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Dividend Smoothing, the Present Value Model, and Negative Autocorrelations of Stock Price Changes

Abstract

A consequence of partial dividend smoothing is that dividends revert (slowly) to their targets and the stock price reverts to the present value of expected future target dividends. This target reverting can cause stock price changes to be negatively autocorrelated. As dividend smoothing increases, the negative autocorrelation becomes less significant. The negative autocorrelation appears to be a "V" shaped function of the length of holding periods. As stock price volatility increases relative to dividend volatility, the negative autocorrelation becomes more significant.

Introduction

Recent research contributions by DeBondt and Thaler (1985), Fama and French (1987), and Poterba and Summers (1988), among others, find that stock price changes (especially in the long run) are NEGATIVELY AUTOCORRELATED. This finding contradicts the long-standing hypothesis in the Finance literature that the stock price is a random walk.

The objective of this paper is to analyze the observed negative autocorrelations of stock price changes. Our analysis is developed upon the present value model and motivated by the well-known fact that corporate managers smooth their dividend payments. The dividend depends in part on a target dividend and in part on previous years' dividends.[1] The degree of dividend smoothing is inversely related to the speed of dividend adjustment to the target. Hence, under partial dividend smoothing, the dividend reverts partially to the target. Holding the discount rate constant, the stock price will also revert to the present value of expected future target dividends. We will show below how the negative autocorrelations of stock price changes are created by the "target reverting" process of stock prices, and how they are affected by the length of holding periods and the degree of dividend smoothing.

I. Analysis

Following Lintner (1956) and Fama and Babiak (1968), we consider a simple model for dividend smoothing:

$$D_t = \gamma(D_t^* - D_{t-1}) + D_{t-1} \quad (1)$$

[1] See Lintner (1956), Brittain (1966), Fama and Babiak (1968), Marsh and Merton (1987), and Choe (1990), among others.

where D_t is the dividend paid for period t, D_t^* is the target dividend for period t, and $\gamma (0 < \gamma \leq 1)$ is the speed of the dividend adjustment toward the target. $(1 - \gamma)$ is the degree of dividend smoothing. When $\gamma = 1$, the dividend immediately reverts to the target level and is not smoothed. If $\gamma = 0$, the dividend will never revert to the target and is completely smoothed.

The present value model is

$$P_t = \sum_{i=1}^{\infty} \frac{E_t D_{t+i}}{(1+k)^i} \tag{2}$$

where P_t is the stock price at the *beginning* of period $t+1$ (or at time t), D_{t+i} is the dividend paid *during* period $t+i$ (or from time $t+i-1$ through time $t+i$), k is the discount rate, which is assumed to be constant, and E_t is the investor's expectations operator conditional upon information available at time t.

Since equation (2) means that $E_t P_{t+1} = (1+k)P_t - E_t D_{t+1}$, from equation (1) we have

$$E_t P_{t+1} = (1+k)P_t - \gamma E_t D_{t+1}^* - (1-\gamma)D_t. \tag{3}$$

Rational investors recognize corporate dividend smoothing and incorporate the dividend smoothing behavior (equation 1) into the present value model (equation 2). This generates

$$P_t = \gamma \sum_{i=1}^{\infty} \frac{E_t D_{t+i}^*}{(1+k)^i} + \left(\frac{1-\gamma}{1+k}\right)(D_t + P_t). \tag{4}$$

We define P_t^* as the present value of expected future target dividends (hereafter referred to as the target price); $P_t^* \equiv \sum_{i=1}^{\infty} \frac{E_t D_{t+i}^*}{(1+k)^i}$. Solving for $(1-\gamma)D_t$ in equation (4) yields

$$(1-\gamma)D_t = (\gamma + k)P_t - \gamma(1+k)P_t^*. \tag{5}$$

In equation (3), we substitute the right hand side of equation (5) for $(1-\gamma)D_t$ and then substitute $E_t(P^*_{t+1} + D^*_{t+1})$ for $(1+k)P^*_t$. This yields

$$E_t P_{t+1} = \gamma E_t P^*_{t+1} + (1-\gamma)P_t. \tag{6}$$

Equation (6) shows that the speed of the stock price adjustment toward the target price is the same as that of the dividend adjustment.

Since $P_{t+1} = E_t P_{t+1} + \eta_{t+1}$, where η_{t+1} is assumed to be a rational stock price forecast error such that $\text{cov}(\eta_t, \eta_{t+i}) = 0$ for all $i \neq 0$, from equation (6) we have (time subscripts are reduced by 1)

$$P_t = \gamma E_{t-1} P^*_t + (1-\gamma)P_{t-1} + \eta_t. \tag{7}$$

If dividends are completely smoothed (i.e., $\lambda = 1$ or $\gamma = 0$), the stock price is a random walk and $\text{cov}(P_{t+2\tau} - P_{t+\tau}, P_{t+\tau} - P_t) = 0$ for all $\tau \geq 1$.

Let τ be the length of holding periods, and $\lambda \equiv 1 - \gamma$ (λ measures the degree of dividend smoothing). The change in stock prices over τ periods is

$$P_{t+\tau} - P_t = (1-\lambda L)^{-1}\{\gamma(E_{t+\tau-1}P^*_{t+\tau} - E_{t-1}P^*_t) + \eta_{t+\tau} - \eta_t\} \tag{8}$$

where L is the backward shift operator.

We assume that $\sigma^2(\eta_t) = \sigma^2_\eta$ for all t. To compute the first-order autocorrelation of $P_{t+\tau} - P_t$ for $0 \leq \lambda < 1$, we need to assume a stochastic process for D^*_t (and thus for P^*_t). We consider two cases: (i) D^* is a white noise around some mean; (ii) D^*_t is a random walk.

A. Case 1: when D_t^* is a white noise around some mean.

We assume that

$$D_t^* = \overline{D} + e_t \tag{9}$$

where $\overline{D}$ is the mean of D_t^*, and e_t is a white noise. It follows that $P_t^* = \overline{D}/k$, and $E_{t+\tau-1}P_{t+\tau}^* - E_{t-1}P_t^* = 0$ for all t.

Changes in stock prices over τ periods are

$$P_{t+\tau} - P_t = \sum_{j=0}^{\tau-1} \lambda^j \eta_{t+\tau-j} - (1-\lambda^\tau)\sum_{j=0}^{\infty} \lambda^j \eta_{t-j}, \tag{10-a}$$

and

$$\begin{aligned} P_{t+2\tau} - P_{t+\tau} &= \Phi_{t+\tau+1}^{t+2\tau} - (1-\lambda^\tau)\sum_{j=0}^{\infty} \lambda^j \eta_{t+\tau-j} \\ &= \Phi_{t+\tau+1}^{t+2\tau} - (1-\lambda^\tau)\sum_{j=0}^{\tau-1} \lambda^j \eta_{t+\tau-j} \\ &\quad - (1-\lambda^\tau)\lambda^\tau \sum_{j=0}^{\infty} \lambda^j \eta_{t-j} \end{aligned} \tag{10-b}$$

where $\Phi_{t+\tau+1}^{t+2\tau}$ denotes the terms with $\eta_{t+2\tau}, \cdots, \eta_{t+\tau+1}$.

For $0 \leq \lambda < 1$, we have

$$\text{var}(P_{t+\tau} - P_t) = \frac{2(1-\lambda^\tau)}{1-\lambda^2}\sigma_\eta^2, \tag{11-a}$$

and

$$\text{cov}(P_{t+2\tau} - P_{t+\tau}, P_{t+\tau} - P_t) = \frac{-(1-\lambda^\tau)^2}{1-\lambda^2}\sigma_\eta^2. \tag{11-b}$$

The first-order autocorrelation of stock price changes over τ periods, $f(\tau, \gamma)$, is

$$f(\tau, \lambda) = \frac{-1}{2}(1-\lambda^\tau). \tag{12}$$

Holding τ constant and for $0 < \lambda < 1$, we find that

$$\frac{\partial f(\tau, \lambda)}{\partial \lambda} > 0. \tag{13}$$

This implies that increased dividend smoothing reduces the magnitude of negative autocorrelations of stock price changes. Fama and French (1987) find that the negative autocorrelations of stock price changes for the 1941–1985 time period are less significant than those for the 1926–1985 time period. Similarly, Kim, Nelson, and Startz (1989) find that negative autocorrelations of stock price changes may not exist during the post-World War II period. These findings could be attributed to temporal shifts in dividend smoothing. In fact, λ during the pre-war period is smaller than that during the post-war period. In particular, after the corporate income tax reform in 1952,[2] the dividend appears to depend mostly on the previous year's dividend. It is found, using S & P annual data, that λ's are about 0.25 to 0.30 and 0.75 to 0.85, respectively, for the 1932–1951 time period and the 1952–1986 time period.[3] We may conjecture that increased dividend smoothing in recent years has reduced negative autocorrelations of stock price changes.

Holding λ constant between 0 and 1, we find that

$$\frac{\partial f(\tau, \lambda)}{\partial \tau} < 0. \tag{14}$$

This result would be consistent with Poterba and Summers' finding that as the length of holding periods increases, the magnitude of negative autocorrelations of stock returns tends to increase. However, Fama and French

[2]In 1952, the statutory corporate income tax rate was raised from 15 percent to 52 percent.

[3]A similar result is found in Fama and French (1988) and Choe (1990).

(1987) observe that the negative autocorrelations reach a maximum for 3- to 5-year stock returns and then decrease toward zero as the length of holding periods increases. The negative sign of $\frac{\partial f(\tau,\lambda)}{\partial \tau}$ may not be the case for all τ.

B. Case 2: when D_t^* is a random walk.

We assume that

$$D_t^* = D_{t-1}^* + \epsilon_t \tag{15}$$

where ϵ_t is a white noise. It follows that

$$D_{t+1} - E_t D_{t+1} = \gamma \epsilon_{t+1} \tag{16}$$

and

$$P_t^* = P_{t-1}^* + \omega_t \tag{17}$$

where $\omega_t = \epsilon_t / k$. Equation (17) generates

$$\begin{aligned} E_{t+\tau-1} P_{t+\tau}^* - E_{t-1} P_t^* &= P_{t+\tau-1}^* - P_{t-1}^*, \\ &= \omega_t + \omega_{t+1} + \cdots + \omega_{t+\tau-1}. \end{aligned} \tag{18}$$

Equation (8), the stock price change over τ periods, becomes

$$P_{t+\tau} - P_t = (1 - \lambda L)^{-1} \{\gamma \omega_{t+\tau-1} + \cdots + \gamma \omega_t + \eta_{t+\tau} - \eta_t\}. \tag{19}$$

For computing the first-order autocorrelation of $P_{t+\tau} - P_t$, we need to understand the relationship between the stock price forecast error (η_{t+1}) and the dividend forecast error ($\gamma \epsilon_{t+1}$). This is seen by substituting $\sum_{i=1}^{\infty} \frac{E_{t+1} D_{t+1+i}}{(1+k)^i}$

for P_{t+1} and $\sum_{i=1}^{\infty} \frac{E_t D_{t+i}}{(1+k)^i}$ for P_t in the present value model, $P_{t+1} = (1+k)P_t - E_t D_{t+1} + \eta_{t+1}$. It follows that

$$\eta_{t+1} = \sum_{i=1}^{\infty} \frac{E_{t+1} D_{t+1+i} - E_t D_{t+1+i}}{(1+k)^i}. \tag{20}$$

By the law of iterative conditional expectations, we have

$$\begin{aligned} E_{t+1} D_{t+1+i} - E_t D_{t+1+i} &= a_i (D_{t+1} - E_t D_{t+1}) + \xi_{i,t+1} \\ &= a_i \gamma \epsilon_{t+1} + \xi_{i,t+1} \end{aligned} \tag{21}$$

where a_i is a regression coefficient, and $\xi_{i,t+1}$ is a regression error such that $\text{cov}(\epsilon_{t+1}, \xi_{i,t+1}) = 0$ for all i. It is convenient to approximate a_i as[4]

$$a_i = \frac{1 - \lambda^{i+1}}{1 - \lambda}. \tag{22}$$

Equation (20) becomes

$$\begin{aligned} \eta_{t+1} &= \epsilon_{t+1} \sum_{i=1}^{\infty} \frac{1 - \lambda^{i+1}}{(1+k)^i} + \sum_{i=1}^{\infty} \frac{\xi_{i,t+1}}{(1+k)^i} \\ &= \left(1 - \frac{\lambda^2 k}{1 + k - \lambda}\right) \omega_{t+1} + \xi_{t+1} \end{aligned} \tag{23}$$

where ϵ_{t+1} is replaced by $k\omega_{t+1}$ (see equation 17), and $\xi_{t+1} \equiv \sum_{i=1}^{\infty} \frac{\xi_{i,t+1}}{(1+k)^i}$. Equation (23) shows that the stock price forecast error is in principle determined by the dividend forecast error ($\omega_{t+1} = \epsilon_t / k$) and "other" forecast errors (ξ_{t+1}). Hereafter,

$$c \equiv \left(1 - \frac{\lambda^2 k}{1 + k - \lambda}\right).$$

[4] From equation (1), we have $E_{t+1} D_{t+1+i} - E_t D_{t+1+i} = \gamma (E_{t+1} D^*_{t+1+i} - E_t D^*_{t+1+i}) + \lambda (E_{t+1} D_{t+i} - E_t D_{t+i}) = \gamma \epsilon_{t+1} (1 + \lambda + \cdots + \lambda^i) = \frac{1-\lambda^{i+1}}{1-\lambda} (D_{t+1} - E_t D_{t+1})$. For the last equality, see equation (16).

Substituting equation (23) into equation (19) generates (see Appendix A)

$$\begin{aligned} P_{t+\tau} - P_t &= (1-\lambda L)^{-1}\left\{ c\omega_{t+\tau} + \gamma \sum_{j=1}^{\tau-1} \omega_{t+\tau-j} + (\gamma - c)\omega_t + \xi_{t+\tau} - \xi_t \right\} \\ &= \sum_{j=0}^{\tau-1} \left\{ (1-\lambda^j) + c\lambda^j \right\} \omega_{t+\tau-j} + (1-c)(1-\lambda^\tau) \sum_{j=0}^{\infty} \lambda^j \omega_{t-j} \\ &\quad + \sum_{j=0}^{\tau-1} \lambda^j \xi_{t+\tau-j} - (1-\lambda^\tau) \sum_{j=0}^{\infty} \lambda^j \xi_{t-j}, \end{aligned} \tag{24-a}$$

and

$$\begin{aligned} P_{t+2\tau} - P_{t+\tau} &= \Phi_{t+\tau+1}^{t+2\tau} + (1-c)(1-\lambda^\tau) \sum_{j=0}^{\infty} \lambda^j \omega_{t+\tau-j} \\ &\quad - (1-\lambda^\tau) \sum_{j=0}^{\infty} \lambda^j \xi_{t+\tau-j} \\ &= \Phi_{t+\tau+1}^{t+2\tau} + (1-c)(1-\lambda^\tau) \sum_{j=0}^{\tau-1} \lambda^j \omega_{t+\tau-j} \\ &\quad + (1-c)(1-\lambda^\tau)\lambda^\tau \sum_{j=0}^{\infty} \lambda^j \omega_{t-j} \\ &\quad - (1-\lambda^\tau) \sum_{j=0}^{\tau-1} \lambda^j \xi_{t+\tau-j} - (1-\lambda^\tau)\lambda^\tau \sum_{j=0}^{\infty} \lambda^j \xi_{t-j} \end{aligned} \tag{24-b}$$

where $\Phi_{t+\tau+1}^{t+2\tau}$ denotes the terms with $\eta_{t+2\tau}, \cdots, \eta_{t+\tau+1}, \xi_{t+2\tau}, \cdots, \xi_{t+\tau+1}$.

We assume that $\text{cov}(\omega_{t+i}, \xi_{t+j}) = 0$ for all i and j, and $\text{var}(\xi_t) = \sigma_\xi^2$ for all t. Since ξ_t also is a rational forecast error, $\text{cov}(\xi_t, \xi_{t+i}) = 0$ for all $i \neq 0$. We express σ_ξ^2 as $q\sigma_\omega^2$, where q is a positive (but unknown) constant, and, without loss of generality, $\sigma_\omega^2 = 1$. For $0 \leq \lambda < 1$, we have[5]

$$\mathrm{var}(P_{t+\tau} - P_t) = \tau + \frac{2(1-\lambda^\tau)\{q-(1-c)(c+\lambda)\}}{1-\lambda^2}, \tag{25-a}$$

and

$$\mathrm{cov}(P_{t+2\tau} - P_{t+\tau}, P_{t+\tau} - P_t) = \frac{-(1-\lambda^\tau)^2\{q-(1-c)(c+\lambda)\}}{1-\lambda^2}. \tag{25-b}$$

The first-order autocorrelation of stock price changes over τ periods is

$$f(\tau, \lambda, q) = \frac{-(1-\lambda^\tau)^2\{q-(1-c)(c+\lambda)\}}{\tau(1-\lambda^2) + 2(1-\lambda^\tau)\{q-(1-c)(c+\lambda)\}}. \tag{26}$$

This autocorrelation can be positive if $\{q-(1-c)(c+\lambda)\}$ is negative. This happens for $0 < q < 1$, because $-(1-c)(c+\lambda)$ decreases from 0 to -1 as λ increases from 0 to 1.

When $\lambda = 0$ (i.e., if dividends revert immediately to the target), we have

$$f(\tau, \lambda = 0, q) = \frac{-q}{\tau + 2q} < 0. \tag{27}$$

This illustrates that stock price forecast errors in the target reverting process cause stock price changes to be negatively autocorrelated.

The sign of $\frac{\partial f}{\partial \tau}$ is not clear. However, unless the dividend is extremely smoothed, it is likely that

$$\frac{\partial f(\tau, \lambda, q)}{\partial \tau} < 0 \text{ for } \tau < \text{ some } \tau^*, \tag{28-a}$$

and

$$\frac{\partial f(\tau, \lambda, q)}{\partial \tau} > 0 \text{ for } \tau > \text{ some } \tau^*. \tag{28-b}$$

[5]It can be shown that $\partial \mathrm{var}(P_{t+\tau} - P_t)/\partial\tau > 0$, and $\mathrm{var}(P_{t+1} - P_t)$ is positive for any $q \geq 0$. Hence, $\mathrm{var}(P_{t+\tau} - P_t)$ is positive for any τ.

This result implies that the pattern of $f(\tau, \lambda, q)$ is a "V" shape with respect to the length of holding periods. To prove this, let $x \equiv \lambda^\tau (0 \leq x \leq \lambda)$ so that $\tau = \frac{\ln x}{\ln \lambda}$, $A \equiv q - (1-c)(c+\lambda)$, which is assumed to be positive for the autocorrelation to be negative,[6] and $B \equiv \frac{1-\lambda^2}{\ln \lambda}$, which is negative. We express equation (26) as

$$f(x) \;=\; \frac{-(1-x)^2 A}{B \ln x + 2A(1-x)}. \tag{29}$$

The sign of $\frac{\partial f}{\partial \tau}$ is the opposite of the sign of $\frac{\partial f}{\partial x}$; as τ increases from 1 to ∞, x decreases from λ to 0. The sign of $\frac{\partial f}{\partial x}$ is the same as the sign of $g(x)$:[7]

$$g(x) \;=\; 2Bx \ln x + 2Ax(1-x) + B(1-x). \tag{30}$$

It follows that

$$g(x=0) \;=\; B \;<\; 0 \tag{31-a}$$

$$g(x=\lambda) \;=\; \lambda(1-\lambda)\left\{(1+\lambda)\left(2+\frac{1}{\ln\lambda}\right)+2A\right\} \;>\; 0(?) \tag{31-b}$$

$$\begin{aligned} \frac{\partial g}{\partial x} &= 2B\ln x + B + 2A(1-2x) \\ &= 2(1-\lambda^2)\tau + \frac{1-\lambda^2}{\ln\lambda} + 2A(1-2\lambda^\tau) \;>\; 0(?) \end{aligned} \tag{31-c}$$

Unless the dividend is extremely smoothed (i.e., if λ is not close to 1), $g(x=\lambda)$ and $\frac{\partial g}{\partial x}$ are likely to be positive. Then, $\frac{\partial f}{\partial \tau} < 0$ (i.e., $\frac{\partial f}{\partial x} > 0$) for small τ (i.e., large x), and $\frac{\partial f}{\partial \tau} > 0$ (i.e., $\frac{\partial f}{\partial x} < 0$) for large τ (i.e., small x).

Assuming that $k = 0.08$ and $q = 1$, Table 1 computes $f(\tau, \lambda)$ for $\tau = 1, 2, \cdots, 10$ and for $\lambda =$ 0.25, 0.50, and 0.75. These λ values would correspond to those of the pre-war period, the 1920s–1980s period, and the

[6] We assume that $q > 1$, which implies that $\sigma_\xi^2 > \sigma_\omega^2$.

[7] $g(x) = 0$ is the first-order condition for the maximum negative autocorrelation.

post-war period, respectively. For $\lambda = 0.25$, 2-year stock price changes have the largest negative autocorrelation. For $\lambda = 0.75$, 6-year stock price changes have the largest negative autocorrelation. The patterns of these computed negative autocorrelations appear to resemble those observed by Fama and French (1987).

Finally, holding τ and λ constant, we find that

$$\frac{\partial f(\tau, \lambda, q)}{\partial q} < 0. \tag{32}$$

As q increases, the magnitude of the negative autocorrelation of stock price changes increases. Equation (23) shows that the variability of stock prices (σ_η^2) is determined by the variability of dividends (σ_ω^2) and the variability of other variables (σ_ξ^2). Since $\sigma_\xi^2 = q\sigma_\omega^2$, from equation (23) we have $c^2 + q = \sigma_\eta^2/\sigma_\omega^2$. Hence, an increase in q means that the variability of stock prices increases relative to the variability of dividends. To the extent that firm size is inversely related to the magnitude of q, our result corroborates Zarowin's (1990) finding that negative autocorrelations of stock price changes are observed mostly for small firms.[8]

II. Summary and Concluding Remarks

When the dividend paid reverts to a target dividend, the stock price also reverts to the present value of expected future target dividends. Rational forecast errors of this target reverting process can create negative autocorrelations of stock price changes. We further find that (1) the significance of the negative autocorrelation is inversely related to the degree of dividend smoothing; (2) the negative autocorrelation appears to be a "V" shaped

[8] See also Chopra, Lakonishok and Ritter (1991).

function of the length of holding periods; (3) as the variability of stock prices increases relative to the variability of dividends, the negative autocorrelation becomes more significant. Future empirical studies can, using cross-section data, test for the characteristics of the behavior of stock price changes which are predicted by our present value model analysis.

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Appndix A: Derivation of Equations (24) and (25)

Equation (24-a): $0 \leq \lambda < 1$.

$$P_{t+\tau} - P_t \;=\; (1-\lambda L)^{-1}\left\{c\omega_{t+\tau} + \gamma\omega_{t+\tau-1} + \cdots + \gamma\omega_{t+1} + (\gamma - c)\omega_t + \xi_{t+\tau} - \xi_t\right\}$$

The right hand side of this equation is rewritten as

$$\begin{array}{cccccccc}
c\omega_{t+\tau} & +c\lambda\omega_{t+\tau-1} & +c\lambda^2\omega_{t+\tau-2} & +\cdots & +c\lambda^{\tau-1}\omega_{t+1} & +c\lambda^{\tau}\omega_t & +c\lambda^{\tau+1}\omega_{t-1} & +\cdots \\
 & +\gamma\omega_{t+\tau-1} & +\gamma\lambda\omega_{t+\tau-2} & +\cdots & +\gamma\lambda^{\tau-2}\omega_{t+1} & +\gamma\lambda^{\tau-1}\omega_t & +\gamma\lambda^{\tau}\omega_{t-1} & +\cdots \\
 & & +\gamma\omega_{t+\tau-2} & +\cdots & & & & \\
 & & & \ddots & & & & \\
 & & & & +\gamma\omega_{t+1} & +\gamma\lambda\omega_t & +\gamma\lambda^2\omega_{t-1} & +\cdots \\
 & & & & & +(\gamma-c)\omega_t & +(\gamma-c)\lambda\omega_{t-1} & +\cdots \\
 & & & & & & & \\
+\xi_{t+\tau} & +\lambda\xi_{t+\tau-1} & & +\cdots & +\lambda^{\tau-1}\xi_{t+1} & +\lambda^{\tau}\xi_t & +\lambda^{\tau+1}\xi_{t-1} & +\cdots \\
 & & & & & -\xi_t & -\lambda\xi_{t-1} & -\cdots
\end{array}$$

This is rearranged as

$$\begin{array}{rl}
 & c\omega_{t+\tau} \\
+ & (\gamma + c\lambda)\omega_{t+\tau-1} \\
+ & (\gamma + \gamma\lambda + c\lambda^2)\omega_{t+\tau-2} \\
+ & (\gamma + \gamma\lambda + \gamma\lambda^2 + c\lambda^3)\omega_{t+\tau-3} \\
\vdots & \\
+ & (\gamma + \gamma\lambda + \cdots + \gamma\lambda^{\tau-2} + c\lambda^{\tau-1})\omega_{t+1} \\
+ & (-c + \gamma + \gamma\lambda + \cdots + \gamma\lambda^{\tau-1} + c\lambda^{\tau})\omega_t \\
+ & (-c + \gamma + \gamma\lambda + \cdots + \gamma\lambda^{\tau-1} + c\lambda^{\tau})\lambda\omega_{t-1} \\
\vdots & \\
+ & \displaystyle\sum_{j=0}^{\tau-1}\lambda^j\xi_{t+\tau-j} - (1-\lambda^{\tau})\sum_{j=0}^{\infty}\lambda^j\xi_{t-j}
\end{array}$$

The coefficient of $\omega_{t+\tau-j}$, for $0 \leq j \leq \tau - 1$, is

$$\begin{array}{rl}
 & \gamma\left(1 + \lambda + \cdots + \lambda^{j-1}\right) + c\lambda^j \\
= & \gamma\dfrac{1-\lambda^j}{1-\lambda} + c\lambda^j \;=\; 1 - \lambda^j + c\lambda^j \text{ (recall } \gamma = 1-\lambda)
\end{array}$$

The coefficient of ω_{t-j}, for $j = 0, 1, \cdots$, is

$$\begin{aligned}
& \left\{-c + \gamma(1 + \lambda + \cdots + \lambda^{\tau-1}) + c\lambda^{\tau}\right\}\lambda^j \\
= & \left\{\gamma\frac{1-\lambda^{\tau}}{1-\lambda} - c(1-\lambda^{\tau})\right\}\lambda^j = (1-c)(1-\lambda^{\tau})\lambda^j
\end{aligned}$$

Equation (24-b):

To compute $P_{t+2\tau} - P_{t+\tau}$, we replace subscript t in equation (24-a) with $t+\tau$ and relegate the terms with $\eta_{t+2\tau}, \cdots, \eta_{t+\tau+1}, \xi_{t+2\tau}, \cdots, \xi_{t+\tau+1}$ to $\Phi_{t+\tau+1}^{t+2\tau}$, which are unnecessary for computing $\mathrm{cov}(P_{t+2\tau} - P_{t+\tau}, P_{t+\tau} - P_t)$.

Equation (25-a): $0 \leq \lambda < 1$, $\sigma_{\omega}^2 = 1$, and $\sigma_{\xi}^2 = q$.

$\mathrm{var}(P_{t+\tau} - P_t) =$

$$\sum_{j=0}^{\tau-1}\left\{(1-\lambda^j) + c\lambda^j\right\}^2 + (1-c)^2(1-\lambda^{\tau})^2\sum_{j=0}^{\infty}\lambda^{2j} + q\left\{\sum_{j=0}^{\tau-1}\lambda^{2j} + (1-\lambda^{\tau})^2\sum_{j=0}^{\infty}\lambda^{2j}\right\}$$

The first sum becomes

$$\begin{aligned}
& \sum_{j=0}^{\tau-1}\left\{(1-\lambda^j) + c\lambda^j\right\}^2 \\
= & \sum_{j=0}^{\tau-1}\left\{(1-\lambda^j)^2 + c^2\lambda^{2j} + 2c(1-\lambda^j)\lambda^j\right\} \\
= & \sum_{j=0}^{\tau-1}\left\{1 - 2(1-c)\lambda^j + (1-c)^2\lambda^{2j}\right\} \\
= & \tau - \frac{2(1-c)(1-\lambda^{\tau})}{1-\lambda} + \frac{(1-c)^2(1-\lambda^{2\tau})}{1-\lambda^2}
\end{aligned}$$

Since $\sum_{j=0}^{\infty}\lambda^{2j} = \frac{1}{1-\lambda^2}$ and $\sum_{j=0}^{\tau-1}\lambda^{2j} = \frac{1-\lambda^{2\tau}}{1-\lambda^2} = \frac{(1-\lambda^{\tau})(1+\lambda^{\tau})}{1-\lambda^2}$, it follows that

$$\begin{aligned}
\mathrm{var}(P_{t+\tau} - P_t) = & \tau - \frac{2(1-c)(1-\lambda^{\tau})}{1-\lambda} + \frac{(1-c)^2(1-\lambda^{\tau})(1+\lambda^{\tau})}{1-\lambda^2} \\
& + \frac{(1-c)^2(1-\lambda^{\tau})^2}{1-\lambda^2} + q\left\{\frac{(1-\lambda^{\tau})(1+\lambda^{\tau}) + (1-\lambda^{\tau})^2}{1-\lambda^2}\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \tau - \frac{2(1-c)(1-\lambda^\tau)}{1-\lambda} + \frac{(1-c)^2(1-\lambda^\tau)(1+\lambda^\tau+1-\lambda^\tau)}{1-\lambda^2} \\
&\quad + \frac{q(1-\lambda^\tau)(1+\lambda^\tau+1-\lambda^\tau)}{1-\lambda^2} \\
&= \tau - \frac{2(1-c)(1-\lambda^\tau)}{1-\lambda} + \frac{2(1-c)^2(1-\lambda^\tau)}{1-\lambda^2} + \frac{2q(1-\lambda^\tau)}{1-\lambda^2} \\
&= \tau + \frac{2(1-\lambda^\tau)\left\{q+(1-c)^2-(1-c)(1+\lambda)\right\}}{1-\lambda^2}
\end{aligned}
$$

This leads to equation (24-a) in the main text.

<u>Equation (25-b)</u>: cov = $\mathrm{cov}(P_{t+2\tau} - P_{t+\tau}, P_{t+\tau} - P_t)$

$$
\begin{aligned}
\mathrm{cov} &= (1-c)(1-\lambda^\tau)\sum_{j=0}^{\tau-1}\left\{1-(1-c)\lambda^j\right\}\lambda^j + (1-c)^2(1-\lambda^\tau)^2\lambda^\tau\sum_{j=0}^{\infty}\lambda^{2j} \\
&\quad - q\left\{(1-\lambda^\tau)\sum_{j=0}^{\tau-1}\lambda^{2j} - (1-\lambda^\tau)^2\lambda^\tau\sum_{j=0}^{\infty}\lambda^{2j}\right\} \\
&= (1-c)(1-\lambda^\tau)\left\{\frac{1-\lambda^\tau}{1-\lambda} - \frac{(1-c)(1-\lambda^\tau)(1+\lambda^\tau)}{1-\lambda^2}\right\} + \frac{(1-c)^2(1-\lambda^\tau)^2\lambda^\tau}{1-\lambda^2} \\
&\quad - q\left\{\frac{(1-\lambda^\tau)(1-\lambda^\tau)(1+\lambda^\tau)-(1-\lambda^\tau)^2\lambda^\tau}{1-\lambda^2}\right\} \\
&= \frac{(1-c)(1-\lambda^\tau)^2}{1-\lambda} - \frac{(1-c)^2(1-\lambda^\tau)^2(1+\lambda^\tau-\lambda^\tau)}{1-\lambda^2} \\
&\quad - \frac{q(1-\lambda^\tau)^2(1+\lambda^\tau-\lambda^\tau)}{1-\lambda^2} \\
&= \frac{(1-c)(1-\lambda^\tau)^2(1+\lambda)-(1-c)^2(1-\lambda^\tau)^2-q(1-\lambda^\tau)^2}{1-\lambda^2} \\
&= \frac{-(1-\lambda^\tau)^2\left\{q+(1-c)^2-(1-c)(1+\lambda)\right\}}{1-\lambda^2}
\end{aligned}
$$

This leads to equation (25-b) in the main text.

Table 1
Computing Autocorrelations of Stock Price Changes
Equation (26)

Length of Holding Periods (τ)	$f(\tau, \lambda, q = 1, k = 0.08)$		
	$\lambda =$ 0.25	$\lambda =$ 0.50	$\lambda =$ 0.75
1	−0.230	−0.140	−0.059
2	−0.233*	−0.182	−0.095
3	−0.202	−0.186*	−0.118
4	−0.172	−0.175	−0.129
5	−0.148	−0.159	−0.134
6	−0.130	−0.144	−0.135*
7	−0.116	−0.131	−0.133
8	−0.105	−0.119	−0.129
9	−0.095	−0.109	−0.124
10	−0.087	−0.100	−0.119

* denotes the largest negative autocorrelation.