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The Earliest Arithmetics in English

edited by
Robert Steele

EARLY ENGLISH TEXT SOCIETY

Extra Series, 118

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EDITED WITH INTRODUCTION

BY

ROBERT STEELE

LONDON :

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INTRODUCTION

THE number of English arithmetics before the sixteenth century is very small. This is hardly to be wondered at, as no one requiring to use even the simplest operations of the art up to the middle of the fifteenth century was likely to be ignorant of Latin, in which language there were several treatises in a considerable number of manuscripts, as shown by the quantity of them still in existence. Until modern commerce was fairly well established, few persons required more arithmetic than addition and subtraction, and even in the thirteenth century, scientific treatises addressed to advanced students contemplated the likelihood of their not being able to do simple division. On the other hand, the study of astronomy necessitated, from its earliest days as a science, considerable skill and accuracy in computation, not only in the calculation of astronomical tables but in their use, a knowledge of which latter was fairly common from the thirteenth to the sixteenth centuries.

The arithmetics in English known to me are :—

- (1) Bodl. 790 G. VII. (2653) f. 146–154 (15th c.) *inc.* “Of augrym ther be IX figures in numbray . . .” A mere unfinished fragment, only getting as far as Duplation.
- (2) Camb. Univ. Ll. IV. 14 (III.) f. 121–142 (15th c.) *inc.* “Al maner of thyngis that proseleth ffro the frist begynnyng . . .”
- (3) Fragmentary passages or diagrams in Sloane 213 f. 120–3 (a fourteenth-century counting board), Egerton 2852 f. 5–13, Harl. 218 f. 147 and
- (4) The two MSS. here printed; Eg. 2622 f. 136 and Ashmole 396 f. 48. All of these, as the language shows, are of the fifteenth century.

THE CRAFT OF NOMBRYNGE is one of a large number of scientific treatises, mostly in Latin, bound up together as Egerton MS. 2622 in the British Museum Library. It measures 7" × 5", 29–30 lines to the page, in a rough hand. The English is N.E. Midland in dialect. It is a translation and amplification of one of the numerous glosses on the *de algorismo* of Alexander de Villa Dei (c. 1220), such as that of

Thomas of Newmarket contained in the British Museum MS. Reg. 12, E. 1. A fragment of another translation of the same gloss was printed by Halliwell in his *Rara Mathematica* (1835) p. 29.* It corresponds, as far as p. 71, l. 2, roughly to p. 3 of our version, and from thence to the end p. 2, ll. 16-40.

The ART OF NOMBRYNG is one of the treatises bound up in the Bodleian MS. Ashmole 396. It measures $11\frac{1}{2}'' \times 17\frac{3}{4}''$, and is written with thirty-three lines to the page in a fifteenth century hand. It is a translation, rather literal, with amplifications of the *de arte numerandi* attributed to John of Holywood (Sacrobosco) and the translator had obviously a poor MS. before him. The *de arte numerandi* was printed in 1488, 1490 (*s.n.*), 1501, 1503, 1510, 1517, 1521, 1522, 1523, 1582, and by Halliwell separately and in his two editions of *Rara Mathematica*, 1839 and 1841, and reprinted by Curze in 1897.

Both these tracts are here printed for the first time, but the first having been circulated in proof a number of years ago, in an endeavour to discover other manuscripts or parts of manuscripts of it, Dr. David Eugene Smith, misunderstanding the position, printed some pages in a curious transcript with four facsimiles in the *Archiv für die Geschichte der Naturwissenschaften und der Technik*, 1909, and invited the scientific world to take up the "not unpleasant task" of editing it.

ACCOMPTYNGE BY COUNTERS is reprinted from the 1543 edition of Robert Record's Arithmetic, printed by R. Wolfe. It has been reprinted within the last few years by Mr. F. P. Barnard, in his work on Casting Counters. It is the earliest English treatise we have on this variety of the Abacus (there are Latin ones of the end of the fifteenth century), but there is little doubt in my mind that this method of performing the simple operations of arithmetic is much older than any of the pen methods. At the end of the treatise there follows a note on merchants' and auditors' ways of setting down sums, and lastly, a system of digital numeration which seems of great antiquity and almost world-wide extension.

After the fragment already referred to, I print as an appendix the 'Carmen de Algorismo' of Alexander de Villa Dei in an enlarged and corrected form. It was printed for the first time by Halliwell in *Rara Mathematica*, but I have added a number of stanzas from

* Halliwell printed the two sides of his leaf in the wrong order. This and some obvious errors of transcription—'ferye' for 'ferthe,' 'lest' for 'left,' etc., have not been corrected in the reprint on pp. 70-71.

various manuscripts, selecting various readings on the principle that the verses were made to scan, aided by the advice of my friend Mr. Vernon Rendall, who is not responsible for the few doubtful lines I have conserved. This poem is at the base of all other treatises on the subject in medieval times, but I am unable to indicate its sources.

THE SUBJECT MATTER.

Ancient and medieval writers observed a distinction between the Science and the Art of Arithmetic. The classical treatises on the subject, those of Euclid among the Greeks and Boethius among the Latins, are devoted to the Science of Arithmetic, but it is obvious that coeval with practical Astronomy the Art of Calculation must have existed and have made considerable progress. If early treatises on this art existed at all they must, almost of necessity, have been in Greek, which was the language of science for the Romans as long as Latin civilisation existed. But in their absence it is safe to say that no involved operations were or could have been carried out by means of the alphabetic notation of the Greeks and Romans. Specimen sums have indeed been constructed by moderns which show its possibility, but it is absurd to think that men of science, acquainted with Egyptian methods and in possession of the abacus,* were unable to devise methods for its use.

THE PRE-MEDIEVAL INSTRUMENTS USED IN CALCULATION.

The following are known :—

- (1) A flat polished surface or tablets, strewn with sand, on which figures were inscribed with a stylus.
- (2) A polished tablet divided longitudinally into nine columns (or more) grouped in threes, with which counters were used, either plain or marked with signs denoting the nine numerals, etc.
- (3) Tablets or boxes containing nine grooves or wires, in or on which ran beads.
- (4) Tablets on which nine (or more) horizontal lines were marked, each third being marked off.

The only Greek counting board we have is of the fourth class and was discovered at Salamis. It was engraved on a block of marble, and measures 5 feet by $2\frac{1}{2}$. Its chief part consists of eleven parallel lines, the 3rd, 6th, and 9th being marked with a cross. Another section consists of five parallel lines, and there are three

* For Egyptian use see Herodotus, ii. 36, Plato, *de Legibus*, VII.

rows of arithmetical symbols. This board could only have been used with counters (*calculi*), preferably unmarked, as in our treatise of *Accomptynge by Counters*.

CLASSICAL ROMAN METHODS OF CALCULATION.

We have proof of two methods of calculation in ancient Rome, one by the first method, in which the surface of sand was divided into columns by a stylus or the hand. Counters (*calculi*, or *lapilli*), which were kept in boxes (*loculi*), were used in calculation, as we learn from Horace's schoolboys (Sat. I. vi. 74). For the sand see Persius I. 131, "Nec qui abaco numeros et secto in pulvere metas scit risisse," Apul. Apolog. 16 (*pulvisculo*), Mart. Capella, lib. vii. 3, 4, etc. Cicero says of an expert calculator "eruditum attigisse pulverem," (*de nat. Deorum*, ii. 18). Tertullian calls a teacher of arithmetic "primus numerorum arenarius" (*de Pallio*, *in fine*). The counters were made of various materials, ivory principally, "Adeo nulla uncia nobis est eboris, etc." (*Juv. XI. 131*), sometimes of precious metals, "Pro calculis albis et nigris aureos argenteosque habebat denarios" (*Pet. Arb. Satyricon*, 33).

There are, however, still in existence four Roman counting boards of a kind which does not appear to come into literature. A typical one is of the third class. It consists of a number of transverse wires, broken at the middle. On the left hand portion four beads are strung, on the right one (or two). The left hand beads signify units, the right hand one five units. Thus any number up to nine can be represented. This instrument is in all essentials the same as the *Swanpan* or *Abacus* in use throughout the Far East. The Russian *stchota* in use throughout Eastern Europe is simpler still. The method of using this system is exactly the same as that of *Accomptynge by Counters*, the right-hand five bead replacing the counter between the lines.

THE BOETHIAN ABACUS.

Between classical times and the tenth century we have little or no guidance as to the art of calculation. Boethius (fifth century), at the end of lib. II. of his *Geometria* gives us a figure of an abacus of the second class with a set of counters arranged within it. It has, however, been contended with great probability that the whole passage is a tenth century interpolation. As no rules are given for its use, the chief value of the figure is that it gives the signs of the

nine numbers, known as the Boethian "apices" or "notae" (from whence our word "notation"). To these we shall return later on.

THE ABACISTS.

It would seem probable that writers on the calendar like Bede (A.D. 721) and Helericus (A.D. 903) were able to perform simple calculations; though we are unable to guess their methods, and for the most part they were dependent on tables taken from Greek sources. We have no early medieval treatises on arithmetic, till towards the end of the tenth century we find a revival of the study of science, centring for us round the name of Gerbert, who became Pope as Sylvester II. in 999. His treatise on the use of the Abacus was written (c. 980) to a friend Constantine, and was first printed among the works of Bede in the Basle (1563) edition of his works, I. 159, in a somewhat enlarged form. Another tenth century treatise is that of Abbo of Fleury (c. 988), preserved in several manuscripts. Very few treatises on the use of the Abacus can be certainly ascribed to the eleventh century, but from the beginning of the twelfth century their numbers increase rapidly, to judge by those that have been preserved.

The Abacists used a permanent board usually divided into twelve columns; the columns were grouped in threes, each column being called an "arcus," and the value of a figure in it represented a tenth of what it would have in the column to the left, as in our arithmetic of position. With this board counters or jetons were used, either plain or, more probably, marked with numerical signs, which with the early Abacists were the "apices," though counters from classical times were sometimes marked on one side with the digital signs, on the other with Roman numerals. Two ivory discs of this kind from the Hamilton collection may be seen at the British Museum. Gerbert is said by Richer to have made for the purpose of computation a thousand counters of horn; the usual number of a set of counters in the sixteenth and seventeenth centuries was a hundred.

Treatises on the Abacus usually consist of chapters on Numeration explaining the notation, and on the rules for Multiplication and Division. Addition, as far as it required any rules, came naturally under Multiplication, while Subtraction was involved in the process of Division. These rules were all that were needed in Western Europe in centuries when commerce hardly existed, and astronomy was unpractised, and even they were only required in the preparation

of the calendar and the assignments of the royal exchequer. In England, for example, when the hide developed from the normal holding of a household into the unit of taxation, the calculation of the geldage in each shire required a sum in division; as we know from the fact that one of the Abacists proposes the sum: "If 200 marks are levied on the county of Essex, which contains according to Hugh of Bocland 2500 hides, how much does each hide pay?"* Exchequer methods up to the sixteenth century were founded on the abacus, though when we have details later on, a different and simpler form was used.

The great difficulty of the early Abacists, owing to the absence of a figure representing zero, was to place their results and operations in the proper columns of the abacus, especially when doing a division sum. The chief differences noticeable in their works are in the methods for this rule. Division was either done directly or by means of differences between the divisor and the next higher multiple of ten to the divisor. Later Abacists made a distinction between "iron" and "golden" methods of division. The following are examples taken from a twelfth century treatise. In following the operations it must be remembered that a figure asterisked represents a counter taken from the board. A zero is obviously not needed, and the result may be written down in words.

(a) MULTIPLICATION. 4600×23 .

Thousands						
Hundreds	Tens	Units	Hundreds	Tens	Units	
		4	6			Multiplicand.
		1	8			600 × 3.
	1	2				4000 × 3.
	1	2				600 × 20.
	8					4000 × 20.
1		5	8			Total product.
				2	3	Multiplier.

* See on this Dr. Poole, *The Exchequer in the Twelfth Century*, Chap. III., and Haskins, *Eng. Hist. Review*, 27, 101. The hidage of Essex in 1130 was 2364 hides.

(b) **DIVISION : DIRECT.** $100,000 \div 20,023$. Here each counter in turn is a separate divisor.

Thousands					
H.	T.	U.	H.	T.	U.
	2			2	3
1	2				
	2				
	1	9	1	9	
				8	
	1	9	9	2	
				1	2
	1	9	9		8
					4

Divisors.

Place greatest divisor to right of dividend.

Dividend.

Remainder.

Another form of same.

Product of 1st Quotient and 20.

Remainder.

Product of 1st Quotient and 3.

Final remainder.

Quotient.

(c) **DIVISION BY DIFFERENCES.** $900 \div 8$. Here we divide by (10-2).

			H.	T.	U.
					2
					8
			*9		
			*1	8	
				2	
			*1		
				2	
					4
					2
				1	
				1	
				9	
			1	1	2

Difference.

Divisor.

Dividend.

Product of difference by 1st Quotient (9).

Product of difference by 2nd Quotient (1).

Sum of 8 and 2.

Product of difference by 3rd Quotient (1).

Product of difference by 4th Quot. (2). **Remainder.**

4th Quotient.

3rd Quotient.

2nd Quotient.

1st Quotient.

Quotient. (Total of all four.)

* These figures are removed at the next step.

DIVISION. $7800 \div 166$.

Thousands						
H.	T.	U.	H.	T.	U.	
				3	4	Differences (making 200 trial divisor).
			1	6	6	Divisors.
		*7	8			Dividends.
		1				Remainder of greatest dividend.
			1	2		Product of 1st difference (4) by 1st Quotient (3).
			9			Product of 2nd difference (3) by 1st Quotient (3).
		*2	8	2		New dividends.
			3	4		Product of 1st and 2nd difference by 2nd Quotient (1).
		*1	1	6		New dividends.
				2		Product of 1st difference by 3rd Quotient (5).
			1	5		Product of 2nd difference by 3rd Quotient (5).
		*3	3			New dividends.
		1				Remainder of greatest dividend.
			3	4		Product of 1st and 2nd difference by 4th Quotient (1).
			1	6	4	Remainder (less than divisor).
					1	4th Quotient.
					5	3rd Quotient.
			1			2nd Quotient.
			3			1st Quotient.
			4	6		Quotient.

* These figures are removed at the next step.

DIVISION. $8000 \div 606$.

Thousands						
H.	T.	U.	H.	T.	U.	
				9		Difference (making 700 trial divisor).
					4	Difference.
			6		6	Divisors.
		*8				Dividend.
		1				Remainder of dividend.
			9	4		Product of difference 1 and 2 with 1st Quotient (1).
		*1	9	4		New dividends.
			3			Remainder of greatest dividend.
				9	4	Product of difference 1 and 2 with 2nd Quotient (1).
		*1	3	3	4	New dividends.
			3			Remainder of greatest dividend.
				9	4	Product of difference 1 and 2 with 3rd Quotient (1).
			7	2	8	New dividends.
			6		6	Product of divisors by 4th Quotient (1).
			1	2	2	Remainder.
					1	4th Quotient.
					1	3rd Quotient.
					1	2nd Quotient.
				1		1st Quotient.
			1	3		Quotient.

* These figures are removed at the next step.

The chief Abacists are Gerbert (tenth century), Abbo, and Hermannus Contractus (1054), who are credited with the revival of the art, Bernellius, Gerland, and Radulphus of Laon (twelfth century). We know as English Abacists, Robert, bishop of Hereford, 1095, "abacum et lunarem compotum et celestium cursum astrorum rimatus," Turchillus Compotista (Thurkil), and through him of Guilielmus R. . . . "the best of living computers," Gislebert, and Simonus de Rotellis (Simon of the Rolls). They flourished most probably in the

first quarter of the twelfth century, as Thurkil's treatise deals also with fractions. Walcher of Durham, Thomas of York, and Samson of Worcester are also known as Abacists.

Finally, the term Abacists came to be applied to computers by manual arithmetic. A MS. Algorithm of the thirteenth century (Sl. 3281, f. 6, b), contains the following passage: "Est et alius modus secundum operadores sive practicos, quorum unus appellatur Abacus; et modus ejus est in computando per digitos et junctura manuum, et iste utitur ultra Alpes."

In a composite treatise containing tracts written A.D. 1157 and 1208, on the calendar, the abacus, the manual calendar and the manual abacus, we have a number of the methods preserved. As an example we give the rule for multiplication (Claud. A. IV., f. 54 vo). "Si numerus multiplicat alium numerum auferatur differentia majoris a minore, et per residuum multiplicetur articulus, et una differentia per aliam, et summa proveniet." Example, 8×7 . The difference of 8 is 2, of 7 is 3, the next article being 10; $7-2$ is 5. $5 \times 10 = 50$; $2 \times 3 = 6$. $50 + 6 = 56$ answer. The rule will hold in such cases as 17×15 where the article next higher is the same for both, *i. e.*, 20; but in such a case as 17×9 the difference for each number must be taken from the higher article, *i. e.*, the difference of 9 will be 11.

THE ALGORISTS.

Algorism (augrim, augrym, algram, agram, algorithm), owes its name to the accident that the first arithmetical treatise translated from the Arabic happened to be one written by Al-Khowarazmi in the early ninth century, "de numeris Indorum," beginning in its Latin form "Dixit Algorismi. . . ." The translation, of which only one MS. is known, was made about 1120 by Adelard of Bath, who also wrote on the Abacus and translated with a commentary Euclid from the Arabic. It is probable that another version was made by Gerard of Cremona (1114-1187); the number of important works that were not translated more than once from the Arabic decreases every year with our knowledge of medieval texts. A few lines of this translation, as copied by Halliwell, are given on p. 72, note 2. Another translation still seems to have been made by Johannes Hispalensis.

Algorism is distinguished from Abacist computation by recognising seven rules, Addition, Subtraction, Duplation, Mediation, Multiplication, Division, and Extraction of Roots, to which were afterwards

added Numeration and Progression. It is further distinguished by the use of the zero, which enabled the computer to dispense with the columns of the Abacus. It obviously employs a board with fine sand or wax, and later, as a substitute, paper or parchment; slate and pencil were also used in the fourteenth century, how much earlier is unknown.* Algorithm quickly ousted the Abacus methods for all intricate calculations, being simpler and more easily checked: in fact, the astronomical revival of the twelfth and thirteenth centuries would have been impossible without its aid.

The number of Latin Algorithms still in manuscript is comparatively large, but we are here only concerned with two—an Algorithm in prose attributed to Sacrobosco (John of Holywood) in the colophon of a Paris manuscript, though this attribution is no longer regarded as conclusive, and another in verse, most probably by Alexander de Villedieu (Villa Dei). Alexander, who died in 1240, was teaching in Paris in 1209. His verse treatise on the Calendar is dated 1200, and it is to that period that his Algorithm may be attributed; Sacrobosco died in 1256 and quotes the verse Algorithm. Several commentaries on Alexander's verse treatise were composed, from one of which our first tractate was translated, and the text itself was from time to time enlarged, sections on proofs and on mental arithmetic being added. We have no indication of the source on which Alexander drew; it was most likely one of the translations of Al-Khowarazmi, but he has also the Abacists in mind, as shewn by preserving the use of differences in multiplication. His treatise, first printed by Halliwell-Phillipps in his *Rara Mathematica*, is adapted for use on a board covered with sand, a method almost universal in the thirteenth century, as some passages in the algorithm of that period already quoted show: "Est et alius modus qui utitur apud Indos, et doctor hujusmodi ipsos [erat quidem nomine Algos. Et modus suus erat in computando per quasdam figuras scribendo in pulvere. . . ." "Si voluerimus depingere in pulvere predictos digitos secundum consuetudinem algorismi . . ." "et sciendum est quod in nullo loco minorum sive secundorum . . . in pulvere debent scribi plusquam sexaginta."

MODERN ARITHMETIC.

Modern Arithmetic begins with Leonardi Fibonacci's treatise "de Abaco," written in 1202 and re-written in 1228. It is modern

* Slates are mentioned by Chaucer, and soon after (1410) Prosdocimo de Beldamandi speaks of the use of a "lapis" for making notes on by calculators.

rather in the range of its problems and the methods of attack than in mere methods of calculation, which are of its period. Its sole interest as regards the present work is that Leonardi makes use of the digital signs described in Record's treatise on *The arte of nombrynge by the hand* in mental arithmetic, calling it "modus Indorum." Leonardo also introduces the method of proof by "casting out the nines."

DIGITAL ARITHMETIC.

The method of indicating numbers by means of the fingers is of considerable age. The British Museum possesses two ivory counters marked on one side by carelessly scratched Roman numerals IIIV and VIIII, and on the other by carefully engraved digital signs for 8 and 9. Sixteen seems to have been the number of a complete set. These counters were either used in games or for the counting board, and the Museum ones, coming from the Hamilton collection, are undoubtedly not later than the first century. Frohner has published in the *Zeitschrift des Münchener Alterthumsvereins* a set, almost complete, of them with a Byzantine treatise; a Latin treatise is printed among Bede's works. The use of this method is universal through the East, and a variety of it is found among many of the native races in Africa. In medieval Europe it was almost restricted to Italy and the Mediterranean basin, and in the treatise already quoted (Sloane 3281) it is even called the Abacus, perhaps a memory of Fibonacci's work.

Methods of calculation by means of these signs undoubtedly have existed, but they were too involved and liable to error to be much used.

THE USE OF "ARABIC" FIGURES.

It may now be regarded as proved by Bubnov that our present numerals are derived from Greek sources through the so-called Boethian "apices," which are first found in late tenth century manuscripts. That they were not derived directly from the Arabic seems certain from the different shapes of some of the numerals, especially the 0, which stands for 5 in Arabic. Another Greek form existed, which was introduced into Europe by John of Basingstoke in the thirteenth century, and is figured by Matthew Paris (V. 285); but this form had no success. The date of the introduction of the zero has been hotly debated, but it seems obvious that the twelfth century Latin translators from the Arabic were

perfectly well acquainted with the system they met in their Arabic text, while the earliest astronomical tables of the thirteenth century I have seen use numbers of European and not Arabic origin. The fact that Latin writers had a convenient way of writing hundreds and thousands without any cyphers probably delayed the general use of the Arabic notation. Dr. Hill has published a very complete survey of the various forms of numerals in Europe. They began to be common at the middle of the thirteenth century and a very interesting set of family notes concerning births in a British Museum manuscript, Harl. 4350 shows their extension. The first is dated Mij.^c lviii., the second Mij.^c lxi., the third Mij.^c 63, the fourth 1264, and the fifth 1266. Another example is given in a set of astronomical tables for 1269 in a manuscript of Roger Bacon's works, where the scribe began to write MCC6. and crossed out the figures, substituting the "Arabic" form.

THE COUNTING BOARD.

The treatise on pp. 52-65 is the only one in English known on the subject. It describes a method of calculation which, with slight modifications, is current in Russia, China, and Japan, to-day, though it went out of use in Western Europe by the seventeenth century. In Germany the method is called "Algorithmus Linealis," and there are several editions of a tract under this name (with a diagram of the counting board), printed at Leipzig at the end of the fifteenth century and the beginning of the sixteenth. They give the nine rules, but "Capitulum de radieum extractione ad algorithmum integrorum reservato, ejus species per cifrales figuras ostenduntur ubi ad plenum de hac tractabitur." The invention of the art is there attributed to Appulegius the philosopher.

The advantage of the counting board, whether permanent or constructed by chalking parallel lines on a table, as shown in some sixteenth-century woodcuts, is that only five counters are needed to indicate the number nine, counters on the lines representing units, and those in the spaces above representing five times those on the line below. The Russian abacus, the "tehatui" or "stehota" has ten beads on the line; the Chinese and Japanese "Swanpan" economises by dividing the line into two parts, the beads on one side representing five times the value of those on the other. The "Swanpan" has usually many more lines than the "stehota," allowing for more extended calculations, see Tylor, *Anthropology* (1892), p. 314.

Record's treatise also mentions another method of counter notation (p. 64) "merchants' casting" and "auditors' casting." These were adapted for the usual English method of reckoning numbers up to 200 by scores. This method seems to have been used in the Exchequer. A counting board for merchants' use is printed by Halliwell in *Rara Mathematica* (p. 72) from Sloane MS. 213, and two others are figured in Egerton 2622 f. 82 and f. 83. The latter is said to be "novus modus computandi secundum inventionem Magistri Thome Thorleby," and is in principle, the same as the "Swanpan."

The Exchequer table is described in the *Dialogus de Scaccario* (Oxford, 1902), p. 38.

The Earliest Arithmetics
in English.

The Crafte of Nombrynge.

Egerton 2622.

¹ **H**ec *algorismus ars presens dicitur; in qua Talibus indorum fruimur bis quinque figuris.*

¹ leaf 136 a.

This boke is called þe boke of algorym, or Augrym after lewder
4 vse. And þis boke tretys þe Craft of Nombryng, þe quych crafte
is called also Algorym. Ther was a kyng of Inde, þe quich heyth
Algor, & he made þis craft. And after his name he called hit
algorym; or els anoþer cause is quy it is called Algorym, for þe
8 latyn word of hit s. *Algorismus* comes of *Algos*, grece, *quid est*
ars, latine, craft on englis, and rides, *quid est numerus*, latine, A
nombur on englys, inde *dicitur Algorismus per addicionem huius*
sillabe *mus* & subtraccionem *d* & *e*, *quasi ars numerandi*. ¶ fforther-
12 more 3e most vnderstonde þat in þis craft ben vsid teen figurys,
as here bene writen for ensampul, ϕ 9 8 7 6 5 4 3 2 1. ¶ Expone
þe too *versus* afore: this present craft ys called *Algorismus*, in þe
quych we vse teen signys of Inde. (Questio. ¶ Why ten) figuris
16 of Inde? Solucio. for as I haue sayd afore þai were fonde fyrst
in Inde of a kynge of þat Cuntre, þat was called Algor.

A derivation
of *Algorismus*.

Another
derivation
of
the word.

¶ *Prima significat unum; duo vero secunda:*

versus [in
margin].

¶ *Tercia significat tria; sic procede sinistre.*

20 ¶ *Donec ad extremam venias, que cifra vocatur.*

¶ *Capitulum primum de significacione figurarum.*

*Expositio
versus.*

In þis verse is notifide þe significacion of þese figuris. And þus
expone the verse. þe first signifiyth one, þe secunde signi²fiyth
24 tweyne, þe thryd signifiyth thre, & the fourte signifiyth 4. ¶ And
so forthe towarde þe lyft syde of þe tabul or of þe boke þat þe
figures bene writene in, til þat þou come to the last figure, þat is

² leaf 136 b.

The meaning
and place of
the figures.

called a cifre. ¶ *Questio.* In quych syde sittes þe first figure? *Solucio,* forsothe loke quich figure is first in þe ryzt side of þe bok or of þe tabul, & þat same is þe first figure, for þou schal write bakeward, as here, 3. 2. 6. 4. 1. 2. 5. The figure of 5. was first write, & he is þe first, for he sittes on þe rizt. syde. And the figure of 3 is last. ¶ *Neuer-þe-les* wen he says ¶ *Prima significat vnum &c.,* þat is to say, þe first betokenes one, þe secunde. 2. & fore-þer-more, he vnderstonde nozt of þe first figure of euery rew. ¶ But he vnderstonde þe first figure þat is in þe nombur of þe forsaid teen figuris, þe quych is one of þese. 1. And þe secunde 2. & so forth.

verse [in margin].

¶ *Quelibet illarum si primo limite ponas,* 12

¶ *Simpliciter se significat: si vero secundo,*

Se decies: sursum procedas multiplicando.

¶ *Namque figura sequens quamuis signat decies plus.*

¶ *Ipsa locata loco quam significat pertinente.* 16

Expositio [in margin].

An explanation of the principles of notation.

1 leaf 137 a.

An example:

units,

tens,

hundreds,

thousands.

¶ *Expone þis verse þus.* Euery of þese figuris bitokens hym selfe & no more, yf he stonde in þe first place of þe rewele / this worde *Simpliciter* in þat verse it is no more to say but þat, & no more. ¶ If it stonde in the secunde place of þe rewle, he betokens tene tymes hym selfe, as þis figure 2 here 20 tokens ten tyme hym selfe, þat is twenty, for he hym selfe betokenes tweyne, & ten tymes twene is twenty. And for he stonde on þe lyft side & in þe secunde place, he betokens ten tyme hym selfe. And so go forth. ¶ ffor euery figure, & he stonde afar a-noper toward the lyft side, he schal betokene ten tymes as mich more as he schul betoken & he stode in þe place þere þat þe figure a-fore hym stonde. loo an ensampulle. 9. 6. 3. 4. þe figure of 4. þat hase þis schape 4. betokens bot hymselfe, for he stonde in þe first place. The figure of 3. þat hase þis schape 3. betokens ten tymes more þen he schuld & he stode þere þat þe figure of 4. stonde, þat is thretty. The figure of 6, þat hase þis schape 6, betokens ten tymes more þan he schuld & he stode þere as þe figure of 3. stonde, for þere he schuld tokyne bot sixty, & now he betokens ten tymes more, þat is sex hundryth. The figure of 9. þat hase þis schape 9. betokens ten tymes more þane he schuld & he stode in þe place þere þe figure of sex stonde, for þen he schuld betoken to 9. hundryth, and in þe place þere he stonde now he betokens 9. þousande. Al þe hole nombur is 9 thousande sex hundryth & foure & thretty. ¶ fforthermore, when

þou schalt rede a nombur of figure, þou schalt begyne at þe last figure in the lyft side, & rede so forth to þe ryzt side as here 9. 6. How to read the number.

3. 4. Thou schal begyn to rede at þe figure of 9. & rede forth þus. 9. ¹thousand sex hundryth thritty & foure. But when þou schalle write, þou schalt be-gynne to write at þe ryzt side. ¹ leaf 137 b.

¶ *Nil cifra significat sed dat signare sequenti.*

Expone þis verse. A cifre tokens nozt, bot he makes þe figure to betoken þat comes aftur hym more þan he schuld & he were away, as þus 1ϕ. here þe figure of one tokens ten, & yf þe cifre were away² & no figure by-fore hym he schuld token bot one, for þan he schuld stonde in þe first place. ¶ And þe cifre tokens nothyng hym selfe. for al þe nombur of þe ylke too figures is bot ten. ¶ Questio. Why says he þat a cifre makys a figure to signifye (tyf) more &c. ¶ I speke for þis worde significatyf, ffor sothe it may happe aftur a cifre schuld come a-nopur cifre, as þus 2ϕϕ. And zet þe secunde cifre shuld token neuer þe more excep he schuld kepe þe order of þe place. and a cifre is no figure significatyf. The meaning and use of the cipher.

¶ *Quam precedentes plus ultima significabit /*

Expone þis verse þus. þe last figure schal token more þan alle þe oþer afore, thouzt þere were a hundryth thousand figures afore, as þus, 16798. þe last figure þat is 1. betokens ten thousand. And alle þe oþer figures ben bot betokene bot sex thousand seynne hundryth nynty & 8. ¶ And ten thousand is more þen alle þat nombur, ergo þe last figure tokens more þan all þe nombur afore. The last figure means more than all the others, since it is of the highest value.

³¶ *Post predicta scias breuiter quod tres numerorum*

³ leaf 138 a.

Distincte species sunt; nam quidam digiti sunt;

Articuli quidam; quidam quoque compositi sunt.

28 ¶ *Capitulum 2^m de triplice divisione numerorum.*

¶ The auctor of þis tretis departyþ þis worde a nombur into 3 partes. Some nombur is called digitus latine, a digit in englys. Digits. Somme nombur is called articulus latine. An Articul in englys. Articles. Some nombur is called a composyt in englys. ¶ Expone þis verse. know þou aftur þe forsayd rewles þat I sayd afore, þat þere ben thre spieces of nombur. Oone is a digit, Anoþer is an Articul, & þe toþer a Composyt. versus. Composites.

36 ¶ *Sunt digiti numeri qui citra denarium sunt.*

¶ Here he telles qwat is a digit, Expone versus sic. Nomburs digitus bene alle nomburs þat ben with-inne ten, as nyne, 8. 7. 6. 5. 4. 3. 2. 1. What are digits.

² In MS 'awiy.'

¶ **Articupli decupli degitorum; compositi sunt
Illi qui constant ex articulis degitisque.**

¶ Here he telles what is a composit and what is an articul.

What are
articles.

Expone sic *versus*. ¶ Articulis ben¹ alle þat may be deuidyt in- 4
to nomburs of ten & nothyng leue ouer, as twenty, thretty, forty,
a hundryth, a thousand, & such oper, ffor twenty may be departyt
in-to 2 nomburs of ten, forty in to foure nomburs of ten, & so forth.

² leaf 138 b.

What
numbers
are com-
posites.

²Compositys ben nomburs þat bene componyt of a digyt & of an 8
articulle as fouretene, fyftene, sextene, & such oper. ffortene is
componyd of foure þat is a digit & of ten þat is an articulle.
ffiftene is componyd of 5 & ten, & so of all oper, what þat þai ben.
Short-lych euery nombur þat be-gynnes with a digit & endyth in a 12
articulle is a composit, as fortene bygennyng by foure þat is a
digit, & endes in ten.

¶ **Ergo, proposito numero tibi scribere, primo
Respicias quid sit numerus; si digitus sit 16
Primo scribe loco digitum, si compositus sit
Primo scribe loco digitum post articulum; sic.**

How to write
a number.

¶ here he telles how þou schalt wrych whan þou schalt write a
nombur. Expone *versum* sic, & fac iuxta exponentis sentenciam; 20
whan þou hast a nombur to write, loke fyrst what maner nombur
it ys þat þou schalt write, whether it be a digit or a composit or an
Articul. ¶ If he be a digit, write a digit, as yf it be seuen, write
seuen & write þat digit in þe first placetoward þe ryght side. If it 24
be a composit, write þe digit of þe composit in þe first place &
write þe articul of þat digit in þe secunde place next toward þe lyft
side. As yf þou schal write sex & twenty. write þe digit of þe
nombur in þe first place þat is sex, and write þe articul next aftur 28
þat is twenty, as þus 26. But whan þou schalt sowne or speke

if it is a
digit;

if it is a
composite.

³ leaf 139 a.

How to read
it.

³or rede an Composit þou schalt first sowne þe articul & aftur þe
digit, as þou seyst by þe comyne speche, Sex & twenty & nouzt
twenty & sex. *versus*. 32

¶ **Articulus si sit, in primo limite cifram,
Articulum vero reliquis inscribe figuris.**

How to write
Articles:

¶ Here he tells how þou schal write when þe nombre þat þou
hase to write is an Articul. Expone *versus* sic & fac *secundum* 36
sentenciam. Ife þe nombur þat þou hast write be an Articul, write
first a cifre & aftur þe eifer write an Articulle þus. 2φ. fforther-
more þou schalt vnderstonde yf þou haue an Articul, loke how

tens,

¹ 'ben' repeated in MS.

mych he is, yf he be *with-yuno* an hundryth, þou schalt write bot one cifre, afore, as here .9ϕ. If þe articulle be by hym-silfe & be an hundrid enene, þen schal þou write .1. & 2 cifers afore, þat he may stonde in þe thryd place, for every figure in þe thryd place schal token a hundrid tymes hym selfe. If þe articul be a thousand or thousandes¹ and he stonde by hym selfe, write afore 3 cifers & so forþ of al oþer.

8 ¶ **Quolibet in numero, si par sit prima figura, Par erit & totum, quicquid sibi continuatur; Impar si fuerit, totum tunc fiet et impar.**

¶ Here he teches a generalle rewle þat yf þe first figure in þe rewle of figures token a nombur þat is enene al þat nombur of figurys in þat rewle schal be enene, as here þou may see 6. 7. 3. 5. 4.

^{To tell an even number}

Computa & proba. ¶ If þe first² figure token an nombur þat is ode, alle þat nombur in þat rewle schalle be ode, as here 5 6 7 8 6 7.

^{2 leaf 139 b. or an odd.}

16 Computa & proba. versus.

¶ **Septem sunt partes, non plures, istius artis;**

¶ **Addere, subtrahere, duplare, dimidiare,**

Sextaque diuidere, sed quinta multiplicare;

20 **Radicem extrahere pars septima dicitur esse.**

¶ Here telles þat þer bene .7. spices or partes of þis craft. The first is called addicioñ, þe secunde is called subtraccioñ. The thryd is called duplacioñ. The 4. is called dimydicioñ. The 5. is called multiplicacioñ. The 6. is called diuision. The 7. is called extraccioñ of þe Rote. What all þese spices bene hit schalle be tolde singillatim in here capitule.

^{The seven rules.}

¶ **Subtrahis aut addis a dextris vel mediabis:**

28 Thou schal be-gynne in þe ryght side of þe boke or of a tabul. loke were þou wul be-gynne to write latyn or englys in a boke, & þat schalle be called þe lyft side of the boke, þat þou writest toward þat side schal be called þe ryght side of þe boke. Versus.

^{Add, subtract, or halve, from right to left.}

32 **A leua dupla, diuide, multiplica.**

Here he telles þe in quych side of þe boke or of þe tabul þou schalle be-gyne to wyreh duplacioñ, diuision, and multiplicacioñ. Thou schal begyne to worch in þe lyft side of þe boke or of þe tabul, but yn what wyse þou schal wyreh in hym dicitur singillatim in sequentibus capitulis et de vtilitate cuiuslibet artis & sic Completur³ prohemium & sequitur tractatus & primo de arte addicionis que prima ars est in ordine.

^{Multiply or divide from left to right.}

^{3 leaf 140.}

¹ In MS. 'thousandes.'

Addere si numero numerum vis, ordine tali
 Incipe; scribe duas primo series numerorum
 Primam sub prima recte ponendo figuram,
 Et sic de reliquis facias, si sint tibi plures. 4

Four things
 must be
 known:

¶ Here by-gynnes þe craft of Addicioñ. In þis craft þou most knowe foure thynges. ¶ Fyrst þou most know what is addicioñ.

Next þou most know how mony rewles of figurys þou most haue.

what it is;

¶ Next þou most know how mony diuers easys happes in þis craft of addicioñ. ¶ And next qwat is þe profet of þis craft. ¶ As for

þe first þou most know þat addicioñ is a castyng to-gedur of two nomburys in-to one nombre. As yf I aske qwat is twene & thre. þou wyl cast þese twene nombres to-gedur & say þat it is fyue. 12

how many
 rows of
 figures;

¶ As for þe secunde þou most know þat þou schalle haue tweyne rewes of figures, one vndur a-nother, as here þou mayst se. 1234

how many
 cases;

¶ As for þe thryd þou most know þat there ben foure diuerse cases. As for þe forthe þou most know þat þe profet of þis craft is 16

what is its
 result.

to telle what is þe hole nombur þat comes of diuerse nomburis. Now as to þe texte of oure verse, he teches there how þou schal worch in þis craft. ¶ He says yf þou wilt cast one nombur to anoþer nombur, þou most by-gynne on þis wyse. ¶ ffirst write 20

1 leaf 140 b.

How to set
 down the
 sum.

1 two rewes of figuris & nombris so þat þou write þe first figure of þe hyer nombur euene vndir the first figure of þe nether nombur, And þe secunde of þe nether nombur euene vndir þe secunde of þe hyer, & so forthe of euery figure of both þe rewes as þou mayst se 123 24
 234.

¶ **Inde duas adde primas hac condicione:**

Si digitus crescat ex addicione priorum;

Primo scribe loco digitum, quicumque sit ille.

Add the first
 figures;

¶ Here he teches what þou schalt do when þou hast write too 28

rewes of figuris on vnder an-oper, as I sayd be-fore. ¶ He says þou schalt take þe first figure of þe heyer nombre & þe fyrst figure of þe neþer nombre, & cast hem to-geder vp-on þis condicion. Thou schal loke qweþer þe number þat comys þere-of be a digit or no. 32

rub out the
 top figure;

¶ If he be a digit þou schalt do away þe first figure of þe hyer nombre, and write þere in his stede þat he stode Inne þe digit, þat comes of þe ylke 2 figures, & so wrich forth on oþer figures yf

write the
 result in its
 place.

þere be ony moo, til þou come to þe ende toward þe lyft side. And 36

lede þe nether figure stonde still euer-more til þou haue ydo. ffor þere-by þou schal wyte wheþer þou hast done wel or no, as I schal

2 leaf 141 a.

þat þou be-gynne to worch in þis Craft of Addicioñ in þe ryzt side, 40

here is an ensampul of þis case ¹²³⁴ Caste 2 to foure & þat wel be sex, do away 4. & write in þe ^{2142.} same place þe figure of sex. Here is an example.

¶ And lete þe figure of 2 in þe nether rewe stonde stil. When þou hast do so, east 3 & 4 to-gedar and þat wel be seuen þat is a digit. Do away þe 3, & set þere seuen, and lete þe neþer figure stonde stille, & so worch forth bakward til þou hast ydo all to-geder.

Et si compositus, in limite scribe sequente

8 **Articulum, primo digitum; quia sic iubet ordo.**

¶ Here is þe secunde ease þat may happe in þis craft. And þe case is þis, yf of þe casting of 2 nomburis to-geder, as of þe figure of þe hyer rewe & of þe figure of þe neþer rewe come a Composyt, how schalt þou worch. þus þou schalt worch. Thou shalt do away þe figure of þe hyer number þat was east to þe figure of þe neþer number. ¶ And write þere þe digit of þe Composyt. And set þe articul of þe composit next after þe digit in þe same rewe, yf þere be no mo figures after. But yf þere be no figuris after þat digit. Suppose it is a Composite, set down the digit, and carry the tens.

12 And þere he schall be rekind for hym selfe. And when þou schalt adde þat ylke figure þat berys þe articulle ouer his hed to þe figure vnder hym, þou schalt east þat articul to þe figure þat hase hym ouer his hed, & þere þat Articul schal token hym selfe. lo an Ensampull ¹of all ^{326.} Cast 6 to 6, & þere-of wil arise twelue. do away þe hyer 6 ²¹⁶ & write þere 2, þat is þe digit of þis composit. Here is an example. 1 leaf 111 b.

And þen write þe articulle þat is ten ouer þe figuris hed of twene as þus ¹³²² Now east þe articulle þat standus vpon þe figuris of twene ^{216.} hed to þe same figure, & reken þat articul bot for one, and þan þere wil arise thre. þan east þat thre to þe neþer figure, þat is one, & þat wul be foure. do away þe figure of 3, and write þere a figure of foure. and lete þe neþer figure stonde stil, & þan worch forth. vnde versus.

¶ **Articulus si sit, in primo limite cifram,**

¶ **Articulum vero reliquis inscribe figuris,**

32 **Vel per se scribas si nulla figura sequatur.**

¶ Here he puttes þe thryde case of þe craft of Addieion. & þe case is þis. yf of Addieion of 2 figuris a-ryse an Articulle, how schal þou do. thou most do away þe heer figure þat was addid to þe neþer, & write þere a cifre, and sett þe articuls on þe figuris hede, yf þat þere come ony after. And wrech þan as I haue tolde þe in þe secunde case. An ensampull ^{25.} Cast 5 to 5, þat wyll be ten. now do away þe hyer 5, & ¹⁵ write þere a cifer. And sette ten vpon þe figuris hed of 2. And reken it but for on þus. lo Suppose it is an Article, set down a cipher and carry the tens.

¹ leaf 142 a.
Here is an
example.

an Ensampulle $\begin{array}{|c|} \hline 1 \\ \hline 2\phi \\ \hline 15 \\ \hline \end{array}$. And ¹þan worch forth. But yf þere come no cifre, write þe articul next hym in þe same rewe as here $\begin{array}{|c|} \hline 5 \\ \hline 5 \\ \hline \end{array}$ cast 5 to 5, and it wel be ten. do away 5. þat is þe hier 5. $\begin{array}{|c|} \hline 5 \\ \hline 5 \\ \hline \end{array}$ and write þere a cifre, & write after hym þe articul as þus $\begin{array}{|c|} \hline 1\phi \\ \hline 5 \\ \hline \end{array}$. And þan þou hast done.

¶ Si tibi cifra superueniens occurrerit, illam

Dele superpositam; fac illic scribe figuram,

Postea procedas reliquas addendo figuras.

8

What to do
when you
have a cypher
in the top
row.

¶ Here he puttes þe fourt case, & it is þis, þat yf þere come a cifer in þe hier rewe, how þou schal do. þus þou schalt do. do away þe cifer, & sett þere þe digit þat comes of þe addicioun as þus 12

An example
of all the
difficulties.

1ϕ84. In þis ensampul ben alle þe foure cases. Cast 3 to foure, 17743 þat wol be seuen. do away 4. & write þere seuen; þan cast 4 to þe figure of 8. þat wel be 12. do away 8, & sett þere 2. þat is a digit, and sette þe articul of þe composit, þat is ten, vpon þe cifers 16 hel, & reken it for hym selfe þat is on. þan cast one to a cifer, & hit wulle be but on, for nozt & on makes but one. þan cast 7. þat stondes vnder þat on to hym, & þat wel be 8. do away þe cifer & þat 1. & sette þere 8. þan go forthermore. cast þe oþer 7 to þe cifer 20 þat stondes ouer hym. þat wul be bot seuen, for þe cifer betokens nozt. do away þe cifer & sette þere seuen, ²& þen go forþermore & cast 1 to 1, & þat wel be 2. do away þe hier 1, & sette þere 2. þan hast þou do. And yf þou haue wel ydo þis nomber þat is sett 24 here-after wel be þe nomber þat schalle aryse of alle þe addicion as here 27827. ¶ Sequitur alia species.

² leaf 142 b.

A numero numerum si sit tibi demere cura
Scribe figurarum series, vt in addicione.

28

Four things
to know
about sub-
traction:

the first;

the second;

the third;

the fourth.

¶ This is þe Chapter of subtraccion, in the quych þou most know foure nessessary thynges. the first what is subtraccion, þe secunde is how many nombers þou most haue to subtraccion, the thryd is how many maners of cases þere may happe in þis craft of subtraccion. The fourte is quat is þe profet of þis craft. ¶ As for þe first, þou most know þat subtraccion is drawynge of one number oute of anoþer number. As for þe secunde, þou most knowe þat þou most haue two rewes of figuris one vnder anoþer, as þou addyst in addicion. As for þe thryd, þou moyst know þat foure maner of diuerse easis mai happe in þis craft. ¶ As for þe fourt, þou most know þat þe profet of þis craft is whenne þou hasse taken þe lasse number out of þe more to telle what þere leues ouer 40

put. & þou most begynne to wyreh in þis craft in þe ryght side of þe boke, as þou diddest in addicion. *Versus.*

¶ **Maiori numero numerum suppone minorem,**

4 ¶ **Siue pari numero supponatur numerus par.**

1 ¶ Here he telles þat þe hier number most be more þen þe neþer, ^{1 leaf 143 a.} or els euend as mych. but he may not be lasse. And þe case is þis, þou schalt drawe þe neþer number out of þe hyer, & þou mayst ^{Put the greater number above the less.} not do þat yf þe hier number were lasse þan þat. for þou mayst not draw sex out of 2. But þou mast draw 2 out of sex. And þou maiste draw twene out of twene, for þou schal leue nozt of þe hier twene *vide versus.*

12 ¶ **Postea si possis a prima subtrahere primam**
Scribens quod remanet.

Here is þe first case put of subtraccion, & he says þou schalt ^{The first case of subtraction.} begynne in þe ryght side, & draw þe first figure of þe neþer rewe out of þe first figure of þe hier rewe. qwhether þe hier figure be more þen þe neþer, or euend as mych. And þat is notified in þe vers when he says "Si possis." Whan þou has þus ydo, do away þe hiest figure & sett þere þat leues of þe subtraccion, lo an Ensampulle ^{Here is an example.} draw 2 out of 4. þan leues 2. do away 4 & write þere 2, & $\begin{array}{r} 234 \\ 122 \end{array}$ latte þe neþer figure stonde stille, & so go for-by oþer figuris till þou come to þe ende, þan hast þou do.

¶ **Cifram si nil remanebit.**

24 ¶ Here he puttes þe secunde case, & hit is þis. yf it happe þat ^{Put a cipher if nothing remains.} qwen þou hast draw on neþer figure out of a hier, & þere leue nozt after þe subtraccion, þus ^{2 leaf 143 b.} þou schalt do. þou schalle do away þe hier figure & write þere a cifer, as lo an Ensampull $\begin{array}{r} 24 \\ 24 \end{array}$. Take foure ^{Here is an example.} out of foure þan leus nozt. þefore do away $\begin{array}{r} 24 \\ 24 \end{array}$ þe hier 4 & set þere a cifer, þan take 2 out of 2, þan leues nozt. do away þe hier 2, & set þere a cifer, and so worch whare so ever þis happe.

Sed si non possis a prima demere primam

32 **Precedens vnum de limite deme sequente,**
Quod demptum pro denario reputabis ab illo
Subtrahere totalem numerum quem proposuisti
Quo facto scribe super quicquid remanebit.

36 Here he puttes þe thryd case, þe quych is þis. yf it happe þat ^{Suppose you cannot take the lower figure from the top one, borrow ten;} þe neþer figure be more þen þe hier figure þat he schalle be draw out of. how schalle þou do. þus þou schalle do. þou schalle borro .i. oute of þe next figure þat comes after in þe same rewe, for þis case may neuer happ but yf þere come figures after. þan þou schalt sett

pat on ouer þe hier figures hed, of the quych þou woldist y-draw
 oute þe neyper figure yf þou haddyst y-myzt. Whane þou hase
 þus ydo þou schalle rekene þat .1. for ten. ¶ And out of þat ten
 þou schal draw þe neypermost figure, And alle þat leues þou schalle 4
 adde to þe figure on whos hed þat .1. stode. And þen þou schalle
 do away alle þat, & sett þere alle that arisys of the addicion of þe
 ylke 2 figuris. And yf yt ¹happe þat þe figure of þe quych þou
 schalt borro on be hym self but 1. If þou schalt þat one & sett it 8
 vppon þe oþer figuris hed, and sett in þat 1. place a cifer, yf þere
 come mony figures after. lo an Ensampul.

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 take 4 out of 2.
 it wyl not be, þefore borro one of þe next figure, þat is 2. and
 sett þat ouer þe hed of þe fyrst 2. & rekene it for ten. and þere þe 12
 secunde stondes write 1. for þou tokest on out of hym. þan take
 þe neyper figure, þat is 4, out of ten. And þen leues 6. cast to 6 þe
 figure of þat 2 þat stode vnder þe hedde of 1. þat was borwed &
 rekened for ten, and þat wylle be 8. do away þat 6 & þat 2, & 16
 sette þere 8, & lette þe neyper figure stonde stille. Whanne þou hast
 do þus, go to þe next figure þat is now bot 1. but first yt was 2, &
 þere-of was borred 1. þan take out of þat þe figure vnder hym, þat
 is 3. hit wel not be. þe-for-þe borowe of the next figure, þe quych is 20
 bot 1. Also take & sett hym ouer þe hede of þe figure þat þou
 woldest haue y-draw oute of þe nether figure, þe quych was 3. &
 þou myzt not, & rekene þat borwed 1 for ten & sett in þe same
 place, of þe quych place þou tokest hym of, a cifer, for he was bot 1. 24
 Whanne þou hast þus ydo, take out of þat 1. þat is reket for ten,
 þe neyper figure of 3. And þere leues 7. ²cast þe ylke 7 to þe figure
 þat had þe ylke ten vpon his hed, þe quych figure was 1, & þat wol
 be 8. þan do away þat 1 and þat 7, & write þere 8. & þan wyrch 28
 forth in oþer figuris til þou come to þe ende, & þan þou hast þe do.
 Versus.

¶ **Facque nonenarios de cifris, cum remeabis**

¶ **Occurant si forte cifre; dum demps-eris vnum** 32

¶ **Postea procedas reliquas demendo figuras.**

¶ Here he puttēs þe fourte case, þe quych is þis, yf it happe þat
 þe neyper figure, þe quych þou schalt draw out of þe hier figure be
 more þan þe hier figur ouer hym, & þe next figure of two or of 36
 thre or of foure, or how mony þere be by cifers, how wold þou do.
 þou wost wel þou most nede borow, & þou mayst not borow of þe
 cifers, for þai haue nozt þat þai may lene or spare. Ergo³ how

³ Perhaps "So."

take the
lower number
from ten;

add the
answer to
the top
number.
¹ leaf 144 a.

Example.

How to
'Pay back'
the borrowed
ten.

² leaf 144 b.

A very hard
case is put.

woldest þou do. Certayn þus most þou do, þou most borow on of þe next figure significatyf in þat rewe, for þis case may not happe, but yf þere come figures significatyf after the cifers. Whan þou 4 hast borowede þat 1 of the next figure significatyf, sett þat on ouer þe hede of þat figure of þe quych þou wold haue draw þe neþer figure out yf þou hadest myzt, & reken it for ten as þou diddest in þe oþer case here-a-fore. Whan þou hast þus y-do loke how 8 mony cifers þere were bye-twene þat figure significatyf, & þe figure of þe quych þou woldest haue y-draw the ¹neþer figure, and of euery ¹ leaf 145 a. of þe ylke cifers make a figure of 9. lo an Ensampulle after.

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 Here is an example.
Take 4 out of 2. it wel not be. borow 1 out of þe next figure 12 significatyf, þe quych is 4, & þen leues 3. do away þat figure of 4 & write þere 3. & sett þat 1 vpon þe figure of 2 hede, & þan take 4 out of ten, & þan þere leues 6. Cast 6 to the figure of 2, þat wol be 8. do away þat 6 & write þere 8. Whan þou hast þus y-do 16 make of euery 0 betweyn 3 & 8 a figure of 9, & þan worch forth in goddes name. & yf þou hast wel y-do þou² schalt haue þis number

¶ **Si subtraccio sit bene facta probare valebis**

Quas subtraxisti primas addendo figuras.

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Sic.

20 ¶ Here he teches þe Craft how þou schalt know, whan þou hast subtrayd, wheþer þou hast wel ydo or no. And þe Craft is þis, ryght as þou subtrayd þe neþer figures fro þe hier figures, ryzt so adde þe same neþer figures to þe hier figures. And yf þou haue 24 well y-wroth a-fore þou schalt haue þe hier nombre þe same þou haddest or þou be-gan to worch. as for þis I bade þou schulde kepe þe neþer figures styлле. lo an ³Ensampulle of alle þe 4 cases togedre. worche welle þis case

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. And yf þou worch welle 28 whan þou hast alle subtrayd

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 þe þat hier nombre here, þis schalle be þe nombre here foloyng whan þou hast subtrayd

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. And þou schalt know þus. adde þe neþer rewe of þe same nombre to þe hier rewe as þus, cast 4 to 4. þat wol 32 be 8. do away þe 4 & write þere 8. by þe first case of addicion. þan cast 6 to 0 þat wol be 6. do away þe 0, & write þere 6. þan cast 6 to 8, þat wel be 14. do away 8 & write þere a figure of 4, þat is þe digit, and write a figure of 1. þat schall be token ten. þat 36 is þe artical vpon þe hed of 8 next after, þan reken þat 1. for 1. & cast it to 8. þat schal be 9. cast to þat 9 þe neþer figure vnder þat þe quych is 4, & þat schalle be 13. do away þat 9 & sett þere 3, & sett a figure of 1. þat schall be 10 vpon þe next figuris hede þe

² 'hali' marked for erasure in MS.

quych is 9. by þe secunde case þat þou hadest in addicion. þan cast 1 to 9. & þat wol be 10. do away þe 9. & þat 1. And write þere a cifer. and write þe articulle þat is 1. betokenynge 10. vpon þe hede of þe next figure toward þe lyft side, þe quych ¹is 9, & so do forth tyl þou come to þe last 9. take þe figure of þat 1. þe quych þou schalt fynde ouer þe hed of 9. & sett it ouer þe next figures hede þat schal be 3. ¶ Also do away þe 9. & set þere a cifer, & þen cast þat 1 þat stondes vpon þe hede of 3 to þe same 3, & þat schalle make 4, þen caste to þe ylke 4 the figure in þe neyþer rewe, þe quych is 2, and þat schalle be 6. And þen schal þou haue an Ensampulle aʒeyn, loke & se, & but þou haue þis same þou hase myse-wrozt.

¹ leaf 146 a.

He works his proof through,

and brings out a result.

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Sequitur de duplacione

12

Si vis duplare numerum, sic incipe primo
Scribe figurarum seriem quamcunque velis tu.

Four things must be known in Duplation.

¶ This is the Chapture of duplacion, in þe quych craft þou most haue & know 4 thinges. ¶ þe first þat þou most know is what is duplacion. þe secunde is how mony rewes of figures þou most haue to þis craft. ¶ þe thryde is how many cases may² happe in þis craft. ¶ þe fourte is what is þe profet of þe craft. ¶ As for þe first. duplacion is a doublynge of a nombre. ¶ As for þe secunde þou most³ haue on nombre or on rewe of figures, the quych called *numerus duplandus*. As for þe thrid þou most know þat 3 diuerse cases may hap in þis craft. As for þe fourte. quat is þe profet of þis craft, & þat is to know what a-risyt of a nombre I-doublyde. ¶ ¶ fforþer-more, þou most know & take gode hede in quych side þou schalle be-gyn in þis craft, or ellis þou mayst spyl alle þi labor þere aboute. certeyn þou schalt begyn in the lyft side in þis Craft. thenke wel ouer þis verse. ¶ ⁴A leua dupla, diuide, multiplica.⁴ The sentens of þes verses afore, as þou may see if þou take hede. As þe text of þis verse, þat is to say, ¶ Si vis duplare. þis is þe sentence. ¶ If þou wel double a nombre þus þou most be-gynn. Write a rewe of figures of what nombre þou welt. *versus*.

Here they are.

³ leaf 146 b.

Mind where you begin.

Remember your rules.

Postea procedas primam duplando figuram

Inde quod exorescit scribas vbi iusserit ordo

Iuxta precepta tibi que dantur in addicione.

How to work a sum.

¶ Here he telles how þou schalt worch in þis Craft. he says, fyrst, whan þou hast writen þe nombre þou schalt be-gyn at þe first

² 'moy' in MS.

⁴ *Subtrahas aut addis a dextris vel mediabis* added on margin of MS.

figure in the lyft side, & doubulle þat figure, & þe nombre þat comes þere-of þou schalt write as þou diddyst in addicion), as ¶ I schal telle þe in þe case. *versus.*

4 ¶ **Nam si sit digitus in primo limite scribas.** ¹ leaf 117 a.

¶ Here is þe first case of þis craft, þe quych is þis. yf of duplacion of a figure arise a digit. what schal þou do. þus þou schal do. do away þe figure þat was doublede, & sett þere þe diget þat comes of þe duplacion), as þus. 23. double 2, & þat wel be 4. do away þe figure of 2 & sett þere a figure of 4, & so worch forth till þou come to þe ende. *versus.*

If the answer is a digit,
write it in the place of the top figure.

¶ **Articulus si sit, in primo limite cifram,**

12 ¶ **Articulum vero reliquis inscribe figuris ;**

¶ **Vel per se scribas, si nulla figura sequatur.**

¶ Here is þe secunde case, þe quych is þis yf þere come an articulle of þe duplacion) of a figure þou schalt do ryzt as þou diddyst in addicion), þat is to wete þat þou schalt do away þe figure þat is doublet & sett þere a cifer, & write þe articulle ouer þe next figuris hede, yf þere be any after-warde toward þe lyft side as þus. 25. begyn at the lyft side, and doubulle 2. þat wel be 4. do away þat 2 & sett þere 4. þan doubl 5. þat wel be 10. do away 5, & sett þere a 0, & sett 1 vpon þe next figuris hede þe quych is 4. & þen draw downe 1 to 4 & þat wolle be 5, & þen do away þat 4 & þat 1, & sett þere 5. for þat 1 schal be rekened in þe drawynge togedre for 1. wen þou hast ydon þou schalt haue þis nombre 50. yf þere come no figure after þe figure þat is addit, of þe quych addicion) comes an articulle, þou schalt do away þe figure þat is doublet & sett þere a 0. & write þe articul next by in þe same rewe toward þe lyft syde as þus, 523. double 5 þat woll be ten. do away þe figure 5 & set þere a cifer, & sett þe articul next after in þe same rewe toward þe lyft side, & þou schalt haue þis nombre 1023. þen go forth & double þe oper numbers þe quych is lyzt y-
nowzt to do. *versus.*

If it is an article,
put a cipher in the place, and 'carry' the tens.

² leaf 147 b.
If there is no figure to 'carry' them to, write them down.

¶ **Compositus si sit, in limite scribe sequente**

Articulum, primo digitum ; quia sic iubet ordo :

Et sic de reliquis faciens, si sint tibi plures.

36 ¶ Here he puttes þe Thryd case, þe quych is þis, yf of duplacion) of a figure come a Composit. þou schalt do away þe figure þat is doublet & set þere a digit of þe Composit, & sett þe articulle ouer þe next figures hede, & after draw hym downe with þe figure ouer whos hede he stondes, & make þere-of an nombre as þou hast done

If it is a Composite,
write down the digit, and 'carry' the tens.

¹ leaf 148 a.
Here is an
example.

afore, & yf *þere* come no figure after þat digit þat þou hast y-write, þan set þe articulle next after hym in þe same rewe as þus, 67 : double 6 þat wel be 12, do away 6 & write *þere* þe digit ¹ of 12, þe quych is 2, and set þe articulle next after toward þe lyft side in þe same 4 rewe, for *þere* comes no figure after. þan dowble þat oþer figure, þe quych is 7, þat wel be 14. the quych is a Composit. þen do away 7 þat þou doublet & sett þe þe diget of hym, the quych is 4, sett þe articulle ouer þe next figures hed, þe quych is 2, & þen draw to hym 8 þat on, & make on nombre þe quych schalle be 3. And þen yf þou hane wel y-do þou schalle haue þis nombre of þe duplacion, 134. *versus*.

¶ Si super extremam nota sit monadem dat eidem

Quod tibi contingat si primo dimidiabis. 12

How to
double the
mark for
one-half.

¶ Here he says, yf ouer þe fyrst figure in þe ryzt side be such a merke as is here made, ", þou schalle fyrst doubulle þe figure, the quych stondes vnder þat merke, & þen þou schalt doubul þat merke þe quych stondes for haluendel on. for too haluedels makes on, & 16 so þat wol be on. cast þat on to þat duplacion of þe figure ouer whos hed stode þat merke, & write it in þe same place *þere* þat þe figure þe quych was doublet stode, as þus 23^v. double 3, þat wol be 6 ; doubul þat halue on, & þat wol be on. cast on to 6, þat wel be 20 7. do away 6 & þat 1, & sett *þere* 7. þan hase þou do. as for þat figure, þan go ² to þe oþer figure & worch forth. & þou schall neuer

² leaf 148 b.
This can only
stand over
the first
figure.

hane such a merk but ouer þe hed of þe furst figure in þe ryght side. And zet it schal not happe but yf it were y-halued a-fore, þus 24 þou schalt vnderstonde þe verse. ¶ Si super extremam &c. Et nota, talis figura ^v significans medietatem, unitatis veniat, i.e. contingat uel fiat super extremam, i.e. super primam figuram in extremo sic versus dextram ars dat : i.e. reddit monadem. i.e. vnitatem eidem. 28 i.e. eidem note & declinat^{ur} hec monos, dis, di, dem, &c. ¶ Quod ergo totum hoc dabis monadem note continget. i.e. eveniet tibi si dimidiasti, i.e. accipisti uel subtulisti medietatem alicuius unius, in cuius principio sint figura numerum denotans imparem primo i.e. principiis. 32

¶ Sequitur de mediacione.

Incepe sic, si vis aliquem numerum mediare :
Scribe figurarum seriem solam, velut ante.

The four
things to be
known in
mediation :

¶ In þis Chapter is tazt þe Craft of mediacion, in þe quych 36 craft þou most know 4 thynges. ffurst what is mediacion. the secunde how many rewes of figures þou most hane in þe wyrehyng of þis craft. þe thryde how many diuerse cases may happ in þis craft.³ ¶ As for þe furst, þou schalt vnderstonde þat mediacion is a 40

the first

³ After 'craft' insert 'the .4. what is þe profet of þis craft.'

takyng out of halfe a number out of a holle number, ¹as yf þou ¹leaf 149 a. wolde take 3 out of 6. ¶ As for þe secunde, þou schalt know þat ^{the second;} þou most haue one rewe of figures, & no moo, as þou hayst in þe 4 craft of duplacion. ¶ As for the thryd, þou most vnderstonde þat ^{the third;} 5 cases may happe in þis craft. ¶ As for þe fourte, þou schalle ^{the fourth.} know þat the profet of þis craft is when þou hast take away þe haluendel of a nombre to telle quat þere schalle leue. ¶ Incipe 8 sic, &c. The sentence of þis verse is þis. yf þou wold medye, þat is to say, take halfe out of þe holle, or halfe out of halfe, þou most begynne þus. Write one rewe of figures of what nombre þou wolte, ^{Begin thus,} as þou dyddyst be-fore in þe Craft of duplacion. *versus.*

12 ¶ **Postea procedas medians, si prima figura
Si par aut impar videas.**

¶ Here he says, when þou hast write a rewe of figures, þou schalt take hede wher þe first figure be euen or odde in nombre, ^{See if the number is even or odd.} & vnderstonde þat he spekes of þe first figure in þe ryzt side. And in the ryght side þou schalle begynne in þis Craft.

¶ **Quia si fuerit par,**

Dimidiabis eam, scribens quicquid remanebit :

20 ¶ Here is the first case of þis craft, þe quych is þis, yf þe first figure be euen. þou schal take away fro þe figure euen halfe, & do away þat figure and set þere þat leues ouer, as þus, 4. take ²halfe out of 4, & þan þere leues 2. do away 4 & sett þere 2. þis is lyght 21 y-nowzt. *versus.* ^{If it is even, halve it, and write the answer in its place. ² leaf 149 b.}

¶ **Impar si fuerit vnum demas mediare**

Quod non presumas, sed quod superest mediabis

Inde super tractum fac demptum quod notat vnum.

28 Here is þe secunde case of þis craft, the quych is þis. yf þe first figure betokene a nombre þat is odde, the quych odde schal not be mediete, þen þou schalt medye þat nombre þat leues, when the odde of þe same nombre is take away, & write þat þat leues as þou 32 diddest in þe first case of þis craft. Whan þou hayst write þat. for þat þat leues, write such a merke as is here ^w vpon his hede, þe quych merke schal betoken halfe of þe odde þat was take away. lo an 36 þe quych is 5, for 5 is odde; þere-fore do away þat þat is odde, þe quych is 1. þen leues 4. þen medye 4 & þen leues 2. do away 4. & sette þere 2, & make such a merke ^w upon his hede, þat is to say ouer his hede of 2 as þus. 242.^w And þen worch forth in þe oþer 40 figures tyll þou come to þe ende. by þe furst case as þou schalt

^{If it is odd, halve the even number less than it.}
^{Then write the sign for one-half over it.}
^{Here is an example.}

NOMBRYNGE.

c

¹ leaf 150 a. vnderstonde þat þou schalt ¹ neuer make such a merk but ouer þe first figure hed in þe ryzt side. Wheþer þe other figures þat comyn after hym be euen or odde. *versus.*

¶ **Si monos, dele; sit tibi cifra post nota supra.** 4

If the first figure is one put a cypher. ¶ Here is þe thryde case, þe quych yf the first figure be a figure of 1. þou schalt do away þat 1 & set þere a cifer, & a merke ouer þe cifer as þus, 241. do away 1, & sett þere a cifer with a merke ouer his hede, & þen hast þou ydo for þat 0. as þus 0^m þen worch forth in þe oþer figurys till þou come to þe ende. for it is lyght as dycche water. *vnde versus.* 8

¶ **Postea procedas hac condicione secunda:
Impar si fuerit hinc vnum deme priori,
Inscribens quinque, nam denos significabit
Monos predictam.** 12

¶ Here he puttes þe fourte case, þe quych is þis. yf it happen the secunde figure betoken odde nombre, þou schal do away on of þat odde nombre, þe quych is significatiue by þat figure. 1. þe quych 1 schall be rekende for 10. Whan þou hast take away þat 1 out of þe nombre þat is signifiede by þat figure, þou schalt medie þat þat leues ouer, & do away þat figure þat is medied, & sette in his styde halfe of þat nombre. ¶ Whan þou hase so done, þou schalt write

² leaf 150 b. Write a figure of five over the next lower number's head. 2 a figure of 5 ouer þe next figures hede by-fore toward þe ryzt side, for þat 1, þe quych made odd nombre, schall stonde for ten, & 5 is halfe of 10; so þou most write 5 for his haluendelle. lo an Example, 4678. begyn in þe ryzt side as þou most nedes. medie 8.

þen þou schalt leue 4. do away þat 8 & sette þere 4. þen out of 7. take away 1. þe quych makes odde, & sett 5. vpon þe next figures hede afore toward þe ryzt side, þe quych is now 4. but afore it was 8. for þat 1 schal be rekenet for 10, of þe quych 10, 5 is halfe, as þou knowest wel. Whan þou hast þus ydo, medye þat þe quych leues after þe takyng away of þat þat is odde, þe quych leuyng schalle be 3; do away 6 & sette þere 3, & þou schalt haue such a nombre 4634. after go forth to þe next figure, & medy þat, & worch forth, for it is lyzt ynovzt to þe certayn.

¶ **Si vero secunda dat vnum.**

Illa deleta, scribatur cifra; priori 36

¶ **Tradendo quinque pro denario mediato;
Nec cifra scribatur, nisi deinde figura sequatur:
Postea procedas reliquos mediando figuras
Vt supra docui, si sint tibi mille figure.** 40

¶ Here he puttes þe 5 case, þe quych is ¹þis: yf þe secunde ^{1 leaf 151 a.}
 figure be of 1, as þis is here 12, þou schalt do away þat 1 & sett ^{If the second}
 þere a cifer. & sett 5 ^{over} þe next figure hede afore toward þe ryzt ^{figure is one,}
 4 side, as þou diddyst afore; & þat 5 schal be haldel of þat 1, þe ^{put a cifer,}
 quych 1 is reKent for 10. lo an Ensampulle, 214. medye 4. þat ^{and write five}
 schalle be 2. do away 4 & sett þere 2. þen go forth to þe next ^{over the next}
 figure. þe quych is bot 1. do away þat 1. & sett þere a cifer. & set
 8 5 vpon þe figures hed afore, þe quych is nowe 2, & þen þou schalt
 haue þis nombre 20², þen worch forth to þe nex figure. And also it
 is no maystery yf þere come no figure after þat on is medyet, þou
 schalt write no 0. ne nowzt ellis, but set 5 ^{over} þe next figure afore
 12 toward þe ryzt, as þus 14. medie 4 then leues 2, do away 4 & sett ^{How to halve}
 þere 2. þen medie 1. þe quych is rekende for ten, þe haluendel þere- ^{fourteen.}
 of wel be 5. sett þat 5 vpon þe hede of þat figure, þe quych is
 now 2, & do away þat 1, & þou schalt haue þis nombre yf þou
 16 worch wel, ⁵ 2. vnde versus.

¶ *Si mediatio sit bene facta probare valebis*

¶ *Duplando numerum quem primo dimediasti*

¶ Here he telles þe how þou schalt know wheþer þou hase wel ^{How to prove}
 20 ydo or no. doubul ²þe nombre þe quych þou hase mediet, and yf ^{your media-}
 þou haue wel y-medyt after þe dupleacion, þou schalt haue þe ^{tion.}
 same nombre þat þou haddyst in þe tabulle or þou began to medye, ^{2 leaf 151 b.}
 as þus. ¶ The furst ensampulle was þis. 4. þe quych I-mediet was
 24 laft 2, þe whych 2 was write in þe place þat 4 was write afore. ^{First}
 Now doubulle þat 2, & þou schal haue 4, as þou hadyst afore. þe ^{example.}
 secunde Ensampulle was þis, 245. When þou haddyst mediet alle ^{The second.}
 þis nombre, yf þou haue wel ydo þou schalt haue of þat mediacion
 28 þis nombre, 122^w. Now doubulle þis nombre, & begyn in þe lyft
 side; doubulle 1, þat schal be 2. do away þat 1 & sett þere 2. þen
 doubulle þat oper 2 & sett þere 4, þen doubulle þat oper 2, & þat wel
 be 4. þen doubul þat merke þat stondes for halue on. & þat schalle
 32 be 1. Cast þat on to 4, & it schalle be 5. do away þat 2 & þat
 merke, & sette þere 5, & þen þou schal haue þis nombre 245. &
 þis was þe same nombur þat þou haddyst or þou began to medye, as
 þou mayst se yf þou take hede. The nombre þe quych þou haddist
 36 for an Ensampul in þe 3 case of mediacion to be mediet was þis
 241. whan þou haddist mediet alle þis nombur truly ³by every ^{The third}
 figure, þou schall haue be þat mediacion þis nombur 120^w. Now ^{example.}
 dowbul þis nombur, & begyn in þe lyft side, as I tolde þe in þe ^{3 leaf 152 a.}
 40 Craft of dupleacion. þus doubulle þe figure of 1, þat wel be 2. do

away þat 1 & sett þere 2, þen doubul þe next figure afore, the quych is 2, & þat wel be 4; do away 2 & set þere 4. þen doubul þe cifer, & þat wel be nozt, for a 0 is nozt. And twyes nozt is but nozt. þerefore doubul the merke aboue þe cifers hede, þe quych be-
 tokens þe haluendel of 1, & þat schal be 1. do away þe cifer & þe merke, & sett þere 1, & þen þou schalt haue þis nombur 241. And þis same nombur þou haddyst afore or þou began to medy, & yf þou take gode hede. ¶ The next ensampul þat had in þe 4 case
 of mediacion was þis 4678. Whan þou hast truly ymedit alle þis nombur fro þe begynnyng to þe endyng, þou schalt haue of þe mediacion þis nombur 2334. Now doubul this nombur & begyn
 in þe lyft side, & doubulle 2 þat schal be 4. do away 2 and sette þere 4; þen doubule 3, þat wol be 6; do away 3 & sett þere 6, þen
 doubul þat oper 3, & þat wel be 6; do away 3 & set þere 16, þen
 doubul þe 4, þat welle be 8; þen doubul 5. þe quych stondes ouer þe hed of 4, & þat wol be 10; cast 10 to 8, & þat schal be 18; do
 away 4 & þat 5, & sett þere 8, & sett that 1, þe quych is an articul of þe Composit þe quych is 18, ouer þe next figures hed toward þe
 lyft side, þe quych is 6. drav þat 1 to 6, þe quych 1 in þe dravyng schal be rekenete bot for 1, & þat 1 & þat 6 togedur wel be 7. do
 away þat 6 & þat 1. the quych stondes ouer his hede, & sett ther 7, & þen þou schalt haue þis nombur 4678. And þis same nombur
 þou hadyst or þou began to medye, as þou mayst see in þe secunde Ensampul þat þou had in þe 4 case of mediacion, þat was þis: when
 þou had mediet truly alle the nombur, a principio usque ad finem. þou schalt haue of þat mediacion þis nombur 102. Now doubul
 1. þat wel be 2. do away 1 & sett þere 2. þen doabul 0. þat will be nozt. þerefore take þe 5, þe quych stondes ouer þe next figures
 hed, & doubul it, & þat wol be 10. do away þe 0 þat stondes betwene þe two figuris, & sette þere in his stid 1, for þat 1 now
 schal stonde in þe secunde place, where he schal betoken 10; þen
 doubul 2, þat wol be 4. do away 2 & sett þere 4. & þou schal haue þus nombur 214. þis is þe same numbur þat þou hadyst or þou
 began to medye, as þou may see. And so do euer more, yf þou wil knowe wheþer þou hase wel ymedyt or no. ¶. doubulle þe numbur
 þat comes after þe mediacion, & þou schal haue þe same nombur þat þou hadyst or þou began to medye, yf þou haue welle ydo. or
 els doute þe nozt, but yf þou haue þe same, þou hase faylide in þe
 Craft.

The fourth
example.

1 leaf 152 b.

The fifth
example.

leaf 153 a.

Sequitur de multiplicatione.

Si tu *per numerum numerum vis multiplicare*
 Scribe duas quascunque velis series numerorum
 Ordo servetur vt vltima multiplicandi
 Ponatur super anteriorem multiplicantis
 A leua relique sint scripte multiplicantes.

¶ Here be-gymnes þe Chaptre of multiplication, in þe quych þou most know 4 thynges. ¶ Ffirst, qwat is multiplicacion. The 8 secunde, how many cases may hap in multiplicacion. The thryde, how many rewes of figures þere most be. ¶ The 4. what is þe profet of þis craft. ¶ As for þe first, þou schal vnderstonde þat multiplicacion is a bryngyunge to-geder of 2 thynges in on nombur, þe quych on nombur contynes so many tymes on, howe 1mony tymes þere ben vnitees in þe nowmbre of þat 2, as twyes 4 is 8. now here ben þe 2 numbers, of þe quych too nowmbres on is betokened be an aduerbe, þe quych is þe worde twyes, & þis worde thryes, & þis worde foure sythes,² & so furth of such other lyke wordes. ¶ And tweyn nombres schal be tokenyde be a nowne, as þis worde foure showys þes tweyn nombres y-broth in-to on hole nombur, þat is 8, for twyes 4 is 8, as þou wost wel. ¶ And þes nombre 8 conteynes as oft tymes 4 as þere ben vnites in þat other nombre, þe quych is 2, for in 2 ben 2 vnites, & so oft tymes 4 ben in 8, as þou wottys wel. ¶ ffor þe secunde, þou most know þat þou most haue too rewes of figures. ¶ As for þe thryde, þou most know þat 8 maner of diuerse case may happe in þis craft. The profet of þis Craft is to telle when a nombre is multiplyed be a noþer, qwat commys þere of. ¶ fforthermore, as to þe sentence of oure verse, yf þou wel multiply a nombur be a-noþer nombur, þou schalt write a rewe of figures of what nomburs so euer þou welt, & þat schal be called Numerus multiplicandus, Anglice, þe nombur the quych to be multiplied. þen þou schalt write a-nother rewe of figures, by þe quych þou schalt multiplie the nombre þat is to be multiplied, of þe quych nombur þe furst figure schal be write vnder þe last figure of þe nombur, þe quych is to be multiplied. And so write forthe toward þe lyft side, as here you may se,

67324
1234

. And þis one nombur schalle be called numerus multiplicans. Any-
 þe nombur multipliynge, for he schalle multiply þe hyer nombur, as þus one tyme 6. And so forth, as I schal telle the afterwarde. And þou schal begyn in þe lyft side. ¶ ffor þere-more þou schalt vnderstonde þat þere is two manurs of multiplicacion; one ys of þe wyrehyunge of þe boke only in þe mynde of a mon. fyrst he

Four things to be known of Multiplication:

the first:

1 leaf 153 b.

the second:

the third:

the fourth.

2 leaf 154 a.

The multiplicand.

How to set down the sum.

Two sorts of Multiplication: mentally.

² After 'sythes' insert '& þis wordes fyue sithe & sex sythes.'

and on paper. teches of þe fyrst maner of duplacion), þe quych is be wyrehyng
of tabuls. Afterwarde he wol teche on þe secunde maner. *vnde*
versus.

In digitum cures digitum si ducere maior

4

1 leaf 154 b.

¹Per quantum distat a denis respice debes

¶ Namque suo decuplo totiens delere minore

Sitque tibi numerus veniens exinde patebit.

How to
multiply two
digits.

¶ Here he teches a rewle, how þou schalt fynde þe noubre þat
comes by þe multiplicacion) of a digit be anoper. loke how many
[vny]tes ben. bytwene þe more digit and 10. And reken ten for ou
vnite. And so oft do away þe lasse noubre out of his owne

Subtract the
greater from
ten;

decuple, þat is to say, fro þat noubre þat is ten tymes so mych is
þe noubre þat comes of þe multiplicacion). As yf þou wol multiply
2 be 4. loke how many vnitees ben by-twene þe quych is þe more
noubre, & be-twene ten. Certen þere wel be vj vnitees by-twene 4

take the less
so many
times from
ten times
itself.

& ten. yf þou reken þere with þe ten þe vnite, as þou may se. so
many tymes take 2. out of his decuple, þe quych is 20. for 20 is þe
decuple of 2, 10 is þe decuple of 1, 30 is þe decuple of 3, 40 is þe

Example.

last y-take so many tymes 2 out of twenty, þe quych is sex tymes, 20
þou schal leue 8 as þou wost wel, for 6 times 2 is twelue. take
[1]2 out of twenty, & þere schal leue 8. bot yf bothe þe digettes

2 leaf 155 a.

ben y-lyech mych as here. 222 or too tymes twenty, þen it is no
fors quych of hem tweyn þou take out of here decuple. als many
tymes as þat is fro 10. but neuer-þe-lesse, yf þou haue hast to
worch, þou schalt haue here a tabul of figures, where-by þou schalt
se a-nom) ryght what is þe noubre þat comes of þe multiplicacion)
of 2 digittes. þus þou schalt worch in þis figure.

28

1	
2	4
3	6 9
4	8 12 16
5	10 15 20 25
6	12 18 24 30 36
7	14 21 28 35 42 49
8	16 24 32 40 48 56 64
9	18 27 36 45 54 63 72 81
1	2 3 4 5 6 7 8 9

How to use it. yf þe figure, þe quych schalle be multiplied, be enene as mych as þe
diget be, þe quych þat oper figure schal be multiplied, as two tymes
twayn), or three tymes 3. or sych other. loke qwere þat figure sittes in

þe lyft side of þe triangle, & loke qwere þe diget sitted in þe neþer most rewe of þe triangle. & go fro hym vpwarde in þe same rewe, be quych rewe gose vpwarde til þou come agaynes þe ober digette þat sitted in þe lyft side of þe triangle. And þat noubre, þe quych þou fyn¹des þere is þe noubre þat comes of the multiplicaciõ of þe 2 digittes, as yf þou wold wete qwat is 2 tymes 2. loke quere sitted 2 in þe lyft side in þe first rewe, he sitted next 1 in þe lyft side al on hye, as þou may se; þe[n] loke qwere sitted 2 in þe lowyst rewe of þe triangle, & go fro hym vpwarde in þe same rewe tyll þou come a-zenenes 2 in þe hyer place, & þer þou schalt fynd ywrite 4, & þat is þe noubre þat comes of þe multiplicaciõ of two tymes tweyn is 4, as þow wotest welle. yf þe diget. the quych is multiplied, be more þan þe ober, þou schalt loke qwere þe more diget sitted in þe lowest rewe of þe triangle, & go vpwarde in þe same rewe tyl² þou come a-nendes þe lasse diget in the lyft side. And þere þou schalt fynde þe noubre þat comes of þe multiplicaciõ; but þou schalt vnderstonde þat þis rewle, þe quych is in þis verse. ¶ In *digitum cures*, &c., noþer þis triangle schalle not serue, bot to fynde þe noubres þat comes of the multiplicaciõ þat comes of 2 articuls or *composites*, þe nedes no craft but yf þou wolt multiply in þi mynde. And þere-to þou schalt haue a craft afterwarde, for þou schall wyrcþ with digettes in þe tables, as þou schalt know afterwarde. *versus*.

The way to use the Multiplication table.

¹ leaf 155 b.

³ leaf 156 a.

¶ **Postea procedas postremam multiplicando**
[Recte multiplicans per cunctas inferiores]
Condicionem tamen tali quod multiplicantes
Scribas in capite quicquid processerit inde
Sed postquam fuit hec multiplicata figure
Anteriores tunc sere multiplicantis
Et sic multiplica velut isti multiplicasti
Qui sequitur numerum scriptum quiscunque figuris.

¶ Here he teches how þou schalt wyrcþ in þis craft. þou schalt multiplye þe last figure of þe noubre, and quen þou hast so ydo þou schalt draw alle þe figures of þe neþer noubre more taward þe ryzt side, so qwen þou hast multiplied þe last figure of þe hyer noubre by alle þe neþer figures. And sette þe noubir þat comes per-of ouer þe last figure of þe neþer noubre, & þen þou schalt sette al þe ober figures of þe neþer noubre more nere to þe ryzt side. ¶ And whan þou hast multiplied þat figure þat schal be multiplied þe next after

How to multiply one number by another.

Multiply the 'last' figure of the higher by the 'first' of the lower number.

² 't1' marked for erasure before 'tyl' in MS.

hym by al þe neþer figures. And worch as þou dyddyst afore til
¹ þou come to þe ende. And þou schalt vnderstonde þat every
 figure of þe hier nounbre schal be multiplied be alle þe figures of the
 neþer nounbre, yf þe hier nounbre be any figure þen one. lo an ⁴
 Ensampul here folowyng. $\begin{array}{r} 2165 \\ \times 232 \\ \hline \end{array}$ þou schalt begyne to multiplie
 in þe lyft side. Multiply 2 be 2, and twyes 2 is 4. set 4
 ouer þe hed of þat 2, þen multiplie þe same hier 2 by 3 of þe nether
 nounbre, as thryes 2 þat schal be 6. set 6 ouer þe hed of 3, þan ⁸
 multiplie þe same hier 2 by þat 2 þe quych stondes vnder hym, þat
 wol be 4; do away þe hier 2 & sette þere 4. ¶ Now þou most
 antery þe nether nounbre, þat is to say, þou most sett þe neþer
 nounbre more towarde þe ryzt side, as þus. Take þe neþer 2 toward ¹²
 þe ryzt side, & sette it euend vnder þe 4 of þe hier nounbre, &
 antery alle þe figures þat comes after þat 2, as þus; sette 2 vnder þe
 4. þen sett þe figure of 3 þere þat þe figure of 2 stode, þe quych
 is now vndur þat 4 in þe hier nounbre; þen sett þe oþer figure of 2, ¹⁶
 þe quych is þe last figure toward þe lyft side of þe neþer number þere
 þe figure of 3 stode. þen þou schalt haue such a nombre $\begin{array}{r} 464465 \\ \times 232 \\ \hline \end{array}$
² ¶ Now multiply 4, þe quych comes next after 6, by þe last
 2 of þe neþer nounbur toward þe lyft side. as 2 tymes 4, þat wel be ²⁰
 8. sette þat 8 ouer þe figure the quych stondes ouer þe hede of þat
 2, þe quych is þe last figure of þe neþer nounbre; þan multiplie þat
 same 4 by 3, þat comes in þe neþer rewe, þat wol be 12. sette þe
 digit of þe composyt ouer þe figure þe quych stondes ouer þe hed of ²⁴
 þat 3, & sette þe articule of þis composyt ouer al þe figures þat
 stondes ouer þe neþer 2 hede. þen multiplie þe same 4 by þe 2 in
 þe ryzt side in þe neþer nounbur, þat wol be 8. do away 4. & sette
 þere 8. Euer more qwen þou multiplies þe hier figure by þat figure ²⁸
 þe quych stondes vnder hym, þou schalt do away þat hier figure, &
 sett þer þat nounbre þe quych comes of multiplicacion of ylke
 digittes. Whan þou hast done as I haue hyde þe, þou schalt haue
 suych an order of figure as is here, $\begin{array}{r} 1 \\ 2 \\ \hline 4648[65] \\ \times 232 \\ \hline \end{array}$. þen take and antery ³²
 þi neþer figures. And sett þe fyrst figure of þe neþer
 figures ³ vndre þe figure of 6. ¶ And draw al þe
 oþer figures of þe same rewe to hym-ward, ⁴ as þou diddyst afore.
 þen multiplie 6 be 2, & sett þat þe quych comes ouer þere-of ³⁶
 ouer al þe oþer figures hedes þat stondes ouer þat 2. þen multi-
 ply 6 be 3, & sett alle þat comes þere-of vpon alle þe figures
 hedes þat standes ouer þat 3; þan multiplie 6 be 2, þe quych

³ Here 'of þe same rew' is marked for erasure in MS.

stondes vnder þat 6, þen do away 6 & write þere þe digitt of þe composit þat schal come þerof, & sette þe articull ouer alle þe figures þat stondes ouer þe hede of þat 3 as here, þen

11
121
828
464825
232

Antery the figures again, and multiply by five:

4 antery þi figures as þou diddyst afore, and multipli 5 be 2, þat wol be 10; sett þe 0 ouer all þe figures þat stonden ouer þat 2, & sett þat 1. ouer the next figures hedes, alle on hye towarde þe lyft side. þen multiplye 5 be 3. þat 8 wol be 15, write 5 ouer þe figures hedes þat stonden ouer þat 3, & sett þat 1 ouer þe next figures hedes toward þe lyft side. þen multiplye 5 be 2, þat wol be 10. do away þat 5 & sett þere a 0, & sett þat 1 ouer þe figures hedes þat stonden ouer 3. And þen

12 þou schalt haue such a nounbre as here stondes aftur.

11
1101
1215
82820
4648
232

¹ leaf 158 a.

¶ Now draw alle þese figures downe togeder as þus, 6.8.1. & 1 draw to-gedur; þat wolle be 16, do away alle þese figures saue 6. lat hym stonde, for þow þou take hym

16 away þou most write þer þe same agene. þerefore late hym stonde, & sett 1 ouer þe figure hede of 4 toward þe lyft side; þen draw on to 4, þat wolle be 5. do away þat 4 & þat 1, & sette þere 5. þen draw 4221 & 1 togedur, þat wol be 10. do away alle 20 þat, & write þere þat 4 & þat 0, & sett þat 1 ouer þe next figures hede toward þe lyft side, þe quych is 6. þen draw þat 6 & þat 1 togedur, & þat wolle be 7; do away 6 & sett þere 7, þen draw 8810 & 1, & þat wel be 18; do away alle þe figures þat stondes ouer þe 24 hede of þat 8, & lette 8 stonde stil, & write þat 1 ouer þe next figuris hede, þe quych is a 0. þen do away þat 0, & sett þere 1, þe quych stondes ouer þe 0. hede. þen draw 2, 5, & 1 togedur, þat wolle be 8. þen do away alle þat, & write þere 8. ¶ And þen þou

Then add all the figures above the line:

28 schalt haue þis nounbre, 571880.

and you will haue the answer.

²¶ *Sed cum multiplicabis, primo sic est operandum, Si dabit articulum tibi multiplicacio solum; Proposita cifra summam transferre memento.*

² leaf 158 b.

32 ¶ Here he puttes þe fyrst case of þis craft, þe quych is þis: yf þere come an articulle of þe multiplicacion ysette before the articulle in þe lyft side as þus

51

, multiplye 5 by 2, þat wol be 10; sette ouer þe hede of þat 2

23

 a 0, & sett þat on, þat is þe 36 articul, in þe lyft side, þat is next hym, þen þou schalt haue þis nounbre

1051

. ¶ And þen worch forth as þou diddist afore. And þou

23

 schalt vnderstonde þat þou schalt write no 0.

What to do if the first multiplication results in an article.

but whan þat place where þou schal write þat 0 has no figure afore 40 hym noþer after. versus.

¶ *Si autem digitus excreuerit articulusque.*

Articulus¹ supraposito digito salit vltra.

What to do
if the result
is a composite
number.

¶ Here is þe secunde case, þe quych is þis : yf hit happe þat þere come a composyt, þou schalt write þe digitte ouer þe hede of þe neþer figure by þe quych þou multiplieth þe hier figure ; and sett þe articulle next hym toward þe lyft side, as þou diddyst afore, as þus

² leaf 159 a.

83
83

 Multiply 8 by 8, þat wol be 64. Write þe 4 ouer 8, þat is to say, ouer þe hede of þe neþer 8 ; & set 6, þe quych² is an articul, next after. And þen þou schalt haue such a noubre as is here,

6483
83

 And þen worch forth.

¶ *Si digitus tamen ponas ipsam super ipsam.* 12

What if it
be a digit.

¶ Here is þe thryde case, þe quych is þis : yf hit happe þat of þi multiplicacioun come a digit, þou schalt write þe digit ouer þe hede of þe neþer figure, by the quych þou multiplieth þe hier figure, for þis nedes no Ensampul. 16

¶ *Subdita multiplica non hanc que [incidit] illi
Delet eam penitus scribens quod prouenit inde.*

The fourth
case of the
craft.

¶ Here is þe 4 case, þe quych is : yf hit be happe þat þe neþer figure schal multiplye þat figure, þe quych stondes ouer þat figures hede, þou schal do away þe hier figure & sett þere þat þat comys of þat multiplicacioun. As yf þere come of þat multiplicacioun an articuls þou schalt write þere þe hier figure stode a 0. ¶ And write þe articuls in þe lyft side, yf þat hit be a digit write þere a digit. yf þat hit be a composit, write þe digit of þe composit. And þe articul in þe lyft side. al þis is lyzt y-nowzt, þere-fore þer nedes no Ensampul. 20

¶ *Sed si multiplicat aliam ponas super ipsam* 28

Adiunges numerum quem prebet ductus earum.

⁴ leaf 159 b.

The fifth case
of the craft.

¶ Here is þe 5 case, þe quych is þis : yf þe neþer figure schul multiplye þe hier, and þat hier figure is not recte ouer his hede. And þat neþer figure hase oþer figures, or on figure ouer his hede by multiplicacioun, þat hase be afore, þou schalt write þat noubre, þe quych comes of þat, ouer alle þe ylke figures hedes, as þus here :

236
234

 Multiply 2 by 2, þat wol be 4 ; set 4 ouer þe hede of þat 2. þen⁵ multiplies þe hier 2 by þe neþer 3, þat wol be 6. set ouer his hede 6, multiplye þe hier 2 by þe neþer 4, þat wol be 8. do away þe hier 2, þe quych stondes ouer þe hede of þe figure of 4,

¹ 'sed' deleted in MS.

² 6883 in MS.

⁵ 'þen' overwritten on 'þat' marked for erasure.

and set *þere* 8. And þou schalt haue þis noubre here $\begin{array}{|l} 46836 \\ \hline 234 \end{array}$. And antery þi figures, þat is to say, set þi *neþer* 4 vnder þe $\begin{array}{|l} 46836 \\ \hline 234 \end{array}$ hier 3, and set þi 2 other figures nere hym, so þat þe *neþer* 2 stonde vndur 4 þe hier 6, þe quych 6 stondes in þe lyft side. And þat 3 þat stondes vndur 8, as þus aftur 3e may se, $\begin{array}{|l} 46836 \\ \hline 234 \end{array}$ Now worch forthermore, And multiplye þat hier 3 by 2, $\begin{array}{|l} 46836 \\ \hline 234 \end{array}$ þat wol be 6, set þat 6 þe quych stondes ouer þe hede of þat 2, And þen worch as I tazt þe 8 afore.

¶ Si supraposita cifra debet multiplicare

¹ leaf 160 a.

Prorsus eam deles & ibi scribi cifra debet.

¶ Here is þe 6 case, þe quych is þis : yf hit happe þat þe figure 12 by þe quych þou schal multiplye þe hier figure, þe quych stondes ryght ouer hym by a 0, þou schalt do away þat figure, þe quych ouer þat cifre hede. ¶ And write *þere* þat noubre þat comes of þe multiplicacion as þus, 23. do away 2 and sett *þere* a 0. vnde 16 versus.

The sixth case of the craft.

¶ Si cifra multiplicat aliam positam super ipsam

Sitque locus supra vacuus super hanc cifram fiet.

¶ Here is þe 7 case, þe quych is þis : yf a 0 schal multiply a 20 figure, þe quych stondes not recte ouer hym, And ouer þat 0 stonde no thying, þou schalt write ouer þat 0 anoper 0 as þus : $\begin{array}{|l} 24 \\ \hline 03 \end{array}$ multiplye 2 be a 0, it wol be nothyng. write *þere* a 0 ouer þe hede of þe *neþer* 0, And þen worch forth til þou come to þe ende.

The seventh case of the craft.

24 ¶ **Si supra² fuerit cifra semper est pretereunda.**

¶ Here is þe 8 case, þe quych is þis : yf *þere* be a 0 or mony cifers in þe hier rewe, þou schalt not multiplie hem, bot let hem stonde. And antery þe figures beneþe to þe next figure sygnifyt

The eighth case of the craft.

28 as þus : $\begin{array}{|l} 00032 \\ \hline 22 \end{array}$. Ouer-lepe alle þese cifers & sett þat ³*neþer* 2 þat stondes toward þe ryght side, and sett hym vndur þe 3, and sett þe *oper* nether 2 nere hym, so þat he stonde vndur þe thrydde 0, þe quych stondes next 3. And þan worch. vnde versus.

³ leaf 160 b.

32 ¶ **Si dubites, an sit bene multiplicacio facta,**

Divide totalem numerum per multiplicantem.

¶ Here he teches how þou schalt know wheþer þou hase wel I do or no. And he says þat þou schalt deuide alle þe noubre þat comes of þe multiplicacion by þe *neþer* figures. And þen þou schalt haue þe same noubur þat þou hadyst in þe begynnyng. but zet þou hast not þe craft of dyuision, but þou schalt haue hit afterwarde.

How to prove the multiplication.

² 'Supra' inserted in MS. in place of 'cifra' marked for erasure.

¶ *Per numerum si vis numerum quoque multiplicare*

¶ *Tantum per normas subtiles absque figuris*

Has normas poteris per versus scire sequentes.

Mental multi-
plication.

¶ Here he teches þe to multiplie þe þowzt figures in þi mynde. 4
And þe sentence of þis verse is þis: yf þou wel multiplie on nounbre
by anoþer in þi mynde, þou schal haue þereto rewles in þe verses
þat schal come after.

¶ *Si tu per digitum digitum vis multiplicare* 8

Regula precedens dat qualiter est operandum.

Digit by digit
is easy.
1 leaf 161 a.

¶ Here he teches a rewle as þou hast afore to multiplie a digit
be anoþer, as yf þou wolde wete qwat is sex tymes 6. þou schalt
wete by þe rewle þat I tazt þe before, yf þou haue mynde þerof. 12

¶ *Articulum si per reliquum reliquum vis multiplicare*

In proprium digitum debet vterque resolui.

¶ *Articulus digitos post se multiplicantes*

Ex digitus quociens retenerit multiplicari 16

Articuli faciunt tot centum multiplicati.

The first case
of the craft.

¶ Here he teches þe furst rewle, þe quych is þis: yf þou wel
multiplie an articul be anoþer, so þat both þe articuls bene with-
Inne an hundreth, þus þou schalt do. take þe digit of bothe the 20
articuls, for euery articul hase a digit, þen multiplie þat on digit by
þat oþer, and loke how many vnytes ben in þe nounbre þat comes
of þe multiplicacion of þe 2 digittes, & so many hundrythes ben in
þe nounbre þat schal come of þe multiplicacion of þe ylke 2 articuls 24
as þus. yf þou wold wete qwat is ten tymes ten. take þe digit of
ten, þe quych is 1; take þe digit of þat oþer ten, þe quych is on.

Article by
article;

an example:

¶ Also multiplie 1 be 1, as on tyme on þat is but 1. In on is but
on vnite as þou wost welle, þerefore ten tymes ten is but a hun- 28
dryth. ¶ Also yf þou wold wete what is twenty tymes 30. take þe
digit of twenty, þat is 2; & take þe digitt of thrytty, þat is 3.

another ex-
ample:

multiplie 3 be 2, þat is 6. Now in 6 ben 6 vnites, ¶ And so many
hundrythes ben in 20 tymes 30², þerefore 20 tymes 30 is 6 hun- 32
dryth euend. loke & se. ¶ But yf it be so þat one articul be with-
Inne an hundryth, or by-twene an hundryth and a thowsande, so
þat it be not a þowsande fully. þen loke how many vnytes ben in
þe nounbur þat comys of þe multiplicacion ³And so many tymes³ 36
of 2 digittes of ylke articuls, so many thowsant ben in þe nounbre,
the quych comes of þe multiplicacion. And so many tymes ten
thowsand schal be in þe nounbre þat comes of þe multiplicacion of

2 leaf 161 b.

2 articuls, as yf þou wold wete qwat is 4 hundryth tymes [two hundryth]. Multiply 4 be 2,¹ þat wol be 8. in 8 ben 8 vnites.

¶ And so many tymes ten thousand be in 4 hundryth tymes <sup>Mental multi-
plication.</sup> 4 [2]¹ hundryth, þat is 80 thousand. Take hede, I schall telle þe a generalle rewle whan þou hast 2 articuls, And þou wold wete qwat comes of þe multiplicacion of hem 2. <sup>Another ex-
ample.</sup> multiplie þe digit of þat on articuls, and kepe þat noubre, þen loke how many cifers schuld go 8 before þat on articuls, and he were write. Als many cifers schuld go before þat other, & he were write of cifers. And haue alle þe ylke cifers togedur in þi mynde, ^{2 leaf 162 a.} 2 a-rowe ychon aftur other, and in þe last plase set þe noubre þat comes of þe multiplicacion of þe 12 2 digittes. And loke in þi mynde in what place he stondes, where in þe secunde, or in þe thryd, or in þe 4, or where ellis, and loke qwat þe figures by-token in þat place; & so mych is þe noubre þat comes of þe 2 articuls y-multiplied to-gedur as þus: yf þou wold <sup>Another ex-
ample.</sup> 16 wete what is 20 thousand tymes 3 þowsande. multiply þe digit of þat articulle þe quych is 2 by þe digitte of þat oþer articul þe quych is 3, þat wol be 6. þen loke how many cifers schal go to 20 thousand as hit schuld be write in a tabul. certainly 4 cifers schuld go to 20 20 þowsant. ffor þis figure 2 in þe fyrst place betokenes twene.

¶ In þe secunde place hit betokenes twenty. ¶ In þe 3. place hit ^{Notation.} betokenes 2 hundryth. ¶ In þe 4 place 2 thousand. ¶ In þe 5 place hit betokenes twenty þowsant. þefore he most haue 4 cifers 24 a-fore hym þat he may stonde in þe 5 place. kepe þese 4 cifers in thy mynde, þen loke how many cifers gon to 3 thousand. Certayn to 3 thousande ^{3 gon} 3 cifers afore. Now cast ylke 4 cifers þat ^{3 leaf 162 b.} 28 schuld go to twenty thousand, And thes 3 cifers þat schuld go afore 3 thousand, & sette hem in rewe ychon after oþer in þi mynde, as þai schuld stonde in a tabulle. And þen schal þou haue 7 cifers; þen sett þat 6 þe quych comes of þe multiplicacion of þe 2 digittes aftur þe ylke cifers in þe 8 place as yf þat hit stode in a 32 tabul. And loke qwat a figure of 6 schuld betoken in þe 8 place. yf hit were in a tabul & so mych it is. & yf þat figure of 6 stonde in þe fyrst place he schuld betoken but 6. ¶ In þe 2 place he schuld betoken sixty. ¶ In the 3 place he schuld betoken sex hundryth.

36 ¶ In þe 4 place sex thousand. ¶ In þe 5 place sixty þowsant. <sup>Notation
again.</sup> ¶ In þe sext place sex hundryth þowsant. ¶ In þe 7 place sex þowsant thousandes. ¶ In þe 8 place sixty þowsant thousandes. þefore sett 6 in octano loco, And he schal betoken sixty þowsant

¹ 4 in MS.

Mental multiplication.

thousantes. And so mych is twenty þowsant tymes 3 thousand,
¶ And þis rewle is generale for alle maner of articuls, Whether
þai be hundryth or þowsant; but þou most know well þe craft of þe
wryrehyngē in þe tabulle¹ or þou know to do þus in þi mynde
aftur þis rewle. Thou most þat þis rewle holdyþe note but where
þere ben 2 articuls and no mo of þe quych ayther of hem hase but
on figure significatyf. As twenty tymes 3 thousand or 3 hundryth,
and such oþur.

¹ leaf 163 a.

4

8

¶ *Articulum digito si multiplicare oportet**Articuli digit[i] sumi quo multiplicatē**Debemus reliquum quod multiplicatur ab illis**Per reliquo decuplum sic summam laterē nequibit.*

12

The third case of the craft;

¶ Here he puttes þe thryde rewle, þe quych is þis. yf þou wel
multiply in þi mynde, And þe Articul be a digitte, þou schalt loke
þat þe digitt be with-Inne an hundryth, þen þou schalt multiply the
digitt of þe Articulle by þe oþer digitte. And every vnite in þe 16
noubre þat schalle come þere-of schal betoken ten. As þus: yf
þat þou wold wete qwat is twyes 40. multiplic þe digitte of 40, þe
quych is 4, by þe oþer diget, þe quych is 2. And þat wolle be 8.
And in þe nombre of 8 ben 8 vnites, & every of þe ylke vnites 20
schuld stonde for 10. þere-fore þere schal be 8 tymes 10, þat wol
be 4 score. And so mony is twyes 40. ¶ If þe articul be a hun-
dryth or be 2 hundryth And a þowsant, so þat hit be notte a
thousand,² worch as þou dyddyst afore, saue þou schalt rekene every 24
vnite for a hundryth.

an example.

² leaf 163 b.¶ *In numerum mixtum digitum si ducere cures**Articulus mixti sumatur deinde resoluas**In digitum post fac respectu de digitis*

28

*Articulusque docet excrecens in diriuando**In digitum mixti post ducas multiplicantem*¶ *De digitis vt norma³ [docet] de [hunc]**Multiplica simul et sic postea summa patebit.*

32

The fourth case of the craft:

Here he puttes þe 4 rewle, þe quych is þis: yf þou multipli
on composit be a digit as 6 tymes 24,⁴ þen take þe diget of þat com-
posit, & multiply þat digitt by þat oþer diget, and kepe þe nombur
þat comes þere-of. þen take þe digit of þat composit, & multiply þat
digit by anoþer diget, by þe quych þou hast multiplied þe diget of
þe articul, and loke qwat comes þere-of. þen take þou þat nounbur,
& cast hit to þat other nounbur þat þou secheste as þus yf þou wel

Composite by digit.

³ docet. deceet MS.⁴ '4 times 4' in MS.

wete quat comes of 6 tymes 4 & twenty. multiply þat articulle of þe composit by þe digit, þe quych is 6, as yn þe thryd rewle þou was tauzt, And þat schal be 6 score. þen multiply þe diget of þe 4 composit, ¹þe quych is 4, and multiply þat by þat other diget, þe quych is 6, as þou wast tauzt in þe first rewle, yf þou haue mynde þerof, & þat wol be 4 & twenty. Cast all ylke nomburs to-gedir, & hit schal be 144. And so mych is 6 tymes 4 & twenty.

Mental multiplication.

¹ leaf 164 a.

8 ¶ **Ductus in articulum numerus si compositus sit**
Articulum purum comites articulum quoque
Mixti pro digitis post fiat [et articulus vt]
Norma iubet [retinendo quod extra dicta ab illis]
 12 **Articuli digitum post tu mixtum digitum duc**
Regula de digitis nec precipit articulusque
Ex quibus excrescens summe tu iunge priori
Sic manifesta cito fiet tibi summa petita.

16 ¶ Here he puttes þe 5 rewle, þe quych is þis: yf þou wel multiply an Articul be a composit, multiplie þat Articul by þe articul of þe composit, and worch as þou was tauzt in þe secunde rewle, of þe quych rewle þe verse begynnes þus. ¶ **Articulum si** The fifth case of the craft:
 20 **per Relicum vis multiplicare.** þen multiply þe diget of þe composit by þat oþir articul aftir þe doctrine of þe 3 rewle. take þerof gode hede, I pray þe as þus. Yf þou wel wete what is 24 tymes ten. Multiplie ten by 20, þat wel be 2 hundryth. þen multiply þe diget of þe 10, þe quych is 1, by þe diget of þe composit, þe quych is 4, & þat ²wol be 4. þen reken enery vnite þat is in 4 for 10, & þat schal be 40. Cast 40 to 2 hundryth, & þat wol be 2 hundryth & 40. And so mych is 24 tymes ten. An example.

Article by Composite.

An example.

² leaf 164 b.

28 ¶ **Compositum numerum mixto si[e] multiplicabis**
Vndecies tredecim sic est ex hiis operandum
In reliquum primum demum duc post in eundem
Vnum post denum duc in tria deinde per vnum
 32 **Multiplicesque demum intra omnia multiplicata**
In summa decies quam si fuerit tibi doces
Multiplicandorum de normis sufficiunt hec.

¶ Here he puttes þe 6 rewle, & þe last of alle multiplicacion), þe quych is þis: yf þou wel multiplie a composit by a noþer composit, þou schalt do þus. multiplie þat on composit, qwych þou welt of the twene, by þe articul of þe toþer composit, as þou were tauzt in þe 5 rewle, þen multiplie þat same composit, þe quych þou hast multiplied by þe oþer articul, by þe digit of þe oþer composit, as

The sixth case of the craft:

Composite by Composite.

Mental multiplication.
An example

þou was tauzt in þe 4 rewle. As þus, yf þou wold wete what is 11 tymes 13, as þou was tauzt in þe 5 rewle, & þat schal be an hundredryth & ten, afterwarde multiply þat same composit þat þou hast multiplied, þe quych is a .11. And multiplie lit be þe digit of þe oþer composit, þe quych is 3, for 3 is þe digit of 13, And þat wel be 30. þen take þe digit of þat composit, þe quych composit þou multiplied by þe digit of þat oþer composit, ¹þe quych is a 11.

¹ leaf 165 a.

of the sixth case of the craft.

¶ Also of þe quych 11 on is þe digit. multiplie þat digitt by þe digitt of þat other composit, þe quych diget is 3, as þou was tauzt in þe first rewle in þe begynnyng of þis craft. þe quych rewle begynnes “In digitum cures.” And of alle þe multiplicacion of þe 2 digitt comys thre, for onys 3 is but 3. Now cast alle þese noubers togedur, the quych is þis, a hundredryth & ten & 30 & 3. And al þat wel be 143. Write 3 first in þe ryght side. And cast 10 to 30, þat wol be 40. set 40 next aftur towarde þe lyft side, And set aftur a hundredryth as here an Ensampulle, 143.

(Cetera desunt.)

The Art of Nombryng.

A TRANSLATION OF

John of Holywood's *De Arte Numerandi*.

[*Ashmole MS.* 396, fol. 48.]

Boys seying in the begynnyng of his *Arsemetrike*:—**A**lle
 thynges that bene fro the first begynnyng of thynges
 have *procedede*, and come forth*e*, And by resoun of
 nombre ben forme*d*; And in wise as they bene, So owethe
 4 they to be knowene; wherfor in vniu*ersall* knowlechyng
 of thynges the Art of nombrynge is best, and most
 operatyf*e*. Fol. 48.

Therfore sithen the science of the whiche at this tyme we
 8 intendene to write of standithe alle and about nombre: The name of
the art.
 first we most se, what is the *propre* name therof*e*, and fro
 whens the name come: Afterwarde what is nombre, And how
 manye spices of nombre ther ben. The name is clepede *Algorisme*.
 12 had*e* out of *Algo*r*e*, other of *Algo*s, in grewe, That is clepide in Derivation of
Algorism.
 englissh*e* art other craft, And of *Rithmus* that is callede nombre.
 So *algorisme* is clepede the art of nombryng, other it is had of*e* en
 or in, and *gogos* that is *introduccioun*, and *Rithmus* nombre, that is Another.
 16 to say *Interduccioun* of nombre. And thirdly it is had*e* of the
 name of a kyng that is clepede *Algo* and *Rythmus*; So callede
Algorismus. Sothely .2. manere of nombres ben notified*e*;
 Formalle,¹ as nombre is vnitees gadrede to-gedres; Materialle,² as Another.
 20 nombre is a colleccioun of vnitees. Other nombre is a multitude
 had*e* out of vnitees, vnitee is that thyng wher-by euery thyng*e* is
 callede oone, other o thyng*e*. Of nombres, that one is cleped*e*
 digitalle, that other*e* Article, Another a nombre componede *oper*
 24 myxt. Another digitalle is a nombre *wit*-in .10.; Article is *part* Kinds of
numbers.
 nombre that may be dyvydede in .10. parties egally, And that there

¹ MS. Materialle.

² MS. Formalle.

The 9 rules
of the Art.

leve no residue; Componede or medlede is that nombre that is come of a digite and of an article. And vndrestande wele that alle nombres betwix .2. articles next is a nombre componede. Of this art bene .9. spices, that is forto sey, numeracioun, addicioun, Subtraccioun, Mediacioun, Duplacioun, Multiplacioun, Dyvysioun, Progressioun, And of Rootes the extraccioun, and that may be hade in .2. maners, that is to sey in nombres quadrat, and in cubices: Amonge the whiche, first of Numeracioun, and afterwarde of þe 8 opers by ordure, y entende to write.

¹ Fol. 48 b.

¹ For-sothe numeracioun is of euery numbre by competent figures an artificialle representacioun.

Figures,
differences,
places, and
limits.

Sothly figure, difference, places, and lynes supposen o thyng 12 other the same, But they ben sette here for dyuers reasons. figure is elepede for *protraccioun* of figuracioun; Difference is callede for therby is shewede euery figure, how it hathe difference fro the figures before them: place by cause of space, where-in me 16 writethe: lynees, for that is ordeynede for the presentacioun of

The 9 figures.

euery figure. And vnderstonde that ther ben .9. lynytes of figures that representen the .9. digitis that ben these. 0. 9. 8. 7. 6.

The cipher.

5. 4. 3. 2. 1. The .10. is elepede theta, or a cercle, other a cifre, 20 other a figure of nought for nought it signyfieth. Nathelesse she holdyng that place giveth others for to signyfie; for withe-out cifre or cifres a pure article may not be write. And sithen that by these .9. figures significatifes Ioynede with cifre or with cifres alle 24 nombres ben and may be representede, It was, nether is, no nede to

The numeration

fynde any more figures. And note wele that euery digite shalle be write with oo figure allone to it aproprede. And alle articles by a cifre, ffor euery article is namede for oone of the digitis as .10. of 28 1. .20. of .2. and so of the others, &c. And alle nombres digitalle owen to be sette in the first difference: Alle articles in the seconde.

of digits,

Also alle nombres fro .10. til an .100. [which] is excludede, with .2. figures mvst be write; And yf it be an article, by a cifre first put, 32 and the figure y-write towarde the lift honde, that signyfieth the digit of the whiche the article is namede; And yf it be a nombre

of composites.

componede, first write the digit that is a part of that componede, and write to the lift side the article as it is seile be-fore. Alle 36 nombre that is fro an hundrede tille a thousande excludede, owithe to be writ by .3. figures; and alle nombre that is fro a thousande

til .x. Mt. must be writ by .4. figures; And so forthe. And vnder-
 stonde wele that every figure sette in the first place signifyeth his
 digit; In the seconde place .10. tymes his digit; In the .3. place an
 4 hundrede so moche; In the .4. place a thousande so moche; In the
 .5. place .x. thousande so moche; In the .6. place an hundrede
 thousande so moche; In the .7. place a thousande thousande. And
 so infynytly multiplying by these .3. 10, 100, 1000. And vnder-
 8 stande wele that competently me may sette vpon figure in the place
 of a thousande, a prike to shewe how many thousande the last figure
 shalle represent. We writene in this art to the left side-ward, as
 arabiane writene, that weren fynders of this science, othere for this
 12 resoun, that for to kepe a custumable ordre in redyng, Sette we
 alle-wey the more nombre before.

The value
due to posi-
tion.

1 Fol. 49.

Numbers are
written from
right to left.

Addiciozn is of nombre other of nombres vnto nombre or to
 nombres aggregaciozn, that me may see that that is come
 16 therof as excessent. In addiciozn, 2. ordres of figures and
 .2. nombres ben necessary, that is to sey, a nombre to be addede
 and the nombre wherto the addiciozn sholde be made to. The
 nombre to be addede is that þat sholde be addede therto, and shalle
 20 be vnderwriten; the nombre vnto the whiche addiciozn shalle be
 made to is that nombre that reseyueth the addicion of þat other,
 and shalle be writen above; and it is convenient that the lesse
 nombre be vnderwrit, and the more addede, than the contrary.
 24 But whether it happe one other other, the same comythe of,
 Therfor, yf þow wilt adde nombre to nombre, write the nombre
 wherto the addiciozn shalle be made in the omest ordre by his
 differences, so that the first of the lower ordre be vndre the first
 28 of the omyst ordre, and so of others. That done, adde the first of
 the lower ordre to the first of the omyst ordre. And of suche
 addiciozn, other þere growith therof a digit, An article, other a
 composede. If it be digitus, In the place of the omyst shalt thou
 32 write the digit exerescyng, as thus:—

Definition.

How the
numbers
should be
written.

The method
of working.

Begin at the
right.

The resultant	[2]
To whom it shal be addede	[1]
The nombre to be adde	[1]

If the article; in the place of the
 omyst put a-way by a cifre writte,
 and the digit transferrede, of þe

The Sum is
a digit,

36 whiche the article toke his name, towarde the left side, and be it
 addede to the next figure folowyng, yf ther be any figure folowyng;
 or no, and yf it be not, leve it [in the] voide, as thus:—

or an article,

The resultant	10
To whom it shalle be addede	7
The nombre to be addede	3

Resultans	2 7 8 2 7
Cui <i>debet</i> addi	1 0 0 8 4
Numerus addendus	1 7 7 4 3

And yf it happe that the figure folowyng wherto the addiciozn shalle be made by [the cifre of] an article, it sette a-side; In his

¹ Fol. 49 b.

place write the ¹[digit of the] Article as thus:—

The resultant	17
To whom it shalle be addede	10
The nombre to be addede	7

And yf it happe that a figure of .9. by the figure that me mvst adde [one] to, In the place of that 9. put a cifre *and* write þe article towarde þe lift honde as bifore, and thus:—

The resultant	10
To whom it shalle be addede	9
The nombre to be addede	1

or a compo-
s.te.

And yf ²[therefrom grow a] nombre composed, ³[in the place of the nombre] put a-way ⁴[let] the digit [be] ⁵writ þat is part of þat compositide, and þan put to þe lift side the article as before, and þus:—

The resultant	12
To whom it shalle be addede	8
The nombre to be addede	4

The trans-
lator's note.

This done, adde the seconde to the seconde, and write above *oper* as before. Note wele þat in addicions and in alle spices folowyng, whan he seithe one the other shalle be written aboue, and me most 16 vse euer figure, as that evely figure were sette by halfe, and by hym-selfe.

Definition of
Subtraction.

Subtraccioun is of .2. *proposede* nombres, the fyndyng of the **S**excesse of the more to the lasse: Other subtraccioun is 20 *ablacioun* of o nombre fro a-nother, that me may see a some left. The lasse of the more, or even of even, may be *withdraw*; The more fro the lesse may neuer be. And sothly that nombre is 24 more that hathe more figures, So that the last be signyficatifes: 24 And yf ther ben as many in that one as in that other, me most deme it by the last, other by the next last. More-ouer in *with-* 28 *drawyng* .2. nombres ben necessary; A nombre to be *withdraw*, And a nombre that me shalle *with-draw* of. The nombre to be *with-draw* shalle be writ in the lower ordre by his differences; The

How it may
be done.

What is re-
quired. i

² 'the' in MS.

³ 'be' in MS.

⁴ 'and' in MS.

⁵ 'is' in MS.

nomb^re fro the whiche me shalle withe-draw in the omyst ordre, so that the first be vnder the first, the seconde vnder the seconde, And so of alle others. Withe-draw therfor the first of the lowere 4 ordre fro the first of the ordre above his hede, and that wolle be

Write the greater number above.

Subtract the first figure if possible.

other more or lesse, *oper* egalle. yf it be egalle or even the figure sette beside, put in his place a 8 cifre. And yf it be more put away *per*fro als many of vnitees the lower figure conteyneth, and writ the residue as thus

The remanent	20
Wherof me shalle <i>with</i> -draw	22
The nombre to be <i>with</i> -draw	2

The remanent	2 2
Wherof me shalle <i>with</i> -draw	2 8
<i>pe</i> nombre to be <i>with</i> -draw	6

12 And yf it be lesse, by-cause the more may not be with-

¹ Fol. 50.

If it is not possible ² borrow ten,

Remanens	2 2 1 8 2 9 9 9 8
A quo sit subtraccio	8 7 2 4 3 0 0 0 4
Numerus subtrahendus	6 5 2 [6] 6

16 draw ther-fro, borow an vnyte of the next figure that is worthe 10. Of that .10. and of the figure that ye wolde have *with*-draw fro be-fore to-gedre Ioyne^d, *with*-draw *pe* figure be-nethe, and put the residue in the place of the figure

and then subtract.

20 put a-side as *pus* :—

And yf the figure wherof me shal borow the vnyte be one,

The remanent	1 8
Wherof me shalle <i>with</i> -draw	2 4
The nombre to be <i>with</i> -draw	0 6

If the second figure is one.

put it a-side, and write a cifre in the place *per*of, lest the figures 24 folowing faile of thaire nombre, and *pan* worche as it sheweth in this figure here :—

And yf the vnyte wherof me shal borow be a cifre, go

The remanent	3 0 9
Wherof me shal <i>with</i> -draw	3 1 2
The nombre to be <i>with</i> -draw	. . 3

If the second figure is a cipher.

28 fether to the figure signy-

ficatife, and ther borow one, and *retournyng* bake, in the place of every cifre *pat* ye passide over, sette figures of .9. as here it is specifiende :—

32 And whan me comethe to the nombre wherof me intendithe, there re-

The remenaunt	2 9 9 9 9
Wherof me shalle <i>with</i> draw	3 0 0 0 3
The nombre to be <i>with</i> -draw	4

A justification of the rule given.

36 *pat* for every cifre left behynde me setteth figures ther of .9. this it is :—If fro the .3. place me borowede an vnyte, that vnyte by respect of the figure that he came fro representith an .C., In the

² 6 in MS.

³ 0 in MS.

place of that cifre [passel over] is left .9., [which is worth ninety], and yit it remayneth as .10., And the same resone wolde be yf me hade borowede an vnyte fro the .4., .5., .6., place, or any other so vpwarde. This done, withdraw the seconde of the lower 4
 ordre fro the figure above his hede of þe omyst ordre, and wirche as before. And note wele that in addicion or in subtraccioun me may wele fro the lift side begynne and ryn to the right side, But it wol be more profitabler to be do, as it is taught. And yf thou 8
 wilt prove yf thou have do wele or no, The figures that thou hast withdraw, adde them ayene to the omyst figures, and they wolde accorde with the first that thou haddest yf thou have labored 12
 wele; and in like wise in addicion, whan thou hast addede alle thy figures, withdraw them that thou first ¹addest, and the same wolde retourne. The subtraccioun is none other but a prouffe of the addicion, and the contrarye in like wise.

Why it is better to work from right to left.

How to prove subtraction,

and addition.

¹ Fol. 50 b.

Definition of mediation.

Where to begin.

If the first figure is unity.

What to do if it is not unity.

Then halve the second figure.

Mediacioun is the fyndyng of the halfyng of euery nombre, 16
 that it may be seyne what and how moche is euery halfe. In halfyng ay oo order of figures and oo nombre is necessary, that is to sey the nombre to be halfede. Therfor yf thou wilt half any nombre, write that nombre by his differences, and 20
 begynne at the right, that is to sey, fro the first figure to the right side, so that it be signyficatife other represent vnyte or eny other digitalle nombre. If it be vnyte write in his place a cifre for the figures folowyng, [lest they signify less], and write that vnyte 24
 without in the table, other resolue it in .60. mynutes and sette a-side half of tho minutes so, and reserve the remenaunt without in the table, as thus .30.; other sette without thus . $\overline{17}$: that kepeth none ordre of place, Nathelesse it hath signyficioum. And yf 28
 the other figure signyfie any other digital nombre fro vnyte forth, oþer the nombre is ode or evene. If it be even, write this half in this wise:—

Halfede	2 2
to be halfede	4 4

And if it be odde, Take the next even vndre 32

hym conteynede, and put his half in the place of that odde, and of þe vnyte that remayneth to be halfede

do thus:—

halfede	2 3	[di]
To be halfede	4 7	

This done, the seconde is to be halfede, yf 36

it be a cifre put it be-side, and yf it be signyficatife, other it is even or ode: If it be even, write in the place of þe nombres wipede out the halfe; yf it be ode, take the next even vnder it contenythe, and in the place of the Impar sette a-side put half of the even: The 40

vnyte that remayneth to be halfede, respect hade to them before, is worthe .10. Dyvide that .10. in .2., 5. is, and sette a-side that one, and adde that other to the next figure

If it is odd, add 5 to the figure before.

4 precedent as here :—

Halfede			
to be halfede			

And yf þe addiciozn sholde be made to a cifre, sette it a-side, and write in his place .5. And vnder this fourme me shalle write and worche,

8 tille the totalle nombre be halfede.

doubled		2		6		8		9		0		10		17		4
to be doubled		1		3		4		4		5		5		8		7

Duplicaciozn is agrvgacion of nombre [to itself] þat me may se the nombre growen. In doublynge ay is but one ordre of figures necessarie. And me most be-gynne with the lift

Definition of Duplution.

12 side, other of the more figure, And after the nombre of the more figure representithe. ¹In the other .3. before we begynne alle way

¹ Fol. 51.

16 fro the right side and fro the lasse nombre, In this spice and in alle other folowyng we wolle begynne fro the lift side, flor and me

Where to begin.

bigon the double fro the first, omwhiie me myght double oo thyng twyes. And how be it that me myght double fro the right, that wolde be harder in techyng and in workyng. Therfor yf thou

Why.

20 wolt double any nombre, write that nombre by his differences, and double the last. And of that doublyng other growithe a nombre digital, article, or componede. [If it be a digit, write it in the place of the first digit.] If it be article, write in his place a cifre

24 and transferre the article towarde the lift, as thus :—

double		10
to be doubled		5

What to do with the result.

And yf the nombre be componede, write a digital that is part of his composiciozn, and sette the article to the

28 lift hande, as thus :—

That done, me most double the last save one, and what growethe þerof me most worche as

doubled		16
to be doubled		8

32 shalle be addede to the cifre, in þe place of þe figure wipede out me most write the nombre to be addede,

as thus :—

doubled		6		0		6
to be doubled		3		0		3

36 In the same wise me shalle wirche of alle others. And this probaciozn : If thou truly double the halvis, and truly half the doubles, the same nombre and figure shalle mete, suche as

How to prove your answer.

thow labourede vpon first, And of the

Doubled		6		1		8
to be doubled		3		0		9

40 contrarie.

Definition of Multiplication.

Multiplicacion of nombre by hym-self other by a-nother, with *proposide* .2. nombres, [is] the fyndyng of the thirde, That so oft conteyneth that other, as ther ben vnytes in the *oper*. In multiplicacion .2. nombres pryncipally ben necessary, that is to sey, the nombre multiplying and the nombre to be multipliede, as here;—twies fyve. [The number multiplying] is designede aduerbially. The nombre to be multipliede reseeyveth a *nominalle* appellacion, as twies .5. 5. is the nombre multipliede, and twies is the nombre to be multipliede.

Multiplier.

Multiplicand.

Resultans	1 1 0	1 3 2	6 6 8	0 0 8
Multiplicandus	. . 5	. . 4	. 3 4	0 0 4
Multiplicans	. 2 2	. 3 3	2 2 2	. . .

Product.

Also me may therupon to assigne the .3. nombre, the whiche is *elepede product* or *provenient*, of takyng out of one fro another: as twies .5 is .10., 5. the nombre to be multipliede, and .2. the multipliant, and .10. as before is come therof. And vnderstonde wele, that of the multipliant may be made the nombre to be multipliede, and of the contrarye, remaynyng euer the same some, and herofe cometh the comen speche, that seithe all nombre is convertede by Multiplying in hym-selfe.

² Fol. 51 b.

There are 6 rules of Multiplication.

(1. Digit by digit.

See the table above.

(2. Digit by article.

And ther ben .6. rules of Multiplicacion; first, yf a digit multiplie a digit, considre how many of vnytees ben betwix the digit by multiplying and his .10. bethe *to-gedre* accomptede, and so oft *with-draw* the digit multiplying, vnder the article of his *denominacion*. Example of grace. If thou wolt wete how moche is .4. tymes .8., ⁴se how many vnytees ben betwix .8.⁵ and .10. *to-geder* rekenede, and it shewith that .2.: *with-draw* ther-for the *quaternary*, of the article of his *denominacion* twies, of .40., And ther remayneth .32., that is, to some of alle the multiplicacion. Wher-upon for more evidence and declaracion the seide table is made. Whan a digit multipliethe an article, thou most bryng the digit into þe digit, of þe whiche the article [has]⁶ his name, and every vnyte

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	10 ²	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	56	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

¹ 2 in MS.

³ sic.

⁴ 'And' inserted in MS.

⁵ '4 the' inserted in MS.

⁶ 'to' in MS.

shalle stonde for .10., and euery article an .100. Whan the digit ^{(3) Composite by digit.} multiplieth a nombre componede, þou most bryng the digit into aþer part of the nombre componede, so þat digit be had into digit 4 by the first rule, into an article by þe seconde rule; and afterwarde Ioyne the produccioun, and þere wol be the some totalle.

Resultans		1		2		6		7		3		6		1		2		0		1		2		0		8
Multiplicandus				2					3		2					6										4
Multiplicans				6		3		2		3					2		0				3		0		2	

Whan an article multiplieth an article, the digit wherof he is ^{(4) Article by article.} namede is to be brought Into the digit wherof the oper is namede, 8 and euery vnyte wol be worthe ¹an .100., and euery article. a .1000. Whan an article multiplieth a nombre componede, thow ¹Fol. 52. most bryng the digit of the article into aither part of the nombre componede; and Ioyne the produccioun, and euery article wol be 12 worthe .100., and euery vnyte .10., and so wolle the some be opene. Whan a nombre componede multiplieth a nombre componede, euery part of the nombre multiplying is to be hade into 16 euery part of the nombre to be multipliede, and so shalle the digit be hade twies, onys in the digit, that other in the article. The article also twies, ones in the digit, that other in the article. Therfor yf thow wilt any nombre by hym-self other by any other 20 multiplie, write the nombre to be multipliede in the ouer ordre by his differences, The nombre multiplying in the lower ordre by his differences, so that the first of the lower ordre be vnder the last of ^{How to set down your numbers.} the ouer ordre. This done, of the multiplying, the last is to be hade into the last of the nombre to be multipliede. Wherof than 24 wolle grow a digit, an article, other a nombre componede. If it be a digit, even above the figure multiplying is hede write his digit ^{If the result is a digit,} that come of, as it appereth here:—

The resultant		6
To be multipliede		3
De nombre multipling		2

And yf an article had be writ ouer the figure multiplying his hede, ^{an article,} 28 put a cifre þer and transerre the article towarde the lift hande, as thus:—

The resultant		1		0
to be multipliede				5
þe nombre multipling				2

And yf a nombre componede be writ ouer the figure multiplying is ^{or a composite.} hede, write the digit in the nombre componede is place, and sette 32 the article to the lift hande, as thus:—

Multiply next
by the last
but one, and
so on.

The resultant	1 2
To be multiplie ^d	4
the nombre multiplying	3

This done, me most bryng the last save one of the multiplying into the last of þe nombre to be multiplie^d, and se what comyth^e therof 4

as before, and so do with alle, till me come to the first of the nombre multiplying, that must be brought into the last of the nombre to be multiplie^d, wherof growithe oper a digit, an article,

¶ Fol. 52 b.

1 other a nombre componde. If it be a digit, In the place of the ouer^e, sette a-side, as here :

Resultant	6 6
to be multiplie ^d	3
the nombre multipliyng	2 2

If an article happe, there put a cifre in his place, and put hym to the lift hande, as here :

The resultant	1 1 0
to be multiplie ^d	5
þe nombre multiplying	2 2

If it be a nombre componde, in the place of the ouer^e sette a-side, write a digit that² is a part of the componde, and sette on the left honde the article, as here :

The resultant	1 3 ³ 2
to be multiplie ^d	4
þe nombre multipliant	3 3

Then entery
the multiplier
one place.

That done, sette forward^e the figures of the nombre multiplying

by oo difference, so that the first of the multipliant be vnder the last save one of the nombre to be multiplie^d, the other by o place sette forward^e. Than me shall^e brynge the last of the multipliant in hym to be multiplie^d, vnder the whiche is the first multipliant.

Work as be-
fore.

And than wolle growe oper a digit, an article, or a componde nombre. If it be a digit, adde hym even above his hede ; If it be an article, transferre hym to the lift side ; And if it be a nombre componde, adde a digit to the figure above his hede, and sette to the lift hande the article. And alle-wayes enery figure of the nombre multipliant is to be brought to the last save one nombre to be multiplie^d, til me come to the first of the multipliant, where me shall^e wirche as it is seide before of the first, and afterwarde to put forward^e the figures by o difference and one till they alle be multiplie^d. And yf it happe that the first figure of þe multipliant be a cifre, and boue it is sette the figure signyficatife, write a cifre in the place of the figure sette a-side, as thus, etc. :

How to deal
with cyphers.

The resultant	1 2 0
to be multiplie ^d	6
the multipliant	2 0

² 'that' repeated in MS.

³ '1' in MS.

And yf a cifre happe in the lower order be-twix the first and the last, and even above be sette the figure signyficatif, leve it vntouchede, as here :—

4 And yf the space above sette be voide, in that place write thow a cifre. And yf the cifre happe betwix þe first and the last to be multipliede, me most sette
8 forwarde the ordre of the figures by thaire differences, for oft of ducceozn of figures in cifres nought is the resultant, as here,¹ wherof it is evident and open, yf that
12 the first figure of the nombre be to be multipliede be a cifre, vndir it shalle be none sette as here :—

The resultant	2	2	6	4	4
To be multipliede			2	2	2
The multipliant	1	0	2		

How to deal with cyphers.

¹ Fol. 53.

Resultant	8	0	0	8
to be multipliede	4	0	0	4
the multipliant	2	.	.	.

Vnder [stand] also that in multiplicacion, division, and of rootis the extractioun, competently me may leve a mydel space betwix .2. ordres of

Leave room between the rows of figures.

16

Resultant	3	2	0
To be multipliede		8	0
The multipliant		4	

figures, that me may write there what is come of addyng other with-drawyng, lest any thyng sholde be over-hippede and sette
20 out of mynde.

For to dyvyde oo nombre by a-nother, it is of .2. nombres proposede, It is forto depart the moder nombre into as many partis as ben of vnytees in the lasse nombre. And note
24 wele that in makyng of dyvysioyn ther ben .3. nombres necessary : that is to sey, the nombre to be dyvydede ; the nombre dyvydyng and the nombre exeant, other how oft, or quocient. Ay shalle the
28 the nombre that is to be dyvydede be more, other at the lest evene with the nombre the dyvysere, yf the nombre shall be made by hole nombres. Therfor yf thow wolt any nombre dyvyde, write the
32 the nombre to be dyvydede in þe overer bordure by his differences, the dyvisere in the lower ordure by his differences, so that the last of the dyviser be vnder the last of the nombre to be dyvyde, the next last vnder the next last, and so of the others, yf it may competently be done ; as here :—

Definition of division.

Dividend, Divisor, Quotient.

How to set down your Sum.

The residne		2	7
The quotient			5
To be dyvydede	3	4	2
The dyvyser		6	3

An example.

¹ Blank in MS.

Examples.

Residuum				8				2	7			2		6								
Quociens			2	1		2			5					9								
Diuidendus		6		8		0		6		6		3		4		2		3		3		2
Diuiser		3		2			3					6		3				3		4		

When the last of the divisor must not be set below the last of the dividend.

And ther ben .2. causes when the last figure may not be sette vnder the last, other that the last of the lower nombre may not be with-draw of the last of the ouerer nombre for it is lasse than the lower, other how be it, that it myght be with-draw as for hym-self fro the ouerer the remenaunt may not so oft of them above, other yf þe last of the lower be even to the figure above his hede, and þe next last oper the figure be fore þat be more þan the figure above sette. ¹These so ordeynede, me most wirche from the last figure of þe nombre of the dyvyser, and se how oft it may be with-draw of and fro the figure above his hede, namely so that the remenaunt may be take of so oft, and to se the residue as here :—

¹ Fol. 53².

How to begin.

The residue			2		6	
The quocient					9	
To be dyvydede		3		3		2
The dyvyser			3		4	

An example.

And note wele that me may not with-draw more than .9. tymes nether lasse than ones. Therfor se how oft þe figures of the lower ordre may be with-draw fro the figures of the ouerer, and the nombre that shewith þe quocient most be writ ouer the hede of þat figure, vnder the whiche the first figure is, of the dyviser; And by that figure me most with-draw alle oper figures of the lower ordir and that of the figures above thaire hedis. This so done, me most sette forwarde þe figures of the diuiser by o difference towardes the right honde and worche as before; and thus :—

Where to set the quociente

Examples.

Residuum																		1		2						
quociens				6		5		4		0			2		0		0		4							
Diuidendus		3		5		5		1		2		2		8		8		6		3		7		0		4
Diuisor			5		4		3				4		4		2		3									

The quocient				6		5		4				
To be dyvydede		3		5		5		1		2		2
The dyvyser		5		4		3						

A special case.

And yf it happo after þe setting forwarde of the figures þat þe last of the divisor may not so oft be with-draw of the figure above his hede, above þat figure vnder the whiche the first of the diuiser is writ me most sette a cifre in ordre of the nombre quocient, and sette the figures forwarde as be fore be o difference alone, and so me shalle do in alle nombres to be dyvydede, for where the dyviser may

not be *with-draw* me most sette there a cifre, and sette forwarde the figures; as here:—

	The residue							1		2	
4	The quocient					2		0		0	
	To be dyvydede		8		8		6		3		7
	The dyvyser		4		4		2		3		

And me shalle not cesse fro Another ex-
ample. suche settinge of figures forwarde, nether of settinge of þe quocient into the dyviser,

neþer of subtraccioun of the dyvyser, till the first of the dyvyser be *with-draw* fro þe first to be dividede. The whiche done, or ought,¹ oþer nought shalle remayne: and yf it be ought,¹ kepe it in the tables, And euer vny it to þe diviser. And yf þou wilt wete how many vnytees of þe divisioun² wol growe to the nombre of the diviser, the nombre quocient wol shewe it: and whan suche divisioun is made, and þou lust prove yf thou have wele done or no, Multiplie the quocient by the diviser, And the same figures wolle come ayene that thou haddest bifore and none other. And yf ought be residue, than *with* addicioun therof shalle come the same figures: And so multiplicacioun provithe divisioun, and divisioun multiplicacioun: as thus, yf multiplicacioun be made, divide it by the multipliant, and the nombre quocient wol shewe the nombre that was to be multipliede, etc.

² Fol. 53^b.
What the
quotient
shows.

How to prove
your division,

or multiplica-
tion.

Progressioun is of nombre after egalle excesse fro oone or twayne take agregacioun. of *progressioun* one is *naturelle* or *contynuelle*, þat oþer broken and *discontynuelle*. *Naturelle* it is, whan me begynnethe *with* one, and kepethe ordure overlepyng one; as .1. 2. 3. 4. 5. 6., etc., so þat the nombre folowyng passithe the other be-fore in one. Broken it is, whan me lepithe fro o nombre till another, and kepithe not the contynuel ordire; as 1. 3. 5. 7. 9, etc. Ay me may begynne *with* .2., as þus; .2. 4. 6. 8., etc., and the nombre folowyng passethe the others by-fore by .2. And note wele, that *naturelle* *progressioun* ay begynnethe *with* one, and *Interse* or broken *progressioun*, omwhile begynnythe *with* one, omwhile *with* twayne. Of *progressioun* naturell .2. rules ther be yove, of the whiche the first is this; whan the *progressioun* naturelle endithe in even nombre, by the half therof multiplie þe next totalle ouerere nombre; Example of grace: .1. 2. 3. 4. Multiplie .5. by .2. and so .10. comethe of, that is the totalle nombre þerof. The seconde rule is *suche*, whan the *progressioun* naturelle endithe in nombre ode. Take the more porcioun of the oddes, and multiplie therby the totalle nombre. Example of grace 1. 2. 3. 4. 5., multiplie

Definition o.
Progression.

Natural Pro-
gression.

Broken Pro-
gression.

The 1st rule
for Natural
Progression.

The second
rule.

¹ 'nought' in MS.

.5. by .3. and thryes .5. shalle be resultant. so the nombre totale
 is .15. Of *progressioun* interseise, ther ben also .2.¹ rules; and þe
 first is þis: When the Interseise progression endithe in even nombre
 by half therof multiplie the next nombre to þat halfe as .2.¹ 4. 6. 4
 Multiplie .4. by .3. so þat is thryes .4., and .12. the nombre of alle
 the *progressioun*, wolle folow. The seconde rule is this: when the
progressioun interseise endithe in ode, take þe more porcioun of alle
 þe nombre, ²and multiplie by hym-selfe; as .1. 3. 5. Multiplie .3. 8
 by hym-selfe, and þe some of alle wolle be .9., etc.

The preamble
 of Broken
 Progression.

The second
 rule.

² Fol. 53^a.

The preamble
 of the extrac-
 tion of roots.

Here folowithe the extraccioun of rotis, and first in nombre
 quadrates. Wherfor me shalle se what is a nombre quadrat,
 and what is the rote of a nombre quadrat, and what it 12
 is to draw out the rote of a nombre. And before other note
 this divisioun: Of nombres one is lyneal, anoþer superficialle,
 anoþer quadrat, anoþer cubike or hoole. lyneal is that þat is con-
 sidrede after the processe, havynge no respect to the direccioun 16
 of nombre in nombre, As a lyne hathe but one dymensioun that
 is to sey after the lengthe. Nombre superficial is þat comethe
 of ledynge of oo nombre into a-nother, wherfor it is callede super-
 ficial, for it hathe .2. nombres notyng or mesuryng hym, as a 20
 superficialle thyng hathe .2. dimensions, þat is to sey lengthe and
 brede. And for bycause a nombre may be hade in a-nother by .2.
 maners, þat is to sey other in hym-selfe, oþer in anoþer, Vnder-
 stonde yf it be had in hym-self, It is a quadrat. ffor dyvisioun 24
 write by vnytes, hathe .4. sides even as a quadrangille. and yf the
 nombre be hade in a-noþer, the nombre is superficial and not
 quadrat, as .2. hade in .3. makethe .6. that is þe first nombre super-
 ficielle; wherfor it is open þat alle nombre quadrat is superficial, 28
 and not *conuertide*. The rote of a nombre quadrat is þat nombre
 that is had of hym-self, as twies .2. makithe 4. and .4. is the first
 nombre quadrat, and 2. is his rote. 9. 8. 7. 6. 5. 4. 3. 2. 1. / The
 rote of the more quadrat .3. 1. 4. 2. 6. The most nombre quadrat 32
 9. 8. 7. 5. 9. 3. 4. 7. 6. / the remenent over the quadrat .6. 0. 8.
 4. 5. / The first caas of nombre quadrat .5. 4. 7. 5. 6. The rote .2.
 3. 4. The seconde caas .3. 8. 4. 5. The rote .6. 2. The thirde
 caas .2. 8. 1. 9. The rote .5. 3. The .4. caas .3. 2. 1. The rote 36
 .1. 7. / The 5. caas .9. 1. 2. 0. 4. / The rote 3. 0. 2. The solide
 nombre or cubike is þat þat comythie of double ledynge of nombre
 in nombre; And it is clepede a solide body that hathe þer-in .3

Linear,
 superficial,
 and solid
 numbers.

Superficial
 numbers.

Square num-
 bers.

The root of a
 square num-
 ber.

Notes of some
 examples of
 square roots
 here interpo-
 lated.

Solid num-
 bers.

¹ 3 written for 2 in MS.

[dimensions] þat is to sey, lengthe, brede, and thiknesse. so þat
 nombre hathe .3. nombres to be brought forthe in hym. But Three di-
 mensions of
 solids.
 nombre may be hade twies in nombre, for other it is hade in hym-
 4 selfe, oþer in a-noþer. If a nombre be hade twies in hym-self, oþer
 ones in his quadrat, þat is the same, þat a cubike ¹is, And is the
 same that is solide. And yf a nombre twies be hade in a-noþer, the
 nombre is clepede solide and not cubike, as twies .3. and þat .2.
 8 makithe .12. Wherfor it is opyne that alle cubike nombre is solide, All cubics
 are solid
 numbers.
 and not *convertide*. Cubike is þat nombre þat comythe of ledynge
 of hym-selfe twyes, or ones in his quadrat. And here-by it is open
 that o nombre is the roote of a quadrat and of a cubike. Natheles
 12 the same nombre is not *quadrat* and cubike. Opyne it is also that
 alle nombres may be a rote to a *quadrat* and cubike, but not alle No number
 may be both
 linear and
 solid.
 nombre *quadrat* or cubike. Therfor sithen þe ledynge of vnyte in
 hym-self ones or twies nought comethe but vnytes, Seithe Boice in
 16 Arsemetrike, that vnyte potencially is al nombre, and none in act.
 And vnderstonde wele also that betwix euery .2. *quadrates* ther is a Unity is not
 a number.

Residuuum			0				4			0				0			
Quadrante	4	3	5	6	3	0	2	9	1	7	4	2	4	1	9	3	6
Duplum	1	2		1	0		2	6				[8]	2				
Subduplum		6		6	5		5	1		3		2	4				4

Examples of square roots.

meene *proporcionalle*, That is openede thus; lede the rote of o
 quadrat into the rote of the oþer quadrat, and þan wolle þe meene
 20 shew. Also betwix the next .2. cubikis, me may fynde a *slouble*
 meene, that is to sey a more meene and a lesse. The more meene
 thus, as to brynge the rote of the lesse into a quadrat of the more.
 The lesse thus, If the rote of the more be brought Into the quadrat
 24 of the lesse.

³**T**o draw a rote of the nombre quadrat it is What-euer nombre be
 proposede to fynde his rote and to se yf it be quadrat. And
 yf it be not quadrat the rote of the most quadrat fynde out, vnder
 28 the nombre *proposede*. Therfor yf thou wilt the rote of any quadrat
 nombre draw out, write the nombre by his differences, and compt
 the nombre of the figures, and wete yf it be ode or even. And yf
 it be even, than most thou begynne worche vnder the last save one. Begin with
 the last odd
 place.
 32 And yf it be ode *with* the last; and forto sey it shortly, al-weyes
 fro the last ode me shalle begynne. Therfor vnder the last in an
 od place sette, me most fynde a digit, the whiche lade in hym-selfe
 it puttithe away that, þat is ouer his hede, oþer as neighe as me

² 7 in MS. ³ runs on in MS.

is] The rote of the most quadrat conteynede vndre the nombre proposede. Therfor yf thow wilt prove yf thow have wele do or no, Multiplie the digit last founde with the vnder-double oper vnder-
 4 doublis, and thow shalt fynde the same figures that thow haddest before; And so that nought be the residue. And yf thow have any residue, than with the addicion perof that is reseruede with-out
 8 before, etc.

How to prove the square root without or with a remainder.

1 Fol. 55.

Heere folowithe the extraccioun of rotis in cubike nombres; wher-for me most se what is a nombre cubike, and what is his roote, And what is the extraccioun of a rote. A
 12 nombre cubike it is, as it is before declarede, that comethe of ledyng of any nombre twies in hym-selfe, other ones in his quadrat. The rote of a nombre cubike is the nombre that is twies hade in hym-selfe, or ones in his quadrat. Wher-thurgh it is open, that
 16 euery nombre quadrat or cubike have the same rote, as it is seide before. And forto draw out the rote of a cubike, It is first to fynde þe nombre proposede yf it be a cubike; And yf it be not, than thow most make extraccioun of his rote of the most cubike
 20 vndre the nombre proposide his rote founde. Therfor proposede some nombre, whos cubical rote þou woldest draw out; First thow most compt the figures by fourthes, that is to sey in the place of
 24 a digit, the whiche lade in hym-self cubikly puttethe a-way that þat is ouer his hede as in respect of hym, other as nyghe as thow maist. That done, thow most trebille the digit, and that triplat is to be put vnder the .3. next figure towarde the right honde,
 28 And the vnder-trebille vnder the trebille; Than me most fynde a digit vndre the next figure bifore the triplat, the whiche with his vnder-trebille had into a trebille, afterwarde other vnder[trebille]² had in his produccioun, puttethe a-way alle that is ouer it in
 32 regarde of³ [the triplat. Then lade in hymself puttethe away that þat is ouer his hede as in respect of hym, other as nyghe as thou maist:] That done, thow most trebille the digit ayene, and the triplat is to be sette vnder the next .3. figure as before, And
 36 the vnder-trebille vnder the trebille: and than most thow sette forwarde the first triplat with his vndre-trebille by .2. differences. And than most thow fynde a digit vnder the next figure before the triplat, the whiche with his vnder-triplat had in his triplat after-

Definition of a cubic number and a cube root.

Mark off the places in threes.

Find the first digit;

treble it and place it under the next but one, and multiply by the digit.

Then find the second digit.

Multiply the first triplate and the second digit, twice by this digit.

² double in MS.

³ 'it hym-selfe' in MS.

yf no digit after the anterioracioun¹ may not be founde, than put there a cifre vndre a cifre yndir the thirde figure, And put forwarde Special case. the figures. Note also wele that yf in the nombre proposede there ben no place of thowsandes, me most begynne vnder the first figure in the extraccioun of the rote. some vsen forto distingue the nombre by threes, and ay begynne forto wirche vndre the first of

The residue								0							1		1											
The cubicandus		8		0		0		0		0		0		0		8		2		4		2		4		1		9
The triple				2		0		0						6														
The vndertuple		[2]				0		0				2			6		2											

Examples.

the last ternary other uncomplete nombre, the whiche maner of 8 operacioun accordethe with that before. And this at this tyme suffisethe in extraccioun of nombres quadrat or cubikes etc.

1 one. 2 x. 3 an. hundrede / a thowsande / x. thowsande / An hundrede
 7 thowsande / A thowsande tymes a thowsande / x. thousande tymes
 12 a thousande / An hundrede thousande tymes a thousande A thousande thousande tymes a thousande / this is the x place etc.

A table of numbers; probably from the Abacus.

[Ende.]

¹ MS. anterioracioun. ² 4 in MS.

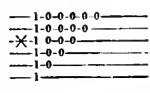
Accomptynge by counters.

¹ 116 b. ¶ The seconde dialoge of accomptynge by counters.

Mayster.



NOwe that you haue learned the comen kyndes of Arithme-
tyke with the penne, you shall se the same art in counters :
whiche feate doth not only serue for them that can not write 4
and rede, but also for them that can do bothe, but haue not at some
tymes theyr penne or tables redye with them. This sorte is in two
fourmes comenly. The one by lynes, and the other without lynes :
in that y^t hath lynes, the lynes do stande for the order of places : 8
and in y^t that hath no lynes, there must be sette in theyr stede so
many counters as shall nede, for eche lyne one, and they shall
supplye the stede of the lynes. *S.* By examples I shuld better



² 117 a perceaue your meanyng. *M.* For example of the ly²nes : Lo here 12

you se .vi. lynes whiche stande for syxe places so 
that the nethermost standeth for y^e fyrst place, and
the next aboute it, for the second : and so vpward tyll

you come to the hyghest, which is the syxte lyne, and standeth for 16
the syxte place. Now what is the valewe of euery place or lyne,

Numeration. you may perceaue by the figures whiche I haue set on them, which
is accordynge as you learned before in the Numeration of figures by
the penne : for the fyrste place is the place of vnities or ones, and 20
euery counter set in that lyne betokeneth but one : *and* the seconde
lyne is the place of 10, for euery counter there, standeth for 10.
The thyrd lyne the place of hundredes : the fourth of thousandes :
and so forth. *S.* Syr I do perceaue that the same order is here of 24

³ 117 b. lynes, as was in the other figures ³ by places, so that you shall not
nede longer to stande about Numeration, excepte there be any other
difference. *M.* Yf you do vnderstande it, then how wyll you set
1543? *S.* Thus, as I suppose.  *M.* You haue set y^e 28
places truely, but your figures be  not mete for this vse :

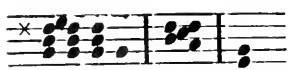
for the metest figure in this behalfe, is the figure of a counter round, as you se here, where I haue expressed that same summe. *S.* So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many digettes as you haue, you set in the lowest lyne: and for euery 10 you set one in the second line: and so of other. But I know not by what reason you set that one counter for 500 betwene two lynes. *M.* you shall remember this, that when so euer you nede to set downe 5, 50, or 500, or 5000, or so forth any other number, whose numerator ¹ is 5, you shall set one counter for it, in the next space about the lyne that it hath his denomination of, as in this example of that 500, because the numerator is 5, it must be set in a voyd space: and because the denominator is hundred, I knowe that his place is the voyde space next about hundredes, that is to say, about the thyrd lyne. And farther you shall marke, that in all workyng by this sorte, yf you shall sette downe any summe betwene 4 and 10, for the fyrste parte of that number you shall set downe 5, & then so many counters more, as there reste numbers about 5. And this is true bothe of digettes and articles. And for example I wyll set downe this summe 287965,  which summe yf you marke well, you nede none other examples for to lerne the numeration of ² this forme. But this shal you marke, that as you dyd in the other kynde of arithmetike, set a pricke in the places of thousandes, in this worke you shall sette a starre, as you se here. *S.* Then I perceave numeration, but I praye you, howe shall I do in this arte to adde two summes or more together? *M.* The easiest way in this arte is, to adde but 2 summes at ones together: how be it you may adde more, as I wyll tell you anone. Therefore when you wyll adde two summes, you shall fyrst set downe one of them, it forseth not whiche, and then by it drawe a lyne crosse the other lynes. And afterward set downe the other summe, so that that lyne may be betwene them, as yf you wolde adde 2659 to 8342,  you must set your summes as you se here. And then yf you lyst, you ³ may adde the one to the other in the same place, or els you may adde them both together in a newe place: which waye, because it is moste playnest, I wyll shoue you fyrst. Therefore wyl I begynne at the vnites, whiche in the fyrst summe is but 2, and in y^e second summe 9, that maketh 11, those do I take vp, and for them I set 11 in the new roume, thus,

¹ 118 a.

² 118 b.

Addition.

³ 119 a.





Then do I take vp all y^e articles vnder a hundred, which in the fyrst summe are 40, and in the seconde summe 50, that maketh 90: or you may saye better, that in the fyrste summe there are 4 articles of 10, and in the seconde summe 5, which make 9, but then take hede that

¹ 119 b.

you sette them in theyr ¹ryght lynes as you se here. Where I

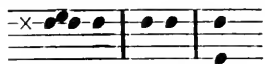


haue taken awaye 40 from the fyrste summe, and 50 from y^e second, and in theyr stede I haue set 90 in the thyrde, whiche I haue set playnely y^t you myght well perceauce it: how be it seyng that 90 with the 10 that was in y^e thyrd roume all redy, doth make 100, I myghte better for those 6 counters set 1 in the thyrde

lyne, thus:  For it is all one summe as you may se, but it is beste,  neuer to set 5 counters in any line, for that may be done with 1 counter in a hygher place. *S.* I iudge that good reason, for many are vnedefull, where one wyll serue.

² 120 a.

M. Well, then ²wyll I adde forth of hundredes: I fynde 3 in the fyrste summe, and 6 in the seconde, whiche make 900, them do I take vp *and* set in the thyrd roume where is one hundred all redy, to whiche I put 900, and it wyll be 1000, therefore I set one counter in the fourth lyne for them all,



as you se here. Then adde I y^e thousandes together, whiche in the fyrst summe are 8000, *and* in y^e second 2000, that maketh 10000: them do I take vp from those two places, and for them I set one counter in the fyfte lyne, and then appereth as



you se, to be 11001, for so many doth amount of the addition of 8342 to 2659. ³*S.* Syr, this I do perceave: but how shall I set one summe to an other, not chaungynge them to a thyrde place? *M.* Marke well how I do it:

I wyll adde together 65436, and 3245,

whiche fyrste I set downe thus. Then



do I begynne with the smailest, which

in the fyrst summe is . that do I take vp, and wold put to the other 5 in the seconde summe, saung that two counters can not be set in a voyd place of 5, but for them bothe I must set 1 in the seconde lyne, which is the place of 10, therefore I take vp the 5 of the fyrst summe, *and* the 5 of the seconde, and for them I set 1 in the second lyne, ⁴as you se here.

⁴ 121 a.

Then do I lyke wayes take vp the 4 counters of the fyrst summe *and*



40

seconde lyne (which make 40) and adde them to the 4 counters of the same lyne, in the seconde summe, and it maketh 80, But as I sayde I maye not conueniently set aboue 4 counters in one lyne, therefore to those 4 that I toke vp in the fyrste summe, I take one also of the seconde summe, and then haue I taken vp 50, for whiche 5 counters I sette downe one in the space ouer y^e second lyne, as here doth appere.



¹and then is there 80, 4 counters, as yf I had set downe y^e other 4 also. Now

¹ 121 b

do I take the 200 in the fyrste summe, and adde them to the 400 in the seconde summe, and it maketh 600, therefore I take vp the 2 counters in the fyrste summe, and 3 of them in the seconde summe, and for them 5 I set 1 in y^e space aboue,



thus. Then I take y^e 3000 in y^e fyrste summe, vnto whiche there are none in the

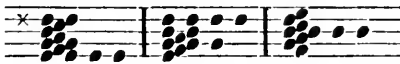
second summe agreynge, therefore I do onely remoue those 3 counters from the fyrste summe into the seconde, as here doth appere.



²And so you see the hole summe, that amounteth of the addytion of 65436 with 3245 to be 6868[1].

² 122 a

And yf you haue marked these two examples well, you nede no farther enstruction in Addition of 2 only summes: but yf you haue more then two summes to adde, you may adde them thus. Fyrst adde two of them, and then adde the thyrde, and y^e fourth, or more yf there be so many: as yf I wolde adde 2679 with 4286 and 1391. Fyrste I adde the two fyrste summes thus.



³And then I adde the thyrde thereto thus.

³ 122 b.

And so of more yf you haue them. *S.* Nowe I thynke beste that you passe forth to Subtraction, except there be any wayes to examyn this maner of Addition, then I thynke that were

good to be knowen nexte. *M.* There is the same profe here that is in the other Addition by the penne, I meane Subtraction, for that onely is a sure waye: but consyderynge that Subtraction must be fyrste knowen, I wyl fyrste teache you the arte of Subtraction, and that by this example: I wolde subtracte 2892 out of 8746. These summes must I set downe as I dyd in Addition: but here it is best ⁴to set the lesser number fyrste,

Subtra^{ti}on.



thus. Then shall I begynne to sub-

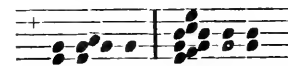
116 a (ete).

tracte the greatest nombres fyrste (contrary to the vse of the penne)

y^t is the thousandes in this example: therefore I fynd amongst the thousandes 2, for which I withdrawe so many from the seconde summe (where are 8) and so remaineth there 6, as this example showeth.



Then do I lyke wayes with the hundredes, of whiche in the fyrste summe I fynde 8, and is the seconde summe but 7, out of whiche I can not take 8, therefore thus muste I do: I muste loke how moche my summe dyffereth from 10, whiche I fynde here to be 2, then must I bate for my summe of 800, one thousande, and set downe the excesse of hundredes, that is to saye 2, for so moche 100[0] is more then I shuld take vp. Therefore from the fyrste summe I take that 800, and from the second summe where are 12 6000, I take vp one thousande, and leue 5000; but then set I downe the 200 unto the 700 y^t are there all redye, and make them 900 thus.




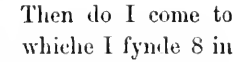
Then come I to the articles of tenes where in the fyrste summe I fynde 90, and in the seconde summe but only 40: Now consydering that 90 can not be bated from 40, I loke how moche y^t 90 doth dyffer from the next summe above it, that is 100 (or elles whiche is all to one effecte, I loke how moch 9 doth dyffer from 10) and I fynd it to be 1, then in the stede of that 90, I do take from the second summe 100: but consyderynge that it is 10 to moche, I set downe 1 in y^e nexte lyne beneth for it, as you se here. Sauynge that here I haue set one counter in y^e space in stede of 5 in y^e nexte lyne. And thus haue I subtracted all saue two, which I must bate from the 6 in the second summe, and there wyll remaine 4, thus.


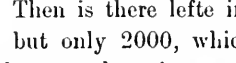


So y^t yf I subtracte 2892 from 8746, the remainer wyll be 5854, And that this is truely wrought, you maye proue by Addition: for yf you adde to this remainer the same summe that you dyd subtracte, then wyll the formar summe 8746 amount agayne. S. That wyll I proue: and fyrst I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus. Then do I adde fyrst y^e 2 to 4, whiche maketh 6, so take I vp 5 of those counters, and in theyr stede I sette 1 in the space, as here appereth.




Then do I adde the 90 nexte aboute the 50, and it maketh 140, therefore I take vp those 6 counters, and for them I sette 1 to the hundredes in y^e thyrde lyne, and 4 in y^e 40


second lyne, thus.  Then do I come to the hundredes, of  whiche I fynde 8 in the fyrst summe, and 9 in y^e second, that maketh 1700, therefore I

4 take vp those 9 counters, and in theyr stede I sette 1 in the .iiii. lyne, and 1 in the space nexte beneth, and 2 in the thyrde lyne, as you se here.  Then is there lefte in the fyrste summe  but only 2000, whiche I


¹ 118 b.


8 shall take vp from thence, and set ¹ in the same lyne in y^e second summe, to y^e one y^t is there all rely: *and* then wyll the hole summe appere (as you may wel se) to be 8746,  which was y^e fyrst grosse summe, *and* therefore

12 I do perceave, that I hadde well subtracted before. And thus you may se how Subtraction maye be tryed by Addition. *S.* I perceave the same order here w^t counters, y^t I lerned before in figures. *M.* Then let me se howe can you trye Addition by

16 Subtraction. *S.* Fyrste I wyl set forth this example of Addition where I haue added 2189 to 4988, and the hole summe appereth to be 7177,  ² Nowe to trye

² 119 a.


20 whether that summe be well added or no, I wyll subtract one of the fyrst two summes from the thyrd, and yf I haue well done y^e remayner wyll be lyke that other summe. As for example: I wyll subtracte the fyrste summe from the thyrde, whiche I set thus 

24 in theyr order. Then do I subtract 2000 of the fyrste summe from y^e second summe, and then remayneth there 5000 thus.  Then in the thyrd lyne, I subtract y^e 100

28 from the second summe, where is onely 100 also, and then in y^e thyrde lyne resteth nothyng. Then in the second lyne with his space ouer hym, I fynde 80, which I shuld subtract ³ from the other summe, then seyng there are but only 70 I must take it out


³ 119 b.

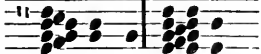
32 of some hygher summe, which is here only 5000, therefore I take vp 5000, and seyng that it is to moch by 4920, I sette downe so many in the seconde roume, whiche with the 70 beyng there all redy do make 4990, & then the summes


36 doth stande thus. Yet remayneth there  in the fyrst summe 9, to be bated from the second summe, where in that place of vnities dothe appere only 7, then I muste bate a hygher summe, that is to saye 10, but seyng that 10 is more then

40 9 (which I shulde abate) by 1, therefore shall I take vp one counter from the seconde lyne, *and* set downe the same in the fyrst ⁴ or

⁴ 120 a.

lowest lyne, as you se here.  And so haue I ended this worke, *and* the summe appereth to be y^e same, whiche was y^e seconde summe of my addition, and therefore I perceauē, I haue wel done. *M.* To stande longer about this, it is but folye: excepte that this you maye also vnderstande, that many do begynne to subtracte with counters, not at the hyghest summe, as I haue taught you, but at the nethermoste, as they do vse to adde: and when the summe to be abatyd, in any lyne appeareth greater then the other, then do they borowe one of the next hygher roume, as for example: yf they shuld abate 1846 from 2378, they set y^e summes thus.


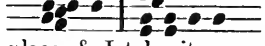



¹ 120 b. ¹ And fyrste they take 6 whiche is in the  12 lower lyne, and his space from 8 in the same roumes, in y^e second summe, and yet there remaineth 2 counters in the lowest lyne. Then in the second lyne must 4 be subtracte from 7, and so remaineth there 3. Then 8 in the thyrde lyne and his space, from 16 3 of the second summe can not be, therefore do they bate it from a hygher roume, that is, from 1000, and bycause that 1000 is to moch by 200, therefore must I sette downe 200 in the thyrde lyne, after I haue taken vp 1000 from the fourth lyne: then is there yet 20 1000 in the fourth lyne of the fyrst summe, whiche yf I withdrawe from the seconde summe, then doth all y^e figures stande in this order.

 So that (as you se) it differeth not greatly whether you begynne subtraction at the hygher lynes, or ²⁴ at ² the lower. How be it, as some menne lyke the one waye beste, so some lyke the other: therefore you now knowyng bothe, may vse whiche you lyst. But nowe touchyng Multiplication: you shall set your numbers in two roumes, as you dyd in those two other ²⁸ kyndes, but so that the multiplier be set in the fyrste roume. Then shall you begyn with the hyghest numbers of y^e seconde roume, and multiply them fyrst after this sort. Take that ouermost lyne in your fyrst workyng, as yf it were the lowest lyne, ³² setting on it some mouable marke, as you lyste, and loke how many counters be in hym, take them vp, and for them set downe the hole multyplyer, so many tymes as you toke vp counters, rekenyng, I saye that lyne for the vnites: *and* when you haue so ³⁶ done with the hygheest number then come to the nexte lyne beneth, *and* do euen so with it, and so with y^e next, tyll you haue done all. And yf there be any number in a space, then for it

² 121 a.
Multiplication.

³ 121 b. ³ shall you take y^e multiplier 5 tymes, and then must you reckon ⁴⁰ that lyne for the vnites whiche is nexte beneth that space: or els


after a shorter way, you shall take only halfe the multiplyer, but then shall you take the lyne nexte about that space, for the lyne of vnites: but in suche workynge, yf chaunce your multiplyer be an 4 odde number, so that you can not take the halfe of it iustly, then muste you take the greater halfe, and set downe that, as if that it were the iuste halfe, and farther you shall set one counter in the space beneth that line, which you reckon for the lyne of vnities, or 8 els only remoue forward the same that is to be multiplied. *S.* Yf you set forth an example hereto I thynke I shal perceau you.


M. Take this example: I wold multiply 1542 by 365, therefore I set y^e numbers thus.  ¹Then fyrste I be-
 12 gonne at the 1000 in  y^e hyghest roume,
 as yf it were y^e fyrst place, & I take it vp, settinge downe for it
 so often (that is ones) the multiplyer, which is 365, thus, as
 you se here:  where for the one
 16 counter taken  ↪ vp from the
 fourth lyne, I  haue sette downe

1 122 a.

other 6, whiche make y^e summe of the multiplyer, rekenynge that fourth lyne, as yf it were the fyrste: whiche thyng I haue marked
 20 by the hand set at the begynnyng of y^e same, *S.* I perceau this well: for in dede, this summe that you haue set downe is 365000, for so moche doth amount ²of 1000, multiplied by 365. *M.* Well
 then to go forth, in the nexte space I fynde one counter which I
 24 remoue forward but take not vp, but do (as in such case I must) set downe the greater halfe of my multiplier (seyng it is an odde number) which is 182, and here I do styll let that fourth place stand, as yf it were y^e

2 122 b.

28 fyrst: as in this fourme  ↪
 you se, where I haue set this multiplication with y^e other: but for the ease of your vnderstandynge, I haue set a lytell lyne betwene them: now shulde they
 32 both in one summe stand thus.

³Howe be it an other fourme  ↪
 to multiplye suche counters in space is this: Fyrst to remoue the fynger to the lyne nexte
 36 benethe y^e space, and then to take vp y^e counter, and to set downe y^e multiplyer .v. tymes, as here you se. Which summes yf you do

3 123 a.

 ↪

adde together into one summe, you shal perceau that it wyll be y^e

1 123 b.

same y^t appeareth of y^e other working before, so that ¹bothe sortes are to one entent, but as the other is much shorter, so this is playner to reason, for suche as haue had small exereyse in this arte. Not withstandynge you maye adde them in your mynde before you sette them downe, as in this example, you myghte haue sayde 5 tymes 300 is 1500, and 5 tymes 60 is 300, also 5 tymes 5 is 25, whiche all put together do make 1825, which you maye at one tyme set downe yf you lyste. But nowe to go forth, I must remoue the hand to the nexte counters, whiche are in the second lyne, and there must I take vp those 4 counters, settinge downe for them my multiplier 4 tymes, whiche thyng other I maye do at 4 tymes seuerally, or elles I may gather that hole summe in my mynde fyrste, and then set it downe: as to saye 4 tymes 300 is 1200: 4 tymes 60 are 240: and 4 tymes 5 make 20: y^t is in all 1460, y^t shall I set downe also: as here you

2 124 a.

se. ² whiche yf I ioyne in one summe with the



3 124 b.

formar numbers, it wyll appeare thus. Then to ende this multiplicacion, I remoue the fynger to the lowest lyne, where are onely 2, them do I take vp, and in theyr stede do I set downe twyse 365, that is 730, for which I set ³one in the space above the thyrd lyne for 500, and 2 more in the thyrd lyne with that one that is there all redye, and the reste in theyr order, and so haue I ended the hole summe thus.

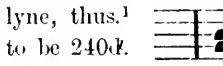
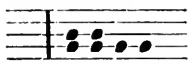
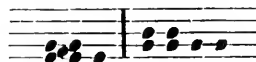


5 125 a.



Wherby you se, that 1542 (which is the number of yeares syth Ch[r]ystes incarnation) beyng multiplyed by 365 which is the number of dayes in one yeare) dothe amounte vnto 562830, which declareth y^e number of daies sith Chrystes incarnation vnto the ende of 1542⁴ yeares. (besyde 385 dayes and 12 hours for lepe yeares). S. Now wyll I proue by an other example, as this: 40 labourers (after 6d. y^e day for eche man) haue wrought 28 dayes, I wold know what theyr wages doth amount vnto: In this case muste I worke doublely: fyrst I must multiplye the number of the labourers by y^e wages of a man for one day, so wyll y^e charge of one daye amount: then secondarely shall I multiply that charge of one daye, by the hole number of dayes, and so wyll the hole summe appeare: fyrst therefore I shall set the summes thus.

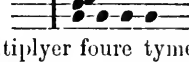
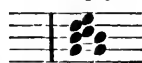
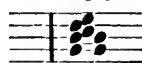
⁴ 1342 in original.

Where in the fyrste space is the multiplyer
 (y^t is one dayes wages for one man) and in
 the second space is set the number of the worke men to be multiplyed: then saye I, 6 tymes 4 (rekenyng that second lync as the lync of vnites) maketh 24, for whiche summe I shulde set 2 counters in the thyrde lync, and 4 in the seconde, therefore do I set 2 in the thyrde lync, and let the 4 stand styll in the seconde

8 lync, thus.¹  So apwereth the hole dayes wages
 to be 240℥.  that is 20s. Then do I multiply
 agayn the same summe by the number of dayes and fyrste I sette
 the numbers, thus.  Then bycause there

¹ 125 b.

12 are counters in  dyuers lynes, I shall
 begynne with the hyghest, and take them vp, settinge for them
 the multiplyer so many tymes, as I toke vp counters, y^t is twyse,
 then wyll y^e summe stande thus.  Then come

16 I to y^e seconde lync, and take  vp those 4
 counters, settinge for them the multiplyer foure tymes, so wyll the
 hole summe appeare thus.²  So is the hole wages
 of 40 workemen, for 28  dayes (after 6℥. eche

² 126 a.


20 daye for a man) 6720℥. that is 560s. or 28 li. *M.* Now if you
 wold proue Multiplication, the surest way is by Dyuision: therefore
 wyll I ouer passe it tyll I haue taught you y^e arte of Diuision,
 whiche you shall worke thus. Fyrste sette downe the Diuisor for


Diuision.

24 feare of forgettyng, and then set the number that shalbe deuided,
 at y^e ryghte syde, so farre from the diuisor, that the quotient may
 be set betwene them: as for example: Yf 225 shepe cost 45 li.
 what dyd euery shepe cost? To knowe this, I shulde diuide the

28 hole summe, that is 45 li. by 225, but that can not be, therefore
 must I fyrste reduce that 45 li. into a lesser denomination, as into
 shyllinges: then I multiply 45 by 20, and it is 900, that summe
 shall I diuide by the number of ³shepe, whiche is 225, these

³ 126 b.

32 two numbers therefore I sette thus.  Then begynne I at the hyghest lync
 of the diuident, and seke how often I may haue the diuisor therein,
 and that maye I do 4 tymes, then say I, 4 tymes 2 are 8, whyche yf

36 I take from 9, there resteth but 1, thus  And bycause I founde the diuisor 4
 tymes in the diuidente, I haue set (as you se) 4 in the myddle
 roume, which ⁴is the place of the quotient: but now must I take

⁴ 127 a.

40 the reste of the diuisor as often out of the remayner: therefore come

I to the seconde lyne of the diuisor, sayeng 2 foure tymes make 8, take 8 from 10, *and* there resteth 2, thus. Then come I to the lowest number, which is 5, and multiply it 4 tymes, so is it 20, that take I from 20, and there remaineth nothyng, so that I se my quotient to be 4, whiche are in valewe shylllynges, for so was the diuident: and therby I knowe, that yf 225 shepe dyd coste 45 l*i*. euery shepe coste 4 s. *S*. This can I do, as you shall perceau by this example: 4

Yf 160 sowldyars do spende euery moneth 68 l*i*. what spendeth eche man? Fyrst ¹bycause I can not diuide the 68 by 160, therefore I wyll turne the poundes into pennes by multiplicacion, so shall there be 16320 d*l*. Nowe muste I diuide this summe by the.12 number of sowldyars, therefore I set them in order, thus. Then begyn I at the highest place of the diuidente, sekyng my diuisor there, whiche I fynde ones, Therefore set I 1 in the nether lyne. *M*. Not in the 16 nether line of the hole summe, but in the nether lyne of that worke, whiche is the thyrde lyne. *S*. So standeth it with reason.

² 128 a. *M*. Then thus do they stande.² Then seke I agayne in the reste, how often I may 20 fynde my diuisor, and I se that in the 300 I myghte fynde 100 thre tymes, but then the 60 wyll not be so often founde in 20, therefore I take 2 for my quotient: then take I 100 twyse from 300, and there resteth 100, out of whiche with the 20 (that maketh 24 120) I may take 60 also twyse, and then standeth the numbers thus,

³ 128 b. ³ where I haue sette the quotient 2 in the lowest lyne: So is euery sowldyars portion 102 d*l*. that is 8 s. 6 d*l*. *M*. But yet bycause you shall perceau 28 iustly the reason of Diuision, it shall be good that you do set your diuisor styll agaynst those nombres *from* whiche you do take it: as by this example I wyll declare. Yf y^e purchase of 200 acres of ground dyd coste 290 l*i*. what dyd one acre coste? Fyrst 32 wyl I turne the poundes into pennes, so wyll there be 69600 d*l*. Then in settinge downe these numbers I shall do thus. Fyrst

⁴ 129 a. ⁴ the diuisor on the lefte hande agaynst 36 those numbers, *from* which I entende to take hym fyrst as here you se, wher I haue set the diuisor two lynes hygher then is their owne place. *S*. This is lyke the order of diuision by the penne. 40

M. Truth you say, and nowe must I set y^e quotient of this worke in the thyrde lyne, for that is the lyne of vnities in respecte to the diuisor in this worke. Then I seke howe often the diuisor 4 maye be founde in the diuident, *and* that I fynde 3 tymes, then set I 3 in the thyrde lyne for the quotient, and take awaye that 60000 from the diuident, and farther I do set the diuisor one line lower, as yow se here.

8¹ And then seke I howe often the



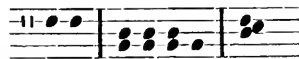
1 129 b.

diuisor wyll be taken from the number agaynste it, whiche wyll be 4 tymes and 1 remaynyng. *S.* But what yf it chaunce that when the diuisor is so remoued, it cannot be ones taken out of the

12 diuident agaynste it? *M.* Then must the diuisor be set in an other line lower. *S.* So was it in diuision by the penne, and therefore was there a cypher set in the quotient: but howe shall that be noted here? *M.* Here nedeth no token, for the lynes do

16 represente the places: onely loke that you set your quotient in that place which standeth for vnities in respecte of the diuisor: but now to returne to the example, I fynde the diuisor 4 tymes in the diuidente, and 1 remaynyng, for 4 tymes 2 make 8, which I take 20 from 9, and there resteth 1, as this figure sheweth: and in the myddle space for the quotient I set 4 in the seconde lyne, whiche is in this worke the place of vnities.²

Then remoue I y^e diuisor to the next



2 130 a.

24 lower line, and seke howe often I maye haue it in the dyuident, which I maye do here 8 tymes iust, and nothyng remayne, as in this fourme,



the hole quoti- ent is 348 d', that is

28 29 s. wherby I knowe that so moche coste the purchace of one aker. *S.* Now resteth the profes of Multiplication, and also of


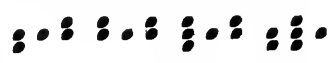
Diuision. *M.* Ther best profes are eche.³ one by the other, for Multiplication is proued by Diuision, and Diuision by Multiplyca-

32 tion, as in the worke by the penne you learned. *S.* Yf that be all, you shall not nede to repete agayne that, y^t was sufficiencytly taughte all redye: and excepte you wyll teache me any other feate, here maye you make an ende of this arte I suppose. *M.* So

36 wyll I do as touchyng hole number, and as for broken number, I wyll not trouble your wytte with it, tyll you haue practised this so well, y^t you be full perfecte, so that you nede not to doubt in any poynte that I haue taught you, and thenne maye I boldly

40 enstruete you in y^e arte of fractions or broken number, wherin I

3 130 b.

wyll also shoue you the reasons of all that you haue nowe learned. But yet before I make an ende, I wyll shoue you the order of commen castyng, wher in are bothe pennes, shylllynges, and poundes, procedyng by no grounded reason, but onely by a reccaueled ¹ 131 a. 4
 Merchants' ¹fourme, and that dyuersly of dyuers men: for marchauntes vse casting. one fourme, and auditors an other: But fyrste for marchauntes fourme marke this example here,  in which I haue expressed this summe 198^li.² 19s. 11d^r. So that 8
 you maye se that the lowest lyne serueth for pennes, the next aboue for shylllynges, the thyrde for poundes, and the fourth for scores of poundes. And farther you maye se, that the space betwene pennes and shylllynges may receaue but one 12
 counter (as all other spaces lyke wayes do) and that one standeth in that place for 6d^r. Lyke wayes betwene the shylllynges and the poundes, one counter standeth for 10s. And betwene the poundes and 20^li. one counter standeth for 10 poundes. But 16
 besyde those you maye see at the left syde of shylllynges, that one counter standeth alone, and betokeneth 5s. ³So agaynste the poundes, that one counter standeth for 5^li. And agaynst the 20
 poundes, the one counter standeth for 5 score poundes, that is 20
 100^li. so that every syde counter is 5 tymes so moch as one of them agaynst whiche he standeth. Now for the accompt of auditors take this example.  where I haue expressed y^e same ³ 131 b. summe 198^li. 24
 19s. 11d^r. But here you se the pennes stande toward y^e ryght hande, and the other encreasyng orderly towarde the lefte hande. Agayne you maye se, that auditours wyll make 2 lynes (yea and more) for pennes, shylllynges, and all other valewes, yf theyr summes extende therto. Also you se, that they set one counter at the ryght ende of eche rowe, whiche so set there standeth for 5 of 28
 that roume: and on ⁴the lefte corner of the rowe it standeth for 10, of y^e same row. But now yf you wold adde other subtracte 32
 after any of both those sortes, yf you marke y^e order of y^e other feate which I taught you, you may easely do the same here without moch teachyng: for in Addition you must fyrst set downe one summe and to the same set the other orderly, and lyke maner yf 36
 you haue many: but in Subtraction you must sette downe fyrst the greatest summe, and from it must you abate that other every denomination from his dewe place. S. I do not doubte but with a

² 168 in original.

lytell practise I shall attayne these bothe : but how shall I multiply
 and diuide after these fourmes? *M.* You can not duely do none
 of both by these sortes, therefore in suche case, you must resort to
 4 your other artes. *S.* Syr, yet I se not by these sortes how to
 expresse hundreddes, yf they excede one hundred, nother yet
 thousandes. *M.* They that vse such accomptes that it excede 200
 1 in one summe, they sette no 5 at the lefte hande of the scores of
 8 poundes, but they set all the hundredes in an other farther rowe
and 500 at the lefte hand therof, and the thousandes they set in a
 farther rowe yet, *and* at the lefte syde therof they sette the 5000,
 and in the space ouer they sette the 10000, and in a hygher rowe
 12 20000, whiche all I haue expressed in this example, which is
 97869*li.* 12*s.* 9*d.* ob. q. for I had not told you before where,
 nother how you shuld set downe farthynges, which
 (as you se here) must be set in a voyde space
 16 sydelynge beneth the pennes : for q one counter :
 for ob. 2 counters : for ob. q. 3 counters : *and*
 more there can not be, for 4 farthynges ²do make
 1*d.* which must be set in his dewe place. And yf you desyre
 20 y^e same summe after audytors maner, lo here it is.

1 132 b.



2 133 a.



But in this thyng, you shall take this for suffycyent, and the reste
 you shall obserue as you maye se by the working of eche sorte : for
 the dyuers wittes of men haue inuented dyuers and sundry wayes
 24 almost vnumerable. But one feate I shall teache you, whiche not
 only for the straungenes and secretnes is moche pleasaunt, but also
 for the good *commoditie* of it ryghte worthy to be well marked.
 This feate hath ben vsed aboue 2000 yeares at the leaste, and yet
 28 was it neuer comenly knowen, especyally in Englysshe it was
 neuer taughte yet. This is the arte of nombrynge on the hand,
 with diuers gestures of the fyngers, expressynge any summe con-
 ceaued in the ³mynde. And fyrst to begynne, yf you wyll expresse
 32 any summe vnder 100, you shall expresse it with your lefte hande :
 and from 100 vnto 10000, you shall expresse it with your ryght
 hande, as here orderly by this table folowyng you may perceauē.

3 133 b.

¶ Here foloweth the table
 of the arte of the
 hande

The arte of nombrynge by the hande.

134

1	10	100	1000
2	20	200	2000
3	30	300	3000
4	40	400	4000
5	50	500	5000
6	60	600	6000
7	70	700	7000
8	80	800	8000
9	90	900	9000

1 134. 1 ¹In which as you may se 1 is expressed by y^e lyttle fynger of y^e
 2 lefte hande closely and harde coked. * [2 is declared by lyke bow-
 yng of the weddyng fynger (whiche is the nexte to the lyttell
 3 fynger) together with the lyttell fynger. [3 is signified by the
 4 myddle fynger bowed in lyke maner, with those other two. [4 is
 declared by the bowyng of the myddle fynger and the ryng

* Bracket ([) denotes new paragraph in original.

fynger, or weddyng fynger, with the other all stretched forth.
 [5 is represented by the myddle fynger onely bowed. [And 6 by ^{5, 6}
 the weddyng fynger only crooked: and this you may marke in
 4 these a certayne order. But now 7, 8, and 9, are expressed *with*
 the bowyng of the same fyngers as are 1, 2, and 3, but after an
 other fourme. [For 7 is declared by the bowyng of the lytell ₇
 8 fynger, as is 1, saue that for 1 the fynger is clasped in, harde *and*
 8 ¹rounde, but for to expresse 7, you shall bowe the myddle ioynte
 of the lytell fynger only, and holde the other ioyntes streyght. ^{1 135 a.}
S. Yf you wyll geue me leue to expresse it after my rude maner,
 thus I vnderstand your meanyng: that 1 is expressed by crookyng
 12 in the lytell fynger lyke the head of a bysshoppes bagle: and 7 is
 declared by the same fynger bowed lyke a gybbet. *M.* So I
 perceaque, you vnderstande it. [Then to expresse 8, you shall bowe ₈
 after the same maner both the lytell fynger and the ryng fynger.
 16 [And yf you bowe lyke wayes with them the myddle fynger, then
 doth it betoken 9. [Now to expresse 10, you shall bowe your ^{9, 10}
 fore fynger rounde, and set the ende of it on the hyghest ioynte of
 the thombe. [And for to expresse 20, you must set your fyngers ²⁰
 20 streyght, and the ende of your thombe to the partition of the ²fore
^{2 135 b.} moste and myddle fynger. [30 is represented by the ioynyng ³⁰
 together of y^e headdes of the foremost fynger and the thombe.
 [40 is declared by settinge of the thombe crossewayes on the fore- ⁴⁰
 24 most fynger. [50 is signified by ryght stretchyng forth of the ⁵⁰
 fyngers ioyntly, and applyenge of the thombes ende to the partition
 of the myddle fynger *and* the ryng fynger, or weddyng fynger.
 [60 is formed by bendyng of the thombe croked and crosseynge it ⁶⁰
 28 with the fore fynger. [70 is expressed by the bowyng of the ⁷⁰
 foremost fynger, and settinge the ende of the thombe between the
 2 foremost or hyghest ioyntes of it. [80 is expressed by settinge ⁸⁰
 of the foremost fynger crossewayes on the thombe, so that 80
 32 dyffereth thus *from* 40, that for 80 the forefynger is set crosse on
 the thombe, and for 40 the thombe is set crosse ouer y^e forefynger.
³[90 is signified, by bendyng the fore fynger, and settinge the ende ⁹⁰ ^{3 136 a.}
 of it in the innermost ioynte of y^e thombe, that is euen at the foote
 36 of it. And thus are all the *numbers* ended vnder 100. *S.* In
 dede these be all the numbers *from* 1 to 10, *and* then all the
 tenthes within 100, but this teacyed me not how to expresse 11, ¹¹
 12, 13, *etc.* 21, 22, 23, *etc.* and such lyke. *M.* You can lytell ^{12, 13, 21, 22,}
 40 vnderstande, yf you can not do that without teachyng: what is ²³

11? is it not 10 and 1? then expresse 10 as you were taught, and 1 also, and that is 11 : and for 12 expresse 10 and 2 : for 23 set 20 and 3 : and so for 68 you muste make 60 and there to 8 : and so
 100 of all other sortes. [But now yf you wolde represente 100 other
 any number aboue it, you muste do that with the ryghte hande,
 after this maner. [You must expresse 100 in the ryght hand,
 with the lytell fynger so bowed as you dyd expresse 1 in the left
 hand.

- ¹ 136 b. 1[And as you expressed 2 in the lefte hande, the same fasshyon
 200 in the ryght hande doth declare 200.
 300 The fourme of 3 in the ryght hand standeth for 300.
 400 The fourme of 4, for 400. 12
 500 Lykewayes the fourme of 5, for 500.
 600 The fourme of 6, for 600. And to be shorte : loke how you did
 expresse single vnities and tenthes in the lefte hande, so must you
 expresse vnities *and* tenthes of hundredes, in the ryghte hande. 16
 900 *S.* I vnderstande you thus : that yf I wold represent 900, I must
 so fourme the fyngers of my ryghte hande, as I shuld do in my
 left hand to expresse 9, And as in my lefte hand I expressed
 1000 10, so in my ryght hande must I expresse 1000. 20
 And so the fourme of euery tenthe in the lefte hande serueth
 to expresse lyke number of thousandes, so y^e fourme of 40 standeth
 4000 for 4000.
 8000 The fourme of 80 for 8000. 24
² 137 a. 2And the fourme of 90 (whiche is
 9000 the greatest) for 9000, and aboue that
 I can not expresse any number. *M.*
 No not with one fynger : how be it,
 with dyuers fyngers you maye expresse 28
 9999, and all at one tyme, and that lac
 keth but 1 of 10000. So that vnder
 10000 you may by your fyngers ex- 32
 presse any summe. And this shal suf-
 fyce for Numeration on the fyngers.
 And as for Addition, Subtraction,
 Multiplication, and Diuision (which 36
 yet were neuer taught by any man as
 farre as I do knowe) I wyll enstruct
 you after the treatyse of fractions.
 And now for this tyme fare well, 40

and loke that you cease not to
practyse that you haue lear
ned. *S.* Syr, with moste
4 hartly mynde I thanke
you, bothe for your
good learnyng, *and*
also your good
8 counsel, which
(god wyllyng) I truste to folow.

Finis.

APPENDIX I.

A Treatise on the Numeration of Algorism.

[From a MS. of the 14th Century.]

To alle suche even nombrys the most have cifrys as to ten. twenty. thirty. an hundred. an thousand and suche other. but ye schal vnderstonde that a cifre tokeneth nothings but he maketh other the more significatyf that comith after hym. Also ye schal vnderstonde that in nombrys composyt and in alle other nombrys that ben of diverse figurys ye schal begynne in the riht syde and to rekene backwarde and so he schal be wryte as thus—1000. the sifre in the riht side was first wryte and yit he tokeneth nothings to the secunde no the thridde but thei maken that figure of 1 the more signyficatyf that comith after hem by as moche as he horn oute of his first place where he schuld yf he stode ther tokene but one. And there he stondith nowe in the ferye place he tokeneth 12 a thousand as by this rewle. In the first place he tokeneth but hymself. In the secunde place he tokeneth ten times hymself. In the thridde place he tokeneth an hundred tymes hymself. In the ferye he tokeneth a thousand tymes hymself. In the fyfthe place 16 he tokeneth ten thousand tymes hymself. In the sexte place he tokeneth an hundred thousand tymes hymself. In the seveth place he tokeneth ten hundred thousand tymes hymself, &c. And ye schal vnderstond that this worde nombre is partyd into thre 20 partyes. Somme is callyd nombre of digitys for alle ben digitys that ben withine ten as ix, viii, vii, vi, v, iv, iii, ii, i. Articules ben alle thei that mow be devyded into nombrys of ten as xx, xxx, xl, and suche other. Composittys be alle nombrys that ben com- 24 ponyd of a digyt and of an articule as fourtene fyftene thrittene and suche other. Fourtene is compond of four that is a digyt

and of ten that is an articule. Fyftene is compond of fyve that is a digyt and of ten that is an articule and so of others

But as to this rewle. In the firste place he tokeneth but himself
4 that is to say he tokeneth but that and no more. If that he stonde
in the secunde place he tokeneth ten tymes himself as this figure 2
here 21. this is oon and twenty. This figure 2 stonddith in the
secunde place and therfor he tokeneth ten tymes himself and ten
8 tymes 2 is twenty and so forye of every figure and he stonde after
another toward the lest syde he schal tokene ten tymes as moche
more as he schuld token and he stode in that place ther that the
figure afore him stonddeth: lo an example as thus 9634. This
12 figure of foure that hath this schape 4 tokeneth but himself for he
stonddeth in the first place. The figure of thre that hath this schape
3 tokeneth ten tyme himself for he stonddeth in the secunde place
and that is thritti. The figure of sexe that hath this schape 6
16 tokeneth ten tyme more than he schuld and he stode in the place
yer the figure of thre stonddeth for ther he schuld tokene but sixty.
And now he tokeneth ten tymes that is sexe hundrid. The figure
of nyne that hath this schape 9 tokeneth ten tymes more than he
20 schulde and he stode in the place ther the figure of 6 stonddeth inne
for thanne he schuld tokene but nyne hundryd. And in the place
that he stonddeth inne nowe he tokeneth nine thousand. Alle the
hole nombre of these foure figurys. Nine thousand sexe hundrid
24 and foure and thritti.

Quolibet in numero, si par sit prima figura, Par erit et totum, quicquid sibi continetur ; Impar si fuerit, totum sibi fiet et impar.	28
Septem ¹ sunt partes, non plures, istius artis ; Addere, subtrahere, duplare, dimidiare ; Sexta est diuidere, set quinta est multiplicare ; Radicem extrahere pars septima dicitur esse. Subtrahis aut addis a dextris vel mediabis ; A leua dupla, diuide, multiplicaque ; Extrahe radicem semper sub parte sinistra.	32
Addere si numero numerum vis, ordine tali Incipe ; scribe duas primo series numerorum Prima sub prima recte ponendo figuram, Et sic de reliquis facias, si sint tibi plures. Inde duas adde primas hac condicione ; Si digitus crescat ex addicione priorum, Primo scribe loco digitum, quicumque sit ille ; Si sit compositus, in limite scribe sequenti Articulum, primo digitum ; quia sic iubet ordo.	36 Addition. 40 44
Articulus si sit, in primo limite cifram, Articulum vero reliquis inscribe figuris ; Vel per se scribas si nulla figura sequatur. Si tibi cifra superueniens occurrerit, illam Deme suppositam ; post illic scribe figuram : Postea procedas reliquas addendo figuras.	48
A numero numerum si sit tibi demere cura, Scribe figurarum series, vt in addicione ; Maiori numero numerum suppone minorem, Siue pari numero supponatur numerus par. Postea si possis a prima subtrahe primam, Scribens quod remanet, cifram si nil remanebit. Set si non possis a prima demere primam ; Procedens, vnum de limite deme sequenti ;	Subtraction. 52 56
¹ En argorisme deuo prendre Vii especes Adision subtracion Doublouison mediacion Monteploie et division Et de radix enstracion A chez vii especes sauoir Doit chaseun en memoire auoir Letres qui figures sont dites Et qui excellens sont ecrites.—MS. <i>Seld. Arch.</i> B. 26.	

	Et demptum pro denario reputabis ab illo, Subtrahe totaliter numerum quem proposuisti.	60
	Quo facto, scribe supra quicquid remanebit, Facque novenarios de cifris, cum remanebis, Occurrant si forte cifre, dum demseris vnum ; Postea procedas reliquas demendo figuras.	64
Proof.	¹ [Si subtraccio sit bene facta probare valebis, Quas subtraxisti primas addendo figuras. Nam, subtractio si bene sit, primas retinebis, Et subtractio facta tibi probat additionem.]	68
Duplation.	Si vis duplare numerum, sic incipe ; solam Scribe figurarum seriem, quamcumque voles que Postea procedas primam duplando figuram ; Inde quod excrescet, scribens, vbi iusserit ordo, Juxta precepta que dantur in addicione.	72
	Nam si sit digitus, in primo limite scribe ; Articulus si sit, in primo limite cifram, Articulum vero reliquis inscribe figuris ; Vel per se scribas, si nulla figura sequatur :	76
	Compositus si sit, in limite scribe sequenti Articulum primo, digitum ; quia sic jubet ordo : Et sic de reliquis facias, si sint tibi plures.	80
	¹ [Si super extremam nota sit, monadem dat eidem, Quod tibi contingit, si primo dimidiabis.]	
Mediation.	Incipe sic, si vis aliquem numerum mediare : Scribe figurarum seriem solam, velud ante ; Postea procedens medias, et prima figura Si par aut impar videas ; quia si fuerit par, Dimidiabis eam, scribens quicquid remanebit ; Impar si fuerit, vnum demas, mediare, Nonne presumas, sed quod superest mediabis ; Inde super tractum, fac demptam quod notat unum ; Si monos, dele ; sit ibi cifra post nota supra. Postea procedas hac condicione secunda : ² Impar ³ si fuerit hic vnum deme priori, Inscribens quinque, nam denos significabit Monos predictam ; si vero secunda dat vnam, Illa deleta, scribatur cifra ; priori	84 88 92 96

¹ From 12 E. 1.² 8 C. iv. inserts Atque figura prior nuper fuerit mediando.³ I. e. figura secundo loco posita.

Tradendo quinque pro denario mediato ; Nec cifra scribatur, nisi inde figura sequatur : Postea proceas reliquas mediando figuras, Quin supra docui, si sint tibi mille figure.	100
¹ [Si mediatio sit bene facta probare valebis, Duplando numerum quem primo dimidiasti.]	
Si tu per numerum numerum vis multiplicare, Scribe duas, quascunque volis, series numerorum ; Ordo tamen seruetur vt vltima multiplicandi Ponatur super anteriorem multiplicantis ;	Multiplication. 104
² [A leua relique sint scripte multiplicantes.] In digitum eures digitum si ducere, major Per quantes distat a denis respice, debes Namque suo decuplo tociens delere minorem ; Sicque tibi numerus veniens exinde patebit.	108
Postea procedas postremam multiplicando, Iuste multiplicans per cunctas inferiores, Condicione tamen tali ; quod multiplicantis Scribas in capite, quicquid processerit inde ; Set postquam fuerit hec multiplicata, figure Anteriores seriei multiplicantis ;	112
Et sic multiplica, velut istam multiplicasti, Qui sequitur numerum scriptum quicumque figuris. Set cum multiplicas, primo sic est operandum, Si dabit articulum tibi multiplicacio solum ; Proposita cifra, summam transferre memento. Sin autem digitus excrescerit articulusque,	120
Articulus supraposito digito salit ultra ; Si digitus tamen, ponas illum super ipsam, Subdita multiplicans hanc que super incidit illi Delet eam penitus, scribens quod provenit inde ; Sed si multiplices illam posite super ipsam, Adiungens numerum quem prebet ductus earum ; Si suprainpositam cifra debet multiplicare, Prorsus eam delet, scribi que loco cifra debet,	124
² [Si cifra multiplicat aliam positam super ipsam, Sitque locus supra vacuus super hanc cifra fiet ;]	128
	132

¹ So 12 E. 1 ; 8 C. iv. inserts—

Si super extremam nota sit monades dat eidem

Quod contingat cum primo dimiabis

Atque figura prior nuper fuerit mediando.

² 12 E. 1 inserts.

	Si supra fuerit cifra semper pretereunda est ;	
	Si dubites, an sit bene multiplicando secunda,	
	Diuide totalem numerum per multiplicantem,	136
	Et reddet numerus emergens inde priorem.	
Mental Multiplica- tion.	¹ [Per numerum si vis numerum quoque multiplicare	
	Tantum per normas subtiles absque figuris	
	Has normas poteris per versus scire sequentes.	140
	Si tu per digitum digitum quilibet multiplicabis	
	Regula precedens dat qualiter est operandum	
	Articulum si per reliquum vis multiplicare	
	In proprium digitum debet uterque resolvi	144
	Articulus digitos post per se multiplicantes	
	Ex digitis quociens teneret multiplicatum	
	Articuli faciunt tot centum multiplicati.	
	Articulum digito si multiplicamus oportet	148
	Articulum digitum sumi quo multiplicare	
	Debemus reliquum quod multiplicaris ab illis	
	Per reliquo decuplum sic omne latere nequibit	
	In numerum mixtum digitum si ducere cures	152
	Articulus mixti sumatur deinde resolvas	
	In digitum post hec fac ita de digitis nec	
	Articulusque docet exerescens in definendo	
	In digitum mixti post ducas multiplicantem	156
	De digitis ut norma docet sit juncta secundo	
	Multiplica summam et postea summa patebit	
	Junctus in articulum purum articulumque	
	² [Articulum purum comittes articulum que]	160
	Mixti pro digitis post fiat et articulus vt	
	Norma jubet retinendo quod egreditur ab illis	
	Articuli digitum post iu digitum mixti due	
	Regula de digitis ut percipit articulusque	164
	Ex quibus exerescens summe tu junge priori	
	Sic manifesta cito fiet tibi summa petita.	
	Compositum numerum mixto sic multiplicabis	
	Vndecies tredecem sic est ex hiis operandum	168
	In reliquum primum demum due post in eundem	
	Unum post deinde due in tercia deinde per unum	
	Multiplices tercia demum tunc omnia multiplicata	
	In summa duces quam que fuerit te dicces	172

¹ 12 E. 1 inserts to l. 174.² 12 E. 1 omits, Eg. 2622 inserts.

Hic ut hic mixtus intentus est operandum Multiplicandorum de normis sufficiunt hec.]	
Si vis dividere numerum, sic incipe primo ;	Division.
Scribe duas, quascunque voles, series numerorum ;	176
Majori numero numerum suppose minorem,	
¹ [Nam docet ut major teneat bis terve minorem ;]	
Et sub supprima supprimam pone figuram,	
Sic reliquis reliquas a dextra parte locabis ;	180
Postea de prima primam sub parte sinistra	
Subtrahe, si possis, quociens potes adminus istud,	
Scribens quod remanet sub tali conditione ;	
Ut totiens demas demendas a remanente,	184
Que serie recte ponentur in anteriori,	
Unica si, tantum sit ibi decet operari ;	
Set si non possis a prima demere primam,	
Procedas, et eam numero suppose sequenti ;	188
Hanc uno retrahendo gradu quo comites retrahantur,	
Et, quotiens poteris, ab eadem deme priorem,	
Ut totiens demas demendas a remanenti,	
Nec plus quam novies quicquam tibi demere debes,	192
Nascitur hinc numerus quociens supraque sequentem	
Hunc primo scribas, retrahas exinde figuras,	
Dum fuerit major supra positus inferiori,	
Et rursus fiat divisio more priori ;	196
Et numerum quotiens supra scribas pereunti,	
Si fiat saliens retrahendo, cifra locetur,	
Et pereat numero quotiens, proponas eidem	
Cifram, ne numerum pereat vis, dum locus illic	200
Restat, et expletis divisio non valet ultra :	
Dum fuerit numerus numerorum inferiore seorsum	
Illum servabis ; hinc multiplicando probabis,	
Si bene fecisti, divisor multiplicetur	204 Proof.
Per numerum quotiens ; cum multiplicaveris, adde	
Totali summae, quod servatum fuit ante,	
Reddeturque tibi numerus quem proposuisti ;	
Et si nil remanet, hunc multiplicando reddet,	208
Cum ducis numerum per se, qui provenit inde	Square Numbers.
Sit tibi quadratus, ductus radix erit hujus,	
Nec numeros omnes quadratos dicere debes,	
Est autem omnis numerus radix alicujus.	212

Quando voles numeri radicem querere, scribi
 Debet ; inde notes si sit locus ulterius impar,
 Estque figura loco talis scribenda sub illo,
 Que, per se dicta, numerum tibi destruat illum, 216
 Vel quantum poterit ex inde delebis eandem ;
 Vel retrahendo duplex retrahens duplando sub ista
 Que primo sequitur, duplicatur per duplicationem,
 Post per se minuens pro posse quod est minuendum. 220
¹ Post his propones digitum, qui, more priori
 Per precedentes, post per se multiplicatus,
 Destruat in quantum poterit numerum remanentem,
 Et sic procedens retrahens duplando figuram, 224
 Preponendo novam donec totum peragatur,
 Subdupla propriis servare docetque duplatis ;
 Si det compositum numerum duplacio, debet
 Inscribi digitus a parte dextra parte propinqua, 228
 Articulusque loco quo non duplicata resessit ;
 Si dabit articulum, sit cifra loco pereunte
 Articulusque locum tenet unum, de duplicata resessit ;
 Si donet digitum, sub prima pone sequente, 232
 Si supraposita fuerit duplicata figura
 Major proponi debet tantummodo cifra,
 Has retrahens solito propones more figuram,
 Usque sub extrema ita fac retrahendo figuras, 236
 Si totum deles numerum quem proposuisti,
 Quadratus fuerit, de dupla quod duplicasti,
 Sicque tibi radix illius certa patebit,
 Si de duplatis fit juncta supprima figura ; 240
 Radicem per se multiplices habeasque
 Primo propositum, bene te fecisse probasti ;
 Non est quadratus, si quis restat, sed habentur
 Radix quadrati qui stat major sub eadem ; 244
 Vel quicquid remanet tabula servare memento ;
 Hoc casu radix per se quoque multiplicetur,
 Vel sic quadratus sub primo major habetur,
 Hinc addas remanens, et prius debes haberi ; 248
 Si locus extremus fuerit par, scribe figuram
 Sub pereunte loco per quam debes operari,
 Que quantum poterit suppressas destruat ambas,

¹ 8 C. iv. inserts—

Hinc illam dele duplans sub ei psalliendo
 Que sequitur retrahens quicquid fuerit duplicatum.

Vel penitus legem teneas operando priorem,	252
Si suppositum digitus suo fine repertus,	
Omnino delet illic scribi cifra debet,	
A leva si qua sit ei sociata figura ;	
Si cifre remanent in fine pares decet harum	256
Radices, numero mediam proponere partem,	
Tali quesita radix patet arte reperta.	
Per numerum recte si nosti multiplicare	
Ejus quadratum, numerus qui pervenit inde	260
Dicetur cubicus ; primus radix erit ejus ;	
Nec numeros omnes cubicatos dicere debes,	
Est autem omnis numerus radix alicujus ;	
Si curas cubici radicem quærere, primo	264 <i>Cube Root.</i>
Inscriptum numerum distinguere per loca debes ;	
Que tibi mille notant a mille notante suprema	
Initiam, summa operandi parte sinistra,	
Illic sub scribas digitum, qui multiplicatus	268
In semet cubice suprapositum sibi perdat,	
Et si quid fuerit adjunctum parte sinistra	
Si non omnino, quantum poteris minuendo,	
Hinc triplans retrahere saltum, faciendo sub illa	272
Que manet a digito deleta terna, figuram	
Illi propones que sub triplo asocietur,	
Ut cum subtriplo per eam tripla multiplicatur ;	
Hinc per eam solam productum multiplicabis,	276
Postea totalem numerum, qui provenit inde	
A suprapositis respectu tolle triplate	
Addita supprimo cubice tunc multiplicetur,	
Respectu ejus, numerus qui progredietur	280
Ex cubito ductu, supra omnes adimetur ;	
Tunc ipsam delens triples saltum faciendo,	
Semper sub ternas, retrahens alias triplicatas	
Ex hinc triplatis aliam propone figuram,	284
Que per triplatas ducatur more priori ;	
Primo sub triplis sibi junctis, postea per se,	
In numerum ducta, productum de triplicatis :	
Utque prius dixi numerus qui provenit inde	288
A suprapositis has respiciendo trahatur,	
Huic cubice ductum sub primo multiplicabis,	
Respectumque sui, removebis de remanenti,	
Et sic procedas retrahendo triplando figuram.	292

Et proponendo nonam, donec totum peragatur, Subtripla sub propriis servare decet triplicatis ; Si nil in fine remanet, numerus datus ante Est cubicus ; cubicam radicem sub tripla prebent,	296
Cum digito juncto quem supprimo posuisti, Hec cubice ducta, numerum reddant tibi primum. Si quid erit remanens non est cubicus, sed habetur Major sub primo qui stat radix cubicam,	300.
Servari debet quicquid radice remansit, Extracto numero, decet hec addi cubicato. Quo facto, numerus reddi debet tibi primus. Nam debes per se radicem multiplicare	304
Ex hinc in numerum duces, qui provenit inde Sub primo cubicus major sic invenietur ; Illi jungatur remanens, et primus habetur, . Si per triplatum numerum nequeas operari ;	308
Cifram propones, nil vero per hanc operare Set retrahens illam cum saltu deinde triplata, Propones illi digitum sub lege priori, Cumque cifram retrahas saliendo, non triplicabis,	312
Namque nihil cifre triplacio dicitur esse ; At tu cum cifram protraxeris aut triplicata, Hanc cum subtriplo semper servare memento :	316
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Set facit hunc scribi per se triplacio prima, Que si det digitum per se scribi facit illum ; Consumpto numero, si sole fuit tibi cifre Triplato, propone cifram saltum faciendo,	324
Cumque cifram retrahe triplam, scribendo figuram, Preponas cifre, sic procedens operare, Si tres vel duo serie in sint, pone sub yma, A dextris digitum servando prius documentum.	328
Si sit continua progressio terminus nuper Per majus melium totalem multiplicato ; Si par, per medium tunc multiplicato sequentem. Set si continua non sit progressio finis :	332
Impar, tunc majus medium si multiplicabis, Si par per medium sibi multiplicato propinquum.	333.

INDEX OF TECHNICAL TERMS¹

algorisme, 33/12; **algorym**, **augrym**, 3/3; the art of computing, using the so-called Arabic numerals.

The word in its various forms is derived from the Arabic *al-Khowarazmi* (i. e. the native of Khwarazm (Khiva)). This was the surname of Ja'far Mohammad ben Musa, who wrote a treatise early in the 9th century (see p. xiv).

The form *algorithm* is also found, being suggested by a supposed derivation from the Greek *ἀριθμός* (number).

antery, 24/11; to move figures to the right of the position in which they are first written. This operation is performed repeatedly upon the multiplier in multiplication, and upon certain figures which arise in the process of root extraction.

anterioracioun, 50/5; the operation of moving figures to the right.

article, 34/23; **articul**, 5/31; **articals**, 9/36, 29/7, 8; a number divisible by ten without remainder.

cast, 8/12; to add one number to another.

'Addition is a *casting* together of two numbers into one number,' 8/10.

cifre, 4/1; the name of the figure 0. The word is derived from the Arabic *sifr* = empty, nothing. Hence *zero*.

A cipher is the symbol of the absence of number or of zero quantity. It may be used alone or in conjunction with digits or other ciphers, and in the latter case, according to the position which it occupies relative to the other figures, indicates the absence of units, or tens, or hundreds, etc. The great superiority of the Arabic to all other systems of notation resides in the employment of this symbol. When the cipher is not used, the place value of digits has to be indicated by writing them in assigned rows or columns. Ciphers, however, may be interpolated amongst the significant figures used, and as they sufficiently indicate the positions of the empty rows or columns, the latter need not be indicated in any other way. The practical performance of calculations is thus enormously facilitated (see p. xvi).

componede, 33/24; **composyt**, 5/35; with reference to numbers, one compounded of a multiple of ten and a digit.

conuertide = conversely, 46/29, 47/9.

cubicede, 50/13; **to be c.**, to have its cube root found.

¹ This Index has been kindly prepared by Professor J. B. Dale, of King's College, University of London, and the best thanks of the Society are due to him for his valuable contribution.

- cublke nombre**, 47/8; a number formed by multiplying a given number twice by itself, *e. g.* $27 = 3 \times 3 \times 3$. Now called simply a cube.
- decuple**, 22/12; the product of a number by ten. Tenfold.
- departys** = divides, 5/29.
- digit**, 5/30; **digitale**, 33/24; a number less than ten, represented by one of the nine Arabic numerals.
- dimydicion**, 7/23; the operation of dividing a number by two. Halving.
- duccioun**, multiplication, 43/9.
- duplacion**, 7/23, 14/15; the operation of multiplying a number by two. Doubling.
- l-mediet** = halved, 19/23.
- intercise** = broken, 46/2; intercise Progression is the name given to either of the Progressions 1, 3, 5, 7, etc.; 2, 4, 6, 8, etc., in which the common difference is 2.
- lede into**, multiply by, 47/18.
- lyneal nombre**, 46/14; a number such as that which expresses the measure of the length of a line, and therefore is not necessarily the product of two or more numbers (*vide* Superficial, Solid). This appears to be the meaning of the phrase as used in *The Art of Nombryng*. It is possible that the numbers so designated are the prime numbers, that is, numbers not divisible by any other number except themselves and unity, but it is not clear that this limitation is intended.
- mediacioun**, 16/36, 38/16; dividing by two (see also **dimydicion**).
- medlede nombre**, 34/1; a number formed of a multiple of ten and a digit (*vide* componede, composyt).
- medye**, 17/8, to halve; **mediete**, halved, 17/30; **ymedit**, 20/9.
- naturelle progressioun**, 45/22; the series of numbers 1, 2, 3, etc.
- produccioun**, multiplication, 50/11.
- quadrat nombre**, 46/12; a number formed by multiplying a given number by itself, *e. g.* $9 = 3 \times 3$, a square.
- rote**, 7/25; **roote**, 47/11; root. The roots of squares and cubes are the numbers from which the squares and cubes are derived by multiplication into themselves.
- significatyf**, significant, 5/14. The significant figures of a number are, strictly speaking, those other than zero, *e. g.* in 3 6 5 0 4 0 0, the significant figures are 3, 6, 5, 4. Modern usage, however, regards all figures between the two extreme significant figures as significant, even when some are zero. Thus, in the above example, 3 6 5 0 4 are considered significant.
- solide nombre**, 46/37; a number which is the product of three other numbers, *e. g.* $66 = 11 \times 2 \times 3$.
- superficial nombre**, 46/18; a number which is the product of two other numbers, *e. g.* $6 = 2 \times 3$.
- ternary**, consisting of three digits, 51/7.
- vnder double**, a digit which has been doubled, 48/3.
- vnder-trebille**, a digit which has been trebled, 49/28; **vnder-triplat**, 49/39.
- w**, a symbol used to denote half a unit, 17/33.

GLOSSARY

- ablacioun, taking away, 36/21
 addyst, haddest, 10/37
 agregacioun, addition, 45/22. (First example in N.E.D., 1547.)
 a-;enenes, against, 23/10
 allgate, always, 8/39
 als, as, 22/24
 and, if, 29/8; &, 4/27; & yf, 20/7
 a-nendes, towards, 23/15
 aproprede, appropriated, 34/27
 apwereth, appears, 61/8
 a-riayt, arises, 14/24
 a-rowe, in a row, 29/10
 arsemetrike, arithmetic, 33/1
 ayene, again, 45/15
- bagle, crozier, 67/12
 bordure = ordure, row, 43/30
 borro, *inf.* borrow, 11/38; *imp. s.* borowe, 12/20; *pp.* borwed, 12/15; borred, 12/19
 boue, above, 42/34
- captule, chapter, 7/26
 certayn, assuredly, 18/34
 clepede, called, 47/7
 competently, conveniently, 35/8
 compt, count, 47/29
 contynes, contains, 21/12; *pp.* contenythe, 38/39
 craft, art, 3/4
- distingue, divide, 51/5
- egalle, equal, 45/21
 excep, except, 5/16
 excludede, excluded, 34/37
 exressent, resulting, 35/16
 exeant, resulting, 43/26
 expone, expound, 3/23
- ferye = ferþe, fourth, 70/12
 figure = figures, 5/1
 for-by, past, 11/21
 fors; no f., no matter, 22/24
 forseth, matters, 53/30
 forye = forþe, forth, 71/8
 fyfye = fyftþe, fifth, 70/16
- grewe, Greek, 33/13
- haluendel, half, 16/16; haldel, 19/4; *pl.* halnedels, 16/16
 hayst, hast, 17/3, 32
 hast, haste, 22/25
 heer, higher, 9/35
 here, their, 7/26
 here-a-fore, heretofore, 13/7
 heyth, was called, 3/5
 hole, whole, 4/39; holle, 17/1; hools, of three dimensions, 46/15
 holdyþe, holds good, 30/5
 how be it that, although, 44/4
- lede = lete, let, 8/37
 lene, lend, 12/39
 lest, least, 43/27
 lest = left, 71/9
 leue, leave, 6/5; *pr.* 3 *s.* leues, remains, 11/19; leus, 11/28; *pp.* laft, left, 19/24
 lewder, more ignorant, 3/3
 lust, desirest to, 45/13
 lyt, easy, 15/31
 lymytes, limits, 31/18; lynes, 34/12; lynees, 34/17; Lat. limes, *pl.* limites.
- maystery, achievement; no m., no achievement, i. e. easy, 19/10
 me, *indef. pron.* one, 42/1
 mo, more, 9/16

- moder** = more (Lat. majorem), 43/22
most, must, 30/3
multipliede, to be m. = multiplying, 40/9
mynvtes, the sixty parts into which a unit is divided, 38/25
myse-wro;t, mis-wrought, 14/11

nether, nor, 34/25
nex, next, 19/9
no;t, nought, 5/7
note, not, 30/5

oo, one, 42/20; **o**, 42/21
omest, uppermost, higher, 35/26; **omyst**, 35/28
omwhile, sometimes, 45/31
on, one, 8/29
opyne, plain, 47/8
or, before, 13/25
or = þe oþer, the other, 28/34
ordure, order, 34/9; row, 43/1
other, or, 33/13, 43/26; **other** . . . or, either . . . or, 38/37
ouerer, upper, 42/15
ouer-hippede, passed over, 43/19

recte, directly, 27/20
remayner, remainder, 56/28
representithe, represented, 39/14
resteth, remains, 63/29
rewarde, regard, 48/6
rew, row, 4/8
rewle, row, 4/20, 7/12; **rewele**, 4/18; **rewles**, rules, 5/33

s. = scilicet, 3/8
sentens, meaning, 14/29
signifye(tyf), 5/13. The last three letters are added above the line, evidently because of the word 'significatyf' in l. 14. But the 'Solucio,' which contained the word, has been omitted.
sithen, since, 33/8
some, sum, result, 40/17, 32
sowne, pronounce, 6/29

singillatim, singly, 7/25
spices, species, kinds, 34/4
spyl, waste, 14/26
stydde, stead, 18/20
subtrahe, subtract, 48/12; *pp.* **subtrayd**, 13/21
sythes, times, 21/16

ta;t, taught, 16/36
take, *pp.* taken; **t. fro**, starting from, 45/22
taward, toward, 23/34
thou;t, though, 5/20
trebille, multiply by three, 49/26
twene, two, 8/11
þow, though, 25/15
þow;t, thought; **þe þ.**, mentally, 28/4
þus = þis, this, 20/33

vny, unite, 45/10

wel, wilt, 14/31
wete, wit, 15/16; **wyte**, know, 8/38; *pp.* 2 s. **wost**, 12/38
wex, become, 50/18
where, whether, 29/12
wher-thurghe, whence, 49/15
worch, work, 8/19; **wrich**, 8/35; **wyrch**, 6/19; *imp. s.* **worch**, 15/9; *pp.* **y-wroth**, 13/24
write, written, 29/19; **y-write**, 16/1
wryrchynge = wyrchynge, working, 30/4
wt, with, 55/8

y-broth, brought, 21/18
ychon, each one, 29/10
ydo, done, added, 9/6
ylke, same, 5/12
y-lyech, alike, 22/23
y-my;t, been able, 12/2
y-now;t, enough, 15/31; **ynov;t**, 18/34
yove, given, 45/33
yt, that, 52/8
y-write, v. write.
y-wroth, v. worch.

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