THE ELASTIC STRENGTH OF GUNS

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ALGER

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BY

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PREFACE

This little book was prepared primarily for use by the midshipmen at the U. S. Naval Academy. It essays to present the subject of the elastic strength of guns as concisely as is consistent with clearness, and to that end treats only of steel guns of modern construction, built-up or wire-wound.

The hypothesis that permanent set will not occur unless the resultant *strain* in some direction exceeds the limit of elastic strain, regardless of what the stresses may be, is adopted. This hypothesis appears to the writer to be the only reasonable one, but it is to be regretted that its truth has never been demonstrated experimentally.

The longitudinal stress is taken to be zero, an assumption made by Claverino in his first treatise on the "Resistance of Hollow Cylinders," published in the "Giornale d'Artiglieria" in 1876, and adopted by Birnie in his exhaustive studies of the resistance and shrinkages of built-up cannon.

The formulæ for wire-wound guns were originally deduced by the writer some twenty years ago, and were then first published in the *U. S. Naval Institute Proceedings.*

A number of illustrative examples are solved in the text, and others, with their answers, follow each chapter.

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THE ELASTIC STRENGTH OF GUNS

CHAPTER I.

INTRODUCTORY.

1. Stress and Strain.—We give the name *stress* to a mutual action between the parts of a body, or between one body and another, causing, or tending to cause them to move relative to one another; it is any pair of equal and opposite actions each of which is what is called a force.

Thus, if a rope be stretched vertically downwards from A to B, we speak of the tension T of the rope as the force T acting downward on A, or as the force T acting upward on B, according as we are considering A or B; but we speak of the action in the rope, which tends to break it, as the stress in the rope.

2. We call the change of volume or figure of any solid or liquid under the action of force a *strain*.

Thus, if a bar is lengthened or shortened, it is strained; a compressed liquid is strained; a stone, a piece of metal, or other part of any structure, is said to experience a strain if it be bent, or twisted, or compressed, or dilated, or in any manner distorted. Furthermore, any change in the configuration of a group of bodies whose relative positions are subject to fixed conditions is called a strain. Thus, any structure is said to strain when its different parts experience relative motion, as, for example, a ship "strains" in a seaway.

3. If we imagine any plane area within a strained body as forming a division between the parts of the body on either side of it, then the force which each of the two parts exerts upon the other is one of the pair of forces which constitute the stress on the area. In other words, the stress on any sectional area is the pair of equal and opposite actions which hold the area in its state of strain.

4. *The intensity of stress* is the number of units of force per unit of area. We shall always express it in tons weight, or pounds weight, per square inch; and, for brevity, we shall use the word stress as meaning "intensity of stress," always applying the term "total stress" to the whole force acting on any area. If the intensity of the stress (p) is the same at all points of a given area (A), the stress on the area is said to be uniformly distributed, and P being the total stress on the area, we have $p = \frac{P}{A}$. If the stress is not uniformly distributed, its intensity at any point is given by $p = \frac{dP}{dA}$, where dP is the total stress on the elementary area dA.

5. Hook's Law.—Every stress is accompanied by a strain, and experiments show that in all solid bodies the strain is proportional to the stress which causes it, provided the stress does not exceed certain limits which vary with the material. This is what is known as *Hook's law,—" ut tensio sic vis "* (*as the extension so the force*).

6. The simplest form of stress is that which exists in a bar of uniform section to which equal and opposite forces are applied axially, tending to lengthen or shorten it. If the forces act to lengthen the bar, the stress is called tension, and if they act to shorten it, the stress is called compression; but mathematically considered compression is merely negative tension.

The strains accompanying tension are an elongation in the direction of the pull and a contraction in all directions perpendicular to it; while the strains accompanying compression are the reverse, *i. e.,* a shortening in the direction of the push and an expansion in all directions perpendicular to it. These strains are elastic, that is, they disappear with the removal of the forces which caused them, so long as the tension—or the compression, as the case may be—does not exceed a value which is called the *elastic limit* of the material. Within that limit the strains follow Hook's law.

7. If P be the total pull (or push) on the bar, and A be the area of its right section, the total stress on any such section is P, and, since it is uniformly distributed, its intensity is $p = \frac{P}{A}$. The *elastic limit* * is the value of p beyond which the strain ceases to

* Some writers use the term elastic limit to denote the greatest elastic *strain* under simple tension or compression, instead of the greatest *stress* causing only elastic strains. We shall use the term *elastic limit of strain* to distinguish the former concept, and shall use *elastic limit* to denote the *elastic limit of stress.*

be wholly elastic; if this value is exceeded, the bar takes a permanent set, *i. e.,* when released it will be found to be longer (or shorter) than it was originally. With some materials, notably cast iron, the elastic limit under compression considerably exceeds that under tension, but in the case of steel the difference, if it exists, is not important. The elastic limit of the steel forgings used in modern gun construction is from 35,000 to 75,000 pounds per square inch.

8. The Modulus of Elasticity.—Within the elastic limit the ratio of stress to strain is, by Hook's law, a constant, and the value of this constant for the case of simple tension or compression is called the ***modulus of elasticity*** and is denoted by E. That is to say, if e is the change of length per unit length under the stress $p = \frac{P}{A}$, then $E = \frac{p}{e}$.

Since e is the relative, not the total, strain, it is an abstract number, being, in the case considered, the total change of length of the bar (due to its tension or compression) divided by its length when free. Consequently E is a quantity of the same kind as p and its value depends upon the units in which p is expressed.

When p is given in pounds per square inch, E has the value 29,000,000 for steel; when p is expressed in tons per square inch, E has the value 13,000.

Evidently E is the stress which would double the length of a bar under tension (if it continued to obey Hook's law to that point), since when $e = 1$, $p = E$.

It must be understood that E is the value of the stress on a right section of the bar divided by the strain perpendicular to that section, or in the direction of the external forces causing the strain; the strains at right angles to the axis of the bar, though proportional to the principal strain, are less in value, their ratio to it, determined by experiment, being, in this work, taken to have the value $\frac{1}{3}$.*

9. Example.—As an example, suppose a round steel bar, 2 inches in diameter and 20 inches long, to be under a tension of 60 tons; then the stress on a right section of the bar is $p = \frac{60}{\pi} = 19.1$ tons

* This quantity is known as "Poisson's ratio" from the great French mathematician. Its value varies for different materials, and for steel has been taken by different authorities as ½, ⅓ and ¼. The best modern experiments assign to it a value in the neighborhood of ⅓.

per square inch; the strain in the direction of the axis of the bar is $\frac{p}{E} = \frac{19.1}{13000} = .00147$; and the strain at right angles to the axis is $\frac{.00147}{3} = .00049$. The length of the bar is increased by the tension $20 \times .00147 = .0294$ inches, making its strained length 20.0294 inches; and its diameter is diminished $2 \times .00049 = .00098$, making its strained diameter 1.99902 inches.

If the force of 60 tons were applied to compress the same bar, it would be shortened .0294 inches and its diameter would be increased .00098 inches.

Under tension the volume of the bar is increased in the ratio 1 to 1.000488; while under compression its volume is diminished in the same ratio.

10. If more than one pair of equal and opposite forces act upon a body, the stress upon any sectional area of the body is the resultant of the stresses which would be caused by the pairs of forces acting separately; and the strain at any point due to the simultaneous action of all the stresses is obtained by simply superposing the strains due to the different stresses taken separately.

Thus, taking a rectangular right prism with equal and opposite forces acting normally upon each pair of its opposite faces, let X, Y and Z be the forces acting per unit area of the respective faces: then the stress on each right section perpendicular to the X axis will be X, the stress on each right section perpendicular to the Y axis will be Y, and the stress on each right section perpendicular to the Z axis will be Z. Also, at each point in the prism, the resulting strains in the directions of the axes will be:

$$\left.\begin{aligned} e_x &= \frac{1}{E}\left(X - \frac{Y}{3} - \frac{Z}{3}\right) \\ e_y &= \frac{1}{E}\left(Y - \frac{Z}{3} - \frac{X}{3}\right) \\ e_z &= \frac{1}{E}\left(Z - \frac{X}{3} - \frac{Y}{3}\right) \end{aligned}\right\} \quad (1)$$

In these expressions e_x, e_y and e_z are the changes of length per unit length in the directions of the X, Y and Z axes, respectively, and are plus when they are lengthenings and minus when they are shortenings, provided the stresses X, Y and Z are given plus signs when they are tensions and minus signs when they are compressions.

11. Evidently if either Y or Z be of opposite sign to X, the strain in the X direction may be greater than $\frac{X}{E}$, and similarly the strains in the Y or Z directions may either be greater or less than $\frac{Y}{E}$ and $\frac{Z}{E}$ respectively, according as X, Y and Z are unlike or like forces. If, for example, $X = 15$ tons per square inch tension, and Y and Z are each 15 tons per square inch compression, we have $e_x = \frac{1}{E}\left(15+\frac{15}{3}+\frac{15}{3}\right)=\frac{25}{13000}=.001923$, and the prism would lengthen .001923 inches per inch of its free length instead of only $\frac{15}{E}=.001154$ inches per inch, as would be the case if the stress X alone acted.

12. In our investigations of the strength of guns we accept the following principle:

The total strain in any direction due to all the stresses is the measure of the tendency to yield in that direction, so that the limit of elastic strength is reached, not when the stress in any direction equals the elastic limit of the material, but when the strain in any direction equals the strain which would be caused by the direct action of a single stress equal to that elastic limit.

If, for example, a steel forging has an elastic limit of 58,000 pounds per square inch, *i. e.*, if 58,000 pounds per square inch is the greatest simple tensile stress which the steel will withstand without permanent lengthening, then for the safe use of such a forging it is necessary, and sufficient, that at no point within it shall the strain at any time exceed $\frac{58000}{E}=\frac{58000}{29000000}=.002$ inches per inch in any direction.

13. At any point in a strained solid there are always three planes, at right angles to one another, upon each of which the stress is wholly normal. These three simple stresses (tensions or compressions) are called the *principal stresses* at the point, and their directions are called the *principal axes of stress.*

In the case we are about to investigate—a hollow cylinder under internal and external fluid pressure—the principal axes of stress are evidently radial, circumferential, and longitudinal (parallel to the cylinder's axis), and the principal stresses, which we denote by p, t and q, are illustrated in Figure 1, where one of the elementary prisms of which we imagine the cylinder to be composed is shown in equilibrium under their joint action.

The strains in the directions of the principal axes of stress are called the *principal strains;* they are simple longitudinal strains (lengthenings or shortenings), and their relations to the principal stresses are those given by equations (1).

14. Since the external pressures with which we are to deal are compressive forces, it will be convenient to call the radial stress (p) plus when it acts to compress the material of the cylinder,

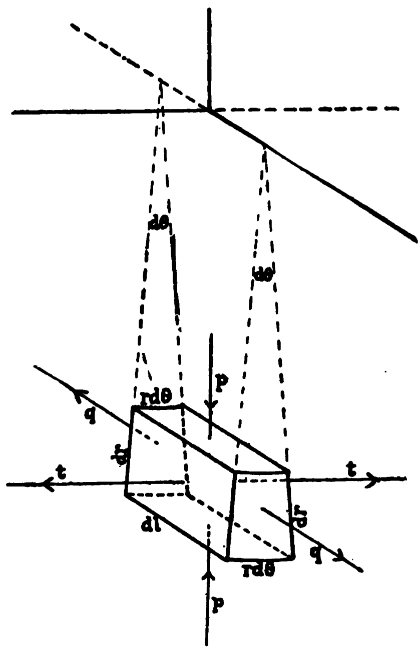


FIG. 1.

though continuing to call the circumferential * stress (t) and the longitudinal stress (q) plus when they produce tension. With this convention, equations (1) become:

$$\left.\begin{aligned} e_t &= \frac{1}{E}\left[t+\frac{p}{3}-\frac{q}{3}\right] \\ e_p &= -\frac{1}{E}\left[p+\frac{q}{3}+\frac{t}{3}\right] \\ e_q &= \frac{1}{E}\left[q-\frac{t}{3}+\frac{p}{3}\right] \end{aligned}\right\} \quad (2)$$

* Tangential stress is synonymous with circumferential stress.

in which e_t is the strain in the direction of the circumference, e_p the strain in the direction of the radius, and e_q the strain in the direction of the axis of the cylinder, in each case a plus value indicating extension and a minus value compression.

In the theory of elasticity it is shown that if an ellipsoid be constructed with semi-axes representing the principal stresses at a point, the stress upon any plane at the point is represented in magnitude and direction by a radius vector of the ellipsoid, which is called the ellipsoid of stress. Evidently, then, one of the three principal stresses acting at each point in a strained solid is the greatest stress at the point. In a similar way it is shown that one of the three principal strains at a point is the greatest strain at the point.

EXAMPLES I.

(1) A round steel rod 1 inch in diameter and 6 feet long is found to stretch .07 inches under a load of 10 tons. What is the intensity of the stress on its transverse section, and what is the value of the modulus of elasticity?

12.73 tons per sq. in.; 13,096 tons.

(2) What length of uniform steel rod, hanging vertically, will just carry its own weight, if the maximum allowable stress is 8 tons per square inch (steel weighs .283 lb. per cu. in.)? 5277 ft.

(3) The ends of a steel I beam whose flanges are 8 inches wide rest on stone supports. If each support takes half the total load of 20 tons, what should the length of bearing surface be, the safe compression stress for stone being 300 lbs. per square inch?

9.3 in.

(4) A bar of steel 2 inches in diameter is bent so that its axis forms the arc of a circle of 372 ft. diameter. What is the greatest strain at any point of the transverse section, and what is the greatest stress? (E for steel is 29,000,000 lbs. in.)

.000448; 12,992 lbs. per sq. in.

(5) A steel bar, 10 inches long and of square section, 1 inch on the side when free, is under 40,000 pounds tension. What are its dimensions under this stress, which is within the elastic limit?

$10.0138 \times \overline{.99954}^2$.

(6) A copper rod of square cross section, 2 inches on the side, and 5 feet long, stretches .0375 inches under a load of 40,000

pounds. What is the modulus of elasticity, and what is the cross-section while the bar is under this stress?

16,000,000 lbs. in.; 3.9983.

(7) A one-inch square steel bar of 32,000 lbs. elastic limit is under a tension of 24,000 lbs.; what pressure per square inch on all of its sides will cause it to lengthen permanently? 12,000 lbs.

(8) If a cube be subjected to equal tensions, or compressions, in each of the three directions normal to its opposite pairs of faces, what relation must exist between the stress of tension, or compression, and the elastic limit of the material in order that the cube may be permanently strained? $p = 3\theta$.

(9) The modulus of elasticity of copper being 16,000,000 (lbs. in.), how much will the length and diameter of a round copper rod, 20 inches long and 3 inches in diameter when free, change under a tensile stress of 9000 lbs. per sq. in.?

.01125 in.; .00056 in.

(10) In order to bring to the vertical opposite walls which have fallen away from each other, round steel rods of 1 in. diameter are stretched from wall to wall and after being heated to 400° C. are set up taut. What pull will each rod exert when its temperature has fallen to 200° C., supposing the walls not to have yielded at all? The coefficient of expansion of steel is .000011 for 1° C.

50,100 lbs.

(11) How much would the steel rod of Example (2), which is 5277 ft. long when free, be increased in length by the stress due to its own weight? 19.56 in.

CHAPTER II.

STRESS AND STRAIN IN SIMPLE HOLLOW CYLINDERS.

15. Consider a horizontal hollow cylinder, open at the ends, which are faced off in planes normal to the axis; and let this cylinder be filled with a fluid which is forced inward by two expanding plungers, the result being a uniform normal pressure upon the entire internal surface of the cylinder. Also let the entire outer cylindrical surface be subjected to a fluid pressure. Then, the ends of the cylinder being free, and there being no longitudinal stress upon its walls, it

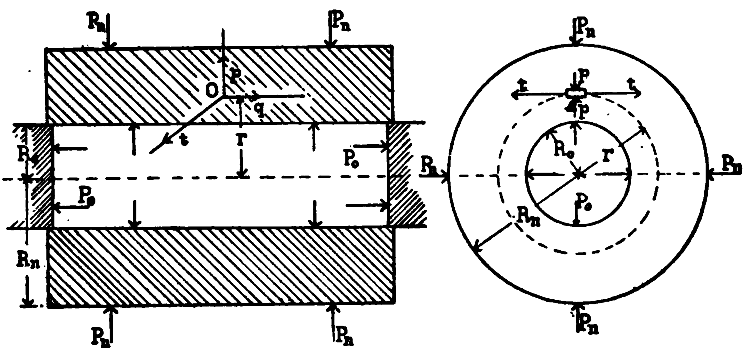


FIG. 2.

is clear that the cylinder will remain a cylinder under the action of the pressures, and that each transverse section normal to the axis will remain a plane normal to the axis. Whatever shortening or lengthening of the cylinder may result from applying internal and external fluid pressure to it must be uniform over its whole cross-section; *i. e.*, the longitudinal strain must, under the stated conditions, be constant throughout the cylindrical walls.

16. Let O be any point (of radius r) within the walls of a cylinder (Figure 2) whose inner and outer radii are R_0 and R_n, and which is subjected to internal and external pressures P_0 and P_n respectively. Also let t, p and q be the circumferential, radial and longitudinal stresses, and e_t, e_p and e_q the circumferential, radial and longitudinal strains, at the point O, E being the modulus of elasticity of the material. And let T_o and T_n be the circumferential tensions at the inner and outer surfaces, or the values of t when $r = R_o$ and when $r = R_n$.

In the strained cylinder, the principal stresses at the point O are evidently the radial pressure p, which varies in value from P_o at the inner to P_n at the outer surface; the circumferential tension t, which varies from T_o at the inner to T_n at the outer surface; and the longitudinal stress q, which is zero in the particular case considered but which might be either tension or compression and either constant or variable. From equations (2), therefore, we obtain as the values of the principal strains,

$$\left.\begin{aligned} e_t &= \frac{1}{E}\left(t + \frac{p}{3}\right) \\ e_p &= -\frac{1}{E}\left(p + \frac{t}{3}\right) \\ e_q &= -\frac{1}{E}\left(\frac{t}{3} - \frac{p}{3}\right) \end{aligned}\right\} \quad (3)$$

and since, under the stated conditions, e_q is constant,

$$* \; t - p = \text{constant} = k \quad (4)$$

If the cylinder is cut by a diametral plane, the whole pressure acting outward upon the section is $2P_oR_o$, and the whole pressure acting inward upon the section is $2P_nR_n$, so that the total force tending to burst the cylinder is $2P_oR_o - 2P_nR_n$. This force must be balanced by the total stress developed in the two sections of the cylinder walls, each of which is $\int_R^{R_n} t dr$.† Thus we have

$$\int_{R_o}^{R_n} t\, dr = P_o R_o - P_n R_n \quad (5)$$

and, assuming $t = f'(r)$, this gives $f(r) \Big]_{R_o}^{R_n} = P_o R_o - P_n R_n$,

from which we see that $f(r) = -pr \pm$ constant, so that the value of $t = f'(r)$ is given by

$$t = -p - r\frac{dp}{dr} \quad (6)$$

* It should be noted that this same result, $t - p =$ constant, follows when q is constant as well as when q is zero.

† We here assume the cylinder to be of unit length.

Combining (6) with (4) we obtain

$$2p+k=-r\frac{dp}{dr} \tag{7}$$

$$\frac{dp}{2p+k}=-\int\frac{dr}{r};\quad \tfrac{1}{2}\log(2p+k)=\log\frac{1}{r}+\log k_1$$

$$\sqrt{2p+k}=\frac{k_1}{r};\quad 2p+k=\frac{k^2{}_1}{r^2} \tag{8}$$

in which k_1 is a constant of integration.

Finally, eliminating k from (8) by means of (4),

$$t+p=\frac{k_1^2}{r^2} \tag{9}$$

17. Equations (4) and (9) express what are known as *Lamé's Laws:* *

1. At any point whatever in a cylinder under fluid pressure the sum of the circumferential tension and the radial pressure varies inversely as the square of the radius.

2. The difference of the circumferential tension and the radial pressure is the same at all points.

These, then, are the equations which express the relation between the circumferential tension and the radial pressure at all points within the cylinder walls:

$$\left.\begin{array}{l} t-p=k=T_o-P_o=T_n-P_n \\ (t+p)r^2=k_1^2=(T_o+P_o)R_o^2=(T_n+P_n)R_n^2 \end{array}\right\} \tag{10}$$

18. Eliminating T_n between the last parts of equations (10), we have

$$T_o=P_o\frac{R_n^2+R_o^2}{R_n^2-R_o^2}-P_n\frac{2R_n^2}{R_n^2-R_o^2}$$

and substituting this value of T_o in the first parts of the same equations, we have, after combining:

$$t=\frac{P_oR_o^2-P_nR_n^2}{R_n^2-R_o^2}+\frac{R_o^2R_n^2(P_o-P_n)}{R_n^2-R_o^2}\frac{1}{r^2} \tag{11}$$

$$p=-\frac{P_oR_o^2-P_nR_n^2}{R_n^2-R_o^2}+\frac{R_oR_n^2(P_o-P_n)}{R_n^2-R_o^2}\frac{1}{r^2} \tag{12}$$

and these equations enable us to determine the values of t and of p at any point.

* As explained, these laws are only strictly true when the longitudinal stress is constant, or zero.

19. To determine the principal strains at any point, we have only to substitute in (3) the values of the principal stresses (t and p) as given in (11) and (12), thus obtaining

$$e_t = \frac{1}{E}\left[\frac{2}{3}\,\frac{P_o R_o^2 - P_n R_n^2}{R_n^2 - R_o^2} + \frac{4}{3}\,\frac{R_o^2 R_n^2 (P_o - P_n)}{R_n^2 - R_o^2}\,\frac{1}{r^2}\right] \quad (13)$$

$$e_p = \frac{1}{E}\left[\frac{2}{3}\,\frac{P_o R_o^2 - P_n R_n^2}{R_n^2 - R_o^2} - \frac{4}{3}\,\frac{R_o^2 R_n^2 (P_o - P_n)}{R_n^2 - R_o^2}\,\frac{1}{r^2}\right] \quad (14)$$

$$e_q = -\frac{1}{E}\left[\frac{2}{3}\,\frac{P_o R_o^2 - P_n R_n^2}{R_n^2 - R_o^2}\right] \quad {}^*(15)$$

The first two of these equations are the fundamental ones from which we shall deduce all the formulæ used in our study of the elastic strength of guns.

The greatest of the three strains given by (13), (14) and (15) for any point in the cylinder walls must not at any time exceed the elastic limit of strain of the material of the cylinder. That is, calling θ the elastic limit of the material, as determined in a testing machine, the limiting value for each of the three strains e_t, e_p and e_q is $\frac{\theta}{E}$.

As e_t and e_p denote the general values of the circumferential and radial strains (at any radius r), we shall distinguish the values of the circumferential and radial strains at radius R_o by $e_t(R_o)$ and $e_p(R_o)$, and those at radius R_n by $e_t(R_n)$ and $e_p(R_n)$.

20. The quantities Ee_t, Ee_p and Ee_q, respectively, equal in value the simple stresses which, acting alone, would cause the strains e_t, e_p and e_q, but these strains are actually caused by the concurrent action of the two stresses p and t. We shall hereafter designate Ee_t, Ee_p and Ee_q as the *true stresses,* circumferential, radial and longitudinal respectively.

21. The distribution of the true stresses throughout the walls of a simple cylinder under fluid pressure is best shown graphically, and we will therefore do this for three cases; first, when the outer pressure (P_n) is zero; second, when the inner pressure (P_o) is zero; and third, when both pressures act and P_o is greater than P_n. In each case we assume a cylinder whose outer is three times its inner radius ($R_n = 3\,R_o$), so that its walls are a caliber thick.

* Note that e_t is a strain while $e_t \times E$ is a stress, so that these equations as they stand represent strain, and may be used to find the stress by multiplying both sides by E.

22. Case I.—No Exterior Pressure.—Putting $P_n = 0$ and $R_n = 3R_o$ in (13), (14) and (15), we obtain as the values of the true stresses:

$$\left.\begin{aligned} Ee_t &= \frac{P_o}{12}\left(1 + \frac{18R_o^2}{r^2}\right) \\ Ee_p &= \frac{P_o}{12}\left(1 - \frac{18R_o^2}{r^2}\right) \\ Ee_q &= -\frac{P_o}{12} \end{aligned}\right\} \quad (16)$$

From these it will be seen that as r increases from R_o to R_n the circumferential true stress diminishes from $\frac{19}{12}P_o$ to $\frac{3}{12}P_o$, its value midway, where $r = 2R_o$, being $\frac{11}{24}P_o$; the radial true stress

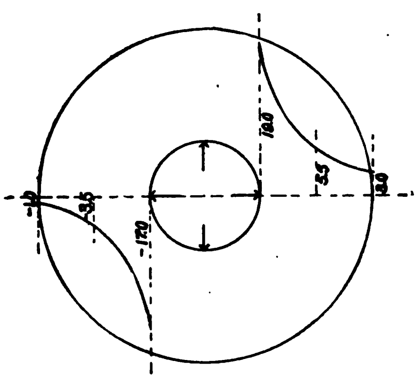


FIG. 3.

diminishes (algebraically it increases) from $-\frac{17}{12}P_o$ to $-\frac{1}{12}P_o$, its value midway being $-\frac{7}{24}P_o$; while the longitudinal true stress has the constant value $-\frac{1}{12}P_o$ throughout the cylinder wall. Figure 3 illustrates the distribution of the tangential and radial true stresses, the former on the right and the latter on the left of the section, the ordinates above the horizontal diameter indicating tensions and those below it indicating compressions. The figures on the inner, middle and outer ordinates are the true stresses in tons per square inch which would result from an internal pressure of 12 tons per square inch.

23. Case II.—No Interior Pressure.—Putting $P_o = 0$ and $R_n = 3R_o$ in (13), (14) and (15) we obtain as the values of the true stresses,

$$\left.\begin{aligned} Ee_t &= -\frac{3P_n}{4}\left(1+\frac{2R_o^2}{r^2}\right) \\ Ee_p &= -\frac{3P_n}{4}\left(1-\frac{2R_o^2}{r^2}\right) \\ Ee_q &= +\frac{3P_n}{4} \end{aligned}\right\} \quad (17)$$

From these it will be seen that as r increases from R_o to R_n, the circumferential true stress diminishes (algebraically it increases)

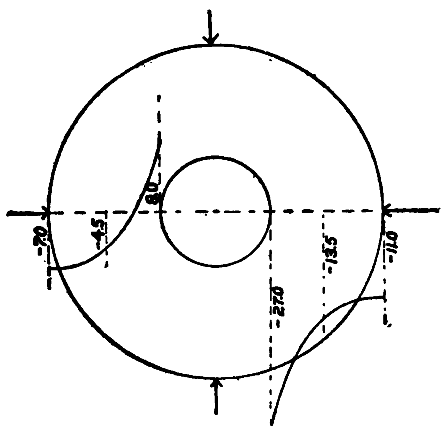


FIG. 4.

from $-\frac{9}{4}P_n$ to $-\frac{11}{12}P_n$, its value midway, where $r = 2R_o$, being $-\frac{9}{8}P_n$; the radial true stress diminishes from $+\frac{3}{4}P_n$ to $-\frac{7}{12}P_n$, its midway value being $-\frac{3}{8}P_n$; while the longitudinal true stress has the constant value $+\frac{3}{4}P_n$. Figure 4 illustrates this, the right-hand curve showing the tangential and the left-hand curve the radial true stress at each point in the wall thickness, ordinates above the horizontal diameter indicating tensions and those below it indicating compressions. The figures on the inner, middle and outer ordinates are the true stresses in tons per square inch which would result from an external pressure of 12 tons per square inch.

24. Case III.—Exterior Pressure One-half the Interior Pressure. —Putting $P_n = \frac{1}{2} P_o$ and $R_n = 3R_o$ in (13), (14) and (15), we obtain as the values of the true stresses:

$$\left.\begin{aligned} Ee_t &= +\frac{P_o}{24}\left(18\frac{R_o^2}{r^2} - 7\right) \\ Ee_p &= -\frac{P_o}{24}\left(18\frac{R_o^2}{r^2} + 7\right) \\ Ee_q &= +\frac{7}{24} P_o \end{aligned}\right\} \quad (18)$$

From these it will be seen that as r increases from R_o to R_n the circumferential true stress diminishes from $\frac{11}{24} P_o$ to $-\frac{5}{24} P_o$, its value midway being $-\frac{5}{48} P_o$; the radial true stress diminishes (algebraically it increases) from $-\frac{25}{24} P_o$ to $-\frac{9}{24} P_o$, its value midway being $-\frac{23}{48} P_o$; while the longitudinal true stress has the constant value $+\frac{7}{24} P_o$. Figure 5 illustrates the distribution of

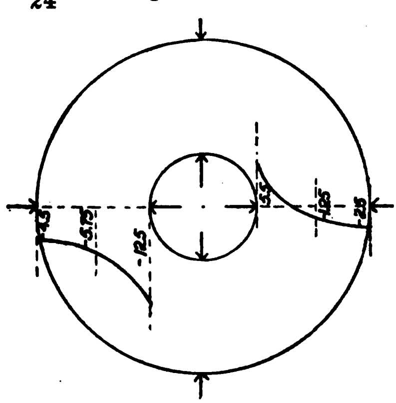


FIG. 5.

the tangential and radial true stresses, the former on the right and the latter on the left of the section, the ordinates above the longitudinal diameter indicating tensions and those below it indicating compressions. The figures on the inner, middle and outer ordinates are the true stresses in tons per square inch which would result from an internal pressure of 12 tons per square inch and an external pressure of 6 tons per square inch.

25. Comparing Figure 5 with Figures 3 and 4, it will be seen that the ordinates of the curves in the former are the algebraic sums of the corresponding ordinates of Figure 3 and half those of Figure 4; the stresses due to 12 tons internal and 6 tons external pressure acting together are the same as the algebraic sums of the stresses due to the same pressures acting separately.

26. Since the strains given by equations (13), (14) and (15) are changes of length per unit length, the change of thickness of the cylinder wall may be determined in any case by integrating $e_p\, dr$ between the limits R_n and R_o. But the change of radius at any point whose radius is r must be re_t, and the difference between the change of the outer radius $(R_n\ e_t(R_n))$ and the change of the inner radius $(R_o\ e_t(R_o))$ must equal the change of thickness Therefore $\int_{R_o}^{R_n} e_p\ dr = R_n\ e_t(R_n) - R_o\ e_t(R_o)$, and it will be found upon trial that this is true of the values of e_p and e_t given by (13) and (14).

27. The hypothesis made in **15** that there is no longitudinal stress, is, of course, not true, as a rule, for actual constructions. In the built-up guns, for example, whose strength we are investigating, one end of the bore is closed by a breech-block which sustains the internal pressure and thus causes a total longitudinal stress $\pi R_o^2 P_o$ which is distributed over the cross-section of one or more of the cylinders of which the gun is composed. This stress may be taken account of by assuming that it is uniformly distributed, but, as will be shown further on, the hypothesis that q is zero accords as well or better with the facts than any other available one.

EXAMPLES II.

(1) Show that in an infinitely thick hollow cylinder $(R_n = \infty)$ subjected only to internal pressure (P_o) the true circumferential and radial stresses at the inner surface are of equal value but opposite sign. What are their values? What is the value of the longitudinal stress?

$$+\frac{4}{3}P_o;\ -\frac{4}{3}P_o;\ 0.$$

(2) What are the true stresses at the inner surface of an infinitely thick hollow cylinder subjected only to external pressure (P_n)?

$$-2P_n; \ +\frac{2}{3}P_n; +\frac{2}{3}P_n.$$

(3) If the external and internal pressures are equal, what is the state of stress in the cylinder walls?

$$Ee_t = Ee_p = -\frac{2}{3}P_o; \ Ee_q = +\frac{2}{3}P_o.$$

(4) What would be the change of thickness of a hollow cylinder one diameter thick under internal pressure alone?

$$-\frac{5}{6}\frac{P_o R_o}{E}.$$

(5) What would be the change of thickness of a hollow cylinder one diameter thick under external pressure only?

$$-\frac{P_n R_o}{2E}.$$

(6) A hollow cylinder half a caliber thick is subjected to an internal pressure of 6 tons per square inch. What is the greatest true stress resulting and where does it occur? What are the true stresses at the outer surface?

$$Ee_t(R_o) = 12 \text{ tons per sq. in.}$$
$$Ee_t(R_n) = 4; \ Ee_p(R_n) = -1\tfrac{1}{3}; \ Ee_q = -1\tfrac{1}{3}.$$

(7) If the cylinder of Example (6) is only one quarter of a caliber thick, what are the true stresses at inner and outer surfaces?

$$\left.\begin{matrix} Ee_t(R_o) = 17\tfrac{3}{5}; \ Ee_p(R_o) = -11\tfrac{1}{5} \\ Ee_t(R_n) = 9\tfrac{3}{5}; \ Ee_p(R_n) = -3\tfrac{1}{5} \end{matrix}\right\} Ee_q = -3\tfrac{1}{5}.$$

(8) Show that, as the thickness of wall of a cylinder under internal pressure is made a smaller and smaller fraction of its inner diameter, the circumferential stress becomes more and more nearly constant throughout the wall. If the circumferential stress were constant, what would be the relation between it and the internal pressure?

$$P_o = \frac{R_n - R_o}{R_o} T.$$

(9) A hollow steel tube, radii 3 in. and 6 in., is subjected to an internal pressure of 13 tons per sq. in. Determine the three principal strains at the inner surface. What is the least elastic limit

of the steel which will permit the application of such a pressure without permanent set?

$$e_t(R_o) = .002;\ e_p(R_o) = -.00156;\ e_q = -.00022.$$

26 tons per sq. in.

(10) With the data of Example (9) determine the three principal strains at the outer surface of the tube.

$$e_t(R_n) = .00067;\ e_p(R_n) = -.00022;\ e_q = -.00022.$$

(11) Show that the change of wall thickness of a cylinder is independent of the value of the external pressure in the case where the outer radius is twice the inner radius.

$$\text{Change} = -\frac{2P_o R_o}{3E}.$$

CHAPTER III.

THE ELASTIC STRENGTH OF SIMPLE HOLLOW CYLINDERS.

28. We will denote the elastic limit under tension of the material of the cylinder by θ and its elastic limit under compression by ρ. In the case of the forged steel used in modern gun construction, these elastic limits are usually taken to be equal, but with some materials, notably cast iron, ρ is considerably greater than θ, and even in the case of steel it is probable that ρ is always somewhat greater than θ.

In accordance with the principle stated in **12**, we consider that the limit of safety is reached whenever either of the principal strains, circumferential, radial or longitudinal, attains the value $\frac{\theta}{E}$ in extension or the value $\frac{\rho}{E}$ in compression; in either case we suppose that the strain ceases to be wholly elastic, and though rupture may not follow, some permanent change of dimensions or distortion will result.

In order, therefore, to determine the maximum pressure which a given cylinder will withstand without permanent set, we have only to equate the greatest strain of extension which results from the pressure to $\frac{\theta}{E}$ and the greatest strain of compression to $\frac{\rho}{E}$ and the least of the pressures given by solving these two equations is the greatest pressure which the cylinder can safely be subjected to. In other words, the limit of the elastic strength of the cylinder is reached when either the greatest true stress of tension equals the elastic limit of the material under simple tension, or the greatest true stress of compression equals the elastic limit of the material under simple compression.

29. Internal Pressure Only.—Putting $P_n = 0$ in (13) we obtain

$$Ee_t = \frac{2P_oR_o^2}{3(R_n^2 - R_o^2)}\left(1 + \frac{2R_n^2}{r^2}\right) \qquad (19)$$

This is always plus, showing that the circumferential true stress is always tension; and its greatest value is when r has its least value R_o. Hence we find the value of P_o which will make the greatest circumferential true stress equal the elastic limit of the material by putting $Ee_t = \theta$ and $r = R_o$ in (19), This gives

$$\theta = \frac{2P_o}{3(R_n^2 - R_o^2)}(R_o^2 + 2R_n^2)$$

$$P_o = \frac{3(R_n^2 - R_o^2)}{4R_n^2 + 2R_o^2}\theta \tag{20}$$

Next putting $P_n = 0$ in (14), we obtain

$$Ee_p = \frac{2P_o R_o^2}{3(R_n^2 - R_o^2)}\left(1 - \frac{2R_n^2}{r^2}\right) \tag{21}$$

This is always negative, showing that the radial true stress is always compression; and its greatest value (numerically) is when $r = R_o$. Hence we find the value of P_o which will make the greatest radial true stress equal the elastic limit of the material by putting $Ee_p = -\rho$ and $r = R_o$ in (21). This gives

$$-\rho = \frac{2P_o}{3(R_n^2 - R_o^2)}(R_o^2 - 2R_n^2)$$

$$P_o = \frac{3(R_n^2 - R_o^2)}{4R_n^2 - 2R_o^2}\rho \tag{22}$$

The determination of the value of the longitudinal true stress is unnecessary, since it can never exceed, and in all practical cases is much less than, one or the other of the other two principal true stresses, the circumferential and the radial.

Now, comparing (20) and (22), since ρ is always equal to or greater than θ, and since the denominator of (20) is greater than the denominator of (22), the value of P_o given by (20) will always be less than the value of P_o given by (22). When P_o reaches the value given by (20), the elastic limit of strain is reached circumferentially, and further increase of P_o is inadmissible.

Consequently the maximum internal pressure allowable in the case of a simple hollow cylinder under no exterior pressure is given by

$$P_o = \frac{3(R_n^2 - R_o^2)}{4R_n^2 + 2R_o^2}\theta \tag{20 bis}$$

in which θ is the elastic limit of the material under tension.

Evidently equation (20) gives not only the relation between the maximum allowable internal pressure and the elastic limit of the material, but equally the relation between any internal pressure and the greatest resulting true stress (within the elastic limit). Moreover, by means of (20) the necessary thickness of a cylinder to safely withstand a given internal pressure is readily determined, since, solving for R_n, we have $R_n = R_o \sqrt{\frac{3\theta + 2P_o}{3\theta - 4P_o}}$.

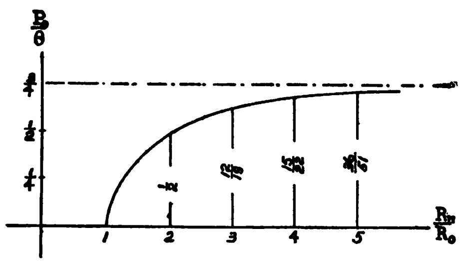


FIG. 6.

Figure 6 shows how the ratio $\frac{P_o}{\theta}$ increases with the ratio $\frac{R_n}{R_o}$, attaining the maximum value $\frac{3}{4}$ when $\frac{R_n}{R_o} = \infty$, and clearly indicates the small effect upon strength of increasing wall thickness beyond a caliber.

30. Examples.—(1) What is the limiting value of the internal pressure which any simple cylinder (regardless of its thickness) will stand without permanent set, the elastic limit of its material being θ? $\frac{3}{4}\theta$.

(2) The walls of a 6-inch steel shell are 1.5 in. thick; if the tensile strength of the steel is 50 tons per sq. in., what powder pressure will burst the shell? 25 tons per sq. in.

(3) What internal pressure will produce a circumferential elongation of .0015 in the case of a simple steel tube of 3 in. interior and 6 in. exterior radius? 9.75 tons per sq. in.

(4) What internal pressure will a cast-steel cylinder of 4 in. internal and 6 in. external radius stand within its elastic limit of 30,000 lbs. per sq. in.? 10,227 lbs. per sq. in.

(5) A nickel-steel cylinder of 7 in. interior radius and 0.5 in. wall thickness has an elastic limit of 70,000 lbs. per sq. in. What internal pressure will it withstand? 4713 lbs. per sq. in.

(6) A cylinder of 7 in. interior diameter has walls 3.5 inches thick. If its elastic limit is 36,000 lbs. per sq. in., what internal pressure will it stand? How much pressure could it withstand if its wall thickness were doubled? if trebled?

18,000; 22,740; 24,550 lbs. per sq. in.

(7) Determine the proper thickness for a cylinder of 6 in. inner radius which is to stand an internal pressure of 3000 lbs. per sq. in., the elastic limit of the material being 28,000 lbs. per sq. in. 0.708 in.

(8) If the radii are 8 in. and 9 in. and the elastic limit is 60,000 lbs. per sq. in., what is the maximum allowable internal pressure? What would it be if the circumferential stress were constant throughout the cylinder walls?

6770; 7500 lbs. per sq. in.

(9) What thickness should a cylinder of 4 in. interior radius have to withstand an internal pressure of 8000 lbs. per sq. in., if the elastic limit is 40,000 lbs. per sq. in.? 0.973 in.

(10) What internal pressure will a cylinder of 6 in. interior radius and 4 in. wall thickness withstand, if the elastic limit is 18 tons per sq. in.? 7.32 tons per sq. in.

31. External Pressure Only.—Putting $P_o = 0$ in (13) we obtain

$$Ee_t = -\frac{2P_n R_n^2}{3(R_n^2 - R_o^2)}\left(1 + \frac{2R_o^2}{r^2}\right) \qquad (23)$$

This is always negative, showing that the circumferential true stress is always compression; and its greatest value is when $r = R_o$. Hence we find the value of P_n which will make the greatest circumferential true stress equal the elastic limit of the material by putting $Ee_t = -\rho$ and $r = R_o$ in (23). This gives

$$\rho = \frac{2P_n R_n^2}{R_n^2 - R_o^2}$$

$$P_n = \frac{R_n^2 - R_o^2}{2R_n^2}\rho \qquad (24)$$

Next, putting $P_o = 0$ in (14), we obtain

$$Ee_p = -\frac{2P_n R_n^2}{3(R_n^2 - R_o^2)}\left(1 - \frac{2R_o^2}{r^2}\right) \qquad (25)$$

This is positive when $r = R_o$ and continues so until r attains the value $R_o\sqrt{2}$, beyond which point it becomes negative; its greatest numerical value, however, is when $r = R_o$. Hence, to find the value of P_n which would make the greatest radial true stress equal the elastic limit of the material, we would put $Ee_p = 0$ and $r = R_o$ in (25). A comparison of (25) with (23), however, will show that, for every value of r, Ee_t is greater than Ee_p, so that the elastic strength of the cylinder depends upon its resistance to circumferential stress and not upon its resistance to radial stress.*

Consequently the maximum external pressure allowable in the case of a simple hollow cylinder under no interior pressure is given by

$$P_n = \frac{R_n^2 - R_o^2}{2R_n^2}\,\rho \qquad (24\text{ bis})$$

in which ρ is the elastic limit of the material under compression.

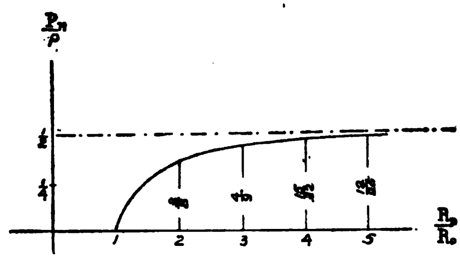


FIG. 7.

Figure 7 shows the increase of the ratio $\frac{P_n}{\rho}$ as wall thickness increases, and clearly indicates how little is gained by going beyond a thickness of one caliber.

Of course (24) expresses the relation between the external pressure and the greatest resulting true stress within as well as at the limit of elastic strain.

32. Examples.—(1) What is the limiting value of the external pressure which any simple hollow cylinder, regardless of its thickness, can withstand without permanent set, the elastic limit of compression being ρ? $\frac{1}{2}\rho$.

* With a material like cast iron, of which the elastic limit of compression greatly exceeds that of tension, the limit of elastic strain radially (in this case extension) may in some cases be reached before the limit of elastic strain circumferentially (in this case compression) is attained.

(2) What external pressure can a tube of 7.5 in. interior radius and 1.75 in. thickness of wall withstand, the elastic limit for compression being 30,000 lbs. per sq. in.? 5139 lbs. per sq. in.

(3) How thick should the tube of Example (2) ($R_o = 7.5$ in.) be to withstand an external pressure of 10,000 lbs. per sq. in.? 5.49 in.

(4) The inner and outer radii of a steel tube are 4 in. and 7 in., and it is to be subjected to an external pressure of 8.3 tons per sq. in. What are the circumferential and radial strains at the inner surface? What is the greatest true stress?

$-.001897$; $+.000632$; 24.65 tons per sq. in.

(5) How thick should the walls of a 6-inch shell be to withstand 6 tons per sq. in. external pressure, without passing the elastic limit of compression of 18 tons per sq. in.? 1.27 in.

(6) What external pressure will a cylinder of 6 in. interior radius and 4 in. wall thickness withstand, if the elastic limit is 18 tons per sq. in.? 5.76 tons per sq. in.

(7) What wall thickness should a cylinder have to withstand 8000 lbs. per sq. in. external pressure, the interior radius being 4 in. and the elastic limit 40,000 lbs. per sq. in.? 1.16 in.

(8) If the interior radius is 8 in., the wall thickness 1 in., and the elastic limit 60,000 lbs. per sq. in., what is the maximum allowable external pressure? What would it be if the circumferential stress were constant throughout the walls?

6297; 6667 lbs. per sq. in.

33. Both Internal and External Pressure.—Which of the true stresses first reaches the elastic limit depends, in this case, upon the relation between the two pressures, and we must consider the three possible cases separately.

$$P_n > P_o$$

In this case both terms of the value of Ee_t, equation (13), are negative, showing that the circumferential true stress is always compression; and its greatest numerical value is when r has its least value R_o. On the other hand, the two terms of the value of Ee_p, equation (14), have opposite signs, showing that the radial true stress may be either tension or compression, according to which term preponderates, and also showing that at each point Ee_t is greater numerically than Ee_p. We therefore obtain an equation between the values of P_o and P_n which will make the greatest true stress resulting from their concurrent action equal the elastic limit of the material by putting $Ee_t = -\rho$ and $r = R_o$ in (13). This gives

$$\rho = \frac{6P_n R_n^2 - 2P_o R_o^2 - 4P_o R_n^2}{3(R_n^2 - R_o^2)}$$

$$P_o = \frac{6P_n R_n^2 - 3(R_n^2 - R_o^2)\rho}{4R_n^2 + 2R_o^2} \tag{26}$$

Consequently, *when P_n exceeds P_o*, the relation between the internal pressure and the maximum allowable external pressure is given by (26), in which ρ is the elastic limit under compression.

$$P_o > P_n \text{ but } P_o R_o^2 < P_n R_n^2.$$

In this case the first term of the value of Ee_t, equation (13), remains negative, while the second term is positive, so that the circumferential true stress may be either tension or compression, according to which term preponderates. But both terms of the value of Ee_p, equation (14), are now negative, showing that the radial true stress is always compression and is numerically greater than Ee_t at each point; moreover, the maximum numerical value of Ee_p is when r has its least value R_o. We therefore obtain an equation between the values of P_o and P_n which will make the greatest true stress resulting from their concurrent action equal to the elastic limit of the material by putting $Ee_p = -\rho$ and $r = R_o$ in (14). This gives

$$\rho = \frac{P_o(4R_n^2 - 2R_o^2) - 2P_n R_n^2}{3(R_n^2 - R_o^2)}$$

$$P_o = \frac{3(R_n^2 - R_o^2)\rho + 2P_n R_n^2}{4R_n^2 - 2R_o^2} \tag{27}$$

Consequently, *when P_o exceeds P_n but at the same time $P_o R_o^2$ is less than $P_n R_n^2$*, the maximum allowable internal pressure is given by (27), in which ρ is the elastic limit of the material under compression.

$$P_o > P_n \text{ and } P_o R_o^2 > P_n R_n^2$$

In this case both terms of the value of Ee_t, equation (13), are positive, showing that the circumferential true stress is always tension; its greatest value occurs when $r = R_o$; and at each point it is numerically greater than Ee_p since the two terms which make up the latter's value are now of different signs. We therefore obtain an equation between the values of P_o and P_n which will make the greatest true stress resulting from their concurrent action equal the elastic limit of the material by putting $Ee_t = \theta$ and $r = R_o$ in (13). This gives

$$\theta = \frac{P_o(4R_n^2 + 2R_o^2) - 6P_n R_n^2}{3(R_n^2 - R_o^2)}$$

$$P_o = \frac{3(R_n^2 - R_o^2)\theta + 6P_n R_n^2}{4R_n^2 + 2R_o^2} \tag{28}$$

Consequently, *when P_o exceeds P_n and at the same time $P_o R_o^2$ exceeds $P_n R_n^2$,* the maximum allowable internal pressure is given by (28), in which θ is the elastic limit of the material under tension.

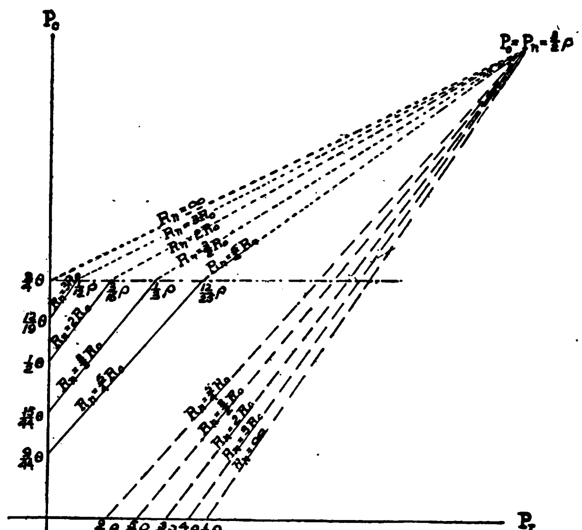


FIG. 8.

Of course equations (26), (27) and (28), each under its appropriate conditions, express the relation between the internal pressure, the external pressure and the greatest resulting true stress within the elastic limit as well as at that limit.

34. The relation between simultaneous values of P_o and P_n which will just bring a given cylinder to the limit of its elastic strength may be graphically shown by drawing the three straight lines represented by equations (26), (27) and (28). In Figure 8, values of P_o are represented by the ordinates, and corresponding values of P_n by the abscissæ, and the cases of five different thicknesses of cylinder wall are illustrated.

Taking (28) first, when $P_n = 0$, the value of P_o is $\frac{3(R_n^2 - R_o^2)}{4R_n^2 + 2R_o^2}\theta$, approaching $\frac{3}{2}\theta$ as a limit when the thickness of the cylinder is indefinitely increased. If P_n is given successively increasing values, the maximum allowable value of P_o also increases, the relation between them being given by (28) and represented by the full lines of the diagram. When P_n attains the value $\frac{3R_o^2}{4R_n^2}\theta$, P_o has the value $\frac{3}{4}\theta$, regardless of the thickness of the cylinder, and with these values of the pressures the inner surface of the cylinder is both at its elastic limit of extension circumferentially and at its elastic limit of compression radially.

Further increase of P_o is allowable if P_n be also increased, but from the point where $P_o = \frac{3}{4}\theta$ the relation between P_o and P_n is given by (27), and represented by the dotted lines of the diagram. When P_o reaches the value $\frac{3}{2}\rho$, P_n must also equal $\frac{3}{2}\rho$, regardless of the thickness of the cylinder wall, and no value of P_n will enable P_o to exceed $\frac{3}{2}\rho$, nor will any value of P_o enable P_n to exceed $\frac{3}{2}\rho$, without the elastic strength of the cylinder being exceeded. At this point, where $P_o = P_n = \frac{3}{2}\rho$, the inner surface of the cylinder is at its elastic limit of compression both radially and circumferentially.

If now P_o be gradually reduced, while P_n is kept as great as allowable, the relation between the values of P_o and P_n will be given by (26) and is represented by the dash lines of the diagram. When P_o has been reduced to zero, the inner surface of the cylinder being maintained at the elastic limit of compression circumferentially, P_n has the value $\frac{R_n^2 - R_o^2}{2R_n^2}\rho$, approaching $\frac{\rho}{2}$ as a limit when the thickness of the cylinder wall is indefinitely increased.

It will be observed that the full and dotted lines represent the relation between P_o and P_n when the former is as great as allowable; while the dash lines represent the relation between P_o and P_n when the latter is as great as allowable.

35. Examples.—(1) If $R_n = 2R_o$ and $P_n = \rho = \theta$, what is the greatest and what the least allowable value of P_o? $\frac{17}{14}\rho$; $\frac{5}{6}\rho$.

(2) If $R_n = \frac{5}{4}R_o$ and $P_n = \rho$, what are the greatest and least allowable values of P_o? $\frac{77}{68}\rho$; $\frac{41}{44}\rho$.

(3) If $R_n = \frac{3}{2}R_o$, what value must P_n have in order that P_o may have the value of $\frac{3}{4}\theta$? $\frac{1}{3}\theta$ or $\frac{8}{9}\theta$.

(4) If $R_n = \frac{5}{4}R_o$, what value must P_n have in order that P_o may have the value $\frac{3}{4}\theta$? $\frac{12}{25}\theta$ or $\frac{21}{25}\theta$.

(5) What internal pressure will a cast-steel cylinder of 4 in. interior and 6 in. exterior radius stand within its elastic limit of 30,000 lbs. per sq. in. if it is under an external pressure of 5000 lbs. per sq. in.? 16,363 lbs. per sq. in.

(6) A nickel-steel cylinder of 7 in. interior radius and 1.5 in. wall thickness has an elastic limit of 70,000 lbs. per sq. in. What external pressure will it withstand if it is under an internal pressure of 10,000 lbs. per sq. in.? 20,190 lbs. per sq. in.

(7) The inner and outer radii of a steel tube are 4 in. and 7 in.; what external pressure will enable it to withstand an internal pressure of 20,000 lbs. per sq. in., if the elastic limit of the steel is 36,000 lbs. per sq. in.? 3390 to 27,630 lbs. per sq. in.

CHAPTER IV.

THE ELASTIC STRENGTH OF COMPOUND CYLINDERS.

36. A reference to Figure 3 will show that the outer portions of a thick simple cylinder play but a small part in resisting internal pressure. A *compound cylinder* is one formed by the superposition of simple cylinders, the object being to utilize to the utmost the contractile power of the outer parts and thus to increase the resistance to internal pressure beyond what it would be if the entire mass were in one piece.

If the elementary cylinders are of the same material, or have equal moduli of elasticity, they must be assembled so that each exerts an initial pressure upon the one within it. This is accomplished by making the interior diameter of each elementary cylinder (before it is put in place) less than the exterior diameter of the cylinder upon which it is to be superposed by a certain quantity which is called the *shrinkage*. A compound cylinder so assembled is said to be under *initial tension*.

If the elementary cylinders are of different materials, and are so arranged that the modulus of elasticity of each is greater than that of the one within it, they may be assembled without shrinkage. Such a cylinder is called a compound cylinder of *variable elasticity*.

These two principles of variable elasticity and of initial tension were formerly often employed in combination, the commonest examples being cast-iron guns with reinforcing hoops of steel, but in modern gun construction, excepting for certain bronze field pieces, steel is now used to the exclusion of other metals, and the principle of initial tension is universally adopted.*

37. In the investigation of the elastic strength of a compound

* Rodman was to some degree successful in applying the principle of initial tension to solid guns, the cast iron smooth bore guns known by his name having been cast hollow and cooled from the interior with the object of securing compression of the bore and tension of the outer parts of the finished gun; and the application of essentially the same process to steel guns, either cast or forged in one piece, has been shown to be feasible and advantageous.

cylinder, it is necessary to consider its state of strain both when the maximum internal pressure is acting and when the internal pressure is zero: the first of these two conditions is called the *state of action* and the second is called the *state of rest.*

In the state of action each cylinder except the outer one is subjected to two pressures, one internal and the other external, while the outer cylinder is subjected to internal pressure only, atmospheric pressure being neglected on account of its insignificant value as compared with the other forces.

In the state of rest the inner cylinder is under external pressure only, the outer cylinder is under internal pressure only, and each of the intermediate cylinders is subjected to both an internal and an external pressure.

38. We adopt the following nomenclature:

R_o and R_1 are the inner and outer radii of the innermost or 1st elementary cylinder, R_1 and R_2 of the next, ... , R_{n-1} and R_n of the outermost or *n*th.

θ_o and ρ_o, θ_1 and ρ_1, ... θ_n and ρ_n are the elastic limits of the material of the elementary cylinders in the same order, from the 1st to the *n*th; and E is their common modulus of elasticity.

P_o, P_1, ... P_n are the radial stresses in the state of action at the successive surfaces of the elementary cylinders, and $\bar{P}_o$, $\bar{P}_1$, ... $\bar{P}_n$ are the radial stresses at the same surfaces in the state of rest; they are always plus, excepting that $\bar{P}_o$, P_n and $\bar{P}_n$, being only atmospheric pressures, are considered to be zero.*

T_o, T_1, ... T_n are the circumferential stresses in the state of action, and $\bar{T}_o$, $\bar{T}_1$, ... $\bar{T}_n$ are the circumferential stresses in the state of rest, at the successive surfaces whose radii are R_o, R_1, ... R_n; they are plus when tensions and minus when compressions.

$e_p(R_o)$, $e_p(R_1)$, ... $e_p(R_n)$ are the radial strains, and $e_t(R_o)$, $e_t(R_1)$, ... $e_t(R_n)$ are the circumferential strains at radii R_o, R_1, ... R_n, in the state of action; the same symbols with a dash over each, as $\bar{e}_p(R_o)$, are the corresponding strains in the state of rest; they are all plus when lengthenings and minus when shortenings.

* This convention that radial stresses which are compressive shall be called positive, is explained in 14: it must be remembered, however, that a radial strain, like all other strains, is called minus when it denotes a decrease of length.

Since the states of stress and strain on either side of the surface of contact of two elementary cylinders may be different (must be if they were assembled with shrinkage), it is necessary to distinguish between them. A prime mark over any letter or symbol indicates that it refers to the outer of the two surfaces which are united by the contact. Thus T_1' is the tension at the inner surface of the second cylinder as distinguished from T_1 which is the tension at the outer surface of the first cylinder; $e_p(R_2')$ is the radial strain in the outer of the two surfaces which meet at R_2; $Ee_t(R_1')$ and $Ee_t(R_1)$ are the circumferential true stresses in the outer and inner of the two surfaces which meet at R_1; and so on. (At R_o and R_n no prime marks are needed, as there is but one surface at each.)

$p_o, p_1, \ldots . p_n$ are the simultaneous changes in the radial pressures $P_o, P_1, \ldots . P_n$ resulting from any cause, such, for example, as the cessation of the internal pressure P_o.

39. Evidently, with any given assemblage of elementary cylinders, the elastic strength to resist internal pressure will be greatest when in the state of action each cylinder is strained to its elastic limit. Moreover, in a compound cylinder so assembled that all the elementary cylinders reach their elastic limits of strain simultaneously under the action of the internal pressure P_o, that pressure must be greater than the pressure P_1 which acts at the surface of contact of the two innermost elementary cylinders; and the pressures at the different surfaces of contact must diminish successively, P_1 being greater than P_2, P_2 greater than P_3, and so on; for the reason that each of these pressures is balanced by the contractile force of only that part of the compound cylinder which is outside of it.

We will first consider a compound cylinder composed of two elementary cylinders so assembled that each reaches the limit of its elastic strength when the internal pressure P_o acts.

Then, since the outer cylinder is at its elastic limit of strain under the sole action of an internal pressure P_1, we have, applying (20),

$$P_1 = \frac{3(R_2^2 - R_1^2)}{4R_2^2 + 2R_1^2}\,\theta_1 \tag{29}$$

And, since the inner cylinder is at its elastic limit of strain under the joint action of an internal pressure P_o and an external pressure

P_1, of which pressures P_o is the greater, we have, applying (27) and (28),

either
$$P_o(\rho) = \frac{3(R_1^2 - R_o^2)\rho_o + 2P_1 R_1^2}{4R_1^2 - 2R_o^2} \tag{30}$$

or
$$P_o(\theta) = \frac{3(R_1^2 - R_o^2)\theta_o + 6P_1 R_1^2}{4R_1^2 + 2R_o^2} \tag{31}$$

of which (30) gives the value of P_o which will bring the inner surface to its elastic limit of strain by *radial compression,* while (31) gives the value of P_o which will bring the inner surface to its elastic limit of strain by *circumferential extension.* The least of these two values of P_o is the true value of the maximum allowable internal pressure, but, since which will be the least depends upon the values of P_1, R_o and R_1, we have to express both values, and we therefore distinguish between them as shown.

40. Having ascertained what maximum internal pressure our assumed compound cylinder will safely withstand, we have next to determine its condition when the internal pressure is removed, for no part of it must be overstrained either in the state of action or in the state of rest.

The state of rest differs from the state of action solely in the cessation of P_o; this must reduce P_1, and consequently the outer cylinder, which is subjected to no other pressure than P_1, must be under less strain after the removal of P_o than while it acts; the inner cylinder, however, while under a less external pressure, is no longer supported by P_o and so may be under greater strain in the state of rest than it was in the state of action. To determine whether this be so, we must find the value of the external pressure to which the inner cylinder is subjected after P_o has been removed.

Putting $r = R_1$ in (13), we obtain for the value of the circumferential strain at the outer surface of the inner cylinder (R_n and P_n becoming R_1 and P_1 in this case),

$$e_t(R_1) = \frac{1}{E}\left[\frac{6P_o R_o^2 - P_1(4R_o^2 + 2R_1^2)}{3(R_1^2 - R_o^2)}\right] \tag{32}$$

Also, remembering that the radii of the outer cylinder are R_1 and R_2, and that it is subjected only to an internal pressure P_1, we obtain for the value of the circumferential strain at the inner surface of the outer cylinder

$$e_t(R_1') = \frac{1}{E}\left[\frac{P_1(2R_1^2 + 4R_2^2)}{3(R_2^2 - R_1^2)}\right] \tag{33}$$

These equations, giving the strains caused by the pressures P_o and P_1, will also give the changes of strain resulting from simultaneous changes of the pressures (p_o and p_1). But the surfaces of contact of the elementary cylinders must contract and expand together, and so the change of circumferential strain at the outer surface of the inner cylinder must equal that which simultaneously occurs at the inner surface of the cylinder embracing it. Hence, substituting p_o for P_o and p_1 for P_1 in the second numbers of (32) and (33), and equating them, we obtain the following relation between simultaneous changes of pressure at $r = R_o$ and $r = R_1$:

$$\frac{6\,p_o R_o^2 - p_1(4R_o^2 + 2R_1^2)}{3(R_1^2 - R_o^2)} = \frac{p_1(2R_1^2 + 4R_2^2)}{3(R_2^2 - R_1^2)}$$

$$3p_o R_o^2 (R_2^2 - R_1^2) = 3p_1 R_1^2 (R_2^2 - R_o^2)$$

$$p_1 = \frac{R_o^2 (R_2^2 - R_1^2)}{R_1^2 (R_2^2 - R_o^2)} p_o \tag{34}$$

Any change of pressure (p_o) at the inner surface, where $r = R_o$, will cause the change of pressure (p_1) at the surface of contact, where $r = R_1$, given by (34); and, vice versa, any change p_1 will cause the change p_o, given by (34). Therefore, putting $p_o = -P_o$ in (34) we have the change in P_1 which results from the suppression of the internal pressure P_o, and so $\overline{P}_1 = P_1 - \frac{R_o^2 (R_2^2 - R_1^2)}{R_1^2 (R_2^2 - R_o^2)} P_o$ is the external pressure to which the inner cylinder is subjected in the state of rest, and this must not exceed $\frac{R_1^2 - R_o^2}{2R_1^2} p_o$, which has been shown in **31** to be the greatest external pressure which, acting alone on the cylinder, is allowable.

41. The Shrinkage.—The excesses of the exterior diameters of the elementary cylinders, before assemblage, over the interior diameters of the cylinders which are to embrace them are called the *shrinkages*, and are designated by S_1, S_2, S_3 etc., S_1 being the shrinkage of the cylinder whose interior radius is R_1, S_2 that of the cylinder whose interior radius is R_2 etc.* The differences of diameter per unit of diameter, $\frac{S_1}{2R_1}$, $\frac{S_2}{2R_2}$, $\frac{S_3}{2R_3}$ etc., are called the *relative shrinkages*, and are designated by ϕ_1, ϕ_2, ϕ_3 etc.

Referring to Figure 9, Oa and Ob represent the inner and outer radii of the inner of two elementary cylinders, and Ob' and Oc the inner and outer radii of the outer one, before assembling, so that $2b'b = S_1$ is the shrinkage; while OA, OB and OC represent the inner radius (R_o), the radius of the surface of contact (R_1) and the outer radius (R_2) after assemblage. When the internal pressure P_o acts, the compound cylinder is expanded, the three radii becoming OA', OB' and OC', respectively, and, by hypothesis, in

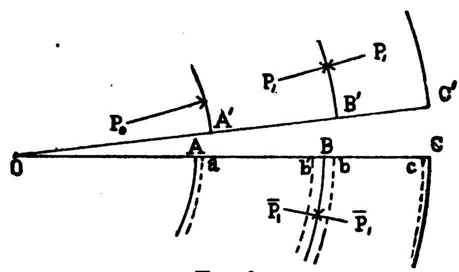


FIG. 9.

this state the inner surface of the outer cylinder is under the circumferential true stress θ_1; *i. e.*, its circumferential strain is $\frac{\theta_1}{E}$. But the change of the inner radius of the outer cylinder from its free state to the state of action is $OB' - Ob'$; therefore $OB' - Ob' = \frac{R_1\theta_1}{E}$. And the change of the outer radius of the inner cylinder from its free state to the state of action is $OB' - Ob$, and this, by (32), is $R_1\, e_t(R_1) = \frac{R_1}{E}\left[\frac{6P_o R_o^2 - P_1(4R_o^2 + 2R_1^2)}{3(R_1^2 - R_o^2)}\right]$. Hence $S_1 = 2b'b = 2[OB' - Ob' - (OB' - Ob)]$ is given by

$$S_1 = \frac{2R_1}{E}\left[\theta_1 - \frac{6P_o R_o^2 - P_1(4R_o^2 + 2R_1^2)}{3(R_1^2 - R_o^2)}\right] \qquad (35)$$

42. The formulæ which we have deduced for this case of a com-

* The shrinkages are so small in comparison with the radii that it is unnecessary to distinguish $R_1 \pm S_1$ from R_1, $R_2 \pm S_2$ from R_2 etc., in the various formulæ.

pound cylinder composed of but two elementary cylinders are grouped together in (36).

$$\left.\begin{aligned}
&(a) \quad P_1 = \frac{3(R_2^2 - R_1^2)}{4R_2^2 + 2R_1^2}\theta_1 \\
&(b) \quad P_o(\theta) = \frac{3(R_1^2 - R_o^2)\,\theta_o + 6P_1 R_1^2}{4R_1^2 + 2R_o^2} \\
&(b') \quad P_o(\rho) = \frac{3(R_1^2 - R_o^2)\,\rho_o + 2P_1 R_1^2}{4R_1^2 - 2R_o^2} \\
&(c) \quad \overline{P}_1 \left(= P_1 - \frac{R_o^2(R_2^2 - R_1^2)}{R_1^2(R_2^2 - R_o^2)} P_o\right) < \frac{R_1^2 - R_o^2}{2R_1^2}\rho_o \\
&(d) \quad S_1 = \frac{2R_1}{E}\left[\theta_1 + \frac{P_1(4R_o^2 + 2R_1^2) - 6P_o R_o^2}{3(R_1^2 - R_o^2)}\right]
\end{aligned}\right\} \quad (36)$$

To apply these formulæ, calculate P_1 and the two values of P_o by (a), (b) and (b′), using for θ_1, θ_o and ρ_o the elastic limits of the material as determined in the testing machine; then, with P_1 and the least of the two values of P_o, determine whether the condition required by (c) is fulfilled; if it is, calculate S_1 with the same values of P_1 and P_o; if it is not, find new values of P_1 and P_o, using the same values of θ_o and ρ_o but a value of θ_1 sufficiently less than the first value assigned it to cause the condition of (c) to be met.

43. As an example, we will determine the strength of a compound cylinder of steel for which $R_o = 3$ in., $R_1 = 5$ in., $R_2 = 8$ in., $\theta_1 = 24$ tons per sq. in., and $\theta_o = \rho_o = 18$ tons per sq. in.

$$P_1 = \frac{3(64 - 25)}{256 + 50} \times 24 = 9.18$$

$$P_o(\theta) = \frac{3(25 - 9) \times 18 + 6 \times 25 \times 9.18}{100 + 18} = 18.99$$

$$P_o(\rho) = \frac{3(25 - 9) \times 18 + 2 \times 25 \times 9.18}{100 - 18} = 16.13$$

An internal pressure of 16.13 tons per sq. in. will bring the radial strain of the inner surface to the elastic limit, and so this is the greatest safe pressure, although the circumferential strain does not reach the elastic limit unless the internal pressure is raised to 18.99

tons per sq. in. We therefore proceed to see if the condition of equation (c) is met with the values $P_1 = 9.18$, $P_o = 16.13$.

$$9.18 - \frac{9(64-25)}{25(64-9)} \times 16.13 < \frac{25-9}{50} \times 18$$

$$9.18 - 4.12 < 5.76$$
$$5.06 < 5.76$$

The external pressure on the inner cylinder in the state of rest is 5.06 tons per sq. in., while it is capable of withstanding 5.76 tons per sq. in. Therefore the values $P_1 = 9.18$ and $P_o = 16.13$ are allowable, and we proceed to determine the shrinkage.

$$S_1 = \frac{10}{13000}\left[24 + \frac{9.18(36+50) - 6 \times 16.13 \times 9}{3(25-9)}\right]$$

$$S_1 = \frac{10}{13000}\left[24 - 1.699\right] = .01715$$

The inner diameter of the outer cylinder must be bored to a diameter .01715 inches less than the outer diameter of the inner cylinder, and, if assembled with this shrinkage, the compound cylinder can be safely subjected to the internal pressure 16.13 tons per sq. in.

44. If the shrinkage used in assembling the compound cylinder be known, the resulting strains and elastic strength are determined as follows:

As shown in **41** and illustrated by Figure 9, the shrinkage is the sum of the contraction of the inner diameter of the outer cylinder and the expansion of the outer diameter of the inner cylinder which would result from disassembling them. In other words, the relative shrinkage is given by $\phi_1 = e_t(R_1') - e_t(R_1)$, in which $e_t(R_1')$ and $e_t(R_1)$ are the circumferential strains at the two surfaces of contact which the pressure between them after assemblage (in this case $\overline{P}_1$) causes. The values of these two strains being obtained by applying (13), we have

$$\frac{1}{E}\left(\frac{\overline{P}_1(2R_1^2 + 4R_2^2)}{3(R_2^2 - R_1^2)}\right) + \frac{1}{E}\left(\frac{\overline{P}_1(4R_o^2 + 2R_1^2)}{3(R_1^2 - R_o^2)}\right) = \phi_1$$

$$\overline{P}_1 = E\frac{(R_1^2 - R_o^2)(R_2^2 - R_1^2)}{2R_1^2(R_2^2 - R_o^2)}\phi_1 \qquad (37)$$

This equation (37) gives the value of the pressure at the surface of contact caused by placing a cylinder of radii R_1 and R_2 over a cylinder of radii R_o and R_1 with the relative shrinkage ϕ_1, and the resulting circumferential strain at R_o being $-\frac{1}{E}\frac{2\overline{P}_1 R_1^2}{R_1^2-R_o^2}$, we have

$$\bar{e}_t(R_o) = -\frac{R_2^2-R_1^2}{R_2^2-R_o^2}\phi_1 \tag{38}$$

by which the relative compression of the bore of the inner cylinder caused by superposing the outer cylinder with the relative shrinkage ϕ_1 may be computed.

Since the only stress at the inner surface in the state of rest is the circumferential compression, the radial strain is one-third the circumferential strain given by (38).

EXAMPLES IV.

(1) Given $R_o = 1.80''$, $R_1 = 2.85''$, $R_2 = 4.50''$, $\theta_o = \rho_o = 18.75$ tons, $\theta_1 = \rho_1 = 21.50$ tons; find $P_o(\theta)$, $P_o(\rho)$ and S_1; also the compression at R_o in the state of rest.

$P_o(\theta) = 17.11$; $P_o(\rho) = 15.58$ tons.
$S_1 = .0074$ in.
$\overline{P}_1 = 3.61$; $E\bar{e}_t(R_o) = -12.02$ tons.

(2) Given $R_o = 2.85''$, $R_1 = 4.70''$, $R_2 = 7.50''$, $\theta_o = \rho_o = 18.75$ tons, $\theta_1 = \rho_1 = 21.5$ tons; find $P_o(\theta)$, $P_o(\rho)$ and S_1; also the compression at R_o in the state of rest.

$P_o(\theta) = 17.88$; $P_o(\rho) = 15.91$ tons.
$S_1 = .0130$ in.
$\overline{P}_1 = 4.03$; $E\bar{e}_t(R_o) = -12.75$ tons.

(3) Given $R_o = 4.00''$, $R_1 = 6.35''$, $R_2 = 8.04''$, $\theta_o = \rho_o = 18.5$ tons, $\theta_1 = \rho_1 = 21.0$ tons; find $P_o(\theta)$, $P_o(\rho)$ and S_1; also the compression at R_o in the state of rest.

$P_o(\theta) = 12.64$; $P_o(\rho) = 13.26$ tons.
$S_1 = .0126$ in.
$\overline{P}_1 = 2.01$; $E\bar{e}_t(R_o) = -6.67$ tons.

(4) Given $R_o = 6.00''$, $R_1 = 8.70''$, $R_2 = 10.46''$, $\theta_o = \rho_o = 18.5$ tons, $\theta_1 = \rho_1 = 21.0$ tons; find $P_o(\theta)$, $P_o(\rho)$ and S_1; also the compression at R_o in the state of rest.

$P_o(\theta) = 10.25$; $P_o(\rho) = 11.91$ tons.
$S_1 = .0148$ in.
$\overline{P}_1 = 1.37$; $E\bar{e}_t(R_o) = -5.22$ tons.

(5) Given $R_o = 4.00''$, $R_1 = 5.80''$, $R_2 = 7.14''$, $\theta_o = \rho_o = 18.5$ tons, $\theta_1 = \rho_1 = 21.0$ tons; find $P_o(\theta)$, $P_o(\rho)$ and S_1; also the compression at R_o in the state of rest.

$$P_o(\theta) = 10.60; \; P_o(\rho) = 12.19 \text{ tons.}$$
$$S_1 = .0105 \text{ in.}$$
$$\overline{P}_1 = 1.53; \; E\overline{e}_t(R_o) = -5.83 \text{ tons.}$$

(6) Given $R_o = 4.00''$, $R_1 = 5.80''$, $R_2 = 7.14''$, if the shrinkage was $S_1 = .0105$, what is the pressure at the surface of contact and what is the compression of the bore (at R_o) in the state of rest? (Compare result with answers to Example (5).)

$$\overline{P}_1 = 1.53; \; E\overline{e}_t(R_o) = -5.83 \text{ tons.}$$

CHAPTER V.

THE ELASTIC STRENGTH OF COMPOUND CYLINDERS.—CONTINUED.

45. The true stresses, circumferential and radial, at the inner and outer surfaces of each of the elementary cylinders are readily calculated by (13) and (14), which, when applied to the case of a compound cylinder of two parts, become

Circumferential True Stresses.

$$\left.\begin{aligned} Ee_t(R_o) &= \frac{P_o(2R_o^2+4R_1^2)-6P_1R_1^2}{3(R_1^2-R_o^2)} \\ Ee_t(R_1) &= \frac{6P_oR_o^2-P_1(4R_o^2+2R_1^2)}{3(R_1^2-R_o^2)} \\ Ee_t(R_1') &= \frac{P_1(2R_1^2+4R_2^2)}{3(R_2^2-R_1^2)} \\ Ee_t(R_2) &= \frac{2P_1R_1^2}{R_2^2-R_1^2} \end{aligned}\right\} \quad (39)$$

Radial True Stresses.

$$(40)\quad \left\{\begin{aligned} Ee_p(R_o) &= \frac{2P_1R_1^2+P_o(2R_o^2-4R_1^2)}{3(R_1^2-R_o^2)} \\ Ee_p(R_1) &= \frac{P_1(4R_o^2-2R_1^2)-2P_oR_o^2}{3(R_1^2-R_o^2)} \\ Ee_p(R_1') &= \frac{P_1(2R_1^2-4R_2^2)}{3(R_2^2-R_1^2)} \\ Ee_p(R_2) &= \frac{-2P_1R_1^2}{3(R_2^2-R_1^2)} \end{aligned}\right.$$

in which, for the state of action, P_o and P_1 have the values used in calculating the shrinkage, and, for the state of rest, P_o is zero and P_1 is the pressure at the surface of contact when $P_o = 0(\overline{P}_1)$.

Applying (39) and (40) to the example worked out in **43**, for which $P_o = 16.13$, $P_1 = 9.18$ and $\overline{P}_1 = 5.06$, we obtain the results illustrated in Figure 10, the right-hand side of which represents circumferential and the left-hand side radial true stresses, full lines indicating the state of action and dotted lines the state of rest.

It will be seen that in the state of action both cylinders are at the elastic limit of strain, the inner one radially and the outer one circumferentially.

46. The fact that the greater the value of P_o used in calculating the shrinkage the less the shrinkage and consequently the less the stresses in the state of rest, suggests an investigation of the results of always using $P_o(\theta)$ in (36d) instead of using $P_o(\rho)$ when it is the smaller of the two values of P_o.

In the example of **43** the shrinkage found by using $P_o(\rho) = 16.13$ tons was 0.01715″; if we had used $P_o(\theta) = 18.99$ tons, we would have found the shrinkage to be 0.01468″, or nearly 0.0025″ less. The true stresses in the states of action and of rest have been computed for the greater shrinkage; we will now determine their values under the same conditions ($P_o = 16.13$ tons and $P_o = 0$), supposing the reduced shrinkage to be used.

With the reduced shrinkage the value $P_1 = 9.18$ corresponds to $P_o = 18.99$, and so we have first to find the change in P_1 which results from reducing P_o from 18.99 to 16.13; this by (34) is -0.73, making the value of P_1 for our assumed state of action $9.18 - 0.73 = 8.45$. Substituting the values $P_o = 16.13$ and $P_1 = 8.45$ in (39) and (40), we obtain the values of the true stresses in the state of action. For the state of rest we find $\overline{P_1} = 4.33$ by

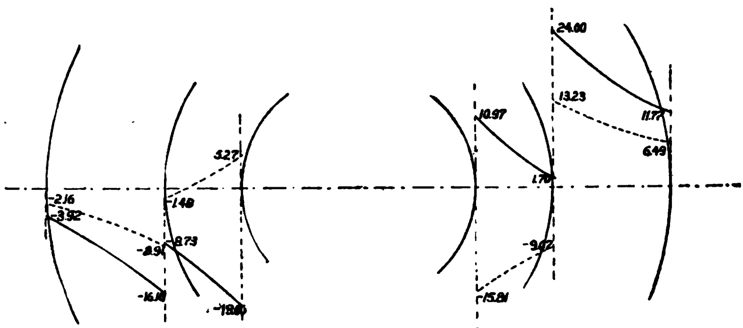


FIG. 10.

Radial True Stresses.
———— State of action.
– – – – – " " rest.

Circumferential True Stresses.
———— State of action.
– – – – – " " rest.

(36c), getting the same result, of course, whether we use $P_o = 18.99$ and $P_1 = 9.18$ or $P_o = 16.13$ and $P_1 = 8.45$; then, putting $P_o = 0$ and $P_1 = \overline{P_1} = 4.33$ in (39) and (40), we get the values of the true stresses in the state of rest.

The following table gives, side by side, the true stresses resulting from the use of the full and the reduced shrinkages:

		Circumferential Stress. Full Shrinkage.	*Circumferential Stress.* Reduced Shrinkage.	*Radial Stress.* Full Shrinkage.	*Radial Stress.* Reduced Shrinkage.
State of Action. $P_o = 16.13$ tons.	Inner cylinder, inner surface	+10.97	+13.25	−18.00	−18.76
	" " outer "	+ 1.70	+ 3.01	− 8.73	− 8.52
	Outer " inner "	+24.00	+22.09	−16.16	−14.88
	" " outer "	+11.77	+10.83	− 3.92	− 3.61
State of Rest.	Inner cylinder, inner surface	−15.81	−13.53	+ 5.27	+ 4.51
	" " outer "	− 9.07	− 7.76	− 1.48	− 1.26
	Outer " inner "	+13.23	+11.83	− 8.91	− 7.62
	" " outer "	+ 6.49	+ 5.55	− 2.16	− 1.85

47. It will be seen that the reduced shrinkage, given by adopting $P_o(\theta)$ instead of $P_o(\rho)$ as the value of P_o, results in a slight loss of elastic strength,* since the internal pressure (16.13 tons) which with full shrinkage just compressed the inner surface to its elastic limit of strain radially, with the reduced shrinkage compresses that surface slightly beyond its elastic limit. As an offset to this, the smaller shrinkage considerably reduces all the stresses in the state of rest, and those of the outer cylinder in the state of action. Moreover, there is reason to suppose that the elastic strength to resist radial compression in the case of a cylinder wall confined by an outer cylinder is greater than would be indicated by the elastic limit of compression of specimens of its material, so that the value of $P_o(\rho)$ may probably be exceeded without producing any permanent set. At all events, it is not radial compression, but circumferential extension, an excessive value of which will cause enlargement and ultimately rupture, and we are therefore adopting a measure of safety when we adjust the shrinkage so as to cause the elementary cylinders to reach their elastic limits of circumferential strain simultaneously, even though it be under a pressure greater than that which will cause the inner one of them to reach its elastic limit of radial strain.

For these reasons the Ordnance Departments of the United States Army and Navy have adopted the practice of disregarding the values of $P_o(\rho)$ and determining the shrinkages for the superposed cylinders of their built-up steel guns by using the values of $P_o(\theta)$.

We will follow the same method, using $P_o(\theta)$ for computing shrinkages, but still regarding $P_o(\rho)$, when it is less than $P_o(\theta)$, as the upper limit of safe internal pressure.

48. In **40**, by equating the simultaneous changes of circumferential strain of the two surfaces in contact at R_1, we found the relation (34) between simultaneous changes of P_o and P_1 in the case of a compound cylinder composed of two elementary cylinders. The same relation might as readily have been found from the consideration that, within the elastic limit, the stresses and strains

* With the reduced shrinkage the internal pressure which will bring the inner surface to its elastic limit of radial strain is given by $P_o = \frac{R_2^2 - R_o^2}{R_1^2 - R_o^2} \cdot \frac{3(R_1^2 - R_o^2)\rho_o + 2\overline{P}_1 R_1^2}{4R_2^2 - 2R_o^2}$, the value of which for the example of 43 is 15.62 tons.

resulting from the application of any force are independent of prior stresses and strains, so that the effect of an internal pressure is exactly the same upon a compound cylinder as it would be upon a simple cylinder of the same dimensions. Thus, putting $P_n = 0$ and substituting R_2 for R_n in (12), we obtain for the pressure at any point in a homogeneous cylinder of radii R_o and R_2 under the sole pressure P_o,

$$P(r) = \frac{P_o R_o^2}{R_2^2 - R_o^2}\left(\frac{R_2^2}{r^2} - 1\right) \tag{41}$$

and, making $r = R_1$ in this, we find

$$P(R_1) = \frac{R_o^2 (R_2^2 - R_1^2)}{R_1^2 (R_2^2 - R_o^2)} P_o$$

which is the same as the relation given by (34).

49. The general principle of which the foregoing is an illustration may be stated as follows:

If any pressure be applied to a compound cylinder, the strain (or stress) at each point will be the algebraic sum of the strain (or stress) at the point before the pressure was applied and the strain (or stress) which the same pressure would cause at the corresponding point in a simple cylinder of the same dimensions as the compound one.

50. An important application of this principle shows that the maximum strength of any compound cylinder to resist internal pressure cannot exceed three-fourths the sum of the elastic limits of tension and compression of its inner elementary cylinder, regardless of the strength of its outer parts. For in the state of rest the pressure upon the inner cylinder due to the outer ones is limited to that which will compress the inner surface circumferentially to its elastic limit of compressive strain $\frac{\rho_o}{E}$; and in the state of action the internal pressure is limited to that which will extend the inner surface circumferentially to its elastic limit of tensile strain $\frac{\theta_o}{E}$; therefore the greatest allowable value of P_o is that which, acting upon a simple cylinder of the same dimensions as the compound one, would cause the circumferential strain $\frac{\rho_o + \theta_o}{E}$ at its inner surface, and, calling the inner and outer radii R_o and R_n, the value of this greatest pressure is by (20),

$$P_o = \frac{3(R_n^2 - R_o^2)}{4R_n^2 + 2R_o^2}(\rho_o + \theta_o) \tag{42}$$

the maximum value of which, when $R_n = \infty$, is $\frac{3}{4}(\rho_o + \theta_o)$, or, when $\theta_o = \rho_o$, $\frac{3}{2}\theta$.

This maximum possible value of the elastic resistance will hereafter be denoted by $[P_o]$, and, since we accept the condition $\rho_o = \theta_o$, it will be written

$$[P_o] = \frac{3(R_n^2 - R_o^2)}{2R_n^2 + R_o^2}\theta_o. \tag{43}$$

This is the maximum possible value of $P_o(\theta)$; $P_o(\rho)$ cannot exceed ρ_o in value.

51. From the formulæ for a compound cylinder of two parts, those for the general compound cylinder (of n parts) may be directly derived, but as the case of three elementary cylinders is the commonest in gun construction, we will deduce the formulæ for that case separately, and explain how they should be used.

We begin by finding the values of the pressures in the state of action (P_2, P_1 and P_o), supposing the cylinders to have been so assembled that they reach their elastic limits of circumferential strain simultaneously.

The outer cylinder being under the sole action of the internal pressure P_2, we have from (20),

$$P_2(\theta) = \frac{3(R_3^2 - R_2^2)}{4R_3^2 + 2R_2^2}\theta_2 \tag{44}$$

The middle cylinder being under the external pressure P_2 and the internal pressure P_1, of which the latter is the greater, we have from (28),

$$P_1(\theta) = \frac{3(R_2^2 - R_1^2)\theta_1 + 6P_2R_2^2}{4R_2^2 + 2R_1^2} \tag{45}$$

And the inner cylinder being under the external pressure P_1 and the internal pressure P_o, of which the latter is the greater, we have from (28),

$$P_o(\theta) = \frac{3(R_1^2 - R_o^2)\theta_o + 6P_1R_1^2}{4R_1^2 + 2R_o^2} \tag{46}$$

Before adopting these values of P_2, P_1 and P_o, we must see that the shrinkages which they require will not over-compress the inner surface in the state of rest. This is most readily done by computing $[P_o]$ by (43) and comparing it with $P_o(\theta)$; if the latter be the greater, the inner surface would be compressed beyond its elastic limit of circumferential strain when in the state of rest, and so less values must be assigned to one or both the elastic limits of the outer cylinders and new values of P_2, P_1 and P_o computed. When the assumed values of θ_2, θ_1 and θ_o are such that $[P_o]$ exceeds $P_o(\theta)$, the inner cylinder will not be too much compressed, and then the values of $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$ given by (42), (43) and (44) may be accepted.

52. The formulæ for the shrinkages are deduced by the same method that was explained in **41**. The inner surface of the outer cylinder when in the state of action is, by hypothesis, under the circumferential strain $\frac{\theta_2}{E}$, so that its diameter is $2R_2\frac{\theta_2}{E}$ greater than when it was free (before assembling). If, then, we find the change of diameter $(2R_2e_t(R_2))$ of the outer surface of the middle cylinder which would result from the simultaneous removal of the outer cylinder and suppression of the internal pressure P_o, the shrinkage with which the outer cylinder was assembled will evidently be given by $S_2 = 2R_2\frac{\theta_2}{E} + 2R_2e_t(R_2)$.

By substituting R_2 for R_n, P_2 for P_n and R_2 for r in (13) we obtain the following expression for the circumferential strain at the outer surface of a cylinder of radii R_o and R_2 under internal pressure P_o and external pressure P_2:

$$e_t(R_2) = \frac{1}{E}\left[\frac{6P_oR_o^2 - P_2(4R_o^2 + 2R_2^2)}{3(R_2^2 - R_o^2)}\right] \qquad (47)$$

But by the principle laid down in **49** the same expression gives the change of strain which the application of the same pressures will cause in a compound cylinder of the same dimensions. Therefore, putting $-P_o$ for P_o and $-P_2$ for P_2 in (47), we obtain the change of circumferential strain at R_2 due to suppressing P_o and P_2, and this multiplied by $2R_2$ will be the change of diameter. Consequently the shrinkage of the outer cylinder is given by

$$S_2 = \frac{2R_2}{E}\left[\theta_2 + \frac{P_2(4R_o^2 + 2R_2^2) - 6P_oR_o^2}{3(R_2^2 - R_o^2)}\right] \qquad (48)$$

Similarly the change of circumferential strain at the outer surface of the inner cylinder due to removing the two outer cylinders (*i. e.,* suppressing P_1) and simultaneously suppressing P_o is found to be

$$e_t(R_1) = \frac{1}{E}\left[\frac{P_1(4R_o^2+2R_1^2)-6P_o R_o^2}{3(R_1^2-R_o^2)}\right] \tag{49}$$

and so the shrinkage of the middle cylinder is

$$S_1 = \frac{2R_1}{E}\left[\theta_1 + \frac{P_1(4R_o^2+2R_1^2)-6P_o R_o^2}{3(R_1^2-R_o^2)}\right] \tag{50}$$

52. We have, finally, to determine the elastic strength to resist internal pressure of the system thus assembled. We know that $P_o(\theta)$ is the pressure which will bring its elementary cylinders simultaneously to their assumed elastic limits of circumferential strain, but a less pressure may bring one or more of them to the elastic limit of radial strain, and, if so, this latter pressure, and not $P_o(\theta)$, should be taken as the maximum safe pressure.

The outer cylinder being under internal pressure only, $P_2(\theta)$ is always less than $P_2(\rho)$, as explained in **29**. Applying (27) to the middle and inner cylinders, we obtain the following values for the respective internal pressures which will bring them to their elastic limits of radial strain:

$$P_1(\rho) = \frac{3(R_2^2-R_1^2)\rho_1 + 2P_2 R_2^2}{4R_2^2-2R_1^2} \tag{51}$$

$$P_o(\rho) = \frac{3(R_1^2-R_o^2)\rho_o + 2P_1 R_1^2}{4R_1^2-2R_o^2} \tag{52}$$

If $P_1(\rho)$ given by (51) is less than the value of $P_1(\theta)$ used in computing the shrinkages, then the former is to be used for P_1 in (52) instead of the latter, and if $P_o(\rho)$ given by (52) is less than the value of $P_o(\theta)$ used in computing the shrinkages, it, and not $P_o(\theta)$, is the maximum safe pressure. That is, with $P_o(\rho) < P_o(\theta)$, the former would be a safe pressure if suitable shrinkages were assigned, but since, for good reasons, we adopt shrinkages based upon the values of $P_1(\theta)$ and $P_o(\theta)$, the actual maximum safe pressure is somewhat less than $P_o(\rho)$. We will call the true maximum safe pressure P_o, thus distinguishing it from $P_o(\rho)$ and $P_o(\theta)$; its value, when it does not equal $P_o(\theta)$, is found as follows:

The pressures in the state of rest are given by (53) and (54), the negative part of each value being the change of pressure due to the suppression of $P_o(\theta)$:

$$\overline{P}_2 = P_2(\theta) - \frac{R_o^2\,(R_3^2 - R_2^2)}{R_2^2\,(R_3^2 - R_o^2)}\,P_o(\theta) \qquad (53)$$

$$\overline{P}_1 = P_1(\theta) - \frac{R_o^2\,(R_3^2 - R_1^2)}{R_1^2\,(R_3^2 - R_o^2)}\,P_o(\theta) \qquad (54)$$

Then by (14) the radial strain at the inner surface of the inner cylinder, in the state of rest, is $\frac{1}{E}\,\frac{2\overline{P}_1\,R_1^2}{3\,(R_1^2 - R_o^2)}$, and the internal pressure which will change this radial strain to $-\frac{\rho_o}{E}$, *i. e.*, which will bring the inner surface to its elastic limit of compression radially, is, by the principle of **49**,

$$P_o = \frac{3\,(R_3^2 - R_o^2)}{2\,R_o^2 - 4R_3^2}\left(-\frac{2\overline{P}_1\,R_1^2}{3\,(R_1^2 - R_o^2)} - \rho_o\right)$$

$$P_o = \frac{R_3^2 - R_o^2}{R_1^2 - R_o^2}\cdot\frac{3\,(R_1^2 - R_o^2)\,\rho_o + 2\overline{P}_1\,R_1^2}{4R_3^2 - 2R_o^2} \qquad (55)$$

The same method applied to the middle cylinder, which in the state of rest is acted on by P_2 externally and P_1 internally, would determine the internal pressure which would bring its inner surface to the elastic limit of compression radially,* but this pressure will practically always be greater than that given by (55), and, accordingly, (55) gives the true elastic strength of the system.

* The formula is

$$P_o = \frac{R_1^2\,(R_3^2 - R_o^2)}{R_o^2\,(4R_3^2 - 2R_1^2)\,(R_2^2 - R_1^2)}\left[3\,(R_2^2 - R_1^2)\,\rho_1 + 2\overline{P}_2\,R_2^2 - \overline{P}_1\,(4R_2^2 - 2R_1^2)\right]$$

54. The formulæ for the case of a compound cylinder composed of three elementary cylinders are grouped together in (56):

$$
\left.
\begin{array}{ll}
(a) & P_2(\theta) = \dfrac{3\,(R_3^2 - R_2^2)\,\theta_2}{4\,R_3^2 + 2R_2^2} \\[2ex]
(b) & P_1(\theta) = \dfrac{3\,(R_2^2 - R_1^2)\,\theta_1 + 6P_2\,R_2^2}{4R_2^2 + 2R_1^2} \\[2ex]
(c) & P_o(\theta) = \dfrac{3\,(R_1^2 - R_o^2)\,\theta_o + 6P_1\,R_1^2}{4R_1^2 + 2R_o^2} \\[2ex]
(d) & P_o(\theta) < \dfrac{3\,(R_3^2 - R_o^2)\,(\theta_o + \rho_o)}{4R_3^2 + 2R_o^2} \\[2ex]
(e) & P_1(\rho) = \dfrac{3\,(R_2^2 - R_1^2)\,\rho_1 + 2P_2\,R_2^2}{4R_2^2 - 2R_1^2} \\[2ex]
(f) & P_o(\rho) = \dfrac{3\,(R_1^2 - R_o^2)\,\rho_o + 2P_1\,R_1^2}{4R_1^2 - 2R_o^2} \\[2ex]
(g) & S_2 = \dfrac{2R_2}{E}\left[\theta_2 + \dfrac{P_2(\theta)\,(4R_o^2 + 2R_2^2) - 6P_o(\theta)\,R_o^2}{3\,(R_2^2 - R_o^2)}\right] \\[2ex]
(h) & S_1 = \dfrac{2R_1}{E}\left[\theta_1 + \dfrac{P_1(\theta)\,(4R_o^2 + 2R_1^2) - 6P_o(\theta)\,R_o^2}{3\,(R_1^2 - R_o^2)}\right] \\[2ex]
(i) & \overline{P}_2 = P_2(\theta) - \dfrac{R_o^2\,(R_3^2 - R_2^2)}{R_2^2\,(R_3^2 - R_o^2)}\,P_o(\theta) \\[2ex]
(j) & \overline{P}_1 = P_1(\theta) - \dfrac{R_o^2\,(R_3^2 - R_1^2)}{R_1^2\,(R_3^2 - R_o^2)}\,P_o(\theta) \\[2ex]
(k) & P_o = \dfrac{R_3^2 - R_o^2}{R_1^2 - R_o^2} \cdot \dfrac{3\,(R_1^2 - R_o^2)\,\rho_o + 2\overline{P}_1\,R_1^2}{4R_3^2 - 2R_o^2}
\end{array}
\right\} \quad (56)
$$

The method of procedure is as follows:

(1) Calculate $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$ by (a), (b) and (c), using for θ_2, θ_1 and θ_o the elastic limits of the materials as determined in a testing machine.

(2) See if the condition (d) is fulfilled. If it is not, find new values of $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$, using values of θ_2 and θ_1 (one or both) sufficiently less than their true values to cause the condition (d) to be met.

(3) Calculate the shrinkages by (g) and (h), using the values of θ_2, θ_1, $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$ which satisfied (d).

(4) Calculate $P_1(\rho)$ and $P_o(\rho)$ by (e) and (f), using for ρ_o

and ρ_1 the true elastic limits of the materials, and for P_1 in (f) putting whichever is least, $P_1(\rho)$ or the value of $P_1(\theta)$ calculated with the true values of θ_2 and θ_1.

(5) If $P_o(\rho)$ is greater than the value of $P_o(\theta)$ used in computing the shrinkages, the latter is the true measure of the elastic strength of the system; if it be less, then P_o, calculated by (k), is the true measure.

55. To find the state of strain at the inner surface (at R_o) caused by superposing the two outer cylinders with relative shrinkages, respectively ϕ_1 and ϕ_2, we have only to apply (38) to this case, the strain resulting from the compressive action of both outer cylinders being merely the sum of the strains caused by their actions considered separately. Thus we have

$$\overline{e}_t(R_o) = -\frac{R_2^2 - R_1^2}{R_2^2 - R_o^2}\phi_1 - \frac{R_3^2 - R_2^2}{R_3^2 - R_o^2}\phi_2 \tag{57}$$

Moreover, the radial strain at the inner surface in the state of rest ($\overline{e}_p(R_o)$) will be one-third the circumferential strain given by (57).

EXAMPLES V.

(1) Given $R_o = 7.0''$, $R_1 = 9.5''$, $R_2 = 15.0''$, $R_3 = 21.0''$; if $\theta_o = \rho_o = 20.0$ tons, what is the greatest possible value of the internal pressure which can be withstood elastically? If $\theta_o = 20.0$, $\theta_1 = 21.4$ and $\theta_2 = 22.3$ tons, find $P_o(\theta)$, S_1 and S_2. What is the true elastic strength after assemblage with the shrinkages based on the value of $P_o(\theta)$?

$[P_o] = 25.26$; $P_o(\theta) = 24.46$ tons.
$S_1 = .0183$; $S_2 = .0386$.
$\overline{P}_1 = 4.28$; $P_o = 18.52$ tons.

(2) Given $R_o = 5.0''$, $R_1 = 9.5''$, $R_2 = 15.0''$, $R_3 = 19.0''$, $\theta_o = \rho_o = 20.0$ tons; what is the limiting value for the internal pressure? If $\theta_o = 20.0$, $\theta_1 = 21.0$, and $\theta_2 = 24$ tons, find $P_o(\theta)$. If assembled with the shrinkages corresponding to the value of $P_o(\theta)$, what would be the compression at R_o in the state of rest?

$[P_o] = 26.99$; $P_o(\theta) = 28.39$ tons.
22.06 tons.

(3) Given $R_o = 6.0''$, $R_1 = 10.3''$, $R_2 = 15.0''$, $R_3 = 17.7''$, $\theta_o = 17.5$ tons, $\theta_1 = 22.0$ tons, $\theta_2 = 22.0$ tons; find $[P_o]$, $P_o(\theta)$, S_1 and S_2.

$[P_o] = 21.97$; $P_o(\theta) = 21.79$ tons.
$S_1 = .0296$; $S_2 = .0400$ in.

(4) Given $R_o = 4.75''$, $R_1 = 7.50''$, $R_2 = 11.375''$, $R_3 = 14.375''$, $\theta_o = 16$ tons, $\theta_1 = 17$ tons, $\theta_2 = 22.2$ tons; find $P_o(\theta)$, S_1 and S_2.

$P_o(\theta) = 20.68$ tons; $S_1 = .0149$; $S_2 = .0327$ in.

(5) Given $R_o = 6.0''$, $R_1 = 11.0''$, $R_2 = 17.0''$, $R_3 = 21.0''$, $\theta_o = 18.0$, $\theta_1 = 19.0$, $\theta_2 = 21$ tons; find $[P_o]$, $P_o(\theta)$, S_1, S_2 and P_o.

$[P_o] = 23.82$; $P_o(\theta) = 23.82$ tons.
$S_1 = .0287$; $S_2 = .0476$ in.
$\overline{P_1} = 6.32$; $P_o = 17.23$ tons.

(6) Given $R_o = 6.0''$, $R_1 = 11.0''$, $R_2 = 17.0''$ and $R_3 = 21.0''$, if the shrinkages were $S_1 = .0287$ and $S_2 = .0476$, find the circumferential and radial true stresses at the inner surface (at R_o) in the state of rest. Then, by the principle of **49**, find the internal pressures which will strain the inner surface to the elastic limit (18 tons) first radially and second circumferentially. (Compare results with answers to example (5).)

$E\bar{e}_t(R_o) = 18.09$; $E\bar{e}_p(R_o) = 6.03$ tons.
$P_o(\theta) = 23.87$; $P_o(\rho) = 17.25$ tons.

CHAPTER VI.

APPLICATIONS TO BUILT-UP GUNS.

56. The modern gun is essentially a compound cylinder, but, being constructed to withstand an internal pressure which diminishes from the breech end to the muzzle, the number of layers and the exterior dimensions are correspondingly decreased for economy of weight, making it necessary to divide the whole length into a number of sections for each of which a separate computation of the elastic strength and shrinkages must be made. In United States guns the inner layer, in which are formed the chamber and bore proper, is called the *tube;* the second layer consists of a *jacket,* in which the breech block is housed, and *chase hoops,* which extend from the front end of the jacket nearly or quite to the muzzle; over that part of the bore in which the maximum powder pressure acts a third and sometimes a fourth layer of *hoops* is placed. With increase of knowledge and of facilities larger and larger steel forgings of assured good quality have become available, and the number of separate parts constituting a built-up gun has tended to diminish, so that at the present time the outer layers, as well as the tube, are sometimes made in one piece.

In one particular, however, there is an important difference between a gun and the compound cylinders with free ends which we have thus far considered; in the latter there is no longitudinal stress, while in a gun the internal pressure, acting upon the breech block as well as upon the cylinder walls, gives rise to a longitudinal stress of very considerable intensity.

57. The Longitudinal Stress.—If we consider a gun recoiling freely under the action of the powder pressure on the bottom of its bore, we see that the total longitudinal stress on any cross-section of the gun must equal the product of the acceleration by the mass forward of the section, so that the said stress diminishes rapidly as we go forward from the front thread of the screw box, where it is a maximum. When recoil is resisted by a brake of any kind, the acceleration is reduced and so, to the same extent, is the longitudinal stress on all cross-sections forward of the point of attachment of the brake to the gun; in rear of that point the longitudinal stress is increased by the action of the recoil brake, the increase diminishing as the cross-section through the front thread of the screw box is approached till, at that point, the total longitudinal stress is practically the same as in free recoil. When, as in most modern United States naval gun mounts, the pistons of the recoil cylinders are attached to a yoke around the breech of the gun, the longitudinal stress is diminished at all sections, its maximum value then being $\frac{M'}{M}(\pi R_0^2 P - F)$, in which M is the whole recoiling mass, M' is that part of it which is forward of the front thread of the screw box, R_0 is the radius of bore and P the maximum powder pressure, and F is the total resistance * of the recoil brake at the instant when P acts.

We do not know how the total longitudinal stress is distributed over the cross-section of the gun. It is not wholly born by the layer in which the breech block houses (the jacket in United States guns), for there is an enormous frictional resistance to the longitudinal motion of any one layer relative to the others; if it were uniformly distributed over the jacket alone, its intensity, even at the section of greatest stress, would seldom exceed 5 or 6 tons per square inch, and if, as many writers assume, it is uniformly distributed over the whole cross-section of the gun, its greatest intensity will not exceed 2 or 3 tons per square inch. Probably the latter assumption is practically true at some distance forward of the breech block and is not very far from the truth at any point forward of the gas check.

Moreover, this maximum intensity of longitudinal stress only exists for the infinitesimally small period of time during which the maximum powder pressure is maintained; during the greater part of the time in which the gun is subjected to internal pressure the longitudinal stress is very small, even at the section where it has its greatest value.

For these reasons, therefore, we are justified in applying to guns the formulæ which we have deduced for cylinders with free ends.

58. If circumferential strain alone had to be considered in the case of a compound cylinder, the greatest strength would be obtained by making the successive radii of the elementary cylinders

* This total resistance of the recoil brake, however, is never more than a small fraction of the maximum total pressure on the bottom of the bore of the gun.

increase in geometrical progression, provided their physical characteristics were the same. Thus, for the case of any one cylinder superimposed upon another, regarding $P_o(\theta)$ in (36b) as a function of R_1 (R_o and R_2 constant, and $\theta_1 = \theta_o$) and putting $\frac{dP_o(\theta)}{dR_1} = 0$, we find, after simplification, $R_1^2 = R_oR_2$, which shows that the maximum value of $P_o(\theta)$ for a given total thickness of a given material occurs when the radius of the common surface is a mean proportional between the inner and outer radii. Very nearly the same proportions will also give the greatest strength as regards radial strain.

In practice, however, other considerations govern in the proportioning of the layers of which guns are composed. In the first place the layer in which the breech block is housed, even though other layers assist it in taking the longitudinal stress, should be of sufficient cross-section to itself safely sustain that stress. Again, the thickness of the tube over the chamber should be sufficient to make relining practicable in case erosion wears away the rifling, and its thickness elsewhere should be sufficient to give ample stiffness. Finally, the necessity for keeping down the weight, which prescribes a decreasing exterior diameter towards the muzzle, and the need for avoiding sudden or great changes of the sections of the different layers, often require dimensions not otherwise desirable.

59. In assigning shrinkages for the different parts of a gun, while as a general rule the maximum attainable strength should be sought at each section, great changes of shrinkage in passing from one section to another must be avoided, as they would cause undesirable inequalities of strain. Not only should each of the parts which make up the outer layers of the gun be assembled so that the strains at its inner surface, both in the state of rest and in that of action, do not change abruptly at any point of its length, but the tube, similarly throughout its length, should be under a compression in the state of rest, and of extension in the state of action, which only gradually varies and at no point changes abruptly. Furthermore, as a rule, slack shrinkages should be preferred to excessive ones, to the end that under the action of an excessive pressure it may be the tube which gives way rather than an outer part.

60. As a simple example of the method of determining the proper shrinkages, and the elastic strength of a gun, we will consider the case of the United States naval 5-inch B. L. R. Mark V, which is shown in Figure 11, with its curves of computed elastic strength and of strains at rest and in action.

The computations are made separately for each of the sections indicated on the drawing, but only those for the most important section, that through the chamber, will be worked out in the text, the final results of the other computations, which are obtained in exactly the same way, being merely stated. As it is always necessary to adjust the shrinkages, in accordance with the principle set forth in **59**, it is most convenient to find their values, as well as the values of the pressures in the state of action, in terms of the elastic limits of the different layers, afterwards assigning suitable values to the elastic limits, always, of course, within their true values as indicated by the testing machine.

5-inch B. L. R Mark V.

Section I.

$R_0 = 3.50$	$R_0^2 = 12.25$	$3(R_3^2 - R_0^2) = 278.43$	log 2.44472
$R_1 = 5.25$	$R_1^2 = 27.56$	$2R_3^2 + R_0^2 = 222.37$	" 2.34707
$R_2 = 8.25$	$R_2^2 = 68.06$		" 0.09765
$R_3 = 10.25$	$R_3^2 = 105.06$	$\theta_0 = 20.0$	" 1.30103
$\theta_0 = \rho_0 = 20.0$ tons*		$[P_0] = 25.04$	" 1.39868
$\theta_1 = \rho_1 = 21.5$ " *			
$\theta_2 = \rho_2 = 22.0$ " *			

That is, 25.04 tons is the greatest possible elastic strength, whatever the qualities of the jacket and hoop.

$3(R_3^2 - R_2^2) = 111.00$	log 2.04532
$4R_3^2 + 2R_2^2 = 556.36$	" 2.74536
A_2	" 9.29996
$\theta_2 = 22.0$	" 1.34242
$P_2(\theta) = 4.389$	" .64238

$$P_2(\theta) = A_2\,\theta_2 = [9.29996]\,\theta_2$$

$3(R_2^2 - R_1^2) = 121.50$	log 2.08458	$6R_2^2 = 408.36$	log 2.61104
$4R_2^2 + 2R_1^2 = 327.36$	" 2.51503		" 2.51503
A_1	" 9.56955	B_1	" .09601
$\theta_1 = 21.5$	" 1.33244	A_2	" 9.29996
7.980	" .90199	$B_1 A_2$	" 9.39597
		$\theta_2 = 22.0$	" 1.34242
5.475			" .73839
$P_1(\theta) = 13.455$			

$$P_1(\theta) = A_1\theta_1 + B_1 A_2\theta_2 = [9.56955]\,\theta_1 + [9.39597]\,\theta_2$$

*** These are the true elastic limits, being the least values given by any of the specimens taken respectively from the tube, jacket and hoop.**

$3(R_1^2-R_o^2)=$ 45.93 . . log 1.66210 $6R_1^2=$165.36 . . log 2.21843 B_1A_2 . . log 9.39597
$4R_1^2+2R_o^2=$134.74 . . " 2.12950 . . . . " 2.12950
A_o . . . . " 9.53260 B_o . . . . " .08893 . . . . . . " .08893
$\theta_o=$ 20.0 . . " 1.30103 A_1 . . . . " 9.56955
6.818 . . " .83363 B_oA_1 . . . . " 9.65848 $B_oB_1A_2$ . . " 9.48490
$\theta_1=$ 21.5 . . " 1.33244
9.793 . . . . " .99092 $\theta_2=$22.0 " 1.34242
6.719 . . . . " .82732
$P_o(\theta)=$23.330

$$P_o(\theta) = A_o\theta_o + B_oA_1\theta_1 + B_oB_1A_2\theta_2 = [9.53260]\,\theta_o + [9.65848]\,\theta_1 + [9.48490]\,\theta_2$$

The value of $P_o(\theta)$ for the true values of the elastic limits being 23.33 tons, while $[P_o] = 25.04$ tons, the inner surface is not too much compressed in the state of rest, and so we proceed to determine the shrinkages.

$6R_o^2=$ 73.50 . . . . log 1.86629
$2R_1=$ 10.50 . . . . . log 1.02119
$E=$13000.0 . . . . . " 4.11394
$\frac{2R_1}{E}=$.0008077 . . . . . . " 6.90725 . . . . " 6.90725
$4R_o^2+2R_1^2=$ 104.12 . . . . . " 2.01753
" 8.92478 " 8.77354
$3(R_1^2-R_o^2)=$ 45.93 . . . . . . " 1.66210 . . . . " 1.66210
" 7.26268 . . . . log 7.26268 " 7.11144 . . . log 7.11144 . . . log 7.11144
A_1 . . . . " 9.56955 A_o " 9.53260
B_1A_2 . . . . " 9.39597 B_oA_1 . . . . " 9.65848
" 6.83223 . . . . " 6.65865 $B_oB_1A_2$ . . . . " 9.48490
log 6.64404 " 6.76992 " 6.59634

$+.0008077\,\theta_1 + .0006796\,\theta_1 + .0004557\,\theta_2 - .0004406\,\theta_o - .0005887\,\theta_1 - .0003948\,\theta_2$
+.0006796 −.0003948
+.0014873 +.0000609
−.0005887
+.0008986

$$S_1 = .0008986\,\theta_1 - .0004406\,\theta_o + .0000609\,\theta_2$$

$6R_o^2=$ 73.50 . . . . log 1.86629
$2R_2=$ 16.50 . . . log 1.21748
$E=$13000.00 . . . " 4.11394
$\frac{2R_2}{E}=$.0012692 . . . " 7.10354 . . . . " 7.10354
$4R_o^2+2R_2^2=$ 185.12 . . . " 2.26745
" 9.37099 . . . . " 8.96983
$3(R_2^2-R_o^2)=$ 167.43 . . . " 2.22383 . . . . " 2.22383
" 7.14716 . . . . " 6.74600 . . . log 6.74600 . . . log 6.74600
A_2 . . . . " 9.29996 A_o " 9.53260
" 6.44712 B_oA_1 . . . . " 9.65848
$B_oB_1A_2$ . . . . " 9.48490
" 6.27860 " 6.40448 " 6.23090

$+.0012692\,\theta_2 + .0002800\,\theta_2 - .0001899\,\theta_o - .0002538\,\theta_1 - .0001702\,\theta_2$
+.0002800
+.0015492
−.0001702
+.0013790

$$S_2 = .0013790\,\theta_2 - .0001899\,\theta_o - .0002538\,\theta_1$$

Now, substituting the values 20.0, 21.5 and 22.0 for θ_0, θ_1 and θ_2, respectively, we have for the shrinkages which will cause tube, jacket and hoop to simultaneously reach their elastic limits of circumferential strain, under the internal pressure $P_0(\theta) = 23.33$ tons, $S_1 = .01185$ and $S_2 = .02108$.

In exactly the same way as shown for Section I, the values of the pressures in the state of action and the corresponding shrinkages are computed for the other sections, the results being as follows:

Section II.

$$\left.\begin{array}{l} R_0 = 2.70 \\ R_1 = 5.25 \\ R_2 = 8.25 \\ R_3 = 10.25 \end{array}\right\} \begin{array}{l} P_2(\theta) = [9.29996]\,\theta_2 \\ P_1(\theta) = [9.56955]\,\theta_1 + [9.39597]\,\theta_2 \\ P_0(\theta) = [9.69769]\,\theta_0 + [9.69170]\,\theta_1 + [9.51812]\,\theta_2 \\ S_1 = .0010376\,\theta_1 - .0002896\,\theta_0 + .0000983\,\theta_2 \\ S_2 = .0013976\,\theta_2 - .0001508\,\theta_0 - .0001488\,\theta_1 \end{array}$$

Section III.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 5.25 \\ R_2 = 8.25 \\ R_3 = 9.00 \end{array}\right\} \begin{array}{l} P_2(\theta) = [8.92618]\,\theta_2 \\ P_1(\theta) = [9.56955]\,\theta_1 + [9.02219]\,\theta_2 \\ P_0(\theta) = [9.71671]\,\theta_0 + [9.69899]\,\theta_1 + [9.15163]\,\theta_2 \\ S_1 = .0009470\,\theta_1 - .0002468\,\theta_0 + .00005599\,\theta_2 \\ S_2 = .0012509\,\theta_2 - .0001919\,\theta_0 - .0001842\,\theta_1 \end{array}$$

Section IV.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 5.25 \\ R_2 = 8.00 \end{array}\right\} \begin{array}{l} P_1(\theta) = [9.54577]\,\theta_1 \\ P_0(\theta) = [9.71671]\,\theta_0 + [9.67521]\,\theta_1 \\ S_1 = .0009391\,\theta_1 - .0002468\,\theta_0 \end{array}$$

Section V.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 5.00 \\ R_2 = 7.00 \end{array}\right\} \begin{array}{l} P_1(\theta) = [9.46639]\,\theta_1 \\ P_0(\theta) = [9.69897]\,\theta_0 + [9.59133]\,\theta_1 \\ S_1 = .0008693\,\theta_1 - .0002564\,\theta_0 \end{array}$$

Section VI.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 5.00 \\ R_2 = 5.50 \end{array}\right\} \begin{array}{l} P_1(\theta) = [8.96428]\,\theta_1 \\ P_0(\theta) = [9.69897]\,\theta_0 + [9.08922]\,\theta_1 \\ S_1 = .0008007\,\theta_1 - .0002564\,\theta_0 \end{array}$$

Section VII.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 5.00 \end{array}\right\} P_0(\theta) = [9.69897]\,\theta_0$$

Section VIII.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 3.8735 \end{array}\right\} P_0(\theta) = [9.55930]\,\theta_0$$

Section IX.

$$\left.\begin{array}{l} R_0 = 2.50 \\ R_1 = 4.50 \end{array}\right\} P_0(\theta) = [9.65244]\,\theta_0$$

61. We adopt as the shrinkages for that part of the gun which is represented by Section I the values $S_1 = .0120$ and $S_2 = .0210$, being (to the nearest thousandth of an inch) those which result from substituting the true values of θ_o, θ_1 and θ_2 in the expressions for S_1 and S_2.

If now we compute the shrinkages for Section II with the same values of θ_o, θ_1 and θ_2, we find $S_1 = .0165$, $S_2 = .0197$ and $P_o(\theta) =$ 27.79 tons, and as the increase of strength over the adjoining section would be valueless, while the great increase of the jacket shrinkage would cause a very undesirable inequality of strains in the state of rest, we see that it will be best to assign less values to θ_1 and θ_2 and to adopt a correspondingly less shrinkage for this section. If, on the other hand, we should adopt the same shrinkages for Section II as for Section I, an internal pressure which would bring the bore to its elastic limit would only cause a circumferential true stress of about 16.6 tons at the inner surface of the jacket, thus causing an undesirable inequality of strains in the state of action, since in the adjoining section the jacket reaches its elastic limit with the tube. We therefore compromise between the two extremes, and adopt the values $S_1 = .0130$ and $S_2 = .0200$ for part of the gun which Section II represents.

Guided by similar considerations, we assign to the shrinkages at the other sections the values stated on the drawing.

62. We might now, by means of the general values of S_1 and S_2 which we have computed for each section, find the values of θ_1 and θ_2 which, in combination with the value 20.0 for θ_o, will give the shrinkages which have been adopted, and then, with those values of θ_o, θ_1 and θ_2, calculate the elastic strength, compression of bore in the state of rest, etc. A better method, however, is to start afresh and with the given shrinkages calculate first, by (57), the circumferential and radial strains at the surface of the bore in the state of rest and then the internal pressure which will increase each of those strains to its greatest allowable value. We will do this for Section II, as an illustration.

$$\left.\begin{array}{ll} R_o = 2.70 & R_o^2 = 7.29 \\ R_1 = 5.25 & R_1^2 = 27.56 \\ R_2 = 8.25 & R_2^2 = 68.06 \\ R_3 = 10.25 & R_3^2 = 105.06 \end{array}\right\} \quad \begin{array}{l} S_1 = .0130; \ \phi_1 = \dfrac{S_1}{2R_1} = .0012381 \\[2ex] S_2 = .0200; \ \phi_2 = \dfrac{S_2}{2R_2} = .0012121 \end{array}$$

$R_2^2 - R_1^2 =$	40.5	....log 1.60746	$R_3^2 - R_2^2 =$	37.0	....log 1.56820
$\phi_1 =$	.0012381....	" 7.09275	$\phi_2 =$	.0012121....	" 7.08354
		" 8.70021			" 8.65174
$R_2^2 - R_o^2 =$	60.77	.... " 1.78369	$R_3^2 - R_o^2 =$	97.77	.... " 1.99021
	.0008251....	" 6.91652			
	.0004587....				" 6.66153
$\bar{e}_t(R_o) =$	$-$ .0012838....	" 7.10850			
$E =$	13000	.... " 4.11394			
$E\bar{e}_t(R_o) =$	$-$ 16.69	.... " 1.22244			
$E\bar{e}_p(R_o) =$	$+$ 5.56				

The true circumferential stress at the surface of the bore in the state of rest is thus found to be — 16.69 tons, while the true radial stress is + 5.56 tons. Therefore, applying the principle laid down in **49**, the internal pressures which will, respectively, bring the inner surface to its elastic limits of strain circumferentially and radially, are found as follows:

$3(R_3^2 - R_o^2) =$	293.31..........	log 2.46733..........	log 2.46733
$\theta_o + 16.69 =$	36.69..........	" 1.56455	
$\rho_e + \ \ 5.56 =$	25.56..........	....	" 1.40756
		" 4.03188	" 3.87489
$4R_3^2 + 2R_o^2 =$	434.82..........	" 2.63831	
$4R_3^2 - 2R_o^2 =$	405.66..........	....	" 2.60816
$P_o(\theta) =$	24.75..........	" 1.39357	
$P_o(\rho) =$	18.48..........	....	" 1.26673

In the same way at each of the other sections the effect upon the bore of superposing the outer cylinders with the adopted shrinkages is first calculated, and thence the elastic strength of the assembled system is determined, the results being as shown by the curves in Figure 11.

63. Since the compression of the bore caused by superposing the hoop with the relative shrinkage ϕ_2 is by (38) $\frac{R_3^2 - R_2^2}{R_3^2 - R_o^2} E\phi_2$, the pressure at the surface of contact in the state of rest must be

$$\overline{P}_2 = \frac{R_2^2 - R_o^2}{2R_2^2} \frac{R_3^2 - R_2^2}{R_3^2 - R_o^2} E\phi_2 \tag{58}$$

and, since the whole compression of the bore in the state of rest is by (57) $\frac{R_2^2 - R_1^2}{R_2^2 - R_o^2} E\phi_1 + \frac{R_3^2 - R_2^2}{R_3^2 - R_o^2} E\phi_2$, we have similarly

$$\overline{P}_1 = \frac{R_1^2 - R_o^2}{2R_1^2}\left(\frac{R_2^2 - R_1^2}{R_2^2 - R_o^2} E\phi_1 + \frac{R_3^2 - R_2^2}{R_3^2 - R_o^2} E\phi_2\right) \quad (59)$$

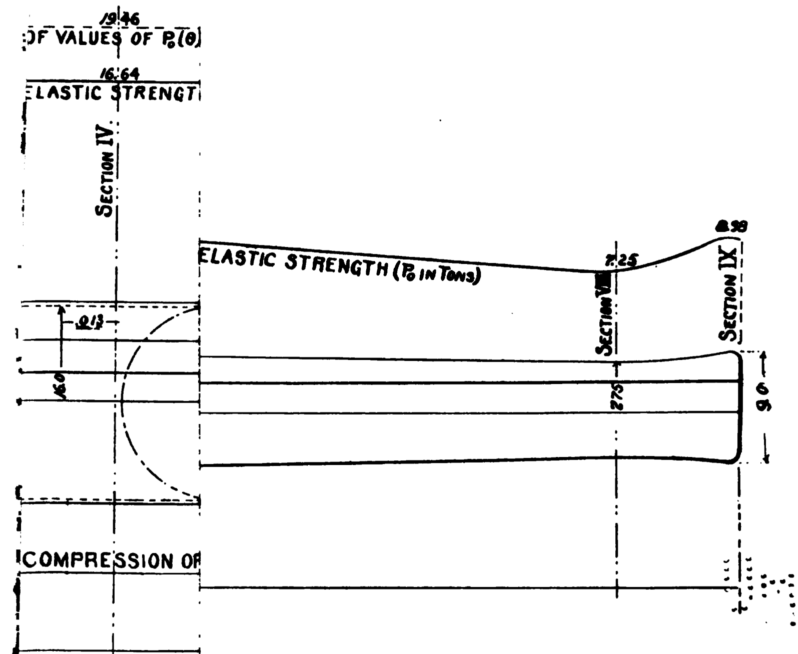
Thus we obtain the values of the pressures in the state of rest at each of the sections of the gun, and from them, together with the known value of P_o, the strains in the state of rest and of action may be found.

The following table gives the results of the calculations for the 5-inch gun shown in Figure 11:

SECTIONS.

	I′	*I*	*II*	*III*	*III′*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
$P_o(\theta)$	23.89	23.40	24.75	22.71	20.84	19.46	17.69	12.63	10.00	7.25	8.98
P_o	18.25	18.15	18.48	17.78	17.12	16.64	16.02	12.63	10.00	7.25	8.98
$\overline{P}_2$	2.75	2.70	2.66	1.24	....	...	....	....	....	....	....
$\overline{P}_1$	5.41	4.83	6.14	5.45	4.54	3.93	3.29	1.28	....	....	....
$E\bar{e}_t(R_o)$	− 16.75	− 17.38	− 16.69	− 14.09	− 11.74	− 10.16	− 8.76	− 3.41	....	....	....
$E\bar{e}_t(R'_1)$	+ 5.32	+ 3.92	+ 7.59	+ 10.52	+ 11.65	+ 11.18	+ 11.22	+ 13.88	....	....	....
$E\bar{e}_t(R'_2)$	+ 13.79	+ 13.53	+ 13.34	+ 14.68	....	....	....	....	....	....	....
$Ee_t(R_o)$	+ 11.32	+ 11.61	+ 10.71	+ 12.60	+ 14.21	+ 15.63	+ 17.28	+ 20.00	+ 20.00	+ 20.00	+ 20.00
$Ee_t(R'_1)$	+ 16.07	+ 17.69	+ 15.51	+ 17.34	+ 18.40	+ 17.96	+ 18.90	+ 21.38	....	....	....
$Ee_t(R'_2)$	+ 18.88	+ 20.09	+ 17.10	+ 18.03	....	....	....	....	....	....	....

64. The method of procedure when there are more than three layers is exactly the same as has been explained for the cases of two and three layers respectively, and the formulæ already deduced are easily extended to cover any number of layers whatever. For the convenience of any one who may wish to use them, the formulæ for the case of four layers are given in full in an appendix.


19.46
OF VALUES OF P₀(θ)
16.64
ELASTIC STRENGT
SECTION IV.
.013
16.0
ELASTIC STRENGTH (P₀ IN TONS)
7.25
SECTION VIII
SECTION IX
2.75
9.0
COMPRESSION OF

CHAPTER VII.

WIRE-WOUND GUNS.

65. A *wire-wound gun* differs from a built-up gun in that one or more of the outer layers of the latter are replaced in the former by steel wire, primarily for the purpose of increasing strength, steel in the form of wire having a higher elastic limit and tensile strength than it is practicable to obtain in large forgings. When first proposed, wire winding was relatively much more advantageous than it is to-day, when sound steel forgings of great size, and not greatly inferior in strength to steel wire, are readily procurable. Moreover, the promise of quicker and cheaper manufacture, often made for wire-wound systems of gun construction, has not as yet been fulfilled in practice. There is little doubt but that a wire-wound gun can be made of greater ultimate strength than a built-up gun of the same weight, or of equal strength, both elastic and ultimate, on less weight, but since the elastic strength after all depends upon the quality of the tube, or inner layer, which is the same in both systems, and since any reduction of weight by increasing the violence of recoil requires an increased weight for the gun mount and its supports, it is difficult to see any great advantage to be gained by substituting wire for solid forgings in gun construction. However, wire guns are in use, and possibly their use will become more extensive as experience in their manufacture increases, and so the principles of wire winding will be briefly discussed.

66. Winding with Constant Tension.—Let R_o and R_1 be the radii of the cylinder upon which the wire is to be wound, R_2 being the outer radius of the layers of wire, and let t_w be the constant tension of winding. Then, if Δr be the thickness of the wire, the application of a layer of wire at radius r will cause the radial pressure $p_r = t_w \dfrac{\Delta r}{r}$ at that radius.

But this pressure, by (24), will cause a circumferential true stress at the surface of the bore given by

$$\Delta E\bar{e}_t(R_o) = -\frac{2r^2 p_r}{r^2 - R_o^2} = -t_w \frac{2r\Delta r}{r^2 - R_o^2} \qquad (58)$$

And the total circumferential compression of the bore due to all the wire will be the sum of the partial compressions given by (58), which, since Δr is small, is given with practical exactness by

$$E\bar{e}_t(R_o) = -\int_{R_1}^{R_2} t_w \frac{2rdr}{r^2 - R_o^2} = -t_w \log_e \left(\frac{R_2^2 - R_o^2}{R_1^2 - R_o^2}\right) \tag{59}$$

But since the greatest elastic strength of the system will result from compressing the inner surface of the tube to its elastic limit, when in the state of rest, the proper tension of winding is given by putting $-\rho_o$ for $E\bar{e}_t(R_o)$ in (59), whence we have

$$t_w = \frac{\rho_o}{\log_e \left(\frac{R_2^2 - R_o^2}{R_1^2 - R_o^2}\right)} = \frac{0.4343\,\rho_o}{\log_{10} \left(\frac{R_2^2 - R_o^2}{R_1^2 - R_o^2}\right)} \tag{60}$$

Then the internal pressure which will bring the inner surface of the tube to its elastic limit of circumferential strain will be

$$P_o(\theta) = \frac{3\,(R_2^2 - R_o^2)}{4\,R_2^2 + 2R_o^2}(\theta_o + \rho_o) \tag{61}$$

and the internal pressure which will bring the inner surface of the tube to its elastic limit of radial strain will be

$$P_o(\rho) = \frac{2\,(R_2^2 - R_o^2)}{2\,R_2^2 - R_o^2}\,\rho_o \tag{62}$$

of which two values, when $\theta_o = \rho_o$, $P_o(\rho)$ will be the smaller and therefore the one to be accepted, if R_2^2 is greater than $\frac{5}{2}\,R_o^2$, which will practically always be the case.*

67. The compression at R_o in the state of rest, due to the wire, being ρ_o, the compression at R_1 will be $\dfrac{R_1^2 + 2\,R_o^2}{3R_1^2}\,\rho_o$, and so the true tension of the inner layer of wire in the state of rest will be $t_w - \dfrac{R_1^2 + 2\,R_o^2}{3R_1^2}\,\rho_o$, while that of the outer layer will, of course, be t_w.

In the state of action, P_o being the internal pressure, the true tension of the inner layer will be increased $\dfrac{2\,P_o\,R_o^2}{3(R_2^2 - R_o^2)}\left(1 + \dfrac{2R_2^2}{R_1^2}\right)$, while the true tension of the outer layer will be increased $\dfrac{2P_oR_o^2}{R_2^2 - R_o^2}$.

* If the compression of the bore is ρ (less than ρ_o), (61) and (62) will still give correct results provided ρ be put for ρ_o in (61) and $\frac{3\rho_o + \rho}{4}$ for ρ_o in (62).

If we suppose $P_o = P_o(\theta) = \frac{3\,(R_2^2 - R_o^2)}{4\,R_2^2 + 2\,R_o^2}\,(\theta_o + \rho_o)$, the true tension of the outer layer of wire in action will be $t_w + \frac{3\,R_o^2}{2\,R_2^2 + R_o^2}$ $(\theta_o + \rho_o)$, and this must not exceed the elastic limit of the wire.

68. As an example we will examine the case of a tube of radii $R_o = 5.0''$ and $R_1 = 8.0''$, with elastic limit $\theta_o = \rho_o = 18.0$ tons, with four inches of wire, for which $\theta = 40.0$ tons, wound upon it, the section of the wire being $0.2''$ wide by $0.1''$ thick.

		0.4343		log 9.63779
		$\rho_o =$ 18.0		" 1.25527
$R_o = 5.0$	$R_o^2 = 25.0$	$R_2^2 - R_o^2 = 119.0$	log 2.07555	
$R_1 = 8.0$	$R_1^2 = 64.0$	$R_1^2 - R_o^2 = 39.0$	" 1.59106	" .89306
$R_2 = 12.0$	$R_2^2 = 144.0$		.48449	" 9.68528
		$t_w =$ 16.14 tons		" 1.20778
				" 3.35025
		= 36143 lbs.		" 4.55803

Therefore, the constant tension of winding which will compress the bore to its elastic limit is 36,143 pounds per square inch, or, the cross-section of the wire being 0.02 sq. in., 723 pounds on the wire.

$\theta_o =$	18.0	log 1.25527	log 1.25527
$R_2^2 - R_o^2 =$	119.0	" 2.07555	" 2.07555
	3.0	" .47712	
	2.0		" .30103
		" 3.80794	" 3.63185
$2R_2^2 + R_o^2 =$	313.0	" 2.49554	
$2R_2^2 - R_o^2 =$	263.0		" 2.41996
$P_o(\theta) =$	20.53	" 1.31240	
$P_o(\rho) =$	16.29		" 1.21189

The least of these two values, $P_o(\rho) = 16.29$ tons, is the true elastic strength of the system.

$\rho_o =$	18.0	log 1.25527
$R_1^2 + 2R_o^2 =$	114.0	" 2.05690
		" 3.31217
$3R_1^2 =$	192.0	" 2.28330
	10.69	" 1.02887

The compression at the outer surface of the tube, and the inner surface of the wire, due to the pressure of the wire in the state of rest, is 10.69 tons. Therefore the true tension of the inner layer of wire at rest is 16.14 — 10.69 = 5.45 tons per square inch.

$2P_o(\rho) =$	32.58	log 1.51295	log 1.51295
$R_o^2 =$	25.0	" 1.39794	" 1.39794
$R_1^2 + 2R_2^2 =$	352 0	" 2.54654	
		" 5.45743	
$3R_1^2 =$	192.0	" 2.28330	
		" 3.17413	" 2.91089
$R_2^2 - R_o^2 =$	119.0	" 2.07555	" 2.07555
	12.55	" 1.09858	
	6.84		" .83534

The increases of true tension at the inner and outer layers of wire caused by the internal pressure $P_o(\rho) = 16.29$ tons, are, respectively,

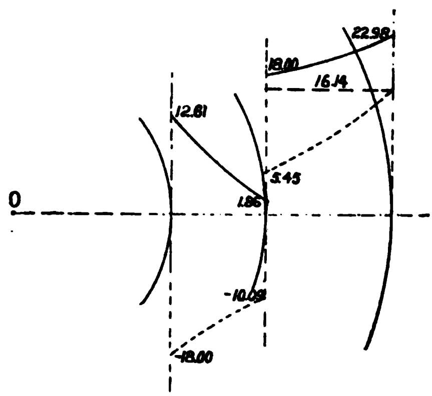


FIG. 12.

12.55 and 6.84 tons, so that the tensions of the inner and outer wires in the state of action are, respectively, 12.55 + 5.45 = 18.0 tons and 16.14 + 6.84 = 22.98 tons per square inch.

Under the internal pressure $P_o(\theta) = 20.53$ tons, the tension of the outer layer of wire would be 24.77 tons, which is still far within its elastic limit.

The circumferential true stresses for the case just discussed are graphically represented in Figure 12, in which the plus ordinates represent tensions and minus ordinates compressions. The dash line shows the tension of winding, the dotted lines represent the state of rest, and the full lines the state of action with $P_o = 16.29$ tons.

69. When, as is usually the case, the number of layers of wire is such that a relatively small tension of winding compresses the tube to its elastic limit, and in the state of action the greatest strain is well within the elastic limit of the wire, a constant tension of winding serves every purpose, and, being easier of accomplishment than a varying tension, is naturally used. If, however, economy of material and weight is an object, it will be attained by winding the wire with a tension which varies from layer to layer in such a manner that in the state of action all the layers simultaneously reach the elastic limit of strain. The tension of winding which will bring about this result is determined as follows:

70. Let R_0 and R_1 be the radii of the tube or cylinder upon which wire is wound to an outer radius R_2, and let T be the constant value of the circumferential true tension of each layer of wire in the state of action, so that $\frac{T}{E}$ is the circumferential strain throughout the mass of wire when P_o acts.

Then at any point r in the wire, the existing extension $\left(\frac{T}{E}\right)$ results from the concurrent action of three forces, namely, the tension of winding (t_w), the pressure of the outer layers of wire (p), and the internal pressure (P_o), and if we find the change of extension at r resulting from the removal of the outer layers of wire and the suppression of the internal pressure, and apply it to the extension $\frac{T}{E}$ which exists under those forces, the result will be the extension which was given to the wire in winding, and this multiplied by E is the desired tension of winding.

Let p and t be the radial pressure and circumferential tension at radius r under the internal pressure P_o. Then, by supposition, the circumferential strain being $\frac{T}{E}$, we have

$$\frac{1}{E}\left(t + \frac{p}{3}\right) = \frac{T}{E}$$

$$t = T - \frac{p}{3} \tag{63}$$

But the product rp must always equal $\int_r^{R_2} t\,dr$, therefore

$$rp = \int_r^{R_2}\left(T - \frac{p}{3}\right)dr \tag{64}$$

from which, by differentiating, we get

$$rdp + pdr = -\left(T - \frac{p}{3}\right)dr$$

$$\frac{dp}{\frac{2}{3}p + T} = -\frac{dr}{r} \tag{65}$$

whence, by integration, knowing that, when $r = R_2$, $p = 0$ and $t = T$,

$$p = \frac{3T}{2}\left[\left(\frac{R_2}{r}\right)^{\frac{2}{3}} - 1\right] \tag{66}$$

and this gives the value of the radial pressure at any point r within the wire in the state of action.

In accordance with the principle laid down in **49**, the change of circumferential strain at radius r, due to simultaneous changes (p_o and p) of the internal and external pressures, will, by (13), be given by

$$e_t = \frac{1}{E}\cdot\frac{6p_oR_o^2 - p(4R_o^2 + 2r^2)}{3(r^2 - R_o)} \tag{67}$$

Therefore, by putting in (67) $-P_o$ for p_o and $-\frac{3T}{2}\left[\left(\frac{R_2}{r}\right)^{\frac{2}{3}} - 1\right]$ for p, we obtain the value of the change of circumferential strain at radius r due to the simultaneous removal of the wire beyond r and suppression of P_o, and $\frac{T}{E}$ plus this change of strain is the extension of the wire in winding, so that the tension of winding is given by

$$t_w = T + \frac{\frac{3T}{2}\left[\left(\frac{R_2}{r}\right)^{\frac{2}{3}} - 1\right](4R_o + 2r^2) - 6P_oR_o^2}{3(r^2 - R_o^2)}$$

$$t_w = \frac{T\left(\frac{R_2}{r}\right)^{\frac{2}{3}}(2R_o^2 + r^2) - R_o^2(3T + 2P_o)}{r^2 - R_o^2} \tag{68}$$

71. To determine the elastic strength of the cylinder when wound with the varying tensions given by (68), since by (66) the external pressure on the tube in the state of action is

$$P_1 = \frac{3T}{2}\left[\left(\frac{R_2}{R_1}\right)^{\frac{2}{3}} - 1\right] \tag{69}$$

we have

$$P_o(\theta) = \frac{3(R_1^2 - R_o^2)\,\theta_o + 6P_1R_1^2}{4R_1^2 + 2R_o^2} \tag{70}$$

and

$$P_o(\rho) = \frac{3(R_1^2 - R_o^2)\,\rho_o + 2P_1R_1^2}{4R_1^2 - 2R_o^2} \tag{71}$$

of which two values of P_o the smaller should be taken.

72. Since (69) gives the value of the external pressure on the tube in the state of action, and since by (34) the change of pressure at R_1 due to the suppression of P_o is $-\frac{R_o^2\,(R_2^2 - R_1^2)}{R_1^2\,(R_2^2 - R_o^2)}\,P_o$, we have as the value of the pressure at R_1 in the state of rest:

$$\overline{P_1} = \frac{3T}{2}\left[\left(\frac{R_2}{R_1}\right)^{\frac{2}{3}} - 1\right] - \frac{R_o^2\,(R_2^2 - R_1^2)}{R_1^2\,(R_2^2 - R_o^2)}\,P_o \tag{72}$$

and this must not exceed $\frac{R_1^2 - R_o^2}{2R_1^2}\,\rho_o$, or the bore will be compressed beyond its elastic limit of circumferential strain.

73. Since by (19) the circumferential true stress at R_1 caused by internal pressure P_o in a cylinder of outer radius R_2 is $\frac{2\,R_o^2\,(R_1^2 + 2\,R_2^2)}{3\,R_1^2\,(R_2^2 - R_o^2)}\,P_o$, the same expression gives the value of the change of stress caused by a change of internal pressure, and therefore the true tension of the inner layer of wire, which in the state of action is T, becomes in the state of rest, when P_o is suppressed,

$$E\bar{e}_t(R_1') = T - \frac{2R_o^2\,(R_1^2 + 2R_2^2)}{3R_1^2\,(R_2^2 - R_o^2)}\,P_o \tag{73}$$

while the tension of the outer layer of wire in the state of rest is, of course, the tension it was wound with, the value of which, found by putting $r = R_2$ in (68), is

$$E\bar{e}_t(R_2) = T - \frac{2P_oR_o^2}{R_2^2 - R_o^2} \tag{74}$$

74. As an example we will consider the case of the tube discussed in **67**, but with two inches of wire, instead of four inches, wound upon it:

$$\left.\begin{array}{lll} R_o = 5.0 & R_o^2 = & 25.0 \\ R_1 = 8.0 & R_1^2 = & 64.00 \\ R_2 = 10.0 & R_2^2 = & 100.00 \end{array}\right\} \left.\begin{array}{l} \theta_o = \rho_o = 18.0 \text{ tons} \\ \theta_1 = \rho_1 = 40.0 \quad \text{"} \end{array}\right\}$$

If the elastic strength of the wire, with its given number of layers, does not allow of compressing the tube to its elastic limit in the state of rest, we take the elastic limit of the wire (θ_1) for the value of T, but when, as in this and most other cases, there is surplus strength in the wire, it is necessary to find a value for T, less than θ_1, such that in the state of rest the tube is at its elastic limit of compression, as thus the greatest elastic strength is given to the system.

Putting $\frac{R_1^2 - R_o^2}{2 R_1^2}\rho_o$ for $\overline{P}_1$ in (72), we obtain the equation

$$P_1 = \frac{R_1^2 - R_o^2}{2R_1^2}\rho_o + \frac{R_o^2(R_2^2 - R_1^2)}{R_1^2(R_2^2 - R_o^2)}P_o$$

which, for this particular case, reduces to $P_1 = .1875\ P_o + 5.484$.

Either (70) or (71), according to which gives the smaller value of P_o, furnishes a second equation between P_1 and P_o, and from the two equations P_1 is found, and then, by (69), T. In this case an examination will show $P_o(\rho)$ to be smaller than $P_o(\theta)$, and (71), after substituting in it the values of R_o, R_1 and ρ_o, reduces to $P_1 = 1.6094\ P_o - 16.453$.

We thus find $P_1 = 8.377$ and $P_o(\rho) = 15.432$:

$$\begin{array}{rll} P_1 = & 8.377 \ldots\ldots\ldots\ldots\ldots & \log .92309 \\ \frac{R_2}{R_1} = & 1.25 \ldots\ldots \log .09691 & \\ \left(\frac{R_2}{R_1}\right)^{\frac{2}{3}} = & 1.1604 \ldots\ldots \tfrac{2}{3} \text{ " } .06461 & \\ \frac{3}{2}\left[\left(\frac{R_2}{R_1}\right)^{\frac{2}{3}} - 1\right] = & .2406 \ldots\ldots\ldots\ldots\ldots & \text{" } 9\ 38130 \\ T = & 34.82 \ldots\ldots\ldots\ldots\ldots & \text{" } \overline{1.54179} \end{array}$$

The proper value for T can readily be found by trial instead of as just shown; thus, if we try $T = \theta_1 = 40.0$, we shall find the compression of the bore in the state of rest to be 21.6 tons, showing that T must be reduced about one-sixth (in order to reduce the compression to 18.0 tons); then, after a second trial, a suitable value can be assigned to T.

We next find the tension of winding by (68), which in this case reduces to $t_w = \dfrac{\dfrac{8080.3}{r^{\frac{2}{3}}} + 161.61\, r^{\frac{4}{3}} - 3383.0}{r^2 - 25}$. Giving r in this equation the successive values 8.0, 8.5, 9.0, 9.5 and 10.0, we find, as the corresponding values of t_w, 31.25, 28.79, 26.96, 25.59 and 24.53. These are the tensions of winding in tons per square inch for the 1st, 5th, 10th, 15th and 20th or outer layer of wire, and, when reduced to pounds on the wire, become 1405, 1290, 1208, 1147 and 1099 pounds. The tensions for the other layers may either be calculated as these were or found by interpolation from them.

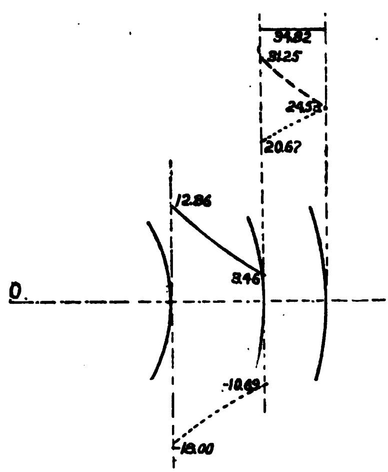


FIG. 13.

To find the tension of the inner wire in the state of rest we apply (73):

$2R_o^2 =$	50.0	..... log	1.69897
$R_1^2 + 2R_2^2 =$	264.0	..... "	2.42160
$P_o =$	15.432	..... "	1.18843
		"	5.30900
$3R_1^2 =$	192.0	..... "	2.28330
		"	3.02570
$R_2^2 - R_o^2 =$	75.0	..... "	1.87506
	14.146	..... "	1.15064

The true tension at R_1 being reduced 14.15 tons when P_o is suppressed, the true tension of the inner layer of wire in the state of rest is $34.82 - 14.15 = 20.67$ tons.

The circumferential true stresses for the case just discussed are represented in Figure 13, in which plus ordinates represent tensions and minus ordinates compressions. The dash line shows the tension of winding, the dotted lines the state of rest, and the full lines the state of action with $P_o = 15.43$ tons.

EXAMPLES VII.

(1) At what constant tension must 20 layers of steel wire 0.1″ thick be wound on a tube for which $R_o = 5.0''$ and $R_1 = 8.0''$ to compress the bore to its elastic limit, 18 tons? Find $P_o(\theta)$ and $P_o(\rho)$ and the true tensions of the inner and outer wires both at rest and when $P_o(\rho)$ acts.

$t_w = 27.53$; $P_o(\theta) = 18.0$; $P_o(\rho) = 15.43$ tons.
16.84 and 27.53 tons, at rest.
30.98 and 37.82 tons, in action.

(2) A thickness of 2.5″ of wire is wound with the constant tension 32.5 tons per sq. in. on a tube for which $R_o = 5.0''$ and $R_1 = 12.5''$. Find the compression of the bore in the state of rest, and the true tensions of the inner and outer wires both at rest and when $P_o(\rho)$ acts, if $\theta_o = \rho_o = 18.0$ tons.

$E\overline{e}_t(R_o) = 13.69$; $P_o(\rho) = 15.93$ tons.
26.48 and 32.50 tons, at rest.
31.63 and 36.48 tons, in action.

(3) A thickness of 1.25″ of wire is wound with the constant tension 15 tons per sq. in. on a tube for which $R_o = 3.0''$, $R_1 = 5.25''$ and $\theta_o = \rho_o = 18$ tons. Find the compression of the bore in the state of rest, and the true tensions of the inner and outer wires both at rest and when $P_o(\rho)$ acts.

$E\overline{e}_t(R_o) = 8.75$; $P_o(\rho) = 13.82$ tons.
10.18 and 15.00 tons, at rest.
20.32 and 22.48 tons, in action.

(4) If, in the case of Example (2), the wire be so wound as to be under the constant true tension 34 tons per sq. in., find the values of $P_o(\theta)$, $P_o(\rho)$, and the compression of the bore and the

true tensions of the inner and outer wires in the state of rest. At what tension must the inner and the outer wires be wound?

$P_o(\theta) = 19.65$; $P_o(\rho) = 15.91$ tons.
$E\bar{e}_t(R_o) = 13.60$; $E\bar{e}_t(R_1') = 28.86$ tons.
$t_w = 34.85$ at R_1; 30.02 at R_2.

(5) Given $R_o = 4.0''$, $R_1 = 5.5''$, $R_2 = 8.0''$, $\theta_o = 30.0$, $\rho_o = 35.0$ and $T = 40.0$ tons (from R_1 to R_2 wire so wound as to have constant true tension of 40 tons when $P_o(\theta)$ acts); find $P_o(\theta)$, compression of bore at rest, true tension of inner and outer wires at rest, and tension of winding inner and outer wires.

$P_o(\theta) = 28.39$; $E\bar{e}_t(R_o) = 31.29$ tons.
$E\bar{e}_t(R_o) = 27.47$; $E\bar{e}_t(R_1') = 7.00$ tons.
$t_w = 25.83$ at R_1; 21.07 at R_2.

CHAPTER VIII.

ELEMENTARY GUN DESIGN.*

75. General Considerations.—The modern high-powered gun is essentially a compound cylinder designed to withstand rapidly varying but not instantaneous internal pressures. The object of the subdivision of the gun into various elements is twofold: 1st, to increase the range through which the metal of the gun may be worked and thus increase the magnitude of the resisting elastic forces by assembling the elements with shrinkage; and 2d, to insure the homogeneity of the metal and thus the safety of the gun by its subdivision into sufficiently small elements. It is a principle of metallurgy, in the present state of the art, that there is a practical limit to the size of cast steel ingots. If this size, which may be determined solely by experience for each kind of steel, is exceeded, the ingot will have unsound areas which no subsequent forging can entirely cure. This unsound metal, in the forms commonly known as segregations, sand-splits, streaks, and blow holes, must be carefully avoided during manufacture if the guns are to merit a proper degree of confidence. Manufacturing processes are undergoing constant improvement, but at the present time two principles must be invariably considered in gun construction: 1st, that in high-powered guns there should be at least two elements resisting stresses whose character is definitely known; and 2d, that a sound forging cannot be obtained if its wall-thickness, its length, and its diameter are all very great. Furthermore, the weight of a gun has an important bearing on its mounting on board ship, and since the weight increases nearly proportionally to the cube of the caliber it is apparent that this fact and the above two considerations tend to limit the caliber and power of naval guns.

*** Written by Lieutenant (j. g.) R. K. Turner, U. S. Navy, at the Naval Gun Factory, February, 1916.**

If a pressure curve is drawn from the formulas of interior ballistics, it is seen that the whole gun in rear of the base of the projectile is subjected to the pressure represented by the successive ordinates passed by the projectile during its travel down the bore. When the base is opposite the maximum ordinate the whole gun in rear of this ordinate is subjected to the maximum pressure and should therefore be cylindrical from the breech to this point. The forward portion of the gun, however, is subjected to continuously decreasing pressures and may therefore continuously decrease in thickness. This decrease in thickness may be theoretically proportional to the decrease in height of the pressure ordinates. For this reason the gun is made smaller at the muzzle than at the breech and thus an economy in weight and cost is effected. The muzzle itself is flared out in the form of a bell because the metal at that point is not supported on the forward side and it is thought that the absence of slightly extra strength might induce splitting. We know that the resistance formulas do not tell the whole truth, since they take into consideration neither the supporting nor the shearing effect due to the continuation of the metal beyond the particular section considered, but experience has shown that the formulas in use give the best approximate mathematical measure of the strength of the gun as a whole, at least relatively to guns of proved worth.

76. Longitudinal Resistance.—In the deduction of the resistance formulas the gun is considered to be undergoing strains in the planes normal to the axis only. This assumption does not accord with the facts, since part of the gun resists for a short time the total gas pressure on the face of the breech block. Suppose that section of the gun which takes this pressure, *i. e.,* those elements to which the block transmits its stresses, have inner and outer radii of R'_o and R'_n, respectively, and the minimum obturator radius is ρ_o, the bore pressure per square inch being P_o. If the gun did not recoil, the section under consideration would sustain a longitudinal stress T in addition to the transverse stresses, such that:

$$\pi\rho_o^2 P_o = \pi(R_n'^2 - R_o'^2) \times T$$

and

$$T = \frac{\rho_o^2 P_o}{R_n'^2 - R_o'^2} \qquad (75)$$

This stress would exist only to the rear of the plane of attachment of the gun to the carriage, which is usually a shoulder turned on the outside near the breech. A yoke to which the piston rods are secured takes against this shoulder.

As a matter of fact, however, the gun recoils, and in doing so relieves this stress to a certain extent. Let W be the weight of the recoiling parts, w_1 the weight to the rear of the longitudinal instantaneous center of pressure of the screw-box liner, v the velocity of recoil, and R_e the constant brake resistance; the total effective thrust, F, on the breech of the gun, neglecting the friction of the projectile in the bore, will be

$$F = ma = \frac{W}{g}\frac{dv}{dt}$$

The total rearward force across any section forward of the breech diminishes proportionally to the decrease of the mass forward of that section. Therefore the maximum stress will be in the plane of the longitudinal instantaneous center of pressure between the screw-box liner and the gun. The force F' at this point will be:

$$F' = m'a = \frac{W - w_1}{g}\frac{dv}{dt}$$

and the ratio between the two forces is

$$\frac{F'}{F} = \frac{W - w_1}{W}, \text{ or } F' = F \times \frac{W - w'}{W}$$

But the total force acting to push the gun to the rear is the difference between the total gas pressure and the constant brake resistance, or

$$F = \pi \rho_0^2 P_0 - R_e$$

and therefore the total stress on the metal of the gun is:

$$F' = \frac{W - w_1}{W}(\pi \rho_0^2 P_0 - R_e)$$

and the unit longitudinal stress is:

$$T = \frac{F'}{\pi(R_n'^2 - R_0'^2)} = \frac{W - w_1}{W} \times \frac{\pi \rho_0^2 P_0 - R_e}{\pi(R_n'^2 - R_0'^2)} \qquad (76)$$

This force acts only in the plane of the instantaneous center of longitudinal pressure of the screw-box liner against the threads of its housing. From this point forward the stress decreases as far as the yoke shoulder. At the yoke shoulder it suddenly changes, however, and the only force acting becomes that of the inertia of the mass forward of any section considered. If this mass is taken equal to $\frac{w_2}{g}$, where w_2 is the weight of the gun forward of the section considered, the total stress is:

$$F = \frac{w_2}{g} \times \frac{dv}{dt}$$

It is useless to attempt to calculate the exact unit stress in any layer because the gun is not a homogeneous tube and we cannot state the relations between the stresses of the various elements. The work is unnecessary, however, because the total force is small and may be neglected.

77. Gun Projects.—The preliminary design of a gun is called a *project.* It includes tentative sketches and rough computations as to maximum strength, muzzle velocity, and chamber capacity.

When it has been decided that a gun of a new type is needed the general requirements of such a type are tentatively fixed and the project commenced. For instance, suppose that a new gun is desired, the progress in artillery having reduced the comparative value of the existing type. Progress being usually along lines of greater power, reduction of erosion, ease of operation, rapidity of fire, or increase in striking energy, it is probable that as many improvements as possible along each of these lines will be incorporated in the new gun. The caliber is first settled upon, and then the approximate length in calibers. In the case of small guns the muzzle velocity is tentatively fixed, but since erosion is proportionately larger for large guns it usually seems more desirable in the case of large calibers to fix the limit of pressure and with that pressure to get as high a velocity as possible. Several sets of computations are made with variations of the chamber capacity and powder characteristics until a proper combination is secured.

Suppose it is required to design a 12-inch 50-caliber gun. With the three elements of caliber, length, and powder pressure several chamber capacities are chosen and calculations made as to the effects of several powders in them. From previous experience as to the limits of allowable densities of loading the weight of powder to be used is approximated and then the various elements varied until several reasonable combinations of chamber capacity, weight of charge, muzzle velocity, and maximum pressure have been obtained.

For several years the allowable densities of loading have risen in value, due to the use of more progressive powders and the tendency toward a reduction in the size of chambers for a given power. It is desirable to have a short chamber so as to lose as little of the travel of the projectile as possible and also to get more uniform ignition, and to have a small chambrage in order that the outside dimensions of the gun need not be too great. As a general rule, though a rule that is departed from without hesitation, it may be stated that the length of the chamber is usually between 6 and 7 calibers, and the chambrage is about 1.20. At least the ratio of chamber length to chambrage is kept near these approximate proportions.

The general design and method of attachment of the screw-box liner is selected. Its length has usually been fixed at about one caliber, but the tendency at present seems to be toward an increase in this dimension. An attempt is made to eliminate defects that may have appeared in previous designs.

Several drawings are now made of the project. The length in all cases is equal to the length in calibers times the caliber plus the length of the screw box.

So many variables enter into a design that experience, based on a sound understanding of the principles of gun construction, can be the only safe guide. The consequences of the bursting of a gun in service are so grave that all possibility of such an accident must be avoided, and yet the gun must not be made excessively heavy nor of a form that cannot be mounted in turrets that have proved the most satisfactory. Experience has shown the general form a gun must take to give the best results with the powders in use at present, and no radical changes in this form can be made without inviting certain disaster. With any new design it is attempted to retain the advantages of previous types and to eliminate any defects that have shown up in service or may seem to be indicated by carefully tested theories. Therefore, in laying down a gun the previous designs are closely followed so far as regards the general outline, thickness and length of elements, mode of attachment of the various parts to each other, manner of assembly and approved practice in general where it appears to answer the purpose. The radical change of too many variables being inadmissible, it follows that progress is necessarily slow, and that at one stroke all previous defects may not be eliminated and a gun produced that will be perfect for all future time.

With these considerations in mind the outline of the new gun will follow closely the outline of a previous gun that seems best adapted to the purpose; changes in the outer dimensions will be made where it seems necessary and thus the form of the gun will be arbitrarily fixed. It may be that a gun of the same caliber will not be chosen as a pattern, but one of a smaller or larger caliber that seems to have fulfilled certain of the requirements for the new type.

For two reasons the breech cylinder over the powder chamber is usually larger than the slide, which is also cylindrical. The first reason is that the chamber diameter under the breech cylinder is larger than the bore diameter under the slide cylinder, and there must therefore be an increased outside diameter for strength. The second reason is that if the gun is heavier at the breech its center of gravity will be farther from the muzzle and a smaller length need be put inside the turret. The gun usually has an approximately constant slope from the slide cylinder to the neck cylinder just in rear of the muzzle; the muzzle bell is also a frustrum of a cone similar to previous types.

The question then arises as to the number of layers of metal to use. Generally large calibers have either four or five layers: four if the tube is later to be bored for the insertion of a liner and five if the liner is to be included in the gun as originally built. This rule is by no means rigid, however, as witness the 14″ Mark IV gun with four layers, liner included. The practice most in favor at the present time is to build five-layer guns with a liner tapered from breech to muzzle for easy removal.

The problem now is to apportion the metal among four layers, the inner and outer radii being given. For the greatest theoretical transverse strength the law of thickness requires that if R_o, r', R_1, R_2, and R_3 are the respective radii from the bore outward, they must be connected together by the following relations:

$$r^2 = R_0 R_1 \quad R_1^2 = R_2 \quad R_2^2 = R_1 R_3$$

These ratios may not be rigidly adhered to for the following reasons:

1. For large caliber guns the breech diameter of the liner must be great enough to allow for at least three shoulders having a height of from 0″.2 to 0″.25 and the proper taper and yet leave sufficient metal at the muzzle for rigidity and for the prevention of creep due to the mandrelling effect of the projectile.

2. It is desirable to have a heavy tube so as to provide rigidity for the gun and so prevent droop of the muzzle.

3. The layer carrying the screw-box liner must have enough additional thickness to provide for taking the longitudinal stresses

without impairing the transverse resistance of the gun. The usual rule is to compute this layer for longitudinal strength and then make it from 2.5 to 3 times as thick as necessary to carry the longitudinal stress. The extra thickness is taken about equally from the contiguous layers on both sides. The calculation for strength is usually made by equation (75).

4. The thickness of the outside layers must not be so great that it will be impossible to get good forgings.

5. Sudden and great changes in the diameter of the gun or its component parts must be avoided.

It is apparent that in the case of a large gun with a large number of elements, as, for instance, the Mark VII 12″ 50-caliber gun, which is in twelve parts, considerable juggling will be necessary before the above conditions can be satisfied and yet obtain sufficient transverse strength.

Having decided upon the various diameters near the breech, at the forward end of the slide cylinder, and at the neck the related questions of the manner of assembly and the character of the joints and shoulders are taken up. The following principles in this connection must be rigidly observed:

1. Joints must be of such a character as to allow the elements to be easily assembled.

2. The tube and liner must be locked to prevent crawl, and all other elements must be locked both ways to prevent movement in either direction.

The tube and liner are so long that ordinarily the shrinkage friction will prevent rearward motion, but shoulders must be provided to keep them from going out at the muzzle.

Locking is accomplished by means of locking rings, locking hoops, and shoulders. Locking rings are relatively short and thin rings either hooked or screwed to the elements of the gun; they are not assembled with shrinkage and do not contribute to the transverse strength. Locking hoops ordinarily attach to the other elements by hook joints and are assembled with shrinkage; they are longer and heavier than the rings.

Shoulders are turned on an element to prevent relative longitudinal movement between it and the element shrunk over it. The distance between shoulders varies as experience dictates. Their height may be from 0″.2 to 1″.0, the usual height being about 0″.5

where possible. As a general rule two shoulders are not put in the same transverse plane, because a plane of rupture is most likely to form at a shoulder, and it is best to scatter the weakest parts so that one plane will not include the weak points of several layers. The same rule is followed in the case of joints.

Butt joints are avoided when it is possible to use a lap joint. The latter are preferable because they distribute the weakness over a greater length, they assist locking, and contribute to the stiffness. Joints at the outside of the gun in particular must be designed so as to prevent droop, as droop is due partly to stretch of the metal and partly to working at the joints.

The several drawings are worked up to embody the various ideas that have been expressed. If there are three drawings, for example, one may show a heavy gun, one a light gun, and one a gun of medium weight, and in each the arrangement will be slightly different. Possibly one drawing will be of a four-layer unlined gun, one of a four-layer lined gun, and one of a five-layer lined gun. Or, in one the joints and layers may be arranged according to previous designs and in one they may be laid down on a new plan. During their construction the drawings are subjected to continuous criticisms and change and new ideas are included as they may occur to those in charge of the project.

Finally, after several weeks' work, when the various projects seem to answer the requirements determined upon, the total weight, location of the center of gravity, and an approximate strength curve are computed for each. They are then submitted for decision and final criticism.

Usually one of the projects is decided upon, though it may be desirable to make a few minor changes in it. The exact chamber is definitely selected and, as a rule, the maximum bore pressure and the muzzle velocity are fixed, together with the desired weight of charge. Orders are then issued for the definite working up of the design, and a decision is made as to whether the batteries of one or more ships are to be built at once or a type gun only. It is the usual practice to build a type gun when a new caliber is in question or when the changes have been numerous and radical as compared with existing guns.

A Mark is then assigned to the design selected.

As a rule, the drawings are worked up in the following order:

1. Shrinkages, strength, velocity, and pressure curves.

2. General arrangement.
3. Details.
4. Chamber and breech.
5. Rough forgings.
6. Shrinkage sheet.
7. Center of gravity for shrinkage pit.
8. Rifling.

Other drawings may sometimes be required, but these drawings are always made, though not always in the above order.

The breech mechanism drawings and computations constitute an entirely separate set.

CHAPTER IX.

GUN COMPUTATIONS.*

78. Preliminary Computations.—A pencil drawing of the gun is laid down and the sections selected for strength computations. These sections vary in number according to the gun; in some cases there are as many as 28. In the case of the gun selected for the purposes of illustration, the Mark VII, Modification 3, 12″ 50-caliber gun, computations were made at 24 sections. The sections are numbered in Roman numerals, the lowest number being near the breech.

The principle governing the selection of sections may be generally stated as follows: "Computations must be made for every plane of the gun having a strength different from that of the planes on either side of it, and where there is reason to believe a sudden change in strength occurs, on both sides of the change and close to it." The plotted results of the computations must give a continuous strength curve from breech to muzzle.

In order to reduce the immense amount of labor involved in the case of a gun of large caliber, the computations are made on printed forms. The Birnie formulas, involving the introduction of subsidiary constants, are used. These are the same as those given in this book by Professor Alger, but arranged for greater convenience and known as the "Reduced Formulas." For a thorough understanding of their meaning it would be necessary to deduce each one from the fundamental equations; this work is not given here as it is easy enough, though long.

In considering these forms we find various methods used that are not those that we have been accustomed to. It is important to know the formulas on which the forms are based. Logarithms are denoted by letters only. An expression such as a_3 (θ_3) indicates that a_3 is to be multiplied by θ_3; therefore their logs are to be added. This is further indicated by a + sign after θ_3. This method is used throughout. Expressions are denoted by letters or numerals and are thereafter always referred to by such letters or numerals. The pressures in the state of rest are denoted by P' instead of $\overline{P}$ as in this text-book.

* Written by Lieutenant (j. g.) R. K. Turner, U. S. Navy, at the Naval Gun Factory, February, 1916.

Sheets 2, 3, and 4 are used for the preliminary computations and Sheet 6 for the final computations.

It sometimes happens that the dimensions of the gun as laid down in the pencil drawing do not given sufficient strength, or that a very sudden break in the curve is caused by an improperly designed joint. To correct these faults new dimensions are tentatively assigned or the faults at the joint in question corrected. The strength is then computed with the new dimensions.

79. Computation Forms.—Sheet 1 of the computations is headed "Constants depending on fixed radii and constant modulus of elasticity" and gives the values of the radii and their various combinations with each other, together with the logarithms.

Sheets 2 and 3 are headed "Computations for reduced formulas and maximum values," and "Computations for reduced formulas and maximum values corrected," respectively, and give the logarithmic forms for computing:

1. The maximum elastic forces $P_m(\theta_m)$ and $P_m(\rho_m)$ (for any layer m).
2. A function $l_m(P_m)$ of the variations of pressures between the states of action and rest.
3. The pressures, state of rest, $P_m{}^1$.
4. The initial limiting pressure on the tube, P_1'.
5. The adopted values of $P_o(\theta_o)$ and $P_o(\rho_o)$ corresponding to the minimum of P_1'.
6. P_m' corresponding to the minimum P_1'.

$P_m(\theta_m)$ is the pressure at any surface that will bring the metal to its limit of elastic tangential strain at that surface, while $P_m(\rho_m)$ is the pressure that will bring the metal to the elastic limit of radial strain. If the pressure is greater than $P_m(\theta_m)$ the metal will actually be permanently stretched tangentially and may even crack if the ultimate strength is passed. On the other hand, if the pressure is greater than $P_m(\rho_m)$ without being greater than $P_m(\theta_m)$ the metal will crush slightly and so enlarge the bore, but its tangential tenacity, upon which the actual stability of the gun depends, will in no way be affected; in other words, $P_m(\rho_m)$ may be exceeded without any other effect than a slight increase in the diameter of the bore, so long as the metal at the outside of the layer is not strained in the same way beyond its elastic limit. This increase in the bore diameter will be very slight and is therefore never considered in the case of the inner layer, so that $P_o(\theta_o)$ is always used instead of $P_o(\rho_o)$, no matter which is the smaller. It will be otherwise with the other layers, however, because it is apparent that any increase in the bore diameter of any layer except the first will reduce the shrinkage of that layer, and will therefore decrease the possible range of working of the inner layers, thus reducing the elastic tangential resistance. The only layer this does not apply to is the outer, since there $P_{n-1}(\rho_{n-1})$ is always less than $P_{n-1}(\theta_{n-1})$. Therefore the following rule is adopted in computing the successive values of $P_m(\theta_m)$ and $P_m(\rho_m)$: "For computing the successive values of $P_m(\theta_m)$ and $P_m(\rho_m)$ always use the smaller of the two quantities, $P_{m+1}(\theta_{m+1})$ and $P_{m+1}(\rho_{m+1})$."

The functions $l_m(p_m)$ is obtained from the formula:

$$l_m(p_m) = \frac{R_m^2(R_n^2 - R_{m+1}^2)}{R_{m+1}^2(R_n^2 - R_m^2)}$$

and is used for the purpose of computing the variations in pressures, p_m. The latter is computed with the formula:

$$p_{m+1} = p_m \times l_m(p_m)$$

and the pressure, state of rest, from

$$P_m' = P_m - p_m$$

The initial limiting pressure on the tube is determined by that pressure, P_1', which the tube will sustain in the state of rest without passing the elastic limit of compression, since it has been shown in the deduction of the formulas (Alger, equation 24) that the dangerous strain, in a tube subjected to external pressure only, occurs at the inner surface and is a tangential compressive strain. If the tube is not to be bored for a liner the first of the two formulas, that for finding ρ_o', is not used, but the second formula only, ρ_o' being taken equal to ρ_o and R_o equal to the inner radius of the tube. Both formulas must be used if the tube is to be bored for a liner.

The "Pressures, state of rest, relieving jacket" give first the computation of the pressures in the state of action at the inner surface of the jacket, using the minimum value of P_1', and then the pressures in the state of rest in the outer layers of hoops that will be required to produce the maximum pressures for the state of action when the variations in pressures have been reduced proportionately to the reduction in P_o. In other words:

$$P_m' = P_m'(\text{max.}) - p_m$$

This will subject the two outer layers to the maximum elastic stress and will reduce the maximum stress in the jacket. Thus all the layers will not participate proportionately in the transverse work, and it may happen, when the working limits on Sheet 4 are figured, that the metal of the jacket will be found to be strained beyond its elastic limit in the state of rest. In this case it will be necessary to reassign values of P_m' to the outer hoops to make the proper adjustments. Therefore this is essentially a trial method and may entail a great deal of additional labor. For this reason the set of approximate formulas under "Pressures, state of rest, corrected, relieving hoops" were adopted and are ordinarily used. These formulas relieve the pressures, P_m', on the outer hoops proportionately to the reduction in P_1' and differ from the theoretically correct pressures by negligible amounts, a small constant term having been omitted in the derivation of each of the formulas. When using this method one may be sure of getting values of P_m' that will not over compress the jacket in the state of rest, though the total maximum resistance of the gun may be very slightly reduced. The jacket is thus made to do its proper share of the work, which is desirable, unless there are special reasons to the contrary, as, for instance, when the screw-box liner is attached only to a comparatively thin jacket.

Sheet 4 gives the "Computations for reduced formulas, shrinkages, and compression of the bore," using the adopted pressures, state of rest, corrected, P_m'. The formulas are self-explanatory.

Sheet 5 is a summary of the reduced formulas and a tabulation of the values of the subsidiary constants a_n, b_n, c_n, etc. This sheet is no longer used, however, as it consumes more time than it saves. In its place has been substituted Sheet 7, with one set of values omitted, viz., the "Relative shrinkages." This space is then used for writing in the "adopted" shrinkages in the adjustment of shrinkages. As Sheet 7 gives the absolute values of all the quantities required, instead of their values in terms of the subsidiary constants, it is much easier to visualize all the conditions obtaining at the various sections and thus gain a clearer viewpoint for the proper adjustment of shrinkages.

In general the preliminary computations may be considered complete with the completion of Sheets 1, 2, 3, and 4.

80. Adjustment of Shrinkages.—When the preliminary computations have been finished the values for all sections are tabulated on Sheet 7 as outlined above. It may be noted that the absolute shrinkages come out to six or eight places of decimals, though it is known that it is necessary to allow a plus or minus tolerance of about .0005 inch, since large machine turning cannot be done more accurately than that. It is obvious, therefore, that the assigned shrinkage can only be given to thousandths of an inch and a total tolerance range allowed of .001 inch.

It will be found that the shrinkages often change their value abruptly when computed for maximum strength, and since it is not desirable to cut a large number of shoulders on the various elements the change must be made gradually in the form of a cone. Also, for the sake of economy and accuracy, it is better to have one shrinkage extend over the greatest possible length of the surface of the element. There are many other practical aspects of the subject of shrinkage, as, for instance, the fact that a heavy shrinkage must not be put on a thin section either for fear of overstraining the metal or because it is obvious that it will not hold the shrinkage until the next envelope is in place. All the various considerations are the result of experience and therefore the assignment of shrinkages can follow no definite rules that will be applicable to all cases.

In general, however, shrinkages are assigned the same value over as long a surface as possible and the value expressed to the nearest thousandth of an inch below the minimum theoretical shrinkage for that surface. The various contact surfaces are considered in order beginning at the inner, and their relation to each other must be understood in order to assign proper values. For instance, if the theoretical shrinkages are:

$$S_1 = .0016 \quad S_2 = .0483 \quad S_3 = .0571$$

it is at once apparent that S_1 must be made greater than .0016, because the tube and jacket will not hold together under so small a pressure as will result from the use of this shrinkage. Therefore a larger value of S_1 is chosen and S_2 and S_3 decreased so that the pressure at the outer surface of the layers in the state of rest will not be too great. In this case shrinkages might be assigned as follows:

$$S_1 = .012 \quad S_2 = .040 \quad S_3 = .047$$

A reassignment will be made if these values are shown to be unsuitable by the computations on Sheet 6.

One other general principle of shrinkage is that it is desirable to work the tube higher than the outer layers; in other words, the tube is to be considered the limiting layer.

81. Final Computations.—The final computations are made on Sheet 6 and the results tabulated on Sheet 7, together with the maximum theoretical pressures found on Sheets 2, 3, and 4. From these tabulated values are constructed the curves of tangential and radial resistance and relative compression of the bore.

Sheet 6 shows the assumed values of the shrinkages and gives the forms for the logarithmic computation of values of the compressions, the pressures P_m' and P_m, and the tangential compression resulting from the use of the assigned shrinkages.

In addition to the formulas for finding the necessary quantities are a considerable number of check formulas obtained by the transposition of the regular formulas. For instance, there are three sets of computations for "Working limits," one for checking the theoretical values of $P_m(\theta_m)$ and $P_m(\rho_m)$, one for checking the assumed values of $P_m(\theta_m)$ and $P_m(\rho_m)$, and one for checking the values of those quantities after the assumed values of the shrinkages have been used. The formulas for "Working limits" give the effective values of θ_m and ρ_m when the various values of $P_m(\theta_m)$ and $P_m(\rho_m)$ are used, and in all cases these values must be equal to or lower than the respective elastic limits of extension and compression for the layer under consideration, except in the case of the inner layer, where ρ_0 may exceed the elastic limit. If, for instance, the true value of θ_m is 60,000 and we get a value of $\theta_m=50{,}000$ from the computation of the working limits, the layer could be replaced by a layer whose elastic limit of extension is 50,000 without reducing the height of the strength curve, and therefore all the total available strength of the layer will not be used when the bore pressure becomes equal to the adopted value of P_0. But if the values of θ_m or ρ_m are greater than the elastic limits either an error has been made or new values of P_m must be chosen.

82. Computations for the Liner.—When a liner is to be originally inserted in the gun it is assembled after the rest of the gun has been built up. In this case computations as to its effect on the other layers are made, the original computations being essentially what would be required for a gun with a bore diameter equal to the inner diameter of the tube.

The formulas are based on the assumption of a two-layer gun, the tube, jacket, and hoops forming the outer layer and the liner the inner layer. The formulas may be deduced from the theoretical formulas for a two layer gun assembled with the shrinkage assigned for the liner. This shrinkage is small because it is desirable to be able to remove the liner and insert a new one without having to bore it out. There is also a possibility of the liner's sticking during assemblage if the shrinkage is very great, since the assembled gun must not be heated to too high a temperature for fear of disassembly of the elements.

The computations are arranged on two sheets, Nos. 8 and 9. It must be understood that so far as the constants are concerned, the liner is treated as a regular layer, R_o being the bore radius and R_1 the outside radius of the liner.

Sheet 8. Assumed Shrinkage $=S_1$.

Pressures, State of Rest, at R_1.—A pressure P_1' on the outside of the liner is caused by putting in the liner with a shrinkage S_1.

$$P_1' = \frac{S_1}{h_1} = S_1 \times \frac{E(R_1^2 - R_o^2)(R_n^2 - R_1^2)}{2R_1^2(R_n^2 - R_o^2) \times D_1}$$

Change of Pressure in State of Rest.—These formulas give the increase of pressure at the various surfaces that result from the insertion of the liner, this causing a pressure of P_1' at the inner surface of the tube where no pressure existed before.

$$p_2' = P_1' l_1 \quad p_3' = p_2' l_2 \quad p_4' = p_3' l_3$$

Pressures, State of Rest.—The addition of the increase of pressures in the state of rest to the *original* pressures before the insertion of the liner gives the new pressures, state of rest, at the contact surfaces.

$$P_2' = p_2' + P_2' (\text{original})$$
$$P_3' = p_3' + P_3' (\text{original})$$
$$P_4' = p_4' + P_4' (\text{original})$$

Tangential Compression of Bore.—$T\rho_o$ is the tangential compressive stress caused at the inner surface of the liner in the state of rest by an outside pressure of P_1'.

$$T\rho_o = \frac{2R_1^2 P_1'}{R_1^2 - R_o^2}$$

Strength of Gun Limited by Liner.—The curve drawn through the plotted values of P_o will be the curve of tangential resistance of the gun when the stress in the liner has a value of θ_o.

$$P_o = \frac{3(R_n^2 - R_o^2)}{4R_n^2 + 2R_o^2}(\theta_o + T\rho_o)$$

Variation of Pressures.—When the gun is fired and a pressure P_o brought into existence in the bore it causes the increase of pressure P_m at the other contact surfaces.

$$p_1 = l_o P_o \quad p_2 = l_1 p_1 \quad p_3 = l_2 p_2 \quad p_4 = l_3 p_3$$

Pressures, State of Action.—P_m is the algebraic sum of the pressures, state of rest, and the increase of pressure p_m. The latter quantity is considered positive in the present case, since it is one of tension with respect to P_m'.

$$P_1=P_1'+p_1 \quad P_2=P_2'+p_2 \quad P_3=P_3'+p_3 \quad P_4=P_4'+p_4$$

Working Limits.—θ_m and ρ_m are the stresses in the various layers when a pressure P_o is caused in the bore.

$$\theta_3=\frac{P_3(\theta_3)}{a_3}$$

$$\theta_2=\frac{P_2(\theta_2)}{a_2}-P_3\times\frac{b_2}{a_2} \qquad \rho_2=\frac{P_2(\rho_2)}{c_2}-P_3\times\frac{d_2}{c_2}$$

$$\theta_1=\frac{P_1(\theta_1)}{a_1}-P_2\times\frac{b_1}{a_1} \qquad \rho_1=\frac{P_1(\rho_1)}{c_1}-P_2\times\frac{d_1}{c_1}$$

$$\theta_o=\frac{P_o(\theta_o)}{a_o}-P_1\times\frac{b_o}{a_o} \qquad \rho_o=\frac{P_o(\rho_o)}{c_o}-P_1\times\frac{d_o}{c_o}$$

When an old gun is to be relined, computations for the liner are made in accordance with a somewhat similar set of formulas, the chief differences being: 1st, that now r represents the outer radius of the liner and R_1 the outer radius of the tube; and 2d, that several additional formulas are necessary to show the changes in the pressures, state of rest, at the various contact surfaces that will result when the tube is bored and the liner inserted.

83. Example of Gun Computations.—The Mark VII, Modification 3, 12″ 50-caliber gun has been selected for the purposes of illustration, the results being given in the case of the section over the chamber, number IV. This is an unlined gun, but provision has been made for the insertion of a conical liner after the inner surface of the tube has been worn out.

It will not be necessary to give the formulas used, as they may be obtained directly from the computation sheets. The results only will be given.

In this case:

$$R_o=15.20 \qquad r=17.083 \qquad R_1=19.75 \qquad R_2=26.0$$
$$R_3=34.0 \qquad R_4=44.0 \qquad E=30{,}000{,}000$$
$$\theta_o=\rho_o=55{,}000 \qquad \theta_1=\rho_1=60{,}000 \qquad \theta_2=\theta_3=\rho_2=\rho_3=65{,}000$$

The elastic limits are the specified values, the actual values not being used because it would be impossible and undesirable to construct strength curves for each individual gun, one set of curves being computed that will apply to all, provided they meet the specifications.

The subscript m will be used to show that a quantity may apply to any layer.

Preliminary Computations.

SHEET 2. COMPUTATIONS FOR REDUCED FORMULAS AND MAXIMUM VALUES.

1. *Maximum Pressures.*—Theoretically possible.

$P_3(\theta_3) = 15{,}056$

$P_2(\theta_2) = 33{,}217$ $P_2(\rho_2) = 39{,}293$ [Use $P_3 = P_3(\theta_3)$]

$P_1(\theta_1) = 53{,}442$ $P_1(\rho_1) = 50{,}095$ [$P_2(\theta_2) < P_2(\rho_2)$ Use $P_2 = P_2(\theta_2)$]

$P_0(\theta_0) = 70{,}948$ $P_0(\rho_0) = 59{,}479$ [$P_1(\rho_1) < P_1(\theta_1)$ Use $P_1 = P_1(\rho_1)$]

2. *Working Limits.*—Check for accuracy of computations for (1).

$\theta_3 = 65{,}000$ $\theta_1 = 60{,}000$ $\rho_1 = 60{,}000$

$\theta_2 = 65{,}000$ $\rho_2 = 65{,}000$ $\theta_0 = 55{,}000$ $\rho_0 = 55{,}000$

3. *Variations of Pressures.*—Computation of $l_m(p_m)$.

$l_0(p_0) = (T.73004)$ $l_1(p_1) = (T.67236)$ $l_2(p_2) = (T.55872)$

SHEET 3. COMPUTATIONS FOR REDUCED FORMULAS AND MAXIMUM VALUES, CORRECTED.

4. *Pressures, State of Rest.*—Theoretically possible.

$p_1 = 38{,}105$ ($P_0(\theta_0)$ has been used for the reasons given in §79.)

$p_2 = 17{,}920$ $p_3 = 6{,}487$

$P_1' = P_1 + (-p_1) = 11{,}990$ $P_2' = P_2 + (-p_2) = 15{,}297$

$P_3' = P_3 + (-p_3) = 8{,}569$

The minimum values of P_m are used for the reasons given in §79. The quantity p_m carries the minus sign because it is negative as compared to P_m.

5. *Initial Limiting Pressure on Tube.*—This is the value of the maximum P_1' that will allow the tube to be bored for the liner without collapsing. The formulas are based on the assumption of a two-layer gun.

$$\rho_0' = 53{,}166 \qquad P_1' = 10{,}838$$

It will be noted that the P_1' given by (4) is greater than that found here, therefore the latter value will be used for correcting the maximum allowable pressures.

6. *P_0 Corresponding to P_{n-1}'.*—Check for "Variations of Pressures" and "Pressures, State of Rest."

$$P_0 = 70{,}948$$

7. *P_0 Corresponding to $P_1 + (-p_1)$.*—Computation of subsidiary constants, check for P_1', and computation of radial resistance when $P_0(\theta_0)$ (theoretical) is used.

$$P_o(\theta_o)=70{,}949$$
$$P_0(\rho_0)=52{,}405$$

It may be noted that the $P_o(\rho_o)$ found here is less than the theoretical $P_o(\rho_o)$; this will always be the case when $P_o(\theta_o)>P_o(\rho_o)$ as found on Sheet 2.

8. *Pressures Corrected.*—This gives the maximum theoretical $P_o(\theta_o)$ and $P_o(\rho_o)$ using the minimum of the two values of P_1', and the maximum variations in pressures corresponding to the adopted value of P_o. If the initial limiting pressure on the tube is not found, this computation is unnecessary.

$P_o(\theta_o)$adopted$=67{,}425$ $P_o(\rho_o)$adopted$=51{,}081$
$p_1=36{,}213$ $P_0(\theta_0)$ (adopted) is used for the reason given in §79.
$p_2=17{,}030$ $p_3=6{,}165$

9. *Pressures, State of Rest, Relieving Jacket.*—See explanation of this and the following set of formulas in §79. It may be noted that only the first of the three following formulas has been used, so as to obtain the pressure, state of action, at the outer surface of the tube corresponding to P_1(min.). The hoops and not the jacket have been relieved in this gun.

$$P_1=47{,}051$$

10. *Pressures, State of Rest, Relieving Hoops.*—These now become the preliminarily adopted values of the pressures in the state of rest on the jacket and *C*-hoop.

$$P_2'=14{,}537 \quad P_3'=8{,}209$$

SHEET 4. COMPUTATIONS FOR REDUCED FORMULAS, SHRINKAGES, AND COMPRESSION OF BORE.

11. *Shrinkages.*—The shrinkages here found are those necessary to give the adopted pressures in the state of rest.

$$S_1=.0092153 \quad S_2=.039499 \quad S_3=.050832$$

12. *Compressions of Bore.*—δ_1, δ_2, and δ_3 are the partial relative bore compressions that result from the successive shrinkage of the three outer layers, and the final relative compression is their sum. The same applies to Δ_m, the absolute compressions.

$$\delta_1=.00029984 \qquad \Delta_1=.0045575$$
$$\delta_2=.00078837 \qquad \Delta_2=.011983$$
$$\delta_3=.00068397 \qquad \Delta_3=.010260$$
$$\delta_o=\delta_1+\delta_2+\delta_3=.00177218$$

13. *P_1' Corresponding to δ_0.*—This is a check for (12).

$$P_1'=10,838$$

14. *Tangential Compression.*

$$\rho=53,166$$

ρ should equal the elastic limit of the metal if the theoretical values of P_1' has been used. If a lower value has been adopted such that the bore of the tube will not be compressed to the elastic limit in the state of rest, ρ will be less than θ_0.

15. *Working Limits.*—The values of θ_m and ρ_m are the *effective* elastic limits as defined in §81. They are introduced as a check on the accuracy of the preceding work, and to find the relative participation of the layers after the outer layers have been relieved so that the tube will not be over-compressed when bored for the liner. It may be noted that θ_0 equals the allowed elastic limit, while ρ_0 greatly exceeds this limit; this is in accordance with what has been said in §79. In the case of the other layers neither θ nor ρ may exceed the proper elastic limits. The ratios of θ_m found here to the allowed θ_m show the relative participation of the various layers.

$$\theta_0=55,000 \quad \rho_0=78,270 \quad \theta_2=61,770 \quad \rho_2=48,649$$
$$\theta_1=41,842 \quad \rho_1=55,770 \quad \theta_3=61,772$$

Sheet 5 is no longer used, but Sheet 7 instead. This will be called Sheet 7a.

Final Computations.

Sheet 7a is for the adjustment of shrinkages. As the shrinkages are adjusted with relation to the other sections, the shrinkages for Sections II to IX are tabulated below to show the method used.

THEORETICAL ABSOLUTE SHRINKAGES.

Shrinkage.	Section.							
	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
S_1	←.024954	←.0092313	.0092153	.010705	.010428→	←.013466	.01258→	←.012470
S_2	.. ↑	←.039574	.039499→	←.039415	.039428	.039075	.039462	.037567→
S_3	.... →	.050773	.050832	.050822→	←.049281	.049194	.049550	.047698

The small arrows indicate the presence of a shoulder between the two sections where they occur. Thus the tube has a shoulder between Sections VI and VII.

From what has been said before it is at once apparent that it would be impossible to obtain these theoretical shrinkages on account of their wide variations and it would therefore be useless and bad practice to assign them. The first thing to do is to examine this table carefully and then by balancing the various considerations governing the adjustment of shrinkages finally arrive at a logical conclusion.

S_1 for Section II, where there are two layers only, may at once be given a value of .025.

From Sections III to VI S_1 varies from .0092313 to .010428. In no case may these shrinkages be exceeded without over-compressing the tube when bored for the liner, so that a proper value of S_1 for these sections seems to be .009. For the same reasons S_1 from Sections VII to IX is given a value of .012.

Proceeding in this way from one surface to another the shrinkages are relieved slightly in every case, until finally the shrinkages as given in the table below are tentatively adopted.

ASSIGNED SHRINKAGES.

Shrinkage.	Section.							
	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
S_1	.025	....	.009	....	....	.012	....	.012
S_2	..	....	.039	....	.037	....	....	....
S_3	....	....	.050	....	....	.047	....	....

The assigned shrinkages must now satisfy the conditions that the tube will not be over-compressed when bored for the liner and that no metal in the gun is strained beyond its elastic limit. Whether it fulfills these conditions is determined in the computations on Sheet 6.

As a matter of fact the shrinkages finally assigned to Section III were $S_1 = .009$, $S_2 = .025$, and $S_3 = .025$, because the cross strains due to the heavy longitudinal stresses at that section made it advisable to relieve the tangential strains that would have occurred if the values given in the table had been used.

When a large change in the assigned shrinkages occurs at a section the change is made gradual by the use of a coned surface. This method is shown in the small figure in the lower left hand corner of the drawing of the strength curves, etc.

SHEET 6. COMPUTATIONS FOR ADJUSTED VALUES.

These formulas are similar to those on Sheets 2, 3, and 4, except that the subsidiary constants do not have to be computed, and the correction and most of the check formulas may be omitted.

16. *Assumed Values.*

$$S_1 = .009 \quad S_2 = .039 \quad S_3 = .050$$

17. *Shrinkages and Compressions.*—Computations are made using the assumed value of S and subsidiary constants from Sheet 4.

$\phi_1 = .00045569$	$\delta_1 = .00029283$	$\Delta_1 = .0044510$
$\phi_2 = .0015000$	$\delta_2 = .00077841$	$\Delta_2 = .011832$
$\phi_3 = .0014706$	$\delta_3 = .00067279$	$\Delta_3 = .010226$
	$\delta_o = .00174403$	

18. *Pressures, State of Rest.*—These are computed from the relative compressions and certain subsidiary constants obtained from Sheet 4.

$$P_3' = 8{,}075 \quad P_2' = 14{,}328 \quad P_1' = 10{,}665$$

19. P_1' *Corresponding to* δ_o.—Check for (17) and (18).

$$P_1' = 10{,}665$$

The value of P_1' must not be greater than the "Initial limiting pressure on tube" found on Sheet 3.

20. *Tangential Compression.*—This must not be greater than the elastic limit of compression.

$$\rho = 52{,}320$$

21. *Pressures, State of Action.*—The first five of these formulas are similar to those in (8); the remainder are those in (4) reversed.

$P_o(\theta_o) = 66{,}898$		$P_o(\rho_o) = 50{,}883$
$p_1 = 35{,}930$	$p_2 = 16{,}897$	$p_3 = 6{,}117$
$P_1 = 46{,}595$	$P_2 = 31{,}225$	$P_3 = 14{,}192$

22. *Working Limits.*—Relative participation of layers.

$\theta_o = 55{,}000$	$\rho_o = 77{,}796$
$\theta_1 = 41{,}610$	$\rho_1 = 55{,}286$
$\theta_2 = 61{,}222$	$\rho_2 = 48{,}164$
$\theta_3 = 60{,}989$	

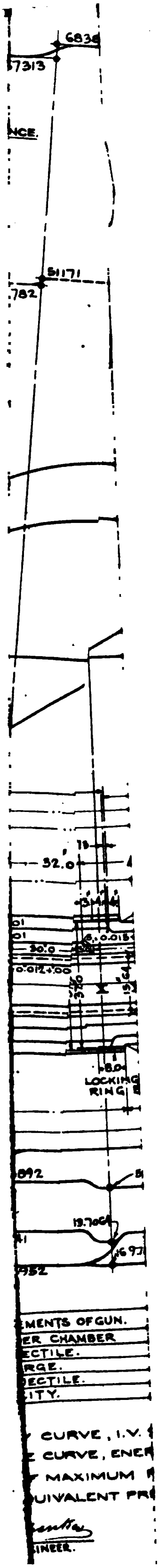
The values of θ_m must not be greater than the elastic limit; θ will be equal to the elastic limit of the tube, while in general the values of θ for the other layers will be less than the elastic limits of those layers. ρ_0 may be greater than the elastic limit of the tube, but the values of ρ for the other layers must not be greater than the elastic limit.

SHEET 7 IS THE TABULATION OF THE COMPUTATIONS AND GIVES THE VALUES OF

Formulas	No. 1.	Maximum pressures.
"	" 4.	Initial maximum compression (P_1' only).
"	" 5.	Limiting pressure on tube.
		Adjusted pressures.
"	" 21.	a. In action.
"	" 18.	b. At rest.
		Shrinkages.
"	" 17.	a. Relative.
"	" 16.	b. Absolute.
		Compressions of bore.
"	" 17.	a. Relative.
"	" 17.	b. Absolute.
"	" 22.	Working limits.
"	" 20.	Tangential compression.

When all the sections have been computed the curves of tangential resistance, radial resistance, and relative compression of the bore are drawn with the values of $P_o(\theta_o)$, $P_o(\rho_o)$, and δ_o, δ_1, δ_2, and δ_3 computed on Sheet 6. The curves of velocities and pressures in the bore are then drawn for purposes of comparison. It will seldom happen that any of the curves will be changed after all the sections have been computed, because the strength actually obtained for each section is compared with the requirements as soon as the computations for the section have been finished.

The figure shows the drawing of the "Shrinkages, strength, velocity, and pressure curves" for the Mark VII, Modification 3. 12″ 50-caliber gun.



6838
7313
NCE.
51171
782
52.0
LOCKING RING
892
19.706
952
MENTS OF GUN.
ER CHAMBER
ECTILE.
RGE.
ECTILE.
ITY.
CURVE, I.V.
CURVE, ENER
MAXIMUM
UIVALENT PR
INEER.

APPENDIX.

FORMULÆ FOR THE CASE OF COMPOUND CYLINDERS OF FOUR LAYERS.

$$(1)\quad P_3(\theta) = \frac{3(R_4^2 - R_3^2)\,\theta_3}{4R_4^2 + 2R_3^2}$$

$$(2)\quad P_2(\theta) = \frac{3(R_3^2 - R_2^2)\,\theta_2 + 6P_3R_3^2}{4R_3^2 + 2R_2^2}$$

$$(3)\quad P_1(\theta) = \frac{3(R_2^2 - R_1^2)\,\theta_1 + 6P_2R_2^2}{4R_2^2 + 2R_1^2}$$

$$(4)\quad P_o(\theta) = \frac{3(R_1^2 - R_o^2)\,\theta_o + 6P_1R_1^2}{4R_1^2 + 2R_o^2} \qquad \text{(A)}$$

$$(5)\quad P_2(\rho) = \frac{3(R_3^2 - R_2^2)\,\rho_2 + 2P_3R_3^2}{4R_3^2 - 2R_2^2}$$

$$(6)\quad P_1(\rho) = \frac{3(R_2^2 - R_1^2)\,\rho_1 + 2P_2R_2^2}{4R_2^2 - 2R_1^2}$$

$$(7)\quad P_o(\rho) = \frac{3(R_1^2 - R_o^2)\,\rho_o + 2P_1R_1^2}{4R_1^2 - 2R_o^2}$$

$$(8)\quad [P_o] = \frac{3(R_4^2 - R_o^2)\,(\theta_o + \rho_o)}{4R_4^2 + 2R_o^2}$$

If $P_o(\theta)$ is greater than $[P_o]$, the tube will be compressed beyond its elastic limit of compression (ρ_o) by shrinkages determined with the values of $P_3(\theta)$ $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$, and so the values of one or more of the assumed elastic limits θ_3, θ_2 and θ_1 must be reduced until $P_o(\theta)$ equals, or is less than, $[P_o]$.

$$(1)\quad S_3 = \frac{2R_3}{E}\left[\theta_3 + \frac{P_3(\theta)\,(4R_o^2 + 2R_3^2) - 6P_o(\theta)\,R_o^2}{3(R_3^2 - R_o^2)}\right]$$

$$(2)\quad S_2 = \frac{2R_2}{E}\left[\theta_2 + \frac{P_2(\theta)\,(4R_o^2 + 2R_2^2) - 6P_o(\theta)\,R_o^2}{3(R_2^2 - R_o^2)}\right] \qquad \text{(B)}$$

$$(3)\quad S_1 = \frac{2R_1}{E}\left[\theta_1 + \frac{P_1(\theta)\,(4R_o^2 + 2R_1^2) - 6P_o(\theta)\,R_o^2}{3(R_1^2 - R_o^2)}\right]$$

In these expressions for the shrinkages, the values of θ_3, θ_2 and θ_1 are not necessarily the real elastic limits, but are the assumed elastic limits with which the finally accepted values of $P_3(\theta)$, $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$ were calculated.

$$\begin{aligned}
&(1)\quad \overline{P}_3 = P_3(\theta) - \frac{R_o^2(R_4^2 - R_3^2)}{R_3^2(R_4^2 - R_o^2)} P_o(\theta) \\
&(2)\quad \overline{P}_2 = P_2(\theta) - \frac{R_o^2(R_4^2 - R_2^2)}{R_2^2(R_4^2 - R_o^2)} P_o(\theta) \qquad \text{(C)} \\
&(3)\quad \overline{P}_1 = P_1(\theta) - \frac{R_o^2(R_4^2 - R_1^2)}{R_1^2(R_4^2 - R_o^2)} P_o(\theta)
\end{aligned}$$

These are the pressures at the surfaces of contact *in the state of rest,* $P_3(\theta)$, $P_2(\theta)$, $P_1(\theta)$ and $P_o(\theta)$ being the values of the pressures in the state of action used in calculating the assigned shrinkages.

$$\bar{e}_t(R_o) = -\frac{R_2^2 - R_1^2}{R_2^2 - R_o^2}\frac{S_1}{2R_1} - \frac{R_3^2 - R_2^2}{R_3^2 - R_o^2}\frac{S_2}{2R_2} - \frac{R_4^2 - R_3^2}{R_4^2 - R_o^2}\frac{S_3}{2R_3} \qquad \text{(D)}$$

This is the circumferential strain at the surface of the bore caused by the superposition of the three outer layers with their respective shrinkages S_1, S_2 and S_3, the successive terms being the three circumferential strains produced by the three successive layers. $2R_o\bar{e}_t(R_o)$ is the change of diameter (contraction) of the bore from its free state to that of complete assemblage of the system, and $-E\bar{e}_t(R_o)$ is the circumferential compression of the bore in the state of rest.

The radial strain at the surface of the bore in the state of rest is $\bar{e}_p(R_o) = -\frac{1}{3}\bar{e}_t(R_o)$, so that it is under a true tension radially one-third as great as its circumferential compression. Therefore the real elastic strength of the system when assembled with shrinkages S_1, S_2 and S_3 is the least of the two following values of P_o:

$$\begin{aligned}
&(1)\quad P_o^{(1)} = \frac{3(R_4^2 - R_o^2)}{4R_4^2 + 2R_o^2}(\theta_o - E\bar{e}_t(R_o)) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(E)} \\
&(2)\quad P_o^{(2)} = \frac{3(R_4^2 - R_o^2)}{4R_4^2 - 2R_o^2}(\rho_o - \tfrac{1}{3}E\bar{e}_t(R_o))
\end{aligned}$$

In these expressions it is important to note that $\bar{e}_t(R_o)$ is a *negative strain,* so that the last factor in each of the two values of P_o is *numerically the sum,* not the difference, of the elastic limits (of tension and compression respectively) and the true stresses at the surface of the bore in the state of rest (circumferential and radial respectively).

The pressures in the state of rest may be computed directly from the shrinkages by the following formulæ:

$$(1)\quad \overline{P}_3 = E\frac{R_3^2 - R_o^2}{2R_3^2}\cdot\frac{R_4^2 - R_3^2}{R_4^2 - R_o^2}\cdot\frac{S_3}{2R_3}$$

$$(2)\quad \overline{P}_2 = E\frac{R_2^2 - R_o^2}{2R_2^2}\left(\frac{R_3^2 - R_2^2}{R_3^2 - R_o^2}\frac{S_2}{2R_2} + \frac{R_4^2 - R_3^2}{R_4^2 - R_o^2}\frac{S_3}{2R_3}\right) \qquad \text{(F)}$$

$$(3)\quad \overline{P}_1 = E\frac{R_1^2 - R_o^2}{2R_1^2}\left(\frac{R_2^2 - R_1^2}{R_2^2 - R_o^2}\frac{S_1}{2R_1} + \frac{R_3^2 - R_2^2}{R_3^2 - R_o^2}\frac{S_2}{2R_2} + \frac{R_4^2 - R_3^2}{R_4^2 - R_o^2}\frac{S_3}{2R_3}\right)$$

The terms in the parentheses are the values of the circumferential strains at R_o caused by the assemblage of the successive layers, their sum with the negative sign being the total compressive strain at the surface of the bore as given by equation (D).

From the pressures in the state of rest, as given by (F), the pressures in the state of action may be found by equations (C), and the true circumferential tension of the inner surface of any layer can then be found by

$$Ee_t(R'_{n-1}) = \frac{P_{n-1}(4R_n^2 + 2R_{n-1}^2) - 6P_nR_n^2}{3(R_n^2 - R_{n-1}^2)} \qquad \text{(G)}$$

In this R_n and R_{n-1} are the outer and inner radii of any layer, P_n and P_{n-1} are the outer and inner pressures (either of action or of rest) and $Ee_t\,(R'_{n-1})$ is the true circumferential tension at the inner surface of the layer resulting from the action of P_n and P_{n-1}.

Similarly, the true radial compression at the inner surface of any layer, either in the state of rest or of action, is given by

$$Ee_p(R'_{n-1}) = \frac{P_{n-1}(4R_n^2 - 2R_{n-1}^2) - 2P_nR_n^2}{3(R_n^2 - R_{n-1}^2)} \qquad \text{(H)}$$