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THE EFFECT OF KEYWAYS ON THE STRENGTH OF SHAFTS

BY

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THE EFFECT OF KEYWAYS ON THE STRENGTH OF SHAFTS

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CONTENTS

THE EFFECT OF KEYWAYS ON THE STRENGTH OF SHAFTS.

I. Introduction.

1. *Preliminary.*—In the transmission of power by means of shafting and pulleys or gears, the common method of fastening the pulley or gear to the shaft, so that the two will rotate together, is by means of a key inserted in a keyway cut in the shaft, and extending into a corresponding keyway cut in the hub of the pulley or gear. The strength and the proper proportioning of keys have been subjects of considerable study and of some experimentation, but the effect of the keyway on the torsional strength of the shaft has apparently been studied but little. Evidently, the keyway must weaken the shaft in which it is cut. It would seem that the sharp corners of the keyway and its location at one side of the shaft might weaken the shaft more than the relatively small size of the keyway would lead us to expect. In view of the very extensive use of shafts with keyways and the small amount of information available on the subject, the effect of keyways on the torsional strength of shafts has seemed to the writer a problem worthy of some experimental study. This bulletin is an account of a brief investigation carried on in the Laboratory of Applied Mechanics by the Engineering Experiment Station of the University of Illinois.

The mathematical analysis of the strength of a shaft with a keyway cut in it is a problem of great complexity. The common theory of stresses in shafts applies only to shafts of circular cross-section. Mathematical researches by Saint Venant and others have developed the theory of square, rectangular, triangular, and elliptical shafts, but, so far as the writer knows, there has been no successful attempt to develop the mathematical theory of the stress in a shaft with a keyway cut in it. However, as the range of sizes of shafts and keys in common use is not very great, it was thought that an experimental study of the effect of keyways on the strength of shafts might lead to formulas which may be safely used in nearly all the cases met by the designer of shafts and keys.

It was found possible to investigate by direct experiment the effect of keyways on the strength of shafts of various sizes, and to study the effect of keyways on the strength of shafts subjected to combined bending and twisting.

For this use in calculation and design, it was thought best to coin a term to permit comparison between a shaft with keyway and an uncut shaft. Adopting a nomenclature similar to that used by many writers on the strength of riveted joints, the ratio of the strength of a shaft with a keyway to the strength of a similar shaft without a keyway is hereafter spoken of as the *efficiency* of the shaft with keyway.

If a shaft with a pulley keyed to it is given a permanent twist, the removal of the pulley is frequently a matter of great difficulty; while if a shaft carries a sleeve or gear with a key sliding in a keyway, any permanent twist practically ruins the shaft. For these reasons the elastic limit of a shaft under torsion is taken as the measure of its strength.

2. *Acknowledgment.*—A considerable part of the experimental work herein described was performed by the following senior students of the College of Engineering of the University of Illinois in the preparation of their graduating theses in Mechanical Engineering:

Mr. F. E. Leidendeker, Class of 1908.

Mr. O. Craig and Mr. J. C. Lund, Class of 1909.

The writer wishes to express his appreciation of the faithful and careful work of the above students. Acknowledgment is also made to the Whitney Manufacturing Company of Hartford, Connecticut, for cutters for keyways of the Woodruff system of keys.

The work was undertaken with the approval of Professor Arthur N. Talbot, head of the department of Theoretical and Applied Mechanics, to whom the writer is indebted for many helpful suggestions, both as to methods of experimentation and to interpretation and arrangement of results.

3. *Notation and Formulas.*—The following notation is used:

d = actual diameter of shaft in inches.

w = width of keyway ÷ diameter of shaft.

h = depth of keyway ÷ diameter of shaft.

T = torsional (twisting) moment on shaft in inch-pounds.

M = bending moment on shaft in inch-pounds.

J = polar moment of inertia of cross-section of shaft (for circular shaft, $J = \frac{d^4}{10.2}$).

f = greatest fiber stress in shaft due to torsion.

Θ = angle of twist of shaft in degrees.

l = length of shaft in inches.

E_s = modulus of elasticity of material of shaft in shear (torsion).

e = efficiency of shaft with keyway.

k = ratio of angle of twist of shaft with keyway to angle of twist of similar uncut shaft.

$H. P.$ = horse-power.

$r. p. m.$ = number of revolutions per minute.

The following formulas are used:

$$T = \frac{2fJ}{d}$$

$$\Theta = \frac{2fl}{E_s d} \times 57.3$$

$$T = 63\,020 \frac{H. P.}{r.p.m.}$$

The first two formulas are based on the following assumptions; (1) that a plane section of the shaft remains plane during torsion; and (2) that the fiber stress varies uniformly from zero at the axis of the shaft to a maximum at the outer fiber, i. e., the modulus of elasticity for shear remains constant. The first assumption is not true for shafts which are not circular in cross-section.

II. Test Pieces, Tests, and Method of Testing.

4. *Test Pieces.*—The principal object of this investigation was to obtain values of the efficiency of shafts with keyways, and as nearly all shafting in common use is cold-rolled, the principal series of tests was made on specimens of cold-rolled steel shafting. The diameters of the test shafts of these series were $1\frac{1}{4}$, $1\frac{9}{16}$, $1\frac{15}{16}$, and $2\frac{1}{4}$ in. Shafts were tested under simple torsion and under torsion combined with bending. The bending moment applied to the shaft was in one case equal to the torsional moment, and in another equal to three-fifths the torsional moment. Table 1 shows the sizes of shafts and the sizes of the keyways cut in them.

TABLE 1.

DIMENSIONS OF SHAFTS AND KEYWAYS.
SERIES OF 1909.

Dia. Shaft inches d	Relative Dimensions of Keyway		Actual Dimensions of Keyway inches	
	w	h	width	depth
1¼	0.25	0.125	$\frac{5}{16}$	0.156
	0.50	0.125	⅝	0.156
	0.25	0.1875	$\frac{5}{16}$	0.234
$1\frac{9}{16}$	0.25	0.125	$\frac{25}{64}$	0.195
	0.50	0.125	$\frac{25}{32}$	0.195
	0.25	0.1875	$\frac{25}{64}$	0.293
$1\frac{15}{16}$	0.25	0.125	$\frac{31}{64}$	0.242
	0.50	0.125	$\frac{31}{32}$	0.242
	0.25	0.1875	$\frac{31}{64}$	0.363
2¼	0.25	0.125	$\frac{9}{16}$	0.281
	0.50	0.125	1⅛	0.281
	0.25	0.1875	$\frac{9}{16}$	0.422

For transmitting power, it is common American practice to use a square key whose width and depth are each equal to about one-fourth the diameter of the shaft (Kent's Pocket-Book, pp. 975 - 976). This means a keyway in the shaft in which $w = 0.25$ and $h = 0.125$. The depth of keyway is measured as shown in Fig. 1.

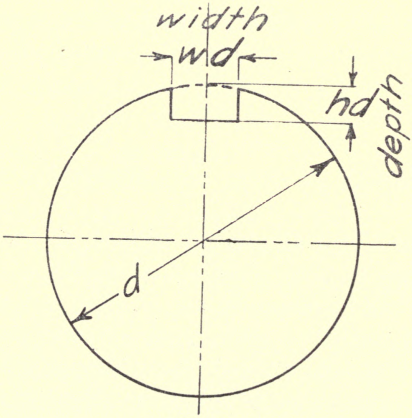

FIG. 1.

Shafts were also tested with keyways for the Woodruff system of keying. The outline of the Woodruff key and its keyway are shown in Fig. 2. In choosing the sizes of Woodruff keyways to be cut in the test shafts, the shearing strengths of various standard sizes of keys were figured, and a standard size was

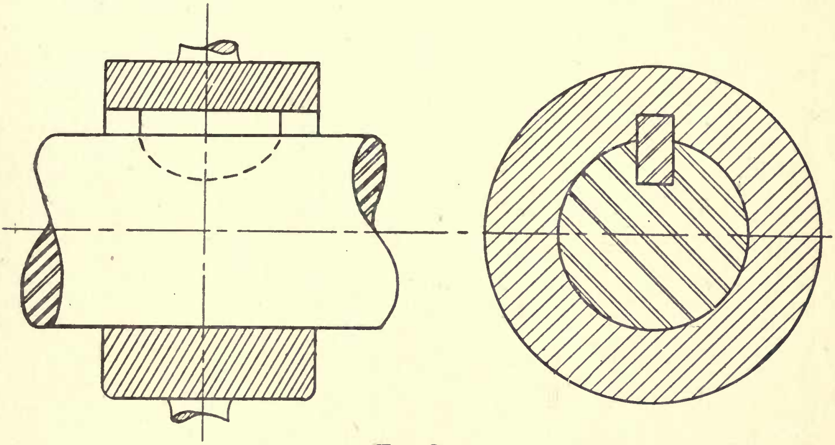

FIG. 2.

chosen such that the shearing strength of two keys equaled, as nearly as possible, the torsional strength of the solid test shaft in question. The sizes of the Woodruff keys chosen are shown in Tables 2 and 4.

In addition to the above tests for effect of single keyways on the strength of cold-rolled shafting, tests were made (principally in the 1908 series) which yielded data on the following subjects: ultimate strength of shafts with keyways; effect of two keyways at right angles; effect of length of keyway; effect of keyways on turned steel shafting.

All keyways, except in the tests for studying the effect of length of keyways, were cut to a length equal to about four times the diameter of the shaft, no keyway being longer than eight inches.

All material for the test shafts was bought in the open market. Both the cold-rolled and the turned shafting were of ordinary soft steel. All tests were planned in duplicate, and with a very few exceptions, all tests were made in duplicate.

5. *Description of Apparatus.*—All shafts tested under simple twisting were tested in the 230 000 in.-lb. Olsen torsion testing machine in the Laboratory of Applied Mechanics of the Univer-

sity of Illinois. To the test shaft were attached long arms in pairs, one arm of a pair carrying a pointer, and the other a scale. The angle of twist between these two arms was measured by the motion of the pointer over the scale. Fig. 3 shows a test shaft

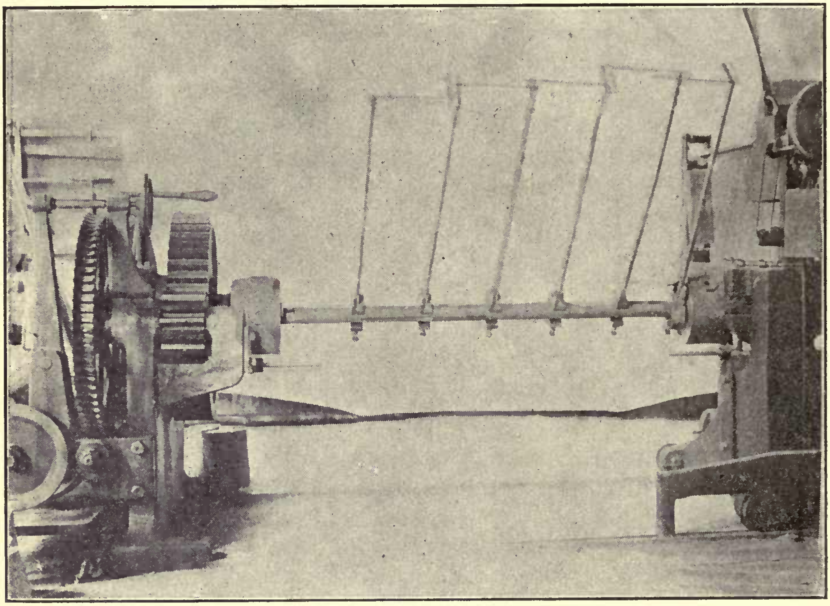

FIG. 3.

of the 1909 series in position with the pointers and scales attached. In this shaft were cut four keyways, each keyway being 90° round the shaft from the adjacent keyways. The angle of twist of the shaft was measured over five portions of its length, four portions of length being occupied by the keyways and one being without keyway. The latter portion was generally at the middle of the shaft.

The apparatus used for studying the effect of combined twisting and bending on shafts with keyways is shown in Fig. 4. To the ends of the test shaft S were keyed arms AA extending at right angles to the shaft. Equal forces *FF* were applied in a vertical direction at points on these arms at a distance p from the axis of the shaft. The test shaft was supported on bearings GG by means of steel balls B, bearing on hardened steel bushings.

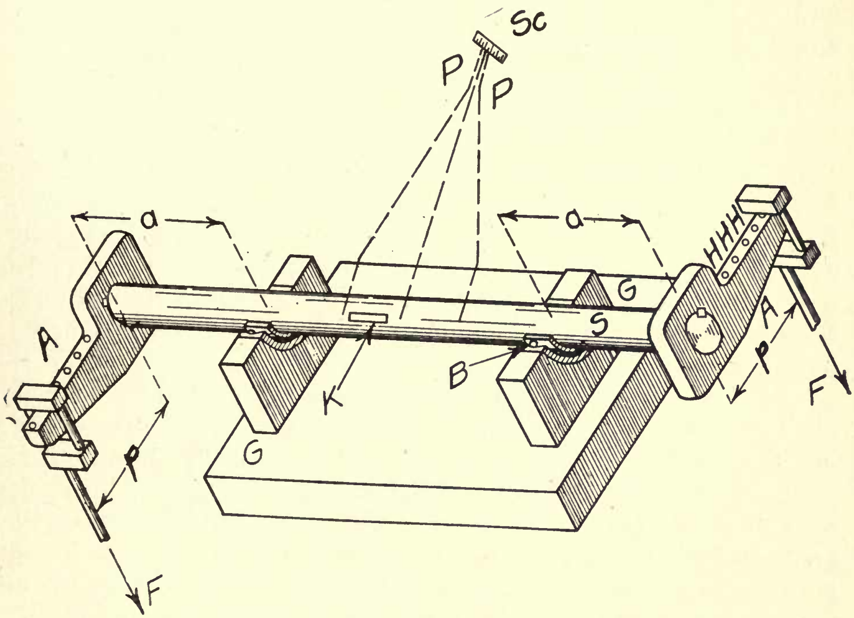

FIG. 4.

The distance a from the arm A to the center of the nearer bearing was the same at the two ends. The test shaft then was subjected to a bending moment Fa over that portion between bearings, and to a twisting moment Fp over its entire length (the very small friction of the ball bearings being neglected). The ratio of the twisting moment to the bending moment equals $\frac{p}{a}$.

The forces FF were applied by the moving crosshead of a testing machine. The entire apparatus shown in Fig. 4 rested on the upper weighing head of the testing machine. The load registered on the weighing table of the machine was equal to $2F$. The force F was transmitted to each of the arms AA through a small spherical pointed knob resting in one of the holes HHHH, the twisting arm being varied by using different holes. The bearings GG could be moved axially along the shaft, thus allowing the bending moment to be varied. In the tests under combined twisting and bending, the keyway cut in the test shaft was located at one side of the center of the shaft, and the angle of twist was measured over the portion of the shaft containing the keyway, and also over the solid portion. The apparatus for measuring angle of twist was the same as in the tests under simple torsion, and is shown in the diagram in Fig. 4, in which K represents the keyway, PP the pointers, and Sc the scale.

6. *Procedure of Tests.*—In the 1908 tests, which were all on shafts under twisting only, the method of conducting the test was to apply torsion continuously until the yield point was passed, frequent readings of twisting moment and of corresponding angle of twist being taken. After the yield point was passed, the twist-measuring apparatus was removed, and the torsion applied until the shaft broke, the maximum twisting moment carried being noted.

In the 1909 tests, both under simple torsion and under combined twisting and bending, the method of procedure was as follows: A small initial load was applied to the shaft, and an initial reading taken on the twist-measuring apparatus; more load was then applied and the angle of twist read; the load was then released to its initial value, and the angle of twist again read, any permanent set being thus detected; a load slightly greater than the previously applied load was then put on the shaft, and this load in turn released to the initial value. This process was repeated with applications of increasing loads until the yield point of the shaft was passed.

III. Data and Results.

7. *Ultimate Strength of Shafts with and without Keyways.*—Table 2 (tests of 1908) shows the results of tests to breaking of shafts with and without keyways. It seems that a shaft with a single keyway of common dimensions has about the same ultimate strength as a shaft without keyway. In the torsional tests to destruction, after the elastic limit of the shaft had been passed, the keyways gradually closed up and at rupture they were entirely closed. The larger keyways and the two keyways 90° apart lowered the ultimate strength somewhat. The variation in strength due to difference in material of the shafting seems to cause more variation in ultimate strength than is caused by different keyways. As previously pointed out, the elastic limit of a shaft is more significant than its ultimate strength.

TABLE 2.

ULTIMATE STRENGTH OF SHAFTS WITH KEYWAYS AND WITHOUT KEYWAYS.

Values are the average results of two tests.

Diameter of Shaft inches	Keyway		Maximum Twisting Moment in.—lb.	Maximum Computed Fiber Stress (solid shaft) lb. per sq. in.	H. P. 100 r.p.m.	Remarks
	Width inches	Depth inches				
1¼ in. cold-rolled	0	0	27 400	70 550	43.4	Shaft without keyway
	¼	¼	27 600	71 000	43.8	
	⅜	¼	30 300	78 000	48.0	
	¼	⅜	24 400	63 000	38.7	
	¼	⅛	27 600	71 000	43.8	
	⅜	⅛	27 200	70 000	43.1	
	No. 10	No. 10	25 500	65 800	40.4	Keyway for No. 10 Woodruff
	No. 15	No. 15	26 200	67 600	41.6	Keyway for No. 15 Woodruff
	¼	¼	25 300	65 200	40.1	2 keyways 90° apart
1¼ in. turned	0	0	25 300	65 200	40.1	Shaft without keyway
	¼	¼	25 800	66 400	40.9	
	¼	⅜	25 500	65 700	40.4	
	No. 10	No. 10	23 700	61 000	37.6	Keyway for No. 10 Woodruff
	No. 15	No. 15	24 100	62 100	38.2	Keyway for No. 15 Woodruff
1⅝ in. cold-rolled	0	0	54 700	65 000	86.8	Shaft without keyway
	11/32	11/64	56 400	67 000	89.6	
2 in. cold-rolled	0	0	103 700	66 000	164.5	Shaft without keyway
	7/16	7/32	102 100	65 000	162.0	
	7/16	7/16	101 500	64 600	161.1	
	7/16	9/16	94 200	60 000	149.1	
	9/16	7/32	104 500	66 500	165.7	
	No. 16	No. 16	105 300	67 000	167.0	Keyway for No. 16 Woodruff
	No. 21	No. 21	100 500	64 000	158.7	Keyway for No. 21 Woodruff
2 in. turned	0	0	100 500	64 000	158.7	Shaft without keyway
	7/16	7/32	94 200	60 000	149.1	
	9/16	7/32	94 200	60 000	149.1	
	No. 16	No. 16	89 800	57 200	142.5	Keyway for No. 16 Woodruff
	No. 21	No. 21	85 000	54 100	134.8	Keyway for No. 21 Woodruff

8. *Effect of Length of Keyway.*—Several special tests were made on the effect of keyways on the strength of shafts. In general, these tests, while too few in number to justify final conclusions, gave suggestive or tentative results.

The keyways in nearly all the shafts tested were cut to a total length of about four times the diameter of the shaft, no keyway being longer than 8 inches; but in several special shafts, keyways were cut 18 inches long. No difference between *strength* of shafts with long keyways and of similar shafts with the usual shorter keyways was observed.

9. *Effect of Two Keyways 90° Apart.*—One test was made of a shaft having cut in it two keyways 90° apart, the two keyways

being located in the same cross-section of the shaft. While the result of this single test is by no means conclusive, it is of interest to note that the reduction in strength at elastic limit of the shaft by these two keyways was nearly three times as great as the reduction in strength at elastic limit of a similar shaft by one such keyway.

10. *Effect of Keyways on Turned Shafting.*—The tests made were mainly on cold-rolled shafting, but in the 1908 series a few tests were made on test specimens of turned shafting. Owing to the imperfect method used in the 1908 tests for locating the elastic limit, these results must be regarded as tentative. In these tests the effect of keyways on the strength of turned shafting at the elastic limit seemed to be about the same as the effect of keyways on the strength of cold-rolled shafting.

11. *Strengthening Effect of Key in Place.*—During the tests, the question arose as to the difference in strength of a shaft with empty keyway and a shaft on which a pulley was keyed in place, the key nearly filling the keyway. It was judged best, however, to test shafts with empty keyways, as there is usually a part of the keyway at either end not filled by the key, and a perfect fit of the key in the keyway is by no means certain, especially after long service and, therefore, for purposes of design the empty keyway determines the strength of the shaft.

TABLE 3.

RATIO OF ANGLE OF TWIST OF SHAFT WITH KEYWAY TO ANGLE OF TWIST OF SIMILAR SHAFT WITHOUT KEYWAY.

Diameter of Shaft inches	Dimensions of Keyway			
	$w = 0.25$ $h = 0.125$	$w = 0.25$ $h = 0.1875$	$w = 0.50$ $h = 0.125$	Woodruff System*
1¼	1.24	1.25	1.27	1.11
1$\frac{9}{16}$	1.14 1.18	1.24 1.21	1.19 1.36	1.11 1.18
1$\frac{15}{16}$	1.16 1.29	1.21 1.48	1.41 1.54	1.11 1.12
2¼	1.10 1.10	1.25 1.28	1.18 1.37	1.05 1.10
Average	1.17	1.27	1.33	1.11

*See Table 4 for sizes of Woodruff Keyways.

12. *Effect of Keyway on Stiffness of Shaft.*—The amount of twist in a shaft transmitting power is frequently of importance. Table 3 gives the ratio of angle of twist of shafts with keyways to angle of twist of shafts without keyways as computed from the data of the torsional tests for stresses within the elastic limit. The results are fairly well represented by the equation

$$k = 1.0 + 0.4\,w + 0.7\,h,$$

in which k = ratio of angle of twist of shaft with keyway to angle of twist of similar shaft without keyway, w = width of keyway ÷ diameter of shaft, and h = depth of keyway ÷ diameter of shaft.

Keyways for two Woodruff keys of shearing strength sufficient to develop the full twisting strength of shaft seemed to reduce the stiffness of the shaft somewhat less than did a keyway for a square key whose side measures one-fourth the diameter of the shaft.

In considering the torsional stiffness of a shaft, it must be remembered that the keyways reduce the stiffness only over that portion of length which they actually occupy.

13. *Efficiency of Shafts with Keyways.*—The efficiency of a shaft with keyway has already been defined as the ratio of strength at elastic limit of a shaft with keyway to the strength at elastic limit of a similar shaft without keyway.

The determination of the elastic limit of a shaft under torsion is somewhat difficult; when the outer fibers are stressed to the elastic limit, the stress is taken more largely by the inner fibers, and the change of angle of twist is not so sudden as is the change of stretch at the elastic limit in a piece under tension, where the fibers are stressed nearly uniformly, and all begin to yield at nearly the same time. In the 1908 tests, each test shaft carried only a single keyway, and comparison between the strength of shafts with keyways and the strength of similar shafts without keyways was made by testing different specimens. This allowed a comparison of the ultimate strengths, which are very clearly defined; but in comparing elastic limits, the variation between the material of different specimens was sufficiently great to throw some doubt on the accuracy of the efficiency of shafts with keyways, as determined by this method. In the 1909 tests, the elastic limit of a section of shaft with keyway was compared with that of an adjacent section without keyway in the same shaft. Thus the error due to difference in material was greatly reduced, but by this method the ultimate strength of only the weakest section of the shaft could be obtained. So while all the results on ultimate strength have been obtained from the 1908 tests, the efficiencies of the shafts with keyways have been obtained entirely from the 1909 tests.

In computing results, J. B. Johnson's method of locating the elastic limit was found most satisfactory*. Fig. 5 to 12 give the deformation and set curves for the 1909 series of tests. The solid lines show the deformation (angle of twist) under load, while the broken lines show the set. The elastic limit as determined by Johnson's method is shown on each curve by a short line drawn across the deformation curve, and it will be noted that the stress at which noticeable permanent set begins is in all cases nearly the same as the stress at the elastic limit as determined by Johnson's method.

Table 4 shows the efficiency of the various test shafts of the 1909 series of tests, using the term efficiency as previously defined. From this table it would appear that for a set of shafts of different sizes having the dimensions of the keyway kept proportional to the diameter of shaft, the efficiency does not depend, in any noticeable degree, on the size of shaft. The efficiency does not seem to be affected by the addition of a bending moment as great as the twisting moment. The efficiency of a shaft with two keyways cut in the same plane for two Woodruff keys, of such size that the strength of solid shaft was equal to the shearing strength of the two Woodruff keys, is about the same as the efficiency of a shaft with a keyway whose width equals one-fourth the diameter of the shaft and whose depth equals one-eighth the diameter of the shaft.

The results of the foregoing tests are fairly well represented by the equation

$$e = 1.0 - 0.2\,w - 1.1\,h$$

in which

e = efficiency of shaft with keyway,
w = width of keyway ÷ diameter of shaft,
h = depth of keyway ÷ diameter of shaft.

*J. B. Johnson's method of locating the elastic limit consists in finding the point on the stress-deformation curve at which the deformation is increasing fifty per cent more rapidly than its initial rate of increase. See Johnson's "Materials of Construction", pp. 18-20.

TABLE 4.

Efficiency of Shafts with Keyways.

$$\left\{ \text{Efficiency} = \frac{\text{elastic strength of shaft with keyway}}{\text{elastic strength of shaft without keyway}} \right\}$$

Dimensions of Keyway	$w = 0.50$ $h = 0.125$	$w = 0.25$ $h = 0.1875$	$w = 0.25$ $h = 0.125$	Woodruff System*
Under simple torsion:				
Cold-rolled shaft, dia. 1¼ in.	0.762	0.760	0.820	0.840
Cold-rolled shaft, dia. 1 9/16 in.	0.803	0.846	0.900	0.860
	0.758	0.817	0.889	0.815
Cold-rolled shaft, dia. 1 15/16 in.	0.748	0.710	0.860	0.826
	0.764	0.750	0.824	0.835
Cold-rolled shaft, dia. 2¼ in.	0.848	0.775	0.839	0.943
	0.705	0.689	0.825	0.861
Under combined torsion and bending:				
1. Twisting moment = Bending moment Cold-rolled shaft, dia. 1¼ in.	0.630	0.636	0.791	0.716
	0.680	0.698	0.803	0.750
Cold-rolled shaft, dia. 1 15/16 in.	0.584	0.697	0.854	0.858
	0.671	0.775	.....	0.840
2. Twisting moment = 5/3 Bending moment Cold-rolled shaft, dia. 1¼ in.	0.895	0.670	0.940	0.930
	0.870	0.735	0.888	0.880
Cold-rolled shaft, dia. 1 15/16 in.	0.740	.....	0.832	0.856
	0.815	.....	0.840	0.810
General Average	0.752	0.735	0.850	0.845

*In 1¼-in. shafts keyways were cut for No. 15 Woodruff keys
1 9/16-in. shafts keyways were cut for No. 25 Woodruff keys
1 15/16-in. shafts keyways were cut for No. S Woodruff keys
2¼-in. shafts keyways were cut for No. U Woodruff keys

This equation gives efficiencies slightly lower than those observed for keyways of small width or depth, and efficiencies about the same as those observed for keyways in which $w = 0.50$ and $h = 0.125$; or $w = 0.25$ and $h = 0.1875$. As this equation is entirely dependent on the results of experiments, it should not be used for points much outside the limits of the experiments. The limits of the above series of tests were keyways having $w = 0.50$ and $h = 0.1875$.

Fig. 13 affords a convenient graphical method of applying the above formula, and is used as follows: To determine the efficiency of a shaft with a given (or proposed) keyway, locate on the diagram a point whose vertical distance from 0 equals the value of h, and whose horizontal distance from 0 equals the value of w. This point will, in general, fall between two lines representing values of efficiency, and the efficiency of the shaft in question may then be estimated with sufficient accuracy. The space within the triangle OAB represents the range covered by the

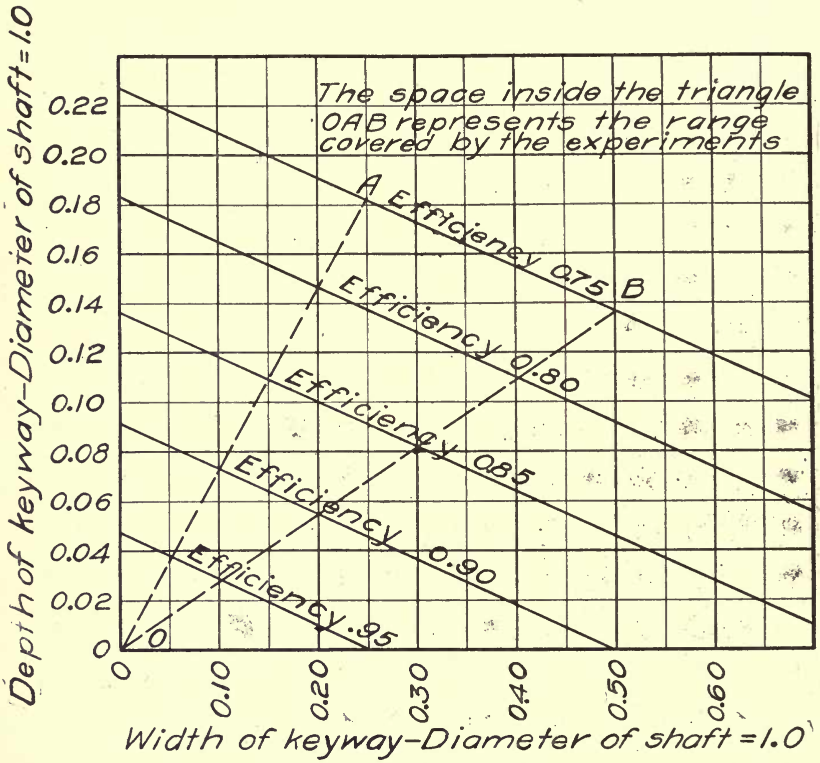

FIG. 13.

tests actually performed, and covers the proportions of keyways commonly used in practice.

14. *Torsional Strength of Shafts with Keyways.*—The object of these tests was to determine ratios of strength and stiffness between shafts with keyways and shafts without keyways. The number of tests was not sufficient to give very much information as to the properties of cold-rolled steel shafting. However, as a matter of general interest, the values found in these tests for the modulus of elasticity in shear (torsion), and of the fiber stress at the elastic limit of the cold-rolled test shafts at sections without keyway, have been tabulated in Tables 5 and 6.

Taking the fiber stress at the elastic limit of cold-rolled steel shafting at 37 500 lb. per sq. in. (a value slightly less than the average found in the tests), and the efficiency of shafts with keyways from the equation $e = 1.0 - 0.2\,w - 1.1\,h$, values for the

TABLE 5.

MODULUS OF ELASTICITY IN SHEAR (TORSION) OF COLD-ROLLED STEEL SHAFTING.

Test No.	Diameter of Shaft inches	Modulus of Elasticity
46	$1\frac{1}{4}$	12 900 000
47	$1\frac{9}{16}$	12 000 000
48	$1\frac{9}{16}$	12 490 000
49	$1\frac{15}{16}$	10 800 000
50	$1\frac{15}{16}$	12 660 000
51	$2\frac{1}{4}$	11 340 000
52	$2\frac{1}{4}$	11 710 000
	Average	11 985 000

TABLE 6.

ELASTIC LIMIT IN TORSION OF COLD-ROLLED STEEL SHAFTING.

Test No.	Diameter of Shaft inches	Fiber Stress lb. per sq. in.
46	$1\frac{1}{4}$	43 300
47	$1\frac{9}{16}$	36 800
48	$1\frac{9}{16}$	38 500
49	$1\frac{15}{16}$	36 800
50	$1\frac{15}{16}$	40 500
51	$2\frac{1}{4}$	36 200
52	$2\frac{1}{4}$	40 500
	Average	38 940

twisting moments and the horse-power at 100 r.p.m., transmitted by cold-rolled shafts stress to the elastic limit, have been computed for various sizes and tabulated in Table 7. These values are for shafts with keyways for square keys whose side measures about one-fourth the diameter of the shaft. In the use of this table, a suitable factor of safety should be allowed.

TABLE 7.

Strength of Shafts with Keyway.

The table gives the calculated twisting moment and horse-power at 100 r. p. m., transmitted in torsion by cold-rolled shafting with keyway when stressed to the elastic limit. Fiber stress is assumed at 37 500 lb. per sq. in. The keyway is cut for a square key whose side measures approximately one-fourth the diameter of the shaft. No allowance is made for bending action. In applying this table, a suitable factor of safety should be used.

Diameter of Shaft inches	Side of Key inches	Twisting Moment in.-lb.	Horse-power at 100 r. p. m.
1	1/4	5 980	9.5
1 1/16	9/32	7 080	11.2
1 1/8	9/32	8 510	13.5
1 3/16	5/16	9 980	15.7
1 1/4	5/16	11 680	18.5
1 5/16	11/32	13 390	21.3
1 3/8	11/32	15 550	24.7
1 7/16	3/8	17 590	27.9
1 1/2	3/8	20 190	32.0
1 9/16	13/32	22 600	35.9
1 5/8	13/32	25 660	40.7
1 11/16	7/16	28 500	45.2
1 3/4	7/16	32 060	50.9
1 13/16	15/32	35 350	56.1
1 7/8	15/32	39 420	62.6
1 15/16	1/2	43 180	68.5
2	1/2	47 860	75.9
2 1/16	17/32	52 140	82.7
2 1/8	17/32	57 900	91.1
2 3/16	9/16	62 210	98.7
2 1/4	9/16	68 160	108.2
2 5/16	19/32	73 490	116.6
2 3/8	19/32	80 120	127.1
2 7/16	5/8	86 080	136.6
2 1/2	5/8	93 470	148.3

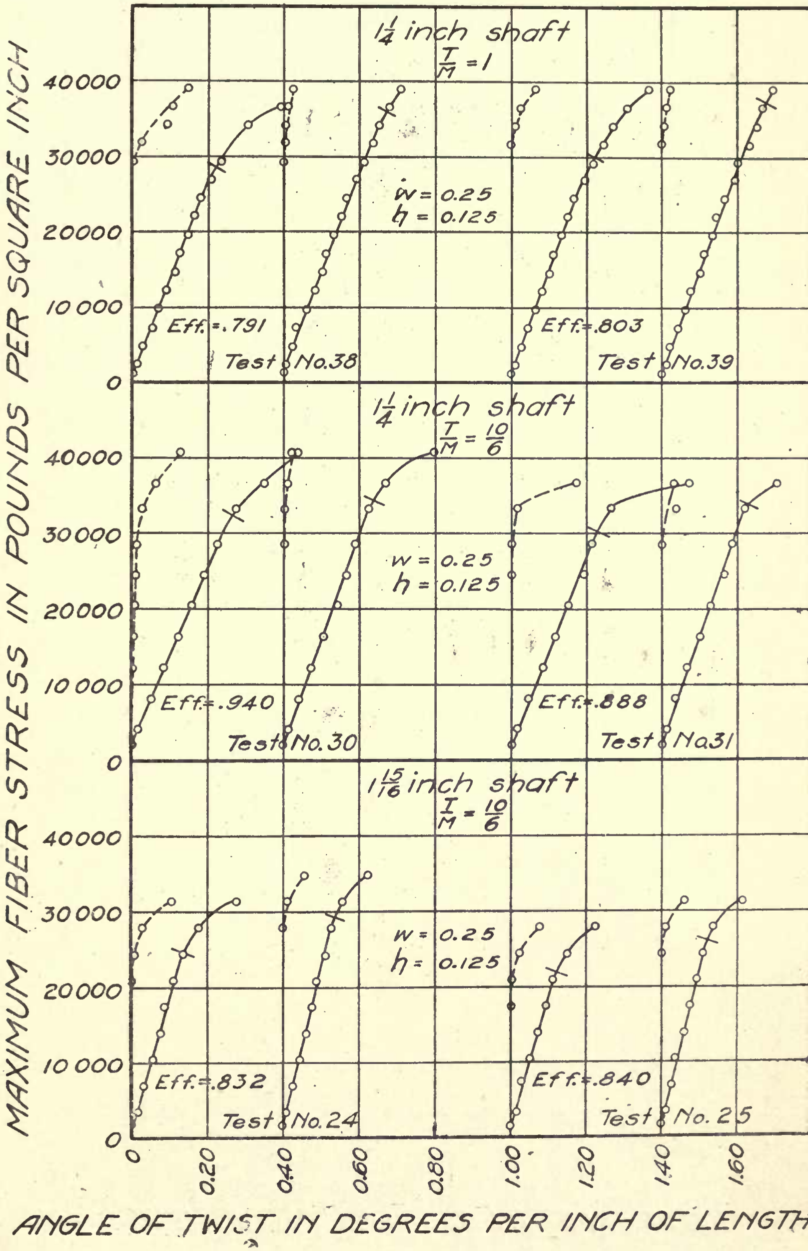

FIG. 5. DIAGRAMS OF TESTS UNDER COMBINED BENDING AND TWISTING.

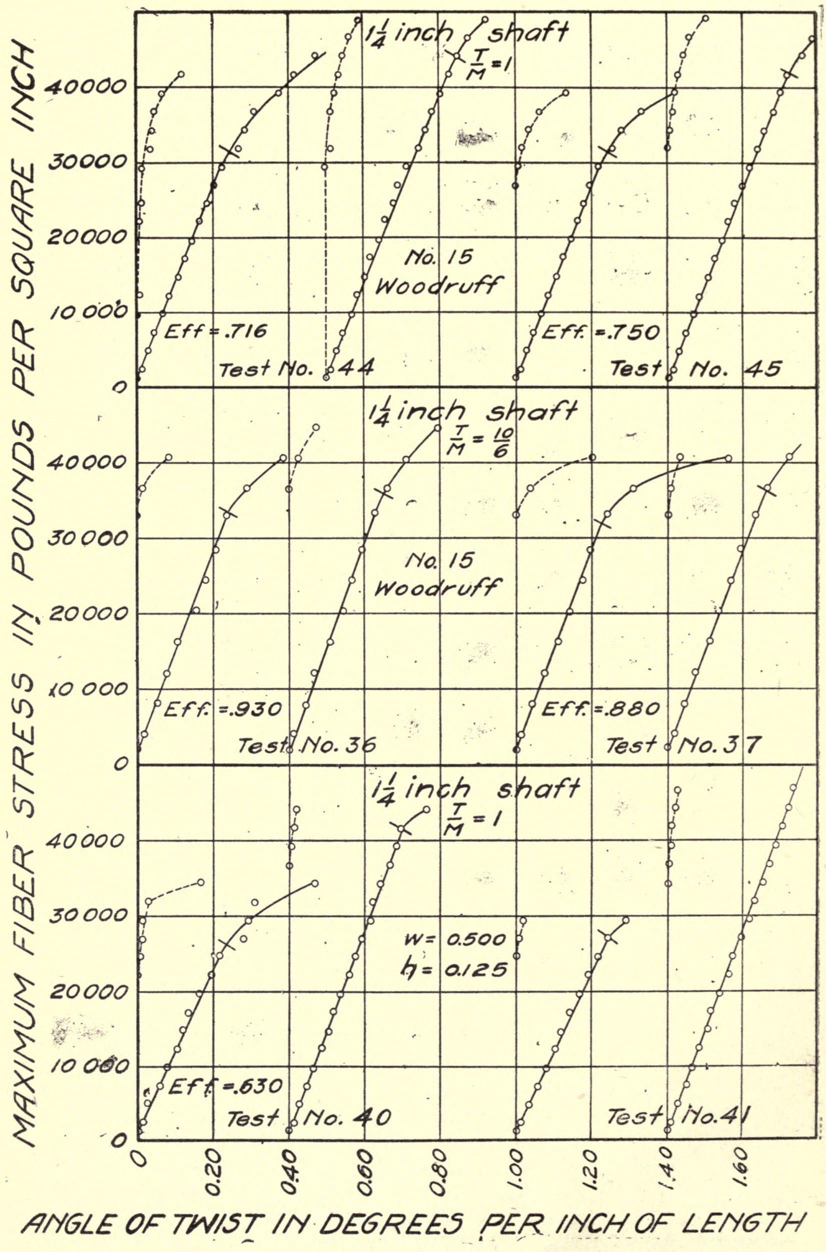

FIG. 6. DIAGRAMS OF TESTS UNDER COMBINED BENDING AND TWISTING.

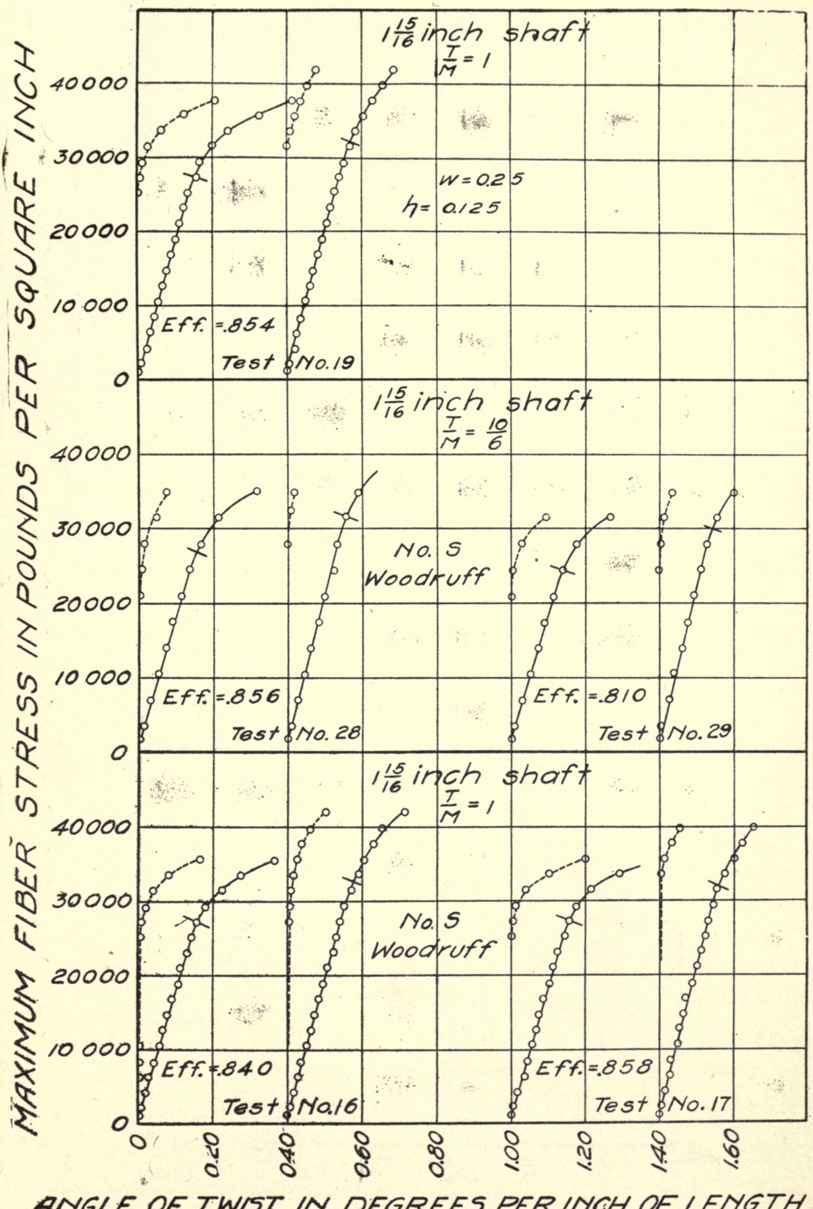

FIG. 7. DIAGRAMS OF TESTS UNDER COMBINED BENDING AND TWISTING.

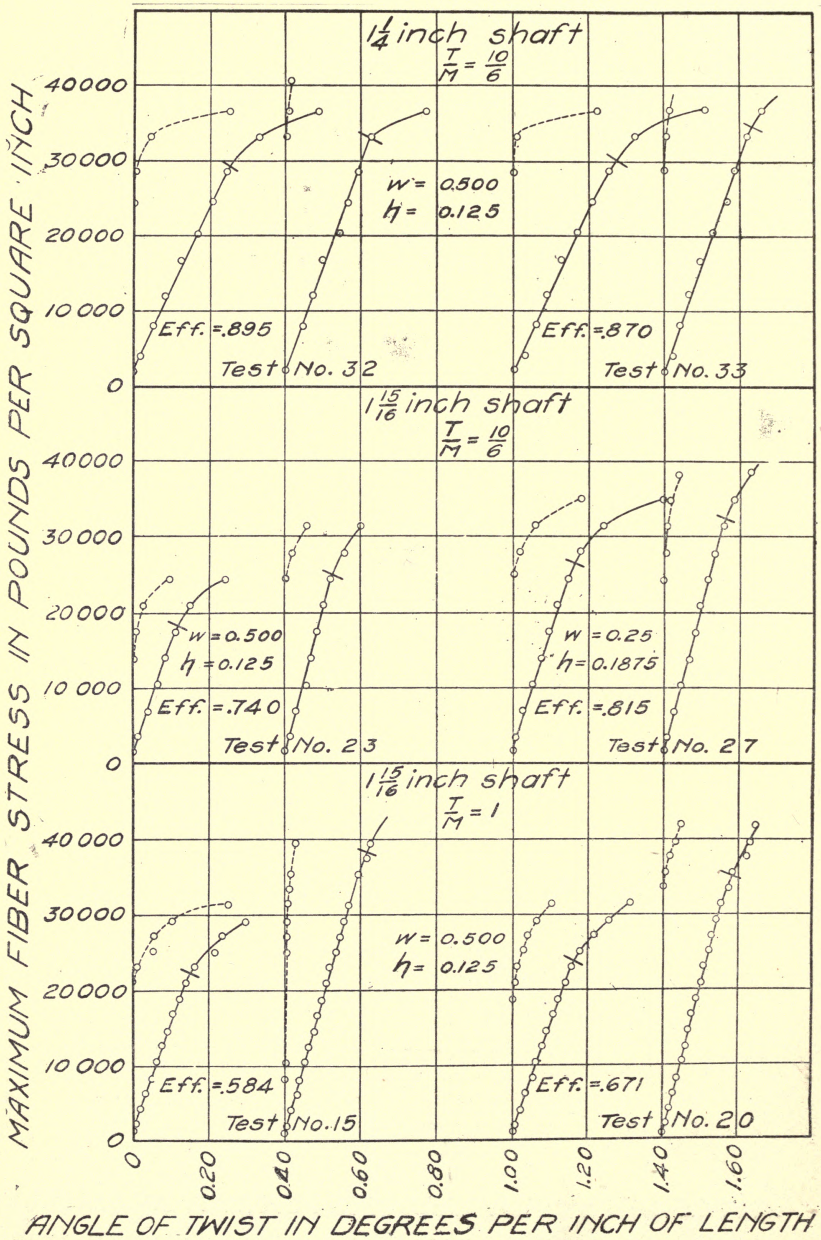

FIG. 8 DIAGRAMS OF TESTS UNDER COMBINED BENDING AND TWISTING

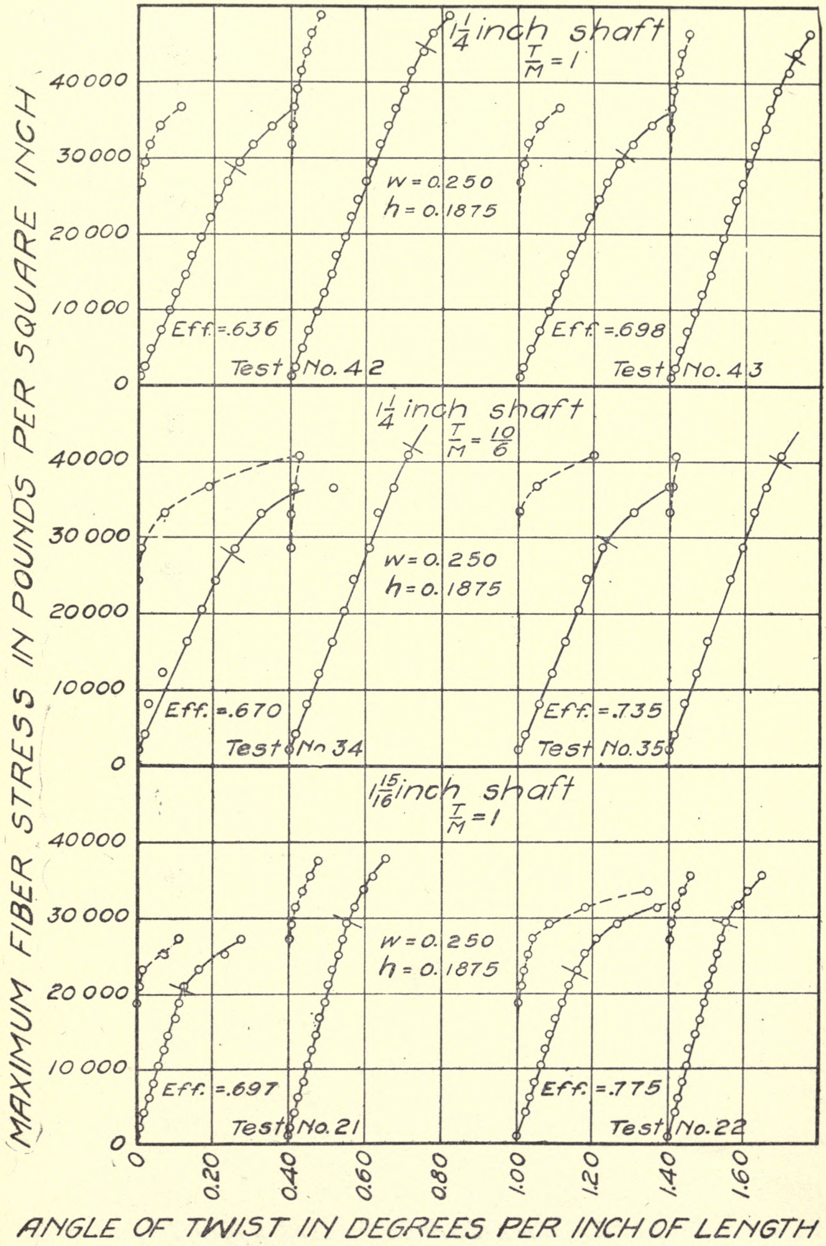

FIG. 9 DIAGRAMS OF TESTS UNDER COMBINED BENDING AND TWISTING.

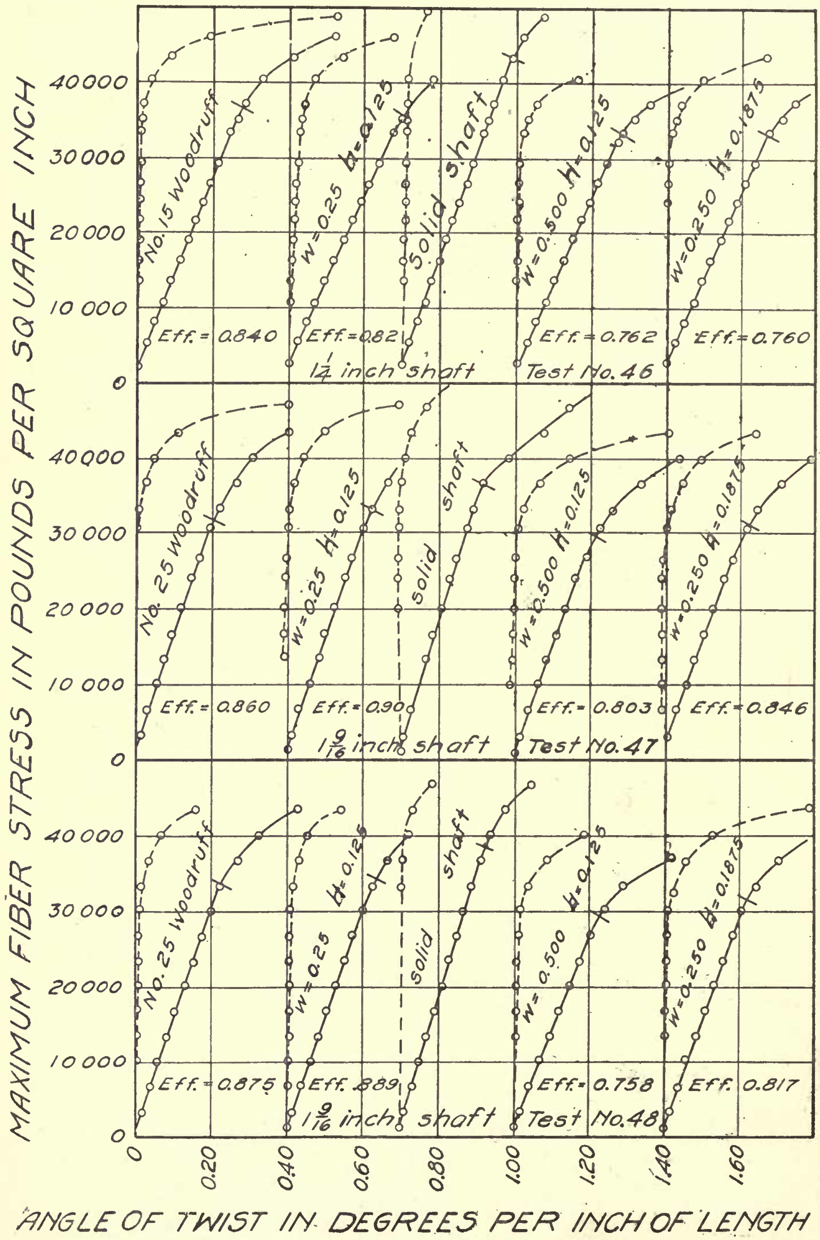

FIG. 10 DIAGRAMS OF TESTS UNDER TWISTING ONLY.

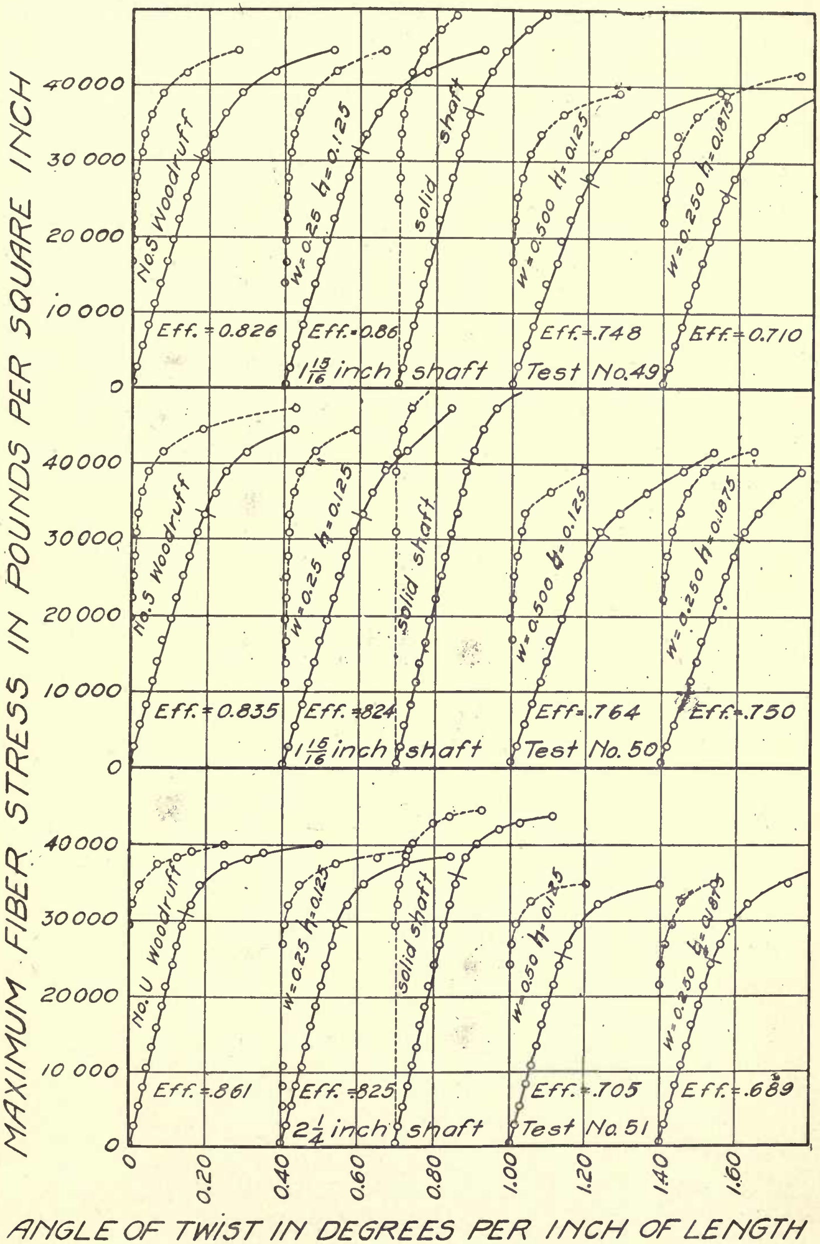

FIG. 11 DIAGRAMS OF TESTS UNDER TWISTING ONLY.

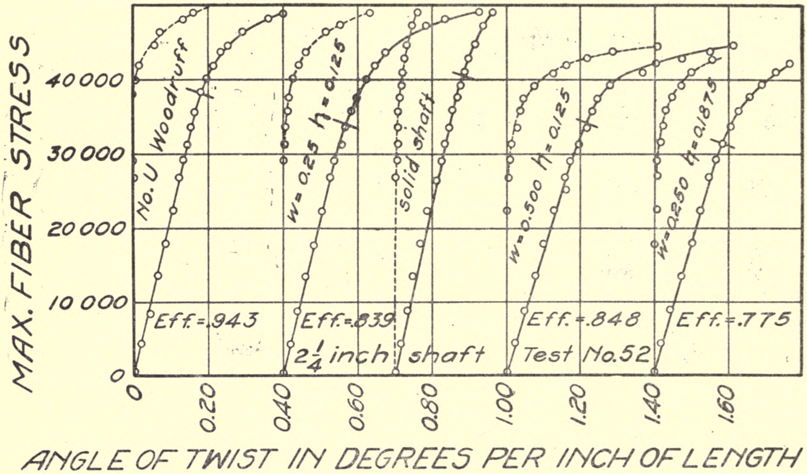

FIG. 12 DIAGRAMS OF TESTS UNDER TWISTING ONLY.

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