Increasing disturbances show a principal maximum in January, a principal minimum in June, and a secondary maximum and minimum in August and November, respectively; decreasing disturbances show a principal maximum in January, a principal minimum in October, and a secondary maximum and minimum in September and May, respectively. It appears, therefore, that upon the whole we observe the same laws as at Toronto, viz: the disturbances increasing the force and those decreasing the force follow the same progressive monthly change, and exhibit maximum values about the equinoxes, and minimum values about the solstices. This last remark also applies to the results of the following table in which the annual inequality of the disturbances is given irrespective of sign.

TABLE VIII.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE TOTAL FORCE IN EACH MONTH AND RATIO OF SUMS.

	Sum.	n	Ratio.
July	.013387	100	0.87
August	0.12132	98	0.79
September	0.16101	115	1.05
October	0.07301	58	0.47
November	0.10659	87	0.70
December	0.19930	146	1.30
January	0.24797	176	1.62
February	0.18620	140	1.22
March	0.19576	154	1.28
April	0.17646	129	1.15
May	0.15161	94	0.99
June	0.08480	68	0.56

TABLE IX.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE TOTAL FORCE IN THE DIFFERENT YEARS OF OBSERVATION.

The ratios of the sums exhibit part of the eleven year inequality.

Year.	Sum.	n	Ratio.
1841—1842	.78836	558	1.72
1842—1843	.51657	379	1.13
1843—1844	.32798	272	0.71
1844—1845	.20499	156	0.44

The minimum of the eleven year period, according to the above ratio, occurred probably in 1845.

TABLE X.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES IN EACH YEAR (JULY TO JULY) ARRANGED FOR DISTURBANCES INCREASING AND DISTURBANCES DECREASING THE FORCE, WITH RATIOS OF SUMS.

	Sum.		Sum.		Ratio.	
Year.	+	n	−	n	+	−
1841—1842	.45299	313	.33537	245	1.75	1.67
1842—1843	.25347	180	.26310	199	0.98	1.32
1843—1844	.19228	158	.13570	114	0.74	0.68
1844—1845	.13870	103	.06629	53	0.53	0.33

The inequality of the eleven year period is equally well marked for disturbances increasing and for disturbances decreasing the total force.

The sum of the positive values is 1.03744, and of the negative values 0.80046; increasing disturbances are therefore preponderating in the ratio of 1.3 to 1. In the year 1842–1843, however, decreasing disturbances were in excess over increasing ones, and at Toronto between 1844 and 1848, the general effect of the larger disturbances of the force was to decrease the total magnetic force more than to increase it. The excess in the different years appears to be rather irregular.

The following tables exhibit the diurnal inequality of the disturbances, the whole series (beginning with February, 1841) is employed. The sums, numbers, and ratios given apply to an epoch 20 minutes later than indicated by the tables.

TABLE XI.—AGGREGATE AMOUNT AND NUMBER OF DISTURBANCES OF THE TOTAL FORCE, DISTRIBUTED OVER THE EVEN HOURS OF THE DAY, AND RATIO SHOWING THE DIURNAL INEQUALITY OF THE SUM.

Hour.	Sum.	n	Ratio.
0	.19733	150	1.19
2	.20046	148	1.22
4	.20018	148	1.21
6	.17163	121	1.04
8	.13858	106	0.84
10	.13691	105	0.83
Noon	.15058	115	0.92
14	.19949	144	1.21
16	.14958	116	0.91
18	.13974	103	0.85
20	.13176	97	0.80
22	.16077	117	0.98

The hourly disturbances exhibit a regular double progression with a principal maximum at 2 A. M., and a principal minimum at 8 P. M., also a secondary maximum about 2 P. M., and a secondary minimum abut 10 A. M. At Toronto these hours were respectively 3 A. M., 11 A. M., and 5 P. M., 9 P. M., showing an exchange of the hours of the principal and secondary minimum; the disturbance at the hour of maximum is about eleven times greater than at the minimum hour, whereas this proportion is but one and a half to one at Philadelphia.

Table XII.—Aggregate amount and number of Hourly Disturbances of the Total Force for increasing and decreasing Values, Ratios, and Mean Effect of Disturbances.

Hour.	Sum. +	n	Sum. —	n	Ratio of sum. +	Ratio of sum. —	Differences of sums.	Average diurnal effect.
0	.10570	83	.09163	67	1.22	1.18	+.01407	+.000011
2	.08589	68	.11457	80	0.99	1.47	—.02868	—.000022
4	.08264	66	.11754	82	0.95	1.51	—.03490	—.000027
6	.08449	57	.08714	64	0.97	1.12	—.00265	—.000002
8	.06261	48	.07597	58	0.72	0.97	—.01336	—.000010
10	.06364	46	.07327	59	0.73	0.94	—.00963	—.000008
Noon	.07094	51	.07964	64	0.82	1.02	—.00870	—.000007
14	.13083	86	.06866	58	1.50	0.88	+.06217	+.000049
16	.08664	66	.06294	50	1.00	0.81	+.02370	+.000018
18	.08880	64	.05094	39	1.02	0.66	+.03786	+.000029
20	.08223	60	.04953	37	0.95	0.64	+.03270	+.000025
22	.09791	71	.06286	46	1.13	0.80	+.03505	+.000027

The ratios of the increasing and decreasing disturbances appear to follow the same law, that is, the values at any hour appear to be complementary to one another, a high plus value corresponding to a low minus value; the phenomenon is, however, not so distinctly brought out as from the longer series at Toronto. The last two columns contain the difference of the sums at each hour, and the average effect of the larger disturbances of the total force. From 1 P. M. to 1 A. M. the larger disturbances augment the total force; from 1 A. M. to 1 P. M. they diminish it; greatest augmentation at 2 P. M., greatest diminution at 4 A. M. The greatest augmentation is nearly twice as great as the greatest diminution, whereas at Toronto the opposite effect was observed.

The distribution of the disturbances of the total force, according to their magnitude for an equal increase of .00090 parts of the force, is as follows:—

Between	Number of disturbances.
.00094 and .00184	1324
.00184 " .00274	122
.00274 " .00364	17
.00364 " .00454	4
.00454 " .00544	2
.00544 " .00634	0
.00634 " .00724	1
beyond	none

PART XI.

SOLAR DIURNAL VARIATION AND ANNUAL INEQUALITY OF THE INCLINATION AND TOTAL FORCE.

SOLAR-DIURNAL VARIATION OF THE DIP AND TOTAL FORCE.

To make the combination of the horizontal and vertical force components complete, there remains the discussion of the regular solar-diurnal variation and its annual inequality, of the resulting dip and total force.

Table III, of Part V, contains the solar-diurnal variation, expressed in parts of the force, of the horizontal component, freed from the larger disturbances; Table III, of Part VIII, contains similar information with regard to the vertical component. The numbers of these tables, however, cannot be combined directly, owing to the eleven year inequality which requires that the two sets of components should cover precisely the same interval of time. In the present case the table of the horizontal force extends from July, 1840, to July, 1845, a five year series; whereas the table of the vertical force extends over four years only; a new table of monthly normals of the horizontal component was therefore prepared, in which the first year's observations were omitted. This was done by the same method as had been followed in the preparation of the former annual values of Part V. The following table differs from that of Table I, of Part V, only in the number of observations employed, and extends from July, 1841, to July, 1845.

TABLE I.—RECAPITULATION OF THE HOURLY NORMALS OF THE HORIZONTAL FORCE (EXPRESSED IN SCALE DIVISIONS) FOR EACH MONTH OF THE YEAR, AND FOR SUMMER, WINTER, AND THE WHOLE YEAR, BETWEEN JULY, 1841, AND JULY, 1845.

1841–45	0^h	1	2	3	4	5	6	7	8	9	10	11^h	$+21\frac{1}{2}^m$
July . .	791	790	789	788	787	786	785	788	796	803	806	804	
August . .	813	813	813	812	811	809	808	815	825	835	837	830	
September .	829	827	830	829	826	824	821	830	842	850	853	852	
October . .	850	846	847	845	844	844	844	848	854	860	865	864	
November .	853	852	850	849	848	846	845	849	853	859	862	865	
December .	883	879	878	877	874	873	871	872	874	878	885	892	
January . .	905	902	902	901	899	899	899	898	900	908	914	918	
February. .	915	914	913	912	910	908	906	908	910	914	920	922	
March . .	919	917	915	915	915	913	912	915	920	927	931	936	
April . .	945	944	944	942	940	938	938	940	948	958	965	966	
May . .	947	946	944	942	942	940	938	944	954	961	960	956	
June . .	972	972	972	972	969	966	962	968	973	980	983	980	
Year . .	885.2	883.5	883.1	882.0	880.4	878.8	877.4	881.2	887.4	894.4	898.4	898.8	
Summer . .	882.8	882.0	882.0	880.8	879.2	877.2	875.3	880.8	889.7	897.8	900.7	898.0	
Winter . .	887.5	885.0	884.2	883.2	881.7	880.5	879.5	881.7	885.2	891.0	896.2	899.5	

The summer months comprise April to September, inclusive
The winter " " October to May, "
One scale division equals 0.0000365 parts of the force.
Increasing numbers denote decrease of force.

1841–1845	12h (Noon)	13	14	15	16	17	18	19	20	21	22	23h	+21½m
July	797	788	782	780	779	783	788	791	793	793	793	793	
August	820	811	803	802	804	809	815	816	817	816	816	815	
September	844	834	829	828	828	832	834	834	832	834	832	832	
October	866	860	856	854	854	854	854	854	855	853	853	852	
November	861	858	853	852	851	853	851	851	852	852	853	853	
December	894	889	885	880	878	878	879	880	882	884	884	883	
January	916	909	905	900	899	902	905	905	904	905	905	904	
February	925	920	915	912	915	915	916	919	921	917	915	915	
March	934	930	923	918	921	923	924	923	921	920	921	920	
April	960	957	947	944	943	944	948	948	949	950	947	947	
May	951	943	938	938	938	939	945	947	949	949	948	945	
June	974	969	962	961	961	965	970	971	971	972	973	973	
Year	895.2	889.0	883.2	880.7	880.9	883.1	885.7	886.5	887.2	887.1	886.7	886.0	
Summer	891.0	883.7	876.8	875.5	875.5	878.7	883.3	884.5	885.2	885.7	884.8	884.2	
Winter	899.3	894.0	889.5	886.0	886.3	887.5	888.2	888.7	889.2	888.5	888.5	887.8	

Subtracting each value from its respective monthly mean value and converting these differences into parts of the force, we obtain the numbers of Table II; a + sign indicates a greater force than the normal value, a — sign a smaller one. The first three decimals are placed at the head of the table.

TABLE II.—REGULAR SOLAR-DIURNAL VARIATION OF THE HORIZONTAL FORCE BETWEEN JULY, 1841, AND JULY, 1845. VALUES OF $\frac{\Delta X}{X}$.

0.000

	0h	1	2	3	4	5	6	7	8	9	10	11h	+21½m
January	—025	+085	+085	+121	+194	+194	+194	+231	+158	—134	—353	—499	
February	—005	+032	+068	+105	+178	+251	+324	+251	+178	+032	—186	—259	
March	+087	+160	+233	+233	+233	+306	+342	+233	+050	—205	—351	—534	
April	+110	+146	+146	+219	+292	+365	+365	+292	000	—365	—620	—657	
May	—036	000	+073	+146	+146	+219	+292	+073	—292	—547	—511	—365	
June	—056	—056	—056	—056	+054	+163	+309	+090	—092	—348	—457	—348	
July	—017	+019	+056	+092	+129	+165	+202	+092	—200	—455	—565	—492	
August	+080	+080	+080	+117	+153	+226	+263	+007	—358	—723	—796	—540	
September	+168	+241	+131	+168	+277	+350	+460	+131	—307	—599	—708	—672	
October	+116	+262	+225	+298	+335	+335	+335	+189	—030	—249	—435	—395	
November	000	+035	+109	+145	+181	+254	+291	+145	000	—220	—329	—439	
December	—091	+055	+091	+128	+237	+274	+347	+310	+237	+091	—164	—420	
Year	+027	+088	+103	+143	+201	+259	+310	+171	—055	—310	—456	—469	
Summer	+043	+072	+072	+116	+174	+247	+317	+116	—209	—505	—610	—512	
Winter	+014	+105	+134	+171	+226	+269	+305	+225	+098	—114	—303	—424	

	0.000												
	(Noon) 12h	13	14	15	16	17	18	19	20	21	22	23h	+21½m
January . .	—427	—171	—025	+158	+194	+085	—025	—025	+011	—025	—025	+012	
February. .	—370	—186	—005	+105	—005	—005	—042	—151	—224	—078	—005	—005	
March . .	—461	—315	—059	+123	+014	—059	—096	—059	+014	+050	+014	+050	
April . .	—438	—328	+036	+146	+182	+146	000	000	—036	—073	+036	+036	
May . .	—182	+109	+292	+292	+292	+256	+036	—036	—109	—109	—073	+036	
June . .	—129	+054	+309	+346	+346	+200	+017	—019	—019	—056	—092	—092	
July . .	—236	+092	+311	+384	+421	+275	+092	—017	—090	—090	—090	—090	
August . .	—175	+153	+445	+482	+409	+226	+007	—029	—066	—029	—029	+007	
September .	—380	—015	+168	+204	+204	+058	—015	—015	+058	—015	+058	+058	
October . .	—468	—249	—103	—030	—030	—030	—030	—030	—067	+006	+006	+043	
November .	—293	—183	000	+035	+072	000	+072	+072	+035	+035	000	000	
December .	—493	—310	—164	+018	+091	+091	+055	+018	—055	—128	—128	—091	
Year . .	—338	—112	+100	+189	+183	+103	+006	—023	—046	—043	—028	—003	
Summer . .	—257	+011	+262	+309	+309	+192	+025	—019	—044	—063	—030	—008	
Winter . .	—416	—236	—059	+068	+057	+014	—012	—030	—048	—023	—023	+004	

To render the preceding table uniform with the similar one of the vertical component, the values at the odd hours will, hereafter, be dropped, their weight is less than one-half of that of the even hours; the small difference in the times (0.9^m) will be disregarded, and the values of the dip and total force, immediately deduced from the horizontal and vertical components, will refer to an epoch 21.1^m after the full hour.

For greater completeness, Table III, of Part VIII, is here inserted.

TABLE III.—REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE BETWEEN JULY, 1841, AND JULY, 1845. VALUES OF $\frac{\Delta Y}{Y}$.

	0.000												
	0h	2	4	6	8	10	Noon	14	16	18	20	22h	+20.6m
January. .	—059	+238	+040	—026	—026	+172	+172	+304	—092	—323	—224	—158	
February .	—198	+033	—132	—165	+099	+231	+264	+396	+198	—264	—330	—132	
March . .	—178	+086	—046	—112	+086	+119	+251	+317	+020	—178	—145	—211	
April . .	—353	—155	—122	—023	+175	+274	+274	+340	+241	—023	—320	—320	
May . .	—412	—346	—214	+049	+247	+280	+346	+445	+313	+016	—313	—412	
June . .	—528	—297	—198	—033	+198	+396	+594	+462	+363	—066	—429	—462	
July . .	—571	—406	—241	+056	+221	+419	+584	+551	+386	—043	—406	—538	
August . .	—485	—386	—287	—089	+109	+406	+736	+571	+340	—023	—386	—485	
September .	—469	—304	—271	—205	+092	+323	+521	+521	+356	+026	—271	—337	
October . .	—099	+066	000	—099	+231	+198	+198	+198	—033	—198	—264	—198	
November .	—043	—043	—142	—109	+086	+122	+254	+221	—010	—076	—109	—109	
December .	—059	+139	+040	—059	+040	+139	+205	+205	—092	—290	—191	—125	
Year . .	—287	—115	—131	—068	+129	+257	+366	+377	+165	—121	—282	—290	
Summer. .	—469	—317	—221	—040	+175	+350	+508	+482	+333	—020	—353	—426	
Winter . .	—106	+086	—040	—096	+082	+162	+224	+274	000	—221	—211	—155	

Solar-Diurnal Variation of the Inclination.—The combination of the component values of $\frac{\Delta X}{X}$ and $\frac{\Delta Y}{Y}$ to form the corresponding values of the dip is effected by the formula

$$\Delta\theta = \sin\theta\cos\theta\left(\frac{\Delta Y}{Y} - \frac{\Delta X}{X}\right)$$

$\Delta\theta$ will be expressed in minutes. $\theta = 71° 59'$. A + sign indicates an augmentation of the north dip; a — sign the converse.

TABLE IV.—REGULAR SOLAR-DIURNAL VARIATION OF THE DIP BETWEEN JULY, 1841, AND JULY, 1845. VALUES OF $\Delta\theta$.

	0h	2	4	6	8	10	Noon	14	16	18	20	22h	+21.1m
Jan'y	—0.'03	+0.'15	—0.'16	—0.'22	—0.'19	+0.'53	+0.'61	+0.'33	—0.'30	—0.'31	—0.'24	—0.'13	
Feb'y	—0.20	—0.04	—0.32	—0.50	—0.08	+0.42	+0.63	+0.40	+0.20	—0.22	—0.11	—0.13	
March	—0.27	—0.15	—0.28	—0.46	+0.04	+0.48	+0.72	+0.38	+0.01	—0.08	—0.16	—0.23	
April	—0.47	—0.30	—0.41	—0.39	+0.18	+0.91	+0.73	+0.31	+0.06	—0.02	—0.28	—0.36	
May	—0.38	—0.43	—0.37	—0.25	+0.54	+0.80	+0.53	+0.15	+0.02	—0.02	—0.20	—0.34	
June	—0.47	—0.24	—0.25	—0.34	+0.30	+0.87	+0.75	+0.16	+0.02	—0.08	—0.40	—0.37	
July	—0.56	—0.46	—0.37	—0.15	+0.43	+1.00	+0.83	+0.24	—0.04	—0.14	—0.32	—0.45	
August	—0.57	—0.47	—0.44	—0.35	+0.47	+1.22	+0.92	+0.13	—0.07	—0.03	—0.32	—0.46	
Sept.	—0.63	—0.43	—0.54	—0.66	+0.41	+1.05	+0.92	+0.36	+0.15	+0.04	—0.33	—0.39	
Oct.	—0.23	—0.17	—0.35	—0.45	+0.25	+0.64	+0.67	+0.30	—0.02	—0.18	—0.21	—0.22	
Nov.	—0.04	—0.15	—0.32	—0.40	+0.09	+0.47	+0.56	+0.22	—0.08	—0.15	—0.14	—0.11	
Dec.	+0.03	+0.05	—0.20	—0.41	—0.20	+0.30	+0.71	+0.37	—0.18	—0.35	—0.14	—0.00	
Year	—0.32	—0.22	—0.33	—0.38	+0.19	+0.72	+0.71	+0.28	—0.02	—0.13	—0.24	—0.27	
Sum'r	—0.51	—0.39	—0.40	—0.36	+0.39	+0.98	+0.78	+0.22	+0.02	—0.04	—0.31	—0.40	
Winter	—0.12	—0.05	—0.27	—0.41	—0.01	+0.47	+0.65	+0.33	—0.06	—0.22	—0.17	—0.14	

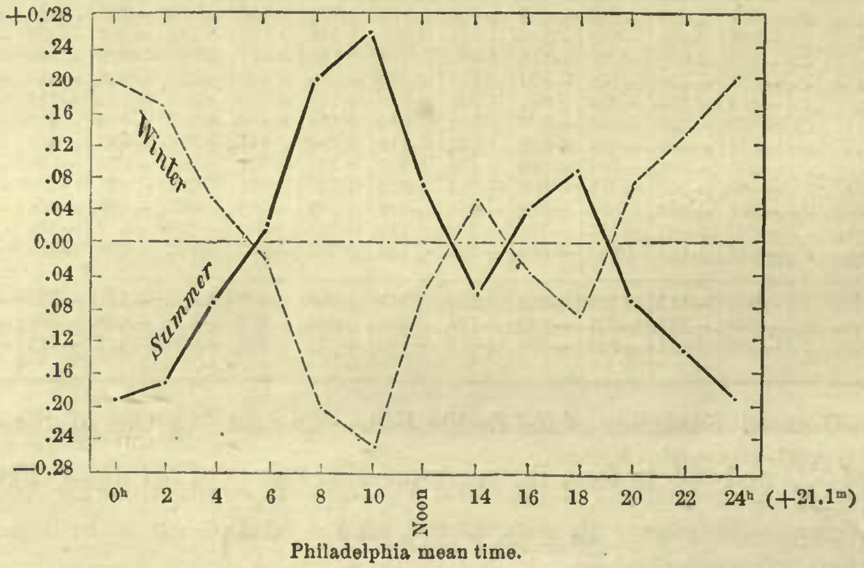
Annual Inequality in the Diurnal Variation of the Dip.—The comparison of the above diurnal variations for summer and winter, with that of the whole year, is given in the following Table.

TABLE V.

	0h	2	4	6	8	10	Noon	14	16	18	20	22h	+21.1m
Summer	—0.'19	—0.'17	—0.'07	+0.'02	+0.'20	+0.'26	+0.'07	—0.'06	+0.'04	+0.'09	—0.'07	—0.'13	
Winter	+0.20	+0.17	+0.06	—0.03	—0.20	—0.25	—0.06	+0.05	—0.04	—0.09	+0.07	+0.13	

The tabular quantities are exhibited in diagram A.

(A). SEMI-ANNUAL INEQUALITY IN THE DIURNAL VARIATION OF THE INCLINATION.



The diagram shows the hours of no semi-annual change as follows: $5\frac{1}{2}$ A. M.; 1 P. M.; 3 P. M., and 7 P. M.; greatest change at 10 A. M., secondary at 6 P. M., with a range of 0.′51 and 0.′18, respectively.

The turning epochs are found by the variation at the hour 10 A. M., when the monthly differences from the annual mean are as follows:—

January	—0.′19	July	+0.′28
February	—0.30	August	+0.50
March	—0.24	September . . .	+0.33
April	+0.19	October	—0.08
May	+0.08	November . . .	—0.25
June	+0.15	December . . .	—0.42

These values are represented by the formula

$$\Delta_a = 0.'35\ sin\ (\theta + 253° 38') + 0.'10\ sin\ (2\theta + 328° 39')$$

the angle θ counting from January 1st. This formula gives a change of sign for the middle of April and the middle of October, or about 25 days after the equinoxes.

ANALYSIS OF THE SOLAR-DIURNAL VARIATION OF THE DIP.

In the following formulæ expressing the solar-diurnal variation of the dip for each month, summer, winter, and year, the angle θ counts from midnight at the rate of 15° an hour; a positive sign indicates increase of north dip, a negative sign the reverse. The expressions are derived directly from Table IV.

For January $\Delta_1 = 0.'24\ sin\ (\theta + 284° 19') + 0.'33\ sin\ (2\theta + 78° 15') + 0.'14\ sin\ (3\theta + 282°)$

For February $\Delta_1 = 0.'34\ sin\ (\theta + 246° 28') + 0.'30\ sin\ (2\theta + 75° 40') + 0.'08\ sin\ (3\theta + 308°)$

For March $\Delta_1 = 0.'40\ sin\ (\theta + 251° 24') + 0.'24\ sin\ (2\theta + 85° 27') + 0.'12\ sin\ (3\theta + 303°)$

For April $\Delta_1 = 0.'55\ sin\ (\theta + 263° 10') + 0.'23\ sin\ (2\theta + 109° 36') + 0.'18\ sin\ (3\theta + 330°)$

For May $\Delta_1 = 0.'50\ sin\ (\theta + 271° 33') + 0.'22\ sin\ (2\theta + 146° 40') + 0.'14\ sin\ (3\theta + 0°)$

For June $\Delta_1 = 0.'53\ sin\ (\theta + 274° 37') + 0.'21\ sin\ (2\theta + 109° 03') + 0.'19\ sin\ (3\theta + 331°)$

For July $\Delta_1 = 0.'64\ sin\ (\theta + 275° 34') + 0.'24\ sin\ (2\theta + 131° 08') + 0.'14\ sin\ (3\theta + 322°)$

For August $\Delta_1 = 0.'68\ sin\ (\theta + 272° 55') + 0.'32\ sin\ (2\theta + 132° 56') + 0.'24\ sin\ (3\theta + 331°)$

For September $\Delta_1 = 0.'70\ sin\ (\theta + 260° 14') + 0.'27\ sin\ (2\theta + 116° 49') + 0.'23\ sin\ (3\theta + 349°)$

For October $\Delta_1 = 0.'41\ sin\ (\theta + 256° 31') + 0.'27\ sin\ (2\theta + 101° 19') + 0.'11\ sin\ (3\theta + 335°)$

For November $\Delta_1 = 0.'28\ sin\ (\theta + 259° 21') + 0.'27\ sin\ (2\theta + 99° 30') + 0.'08\ sin\ (3\theta + 328°)$

For December $\Delta_1 = 0.'20\ sin\ (\theta + 258° 09') + 0.'37\ sin\ (2\theta + 77° 45') + 0.'12\ sin\ (3\theta + 258°)$.

We have also:—

For Summer $\Delta_1 = 0.'60\ sin\ (\theta + 269° 37') + 0.'25\ sin\ (2\theta + 125° 02') + 0.'18\ sin\ (3\theta + 335°)$

For Winter $\Delta_1 = 0.'30\ sin\ (\theta + 260° 57') + 0.'29\ sin\ (2\theta + 85° 12') + 0.'10\ sin\ (3\theta + 300°)$

For Year $\Delta_1 = 0.'45\ sin\ (\theta + 266° 47') + 0.'25\ sin\ (2\theta + 103° 19') + 0.'13\ sin\ (3\theta + 322°)$

Summer is comprised of the months from April to September; winter of the months from October to March.

The following comparison of the observed and computed values for August shows about average differences; in general the summer values are a little better represented than the winter values.

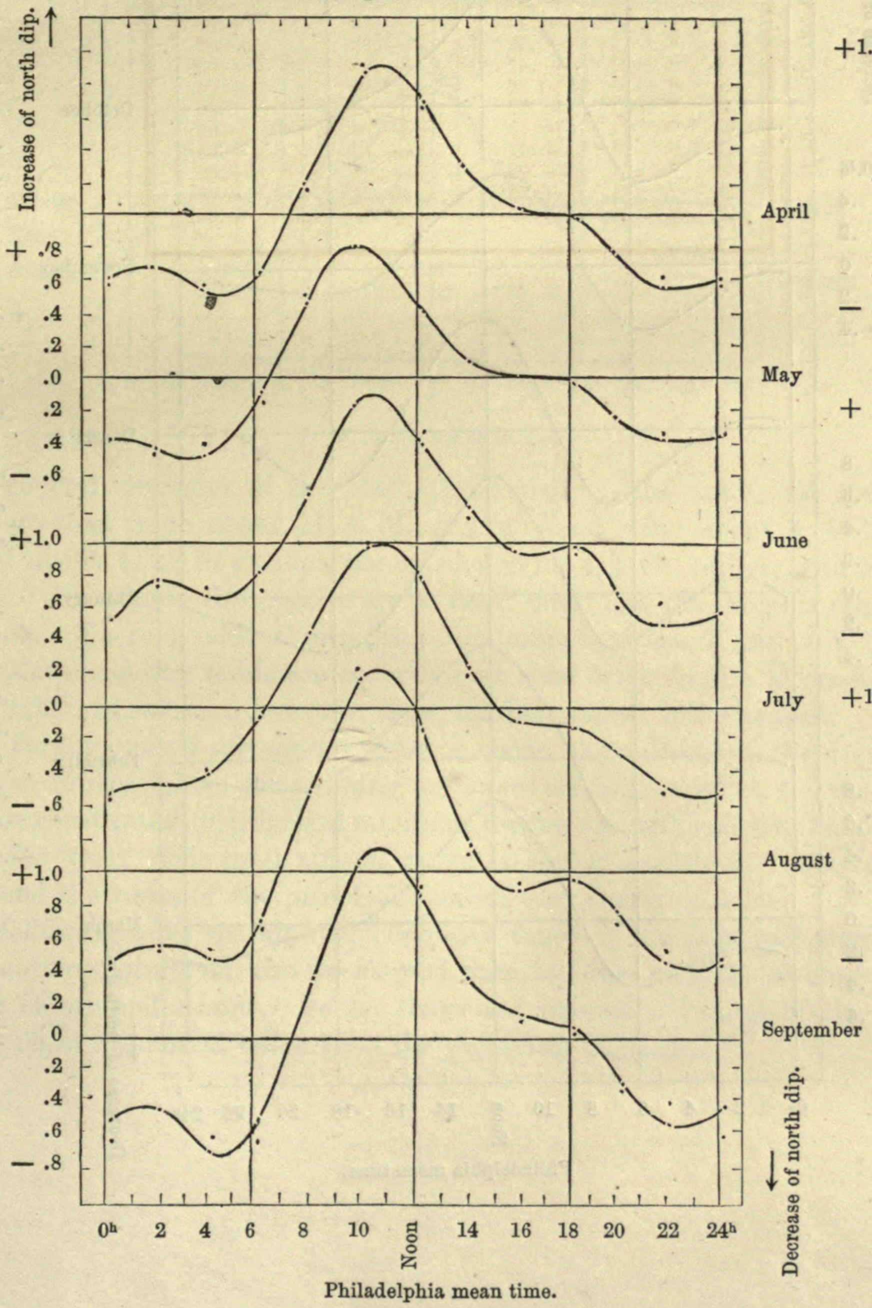
COMPARISON FOR AUGUST.

Time.	Observed.	Computed.	O—C.
0^h 21.1^m	—′.57	—′.53	—′.04
2 "	— .47	— .43	—.04
4 "	— .44	— .52	+.08
6 "	— .35	— .33	—.02
8 "	+ .47	+ .50	—.03
10 "	+1.22	+1.18	+.04
12 "	+ .92	+ .92	.00
14 "	+ .13	+ .18	—.05
16 "	— .07	— .12	+.05
18 "	— .03	— .05	+.02
20 "	— .32	— .24	—.08
22 "	— .46	— .55	+.09

Diagram (B) exhibits the observed and computed diurnal variation for the summer months, and diagram (C) for the winter months.

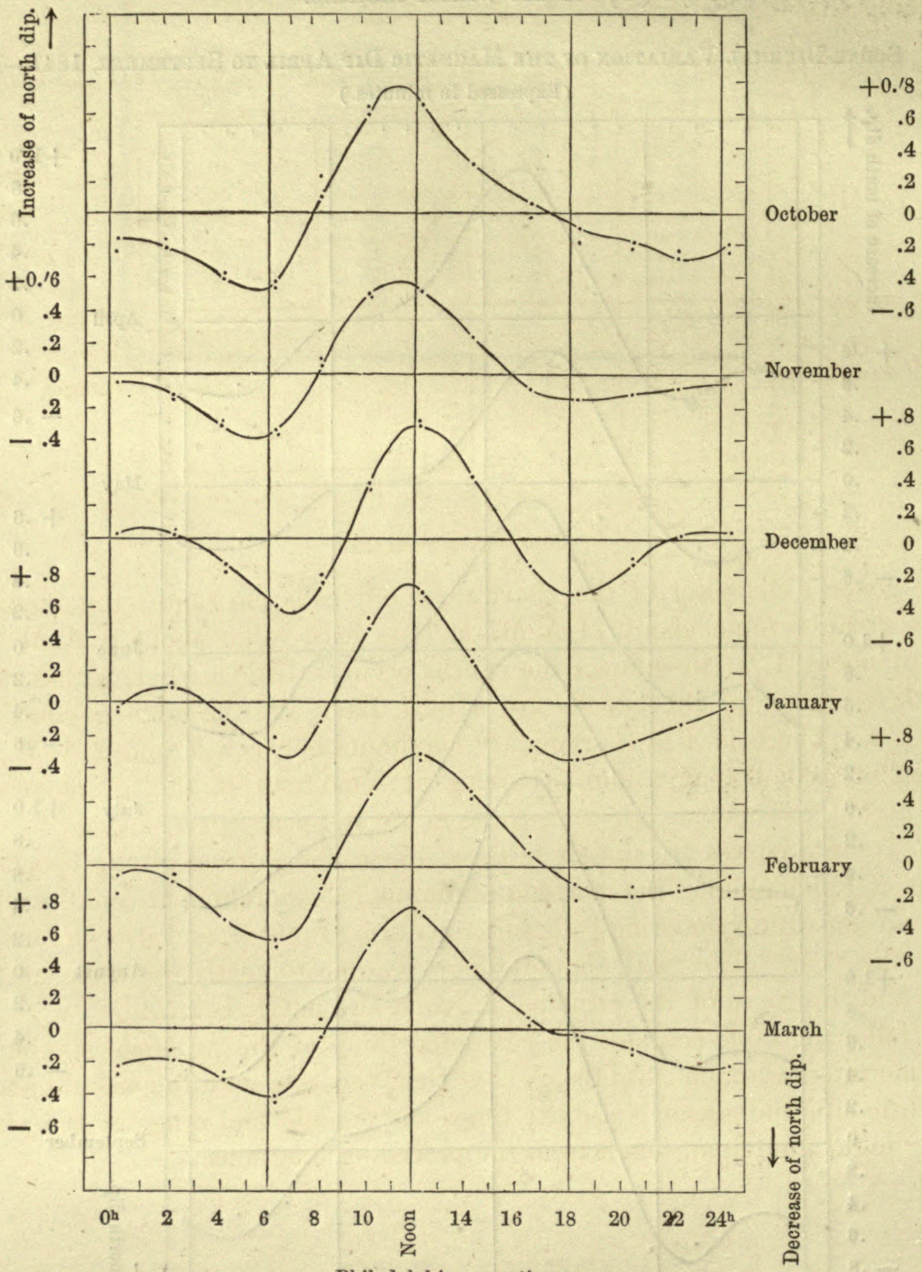
(B). SOLAR-DIURNAL VARIATION OF THE MAGNETIC DIP APRIL TO SEPTEMBER, 1841—1845.

(Expressed in minutes.)



(C). Solar-Diurnal Variation of the Magnetic Dip October to March, 1841—1845.

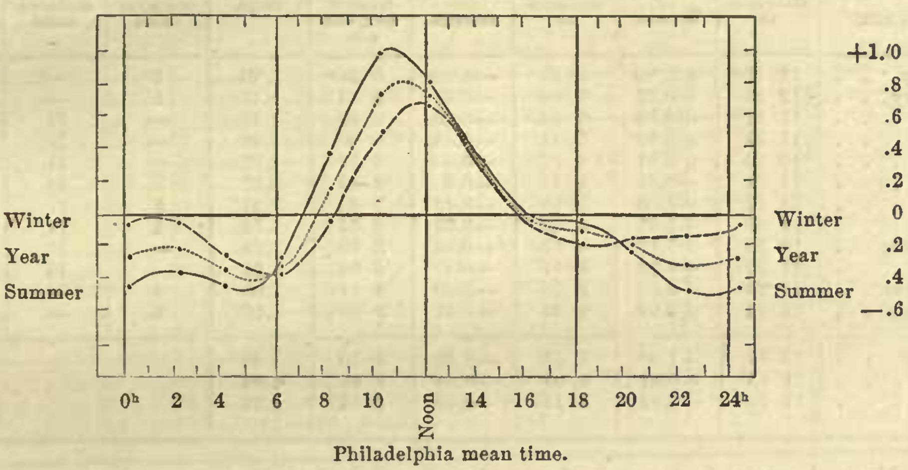
(Expressed in minutes.)



Philadelphia mean time.

Diagram (D) exhibits the diurnal variation of the dip for summer, winter, and the whole year.

(D). Solar-Diurnal Variation of the Magnetic Dip between 1841 and 1845.



The general character of the above three curves is the same; the annual curve has its greatest value about 11 A. M., and its least value about 5 A. M., with a range of about 1.′2; in summer the epochs occur a little earlier, with a range of about 1.′5; in winter the epochs are a little later, and the range is contracted to about 1.′0. There is also a secondary maximum between 1 and 2 A. M., with a less regular secondary minimum occurring at some hour in the afternoon or early in the night; in summer, however, these minima appear interchanged.

The Toronto curves are similar to those above, the shifting of the epochs is the same as at Philadelphia; the morning minimum is less prominent, and the afternoon minimum constitutes the principal minimum during summer, and also, on the average, during the year. The total annual range is almost exactly the same at the two places, and the times of the principal maxima also nearly coincide.

The following table contains the computed times of the principal maximum and of the morning minimum, also the elapsed time, together with the amount and range for each month and season; also the times and amount and range of the afternoon or early night minimum, taken from the preceding diagrams.

Table VI.

1841 to 1845.	Principal maximum at	Amount in minutes.	A. M. minimum at	Amount in minutes.	Elapsed time between max. and min.	A. M. range.	Afternoon minimum about	Early night minimum about	Range of afternoon minimum & principal maximum.
January	11^h 59^m	+0.′69	6^h 39^m	—0.′32	5^h 20^m	1.′01	6^h	—^{h}	1.′0
February	12 15	+0.67	6 04	—0.48	6 11	1.15	8	—	0.9
March	11 57	+0.72	5 54	—0.43	6 03	1.15	—	11	—
April	11 11	+0.92	5 11	—0.48	6 00	1.40	—	10	—
May	10 24	+0.81	4 07	—0.46	6 17	1.27	—	11	—
June	11 01	+0.91	5 16	—0.36	5 45	1.27	5	10	1.0
July	10 57	+1.00	3[1] 15	—0.43	7 42	1.43	5	11	1.1
August	10 49	+1.21	4 45	—0.53	6 04	1.74	4	11	1.3
September	10 56	+1.10	4 57	—0.69	5 59	1.79	—	10	—
October	11 25	+0.73	5 17	—0.49	6 08	1.22	—	10	—
November	11 29	+0.60	5 28	—0.40	6 01	1.00	6	—	0.7
December	12 24	+0.69	6 45	—0.41	5 39	1.10	6	—	1.0
Summer	10 55	+1.00	4 45	—0.48	6 10	1.48			
Winter	11 55	+0.67	6 00	—0.39	5 55	1.06			
Year	11 22	+0.81	5 18	—0.40	6 04	1.21			

The diurnal range is greater about the time of the equinoxes than at any other time, and in general less in winter and greater in summer. The afternoon minimum disappears about the time of the equinoxes, and is best marked about the solstices; the early night minimum only disappears about the time of the winter solstice.

Table VII.—Principal Epochs of Normal Dip.

The morning epoch is computed to the nearest minute or two, the afternoon epoch is taken from the diagrams.

1841—1845.	A. M.	P. M.
January	8^h 38^m	$3\frac{1}{4}^h$
February	8 49	5
March	8 35	5
April	7 46	$5\frac{3}{4}$
May	6 52	$5\frac{1}{2}$
June	7 11	$3\frac{3}{4}$
July	6 55	$3\frac{1}{4}$
August	7 16	3
September	7 37	$6\frac{3}{4}$
October	8 03	5
November	8 10	$3\frac{1}{2}$
December	9 14	$3\frac{1}{2}$
Summer	7 22	4
Winter	8 33	$3\frac{1}{2}$
Year	7 50	$3\frac{3}{4}$

Solar-Diurnal Variation of the Total Force.—The combination of the component values of $\frac{\Delta X}{X}$ and $\frac{\Delta Y}{Y}$ to form the corresponding values of the total force is effected by the formula

$$\frac{\Delta\phi}{\phi} = sin\ ^2\theta\ \frac{\Delta Y}{Y} + cos\ ^2\theta\ \frac{\Delta X}{X}$$

the result being expressed in parts of the force. A + sign indicates an augmentation of the force, a — sign a diminution.

[1] About $3\frac{1}{4}$ A. M.; minimum not distinctly marked.

TABLE VIII.—REGULAR SOLAR-DIURNAL VARIATION OF THE TOTAL FORCE BETWEEN JULY, 1841, AND JULY, 1845.

Values of $\frac{\Delta\phi}{\phi}$ to six places of decimals, the first three are placed outside the table.

0.000

	0h	2	4	6	8	10	Noon	14	16	18	20	22h	+21.1m
January . .	—056	+224	+055	—004	—007	+122	+115	+273	—064	—294	—202	—144	
February . .	—178	+038	—100	—115	+108	+190	+193	+358	+178	—241	—319	—119	
March . . .	—152	+099	—019	—068	+082	+072	+183	+280	+019	—169	—130	—189	
April . . .	—308	—126	—081	+015	+158	+188	+205	+311	+236	—021	—292	—286	
May . . .	—375	—305	—179	+073	+194	+203	+294	+431	+312	+017	—294	—379	
June . . .	—483	—274	—175	—001	+169	+312	+523	+447	+362	—058	—391	—428	
July . . .	—519	—363	—206	+069	+182	+323	+504	+527	+387	—031	—377	—496	
August . .	—433	—343	—246	—056	+062	+289	+646	+557	+345	—022	—357	—444	
September .	—408	—263	—218	—142	+054	+226	+436	+488	+343	+022	—240	—299	
October . .	—077	+082	+033	—056	+208	+138	+135	+169	—032	—180	—244	—177	
November .	—043	—033	—114	—074	+075	+075	+197	+196	—006	—065	—099	—102	
December .	—059	+137	+062	—017	+062	+112	+142	+173	—071	—254	—174	—122	
Year . . .	—257	—094	—099	—031	+112	+187	+298	+351	+167	—108	—260	—265	
Summer . .	—421	—279	—184	—007	+137	+257	+435	+460	+331	—016	—325	—389	
Winter . . .	—094	+091	—014	—056	+088	+118	+161	+241	+004	—200	—195	—142	

Annual Inequality in the Diurnal Variation of the Total Force.—The comparison of the above diurnal variations for summer and winter, with that of the whole year, is given in the following table:—

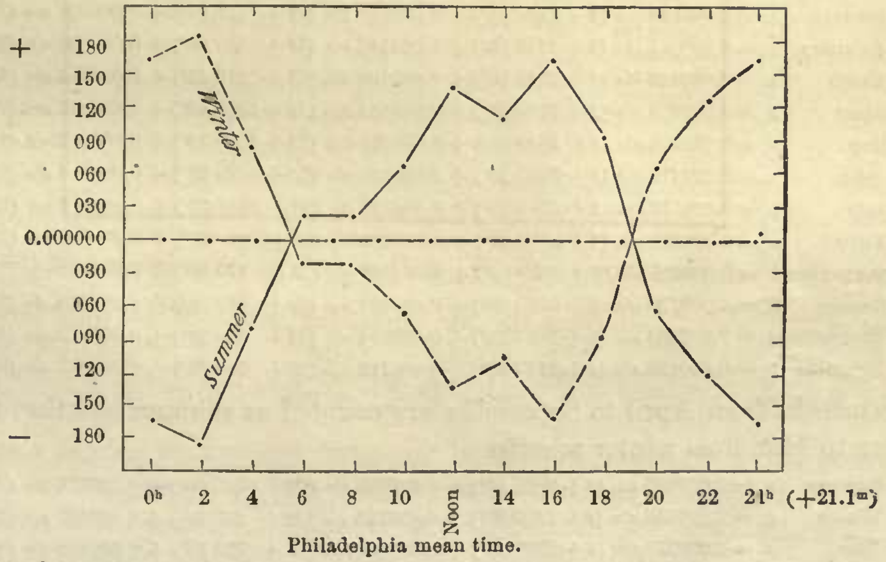
TABLE IX.

0.000

	0h	2	4	6	8	10	Noon	14	16	18	20	22h	+21.1m
Summer . .	—164	—185	—085	+024	+025	+070	+137	+109	+164	+092	—065	—124	
Winter . .	+163	+185	+085	—025	—024	—069	—137	—110	—163	—092	+065	+123	

These tabular quantities are exhibited in diagram E, which closely resembles diagram B, Part VIII, of the Vertical Force.

(E). SEMI-ANNUAL INEQUALITY IN THE DIURNAL VARIATION OF THE TOTAL FORCE.



The hours of no semi-annual change are 6 A. M. and 7 P. M.; the greatest changes take place about 2 A. M. and 4 P. M., with a range of .000370 and .000328 parts of the force respectively.

The turning epochs are found from the variations at the hours 6 A. M. and 7 P. M. The following numbers are the differences from the respective annual means. For 20^h the sign has been changed.

	6 A. M.	Mean 18 and 20.	Mean 6 A. M. 7 P. M.
January	+027	—122	—048
February	—084	—037	—060
March	—037	—095	—066
April	+046	+060	+053
May	+104	+080	+092
June	+030	+090	+060
July	+100	+097	+098
August	—025	+091	+033
September	—111	+055	—028
October	—025	—044	—034
November	—043	—059	—051
December	+014	—116	—051

All expressed in units of the sixth place of decimals.

The values in the last column are represented by the formula:—

$$\Delta_a = 0.000075 \; sin \; (\theta + 280°) + 0.000025 \; sin \; (2\theta + 131°)$$

θ counting from January 1st. The change of sign occurs about April 4th and about September 12th, on the average, therefore, the change takes place about three days after the equinoxes.

ANALYSIS OF THE SOLAR-DIURNAL VARIATION OF THE TOTAL FORCE.

In the following expressions of the solar-diurnal variation of the total force, θ is counted from midnight at the rate of 15° an hour; a positive sign indicates increase of total force, a negative sign the reverse. The coefficients are expressed in parts of the force. The formulæ are deduced directly from Table VIII.

For January $\Delta_t = 0.000139 \; sin \; (\theta + 318° \; 14') + 0.000157 \; sin \; (2\theta + 37° \; 57') + 0.000020 \; sin \; (3\theta + 217°)$

For February $\Delta_t = 0.000215 \; sin \; (\theta + 276° \; 00') + 0.000143 \; sin \; (2\theta + 41° \; 01') + 0.000060 \; sin \; (3\theta + 96°)$

For March $\Delta_t = 0.000148 \; sin \; (\theta + 284° \; 00') + 0.000106 \; sin \; (2\theta + 31° \; 27') + 0.000014 \; sin \; (3\theta + 242°)$

For April $\Delta_t = 0.000283 \; sin \; (\theta + 270° \; 27') + 0.000084 \; sin \; (2\theta + 342° \; 03') + 0.000042 \; sin \; (3\theta + 74°)$

For May $\Delta_t = 0.000383 \; sin \; (\theta + 264° \; 08') + 0.000084 \; sin \; (2\theta + 325° \; 13') + 0.000062 \; sin \; (3\theta + 108°)$

For June $\Delta_t = 0.000470 \; sin \; (\theta + 266° \; 26') + 0.000108 \; sin \; (2\theta + 2° \; 57') + 0.000016 \; sin \; (3\theta + 96°)$

For July $\Delta_t = 0.000513 \; sin \; (\theta + 265° \; 24') + 0.000103 \; sin \; (2\theta + 344° \; 53') + 0.000036 \; sin \; (3\theta + 126°)$

For August $\Delta_t = 0.000500 \; sin \; (\theta + 259° \; 41') + 0.000109 \; sin \; (2\theta + 29° \; 29') + 0.000025 \; sin \; (3\theta + 193°)$

For September $\Delta_t = 0.000405 \; sin \; (\theta + 252° \; 37') + 0.000097 \; sin \; (2\theta + 15° \; 05') + 0.000010 \; sin \; (3\theta + 163°)$

For October $\Delta_t = 0.000170 \; sin \; (\theta + 307° \; 00') + 0.000084 \; sin \; (2\theta + 45° \; 35') + 0.000033 \; sin \; (3\theta + 44°)$

For November $\Delta_t = 0.000122 \; sin \; (\theta + 266° \; 37') + 0.000074 \; sin \; (2\theta + 68° \; 28') + 0.000010 \; sin \; (3\theta + 119°)$

For December $\Delta_t = 0.000136 \; sin \; (\theta + 317° \; 52') + 0.000112 \; sin \; (2\theta + 46° \; 38') + 0.000017 \; sin \; (3\theta + 234°)$

The months from April to September are counted as summer months; those from October to March as winter months.

For Summer $\Delta_t = 0.000426 \; sin \; (\theta + 262° \; 51') + 0.000095 \; sin \; (2\theta + 358° \; 00') + 0.000026 \; sin \; (3\theta + 114°)$

For Winter $\Delta_t = 0.000146 \; sin \; (\theta + 292° \; 00') + 0.000110 \; sin \; (2\theta + 43° \; 20') + 0.000007 \; sin \; (3\theta + 119°)$

For Year $\Delta_t = 0.000277 \; sin \; (\theta + 270° \; 45') + 0.000095 \; sin \; (2\theta + 22° \; 29') + 0.000018 \; sin \; (3\theta + 115°)$

The following comparison of the observed and computed values for September shows about average differences; in general the summer values are better represented or less irregular than the winter values.

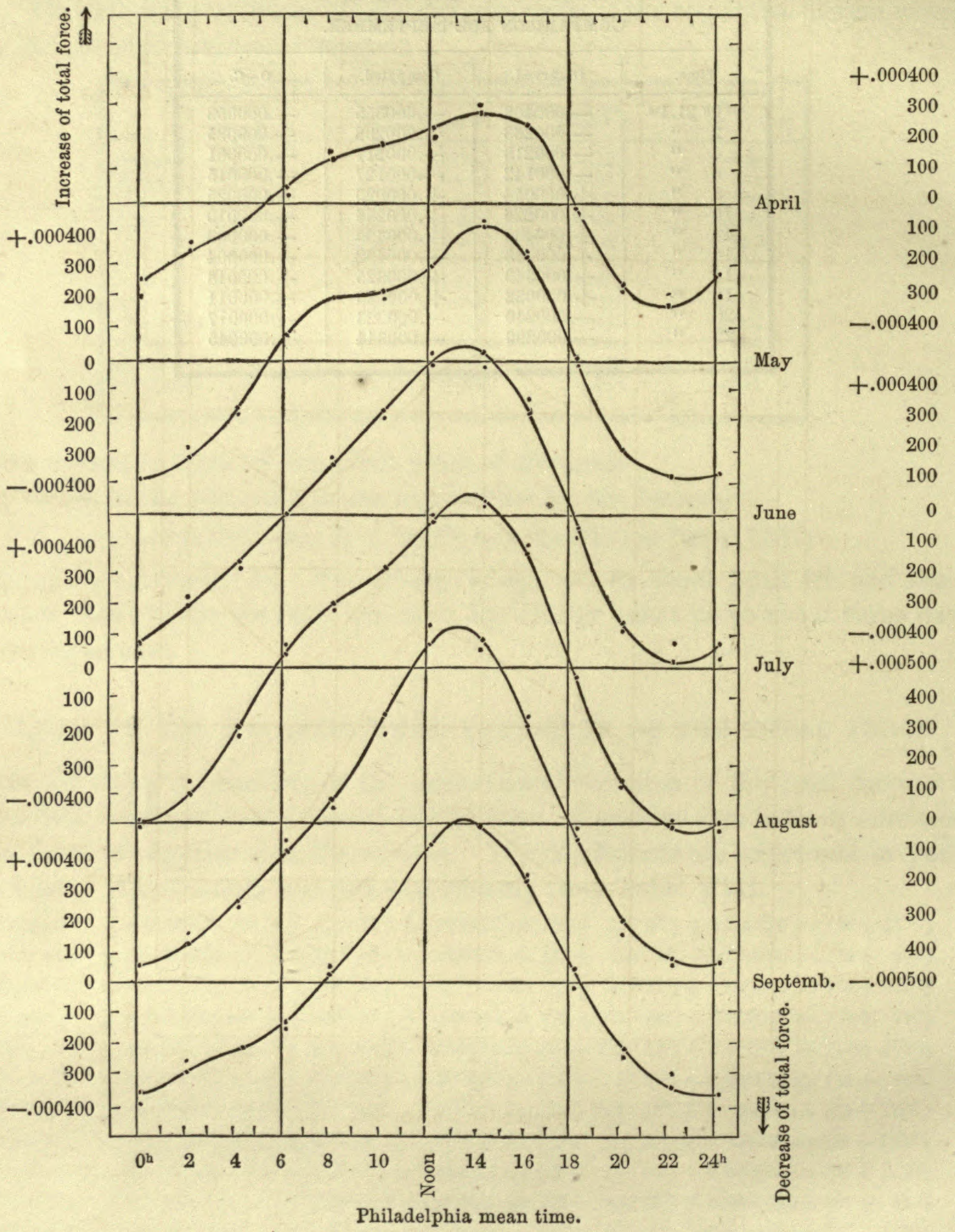
Comparison for September.

Time.	Observed.	Computed.	O—C.
0^h 21.1^m	—.000408	—.000355	—.000053
2 "	—.000263	—.000298	+.000035
4 "	—.000218	—.000217	—.000001
6 "	—.000142	—.000127	—.000015
8 "	+.000054	+.000029	+.000025
10 "	+.000226	+.000236	—.000010
12 "	+.000436	+.000439	—.000003
14 "	+.000488	+.000492	—.000004
16 "	+.000343	+.000325	+.000018
18 "	+.000022	+.000033	—.000011
20 "	—.000240	—.000223	—.000017
22 "	—.000299	—.000344	+.000045

Diagram (F) exhibits the observed and computed diurnal variation for the summer months, and diagram (G) for the winter months.

(F). SOLAR-DIURNAL VARIATION OF THE MAGNETIC TOTAL FORCE APRIL TO SEPTEMBER, 1841—1845.

(Expressed in parts of the force.)



(G). SOLAR-DIURNAL VARIATION OF THE MAGNETIC TOTAL FORCE OCTOBER TO MARCH, 1841—1845.

(Expressed in parts of the force.)

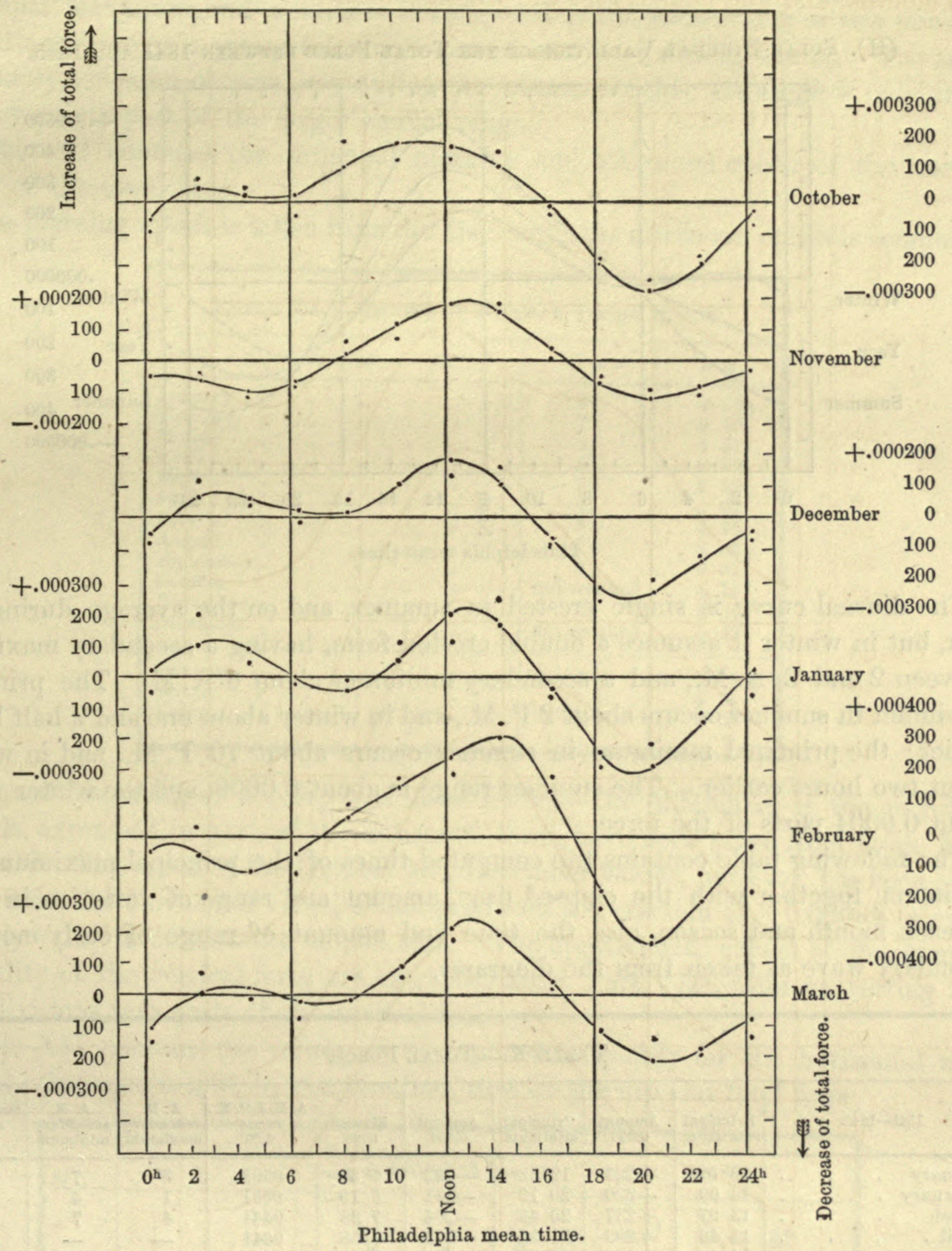
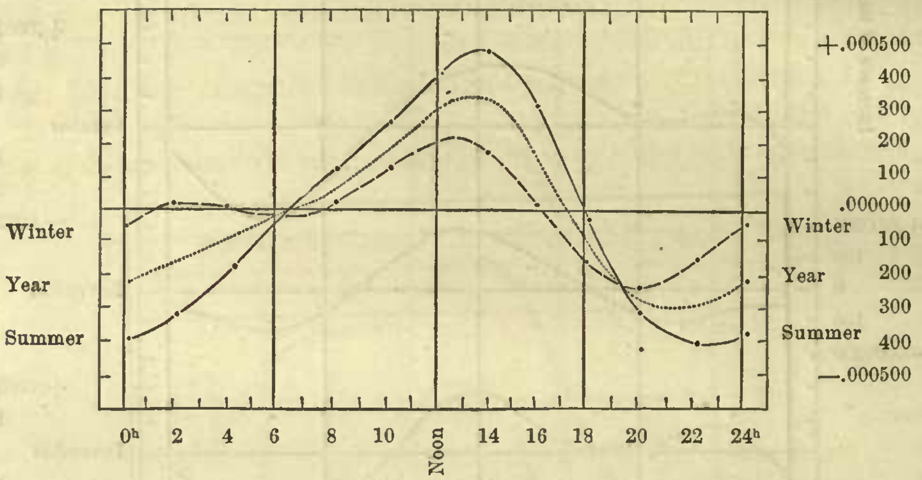


Diagram (H) exhibits the diurnal variation of the total force for summer, winter, and the whole year.

(H). SOLAR-DIURNAL VARIATION OF THE TOTAL FORCE BETWEEN 1841 AND 1845.



Philadelphia mean time.

The diurnal curve is single crested in summer, and on the average during the year, but in winter it assumes a double crested form, having a secondary maximum between 2 and 3 A. M., and a secondary minimum about 6 A. M. The principal maximum in summer occurs about 2 P. M., and in winter about one and a half hours earlier; the principal minimum in summer occurs about 10 P. M., and in winter about two hours earlier. The summer range is about 0.0009, and the winter range about 0.0004 parts of the force.

The following table contains the computed times of the principal maximum and minimum, together with the elapsed time, amount and range of variation in force for each month and season, also the time and amount of range of early morning secondary wave as taken from the diagrams.

TABLE X.—TOTAL FORCE.

1841—1845.	Principal maximum.	Amount 0.000	Principal minimum.	Amount 0.000	Elapsed time.	A. M. & P. M. range 0.00	A. M. secondary maximum.	A. M. secondary minimum.	Secondary range .000
January	13h 02m	+225	19h 38m	—283	6h 36m	0508	3h	7½h	15
February	14 00	+320	20 19	—331	6 19	0651	1	4	07
March	13 07	+237	20 45	—204	7 38	0441	4	7	04
April	14 46	+299	21 54	—342	7 08	0641	—	—	—
May	14 46	+422	22 57	—404	8 11	0826	—	—	—
June	13 41	+521	22 32	—486	8 51	1007	—	—	—
July	14 04	+560	23 05	—522	9 01	1082	—	—	—
August	13 23	+618	23 42	—447	10 19	1065	—	—	—
September	13 54	+498	23 33	—360	9 39	0858	—	—	—
October	11 00	+190	20 30	—266	9 30	0456	2½	5	04
November	12 50	+199	20 02	—118	7 12	0317	0½	4½	04
December	12 33	+193	19 29	—231	6 56	0424	8	7	05
Summer	14 02	+485	22 38	—421	8 36	0906	—	—	—
Winter	12 58	+220	20 11	—229	7 13	0449	2½	6	04
Year	13 36	+344	21 28	—296	7 52	0640	—	—	—

The amount and range are expressed in parts of the force. The time of the principal maximum and minimum is computed to the nearest one or two minutes. The diurnal range is greatest during summer, and least during winter. The small secondary inflexion obtains only during the winter months; its range is only about the seventieth part of the larger annual range.

Table XI contains the principal morning and afternoon epoch of the normal value of the total force.

The morning epoch is taken from the diagrams; the afternoon epoch is computed.

TABLE XI.—EPOCHS OF NORMAL TOTAL FORCE.

1841—1845.	A. M.	P. M.
January	9^h	$4^h\ 02^m$
February	7	5 16
March	8½	4 51
April	5½	6 15
May	5½	6 22
June	6¼	6 07
July	5¾	6 13
August	7¼	6 03
September	8	6 35
October	not reached	4 18
November	7¾	4 48
December	not reached	3 42
Summer	6.5	6 15
Winter	7.4	4 35
Year	6.6	5 38

Annual Inequality of the Dip and Total Force.—The differences of the monthly normals, expressed in parts of the horizontal force, have been taken from Table XII of Part V. The mean of the October and December values, however, has been substituted for the November value, which produced a correction of +0.00004 to each monthly value to balance the annual sum. The approximate values of the annual inequality of the vertical force are extracted from Part VIII, after converting the scale divisions into parts of the vertical force.

Table XII contains the values of the annual inequality for the horizontal and vertical components, and, by combination, that of the dip and total force.

TABLE XII.

	$\frac{\Delta X}{X}$	$\frac{\Delta Y}{Y}$	$\Delta\theta$	$\frac{\Delta\phi}{\phi}$
January	—0.00045	—0.00003	+0.′4	—0.00008
February	—0.00015	—0.00053	—0.4	—0.00050
March	+0.00023	0.00000	—0.2	+0.00002
April	—0.00003	—0.00003	0.0	—0.00003
May	+0.00039	+0.00036	0.0	+0.00036
June	+0.00006	+0.00053	+0.5	+0.00049
July	+0.00043	+0.00017	—0.3	+0.00019
August	+0.00005	+0.00020	+0.2	+0.00019
September	—0.00020	—0.00020	0.0	—0.00020
October	—0.00013	—0.00013	0.0	—0.00013
November	—0.00012	—0.00040	—0.3	—0.00037
December	—0.00011	—0.00003	+0.1	—0.00004

From what has been said of the annual inequality of the horizontal and vertical force, it could not be expected that this inequality should appear in any decided manner in the dip and total force. With reference to the dip, all that can be concluded is, that the inequality probably does not exceed half a minute. At Toronto, where the dip is greater, it is between 0.′8 and 0.′9; lower in June and July than in January and December; range 1.′7. With respect to the total force, the inequality seems to be about 0.0003 parts of the force, which gives nearly the same range as that found at Toronto, with this difference, however, that at Philadelphia the force is greater in the summer months and less in the winter months, the reverse of what has been found at the other station.

The next and last part of the discussion of the Girard College Magnetic Observations will contain the absolute values of the magnetic declination, dip and intensity.

PART XII.

DISCUSSION

OF THE

MAGNETIC INCLINATION, AND TABLE OF ABSOLUTE VALUES OF THE DECLINATION, INCLINATION, AND INTENSITY BETWEEN 1841 AND 1845.

5 December, 1864.

RESULTS AND DISCUSSION

OF THE

OBSERVATIONS FOR MAGNETIC INCLINATION, TAKEN AT GIRARD COLLEGE, PHILADELPHIA, IN 1842, 1843, AND 1844.

THE dip circle was made by Robinson, of London, in 1836, and is six inches in diameter.[1] The needles used are No. 1 and No. 2, and the poles were reversed in each set of observations. The ends of the needles are marked A and B. The instrument was mounted upon a marble pier, about twenty feet to the southeast of the Observatory.

The observations for dip were made once each week, and were commenced in January, 1842; they terminate in July, 1844. There are some interruptions, however, in the series of observation, as will be noticed in looking over the results of Table No. 1. This table contains an abstract of the results taken directly from the record; it has also been compared with the synopsis of the resulting dips in Volume III of the Record. The time is observatory mean time counted for convenience from 0^h to 24^h. Each mean result consists of 24 separate measures, with face of instrument west and east; marked side of the needle west and east, and with polarity north and south. Three readings have been taken in each position of face of the needle. With but a few exceptions, needle No. 1 was employed throughout the series; in the exceptional cases, where needle No. 2, or one of the Lloyd needles No. 1 or No. 3 were used, special corrections to refer their indications to the result by needle No. 1 have been deduced and applied.

TABLE I.—ABSTRACT OF RESULTS OF MAGNETIC DIP OBSERVED AT GIRARD COLLEGE OBSERVATORY BETWEEN 1842 AND 1844.

Year.	Month.	Day.	Hour.	Minute.	Dip. Needle No. 1. A north.	B north.	Mean.	Mean monthly dip.
1842	January	4	10	0	71° 56.′4	71° 55.′2	71° 55.′8	71° 57.′5
		11	10	0	71 58.6	71 53.6	56.1	
		18	10	0	72 00.9	71 55.0	57.9	
		25	10	0	71 56.3	71 04.4	60.3	
	February	1	10	0	72 01.9	71 57.1	59.5	71 58.4
		8	10	0	72 03.6	71 55.1	59.4	
		15	10	0	71 58.0	71 53.2	55.6	
		22	10	0	72 02.0	71 56.7	59.3	

[1] It is the same instrument with which I made the observations at stations in Europe (Amer. Phil. Trans. Vol. VII, Part I, 1840), and those in the magnetic survey of Pennsylvania, in 1840 and 1841, and at other stations farther northward and eastward, in 1843.

Year.	Month.	Day.	Hour.	Minute.	Dip. Needle No. 1. A north.	Dip. Needle No. 1. B north.	Dip. Needle No. 1. Mean.	Mean monthly dip.
1842	March	1	10	0	72° 08.′1	71° 49.′5	71° 58.′8	71° 59.′5
		8	10	0	72 04.6	71 51.0	57.8	
		15	10	0	72 07.5	71 52.7	60.1	
		22	10	0	72 08.2	71 55.7	62.0	
		29	10	0	72 02.6	71 54.9	58.7	
	April	5	10	0	71 46.4	71 55.5	51.0	71 59.0
		12	10	30	72 06.2	71 57.0	61.6	
		19	9	35	72 08.5	71 55.3	61.9	
		26	9	58	72 10.5	71 52.4	61.4	
	May	3	10	19	72 11.4	71 56.1	63.7	71 61.3
		17	9	29	72 09.4	71 59.4	64.4	
		17	10	43	71 54.2	72 02.6	[1] 58.4	
							+ 1.7	
		24	10	0	72 11.6	71 51.3	61.4	
		31	10	1	72 06.7	71 48.9	57.8	
	June	7	10	15	72 11.8	71 53.4	62.6	71 60.7
		14	9	46	72 09.1	71 52.9	61.0	
		21	9	56	71 57.1	71 59.1	58.1	
		28	9	24	72 07.8	71 54.3	61.0	
	July	13	9	11	72 10.2	71 58.3	64.3	71 57.2
		19	9	52	71 54.2	71 53.1	53.6	
		26	9	34	71 49.2	71 57.9	53.6	
	August	3	9	42	72 01.4	72 00.5	61.0	71 60.2
		9	10	1	72 06.3	71 53.5	59.9	
		16	9	57	72 04.2	71 54.8	59.5	
		30	9	40	72 05.0	71 56.2	60.6	
	September	6	9	4	72 10.0	71 58.7	64.3	71 61.2
		13	9	55	71 56.7	71 58.8	57.8	
		27	9	43	72 08.6	71 54.6	61.6	
	October	4	10	12	72 07.0	71 55.4	61.2	71 60.6
		11	10	8	72 06.4	71 55.6	61.0	
		11	11	17	71 51.3	72 03.2	[1] 57.2	
							+ 1.7	
	November	1	10	6	72 11.9	72 01.5	66.7	71 64.1
		15	9	48	72 08.3	71 52.3	60.3	
		22	10	13	72 13.3	71 57.1	65.2	
	December	27	9	16	72 08.2	71 55.3	61.7	71 61.7
1843	April	11	10	53	71 58.0	71 42.5	71 50.3	71 54.9
		18	13	10	72 08.2	71 48.9	58.5	
		25	10	0	71 59.7	71 52.0	55.8	
	May	2	9	40	72 11.7	71 44.7	58.2	71 63.2
		9	9	25	72 11.8	72 08.2	70.0	
		16	10	22	72 00.3	72 14.9	67.6	
		23	9	50	71 59.2	72 12.9	66.1	
		30	9	55	72 05.6	71 42.7	54.1	
	June	6	10	22	72 09.5	71 54.5	62.0	71 57.7
		13	10	20	72 10.7	71 57.8	64.2	
		20	10	5	71 56.5	71 45.4	51.0	
		27	9	55	71 58.8	71 48.2	53.5	
	July	4	10	25	72 01.4	71 56.3	58.8	71 57.5
		11	10	35	72 07.9	71 44.3	56.1	
		18	10	5	72 04.9	71 51.5	58.2	
		18	11	20	71 58.8	71 55.5	[1] 57.2	
							+ 1.7	
		20	16	37	[2] 71 56.2	[3] 71 45.3	56.7	
					+1.9	+10.0		
	August	22	10	5	[4] 71 53.4	[5] 72 12.3	64.1	71 60.5
					+4.3	— 1.8		
		24	11	17	72 02.0	71 50.7	56.3	
		24	12	0	72 01.6	72 00.2	[1] 60.9	
							+ 1.7	
		26	17	46	[2] 71 54.0	[3] 71 51.2	58.6	
					+1.9	+10.0		
		29	10	25	72 05.1	71 55.5	60.3	
		29	11	55	[2] 71 56.3	[3] 71 50.3	59.3	
					+1.9	+10.0		

[1] Needle No. 2. [2] Lloyd needle No. 1, A end north. [3] Lloyd needle No. 3, A end north.
[4] Needle No. 1, B end north. [5] Needle No. 2, B end north.

Year.	Month.	Day.	Hour.	Minute.	Dip. Needle No. 1. A north.	B north.	Mean.	Mean monthly dip.
1843	September	5	9	42	72° 07.′3	71° 52.′0	71° 59.′6	71° 60.′7
		5	11	12	[2]71 58.9	[3]71 49.3	60.0	
					+ 1.9	+10.0		
		12	9	56	72 02.7	71 53.7	58.2	
		12	11	17	71 54.8	71 57.0	[1] 55.9	
							+ 1.7	
		12	13	5	[2]71 57.1	[3]71 48.4	58.7	
					+ 1.9	+10.0		
		19	9	40	72 02.9	71 59.6	61.3	
		26	9	52	72 04.1	72 03.1	63.6	
	October	3	9	25	71 55.2	71 49.0	52.1	71 54.8
		10	9	32	71 56.9	71 55.2	56.0	
		17	9	42	71 50.8	72 00.7	55.7	
		24	9	40	72 00.1	71 53.4	56.8	
		31	9	32	71 50.3	71 51.0	53.6	
	November	7	9	45	72 01.2	71 51.5	56.4	71 56.4
		14	10	17	72 00.7	71 57.5	59.1	
		21	10	10	72 02.1	71 47.7	54.9	
		28	10	25	72 02.5	71 48.1	55.3	
	December	5	10	20	72 01.9	71 51.7	56.8	71 57.7
		12	10	42	72 05.7	71 58.3	62.0	
		19	10	12	71 59.0	71 51.8	55.4	
		26	10	40	72 00.5	71 52.6	56.6	
1844	January	2	10	20	71 57.6	71 53.8	71° 55.′7	71 56.2
		9	9	52	72 04.0	71 54.0	59.0	
		16	10	40	72 01.2	71 52.8	57.0	
		23	10	12	71 57.8	71 50.5	54.1	
		30	10	17	71 58.9	71 51.7	55.3	
	February	6	10	25	71 59.3	71 57.9	58.6	71 59.5
		13	10	22	72 01.0	71 59.1	60.0	
		20	10	12	71 59.1	72 01.2	60.1	
		27	10	47	71 59.7	71 58.5	59.1	
	March	5	11	10	71 56.6	71 59.1	57.8	71 58.6
		12	9	56	71 56.6	71 58.7	57.7	
		19	10	22	72 01.5	72 02.2	61.9	
		26	9	55	71 58.6	71 55.3	57.0	
	April	2	10	57	72 04.1	71 52.7	58.4	71 55.9
		9	10	1	71 55.2	71 54.8	55.0	
		16	10	7	71 59.1	71 55.9	57.5	
		23	10	18	71 55.7	71 49.4	52.6	
		30	9	55	72 00.6	71 51.5	56.1	
	May	7	9	50	71 57.2	71 54.5	55.8	71 57.1
		14	9	49	71 59.0	71 54.4	56.7	
		21	10	9	71 57.5	71 57.0	57.3	
		28	9	50	71 59.5	71 57.6	58.5	
	June	18	11	47	71 59.5	71 55.1	57.3	71 57.3
	July	4	12	53	71 56.5	71 56.6	56.6	71 58.9
		16	10	37	72 02.1	72 06.7	64.4	
		23	10	48	71 56.3	71 57.7	57.0	
		30	10	40	72 00.4	71 54.8	57.6	

Determination of corrections to results by needle No. 2, by Lloyd needles Nos. 1 and 3, and for want of reversal of polarity for needles Nos. 1 and 2 on August 22, 1843. Needle No. 1 being that ordinarily used, the exceptional readings with the other three needles have been referred to the indications of needle No. 1.

[1] Needle No. 2. [2] Lloyd needle No. 1, A end north. [3] Lloyd needle No. 3, A end north.

The index error to needle No. 2 we find by direct comparison with needle No. 1 on the following dates:—

May 17, 1842	correction	+6.′0	Mean +1.′7
October 11, 1842	"	+3.8	
July 18, 1843	"	+1.0	
August 24, 1843	"	—4.6	
September 12, 1843	"	+2.3	

The correction to the Lloyd needles No. 1 and No. 3, A end north, we obtain also by direct comparison, viz:—

August 29, 1843, correction to Lloyd No. 1,	+4.′0	to Lloyd No. 3, +10.′0
September 5, 1843, " " "	+0.7	" " +10.3
September 12, 1853, " " "	+1.1	" " + 9.8
Mean correction,	+1.9	" " +10.00

The corrections for polarity to needles 1 and 2 in 1843, are determined as follows:—

For needle No. 1. Mean dip in 1843 from 34 results, A north,	71° 62.′4
" " " " " B north,	71 53.8
Mean dip	71 58.1

Hence correction to needle 1, A north, —4.′3, and B north + 4.′3

For needle No. 2 we have the following differences:—

May 17, 1842, A north —B north	— 8.′4	Mean —3.′6
October 11, 1842, " "	—11.9	
July 18, 1843, " "	+ 3.3	
August 24, 1843, " "	+ 1.4	
September 12, 1843, " "	— 2.2	

Hence correction to needle 2, A north +1.′8, B north, —1.′8

The above corrections have been applied.

RECAPITULATION OF MONTHLY MEANS OF THE INCLINATION.

Month.	1842 71°+	1843 71°+	1844 71°+
January	57.′5	- - -	56.′2
February	58.4	- - -	59.5
March	59.5	- - -	58.6
April	59.0	54.′9	55.9
May	61.3	63.2	57.1
June	60.7	57.7	57.3
July	57.2	57.5	58.9
August	60.2	60.5	- - -
September	61.2	60.7	- - -
October	60.6	54.8	- - -
November	64.1	56.4	- - -
December	61.7	57.7	- - -
Mean	60.1	58.2	57.6

The preceding results indicate an annual diminution of the dip of 1.′2. To complete the dip for the year 1843, the values for January, February, and March have been interpolated by taking the means of 1842 and 1843 of these months respectively. The interpolated dips are 71° 56.′9, 58.′9, and 59.′0. We have the final values:—

Dip for 1842.5	72° 01.′1
" 1843.5	71 58.2
" 1844.4	71 57.6

If we divide the monthly means (inclusive of the interpolated dips for January, February, and March, 1843) into two parts, we find the values:—

From January, 1842, to April, 1843 (inclusive) . . . 71° 59.′4
" April 1843, to July, 1844 " . . . 71 57.9

The corresponding epochs are September 1, 1842, and December 1, 1843, which again give an annual decrease of 1.′2.

It is desirable, however, to extend the investigation of the annual effect of the secular change of the dip beyond the years above stated. In the Coast Survey Report for 1856,[1] Assistant Schott discussed the secular change of the dip at various places, and finds that the middle of the year 1842 (1842.7 ±0.7 years) was an epoch of minimum dip for places between Cambridge, Mass., Toronto, Canada, and Washington, D. C. The expression for the secular change for Philadelphia (page 241 of the 1856 report) is derived from 19 observations, contracted to 8 normals, between 1834 and 1855.

The dips extracted from a manuscript paper on my magnetic surveys in various parts of the Northeastern States during the years 1834–5, 1840 1841, and 1843, are as follows:—

Observed dip at Philadelphia, July 21, 1840 . . . 71° 52.′6
" " October 28, 1840 . . . 71 53.0
" " April 26, 1841 . . . 72 00.6
" " July 20, 1841 . . . 71 57.0
" " October 9, 1841 . . . 71 58.2
" " November 1, 1841 . . . 71 59.1

A collection and combination of all the observed values for dip at Philadelphia (as far as they have come to my notice) are given in the following table:—

Date.	Observer.	Dip.	Mean dip.	Mean epoch.
July, 1834	Prof. Bache and Prof. Courtenay	71° 60.′2	72° 00.′2	1834.5
-------- 1838	Prof. Bache	71 43.9	excluded	
September, 1839	Prof. Loomis	71 67.1	72 07.1	1839.7
July, 1840	Prof. Bache	71 52.6		
September, 1840	Prof. Bache	71 53.3	71 53.0	1840.7
October 1840	Prof. Bache	71 53.0		
March, 1841	--------------------	71 60.7		
April, 1841	Prof. Bache	71 58.2		
April, 1841	Prof. Bache	71 60.6		
April, 1841	Prof. Bache	71 59.0		
June, 1841	Major Graham	71 54.5	71 58.4	1841.5
July, 1841	Prof. Bache	71 57.0		
October, 1841	Prof. Bache	71 58.2		
November, 1841	Prof. Bache	71 59.1		
-------- 1842	Dr. Locke	71 60.1		
-------- 1842	Captain Lefroy . . .	71 59.0	71 59.7	1842.5
January to December, 1842	Prof. Bache	71 60.1		
April to December, 1843	Prof. Bache	71 58.2	71 58.2	1843.6
-------- 1844	Major Graham	71 61.8		
April, 1844	Dr. Locke	71 59.3	72 02.0	1844.3
May, 1844	--------------------	71 69.2		
January to July, 1844	Prof. Bache	71 57.6		
May, 1846	Dr. Locke	71 61.0	72 01.0	1846.4
September, 1855	Ass't Schott	71 77.7	72 17.7	1855.7
August, 1862	Ass't Schott	71 65.8	72 05.8	1862.6

[1] Appendix No. 32, p. 235. Discussion of the secular variation of the magnetic inclination in the Northeastern States.

The annual inequality in the dip need not here be considered, as the index error of the various needles and the observing error are much greater than the maximum amount of that inequality which, according to the Toronto observations, hardly exceeds $\pm 1'$.

Collecting the mean dips and mean epoch, and adopting the expression

$$\theta = \theta_1 + x + y\,(t - t_0) + z\,(t - t_0)^2$$

where

θ = resulting dip at any time between 1830 and 1860
θ_1 = assumed dip at epoch, x its correction, $\theta_1 = 72^\circ.00$
t_0 = epoch or 1840.0
t = any other time between the above limits.

We obtain from the following combination of the observations by the method of least squares, the values of x, y, and z.

		Mean dip.	Mean year.
Group I	3 results	72.°00	1838.3
" II	5 "	72.00	1843.5
" III	2 "	72.20	1859.2

whence $\theta = +72.^\circ00 - 0.00011\,(t - 1840) + 0.00060\,(t - 1840)^2$

The observed and computed dips compare as follows:—

Epoch.	Observed dip.	Computed dip.	Obs'd—comp'd.
1834.5	72.°00	72.°02	—0.°02
1839.7	72.12	72.00	+0.12
1840.7	71.88	72.00	—0.12
1841.5	71.97	72.00	—0.03
1842.5	72.00	72.00	0.00
1843.6	71.97	72.01	—0.04
1844.3	72.03	72.01	+0.02
1846.4	72.02	72.03	—0.01
1855.7	72.29	72.15	+0.14
1862.6	72.10	72.31	—0.21

The probable error of any one representation is $\pm 4.'8$.

The minimum dip, according to the above formula, occurred in January, 1840; at Toronto this minimum occurred in 1843.

By means of the formula we find the dip for the middle of each year:—

Year.	Computed dip.	Observed dip.	Adopted dip.
1840.5	72.°00		71° 59′
1841.5	72.00		71 59
1842.5	72.00	72.°00	72 00
1843.5	72.01	71.97	71 58
1844.5	72.01	71.97	71 58
1845.5	72.02		72 01

We may now collect in one table the numerical values of the magnetic elements as found in the preceding discussion. The units for the force are feet and grains. + indicates west declination and north dip.

Epoch.	Girard College, Philadelphia.				
	ψ	θ	X	Y	ϕ
January, 1841	+3° 23′	+71° 59′	4.178	12.05	13.51
" 1842	3 28	71 59	4.175	12.84	13.50
" 1843	3 32	71 59	4.173	12.83	13.49
" 1844	3 36	71 58	4.170	12.81	13.47
" 1845	3 41	72 00	4.168	12.83	13.49
Mean: January, 1843	+3 32	+71 59	4.173	12.83	13.49

The latitude of the Observatory is . . . . . . 39° 58′.4
And its longitude west of Greenwich 75 10.1
Or, in time 5h 00m 40.s3