BEBR

FACULTY WORKING PAPER NO. 1260

Dividend Policy Under Conditions of Capital Market and Signaling Equilibria

Dong Han
Cheng F. Lee

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 1260

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

June 1986

Dividend Policy Under Conditions of Capital
Market and Signaling Equilibria

Dong Han
Arizona State University

Cheng F. Lee, Professor
Department of Finance

Abstract

This study develops a dividend signaling capital market equilibrium model under the assumption of the asymmetric information between corporate insiders and outside investors being resolved through dividends. The generalized capital asset pricing model is shown to satisfy the condition of the dividend signaling equilibrium through analyzing the costs and benefit of paying dividends. The model provides a theoretical framework for testing the existence of the market moral hazard penalty rate. If dividends serve as a signal and the penalty rate is positive, paying higher dividends would result in higher systematic risk. Furthermore the model can identify the agency cost occurring between current and new shareholders, assuming that managers' objective is to maximize the current firm value.

DIVIDEND POLICY UNDER CONDITIONS OF CAPITAL MARKET AND SIGNALING EQUILIBRIA

I. Introduction

The effect of dividend policy on stock prices still remains as one of puzzling issues in finance theory. The traditional studies can be summarized into three established major contending hypotheses about the dividend effects. The first is the view that risk-adverse investors are likely to perceive current dividends as less risky than future ones. Hence increasing current dividends will result in increasing share prices and vice versa [Gordon, 1963]. The second view is that within a perfect capital market dividend policy is irrelevant to the share prices, provided the investment decision is independent of dividend policy [Miller and Modigliani, 1961 (MM)]. The irrelevance proposition is preserved even under the first hypothesis [Higgins, 1972] as well as in a world where dividends receive tax penalties relative to capital gains [Black and Scholes, 1974; Miller and Scholes, 1978, 1982]. The last contrary view is that the market requires higher returns and hence lower current prices on high dividend yielding stocks to compensate for the tax disadvantage of dividend income [Brennan, 1973; Litzenberger and Ramaswamy, 1979]. As shown in the recent theoretical financial literature the first traditional view has not received much support. However the other two hypotheses cannot explain the nearly universal policy of paying substantial dividends, considering the obvious cost of paying dividends to the firms involved.[1]

One possible resolution of the "puzzle" is that dividends can convey information about a firm's future level and growth of real income to the capital market, if the perfect information assumption is relaxed [MM, 1961].[2] A number of studies have tested the MM's information content of dividends (ICD) hypothesis by examining the abnormal returns during the period surrounding the dividend announcement date. However the results are mixed.[3] A common difficulty in testing the ICD hypothesis is to measure the unexpected portion of the dividends announcement, since the expected portion would be already incorporated in the announcement day stock prices. Asquith and Mullins [1983] investigate the impact of initiating dividend payments on share prices, assuming that initial dividend payments are totally unexpected by the market. They find that the positive excess returns associated with initiating dividends are larger than in any other studies, which strongly supports the ICD hypothesis.

Based on the ICD hypothesis, Bhattacharya [1979] develops a dividend signaling equilibrium model in which cash dividends function as a signal of expected cash flows of firms in an imperfect information setting. The promised dividends are assumed to resolve information asymmetries which exist between corporate managers who possess superior information about the future profitability of the firm's assets and outside investors. Under the assumptions of a risk-neutral world and a uniform distribution of future cash flows (among other assumptions) he derives an equilibrium optimal dividend function. By assuming risk neutrality he avoids the capital market equilibrium. Development of a dividend signaling theory under condition of capital market equilibrium would enhance the understanding of not only dividend policy but also the risk structure in a real world.

Talmor [1981] extends the Bhattacharya model to employ the more plausible assumptions of normal distribution of cash flows and a risk-adverse world. He develops a general signaling theory in which multi-financial instruments serve as signaling devices for multi-unknown valuation parameters. Applying general signaling theory into a specific example, he shows the feasibility of the optimal function.[4] No empirical hypotheses could be derived. Furthermore Talmor employs the certainty equivalent (CEQ) of the firm's expected earnings using the traditional CAPM in order to incorporate the risk of the future earnings into his model. However, he fails to get an accurate CEQ since the market does not assess the appropriate risk from the distribution of the before-dividend earnings. The appropriate risk should come from the distribution of the after-dividend earnings which are supposed to be discounted to assess the current value of the firm in the market.

There has been only one study directly related to testing the dividend signaling theory [Eades, 1982].[5] Instead of deriving an optimal dividend function, Eades indirectly shows a negative relationship between equilibrium optimal dividends and variance of future cash flows. The results seem to support the implied negative relationship.[6] A major drawback of the study is that it does not provide a theoretical background for explicitly testing the feasibility of the dividend signaling equilibrium.[7]

This study develops a dividend signaling theory under condition of capital market equilibrium. The paper extends the work of Bhattacharya [1979], Brennan [1973] and Litzenberger and Ramaswamy [1979] [LR] by achieving the general capital market equilibrium in Section II. The general capital market equilibrium model can be possibly derived under the condition that the signaling equilibrium is achieved. Based on the capital market equilibrium model derived in Section II, the signaling equilibrium condition is examined in Section III. It is shown that the capital market equilibrium model satisfies the Spence-type [1974] signaling equilibrium condition. Other important theoretical finding is that the firm's systematic risk would be higher when the firm pays dividends than when they would not, if dividends serve as a signal of the firm's uncertain future cash flows. The dividend signaling capital market equilibrium model can identify the agency cost, between current and future stockholders, occurring in a way of resolving the informational asymmetries about the firm's future earnings. In Section IV, major results are summarized and concluding remarks are indicated.

II. Capital Market Equilibrium Model in a Imperfect-Information Setting

This section derives an equilibrium certainty-equivalent (CEQ) formula for the market value of the firm under the assumption of the asymmetric information between corporate insiders and outside investors only about the future profitability of firms. As MM [1961] implicitly show what is valued in the marketplace is the perceived stream of expected cash flows for the firm. If the market agrees that corporate managers know better the future income stream than the market and that they have the proper incentive to signal the true income stream to the market, the market will try to adapt its perception to the signaled future income stream by the firm. The dividend level set by each firm is assumed to function as a signal through which the uncertain future cash flows of the firm can be unambiguously revealed to the market. However, a dividend signaling equilibrium is required in order to make the dividend level set by the firm serve as a signal. In other words there should be costs involved in paying dividends that are sufficient enough to prohibit firms from sending false signals, the so called moral hazard problem discussed by Ross [1977] and Bhattacharya [1979]. The signaling equilibrium will be discussed in detail in Section III. In this section the signaling equilibrium is assumed to be achieved.

Let $\bar{X}^P$ be the perceived expected future cash flow in the market through the announced dividend level of the firm. Then $\bar{X}^P$ would be the conditional expected value of the firm's future cash flow, given the announced dividend (D). In equation $\bar{X}^P$ can be expressed as,

$$\bar{X}^P = G(D). \tag{1}$$

The dividend function, G(D), is assumed to be known to be the market. The market uses past experience to estimate the uncertain future cash flow, given the dividend signal. As the acutal cash flow is revealed ex post, the market will revise the function G(D). Thus in equilibrium G(D) is to be known to the market. However, if G(D) turns out to be not equal to the profitability of the firm as signaled by the dividend level ex post, the dividend cannot serve as a signal. Thus, in equilibrium $\bar{X}^P$ should equal the actual expected future cash flow $\bar{X}$, that is

$$\bar{X}^P = \bar{X} \tag{2}$$

for every D. The conditions for the signaling equilibrium are examined in Section III. Since it is assumed that the signaling equilibrium is achieved, the superscript of $\bar{X}$ will be dropped in this section. In summary, the signaling equilibrium allows the market to interpret the dividend signal homogeneously and to get rid of the informational asymmetry on the firm's future expected profitability.

In order to clearly identify the benefits and the costs of paying dividends and to simplify the model structure in a two period context, it is assumed that each firm i generates perpetuity of uncertain net after-tax operating cash flow ($\tilde{X}_i$) and have a normal distribution, and the market return ($\tilde{R}_m$) on all assets in the economy is stable through time. Investors' risk preferences and tax rates are also assumed to remain constant through time. The market convention on the dividend policy, managerial reluctance to cut dividends, is embodied in this model in the following way. At the beginning of the period managers send a signal by promising a certain level of dividends to be paid at the end of each period based on their expectation ($\bar{X}$) on the firm's uncertain cash flows at the end of the period. The cash flows are perpetual streams which are intertemporally independently identically distributed. The announced dividends are supposed to be paid at all periods in the future. Under this setting, a market moral hazard penalty rate (γ) is introduced, as in Bhattacharya [1979]. The penalty rate is designed to prevent 'poor' firms from sending good signals which should be sent by 'good' firms. That is, if $X < D$, the short fall should be financed from other sources of funds. In the way of financing the difference, the additional costs incurred to current stockholders are defined as $\gamma(D-X)$ compared with the case of not paying dividends. The penalty rates could be transaction costs or dissipative costs for additional financing.[8] It is assumed that market homogeneously assesses the rate in an ex ante sense and the rate is commonly applied to all firms. Thus we term the rate γ as the market moral hazard penalty rate.

The above set of assumptions (i.e., perpetual operating cash flows, the market moral hazard penalty, and the dividend signaling equilibrium) can lead to the uncertain end-of-period value of the firm, $\tilde{V}_1$, after dividends have been paid to equal the certain beginning-of-period value, V_0, plus the uncertain after-dividend cash flow:

$$\tilde{V}_1 = V_0 + (\tilde{X} - D)(1 + \gamma z), \tag{3}$$

where the dummy variable, z, is

$$z=0 \text{ if } X \geq D,$$

$$z=1 \text{ if } X<D.$$

V_0 will be constant through time, because of the set of assumptions. The expected value of the firm at the end of each period will be

$$E(\tilde{V}_1) = V_0 + E[(\tilde{X} - D)(1 + \gamma z)], \text{ or} \tag{4}$$

$$E(\tilde{V}_1) = V_0 + \int_D^{\infty} (\tilde{X} - D)f(X)dX + \int_{-\infty}^{D} (1 + \gamma)(\tilde{X} - D)f(X)dX. \tag{5}$$

The promised dividend level will be a truncated point because the market assesses the penalty for the case when actual cash flows (X) are less than the promised dividend (D).

The effect of dividend policy on the expected return of equity securities is investigated in detail in a perfect information setting by Brennan [1973] and LR [1979]. The foregoing arguments will be integrated with LR type of CAPM. In order to examine the capital market equilibrium relationship between the value of the firm and the promised dividend level, the CEQ form of CAPM under the imperfect information assumption will be developed, employing the additional assumptions in the LR study.[9]

The notations used in the model are:

V_{0i} = the value of the ith firm at the beginning of period;

V_{1i} = the value of the ith firm at the end of period;

D_i = total dividend payments promised by the ith firm and known with certainty at the beginning of period;

x_i^k = the fraction of the ith firm held by the kth individual;

B^k = total dollar amount of money invested in the riskless asset by the kth individual (a negative value indicates borrowing);

V_{0m} = the market value of the firms in the market at the beginning of period;

V_{1m} = the market value of the firms in the market at the end of period;

W_0^k = the kth individual's initial wealth;

W_0^m = the market's initial wealth;

$U^k(\mu_k, \sigma_k^2)$ = the kth individual's expected utility function defined on the mean and variance of the after-tax portfolio wealth, respectively;

α = the margin requirement in the market;

Y_1^k = the kth individual's taxable income at the end of period;

t^k = the kth individual's average tax rate ($t_k = g(Y_1^k)$);

T^k = the kth individual's marginal tax rate ($T_k = dt^k Y_1^k / dY_1^k = t^k + Y_1^k g'(Y_1^k)$);

θ^k = the kth individual's global risk tolerance;

θ^m = the market's global risk tolerance;

R_m = the market rate of return.

The kth individual's taxable income at the end of the period is

$$Y_1^k = \sum_i x_i^k D_i + r_f B^k. \tag{6}$$

The mean after-tax value of the kth individual's portfolio, under the assumption of the signaling equilibrium, is

$$\mu_k = \sum_i x_i^k (E(\tilde{V}_{1i}) + D_i) + (1 + r_f)B^k - t^k(\sum_i x_i^k D_i + r_f B^k). \tag{7}$$

Substituting equation (5) for $E(\tilde{V}_{1i})$, equation (7) becomes

$$\mu_k = \sum_i x_i^k [V_{0i} + \int_{D_i}^{\infty} (\tilde{X}_i - D_i) f(X_i) dX_i + \int_{-\infty}^{D_i} (1 + \gamma)(\tilde{X}_i - D_i) f(X_i) dX_i + D_i]$$
$$+ (1 + r_f) B^k - t^k(\sum_i x_i^k D_i + r_f B^k). \tag{8}$$

The variance of the after-tax value of the kth individual's portfolio is

$$\sigma_k^2 = \sum_{ij}\sum x_i x_j \mathrm{cov}[V_{0i} + (\tilde{X}_i - D_i)(1 + \gamma z_i),\ V_{0j} + (\tilde{X}_j - D_j)(1 + \gamma z_j)]. \tag{9}$$

The above equation can be written as

$$\sigma_k^2 = \sum_{ij}\sum x_i x_j \mathrm{cov}[(\tilde{X}_i - D_i)(1 + \gamma z_i),\ (\tilde{X}_j - D_j)(1 + \gamma z_j)]. \tag{10}$$

The budget constraint is

$$\sum_i x_i^k V_{0i} + B^k = W_0^k. \tag{11}$$

The income constraint on borrowing is

$$\sum_i x_i^k D_i + r_f B^k > 0.^{10} \tag{12}$$

The margin constraint on borrowing is

$$(1 - \alpha)\sum_i x_i^k V_{0i} + B^k > 0, \tag{13}$$

where α, $0 < \alpha < 1$, is the margin requirement imposed on the individual investor.[11]

The kth individual's objective is to find the optimal weight (x_i^k) and borrowing amount (B^k) which maximize his/her expected end-of-period utility subject to his/her constraints, i.e., equation (11), (12), and (13):

$$\text{MAX } EU(\mu_k, \sigma_k^2)$$

subject to

$$\sum_i x_i^k V_{0i} + B^k = W_0^k,$$

$$\sum_i x_i^k D_i + r_f B^k > 0, \text{ and}$$

$$(1 - \alpha) \sum_i x_i^k V_{0i} + B^k > 0. \tag{14}$$

Assuming that all investors have homogeneous expectations regarding μ_k and σ_k^2 after the dividend announcement, the kth individual's constrained optimization can be solved by forming the Lagrangian, Z^k:

$$Z^k = EU^k(\mu_k, \sigma_k^2) + \lambda_1^k(W_0^k - \sum_i x_i^k V_{0i} - B^k)$$
$$+ \lambda_2^k(\sum_i x_i^k D_i + r_f B^k - S_2^k) + \lambda_3^k((1 - \alpha)\sum_i x_i^k V_{0i} + B^k - S_3^k), \tag{15}$$

where λ_1^k, λ_2^k, λ_3^k are the Lagrange multipliers, and S_2^k and S_3^k are nonnegative slack variables. Differentiating partially with respect to x_i^k and B^k, and setting these derivatives equal to zero, an equilibrium relationship for all individuals can be derived. The equilibrium relationship can be summed over all individuals by using the market equilibrium condition (all assets must be held by investors). Then the equilibrium value of the firm (V_{0i}) can be expressed by

$$V_{0i}(D_i) = (1/1+a+(1-c)r_f)[V_{0i} + \int_{-\infty}^{D_i} (1+\gamma)(\tilde{X}_i - D_i)f(X)dX_i + \int_{D_i}^{\infty} (\tilde{X}_i - D_i)f(X)dX_i$$
$$+ D_i(1-c) - \lambda \text{cov}((\tilde{X}_i - D_i)(1+\gamma z_i), \tilde{R}_m)]. \tag{16}$$

where

$$a = \alpha(\sum_k \theta^k/\theta^m)(\lambda_3{}^k/U_1'),$$

$$c = (\sum_k \theta^k/\theta^m)(T^k-\lambda_2{}^k/U_1'),$$

$$\lambda = (W_0{}^m/\theta^m)(1/V_{0m}),$$

$$U_1' = \alpha U(\mu_k,\sigma_k{}^2)/\alpha\mu^k.$$ [12]

The term 'c' presents weighted average of investors' marginal tax rates if the income constraint is not binding. ($\lambda_2{}^k = 0$).[13] The weights (θ^k/θ^m) will depend on individuals' global risk tolerances. The term 'a' is related to the wealth constraint. If the wealth constraint is binding ($\lambda_3{}^k \neq 0$) and when the margin requirement is positive, 'a' would be positive. If the wealth constraint is not binding ($\lambda_3{}^k=0$) or when the margin requirement is zero, 'a' would be zero. And the term 'λ' is a scaling factor. However we must evaluate the covariance term in equation (16) as the expectation over all X in the following way (~ is dropped for convenience sake):

$$\text{cov}[(X-D)(1+\gamma z),R_m] = \text{cov}(X,R_m) + \gamma\text{cov}(zX,R_m) - \gamma D\text{cov}(z,R_m) \tag{17}$$

$$= \text{cov}(X,R_m)\ (1 + \gamma F(D)), \tag{18}$$

where F(D) is the cumulative normal density function at D.[14]

Then the equilibrium value of firm i at the beginning of the period in equation (16) will be rewritten in the form of the following equation, using equation (18):

$$V_{0i}(D_i) = (1/(1+a+(1-c)r_f))[V_{0i} + \int_{-\infty}^{D_i} (1+\gamma)(X_i-D_i)f(X)dX_i$$

$$+ \int_{D_i}^{\infty} (X_i - D_i) f(X) dX_i + D_i \ (1-c) - \lambda \text{cov}(X_i, R_m)(1+\gamma F(D_i))], \tag{19}$$

which is the capital market equilibrium value of firm i under the assumption that the dividend signaling equilibrium is achieved.

Equation (19) reduces to the traditional CEQ CAPM form under the assumptions of the perfect information, no tax, no income and margin constraints, i.e.,

$$V_{0i} = (1/(1+r_f))[E(V_{1i}) + D - \lambda \text{cov}(X_i, R_m)], \tag{20}$$

where $E(V_{1i}) = V_{0i} + E(X - D)$, the expected value of the firm after paying dividends at the end of the period. Since $E(V_{1i}) + D$ is same regardless of the amount of dividends paid, dividend policy is irrelevant to share prices according to the traditional CEQ CAPM.

If there are no informational asymmetries between corporate insiders of firm i and investors about the future profitability of firm i, the equilibrium value of firm i, equation (19), reduces to

$$V_{0i} = (1/(1+a+(1-c)r_f))[E(V_{1i}) + D_i(1-c) - \lambda \text{cov}(X_i, R_m)], \tag{21}$$

under the assumptions of progressive tax scheme, known dividends, and the income and margin constraints. Equation (21) will be the LR's type of CEQ CAPM. As noted earlier, paying a dividend will decrease the firm value by the amount of discounted tax penalty (i.e., $cD_i/1+a+(1-c)r_f$). Thus the expected return will increase as dividends increase to compensate the tax penalty under the LR's CAPM.

In order to compare the equilibrium value in the imperfect information setting with the LR's type, let

$$E(V_{1i}(D)) = V_{0i} + \int_{-\infty}^{D_i} (1+\gamma)((X_i - D_i) f(X) dX_i + \int_{D_i}^{\infty} (X_i - D_i) f(X) dX_i \qquad (22)$$

where $E(V_{1i}(D))$ is the expected value of the firm after paying dividends at the end of the period and is a function of the announced dividends. In other words, $E(V_{1i}(D))$ is signaled by announcing dividends at the beginning of the period. Then equation (19) becomes

$$V_{0i}(D_i) = (1/(1+a+(1-c)r_f))[E(V_{1i}(D_i))+D_i(1-c)$$
$$-\lambda \mathrm{cov}(X_i, R_m)(1+\gamma F(D_i))]. \qquad (23)$$

In equation (23) paying dividends will decrease the firm value by $(cD+\lambda \mathrm{cov}(X_i,R_m)\gamma F(D_i))/(1+a+(1-c)r_f)$ without considering the benefit of paying dividends. Compared to the cost of paying dividends in LR, the costs in this model appear to be the added discounted covariance risk as well as the discounted tax penalty on dividends.[15] However if the managers' objective is to maximize the present firm value, they will not pay dividends unless $E(V_{1i}(D))$ increases more than the costs of paying the dividend. The benefit of paying dividends is reflected in $E(V_{1i}(D))$. Thus, under the dividend signaling theory, paying dividends should result in increasing the current firm value, which is in contradiction to the LR's result.[16]

Equation (23) can be converted into a rate of return form, using equation (18), if we define the covariance term in equation (23) as

$$\begin{aligned}
& \lambda \mathrm{cov}(X_i, R_m)(1+\gamma F(D_i)) \\
= \; & \lambda \mathrm{cov}[(X_i - D_i)(1+\gamma z_i), R_m] \\
= \; & \lambda \mathrm{cov}[V_{0i} + (X_i - D_i)(1+\gamma z_i), R_m] \\
= \; & \lambda V_{0i} \mathrm{cov}[\{V_{0i} + (X_i - D_i)(1+\gamma z_i) + D_i - V_{0i}\}/V_{0i}, R_m] \\
= \; & \mathrm{var}(R_m) \lambda V_{0i} \mathrm{cov}(R_i, R_m)/\mathrm{var}(R_m) \\
= \; & \mathrm{var}(R_m)(W_0^m/\theta^m)(V_{0i}/V_0^m)\beta_i . \qquad (24)
\end{aligned}$$

Then equation (23) equals

$$V_{0i}(D_i) = (1/(1+a+(1-c)r_f))[E(V_{1i}(D)) + D_i - cD_i - \mathrm{var}(R_m)(W_0^m/\theta^m)(V_{0i}/V_0{}^m)\beta_i]. \tag{25}$$

Using $E(R_i)=[E(V_{1i}(D_i))+D_i-V_{0i}]/V_{0i}$, we now have the capital market equilibrium model under the condition of the signaling equilibrium, that is

$$E(R_i) - r_f = a + b\beta_i + c(d_i - r_f), \tag{26}$$

where

$$b = \mathrm{var}(R_m)(W_0^m)/(\theta^m V_0{}^m),$$

$$\beta_i = \mathrm{cov}(R_i, R_m)/\mathrm{var}(R_m), \text{ and}$$

$$d_i = D_i/V_{0i},$$

other notations have been defined in equation (16). The functional form of equation (26) under the signaling theory is exactly the same as LR's. However the interpretation is different. The expected return increases as dividend increases, because the expected value of the firm at the end of the period $(E(V_{1i}(D)))$ increases more than the increase in costs of paying dividends, not because the present value of the firm decreases with $E(V_{1i})$ given as LR says. Thus under the signaling theory paying dividend has a positive impact on the current value of the firm as well as on the expected stock return, since paying dividends result in increasing the market's perceived value at the end of period under the signaling equilibrium.

From the equation (24), the systematic risk (β_i) of the dividend signaling CAPM, equation (26), can be written as

$$\beta_i = \beta_{Pi}(1 + \gamma F(D_i)), \tag{27}$$

where

β_i = the firm i's systematic risk when the market is informationally imperfect and the informational asymmetries can be resolved by dividends,

β_{Pi} = the firm i's systematic risk when the market is informationaly perfect.[17]

Under the traditional and LR's CAPM, $\beta_i = \beta_{Pi}$, since the true expected cash flows are assumed to be revealed to the market without costs (i.e., $\gamma = 0$). It is difficult to find empirically the difference between β_i and β_{Pi}, because what we can estimate ex post is β_i not β_{Pi}, regardless of the assumptions about the information market. However, equation (27) implies β_i with D_i^A is larger than β_i with D_i^B, <u>ceteris paribus</u> if $D_i^A > D_i^B$ and the market moral hazard penalty rate is positive. Thus a direct test for the dividend signaling theory could be designed to show whether the penalty rate (γ) is positive, based on the theoretical finding of equation (27).

One more observation from this section is that we can identify the cost of informational asymmetries from equation (21) and (23). Equation (21) indicates the firm value when the true $\overline{X}$ is known to the market, while equation (23) shows the firm value when the true $\overline{X}$ is signaled through dividends. It is obvious that V_{0i} of (21) is larger than $V_{0i}(D_i)$ of (23), because of the positive market moral hazard penalty rate. Thus the difference between equation (21) and (23) can be defined as the cost of resolving the informational asymmetries, which is born by current shareholders.[18] The difference would be the agency cost occurring from the conflict between current and new shareholders.

III. Dividend Signaling Equilibrium

Once dividends are announced, the firm's perceived market value at the end of the period will be valued according to equation (19) under the assumption of the signaling equilibrium. The signaling equilibrium can be said to be achieved when the market perceived expected value of cash flows equals the true expected value of cash flows. In other words when the true expected value is signaled by announced dividends, the market homogeneously believes that the signaled expected value is the true expected value. In this section the necessary condition for the signaling equilibrium is examined.

The main reason for dividends to serve as signals is the signaling costs. In order to identify the signaling costs explicitly, equation (19) can be rewritten as

$$V_{0i} = (1/(1+a+(1-c)r_f)[V_i + \bar{X} - cD - \gamma\int_{-\infty}^{D} F(X)dX$$
$$-\lambda cov(X,R_m)(1+\gamma F(D)]. \tag{28}$$

The costs of paying dividend are, from equation (42),

$$cD + \gamma\int_{-\infty}^{D} F(X)dX + \lambda cov(X,R_m)\gamma F(D). \tag{29}$$

The first term can be considered as the tax penalty for cash dividends because the ordinary income tax rate is imposed on cash dividends, whereas no tax is assumed on capital gains. The second term is the market expected penalty amount for the firm that should finance the difference when X<D. The market penalty can be inferred from the observed market convention that firms usually maintain the promised dividend level. When net operating cash flows are less than the promised dividend level, financing for paying the promised dividend level will incur additional costs to current stock holders. Thus investors will assess the

possibility of actual cash flows being less than the promised dividends by imposing the market's expected penalty. The last term in equation (29) is the added covariance risk by paying dividends. The risk results from the covariance between the truncated after-dividend cash flows with the market return.

So far the promised dividend level is treated as an exogenous variable in determining the equilibrium market value of the firm. Managers who have inside information on the firm's future cash flows are assumed to maximize the equilibrium firm value by choosing an optimal dividend level. Because the managers also recognize the cost structure of paying dividends they will compare the benefits and costs of paying dividends when they signal the firm's future profitability to the market. As in Bhattacharya [1979], the signaling benefits would be the increase in liquidation value at the end of period. The liquidation value will be V_i from equation (28). Then the equilibrium market value of the firm is

$$V_{0i}(D) = (1/(1+a+(1-c)r_f))[V_i(D) + \overline{X} - cD - \gamma\int_{-\infty}^{D} F(X)dX$$
$$-\lambda \text{cov}(X,R_m)(1+\gamma F(D)], \tag{30}$$

from the managers' point of view, because the dividend level is an endogenous variable. In order to maximize the firm value, the managers will adjust the dividend level up to the optimal level where the marginal costs and the marginal benefits of paying dividends are same.

The signaling equilibrium condition can be tested under the costs and benefits structure of the dividend signaling model. According to Spence (1974) the signaling equilibrium condition is that the marginal signaling costs should be negatively related to the quality of the sender. When the condition is not satisfied, the signal does not deliver any information to the market in an equilibrium. In our model the marginal signaling cost will be

$$d(\mathrm{Eq.}(29))/dD = c + \gamma F(D) + \lambda \mathrm{cov}(X,R_m)\gamma f(D), \tag{31}$$

which is obviously positive. The quality of the sender (firm) is assumed to depend on the level of the firm's future expected cash flows ($\bar{X}$). The relationship between the marginal cost and the quality of the firm can be found by differentiating the marginal cost with respect to $\bar{X}$:

$$\begin{aligned} & d(\mathrm{Eq.}(31))/d\bar{X} \\ = \quad & \gamma F_{\bar{X}}'(D) + \lambda \mathrm{cov}(X,R_m)\gamma f_{\bar{X}}'(D) \\ = \quad & -\gamma f(D) - \lambda \mathrm{cov}(X,R_m)\gamma f(D)((\bar{X}-D)/\sigma_X^{\ 2}). \end{aligned} \tag{32}$$

$\bar{X}$ is assumed to be larger than D, because it is unreasonable to set the dividend level higher than the expected net cash flow. Then equation (32) is strictly negative. Thus the critical condition for the dividend signaling equilibrium is satisfied.[19] Furthermore, based on Spence [1974], the signaling equilibrium can be defined by the pair of equations:

the marginal signaling costs = the marginal signaling benefits,[20] (33)

$$\bar{X}^P = \bar{X}. \tag{34}$$

In other words, under the signaling equilibrium, every firm chooses the optimal dividend level to maximize the firm value and the market's perceived firm's cash flows equal the true firm's ex ante cash flows. Therefore the derived capital market equilibrium value of the firm expressed in equation (19) can be justified, which is derived under the assumption of the signaling equilibrium, because the condition for the signaling equilibrium is satisfied.

IV. Summary & Conclusions

The dividend "puzzle" can be solved under the dividend signaling capital market equilibrium model. The major finding on the dividend puzzle is that the announced dividend will increase the market perceived value of the firm at the end of period more than the cost involved in paying dividends, because the managers who have superior information on the firm's future cash flow only pay dividends when the benefits is greater than the costs. Thus the announced dividend has an positive effect on the current value of the firm. But the required rate of return in an imperfect information setting has the same form as in the perfect information setting, since the perceived expected return is based on the perceived end-of-period value of the firm under the signaling theory. However, the beta in the dividend signaling CAPM is positively dependent on the announced dividends, if the market moral hazard rate is positive. This finding can provide a theoretical model for estimating the market moral hazard penalty rates on which the dividend signal model is largely dependent.

The capital market equilibrium value is derived from the assumption of the signaling equilibrium. Thus the existence condition of the signaling equilibrium is examined in detail. The negative relationship between the marginal cost of dividends and the quality of the firm can justify the dividend signaling capital market equilibrium model.

Finally determining the validity of the dividend signaling CAPM is of great importance. Since the dividend signaling CAPM is based on the traditional CAPM, this theory is subject to the same criticism that the traditional CAPM. However if we can give more attention to the initial effort to develop a general equilibrium model for explaining the unsolved dividend effects on share prices, the theory seems to be worth while. Especially this study is the first to document the theoretical background for directly testing the validity of the dividend signaling model and adds a new dimension to explaining dividend behavior in the U.S.

References

1. Aharony, J., and Swary, I. "Quarterly Dividend and Earnings Announcements and Stockholders' Returns: An Empirical Analysis." Journal of Finance, Vol. 35, No. 1 (March 1980), pp. 1-12.

2. Akerlof, G.A. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." Quarterly Journal of Economics, Vol. 84. No. 3 (August 1970), pp. 488-500.

3. Ang. J.S. "Dividend Policy: Informational Content or Partial Adjustment?" Review of Economics and Statistics, Vol. 57, No. 1 (February 1975), pp. 65-70.

4. Asquith, P., and Mullins, D.W. "The Impact of Initiating Dividend Payments on Shareholders' Wealth." Journal of Business, Vol. 56, No. 1 (January 1983), pp. 77-96.

5. Ball, R., "Anomalies in Relationships Between Securities' Yields and Yield-Surrogate." Journal of Financial Economics (June/September 1978), pp. 103-126.

6. Bhattacharya, S. "Imperfect Information, Dividend Policy, and the Bird in the Hand' Fallacy." Bell Journal of Economics, Vol. 10, No. 1 (Spring 1979), pp. 259-270.

7. ______________ "Nondissipative Signaling Structures and Dividend Policy." Quarterly Journal of Economics, Vol. 95, No. 1 (August 1980), pp.1-24.

8. Black, F. "The Dividend Puzzle." The Journal of Portfolio Management (Winter 1976), pp. 5-8.

9. Black, F., Jensen, M., and Scholes, M. "The Capital Asset Pricing Model: Some Empirical Tests." In Jensen (ed.), Studies in the Theory of Capital Markets, Praegar, New York (1972), pp. 79-121.

10. Black, F., and Scholes, M. "The Effects of Dividend Yield and Dividend Policy on Common Stock Prices and Returns." Journal of Financial Economics, Vol. 1, No. 1 (January 1974), pp. 1-22.

11. Brennan, M.J. "Taxes, Market Valuation and Corporate Financial Policy." National Tax Journal, Vol. 23, No. 4 (December 1973), pp. 417-427.

12. DeAngelo, H., and Masulis, R.W. "Optimal Capital Structure Under Corporate and Personal Taxation." Journal of Financial Economics, Vol. 8, No. 1 (March 1980), pp. 3-29.

13. Eades, K.M. "Empirical Evidence on Dividends as a Signal of Firm Value." Journal of Financial and Quantitative Analysis, Vol. 17, No. 4 (March 1982), pp. 473-501.

14. ____________, Hess, P.J., and Kim, E.H. "On Interpreting Security Returns During the Ex-Dividend Period." Journal of Financial Economics, Vol. 13, No. 3 (December 1984), pp. 3-34.

15. Easterbrook, F. H. "Two Agency-Cost Explanations of Dividends." American Economic Review, Vol. 74 (September 1984), pp. 650-659.

16. Feldstein, M., and Green, J. "Why Do Companies Pay Dividends?" American Economic Review, Vol. 73, No. 1 (March 1983), pp. 17-30.

17. Frankel, A. W., and Hawkins, C.A. "Beta Coefficients for Convertible Bonds." Journal of Finance, Vol. 30, No. 1 (March 1975), pp. 207-210.

18. Gonedes, N. "Corporate Signaling, External Accounting, and Capital Market Equilibrium: Evidence on Dividends, Income, and Extraordinary Items." Journal of Accounting Research, Vol. 7, No. 1 (Spring 1978).

19. Gordon, M.J. "Dividends, Earnings, and Stock Prices." Review of Economics and Statistics, Vol. 41, No. 2 (May 1959).

20. __________ "Optimal Investment and Financing Policy." Journal of Finance, Vol. 18, No. 2 (May 1963), pp. 264-272.

21. Hakansson, N.H. "To Pay or Not to Pay Dividend." Journal of Finance, Vol 37, No. 2 (May 1982), pp. 415-428.

22. Hamada, R.S. "Portfolio Analysis, Market Equilibrium, and Corporation Finance." Journal of Finance, Vol. 23, No. 1 (March 1969), pp. 13-31.

23. ______________ "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks." Journal of Finance, Vol. 27, No. 2 (May 1982), pp. 435-452.

24. Hess, P.J. "The Ex-Dividend Day Behavior of Stock Returns: Further Evidence on Tax Effects." Journal of Finance, Vol. 37, No. 2 (May 1982), pp. 445-456.

25. ______________ "Test for Tax Effects in the Pricing of Financial Assets." Journal of Business, Vol. 56, No. 4 (October 1983), pp. 537-554.

26. Higgins, R.C. "Dividend Policy and Increasing Discount Rates: A Clarification." Journal of Financial and Quantitative Analysis, Vol. 7 No. 3 (June 1972), pp. 1757-1762.

27. John, K., and Kalay, A. "Costly Contracting and Optimal Payout Constraints." Journal of Finance, Vol. 37, No. 2 (May 1982), pp. 457-470.

28. ________ and Williams, J. "Dividends, Dilution, and Taxes: A Signaling Equilibrium." Journal of Finance, Vol. 40, No. 3 (September 1985).

29. Kalay, A. "Signaling, Information Content, and the Reluctance to Cut Dividends." Journal of Financial and Quantitative Analysis, Vol. 15, No. 4 (november 1980), pp. 855-869.

30. Kim, E. H. "A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity." Journal of Finance, Vol. 33, No. 1 (May 1978), pp. 45-63.

31. __________ "Miller's Equilibrium, Shareholder Leverage Clienteles, and Corporate Debt Capacity." Journal of Finance, Vol. 37, No. 1 (may 1982), pp. 301-319.

32. Laub, P. M. "On the Informational Content of Dividends." Journal of Business, Vol. 49, No. 1 (January 1976).

33. Lee, C. F. "Functional Form and the Dividend Effect in the Electric Utility Industry." Journal of Finance, Vol. 31, No. 5 (December 1976), pp. 1481-1486.

34. Leland, H. E., and Pyle, D. H. "Informational Asymmetries, Financial Structure, and Financial Intermediation." Journal of Finance, Vol. 32, No. 2 (May 1977), pp. 371-387.

35. Lintner, J. "Distribution of Income of Corporation Among Dividends, Retained Earnings, and Taxes." American Economic Review, Vol. 46, No. 1 (May 1056), pp. 97-113.

36. __________ "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." Review of Economics and Statistics, Vol. 47, No. 1 (February 1965a), pp. 13-37.

37. __________ "Security Prices, Risk and Maximal Gains from Diversification." Journal of Finance, Vol. 20, No. 4 (December 1965b), pp. 587-616.

38. __________ "Risk and Return in Finance." Edited by Irwin Friend and James L. Bicksler, Ballinger Publishing Company, Cambridge, Massachusetts (1977), pp. 1-128.

39. Litzenberger, R.H., and Ramaswamy, K. "The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Empirical Evidence." Journal of Financial Economics, Vol. 7, No. 2 (June 1979), pp. 163-195.

40. __________ "Dividends, Short Selling Restrictions, Tax-Induced Investor Clientele and Market Equilibrium." Journal of Finance, Vol. 35, No. 2 (May 1982), pp. 469-444.

41. __________ "The Effects of Dividends on Common Stock Prices Tax Effects or Information Effects?" Journal of Finance, 37, No. 2 (May 1982), pp. 429-444.

42. Long, J.B. "The Market Valuation of Cash Dividends: A Case to Consider." Journal of Financial Economics, Vol 6 (1978), pp. 235-264.

43. Miller, M.H., and Modigliani, F. "Dividend Policy, Growth, and the Valuation of Shares." Journal of Business, Vol. 36, No. 4 (October 1961), pp. 411-433.

44. __________, and Rock, K. "Dividend Policy under Asymmetric Information." Journal of Finance, Vol 40, No. 3 (September 1985).

45. __________, and Scholes, M. "Dividends and Taxes." Journal of Financial Economics, Vol. 6, No. 4 (December 1978), pp. 333-364.

46. ______________________ "Dividends and Taxes: Empirical Evidence." Journal of Political Economy, Vol. 90, No. 6 (December 1982), pp. 1118-1141.

47. Morgan, I.G. "Dividends and Capital Asset Prices." Journal of Finance, Vol. 37, No. 4 (September 1982), pp. 1071-1087.

48. Mossin, J. "Equilibrium in a Capital Asset Market." Econometrica, Vol. 34 (October 1966), pp. 768-783.

49. Pettit, R.R. "Dividend Announcements, Security Performance, and Capital Market Efficiency." Journal of Finance, Vol. 27, No. 5 (December 1972), pp. 993-1007.

50. __________ "The Impact of Dividend and Earnings Announcements: A Reconciliation." Journal of Business, Vol. 49, No. 1 (January 1976), pp. 86-96.

51. __________ "Taxes, Transactions Costs and the Clientele Effects of Dividends." Journal of Financial Economics, Vol. 5, No. 3 (December 1977), pp. 419-436.

52. Ross, S.A. "The Determination of Financial Structure: The Incentive-Signaling Approach." Bell Journal of Economics, Vol. 8, No. 1 (Spring 1977), pp. 23-40.

53. Sharpe, W.F. "Capital Asset Prices: A Theory of Market Equilibrium Under Condition of Risk." Journal of Finance, Vol. 19, No.3 (September 1964), pp. 425-442.

54. Spence, M. "Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution." Journal of Economic Theory, Vol. 7, No. 3 (March 1974), pp. 296-332.

55. Talmor, E. "Asymmetric Information, Signaling, and Optimal Corporate Financial Decisions." Journal of Financial and Quantitative Analysis, Vol. 16, No. 4 (November 1981), pp. 191-211.

56. Watts, R. "The Information Content of Dividends." Journal of Business, Vol. 46, No. 2 (April 1973), pp. 191-211.

57. __________ "Comments on 'On The Informational Content of Dividends'." Journal of Business, Vol. 49. No. 1 (January 1976a), pp. 81-85.

58. __________ "Comments on 'The Impact of Dividend and Earnings Announcements: A reconciliation'." Journal of Business, Vol. 49, No. 1 (January 1976b), pp. 97-106.

59. Winkler, R.L., Roodman, G., and Britney, R. "The Determination of Partial Moments." Management Science, Vol. 19, No. 3 (November 1972), pp. 290-296.

60. Woolridge, J.R. "The Information Content of Dividend Changes." Journal of Financial Research, Vol. 5, No. 3 (Fall 1982), pp. 237-247.

Endnotes

[1]Long [1978] finds a premium in the market price of a stock with cash dividends over a stock with stock dividends by examining two classes of common stock which are identical in all respects except dividend payout.

[2]Other explanations for the dividend puzzle can be found in Feldstein and Green [1983] and Easterbrook [1984].

[3]Pettit [1976], Griffin [1976], and Laub [1976] support the ICD hypothesis, while Watts [1976a and 1976b], Ang [1975], and Gonedes [1978] interpret their findings as against the hypothesis. Recently Aharony and Swary [1980], Woolridge [1982] and Asquith and Mullins [1983] employ new approaches to the issue and their results strongly support the hypothesis.

[4]In the Talmar's example, there are two unknown parameters which are a mean and a variance of a normally distributed future cash flows. Two signaling devices are assumed to be the firm's capital structure and its dividend policy in the example.

[5]For an empirical test of signaling hypotheses for unseasoned new issues, see Downes and Heinkel [1982].

[6]The empirical analog of the negative relationship is defined as an implied negative relationship between dividend yield and a stock's own variance.

[7]This argument can be also applicable to other dividend signaling studies in the finance literature. It will be described in detail in this study.

[8]If we assume only the real asset market is imperfect, the penalty rates could be transaction costs. However both markets (real and financial market) are assumed perfect, the penalty rates could be dissipative costs. Bhattacharya [1979] defines the dissipative costs as costs of selling real assets, opportunity costs of postponing positive net present value investments, and costs of holding buffer stocks earning less than firms' costs of capital.

[9]The major assumptions in this study are summarized in Appendix A.

[10]See assumption 17i in Appendix A.

[11]See assumption 17ii in Appendix A.

[12]The derivation of equation (16) can be provided upon request.

[13]According to Feenburg [1981] only 2.5 percent of dividend income goes to constraint taxpayers. Thus we interpret 'c' as investors' average marginal tax rate from now on, assuming $\lambda_2^k = 0$. If $\lambda_2^k \neq 0$, the sign of 'c' will be dependent on the proportion of investors whose income constraints are binding.

[14]The derivation of equation (18) is shown in Appendix B.

[15]Actually some costs involved in paying dividends are hidden in the expression of $E(V_{1i}(D))$ in (22). Exact costs of paying dividends will be discussed in section III.

[16]John and Williams [1985] have relied upon the same argument to obtain their signaling equilibrium. However, their models are not in terms of CAPM framework as are in this paper.

[17]β_{Pi} could be the firm i's systematic risk when the firm does not pay dividend in the imperfect-information setting, if we change the basic assumption on $\overline{X}^P$. Investors can be assumed to revise their expectation on $\overline{X}$ based on announced dividends, then equation (1) can be changed to $\overline{X}^P = \overline{X}^I + G(D)$, where $\overline{X}^I$ = the investors' homogeneous expectation when D=0.

[18]The cost can be shown as

$$V_o - V_o(D) = [\{\gamma \int_{-\infty}^{D} F(X)dX + \lambda \mathrm{cov}(X,R_m)\gamma F(D)\}/\{a + (1 - c)r_f\} +$$

$$\gamma \int_{-\infty}^{D} F(X)dX + \lambda \mathrm{cov}(X,R_m)\gamma F(D)]/\{1 + a + (1 - c)r_f\},$$

which is obviously positive.

[19]The necessary condition for the dividend signaling equilibrium can be achieved even with $c = 0$.

[20]The marginal signaling benefits are $dV_iC(D)/dD$,

$$\text{where } V_i(D) = [\bar{X} - CD - \gamma \int_{-\infty}^{D} F(X)\, dX$$

$$- \lambda \text{cov}(X, R_m)(1 + \gamma F(D^*))]/\{a + (1 - c)\, r_f\}.$$

The optimal dividend (D*) can be achieved when the marginal signaling benefits equal the marginal signaling costs, assuming the second order condition is satisfied. The derivation for D* can be provided upon request.

Appendix A

The major assumptions in the model are:

1) The market is perfect except that there is asymmetric information between firms' managers and investors about firms' future cash flows.

2) Dividends on stocks are paid at the end of each period and are announced at the beginning of each period. The announced dividends are believed to be paid through time.

3) There are market penalties if actual cash flows are less than promised dividends. The market penalty rates are constant through time.

4) Dividends serve as a signal for firms' future profitability.

5) Dividend signaling equilibrium is reached.

6) Investors assess the expected value of each firm at the end of period based on the announced dividends (i.e., $\bar{X}^P = G(D) = \bar{X}$ and G(D) is known).

7) Investors have a single period investment horizon.

8) Firms generate cash flows that are perpetual streams which are intertemporally independently identically distributed.

9) After-tax operating cash flows of firms have a multivariate normal distribution.

10) Investors' risk preferences are constant through time.

11) Investors' utility functions are continuously increasing concave functions of after-tax end of period wealth.

12) Individuals have homogeneous expectations after the signaling equilibrium is reached.

13) All assets are marketable.

14) There is a riskless asset, producing a constant rate, r_f, through time.

15) A progressive tax scheme is applied to dividends and interest income, and the marginal tax rate is a function of taxable income which is differentiable. Individuals' tax rates are constant over time.

16) Taxes on capital gains are zero. But ordinary income tax rate is applied to dividend income.

17) Two constraints on individuals borrowings are i) the interest payments on borrowing should be less than or equal to dividend income (income constraint), ii) the individual's net worth should be larger than or equal to a given fraction (α) of the market value of his/her holdings of risky securities (margin constraint).

Assumptions 1) through 6) are made in order to link the signaling equilibrium to the capital market equilibrium. Assumptions 7) through 14) are same as in the traditional CAPM. Assumptions 15) through 17) are from LR.

Appendix B

The second and third terms in (17) can be expressed in terms of $cov(X,R_m)$ by employing techniques used in Lintner [1977] and Kim [1978]. The second covariance equals, by definition,

$$\begin{aligned}
&cov(zX,R_m) \\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(zX-E(zX))(R_m-E(R_m))f(X,R_m)dXdR_m \\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{D}(X-E(zX))(R_m-E(R_m))f(X,R_m)dXdR_m \\
&\qquad\qquad +\int_{-\infty}^{\infty}\int_{D}^{\infty}(0-E(zX))(R_m-E(R_m))f(X,R_m)dXdR_m. \qquad (A1)
\end{aligned}$$

The first term in equation (A1) equals

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}XR_m f(X,R_m)dXdR_m - E(R_m)\int_{-\infty}^{\infty}\int_{-\infty}^{D} Xf(X,R_m)dXdR_m \\
&\qquad - E(zX)\int_{-\infty}^{\infty}\int_{-\infty}^{D} R_m f(X,R_m)dXdR_m + E(zX)E(R_m)\int_{-\infty}^{\infty}\int_{-\infty}^{D} f(X,R_m)dXdR_m. \qquad (A2)
\end{aligned}$$

The following relationships can be found in Winkler, Roodman, and Britney, [1972]; and Mood and Graybill [1963]:

$$E(zX) = \int_{-\infty}^{D} Xf(X)dX, \qquad (A3)$$

$$f(x,R_m) = f(X)g(R_m|X), \qquad (A4)$$

$$\int_{-\infty}^{\infty}R_m g(R_m|X)dR_m = E(R_m) + cov(X,R_m)(X-E(X))/\sigma^2, \qquad (A5)$$

$$\int_{-\infty}^{D}Xf(X)dX = E(X)F(D) - \sigma^2 f(D), \text{ and} \qquad (A6)$$

$$\int_{-\infty}^{D}X^2 f(X)dX = -\sigma^2 Df(D) + \sigma^2 F(D) + E(X)(E(X)F(D)-\sigma^2 f(D)). \qquad (A7)$$

Substituting equations (A3) - (A7) into (A2), the first term in (A1) equals

$$cov(X,R_m)[F(D) - Df(D) + E(X)F(D)f(D) - \sigma^2(f(D))^2], \qquad (A8)$$

where f(D) is the normal density function at D and F(D) is the cumulative normal density function at D. The second term in (A1) can be written as

$$E(zX)E(R_m)\int_{-\infty}^{\infty}\int_{D}^{-\infty}f(X,R_m)dXdR_m - E(zX)\int_{-\infty}^{\infty}\int_{D}^{-\infty}R_m f(X,R_m)dXdR_m. \tag{A9}$$

Substituting equations (A3)-(A7) into (A9), the second term in (A1) is reduced to

$$- \operatorname{cov}(X,R_m)[E(X)F(D)f(D) - \sigma^2(f(D))^2]. \tag{A10}$$

Therefore, the second covariance term, $\operatorname{cov}(zX,R_m)$, in equation (17) is the sum of equation (A8) and (A10):

$$\operatorname{cov}(zX,R_m) = \operatorname{cov}(X,R_m)(F(D) - Df(D)). \tag{A11}$$

Similarly the third covariance term, $\operatorname{cov}(z,R_m)$, in equation (17) can be expressed as, by definition,

$$\begin{aligned}
&\operatorname{cov}(z,R_m) \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(z-E(z))(R_m-E(R_m))f(X,R_m)dXdR_m \\
&= \int_{-\infty}^{\infty}\int_{D}^{\infty} (0-E(z))(R_m-E(R_m))f(X,R_m)dXdR_m \\
&\qquad +\int_{-\infty}^{\infty}\int_{\infty}^{D}(1-E(z))(R_m-E(R_m))f(X,R_m)dXdR_m,
\end{aligned} \tag{A12}$$

where

$$E(z)= \int_{-\infty}^{D}f(X)dX = F(D). \tag{A13}$$

Substituting equation (A4), (A5), (A6), and (A13) into equation (A12), the third covariance term in equation (17) becomes

$$\text{cov}(z,R_m) = -\ \text{cov}(X,R_m)f(D). \tag{A14}$$

Therefore equation (17) can be expressed in terms of $\text{cov}(X,R_m)$, using equations (A11) and (A14):

$$\begin{aligned}
&\text{cov}[(X-D)(1+\gamma z),R_m] \\
&= \text{cov}(X,R_m) + \gamma\text{cov}(zX,R_m) - \gamma D\text{cov}(z,R_m) \\
&= \text{cov}(X,R_m) + \gamma\text{cov}(X,R_m)(F(D)-Df(D))+\gamma D\text{cov}(X,R_m)f(D) \\
&= \text{cov}(X,R_m)(1+\gamma F(D)).
\end{aligned}$$

Q.E.D.