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Effects of Currents on Waves

by

Barry E. Herchenroder

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This report presents ways in which a horizontal current influences surface gravity waves and their measurement. Relatively simple hand-calculation methods are described which provide a means to estimate (a) the wavelength modification due to a current, (b) whether a current can prevent waves from reaching a particular location, (c) the correction needed to compensate for a current when observed bottom pressure fluctuations are used to estimate wave heights, and (d) the range of periods (if any) where the effects of currents can be neglected when wave heights are estimated from bottom pressure fluctuations.

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PREFACE

This report presents some ways in which a horizontal current influences surface gravity waves and their measurement. Relatively simple hand-calculation methods are described which provide ways to estimate how much currents affect the wavelength and measured height of waves. These methods are the initial results of a research effort designed to furnish practical guidance on how to account for the effects of currents on waves. Such guidance fills an information gap in the Shore Protection Manual (SPM). The work was carried out under the waves on currents program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Dr. Barry E. Herchenroder, Oceanographer, under the general supervision of Dr. C.L. Vincent, Chief, Coastal Oceanography Branch.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

CONTENTS

TABLE

FIGURES

CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1]To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

$\bar{d}_T$ time-averaged water depth

$\bar{d}_{T,L}$ largest time-averaged water depth expected

$\bar{d}_{T,S}$ smallest time-averaged water depth expected

F effective Froude number (eq. 3)

FM minimum effective Froude number for which waves with dimensionless frequency Ω can propagate (Table)

FL largest F that can be expected (eq. 15)

FS smallest F that can be expected (eq. 16)

g acceleration of gravity = 32.2 ft (9.8 m)/s^2

H height of a monochromatic wave

H_A height of monochromatic wave when current is absent (eq. 10)

H_V height of monochromatic surface gravity wave when current is present (eqs. 9 and 12)

$\underline{K}$ horizontal wave vector

L wavelength of monochromatic wave

L_A wavelength of monochromatic wave when current is absent (solution of eq. 1)

L_V wavelength of monochromatic wave when current is present (eq. 7 and solution of eq. 2)

P bottom pressure fluctuation magnitude accompanying monochromatic surface gravity wave

R_L dimensionless wavelength factor = L_V/L_A

R_H dimensionless wave height factor = H_V/H_A (eq. 11)

$R_{H,L}$ largest value of R_H expected

$R_{H,S}$ smallest value of R_H expected

T period of monochromatic wave

TS smallest wave period of interest

TS' smallest wave period for which currents can be neglected when observed bottom pressures are used to estimate wave heights (eq. 17)

V horizontal current speed

$\underline{V}$ horizontal current velocity vector

VL	largest value of V expected
VS	smallest value of V expected
VST	"stopping velocity" of a wave (eq. 6)
π	3.14159...
θ	angle between $\underline{V}$ and $\underline{K}$
θL	largest value of θ expected
θS	smallest value of θ expected
ρ_w	water density = 2.00 slugs/ft^3 (1,031 kg/m^3)
Ω	dimensionless frequency (eq. 4)
ΩA,B,C,D	defined in equation (19)
ΩL	largest value of Ω that can be expected (eq. 14)
ΩS'	defined in equation (18)

EFFECTS OF CURRENTS ON WAVES

by
Barry E. Herchenroder

I. INTRODUCTION

A horizontal current can modify surface gravity waves in several ways. One modification is to the wavelength. A current can locally stretch or shrink features in a wave train. This wave-train distortion produces a "Doppler shift" in the wave period. As a result, a stationary observer measures a different wave period than an observer moving with the local current velocity. To a good approximation, the period of a monochromatic wave is fixed in the stationary coordinate system. To an observer moving with the local current velocity, the measured period of this wave changes in time and at each point in response to the changing current.

The wave orthogonal, crest, and ray directions are also modified. An orthogonal is a line perpendicular to the local wave crest direction. A ray is a line parallel to the local group velocity vector, i.e., tangent to the local direction of wave energy flow. In the absence of a current, rays are parallel to orthogonals. When a current is present, orthogonals (by definition) are still perpendicular to the local crest orientation but rays are not parallel to the orthogonals unless the wave is propagating in the same direction as the current.

Another way a current modifies surface waves is to change the wave energy by causing an exchange of energy between wave and current. Energy per unit length of wave crest is no longer approximately conserved between rays, but rather a quantity called "wave action" is conserved. This new quantity, a generalization of wave energy, is given by the energy per unit length of wave crest divided by the "intrinsic angular frequency" of the wave, the latter being the Doppler-shifted angular frequency seen by an observer moving with the local current velocity. Since wave action rather than energy is conserved between rays, enough wave energy is gained from the current or lost to the current to keep the wave action the same as the wave propagates.

A modification also occurs in the pressure field accompanying the wave. This change can be an appreciable source of error in measuring wave characteristics if an existing current is not accounted for. In particular, a significant error can sometimes arise if bottom pressure measurements are used to determine surface wave heights and lengths.

Prediction of current-modified wave energy, heights, directions, and pressures usually involves the use of complex numerical models and computer programs. Such models and programs are under development and will be available over the next few years. However, relatively easy but useful calculations can be done without a computer. This report presents some of these simpler calculations, such as methods to determine (a) the wavelength modification due to a current, (b) whether a current can stop a wave from reaching a particular location, (c) a correction for the presence of a current when bottom pressure measurements are used to determine wave heights, and (d) the range of periods (if any) for which currents can be neglected when these pressure measurements are used to determine wave heights.

II. WAVELENGTH MODIFICATION AND THE STOPPING OF WAVES

When a current is absent, Peregrine (1976)[1] and the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977)[2] indicate that the wavelength, L, of a monochromatic wave is given by $L = L_A$, where L_A is computed by solving the "dispersion relation"

$$\frac{2\pi}{T^2 g} = \frac{1}{L_A} \tanh\left(\frac{2\pi \bar{d}_T}{L_A}\right) \tag{1}$$

and

tanh = the hyperbolic tangent

T = the wave period

g = the acceleration of gravity 32.2 feet (9.8 meters) per second squared

$\bar{d}_T$ = the time-averaged (over enough wave periods to filter out waves) water depth

π = the factor 3.14159...

When a current is present, Peregrine (1976)[3] indicates that L is no longer equal to L_A but to L_V, where L_V is given by solving the equation

$$2\pi \frac{[(1/T) - (V \cos \theta / L_V)]^2}{g} = \frac{1}{L_V} \tanh\left(\frac{2\pi \bar{d}_T}{L_V}\right) \tag{2}$$

where cos is the cosine, V the horizontal current speed, and θ the angle between the horizontal current vector, $\underline{V}$, and the horizonal wave vector, $\underline{K}$. The angle, θ, is taken to be greater than or equal to zero and less than or equal to 180° (π radians). The vector $\underline{K}$ has a magnitude of $2\pi/L_V$ and points along the *wave orthogonal* in the direction of wave crest and trough propagation. The relationship between horizontal current velocity, $\underline{V}$, horizontal wave vector, $\underline{K}$, wave orthogonal, wave crest, wave ray, and angle, θ, is shown in Figure 1. The figure shows that when a current is present, wave rays are not parallel to wave orthogonals and hence not perpendicular to wave crests.

Figure 2 shows a set of curves which represents the dimensionless solution of equation (2). The F axis represents various values of the effective Froude number

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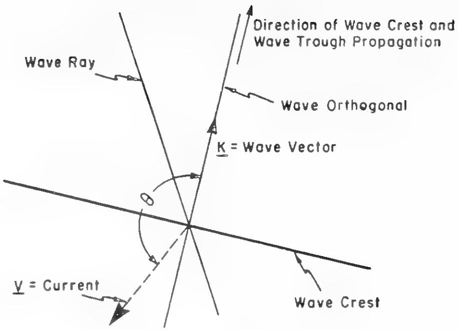
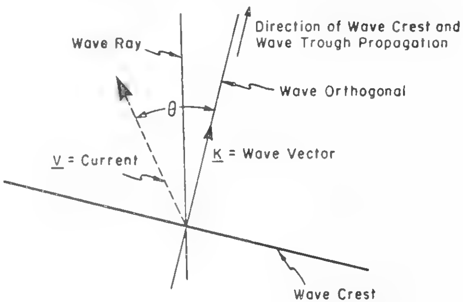
[1]PEREGRINE, D.H., "Interaction of Water Waves and Currents," *Advances in Applied Mechanics*, Vol. 16, Academic Press, New York, 1976, pp. 9-117.

[2]U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 3d ed., Vols. I, II, and III, Stock No. 008-022-00113-1, U.S. Government Printing Office, Washington, D.C., 1977, 1,262 pp.

[3]PEREGRINE, D.H., op. cit.



a. $90° \leq \theta \leq 180° (\pi/2 \text{ Rad} \leq \theta \leq \pi \text{ Rad})$



b. $0° \leq \theta \leq 90° (0 \text{ Rad} \leq \theta \leq (\pi/2) \text{ Rad})$

Figure 1. Relationship between horizontal current velocity, $\underline{V}$, horizontal wave vector, $\underline{K}$, wave orthogonal, wave crest, wave ray, and angle, θ.

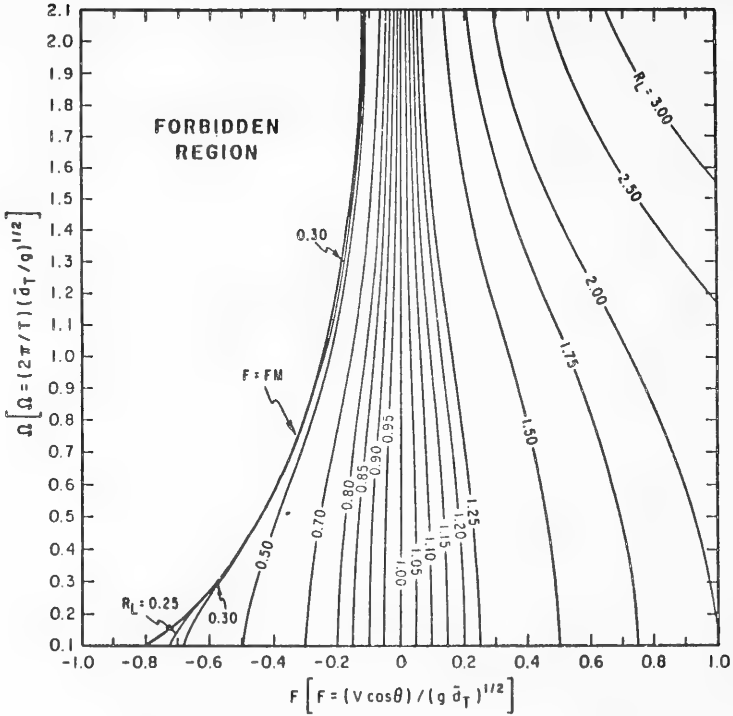


Figure 2. Contours of dimensionless wavelength factor, R_L, given by equation (7); curves represent the dimensionless solution of equation (2). Waves cannot propagate for values of F and Ω which lie in the forbidden region (boundary line F = FM).

$$F = \frac{(V \cos \theta)}{(g\bar{d}_T)^{1/2}} \tag{3}$$

where the Ω axis represents values of dimensionless frequency

$$\Omega = \frac{(2\pi)}{T}\left(\frac{\bar{d}_T}{g}\right)^{1/2} \tag{4}$$

Each curve is a fixed value of wavelength factor, R_L, where

$$R_L = \left(\frac{L_V}{L_A}\right) = \frac{\text{wavelength with current}}{\text{wavelength without current}} \tag{5}$$

Negative values of F correspond to waves with a component of phase velocity in the direction opposite to that of the current. The phase velocity is the velocity that a point on the crest or trough is moving. The curve $R_L = 1.0$ is the case where a current is absent; i.e., where $L = L_A$, L_A fulfilling the usual dispersion relationship (eq. 1).

The curve F = FM in Figure 2 represents the limiting effective Froude numbers for which waves can propagate. If F is less than FM for a particular Ω, then waves cannot propagate against the current for the particular Ω and F values. In other words, *the current acts as a filter under certain conditions and prevents waves from ever reaching a given point.* The limiting value of "effective current velocity," $V \cos \theta$, for which waves of a given period, T, in a mean depth of water, $\bar{d}_T$, can propagate is the "stopping velocity," VST, given by

$$VST = (FM)\,(g\bar{d}_T)^{1/2} \tag{6}$$

VST is negative (since FM is negative) and depends on dimensionless frequency, Ω (since FM, as shown in Fig. 2, depends on Ω), and mean depth, $\bar{d}_T$ (through the $(g\bar{d}_T)^{1/2}$ factor). If a wave is propagating against the current (i.e., $\cos \theta$ is less than zero), then the wave cannot reach any area where the current speed V is greater than the local value of VST. Such an unreachable area in physical space or on a plot like Figure 2 is called the "forbidden region." Waves traveling with the current (i.e., with a $\cos \theta$ greater than or equal to zero) have no forbidden region.

The Table gives the minimum effective Froude number, FM, and the ratio

$$\frac{L_A}{\bar{d}_T} = \frac{\text{wavelength without current}}{\text{average (in time) depth}}$$

for various values of dimensionless frequency, Ω. *For a particular* Ω *value,* this table (used in conjunction with Fig. 2 and linear interpolation) allows the following to be found: (a) whether a wave can propagate against the current (i.e., is F less than FM), and (b) if a wave can propagate, the value of its current modified wavelength, L_V. Using the value of $(L_A/\bar{d}_T)$ from the table and R_L from Figure 2, L_V is computed from

Table. $(L_A/\bar{d}_T)$ and FM for various values of Ω.

Ω	$L_A/\bar{d}_T$	FM	Ω	$L_A/\bar{d}_T$	FM
0.10	62.73	-0.787	1.12	4.45	-0.223
0.12	52.23	-0.761	1.14	4.33	-0.219
0.14	44.73	-0.737	1.16	4.22	-0.216
0.16	39.10	-0.714	1.18	4.11	-0.212
0.18	34.72	-0.693	1.20	4.00	-0.208
0.20	31.21	-0.673	1.22	3.90	-0.205
0.22	28.33	-0.653	1.24	3.80	-0.202
0.24	25.93	-0.635	1.26	3.70	-0.198
0.26	23.89	-0.617	1.28	3.61	-0.195
0.28	22.15	-0.600	1.30	3.52	-0.192
0.30	20.63	-0.584	1.32	3.43	-0.189
0.32	19.30	-0.568	1.34	3.34	-0.187
0.34	18.12	-0.553	1.36	3.26	-0.184
0.36	17.08	-0.538	1.38	3.18	-0.181
0.38	16.14	-0.524	1.40	3.10	-0.179
0.40	15.29	-0.510	1.42	3.02	-0.176
0.42	14.52	-0.497	1.44	2.95	-0.174
0.44	13.82	-0.484	1.46	2.87	-0.171
0.46	13.18	-0.472	1.48	2.80	-0.169
0.48	12.59	-0.459	1.50	2.74	-0.167
0.50	12.04	-0.448	1.52	2.67	-0.164
0.52	11.54	-0.436	1.54	2.61	-0.162
0.54	11.07	-0.425	1.56	2.55	-0.160
0.56	10.63	-0.415	1.58	2.49	-0.158
0.58	10.22	-0.404	1.60	2.43	-0.156
0.60	09.84	-0.394	1.62	2.37	-0.154
0.62	09.48	-0.385	1.64	2.32	-0.152
0.64	09.14	-0.375	1.66	2.26	-0.151
0.66	08.83	-0.366	1.68	2.21	-0.149
0.68	08.52	-0.357	1.70	2.16	-0.147
0.70	08.24	-0.349	1.72	2.11	-0.145
0.72	07.97	-0.340	1.74	2.07	-0.144
0.74	07.71	-0.332	1.76	2.02	-0.142
0.76	07.47	-0.324	1.78	1.98	-0.140
0.78	07.24	-0.317	1.80	1.93	-0.139
0.80	07.01	-0.310	1.82	1.89	-0.137
0.82	06.80	-0.303	1.84	1.85	-0.136
0.84	06.60	-0.296	1.86	1.81	-0.134
0.86	06.40	-0.289	1.88	1.77	-0.133
0.88	06.22	-0.283	1.90	1.74	-0.132
0.90	06.04	-0.277	1.92	1.70	-0.130
0.92	05.86	-0.271	1.94	1.67	-0.129
0.94	05.70	-0.266	1.96	1.63	-0.128
0.96	05.54	-0.260	1.98	1.60	-0.126
0.98	05.39	-0.255	2.00	1.57	-0.125
1.00	05.24	-0.250	2.02	1.54	-0.124
1.02	05.09	-0.245	2.04	1.51	-0.123
1.04	04.96	-0.240	2.06	1.48	-0.121
1.06	04.82	-0.236	2.08	1.45	-0.120
1.08	04.69	-0.231	2.10	1.42	-0.119
1.10	04.57	-0.227			

$$L_V = (R_L)\left(\frac{L_A}{\bar{d}_T}\right)(\bar{d}_T) \tag{7}$$

III. CURRENT-INDUCED CORRECTIONS WHEN BOTTOM PRESSURE MEASUREMENTS ARE USED TO DETERMINE SURFACE WAVE HEIGHTS

Peregrine (1976)[4] and Jonsson, Skovgaard, and Wang (1970)[5] indicate the errors involved in not correcting for the current when using bottom pressure measurements to determine wave heights. They give numerical results which cover some of the conditions encountered in practice. A more extensive set of correction curves is presented in this section. These curves allow an estimate of current corrections to be made for the range of circumstances which usually occur in the field.

Peregrine (1976)[6] indicates that if a monochromatic surface wave height, H, has a bottom pressure fluctuation magnitude, P, then H can be estimated from P by

$$H = H_V \tag{8}$$

where

$$H_V = 2\left[\cosh\left(\frac{d\pi\bar{d}_T}{L_V}\right)\right]\frac{P}{\rho_w g} \tag{9}$$

with cosh the hyperbolic cosine, ρ_w the water density (2.00 slugs per cubic foot or 1,031 kilograms per cubic meter); g, $\bar{d}_T$, and L_V are the same as previously defined. Equation (9) takes the current into account since the wavelength L_V for a nonzero current rather than L_A for no current is used. Usually the current is not taken into account when estimating H from P. Rather, the equation analogous to (9) but with L_A instead of L_V is used to estimate H from P; i.e., $H = H_A$ is usually used where

$$H_A = 2\left[\cosh\left(\frac{2\pi\bar{d}_T}{L_A}\right)\right]\frac{P}{\rho_w g} \tag{10}$$

The wave height factor, R_H, where

$$R_H = \frac{H_V}{H_A} = \frac{\cosh(2\pi\bar{d}_T/L_V)}{\cosh(2\pi\bar{d}_T/L_A)} \tag{11}$$

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[4] PEREGRINE, D.H., op. cit., p. 10.

[5] JONSSON, I.G., SKOVGAARD, C., and WANG, J.D., "Interaction Between Waves and Currents," *Proceedings of the 12th Conference on Coastal Engineering*, American Society of Civil Engineers, Vol. 1, 1970, pp. 489-509.

[6] PEREGRINE, D.H., op. cit.

allows the "correct" wave height value, H, to be determined from the uncorrected height, H_A, by using the simple equation

$$H = (R_H)\ H_A \tag{12}$$

Contours of R_H are shown in Figure 3. The F and Ω axes are the same as in Figure 2. The boundary of the forbidden region is again indicated by the curve labeled F = FM. Figure 3 shows that R_H is always greater than 1.0 for F less than zero (for "adverse" currents where waves propagate against the current) and R_H is less than 1.0 for F greater than zero (for "following" currents where waves propagate with the current). Figure 3 also shows that for a given value of F, adverse currents have a larger effect (i.e., give R_H values further removed from 1.0) than following currents.

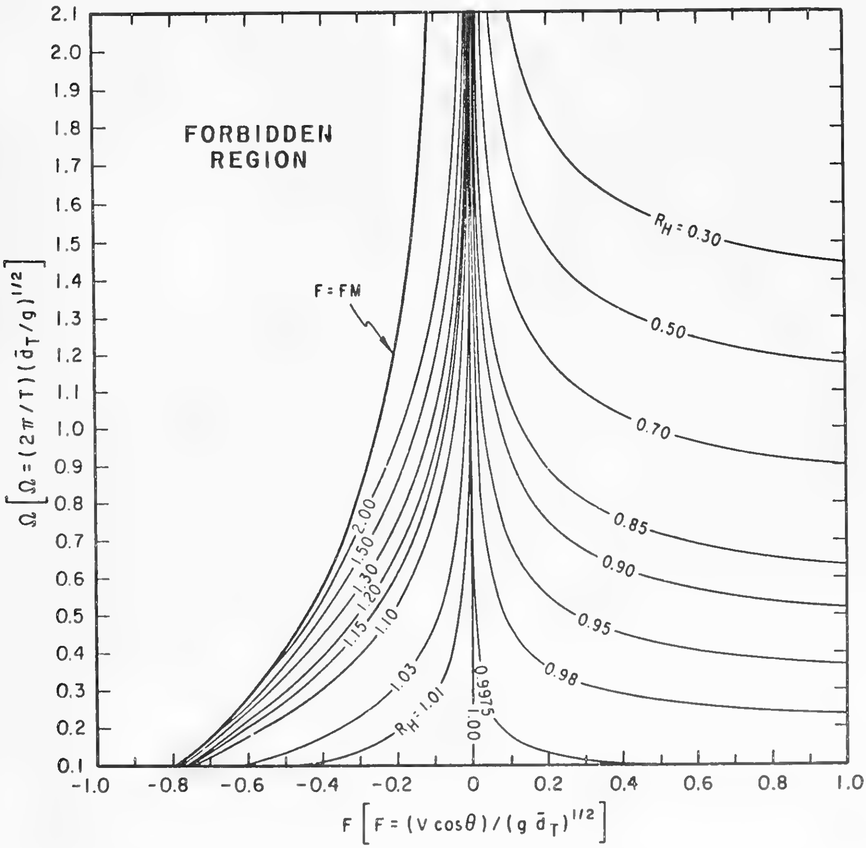

Figure 3. Contours of dimensionless wave height factor, R_H, given by equation (11). Waves cannot propagate for values of F and Ω which lie in the forbidden region (boundary line F = FM).

Knowledge of R_H can also be used to estimate whether current effects are great enough to warrant being taken into account when using an observed bottom pressure fluctuation magnitude, P, to get a wave height, H. For the range of current speeds, average depths, angles between the current vector and wave orthogonal, and wave periods, the largest and smallest values of R_H (denoted by $R_{H,L}$ and $R_{H,S}$, respectively) are estimated. If both are within the acceptable range, then the current correction can be neglected and the wave height estimated from

$$H = H_A \tag{13}$$

where H_A is given by equation (10). If either $R_{H,L}$ or $R_{H,S}$ or both are out of range, then equation (12) must be used to compute H. Practical limits on the acceptable range for both $R_{H,L}$ and $R_{H,S}$ are 1.15 and 0.85; **i.e.,** *if the wave current interaction produces* R_H *values which differ from* 1.0 *by* ±0.15 *or less, the effect of the current can be neglected.* Figure 3 shows that $R_{H,L}$ is the R_H value corresponding to the *smallest* F and *largest* Ω that is expected. Analogously, $R_{H,S}$ is the R_H value corresponding to the *largest* F and *largest* Ω that is expected. The largest expected Ω, ΩL, is given approximately by

$$\Omega L = \frac{2\pi}{TS}\left(\frac{\bar{d}_{T,L}}{g}\right)^{1/2} \tag{14}$$

where TS is the smallest wave period of interest and $\bar{d}_{T,L}$ the largest time-averaged water depth expected (i.e., largest value of $\bar{d}_T$ expected). The largest F, FL, and the smallest F, FS, that can be expected are given approximately by

$$FL = \frac{(VL)\cos(\theta S)}{\left(g\bar{d}_{T,S}\right)^{1/2}} \quad \text{if} \quad 0° \le \theta S \le 90° \left(0\ \text{rad} \le \theta S \le \frac{\pi}{2}\ \text{rad}\right) \tag{15a}$$

$$FL = -\frac{(VS)\cos(\theta S)}{\left(g\bar{d}_{T,L}\right)^{1/2}} \quad \text{if} \quad 90° < \theta S \le 180° \left(\frac{\pi}{2} \le \theta S \le \pi\ \text{rad}\right) \tag{15b}$$

and

$$FS = \frac{-(VL)|\cos(\theta L)|}{\left(g\bar{d}_{T,S}\right)^{1/2}} \quad \text{if} \quad 90° < \theta L \le 180° \left(\frac{\pi}{2}\ \text{rad} < \theta L \le \text{rad}\right) \tag{16a}$$

$$FS = \frac{(VS)\cos(\theta L)}{\left(g\bar{d}_{T,L}\right)^{1/2}} \quad \text{if} \quad 0° \le \theta L \le 90° \left(0\ \text{rad} \le \theta L \le \frac{\pi}{2}\ \text{rad}\right) \tag{16b}$$

where (VL, VS) are the (largest, smallest) current speeds expected, (θL, θS) are the (largest, smallest) values of the angle θ expected, $\bar{d}_{T,S}$ is the smallest time-averaged water depth expected (i.e., smallest value of $\bar{d}_T$ expected), and $\bar{d}_{T,L}$ has already been defined. If FS is less than the FM value corresponding to Ω = ΩL, then FS is reset equal to FM.

In computing the wave height elevation, H, from the fluctuating bottom pressure magnitude, P, there may be a range of wave periods over which a current can be neglected (H is given approximately by H_A where H_A is defined by eq. 10) and another range over which the current must be accounted for (H is given by $(R_H) H_A$ where R_H is defined by eq. 11). Figure 3 and the values of FL and FS, once computed, can be used to determine the smallest wave period beyond which currents can be neglected, TS'. For all wave periods greater than or equal to TS', the current can be neglected when computing H from P. TS' is given by

$$TS' = \frac{2\pi}{\Omega S'}\left(\frac{\bar{d}_{T,L}}{g}\right)^{1/2} \quad \text{for} \quad \Omega S' \neq 0 \tag{17a}$$

$$TS' = \infty \text{ (i.e., currents are never important) for } \Omega S' = 0 \tag{17b}$$

where

$$\Omega S' = \mathrm{MIN}(\Omega A, \Omega B, \Omega C, \Omega D) \tag{18}$$

MIN( ) means take the minimum value of the numbers in parentheses, and the factors ΩA, ΩB, ΩC, and ΩD are estimated from Figure 3 by using the following definitions:

ΩA = the value of Ω which corresponds to $R_H = 0.85$ when $F = FL$ ($\Omega A = 0$ if FL is less than or equal to 0)

ΩB = the value of Ω which corresponds to $R_H = 1.15$ when $F = FL$ ($\Omega B = 0$ if FL is greater than or equal to 0)

ΩC = the value of Ω which corresponds to $R_H = 0.85$ when $F = FS$ ($\Omega C = 0$ if FS is less than or equal to 0) (19)

ΩD = the value of Ω which corrresponds to $R_H = 1.15$ when $F = FS$ ($\Omega D = 0$ if FS is greater than or equal to 0)

Figure 4 gives a schematic representation of how ΩA and ΩD are related to FL and FS for the example problem presented in the next section (in the problem, $\Omega B = \Omega C = 0$).

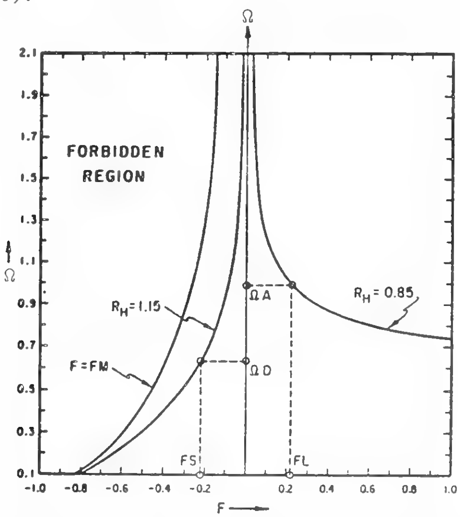

Figure 4. Schematic representation of how ΩA and ΩD are related to FL and FS for the example problem in Section IV. For the problem, $\Omega B = \Omega C = 0$.

IV. EXAMPLE PROBLEM

This example illustrates the method used to estimate the maximum and minimum wave height factors, $R_{H,L}$ and $R_{H,S}$, whether the current can be neglected completely in computing wave height, H, from the fluctuating bottom pressure magnitude, P, and if not what range (if any) the current can be neglected.

GIVEN: A bottom-mounted wave pressure gage is located near an inlet at an elevation of -19.68 feet (-6 meters) referenced to the 1929 National Geodetic Vertical Datum (NGVD). The tide elevation ranges from -3.28 to 3.6 feet (-1 to 1.1 meters) NGVD. Current speeds from 0 to 4.92 feet (1.5 meters) per second are known to occur at the location of the gage. The wave climatology indicates that the dominant waves have periods between 6 and 15 seconds and can travel at any angle with respect to the current.

FIND:

(a) The largest and smallest values of R_H ($R_{H,L}$ and $R_{H,S}$) for the conditions expected and the range of wave periods of interest (periods between 6 and 15 seconds).

(b) Whether the currents can be neglected for the entire range of wave periods of interest when bottom pressure observations are used to determine wave heights.

(c) The range of wave periods over which currents can and cannot be neglected if they cannot be neglected over the entire range of interest.

SOLUTION:

Step 1. Set θS and θL. Since the angle between the direction of wave travel and the current can take on any value,

$$\theta S = 0; \ \theta L = 180° = \pi \text{ rad}$$

Step 2. Set VL, VS, and TS. From the given values of the problem,

$$VL = 4.92 \text{ ft } (1.5 \text{ m})/\text{s}$$

$$VS = 0$$

$$TS = 6 \text{ s}$$

Step 3. Compute $\bar{d}_{T,S}$ and $\bar{d}_{T,L}$. From the tide elevation and bottom elevation information given,

$$\bar{d}_{T,S} = 19.68 - 3.28 = 16.4 \text{ ft } (5 \text{ m})$$

$$\bar{d}_{T,L} = 19.68 + 3.60 = 23.28 \text{ ft } (7.1 \text{ m})$$

Step 4. Compute ΩL using equation (14).

$$\Omega L = \frac{2(3.14159)}{6 \text{ s}} \left(\frac{23.2 \text{ ft}}{32.2 \text{ ft/s}^2} \right)^{1/2} = 0.89 \text{ (dimensionless)}$$

Step 5. Compute FL using equation (15) and FS using equation (16). Since θS is zero, equation (15a) is used for FL, giving

$$FL = \frac{(4.92 \text{ ft/s}) (1.0)}{[(32.2 \text{ ft/s}^2) \ 16.40 \text{ ft}]^{1/2}} = 0.214 \text{ (dimensionless)}$$

Since θL is 180° (π rad), equation (16a) is used for FS, giving

$$FS = \frac{-(4.92 \text{ ft/s}) (1.0)}{[(32.2 \text{ ft/s}^2) \ 16.40 \text{ ft}]^{1/2}} = -0.214 \text{ (dimensionless)}$$

Step 6. In Figure 3, find the value of R_H corresponding to FL and ΩL. This value of R_H is $R_{H,S}$ given approximately by

$$R_{H,S} = 0.77 \text{ (dimensionless)}$$

Step 7. In Figure 3, find the value of R_H corresponding to FS and ΩL. This value of R_H is $R_{H,L}$, given approximately by

$$R_{H,L} = 1.75 \text{ (dimensionless)}$$

Steps 1 through 7 solve part 1 under "Find."

Step 8. To answer part 2 under "Find," note that $R_{H,S}$ is less than 0.85 and $R_{H,L}$ is greater than 1.15. As a result, the current *cannot* be neglected for all wave periods between 6 and 15 seconds.

Step 9. To solve part 3 under "Find," Figure 3 is first used to determine the factors ΩA, ΩB, ΩC, and ΩD. Invoking the definitions for these parameters given in equation (19), it is found that approximately (see Fig. 4)

$$\Omega A = 0.80; \ \Omega B = \Omega C = 0; \ \Omega D = 0.59 \text{ (all dimensionless)}$$

Step 10. Compute ΩS' using equation (18). The result is

$$\Omega S' = \text{MIN}(0.80, 0.0, 0.0, 0.59) = 0.59 \text{ (dimensionless)}$$

Step 11. Compute TS' using equation (17) and the value of $\bar{d}_{T,L}$ found in step 3. The result is

$$TS' = \frac{[2(3,14159)]}{0.59} \left(\frac{23.28 \text{ ft}}{32.2 \text{ ft/s}^2}\right)^{1/2} = 9.1 \text{ s}$$

This value of TS' shows that only for waves between 9.1 and 15 seconds can the current be neglected in computing H from P. Waves between 6 and 9.1 seconds must have the influence of the current taken into account when using bottom pressure measurements to compute wave heights.

V. SUMMARY

Some of the ways in which a horizontal current influences surface gravity waves and their measurement have been presented. Relatively simple hand-calculation methods have been described which provide a means to estimate (a) the wavelength modification due to a current, (b) whether a current can prevent waves from reaching a particular location, (c) the correction needed to compensate for a current when observed bottom pressure fluctuations are used to estimate surface gravity wave heights, and (d) the range of periods (if any) where the effects of currents can be neglected when surface wave heights are estimated from bottom pressure fluctuations.

Herchenroder, Barry E.
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