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**THE ELASTICITY OF SUBSTITUTION BETWEEN
LAND AND CAPITAL IN SINGLE-FAMILY HOUSING**

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#375

**College of Commerce and Business Administration
University of Illinois at Urbana-Champaign**



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February 11, 1977

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THE ELASTICITY OF SUBSTITUTION BETWEEN
LAND AND CAPITAL IN SINGLE-FAMILY HOUSING

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ABSTRACT

Many of the models which have been developed to explain urban spatial structure and land-use patterns rest on the properties of production functions. Differing factor price ratios within urban areas, particularly land prices, result in capital-land ratios exemplified by high-rise apartments to single-family dwellings. The purpose of this paper is to provide some empirical evidence on the elasticity of substitution between land and capital in single-family housing. Estimates are made for both the constant elasticity of substitution and the variable elasticity of substitution production functions.

THE ELASTICITY OF SUBSTITUTION BETWEEN LAND AND CAPITAL IN SINGLE-FAMILY HOUSING

I. Introduction

The literature on urban economics has expanded rapidly in recent years.¹ Recent articles by Mills (1967), Henderson (1974), and Schuler (1974) have investigated the problems associated with urban structure. Many of the models which have been developed to explain urban structure and land-use patterns rest on the properties of production functions.² Differing factors price ratios within urban areas, particularly land prices, result in capital-land ratios exemplified by high-rise apartments to single family dwellings. To understand the effects of changing factor prices and the corresponding change in factor ratios, it is necessary to make assumptions about the elasticity of substitution between capital and land. This has been done in the literature by assuming a unitary elasticity parameter (Cobb-Douglas) as well as direct estimates of the substitution parameter by assuming that it is constant over the range of observed data (CES production function).

Previous literature has failed to provide adequate evidence on the substitution between land and capital in the urban housing production function which is an important aspect of urban models.³ Obviously, the lack of knowledge about this important parameter leads to serious bias in the analysis of the urban housing market. For example, Kau and

Lee (1976) have demonstrated that the magnitude of the elasticity of substitution will affect urban structure and growth of a city. The purpose of this paper is to provide some empirical evidence on this important aspect of urban structure.

Using data for a single-family housing market, this paper provides a test of the functional form of the production relationships used in analyzing urban spatial structure.

II. The Model

The model in this paper is similar to that used by Muth (1969) and Koenker (1972). For a representative firm in the housing industry, the following can be written:

$$(1) \quad Q = F(L, K)$$

$$(2) \quad pF_L = r$$

$$(3) \quad pF_K = n$$

$$(4) \quad pQ = rL + nK$$

Equation (1) is the production function; equations (2) and (3) are the profit maximizing conditions and equation (4) is a condition of competitive equilibrium for the industry. The physical quantities of output, land, and a composite factor, capital, are represented by Q , L , and K ; p , r , and n are the respective prices of output and input factors and the subscripts denote marginal products.

Assuming a constant elasticity of substitution production function, the relationship between a percentage

change in the intensity of land use, measured by the dollar value of physical structure per square foot of site, and a percentage change in the prices of land and non-land factors can be written as

$$(5) \quad \ln\left(\frac{K}{L}\right) = \alpha + \sigma(\ln r)$$

Equation (5) assumes that the price of all non-land factor prices are invariant with respect to location (Muth 1969 [pp. 52-53]). In equation (5), σ represents the elasticity of substitution.

III. Empirical Estimates: CES and VES Functions

Equation (5) can be estimated in the following stochastic form:

$$(6) \quad \ln\left(\frac{K}{L}\right) = \alpha + \sigma(\ln r) + u_i$$

where u_i is a randomly distributed error term with mean of zero and constant variance. A test of the model is based on a sample of single-family housing data for Santa Clara County, California.⁴ This data was used in the study of land values by Wendt and Goldner (1966). The data contains 98 observations which included the average lot value, average value of properties and the average size of the lot of the various Census tracts.

Estimating equation (6) yields:

$$(7) \quad \ln\left(\frac{K}{L}\right) = 1.080 + .860(\ln r)$$

(.004) (.052)

Standard errors are shown in parentheses below the coefficients. The variation in land prices explains 75 percent of the variation in the intensity of land use. This result indicates that a one percent increase in relative factor prices will induce a .86 percent increase in the intensity of land use. A 95 percent confidence interval for the elasticity of substitution is (.756, .962).

The estimated elasticity in equation (6) indicates that substitution is "easier" in this single family housing market as compared to the multi-family market data used by Koenker (1972).⁵ This result is not unexpected since, in general, multi-family housing tends to occupy more expensive land, i.e., nearer the city center, and the substitution becomes "harder" nearer the center due to technological and other constraints on capital production.⁶ The estimated elasticity is less than and significantly different from one indicating the inappropriateness of assuming a Cobb-Douglas production function. This result tends to support the CES function as suggested by Kau and Lee (1976).

However, as indicated by Hicks (1948) and Allen (1956) and discussed by Revankar (1971), the elasticity of substitution can be variable depending upon output and/or factor combinations and can thus assume any value between zero and infinity. Hence, any a priori constraint as to the value of σ can lead to possible specification bias. It is thus important to determine empirically if the log-linear relationship in equation (6) is the correct functional form. Several recent

articles by Lovell (1971), Lu and Fletcher (1972), and Revankar (1971) have suggested variable elasticity of substitution (VES) production functions which are designed to capture the relationship between the elasticity of substitution and output and/or factor combinations.

Revankar (1971 [pp. 67-68]) has recently shown that the marginal conditions for the VES function is the linear counterpart to the log-linear relation derived from the CES specification. Thus, equation (5) becomes

$$(8) \quad \frac{K}{L} = \beta_0 + \beta_1(r)$$

The elasticity of substitution can be determined by (Revankar 1971 [pp. 65-66]),

$$(9) \quad \sigma = 1 - \beta_0 \left(\frac{L}{K} \right)^*$$

where $\left(\frac{L}{K} \right)^*$ is the mean of the land-capital ratios.

Equation (8) was estimated, by adding a disturbance term of the ordinary sort, with the following results:

$$(10) \quad \frac{K}{L} = \begin{matrix} .285 + 2.710(r) \\ (.116) \quad (.194) \end{matrix}$$

where the standard errors are in parentheses. Equation (10) had an adjusted coefficient of determination of .70. Since $\beta_0 \neq 0$, the Cobb-Douglas case can be rejected. Using the mean value of the L/K ratio, the elasticity can be calculated using equation (9) as,

$$(11) \quad \sigma = 1 - .285(.543) = .845$$

The VES estimate of the elasticity at the mean L/K ratio is very similar to that of the CES function. It is important, however, to discriminate between the CES and VES specifications. The next section provides test of the functional form of the relationship between the intensity of land-use and relative factor prices.

IV. Functional Form Analysis

The problem of selecting between the VES and CES function can be viewed as one of discriminating between two specifications of a postulated relationship between intensity of land-use and factor-price ratios, where the price of capital is invariant. The CES specification is log-linear, the VES is linear, and there is no a priori economic rationale for preference of one over the other. Discrimination on goodness-of-fit is inappropriate since the dependent variables are different.

Box and Cox (1964) have recently suggested a technique to discriminate between linear and log-linear functional forms.⁷ Consider the following relation between intensity of land-use and the factor-price ratio

$$(12) \quad \left(\frac{K}{L}\right)^\lambda = a_0 + a_1(r)^\lambda$$

where λ is the parameter of the power transformation on the variables. As noted by Lovell (1973), this differential equation defines a whole class of production functions, two of which are the CES and the VES. When $\lambda \rightarrow 0$, then equation (12) approaches equation (5), the CES case, and if $\lambda \rightarrow 1$, then (12) reduces to (8), the VES form.

Equation (12) can be written

$$(13) \quad \frac{\left(\frac{K}{L}\right)^\lambda - 1}{\lambda} = a'_0 + a'_1 \frac{(r)^\lambda - 1}{\lambda} + u_i$$

where u_i is the disturbance term that is normally distributed with zero mean and constant variance, σ^2 . Equation (13) can be estimated using maximum likelihood techniques.

The logarithm of the likelihood function is maximized with respect to a'_0 , a'_1 , and σ^2 given λ . For the given λ , the maximum likelihood estimate of σ^2 is given by the estimated variance of the disturbances of the regression on $\frac{K}{L}^\lambda$ and $(r)^\lambda$. Box and Cox (1964) have derived a maximum logarithmic likelihood for determining the functional form parameter, except for a constant, as

$$(14) \quad L_{\max}(\lambda) = -\left(\frac{n}{2}\right) \ln \sigma^2(\lambda) + (\lambda-1) \sum_{i=1}^n \ln \frac{K}{L}$$

which may be calculated for different values of λ to find the maximized log likelihood over the entire parameter space. An approximate 95 percent confidence region for λ can be obtained from

$$(15) \quad L_{\max}(\hat{\lambda}) - L_{\max}(\lambda) < 1/2 \chi_1^2(.05) = 1.92$$

Using this approach, equation (13) was estimated using λ values between -1.0 and 1.5, at intervals of .1 to transform the variables. This results in twenty-six equations for which the $L_{\max}(\lambda)$'s are plotted in Figure 1.

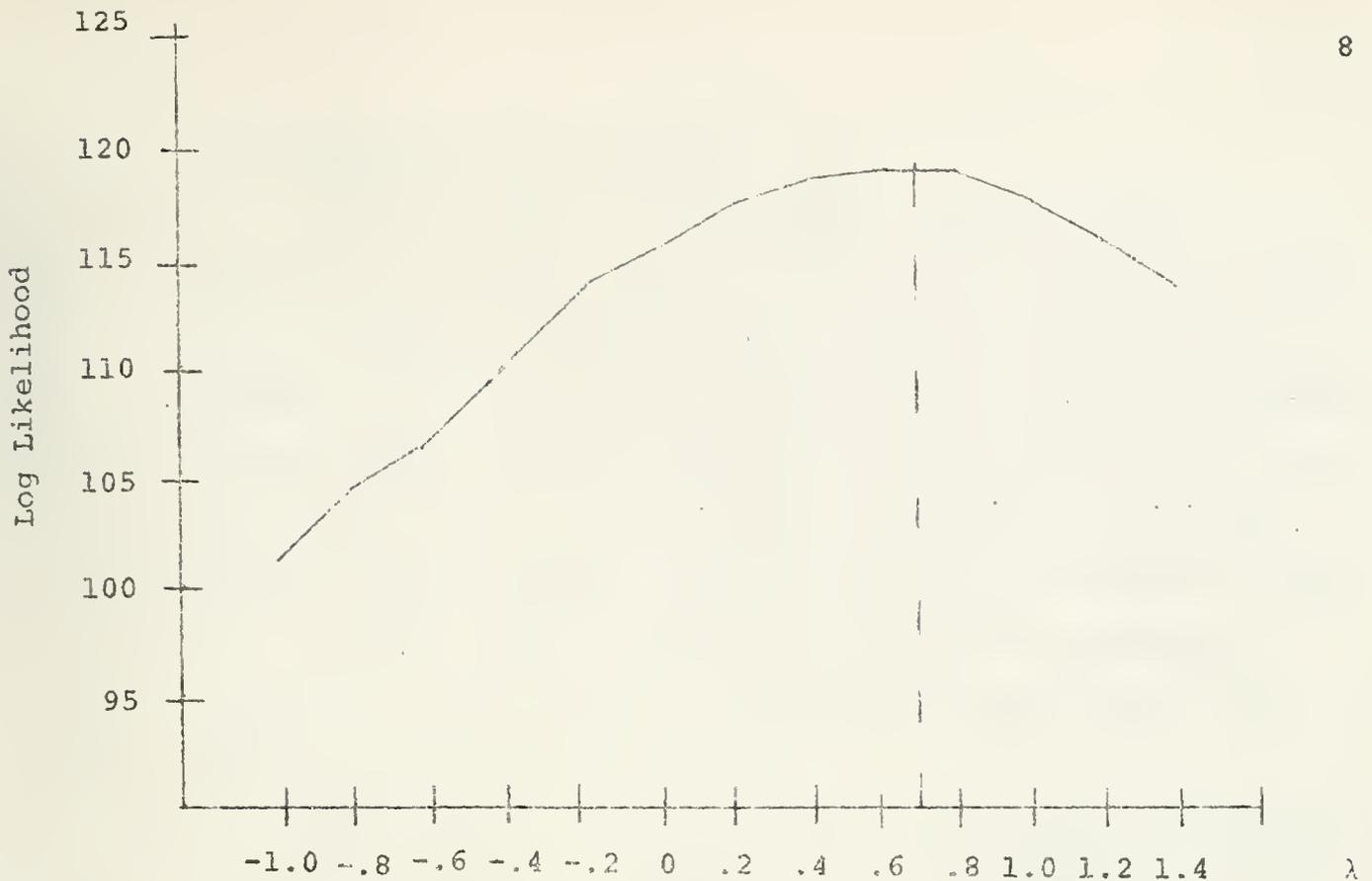


Figure 1
Log Maximum Values for Alternative λ 's

The results for equation (13) indicate that the log maximum of the likelihood function is 119.1 at $\lambda = .7$. Using equation (15), a 95 percent confidence interval around λ is (.3, 1.1). Thus it includes the VES function. The hypotheses that the CES is the appropriate function cannot be accepted. This result tends to indicate that the elasticity of substitution is not constant within this sample of a single-family housing market and the acceptance of the CES estimate would lead to specification bias. The result also indicates the need for analyzing the elasticity of substitution between land and non-land input by using a production function which would allow the elasticity to vary with output and/or factor proportions.

V. Summary and Conclusions

The elasticity of substitution between land and non-land factors of production has become an important aspect in the analysis of urban spatial structure. There has been, however, little empirical evidence on this important parameter in the urban economics literature. The purpose of this paper has been to provide some empirical estimates for a single-family housing market. Recent literature has shown the need for analyzing urban structures using the CES production function. This paper has also provided some insight into this aspect of urban economics.

The results for the CES production function indicated that a one percent increase in the factor-price ratio would lead to a .86 percent increase in the intensity of land use as measured by the dollar value of physical structure per square foot of site. This result indicates that substitution is "easier," as expected, in the single family market as compared to the multi-family market studied by Koenker (1972). The test of the functional form of the relationship between the intensity of land use and the relative factor prices indicated that the CES form is possibly not the correct specification. The results implied that a more correct specification of this relationship would be through the use of a variable elasticity of substitution (VES) production function. This result should be explored in greater detail in future research on urban spatial structure.

FOOTNOTES

¹See, for example, Goldstein and Moses (1974) and Mills (1972).

²See, for example, the urban models by Muth (1969), Mills (1972), Kau and Lee (1976), and Fallis (1975). Atkinson (1975) has recently provided a discussion of the empirical estimates of the elasticity of factor substitution. It is interesting to note that he fails to consider the importance of this parameter in models of urban structure.

³The only existing evidence, according to this author's knowledge, are those by Koenker (1972), Muth (1969), Fallis (1975), and Kau, Lee and Sirmans (1976).

⁴See Wendt and Goldner (1966) for a more complete discussion of the data.

⁵Koenker (1972) found in his multi-family market that the elasticity was .71.

⁶Kau, Lee and Sirmans (1976) have recently provided a detailed discussion of the importance of the variation in the elasticity of substitution within an urban area.

⁷This technique has been used by Zarembka (1968) to examine the money demand function, Kau and Lee (1976) to examine the population density gradient, and by Kau and Sirmans (1976) to determine the price elasticity of demand for housing.

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