Faculty Working Papers

DIVERSIFICATION IN OPTIONS

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#721

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Summary

This paper analyzes the return distribution parameters of alternative option portfolio strategies. The findings indicate that the diversification process for options operates in much the same manner as common stock -- the distribution parameters approach their assumptotic levels at approximately the same level of diversification. Furthermore, the relative systematic levels of the parameters across the strategies are in accordance with theory.

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Diversification In Options

Introduction

A recent paper by Merton, Scholes and Gladstein [MSG, 9] reported that a portfolio consisting of 90% commercial paper and 10% options provided a higher average return and a lower variance of return than a stock portfolio comprised of the underlying securities. Two stock samples were included in the study:

1. The 136 securities on which listed options were available in 1975.
2. The Dow Jones Industrials.

The time period examined extended from January, 1963 thru December, 1975.

Since the results reported in [9] were for the entire sample (as a portfolio), an important issue is "how large a portfolio of options is needed in order to achieve the results similar to those in MSG?" Statistically, how does changing the size of an option portfolio affect the distribution parameters of average return, variance of return and skewness of return? Or, alternatively, what are the diversification effects upon the return distribution parameters of option portfolios? A related issue, given the scenario in [9], is whether or not the inclusion of commercial paper (or some other fixed-income instrument) alters the diversification process in any way.

Previous research with stocks, [7], indicates that randomly selected portfolios of sixteen to twenty securities produces a variance of return on the market itself, since at this level of diversification, nearly all of the diversifiable risk is removed. In addition, evidence exists that covered option writing portfolios achieve comparable risk reduction at a level of diversification of only five securities [5]. However, Klemkosky and Martin [7] have shown that an inverse relationship exists between security volatility and the level of risk reduction achievable in a portfolio. Thus, since options are extremely volatile instruments, it might be surmised that a large number of options will be required to replicate the distribution statistics as reported in [9]. Furthermore, since alternative option strategies (e.g., in the money vs. out of the money) differ in return volatility, the diversification effects upon return distribution parameters may vary across differing option portfolio strategies.

Concerning the above remarks, this research seeks to address the following issues:

1. What are the diversification effects upon the return distribution parameters of alternative option portfolio strategies?

2. What are the relationships between the systematic parameters of alternative option strategies and is the relationship between exercise price and stock price (at the time of option purchase) important in explaining the impact that diversification has upon the return distribution parameters?

The first section will develop the relevant hypotheses concerning these two issues. This will be followed by a discussion of the data base and the methodology employed. The paper concludes with a report of the results and a brief summary.

Analytics of the Issues

Diversification and its Effects on the Distribution Parameters of Option Portfolios

Conventional wisdom suggests that the return on any asset i (e.g., stocks, options) can be expressed in the following manner:

(1) $$\tilde{r}_i = u_i + \tilde{s}_i + \tilde{e}_i$$

where:

$\tilde{r}_i$ = the asset's total return

u_i = a unique return portion

$\tilde{s}_i$ = a systematic (market factor) return portion

$\tilde{e}_i$ = an error term

$\tilde{}$ = denotes random variables

The expectation of (1) is simply:

(2) $$E[\tilde{r}_i] = E[u_i+\tilde{s}_i+\tilde{e}_i]$$

Subtracting (2) from (1), squaring, and then taking expectations yields the following:

(3) $$E[\tilde{r}_i-E(\tilde{r}_i)]^2 = E[\tilde{s}_i-E(\tilde{s}_i)]^2 + E[\tilde{e}_i-E(\tilde{e}_i)]^2$$
$$+ 2 \cdot E[(\tilde{s}_i-E(\tilde{s}_i)(\tilde{e}_i-E(\tilde{e}_i)]$$

Equation (3) simply states that the variance about the return on any asset can be decomposed into its systematic and unsystematic components as well as a covariance term.

In similar fashion, subtracting (2) from (1), cubing, and then taking expectations yields the following:

(4) $$E[\tilde{r}_i-E(\tilde{r}_i)]^3 = E[\tilde{s}_i-E(\tilde{s}_i)]^3 + E[\tilde{e}_i-E(\tilde{e}_i)]^3$$
$$+ 3 \cdot E[(\tilde{s}_i-E(\tilde{s}_i))^2(\tilde{e}_i-E(\tilde{e}_i))]$$
$$+ 3 \cdot E[(\tilde{s}_i-E(\tilde{s}_i))(\tilde{e}_i-E(\tilde{e}_i))^2]$$

Equation (3) states that the skewness about the return on any asset can be decomposed into its systematic and unsystematic components as well as two cross-product terms.

To analyze the effects of diversification upon the above return distribution parameters, it is necessary to restate the results in terms of portfolios. For a portfolio of m assets, (1) becomes:

$$\text{(1a)} \quad \tilde{r}_m = u_m + \tilde{s}_m + \tilde{e}_m$$

Letting x_i (i=1, ..., m) represent the proportion of the portfolio invested in asset i, (1a) can be redefined as:

$$\text{(1b)} \quad \tilde{r}_m = \sum_{i=1}^{m} x_i(u_i+\tilde{e}_i) + \sum_{i=1}^{m} x_i\tilde{s}_i$$

By refining the second component of (1b), the elements of portfolio return can be analyzed. Thus, let:

$$\text{(1c)} \quad \sum_{i=1}^{m} x_i\tilde{s}_i = \sum_{i=1}^{m} x_i a_i(u_s^{\sim}+\tilde{e}_s^{\sim})$$

where:

a_i = denotes the responsiveness of asset i to movements in the systematic (market) factor

$u_s^{\sim}$ = a measure of the expected return on the systematic factor

$\tilde{e}_s^{\sim}$ = random fluctuation about $u_s^{\sim}$

Thus, $\tilde{s}_i$ has been decomposed into its unique effect upon each $\tilde{r}_i$ (the a_i) and its expected return and uncertainty (the $u_s^{\sim}$ and $\tilde{e}_s^{\sim}$, which are common to all $\tilde{r}_i$). Letting $\sum_{i=1}^{m} x_i a_i = X_{m+1}$, $u_s^{\sim} = u_{m+1}$, and $\tilde{e}_s^{\sim} = \tilde{e}_{m+1}$, (1c) becomes:

$$(1d)\quad \tilde{r}_m = \sum_{i=1}^{m+1} x_i(u_i+\tilde{e}_i)$$

Taking the expectation of (1d) yields the portfolio equivalent of (2):

$$(2a)\quad E[\tilde{r}_m] = \sum_{i=1}^{m+1} x_i \cdot E[u_i+\tilde{e}_i]$$

Subtracting (2a) from (1d), squaring, and then taking expectations, provides the portfolio equivalent of (3):

$$(3a)\quad E[\tilde{r}_m-E(\tilde{r}_m)]^2 = x_{m+1}^2 \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]^2$$

$$+ \sum_{i=1}^{m} x_i^{\,2} \cdot E[\tilde{e}_i-E(\tilde{e}_i)]^2$$

$$+ \sum_{i=1}^{m+1} \sum_{j\neq i}^{m+1} x_i x_j \cdot E[(\tilde{e}_i-E(\tilde{e}_i))(\tilde{e}_j-E(\tilde{e}_j))]$$

In similar fashion, the portfolio equivalent of (4) can be obtained by subtracting (2a) from (1d), cubing, and then taking expectations:

$$(4a)\quad E[\tilde{r}_m-E(\tilde{r}_m)]^3 = x_{m+1}^3 \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]^3$$

$$+ \sum_{i=1}^{m} x_i^3 \cdot E[\tilde{e}_i-E(\tilde{e}_i)]^3$$

$$+ \sum_{i=1}^{m+1} \sum_{j\neq i}^{m+1} \sum_{k\neq i}^{m+1} x_i x_j x_k \cdot E[(\tilde{e}_i-E(\tilde{e}_i))(\tilde{e}_j-E(\tilde{e}_j))(\tilde{e}_k-E(\tilde{e}_k)$$

Analyzing the return equation (2a), it is a well-accepted principle that diversification should not, in and of itself, increase or decrease the rate of return expected on a portfolio of assets. The expected return should be a function of the systematic levels of relevant higher moments (e.g., risk, and possibly others) since the unsystematic portions of these factors can be eliminated via portfolio diversification. Therefore, one should expect the average value of expected return to remain constant (aside from sampling errors) throughout diversification for an option strategy containing a given level of relevant systematic factors.

Focusing upon the variance equation (3a) first, it is noted that since the systematic variance term, $x_{m+1}^2 \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]^2$, remains constant (and represents the component of portfolio variance which is attributable to movements in the market factor) as diversification occurs (or as $m \to M$, the total number of possible options in a given category), the benefits of increased portfolio size will depend upon what happens to the last two expressions in (3a). Concerning the unsystematic variance component, $\sum_{i=1}^{m} x_i^2 \cdot E[\tilde{e}_i-E(\tilde{e}_i)]^2$, its average value $\to 0$ as $m \to M$.[1] Furthermore, since its value is positive, diversification reduces portfolio variance via elimination of this term. However, the magnitude of this gain depends upon the behavior of the third term in (3a) as diversification occurs. If the average value of this term (as $m \to M$) increases, stays the same, or decreases, then the benefits of unsystematic risk elimination will be reduced, unchanged, or increased.

The third term captures the covariances among individual assets' error terms as well as the covariances between an asset's error term and the return which is attributable to movements in the market factor. Since this term may be negative, zero, or positive, it is difficult to tell, without measuring, whether the benefits of option diversification will arise solely from the elimination of the unsystematic component.[2]

Traditional analyses of common stocks have estimated the responsiveness factors (the a_i's) via the popular "market model" where the resultant a_i's denote stock betas. Hence, under this framework, x_{m+1} would represent the portfolio beta. The employment of this approach most nearly always assumes away the last term of (3a) by invoking:

$$(5) \quad E(\tilde{e}_i) = 0 \quad i = 1, \ldots, m + 1$$

$$(6) \quad E[(\tilde{e}_i - E(\tilde{e}_i))(\tilde{e}_j - E(\tilde{e}_j))] \quad i \neq j \quad i, j = 1, \ldots, m + 1$$

However, there is some evidence (see [7]) that these assumptions (especially (6)) may not hold.[3] The point of all this is that what has been observed is that portfolio risk approaches the level of risk attributable to movements in the market factor, as portfolio size is increased. What is not clear is whether all of the risk reduction arises solely from the elimination of the unsystematic element, or whether some of it arises from elimination of the covariance term.[4] In any event, for common stocks it has been observed, [7], that substantial reductions in portfolio variance occurs with moderate levels of diversification.

Based upon the analytic arguments given, one should expect diversification to reduce the level of risk inherent in option portfolios, with the asymptotic variation approaching the systematic variation inherent in options.

Shifting the focus to (4a), the effects of diversification upon the skewness of an option portfolio are less clear than for portfolio variance. Again, since systematic skewness, $X_{m+1}^3 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1})]^3$, remains constant as diversification occurs, the effects of increased portfolio size will depend upon what happens to the last two expressions in (4a). The unsystematic term, $\sum_{i=1}^{m} x_i^3 \cdot E[\tilde{e}_i - E(\tilde{e}_i)]^3$, will as before, approach zero as portfolio size becomes large.[5] But, whether or not this tendency increases, leaves unchanged, or decreases portfolio skewness depends upon whether the unsystematic term's sign is negative, zero, or positive. Unlike the case for unsystematic variance, the sign of unsystematic skewness cannot be determined ex ante (since its value is in cubic terms, rather than squared terms). In addition, the magnitude of this effect will depend upon the behavior of the third term of (4a) as $m \to M$. If the average value of this term increases, stays the same, or decreases, then the effects of diversification upon the elimination of unsystematic skewness will be reduced, unchanged, or increased.[6]

For common stocks, it has been shown [12] that the signs of the last two terms in (4a) are both positive. Furthermore, the magnitude of the unsystematic component was much larger than the covariance component (implying that the majority of skewness reduction comes from elimination of the unsystematic component). Since stock price movements and option price movements are highly correlated, option skewness should behave in the same manner as stock skewness. Thus, it is hypothesized that the last two terms of (4a) should both be positive, implying that diversification should reduce the skewness inherent in option portfolios, with the limiting value being the systematic component of skewness.[7] Intuitively, this seems reasonable since as the portfolio size is increased, portfolio return will approach its expected value conditional upon the market factor's return.

Systematic Levels of Return Distribution Parameters: Stocks vs. Options; Options vs. Options

Now that the hypothesized effects of diversification upon the return distribution parameters of option portfolios have been developed, the analysis turns to the relationships one should expect between the systematic levels of the distribution parameters across alternative asset portfolios. In particular, the focus will be on two types of portfolios:

1. a portfolio of the underlying stocks.
2. portfolios of options similar in all respects except for the relationship of stock price (at time of option purchase) and the exercise price.

For some time now, research in finance has focused on the concept that the expected return on a financial asset should be a positive function of its underlying systematic risk. This notion has led to the now famous "Capital Asset Pricing Model" (CAPM) which not only formulates this idea, but also provides a mechanism for the pricing of financial assets under specified equilibrium conditions (see [5]).

An important issue is whether or not this systematic risk-expected return hypothesis holds on an ex-post basis among alternative option strategies as well as among options and stocks. In other words, is there a positive (ex-post) relationship between the return and systematic risk as exhibited between alternative option/stock strategies? Alternatively, do option/stock strategies which exhibit higher levels of systematic risk also earn higher returns?[8]

To gain some insight into what one could expect after diversification runs its course, consider the well-tested option pricing model as developed in a seminal piece by Black and Scholes [2]. Their formula for the value of a call option is:

$$C = P \cdot N(D1) - K \cdot e^{-rt^*} N(D2) \tag{7}$$

where: P = stock price at time of option purchase

K = exercise price for the option

t^* = time to maturity

r = risk-free rate

v^2 = variance rate about the stock return

$N(\cdot)$ = cumulative normal density function

$D1 = [\log(P/K) + (r+\frac{1}{2}v^2)t^*]/v\sqrt{t^*}$

$D2 = D1-v\sqrt{t^*}$

The assumptions and development of this model are well-known and need not be stated here (see [1], [2], and [5]). The general conclusion regarding this model is that prices generated via its formula conform very well with actual price options (see [9]).

What is important, for our purposes, is that in developing the valuation formula, an expression for the systematic risk of an option is determined. The formula is:[9]

(8) $$B_c = (P \cdot N(D1)/C) \cdot B_p$$

where:

B_c = systematic risk of the option

B_p = systematic risk of the underlying stock

The problem with formula (8), in terms of implementation, is that (8) is specified for instantanous relationships. In other words, (8) changes from instant to instant depending upon the parameters affecting C (particularly K and t*). Even so, the model can be employed to predict what theory would suggest to be the ranking among alternative option/stock strategies' systematic risks and average returns.

Using (7) to make substitutions into (8) and simplifying:

(9) $$B_c = \left[\frac{1}{\left[1 - \frac{K \cdot e^{-rt^*} N(D2)}{P \cdot N(D1)}\right]} \right] \cdot B_p$$

Because C is non-negative ($P \cdot N(D1) \geq K \cdot e^{-rt^*} N(D2)$), thus making the ratio in the denominator ≤ 1), the denominator is ≥ 0 and ≤ 1. Thus $B_c \geq B_p$. Hence, since nearly all stocks have positive systematic risks, any option on a particular stock should have a level of systematic risk at least as great as the systematic risk of the underlying stock.

Since this relationship holds on a one-for-one basis, it should also hold on a portfolio basis. That is, a portfolio of options should have a level of systematic risk that is at least as great as the systematic risk level of the underlying stocks <u>and</u> this relationship should hold for every instant in time.[10] Hence, if $B_{c,i} \geq B_{p,i}$; for all i (i=1, ..., m), then:

$$(10) \qquad \sum_{i=1}^{m} x_i \cdot B_{c,i} \geq \sum_{i=1}^{m} x_i \cdot B_{p,i}$$

<u>But</u> if $B_{c,i} > B_{p,i}$ for any i, then

$$(11) \qquad \sum_{i=1}^{m} x_i B_{c,i} > \sum_{i=1}^{m} x_i \cdot B_{p,i}$$

where $B_{c,i}$ and $B_{p,i}$ are used as proxies for the systematic risks inherent in options and stocks. Thus, as diversification runs its course (as $m \to M$, and thus the only element remaining is the systematic component), the empirical results should demonstrate that:

$$(3b) \qquad x_{m+1}^2 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1})]_C^2 \geq x_{m+1}^2 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1})]_P^2$$

where the C and P subscripts denote portfolios of call options and stocks at a fully diversified level.

To evaluate systematic risk among alternative option strategies, consider two options that are similar in <u>every</u> respect, except for exercise price (K). To make the comparison more vivid, assume that the price of the underlying stock is such that one of the options is "in the money" (I) and the other option is "out of the money"(0).[10]

Examination of (8) reveals the factors which would cause $B_c(I)$ to differ from $B_c(0)$:

(12) $$\frac{N(D1)(I)}{C(I)} \gtreqless \frac{N(D1)(0)}{C(0)}$$

The determination of which side of (12) has the greater value will indicate which option has the greater level of systematic risk. By inspection, $N(D1)(I) > N(D1)(0)$ and $C(I) > C(0)$. But this tells us nothing. What is of interest is how the above ratio changes with respect to changes in K (the exercise price). Differentiating $\frac{N(D1)}{C}$ with respect to K reveals that the ratio is a positive function of K (see [4]). What this means is that the higher the exercise price (or the more "out of the money") of the option, the greater should be the level of systematic risk:[11]

(13) $$B_c(0) > B_c(I)$$

Since this relationship holds on a one-for-one basis, it should also hold on a portfolio basis. Thus, if $B_{c,i}(0) > B_{c,i}(I)$ for every $i(i=1, \ldots, m)$, then:

(14) $$\sum_{i=1}^{m} x_i \cdot B_{c,i}(0) > \sum_{i=1}^{m} x_i \cdot B_{c,i}(I)$$

Thus, when comparing two option portfolios which are similar in all respects except one portfolio is constructed with higher underlying exercise prices, the portfolio which is more out of the money should exhibit a higher level of systematic risk. Thus, at a fully diversified level:

(3c) $$x_{m+1}^2 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1})]_0^2 > x_{m+1}^2 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1})]_I^2$$

where the 0 and I subscripts indicate fully diversified out of the money and in the money portfolios.

Therefore, systematic risk (the amount remaining after diversification) should increase as the portfolio strategy moves from stocks to in the money to successively more out of the money options. Furthermore, if return is positively related to systematic risk (ala CAPM framework), then the average returns earned by the alternative option/stock strategies should increase in the same manner.

One final consideration, with respect to risk, is the relationship (if any) that exists between systematic risk and the last two components of (3a). Klemkosky and Martin [8] have shown that a positive relationship exists between the <u>level</u> of systemtic risk and the <u>level</u> of the diversifiable components. That is, stock portfolios containing greater amounts of systematic risk also contain greater amounts of risk to be diversified. Further, they demonstrate that the <u>rate</u> at which these divesifiable elements are eliminated (via diversification) is different (diversifiable risk in a low "beta" portfolio is eliminated at a faster rate—a lower level of diversification).

Thus, if these phenomena are also present in options, the empirical results will demonstrate that:

$$\sum_{i=1}^{m} x_i^2 \cdot E[\tilde{e}_i - E(\tilde{e}_i)]_0^2 + \sum_{i=1}^{m+1} \sum_{j \neq i}^{m+1} x_i x_j E[(\tilde{e}_i - E(\tilde{e}_i))(\tilde{e}_j - E(\tilde{e}_j))]_0 \tag{15}$$

$$>$$

$$\sum_{i=1}^{m} x_i^2 E[\tilde{e}_i - E(\tilde{e}_i)]_I^2 + \sum_{i=1}^{m+1} \sum_{j \neq i}^{m+1} x_i x_j \cdot E[\tilde{e}_i - E(\tilde{e}_i))(\tilde{e}_j - E(\tilde{e}_j))]_I$$

and:

$$(16)\qquad \left(\sum_{m=2}^{m} \frac{\Delta E[\tilde{r}_m - E(\tilde{r}_m)]^2}{E[\tilde{r}_M - E(\tilde{r}_M)]^2 - E[\tilde{r}_1 - E(\tilde{r}_1)]^2}\right)_0$$

$$<$$

$$\left(\sum_{m=2}^{m} \frac{\Delta E[\tilde{r}_m - E(\tilde{r}_m)]^2}{E[\tilde{r}_M - E(\tilde{r}_M)]^2 - E[\tilde{r}_1 - E(\tilde{r}_1)]^2}\right)_I$$

On a one-for-one basis, an option should possess a higher level of systematic skewness vis a vis its underlying stock due to the fact that the relationship between exercise price and the stock price at option maturity produces a truncated distribution of possible returns for the option holder. Furthermore, this truncation effect exists for every time period (e.g., every six-month period) and thus introduces the element of positive systematic skewness in the option's time series of returns. This truncation effect results in a larger range of possible returns for the option vis a vis the underlying stock. This increased return range is non-symmetric since the maximum loss is -100% (when $P \leq X$), whereas the maximum percentage return can be very large depending upon the cost of the option and the value of $P - K$ at maturity. Obviously, the magnitude of this effect is not as great for the underlying stock. Thus, given movement in the market factor, the option holder possesses a positive systematic skewness advantage vis a vis the stock holder.

In a portfolio context, the situation is even more evident. For the influence of the market factor in determining the movement of stock prices can be such that negative stock price movements may offset positive price movements to produce a zero portfolio return. <u>But</u>, in the corresponding option portfolio, the influence of the market factor will, at worst, produce the same "washing out" result due to the truncation effect. Hence, the return on the option portfolio can be positive even when the stock portfolio has a zero return (see [9] for an example). Thus, if $\gamma_{c,i}$ and $\gamma_{p,i}$ represent the systematic skewness' of a call option and its underlying stock, then since $\gamma_{c,i} > \gamma_{p,i}$; for all i (i=1, ..., m):

$$(17) \qquad \sum_{i=1}^{m} x_i\gamma_{c,i} > \sum_{i=1}^{m} x_i\gamma_{p,i}$$

Thus, as diversification runs its course (or as skewness approaches its systematic level), the empirics should show that:

$$(4b) \qquad x^3_{m+1} \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]^3_C > X^3_{m+1} \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]^3_P$$

Concerning the levels of systematic skewness present in alternative option strategies, the higher the underlying exercise price, the greater is the systepatic skewness. Since out of the money options involve smaller dollar amounts, any movement in the market factor has the potential of producing very large returns in the option vis a vis options with higher exercise prices. However, the market movement must be substantially larger in order that its effect produces a positive option value at maturity. Hence, increasing the exercise price also increases the probability of a zero option value at maturity. Thus, options carrying higher exercise prices will experience a greater number of

-100% returns, but also a greater number of significantly large returns relative to options with lower exercise prices--thus, a greater amount of positive systematic skewness. Thus, if $\gamma_{c,i}(0) > \gamma_{c,i}(I)$ for every $i(i=1, \ldots, M)$, then:

$$\sum_{i=1}^{m} x_i \gamma_{c,i}(0) > \sum_{i=1}^{m} x_i \gamma_{c,i}(I) \tag{18}$$

Therefore, at a fully diversified level, our results should demonstrate that:

$$x_{m+1}^3 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1})]_0^3 > x_{m+1}^3 \cdot E[\tilde{e}_{m+1} - E(\tilde{e}_{m+1}]_I^3 \tag{4c}$$

Therefore, systematic skewness (the amount remaining after diversification) should increase as the analysis moves from stocks to options with increasingly higher exercise prices.

Finally, an empirical question is whether or not option portfolios which contain higher levels of systematic skewness also contain greater amounts of diversifiable skewness. Furthermore, is the rate at which non-systematic skewness is eliminated an inverse function of the level of systematic skewness. Thus, as for risk, will the following be observed:[12]

$$\sum_{i=1}^{m} x_i^3 \cdot E[\tilde{e}_i - E(\tilde{e}_i)]_0^3 + \sum_{i=1}^{m+1} \sum_{j \neq i}^{m+1} \sum_{k \neq i}^{m+1} x_i x_j x_k \cdot E[(\tilde{e}_i - E(\tilde{e}_i))(\tilde{e}_j - E(\tilde{e}_j))(\tilde{e}_k - E(\tilde{e}_k)]_0 \tag{19}$$

$$>$$

$$\sum_{i=1}^{m} x_i^3 \cdot E[\tilde{e}_i - E(\tilde{e}_i)]_I^3 + \sum_{i=1}^{m+1} \sum_{j \neq i}^{m+1} \sum_{k \neq i}^{m+1} x_i x_j x_k \cdot E[(\tilde{e}_i - E(\tilde{e}_i))(\tilde{e}_j - E(\tilde{e}_j))(\tilde{e}_k - E(\tilde{e}_k))]_I$$

and:

$$(20) \qquad \left[\sum_{m=2}^{m} \frac{\Delta E[\tilde{r}_m - E(\tilde{r}_m)]^3}{E[\tilde{r}_M - E(\tilde{r}_M)]^3 - E[\tilde{r}_1 - E(\tilde{r}_1)]^3} \right]_0 < \left[\sum_{m=2}^{m} \frac{\Delta E[\tilde{r}_m - E(\tilde{r}_m)]^3}{E[\tilde{r}_M - E(\tilde{r}_M)]^3 - E[\tilde{r}_1 - E(\tilde{r}_1)]^3} \right]_I$$

The Data and Methodology

The Stock Sample

To carry out the tests of the aforementioned diversification issues, the sample of stocks on which options would be purchased must be specified. The initial sample in this study includes the 136 stocks on which listed options were available as of December 31, 1975. Unfortunately, thirty-four of the stocks in the data base (to be discussed below) did not have continuous price and/or dividend information over the sample period. For this reason, they are not included in the study. Even so, the remaining sample represents a wide range of risk levels and dividend yields. Therefore, portfolios based upon these stocks (as well as their underlying options) should be well diversified.

Unfortunately (as noted in [9]), the selection of this particular stock group introduces a selection bias. Although the sample was not chosen on the basis of past performance, past performance was probably a consideration in their selection by the various options exchanges.

Therefore, these securities can be expected to outperform a randomly chosen group of stocks over the period investigated in this study.

On the other hand, these are the securities upon which options may be purchased. Thus, in this sense, there is no selection bias in the sample.

Period of Study

The period investigated by this study extends from July 1, 1963 to December 31, 1978. This period was marked by a variety of up and down market periods. For all of the portfolio strategies examined, a six-month holding period was assumed. Thus, to be consistent with this structure, all of the return distribution statistics reported are on a semiannual basis.

To generate a time series of returns for the various option portfolio strategies, it is necessary to have option prices. For this study, a formula, (7), was used to generate option prices at the beginning of each six-month period. The reasons for this approach are several.

First, to generate a representative pattern of returns requires a period long enough to encompass varying market environments. Prior to 1973, however, all options transactions took place through options dealers on an individual trade basis, with little standardization with respect to exercise price and/or maturity date. Aside from obtaining a dealer's book, the only reasonable source was the advertisements by dealers in financial newspapers.

Second, there is the problem with having enough stocks on which options are available. Since this study is concerned with diversification (e.g., the effects of increased portfolio size), a sample of "reasonable" size is needed. Unfortunately, it was not until 1975, that a significantly large number of stocks carried options.

Third, to compare the diversification effects upon the return distribution parameters of alternative option portfolios, the options within a given portfolio should carry the same initial stock price/ exercise price ratio continuously through the period of examination. Such price data are not available prior to 1973.

Fourth, the terms of options bought and sold through options dealers provided for adjustments to the exercise price when cash dividends were paid on the stock. These adjustments are not made for listed call options.

Finally, the sensitivity of option prices and option return distributions to the use of model prices was conducted in [9]. Essentially, MSG compare the prices and return distributions of actual option prices versus the prices and return distributions of model prices. The differences they found were quite small.

Thus, we believe that the results in this study should conform quite well to an analysis using actual option prices (when the data availability requirement is met). Furthermore, this study is not concerned with the actual levels of returns, per se; but, rather, with the effects of diversification upon the return distribution parameters and relative levels of the parameters for alternative option portfolio strategies.

The Option Prices

The option pricing formula used in our study is the formula given by equation (7). The variance rate, v^2, is the only input that requires estimation. Since (7) is derived for non-dividend paying stocks, the formula requires some modification for dividend paying stocks; hence, the dividends on relevant stocks are required as an additional input. The adjustment for dividends in the option valuation formula is done in accordance with the procedure as described in [1].

Since all option positions are maintained until maturity, the valuation formula is only required at the beginning of each of the six-month periods. Relevant price and dividend information used to compute the option prices comes from the CRSP and Compustat data files. The yield on six-month commercial paper, as reported in selected issues of the Wall Street Journal, is used as the proxy for the risk-free rate. Finally, the variance rate is estimated from the sample variance of the previous six-months of daily logarithmic stock price changes.

Asset Return Calculations

Given the basic data set along with the beginning of the period (six-months) option prices, returns were computed for the assets under consideration on a semiannual basis. For common stocks, the standard holding period return formulation was employed, where:

$$(21) \qquad HPR_{i,t} = \frac{P_{i,t} + D_{i,t}}{P_{i,t-1}}$$

where:

$P_{i,t}$ = six-month ending price for stock i and time period t

$D_{i,t}$ = dividends paid on stock i during time period t

$t = 1, \ldots, 31$ (thirty-one six-month periods)

$i = 1, \ldots, 102$

For the call options, the following holding period return formulation was used:

$$(22) \qquad HPRC_{i,j,t} = \frac{V_{i,j,t} - C_{i,j,t-1}}{C_{i,j,t-1}}$$

where:

$V_{i,j,t}$ = value of the call at exercise price j on stock i at the end of period t (expiration)

$= \max\,[0, P_{i,t} - X_{i,j,t}]$

$C_{i,j,t-1}$ = value of the call, at the beginning of period t, on stock i with exercise price j

$t = 1, \ldots, 31$

$j = 1, \ldots, 5$; five alternatie exercise prices were chosen such that the initial stock price/exercise price ratio was .90, .95, 1.00, 1.05, and 1.10

$i = 1, \ldots, 102$

Portfolio Return, Dispersion, and Skewness Calculations

After the individual returns for stocks and calls (across the five stock price/exercise price ratios) were computed for each time period, 200 portfolios of size one, two, ..., fifty were selected by random sampling from the population of securities.[13] Thus, 200 portfolios of size one were selected, then 200 portfolios of size two, and so on. In all, 10,000 portfolios were selected for examination.

The portfolio return for any period t was defined as the average of the m component stock or option returns in period t. For stocks:

(23) $$\overline{HPR}_t = \sum_{i=1}^{m} x_i HPR_{i,t}$$

where:

$t = 1, \ldots, 31$

m = the number of securities in the portfolio ($m=1, \ldots, 50$)

$x_i = \frac{1}{m}$, an equal weighting scheme was assumed throughout

For the call options:

(24) $$\overline{HPRC}_{j,t} = \sum_{i=1}^{m} x_i \cdot HPRC_{i,j,t}$$

where:

$t = 1, \ldots, 31$

m = the number of call options in the portfolio ($m=1, \ldots, 50$)

j = the stock price/exercise price classification scheme ($j=1, \ldots, 5$ for the ratios of .90, .95, 1.00, 1.05, and 1.10)

$x_i = \frac{1}{m}$, an equal weighting scheme was assumed

An average of the thirty-one semiannual portfolio returns was then computed to obtain the mean portfolio return for each of the 10,000 portfolios over the entire period:

(25) $$\overline{HPR}_m = \frac{1}{31} \sum_{t=1}^{31} \overline{HPR}_t$$

(26) $$\overline{HPRC}_{j,m} = \frac{1}{31} \sum_{t=1}^{31} \overline{HPRC}_{j,t}$$

Next, the variance and raw skewness, about the portfolio returns, was computed:

(27) $$\sigma^2(\overline{HPR}_m) = \frac{1}{30}\sum_{t=1}^{31}(\overline{HPR}_t-\overline{HPR}_m)^2$$

(28) $$\sigma^2(\overline{HPR}_m) = \frac{1}{30}\sum_{t=1}^{31}(\overline{HPR}_t-\overline{HPR}_m)^3$$

(29) $$\sigma^2(\overline{HPRC}_{j,m}) = \frac{1}{30}\sum_{t=1}^{31}(\overline{HPRC}_{j,t}-\overline{HPRC}_{j,m})^2$$

(30) $$\sigma^2(\overline{HPRC}_{j,m}) = \frac{1}{30}\sum_{t=1}^{31}(\overline{HPRC}_{j,t}-\overline{HPRC}_{j,m})^3$$

Finally, the mean values of equations (25) - (30) were struck across the 200 sample values for each portfolio size m (m=1, ..., 50). Thus, the average return, variance and raw skewness were computed for portfolios of size m.

The Results

The return distribution statistics, along with some supplementary data, are presented in Tables I-VI for the various stock/option portfolio strategies. These results will now be reviewed in light of the issues raised in this research.

The Return Distribution Parameters and the Effects of Diversification

Reading down the return column of each of Tables I-VI, the average return for any particular portfolio strategy remains, for all practical purposes, constant as it should under a random selection scheme.[14,15] Thus, there is no discernible impact of diversification upon portfolio return, within a given portfolio strategy. As can be seen the average

Distributional Measures Averaged over Stock Portfolios of Varying Sizes

Portfolio Size	Holding Period Return Minus One	Variance	Percentage of Non-Systematic Variation-Eliminated	Raw Skewness	Percentage of Non-Systematic Skewness Eliminated	Percentage Return to Standard Deviation	Cube Root of Raw Skewness to Standard Deviation
1	0.06449	0.07324	0.0	0.03442	0.0	0.25626	0.7979
2	0.06777	0.05193	0.49242	0.01648	0.63314	0.31128	0.8381
3	0.06551	0.04034	0.76037	0.00963	0.87515	0.33350	0.9337
4	0.06672	0.04042	0.75848	0.00972	0.87193	0.33893	0.9692
5	0.06577	0.03730	0.83071	0.00875	0.90610	0.34746	0.9973
6	0.06488	0.03516	0.88001	0.00774	0.94191	0.35103	0.9772
7	0.06677	0.03502	0.88327	0.00743	0.95275	0.36134	0.9901
8	0.06875	0.03529	0.87856	0.00778	0.94047	0.37067	0.9939
9	0.06661	0.03304	0.92909	0.00678	0.97584	0.37063	0.9972
10	0.06684	0.03360	0.91606	0.00731	0.95692	0.36862	1.0233
11	0.06734	0.03356	0.91719	0.00700	0.96775	0.37183	0.9983
12	0.06610	0.03239	0.94406	0.00697	0.96901	0.37113	1.0256
13	0.06804	0.03354	0.91742	0.00715	0.96250	0.37492	1.0173
14	0.06628	0.03243	0.94308	0.00706	0.96574	0.37439	1.0388
15	0.06750	0.03188	0.95580	0.00661	0.98181	0.38116	1.0228
16	0.06664	0.03120	0.97154	0.00618	0.99681	0.38028	1.0114
17	0.06612	0.03090	0.97860	0.00652	0.98496	0.37910	1.0368
18	0.06745	0.03190	0.95548	0.00671	0.97816	0.38024	1.0299
19	0.06607	0.03052	0.98729	0.00624	0.99471	0.38051	1.0289
20	0.06727	0.03146	0.96564	0.00656	0.98360	0.38146	1.0363
21	0.06710	0.03080	0.98080	0.00623	0.99495	0.38465	1.0269
22	0.06641	0.03075	0.98201	0.00618	0.99692	0.38061	1.0273
23	0.06710	0.03076	0.98178	0.00616	0.99763	0.38446	1.0288
24	0.06648	0.03062	0.98502	0.00613	0.99870	0.38204	1.0286
25	0.06628	0.03038	0.99050	0.00622	0.99541	0.38197	1.0392
26	0.06744	0.03073	0.98241	0.00620	0.99628	0.38638	1.0346
27	0.06713	0.03063	0.98478	0.00621	0.99582	0.38527	1.0324
28	0.06652	0.03027	0.99300	0.00622	0.99545	0.38378	1.0416
29	0.06674	0.03018	0.99510	0.00612	0.99910	0.38590	1.0355
30	0.06664	0.03027	0.99301	0.00611	0.99944	0.38442	1.0355
31	0.06667	0.03023	0.99400	0.00614	0.99843	0.38479	1.0373
32	0.06665	0.02985	1.00266	0.00610	0.99953	0.38716	1.0417
33	0.06673	0.02983	1.00323	0.00590	1.00690	0.38775	1.0325
34	0.06696	0.03030	0.99240	0.00609	1.00009	0.38625	1.0343
35	0.06660	0.03008	0.99746	0.00615	0.99783	0.38517	1.0471
36	0.06645	0.03006	0.99802	0.00621	0.99569	0.38468	1.0474
37	0.06602	0.02961	1.00834	0.00596	1.00459	0.38500	1.0403
38	0.06726	0.03030	0.99244	0.00626	0.99385	0.38776	1.0455
39	0.06760	0.03019	0.99480	0.00616	0.99749	0.39014	1.0433
40	0.06663	0.02966	1.00715	0.00598	1.00385	0.38804	1.0442
41	0.06721	0.03011	0.99686	0.00606	1.00117	0.38846	1.0389
42	0.06699	0.03019	0.99480	0.00635	0.99095	0.38650	1.0541
43	0.06741	0.03017	0.99530	0.00611	0.99934	0.38505	1.0441
44	0.06627	0.02949	1.01104	0.00596	1.00473	0.38765	1.0433
45	0.06729	0.02999	0.99962	0.00591	1.00647	0.38954	1.0357
46	0.06630	0.02958	1.00897	0.00590	1.00686	0.38652	1.0417
47	0.06682	0.02972	1.00585	0.00603	1.00199	0.38881	1.0436
48	0.06583	0.02940	1.01318	0.00585	1.00841	0.38484	1.0429
49	0.06747	0.03016	0.99568	0.00618	0.99695	0.38960	1.0474
50	0.06689	0.02997	1.00000	0.00609	0.99999	0.38744	1.0444

Table II

Distributional Measures Averaged Over Option Portfolios of Varying Sizes, where the Relationship Between Initial Stock Price and Exercise Price Equals 1.0

Portfolio Size	Holding Period Return Minus One	Variance	Percentage of Non-Systematic Variation Eliminated	Raw Skewness	Percentage of Non-Systematic Skewness Eliminated	Percentage Return to Standard Deviation	Cube Root of Raw Skewness to Standard Deviation
1	0.15499	1.72177	0.0	5.15337	0.0	0.12439	1.0118
2	0.16416	1.18521	0.48112	2.04559	0.66612	0.15541	0.9854
3	0.15920	0.94984	0.69217	1.27086	0.83218	0.16620	0.9932
4	0.16533	0.87592	0.75846	1.02038	0.88586	0.17923	0.9882
5	0.15770	0.80947	0.81804	0.95029	0.90089	0.17784	1.0111
6	0.15644	0.74450	0.87630	0.71818	0.95064	0.18299	0.9771
7	0.16574	0.75763	0.86553	0.75283	0.94321	0.19207	0.9866
8	0.17250	0.73758	0.88251	0.72984	0.94814	0.20258	0.9866
9	0.16399	0.69625	0.91956	0.62935	0.96968	0.19795	0.9840
10	0.16634	0.69850	0.91755	0.67659	0.95955	0.20060	1.0037
11	0.16983	0.70707	0.90986	0.67563	0.95976	0.20364	0.9837
12	0.15892	0.67215	0.94118	0.62428	0.97076	0.19546	0.9958
13	0.17274	0.70058	0.91568	0.67247	0.96043	0.20769	1.0063
14	0.16522	0.67543	0.93823	0.62866	0.96982	0.20217	1.0111
15	0.16669	0.66389	0.94858	0.59534	0.97697	0.20573	1.0009
16	0.16665	0.65366	0.95776	0.57300	0.98176	0.20739	0.9916
17	0.16101	0.63422	0.97518	0.55313	0.98601	0.20331	0.9983
18	0.16786	0.65076	0.96036	0.56714	0.98301	0.20913	0.9962
19	0.16157	0.63559	0.97396	0.53961	0.98891	0.20345	0.9990
20	0.16824	0.64247	0.96778	0.55200	0.98626	0.21081	0.9970
21	0.16462	0.63469	0.97476	0.53184	0.99058	0.20751	0.9924
22	0.16634	0.63741	0.97233	0.54409	0.98795	0.20915	0.9980
23	0.16554	0.63041	0.97860	0.53355	0.99021	0.20922	1.0020
24	0.16289	0.62787	0.98088	0.52200	0.99269	0.20635	0.9967
25	0.16350	0.62584	0.98270	0.52977	0.99102	0.20735	1.0011
26	0.16728	0.62841	0.98039	0.51608	0.99395	0.21240	0.9966
27	0.16765	0.62281	0.98004	0.51993	0.99313	0.21207	0.9935
28	0.16352	0.61789	0.98983	0.50995	0.99527	0.20860	0.9994
29	0.16396	0.61820	0.98901	0.51032	0.99519	0.20907	0.9958
30	0.16600	0.61712	0.99052	0.49859	0.99770	0.21057	0.9945
31	0.16530	0.61955	0.98834	0.51340	0.99453	0.21055	1.0006
32	0.16408	0.61029	0.99610	0.50315	0.99673	0.21045	1.0028
33	0.16616	0.61340	0.99385	0.49764	0.99791	0.21277	0.9953
34	0.16692	0.61742	0.99025	0.49700	0.99804	0.21302	0.9911
35	0.16415	0.61289	0.99431	0.50050	0.99729	0.21012	1.0021
36	0.16379	0.61465	0.99270	0.51299	0.99462	0.20946	1.0050
37	0.16204	0.60727	0.99935	0.49047	0.99944	0.20845	0.9985
38	0.16687	0.61502	0.99240	0.50813	0.99566	0.21335	1.0014
39	0.16439	0.61223	0.99491	0.49497	0.99848	0.21694	0.9994
40	0.16466	0.60311	1.00308	0.48278	1.00109	0.21250	0.9980
41	0.16794	0.61596	0.99156	0.50446	0.99644	0.21440	1.0018
42	0.16479	0.61235	0.99480	0.50620	0.99607	0.21099	1.0074
43	0.16811	0.60813	0.99258	0.49164	0.99919	0.21600	1.0009
44	0.16305	0.59759	1.00803	0.47268	1.00325	0.21135	0.9965
45	0.16882	0.61073	0.99624	0.48537	1.00054	0.21641	0.9966
46	0.16328	0.60427	1.00204	0.48164	1.00133	0.21045	0.9980
47	0.16533	0.60345	1.00278	0.48498	1.00062	0.21327	0.9995
48	0.15552	0.60189	1.00418	0.48249	1.00115	0.20596	1.0011
49	0.16373	0.61264	0.99453	0.50006	0.99739	0.21593	1.0050
50	0.16612	0.60655	1.00000	0.48788	1.00000	[illegible]	[illegible]

Table III

Distributional Measures Averaged Over Option Portfolios of Varying Sizes, where the Relationship Between Initial Stock Price and Exercise Price Equals 1.05

Portfolio Size	Holding Period Return Minus One	Variation	Percentage of Non-Systematic Variation Eliminated	Raw Skewness	Percentage of Non-Systematic Skewness Eliminated	Percentage Return to Standard Deviation	Cube Root of Raw Skewness to Standard Deviation
1	0.16136	2.35422	0.0	10.04410	0.0	0.11144	1.0761
2	0.17195	1.57866	0.48868	3.57493	0.69471	0.14129	1.0341
3	0.16691	1.26637	0.68545	2.26831	0.83503	0.15137	1.0303
4	0.17474	1.14244	0.76354	1.69957	0.89610	0.16607	1.0210
5	0.16506	1.06042	0.81521	1.62023	0.90462	0.16299	1.0411
6	0.16350	0.96190	0.87729	1.15632	0.95444	0.16846	1.0018
7	0.17533	0.98163	0.86486	1.22853	0.94669	0.17866	1.0212
8	0.18192	0.94844	0.88577	1.16844	0.95314	0.18859	1.0151
9	0.17225	0.89555	0.91910	1.00426	0.97077	0.18350	1.0098
10	0.17525	0.89959	0.91659	1.08699	0.96189	0.18646	1.0275
11	0.18073	0.91072	0.90954	1.09806	0.96070	0.19119	1.0088
12	0.16635	0.85935	0.94191	0.98245	0.97311	0.18117	1.0152
13	0.18340	0.90160	0.91528	1.07398	0.96328	0.19453	1.0306
14	0.17506	0.86556	0.93799	0.98324	0.97303	0.18934	1.0320
15	0.17585	0.84877	0.94857	0.93157	0.97858	0.19206	1.0222
16	0.17699	0.83848	0.95506	0.91702	0.98014	0.19466	1.0160
17	0.16835	0.80705	0.97486	0.85498	0.98680	0.18860	1.0175
18	0.17771	0.82755	0.96194	0.87330	0.98483	0.19647	1.0158
19	0.17024	0.81197	0.97176	0.83981	0.98843	0.18974	1.0208
20	0.17755	0.81741	0.96834	0.84882	0.98746	0.19735	1.0158
21	0.17326	0.80726	0.97473	0.81674	0.99091	0.19374	1.0122
22	0.17635	0.81473	0.97002	0.85228	0.98709	0.19627	1.0192
23	0.17454	0.80276	0.97757	0.82032	0.98945	0.19561	1.0240
24	0.17120	0.79613	0.98174	0.79147	0.99362	0.19266	1.0152
25	0.17243	0.79628	0.98165	0.81518	0.99107	0.19398	1.0201
26	0.17774	0.79797	0.98058	0.78563	0.99425	0.19962	1.0139
27	0.17759	0.79785	0.98066	0.79160	0.99361	0.19950	1.0119
28	0.17236	0.78454	0.98905	0.77276	0.99563	0.19520	1.0166
29	0.17275	0.78475	0.98891	0.77368	0.99553	0.19567	1.0133
30	0.17456	0.78216	0.99054	0.75115	0.99795	0.19792	1.0118
31	0.17481	0.78850	0.98655	0.78630	0.99418	0.19744	1.0198
32	0.17267	0.77494	0.99509	0.76236	0.99675	0.19668	1.0213
33	0.17566	0.77950	0.99222	0.76338	0.99664	0.19964	1.0149
34	0.17627	0.78231	0.99045	0.75077	0.99799	0.19991	1.0087
35	0.17290	0.77587	0.99451	0.75336	0.99771	0.19676	1.0189
36	0.17273	0.78017	0.99179	0.77902	0.99496	0.19614	1.0226
37	0.17079	0.76976	0.99836	0.74212	0.99892	0.19521	1.0168
38	0.17616	0.77843	0.99290	0.76411	0.99656	0.20027	1.0177
39	0.17551	0.77531	0.99486	0.74422	0.99869	0.20434	1.0161
40	0.17350	0.76258	1.00288	0.72297	1.00096	0.19892	1.0134
41	0.17801	0.78193	0.99069	0.76887	0.99605	0.20175	1.0204
42	0.17383	0.77417	0.99557	0.75592	0.99744	0.19797	1.0228
43	0.17795	0.76812	0.99939	0.73862	0.99929	0.20350	1.0176
44	0.17182	0.75489	1.00772	0.70783	1.00260	0.19821	1.0133
45	0.17873	0.77368	0.99585	0.73251	0.99995	0.20361	1.0145
46	0.17238	0.76496	1.00138	0.72494	1.00076	0.19752	1.0146
47	0.17440	0.76306	1.00258	0.72699	1.00054	0.20011	1.0162
48	0.16783	0.76155	1.00353	0.72370	1.00090	0.19267	1.0175
49	0.17862	0.77445	0.99540	0.74678	0.99842	0.20333	1.0209
50	0.17581	0.76715	1.00000	0.73207	1.00000	0.20116	1.0157

Table IV

Distributional Measures Averaged Over Option Portfolios of Varying Sizes, where the Relationship Between Initial Stock Price and Exercise Price Equals 1.00

Portfolio Size	Holding Period Return Minus One	Variation	Percentage of Non-Systematic Variation Eliminated	Raw Skewness	Percentage of Non-Systematic Skewness Eliminated	Percentage Return to Standard Deviation	Cube Root of Raw Skewness to Standard Deviation
1	0.17023	3.34527	0.0	22.49785	0.0	0.10019	1.1472
2	0.18106	2.15624	0.50156	6.94780	0.72824	0.12808	1.0949
3	0.17751	1.72051	0.66536	4.44657	0.84538	0.13904	1.0773
4	0.18625	1.51658	0.77139	3.08802	0.90900	0.15419	1.0618
5	0.17465	1.42119	0.81165	3.04022	0.91124	0.14993	1.0781
6	0.17118	1.26103	0.87918	2.01851	0.95909	0.15455	1.0344
7	0.18776	1.29115	0.86648	2.15694	0.95261	0.16725	1.0590
8	0.19269	1.23453	0.89036	2.00435	0.95975	0.17555	1.0484
9	0.18196	1.16602	0.91926	1.71092	0.97350	0.17029	1.0441
10	0.18680	1.17609	0.91501	1.87760	0.96569	0.17435	1.0590
11	0.19419	1.18973	0.90926	1.92617	0.96341	0.18026	1.0421
12	0.17521	1.10661	0.94349	1.65372	0.97617	0.16845	1.0435
13	0.19757	1.12559	0.91522	1.83166	0.96783	0.18392	1.0640
14	0.18768	1.12061	0.93633	1.63046	0.97726	0.17667	1.0623
15	0.18709	1.09509	0.94918	1.54268	0.98136	0.18019	1.0516
16	0.18978	1.08629	0.95205	1.54785	0.98020	0.18364	1.0469
17	0.17661	1.03502	0.97452	1.39883	0.98811	0.17502	1.0448
18	0.18972	1.06039	0.96385	1.41909	0.98712	0.18558	1.0441
19	0.18185	1.04580	0.96997	1.38221	0.98869	0.17886	1.0517
20	0.18862	1.04950	0.96841	1.37526	0.98921	0.18527	1.0427
21	0.18436	1.03369	0.97504	1.31962	0.99182	0.18241	1.0412
22	0.18870	1.05146	0.96755	1.42133	0.98706	0.18517	1.0507
23	0.18562	1.03149	0.97580	1.36965	0.98948	0.18375	1.0554
24	0.18114	1.01447	0.98697	1.25139	0.99502	0.18067	1.0420
25	0.18326	1.02000	0.98065	1.32139	0.99174	0.18238	1.0483
26	0.18955	1.01971	0.98098	1.25182	0.99500	0.18853	1.0404
27	0.19014	1.01872	0.98139	1.25041	0.99459	0.18921	1.0384
28	0.18310	1.00162	0.98665	1.22321	0.99634	0.18368	1.0426
29	0.18344	1.00059	0.98904	1.22491	0.99626	0.18419	1.0388
30	0.18652	0.99676	0.99066	1.17986	0.99836	0.18744	1.0384
31	0.18703	1.01119	0.98460	1.26610	0.99432	0.18673	1.0402
32	0.18132	0.98679	0.99402	1.20659	0.99711	0.18478	1.0490
33	0.18239	0.99716	0.99049	1.22705	0.99615	0.18852	1.0432
34	0.18780	0.99523	0.99130	1.17973	0.99837	0.18897	1.0349
35	0.18330	0.98610	0.99515	1.17839	0.99843	0.18514	1.0445
36	0.18414	0.99649	0.99077	1.23671	0.99561	0.18518	1.0489
37	0.18165	0.98080	0.99739	1.17045	0.99891	0.18407	1.0442
38	0.18770	0.98901	0.99393	1.19416	0.99720	0.18946	1.0425
39	0.19205	0.98725	0.99467	1.16655	0.99859	0.19383	1.0420
40	0.18356	0.96791	1.00283	1.12265	1.00104	0.18713	1.0368
41	0.19109	0.99869	0.98989	1.22749	0.99613	0.19172	1.0482
42	0.18527	0.98290	0.99650	1.17181	0.99674	0.18738	1.0465
43	0.18995	0.97518	0.99876	1.15700	0.99943	0.19293	1.0430
44	0.18236	0.95724	1.00733	1.10180	1.00202	0.18691	1.0389
45	0.19115	0.98437	0.99568	1.14907	0.99981	0.19315	1.0416
46	0.18372	0.97266	1.00082	1.13570	1.00043	0.18680	1.0396
47	0.18569	0.96038	1.00253	1.13231	1.00059	0.18923	1.0417
48	0.17799	0.96794	1.00281	1.12562	1.00089	0.18133	1.0423
49	0.19071	0.98259	0.99663	1.15591	0.99948	0.19283	1.0453
50	0.18749	0.97461	1.00000	1.14496	1.00000	0.19044	1.0410

Table V

Distributional Measures Averaged Over Option Portfolios of Varying Sizes, where the Relationship Between Initial Stock Price and Exercise Price Equals .95

Portfolio Size	Holding Period Return Minus One	Variation	Percentage of Non-Systematic Variation Eliminated	Raw Skewness	Percentage of Non-Systematic Skewness Eliminated	Percentage Return to Standard Deviation	Cube Root of Raw Skewness to Standard Deviation
1	0.17959	5.08395	0.0	59.75667	0.0	0.08866	1.2227
2	0.19166	3.08264	0.52241	15.66918	0.76216	0.11503	1.1608
3	0.18566	2.43366	0.69192	9.89574	0.86197	0.12391	1.1320
4	0.19765	2.08904	0.76178	6.41060	0.92222	0.14069	1.1128
5	0.18676	1.98690	0.80844	6.54282	0.91994	0.13702	1.1227
6	0.17630	1.70067	0.88316	3.99371	0.96400	0.13812	1.0768
7	0.20163	1.75295	0.86951	4.23533	0.95983	0.15511	1.1034
8	0.20355	1.65107	0.89611	3.83389	0.96677	0.16136	1.0898
9	0.19169	1.55401	0.92144	3.20764	0.97259	0.15622	1.0861
10	0.20065	1.58178	0.91419	3.60821	0.97067	0.16255	1.0975
11	0.20881	1.59741	0.91011	3.77062	0.96786	0.16840	1.0835
12	0.18292	1.46405	0.94493	3.09332	0.97957	0.15393	1.0797
13	0.21360	1.57335	0.91640	3.44124	0.97355	0.17272	1.1060
14	0.20156	1.48535	0.93823	2.95953	0.98188	0.16725	1.1020
15	0.19932	1.44376	0.95022	2.79795	0.98468	0.16775	1.0896
16	0.20440	1.44589	0.94967	2.95297	0.98200	0.17244	1.0909
17	0.18447	1.35325	0.97385	2.50620	0.98972	0.16052	1.0806
18	0.20220	1.38549	0.96543	2.51937	0.98958	0.17366	1.0813
19	0.19476	1.37393	0.96845	2.47706	0.99022	0.16762	1.0917
20	0.20077	1.37130	0.96914	2.40800	0.99142	0.17304	1.0787
21	0.19622	1.34703	0.97547	2.30922	0.99312	0.17057	1.0795
22	0.20135	1.38611	0.96592	2.60833	0.98795	0.17274	1.0939
23	0.19819	1.35370	0.97373	2.46490	0.99043	0.17185	1.0974
24	0.19120	1.31263	0.98445	2.11636	0.99646	0.16804	1.0781
25	0.19392	1.32902	0.98017	2.32078	0.99295	0.16957	1.0864
26	0.20256	1.32327	0.98167	2.14123	0.99603	0.17719	1.0757
27	0.20317	1.32033	0.98244	2.15136	0.99585	0.17798	1.0763
28	0.19476	1.29535	0.98897	2.06905	0.99726	0.17212	1.0789
29	0.19448	1.29320	0.98952	2.08232	0.99705	0.17216	1.0738
30	0.19873	1.28807	0.99086	1.98376	0.99875	0.17593	1.0768
31	0.20060	1.31600	0.98305	2.19942	0.99502	0.17581	1.0874
32	0.19461	1.28036	0.99286	2.04033	0.99777	0.17206	1.0867
33	0.19918	1.29572	0.98887	2.11926	0.99641	0.17620	1.0818
34	0.19335	1.28155	0.99252	1.97226	0.99895	0.17238	1.0712
35	0.19332	1.26586	0.99562	1.95956	0.99917	0.17230	1.0802
36	0.19665	1.29368	0.98940	2.11960	0.99640	0.17393	1.0859
37	0.19202	1.26629	0.99655	1.96998	0.99893	0.17151	1.0826
38	0.19847	1.27061	0.99542	1.98234	0.99877	0.17704	1.0775
39	0.20496	1.27589	0.99430	1.95064	0.99932	0.18227	1.0788
40	0.19378	1.24205	1.00286	1.84460	1.00115	0.17463	1.0700
41	0.20500	1.29623	0.98873	2.10609	0.99663	0.18069	1.0866
42	0.19736	1.26294	0.99745	1.92840	0.99971	0.17633	1.0802
43	0.20233	1.25429	0.99968	1.93151	0.99965	0.18152	1.0781
44	0.19330	1.22821	1.00649	1.82677	1.00146	0.17515	1.0752
45	0.20480	1.26783	0.99614	1.91189	0.99999	0.18256	1.0796
46	0.19561	1.25266	1.00010	1.89230	1.00023	0.17554	1.0748
47	0.19700	1.24319	1.00259	1.87534	1.00062	0.17743	1.0780
48	0.18856	1.24361	1.00247	1.85340	1.00100	0.16968	1.0773
49	0.20365	1.26152	0.99779	1.89448	1.00029	0.18191	1.0798
50	0.20032	1.25307	1.00000	1.91152	1.00000	0.17923	1.0771

Table VI

Distributional Measures Averaged Over Option Portfolios of Varying Sizes, where the Relationship Between Initial Stock Price and Exercise Price Equals .90

Portfolio Size	Holding Period Return Minus One	Variation	Percentage of Non-Systematic Variation Eliminated	Raw Skewness	Percentage of Non-Systematic Skewness Eliminated	Percentage Return to Standard Deviation	Cube Root of Raw Skewness to Standard Deviation
1	0.20774	8.49784	0.0	191.48610	0.0	0.08470	1.2986
2	0.21662	4.72609	0.55165	42.86905	0.79086	0.10830	1.2326
3	0.20749	3.61371	0.71434	25.23169	0.88472	0.11659	1.1936
4	0.22171	3.05908	0.79546	15.94052	0.93416	0.13303	1.1708
5	0.21681	2.97563	0.80767	16.90575	0.92902	0.13315	1.1768
6	0.19299	2.41225	0.89006	9.47912	0.96855	0.12926	1.1279
7	0.22973	2.50985	0.87579	9.81533	0.96676	0.14982	1.1572
8	0.22690	2.31258	0.90464	8.60294	0.97321	0.15407	1.1394
9	0.21359	2.16525	0.92613	6.98029	0.98184	0.14925	1.1353
10	0.22881	2.24093	0.91512	8.12356	0.97576	0.15788	1.1450
11	0.23759	2.24939	0.91389	8.63060	0.97306	0.16382	1.1347
12	0.20177	2.01795	0.94774	6.74721	0.98308	0.14642	1.1228
13	0.24546	2.20983	0.91967	7.43165	0.97944	0.16918	1.1574
14	0.23007	2.05657	0.94209	6.11257	0.98643	0.16357	1.1504
15	0.22508	1.98543	0.95293	6.81269	0.98806	0.16310	1.1364
16	0.23383	2.00983	0.94893	5.40063	0.98493	0.16913	1.1400
17	0.20583	1.83466	0.97454	5.16060	0.99153	0.15525	1.1247
18	0.22807	1.88778	0.96855	5.07333	0.99133	0.16951	1.1271
19	0.22249	1.87741	0.96828	5.04733	0.99213	0.16502	1.1394
20	0.22569	1.86125	0.97056	4.80492	0.99342	0.16882	1.1248
21	0.22121	1.83536	0.97578	4.59940	0.99451	0.16659	1.1280
22	0.22831	1.90720	0.97433	5.49880	0.98973	0.16832	1.1483
23	0.22546	1.84852	0.97256	5.06576	0.99203	0.16653	1.1496
24	0.21356	1.75220	0.98660	3.90909	0.99776	0.16323	1.1248
25	0.21750	1.79725	0.98002	4.62657	0.99437	0.16461	1.1351
26	0.22954	1.77489	0.98325	4.13335	0.99700	0.17423	1.1225
27	0.22833	1.76633	0.98454	4.10369	0.99715	0.17375	1.1249
28	0.22044	1.72650	0.99036	3.89466	0.99626	0.16945	1.1266
29	0.21979	1.71903	0.99041	4.00667	0.99766	0.16654	1.1201
30	0.22507	1.70031	0.99146	3.72237	0.99916	0.17304	1.1280
31	0.22551	1.71113	0.98245	3.31223	0.99604	0.17394	1.1387
32	0.22043	1.73838	0.99261	3.83398	0.99859	0.16947	1.1358
33	0.22420	1.69676	0.98663	4.08101	0.99727	0.17211	1.1307
34	0.22433	1.67624	0.99477	3.64936	0.99957	0.17301	1.1197
35	0.22722	1.73843	0.99622	3.50063	0.99863	0.16853	1.1275
36	0.22351	1.66225	0.99041	4.07408	0.99731	0.17154	1.1347
37	0.22153	1.67417	0.99649	3.67101	0.99945	0.16746	1.1330
38	0.22210	1.69584	0.99802	3.61923	0.99973	0.17325	1.1232
39	0.23129	1.75855	0.99485	3.61339	0.99976	0.17887	1.1282
40	0.21607	1.61329	1.00414	3.33086	1.00126	0.17036	1.1153
41	0.22338	1.74522	0.99805	4.07135	0.99733	0.17854	1.1375
42	0.21722	1.66457	0.99946	3.50162	1.00035	0.17309	1.1263
43	0.22314	1.66104	0.99993	3.58949	0.99989	0.17275	1.1249
44	0.21753	1.62204	1.00552	3.37111	1.00105	0.17205	1.1235
45	0.22324	1.68022	0.99713	3.51052	1.00031	0.18032	1.1310
46	0.22133	1.65911	1.00022	3.51945	1.00026	0.17320	1.1210
47	0.22116	1.63791	1.00035	3.40998	1.00084	0.17407	1.1255
48	0.21336	[illegible]	1.00245	3.36568	1.00107	0.16756	1.1249
49	0.23005	1.66672	0.99910	3.43375	1.00071	0.17922	1.1279

return increases as the portfolio strategy moves from stocks to in the money options to out of the money options, where all values are measured for six-month holding periods.

Shifting the focus to the variance and skewness measures, we note that in all strategies, diversification reduces the value of these parameters via the elimination of the non-systematic components (the last two items in equations (3a) and (4a). Thus, these results support the aforementioned hypotheses concerning the effects that diversification should have upon portfolio variation and skewness. In particular, the evidence indicates that the sign of the combination of the last two terms in each of (3a) and (4a) are significantly positive at the one security level to produce such dramatic diversification effects upon these parameters. Furthermore, if we assume that a portfolio of size fifty is well-diversified in the sense that the variance and skewness amounts (at this portfolio size) approximate their asymptotic (systematic) levels, Tables I-VI demonstrate that diversification has a different impact upon variation vis a vis skewness. Columns (4) and (6) of each of the tables represent the rate at which the non-systematic amounts of variance and skewness are eliminated.[16] The results reveal that skewness approaches its systematic level at a faster rate than variance.

The Systematic Levels of Variance and Skewness for Alternative Stock/Option Strategies

Tables I-VI demonstrate that systematic variance (risk) and skewness increase as one moves from stocks to options with increasingly higher exercise prices. Thus, the hypotheses set forth in (3b), (3c), (4b), and (4c) are supported by the empirical results. In particular, there is a dramatic jump in the systematic levels of these distribution parameters as one moves from stocks to options that are in the money by ten percent.

Since the results reported in Tables I-VI represent average values for 200 portfolios of a given size, the beginning and ending amounts in the variance and skewness columns can be interpreted as the total and systematic levels of risk and skewness for a typical stock/option. Thus, since an equal weighting scheme has been assumed, the systematic (size fifty) levels found in Tables I-VI can be used to compute the implied variation and skewness responsiveness factors inherent in the typical option of a given stock price/exercise price classification scheme. These factors would represent the systematic risk (beta) and systematic skewness (gamma) inherent in the typical option of a given classification scheme. The greater the values of the factors, the greater is the systematic risk and systematic skewness of the option. Thus, the average beta of an option in the jth classification scheme can be found by solving:

$$(31) \qquad E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]_J^2 = x_{m+1}^2 \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]_P^2$$

for x_{m+1}, where $x_{m+1} = \sum_{i=1}^{50} x_i \cdot b_i(J)$ and $E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]_p^2 = .02997$.

Also, the average gamma (co-skewness) of an option in the jth classification scheme can be found by solving:

$$(32) \qquad E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1}]_J^3 = x_{m+1}^3 \cdot E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]_p^3$$

for x_{m+1}, where $x_{m+1} = \sum_{i=1}^{50} x_i \gamma_i(J)$ and $E[\tilde{e}_{m+1}-E(\tilde{e}_{m+1})]_p^3 = .00609$

These results are presented in Table VII and indicate, on an average basis, how the systematic levels of risk and skewness increase through the alternative asset categories.[17]

Finally, there is the issue of whether or not some portfolio strategies are more effective in diversification than others. In Klemkosky and Martin [8], it was found that portfolios containing greater levels of systematic risk also contained greater amounts of non-systematic risk at any level of diversification. Futhermore, the rate at which the non-systematic risk was eliminated was slower for the portfolios with greater systematic risk. The conclusion was (for stocks) that portfolios with greater systematic risk were less diversified at any given level of diversification.

Examination of the variance and skewness columns of Tables I-VI clearly indicates that portfolio strategies containing greater amounts of systematic risk and systematic skewness also contain greater <u>amounts</u> of the non-systematic components of these parameters at any level of diversification.[18] Thus, our results indicate that (15) and (19) hold.

However, columns (4) and (6) in Tables I-VI reveal that the <u>rates</u> at which the non-systematic components are eliminated does not vary in any detectable fashion across alternative stock/option strategies. Hence, (16) and (20) do not hold. Thus, an interesting question arises as to what is the appropriate measure of whether one strategy is more diversified than another--the amount of non-systematic risk/skewness

Table VII

Average Betas and Gammas Inherent
in Alternative Stock/Option Strategies

Strategy	Beta	Gamma
Stocks	1.0000	1.0000
In the Money, 10%	4.4987	4.3109
In the Money, 5%	5.0594	4.9353
At the Money	5.7026	5.7287
Out of Money, 5%	6.4661	6.7960
Out of Money, 10%	7.4437	8.3678

or the rate at which non-systematic risk/skewness is eliminated? The answer is not clear since the amounts of all of the relevant components differ across the portfolio strategies.[19] On the one hand, the levels effect may be more important simply because the greater the amount of non-systematic variation, the greater the amount of total variation (since the systematic amount is constant) and hence the greater the risk. On the other hand, if the rate of reduction is the same, then the systematic levels will be reached at the same portfolio size.

Diversification—Is It Worth It?

Traditional two-parameter asset pricing theory suggests that relating the average return to standard deviation provides a useful measure of the performance and, hence, attractiveness of a particular portfolio strategy. Column (7) of each of Tables I-VI indicates that this ratio is increasing with diversification, as it should, for all of the portfolio strategies. Furthermore, the values decrease as one moves into successively more "risky" strategies. Thus, stocks were more "efficient" than options in the sense of providing more return relative to the risk borne. Alternatively, in a systematic risk-return framework, the size fifty values of this ratio indicates that the additional return earned by options was outbalanced by the increased level of systematic risk.

Now suppose investors base their investment decisions upon the first three moments of the return distribution. In this framework, the last column of Tables I-VI provides another measure of asset performance. If skewness is desirable, then investors desire higher values of (skewness)$^{1/3}$/variance)$^{1/2}$. For this sample of stocks, this ratio is also increasing, thus implying that diversification is desirable.[20] However, for options, the opposite result occurs. This raises the question of whether or not diversification is desirable if option investors prefer skewness.

Conclusion

This study has analyzed the issue of diversification in the options market. The results reported, for the most part, confirm the hypotheses stated. In particular, diversification does indeed reduce the levels of risk and skewness inherent in alternative option portfolio strategies. Furthermore, it appears that the additional systematic risk borne by option holders is not compensated for by the additional return. Finally, the desirability of diversification is questioned somewhat in light of the presumed preference of option investors for positive skewness.

Footnotes

[1]Assuming equal weighting.

[2]As was the case for the unsystematic component, with an equal weighting scheme, the average value of this term (as $m \to M$) goes to zero.

[3]Under the technique of ordinary least squares, these assumptions are usually invoked to protect the properties of the estimated coefficients. Whether or not these assumptions hold is another matter. Empirical evidence (see [8]) seems to indicate that the last term, for common stocks, is positive. Thus, risk reduction would appear to result from elimination of both the last two terms in (3a).

[4]The specification of an "option market model" is necessarily more difficult for at least two reasons. First, since an option is a short-lived asset, its rate of return can vary dramatically over its life due to changes in the parameters (especially time to maturity and the price of the underlying stock) affecting its value. Hence, there can be a great deal of heterogeniety among an option's daily or weekly returns which in turn can create a heteroskedastic element in return variance. The result is that the systematic risk (proxied by beta) of an option is apt to be very nonstationary (see [2]).

Second, "the" appropriate market factor requires careful consideration. Since options are derivative assets, some would argue that an index of stocks is the correct choice. However, some will argue that individual assets and indexes be of the same type—thus, implying that an index of options is more appropriate. In any event, the issue is not clear and any market specification is apt to be incorrect (see [11]). These two issues are not dealt with in this study, but are the subject of another forthcoming paper by the authors.

[5]Assuming equal weighting, this term $\to 0$ as $m \to M$.

[6]The value of this term should also $\to 0$ as $m \to M$.

[7]As discussed in footnote 4, the quantification of these two factors is the subject of another forthcoming paper.

[8]It is important to keep in mind that the theory is formulated in "expected" terms; whereas most of the empirical tests relate to "ex-post" or realized results. Researchers studiously avoid the issue of whether or not ex-post realizations are what investors expect ex-ante. In any event, the purpose of this paper is not to specify or test a particular theory about the pricing of option assets. Rather, our purpose is to lend credence to the empirical results in light of generally accepted finance theory.

[9] In words, (8) says that the responsiveness of an option to movements in the market factor, B_c, equals the product of the responsive of an option to movements in the underlying stock, $(P \cdot N(D1)/C)$, and the responsive of the stock to movements in the market factor, B_p. Here, "betas" are used as proxies for the underlying systematic risks.

[10] The fact that one option is in the money and the other option is out of the money is not critical in the establishment of the ensuing analytical argument. It simply distinguishes the two options in the mind of the reader. The critical thing is that the exercise prices of the two options are different.

[11] This result assumes that $B_x > 0$, which, in light of empirical evidence, seems reasonable.

[12] A paper is forthcoming which tests the importance of skewness in explaining option returns.

[13] Random samples (with replacement) were generated via a random number generator. Alternative initial seed specifications produced strikingly similar results. Thus, we believe our results are not significantly biased in any detectable fashion.

[14] Sampling error explains the fluctuation in the return columns as well as the nonmonotonic nature of the variance and skew as they approach their asymptotic values. Similar phenomena occur in Evans and Archer [7] and Simkowitz and Beedles [12].

[15] Some of the prior studies in diversification, [7] and [8], have employed logarithms because of the desirable properties of logs. However, in this study, logs cannot be used due to the fact that for some strategies, the portfolio return was zero for a given time period and portfolio chosen. Therefore, the results presented in Tables I-VI are in terms of the raw data. This, of course, does introduce some additional skewness into the figures.

[16] Columns (4) and (6) of each of Tables I-VI were tabulated via the formulas in (16) and (20), where $M = 50$. Thus, the non-systematic levels inherent in a given portfolio strategy were taken as the difference between the levels at size one (total risk or skewness) and size fifty (systematic risk or skewness).

[17] It is important to remember that the beta and gamma values presented are on an <u>average</u> basis. That is, on average, the beta of an option that is 10 percent in the money is 4.4987; on average, the gamma of an option that is 10 percent in the money is 4.3109. Individually, option betas and gammas will fluctuate about these levels. The examination of these values on an individual basis, as well as their importance in explaining option returns are being investigated in forthcoming papers (see footnotes 4 and 12).

[18] Subtracting the systematic levels of these parameters from the corresponding risk and skewness values at a given portfolio size and then relating these differences (the non-systematic amounts remaining) across alternative portfolio strategies indicates that the amount of the non-systematic levels increases in the direction of the strategies containing higher systematic levels of these two parameters.

[19] It is important to note that the comparisons are being made across different portfolio strategies stratified, in the case, according to the systematic levels of relevant variables. A separate issue is whether or not the systematic level of a given parameter is important in explaining the effectiveness of diversification within a given portfolio strategy. This second issue is being examined in another forthcoming paper.

[20] We are somewhat puzzled as to why this ratio increases with diversification. Our results are in conflict with the sample results in [12]. One possible explanation is that our sample is very special and as a group enjoyed "above average" performance over the period analyzed. Thus, the persistance of positive skewness probably indicates one reason for this sample's desirability in option exchange selection.

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