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Dynamic Specification Error in Cost Function and Factor Demand Estimation

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ABSTRACT

This paper concerns the problem of properly specifying the dynamic structure of models of industry costs and factor demands. The paper compares three common frameworks: long-run costs with all factors assumed in equilibrium (Full Static Equilibrium), short-run costs with variable factors in short-run equilibrium (Partial Static Equilibrium) followed by computation of long-run costs, and short-run costs including internal capital adjustment costs (Partial Dynamic Equilibrium). The approach of the paper is to estimate a capital-labor-fuel-electricity model for six OECD countries (G7 less Italy) for the 1960-1989 period. Using the three different "dynamic" specifications, we obtain substantially different results in terms of factor demand, cross-price effects and technical change. The implication is that proper dynamic specification is critical. The Partial Dynamic Equilibrium model appears to behave most consistently across the cross-section.

I. INTRODUCTION

The analysis of industrial factor demand, particularly energy, has been a popular subject for applied economists. Early studies include Fisher and Kaysen's (1962) and Balestra and Nerlove's (1966) studies of electricity and gas demand. A plethora of studies appeared during the last two decades, spurred by the OPEC price action of 1973 (see Bohi, 1981, or Kolstad, 1987, for surveys).

A major development in this literature has been a movement towards a more system-wide analysis, recognizing that demand for one factor such as energy is influenced by prices and demand for a variety of other factors, such as capital and labor. Motivated by the Berndt and Wood (1975) model of factor demand in U.S. manufacturing, a number of studies have estimated complete cost functions for the entire manufacturing sector from which an estimate of labor, capital and fuel demand is derived.

Most of the work over the past decade or two has involved understanding and representing how firms adjust quasi-fixed factors in response to price changes. The early multi-factor models such as those of Berndt and Wood (1975), Pindyck (1979) and Griffin (1979) implicitly assumed factors were in long-run equilibrium. A second, more recent approach has been to estimate short-run cost functions (Brown and Christensen, 1981; Berndt and Hesse, 1986; Kulatilaka, 1987; Morrison, 1988). This yields an estimate of short-run factor demand. Then using long-run equilibrium conditions on capital as a variable factor, one can obtain estimates of long-run factor demand as well. The advantage of this approach is that the dynamic structure of production is inferred from economic optimizing behavior, not ad-hoc distributed lag structures. The disadvantage is that we only deduce short-run and long-run demand and infer nothing of the path of adjustment between the short- and long-run. A third type of model, which has its origins in the work of Lucas (1967), views the capital stock as quasi-fixed but costly to change. Thus costs include a cost-of-adjusting capital (see Pindyck and Rotemberg, 1983; Morrison, 1988; Berndt et al., 1980).

The purpose of this paper is to investigate the significance of the specification (or misspecification) of the dynamic structure of costs and demand. We examine the three most common dynamic specifications: direct estimation of the long-run cost function (as done by Berndt and Wood, 1975, among others), estimation of the short-run restricted cost function (as done by Brown and Christiansen, 1981, among others), and estimation of the short-run restricted cost function along with an Euler equation governing the rate at which the capital stock is changed. Kulatilaka (1987) refers to the first as full static equilibrium and the second as partial static equilibrium. We refer to the third as partial dynamic equilibrium. Using a data set for 1960-1989 in six major OECD countries, we estimate all three types of models and compare the results, particularly estimates of long-run price elasticities of demand and rates of technical change. We find that there are very significant differences in estimated elasticities and rates of cost diminuation among these three approaches.

The remainder of this paper consists of three basic sections. The next section presents the KLFE (capital, labor, fuel and electricity) model of the manufacturing sector of six OECD countries. This is followed by a discussion of the estimation of the three models. The last section considers results and is followed by conclusions.

II. MODELS OF INDUSTRIAL FACTOR DEMAND

Probably the most significant issue that has dominated the economics of industrial factor demand is that of dynamics (see Slade et al., 1992, or Berndt et al., 1981). Much of this work has focused on energy. Demand for energy is generally a derived demand, based in large part on the existing stock of energy-using capital. Thus how demand changes over time depends on how the capital stock changes in size and composition over time. As Fisher and Kaysen (1962) noted in their seminal study of electricity demand in the U.S., "in both households and industry, the use of electricity is complementary to the use of a stock of electricity using equipment."

Most early models of energy demand (and even some that have been generated recently) rely on relatively ad hoc representations of dynamics.[1] In particular, either demand is assumed to adjust according to a partial adjustment framework or price expectations are assumed to adjust according to a similar, adaptive expectations framework. In both cases, prices or other variables determine a consumer's demand or eventual equilibrium price, P^*. The consumer adjusts his expectations or actual demand according to $P(t) = P(t-1) + \theta(P^*-P(t-1))$. If $\theta = 1$, adjustment is instantaneous; if $\theta < 1$, adjustment is noninstantaneous. There are a large number of such partial adjustment models including the

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[1] While distributed lags are usually adopted on ad hoc grounds, this is not always the case; see Treadway (1971, 1974) and Nadiri and Rosen (1969).

natural gas models of Balestra and Nerlove (1966) and Taylor et al., (1982) and the electricity model of Fisher and Kaysen (1962).

A closely related class of models consists of those that assume the market is in long-run equilibrium (e.g., Berndt and Wood, 1975). Of course if the market is in equilibrium, as Pindyck (1979) has argued for his international pooled cross-section work, then this is a perfectly legitimate characterization of costs and demand. Making the assumption of long-run equilibrium, all factors are assumed variable and thus the estimated cost function is dependent on prices of factors and output. We term this class of models full static equilibrium models.[2]

A third type of model consists of short-run models of energy demand with capital (usually) as a fixed stock: a series of variable cost functions are estimated; using this information and duality relationships between variable costs and long-run costs; both short-run and long-run (but nothing in between) demand functions can be computed. This is shown in Figure 1 where k is assumed to be fixed in the short-run and variable in the long-run: the lower boundary of a set of short-run average cost functions (holding the prices of energy and labor constant) delimits the long-run average cost function which in turn permits us to compute long-run factor demands. With capital as the quasi-fixed factor in the short-run cost function, we can compute a long-run cost function with capital as a variable factor, using the rental price of capital as an explanatory variable. An example of this type of model is the aggregate energy demand model of Mork (1978) and the OECD model of Berndt and Hesse (1986) and the U.S./Japanese model of

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[2]Kulatilaka (1987) uses similar terminology.

Morrison (1988). We term this class of models partial static equilibrium models.

A fourth type of model explicitly focuses on the evolution of the capital stock. Such models have their origins in the investment model of Lucas (1967) where it is hypothesized that it is costly to change the capital stock and the more rapidly the change is made the more costly it is. Thus, a firm may know that it needs to buy new energy efficient equipment but it does so slowly because the cost of doing it rapidly (e.g., information costs, plant shutdown costs) are high. Unfortunately, such adjustment cost models are extremely complex and must be quite simple in order to be tractable. Examples of adjustment cost models of energy demand are the Berndt et al. (1980) model of U.S. energy demand, the Pindyck and Rotemberg (1983) model, also of U.S. energy demand, and Morrison's (1988) model of U.S. and Japanese demand. We term this class of models partial dynamic equilibrium models.

We consider the last three of these four types of models, examining the manufacturing sector, viewed as using as inputs labor, capital, electricity and a fuel aggregate consisting of oil, gas and coal; costs also depend on the state of technology and manufacturing output. One model we consider treats all factors as variable and thus costs depend on factor prices. In the second type of model, the quantity of capital in use in the manufacturing sector of the economy is assumed to be fixed and thus the price of capital of no consequence to the estimation of short-run costs. The third type of model we examine assumes that capital adjustment costs are part of the observed variable costs. Thus variable costs depend on the rate of change of the capital stock.

A. <u>A General Model of Industrial Factor Demand</u>

Consider the manufacturing sector in a particular country. Let the sector produce output y, with technology (time) t, using a set of inputs which we will partition into variable factors (x), including fuels, and a quasi-fixed factor, capital (K). We can hypothesize the aggregate production function for the sector to be:

$$y = f(x, K, t). \tag{1}$$

In long-run equilibrium, all factors are considered variable; thus the long-run total cost function is given by

$$C_T = T(P_x, P_K, y, t) \tag{2}$$

In the short-run, we will assume the variable inputs are labor, electricity and a fuel aggregate (oil, gas and coal).

Thus in the short-run, the production function in (1) is dual to a restricted cost function giving the variable costs of producing a specified level of output with a specified level of capital as a function of prices of variable inputs:

$$C_V = V(P_x, K, y, t). \tag{3}$$

At any point in time, the shadow value of capital (ρ_K) will be the derivative of variable costs with respect to K; i.e., how much of variable costs could be saved if K were increased by one unit:

$$\rho_K = - \frac{\partial V}{\partial K}. \tag{4}$$

If variable costs exhibit constant returns, Eqn. (3) will be homogeneous of degree 1 in capital and output. Thus an application of Euler's theorem to Eqn. (3) yields

$$C_V = K\frac{\partial V}{\partial K} + y\frac{\partial V}{\partial y}. \tag{5}$$

If the market is competitive, marginal costs are equal to the price of output; thus (5) can be rewritten, using (4), to obtain

$$\rho_K K = P_y y - C_V. \tag{6}$$

where P_y is the price of output. Eqn. (6) states that the difference between the value of output and variable costs is returns to capital, which is generally the operating surplus in national accounts (see Berndt and Hesse, 1986).

Of course the shadow value of capital (ρ_K) may not equal the rental price of capital, and in fact will not unless firms are in long-run equilibrium. In long-run equilibrium, we know (Samuelson, 1953) that the change in variable cost associated with one less unit of capital (the shadow value) must equal the rental price of capital; i.e.,

$$\frac{\partial V}{\partial K}(K*) = -P_K. \tag{7}$$

This equation implicitly defines a long-run equilibrium level of capital as a function of prices, output and technology: $K*(P_X, P_K, y, t)$. Thus long-run total costs are given by

$$C_T = T(P_x, P_K, y, t) = V(P_x, K*(P_x, P_K, y, t), y, t) + K*(P_x, P_K, y, t) \cdot P_K. \tag{8}$$

Unfortunately, while eqn. (7) defines the long-run equilibrium capital stock, it says nothing of how one moves from the current K to K*. If one assumes that there is a cost to adjusting the capital stock (the more rapid the adjustment, the higher the variable costs), then the path from K to K* becomes determinant. After Morrison (1988) assume that capital adjustment costs are internal to the short-run decisions of the firm.[3] As a consequence, C_T (eqn. 2) and C_V (eqn. 3) are assumed to depend on $\dot{K}$ as well as the other parameters. The Euler first order condition (Morrison, 1988) on the optimal $\dot{K}$ are

$$-C_K - rC_{\dot{K}} - P_K + C_{\dot{K}\dot{K}}\ddot{K} + C_{K\dot{K}}\dot{K} = 0 \tag{9}$$

where r is the interest rate, $C = C_V$ and the subscripts refer to derivatives. Thus short-run costs depend not only on the prices of variable factor and the stock of capital but the rate of change of the

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[3]Internal adjustment costs are contained within variable costs. External adjustment costs are in addition to variable costs. Berndt et al. (1980) adopt an internal adjustment cost approach whereas Pindyck and Rotemberg (1983) assume external adjustment costs.

capital stock, $\dot{K}$. In long-run equilibrium, we would expect

$\dot{K}^* = \ddot{K}^* = 0$.

B. Factor Demands, Substitutability and Price Elasticities

Let C denote a cost function--either variable (short-run) or total (long-run). Demand for factor i is given by Shepherd's lemma:

$$X_i^* = \frac{\partial C}{\partial P_i}. \tag{10}$$

The Allen elasticity of substitution between factors i and j is given by (see Uzawa, 1962)

$$\sigma_{ij} = \frac{C \dfrac{\partial^2 C}{\partial P_i \partial P_j}}{\dfrac{\partial C}{\partial P_i} \dfrac{\partial C}{\partial P_j}} \tag{11}$$

Finally, the elasticity of demand for factor i with respect to the price of factor j is given by

$$\varepsilon_{ij} = \frac{\partial X_i^*}{\partial P_j} \frac{P_j}{X_i^*} \tag{12a}$$

$$= \frac{\dfrac{\partial^2 C}{\partial P_i \partial P_j} P_j}{\dfrac{\partial C}{\partial P_i}} \tag{12b}$$

where the second equality is obtained using Eqn. (10).

Whether x_i^*, σ_{ij} and ε_{ij} are short-run or long-run depends on whether C is a short-run variable cost function or a long-run total cost function.

Evaluating Eqn. (10-12) is straightforward if C is defined explicitly. If, however, one is interested in deriving long-run effects from a restricted cost function, evaluation of these equations is more difficult. In this case, taking the first derivative of Eqn. (8), one obtains

$$\frac{\partial C_T}{\partial P_i} = \begin{cases} \dfrac{\partial V}{\partial P_i} + \left[\dfrac{\partial V}{\partial K}\dfrac{\partial K*}{\partial P_i} + \dfrac{\partial K*}{\partial P_i}P_K\right] = \dfrac{\partial V}{\partial P_i} & \text{for } i \text{ variable} \quad (13a) \\ \left[\dfrac{\partial V}{\partial K}\dfrac{\partial K*}{\partial P_K} + \dfrac{\partial K*}{\partial P_K}P_K\right] + K* = K* & \text{for } i = K. \quad (13b) \end{cases}$$

Note from Eqn. (7) that the expressions in brackets above are both zero. In order to take the second derivatives of C_T, it is necessary to find the derivatives of K*. These can be obtained by totally differentiating the long-run capital stock efficiency condition, Eqn. (7):

$$\sum_i \frac{\partial^2 V}{\partial P_i \partial K}dP_i + \frac{\partial^2 V}{\partial K^2}dK + \frac{\partial^2 V}{\partial K \partial y}dy + \frac{\partial^2 V}{\partial K \partial t}dt = -dP_K \quad (14)$$

where i indicates a variable factor. This equation can be used to find the following partial derivatives by setting dy = dt = 0.

$$\frac{\partial K^*}{\partial P_i} = \begin{cases} -\dfrac{\partial^2 V}{\partial P_i \partial K} \Big/ \dfrac{\partial^2 V}{\partial K^2} & \text{for i variable} \quad (15a) \\ -\dfrac{1}{\dfrac{\partial^2 V}{\partial K^2}} & i = K. \quad (15b) \end{cases}$$

The second derivatives of C_T can be taken from (13) and combined with (15) to obtain

$$\frac{\partial^2 C_T}{\partial P_i \partial P_j} = \begin{cases} \dfrac{\partial^2 V}{\partial P_i \partial P_j} - \dfrac{\partial^2 V}{\partial P_i \partial K} \dfrac{\partial^2 V}{\partial P_j \partial K} \Big/ \dfrac{\partial^2 V}{\partial K^2} & \text{i,j variable} \quad (16a) \\ -\dfrac{\partial^2 V}{\partial P_i \partial K} \Big/ \dfrac{\partial^2 V}{\partial K^2} & \text{i variable, j = k} \quad (16b) \\ -\dfrac{1}{\dfrac{\partial^2 V}{\partial K^2}} & \text{i,j = K} \quad (16c) \end{cases}$$

To compute long-run price and substitution elasticities, and factor demands, from a variable cost function it is necessary to compute $K^*(P,P_k,y,t)$ from Eqn. (7). This must generally be done numerically.[4]

III. ESTIMATION

We turn now to estimating the cost function under three "dynamic specifications": long-run total costs (full static equilibrium), short-run restricted costs (partial static equilibrium) and internal

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[4]The Generalized Lentieff flexible function form allows the explicit solution of eqn (7) for K* (see Morrison, 1988).

adjustment costs (partial static equilibrium). We have chosen to estimate the cost function assuming constant returns to scale.

The full static equilibrium case amounts to assuming Eqn. (2) applies at all points in time. Partial static equilibrium amounts to assuming Eqns. (3-4) apply at all points in time. Partial dynamic equilibrium amounts to assuming Eqns. (3) and (9) apply at all points in time.

In our empirical implementation, we assume a translog cost function.[5] We are also faced with the question of whether to impose constant returns to scale (CRTS). It is well-known that there are computational and estimation problems with nonconstant returns, partial static equilibrium models. Morrison (1988) identifies some of these problems and thus adopts the CRTS assumption. One problem, which was discussed in the context of Eqns. (5-6) above, is that without CRTS, the returns to capital are unobservable and thus Eqns. (4-6) cannot be used in the estimation. In particular, we found that without the equation in the estimation, ρ_K can go negative, in which case there is no solution to the long-run capital stock in Eqn. (7). Kulatilaka (1987) tests for CRTS in a partial static model for the U.S. and cannot reject CRTS. Thus he, too, adopts the CRTS assumption. However, other authors (e.g., Pindyck and Rotemberg, 1983) have been able to reject CRTS. Thus the evidence is ambiguous but suggests that it is not unreasonable to adopt CRTS, which we do.

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[5]In the partial dynamic equilibrium model we use K rather than log K since K can be negative. Similarly, time/technology, t, enters in non-logarithmic form.

The models we estimate involve capital (K), labor (L), electricity (E) and nonelectric fuel (F), as well as time (t). In all cases, factor share equations (with electricity shares assumed redundant) are estimated. In the case of the partial dynamic equilibrium model, the Euler equation (9) is also estimated, using first and second differences of the capital stock for $\dot{K}$ and $\ddot{K}$. The model is estimated for six OECD countries (Canada, United States, Japan, France, West Germany and United Kingdom) for the 1960-89 period. Data is documented in the appendix and estimation was via full information maximum likelihood as implemented in TSP4.2. Thus the estimated equations are

Model FSE: Eqn. (2) with share equations for L,F,K.

Model PSE: Eqn. (3) with share equations for L,F,K; share equation for K is from Eqn. (6).

Model PDE: Eqn. (3) with share equations for L,F, and Euler Eqn. (9).

Errors are assumed additive and endogenous variables are the costs, shares and in the case of Model PDE, K.

There are many different ways of pooling time-series and cross-section models. As pointed out by Griffin (1979) and Johnston (1984), the method of using different intercepts for the share equations for different countries is probably the most appropriate method when one wishes to estimate short-run demand. However, they also point out that the use of varying intercepts can diminish the explanatory power of the exogenous variables. We adopt a simple fixed effects model. Different zero and first order terms of the cost function are assumed for different countries with the same second order terms assumed across countries. This implies that the share equations have country-dependent intercepts but constant slopes.

Thus the model estimated is the translog version of Eqn. (2-3):

$$\log C = \alpha + \beta x + \frac{1}{2}x\Gamma x \tag{17}$$

$$\text{where } x = \begin{cases} (\log P_L, \log P_F, \log P_E, \log P_K, t) & \text{for FSE model} \\ (\log P_L, \log P_F, \log P_E, \log K, t) & \text{for PSE model} \\ (\log P_L, \log P_F, \log P_E, \log K, \Delta K, t) & \text{for PDE model} \end{cases}$$

and where C and K are normalized by value added plus energy costs (the value of labor, energy and capital service inputs).

Table I presents estimates of the parameters for all three models along with standard errors. Not all coefficients are reported since homogeneity in prices reduces the number of coefficients. In all three cases, nearly all parameters are highly significant.[6]

V. RESULTS

The basic purpose of this paper is to compare the three different formulations of the dynamic structure of costs. Hypothesis testing is difficult because of the substantially different structures of the three models as well as their non-nested nature. All three models purport to be able to measure long-run behavior. Consequently, we will contrast

[6]For the FSE model, all variables except γKT, γFT and γTT were significant at the 98% level based on t-statistics. For the PSE, all except γTT were significant, and for the PDE model, all but ßDCA, ßDUS, ßDJA, ßDWG, ßKFR, ßKWG, γDD and γDt where D refers to $\dot{K}$.

and compare long-run price effects as well as rates of technical change and the extent of disequilibrium in the capital stock (K*/K).

Table II presents long-run elasticities of factor demand, rates of cost diminuation and K*/K, along with t-statistics,[7] based on the 1960-89 average values of the cost function arguments. Results from all three models are shown--the full static equilibrium model (FSE), the partial static equilibrium model (PSE), and the partial dynamic equilibrium model (PDE). Note first of all that all three models seem relatively well-behaved regarding downward sloping demand. Even though curvature was not imposed, own elasticities are negative except for a few own price elasticities of demand for energy which are significantly positive.

Note that demand for aggregate fuel is somewhat more inelastic that has been reported in some other international studies (eg, Pindyck, 1979 and Griffin, 1979, report figures for energy on the order of -0.7 to -0.8). In fact the reported price elasticities for the three models, generally less than -0.5, are more consistent with national studies such as Berndt and Wood (1975) or Kulatilaka (1987). The results are also consistent with the US-Japan study of Morrison (1988).

While there are many observations that can be made about the magnitudes of the various elasticities, let us focus on the similarities and differences among the three models. One thing that stands out is that own-price elasticities are generally much more inelastic for the

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[7]Standard errors are obtained by second order expansions of the functions defined the various elasticities and other measures, around the mean parameter values. t-statistics are the ratio of parameter values to standard errors.

PDE model than for the other two. This suggests that what is attributed to a price effect by the PSE and FSE models is considered more of an adjustment cost in the PDE model. The remarkable thing is that the only difference between the PSE and PDE models is a single equation--a capital share equation (value added less the wage bill equals the capital share) in the case of the PSE model and the Euler equation in the PDE model. It is also quite remarkable that this one difference makes such a big difference in K*/K. For the PSE model, actual capital stocks are much lower than their long-run optimum, given factor prices. However, in the case of the PDE model, K and K* are quite similar.

Focusing on the rate of cost diminuation, the results are remarkably significant and consistent among the models. However, in all cases, costs are declining at the most rapid rate for the PDE model. In fact the PDE model indicates that costs were declining in Japan at a rate exceeding 5% per year; the PSE suggests a figure for Japan closer to 3%. The results for the other countries are less dramatic but the pattern remains. Only for the US do the three models give basically the same result for the rate of cost diminuation.

Finally note that capital and fuel are complements based on all three models. Furthermore, consistent with Fisher and Kaysen's (1962) contention 30 years ago, the same is true for capital and electricity with a few exceptions: Japan and Canada with the FSE model and Japan with the PSE model. These anomalies do not occur with the PDE model. For the PDE model, these cross-price elasticities are all negative, and largely significantly different from zero. One cannot, of course, conclude from this that the PDE model gives better results although fewer discrepancies exist in our case.

In fact on curvature, consistency among countries and closeness of K* to K, the PDE model appears to perform best. On the other hand, there is no reason a priori why there should be consistency among countries nor why K* and K should be similar.

VI. CONCLUSIONS

The issue of properly representing dynamics in models of firms has dominated empirical analysis of factor choice over the past two decades. Several approaches have been offered to improve the representation of dynamics in such models. In this paper we have explored the significance of properly specifying dynamics in a model of the firm. We have shown that very substantial differences in estimates of the structure of costs and demand can arise by making different assumptions about the extent to which the firm is in long-run equilibrium. The three assumptions we contrasted were that the firm is in long-run equilibrium vs. the firm is in temporary equilibrium, just measuring the short-run variable cost function vs. the firm is in temporary equilibrium but some variable costs are capital adjustment costs.

We found that own price elasticities of demand were generally closer to zero for the adjustment cost model (PDE) than the other two models. We also found that the PDE model reported consistently higher rates of technical change (cost diminuation). The PDE model yields the most consistent results across the cross-section. With few exceptions, all three models report negative cross price elasticities of demand for capital relative to the price of energy, suggesting complementarity.

The implication of this work is that the quest for the appropriate specification of dynamics is extremely important. Misspecification of dynamics can lead to dramatic error in estimating the underlying structure of demand and costs.

APPENDIX; DATA

Data coverage for this study is 1960-1989 for Canada, the United States, Japan, France, West Germany and the United Kingdom and is an extension of a data set reported in Kolstad et al. (1986). In large part, data for the model were obtained from OECD sources. Output of the manufacturing sector was obtained from various editions of OECD's National Accounts except for Japan and France, for which national statistical yearbooks were also used.

Gross capital formation from 1955-1989 for France and the UK, and 1950-1989 for Canada, West Germany and the US by category (structures and equipment are considered separately) for the manufacturing sector were obtained from the OECD Flows and Stocks of Fixed Capital Stock (1991). For Japan 1946-1989, this was obtained from Sawa and Mori's (1989) database for 1929-1987 and was extrapolated to 1989 using the Japan Statistical Yearbook (1991). Earlier investment data for West Germany (1950-1959) and Canada (1950-1955) were taken from Berndt and Hesse (1986) and that of the US (1950-1955) was obtained from National Income Production Account. The net capital stock was calculated by a perpetual inventory method using depreciation rates from Berndt and Hesse (1986). Although stock figures were available for most OECD countries, this perpetual inventory method, assuming similar depreciation rates, was chosen for consistency.

Base year capital stock values (necessary for a perpetual inventory computation of the stock) were taken from OECD's Flows and Stocks of Fixed Capital except for the cases of Japan where these values were inferred from Ward (1976). The base year was 1955 except for

Germany for which a base of 1959 was used. The capital stock was computed in constant 1975 units of local currency and then converted to 1975 U.S. dollars using the multilateral purchasing power parity (either for nonresidential buildings, producer's durables or capital formation generally) found in Kravis et al. (1982). For Canada (which was not considered in the Kravis et al. study), conversion rates from the database used by Pindyck (1979) and discussed in Carson (1978) was used.

Christiansen and Jorgenson (1969) have presented formulae for computing the price of capital services, based on tax rates, asset prices, depreciation rates and the cost of money. Two factors complicate the application of their method to our international cross-section. One is tax policy. Tax policy in the U.S. and many industrial countries tends to decrease the capital service price. As Berndt and Wood (1981) have shown, ignoring tax policy can significantly distort results of a study such as ours. However, tax information is simply not readily available on all of the countries we consider; thus, we are forced to ignore it.

A second complicating factor is capital gains. In a study of the Netherlands, Magnus (1979) argues that ignoring capital gains gives a more realistic and less volatile estimate of the service price. Ignoring capital gains serves to decrease the computed services price. Since ignoring taxes has the opposite effect from ignoring capital gains, one could argue that ignoring both is better than only ignoring one. Griffin (1979) and Berndt and Hesse (1986) take this approach. Pindyck (1979) chooses to ignore taxes only. We follow Pindyck (1979), using the following formula for the real service price:

$$P(t) = 1 + r(t) - [1-\delta(t)]\left(\frac{q(t)}{q(t-1)}\right) \qquad \text{(A-1)}$$

where r(t) is the long-term government bond yield (IMF, 1991), q(t) is the asset price index and δ(t) is the depreciation rate. Unfortunately, our approach to computing capital service prices can lead to negative values in periods of high inflation due to interest rates being significantly less than the rate of asset appreciation. We compute separate service prices for manufacturing structures and equipment for all countries and then aggregate prices based on relative stock of these two types of capital. In cases where the aggregate price of capital was less than 10^{-3}, we truncated it to 10^{-3} to avoid numerical problems taking logarithms of negative numbers. Depreciation rates were as used in the computation of capital stock and the gross fixed capital formation price index was used for the asset price index.

The quantity of labor in hours per year per country is the product of average hours worked times the number of employees in manufacturing. Average hours worked was obtained from the International Labor Office's Yearbook. Number of employees was taken from the OECD's Labor Force Statistics. Total compensation of employees was taken from the OECD's National Accounts or the U.N. National Accounts. The wage rate is the quotient of the total compensation and hours worked per year. Wages were converted to constant 1975 units of local currency and then converted to 1975 U.S. dollars using the Kravis et al. (1982) multilateral purchasing power parties for GDP or, for Canada, the equivalent conversion rate from Carson (1978).

Energy consumption was taken from the International Energy Agency's Energy Balances and Energy Statistics. The figure used was the total industrial consumption of energy. Energy prices through 1977 were taken from the U.S. Department of Energy (1981). Prices from 1978 were taken from the IEA's periodical "Energy Prices and Taxes." Aggregate oil prices were computed by averaging the light fuel oil price and the heavy fuel oil price, based on the consumption quantities in the IEA Energy Statistics. Converting the units for prices to compatible units for quantities was straightforward except for the case of coal where the heating value for "brown coal" in the IEA Energy Balances was used, reflecting the supposition that lower grade coal would be used mostly for electricity generation. Energy prices were converted to constant 1975 units of local currency using the GDP price deflator and then converted to 1975 U.S. dollars using the multilateral GDP purchasing power parties of Kravis et al. (1982) or Carson (1978). Finally, the aggregate energy price was computed by estimating the share equations for a translog price aggregator function using the 1975 U.S. prices and quantities as a base. This is the approach suggested by Pindyck (1979). Coefficients did not vary by country. Estimation of the oil and gas share equations was by an interative Zellner approach as implemented in TSP 4.2.

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Figure 1: Short-run and Long-run Costs

Table I: Coefficient Estimates

Parameter		-----PSE Model---- Value	Std Err	------FSE Model------ Value	Std err	-----PDE Model------- Value	Std err
α	CA	-1.40178	0.071421	0.505942	0.160211	-1.3993	0.058384
βL	CA	0.865643	0.00918624	0.392366	0.049841	0.863875	0.008857
βF	CA	0.160295	0.011286	0.059868	0.009105	0.162178	0.011202
βK	CA	-0.555033	0.038662	0.591566	0.055645	-0.15975	0.055711
βD	CA					0.337316	0.385413
βT	CA	-0.034839	0.00535506	-0.01729	0.007247	-0.035	0.004292
α	US	-1.58205	0.054708	0.424452	0.18711	-1.44507	0.045591
βL	US	0.844184	0.01251	0.415077	0.05007	0.847675	0.017589
βF	US	0.194439	0.01468	0.066577	0.009761	0.194473	0.01557
βK	US	-0.317061	0.063872	0.584478	0.056068	0.356661	0.143712
βD	US					-0.52709	0.521314
βT	US	-0.043185	0.00458497	-0.02407	0.00666	-0.04015	0.002928
α	JA	-0.820858	0.061351	1.41813	0.161477	-0.61672	0.067033
βL	JA	0.95017	0.011121	0.337664	0.039959	0.943116	0.012548
βF	JA	0.114604	0.012602	0.025879	0.008675	0.118374	0.013899
βK	JA	-1.20292	0.034488	0.710403	0.043635	-0.27422	0.071505
βD	JA					0.246891	0.33295
βT	JA	-0.067282	0.00726628	-0.0369	0.008215	-0.07681	0.005959
α	FR	-1.12618	0.051988	0.705683	0.170259	-0.99322	0.036619
βL	FR	0.942404	0.00967226	0.465319	0.04547	0.940527	0.010413
βF	FR	0.13503	0.012049	0.04023	0.008901	0.137026	0.012016
βK	FR	-0.506429	0.048881	0.573657	0.049587	0.052733	0.097185
βD	FR					-0.12182	0.447628
βT	FR	-0.052339	0.00481756	-0.03442	0.007375	-0.05567	0.003445
α	WG	-1.25621	0.052628	0.712717	0.170285	-1.13615	0.044215
βL	WG	0.978856	0.01074	0.445723	0.04664	0.975739	0.011914
βF	WG	0.11021	0.013341	0.025052	0.009168	0.112315	0.012945
βK	WG	-0.657078	0.038747	0.613096	0.052131	-0.10639	0.080557
βD	WG					0.239688	0.363753
βT	WG	-0.042498	0.00547298	-0.0216	0.007064	-0.04603	0.003204
α	UK	-0.446702	0.075552	0.863447	0.166386	-0.50726	0.041278
βL	UK	1.04107	0.00908819	0.505272	0.045619	1.03149	0.0148
βF	UK	0.0875	0.011735	0.029621	0.009282	0.091695	0.013781
βK	UK	-0.756106	0.045757	0.553803	0.04908	-0.74905	0.133332
βD	UK					1.5516	0.373117
βT	UK	-0.047593	0.00600311	-0.02149	0.007554	-0.04678	0.003233
γLL		0.093398	0.0047474	0.087191	0.01176	0.091272	0.006129
γLF		-0.049634	0.00361752	-0.01952	0.002634	-0.04943	0.004228
γLK		-0.114488	0.017308	-0.05234	0.013642	-0.09816	0.028222
γLD						0.073338	0.030893
γLT		-0.00291688	0.00029529	-0.00274	0.000803	-0.00281	0.000236
γFF		0.055105	0.00413098	0.036145	0.003	0.055663	0.004264
γFK		0.056955	0.013741	-0.01347	0.002824	0.050316	0.020416
γFD						-0.04001	0.022238
γFT		0.000940266	0.00021178	0.000164	0.000173	0.000892	0.000193
γKK		0.481929	0.10154	0.075788	0.01441	1.22752	0.313989
γKD						-2.54609	0.658205
γKT		0.00441033	0.00117778	0.001757	0.000918	0.003456	0.001876
γDD						-0.16137	0.235492
γDT						-0.01433	0.014826
γTT		0.000313527	0.00033749	0.000112	0.000342	0.000478	0.000201

Note: L=labor; F=fuel; K=capital; D=ΔK; T=time.
Estimate of Costs, shares and Euler equations using FIML (refer to eqn 17).
Annual observations, 1960-89.
CA=Canada; US=United States; JA=Japan; FR=France; WG=West Germany; UK=United Kingdom

Table II: Selected Price Elasticities, Rates of Cost Diminuation and K*/K

	FSE Model		PSE Model		PDE Model	
CANADA	Value	t-stat	Value	t-stat	Value	t-stat
∂ln C/∂t	-0.021194	-4.69167	-0.02051	-6.95221	-0.02252	-10.2298
ηLL	-0.288364	-12.4254	-0.27719	-6.39286	-0.0888	-5.97401
ηFF	-0.108764	-1.42749	-0.30148	-5.56518	-0.11682	-1.65395
ηEE	-0.289109	-5.03556	-0.36842	-8.10865	-0.22729	-3.80031
ηKF	0.00523759	0.636572	-0.02517	-1.81491	-0.02467	-1.8863
ηKE	0.0149	10.1041	-0.02568	-3.19173	-0.02301	-1.94061
ηKK	-0.429282	-10.0693	-0.37139	-14.9996	-0.14	-4.62976
K*/K			1.47352		1.00169	
US						
∂ln C/∂t	-0.028549	-7.84804	-0.02974	-8.62617	-0.0296	-14.883
ηLL	-0.25412	-10.0463	-0.21146	-4.4532	-0.04086	-3.77333
ηFF	-0.072843	-0.79829	-0.30513	-5.22793	-0.04533	-0.52851
ηEE	0.00467999	0.055844	-0.26585	-3.9122	0.122872	1.09911
ηKF	-0.00011078	-0.0128	-0.04203	-2.41331	-0.03056	-2.31735
ηKE	-0.00113115	-0.53324	-0.04822	-4.22093	-0.03046	-2.48393
ηKK	-0.44055	-10.0677	-0.32081	-13.6579	-0.09722	-4.35018
K*/K			1.62241		1.00107	
JAPAN						
∂ln C/∂t	-0.038357	-9.00043	-0.03129	-16.3795	-0.05249	-13.948
ηLL	-0.368649	-11.5919	-0.46466	-12.151	-0.12901	-5.61852
ηFF	0.159703	1.23819	-0.35044	-7.09245	-0.1047	-1.72807
ηEE	-0.39136	-8.16987	-0.51343	-19.9255	-0.43362	-12.0691
ηKF	0.00455169	0.79327	0.010782	1.77796	-0.02261	-1.90485
ηKE	0.030658	38.9479	0.026136	5.72452	-0.01521	-1.61225
ηKK	-0.355602	-11.6878	-0.4174	-19.3814	-0.15707	-5.00498
K*/K			1.89961		1.03260	
FRANCE						
∂ln C/∂t	-0.037313	-8.68007	-0.03331	-14.1326	-0.03941	-29.2356
ηLL	-0.257807	-12.7888	-0.22994	-5.44952	-0.04302	-2.07518
ηFF	0.0612	0.497678	-0.2216	-3.16566	0.052089	0.528359
ηEE	0.381202	3.26203	-0.08874	-0.8509	0.471573	2.01763
ηKF	-0.00346461	-0.39502	-0.03313	-2.25805	-0.02744	-2.18001
ηKE	-0.00775744	-5.43657	-0.04151	-5.15904	-0.02863	-2.45691
ηKK	-0.434226	-10.203	-0.36163	-13.5422	-0.13095	-4.56627
K*/K			1.51896		1.00907	
WEST GERMANY						
∂ln C/∂t	-0.024093	-6.58091	-0.02385	-9.48578	-0.02998	-16.699
ηLL	-0.281203	-12.1594	-0.26146	-5.66911	-0.05113	-2.5114
ηFF	0.26803	1.73408	-0.18557	-2.23742	0.183686	1.40221
ηEE	0.023488	0.274812	-0.26317	-4.44107	0.046587	0.454865
ηKF	-0.0064515	-0.86739	-0.02881	-2.26018	-0.02807	-2.23094
ηKE	0.00220041	1.57496	-0.02953	-3.81992	-0.0265	-2.4344
ηKK	-0.421532	-10.3608	-0.37798	-20.0181	-0.13326	-4.76222
K*/K			1.60330		1.02679	
UK						
∂ln C/∂t	-0.0239	-5.98673	-0.02779	-9.45062	-0.02859	-20.219
ηLL	-0.23821	-11.3985	-0.21322	-5.24631	-0.06241	-2.58736
ηFF	-0.047293	-0.4935	-0.22536	-3.19855	-0.02285	-0.27878
ηEE	-0.086146	-1.13747	-0.21689	-3.2072	0.012589	0.135814
ηKF	-0.00432852	-0.46684	-0.04066	-2.59986	-0.02659	-2.15359
ηKE	-0.00026326	-0.2072	-0.04495	-5.02805	-0.02631	-2.44627
ηKK	-0.446205	-9.49827	-0.33812	-10.7718	-0.13278	-4.6979
K*/K			1.42819		1.07502	

Note: t-statistics are estimated parameter values divided by standard errors. Critical values are 1.6 (95% confidence) and 2.3 (99% confidence). Price elasticities (η) are long-run elasticities of demand of first argument with respect to price of second argument. K*/K standard errors are not available due to the numerical methods employed to compute K*.