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ROBERT TURNER HUDSPETH

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THE EFFECT OF A SINGLE RESONANT EXPANSION CHAMBER ON THE PROPAGATION OF LONG WAVES IN A CHANNEL

by

ROBERT TURNER HUDSPETH

A thesis submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE

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PREFACE

The study of the effect of resonant expansion systems on the propagation of long waves is presented in this thesis as a design problem in coastal hydraulics. Two previous studies which have been made on similar systems are presented in Chapter I to provide background material on the subject. The first study presented was made by Lamb (1916); and it demonstrates that reflection of long waves can be obtained by an expansion system and that the amount of wave energy that is reflected is a function of the geometry of the system. The second study presented was made by Valembois (1953) and is based on a hydrodynamic impedance theory. An important point to note in Chapter I is that both of these studies use a scalar equation of pressure continuity and a vector equation for the conservation of mass to obtain a solution. A new solution is derived in Chapter 2 by formulating a boundary value problem which incorporates the same type of boundary condition equations noted earlier. The problem is formulated in considerable detail to demonstrate the type of problem solution technique which is required to solve problems arising in coastal hydraulics. The details given in the problem statement become important later when the effect of geometrical changes of the resonant system are evaluated. Chapter 3 discusses the interdependance required between Civil

Engineering and Oceanography to effect a complete solution to a design problem in coastal hydraulics. Several methods for presenting wave data are given, and one of these methods is selected for the design problem being considered. Finally, a linear analysis is employed in Chapter 4 to evaluate the effect of the resonator on the propagation of long waves by combining the solution derived in Chapter 2 with the spectral method of wave data presentation selected in Chapter 3. The results of the linear analysis are presented in graphical form as a measure of the amount of wave energy that is reflected and transmitted by the resonator.

ABSTRACT

The response of a single resonant expansion chamber to the periodic pressure fluctuations of ocean waves is to act as a rigid vertical reflecting surface. The effective frequency band-width over which reflection is significant may be determined for a given harbor geometry by means of a graphical linear analysis incorporating the reflection or transmission coefficient and wave data presented by a Bretschneider Power Sprectrum. The most critical dimension of the resonator was determined to be the length of the expansion, ℓ_R, measured transverse to the centerline of the channel; the effect of the resonator was maximized when this dimension was either one-quarter or three-quarters of the design wavelength, L_*. The influence of the width of the resonator, b_R, measured parallel to the centerline of the channel was determined to be maximal when this dimension was one-half of the design wavelength, L_*. The effectiveness of the resonator is also a function of the width of the main channel, b_M, and this parameter was incorporated in the expressions for the coefficients.

Because of its analogy to electrical band-stop filters, the possibility for improving the effect of the resonator by constructing several resonators in cascade appears good and offers an opportunity for extending the results obtained here.

1. INTRODUCTION

As the logistical requirements of nations grow and expand, new techniques for designing harbors will be required to accomodate the expansion of littoral ports of communications. The port areas which are sheltered naturally from the ocean forces and which require little or no additional protective structures have already been exploited. Designers, therefore, must address themselves to the problems of the less protected areas where this expansion must occur. One of the major concerns in an unsheltered area is the design of breakwaters.

The purpose of breakwaters is to cause a reduction of wave heights in its lee (Wiegel, 1964). The conventional methods used for sheltering the entrance of a harbor between breakwaters may be divided into three general types (Figure 1.1). All three methods shown require shipping vessels to maneuver in order to enter or to leave the harbor. This required maneuvering is both expensive and inconvenient to the user. In addition, the third method shown requires a deep shipping channel close to the shore, and may require the additional expense of periodic dredging. The alternatives which have been considered to provide a direct access to the shipping channels from the harbor have included pneumatic and hydraulic devices as well as resonant expansion chambers (Wiegel, 1964). The development of the latter is considered here.

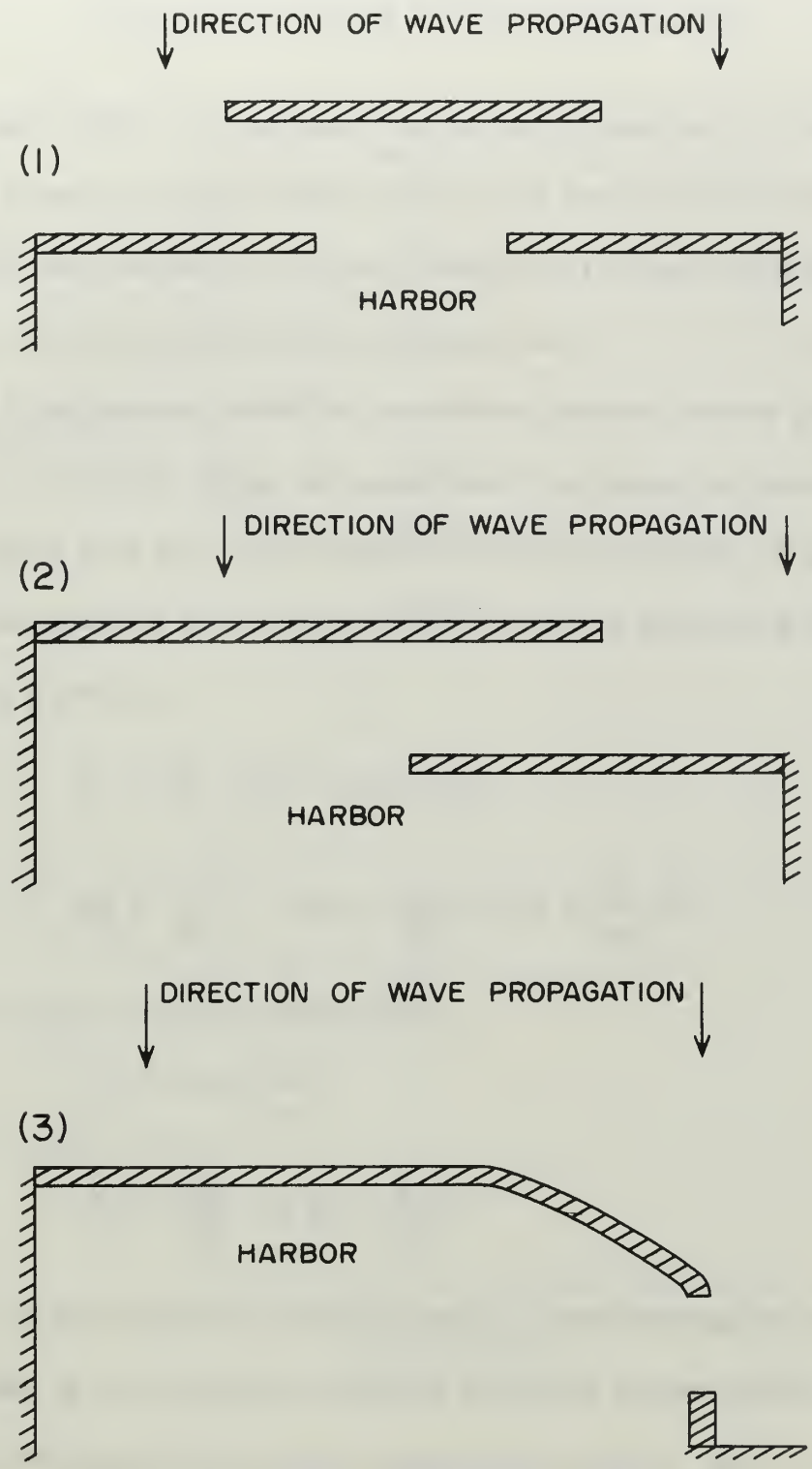


Figure 1.1. Three conventional methods used to protect breakwater entrances.

Wave Reflection by an Abrupt Expansion

Lamb (1916) investigated the partial reflection of a wave resulting from an abrupt change in the cross section of a channel. His results were based on a linear solution to the wave equation. (A linear solution is presented in Chapter 2).

A right-handed cartesian coordinate system (z-axis positive upwards) is oriented along the centerline of a channel at the mean-water surface with the origin located at the discontinuity (Figure 1.2). The wave amplitudes and relative velocities to the left of the discontinuity are given by

$$\eta_1 = F(t - \frac{x}{c_1}) + f(t + \frac{x}{c_1}) \tag{1.1}$$

$$\vec{u_1} = \frac{g}{c_1} \cdot [F(t - \frac{x}{c_1}) - f(t + \frac{x}{c_1})] \tag{1.2}$$

and to the right of the discontinuity by

$$\eta_2 = \phi(t - \frac{x}{c_2}) \tag{1.3}$$

$$\vec{u_2} = \frac{g}{c_2} \cdot \phi(t - \frac{x}{c_2}) ; \tag{1.4}$$

where F is an arbitrary periodic function representing the incident wave; f and ϕ are arbitrary periodic functions representing the reflected and transmitted wave, respectively; and c_1 and c_2 are the wave celerities of the wave systems to the left and to the right of the discontinuity, respectively. The linear superposition theory is

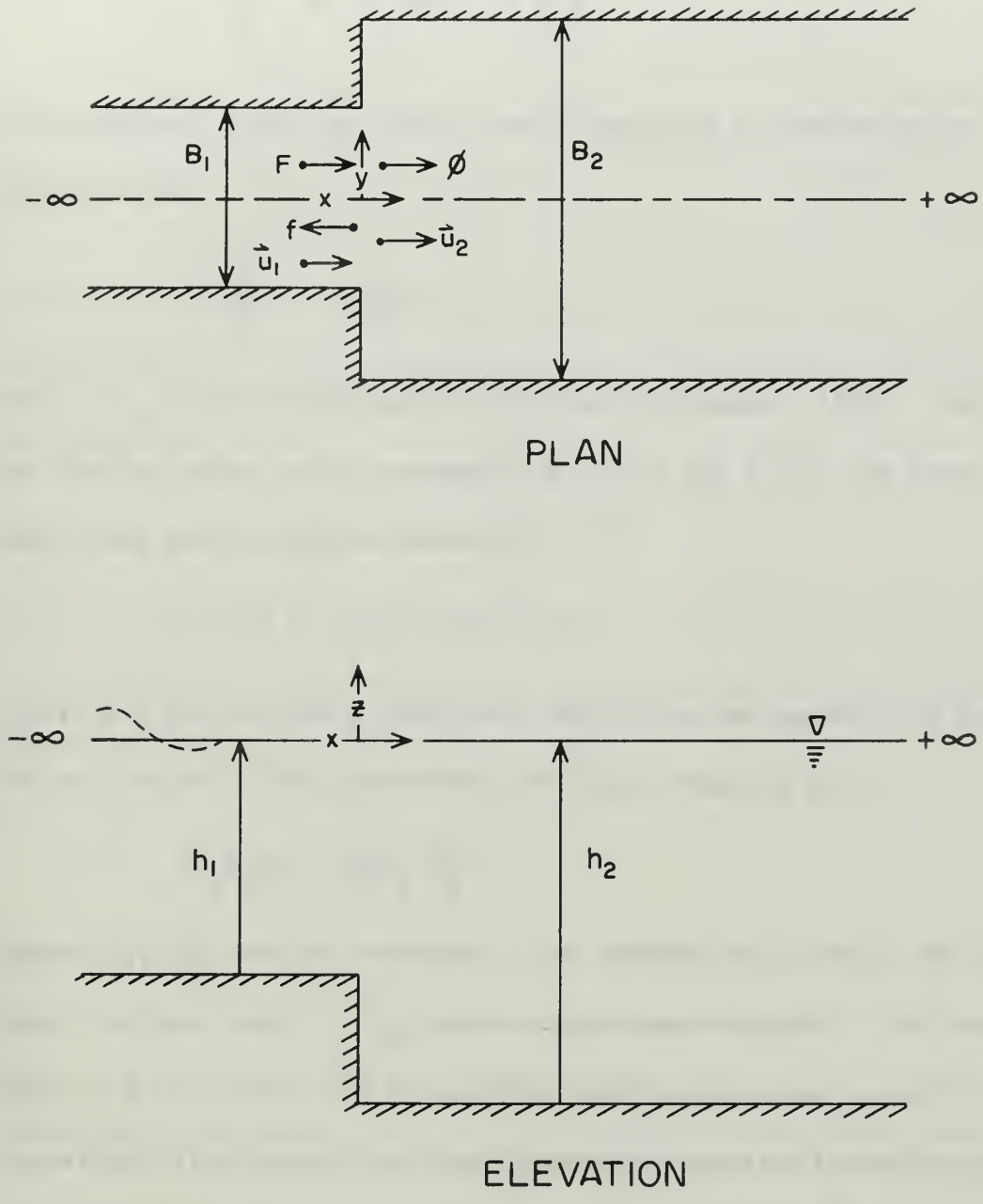


Figure 1.2. Partial wave reflection by an abrupt expansion in a channel.

valid provided that the small amplitude assumption holds; i. e.,

$$\frac{\eta_i}{h_i} \ll 1 \quad \text{for } i = 1, 2. \tag{1.5}$$

The assumption that the waves are propagating in shallow water requires that

$$\frac{h_i}{L_d} \leq \frac{1}{20}\ ; \tag{1.6}$$

where L_d is the wavelength in deepwater (Kinsman, 1965). By invoking the shallow water wave assumption given in Eq. (1.6), the wave celerities may be approximated by

$$c_i^2 = g\, h_i \quad \text{for } i = 1, 2. \tag{1.7}$$

There are two boundary conditions which must be satisfied at the origin (x=y=0). The conservation of mass requires that

$$B_1 h_1 \vec{u_1} = B_2 h_2 \vec{u_2}\ ; \tag{1.8}$$

where B_1, B_2 are the breadths of the channel measured at the mean-water surface, and h_1, h_2 are the mean-water depths. The assumption that the fluid motion will be sensibly uniform along and parallel to the centerline of the channel for small distances (compared to a wavelength) on either side of the discontinuity requires that there be no sensible change in the mean-water surface across the discontinuity. This condition requires that the continuity of pressure across the abrupt change be

$$\eta_1 = \eta_2 . \tag{1.9}$$

Substituting Eqs. (1.2) and (1.4) into (1.8) gives

$$g \frac{B_1 h_1}{c_1} [F(t) - f(t)] = g \frac{B_2 h_2}{c_2} \cdot \phi(t) \quad \text{at } x = 0 ; \tag{1.10}$$

and the substitution of Eqs. (1.1) and (1.3) into (1.9) yields

$$F(t) + f(t) = \phi(t) \quad \text{at} \quad x = 0. \tag{1.11}$$

The ratios of the reflected wave and of the transmitted wave to the incident wave determine the reflection and transmission coefficients, respectively; i.e.,

$$K_R = \frac{f}{F} \quad \text{and} \quad K_T = \frac{\phi}{F} .$$

These coefficients are determined by the simultaneous solution of Eqs. (1.10) and (1.11). By means of Eq. (1.7), the reflection and transmission coefficients, respectively, are

$$K_R = \frac{B_1 c_1 - B_2 c_2}{B_1 c_1 + B_2 c_2} \tag{1.12}$$

and

$$K_T = \frac{2 B_1 c_1}{B_1 c_1 + B_2 c_2} . \tag{1.13}$$

The energy contained in the reflected and in the transmitted waves is equal to the energy of the incident wave if the sum of the squares of the two coefficients is equal to unity; i.e.,

$$K_R^2 + K_T^2 = 1. \tag{1.14}$$

Equations (1.12) and (1.13) establish that the effectiveness of the

abrupt expansion for reflecting waves is dependent on the geometry of the channel.

A similar procedure to the above approximation given by Lamb (1916) could be employed to evaluate the reflection and transmission resulting from an abrupt contraction of a channel and the two solutions superimposed in a limiting case. However, the basic assumption that uniform conditions exist parallel to the centerline of the channel would now introduce a greater error of approximation, as the dimension of the length of the expansion measured parallel to the centerline of the channel approached small multiples of the incident wavelength. These results obtained by Lamb (1916) indicate that substantial reflection occurs from an abrupt expansion in a channel and that a further investigation of an expansion system is justified.

The Resonant Expansion Chamber

The resonant expansion chamber (resonator) takes advantage of the periodic character of the surface disturbances and of the pressure fluctuations produced by a wave (Valembois, 1953). The solution derived by Valembois includes a wave system transverse to the centerline of the channel which is induced in the resonator; this wave system was neglected by Lamb in the preceding example. The derivation again assumes a linear solution to the wave equation but provides an additional parameter for evaluating any energy dissipation which may result. The

measure of energy dissipation utilizes the Neumann impedance theory for a hydrodynamic oscillatory system; this theory is analogous to the impedance theory for electrical oscillatory systems (Defant, 1961).

The hydraulic impedance of the discontinuity (Figure 1.3) is in accordance with Ohm's law for the analogous electrical oscillatory system. The effect of the resonator is to alternately withdraw from and discharge into the main channel a quantity of fluid which is proportional to the surface distortion associated with the incident wave system. The constant of proportionality is a measure of the hydraulic impedance of the resonator; i.e.,

$$Z = \frac{H_A}{v} \tag{1.15}$$

A right-handed cartesian coordinate system (z-axis positive upwards) is oriented at the orthogonal intersection of the centerlines of the main channel and of the resonator at the mean-water depth. The linear shallow water approximation for the wave celerity may be written as in the preceding example as

$$c = \sqrt{gh} \; ; \tag{1.16}$$

where h is the mean-water depth of the channel. The relative velocity $U(x, t)$ and superelevation $H(x, t)$ in the section to the left of the discontinuity at A are given by the following relationships:

$$\vec{U}(x, t) = F\left(t - \frac{x}{c}\right) - f\left(t + \frac{x}{c}\right) \tag{1.17}$$

$$H(x, t) \cdot \frac{g}{c} = F\left(t - \frac{x}{c}\right) + f\left(t + \frac{x}{c}\right) , \tag{1.18}$$

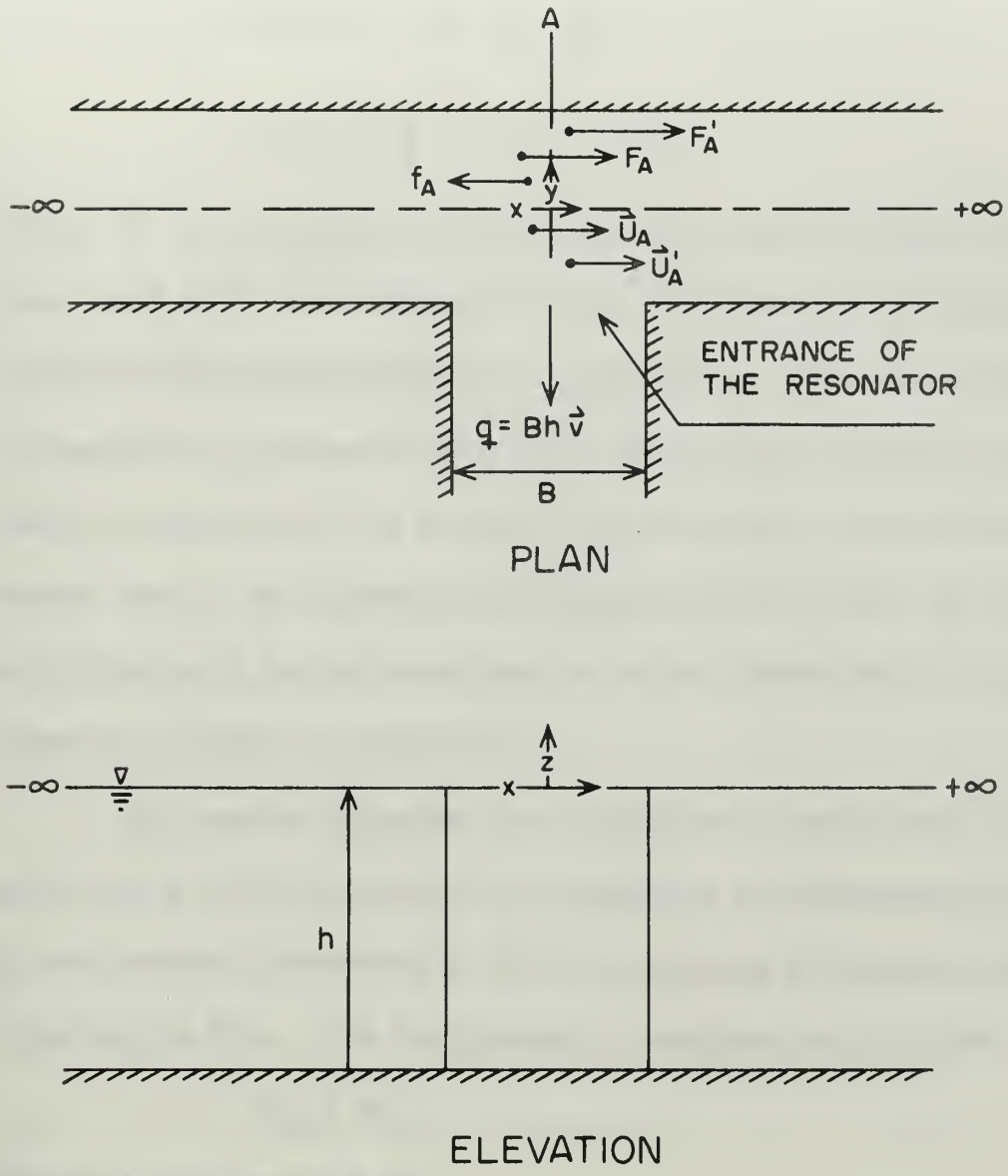


Figure 1.3. Action of a resonator on the propagation of a gravity wave. (after Valembois, 1953)

and to the right of A by the following:

$$\vec{U'}(x, t) = F'\left(t - \frac{x}{c}\right) \tag{1.19}$$

$$H'(x, t) \cdot \frac{g}{c} = F'\left(t - \frac{x}{c}\right); \tag{1.20}$$

where F is an arbitrary periodic function representing the incident wave; and f, F′ are arbitrary periodic functions representing the reflected and transmitted waves, respectively. Since the medium of propagation is assumed to be at rest, all relative velocities will denote absolute velocities. The channel is also assumed to be infinite in extent, both in the upstream and downstream directions; therefore, no reflection of the reflected wave or of the transmitted wave by a terminal boundary is considered.

The desired reflection and transmission coefficients will be determined exactly as before by considering the simultaneous solution of two boundary conditions at A: the continuity of pressure and the continuity of flow. The first boundary condition requires that

$$H_A = H'_A, \tag{1.21}$$

and the latter requires that

$$\vec{U}_A = \vec{U}'_A + \vec{v}. \tag{1.22}$$

Substituting Eqs. (1.18) and (1.20) into (1.21) yields

$$F_A + f_A = F'_A = \frac{g}{c} \cdot H_A, \tag{1.23}$$

and substituting Eqs. (1.17) and (1.19) into (1.22) yields

$$F_A - f_A = F'_A + v \,. \tag{1.24}$$

Substituting Eq. (1.15) for v and designating the resulting ratio by

$$\alpha = \frac{g}{c} \cdot Z \,, \tag{1.25}$$

the simultaneous solutions of Eqs. (1.23) and (1.24) result in the following values for the reflection and transmission coefficients, respectively:

$$K_R = \left| \frac{-1}{2\alpha + 1} \right| \tag{1.26}$$

and

$$K_T = \frac{2\alpha}{2\alpha + 1} \,. \tag{1.27}$$

The sum of the squares of the two coefficients is not equal to unity as in the previous example by Lamb (1916), except when the value of α (and hence Z) is exactly equal to zero. The requirement that the value of Z given by Eq. (1.15) be equal to zero corresponds to no surface elevation in the resonator (e.g., the resonator discharges into a large reservoir) or to an infinitely high velocity of discharge. The hydraulic impedance may be written in a form analogous to the electrical impedance; i.e.,

$$Z = r + j \left(\ell \sigma - \frac{1}{\gamma \sigma} \right); \tag{1.28}$$

where r is proportional to the energy dissipative forces; and ℓ, γ are proportional to the inertial and potential (e.g., gravity) forces. Requiring Z to vanish is analogous to a tuned resonant circuit in which

$$\sigma_\ell = \frac{1}{\gamma \sigma} ; \qquad (1.29)$$

where σ is the angular frequency of oscillation. The value of Z will vanish at resonance only in an inviscid fluid (or in an analogous conservative system). Equations (1.26) and (1.27) may be used for evaluating the amount of energy dissipation and the corresponding amount of reflection of waves propagating in a real fluid only when a value for r is known or may be determined.

The difficulties encountered in the application of the results of Valembois (1953) lie in the evaluation of the parameter r and in the criteria for dimensioning a tuned resonator. The values of the hydraulic resistance and reactances are very small and are usually determined empirically. Secondly, an important omission in the theory is a rigorous analytical procedure for dimensioning the geometry of the resonator. The only reference given to a dimension was the empirically determined value for the length of the lateral expansion which was equal to one-quarter of a wavelength. Since surface gravity waves have no natural frequency of oscillation to which the natural frequency of the resonator may be matched, the electrical analogy cannot be employed as a means for dimensioning the resonator.

This thesis is addressed to the evaluation of a reflection and of a transmission coefficient which are independent of any empirical data required for measuring the small dissipative forces and which relate any geometric changes in the resonator to its effectiveness for

reflecting waves in terms of the characteristic parameters of the incident wavelength.

2. THE SOLUTION

Formulation of the Boundary Value Problem

The derivation of the desired coefficients will proceed in a manner similar to that of the examples presented in Chapter 1 by the formulation of a boundary-value problem. A right-handed cartesian coordinate system (z-axis positive upwards) is oriented at the orthogonal intersection of the centerlines of the channel and of the resonator at the mean-water level (Figure 2.1). The linear second-order homogeneous hyperbolic partial differential equation which approximately describes the instantaneous surface elevation of the fluid in the domain mutually occupied by the channel and by the resonator is the wave equation; i.e.,

$$\frac{\partial^2 \eta}{\partial t^2} = c^2 \frac{\partial^2 \eta}{\partial x_k^2 \, \partial x_k} \; ; \tag{2.1}$$

where k has a range of values from unity to three. The wave form propagates at a constant celerity of

$$c = \frac{\sigma}{k} \; . \tag{2.2}$$

Neglecting the non-linear effects of the boundary conditions requires that the following assumptions be made (McLellan, 1965):

(1) The amplitude of the surface disturbance is small compared both to the wavelength and to the depth of the fluid.

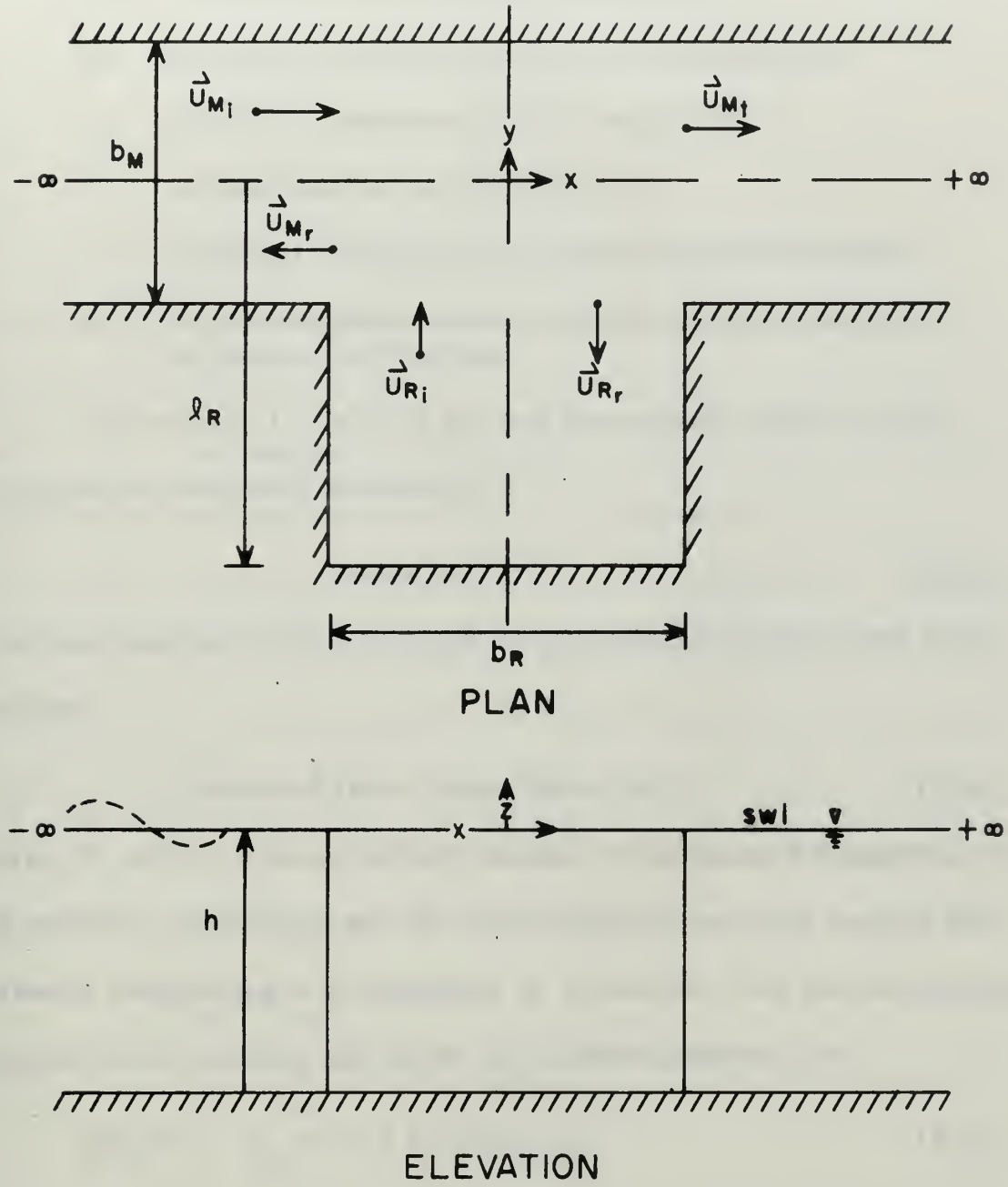


Figure 2.1. The wave systems considered in evaluating the effect of the resonator on long waves.

(2) The channel is of uniform depth.

(3) The fluid is inviscid and irrotational.

(4) The fluid is incompressible and homogeneous.

(5) Coriolis acceleration may be neglected.

(6) Surface tension may be neglected.

(7) The rigid boundaries are smooth and impermeable.

(8) The atmospheric pressure along the water surface is constant and uniform.

The solution to Eq.(2. 1) for one dimensional space may be expressed in functional notation by

$$\eta = \eta (x, t) . \tag{2. 3}$$

The most general solution is given by D'Alembert's Theory and is of the form

$$\eta (x, t) = F (kx - \sigma t) + G (kx + \sigma t); \tag{2. 4}$$

where F is an arbitrary periodic surface disturbance propagating in the positive x direction and G is an arbitrary periodic surface disturbance propagating in the negative x direction. One simple-harmonic function which satisfies Eq. (2. 4) is a cosine function; i. e. ,

$$\eta_k (x_j, t) = A_k \cos (k x_j \pm \sigma t + \epsilon_k) ; \tag{2. 5}$$

where k has a range from unity to five corresponding to the components shown in Figure 2. 1 and j has a range from unity to two.

The numerical indices for k are now replaced by alphabetical

indices shown in Figure 2. 1 in the following manner:

1 = Mi

2 = Mr

3 = Mt

4 = Ri

5 = Rr;

where the upper case letters M, R correspond to the main channel and to the resonator components, respectively; and the lower case letters i, r, t correspond to the incident, reflected, and transmitted wave components, respectively. The x_j components are replaced by x and y for the coordinate system shown in Figure 2. 1. Equation (2. 5) is a set of five wave component equations containing ten unknown constants: viz., the five values for the wave amplitudes A_{Mi}, A_{Mr}, A_{Mt}, A_{Ri}, A_{Rr} and their associated phase angles ϵ_{Mi}, ϵ_{Mr}, ϵ_{Mt}, ϵ_{Ri}, ϵ_{Rr}. The set of equations expressed in Eq. (2. 5) are the following:

$$\eta_{Mi} = A_{Mi} \cos (k x - \sigma t + \epsilon_{Mi}) \qquad (2.6{:}1)$$

$$\eta_{Mr} = A_{Mr} \cos (k x + \sigma t + \epsilon_{Mr}) \qquad (2.6{:}2)$$

$$\eta_{Mt} = A_{Mt} \cos (k x - \sigma t + \epsilon_{Mt}) \qquad (2.6{:}3)$$

$$\eta_{Ri} = A_{Ri} \cos (k y - \sigma t + \epsilon_{Ri}) \qquad (2.6{:}4)$$

$$\eta_{Rr} = A_{Rr} \cos (k y + \sigma t + \epsilon_{Rr}). \qquad (2.6{:}5)$$

In a well-stated boundard-value problem, the unknowns contained in Eqs. (2. 6: i) may be evaluated by a given set of boundary and initial

conditions (Tikhonov and Samarskii, 1963). In the stating of the conditions required by the above to comply with the criteria for a well-stated boundary-value problem, the following assumptions are made:

(9) The fluid medium of propagation is at rest (i.e., all velocities are absolute).

(10) The terminal boundaries of the centerline of the main channel are sufficiently removed from the proximity of the resonator and, therefore, are not required boundary conditions.

(11) Steady state conditions exist.

(12) All geometric dimensions considered are small compared to wavelengths (i.e., long waves).

(13) The cross sectional area of the channel and of the resonator consists of a horizontal bottom and parallel, vertical sides.

(14) Shallow water conditions exist.

Assumption (11) negates the requirement that the initial conditions be stated. Assumption (14) permits the use of the shallow water approximation for the wave celerity; i.e.,

$$c^2 = g\,h. \tag{2.7}$$

The relative velocities shown in Figure 2.1 are related to Eqs. (2.6:i) in the following manner:

$$\vec{U}_{Mi} = \frac{A_{Mi}\,c_M}{h_M} \cdot \cos\,(k\,x - \sigma t + \epsilon_{Mi}) \tag{2.8:1}$$

$$\vec{U}_{Mr} = -\frac{A_{Mr}\,c_M}{h_M} \cdot \cos\,(k\,x + \sigma t + \epsilon_{Mr}) \tag{2.8:2}$$

$$\vec{U}_{Mt} = \frac{A_{Mt}\, c_M}{h_M} \cdot \cos(k x - \sigma t + \epsilon_{Mt}) \qquad (2.8{:}3)$$

$$\vec{U}_{Ri} = \frac{A_{Ri}\, c_R}{h_R} \cdot \cos(k y - \sigma t + \epsilon_{Ri}) \qquad (2.8{:}4)$$

$$\vec{U}_{Rr} = -\frac{A_{Rr}\, c_R}{h_R} \cdot \cos(k y + \sigma t + \epsilon_{Rr}) . \qquad (2.8{:}5)$$

The boundary conditions will be prescribed along the free surface in terms of Eqs. (2.6:i) and along the vertical boundaries in terms of Eqs. (2.8:i).

The free surface boundary condition prescribes the instantaneous continuity of pressure at the origin for all time:

$$\eta_{Mi} + \eta_{Mr} = \eta_{Mt} = \eta_{Ri} + \eta_{Rr}. \qquad (2.9)$$

The vertical boundary conditions require two statements. The first is that the continuity of flow into tne domain is expressed by

$$[\vec{U}_{Mi} + \vec{U}_{Mr} - \vec{U}_{Mt}]\; b_M h_M + [\vec{U}_{Ri} + \vec{U}_{Rr}]\; b_R h_R = 0; \qquad (2.10)$$

and the second is that the no flow condition across the rigid terminal boundary of the resonator is given by

$$[\vec{U}_{Ri} + \vec{U}_{Rr}]\; b_R\, h_R = 0. \qquad (2.11)$$

The above three boundary conditions are evaluated at the following coordinates:

Eq. (2.9) at the origin (i.e., $x = y = 0$);
Eq. (2.10) at $x = \pm\frac{b_R}{2}$, $y = -\frac{b_M}{2}$;
and Eq. (2.11) at $y = -\ell_R$, $x = 0$.

Using the Boundary Conditions to Obtain a Solution

The main channel incident wave may be arbitrarily selected as the reference wave profile by assumption (11) without any loss in generality; therefore, its associated phase angle is defined as

$$\epsilon_{Mi} = 0. \tag{2.12}$$

Eight independent boundary equations will result by the substitution of Eqs. (2.6:i) and (2.8:i) into the three boundary conditions prescribed in Eqs. (2.9), (2.10) and (2.11) when the expressions obtained after substitution are expanded by means of the trignometric identities for the sine and cosine of the sums and differences of angles. The time dependence of the boundary conditions may be eliminated by equating the sine and cosine terms which result from the above expansion. The ten equations which result from the preceding algebra are the following:

$$A_{Mi} + A_{Mr} \cos \epsilon_{Mr} = A_{Mt} \cos \epsilon_{Mt} \tag{2.13:1}$$

$$- A_{Mr} \sin \epsilon_{Mr} = A_{Mt} \sin \epsilon_{Mt} \tag{2.13:2}$$

$$A_{Mi} + A_{Mr} \cos \epsilon_{Mr} = A_{Ri} \cos \epsilon_{Ri} + A_{Rr} \cos \epsilon_{Rr} \tag{2.13:3}$$

$$- A_{Mr} \sin \epsilon_{Mr} = + A_{Ri} \sin \epsilon_{Ri} - A_{Rr} \sin \epsilon_{Rr} \tag{2.13:4}$$

$$A_{Mt} \cos \epsilon_{Mt} = A_{Ri} \cos \epsilon_{Ri} + A_{Rr} \cos \epsilon_{Rr} \tag{2.13:5}$$

$$A_{Mt} \sin \epsilon_{Mt} = A_{Ri} \sin \epsilon_{Ri} - A_{Rr} \sin \epsilon_{Rr} \tag{2.13:6}$$

$$A_{Ri} \cos (k \ell_R - \epsilon_{Ri}) = A_{Rr} \cos (k \ell_R - \epsilon_{Rr}) \tag{2.13:7}$$

$$-A_{Ri} \sin (k\ell_R - \epsilon_{Ri}) = A_{Rr} \sin (k\ell_R - \epsilon_{Rr}) \tag{2.13:8}$$

$$c_M b_M [A_{Mi} \cos(k\frac{b_R}{2}) - A_{Mr} \cos(k\frac{b_R}{2} - \epsilon_{Mr}) - A_{Mt} \cos(k\frac{b_R}{2} + \epsilon_{Mt})] \tag{2.13:9}$$

$$= c_R b_R [-A_{Ri} \cos(k\frac{b_M}{2} - \epsilon_{Ri}) + A_{Rr} \cos(k\frac{b_M}{2} - \epsilon_{Rr})]$$

$$-c_M b_M [+A_{Mi} \sin(k\frac{b_R}{2}) + A_{Mr} \sin(k\frac{b_R}{2} - \epsilon_{Mr}) + A_{Mt} \sin(k\frac{b_R}{2} + \epsilon_{Mt})] \tag{2.13:10}$$

$$= c_R b_R [A_{Ri} \sin(k\frac{b_M}{2} - \epsilon_{Ri}) + A_{Rr} \sin(k\frac{b_M}{2} - \epsilon_{Rr})] \ .$$

By means of Eqs. (2.13:7) and (2.13:8), it may be shown that

$$A_{Ri} = A_{Rr} \tag{2.14}$$

and that

$$+2 k\ell_R = \epsilon_{Ri} + \epsilon_{Rr} . \tag{2.15}$$

By squaring and adding Eqs. (2.13:5) and (2.13:6), Eq. (2.14) may be rewritten in the form

$$A_{Ri} = \frac{A_{Mt}}{|2 \cos(k\ell_R)|} . \tag{2.16}$$

Equation (2.7) may be employed to equate the two celerities; i.e.,

$$c_R = c_M . \tag{2.17}$$

Following the substitution of the known relationships and applying some additional trignometric identities, the reflection and transmission coefficients may be obtained by squaring and adding Eqs. (2.13:9) and

(2.13:10). The reflection coefficient may be expressed by

$$K_R = \frac{\sin(k\frac{b_R}{2}) + (\frac{b_M}{2b_M})\left[\sin(k\frac{b_M}{2}) - \left|\tan(k\ell_R)\right| \cos(k\frac{b_M}{2})\right]}{\left[1+(\frac{b_R}{b_M})\sin(k\frac{b_R}{2})\left[\sin(k\frac{b_M}{2}) - \left|\tan(k\ell_R)\right| \cos(k\frac{b_M}{2})\right] + (\frac{b_R}{2b_M})^2\left[\sin(k\frac{b_M}{2}) - \left|\tan(k\ell_R)\right| \cos(k\frac{b_M}{2})\right]^2\right]^{\frac{1}{2}}} \tag{2.18}$$

and the transmission coefficient by

$$K_T = \frac{\cos(k\frac{b_R}{2})}{\left[1+(\frac{b_R}{b_M})\sin(k\frac{b_R}{2})\left[\sin(k\frac{b_M}{2}) - \left|\tan(k\ell_R)\right| \cos(k\frac{b_M}{2}) -\right] + (\frac{b_R}{2b_M})^2\left[\sin(k\frac{b_M}{2}) - \left|\tan(k\ell_R)\right| \cos(k\frac{b_M}{2})\right]^2\right]^{\frac{1}{2}}} \tag{2.19}$$

The sum of the squares of Eqs. (2.18) and (2.19) is equal to unity and verifies that the energy in the system is conserved.

Evaluating the Geometric Parameters

The criteria for dimensioning the resonator are the maximizing and minimizing of Eqs. (2.18) and (2.19), respectively. This minimizing and maximizing process requires the evaluation of the arguments of the trignometric functions involving ℓ_R, b_R, and b_M in terms of the tuned wave length, L_*, and wave period, T_o, of the resonator.

The dimension which has the most critical influence on the value of the coefficients is the transverse length of the resonator, ℓ_R, which appears in the argument for the tangent function. The tangent function becomes discontinuous when its argument approaches a value

of $(2n - 1)\frac{\pi}{2}$ radians, where n is any positive integer. The argument reaches this value when

$$\ell_R = (2n - 1) \frac{L_*}{4} ; \qquad (2.20)$$

where n must be equal to unity or two by assumption (12). Eq. (2.20) agrees with the results obtained by Valembois (1953) when the value of one-half of the width of the main channel, b_M, is either a very small per cent of L_* or equal to one-half L_*. The argument containing ℓ_R may be expressed in terms of the angular frequency, σ, by means of the wave celerity equation; i. e.,

$$k\ell_R = \frac{\sigma}{c} \ell_R . \qquad (2.21)$$

Decreasing the dimension ℓ_R in Eq. (2.21) to a value less than $\frac{L_*}{4}$ increases the tuned resonant frequency, σ_o, by a corresponding factor; while increasing ℓ_R to a value greater than $\frac{L_*}{4}$ decreases the tuned resonant frequency, σ_o, also by a corresponding factor; i. e.,

$$\text{if } \ell_R < \frac{L_*}{4} , \text{ then } \sigma > \sigma_o$$

$$\text{and if } \ell_R > \frac{L_*}{4} , \text{ then } \sigma < \sigma_o ;$$

where σ is the tuned resonant frequency for maximum reflection. It may be seen from Eq. (2.16) that the resonator wave amplitude would become infinite at resonance for any finite value of the transmitted wave height, A_{Mt}. Since an infinite wave amplitude does not seem to be physically valid, the transmitted wave height, A_{Mt}

must be identically zero.

The influence of the width of the resonator, b_R, on the value of the coefficients is most easily evaluated by means of Eq. (2.19). Minimizing the transmission coefficient requires that the value of the cosine function in the numerator approach zero. The cosine function approaches zero when its argument approaches $(2n-1)\frac{\pi}{2}$, where n is any positive integer. Therefore, the value of the width of the resonator must be

$$b_R = (2n-1)\frac{L_*}{2} ; \qquad (2.22)$$

where n must be unity by assumption (12). If assumption (12) is valid, the value of b_R determined by Eq. (2.22) is a maximum; hence, b_R may only decrease from this maximum value. The argument containing b_R may be expressed in terms of the angular frequency as before by

$$k\frac{b_R}{2} = \frac{\sigma}{c}\frac{b_R}{2} . \qquad (2.23)$$

Decreasing b_R from its maximum value given by Eq. (2.22) increases the tuned resonant frequency σ_o.

Finally, a critical value for the width of the main channel, b_M, is not easily evaluated by means of the equations for the coefficients; however, because the preceding derivation was based on the assumption that all dimensions were small compared to a wavelength, it may be determined that the width of the main channel must be a small per cent

of the tuned wavelength, L_*.

The evaluation of the geometry of the resonator system has been discussed by considering the effect of a geometrical change on the ability of the resonator to reflect an incident wave of a single frequency. Assumption (11) does not allow the application of the coefficients to a constantly varying input inducing a transitory response. However, the experimental results obtained by Valembois (1953) indicate that appreciable reflection does occur over a range of frequencies. In fact, the response curves obtained by Valembois are similar to the response curves for electrical filter circuits (Olson, 1958). The analogy of these two oscillatory systems introduces the possibility of determining an effective frequency band-width for which reflection may be significant.

3. PRESENTATION OF WAVE DATA

The derivation of the reflection and transmission coefficients in Chapter 2 and their subsequent discussions dealt with an incident wave of a single frequency and wavelength. An analysis of an incident wave record reveals that waves in nature are composed of an infinite and continuous number of frequencies resulting from a complex phenomenon. Since the resonant structure is static, some means of determining a single frequency which is a characteristic measure of the complicated group phenomenon contained in the waves must be made as well as some means for evaluating the effect of the resonator when subjected to an input composed of an infinite series of incident frequencies. Fortunately, the solution to the first problem provides a means for resolving the second.

The problems encountered in recording and in analyzing surface waves are currently under research by the discipline of Oceanography. A complete or even adequate treatment of these complex problems is beyond the scope of this thesis. The design and construction of the resonant breakwater serves to illustrate the interdependance required between Civil Engineering and Oceanography in oceanographic construction. The following discussion is intended only to provide some background and continuity for the use of the oceanographic data required

to effect a complete solution to the design problem posed. The references found in the Bibliography will provide a means for initiating a more detailed study concerning spectral analysis of ocean waves.

Recording Wave Data

The answer to the rhetorical question of how to extract a characteristic frequency from those occurring in a natural phenomenon is affected by the methods available for recording the actual physical profile of the surface disturbance. The various recording devices currently in use all have the common failure of a lack of a standard measurement and of distortions inherent in the recording apparatus. The effect of the recording distortions becomes more pronounced if small non-linear components must be recorded accurately; but the presence of these inherent distortions will influence the shape of any wave profile recorded and will, therefore, be incorporated in the data for a nonlinear analysis as well. Because the recording process is quite complicated, the author can only refer to oceanographic literature and follow Kinsman (1965) by assuming that for the case in question an adequate and representative wave record is available to the designer from the oceanographer.

The Wave Spectrum

The assumed wave record obtained from the oceanographer may also provide an answer to the again rhetorical question of how to extrapolate a characteristic design frequency from the natural wave phenomenon. The continuous distribution of the frequencies in a wave train suggest that a statistical technique may be employed to extract the characteristic frequency. The Sverdrup and Munk Theory, for example, describes the sea surface by a single sinusoidal component called the "significant wave" (Kinsman, 1965). The next level of complexity involves a Fourier analysis.

In a Fourier or harmonic analysis, the wave record is mechanically transformed into a spectrogram (McLellan, 1965). If the characteristic solutions to Eq. (2.1) are orthogonal with respect to the weighting function (which arises from the Sturm-Liouville problem generated by the separation of variables) and are piecewise differentiable in the recording interval τ (Hildebrand, 1962), the solutions may be represented by a product series of the form

$$\eta(t) = \sum_{n=1}^{\infty} \eta_n (t). \tag{3.1}$$

The solution determined in Chapter 2 was a cosine function. Since this function is symmetrical with respect to the z-axis, it may be expressed in a Fourier series for an even function; i.e.,

$$\eta_n(t) = \sum_{n=1}^{\infty} \frac{H_n}{2} \cos (\sigma_n t - \epsilon_n) . \qquad (3.2)$$

The coefficients H_n are determined by multiplying each side of Eq. (3.2) by the weighting function and by the k^{th} solution, η_k, given in Eq. 3.1, and then integrating over the truncated recording interval, τ.

$$H_n = \frac{2}{\tau} \int_0^{\infty} \eta(t) \cdot \cos (\sigma_n t - \epsilon_n). \qquad (3.3)$$

The value given by Eq. (3.3) is equal to twice the average value of the product $\eta(t) \cdot \cos (\sigma_n t - \epsilon_n)$ over the recording interval τ. The wave energy spectrum may also be derived from Eq. (3.2) by squaring both sides of the equation and integrating over the recording interval τ. The harmonic analysis of the wave record does not provide any information regarding the shape of the surface disturbance when

$$| t | > \tau \quad \text{(Kinsman, 1965)} . \qquad (3.4)$$

The next logical progression from this discrete spectrum is a representation of the wave record by a continuous spectrum.

The Bretschneider Continuous Spectrum

The purpose of this discussion on the methods of presentation of wave data is to enable the designer either to request wave data in a particular form or to transform a wave record available from the

oceanographer. One such method of wave data presentation currently in use is the Bretschneider Power Spectrum. This spectrum requires that the average height of the highest one-third waves, $H_{1/3}$, as determined from a Rayleigh probability distribution of wave heights, be known (Dean, 1966). From the value of the significant wave height, $H_{1/3}$, a parameter which is equal to twice the total energy contained in the waves is defined by

$$H_{1/3} = 2.83 \sqrt{E_f} . \tag{3.5}$$

Bretschneider determined that the spectral distribution of the wave energy, S_{H^2}, as a function of frequency σ could be represented by

$$S_{H^2}(\sigma) = \frac{5 E_f}{\sigma_o} \left(\frac{\sigma_o}{\sigma}\right)^5 e^{-1.25 \left(\frac{\sigma_o}{\sigma}\right)^4} \tag{3.6}$$

The parameter σ_o is the frequency at which the peak energy occurs and is defined in terms of the significant wave period, $T_{1/3}$, by

$$\sigma_o = \frac{2\pi}{1.17 \cdot T_{1/3}} . \tag{3.7}$$

Integrating Eq. (3.6) from zero to infinity yields twice the energy contained by the waves E_f.

A Functional Design

The design of the resonant breakwater considered in this thesis is an example of a functional design discussed by Wiegel (1964);

however, only part of the protection function (which is but one aspect of the over-all function of the breakwater) is developed. The functional criteria also affect the method required for presenting the wave data. The purpose for which the harbor is being constructed dictates both the functional requirements of the resonator and the required method of wave data presentation.

As discussed previously, the wave height and wave period parameters normally available from the oceanographer are those of the significant wave, $H_{1/3}$ and $T_{1/3}$. The design criteria will normally require protection from the effects of the highest waves present at the structure. The significant wave height, $H_{1/3}$, has been determined to be an accurate index to structural damage by waves (Wiegel, 1964).

Harbor oscillations or seiching will also normally be found in the design criteria for a harbor. Seiching is a complicated response of the harbor to a periodic force. This response is a function both of the geometry of the harbor and of the period of the force. Wave data for designing protection from seiching, therefore, must be a function of frequency.

One solution to the problem posed at the beginning of this chapter of how to determine a single design frequency from the continuous spectrum of frequencies occurring in the natural phenomenon appears to lie with the statistical significant wave period, $T_{1/3}$.

The Bretschneider Power Spectrum derived from the significant wave height, $H_{1/3}$, will then provide a means of evaluating the effectiveness of the resonator to an input composed of an infinite and continuous spectrum of frequencies. By means of this power spectrum, the effect of a resonator on a wave system occurring in nature may now be evaluated.

4. AN EXAMPLE OF A FUNCTIONAL DESIGN

The Bretschneider Power Spectrum offers a solution to the design problems posed at the beginning of Chapter 3. Evaluating the effectiveness of the resonator over a spectrum of frequencies by combining this power spectrum with the reflection and transmission coefficients represents a linear analysis. The analysis is linear since both the power spectrum and the expressions for the coefficients were derived from linear solutions.

A Linear Analysis

In a linear process, the output, 0, is a linear function of the input, I; i.e.,

$$0 = R(\sigma) \cdot I . \qquad (4.1)$$

$R(\sigma)$ is the linear transfer function and is a function of frequency, σ. The essential features of a linear process are the following (Dean, 1966):

1. Changing the amplitude of the input function, I, changes the amplitude of the output function, 0, proportionally.

2. The constant of proportionality is the linear transfer function and is a function of frequency, σ, only (i.e., the transfer function is independent of the amplitude).

3. Input functions may be superimposed.

The effectiveness of the resonator at a given frequency, therefore,

may be determined from the product of the square of Eq. (2.18) or (2.19) and the value of the ordinate $S_{H^2_{Mi}}$ from Eq. (3.6) for the same corresponding frequency; i.e.,

$$S_{H^2_{Mr}}(\sigma_i) = K_R^2(\sigma_i) \cdot S_{H^2_{Mi}}(\sigma_i) \tag{4.2}$$

or

$$S_{H^2_{Mt}}(\sigma_i) = K_T^2(\sigma_i) \cdot S_{H^2_{Mi}}(\sigma_i). \tag{4.3}$$

Wave Data and a Linear Analysis

As an illustrative example of a linear analysis, it is assumed that a previously undeveloped coastal area has been selected for the construction of a new harbor facility. The harbor has been designed to accomodate large commercial vessels (Hennes and Ekse, 1955). The proposed entrance to the harbor between the breakwaters is to have a depth of fifty (50) feet and a width at the mean-water level of two-hundred (200) feet (Figure 4.1). The effectiveness of the construction of a single resonator on the local wave conditions is to be measured.

The preliminary engineering report includes the collection of extensive oceanographic data on the proposed location. From this report, the significant wave height and period were determined to be the following:

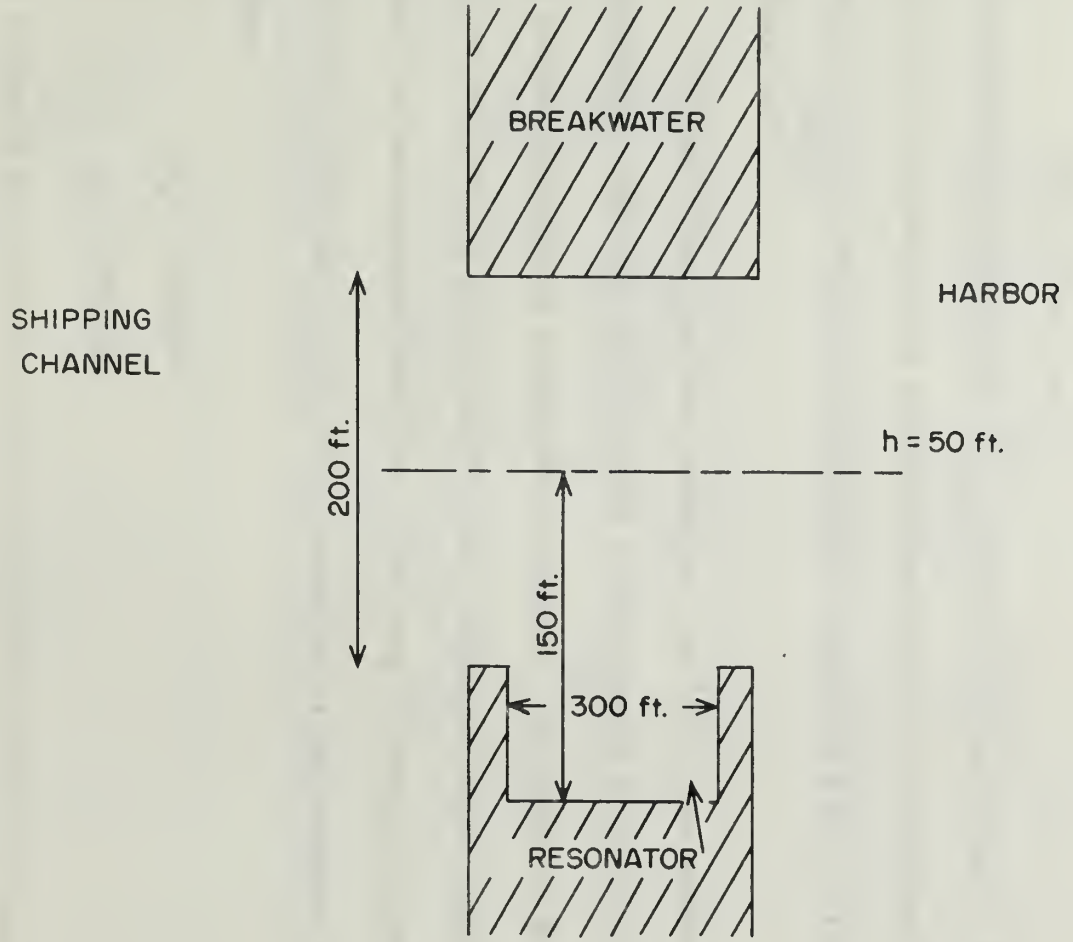


Figure 4.1. Proposed harbor entrance design.

$$H_{1/3} = 12 \text{ feet}$$

$$T_{1/3} = 12.7 \text{ sec}$$

The total energy contained in the waves is determined from Eq. (3.5) to be

$$E_f = 18 \text{ sq. ft.}$$

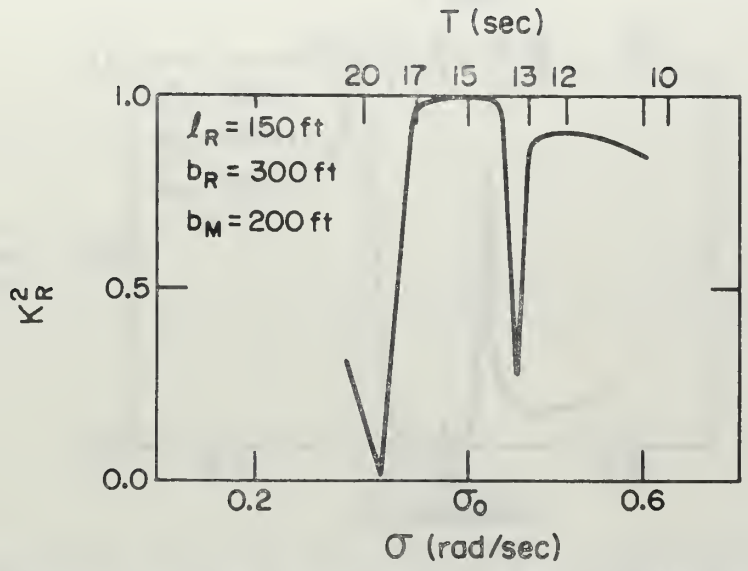
The significant wave period, $T_{1/3}$, determines the value of σ_o in Eq. (3.7) from which the spectral distribution of the incident wave energy, $S_{H_{Mi}}^2$, may be determined by means of Eq. (3.6) (Figures 4.2 and 4.3).

By satisfying the requirement of Eq. (1.6), the shallow water approximation for the wave celerity is determined to be forty (40) feet per second through the breakwater entrance. By means of Eq. (2.2) and the value of the wave celerity, the significant wave length, L_*, is found to be six-hundred (600) feet. Utilizing the design parameters from Eq. s (2.20) and (2.22), the dimensions of the resonator are as follows:

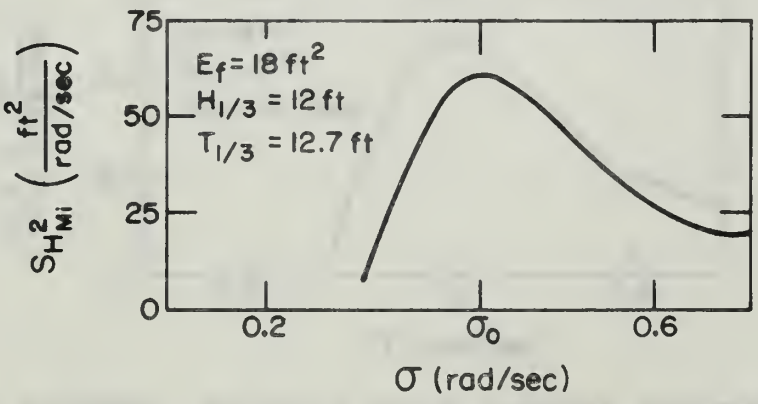
$$\ell_R = 150 \text{ feet}$$

$$b_R = 300 \text{ feet}$$

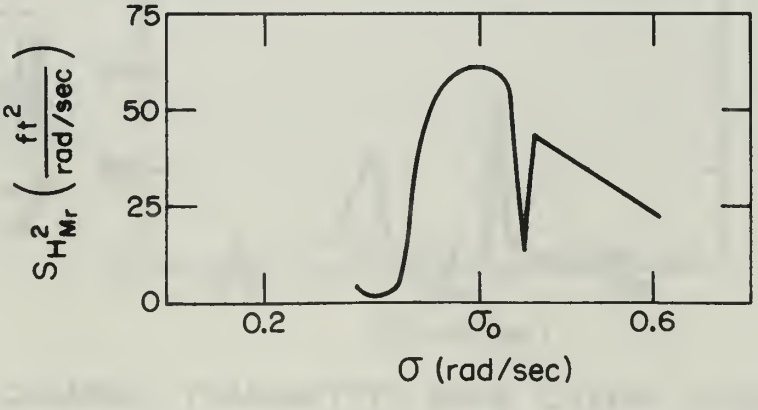
The values of the reflection and transmission coefficients may now be determined as a function of frequency from Eqs. (2.18) and (2.19), respectively. The frequency dependence of the square of these coefficients is also shown in Figures 4.2 and 4.3.



SQUARE OF THE REFLECTION COEFFICIENT

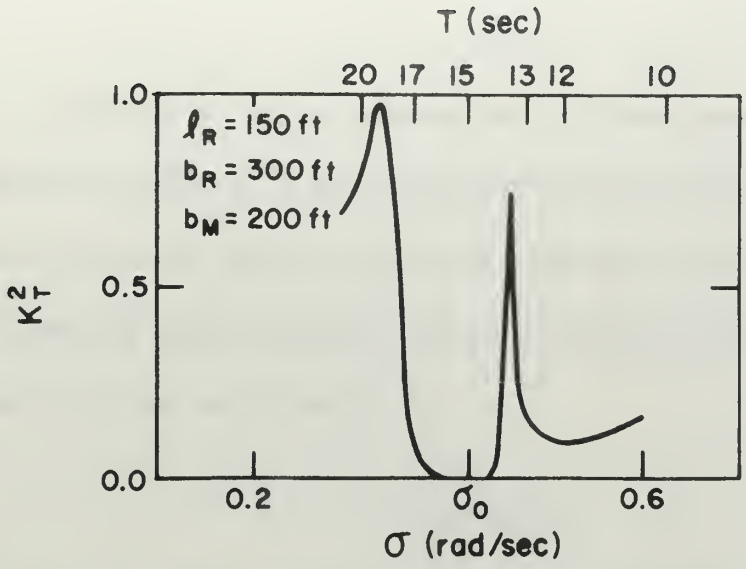


MAIN CHANNEL INCIDENT WAVE ENERGY SPECTRUM

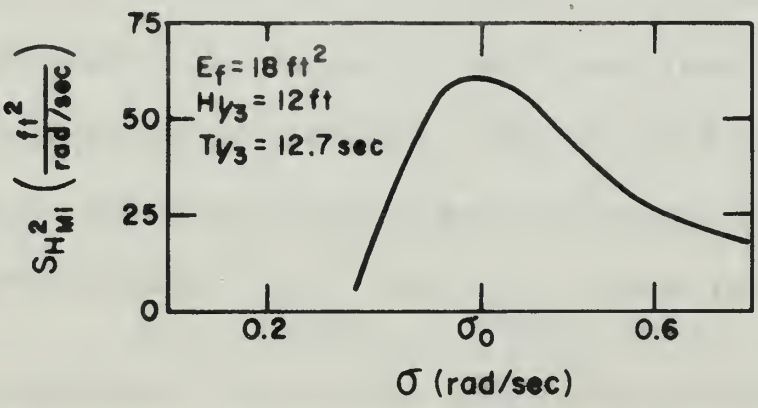


MAIN CHANNEL REFLECTED WAVE ENERGY SPECTRUM

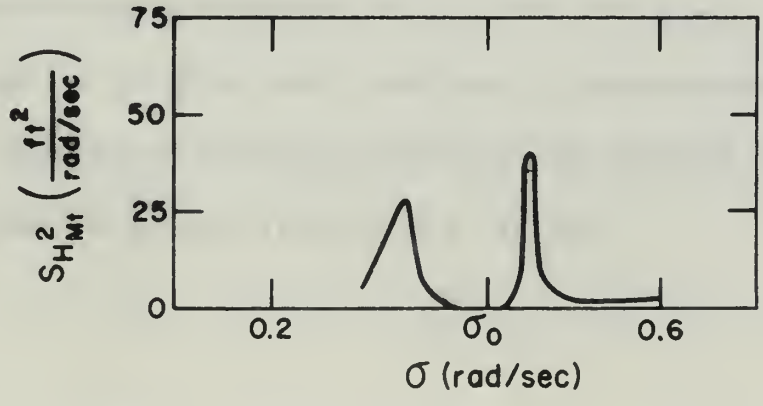
Figure 4.2. Determination of the effective frequency band-width of the resonator.



SQUARE OF THE TRANSMISSION COEFFICIENT



MAIN CHANNEL INCIDENT WAVE ENERGY SPECTRUM



MAIN CHANNEL TRANSMITTED WAVE ENERGY SPECTRUM

Figure 4.3. Determination of the effective frequency band-width of the resonator.

The reflective and transmissive effectiveness of the resonator at a given frequency is determined from the product of the ordinate of the incident wave spectrum and the ordinate of the transfer coefficient. The resulting spectral distributions of these two linear analyses are shown on Figures 4. 2 and 4. 3.

Discussion

The resonator appears to have a frequency band-width ranging from 13 to 17 seconds in which reflection is approximately 100%. The effectiveness of the resonator is determined from the amount of energy which it reflects and transmits. This amount of energy is measured from the area under the power spectrum curves for the reflected and transmitted waves, $S_{H^2_{Mr}}$ and $S_{H^2_{Mt}}$, shown in Figures 4. 2 and 4.3, respectively. The narrow spike occurring between the periods of 13 and 14 seconds prevents the resonator from being greater than 85% effective in reflecting the incident wave energy. The proposed design for the given wave conditions is approximately 75% effective for reflecting the energy contained in the incident wave spectrum between the periods from 10 to 17 seconds.

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