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DYNAMOS, ALTERNATORS, AND TRANSFORMERS.

DYNAMOS, ALTERNATORS,

AND

TRANSFORMERS.

BY

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PREFACE.

In the present work it has been my object to place before the reader an exposition of the general principles underlying the construction of dynamo-electric apparatus, and to do this without the use of high mathematics and complicated methods of investigation. To avoid mathematics altogether in a book treating of any engineering subject is, of course, impossible; but I have endeavoured to restrict the use of mathematics within such limits as will enable the average engineering student and the average practical engineer, even if he have no previous knowledge of electrical science, to follow the subject.

GISBERT KAPP.

31, Parliament Street, Westminster, 1893.

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DYNAMOS, ALTERNATORS, AND TRANSFORMERS.

CHAPTER I.

Definition.—Efficiency of Dynamo-Electric Apparatus.—Measurement of Electric Energy.—Principal Parts of Dynamo.—Distinction between Dynamo and Alternator. Use and Power of these Machines.

In its broadest sense the term dynamo-electric machine denotes an apparatus in which by the agency of electromagnetic induction mechanical energy of rotation is converted into the energy of an electric current, or inversely the energy of such currents is converted into mechanical energy of rotation. This definition holds good whether the current given out by the machine, when driven by power from a prime mover, is always flowing in the same direction or is alternately flowing in opposite directions; it also holds good for a machine which is driven by a current supplied to it from some external source, whether the current is always flowing in the same direction or whether the direction of flow is periodically reversed. The qualification that the mechanical energy forming the starting or finishing point of the process must be energy of rotation is of importance in order to exclude a class of apparatus which have this in common with dynamo-electric machines, that their action is based on electromagnetic induction. Thus any ordinary electric bell, a Morse telegraph apparatus, or the Timmis-Forbes electric railway brake, are all instruments in which the energy of electric currents is transformed into mechanical energy, but they are obviously not dynamo-electric machines. On the other hand, the Wimshurst electric machine is also excluded by the above definition, since in it the agency by which mechanical energy of rotation is converted into electric currents is not electromagnetic, but electrostatic induction. The limits within which the term dynamo-electric machine is applicable are even with these restrictions still inconveniently wide, and for practical purposes a subdivision is necessary. The basis for this subdivision is twofold: first, whether the conversion is from mechanical into electric energy or the opposite; secondly, whether the currents are direct—that is, flowing always in the same direction—or alternating, that is, flowing alternately in opposite directions. We obtain thus four classes of machines. These are:

1. **The dynamo,** in which mechanical energy of rotation is converted into the energy of a direct current.
2. **The alternator,** in which mechanical energy of rotation is converted into the energy of an alternating current.
3. **The motor,** in which the energy of a direct current is converted into mechanical energy of rotation.
4. **The alternate current** motor, in which the energy of one or more alternating currents is converted into mechanical energy of rotation.

Thus, either of these four types of apparatus has for its object the conversion of energy from one form into another form, and it is self-evident that the commercial value of the apparatus must depend, to a certain extent, on the efficiency of conversion—that is, the ratio between the amount of energy supplied to the machine in one form and the amount obtained from it in the other form. The smaller the loss incurred in the process of conversion the better is the machine. That some loss must take place in dynamo-electric machines may be expected from analogy with other mechanical appliances, for there never has been any machine devised which works without loss, but in the class of apparatus we are now considering, the loss is smaller than in most other mechanical appliances. Thus it is by no means difficult to build dynamos which shall have an efficiency of 90 per cent., whereas the best centrifugal pumps scarcely reach 70 per cent. efficiency, and the best turbines 85 per cent., whilst in steam engines an efficiency of 75 per cent. is exceptionally high. If we except such simple mechanical devices as spur or belt gearing and the like appliances which serve for the transmission, as distinguished from the transformation of energy, then the dynamo-electric machine is unquestionably the most efficient apparatus at present known in mechanics.

In this connection the question naturally arises how the efficiency of a dynamo or motor is to be determined. The efficiency is the ratio of two energies, that supplied, and that obtained from the machine. With one of the forms in which energy enters into the process every engineer is familiar, and there is no special difficulty in

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accurately measuring it. If the dynamo is driven by a steam engine we can take full load and friction diagrams, and thus ascertain, with a fair amount of accuracy, what power is actually supplied to the dynamo, or better still, we can measure the power by some form of transmission dynamometer, and thus eliminate any slight error which might be due to the difference in engine friction when running light and loaded. Such measurements are perfectly familiar to mechanical engineers, but when we come to the electrical measurements required at the other end of the process we enter upon new ground. The interdependence of magneto-electric and purely mechanical forces will be considered in chapter IV., but for our present purpose it will suffice if we consider merely one method of measuring electric energy. If a current be sent through a wire we observe that the wire becomes heated. The heat developed is due to the energy given off by the current in overcoming the resistance of the wire, and since the principle of the conservation of energy must hold good in electrical as well as in purely mechanical or thermo-dynamic processes, we conclude that the heat given off by the wire is an exact measure of the electric energy given off by the current. The heat developed per second can be measured in a calorimeter, and its mechanical equivalent in foot-pounds, kilogramme-metres, or horse-power ascertained, and the energy which we thus obtain must obviously be that given out by the current in the conductor. If at the same time we measure the current and the potential difference between the ends of the conductor, we find that with a direct and steady current the product of the two readings is proportional to the number of calories liberated per unit-time. We may therefore substitute for the somewhat cumbersome and difficult calorimetric method of measurement the far simpler electrical method, and say that the energy developed by a direct, steady current in a conductor is measured by the product of strength of current and difference of potential between the ends of the conductor. Thus the energy absorbed by a glow lamp would be found by multiplying the voltage at which the lamp is run by the current passing through it. In order that the measurement may be accurate, it is necessary that the conductor shall not be under any other electrodynamic influence. It must therefore not be moved in the neighbourhood of a magnet, nor must a magnet be moved near it. The reason is that such relative movement between a magnet and a conductor tends to set up in the latter a current which may either assist or retard the original current of which the energy is to be measured, and the measurement would therefore be erroneous by the amount of energy expended in or obtained by the movement. For the same reason the measurement of the energy of an alternating current cannot always be made in the simple manner above explained for direct currents. Under certain circumstances an alternating current acts upon its own conductor somewhat in the same way as a magnet in motion, and in such cases the product of voltage and current strength is greater than the true energy transformed into heat. To obtain the true energy when the current is alternating and the conductor has the property of reacting upon itself, a property commonly designated by the term "self-induction," certain accessory measurements must be made, but it is not necessary to enter into this matter at present, since it will be dealt with at some length in a subsequent chapter. Suffice it to say that for steady direct currents the product of potential difference and current strength is a true measure of the energy given off. Potential difference or electric pressure is measured in volts, and current strength is measured in amperes, the product being volt-amperes, or watts. The interdependence of the watt and the other units of energy will be explained later on, for the present we need only note the following relations:

One foot-pound per second = 1·36 watts.
One kilogramme-metre ,, = 9·81 watts.
One English horse-power = 746 watts.
One metric horse-power = 736 watts.

By the aid of these equivalents we can therefore express the power of a dynamo in any convenient system of mechanical units. We measure the electric energy in watts and convert the latter into horse-power if the energy supplied to the machine is given in that measure. For instance, let the current from a dynamo be measured by an ampere-meter placed in circuit. Let a voltmeter be connected close to the dynamo across the main wires, through which current is being supplied to a number of lamps, each lamp being also connected across the mains. Then by taking simultaneous readings on the two instruments we can determine what energy is being used in the mains and lamps. The arrangement is diagrammatically shown in Fig. 1, where D represents the dynamo, which is connected by the conductors, B_1 B_2, technically termed "brushes," with the main wires, M_1 M_2. The main M_2 is interrupted and an ampere-meter, A, inserted into the gap, thus forcing the current to flow through the instrument and be thereby registered. A voltmeter, V, is connected by a couple of wires across the mains, and shows the pressure under which the current is being sent into the mains and through the group of lamps, L. Say the ampere-meter registers 140 amperes and the voltmeter 105 volts, then we conclude that the energy

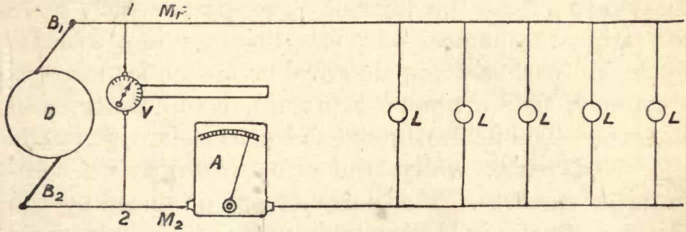

FIG. 1.

consumed in the circuit beyond the points 1 and 2 is $105 \times 140 = 14{,}700$ watts, or 19·72 horse-power. In the same way would be measured the energy supplied to a motor. In this case D would represent the motor receiving current from some source—for instance, a set of batteries, which would take the place of the lamps, L; and the energy which is being supplied to the motor in the form of electric current, flowing under a certain potential difference or pressure, can be computed from the indications of the two instruments A and V.

We have here shown how the electric energy given off by a dynamo or supplied to a motor can be measured. To make such a measurement no knowledge of the construction of the machine is required, the experimenter having simply to read the indications of two instruments and make a very simple calculation. We may, however, now enter upon the construction of these machines in a general way and give an account of their principal parts. For the sake of brevity, the description will be given with reference to dynamos, leaving it to be understood by the reader that the different parts are substantially the same in motors. Leaving aside for the present those parts which serve for purely mechanical purposes, there are in a dynamo four main parts serving electrical or magnetic purposes —namely, field-magnets, armature, commutator, and brushes. The field-magnets and brushes are generally the fixed parts, whilst the armature with its commutator revolves. The currents are produced by the electromagnetic induction taking place in certain wires which are moved in front of magnet poles. These wires form part of the armature and are so interconnected that the single current impulses are added. They are also connected with the commutator upon which the brushes rub, and by this means the current is allowed to flow out of and return to the armature. The object and function of each of the four principal parts is, therefore, as follows: The field-magnets produce the poles, which form the starting point of the process. In front of these poles or between them revolves the armature carrying wires in which currents are generated. These currents are grouped and directed by the commutator. The brushes finally have the office of establishing suitable connections between the fixed terminal points of the external circuit and the revolving commutator. The above description will be made clearer by reference to the drawing of a dynamo, and for this purpose we select the "Victoria" machine represented in Fig. 2.

This dynamo belongs to a class of machines usually comprised under the title "disc machines," on account of the fact that the armature is shaped somewhat like a disc or cylindrical ring of comparatively large diameter and small axial length. In the side elevation, Fig. 2, the armature is shown in section, A representing the core and W the winding. The core is composed of thin sheet iron strip, wound upon a stouter iron ring, R, which in its turn is supported by arms and hub, H, on the shaft. The torque is transmitted from the shaft to the wheel, H, by a key in the usual way, and from the wheel to the core, A, by the flat arms which are arranged in halves and drawn together by screw bolts, thus pressing the core between them. For certain reasons, which need at present not be detailed, the core is formed not of one wide iron strip, but of several narrow strips wound side by side. When completed, the core is insulated over its whole surface, and the armature conductor, consisting of cotton covered wire, is wound on it, the convolutions passing round the outside and through the inside between the flat arms. The armature winding forms a closed spiral which, for convenience of manufacture, is subdivided into sections, and the points of juncture between neighbouring sections are connected with neighbouring plates in the commutator C. These plates are insulated from each

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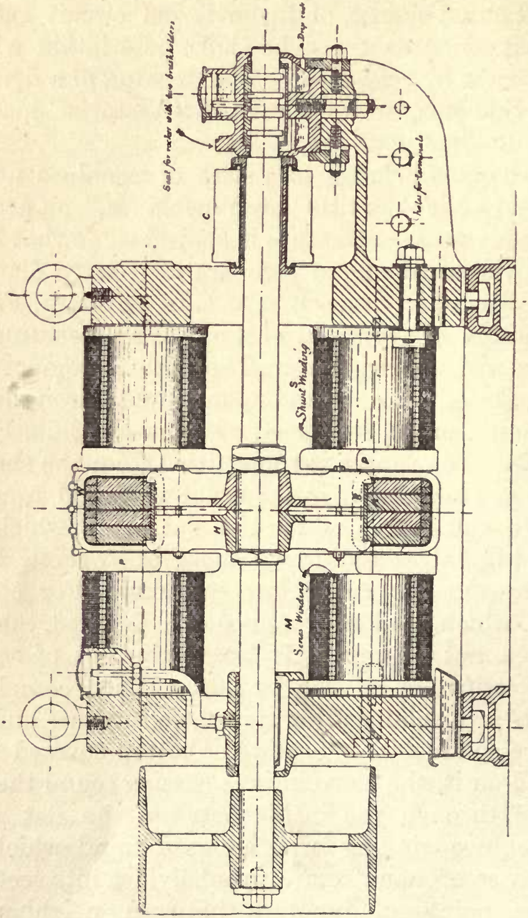

FIG. 2.—Mordey-Victoria Dynamo. Side Elevation.

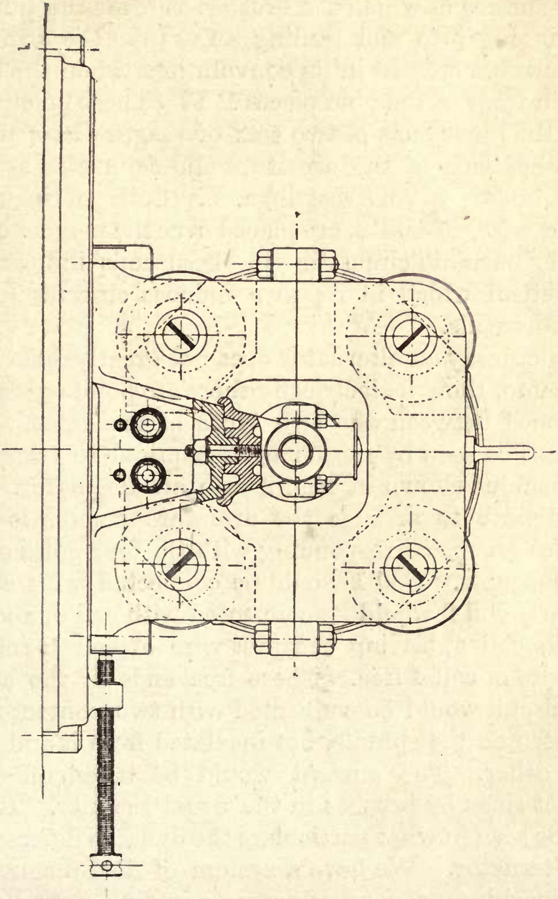

Fig. 2.—Mordey Victoria-Dynamo. End View.

other, and form together a cylindrical body upon the outer surface of which the brushes rub for the purpose of bringing into and leading away from the armature the currents created in its convolutions when the latter pass in front of the pole pieces P P. These pole pieces form the inner ends of two sets of magnet bars placed on either side of the armature and connected at their outer ends by yoke castings Y. Coils of insulated copper wire, M and S, are placed over the magnet cores, and the currents circulating in these coils induces the magnetism which in its turn induces currents in the armature coils.

Structurally an alternator does not greatly differ from a dynamo, though electrically there are points of radical difference between the two types of machines. This can best be seen by the changes required to transform the machine shown in Fig. 2 into an alternator. We would have to arrange the armature coils into four distinct groups corresponding with the four poles of the field-magnets. Coil 1 would be connected with coil 2, similarly coil 2 would be connected with coil 3, and coil 3 with coil 4, leaving the first wire of coil 1 and the last wire of coil 4 free. These free ends of the armature circuit would be connected with two contact rings mounted on the spindle, but insulated from it and from each other. The current would be taken off these contact rings by brushes in the usual manner. It will now be seen in what particulars the dynamo differs from the alternator. We have a system of field-magnets in both machines, also an armature containing wire coils, but whereas in the dynamo these coils are numerous and each contains only a few turns, and in many cases only one turn of wire, in the alternator the number of coils is only that of the field-magnet poles, but each coil contains a large number of turns of wire. In the dynamo the coils are continuously connected with each other and with the sections of the commutator. In the alternator there is no commutator, but merely a pair of contact rings forming the terminals of the armature circuit. In the dynamo one coil after another comes, so to speak, gradually into action and as gradually out of action, whereas in the alternator all the coils come simultaneously and more abruptly into and out of action. There are other differences between the two types of machines, but we must leave the consideration of these to future chapters, where we shall deal with the theory and practice of these machines more in detail. For the present it will suffice if we point out the various purposes for which dynamos and alternators are used. As regards the former they are used for electric lighting, electro-chemical work, thermo-electric work, and electric transmission of energy. Alternators are as yet principally used for lighting purposes, though the transmission of energy can also be affected by their agency. For electro-chemical work they have hitherto not been used, and as regards thermo-electric work, such as the manufacture of aluminium alloys, there is no reason to doubt that alternating currents could be successfully employed, though the bulk of the work has hitherto been done by direct currents.

A point of considerable practical interest is that concerning the power of dynamos and alternators. It is scarcely ten years since engineers in this and other countries began to turn their attention to these machines, and within that time there has been a steady growth in the size and power of them. Prior to the Paris Electrical Exhibition of 1881 there were but very few manufacturers of dynamo-electric machines, and the apparatus turned out by these few firms was rather of the kind suited for the laboratory than for the workshop. The machines were of small power and imperfect design, both electrically and mechanically; they were, in fact, made not by engineers, but by electricians accustomed to manufacture all kinds of small electrical apparatus, and who evidently did not in those days realise the magnitude of the mechanical forces with which they might have to deal in a properly designed dynamo. Yet the dynamo as a laboratory toy was then already an old invention. Shortly after Faraday had, in 1831, announced his great discovery of electromagnetic induction, Pixii produced magneto-electric machines for alternating and for direct currents. He was followed by a large number of physicists and scientific instrument makers, whose improvements in matters of detail form an almost continuous record up to the year 1864, when Pacinotti made his great invention of the closed armature circuit and commutator as now used; Gramme in 1870 re-invented the same thing, and being a skilled workman was able to at once give practical shape to his invention in the dynamo which bears his name, and which has become the prototype of all dynamos with closed coil armatures. The Gramme machine was shown in 1873 at the Vienna Exhibition, where there were also on view alternators for use in lighthouses, but these machines attracted the attention of engineers to only a limited extent. It was after the invention of the glow lamp, and after the Paris Exhibitions of 1878 and 1881, when the engineering profession began to realise that there was an enormous field for the commercial application of apparatus which had until then only been used for scientific purposes in the laboratory, or practically used in isolated instances, that engineers took up the manufacture of dynamos as a regular trade. The power of machines was at first very small, since they were generally required for isolated private electric light installations, which were naturally of a limited character. Gradually, however, when confidence in the electric light brought about its adoption in mills, factories, and other large establishments, the size of machines became larger, and this tendency has been further strengthened by the establishment of central electric light stations. The power of modern dynamos and alternators is reckoned by hundreds of horse-power, and in some cases by thousands. Thus at the Deptford station Mr. Ferranti built alternators designed to give out 1,500 h.p. of electric energy, and still larger machines intended for an output of 10,000 h.p. were projected. In some of the Berlin central stations there are in use dynamos of 500 h.p., and it seems highly probable that for central stations generally machines of even larger power will in future be required. The power of machines required for other than lighting purposes is also constantly on the increase. As an instance, we may take the application of the dynamo to the transmission of energy. The transmission plant of 50 h.p., between Kriegstetten and Solothurn, Switzerland, which was erected some years ago, was then considered an undertaking of considerable magnitude, but now there has been erected at Schaffhausen a plant of 600 h.p., whilst in America there are many generating stations for the supply of current to electric tramways and railways, the capacity of which is reckoned by many hundreds of horse-power.

Similarly, we find that the machines required for electro-chemical and thermo-electric work are of considerable size. When storage batteries are used in central station lighting, the charging dynamos are seldom of less than 200 h.p. output, while large machines are also required in the manufacture of storage cells, the purification of copper and other processes. As regards thermo-electric work, it is interesting to note that the current required in the electric furnace represents a very considerable energy. Thus, at the Cowles Aluminium Works, at Milton, the energy necessary to work the furnace is about 400 h.p., whilst that required for the Heroult Furnace, at the Neuhausen Works, is between 300 and 400 h.p. We see, therefore, that in all branches of heavy electrical engineering the tendency of the present time is in the direction of large and powerful machines, such as can only be produced in engineering shops fitted with large tools and modern appliances. We also see that to turn out apparatus of such magnitude by the use of haphazard or rule of thumb methods of construction is out of the question.

To ensure success the electrical engineer of our day must thoroughly understand the scientific principles involved in dynamo-electric machines, and must be careful to employ only the best materials and workmanship.

CHAPTER II.

Scope of Theory.—The Magnetic Field.—Strength of Field.—Units of Measurement.—Physical and Mathematical Magnets.—Field of a Mathematical Pole.

General Scope of the Theory of Dynamo-Electric Machines.

The working of dynamos, alternators, and transformers is based on electromagnetic induction, and before we attempt to establish a working theory of any such machine it is necessary to investigate the general principles on which the theory must be based. We must for this purpose consider the interaction of magnets and electric currents, the general character and magnitude of the mechanical forces resulting from or causing such interaction, and the general relations existing between mechanical and electromagnetic forces and energies. It would be beyond the scope of the present book, which is primarily intended for practical engineers, to give a complete theory of electrodynamics. For this the reader must go to the works of Clerk Maxwell, Mascart and Joubert, Lord Kelvin, Lord Rayleigh, Oliver Lodge, Oliver Heaviside, Wiedemann, Wüllner, and others. No single author has treated the subject completely, but by referring to the writings of several, the reader may be able to collect what may be called a tolerably complete theory of the subject. This would of course involve a considerable amount of labour, and would require a degree of mathematical skill not usually possessed by practical engineers. It is, however, fortunately not necessary to establish or understand a theory in all its minute details in order to apply it to a useful purpose. Excellent steam engines have been and are being built by men who have never read a word of the writings of Carnot, Clapeyron, Clausius, Joule, or Helmholtz, but who have nevertheless thoroughly grasped the chief thermo-dynamic principles, and have understood how to apply them. It is the same with electro-dynamics. Very few, if any, of the successful designers of dynamos have found it necessary to first master the writings of Maxwell before attempting the construction of machines. They nevertheless profited by the thoughts of Maxwell and similar men, but only after the information contained in their books had filtered down to them through the medium of more popular expounders of the theory aided by practical experiment. In attempting to establish a working theory of dynamo-electric machinery, or rather in setting forth the rules and formulæ now used by the designers of such machines, we shall therefore not follow the lead of the pioneers in science so much as that of their more popular expounders and that of practical experience. The treatment will thus necessarily lack that mathematical elegance of which the scholastic mind is so fond, but on the other hand it will be more easily grasped and adopted by the practical engineer who works as much by the aid of his mechanical instinct as by that of science.

The Magnetic Field.

If we lay a straight bar magnet upon the table and explore the space surrounding it by means of a compass

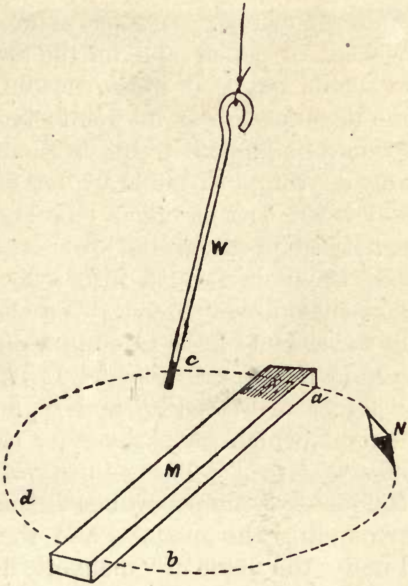

FIG. 3.

needle, N, Fig. 3, we find that the needle takes up at every point in the vicinity of the magnet a perfectly definite position. In the diagram the north pole of the bar (that is, the end which, if the bar were freely suspended, would point to the geographical north) is shaded, as is also the north pole of the needle. The direction at which the needle sets itself in any point is such as to make its south pole point more or less directly to the north pole of the bar, whilst its north pole points more or less directly to the south pole of the bar, the exact position being, so to speak, the best compromise the needle can make in order to satisfy the different attractions and repulsions. Let us assume the bar placed on a sheet of paper, and on the paper a line, *a b*, drawn in such a way that if the needle be shifted along that line its axis shall at all points be tangential to it. As it would be difficult to hit off the correct line without having anything to guide us, let us resort to the following device. Let us take a long and thin steel wire, W, magnetised north at its lower and south at its upper end, which is formed into a loop for convenience of suspension by a thread. We shall now be able to let the lower end of the wire mark on the paper curves of the kind above mentioned. It is as though the magnetic forces of attraction and repulsion acted along these curves, which have therefore received the name of "lines of force." These lines will be found not only in the plane of the paper, but in the whole of the space surrounding the magnet, and their entirety is comprised under the name of "magnetic field." We therefore define the magnetic field as a space within which can be traced magnetic lines of force. The magnetic field of a steel magnet has an inner boundary formed by the surface of the magnet, but it has no definite outer boundary. The action on our exploring wire, W, becomes weaker and weaker as we go away from the magnet, but there is no definite limit within which lines of force could not be detected provided the exploring apparatus were delicate enough. Another method of rendering the lines of force visible consists in placing a sheet of paper over the magnet and sprinkling iron filings upon it. We then see that the filings

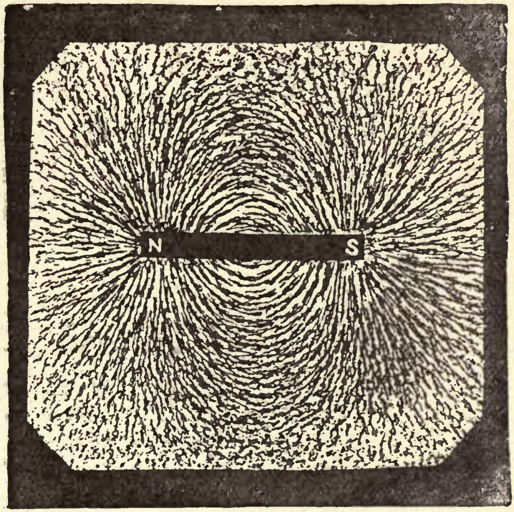

FIG. 4.

arrange themselves as in Fig. 4. They are densest near the poles and become more sparse at a distance from them. We find accordingly that the lower end of our exploring wire is strongly repelled when close to the north pole, and equally strongly attracted when close to the south pole of the magnet, whilst the forces acting on it in intermediate positions are smaller. The

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end of the wire in moving along the line, *c d*, is doing mechanical work, and the amount of work performed by a unit pole in the transition from one point of the curve to the other represents the difference of magnetic potential between these two points. It is useful to note that the amount of work is independent of the path traversed by the exploring pole. If the latter be constrained to move in any circuitous route, work may be

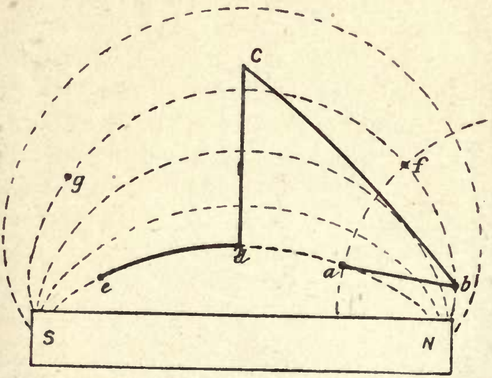

Fig. 5.

absorbed and given off at different parts of its journey; but if we deduct the work absorbed from that given off, we find that the difference represents exactly the work which would have been given off had we permitted the exploring pole to travel along the line of force on which it was originally set. Thus let, in Fig. 5, the exploring pole be set down on the point *a* in the line of force, N, *a*, *d*, *e*, S, and instead of allowing it to move along this line constrain it to move towards *b*, then to *c*, *d*, and finally to *e*. The journey *a b* will be performed against the repulsion of N upon the exploring pole, and work will therefore have to be done upon the latter. The journey *b c*, on the other hand, will yield work, for the point moves more or less in the direction in which it is impelled by the magnet. During the journey *c d* work is neither done upon the exploring pole nor given off by it, since it moves at right angles to all the lines of force which it passes. That this must be so can easily be seen from the consideration that work is the product of two factors—namely, force and movement in the direction or exactly opposite to the direction of the force. In other words, in calculating the work we must only take account of that component of the motion which coincides at any moment with the direction of the force, and when the motion throughout takes place at right angles to all the forces, then the component which forms one factor in the work is zero, and therefore the whole work is zero. If no work is done or obtained in the transition between two points of the field, these two points must obviously have the same magnetic potential—that is, must be equipotential points. We can imagine an infinite number of such points forming a surface which cuts all the lines of force at right angles, and an exploring pole moved in any way along such a surface will neither absorb nor give off work. Such a surface cutting all the lines of the field at right angles is called an equipotential surface.

Thus, in a dynamo, the surface of the pole-piece or that of the armature core as far as it lies within the pole-piece are equipotential surfaces. We can move an exploring pole along either surface or along any intermediate surface equidistant from the polar or armature surface without receiving or doing work. If, however, we move the exploring pole from one surface to the other, we must either do work or obtain work. The space between the poles and armature core of a dynamo, even before the core is wound, is so confined that the experiment here described cannot easily be performed, to say nothing of the additional difficulty that in so intense a field the polarity of our exploring magnet would easily be reversed. In a somewhat more imperfect or rough and ready manner the experiment can, however, on any dynamo be very easily performed as follows: Take a key or spanner in your hand, and approach with it the back of one of the pole-pieces. The piece of iron will be strongly attracted to the surface of the pole, and stick out from it self-sustained if allowed to actually come into contact. The piece of iron has become an exploring magnet, the end touching the pole-piece of the machine, assuming the opposite polarity to it, and the end pointing away the same polarity. If we rotate the iron bar about its point of contact, so as to bring its outer end nearer to the surface of the machine pole, we find that energy must be expended, which the bar will give out again if allowed to return into its natural position of sticking out straight from the surface. If, however, we displace the bar parallel to itself, we find that apart from friction there is no resistance to the movement. Either end of the bar is in this case moved along an equipotential surface, and consequently no work is being expended or received.

Returning now to Fig. 5, we have seen that no work is performed in the journey from *c* to *d*. In the journey from *d* to *e* the exploring pole gives off work because moving along a line of force. Now it can easily be shown that the total amount of work given off by the exploring pole during its journey from *a* to *e* is independent of the particular route traversed, and only depends on the difference of magnetic potential between the two points *a* and *e*. We need for this purpose only imagine that a movement taken slantingly across the lines of force consists in a large number of minute steps taken alternately along and at right angles to the lines of force. The steps at right angles do not count as far as the performance of work is concerned, and the final result is the same as if all the steps had been taken along lines of force. Thus, the work done by the exploring pole in moving in a straight line from *a* to *g* is precisely the same as that which would be done if the pole were first to travel along the equipotential line *a f*, and then from *f* to *g* along a line of force. If the exploring pole be of unit strength, this work represents the difference of potential between *a* and *g*.

Strength of Field.

In exploring the field of a magnet, as shown in Fig. 3, we find that the forces acting upon the exploring pole vary with its position. The nearer the point of the wire, W, is to one of the poles or ends of the bar, M, the greater is the attractive force, and the nearer it is to the other end the greater is the repulsive force. Thus, although the exploring pole may travel from the north to the south pole of the bar along the same line of force, the actual amount of the force exerted upon it varies from point to point. If we investigate the variation of magnetic force with reference to Fig. 4, which shows the lines of force as revealed by the natural arrangement of iron filings, we find that the lines are densest in the neighbourhood of the poles, and the less dense the farther away we go from the poles. The density of the lines is, in fact, a measure for the force acting upon the exploring pole in different parts of the field. This relation between density of lines and magnetic force has led to the conclusion that the one is a function of the other; and it has become usual to express the force exerted upon an exploring pole in a given part of a magnetic field as due to a density of so many lines of force per square centimetre of the cross-section taken at right angles to the flow of lines at that part of the field. When we define the strength of field between the poles and armature of a dynamo as 5,000 C.G.S. units, we mean thereby that through each square centimetre of the intervening space there pass 5,000 times as many lines as pass through each square centimetre of a space in which unit force is exerted upon a unit exploring pole. It is, therefore, only necessary to agree upon the units to be adopted, and we are at once able to numerically define the strength of a magnetic field at any point.

In this connection it is, however, necessary to guard against a misconception which might arise from a too narrow or strictly literal interpretation of the lines of force theory. This theory, as far as it applies to magnetism, is due to Faraday, who adopted it as a natural and simple way of accounting for magnetic phenomena, without, however, ascribing to the lines any actual physical existence. With this reservation there is no danger of misconstruing Faraday's conception, but if we look upon lines of force as if each were a physical ntity, having a definite dimension, occupying a definitee position and exerting a definite force, the theory breaks down altogether. To show that this is so we have only to consider what must be the arrangement of lines in, say, a unit field. According to the theory there would in such a field be one line to each square centimetre of sectional area of the field, each line exerting unit mechanical force upon unit pole placed on it. The unit pole being a mathematical point, and the line of force having no other dimension than length, mathematical precision would be required to get the pole exactly on to the line. To one such position of coincidence there is an infinite number of positions in which the two will not coincide if we assume that the position of each line in space is fixed at the centre of its own square. Experiment shows that the magnetic force is exerted upon the pole not only at certain points, but at all points of the field, and to explain this we would have to assume that each line of force, though an entity and limited to its own square, is free to shift within that square in any way necessary to pick up our unit pole. This explanation, clumsy as it is, would suffice to account for the properties of the field if investigated by one unit pole only, but it breaks down if we imagine the investigation made by the aid of two poles less than one centimetre apart, for both poles are equally influenced by the field. The idea that a line of force is a physical entity, pulling at a magnet pole as an elastic thread may pull at a heavy body, is therefore quite untenable, and if we wish to represent a magnetic field in some way by a mechanical model, we must abandon the idea of constructing such a model by means of threads to represent the lines of force.

A more satisfactory, though by no means complete, mechanical representation of the magnetic field is by means of a liquid mass in motion. Imagine the magnet represented by a tube, in the centre of which there is a screw pump, and let the tube be immersed in water whilst the pump is rotated. The water will issue at one end, flow in curved stream-lines and with varying velocity round the tube, and enter it again at the other end. The unit exploring pole we replace by a disc of unit surface, which we place into various positions within the space surrounding the tube, and thus measure the force of the stream at any point. This analogy is imperfect, because the force exerted by the water varies not as the velocity, but as the square of the velocity. Assuming, however, that the former be the case, then such a model can in a somewhat crude fashion be made to represent the magnetic field. We may think of the lines of force not as a definite number of fixed lines threading through the space which separates the two poles of the magnet, but as the stream-lines of a kind of magnetic fluid circulating through this space. Near the poles of the magnet the stream is contracted, and the velocity therefore great. In these places the force of impact of the magnetic fluid upon the exploring pole is a maximum, whilst farther away from the poles where the velocity, consequent upon the expansion of the stream, is less, the force of impact is also less. In this manner can be explained the variation of magnetic force as we move the exploring pole into different parts of the field, and the fact that a magnetic field taken by itself represents a definite amount of stored up energy. The conception of magnetic stream-lines is thus preferable to that of the rigid lines of force, and is indeed now generally adopted. Though we speak of a field of so many lines per square centimetre, we understand by this that the flow of force is so many times that existing in a unit field, and the reader is asked to put this interpretation upon the term "lines of force" whereever this term occurs in this book.

Units of Electro-Magnetic and Dynamic Measurements.

Force.—Every system of physical measurement must be based upon the three fundamental units of mass, length and time, and the different systems vary only in so far as the absolute magnitude of these fundamental units and their corresponding numerical values may differ. Thus, in the English system of measurement, a force of one pound is that force which if applied for one second to the mass of one pound will give it an acceleration equal to that of gravity, or, say, 32·2ft. per second. In the metric system the force of one kilogramme is similarly defined as that force which, if applied for one second to the mass of one kilogramme, will give it an acceleration equal to that of gravity, or, say, 9·81 metres per second. In both systems the term force is similarly defined, but the units are of different magnitude, though identical in kind. For electro-dynamic measurements it is customary to reckon forces in a much smaller unit than either the pound or the kilogramme. This unit force is obtained by adopting the centimetre as the fundamental unit of length, the gramme as the fundamental unit of mass, and the second as the fundamental unit of time. Measurements of forces and all other physical quantities which are based upon these fundamental units are said to be given in the centimetre, gramme, second, or, briefly, in the C.G.S. system. Thus, if we are told that a certain force has the value 20 in the C.G.S. system, we know that it is a force which, acting for one second upon the mass of

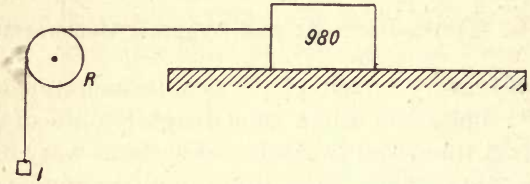

Fig. 6.

one gramme, will give it an acceleration of 20 cetimetres per second, or, if acting for one second upon the mass of four grammes, will give it an acceleration of five centimetres per second, or, if acting for the twentieth part of a second upon the mass of one gramme, will give it an acceleration of one centimetre per second. A force of one, or unit force, will similarly be defined as that force, which, acting upon the mass of one gramme for one second, will accelerate its velocity by one centimetre per second. Fig. 6 may serve to clearly explain this relation. Let a weight of 980 milligrammes slide absolutely without friction upon a table. Attach to this weight a perfectly weightless and flexible cord, which is taken over a pulley, R, and to the lower end of the cord attach a weight of one milligramme. Let the pulley have no mass, and turn without friction. The only force acting upon the system is that of gravity, which tends to pull the small weight down and the large weight forward on the table. If the large weight were not attached to the cord, the small weight would fall with an acceleration of 981 centimetres per second, but as the total mass to be set in motion is 981 times as great, the acceleration will only be one centimetre per second. Now let us increase both weights in the same proportion, namely, the large weight from 980 milligrammes to 1,000 milligrammes—that is, to one gramme—and the small weight from one milligramme to 1,000; 980 = 1·0204 milligrammes. By the proportional increase of both weights we have not altered the acceleration, which is still one centimetre, and as the force which produces this acceleration in the large weight passes through the cord, we find that the tension in this cord represents exactly unit force in the C.G.S. system. Engineers are in the habit of expressing forces, not as the cause of acceleration in a given mass, but simply as so many pounds, or grammes, or tons whichever unit is most convenient. To express the magnitude of unit force in our cord in this manner, we have only to ascertain how much of the small weight is actually transmitted in the shape of a pull in the cord. It is obvious that the whole of the small weight cannot be so transmitted unless the small weight is at rest, but as the small weight is moving downwards with an accelerated speed, the force corresponding to the acceleration is, so to speak, taken off the cord, and only the difference between the weight and the force required to accelerate that weight is transmitted. The small weight is 1·0204 milligrammes, and its acceleration is one centimetre per second, whilst that due to gravity is 981 centimetres per second. The force which reaches the cord is therefore that corresponding to an acceleration of 980 centimetres, or

$$1{\cdot}0204 \times \frac{980}{981} = 1{\cdot}019359 \text{ milligrammes.}$$

We thus find that in a locality where the acceleration of gravity is 981 centimetres, unit force may be represented by the weight of 1·019359 milligrammes. The reference to gravity is necessary, as can easily be seen if we go through a calculation similar to the above, but made on the supposition that the acceleration due to gravity is different. As a matter of fact, such differences exist even on our planet, but the differences are small. Suppose, however, that there were on this earth a spot in which the acceleration of gravity is only half that assumed in our previous calculation. This would not alter the magnitude of unit force as defined by unit mass and unit acceleration, and which could be recorded in precisely the same manner by a spring balance at all points of the earth, but it would alter the equivalent weight. We should find that unit force in this case would be represented by the downward pull of a weight of 2·038718 milligrammes, that is twice the weight as before. In expressing unit force as the dead weight of 1·019359 milligrammes, it is therefore necessary to remember that the relation only holds good for those localities in which the acceleration of gravity is 981 centimetres per second.

Unit force as thus defined is called a "DYNE," and we may therefore say that the force of a dyne is in our latitudes represented by the weight of 1·019359 milligrammes, or approximately by a weight by two per cent greater than that of a milligramme. From this we find the following relations:

One gramme ...	=	981 dynes.
One kilogramme	=	981,000 dynes.
One pound ...	=	444,980 dynes.
One ton	=	996,752,240 dynes.

Activity or Power.—Having now defined the unit of force, we must next go through a similar process to define the unit of activity or power. Obviously, unit work is done if the force of one dyne acts through the distance of one centimetre, and if this work be performed in unit time—that is, in one second—we have unit rate of doing work. For this we use the term unit energy. The name given to the unit of work is the "ERG," representing the work performed in overcoming a force of one dyne through a distance of one centimetre, and if this be done in one second we have the unit of power or activity. Some very simple arithmetical operations, which need not be given in detail, show that the following relations exist:

1 gramme-centimetre per sec	=	981 ergs per sec.	
1 kilogrammetre	,, ,, =	98,100,000 ergs	,, ,,
1 foot-pound ...	,, ,, =	13,562,859 ergs	,, ,,
1 English horse-power	=	7,459,571,687 ergs	,, ,,
1 metric horse-power	=	7,357,500,000 ergs	,, ,,

These figures are inconveniently large, and for practical work a larger unit than the erg-second is generally adopted. This is called the "WATT," and is equivalent to 10,000,000 ergs. By introducing this unit we have the following relations:

One English horse-power = 745·957168, or very nearly 746 watts.

One metric horse-power = 735·75, or very nearly 736 watts.

In future, when speaking of horse-power, it will be understood that the horse-power is reckoned as equivalent to 746 watts.

Work.—We have yet to define the unit of work. This as already stated is the erg, and is of course to be considered irrespectively of the time in which it has been performed, but as engineers are more familiar with the idea of power than work, unit work is sometimes defined with reference to unit power. Unit work is obviously represented by the work done by unit power in unit time. This unit is also inconveniently small, and for practical purposes it is customary to employ a unit 10,000,000 times as great—namely, the "WATT-SECOND" or "JOULE." If we raise a weight from the floor and place it upon the table we have done work, and the amount of this work is independent of the time it has taken us to raise the weight. The rate at which the work has been done (that is, the power or activity) is inversely proportional to the time, but the work itself is a constant, and may be expressed as the product of the weight and the height to which the weight has been raised. We may thus express the work by using the foot-pound or the kilogrammetre as a unit, but as these terms are generally also used to express rate of doing work or activity, it is preferable to adopt another way of reckoning work. We can for this purpose use the thermo-dynamic equivalent and reckon work not as so many foot-pounds, but as so many heat units. Thus, if we lift 772 lb. 1ft. we have done 772 foot-pounds, or the work represented by one British heat unit. Similarly, if we raise one kilogramme to a height of 424 metres we have done the work which is equivalent to one calorie. The calorie is the heat required to raise the temperature of one kilogramme of water by one deg. C. Adopting the thermo-dynamic equivalent as the basis for reckoning work, we find by a simple arithmetical operation, which need not be given at length, that the following relations exist:

One British Fahrenheit heat unit = 1,047·053 joules or watt-seconds.

One calorie = 4,159·44 joules or watt-seconds.

The use of these units may be illustrated by the following example: Glow lamps are often used under water for decorative purposes. Assume that a lamp absorbing energy at the rate of 60 watts is placed into a vessel containing one litre or, say, one kilogramme of water at 20 deg. C. Supposing the energy which in the lamp is transformed into heat is all communicated to the water and that there is no radiation of heat How long will it take until the water is raised to boiling point? Boiling point will be reached when 80 calories have been given to the water. This will be the case when the lamp has dissipated into the mass of water surrounding it 80 × 4,159·44 = 332,755 joules. Since 60 joules are given off by the lamp in one second, or 3,600 joules in one minute, it will take 92·43 minutes, or, say, about one and a half hour, to bring the water to boiling point. In reality it will take somewhat longer, as we are not able to entirely prevent radiation from the vessel containing the water.

Mathematical and Physical Poles.

In the same manner as we distinguish between mathematical and physical points must we also distinguish between mathematical and physical magnet poles. The magnets shown in Figs. 3, 4, and 5 have physical poles—that is, poles of finite dimensions. The poles are those parts of the magnet from which lines of force emanate, and as shown in the figures those parts occupy some space. In Fig. 4 it is, in fact, difficult to distinguish between the poles and other parts of the bar, since the lines of force emanate from almost the whole of its surface. They are, however, densest at the ends, and we therefore call the ends of the bar conventionally its poles without fixing any very definite limits to their extent. This indefinite arrangement of lines is obviously inconvenient for mathematical treatment, and in order to get over the difficulty we imagine the physical magnet replaced by an imaginary or mathematical magnet, consisting of a middle part wholly free from lines and two mathematical points for its poles from which all the lines of force emanate. A single pole is impossible in nature, but by making our imaginary magnet long enough we can separate its two poles sufficiently far to obtain round each almost the same effects as might be expected from single poles. The strength of a magnet, whether it be a physical or mathematical magnet, can be expressed as the product of its length—that is, the distance between its poles and the amount of free magnetism at either pole. This product is called the "MAGNETIC MOMENT." We assume that in each pole there is concentrated a definite amount of what may be called magnetic matter, from which the flow of force emanates. This matter, though of the same kind at both poles, must be supposed to differ in its sign. At one end of the magnet we have positive or north magnetic matter, and at the other end we have negative or south magnetic matter. Supposing the flow of force to be taken as proceeding from the north pole to the south pole through air, we can also say that the north magnetic matter sends out and the south magnetic matter absorbs the stream-lines of magnetic force. We take in this definition the direction of the stream-line to be that in which a free north pole would be urged through the field. Whether such a thing as magnetic matter actually exists or not is of no practical importance. The term magnetic matter is merely a convenient way of expressing a certain property of magnet poles, and may be retained without in the slightest degree contradicting experimental facts. Under this conception the attractive force of a magnet must be assumed to be proportional to the amount of magnetic matter, or, as we may also say, to the amount of free magnetism concentrated in its poles, and similarly the strength of field must be assumed to be directly proportional to the amount of free magnetism at the poles.

The Magnetic Field of a Mathematical Pole.

Let, in Fig. 7, M represent the north pole of a mathematical magnet of such length that we may leave its south pole out of consideration. Let the quantity of magnetic matter concentrated in this pole be also

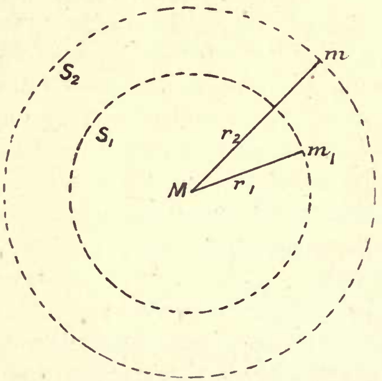

FIG. 7.

denoted by M, and place another north pole containing m units of magnetism at the distance r_2 from M. According to a well-known law the repulsion between the two poles is given by the expression $\frac{m\,M}{r_2^2}$ Now describe with radius, r_2 a sphere, S_2, round M, and imagine the pole m placed at various points on this sphere, then the repulsion between M and m, though varying in direction, will for all these positions have the same magnitude. The sphere, S_2, is, in fact, a surface of constant magnetic potential. The lines of force constituting the field of M are radii cutting S_2 at right angles. Now move the pole m from its position on the equipotential surface S_2 to the position m_1 on the equipotential surface S_1. The work done upon m in transit is obviously

$$\int_{r_2}^{r_1} \frac{M\ m}{r^2} d\,r = -\,M\,m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

the negative sign denoting that such work has been expended. This expression gives the difference of magnetic potential between any two points of the surfaces S_1 and S_2 if we suppose the pole m to contain unit of magnetism. Now let r_2 be infinitely large as compared with r_1, or, in other words, let the pole be brought from a point as far away from M as to be beyond its field, then the work done upon unit pole in bringing it to a point on the sphere S_1 is obviously

$$P = -\frac{M}{r_1},$$

and this expression gives the potential of the surface S_1. *We may thus define magnetic potential at any point of a magnetic field as the work which must be done upon unit pole to bring it from a place beyond the field where the potential is zero to that point of the field.* The numeric value of the magnetic potential must naturally depend upon the units we choose for expressing it. We define unit magnetism as *that amount of magnetic matter which, if concentrated in a point, will repel an equal amount of magnetic matter concentrated in another point one centimetre distant with the force of one dyne.*

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Thus let, in Fig. 7, both M and m be poles containing unit magnetism, or, briefly, unit poles, and let r_1 be one centimetre. Imagine the poles tied at that distance to the ends of a cord, then the tension in that cord will be one dyne, and this tension will be the same if m is moved to any point of the sphere S_1. Now we have previously defined unit strength of field as that flow of magnetic lines which will exert unit of mechanical force upon unit pole. The unit of mechanical force is the dyne, and the unit of field strength is a density of one line per square centimetre. In order to obtain unit of repulsive force there must therefore pass such a flow of magnetic force through the sphere S_1 as can be expressed by a density of one line of force per square centimetre. Now a sphere of the radius 1 has a surface of 4π square centimetres, and we find therefore that the pole M is sending out a total of 4π lines of force. A pole of twice the strength will obviously send out twice as many lines, and generally a pole of the strength M will send out 4π M lines. Calling F the total field strength, expressed as the number of lines of force, or, as it is also termed, the total induction, emanating from a pole containing M units of magnetic matter, we have therefore the following relation between these quantities

$$F = 4\pi M \quad . \quad . \quad . \quad . \quad (1)$$

$$M = \frac{F}{4\pi}$$

CHAPTER III.

Magnetic Moment.—Measuring Weak Magnetic Fields.—Attractive Force of Magnets.—Practical Examples.

Magnetic Moment.

It has already been mentioned that the magnetic moment of a magnet is the product of the strength of pole, M, and length, L. Replacing M by its equivalent value here given, we have

$$\text{Magnetic moment} = \frac{LF}{4\pi}$$

Now F is the number of unit lines of force going out at one end and in at the other end of the bar. If we denote by B the number of lines flowing through each square centimetre of cross-section of bar and by A the area of cross-section, we may also write

$$\text{Magnetic moment} = \frac{LAB}{4\pi}$$

The symbol B indicates the density of lines within the bar and is commonly called the "specific induction," or, briefly, the "induction." Since LA is the volume of the bar, we can also say that the magnetic moment of a straight bar magnet is equal to the volume multiplied by the specific induction and divided by 4π. Now imagine our bar magnet suspended in a magnetic field, in which the induction is H, and let the lines of this field be all horizontal and at right angles to the axis of the bar. The north pole of the bar will be pulled forward—that is, in the direction in which the lines of the field flow—and the south pole will be pulled in the opposite direction, the two forces producing a certain torsional moment, which is given by the expression

$$\text{Torque} = M\,L\,H.$$

$$\text{Torque} = \frac{L\,A\,B\,H}{4\,\pi}$$

The torque is of course here given in dyne-centimetres. To obtain it in gramme-centimetres we have to divide by 981. An example may serve to give an idea of the kind of forces we have to deal with in magnetism. Let us suppose that we magnetise a large steel bar and suspend it in the field of the earth, in fact, that we make a gigantic compass needle and measure the torsional moment which is required to keep this compass needle in an east-west position. Let the magnet be one metre long and ten centimetres square. If strongly magnetised by suitable means we shall be able to concentrate on each square centimetre of end face about 400 units of magnetic matter, corresponding to an induction of about 5,000 lines per square centimetre of cross-section. The field of the earth may be taken as ·18 C.G.S. units. Inserting these values in the above equation, we find that the field of the earth will exert upon our bar magnet a torque of 730 gramme-centimetres. To keep the bar in its east to west position we must therefore apply to one end of it a force of

14·60 grammes, or, say, a little over half an ounce. This, it will be seen, is a very small force for so large an apparatus, the dead weight of which would be about 180lb., but then it must be remembered that though the magnet taken by itself is powerful, the field in which it is placed is very weak. Had the strength of field, H, been such as can easily be produced in air by means of coils of wire through which currents pass, the torque exerted by the bar would have been enormously greater. A field of 500 C.G.S. units can easily be produced between two coils placed parallel to each other at a distance about equal to their radius. Now, if we suspend our steel magnet in such a field the torque in gramme-centimetres will be

$$\text{Torque} = 100 \times 100 \times \frac{5,000}{4\,\pi} \times 500 \times \frac{1}{981} = 2,030,000.$$

To reduce this to kilogrammetres we divide by 1,000 × 100 = 100,000, since the kilogramme contains 1,000 grammes and the centimetre is the hundredth part of the metre. We thus obtain

$$\text{Torque} = 20{\cdot}3 \text{ kilogrammetres.}$$

To keep the bar in its position parallel to the plane of the coils we would, therefore, have to apply at each end a force of 20·3 kilogrammes, or 45lb., the direction of these forces being at right angles to the axis of the bar.

Measuring Weak Magnetic Fields.

The calculation here given is only correct under the supposition that the magnetism of the bar remains unaltered when the bar is placed into the field. In reality, however, this is not the case when the field is strong. A field of 500 C.G.S. units is already a very strong field, and would alter the magnetisation of the bar even if the latter be made of the very hardest steel.

The calculation of torque must therefore be taken as being only an approximation, and has been given merely to show the kind of forces coming into play in such cases. In a weak field the magnetism of a strongly magnetised steel bar is not changed, and is, in fact, what it professes to be by name, that is "permanent." The magnetic moment of the bar may therefore be regarded as constant for all its positions in the weak field, and this fact is made use of in the determination of the strength of magnetic fields. It might appear at a first glance that we if knew the magnetic moment of the bar, the strength of the field could be easily found by measuring the mechanical couple required to keep the bar at right angles to the lines of the field, but such a measurement could not be made with any accuracy. In the first place, the couple with a bar of moderate dimensions and a weak field is exceedingly small, and therefore difficult to determine exactly, and, in the second place, the determination of the magnetic moment is in itself a more difficult operation than the determination of the strength of a magnetic field, which, indeed, generally precedes it. The method commonly used for the determination of weak magnetic fields, and more especially for the field of the earth, consists in making two distinct tests with the same magnet. In the first test the magnet is so placed as to deflect a compass needle, and from the relative position and distance between needle and magnet, and the deflection of the former, the ratio between M, the magnetic moment of the latter, and H, the field strength, can be calculated. We thus obtain $\frac{H}{M}$. Next we set the magnet swinging, and note the time of vibration. According to a well-known law, the time of vibration is proportional to the square root of the moment of inertia (which for a cylindrical bar can be easily calculated), and inversely proportional to the square root of M H, the force under the influence of which the bar swings. By multiplying the two values we obtain H^2, and by dividing one by the other we obtain M^2, so that the two observations suffice for the determination of the strength of field as the magnetic moment of the bar. As this method is to be found described in every text-book on magnetism it is not necessary to enter into its details in this place, the more so as for the determination of the strong fields with which the electrical engineer is mostly concerned, it is only of value in so far as it gives us a point of comparison. Strong fields are generally measured by another method based upon electromagnetic induction, the apparatus used consisting of wire coils and a ballistic galvanometer. One of the wire coils is placed under the influence of the field of the earth, and the other under that of the field to be measured, whilst the deflection of the galvanometer in both cases enables us to compare the two fields. The subject must, however, be left to a later chapter, in which we shall deal with the interaction between magnetic fields and electric currents.

The Attractive Force of Magnets.

The formulæ given in the preceding and present chapter enable us to calculate the mechanical force in dynes, or grammes, or pounds with which magnets attract each other, or a magnet attracts a piece of iron it has magnetised by induction. When the distance between the attracting (or repelling) poles is large in comparison with the dimensions of the magnets, the problem is simple enough. We can in this case imagine the physical magnets replaced by their equivalent mathematical magnets with their poles concentrated in mathematical points, and by applying the law of inverse squares, in the manner to be found in every text-book on magnetism, obtain perfectly definite expressions for the forces acting between the poles and the resulting couples. The problem in this form has, however, no interest for the designer of dynamos, and need, therefore, not be further considered in this place. What interests the dynamo builder is the attraction between magnetised surfaces of large extent, as compared with their distance apart, and in these cases the law of inverse squares ceases to be applicable. When investigating the attraction between the pole-pieces of a dynamo and the surface of its armature, we are not dealing with magnetism concentrated in mathematical points, but with magnetism distributed over definite surfaces. The forces resulting from this may become under certain conditions very considerable, and as they must directly affect the armature shaft, bearings, and other parts, it is necessary to investigate the matter so as to be able to take these forces into consideration in designing the mechanical portion of the machine. Before proceeding to the theoretical consideration of the subject, it will be useful to show its practical bearing with reference to a definite case. Fig. 8 represents diagrammatically the field, F, and armature core, A, of an ordinary dynamo of the so-called "upright" type. The flow of lines takes place from the left, or north

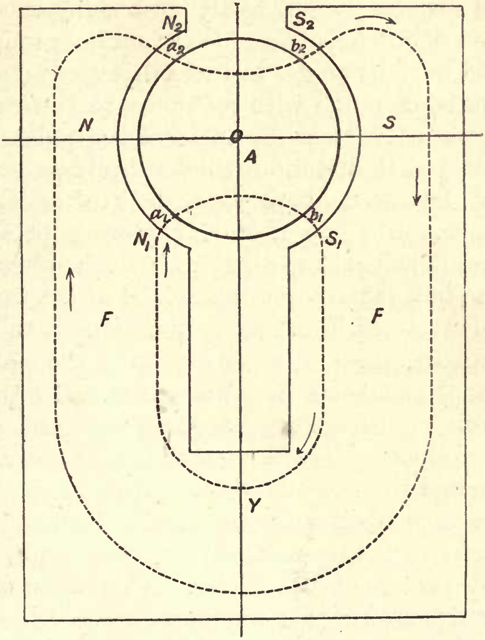

FIG. 8.

pole-piece, N_1 N N_2, through the small air space, a_1, a_2, into the armature core, A, then out on the other side, where the flow of force again leaps across the air space, b_1 b_2, and enters the pole-piece, S_1 S S_2, returning by the yoke, Y, to the north pole-piece, and thus forming a closed magnetic circuit. Two such circuits are shown in the figure by dotted lines, the direction of flow being indicated by arrows. It has already been pointed out in the previous chapter that the surface of the pole-piece and that of the armature must be equipotential surfaces, a fact easily verified by experiment. The lines of force between these surfaces must therefore stand at right angles to them in every point, or, in other words, be radial with reference to the centre, O, of the armature. Near the edges of the polar surfaces their true radial direction will naturally be somewhat disturbed, but we deliberately neglect the effect of this disturbance. Imagine a unit exploring pole placed at N_2, and it will be repelled from the surface of this pole-piece in a radial direction, whilst at the same time it will be attracted in the same direction, by the surface of the armature, the force acting upon the pole being the sum of repulsion and attraction. On the other side, a unit exploring pole placed on the surface of the armature opposite S will be repelled by the armature and attracted by the field pole. Now let, in the first case, the exploring pole be rigidly fastened to the surface of the field pole at N_1, or, better still, let it be part and parcel of the latter. This assumption is equivalent to saying that we consider the forces acting upon an element of the polar surface, N_1 N N_2, of such extent that it contains unit quantity of magnetic matter. Obviously this element of surface cannot be repelled from the rest of the surface, since it forms an integral part of it; and one of the forces which we found above as acting upon a free unit pole is now eliminated. The other force, that of attraction towards the surface of the armature, remains, however, unchanged. Each element of the polar surface is thus attracted towards the armature, and since action and reaction must be equal and opposite, the armature as a whole is attracted by the polar surface. The same reasoning applies to the other side of the machine. If we imagine the unit exploring pole to form part and parcel of the armature surface opposite S_1, we find that although the repulsion towards the right has ceased, the attraction towards the right remains, and as this applies to every element of armature surface within the pole-piece, $S_1 S_2 S$, the armature as a whole experiences an attraction to the right. If the arrangement of the armature in the field is perfectly symmetrical, the attraction to the left balances the attraction to the right, and there is no side thrust on the bearings, though there may be up or down thrust, if the arrangement is unsymmetrical with reference to the diameter, N O S. Imagine, for instance, the upper half of both pole-pieces removed. The armature would in this case be attracted by the north pole-piece, not only to the left, but also downwards. Similarly, the attraction on it from the south pole-piece would be to the right and downwards. The two horizontal components balance each other, but the vertical components are added, and produce a down thrust on the bearings in addition to the thrust due to the weight of the armature. The same effect, but to a lesser degree, must result from any minor inequality between the upper and lower halves of the pole-pieces, and as it is not always practicable to ensure absolute symmetry in all directions, it becomes important to be able to calculate the mechanical forces and strains resulting from such want of symmetry. We now proceed to investigate this matter from a more general point of view.

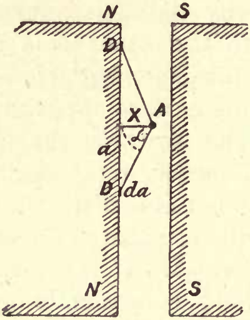

FIG. 9.

Let, in Fig. 9, N N and S S represent the polar end faces of two straight magnets, of so great a length that the influence of their other poles upon an exploring pole

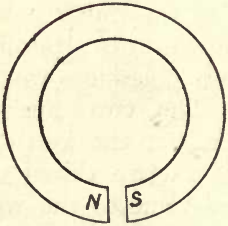

FIG. 10.

placed into the gap at A may be neglected; or let N S be the poles of a magnet bent into ring form as shown in Fig. 10. In such a magnet the only accessible field

$I = \frac{ml}{V} = \frac{ml}{al} = \frac{m}{a}$

is that existing in the narrow gap between the poles, and an exploring pole placed in this space cannot be affected by any other lines of force but those leaping across the gap. The amount of magnetic matter contained on each polar surface, divided by the surface, is, in a straight bar magnet, obviously equal to the magnetic moment divided by the volume, and is called the intensity of magnetisation. In a curved magnet this definition does, however, not hold good, as can easily be seen if we imagine the originally straight magnet bent into a circle until the poles nearly touch. The magnetic moment, which is the product of pole strength and distance between the poles, has now decreased, but the amount of magnetic matter on each pole has not decreased. To make the definition fit the case of bent magnets, we must consider not the ratio between the magnetic moment of the magnet taken as a whole and divided by its whole volume, but the magnetic moment of a cubic centimetre cut out and separated from the rest of the mass. It is, however, more simple to abstract altogether from the idea of intensity of magnetisation and substitute that of "density of magnetic matter." Thus we may assume that the magnetic matter is uniformly spread over the polar surfaces with a density m, meaning thereby that on each square centimetre of surface there are m absolute or C.G.S. units of magnetic matter. Each particle of magnetic matter on the surface N N repels the point A according to the law of inverse squares, but the direction of these forces and their intensity varies. The total force is found by integrating the elementary forces, as we now proceed to show.

The horizontal component, d P, of the force exerted by an elementary particle, σ, of the polar surface at D, upon the unit pole placed at A, is evidently $\frac{m\,\sigma}{a^2 + x^2}$ cos. α, and if we imagine a complete ring, D D, of such elementary particles of width, $d\ a$, the force exerted by the ring is

$$d\,\mathrm{P} = \frac{m\ 2\ \pi\ a\ d\ a}{x^2 + a^2} \cos.\ \alpha.$$

It will be seen from the diagram that $a = x$ tan. α, and for $d\ a$ we can therefore write $x \frac{d\ \alpha}{\cos.^2\ \alpha}$; so that the above expression for the horizontal force becomes

$$d\,\mathrm{P} = \frac{m\ 2\ \pi\ a}{(x^2 + a^2)\cos.^2\alpha}\ x \cos.\ \alpha\ d\ \alpha.$$

But since $(x^2 + a^2) \cos.^2\ \alpha = x^2$, we have also

$$d\,\mathrm{P} = m\ 2\ \pi \frac{a}{x} \cos.\ \alpha\ d\ \alpha.$$

$$\text{Since } \frac{a}{x} = \frac{\sin.\ \alpha}{\cos.\ \alpha} \text{ we find}$$

$$d\,\mathrm{P} = 2\ \pi\ m \sin.\ \alpha\ d\ \alpha,$$

and by integrating between the limits $\alpha = 0$, and $\alpha = \alpha$, we find the total repulsive force exerted by the polar surface upon a unit pole,

$$\mathrm{P} = 2\ \pi\ m\ (1 - \cos.\ \alpha).$$

Now imagine the surface very large in comparison with the distance x of the point A. In this case the lines joining A with the edges of the polar surface will be sensibly parallel to it. We have therefore $a = \frac{\pi}{2}$. Since the cosine of $\frac{\pi}{2}$ is 0, we have

$$P = 2\,\pi\,m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The unit pole is not only repelled by the surface N N, but it is at the same time attracted by the surface S S, and a similar calculation shows that the attractive force of this surface upon it is also $2\,\pi\,m$, so that the total force exerted upon the unit pole in the gap between N N and S S is

$$2\,P = 4\,\pi\,m.$$

This expression enables us to calculate the strength of the field within the gap. It has been previously stated that, according to a conventionally adopted measure, we call that a unit strength of field in which there is unit flow of force per square centimetre, or in which a unit pole is impelled with the force of 1 dyne. If the impelling force is $4\,\pi\,m$ dynes, the flow of force per square centimetre is $4\,\pi\,m$. This is commonly called the "induction," and denoted by the symbol $\mathfrak{B}$. We thus find that

$$\mathfrak{B} = 4\,\pi\,m.$$

Let S denote the number of square centimetres in each polar surface, then S $\mathfrak{B}$ is the total flow of force or field strength, F, expressed in number of unit lines of force; and S m is the total pole strength, or amount of magnetic matter, M, spread over each of the polar surfaces. We find therefore

$$F = 4\,\pi\,M.$$

that is to say, *the total field is* 4π *times the total pole strength*, a result which has already been obtained in the previous chapter for a single pole. In that case, however, the field surrounded the pole on all sides, and it was not immediately obvious that the expression would also hold good in cases where the field is, so to speak, one sided—that is, emanating from the pole in one direction only. This we now see is the case, and the formula $F = 4\pi M$ is universally applicable.

Returning now to formula (2) we have seen that the repulsion of the surface N N upon unit north pole placed close to it is $2\pi m$. Had the exploring pole been a unit south pole, we would have found the same expression, but with a negative sign, showing that the force was one of attraction—that is, opposite in direction. Now let it be part of the surface of the south pole, and we see immediately that every unit of magnetic matter spread over the surface S S is attracted by N N with a force of $2\pi m$ dynes; and since there are $m\,s$ such units on this surface, the total force is $2\pi m^2 s$. It will be convenient to bring this expression into another form by introducing the induction, $\mathfrak{B}$.

Since $m = \frac{\mathfrak{B}}{4\pi}$, we have $m^2 = \frac{\mathfrak{B}^2}{16\pi^2}$, and

$$2\pi m^2 s = \frac{s\,\mathfrak{B}^2}{8\pi}.$$

This formula, it should be remembered, is only correct if the distance between the polar surfaces is so small in comparison with their extent that the disturbance near the edges, where the upper limit for the angle a is less than $\frac{\pi}{2}$, may be neglected; or where, though the distance between the polar faces is sensible, as in dynamo machines, one of the surfaces extends considerably beyond the other. When the surfaces are in actual contact—as for instance, in a magnet and its keeper—the formula may be applied without correction, but when the distance is sensible and the surfaces are small, a certain allowance for the influence of the edges must be made. For practical purposes it is, however, not generally necessary to calculate the attractive force with very great accuracy, and the formula,

$$\text{Attractive force in dynes} = \frac{s\,\mathfrak{B}^2}{8\,\pi} \quad . \quad . \quad (3)$$

may be used as a fair approximation.

It will be useful to show the application of this formula by a few examples. Let, in Fig. 11, M represent a magnet provided with a keeper, K. The magnet limbs are 3 centimetres square, and the induction is $\mathfrak{B} = 20{,}000$. The attractive force of each limb on the keeper in dynes is therefore $\frac{9 \times 20{,}000^2}{8 \times 3{\cdot}14}$. To obtain it in kilogrammes we divide by 981,000 and find the attractive force of both limbs $= \frac{9 \times 400 \times 10^6}{25{\cdot}12 \times 981{,}000} \times 2.$

$$Q = 292 \text{ kilogrammes.}$$

A magnet of the dimensions here given would weigh a little under 2 kilogrammes, so that its attractive force is approximately 150 times its dead weight.

As another example we may take the dynamo shown in Fig. 8. Let the armature core be 30 cm. (nearly

12in.) diameter, and 40 cm. (nearly 16in.) long. Let the angle embraced by the pole-pieces on each side be 120 deg., and assume a mean induction of $\mathfrak{B} = 5{,}000$ across the polar diameter, N O S. It will be shown later on that the induction is not constant over the whole extent of air gap, but that it has a greater than

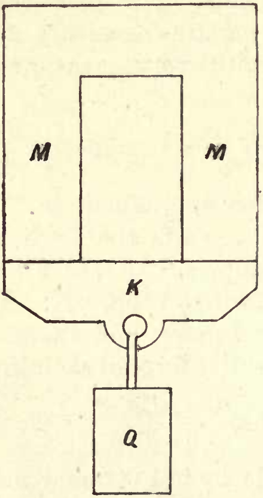

FIG. 11.

its mean value below and a smaller than its mean value above the polar diameter. The reason for this variation cannot be given now, as it has to do with the interaction between electric currents and magnets, which will be treated in subsequent chapters. For the present we shall simply assume that such a difference exists, and that numerically it may be expressed by saying that the mean field induction above the polar diameter is 4,800 and below it 5,200 C.G.S. units. The attraction on the upper right-hand quarter of the armature may, with a polar angle of 120 deg., be represented by

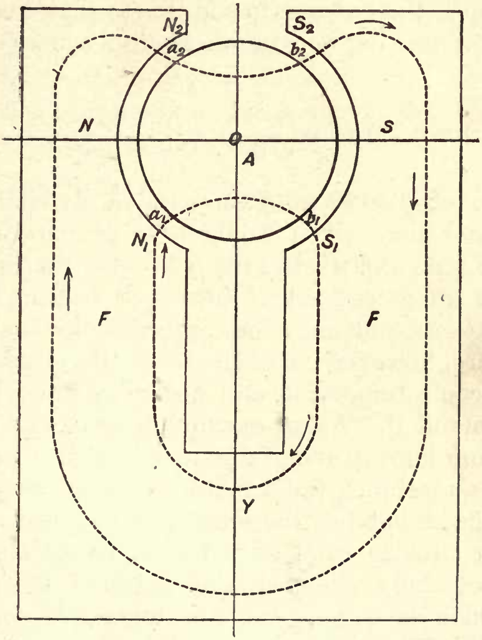

Fig. 8.

a force acting at 30 deg. to the polar diameter, and its vertical component is obviously sin. 30 deg. = ·5 of its numerical value. Similarly with the left-hand upper quarter. The total vertical force acting upwards upon the upper half of the armature is therefore equal to the attraction of the upper half of one pole-piece. In the same way we find the total vertical force acting downwards upon the lower half of the armature equal to the attraction of the lower half of one pole-piece. The difference between these two forces represents the down thrust upon the bearings in addition to that due to dead weight. Inserting the numbers into equation (3) we find:

$$\text{Down thrust} = \left(\frac{\pi\,30}{6} \times 40\right)\frac{5{,}200^2 - 4{,}800^2}{25 \cdot 12},$$

which is about 100 kilogrammes. A dynamo of the dimensions here given would be a machine of about 40 or 50 h.p., and it will readily be seen that in such a machine an extra load of 1cwt. per bearing is quite unimportant and may be neglected in the design. There are, however, cases in which the magnetic pull may become important, and provision must be made to withstand it. As an example may be cited a class of dynamo known under the name of disc machines. In these machines the armature core is not of cylindrical shape, but has the form of a flat disc revolving between circular rows of poles on either side of it. When carefully adjusted, the width of the gaps on either side is equal, and the attractive forces are balanced. The armature is kept in this position by a thrust bearing on its shaft, and as long as there is no wear the load on the thrust bearing is very small. If, however, wear takes place, or from some other cause the armature be allowed to run nearer to one ring of pole faces than the other, the field on that side is stronger, and a considerable magnetic pull is the result.

Let, in a 100-h.p. machine, the sum of the polar surfaces on one side of the disc be 2,000 square centimetres, and let the induction for which the machine was originally built be 4,600, the air gap being on either side 20 mm., a little over ¾in. Now, suppose that from some cause the armature is allowed to shift by 2 mm., then, roughly speaking, the induction on the side of the reduced gap will have become 5,000, and that on the side of the widened gap 4,200. We shall then have an unbalanced magnetic thrust of

$$\frac{2{,}000 \times (5{,}000^2 - 4{,}200^2)}{25 \cdot 12 \times 981{,}000} \text{ kilogrammes,}$$

or, say, about 600 kilogrammes, which is a force of sufficient magnitude to be taken into account when designing the mechanical details of the machine.

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CHAPTER IV.

Action of Current upon Magnet—Field of a Current. Unit Current—Mechanical Force between Current and Magnet—Practical Examples—English System of Measurement.

Directive Action of a Current upon a Magnet.

If we lay a case containing a compass needle on the table and stretch a wire across the top of it, we find that upon sending a current through the wire the needle tends to set itself at right angles to the wire. If the needle is not under the influence of any other force, or if the current is very strong, the position assumed is exactly perpendicular to the wire, and if other forces act on the needle, the position assumed by it indicates the direction of the resultant of these other forces and the deflecting force due to the current. We are thus able by observing the degree to which the needle is deflected to form an estimate of the deflecting force exerted by the current. We find in this way that the force is diminished if we raise the wire parallel to itself a certain distance from the needle, also that the direction of this force is reversed if we place the wire below instead of above the needle, and that for all positions the force increases with the current.

The Magnetic Field of a Current.

It will be obvious from these experiments that a wire carrying a current is surrounded by circular lines of force over its whole length, and that the lines are densest near the wire and less dense at a distance.

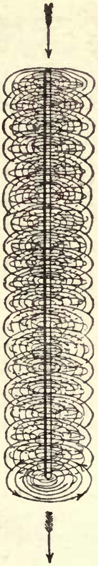

FIG. 12.

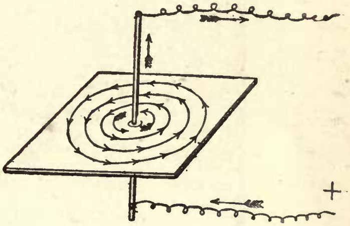

FIG. 13.

There is, in fact, a kind of magnetic whirl round the wire, as shown in Fig. 12, the lines forming concentric rings. This will be more clearly seen in Fig. 13, where the wire is supposed to pierce a sheet of paper on which the lines are traced. According to Ampère's well-known rule, the direction in which the north pole of a needle is deflected can be ascertained in the following manner: Imagine a man swimming in and with the current and looking at the north pole of the needle, then the latter will be deflected to the left of the swimmer. We have in a previous chapter defined the

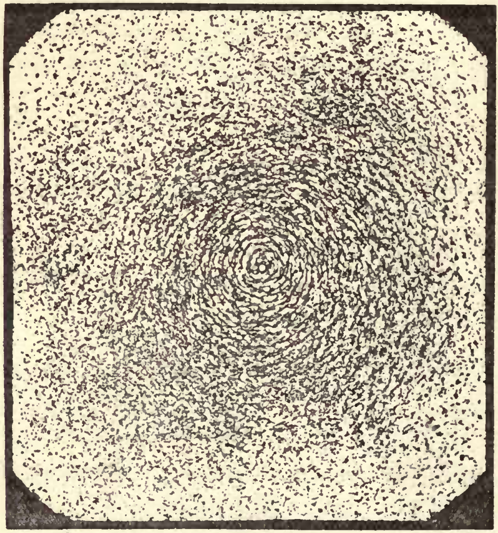

FIG. 14.

direction of flow of force in a magnetic field as that in which a free north pole is urged to travel, and this definition, combined with Ampère's rule, enables us at once to determine the sense in which the magnetic whirl takes place. If the current flows upwards through the wire, Fig. 13, the flow of force must be in the sense as shown by the arrows; or, to put it in another way, *if we look in the direction in which the current flows, the flow of force is clockwise.* That the lines of force arrange themselves in concentric circles can easily be shown by a modification of the experiment with iron filings, which we described in connection with the field of a magnet. We take a glass plate, Fig. 14, and drill a hole through its centre. The plate is covered with a thin film of paraffin, and a wire is threaded through the hole. Now send a current through the wire, and sprinkle iron filings, at the same time gently tapping the plate to facilitate the arrangement of the filings. The latter will assume the position shown in the diagram. To retain the filings we have only to gently heat the plate, when the paraffin melts, and the iron filings become embedded, and are thus fixed in position after the plate has cooled again.

Strength of Field Produced by a Current.

Having thus found that a current is surrounded by a magnetic whirl, constituting the field of the current, the next thing to do is to ascertain the strength of field at any point in the space surrounding the wire. From the circular arrangement of the iron filings we at once conclude that the force at any point is exerted at right angles to the plane laid through that point and the part of the wire, the influence of the current in which we desire to measure. It is, however, impossible to measure by itself the force exerted upon an exploring pole by the current in a short bit of wire, since the current must be brought to and led from this short bit by other wires, the current in which must also influence our pole, and thus mask the effect of the particular piece investigated. We can only obtain a current in a closed circuit, and the exploring pole must necessarily be under the influence of the whole of the circuit. To investigate the law experimentally, it is therefore necessary to take a circuit of such simple shape that the observed effects due to the whole circuit may enable us to draw a conclusion as to the effect of any single part. The most simple imaginable arrangement is that of a circular current with the exploring pole placed in the plane and centre of the circle. In this case, however, all parts of the circuit are equidistant from the pole, and the direction of the current in any element of the circuit is at right angles to the line joining the element and the pole. The results obtained with such an apparatus will therefore not be applicable without further verification to circuits in which the elements are not equidistant from the pole and form other than right angles with the lines joining them to the pole. By using a circular current we find that the force exerted upon unit pole placed in the centre of the circle is proportional to the circumference of the circle, to the current strength, and inversely proportional to the square of the radius of the circle. We may thus reasonably conclude that the force exerted by an element is proportional to the length of this element, the current, and inversely proportional to the square of the distance of the element from the unit pole, but only if the element forms a right angle with the line joining it to the pole. Where this condition is not fulfilled the experiment leaves us without information. Here it is necessary to make an assumption, and to verify it by a subsequent experiment.

The assumption we make is that if the element forms an angle of less than 90 deg. with the line joining it with the pole—that is, when the element is not seen in its full length, or, so to speak, broad side on from the pole—the force is diminished in the ratio of one to the sine of the angle. In other words, instead of taking the length of the element itself, we take that of its projection in the line of view from the pole. An element lying wholly in that line will therefore exert no force on the pole. Whether our assumption is correct can be verified by experiment, and for this purpose we chose an infinite straight current flowing down the wire, W W, Fig. 15. In reality we can, of course, not have an infinitely long wire, but by taking a wire of considerable length as compared with the distance of the pole N from the wire, we shall approach very closely to the theoretical condition, especially if we take care to carry the other parts of the circuit as far away from the pole as possible. The current, c, in the element A B exerts a force which in absolute measure is given by the expression $\frac{c\,\mathrm{A\,B}}{a^2}$; and that exerted by the element C D is according to our assumption given by the expression $\frac{c\,\mathrm{C\,E}}{b^2}$.

If we now integrate these elementary forces over the whole length of the wire, and find that the resulting force is that found by experiment, we naturally conclude that our assumption was correct. It is for this purpose not even necessary to make a quantitative experiment, that is, actually weigh the force, since the absolute magnitude of such forces has already been found by the experiment with the circular current.

All we now care to know is the law according to which the force varies with the distance a, and this can be found by a very simple experiment. Let, in Fig. 16,

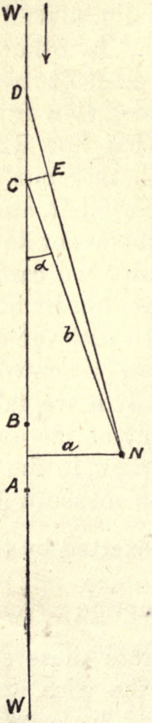

Fig. 15.

W W again represent the wire, and suppose there is suspended round it a ring-shaped wooden disc, D, on which we place a magnet, N S, in any position. We find that there is no tendency for the disc to rotate, though the magnet taken by itself tends, as we have already seen, to set itself at right angles to the wire. With the direction of current as indicated by the arrow,

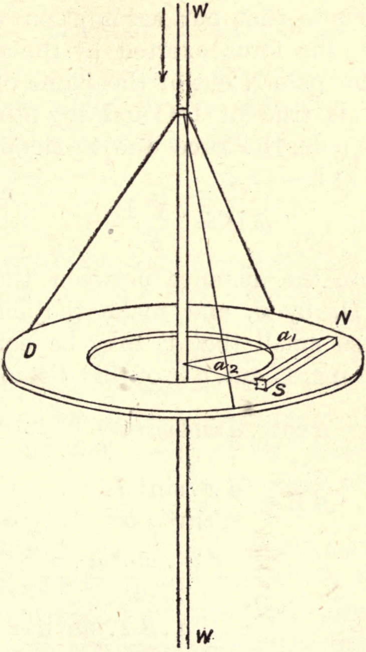

FIG. 16.

the north pole of the magnet will tend to move forward in clockwise direction, as seen from above, whilst the south pole will tend to move in the opposite direction; and since each force taken by itself would produce rotation of the disc, we conclude that the corresponding turning moments exerted upon the disc are equal and opposite. The forces exerted upon the poles must therefore be inversely as their distances, a_1, a_2, from the wire; and if we find that the integration of the elementary forces in Fig. 15 give us this result, we naturally conclude that our assumption was correct. In this figure, the force exerted by the element A B tends to lift the pole N out of the plane of the paper, and the same is true for D C and any other element. If N be a unit pole, the force due to the current, c, in the element C D is—

$$d\,P = \frac{c\,C\,E}{b^2}.$$

If we denote the distance between the elements, A B and C D, by x, and make the length of the element C D so small that it may be considered an infinitely small increment, $d\,x$, of this distance, we have

$$x = a \operatorname{cotg.} \alpha \text{ and } d\,x = \frac{a\,d\,\alpha}{\sin^2 2\,\alpha}$$

$$d\,P = \frac{d\,x\,.\sin^3 \alpha\,.\,c}{\sin^2 \alpha\ b^2}$$

$$= \frac{d\,x\ .\ \sin^3 \alpha\ .\ c}{a^2}$$

$$= -\frac{a\,.\,d\,\alpha\,.\sin^3 \alpha\,.\,c}{a^2 \sin^2 \alpha}$$

$$= -\frac{c \sin \alpha\, d\,\alpha}{a}$$

which, integrated over the whole length of the wire—that is, from $\alpha = 0$ to $\alpha = \pi$, gives the total force

$$P = \frac{2\,c}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We see, therefore, that the force tending to lift the unit pole, N, out of the plane of the paper is indeed inversely proportional to a, the distance of the pole from the wire, and that, therefore, our above assumptions are correct.

Definition of Unit Current.

Since the force exerted upon unit pole is in the C.G.S. system equal to the induction in air, or the strength of field, H, we can also write

$$H = \frac{2\,c}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and say that *the induction at a point of the field of a current flowing along a very long straight wire is two times the current divided by the distance of the point from the wire.* This relation gives us at once the definition of unit current. *It is that current which, flowing along a straight wire of infinite length, produces a field of* 2 *C.G.S. units at all points* 1 *centimetre from the wire, or unit field strength at a distance of* 2 *centimetres from the wire.*

This definition, although perfectly correct, is not the one generally given in text-books. It is customary to define unit current in relation to a circular conductor of 1 centimetre radius. Obviously the force exerted by a current, c, upon a unit pole placed in the centre of the circle is $2\,\pi\,c$, and if the current is unity the force is $2\,\pi$. Hence we may also define unit current as *that current which, flowing in a thin wire forming a circle of 1 centimetre radius, acts upon a unit pole placed in the centre with a force of* $2\,\pi$ *dynes.* The unit of current thus defined, although of convenient magnitude, has not been adopted in practice. It is customary to measure currents in a unit only one-tenth the magnitude of that here defined, and to call this practical unit the AMPERE. Thus a current of 25 amperes is the same as a current of 2·5 units in the C.G.S. system.

Mechanical Forces between Currents and Magnets.

To a certain extent we have already in the preceding paragraphs considered the mechanical forces between conductors and magnets, but this was done principally with the object of determining the properties of the magnetic field of a current. We must now approach the subject more from the engineer's point of view, and consider in detail the mechanical forces between conductors carrying currents and magnet poles or magnetic fields. It will at once be evident that if in our previous investigations we had assumed the strength of the exploring pole to be M instead of unity, all the forces would have been M times larger. Similarly, if the radius of the coil had been r centimetres instead of 1 centimetre, the force exerted upon the pole would have been smaller in the ratio of 1 to r. Let, in Fig. 17, N S represent a magnet of the pole strength, M, and place the north end into the centre of a circular wire of radius, r, which is supplied with a current, c, from a cell, C. Let the magnet be so long that the influence of the current upon its south pole may be neglected, then the force with which the north pole will be drawn to the left is given by the expression

$$P = \frac{M\,2\,\pi\,c}{r} \quad . \quad . \quad . \qquad (6)$$

From the pole N, lines of force emanate in all directions, and, as was shown in Chapter II., equation (1), the whole flow of force is $4\pi M$.

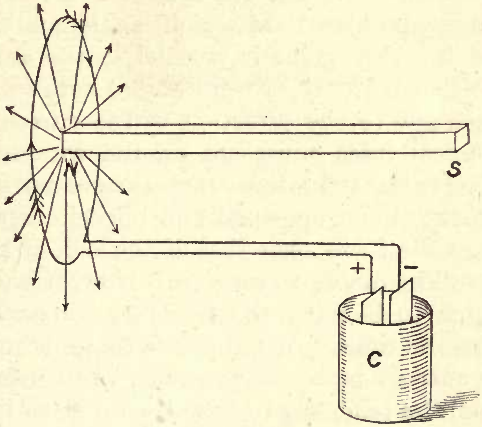

FIG. 17.

Imagine a sphere of radius, r, laid round the pole: then the density of lines on the surface of this sphere is $\frac{4\pi M}{4\pi r^2} = \frac{M}{r^2}$. The whole of the circular wire lies, therefore, in a field of the strength $\frac{M}{r^2}$, for which we write the symbol $\mathfrak{B}$, meaning thereby the induction or flow of force through the space close round the wire. It is customary to use the symbol $\mathfrak{B}$ for the induction through iron, as will be shown later on, and to express the strength of a field by the symbol H. But as we shall use this symbol to denote the line integral of magnetic force, it will be best to retain the symbol $\mathfrak{B}$ in all cases where we deal with induction or flow of force per square centimetre, whether this flow takes place through iron or any other substance. In the present case the substance is air, and the lines of force cut the wire in every point of it at a right angle, and the force produced in every point is parallel to the axis of the magnet—that is to say, at right angles both to the lines of the field and to the direction of the current. That a mechanical force must act on the wire is evident from the consideration that we can have no action on the magnet without an equal and opposite reaction on the wire. We have seen that if the coil is fixed, the magnet will be drawn to the left. Now, if we imagine the magnet to be fixed, the tendency will be to draw the coil to the right. We thus see that a wire through which a current passes has, when placed into a magnetic field, the tendency to move parallel to itself and at right angles to the lines of the field. The force producing this tendency is, in the case represented by Fig. 17, given by the equation—

$$P = 2 \pi r c \mathfrak{B}.$$

Now, $2 \pi r$ is the length, l, of the circular wire, and we find, therefore, that the mechanical force in dynes acting upon the wire is given by the product of current strength, length of conductor within the influence of the field, and strength of field, or in symbols

$$P = l c \mathfrak{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this expression the current is, of course, given in C.G.S. measure. If it be given in amperes we have

$$P = l c \mathfrak{B} \, 10^{-1}.$$

To get the forces in kilogrammes we divide by 981,000

$$P \text{ kilogrammes} = \frac{l\, c\, \mathfrak{B}}{9{,}810{,}000} \quad . \quad . \quad . \ (8)$$

Or for convenience

$$P \text{ kilogrammes} = 10 \cdot 1937\ l\, c\, \mathfrak{B}\ 10^{-8} \ (9)$$

Practical Examples.

Thus, a wire carrying 100 amperes, and passing for the length of 1 metre through a field of 1,000 C.G.S. units, is acted upon with a force of 1·01937 kg., or very nearly $2\frac{1}{4}$lb.

It is this mechanical force due to the interaction of magnetic fields and electric currents which in dynamos has to be overcome by the power of the prime mover, and in motors gives the turning moment or torque to the spindle of the armature.

This will be clearly seen from Fig. 18, which is a diagrammatic representation of a motor or dynamo. For the sake of simplicity only one loop of wire, A B C D, is shown on the armature, and the field magnets are shown in dotted lines. Through the narrow space included between the inner surface of the pole-pieces and the outer surface of the armature core (variously called air space, interpolar space, air gap, etc.) there exists a strong magnetic field—that is to say, the space is traversed by lines of force, all flowing radially inwards from the north pole-piece into the armature core, and radially outwards from the armature core to the south pole-piece on the other side. An element of the armature conductor, A B, is therefore in exactly the same relation to the lines emanating from the north pole-piece as in an element of the circular conductor, Fig. 17, in relation to the lines emanating from N. Each element of this circular conductor, and therefore the whole conductor, is pushed to the right (the right hand of a swimmer in the current looking at the north pole, so that the lines would pierce him in front), and if we imagine a swimmer

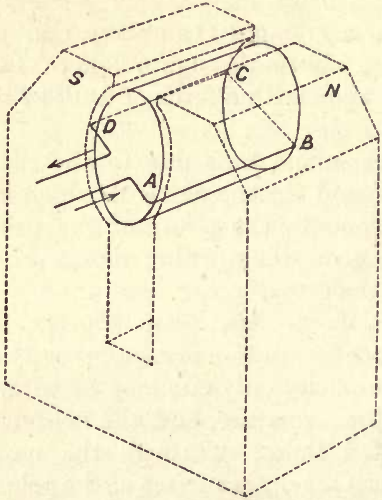

Fig. 18.

in the conductor A B it is easy to see that the tendency will be to push the conductor upwards, giving the armature a counter clockwise rotation, as seen from the end at which the current enters. If, in Fig. 17, we reverse the magnet so that the south pole, S, occupies the centre of the circle, and imagine, again, a man swimming in and with the current whilst looking to the south pole, so that the lines of force pierce him at the back, then the conductor will be pushed to the left. By applying the same reasoning to the conductor C D, in Fig. 18, we find that it will be pushed downwards, which will also produce a counter clockwise rotation.

The application of formula (9) may be shown by the following example: In modern dynamos and motors the strength of field in the interpolar space may be taken as about 5,000 C.G.S. units. Assume that a current of 100 amperes flows through the wire A B, in Fig. 18, then the force acting upon, say, 10 centimetres of wire is given by formula (9) as ·5097 of a kilogramme, or 1·12lb. This corresponds in English measure to about 3·4lb. per foot of wire if the current is 100 amperes. We may thus say that with the strength of field customary in ordinary dynamos and motors every foot of wire under the influence of the field is subjected to a force of a little less than 3½lb. when the current passing through the wire is 100 amperes. With larger or smaller currents this force will, of course, be proportionately larger or smaller.

The English System of Measurement.

The metric system of measurement, which we have up to the present employed, although the most rational and simple for purely scientific work, is not always convenient for the workshop, and for such purposes another system, which has received the name of the English system of measurement, is often employed. In this system we reckon forces in pounds, lengths in inches, and magnetic induction in a unit 6,000 times as great as the corresponding C.G.S. unit. This particular relation of 1 to 6,000 has been adopted on account of certain calculations required in the design of dynamos, as will be explained in a later chapter. For the present it suffices to note that a total flow of force of 6,000 C.G.S. lines is equivalent to a total flow of force represented by one English line; that 120,000 C.G.S. lines are equivalent to 20 English lines and so on. The induction per square centimetre, or what is briefly called "induction," in the C.G.S. system can be expressed by its equivalent in the English system as a density of so many English lines per square inch. Calling B the induction in English measure, and $\mathfrak{B}$ the induction in the C.G.S. measure, we find by a simple calculation that the two are related as follows—

$$\mathsf{B} = \frac{\mathfrak{B}}{930 \cdot 04} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

By introducing this term and substituting inches for centimetres in equation (9), we find

$$\text{P pounds} = 531 \, c \, l \, \mathsf{B} 10^{-6} \quad . \quad . \quad . \quad (11)$$

By the aid of equations (9) or (11), we can at once find the torque or turning moment of an armature of the kind represented in Fig. 18. Let, by way of example, the diameter of the armature measured to the centre of the conductors be 12in. and its length 20in. Assume that there are 120 conductors on the outside of the armature, and let the pole-pieces embrace an angle of 120 deg. on each side. Then there will be under each pole-piece—that is, within the field on each side—

40 wires. Let the current passing through the armature be 400 amperes. It will be shown in a subsequent chapter that the current passing through an armature of this description splits at a certain point of the commutator into two branches, each branch or half going successively through half the wires and the two branches uniting again at the opposite point of the commutator. Thus, in our case, each wire will carry a current of 200 amperes, and the direction of these currents is towards the commutator in all wires lying to one side of the vertical (the so-called neutral) diameter of the armature, and from the commutator on the other side of that diameter. Thus the currents in both branches conspire to produce torque.

Assuming the strength of field to be equivalent to an induction of five lines (English measure), then the force exerted by one of the wires under a pole-piece is from (11)

$$P \text{ pounds} = 531 \times 200 \times 20 \times 5 \times 10^{-6}$$

$$P = 10{\cdot}62\text{lb.}$$

As there are 40 wires on each side we have a tangential force on the surface of the armature of $2 \times 40 \times 10{\cdot}62$lb., or

849·6lb.,

which force, acting on a radius of ·5ft., produces a torque of

424·8 foot-pounds.

If we know the speed at which the armature is driven, we can at once find the energy required to overcome the torque due to the electromagnetic interaction between the field and the armature current. The work done in one revolution is obviously $849 \cdot 6 \times 3 \cdot 14 = 2,670$ foot-pounds. If the armature makes 500 revolutions per minute the energy is $\frac{500 \times 2,670}{33,000} = 40 \cdot 5$ h.p.

From what has been said in the first chapter it will be clear that this energy must represent the product of current and electromotive force divided by 746, and since we know the current, which was assumed to be 400 amperes, we can calculate the electromotive force, E, from the equation

$$40 \cdot 5 \times 746 = 400 \text{ E}$$

$$\text{E} = 75 \cdot 5 \text{ volts.}$$

In a subsequent chapter it will be shown how the electromotive force of an armature can be calculated without the preliminary process of determining the torque.

CHAPTER V.

The Electromagnet—The Solenoid—Magnetic Permeability—Magnetic Force—Line Integral of Magnetic Force — Total Field — Practical Example — Extension of Theory to Solenoidal Electromagnets—Magnetic Resistance.

The Electromagnet.

Up to the present, we have spoken of magnets and the field surrounding their poles without enquiring how the magnets were produced. It was, indeed, immaterial, for our purpose, to know whether the magnet was a so-called "permanent" steel magnet or an electromagnet, so long as it had the required strength of magnetisation. In practice, however, it is not possible to obtain this strength of magnetisation by the well-known process of stroking a steel bar with a loadstone, and we are forced to employ electromagnets—that is, pieces of soft iron which have been rendered magnetic by passing an electric current round them. The action of dynamo machines, so far as we have, up to the present, investigated it, depends entirely upon the strength of the magnets employed; and before we can establish a working theory of such machines we must answer the question as to how such magnetism will be developed in a given piece of iron by passing a certain current so many times round it. In other words, we must find the relation between current-turns, or, as they are commonly called, "ampere-turns," and the total flow of magnetic force which they produce. It will at once be evident that the exact manner in which the product of turns and current is made up is, for our present purpose, quite immaterial. Whether we have a small current going round a piece of iron many times, or a large current going round the same piece of iron only a few times, the resulting magnetism will be the same in both cases, provided the space occupied by the wire and the product of current and number of turns are the same. That this must be so can easily be seen if we imagine two, three, or more of the convolutions of the small wire carrying the small current grouped into one, which would then carry a current two, three, or more times the strength of the small current; and by applying this substitution over the whole coil, we can pass from one case to the other without having changed the final result.

Fig. 19 represents an electromagnet of the form usually found in physical laboratories, and intended to show that a piece of soft iron can be made magnetic by passing a current round it. A bar of round iron is bent into the rectangular shape shown, and forms the magnet, N Y S. Over the vertical limbs are placed two coils of insulated copper wire, C C, and across the poles of the magnet is laid an armature, A, to which a weight, Q, is attached to show the power of the magnet. As long as the current flows in the direction indicated by the arrows the weight is supported, but if the current is interrupted the weight generally falls. We use the word "generally" advisedly, because, as a matter of fact, the weight does not always fall upon stopping the current, the reason being that in a closed magnetic circuit the flow of force persists to a certain extent

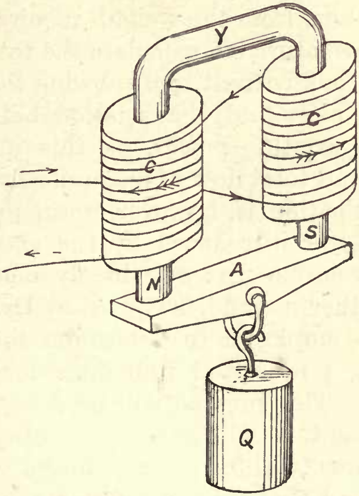

FIG. 19.

after the cause which created this flow in the first instance has ceased to act. When therefore the weight is much smaller than the electromagnet could support the residual flow of force may be sufficient to keep the weight up, but as this is merely a secondary effect we need not further consider it, and can assume that the ability to support the weight is lost as soon as the current is interrupted. Experiment shows, also, that the amount of weight which a magnet can support increases up to a certain point with the current. It was shown on page 62 that between the sustaining power of a magnet and its strength of pole, or rather the induction passing through its polar surface, there exists a definite relation, and by making use of the formula then given we might thus, from the weight required in each case to tear the armature off, calculate the total induction produced by each current, and knowing the number of turns in the coils find the relation between the ampere-turns (or exciting power, as this quantity is generally called) and total flow of magnetic force. This method of investigation is, however, unsatisfactory, on account of the error introduced by the effect of the edges of the polar surfaces, as was already mentioned on page 63, and another method, first used by Dr. Hopkinson, is generally employed to determine the relation between exciting power and induction for different samples of iron. This method will be described later on; for the present it will suffice to note that the induction increases within certain limits with the exciting power, and that for a given exciting power it is the greater the softer the iron.

The Solenoid.

We have seen that a single circular loop of wire traversed by a current, Fig. 17, acts upon a magnet as if it were a magnet itself, and if such a loop is delicately suspended so as to be freely capable of rotation round its vertical diameter it will behave very much like a compass needle—that is to say, it will set itself into such a position that the plane of the loop points east-west, the top half circle being traversed by a current in a west-east and the lower half circle by a current in an east-west direction. Now let us imagine that instead of a single circular loop we have a succession of spiral

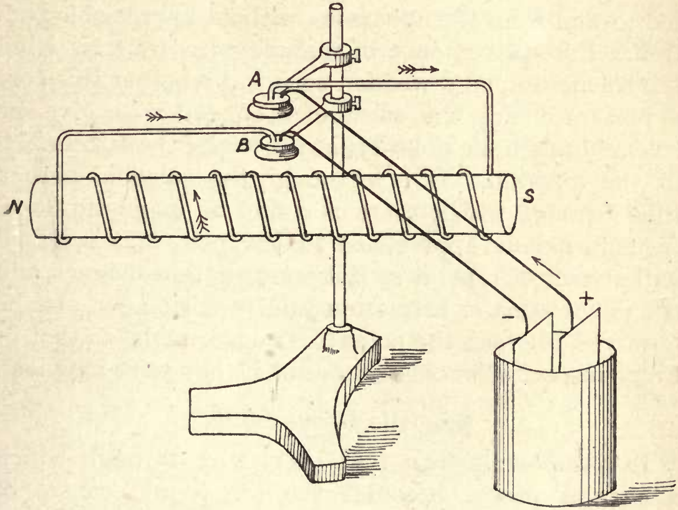

Fig. 20.

loops, as shown in Fig. 20, forming together a cylindrical coil. Such a coil is called a solenoid, and with it we can reproduce all the effects of a straight bar magnet. If we lay a sheet of paper over it and sprinkle iron filings we find the same arrangement of lines of force as with a real steel magnet, and if we suspend it in a magnetic field (that of the earth, for instance) its axis will point north-south as if it were a compass needle. This experiment can be performed with the apparatus shown diagrammatically in Fig. 20. The ends of the wire are bent as shown, and dip into mercury cups supported on the horizontal arms of a stand, so as to allow of free rotation. The mercury in the cups are connected by wires with the terminals of a cell, and the current is thus led to and from the apparatus without appreciable friction. For convenience of manufacture we may wind the solenoid upon a wooden core, but whether this core is present or not, the effect of a current is to give the solenoid magnetic polarity, as shown by the letters N S in the diagram. It is as though the solenoid were a tube through which there is a flow of magnetic force. A magnet pole approached to the pole of a solenoid will attract or repel it as if it were another magnet, and the phenomena of attraction and repulsion can also be observed between the poles of two solenoids, so that in every respect solenoids behave as if they were magnets.

Magnetic Permeability.

It has been shown that every wire through which a current passes becomes the origin and centre of a magnetic whirl, and in a solenoid where all the wires lie close together these separate whirls merge into one common flow of magnetic force which passes into the centre cylindrical space at or near the south pole of the solenoid, traverses its length in stream-lines which are more or less parallel to the axis of the cylinder, and passes out at the north pole. In the space surrounding the solenoid the lines of force curve round from the north to the south pole, as is easily seen by the iron filings. If we now insert a bar of iron into the solenoid in lieu of the bar of wood, we find that the external field has become much stronger. The filings arrange themselves in denser lines, the directive power of the suspended solenoid, Fig. 20, has become much stronger, and the forces of attraction and repulsion are more considerable. We conclude that the presence of the iron has induced a flow of force which is many times as strong as was previously the case. It is as though the iron offered additional facility to the path of the lines, or were more permeable to them than air. This property of increasing the flow of force is accordingly called the "permeability" of the iron, and different samples of iron are distinguishable by the greater or lesser degree in which they have this property. The permeability is, therefore, a numerical coefficient denoting the ratio in which the presence of iron multiplies the number of lines of force previously existing in a magnetic field. It is generally denoted by the symbol μ; and if we call the original strength of field (that is, the induction through each square centimetre of field) H, and the induction through the iron after it has been placed into the field, $\mathfrak{B}$, we have the following relation:

$$\mu = \frac{\mathfrak{B}}{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Conversely, if we know the permeability we can find the induction by the formula

$$\mathfrak{B} = \mu H \quad . \quad . \quad . \quad , \quad . \quad (11)$$

The permeability of a given sample of iron is not a constant, but varies with the induction in a manner which cannot be accurately expressed by any mathematical formula, but must be determined experimentally for each sample. Now, what we require to know in the designing of dynamos is the induction which we shall obtain with a given system of field magnets excited by a certain number of ampere-turns, and we see that the induction is the product of two factors—one the permeability, which must be found by experiment, and the other the original flow of force, which induces the magnetism in the iron, and which we therefore call "magnetising force" or "magnetic force." Between the latter and the exciting power given in ampere-turns there exist definite relations which are capable of being represented by formulæ, and our problem has therefore been brought down to the question, What are the laws determining the relation between exciting power and magnetising force, and how are these laws affected by the size and shape of the magnets?

Magnetic Force.

Imagine a uniform magnetic field of the strength H. In such a field the lines of force are all straight parallel lines, and there are H unit lines per square centimetre of cross-section. Define in this field a cylindrical space of length l and area A, the axis of the cylinder being parallel to the lines of force. Now let a unit exploring pole be moved from a point on one of the end faces of this cylinder to a point on the other end face. The work which must be done in transit is obviously H l, and, as was shown in Chapter II., it is independent of the route over which we carry the exploring pole. This work is equal to the difference of magnetic potential between the two points. We thus find that $\mathrm{H}\,l$ is simply the difference of magnetic potential between the two end faces of our cylindrical space. We can now, by multiplying and dividing the right-hand side of equation (11) by l, bring it into the form

$$\mathfrak{B} = \frac{\mu\,\mathrm{H}\,l}{l}$$

which shows that the induction is proportional to the difference of magnetic potential, to the permeability, and inversely proportional to the length of the space. Now imagine a series of such cylindrical spaces, each of a different length, but all of the same cross-section, a, and let these spaces be filled by materials of different permeabilities. Let the lengths be respectively l_1, l_2, l_3, etc., and the permeabilities μ_1, μ_2, μ_3, etc., then we can establish the following equations:

$$\frac{\mathfrak{B}\,l_1}{\mu_1} = \mathrm{H}\,l_1$$

$$\frac{\mathfrak{B}\,l_2}{\mu_2} = \mathrm{H}\,l_2$$

$$\frac{\mathfrak{B}\,l_3}{\mu_3} = \mathrm{H}\,l_3.$$

By adding these equations we obtain

$$\mathfrak{B}\left(\frac{l_1}{\mu_1} + \frac{l_2}{\mu_2} + \frac{l_3}{\mu_3} + \quad . \quad . \quad \right) = \mathrm{H}\,(l_1 + l_2 + l_3 +) \quad . \quad . \quad (12)$$

Line Integral of Magnetic Force.

It will be seen that the term on the right of this equation is simply the work which must be done upon

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unit pole in bringing it from one end to the other of our chain of cylinders. Nothing would have been altered in our reasoning if instead of a field of straight lines we had assumed one of curved lines, provided we had suitably altered the shape of our cylindrical spaces.

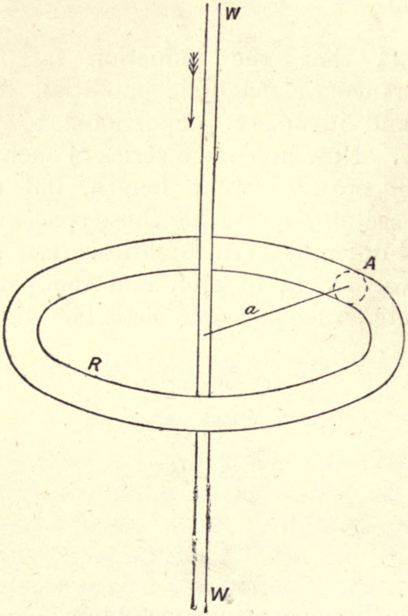

FIG. 21.

The spaces filled by materials of different permeability might in this case form a complete chain closed on itself, and our exploring pole would then start from and arrive at the same point, but the journey could no longer be taken along any arbitrary route. It must now be taken along a path, looping once round the conductor, the current in which produces the field, and for convenience we may choose a route along the lines of force. The work done is therefore the line integral of the magnetic force taken once round the closed magnetic circuit, and if we divide this quantity by the term $\left(\frac{l_1}{\mu_1} + \frac{l_2}{\mu_2} + \frac{l_3}{\mu_3} + \quad . \quad . \quad . \right)$ we obtain the induction. An example will make this matter clear. Let, in Fig. 21, W W be a straight wire of very great length, through which flows a current, c, in the direction indicated by the arrow. Define round this wire a ring-shaped space, R, of section A and radius a. The flow of magnetic force within this space, as seen from above, has a clockwise direction, so that if we carry a unit north pole once round the ring in a counter clockwise direction we must perform work. It has been shown in equation (5) that the induction in air or the strength of field of such a current is $\frac{2c}{a}$, which represents the force resisting the movement of the exploring pole at every point of its journey. To find the work done in carrying the pole once round the ring we multiply this force with the distance traversed, which is $2\pi a$, and find

$$\text{Line integral of magnetic force} = 4\pi c. \quad . \quad (13)$$

The induction can now be found from equation (12).

$$\mathfrak{B} = \frac{4\pi c}{\frac{l_1}{\mu_1} + \frac{l_2}{\mu_2} + \frac{l_3}{\mu_3} + \quad . \quad . \quad .} \quad . \quad . \quad . \quad . \quad (14)$$

whilst the total field strength of flow of lines is

$$F = A\,\mathfrak{B}.$$

It should be noted that the radius of the ring has vanished from the equation. It is implicitly contained in the terms which denote the lengths of the different sections, for the larger the radius the larger will also be these terms, but except in this way the radius has no influence on the induction. We conclude from this fact that the true circular shape of the ring is not essential,

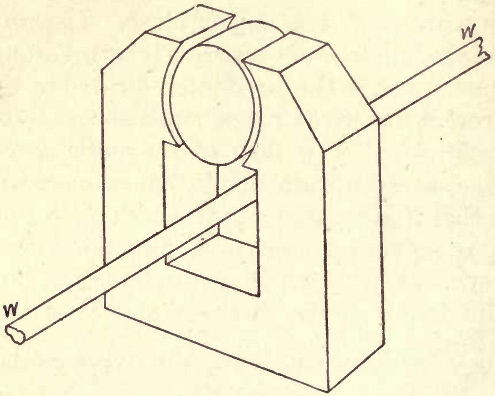

FIG. 22.

and that a ring of any shape would give the same induction provided the length of the different sections were the same as in the circular ring. This conclusion also follows from the fact that the work done upon unit pole is independent of the exact route traversed, provided this route loops once round the conductor, the current in which produces the field. Instead of grouping our materials so as to form a truly circular ring round the conductor, we may therefore group them in any other way, the only essential condition being that they shall form a closed circuit round it. An arrangement, such as is shown in Fig. 22, will therefore be magnetically equivalent to that shown in Fig. 21. We have here the field-magnet system and armature of a dynamo, which form together a closed magnetic circuit round the wire, W W, through which the current, c, flows. If the cross-section of the magnetic circuit were the same at every point, formula (14) could at once be used to determine the induction across the armature in this dynamo, but for reasons which will be explained later on, it is not customary to make the cross-section of the different parts in a machine the same, and before we can use the formula in this case, we must alter it so as to be applicable to magnetic circuits of varying cross-section.

Total Field.

Let A_1, A_2, A_3, etc., denote the cross-sections of the various parts, and $\mathfrak{B}_1$, $\mathfrak{B}_2$, $\mathfrak{B}_3$, etc., the corresponding inductions, then the total flow of lines, which must obviously be a constant for the whole magnetic circuit, is given by the expressions $A_1 \mathfrak{B}_2$, $A_2 \mathfrak{B}_2$, etc.

Formula (11), which we have brought into the form

$$\mathfrak{B} = \frac{\mu H l}{l},$$

can now also be written as follows:

$$F \left(\frac{l_1}{\mu_1 A_1} \right) = H l_1.$$

$$F \left(\frac{l_2}{\mu_2 A_2} \right) = H l_2.$$

$$F \left(\frac{l_3}{\mu_3 A_3} \right) = H l_3.$$

And by adding these equations we obtain

$$F\left(\frac{l_1}{A_1}\frac{1}{\mu}+\frac{l_2}{A_2}\frac{1}{\mu_2}+\frac{l_3}{A_3}\frac{1}{\mu_3}+\ .\ .\right) = H\,(l_1+l_2+l_3+\ \ .\ \ .\ )$$

The term on the right is, as shown above, the line integral of the magnetic force emanating from the current in the wire, W W, and is given by the expression $4\,\pi\,c$, so that we obtain for the total flow of lines the expression

$$F = \frac{4\,\pi\,c}{\frac{l_1}{A_1}\frac{1}{\mu_1}+\frac{l_2}{A_2}\frac{1}{\mu_2}+\frac{l_3}{A_3}\frac{1}{\mu 3}+\ .\ .\ .} \quad . \quad . \quad (15)$$

$$F = \frac{4\,\pi\,c}{\Sigma\,\frac{l}{A}\,\frac{1}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Practical Example.

The application of formula (16) can best be shown by an example. For this purpose we take a dynamo having an armature of 30 centimetres diameter and 50 centimetres length. We wish to find what strength of current will be required in the straight wire, W W, Fig. 22, in order to obtain a total flow of 1,000 English lines ($6\,.\,10^6$ C.G.S. lines) through the armature. The magnetic circuit may be divided into three parts—namely, the field magnets the air spaces, and the armature—the lengths of which we assume to be respectively 140, $2 \times 2 = 4$, and 30 centimetres. The length of each air space we take as 2 centimetres, but as there are two such spaces we must double this figure. Let the cross-sectional area of the field magnets be

800 square centimetres, that of the air spaces 1,800 square centimetres, and that of the armature 500 square centimetres. The permeability of the armature core we assume as 1,000, and that of the field magnets as 2,000. The permeability of the air space we take as unity. Inserting these values in equation (16), we find

$$6 \times 10^6 = \frac{1{\cdot}256\, c}{\dfrac{4}{1800} + \dfrac{140}{800 \times 2000} + \dfrac{30}{500 \times 1000}}$$

if c is to be given in amperes. We find from this in round numbers

$$c = 11{,}400 \text{ amperes.}$$

If we lay a long straight wire through the limbs of the field magnets of this machine, and send a current of 11,400 amperes through this wire, we shall obtain the desired total field of six million C.G.S. lines. It would of course be quite impracticable to employ such an enormous current to excite the field magnets, and to get over this difficulty we would naturally employ, not a single conductor carrying the whole of the current, but a large number of conductors laid side by side, each carrying a portion of the total current. We might, further, loop the ends of the different conductors together so that the same current shall traverse them successively, and by making the loops as short as possible, so as to save wire and reduce the resistance of the coil, arrive at the usual form of field-magnet coils. Instead of a straight current of 11,400 amperes we would thus have a coil of 11,400 ampere-turns. Now, however, the question presents itself whether such a coil, wound closely round the magnets, is really equivalent to the long straight wire passing between the limbs. At first sight this would not seem to be the case, because the equation (5), on which we had based our calculations, is, strictly speaking, only correct for a wire infinitely long, a condition not even approximately fulfilled by a coil of finite perimeter. Before we can accept equation (16) for the calculation of dynamos we must, therefore, verify it in reference to the usual form of field-magnet coils.

Extension of Theory to Solenoidal Electromagnets.

We have seen that a solenoid provided with an iron core becomes an electromagnet as soon as we send

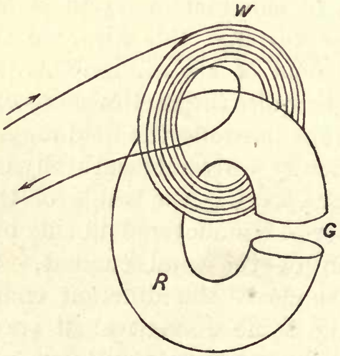

FIG. 23.

a current through the wire coils of the solenoid. Hitherto we have supposed the core to be straight, and of about the same length as the solenoid, but as in dynamos we must have a closed magnetic circuit, we shall now suppose the core bent so as to form a ring interlinked with the ring formed by the wire coil. This arrangement is shown in Fig. 23, where W represents the coil of insulated wire receiving current in the direction indicated by the arrows from some source, and R a ring which may be composed of

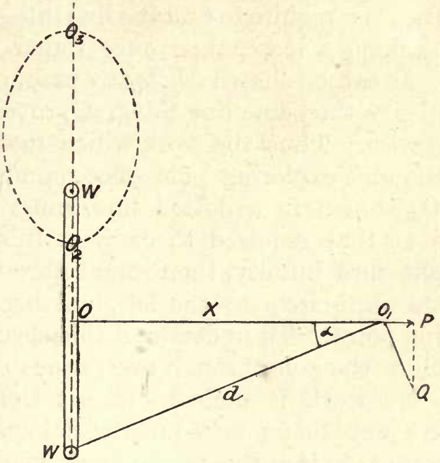

FIG. 24.

different sections of different permeabilities. One of these sections may be an air gap, G, in which the permeability is 1, and this corresponds in a dynamo to the spaces between the polar and armature surfaces. The problem now is to find the line integral of magnetic force due to the current in the coil, W, the integration being performed once round the magnetic circuit, R.

Let, in Fig. 24, W W be the section through one turn of wire in the coil, taken at right angles to its plane, and let a be the radius of the coil. The magnetic whirl round the wire consist of lines which cut the plane of the coil at right angles. The plane of the coil must therefore be an equipotential surface, and no work is performed in moving an exploring pole over the plane surface encircled by the wire, either within or without. We require to find the line integral of magnetic force along a loop taken once round any part of the wire. It can be shown that any path, of whatever shape, will give the same line integral, provided it loops round the wire. Thus the work which must be done to carry the unit exploring pole once round the oblong path, $O_2 O_3$, shown in a dotted line, must be exactly the same as that required to carry it from O, away to the right into infinity, then round through an infinitely large semi-circle to the left, and back again to the starting point. To understand this clearly we have only to follow the pole through the various stages of its journey. No work is done in transit from O_2 to O, since both points lie on an equipotential surface. The journey from O to infinity absorbs work, whilst no work is absorbed in any movement which the exploring pole makes at an infinite distance. Thus the quarter circle through which we must carry the pole until it meets the plane $O O_2 O_3$ absorbs no work, and as this plane is itself an equipotential surface, we find that the magnetic potential at O_3 is exactly the same as that at a point on the line $O O_1$ at an infinite distance from O. To carry our unit exploring pole from O to infinity to the right requires, therefore, exactly the same amount of work as to carry it from O_2 to O_3 along the finite path shown by the dotted line. Similarly, to carry it from infinity on the left back to O requires the same amount of work as is consumed in the second part of the dotted circuit—namely, from O_3 to O_2. As the same reasoning applies to any closed circuit looping round the wire we see that the line integral along any such circuit is equal to that found for the journey of unit pole from an infinite distance on the left through the circle W W, and out to an infinite distance to the right. This integral can easily be found as follows: Assume the pole arrived at the point O_1. An element, l, of the conductor acts upon it with a force,

$$Q = \frac{l c}{d^2}$$

in the direction $O_1 Q$. The horizontal component of this is

$$P = \frac{l c}{d^2} \sin \alpha,$$

and as this equation applies to every element, we find the total force due to the whole circular conductor

$$P = 2 \pi a c \frac{\sin \alpha}{d^2},$$

since $d = \frac{a}{\sin \alpha}$, we have also

$$P = \frac{2 \pi c}{a} \sin^3 \alpha.$$

The work done in a small displacement, $d x$, of the exploring pole is

$$P \, d x = \frac{2 \pi c}{a} \sin^3 \alpha \, d x.$$

Now $x = a$ cotg. α and $d\,x = -\dfrac{a\,d\,\alpha}{\sin^2\alpha}$, which inserted gives

$$P\,d\,x = -2\,\pi\,c\sin\alpha\,d\,\alpha.$$

Integrating this equation between the limits of $\alpha = 0$ and $\alpha = \pi$ we find

Line integral of magnetic force $= 4\,\pi\,c$ . . . (17)

exactly the same expression as we found in (13) for the straight conductor infinitely long. We thus find that the expression (16)

$$F = \frac{4\,\pi\,c}{\Sigma\,\frac{l}{A}\,\frac{1}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

is also applicable to solenoidal electromagnets. It is obviously immaterial whether the coil in Fig. 24 consists of only one circular wire, or of a wire making many turns, since neither the diameter nor the thickness of the coil enter into equation (17), and the integral has been taken over a line infinitely long. We may thus subdivide the whole current into a number of wires lying either closely side by side, or spread over a certain length of coil, and the result will be the same, provided we take the magnetic circuit through all the turns of the coil. We see, therefore, that the magnetic effect is independent of the shape of the coil. If the coil consists of several wires the magnetic effect will of course be due to the product of the number of turns and the current passing through each turn; it will, in fact, be due to the ampere-turns. Let, now, c be the current in each wire, and τ the number of turns in the coil, then equation (16) must be written as follows:

$$F = \frac{4 \pi c \tau}{\Sigma \frac{l}{A} \frac{1}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

It must not be forgotten that the current in this formula is to be taken in C.G.S. units. If it is taken in amperes we must divide by 10, and have

$$F = \frac{\cdot 4 \pi c \tau}{\Sigma \frac{l}{A} \frac{1}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This expression may conveniently be brought into another form. Suppose the magnetic circuit is composed of three sections of different length, area, and permeability, which we distinguish by the indices 1, 2, and 3, then we can also write

$$\cdot 4 \pi c \tau = \frac{F\, l_1}{A_1 \mu_1} + \frac{F\, l_2}{A_2 \mu_2} + \frac{F\, l_3}{A_3 \mu_3} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The term on the left is the line integral of the magnetic force taken once round the magnetic circuit, or the total difference of magnetic potential under which the flow of force, F, is produced; the terms on the right show how this total potential difference is divided between the different sections of the magnetic circuit. They are of the character: flow multiplied by an expression which contains the length of the section in the nominator, and the product of area and permeability in the denominator.

It will be at once apparent that a very remarkable analogy exists between formula (20) as expressing the property of a magnetic circuit and Ohm's law expressing the property of an electric circuit. To see this clearly, we have only to substitute for the flow of magnetic force strength of current, for the permeability the specific conductivity or the reciprocal of the specific resistance, and for $\cdot 4 \pi c \tau$ the electromotive force.

According to this analogy the terms of the form $\frac{l}{A \mu}$ must be regarded as the magnetic resistances of those parts of the magnetic circuit to which they refer, and we may translate Ohm's law from the electric into the magnetic circuit as follows: *The magnetomotive force (line integral of the magnetic force) is equal to the product of the total flow of magnetic force multiplied with the total magnetic resistance.*

The conception of magnetic resistance is a very convenient one, and helps to greatly simplify the calculation of dynamo-electric apparatus; but on strictly scientific grounds it is open to some objections. As we shall in future make frequent use of the term magnetic resistance, it is desirable to at once state what these objections are, and how far they are justified. The principal objections are that the overcoming of magnetic resistance, unlike that of electric resistance, does not necessitate the expenditure of energy, and that the magnetic resistance is not constant, but varies with the degree of induction—that is, with the total flow of magnetism. As regards the first objection, this no doubt is justified. If we apply an electromotive force to the ends of a conductor in such way as to cause a current to flow, the conductor becomes heated, and under no conceivable arrangement can the corresponding loss of energy be avoided. With the magnetic circuit the case is quite different. It is true that the exciting coil through which we pass the magnetising current must have some resistance, and to that extent energy will be required to make the current flow through it, but we can reduce this energy to any desired extent by making the wire of large size, without in any way altering the magnetic flow. Moreover, we can produce such a flow without the use of any wire coil at all—if we chose, as the source of magnetism, a permanent steel magnet. It is therefore necessary, when speaking of magnetic resistance, to always bear in mind that it is not a resistance in the ordinary sense of the word—that is, one which can only be overcome by the expenditure of energy—but rather of the nature of that resistance which bodies offer to forces tending to produce deformation.

The second objection mentioned above is not so well founded, because the electric resistance of a circuit is also liable to vary with the current passing through it. The specific resistance of all metals increases with temperature, and as the temperature rises as the current is increased, it follows that the larger the current the higher will be the electric resistance of those parts of the circuit which are of metal. This is precisely the relation between flow of magnetism and magnetic resistance. The greater the flow the smaller is the permeability, and the greater its reciprocal, which is a measure for the magnetic resistance, so that in this respect there is only a difference in degree, but not one in kind between the electric and magnetic circuit.

From the structure of formula (20), it will at once be apparent that the total magnetomotive force, acting in a given magnetic circuit, is the sum of the magnetomotive forces required in the different parts of the circuit, and that, in fact, Ohm's law applies not only to the circuit as a whole, but also to every part of it. We may therefore establish the general proposition: "*The flow of magnetism through any section of a magnetic circuit is the quotient of the magnetomotive force in that section (difference of magnetic potential between the beginning and the end of the section) and its magnetic resistance.*"

From this proposition it follows that where several paths are offered to the flow of magnetism under the same magnetomotive force, the flow will divide amongst them in the inverse ratio of their magnetic resistances. Thus, in a dynamo, there is between the pole-pieces a multiplicity of paths for the flow of magnetism. One path is through the armature, and the lines taking this path are the only lines of use in the working of the machine. In addition, there are, however, other lines which take a path through air from one pole-piece to the other, and these lines, although, like the others, created by and passing through the exciting coils, are lost to the purpose for which the machine is constructed. We shall revert to this subject when we come to deal with the leakage or waste field of dynamo machines.

For the practical application of formula (20), it is convenient to bring it into a slightly different form by dividing both sides by $\cdot 4\,\pi$, so as to obtain directly the exciting power in ampere-turns

$$c\,\tau = \mathrm{F}\left(\frac{1}{1\cdot256}\,\frac{l_1}{\mathrm{A}_1}\,\frac{1}{\mu} + \frac{1}{1\cdot256}\,\frac{l_2}{\mathrm{A}_2}\,\frac{1}{\mu_2} + \frac{1}{1\cdot256}\,\frac{l_3}{\mathrm{A}_3}\,\frac{1}{\mu_3}\right) \quad . \quad (21)$$

or, briefly,

$$c\,\tau = \mathrm{F}\,\mathrm{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where

$$\mathrm{R} = \Sigma\,\frac{1}{1\cdot256}\,\frac{l}{\mathrm{A}}\,\frac{1}{\mu} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

which we conventionally call the sum of all the magnetic resistances. When, in future, speaking of magnetic resistance, it will be understood that the term given in equation (23) is meant.

In equation (21) the flow of lines is given in the C.G.S. system, the lengths are in centimetres, and the areas in square centimetres. For the use in the workshop it is, however, convenient to give dimensions in inches, and to employ the English unit for the flow of magnetism. Let Z be the flow in English measure, and l be given in inches, and A in square inches, then a simple arithmetical operation shows that formula (21) becomes

$$c\,\tau = \mathrm{Z}\left(1880\,\frac{l_1}{\mathrm{A}_1}\,\frac{1}{\mu_1} + 1880\,\frac{l_2}{\mathrm{A}_2}\,\frac{1}{\mu_2} + 1880\,\frac{l_3}{\mathrm{A}_3}\,\frac{1}{\mu_3}\right) \quad . \quad (24)$$

In the English system of measurement the magnetic resistance is therefore given by the expression

$$\mathrm{R} = \Sigma\,1880\,\frac{l}{\mathrm{A}}\,\frac{1}{\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

When the permeability is 1, as in the interpolar space of a dynamo or motor, this formula becomes

$$\mathrm{R}a = 1880\,\frac{l}{\mathrm{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where we use the index a to show that the resistance refers to air and not to iron. The coefficient 1880 is only valid if the permeability of the copper, which partly fills the interpolar space, is equal to that of air, which is unity. Commercial copper is sometimes slightly magnetic, and then a somewhat smaller coefficient must be used. It should also be noted that the area A is not only the area of the pole-piece, but this area with an addition of a certain fringe due to the spreading of the lines near the edges of the pole-piece, a subject to which we shall return in a later chapter.

CHAPTER VI.

Magnetic Properties of Iron—Experimental Determination of Permeability—Hopkinson's Method—Energy of Magnetisation—Hysteresis.

Magnetic Properties of Iron.

According to the definition given on page 93, magnetic permeability is a numerical coefficient denoting the ratio in which the presence of iron multiplies the number of lines of force previously existing in a magnetic field. This multiplying power is different with different samples of iron, and it also varies for the same sample with the strength of the original field, or, what comes to the same thing, with the magnetising force, and therefore also with the induction produced. It is customary to state the permeability as a function of the induction, or of the magnetising force, and thus the magnetic quality of any sample of iron can be expressed in the shape of a table or a curve. These curves may be constructed to represent the following relations: Magnetising force and permeability. Induction and permeability, or magnetising force and induction. The latter is the most directly obtainable and useful curve.

If the magnetising force be removed the iron does not return to its original state—that is, to the state in which it was before any magnetising force had been applied; but it retains a certain amount of magnetisation called the "residual magnetisation," and which may be numerically expressed by the corresponding induction. To each magnetising force and consequent induction corresponds, therefore, a definite residual induction, which can also be represented by a curve. If the magnetising forces be plotted as abscissæ and the inductions as ordinates, the curve of residual induction is found to have a similar shape with the curve of induction, but to lie wholly below it. It may here be mentioned that if the sample, whilst being tested for its magnetic properties, is subjected to mechanical vibrations or strains, the curve of induction is slightly raised and the curve of residual induction is considerably lowered.

Another point which should be noted is the difference between an ascending and a descending curve of magnetisation. If we first test a sample by applying gradually higher and higher magnetising forces, taking care to determine the induction at each step, and plot the results, we obtain an ascending curve of magnetisation. As the induction becomes larger its increment for equal increments of magnetising force becomes smaller and smaller, the curve becoming more and more flat, until a point is reached when no increase of magnetising force produces any increase in the induction. When in this condition the iron is said to have attained a point of saturation, its permeability being reduced to zero. In what follows we assume that the magnetisation has not been carried so far, but only to a certain point.

If we now gradually reduce the magnetising force, and again plot the induction at each step, we obtain a descending curve of magnetisation, which lies wholly outside of the former curve, and passes through the axis of ordinates (corresponding to zero magnetising force) at a point above the centre of co-ordinates. The distance of this point of intersection from the centre of co-ordinates represents the induction which is still left in the sample after the magnetising force has been gradually reduced to zero, and this induction is called the " retentiveness " of the sample. Now suppose that when the magnetising force has been gradually brought down to zero, we reverse it so as to demagnetise the sample beyond its point of retentiveness, and gradually increase the magnetising force which is now acting in a negative direction, until the previous induction but in the reverse sense is reached, we obtain the ascending negative branch of the curve of magnetisation. Decreasing now the negative or reverse magnetising force to zero, and reversing it a second time so as to make it positive, and gradually increasing it to the previous value, will give us, first, the descending negative, and, finally, the ascending positive branch of the curve of magnetisation, which brings us again to the point from which the descending curve of magnetisation starts. We have thus carried the iron through a complete cycle of magnetisation from a certain positive induction, through zero to an equal negative induction, and back through zero to the starting point. The closed curve representing this cycle cuts the axis of co-ordinates in four points—namely, the axis of ordinates above and below the centre in points, which give the retentiveness of the sample, and the axis of abscissæ in points to the right and left of the centre, which show what amount of reversed magnetising force is required to reduce the induction of the sample to zero. Dr. Hopkinson, in his celebrated paper on the "Magnetisation of Iron," read before the Royal Society in 1885, has suggested that this reverse magnetising force should be called the "coercive force" of the sample, if the curves have been obtained by extreme magnetising forces in both directions.

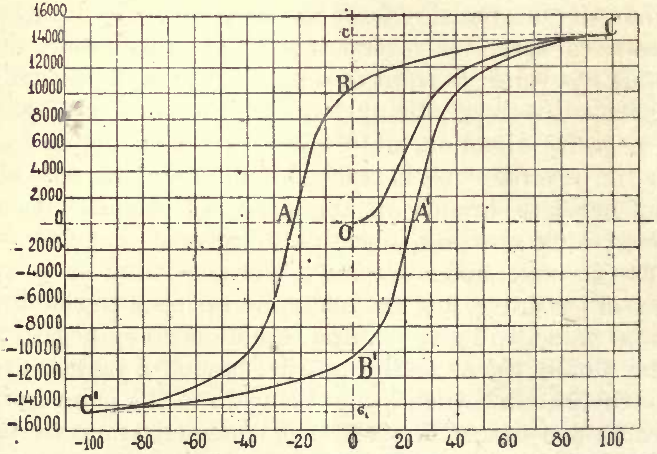

FIG. 25.

It will be convenient to illustrate the various terms here introduced by a diagram. Let, in Fig. 25, the curve, O C, represent the magnetising curve of a certain sample of iron which has not previously been subjected to any magnetising force. Arrived at C, we gradually reduce the magnetising force to zero, and obtain the portion, C B, of the positive descending curve of magnetisation. We next reverse the magnetising force until it has reached the negative value, O A, when the remainder, B A, of the positive descending curve of magnetisation is obtained. By still further increasing the negative magnetising force we obtain the negative ascending curve of magnetisation, A C^1, and by gradually reducing the magnetising force to zero and increasing it to O A^1, we can plot the negative descending curve of magnetisation, C^1 A^1. A further increase of magnetising force gives us, finally, the ascending curve of magnetisation, A^1 C. The diagram gives us now

$$O B = O B^1 = \text{Retentiveness.}$$

$$O A = O A^1 = \text{Coercive force.}$$

It is especially the latter quantity which plays an important part in dynamo machines and similar apparatus, since upon the coercive force depends to a certain degree the energy which is transformed into heat when the iron undergoes a cyclic change of magnetisation, a point to which we shall return later on.

Experimental Determination of Permeability.

In the above description of the magnetic behaviour of a sample of iron we have assumed that the magnetising force and corresponding induction are known at every instant, and it will therefore now be necessary to show how this knowledge is obtained, or, in other words, how the relations between magnetic force, induction, and permeability can be experimentally determined. Various methods have been used for this purpose. In some of the earlier methods the sample was in the form of a short rod or piece of straight wire inserted into a solenoid and there magnetised. A magnetometer was used to determine the magnetic moment of the sample corresponding to each magnetising current, the precaution having been used to either separately determine and allow for the action of the solenoid itself on the magnetometer, or to compensate for it by the use of certain compensating coils so arranged as to produce an exactly equal but opposite magnetic effect. The deflection of the magnetometer could then be used to calculate the magnetic moment, intensity of magnetisation, and induction of the sample. This method has been used with good effect by Prof. J. A. Ewing, who described it, as well as other methods, in his paper, "Experimental Researches in Magnetism," presented to the Royal Society in 1885. Ewing points out that the method is only reliable, especially as regards the determination of retentiveness, if the sample be very long in comparison with its diameter. Where this is not the case the free magnetism at the ends exercises a self-demagnetising force upon the interior and middle parts of the bar or wire, so that the induction determined for short bars is lower than that determined for long bars. The same difficulty accompanies, of course, all methods in which the sample experimented on is in the form of a short bar with free ends; and with a view to eliminate the error likely to arise from the action of the ends, which cannot be exactly estimated, Stoletow and Rowland used samples formed into closed rings, and the latter also straight bars of very great length.

Ewing found that if the length of the bar is over 300 diameters the demagnetising effect of the free ends becomes negligible.

With samples of the closed ring form the use of any magnetometer method is, however, excluded, since there is, or, at least, should not be any free magnetism to affect the magnetometer. Another method of measuring the induction must therefore be adopted. The most generally used is the ballistic method, which is based upon the fact that any change in the total flow of lines within the sample produces an electromotive force in a coil of wire surrounding the sample. This so-called "exploring coil" is connected with a ballistic galvanometer, the deflection of which indicates the integral of electromotive force multiplied by time; and since this integral is proportional to the variation in the total number of lines of force passing through the coil, the deflection of the ballistic galvanometer is also proportional to the variation in the induction of the sample of iron under experiment.

A consideration in detail of the apparatus required in these experiments, and of the corrections to be made and precautions to be observed, would be beyond the scope of this book. Suffice it to say that from the resistance of exploring coil and ballistic galvanometer, the period and logarithmic decrement of the latter and other electrical data, the change of induction corresponding to any observed deflection can be calculated. The constant of the ballistic galvanometer can also be found experimentally by the use of what Ewing terms an "earth coil." This is a flat coil of known area and number of turns connected with the ballistic galvanometer. The earth coil is laid upon a horizontal table and then suddenly turned over. During this movement the vertical component of the earth's lines of force are, so to speak, first withdrawn (when the earth coil gets into a vertical position) and then reinserted in the opposite direction when this coil again becomes horizontal. We have thus a change in the total flow of lines amounting to twice the number of vertical lines of force passing through the coil when it is horizontal. This number can be calculated from the dimensions of the earth coil and the intensity of the vertical component of the earth's magnetism, which latter can be determined by a magnetometer, as explained in Chapter III., p. 52. Having thus experimentally determined the relation between a given change in the flow of force and the corresponding deflection of the ballistic galvanometer, this relation is used to determine the change in the flow of force corresponding to other observed deflections.

Hopkinson's Method of Investigating the Magnetic Properties of Metals.

From a practical point of view it is important to determine the magnetic properties of different brands of iron on samples which are of such a shape that they may be readily obtained and not differ in their properties from the bulk of the metal, the properties of which they are supposed to represent. Assume, for instance, that we wish to test a certain brand of wrought iron for its suitability for field magnets. In such a case it would be useless to draw the iron into wire and experiment upon samples of this wire, since the very manipulation of wire-drawing may have so altered the iron as to make the subsequent magnetic tests misleading. What we would do in this case is to make a small forging and treat it (as to annealing and machining) as much as possible in the same manner as we would treat the real field-magnet forging. A method of testing which satisfies the requirements of practical engineers has been devised by Dr. John Hopkinson. This apparatus is adapted for the testing of samples in the shape of bars which are cast or forged, and then turned up true to a diameter

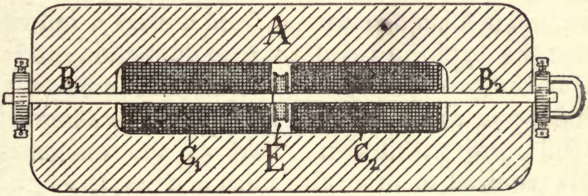

FIG. 26.

of ½in. The length of the bars need only be moderate, as by the peculiar arrangement to be described presently there is no free magnetism at the ends. Samples of this shape are easily obtained, and accuracy of diameter ensured, by using the ordinary gauges found in all engineering workshops.

The apparatus itself is very simple. It consists of a block, A, Fig. 26, of annealed wrought iron 18in. long, 6in. wide, and 2in. deep. In the middle a space is cut out as shown to receive the magnetising coils, C_1 C_2, and an exploring coil, E, which is connected with the ballistic galvanometer, not shown in the drawing. The

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ends of the block are bored to receive the sample rods, B_1 B_2, the diameter of the holes being just sufficiently larger than that of the rods to give an easy fit. The magnetising current is measured, and from its observed value, together with the number of turns in the magnetising coils and the dimensions of the apparatus, the magnetising force can be calculated. Two sample bars are used which meet with their accurately turned end faces at or near the centre of the apparatus, and one of the bars can be suddenly withdrawn by means of the shackle shown. The exploring coil is in this case set free and pulled suddenly out of its position by a spring. Thus there is a sudden change from a certain induction through the exploring coil to zero induction, and the throw of the galvanometer can be used to indicate what the induction was the moment before the sample bar and coil were withdrawn. Part of the magnetising force is required to drive the lines of force through the sample bars and part to drive them through the block, the arrangement representing a case to which formula (20), page 107, applies. If the magnetising current, c, be measured in amperes, and τ represent the number of effective turns of wire in both magnetising solenoids, we have

$$\cdot 4\, \pi\, c\, \tau = \mathrm{F} \left(\frac{l_1}{\mathrm{A}_1 \mu_1} + \frac{l_2}{\mathrm{A}_2 \mu_2} \right)$$

where the index 1 refers to the sample bars and the index 2 to the block. The latter being of soft annealed iron, μ_2 has a high value. At the same time the cross-section of the block A_2 is very great in comparison with that of the sample bar, so that the fraction $\frac{l_2}{\mathrm{A}_2 \mu_2}$

has a very small value and may be altogether neglected, so that the above formula becomes

$$\cdot 4 \pi c \tau = \frac{F l_1}{A_1 \mu_1}.$$

Since the area of the sample bar is known, we can calculate the induction, $\mathfrak{B}$, from the total flow of lines, F, and also the permeability. The magnetising force is $H = \frac{\cdot 4 \pi c \tau}{l}$, and this is calculated from the magnetising current. The total flow of force, F, is found from the throw of the ballistic galvanometer, and by dividing by the area of the sample bar, we find the induction, $\mathfrak{B}$, and finally we find the permeability by dividing induction by magnetising force—

$$\mu = \frac{\mathfrak{B}}{H}.$$

Dr. Hopkinson has in the paper already mentioned, "Magnetisation of Iron," given the result of experiments upon a great variety of samples of iron. The most important are of course annealed wrought iron and grey cast iron, because these materials are used in dynamo machines. The curves in Figs. 27 and 28 give average values compiled from Dr. Hopkinson's tables and diagrams. It should be noted that each brand of iron, although it may be generally classified as annealed wrought iron or grey cast iron, may, and generally does, show curves differing somewhat from the curves here given, and the latter must therefore be regarded merely as convenient approximations. It should also be noted that no distinction is made between the

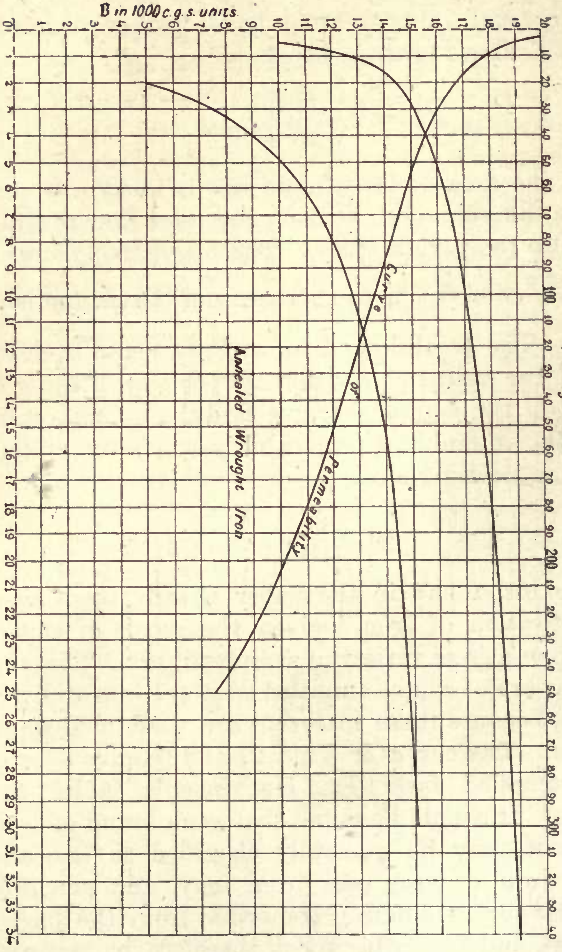

FIG. 27.

ascending and the descending curves of magnetisation, for the simple reason that with dynamo machines, for

$H = (\frac{1}{\mu})B$

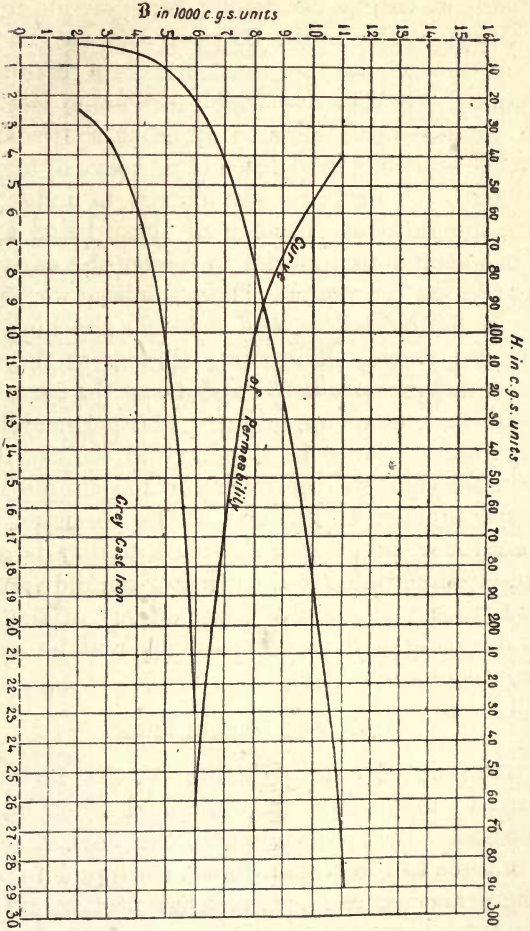

Fig. 28.

the calculation of which these curves are primarily intended, we reach a certain induction as often with an increasing as with a decreasing magnetising force, and that therefore the mean between the two curves will give on the whole the most accurate results. The mechanical vibrations to which a dynamo is subjected when working also tends to diminish the interval between the ascending and descending curve of magnetisation. On each diagram two curves of induction are shown, the ratio of abscissæ of these being as 1 : 10. This has been done in order to render the early part of the curves better visible. The abscissæ for the lower curve are figured below, and those for the higher curve are figured above. The curve sloping to the right on each diagram represents by its abscissæ the permeability, the ordinates being induction. The permeability is a simple numeric given in the diagram for cast iron by the tenfold value of the lower or the simple value of the upper figures on the axis of the abscissæ. In the diagram for wrought iron the permeability is given by the hundredfold value of the lower or tenfold value of the upper figures. The use of these curves in the designing and testing of dynamo machines will be explained in a subsequent chapter.

Energy of Magnetisation.

It was pointed out in Chapter V. that no energy is required to maintain a magnetic field when once it is established. We have compared the flow of magnetic lines of force to the stream-lines of a liquid in motion, and here also there is, apart from friction, no energy required to keep the liquid in motion. Energy is, however, required to set the liquid in motion, and this energy is stored in the liquid, and can be recovered by arresting again its motion. If our analogy between a liquid in motion and a magnetic field is generally correct, we should expect to find that a magnetic field can only be established by the expenditure of a definite amount of mechanical energy which remains, so to speak, stored in the field, and can be recovered by allowing the field to vanish again. This is indeed the case, as can easily be shown. It has been explained in Chapter V. that the line integral of magnetic force or difference of magnetic potential between two points of a magnetic field is the energy which must be expended or can be obtained by moving unit pole from one to the other of these points. If these two points are 1 centimetre apart, the difference of magnet potential between them is H, the magnetic force. Imagine now a space of 1 cubic centimetre, delineated in the field in such a position that the lines of force pass at right angles through two opposite sides of the cube, and let the induction be uniform over their surfaces, its value in absolute measure being denoted by $\mathfrak{B}$. The value of $\mathfrak{B}$ will of course depend upon the permeability of the substance which fills our cube. If the substance be air or another non-magnetic material, the induction will be H; if it be iron of permeability μ, it will be $= \mu$ H. Whatever the material occupying the field, there will be a definite induction for each magnetising force. Now assume that we increase the magnetising force by an infinitesimal amount. The induction will in this case also increase by an infinitesimal amount $d\,\mathfrak{B}$. Previous to the change the amount of magnetic matter on the end faces of our cube was $\frac{\mathfrak{B}}{4\pi}$. After the change it will be

$\frac{\mathfrak{B}}{4\pi} + \frac{d\,\mathfrak{B}}{4\pi}$; that is to say, $\frac{d\,\mathfrak{B}}{4\pi}$ units of magnetic matter have been transferred from one end face of the cube to the other under a magnetic force which varied during the process from H to $\mathsf{H} + d\,\mathsf{H}$. We may neglect the term $d\,\mathsf{H}$ as infinitely small, and compute the energy represented by the transfer as $\mathsf{H}\frac{d\,\mathfrak{B}}{4\pi}$. We may now imagine the magnetic force again increased by an infinitely small step, and the process repeated an infinite number of times until a finite increase of magnetic force and induction has been obtained. The total energy will obviously be the integral of the above expression taken between the limits of induction produced, or in symbols

$$\text{Ergs} = \int_{\mathfrak{B}_1}^{\mathfrak{B}_2} \mathsf{H}\frac{d\,\mathfrak{B}}{4\pi},$$

if by $\mathfrak{B}_1$ and $\mathfrak{B}_2$ we denote the limits between which the induction has been altered. The total energy of 1 cubic centimetre of the magnetised substance raised to the induction $\mathfrak{B}$ is obtained by adopting zero as one and $\mathfrak{B}$ as the other limit, or in symbols

$$\text{Total energy in ergs} = \int_{0}^{\mathfrak{B}} \mathsf{H}\frac{d\,\mathfrak{B}}{4\pi} \quad . \quad . \quad . \quad . \quad (27)$$

The application of this formula to the magnetisation of iron will be clear from Fig. 29. The curve shown represents the relation between magnetic force and induction. Let A_1 A_2 represent two conditions of the magnetisation sufficiently near each other to be attributed without any great error to the same magnetic force H, and let $\mathfrak{B}_1$ $\mathfrak{B}_2$ represent the corresponding values of the induction. The increment of magnetic matter transferred under the magnetic force, H, is graphically represented by the length of the line $\mathfrak{B}_1$ $\mathfrak{B}_2$, divided by 4π, and the corresponding energy by the area A_1 A_2 $\mathfrak{B}_2$ $\mathfrak{B}_1$, divided by 4π.

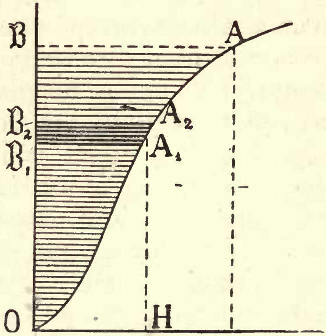

FIG. 29.

The total energy in ergs represented by the magnetisation of 1 cubic centimetre of iron from the induction zero to the induction $\mathfrak{B}$, is therefore given by the area contained between the axis of ordinates and the curve of magnetisation divided by 4π.

This is the shaded area, A O $\mathfrak{B}$, in the diagram. It will thus be seen that if the curve of magnetisation of any particular brand of iron is known, the energy which can be magnetically stored in a given volume, or weight of this iron at different inductions, can readily be calculated.

Hysteresis.

The theory of energy of magnetisation which has been here given, is of practical importance in two respects. One is the construction of choking coils for use with alternating currents, and the other alternating-current apparatus generally. A choking coil consists of an iron core surrounded by a solenoid, which is inserted into an alternate-current circuit. During the growth of the current from zero to its maximum value, a certain amount of energy is magnetically stored in the iron core, and thus prevented from reaching the lamp or other apparatus upon which the current operates. During the period of decline of current, the energy is again given off, and opposes to a certain extent the reversal of current, and thus the choking coil acts as a kind of elastic buffer between the source of current and the lamp, reducing the effective electromotive force at the terminals of the latter. It is not necessary to enter in this place more minutely into the theory and construction of choking coils, and we have only mentioned these appliances to show that the seemingly abstruse calculations concerning the energy of a magnetic field are by no means without practical value in electrical engineering.

The other application mentioned at the beginning of this paragraph is even of more importance. It concerns alternate-current apparatus generally, and is known under the name of "hysteresis," given to it by Professor Ewing, who has made a special study of the subject. The name implies a lagging behind, and refers more particularly to the lag of induction behind magnetic force, as shown by the difference between the ascending and descending curves of magnetisation, Fig. 25.

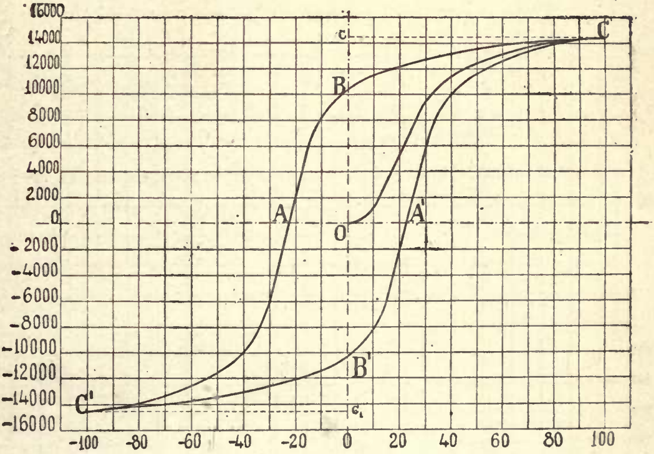

FIG. 25.

Starting with a magnetic force of zero, and increasing it to its maximum positive value, we obtain the curve $B^1 A^1 C$. When the point C has been reached each cubic centimetre of iron has absorbed an amount of energy, which is given in ergs, by the area enclosed between the curve $B^1 A^1 C$ and $B^1 c$ on the axis of ordinates divided by 4π. If we now decrease the magnetic force again to zero, we ought to recover the whole of the energy previously absorbed if the iron acted as a perfect storing appliance. This, however, is not the case. We only obtain the energy corresponding to the area comprised between B C and *c* B. The difference—namely, the energy represented by the area of the figure $B^1 A^1$ C B O B^1—has been lost, or, rather, transformed into heat, which is dissipated. The same reasoning applies to

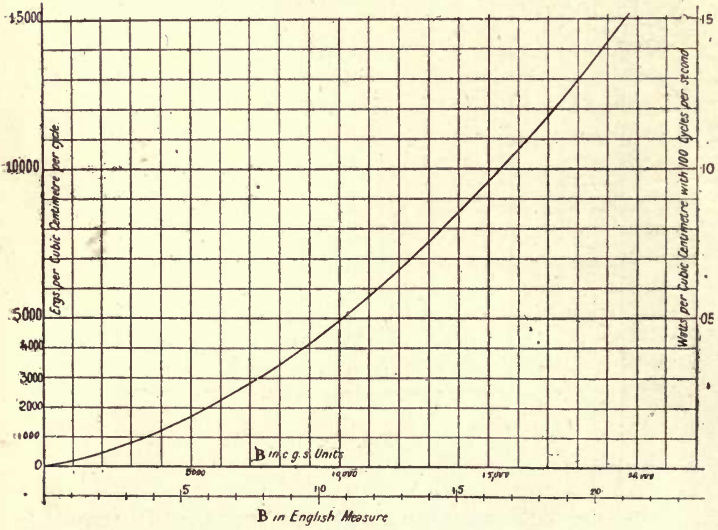

FIG. 30.

negative magnetic forces, with the result that if we carry the iron once through a complete magnetic cycle we waste an amount of energy, which is given in ergs, by the area of the distorted lozenge-shaped figure, C B A $C^1 B^1 A^1$ C, divided by 4π. The energy thus lost in hysteresis not only reduces the efficiency of alternate-

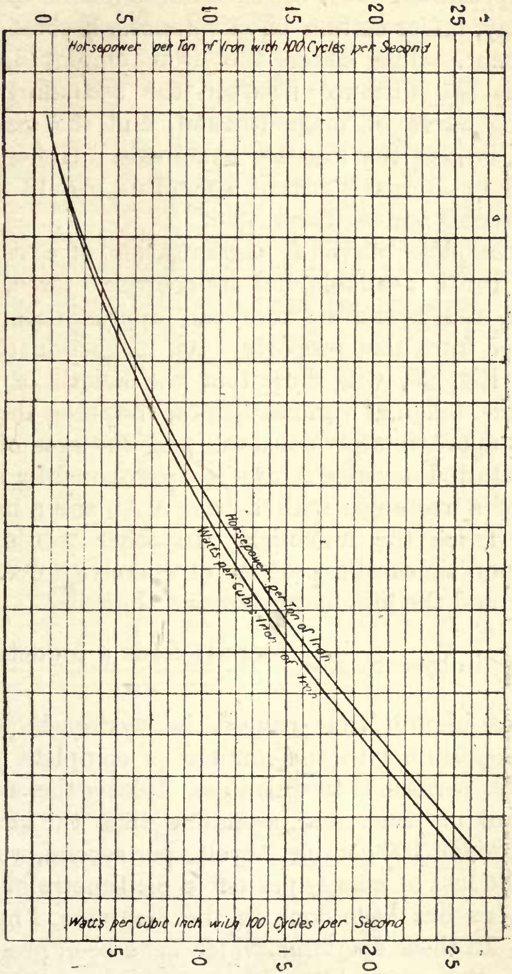
0
5
10
15
20
25
Horsepower per Ton of Iron with 100 Cycles per Second
Horsepower per Ton of Iron
Watts per Cubic Inch of Iron
Watts per Cubic Inch with 100 Cycles per Second
5
10
15
20
25

FIG. 30A

current apparatus, but it causes a development of heat which may, under certain circumstances, become very inconvenient. The softer the iron employed, the smaller is the distance between the ascending and descending curve of magnetisation, and the smaller, therefore, is also the loss by hysteresis. Hence, the iron used in alternate-current apparatus should be as soft as possible and well annealed.

If no complete curve of magnetisation of a certain brand of iron is available, but its coerceive force, O A, is known, the hysteresis loss can approximately be determined from this property. An inspection of the diagram, Fig. 25, will show that the length of horizontal lines, taken at various heights between the two curves, is approximately constant, and the area of the whole distorted lozenge figure is approximately equal to that of a rectangle with a base A A^1, and a height equal to twice the induction. In other words, the area is four times the product of coercive force and induction and the loss by hysteresis; therefore

$$\text{Ergs lost in hysteresis} = \frac{\text{coercive force} \times \text{induction}}{\pi}.$$

The loss in unit time caused by hysteresis is, of course, proportional to the number of complete magnetic cycles performed in unit time. Taking the second as the unit of time, and assuming that the iron is carried through 100 complete cycles per second, we can express the loss of energy per cubic centimetre of iron in watts, as has been done in the diagrams, Figs. 30 and 30A. In these diagrams, which have been prepared from Prof. Ewing's results with annealed wrought iron, the curve Fig. 30 represents on the left the hysteresis loss in ergs per cycle per cubic centimetre of iron, and on the right the hysteresis loss in watts with 100 cycles per second per cubic centimetre of iron for different degrees of induction. With another number of cycles than 100 the loss is proportionately altered. For convenience of practical work the two curves Fig. 30A have been added, giving the hysteresis loss with 100 cycles per second in watts and English horse-powers per cubic inch and ton of iron respectively. A scale showing induction in English measure has also been added to Fig. 30.

CHAPTER VII.

Induced Electromotive Force—Cutting or Threading of Lines—Value of Induced Electromotive Force—C.G.S. Unit of Resistance—Fleming's Rule—Electromotive Force of Two Pole-Armature.

Induced Electromotive Force.

In the preceding chapters it was shown how an electric current produces a magnetic field in the space surrounding the conductor through which it flows, how a piece of soft iron when placed into the field so produced becomes magnetic, and how currents and magnets act upon each other with measurable mechanical forces; we have thus seen that a current always produces magnetism, and the question which now presents itself is whether the converse is also true, in other words, whether a magnet can produce a current. Experiment shows that this is only the case under certain conditions. If we wind a conductor over a magnet and leave the system at rest, the most delicate galvanometer connected to the free ends of the conductor fails to show a current. If, however, we displace the conductor in relation to the magnet we at once obtain a current. We thus see that although no motion is required to produce magnetism by a current, in the converse process relative motion between the magnet and the conductor is essential to produce a current in the latter. That this is the case can easily be shown by an experiment. Let, in Fig. 31, C be a conductor wound in the form of a reel or coil, and placed over the straight bar magnet, N S. The free ends of the conductor are connected with a galvanometer, G, and we assume the connecting wires to be of sufficient length to prevent the galvanometer being disturbed by the magnet. In the position shown, when the reel is near the middle of the bar, the

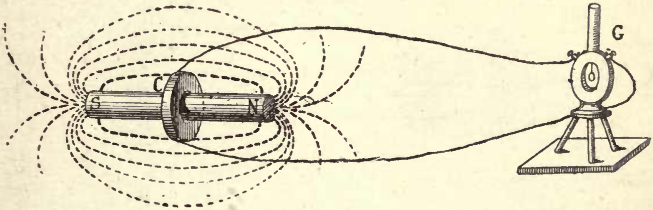

FIG. 31.

whole flow of the magnetic force passes through, but as long as the reel remains in that position the galvanometer shows no deflection. If we now slide the reel off the magnet the galvanometer shows a deflection which, generally speaking, will be the greater, the quicker the motion. We also find that the sense of the deflection is different according to the direction in which we slide off the reel, so that if by sliding the reel off over the north pole we get a deflection in one sense, by sliding it off over the south pole we get a deflection in the opposite sense. By sliding the reel on again, we also get deflections the sense of which is in every case opposite to that produced by sliding off. Since the same effects can be produced by holding the reel in place and moving the magnet, we gather from the experiments the broad fact that a current is produced by the relative movement between the magnet and the conductor. Now, in order to produce a current in any conductor there must be acting in it an electromotive force, and we thus come to the conclusion that an electromotive force is produced by the relative movement between the magnet and conductor. In the position

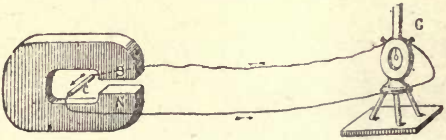

Fig. 32.

shown in Fig. 31 the number of lines of force passing through the reel is a maximum, whilst after it has been drawn off the bar this number is a minimum, and we therefore attribute the electromotive force to the variation in the number of lines of force which pass through the reel, or generally through the circuit, for the experiment succeeds also if made in the manner represented by Fig. 32. It has already been pointed out that the deflection of the galvanometer is the greater the quicker we move the conductor, and we conclude from this that not the variation in the number of lines as such, but the rate at which this variation takes place determines the electromotive force. In other words, that the electromotive force is proportional tc the positive or negative increment per unit time in the number of lines of force which pass through the circuit. The growth of diminution of lines of force induces an electromotive force in the circuit through which these lines pass. This is a broad experimental fact which can be proved in a variety of ways, but for which we have no explanation. Two ways of proving it are shown in Figs. 31 and 32. In both cases, and, in fact, in all cases where induction occurs, the magnetic and electric circuits are interlinked—that is to say, the magnetic lines of force are threaded through the electric circuit.

Cutting or Threading of Lines.

It will be seen that both in Fig. 31 and Fig. 32 the field itself undergoes no change, though the number of lines passing through the electric circuit vary. In these cases the "threading" of lines is necessarily accompanied by "cutting" of lines, and it will be equally correct if we attribute the generation of an electromotive force in a conductor to the cutting of lines of force by the conductor. This view, although it does not bring us any nearer to an explanation of the process under which the generation of electromotive force actually takes place, has the advantage of greater directness, since the conductor comes actually in contact with the agent producing the electrical effect. It has, moreover, the other advantage of lending itself better than the other view to mathematical treatment. On the other hand, there are cases where an electromotive force is undoubtedly the result of threading of lines, but where it is not so obvious that the threading is also accompanied by cutting. Such a case is represented in Fig. 33. R is an iron ring wound with two circuits, C_1 and C_2. A current sent through C_1 will produce a flow of lines in the ring, the induction increasing from zero, its value before the current was started, to a maximum which can be calculated from the electric and magnetic data of the apparatus, according to formula (16). During the period of growth of induction an electromotive force is induced in circuit C_2, through which the lines thread.

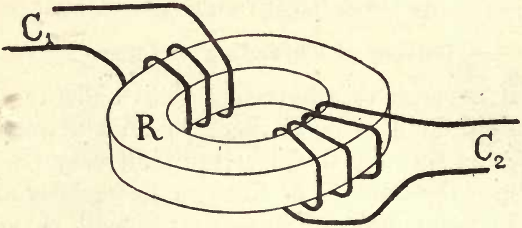

FIG. 33.

In this case it is not very obvious how cutting can be combined with threading, except on the supposition that all the lines start as infinitely small closed circles from a point somewhere in the centre of the ring, and expand into larger and larger circles until they enter the iron of the ring and at that moment cut the coils of the conductor. It is, however, not a matter of any practical importance to determine whether in any particular case cutting or threading more correctly describes the interaction between the magnetic and electric circuits. Either view is correct and leads to the same result. We shall therefore make use of either one or the other conception, whichever is more natural to the case in point.

Value of Induced Electromotive Force.

Let, in Fig. 34, R_1 R_2 represent two straight horizontal and parallel metallic rails united at one end by a galvanometer, G. Across these rails and at right angles to them we lay a third metallic rail, C, and assume the whole apparatus situated in a uniform magnetic field, the lines of which are vertical. If the rail C, which we

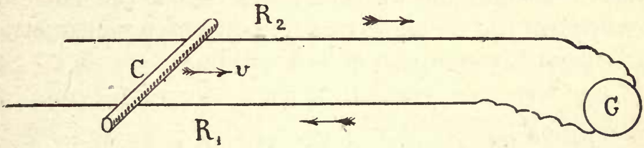

FIG. 34.

term the "slider," be moved along the fixed rails at a velocity, v, it will cut the lines of the field, and thus there will be created in it an electromotive force which will cause a current to flow through the circuit. If we know the electromotive force and the resistance we can calculate the current, and conversely, if we know the resistance and measure the current we can calculate the electromotive force. The latter depends of course on the velocity of the movement, the strength of the field, and the length of the slider which is comprised between the fixed rails. By a series of experiments carefully made it would thus be possible to determine the law which governs the production of electromotive force. It is, however, not necessary to make such experiments, since we can find this law by simply applying the principle of the conservation of energy to the present case. The current flowing through the circuit under the induced electromotive force represents a certain energy, which must obviously be equal to that required to pull the slider across the lines of the magnetic field. It was shown in Chapter IV. that the mechanical force, P, acting upon a conductor in a field of $\mathfrak{B}$ lines per square centimetre is (C.G.S. measure)

$$P = l c \mathfrak{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where l is the length of conductor and c the current. If we move the slider with a velocity of v centimetres per second the energy required will be

$$P v = l c \mathfrak{B} v \text{ ergs.}$$

The energy represented by the current is $e c$, if by e we denote the electromotive force expressed in a suitable measure. We have therefore the equation

$$e c = l c \mathfrak{B} v \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

or

$$e = l \mathfrak{B} v \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

from which we find that the induced electromotive force is given by the product of length of conductor, strength of field, and velocity, all in C.G.S. measure. Formula (28) gives the energy in ergs. To obtain it in watts we divide by 10,000,000, and have

$$\text{Watts} = l c \mathfrak{B} v 10^{-7}.$$

In this formula c is given in C.G.S. measure, but if we give c in amperes we must divide by 10, and obtain

$$\text{Watts} = c l \mathfrak{B} v 10^{-8}$$

The term $l \mathfrak{B} v 10^{-8}$ represents therefore the electromotive force in volts, and by comparing this with equation (28), we find that the C.G.S. unit of electromotive force is the one hundred millionth part of a volt. Unit electromotive force is produced in a slider 1 centimetre long, moving with a velocity of 1 centimetre at right angles across the lines of unit magnetic field—namely, a field containing one line per square centimetre.

The expression for calculating the electromotive force in volts is

$$\text{Volts} = l \mathfrak{B} v 10^{-8} \quad . \quad . \quad . \quad . \quad (30)$$

It will be readily seen that in this expression $l v$ is the area swept by the slider in one second, and therefore $l v \mathfrak{B}$, the increment or decrement per second of the total number of lines of force passing through the circuit. An increment, $d\text{F}$, in the total strength of the field in the time $d t$ will therefore produce in a simple circuit $\frac{d\text{F}}{d t} 10^{-8}$ volts, and when the circuit contains τ turns of wire, the electromotive force will be

$$\text{Volts} = \tau \frac{d\text{F}}{d t} 10^{-8} \quad . \quad . \quad . \quad . \quad (31)$$

This formula is convenient for the calculation of transformers; for dealing with dynamos, formula (30) is more convenient. An example may serve to show its application. Let in a dynamo the strength of the field in the interpolar space be 5,000 C.G.S. units, and let the velocity of the wires on the armature be 1,500 centimetres per second (about 3,000ft. per minute), then the electromotive force generated in a piece of armature conductor 10 centimetres long, will be $\frac{10 \times 5{,}000 \times 1{,}500}{100{,}000{,}000} = \cdot 75$, or about $2\frac{1}{4}$ volts per foot.

C.G.S. Unit of Resistance.

The experiment with the slider, illustrated in Fig. 34, gives us a convenient means of determining the relation between the absolute or C.G.S. unit of resistance and the ohm. The unit of resistance in the C.G.S. system is obviously the resistance of a conductor through which unit electromotive force produces a flow of unit current. Let the distance between the fixed rails be 1 centimetre; let the strength of the field be $\mathfrak{B}=1$, and let the slider be moved with a velocity of 1 centimetre per second. The electromotive force will then be one C.G.S. unit, or 10^{-8} volts. If the total resistance of the circuit be unity, there will flow unit current, or 10 amperes. Now imagine the velocity of the slider increased to 10,000 kilometres $= 10^9$ centimetres per second. This would give an electromotive force of 10 volts. If we desire that under this increased velocity the current shall retain its former value, we must increase the resistance in the same ratio, that is, instead of one C.G.S. unit of resistance the circuit must have 10^9 C.G.S. units of resistance. But since the electromotive force is 10 volts, and the current is 10 amperes, the resistance is 1 ohm, and we thus find that the C.G.S. unit of resistance is the $\frac{1}{1{,}000{,}000{,}000}$ part of the ohm, or,

One ohm $= 10^9$ C.G.S. units of resistance.

We have seen that in order to keep the current constant the resistance must be increased proportionately to the velocity, and thus the velocity of the slider may be taken as a measure of the resistance. According to this view the resistance of 1 ohm is given by the velocity of 10^9 centimetres = 10,000 kilometres = very nearly to the length of one earth-quadrant per second.

Fleming's Rule.

Before applying the laws here explained to dynamos it will be convenient to give a simple rule by which the

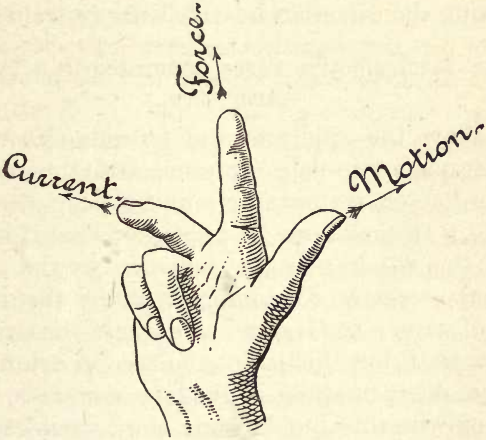

FIG. 35.

direction of electromotive force can be found. Since the principle of the conservation of energy must also apply to the phenomena of electromagnetic induction, it will be immediately apparent that the direction in which the electromotive force acts and the current is generated must be such as to oppose the motion which produced it. By applying this rule, the direction in which the electromotive force acts can in every case be determined, but to facilitate this determination Dr. Fleming has devised a simple method which is easily remembered. He uses for this purpose the right hand, Fig. 35, pointing with the thuMb in the direction of Motion, and with the Forefinger in the direction of flow of Force. The electromotive force is generated at right angles to the plane laid through forefinger and thumb, and its sense is given by the middle or Central finger indicating the direction in which the Current flows.

Total Electromotive Force Generated in a Two-Pole Armature.

To show the application of formula (30) we take as an example a two-pole Gramme armature containing, as counted on its outer circumference, τ active conductors. It may here be remarked that it is better to define the winding of an armature by the number of external or active conductors than by the number of turns of wire. In Gramme armatures the two numbers are identical, but this is not the case in drum and some forms of disc armature, so that if we were to introduce turns of wire into our formula they would cease to be universally applicable. The construction of the Gramme armature is so well known that only a few words of general explanation need be given here. The armature consists of a cylindrical core built up of iron discs or wire, and wound with insulated copper wire, which forms a spiral closed on itself. On the outer and inner surface of the cylinder the wire is parallel to the spindle, and at the ends it occupies a more or less radial position. At equal intervals all round this winding the coils are in metallic connection with conductors leading to the plates of a commutator. For convenience of manufacture the winding is not put on in a continuous length, but in the form of coils containing one, two, or more turns, and the points of junction between neighbouring coils are utilised for making connection with the commutator plates. A current entering at one commutator plate splits into two branches, one flowing successively through all the coils on one half of the cylinder and the other successively through all the coils on the other half of the cylinder, the two branches uniting again and leaving the armature at the commutator plate opposite that at which it entered. It will thus be seen that half the active conductors are at any time in series connection, and the two halves are in parallel connection. The armature revolves between the poles of the field magnet, and the conductors are thus forced to cut the lines of the field, whereby an electromotive force is generated in each conductor. On account of the series connection the electromotive forces in all the wires which at any time are under the influence of the same field pole are added, and the sum makes up the total electromotive force of the armature, which we are now about to determine.

Let $\mathfrak{B}_a$ represent the induction through the interpolar space, l the length of the armature, D its diameter, and n the speed in revolutions per minute. Let ω be the angle embraced by each pole-piece, and assume the induction uniform over that portion of the armature defined by this angle. The induction in the inter-

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mediate portions we assume to be zero. Strictly speaking, this is not correct. In reality there is no abrupt change in the induction at the edges of the pole-pieces, but a gradual shading off. For the determination of the electromotive force a knowledge of the exact distribution of the field is, however, not required, since we are not concerned in knowing the electromotive force in each wire, but its sum computed from all the wires, and if in consequence of an uneven distribution of field one wire does less than its proper share of the work, another wire must do more, leaving the total electromotive force exactly the same as if the distribution of field were as above assumed. The number of active wires under the influence of one pole-piece is at any time $\frac{\omega}{2\pi}\tau$. The electromotive force in volts generated in each of these wires is by formula (30) $l \mathfrak{B} a \pi D \frac{n}{60} 10^{-8}$, where we write $\pi \frac{D n}{60}$ for v, the linear speed of the wires across the lines of the field. The total electromotive force is therefore

$$E = \frac{\omega}{2\pi} \tau l \mathfrak{B} a \frac{D n}{60} 10^{-8}.$$

$$E = \tau \omega \frac{D}{2} \mathfrak{B} a l \frac{n}{60} 10^{-8}.$$

Now, the term $\omega \frac{D}{2} \mathfrak{B} a l$ is the total flow of lines or strength of field, F, in C.G.S. measure, which can be determined from the constructive data of the machine and the exciting power applied according to formula (19), Chapter V. Inserting this value we have

$$E = F \tau \frac{n}{60} 10^{-8} \quad . \quad . \quad . \quad . \quad (32)$$

Calling Z the strength of field in English measure (one English line of force is equivalent to 6,000 C.G.S. lines, Chapter IV.) we also obtain

$$E = Z \tau n \, 10^{-6}. \quad . \quad . \quad . \quad . \quad . \quad (33)$$

CHAPTER VIII.

Electromotive Force of Armature — Closed-Coil Armature Winding—Bi-Polar Winding—Multipolar Parallel Winding—Multipolar Series Winding—Multipolar Series and Parallel Winding.

Electromotive Force of Armature.

At the end of the preceding chapter a formula was given for the electromotive force of an armature taken as a whole. The formula was obtained on the supposition that the field is absolutely uniform, or, in other words, that the induction through the interpolar space (the space between the iron of the armature and that of the pole faces) is constant. It was also stated that this supposition is not correct, but that any variation in the induction between one part and another part of the interpolar space may be left out of account, as the excess of electromotive force in some wires would be counterbalanced by the deficiency of electromotive force in others, and thus the electromotive force of the armature taken as a whole would be the same as if the field were absolutely uniform. This is a plausible explanation, but not a scientific proof, and before proceeding further it will be expedient to give the strict proof for formulæ (32) and (33). Fig. 36 represents a transverse section through an armature and its field poles, N S. The distribution of magnetic lines of force within the interpolar space depends on so many circumstances that we cannot possibly map out the field merely by drawing, but if we wanted to know the distribution of magnetism we would have to determine it experimentally by means of an exploring coil, or iron filings, or in some other way. It is, however, for our present purpose, not at all necessary that we should know the exact distribution of lines of force; all we need know is the total magnetic flux which enters the armature on the left of the neutral line, A B, and issues from it on the right of that line. The winding on the armature is so arranged that the electromotive forces induced in all the wires lying to one side of the line of commutation, A B, add up, and that

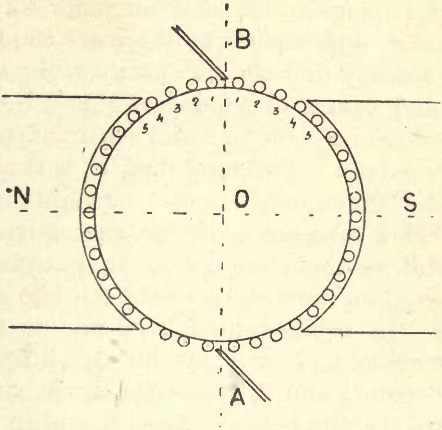

FIG. 36.

the total electromotive force on the left of this line is equal to the total electromotive force on the right. Retaining the previous notation, we have τ active conductors on the armature, and the total electromotive force induced is that due to $\frac{\tau}{2}$ wires in series. If we number the conductors from the top right and left 1, 2, 3, and so on, and consider the centre line of each, we can imagine the total magnetic flux, F, subdivided into as many parts as there are conductors on each side, and say that the flux between the top of the armature and centre of conductor 1 is ΔF_1; the flux between centre of conductor 1 and centre of conductor 2 is ΔF_2, and so on. The total flux, F, is thus the sum of all the ΔF taken over the left or right half of the armature. Now let us consider two successive positions of the armature separated by an angle equal to that corresponding to the distance between two successive conductors. In being shifted from one to the other position, conductor 1 will cut all the lines of force passing between 1 and 2, conductor 2 will cut all the lines of force passing between 2 and 3, and so on. The electromotive force created in conductor 1 is, therefore, $\frac{\Delta F_2}{t}$, that in wire 2 is $\frac{\Delta F_3}{t}$, and so on, t being the time in which the armature rotates through the small angle corresponding to the distance of one conductor to its neighbour. The total electromotive force generated in the $\frac{\tau}{2}$ conductors on one side of the diameter ot commutation, A B, is therefore $\frac{\Sigma \Delta F}{t} = \frac{F}{t}$.

Let D be the diameter of the armature in centimetres and n the number of revolutions per second, then the circumferential speed is $\frac{n}{60}\pi D$, and the distance between two neighbouring conductors is $\frac{\pi D}{\tau}$.

Hence $$t\frac{n}{60}\pi D = \frac{\pi D}{\tau},$$

and $$\frac{1}{t} = \tau\frac{n}{60},$$

which, inserted into the above equation, gives

$$E = F\tau\frac{n}{60}.$$

The electromotive force is here given in C.G.S. units. To obtain it in volts we must divide by 10^{-8}, and have

$$E = F\tau\frac{n}{60}10^{-8} \quad . \quad . \quad . \quad . \quad (32)$$

which is the same expression as was given in the preceding chapter. We see, therefore, that the electromotive force depends simply on the total flux, F, but is quite independent of the more or less regular distribution of the flux throughout the interpolar space.

The total flux is of course that emanating from one pole-piece. We have thus proved that the formulæ (32) and (33), given at the end of the last chapter, are rigorously correct, but as they refer only to bi-polar machines we must yet investigate what modification, if any, will be required in the formula for armature electromotive force in order to become applicable to multipolar machines. Take, for instance, a machine with four poles, as shown in Fig. 37. The flux emanating from each pole divides so as to reach the two neighbouring poles, as shown by the dotted lines. If we assume that each pole supplies a flux of F lines,

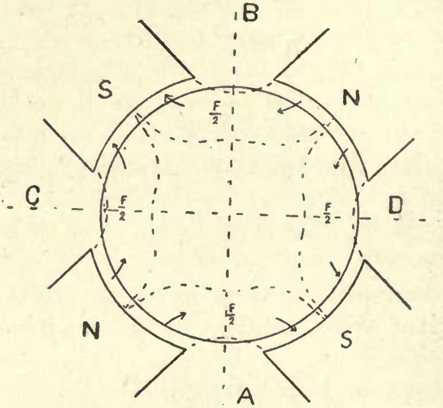

FIG. 37.

there will be four bands of induction through the armature, each carrying $\frac{F}{2}$ lines. In order to facilitate the comparison with a bi-polar machine, let us assume that we have in Fig. 37 the same armature as in Fig. 36. The number of conductors under the influence of each pole is now $\frac{\tau}{4}$ instead of $\frac{\tau}{2}$ as formerly; but if we assume that the total flux from each of the four poles is the same as that emanating from each of the two poles in Fig. 36, then the number of lines passing between neighbouring conductors will be greater than it is in the two-pole machine. In other words, although now we have fewer Δ F, each Δ F, taken individually, represents more lines, and the induction within the interpolar space is stronger. By adopting the same reasoning as before, we find that also in the case of a multipolar machine the total electromotive force of the conductors under the influence of one pole-piece is represented by

$$E = \frac{F}{t},$$

$$E = F \tau \frac{n}{60} \text{ C.G.S. units,}$$

and is therefore the same as in a bi-polar machine. It is important to note that the formula here found refers to the electromotive force of that part of the armature which lies between two successive points of commutation, and which for brevity we shall call armature sections. In a bi-polar machine these points are diametrically opposite, and the electromotive force is that due to one-half the armature conductors on either side of the diameter of commutation. In this case there are two armature sections coupled in parallel, and the electromotive force of one section is the same as that of the armature taken as a whole. If we had to do with a machine having a four-pole field the armature winding would have to be considered as consisting of four sections, each embracing 90 deg. If the machine had six poles there would be six armature sections, each embracing 60 deg., and so on. The formula

$$E = F \tau \frac{n}{60} 10^{-8} \text{ volts} \quad . \quad . \quad . \quad (32)$$

gives the electromotive force of each section, and although in a bi-polar machine this is the same as the electromotive force of the armature taken as a whole, this is not necessarily the case in a multipolar machine. It will be the case if the winding of the armature is such that all the sections are coupled in parallel, but if a method of winding be employed whereby two, three, or more sections are coupled in series, then the electromotive force of the armature taken as a whole will be two, three, or more times that given by formula (32). Thus in a four-pole machine with series-wound armature the total electromotive force will be twice that of a bi-polar machine with equal field strength and equal number of armature conductors. In a six-pole machine with series-wound armature the electromotive force will be three times that of a bi-polar machine, and so on.

The current given by a bi-polar machine is twice the current passing through each armature conductor. In a four-pole machine with parallel-wound armature the current is four times that passing through each armature conductor; in a six-pole machine with parallel-wound armature it is six times that passing through each armature conductor, and so on. The following table will render the relations between number of poles, electromotive force, current, and output clear.

In this table, E represents the electromotive force of one armature section as given by formula (32), and $\frac{C}{2}$ is the current in each armature conductor.

Number of Poles.	Total E.M.F.		Total Current.		Total Output.
	Parallel winding.	Series winding.	Parallel winding.	Series winding.	Parallel or series winding.
2	E	E	C	C	E C
4	E	2 E	2 C	C	2 E C
6	E	3 E	3 C	C	3 E C
8	E	4 E	4 C	C	4 E C
2*n*	E	*n* E	*n* C	C	*n* E C

This table shows that the output obtainable with any given size and weight of armature increases as the number of poles is increased, and it would thus appear that multipolar machines must, under all circumstances, be better than bi-polar machines. It should, however, be borne in mind that the electromotive force of one armature section depends on the strength of the field, and if we put more magnets round a given armature each pole-piece will become smaller, and we must either provide for a vastly greater density of lines in the interpolar space—which, for reasons that will be explained later on, is not always feasible—or we must be satisfied with a weaker field, and this may counteract the advantage which would otherwise result from the multipolar arrangement. Generally speaking, an armature designed for a bi-polar field will probably not work well in a multipolar field, and when designed for a multipolar field will certainly work badly in a bi-polar field; but if we are free to vary the dimensions and winding of the armature to suit either one or the other type of field, then we find that the bi-polar field is best for small and the multipolar for large machines. This point will be found fully considered in Chapter XI. For the present we are only concerned with the electromotive force that may be obtained with a given armature winding and field. If we have to do with a multipolar machine the winding of the armature may, as already stated, be on the parallel or series system. It is, however, also possible to adopt a combination of the two methods and to wind, for instance, a 12-pole armature in such way that three sections are coupled in series and four in parallel.

Armature Windings.

It is now necessary to explain some of the methods of winding by which armature sections may be put in series or in parallel.* Bi-polar machines of the open-coil armature type have the sections coupled in series, and these will be found described in Chapter IX. In bi-polar machines of the closed-coil type the parallel arrangement is the only feasible one, and, for the sake

* Space permits only to give some of the windings more generally used. Those who require more detailed information on the subject should consult Dr. Arnold's excellent little book, "Die Ankerwickelungen der Gleichstrom—Dynamomaschinen"; Berlin, Julius Springer, 1891. Dr. Arnold treats the subject in a most comprehensive manner, and shows how any given winding can be represented and new windings discovered by the use of algebraical formulæ and diagrams. If I do not follow Arnold's method it is not that I underestimate its scientific merit, but simply because I think that within the limited space at my disposal explanation by means of examples and winding tables are more easily given.

of simplicity, we shall begin the investigation with them. We shall then proceed to the consideration of multipolar windings, taking the parallel arrangement first, the series arrangement next, and finally, the combination of the two.

Bi-polar Winding.

Let, in Fig. 38, the circle represent the cross-section

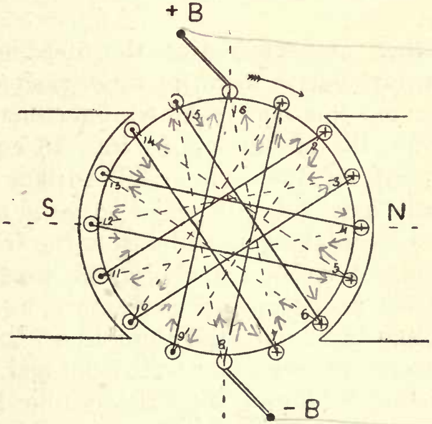

FIG. 38.

of the armature revolving clockwise between the field poles, N S, and the 16 small circles on the circumference the conductors. Applying Fleming's rule of Chapter VII., we find that in all the conductors between the armature and the N pole-piece an electromotive force will be induced downwards—that is, away from the observer—and in all the conductors under the influence of the S pole-piece the electromotive force will be upwards, or towards the observer. We represent the latter direction by a dot in the middle of the wire, being the point of the arrow which denotes the direction of the current. Similarly a downward current may be represented by an arrow flying away from the observer, who will then see, not its point, but its wings at the back. We therefore represent the downward current by a small cross inscribed in the wire.

This method of representing the direction of currents, or electromotive force, in wires seen end on is used throughout this book. In winding this armature we first divide the circumference into 16 equal parts, and mark out on the cylindrical surface 16 lines parallel to the axis. Let us begin to wind at line 16, laying the wire on along this line from the front end to the back end. Arrived at the back end, we stretch the wire across the back face of the armature, as shown by the dotted line 16-7, and wind along line 7 to the front. Next we stretch the wire along the front end, as shown by the full line 7-14, and wind down line 14. Then across, back, and up line 5, and so on, until we close the winding on itself with the front connection 9-16. It is characteristic for this winding, commonly known under the term "drum winding," that we complete each turn, not with the next, but the next but one wire to that with which the turn has begun. This winding may be represented by the following winding table, in which the numbers in the columns headed D represent wires wound downwards, and those in the columns marked U wires wound upwards, the letters B and F representing back and front connections respectively.

WINDING TABLE FOR DRUM ARMATURE WITH SIXTEEN ACTIVE CONDUCTORS.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
—	16	—	7	—	14	—	5	—	12	—	3	—	10	—	1
—	8	—	15	—	6	—	13	—	4	—	11	—	2	—	9
—	16														

The number of columns in the table is quite immaterial, and we could also arrange the table as follows:

F.	D.	B.	U.
—	16	—	7
—	14	—	5
—	12	—	3
—	10	—	1
—	8	—	15
—	6	—	13
—	4	—	11
—	2	—	9
—	16	—	7

This method of representing the winding is preferable to a diagram on account of its greater clearness, especially if the number of conductors is large, as in this case the diagram becomes confused on account of the many lines crossing each other.

We must now investigate the distribution of electromotive force between the different armature wires. It will be seen from the diagram that the electromotive force is as follows in the different wires:

15, 16, 1	no E.M.F.
2, 3, 4, 5, 6	downward E.M.F.
7, 8, 9, ...	no E.M.F.
10, 11, 12, 13, 14	upward E.M.F.

The brushes are shown touching the wires at the diameter of commutation, −B being the negative brush, where the current enters, and +B the positive brush, where it leaves the armature. In reality, the brushes do not touch the wires directly, but a commutator, the plates of which (numbering in this case eight) are attached to each alternate wire. For clearness of illustration, the commutator has, however, been omitted from the figure. For the sake of simplicity, let us assume that the same electromotive force is generated in each of the wires 2 to 6 and 10 to 14, and let us call this the unit of electromotive force, and designate it by 1. Let us also assume the absolute potential of −B to be 0. Then the absolute potential of wire 1 and of the back connection 1-10 will be zero, but that of the front connection 11-3 will be 1, 10 being an active wire, in which an electromotive force of 1 is produced. When we reach the back end of 3 another unit of electromotive force has been added, so that the absolute potential of the back connection 3-12 will be 2. Similiarly the absolute potential of the front connection 12-5 will be 3, and so on. We may represent this accumulation of potential in our winding table by inserting under the columns B and F, instead of the dash representing the back or front connections, numbers denoting the units of potential in each of these connections. We arrive thus at the following table :

F.	D.	B.	U.
5	16	*5*	7
5	14	*4*	5
3	12	*2*	3
1	10	*0*	1
0 – B	8	*0*	15
0	6	*1*	13
2	4	*3*	11
4	2	*5*	9
5 + B	16	*5*	7

The negative brush is in contact with the front connection 8-1, and the positive brush with the front connection 16-9, and by reference to the table we find that the total difference of potential between the two brushes is 5 units. We also see that the difference of potential between adjacent wires in the neighbourhood of the points of commutation is 5 units—*i.e.*, the full electromotive force of the machine. This is an important point in the practical construction of armatures, and in order to bring it more fully into light a winding table is here given for a machine with a larger number of armature conductors. We have in the previous examples assumed that the machine has only 16 armature conductors, in order that the winding diagram may not become too complicated, but in actual practice the number of conductors is much larger, and it is expedient to study the distribution of potential in a machine as actually built. For this purpose we take a 200-volt machine, having 100 conductors on the armature—that is, 50 on each side of the diameter of commutation. Of these there will be about 40 under the influence of each pole-piece, so that the unit of electromotive force—that is, the electromotive force in each wire—will be 5 volts. In passing from one wire to the next we have, therefore, to add 5 volts at each step. This progression of electromotive force from wire to wire is clearly shown by the figures inserted in the columns B and F. Some of the numbers in these columns are underlined, and these represent the connections which are attached to those commutator bars that are in contact with the brushes. Thus, when the brushes touch two commutator segments on each side, the current enters the winding at the front ends of wires 49, 51, 98, and 100, and leaves it at the front ends of 48, 50, 99, and 1. It runs, therefore, down in the former and up in the latter wires. An instant later, when the armature has turned through a small angle, the commutator segment connected with 51 and 100 has passed beyond the brush, and the direction of current in these two wires is upwards, whilst the current in 50 and 1 is similarly reversed, running now downwards.

By reference to the winding table, it will be seen that there is no electromotive force acting in any of these eight wires, and if the brushes were set as represented by the table, the change in the direction of the current would have to take place in each wire suddenly upon its emerging from under the brush.

Winding and Potential Table for Drum Armature.

100 Conductors, 200 Volts, 5 Volts per Active Conductor.

F.	D.	B.	U.	F.	D.	B.	U.	F.	D.	B.	U.	F.
0	100	0	49	0	98	0	47	0	96	0	45	0
0	94	0	43	5	92	10	41	15	90	20	39	25
25	88	30	37	35	86	40	35	45	84	50	33	55
55	82	60	31	65	80	70	29	75	78	80	27	85
85	76	90	25	95	74	100	23	105	72	110	21	115
115	70	120	19	125	68	130	17	135	66	140	15	145
145	64	150	13	155	62	160	11	165	60	170	9	175
175	58	180	7	185	56	190	5	195	54	200	3	200
200	52	200	1	200	50	200	99	200	48	200	97	200
200	46	200	95	200	44	195	93	190	42	135	91	180
150	40	175	89	170	38	165	87	160	36	155	85	150
150	34	145	83	140	32	135	81	130	30	125	79	120
120	28	115	77	110	26	105	75	100	24	95	73	90
90	22	85	71	80	20	75	69	70	18	65	67	60
60	16	55	65	50	14	45	63	40	12	35	61	30
30	10	25	59	21	8	15	57	10	6	5	55	0
0	4	0	53	0	2	0	51	0	100			

Idle wires, 94 to 4 and 45 to 53; 5 volts down in each of the wires 5 to 44; 5 volts up in each of the wires 54 to 93.

This would lead to sparking, and to avoid this fault the brushes must be shifted slightly forward, so that a slight electromotive force may act in the wires under commutation, whereby the previous current is gradually stopped and the reverse current as gradually induced, even before the wire emerges from under the brush. This point will be found more fully dealt with in Chapter XI. For the present, it interests us only to study the distribution of electromotive force between the different wires which is not materially affected by a slight displacement of the brushes; and on reference to the table it will be seen that there is a potential difference of 200 volts between adjacent wires in the neighbourhood of the points of commutation. This can be shown more clearly by writing the wires down, not in their order of winding as in the table, but in the order of their numerical succession, and writing under each pair the average potential difference as extracted from columns B and F of the table. We select for this purpose the quarter of the armature containing wires 100 to 50, the distribution of potential in the other quarters being symmetrical. We thus obtain the following :

Wires..............	100		1		2		3		4		5		6
Potential difference		200		200		200		200		192½		185	
Wires..........	6		7		8		9		10		11		
Potential difference		175		165		155		145		135			
Wires.................	11		12		13		14		15		16		
Potential difference		125		115		105		95		85			
Wires..........	16		17		18		19		20		21		
Potential difference		75		65		55		45		35			
Wires.................	21		22		23		24		25				
Potential difference		25		15		5		5					

This table shows clearly that the full voltage stress comes on the insulation of the idle wires, and that as we pass along the active wires to the polar diameter this stress is gradually reduced. As, however, all the wires must successively pass through the region of commutation, it is necessary that the insulation between every wire and its neighbour should be good enough to sustain the full voltage of the machine. It need hardly be mentioned that each coil wound on the armature may consist, not only of one turn, as in our present example, but of any number of turns. Thus we might, for instance, use five turns for each coil, and in this case the total electromotive force of the machine would become 1,000 volts, and we should have to insulate neighbouring coils for a pressure of 1,000 volts. This is a somewhat difficult matter, and presents a certain risk of breakdown, so that the drum winding is not used for very high voltages. The limit at which this type of winding is yet safe may be taken at 1,000

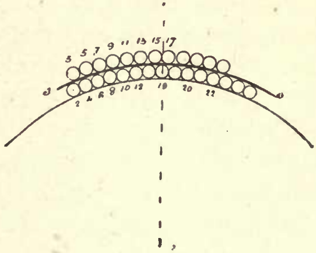

FIG. 39.

volts in certain exceptional cases, but more generally at about 600 volts.

One means for preventing the strain on the insulation between neighbouring wires which has been employed by C. E. L. Brown and others is to put the winding on in two layers, separated by a stout sheet of insulating material, as shown in Fig. 39. Here all the wires of even number are first placed on the armature core all round, the ends which are to form the return winding being left projecting at the back. Then a sheet of specially strong insulation, *s s*, is put on, and the wires are brought forward over it to form the conductors of odd numbers. In this manner the voltage difference between adjacent wires is kept down to a very moderate amount, whilst that between superimposed wires, although equal to the full voltage of the machine, can be safely borne owing to the extra insulation of the insertion, *s s*.

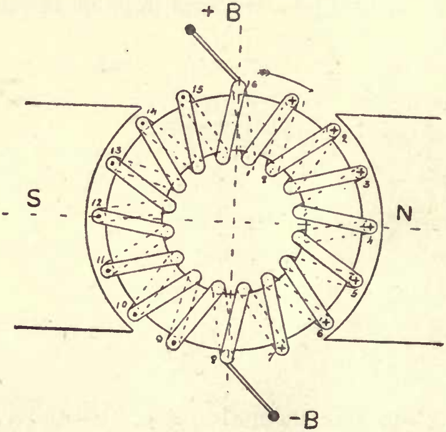

FIG. 40.

Another point which requires careful attention in drum armatures is the arrangement and insulation of the end connections. A glance at Fig. 38 will show that the wires at the end faces of the armature cross each other at various angles. Now, it is comparatively easy to insulate two wires laid parallel side by side, but when the wires cross there is great risk of the insulation being cut through, to avoid which the insertion of strong felt or other insulating material becomes necessary. In larger machines, which are not wound with wire but with bars, the end connections are generally formed by specially-shaped plates so arranged as to avoid any large difference of potential between adjacent plates. Such constructions will be found more fully described in the examples of machines given later on.

Another method of winding two-pole armatures is shown in Fig. 40. This is known under the term ring winding, or Gramme winding, although first used by Pacinotti in his electromotor. In this arrangement the core of the armature forms a hollow cylinder, and the winding is carried spirally round it, being, of course, closed on itself. In the figure there are shown 16 conductors. Starting the winding at the top, we would wind down 16, then up 1′, through the interior, then down 1, up 2′, down 2, up 3′, and so on.

The winding table is in this case simple. Writing 1′, 2′, 3′ for the inner turns of coils 1, 2, 3, we have

F.	D	B.	U.	F.	D.	B.	U.	F.	D.	B.	U.
—	16	—	1′	—	1	—	2′	—	2	—	3′
—	3	—	4′	—	4	—	5′	—	5	—	6′
—	6	—	7′	—	7	—	8′	—	8	—	9′
—	9	—	10′	—	10	—	11′	—	11	—	12′
—	12	—	13′	—	13	—	14′	—	14	—	15′
—	15	—	16′	—	16	—		—		—	

In this case the back and front connections are much shorter than in the drum armature, their length being, in fact, only slightly greater than the radial depth of the armature core, or, say, about one-third the diameter, whilst in drum armatures the length of the end connections is from $1\frac{1}{4}$ to $1\frac{3}{4}$ times the diameter. This is a distinct advantage of the ring winding; but, on the other hand, the greater number of wires is a disadvantage as compared with the drum winding. On comparing the above winding table with that given on page 161, it will be seen that of those wires which are parallel to the iron there are twice as many, and of end connections there are also twice as many. It is, of course, advantageous to produce the required voltage with as short a length of wire as possible, not only on account of the saving in material, but also because the resistance of the armature will thereby be reduced. In comparing, therefore, the merits of two methods of winding armatures, one of the points to be taken into account is the length of wire required to produce a given voltage, or, in other words, the ratio of the length of wire under the influence of the field magnet to the total length of wire in the winding. In both kinds of winding only the outer conductors are under the influence of the field poles, and are, therefore, producing electromotive force; the end connections in ring and drum, and the internal wire in the ring winding, do not contribute to the electromotive force, and must, therefore, be regarded as idle wires. The ratio of useful to total winding depends, of course, on the general proportions of the wire and on the skill with which the designer contrives to arrange the wires with the least waste of space. The latter consideration depends again on the size, speed, and voltage of the machine, as there will obviously be less waste of space where the machine is large and the wires stout, than where the machine is small and has to be wound with fine wire, the space occupied by insulation being in the latter case large in proportion to the space occupied by copper. Taking, however, average condition of winding, we can make a rough comparison between the ring and drum type of armature, assuming for this purpose that the end connections in the ring are ·4 of the diameter of the core, and in the drum 1·6. We must also assume a certain proportion between the diameter and length of core. If, for instance, the length is equal to the diameter, we would have in the ring each turn equal to 2·8 times the diameter, and the efficiency of the winding would be $\frac{1}{2{\cdot}8} = {\cdot}356$. In the drum each turn would be 5·2 times the diameter of the core, and the efficiency of winding $\frac{2}{5{\cdot}2} = {\cdot}385$. The following table shows the efficiency of winding calculated in this manner for different ratios of length to diameter of core:

$\frac{\text{Length}}{\text{Diameter.}}$	Efficiency of Winding for	
	Ring.	Drum.
·5	·278	·238
1·0	·356	·385
1·5	·395	·484
2·0	·416	·555

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This table shows clearly that the longer the armature in comparison with its diameter, the better is the total length of winding utilised. It also shows that for all but very short armatures the drum winding is more efficient than the ring winding, though for the usual proportions, when the length is from one to $1\frac{1}{2}$ times the diameter, the difference between the efficiency of the two windings is not very great. There is, however, another point in favour of drum winding which does not appear in the above table. This table gives simply the length of useful winding, but says nothing as to resistance of winding. When an armature is wound with wire the total armature resistance is, of course, directly proportional to the length of wire comprised in the winding, but when a bar winding is used the section of its different parts (outer bars, cross-connections, and inner bars, if any) can be made different, so as to utilise the available winding space in the best possible way, or reduce the armature resistance as far as possible. Now, in a ring, the part where the winding is most difficult to house is the inside, as the space there is necessarily limited, and for this reason it is hardly ever found possible to increase the section of the inside bars as compared to that of the outside or active bars. On the other hand, with a drum, the winding space at the ends is not so restricted, and we can generally manage to use cross-connections of larger area than that of the active bars. We thus find that not only does the drum have a shorter winding circuit than the ring, but parts of this circuit may be made of larger section than is possible in the ring, with the result that the resistance is sensibly reduced. A lower resistance means, of course, that we may pass a larger current through the armature, and obtain a larger output from a given weight and size of armature. In practice it is found that this increase of output amounts to from 30 to 50 per cent.

Thus far the advantage would seem to lie entirely with the drum winding, but this is to some extent balanced by the greater difficulty of the insulation and support of the coils. In small armatures which are wound with wire, the coils are more difficult to hold in place when wound drum fashion than when wound ring fashion, as in the latter case the inner turns and short end connections help to keep the outer turns in their position. The same thing applies to large machines for high voltage, with the additional difficulty of insulating neighbouring coils, which is absent in ring armatures. By referring to Fig. 40, it will be seen that the current passes successively through coil after coil in the order in which the coils succeed each other, so that the difference of potential between adjacent coils is equal to the electromotive force generated in one coil. It is therefore an easy matter to insulate the coils against each other, and on account of this advantage we find that for machines of high voltage the ring winding is generally preferred. In medium-sized and large machines of moderate voltage, the difficulties above mentioned can, however, easily be met, and in these cases the drum winding is certainly preferable to the ring winding.

Multipolar Parallel Winding.

We have now to enquire how either method of winding may be extended to multipolar machines. The simplest case is a parallel-wound ring armature, and this we take first. Fig. 41 shows such an armature in a six-pole field. It will be seen that the winding is carried spirally round the armature core in

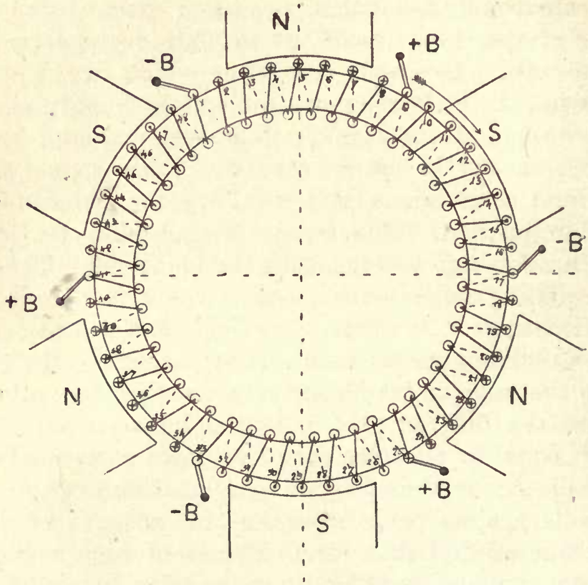

FIG. 41.

precisely the same manner as would be adopted in a two-pole machine. The direction in which the electromotive force is induced in the different wires is shown by dots and crosses as before. In each group of seven wires under the N poles the direction is downwards, in each group under the S poles it is upwards. Assuming each of the conductors to generate 1 volt and the current to enter at the brush − B to the left of the uppermost N pole, where we may regard the absolute value of the potential to be zero, there will be an absolute potential of 7 volts in wire 9. From wire 10 onwards the electromotive force is upwards—that is to say, we have to deduct 1 volt for each successive wire, so that by the time we arrive at 17 the absolute potential has again been reduced to zero. The two brushes marked − B in the diagram are, therefore, at the same potential, and may be joined by an external conductor. The same consideration applies to the rest of the armature and to the positive brushes, so that we may connect externally the three negative brushes together and the three positive brushes together. The voltage between the negative and positive brushes is, of course, that due to one section of the armature winding, and the total current is six times that passing through each conductor.

In the type of winding shown in Fig. 41, we require, therefore, six brushes spaced equally round the commutator. This is with certain constructions of machine an inconvenient disposition, and has the further disadvantage that we have to adjust six brushes instead of two. It is, however, possible to reduce the number of brushes to two by adding to the winding internal cross-connections. The figure represents an armature with 48 wires, and in the position shown the three negative brushes connect coils 1, 17, and 33, whilst at the same time the three positive brushes connect coils 9, 25, and 41. Now, it will be clear that if we wish to remove four out of the six brushes, we must replace the external connections made between the two sets of three brushes by internal connections made between the two sets of three coils above mentioned. This would, of course, have to be done for all sets of three coils, and might be represented by the following winding table, in which the vertical columns, read downwards, represent the successive spirals of the usual ring winding, and the horizontal lines between the columns the internal cross-connections. Those of the latter which at the moment are in immediate connection with the brushes are shown by thicker lines.

Fig. 42 represents diagrammatically a four-pole cylinder armature with cross-connections. In order to keep the illustration clear, the armature is supposed to have only 16 coils, and the cross-connections are shown in concentric circles, though in practice they are generally arranged spirally round a cylindrical sleeve behind the commutator, or they are housed within the commutator. Cross-connections of this kind have first been used by Mr. Mordey, in his Victoria dynamos.

Winding Table for Parallel Six-Pole Ring Armature with Internal Cross-Connections.

1 — 17 — 33 —	10 — 26 — 42 —
2 — 18 — 34 —	11 — 27 — 43 —
3 — 19 — 35 —	12 — 28 — 44 —
4 — 20 — 36 —	13 — 29 — 45 —
5 — 21 — 37 —	14 — 30 — 46 —
6 — 22 — 38 —	15 — 31 — 47 —
7 — 23 — 39 —	16 — 32 — 48 —
8 — 24 — 40 —	17 — 33 — 1 —
9 — 25 — 41 —	

The advantages of the multipolar ring winding are that we can use conductors of smaller section, which can be more easily handled, and that the commutated currents are smaller, so that the danger of sparking at the brushes can be more easily avoided. There is also the further advantage that no great difference of potential can ever exist between adjacent coils. On the other

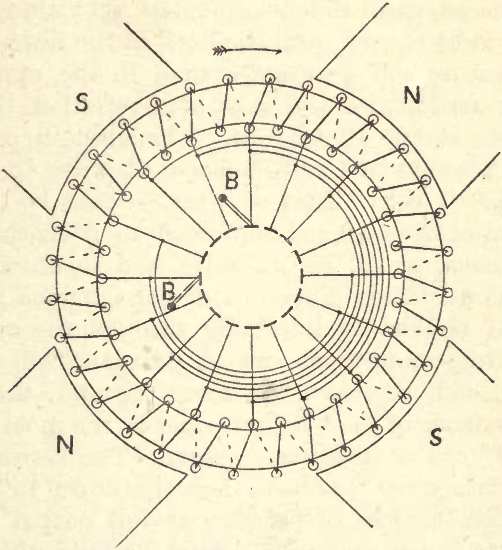

FIG. 42.

hand, there is the danger of internal currents, heating, and waste of power due to the following cause: Suppose that, owing to careless lining out in the erection of the machine, or to wear in the bearings, the armature is not quite in the centre of the field, but a little lower, the air space between its core and the three lower field poles will be less, and that between the core and the three upper field poles, Fig. 41, will be more than the normal amount. It will be obvious that the total flux of lines depends, amongst other things, on the air space; being greater for a small and smaller for a large air space. The eccentric position of the armature results, therefore, in an inequality of magnetic flux from the different pole-pieces with the consequence that the electromotive force of each individual coil in the lower half of the armature will be greater than in the upper half. Let us assume, for the sake of illustration, that the difference is only 10 per cent. The absolute potential of wire 2 would be 1 volt, and that of wires 18 and 34 would be 1·1 volt. That of wire 3 would be 2 volts, and that of wires 19 and 35, joined to it by the cross-connections, would be 2·2 volts, and so on with the other wires. This difference of voltage must produce currents circulating internally through the coils and their cross-connections, which currents will be the greater, and, therefore, the more harmful, the lower the resistance of the winding—that is, the more perfect the armature is in other respects. The resistance of an armature can easily be brought down to such a value that the loss of pressure at full output is only from 3 to 3½ per cent. of the total electromotive force. If such an armature should be out of centre to the extent assumed (*i.e.*, producing a difference of flux of 10 per cent.), then the wasteful internal currents would be about 1½ times as strong as the normal currents, and being, as it were, superimposed on the latter, the result would be that those coils which are in the strong field would carry 2½ times the normal current, and those coils which are in the weak field would carry a negative current of half the normal strength. Owing to armature reaction, the inequality in the current in the different coils will, in reality, not be so great as here indicated, but even if we assume that the armature reactions would entirely prevent any reversal of

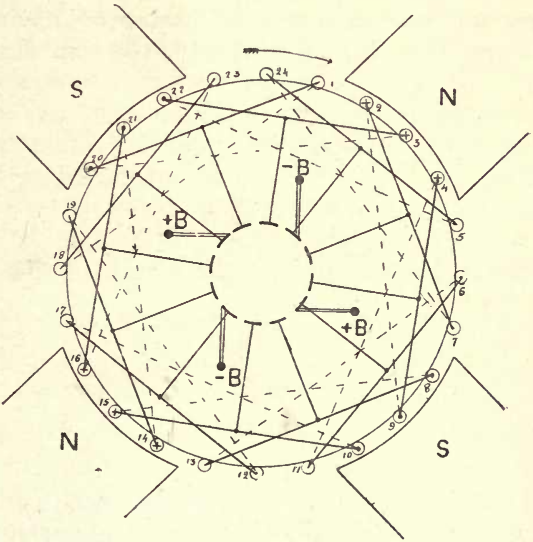

FIG. 43.

current in the top coils, this would still leave double the normal current to be carried by the lower coils, and consequently there would be double the voltage loss over the armature. There would also be a great tendency to sparking, owing to the want of symmetry in the field and the one-sided load on the armature.

For these various reasons it is important, if a parallel method of winding be employed, to take great care to have the armature properly centred, and the field poles all of the same strength. This applies, of course, equally to multipolar drums.

We have now to investigate the parallel method of drum winding for multipolar machines, and for this purpose we take a four-pole armature having 24 conductors, Fig. 43. Electrically, such an armature

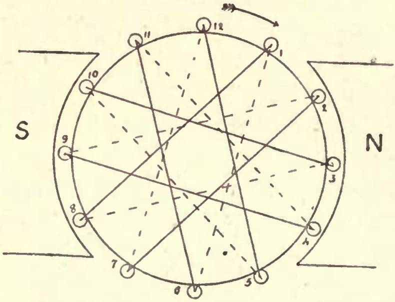

Fig. 44.

is equivalent to a pair of armatures, each having 12 conductors, and each taking half the total current. In order to find the winding for the four-pole drum, we need therefore only copy the connections which would be required in the two-pole drum. Thus, beginning the winding in the latter at wire 2, Fig. 44, we wind down 2, then across the back and up 9, then across the front and down 4, and so on. Precisely the same sequence of winding has to be used in the four-pole drum, Fig. 43, but since here the angular distance between adjacent conductors is half of the corresponding value in the two-pole armature, the cross-connections span only about one quarter instead of one half the circumference. It should also be noted that the front and back cross-connections are not equal in length. Thus the connection 2-9 spans seven wires, whilst that 9-4 spans only five wires. The mean length between the two would be a connection spanning six wires—that is, exactly one-quarter the circumference. If we continue in Fig. 43 the sequence of winding here indicated, we arrive again at the starting point, and so obtain a closed winding. This is technically known under the term "lap winding," from the fact that successive turns, like 2 B 9 F, 4 B 11 F, etc., overlap each other. The following is the winding table for the armature shown in Fig. 43:

F.		B.		F.		B.		F.		B.		F.		B.		F.		B.	.
0	24	*0*	0	*0*	2	*1*	9	*2*	4	*3*	11	$\underline{\underline{3}}$	6	*3*	13	*3*	8	*2*	15
1	10	*0*	17	$\underline{0}$	12	*0*	19	*0*	14	*1*	21	*2*	16	*3*	23	$\underline{\underline{3}}$	18	*3*	1
3	20	*2*	3	*1*	22	*0*	5	$\underline{0}$	24	*0*	7	*0*	2	*1*	9	*2*	4	*3*	11

The letters D and U are omitted because superfluous, as it is immaterial whether we start at any given wire by winding downwards or upwards. The result must be in either case the same. The letters F and B represent, as before, cross-connections, and the smaller numbers in the F columns represent the absolute potential in that part of the winding given in any convenient unit. To determine the potential at any point of the winding we have only to start at the negative brush (assumed to be a zero potential) and follow the winding, adding for each active conductor the number of volts it produces. The direction of electromotive force is shown, in Fig. 43, in the usual way by dots and crosses, and this corresponds to a disposition of field where the magnets are set at an angle of 45 deg., as shown. We thus have

No electromotive force in wires 23, 24, 1; 5, 6, 7; 11, 12, 13; 17, 18, 19.

Downwards electromotive force in wires 2, 3, 4; 14, 15, 16.

Upwards electromotive force in wires 8, 9, 10; 20, 21, 22.

Assuming, for the sake of easy calculation, that each wire adds 1 volt, and that the negative brush is touching the commutator bar corresponding to the front connection 24-5, then we shall have 1 volt in connection 22-3, and 3 volts in connections 20-1 and 18-23. The next connection is at the back 23-16, and in it the potential will still be 3 volts, but if we pass through wire 16 to the front we lose 1 volt, as the electromotive force in this wire is downwards. The potential of the front connection 16-21 is therefore only 2 volts. As our object is to tap the armature at the point where the potential is highest, it follows that we must place the positive brush at a point which is beyond wire 20 (which is the last wire adding electromotive force), and before wire 16 is reached. As the point must be on a front connection, its locality is restricted to either 20-1 or 18-23. Let us select the latter position, as that will place the brushes −B and +B, Fig. 43, exactly 90 deg. apart. We have seen that the current entering at −B and passing down wire 5, up 22, and so on, finds its way quickly out at brush +B. How about the other branch of the current, which passes down 24? If we follow this in the winding table, we find that it has to pass 18 wires before reaching the positive brush, 23-18—that is, three times as many wires as the former current. Moreover, the potential will rise to 3 volts when we reach 11-6, then fall to zero when we reach 12-17, and then rise again to 3 volts when we reach 18-23. Clearly, it is advantageous to let the current out as soon as it has reached the 3 volts for the first time, and for this reason we must place a brush on the commutator section corresponding to the front connection 11-6, indicated in the diagram by +B′. Similarly, we must place a negative brush, −B′, on the commutator bar corresponding with the front connection 12-17. In the winding table the position of the brushes is indicated by underlining the figures giving the voltage in the columns F, single underlining being used to denote the negative, and double underlining the positive brush. In winding an armature in this manner we lay on successive laps, with an advance of two between each lap and the following lap, and continue till we have gone once round the armature. The only conditions which must be fulfilled in order that the winding may close on itself is that there be an even number of bars if counted all round the armature, and that the distance between the two bars forming one lap should be represented by an odd number. Thus, starting with the last bar, which must have an even number, we wind down this and cross the back in a forward direction till we get to, say, the twenty-first bar. Here we wind up and cross the face in a backward direction to bar 2. We might thus say that the winding is laid on with a forward "pitch" of 21 and a backward "pitch" of 19. Or we might have a forward pitch of 17 and a backward pitch of 15, or any other combination in which the forward and backward pitch are odd numbers differing by 2. The pitch must, of course, be such as to embrace a little more than the angular width of the pole-piece, in order to fully utilise the field. If the pitch be chosen larger than necessary, the winding can still be used, but there is waste of copper in the extra length of cross-connections, and an increased armature resistance. An excessively large pitch, which would bring the two bars of one lap simultaneously under the influence of two equal poles, would not only reduce the voltage, but also increase the sparking difficulty.

It is characteristic for this type of drum winding that precisely the same armature may be used in fields having different numbers of poles, the only alteration required being in the number of brushes, just the same as with a ring armature. Thus a drum with 24 conductors having a forward pitch of 7 and a backward pitch of 5 will work perfectly well in a four-pole field, provided the angular width of the pole-pieces does not exceed the space occupied by three bars. Precisely the same armature can be used in a two-pole field with pole-pieces of the same dimensions. The electromotive force would be the same in both cases, but the

WINDING AND POTENTIAL TABLE FOR SIX-POLE PARALLEL DRUM, 120 CONDUCTORS.

F		B.		F.		B.		F.		B.		F.		B.		F.		B.		F.
9	120	*8*	21	*7*	2	*6*	23	*5*	4	*4*	25	*3*	6	*2*	27	*1*	8	*0*	29	*0* –
0 –	10	*0*	31	*0* –	12	*0*	33	*1*	14	*2*	35	*3*	16	*4*	37	*5*	18	*6*	39	*7*
7	20	*8*	41	*9*	22	*10*	43	*11*	24	*12*	45	*13*	26	*14*	47	*15*	28	*16*	49	*16* =
16 =	30	*16*	51	*16* =	32	*16*	53	*15*	34	*14*	55	*13*	36	*12*	57	*11*	38	*10*	59	*9*
9	40	*8*	61	*7*	42	*6*	63	*5*	44	*4*	65	*3*	46	*2*	67	*1*	48	*0*	69	*0* –
0 –	50	*0*	71	*0* –	52	*0*	73	*1*	54	*2*	75	*3*	56	*4*	77	*5*	58	*6*	79	*7*
7	60	*8*	81	*9*	62	*10*	83	*11*	64	*12*	85	*13*	66	*14*	87	*15*	68	*16*	89	*16* =
16 =	70	*16*	91	*16* =	72	*16*	93	*15*	74	*14*	95	*13*	76	*12*	97	*11*	78	*10*	99	*9*
9	80	*8*	101	*7*	82	*6*	103	*5*	84	*4*	105	*3*	86	*2*	107	*1*	88	*0*	109	*0* –
0 –	90	*0*	111	*0* –	92	*0*	113	*1*	94	*2*	115	*3*	96	*4*	117	*5*	98	*6*	119	*7*
7	100	*8*	1	*9*	102	*10*	3	*11*	104	*12*	5	*13*	106	*14*	7	*15*	108	*16*	9	*16* =
16 =	110	*16*	11	*16* =	112	*16*	13	*15*	114	*14*	15	*13*	116	*12*	17	*11*	118	*10*	19	*9*

E.M.F. upwards in ... 33 — 48; 73 — 88; 113 — 8

E.M.F. downwards in 13 — 28; 53 — 68; 93 — 108

Idle wires.. 29 — 32; 49 — 52; 69 — 72; 89 — 92; 109 — 112; 9 — 12

current with a two-pole field would be half that obtainable with a four-pole field. To make this point clear, a winding table is here given for a six-pole parallel drum, having 120 bars, pitch forward 21 and backward 19. The position of the six brushes is indicated by underlining the corresponding numbers in the voltage

WINDING AND POTENTIAL TABLE FOR FOUR-POLE PARALLEL DRUM, 120 CONDUCTORS.

F.		B.		F.		B.		F.		B.		F.		B.		F.		B.		F.
4	120	*4*	21	*3*	2	*3*	23	*2*	4	*2*	25	*1*	6	*1*	27	*0*	8	*0*	29	*0*
0	10	*0*	31	*0*	12	*0*	33	*0*	14	*1*	35	*1*	16	*2*	37	*2*	18	*3*	39	*3*
3	20	*4*	41	*4*	22	*5*	43	*6*	24	*7*	45	*8*	26	*9*	47	*10*	28	*11*	49	*12*
12	30	*12*	51	*13*	32	*13*	53	*14*	34	*14*	55	*15*	36	*15*	57	*16*	38	*16*	59	*16*
16	40	*16*	61	*16*	42	*16*	63	*16*	44	*15*	65	*15*	46	*14*	67	*14*	48	*13*	69	*13*
13	50	*12*	71	*12*	52	*11*	73	*10*	54	*9*	75	*8*	56	*7*	77	*6*	58	*5*	79	*4*
4	60	*4*	81	*3*	62	*3*	83	*2*	64	*2*	85	*1*	66	*1*	87	*0*	68	*0*	89	*0*
0	70	*0*	91	*0*	72	*0*	93	*0*	74	*1*	95	*1*	76	*2*	97	*2*	78	*3*	99	*3*
3	80	*4*	101	*4*	82	*5*	103	*6*	84	*7*	105	*8*	86	*9*	107	*10*	88	*11*	109	*12*
12	90	*12*	111	*13*	92	*13*	113	*14*	94	*14*	115	*15*	96	*15*	117	*16*	98	*16*	119	*16*
16	100	*16*	1	*16*	102	*16*	3	*16*	104	*15*	5	*15*	106	*14*	7	*14*	108	*13*	9	*13*
13	110	*12*	11	*12*	112	*11*	13	*10*	114	*9*	15	*8*	116	*7*	17	*6*	118	*5*	19	*4*

E.M.F. upwards	43 — 58	103 — 118	Idle wires	119 — 12	29 — 42	
E.M.F. downwards ...	13 — 28	73 — 88		59 — 72	89 — 102	

columns. It will also be seen by reference to these columns that the full difference of potential exists between adjacent bars, just as in the ordinary two-pole drum, but as the multipolar parallel winding is generally used for large currents and moderate voltages, the difficulties of insulation are not serious.

In the winding table it is assumed that each of the six brushes touches two commutator segments, the negative brushes being at *0*, and the positive brushes at *16*.

If we now take the same armature and place it in a four-pole field we obtain a perfectly feasible combination, the angular width of the pole-pieces being, of course, the same as before. The winding table for this arrangement is given on opposite page.

By reducing the number of poles from six to four, we have gained nothing in electromotive force, but we have lost one-third of the current, there being now only four circuits through the armature instead of six, as before. To employ a four-pole field to advantage we should have to increase the angular width of the poles and the total flux from each, and we should also have to make the pitch greater, say, 29 forward and 27 backward.

Multipolar Series Winding.

When discussing the multipolar parallel winding, we selected a ring armature as the starting point of our investigation, for the reason that its explanation is rather more simple than that of a parallel-wound drum. With series winding the case is reversed, the drum winding being more easily explained than the ring winding, and for this reason we shall begin the investigations with the former, taking as our first example a four-pole drum. The characteristic feature of all drum armatures is that no wires of any kind pass through the interior; hence to get from one bar to the other we cannot admit any other kind of connections but those that lie entirely on the back or front face of the armature core. The necessary consequence of this condition is that when we join two bars we can only join the back end of one bar to the back end of the other, or the front end of the one to the front end of the other, but in no case the back end of one bar to the front end of the other. Since the direction of electromotive force changes with the sign of the magnet pole, and since our object is to so couple up the bars that their electromotive forces shall add up, it follows that the length of front and back connections must correspond to the angular distance between the poles, or, in other words, that the pitch, y, must be about equal to the total number of bars divided by the number of poles, p. We say advisably "about" equal, because, as will be shown presently, the total number of bars can never be an exact multiple of the pitch. A four-pole series armature may be considered to result from the combination of two two-pole armatures in such a way that the electromotive forces are added. Let us then suppose the two-pole armatures cut open and stretched into semicylinders, which we place together so as to form an armature of double the original diameter. The successive bars, which in the two-pole armatures were opposite (or 180 deg. apart), will now be only 90 deg. apart, so that in passing through four bars in their order of connection we shall go once round the armature. The pitch, in other words, will now not be forward and backward as in the parallel method of winding, but always forward. It is also clear that the pitch must be an odd number, because if it were an even number we should never get any bars at all into the places distinguished by odd numbers. The distance between two successive bars wound downwards is, therefore, an even number, being twice the pitch, and in following the winding once round the armature we find that the bars coming under north poles have, say, even numbers, and the bars coming under south poles have odd numbers. Starting, then, with an even-numbered bar under one of the north poles, we arrive, after going once round, at a bar under the same pole, and this must also have an even number, though not, of course, the same number as the bar with which we started, as that would at once close the winding. By analogy with the two-pole drum we conclude that in going once round we must arrive at a bar either two in front or two behind that from which we started. The relation between the number of poles, p, total number of bars, τ, and pitch, y, is therefore given by the formula—

$$\tau = p\,y \pm 2,$$

y being an odd number.

Thus in a four-pole drum, with a pitch of 7, the number of bars may either be 30 or 26, but not 28, which would be the exact multiple of the pitch. With a pitch of 5 the number of bars would similarly be 18 or 22.

We have in the foregoing assumed that the length of the connectors in front is the same as that of the connectors at the back, but this is not absolutely necessary. By having the same pitch at both ends we obtain a perfectly symmetrical winding, and the designer would, for this reason, naturally adopt such a winding where possible. It is, however, not an absolute necessity to have the same pitch back and front, and it might, under certain circumstances, even be advantageous to abandon the perfectly symmetrical winding for one that is slightly unsymmetrical. Suppose, for instance, we made in the four-pole machine the back connectors with a pitch of 7, and the front connectors with a pitch of 5, then we could employ 26 bars, the winding being 26-7-12-19-24-5-10, etc. Or we could make the back connectors with a pitch of 9, and the front connectors with a pitch of 7, when we could wind a 30-bar armature as follows:

30-9-16-25-2-11-18, etc.

Electrically, either of these armatures is equivalent to the corresponding armatures ($\tau = 26$ and $\tau = 30$), which we obtained by making the pitch of the front and back connectors both 7.

To include cases where the pitch, front and back, differs by 2, we must write our formula for the number of bars as follows:

$$\tau = \frac{p}{2}(2\,y + 2) \pm 2,$$

y being the smaller of the two pitches and an odd number. We could thus wind a six-pole 50-bar armature with a pitch of 9 at the back and 7 in front.

$$50 = \frac{6}{2}\,(2 \times 7 + 2) + 2.$$

It is not necessary to give the whole winding table for such an armature, as a few figures suffice to show the sequence—thus :

50-9-16-25-32-41
48-7-14-23, etc.

If we assume the electromotive force to be downwards in bars 6 to 10, 23 to 27, 40 to 44, and upwards in bars 48 to 2, 14 to 18, 31 to 35, we find that the negative brush must touch the commutator segments connected to the front ends of bars 5, 21, or 37, and the positive brush the segments connected with the front ends of bars 47, 13, or 29, the distance between the two brushes being either 60 deg. or 180 deg. The advantage of using unequal pitch for the front and back connectors is that we are not so restricted in the choice of the number of bars. Thus in a six-pole armature with an equal pitch of 7, front and back, we could not get more than 44 bars, whereas with an equal pitch of 9, front and back, we could not get less than 52 bars.

Supposing, now, that when designing the machine we found that 44 bars would not give enough electromotive force, and that with 52 bars the electromotive force would be too great, then we can help ourselves by making the first connectors with a pitch of 7 and the back connectors with a pitch of 9. The number of bars will then be either 46 or 50. The adoption of unequal pitch gives us, therefore, greater choice in the number of bars which can be adopted. This will be seen more closely from the following table, which refers to six-pole machines.

Pitch.		Possible number of bars.	Pitch.		Possible number of bars.
Front.	Back.		Front.	Back.	
7	7	40 and 44	19	21	118 and 122
7	9	46 ,, 50	21	21	124 ,, 128
9	9	52 ,, 56	21	23	130 ,, 134
9	11	58 ,, 62	23	23	136 ,, 140
11	11	64 ,, 68	23	25	142 ,, 146
11	13	70 ,, 74	25	25	148 ,, 152
13	13	76 ,, 80	25	27	154 ,, 158
13	15	82 ,, 86	27	27	160 ,, 164
15	15	88 ,, 92	27	29	166 ,, 170
15	17	94 ,, 98	29	29	172 ,, 176
17	17	100 ,, 104	29	31	178 ,, 182
17	19	106 ,, 110	31	31	184 ,, 188
19	19	112 ,, 116	31	33	190 ,, 194

Similarly, the possible numbers of bars for eight-pole machines is given in the following table.

Pitch.		Possible number of bars.	Pitch.		Possible number of bars.
Front.	Back.		Front.	Back.	
11	11	86 and 90	23	25	190 and 194
11	13	94 ,, 98	25	25	198 ,, 202
13	13	102 ,, 106	25	27	206 ,, 210
13	15	110 ,, 114	27	27	214 ,, 218
15	15	118 ,, 122	27	29	222 ,, 226
15	17	126 ,, 130	29	29	230 ,, 234
17	17	134 ,, 138	29	31	238 ,, 242
17	19	142 ,, 156	31	31	246 ,, 250
19	19	150 ,, 154	31	33	254 ,, 258
19	21	158 ,, 162	33	33	262 ,, 266
21	21	166 ,, 170	33	35	270 ,, 274
21	23	174 ,, 178	35	35	278 ,, 282
23	23	182 ,, 186	35	37	286 ,, 290

To summarise: If the pitch at front and back are equal, the number of bars is given by

$$\tau = p\, y \pm 2.$$

If there is a difference of 2 between the front and back pitch, the formula becomes

$$\tau = p\ (y+1) \pm 2,$$

y being the smaller pitch, and in either case an odd number.

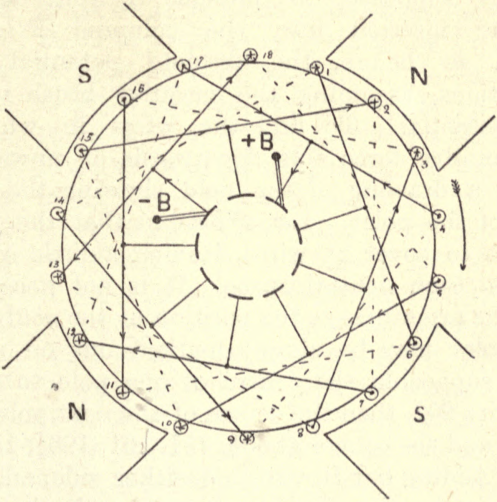

FIG. 45.

Having now settled the question concerning the possible number of bars, we return to our example of a four-pole machine.

The diagram, Fig. 45, shows the winding for a four-pole drum having 18 conductors. The current enters at the negative brush, − B, and issues at the positive brush, + B, the branch passing down 18 receiving electromotive force from the wires 15, 2, 7, 12, and the branch passing down 13 receiving electromotive force from the wires 3, 16, 11, 6.

This kind of winding is, of course, applicable to any number of poles. The following table gives the winding for an eight-pole drum having 202 conductors and a pitch of 25, front and back. Each active wire is supposed to produce 1 volt, and the numbers inserted into the columns F and B denote, as before, the absolute potential of the connections, assuming the negative brush to be at zero potential. To find the wires in which the electromotive force acts downwards or upwards, we require a drawing of the field showing the angular width of the poles. Let us assume that the latter is such as to cover 21 wires, leaving a little over four wires in each neutral space. It is not necessary to draw the armature, as the position of the centre of the field poles may be simply marked out on a circle. Thus, supposing the centre of one pole to coincide with wire 202, then the centre of the next pole will be at $25\frac{1}{4}$, and the others at $50\frac{1}{2}$, $75\frac{3}{4}$, 101, $126\frac{1}{4}$, $151\frac{1}{2}$, and $176\frac{3}{4}$. Adding our 10 wires on either side, and rounding off the fractions, we arrive at the following result:

E.M.F. downwards in	192 — 10		Idle wires	11 — 14
	40 — 60			36 — 39
	91 — 111			61 — 65
	141 — 161			87 — 90
E.M.F. upwards in ...	15 — 35			112 — 115
	66 — 86			137 — 140
	116 — 136			162 — 166
	167 — 187			188 — 191

By reference to these figures, it is now an easy matter to insert the potential in the columns F and B, which has been done in the winding table here given.

WINDING AND POTENTIAL TABLE FOR EIGHT-POLE SERIES-WOUND DRUM. 202 CONDUCTORS; PITCH 25.

F.		B.		F.		B.		F.		B.		F.		B.		F.
41	202	42	25	43	50	44	75	45	100	46	125	47	150	48	175	49
49	200	50	23	51	48	52	73	53	98	54	123	55	148	56	173	57
57	198	58	21	59	46	60	71	61	96	62	121	63	146	64	171	65
65	196	66	19	67	44	68	69	69	94	70	119	71	144	72	169	73
73	194	74	17	75	42	76	67	77	92	78	117	79	142	80	167	81
81	192	82	15	83	40	84	65	84	90	84	115	84	140	84	165	84
84	190	84	13	84	38	84	63	84	88	84	113	84	138	84	163	84
84	188	84	11	84	36	84	61	84	86	83	111	82	136	81	161	80
80	186	79	9	78	34	77	59	76	84	75	109	74	134	73	159	72
72	184	71	7	70	32	69	57	68	82	67	107	66	132	65	157	64
64	182	63	5	62	30	61	55	60	80	59	105	58	130	57	155	56
56	180	55	3	54	28	53	53	52	78	51	103	50	128	49	153	48
48	178	47	1	46	26	45	51	44	76	43	101	42	126	41	151	40
40	176	39	201	38	24	37	49	36	74	35	99	34	124	33	149	32
32	174	31	199	30	22	29	47	28	72	27	97	26	122	25	147	24
24	172	23	197	22	20	21	45	20	70	19	95	18	120	17	145	16
16	170	15	195	14	18	13	43	12	68	11	93	10	118	9	143	8
8	168	7	193	6	16	5	41	4	66	3	91	2	116	1	141	0
0	166	0	191	0	14	0	39	0	64	0	89	0	114	0	139	0
0	164	0	189	0	12	0	37	0	62	0	87	0	112	0	137	0
0	162	0	187	1	10	2	35	3	60	4	85	5	110	6	135	7
7	160	8	185	9	8	10	33	11	58	12	83	13	108	14	133	15
15	158	16	183	17	6	18	31	19	56	20	81	21	106	22	131	23
23	156	24	181	25	4	26	29	27	54	28	79	29	104	30	129	31
31	154	32	179	33	2	34	27	35	52	36	77	37	102	38	127	39
39	152	40	177	41	202	42										

It will be noticed that there are no less than nine front connections which are at potential zero, and nine front connections which are all at the same potential of 84 volts. We might place the negative brush on any of the former and the positive brush on any of the latter. Selecting, however, in each case the connection equally distant on both sides from active wires, we find the position for the negative brush on that commutator segment which is joined with the connection 139-164, and the positive brush on the segment corresponding to connection 63-88. The two brushes will then be 135 deg. apart. It would, however, be equally correct to place, say, the positive brush on 115-140, when its distance from the negative brush would be 45 deg. In fact, if the angular width of the poles is sufficiently small so as to leave a large number of wires idle, eight brushes may be used spaced 45 deg. apart, of which four would be positive and four negative. This arrangement may be advantageous when it becomes important to reduce the length of the commutator without cutting down the brush surface. The number of commutator bars required is 101, or half the number of conductors. The displacement of the commutator sections relatively to the brushes may be represented in the winding table by drawing a pencil down the first and third F column. Thus, taking the positive brush, we may assume that at the moment to which the table refers the positive brush has just left the segment corresponding to connection 65-90, and is now only touching the segment corresponding to 63-88, as shown by the double underlining. A moment later it will also touch the segment corresponding to 61-86, and finally leave 63-88. To truly represent the action going on in the armature, all the numbers in the winding table must be considered to move downwards, so that the effect will be the same as if the numbers stood still and the brush oscillated up and down over the distance of two lines. It will be seen that the current must be reversed simultaneously in eight wires, but there are also eight magnet poles to produce the reversal.

With this type of winding the difficulty of insulating adjacent conductors from each other is magnified. From what has been already said on the subject of parallel winding, it will be clear that had this armature been wound parallel the greatest difference of potential between adjacent conductors would have been 21 volts. It is now 84 volts, or four times as great. The insulation between adjacent bars must be strong enough to resist the full voltage of the machine, and for this reason the winding here described is only used for moderate pressures. For ordinary central station work on the three-wire system, where a pressure of 250 is about the maximum required, this winding is perfectly safe, and it has also been used successfully in power transmission, and for arc lighting up to 600 volts, but beyond this pressure the series ring winding is preferable.

When discussing the multipolar parallel winding we found that an inequality in the strength of the fields must cause wasteful internal currents. This defect is entirely absent in the multipolar series winding. On reference to the winding table it will be seen that if there exists such an inequality, it must affect both branches of the current within the armature to the same extent, so that the balance between them is not disturbed, and no wasteful currents can be generated. This is an important advantage, not only of this particular winding, but of all methods of series winding.

We shall now proceed to investigate the multipolar series ring winding. The transition from the drum to the ring is most easily made if we replace each bar by a coil wound over the ring in the usual Gramme fashion. In order, however, to leave the connectors where they were we must reverse the direction in which each alternate coil is wound. Thus in the four-pole armature, Fig. 45, we would wind the coil corresponding to bar 18, say, down on the outside and up through the inside of the ring, and the same with coils 2, 4, 6, etc. On the other hand, coils 1, 3, 5, etc., would be wound up on the outside and down through the inside of the ring. Such a winding is shown in Fig. 46, but to avoid overlapping and keep the illustration clear, the number of coils is assumed to be 22, instead of 18. Beginning the winding at coil 22, we wind this down on the outside and finish at the outside on the back. Coil 5 we wind up on the outside, down on the inside, and finish at the outside in front. The object of winding the coils alternately up and down is merely to get the connectors of the same length; where that is not of importance the coils may all be wound the same way, the beginning and finish being both left on outside wires. We now connect the back of 22 with the back of 5, the front of 5 with the front of 10, the back of 10 with the back of 15, and so on, exactly the same as in a drum armature. This winding has also the same fault, inasmuch as the difference of potential between adjacent coils is equal to the full pressure generated, and, as far as the author is aware, it has never been used in practice. The fault here mentioned can, however, easily be removed, and on removing it we arrive at a winding which (originally invented by Ayrton and

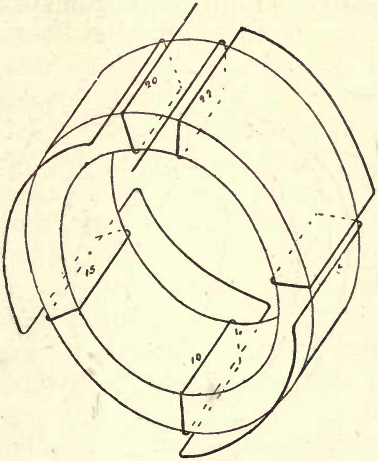

FIG. 46.

Perry, and first used by Andrews) is now extensively used for high-pressure work. The potential difference between coil 4 and coil 5 is great; it is also great between 5 and 6, but between 4 and 6, 6 and 8, 8 and 10, etc., it is small. If we therefore omit from the winding all unevenly numbered coils (1, 3, 5, etc.), we at once obtain a winding in which the potential difference is nowhere great, and which may be used up to

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any pressure for which the ordinary two-pole ring winding is safe. Now if we wish to omit coil 5 we can do so, provided we supply a connection between 22 and 10. We would therefore have to join the back outside end of coil 22 with the front outside end of coil 10. The connection would then pass from back to front through the interior of the armature, at the same time crossing over to the opposite point of the diameter.

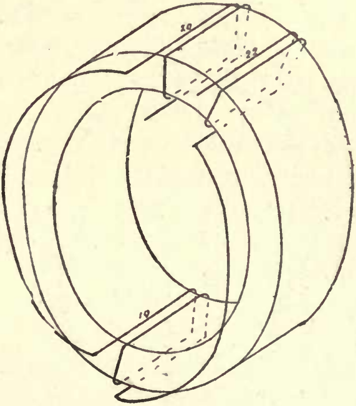

Fig. 47.

Such an arrangement would, however, be inconvenient. To avoid it, we need only increase coil 22 by half a turn by bringing the wire once more forward on the inside. This finishes the coil with an inside end in front. The connector lies now entirely on the front face of the armature, as shown in Fig. 47. In the same way we may put half a turn on the inside to the coil numbered 10, and thus finish it also in front.

The connector 10-20 will therefore likewise lie in front, and treating all the even-numbered coils in the same way, we find that all the connectors come to the front, and the winding becomes perfectly symmetrical. The winding includes, however, only the even-numbered coils and misses the coils of odd numbers. Thus, instead of a drum containing 22 bars, we have obtained a ring with only 11 coils; but if we give two turns to each coil we shall still have 22 external conductors, and therefore the same electromotive force as before. Counting, however, coils and not conductors, and giving the coils consecutive numbers, we can describe the winding by saying that the inside end of No. 11 is joined to the outside end of No. 5, the inside of the latter to the outside of 10, the inside of 10 to the outside of 4, and so on. The pitch in this case is 5, or half the sum of a front and back pitch of the equivalent drum winding. In the case described the two were equal, but they might also have differed by 2, and then the pitch of the ring winding, instead of being an odd number, would have become an even number. Calling y_f and y_b respectively the front and back pitch in a drum armature, the total number of bars is given by

$$\tau = \frac{p}{2}(y_f + y_b) \pm 2.$$

The equivalent ring armature has half the number of coils, and calling y the pitch for the ring, we have

$$y = \frac{y_f + y_b}{2},$$

and the number of coils in the ring $\tau = \frac{p}{2} y \pm 1$.

We have seen that in a drum armature the pitch must always be an odd number. In a ring armature, on the other hand, it may be either an odd or an even number. It will be an odd number if the front and back pitch of the equivalent drum (from which we may consider the ring to have been evolved) are equal, and it will be an even number if the front pitch is either greater or smaller by 2. It is convenient to tabulate the formulæ for armatures wound for different numbers of poles; we thus obtain the following table:

The number of coils must be equal to—	Machine has					
	4 poles	6 poles	8 poles	10 poles	12 poles	14 poles
	$2\,y \pm 1$	$3\,y \pm 1$	$4\,y \pm 1$	$5\,y \pm 1$	$6\,y \pm 1$	$7\,y \pm 1$

The pitch, y, being any even or odd number. It will be seen that whether the pitch is even or odd, the number of coils in machines having 4, 8, or 12 poles must always be an odd number. It must also be an odd number in machines having 6, 10, and 14 poles if the pitch is even, but if the pitch is odd the number of coils in machines having 6, 10, and 14 poles must be even.

We have now found the law which governs the number of coils that can be used in series-wound multipolar ring armatures, and must now find a way to represent the winding by means of a table analogous to that we have adopted with drum armatures. For this purpose we must agree on some method of distinguishing in the table between the outside and inside ends of the coils. We might, for instance, agree that the outside wire of a coil is on the left and the inside wire on the right of the number representing the coil in the winding table. Thus, if we write 31-62-30 it shall mean that the outside of 30 is connected to the inside of 62 and the outside of 62 to the inside of 31. This convention may be represented diagrammatically thus:

Outside Wire {Number of Coil} Inside Wire

The following is the winding table for a four-pole ring armature having 63 coils ($63 = 2 \times 31 + 1$).

WINDING AND POTENTIAL TABLE FOR FOUR-POLE SERIES RING, 63 COILS.

Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.	Coil.	Volts.
0	63	*0*	31	*0*	62	*0*	30	*0*	61	*0*	29	*5*	60	*10*	28	*15*	59	*20*	27	*25*
25	58	*30*	26	*35*	57	*40*	25	*45*	56	*50*	24	*55*	55	*60*	23	*65*	54	*70*	22	*75*
75	53	*80*	21	*85*	52	*90*	20	*95*	51	*100*	19	*105*	50	*105*	18	*105*	49	*105*	17	*105*
105	48	*105*	16	*105*	47	*105*	15	*105*	46	*105*	14	*105*	45	*105*	13	*100*	44	*95*	12	*90*
90	43	*85*	11	*80*	42	*75*	10	*70*	41	*65*	9	*60*	40	*55*	8	*50*	39	*45*	7	*40*
40	38	*35*	6	*30*	37	*25*	5	*20*	36	*15*	4	*10*	35	*5*	3	*0*	34	*0*	2	*0*
0	33	*0*	1	*0*	32	*0*														

E.M.F. is directed .... { downward inside } 3 — 13; { upward outside } 35 — 44

E.M.F. is directed .... { downward outside } 19 — 29; { upward inside } 51 — 60

No. E.M.F. in coils .... { 61 — 2; 14 — 18; 30 — 34; 45 — 50

We suppose each wire on the outer circumference of the armature to produce 1 volt, and each coil to have five turns, so that the electromotive force of each coil will be 5 volts. It is evident that each connector must be joined to one segment of the commutator, and as there are as many connectors as there are coils, we must have as many segments in the commutator as there are coils on the armature core. We may thus number the segments in the same way as we number the coils. The segments so numbered must, however, be connected either all to the inside wires or all to the outside wires of the coils, but not some segments to inside and some to outside wires.

It will be seen from this table that there is in no part of the winding a greater difference of potential than 5 volts between two adjacent coils. The negative brush may be placed on any commutator segment between 30 and 33 on one side, and between 62 and 2 on the opposite side. The positive brush may be placed either on any segment between 14 and 17 on one side, or on any segment between 46 and 49 on the opposite side. Two brushes only are necessary, placed 90 deg. apart; but four may be employed to get increased brush surface if required. In this respect the series ring resembles the series drum, though in the ring the possibility of placing additional brushes is not of so much advantage, since the ring winding would naturally only be employed in cases where the voltage is high and the current low or moderate, so that the brush surface need not be very large.

The question as to the angular distance between the positive and negative brushes is of considerable practical importance. If accessibility, ease of supervision, and compactness of design were the only considerations involved, we would naturally place the brushes as near together as the character of the winding permits, but from an electrical point of view this is not a good arrangement. In the first place there is the danger that both brushes may at the same time be accidentally touched, and in the second place there is greater probability of flashing over from one brush to the other if the distance between the brushes is small. For these reasons it is safer to put the brushes as far apart as the character of the winding will permit. The law which regulates the relative position of the brushes is very simple. We have seen that there are half as many equidistant positions for the negative brush as there are poles, and the same number of intermediate positions for the positive brush. Supposing now that we place brushes all round occupying these positions, and then see which of these brushes we can omit. Let us, for example, retain two neighbouring brushes and take away all the others. This will give us the minimum distance between the positive and negative brush, and this must obviously be equal to the angular distance between neighbouring poles. Thus, in a four-pole machine the distance would be 90 deg., in a six-pole machine 60 deg., in an eight-pole machine 45 deg., and so on. If we wish to increase this distance we can advance one of the brushes by an amount corresponding to twice the polar angle, or four times or six times the polar angle. To advance the brush by one, three, or five times the polar angle would obviously not do, as we should then occupy positions of the same potential as that of the brush which has not been moved. The advance of one brush would, of course, only be adopted if it resulted in an increase of distance between the two brushes. Thus, in a four-pole machine, the advance through, say, twice the polar angle would be useless, as that would bring the brush again within a distance of 90 deg. from the fixed brush, only on the other side of it. Similarly, in a six-pole machine we should advance through twice, but not through four times the polar angle, and so on. The angular distance between the two brushes must therefore be an odd multiple of the polar angle. For convenience of reference the following table is given:

Number of Poles.	Angular Distance between Brushes.				
2	180	—	—	—	—
4	90	—	—	—	—
6	60	180	—	—	—
8	45	135	—	—	—
10	—	108	180	—	—
12	—	90	150	—	—
14	—	77	128	180	—
16	—	—	112	158	—
18	—	—	100	140	180
20	—	—	90	126	162

Multipolar Series and Parallel Winding.

It is possible to combine the series and parallel method of winding in the same armature. For instance, we could wind a 12-pole drum with three independent circuits, starting at points 60 deg. or 120 deg. apart, each representing a four-pole series winding. We could then add a set of internal connectors, joining the bars of equal potential. The disadvantages of such an arrangement are that the front and back connectors become thrice as long as with the ordinary 12-pole series winding (spanning 90 deg. instead of 30 deg.), and that the internal cross-connections have to be added if we wish to avoid the use of 12 brushes. A better way is to wind the independent circuits side by side, each in the usual 12-pole series manner. The front and back connectors remain short, and no additional internal connectors are required, provided we make the two brushes wide enough to cover each at least as many segments as there are independent circuits. This method also leaves us free to choose as many independent circuits as may be convenient.

The object of employing a combined series and parallel winding is to obtain bars of convenient sectional area. Suppose, for instance, we have to design a six-pole machine for 1,000 amperes. If we wind the armature series, each bar would have to be large enough to carry 500 amperes, and the connections of large bars are difficult to make. There is, moreover, the difficulty of having to commutate the large current of 500 amperes. On the other hand, if we wind the armature parallel, we have to employ three times as many bars (each one-third the former section), and to make three times the number of joints. The space occupied by insulating material becomes larger, and the armature more expensive. There is the further danger of internal currents and waste of power, as previously explained. Neither method of winding employed alone is in this case quite satisfactory, but if we combine both we can obtain a perfectly satisfactory winding. Say, that to get the required electromotive force we want about 150 bars on the armature. We would naturally employ 152 bars, being $6 \times 25 + 2$, but as 500 amperes in each bar is too large we double the number of bars, and thus reduce the current to be commutated to 250 amperes. We would thus have 304 bars, and put these on in two series windings; one series running:

304-50-100-150-200-250-300-46-96, etc.

and the other

1-51-101-151-201-251-301-47-97, etc.

The brushes must in this case be wide enough to cover each at least two segments of the commutator.

CHAPTER IX.

Open-Coil Armatures—The Brush Armature—The Thomson-Houston Armature.

Open-Coil Armatures.

The simplest example of an open-coil armature is the so-called shuttle-wound armature represented in Fig. 48. It consists of a cylindrical iron core, in which two grooves are planed out for the reception of a coil, the ends of which are attached to the two semicircular segments of a commutator. In the figure the wires are shown passing behind the commutator, though in armatures as actually made they must, of course, be grouped to the right and left of it to make room for the hub of the commutator and the spindle of the armature. In the position shown, when the maximum number of lines of force passes through the coil, the electromotive force is zero, and the brushes short-circuit the two sections of the commutator. As the armature revolves, the flux through the coil diminishes until it is zero, when the armature occupies a position at right angles to that shown, and the electromotive force has attained its maximum value, and then the flux increases again to a maximum, whilst the electromotive force decreases to zero. Owing to the action of the commutator, the connection between the external circuit and the coil is reversed each time that the electromotive force in the latter is reversed, so that the direction of electromotive force in the external circuit remains the same, though the electromotive force changes or pulsates between zero and a maximum. If we represent the electromotive force as a function of the time, or the

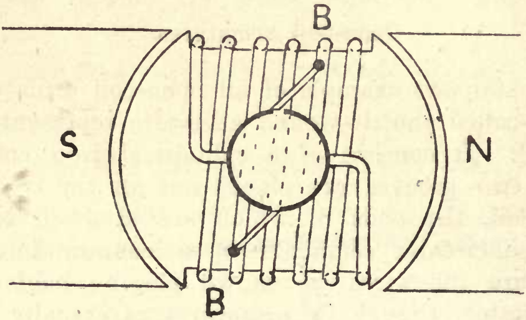

FIG. 48.

angular position of the armature, we obtain a curve, as shown in Fig. 49 by the full line. Had the armature been provided with two contact rings, instead of a two-part commutator, the electromotive force at the brushes, which are the terminals of the external circuit, and, therefore, the current in the latter, would have been alternating, as shown by the full and dotted curve in Fig. 49. The part of this curve above the horizontal is the same as before, but every alternate impulse is now negative. By using a commutator we obtain impulses which are all in the same direction, and this holds good, not only for the shuttle-wound armature shown in Fig. 48, but for other types. We could, for instance, wind the coil over a portion of a ring-shaped core, as shown in Fig. 50, but in this arrangement only one side of the ring is active, and it would obviously be an improvement to place a coil

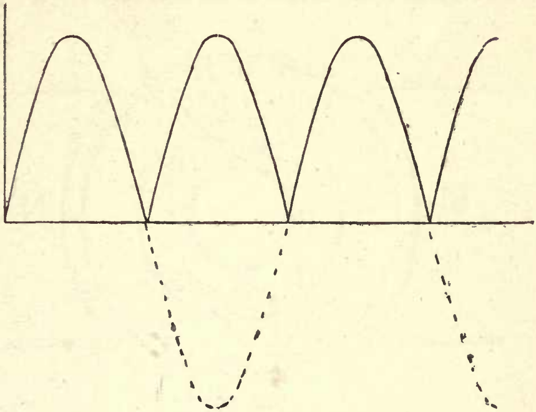

FIG. 49.

also on the opposite side of the ring, as shown in Fig. 51. The two coils must, of course, be coupled in series by having their inner ends joined across and their outer ends each to one section of the commutator. This armature is electrically equivalent to that shown in Fig. 48, though mechanically it is an improvement over the shuttle type, because the winding support and insulation of the coils are easier and the armature is ventilated. The current will, however, be equally pulsating as that from the shuttle-type armature. A current fluctuating as much as shown in Fig. 49 would be useless for lighting purposes, and would also, by virtue of the self-induction in the various portions of the circuit, cause severe strains on the insulation.

The question therefore is, how can we prevent the electromotive force from fluctuating between

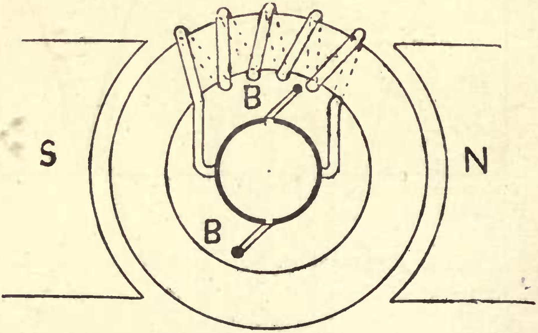

FIG. 50.

such wide limits? Starting from the position shown in Fig. 51, and calling it 0 in the diagram Fig. 52, we find that when the armature has turned through 90 deg., we get the first maximum of electromotive force, we get zero again at 180 deg., the second maximum at 270 deg., and so on. Thus, the best action will be within the limits of about 45 deg. and 135 deg., and 225 deg. and 315 deg. respectively, corresponding to the parts of the curve shown by thicker lines. If, then, we wish to avoid too great a fluctuation in the electromotive force, we would have to utilise only that part of the electromotive force curve in Fig. 52 which lies above the line *y y*. This can be done by reducing the length of the segments on the commutator from 180 deg. to 90 deg., but now a new difficulty crops up. It is true that during the time that the brushes make contact with the segments of the commutator the electromotive force does not vary

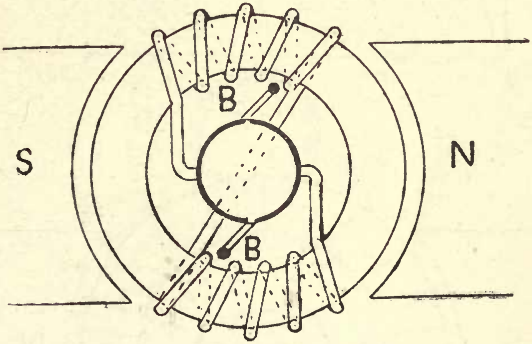

FIG. 51.

as much as before, but the contact is totally interrupted twice in each revolution, so that as regards continuity of current we are now really worse off than before. The remedy is, however, simple. We need only put another pair of coils on the ring at right angles to the first pair, and another commutator side by side with the first. Then, if we make the brushes wide enough to cover both commutators, the current can never be interrupted, because as the segment of one commutator leaves the brush the corresponding segment of the other commutator begins to make contact. The

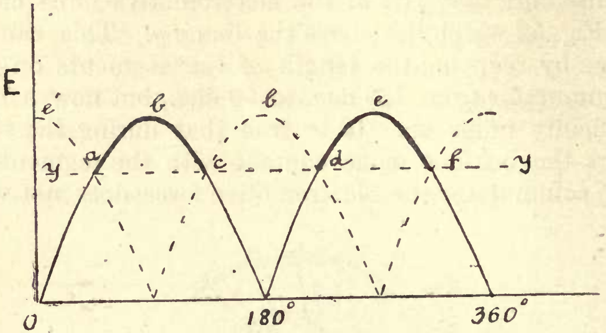

FIG. 52.

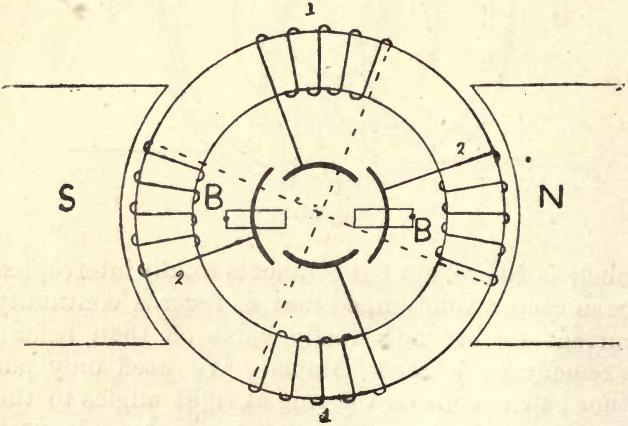

FIG. 53.

second pair of coils serves to bridge over the break between *f* and *a* and *c* and *d*, Fig. 52. The winding is shown in Fig. 53, but for clearness of illustration the segments of the two commutators are shown as concentric circles. The electromotive force curve of the pair of coils marked 1 1 is shown in Fig. 52 by the full line, that of the other pair of coils, marked 2 2, by the dotted line. The resultant electromotive force is therefore represented by the curve *e′ a b c b′ d e f*.

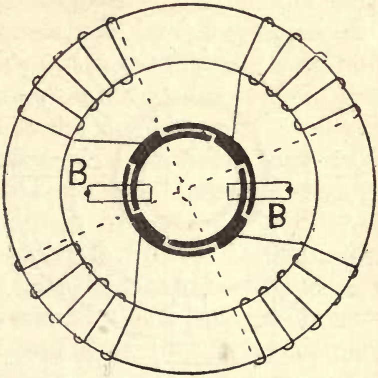

FIG. 54.

There is yet another slight improvement possible. We have assumed that each segment of the commutator is only 90 deg., or a little more than 90 deg., long to ensure continuity of contact. The brushes would therefore have to rub alternately over insulating material and over metal. This would cause unequal wear and jumping of brushes. To avoid this defect, we can provide each segment with an extension projecting into the space between the two neighbouring segments, and thus reduce the width of the insulation so much that the brushes rub only on metal, as is shown diagrammatically in Fig. 54.

The Brush Armature.

The construction of armature at which we have here arrived is that of the well-known Brush dynamo. It

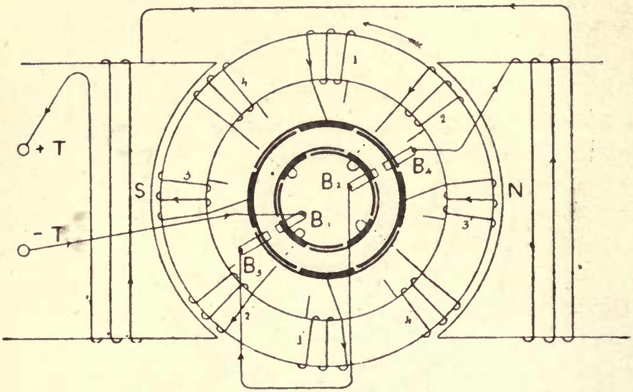

FIG. 55.

is, of course, possible to double or treble the number of coils and commutators, thus making an eight-coil or twelve-coil armature. The different sets of four coils are in this case coupled in series by the brush connections, and thus a greater degree of uniformity of electromotive force is obtained. Fig. 55 shows diagrammatically the winding and coupling up of an eight-coil Brush armature.

The two sets of coils are marked 1 1′, 3 3′, and 2 2′, 4 4′, respectively, but the cross-connections are omitted to avoid complication of drawing. The commutators are shown, as before, arranged in concentric circles, though in reality they are placed side by side. If the current in the armature coils had no magnetic effect on the armature core the diameter of commutation would be vertical, and the flux through 1 and 1′ would in the position shown be a maximum. But the current in the armature produces a flux of its own, which is superimposed upon the flux emanating from the field magnets, and by following the direction of the currents it is easy to see that the resultant flux within the core becomes a maximum at some place to the left of coil 1 and to the right of coil 1′. The true diameter of commutation will therefore not be vertical, but inclined in the sense of rotation. Consequently the electromotive force in coils 4 4′ will be either zero or very feeble, whilst it will be a maximum in coils 2 2′. In the other coils the electromotive force will have an intermediate value. The current enters at the brush marked $-B_1$, which at the moment touches only the central portion of the segment belonging to coil 2′. There is thus only one path open to the current—namely, through 2′, then across to coil 2, and out by the brush $+B_2$. From here the current flows by an external wire to brush B_3, which touches simultaneously two commutator segments—namely, those belonging to coils 3 and 1′. The current splits between these coils, and the two branches, after passing through 3′ and 1, finally unite again and leave the armature at the brush B_4. From here the current is led round the field magnets as shown, and to the external circuit by the terminal, $+T$. By this arrangement the coils of weakest action are entirely cut out, those of medium action are coupled in parallel series, and those of strongest action in single series, each coil entering and leaving the circuit twice per revolution, thus :

$2'-2 < \substack{3-3' \\ 1'-1} >$ .... 4 and 4′ out.

$< \substack{2'-2 \\ 4-4'} > 3-3'$ .... 1 and 1′ out.

$4-4' < \substack{1-1' \\ 3-3'} >$ .... 2 and 2′ out.

$< \substack{4-4' \\ 2-2'} > 1-1'$ .... 3 and 3′ out.

$2-2' < \substack{1-1' \\ 3-3} >$ .... 4 and 4′ out.

For clearness of illustration, the magnet poles have been shown with cylindrical faces, but, in reality, as the armature is a short flat ring, the magnets are set with their axes parallel to the spindle on either side of the armature.

The Thomson-Houston Armature.

Fig. 56 shows another type of open-coil armature. It is that employed by Profs. Thomson and Houston in the arc-light machine which bears their name. As actually made, the armature belongs to the drum type, though spherical in shape, but for clearness of illustration the armature is shown as a ring in the diagram. We have here only three coils, the inner ends of which are connected together at 0, whilst the outer ends are connected each to the corresponding segment of a three-part commutator. In the position shown, coil D has the maximum of electromotive force generated in it, coil C has less, and coil A has very little or none at all. If the current were allowed to pass through the latter coil when it occupies the position as shown, the coil would add nothing to the electromotive force, but absorb some electromotive force by reason of its resistance. The brushes are, therefore, so set that the coil of weakest action is always cut out of circuit, the current passing through the other two coils in series. The diagram shows the armature in the position when the

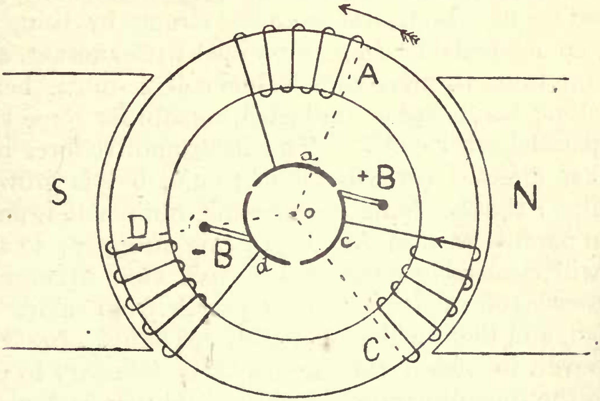

FIG. 56.

positive brush has just left the segment *a*. A moment before, coils A and C were in parallel. Now, A will remain out of circuit during a sixth part of a revolution and then it will come into parallel with D, but only for an instant—namely, whilst the negative brush bridges the sections *a* and *d*. Then D will be cut out and A will advance into the position of maximum action, while C will begin to recede from that position on the other side, and so on. During one revolution each coil is twice in circuit for a third of a revolution, and twice out of circuit for a sixth of a revolution. If the thickness of the brushes is only sufficient to bridge the gap between two commutator sections the parallel grouping of any two coils is only momentary, and this would obviously be inadmissible. The current would in this case have to change instantly, and heavy sparking would result. To avoid sparking, it is essential that each coil should, so to speak, be gradually prepared for its withdrawal from the circuit by being left for an appreciable time in parallel with another, and, for the time, stronger coil. Thus coil A must, before reaching the position indicated, remain for some time in parallel with coil C. The electromotive force in A is then directed towards the section *a*, but is growing weaker; the electromotive force in C is directed towards *c* (in parallel with *a*), but is growing stronger; so that A will eventually overpower C, and stop its current at about the moment when *a* passes from under the brush, and there will be but little sparking. In order, however, to obtain this action it is necessary to prolong the time of parallel grouping, and that is done by employing two brushes on each side, connected together, but set one with a certain angular advance upon the other. By increasing the angle (shifting the leading brush forward and the trailing brush backward), the time during which a weak coil is in parallel with a strong coil can be increased, with the result of decreasing the joint electromotive force of these coils. The electromotive force of the machine may thus be regulated within wide limits by a suitable displacement of the brushes.

CHAPTER X.

Field Magnets—Two-Pole Fields—Multipolar Fields Weight of Fields—Determination of Exciting Power Predetermination of Characteristics.

Field Magnets.

The magnetic field within which the armature revolves may be produced either by the use of permanent steel magnets or electromagnets. The former are not so effective as the latter, and are only used in exceptional cases, notably in the older forms of machines for lighthouses and in very small dynamos, where simplicity of construction is of more importance than small weight—such as mine exploders, medical machines, signalling apparatus, and machines for laboratory work. There is, besides simplicity, a further reason for using permanent steel magnets in preference to electromagnets for very small machines, and this is that the energy required for exciting the magnets becomes inordinately great when the size of the machine is reduced beyond a certain limit, as will be shown later on. Machines with permanent steel magnets are known under the name of "magneto machines," whilst the term "dynamos" is more particularly applied to machines in which the field is produced by electromagnets. Since magneto machines have only a very limited sphere of application, we pass at once to the consideration of the field magnets of dynamos. The number of types of field magnets which have been used or proposed for dynamos is exceedingly great, but the difference between many of these is more apparent than real. It will therefore be best not to attempt to give a complete list of all the various designs of magnets, but rather select a few representative types for purposes of comparison. In any electromagnet we have to distinguish between two circuits, the electric and the magnetic circuit. These two must be interlinked, so that the current through the electric circuit may produce a flux of lines of force through the magnetic circuit, and the difference in type of dynamo field magnets is due to the more or less suitable arrangement of these two circuits.

Two-Pole Fields.

The most simple arrangement is that shown in Fig. 23, Chapter V. Here we have a coil of wire, W, interlaced with a ring of iron, R, cut open at G. If the gap G be made of cylindrical or tunnel-like shape, it may receive a cylindrical armature, and thus Fig. 23 may be considered as representing the field magnet of a dynamo machine, but on the whole not a good arrangement. In the first place, the length of wire in the coil is unnecessarily large, and this can be reduced by lapping the wire more closely round the iron ring, and spreading it over a greater portion of it. In the next place, the curved form of magnet is bad from a practical point of view partly because a forging of this kind is difficult to produce and to fix in the frame of the machine, and partly because it cannot be wound in the lathe. It has been shown in Chapter V. that neither the shape of the core nor the disposition of the exciting wire have a direct influence on the magnetic flux produced by a given exciting power, and we are therefore free to alter the shape and arrangement of the magnetic and electric circuit in such manner as may be convenient. Instead of a hank of wire we may thus use a cylindrical coil wound on a former in a lathe, and instead of the curved iron core we may use a core consisting of straight pieces, which are more easily forged, machined, and put together. We may also make the pole-pieces detachable from the magnet core proper if by doing so we obtain some advantage as regards manufacture, but in this case we must take care to fit the different parts of the magnetic circuit properly together so as not to impede the flow of lines when it passes from one part to the next. In this way we arrive at something like the design shown in Fig. 57a M is a straight cylindrical magnet core of wrought iron shouldered into the cast-iron pole-pieces, P P, and C is the exciting coil.

It will be seen at a glance that this arrangement is electrically and magnetically equivalent to that shown in Fig. 23, but mechanically it is a great improvement. The design is simple and substantial, all the machining can be done on a lathe or boring machine, and the coil may be wound on a separate frame and slipped on when the machine is put together. This winding of magnet coils separately is important, not only because of facility for repairs, but chiefly because

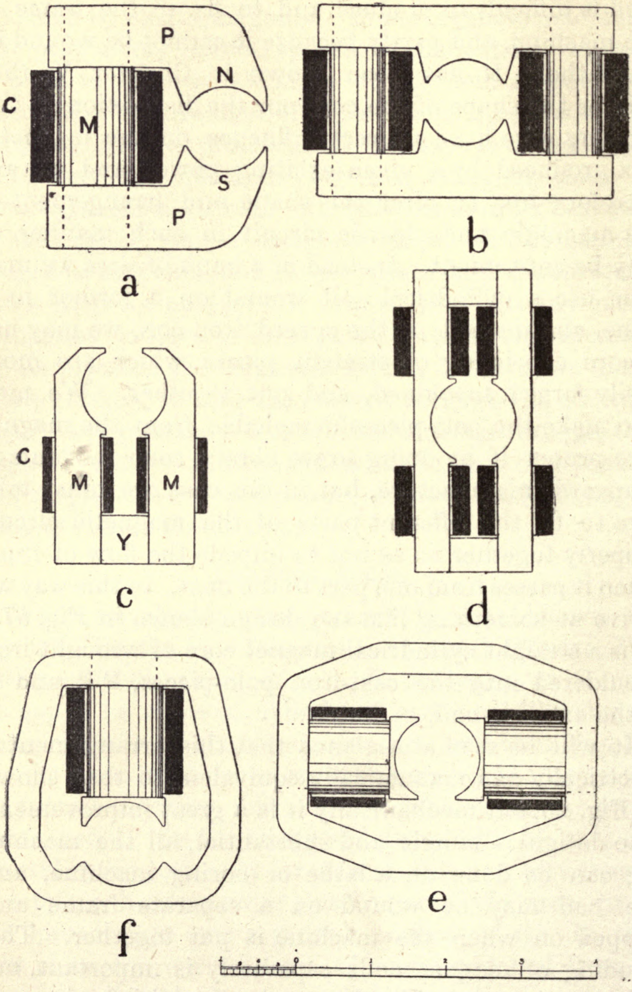

FIG. 57

it is possible to keep the electrical and the mechanical part of the work in distinct departments. If the coil is wound directly upon the magnet core a much greater weight has to be handled, and there is a risk of the insulation being injured by metal chips or filings, which are necessarily present in a shop where machine tools are at work and fitting is going on. For this reason it is best to do the winding and other electrical work in a separate shop.

The design of magnet shown in Fig. 57a, although, as was already said, perfectly practical, is still capable of improvement in two ways. In the first place, the magnet being on one side of the armature, the field is slightly unsymmetrical, and in the next place the arrangement is very heavy. Both of these defects can be remedied by duplicating the magnetic circuit, as shown in Fig. 57b. We require now two exciting coils and more wire, but we obtain, on the whole, a lighter machine, and one in which the field is perfectly symmetrical.

The field magnet, Fig. 57a, has another defect, inasmuch as the coil is short, and has therefore only a small external surface through which the heat generated by the passage of the current can escape. Practical experience has shown that for every watt absorbed by the resistance of the coil there must be provided a certain area of cooling surface, if the temperature of the coil is to be kept down at a safe limit. Authorities differ as to the exact number of square inches of cooling surface required per watt of energy dissipated, and it is obviously impossible to lay down a hard and fast rule, as the disposition of the machine, with regard to the fanning action of the armature and the locality where the machine is used, must necessarily influence the rate at which the coil can dissipate heat, but, generally speaking, the cooling surface should not be less than 1 square inch and need not be more than 4 square inches per watt. To prevent the coil, in Fig. 57a, from becoming too hot, we must, therefore, either increase its external surface by making it longer and shallower, or we must put more copper into it. The first expedient is of doubtful value, as it leads to a much heavier field, and the second is expensive. We can, however, alter the design altogether so as to obtain enough cooling surface without increasing the weight of the field. We need only treat the part marked M as the yoke and put coils on the two limbs marked P P in Fig. 57a. In this way we obtain the design shown in Fig. 57c, which is a very favourite type. By putting two exciting coils on the magnet limbs, M M, we have not only increased the cooling surface, but have also materially decreased the whole weight of the machine.

This design is known as the "overtype" field. By reversing it—that is, putting the armature below and the yoke, Y, at the top—we get the "undertype" field, which is also much in use, and is specially adapted for direct-driven machines, where it is important to get the spindle low down to correspond with the position of the engine shaft. In this case the machine is supported from its pole-pieces by brackets or packing pieces of non-magnetic material. In the overtype these pieces are not required, and the yoke may be either bolted direct to the bed-plate or may be cast in one piece with it.

This type of field, although lighter than the previously described types, is still rather heavy if the diameter of the armature is large in comparison with its length. If such an armature must be employed, and if it is important to save weight, we may duplicate the field, and thus we obtain the type shown in Fig. 57d. This contains less iron than the overtype field, but more copper, and, although, on the whole, it is considerably lighter, it is also more expensive.

Figs. 57e and 57f show fields of the "iron-clad" type. Their characteristic feature is that the yokes surround the magnets completely. There is consequently no stray magnetic field. Fig. 57e is very heavy, but requires little wire, whilst Fig. 57f is not quite so heavy, but requires more wire.

To give at a glance an approximate idea of the amount of copper required in each type of field, the space occupied by the coils is shown in black. The fields are all designed to take the same size of armature—namely, a drum 12in. diameter by 15in. long.

Multipolar Fields.

An example of a multipolar field has already been given in Fig. 2, Chapter I. This is the double four-pole magnet used by the Brush Company in their "Victoria" dynamo. The armature is a ring of large diameter as compared to its length, and the poles are presented to the end faces from either side. We require thus eight magnet cores with their axes parallel to the spindle, four on either side. The outer ends of the cores are joined by two massive cast-iron yokes. Fields of this type are frequently used in alternators.

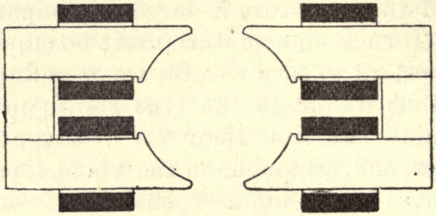

FIG. 58.

In machines with cylindrical armatures (whether dynamos or alternators) the pole-pieces are necessarily

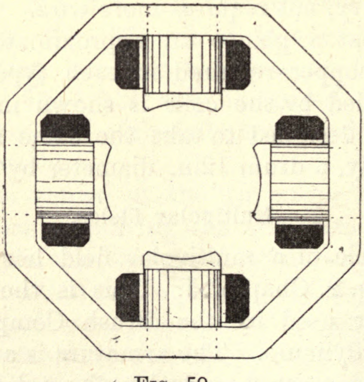

FIG. 59.

parts of a cylindrical surface, and the axes of the magnet cores are generally at right angles to the spindle. Any multipolar field may be considered as a combination of bi-polar fields. Thus, by taking two fields of the type 57c, we can produce a four-pole field of the type 58. In a similar way Fig. 59 may be considered to result from Fig. 57e if we increase the curvature of the yoke so as to get room for another pair of magnets. The coupling up of the coils must

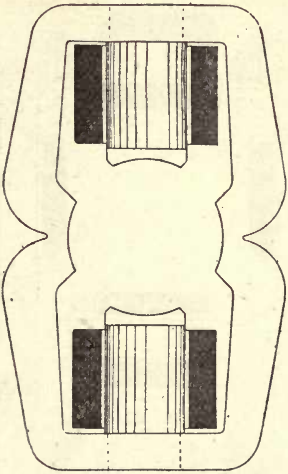

Fig. 60.

in this case be reversed, so that diametrically opposed poles have the same, and neighbouring poles the opposite, sign. By duplicating Fig. 57f, we obtain the field shown in Fig. 60. In this design four poles are produced by the use of only two exciting coils. Fig. 61 may be considered to result from the combination of four fields of the type shown in Fig. 57a.

If a field of more than four poles be required, we may produce it by combining three or more fields of the 57c type; but this presents considerable mechanical difficulties iu supporting the magnets, and is also, for other reasons, less advantageous than an expansion of the arrangement, Fig. 59, which is shown in Fig. 62 as applied to a 10-pole machine. Fig. 61 may also be

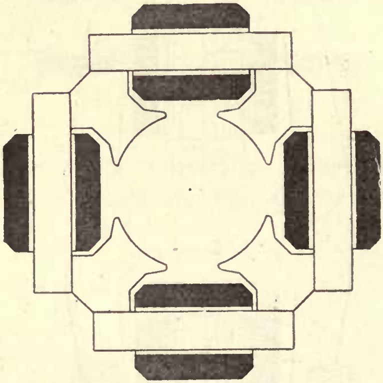

FIG. 61.

expanded into a field of six, eight, or more poles, and is with continental makers a favourite type. Another type exclusively used on the Continent is the reverse of Fig. 59, the poles being placed inside the armature, which must in this case be overhung, the active conductors being on the inside. Fig. 63 shows a 10-pole field of this kind. To give an idea of the relative weight of armature in Figs. 62 and 63, the outlines of the armature core have been inserted in the diagrams, which represent machines of equal output and equal speed. The field of Fig. 63 is about half the weight of Fig. 62, but this advantage is bought at the cost of a more difficult mechanical construction, both as regards the support of the armature core and the positive driving of the armature conductors.

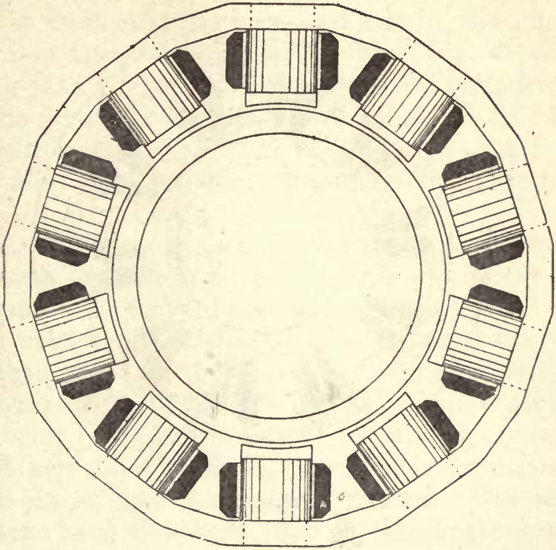

FIG. 62.

There is no hard-and-fast rule by which we can judge the merit or otherwise of any of these types of field. The voltage, size, and speed of the machine, the greater or lesser importance of small weight, the possibility of obtaining soft steel castings, the relative cost of copper and iron, the energy permitted for excitation, the temperature rise allowed, and, last, but not least, the skill of the designer, are all items on which the value of any type depends; but, as a general guide, a few facts may here be usefully stated. If pole-plates are used with Fig. 58, whereby it is possible to make the section of magnets not too much different from a

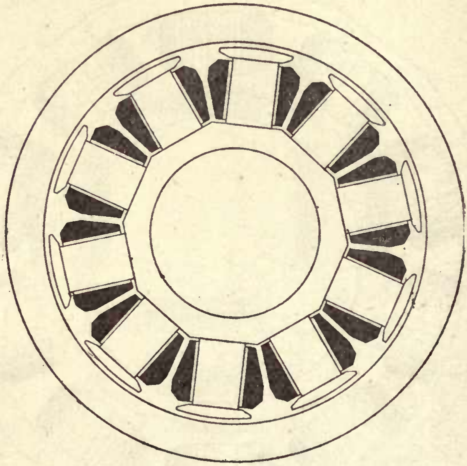

FIG. 63.

square, or if the armature is fairly short, whereby the magnets' section naturally approaches a square, the amount of exciting wire required is not excessive, and the total weight of field is very moderate. As far as iron and copper are concerned, this type of field is fairly cheap, but the mechanical support of the magnets is somewhat expensive because it must be formed entirely of gunmetal brackets or chairs. Another drawback is that there is little ventilation for the armature and less for the inside of the field coils, so that the heating will be greater than in machines of less compact design. Fig. 59 requires the same amount of magnet wire or possibly a little less than Fig. 58, but the whole field is much heavier if the yoke be made of cast iron. If the yoke be made of soft cast steel it need only be from one-half to one-third the section of the cast-iron yoke, and then Fig. 59 becomes lighter than Fig. 58. There is the further advantage that no gunmetal supports are required, the field being of the iron-clad type. The whole design is more open than Fig. 58, and the ventilation of armature and magnet coils is better.

Fig. 60 is a very simple design and requires about the same amount of copper as Figs. 58 and 59; it is, however, very heavy if cast iron be used for the yoke. With cast steel the weight may be brought down to less than either of the previous designs, especially in small machines. The fact of the armature and field coils being protected by the surrounding yoke renders this design suitable for machines which are exposed to rough usage, such as tramway motors. Machines of this type have also been used for shiplighting, where an iron-clad field has the advantage of not disturbing the compasses. The field shown in Fig. 61 is heavy and expensive. It requires gunmetal supports and rather more wire than Fig. 58, but the cooling surface of the coils is large and the ventilation very good. This type is used for 10 and more poles in the newest design of Edison central-station machines, and an

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example is also furnished by the new generator of 300 kilowatts (2,400 amperes at 125 volts) used for the distribution of electrical energy for motive power throughout a small-arms factory near Liège.* In this machine, which is more remarkable for its dimensions than its output, the armature is 15ft. 9in. diameter, and the field has 20 poles. The magnet cores and poles are of soft cast steel and weigh 10 tons, whilst two tons of exciting wire are used. The speed of the machine (which is direct driven by a Van der Kerchove engine) is 166 revolutions per minute, giving the armature the remarkably high peripheral speed of over 8,000ft. per minute. At this speed the weight of field is about 90lb. per kilowatt output. In machines of the type, Fig. 58, the weight of field is under 100lb. per kilowatt for a peripheral speed of armature of 2,000ft. per minute. This is three and a half times better than the condition of the Liège machine, the field of which is of the type shown in Fig. 61, but expanded to 20 poles.

Weight of Fields.

A rough comparison of the different types of field as regards their weight has been made above, but in order to bring this important matter more clearly into view, it appeared to me desirable to give the reader some examples actually worked out so that he could compare figures rather than mere general statements. The figures given refer, of course, only to the particular cases selected, and their relative value would come out differently if we made the designs for a larger or smaller output, or a different speed or

* *L'Electricien,* vol. iv., No. 39, p. 177, September 10, 1892.

different armatures. In order to get representative figures it was, therefore, necessary to select fairly representative cases, and I have chosen for two-pole machines an output of 25 kilowatts, at 550 revolutions, as being about midway in the range of output and speed for which two-pole machines would usually be adopted. The armatures are 12in. diameter by 15in. long, and have, therefore, a peripheral speed of 1,730ft. In two cases the armatures are 15in. by 15in., and have a peripheral speed of 2,150ft. For the fields having four poles I have chosen an output of 80 kilowatts, at a speed of 380 revolutions, as being fair average conditions for which four-pole machines would be employed. The armatures are in all cases 24in. diameter by 20in. long, and have a peripheral speed of 2,380ft.

Before giving the results of this investigation it is necessary to say a few words on the methods employed in designing the fields. The laws which govern the amount of exciting power or ampere-turns for any given configuration of magnets, will be found below in this and the next chapter. For the present it is only necessary to state that these laws have been followed in determining the amount of field wire required, and that due regard has been paid to armature reaction (see Chapter XI.), heating limit, and percentage of exciting energy. Where advisable, pole-horns or pole-plates have been added to reduce exciting power or length of wire; and in two cases (Figs. 64 and 65) the poles have been cut so as to reduce armature reaction and permit the use of a lighter field. The armature in these two cases has been increased to 15in., and is, of course, heavier and more expensive than the 12in. armature, which is used throughout the other two-pole fields. In all

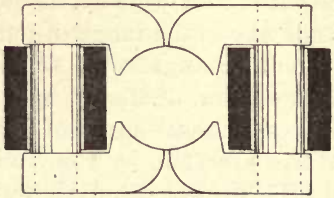

Fig. 64.

the two-pole machines the armature is designed for a copper loss of $3\frac{3}{4}$ per cent. Precisely the same armature (24in. diameter by 20in. long) is used in

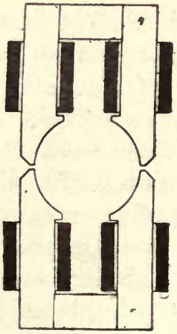

Fig. 65.

all the four-pole fields, and it has been designed for a copper loss of $2\frac{1}{2}$ per cent.

As regards the copper loss in the fields this has been limited to $3\frac{1}{2}$ per cent. in the two-pole and to 2 per cent. in the four-pole fields, except in those cases where the heating limit made it desirable to work with a smaller expenditure of power in the field. The temperature rise has been determined in every case, and will be found in the table below. The weights given are simply for the iron in the magnetic circuit and for the copper wire, but do not include the weight of any formers, terminals, chairs, and support for the magnets. For convenience of comparison I have added the weight of magnets per kilowatt output, on the supposition that the peripheral speed of the armature is in all cases 2,000ft. per minute. The magnet cores are in all cases of wrought iron; the poles are of cast iron in 57a, 57b, and 64. The yokes are of cast iron in 57c, 57e, 57f, 59, and 60.

Field Magnets for Two-Pole Machines.—25 kilowatts output, 550 revolutions.

Type of Field. Fig.—	57a.	57b.	57c.	57d.	57e.	57f.	64.	65.
Total weight of field, pounds	5,090	4,670	3,020	2,790	6,750	5,070	3,670	2,400
Weight of iron, pounds ..	4,600	3,600	2,600	1,800	6,400	4,600	3,000	1,750
Weight of copper, pounds..	490	1,070	420	990	350	470	670	650
Percentage of output required for excitation......	3	3·5	3·5	3·5	3·25	2·8	3·5	3·5
Temperature rise, degrees centigrade	33	25	33	20	33	33	28	22
Weight of field in pounds per kilowatt at 2,000ft. peripheral armature speed ...	175	160	104	96	230	175	155	104

FIELD MAGNETS FOR FOUR-POLE MACHINES.—80 kilowatts output 380 revolutions.

Type of Field. Fig.—	58	59	60	61
Total weight of field, pounds	6,070	9,880	10,980	7,300
Weight of iron, pounds....................	4,400	8,700	9,600	4,700
Weight of copper, pounds.....	1,670	1,180	1,380	2,600
Percentage of output required for excitation	2	2	1·94	3
Temperature rise, degrees centigrade	30	36	33	25
Weight of field in pounds per kilowatt output at 2,000ft. peripheral armature speed	90	147	163	109

Determination of Exciting Power.

The law which governs the production of magnetic flux by the application of a certain magnetic force has already been given in Chapter V., and it is by applying this law to special cases that we find the total exciting power measured in ampere-turns that is required to produce in dynamo magnets a definite flux, or total field. In what follows, the letter F will be used to denote the total flux in C.G.S. measure, and the letter Z will be used to denote the same quantity in English measure—or, F = 6,000 Z. The exciting power in ampere-turns we denote by the letter X, so that the general equation (22) can also be written

$$X = F R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

R being the magnetic resistance as defined in Chapter V., and being found by the formulæ

$$\left.\begin{aligned} R &= \Sigma \frac{1}{1{\cdot}256} \frac{L}{A} \frac{1}{\mu} \\ \text{or, } R &= \Sigma \ {\cdot}8 \ \frac{L}{A} \frac{1}{\mu} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (23)$$

Or, in English measure,

$$X = Z\,R \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where $$R = \Sigma\, 1{,}880 \frac{L}{A}\frac{1}{\mu} \quad . \quad . \quad . \quad . \quad (25)$$

In this expression the length of the circuit, L, must be inserted in inches and the area, A, in square inches; μ, being merely a numeric, remains the same in both systems of measurement. In the special case that the part of the magnetic circuit under consideration contains only air or other non-magnetic substance μ becomes 1, and we have

$$R = \cdot 8 \frac{L}{A} \qquad\qquad R = 1{,}880 \frac{L}{A},$$

since $F = \mathfrak{B}\,A$ and $Z = B\,A$, we find the ampere-turns required to produce the flux, F, in air by $X = \mathfrak{B}\,A \times \cdot 8 \frac{L}{A}$

$$X = \cdot 8\, \mathfrak{B}\, L \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and similarly for English measure

$$X = 1{,}880\, B\, L \quad . \quad . \quad . \quad . \quad (36)$$

It is generally convenient to determine the exciting power separately for each part of the magnetic circuit, because the flux is not the same in all its parts. In a dynamo machine we have to distinguish between the flux through the armature, that through the air space (assumed to be the same), and that through the pole-pieces, joints, magnet cores and yokes, which is always larger than the armature flux, because a certain proportion of the lines generated within the magnet coils never passes through the armature, but produce what is termed a leakage field through the air surrounding the exciting coils. Since leakage is simply a magnetic flux through air, and must follow the general law $F = \frac{X}{R}$, it will be seen that the amount of leakage must depend on the extent of the surfaces which are under different magnetic potentials, their distance apart, and on the difference of magnetic potential or magnetomotive force. Generally speaking, the leakage will be the greater, the greater the exciting power the larger the external surface of pole-pieces, and the less the distance between poles of opposite sign or between poles and yokes.

The magnetic resistance of joints is generally neglected, and in well-made machines, where the joints are properly faced and strongly pressed together mechanically, their resistance is quite insignificant compared to that of other parts of the circuit. Professor Ewing* has experimentally investigated the magnetic resistance of joints by observing the decrease of induction with the same magnetising power in a bar which had been successively cut into two, four, and eight pieces. He found that by applying mechanical pressure to the joints their resistance was diminished, and since in a well-made machine the joints are either strongly bolted up or are a good driving fit, we may take it that the necessary mechanical pressure for a good magnetic fit is obtained. Ewing gives the resistance of the joint in comparison with an equivalent layer of air between the surfaces. For $H = 30$, and $\mathfrak{B}$ varying from 14,550 to 9,800, the equivalent layer of air is ·002 centimetre

* *Phil. Mag.*, Sept., 1888.

thick. For H = 50, and 𝕭 varying from 15,950 to 13,300, it is ·0013 centimetre thick, and for H = 70, and 𝕭 varying from 16,820 to 15,200, it is ·0009 centimetre thick. If we calculate from Ewing's figures what exciting power is required to overcome in each case the resistance of the joint, we come to the remarkable result that with an increasing induction the exciting power actually diminishes. We find for

𝕭 = 12,175 {ampere-turns required to overcome resistance of joint} = 19·6
𝕭 = 14,625 „ „ „ = 17
𝕭 = 16,020 „ „ „ = 11·6

This result may possibly be due to the fact that with the greater induction the magnetic attraction is also increased, and thus the contact of the joints in Ewing's experiments may have become better as 𝕭 became larger. The conditions of the experiment were therefore not exactly those obtaining in a system of heavy field magnets, where the joints are in any case under great mechanical pressure. For the rest it is a matter of indifference whether we allow 10, or 50, or 100 ampere-turns for the resistance of each joint, because the total exciting power is reckoned not by tens, but by thousands of ampere-turns, and we are thus justified in entirely neglecting the resistance.

There remain thus only three parts of the magnetic circuit to be separately dealt with—namely, armature, air spaces, and magnets—including poles, cores, and yoke. To find the total exciting power we have merely to determine separately the exciting power, X_a, required to drive the flux through the armature, that required for the air space (which we call X_a), that required for the magnet cores and poles (X_m), and that required for the yoke (X_y). We shall use the same indices, *a*, *a*, *m*, *y* for F or Z, and or B to distinguish respectively the flux and induction through the various parts of the circuit. The following symbols will also be used :

L_a = average length of path of lines through armature core ;

δ = length of air gap or interpolar space ;

A_a = net area occupied by iron in armature core ;

A_a = area of air gap ;

A_m = cross-sectional area of magnet cores ;

A_y = cross-sectional area of yoke ;

L_m and L_y = length of paths through magnets and yoke respectively.

The magnetic force required to produce an induction of $\mathfrak{B}_a$ lines through the armature, Fig. 66, is $H_a = \frac{\mathfrak{B}_a}{\mu}$, where

$$H_a = \frac{\cdot 4\,\pi X_a}{L_a}.$$

We have, therefore, $\mathfrak{B}_a = \mu \frac{\cdot 4\,\pi X_a}{L_a}$;

and in order to determine X_a, the ampere-turns required for the armature alone, we must know the value of the permeability at the particular induction chosen. This can be found experimentally (as was first done by Dr. Hopkinson in connection with the design of dynamo field magnets) by testing the particular sample of iron used for armature plates and determining the permeability curve, or better still, the magnetisation curve, representing $\mathfrak{B}$ as a function of H. We know what total flux we require to give the desired electromotive force, and dividing this by the area of the armature core we find $\mathfrak{B}_a$. Then, referring to the magnetisation curve we find the corresponding H_a. The ampere-turns required for the armature are then given by

$$X_a = \frac{H_a}{\cdot 4\pi} L_a$$

$$X_a = \cdot 8\, H_a L_a.$$

In order to avoid the coefficient ·8 it is convenient to compile once for all a table from the magnetisation curve in which the values of ·8 H and $\mathfrak{B}_a$ are inserted. We have then only to refer to the table and find the number with which the mean length of the lines through the armature must be multiplied to give X_a. In other words, the numbers of the table give the ampere-turns per centimetre of the path which the l nes take through the armature.

The following two tables (one for C.G.S. and the other for English measure) may be used when the armature core is composed of best charcoal iron discs.

The use of these tables can best be explained by an example. Say that in the armature shown in Fig. 66 the average length of path is L_a = 11in., and that we desire to have an induction of B = 16·5. By referring to the table we find that 75 ampere-turns are required per inch of path, so that out of the total exciting power applied to the field magnets 11 × 75 = 825 ampere-turns will be required for producing the flux through the armature.

EXCITING POWER IN AMPERE-TURNS REQUIRED PER CENTIMETRE AND PER INCH OF ARMATURE PATH.

C.G.S. Measure.		English Measure.	
𝕭.	$\frac{Xa}{La}$	B.	$\frac{Xa}{La}$
5,000	1·80	5	4·25
10,000	3·60	10	8·5
11,000	4·40	11	9·5
12,000	5·83	12	11·3
13,000	8·40	13	15
13,500	10·40	13·5	18·2
14,000	12·95	14	21·3
14,500	17·60	14·5	26·2
15,000	22·40	15	32·4
15,500	30·50	15·5	42·5
16,000	40	16	55·7
16,500	56	16·5	75
17,000	72	17	97
17,500	88	17·5	130
18,000	104	18	164
18,500	128	18·5	203
19,000	160	19	240
19,500	220	19·5	285
20,000	280	20	350
—	—	20·5	450
—	—	21	570
—	—	21·5	710
—	—	22	1,000

To determine that part of the exciting power which is required to drive the flux through the air space, we use formulas (35) and (36). The dimensions of the polar surfaces we take from the drawing of the machine, but it is important to note that the average area through which the flux passes is slightly larger than the polar area, since the flux spreads out into a fringe at the corners of the pole-pieces, as shown to an enlarged scale to the right of Fig. 66. In practice it is generally assumed that the total width of fringe may

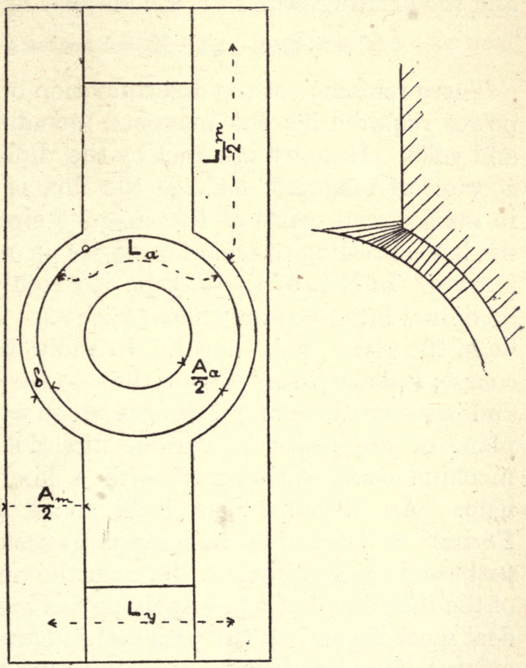

FIG. 66.

be taken as equal to the air space, and if λ is the length of the polar area, and l the length of the armature, the mean area of interpolar space is

$$A_a = l(\lambda + \delta) \quad . \quad . \quad . \quad . \quad . \quad (37)$$

The average induction in that space is

$$\mathfrak{B}_a = \frac{F_a}{A_a} \quad . \; . \; . \; . \; . \; . \; . \quad (38)$$

and the exciting power for the air space is

$$X = {\cdot}8\, \mathfrak{B}_a\, 2\, \delta \text{ and } X = 1{,}880\, B_a\, 2\, \delta \quad . \quad (39)$$

There remains yet the determination of the exciting power required for the magnets, including pole-pieces and yoke. Here we are met by the difficulty that on account of magnetic leakage the flux is not the same in the different parts of the magnet circuit, and that the law according to which it varies is not accurately known. In Fig. 67 the leakage is roughly represented by dotted lines, but only those paths are shown which lie in the plane of the paper. In addition there are, of course, leakage lines between the side faces of the poles and between these and the yokes which come out of the plane of the paper in various directions, the whole machine being surrounded with a kind of magnetic halo. An attempt has been made by Professor Forbes* to determine the leakage by assuming certain paths and integrating the flux over the various surfaces of the machine, but as in applying his methods a great deal must depend on the personal judgment of the calculator, it is generally found more accurate to calculate the leakage of a new machine from the experimental results previously obtained with machines of a similar type but of different size. Such experiments are easily performed. Referring to Fig. 67, we may assume with tolerable certainty that the flux will be a maximum at

* *Journal*, Society of Telegraph Engineers, xv., 551, 1886.

M, being in the middle of the excited part of the magnet circuit, and a minimum at A in the armature. We need then only place exploring coils round the magnet and armature in the two positions, and connect them with a ballistic galvanometer. We note the deflections on making and breaking the exciting

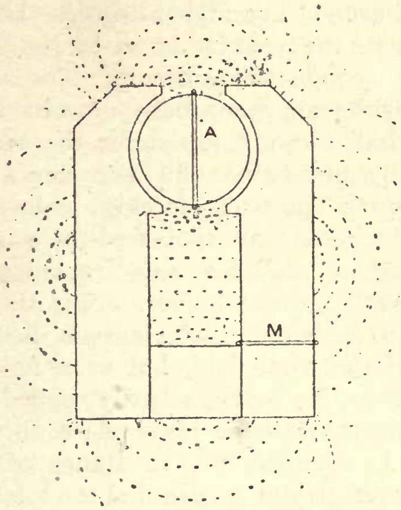

Fig. 67.

circuit, and determine thus the total flux in these two points and their ratio. As a further check we can couple the two exploring coils in series, and by reversing the connections determine $F_a + F_m$ and $F_m - F_a$. Mr. Esson has published leakage tests made with various machines,† and has found that the ratio F_m/F_a varies

† *Journal* Inst. Electrical Engineers, xix., 122, 1890.

from 1·3 to 2. The leakage is thus represented for each type of machine as a coefficient, but such a way of representation cannot be correct for all cases, as will be seen from the following consideration. Suppose we have found a certain leakage coefficient when testing a machine with a moderately strong field. The same coefficient cannot possibly be right for the same machine if excited to a higher degree. Let us assume that we excite the machine so as to get 30 per cent. more flux through the armature. The ampere-turns required for the air space have now increased by 30 per cent., but those required for the armature have increased by more than 30 per cent., so that the total magnetic pressure forcing leakage lines out of the pole-pieces has increased in a greater ratio than the flux, and for this reason the leakage coefficient will now be higher. It is therefore more accurate to determine the leakage field not as a function of the total field, but as a function of the exciting power, $X_a + X_a$, actually applied to the armature. Since the leakage takes place through air, for which $\mu = 1$, the magnetic resistance of the leakage paths is constant, and we can find the total waste field, $\xi = Z_m - Z_a$ by simply dividing the armature exciting power by the magnetic resistance, ρ, of the leakage paths.

$$\xi = \frac{X_a + X_a}{\rho}.$$

The coefficient ρ depends, of course, on the size and general configuration of the machine. A machine with large exposed poles will naturally have a smaller ρ than one in which the poles are small. Thus Fig. 57b will have more waste field than 57d, the latter will have more than 57c, and this will have more than Fig. 59. Again, if we invert 57c so that the poles come near the bed-plate, the leakage will also increase.

We have now to consider what influence the size of a machine has on the leakage resistance ρ. By doubling the linear dimensions we quadruple the surfaces from which the leakage takes place, but at the same time we double the average length of the leakage paths so that the total leakage resistance will be halved. The leakage resistance for two machines of the same type will, therefore, vary inversely as the linear dimensions. A convenient way of stating the linear dimensions of a machine is to give the size of its armature, and in order to permit of slight variations in the ratio of length and diameter of armature, we may conveniently state the linear dimensions of the machine, not in terms of either the diameter or the length of the armature, but in terms of the square root of their product. The leakage resistance will then be given by the expression

$$\rho = \frac{K}{\sqrt{l\,d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where l and d are the length and diameter of the armature core, and K is a coefficient depending on the type of the machine, but not on its size.

In overtype machines K may be taken at ·29 in C.G.S., and 680 in English measure, and in undertype machines at ·21 and 460 respectively. By adding the leakage lines to the useful flux we find the total flux through the magnets, and therefore the induction. To find the corresponding exciting power we proceed in the same way as with the armature, by the use of a table giving ampere-turns per centimetre or per inch as a function of the induction. There is, however, this difference, that an error in the estimation of the permeability of the armature core is not of very great importance, the exciting power required for the armature being, as a rule, comparatively small, whereas an error in the estimation of the permeability of the field magnets may have great importance, as it affects a much larger part of the total exciting power. The difference in the permeability between various samples of iron is generally greater for high than for low inductions, so that if for constructive reasons we are compelled to work at a high induction in the field magnets, it becomes all the more important to test the magnetic properties of the particular brand of iron employed. On the other hand, there is less danger of error if we work with a moderate induction, a course which we would adopt wherever possible, with a view to economise exciting wire. In such cases the exciting power required for the magnets is relatively smaller, and an error in this smaller quantity is not of so much importance—to say nothing of the fact that the error itself is not so likely to occur. In ordinary work we may, therefore, neglect to test a sample of the metal in each case, and use a table of $\mathfrak{B}$ and $\frac{X}{L}$ prepared, once for all, from tests made with iron of average quality. The usual materials for field magnets are wrought iron, cast steel, and cast iron. The latter should be only employed in those parts of the circuit which are not surrounded by exciting wire, such as yokes and pole-pieces, because, otherwise, the cost of copper owing to the larger sectional area surrounded by wire becomes too great. Wrought iron and mild cast steel are generally used for the magnet cores proper, and are about equivalent, magnetically.

EXCITING POWER IN AMPERE-TURNS REQUIRED PER CENTIMETRE AND PER INCH OF PATH THROUGH WROUGHT IRON.

C.G.S. Measure.				English Measure.			
𝔅.	$\frac{X_m}{L_m}$	𝔅.	$\frac{X_m}{L_m}$	B.	$\frac{X_m}{L_m}$	B.	$\frac{X_m}{L_m}$
5,000	1·92	16,100	60·0	5	4·85	16·1	74
10,000	4·25	16,200	63·5	10	9·7	16·2	78·5
11,000	5·20	16,300	67·2	11	11·3	16·3	83·5
11,500	5·91	16,400	71·0	11·5	12·3	16·4	89
12,000	6·80	16,500	76	12	13·43	16·5	95
12,500	7·84	16,600	81	12·5	15·2	16·6	101
12,750	8·47	16,700	86	13·0	17·2	16·7	107·2
13,000	9·27	16,800	91	13·2	18·2	16·8	113·6
13,200	10·12	16,900	97	13·4	19·4	16·9	120·6
13,400	11·08	17,000	103	13·6	20·6	17	127·5
13,600	12·1	17,100	112	13·8	22	17·1	136
13,800	13·5	17,200	130	14	23·5	17·2	145
14,000	15·1	17,300	150	14·2	25·5	17·3	155
14,200	17·2	17,400	170	14·4	27·7	17·4	165
14,400	19·4	17.500	195	14·6	30·3	17·5	175
14,600	22·3	—	—	14·8	33·2	17·6	185
14,800	25·6	—	—	15	37·3	17·7	196
15,000	29·7	—	—	15·2	41·8	17·8	207
15,200	34·3	—	—	15·4	46·8	17·9	218
15,400	39·0	—	—	15·6	54	18	230
15,600	44·4	—	—	15·8	61·5	18·1	243
15,800	50·3	—	—	16	70	18·2	257
16,000	56·0	—	—	—	—	18·3	272
—	—	—	—	—	—	18·4	288
—	—	—	—	—	—	18·5	305
—	—	—	—	—	—	18·6	324
—	—	—	—	—	—	18·7	344

Out of 13 samples of cast steel which I tested in the magnetometer, three only were of sensibly lower permeability than ordinary wrought iron as used for magnet forgings, and the 10 others came out nearly equivalent to Lowmoor iron, and equal to, or even slightly better than, ordinary good wrought iron.

EXCITING POWER IN AMPERE-TURNS REQUIRED PER CENTIMETRE AND PER INCH OF PATH THROUGH CAST IRON.

C.G.S. Measure.		English Measure.	
𝔅.	$\frac{Xy}{Ly}$	B.	$\frac{Xy}{Ly}$
3,000	4·00	3	8·3
4,000	5·92	4	13·2
4,500	7·23	4·5	16·1
5,000	8·59	5	19·4
5,200	9·20	5·2	20·6
5,400	10·10	5·4	22·3
5,600	11·10	5·6	24·1
5,800	12·10	5·8	26
6,000	13·60	6	28·3
6,200	15·30	6·2	30·7
6,400	17·60	6·4	34
6,600	20·60	6·6	37·2
6,800	24·15	6·8	42·5
7,000	28·85	7	48·5
7,200	34·40	7·2	56
7,400	40	7·4	66·2
7,600	47·5	7·6	77
7,800	53	7·8	91
8,000	60	8	105
8,500	75	8·2	120
9,000	90	8·4	135
9,500	105	8·6	149
10,000	121	8·8	163
—	—	9	177
—	—	9·5	216
—	—	10	252
—	—	10·5	290
—	—	11	322

The table given on page 251 may be used in designing machines with magnets of good wrought iron or special mild cast steel. For the cast-iron parts of the magnetic circuit the table given on page 252 may be used.

Predetermination of Characteristics.

Generally speaking, a characteristic of a dynamo is a curve giving the relation between two variables, such as current and terminal pressure for constant speed, current and resistance of external circuit for constant speed, speed and current for constant external resistance, torque and current, or any other relation between two quantities depending on each other. Amongst these relations one of the most important is that of the interdependence of exciting power and total armature flux. This can be represented by a characteristic, usually called the curve of magnetisation of the machine, in which the exciting power is plotted on the horizontal and the total armature flux on the vertical. This curve is important, because by its aid we can determine the winding of the field not only for one working point, but for any given range of working conditions, such as may occur in compound-wound dynamos, railway or tramway motors, and generally in problems connected with the electric transmission of power. The problem before us is how different points on the characteristic of magnetisation can be determined, the drawing of the machine being given. The solution of this problem is, in reality, contained in what has been already said on the calculation of the exciting power required for the different parts of the magnetic circuit, but for the convenience of the reader I recapitulate the subject by giving an example. For the sake of avoiding large numbers, we use the English system of measurement.

Let us assume that the machine has a ring armature 18in. diameter by 14in. long, core discs $3\frac{1}{2}$in. deep, and that 82 per cent. of the core space is actually filled by iron. Output of machine, 40 amperes at 1,000 volts at 500 revolutions; resistance of armature, ·95 ohm; loss, 3·8 per cent. For reasons which will be found in the next chapter, we would choose for an armature of this kind a double-horseshoe field of the type shown in Fig. 65. The area of iron in the armature is 80 square inches, and the useful flux is therefore 80 times the induction. Let us assume that by reference to a drawing of the machine we find the following measurements:

Length of polar arc,	$\lambda = 23$;
Air gap	$\delta = \cdot 9$;
Area of magnets and yokes,	$A = 65 \times 2$, all of wrought iron;
Path through armature	$L_a = 16$;
Path through magnets and yoke	$L = 66$.

Since in this case the yokes are of the same material and have the same sectional area as the magnet cores, we need not consider them separately, and can write A_m for A_y and L_m for the sum of the paths through magnets and yoke. Similarly B_m stands for the induction through the yoke as well as through the magnet cores.

Assume that the armature has 1,440 external conductors, then the flux required for the full output will be about 1,500 lines. In determining the characteristic we must, therefore, find points from the origin to $Z = 1{,}500$, and preferably a little beyond, so as to see the shape of the curve in the neighbourhood of the working point. The process is as follows: We assume a certain armature flux, and find the exciting power required to produce it. The corresponding point we plot. We then assume another flux, and again calculate the corresponding exciting power, and so on, for as many points as may be required to get the curve properly plotted.

The polar area (formula 37) is $(\lambda + \delta)\, l$, in this case $(23 + \cdot 9)\, 14$

$$A_a = 335.$$

Assuming $Z = 500$ for the first point on the characteristic, we find the armature induction by dividing by 80 (formula 38)

$$B_a = 6{\cdot}25.$$

The nearest figure in the table for armature plates is $B = 5$, for which 4·25 ampere-turns are required per inch of path. By interpolation we find that, with an induction of 6·25, the corresponding number is 5·3, and as the average length of path through the armature is 16in., the corresponding exciting power is

$$X_a = 16 \times 5{\cdot}3 = 85.$$

The exciting power for the air space we find from (39)

$$X_a = 1{\cdot}8 \times 1{,}880\, B_a.$$

$$X_a = 3{,}384\, B_a.$$

The flux is 500, and the area of air space is 335 square inches. The induction in the air space is therefore

$$B_a = \frac{500}{335} = 1{\cdot}49.$$

$$X_a = 3{,}384 \times 1{\cdot}49 = 5{,}042.$$

The total exciting power required for the armature and air space is therefore 85 + 5,042 = 5,127 ampere-turns. This is also the exciting power which drives the leakage lines through the air surrounding the machine.

The next step is to find the amount of leakage or waste field, and for this purpose we must know the value of ρ. This we find from (40), if we know the value of K for the type of machine under consideration. This may, with double-horseshoe machines of the type represented by Fig. 65, be taken as 500, so that we find ρ from the equation

$$\rho = \frac{500}{\sqrt{l\,d}} = \frac{500}{\sqrt{14 \times 18}}.$$

$$\rho = 31{\cdot}5.$$

The leakage field we find by dividing 5,127 by ρ, which gives

$$\xi = 163.$$

The field magnets have, therefore, not only to produce the 500 lines which are utilised in the armature, but also 163 lines which are dissipated through space; and the induction through the magnets must, therefore, be calculated on the basis of 663 lines passing through 130 square inches of iron (65 in each horseshoe magnet):

$$B_m = \frac{663}{130} = 5{\cdot}10.$$

By referring to the table for magnet iron, we find that the nearest number of B is 5, to which correspond 4·85 ampere-turns per inch of path. Interpolating, we find that to B = 5·10 correspond 4·95 ampere-turns, and as the average length of path of the lines through magnets and yoke is 66in., the magnet exciting power is

$$X_m = 4{\cdot}95 \times 66 = 326.$$

We now find the total exciting power by adding up its component parts. Thus:

The armature core requires	85	ampere-turns
The air spaces require	5,042	,,
The magnets require	326	,,
Total	5,453	,,

The calculation here explained gives us, incidentally, also the leakage coefficient, but only for the particular flux of 500 lines. We have seen that 163 lines are wasted over and above the 500 utilised. The percentage of waste is therefore 163/500 = 32·6 per cent., and the leakage coefficient is 1·326.

To find other points on the characteristic, we repeat the calculations, assuming a flux of, say, 800, 1,000, 1,100, 1,200, 1,300, 1,400, 1,500, 1,600, and 1,650 lines. These calculations may be conveniently made in tabular form, so that clerical errors may be more easily detected and labour economised. At the head of the table we write the constructive data of the machine to facilitate reference.

$A_a = 80$ $A_a = 335$ $A = 130$ $\rho = 31{\cdot}5$

$L_a = 16$ $\delta = {\cdot}9$ $L_m = 66$ $2\,\delta\,1{,}880 = 3{,}384$

Z_a ξ Z_m	B_a B_a B_m	X X_a —	— $X_a + X_a$ X_m	— — X
500	6·25	8·5	—	—
163	1·49	504·2	5,127	—
663	5·10	—	326	5,453
800	10	136	—	—
261	2·385	8,100	8,236	—
1,061	8·17	—	543	8,779
1,000	12·5	210	—	—
328	2·98	10,100	10,310	—
1,328	10·20	—	680	10,990
1,100	13 75	313	—	—
364	3·29	11,150	11,463	—
1,464	11·23	—	780	12,243
1,200	15	515	—	—
402	3·58	12,150	12,665	—
1,602	12·31	—	942	13,607
1,300	16·23	1,000	—	—
448	3·88	13,150	14,150	—
1,748	13·44	—	1,290	15,440
1,400	17·50	2,075	—	—
516	4·18	14,170	16,245	—
1,916	14·7	—	2,070	18,315
1,500	18·75	3,520	—	—
594	4·48	15,200	18,720	—
2,094	16·1	—	4,880	23,600
1,600	20	5,600	—	—
691	4·775	16,200	21,800	—
2,291	17 6	—	11,550	33,350
1,650	20·6	7,600	—	—
770	4·92	16,650	24,250	—
2,420	18·6	—	21,300	45,550

If we now determine the leakage coefficient for a flux of 1,500 lines we find it to have risen to 1 + 594/1,500 = 1·394. For Z = 1,600 it is still greater—namely, 1 + 691/1,600 = 1·432, whilst the initial value was only 1·326. It will thus be seen that the leakage coefficient can only give the approximate value of the waste field. Fig. 68 shows the characteristic as

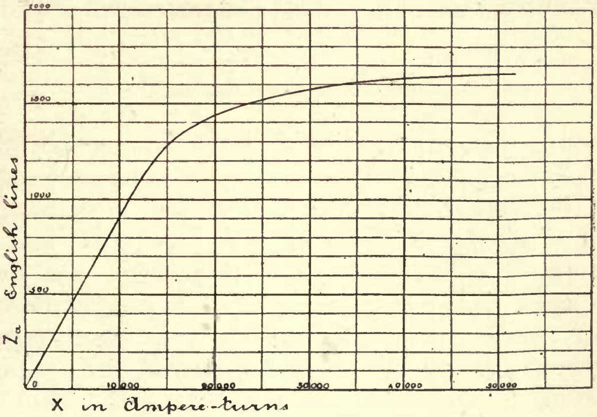

Fig. 68.

plotted from the above table. By the use of this curve we can at once read off the exciting power required to produce any desired flux through the armature within the limits of its capacity to pass the flux. The electromotive force produced being proportional to the flux and the speed, we see that the curve, by suitably choosing the scale of ordinates, may also be used to represent the electromotive force on open circuit for constant speed as a function of the exciting power. It is, however, important to observe that the curve only refers to the electromotive force on open circuit. If a current is permitted to flow through the armature it produces a certain reaction on the field, and this must be taken into account, as will be shown in the following chapter.

CHAPTER XI.

Static and Dynamic Electromotive Force—Commutation of Current—Armature Back Ampere-Turns—Dynamic Characteristic—External Characteristic—Armature Cross Ampere-Turns—Sparkless Collection.

Commutation of Current.

By the method detailed in the foregoing chapters we can determine the electromotive force generated in the armature when the constructive data of the machine are given. The electromotive force is that which can be measured at the brushes when the external circuit is open—that is to say, when no current flows through the armature. Under this condition, the machine produces, so to say, merely a static electric pressure, comparable to the hydrostatic pressure of a head of water close behind the orifice of the discharge pipe from a reservoir when the orifice is closed. But as soon as we open the orifice only very slightly, the water rushes out, and the pressure within the pipe becomes less than it previously was. In the same way we observe that the pressure between the brushes of a dynamo falls slightly as soon as we close the external circuit and allow a current to flow through the armature. We have, therefore, to distinguish between the "static electromotive force"—that

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is, the electromotive force generated in the armature, and directly measurable on the brushes if the machine is working on open circuit—and the "dynamic electromotive force"—that is, the electromotive force generated in the armature when the machine is working on a closed circuit. The dynamic electromotive force is not directly measurable at the brushes, but can be found by adding to the brush electromotive force the loss of pressure due to the product of armature resistance and current. The static electromotive force depends for any given machine simply on the strength of field and the speed, but the dynamic electromotive force depends not only on these two quantities, but also on the current, being the lower, the more the current is increased.

This lowering of electromotive force is due to a group of secondary effects, commonly comprised under the term "armature reaction," and these effects must now occupy our attention. The first of them to be considered is that of the commutation of current in the armature wires passing under the brushes. The subject may be profitably approached in the first place by experiment. Let the reader take an armature and put it into a lathe, fixing the brushes in their proper position so that a current may be sent through the armature. On starting the lathe, and turning on even a very moderate current, he will find violent sparking at the brushes. No work is done except the small amount represented by the overcoming of the ohmic resistance of the armature. Yet there is bad sparking, whilst the same armature running in its own field and doing full work will run without sparking.

How is this difference to be explained? Obviously, by the difference in the working conditions. In the one case the armature is running in air, and is not under the influence of a magnetic field (the field of the earth being not only inappreciably weak, but possibly also applied regardless of the position of brushes), and in the other case it is running in a strong magnetic field, and the brushes are properly set with regard to the direction of the field. The natural conclusion to be

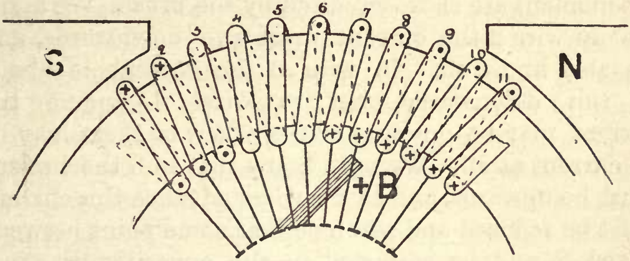

FIG. 69.

drawn from this experiment is, then, that the presence of a field is necessary to prevent sparking; and having arrived at this point experimentally, we can start on the theoretical investigation.

Fig. 69 shows a portion of a ring-wound armature with the positive brush resting on two sections of the commutator fully, and just leaving a third section. With field magnets of the polarity indicated and the movement counter clockwise, the induced electromotive force will be directed upwards in all the wires to the right of the vertical (from No. 6 upwards), and will be directed downwards in all the wires to the left of this line. What the direction of current in each external wire will be depends partly on that of the induced electromotive force, but also on the position of the brush, + B. Since the current must flow out of this brush it is obvious that in all the external wires to the left of it the direction of current must be downwards, and in all those to the right of it the direction must be upwards, but it is not at the first glance obvious what the direction of current will be in those wires which at the moment are short-circuited by the brush. We know that in wire 2 the current must run downwards, and we also know that in wire 3 the current is about to run downwards, but in wires 4 and 5 the current may be one way or the other, or there may be no current at all. We only know that in 6 the current must be upwards, and as the wires advance this current must be reduced and reach zero at some point between 6 and 2, and be restarted in the opposite direction, increasing to the full strength by the time the wire has reached the position 2. The current in each wire must, therefore, be stopped and restarted in the opposite direction during the time that the wire passes under the brush, and this holds good equally for the armature working in its proper field and running idle in air. In the latter case there is sparking—that is to say, the current, or at least a portion of it, refuses to flow through wires 2 and 3, and prefers to leap across the corresponding commutator plates to the toe of the brush. When we say the current refuses, we use a figure of speech; it would be more correct to say that there is something which prevents the current from flowing down these wires, and compels it to leap across the insulating spaces on the commutator. This something can obviously be nothing else than an electromotive force, and the reason for its existence when the armature revolves in air will be easily seen. It will be clear that with current flowing through the wires as indicated by dots and crosses, the region between 2 and 6 will become a south pole—that is to say, lines of force will flow into the iron core of the armature in this space. As a matter of fact, the influx is not limited to this space, but extends a considerable distance on either side, but what goes on beyond wires 2 and 6 does not concern us at present. The inflowing lines are, of course, being cut by wires 2, 3, 4, and 5, and in these wires an upward electromotive force is accordingly set up. Consider now what happens to wire 3. As soon as its commutator segment emerges from under the toe of the brush, it ought to carry a downward current, but it cannot do so, because in it acts an upward electromotive force which stems back the flow of current. The result is that the current is, so to speak, squeezed out at the commutator segment and forced to leap across the air into the brush, thus producing the sparking at the brush. This sparking is therefore directly due to the field produced by the collective action of the armature wires; but this is not all. In addition to this field, each wire produces individually a kind of magnetic whirl round itself (Fig. 12, page 69), which acts in the same way as the collective field. In other words, each wire has a certain amount of self-induction, which opposes any sudden change in the strength or direction of current. The only remedy for sparking is therefore to devise some means by which these changes can take place gradually—that is to say, by which each wire shall be prepared, whilst under the brush, for the current it will have to carry when it emerges from under the brush. The natural means of accomplishing this object is to let the short-circuited wires be acted upon by a downward electromotive force. Thus, if wire 3 at the moment to which Fig. 69 refers is already carrying the same current as wire 2, there will, at the moment of separation, be no electromotive force between the toe of the brush and the commutator segment belonging to wire 3, and there will be no sparking. The downward electromotive force is produced by lines coming out of the armature—that is to say, by a flux of the same nature as is produced by the south pole-piece. All we have to do, therefore, to prevent sparking is to advance the brush in the direction of rotation, so far as to bring the commutated wires sufficiently under the influence of the south pole-piece. We have thus arrived at the following result, which is easily verified by experiment. If we leave the brushes exactly midway between the pole-pieces we have a commutation of the same kind, though not quite as bad, as with an armature revolving in air, and there will be sparking, more or less. If we shift the brushes forward, and if the magnet field is strong enough in comparison with the field produced by the armature current, we can find a position where there will be sparkless collection. The proviso that the magnet field must be strong enough is of importance, as will be shown later on. For the present it suffices to note that the brushes must be shifted forwards in a dynamo, and from what has been said above it will be obvious that the amount of forward displacement or "lead," as it is technically termed, must increase with the current. The amount of lead depends on the relative strength of armature and magnet field, on the shape of the pole-pieces, the number of armature conductors to each section of the commutator, and on the winding of the armature. It is quite possible to so design a machine that the brushes need not be shifted at all when the current is varied, but this does not prove the absence of lead. It only proves that the lead is less than the angular width of the brushes.

In a motor the direction of current through the armature is reversed, and consequently, to get sparkless collection, we must shift the brushes backward.

In the above explanation of the process of commutation no account was taken of the internal wires of the ring. We have only spoken of the collective field produced by the outside armature conductors and the magnetic whirl in each conductor. This is all that has to be considered in a drum where there are only outside conductors, but in a ring we have, besides these, also inside conductors, and we have therefore an inside and an outside field, Fig. 70, and self-induction in the inside and outside wires. There is consequently greater tendency to sparking in a ring than in a drum, and a greater lead is required to bring the brushes into the sparkless position.

The grouping of the winding has also an influence on the tendency to sparking, and consequently on the lead. If to each commutator segment there corresponds not one turn but a coil of many turns of wire, the self-induction of the coil will be very much greater than that of a single turn, and consequently we shall

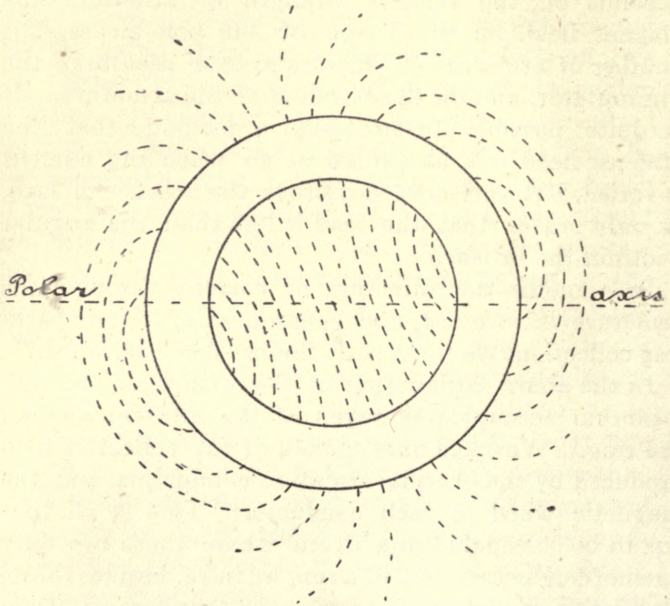

FIG. 70.

require a stronger field for reversal. The modern practice is therefore as far as possible to reduce the number of wires or turns controlled by each segment of the commutator; in other words, to employ a large number of sections in the commutator.

It is useful to enquire what part the element of time plays in the process of commutation. We have seen that the current in each wire must be stopped and restarted whilst the wire is advancing from the position 6 to the position 2, Fig. 69. The whole process of commutation must, therefore, be completed within the time occupied in the transition of the wire from the one to the other position; and as this time will be the shorter the higher the speed of the armature, it might at first sight appear more difficult to get sparkless collection at high than at low speeds, or that the lead must be greater at high than at low speeds. This, however, is not the case. It is true that the electromotive force required to kill off the old and start the new current in each wire must be the greater the shorter the time available for the whole process, but then we must remember that this electromotive force is produced by the cutting of lines, and is, therefore, directly proportional to the product of the speed and the density of flux in the region between 2 and 6. The higher the speed the greater is the electromotive force producing commutation, and that is just what is required, so that the tendency to sparking is not directly dependent on the speed. If we leave the strength of the magnet field constant, and so adjust the resistance of the external circuit that the current remains constant, then we may either work at a low speed (producing a low electromotive force), or a high speed (producing a high electromotive force), and if there is no sparking in one case there will be none in the other. But if we have adjusted the machine for low speed and low electromotive force and wish to work it at high speed without altering the electromotive force, then it becomes necessary to weaken the magnet field, and in this case it may happen that the machine will spark, and the sparking will be due, not to the high speed in itself, but to the fact that the high speed necessitates the employment of a weaker field, which may be insufficient to produce proper commutation.

Armature Back Ampere-Turns.

We have seen that a certain part of the magnet field—namely, the fringe of it at the leading polar edge—is required simply and solely for commutation. This part of the field contributes nothing to the total armature electromotive force, and must therefore be considered as a loss so far as the useful work of the machine is concerned. This will help us to understand the difference in the working of the machine when running idle and loaded. In the former case the brushes are set midway between the poles, and the whole of the flux is producing electromotive force. When the machine is running loaded we are obliged to put the brushes forward, and thus we lose some of the flux and the electromotive force produced is smaller. This accounts for the fact that the static electromotive force is always higher than the dynamic electromotive force. In addition to this difference there must also be taken into account the loss of electromotive force due to armature resistance, so that the pressure measured at the brushes when the machine is doing work is perceptibly lower than when running on open circuit.

The reduction of voltage due to armature reaction may also be represented in another way.* Fig. 71 shows the direction of current in the different armature wires and the position of the brushes, which, for the sake of simplicity, are supposed to lie directly on the

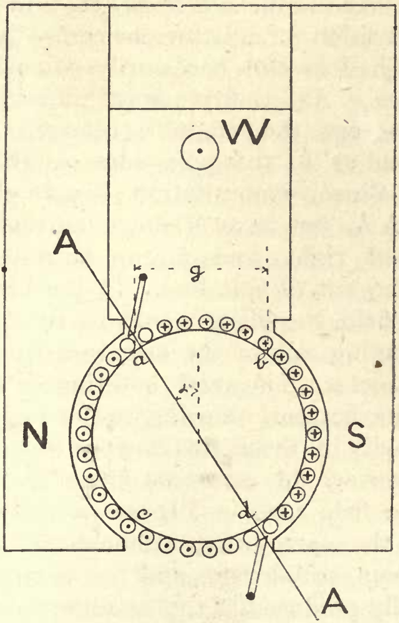

FIG. 71.

wires, the commutator being omitted in the illustration. We have seen that in order to obtain sparkless commutation it is necessary to shift the brushes so far

* Esson and Swinburne. *Journal* of the Institute of Electrical Engineers, vol. xv., 1886; vol. xix., 1890; and vol. xx., 1891.

forward that the short-circuited wires come within the influence of the leading edges of the poles. Whether the position thus defined be exactly under the polar edges or at a little distance from them, depends upon a variety of considerations into which we need not enter at present. It is sufficient to say that, in practice, the sparkless position for full current comes generally very near, though does not necessarily coincide with, the polar edges. As a first approximation, we may therefore accept the angle a between the vertical and the radius to the pole edge as the true angle of lead. Since commutation takes place on the diameter A A, we have a down current in all the wires to the right, and an up current in all the wires to the left of this line. If the armature were out of its field, the effect would be to produce a flux of lines coming out of the armature core at the top of the diameter, A A, and entering it at the right hand at the bottom, showing respectively north and south polarity in these two regions (see also Fig. 70). Since, however, the armature is, so to speak, in the grip of the field magnets, these polarities cannot be developed the same way, although the tendency to develop them still exists, and we may consider the flux actually produced as the resultant of the flux due to the magnets alone and that due to the armature alone. Following Esson's method of representing the excitation which produces the latter, we may consider the magnetising action of the armature wires as resulting from two groups of coils placed at right angles to each other—namely, a vertical coil comprising the wires a to b at the top and c to d at the bottom of the armature, and a horizontal coil comprising the wires *a* to *c* on the left and *b* to *d* on the right-hand side of the armature. The former coil tends to produce a flux which is directly opposed to that produced by the field magnets, and the latter a flux at right angles to it. Esson calls the corresponding turns on the armature "back turns" and "cross turns." The flux due to the field magnets may be considered as produced by a current, X, equal to the total field exciting power flowing up in the single wire, W, passing between the limbs of the magnets (compare Figs. 22 and 23). It was shown in Chapter V. that such a current has the same effect as that in coils surrounding the magnet limbs, and this effect is independent of the precise position of the wire, W. So long as this wire passes between the two limbs within the space bounded by the yoke on the top and the armature below, it will produce the same flux. It is obvious that we may consider all the wires from *a* to *b* magnetically in the same condition as wire W, for they, too, are within the space bounded by the armature limbs and yoke, but the current in these wires is in the opposite direction to that in W. The total exciting power applied to the magnets is therefore X minus the product of back turns and current flowing through them, technically termed "armature back ampere-turns." Using for the latter the symbol X_b, and retaining the previous notation, we have

$$X_b = \tau c \frac{\alpha}{\pi},$$

c being the current through one armature conductor. This formula is applicable to bi-polar and multipolar machines, but can be more conveniently used if we replace the angle of lead by the gap g between opposite polar edges. Calling d the diameter of the armature, the number of wires within the gap is $\tau \frac{g}{\pi d}$, and the back ampere-turns are given by the expression

$$X_b = \tau c \frac{g}{\pi (d + 2 \delta)} \quad . \quad . \quad . \quad . \quad (41)$$

Dynamic Characteristic.

The actual field produced is that due, not to X alone, but to $X - X_b$, and this correction must be taken into account when plotting the dynamic characteristic of magnetisation. It is obvious that such a characteristic can only be plotted for a definite armature current and that we require a separate curve for each current. It must also be pointed out that formula (41) will give rather too large a value for the back ampere-turns on the armature (and consequently too low a characteristic) especially when the current is small, the reason being that the diameter of commutation in good machines is, even with full current, not quite in line with the polar edges, and with a small current the lead must be still further reduced. The error may be corrected by assuming for g a smaller value than measured from the drawing; and as this is a matter which must be left to the judgment of the calculator, the predetermination of the dynamic characteristic cannot be made with the same accuracy as that of the static characteristic. The probable errors are, however, very small. The armature back ampere-turns are for full current generally from one-tenth to one-fifth of the field exciting power. For half current they range from one-twentieth to one-tenth, or, say, an average of 8 per cent. If, then, in estimating g we should make an error, even so large as 50 per cent., it would only cause an error of 4 per cent. in the determination of the total exciting power, and owing to the shape of the curve, a very much smaller error in the determination of the useful field and armature electromotive force. For practical purposes this method of

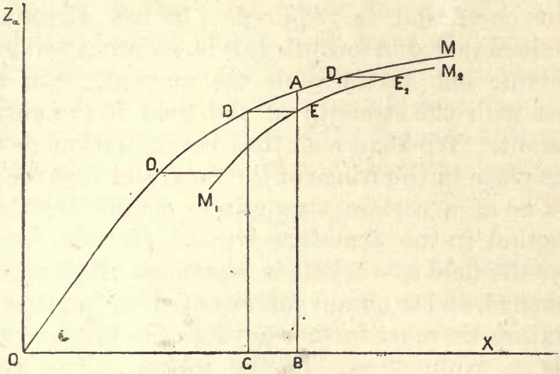

FIG. 72.

calculation, which has been independently devised by Mr. Esson and Mr. Swinburne, is, therefore, sufficiently accurate.

The plotting of the dynamic characteristic from the static is a very simple process. Let O M in Fig. 72 be the static characteristic and A one point on it. Then O B = X represents the exciting power corresponding to the flux B A. Measure back from B a distance B C = X_b representing the back ampere-turns of the armature, then $O\,C = X - X_b$ represents the actual exciting power producing the useful flux, C D. Drawing a horizontal through D to the point of intersection with B A, we obtain the point E on the dynamic characteristic. We can obtain any number of such points by simply drawing horizontal lines through the static characteristic and measuring off to the right on each a distance representing the back ampere-turns of the armature. We thus obtain the curve, M_1 M_2. Here, again, a slight correction is required. It has already been explained that g in formula (41) is, strictly speaking, not constant, but varies with the current. But it also varies with the strength of the field if the current is constant. We have seen that commutation generally takes place in the fringe of the field, and that the fringe must be of a certain strength to counteract the self-induction in the armature wires. If, then, from any cause the field as a whole is weakened, its fringe is also weakened, and to obtain sufficient field strength for commutation we must further advance the brushes so as to utilise a stronger part of the fringe. This increases the armature back ampere-turns and further weakens the field, necessitating a further advance of brushes, and this may go on until the brushes are actually under the polar edges. After this point has been reached, no further advance of the brushes can increase the back ampere-turns, since the pole-pieces shield the magnet limbs from the action of the armature wires. Commutation then takes place, not in the fringe of the field, but within the field itself, and by so shaping the magnets and pole-pieces that the induction is the same all through the polar cavity, we can obtain sparkless collection at all points from the edge to the centre of the pole-pieces. This property is taken advantage of in the construction of arc light machines designed for constant current and variable voltages, the variation in voltage being produced by an automatic relay gear, which shifts the brushes forward on a slight increase and backward on a slight decrease of current.

Returning now to the ordinary condition of working, we have seen that the nearer the diameter of commutation comes to the polar edges the greater is the demagnetising effect of the back turns, and since we must give more lead as the field strength is diminished, it follows that in plotting the dynamic from the static characteristic we must increase the distance D E when working on the lower part of the curve, and reduce it on the higher part

$$D_1 E_1 > D E > D_2 E_2.$$

This is again a matter which must be left to the judgment of the calculator. In constant-voltage machines where it is for commercial reasons advisable to work over a region fairly high up on the characteristic, the two curves come so near together that an error in estimating the variation in the length D E has little influence on the final result. Moreover, it should be remembered that the formula (41) gives us the greatest possible amount of armature back ampere-turns if we insert the gap distance measured on the drawing, so that if we neglect the correction altogether the voltage of the machine may come out slightly too high, but never too low. A fault of this kind can of course be very easily compensated in the finished machine.

Fig. 73 shows the static curve of Fig. 70 with the dynamic added, the latter being drawn from the lowest point at which sparkless collection is just possible, and when the diameter of commutation coincides with the polar edges to the highest point when half this lead is assumed to be necessary for sparkless collection.

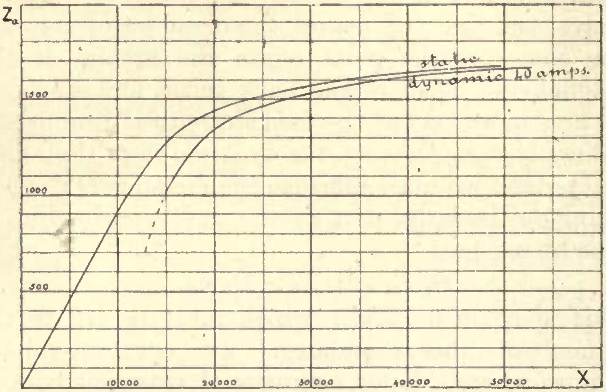

FIG. 73.

External Characteristic.

An interesting case is that of a series machine. In such a machine the armature current traverses the field winding, and the exciting power is, therefore, strictly proportional to the current. The total field strength, and, therefore, also the strength of the fringe producing commutation, increases with the armature back turns; and by properly designing the machine, it is possible to keep the lead constant over a fairly large range of output. In this case the exciting power is proportional to

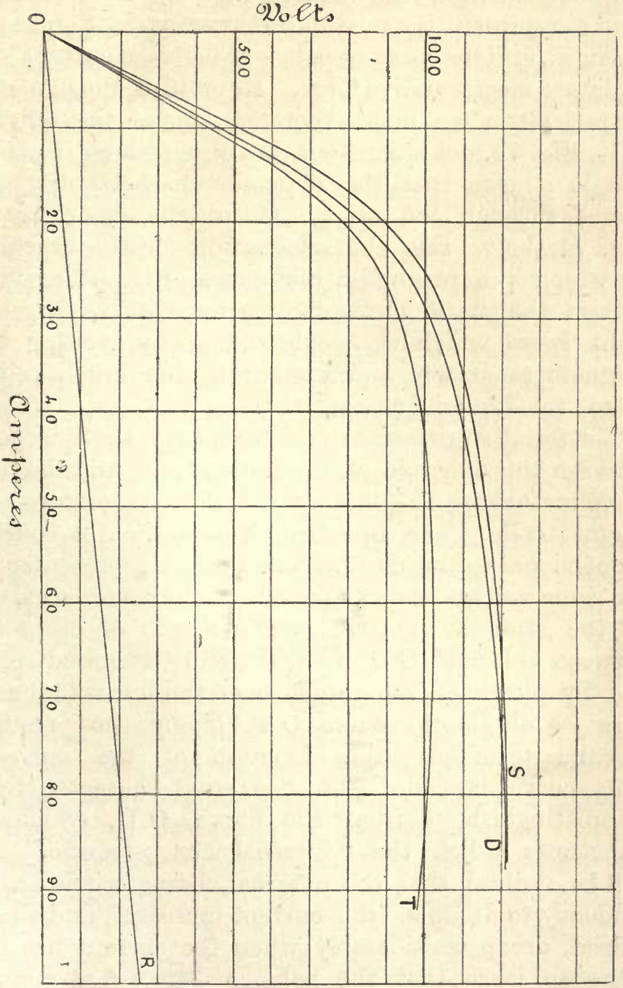

FIG. 74.

the main current, and we may draw the characteristic so as to represent the relation between current and field strength, or if the speed be constant between current and armature electromotive force. In plotting the dynamic characteristic we must, therefore, make the length, D E, Fig. 72, not a constant, but proportional to O B, and it follows that the dynamic characteristic now passes through the origin, O, of the co-ordinates. Fig. 74 shows this characterisation for the machine to which the magnetisation curve, Fig. 70, refers. O S is the static electromotive force curve for constant speed which we would obtain by exciting the magnets separately and measuring the brush volts; O D is the dynamic characteristic.

The terminal pressure of the machine is the difference between the dynamic electromotive force and the loss of volts through the ohmic resistance of armature and magnet coils. The pressure thus lost is, of course, proportional to the current, and may be represented to the same voltage scale as the electromotive force curves by the straight line, O R. The length of ordinates between O R and O D gives, therefore, terminal volts, and by plotting these values over the horizontal as a base we obtain the curve O T, giving the pressure at the terminals as a function of the current. This curve is called the "external characteristic," to distinguish it from the curve O D, which is sometimes called the "internal characteristic." It will be noticed that the external characteristic has a tendency to droop as the current increases, and does, indeed, droop considerably when the current has become so large that the balance between armature reaction and the strength of the fringe of the field producing commutation is no longer maintained, and it becomes necessary to advance the brushes (dotted part of the curves) in order to get sparkless collection. This drooping of the terminal electromotive force curve is especially noticeable in machines of older construction, in which both the resistance and armature reactions are large. In modern machines having comparatively strong fields and small armatures, the armature reaction is slight, and there is but little loss of electromotive force through resistance. With such machines, unless considerably overloaded, there is no droop in the characteristic. An exception to this rule is, however, formed by the various types of open-coil armatures used for arc lighting. In these machines the armature reaction is enormous, producing a very decided droop in the characteristic, which is, however, a positive advantage, as it protects the machine from excessive strains when overloaded or short-circuited.

Armature Cross Ampere-Turns.

We have now to consider the part played in the working of a machine by the cross turns on the armature—namely, the wires a to c and b to d (Fig. 71). Each group is obviously equivalent to a sheet of current flowing between two parallel iron surfaces of breadth λ and the distance δ apart, the total strength of current being $\tau\, c\, \frac{\lambda}{\pi\, d}$, whilst the current density per inch or centimetre is $\frac{\tau\, c}{\pi\, d} = \gamma$.

To determine the effect of the sheet of current on the induction between the two surfaces, we suppose the latter to be straightened out into a plane, Fig. 75, when A A represents the surface of the armature, P P that of the pole, and C C the sheet of current. Selecting any point p on the pole face at a distance a from the centre, we find that the induction within the air

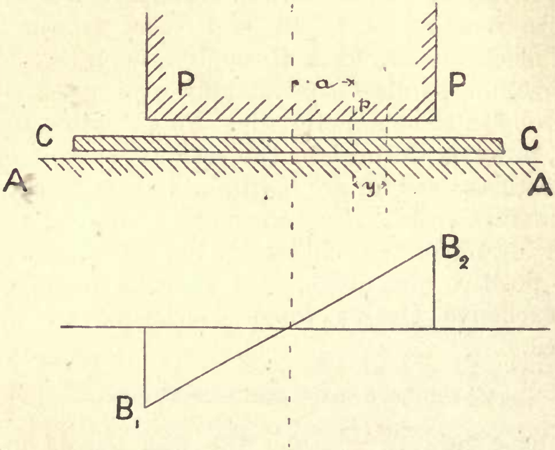

Fig. 75.

space at p is due to the action of all the current elements to the right and left of that point, the integration being extended to the edges of the polar face. A current element, $\gamma\,d\,y$, at the distance y from p produces a magnetising force $H = \frac{\cdot 4\,\pi\,\gamma\,d\,y}{2\,\delta}$, and this integrated over all the elements to the right of p gives the induction through p due to that part of the current sheet which lies to the right of p. Neglecting the comparatively very small magnetic resistance of the iron part of the path of lines, this induction is $\frac{\cdot 4\,\pi\,\gamma}{2\,\delta}\left(\frac{\tau}{2}-a\right)$. In a similar manner we find the induction due to that part of the current sheet which lies to the left of p, or $\frac{\cdot 4\,\pi\,\gamma}{2\,\delta}\left(\frac{\tau}{2}+a\right)$. This is obviously of the opposite sign, and the resultant induction is the algebraical sum of these two values—namely:

$$\frac{\cdot 4\,\pi\,\gamma}{2\,\delta}\,2\,a.$$

For $a=0$—that is, for the centre of the pole-piece—the induction is zero, and for $a=\frac{\lambda}{2}$—that is, for the edges of the pole-piece—it is a maximum, being positive for one and negative for the other edge, as shown by the sloping line, B B. Its value is $\frac{1{\cdot}256\,\gamma\,\tau}{2\,\delta}=\frac{1{\cdot}256}{2\,\delta}\,\tau\,c\,\frac{\lambda}{\pi\,d}$.

This is the induction due to the armature cross turns only, but in addition there is the induction due to the exciting coils on the field magnets, and to find the true induction within the air space we must add these two values. In Fig. 76 is reproduced the line B_1, B_2, but with the ends joined to the axis of abscissæ by sloping lines, $B_0\,B_1$ and $B_3\,B_2$, as it is obvious that the induction cannot abruptly change from nothing to a maximum at the polar edges. There must be a kind of fringe also to the field of induction produced by the armature current, as there is a fringe to the field of induction produced by the exciting coils. The field induction is, of course, constant over the whole of the polar face, and is represented in Fig. 76 by the horizontal line, $P_1 P_2$, whilst the fringes are represented by the sloping lines, $P_0 P_1$ and $P_3 P_2$. The

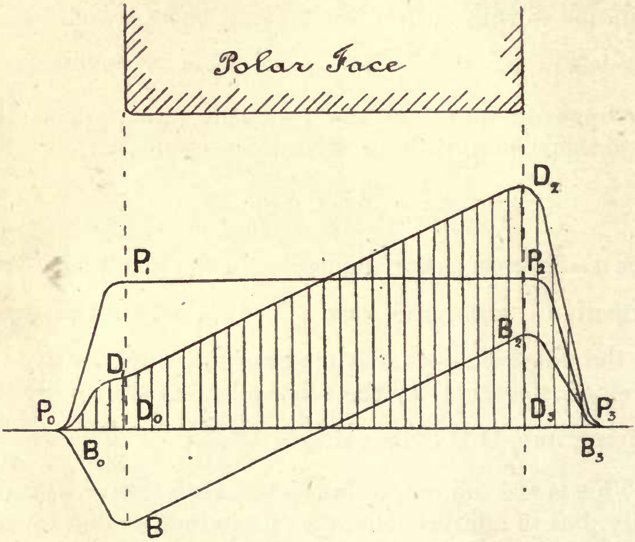

FIG. 76.

true induction is found by combining the two diagrams, which gives us the line P $D_1 D_2 P_3$.

This curve can also be obtained experimentally.* Let, in Fig. 77, C represent the commutator and A a piece of fibre or other insulating material through which two

* First suggested by Prof. S. P. Thompson, a voltmeter being used instead of condenser key and galvanometer.

holes have been drilled for the reception of two pointed wires, bent down on to the commutator. Care must be taken to make the distance between the points equal to the pitch of the sections, and to keep the points fairly sharp, so that the surface of contact of each shall be less than the width of the insulation between the sections. Otherwise there would be flashing over

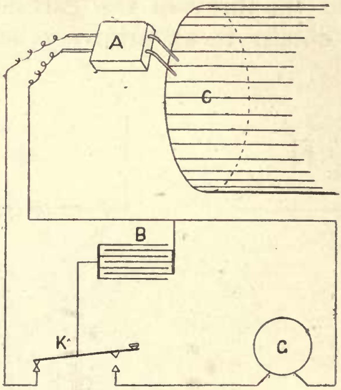

FIG. 77.

from one section to the next. The back ends of the two wires are connected with a delicate voltmeter, or, better still, with a condenser, B, discharging key, K, and ballistic galvanometer, G. The fibre piece, A, is mounted on a pin, which can be set in any desired position round a graduated circle (not shown in the diagram) so as to bring the points of contact successively into various positions with regard to the polar face. Whilst the key remains in the position shown, the condenser receives a charge which is proportional to the induction (ordinate of the line $D_1 D_2$) in that part of the field to which the then position of the piece A corresponds. On pressing down the key, we discharge the condenser through the galvanometer and obtain a deflection, which is, of course, also proportional to the induction. By plotting the angular position of A, which we read off on the graduated circle, on the horizontal and the throw of the galvanometer on the vertical, we obtain to an arbitrary scale the curve $P_0 D_1 D_2 P_3$.

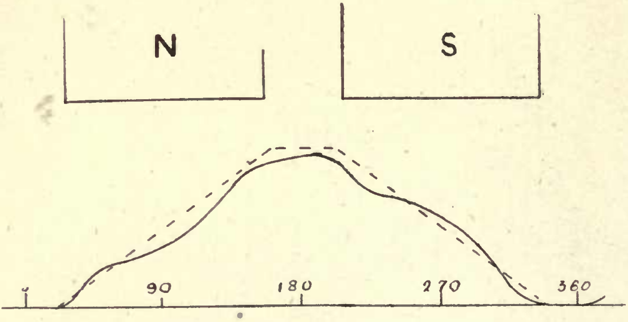

Fig. 78.

We may also use the same instrument to determine the variation of electromotive force round the commutator* by using only one contact wire—for instance, that connected directly to the condenser; and connecting the back contact of the key to one of the brushes. In this case we obtain a curve of electromotive force of the general shape represented in Fig. 78. If there

* First done by Mr. W. Mordey.

were no armature reaction—that is, if the readings were taken when no current is permitted to flow through the armature—the curve would be of the general character shown by the dotted line.

An armature giving a diagram of the general character shown in Fig. 76 will run without sparking. The brush would have to be placed somewhere in the region between P_0 and D_0, the exact spot depending on

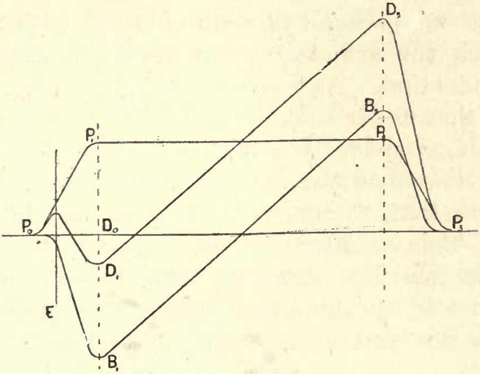

Fig. 79.

the amount of induction required to balance the self-induction of the armature wires; but there is, at any rate, a sufficiently strong field to produce commutation. In other words, the armature cross-induction is small as compared with the forward induction produced by the field winding. Now let us enquire how the matter stands if the armature cross-induction is comparatively large. Let us, for instance, assume that D_0B_1 is larger than D_0P_1. In this case the point D_1, Fig. 79, falls below the axis, and the field under the influence of which commutation takes place becomes negative. There may or may not be the small hump near P_0. This depends on the shape of the polar edges. It may be just possible to obtain sparkless collection by placing the brush into the position E, but this would be merely a chance success upon which no prudent designer should rely. I only mention this as affording a possible explanation of the few cases in which sparkless collection has been obtained with machines in which the armature cross-induction exceeded the field induction. As a general rule, we may, however, take it that under such circumstances sparkless collection is impossible. In order to get sparkless collection it is obvious that the point D_1 must remain above the axis, and that, therefore, $D_0 B_1$ must be smaller than $D_0 P_1$. This condition will be obtained if the ampere-turns on the field magnets required to overcome the resistance of the air space exceed the cross ampere-turns of the armature. Calling the latter X_λ we have

$$X_a > X_\lambda,$$

where $X_a = \cdot 8 \times 2\,\delta\,\mathfrak{B}_a \qquad X_a = 1{,}880 \times 2\,\delta\,B,$

and $X_\lambda = \tau c \dfrac{\lambda}{\pi d} \qquad X_\lambda = \tau c \dfrac{\lambda}{\pi d}.$

The induction under the leading polar edge is

$$\mathfrak{B}_a{}' = \frac{1\cdot 256\,(X_a - X_\lambda)}{2\,\delta},$$

and under the trailing edge it is

$$\mathfrak{B}_a{}'' = \frac{1\cdot 256\,(X_a + X_\lambda)}{2\,\delta}.$$

For practical work it is convenient to express the induction under the leading and that under the trailing polar edge as a function of the average induction, since the latter has to be determined in any case when working out the characteristic of magnetisation. We have

$$\mathfrak{B}_a' = \mathfrak{B}_a \left(\frac{X_a - X_\lambda}{X_a}\right)$$ minimum induction under leading edge.

$$\mathfrak{B}_a'' = \mathfrak{B}_a \left(\frac{X_a + X_\lambda}{X_a}\right)$$ maximum induction under trailing edge.

Sparkless Collection.

The condition for sparkless collection is that $\mathfrak{B}_a'$ shall be above a certain limit. Authorities differ what this limit should be, and it is indeed not possible to fix a limit that shall be applicable to all cases. Thus, in a drum-wound machine, we may allow a smaller value for $\mathfrak{B}_a'$ than in a ring-wound machine, the reason being that the greater self-induction in the coils of the latter requires naturally a stronger reversing field. Again, in a machine where each section of the commutator controls only one turn of wire, we may allow a smaller $\mathfrak{B}_a'$ than in one where to each section correspond many turns. The shape of the pole-pieces, especially that of their leading edges, has also an influence on the minimum value which may be safely allowed for $\mathfrak{B}_a'$. Other elements affecting the sparking limit are the type of field winding (whether shunt, series, or compound), the use of the machine either as generator or as motor, the amount of attention which it will receive in working, and many other

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points. Where there are so many considerations affecting the design, and especially where some of them cannot be known beforehand, it would obviously not be prudent to cut the reversing field down too low, although by so doing we obtain a machine of large output in comparison to its weight. The best modern practice is to keep the reversing field fairly strong, say, at a minimum of 2,000 C.G.S., or 2·2 English lines. There is also another reason for limiting the difference of field strength under the leading and trailing polar edges, and this is that when the field under the trailing edge becomes very strong there occur certain losses, resulting in heating and waste of power, as will be shown in Chapter XIII.

We may, however, be obliged in certain cases to employ an armature having a large cross-induction, and the question now arises, how in spite of this the reversing field can be kept strong enough. Take the case of Fig. 79, which represents a machine that will certainly spark at full load. In determining the cross-induction under the polar edges $\left(\frac{1{\cdot}256}{2\,\delta}\,\tau\,c\,\frac{\lambda}{\pi\,d}\right)$ we made the assumption that the magnetic resistance of the iron parts in the path of the lines of cross-induction may be neglected. This is strictly true for machines of the type Fig. 57a, b, or c, or Fig. 59, in all of which cases there is so much iron right through the pole-piece that practically no magnetising force is required to drive the lines from one polar edge to the other. But if we have a machine of the type Fig. 57d, this assumption may not be strictly true. The path for the cross-lines through the pole-pieces is no longer quite free, since the smaller area at the centre causes a certain crowding and consequent throttling of the flux; and if the restriction thus introduced into the path of the cross-lines is sufficiently great it will sensibly reduce the cross-induction. In this respect the field, Fig. 57d, is better than any of the three other fields, but we can go a step further. We can cut the pole-pieces completely through on the polar diameter, and introduce an air gap as shown in Figs.

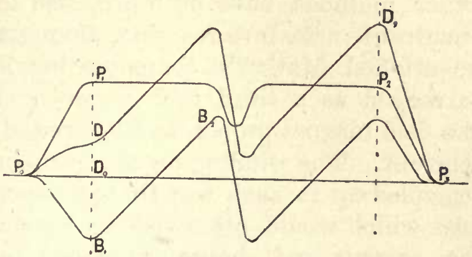

FIG. 80.

64 and 65. By this means we can almost completely prevent the flow of lines along the pole-piece from one edge to the other, and restrict it to a flow through each half of the pole-piece. This will reduce the cross-induction at the extreme edges to half its former amount, and we are thus able to keep the resultant induction under the leading edge above the sparking limit. A glance at Fig. 80 will make this matter clear. This diagram refers to the same machine as Fig. 78, with the only difference that the pole-pieces are cut in the middle, and contain a gap sufficiently

wide to prevent the lines flowing across. Instead of a single line of cross-induction, we now have two lines of saw-tooth shape and of half the amplitude. The line $P_1 P_2$, representing forward field induction, will have a depression in the middle owing to the gap in the pole-piece, but its height at either end remains, of course, the same. The resultant induction, as seen by the line $P_0 D_1 D_2 P_3$, is now positive throughout, and sparkless commutation will be possible if the brush be placed somewhere in the region between P_0 and D_0. Various other methods have been proposed to either reduce or neutralise armature reaction, amongst which may be mentioned Mather's compensating magnet, which is arranged as a third limb between the two limbs of the field magnet proper, and is excited by the armature current. The winding on the compensating magnet is coupled up in such way that it opposes the flow of lines which would otherwise be produced by the armature current, and commutation can therefore take place exactly midway between the poles. The result is that the width of the vertical belt of back turns, Fig. 71, is reduced to nothing; in other words, that there are no back turns, and, consequently, that the dynamic electromotive force has the same value as the static electromotive force whatever the current may be. Swinburne suggested the use of an auxiliary reversing pole formed by cutting a groove in the field pole-piece behind the leading polar edge, and applying to the part thus separated an exciting coil in series with the armature circuit. This arrangement does, of course, not reduce the width of the belt of back turns as with Mather's compensating magnet, but it tends to keep the lead constant for all loads. Another arrangement for balancing cross-induction is that devised by H. J. Ryan,* in which the pole-pieces are provided with perforations parallel to the axis of the armature, and coils are wound through these so as to compensate the cross-turns on the armature.

* *Electrical World*, vol. xx., No. 21, November 19, 1892.

CHAPTER XII.

Influence of Linear Dimensions on the Output—Very Small Dynamos—Critical Conditions—Large Dynamos—Limits of Output—Advantage of Multipolar Dynamos.

Influence of Linear Dimensions on the Output.

In self-exciting dynamos a certain portion of the output given by the armature is required for the excitation of the field magnets, and only the rest is available for external work. The larger this amount is in comparison with the energy required for excitation the better, but as in mechanical appliances generally size has a great influence on the efficiency of the apparatus, it is only reasonable to expect the same conditions to hold good with dynamos. A toy steam engine cannot possibly work with the same efficiency as a steam engine of 100 h.p., and a very small dynamo must be a more wasteful machine than a large one. The question now presents itself whether by reducing the size of the dynamo more and more, we should at last arrive at a point when the waste exceeds the total output of the armature—that is to say, when the machine ceases to work altogether. In order to find an answer to this question, let us compare two machines of exactly the same type, but of different size. One of these we consider the normal machine, designed on scientific principles, and with due regard to the work it has to do, and the other we consider to be simply a model of the normal machine, the dimensions of all the parts being enlarged or reduced in the same ratio. Mathematically speaking, the linear dimensions of the model machine will then be q times those of the normal machine, the coefficient q being greater than unity if the model machine is larger, and less than unity if it is smaller than the normal machine. It is important to note that if $q < 1$, the, condition of proportionality cannot be strictly carried out. This is on account of certain practical difficulties. Thus we cannot reduce the thickness of the cotton covering on the wires in the same ratio as we reduce the diameter of the naked wire, and the consequence is that in the model machine the area of our wires will be rather less than the q^2 part of the area of the corresponding wires in the normal machine. With the air space it is the reverse. A machine of half the linear dimensions requires rather more than half the space for insulation and clearance, so that the air resistance will be rather more than would obtain in a machine having all parts reduced to the same extent. The precise influence of these practical considerations depends, of course, on the size and design of the normal machine with which we start, and the degree of reduction of the model; and in order not to complicate the investigation, we shall entirely neglect these considerations. We must, however, remember that if we find that there exists a theoretical limit of reduction beyond which a model will not work, the practical limit will be reached sooner.

Very Small Dynamos.

It is immaterial whether we apply the investigation to a series, shunt, or compound machine; the result is in all cases the same, but for the sake of simplicity we take the series machine. To put the normal machine and the model mechanically into the same condition, we must make the speed of the model $\frac{1}{q}$ times that of the normal machine. This will give us the same peripheral speed for the armature wires and the same stress due to centrifugal force. The number of active wires on the armature, τ_a, and the number of turns in the field coils, τ_f, remain, of course, the same in both machines. Let r and r' be the resistances of the external circuit for the normal machine and model respectively, and r_o and r'_o the internal resistances of the two machines, then we have for the lower or straight part of the external characteristic (in Fig. 68) the following relation: $Z = \frac{X}{R}$; $r + r_o = \frac{X}{R} \frac{\tau_a n 10^{-6}}{C}$, where $X = \tau_f C$, and R the total magnetic resistance.

$$r + r_o = \frac{\tau_f \tau_a n 10^{-6}}{R}.$$

For the higher points on the characteristic the increased resistance of the iron part of the magnetic circuit makes it necessary to employ rather a larger exciting power than corresponds to the formula $X = Z R$, and we have therefore generally

$$r + r_o \underset{<}{=} \frac{\tau_f \tau_a n 10^{-6}}{R}.$$

Critical Conditions.

In this equation all the terms, except r_o and n, are constants fixed by the design of the machine. We may vary the external resistance, r, and we may vary the speed, n, but only within certain limits, the conditions being that r must not be made larger and n must not be made smaller than corresponds to this equation. If these limits be passed, the machine fails to excite itself, and ceases to work as a dynamo. When there is equality between the two sides of the above expression, the machine is in a kind of critical condition, and the current is indeterminate. Let $K = \frac{\tau_f \tau_a 10^{-6}}{R}$, then the critical resistance for a given speed, n, is given by

$$r_o = K n - r,$$

and the critical speed for a given resistance, r_o, is given by

$$n = \frac{r + r_o}{K}.$$

Returning now to the model, we have at first to enquire how the various quantities depend on q. The length of wire in the model will be q times that in the normal machine, but as its sectional area will be q^2 times that in the normal machine, the resistance will be $r' = \frac{r}{q}$. The magnetic resistance of the interpolar space will be similarly altered. Since $\delta' = q \delta$ and $A_a' = q^2 A_a$, the air resistance will be reduced in the ratio of $1 : q$, and if we neglect the small correction due to permeability—which is permissible, since at low magnetisation, to which the critical condition refers, the characteristic is very nearly a straight line—we find that all the other magnetic resistances follow the same law. We have therefore

$$R' = \frac{R}{q} \text{ and } K' = K\,q.$$

The critical resistance—that is, the highest value of the external resistance through which the model machine is just able to send a current—will, therefore, be

$$r_o' = K' n' - r'$$

$$r_o' = K\ n - \frac{r}{q}.$$

The first term on the right is the same as for the normal machine, but the second term is different. If $q > 1$—that is to say, if we enlarge the dimensions of the normal machine—the second term becomes smaller and the critical resistance larger. This is quite natural. The larger machine is more powerful, and can, therefore, force current through an external circuit of increased resistance. If, however, $q < 1$—that is to say, if we reduce the linear dimensions of the normal machine so as to get a small model—the second term on the right becomes larger and the critical resistance smaller. It is then only a question of how much we must reduce the dimensions to get a model which will only work on an external circuit of no resistance, and if we go a fraction below this size, the model will not even excite itself. This limit will obviously be reached if $K n = \frac{r}{q}$

$$q = \frac{r}{K n}.$$

To show the application of this formula, let us take as an example the machine to which the external characteristic, Fig. 74, refers, and assume it to be the normal machine. The total internal resistance of such a machine would be about $2\frac{1}{2}$ ohms, and the critical resistance of the external circuit about 40 ohms. This gives $Kn = 42{\cdot}5$. Let us now make a model to one-fifth size. The largest resistance through which this model would just be able to send a current would then be $42{\cdot}5 - \frac{2{\cdot}5}{0{\cdot}2} = 30$ ohms. The normal machine had an armature 18in. diameter, running at 500 revolutions per minute.

The model would therefore have an armature 3·6in. diameter, and this would run at 2,500 revolutions. If we now still further reduce the size of the model, and run proportionately faster, we arrive finally at a model which will not work at all. This limit will be reached if

$$q = \frac{2{\cdot}5}{42{\cdot}5},$$

that is, if the model is made to a scale of $\frac{1}{17}$th. The diameter of the armature would be a little over 1in., and the speed 8,500 revolutions. This is, however, merely the theoretical limit, and could not possibly be reached in practice. The wire would be so fine that it could scarcely be handled and the insulating covering would only be half a mil thick. Moreover, the clearance would have to be reduced to something like $\frac{1}{170}$th of an inch, and this at the high speed of 8,500 revolutions would not be mechanically safe. To obtain a model that will work, we should therefore have to make it to a scale considerably larger than $\frac{1}{17}$th.

As regards motors, there is no such limit to their size. The power being supplied to the field and armature electrically from an outside source, we can always make the model work if we expend enough power on it.

From what has been said above on the question of reducing the size of dynamos, it will be obvious that the design and manufacture of very small machines to work as generators presents considerable practical difficulties, and this is the reason why such machines are, as a rule, not made self-exciting at all, but are provided with field magnets of hardened steel, as was already stated in Chapter X.

Large Dynamos.

A far more important question, however, is that of an increase in the linear dimensions of any given type of machine. The modern tendency, especially in connection with central station work, electric traction, and power transmission, is in the direction of larger and still larger machines, and the question arises whether the demand for such machines can be satisfied by simply increasing the linear dimensions of any type which has proved successful when built for a small or moderate output, or whether it becomes expedient to change the type when the output exceeds a certain limit. To find an answer to this question let us first investigate how the output increases with an increase of linear dimensions in a two-pole dynamo this being the type which is generally employed for machines of moderate size. Leaving aside for a moment the part played by the field magnets, the output of such a machine is limited by three conditions.

Limits of Output.

1. The efficiency of the armature—that is, the ratio of watts generated to watts available at the brushes.
2. The heating limit—that is, the ratio of watts wasted in the armature to total cooling surface, due account being taken of the provision for ingress and egress of air.
3. The sparking limit as given by the minimum induction in the air space under the leading polar edge.

In order to put the large machine mechanically into the same condition as the small machine, we assume that its armature has the same peripheral speed. The rotary speed must therefore be reduced in the same proportion as the linear dimensions are increased. It will be shown presently that the limits of output imposed by these three conditions are different, and we shall for this reason take them separately.

First, as to efficiency. If the large machine has q times the linear dimensions of the small machine, the resistance of the armature will be $\frac{1}{q}$ times that of the small machine, or a little less, because there is proportionately less space occupied by insulation and more by copper. Suppose we work both armatures at the same induction, then the useful field will be increased in the ratio of q^2, and the electromotive force in the ratio of q.

$$E_a' = E_a q,$$

while the armature resistance will be

$$R_a' = R_a \frac{1}{q},$$

or a little less.

The waste of power per cubic inch of iron in the armature core due to hysteresis will be reduced in the ratio of $\frac{1}{q}$, the frequency of reversal being reduced in this proportion, but as there are q^3 as many cubic inches, the hysteresis loss will be increased in the ratio of q^2. In addition to this loss, we have to consider the loss of power by eddy currents in the iron and copper. As regards eddy currents in the iron, the loss per cubic inch of core will be the same if we employ the same thickness of plates (generally 20 to 25 mils), and the total loss will therefore be increased in the ratio of q^3. The loss by eddy currents in the copper is more difficult to determine. If we simply increase the linear dimensions of the armature bars without further subdivision, it will increase much faster than q^3, but if we make up the bars of the large machine out of strips or wires of the same cross-sections as in the small machine, only using more of them, this loss will be proportional to q^3. It must, however, be noticed that the greater subdivision necessary to keep the eddy-current losses down implies the use of more insulating material, and thus prevents us from taking full advantage of the available winding space. The resistance of the armature will therefore not be much smaller than $\frac{R_a}{q}$. Let W_h be the hysteresis loss, and W_f the eddy-current loss in iron and copper in the small machine, then the corresponding values in the large machine will be $q^2 W_h$ and $q^3 W_f$. Let C′ be the current in the large armature, then the efficiencies of the two armatures will be equal if

$$\frac{W_h + W \quad + C^2 R_a}{E_a C} = \frac{q^2 W_h + q^3 W_f + C'^2 \frac{R_a}{q}}{E_a q C'} \quad . \quad (42)$$

If we know the losses in the small machine, we can from this formula calculate the current of the large machine. Suppose it were permissible to neglect Foucault and hysteresis losses, the $C' = q^2 C$ and the output of the large machine would be q^3 times that of the small machine. These losses are, however, by no means negligible. They vary according to the more or less perfect design of the machine, and it is therefore impossible to solve the above equation in a general way.

We may, however, do so by taking an example selected so as to represent fairly average conditions. Say that in a 20-kilowatt machine (200 amperes, 100 volts) we have ·5 per cent. hysteresis, 1 per cent. eddy-current, and 3½ per cent. resistance loss, and that we make a machine twice the size, the above equation becomes then—

$$\frac{100 + 200 + 700}{100 \times 200} = \frac{4 \times 100 + 8 \times 200 + C'^2 \times \cdot 00875}{100 \times 2 \times C'}$$

$$\frac{1{,}000}{100} = \frac{2{,}000 + C'^2 \times \cdot 00875}{C'},$$

which gives two values for C′—namely, 260 and 880 amperes. We need, of course, only consider the higher value, giving an output of 176 kilowatts, or slightly more than eight times that of the small machine. Had the eddy currents in the small machine absorbed 2 per cent. of the power, its efficiency would have been only 94 per cent. instead of 95 per cent., and the large machine would have the same efficiency if the current were 925 amperes, the output being twelve times that of the small machine. The same holds good with hysteresis loss. The greater this loss to begin with in the small machine, the lower its efficiency, and to get down to the same efficiency in the large machine we must pass more current, and thus obtain an output rather greater than q^3 times the output of the small machine. Let W be the output of the small and W′ that of the large machine, we have therefore by the condition of equal efficiency*

$$W' \geqq W q^3.$$

* For the sake of simplicity, and not to burden the text with too much algebra, I have above given the proof by means of an example. The general proof is, however, easily given as follows: We assume for the moment that $W' = q^3 W$, and inserting the corresponding value of C′ into (42), ascertain whether the right-hand term becomes thereby larger or smaller than the left-hand term.

$$W_h + W_f + C^2 R_a \gtreqless E_a C \frac{q^2 W_h + q^3 W_f + q^3 C^2 R_a}{E_a q^3 C},$$

$$W_h + W_f + C^2 R_a \gtreqless \frac{W_h}{q} + W_f + C^2 R_a$$

$$W_h \gtreqless \frac{W_h}{q}.$$

Since $q > 1$ it is obvious that only the upper sign is possible, and we find, therefore, that by inserting for C′, q^2 C (in 42) we make the right-hand term too small. Now to obtain equality—that is to say, to comply with the condition of equal efficiency of the two armatures—we must so alter

We have next to enquire how the output is limited by heating. The cooling surface varies as the square of the linear dimensions, and to get the same temperature rise (the peripheral speed and, therefore, the efficiency of ventilation being equal) the waste of power in the large armature may be q^2 times that in the small armature, or in symbols

$$(W_h + W_f + C^2 R_a)\, q^2 = q^2 W_h + q^3 W_f + C'^2 \frac{R_a}{q}$$

$$(W_f + C^2 R_a\ q^2 = q^3 W_f + C'^2 \frac{R_a}{q} \quad . \quad . \quad . \quad (43)$$

As regards limit of output by heating, hysteresis has no influence. This is quite natural, since both the hysteresis loss and the cooling surface are proportional to the square of the linear dimensions, but as regards Foucault losses this proportionality does no longer exist. The latter increase as the cube of the linear dimensions, and become therefore relatively of more importance as the dimensions are increased. If it were permissible to neglect them altogether, the current would be

$$C' = C\, q^{\frac{3}{2}}.$$

C′ as to make the right-hand term larger. This will be the case if we assume $C' > q^2 C$. Equation (42) may also be written thus :

$$\frac{W_h}{C} + \frac{W_f}{C} + C\, R_a = q \frac{W_h}{C'} + q^2 \frac{W_f}{C'} + C' \frac{R_a}{q^2}.$$

$q \frac{W_h}{C'}$ is obviously smaller than $q \frac{W_h}{q^2 C}$, and this is again smaller than $\frac{W_h}{C}$, so that the first term on the right is smaller than the first term on the left. Similarly, the second term on the right is obviously smaller than the second term on the left. If equality is to be preserved, the third term on the right must, therefore, be larger than the third term on the left—that is, C′ must be larger than q^2 C.

Let us now see whether the pressure of eddy currents raises or lowers the limit of output. Equation (43) may also be written thus :

$$W_f \ (q^2 - q^3) = C'^2 \frac{R_a}{q} - C^2 \ R_a \ q^2.$$

Since q is greater than unity, the left side of the equation is negative, and $C^2 \ R_a \ q^2$ must therefore be greater than $C'^2 \frac{R_a}{q}$, or

$$C' < C \ q^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (44)$$

We learn from this expression that if the eddy currents in the small machine are sufficiently great to be taken into account, and if we use in the large machine bars made up of the same size of wire or strip, the current given by the large machine will be rather less than $C \ q^{\frac{3}{2}}$—that is, less than would be the case if there were no eddy currents in either armature. In other words, the presence of eddy currents is more detrimental to the large than to the small machine as regards heating. To see this clearly, let us take the previous example of a 20-kilowatt machine, and let us double its linear dimensions. We had $W_{f} = 200$, $R_a = \cdot 0175$. If $q = 2$, $q^2 - q^3 = -4$, and

$$4 \times 200 = 200^2 \times \cdot 0175 \times 4 - C'^2 \ . \ \cdot 00875,$$

from which $C' = 477.$

If there were no eddy currents, C' would be 570. The output of the large machine is

$$W' = C' \, 2 \, E_a$$

$$W' < C \ q^{\frac{5}{2}}.$$

If there were no eddy currents, the output of the large machine would be 104 kilowatts, but owing to eddy currents the output obtainable with the same temperature rise as in the small machine is only 95·4 kilowatts.

The third condition which limits the output is the sparking point, as defined by the minimum induction under the leading pole edge. Assuming, again, the same induction per square centimetre of armature core, the exciting power required for the air space will in the large machine be q times the corresponding value in the small machine. The exciting power required for the armature path will also be q times that of the small machine; so that to get the same value for the resultant induction under the leading pole edge we must limit the load on the large armature to q times the current carried by the small armature. The output will therefore increase as q^2. This is the theoretical limit. In practice, it is generally possible to get a little more output by working the large machine at a higher induction, or allowing a lower induction in the air under the leading pole edge. The first expedient increases the electromotive force, the second the current. Since the limit of output imposed by heating is, as a rule, above that corresponding to q^2 (though, as was shown above, less than that corresponding to $q^{\frac{5}{2}}$), we may increase the induction per square centimetre of air and iron, and thus work at an electromotive force slightly larger than q E. This by itself increases the output beyond q^2 W. In addition it must be noted that the higher induction requires more exciting power, and permits, therefore, a larger number of cross ampere-turns without overstepping the sparking limit, so that both the electromotive force and the current may be increased. The result is that the output is rather larger than q^2 W.

$$W' > q^2 W.$$

This will best be seen by an example. Taking the design of the machine, Fig. 57c, as the small machine, and doubling all its dimensions, let us see what output we may expect from the enlarged machine. The armature of the small machine is 12in. diameter by 15in. long. At 550 revolutions per minute the output is 25 kilowatts, with a total armature loss of 1,200 watts. The large machine would have an armature 24in. diameter by 30in. long, and having four times the cooling surface we may work it with a total loss of 4,800 watts. It is not necessary to give the calculation in detail, but the result may be thus stated, the first figure referring to the small and the second to the large machine. Armature resistance, ·015 and ·006; exciting power, 23,000 and 60,000; weight of iron plates in armature, 300lb. and 2,600lb.; weight of iron in field, 2,600lb. and 21,500lb.; power required for excitation, 3½ per cent. and 1·95 per cent.; speed, 550 and 275 revolutions; output, 250 amperes at 100 volts and 700 amperes at 230 volts. The heating and sparking limits are the same in both machines.

If the relation $W' = q^2 W$ held good in practice, the output of the large machine would only be $4 \times 25 = 100$ kilowatts. In reality it is $\frac{700 \times 230}{1,000} = 161$ kilowatts, but this result has been obtained by slightly departing from the strict proportionality in the dimensions between the two machines, with the result that the large machine is rather heavy in comparison with its size. It is also heavy in comparison with its output. From the figures given above, it will be seen that the iron weight in the small machine is 2,900lb., and in the large machine 24,100lb.—a ratio of 1 : 8·3; whereas the ratio in output is 25 : 161, or 1 : 6·45. To put it in another way, the small machine weighs 116lb. per kilowatt, and the large machine weighs 150lb. per kilowatt. As regards weight and cost, the large machine is therefore not as good as the small machine, and this shows that the same type of machine is not equally suitable for all sizes.

It may be useful to recapitulate here the results obtained above as to the limits of output depending on the three conditions of efficiency, heating, and sparking:

Output of large machine for same efficiency as small machine	$W' \geqq W q^3$.
Output of large machine for same heating as small machine	$W' \leqq W q^{\frac{5}{2}}$.
Output of large machine for same sparking as small machine	$W' \geqq W q^2$.

The limit of output as determined by the condition of equal efficiency is, as a rule, not reached. We naturally expect the large machine to have a higher efficiency than the small machine, and since the limit of output refers to equal efficiency it cannot be reached in cases where we demand a higher efficiency. Moreover, the limit of output dependent on the efficiency condition is far higher than that due to the other two conditions, so that in any case we could not take full advantage of it without incurring the risk of heating and sparking. The output of the large machine will therefore lie between W q^2 and W $q^{\frac{5}{2}}$, whereas its weight will, of course, be q^3 times that of the small machine. It would appear from this investigation that the larger the machine the heavier and more expensive does it become relatively to its output, but it should be remembered that this conclusion only holds good under the circumstances to which our investigation was applied, and which are: (1) That the small machine is already working right up to the limit of output imposed by the condition of heating and sparking; (2) that precisely the same limits are observed in the large machine; and (3) that it is in every detail a faithful but enlarged copy of the small machine. In practice, however, the output of machines which may be called small (say, under 15 kilowatts) is generally not so much limited by heating or sparking as by the efficiency condition, and we may therefore venture to raise the sparking and heating limits in the large machine so that its output will be rather larger than given by the above expressions. The case is different if the small machine from which we start is already itself of a sufficient size to make its output dependent more on the sparking and heating limit than on the efficiency limit. In this case we have no margin to work upon, and a still larger machine, although designed precisely on the same lines as the successful machine of moderate size, may and probably will turn out to be an unsuccessful design. Thus there is no difficulty in producing very good designs of two-pole machines for 50 or even 100 kilowatts output, but if we attempt to apply the same designs to machines of 300 or 600 kilowatts output, we shall find that these larger machines are more than six times as heavy and expensive than their smaller prototypes. We conclude from this that the two-pole type is not suitable for large machines.

Advantage of Multipolar Dynamos.

Large machines must, however, be made, and the question now arises, how should they be made in order to be at least not worse, and, if possible, better, than small and moderate size two-pole machines in point of weight and cost? Practical experience has answered this question in favour of multipolar machines. Whereas for small and moderate size machines the bi-polar type is undoubtedly the best, there is a limit of output beyond which a four-pole machine is preferable. If we still further increase the output, we find that a point is eventually reached where a six-pole machine is better than a four-pole, and so on, the number of poles increasing with the output. The precise points at which a change from two to four or from four to six poles, and so on, becomes expedient, depends on a variety of circumstances, and no hard-and-fast rule can be given, but that the value of a design depends on the appropriate choice of type the reader can easily find out for himself by making comparative designs for various sizes of machine. Without, however, going so far as to prepare a whole series of designs, we may show the effect of increasing the number of poles by an example, and for this purpose we take the 25-kilowatt machine (250 amperes, 100 volts, 550 revolutions) design, Fig. 57c, particulars of which were given in the table on page 66. The weight of the iron plates in the armature is 300lb., that of the iron in the field is 2,600lb. The diameter of the armature is 12in., and its length 15in. Let us now make a four-pole machine having an armature of double the diameter, but the same radial depth of iron (in this case $3\frac{3}{4}$in.) and the same length. Assuming that we employ the type of field shown in Fig. 58, page 228, its weight will be very nearly double that of Fig. 57c, provided we work with the same total induction. The number of turns on the armature and its resistance will be doubled, and if we work at half the speed, the electromotive force will be doubled, but the current will remain the same. The four-pole machine, with an armature of 24in. by 15in., and running at 275 revolutions, will have the same efficiency and the same sparking limit as the two-pole machine with an armature of 12in. by 15in., and running at 550 revolutions. The output will, however, be doubled. Let us now see by how much we would have to increase the linear dimensions of the two-pole machine to get double the output. Without going into complicated calculation, we may assume that $W' = W\,q^{2 \cdot 25}$, since we know that q^2 will give rather too little and $q^{2 \cdot 5}$ rather too much for the output of the large machine. In order that the output may be doubled, we have $q^{2 \cdot 25} = 2$, from which $q = 1 \cdot 36$. The 50-kilowatt machine must therefore have an armature $16\frac{1}{4}$in. in diameter by $20\frac{1}{2}$in. long, and must run at a speed of $550/1{\cdot}36 = 405$ revolutions per minute. The weight will be increased in the ratio of $1 : 1{\cdot}36^3 = 1 : 2{\cdot}5$; but the efficiency will be slightly better. The weight of iron in the four-pole machine is: Armature, 710lb.; field, 5,190lb.; total, 5,900lb.; or 2·05 times that of the small two-pole machine. The following table shows these various quantities, all being referred to the small two-pole machine:

	Small Machine.	Large Machine.	
	Two-pole.	Two-pole.	Four-pole.
Output	25 kilowatts	double	double
Heating	—	the same	slightly less
Sparking	—	the same	the same
Efficiency	—	slightly more	the same
Speed	550	405	275
Weight	—	2·5 times	2·05 times

It will be seen at a glance from this table that the four-pole machine is not only lighter than the two-pole machine, but runs at a considerably lower speed. It will heat a little less in the armature owing to the more open construction, but its efficiency will be slightly less. Apart from this defect, which can, however, easily be avoided by a slight increase of speed, the four-pole machine is therefore decidedly the better design of the two.

CHAPTER XIII.

Loss of Power in Dynamos—Eddy Currents in Pole-Pieces—Eddy Currents in External Conductors—Eddy Currents in the Armature Core—Eddy Currents in the Interior of Ring Armatures—Experimental Determination of Losses.

Loss of Power in Dynamos.

There are various causes which produce loss of power in a dynamo machine; some of these are purely mechanical, and others are electrical or magnetic. In the field-magnet system of a dynamo the loss of power is entirely electrical—namely, that represented by the product of exciting current and difference of potential between the terminals of the exciting coil. The determination of this loss is so simple and elementary a matter that it need not further occupy us, but in calculating the loss due account must be taken of the increase of resistance with rise of temperature.

In the armature the losses are of a more complicated nature, and not so easily determined. We have first the loss by friction in the bearings and that between brushes and commutator. The latter is very small, and may be neglected. The former can be easily calculated, the required formulas being found in every mechanical text-book. Care must, however, be taken to take into account not only the weight of the armature, but also the pull of the belt, if any, and magnetic attraction, if this should not be perfectly balanced. Examples showing the influence of unbalanced magnetic attraction have been given in Chapter III. Then there is a small loss of power due to air resistance. The armature being a rapidly-revolving body acts in a certain degree as a fan, and in properly-designed machines this fan action is made use of for ventilating and cooling, but a small amount of power is, of course, required to drive the air through the armature. All these losses are, however, small in comparison with the electric and magnetic losses.

These are occasioned by hystersis, eddy currents, and armature resistance, including the resistance of brush contacts. As regards loss of power by armature resistance, this can be found by Ohm's law, and need not occupy our attention further, but the hysteresis and eddy-current losses cannot be so easily determined. If the magnetisation curve of the particular quality of iron used for armature plates be known, the hysteresis loss may be calculated approximately. We know the average induction and the number of ergs lost per cubic centimetre per cycle at that induction, and from the speed and number of poles we can calculate the frequency. The total volume of iron is, of course, also known, and we have therefore all the elements required to calculate the total hysteresis loss, but, as was already said, the result is only an approximation. The reason is this: we know the average induction, but we cannot be

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certain that this is the true induction in every part of the cross-section of the armature core. On the contrary, it is highly probable that the induction will be greater at some places than at others, partly because the length of path varies, causing a greater crowding of lines into the short paths and a thinning out in the long paths, and partly because the armature current itself must disturb the even flow of lines. By reference to the hysteresis curves (Fig. 30, Chapter VI.) it will be seen that the hysteresis loss increases faster than the induction, and, consequently, the increase of hysteresis loss in those parts where the induction is above the average value, will outweigh the reduction of hysteresis loss in those parts where the induction is below the average value, with the result that the total loss will be greater than would be the case were the induction uniform.

This is borne out by experiment. It will presently be shown how the eddy-current losses can be determined by a simple experiment. Deducting these from the total loss we obtain the losses due to friction, air resistance, and hysteresis. Deducting again the frictional loss, which can be calculated with fair accuracy, and neglecting air-resistance losses (technically termed windage), which is permissible, we obtain the hysteresis loss alone. It is invariably found that this is greater than corresponds to Ewing's curves, Fig. 30, the reason of the increase being that stated above.

The Foucault losses are of a very complicated character. There are eddy currents in the iron of the core discs, there may be eddy currents in the pole-pieces, there are eddy currents in the active conductors, and there may be eddy currents in the shaft, metal support of the armature core, and the non-active parts of the conductors, such as end connections or internal conductors in a ring-wound armature. To determine these losses by calculation based on theory is, of course, quite hopeless. We know in a general way how the eddy currents in each part of the machine are caused, and we can approximately guess their direction. But this does not help us to determine the power wasted. All it helps us to do is to so design the machine that this waste shall be as small as possible; and having done this we must fall back on experiment if we wish to know the exact amount of power wasted in eddy currents.

Before describing the kind of experiment required for this purpose it will be useful to enquire into the general cause of eddy currents, so that we may be able to form a mental picture of what is going on in those parts of the machine which are likely to become the seat of eddy currents. Take, for instance, the case of a Brush arc light machine. The most cursory examination reveals at once the fact that after the machine has been at work for some time the trailing polar horns are much hotter than the leading polar horns. It might, perhaps, be thought that this difference is due to hot air being thrown forward by the armature, but such an explanation is negatived by the fact that if the Brush machine be worked as a motor, the leading polar horns become hotter than the trailing horns. It is clear that the heat in the pole-pieces cannot be heat communicated from the armature by air currents, but must be heat generated in each pole-piece itself, or, in other words, must be the effect of eddy currents. This view is confirmed by the difference in temperature between the leading and trailing horn. Eddy currents being due to change of induction, it is clear that the higher the induction and the greater the change, the greater will be the heating, and this is precisely what we find in the polar horns of the Brush machine. Owing to the armature reaction, which in this type of machine is extremely great, the induction is piled up towards the trailing horn (compare Fig. 76), but is not kept steadily at its maximum value, because of the gaps in the core of the armature. There are thus violent fluctuations which cause eddy currents. Such fluctuations take, of course, also place in the leading horns, but to a smaller extent; hence the heating is less.

Eddy Currents in Pole-Pieces.

The way in which eddy currents are generated in pole-pieces will be seen by reference to Fig. 81, in which A B C D is the surface of a pole-piece straightened out, and the shaded narrow rectangles represent teeth on the armature core. On account of these teeth coming very near to the surface of the pole-piece, the induction is mainly concentrated within the shaded spaces, and as the armature revolves (movement from right to left in the figure), the groups of bunched lines sweep along the polar surface, and induce under each tooth an electromotive force in the metal of the pole-piece parallel to the direction of the spindle. The electromotive force generated in the intermediate spaces is much smaller, and the result is that currents flow in the direction of the arrows. It is obvious that the greater the area covered by the teeth and the wider these are apart, the greater will be the difference of electromotive force and the more room will there be for eddy currents. The latter reacting upon the teeth produce a resisting torque, absorbing power, and this power reappears in the shape of heat generated in the teeth and pole-pieces. In

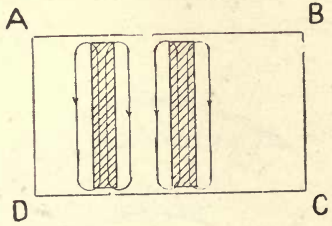

FIG. 81.

order to minimise the loss from this cause, we must, therefore, restrict the area through which eddy currents can flow by providing the armature core with narrow teeth set close together. This construction has also the advantage that it tends to reduce the difference of induction under and between the teeth, thus reducing the difference of electromotive force which causes eddy currents. The modern practice in toothed armatures is, therefore, to make the slots narrow and deep. Other methods to reduce eddy currents are to close the top of the slot completely—in other words, lay the wire into holes instead of grooves, or to make the slots just wide enough at the top to permit winding, and wider below, so that a number of wires may be laid side by side, or to employ parallel slots, and close them after winding by a serving of iron wire. These constructions are shown in Fig. 82.

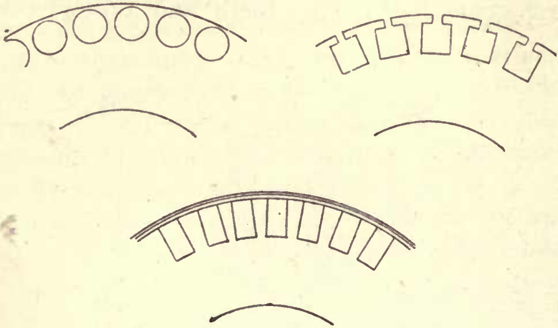

Fig. 82.

Eddy Currents in External Conductors.

If a solid conductor be moved parallel to itself in a perfectly uniform field, no eddy currents whatever are produced, because the electromotive force in every part of the conductor is the same. If, however, the field is not uniform, then some parts of the conductor at any moment may be in a stronger field than other parts, and there will be a difference of electromotive force between them, causing eddy currents. Fig. 83 will make this matter clear. A represents a section through the armature core, P a section through the pole, and *a b* the cross-section of armature conductors, supposed to be solid bars. It will be easily seen that in the position shown in the figure, the part *a* of the left conductor is in a strong field, and the part *b* in a weak field, being merely the fringe projecting out from the polar edge. The electromotive force in *a* will therefore be greater than in *b*, and the result will be a current running up in the right-hand part of the conductor, and down in the left-hand part. When the conductor has passed the edge of the pole-piece (as

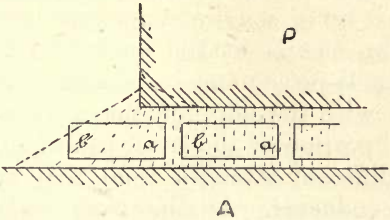

FIG. 83.

shown by the rectangle *a b* on the right), and provided the field under the pole-piece is uniform, there will be no eddy currents, because the induction in *a* will then be the same as that in *b*. The field can, however, only be uniform if the machine is working on open circuit. The induction for the whole length of the pole-piece is then represented by the horizontal line P_1 P_2, Fig. 76, page 284, and eddy currents occur only under the leading and trailing polar edges, but not under the pole-pieces themselves. If the machine works on a closed circuit, the line representing induction is distorted, as shown by D_1 D_2, and there are eddy currents, not only in those conductors which at any given time are under the polar edges, but in all the intermediate conductors as well. Moreover, the eddy currents at the trailing edge are increased by reason of the larger induction. We conclude from this that the loss of power by eddy currents in a machine working on closed circuit will be greater than in the same machine working on open circuit.

In order to reduce this loss of power various means may be adopted. The most obvious remedy is, of course, to laminate the conductor. This may be done by building it up of narrow strips, insulated from each other, but in contact at the ends. In this case the edge of the pole-piece must be of such a shape as not to coincide with the edge of the conductor, so that only a small portion of the latter can at any time be in a field of different strength from that acting on the rest of the conductor. Another way of laminating is by building the conductor up of cable with insulated strands pressed into the desired shape. We may also chamfer the polar edges, as shown by the dotted line in Fig. 83, to make the transition from the neutral space to the strong field more gradual; or we may place the conductors in grooves, which cause the lines of induction to snap across so suddenly as to give no time for the generation of eddy currents. Other things being equal, or proportionate, it will be clear that the electromotive force causing eddy currents is directly proportional to the average induction; the eddy currents themselves are proportional to the electromotive force generating them, and the waste of power is therefore proportional to the square of the induction.

Eddy Currents in the Armature Core.

The eddy currents in the armature core itself follow very much the same law as those in the external conductors. When the core is turned up in a lathe there is danger of the tool burring over the edges of the plates, and bringing them into contact, notwithstanding the paper insulation; and if this happens, the armature core becomes coated with a thin film of more or less continuous metal in which eddy currents can circulate. In addition to these there are also eddy currents in the body of each plate, but with thin plates these are exceedingly small. With careful workmanship it is also possible to almost completely avoid contact between the external edges of the plates, so that, as a rule, the loss of power by eddy currents in the armature core may be reduced to a negligible quantity.

Eddy Currents in the Interior of Ring Armatures.

In addition to the losses detailed above, there is in ring armatures another loss caused by eddy currents in the internal conductors and metal parts within the armature core. If the sectional area of the core be sufficient, there is, of course, no internal field in a machine working on open circuit. But as soon as a current flows an internal field is produced (see Fig. 70), and since the lines of this field are stationary in space they must be cut by the internal conductors, the shaft, hub, and supporting arms of the armature core, as will be seen by Fig. 84. The internal field due to the armature current is shown by dotted lines, and a few of the internal conductors, C, are also shown. If the hub, with the arms, were made of iron, the internal field would become much stronger and the losses greater. To minimise this loss, the best modern ring machines are made with a hub and arms of gun-

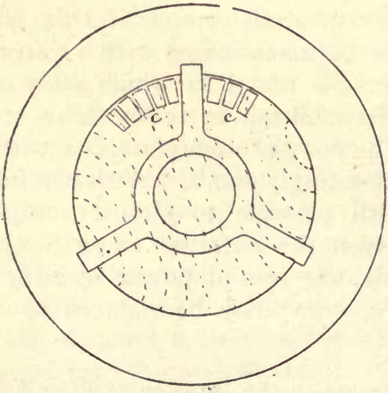

FIG. 84.

metal. This is, however, only a palliative. The loss cannot be entirely avoided, and it is important to note that it increases with the output. This kind of loss does, of course, not occur in a drum armature, since there are no internal conductors, and it follows that, other things being equal, the drum machine must have a slightly higher efficiency than the ring machine.

Experimental Determination of Losses.

The determination of the total losses in a dynamo, when working on open circuit, can be made very accurately by running the machine as a motor and observing the power supplied. Care must, of course, be taken to so adjust the supply of power that the machine runs at its normal speed and with the normal brush voltage. If this be the case, the strength of field and the field excitation will be approximately the same as when the machine works as a dynamo. They cannot, of course, be exactly the same because of armature reaction and resistance, but as the effect of these disturbing influences is approximately known, it is easy to so alter the excitation as to fairly represent the actual working conditions. The measurement of electric power supplied to the armature can be made with very great accuracy, and the power supplied for field excitation is also easily found. All we require for the experiment is a speed counter, a voltmeter, and an ampere-meter. The result of such an experiment is, however, not sufficient for practical work. It is no doubt of some value to know exactly how much power is wasted in the field and how much is wasted in the armature, but as regards the latter we want something more. In addition to knowing the total waste, we want to know how this total is made up. We wish, in other words, to separate the total loss into its component parts, so that we may see in what direction improvements may be possible, or what the effect of any alteration in design has been. Take, as an example, the question of how far the conductors should be stranded or otherwise subdivided. The greater the amount of subdivision, the more space is wasted in insulation and the more expensive becomes the machine. On the other hand, the more we subdivide, the smaller becomes the eddy-current loss. The design of machine actually adopted is therefore a compromise between that which is theoretically perfect and that which is commercially feasible, and in order that the designer may be able to strike the balance between these conflicting conditions properly, he must know up to what point subdivision of conductors is of importance. This point he can only determine if he is able to measure the loss occasioned by imperfect subdivision in any type of conductor; in other words, if he can separate eddy-current losses from the total loss.

For the same strength of field the hysteresis loss is obviously proportional to the speed, and the same holds good for the frictional losses, provided the speed be not reduced too much. The eddy-current losses being proportional to the square of the electromotive forces which produce them, must, for the same strength of field, be proportional to the square of the speed. Taking advantage of the fact that the two kinds of losses follow different laws, we can separate them as follows: We excite the machine under test from an independent source, and keep the excitation constant. We also send a current through the armature, whereby the latter is set revolving, and we vary the voltage applied so as to get a variation in speed. The current required to run an armature light is so small that we may neglect armature reaction and resistance, and consider the measured brush voltage to be equal to the armature electromotive force. We now take three readings—namely, speed (n), current (c), and voltage (e). If we increase the voltage, we increase all the readings; and by suitable arrangements for the purpose we can very rapidly take a large

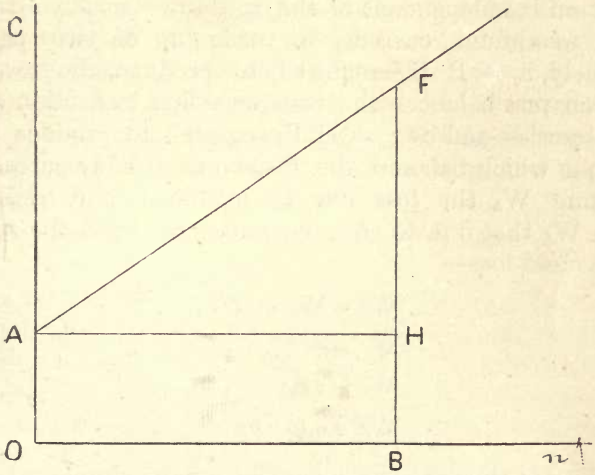

FIG. 85.

number of readings. If we plot the current as a function of the speed, we obtain a sensibly straight line, Fig. 85, and the point A, where this line cuts the vertical, corresponds to the current at which the armature would just start, provided the frictional resistance were not increased at a very slow speed or at rest. Since the coefficient of friction does, however, increase, it would be incorrect to determine the point A by measuring the current at starting, but we can find it by measuring the current for a moderate speed and projecting the line backwards. The length O A = c_0 represents, then, the initial current at no speed, and the length B F = c represents the maximum current at the normal speed, n = O B. Since the resistance due to hysteresis and friction is independent of the speed we may consider the maximum current, c, made up of two parts, namely, c_0 = B H—required to produce the torque which just balances the resistance due to friction and hysteresis—and $c - c_0$ = H F, required to produce the torque which balances the resistance of eddy currents. Calling W_h the loss due to hysteresis and friction, and W_f that due to eddy currents, we have the total measured loss—

$$W = W_h + W_f;$$

$$W = e\,c;$$

$$W_h = e\,c_0;$$

$$W_f = e\,(c - c_0).$$

We are thus able by means of a very simple experiment to determine the eddy-current loss, but it should be remembered that this determination is only valid for the machine working on open circuit. When the machine is working on closed circuit, the eddy-current loss is increased for the reasons above stated.

It is, however, possible to adapt the method here described for the measurement of eddy-current losses under full load. We require for this purpose two machines of equal size and type, and a third machine of smaller power, but giving the same current. The two machines to be tested are rigidly coupled, and their armatures are placed in series with each other and with the small machine. The fields are so arranged that one machine is working as a generator and the other as a motor, the power to keep the combination at work being supplied by the small machine. By suitably adjusting the field excitation of the two machines and the electromotive force of the small machine, we can keep the current fairly constant over large variations of speed, and thus obtain a series of readings which enable us to separately determine the various losses.

CHAPTER XIV.

Examples of Dynamos—Ronald Scott's Dynamo—Johnson and Phillips's Dynamo—Oerlikon Dynamo—Other Dynamos.

Examples of Dynamos.

To give an even approximately complete collection of drawings and descriptions of the various types of dynamos now in the market would extend this book far beyond its proper limits. As it will, however, be useful to show at least in a few cases how the general principles of dynamo design set forth in the preceding chapters are carried out in practice, I give in Figs. 86, 87, and 88, illustrations of dynamos belonging to three distinct and representative types. Fig. 86 shows a bi-polar undertype machine suitable for belt or direct driving. Fig. 87 shows an overtype machine for belt-driving, and Fig. 88 a multipolar machine for large output and slow speed, arranged for direct driving by a turbine. The descriptions of these machines are here given, each under the name of its maker, to whom I am indebted for the particulars of construction.

Ronald Scott's Dynamo.

This machine, Fig. 86, is designed for an output of 200 amperes at 80 volts when driven at 600 revolutions

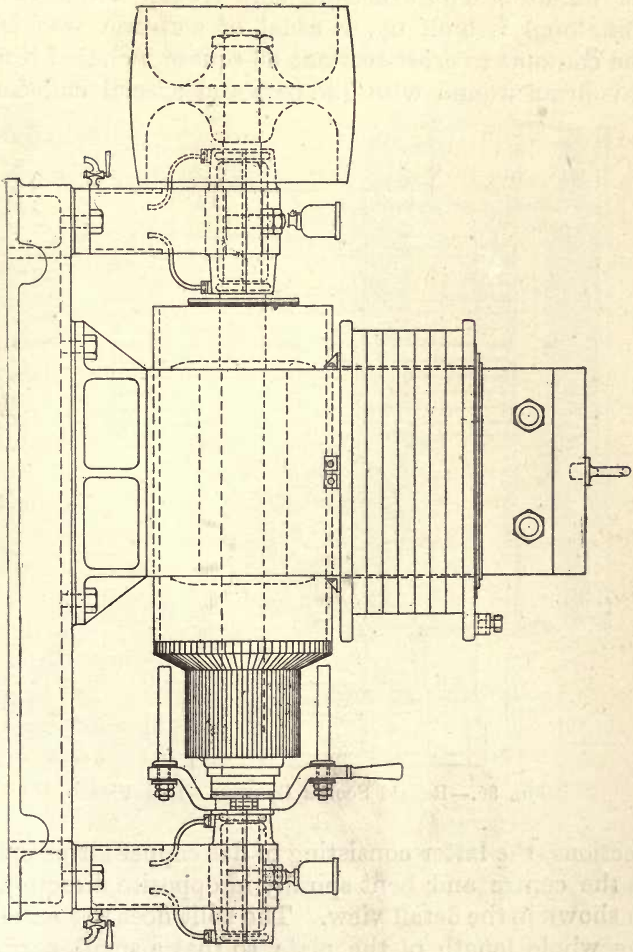

Fig. 86.—Ronald Scott's Dynamo.—Side View.

per minute. The armature core ($10\frac{1}{2}$in. diameter by 13in. long) is built up, as usual, of soft-iron washers, and contains in cross-sections 66 square inches of iron. It is drum wound with 136 bars and special end con-

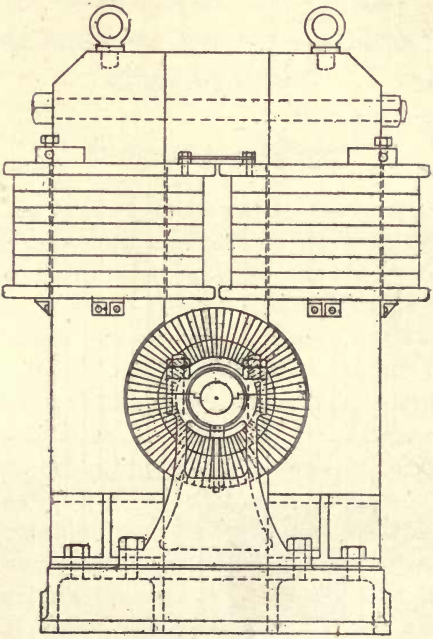

Fig. 86.—Ronald Scott's Dynamo.—End View.

nections, the latter consisting of flat copper strips split in the centre and bent spirally in opposite directions, as shown in the detail view. The split does not extend the whole length of the plate, so that a small part of the plate at one end is left the full width. This part

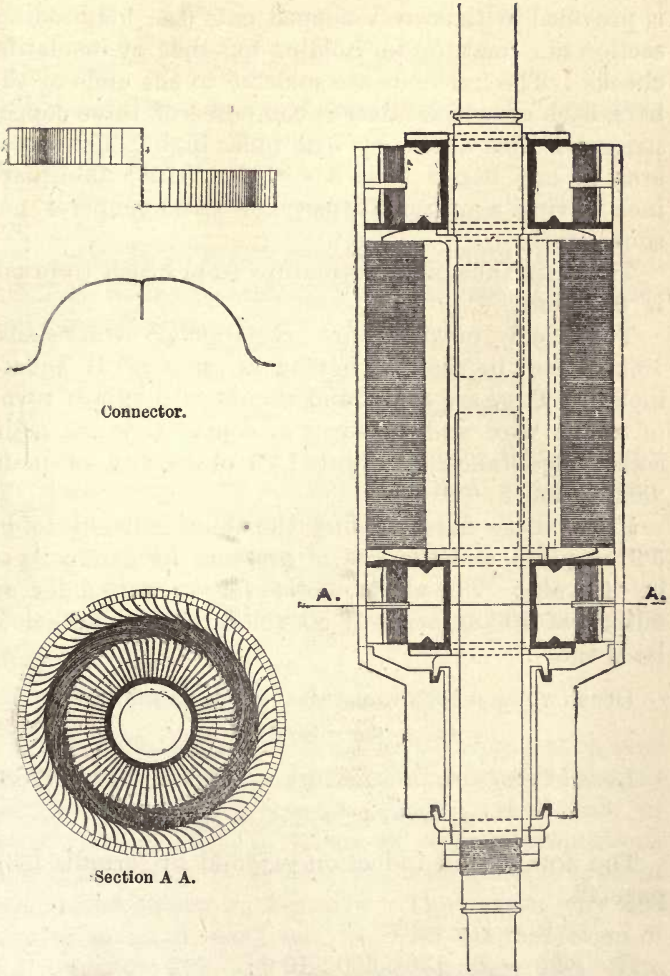

FIG. 86.—Details of Ronald Scott's Dynamo.

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is provided with two V-shaped cuts (see longitudinal section of armature) for holding together by insulating cheeks. The free ends are soldered to the ends of the bars, each of which latter is composed of three copper strips 64 mils thick by 312 mils high. The total area of one bar is thus $3 \times {\cdot}064 \times {\cdot}312 = {\cdot}06$ square inch, giving a current density of 1,720 amperes per square inch.

The resistance of the armature from brush to brush is ·014 ohm.

The field magnets are rectangular iron slabs, 13in. × 7in. in section, having an area of 91 square inches. They are compound wound with 2,040 turns of shunt wire and 14 turns of copper tape for main coils. Resistance of shunt 13·5 ohms, and of main ·00213 ohm.

From these data we find the shunt current to be 5·9 amperes, and the loss of pressure in main coils to be ·42 volts. The electromotive force required for an output of 200 amperes at 80 volts can now be calculated thus :

$$\text{Brush volts} = \text{terminal volts} + \text{loss in main coils.}$$
$$= 80 + {\cdot}42 = 80{\cdot}42.$$

$$\text{Loss of pressure in armature} = 205{\cdot}9 \times {\cdot}014 = 2{\cdot}88$$
$$E = 80{\cdot}42 + 2{\cdot}88 = 83{\cdot}3.$$

The total useful induction is found by formula (33), page 42.

$$E = Z \tau n 10^{-6};$$
$$83{\cdot}3 = Z \,.\, 136 \,.\, 600 \,.\, 10^{-6};$$
$$83{\cdot}3 = Z \,{\cdot}0816;$$

$$Z = 1{,}022 \text{ in English measure ; or}$$
$$F = 6{,}132{,}000 \text{ in C.G.S. measure.}$$

The average density of induction in the armature core is $B_a = \frac{1{,}022}{66}$;

$$B_a = 15{\cdot}5 \text{ in English measure ; or}$$
$$\mathfrak{B}_a = 14{,}500 \text{ in C.G.S. measure ;}$$

and if we assume a leakage of 33 per cent., the induction through magnets is

$$B_m = 14{\cdot}7 \text{ in English measure, or}$$
$$\mathfrak{B}_m = 13{,}700 \text{ in C.G.S. measure.}$$

The total exciting power is made up of that given by the shunt coils ($5{\cdot}9 \times 2{,}400$) and that given by the main coils (200×14), or in all

$$X = 16{,}960.$$

The electrical efficiency of this machine is given by the maker as 92·5 per cent.

Johnson and Phillips's Dynamo.

The machine illustrated in Fig. 87 represents a type which is with English makers a favourite one for small and medium-sized dynamos. Each maker has, of course, his own special design as regards proportions and construction of details, but the general type of field is that shown in Fig. 57c. The reason why this type is so much used will be clear on inspection of Fig. 57, page 224, and the table on page 237. The construction is mechanically strong and simple, the

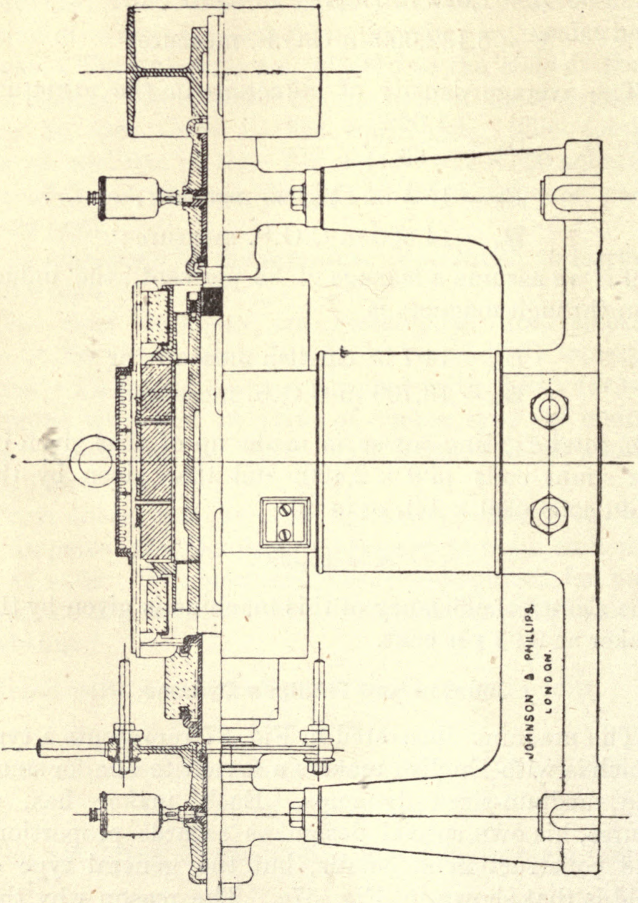

Fig. 87.—Johnson and Phillips's Dynamo.

amount of exciting copper is very moderate, and the commutator and bearings are at a convenient height.

The fact that the bed-plate of the machine may be utilised as yoke, and that no gunmetal supports are required for the field, is an additional advantage, because tending to reduce cost.

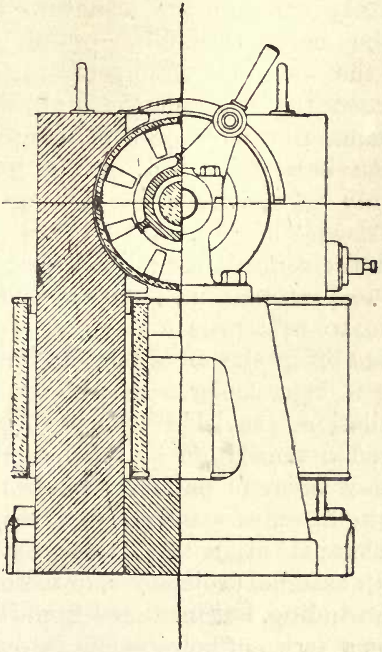

FIG. 87.—Johnson and Phillips's Dynamo.

The particular machine illustrated is designed for an output of 15 kilowatts when driven at 870 revolutions per minute. Terminal pressure 140 volts. The armature core is made up in the usual way of soft iron washers, insulated from each other, and supported on the three wings of a central hub. The washers are pressed together by end checks, also provided with wings and hub, so that the spaces between the wings remain open from end to end, and air can pass freely through the armature between the hub and the core, the flow of air being promoted by the fanning action of the connecting strips between the commutator and the armature bars. In armatures which are completely closed at the ends the heat generated in the iron core can obviously only be dissipated by passing through the external surface—that is, through the copper conductors and their insulation; but if proper provision is made for end ventilation, a considerable portion of the heat due to hysteresis and eddy currents in the core is carried off by the air direct, and to that extent the winding is kept cooler. In addition to this end-to-end ventilation, provision is made for a moderate amount of radial ventilation by the insertion between the thin iron washers of pairs of stouter iron washers, kept a definite distance apart by fibre distance-pieces, so that air channels are left. These stout washers are provided with external projections, or teeth, penetrating through the winding, but insulated from it by fibre and mica. Thus a series of holes is left, through which air can escape radially, and this also helps to some slight extent to keep the armature cool. The chief reason for the employment of these stout washers with projecting teeth is, however, a mechanical one—namely, to transmit the driving power from the spindle to the external conductors in a positive and reliable manner. It has been shown, in Chapter IV., p. 81, how the force can be calculated that is required to move a conductor carrying a given current through a magnetic field of given intensity. It was there shown that with a current of 100 amperes flowing through the wire, each foot of wire is subjected, in a field of 5,000 C.G.S. units, to a drag of about 3½lb. In a stronger field the force would be greater, and in a weaker field smaller, but, for a rough calculation, we may take a force of 3½lb. per 100 ampere-feet, or ·035lb. per ampere-foot of conductor as a fair average value. Thus in a two-pole machine, with an armature 12in. long, each of the conductors under the pole-pieces (or about 75 per cent. of all the conductors) would be subjected to a drag of 3½lb. when the total armature current is 200 amperes, or 1¾lb. when the total armature current is 100 amperes. This, taken for one conductor alone, is not a very large force; but if it be remembered that the number of conductors is counted by hundreds, it will be seen that the aggregate effect is one of considerable magnitude.

It is also necessary to bear in mind that a dynamo machine may at some time or other be subjected to rough usage, such as an accidental short-circuit, when the current, and consequently the mechanical strain on the conductors, will be much greater than during regular work, and for this reason it is very important to make ample provision for positive driving.

In the machine illustrated there are two sets of stout washers, or driving discs, each having four driving horns. The total force required for pushing the conductors through the field is therefore divided between eight driving horns. What is the force which each horn must exert in regular work? Assuming for the purpose of this calculation, which need obviously only be an approximation, that the machine has an efficiency of 85 per cent., we find that the total power put into the spindle at 870 revolutions per minute will be 15/0·85 = 17·6 kilowatts, or about 23·6 h.p. A small portion of this power is absorbed in losses occurring in the bearings and armature core, and does, therefore, not reach the armature conductors, but it would be pushing scientific accuracy beyond the limit of practical work, if we were to make a deduction on that account, especially as there may be initial stresses in the driving horns, due to imperfect workmanship in laying the conductors on, which are quite beyond the reach of calculation. We have, therefore, to do 23·6 h.p. by means of eight horns, or very nearly 3 h.p. per horn. The core is 10in. diameter by 12in. long, the discs being 2in. wide. The speed at which the driving energy is transmitted is therefore $\frac{31{\cdot}4}{12} \times 870 = 2{,}270$ft. per minute, and the force is $P = \frac{33{,}000 \times 3}{2{,}270}$, or in round figures P = 43lb.

The net cross-sectional area in the armature core is $A_a = 39{\cdot}5$ square inches, and there are 216 armature bars, the end connections being in the shape of semi-circular copper plates, with tags at each end. These plates are separately insulated and placed spirally side by side into the insulated channel of a cast-iron carrier. As will be seen from the longitudinal section of the machine, the tags are bent at right angles to the surface of the plates, and thus form at each end of the carrier a row of connecting-pieces, to which the ends of the corresponding bars are soldered.

The resistance of the armature, warm, is ·0507 ohm, and that of the shunt coils (1,452 turns on each limb) is 26·05 ohms. The magnets are formed by wrought-iron slabs $11\frac{1}{4}$in. wide by $5\frac{1}{2}$in. thick, and their area is

$$A_m = 62 \text{ square inches.}$$

At 140 volts the armature current is 107 amperes for the external circuit, and 140/26·05 = 5·37 amperes for excitation, or a total of 112·37 amperes, causing a loss of 5·7 volts in the armature. The total useful field is therefore found from the formula—

$$145{\cdot}7 = Z \times 216 \times 870 \times 10^{-6},$$

$$Z = 773.$$

The electrical efficiency of the machine is the ratio of the output to the electrical energy generated in the armature conductors, or

$$\eta = \frac{140 \times 107}{145{\cdot}7 \times 112{\cdot}37};$$

$$\eta = \frac{14{,}980}{16{,}372};$$

$$\eta = 91{\cdot}5 \text{ per cent.}$$

In the two examples of dynamos here quoted, the electrical efficiency has been given to show the method of calculating it, but from a practical point of view it is not the electrical, but the mechanical efficiency (sometimes also called the commercial efficiency) which is of importance, and it will, therefore, be useful to cite the example of a dynamo, also made by Messrs. Johnson and Phillips, for which the mechanical efficiency has been determined by the method described in the last chapter. The machine is of the same type as shown in Fig. 87, but larger. Armature core, 14in. diameter by 19in. long. Radial depth of core, 3in. Output, 42 kilowatts (600 amperes by 70 volts) at 470 revolutions per minute. Field, compound wound for constant terminal pressure. Shunt exciting power, 20,000 ampere-turns; main exciting power, 10,000 ampere-turns. Loss of pressure over main coils, 1 volt; loss of current in shunt coils, 13·62 amperes. The armature contains 84 subdivided bars, and the cross-sections of connectors exceeds that of the bars by 70 per cent. Total armature resistance from brush to brush, ·0036 ohm when warm. From these data we find

Loss of energy in shunt coils	970 watts.
Loss of energy in main coils	600 watts.
Loss of energy due to armature resistance	1,358 watts.
Total	2,928 watts.

The electrical efficiency of this machine is therefore

$$\frac{42,000}{42,000+2,928} = 93 \cdot 438 \text{ per cent.}$$

The mechanical efficiency is, of course, lower, because, in addition to the 2,928 watts absorbed by resistance in the field and armature, we must provide sufficient energy to cover the losses due to magnetic and mechanical friction and eddy currents. These losses were determined in the manner detailed in the last chapter.

The field of the machine was separately excited, and a current sent through the armature so as to run the machine light as a motor. By plotting the current as a function of the speed we obtain a straight line, which cuts the axis of ordinates at the point corresponding to 9·2 amperes. This, therefore, is the current required at that particular excitation for overcoming all frictional resistances. When the pressure was raised to 73 volts the speed was 464 revolutions per minute, and the current was 17 amperes. We have therefore

Total losses	$W = 17 \times 73 =$	1,241 watts.
Frictional losses	$W_h = 9{\cdot}2 \times 73 =$	671·6 watts.
Eddy-current losses	$W_f = 7{\cdot}8 \times 73 =$	569·4 watts.

These losses refer, of course, only to the speed of 464 revolutions per minute, and for a different speed other values would be found. It is, however, not necessary to repeat the experiment for different speeds, since the law of these losses is known. It was shown in the last chapter that the frictional losses vary as the speed and the eddy-current losses as the square of the speed. It is, therefore, possible to represent each group of losses by a simple expression—thus :

$$W_h = h . n,$$
$$W = f n^2,$$

where n is the speed in revolutions per minute, and h and f are coefficients for frictional and eddy-current losses respectively. To avoid large numbers it is convenient to insert, not the speed, but the speed divided by 100—thus :

$$W_h = h \frac{n}{100}$$

$$W_f = f \left(\frac{n}{100}\right)^2$$

We can now determine the coefficients h and f from the observed values for W_h and W_f, and find

$$h = 144{\cdot}2 \qquad f = 26{\cdot}5.$$

The losses at 470 revolutions per minute are then found to be

$$W_h = 144{\cdot}2 \;.\; 4{\cdot}70 = 680,$$

$$W_f = 26{\cdot}5 \;.\; (4{\cdot}70)^2 = 583\,;$$

or a total of 1,263 watts for the machine running light at 470 revolutions per minute. If we allow an increase of 30 per cent. in the eddy-current losses when running at full load, we find that 1,439 watts are wasted over and above the 2,928 watts absorbed by resistance in the field and armature, bringing the total loss up to 4,367 watts. The mechanical efficiency of the machine when working at full load is therefore

$$\eta = \frac{42,000}{46,367},$$

$$\eta = 90\tfrac{1}{2} \text{ per cent.}$$

Oerlikon Dynamo.

The machine shown in Fig. 88 is a very interesting illustration of the best modern practice in large slow-speed machines for high voltage. It is the generator for a power transmission plant at Innsbruck, designed and built by the Oerlikon Engineering Works, Switzerland. The machine is of the 10-pole type, with vertical spindle, arranged for direct coupling to the shaft of the turbine, and its output is 240 kilowatts at 1,550 volts, the speed being 230 revolutions per minute. The drawing is to one-twenty-fourth of full size, or $\frac{1}{2}$in. to the

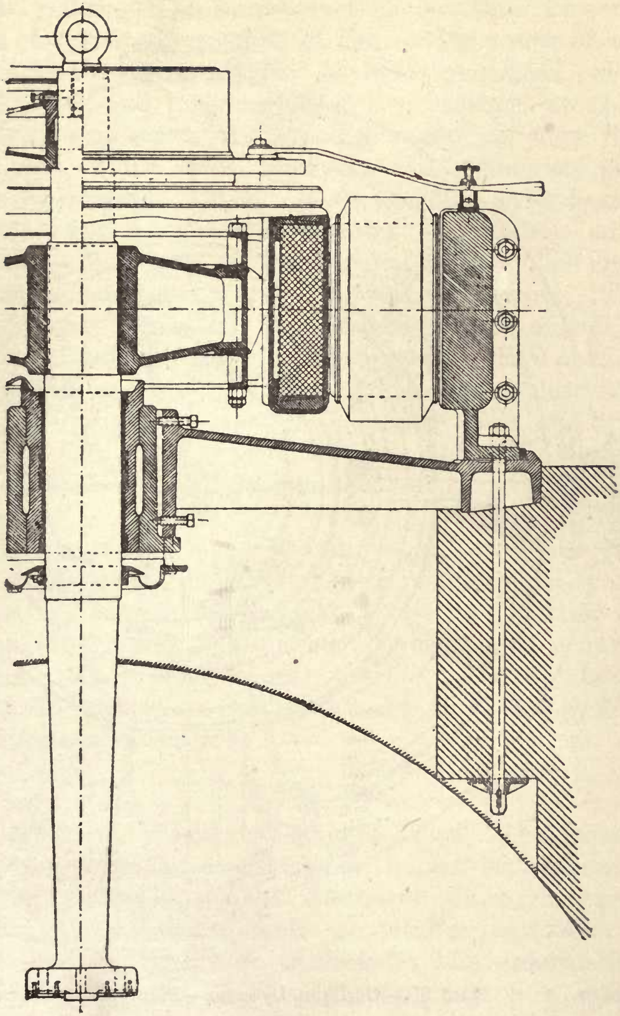

FIG. 88.—Oerlikon Dynamo.—Sectional Elevation.

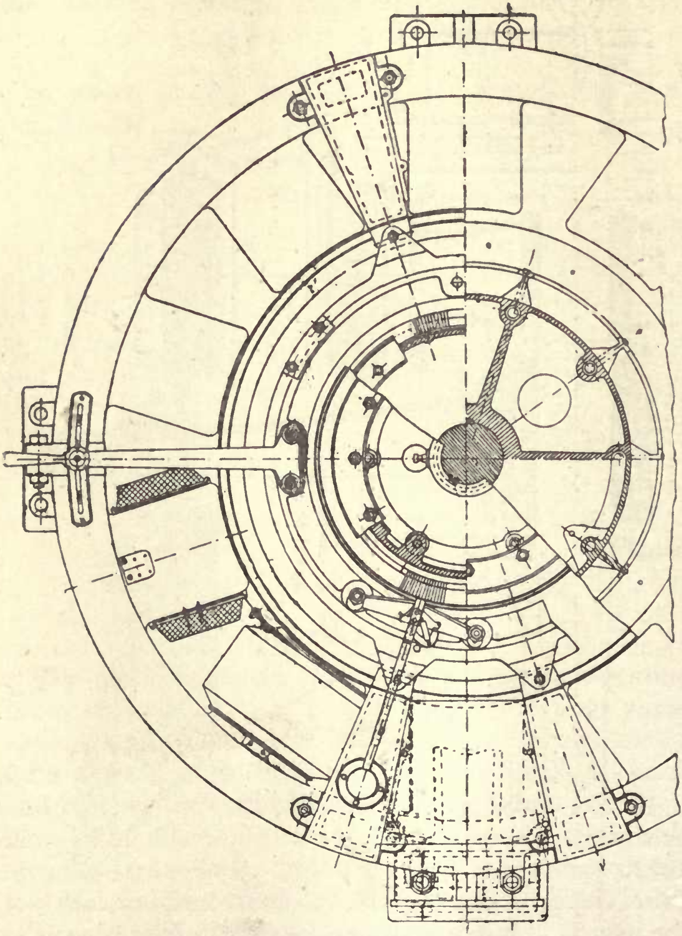

Fig. 88.—Oerlikon Dynamo.—Plan.

foot, and the dimensions can thus be taken by scale, but for the convenience of the reader a few of the principal dimensions may be here mentioned. The yoke ring and magnets are in two castings, no pole-shoes being used. External diameter of yoke ring 8ft. 5in., width 24in., thickness $6\frac{3}{8}$in. Magnets projecting inwards $13\frac{3}{4}$in.; of rectangular cross-section 21in. × $13\frac{3}{4}$in. External diameter of armature core, 59in. by 21in. long, and $5\frac{1}{2}$in. radial depth. This gives a circumferential velocity of 3,550ft. per minute. The air-space is ·79in., and the clearance between outside of binding wire and poles is only about $\frac{1}{8}$in. It is obvious that with so large a diameter and so small a clearance, the workmanship must be extremely good, and that special care must be given to this point in the design. How this has been done will be seen in the vertical section.

The armature is series ring-wound, and the end connections are placed immediately above the armature, the commutator, which is 36in. in diameter by 10in. long, being again placed above the end connections. There is no bearing outside the commutator, and the brushes are carried on a ring supported by four brackets from the yoke.

Other Dynamos.

Mr. R. W. Weekes, in his articles on "The Direct-Current Dynamos at the Crystal Palace Exhibition of 1892," published in the *Electrical Engineer*, April, 1892, gave a series of tables containing particulars of the various machines exhibited. The information compiled by Mr. Weekes should prove very useful for reference, and for this reason it is reproduced here. It should be remembered that the figures in these tables are based upon statements obtained from the makers themselves, and as different makers adopt different limits as regards temperature rise and sparking in fixing the output, it follows that the comparison between the different machines can only be an approximation. It is, however, sufficiently near the truth to give a correct idea of the relative merits of the different designs.

In comparing machines of different type it is, of course, essential to take account of the circumferential velocity of the armature, and this has been done in the first table by adding a column, in which the output is given on the supposition that the circumferential speed of the armature in all machines is 2,000ft. per minute, the output being considered directly proportional to the speed. The figures in this column have been used to determine the output per ton weight and per square foot of floor space in the second table.

The type of each machine is indicated in the first table by reference to the illustrations in the previous chapters. The second and third tables contain particulars of armature, commutator, and field magnets, and also columns giving total field strength and induction. The latter is given as useful induction through armature and field magnets, as it was impossible to obtain correct information regarding magnetic leakage in each case. The true induction through the field magnets is therefore in all cases greater than shown in the table for the useful induction.

LIST OF DETAILS OF DIRECT-CURRENT DYNAMOS AT THE CRYSTAL PALACE, 1892.

Reference letter.	Maker, or name of exhibitor.	Normal			Output in kilowatts at		Type		Number of poles.
		Volts.	Amps.	Speed	normal speed.	when circumferential speed of core = 2,000ft. per minute.	of field magnets, and reference to illustrations.	of armature	
A1	Brush Co.	65	185	285	12	—	Victoria Brush, Fig. 2	Disc	4
A2	" "	100	360	550	36	—	" " "	"	4
A3	" "	3,000	10	800	30	—	Double horseshoe	Open circuit	2
B1	Crompton and Co.	225	600	350	130	101	Fig. 58	Drum	4
B2	" "	225	500	350	112	123	Fig. 57d	"	2
B3	" "	1,450	25	800	36	18·2	"	Ring	2
B4	" "	110	500	890	55	—	"	Drum	2
C1	Easton and Anderson	100	150	350	15	28·2	Manchester type, Fig. 64	"	2
C2	" "	100	150	800	15	15·5	" " "	"	2
D	Ernest Scott and Mountain	105	52	1,050	5·5	3·5	Single horseshoe	Disc	2
E1	Electrical Construction Corporation	110	240	600	26	24	Fig. 57c	Drum	2
E2	" " "	110	370	500	40	70	"	"	2
		1,000	—	—	40	—			
F	Goolden and Co.	115	142	850	16	10	"	"	2
G	Gulcher Co.	65	600	750	40	17·1	Two double horseshoes	Disc	4
H	J. H. Holmes and Co.	65	150	250	10	23	Under type	Ring	2
J1	Johnson and Phillips	210	620	130	130	160	Eight-pole, Fig. 62	Drum	8
J2	" "	140	100	700	14	14	Over type	"	2
J3	" "	600	30	900	8	6	Four-pole, Fig. 59	"	4
J4	" "	105	62	950	6·5	6	Over type	Ring	2
K1	Laurence, Scott, and Co.	50	130	320	6·5	12·5	Four-pole	Drum	4
K2	" " "	100	110	700	11	14	Over type	"	2
L	Laing, Wharton, and Down	110	300	900	33	—	Four-pole	—	4
M	Newton Electrical Engineering Co.	80	200	350	16	27	Over type	Ring	2
N	Roper Electrical Engineering Co.	65	175	750	11	10	Under type	"	2
O	Ronald Scott and Co.	80	165	280	13	28·5	" "	Drum	2
P1	Siemens Bros. and Co.	120	1,500	350	180	165	" "	"	2
P2	" " "	106	220	200	23	63	" "	"	2
P3	" " "	120	200	320	24	38	" "	"	2
P4	" " "	120	400	450	48	51	" "	"	2

LIST OF DETAILS OF DIRECT-CURRENT DYNAMOS AT THE CRYSTAL PALACE, 1892.

Reference letter.	Weight in tons.		Floor space.		Kilowatts per ton at 2,000ft. circumferential speed.	Kilowatts per square foot of floorspace at 2,000ft. circumferential speed.	Armature.					
							Core.			Conductor.		
	Belt driven.	As supplied for coupling direct.	Belt driven.	Direct driven.			Diam. inches.	Length in ins.	Depth in ins.	No. of turns.	Section of conductor.	Description.
			ft.in.ft.in.	ft.in.ft.in.								
A1	—	—	—	4 6×2 8	—	—	—	—	—	—	—	—
A2	—	—	4 9×2 8	—	—	—	—	—	—	—	—	—
A3	—	—	6 8×2 8	—	—	—	—	—	—	—	—	—
B1	—	5	—	4 6×7 6	20	3·01	28	16	—	214	—	Crompton bars
B2	—	—	—	—	—	—	20	36	—	128	—	"
B3	1·65	—	3 3×5 0	—	11	1·11	20	5	3½	—	—	Wire
B4	3·25	—	—	—	—	—	—	—	—	—	—	Crompton bars
C1	—	1 5	—	5 0×2 10	18·8	2·01	11¾	20	—	178	—	Laminated strip
C2	—	—	4 0×2 2	—	—	1·78	9⅜	10	—	120	—	Rectangular bars
D	—	—	3 0×2 4	—	—	0·5	12	4½	—	—	—	Wire
E1	2 3	—	5 8×2 0	—	10·5	2·14	14	14	4½	120	·05 sq. in.	Rectangular bars
E2	{ 6·1	—	7 0×3 3	—	11·5	3·09	18	20	6	72	·075 sq. in.	"
	—	—	—	—	—	—	—	—	—	324	·0113 sq. in.	—
F	—	—	4 3×2 0	—	—	1·18	10½	14	—	—	—	—
G	—	—	6 6×2 9	—	—	0·96	24	5	5¼	160	—	Rectangular bars
H	—	1·15	—	2 8×2 3	20	3·83	13¼	11¾	2¾	272	—	Laminated strip
J1	—	10	—	5 3×6 9	16	4·52	48	18	3½	362	—	Crompton bars
J2	1·5	—	3 8×2 0	—	9·4	1·91	11	16	2⅜	162	3(·270×·035)	Laminated strip
J3	—	—	3 6×2 8	—	—	0·65	11	11	2⅜	102	—	—
J4	0·62	—	2 8×1 6	—	9·7	1·51	9	9	1¾	228	(·170×·105)	—
K1	—	1·0	—	2 8×2 6	12·5	1·88	12½	8	—	—	—	—
K2	0·6	—	3 3×2 0	—	23	2·15	8½	7½	—	—	—	—
L	—	—	—	—	—	—	—	—	—	—	—	—
M	—	—	—	4 6×2 6	—	2·40	13	16½	—	—	—	—
N	1·25	—	3 3×2 4	—	8	1·31	11	9	—	144	—	—
O	—	1·75	—	4 3×2 0	16·2	3·65	11½	18	—	192	—	Laminated strip
P1	—	13 6	—	6 9×6 0	12·2	4·08	24	34	—	—	—	Crompton bars
P2	—	2·9	—	4 6×2 10	22	5·00	14	20	—	—	—	—
P3	—	2·8	—	4 0×2 10	13·6	3·40	15	19	—	—	—	—
P4	—	3·2	—	4 3×4 0	16	3·01	16	20	—	—	—	Crompton bars

LIST OF DETAILS OF DIRECT-CURRENT DYNAMOS AT THE CRYSTAL PALACE, 1892.

Reference letter	Commutator.		Brushes.			Field magnets. Section of bars.		Field magnets.	Induction in C.G.S. lines per square centimetre.			Remarks.
	No. of segments.	Material.	Number.	Size.	Material.	length.	thickn'ss	—	Total lines from one pair of poles.	Induction in the armature B_a.	Useful induction in the magnets B_m.	
				in. in.		in.	in.					
A1	—	Copper	2	$1\frac{5}{8} \times \frac{1}{4}$	Copper gauze	—		W.I.*	—	—	—	—
A2	—	”	2	$2 \times \frac{1}{4}$	”	—		”	—	—	—	—
A3	—	”	—	—	Copper plates	—		”	—	—	—	A.
B1	107	Cast copper	3	$2 \times \frac{3}{8}$	”	14 ×	12	W.I.	9,400,000	12,500	8,700	—
B2	64	”	—	—	—	6 bars 6 ×	6	”	31,500,000	13,000	11,200	—
B3	90	”	2	$1\frac{1}{2} \times \frac{3}{8}$	Copper plates	$7\frac{1}{2}$ ×	4	”	—	—	—	B.
B4	—	—	—	—	—	—		—	—	—	—	—
C1	89	Drawn copper	3	$2 \times \frac{3}{8}$	Copper gauze	—		W.I.*	9,910,000	—	—	C.
C2	60	”	3	$1\frac{1}{2} \times \frac{3}{8}$	”	—		”	6,450,000	—	—	C.
D	—	”	2	$1\frac{1}{2} \times$ 3-16	Brass gauze	—		”	—	—	—	—
E1	60	Cast copper	3	$2 \times$ 3-16	Copper gauze	14 ×	7	W.I.	9,500,000	15,000	—	—
E2	36	”	3	$2 \times \frac{3}{8}$	”	20 ×	12	”	19,000,000	15,000	12,400	D.
	162	”	—	—	—	—			—	—	—	
F	—	”	2	$2 \times \frac{1}{4}$	Copper gauze	—		W.I.	—	—	—	—
G	80	Brass	2	$2 \times \frac{1}{4}$	Brass gauze	circular bars		W.I.*	17,100,000	12,700	—	—
H	74	Drawn copper	—	—	Copper gauze	10 diameter		”	6,000,000	16,300	11,800	—
J1	181	”	4	$1\frac{3}{4} \times$ 15-16	Brass gauze	16 diameter		C.I.*	7,000,000	10,600	5,400	—
J2	81	”	2	—	”	15 ×	$5\frac{3}{4}$	W.I.	7,600,000	15,300	13,800	—
J3	51	”	1	$1\frac{3}{4} \times$ 3-16	”	$10\frac{1}{4}$ ×	5	C.I.	2,040,000	—	6,100	E.
J4	57	”	2	$1 \times \frac{1}{4}$	”	$8\frac{1}{4}$ ×	5	W.I.	3,000,000	16,400	11,300	—
K1	—	Cast copper	2	$1\frac{1}{4} \times \frac{1}{4}$	Copper gauze	—		C.I.	—	—	—	C.
K2	60	”	—	—	”	—		W.I.*	—	—	—	C.
L	—	—	—	—	—	—		—	—	—	—	—
M	—	—	2	—	Copper gauze	$15\frac{1}{4}$ ×	7	W.I.	—	—	—	—
N	72	Cast copper	3	$1\frac{1}{2} \times \frac{1}{4}$	”	9 ×	9	C.I.	3,700,000	—	7,100	—
O	96	Castphosphorbronze	2	$2 \times$ 3-16	Brass gauze	19 ×	7	W.I.	9,250,000	—	10,800	—
P1	—	—	3	$4(2 \times \frac{1}{4})$	Copper gauze	34 ×	$16\frac{1}{2}$	”	—	—	—	—
P2	—	—	—	—	Copper wire	—		—	—	—	—	—
P3	—	—	—	—	—	—		—	—	—	—	—
P4	—	—	—	—	Copper wire	19 ×	$9\frac{1}{2}$	W.I.	—	—	—	—

A Brush open-coi arc-lighting dynamo. B. Arc-lighting dynamo, constant current. C. Armature with toothed core. D. Motor-generator. E. Arc-lighting dynamo. * With cast-iron pole-pieces.

CHAPTER XV.

Elementary Alternator—Measurement of Electromotive Force—Fawcus and Cowan Dynamo—Electromotive Force of Alternators—Self-Induction in Armatures of Alternators—Clock Diagram—Power in Alternating-Current Circuit—Conditions for Maximum Power—Application to Motors.

Elementary Alternator.

A closed conductor revolving in a magnetic field in such way as to cut lines of force becomes the seat of an alternating electromotive force, and will be traversed by an alternating current. A machine constructed for the purpose of producing this effect is an alternating-current generator, or alternator, and the simplest form in which we can conceive such a machine is a metallic ring or coil of wire revolving round a vertical diameter in the field of the earth. When the plane of the coil is at right angles to the magnetic meridian (east-west position) no lines are cut, whilst at the moment the plane of the coil passes through the magnetic meridian (north-south position) the rate of cutting lines is a maximum, and the electromotive force generated in the coil is also a maximum. The electromotive force, which relatively to the coil is of course alternating, may be used to produce an alternating current through any conductor joined to the terminals of the coil. We could, for instance, imagine the two ends of the wire forming the coil attached to the filament of an incandescent lamp, Fig. 89, and this combination would form a very simple electric light plant, provided it were possible to work the

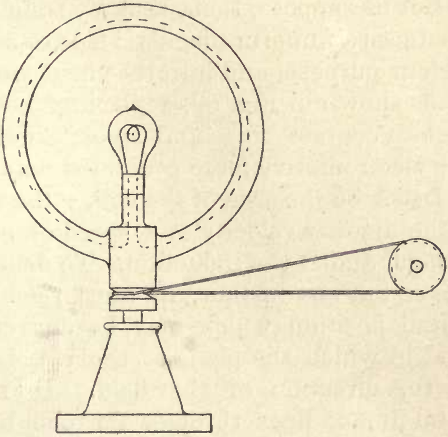

FIG. 89.

apparatus at the required speed. This is, however, totally impracticable, as can be seen from the following figures. Assuming the coil to be 1 metre in diameter, and to contain 1,000 turns of wire, it would have to be worked at the rate of 20,000 revolutions per minute in order to light up a 100-volt lamp. With the axis horizontal and at right angles to the magnetic meridian, the speed corresponding to 100 volts would be 8,000 revolutions per minute, the decrease of speed being due to the fact that in the latter arrangement we make use of the total intensity of the earth's field, and not only of its horizontal component, as in the former arrangement. But yet the field is far too weak for any practical work, and to obtain a serviceable machine we must employ a field artificially produced.

Let us suppose, then, that by some means we have produced a uniform magnetic field of sufficient strength for our purpose, and into that field we place the apparatus shown in Fig. 89, replacing, however, the lamp by two contact rings and an external circuit, so that the electromotive force generated may be measured.

Let A be the area of the coil, τ the number of turns, ω the angular velocity at a speed of n revolutions per second, and B the induction. To define the position of the coil at any instant, we must refer to some definite initial position which may be conveniently taken as that in which the plane of the coil is at right angles to the direction of the field. In this position the total flux of lines through the coil has its maximum value, $F = A\,B$, and the electromotive force is zero. Let the coil have advanced in the time t through an angle, α, then the flux is $f = F \cos \alpha$, and the instantaneous electromotive force corresponding to this position is $\tau \frac{d f}{d t} = - \tau F \frac{d}{d t} \sin \alpha$. Since $\alpha = \omega t$ and $\omega = 2 \pi n$, we can also write for the instantaneous electromotive force

$$e = \tau \pi n F \sin \alpha.$$

This is, of course, a variable quantity. It is zero, for $a = 0$, and for $a = \frac{\pi}{2}$ it is a maximum, which we may denote by E,

$$E = \tau 2 \pi n F.$$

The instantaneous electromotive force may then be represented as a sine function of the maximum electromotive force by any of the following expressions, T being the time of a complete cycle:

$$e = E \sin a;$$

$$e = E \sin (2 \pi n t);$$

$$e = E \sin \left(2 \pi \frac{t}{T}\right).$$

It is, however, not the instantaneous, but the effective* electromotive force which is of practical importance. By the term effective voltage of an alternating current we mean such an alternating voltage as will produce in a conductor the same heating effect as a continuous and constant difference of potential. Since the indications of a Cardew voltmeter depend directly on the heating effect on its wire, it follows that such an instrument applied to our alternating-current circuit will give its effective voltage. Or to put this matter in another way. If we have two conductors between which an effective pressure of 100 volts is maintained, and we connect them by an incandescence lamp, we shall get from this lamp exactly the same light effect as if the lamp were placed on a 100-volt continuous-current circuit.

* The term "effective," as applied to pressure and current, has been adopted at the Paris Congress of 1889.

Measurement of Electromotive Force.

The point to be determined now is what relation the effective electromotive force bears to the maximum electromotive force. In order to get the same amount of light from the lamp with an alternating as with a continuous current, it is obviously necessary that the same amount of energy shall be expended per unit of time in the two cases. The resistance of the filament depends on its temperature, and the latter on the current, so that with an alternating current the temperature and resistance must, to a certain extent, vary. But when the reversals are rapid (at the rate of 100 or 200 per second), the variation of resistance becomes negligible, since there is, between two current waves, not sufficient time for the filament to cool; and we may therefore assume that the resistance is constant, and equal to that which the filament has when the current is continuous. Let r be this resistance, and take as unit-time the time T, in which the alternating electromotive force passes through a complete cycle. The work done by the continuous current of electromotive force, e, in this time is obviously

$$T \frac{e^2}{r} \text{ watt-seconds.}$$

The work done by the alternating current is

$$\int_0^T \frac{E^2}{r} \sin^2\left(2\pi \frac{t}{T}\right) d\,t = T\,\tfrac{1}{2}\frac{E^2}{r}.$$

The effective voltage is, therefore, equal to the maximum voltage divided by the square root of 2.

$$e = \frac{E}{\sqrt{2}}.$$

Another proof of this law, due to Mr. Blakesley,* is as follows: To obtain the work done in one revolution of the coil, Fig. 89, we imagine the cycle subdivided into a large number of small increments, and add the work done during successive increments. If instead of considering each position singly, we consider it jointly with a position 90 deg. in advance, we shall obtain twice the work. The activity of the coil at the instant it occupies the position α is $\frac{E^2}{r} \sin^2 \alpha$, and that corresponding to the conjugate position $\alpha + \frac{\pi}{2}$ is $\frac{E^2}{r} \cos^2 \alpha$. The sum of the two is obviously $\frac{E^2}{r}$, and as this holds good for every position, we find that $\frac{E^2}{r}$ represents twice the activity of the coil. The activity is therefore

$$\frac{e^2}{r} = \tfrac{1}{2} \frac{E^2}{r}$$

and

$$e = \frac{1}{\sqrt{2}} E,$$

the same expression as already obtained. The effective volts of our alternator are, therefore,

$$e = \frac{2\pi}{\sqrt{2}} n F \tau 10^{-8}.$$

It is convenient to bring this equation into the same form as that giving the electromotive force of a two-pole continuous-current machine. In that case we

* "Alternating Currents of Electricity," 1885.

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did not count the conductors on the armature as so many complete turns, but as so many active armature wires. As each turn comprises two active wires, the τ in the above equation represents in reality 2τ active wires; or if we denote, as in continuous-current machines, the number of active wires by τ, we must insert $\frac{\tau}{2}$ in the above equation. It is also convenient to reckon the speed, not in revolutions per second, but per minute, so that the effective electromotive force, in volts, of the alternator is—

$$e = \frac{6{\cdot}28}{1{\cdot}41} \frac{n}{60} \mathrm{F} \frac{\tau}{2} 10^{-8};$$

$$e = 2{\cdot}22 \, \mathrm{F} \, \tau \frac{n}{60} 10^{-8} \quad . \quad . \quad . \quad (45)$$

or, in English measure—

$$e = 2{\cdot}22 \, \mathrm{Z} \, \tau \, n \, 10^{-6} \quad . \quad . \quad . \quad (46)$$

Although no pole-pieces were shown in the simple apparatus represented by Fig. 89, it is evident that the formulæ apply to bi-polar alternators, provided the field between the poles is perfectly uniform. A machine of this kind is shown in Fig. 90, and the only difference between it and Fig. 89 consists in the employment of an artificial field, whereby we are able to increase the electromotive force. The arrangement, is, however, not yet perfect. The distance between the N and S polar faces is necessarily great, and the field, although stronger than that of the earth, is yet much weaker than obtains in dynamos.

To improve the machine in this respect we must adopt the same expedients as in dynamos—namely, use an iron core in the armature, and shape the pole-pieces so as to reduce the air path of the lines

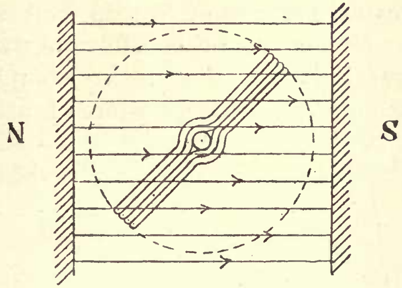

FIG. 90.

to a minimum. We thus arrive at the construction shown in Fig. 91, but the question now presents itself whether formula (45) will give correctly the electromotive force of such a machine. This formula was

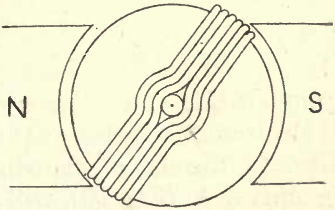

FIG. 91.

deduced on the supposition that the change in the number of lines flowing through the coil was perfectly gradual, and that the total induction followed a sine law. That this cannot be the case in the machine represented by Fig. 91 is obvious. The field through which the coil cuts is now restricted to the space covered by the pole-pieces, and whilst the coil remains completely within this space the electromotive force must be sensibly constant, whilst during the time that the coil remains completely outside this space the electromotive force must be zero.

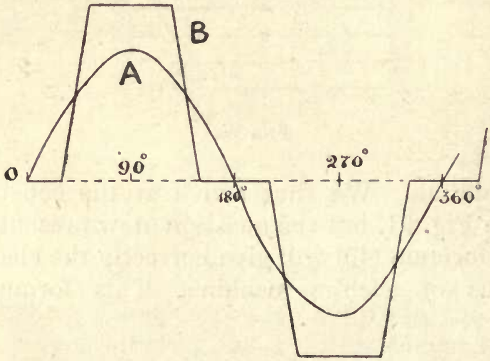

FIG. 92.

During entry and exit there must be a rapid increase and decrease of electromotive force, so that the electromotive force diagram, instead of showing the smooth and undulating curve, A (Fig. 92), will now show a broken line, B. In this diagram the angular position of the coil is measured on the horizontal, and the electromotive force on the vertical, the initial position of the coil being vertical in both cases, when the electromotive force is zero.

The exact shape of the broken line depends on the width of the coil and the angular width of the pole-pieces, and can in any special case be easily found. If that is done, the effective electromotive force can be determined by plotting a second line, the ordinates of which represent the square of the ordinates of B, and measuring the area enclosed between this line and the horizontal. The height of a rectangle of equal area and base represents the square of the effective electromotive force, and the latter itself can thus be found.

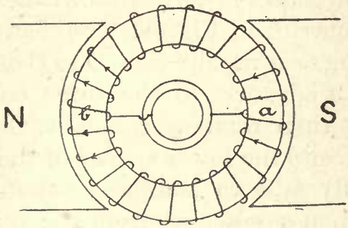

FIG. 93.

It would be tedious and certainly unnecessary to go through this process in every case. If we have once for all ascertained the relation of the effective to the maximum electromotive force for a machine, in which the width of coil and width of pole bear a certain proportion to each other, then the same relation must obviously hold good in all other machines similarly constructed, and it will thus suffice to investigate a few typical cases which shall more or less accurately represent types of machines found in practice. It has been assumed hitherto that the armature of the alternator is of the drum type, but a glance at Fig. 93 will show that a ring armature may equally well be employed. We need only connect two opposite points, *a b*, of the winding with two contact rings in order to take off alternating currents from these rings. When the armature is in the position shown, the alternating electromotive force at the contact rings is zero, but whereas the zero period in Fig. 91 extends over an appreciable time it is in Fig. 93 instantaneous; that is to say, the electromotive force merely passes through zero. The B line in Fig. 94 cuts through the horizontal without the break of continuity occurring in Fig. 92. The electromotive force of the machine rises rapidly from O to C during the time that the coil *a* advances to the lower corner of the S pole-piece; it then remains constant, C to D, until *a* has come opposite the lower corner of the N pole-piece, then it rapidly falls to zero and attains a negative maximum, when *a* emerges from the upper corner of the N pole-piece, and so on.

Fawcus and Cowan Dynamo.

Now let us suppose that, in addition to the contact rings already mentioned, the armature is provided with a commutator as usual; we shall then be able to take from it simultaneously a continuous current of an electromotive force equal to the maximum alternating electromotive force, and an alternating current of a lower effective electromotive force. The continuous electromotive force is shown in Fig. 94 by the horizontal line VV, and the alternating electromotive force by the line B, the negative part of which is for convenience reproduced by a dotted line about the axis. The triangular spaces, D E F, G K H, show the gaps in the voltage line by which the alternating pressure falls short of the continuous pressure. The effective alternating volts must obviously be less than the continuous volts, the ratio between the two depending on the angular spaces covered by the pole-pieces. With a polar angle of 90 deg., the ratio is 1 to ·765. This property of the machine to give two currents of different voltage is ingeniously utilised by Messrs. Fawcus

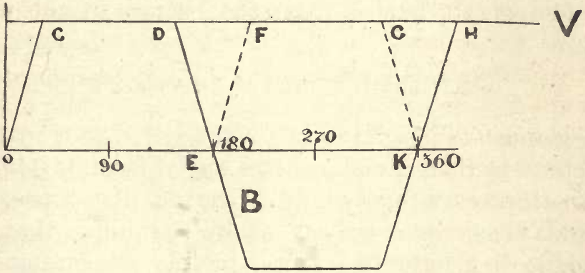

FIG. 94.

and Cowan for lighting lamps and charging a battery at the same time.

The machine designed by Fawcus and Cowan is intended for use in electric light installations where it is desired to charge the battery during lighting hours. Two opposite sections of the commutator are connected with the two contact rings already mentioned, and a switch is provided by which the lamp circuit can be switched on to the brushes bearing on these contact rings. The lamps are then fed by an alternating current of 100 volts. The voltage at the brushes of the commutator is at that time larger than 100 in the ratio of 1 : 0·765, or whatever ratio may correspond to the particular construction of pole-pieces. The electromotive force of the continuous current will therefore be 100/0·765 = 130, which is sufficient to fully charge a 100-volt battery. By weakening the field the continuous electromotive force can, of course, be reduced to 100 volts, so that the lamp circuit may also be worked with a 100-volt continuous current, and when the machine is stopped the lamps can be fed from the battery in the usual way.

Electromotive Force of Alternators.

A glance at Figs. 92 and 94 will show that we cannot without further investigations apply formula (45) to alternators as actually built. In the first place, this formula has been arrived at by assuming that the machine has only two poles, but as alternators are generally made multipolar, the formula cannot be used without such modification as will take account of the number of poles and the manner in which the armature coils are joined up. In the next place, we have assumed the coil to revolve, whereas in a multipolar machine the angular movement of the coil in passing from one pole to the next is small as compared with its movement of translation, and if the poles are set radially, as is always the case, the angular movement need not be considered at all, and we have merely to consider the linear shifting of the coil from one pole to the next. Then we must take into account the shape and dimensions of the pole-pieces, and the space occupied by the coils relatively thereto. On these details will depend the shape of the line B, Fig. 92, from which the effective electromotive force can be calculated. It is obvious that if we have only a single conductor on the armature occupying no appreciable space, the electromotive force in this conductor will retain its maximum value all the time that it remains within the polar area, and then the sloping parts of line B would become vertical with a corresponding increase of electromotive force. It is also clear that if we could abolish the idle periods between a positive and negative electromotive force, the effective electromotive force would attain the highest possible value, and become equal to the maximum electromotive force. This latter condition can, however, not be fulfilled in practice because it implies the north and south pole-pieces to be absolutely contiguous. The nearest approach to this condition is in the Mordey machine, where fields of the same polarity alternate with blank or neutral spaces. Let, in Fig. 95, N S represent the poles which are supposed to be rectangular, and *a* and *b* two positions of the armature coil, consisting of a single loop of wire of the same shape as the contour of the pole-piece. In the position *a* the whole field passes through the loop, and the electromotive force is zero. Immediately after, the left wire of the loop has entered the field and the electromotive force leaps up to its maximum value, which it retains until the left wire has reached the end of the field. At this instant the electromotive force is zero, but immediately after, the right wire of the loop enters the next field, and the electromotive force at once assumes its maximum negative value, and so the action continues, either the right or the left wire of the loop being active, but never both at the same time. The effective electromotive force of the loop in C.G.S. measure is given by the product of induction, B a, length of wire, and linear speed.

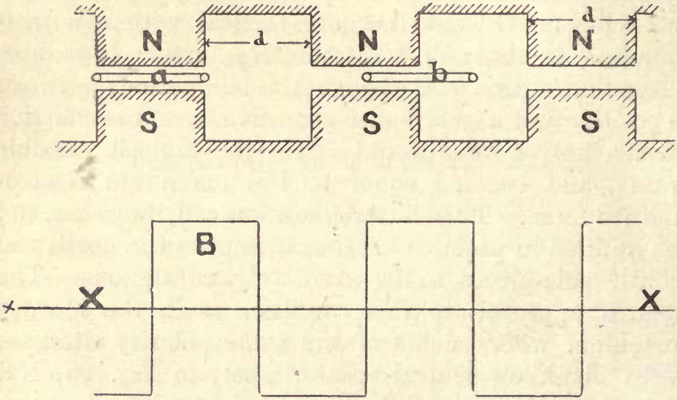

FIG. 95.

If p represents the number of poles on one side, and D the mean diameter of the armature, then $2\,p\,d = \pi D$, and the linear speed is $\pi D \frac{n}{60} = 2\,p\,d \frac{n}{60}$. The electromotive force in volts can then also be written thus:

$$e = F\,2\,p \frac{n}{60}\,10^{-8}.$$

If instead of one loop only we had $2p$ loops all joined in series, we would have $4p = \tau$ active wires, and the electromotive force would be

$$e = p \, F \, \tau \frac{n}{60} 10^{-8}.$$

It is instructive to compare this expression with formula (32), giving the electromotive force of a continuous-current two-pole dynamo. It has been pointed out that with a series-wound multipolar armature the electromotive force is increased in the same ratio as the number of pairs of poles is increased. Thus, in a four-pole machine we have two pairs of poles and double the electromotive force; in a six-pole machine, we have three pairs of poles and three times the electromotive force, and so on. To get the total electromotive force of such an armature, we have therefore to multiply the electromotive force, as given in formula (32), by the number of poles of equal sign employed. If the machine has p north poles and p south poles, we thus find the total electromotive force to be

$$E = p \, F \, \tau \frac{n}{60} 10^{-8},$$

or precisely the same expression as above. We thus find that our alternator, having an armature with $2p$ single loops, or $\tau = 4p$ active wires, and a field containing a total flux of p F lines, produces the same effective electromotive force as a continuous-current dynamo in which τ, p, and F are the same. There is, however, this difference, that whereas in the dynamo there are two parallel circuits through the armature, there is only one such circuit in the alternator; and if we allow the same current in each conductor in both cases, the output of the alternator will only be half that of the dynamo.

This result has been obtained on the supposition that the adjacent wires of two neighbouring coils occupy no appreciable space, a condition which cannot be fulfilled in practice. We may approximate to it by employing a toothed armature once, but then only imperfectly. Let us now see how the case stands if we cover an appreciable space on the armature with coils, as would naturally be done in order to increase τ and the electromotive force. The electromotive force line would then assume a shape somewhat as shown in Fig. 94. To determine the effective electromotive force we must assume a certain proportion between the width of poles, d, and the internal or blank space of the coils. The external width of the coil would naturally be made equal to the width of poles, so that no winding space shall be wasted. Assume, then, that the band of conductors on each side of the coil occupies a space equal to one quarter the polar width. This will leave a blank space inside the coil equal to half the polar width, and the maximum electromotive force, represented by the sections C D and F G of the line B, will be kept up during half the time of each cycle, the changes occupying the other half, as shown by the sloping lines. We can now find by integration the square root of the mean square of the electromotive force line, an operation which is so simple that it need not be given at length. The result is

$$e = \cdot 817\, p\, F\, \tau \frac{n}{60} 10^{-8} \quad . \quad . \quad . \quad (47)$$

The electromotive force of this alternator is only 81·7 per cent. of the electromotive force of a dynamo with the same number of conductors on the armature and the same field.

Let us next investigate the case where poles of alternate sign follow each other on the same side of the armature, such as in the field of the Siemens and

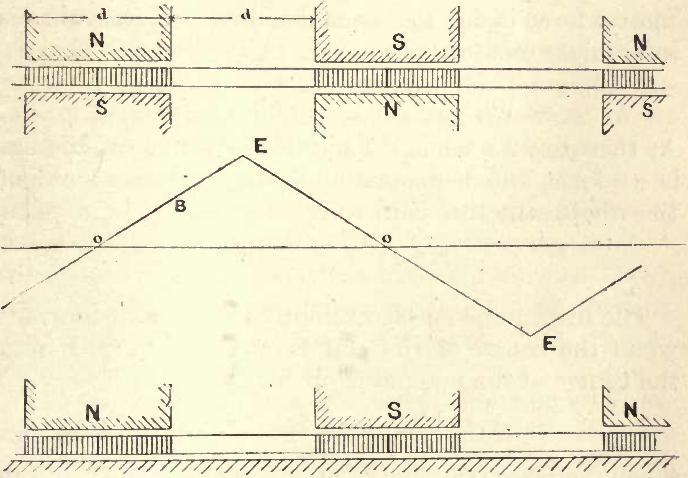

Fig. 96.

Ferranti alternators, and let us assume that the distance between the poles is equal to their width, d. The pitch from pole to pole will now be $2d$, or twice as great as in the former example, and the outside width of the coil will naturally be made equal to the pitch so that no winding space between adjacent coils shall be wasted. As regards the width of the blank space within the coil, designs vary, but we may assume as a fair average that it is about equal to half the polar width.

In Fig. 96 the coil is shown in the position of maximum electromotive force, which occurs only at one instant, and the electromotive force line B assumes the zigzag shape O E O E as shown. If each side of the coil contains w active conductors, the electromotive force is due to $2\,w$ conductors, and its value is in absolute measure

$$E = 2\,w\,F\,4\,p\,\frac{n}{60}.$$

As there are $2\,p$ coils, the number of active conductors is $\tau = 4\,p\,w$, and the maximum electromotive force of the whole armature is in volts

$$E = 4\,p\,F\,\tau\,\frac{n}{60}\,10^{-8}. \quad . \quad . \quad . \quad (48)$$

The instantaneous electromotive force at the moment when the centre of the coil is at the distance x from the centre of the magnet is obviously

$$e = \frac{x}{d}\,E$$

taken between the limits of $x = 0$ and $x = d$; and to find the area of the curve giving the squares of the instantaneous volts we must integrate $e^2\,d\,x$ between these limits. The result is: area $= \frac{d\,E^2}{3}$; and as the base of this area is d, we find the

$$\text{mean ordinate} = \frac{E^2}{3},$$

and the effective electromotive force $= \frac{1}{\sqrt{3}}\,E$.

To get the effective voltage we must therefore divide the expression (48) by the square root of 3:

$$e = 2{\cdot}31\, p\, F\, \pi \frac{n}{60} 10^{-8} \quad . \quad . \quad . \quad (49)$$

The electromotive force of this alternator is therefore 2·31 times that of a dynamo having the same number of conductors on the armature and a field of equal strength. It is obviously not necessary to have poles on both sides of the armature. We could, for instance, as shown in the lower figure, replace the poles on one side by the smooth iron core of an armature. In this case it would only be necessary to double the exciting power on the remaining set of magnets to get the same total field strength as before, and the alternator becomes thus directly comparable to an ordinary multipolar drum-wound dynamo. Some types of alternators are indeed so constructed, notably the Westinghouse machine, and that built by the Electric Construction Corporation.

The coefficient 2·31 is of course only applicable to machines in which the width of coil, width of pole, and pitch are in the ratio indicated. Had we assumed different proportions, the coefficient would also have been different. To show the effect of a variation in these proportions we may take the case of a ring-wound alternator in which the pitch is 8in., width of coil 4in., and width of pole 5in. The advantage of widening the pole is that the maximum electromotive force, instead of being only momentary, as in the previous case, is maintained for an appreciable time, and thus the effective electromotive force is raised without increase of maximum electromotive force. A high maximum electromotive force, although it may only be momentary, throws, nevertheless, a great strain upon the insulation, and to avoid this it is advisable to make the width of the active part of the coil either larger or smaller than the width of the pole, so as to avoid the sharp peak in the electromotive force line. In the construction shown in Fig. 97 this has

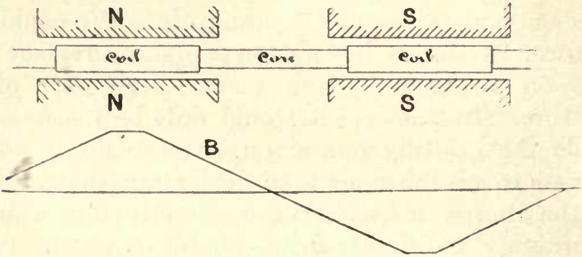

FIG. 97.

been done by making the coil 1in. narrower than the pole, and the maximum electromotive force is thus kept up during the time that the armature travels 1in. The maximum electromotive force is now only

$$E = \frac{16}{5} p \, F \, \tau \frac{n}{60} \, 10^{-8},$$

or by 25 per cent. smaller than in the previous case, whereas the effective electromotive force is 64·6 per cent. of the maximum, instead of only 57·8 per cent. as before. The coefficient now works out to $\frac{16}{5} \times {\cdot}646 = 2{\cdot}06$.

$$e = 2{\cdot}06 \, p \, F \, \tau \frac{n}{60} \, 10^{-8} \quad . \quad . \quad . \quad (50)$$

Thus, while we have lowered the electromotive force by only 11 per cent., we have lowered the strain on the insulation by 25 per cent. With an effective pressure of 2,000 volts the insulation of the machine would be subjected to a strain of $\frac{2,000}{0 \cdot 646} = 3,100$ volts, whereas in the previous machine the strain would be

$$\frac{2,000}{0 \cdot 587} = 3,400 \text{ volts.}$$

It would, of course, be possible to shape the contour of the polar faces so that the electromotive force line becomes a true sine curve, in which case the strain on the insulation would be reduced to

$$2,000 \sqrt{2} = 2,828 \text{ volts.}$$

From the examples above given, it will be seen that it is possible in every case to determine the effective electromotive force beforehand, provided the configuration of poles and coils is known. For every type of alternator we thus find a certain coefficient giving the ratio of the effective electromotive force of the alternator to the electromotive force of a continuous-current dynamo having the same field and the same number of conductors on the armature. The electromotive force of the alternator is thus given by the formula

$$e = \text{K} \, . \, p \, \text{F} \, \tau \, \frac{n}{60} 10^{-8} \quad . \quad . \quad . \quad (51)$$

or in English measure

$$e = \text{K} \, p \, \text{Z} \, \tau \, n \, 10^{-6} \quad . \quad . \quad . \quad (52)$$

the coefficient K depending on the particular type of alternator under consideration.

In the following table are given the values of K for different cases, including those referred to at some length above. The width of poles and space occupied by the armature winding are expressed as a fraction of the pitch, the pitch being the distance between two poles of opposite sign on the same side of the armature. Thus, in the Siemens, Ferranti, or Westinghouse machines, the pitch is simply the distance between the centres of two neighbouring polar faces. In the Mordey machine the pitch is half that distance.

Reference Number.	Pitch Ratio of		K.
	Poles.	Winding.	
1	1	0·00	1·000
2	1	1·00	·580
3	1	·50	·817
4	·62	·50	2·060
5	·50	1·00	1·635
6	·50	·50	2·310
7	·33	·33	2·830
8	Sine Function.		2·220

In comparing different designs it should be remembered that the coefficient K is the ratio of electromotive force between the alternator and the equivalent continuous-current dynamo. Thus, the equivalent to a Mordey nine-pole machine for which K may assume either of the values given under 1, 2, or 3, is an 18-pole drum dynamo, and it is the electromotive force of such a dynamo which must be multiplied with K to get the effective electromotive force of the alternator. In the machines to which cases 4, 5, 6, and 7 refer, poles of opposite sign succeed each other on the same side of the armature, and their number, which must, of course, be even, is the same as that of the equivalent dynamo. The cases given under 4 and 6 approach most nearly to designs as actually found in practice, and it will be seen that either does not materially differ from the value of K which was obtained on the supposition that the electromotive force line is a true sine curve. It should further be remembered that the values for K were obtained on the assumption that the field is sharply defined, but this is in reality not the case. There must at the polar edges be fringes where the induction shades off to zero, and thus all the sharp corners in the line of electromotive force B (Figs. 92, 94, 95, 96, and 97), become rounded off, and the two last-mentioned lines will therefore more or less approach to sine curves. In consideration of these circumstances we shall not introduce any appreciable error if we assume that any ordinary commercial alternator has an electromotive force line of sinuisoidal form; and as this assumption considerably facilitates calculations in connection with alternators and transformers, it will in the following be made where convenient.

Self-Induction in Armatures of Alternators.

Any electric circuit of such shape that an electromotive force can be generated in it by electromagnetic induction, must also have electromagnetic inertia or self-induction. That this cannot be otherwise can easily be seen by the following consideration. In order that an electromotive force may be generated, the circuit must be threaded by a varying number of lines of force, and in order that threading may be possible the circuit must necessarily be in the form of one or more loops. But if we send a current through a loop of wire there is produced a magnetic whirl all round the wire, Fig. 12, the lines of which pass in the same sense through the loop. In other words, the current becomes interlinked with the lines of force produced by itself, and any change in the current strength is accompanied by a corresponding change in the total induction produced by the current. The change in the induction produces in its turn an electromotive force in the conductor commonly called the electromotive force of self-induction. It will thus be seen that it is a physical impossibility to construct an alternator which shall have no self-induction.

The electromotive force of self-induction is obviously proportional to three quantities: (*a*) the induction, (*b*) the number of loops, and (*c*) the rate at which the induction changes, or the number of reversals in unit time. The induction, again, is proportional to the current, provided the permeability of the medium surrounding the coil may be assumed to be constant, so that the electromotive force of self-induction is proportional to the current. Let f be the field produced by unit current (10 amperes) and per turn of the coil, then with a current of I and τ turns in the coil the field will be $I \tau f$. Let the current be reversed $2n$ times per second, giving a frequency of n complete cycles per second, then the maximum electromotive force of self-induction at the moment when the current passes through zero will be $2 \pi n \mathrm{I} \tau f$ for each loop or turn, and $\mathrm{E}_s = 2 \pi n \tau^2 f \mathrm{I}$ for the whole coil of τ turns. The product $\tau^2 f$ is called the coefficient of self-induction, and is usually designated by the letter L. Since the self-induced field depends on the permeability of the medium, and since the useful field, F, produced by the magnets depends also on this quantity, it is obvious that we cannot afford to make f small. On the contrary, we must aim at making the permeability large by reducing the air-space and widening the polar surfaces as much as possible, so as to produce a high effective electromotive force with a moderate exciting power. It follows that if we wish to reduce the electromotive force of self-induction we must decrease τ, and increase the strength of the magnet field. This means that we must employ very strong field magnets and few turns of wire on the armature. A machine with very small self-induction must therefore be large, heavy, and expensive in comparison to its output. There is, however, no advantage gained by reducing the self-induction beyond a certain limit, and we find, therefore, that all commercial alternators have an appreciable self-induction.

It is an easy matter to test an alternator for its self-induction. We need only, while the machine is at rest, send a measured current of the normal frequency through the armature and measure the electromotive force at its terminals. A certain correction must, of course, be made to eliminate the effect of armature resistance, but this correction is so obvious and easily applied as to need no further description. The coefficient of self-induction for one armature coil is

$\tau^2 f$, and as there are $2p$ coils in series, the coefficient of self-induction for the whole armature is $L = 2p\tau^2 f$. The electromotive force required to drive the current through the armature is therefore

$$E_s = 2\pi n L I,$$

all quantities being taken in absolute measure. The current in the above formula is the maximum or crest of the current wave, but since the current measurement by any instrument such as a Siemens dynamometer, Kelvin balance, or any alternate-current ampere-meter, gives the effective and not the maximum current, we must multiply the current reading, i, in amperes by $\frac{1 \cdot 41}{10}$ to get the value of I in absolute units. To get the electromotive force in volts, we must multiply by 10^{-8}; and if we wish to express the coefficient of self-induction in quadrants instead of centimetres we must multiply by 10^9, so that the above formula in practical units becomes:

$$E_s = 2\pi n L 10^9 \times 1 \cdot 41 i 10^{-1} \times 10^{-8},$$

or maximum volts of self-induction $E_s = 2\pi n L 1 \cdot 41 i$.

If we call the effective electromotive force of self-induction e_s, then $E_s = 1 \cdot 41 e_s$, and the above formula may also be written thus:

$$e_s = 2\pi n L i \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Since the frequency n is known, and i and e_s can be measured, we can find the coefficient of self-induction; or we may measure directly the coefficient of self-induction by means of a secohmmeter, and from it find the value of e_s or E_s corresponding to any armature current i. It should be noted that L is not constant, but varies according to the position which the armature occupies in the field, as will be seen from Fig. 98, which represents a Mordey armature in two positions, the upper diagram A showing position of maximum electromotive force, and the lower, B, of zero electromotive force. When the armature is in the position of maximum electromotive force, one half of each of the coils a, b, c is active, and the field magnets form somewhat imperfect cores to all the armature coils.

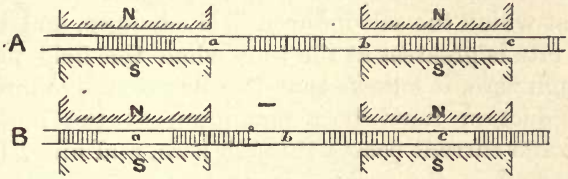

FIG. 98.

In the position of zero electromotive force the field magnets form perfect cores to the coils a and c, but the intermediate coil, b, has no core at all. The permeability of the medium surrounding these intermediate coils (mostly air) will therefore be a minimum, but to make up for this the permeability of the medium surrounding the other coils (mostly iron) will be a maximum, whereas in the other position the permeability has an intermediate value, and is the same for all the coils. It is impossible to say at a glance whether the self-induction in position A or in position B will be the greater. Tests made by Prof. Ayrton on a Mordey machine showed that the difference is not very great,* the coefficient of self-induction being ·038 quadrant for position A, and ·036 quadrant for position B. He also found that both values decreased by about 14 per cent. when the magnets were fully excited, which is quite natural, since the permeability of the magnet cores must decrease when there is an initial induction passing through them.

Clock Diagram.

It has already been pointed out that the electromotive force of self-induction is proportional to the rate at which the self-induced field changes, and since this rate is greatest at the time when the field passes through zero, it follows that the electromotive force of self-induction must be a maximum at the moment when the current passes through zero, and must itself be zero when the current is a maximum. This interdependence between these two quantities can easily be represented by an algebraical formula, and is indeed so represented in all the text-books on alternating currents, but for practical work a graphic representation is preferable, because the variations in the different quantities can be more easily followed and comprehended. One of the first to employ graphic methods in connection with alternating-current problems was Mr. Thomas Blakesley,† and the diagrams which will in future be used, although different from his diagrams, are an application of the method of treatment originally indicated by him.

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* Discussion on Mordey's paper on "Alternate-Current Working" at the Institution of Electrical Engineers, May 30, 1889.

† "Alternating Currents of Electricity," *Electrician*, 1885.

Let us now see how we can represent an alternating current graphically. If we suppose the line O I, Fig. 99, to revolve round O as a centre n times per second we shall, when looking at it from a distance, in the direction X O, see its projection on the vertical, O Y, continually expanding and shrinking alternately

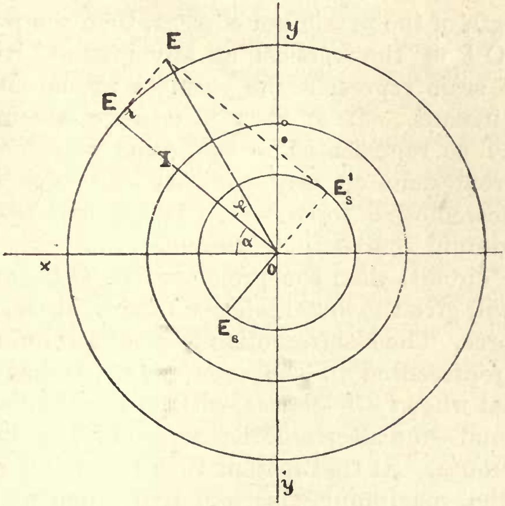

FIG. 99.

above and below the horizontal. The length of the projection at any instant is O I sin α, and this expression is of the same character as that which we found in the beginning of this chapter for the instantaneous value of the electromotive force in the coil of our elementary alternator, Fig. 89. It was then shown that the instantaneous electromotive force is $e = \mathrm{E} \sin \alpha$, and if we suppose that no other electromotive force acts in the circuit, then the maximum current by Ohm's law would be $I = \frac{E}{r}$, and the instantaneous current would be $i = \frac{e}{r}$, or $i = I \sin \alpha$. If we suppose the line O I to represent to any arbitrary scale the strength of the maximum current, then the projection of O I on the vertical at any instant will to the same scale represent the strength of the current at that instant. It is clear that an electromotive force may be represented in the same way. Thus, if O E_r represents to any arbitrary scale the maximum electromotive force which is required to force the maximum current, I, through the resistance, r, of the circuit, then the projection of O E_r on the vertical will give the instantaneous value of the electromotive force. The electromotive force of self-induction may be represented in the same way. It has been shown that when $i = 0$—that is, when O I in the diagram is horizontal—the electromotive force of self-induction is a maximum. At that instant then the radius representing the maximum electromotive force of self-induction must stand vertically—that is, at right angles to the current radius. The only question to be considered is whether the radius of self-inductive electromotive force will point upwards or downwards. Suppose that the current radius revolves clockwise (hence the term clock diagram), and that I is to the left of O. At that moment the current passes through zero and begins to increase. By Lenz's law the self-induction must tend to prevent the increase of current—that is to say, the radius of self-induction must point downwards. Let $O\,E_s$ represent the maximum value of electromotive force of self-induction drawn to the same scale as $O\,E_r$, then the projection of $O\,E_s$ on the vertical will give us the instantaneous value of the electromotive force of self-induction. The lines O I and $O\,E_s$ revolve round O together, always preserving their mutual right angular positions.

In determining the position of I and E_r it has been assumed that no other electromotive force but E_r acts in the circuit. This assumption we now see was incorrect, since, in addition to the electromotive force E_r which is required to overcome the resistance of the circuit, there is also active the electromotive force of self-induction, E_s. If, then, we wish to preserve the current at its assumed strength, we must introduce a new electromotive force to counteract the electromotive force of self-induction. This must obviously be opposed to it and of the same magnitude, as shown by the dotted line $O\,E_s{}^1$. The alternator must, therefore, not only give the electromotive force E_r, but also the electromotive force $E_s{}^1$, or, in other words, it must give the resultant electromotive force, E.

It will be seen that E must under all circumstances be larger than E_r. Now imagine the field magnets of the alternator excited and the external circuit open. The pressure measured at the terminals of the armature will be $\frac{E}{\sqrt{2}}$. If we close the external circuit so as to obtain the current I, we find that the electromotive force producing this current has now fallen to the value E_r, the difference being due to self-induction. It will be seen from the diagram that $E^2 = E_r^{\,2} + E_s^{\,2}$, and this formula gives us a new way of determining the average self-induction of an alternator.

We need only determine for the same speed and excitation the terminal voltage at full current and when running on open circuit. In the latter case we get what may be termed the static voltage, E, and in the former the dynamic voltage less the small percentage wasted in armature resistance. If R be the resistance of the armature, and E_t the terminal voltage, then $E_r = E_t + RI$.

The two measurements give us therefore at once—

$$E_s = \sqrt{E^2 - E_r^{\,2}} \quad . \quad . \quad . \quad . \quad (54)$$

and from this we find by formula (53) the coefficient of self-induction, L.

An example will make this matter clear. A 30-kilowatt alternator of the author's design, when running at a frequency of 70 cycles per second was excited to give 2,100 volts at the terminals with a current of 15 amperes. I in this case is therefore 21·1 and $E_t = 2{,}960$. The armature resistance is 7 ohms, causing a loss of 148 volts. The dynamic electromotive force, E_r, is therefore $2{,}960 + 148 = 3{,}108$ volts. When the external circuit was interrupted the terminal pressure rose to 2,295 volts, corresponding to $E = 3{,}230$. The electromotive force of self-induction is therefore $E_s = \sqrt{3{,}230^2 - 3{,}108^2}$, or

$$E_s = 908.$$

By formula (53), if substituting maximum for effective values, we have $908 = 2\,\pi\,70\,L\,21{\cdot}1$;

$$L = {\cdot}0977 \text{ quadrant.}$$

A correction is, however, here necessary. It will be shown in the following chapter that a lagging current tends to weaken the field, and consequently to reduce the value of E_r below the value due to self-induction only. The coefficient of self-induction obtained above is therefore greater than its true value, such as can be measured with a secohmmeter, or determined by the previously-described method.

Power in Alternating-Current Circuit.

The rate at which work is being done by any current, or, as it is also called, the activity or power of the current, is the integral of the product of pressure and current into time, divided by the total time over which the integration extends. When the current is continuous and constant, the integration is a very simple operation, and the power is found by multiplying current and electromotive force. The same holds good with an alternating current, provided current and electromotive force are in the same phase, but if this is not the case then there are periods when the current is positive and the electromotive force negative, or *vice versâ*, and the activity during these periods is a negative quantity, meaning that the circuit, instead of absorbing power, gives out part of the power previously absorbed, and the power actually absorbed during a complete cycle is less than would be the case if the current and electromotive force phases coincided.

Let the electromotive force and current occupy the positions shown, in Fig. 100, by the lines O E and O I, then the activity at that instant would be given by the product of the lines *o e* and *o i*.

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After a quarter period, E will have advanced into the position E^1, and I into the position I^1. Let us now determine the power corresponding to each position and take the mean:

$$e\,i = \mathrm{E\,I} \cos \alpha \cos \beta$$
$$e^1 i^1 = \mathrm{E\,I} \sin \alpha \sin \beta.$$

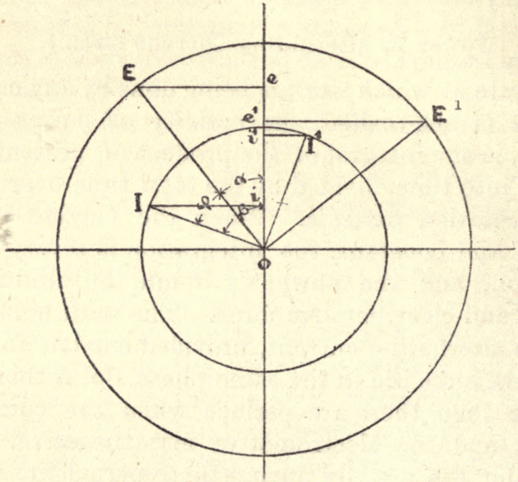

FIG. 100.

$$\text{Mean power} = \frac{e\,i + e^1 i^1}{2} = \frac{\mathrm{E\,I}}{2} (\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= \frac{\mathrm{E\,I}}{2} \cos (\beta - \alpha).$$

Now, the difference between β and α is, as will be seen from the diagram, simply the angular distance between the electromotive force radius and the current radius, or the angle by which the current lags behind the electromotive force. The mean power between the two positions is therefore given in watts by half the product of maximum volts and maximum amperes into the cosine of the angle of lag:

$$W = \frac{E\,I}{2} \cos \phi \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Had we chosen any other pair of conjugate positions the result would have been the same, and we thus find that the mean power for any two positions, and therefore for the whole cycle, is given by the formula (55). Since $E = e \sqrt{2}$ and $I = i \sqrt{2}$, we have also:

$$W = e\,i \cos \phi. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

To get the power put by an alternator into any circuit we must therefore measure the effective volts and effective amperes, and multiply the product by the cosine of the angle of lag. The diagram Fig. 100 has been drawn on the supposition that the radii O E and O I represent the maximum of electromotive force and current respectively; but it is obvious that we may draw the diagram to represent effective volts and amperes, when the projection of the ampere line on the volt line, multiplied by the length of the volt line, will represent true watts.

Conditions for Maximum Power.

The immediate result of self-induction in the armature of an alternator, or anywhere else in the circuit, is a reduction in the output. The machine induces a greater electromotive force than can reach the part of the circuit where the power is required, and consequently we require a larger machine than would suffice if current and electromotive force were in the same phase. The product of effective pressure and effective current is sometimes called the apparent power, and the ratio between the apparent and the true power gives, in a rough-and-ready way, an indication as to the extent to which the material used in the construction of the machine is usefully employed. This ratio has received the name "plant efficiency." Now, a large amount of self-induction in an alternator reduces its plant efficiency, and increases its weight and cost per kilowatt output, though it need not necessarily decrease the mechanical efficiency of the machine. On the other hand, a certain amount of self-induction is necessary in machines intended to be worked in parallel, and for power transmission, as will be shown in Chapter XVI. The question which interests us for the moment is, however, as to the conditions under which a given electromotive force will produce a maximum of power in a given circuit containing a certain amount of self-induction either in the armature or elsewhere. Let A in Fig. 101 be an alternator, L a part of the circuit having self-induction, and R an inductionless resistance in which it is desired to take up the maximum possible amount of power, with a given induced electromotive force in the armature of the alternator. What value must we give to the resistance, R, in order to absorb in it the largest possible number of watts. Not to complicate the problem needlessly, we assume that neither the armature nor any other part of the circuit except R has resistance.

If we had to do with a continuous current we should get the more power the more we reduced the resistance R, but with an alternating current this is obviously not the case. For as the current increases, so does also increase the electromotive force of self-induction in the coil L, and less electromotive force remains available for supplying power to R. On the other hand, if we increase R, the current will be reduced, and less of its electromotive force will be choked back by L, so that more electromotive force remains available for R, but then the current is also smaller, and the power taken up

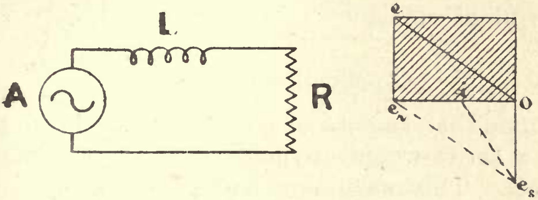

FIG. 101.

may be again reduced. It is evident that there must exist one definite value for the resistance of R at which the power taken up is a maximum. This value can easily be determined. Let, in Fig. 101, $o\,e_s$ represent the electromotive force of self-induction corresponding to the current $o\,i$, and let $o\,e$ represent the electromotive force of the machine. The electromotive force available for R is then $o\,e_r$, and the problem is to make the product of i with e_r a maximum. Now i is proportional to e_s, and hence the problem may also be stated in these terms: Find that value of R for which the product of electromotive force used up in R and electromotive force required for L becomes a maximum. The product $e_s \times e_r$ is given by the area of the shaded rectangle, and it is at once obvious that the area will be greatest when the rectangle becomes a square, that is when

$$e_r = e_s ,$$

and when the angle of lag is 45 deg.

The actual resistance of R is now found by applying formula (53).

$$\text{R}\, i = 2\, \pi\, n\, \text{L}\, i\, ;$$

$$\text{R} = 2\, \pi\, n\, \text{L}.$$

The plant efficiency in this case is $1/\sqrt{2} = 71$ per cent.

Application to Motors.

Assume that, instead of a resistance, we had placed at R a series-wound dynamo with laminated field magnets. This machine will offer a threefold opposition to the passage of the current. First, by virtue of its ohmic resistance; secondly, on account of its self-induction; and thirdly, because when running it will produce a counter electromotive force in the same way as with a continuous current. Suppose the field magnets to be worked at a low degree of magnetisation (few series turns) then the counter electromotive force will be very nearly proportional to the current, and will be of the same order as the electromotive force required to overcome an inductionless resistance—that is to say, it will be correctly represented by the product of a constant into the current, and the above investigation becomes at once applicable to this case.

Neglecting the ohmic resistance of the motor, we thus find that the power it can develop will be a maximum if the volts required to overcome its self-induction equal the volts required to overcome its counter electromotive force. As with high frequencies the electromotive force of self-induction of such a motor is very much greater than any counter electromotive force that could be developed at a reasonable speed, it will be seen that a low frequency is essential if the condition of maximum power is to be fulfilled or even approached. The above investigation is of importance in some forms of self-starting single-phase alternate-current motors.

CHAPTER XVI.

Working Conditions—Effect of Self-Induction—Effect of Capacity—Two Alternators working on same Circuit—Armature Reaction—Condition of Stability—General Conclusions.

Working Conditions.

When we have to do with a continuous-current dynamo, the question whether its electromotive force is used up to overcome an ohmic resistance merely, or also a counter electromotive force, is of no importance, and so long as the external circuit is adjusted in such way as to take the same current at the same voltage the working of the machine is not altered, whether work is done on arc lamps, incandescent lamps, batteries, or electromotors. We may, without altering the working condition, substitute one apparatus for the other requiring the same current and voltage. With an alternator this is not so. The condition under which the machine works depends not only on the terminal voltage and current, but very largely also on the kind of work the current is doing. Thus, when lighting incandescent lamps the self-induction in the external circuit will be very small, and the lag of current behind terminal electromotive force will be very nearly zero, while the lag of current behind induced electromotive force will be limited to the small amount resulting from the self-induction of the armature. On the other hand, when lighting an arc lamp the solenoid in the lamp increases the total self-induction in the circuit considerably, and we shall now have, not only a lag of current behind the induced electromotive force, but also behind the terminal electromotive force. Although apparently the current and terminal voltage may be the same, the machine in these two cases works under very different conditions. These conditions will again be altered if we introduce a condenser (such, for instance, as a few miles of concentric cable) into the circuit, or if there is a second source of alternating electromotive force working on the same circuit. It is necessary to consider these various cases somewhat in detail.

Effect of Self-Induction.

One effect of self-induction—namely, that of retarding the current and lowering the plant efficiency and power put into the circuit—has already been explained in the previous chapter, and need not be repeated here. There is, however, another effect produced which must be taken into consideration, and this is the reaction of the armature current on the field. Let, in Fig. 102, W represent one loop of an armature coil moving from left to right between the field poles, N S. When this loop is in the position shown at A, the electromotive force is a maximum, and is induced downwards in the left and upwards in the right wire. If there were no self-induction, the current would at that moment also be a maximum, and would, owing to the symmetrical position of the wires, neither strengthen nor weaken the field. But, on account of the retarding influence of self-induction, the current will only become a maximum when the loop has moved some distance to the right, as shown at B, and the result is that it will weaken the field. What is true for a single loop is more or less true for the whole armature, so that, generally speaking, the effect of self-induction is not only to reduce the power which can be put into the

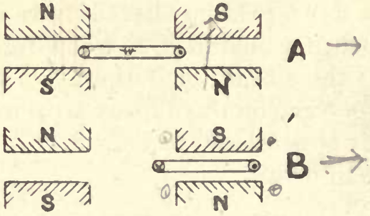

FIG. 102.

circuit under a given voltage, but also to reduce the induced voltage itself. The drop in terminal pressure from open circuit to full load is, therefore, slightly greater than can be accounted for by self-induction only.

Effect of Capacity.

Imagine an alternator supplying a bank of incandescent lamps at the end of a concentric main several miles long. The main will act as a condenser placed in parallel with the bank of lamps. Not to complicate the problem needlessly, we will assume that the ohmic resistance and self-induction of the main and of the alternator are negligible. The induced electromotive force will then be the same as the terminal electromotive force, and the two will be in the same phase. Let, in Fig. 103, O E represent this electromotive force, and O I_o the current which flows when the bank of lamps has a certain resistance. In addition to this working current, there will also be a current flowing in

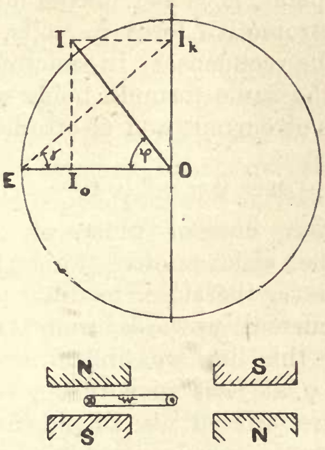

FIG. 103.

and out of the condenser. If K be the capacity of the condenser, and e the instantaneous potential difference between the two surfaces (in our case equal to the instantaneous value of the machine electromotive force), then $i_k = K \frac{d\,e}{d\,t}$ is the instantaneous value of the current, and since $\frac{d\,e}{d\,t} = 2\,\pi\,n\,E \cos\alpha$, we find the maximum value of the condenser current to be

$$I_k = 2 \pi n E K,$$

and $$i_k = I_c \cos \alpha, \text{ in C.G.S. units,}$$

or, in practical units,

$$I_k = 2 \pi n E K 10^{-6}. \quad . \quad . \quad . \quad (56)$$

where I_k is the maximum value of the condenser current in amperes, E is the maximum value of the condenser electromotive force in volts, and K is the capacity of the condenser in microfarads. It is obvious that the same formula holds good when the effective values of current and electromotive force are inserted—

$$i_k = 2 \pi n e K 10^{-6} \quad . \quad . \quad . \quad (57)$$

The condenser current must, of course, be in advance of the electromotive force by a quarter period. We have, therefore, to draw the line representing this current upwards from O in Fig. 103. The length of this line we find from formula (56), and the angle, γ, at E is such that $\gamma = 2 \pi n$ K 10^{-6}. Thus if E were altered we should find the corresponding points, I_k, by drawing lines parallel to the dotted line E I_k. The machine has to give the current O I_o to the lamps, and also the current O I_k to the condenser; in other words, it has to give the resultant current, O I. This, as will be seen from the diagram, is in advance of the electromotive force. Instead of a lag we have now a lead, ϕ, and the position of the loop of wire, W, in which the current becomes a maximum will be reached before the loop has advanced into the position of maximum electromotive force shown in Fig. 102A. The armature current will therefore strengthen the field. Mr. Swinburne has proposed to make use of this property of condensers to produce a leading current, in order to obviate the necessity of applying any exciting circuit to the field magnets of an alternator. He suggested to simply place a condenser across the terminals of the armature. The residual magnetism of the field magnets would suffice to induce a weak electromotive force which would cause a condenser current to flow. This, in its turn, would strengthen the field, and raise the electromotive force, producing more condenser current, and so on until the full voltage had been attained, the action being the analogue of that on which depends the self-excitation in dynamos. It would thus be possible to obtain a self-exciting alternator. Although a machine of this kind could doubtless be made and worked, the advantage of doing without an exciter would, however, hardly compensate for the increased cost. It must be remembered that the omission of exciting coils in such a machine would only be apparent. In reality it would have exciting coils, but they would be part and parcel of the armature. This means that we would have to apply our exciting coils in a part of the machine where room is very limited, instead of on the magnets, where any required room can easily be obtained. The increased number of armature conductors would also augment self-induction, and thus partly neutralise the effect of the condenser.

Two Alternators working on the same Circuit.

The question of the behaviour of alternators when two or more are working on the same circuit is one of great importance not only in central-station work, but also in transmission of power. First, as regards central stations. There can be no doubt that, for economical working, it is necessary to reduce or increase the amount of plant in action at any time so as to correspond as nearly as possible to the demand for current at that time. If the station contained only two large alternators each capable of bearing the maximum load, we should not only have a needlessly large percentage of spare plant, but we should be working the machinery at a very low average output, and consequently at low efficiency. To avoid these defects we must instal a reasonable number of smaller machines, and in order to avoid complication in the switch gear, it is desirable that these machines should be capable of working together on the same external circuit.

As regards transmission of power, it is of course essential that the two alternators—namely, the generator and the motor—should be coupled to the same circuit. To facilitate the investigation, we start with the assumption that one of the two coupled machines is so large, and has so little self-induction and resistance, that its working condition shall not be altered whatever changes may take place in the circuit, or however much the working condition of the other machine may change. This condition of things would very nearly obtain in a central station where a large number of machines is working on to the same pair of omnibus bars, and one small machine has to be added or withdrawn. In this case the change will scarcely affect the group of machines, and we may assume that the voltage on the omnibus bars remains the same whatever may be the current given to or taken from them by the small machine. In making this assumption we limit the investigation to the small machine, and avoid the complications which arise in considering the behaviour of two machines simultaneously.

We have then to consider the following combination: A large machine without resistance and without self-induction supplying a large amount of power to an external circuit, and also coupled to a small machine having resistance and self-induction, both machines running with the same frequency, and having the same terminal pressure. Various problems present themselves in practical work with machines so coupled, and amongst these one of the most important is the following: Let the machine be supplied with mechanical power by its own engine, and determine the working conditions under which it will give a definite electric output with the highest possible plant efficiency. This problem may also be stated in another way—namely, how must a number of alternators in a central station be worked in order that, when coupled to the omnibus bars, each may give, not only the same current, but also the same power.

Since the losses in the machine can only be comparatively small, any variation in the working condition cannot alter the total efficiency very much, and consequently the conditions of equal output between the different machines will approximately be attained if we take care to have the brake power of the engines as near as possible alike. If the engines are fitted with sensitive governors of the usual type—namely, arranged to keep the speed constant—this condition would not be fulfilled, because the speed is already controlled by the frequency, and must be the same for all the engines. Suppose, however, that the governor is so adjusted that it shall not come into action at the normal speed, but merely in the event of the engine racing, then the power supplied by the engine per revolution will only depend on the steam pressure and the cut-off, and may be considered to be constant. As the speed is fixed, the total power put into the alternator will therefore be constant, and its electric output will be approximately constant. The condition of constant output can thus be very nearly attained by running the engine without a governor and with a fixed cut-off. Should it be necessary to vary the power this may be done by a governor regulating the pressure in the main steam-pipe, the regulation affecting all the engines simultaneously.

After this digression into the engine part of the problem let us go back to the electrical part, as stated above. We have there an alternator whose armature is impelled by a constant torque, and the problem is to find the relations between the output, current, lag, and exciting power. A convenient way of stating the excitation is to state the voltage which the machine would give on open circuit. Thus, if we say the machine is excited to 2,100 volts we do not mean that its terminal pressure when a current is flowing is 2,100 volts, but that the field is excited to such a degree that when the machine is running on open circuit the pressure at the terminals is 2,100 volts. We may therefore substitute for the term strength of field the term armature voltage, and the problem may now be stated in these terms: Given a definite driving power and omnibus or terminal voltage, how does the current and lag depend on the armature voltage? Not to complicate the problem too much, we neglect at first armature reaction. It will be shown later on to what extent the armature current

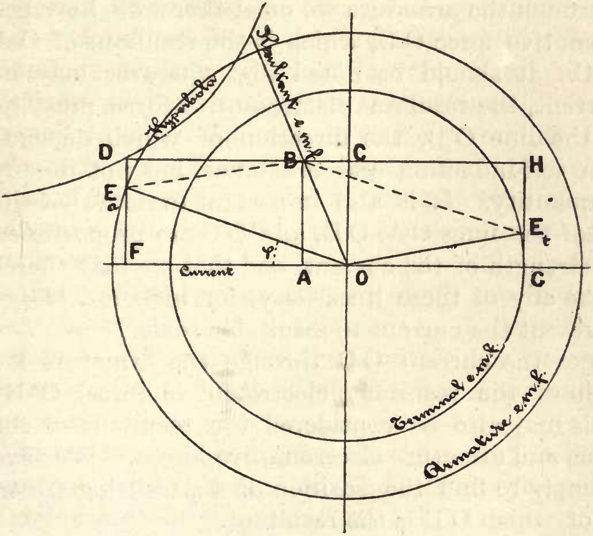

FIG. 104.

adds or detracts from the field-exciting current, and it is thus an easy matter to correct the excitation corresponding to the assumed armature voltage when current and lag are known.

Let, in the clock diagram, Fig. 104, the inner circle represent the terminal or omnibus voltage, and the outer circle the armature voltage, and let the current line be drawn to the left from O. The loss by ohmic resistance for any given current can be calculated and marked off on the current line. We thus get point A. The electromotive force of self-induction acts downwards, and must be balanced by an electromotive force acting upwards. Let this be O C. To drive the current through the armature we must therefore have the electromotive force O B, which is the resultant of O A and O C. It should be noted that whatever may be the current, the resultant electromotive force must lie along the line O B, the direction of which depends only on self-induction and resistance, but not on any other quantity. It is also important to note that the length of the lines O A, O B, and O C are proportional to the strength of the current, and that we may therefore take any of these lines—say, for instance, O C—to represent the current to a suitable scale.

To get the current O C through the armature we must have the resultant electromotive force, O B; and this may also be considered the resultant of the omnibus and armature electromotive forces. We have now simply to find the position of a parallelogram of forces of which O B is the resultant.

Two such parallelograms are possible, and only two. In one of these the armature electromotive force lies to the right of the vertical, which means that the armature current flows in opposition to the armature electromotive force, and must therefore give power to the machine. With this parallelogram, which corresponds to the machine when working as a motor, we shall not deal at present. The other possible parallelogram shows the action of the machine when working as a generator, and is drawn in the diagram. The armature electromotive force is represented by the line O E_a, and the omnibus electromotive force by the line O E_t, which is, of course, equal and opposite to the electromotive force at the armature terminals. The angle of lag is ϕ, and the power put into the machine (including loss by resistance, but irrespective of frictional, hysteresis and eddy-current losses) is found by multiplying the current with the armature electromotive force, and with the cosine of ϕ. In other words, we must project E_a on to the current line, which gives the point F, and multiply O F with the current. It is, of course, supposed that the effective values, and not the maximum values, of current and electromotive force are plotted.

The multiplication may be done graphically. It was shown above that O C may to a suitable scale represent the current, consequently the area of the rectangle O C D F represents the power put into the machine.

In the same way the area of the rectangle O C H G represents the power obtained from the machine, and the area of the rectangle O C B A represents the power wasted in armature resistance. Since the power put in is a constant, the product of D C and D F must be constant, and the point D must lie on a rectangular hyperbola as shown. It is now easy to draw the diagram for other working conditions. Say, for instance, we wish to have a larger armature current. The point B will then be shifted higher up on the line of resultant electromotive force. In Fig. 105 this line and the hyperbola are reproduced from Fig. 104. The problem is to find what armature electromotive force will be required in order to increase the armature current to the desired extent. Say O C represents the current selected. Draw a horizontal till it meets the line of resultant electromotive force at B and the hyperbola at D. The end of the radius, representing

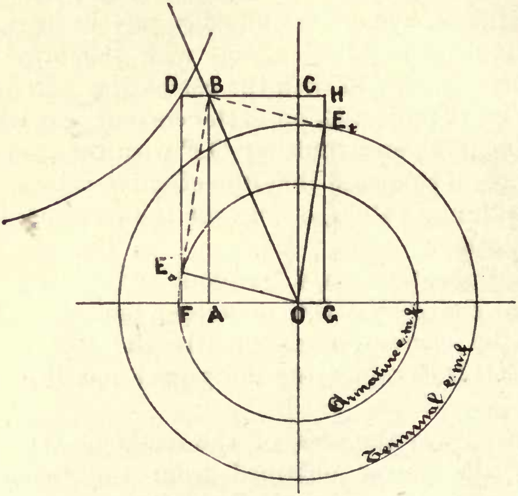

Fig. 105.

armature electromotive force, must be somewhere on the vertical through D, the point F being its projection on the horizontal, as before. To find the point E_a, intersect the line D F with an arc drawn from B as a centre with a radius equal to the terminal electromotive force. The armature electromotive force is now O E_a, and is much smaller than before.

The mechanical power absorbed by the machine is of course the same as before, but the power delivered is much smaller, whilst that wasted in armature resistance is much larger.

To reduce the waste power to a minimum it is, of

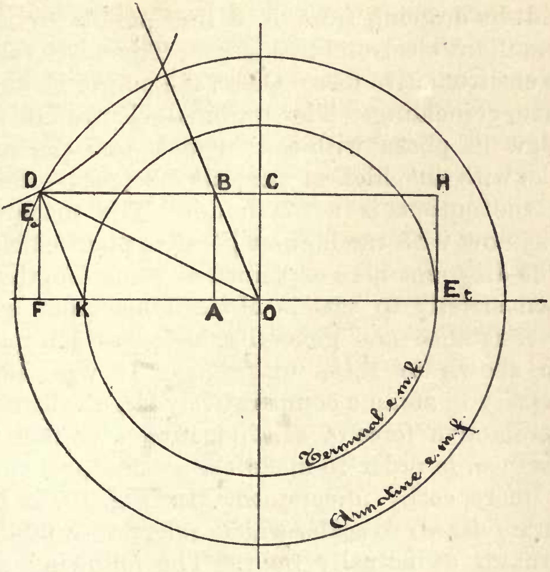

Fig. 106.

course, necessary to excite the machine to such a degree that with the same amount of power absorbed the armature current shall be a minimum. That is to say, the point D should come down on the hyperbola, and B on the line of resultant electromotive force as far as possible.

Now the lowest possible position of B will be reached when B E_a becomes horizontal. If we chose B still lower, the radius of terminal electromotive force will not reach the vertical through D, and this shows that the machine cannot possibly absorb the power when giving so small a current. In Fig. 106 the point B is found by drawing from K a line parallel to the line of resultant electromotive force. This gives the armature electromotive force O E_a, the points E_a and D, of course, coinciding. The terminal electromotive force is now in phase with the current, and the machine works with the highest possible efficiency—that is to say, the output is a maximum. The machine also works now with the highest possible plant efficiency.

The diagrams here explained at some length are not drawn strictly to scale. Modern machines have far less resistance and generally, also, less self-induction than shown in these diagrams. It was, however, necessary to assume comparatively large values for the electromotive force of self-induction and that lost in resistance, in order to make the geometrical construction more easily understood. In Fig. 107 is given a diagram drawn to scale, which refers to a 60-kilowatt alternator as actually built. The following are the particulars of this machine:

Terminal electromotive force...	2,000 volts.
Current	30 amperes.
Frequency.............	60 cycles per second.
Armature resistance	1·94 ohms when warm.
$L = \cdot 069.$	$e_s = 26\,i.$

To make the diagram clear the lines of construction are omitted, and the points giving the position of the radius of armature electromotive force are joined by a curve. These points are marked 1, 2, 3, etc., and the corresponding points on the circle of omnibus electromotive force, which is, of course, diametrically opposed to the terminal electromotive force, are similarly marked. This diagram shows at a glance the current corresponding to any armature electromotive force, and the lag of current with regard to armature and terminal electromotive force. The resultant electromotive force is measured on the sloping line next to the axis of ordinates. The current line is always horizontal and to the left of the centre. The lines representing armature volts are partly above the current line (points 1 to 5), and partly below (points 6 to 9). In the former case the current lags and in the latter the current leads. It will be noticed that the armature voltage may vary within very wide limits (this variation being produced by varying the exciting current), and yet the power absorbed by the machine remains constant—in this case 60 kilowatts. The only difference is that with either too strong a field or too weak a field the current becomes excessive, and the efficiency and output drop, because of the greater amount of power wasted in overcoming the ohmic resistance of the armature. The best working point is at 5, when the armature current is in phase with the omnibus voltage and the machine is excited to give, on open circuit, 2,200 volts.

Although diagram Fig. 107 gives us all the information required, it is not in such a form as to show at a glance the interdependence between field strength, or armature voltage and current. In this respect the diagram devised by Mr. R. W. Weekes, and shown in

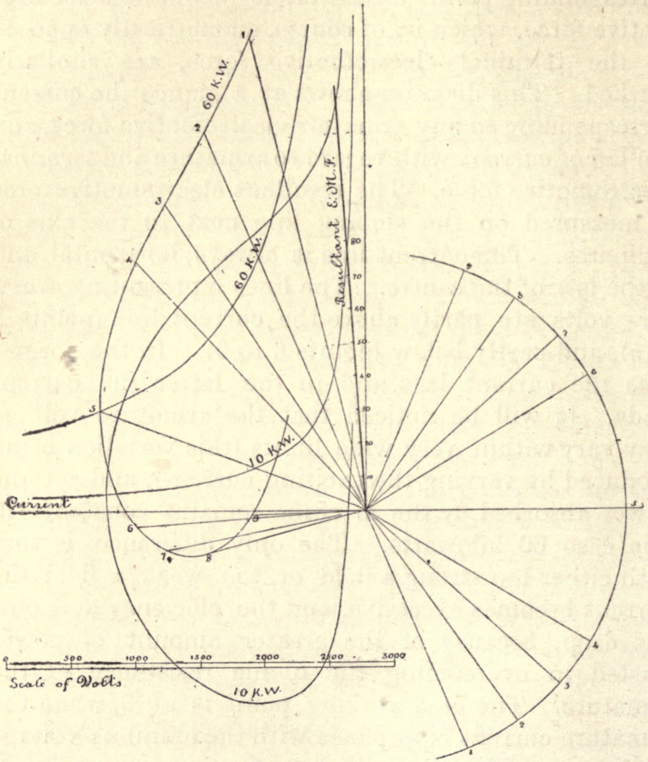

Fig. 107.

Fig. 108, is preferable. It is constructed from Fig. 107 by plotting armature voltage on the horizontal and current (which can be read off in Fig. 107 on the line of resultant electromotive force) on the vertical. Thus we obtain a volt-ampere curve which shows at a glance what current will be given by the machine when the excitation is varied, but the driving power kept constant at 60 kilowatts. We can thus construct a series of volt-ampere curves, each curve corresponding to a definite driving power. In the diagram two only are shown—namely, for 60 and for 10 kilowatts. These curves, shown in thin lines, have been obtained by the

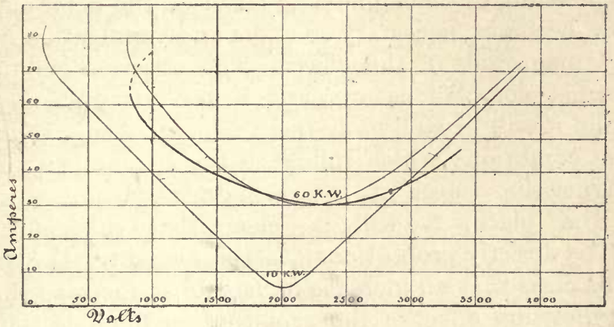

FIG. 108.

construction explained above with reference to Fig. 107, and no correction has been made for armature reaction. The curve shown by a thick line includes the effect of armature reaction, as will be explained presently. The point of importance to be noted is that to each driving power (or load on the machine) there corresponds one particular excitation at which the current is a minimum and the efficiency a maximum. If we excite more the current will be increased, and if we excite less the current will also be increased, the efficiency in both cases being diminished. It follows that the output cannot be controlled by adjusting the excitation, but must be controlled by adjusting the driving power. The best excitation for 60 kilowatts is, however, not very different from the best excitation for 10 kilowatts, the two curves having the lowest point near each other.

Armature Reaction.

It has already been pointed out that in all cases where the armature current leads or lags, it produces a magnetising or demagnetising effect on the field, and it now becomes necessary to make an investigation as to the magnitude of this effect. The question is of a somewhat complex nature, and before attempting a solution it will be well to state in general terms some of the conditions which influence the magnetising or demagnetising effect of the armature current. In the first place, it will be clear that this effect must be directly proportional to the current. It will also be clear that an increase of lag must increase the demagnetising effect of the armature on the field, but it is not at first sight possible to say whether this effect will be simply proportional to the lag. The shape of the coils and pole-pieces and the shape of the current curve must naturally affect this relation. It will also be obvious that the effect cannot be a constant one, but must be a periodic function of the time. Since, however, the pole-pieces and magnet cores are generally solid, and since the exciting circuit has necessarily a large self-induction, the resultant field cannot vary within very wide limits, and may, in fact, without committing any great error, be considered constant. The demagnetising effect, or armature back ampere-turns (to adopt the term used in connection with dynamos), can therefore be arrived at by integrating the momentary effect over the time of a complete period. Let X represent the back ampere-turns at any instant, then the exciting power which must be added to the field in order to compensate for the back exciting power of the armature will be $\int_0^T X\,d\,t$ ampere-turns. In attempting a solution of this integral, it must be remembered that X is the product of instantaneous current multiplied, not necessarily with the total number of turns in the armature coil, but only with the number of turns occupying at that instant such a position that the current can exercise a demagnetising effect. The solution is therefore somewhat complicated, and, in certain cases, impossible. But in the simple case when the width of poles and coils is equal to half the pitch, which corresponds more or less with the majority of machines as actually built, an approximate solution is possible. Calling ϕ the angle of lag, and w the total number of active wires in one coil, we have

$$X = \frac{w\,I}{\pi}\left(\frac{\pi}{2} - \phi - \alpha\right) \sin \alpha,$$

and by substituting for sin α an exponential series, we arrive at the approximate solution that the average or effective back exciting power of one armature coil, and, therefore, of the whole armature, is

$$X_b = w\,i\,\sqrt{2}\,\frac{2\,\phi}{\pi} \quad . \quad . \quad . \quad . \quad (58)$$

the effective value of the current being taken.

This formula is only an approximation, and gives the back ampere-turns slightly too large, but the error is so small that in practical work it may be neglected. If we wish to express the lag, ϕ, in degrees, the formula may also be written thus:

$$X_b = {\cdot}0156\, w\, i\, \phi^\circ \quad . \quad . \quad . \quad . \qquad 59)$$

To take an example, let us suppose that in a 100-kilowatt machine each armature coil contains 80 active conductors and carries 50 amperes. Then $w\,i$ would be 4,000. Let the lag as found from the working diagram be 20 deg., then the total back ampere-turns will be $\cdot0156 \times 4{,}000 \times 20 = 1{,}248$.

In order that the machine may work under the conditions corresponding to the working diagram, we would, therefore, have to apply to the field magnets 1,248 ampere-turns of exciting power over and above the exciting power corresponding to the armature voltage shown on the diagram. This correction has been made in Fig. 108, where the volt-ampere curves, shown in fine lines, represent the working conditions for 60 and 10 kilowatts respectively when armature reaction is neglected, and the full-line curve for 60 kilowatts when armature reaction is taken into account. In making the correction, it is of course necessary to know the characteristic of the machine when working on open circuit, or, as we have called it in connection with dynamos, the static characteristic. This is found in the same way. The machine is run light at about its normal speed, and the exciting current is varied. Readings are taken of speed, exciting current, and terminal voltage. This gives us the armature voltage at the normal speed as a function of the exciting current, Fig. 109, and the curve representing this relation is the static characteristic of the machine. The working diagram, Fig. 107, gives us the current and angle of lead or lag corresponding to any armature voltage. The corresponding exciting power is then found from the static characteristic. We next calculate from (59) the armature reaction, which in the case

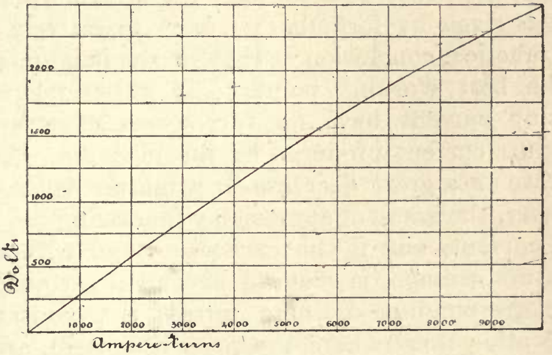

FIG. 109.

of a leading current must be deducted, and in the case of a lagging current must be added to the field ampere-turns found from the characteristic. This gives us the exciting power which must actually be applied to the field, and the corresponding armature voltage which would result if the machine were running on open circuit. It is this value which has been taken for the abscissæ of the volt-ampere curves, shown in the heavy line in Fig. 108. It need hardly be said that this diagram could also be drawn by plotting as abscissæ, not the armature voltage, but the exciting current.

It will be noticed that the divergence between the two volt-ampere curves corresponding to an output of 60 kilowatts is not very great. This is due to the fact that the diagram was constructed for a machine having a reasonable amount of self-induction. The result, as will be seen from Fig. 107, is that even within wide limits of armature voltage the angle of lag or lead remains moderate, and the amount of armature reaction, as given by formula (59), is comparatively small. The practical conclusion is that in the neighbourhood of the best working point small variations in the exciting current have no very great effect on the armature current produced by the machine. A great increase or a great decrease in armature voltage has, however, the result of appreciably increasing the armature current, and if the excitation, and with it the armature voltage, is reduced beyond a certain limit, the corresponding ordinate misses the volt-ampere curve altogether (where the latter is dotted), and this shows that the machine cannot any more hold the engine in step and racing must ensue. When working with a normal field the combined steam-alternator is, however, perfectly stable, thanks to the appreciable amount of self-induction. The case would, however, be entirely different had we assumed the machine to possess extremely little self-induction—say, for instance, so little that the electromotive force of self-induction would be equal to the electromotive force lost in resistance.

Although it is, on account of excessive cost, quite impracticable to build a machine in which this condition is fulfilled, the consideration of such a case is interesting, and the reader should construct for himself the respective working diagram and volt-ampere curves. He will find that the smaller the self-induction the greater is the variation in the angle of lag or lead produced even with very small changes of exciting current, and the more pointed becomes the volt-ampere curve when armature reaction is neglected.

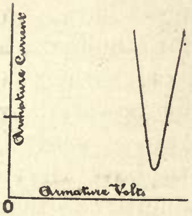

FIG. 110.

If it is taken into account the tendency is to slightly widen the two limbs of the curve, but not materially, because the condition on which the design of the machine is based preclude the possibility of a strong armature reaction. The machine is to have extremely small self-induction. This means that we must provide an extremely strong field and an armature with very few conductors. The magnetising or demagnetising power of such an armature is necessarily very feeble, and it will be the less felt as it has to be exerted upon a field which itself is initially very strong. The result will be that the volt-ampere curve, even when armature reaction is taken into account, will present the shape of a very narrow and pointed V, as shown in Fig. 110. This is, roughly, the curve for a machine in which the electromotive force of self-induction at full load only amounts to a few per cent. of the armature electromotive force.

It has already been pointed out that such a machine must have a very strong field, and would be very expensive to build. Apart, however, from the question whether it would be commercially practicable, we have to consider the question whether such a machine would be desirable for central-station work.

Let us go back for a moment to the case (described a few pages previously) when several machines, each absorbing the same power, deliver current to a pair of omnibus bars at a central station. It is, of course, desirable to have all the machines working under the same conditions—that is, giving the same current and the same output in power. To attain this object the machines must be so excited as to give the same armature volts. But how are we to know when the excitation is right? The ampere-gauge in the exciting current only tells us what exciting power we apply, but as there may be slight differences in the air-space and other constructive details, equality of exciting current does not necessarily mean equality of armature volts. There is, further, the difficulty of getting all the ampere-gauges to give absolutely reliable readings and of adjusting the field rheostats with great nicety. On paper it is easy enough to assume the condition that all the machines shall be excited to such a degree as to give absolutely the same armature volts, but in practice this is not possible. We must expect to have certain variations in voltage. Now, let us see what such a variation means in the two cases of (*a*) machines with a reasonable amount of self-induction and (*b*) with machines of extremely little self-induction. In the former case, to which Fig. 108 refers, a considerable variation in armature volts means a very small variation in armature current, as the volt-ampere curve has the shape of a very wide and rounded V. If, then, we adjust the exciting currents of the various machines to be only approximately equal, we can be certain that the machines, if provided with equal driving power, will give not only equal output, but also equal current. Conversely, we shall be able to adjust the driving power by the ampere-gauge in the armature circuit of each machine.

This is not possible in the latter case. An inspection of Fig. 110 will show that with constant driving power a very small variation in field strength produces a very large variation in armature current. The ampere-gauge in the armature circuit is therefore no guide whatever as to the output of the machine, and the driving power cannot be adjusted thereby. In other words, it is extremely difficult to get the load equally divided between the various machines. The difficulty increases as the self-induction of the machine is diminished, and if it were possible to abolish self-induction altogether, it would be impossible to work the machine on any circuit in which there is another electromotive force active. In this case the volt-ampere curve would shrink to a point, and to work at that precise point it would be necessary to adjust the field with mathematical accuracy, which is, of course, utterly impossible in practical work. Self-induction, so far from being an objectionable feature in alternators, is, in reality, a very valuable property, and it is only by virtue of this property that machines can be worked in parallel and used for the transmission of power.

Condition of Stability.

We have up to the present dealt with machines working on the same circuit without specially considering the question whether they would work in parallel or in series. Parallel working is, of course, implied in all the working diagrams hitherto given, because in all cases the lines of armature electromotive force and omnibus electromotive force included an angle of more than 90 deg.; but as this question is one of considerable practical importance it is worth while to investigate it further. The problem may be stated thus: Given a certain omnibus electromotive force and armature electromotive force, how does the output of the machine vary with the angle of lag between the two electromotive forces? To explain the practical bearing of this problem, let us assume that the armature voltage leads over the omnibus voltage, and that the machine works steadily, the engine supplying a certain amount of power. Now, suppose that from some cause the power of the engine increases. The immediate effect will be to push on the machine so as to increase its lead. If this increase of lead is accompanied by a sufficiently large increase of output, the steam-engine and its alternator will settle down into a new working condition which will be stable. If, however, the increase of lead should result in a decrease of output, then the working will be unstable, and the engine will begin to race.

To simplify the explanation, we shall assume that the omnibus voltage is produced by a very large machine, the self-induction and resistance of which may be neglected. Let the large and small machine be mechanically coupled, and let the coupling be adjustable, so that the two armatures may be set relatively to each other at various angles. In one position of the coupling the maximum electromotive force occurs in both machines simultaneously; with other settings the maximum electromotive force shall occur in the small machine sooner than in the large one, when the small machine may be said to lead, or later, when it may be said to lag.

In drawing the clock diagram for each case, it is necessary that we should be quite clear as to the direction in which each electromotive force acts. Take the case of the large and small machines being coupled in parallel, and assume that in the clock diagram of the large machine the electromotive force would, at a given moment, be shown by a line running from the centre vertically downwards. Since this electromotive force must be opposed to that of the small machine, it would have to be represented in the clock diagram of the latter by a line running from the centre vertically upwards. In the clock diagram of the small machine the omnibus electromotive force must be diametrically opposed to the terminal electromotive force and of equal magnitude, the resistance of the connecting cables between the two machines being considered negligible.

When the two machines are coupled in series, the electromotive force line in the clock diagram of the large machine retains, of course, its direction when transferred to the clock diagram of the small machine.

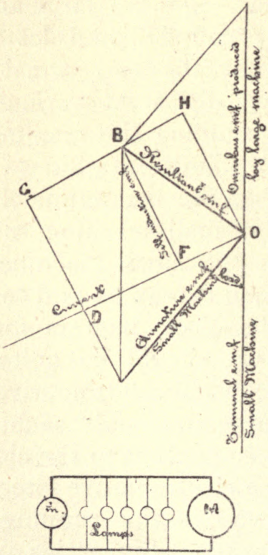

FIG. 111.

Parallel coupling is shown diagrammatically in Fig. 111, where m is the small and M the large machine with the lamps coupled across their common leads. The clock diagram above is so simple that a few words of explanation will suffice. At the moment to which the diagram refers the large machine produces through its armature a downward electromotive force, which, transferred to the small machine, becomes an upward electromotive force, as shown in the clock diagram. This is opposed to the downward electromotive force produced at the terminals of the small machine at that moment. Suppose we have coupled up the two armatures mechanically in such a position that the small machine leads by the amount shown in the diagram. The omnibus electromotive force and armature electromotive force (which in the case illustrated is the smaller of the two) combine to produce the resultant electromotive force, O B. This is, then, the electromotive force which drives the current through the small machine, and has to overcome, first, the armature resistance, O F, and, secondly, the electromotive force of self-induction, B F. Whatever may be the resultant electromotive force, the ratio between its two components is the same. In other words, since B F is vertical to O F the angle B O F is a constant, and will the more approach a right angle the less electromotive force is lost in armature resistance. In modern machines the resistance of the armature is very small, and the angle at B is therefore very acute, and that at O very nearly a right angle. In the diagram it has, however, been shown sensibly smaller than a right angle in order to make the construction clear.

The mechanical power given to the small machine is found by multiplying the current with the projection of the armature electromotive force on the current line. Since the electromotive force is proportional to the current, we can also represent the power by the area of a rectangle the base of which is the projection of armature electromotive force on the current line, and the height of which is the electromotive force of self-induction. This gives the figure O D G H. The rectangle F D G B represents the power supplied to the omnibus bars, and O F B H that wasted in the armature.

Now let us change the coupling so as to work with a different lead, and repeat the geometrical construction above explained. We shall thus get another value for the power, and we may in this way determine the power for any given lead. If the results be plotted in polar co-ordinates, we obtain a curve of the shape shown in Fig. 112 on the left, and marked "Generator." This curve for modern alternators having small armature resistance is nearly a circle, and if armature resistance may be neglected, it becomes a true circle. The power given by the engine to the machine is represented by the length, O P, cut off on the radius, O R, drawn to represent the lead of the small machine over the large machine. It will be seen that if the lead be zero, or very small, the power is also very small, but increases rapidly as the lead increases. We have up to the present always supposed the small machine to be rigidly coupled with the large machine. Let us now imagine the coupling bolts to be suddenly slipped out, and see what happens. Let, at the moment of uncoupling, the lead and power be represented by the line O P, and suppose that from some cause the power of the engine is diminished. The engine will slightly hang back, and the lead will be diminished. This will cause the point P to run back on the power curve nearer to O—that is to say, the engine will be required to give less power than before. On the other hand, if by increasing the steam pressure or altering the cut-off we let the engine do more power, it will push

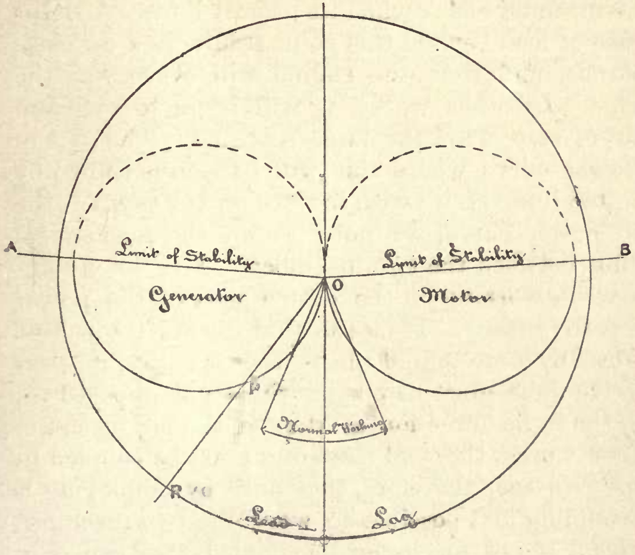

FIG. 112

on, and thus increase the lead, bringing the point, P, farther away from O on the power curve, and increasing the power required by the alternator. It will thus be seen that the working is perfectly stable. Any tendency of the engine to push on too far is immediately checked by the greater output, and any

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tendency to lagging on the part of the engine is counteracted by the decrease of output. There is, however, a limit to this automatic adjustment of load and power in either direction.

If the engine pushes on until the radius O R coincides with O A, the power which can possibly be taken up by the machine has reached its utmost limit. A slight increase of lead beyond this point results in a decrease of output, and thus the engine will overpower the machine. In other words, it will begin to race and get out of step with the large machine. The part of the power curve where this will happen is shown by the dotted line. If we wish to work on any point of the dotted power curve, we must retain the mechanical coupling between the two machines. Now let us enquire what working on the dotted part of the power curve really means. It means that the small machine must lead by more than 90 deg.—that is to say, its electromotive force must be on the whole not opposed to, but in the same direction as that of the big machine. In other words, the two machines must be coupled in series. We see, therefore, that only by employing a rigid coupling is it possible to work the two machines in series. If not mechanically coupled, they can only work in parallel.

The diagram contains also a power curve on the right marked "Motor." This refers to the working of the small machine as a motor, when its armature must lag behind that of the big machine. The more we load the motor the more the lag increases, and an increase of lag will at first produce a very rapid increase in the power given out. As the lag approaches the line O B the increase of power becomes less rapid, and if we load the motor so much as to produce this lag the working will be unstable—that is to say, the slightest further increase of load will throw the machine out of step. Within the range of the power curve shown in full, the working of the motor will, therefore, be stable; and to provide for the possibility of any sudden and accidental increase of load it is best to work normally with a small lag, which is only another way of saying that the motor should be worked well within its power.

The range within which the machine may safely be worked either as a generator or as a motor is shown in the diagram by the angle marked "Normal working." This is an angle of about 45 deg. between the two extreme positions of the armature when the machine changes from being a generator giving full normal output to a motor producing full normal power.

Taking the machine as a generator only, the danger of being at any time supplied with too much power is, of course, not nearly so great as that of being overloaded when working as a motor, and we may therefore work with a greater lead than shown in the diagram. Say that we work with a lead of 40 deg., which gives a power margin of 60 or 70 per cent. In this case the range from full output to zero is comprised within an angle of from 40 deg. to zero. This refers, ot course, to a two-pole machine. For a 20-pole machine the range measured on the crank-pin circle of the engine is only 4 deg. to zero. Say we have two steam-alternators working in parallel. If the crank-pin of one engine is by 4 deg. in advance of that of the other engine, the former will at that moment give the whole output. If the advance is more than 4 deg., it will give not only the full output but also some power for driving the other machine. This would cause a surging of power between the two machines, which may make parallel working impossible. It must be remembered that, although the engine of the leading machine cannot permanently supply the power required for the whole of the output, it may do so for an instant by virtue of the energy stored in its flywheel, and the momentary overload may be sufficient to throw the machine out of step. The remedy for this trouble is, of course, to ensure that no engine shall have a tendency to lead or lag behind the other at any time, and for this reason high-speed direct-coupled engines are preferable to low-speed belted engines. If the latter must be used, it is well to make arrangements by which it is possible to time the switching-on of machines so that it shall take place when the engine is in the same part of the stroke as the other engines already working. We must, in other words, synchronise, not only the alternators, but also the engines.

The method of coupling two machines shown in Fig. 111, although it allows a current to run through the two machines in series, is not the one usually understood by the term series coupling. In Fig. 111 there may be a current circulating between the two machines, but, in addition, there must be a current flowing through the lamps. Now, in true series working, the lamp current must flow in series through the two machines, and there is no other current possible. In Fig. 111 the series current through the two machines is, so to speak, a mere accidental effect which may or

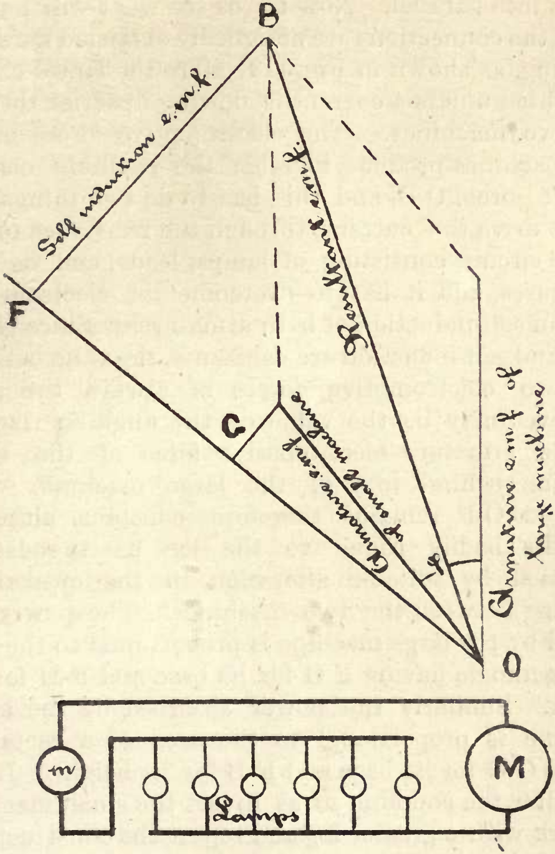

FIG. 113.

may not take place, and, as was shown, it cannot take place when the machines are running free of mechanical control. The machines in this case simply put themselves into parallel. Now let us see what will happen when the connections are designedly arranged for series working, as shown in Fig. 113. Here the lamps cannot be lighted unless a current is flowing in series through the two machines. The electromotive forces of the two machines produce together the resultant electromotive force, O B, and this has to do two things. It has to drive the current through the resistance of the whole circuit, consisting of lamps, leads, and the two armatures, and it has to overcome the electromotive force of self-induction of both armatures. Since resistance and self-induction are constants, the ratio between the two electromotive forces is always the same whatever may be the value of the angle of lag, ϕ, of the armature electromotive force of the small machine behind that of the large machine. The angle B O F remains therefore constant, although it shifts bodily round to the left as the lag is increased by suitable alteration in the mechanical coupling between the two machines. The power absorbed by the large machine is proportional to the area of a rectangle having F G for its base and F B for its height. Similarly the power absorbed by the small machine is proportional to the area of a rectangle having O G for its base and F B for its height. If we now alter the coupling so as to set the small machine to work with a greater lag and repeat the construction, we find the power corresponding to the new angle of lag. By plotting the result in polar co-ordinates we obtain power curves as before, but these are of an entirely different character.

In Fig. 114 the power curves are shown for two machines of equal electromotive force, called A and B. The machines will work with maximum power if they are so coupled up mechanically that the lag of A behind B is zero—that is to say, that the maximum electromotive force occurs in both simultaneously. This con-

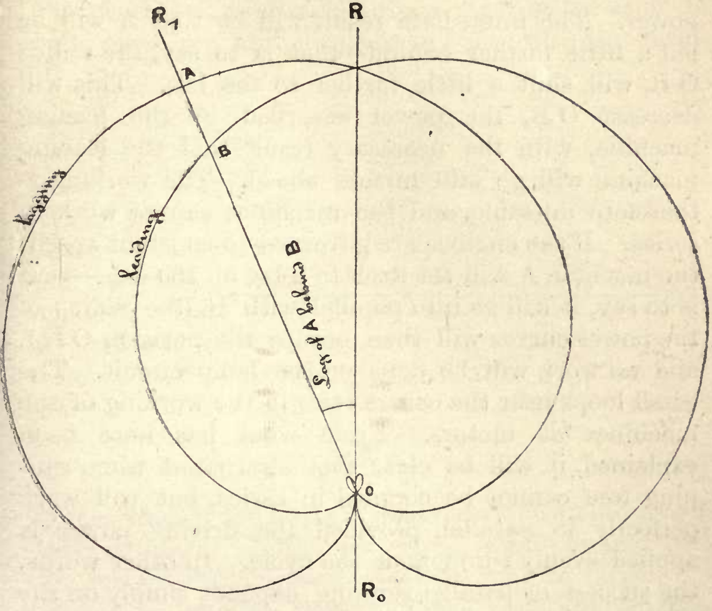

FIG. 114.

dition of working corresponds in the diagram to the radius line O R. If we now set the machine A back to O R_1, it will absorb the power O A, and the machine B will absorb the power O B, and so on for any other setting of the coupling.

When dealing with machines coupled parallel, we have seen that even after slipping the coupling bolts the machines would run on in a perfectly stable condition. This is not so in the present case. For imagine that after the coupling bolts are slipped, the engine of B, which is leading, gives a little more driving power. The immediate result will be that A will be left a little further behind—that is to say, the radius $O R_1$ will shift a little further to the left. This will decrease O B, the power absorbed by the leading machine, with the necessary result that the leading machine will go still further ahead. The working is therefore unstable, and the machines cannot work in series. If the engines are governed to constant speed, the machine A will set itself to a lag of 180 deg.—that is to say, it will go into parallel with B (the radius of the power curves will then occupy the position $O R_0$), and no work will be done on the lamp circuit. The small loops near the centre refer to the working of the machines as motors. From what has here been explained, it will be clear that alternators when running free cannot be coupled in series, but will work perfectly in parallel, provided the driving power is applied evenly throughout the cycle. In other words, the success of parallel running depends simply on the engines, and the way they are governed.

When dealing with the working of one alternator on to a pair of omnibus bars, we have assumed that the voltage on the bars is kept constant by a large machine or a number of small machines, so that the terminal voltage of the machine under consideration is fixed beforehand. It remains to investigate the case of two machines working in parallel, neither of them so much more powerful than the other as to completely control the terminal voltage. Let us take two machines, A and B, driven by engines of equal power, and let us see what will happen if the armature voltage is not the same in both machines. In the first place, it will be obvious that the terminal voltage must be the same in both machines, and must be in phase with the resul-

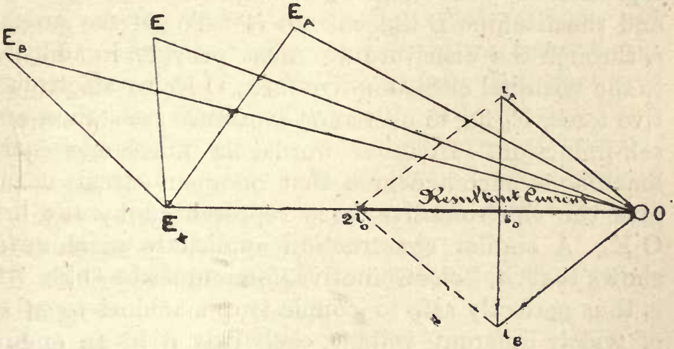

FIG. 115.

tant current if the latter is doing work on an inductionless resistance. In the next place, it will be seen that the current through each machine may be larger, but cannot be smaller, than half the resultant current. Since the driving power on both machines is the same, the output must be the same, and therefore the current must be the same.

Let us first excite both machines to the same degree, so that their armature electromotive forces shall be exactly equal, and let this be O E, giving a terminal electromotive force, O E_t , Fig. 115. The machines will now each give a minimum current, i_0, or together, $2 i_0$, and work with maximum efficiency. The line E E_t represents the electromotive force required to overcome self-induction and armature resistance, and corresponds to line B K in Fig. 106. Let us now see how we must alter the armature electromotive forces in order that the machine A may give the current represented by the line O i_A in magnitude and direction, and the machine B the current i_B. To get the current i_A through the armature we must provide, in addition to the terminal electromotive force O E_t, an electromotive force, E_t E_A, to overcome armature resistance and self-induction. In other words, we must excite the machine to such a degree that on open circuit it will give the electromotive force represented by the line O E_A. A similar construction applied to machine B shows that its electromotive force must be O E_B. It is thus perfectly safe to couple two machines together of widely different voltage, each driven by an engine exerting the same driving power. A similar construction to Fig. 115 can be applied in the case that the two engines do not exert the same driving power, the only difference being that the two currents will then be unequal, and the points i_A, i_B will not be on the same vertical. We might thus have, say, 2,500 volts in B and 1,500 volts in A before coupling up, and after the machines are coupled up they would both settle down to about 2,000 volts terminal pressure.

General Conclusions.

It will be useful to recapitulate briefly some of the conclusions of the above investigations regarding the working conditions of alternators.

All commercial alternators have an appreciable amount of self-induction. Alternators with very small self-induction must necessarily be very large, heavy, and expensive, and could not safely be used on any circuit to which there is connected another source of alternating electromotive force.

The primary effect of self-induction is to produce a lagging current and lower the terminal voltage. Its secondary effect is to place the armature current into such a phase that it produces a certain demagnetising action on the field. Thus both self-induction and armature reaction tend to lower the terminal voltage.

The effect of capacity is to produce a leading current and raise the terminal voltage, and this effect is enhanced by armature reaction.

It is impossible to couple free-running alternators in series.

It is possible to couple free-running alternators in parallel. Armature resistance does not assist in parallel working, the lower the armature resistance the better.

A certain amount of self-induction is indispensable for parallel working. If there is too much self-induction, parallel working is still possible, but the plant efficiency is needlessly lowered. If there is too little self-induction, it becomes very difficult to adjust the excitation so as to obtain the same output from all the machines.

For safe working the excitation should be pushed beyond the lowest point of the volt-ampere curve, so that an increase of excitation will produce an increase of armature current.

Two alternators of different voltage may be safely coupled in parallel, and will then give an intermediate terminal voltage.

CHAPTER XVII.

Elementary Transformer—Shell and Core Type—Effect of Leakage—Open-Circuit Current—Working Diagrams.

Elementary Transformer.

The electromotive force generated in the armature coil of an alternator is due to the change in the magnetic flux passing through the coil, the change being produced by relative movement between field and armature. If it were possible to produce a changing flux in any other way the result would, of course, be the same. We might, for instance, place a second coil close to the first in such a position that the whole or a part of the self-induced flux of one coil passes through the other, and induces, therefore, in it an electromotive force. Such an apparatus is known under the name of transformer, the coil through which we send the alternating current being called the primary or driving coil, and the other coil from which we obtain electromotive force and current being called the secondary or driven coil. The object of using a transformer is to obtain the secondary current of any desired voltage, and if the construction of the apparatus is such that the same flux must under all circumstances pass through both coils, it will be at once obvious that the electromotive forces induced must be directly proportional to the number of turns of wire contained in each. If we call F the total flux in C.G.S. measure, n the frequency, and τ the number of turns, the maximum electromotive force occurring at the moment when the flux passes through zero is

$$E = 2\,\pi\,n\,F\,\tau \quad . \quad . \quad . \quad . \quad (60)$$

as will be obvious from what was said on page 355.

The effective electromotive force in volts is

$$E = \frac{2\,\pi\,n}{\sqrt{2}}\,F\,\tau\,10^{-8} \quad . \quad . \quad . \quad . \quad (61)$$

$$E = 4{\cdot}45\,n\,F\,\tau\,10^{-8} \quad . \quad . \quad . \quad (62)$$

Conversely, if we impress an electromotive force of E volts on the coil, the current which will flow must be such as to produce a field of

$$F = \frac{E\,10^8}{4{\cdot}45\,n\,\tau} \text{ lines of force} \quad . \quad . \quad (63)$$

In this formula we neglect, for the sake of simplicity, the ohmic resistance of the coil. An inspection of (62) and (63) will show that the larger F, the smaller may be τ for a given voltage—that is to say, the less copper is needed for the coil. In order to reduce the cost and size of the coil to withstand a given voltage, we must, therefore, so design it that a moderate current may produce a strong self-induced field; in other words, we must provide the coil with an iron core.

Shell and Core Type.

We arrive thus at the apparatus shown in Fig. 116, where A is an alternator supplying current to the primary coil, P, whilst a lamp, L, obtains current from the secondary coil, S, the magnetic connecting link between the two coils being formed by the iron core, C. Some of the flux induced by the driving coil in C passes through the driven coil, but not all. In the first place, there is necessarily some lateral leakage of lines all along the surface of such a core, even if one coil only is acting, and in the next place this leakage is very much increased by the action of the driven coil. The reason lies in this, that by Lenz's law the magnetising action

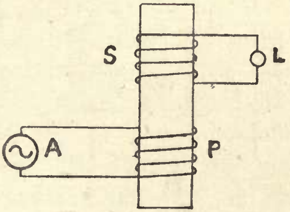

Fig. 116.

of the driven coil must necessarily be opposed to that of the driving coil, resulting in a tendency to the formation of poles in the middle and at the two ends of the core. The immediate result of this leakage is that the electromotive force generated in S must be lower than would be the case if there were no leakage. There is the further defect that the magnetic circuit is only partially formed by iron, the rest being air, which offers great resistance to the flow of lines and necessitates the expenditure of a very large driving current. This defect is, of course, easily remedied. We need only provide a continuous iron path for the lines of force, or, in other words, provide the transformer with a closed magnetic circuit.

This may be done in a variety of ways. One very obvious way is to use a core of the kind customary in Gramme armatures, and to wind the primary and secondary coils on it in Gramme fashion, or we might reverse the position of iron and copper, forming the coils of two hanks of wire and winding iron wire at right angles to and over the copper coils so as to

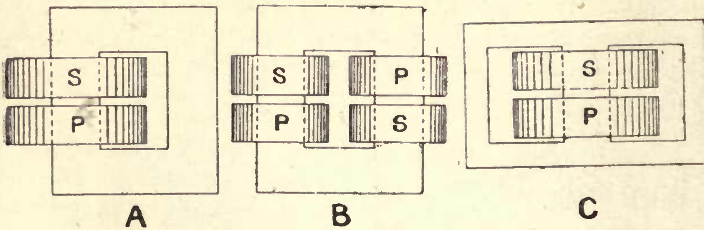

FIG. 117.

envelope them in a kind of iron shell. The former are called core transformers and the latter shell transformers. When the iron part of the transformer is made up of plates, a construction adopted in all modern designs, this distinction between the core and shell type is not always equally clear, as anything like a complete iron envelope or shell to the coils is impossible with plates. We may, however, define a shell transformer as having a double or multiple magnetic circuit, and a core transformer as having a single magnetic circuit.

In Fig. 117 are shown three typical designs, A and B being of the core type and C of the shell type. The primary and secondary coils are shown side by side, and the area of the iron core is the same in all cases. The sectional area of the coils is also the same throughout, so that, roughly speaking, the currents and voltages of all three transformers may be taken as the same. Our aim in designing a transformer must, of course, be to obtain maximum output and efficiency with a minimum weight and cost of materials. The quantity of iron in A is the same as in B, but in C it is less. Again, the amount of copper in A and C is the same, but in B it is less. We find thus that A has no advantage over either of the two other types. Of these the core type contains less copper than the shell type, but to make up for this the shell type contains less iron; and it is not possible to say offhand which is the best. To answer this question it is necessary to work out the designs for each type in detail with due regard to the magnetic properties of the iron to be used in the transformer.

Effect of Leakage.

It has already been pointed out that the driving and driven coil tend to magnetise the core in opposite directions. These magnetising forces are of course proportional to the ampere-turns in the two coils, whilst the resultant magnetisation or the flux actually passing through both coils is due to the combined action of the two coils. We have therefore to distinguish between three fields—the main field, F, comprising all the lines of force which pass through both coils, the leakage field, F_1, which passes only through the primary, and the leakage field, F_2, which passes only through the secondary coil. Strictly speaking, the leakage field is not the same for all the turns of a coil, as some leakage lines ooze out of the core before they reach the end of the coil; but the general effect may nevertheless be represented by a well-defined leakage field affecting the whole of the coil. To explain the effect of leakage, let us assume that it

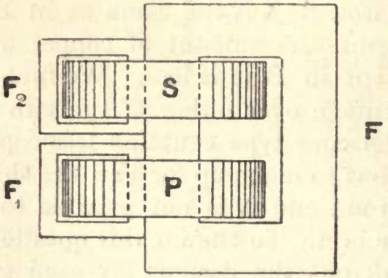

FIG. 118.

were possible to find some medium of zero permeability in which the transformer can be placed, and that a leakage path for each coil is specially provided by a closed iron circuit of such dimensions as to produce the same leakage effect as takes place if the transformer is placed in air. In Fig. 118 the main field, F, passes through both coils, the leakage field F_1 through the primary only, and the leakage field F_2 through the secondary only. The electromotive force due to the main field is in the primary,

$$E_1 = 4{\cdot}45\, n\, F\, \tau_1\, 10^{-8},$$

and in the secondary

$$E_2 = 4{\cdot}45\ n\ F\ \tau_2\ 10^{-8},$$

τ_1 and τ_2 being respectively the number of turns in the two coils. In addition, there is in the primary coil an electromotive force due to the leakage field of

$$e_1 = 4{\cdot}45\ n\ F_1\ \tau_1\ 10^{-8} \text{ volts},$$

and in the secondary an electromotive force of

$$e_2 = 4{\cdot}45\ n\ F_2\ \tau_2\ 10^{-8} \text{ volts}.$$

The effect of leakage will evidently be the same as if coils having self-induction were inserted outside the transformer in the primary and secondary circuits.

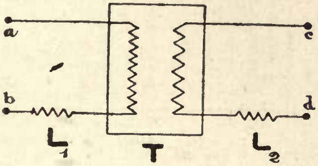

FIG. 119.

Diagrammatically, this is represented in Fig. 119, where T represents a transformer having no leakage, and L_1 and L_2 choking coils of such self-induction as to be equivalent to the apparatus shown in Fig. 118. The terminals *a*, *b* and *c*, *d* are the terminals of the transformer, and the electromotive force measured between each set of terminals is now influenced by the action of the choking coils in the same manner as in a real transformer it is influenced by magnetic leakage.

It will be shown presently that under certain conditions the effect of leakage is to lower the voltage on the secondary terminals as the output of the transformers is increased. If the primary terminals are joined to mains in which the pressure is kept constant, we get a certain electromotive force on the secondary terminals when the secondary circuit is open. If we now close the secondary circuit by the insertion of lamps we find a drop in voltage. This is partly due to the ohmic resistance of the driving and driven coil, and partly to magnetic leakage. The more lamps we put on, the more is the pressure lowered, and it becomes

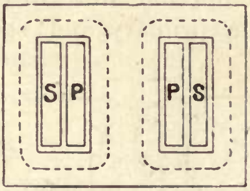

Fig. 120.

important to so design the transformer that this "voltage drop" shall remain within reasonable limits. In other words, we must reduce the resistance of the windings as far as possible, and so group the coils as to have as small a leakage field as possible. In the design shown in Fig. 117c, the leakage field will be considerable, but if we group the coils as shown in Fig. 120, it will be very much reduced, because the magnetic resistance of the leakage path is increased. In the same way is it possible to reduce the leakage of Fig. 117b by placing the coils, not side by side, but one within the other.

Open-Circuit Current.

A transformer working on open secondary circuit must take from the primary mains sufficient current to produce that field which will just balance the primary voltage. This current is called the open-circuit current, and it is, of course, desirable that it should be as small as possible. If the magnetic properties of the iron are known, we can easily calculate the magnetising current. This current has to do two things: it must supply the power wasted in hysteresis and eddy currents, and it must produce the magneto-motive force required to drive the total flux round the magnetic circuit. Since flux and electromotive force are in quadrature, no power is spent in producing the flux, and the corresponding component of the magnetising current is a wattless current, whilst the other component giving power must, of course, be in phase with the electromotive force, and is a watt current. Thus, if i_o is the total open-circuit current, and i_h and i_μ its two right-angular components required respectively for the supply of power and magneto-motive force, we have

$$i_o^2 = i_h^2 + i_\mu^2.$$

The apparent watts supplied on open circuit are $i_o E_1$, and the true watts $i_h E_1$, the ratio between the two being called the power factor* $= \frac{i_h}{i_o}$. This is a coefficient which approaches the nearer to unity, the smaller i_μ is in comparison to i_h, and is therefore a rough kind of measure for the greater or lesser conduc-

* Fleming, *Proceedings* Institution of Electrical Engineers, vol. XXI.

tivity of the magnetic circuit. In modern transformers the power factor varies from 70 to 80 per cent. If the power factor is 71 per cent., it means that the number of ampere-turns required to produce the field is the same as the number required for supplying the waste power. If the power factor is larger than 71 per cent., it means that the power wasted requires more current than the production of the field.

We can now proceed to calculate the open-circuit current. Let μ be the permeability of the iron (generally between 1,000 and 3,000), l the length of the magnetic circuit shown by the dotted lines in Fig. 120, and A the cross-sectional area of this circuit, then to produce the induction, B, a magnetic force, $H = \frac{B}{\mu}$ is required. Since $H = \frac{4 \pi i_\mu \tau_1}{l}$, we have in absolute measure

$$i_\mu = \frac{B}{\mu} \frac{l}{4 \pi \tau_1}.$$

This refers, of course, to the crest of the wave. If we wish to obtain the effective current in amperes we must multiply by 10 and divide by $\sqrt{2}$, which gives

$$\text{Magnetising current in amperes, } i_\mu = \cdot 565 \frac{B}{\mu} \frac{l}{\tau_1} \quad (64)$$

If w represents the loss in watts per cubic centimetre of iron per cycle at the induction, B, on account of hysteresis and eddy currents, then the total loss is

$$i_h E_1 = w \, l \, A \, n.$$

Inserting E_1 from (62) we have

$$\text{Watt current in amperes } i_h = \cdot 225 \frac{w \, 10^8}{B} \frac{l}{\tau_1} \quad . \quad (65)$$

The value of w can be found from the hysteresis curve, page 132, with a suitable addition for eddy currents, which has to be determined by experience for every type of transformer, but is generally very small, especially if the thickness of plates does not exceed 10 mils.

It is, of course, very important that the open-circuit current shall be small. When transformers are used in central-station work they must necessarily be connected to the mains for a much longer time than corresponds to their period of full or even moderate output. Indeed, in the majority of stations as at present arranged, the transformers are continuously under pressure, but are only used for some few hours each day. During the rest of the time the transformer simply wastes primary current and power, and to reduce this waste to a minimum is one of the points aimed at by the makers of modern transformers. Formulas (64) and (65) show us how this may be done. We must work at a low induction, have a short magnetic circuit, and employ the very softest iron that is procurable. One method of reducing the induction is to increase the frequency, and we accordingly find in practice that the same transformer worked at a higher frequency has a smaller open-circuit current and a smaller loss.

It must, however, be pointed out that the length of the magnetic circuit (the dotted lines in Fig. 120) cannot be reduced without cramping the space available for winding, whilst, on the other hand, too great a reduction in B (by increasing the area of the core) lengthens both the perimeta of the coils and the magnetic circuit, so that the best design for any given type of transformer must always be a sort of compromise between a number of contradictory conditions. A low electric resistance is important on account of voltage drop and heating at full load; a low magnetic resistance is of importance on account of open-circuit current and waste of power at no loads or light loads. So both the electric and the magnetic circuit should be as short as possible.

Working Diagrams.

We may use the clock diagram to explain the working of a transformer, and in order to begin with a simple case we shall assume that the primary and secondary coils are so well interlaced that there is left no space between them for the passage of leakage lines; in fact, that there is no magnetic leakage. It will also be convenient to assume a transforming ratio of 1 : 1 in this and in the following cases. There is no need to draw the diagram for any other ratio, since we can always imagine one of the windings to be afterwards so changed as to produce any other ratio. All pressure effects in the altered coil will then change in the same ratio as the winding, and all current effects in the inverse ratio. Thus if we double the number of turns we double the electromotive force and halve the current; we quadruple the resistance and double the loss of pressure by resistance. The total ampere-turns produced by the coil remain, however, the same. Not only can we imagine both coils to have an equal number of turns, but we may assume that this number is 1—namely, that the primary and secondary each consist of one turn of wire, having a sectional area equal to the aggregate area of the wires in a coil of many turns. This assumption, as will be seen presently, is made so that we may use the same scale for amperes and ampere-turns in the clock diagram.

We assume constant pressure at the primary terminals and a variable resistance in the secondary circuit—this resistance to be non-inductive. Let, in Fig. 121,

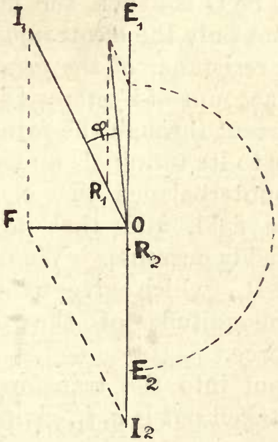

FIG. 121.

$O I_2 = i_2$ represent the effective current in the secondary circuit produced by the impressed electromotive force, $O E_2$. Let the electromotive force required to overcome the resistance of the coil itself be represented by the length $O R_2$, then the terminal pressure available for doing work in the external circuit is $e_2 = R_2 E_2$. Let O F represent the effective ampere-turns required to produce such a field as will give the impressed electromotive force e_2. The line O F must be at right angles to O E_2, and to find the position of the primary current we must draw such a parallelogram as will give O F as the resultant. Thus we find O $I_1 = i_1$, the primary current. The power wasted has to be supplied by the primary current, and we may represent it as the product of this current with a certain electromotive force. Let this be O R_1, then the length of this line will represent not only the electromotive force lost in overcoming the resistance of the primary coil, but also that wasted on account of hysteresis and eddy currents. To drive the current through the primary coil, we must therefore supply to its terminals an electromotive force which must counterbalance the electromotive force induced by the field, and that lost in resistance, hysteresis, and eddy currents. We make O $E_1{}^1$ = O E_2 and $E_1{}^1 E_1$ = O R_1, which give us O $E_1 = e_1$ as the position and magnitude of the primary terminal electromotive force.

The power put into the transformer is $e_1 \, i_1 \cos \phi$, and the power taken out is $e_2 \, i_1$, giving the efficiency,

$$\eta = \frac{e_2 \, i_2}{e_1 \, i_1 \cos \phi}.$$

All the quantities plotted in Fig. 121 are obtained from the formulæ previously given; O F being the effective ampere-turns calculated from (64). The transforming ratio is e_2/e_1, and if there were no losses this should be unity. It is, however, obvious from the diagram that $e_1 > e_2$, making the actual transforming ratio on full load less than unity. If we now increase the external resistance in the secondary circuit and repeat the construction, we shall find that the line representing the primary current becomes shorter and less inclined, whilst e_1 becomes slightly reduced. Since, however, e_1 was to be kept constant, we must lengthen all the lines in the diagram proportionately to satisfy this condition. It is obvious that, partly for this reason and partly because of the reduction in $O R_2$, the pressure on the secondary terminals must increase—that is to say, as we reduce the load on the transformer the secondary voltage rises. At the same time the lag, ϕ, increases. The diagram has been drawn out of scale to make the construction clear. In reality, the line O F is very much shorter in comparison with $O I_2$ than shown. $O I_2$ represents several thousand ampere-turns, and O F only some hundreds, or even less than one hundred, so that the inclined lines all become much steeper. The voltage losses, $O R_2$ and $O R_1$, are also smaller than shown, the result being that the total voltage drop becomes limited to a few per cent. of the terminal pressure.

We have next to investigate the effect of magnetic leakage. As it has already been shown how resistance and other losses can be treated in the clock diagram, it will not be necessary to include these losses in the following investigation. To do so would needlessly complicate an already intricate problem. We shall, therefore, assume that we have to do with a transformer which, except for leakage, is perfect—that is to say, which has neither resistance nor hysteresis nor eddy-current losses.

Those who have to do with transformers have long

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ago discovered that although a transformer when working on incandescent lamps may have an exceedingly small voltage drop, the same transformer when working on arc lamps or motors shows a very much greater drop. Again, a transformer when working on a condenser—such as a water resistance, or, better still, a long line of concentric cable—may show a negative drop—that is to say, may actually give a voltage rise when loaded. In what follows no attempt will be made to give a quantitative solution of the problems connected with the working of transformers under various conditions. It will suffice to show in a general way the reasons why transformers show a drop or a rise under certain circumstances, and the reader will then be able to work out specific cases quantitatively for himself.

The first case that we take is one which occurs very frequently in practice—namely, when the transformer works on an inductionless resistance, such as a bank of incandescent lamps. In Fig. 122, T is the transformer, and R the inductionless resistance. In the working diagram above, the same letters as occur in Fig. 121 refer to the same quantities, and need no further explanation. Let O E_{s2} be the electromotive force of self-induction produced by the leakage field F_2, then O $E_2{}^1$ must obviously be the induced electromotive force in the driven coil. The resultant ampere-turns producing the main field must be at right angles to it and a quarter period in advance. This gives the line O F, and we can now find the line of primary current O I_1. Now the primary current has its own leakage field, F_1, producing an electromotive force of self-induction which must be counteracted by the electromotive force O E_{s1}, a quarter period ahead of O I_1. O E_1^1, the induced electromotive force in the primary, must be equal and opposite O E_2^1, and to find

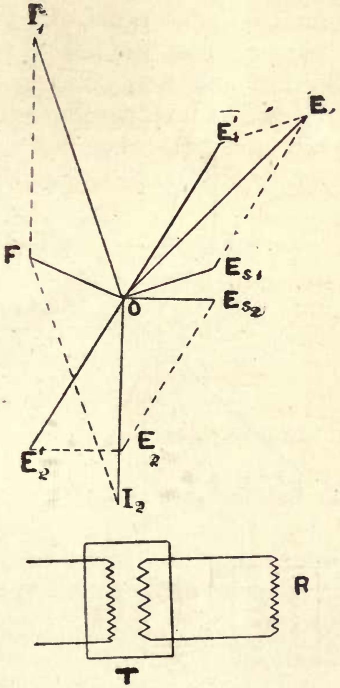

FIG. 122.

the terminal electromotive force of the driving coil we combine O E_{s1} and O E_1^1, which gives us O E_1. An inspection of the diagram shows at a glance that the primary terminal electromotive force must be greater than the secondary terminal electromotive force. It is obvious, also, that if we do not allow a secondary current to flow, the two terminal electromotive forces

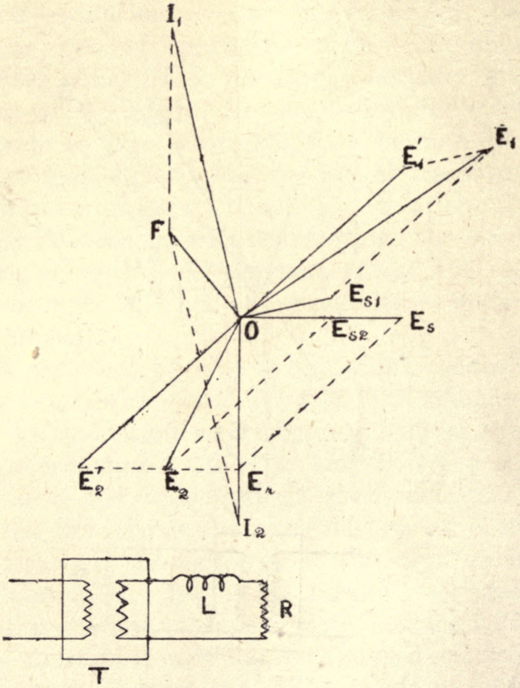

FIG. 123.

are equal. We see, therefore, that the transformer, although giving 100 per cent. efficiency, has a voltage drop when working on incandescent lamps.

We next take the case of there being in the external circuit a self-induction in series with the resistance, Fig. 123. Such a case arises if arc lamps or motors only are worked by the transformer. O E_{s2} represents the voltage due to the leakage field in the driven coil as before, and the length E_{s2}, E_s represents the voltage absorbed by the self-induction, L. The voltage absorbed by R is O E_r, and the terminal voltage is O E_2. Adding the voltage due to leakage on the left we find $E_2{}^1$, the induced voltage in the driven coil. That in the driving coil is O $E_1{}^1$, the rest of the

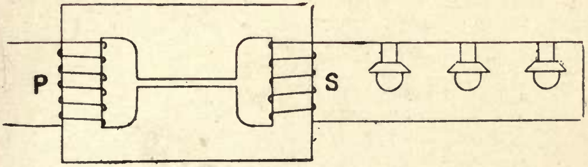

FIG. 124.

diagram being constructed as before. The voltage drop is now greater.

(Transformers for series arc lighting are made purposely with a large leakage field so as to give a heavy voltage drop, and consequently an approximately constant current. The design is shown diagrammatically in Fig. 124.)

It is sometimes necessary to work incandescent lamps and arc lamps or motors in parallel off the same transformer. This arrangement and working diagram are shown in Fig. 125. The inductive circuit must, of course, have some resistance, and will generally also have some counter electromotive force in phase with the current. The latter must therefore lag

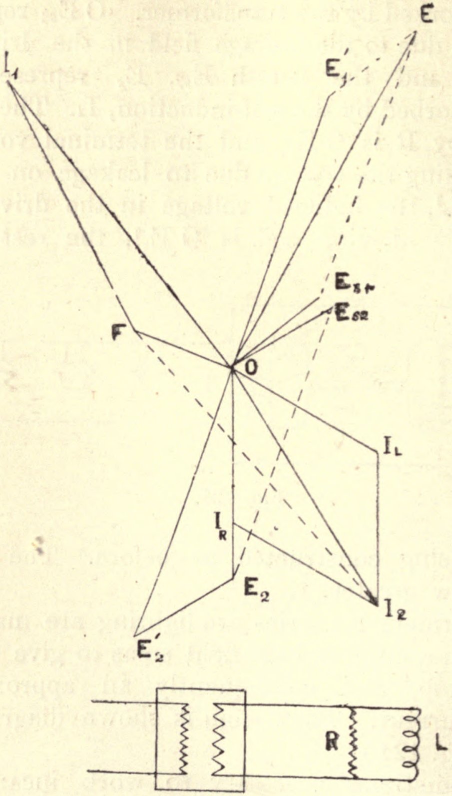

Fig. 125.

behind the terminal electromotive force of the transformer by less than a quarter phase. The current passing through R is O I_R, and that passing through L is O I_L, giving the resultant current O I_2. The rest of the diagram is constructed as before. It will be seen that the voltage drop is very considerable, and this conclusion is borne out in actual work. Central-station engineers know that if arc lamps or motors are

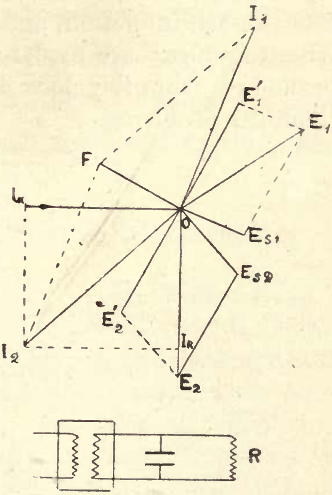

FIG. 126.

added to an incandescent light circuit fed by a transformer the voltage drop is increased.

The last case we take occurs when a transformer has to supply current through a long concentric main of considerable capacity. Here the current passing through the driven coil is the resultant of the watt current which is in step with the terminal electromotive force, and the capacity current which leads by a quarter period. In Fig. 126 the previous lettering has been retained, and O I_K represents the capacity current. The construction is so simple as to need no further explanation, and it will be seen that, instead of there being a drop, there is actually a rise of terminal voltage when the main is switched on. Various other combinations of resistance, self-induction, and capacity could be given, but the examples here explained will suffice to show the method of using the clock diagram for the solutions of all similar problems.

CHAPTER XVIII.

Examples of Alternators—The Siemens Alternator—The Ferranti Alternator—Johnson and Phillips's Alternator—The Electric Construction Corporation's Alternator—The Gulcher Company's Alternator—The Mordey Alternator—The Kingdon Alternator.

Examples of Alternators.

The following is a slightly condensed reprint of a series of articles published by Mr. R. W. Weekes in the *Electrical Engineer* in July and August, 1892, descriptive of the alternators which were on view at the Crystal Palace Electrical Exhibition held in the beginning of that year. The information then collected is quite up to date now, and will be found useful for reference.

The alternators made at present can be classified into two characteristic divisions: First, those in which the magnetic lines have a fixed path of constant resistance, and the electromotive force is produced by conductors intersecting this path; and second, those in which the resistance and shape of the magnetic circuit is varied, and the electromotive force is produced by the change of induction caused by such variations.

The first class includes all the older designs, and may again be subdivided into two divisions—*i.e.*, those alternators which have moving armatures and fixed magnets, and those in which the field magnets revolve and the armature is fixed. In all these machines collecting rings are needed either for the armature or the exciting circuit, but as no commutation is required the collectors can be easily designed to run cool and sparkless.

In the second class of alternators the collectors can be dispensed with, and the manufacturers of these machines make this fact one of their claims for support.

Another way in which alternators have been classified is as to whether they have iron in their armatures or not. The disadvantage of the use of iron is loss from hysteresis, but owing to the low induction used in the armature iron and to the small amount required, this loss can be reduced to about 1 per cent. of the output by careful design. On the other hand, the presence of an iron core makes the armature mechanically stronger.

The following are the principal requirements which have to be considered in a good alternating-current dynamo to be worked at a high pressure: (*a*) Perfect mechanical design to ensure that the machine can be run continuously; (*b*) strength and stability in the armature; (*c*) perfect insulation in the armature coils, and arrangements so that little difference of potential shall exist between two adjacent parts; (*d*) ease of repair of a defective coil when needed; (*e*) the collecting gear should give no trouble; (*f*) efficiency.

The Siemens Alternator.

The magnets are constructed with wrought-iron cores, fitting into the cast-iron frame, which is cast in two halves and clamped together. The exciting coils are wound on brass bobbins, and so connected that the

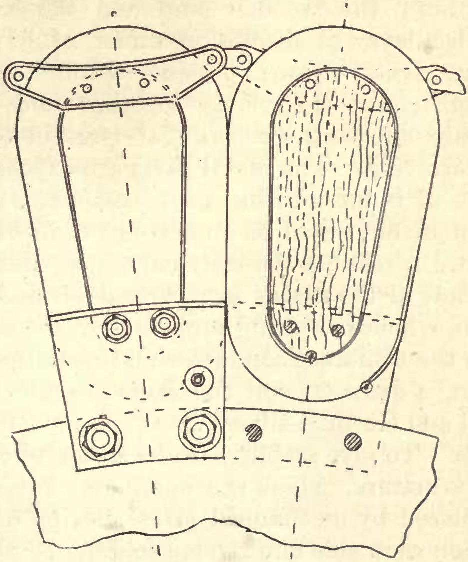

Fig. 127.

adjacent poles round the frame have always opposite polarity, and also that any two magnets facing each other are of opposite polarity. Thus, the lines of force pass always through the armature at right angles to it.

The details of the construction of the armature can be well seen in Fig. 127. The centres of the bobbins are made of hard wood and bound with brass. Then a layer of insulation, such as fibre or press-spahn, is cemented on, and the conductor is wound tightly round this. The connections are made by two wires, which are led through well-insulated holes in the supporting plates. These plates are made of German silver, and clamp both the wooden core and the conductors. There is a layer of insulation under each plate, and the two bolts shown passing through the wood core both clamp the plates together and also take the strain due to the centrifugal force in the bobbin when revolving. The metal parts are exposed to some changes of induction due to the magnets, and hence German silver is used, as on account of its high specific resistance it reduces the eddy currents generated. The other ends of the plates are slipped on and bolted to the hub, which has a ring projecting of the same thickness as the armature conductor and insulation. When renewing a damaged coil the bolts are taken out, and the coil and German-silver plates, etc., are removed as a whole. To give stability to the outer circumference of the armature, where the conductor might tend to be displaced by mechanical stress, strips of brass are placed on each side and riveted together. The ends of these strips are linked together and thus form a chain completely round armature.

To reduce noise the spaces between the polar faces are filled in with wood, so that the surface presented to the armature is continuous. The collectors are two copper rings, and a pair of ordinary copper-wire brushes are used to take the current off each.

The following particulars refer to a low-pressure alternator of this type: Output, 80 volts 500 amperes at 400 revolutions; complete cycles, 66 per second; number of poles, 20; weight complete, 2 tons; floor space, 3ft. 6in. by 5ft. 6in.

The Ferranti Alternator.

This is in first principles like the Siemens machine described above, but the constructive details are different, especially as regards the armature. The cores round which the conductor is wound are made of laminations of brass and asbestos, Figs. 128 and 129. The radial brass strips have a longitudinal corrugation pressed in them, so that when placed together these form keys to prevent any individual strip being displaced. The thickness of the asbestos between the laminations is increased along the radius, so as to give the necessary angle to the core. When the core is built up to the correct shape, it is clamped firmly, and a brass connection is burnt on to the thin end. This is done by running molten metal over the ends of the strip till they fuse together. The core is then machined at both ends to the proper shape, and the solid brass end is drilled for the bolt, which acts as an electrical and mechanical connection. The inside end of the copper conductor is brazed on to this solid end. This copper strip also has a corrugation in it to prevent side displacement, and is wound bare with a strip of fibre as insulation between the succeeding turns. A large strain is kept on the strip while being wound, and this forces the insulation well into the groove, which securely keys the turns together. In mounting the coils, one carrier is provided for each pair of coils, as shown in Fig. 128. There is a sheet of fibre insulation on each side of the coil when placed in the carrier, but the bolt which secures the coil *in situ* also connects the inner end of the conductor to the carrier. Hence the carrier connects the two inside ends of the coils it holds. The outside ends of the conductors are joined where the coils in adjacent carriers touch. This

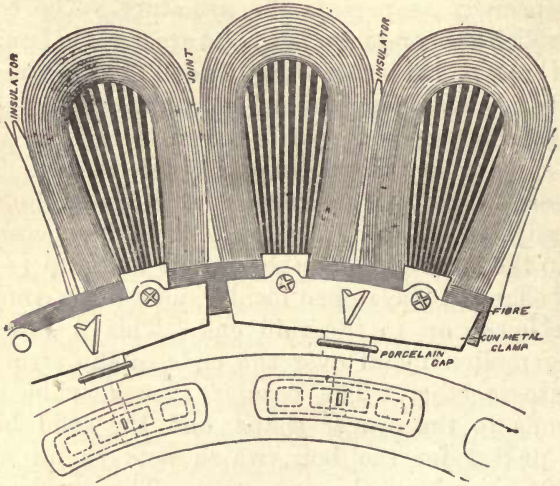

Fig. 128.

is done by brazing the two outside ends together before the coils are fixed into position. With this system of connection, it is clear that the individual carriers must be well insulated both from the frame and from each other. The shank of the carrier is first insulated with porcelain where it passed into the hole in the driving ring. This ring is hollow inside, and a large rectangular nut is then keyed on to the shank so that it leaves a small space all round. Sulphur compound is then run into this space, and both firmly clamps the nut by expansion and also insulates it. The porcelain insulation is used to give a greater surface insulation, and also for fear a spark should ignite the sulphur if it

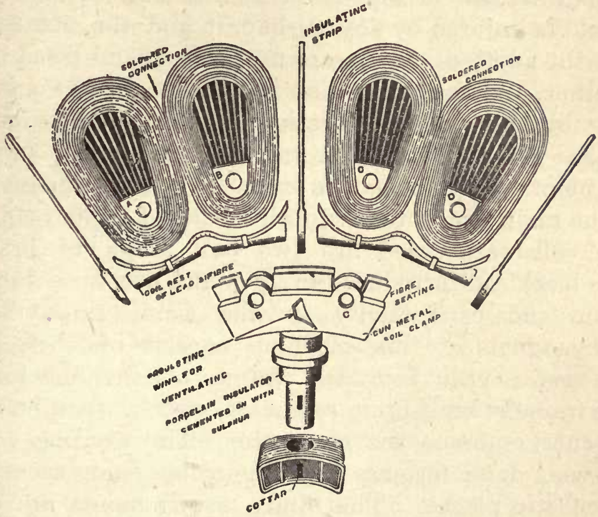

FIG. 129

was exposed. The two halves of the armature are connected in parallel so as to reduce the maximum voltage between any two coils.

In a 245-kilowatt machine, when working at 2,400 volts, we get 200 volts produced in each bobbin, and hence a maximum difference of potential of 400 volts between the two adjacent bobbins in one carrier. At this place ebonite strips are introduced to tighten up the armature coils, and these strips thus give special insulation where it is needed. The lower ends of the coils are blocked up in the carrier by means of insulated metallic segments, shown in Fig. 129. This method of connecting the armature is exceedingly convenient in case of repairs being needed. If one coil should be injured by any mishap, it and the one next it in the adjacent carrier are unclamped and lifted out together, and the connection is completed to two new coils by the simple operation of bolting them into place. The connections from two diametrically opposite points on the armature are taken through the inside of the main shaft to two well-insulated copper rings. The collectors used are two half rings of brass, with blacklead introduced to give the necessary lubrication and conductivity at the same time. The field magnets of this machine consist of wrought-iron slabs cast into the frame of the machine. The frame is built up in segments, which, when bolted together, embrace the armature. The exciting coils are wound on formers and slipped on, being securely fastened in places. The oiling arrangements are exceedingly well devised. The oil is forced up in the bearing at the underside of the shaft, and so tends to float it. The oil pumps at either end are worked by means of eccentrics fixed to the shaft.

The efficiency of the machine should be high, but although the lamination of the core will reduce the Foucault currents in the brass strip, it is probable that this loss will still be higher than hysteresis loss in an iron-cored armature. The brass, however, gives exceptional stability to the armature. The power required to excite this machine is supplied by a current of 150 amperes at 30 volts, which is equal to 1·85 per cent. of the total output.

The following are the details of the alternator: Volts, 2,400; amperes, 100; speed, 335; complete periods, 66 per second; number of bobbins, 24; conductor, 40 mils by $\frac{5}{8}$in. wide; number of turns per bobbin, 40; insulation, vulcanised fibre, 20 mils thick; weight of armature conductor, 250lb.; area of pole face, 126 square inches; exciting coils wound with 522 turns of 160 mils wire; weight of whole machine, 18 tons 7 cwt.; floor space, 9ft. 9in. by 13ft. 8in.; height, 9ft. 3in.

Johnson and Phillips's Alternator.

In this machine, which has been designed by the author, the arrangement of the field magnets is different from the two last described, as the opposite poles have in this case the same polarity. The magnetic lines of force pass from one magnet to the next on the same side of the machine through the iron core of the armature. So, though a set of poles is required on each side of the armature to give magnetic and mechanical balance, yet the magnetic lines generated by the set of poles on each side of the armature are distinct in their action.

The magnet cores are made of wrought iron, with expanded polar faces, and inlet into cast-iron frames, as seen in Fig. 130. The pole face is made nearly rectangular, so that there shall be an equal number of lines of force entering the armature core at all distances from the centre. This is to prevent as much as possible the flow of lines from one lamination of the core to the next, which would produce eddy currents

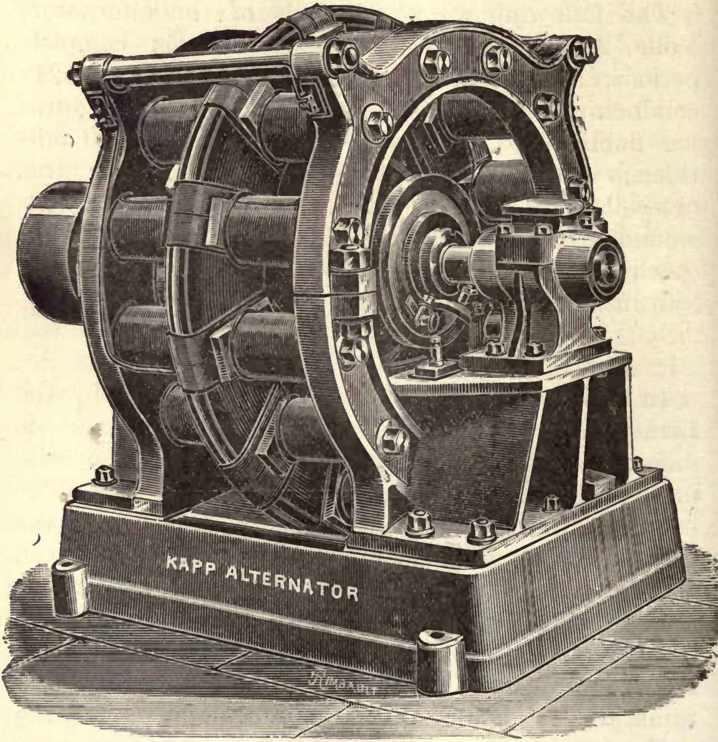

FIG. 130.

in the iron. The cast-iron frame into which the cores are fastened is made in two halves, and bolted together to facilitate the removal of the armature when needed.

In the large machines, each ring of magnets is arranged to withdraw along the axis of the armature by means of a rack and pinion gear to enable the armature to be examined *in situ.* The armature core is built up by winding charcoal-iron strip with paper insulation of the full width of the core on to a cast-iron spider ring till the desired depth is obtained. The number of arms in the spider is half the number of poles in the machine, so that two coils are wound in each space between the arms. The outside of the core is then made up with hard wood to the desired contour where the coils have to be wound. The wood blocks all round are secured in position by bolts, and bound on by means of steel wire wound in the circumferential groove left for this purpose. The coils are direct driven by means of shoulders on the wood and the spider arms. The core is insulated with mica and two layers of vulcanised fibre where the conductor has to be wound, and the cast-iron spider is fitted with ebonite caps where needed for insulation.

In the small machines all the armature coils are connected in series, but in the larger central-station alternators the two halves of the armature are connected in parallel to reduce the voltage between any two adjacent coils. The connections are made to bolts in the spider ring, which are well insulated from the spider itself. The current is collected by ordinary copper brushes, of which there are two to each collecting ring. This ring has its collecting surface vertical, and not horizontal, as is the case in the other alternators. This is done to enable the collector to be placed well inside the magnet frame, so as to render the dangerous parts less exposed. The magnet poles are filled in to prevent noise, as already explained.

The following particulars refer to the two machines given in the list. The 15-kilowatt alternator is designed to give 2,000 volts at 900 revolutions. The weight complete is 2 tons 5 cwt., and floor space 4ft. 9in. by 3ft. It has 10 poles, and hence gives 75 complete cycles per second. The 120-kilowatt machine gives 1,000 volts and 120 amperes at 600 revolutions. Weight complete 6 tons 3 cwt., and floor space 5ft. 9in. by 6ft. 9in., height 5ft. 6in. It has 20 poles and gives 100 complete periods per second. The power used to excite the field is 1·8 per cent. of the output. The weight of the machine is made up as follows:

	cwt.	qr.	lb.
Bed-plate	47	1	23
Armature	17	1	26
Two magnet rings	17	3	14
Forty magnet cores	17	2	20
Copper on magnets	13	3	0
Pulley	6	3	14
Other parts	2	0	18
Total	123	1	3

The Electric Construction Corporation's Alternator.

The 30-kilowatt alternator made by this firm has its field magnets revolving inside a fixed armature. The construction of the field magnets is as follows: The 18 poles, which are made of wrought iron of 3in. by 6in. section, are inlet radially into a solid ring of wrought iron, which is shrunk on to a cast-iron hub or spider. This hub is securely keyed on to the shaft, and the ends are closed by thin iron plates to prevent loss of power due to the air current caused by the supporting arms. The exciting coils are wound on sheet-iron formers, which are securely fastened on to the radial magnets. This requires careful attention, as the centrifugal force at normal speed tending to throw a

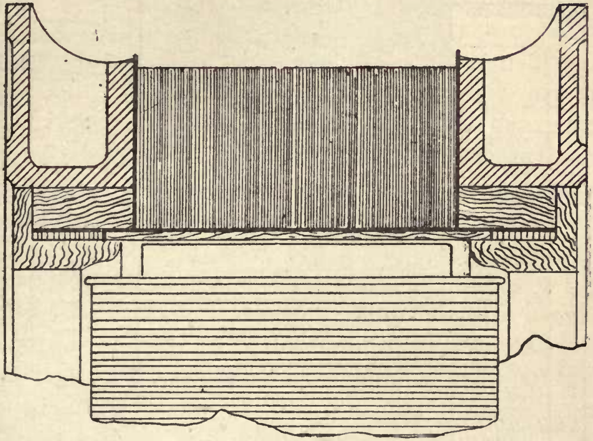

FIG. 131.

bobbin off the magnets, is nearly 150 times the weight of the bobbin. The coils are connected up in series, and the two ends are led to collecting rings fixed on the shaft. The exciting current is supplied to these two rings by means of two pairs of copper-gauze brushes. This enables one brush to be adjusted and bedded without affecting the working of the machine. The iron core of the armature is in this case external to the conductors, and consists of a ring built up of thin charcoal-iron segments, the lamination being at right angles to the axis of the machine, Figs. 131 and 132. These segments are clamped

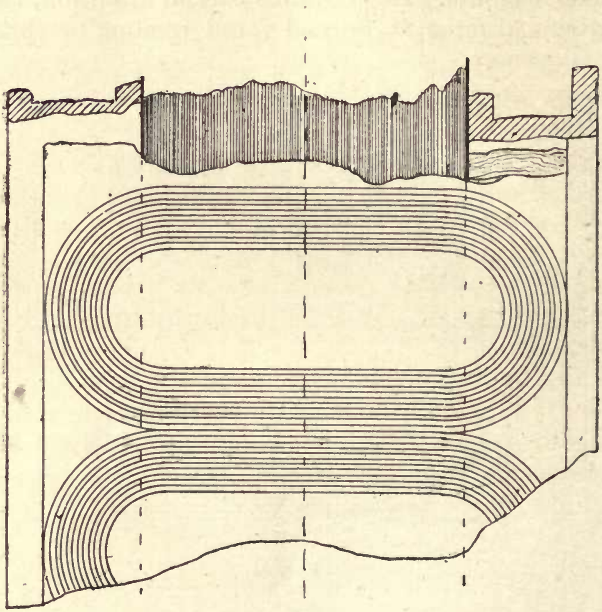

FIG. 132.

between the two halves of the frame, which are made of cast iron. Their internal diameter is larger than that of the wrought iron, and the spaces so left at the ends are filled in with wood, which forms the insulating support for the armature coils. These coils are wound flat on wooden cores, and then laid on the inner surface of the wrought iron with a sheet of insulating material between. Another wooden ring at each end keeps the coils in position, and securely clamps both the wooden core and the conductors. This ring is made in segments, so that when it is necessary to take out a coil, one segment only need be displaced. The breadth of the wooden cores is about equal to twice the breadth of the winding. Thus on the armature surface the cores and the conductors occupy equal space alternately, as shown in Fig. 132. The connections between the armature coils are made in the channels cast in the frame. There are spaces left in casting for the conductors to come through, and the joints are well protected by the external lagging of wood. As the armature is fixed, the two ends, after all the coils are connected in series, are led to two terminals, which also are usually placed under the wood lagging. This ensures that no dangerous shock can be got from the machine, as the high-tension parts are inaccessible. The magnets for the machine given in the list are excited from a 100-volt circuit, and take 16 amperes; thus 5·3 per cent. of the total power given out is required to excite the field.

The details of this alternator are as follows: Output, 1,000 volts 30 amperes at 600 revolutions; 60 complete periods per second; weight complete, 2 tons 15 cwt.; floor space, 4ft. 6in. by 4ft.; height, 4ft. 4in.; armature, 36in. diameter, 6in. active length; conductors, ·015 square inch section, wound in 12 coils of 34 turns each; 12 poles of wrought iron, 3in. by 6in.

The Gulcher Company's Alternator.

In this machine, designed by Mr. G. Fricker, the field-magnet details are somewhat similar in shape to those of the alternator just described, but there the similarity ends. The magnets consist of a heavy star-shaped casting, having 12 radial arms, Figs. 133 and

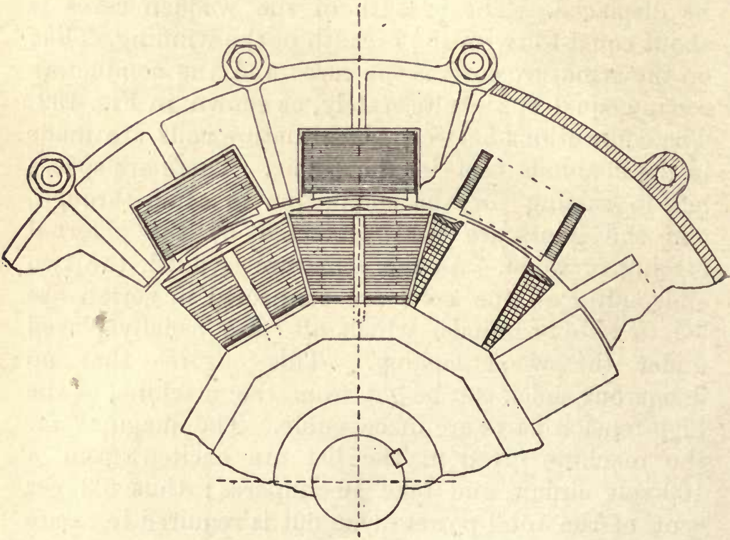

Fig. 133.

134. This casting is mounted on the spindle. The exciting coils are wound on bobbins made of sheet iron with brass flanges, and the depth of winding increases with the radius, so as to get as much wire on as possible. The bobbins are slipped on and held in position by two iron bolts which screw into the

hub, and are riveted into the upper flanges of the bobbin. These bolts take all the centrifugal strain, which tends to throw the bobbins off. The power required to excite this machine is 2·33 per cent. at three-quarter full load. This is lower than in the last-described machine having the same type of field

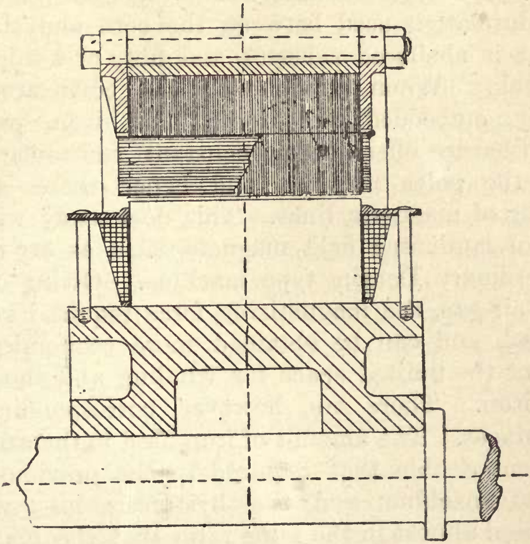

FIG. 134.

magnets, owing to the small air-gap used. The armature can best be considered as an improvement on the Lontin pole type, the improvement consisting in filling up the spaces between the poles with iron, to prevent the variations of magnetic flux. In this machine the armature is built up of charcoal-iron plates ·016in. thick, placed radially, and so shaped that, when completed, they form a complete ring with 24 slots in it, into which the armature coils can be placed. These plates are clamped together in a frame by bolts passing through the cast-iron end rings. The armature conductor used is copper strip ·162in. by ·08in., and 25½ turns of this are wound on edge on a former. The insulation used between the core and the conductor is shellaced asbestos and fibre or a thin layer of teak. When completed, the coils are completely embedded in iron. The iron path in the armature offers nearly constant resistance wherever the poles may be, and hence there is little surging of magnetic lines. This does away with the need of laminated field magnets, such as are used in the ordinary Lontin type machine. Owing to the short air-gap, the magnetising force required is much reduced, and can be obtained more economically in spite of the limited space for winding and the use of cast iron. There are, however, corresponding disadvantages. The amount of iron used in the armature is about double that required in the previously described machine, and the hysteresis loss will be increased almost in the same ratio that the magnetising power is decreased. The other objection is, that in machines with embedded conductors, the armature reaction is much increased owing to the small air-gap. Thus, the exciting current has to be varied considerably to prevent alteration in the terminal pressure when the load is varied. Also with high voltages the insulation of the embedded coils would be a rather difficult matter. The coils can be removed individually if one should become damaged, and to facilitate this the whole armature is placed on slides, and can in a short space of time be moved along parallel to the axis till clear of the armature field. A multiple threaded screw is used to obtain the necessary force, and at he same time a fairly rapid motion. The slots in the armature iron into which the coils are placed cause a slight variation of induction at the surface of the magnets. To prevent eddy currents in the iron, grooves about $\frac{1}{16}$in. broad are turned in it $\frac{1}{4}$in. deep. The machine makes very little noise when working.

The following are the details of the 30-unit alternator given in the list: Output, 100 volts 300 amperes at 700 revolutions; 70 complete periods per second; weight complete, 30cwt.; floor space, 4ft. 10in. by 3ft.; height, 4ft.; armature conductor, ·08in. by ·162in., wound in 12 coils of $25\frac{1}{2}$ turns each; weight of conductor, 39lb. 12oz.; magnets of cast iron, of 10in. by $5\frac{1}{4}$in. section.

The Mordey Alternator.

This machine differs in both principle and detail from any before described. In this alternator the direction of the lines of force through the armature coils is never reversed, as in all previously described machines, but the electromotive force is produced by a variation of the magnetic field through the coil from the maximum to practically zero. This is done by having twice as many coils as there are poles. Then when one coil is directly opposite a pole, and hence has the maximum field throughout it, the adjacent coils are midway between two poles, and have practically no magnetic flux passing through them. The field-magnet design for obtaining a number of consecutive poles of the same polarity is simple and easy to manufacture. The magnetic circuit consists practically of a short bar of cast iron excited by one large coil, and with inverted claw pole-pieces fastened on either end to form the returning path of the magnetic lines. In the alternator, the centre core, of cast iron, is keyed to the shaft, and the star-shaped end castings having the number of poles required are bolted against each face of this. The exciting coil is wound on a strong bobbin, which slips on the core. In the centre of the coil there is a space left in the winding to clear the armature coils. The advantage of thus causing the armature to project into the coil is that the magnetic circuit can be then shortened radially. The exciting current required is taken by two brass rings connected to the exciting coils, but the method of connecting to the revolving ring is better than the ordinary brush arrangement. It is done by means of a flexible band of copper gauze so folded as to give a rectangular section. The one end of the strip is secured to the terminal and the other has a weight attached. Thus, when hanging over the brass collecting ring, the weight gives the tension required to ensure perfect contact. The armature coils are wound on cores of porcelain, which gives stiffness and good insulation without the disadvantages of the metallic core—*i.e.*, hysteresis or eddy currents. The conductor is a copper strip, which is wound bare with a strip of insulating material between each layer. The inside and outside ends respectively are connected by flexible conductors through

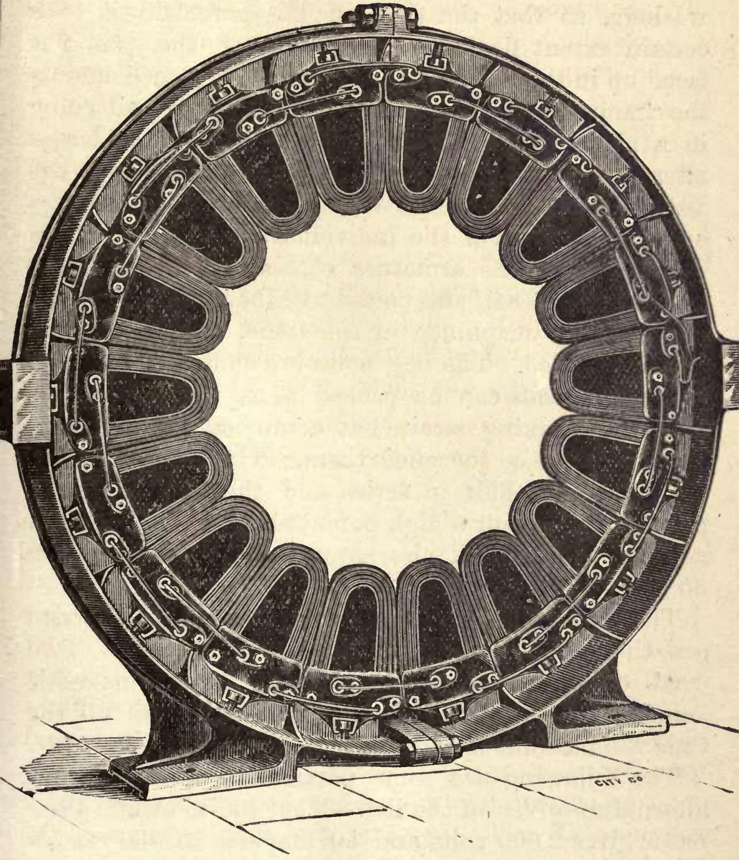

FIG. 135.

German-silver clamps, which hold the outer end of the coil. The bolts holding these plates of German silver on to the coil are all provided with spring washers, so that the grip on the porcelain is to a certain extent flexible. One side of the clamp is faced up in the lathe, so that when it is placed against the frame of the armature it ensures the coil being in a plane perpendicular to the axis. In the larger alternators the inside end of each armature coil has also a small brass clamp over the conductors to prevent any displacement of the individual strips taking place. The frame of the armature consists of a large ring usually cast in half and bolted together, Fig. 135. One radial face is machined in the lathe, and to this the coils are bolted. The bolt holes are elongated radially, so that the coils can be packed more tightly together after the working strain has compressed the various parts, and hence loosened them. The armature coils are connected half in series and the two halves in parallel to prevent a high potential difference between adjacent coils in all alternators having an output of 50 kilowatts and over.

The mechanical details are well designed and carried out, the oiling arrangements being automatic. Two small oil-pumps are driven by belts off the magnetic spindle, and ensure a good circulation of oil all the time the machine is at work.

The following are some particulars of the Mordey alternators given in the list. The 100-kilowatt alternator gives 2,000 volts and 50 amperes at 430 revolutions. The weight complete is 9 tons 1 cwt., and the floor space occupied is 8ft. $3\frac{1}{4}$in. by 6ft. $4\frac{3}{4}$in. The armature ring is 5ft. 10in. diameter and has 28 bobbins. There are 14 poles to each magnet casting, thus giving 100 complete periods per second. The magnets are

4ft. 8in. mean diameter, and the armature ring is 5ft. 10in.

The details of the weight are:

	tons	cwt.	qr.	lb.
Shaft	0	8	3	18
Bed-plate	2	10	2	0
Armature..............	0	10	1	12
Each magnet casting............	1	16	2	0

The 50-kilowatt machine gives 2,000 volts and 25 amperes at a speed of 600; weight complete, 4 tons; floor space, 6ft. 7½in. by 5ft. 6¾in.; the armature ring has 20 bobbins, and is 4ft. 6in. diameter. The magnets are 3ft. 4⅝in. diameter, and each pole has 10 pole-pieces.

Details of weight:

	tons	cwt.	qr.	lb.
Each pole	0	15	1	20
Magnet spool	0	13	1	22
Armature ring complete	0	6	0	20
Shaft	0	2	3	15
Bed-plate and bearings	1	5	3	0

The Kingdon Alternator.

This machine belongs to the class of alternators in which the electromotive force is produced by changes in the magnetic path. These changes cause a fluctuation of lines of force in certain definite places where the armature coils are situated. The diagrammatic sketches, Figs. 136, 137, and 138, will help to show the principle of action.

The iron parts are all built up of charcoal-iron plates. The outer ring, which has a number of projections on the inside, forms the core for both the armature and exciting coils. The latter are placed over the projections marked S N, and the armature coils are placed over the intermediate projections marked A. The revolving masses of iron which

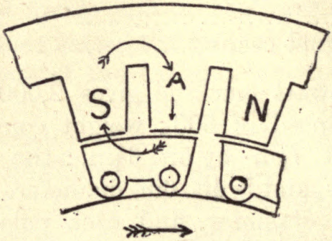

FIG. 136.

complete the magnetic circuit are also laminated, and are clamped in position by bolts passing through two steel discs.

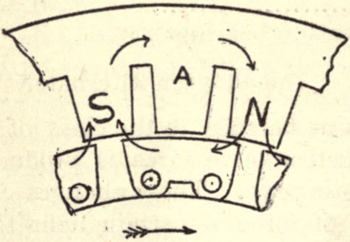

FIG. 137.

In Fig. 136 the keeper bridges cross the armature and the south pole of the field magnet, so that the lines of force pass as shown by arrows. Fig. 137 shows the condition of the magnetic circuits when the keeper has passed on till it is exactly opposite the armature core. Then the actions of the north and south pole tend to produce equal and opposite magnetic flux in the armature core, and hence neutralise each other, so at that instant no lines of force pass through the armature core. In position Fig. 138 the keeper unites the north pole to the armature, so that the flux is again a maximum, and in the opposite direction to that in Fig. 136. The keeper has now been moved through one thirty-second of a revolution, as there are 16 poles,

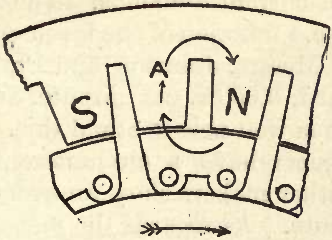

FIG. 138.

and the electromotive force induced will have passed through half a complete cycle.

On comparing Figs. 136 and 137 with respect to the magnetic resistance of the iron circuits, it will be seen that in Fig. 137 the resistance offered to the magnetic flux is much greater than in Fig. 136, whilst the magneto-motive force is doubled. In an intermediate position the resistance will have increased without a corresponding increase of magneto-motive force. This will cause a variation of the flux in the magnets and tend to produce an alternating current in the exciting coils. This effect, and the large masses of iron exposed to changes of induction, and hence causing a large hysteresis loss, are the objections to this class of machine, but they may be reduced considerably by careful design.

The adjoining list of alternators has been compiled in the same way as that previously given for dynamos; and the same general remarks apply to it.

The mean circumferential speed of the armature or moving magnets is in each case much higher than that adopted in direct-current dynamos. The limiting safe speed is, of course, a function of the mechanical strength of the design. Messrs. Johnson and Phillips work at the highest speed, 8,000ft. per minute, and their construction of core is well able to stand this. The Brush Company's magnets have a circumferential speed of 6,300, and the other makers show an average of about 5,000ft. per minute. As regards the material used for the core, four of the seven makers use iron, and the others use respectively porcelain, laminated brass, and wood.

COMPARATIVE LIST OF ALTERNATORS.

Reference number.	Name of maker.	Name of designs.	Normal. Revs. per minute.	Normal. Volts.	Normal. Amperes.	Normal. Kilowatts.	Complete periods per second.	Weight in tons.	Floor space. Length × Breadth. (ft. in. × ft. in.)	Height. (ft. in.)	Kilowatts per ton.	Kilowatts per square foot floor space.
1..	The Brush Electrical Engineering Company	Mordey	430	2,000	50	100	100	9·05	8 3 × 6 4	7 0	11·1	1·92
		Mordey	600	2,000	25	50	100	4·00	6 7 × 5 6	5 3	12·5	1·39
2..	The Electric Construction Corporation..	Elwell-Parker	600	1,000	30	30	60	2·75	4 6 × 4 0	4 4	10·9	1·66
3..	The Gulcher Company	Fricker	700	100	300	30	70	1·5	4 8 × 3 0	4 0	20	2·15
4..	Johnson and Phillips	Kapp	900	2,000	7·5	15	75	2·25	4 9 × 3 0	3 9	6·7	1·08
5..	Johnson and Phillips	Kapp	600	1,000	120	120	100	6·15	5 9 × 6 9	5 6	19·5	3·10
6..	Ferranti	Ferranti	335	2,400	93	225	67	18·35	9 9 ×13 8	9 3	12·3	1·68
7..	Siemens Bros..............................	Siemens	400	80	500	40	66	2·00	3 6 × 5 6	—	20	2·07
8..	Woodhouse and Rawson..................	Kingdon	375	110	640	70	100	7·00	3 9 × 6 9	—	10·0	2·76

Reference number.	Revolving parts. Mean diameter. (ft. in.)	Revolving parts. Circumferential speed, feet per minute	Magnets. Material.	Magnets. Number of poles.	Magnets. Section. (in. in.)	Magnets. Per cent. of output for exciting.	Armature. Material used for core.	Armature. Number of coils.	Armature. Conductor.	Armature. Number of turns per coil.	Armature. Total number of turns in series.	Armature. Number of parallel circuits.	Armature. Weight of conductors. (lb. oz.)
1..	4 8	6,300	cast iron	14	—	2·0	porcelain	28	copper tape	—	—	2	—
	3 4½	6,350	cast iron	10	8×4	—	porcelain	20	copper tape	—	—	1	—
2..	3 0	5,550	wrought iron	12	3×6	5·3	iron	12	·015 sq. in.	34	408	1	—
3..	2 1	4,560	cast iron	12	10×5¼	2 3	iron	12	·08in. × ·162in.	25½	25½	12	39 12
4..	2 6	7,100	wrought iron*	10	5¾×4	—	iron	10	circular wire	—	—	—	—
5..	4 3	8,000	wrought iron*	20	4½ dia.	1 8	iron	20	2(·075 × ·230)	20	200	2	124 0
6..	5 6	5,770	wrought iron†	24	126 sq.	2·0	brass and asbestos	24	·0394in. × ·625in.	40	480	2	251
7..	4 4	5,420	wrought iron*	20	—	—	wood	20	compressed strand	—	—	—	—
8..	4 6	4,930	laminated iron	16	—	6·8	iron	16	·197in. × ·063in.	44	44	16	—

* The cores only are of wrought iron fitting into cast-iron frames. † Cores of wrought iron cast in the frames.

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