ELECTRICAL CHARACTERISTICS of TRANSMISSION CIRCUITS

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ELECTRICAL CHARACTERISTICS OF TRANSMISSION CIRCUITS

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PREFACE

THE rapid expansion in distributing and transmission systems will continue unabated until the natural power resources will have been fully developed. This expansion will necessitate a tremendous amount of arithmetical labor in connection with the proper solution and calculation of performance of projected transmission and distributing circuits. It will demand much valuable time and energy in the education of the younger engineers now going thru the technical schools and others who will follow them. It was primarily to assist these younger engineers by making their work more easy and less liable to error, and providing them with all necessary tools that the data in this book have been compiled.

Many articles each pertaining to some particular method of solution of transmission circuits have been published from time to time. This book constitutes a review of each of numerous methods perviously proposed by different authors with examples illustrating each method of solution and the accuracy which may be expected by its use. Thus by permission of various authors the reader of this book is provided with a choice of numerous methods ranging between the most simplified graphical forms of solutions and complete mathematical solutions. He is also provided with numerous and extensive tables of circuit and other constants making it unnecessary for him to lose time and risk making mistakes in calculating constants for each case in question. Much effort has been expended with a view of simplifying explanations by the aid of supplementary diagrams and tabulations. The engineer upon whose lot it only occasionally falls to determine the size of conductors and performances of circuits appreciates how easy it is to make errors in calculations which may prove very serious and should find the quick estimating tables very useful particularly for short line solutions.

For those preferring to avoid the more mathematical solutions the all graphical methods for solving long line problems including the Wilkinson & Kennelly charts for obtaining graphically the auxiliary constants should prove helpful.

When borrowed material has been used in this book full credit has been given the author at the place the material is used. It is desired, however, at this place to mention the high appreciation of assistance given by Ralph W. Atkinson, Herbert B. Dwight, Dr. A. E. Kennelly, Dr. A. S. McAllister, Ralph D. Mershon, F. W. Peak Jr., J. F. Peters, Charles R. Riker and T. A. Wilkinson.

Wm Nesbit

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ELECTRICAL CHARACTERISTICS *of* TRANSMISSION CIRCUITS

CHAPTER I
RESISTANCE—SKIN EFFECT—INDUCTANCE

THE transmission of alternating-current power involves three separate circuits, one of which is composed of the wires forming the transmission line, while the others lie in the medium surrounding the wires. The constants of these circuits are interdependent; although any one may vary greatly from the others in magnitude.* There is first the electric circuit through the conductors. Then since all magnetic and dielectric lines of force are closed upon themselves forming complete circuits there is a magnetic and a dielectric circuit. The magnetic circuit consists of magnetic lines of force encircling the current carrying conductors and the dielectric circuit the dielectric lines of force terminating in the current carrying conductors. The close analogy of these is given in Table A, a careful study of which will help those not familiar with the subject to a clearer understanding of what happens in an alternating-current transmission circuit.

For a unidirectional constant current the magnetic field remains constant, and similarly for a unidirectional constant voltage the dielectric field is constant. With both the current and the voltage unidirectional and constant, the electric circuit alone enters into the calculations. A changing magnetic flux introduces a voltage into the electric circuit which modifies the initial or impressed voltage. This effect of the magnetic circuit, which is measured by the inductance L, storing the energy $0.5i^2L$, is a function of the current, and hence is of most importance in dealing with heavy current circuits. Similarly a changing electrostatic flux adds (vectorially) a current to the main power current. This effect of the dielectric circuit, which is measured by the capacitance, storing the energy $0.5e^2C$, is a function of the voltage, and hence is of most importance in dealing with high-voltage circuits.

In an alternating-current circuit, both the voltage and the current are continually varying in magnitude, and morever, reversing in direction for each successive half cycle. Therefore, with alternating currents, energy changes occur continuously and simultaneously in the interlinked magnetic, dielectric and electric circuits.

Figs. 1 to 5 inclusive illustrate the magnetic and dielectric field surrounding conductors carrying current. Figs. 1 and 3 represent respectively the magnetic and dielectric circuits when the conductors are far apart and Figs. 2 and 4 when they are close together. Fig. 5 represents the resultant of the superimposed magnetic and dielectric fields.

The magnetic field surrounding a conductor which is not influenced by any other field is represented by concentric circles. This field is strongest at the surface of the conductors and rapidly decreases with increasing distance from the conductor as indicated by the spacing of the lines of Figs. 1 and 2.

The dielectric stresses surrounding conductors are represented by lines drawn radially from the conductor. The strength of the dielectric field likewise decreases with the distance from the conductor as is indicated by the widening of the space between the lines. The magnetic and the dielectric lines of force always cross each other at right angles, as shown in Fig. 5.

*For a further description of these circuits see "Alternating Currents" by Prof. Carl E. Magnusson, from which Figs. 1 to 5 are reproduced with the permission of the author.

RESISTANCE OF COPPER CONDUCTORS

In Table I the resistance per thousand feet is listed and in Table II per mile of single conductor. Values are given for both solid and stranded copper conductors at both 100 and 97.3 percent conductivity and corresponding to various temperatures between zero and 75 degrees C. The foot notes with these tables cover all of the pertinent data upon which the values are based.

The resistance values in Table I corresponding to temperatures of 25 and 65 degrees C. were taken from

FIG 1
MAGNETIC FIELD OF SINGLE CONDUCTOR

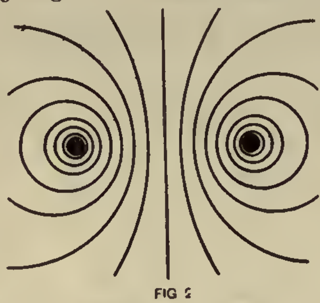


FIG 2
MAGNETIC FIELD OF CIRCUIT

THE MAGNETIC CIRCUIT

FIG. 3
DIELECTRIC FIELD OF SINGLE CONDUCTOR

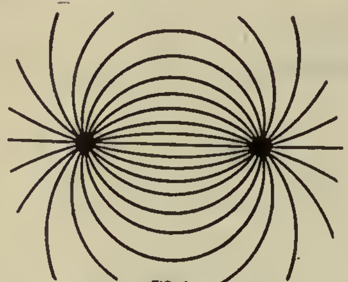


FIG. 4
DIELECTRIC FIELD OF CIRCUIT

THE DIELECTRIC CIRCUIT

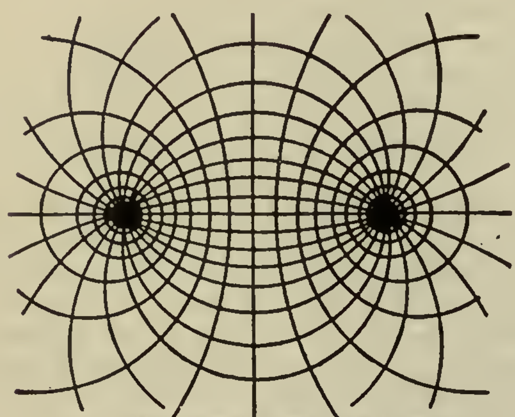


FIG. 5. COMBINED DIELECTRIC AND MAGNETIC CIRCUITS

THE ELECTRIC CIRCUIT

Bulletin 31 of the Bureau of Standards issued April 1st, 1912. The resistance values (taking into account the expansion of the metal with rise in temperature) for the other temperatures were calculated in accordance with the following rule from page 10 of Bulletin No. 31.

> The change of resistivity of copper per degree C. is a constant, independent of the temperature of reference and of the sample of copper. This resistivity-temperature constant may be taken for general purposes as 0.0409 ohm (mil foot).

As an illustration:—A 2 000 000 circ. mil stranded copper conductor at 100 percent conductivity, has a resistance of 0.00623 ohm per 1000 feet at 65 degrees C. Required to calculate its resistance at zero degrees C.

$65 \times 0.0409 = 2.6585$ ohms (mil-foot) temperature correction or 2658.5 ohms (mil, 1000 feet). $\frac{2658.5}{2\,000\,000} = 0.00133$ ohm change in resistance. $0.00623 - 0.00133 = 0.0049$ ohm resistance at zero degrees C.

It has been customary to publish tables of resistance values based upon a temperature of 20 degrees C. and 100 percent conductivity. The operating temperatures of conductors carrying current is usually considerably higher than 20 degrees C. and therefore calculations based upon this temperature do not often represent operating conditions. Neither does copper of 100 percent conductivity represent the usual condition for transmission circuit copper, whose average conductivity is probably nearer 97.3 percent. The values in Tables I and II furnish a comparison of resistance for annealed and hard drawn copper of stranded and solid conductors at various temperatures based upon the new "Annealed Copper Standard".

SKIN EFFECT

A solid conductor may be considered as made up of separate filaments, just as a piece of wood is made up of separate fibres. As a stranded conductor is actually made up of a number of separate wires, such a conductor will be considered in the following explanation. The inductance of the various wires of the cable will be different, due to the fact that those wires near the center of the cable will be linked by more flux lines than are the wires near the outer surface. The self-induced back e.m.f. will therefore be greater in the wires located near the center of the cable. The higher reactance of the inner wires causes the current to distribute in such a manner that the current density will be less in the interior than at the surface. This crowding of the current to the surface or "skin" of the wire is known as "skin effect".

Since the self-induced e.m.f. is proportional to the frequency as well as to the total flux linked, the skin effect becomes more pronounced at higher frequencies of the impressed e.m.f. It also becomes greater the larger the cross-section, the greater the conductivity and the greater the permeability of the conductor.

As a result the effective resistance of a conductor to alternating current is greater than to direct current. The effective resistance of nonmagnetic conductors to alternating current may be obtained by increasing their direct-current resistances by the percentages in Table B, which were derived by the formulas in Pender's Handbook. Thus the ohmic resistance of a 1 000 000 circ. mil cable is approximately 8.4 percent greater at 60 cycles than its resistance to direct current at a temperature of 25°C. If the temperature of the conductor is 65°C, its 60 cycle ohmic resistance will be approximately 6.4 percent greater than its direct-current resistance. The practical result of skin effect is to reduce the carrying capacity of large cables. As indicated by the values in Table B, skin effect may be neglected when employing non-magnetic conductors except in the use of very large diameters. It is usual to manufacture cables of very large diameter, especially for service at high frequencies, with a non-conducting core. In case of magnetic conductors, such as steel wire or cable, as is some times used for long spans or short high voltage feeders, skin effect must be carefully considered.*

TABLE A—COMPARISON OF THE THREE CIRCUITS

THE ELECTRIC CIRCUIT	THE MAGNETIC CIRCUIT	THE DIELECTRIC CIRCUIT
Current I Voltage $E=RI$ Electric Power	Magnetic Flux ϕ Magnetomotive Force $F=n\ i$ Magnetic Energy	Dielectric Flux ψ Electromotive Force $E=Q/C$ Dielectric Energy
Resistivity Resistance $R=W/I^2$	Reluctivity Reluctance R Inductance $L=\phi/i$	Elastivity $1/K$ Elastance S Capacitance $C=\psi/E$
	Reactance $x=\omega L-1/\omega C$	
Impedance $z=\sqrt{r^2+x^2}$		
Conductivity γ Conductance $g=W/E^2$, $g=r/z^2$	Permeability $\mu=B/H$ Permeance $M=\phi/4\pi F$	Permittivity K Permittance (Capacitance) C
	Susceptance $b=x/z^2$	
Admittance $y=1/z=g\mp j\,b=\sqrt{g^2+b^2}$		

INDUCTANCE

Any moving mass, for instance a flywheel in motion, will resist a change in velocity. That is, the inertia of the moving mass will tend to keep the mass moving when disconnected from the source of power. On the other hand the inertia will oppose any effort to speed up the movement of the mass.

In a similar manner, the inductance of an electric circuit resists a change in current. The cause of inductance in an electric circuit is the magnetic field which surrounds the circuit. When the current changes this magnetic field changes correspondingly, and in effect cuts the conductor, producing an e.m.f. in it. This e.m.f. of self induction has such a direction as to resist the change in current. While the current is increasing, energy is stored in the magnetic field and while the current decreases, the magnetic stored energy is returned to the electric circuit. This effect of the electric current on the surrounding space is termed magnetic induction.

Unit of Inductance—When a rate of change of current of one ampere per second produces an e.m.f. of one volt, the circuit is said to have a unit of inductance called a henry. The henry being inconviently large, a thousandth part of it, called the millihenry, is the usual practical unit. This unit is the coefficient of self-induction and is represented by the letter L.

DISTRIBUTION OF FLUX

When current flows through a conductor, a magnetomotive force (m.m.f.) is established of a value proportional to the current. This m.m.f. is of zero value at the center of the conductor and increases as the square of the distance from the center until the surface is reached. (This statement as well as those following is based upon the assumption of a uniform distribution of current throughout the conductor, the conductor being of non-magnetic material and located in non-magnetic surroundings, such as air). At the surface it becomes maximum for a given current and remains at this maximum value for all distances beyond the surface. It is customary to think of the magnetic field surrounding conductors as concentric circles of lines of force.

A physical picture of the magnetic field density surrounding a current carrying conductor A is shown by Chart I. The magnetic density due to the return circuit (conductor B) is indicated in outline by broken lines. The horizontal divisions represent the distance from the center of conductor A and the height of the curve measured vertically the intensity of the field at the corresponding distance. The radius of the conductor has been assumed as unity, and maximum field density (always at the surface of the conductor) as 100 percent.

The intensity of the magnetic field starts at zero at the conductor center, and increases (with uniform distribution of current in the conductor) directly as the

TABLE B—INCREASE OF EFFECTIVE RESISTANCE DUE TO SKIN EFFECT.

For various sizes of solid copper rods. For stranded conductors of equivalent cross sectional area the skin effect is practically the same as for the solid conductor.

Area in Circ. Mils.	Diameter in Inches of Stranded Conductor	Diameter in Inches of Solid Rod	Percent Increase of Copper Wires Above the Direct-Current Resistance Due to Alternating-Currents of Different Frequencies									
			Based Upon Direct-Current Resistance at 25 Degrees C. (77 Degrees F.)					Based Upon Direct-Current Resistance at 65 Degrees C. (149 Degrees F.)				
			15 Cycles	25 Cycles	40 Cycles	60 Cycles	133 Cycles	15 Cycles	25 Cycles	40 Cycles	60 Cycles	133 Cycles
2 000 000	1.631	1.414	2.2	6.0	14.1	28.0	78.6	1.7	4.5	10.9	22.1	67.0
1 800 000	1.548	1.342	1.8	4.0	11.7	23.7	70.4	1.3	3.7	9.0	18.5	60.0
1 600 000	1.459	1.265	1.4	3.9	9.4	19.4	61.4	1.1	3.0	7.3	15.0	51.8
1 500 000	1.412	1.225	1.3	3.4	8.4	17.4	57.3	0.9	2.6	6.4	13.5	47.4
1 200 000	1.263	1.096	0.8	2.1	5.5	11.7	42.7	0.6	1.7	4.1	9.0	34.8
1 000 000	1.152	1.000	0.6	1.5	3.8	8.4	33.8	0.4	1.1	3.0	6.4	26.2
750 000	0.998	0.866	0.3	0.9	2.2	4.9	20.6	0.2	0.7	1.7	3.7	16.4
500 000	0.815	0.707	0.1	0.4	1.0	2.2	10.1	0.1	0.3	0.7	1.7	7.7
250 000	0.575	0.500	0.0	0.1	0.3	0.6	2.7	0.0	0.1	0.2	0.4	2.0

*References:—For a bibliography on the subject of skin effect see article "Experimental Researches on Skin Effect in Conductors" by A. E. Kennelly, F. A. Laws, and P. H. Pierce in *A. I. E. E. Trans.*, Vol. 34, Part II of Sept. 1915. This article ends with a bibliography on the subject embracing a very complete list of articles.

"Calculation of Skin Effect in Strap Conductors" by H. B. Dwight in *Electrical World*, March 11, 1916.

"Skin Effect in Tubular and Flat Conductors" by H. B. Dwight in *A. I. E. E. Trans.* for 1918.

distance from its center until its surface is reached, where it becomes maximum. For distances beyond the surface of the conductor, the field intensity varies inversely as the distance from its center.

The intensity of the magnetic field at any point is proportional to the m.m.f. acting at that point and inversely proportional to the length of its circular path (magnetic reluctance). Thus at the surface of the

TABLE I—RESISTANCE PER 1000 FEET, OF COPPER CONDUCTORS AT VARIOUS TEMPERATURES

STRANDED CONDUCTORS

B & S No.	Area Circular Mils	Ohms per 1000 Feet of Single Conductor: Annealed Copper, 100% Conductivity, 0°C 32°F	15°C 59°F	20°C 68°F	25°C 77°F	35°C 95°F	50°C 122°F	65°C 149°F	75°C 167°F	Hard Drawn Copper, 97.3% Conductivity, 0°C 32°F	15°C 59°F	20°C 68°F	25°C 77°F	35°C 95°F	50°C 122°F	65°C 149°F	75°C 167°F
	2 000 000	.00487	.00518	.00528	.00539	.00559	.00591	.00623	.00643	.00500	.00533	.00544	.00554	.00574	.00607	.00640	.00660
	1 900 000	.00512	.00546	.00556	.00568	.00590	.00623	.00656	.00678	.00526	.00561	.00570	.00574	.00606	.00640	.00674	.00697
	1 800 000	.00541	.00577	.00588	.00599	.00622	.00657	.00692	.00716	.00556	.00593	.00605	.00615	.00640	.00675	.00711	.00735
	1 700 000	.00573	.00610	.00622	.00635	.00659	.00695	.00733	.00758	.00590	.00626	.00640	.00652	.00677	.00714	.00753	.00780
	1 600 000	.00609	.00647	.00660	.00674	.00700	.00740	.00779	.00805	.00626	.00665	.00678	.00693	.00720	.00760	.00800	.00827
	1 500 000	.00650	.00690	.00704	.00719	.00746	.00787	.00830	.00858	.00668	.00709	.00724	.00739	.00766	.00808	.00853	.00882
	1 400 000	.00696	.00741	.00755	.00771	.00800	.00845	.00890	.00920	.00715	.00761	.00775	.00792	.00822	.00868	.00915	.00945
	1 300 000	.00749	.00798	.00813	.00830	.00862	.00910	.00958	.00990	.00770	.00820	.00836	.00853	.00885	.00935	.00985	.0102
	1 200 000	.00812	.00864	.00880	.00899	.00933	.00985	.0104	.0107	.00835	.00888	.00905	.00924	.00958	.0101	.0107	.0110
	1 100 000	.00886	.00942	.00960	.00981	.0102	.0108	.0113	.0117	.00910	.00968	.00986	.0101	.0105	.0111	.0116	.0120
	1 000 000	.00974	.0104	.0106	.0108	.0112	.0118	.0125	.0129	.0100	.0107	.0109	.0111	.0115	.0121	.0128	.0132
	950 000	.0102	.0109	.0111	.0114	.0118	.0124	.0131	.0135	.0105	.0112	.0114	.0117	.0121	.0127	.0134	.0138
	900 000	.0108	.0115	.0117	.0120	.0124	.0131	.0138	.0142	.0111	.0118	.0120	.0123	.0127	.0134	.0142	.0146
	850 000	.0115	.0122	.0124	.0127	.0132	.0139	.0147	.0152	.0118	.0125	.0127	.0130	.0135	.0143	.0151	.0156
	800 000	.0122	.0130	.0132	.0135	.0140	.0148	.0156	.0161	.0125	.0133	.0136	.0139	.0144	.0152	.0160	.0165
	750 000	.0130	.0138	.0140	.0144	.0149	.0157	.0166	.0171	.0134	.0142	.0144	.0148	.0153	.0161	.0170	0.75
	700 000	.0139	.0148	.0151	.0154	.0160	.0169	.0178	.0184	.0143	.0152	.0155	.0158	.0164	.0173	.0183	.0189
	650 000	.0150	.0160	.0163	.0166	.0172	.0182	.0192	.0199	.0154	.0164	.0167	.0170	.0176	.0187	.0197	.0204
	600 000	.0162	.0173	.0176	.0180	.0187	.0197	.0208	.0215	.0166	.0178	.0181	.0185	.0192	.0202	.0214	.0221
	550 000	.0177	.0188	.0191	.0196	.0203	.0214	.0226	.0234	.0182	.0193	.0196	.0202	.0209	.0220	.0232	.0240
	500 000	.0195	.0207	.0211	.0216	.0224	.0236	.0249	.0258	.0200	.0213	.0217	.0222	.0230	.0242	.0256	.0265
	450 000	.0216	.0230	.0234	.0240	.0249	.0263	.0277	.0286	.0222	.0236	.0240	.0247	.0256	.0270	.0285	.0294
	400 000	.0243	.0259	.0264	.0270	.0280	.0296	.0311	.0322	.0250	.0266	.0271	.0277	.0288	.0304	.0319	.0331
	350 000	.0278	.0297	.0303	.0308	.0319	.0337	.0356	.0368	.0286	.0305	.0312	.0316	.0328	.0346	.0366	.0378
	300 000	.0324	.0346	.0353	.0360	.0373	.0394	.0415	.0428	.0333	.0356	.0363	.0370	.0383	.0405	.0427	.0440
	250 000	.0390	.0415	.0423	.0432	.0448	.0473	.0498	.0515	.0400	.0426	.0435	.0444	.0460	.0487	.0512	.0530
0000	211 600	.0460	.0490	.0500	.0510	.0529	.0559	.0589	.0609	.0473	.0503	.0514	.0525	.0544	.0573	.0605	.0626
000	167 772	.0580	.0618	.0630	.0644	.0668	.0706	.0742	.0767	.0596	.0635	.0648	.0662	.0687	.0725	.0762	.0788
00	133 079	.0732	.0778	.0795	.0811	.0841	.0888	.0936	.0967	.0752	.0800	.0815	.0834	.0865	.0900	.0962	.0995
0	105 560	.0922	.0982	.100	.102	.106	.112	.118	.122	.0948	.101	.103	.105	.109	.115	.121	.125
1	83 694	.116	.124	.126	.129	.134	.141	.149	.154	.119	.127	.129	.132	.138	.145	.153	.158
2	66 358	.147	.156	.159	.163	.169	.178	.188	.194	.151	.160	.163	.167	.173	.183	.193	.199
3	52 624	.185	.197	.201	.205	.213	.225	.237	.245	.190	.202	.207	.211	.219	.231	.244	.252
4	41 738	.233	.248	.253	.259	.269	.284	.298	.308	.239	.255	.260	.266	.276	.292	.306	.316
5	33 078	.294	.314	.320	.327	.339	.358	.376	.388	.302	.323	.328	.336	.348	.368	.386	.399
6	26 244	.371	.395	.403	.412	.427	.452	.475	.491	.381	.406	.415	.423	.438	.464	.488	.504
7	20 822	.468	.497	.507	.519	.538	.569	.598	.618	.482	.512	.520	.533	.553	.585	.615	.635
8	16 512	.590	.628	.640	.654	.678	.716	.755	.781	.607	.646	.658	.672	.697	.736	.775	.802
SOLID CONDUCTORS																	
0000	211 600	.0451	.0480	.0490	.0500	.0519	.0548	.0577	.0596	.0463	.0493	.0503	.0514	.0533	.0563	.0592	.0612
000	167 772	.0569	.0606	.0618	.0630	.0654	.0691	.0727	.0752	.0585	.0623	.0635	.0647	.0672	.0710	.0746	.0772
00	133 079	.0718	.0764	.0779	.0795	.0826	.0871	.0917	.0948	.0738	.0785	.0800	.0817	.0850	.0895	.0942	.0974
0	105 560	.0905	.0963	.0983	.100	.104	.110	.116	.120	.0930	.0988	.101	.103	.107	.113	.119	.123
1	83 694	.114	.121	.124	.126	.131	.139	.146	.151	.117	.124	.127	.129	.134	.143	.150	.155
2	66 358	.144	.153	.156	.159	.165	.175	.184	.190	.148	.157	.160	.163	.170	.180	.189	.195
3	52 624	.181	.193	.197	.201	.209	.220	.232	.240	.186	.198	.202	.207	.215	.226	.238	.246
4	41 738	.229	.244	.248	.253	.263	.278	.293	.302	.235	.250	.255	.260	.270	.286	.301	.310
5	33 088	.289	.307	.313	.319	.331	.350	.368	.381	.297	.315	.321	.328	.340	.360	.378	.391
6	26 244	.364	.387	.395	.403	.418	.442	.465	.481	.374	.398	.407	.415	.430	.454	.477	.494
7	20 822	.459	.488	.498	.508	.528	.557	.586	.606	.472	.502	.512	.523	.543	.572	.602	.623
8	16 512	.579	.616	.628	.640	.665	.702	.739	.764	.595	.633	.645	.657	.685	.722	.759	.785

These resistance values do not take into account skin effect. This should be considered when the larger conductors are used, particularly at the higher frequencies. No allowance has been made for increased length due to sag when the conductors are suspended. The resistance values for the stranded conductors are two percent greater than for a solid rod of cross-section equal to the total cross-section of the wires of the cable.

The change of resistivity of copper per degree C. is a constant independent of the temperature of reference and of the sample of copper. This resistivity-temperature constant is 0.0409 ohm (mil, foot). The fundamental resistivity used in calculating this table is the annealed copper standard, viz. 0.15328 ohm (meter, gram) at 20 degrees C.

For sizes not given in the table computations may be made by the following formulas which were used in calculating the above table:—

Ohms per 1000 feet of annealed copper at 25 degrees C $= \frac{10787}{\text{Circ. mils}}$; at 65 degrees C $= \frac{12457}{\text{Circ. mils}}$

conductor the m.m.f. reaches its maximum because all of the current of the conductor is acting to produce m.m.f. at this and all points beyond. On the other hand the circular path subject to this maximum m.m.f. is shortest at the surface, the reluctance a minimum and consequently the field intensity is greatest. For points beyond the surface the length of the circular path through air is proportional to the distance from the center of the conductor. Thus at a distance of *2* from the center the circular path is twice as long as at

TABLE II—RESISTANCE PER MILE

OF COPPER CONDUCTORS AT VARIOUS TEMPERATURES

STRANDED CONDUCTORS

B & S No.	Area Circular Mils	Ohms per Mile of Single Conductor — Annealed Copper 100% Conductivity								Hard Drawn Copper 97.3% Conductivity							
		0°C 32°F	15°C 59°F	20°C 68°F	25°C 77°F	35°C 95°F	50°C 122°F	65°C 149°F	75°C 167°F	0°C 32°F	15°C 59°F	20°C 68°F	25°C 77°F	35°C 95°F	50°C 122°F	65°C 149°F	75°C 167°F
	2 000 000	.0258	.0274	.0279	.0285	.0295	.0312	.0329	.0340	.0265	.0282	.0288	.0293	.0304	.0321	.0337	.0349
	1 900 000	.0271	.0289	.0294	.0301	.0312	.0330	.0347	.0359	.0278	.0296	.0301	.0304	.0320	.0338	.0356	.0368
	1 800 000	.0286	.0305	.0311	.0317	.0329	.0347	.0366	.0379	.0294	.0314	.0320	.0325	.0338	.0357	.0375	.0389
	1 700 000	.0303	.0323	.0329	.0336	.0348	.0368	.0388	.0400	.0312	.0331	.0339	.0344	.0358	.0377	.0398	.0412
	1 600 000	.0322	.0342	.0349	.0357	.0370	.0391	.0412	.0425	.0331	.0352	.0358	.0367	.0381	.0402	.0422	.0438
	1 500 000	.0344	.0365	.0373	.0380	.0394	.0417	.0438	.0454	.0353	.0375	.0382	.0391	.0405	.0427	.0451	.0467
	1 400 000	.0368	.0391	.0399	.0408	.0423	.0447	.0470	.0487	.0378	.0402	.0410	.0418	.0435	.0459	.0484	.0500
	1 300 000	.0396	.0422	.0430	.0439	.0456	.0482	.0507	.0523	.0407	.0433	.0442	.0451	.0468	.0495	.0521	.0539
	1 200 000	.0429	.0457	.0465	.0475	.0493	.0520	.0550	.0565	.0442	.0470	.0478	.0489	.0507	.0534	.0565	.0582
	1 100 000	.0467	.0498	.0507	.0518	.0539	.0572	.0597	.0618	.0482	.0512	.0521	.0533	.0555	.0587	.0615	.0634
	1 000 000	.0515	.0550	.0560	.0570	.0592	.0623	.0660	.0682	.0528	.0565	.0577	.0587	.0608	.0640	.0675	.0699
	950 000	.0538	.0577	.0587	.0603	.0624	.0656	.0693	.0713	.0555	.0593	.0603	.0618	.0640	.0672	.0710	.0730
	900 000	.0571	.0608	.0618	.0635	.0655	.0693	.0730	.0751	.0587	.0623	.0635	.0650	.0672	.0708	.0750	.0772
	850 000	.0608	.0645	.0655	.0672	.0698	.0735	.0778	.0803	.0623	.0660	.0672	.0688	.0713	.0755	.0795	.0825
	800 000	.0645	.0687	.0698	.0713	.0740	.0783	.0825	.0851	.0660	.0703	.0718	.0735	.0762	.0803	.0845	.0873
	750 000	.0688	.0729	.0740	.0761	.0788	.0830	.0878	.0905	.0708	.0751	.0762	.0782	.0808	.0850	.0900	.0925
	700 000	.0735	.0783	.0798	.0814	.0846	.0894	.0940	.0973	.0756	.0803	.0819	.0835	.0866	.0915	.0965	.100
	650 000	.0793	.0846	.0861	.0878	.0910	.0962	.102	.105	.0815	.0867	.0883	.0900	.0930	.0990	.104	.108
	600 000	.0857	.0915	.0930	.0952	.0988	.104	.110	.114	.0878	.0940	.0957	.0978	.102	.107	.113	.117
	550 000	.0935	.0995	.101	.104	.107	.113	.121	.124	.0963	.102	.104	.107	.111	.116	.122	.127
	500 000	.103	.110	.112	.114	.119	.125	.132	.136	.106	.113	.115	.117	.122	.128	.135	.140
	450 000	.114	.122	.124	.127	.132	.139	.146	.151	.118	.125	.127	.131	.136	.143	.150	.156
	400 000	.129	.137	.140	.143	.148	.157	.165	.170	.132	.141	.144	.147	.152	.161	.168	.175
	350 000	.147	.157	.160	.163	.169	.178	.188	.195	.151	.162	.165	.167	.174	.183	.193	.200
	300 000	.171	.183	.187	.190	.197	.208	.220	.226	.176	.188	.192	.196	.203	.214	.226	.233
	250 000	.206	.219	.224	.228	.237	.250	.263	.272	.211	.225	.230	.235	.243	.258	.270	.280
0000	211 600	.243	.259	.264	.269	.280	.296	.311	.322	.249	.266	.272	.277	.288	.303	.320	.330
000	167 772	.306	.326	.333	.341	.353	.372	.392	.405	.315	.335	.342	.350	.363	.383	.402	.416
00	133 079	.387	.412	.420	.428	.444	.470	.495	.512	.398	.423	.432	.442	.457	.476	.510	.527
0	105 560	.488	.520	.528	.540	.560	.592	.624	.645	.502	.535	.545	.555	.576	.608	.640	.661
1	83 694	.612	.655	.665	.682	.708	.745	.787	.815	.630	.672	.682	.697	.730	.766	.810	.835
2	66 358	.777	.825	.840	.862	.895	.942	.995	1.03	.798	.845	.862	.883	.915	.968	1.02	1.05
3	52 624	.978	1.04	1.07	1.09	1.13	1.19	1.25	1.30	1.01	1.07	1.10	1.12	1.16	1.22	1.29	1.33
4	41 738	1.23	1.31	1.34	1.37	1.42	1.51	1.58	1.63	1.27	1.35	1.38	1.41	1.46	1.55	1.61	1.67
5	33 078	1.56	1.66	1.69	1.73	1.80	1.89	1.99	2.05	1.60	1.71	1.73	1.78	1.84	1.95	2.04	2.11
6	26 244	1.96	2.09	2.13	2.17	2.26	2.39	2.51	2.59	2.01	2.14	2.20	2.24	2.32	2.45	2.58	2.66
7	20 822	2.48	2.63	2.68	2.74	2.84	3.01	3.16	3.27	2.55	2.71	2.75	2.82	2.93	3.09	3.25	3.35
8	16 512	3.12	3.32	3.38	3.46	3.58	3.78	3.99	4.13	3.21	3.41	3.48	3.55	3.69	3.89	4.10	4.24
SOLID CONDUCTORS																	
0000	211 600	.238	.254	.259	.264	.274	.289	.305	.315	.245	.261	.266	.272	.282	.298	.312	.323
000	167 772	.301	.320	.327	.333	.346	.365	.384	.397	.309	.329	.336	.342	.355	.375	.395	.408
00	133 079	.380	.404	.412	.420	.436	.460	.485	.501	.390	.415	.423	.432	.450	.473	.497	.515
0	105 560	.478	.509	.520	.528	.550	.582	.613	.635	.492	.522	.535	.545	.565	.597	.628	.650
1	83 694	.603	.640	.655	.666	.693	.735	.772	.798	.618	.655	.672	.680	.708	.755	.793	.820
2	66 358	.760	.808	.825	.840	.872	.925	.972	1.01	.783	.830	.845	.862	.900	.950	1.00	1.03
3	52 624	.955	1.02	1.04	1.06	1.11	1.16	1.23	1.27	.983	1.05	1.07	1.10	1.14	1.19	1.26	1.30
4	41 738	1.21	1.29	1.31	1.34	1.39	1.47	1.55	1.60	1.24	1.32	1.35	1.38	1.43	1.51	1.59	1.64
5	33 088	1.53	1.62	1.66	1.69	1.75	1.85	1.95	2.02	1.57	1.67	1.70	1.73	1.80	1.90	2.00	2.07
6	26 244	1.93	2.05	2.09	2.14	2.21	2.33	2.46	2.54	1.98	2.10	2.15	2.20	2.27 (struck through)	2.40	2.52	2.61
7	20 822	2.43	2.58	2.63	2.69	2.79	2.94	3.10	3.20	2.49	2.65	2.71	2.77	2.87	3.02	3.18	3.29
8	16 512	3.06	3.26	3.33	3.39	3.51	3.71	3.90	4.04	3.14	3.35	3.41	3.47	3.62	3.82	4.02	4.15

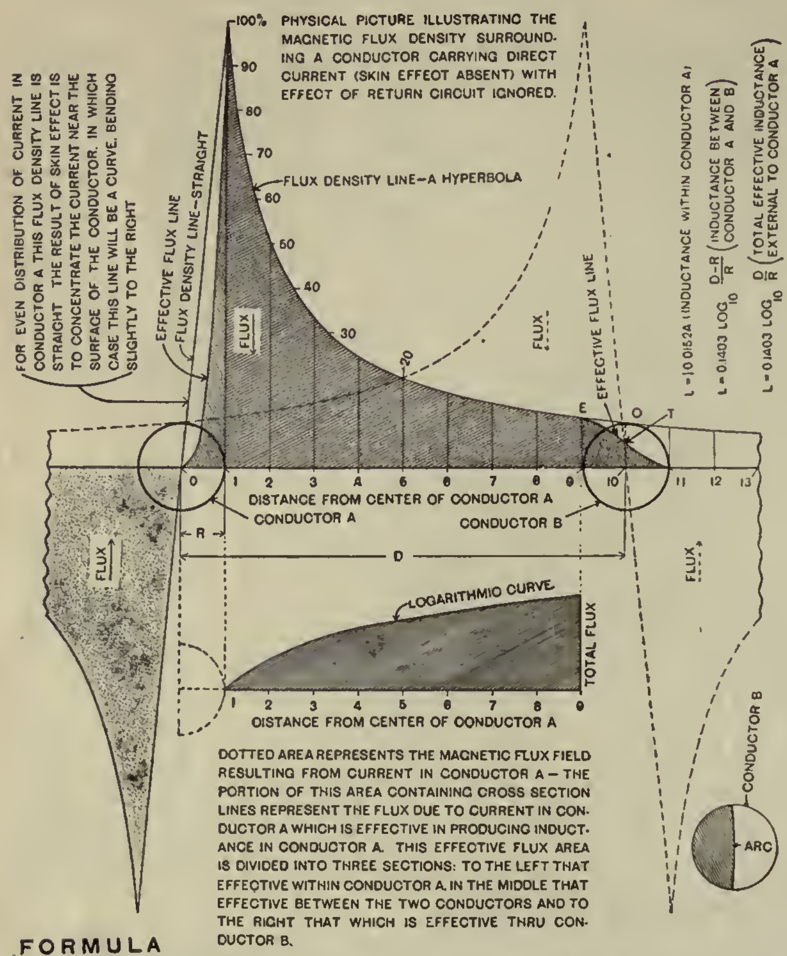
These resistance values do not take into account skin effect. This should be considered when the larger conductors are used, particularly at the higher frequencies. No allowance has been made for increased length due to sag when the conductors are suspended.. The resistance values for the stranded conductors are two percent greater than for a solid rod of cross-section equal to the total cross-section of the wires of the cable.

The change of resistivity of copper per degree C. is a constant independent of the temperature of reference and of the sample of copper. This resistivity-temperature constant is 0.0409 ohm (mil, foot). The fundamental resistivity used in calculating this table is the annealed copper standard, viz. 0.15328 ohm (meter, gram) at 20 degrees C.

a distance of *1* (its surface) and consequently, although the m.m.f. is the same the reluctance is double, permitting only one-half as great a flux to flow as at the surface. For a similar reason the density of the field at a distance of *10* is one-tenth the surface density; at *50* it is one-fiftieth, etc. The curve of field density beyond the surface of the conductor therefore assumes the form of a hyperbola.

Inside conductor A the field density is represented by a straight line joining the center of the conductor to the apex of the density curve, represented as 100 percent. Suppose it is desired to determine the field density at a point midway between the center and surface of the conductor. At this point the length of the circular path is one half its length at the conductor surface. Since the current distributes uniformly throughout the cross-section of the conductor, at a point midway between the center and its surface, one-fourth of the total current would be embraced by the circle. The m.m.f. corresponding to this point would therefore be one-fourth its value at the surface. With one-fourth m.m.f. and one-half the surface reluctance the resulting density will be one-half of its surface density as shown by this value falling on the straight line at this distance from the center.

CHART I—INDUCTANCE



FORMULA

TOTAL INDUCTANCE IN MILLIHENRIES (L) PER 1000 FEET OF EACH CONDUCTOR $L = 0.01524 + 0.1403 \log_{10} \frac{D}{R}$

The m.m.f. resulting from equal currents is the same for all sizes of conductors. Thus the field density at points equally distant from the center of different sizes of conductors carrying equal currents is equal provided these points lie beyond the surface of the larger conductor. For points equally distant from the center of different size conductors which lie inside the conductors the density will be different. Thus if the conductor diameter carrying equal current be reduced to one half, the m.m.f. at its surface will remain the same, but since the flux path at the surface is now only one-half as long, the flux density at the surface will be twice as great. In other words, the magnetic field density at the surface of conductors having different diameters but carrying the same currents is inversely proportional to their radii.

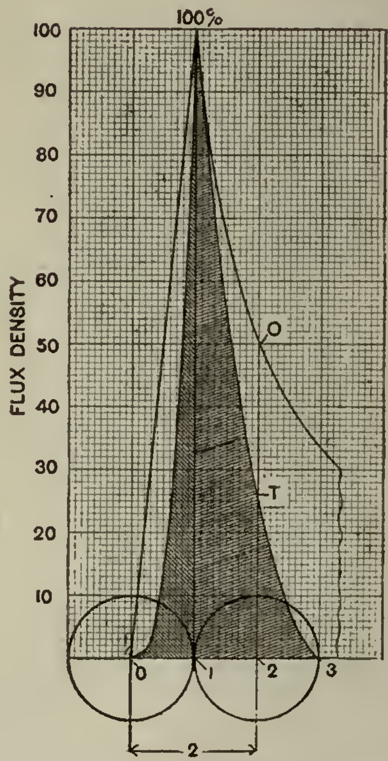


FIG. 6

The area indicated by cross-sectional lines on the inductance chart represents the amount of inductance effective in conductor A resulting from current in this conductor. It will be seen that the total area between the adjacent surfaces of the conductors *1* to *9* below the flux density line is effective. This part of the inductance follows a logarithmic curve as illustrated on the chart and is represented by the formula.

$$L = 0.14037 \log_{10} \frac{D-R}{R} \quad \ldots\ldots (1)$$

Where L is in millihenries per 1000 feet of single conductor.

The effective flux area departs from the flux density line at E dropping down in the form of a reverse curve and terminating in zero at *11*. All flux to the right of *11* cuts the whole of both conductors producing the same amount of inductance in both of them in such a direction as to oppose or neutralize each other.

The flux cutting conductor B from *9* to *11* has its full value of effectiveness in producing inductance in conductor A. On the other hand it also produces to a less extent inductance in conductor B but in a direction to oppose that which it produces in conductor A. The difference between that produced in conductors A and B is the effective flux producing inductance in the circuit and is represented by the shaded portion through conductor B within the area *E-9-11-T-E*. To illustrate how the effective flux curved line *E-T-11* was determined, suppose it is required to determine the effective flux at the distance *10* (center of conductor B). At this point the flux density is ten percent, but since these flux density lines are actually concentric circles, having their center at the middle of conductor A this flux density curve cuts conductor B in the form of an arc (see lower right hand corner of inductance chart). The area of the shaded portion between the two arcs is a measure of the inductance in conductor B at its center. The difference between this shaded area, and the whole area of B, or the clear part to the right of the shaded portion, is a measure of the difference in inductance of the two conductors. In other words, for the spacings shown, approximately 55 percent of ten or 5.5 percent is the value of the effective flux at distance of *10* from conductor A.

If in place of $L = 0.14037 \log_{10} = \frac{D-R}{R}$ .... (*1*)

we take $L = 0.14037 \log_{10} \frac{D}{R}$ .... (*2*)

we include all of the inductance area out to the vertical line *O-10*. This would include the area *E-O-T* but not the area *T-10-11*. Since these two areas are equal, the omission of one is balanced by including the other and therefore formula (2) correctly takes into account all of the effective inductance beyond the surface of conductor A.

The inductance within conductor A is determined as follows:—At a point midway between the center and its surface the flux density is 50 percent as indicated by the straight flux density line of the chart. However at this point only one-fourth of the conductor area is enclosed, so that, measured in terms of its effect if outside the conductor, its effectiveness would be only one-fourth of 50 or 12.5 percent. This is the reason that the so-called effective flux line is curved and falls to the right of the straight flux density line. The area of the triangular section *O-1-100* is a measure of the effective inductance within conductor A. This is a constant for all sizes of solid conductors and is represented by the constant 0.01524 of the inductance formula based upon 1000 feet of conductor.

The fundamental formula for the total effective inductance (within and external to conductor A) of a single solid non-magnetic conductor suspended in air is therefore:

$$L = 0.01524 + 0.14037 \log_{10} \frac{D}{R} \text{ per 1000 ft.} \ldots\ldots (3)$$

or

$$L = 0.08047 + 0.74115 \log_{10} \frac{D}{R} \text{ per mile} \ldots\ldots\ldots (4)$$

It may be interesting to note here that the above described graphical solution for inductance produces results in close agreement with these obtained by the fundamental formula for inductance. That is, lay out such a chart on cross section paper to a large scale and count the number of squares or area representing the internal and the external inductance due to current in conductor A. It will be seen that the relative values of the external and internal flux areas conform with the relative values as determined by the formula. This will also be true in the case of the conductors when so placed as to give zero separation, as illustrated by Fig. 6.

VARIATIONS FROM THE FUNDAMENTAL INDUCTANCE FORMULA

It has been proven mathematically by the Bureau of Standards and others that the fundamental formula (3) for determining inductance will give exact results for solid, round, straight, parallel conductors, provided skin and proximity effects are absent. Proximity effect is the crowding of the current to one side of a conductor, due to the proximity of another current carrying conductor. It is similar to skin effect in that it increases the resistance and decreases the inductance. Proximity effect as well as skin effect changes only the inductance due to the flux inside the conductor. Proximity effect is more pronounced for large conductors, high frequencies and close proximity.

For No. 0000 solid conductors at zero separation and 60 cycles, the error in the results (as determined by the fundamental inductance formula) due to skin effect is less than one-tenth of one percent. This error, however, increases rapidly as the size of the conductor increases. Proximity effect cannot be calculated but it is believed to be less than two percent in the above case.

Should skin and proximity effect combined, be sufficient to force all of the current out to within a very thin annulus at the surface of the conductor (a condition obviously never obtained at commercial frequencies) their combined effect would be a maximum. In such a case there would be no inductance within the conductors and the first constant 0.01524 would disappear from formula (3).

Skin and proximity effect are so small in the case of the greater spacings of conductors required for high-tension aerial transmission circuits that they may in such cases be ignored. Even in the case of the close

TABLE III—INDUCTANCE PER 1000 FEET OF SINGLE CONDUCTOR

INDUCTANCE IN MILLIHENRIES (L) PER 1000 FEET OF EACH CONDUCTOR OF A SINGLE-PHASE OR OF A SYMMETRICAL 3 PHASE CIRCUIT. THE TABLE VALUES WERE DERIVED FROM THE EQUATION $L = 0.01524 + 0.1403 \log_{10} \frac{D}{R}$ WHEN R IS THE RADIUS OF CONDUCTOR AND D DISTANCE BETWEEN CENTERS OF CONDUCTORS EXPRESSED IN SAME TERMS AS R. FOR STRANDED CONDUCTORS D WAS TAKEN AS THE DIAMETER OF A SOLID ROD OF EQUIVALENT CROSS-SECTION.

Material	Type	Diameter in Inches	B & S No.	Area in Circular Mils	½"	1"	2"	3"	4"	5"	6"	8"	12"	18"	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet	11 Feet	13 Feet	15 Feet	17 Feet	19 Feet	21 Feet	23 Feet	25 Feet
COPPER	STRANDED	1.631		2 000 000			.079	.104	.121	.135	.146	.163	.188	.212	.230	.255	.272	.283	.296	.307	.315	.322	.334	.344	.352	.361	.367	.373	.379	.383
		1.548		1 800 000			.082	.107	.124	.138	.149	.166	.191	.215	.233	.258	.276	.289	.300	.309	.317	.325	.338	.347	.355	.364	.370	.377	.382	.387
		1.504		1 700 000			.084	.109	.126	.139	.151	.168	.193	.217	.235	.259	.277	.291	.301	.311	.320	.326	.339	.349	.357	.365	.372	.378	.383	.388
		1.459		1 600 000			.086	.110	.128	.141	.153	.170	.195	.219	.237	.261	.279	.294	.303	.314	.321	.329	.341	.351	.359	.368	.373	.381	.385	.391
		1.412		1 500 000			.088	.112	.130	.143	.155	.172	.196	.222	.238	.264	.281	.295	.306	.315	.323	.331	.343	.353	.361	.369	.376	.382	.388	.393
		1.364		1 400 000			.089	.114	.132	.145	.156	.174	.198	.223	.241	.265	.283	.297	.308	.318	.325	.333	.345	.355	.364	.371	.378	.384	.390	.395
		1.263		1 200 000			.094	.119	.136	.150	.161	.179	.203	.227	.246	.270	.288	.301	.314	.322	.330	.338	.350	.359	.368	.377	.384	.389	.395	.399
		1.209		1 100 000			.097	.121	.139	.153	.164	.181	.206	.230	.248	.273	.290	.305	.315	.324	.333	.340	.352	.362	.371	.379	.385	.392	.396	.403
		1.152		1 000 000			.100	.125	.142	.156	.167	.184	.209	.234	.251	.276	.294	.307	.318	.327	.336	.344	.355	.365	.374	.381	.388	.394	.399	.405
		1.123		950 000			.101	.126	.143	.157	.168	.186	.211	.235	.252	.277	.295	.309	.319	.329	.338	.345	.356	.366	.376	.384	.390	.396	.402	.407
		1.093		900 000			.103	.128	.145	.159	.170	.187	.212	.236	.255	.278	.296	.311	.321	.330	.339	.346	.358	.368	.377	.385	.391	.398	.404	.408
		1.062		850 000			.105	.130	.147	.161	.172	.189	.214	.238	.256	.281	.299	.312	.323	.333	.341	.348	.360	.370	.379	.386	.393	.399	.405	.410
		1.031		800 000			.107	.131	.149	.163	.174	.191	.215	.241	.258	.282	.300	.314	.324	.334	.343	.350	.362	.372	.381	.389	.395	.401	.407	.412
		.998		750 000		.066	.108	.133	.151	.164	.176	.193	.218	.243	.260	.285	.302	.316	.327	.337	.345	.351	.364	.374	.383	.390	.397	.403	.409	.414
		.964		700 000		.068	.111	.135	.153	.167	.178	.195	.219	.244	.262	.286	.304	.318	.329	.339	.347	.353	.366	.375	.385	.392	.398	.405	.411	.416
		.929		650 000		.071	.113	.138	.155	.169	.180	.197	.222	.247	.264	.289	.306	.321	.331	.341	.349	.356	.368	.378	.387	.395	.402	.407	.413	.418
		.893		600 000		.073	.116	.140	.158	.170	.182	.200	.224	.249	.266	.291	.309	.322	.334	.343	.352	.358	.370	.380	.390	.397	.403	.410	.415	.420
		.855		550 000		.076	.118	.143	.160	.173	.185	.202	.227	.252	.269	.293	.312	.325	.337	.346	.354	.361	.373	.383	.392	.400	.407	.413	.418	.423
		.815		500 000		.079	.121	.146	.163	.176	.188	.206	.230	.255	.272	.297	.315	.328	.339	.347	.357	.364	.376	.386	.395	.402	.409	.415	.422	.426
		.772		450 000		.082	.124	.149	.166	.180	.191	.208	.233	.257	.275	.300	.318	.331	.342	.351	.360	.367	.380	.389	.398	.406	.413	.418	.425	.430
		.728		400 000		.086	.128	.153	.170	.183	.195	.213	.237	.261	.279	.304	.322	.335	.346	.356	.363	.371	.383	.393	.402	.409	.417	.422	.428	.433
		.681		350 000		.090	.133	.157	.174	.187	.199	.216	.241	.266	.283	.308	.326	.339	.350	.360	.368	.375	.387	.398	.406	.413	.420	.427	.432	.437
		.630		300 000		.094	.137	.161	.179	.192	.203	.220	.245	.270	.288	.312	.330	.343	.355	.364	.372	.380	.392	.402	.410	.418	.425	.432	.437	.442
		.575		250 000		.100	.142	.167	.184	.197	.209	.226	.251	.276	.294	.318	.336	.349	.361	.369	.378	.385	.397	.408	.416	.425	.432	.436	.441	.448
		.528	0000	211 600		.105	.147	.172	.189	.203	.214	.231	.256	.281	.299	.323	.341	.354	.366	.374	.383	.389	.401	.412	.421	.429	.435	.442	.448	.453
		.470	000	167 772	.070	.112	.154	.179	.196	.210	.221	.238	.263	.288	.306	.330	.348	.362	.373	.381	.390	.396	.409	.419	.429	.436	.442	.449	.455	.460
		.418	00	133 079	.077	.119	.161	.186	.203	.217	.229	.245	.271	.295	.313	.338	.355	.369	.380	.388	.397	.403	.416	.426	.436	.443	.450	.457	.462	.467
		.373	0	105 560	.084	.126	.168	.193	.210	.224	.235	.252	.278	.302	.320	.345	.362	.376	.387	.395	.404	.411	.423	.433	.443	.450	.454	.463	.469	.474
		.332	1	83 694	.091	.133	.175	.200	.217	.231	.242	.259	.285	.309	.327	.352	.370	.383	.394	.402	.411	.418	.430	.440	.450	.458	.465	.470	.477	.481
		.292	2	66 358	.098	.140	.182	.207	.224	.238	.249	.266	.292	.316	.334	.359	.376	.390	.401	.409	.418	.425	.437	.447	.457	.464	.472	.477	.482	.488
		.269	3	52 624	.105	.147	.189	.214	.231	.245	.256	.273	.298	.323	.340	.365	.382	.396	.407	.416	.425	.432	.444	.454	.463	.472	.478	.485	.492	.495
		.232	4	41 738	.112	.154	.196	.221	.238	.252	.263	.280	.306	.331	.348	.373	.390	.404	.415	.423	.433	.439	.451	.461	.471	.478	.485	.492	.497	.502
		.206	5	33 088	.119	.161	.203	.228	.245	.259	.270	.288	.312	.337	.354	.379	.396	.410	.421	.431	.439	.446	.458	.468	.477	.485	.492	.499	.505	.510
	SOLID	.460	0000	211 600	.062	.105	.147	.172	.189	.203	.214	.231	.256	.281	.299	.323	.341	.354	.366	.374	.383	.389	.401	.412	.421	.429	.435	.442	.448	.453
		.410	000	167 772	.070	.112	.154	.179	.196	.210	.221	.238	.263	.288	.306	.330	.348	.362	.373	.381	.390	.396	.409	.419	.429	.436	.442	.449	.455	.460
		.365	00	133 079	.077	.119	.161	.186	.203	.217	.229	.245	.271	.295	.313	.338	.355	.369	.380	.388	.397	.403	.416	.426	.436	.443	.450	.457	.462	.467
		.325	0	105 560	.084	.126	.168	.193	.210	.224	.235	.252	.278	.302	.320	.345	.362	.376	.387	.395	.404	.411	.423	.433	.443	.450	.457	.463	.469	.474
		.289	1	83 694	.091	.133	.175	.200	.217	.231	.242	.259	.285	.309	.327	.352	.370	.383	.394	.402	.411	.418	.430	.440	.450	.458	.465	.470	.477	.481
		.258	2	66 358	.098	.140	.182	.207	.224	.238	.249	.266	.292	.316	.334	.359	.376	.390	.401	.409	.418	.425	.437	.447	.457	.464	.472	.477	.482	.488
		.229	3	52 624	.105	.147	.189	.214	.231	.245	.256	.273	.298	.323	.340	.365	.382	.396	.407	.416	.425	.432	.444	.454	.463	.472	.478	.485	.492	.495
		.204	4	41 738	.112	.154	.196	.221	.238	.252	.263	.280	.306	.331	.348	.373	.390	.404	.415	.423	.433	.439	.451	.461	.471	.478	.485	.492	.497	.502
		.182	5	33 088	.119	.161	.203	.228	.245	.259	.270	.288	.312	.337	.354	.379	.396	.410	.421	.431	.439	.446	.458	.468	.477	.485	.492	.499	.505	.510
ALUMINUM	STEEL REINFORCED	.953		605 000		.073	.115	.140	.157	.170	.182	.200	.225	.248	.266	.291	.309	.323	.333	.343	.351	.358	.371	.381	.389	.397	.405	.410	.415	.420
		.904		500 000		.079	.121	.145	.163	.178	.188	.206	.230	.255	.272	.297	.315	.329	.340	.348	.357	.365	.375	.386	.395	.403	.410	.415	.422	.428
		.741		336 420		.091	.133	.158	.175	.188	.200	.218	.242	.267	.285	.309	.327	.340	.351	.361	.369	.375	.388	.399	.407	.415	.422	.428	.433	.440
		.635		266 800		.098	.141	.165	.182	.195	.207	.225	.249	.274	.292	.316	.335	.347	.359	.367	.375	.384	.395	.405	.415	.422	.429	.435	.440	.447
		.564		211 950		.105	.147	.171	.190	.202	.214	.231	.256	.281	.299	.323	.340	.355	.365	.375	.383	.390	.401	.411	.422	.430	.437	.442	.447	.453
		.501		167 800		.112	.154	.179	.196	.210	.221	.239	.264	.287	.305	.331	.347	.361	.373	.382	.390	.397	.410	.420	.429	.437	.443	.449	.455	.460
		.447		133 200	.077	.119	.161	.186	.204	.216	.228	.246	.271	.295	.313	.337	.354	.369	.379	.389	.397	.405	.417	.427	.435	.445	.450	.457	.462	.467
		.398		105 530	.084	.126	.169	.192	.210	.225	.235	.252	.278	.302	.320	.344	.362	.375	.386	.396	.405	.411	.425	.435	.443	.450	.458	.464	.469	.475
		.355		83 640	.091	.133	.175	.200	.217	.232	.242	.260	.285	.309	.327	.351	.367	.382	.393	.404	.410	.420	.430	.441	.450	.457	.465	.470	.475	.480
		.316		66 370	.098	.140	.182	.207	.224	.239	.249	.267	.291	.316	.335	.358	.375	.389	.400	.411	.418	.425	.437	.448	.457	.465	.472	.477	.485	.487
		.281		52 630	.105	.147	.189	.213	.231	.245	.256	.274	.299	.324	.341	.365	.383	.397	.407	.417	.425	.433	.445	.455	.465	.471	.477	.485	.490	.495
		.250		41 740	.112	.154	.197	.221	.239	.253	.264	.281	.306	.331	.347	.373	.389	.404	.415	.425	.433	.440	.452	.462	.471	.478	.485	.492	.497	.502

A	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
B	.003	.006	.008	.011	.014	.016	.018	.020	.023	.025	.027	.029	.030	.032	.034	.036	.037	.039	.041	.042

X The inductance for any distance D not given in the table can be found as follows: Let E = the nearest smaller distance in the table. Divide D by E and, taking a value of A nearest to the quotient, find the corresponding value of B which must be added to the inductance corresponding to the size of conductor and distance E.

For three-phase regular flat spacing use D = 1.26 A. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$. For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

spacings required for three conductor cables these combined effects are usually less than four percent.

EFFECT OF STRANDING ON INDUCTANCE

The fundamental formula (3) for determining inductance is based upon a solid conductor, R being taken as the radius of the conductor. In stranded cables the effective value for R lies between the actual radius and that of a solid rod having an equivalent cross-section to that of the cable. The effective value for R varies with the stranding of the cable employed.

Formulas for use in determining the inductance of stranded cables when used for high-tension aerial transmission have been calculated by Mr. H. B. Dwight as follows:—

For a 7-wire cable, $L = 0.741 \log_{10} \frac{2.756\ D}{d}$ ...... (5)

For a 19-wire cable, $L = 0.741 \log_{10} \frac{2.640\ D}{d}$ ...... (6)

For a 37-wire cable, $L = 0.741 \log_{10} \frac{2.605\ D}{d}$ ...... (7)

For a 61-wire cable, $L = 0.741 \log_{10} \frac{2.590\ D}{d}$ ...... (8)

where L is in millihenries per mile of a single conductor, D is the spacing between centers of cables, and d is the outside diameter of the cables measured in same units as D.

SPIRALING EFFECT UPON INDUCTANCE

Spiraling of the strands of a cable and spiraling of the conductors of a three-conductor cable tend to increase the inductance. It is difficult to calculate the effect of spiraling for the various cases, but it may be considered negligible for high-tension aerial transmission circuits using non-magnetic conductors. For three-conductor cables the effect of spiraling is probably in the neighborhood of two percent.

Values for inductance per thousand feet of single conductor are given in Table III, for commercial sizes of copper and steel reinforced aluminum conductors. The formula by which the values were derived are:—

$$L = 0.01524 + 0.1403 \log_{10} \frac{D}{R} \quad\ldots\ldots\ldots\ldots\ldots\ (3)$$

where L = Millihenries per 1000 feet of single conductor of a single phase, or of a symetrical three-phase circuit.
D = Distance between centers of conductors.
R = Radius (to be measured in same units as D) of solid conductor. In the case of stranded conductors, R was taken as the radius of a solid rod of equivalent cross-section to that of the stranded conductors.

Table III has been carried out to three figures only. This would seem sufficiently accurate for working values when it is considered that there are numerous sources of variation from the calculated values. In the first place formulas are based upon a uniform distribution of current throughout the cross section of the conductors, whereas the current is seldom uniform and in the larger conductors, especially at 60 cycles, may be to a large extent crowded to the outer strands as a result of skin effect. This condition is further modified when the conductors are placed close together, by the proximity effect. Stranded conductors made up of various stranding combinations result in variation of inductance of several percent. In practice the length and spacing of conductors will vary more or less from those assumed when determining the calculated values.

The values for inductance of stranded conductors in Table III, as stated above, were derived by taking R as the radius of a solid rod having an equivalent cross-section area to that of the stranded conductors. Thus for 1 000 000 circ. mil cable the outside diameter is 1.152 in. and that of an equivalent solid iron is 1.0 in. R was therefore in this case taken as 0.5 in. The effective radius is really slightly greater than that of the solid rod and less than that of the cable, varying with the stranding employed. The actual inductance of cables will therefore be slightly less (usually two or three percent) than those indicated in the table for solid rods. The table values are therefore conservative.

The steel core of steel reinforced aluminum cables carries so little current on account of its relatively greater resistance that for practical purposes it has been customary to ignore its presence and to consider such conductors as solid rods of same area as that of the aluminum strands. In the absence of accurate data this practice was followed in determining the values for inductance of such cables in Table III.

The minimum value for inductance occurs when the conductors have zero separation $\frac{D}{R} = 2$, (Fig. 6). In this case the inductance in millihenries is independent of the size of the conductor. As given by formula (3) it is $L = 0.05124 + 0.1403 \log_{10} 2 = 0.0575$ millihenries per 1000 feet of each conductor. Obviously insulation requirements will not permit of such a low value for inductance although it will be closely approached in low voltage cables.

Any given percentage difference in distance between centers of conductors represents a definite and constant value in inductance regardless of their size. These values are given in column B at the bottom of the table for various percentages increase in spacings. Thus if the distance between conductor centers is increased 50 percent the corresponding increase in inductance is 0.025 as indicated in column B, under the A values of 1.50. Likewise doubling the distance increases the inductance by an amount of 0.042. For instance the table value for inductance of No. 0 solid copper conductor is for one-half inch spacing 0.084, and for one inch spacing 0.126 (an increase of 0.042.) For four foot spacing the table value is 0.362, and for eight foot spacing 0.404, also a difference of 0.042.

References:—An article by Prof. Charles F. Scott, "Inductance in Transmission Circuits" in The Electric Journal for Feb. 1906 very clearly covers the field of self and mutual inductance external to the conductors.
H. B. Dwight, "Transmission Line Formulas."
V. Karapetoff, "The Magnetic Circuit" p. 189.

CHAPTER II
REACTANCE—CAPACITANCE—CHARGING CURRENT

REACTANCE

A CONDUCTOR carrying an electric current is surrounded by a magnetic flux, whose value is proportional to the current. If the current varies, this flux also changes, thereby inducing an electromotive force in a direction which opposes the change. This counter e.m.f. is proportional to the rate of change and hence in alternating current is proportional to the frequency. It can be expressed in ohms per mile of each conductor of a single-phase or of a symmetrical three-phase circuit as follows:—

Ohms Reactance $= 2 \pi f L$ (9)

When $f =$ Frequency in cycles per second

$L =$ Henries per mile of single conductor.

The value for $2 \pi f$ are as follows:—

Frequency	$2 \pi f$
1	6.28
15	94.25
25	157.1
40	251.3
60	377.0
133	835.7

Tables IV and V indicate the reactance in ohms per mile, of a single conductor at 25 and 60 cycles respectively for various spacings of conductors. The foot notes to these tables cover the pertinent points relating to them. The resistance per 1000 feet, and per mile of single conductor at 25 degrees C. (77 degrees F) is given in parallel columns as a convenience for comparison of the resistance and reactance values. The resistance corresponding to other temperatures when desired may be taken from Tables I and II.

Tables VI and VII indicate the relative importance of reactance and resistance. In some cases of short lines and large single conductors, the reactance and not the resistance may determine the size and number of cables necessary. In other words, it may be necessary to keep the resistance abnormally low so that the reactance will not be so high as to result in an abnormal voltage drop in the circuit. In such cases the values in Tables VI and VII may be used for determining the permissible resistance in order not to exceed the desired reactance.

Example:—It is desired to use 1 000 000 circ. mil single conductor cables at 60 cycles, spaced two feet apart; from Table VII it is seen that the reactance drop under these conditions is 8.52 times the ohmic drop at 25 degrees C. If an ohmic drop of five percent at 25 degrees C is suggested the corresponding reactive drop would be 5 × 8.52 or 42.6 percent which would be excessive. If it is desired to limit the reactive drop to 10 percent in this case, the ohmic drop at 25 degrees C must be 10 ÷ 8.52 or 1.18 percent.

Probably a more important use for Tables VI and VII is for determining the reactance of a conductor directly from its resistance. To do this it is only necessary to multiply its resistance (at 25 degrees C) by the ratio value in table VI or VII corresponding to the conductor and spacing desired.

UNSYMMETRICAL SPACING

The inductance and capacitance per conductor of a three-phase circuit for symmetrical spacing of conductors is the same as the inductance and capacitance per conductor of a single-phase circuit for the same size conductor and the same spacing. For irregular spacing of conductors, the inductance and capacitance will be different. When the three conductors are placed in the same plane (flat spacing), the inductance of each of the outside conductors is greater than that of the middle conductor. By properly transposing the conductors, the inductance and capacitance may be equalized in all three conductors. However, the effect of flat spacing is equivalent to that of a symmetrical arrangement of greater spacing.

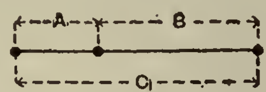


FIG. 7

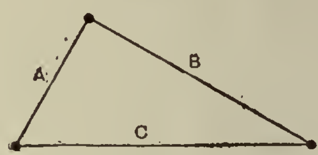


FIG. 8

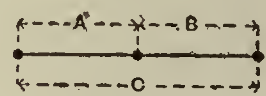


FIG. 9

Conductor Spacings.

For three-phase irregular flat or triangular spacing (Figs. 7 and 8) use $D = \sqrt[3]{A\ B\ C}$.

For three-phase regular flat spacing Fig. 9 use $D = 1.26\ A$.

For two-phase line the spacing is the average distance between centers of conductors of the same phase. It makes no difference whether the plane of the conductors with flat spacing is horizontal, vertical or inclined.

Various arrangements of conductors are indicated in Figs. 7, 8 and 9. Many three-phase high tension circuits have the three conductors regularly spaced in a common plane (regular flat spacing) Fig. 9. Beneath these figures are placed statements indicating the determination of "effective spacings" for any arrangement of conductors.

Since the so called "effective spacing" corresponding to unsymmetrical arrangements of conductors is usually a fractional number, the line constants for such effective spacing can usually not be taken directly from the tables but may be obtained by the use of the values in columns A and B at the foot of these tables.

Example:—It is desired to determine the 60 cycle reactance per mile of a single conductor for flat spacing of 11 ft. between adjacent 0000 solid copper conductors. The effective spacing is 1.26 × 11 or 13.8 feet. The reactance (Table V) for this conductor at 13 feet symmetrical spacing is 0.820 ohm. The value for A, (bottom of Table V) = 13.8 ÷ 13 = 1.06. The value of B corresponding to the value for A of 1.06 is approximately 0.006 which, added to 0.820 gives a reactance of 0.826 ohm for the effective spacing of 13.8 feet. The values of reactance for all effective spacings not included in the Table may be determined in a similar manner.

With an unsymmetrical arrangement of conductors there must be a sufficient number of transpositions of conductors to obtain balanced electrical conditions along the circuit.

CAPACITANCE

When mechanical force is exerted against a liquid or a solid mass, a displacement takes place proportional to the force exerted and inversely proportional to the resistance offered by the liquid or solid mass subjected to the force. If the mass consists of some elastic material, such as rubber, the displacement will be greater than if it consists of a more solid material, such as metal.

In a similar manner when an e.m.f. is applied to a condenser, a certain quantity of electricity will flow into it until it is charged to the same pressure as that of the applied circuit. A condenser consists of plates of conducting material separated by insulating material known as the dielectric. All electric circuits consist of conductors separated by a dielectric (usually air) and therefore act to a greater or less extent as condensers. The ability of a condenser or any electric circuit to receive the charge is a measure of its "capacity" more properly known as its "capacitance". Just as the rubber mass referred to above will, for a given force, permit of greater displacement so will circuits of greater capacitance permit more current to flow into them for a given e.m.f. impressed.

The process of charging a dielectric consists of setting up an electric strain in it similar to the mechanical strain in a liquid or mass referred to above. If an alternating voltage is impressed upon the terminals of a circuit containing capacitance, the charging current will vary directly with the impressed e.m.f. There is current to the condenser during rising and from the condenser during decreasing e.m.f. Thus the condenser is charged and then discharged in the opposite direction during the next alternation, making two complete charges and discharges for each cycle of impressed e.m.f. (Fig. 10). As long as the e.m.f. at the terminals is changing, the condenser will continue to receive or give out current. The current flowing to and from the condenser, assuming negligible resistance, leads the impressed e.m.f. by 90 electrical degrees.

DEFINITION

The capacitance of a circuit or condenser is said to be one farad when a rate of change in pressure of one volt per second at the terminals produces a current of one ampere. Stated another way, its capacitance in farads is numerically equal to the quantity of electricity in coulombs which it will hold under a pressure of one volt. The farad being an inconveniently large unit, one millionth part of it, the microfarad, is the usual practical unit.

CAPACITANCE FORMULA

An exact formula for the capacitance between parallel conductors must take into account the nonuniformity of the distribution of charge around the conductors. Such a formula* is formed by considering the charges as concentrated at the inverse points of the conductors; thus,—

$$C = \frac{0.008467}{\cosh^{-1}\frac{D}{d}} \qquad (10)$$

Where C equals the microfarads per 1000 feet of conductor between two parallel bare conductors in air, D, the distance between centers of the conductors and d, the diameter and R the radius of the conductors measured in the same units as D.

$$\text{Since } \cosh^{-1} X = \log_\epsilon \left(X + \sqrt{X^2 - 1}\right) \qquad (11)$$

$$C = \frac{0.008467}{\log_\epsilon \left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)} \qquad (12)$$

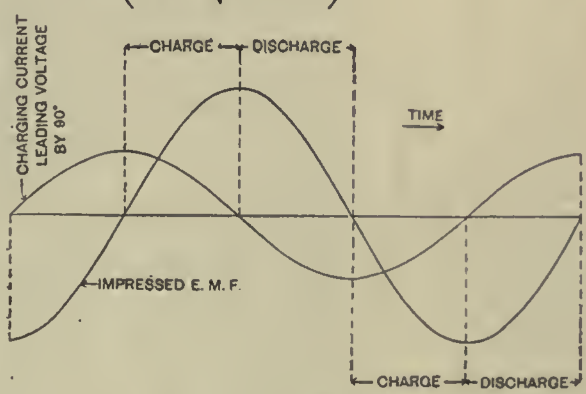


FIG. 10—CHARGING CURRENT

Reducing to common logarithms and capitance to neutral,—

$$C = \frac{0.007354}{\log_{10} \left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)} \qquad (13)$$

Microfarads per 1000 feet of single conductor to neutral.

or

$$C = \frac{0.038829}{\log_{10} \left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)} \qquad (14)$$

Microfarads per mile of single conductor to neutral.

When D is greater than 10 d, which is always the case in high-tension transmission lines employing bare conductors, the following simplified formulas may be used with negligible error.—

$$C = \frac{0.007354}{\log_{10} \frac{D}{R}} \qquad (15)$$

*See article by Pender & Osborne in *Electrical World* of Sept. 22, 1910, Vol. 56.

TABLE IV—RESISTANCE AND 25 CYCLE REACTANCE

OHMS PER MILE OF SINGLE CONDUCTOR

CONDUCTORS — Material, Type, Diameter in inches, B & S No., Area in circular mils.

RESISTANCE OF A SINGLE CONDUCTOR IN OHMS AT 25° C (77° F) XX.

REACTANCE IN OHMS PER MILE OF EACH CONDUCTOR OF A SINGLE PHASE, OR OF A SYMMETRICAL-3 PHASE CIRCUIT. FOR OTHER ARRANGEMENTS OF CONDUCTORS SEE FOOT NOTES. X. THE TABLE VALUES WERE DERIVED FROM THE EQUATION—OHMS REACTANCE $= 2\pi FL$ (L BEING EXPRESSED IN HENRIES PER MILE OF SINGLE CONDUCTOR.) THE REACTANCE FOR OTHER FREQUENCIES IS $\frac{F}{25}$ THE TABLE VALUES.

DISTANCE (D) BETWEEN CENTERS OF CONDUCTORS X

Material	Type	Diameter in Inches	B & S No.	Area in Circular Mils	Resistance per 1000 Feet	Resistance per Mile	½"	1"	2"	3"	4"	5"	6"	8"	12"	18"	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet	11 Feet	13 Feet	15 Feet	17 Feet	19 Feet	21 Feet	23 Feet	25 Feet
COPPER	STRANDED	1.631		2 000 000	.00554	.0293			.065	.086	.100	.112	.121	.135	.156	.175	.191	.211	.225	.234	.245	.254	.261	.266	.277	.285	.292	.299	.304	.309	.314	.317
		1.548		1 800 000	.00615	.0325			.068	.089	.103	.114	.123	.138	.158	.178	.193	.214	.228	.240	.248	.256	.262	.269	.279	.287	.294	.301	.306	.312	.316	.320
		1.504		1 700 000	.00652	.0344			.070	.090	.104	.115	.125	.139	.160	.180	.195	.215	.230	.241	.250	.258	.265	.270	.281	.289	.296	.302	.308	.314	.317	.321
		1.459		1 600 000	.00693	.0367			.071	.091	.106	.117	.127	.141	.161	.181	.196	.216	.231	.244	.251	.260	.266	.272	.282	.290	.297	.304	.309	.315	.319	.324
		1.412		1 500 000	.00739	.0391			.073	.093	.108	.119	.128	.142	.162	.183	.197	.219	.233	.244	.253	.261	.267	.274	.284	.292	.299	.306	.311	.316	.322	.325
		1.364		1 400 000	.00792	.0418			.074	.095	.109	.120	.129	.144	.164	.185	.200	.220	.234	.246	.255	.263	.269	.276	.285	.294	.301	.307	.313	.318	.323	.327
		1.263		1 200 000	.00924	.0489			.078	.099	.113	.124	.133	.148	.168	.188	.204	.224	.238	.250	.260	.266	.274	.280	.290	.297	.305	.312	.318	.322	.327	.330
		1.209		1 100 000	.0101	.0533			.080	.100	.115	.127	.136	.150	.171	.190	.205	.226	.240	.253	.261	.268	.276	.282	.291	.300	.307	.314	.319	.325	.328	.334
		1.152		1 000 000	.0111	.0588			.083	.104	.117	.129	.138	.152	.173	.194	.208	.229	.244	.255	.263	.271	.279	.285	.294	.302	.311	.315	.322	.326	.330	.336
		1.123		950 000	.0117	.0618			.084	.105	.118	.130	.139	.154	.175	.195	.209	.229	.245	.256	.264	.272	.280	.286	.295	.303	.312	.318	.323	.328	.332	.337
		1.093		900 000	.0123	.0650			.086	.106	.120	.132	.141	.155	.176	.196	.211	.230	.246	.258	.265	.273	.281	.286	.296	.305	.312	.319	.323	.330	.334	.338
		1.062		850 000	.0130	.0688			.087	.108	.122	.134	.143	.157	.177	.197	.212	.233	.247	.258	.267	.276	.282	.288	.298	.306	.314	.320	.325	.330	.335	.340
		1.031		800 000	.0139	.0735			.089	.109	.124	.135	.144	.158	.178	.200	.214	.233	.248	.260	.268	.277	.284	.290	.300	.308	.315	.322	.327	.332	.337	.341
		.998		750 000	.0148	.0782		.055	.090	.110	.125	.136	.146	.160	.181	.201	.216	.236	.251	.262	.271	.279	.286	.291	.301	.310	.318	.323	.328	.334	.339	.344
		.964		700 000	.0158	.0835		.057	.092	.112	.127	.138	.148	.161	.182	.202	.217	.238	.252	.264	.273	.281	.287	.292	.303	.311	.319	.325	.330	.335	.340	.345
		.929		650 000	.0170	.0900		.059	.094	.114	.129	.140	.149	.163	.183	.205	.218	.239	.254	.266	.274	.282	.289	.294	.304	.313	.320	.327	.333	.337	.342	.346
		.893		600 000	.0185	.0978		.061	.096	.116	.131	.141	.151	.166	.186	.206	.220	.241	.256	.266	.276	.284	.291	.296	.306	.315	.323	.329	.334	.340	.344	.348
		.855		550 000	.0202	.107		.063	.098	.118	.133	.143	.153	.167	.188	.208	.223	.243	.258	.269	.280	.286	.293	.299	.309	.317	.325	.331	.337	.342	.346	.351
		.815		500 000	0222	.117		.065	.100	.121	.135	.146	.156	.171	.191	.212	.226	.247	.261	.272	.282	.288	.296	.302	.312	.319	.328	.333	.339	.344	.349	.354
		.772		450 000	.0247	.131		.068	.103	.124	.137	.149	.158	.173	.193	.213	.228	.248	.263	.274	.283	.291	.298	.304	.315	.322	.329	.336	.342	.347	.352	.356
		.728		400 000	.0277	.147		.071	.106	.127	.141	.152	.162	.176	.196	.216	.231	.252	.266	.278	.286	.295	.301	.307	.317	.325	.332	.339	.345	.349	.354	.359
		.681		350 000	.0316	.167		.074	.110	.130	.144	.155	.165	.179	.200	.220	.235	.255	.270	.281	.291	.299	.305	.311	.321	.330	.337	.342	.348	.354	.358	.363
		.630		300 000	.0370	.196		.078	.113	.133	.148	.159	.168	.182	.203	.224	.238	.258	.274	.284	.294	.302	.308	.315	.324	.333	.339	.346	.352	.358	.362	.366
		.575		250 000	.0444	.235		.083	.118	.138	.153	.163	.173	.187	.209	.229	.244	.264	.279	.290	.299	.306	.314	.319	.329	.338	.345	.352	.358	.360	.365	.371
		.528	0000	211 600	.0525	.277		.087	.122	.143	.157	.168	.177	.192	.213	.233	.248	.268	.283	.294	.303	.310	.318	.322	.332	.341	.350	.355	.360	.365	.371	.376
		.470	000	167 772	.0662	.350	.058	.093	.128	.148	.163	.174	.183	.197	.219	.239	.254	.274	.289	.300	.309	.316	.324	.328	.339	.347	.356	.361	.366	.372	.376	.382
		.418	00	133 079	.0834	.442	.064	.099	.133	.154	.168	.180	.190	.203	.224	.245	.259	.280	.294	.306	.315	.321	.330	.334	.344	.352	.362	.367	.372	.378	.382	.387
		.373	0	105 560	.105	.555	.069	.104	.139	.160	.174	.185	.195	.209	.230	.251	.265	.286	.300	.312	.321	.327	.336	.340	.350	.359	.367	.373	.378	.384	.388	.393
		.332	1	83 694	.132	.697	.075	.110	.145	.166	.180	.192	.200	.214	.236	.257	.271	.292	.306	.318	.327	.333	.341	.346	.356	.364	.373	.380	.385	.390	.395	.399
		.292	2	66 358	.167	.883	.081	.116	.151	.172	.186	.197	.216	.220	.242	.263	.277	.298	.312	.323	.333	.339	.347	.352	.362	.370	.379	.384	.390	.394	.399	.405
		.260	3	52 624	.211	1.12	.087	.122	.157	.177	.191	.203	.212	.226	.247	.268	.282	.302	.316	.328	.337	.344	.352	.358	.368	.376	.384	.391	.396	.402	.408	.410
		.232	4	41 738	.266	1.41	.093	.127	.163	.183	.198	.208	.218	.232	.254	.274	.289	.309	.324	.335	.344	.351	.359	.364	.374	.382	.391	.396	.402	.407	.412	.417
		.206	5	33 088	.336	1.78	.099	.133	.168	.189	.203	.214	.224	.238	.259	.279	.294	.314	.328	.339	.349	.356	.364	.370	.380	.388	.395	.402	.407	.413	.418	.422
	SOLID	.460	0000	211 600	.0514	.272	.052	.087	.122	.143	.157	.168	.177	.192	.213	.233	.248	.268	.283	.294	.303	.310	.318	.322	.332	.341	.350	.355	.360	.365	.371	.376
		.410	000	167 772	.0647	.342	.058	.093	.128	.148	.163	.174	.183	.197	.219	.239	.254	.274	.289	.300	.309	.316	.324	.328	.339	.347	.356	.361	.366	.372	.376	.382
		.365	00	133 079	.0817	.432	.064	.099	.133	.154	.168	.180	.190	.203	.224	.245	.259	.280	.294	.306	.315	.321	.330	.334	.344	.352	.362	.367	.372	.378	.382	.387
		.325	0	105 560	.103	.545	.070	.104	.139	.160	.174	.185	.195	.209	.230	.251	.265	.286	.300	.312	.321	.327	.336	.340	.350	.359	.367	.373	.378	.384	.388	.393
		.289	1	83 694	.129	.680	.075	.110	.145	.166	.180	.192	.200	.214	.236	.257	.271	.292	.306	.318	.327	.333	.341	.346	.356	.364	.373	.380	.385	.390	.395	.399
		.258	2	66 358	.163	.862	.081	.116	.151	.172	.186	.197	.206	.220	.242	.263	.277	.298	.312	.323	.333	.339	.347	.352	.362	.370	.379	.384	.390	.394	.399	.405
		.229	3	52 624	.207	1.10	.087	.122	.157	.177	.191	.203	.212	.226	.247	.268	.282	.302	.316	.328	.337	.344	.352	.358	.368	.376	.384	.391	.396	.402	.408	.410
		.204	4	41 738	.260	1.38	.093	.127	.163	.183	.198	.208	.218	.232	.254	.274	.289	.309	.324	.335	.344	.351	.359	.364	.374	.382	.391	.396	.402	.407	.412	.417
		.182	5	33 088	.328	1.73	.099	.133	.168	.189	.203	.214	.224	.238	.259	.279	.294	.314	.328	.339	.349	.356	.364	.370	.380	.388	.395	.402	.407	.413	.418	.422
ALUMINUM	STEEL REINFORCED	.953		605 000	.0291	.154		.060	.095	.116	.130	.142	.151	.166	.186	.206	.220	.241	.256	.267	.276	.284	.291	.296	.308	.315	.322	.328	.335	.340	.344	.348
		.904		500 000	.0352	.186		.065	.100	.120	.135	.147	.156	.171	.190	.211	.226	.246	.261	.272	.282	.288	.296	.302	.310	.319	.327	.334	.340	.344	.350	.355
		.741		336 420	.0525	.277		.075	.110	.131	.145	.156	.166	.181	.200	.221	.236	.256	.271	.282	.291	.299	.305	.311	.321	.330	.337	.344	.349	.354	.358	.364
		.635		266 800	.0662	.350		.081	.117	.137	.151	.162	.172	.186	.206	.227	.242	.262	.278	.288	.298	.304	.311	.318	.327	.335	.344	.350	.355	.360	.364	.370
		.564		211 950	.0834	.442		.087	.122	.142	.157	.167	.177	.191	.212	.233	.247	.268	.282	.294	.302	.311	.317	.323	.332	.340	.349	.356	.362	.366	.370	.375
		.501		167 800	.105	.555		.093	.127	.148	.163	.174	.183	.198	.219	.238	.252	.274	.287	.299	.309	.316	.323	.329	.340	.348	.355	.362	.366	.372	.376	.381
		.447		133 200	.132	.697	.064	.098	.134	.154	.169	.179	.189	.204	.224	.244	.259	.279	.294	.305	.314	.322	.329	.335	.345	.353	.360	.368	.373	.378	.382	.386
		.398		105 530	.167	.883	.070	.104	.140	.159	.174	.184	.195	.209	.230	.250	.264	.284	.300	.310	.320	.328	.335	.340	.352	.360	.367	.373	.380	.384	.389	.393
		.355		83 640	.211	1.12	.075	.110	.145	.166	.180	.192	.200	.216	.236	.256	.271	.291	.304	.316	.326	.334	.340	.348	.356	.365	.373	.379	.385	.390	.393	.397
		.316		66 370	.266	1.41	.081	.116	.151	.172	.186	.198	.206	.221	.241	.262	.278	.296	.310	.322	.331	.340	.347	.352	.362	.371	.378	.385	.390	.395	.402	.403
		.281		52 630	.336	1.78	.087	.122	.157	.176	.191	.203	.212	.227	.248	.268	.282	.303	.317	.329	.337	.346	.352	.358	.368	.377	.385	.390	.395	.402	.405	.410
		.250		41 740	.423	2.24	.093	.128	.163	.183	.198	.210	.219	.233	.254	.274	.288	.310	.322	.334	.344	.352	.359	.365	.374	.382	.390	.396	.402	.407	.412	.415

A	105	1.10	115	1.20	125	1.30	1.35	140	1.45	1.50	155	160	165	1.70	175	1.80	1.85	1.90	1.95	2.00
B	.002	005	.007	.009	.011	.013	.015	.016	.019	020	:022	024	025	.027	028	.030	.031	.032	.034	.035

x The reactance for any distance D not given in the table can be found as follows: Let E = the nearest smaller distance in the table. Divide D by E and taking a value of A nearest to the quotient find the corresponding value of B, which must be added to the reactance corresponding to the size of conductor and distance E.

For three-phase regular flat spacing use D = 1.26 A. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{A\,B\,C}$. For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

xx At a temperature of 65° C (149° F) these resistance values would be increased by 15 percent. They are based upon a conductivity for copper of 97.3—for aluminum of 61 percent. They do not take into account skin effect; this should be considered when the larger conductors are used, particularly at the higher frequencies. No allowance has been made for increased length due to sag when the conductors are two percent greater than for a solid rod of cross section equal to the total cross section of the wires of the cable.

For stranded conductors D was taken as the diameter of a solid rod of equivalent cross sectional area. Actually D for stranded conductors is slightly greater, resulting in slightly less reactance than the table values. The table values are therefore conservative. In calculating the reactance values for the steel reinforced aluminum cable the presence of the steel strands was ignored.

TABLE V—RESISTANCE AND 60 CYCLE REACTANCE

OHMS PER MILE OF SINGLE CONDUCTOR

CONDUCTORS — RESISTANCE OF A SINGLE CONDUCTOR IN OHMS AT 25° C (77° F) xx

REACTANCE IN OHMS PER MILE OF EACH CONDUCTOR OF A SINGLE PHASE, OR OF A SYMMETRICAL 3 PHASE CIRCUIT. FOR OTHER ARRANGEMENTS OF CONDUCTORS SEE FOOT NOTES. x THE TABLE VALUES WERE DERIVED FROM THE EQUATION—OHMS REACTANCE $= 2\pi FL$ (L BEING EXPRESSED IN HENRIES PER MILE OF SINGLE CONDUCTOR). THE REACTANCE FOR OTHER FREQUENCIES IS $\frac{F}{60}$ THE TABLE VALUES.

DISTANCE (D) BETWEEN CENTERS OF CONDUCTORS x

Material	Type	Diameter in Inches	B & S No.	Area in Circular Mils	Resistance per 1000 Feet	Resistance per Mile	½"	1"	2"	3"	4"	5"	6"	8"	12"	18"	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet	11 Feet	13 Feet	15 Feet	17 Feet	19 Feet	21 Feet	23 Feet	25 Feet
COPPER	STRANDED	1.631		2 000 000	.00554	.0293			.157	.207	.241	.268	.290	.324	.374	.422	.458	.507	.540	.563	.589	.610	.627	.640	.665	.684	.700	.720	.730	.742	.755	.761
		1.548		1 800 000	.00615	.0325			.163	.212	.246	.275	.297	.330	.380	.428	.463	.514	.550	.575	.597	.615	.630	.647	.673	.690	.707	.724	.736	.750	.760	.770
		1.504		1 700 000	.00652	.0344			.166	.216	.251	.277	.300	.334	.384	.432	.468	.515	.552	.580	.599	.620	.637	.648	.675	.695	.710	.725	.740	.752	.763	.773
		1.459		1 600 000	.00693	.0367			.170	.219	.255	.281	.304	.338	.388	.435	.472	.520	.555	.585	.603	.625	.640	.655	.678	.698	.715	.732	.742	.758	.765	.778
		1.412		1 500 000	.00739	.0391			.174	.223	.259	.284	.309	.342	.390	.442	.474	.525	.560	.588	.609	.627	.642	.659	.683	.702	.718	.735	.749	.760	.770	.780
		1.364		1 400 000	.00792	.0418			.177	.226	.262	.288	.310	.346	.394	.444	.480	.527	.563	.591	.613	.633	.648	.663	.687	.707	.723	.738	.752	.765	.775	.786
		1.263		1 200 000	.00924	.0489			.187	.237	.270	.299	.320	.356	.404	.451	.490	.537	.574	.598	.625	.642	.657	.672	.696	.715	.732	.750	.765	.773	.785	.794
		1.209		1 100 000	.0101	.0533			.192	.241	.277	.304	.326	.360	.410	.458	.494	.543	.577	.607	.627	.645	.663	.676	.700	.720	.739	.755	.767	.780	.787	.803
		1.152		1 000 000	.0111	.0587			.199	.248	.282	.310	.332	.366	.416	.465	.500	.550	.585	.612	.634	.650	.669	.685	.707	.725	.745	.758	.773	.783	.793	.807
		1.123		950 000	.0117	.0618			.201	.251	.284	.312	.335	.370	.420	.467	.502	.552	.587	.615	.635	.655	.673	.687	.708	.728	.748	.765	.776	.788	.800	.810
		1.093		900 000	.0123	.0650			.205	.255	.288	.317	.338	.372	.422	.470	.508	.553	.590	.619	.638	.657	.675	.689	.712	.732	.750	.765	.778	.792	.803	.813
		1.062		850 000	.0130	.0688			.209	.259	.292	.320	.342	.376	.426	.474	.510	.560	.595	.622	.642	.663	.678	.692	.717	.735	.755	.767	.782	.793	.805	.815
		1.031		800 000	.0139	.0735			.213	.261	.296	.324	.346	.380	.428	.480	.514	.562	.597	.625	.645	.667	.682	.696	.720	.740	.758	.775	.786	.795	.810	.820
		.998		750 000	.0148	.0782		.132	.216	.265	.300	.327	.350	.384	.434	.483	.518	.567	.602	.629	.651	.670	.686	.697	.725	.744	.762	.775	.790	.800	.814	.824
		.964		700 000	.0158	.0835		.136	.221	.268	.305	.332	.354	.388	.436	.486	.522	.570	.605	.633	.655	.675	.690	.703	.728	.745	.765	.780	.792	.805	.815	.826
		.929		650 000	.0170	.0900		.141	.225	.275	.308	.336	.358	.392	.442	.492	.525	.575	.609	.638	.659	.678	.695	.708	.732	.752	.770	.785	.800	.810	.822	.833
		.893		600 000	.0185	.0978		.145	.231	.279	.314	.339	.362	.398	.445	.495	.530	.580	.615	.640	.665	.682	.700	.712	.736	.755	.777	.790	.803	.815	.825	.835
		.855		550 000	.0202	.107		.151	.235	.284	.319	.344	.368	.402	.452	.501	.535	.583	.620	.647	.670	.688	.705	.718	.742	.762	.780	.795	.808	.822	.833	.843
		.815		500 000	.0222	.117		.157	.241	.290	.325	.350	.374	.410	.458	.508	.542	.592	.626	.654	.676	.690	.711	.724	.748	.769	.787	.800	.813	.825	.840	.849
		.772		450 000	.0247	.131		.163	.247	.296	.330	.358	.380	.414	.463	.512	.548	.597	.633	.660	.680	.698	.716	.730	.755	.773	.792	.807	.822	.833	.845	.855
		.728		400 000	.0277	.147		.170	.255	.304	.338	.364	.388	.423	.472	.520	.555	.605	.640	.667	.688	.709	.722	.738	.762	.782	.800	.814	.828	.840	.852	.860
		.681		350 000	.0316	.167		.178	.264	.312	.346	.372	.396	.430	.480	.529	.564	.613	.648	.675	.697	.716	.732	.746	.770	.792	.809	.822	.835	.850	.858	.871
		.630		300 000	.0370	.196		.187	.273	.320	.356	.382	.403	.437	.488	.537	.573	.620	.657	.682	.706	.725	.740	.756	.780	.798	.815	.832	.845	.860	.868	.880
		.575		250 000	.0444	.235		.199	.282	.332	.366	.392	.416	.450	.500	.550	.585	.634	.669	.696	.718	.734	.753	.765	.790	.812	.829	.845	.860	.865	.875	.891
		.528	0000	211 600	.0525	.277		.209	.292	.342	.376	.403	.426	.460	.510	.560	.595	.644	.679	.706	.728	.745	.763	.774	.797	.820	.839	.853	.865	.880	.892	.901
		.470	000	167 772	.0662	.350	.139	.223	.306	.356	.390	.418	.441	.475	.525	.574	.609	.658	.693	.721	.742	.757	.777	.788	.814	.833	.853	.866	.878	.893	.905	.915
		.418	00	133 079	.0834	.442	.153	.237	.320	.370	.403	.432	.454	.488	.539	.588	.622	.672	.707	.734	.756	.772	.791	.802	.827	.846	.867	.880	.895	.908	.919	.929
		.373	0	105 560	.105	.555	.167	.251	.334	.384	.418	.445	.469	.502	.552	.602	.637	.686	.721	.748	.770	.787	.805	.817	.843	.862	.881	.895	.908	.920	.935	.943
		.332	1	83 694	.132	.697	.181	.265	.348	.399	.432	.460	.483	.515	.567	.616	.651	.700	.735	.762	.784	.800	.819	.832	.855	.875	.895	.910	.925	.935	.950	.957
		.292	2	66 358	.167	.883	.195	.279	.362	.412	.445	.473	.497	.530	.581	.630	.665	.714	.749	.776	.798	.813	.833	.845	.870	.890	.910	.923	.938	.950	.959	.971
		.260	3	52 624	.211	1.12	.209	.292	.377	.425	.460	.488	.510	.543	.594	.643	.677	.727	.760	.787	.810	.828	.845	.860	.885	.903	.922	.940	.952	.965	.978	.985
		.232	4	41 738	.266	1.41	.223	.307	.390	.440	.475	.502	.525	.558	.609	.658	.693	.742	.777	.804	.826	.840	.861	.873	.895	.915	.937	.950	.964	.980	.990	.999
		.206	5	33 088	.336	1.78	.237	.320	.403	.453	.487	.515	.538	.573	.620	.670	.705	.753	.789	.815	.837	.857	.875	.888	.910	.930	.950	.965	.980	.995	1.001	1.013
	SOLID	.460	0000	211 600	.0514	.272	.123	.209	.292	.342	.376	.403	.426	.460	.510	.560	.595	.644	.679	.706	.728	.745	.763	.774	.797	.820	.839	.853	.865	.880	.892	.901
		.410	000	167 772	.0647	.342	.139	.223	.306	.356	.390	.418	.441	.475	.525	.574	.609	.658	.693	.720	.742	.757	.777	.788	.814	.833	.853	.866	.878	.893	.905	.915
		.365	00	133 079	.0817	.432	.153	.237	.320	.370	.403	.432	.454	.488	.539	.588	.622	.672	.707	.734	.756	.772	.791	.802	.827	.846	.867	.880	.895	.908	.919	.929
		.325	0	105 560	.103	.545	.167	.251	.334	.384	.418	.445	.469	.502	.552	.602	.637	.686	.721	.748	.770	.787	.805	.817	.843	.862	.881	.895	.708	.920	.935	.943
		.289	1	83 694	.129	.680	.181	.265	.348	.399	.432	.460	.483	.515	.567	.616	.651	.700	.735	.762	.784	.800	.819	.832	.855	.875	.895	.910	.925	.935	.950	.957
		.258	2	66 358	.163	.862	.195	.279	.362	.412	.445	.473	.497	.530	.581	.630	.665	.714	.749	.776	.798	.813	.833	.845	.870	.890	.910	.923	.938	.950	.959	.971
		.229	3	52 624	.207	1.10	.209	.292	.377	.425	.460	.488	.510	.543	.594	.643	.677	.727	.760	.787	.810	.828	.845	.860	.885	.903	.922	.940	.952	.965	.978	.985
		.204	4	41 738	.260	1.38	.223	.307	.390	.440	.475	.502	.525	.558	.609	.658	.693	.742	.777	.804	.826	.840	.861	.873	.895	.915	.937	.950	.964	.980	.990	.999
		.182	5	33 088	.328	1.73	.237	.320	.403	.453	.487	.515	.538	.573	.620	.670	.705	.753	.789	.815	.837	.857	.875	.888	.910	.930	.950	.965	.980	.995	1.001	1.013
ALUMINUM	STEEL REINFORCED	.953		605 000	.0291	.154		.145	.229	.279	.312	.340	.362	.398	.447	.493	.530	.580	.615	.642	.662	.683	.698	.713	.738	.758	.774	.790	.805	.815	.825	.835
		.904		500 000	.0352	.186		.157	.240	.288	.324	.354	.374	.410	.458	.507	.541	.591	.627	.655	.676	.692	.710	.725	.745	.768	.785	.802	.815	.825	.839	.852
		.741		336 420	.0525	.277		.181	.265	.315	.348	.374	.398	.434	.482	.530	.567	.615	.650	.677	.698	.718	.734	.745	.770	.795	.810	.825	.840	.852	.860	.875
		.635		266 800	.0662	.350		.195	.280	.328	.362	.388	.412	.448	.495	.545	.582	.630	.666	.690	.715	.730	.746	.763	.786	.805	.825	.840	.853	.865	.875	.890
		.564		211 950	.0834	.442		.209	.292	.340	.378	.402	.427	.460	.510	.560	.595	.643	.676	.707	.726	.747	.762	.775	.795	.816	.840	.855	.870	.879	.890	.902
		.501		167 800	.105	.555		.222	.307	.356	.390	.418	.440	.476	.525	.573	.608	.660	.690	.718	.743	.760	.775	.790	.815	.836	.853	.870	.880	.892	.905	.915
		.447		133 200	.132	.697	.152	.236	.321	.370	.406	.430	.454	.490	.540	.588	.623	.671	.705	.735	.755	.774	.790	.805	.830	.850	.865	.885	.895	.908	.918	.930
		.398		105 530	.167	.883	.167	.250	.336	.382	.418	.448	.468	.501	.554	.600	.637	.685	.720	.745	.768	.787	.805	.817	.845	.865	.882	.895	.910	.925	.932	.945
		.355		83 640	.211	1.12	.181	.264	.348	.398	.432	.462	.482	.518	.568	.615	.650	.698	.730	.760	.782	.803	.815	.835	.855	.876	.895	.908	.925	.935	.945	.955
		.316		66 370	.266	1.41	.195	.278	.362	.412	.445	.475	.496	.531	.580	.630	.666	.713	.746	.775	.795	.817	.833	.845	.869	.892	.908	.925	.940	.950	.965	.970
		.281		52 630	.336	1.78	.209	.292	.376	.424	.460	.488	.510	.545	.595	.645	.678	.725	.763	.790	.810	.830	.845	.862	.885	.905	.925	.938	.950	.965	.975	.985
		.250		41 740	.423	2.24	.222	.306	.392	.440	.476	.503	.525	.558	.610	.659	.690	.743	.775	.805	.827	.845	.862	.875	.900	.920	.937	.952	.965	.980	.990	.993

A	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
B	.006	.011	.017	.022	.027	.032	.036	.040	.045	.049	.053	.057	.061	.065	.068	.072	.075	.078	.081	.084

x The reactance for any distance D not given in the table can be found as follows: Let E = the nearest smaller distance in the table. Divide D by E and taking a value of A nearest to the quotient find the corresponding value of B, which must be added to the reactance corresponding to the size of conductor and distance E.

For three-phase regular flat spacing use $D = 1.26\ A$. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{A\ B\ C}$. For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

xx At a temperature of 65° C (149° F) these resistance values would be increased by 15 percent. They are based upon a conductivity for copper of 97.3—for aluminum of 61 percent. They do not take into account skin effect; this should be considered when the larger conductors are used, particularly at the higher frequencies. No allowance has been made for increased length due to sag when the conductors are two percent greater than for a solid rod of cross section equal to the total cross section of the wires of the cable.

For stranded conductors D was taken as the diameter of a solid rod of equivalent cross sectional area. Actually D for stranded conductors is slightly greater, resulting in slightly less reactance than the table values. The table values are therefore conservative. In calculating the reactance values for the steel reinforced aluminum cable the presence of the steel strands was ignored.

TABLE VI—RATIO OF 25 CYCLE REACTANCE TO RESISTANCE AT 25° C

THE RESISTANCE VOLTS HAVING BEEN DETERMINED (AT 25° C) THE REACTANCE VOLTS MAY BE FOUND BY MULTIPLYING THE RESISTANCE VOLTS BY THE CONSTANTS GIVEN IN TABLE BELOW FOR THE SPACING AND SIZE OF CONDUCTORS CONTEMPLATED. THE RATIO FOR OTHER FREQUENCIES IS $\frac{F}{25}$ TIMES THE TABLE VALUES. FOR A TEMPERATURE OF 65° C (149° F) MULTIPLY TABLE VALUES BY .87

CONDUCTORS					RESISTANCE OF A SINGLE CONDUCTOR IN OHMS AT 25° C (77° F) XX		DISTANCE (D) BETWEEN CENTERS OF CONDUCTORS X																									
MATERIAL	TYPE	DIAMETER IN INCHES	B & S NO.	AREA IN CIRCULAR MILS	PER 1000 FEET	PER MILE	½"	1"	2"	3"	4"	5"	6"	8"	12"	18"	2 FEET	3 FEET	4 FEET	5 FEET	6 FEET	7 FEET	8 FEET	9 FEET	11 FEET	13 FEET	15 FEET	17 FEET	19 FEET	21 FEET	23 FEET	25 FEET
COPPER	STRANDED	1.631		2 000 000	.00554	.0293			2.22	2.93	3.42	3.82	4.12	4.60	5.32	5.98	6.52	7.20	7.68	8.00	8.35	8.67	8.92	9.08	9.45	9.73	9.95	10.20	10.40	10.50	10.70	10.80
		1.54		1 800 000	.00615	.0325			2.09	2.74	3.17	3.50	3.79	4.25	4.86	5.47	5.95	6.60	7.02	7.40	7.64	7.88	8.06	8.30	8.60	8.83	9.05	9.27	9.42	9.60	9.72	9.85
		1.504		1 700 000	.00652	.0344			2.02	2.62	3.02	3.34	3.64	4.04	4.65	5.24	5.67	6.25	6.70	7.02	7.27	7.50	7.70	7.85	8.17	8.40	8.60	8.78	8.95	9.15	9.22	9.34
		1.459		1 600 000	.00693	.0367			1.94	2.48	2.89	3.19	3.46	3.84	4.38	4.93	5.35	5.90	6.30	6.65	6.85	7.10	7.25	7.42	7.69	7.90	8.10	8.30	8.43	8.60	8.70	8.83
		1.412		1 500 000	.00739	.0391			1.87	2.38	2.76	3.04	3.27	3.63	4.15	4.68	5.05	5.60	5.96	6.24	6.48	6.68	6.83	7.00	7.25	7.48	7.65	7.82	7.97	8.10	8.25	8.32
		1.364		1 400 000	.00792	.0418			1.77	2.27	2.60	2.87	3.08	3.44	3.92	4.42	4.77	5.25	5.60	5.88	6.10	6.30	6.43	6.60	6.80	7.02	7.20	7.34	7.50	7.60	7.72	7.80
		1.263		1 200 000	.00924	.0489			1.60	2.02	2.31	2.54	2.72	3.02	3.44	3.85	4.18	4.58	4.87	5.12	5.32	5.45	5.60	5.73	5.93	6.08	6.25	6.39	6.50	6.60	6.70	6.75
		1.209		1 100 000	.0101	.0533			1.50	1.87	2.15	2.38	2.55	2.81	3.20	3.56	3.85	4.24	4.50	4.75	4.90	5.03	5.17	5.30	5.46	5.62	5.75	5.89	5.98	6.10	6.15	6.25
		1.152		1 000 000	.0111	.0587			1.42	1.78	2.00	2.20	2.36	2.60	2.95	3.31	3.5[illegible]	3.91	4.16	4.35	4.48	4.63	4.76	4.86	5.02	5.15	5.30	5.37	5.50	5.56	5.62	5.73
		1.123		950 000	.0117	.0618			1.36	1.70	1.91	2.10	2.25	2.49	2.83	3.16	3.38	3.70	3.96	4.14	4.27	4.40	4.53	4.62	4.77	4.90	5.05	5.15	5.25	5.30	5.37	5.45
		1.09[illegible]		900 000	.0123	.0650			1.32	1.63	1.85	2.03	2.17	2.38	2.71	3.02	3.25	3.54	3.79	3.97	4.08	4.20	4.32	4.40	4.55	4.70	4.80	4.90	4.97	5.08	5.15	5.20
		1.062		850 000	.0130	.0688			1.26	1.57	1.77	1.95	2.08	2.28	2.57	2.86	3.08	3.39	3.60	3.75	3.89	4.01	4.10	4.20	4.33	4.45	4.56	4.65	4.72	4.80	4.87	4.95
		1.031		800 000	.0139	.0735			1.21	1.48	[illegible]	1.84	1.96	2.15	2.42	2.72	2.91	3.17	3.38	3.54	3.65	3.77	3.87	3.95	4.08	4.19	4.28	4.38	4.45	4.52	4.58	4.65
		.99[illegible]		750 000	.0148	.0782		.70	1.15	1.41	1.60	1.74	1.87	2.05	2.32	2.57	2.77	3.02	3.21	3.36	3.47	3.57	3.66	3.73	3.85	3.97	4.08	4.15	4.20	4.27	4.35	4.40
		.964		700 000	.0158	.0835		.68	1.10	1.34	1.52	1.65	1.77	1.93	2.18	2.42	2.60	2.85	3.02	3.16	3.27	3.36	3.44	3.50	3.63	3.73	3.82	3.89	3.97	4.02	4.07	4.13
		.929		650 000	.0170	.0900		.66	1.04	1.27	1.43	1.56	1.66	1.81	2.04	2.28	2.42	2.66	2.83	2.96	3.04	3.14	3.22	3.27	3.38	3.48	3.56	3.64	3.70	3.75	3.80	3.85
		.893		600 000	.0185	.0978		.62	.98	1.19	1.34	1.44	1.55	1.70	1.90	2.11	2.25	2.47	2.62	2.72	2.82	2.90	2.98	3.03	3.13	3.22	3.30	3.37	3.42	3.48	3.52	3.56
		.855		550 000	.0202	.107		.59	.92	1.10	1.24	1.34	1.43	1.56	1.76	1.94	2.08	2.27	2.41	2.51	2.62	2.68	2.74	2.80	2.89	2.96	3.04	3.10	3.15	3.20	3.24	3.28
		.815		500 000	.0222	.117		.56	.85	1.03	1.15	1.25	1.33	1.46	1.63	1.81	1.93	2.12	2.23	2.33	2.42	2.46	2.53	2.58	2.67	2.73	2.80	2.85	2.90	2.95	2.99	3.02
		.772		450 000	.0247	.131		.52	.79	.95	1.05	1.14	1.21	1.32	1.47	1.63	1.75	1.90	2.01	2.09	2.16	2.22	2.28	2.32	2.41	2.48	2.51	2.57	2.61	2.65	2.69	2.72
		.728		400 000	.0277	.147		.48	.72	.86	.96	1.03	1.10	1.20	1.33	1.47	1.57	1.72	1.81	1.89	1.95	2.01	2.05	2.09	2.16	2.22	2.26	2.31	2.35	2.38	2.41	2.44
		.681		350 000	.0316	.167		.44	.66	.78	.86	.93	.99	1.07	1.20	1.32	1.41	1.53	1.62	1.68	1.75	1.80	1.83	1.87	1.92	1.98	2.02	2.05	2.09	2.12	2.15	2.18
		.630		300 000	.0370	.196		.40	.58	[illegible]	.76	.81	.86	.93	1.04	1.15	1.22	1.32	1.40	1.45	1.50	1.54	1.57	1.61	1.65	1.70	1.73	1.77	1.80	1.83	1.85	1.87
		.575		250 000	.0444	.235		.35	.50	[illegible]	.65	.69	.74	.80	.89	.97	1.04	1.12	1.19	1.23	1.27	1.30	1.34	1.35	1.40	1.44	1.47	1.50	1.52	1.53	1.56	1.58
		.528	0000	211 600	.0525	.277		.31	.44	.51	.57	.61	.64	.69	.77	.84	.90	.97	1.02	1.06	1.09	1.12	1.15	1.16	1.20	1.23	1.26	1.28	1.30	1.32	1.34	1.36
		.470	000	167 772	.0662	.350	.16	.26	.37	.42	.46	.50	.52	.56	.62	.68	.73	.79	.83	.86	.88	.90	.93	.94	.97	.99	1.02	1.03	1.05	1.07	1.08	1.09
		.41[illegible]	00	133 079	.0834	.442	.14	.22	.30	.35	.38	.41	.43	.46	.51	.55	.59	.63	.66	.69	.71	.73	.75	.76	.78	.80	.82	.83	.84	.86	.87	.88
		.373	0	105 560	.105	.555	.12	.19	.25	.29	.31	.33	.35	.38	.41	.45	.48	.51	.54	.56	.58	.59	.60	.61	.63	.65	.66	.67	.68	.69	.70	.71
		.332	1	83 694	.132	.697	.11	.16	.21	.24	.26	.28	.29	.31	.34	.37	.39	.42	.44	.46	.47	.48	.49	.50	.51	.52	.54	.55	.55	.56	.57	.57
		.292	2	66 358	.167	.883	.09	.13	.17	.19	.21	.22	.24	.25	.27	.30	.31	.34	.35	.36	.38	.38	.39	.40	.41	.42	.43	.43	.44	.44	.45	.46
		.260	3	52 624	.211	1.12	.08	.11	.14	.16	.17	.18	.19	.20	.22	.24	.25	.27	.28	.29	.30	.31	.31	.32	.33	.33	.34	.35	.35	.36	.36	.37
		.232	4	41 738	.266	1.41	.06	.09	.11	.13	.14	.15	.15	.16	.18	.19	.20	.22	.23	.24	.24	.25	.25	.26	.26	.27	.28	.28	.28	.29	.29	.30
		.206	5	33 088	.336	1.78	.05	.07	.09	.10	.11	.12	.12	.13	.14	.16	.16	.18	.18	.19	.19	.20	.20	.21	.21	.22	.22	.23	.23	.23	.23	.24
	SOLID	.460	0000	211 600	.0514	.272	.19	.32	.45	.52	.58	.62	.65	.71	.78	.86	.91	.99	1.04	1.08	1.12	1.14	1.17	1.19	1.21	1.25	1.29	1.31	1.32	1.35	1.36	1.38
		.410	000	167 772	.0647	.342	.17	.26	.37	.43	.48	.51	.54	.58	.64	.70	.74	.80	.84	.88	.90	.92	.95	.96	.99	1.02	1.04	1.06	1.07	1.09	1.10	1.12
		.365	00	133 079	.0817	.432	.14	.23	.31	.36	.39	.42	.44	.47	.52	.57	.60	.65	.68	.71	.73	.74	.76	.77	.80	.82	.84	.85	.86	.88	.89	.90
		.325	0	105 560	.103	.545	.13	.19	.25	.29	.32	.34	.36	.38	.42	.46	.49	.52	.55	.57	.59	.60	.62	.63	.64	.66	.67	.68	.69	.71	.71	.72
		.289	1	83 694	.129	.680	.10	.16	.21	.24	.26	.28	.29	.31	.35	.38	.40	.43	.45	.47	.48	.49	.50	.51	.52	.53	.55	.56	.56	.58	.58	.59
		.258	2	66 358	.163	.862	.09	.13	.17	.20	.22	.23	.24	.25	.28	.30	.32	.35	.36	.37	.39	.39	.40	.41	.42	.43	.44	.44	.45	.46	.46	.47
		.229	3	52 624	.207	1.10	.08	.11	.14	.16	.17	.18	.19	.20	.22	.24	.25	.27	.29	.30	.31	.31	.32	.32	.33	.34	.35	.35	.36	.36	.37	.37
		.204	4	41 738	.260	1.38	.07	.09	.12	.13	.14	.15	.16	.17	.18	.20	.21	.22	.23	.24	.25	.25	.26	.26	.27	.28	.28	.29	.29	.29	.30	.30
		.182	5	33 088	.328	1.73	.06	.08	.09	.11	.13	.12	.13	.14	.15	.16	.17	.18	.19	.20	.20	.21	.21	.21	.22	.22	.23	.23	.24	.24	.24	.24
ALUMINUM	STEEL REINFORCED	.953		605 000	.0291	.154		.39	.62	.75	.85	.92	.98	1.08	1.20	1.33	1.43	1.56	1.66	1.73	1.80	1.85	1.89	1.92	2.00	2.05	2.09	2.13	2.18	2.21	2.24	2.27
		.904		500 000	.0352	.186		.35	.54	.62	.70	.79	.81	.92	1.02	1.13	1.22	1.32	1.40	1.46	1.52	1.55	1.59	1.62	1.67	1.72	1.76	1.80	1.83	1.85	1.88	1.91
		.741		336 420	.0525	.277		.27	.40	.47	.52	.56	.60	.65	.72	.80	.85	.92	.98	1.02	1.05	1.08	1.10	1.12	1.16	1.19	1.22	1.24	1.26	1.28	1.29	1.31
		.635	-	266 800	.0662	.350		.23	.33	.39	.43	.46	.49	.53	.59	.65	.69	.75	.80	.82	.85	.87	.89	.91	.93	.96	.98	1.00	1.02	1.03	1.04	1.06
		.564		211 950	.0834	.442		.20	.28	.32	.35	.38	.40	.43	.48	.52	.56	.60	.63	.66	.68	.70	.71	.73	.75	.77	.79	.80	.82	.83	.84	.85
		.501		167 800	.105	.555		.17	.23	.27	.30	.31	.33	.36	.40	.43	.45	.50	.51	.54	.56	.57	.58	.59	.61	.63	.64	.65	.66	.67	.68	.69
		.447		133 200	.132	.697	.09	.14	.19	.22	.24	.26	.27	.26	.32	.35	.37	.40	.42	.44	.45	.46	.47	.48	.50	.51	.52	.53	.54	.54	.55	.55
		.398		105 530	.167	.883	.08	.12	.16	.18	.20	.21	.22	.24	.26	.28	.30	.32	.34	.35	.36	.37	.38	.39	.40	.41	.41	.42	.43	.44	.44	.45
		.355		83 640	.211	1.12	.07	.10	.13	.15	.16	.17	.18	.19	.21	.22	.24	.26	.27	.28	.29	.30	.30	.31	.32	.32	.33	.34	.34	.35	.35	.35
		.316		66 370	.266	1.41	.06	.08	.11	.12	.13	.14	.15	.16	.17	.18	.20	.21	.22	.23	.24	.24	.25	.25	.26	.26	.27	.27	.28	.28	.29	.29
		.281		52 630	.336	1.78	.05	.07	.09	.10	.11	.11	.12	.12	.13	.15	.16	.17	.18	.18	.19	.19	.19	.20	.21	.21	.21	.22	.22	.22	.23	.23
		.250		41 740	.423	2.24	.04	.06	.07	.08	.09	.09	.10	.10	.11	.12	.13	.14	.14	.15	.15	.16	.16	.16	.17	.17	.17	.18	.18	.18	.18	.19

xFor three-phase regular flat spacing use D = 1.26 A. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$ For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

xxAt a temperature of 65° C (149° F) these resistance values would be increased by 15 percent. They are based upon a conductivity for copper of 97.3—for aluminum of 61 percent. They do not take into account skin effect; this should be considered when the larger conductors are used, particularly at the higher frequencies. No allowance has been made for increased length due to sag when the conductors are suspended. The resistance values for the stranded conductors are two percent greater than for a solid rod of cross section equal to the total cross section of the wires of the cable.

For stranded conductors D was taken as the diameter of a solid rod of equivalent cross sectional area. Actually D for stranded conductors is slightly greater, resulting in slightly less reactance than the table values. The table values are therefore conservative. In calculating the reactance values for the steel reinforced aluminum cable the presence of the steel strands was ignored.

TABLE VII—RATIO OF 60 CYCLE REACTANCE TO RESISTANCE AT 25° C

THE RESISTANCE VOLTS HAVING BEEN DETERMINED (AT 25° C) THE REACTANCE VOLTS MAY BE FOUND BY MULTIPLYING THE RESISTANCE VOLTS BY THE CONSTANTS GIVEN IN TABLE BELOW FOR THE SPACING AND SIZE OF CONDUCTORS CONTEMPLATED. THE RATIO FOR OTHER FREQUENCIES IS $\frac{F}{60}$ TIMES THE TABLE VALUES. FOR A TEMPERATURE OF 65° C (149° F) MULTIPLY TABLE VALUES BY .87

Conductors: Material, Type, Diameter in Inches, B & S No., Area in Circular Mils. Resistance of a single conductor in ohms at 25° C (77° F) xx: Per 1000 Feet, Per Mile. Distance (D) between centers of conductors X (inches):

Material	Type	Diameter in Inches	B & S No.	Area in Circular Mils	Per 1000 Feet	Per Mile	½"	1"	2"	3"	4"	6"	6"	8"	12"	18"
COPPER	STRANDED	1.631		2 000 000	.00554	.0293			5.36	7.07	8.23	9.16	9.90	11.10	12.80	14.4
		1.548		1 800 000	.00615	.0325			5.02	6.52	7.58	8.46	9.15	10.20	11.70	13.2
		1.504		1 700 000	.00652	.0344			4.83	6.30	7.30	8.05	8.72	9.70	11.20	12.6
		1.457		1 600 000	.00693	.0367			4.63	5.97	6.96	7.67	8.30	9.22	10.60	11.9
		1.412		1 500 000	.00739	.0391			4.45	5.70	6.62	7.25	7.90	8.75	9.95	11.3
		1.364		1 400 000	.00792	.0418			4.23	5.40	6.26	6.90	7.40	8.28	9.43	10.6
		1.263		1 200 000	.00924	.0489			3.82	4.84	5.52	6.10	6.55	7.28	8.26	9.20
		1.209		1 100 000	.0101	.0533			3.60	4.51	5.20	5.70	6.11	6.75	7.70	8.60
		1.152		1 000 000	.0111	.0587			3.40	4.22	4.81	5.29	5.65	6.25	7.10	7.92
		1.123		950 000	.0117	.0618			3.25	4.05	4.60	5.05	5.42	6.00	6.80	7.55
		1.093		900 000	.0123	.0650			3.15	3.92	4.43	4.87	5.20	5.75	6.50	7.24
		1.062		850 000	.0130	.0688			3.04	3.76	4.24	4.65	4.97	5.46	6.20	6.90
		1.031		800 000	.0139	.0735			2.90	3.55	4.03	4.42	4.70	5.17	5.83	6.55
		.997		750 000	.0148	.0782		1.69	2.76	3.39	3.84	4.18	4.48	4.91	5.55	6.18
		.964		700 000	.0158	.0835		1.63	2.65	3.21	3.66	3.98	4.25	4.65	5.23	5.82
		.929		650 000	.0170	.0900		1.56	2.50	3.06	3.42	3.73	3.98	4.35	4.90	5.46
		.893		600 000	.0185	.0978		1.48	2.36	2.85	3.21	3.47	3.70	4.07	4.55	5.05
		.855		550 000	.0202	.107		1.41	2.20	2.66	2.98	3.22	3.44	3.76	4.22	4.68
		.815		500 000	.0222	.117		1.34	2.06	2.48	2.78	2.99	3.20	3.51	3.92	4.35
		.772		450 000	.0247	.131		1.24	1.88	2.26	2.52	2.74	2.90	3.16	3.54	3.90
		.728		400 000	.0277	.147		1.16	1.74	2.07	2.30	2.48	2.64	2.88	3.21	3.54
		.681		350 000	.0316	.167		1.06	1.58	1.87	2.08	2.23	2.37	2.58	2.87	3.17
		.630		300 000	.0370	.196		.96	1.40	1.63	1.81	1.95	2.06	2.23	2.49	2.74
		.575		250 000	.0444	.235		.85	1.20	1.41	1.56	1.67	1.76	1.91	2.13	2.34
		.528	0000	211 600	.0525	.277		.76	1.05	1.23	1.36	1.46	1.54	1.66	1.84	2.02
		.470	000	167 772	.0662	.350	.40	.64	.87	1.02	1.11	1.20	1.26	1.36	1.50	1.64
		.418	00	133 079	.0834	.442	.34	.54	.72	.84	.91	.98	1.03	1.11	1.22	1.33
		.373	0	105 560	.105	.555	.30	.45	.60	.69	.75	.80	.84	.90	1.00	1.08
		.332	1	83 694	.132	.697	.26	.38	.50	.57	.62	.66	.70	.74	.81	.88
		.292	2	66 358	.167	.883	.22	.32	.41	.47	.50	.53	.56	.60	.65	.71
		.260	3	52 624	.211	1.12	.19	.26	.34	.38	.41	.44	.45	.48	.53	.58
		.232	4	41 738	.266	1.41	.16	.22	.28	.31	.34	.35	.37	.40	.43	.47
		.206	5	33 088	.336	1.78	.13	.18	.22	.25	.27	.29	.30	.32	.35	.37
	SOLID	.460	0000	211 600	.0514	.272	.45	.76	1.06	1.25	1.38	1.48	1.56	1.69	1.87	2.06
		.410	000	167 772	.0647	.342	.40	.65	.90	1.04	1.14	1.22	1.29	1.39	1.53	1.68
		.365	00	133 079	.0817	.432	.35	.55	.73	.85	.94	1.00	1.05	1.13	1.25	1.36
		.325	0	105 560	.103	.545	.31	.46	.61	.70	.77	.82	.86	.92	1.01	1.11
		.289	1	83 694	.129	.680	.26	.39	.51	.59	.63	.67	.71	.76	.83	.90
		.258	2	66 358	.163	.862	.22	.32	.42	.48	.52	.55	.57	.61	.67	.73
		.229	3	52 624	.207	1.10	.19	.26	.34	.39	.42	.44	.46	.49	.54	.58
		.204	4	41 738	.260	1.38	.16	.22	.28	.32	.34	.36	.38	.40	.44	.48
		.182	5	33 088	.328	1.73	.14	.18	.23	.26	.28	.30	.31	.33	.36	.39
ALUMINUM	STEEL REINFORCED	.953		605 000	.0291	.154		.94	1.49	1.80	2.02	2.20	2.34	2.58	2.90	3.20
		.904		500 000	.0352	.186		.84	1.29	1.55	1.75	1.90	2.01	2.21	2.47	2.73
		.741		336 420	.0525	.277		.65	.95	1.14	1.25	1.35	1.44	1.57	1.74	1.91
		.635		266 800	.0662	.350		.55	.80	.94	1.03	1.10	1.17	1.28	1.41	1.56
		.564		211 950	.0834	.442		.47	.66	.77	.85	.91	.97	1.04	1.15	1.26
		.501		167 800	.105	.555		.40	.55	.64	.70	.75	.79	.85	.95	1.03
		.447		133 200	.132	.697	.22	.34	.46	.53	.58	.62	.65	.70	.77	.84
		.398		105 530	.167	.883	.19	.28	.38	.43	.47	.50	.53	.56	.62	.68
		.355		83 640	.211	1.12	.16	.24	.31	.35	.39	.41	.43	.46	.50	.55
		.316		66 370	.266	1.41	.14	.20	.25	.29	.31	.34	.35	.37	.41	.45
		.281		52 630	.336	1.78	.11	.16	.21	.24	.26	.27	.28	.30	.33	.36
		.250		41 740	.423	2.24	.10	.13	.17	.19	.21	.22	.23	.25	.27	.29

Distance (D) between centers of conductors X (feet), continued:

Material	Diameter in Inches	Area in Circular Mils	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet	11 Feet	13 Feet	15 Feet	17 Feet	19 Feet	21 Feet	23 Feet	26 Feet
COPPER STRANDED	1.631	2 000 000	15.7	17.3	18.5	19.2	20.1	20.8	21.4	21.9	22.7	23.4	23.9	24.6	25.0	25.3	25.8	26.0
	1.548	1 800 000	14.3	15.8	17.0	17.7	18.4	19.0	19.4	20.0	20.7	21.3	21.8	22.3	22.6	23.1	23.4	23.7
	1.504	1 700 000	13.6	15.0	16.1	16.9	17.4	18.0	18.5	18.9	19.6	20.2	20.6	21.1	21.5	21.9	22.2	22.5
	1.457	1 600 000	12.9	14.2	15.2	16.0	16.4	17.0	17.5	17.9	18.5	19.0	19.5	20.0	20.2	20.7	20.9	21.2
	1.412	1 500 000	12.1	13.4	14.3	15.0	15.6	16.0	16.4	16.8	17.5	18.0	18.4	18.8	19.1	19.4	19.7	19.9
	1.364	1 400 000	11.5	12.6	13.5	14.1	14.7	15.2	15.5	15.9	16.4	16.9	17.3	17.6	18.0	18.3	18.5	18.8
	1.263	1 200 000	10.0	11.0	11.7	12.2	12.8	13.1	13.4	13.7	14.2	14.6	15.0	15.3	15.6	15.8	16.1	16.2
	1.209	1 100 000	9.25	10.2	10.8	11.4	11.8	12.1	12.4	12.7	13.1	13.5	13.9	14.1	14.4	14.6	14.8	15.1
	1.152	1 000 000	8.52	9.37	9.96	10.4	10.8	11.1	11.4	11.7	12.1	12.4	12.7	12.9	13.2	13.4	13.5	13.8
	1.123	950 000	8.12	8.95	9.50	9.95	10.3	10.6	10.9	11.1	11.4	11.8	12.1	12.4	12.5	12.7	12.9	13.1
	1.093	900 000	7.82	8.50	9.10	9.52	9.82	10.1	10.4	10.6	10.9	11.3	11.5	11.8	12.0	12.2	12.4	12.5
	1.062	850 000	7.40	8.15	8.65	9.03	9.33	9.65	9.85	10.0	10.4	10.7	11.0	11.1	11.3	11.5	11.7	11.8
	1.031	800 000	7.00	7.65	8.15	8.50	8.80	9.10	9.30	9.50	9.80	10.1	10.3	10.6	10.7	10.8	11.0	11.2
	.997	750 000	6.63	7.25	7.70	8.05	8.35	8.60	8.78	8.92	9.28	9.52	9.75	9.92	10.1	10.2	10.4	10.5
	.964	700 000	6.25	6.83	7.25	7.60	7.85	8.10	8.27	8.43	8.72	8.95	9.17	9.35	9.50	9.65	9.77	9.90
	.929	650 000	5.85	6.40	6.77	7.10	7.32	7.55	7.73	7.88	8.15	8.35	8.55	8.73	8.90	9.00	9.15	9.28
	.893	600 000	5.42	5.93	6.30	6.55	6.80	6.98	7.15	7.27	7.52	7.72	7.95	8.08	8.20	8.35	8.45	8.55
	.855	550 000	5.00	5.45	5.80	6.05	6.27	6.45	6.60	6.72	6.95	7.12	7.30	7.45	7.57	7.70	7.80	7.90
	.815	500 000	4.63	5.05	5.35	5.60	5.78	5.90	6.08	6.20	6.40	6.58	6.73	6.85	6.96	7.05	7.18	7.25
	.772	450 000	4.18	4.55	4.84	5.04	5.20	5.32	5.47	5.58	5.77	5.90	6.05	6.15	6.28	6.35	6.45	6.55
	.728	400 000	3.77	4.12	4.35	4.55	4.68	4.82	4.91	5.02	5.18	5.32	5.45	5.55	5.63	5.72	5.80	5.85
	.681	350 000	3.38	3.68	3.88	4.05	4.17	4.30	4.39	4.47	4.62	4.75	4.85	4.92	5.00	5.10	5.15	5.22
	.630	300 000	2.92	3.16	3.34	3.48	3.60	3.70	3.77	3.86	3.98	4.08	4.17	4.25	4.32	4.40	4.44	4.50
	.575	250 000	2.50	2.70	2.85	2.97	3.06	3.13	3.20	3.26	3.36	3.46	3.53	3.60	3.66	3.68	3.72	3.80
	.528	211 600	2.15	2.33	2.46	2.55	2.63	2.69	2.76	2.80	2.88	2.97	3.03	3.08	3.13	3.18	3.22	3.26
	.470	167 772	1.74	1.88	1.98	2.06	2.12	2.16	2.22	2.25	2.33	2.38	2.44	2.47	2.51	2.56	2.60	2.62
	.418	133 079	1.41	1.52	1.60	1.66	1.71	1.75	1.79	1.81	1.87	1.92	1.96	2.00	2.03	2.06	2.08	2.10
	.373	105 560	1.15	1.23	1.30	1.35	1.39	1.42	1.45	1.47	1.52	1.55	1.59	1.61	1.64	1.66	1.68	1.70
	.332	83 694	.94	1.00	1.06	1.10	1.13	1.15	1.17	1.20	1.23	1.26	1.28	1.31	1.33	1.34	1.36	1.38
	.292	66 358	.75	.81	.85	.88	.90	.92	.95	.96	.98	1.01	1.03	1.05	1.06	1.08	1.08	1.10
	.260	52 624	.60	.65	.68	.70	.72	.74	.75	.77	.79	.81	.82	.84	.85	.86	.87	.88
	.232	41 738	.49	.53	.55	.57	.59	.60	.61	.62	.63	.65	.66	.67	.68	.69	.70	.71
	.206	33 088	.40	.42	.44	.46	.47	.48	.49	.50	.51	.52	.53	.54	.55	.56	.56	.57
COPPER SOLID	.460	211 600	2.18	2.37	2.50	2.60	2.68	2.72	2.81	2.84	2.93	3.02	3.08	3.13	3.18	3.24	3.28	3.31
	.410	167 772	1.78	1.92	2.04	2.10	2.17	2.20	2.27	2.30	2.38	2.43	2.50	2.53	2.37	2.62	2.66	2.68
	.365	133 079	1.44	1.55	1.64	1.70	1.75	1.79	1.84	1.86	1.91	1.96	2.00	2.04	2.08	2.01	2.13	2.15
	.325	105 560	1.17	1.26	1.32	1.37	1.41	1.45	1.48	1.50	1.55	1.58	1.61	1.64	1.66	1.69	1.71	1.73
	.289	83 694	.95	1.03	1.08	1.12	1.15	1.17	1.20	1.22	1.26	1.28	1.32	1.34	1.36	1.38	1.40	1.41
	.258	66 358	.77	.83	.87	.89	.92	.94	.96	.98	1.01	1.03	1.05	1.07	1.09	1.10	1.11	1.12
	.229	52 624	.61	.66	.69	.71	.74	.75	.77	.78	.80	.82	.84	.86	.87	.88	.88	.90
	.204	41 738	.50	.53	.56	.58	.60	.61	.62	.63	.65	.66	.68	.69	.70	.71	.72	.72
	.182	33 088	.40	.43	.45	.47	.48	.50	.50	.51	.52	.54	.55	.56	.57	.57	.58	.58
ALUMINUM STEEL REINFORCED	.953	605 000	3.44	3.77	4.00	4.17	4.30	4.42	4.53	4.64	4.78	4.92	5.02	5.12	5.22	5.30	5.36	5.42
	.904	500 000	2.90	3.18	3.38	3.53	3.64	3.72	3.82	3.90	4.00	4.13	4.23	4.32	4.38	4.45	4.52	4.60
	.741	336 420	2.04	2.22	2.35	2.45	2.52	2.59	2.66	2.70	2.78	2.87	2.93	2.98	3.03	3.08	3.10	3.16
	.635	266 800	1.66	1.80	1.90	1.97	2.04	2.08	2.12	2.18	2.24	2.30	2.35	2.40	2.43	2.47	2.50	2.54
	.564	211 950	1.35	1.45	1.53	1.60	1.64	1.69	1.72	1.75	1.80	1.85	1.90	1.94	1.97	2.00	2.02	2.04
	.501	167 800	1.10	1.19	1.24	1.30	1.34	1.37	1.40	1.43	1.47	1.50	1.54	1.57	1.59	1.61	1.63	1.65
	.447	133 200	.89	.96	1.01	1.05	1.08	1.11	1.13	1.16	1.19	1.22	1.24	1.26	1.28	1.30	1.32	1.34
	.398	105 530	.72	.77	.82	.84	.87	.89	.91	.92	.95	.98	1.00	1.01	1.03	1.05	1.05	1.07
	.355	83 640	.58	.62	.65	.68	.70	.72	.73	.74	.76	.78	.80	.81	.82	.83	.84	.85
	.316	66 370	.47	.50	.52	.55	.56	.58	.59	.60	.61	.63	.64	.65	.67	.67	.69	.69
	.281	52 630	.38	.40	.43	.44	.45	.46	.47	.48	.50	.50	.52	.52	.53	.54	.54	.55
	.250	41 740	.31	.33	.34	.36	.37	.38	.39	.39	.40	.40	.42	.42	.43	.44	.44	.45

For three-phase regular flat spacing use D = 1.26 A. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$ For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

xx At a temperature of 65° C (149° F) these resistance values would be increased by 15 percent. They are based upon a conductivity for copper of 97.3—for aluminum of 61 percent. They do not take into account skin effect; this should be considered when the larger conductors are used, particularly at the higher frequencies. No allowance has been made for increased length due to sag when the conductors are suspended. The resistance values for the stranded conductors are two percent greater than for a solid rod of cross section equal to the total cross section of the wires of the cable.

For stranded conductors D was taken as the diameter of a solid rod of equivalent cross sectional area. Actually D for stranded conductors is slightly greater, resulting in slightly less reactance than the table values. The table values are therefore conservative. In calculating the reactance values for the steel reinforced aluminum cable the presence of the steel strands was ignored.

TABLE VIII—CAPACITANCE TO NEUTRAL PER 1000 FEET OF SINGLE BARE CONDUCTOR

CAPACITANCE (C) TO NEUTRAL IN MICROFARADS PER 1000 FEET OF EACH CONDUCTOR OF A SINGLE-PHASE OR OF A SYMMETRICAL THREE-PHASE LINE. THE VALUES FOR CAPACITANCE WERE DERIVED FROM THE EQUATION $C = \frac{.007354}{\log_{10}\left[\frac{D}{2R}+\left(\left(\frac{D}{2R}\right)^2-1\right)^{\frac{1}{2}}\right]}$ R BEING THE RADIUS OF THE CONDUCTOR EXPRESSED IN SAME TERMS AS D X X THE CAPACITANCE BETWEEN CONDUCTORS EQUALS ONE HALF THE TABLE VALUES

CONDUCTORS: MATERIAL	TYPE	DIAMETER IN INCHES	B & S NO.	AREA IN CIRCULAR MILS	DISTANCE (D) BETWEEN CENTERS OF CONDUCTORS X: ½"	1"	2"		4"	5"	6"	8"	12"	18"	2 FEET	3 FEET	4 FEET	5 FEET	6 FEET	7 FEET	8 FEET	9 FEET	11 FEET	13 FEET	15 FEET	17 FEET	19 FEET	21 FEET	23 FEET	25 FEET
COPPER	STRANDED	1.631		2 000 000			.0257	.0129	.0111	.00953	.00857	.00745	.00630	.00547	.00501	.00447	.00418	.00396	.00378	.00366	.00356	.00346	.00332	.00322	.00314	.00306	.00300	.00294	.00290	.00286
		1.548		1 800 000			.0228	.0132	.0106	.00920	.00835	.00728	.00618	.00538	.00493	.00442	.00412	.00392	.00372	.00362	.00352	.00342	.00330	.00320	.00310	.00304	.00298	.00294	.00288	.00284
		1.504		1 700 000			.0214	.0129	.0104	.00905	.00822	.00718	.00613	.00535	.00490	.00438	.00408	.00388	.00370	.00360	.00348	.00340	.00328	.00318	.00310	.00302	.00296	.00292	.00288	.00284
		1.459		1 600 000			.0203	.0126	.0102	.00890	.00810	.00707	.00605	.00532	.00488	.00435	.00406	.00386	.00370	.00358	.00346	.00338	.00326	.00316	.00308	.00300	.00296	.00290	.00286	.00284
		1.412		1 500 000			.0192	.0123	.00995	.00872	.00796	.00700	.00600	.00528	.00480	.00430	.00404	.00384	.00368	.00354	.00344	.00336	.00324	.00314	.00306	.00300	.00294	.00288	.00284	.00280
		1.364		1 400 000			.0182	.0119	.00972	.00858	.00783	.00690	.00590	.00518	.00475	.00427	.00400	.00380	.00362	.00352	.00342	.00334	.00322	.00312	.00304	.00296	.00290	.00286	.00282	.00278
		1.263		1 200 000			.0164	.0112	.00932	.00825	.00755	.00668	.00577	.00507	.00465	.00418	.00394	.00374	.00356	.00346	.00338	.00330	.00316	.00308	.00300	.00294	.00288	.00284	.00280	.00274
		1.209		1 100 000			.0155	.0109	.00907	.00808	.00740	.00657	.00572	.00500	.00460	.00415	.00390	.00368	.00354	.00344	.00336	.00326	.00314	.00306	.00298	.00290	.00286	.00280	.00278	.00274
		1.152		1 000 000			.0147	.0105	.00883	.00790	.00725	.00645	.00558	.00492	.00454	.00410	.00383	.00365	.00351	.00340	.00331	.00324	.00312	.00302	.00295	.00288	.00282	.00280	.00274	.00271
		1.123		950 000			.0143	.0104	.00872	.00778	.00717	.00638	.00552	.00488	.00450	.00407	.00382	.00362	.00348	.00338	.00330	.00322	.00310	.00302	.00294	.00288	.00282	.00278	.00274	.00270
		1.093		900 000			.0140	.0102	.00858	.00770	.00710	.00632	.00548	.00483	.00448	.00403	.00380	.00360	.00346	.00336	.00328	.00320	.00308	.00298	.00292	.00286	.00280	.00276	.00272	.00268
		1.062		850 000			.0136	.0100	.00845	.00760	.00700	.00623	.00545	.00480	.00445	.00402	.00378	.00358	.00344	.00334	.00326	.00318	.00306	.00298	.00290	.00284	.00280	.00276	.00270	.00268
		1.031		800 000			.0132	.00978	.00832	.00750	.00693	.00618	.00536	.00477	.00440	.00399	.00374	.00354	.00342	.00332	.00324	.00316	.00306	.00296	.00290	.00284	.00278	.00274	.00270	.00266
		.998		750 000			.0126	.00958	.00820	.00738	.00683	.00612	.00533	.00472	.00437	.00396	.00371	.00353	.00341	.00330	.00322	.00316	.00304	.00296	.00288	.00282	.00278	.00272	.00268	.00265
		.964		700 000		.0618	.0125	.00942	.00807	.00727	.00672	.00602	.00530	.00468	.00435	.00393	.00370	.00352	.00338	.00328	.00320	.00312	.00302	.00292	.00286	.00280	.00274	.00270	.00266	.00264
		.929		650 000		.0432	.0120	.00920	.00792	.00715	.00663	.00597	.00522	.00462	.00430	.00389	.00366	.00348	.00336	.00326	.00318	.00312	.00300	.00292	.00284	.00278	.00274	.00270	.00266	.00262
		.893		600 000		.0300	.0117	.00900	.00777	.00703	.00652	.00588	.00515	.00458	.00425	.00384	.00362	.00346	.00334	.00324	.00316	.00308	.00298	.00290	.00292	.00276	.00272	.00268	.00264	.00260
		.855		550 000		.0295	.0113	.00877	.00761	.00690	.00643	.00578	.00507	.00454	.00420	.00381	.00358	.00342	.00330	.00322	.00312	.00306	.00296	.00286	.00280	.00274	.00270	.00266	.00262	.00260
		.813		500 000		.0255	.0109	.00856	.00745	.00675	.00630	.00570	.00501	.00447	.00415	.00378	.00355	.00339	.00327	.00318	.00310	.00304	.00297	.00284	.00278	.00274	.00268	.00264	.00260	.00257
		.772		450 000		.0224	.0105	.00832	.00727	.00662	.00618	.00560	.00493	.00440	.00411	.00373	.00352	.00336	.00324	.00314	.00306	.00300	.00290	.00282	.00276	.00270	.00266	.00260	.00258	.00254
		.728		400 000		.0202	.0101	.00808	.00706	.00648	.00608	.00548	.00483	.00435	.00403	.00369	.00348	.00332	.00320	.00312	.00304	.00298	.00288	.00280	.00272	.00268	.00264	.00260	.00256	.00252
		.681		350 000		.0181	.00974	.00783	.00688	.00631	.00591	.00537	.00476	.00427	.00398	.00363	.00342	.00328	.00316	.00308	.00300	.00294	.00284	.00276	.00270	.00264	.00260	.00258	.00252	.00250
		.630		300 000		.0164	.00930	.00753	.00663	.00612	.00575	.00523	.00465	.00418	.00391	.00357	.00338	.00322	.00312	.00304	.00296	.00290	.00280	.00272	.00266	.00262	.00256	.00254	.00250	.00246
		.575		250 000		.0147	.00883	.00725	.00645	.00595	.00558	.00511	.00454	.00409	.00383	.00351	.00331	.00317	.00307	.00298	.00292	.00286	.00276	.00268	.00263	.00258	.00254	.00250	.00246	.00244
		.528	0000	211 600		.0136	.00843	.00700	.00623	.00574	.00542	.00497	.00444	.00401	.00376	.00345	.00326	.00312	.00302	.00294	.00287	.00282	.00272	.00266	.00260	.00266	.00250	.00246	.00244	.00240
		.470	000	167 772	.0477	.0122	.00795	.00667	.00598	.00557	.00523	.00480	.00431	.00390	.00366	.00337	.00318	.00306	.00296	.00288	.00282	.00276	.00268	.00260	.00255	.00250	.00246	.00242	.00240	.00237
		.418	00	133 079	.0275	.0112	.00756	.00637	.00575	.00532	.00504	.00466	.00418	.00380	.00357	.00329	.00312	.00299	.00290	.00282	.00276	.00272	.00262	.00256	.00251	.00246	.00242	.00238	.00236	.00233
		.373	0	105 560	.0213	.0103	.00716	.00610	.00552	.00518	.00488	.00451	.00407	.00371	.00349	.00322	.00305	.00293	.00284	.00278	.00271	.00266	.00258	.00252	.00247	.00242	.00238	.00224	.00222	.00229
		.332	1	83 694	.0175	.00960	.00682	.00586	.00532	.00497	.00472	.00438	.00396	.00361	.00341	.00315	.00299	.00288	.00279	.00272	.00266	.00260	.00254	.00246	.00242	.00238	.00234	.00232	.00228	.00226
		.292	2	66 358	.0150	.00890	.00648	.00560	.00512	.00480	.00456	.00422	.00384	.00352	.00332	.00308	.00292	.00281	.00273	.00266	.00261	.00256	.00248	.00242	.00238	.00234	.00230	.00228	.00226	.00222
		.260	3	52 624	.0133	.00857	.00622	.00538	.00495	.00465	.00442	.00410	.00376	.00344	.00326	.00302	.00286	.00276	.00268	.00262	.00256	.00252	.00244	.00238	.00234	.00230	.00226	.00224	.00222	.00218
		.232	4	41 738	.0121	.00792	.00596	.00521	.00478	.00451	.00429	.00400	.00365	.00336	.00318	.00295	.00281	.00271	.00263	.00256	.00252	.00248	.00240	.00234	.00230	.00226	.00224	.00220	.00218	.00215
		.206	5	33 088	.0111	.00746	.00572	.00503	.00463	.00438	.00417	.00390	.00356	.00328	.00312	.00288	.00276	.00266	.00260	.00252	.00248	.00244	.00236	.00232	.00226	.00224	.00220	.00218	.00214	.00212
	SOLID	.460	0000	211 600	.0412	.0120	.00708	.00654	.00593	.00552	.00519	.00477	.00428	.00388	.00364	.00335	.00317	.00304	.00295	.00288	.00281	.00276	.00266	.00260	.00254	.00248	.00246	.00242	.00228	.00226
		.410	000	167 772	.0260	.0110	.00747	.00632	.00570	.00532	.00501	.00462	.00416	.00378	.00356	.00328	.00310	.00298	.00289	.00282	.00275	.00270	.00262	.00256	.00250	.00244	.00242	.00228	.00224	.00232
		.365	00	133 079	.0203	.0102	.00710	.00605	.00550	.00510	.00485	.00448	.00405	.00369	.00347	.00320	.00304	.00292	.00283	.00276	.00270	.00266	.00258	.00250	.00246	.00240	.00236	.00224	.00230	.00229
		.325	0	105 560	.0170	.00946	.00675	.00582	.00528	.00493	.00469	.00435	.00394	.00360	.00339	.00314	.00298	.00286	.00278	.00270	.00265	.00260	.00252	.00246	.00242	.00236	.00234	.00230	.00228	.00225
		.289	1	83 694	.0148	.00886	.00647	.00558	.00510	.00477	.00455	.00422	.00383	.00351	.00331	.00307	.00292	.00281	.00273	.00266	.00261	.00256	.00248	.00242	.00228	.00232	.00230	.00226	.00224	.00222
		.258	2	66 358	.0132	.00835	.00620	.00538	.00493	.00462	.00441	.00410	.00374	.00343	.00324	.00301	.00286	.00276	.00268	.00262	.00256	.00252	.00244	.00238	.00234	.00230	.00226	.00222	.00220	.00218
		.229	3	52 624	.0120	.00787	.00592	.00520	.00477	.00448	.00427	.00400	.00364	.00336	.00318	.00294	.00280	.00270	.00262	.00256	.00252	.00248	.00240	.00234	.00230	.00226	.00222	.00220	.00218	.00214
		.204	4	41 738	.0110	.00745	.00570	.00500	.00462	.00435	.00416	.00389	.00355	.00327	.00310	.00289	.00275	.00266	.00258	.00252	.00247	.00244	.00236	.00232	.00227	.00222	.00220	.00216	.00214	.00212
		.182	5	33 088	.0102	.00708	.00548	.00485	.00448	.00423	.00403	.00379	.00348	.00320	.00304	.00284	.00270	.00260	.00254	.00248	.00244	.00240	.00232	.00228	.00222	.00220	.00216	.00212	.00212	.00208
ALUMINUM	STEEL REINFORCED	.953		605 000		.0532	.0124	.00932	.00800	.00723	.00675	.00602	.00523	.00466	.00433	.00391	.00368	.00350	.00336	.00326	.00320	.00312	.00302	.00294	.00286	.00280	.00274	.00270	.00266	.00262
		.904		500 000		.0371	.0118	.00912	.00780	.00705	.00657	.00588	.00518	.00460	.00427	.00386	.00364	.00346	.00334	.00324	.00316	.00310	.00299	.00290	.00282	.00278	.00272	.00268	.00264	.00260
		.741		336 420		.0207	.0102	.00817	.00714	.00652	.00610	.00553	.00488	.00435	.00406	.00370	.00348	.00332	.00320	.00312	.00304	.00298	.00288	.00280	.00274	.00270	.00264	.00260	.00256	.00254
		.636		266 800		.0165	.00935	.00757	.00668	.00615	.00580	.00525	.00465	.00420	.00391	.00359	.00336	.00322	.00312	.00309	.00296	.00290	.00280	.00274	.00266	.00262	.00258	.00254	.00250	.00248
		.564		211 950		.0144	.00874	.00717	.00640	.00590	.00552	.00507	.00451	.00407	.00381	.00349	.00330	.00318	.00306	.00296	.00290	.00284	.00276	.00268	.00262	.00258	.00252	.00250	.00246	.00244
		.501		167 800		.0129	.00822	.00682	.00612	.00565	.00533	.00488	.00437	.00395	.00371	.00340	.00322	.00310	.00300	.00292	.00284	.00280	.00270	.00264	.00258	.00252	.00248	.00246	.00242	.00238
		.447		133 200	.0348	.0118	.00778	.00654	.00588	.00545	.00521	.00475	.00425	.00385	.00362	.00334	.00316	.00302	.00294	.00286	.00280	.00274	.00266	.00260	.00252	.00248	.00246	.00242	.00238	.00226
		.398		105 530	.0240	.0108	.00738	.00625	.00565	.00525	.00497	.00458	.00413	.00376	.00354	.00326	.00310	.00296	.00288	.00280	.00274	.00268	.00260	.00254	.00248	.00244	.00242	.00238	.00234	.00222
		.355		83 640	.0193	.00997	.00702	.00600	.00543	.00507	.00480	.00445	.00402	.00366	.00342	.00320	.00304	.00290	.00282	.00276	.00270	.00264	.00256	.00250	.00246	.00242	.00236	.00234	.00230	.00228
		.316		66 370	.0165	.00933	.00668	.00577	.00523	.00490	.00465	.00431	.00394	.00354	.00338	.00312	.00296	.00286	.00276	.00270	.00264	.00260	.00252	.00246	.00242	.00236	.00232	.00220	.00226	.00224
		.281		52 630	.0143	.00872	.00638	.00555	.00505	.00474	.00451	.00420	.00382	.00350	.00330	.00306	.00290	.00280	.00272	.00264	.00260	.00254	.00248	.00240	.00236	.00232	.00220	.00226	.00224	.00222
		.250		41 740	.0129	.00822	.00612	.00533	.00488	.00460	.00437	.00407	.00368	.00338	.00322	.00300	.00284	.00274	.00266	.00260	.00254	.00250	.00244	.00238	.00232	.00228	.00226	.00222	.00220	.00218

xFor three-phase regular flat spacing use $D = 1.26\ A$. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$ For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

TABLE IX—25 CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL PER MILE OF SINGLE BARE CONDUCTOR

MICROMHOS PER MILE OF EACH CONDUCTOR OF A SINGLE-PHASE OR OF A SYMMETRICAL THREE-PHASE LINE—THE SUSCEPTANCE VALUES WERE DERIVED FROM THE EQUATION $b = 2\pi F C$ THE CHARGING CURRENT IN AMPERES PER MILE OF SINGLE CONDUCTOR TO NEUTRAL — THE (SUSCEPTANCE FROM TABLE) X (VOLTS TO NEUTRAL) X 10^{-6}. THE SUSCEPTANCE BETWEEN CONDUCTORS EQUALS ONE HALF THE TABLE VALUES

Material	Type	Diameter in Inches	B & S No	Area in Circular Mils	½"	1"	2"	3"	4"	5"	6"	8"	12"	18"	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet	11 Feet	13 Feet	15 Feet	17 Feet	19 Feet	21 Feet	23 Feet	26 Feet
COPPER	STRANDED	1.631		2 000 000			213	115	920	790	7.10	6.18	523	4.53	415	3.70	346	328	313	303	2.94	286	275	267	260	2.54	2.49	2.44	241	2.37
		1.548		1 800 000			199	110	8.78	763	692	6.03	5.13	446	408	3.56	341	3.25	308	300	292	283	274	2.65	257	2.52	247	2.44	2.39	236
		1.504		1 700 000			177	107	8.62	750	681	595	5.08	443	406	364	338	3.22	306	298	288	282	272	264	257	250	246	2.42	239	236
		1.459		1 600 000			168	10.5	845	7.38	6.71	585	502	4.40	4.04	360	3.36	320	3.06	296	286	280	271	262	255	249	2.46	2.41	237	2.36
		1.412		1 500 000			15.9	103	825	723	6.60	5.80	4.98	435	398	356	3.35	318	304	293	285	278	269	260	254	249	2.44	2.39	2.36	232
		1.369		1 400 000			151	986	8.05	710	648	5.73	489	429	3.94	353	332	315	300	291	282	276	267	259	252	2.45	2.41	237	234	230
		1.263		1 200 000			136	928	772	683	626	554	478	4.20	385	347	326	310	295	287	280	2.73	262	2.56	249	244	2.49	236	232	227
		1.209		1 100 000			129	9.03	7.50	6.70	613	545	4.74	415	382	3.44	323	305	293	285	278	270	261	254	247	241	2.47	232	220	2.27
		1.152		1 000 000			12.2	870	732	655	601	535	462	408	377	339	317	303	290	282	275	268	259	250	245	239	234	232	227	225
		1.123		950 000			119	862	723	645	594	529	457	405	373	337	3.16	3.00	283	280	273	266	257	250	244	239	234	2.31	2.27	2.24
		1.093		900 000			11.6	845	712	638	588	524	454	400	371	334	315	298	286	278	2.72	265	255	247	242	2.37	232	229	2.26	222
		1.062		850 000			113	828	700	630	580	517	452	397	368	333	313	296	285	276	270	263	254	246	241	236	232	229	224	222
		1.021		800 000			110	810	690	6.22	575	513	444	395	364	331	310	293	283	275	268	262	254	246	241	236	231	227	224	2.21
		.998		750 000			104	794	680	612	567	508	4.41	391	363	328	3.08	293	283	273	267	262	252	246	239	2.34	2.31	226	222	2.20
		.964		700 000		512	104	780	669	603	557	499	439	388	360	325	306	292	280	272	265	258	250	242	237	232	227	224	221	2.19
		.929		650 000		358	995	763	657	593	550	4.94	433	383	356	3.22	303	288	278	270	263	258	249	242	236	231	227	2.24	221	2.18
		.893		600 000		248	970	745	643	583	540	488	4.27	380	352	320	300	287	276	268	262	255	247	241	2.34	229	226	222	2.19	2.16
		.853		550 000		244	935	7.27	630	572	533	479	420	376	348	316	297	283	273	267	259	253	246	237	232	227	224	221	217	216
		.815		500 000		211	903	710	618	5.60	523	4.72	414	371	3.44	314	295	281	272	263	257	252	246	235	231	227	222	219	216	2.12
		.772		450 000		18.6	8.70	690	6.03	548	5.13	4.64	4.09	3.65	340	309	292	278	268	260	253	248	241	234	229	224	221	2.16	2.14	2.11
		.728		400 000		167	837	670	585	5.37	5.04	454	4.01	360	334	306	288	275	265	259	252	247	239	232	226	222	219	216	212	2.09
		.681		350 000		150	807	648	570	523	490	445	3.94	353	3.30	3.02	284	272	262	255	2.50	243	236	229	225	2.19	2.16	2.14	2.09	2.07
		.630		300 000		136	770	625	550	507	477	434	3.85	347	3.24	296	280	267	258	252	245	240	2.32	227	221	2.17	212	211	2.07	204
		.575		250 000		122	732	601	5.35	493	463	423	377	339	317	290	275	264	254	247	2.42	237	229	222	218	214	211	208	204	203
		.528	0000	211 600		113	698	579	517	475	449	413	367	333	311	286	270	259	251	243	239	233	226	221	215	212	2.07	204	202	199
		.470	000	167 772	39.5	10.1	658	553	497	462	433	3.98	357	324	303	279	264	253	245	238	234	228	222	215	212	207	2.04	201	199	1.96
		.418	00	133 079	22.8	9.28	625	5.28	477	4.40	418	386	347	316	297	273	259	248	2.40	234	229	225	217	212	207	204	201	1.97	196	193
		.373	0	105 560	176	854	593	506	458	429	4.05	374	336	3.08	289	267	253	243	236	2.30	225	220	214	209	2.09	2.01	1.97	194	193	1.90
		.332	1	83 694	145	794	565	485	440	412	391	362	3.28	300	283	261	248	2.39	231	225	222	215	211	204	201	197	194	192	189	187
		.292	2	66 358	124	7.38	5.38	4.65	424	398	379	3.50	319	292	275	254	242	234	226	220	217	212	206	201	198	194	191	189	188	184
		.260	3	52 624	111	710	516	4.47	410	3.85	366	340	3.12	285	270	250	237	229	222	217	212	209	203	197	194	191	187	186	184	181
		.232	4	41 738	100	655	494	432	396	3.74	357	332	3.03	278	264	2.45	232	225	218	212	209	205	199	194	192	188	186	183	181	1.79
		.206	5	33 088	92	618	4.74	418	384	364	346	323	295	272	259	239	229	220	215	209	205	202	196	193	188	186	182	181	178	176
	SOLID	.460	0000	211 600	341	995	653	548	492	4.57	430	395	3.55	322	302	277	263	252	244	238	232	228	221	216	2.10	206	2.09	201	198	195
		.410	000	167 772	20.6	9.11	619	524	473	4.42	415	383	3.45	313	294	271	257	247	239	233	228	224	217	212	207	203	201	197	194	1.92
		.365	00	133 079	168	845	589	502	456	423	402	372	3.35	305	287	265	252	242	235	228	224	220	214	207	203	199	196	194	191	189
		.325	0	105 560	141	7.84	560	482	438	408	389	360	326	298	281	260	247	237	230	223	220	215	209	204	200	196	194	191	189	186
		.289	1	83 694	123	7.32	537	463	423	3.95	3.77	350	318	291	2.74	254	241	233	226	220	216	212	206	201	197	193	191	187	186	184
		.258	2	66 358	109	691	514	446	408	383	365	340	309	284	268	249	237	228	222	217	212	208	203	197	194	191	188	184	183	181
		.229	3	52 624	99	6.52	490	431	393	3.72	3.54	332	302	279	263	243	232	224	217	2.12	208	205	199	194	191	187	184	182	181	177
		.204	4	41 738	91	618	4.73	4.15	384	361	344	322	294	271	257	239	228	220	214	209	205	202	196	192	187	184	182	179	178	176
		.182	5	33 088	86	587	454	403	372	3.51	335	314	288	2.65	252	235	224	216	210	205	202	198	193	189	184	182	179	176	176	173
ALUMINUM	STEEL REINFORCED	.953		605 000		44.5	103	7.72	7.50	6.00	560	498	4.34	386	3.59	3.24	305	290	278	2.70	265	258	251	244	237	232	227	224	221	2.18
		.904		500 000		30.7	9.78	7.55	6.47	5.84	5.44	488	4.20	3.81	3.54	320	3.02	287	276	268	262	2.56	248	241	2.34	231	226	222	2.19	216
		.741		336 420		17.2	845	677	592	5.40	505	4.59	4.05	3.61	336	3.06	288	275	265	258	252	247	239	232	227	2.24	219	216	212	211
		.635		266 800		13.7	775	628	559	510	480	425	385	3.48	324	297	278	267	259	252	245	240	232	227	221	2.17	214	211	208	2.06
		.564		211 950		11.9	724	594	5.30	487	457	420	373	3.37	316	289	273	263	253	245	240	235	229	222	217	2.14	209	207	204	2.02
		.501		167 800		10.7	681	565	508	468	442	405	362	3.27	308	282	267	257	248	242	235	232	224	218	214	209	206	204	201	197
		.447		123 200	288	978	645	5.42	488	452	431	394	3.52	3.19	300	2.77	262	2.50	243	237	232	227	221	216	209	206	204	201	1.98	1.96
		.398		105 530	198	895	612	518	468	435	412	380	3.42	3.12	294	2.70	257	245	238	232	227	222	216	211	206	203	201	197	194	1.92
		.355		83 640	160	875	582	4.98	450	4.20	398	369	333	3.03	288	265	252	240	233	228	224	218	213	208	2.04	201	196	194	191	189
		.316		66 370	137	773	553	478	4.34	407	386	357	3.26	293	280	2.59	2.45	237	228	224	218	216	209	204	201	196	192	191	187	186
		.281		52 630	118	722	529	460	418	393	373	348	3.17	290	274	2.54	2.40	232	225	218	215	210	206	199	196	192	191	1.88	1.84	184
		.250		41 740	107	681	507	442	406	381	362	338	305	280	2.67	249	235	227	220	215	210	207	203	197	192	1.89	188	184	183	181

x For three-phase regular flat spacing use D = 1.26 A. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$. For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

TABLE X—60 CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL PER MILE OF SINGLE BARE CONDUCTOR

MICROMHOS PER MILE OF EACH CONDUCTOR OF A SINGLE-PHASE OR OF A SYMMETRICAL THREE-PHASE LINE—THE SUSCEPTANCE VALUES WERE DERIVED FROM THE EQUATION $b = 2\pi FC$. THE **CHARGING CURRENT** IN AMPERES **PER MILE** OF SINGLE CONDUCTOR TO NEUTRAL = THE (SUSCEPTANCE FROM TABLE) X (VOLTS TO NEUTRAL) X 10^{-6} THE SUSCEPTANCE BETWEEN CONDUCTORS EQUALS ONE HALF THE TABLE VALUES.

Conductors: Material	Type	Diameter in Inches	B & S No.	Area in Circular Mils	DISTANCE (D) BETWEEN CENTERS OF CONDUCTORS x: 1/2"	1"	2"	3"	4"	5"	6"	8"	12"	18"	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet	11 Feet	13 Feet	15 Feet	17 Feet	19 Feet	21 Feet	23 Feet	25 Feet
COPPER	STRANDED	1.631		2 000 000			51.2	27.7	22.1	19.0	17.1	14.8	12.5	10.9	9.95	8.90	8.33	7.88	7.53	7.29	7.08	6.89	6.60	6.40	.27	6.08	5.98	5.85	5.77	5.69
		1.598		1 800 000			45.3	26.3	21.1	18.3	16.6	14.5	12.3	10.7	9.82	8.80	8.20	7.80	7.40	7.20	7.00	6.80	6.57	6.38	6.17	6.05	5.93	5.85	5.73	5.65
		1.504		1 700 000			42.7	25.7	20.7	18.0	16.4	14.3	12.2	10.6	9.75	8.72	8.12	7.73	7.37	7.17	6.93	6.77	6.53	6.33	6.17	6.00	5.88	5.81	5.73	5.65
		1.459		1 600 000			40.4	25.1	20.3	17.7	16.2	14.1	12.1	10.6	9.72	8.67	8.08	7.68	7.37	7.13	6.90	6.73	6.48	6.30	6.13	5.97	5.88	5.78	5.68	5.65
		1.412		1 500 000			38.3	24.5	19.8	17.4	15.8	13.9	11.9	10.5	9.55	8.56	8.05	7.65	7.33	7.05	6.85	6.69	6.45	6.25	6.08	5.97	5.85	5.73	5.65	5.57
		1.364		1 400 000			36.2	23.7	19.3	17.1	15.6	13.8	11.7	10.3	9.45	8.50	7.97	7.57	7.20	7.00	6.81	6.65	6.42	6.20	6.05	5.88	5.78	5.70	5.62	5.53
		1.263		1 200 000			32.7	22.3	18.6	16.4	15.1	13.3	11.5	10.1	9.27	8.33	7.84	7.45	7.10	6.90	6.73	6.51	6.28	6.13	5.97	5.85	5.73	5.65	5.57	5.45
		1.209		1 100 000			30.9	21.7	18.1	16.1	14.7	13.1	11.4	9.95	9.15	8.26	7.76	7.33	7.05	6.85	6.70	6.50	6.25	6.09	5.93	5.77	5.68	5.57	5.53	5.44
		1.152		1 000 000			29.3	20.9	17.6	15.8	14.4	12.9	11.1	9.80	9.05	8.14	7.62	7.28	6.97	6.78	6.60	6.45	6.22	6.01	5.88	5.73	5.62	5.57	5.45	5.39
		1.123		950 000			28.5	20.7	17.4	15.5	14.3	12.7	11.0	9.72	8.96	8.10	7.60	7.20	6.93	6.73	6.57	6.42	6.17	6.01	5.85	5.73	5.62	5.53	5.45	5.37
		1.093		900 000			27.8	20.3	17.1	15.3	14.1	12.6	10.9	9.62	8.92	8.05	7.57	7.17	6.89	6.69	6.53	6.38	6.13	5.93	5.81	5.69	5.58	5.49	5.42	5.33
		1.062		850 000			27.1	19.9	16.8	15.1	13.9	12.4	10.8	9.55	8.85	8.00	7.52	7.13	6.85	6.65	6.50	6.33	6.09	5.93	5.77	5.65	5.58	5.49	5.38	5.33
		1.031		800 000			26.3	19.5	16.6	14.9	13.8	12.3	10.7	9.50	8.75	7.94	7.45	7.05	6.80	6.62	6.45	6.29	6.09	5.89	5.77	5.65	5.53	5.45	5.38	5.29
		.998		750 000			25.1	19.1	16.3	14.7	13.6	12.2	10.6	9.39	8.71	7.88	7.39	7.05	6.79	6.57	6.41	6.29	6.05	5.89	5.73	5.62	5.53	5.42	5.33	5.28
		.964		700 000		230	24.9	18.8	16.1	14.5	13.4	12.0	10.5	9.33	8.65	7.83	7.36	7.00	6.73	6.53	6.38	6.22	6.00	5.81	5.68	5.57	5.45	5.38	5.30	5.25
		.929		650 000		86.0	23.9	18.3	15.8	14.3	13.2	11.9	10.4	9.20	8.57	7.74	7.29	6.93	6.69	6.50	6.33	6.22	5.98	5.81	5.65	5.53	5.45	5.38	5.30	5.22
		.893		600 000		59.8	23.3	17.9	15.5	14.0	13.0	11.7	10.3	9.12	8.45	7.68	7.20	6.88	6.65	6.45	6.29	6.13	5.94	5.77	5.61	5.49	5.42	5.33	5.25	5.17
		.855		550 000		58.7	22.5	17.5	15.2	13.8	12.8	11.5	10.1	9.04	8.37	7.58	7.13	6.80	6.57	6.40	6.21	6.09	5.90	5.69	5.57	5.45	5.38	5.30	5.22	5.17
		.815		500 000		50.7	21.7	17.0	14.8	13.4	13.6	11.4	9.95	8.90	8.26	7.54	7.09	6.75	6.52	6.33	6.18	6.05	5.92	5.65	5.54	5.45	5.33	5.26	5.18	5.09
		.772		450 000		44.6	20.7	16.6	14.5	13.2	12.3	11.2	9.82	8.75	8.17	7.43	7.00	6.68	6.45	6.25	6.09	5.97	5.78	5.61	5.50	5.37	5.30	5.17	5.13	5.05
		.728		400 000		40.2	20.1	16.1	14.1	12.9	12.1	10.9	9.62	8.65	8.03	7.35	6.93	6.60	6.37	6.22	6.05	5.93	5.73	5.57	5.42	5.33	5.25	5.17	5.08	5.02
		.681		350 000		36.0	19.3	15.6	13.7	12.6	11.8	10.7	9.46	8.48	7.92	7.24	6.82	6.52	6.30	6.13	5.99	5.85	5.66	5.48	5.39	5.25	5.18	5.13	5.02	4.98
		.630		300 000		32.6	18.5	15.0	13.2	12.2	11.5	10.4	9.25	8.35	7.79	7.10	6.73	6.42	6.21	6.05	5.90	5.78	5.58	5.42	5.28	5.22	5.10	5.05	4.97	4.90
		.575		250 000		29.3	17.6	14.4	13.8	11.9	11.1	10.2	9.05	8.14	7.62	6.97	6.60	6.33	6.11	5.93	5.81	5.70	5.50	5.33	5.24	5.13	5.05	4.98	4.90	4.86
		.528	0000	211 600		27.0	16.8	13.9	12.4	11.4	10.8	9.89	8.82	7.99	7.46	6.86	6.48	6.22	6.03	5.85	5.73	5.62	5.42	5.28	5.16	5.10	4.97	4.90	4.85	4.79
		.470	000	167 772	95.0	24.3	15.8	13.3	11.9	11.1	10.4	9.55	8.56	7.77	7.28	6.71	6.33	6.07	5.88	5.73	5.62	5.50	5.34	5.18	5.09	4.97	4.90	4.82	4.78	4.71
		.418	00	133 079	54.8	22.3	15.1	12.7	11.5	10.6	10.0	9.27	8.33	7.58	7.13	6.56	6.22	5.96	5.77	5.63	5.50	5.41	5.22	5.08	4.98	4.90	4.82	4.74	4.70	4.64
		.373	0	105 560	42.4	20.5	14.3	12.1	11.0	10.3	9.73	8.97	8.07	7.39	6.94	6.40	6.07	5.84	5.66	5.53	5.39	5.30	5.14	5.02	4.90	4.82	4.73	4.66	4.62	4.56
		.332	1	83 694	34.9	19.1	13.6	11.6	10.6	9.90	9.39	8.73	7.88	7.20	6.79	6.26	5.96	5.73	5.54	5.42	5.32	5.18	5.06	4.89	4.83	4.74	4.65	4.62	4.53	4.49
		.292	2	66 358	29.9	17.7	12.9	11.2	10.2	9.55	9.09	8.40	7.65	7.01	6.60	6.11	5.81	5.62	5.43	5.30	5.20	5.10	4.94	4.82	4.75	4.65	4.57	4.54	4.50	4.41
		.260	3	52 624	26.5	17.1	12.4	10.7	9.85	9.25	8.80	8.17	7.48	6.86	6.50	6.02	5.70	5.50	5.33	5.22	5.10	5.02	4.86	4.74	4.66	4.57	4.50	4.47	4.42	4.34
		.232	4	41 738	24.1	15.7	11.9	10.4	9.50	8.97	8.56	7.96	7.28	6.67	6.33	5.88	5.58	5.39	5.24	5.10	5.01	4.94	4.78	4.66	4.60	4.50	4.45	4.38	4.34	4.30
		.206	5	33 088	22.1	14.9	11.4	10.0	9.22	8.73	8.30	7.77	7.10	6.53	6.22	5.73	5.50	5.30	5.18	5.02	4.94	4.86	4.70	4.62	4.50	4.46	4.38	4.34	4.26	4.22
	SOLID	.460	0000	211 600	82.0	23.9	15.7	13.1	11.8	11.0	10.33	9.50	8.52	7.73	7.25	6.67	6.31	6.06	5.87	5.73	5.59	5.50	5.30	5.18	5.06	4.94	4.90	4.82	4.74	4.70
		.410	000	167 772	52.6	21.9	14.9	12.6	11.4	10.6	9.98	9.20	8.28	7.53	7.08	6.52	6.18	5.93	5.75	5.62	5.48	5.38	5.22	5.09	4.97	4.86	4.82	4.73	4.66	4.63
		.365	00	133 079	40.4	20.3	14.2	13.0	10.9	10.2	9.65	8.92	8.05	7.34	6.91	6.38	6.05	5.82	5.64	5.50	5.38	5.30	5.13	4.98	4.89	4.78	4.70	4.66	4.58	4.55
		.325	0	105 560	33.8	18.8	13.4	11.6	10.5	9.83	9.34	8.65	7.83	7.16	6.75	6.24	5.93	5.70	5.53	5.38	5.28	5.18	5.02	4.90	4.81	4.70	4.65	4.58	4.54	4.48
		.289	1	83 694	29.5	17.6	12.9	11.1	10.2	9.50	9.05	8.40	7.63	6.99	6.59	6.11	5.81	5.59	5.43	5.30	5.19	5.10	4.93	4.82	4.73	4.62	4.57	4.50	4.46	4.42
		.258	2	66 358	26.3	16.6	12.4	10.7	9.83	9.20	8.78	8.17	7.43	6.82	6.45	5.98	5.69	5.49	5.33	5.22	5.10	5.02	4.87	4.74	4.66	4.58	4.50	4.42	4.38	4.35
		.229	3	52 624	23.9	15.7	11.8	10.4	9.50	8.92	8.50	7.97	7.25	6.70	6.33	5.85	5.57	5.38	5.22	5.10	5.02	4.93	4.78	4.66	4.58	4.50	4.42	4.38	4.34	4.26
		.204	4	41 738	21.9	14.8	11.4	9.97	9.20	8.67	8.28	7.75	7.07	6.52	6.18	5.75	5.48	5.29	5.14	5.02	4.92	4.86	4.70	4.62	4.51	4.42	4.38	4.30	4.26	4.22
		.182	5	33 088	20.4	14.1	10.9	9.65	8.92	8.43	8.03	7.55	6.93	6.38	6.06	5.65	5.38	5.18	5.06	4.93	4.86	4.78	4.62	4.54	4.42	4.38	4.30	4.22	4.22	4.14
ALUMINUM	STEEL REINFORCED	.953		605 000		107.0	24.7	18.6	17.9	14.4	13.5	12.0	10.4	9.28	8.62	7.78	7.33	6.97	6.69	6.50	6.37	6.22	6.01	5.85	5.68	5.58	5.45	5.37	5.30	5.21
		.904		500 000		74.0	23.5	18.2	15.6	14.0	13.1	11.7	10.3	9.16	8.50	7.68	7.24	6.90	6.65	6.45	6.29	6.17	5.95	5.77	5.62	5.53	5.42	5.33	5.25	5.17
		.741		336 420		41.2	20.3	16.3	14.2	13.0	12.2	11.0	9.72	8.65	8.08	7.37	6.93	6.61	6.38	6.22	6.05	5.93	5.73	5.57	5.45	5.37	5.25	5.17	5.08	5.05
		.635		266 800		32.9	18.6	15.1	13.3	12.3	11.6	10.5	9.25	8.35	7.79	7.15	6.69	6.40	6.22	6.05	5.90	5.78	5.58	5.45	5.30	5.22	5.13	5.05	4.97	4.93
		.564		211 950		28.7	17.4	14.3	12.8	11.8	11.0	10.1	8.96	8.10	7.58	6.95	6.58	6.33	6.08	5.90	5.78	5.65	5.50	5.34	5.21	5.13	5.02	4.97	4.90	4.86
		.501		167 800		25.7	16.4	13.6	12.2	11.3	10.6	9.72	8.70	7.86	7.38	6.77	6.41	6.18	5.97	5.82	5.65	5.58	5.38	5.25	5.13	5.02	4.94	4.90	4.82	4.74
		.447		133 200	69.3	23.5	15.5	13.0	11.7	10.9	10.4	9.45	8.45	7.66	7.20	6.65	6.29	6.02	5.85	5.70	5.57	5.45	5.30	5.17	5.02	4.94	4.89	4.82	4.74	4.70
		.398		105 530	47.8	21.5	14.7	12.5	11.3	10.4	9.90	9.12	8.23	7.49	7.04	6.50	6.17	5.90	5.74	5.58	5.45	5.34	5.18	5.05	4.93	4.85	4.82	4.74	4.65	4.62
		.355		83 640	38.4	19.8	14.0	11.9	10.8	10.1	9.57	8.85	8.00	7.29	6.82	6.38	6.05	5.77	5.62	5.50	5.38	5.25	5.10	4.97	4.90	4.82	4.70	4.66	4.58	4.53
		.316		66 370	32.9	18.6	13.3	11.5	10.4	9.75	9.27	8.58	7.83	7.05	6.73	6.22	5.90	5.70	5.50	5.38	5.25	5.18	5.02	4.90	4.82	4.70	4.62	4.58	4.50	4.46
		.281		52 630	28.5	17.4	12.7	11.1	10.1	9.45	8.98	8.37	7.60	6.97	6.57	6.10	5.77	5.57	5.42	5.25	5.18	5.06	4.94	4.78	4.70	4.62	4.58	4.50	4.46	4.42
		.250		41 740	25.7	16.4	12.2	10.6	9.72	9.15	8.70	8.10	7.32	6.73	6.41	5.97	5.66	5.45	5.29	5.18	5.06	4.98	4.86	4.73	4.62	4.53	4.50	4.42	4.38	4.33

x For three-phase regular flat spacing use $D = 1.26\ A$. For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$ For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

TABLE XI—CHARGING K.V.A. IN THREE-PHASE CIRCUITS PER MILE OF THREE-BARE CONDUCTORS

CHARGING CURRENT PER MILE (EXPRESSED IN KVA 3 PHASE) FOR A SYMETRICAL 3 PHASE CIRCUIT AT THE AVERAGE VOLTAGES AND SPACINGS OF CONDUCTORS STATED. FOR OTHER ARRANGEMENTS OF CONDUCTORS SEE FOOT NOTES. x• FOR OTHER SPACINGS THESE VALUES WILL VARY DIRECTLY AS THE SUSCEPTANCE VALUES OF THE SPACINGS COMPARED. THE CHARGING K.V.A. 3 PHASE = (CHARGING CURRENT IN AMPERES TO NEUTRAL) X (VOLTS TO NEUTRAL) X 3.

IN DETERMINING THE CHARGING K.V.A. FOR THE ENTIRE HIGH TENSION SYSTEM THE LENGTH OF ALL BRANCH CIRCUITS MUST BE INCLUDED AS WELL AS THAT OF THE MAIN CIRCUITS.

25 CYCLES

MATERIAL	TYPE	DIAMETER IN INCHES	B & S NO.	AREA IN CIRCULAR MILS	20 KV 4 FT.	30 KV 4 FT.	40 KV 5 FT.	50 KV 6 FT.	60 KV 6 FT.	70 KV 7 FT.	80 KV 8 FT.	100 KV 9 FT.	110 KV 10 FT.	120 KV 11 FT.	140 KV 13 FT.	160 KV 17 FT.	200 KV 21 FT.
COPPER	STRANDED	1631		2 000 000	1.38	311	5.25	782	11.3	148	16.8	286	34.0	396	52.3	65.0	97.6
		1548		1 800 000	1.36	307	5.20	7.70	11.1	14.7	18.7	28.3	33.7	394	51.9	64.5	97.6
		1504		1 700 000	1.35	3.04	515	765	11.0	146	18.4	28.2	33.5	392	51.7	640	96.8
		1458		1 600 000	1.34	302	512	7.65	11.0	145	18.3	28.0	33.4	39.0	51.3	63.8	96.4
		1412		1 500 000	134	3.01	508	760	10.9	143	18.3	27.8	33.2	387	51.0	638	95.5
		1364		1 400 000	133	2.98	503	750	10.8	14.2	180	276	329	384	507	62.7	94.8
		1263		1 200 000	130	293	4.95	7.37	10.6	141	17.9	27.3	32.5	37.7	50.2	62.5	943
		1209		1 100 000	129	291	4.88	733	10.6	14.0	17.8	270	32.2	376	498	61.7	92.8
		1152		1 000 000	127	285	485	725	10.4	138	17.6	268	31.9	373	490	612	92.8
		1123		950 000	126	284	4.80	720	10.4	137	17.5	26.6	31.7	37.0	49.0	61.2	925
		1093		900 000	126	284	476	7.15	10.3	13.6	17.4	26.5	31.6	36.7	485	60.7	91.5
		1062		850 000	125	282	473	7.12	10.3	13.5	17.3	26.3	31.3	366	482	60.5	915
		1031		800 000	124	279	468	707	102	13.5	17.1	26.2	31.2	366	48.2	60.5	90.7
		.998		750 000	123	277	468	707	10.2	13.4	17.1	26.2	31.2	363	48.2	598	90.3
		.964		700 000	1.22	275	466	700	10.1	13.3	17.0	25.8	30.7	360	47.5	594	895
		.929		650 000	1.21	273	460	695	10.0	13.2	16.8	258	30.2	35.8	49.5	59.1	895
		893		600 000	120	2.70	458	690	993	13.1	168	255	304	356	47.2	586	888
		855		550 000	119	267	452	682	982	13.1	165	253	30.1	354	465	581	884
		815		500 000	1.18	265	4.49	660	978	129	16.5	25.2	30.0	354	460	581	875
		772		450 000	1.17	263	445	670	965	12.7	16.2	24.8	29.5	34.7	458	57.3	86.3
		.728		400 000	115	258	440	662	954	127	161	24.7	29.4	34.4	45.5	568	863
		681		350 000	114	255	435	6.55	9.43	12.5	160	24.3	29.0	340	448	560	85.6
		630		300 000	112	252	427	6.45	9.28	12.3	15.7	24.0	28.6	334	445	555	84.5
		575		250 000	1.10	247	422	635	915	12.1	15.5	23.7	28.2	330	435	547	832
		528	0000	211 600	1.08	243	4.13	627	903	11.9	153	23.3	27.7	32.5	433	54.3	81.5
		.470	000	167 772	106	237	404	6.13	881	11.6	15.0	22.8	27.2	32.0	42.1	530	80.5
		418	00	133 079	104	233	396	600	864	11.5	147	225	26.8	31.2	41.5	522	788
		.373	0	105 560	102	228	388	590	8.50	113	14.4	22.0	26.2	308	41.0	51.4	776
		332	1	83 694	.99	223	3.82	5.78	8.32	11.0	142	215	25.6	30.4	40.0	504	768
		292	2	66 358	.97	218	374	565	813	108	13.9	212	25.3	297	394	496	75.7
		260	3	52 624	.95	213	366	555	8.00	10.6	136	20.9	249	29.2	386	489	745
		232	4	41 738	.93	209	360	5.45	784	10.4	13.4	20.5	24.4	287	38.0	481	732
		.206	5	33 088	.92	206	352	5.37	7.74	102	13.1	20.2	24.1	282	37.8	47.6	72.3
	SOLID	.460	0000	211 600	1.05	237	403	610	878	116	148	228	27.1	31.9	42.3	52.7	803
		410	000	167 772	1.03	231	3.95	5.98	8.60	114	14.6	224	26.7	31.2	415	520	788
		365	00	133 079	1.01	227	386	587	845	112	143	22.0	262	308	40.5	51.0	776
		.325	0	105 560	.99	222	379	575	828	109	141	21.5	25.6	301	400	50.2	763
		289	1	83 694	.96	217	372	5.65	813	108	138	212	253	297	394	49.3	748
		.258	2	66 358	.95	213	364	555	800	10.6	136	209	24.9	292	386	489	73.7
		229	3	52 624	.93	209	358	5.42	780	10.4	133	20.5	244	286	380	478	728
		204	4	41 738	.91	205	352	535	770	10.2	13.1	202	241	282	376	47.2	716
		182	5	33 088	90	202	346	525	7.55	10.0	129	198	236	278	370	465	70.4
ALUMINUM	STEEL REINFORCED	953		605 000	122	2.74	463	695	100	13.2	169	258	30.7	361	47.8	59.4	89.5
		904		500 000	121	272	458	690	993	131	168	256	30.5	357	472	590	888
		741		336 420	115	259	439	663	953	126	161	24.7	29.4	344	455	573	86.5
		635		266 800	111	2.50	4.27	648	932	123	157	240	28.6	334	44.5	55.5	84.5
		564		211 950	109	246	420	633	910	120	15.4	235	280	329	43.7	54.8	828
		501		167 800	107	240	410	620	893	118	150	232	276	322	429	53.5	81.6
		447		133 200	105	236	399	608	874	116	148	22.7	270	318	423	527	80.5
		398		105 530	103	231	392	595	857	114	145	222	265	311	41.3	520	788
		355		83 640	101	227	383	582	838	112	143	218	260	307	408	51.5	776
		316		66 370	98	220	378	570	820	11.0	140	216	25.7	30.1	400	50.2	765
		281		52 630	96	216	371	562	810	107	137	210	250	296	390	491	752
		250		41 740	94	212	363	550	792	105	134	207	246	292	386	484	737

60 CYCLES

MATERIAL	TYPE	DIAMETER IN INCHES	B & S NO.	AREA IN CIRCULAR MILS	20 KV 4 FT.	30 KV 4 FT.	40 KV 5 FT.	50 KV 6 FT.	60 KV 6 FT.	70 KV 7 FT.	80 KV 8 FT.	100 KV 9 FT.	110 KV 10 FT.	120 KV 11 FT.	140 KV 13 FT.	160 KV 17 FT.	200 KV 21 FT.
COPPER	STRANDED	1631		2 000 000	3.33	7.50	12.6	188	271	35.6	45.3	68.9	81.5	95.0	125	156	234
		1548		1 800 000	3.28	7.39	12.5	185	266	35.2	44.7	68.0	80.5	94.5	125	155	234
		1504		1 700 000	3.25	730	12.4	184	26.5	35.1	44.3	67.7	80.2	94.0	124	154	232
		1458		1 600 000	323	7.28	12.3	184	265	34.9	441	67.3	80.0	93.3	123	153	231
		1412		1 500 000	3.22	7.25	12.2	18.3	264	34.5	43.7	66.9	79.5	92.8	122	153	229
		1364		1 400 000	3.18	715	121	180	25.9	343	43.5	66.5	79.0	925	121	151	228
		1263		1 200 000	3.14	7.07	11.9	17.8	25.5	33.8	431	65.1	77.0	90.5	120	150	226
		1209		1 100 000	3.10	697	11.7	17.6	254	33.5	42.8	650	770	900	119	148	223
		1152		1 000 000	3.05	6.85	11.6	17.4	251	33.2	42.2	64.5	765	89.5	118	147	223
		1123		950 000	3.04	6.83	11.5	17.3	24.9	329	420	642	760	888	118	147	221
		1093		900 000	3.02	6.80	11.5	17.2	247	32.8	41.8	63.8	75.7	883	116	146	220
		1062		850 000	3.01	6.77	114	17.1	24.6	326	41.5	63.3	75.0	876	116	145	220
		1031		800 000	2.98	6.70	11.3	17.0	244	324	41.3	62.9	74.5	87.6	115	145	218
		.998		750 000	2.95	6.65	113	17.0	244	32.1	41.0	629	74.5	87.0	115	144	217
		.964		700 000	294	662	112	16.8	24.2	320	408	622	73.7	86.4	114	143	215
		.929		650 000	291	655	11.1	16.7	24.0	318	40.5	622	73.7	861	114	142	215
		893		600 000	2.88	648	11.0	166	239	31.6	402	61.3	72.8	855	113	141	213
		855		550 000	2.85	6.42	109	16.4	236	31.3	39.7	609	722	850	112	140	212
		815		500 000	2.84	6.38	10.8	16.3	23.4	31.0	396	60.5	71.7	850	111	140	210
		772		450 000	2.80	6.30	107	16.1	232	306	389	59.7	71.0	833	110	138	207
		.728		400 000	277	623	106	15.9	22.9	30.5	387	593	70.4	825	109	137	207
		681		350 000	2.72	613	104	15.7	22.6	300	383	58.5	69.4	81.5	107	135	205
		630		300 000	2.69	6.05	10.3	15.5	22.3	29.6	377	57.8	68.5	80.3	106	134	202
		575		250 000	264	5.94	10.1	15.3	220	290	37.1	37.0	67.5	792	105	132	199
		528	0000	211 600	259	5.83	993	15.1	21.7	286	366	56.2	66.7	780	104	131	196
		.470	000	167 772	253	5.70	9.70	14.7	21.2	281	359	55.0	65.2	770	102	127	193
		418	00	133 079	249	5.59	953	144	207	276	352	54.1	640	752	100	126	190
		.373	0	105 560	243	5.45	935	14.1	20.4	27.1	345	530	62.7	740	983	123	186
		332	1	83 694	2.38	5.36	918	139	199	265	340	518	61.4	728	958	121	185
		292	2	66 358	232	522	898	13.6	195	260	33.3	510	60.5	712	944	119	182
		260	3	52 624	228	5.13	8.80	13.3	192	255	32.6	50.2	59.6	699	930	117	179
		232	4	41 738	224	502	863	13.1	188	250	320	494	58.7	688	912	115	175
		.206	5	33 088	220	4.95	847	13.0	186	246	31.6	486	576	677	904	114	174
	SOLID	.460	0000	211 600	252	567	9.70	14.7	21.1	281	35.8	55.0	650	763	102	127	193
		410	000	167 772	247	566	9.48	144	207	27.5	35.1	53.8	638	751	995	124	189
		365	00	133 079	2.42	545	9.30	14.1	203	269	344	530	627	74.0	975	123	186
		.325	0	105 560	2.37	5.33	912	138	198	26.3	33.8	51.8	61.5	723	959	120	183
		289	1	83 694	232	522	895	136	196	260	332	51.0	60.5	710	943	118	180
		.258	2	66 358	227	5.12	8.78	13.3	192	256	326	50.2	595	700	929	117	177
		229	3	52 624	223	5.02	860	131	188	250	321	49.3	58.5	688	912	115	175
		204	4	41 738	2.19	492	846	12.9	18.5	246	31.4	486	57.7	677	905	113	172
		182	5	33 088	2.15	484	829	126	18.2	24.2	31.1	478	56.7	665	890	112	168
ALUMINUM	STEEL REINFORCED	953		605 000	293	659	11.2	167	24.1	31.8	40.7	622	73.7	865	115	143	215
		904		500 000	289	652	11.1	166	23.9	315	40.2	61.7	73.0	857	113	142	212
		741		336 420	2.77	6.23	106	159	22.9	30.4	387	59.3	703	825	109	138	207
		635		266 800	2.67	6.01	10.3	15.5	224	296	37.7	578	68.5	805	107	134	202
		564		211 950	2.63	5.91	102	152	21.8	289	370	56.5	670	792	105	132	198
		501		167 800	256	5.76	988	149	21.5	284	362	558	66.2	775	103	129	196
		447		133 200	2.51	5.65	963	14.6	210	279	356	545	64.5	762	101	127	193
		398		105 530	2.47	5.55	943	14.3	207	273	34.9	534	63.2	747	988	124	190
		355		83 640	2.42	544	922	141	202	269	344	525	62.2	735	973	123	186
		316		66 370	236	5.30	910	13.7	19.8	264	336	518	61.5	723	960	120	183
		281		52 630	2.31	5.18	890	135	195	257	331	506	600	712	935	118	180
		250		41 740	226	509	872	132	190	254	323	498	59.2	700	926	116	177

x For three-phase regular flat spacing use D = 1.26 A. • For three-phase irregular flat or triangular spacing use $D = \sqrt[3]{ABC}$

Microfarads per 1000 feet of single conductor to neutral.

or

$$C = \frac{0.03883}{\log_{10} \frac{D}{R}} \quad \ldots\ldots\ldots\ldots\ldots \quad (16)$$

Microfarads per mile of single conductor to neutral.

The above formulas are only applicable to ordinary overhead circuits when the distance from the conductor to other conductors, particularly the earth, is large compared to their distance apart. However, since the effect of the earth is usually small in most practical cases, the formulas give a very close approximation to the actual capacitance of overhead circuits.

The values of capacitance in Table VIII were derived by using formula (13). For calculating the capacitance for the stranded conductors, the actual overall diameter of the cable was taken. This introduces a small error which is negligible except for very close spacings not used in high tension transmission lines employing bare conductors.

CHARGING CURRENT

RELATION OF CHARGING CURRENTS OF SINGLE AND THREE-PHASE SYSTEMS

The diagrams (Fig. 11) may assist in forming a clear understanding of the relation of charging current

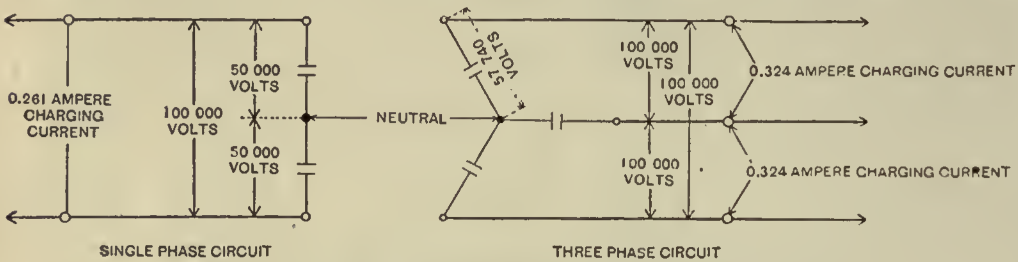


FIG. 11—CHARGING CURRENT IN SINGLE AND THREE-PHASE CIRCUITS

to susceptance for single and three-phase circuits. In the following consideration No. 0000 stranded copper conductors will be assumed as spaced nine feet between any two conductors, frequency 60 cycles, voltage 100 000 volts between conductors. Voltage to neutral will therefore be, for single phase circuit, 50 000 volts and for three-phase circuit 57 740 volts. Distance of transmission one mile. From Table VIII, a capacitance to neutral of 0.00282 microfarads per 1000 feet is obtained which is equivalent to 0.0149 microfarads per conductor to neutral for this one mile of circuit. The susceptance will therefore be as follows:—

Per conductor to neutral $2\pi f C_n = 5.62$ microhms
Between conductors $2\pi f C_{12} = 2.81$ microhms

For Single-Phase Circuit—To neutral $5.62 \times 50\,000 \times 10^6 = 0.281$ amperes or between conductors $2.81 \times 100\,000 \times 10^6 = 0.281$ amperes therefore charging k.v.a. is $0.281 \times 50\,000 \times 2 = 28.1$ k.v.a. single phase or $0.281 \times 100\,000 = 28.1$ k.v.a. single phase.

For a Three-Phase Circuit—To neutral $5.62 \times 57\,740 \times 10^6 = 0.324$ amperes. Therefore charging k.v.a. is $0.324 \times 57\,740 \times 3 = 56.2$ k.v.a. three-phase.

It will be seen from the above that the charging current per conductor in the three-phase symmetrical system is 15.5 percent greater than in the single-phase system, and the resulting charging k.v.a. is just double that of the single-phase system. The charge on any particular conductor is in phase with the voltage between that conductor and the neutral and the charging current for that conductor is 90 degrees ahead of the voltage drop from that conductor to neutral.

Grounding of the neutral point of a system has no effect upon the charging current when the system is in static balance. In determining the total charging current to be supplied by a given generating station, it should be remembered that in cases of duplicate transmission circuits, when both circuits are excited, the charging current will be approximately double what it would be if only one of the circuits were in use.

Tables IX and X contain values for capacitance susceptance to neutral in micromhos per mile of conductor. As indicated, the charging current in amperes per mile of single conductor to neutral = the (susceptance from table) $\times$ (volts to neutral) $\times$ 10^{-6}. Thus in a three-phase, 60 cycle, 100 000 volt, (57 740 volts to neutral), symmetrical circuit, the No. 0000 stranded conductors being arranged at the corners of an equilateral triangle spaced nine feet apart, the charging current per mile would be determined as follows:—

$5.62 \times 57\,740 \times 10^{-6} = 0.3245$ amperes to neutral

or $0.3245 \times 57\,740 = 18.737$ k.v.a. to neutral

$18.737 \times 3 = 56.2$ K.v.a. total three phase

Table XI is an extension of Tables IX and X from which values in k.v.a., three-phase for charging current have been calculated for certain assumed spacings and average voltages. In the case cited above it was found that the charging current would be 56.2 k.v.a., three-phase per mile. Table XI gives this value directly for the conditions specified.

CHARGING CURRENT AT ZERO LOAD

The term charging current of a transmission circuit refers to the amount of current which flows into the circuit at the supply end with normal voltage held at the receiver end at *zero load.* If the circuit is long, its capacitance will be high and therefore the voltage at the supply end may be considerably less than at the receiver end. For instance a 60 cycle circuit 300 miles long, having certain constants will, with 100 000 volts maintained at the receiver end, have a voltage of only 80 000 volts at the supply end at zero load. This same circuit will at full load and 100 000 volts maintained at the receiver end, require 120 000 volts at the supply end. It is evident therefore that, since the charging current varies with the voltage, if the circuit has much capacitance the voltage along the circuit, and particularly near the supply end, will vary to a large extent and consequently the charging current of the circuit will be different for different loads.

In case of the 300 mile circuit referred to above, the charging current at zero load will be very much less than it is at full load, because the average voltage at zero load is less than the average voltage at full load. At zero load the average voltage is less and at full load it is greater than the receiver end voltage.

It is customary to calculate the total charging current for the circuit by multiplying the total susceptance by the receiver end voltage. This would be correct if the voltage throughout the length of the circuit were held constant and of the same value as at the receiver end. This condition is approximately met within commercial lines and this method of determining the total charging current is therefore sufficiently accurate for most practical purposes.

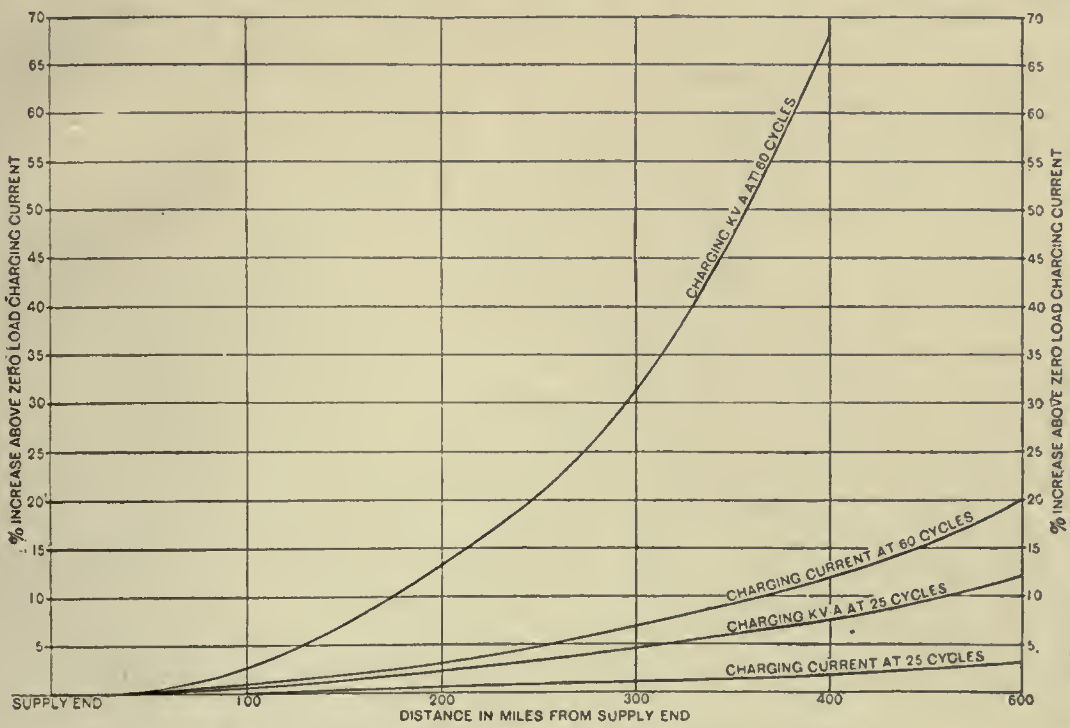


FIG. 12—CHARGING CURRENT AT ZERO LOAD FOR VARIOUS LENGTHS

At zero load the voltage (on account of the effect of capacitance) decreases as the supply end of the circuit is approached. The charging current at points along the circuit decreases directly as the voltage. If the charging current for zero load is estimated by the approximate method based upon the receiver voltage being maintained throughout the length of the circuit the result will be too high. The error will increase as the length of the circuit is increased; it will also increase rapidly as the frequency is raised. The error in the resulting K.V.A. required to charge the circuit will therefore increase very rapidly with an increase in distance or frequency. The curves below represent an approximation of this error.

For the purpose of making exact calculation of the total current at the supply end of long circuits, the charging current must be calculated by mathematical formulas which accurately take into account the change in voltage along the circuit at zero load. This will be taken up in a later article. It may be interesting to note approximately, however, how the charging current and charging k.v.a., as determined by the above method, varies from what it would be if calculated by the rigorous formula. The curves in Fig. 12 represent an approximation to the error when calculating the charging current at zero load by multiplying the total susceptance by the receiver voltage. For a circuit 300 miles long the error in charging current is only two percent for 25 cycles and seven percent for 60 cycle circuits. The error in charging k.v.a. is four percent for 25 cycle and 32 percent for 60 cycle circuits.

RELATION OF INDUCTANCE TO CAPACITANCE

As conductors are brought closer together, the inductance decreases and the capacitance increases. These values change with changes in spacings between conductors in such a manner that their product $L \times C$ is practically a constant for all spacings (except very close spacings such as used in low-voltage service and lead-covered cables) and for all sizes of conductors. If there were no losses encountered by the electric propagation in the conductors themselves the product of *L* and *C* would be a constant for all spacings and sizes of conductors.

In Table C is indicated the relation of the total inductance and capacitance, and their product, in two bare parallel conductors in air for a circuit one mile long. The values for *L* are in millihenries and for *C* in microfarads. Since the formulas by which *L* and *C* were calculated account for the flux within the conductors themselves, the product *LC* is not a constant, as will be seen by the tabulated values, although for the larger spacings such as used in high-tension transmission the product is nearly a constant.

TABLE C—PRODUCT OF (TOTAL) L AND (TOTAL) C

Solid Conductors		Spacing Inches	Inductance *L* Formula (4)	Capacitance *C* Formula (14)	Product *LC*
Size	Diam. Inches				
1 000 000	1.00	2	1.053	0.03395	0.03575
1 000 000	1.00	24	2.653	0.01155	0.03064
1 000 000	1.00	300	4.279	0.00695	0.02974
0000	0.46	2	1.553	0.02079	0.03228
0000	0.46	24	3.153	0.00961	0.03030
0000	0.46	300	4.779	0.00623	0.02977

RELATION OF INDUCTANCE AND CAPACITANCE TO LIGHT VELOCITY

The propagation of the electric and the magnetic fields in a dielectric, such as air, is the same as that of light. Along a transmission line it is retarded only slightly due to losses or the fact that the current is not confined to the surface of the conductors. If the inductance inside the conductors is negligible, then the velocity of the electric and the magnetic fields is the same as light, that is approximately 186 000 miles per second or approximately 3×10^{10} cm. per second. For high-tension transmission lines of large spacings, the inductance inside the conductor is relatively small, so that the speed of the electric field is practically that of light.

The following relation exists between inductance L in henries, capacity C in farads and velocity of light V per second:—

$$LC \text{ (in air)} = \frac{1}{V^2} \text{ or, } V = \frac{1}{\sqrt{LC}} \quad \ldots\ldots\ldots\ldots \text{ (17)}$$

Thus it will be seen that if either L or C is known, the other may be determined since the velocity of light V is known. If values for L and C are taken which include the inductance inside the conductors, particularly if the conductors are very close together, it would be necessary to assume a velocity of electric propagation somewhat less than that of light. If, on the other hand, the values for L and C external to the conductors are taken, then the above equation is rigidly correct.

In Table C, it was shown that for No. 0000 conductors, 300 inch spacing, the total values of L and C were for a single-phase line,—

$L = 0.004\,779$ henries per mile of circuit.
$C = 0.000\,000\,006\,23$ farads per mile of circuit.

therefore, $V = \dfrac{1}{\sqrt{0.004\,779 \times 0.000\,000\,006\,23}} =$
183 000 miles per second .... (18)
which is less than the speed of light.

If we take the inductance in the air space between the conductors, Formula (2); we arrive at the values,—

$L = 0.004\,617\,9$ henries per mile of circuit.
$C = 0.000\,000\,006\,23$ farads per mile of circuit.

therefore $V = \dfrac{1}{\sqrt{0.004\,617\,9 \times 0.000\,000\,006\,23}} =$
186 000 miles per second .... (19)
which is approximately the speed of light.

CHAPTER III
QUICK ESTIMATING TABLES

FOR every occasion where a complete calculation of a long distance transmission line is made, there are many where the size of wire needed to transmit a given amount of power economically is required quickly. This knowledge is, moreover, the basis for all transmission line calculations, as all methods of calculating regulation presuppose that the size of wire is known. To determine quickly and with the least possible calculation the approximate size of conductor corresponding to a given I^2R transmission loss for any ordinary voltage or distance, is the function of Tables XII to XXI inclusive. By including so many transmission voltages it is not intended to indicate that any of them might equally well be selected for a new installation. On the contrary it is very desirable in the consideration of a new installation, to eliminate consideration of some of the voltages now in use. This point will be considered later.

Since both the power-factor of the load, and the charging current of the circuit, as well as any change in the resistance of the conductors, will alter the I^2R loss, it is evident that it is impractical to present tables which will take into account the effect of all of these variables. The accompanying tables do, however, give the percentage I^2R loss corresponding to the two temperatures (25 and 65 degrees C) ordinarily encountered in practice and the usual load power-factors of unity and 80 percent lagging, upon which the k.v.a. values of the tables are based. The effect, however, of charging current, corona or leakage loss is not taken into account in these table values. The latter two (corona and leakage) are usually small and need not be considered here. The effect of charging current, may, however, with long circuits be material and will be discussed.

The values of k.v.a. in these tables are based upon the following percentage I^2R loss in transmission (neglecting the effect of charging current) :—

	Percent Loss At 25°C	Percent Loss At 65°C
Load at 100 percent P-F.	8.66	10.0
Load at 80 percent P-F.	10.8	12.5

These loss values are based upon the power delivered at the end of the circuit as 100 percent, and not upon the power at the supply end. If raising or lowering transformers are employed, the loss and voltage drop in them will, of course, be in addition to the above.

At first glance, some of these tables may appear to have been carried to extremes of k.v.a. values for the conductor sizes. This is because the tables are calculated for ten percent loss, (at 100 percent power-factor and 65 degrees C) whereas the permissible loss is frequently much less than ten percent. As the loss is directly proportional to the load, the permissible loads for a given size wire and distance can be read almost directly for any loss. Thus for a two percent loss the permissible k.v.a. will be two-tenths the table values. Conversely, the size of wire to carry a given k.v.a. load at two percent loss will be the same as will carry five (10÷2) times the k.v.a. at ten percent loss. In other words to find the size of wire to carry a given k.v.a. load at any desired percent loss, find the ratio of the desired I^2R loss to the I^2R loss upon which the table values are based (corresponding of course to the temperature and the load power-factor). Divide this ratio into the k.v.a. to be transmitted. The result will be the table k.v.a. value corresponding to the desired I^2R loss.

For example:—Assume 400 k.v.a. is to be delivered a distance of 14 miles at 6000 volts, three-phase, and 80 percent power-factor lagging, at an assumed temperature of 25 degrees C. Table XV indicates that this condition will be met with an I^2R loss of 10.8 percent if No. 0 copper or 167 800 circ. mil aluminum conductors are used.

Now assume that the I^2R loss should not exceed 5.4 percent, in place of 10.8 percent (upon which the table values are based). 5.4 ÷ 10.8 = 0.5 and 400 ÷ 0.5 = 800 k.v.a. as the table value corresponding to an I^2R loss of 5.4 percent. The conductors corresponding to 800 k.v.a. table value (5.4 percent I^2R loss) will be seen to be No. 0000 copper or 336 420 circ. mil aluminum.

If conductors corresponding to 15 percent I^2R loss are desired the same procedure will be followed:—15÷10.8 = 1.39 and 400÷1.39 = 287 k.v.a. table value. This table value corresponds to approximately No. 1 copper or 133 220 circ. mil aluminum conductors.

The table k.v.a. values have been tabulated for various distances. Should the actual distance be different from the table values and it is desired to obtain k.v.a. values corresponding to the losses upon which the table k.v.a. values have been calculated, the following procedure may be followed:—

For a given I^2R loss in a given conductor (effect of charging current neglected) the k.v.a. × feet or the k.v.a. × miles is a constant. Thus Table XII indicates that for 2 000 000 circ. mil cable, 756 000 k.v.a. × feet is the constant; that is 756 k.v.a. may be transmitted 1000 feet; 378 k.v.a., 2000 feet, and so on. If the actual distance to be transmitted is 1300 feet the corresponding k.v.a. value will be 756 000÷1300 or 581 k.v.a. Usually the k.v.a. value can readily be approximated for any distance with sufficient accuracy for the purpose for which these quick estimating tables are presented. One way of dong this would be as follows:— The k.v.a. value corresponding to 2500 ft. is 302 k.v.a.

TABLE XII—QUICK ESTIMATING TABLE

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.65% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

220 VOLTS DELIVERED

B & S NO.	CONDUCTORS: COPPER AREA IN CIRCULAR MILS	CONDUCTORS: ALUMINUM AREA IN CIRCULAR MILS	50 FEET	100 FEET	150 FEET	200 FEET	250 FEET	300 FEET	400 FEET	500 FEET	600 FEET	750 FEET	1000 FEET	1500 FEET	2000 FEET	2500 FEET	3500 FEET	5000 FEET	1 MILE
	2 000 000		15125	7562	5042	3781	3025	2521	1890	1512	1260	1008	756	504	378	302	216	151	143
	1 800 000		13730	6865	4577	3432	2746	2288	1716	1373	1144	915	686	458	343	274	196	137	130
	1 700 000		12821	6410	4274	3205	2564	2137	1602	1282	1068	855	641	427	320	256	183	128	121
	1 600 000		12100	6050	4033	3025	2420	2016	1512	1210	1008	806	605	403	302	242	173	121	114
	1 500 000		11321	5660	3774	2830	2264	1887	1415	1132	943	755	566	377	283	226	162	113	107
	1 400 000		10579	5289	3526	2644	2116	1763	1322	1058	881	705	529	353	264	211	151	106	100
	1 200 000		9047	4523	3015	2262	1809	1507	1131	905	753	603	452	301	226	181	129	90	85
	1 100 000		8345	4172	2782	2086	1669	1391	1043	834	695	556	417	278	209	167	119	83	78
	1 000 000	1 590 000	7562	3781	2521	1890	1512	1260	945	756	630	504	378	252	189	151	108	76	72
	950 000	1 515 000	7224	3612	2408	1806	1445	1204	903	722	602	482	361	241	181	144	103	72	68
	900 000	1 431 000	6817	3408	2272	1704	1363	1136	852	682	568	454	341	227	170	136	98	68	64
	850 000	1 351 500	6410	3205	2136	1602	1282	1068	801	641	534	427	320	214	160	128	91	64	61
	800 000	1 272 000	6050	3025	2017	1512	1210	1008	756	605	504	403	302	202	151	121	86	60	57
	750 000	1 192 500	5678	2839	1893	1420	1135	947	710	568	473	378	284	189	142	113	81	57	54
	700 000	1 113 000	5290	2645	1763	1322	1058	881	661	529	440	353	264	176	132	105	75	53	50
	650 000	1 033 500	4914	2457	1638	1228	983	819	614	491	409	328	246	164	123	98	70	49	45
	600 000	954 000	4523	2262	1507	1131	905	753	565	452	376	302	226	151	113	90	64	45	43
	550 000	874 500	4173	2086	1391	1043	834	695	522	417	347	278	209	139	104	83	59	42	39
	500 000	795 000	3781	1890	1260	945	756	630	472	378	315	252	189	126	94	75	54	38	36
	450 000	715 500	3396	1698	1132	849	679	566	425	340	283	226	170	113	85	68	49	34	32
	400 000	636 000	3034	1517	1011	758	607	505	379	303	252	202	152	101	76	60	43	30	29
	350 000	556 500	2645	1322	882	661	529	441	330	264	220	176	132	88	66	53	38	26	25
	300 000	477 000	2267	1133	755	567	453	378	283	227	189	157	113	75	57	45	32	23	21
	250 000	397 500	1898	949	633	474	379	316	237	190	158	126	95	63	47	38	27	19	18
0000	211 600	336 420	1600	800	533	400	320	266	200	160	133	107	80	53	40	32	23	16	15
000	167 772	266 800	1274	637	425	318	255	212	159	127	106	85	64	42	32	25	18	13	12
00	133 079	211 950	1008	504	336	252	202	168	126	101	84	67	50	34	25	20	14	10	9
0	105 560	167 800	800	400	266	200	160	133	100	80	66	53	40	27	20	16	11	8	8
1	83 694	133 220	632	316	211	158	126	106	79	63	53	42	32	21	16	12	9	6	6
2	66 358	105 530	501	250	167	125	100	83	62	50	41	33	25	17	12	10	7	5	5
3	52 624	83 640	396	198	132	99	79	66	49	40	33	26	20	13	10	8	6	4	4
4	41 738	66 370	316	158	105	79	63	52	39	32	26	21	16	10	8	6	5	3	3
5	33 088	52 630	251	125	84	62	50	42	31	25	21	16	12	8	6	5	4	2	2
6	26 244	41 740	198	99	66	50	40	33	25	20	16	13	10	6	5	4	3	2	2
7	20 822		157	78	52	39	31	26	20	16	13	10	8	5	4	3	2	1	1
8	16 512		125	62	42	31	25	21	15	12	10	8	6	4	3	2	2	1	1

440 VOLTS DELIVERED

B & S NO.	CONDUCTORS: COPPER AREA IN CIRCULAR MILS	CONDUCTORS: ALUMINUM AREA IN CIRCULAR MILS	50 FEET	100 FEET	150 FEET	200 FEET	250 FEET	300 FEET	400 FEET	500 FEET	600 FEET	750 FEET	1000 FEET	1500 FEET	2000 FEET	2500 FEET	3500 FEET	5000 FEET	1 MILE
	2 000 000		60500	30250	20166	15125	12100	10083	7562	6050	5042	4033	3025	2017	1512	1210	864	605	573
	1 800 000		54922	27461	18307	13730	10988	9153	6865	5492	4576	3648	2746	1831	1373	1094	784	549	520
	1 700 000		51285	25642	17095	12821	10257	8547	6410	5128	4273	3419	2564	1710	1282	1026	732	513	485
	1 600 000		48400	24200	16133	12100	9680	8066	6050	4840	4033	3226	2420	1613	1210	968	691	484	458
	1 500 000		45286	22643	15095	11322	9057	7547	5661	4528	3773	3019	2264	1510	1132	906	646	453	429
	1 400 000		42317	21158	14105	10579	8463	7052	5289	4231	3526	2821	2115	1410	1057	846	604	423	400
	1 200 000		36187	18093	12062	9047	7237	6031	4523	3618	3015	2412	1809	1206	904	724	517	362	343
	1 100 000		33379	16687	11126	8344	6676	5563	4172	3337	2782	2225	1668	1113	834	668	477	334	315
	1 000 000	1 590 000	30250	15125	10083	7562	6050	5042	3781	3025	2521	2016	1512	1008	756	605	432	302	287
	950 000	1 515 000	28896	14448	9632	7224	5779	4816	3612	2889	2408	1926	1444	963	722	578	412	289	273
	900 000	1 431 000	27268	13634	9089	6817	5453	4544	3408	2726	2272	1817	1363	909	682	545	389	273	258
	850 000	1 351 500	25642	12821	8547	6410	5128	4273	3205	2564	2136	1709	1282	855	641	513	366	256	243
	800 000	1 272 000	24200	12100	8066	6050	4840	4033	3025	2420	2016	1613	1210	807	605	484	346	242	226
	750 000	1 192 500	22710	11355	7570	5677	4542	3785	2838	2271	1892	1514	1135	757	567	454	324	227	215
	700 000	1 113 000	21158	10579	7053	5289	4231	3527	2644	2115	1763	1410	1057	705	528	423	302	211	201
	650 000	1 033 500	19655	9827	6552	4913	3931	3276	2456	1965	1638	1310	982	655	491	393	281	196	186
	600 000	954 000	18093	9046	6031	4523	3618	3015	2261	1809	1507	1206	904	603	452	362	258	181	171
	550 000	874 500	16690	8345	5563	4172	3338	2781	2086	1669	1390	1113	834	556	417	334	238	167	158
	500 000	795 000	15125	7562	5042	3781	3025	2521	1890	1512	1260	1008	756	504	378	302	216	151	143
	450 000	715 500	13586	6793	4529	3396	2717	2264	1698	1358	1132	905	679	453	340	272	194	139	129
	400 000	636 000	12138	6069	4046	3034	2427	2023	1517	1213	1012	809	607	405	303	243	173	121	115
	350 000	556 500	10579	5289	3526	2644	2115	1763	1322	1058	881	705	529	353	264	211	151	106	100
	300 000	477 000	9068	4534	3023	2267	1813	1512	1133	906	756	604	453	302	226	181	129	91	86
	250 000	397 500	7592	3796	2531	1898	1518	1265	949	759	632	506	379	253	189	152	108	76	72
0000	211 600	336 420	6400	3200	2133	1600	1280	1067	800	640	533	426	320	213	160	128	91	64	61
000	167 772	266 800	5095	2547	1698	1273	1019	849	636	509	425	339	254	170	127	102	73	51	48
00	133 079	211 950	4033	2016	1344	1008	806	672	504	403	336	268	202	134	101	81	58	40	38
0	105 560	167 800	3200	1600	1066	800	640	533	400	320	266	213	160	107	80	64	46	32	30
1	83 694	133 220	2530	1265	843	632	506	422	316	253	211	169	127	84	63	51	36	25	24
2	66 358	105 530	2006	1003	669	501	401	334	250	200	167	134	100	67	50	40	28	20	19
3	52 624	83 640	1586	793	529	396	317	264	198	158	132	105	79	53	40	32	22	16	15
4	41 738	66 370	1265	632	423	316	253	212	158	126	106	84	63	42	32	25	18	13	12
5	33 088	52 630	1003	502	334	253	201	167	125	100	83	67	50	33	25	20	14	10	9
6	26 244	41 740	791	395	264	197	158	132	98	79	66	53	39	26	20	16	11	8	8
7	20 822		629	314	210	157	126	105	78	63	52	41	32	21	16	15	9	6	6
8	16 512		500	250	166	125	100	83	62	50	42	33	25	17	12	10	7	5	5

The heating limitations may, for the shorter distances, particularly if insulated or concealed conductors are employed, necessitate the use of larger conductors, resulting in a correspondingly less transmission loss. In the case of insulated or concealed conductors, should the k.v.a. values fall near or to the left of the heavy line, consult Table XXV for insulated or Table XXIII for bare conductors. The reactance for the larger conductors may be excessive, particularly for 60-cycle service, producing excessive voltage drop. This may be obviated by installing two or more parallel circuits or using three-conductor cables. For single-phase circuits the k.v.a. will be one-half the table values.

Hence the value corresponding to half this distance (1250 ft.) is 604 k.v.a., which is sufficiently accurate for practical purposes.

REACTANCE LIMITATIONS

The k.v.a. value of the tables naturally do not take into account the reactance of the circuit. It will be

TABLE XIII—QUICK ESTIMATING TABLE

CONDUCTORS

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.66% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

550 VOLTS DELIVERED

B & S NO.	COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	50 FEET	100 FEET	150 FEET	200 FEET	250 FEET	300 FEET	400 FEET	500 FEET	600 FEET	750 FEET	1000 FEET	1500 FEET	2000 FEET	2500 FEET	3500 FEET	5000 FEET	1 MILE
	2 000 000		94531	47266	31510	23633	18906	15755	11816	9453	7877	6302	4727	3151	2363	1891	1350	945	896
	1 800 000		85815	42907	28605	21453	17163	14302	10727	8581	7151	5721	4291	2860	2145	1716	1226	858	812
	1 700 000		80132	40066	26711	20033	16026	13355	10016	8013	6677	5342	4007	2671	2003	1603	1144	801	758
	1 600 000		75625	37812	25208	18906	15125	12604	9453	7562	6302	5042	3781	2521	1891	1512	1081	756	716
	1 500 000		70760	35380	23587	17690	14152	11793	8845	7076	5896	4717	3538	2353	1769	1415	1011	708	670
	1 400 000		66120	33060	22040	16530	13224	11020	8265	6612	5510	4408	3306	2204	1653	1322	945	661	625
	1 200 000		56542	28271	18847	14135	11308	9423	7067	5654	4711	3769	2827	1885	1413	1131	808	565	535
	1 100 000		52155	26077	17385	13038	10431	8692	6519	5215	4346	3477	2608	1738	1304	1043	745	522	492
	1 000 000	1 590 000	47265	23632	15755	11816	9453	7877	5908	4727	3938	3151	2363	1576	1182	945	675	473	448
	950 000	1 515 000	45149	22574	15050	11287	9030	7525	5643	4515	3762	3010	2257	1505	1129	903	645	452	426
	900 000	1 431 000	42606	21303	14202	10652	8521	7101	5326	4260	3550	2840	2130	1420	1065	852	608	426	403
	850 000	1 351 500	40066	20033	13355	10016	8013	6677	5008	4007	3338	2671	2003	1336	1002	801	572	401	380
	800 000	1 272 000	37813	18907	12604	9453	7562	6302	4727	3781	3151	2521	1891	1260	945	756	540	378	358
	750 000	1 192 500	35484	17742	11828	8871	7097	5914	4435	3548	2957	2366	1774	1183	887	708	507	355	335
	700 000	1 113 000	33060	16530	11020	8265	6612	5510	4132	3306	2755	2204	1653	1102	827	661	472	331	313
	650 000	1 033 500	30710	15355	10237	7677	6142	5118	3838	3071	2559	2047	1535	1024	868	614	439	307	291
	600 000	954 000	28271	14135	9424	7067	5654	4712	3533	2827	2356	1884	1414	942	707	565	404	283	268
	550 000	874 500	26078	13039	8693	6520	5215	4346	3260	2607	2173	1738	1304	869	652	522	372	261	247
	500 000	795 000	23633	11816	7878	5908	4726	3939	2954	2363	1969	1575	1182	788	591	473	337	236	224
	450 000	715 500	21228	10614	7076	5307	4246	3538	2653	2123	1769	1415	1061	708	531	425	303	212	201
	400 000	636 000	18965	9482	6322	4741	3793	3161	2370	1896	1580	1264	948	632	474	379	271	190	180
	350 000	556 500	16530	8265	5510	4132	3306	2755	2066	1653	1377	1103	826	551	413	331	236	165	156
	300 000	477 000	14168	7084	4723	3542	2823	2361	1771	1416	1180	944	708	472	354	283	202	142	134
	250 000	397 500	11863	5932	3954	2966	2372	1977	1483	1186	988	791	593	395	297	237	170	119	112
0000	211 600	336 420	10000	5000	3333	2500	2000	1666	1250	1000	833	666	500	333	250	200	143	100	95
000	167 772	266 800	7960	3980	2653	1990	1592	1327	995	796	663	531	398	265	199	159	113	80	75
00	133 079	211 950	6302	3151	2101	1575	1260	1050	787	630	525	420	315	210	158	126	90	63	60
0	105 560	167 800	5000	2500	1666	1250	1000	833	625	500	416	333	250	166	125	100	71	50	47
1	83 694	133 220	3954	1977	1318	988	791	659	494	395	329	264	198	132	99	79	56	40	38
2	66 358	105 530	3134	1567	1045	783	627	522	392	313	261	209	157	105	78	63	45	31	30
3	52 624	83 640	2479	1239	826	620	496	413	310	248	207	165	124	83	62	50	35	25	24
4	41 738	66 370	1977	988	659	494	395	329	297	197	164	132	99	66	49	40	28	20	19
5	33 088	52 630	1567	783	522	392	313	261	196	156	130	104	78	52	39	31	22	16	15
6	26 244	41 740	1239	619	413	310	247	206	155	123	103	82	61	41	31	25	18	12	12
7	20 822		984	492	328	246	197	164	123	98	82	65	49	33	25	20	14	10	9
8	16 512		780	390	260	195	156	130	97	78	65	52	39	26	20	16	11	8	7

1100 VOLTS DELIVERED

B & S NO.	COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	100 FEET	200 FEET	300 FEET	500 FEET	750 FEET	1000 FEET	2500 FEET	4000 FEET	1 MILE	1¼ MILES	1½ MILES	2 MILES	2½ MILES	3 MILES	3½ MILES	4 MILES	5 MILES
	2 000 000		189062	94531	63021	37812	25208	18906	7562	4726	3580	2865	2388	1790	1432	1194	1023	895	716
	1 800 000		171631	85815	57210	34326	22884	17163	6865	4291	3250	2600	2166	1625	1300	1083	928	812	650
	1 700 000		160265	80132	53422	32053	21368	16026	6410	4006	3035	2428	2023	1517	1214	1012	867	758	607
	1 600 000		151250	75625	50416	30250	20166	15125	6050	3781	2865	2291	1910	1432	1145	955	818	716	572
	1 500 000		141520	70760	47173	28304	18868	14152	5661	3538	2680	2144	1786	1340	1072	893	766	670	536
	1 400 000		132240	66120	44080	26448	17631	13224	5289	3306	2505	2003	1669	1252	1002	834	716	626	501
	1 200 000		113084	56542	37694	22617	15078	11308	4523	2827	2141	1713	1428	1070	856	714	612	535	428
	1 100 000		104310	52155	34770	20862	13908	10431	4172	2608	1976	1580	1317	988	790	658	565	494	395
	1 000 000	1 590 000	94531	47265	31510	18906	12604	9453	3781	2363	1790	1432	1194	895	716	597	512	447	358
	950 000	1 515 000	90298	45149	30099	18059	12040	9030	3612	2257	1710	1368	1140	855	684	570	489	427	342
	900 000	1 431 000	85211	42605	28404	17042	11361	8521	3408	2130	1613	1291	1076	807	645	538	461	403	322
	850 000	1 351 500	80132	40066	26711	16026	10684	8013	3205	2003	1518	1214	1012	759	607	506	434	379	303
	800 000	1 272 000	75625	37814	25209	15126	10083	7563	3025	1891	1432	1146	955	716	573	477	409	358	286
	750 000	1 192 500	70968	35484	23656	14193	9462	7097	2839	1774	1344	1075	896	672	537	448	384	336	268
	700 000	1 113 000	66120	33060	22040	13224	8816	6612	2645	1653	1252	1002	835	626	501	417	358	313	250
	650 000	1 033 500	61421	30710	20474	12284	8189	6142	2457	1535	1163	930	775	582	466	388	332	291	232
	600 000	954 000	56542	28271	18847	11308	7539	5654	2261	1413	1071	856	714	535	428	357	306	267	214
	550 000	874 500	52155	26077	17385	10431	6954	5215	2086	1304	988	790	658	494	395	329	282	247	197
	500 000	795 000	47265	23632	15755	9453	6302	4726	1890	1181	895	716	597	447	358	298	256	223	179
	450 000	715 500	42456	21228	14152	8491	5661	4246	1698	1061	804	643	536	402	322	268	229	201	161
	400 000	636 000	37931	18965	12644	7586	5057	3793	1517	948	718	575	479	359	287	239	205	179	143
	350 000	556 500	33060	16530	11020	6612	4408	3306	1322	826	626	501	417	313	250	208	179	156	125
	300 000	477 000	28337	14168	9446	5667	3778	2833	1133	708	536	430	358	268	215	179	153	134	107
	250 000	397 500	23725	11862	7908	4745	3163	2372	949	593	449	359	300	224	179	150	128	112	89
0000	211 600	336 420	20000	10000	6666	4000	2666	2000	800	500	378	303	252	189	152	126	109	94	76
000	167 772	266 800	15921	7960	5307	3184	2123	1592	637	398	301	241	201	150	120	100	86	75	60
00	133 079	211 950	12604	6302	4201	2521	1680	1260	504	315	238	191	159	119	95	79	68	59	47
0	105 560	167 800	10000	5000	3333	2000	1333	1000	400	250	190	151	126	95	75	63	54	47	37
1	83 694	133 200	7908	3954	2636	1581	1054	791	316	198	150	120	100	75	60	50	42	37	30
2	66 358	105 530	6269	3134	2089	1254	835	627	251	157	118	95	79	59	47	39	34	29	23
3	52 624	83 640	4958	2479	1653	991	661	496	198	124	94	75	62	47	37	31	26	23	18
4	41 738	66 370	3954	1977	1318	791	527	395	158	99	75	60	50	37	30	25	21	18	15
5	33 088	52 630	3134	1567	1045	627	418	313	125	78	59	47	40	29	23	20	17	14	12
6	26 244	41 740	2479	1239	826	496	330	248	99	62	47	37	31	23	18	15	14	12	9
7	20 822		1967	983	655	393	262	197	78	49	37	30	25	18	15	12	10	9	7
8	16 512		1561	780	520	312	208	156	62	39	30	24	20	15	12	10	8	7	6

The heating limitations may, for the shorter distances, particularly if insulated or concealed conductors are employed, necessitate the use of larger conductors, resulting in a correspondingly less transmission loss. In the case of insulated or concealed conductors, should the k.v.a. values fall near or to the left of the heavy line, consult Table XXV for insulated or Table XXIII for bare conductors. The reactance for the larger conductors may be excessive, particularly for 60-cycle service, producing excessive voltage drop. This may be obviated by installing two or more parallel circuits or using three-conductor cables. For single-phase circuits the k.v.a. will be one-half the table values.

necessary in some cases of low voltage and single conductors (where the reactance is high) to use lower values of k.v.a. or even in some cases to multiple circuits in order to keep the reactance within satisfactory operating limits. This will be considered later by examples on voltage regulation.

TABLE XIV—QUICK ESTIMATING TABLE

CONDUCTORS

KILOVOLT-AMPERES. 3 PHASE. WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.66% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

2200 VOLTS DELIVERED

B & S NO.	COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	100 FEET	200 FEET	300 FEET	500 FEET	750 FEET	1000 FEET	2500 FEET	4000 FEET	1 MILE	1¼ MILES	1½ MILES	2 MILES	2½ MILES	3 MILES	3½ MILES	4 MILES	5 MILES
	2 000 000		756 000	378 000	252 000	151 000	101 000	75 600	30 200	18 900	14 300	11 460	9550	7160	5730	4770	4090	3580	2860
	1 800 000		686 000	343 000	229 000	137 000	91 500	68 600	27 400	17 100	13 000	10 400	8670	6500	5200	4330	3710	3250	2600
	1 700 000		641 000	320 000	214 000	128 000	85 500	64 100	25 600	16 000	12 100	9710	8100	6070	4850	4050	3470	3040	2430
	1 600 000		605 000	302 000	202 000	121 000	80 700	60 500	24 200	15 100	11 400	9170	7640	5730	4580	3820	3270	2860	2290
	1 500 000		566 000	283 000	189 000	113 000	75 500	56 600	22 600	14 100	10 700	8580	7150	5360	4290	3570	3060	2680	2140
	1 400 000		529 000	264 000	176 000	106 000	70 500	52 900	21 100	13 200	10 000	8010	6680	5010	4010	3340	2860	2500	2000
	1 200 000		452 000	226 000	151 000	90 500	60 300	45 200	18 100	11 300	8560	6850	5710	4280	3430	2850	2450	2140	1710
	1 100 000		417 000	208 000	139 000	83 400	55 600	41 700	16 700	10 400	7900	6320	5270	3950	3160	2630	2260	1970	1580
	1 000 000	1 590 000	378 000	189 000	126 000	75 600	50 400	37 800	15 100	9450	7160	5730	4770	3580	2860	2390	2040	1790	1430
	950 000	1 515 000	361 000	180 000	120 000	72 200	48 100	36 100	14 400	9030	6840	5470	4560	3420	2740	2280	1950	1710	1370
	900 000	1 431 000	341 000	170 000	114 000	68 100	45 400	34 100	13 600	8520	6450	5160	4300	3230	2580	2150	1840	1610	1290
	850 000	1 351 500	320 000	160 000	107 000	64 100	42 700	32 000	12 800	8010	6070	4860	4050	3030	2430	2020	1730	1520	1210
	800 000	1 272 000	302 000	151 000	101 000	60 500	40 300	30 200	12 100	7560	5730	4580	3820	2860	2290	1910	1640	1430	1146
	750 000	1 192 500	284 000	142 000	94 600	56 800	37 800	28 400	11 300	7100	5370	4300	3580	2700	2150	1790	1540	1340	1070
	700 000	1 113 000	264 000	132 000	88 100	52 900	35 200	26 400	10 600	6610	5010	4010	3340	2500	2000	1670	1430	1250	1000
	650 000	1 033 500	246 000	123 000	81 900	49 100	32 700	24 500	9800	6140	4650	3720	3100	2330	1861	1550	1330	1160	931
	600 000	954 000	226 000	113 000	75 400	45 200	30 100	22 600	9050	5650	4280	3430	2850	2140	1710	1430	1220	1070	857
	550 000	874 500	208 000	104 000	69 500	41 700	27 800	20 800	8350	5210	3950	3160	2630	1970	1580	1310	1130	987	790
	500 000	795 000	189 000	94 500	63 000	37 800	25 200	18 900	7560	4730	3580	2860	2380	1790	1430	1190	1020	895	716
	450 000	715 500	170 000	84 900	56 600	33 900	22 600	17 000	6800	4240	3220	2570	2140	1610	1290	1070	920	804	643
	400 000	636 000	152 000	75 800	50 600	30 300	20 200	15 200	6070	3790	2870	2300	1910	1440	1150	958	824	718	575
	350 000	556 500	132 000	66 100	44 100	26 400	17 600	13 200	5290	3310	2500	2000	1670	1250	1000	835	716	626	501
	300 000	477 000	113 000	56 700	37 800	22 700	15 100	11 300	4530	2830	2150	1720	1430	1073	860	715	613	536	429
	250 000	397 500	94 900	47 400	31 600	19 000	12 600	9500	3800	2370	1800	1440	1200	900	720	600	513	450	359
0000	211 600	336 420	80 000	40 000	26 700	16 000	10 700	8000	3200	2000	1510	1210	1010	757	606	505	433	378	303
000	167 772	266 800	63 700	31 800	21 200	12 700	8490	6370	2550	1590	1200	965	804	603	482	402	345	301	241
00	133 079	211 950	50 400	25 200	16 800	10 100	6720	5040	2000	1260	955	760	636	477	382	318	273	238	191
0	105 560	167 800	40 000	20 000	13 300	8000	5330	4000	1600	1000	757	606	505	380	303	252	217	189	151
1	83 694	133 220	31 600	15 800	10 500	6330	4220	3160	1260	790	600	480	400	300	239	200	171	150	120
2	66 358	105 530	25 100	12 500	8360	5010	3340	2510	1000	630	475	380	317	237	190	158	136	118	95
3	52 624	83 640	19 800	9900	6610	3970	2640	1980	790	500	375	300	250	188	150	125	107	94	75
4	41 738	66 370	15 800	7910	5270	3160	2110	1580	630	395	300	240	200	150	120	100	86	75	60
5	33 088	52 630	12 500	6270	4180	2510	1670	1250	500	313	237	190	159	118	95	79	68	59	47
6	26 244	41 740	9900	4960	3310	1980	1320	990	400	250	188	150	125	94	75	63	54	47	37
7	20 822		7900	3930	2620	1570	1050	790	310	197	149	119	99	74	59	49	42	37	30
8	16 512		6250	3120	2080	1250	830	620	250	156	118	95	79	59	47	39	34	30	24

4000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	1 MILE	1½ MILES	2 MILES	2½ MILES	3 MILES	3½ MILES	4 MILES	5 MILES	6 MILES	7 MILES	8 MILES	9 MILES	10 MILES	11 MILES	12 MILES	13 MILES	14 MILES
	650 000	1 033 000	15400	10250	7700	6150	5120	4400	3850	3080	2560	2200	1920	1710	1540	1400	1280	1180	1100
	600 000	954 000	14200	9460	7100	5670	4750	4050	3550	2840	2370	2030	1775	1580	1420	1290	1180	1090	1020
	550 000	874 500	13100	8750	6550	5250	4370	3750	3280	2620	2180	1870	1640	1460	1310	1190	1090	1010	940
	500 000	795 000	11900	7950	5950	4750	3960	3400	2980	2380	1980	1700	1490	1320	1190	1080	990	915	850
	450 000	715 500	10700	7120	5350	4270	3560	3060	2670	2140	1780	1530	1330	1190	1070	970	890	822	763
	400 000	636 000	9490	6330	4750	3800	3165	2713	2373	1900	1582	1357	1186	1055	949	863	791	736	678
	350 000	556 500	8270	5510	4130	3307	2756	2362	2067	1653	1378	1181	1033	919	827	752	689	636	590
	300 000	477 000	7080	4720	3540	2832	2359	2023	1769	1416	1179	1011	884	786	708	644	589	545	506
	250 000	397 500	5920	3950	2960	2370	1975	1693	1481	1185	987	846	740	658	593	539	493	456	423
0000	211 600	336 420	5000	3330	2500	2000	1666	1428	1250	1000	833	714	625	555	500	454	416	385	357
000	167 772	266 800	3980	2650	1990	1592	1326	1137	995	796	663	568	497	442	398	362	331	306	284
00	133 079	211 950	3140	2090	1570	1255	1046	896	783	627	523	448	391	349	313	285	261	241	224
0	105 560	167 800	2500	1670	1250	1000	833	713	625	500	416	357	312	277	250	227	208	192	178
1	83 694	133 220	1970	1320	987	790	658	563	493	395	329	282	247	219	197	179	164	152	140
2	66 358	105 530	1570	1045	783	627	522	448	392	313	261	224	196	174	157	142	130	120	112
3	52 624	83 640	1240	826	620	496	413	354	310	248	207	177	155	138	124	113	103	95	88
4	41 738	66 370	990	661	495	396	330	283	247	198	167	141	123	110	99	90	82	76	70
5	33 088	52 630	786	524	393	314	262	225	196	157	131	112	98	87	78	71	65	60	56

4400 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	1 MILE	1½ MILES	2 MILES	2½ MILES	3 MILES	3½ MILES	4 MILES	5 MILES	6 MILES	7 MILES	6 MILES	9 MILES	10 MILES	11 MILES	12 MILES	13 MILES	14 MILES
	650 000	1 033 000	18600	12400	9300	7450	6200	5300	4650	3720	3100	2660	2330	2070	1860	1690	1550	1430	1330
	600 000	954 000	17100	11400	8550	6850	5700	4880	4270	3420	2850	2440	2140	1900	1710	1550	1430	1320	1220
	550 000	874 500	15900	10600	7950	6370	5300	4550	3980	3180	2650	2270	1990	1770	1590	1450	1330	1230	1140
	500 000	795 000	14300	9530	7150	5720	4770	4080	3570	2870	2380	2050	1780	1590	1430	1300	1190	1100	1020
	450 000	715 500	12900	8600	6450	5150	4300	3690	3230	2580	2150	1840	1620	1430	1290	1180	1080	995	920
	400 000	636 000	11500	7650	5750	4600	3830	3290	2880	2300	1920	1640	1440	1280	1150	1050	960	887	825
	350 000	556 500	10000	6670	5000	4000	3330	2860	2500	2000	1670	1430	1250	1110	1000	910	832	770	715
	300 000	477 000	8590	5720	4290	3440	2860	2450	2150	1720	1430	1230	1070	950	858	780	715	660	613
	250 000	397 500	7170	4780	3585	2870	2390	2050	1800	1430	1200	1030	900	800	717	653	600	552	513
0000	211 600	336 420	6050	4030	3025	2420	2020	1730	1510	1210	1010	865	755	673	605	550	505	467	433
000	167 772	266 800	4820	3220	2410	1930	1610	1380	1210	965	805	688	603	535	482	438	402	371	345
00	133 079	211 950	3790	2530	1895	1520	1260	1080	950	758	632	543	475	422	379	345	317	292	271
0	105 560	167 800	3030	2020	1515	1220	1010	867	758	607	506	433	379	337	303	276	253	233	217
1	83 694	133 220	2390	1590	1195	955	795	683	598	478	398	341	299	266	239	217	199	183	171
2	66 358	105 530	1900	1270	950	760	633	542	475	380	317	271	237	211	190	173	158	146	136
3	52 624	83 640	1500	1000	750	600	500	428	375	300	250	214	188	167	150	136	125	116	107
4	41 738	66 370	1200	800	600	480	400	342	300	240	200	172	150	133	120	109	100	92	86
5	33 088	52 630	950	633	475	380	318	272	237	190	158	136	119	106	95	87	80	73	[illegible]

The heating limitations may, for the shorter distances, particularly if insulated or concealed conductors are employed, necessitate the use of larger conductors, resulting in a correspondingly less transmission loss. In the case of insulated or concealed conductors, should the k.v.a. values fall near or to the left of the heavy line, consult Table XXV for insulated or Table XXIII for bare conductors. The reactance for the larger conductors may be excessive, particularly for 60-cycle service, producing excessive voltage drop. This may be obviated by installing two or more parallel circuits or using three-conductor cables. For single-phase circuits the k.v.a. will be one-half the table values.

TABLE XV—QUICK ESTIMATING TABLE

CONDUCTORS

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED, BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.86% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

6000 VOLTS DELIVERED

B & S No.	Copper Area in Circular Mils	Aluminum Area in Circular Mils	1 Mile	1½ Miles	2 Miles	2½ Miles	3 Miles	3½ Miles	4 Miles	5 Miles	6 Miles	7 Miles	8 Miles	9 Miles	10 Miles	11 Miles	12 Miles	13 Miles	14 Miles
	650000	1033000	34600	23100	17300	13800	11500	9900	8650	6930	5780	4950	4320	3850	3460	3150	2890	2670	2470
	600000	954000	31800	21200	15900	12700	10600	9100	7950	6370	5300	4550	3980	3540	3180	2890	2650	2450	2270
	550000	874500	29500	19700	14700	11800	9850	8430	7370	5900	4920	4220	3690	3280	2950	2680	2460	2270	2110
	500000	795000	26600	17700	13300	10600	8860	7600	6670	5320	4450	3800	3330	2960	2660	2420	2220	2050	1900
	450000	715500	23900	16000	11900	9550	8000	6880	5900	4770	4000	3440	2950	2660	2390	2170	2000	1840	1720
	400000	636000	21300	14200	10700	8540	7120	6100	5340	4270	3561	3050	2670	2370	2130	1940	1780	1640	1530
	350000	556500	18600	12400	9300	7440	6200	5310	4650	3720	3100	2660	2320	2070	1860	1690	1550	1430	1330
	300000	477000	15900	10600	7960	6370	5310	4550	3980	3190	2650	2270	1990	1770	1590	1450	1330	1220	1130
	250000	397500	13300	8890	6670	5330	4440	3810	3330	2670	2220	1900	1670	1480	1330	1210	1110	1020	950
0000	211600	336420	11200	7500	5620	4500	3750	3210	2810	2250	1870	1610	1410	1250	1120	1020	937	866	800
000	167772	266800	8950	5970	4480	3580	2980	2560	2240	1791	1490	1280	1120	995	895	814	746	689	640
00	133079	211950	7060	4710	3530	2820	2350	2020	1760	1410	1180	1010	882	784	706	642	588	543	500
0	105560	167800	5620	3750	2810	2250	1870	1610	1410	1120	937	803	703	625	562	515	468	433	400
1	83694	133220	4440	2960	2220	1780	1480	1270	1110	890	741	634	555	494	444	404	370	342	317
2	66358	105530	3530	2350	1760	1410	1170	1010	882	706	588	504	441	392	353	321	294	271	250
3	52624	83640	2790	1860	1390	1110	930	797	697	558	465	398	348	310	279	253	232	215	200
4	41738	66370	2230	1490	1110	891	743	637	556	446	371	318	278	247	223	202	185	171	160
5	33088	52630	1770	1180	884	708	590	505	442	354	295	252	221	196	177	161	146	136	126

6600 VOLTS DELIVERED

B & S No.	Copper Area in Circular Mils	Aluminum Area in Circular Mils	1 Mile	1½ Miles	2 Miles	2½ Miles	3 Miles	3½ Miles	4 Miles	5 Miles	6 Miles	7 Miles	8 Miles	9 Miles	10 Miles	11 Miles	12 Miles	13 Miles	14 Miles
	650000	1033000	41900	28000	20900	16800	14000	12000	10500	8400	7000	6000	5250	4670	4190	3830	3500	3230	3000
	600000	954000	38500	25700	19300	15400	12800	11000	9630	7700	6400	5500	4810	4270	3850	3500	3210	2960	2750
	550000	874500	35700	23800	17800	14300	11900	10000	8930	7140	5950	5100	4460	3960	3570	3250	2980	2750	2550
	500000	795000	32300	21500	16100	12900	10800	9250	8075	6460	5400	4620	4030	3600	3230	2940	2690	2480	2310
	450000	715500	29100	19400	14500	11600	9700	8820	7270	5820	4850	4150	3640	3230	2910	2640	2420	2240	2080
	400000	636000	25900	17300	12900	10300	8650	7400	6470	5180	4320	3700	3240	2880	2590	2360	2160	1990	1850
	350000	556500	22600	15000	11300	9040	7520	6450	5650	4520	3770	3220	2820	2510	2260	2050	1880	1740	1610
	300000	477000	19300	12900	9650	7720	6440	6520	4820	3860	3220	2760	2420	2150	1930	1760	1610	1480	1380
	250000	397500	16200	10800	8100	6480	5400	4620	4050	3240	2700	2310	2020	1800	1620	1470	1350	1240	1160
0000	211600	336420	13600	9050	6800	5440	4530	3890	3400	2720	2260	1940	1700	1510	1360	1230	1130	1040	970
000	167772	266800	10800	7200	5400	4320	3600	3080	2700	2160	1800	1540	1350	1200	1080	980	900	830	770
00	133079	211950	8550	5700	4270	3420	2850	2440	2140	1710	1420	1220	1070	950	855	778	713	657	610
0	105560	167800	6800	4520	3400	2720	2260	1940	1700	1360	1130	970	850	755	680	617	565	522	486
1	83694	133220	5380	3590	2690	2150	1790	1530	1340	1070	895	767	672	597	538	490	448	413	385
2	66358	105530	4270	2850	2130	1710	1420	1220	1070	854	710	610	533	477	427	390	356	329	306
3	52624	83640	3380	2250	1690	1350	1130	965	845	676	565	483	422	376	338	307	282	260	241
4	41738	66370	2710	1800	1350	1080	900	775	677	542	450	388	339	300	271	246	226	209	193
5	33088	52630	2140	1420	1070	860	712	612	535	428	356	306	268	238	214	195	178	165	151

10 000 VOLTS DELIVERED

B & S No.	Copper Area in Circular Mils	Aluminum Area in Circular Mils	6 Miles	7 Miles	8 Miles	9 Miles	10 Miles	11 Miles	12 Miles	13 Miles	14 Miles	15 Miles	16 Miles	18 Miles	20 Miles	22 Miles	24 Miles	26 Miles	28 Miles
	650000	1033000	16000	13800	12100	10700	9620	8750	8020	7400	6870	6420	6000	5350	4820	4380	4020	3700	3440
	600000	954000	14800	12700	11100	9820	8850	8050	7350	6800	6320	5900	5530	4920	4420	4020	3680	3410	3160
	550000	874500	13700	11700	10200	9100	8200	7450	6820	6300	5850	5450	5120	4550	4100	3720	3410	3150	2920
	500000	795000	12300	10600	9250	8250	7400	6720	6150	5700	5280	4920	4620	4100	3700	3360	3090	2850	2640
	450000	715500	11100	9550	8450	7420	6680	5080	5560	5130	4770	4450	4170	3700	3340	3040	2780	2570	2390
	400000	636000	9890	8480	7420	6594	5930	5390	4940	4560	4240	3950	3710	3300	2970	2700	2470	2280	2120
	350000	556500	8610	7380	6460	5742	5170	4700	4310	3970	3690	3440	3230	2870	2580	2350	2150	1940	1840
	300000	477000	7370	6320	5530	4910	4420	4020	3690	3400	3160	2950	2760	2460	2210	2010	1840	1700	1580
	250000	397500	6170	5290	4630	4110	3700	3370	3090	2850	2640	2470	2310	2060	1850	1680	1540	1420	1320
0000	211600	336420	5210	4460	3910	3470	3120	2840	2600	2400	2230	2080	1950	1740	1560	1420	1300	1200	1110
000	167772	266800	4140	3550	3110	2760	2490	2260	2070	1910	1780	1660	1550	1380	1240	1130	1040	957	890
00	133079	211950	3270	2800	2450	2180	1960	1780	1630	1510	1400	1310	1220	1090	980	891	817	754	700
0	105560	167800	2600	2230	1950	1740	1560	1420	1300	1200	1110	1040	976	868	781	710	651	601	558
1	83694	133200	2060	1760	1540	1370	1230	1120	1030	949	882	823	771	686	617	561	514	474	441
2	66358	105530	1633	1400	1220	1090	980	891	816	754	700	653	612	544	490	445	408	377	350
3	52624	83640	1290	1110	969	861	775	705	646	596	554	517	484	431	387	352	323	298	277
4	41738	66370	1030	884	774	688	619	563	516	476	442	413	387	344	309	281	258	238	221
5	33088	52630	819	702	614	546	491	447	409	378	351	327	307	273	245	223	204	189	175

11 000 VOLTS DELIVERED

B & S No.	Copper Area in Circular Mils	Aluminum Area in Circular Mils	6 Miles	7 Miles	8 Miles	9 Miles	10 Miles	11 Miles	12 Miles	13 Miles	14 Miles	15 Miles	16 Miles	18 Miles	20 Miles	22 Miles	24 Miles	26 Miles	28 Miles
	650000	1033000	19400	16600	14500	12900	11600	10600	9700	8920	8300	7750	7250	6450	5800	5270	4850	4450	4150
	600000	954000	17800	15300	13400	11900	10700	9750	8900	8230	7650	7150	6680	5950	5350	4870	4450	4120	3820
	550000	874500	16500	14200	12400	11100	9920	9000	8250	7620	7080	6600	6200	5500	4960	4500	4120	3810	3540
	500000	795000	14900	12800	11200	10000	8970	8170	7450	6900	6400	5980	5600	5000	4480	4080	3720	3450	3200
	450000	715500	13500	11500	10100	8950	8070	7350	6750	6200	5770	5380	5050	4470	4030	3670	3370	3100	2880
	400000	636000	12000	10300	9000	8000	7200	6550	6000	5550	5150	4800	4500	4000	3600	3270	3000	2780	2570
	350000	556500	10500	8950	7820	6960	6270	5680	5250	4830	4470	4180	3910	3480	3130	2840	2620	2410	2230
	300000	477000	8920	7650	6700	5980	5350	4870	4460	4120	3830	3560	3350	2990	2670	2430	2230	2060	1910
	250000	397500	7470	6400	5600	4980	4480	4070	3730	3440	3200	2990	2800	2490	2240	2030	1870	1720	1600
0000	211600	336420	6300	5400	4730	4200	3780	3440	3150	2910	2700	2530	2360	2100	1890	1720	1570	1450	1350
000	167772	266800	5020	4300	3760	3340	3010	2740	2510	2310	2150	2010	1880	1670	1500	1370	1250	1150	1070
00	133079	211950	3960	3390	2960	2640	2370	2150	1980	1820	1700	1580	1480	1320	1180	1070	990	910	850
0	105560	167800	3150	2700	2370	2100	1890	1720	1570	1450	1350	1260	1180	1050	945	860	787	725	675
1	83694	133220	2490	2140	1870	1670	1490	1360	1240	1150	1070	1000	935	835	747	680	622	575	535
2	66358	105530	1980	1700	1480	1320	1180	1080	990	910	845	790	740	660	592	540	495	455	422
3	52624	83640	1570	1340	1170	1040	940	850	785	720	670	627	585	520	469	425	392	360	335
4	41738	66370	1250	1070	940	836	750	685	625	578	537	503	471	418	376	342	312	289	268
5	33088	52630	990	850	740	660	590	540	495	457	424	395	371	330	296	270	247	228	212

The heating limitations may, for the shorter distances, particularly if insulated or concealed conductors are employed, necessitate the use of larger conductors resulting in a correspondingly less transmission loss. In the case of insulated or concealed conductors should the k.v.a values fall near or to the left of the heavy line consult Table XXV for insulated or Table XXIII for bare conductors. The reactance for the larger conductors may be excessive particularly for 60-cycle service, producing excessive voltage drop. This may be obviated by installing two or more parallel circuits or using three-conductor cables. For single-phase circuits the k.v.a. will be one-half the table values.

TABLE XVI—QUICK ESTIMATING TABLE

CONDUCTORS

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED, BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.66% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.6 LOSS

12 000 VOLTS DELIVERED

B & S NO.	COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	6 MILES	7 MILES	8 MILES	9 MILES	10 MILES	11 MILES	12 MILES	13 MILES	14 MILES	15 MILES	18 MILES	18 MILES	20 MILES	22 MILES	24 MILES	26 MILES	28 MILES
	650 000 600 000 550 000	1 033 000 954 000 874 500	23100 21200 19700	19800 18200 16900	17300 15900 14800	15400 14200 13200	13900 12700 11800	12600 11600 10800	11600 10600 9850	10700 9800 9100	9900 9100 8450	9250 8500 7900	8650 7950 7400	7700 7100 6600	6950 6350 5900	6300 5800 5400	5800 5300 4920	5350 4900 4550	4950 4550 4220
	500 000 450 000 400 000	795 000 715 500 636 000	17800 16000 14200	15300 13700 12200	13300 12000 10700	11900 10600 9490	10700 9600 8540	9700 8600 7770	8900 8000 7120	8200 7380 6570	7600 6850 6100	7100 6400 5700	6650 6000 5340	5900 5300 4750	5350 4800 4270	4850 4300 3880	4450 4000 3560	4100 3690 3290	3800 3420 3050
	350 000 300 000 250 000	556 500 477 000 397 500	12400 10600 8900	10600 9100 7620	9300 7960 6670	8270 7080 5920	7440 6370 5330	6760 5790 4850	6200 5310 4440	5720 4900 4100	5300 4550 3810	4960 4250 3550	4650 3980 3330	4130 3540 2960	3720 3190 2670	3380 2890 2420	3100 2650 2220	2860 2450 2050	2660 2270 1900
0000 000 00	211 600 167 772 133 079	336 420 266 800 211 950	7500 5970 4710	6430 5120 4033	5620 4480 3530	5000 3980 3140	4500 3580 2820	4090 3250 2560	3750 2990 2350	3460 2750 2170	3210 2560 2010	3000 2390 1880	2810 2240 1760	2500 1990 1570	2250 1791 1410	2040 1630 1280	1870 1490 1180	1730 1380 1090	1610 1280 1010
0 1 2	105 560 83 694 66 358	167 800 133 220 105 530	3750 2960 2350	3210 2540 2010	2810 2220 1760	2500 1970 1570	2250 1780 1410	2050 1610 1280	1870 1480 1170	1730 1370 1090	1610 1270 1010	1500 1180 940	1410 1110 880	1250 990 780	1120 890 700	1020 810 640	930 740 590	865 683 543	803 635 504
3 4 5	52 624 41 738 33 088	83 640 66 370 52 630	1860 1490 1180	1590 1270 1010	1390 1110 884	1240 990 780	1110 890 710	1010 810 640	930 740 590	860 680 540	790 640 500	740 590 472	700 560 440	620 490 390	560 440 350	510 400 320	460 370 295	430 343 272	400 318 252

13 200 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	8 MILES	7 MILES	8 MILES	9 MILES	10 MILES	11 MILES	12 MILES	13 MILES	14 MILES	15 MILES	18 MILES	18 MILES	20 MILES	22 MILES	24 MILES	26 MILES	28 MILES
	650 000 600 000 550 000	1 033 000 954 000 874 500	27900 25700 23900	24000 22000 20400	20900 19300 17900	18600 17100 15900	16700 15400 14300	15200 14000 13000	14000 12800 11900	12900 11800 11000	12000 11000 10200	11200 10300 9550	10400 9650 8950	9300 8550 7950	8370 7700 7150	7600 7000 6500	7000 6400 5950	6450 5920 5500	6000 5500 5100
	500 000 450 000 400 000	795 000 715 500 636 000	21500 19300 17300	18500 16600 14800	16100 14500 12900	14300 12900 11500	12900 11600 10300	11700 10600 9420	10700 9700 8650	9920 8930 8000	9220 8280 7400	8600 7750 6900	8050 7250 6450	7150 6450 5750	6450 5800 5170	5850 5300 4710	5350 4850 4320	4960 4460 4000	4610 4140 3700
	350 000 300 000 250 000	556 500 477 000 397 500	15100 12900 10800	12900 11000 9250	11300 9650 8080	10000 8570 7180	9020 7720 6450	8200 7020 5870	7520 6430 5380	6950 5930 4960	6450 5520 4620	6020 5150 4300	5650 4820 4040	5000 4280 3600	4510 3860 3220	4100 3510 2930	3760 3210 2700	3470 2960 2480	3220 2760 2310
0000 000 00	211 600 167 772 133 079	336 420 266 800 211 950	9100 7220 5700	7800 6200 4900	6850 5420 4280	6070 4820 3800	5450 4330 3420	4970 3940 3110	4550 3620 2850	4200 3330 2630	3900 3100 2450	3690 2890 2280	3420 2710 2140	3030 2410 1900	2720 2160 1710	2480 1970 1550	2270 1810 1420	2100 1660 1310	1950 1550 1220
0 1 2	105 560 83 694 66 358	167 800 133 220 105 530	4550 3590 2850	3900 3070 2440	3410 2690 2140	3030 2390 1900	2730 2150 1710	2480 1960 1560	2270 1800 1430	2100 1660 1320	1950 1540 1220	1820 1430 1140	1700 1340 1070	1510 1200 950	1360 1070 850	1240 980 780	1130 900 710	1050 830 660	975 770 610
3 4 5	52 624 41 738 33 088	83 640 66 370 52 630	2250 1800 1420	1930 1550 1220	1680 1360 1070	1500 1200 950	1350 1080 850	1220 985 778	1120 900 710	1030 830 660	960 770 610	900 720 570	840 680 530	750 600 470	670 540 430	610 490 390	560 450 360	510 420 330	480 390 300

15 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	6 MILES	7 MILES	8 MILES	9 MILES	10 MILES	11 MILES	12 MILES	13 MILES	14 MILES	15 MILES	18 MILES	18 MILES	20 MILES	22 MILES	24 MILES	26 MILES	28 MILES
	650 000 600 000 550 000	1 033 000 954 000 874 500	36000 33200 30600	31000 28500 26300	27000 24900 23000	24000 22200 20500	21600 19900 18400	19800 18200 16700	18000 16600 15300	16600 15300 14100	15400 14200 13100	14400 13300 12300	13500 12400 11500	12000 11100 10200	10800 9950 9200	9900 9100 8350	9000 8300 7650	8300 7650 7050	7700 7100 6550
	500 000 450 000 400 000	795 000 715 500 636 000	27800 25000 22200	23900 21400 19100	20800 18700 16700	18600 16700 14800	16700 15000 13300	15200 13600 12100	13900 12500 11100	12800 11500 10200	11900 10700 9540	11100 10000 8900	10400 9350 8340	9300 8350 7420	8350 7500 6670	7600 6800 6070	6950 6250 5560	6400 5750 5130	5950 5350 4770
	350 000 300 000 250 000	556 500 477 000 397 500	19400 16600 13900	16600 14200 11900	14500 12400 10400	12900 9260 7810	11600 9960 8330	10600 9050 7570	9690 8290 6940	8940 7660 6410	8300 7100 5950	7750 6640 5560	7260 6220 5210	6460 5530 4630	5810 5000 4160	5280 4520 3790	4840 4150 3470	4470 3830 3200	4150 3550 2980
0000 000 00	211 600 167 772 133 079	336 420 266 800 211 950	11700 9330 7350	10000 8000 6300	8790 7000 5510	6220 4900 3910	7030 5600 4410	6390 5090 4010	5860 4660 3670	5410 4300 3390	5020 4000 3150	4680 3730 2940	4390 3500 2760	3910 3110 2450	3510 2800 2210	3200 2540 2000	2930 2330 1840	2700 2150 1700	2510 2000 1570
0 1 2	105 560 83 694 66 358	167 800 133 220 105 530	5860 4630 3670	5000 3970 3150	4390 3470 2760	3090 2450 1940	3510 2780 2210	3170 2520 2000	2930 2310 1840	2700 2140 1690	2510 1980 1570	2340 1850 1470	2200 1740 1380	1950 1540 1220	1760 1390 1100	1600 1260 1000	1460 1160 920	1350 1070 850	1250 990 780
3 4 5	52 624 41 738 33 088	83 640 66 370 52 630	2910 2320 1840	2490 1990 1580	2180 1740 1380	1550 1230 970	1740 1390 1110	1590 1270 1000	1450 1160 920	1340 1070 850	1240 990 790	1160 930 730	1090 870 690	970 774 614	870 690 430	790 630 500	730 580 460	670 530 420	620 490 390

16 500 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	6 MILES	7 MILES	8 MILES	9 MILES	10 MILES	11 MILES	12 MILES	13 MILES	14 MILES	15 MILES	18 MILES	18 MILES	20 MILES	22 MILES	24 MILES	26 MILES	28 MILES
	650 000 600 000 550 000	1 033 000 954 000 874 500	45400 40200 37200	38900 34500 31900	34000 30200 27900	30300 26800 24800	27200 24100 22300	24700 21900 20300	22700 20100 18600	21000 18600 17200	19500 17700 15900	18200 16100 14800	17000 15100 13900	15100 13400 12400	13600 12000 11100	12300 10900 10100	11300 10000 9300	10500 9300 8600	9750 8600 7950
	500 000 450 000 400 000	795 000 715 500 636 000	33700 30400 27000	28900 26000 23100	25300 22800 20200	22500 20300 18000	20200 18200 16200	18300 16600 14700	16800 15200 13500	15500 14000 12400	14400 13000 11600	13500 12100 10800	12600 11400 10100	11200 10100 9000	10100 9100 8100	9150 8300 7350	8400 7600 6750	7750 7000 6200	7200 6500 5800
	350 000 300 000 250 000	556 500 477 000 397 500	23700 20200 16800	20300 17200 14400	17700 15100 12600	15800 13400 11200	14200 12100 10100	12900 11000 9200	11800 10100 8450	10900 9300 7780	10200 8650 7220	9480 8060 6720	8850 7550 6300	7900 6700 5600	7100 6050 5050	6450 5500 4600	5900 5050 4220	5450 4650 3890	5100 4320 3610
0000 000 00	211 600 167 772 133 079	336 420 266 800 211 950	14200 11300 8920	12200 9700 7650	11700 8500 6700	9500 7550 5950	8520 6780 5370	7770 6170 4870	7100 5670 4460	6560 5220 4120	6080 4860 [illegible]	5700 4530 3580	5850 4250 3350	4750 3770 2970	4260 3390 2670	3880 3080 2430	3550 2830 2230	3280 2610 2060	3040 2430 1910
0 1 2	105 560 83 694 66 358	167 800 133 220 105 530	7100 5620 4450	6100 4820 3820	5320 4220 3340	4720 3750 2970	4260 3370 2670	3860 3070 2430	3540 2810 2230	3270 2590 2060	3040 2410 1910	2840 2250 1780	2660 2110 1670	2360 1870 1480	2130 1680 1330	1930 1530 1210	1770 1400 1110	1630 1290 1030	1520 1200 950
3 4 5	52 624 41 738 33 088	83 640 66 370 52 630	3540 2820 2220	3030 2420 1900	2650 2120 1670	2360 1880 1480	2120 1690 1330	1930 1530 1210	1760 1410 1120	1630 1300 1020	1510 1210 950	1410 1130 890	1320 1060 830	1180 940 740	1060 840 670	960 760 600	880 700 560	810 650 510	750 600 480

The heating limitations may, for the shorter distances, particularly if insulated or concealed conductors are employed, necessitate the use of larger conductors, resulting in a correspondingly less transmission loss. In the case of insulated or concealed conductors, should the k.v.a values fall near or to the left of the heavy line, consult Table XXV for insulated or Table XXIII for bare conductors. The reactance for the larger conductors may be excessive, particularly for 60-cycle service, producing excessive voltage drop. This may be obviated by installing two or more parallel circuits or using three-conductor cables. For single-phase circuits the k.v.a. will be one-half the table values.

TABLE XVII—QUICK ESTIMATING TABLE

CONDUCTORS: B. & S. No.; COPPER area in circular mils; ALUMINUM area in circular mils.

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.66% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

20 000 VOLTS DELIVERED

B. & S. No.	Copper	Aluminum	7 Miles	8 Miles	9 Miles	10 Miles	11 Miles	12 Miles	13 Miles	14 Miles	15 Miles	16 Miles	18 Miles	20 Miles	22 Miles	24 Miles	26 Miles	28 Miles	30 Miles
	650000	1033000	55000	47200	42700	38500	34900	32000	29600	27500	25700	23600	21300	19200	17400	16000	14800	13400	12800
	600000	954000	50500	44200	39400	35400	32200	29500	27200	25300	23600	22100	19700	17700	16100	14700	13600	12700	11800
	550000	874500	46800	41000	36500	32800	29800	27300	25200	23400	21800	20500	18200	16400	14900	13600	12600	11700	10900
	500000	795000	42500	37000	33000	29700	27000	24700	22800	21200	19800	18500	16500	14800	13500	12300	11400	10600	9900
	450000	715500	38200	33400	29600	26600	24300	22200	20500	19000	17800	16700	14800	13300	12100	11100	10200	9500	8900
	400000	636000	33900	29600	26400	23700	21600	19800	18200	16900	15800	14800	13200	11800	10800	9890	9130	8480	7910
	350000	556500	29500	25800	23000	20700	18800	17200	15900	14700	13800	12900	11500	10300	9400	8610	7950	7380	6890
	300000	477000	25300	22100	19700	17700	16100	14700	13600	12600	11800	11000	9830	8850	8050	7370	6810	6320	5900
	250000	397500	21100	18500	16400	14800	13400	12300	11400	10600	9880	9260	8230	7410	6730	6170	5700	5290	4940
0000	211600	336420	17800	15600	13900	12500	11300	10400	9610	8930	8330	7810	6940	6250	5680	5210	4810	4460	4170
000	167772	266800	14200	12400	11000	9950	9040	8280	7650	7110	6630	6220	5530	4970	4520	4150	3830	3550	3310
00	133079	211950	11200	9800	8710	7840	7130	6530	6030	5600	5230	4900	4360	3920	3560	3270	3010	2800	2610
0	105560	167800	8930	7810	6940	6250	5680	5210	4810	4460	4170	3910	3470	3120	2840	2600	2400	2230	2080
1	83694	133220	7050	6170	5490	4940	4490	4110	3810	3530	3290	3090	2740	2470	2240	2060	1900	1760	1640
2	66358	105530	5600	4900	4360	3920	3560	3270	3020	2800	2610	2450	2180	1960	1780	1630	1510	1400	1310
3	52624	83640	4430	3870	3440	3100	2820	2570	2380	2210	2070	1940	1720	1550	1410	1290	1190	1110	1030
4	41738	66370	3540	3100	2750	2480	2250	2060	1900	1770	1650	1550	1380	1240	1120	1030	950	884	820
5	33088	52630	2810	2460	2180	1960	1790	1640	1510	1400	1310	1230	1090	980	890	820	750	700	650

22 000 VOLTS DELIVERED

B. & S. No.	Copper	Aluminum	7 Miles	8 Miles	9 Miles	10 Miles	11 Miles	12 Miles	13 Miles	14 Miles	15 Miles	16 Miles	18 Miles	20 Miles	22 Miles	24 Miles	26 Miles	28 Miles	30 Miles
	650000	1033000	66500	58200	51800	46600	42200	38800	35800	33300	31000	29100	25900	23300	21100	19400	17900	16600	15500
	600000	954000	61200	53500	47600	42800	38900	35600	32900	30600	27500	26700	23800	21400	19400	17800	16400	15300	13700
	550000	874500	56800	49600	44200	39700	36100	33100	30500	28400	26600	24800	22100	19800	18000	16500	15200	14200	13200
	500000	795000	51300	44800	40000	35900	32600	29900	27600	25700	23900	22400	20000	17900	16300	14900	13800	12800	11900
	450000	715500	46000	40200	35800	32200	29300	26900	24800	23000	21500	20100	17900	16100	14600	13400	12400	11500	10700
	400000	636000	41200	36000	32000	28800	26200	24000	22200	20600	19200	18000	16000	14400	13100	12000	11100	10300	9600
	350000	556500	35800	31400	27900	25100	22800	20900	19300	17900	16700	15700	13900	12550	11400	10400	9650	8950	8350
	300000	477000	30600	26700	23800	21400	19400	17800	16500	15300	14200	13300	11900	10700	9700	8900	8250	7650	7100
	250000	397500	25600	22400	19900	17900	16300	14900	13800	12800	12000	11200	9950	8950	8150	7450	6900	6400	6000
0000	211600	336420	21500	18900	16800	15100	13700	12600	11600	10800	10100	9450	8400	7550	6850	6300	5800	5400	5050
000	167772	266800	17300	15100	13400	12100	11000	10100	9300	8650	8080	7550	6700	6050	5500	5050	4650	4320	4040
00	133079	211950	13600	11800	10600	9500	8650	7920	7300	6780	6330	5900	5300	4750	4320	3960	3650	3390	3160
0	105560	167800	10800	9450	8380	7550	6860	6300	5820	5400	5030	4720	4190	3770	3430	3150	2910	2700	2510
1	83694	133220	8520	7480	6650	5970	5420	4980	4600	4270	3980	3740	3320	2980	2710	2490	2300	2130	1980
2	66358	105530	6780	5950	5280	4750	4320	3960	3660	3400	3180	2970	2640	2370	2160	1980	1830	1700	1590
3	52624	83640	5360	4680	4170	3750	3410	3130	2890	2680	2500	2340	2080	1870	1700	1560	1440	1340	1250
4	41738	66370	4300	3770	3340	3010	2740	2510	2320	2150	2010	1880	1670	1500	1370	1250	1160	1070	1000
5	33088	52630	3390	2970	2640	2370	2160	1980	1820	1690	1580	1480	1320	1180	1080	990	910	840	790

30 000 VOLTS DELIVERED

B. & S. No.	Copper	Aluminum	12 Miles	14 Miles	16 Miles	18 Miles	20 Miles	22 Miles	24 Miles	26 Miles	28 Miles	30 Miles	32 Miles	36 Miles	40 Miles	44 Miles	48 Miles	52 Miles	56 Miles
	650000	1033000	72200	62000	54200	48200	43200	39400	36100	33300	30900	28900	27100	24100	21600	19700	18000	16600	15400
	600000	954000	66200	56800	49700	44200	39800	36200	33200	30500	28400	26500	24800	22100	19900	18100	16600	15200	14200
	550000	874500	61500	52800	46200	41000	36900	33500	30800	28400	26400	24700	23100	20500	18400	16700	15400	14200	13200
	500000	795000	55600	47800	41800	37100	33400	30400	27900	25700	23900	22300	20900	18500	16700	15200	13900	12800	11900
	450000	715500	50000	42800	37500	33400	30000	27300	25100	23100	21500	20000	18700	16700	15000	13600	12500	11500	10700
	400000	636000	44500	38100	33400	29700	26700	24300	22200	20500	19100	17800	16700	14800	13300	12100	11100	10200	9540
	350000	556500	38700	33200	29100	25800	23200	21100	19400	17900	16600	15500	14500	12900	11600	10600	9690	8940	8300
	300000	477000	33200	28400	24900	22100	19900	18100	16600	15300	14200	13200	12400	11000	9950	9050	8290	7660	7100
	250000	397500	27800	23800	20800	18500	16700	15100	13900	12800	11900	11100	10400	9260	8330	7570	6940	6400	5950
0000	211600	336420	23400	20100	17600	15600	14100	12800	11700	10800	10000	9370	8790	7810	7030	6390	5860	5410	5020
000	167772	266800	18600	16000	14000	12400	11200	10200	9330	8610	8000	7460	7000	6220	5600	5090	4660	4300	4000
00	133079	211950	14700	12600	11030	9800	8820	8020	7350	6790	6300	5880	5510	4900	4410	4010	3670	3390	3150
0	105560	167800	11700	10000	8800	7810	7030	6390	5860	5400	5020	4690	4390	3910	3510	3190	2930	2700	2510
1	83694	133220	9260	7940	6940	6170	5550	5050	4630	4270	3970	3700	3470	3090	2780	2520	2310	2140	1980
2	66358	105530	7350	6300	5510	4900	4410	4010	3670	3390	3150	2940	2760	2450	2210	2000	1840	1690	1570
3	52624	83640	5810	4980	4360	3880	3490	3170	2910	2680	2490	2330	2180	1940	1740	1580	1450	1340	1240
4	41738	66370	4640	3980	3480	3100	2790	2530	2320	2140	1990	1860	1740	1550	1390	1270	1160	1070	990
5	33088	52630	3681	3160	2760	2450	2210	2010	1840	1700	1580	1470	1380	1230	1100	1000	920	850	790

33 000 VOLTS DELIVERED

B. & S. No.	Copper	Aluminum	12 Miles	14 Miles	16 Miles	18 Miles	20 Miles	22 Miles	24 Miles	26 Miles	28 Miles	30 Miles	32 Miles	36 Miles	40 Miles	44 Miles	48 Miles	52 Miles	56 Miles
	650000	1033000	87200	74700	65300	58200	52200	47600	43700	40200	37400	34800	32650	29100	26100	23800	21800	20100	18700
	600000	954000	80200	68800	60200	53600	48200	43800	40200	37100	34400	32100	30100	26800	24100	21900	20100	18500	17200
	550000	874500	75500	64800	56700	50400	45300	41200	37700	34800	32400	30300	28300	25200	22600	20600	18800	17400	16200
	500000	795000	67300	57800	50600	45000	40400	36800	33700	31100	28900	27000	25300	22500	20200	18400	16800	15500	14400
	450000	715500	60400	51800	45300	40200	36300	33000	30200	27900	25900	24100	22600	20100	18100	16500	15100	13900	12900
	400000	636000	54000	46300	40600	36000	32400	29500	27000	25000	23200	21600	20300	18000	16200	14700	13500	12500	11600
	350000	556500	47000	40300	35300	31400	28200	25700	23500	21700	20200	18800	17600	15700	14100	12800	11700	10800	10100
	300000	477000	40200	34500	30200	26900	24100	21900	20100	18600	17300	16100	15100	13400	12000	10900	10000	9300	8650
	250000	397500	33700	28800	25200	22400	20200	18300	16800	15500	14400	13400	12600	11200	10100	9150	8400	7750	7200
0000	211600	336420	28300	24300	21200	18900	17000	15400	14200	13100	12200	11400	10600	9450	8500	7700	7100	6550	6100
000	167772	266800	22600	19300	16900	15000	13500	12300	11300	10500	9680	9050	8450	7500	6750	6150	5650	5200	4840
00	133079	211950	17800	15200	13300	11800	10700	9700	8900	8220	7620	7120	6650	5900	5350	4850	4450	4110	3810
0	105560	167800	14200	12200	10600	9460	8520	7750	7100	6550	6080	5680	5300	4730	4260	3870	3550	3270	3040
1	83694	133220	11200	9600	8400	7480	6720	6120	5600	5180	4820	4500	4200	3740	3360	3060	2800	2590	2410
2	66358	105530	8900	7620	6670	5920	5330	4850	4450	4100	3810	3560	3340	2960	2670	2430	2230	2050	1910
3	52624	83640	7050	6050	5300	4700	4230	3850	3520	3250	3020	2820	2650	2350	2110	1920	1760	1620	1510
4	41738	66370	5650	4850	4230	3760	3390	3080	2820	2610	2420	2260	2110	1880	1690	1540	1410	1300	1210
5	33088	52630	4430	3800	3330	2960	2670	2420	2220	2050	1900	1770	1660	1480	1330	1210	1110	1020	950

The heating limitations may, for the shorter distances, particularly if insulated or concealed conductors are employed, necessitate the use of larger conductors, resulting in a correspondingly less transmission loss. In the case of insulated or concealed conductors, should the k.v.a. values fall near or to the left of the heavy line, consult Table XXV for insulated or Table XXIII for bare conductors. The reactance for the larger conductors may be excessive, particularly for 60-cycle service, producing excessive voltage drop. This may be obviated by installing two or more parallel circuits or using three-conductor cables. For single-phase circuits the k.v.a. will be one-half the table values.

TABLE XVIII—QUICK ESTIMATING TABLE

CONDUCTORS

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.66% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

40 000 VOLTS DELIVERED

B & S NO.	COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	14 MILES	16 MILES	18 MILES	20 MILES	22 MILES	24 MILES	26 MILES	28 MILES	30 MILES	32 MILES	36 MILES	40 MILES	44 MILES	48 MILES	52 MILES	56 MILES	60 MILES
	650000	1033000	110000	96000	85300	76800	70000	64200	59000	55000	51300	48000	42600	38400	35000	32100	29500	27500	25600
	600000	954000	101000	88700	78800	70800	64500	59000	54500	50700	47300	44300	39400	35400	32200	29500	27200	25300	23600
	550000	874500	93800	82000	73000	65500	59700	54700	50500	46800	43700	41000	36500	32700	29800	27300	25200	23400	21800
	500000	795000	84600	74000	66000	59200	54000	49400	45600	42300	39500	87000	33000	29600	27000	24700	22800	21100	19700
	450000	715500	76300	66700	59200	53500	48500	44500	41000	38100	35600	33300	29600	26700	24200	22200	20500	19000	17800
	400000	636000	67800	59300	52700	47500	43100	39500	36500	33900	31600	29700	26400	23700	21600	19800	18200	16900	15800
	350000	556500	59000	51700	45900	41300	37600	34400	31800	29500	27500	25800	23000	20700	18800	17200	15900	14700	13800
	300000	477000	50600	44200	39300	35400	32200	29500	27200	25300	23600	22100	19700	17700	16100	14700	13600	12600	11800
	250000	397500	42300	37000	32900	29600	26900	24700	22800	21100	19700	18500	16400	14800	13500	12300	11400	10600	9880
0000	211600	336420	35700	31200	27800	25000	22700	20800	19200	17800	16700	15600	13900	12500	11400	10400	9610	8930	8330
000	167772	266800	28400	24800	22100	19900	18100	16600	15300	14200	13200	12400	11000	9950	9040	8230	7650	7110	6630
00	133079	211950	22400	19600	17400	15700	14200	13100	12100	11200	10400	9800	8710	7840	7130	6530	6030	5600	5230
0	105560	167800	17800	15600	13900	12500	11300	10400	9620	8930	8330	7810	6940	6250	5680	5210	4810	4460	4170
1	83694	133220	14100	12300	10900	9880	8980	8230	7590	7050	6580	6170	5490	4940	4490	4110	3800	3530	3290
2	66358	105530	11200	9800	8710	7840	7130	6540	6030	5600	5230	4900	4360	3920	3560	3270	3020	2800	2610
3	52624	83640	8860	7750	6880	6200	5640	5170	4770	4430	4130	3870	3440	3100	2820	2580	2380	2210	2070
4	41738	66370	7070	6190	5500	4950	4500	4130	3810	3540	3300	3090	2750	2480	2250	2060	1910	1770	1650
5	33088	52630	5610	4910	4370	3930	3570	3280	3020	2810	2620	2460	2180	1960	1790	1640	1510	1400	1310

44 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	14 MILES	16 MILES	18 MILES	20 MILES	22 MILES	24 MILES	26 MILES	28 MILES	30 MILES	32 MILES	36 MILES	40 MILES	44 MILES	48 MILES	52 MILES	56 MILES	60 MILES
	650 000	1 033 000	133000	116000	103000	93000	84500	77500	71500	66500	62000	58000	51500	46500	42200	38700	35700	33200	31000
	600 000	954 000	122000	107000	95000	85500	77800	71500	66000	61200	57200	53500	47500	42700	38900	35700	33000	30600	28600
	550 000	874 500	113000	99000	88000	79200	72000	66000	61000	56500	52800	49500	44000	39600	36000	33000	30500	28200	26400
	500 000	795 000	102000	89700	79700	71800	65300	59800	55200	51300	47800	44800	39800	35900	32600	29800	27600	25600	23900
	450 000	715 500	92200	80700	71600	64500	58700	53800	49700	46200	43000	40300	35800	32200	29300	26900	24800	23100	21500
	400 000	636 000	82200	72000	64000	57700	52300	48000	44200	41200	38400	36000	32000	28800	26100	24000	22100	20600	19200
	350 000	556 500	71800	62800	55800	50200	45600	41800	38700	35800	33500	31400	27900	25100	22800	20900	19300	17900	16700
	300 000	477 000	61300	53600	47700	42800	39000	35700	33000	30600	28600	26800	23800	21400	19500	17850	16500	15300	14300
	250 000	397 500	51200	44800	39800	35800	32600	29900	27600	25600	23900	22400	19900	17900	16300	14900	13800	12800	11900
0000	211 600	336 420	43200	37900	33700	30300	27500	25200	23300	21600	20200	18950	16800	15100	13700	12600	11600	10800	10100
000	167 772	266 800	34400	30100	26800	24100	21900	20100	18500	17200	16000	15050	13400	12000	10900	10000	9250	8600	8000
00	133 079	211 950	27100	23700	21100	19000	17300	15800	14600	13500	12600	11860	10500	9500	8650	7900	7300	6750	6300
0	105 560	167 800	21700	18900	16800	15100	13800	12600	11700	10800	10100	9450	8400	7550	6900	6300	5850	5400	5050
1	83 694	133 200	17100	14900	13300	11900	10800	9970	9200	8550	7970	7450	6650	5900	5400	4980	4600	4270	3980
2	66 358	105 530	13500	11800	10500	9470	8600	7880	7280	6770	6320	5900	5250	4730	4300	3940	3640	3380	3160
3	52 624	83 640	10700	9400	8350	7520	6830	6250	5780	5360	5000	4700	4170	3760	3410	3120	2890	2680	2500
4	41 738	66 370	8600	7500	6680	6000	5470	5000	4620	4300	4000	3750	3340	3000	2730	2500	2310	2150	2000
5	33 088	52 630	6780	5930	5270	4750	4320	3960	3660	3400	3170	2960	2630	2370	2160	1980	1830	1700	1580

50 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	20 MILES	24 MILES	28 MILES	32 MILES	36 MILES	40 MILES	44 MILES	48 MILES	52 MILES	56 MILES	60 MILES	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES
	650 000	1 033 000	120000	100000	86000	75000	66800	60000	54800	50200	46200	43000	40000	37500	33400	30000	27400	25100	23100
	600 000	954 000	111000	92500	79200	69200	61500	55500	50300	46200	42500	39600	37000	34600	30700	27700	25100	23100	21200
	550 000	874 500	103000	85500	73500	64200	57000	51300	46600	42800	39500	36700	34300	32100	28500	25600	23300	21400	19700
	500 000	795 000	92800	77200	66200	58000	51500	46400	42200	38700	35600	33100	30900	29000	25700	23200	21100	19300	17800
	450 000	715 500	83800	69500	59800	52200	46500	41800	38000	35000	32200	29900	27900	26100	23200	20900	19000	17500	16100
	400 000	636 000	74200	61800	53000	46300	41200	37100	33700	30900	28500	26500	24700	23200	20600	18500	16900	15400	14200
	350 000	556 500	64600	53800	46100	40400	35900	32300	29300	26900	24800	23100	21500	20200	17900	16100	14700	13400	12400
	300 000	477 000	55300	46100	39500	34600	30700	27600	25100	23000	21300	19700	18400	17300	15400	13800	12600	11500	10600
	250 000	397 500	46300	38600	33100	28900	25700	23100	21000	19300	17800	16500	15400	14500	12900	11600	10500	9640	8900
0000	211 600	336 420	39100	32500	27900	24400	21700	19500	17800	16200	15000	13900	13000	12200	10800	9760	8880	8140	7510
000	167 772	266 800	31100	25900	22200	19400	17300	15500	14100	13000	11900	11100	10400	9720	8640	7770	7070	6480	5980
00	133 079	211 950	24500	20400	17500	15300	13600	12200	11100	10200	9430	8750	8170	7660	6810	6130	5570	5100	4710
0	105 560	167 800	19500	16300	13900	12200	10800	9760	8880	8140	7510	6970	6510	6100	5420	4880	4440	4070	3760
1	83 694	133 220	15400	12900	11020	9640	8570	7710	7010	6430	5930	5510	5140	4822	4290	3860	3510	3210	2970
2	66 358	105 530	12200	10200	8751	7660	6810	6130	5570	5100	4710	4370	4080	3833	3400	3060	2780	2550	2350
3	52 624	83 640	9690	8070	6920	6060	5380	4840	4400	4040	3730	3460	3230	3030	2690	2420	2200	2020	1860
4	41 738	66 370	7740	6450	5530	4840	4300	3870	3520	3220	2980	2760	2580	2420	2150	1930	1760	1610	1490
5	33 088	52 630	6140	5120	4390	3840	3410	3070	2790	2560	2360	2190	2050	1920	1710	1530	1400	1280	1180

60 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	20 MILES	24 MILES	28 MILES	32 MILES	36 MILES	40 MILES	44 MILES	48 MILES	52 MILES	56 MILES	60 MILES	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES
	650 000	1 033 000	173000	144000	124000	108000	96000	86600	78800	72200	66500	61800	57700	54000	48000	43300	39400	36100	33200
	600 000	954 000	159000	133000	114000	99500	88500	79600	72500	66500	61300	57000	53200	49700	44200	39800	36200	33200	30700
	550 000	874 500	148000	123000	106000	92500	82200	74000	67300	61800	57000	52800	49400	46200	41100	37000	33600	30900	28500
	500 000	795 000	133000	111000	95000	83500	74000	66600	60500	55500	51200	47500	44400	41700	37000	33300	30200	27700	25600
	450 000	715 500	120000	100000	85700	75200	66700	60000	54700	50200	46300	42800	40000	37600	33300	30000	27300	25100	23100
	400 000	636 000	107000	89000	76300	66700	59300	53400	48500	44500	41100	38100	35600	33400	29700	26700	24200	22200	20500
	350 000	556 500	93000	77500	66400	58100	51700	46500	42300	38700	35800	33200	31000	29100	25800	23200	21100	19400	17900
	300 000	477 000	79600	66300	56900	49800	44200	39800	36200	33100	30600	28400	26500	24900	22100	19900	18100	16600	15300
	250 000	397 500	66700	55500	47600	41700	37000	33300	30300	27800	25600	23800	22200	20800	18500	16700	15100	13900	12800
0000	211 600	336 420	56200	46900	40200	35100	31200	28100	25500	23400	21600	20100	18700	17600	15600	14100	12800	11700	10800
000	167 772	266 800	44800	37300	32000	28000	24800	22400	20300	18600	17200	16000	14900	14000	12400	11200	10200	9330	8610
00	133 079	211 950	35300	29400	25200	22000	19600	17600	16000	14700	13600	12600	11700	11000	9800	8820	8020	7350	6790
0	105 560	167 800	28100	23400	20100	17600	15600	14000	12800	11700	10800	10000	9370	8790	7810	7030	6390	5860	5410
1	83 694	133 220	22200	18500	15800	13900	12300	11100	10100	9260	8550	7930	7410	6940	6170	5550	5050	4630	4270
2	66 358	105 530	17600	14700	12600	11000	9800	8820	8020	7350	6790	6300	5880	5510	4900	4410	4010	3670	3390
3	52 624	83 640	13900	11600	9970	8720	7750	6980	6340	5810	5370	4980	4650	4360	3880	3490	3170	2910	2680
4	41 738	66 370	11100	9290	7960	6970	6190	5570	5070	4640	4290	3980	3710	3480	3100	2790	2530	2320	2140
5																			

The loss due to corona will not be excessive with any of the above conductors used at sea level for the voltages stated. For elevations above sea level, check the values with Table XXII, especially for the smaller conductors. On long circuits of high voltage, the effect of charging current (also corona and leakage losses) will be to increase or decrease the I^2R loss, depending on the amount of load and its power-factor. See Fig. 18

TABLE XIX—QUICK ESTIMATING TABLE

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.65% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

66 000 VOLTS DELIVERED

B & S No.	Copper Area in Circular Mils	Aluminum Area in Circular Mils	20 Miles	24 Miles	28 Miles	32 Miles	36 Miles	40 Miles	44 Miles	48 Miles	52 Miles	56 Miles	60 Miles	64 Miles	72 Miles	80 Miles	88 Miles	96 Miles	104 Miles
	650 000	1 033 000	210000	174000	150000	131000	116000	105000	95300	87500	80800	75000	69800	65500	58000	52500	47600	43700	40400
	600 000	954 000	193000	161000	138000	120000	107000	96200	87500	80200	74200	68800	64300	60000	53500	48100	43700	40100	37100
	550 000	874 500	178000	148000	127000	111000	99000	89200	81000	74200	68700	63800	59500	55500	49500	44600	40500	37100	34300
	500 000	795 000	161000	134000	115000	101000	89500	80500	73300	67000	62000	57600	53700	50500	44700	40200	36600	33500	31000
	450 000	715 500	145000	121000	104000	91000	80700	72700	66000	60500	56000	51800	48500	45500	40300	36300	33000	30200	28000
	400 000	636 000	130000	108000	92700	81000	72000	65000	59000	54000	50000	46300	43300	40500	36000	32500	29500	27000	25000
	350 000	556 500	113000	94000	80600	70500	62500	56500	51500	47200	43500	40500	37700	35200	31200	28200	25700	23600	21700
	300 000	477 000	96000	80000	68700	60000	53500	48000	43700	40000	37000	34300	32000	30000	26700	24000	21800	20000	18500
	250 000	397 500	80500	67200	57600	50500	44700	40300	36700	33600	31000	28800	26900	25200	22300	20100	18300	16800	15500
0000	211 600	336 420	68000	56700	48700	42500	37700	34000	31000	28300	26100	24300	22700	21200	18800	17000	15500	14100	13000
000	167 772	266 800	54000	45000	38600	33900	30100	27100	24600	22500	20800	19300	18100	16900	15000	13500	12300	11200	10400
00	133 079	211 950	42500	35500	30400	26600	23700	21300	19300	17700	16300	15200	14200	13300	11800	10600	9650	8850	8150
0	105 560	167 800	34000	28400	24300	21300	18700	17000	15500	14200	13100	12200	11300	10600	9450	8500	7750	7100	6550
1	83 694	133 220	27000	22500	19300	16800	15000	13500	12200	11200	10400	9600	8900	8400	7500	6750	6100	5600	5200
2	66 358	105 530	21300	17800	15200	13300	11800	10600	9700	8900	8200	7600	7100	6650	5900	5300	4850	4450	4100
3	52 624	83 640	16800	14000	12000	10500	9300	8400	7600	7000	6400	6000	5600	5250	4670	4200	3820	3500	3240
4	41 738	66 370	13500	11200	9600	8400	7500	6700	6100	5600	5200	4800	4500	4210	3750	3375	3070	2810	2600

70 000 VOLTS DELIVERED

B & S No.	Copper	Aluminum	36 Miles	40 Miles	44 Miles	48 Miles	52 Miles	56 Miles	60 Miles	64 Miles	72 Miles	80 Miles	88 Miles	96 Miles	104 Miles	112 Miles	120 Miles	128 Miles	144 Miles
	650 000	1 033 000	130000	118000	107000	98000	90600	84000	78600	73500	65500	59000	53500	49000	45300	42000	39300	36700	32700
	600 000	954 000	120000	108000	98500	90500	83500	77500	72300	67700	60300	54000	49200	45200	41700	37700	36100	33800	30100
	550 000	874 500	112000	100000	91200	83500	77500	71700	67000	62600	55800	50000	45600	41700	37700	35800	33200	31300	27900
	500 000	795 000	101000	90500	82500	75500	69800	64800	60500	56700	50500	45200	41200	37700	34900	32400	30200	28300	25200
	450 000	715 500	90800	81600	74200	68000	62800	58500	54500	51000	45500	41800	37100	34000	31400	29200	27200	25500	22700
	400 000	636 000	80800	72700	66100	60600	55900	51900	48400	45400	40400	36300	33000	30300	28000	25900	24200	22700	20200
	350 000	556 500	70300	63300	57500	52700	48700	45200	42200	39500	35100	31600	28800	26400	24300	22600	21100	19700	17600
	300 000	477 000	60200	54200	49200	45100	41700	38700	36100	33900	30100	27100	24600	22600	20800	19300	18000	16900	15000
	250 000	397 500	50400	45300	41200	37800	34900	32400	30200	28300	25200	22700	20600	18900	17400	16200	15100	14200	12600
0000	211 600	336 420	42500	38300	34800	31900	29400	27300	25500	23900	21200	19100	17400	15900	14700	13600	12600	11900	10600
000	167 772	266 800	33800	30400	27700	25400	23400	21700	20300	19000	16900	15200	13800	12700	11700	10900	10100	9520	8460
00	133 079	211 950	26700	24000	21800	20000	18400	17100	16000	15000	13300	12000	10900	10000	9240	8580	8000	7500	6670
0	105 560	167 800	21200	19100	17400	15900	14700	13600	12700	12000	10600	9570	8700	7970	7360	6830	6380	6000	5310
1	83 694	133 220	16800	15100	13700	12600	11600	10800	10100	9450	8400	7560	6870	6300	5820	5400	5040	4720	4200
2	66 358	105 530	13300	12000	10900	10000	9240	8580	8000	7500	6670	6000	5460	5000	4620	4290	4000	3750	3330
		83 640	10500	9500	8630	7920	7300	6780	6330	5900	5270	4750	4320	3960	3670	3390	3160	2970	2640

80 000 VOLTS DELIVERED

B & S No.	Copper	Aluminum	36 Miles	40 Miles	44 Miles	48 Miles	52 Miles	56 Miles	60 Miles	64 Miles	72 Miles	80 Miles	88 Miles	96 Miles	104 Miles	112 Miles	120 Miles	128 Miles	144 Miles
	650 000	1 033 000	171000	154000	140000	128000	118000	110000	102000	96000	85500	77000	70000	64000	59000	55000	51000	48000	42700
	600 000	954 000	157000	142000	128000	118000	109000	101000	94500	88500	78500	71000	64000	59000	54500	50500	47700	44200	39200
	550 000	874 500	146000	131000	119000	110000	101000	93600	87600	82000	72800	65500	59500	55000	50500	46800	43800	41000	36400
	500 000	795 000	132000	119000	108000	99000	91500	85000	79000	74200	66000	59500	54000	47500	45700	42500	39500	37100	33000
	450 000	715 500	118000	107000	97000	88800	82000	76200	71000	66700	59200	53500	48500	44400	41000	38100	35500	33300	29600
	400 000	636 000	105000	94900	86300	79100	73000	67800	63300	59300	52700	47400	43100	39500	36500	33900	31600	29600	26300
	350 000	556 500	91800	82600	75100	68900	63600	59000	55100	51600	45900	41300	37500	34400	31800	29500	27500	20800	22900
	300 000	477 000	78600	70800	64300	59000	54400	50500	47200	44200	39300	35400	32100	29500	27200	25300	23600	22100	19600
	250 000	397 500	65800	59200	53800	49400	45600	42300	39500	37000	32900	29600	26900	24700	22800	21100	19700	18500	16400
0000	211 600	336 420	55500	50000	45400	41700	38400	35700	33300	31200	27800	25000	22700	20800	19200	17800	16700	15600	13900
000	167 772	266 800	44200	39800	36200	33100	30600	28400	26500	24800	22100	19900	18100	16600	15300	14200	13200	12400	11000
00	133 079	211 950	34800	31300	28500	26100	24100	22400	20900	19600	17400	15600	14200	13100	12100	11200	10400	9800	8700
0	105 560	167 800	27800	25000	22700	20800	19200	17800	16700	15600	13900	12500	11300	10400	9610	8930	8330	7810	6940
1	83 694	133 220	21900	19700	17900	16400	15200	14100	13100	12300	10900	9880	8980	8230	7600	7050	6580	6170	5490
		105 530	17400	15700	14200	13100	12100	11200	10400	9800	8710	7840	7130	6530	6030	5600	5230	4900	4360

88 000 VOLTS DELIVERED

B & S No.	Copper	Aluminum	36 Miles	40 Miles	44 Miles	48 Miles	52 Miles	56 Miles	60 Miles	64 Miles	72 Miles	80 Miles	88 Miles	96 Miles	104 Miles	112 Miles	120 Miles	128 Miles	144 Miles
	650 000	1 033 000	207000	186000	169000	155000	143000	133000	124000	116000	103000	93000	84500	77500	71500	66500	62000	58000	51500
	600 000	954 000	191000	171000	156000	143000	132000	122000	114000	107000	95500	85500	78000	71500	66000	61000	57000	53500	47700
	550 000	874 500	177000	159000	144000	132000	122000	113000	106000	99500	88200	79500	72000	66000	61000	56500	53000	49700	44100
	500 000	795 000	159000	143000	131000	120000	111000	102000	96000	90000	80000	71500	65500	60000	55500	51000	48000	45000	40000
	450 000	715 500	143000	129000	117000	107000	99000	92000	86000	80600	71500	64500	58500	53500	49500	46000	43000	40300	35700
	400 000	636 000	128000	115000	105000	96000	88700	82500	76800	72000	64000	57500	52500	48000	44300	41200	38400	36000	32000
	350 000	556 500	111000	100000	91200	83500	77200	71600	67000	62700	55800	50000	45600	41750	38600	35800	33500	31300	27900
	300 000	477 000	95000	85500	78000	71500	66000	61200	57200	53600	47700	42700	39000	35700	33000	30600	28600	26800	23800
	250 000	397 500	79500	71500	65000	59800	55000	51000	47800	44800	39800	35700	32500	29900	27500	25500	23900	22400	19900
0000	211 600	336 420	67200	60500	55000	50500	46200	43200	40300	37800	33600	30200	27500	25200	23200	21600	20100	18900	16800
000	167 772	266 800	53500	48200	43800	40200	37000	34400	32200	30200	26800	24100	21900	20100	18500	17200	16100	15100	13400
00	133 079	211 950	42300	38000	34600	31700	29300	27200	25400	23800	21200	19100	17300	15800	14600	13600	12700	11900	10600
0	105 560	167 800	33700	30300	27600	25300	23300	21700	20200	19000	16800	15100	13800	12600	11600	10800	10100	9500	8400
1	83 694	133 220	26500	23900	21700	19900	18300	17000	15900	14900	13200	11900	10850	9900	9100	8500	7900	7400	6600
		105 530	21100	19000	17200	15800	14600	13500	12600	11900	10500	9500	8400	7900	7300	6700	6300	5900	5200

The loss due to corona will not be excessive with any of the above conductors used at sea level for the voltages stated. For elevations above sea level, check the values with Table XXII, especially for the smaller conductors. On long circuits of high voltage, the effect of charging current (also corona and leakage losses) will be to increase or decrease the I R loss, depending on the amount of load and its power-factor. See Fig. 13

TABLE XX—QUICK ESTIMATING TABLE

CONDUCTORS

KILOVOLT-AMPERES, 3 PHASE, WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

	AT 25° C	AT 65° C
FOR LOAD POWER-FACTOR OF 100%	8.66% LOSS	10.0% LOSS
FOR LOAD POWER-FACTOR OF 80%	10.8% LOSS	12.5 LOSS

100 000 VOLTS DELIVERED

B & S NO.	COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	52 MILES	56 MILES	60 MILES	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES
	650 000	1 033 000	185000	172000	160000	150000	133000	120000	109000	100000	92500	86000	80000	75000	66500	60000	54500	50000	46200
	600 000	954 000	170000	158000	148000	138000	123000	110000	101000	92500	85500	79000	74000	69000	61500	55000	50500	46200	42700
	550 000	874 500	158000	147000	137000	128000	114000	103000	93500	85500	79000	73500	68500	64000	57000	51500	46700	42700	39500
	500 000	795 000	143000	132000	124000	116000	103000	93000	84500	77500	71500	66000	62000	58000	51500	46500	42200	38700	35700
	450 000	715 500	128000	119000	111000	104000	93000	83500	76000	69500	64200	59500	55500	52000	46500	41700	38000	34700	32100
	400 000	636 000	114000	106000	98900	92700	82400	74200	67400	61800	57000	53000	49400	46300	41200	37100	33700	30900	28500
	350 000	556 500	99400	92300	86100	80700	71800	69600	58700	53800	49700	46100	43000	40300	35900	32300	29300	26900	24800
	300 000	477 000	85100	79000	73700	69100	61400	55300	50300	46100	42500	39500	36800	34500	30700	27600	25100	23000	21200
	250 000	397 500	71200	66100	61700	57800	51400	46300	42100	38600	35600	33000	30800	28900	25700	23100	21000	19300	17800
0000	211 600	336 420	60100	55800	52100	48800	43400	39000	35500	32500	30000	27900	26000	24400	21700	19500	17700	16200	15000
000	167 772	266 800	47800	44400	41400	38800	34500	31100	28200	25900	23900	22200	20700	19400	17200	15500	14100	12900	11900
00	133 079	211 950	37700	35000	32700	30600	27200	24500	22300	20400	18800	17500	16300	15300	13600	12200	11100	10200	9400
0	105 560	167 800	30000	27900	26000	24400	21700	19500	17700	16300	15000	13900	13000	12200	10800	9800	8900	8100	7500
		133 220	23700	22000	20600	19300	17100	15400	14000	12900	11900	11000	10300	9600	8600	7700	7000	6400	5900
		105 530	18800	17500	16300	15300	13600	12200	11100	10200	9400	8700	8200	7600	6800	6100	5600	5100	4700

110 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	52 MILES	56 MILES	60 MILES	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES
	650 000	1 033 000	224000	208000	194000	182000	162000	146000	133000	121000	112000	104000	97000	91000	81000	73000	66500	60500	56000
	600 000	954 000	206000	191000	178000	167000	148000	134000	122000	111000	103000	95500	89000	83500	74000	67000	61000	55500	51500
	550 000	874 500	191000	177000	165000	155000	138000	124000	113000	103000	95500	88500	82500	77500	69000	62000	56500	54500	47700
	500 000	795 000	173000	160000	150000	140000	125000	112000	102000	93500	86000	80000	75000	70000	62500	56000	51000	46700	43000
	450 000	715 500	155000	144000	139000	126000	112000	101000	91600	84000	77600	72000	67000	63000	56000	50500	45800	42000	38800
	400 000	636 000	138000	128000	119000	112000	99700	89700	81600	74800	69000	64100	59800	56100	49800	44900	40800	37400	34500
	350 000	556 500	120000	111000	104000	97700	86800	78100	71000	65100	60100	55800	52100	48800	43400	39100	35500	32500	30000
	300 000	477 000	103000	95600	89200	83600	74300	66900	60800	55700	51500	47800	44600	41800	37210	33400	30400	27900	25700
	250 000	397 500	86100	80000	74700	70000	62200	56000	50600	46600	43100	40000	37300	35000	31100	28000	25300	23300	21500
0000	211 600	336 420	72700	67500	63000	59100	52500	47200	42900	39400	36300	33700	31500	29500	26200	23600	21500	19700	18200
000	167772	266 800	57800	53700	50100	47000	41800	37600	34200	31300	28900	26800	25100	23500	20900	18800	17100	15600	14400
00	133 079	211 950	45600	42300	39500	37000	32900	29600	26900	24700	22800	21100	19800	18500	16400	14800	13500	12300	11400
		167 800	36300	33700	31500	29500	26200	23600	21400	19700	18100	16900	15700	14700	13129	11800	10700	9800	9100
		133 220	28700	26600	24900	23300	20700	18600	16900	15500	14300	13300	12400	11670	10300	9300	8500	7800	7200

120 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES	224 MILES	240 MILES	256 MILES
	650 000	1 033 000	216000	192000	173000	157000	144000	133000	123000	115000	108000	96000	86500	78500	72000	66500	61500	57500	54000
	600 000	954 000	199000	177000	160000	145000	133000	123000	114000	106000	100000	88500	80000	72500	66500	61500	57000	53000	50000
	550 000	874 500	184000	164000	148000	134000	123000	113000	105000	98500	92500	82000	74000	67000	61500	56500	52500	49200	46200
	500 000	795 000	167000	148000	133000	121000	111000	102000	95000	89000	83500	74000	66500	60500	55500	51000	47500	44500	41700
	450 000	715 500	150000	133000	120000	109000	100000	92500	86000	80000	75000	66500	60000	54500	50000	46200	43000	40000	37500
	400 000	636 000	133000	119000	107000	97100	89000	82100	76300	71200	66700	59300	53400	48500	44500	41100	38100	35600	33300
	350 000	556 500	116000	103000	93000	84500	77500	71500	66400	62000	58100	51600	46500	42200	38700	35800	33200	31000	29100
	300 000	477 000	99000	88500	79600	72400	66400	61200	56900	53100	49800	44200	39800	36200	33100	30600	28400	26500	24800
	250 000	397 500	83000	74100	66700	60600	55500	51200	47600	44400	41600	37000	33300	30300	27700	25600	23800	22200	20800
0000	211 600	336 420	70300	62500	56200	51100	46800	43200	40100	37500	35100	31200	28100	25500	23400	21600	20100	18700	17500
000	167772	266 800	56000	49700	44800	40700	37300	34400	32000	29800	28000	24800	22300	20300	18600	17200	16000	14900	14000
		211 950	44100	39200	35300	32100	29400	27100	25200	23500	22000	19600	17600	16000	14700	13500	12600	11700	11000
		167 800	35100	31200	28100	25600	23400	21600	20100	18700	17500	15600	14000	12700	11700	10800	10000	9300	8800

132 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES	224 MILES	240 MILES	256 MILES
	650 000	1 033 000	262000	233000	210000	191000	175000	161000	150000	140000	131000	116000	105000	95500	87500	80500	75000	70000	65500
	600 000	954 000	242000	215000	193000	176000	161000	148000	138000	129000	121000	107000	96500	88000	80500	74000	69000	64500	60500
	550 000	874 500	223000	198000	178000	162000	149000	137000	128000	109000	112000	99000	89000	81000	74500	68500	64000	54500	56000
	500 000	795 000	202000	179000	161000	147000	134000	124000	115000	107000	101000	89500	80500	73500	67000	62000	57500	53500	55050
	450 000	715 500	182000	161000	145000	132000	121000	112000	104000	96800	90800	80500	72500	66000	60500	56000	52000	48400	45400
	400 000	636 000	162000	144000	129000	118000	108000	100000	92700	86500	81000	72000	64500	59000	54000	50000	46300	43200	40500
	350 000	556 500	140000	125000	112000	102000	93500	86500	80500	75000	70300	62500	56000	51000	46750	43200	40200	37500	35100
	300 000	477 000	120000	107000	96800	87500	80000	74000	68700	64000	60000	53500	48000	43700	40000	37000	34300	32000	30000
	250 000	397 500	100000	89300	80500	73000	67000	62000	57500	53700	50200	44600	40200	36500	33500	31000	28700	26800	25100
0000	211 600	336 420	85000	75500	68000	61800	56700	52300	48500	45300	42500	37700	34000	30900	28300	26100	24200	22600	21200
		266 800	67700	60200	54200	49300	45200	41700	38700	36100	33900	30100	27100	24600	22600	20800	19300	18000	16900
		211 950	53500	47500	42700	38800	35600	32900	30500	28500	26800	23700	21300	19400	17800	16400	15200	14200	13400

140 000 VOLTS DELIVERED

B & S NO.	COPPER	ALUMINUM	64 MILES	72 MILES	80 MILES	88 MILES	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES	224 MILES	240 MILES	256 MILES
	650 000	1 033 000	295000	262000	236000	215000	196000	181000	168000	157000	147000	131000	118000	107000	98000	90500	84000	78500	73500
	600 000	954 000	272000	242000	217000	197000	181000	167000	156000	145000	136000	121000	108000	98500	90500	83500	78000	72500	68000
	550 000	874 500	251000	223000	201000	182000	167000	155000	143000	134000	126000	111000	100000	91000	83500	77500	71500	67000	63000
	500 000	795 000	226000	201000	181000	165000	151000	140000	130000	121000	113000	101000	90700	82500	75600	69800	64800	60500	56700
	450 000	715 500	203000	181000	163000	148000	135000	125000	116000	108000	102000	90400	81400	74000	67800	62600	58100	54200	50800
	400 000	636 000	181000	161000	145000	132000	121000	112000	104000	96900	90800	80800	72700	66100	60600	55900	51900	48400	45400
	350 000	556 500	158000	140000	127000	115000	105000	97400	90400	84400	79100	70300	63300	57500	52700	48700	45200	42200	39600
	300 000	477 000	135000	120000	108000	98500	90300	83400	77400	72200	67700	60200	54200	49300	45100	41700	38700	36100	33800
	250 000	397 500	113000	101000	90700	82500	75600	69800	64800	60500	56700	50400	45400	41200	37800	34900	32400	30200	28300
0000	211 600	336 420	95700	85000	76500	69600	63800	58900	54700	51000	47800	42500	38300	34800	31900	29400	27300	25500	23900
		266 800	76200	67700	61000	55400	50800	46900	43600	40600	38100	33800	30500	27700	25400	23400	21800	20300	19000
		211 950	60000	53400	48000	43700	40000	36900	34300	32000	30000	26700	24000	21800	20000	18500	17100	16000	15000

The loss due to corona will not be excessive with any of the above conductors used at sea level for the voltages stated. For elevations above sea level, check the values with Table XXII, especially for the smaller conductors. On long circuits of high voltage, the effect of charging current (also corona and leakage losses) will be to increase or decrease the I^2R loss, depending on the amount of load and its power-factor. See Fig. 13

HEATING LIMITATIONS

The k.v.a. values given in these tables do not take into account the heating and consequently carrying capacity of the conductors. This may be ignored in the case of the longer overhead high-voltage transmission circuits. For very short circuits (especially for the lower voltages and particularly for insulated or concealed conductors) the carying capacity (safe heating limits) of the conductors must be carefully considered. approximately the point at which the carrying capacity of that particular conductor is reached if insulated and installed in a fully loaded four duct line. If the conductor is to be installed in a duct line having more than four ducts its capacity will be still further reduced. The position of this line is based upon the use of lead covered, paper insulated, three conductor, copper cables for sizes up to 700 000 circ. mils and of lead covered, paper insulated, single conductor, copper cables for the larger sizes. In other words, the position of this heavy

TABLE XXI—QUICK ESTIMATING TABLE

CONDUCTORS — KILOVOLT-AMPERES. 3 PHASE. WHICH MAY BE DELIVERED AT THE FOLLOWING VOLTAGES OVER THE VARIOUS CONDUCTORS FOR THE DISTANCES STATED. BASED UPON THE FOLLOWING I^2R LOSS (EFFECT OF CHARGING CURRENT NEGLECTED)

FOR LOAD POWER-FACTOR OF 100%—8.66% LOSS AT 25° C — 10.0% LOSS AT 65° C
FOR LOAD POWER-FACTOR OF 80%—10.8% LOSS AT 25° C — 12.5 LOSS AT 65° C

154,000 VOLTS DELIVERED

COPPER AREA IN CIRCULAR MILS	ALUMINUM AREA IN CIRCULAR MILS	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES	224 MILES	240 MILES	256 MILES	288 MILES	320 MILES	352 MILES	384 MILES
650 000	1 033 000	237 500	219 500	203 500	190 000	178 500	158 500	142 500	129 500	118 500	109 500	102 000	95 000	89 000	79 200	71 200	64 700	59 300
600 000	954 000	219 000	202 000	187 500	175 000	164 000	145 500	131 500	119 500	109 500	101 000	93 500	87 500	82 000	73 000	65 700	59 700	54 700
550 000	874 500	202 000	187 000	173 000	161 500	151 500	134 500	121 300	110 200	101 000	93 300	86 600	80 800	75 700	67 300	60 700	55 200	50 500
500 000	795 000	183 500	169 000	157 000	146 500	137 500	122 000	109 500	99 500	91 500	84 500	78 500	73 200	68 700	61 000	55 000	50 000	45 700
450 000	715 500	164 500	152 000	141 000	131 600	123 500	109 500	98 800	89 800	82 300	76 000	70 600	65 800	61 700	54 800	49 300	44 900	41 200
400 000	636 000	147 000	136 000	126 500	117 500	110 500	98 000	88 300	80 200	73 500	67 800	63 000	58 800	55 200	49 000	44 200	40 200	36 800
350 000	556 500	128 200	118 500	109 500	102 500	96 000	85 300	76 800	69 800	64 000	59 200	54 800	51 200	48 000	42 700	38 500	35 000	32 100
300 000	477 000	109 500	102 000	94 000	87 500	82 000	73 000	65 700	59 700	54 700	50 500	46 800	43 700	41 000	36 500	32 800	29 800	27 300
250 000	397 500	91 500	84 500	78 500	73 200	68 600	61 000	54 800	49 800	45 700	42 200	39 200	36 600	34 300	30 500	27 400	24 900	22 900
	336 420	77 200	71 200	66 200	61 600	57 700	51 500	46 200	42 000	38 500	35 600	33 000	30 800	28 900	25 700	23 200	21 000	19 200
	266 800	61 500	56 800	52 700	49 200	46 100	41 000	36 900	33 600	30 700	28 400	26 400	24 600	23 100	20 500	18 400	16 800	15 400

187,000 VOLTS DELIVERED

COPPER	ALUMINUM	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES	224 MILES	240 MILES	256 MILES	288 MILES	320 MILES	352 MILES	384 MILES
650 000	1 033 000	350 000	323 000	300 000	280 000	263 000	234 000	210 000	191 000	175 000	161 500	150 000	140 000	131 500	116 500	105 000	95 200	87 500
600 000	954 000	323 000	297 000	277 000	258 000	242 000	215 000	193 500	176 000	161 500	149 000	138 000	129 000	121 000	107 500	97 000	88 000	80 700
550 000	874 500	299 000	275 000	256 000	239 000	224 000	199 000	179 000	162 500	149 000	137 500	128 000	119 500	112 000	99 500	89 500	81 500	74 600
500 000	795 000	270 000	250 000	232 000	216 000	203 000	180 000	162 000	147 500	135 000	125 000	115 500	108 000	101 000	90 000	81 000	73 700	67 600
450 000	715 500	243 000	225 000	209 000	194 500	182 000	162 000	145 500	132 500	121 500	112 000	104 000	97 200	91 000	81 000	73 000	66 200	60 700
400 000	636 000	217 000	200 000	185 500	173 500	162 500	144 500	130 000	118 500	108 500	100 000	93 000	86 700	81 300	72 200	65 000	59 000	54 200
	556 500	189 000	174 500	162 000	151 000	141 500	126 000	113 500	103 000	94 500	87 000	81 000	75 500	70 800	63 000	56 700	51 500	47 200
	477 000	161 000	149 000	138 000	129 000	121 000	107 500	96 500	88 000	80 500	74 300	69 000	64 500	60 500	53 700	48 300	44 000	40 300
	397 500	134 500	124 500	115 500	107 500	101 000	89 800	80 800	73 500	67 300	62 200	57 800	53 800	50 500	44 800	40 400	36 800	33 700
	336 420	114 000	105 000	97 500	91 000	85 200	75 800	68 200	62 000	57 000	52 600	48 800	45 500	42 700	38 000	34 200	31 000	28 500

220,000 VOLTS DELIVERED

COPPER	ALUMINUM	96 MILES	104 MILES	112 MILES	120 MILES	128 MILES	144 MILES	160 MILES	176 MILES	192 MILES	208 MILES	224 MILES	240 MILES	256 MILES	288 MILES	320 MILES	352 MILES	384 MILES
650 000	1 033 000	485 000	447 000	417 000	388 000	364 000	323 000	291 000	265 000	243 000	224 000	208 000	194 000	182 000	162 000	145 500	132 500	121 500
	954 000	447 000	413 000	383 000	358 000	336 000	298 000	268 000	244 000	224 000	207 000	192 000	179 000	168 000	149 000	134 500	122 000	112 000
	874 500	413 000	382 000	354 000	331 000	310 000	276 000	249 000	226 000	207 000	191 000	177 500	165 500	155 000	138 000	124 000	112 800	103 500
	795 000	374 000	345 000	321 000	299 000	281 000	249 000	224 000	204 000	187 000	173 000	160 000	149 500	140 500	124 500	112 500	102 000	93 500
	715 500	336 000	310 000	288 000	269 000	252 000	224 000	202 000	183 500	168 000	155 000	144 000	134 500	126 000	112 000	101 000	92 000	84 200
	636 000	300 000	277 000	257 000	240 000	225 000	200 000	180 000	163 500	150 000	138 500	128 500	120 000	112 500	100 000	90 000	82 000	75 000

The loss due to corona will not be excessive with any of the above conductors used at sea level for the voltages stated. For elevations above sea level, check the values with Table XXII, especially for the smaller conductors. On long circuits of high voltage, the effect of charging current (also corona and leakage losses) will be to increase or decrease the I^2R loss, depending on the amount of load and its power-factor. See Fig. 13

For circuits of short length the carrying capacity of conductors will frequently determine these sizes and not the economic transmission loss. The carrying capacity of bare copper conductors suspended in air and of insulated copper conductors in duct lines are given in tables XXIII and XXIV, both of which are to appear in subsequent articles.

Running diagonally across each table from XII to XVII inclusive, is a heavy line. The point at which this heavy line intersects the horizontal line containing the k.v.a. values for a given size of conductor indicates line is based upon the k.v.a. values for carrying capacity given in Table XXIV and is placed upon the tables as a warning that the heating limit capacity of the conductors must be considered. To illustrate, suppose 220 volts is to be delivered, over 1 000 000 circ. mil, insulated, single conductor, copper cables in a fully loaded four duct conduit. Table XII indicates that 189 k.v.a. can be transmitted over these conductors a distance of 2000 ft. without overheating the cable. If it is desired to transmit 378 k.v.a. a distance of 1000 feet, the fact that this value occurs to the left of the heavy line, indicates that it is beyond the safe carrying capacity for this size conductor in a four duct line. Reference to Table XXIV will show that 297 k.v.a. is the maximum capacity of this cable under the conditions stated. In this case, either a larger conductor, or two or more smaller conductors must be used to prevent overheating. This will result in a less loss than those upon which the table k.v.a. values are based, and in this case the heating of the cable will probably determine the size to use.

EFFECT OF CHARGING CURRENT IN ABOVE I^2R LOSS VALUES

As stated previously, the percent I^2R losses in the quick estimating tables are based upon the load current and therefore do not take into account the effect of the charging current which is of a distributed nature and superimposed upon the load current. The effect of the charging current is to increase or decrease the current in the circuit by an amount depending upon the relative values of the lagging and leading quadrature components of the current in the circuit.

For instance assume that the power-factor of the load is unity. In such case there is no quadrature component in the load current. If, however, the circuit is of considerable length, and particularly if the frequency is 60 cycles, there will be an appreciable amount of charging current (quadrature leading component) added vectorially to the load current. The sum of these two currents in quadrature with each other will result in an increase of current in the circuit with a consequent increase in the I^2R loss. Thus the effect of charging current in a circuit delivering a load of 100 percent power-factor will always be to increase the I^2R loss.

If, however, the power-factor of the load is lagging, there will be a lagging component in the load current. The charging or leading current will be practically in opposition to the lagging component of the load current and will therefore tend to cancel or neutralize the lagging component of the load current. The result will be a reduction of the current in the circuit and consequently in the I^2R loss. But if the circuit is very long, particularly if the frequency is 60 cycles and the load power-factor is near unity (lagging component in load current small) the comparatively large leading component (charging current) will not only neutralize the lagging component of the load current, but will produce a leading power-factor at points along the circuit. If the charging current is sufficiently high it will increase the current, causing an increase in the I^2R loss. Thus the effect of charging current in circuits delivering a lagging load is to decrease the I^2R loss up to a certain amount and then, if the charging current is sufficiently large, to increase I^2R loss.

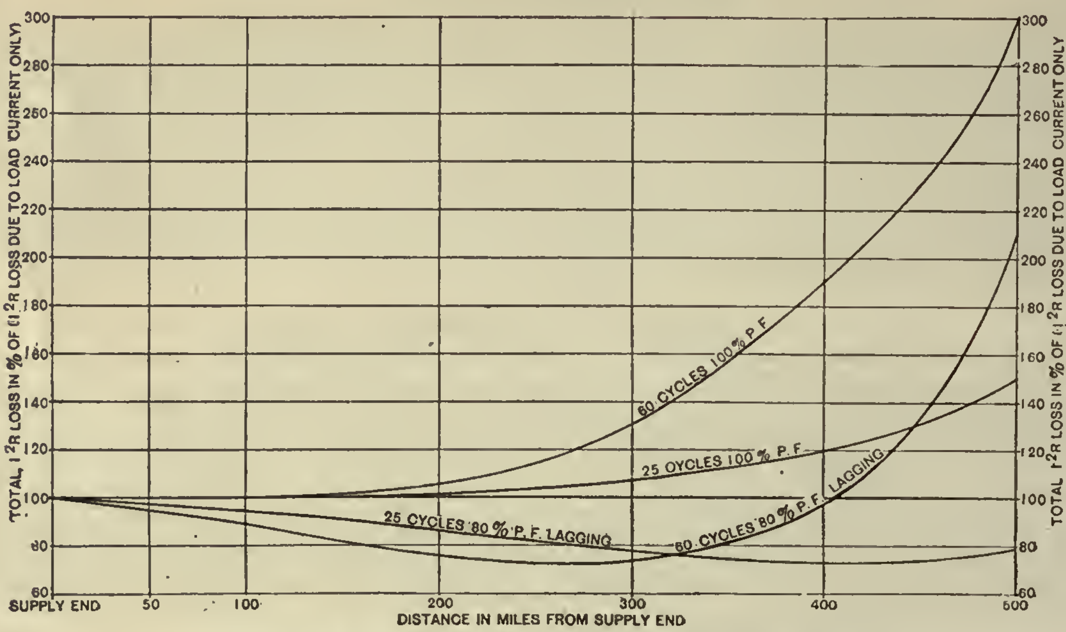


FIG. 13—EFFECT OF CHARGING CURRENT ON I^2R TRANSMISSION LOSS

The curves represent (for certain circuits) an approximation of the resultant I^2R loss, compared to what it would have been if there were no charging current present in the circuit. The effect of the charging current superimposed upon the receiver current is either to increase or to decrease the I^2R loss of the circuit depending principally upon the relative amount of the leading and lagging components of the current in the circuit.

The curves in Fig. 13 show this effect for 25 and 60 cycle circuits delivering loads of unity power-factor; also loads of 80 percent lagging power-factor for circuits up to 500 miles long. It will be seen that for circuits 300 miles long the effect of charging current will be to reduce the I^2R loss by approximately 25 percent if the load is 80 percent lagging. If the load power-factor is unity the I^2R loss will be increased approximately 10 percent for these particular problems if the frequency is 25, and 30 percent if the frequency is 60 cycles.

The curves in Fig. 13 show that for circuits 500 miles long, in which the entire charging current is furnished from one end of the circuit, the effect of this charging current is to increase the I^2R loss by 300 percent if the frequency is 60 cycle and the load power-factor 100 percent. In other words a large part of the current in the circuit for such a long 60 cycle circuit is charging current so that the effect of the load current on the I^2R loss is comparatively small. Of course such a long circuit, unless fed from two or more generating stations located at widely separated points along the transmission line, would not be commercially practical.

CHAPTER IV
CORONA EFFECT

In 1898 Dr. Chas. F. Scott presented a paper before the A.I.E.E. describing experimental tests (made during several years previous) relating to the energy loss between conductors due to corona effect. These investigations began at the Laboratory at Pittsburgh and were continued at Telluride, Colorado, in conjunction with the engineers of the Telluride Power Company. Preliminary observations were made by Mr. V. G. Converse and were continued in notable measurements by Mr. R. D. Mershon. These investigations were later followed by the work of Professor Ryan, by Mr. R. D. Mershon, Mr. F. W. Peek, Jr., Dr. J. B. Whitehead, Mr. G. Faccioli and others. The electrical profession is particularly indebted to Mr. Peek and Dr. Whitehead for the large amount of both practical and theoretical data which they have presented to the electrical profession on the subject. Mr. Peek developed and presented the empirical formulas which follow, for determining the disruptive critical voltage, the visual critical voltage and the power loss due to corona effect. The close accuracy of Mr. Peek's formulas has been confirmed by various investigators in different sections of the country. The following deductions concerning corona have to a large extent been previously presented by Mr. Peek.

CORONA, manifesting its presence usually by an electrostatic glow or luminous discharges, and audibly by a hissing sound, was clearly observed and studied in connection with electrostatic machines. It did not become a serious factor to be considered in connection with the design of commercial electrical apparatus until the increasing generator and transmission voltage emphasized its importance.

Although it is usual to think of corona effect only in connection with high-voltage transmission lines, it has received not a little thought of late by the designers of high-voltage generators and motors, notably large, high-voltage turbogenerators. By effectively insulating the portion of the conductor embedded in the iron of the armatures of alternating-current machines, particularly with mica, punctures to ground due to corona effect are not likely to occur. However, at the ends of the armature coils (where it is difficult to employ mica for insulating), where air is partially depended upon as an insulating medium between coils and ground, corona may appear. The presence of these corona stresses results in disintegrating and weakening some kinds of insulating materials, causing them to break down after a period of service. This deterioration of insulation may be due to local heating, mechanical vibration or chemical formations in the overstressed air, such as ozone, nitric acid, etc.

Higher voltages are being chosen as an economic means for reducing loss in transmission. These higher voltages may result in corona loss far in excess of the saving in transmission loss due to the adaptation of the higher voltages. It is, therefore, pertinent that particular consideration be given to the limitation of corona loss when the choice of conductors is made. This consideration will sometimes make it desirable to take advantage of the higher critical voltage limits of aluminum conductors (with steel reinforced centers) of an equivalent resistance, due to their greater diameter; or it may be desirable to obtain the necessary larger diameter by the use of copper conductors having some form of nonconducting centers or, for still larger diameters, of aluminum conductors having such centers, in order to avoid skin effect. The use of copper conductors having hemp centers has in some instances given mechanical trouble.

The critical voltage at which corona becomes manifest, is not constant for a given line, but is somewhat dependent upon atmospheric conditions. Assuming a line employing conductors just within the critical voltage limitations for the conditions to be met, the corona loss in such a line would be almost negligible during fair weather, but during stormy weather, (particularly during snowstorms) this corona loss would be many times what it is during fair weather. On the other hand, since the storm will usually not appear over the whole length of lines at the same time and since storms occur only at intervals, it may often be economical to allow this loss to reach fairly high values during storms. Fog, sleet, rain and snowstorms lower the critical voltage and increase the losses. The effect of snow is greater than any other weather condition. Increase in temperature or decrease in barometric pressure lowers the voltage at which visual corona starts.

The critical voltage increases with both the diameter of conductors and their distance apart. This sometimes makes it desirable to use aluminum conductors as previously stated. It also increases with the horizontal or vertical arrangement of conductors, due to the fact that the two outside conductors considered as a pair are twice as far apart as are the other pairs. The same general rules apply to stranded conductors as to solid conductors, the actual diameter of the former being considered as the effective diameter of the conductor.

The losses due to corona effect increase very rapidly with increase in voltage after the critical voltage has been reached. A long transmission line having considerable capacitance may deliver a higher voltage than appears at the generator end of the line due to capacitance effect. The corona loss would in this case be greater per mile at the receiving end than at the sending end of the line.

The magnitude of the losses, as well as the critical voltage, is affected by atmospheric conditions;—hence they probably vary with the particular locality and the season of the year. Therefore, for a given locality, a voltage which is normally below the critical point, may at times be above the critical voltage, depending upon changes in the weather.

The material of the conductors does not seem to affect the losses. Sometimes the conductors of new transmission lines, when first placed in service will show visual corona, which may entirely disappear after a few hours or weeks of service. This may be due to scratches, particles of foreign substances, etc., on the conductors which are eliminated after the voltage stress has been kept on the conductors for a short time. Under such conditions the corona loss will also become less as the visual effect disappears.

The loss of power due to corona effect increases with frequency and increases as the square of the excess voltage above a certain critical voltage referred to as the "disruptive critical voltage" e_0. This disruptive critical voltage is that voltage, at which a certain definite and constant potential gradient is reached at the conductor surface. This gradient g_0 is 30 kv maximum (21.1 kv effective) per centimeter, or 76.2 kv maximum (53.6 kv effective) per inch. These values are based upon an air density at sea level (25° C., 29.92 inches or 76 cm. barometer). This gradient is independent of the size of conductors and their distance apart, but is proportional to the air density, that is to the barometric pressure and the absolute temperatures. It may be considered as the dielectric strength of air. The presence of corona at a certain point of the system shows that a critical electric stress has been exceeded at that point. The corona loss is also proportional to the square root of the conductor radius r and inversely proportional to the square root of the conductor spacing.

The law by which corona losses increase with the voltage does not give a very steep curve, but a rather mild curve following the quadratic law at and above the critical limit. In other words there is no sharp elbow in the curve above which the losses increase very rapidly with the voltage and which could be adopted as the normal operating point of the circuit.

Table XXII, indicating the voltage limitations due to corona effect, has been worked up from Mr. F. W. Peek's formula as indicated at the bottom of the table. The values in this table are conservative and may in many cases be exceeded. They are the effective e_0 disruptive critical voltage between conductors for fair weather based upon δ values for 25 degrees C. (77 degrees F) and m_0 values of 0.87 for cable and 0.93 for wire. With these table values, corona loss should not be excessive during storms. If the values of Table XXII indicate that the conductors contemplated are close to the limit due to corona effect, a careful check should be made by the formula to determine definitely the corona loss for such conductors under storm operating conditions.

F. W. PEEK'S CORONA FORMULAE

Disruptive Critical Volts, Fair Weather (parallel wires)

$$e_0 = 2.302\, m_0 g_0 \,\delta\, r \log_{10}\frac{s}{r} \quad \ldots\ldots (20)$$

effective kv to neutral,—

Visual Critical Volts—Fair Weather (parallel wires)

$$e_v = 2.302\, m_v g_0 \,\delta\, r \left(1 + \frac{0.189}{\sqrt{r\,\delta}}\right) \log_{10}\frac{s}{r} \quad \ldots (21)$$

effective kv to neutral

Power Loss (fair weather)—

$$P = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{s}}\,(e - e_0)^2\, 10^{-5} \quad \ldots\ldots (22)$$

kw per mile of each conductor

Power Loss (Storm)—Storm power loss is higher and can generally be found with fair approximation by assuming e_0 = 0.80 times fair weather e_0. It generally works out in practice that the e_0 voltage is the highest that should be used on transmission lines .... (22A)

All of the above voltages are to neutral. To find voltages between lines multiply by 1.73 for three-phase, and by 2 for single phase.

Notation—

e = Effective applied voltage in kv to neutral. (This will vary at different points of the circuit and at different loads. At low loads and long lines of high voltage it may be higher at the receiving end than at the generator end due to inductive capacitance)

e_0 = effective disruptive critical voltage in kv to neutral. It is the voltage that gives a constant break down gradient for air of 76 kv maximum per inch, the "elastic limit" at which the air breaks down. Visual corona does not start at the disruptive critical voltage, but at a higher voltage e_v.

e_v = effective visual critical kv to neutral (voltage at which visual corona starts)

P = power loss in fair weather in kw per mile of *single* conductor.

δ = $\frac{17.9b}{459 + t}$. This takes care of the effect of altitude and temperature, (air density). It is 1 at 25 degrees C. (77 degrees F.) and 29.92 inches (76 cm.), barometric pressure.

g_0 = 53.6 kv per inch effective (disruptive gradient of air)

b = barometric pressure in inches.

t = maximum temperature in degrees F.

f = frequency in cycles per second.

m_0 = irregularity factor.
= 1 for polished wires.
= 0.98 to 0.93 for roughened or weathered wire.
= 0.87 to 0.83 for cables.

m_v = m_0 for wires (1 to 0.93)
= 0.72 for local corona all along cables (7 strands)
= 0.82 for decided corona all along cables (7 strands)

r = radius of conductor in inches.

s = spacing in inches between conductor centers, based upon the assumption of a symmetrical triangular arrangement. For three-phase irregular flat or triangular spacing take $s = \sqrt[3]{ABC}$. For three-phase regular flat spacing take $s = 1.26A$.

Theoretically, if the conductors were perfectly smooth, no loss would occur until the critical voltage, e_v is reached, when the loss should suddenly take a definite value, equal to that calculated by quadratic law, with e_v as the applied voltage and e_0 as the critical voltage in the equation. It should then follow the quadratic law for all higher voltages. On the weathered conductors used in practice, the quadratic law is followed over the whole range of voltage, starting at e_0.

Example:—In order to show the variation in corona loss at different voltages and for different weather conditions, Table E has been calculated for No. 0 stranded copper conductors (105 560 circ. mils, 0.373 in. diameter) and for steel reinforced aluminum conductors (167 800 circ. mils, 0.501 in. diameter) having an quivalent resistance but greater diameter. F. W. Peek's formulas were used and the following assumptions were made:—

f = 60 cycles.
m_0 = 0.87
m_v = 0.72
g_0 = 53.6

r = 0.186 in. for copper = 0.250 in. for aluminum.
s = 144 inches (delta arrangement of conductors).
b = 28.9 corresponding to an altitude of 1000 feet.
t = 77 degrees F. δ therefore = 0.967.

$\frac{s}{r}$ = 774 for copper = 576 for aluminum

$\log_{10} 774 = 2.89$ and $\log_{10} 576 = 2.76$

$\sqrt{\frac{r}{s}}$ = 0.036 for copper and 0.0415 for aluminum.

DISRUPTIVE CRITICAL VOLTAGE—Fair Weather

$$e_o = 2.302\, m_o\, g_o\, \delta\, r \log_{10} \frac{s}{r} \quad (20)$$

effective kv to neutral

For the Copper Conductors

$$e_o = 2.302 \times 0.87 \times 53.6 \times 0.967 \times 0.186 \times 2.89$$

= 55.8 kv to neutral (96 500 volts between conductors).

Table XXII gives, by interpolation, the limitation of e_o for above conditions, as 96 500 volts between conductors. To find e_o to neutral for any other altitude or temperatures insert the corresponding values of δ for the altitude and temperature in the formula.

TABLE D—WORKING TABLE—δ (DENSITY) VALUES

Altitude and Temperature Correction Factors

$\delta = \frac{17.9b}{459 + t}$ where b = barometric pressure in inches and t = temperature in degrees F.

Altitude in Feet	Barometer		δ Values for Different Temp.		
	In Inches	In Cm.	0° C. (32° F.)	25° C. (77° F.) *	50° C. (122° F.)
Sea Level	30.0	76.2	1.09	1.00	0.925
500	29.45	74.8	1.07	0.985	0.910
1000	28.90	73.3	1.05	0.967	0.892
1500	28.30	71.8	1.03	0.947	0.873
2000	27.80	70.7	1.01	0.932	0.860
2500	27.25	69.2	0.955	0.912	0.841
3000	26.80	68.0	0.980	0.897	0.827
4000	25.75	65.3	0.940	0.860	0.793
5000	24.70	62.7	0.902	0.827	0.762
6000	23.90	60.7	0.875	0.800	0.738
7000	22.95	58.3	0.840	0.770	0.710
8000	22.05	56.0	0.805	0.738	0.682
9000	21.30	54.1	0.778	0.712	0.657
10 000	20.50	52.1	0.750	0.687	0.633
12 000	19.00	48.3	0.697	0.637	0.588
14 000	17.55	44.7	0.643	0.588	0.543
15 000	16.90	42.9	0.618	0.566	0.522

*This column contains the values for δ which were used in determining the values of e_o in Table XXII. That is, the values for sea level in Table XXII multiplied by these δ values gives the e_o values of the table for the higher altitudes.

For the Aluminum Conductors

$$e_o = 2.302 \times 0.87 \times 53.6 \times 0.967 \times 0.25 \times 2.76$$

= 71.5 kv to neutral (123 500 volts between conductors).

Table XXII gives (by interpolation) the limitation for above conditions as 123 500 volts between conductors.

To find e_o to neutral for any other altitude or temperature insert the corresponding value of δ for that altitude and temperature in the formula.

DISRUPTIVE CRITICAL VOLTAGE—Stormy Weather

e_o during storm = approximately 80 percent e_o during fair weather.

For the Copper Conductors

e_o for storm = 55.8 × 0.80 = 44.6 kv to neutral or 77 000 volts between conductors.

For the Aluminum Conductors

e_o for storm = 715 × 0.80 = 57.2 kv to neutral or 98 800 volts between conductors.

VISUAL CRITICAL VOLTAGE—Fair Weather

$$e_v = 2.302\, m_v\, g_o\, \delta\, r \left(1 + \frac{0.189}{\sqrt{r\,\delta}}\right) \log_{10} \frac{s}{r} \ldots (21)$$

effective kv to neutral

For Copper Conductors

$$e_v = 2.302 \times 0.72 \times 53.6 \times 0.967 \times 0.186 \left(1 + \frac{0.189}{0.424}\right) 2.89$$

= 66.4 kv to neutral (115 000 volts between conductors).

To find e_v to neutral for any other altitude and temperature, insert the corresponding values of δ for that altitude and temperature in the formula above.

For the Aluminum Conductors

$$e_v = 2.302 \times 0.72 \times 53.6 \times 0.967 \times 0.25 \left(1 + \frac{0.189}{0.492}\right) 2.76$$

= 82 kv to neutral (141 500 volts between conductors).

To find e_v to neutral for any other altitude and temperature, insert the corresponding values of δ for that altitude and temperature in the formula above.

POWER LOSS

$$P = \frac{390}{\delta} (f + 25) \sqrt{\frac{r}{s}} (e - e_o)^2\, 10^{-5} \ldots\ldots\ldots (22)$$

kw per mile of each conductor

The corona power loss corresponding to various conditions for the above circuit has been calculated by formulae (22) and (22A). They are given in Table E. However, in order to illustrate the application of the power loss formula the losses for the following conditions are determined below. Assuming that the No. 0 stranded copper conductors will be operated at 105 kv between conductors (60.7 kv to neutral).

For Fair Weather—Max. Temp. 50 degrees C. (122 degrees F.)—E_o = 51.3 *kv.*

$$P = \frac{390}{\delta} (f + 25) \sqrt{\frac{r}{s}} (e - e_o)^2\, 10^{-5} \ldots\ldots\ldots (22)$$

kw per mile of each conductor

$$P = \frac{390}{0.892} (60 + 25) \times 0.036\, (60.7 - 51.3)^2\, 10^{-5}$$

= 1.2 kw per mile of each conductor or 3.6 kw per mile for three conductors.

For Stormy Weather—Max. Temp. 25 degrees C. (77 degrees F.)—E_o = 55.8 × 0.8 = 44.6 *kv.*

$$P = \frac{390}{0.967} (60 + 25) \times 0.036\, (60.7 - 44.6)^2\, 10^{-5} \ (22A)$$

= 3.2 kw per mile of each conductor or 9.6 kw per mile for three conductors.

By applying formula (20) to the above case it develops that the fair weather values of e_o are for 25 degrees C. (77 degrees F.) 96 500 kv and for 50 degrees C. (122 degrees F.) 88 800 kv between conductors. Table XXII values for 25 degrees C. (77 degrees F.) confirm this.

Table E values for corona loss indicate that No. 0 copper conductors can, with 144 inch delta arrangement of conductors and 1000 ft. elevation be used at line voltages as high as 100 000 volts without excessive corona loss during stormy weather. At 100 000 volts and assuming a 25 degrees C. (77 degrees F) temperature during fair weather and storm conditions, the corona losses would be 0.1 kw per mile for fair weather and 6.5 kw per mile for stormy weather. If the transmission is single circuit 100 miles long and without branches, has an average altitude of 1000 feet and the storm condition existed throughout the length of the circuit, the power loss due to corona would be 6.5 × 100 or 650 kw. The capacity of such a circuit at 100 000 volts (see Table XX) would be roughly 15 000 kw at ten percent I^2R loss. The storm corona loss therefore would represent $\frac{650}{15000}$ or 4.3 percent. This, in addition to ten percent I^2R loss, would represent approximately 14 percent loss in transmission during the storm conditions.

In the above case it would probably be considered good engineering (so far as corona loss is concerned)

TABLE XXII—APPROXIMATE VOLTAGE LIMITATIONS RESULTING FROM CORONA

STRANDED COPPER CONDUCTORS

B & S No. and Circular Mils	Diameter in Inches	Elevation in Feet	Limit in Kilovolts between Conductors 3 Phase for Various Spacings X											
			3 Ft.	4 Ft.	5 Ft.	6 Ft.	7 Ft.	8 Ft.	9 Ft.	11 Ft.	13 Ft.	15 Ft.	19 Ft.	25 Ft.
4	.232	Sea Level	54	56	58	60	62	63	64	66	67	69	71	73
		1000	52	54	56	58	60	61	62	64	65	67	69	71
		2000	50	52	54	56	58	59	60	62	63	65	66	68
		4000	46	48	50	51	53	54	55	57	58	59	61	63
		6000	43	45	46	48	49	50	51	52	53	55	56	58
		8000	40	41	43	44	46	46	47	49	49	51	52	54
		10 000	37	38	40	41	42	43	44	45	46	47	48	50
		12 000	34	35	37	38	39	40	40	42	42	44	45	46
		14 000	32	33	34	35	36	37	37	39	39	40	42	43
3	.260	Sea Level	59	62	64	66	68	69	70	72	74	76	78	81
		1000	57	60	62	64	66	67	68	70	72	74	76	79
		2000	55	58	60	62	63	64	65	67	69	71	73	76
		4000	51	53	55	57	58	59	60	62	64	65	67	70
		6000	47	49	51	53	54	55	56	57	59	61	62	64
		8000	43	46	47	49	50	51	51	53	54	56	57	60
		10 000	40	42	44	45	46	47	48	49	51	52	53	55
		12 000	37	39	40	42	43	44	44	46	47	48	49	51
		14 000	34	36	37	39	40	40	41	42	43	45	46	47
2	.292	Sea Level	65	68	71	73	75	77	78	80	82	84	87	90
		1000	63	66	69	71	73	75	76	78	79	81	84	87
		2000	61	64	66	68	70	72	73	75	76	78	81	84
		4000	56	58	61	63	65	66	67	69	70	72	75	77
		6000	52	54	57	58	60	61	62	64	65	67	69	72
		8000	48	50	52	54	55	57	57	59	60	62	64	66
		10 000	44	46	49	50	51	53	53	55	56	57	60	62
		12 000	41	43	45	46	47	49	49	51	52	53	55	57
		14 000	38	40	42	43	44	45	46	47	48	49	51	53
1	.332	Sea Level	72	76	79	81	83	85	87	89	91	93	96	100
		1000	70	73	77	78	80	82	84	86	88	90	93	97
		2000	67	71	74	76	77	79	81	83	85	87	90	93
		4000	62	65	68	69	71	73	75	77	78	80	82	86
		6000	57	60	63	65	66	68	69	71	73	74	77	80
		8000	53	56	58	60	61	62	64	65	67	68	71	74
		10 000	49	52	54	55	57	58	60	61	62	64	66	68
		12 000	45	48	50	51	52	54	55	56	58	59	61	63
		14 000	42	44	46	47	49	50	51	52	53	54	56	59
0	.373	Sea Level	79	83	87	89	92	94	96	98	101	103	107	111
		1000	77	80	84	86	89	91	93	95	98	100	104	107
		2000	74	77	81	83	86	88	89	91	94	96	100	104
		4000	68	71	75	77	79	81	82	84	86	88	92	95
		6000	63	66	69	71	73	75	76	78	81	82	85	89
		8000	58	61	64	66	68	69	71	72	74	76	79	82
		10 000	54	57	60	61	63	64	66	67	69	70	73	76
		12 000	50	52	55	56	58	59	61	62	64	65	68	70
		14 000	46	49	51	52	54	55	56	57	59	60	63	65
00	.418	Sea Level	87	91	95	98	101	103	105	109	111	114	118	122
		1000	84	88	92	95	98	100	102	105	107	110	114	118
		2000	81	85	88	91	94	96	98	102	104	107	110	114
		4000	75	78	82	84	87	89	90	94	95	98	101	105
		6000	69	73	76	78	80	82	82	87	89	91	94	97
		8000	64	67	70	72	74	76	77	80	82	84	87	90
		10 000	59	62	65	67	69	71	72	75	76	78	81	84
		12 000	55	58	60	62	64	65	66	69	70	72	75	77
		14 000	51	53	56	57	59	61	62	64	65	67	69	71
000	.470	Sea Level	95	101	105	108	112	114	116	120	123	125	130	135
		1000	92	98	102	104	108	110	112	116	119	121	126	131
		2000	89	94	98	101	104	106	108	112	115	116	121	126
		4000	81	87	90	93	96	98	100	103	106	108	112	116
		6000	76	80	84	86	89	91	92	96	98	100	104	108
		8000	70	74	77	80	83	84	85	88	90	92	96	99
		10,000	65	69	72	74	77	78	79	82	84	86	89	93
		12,000	60	64	67	68	71	72	73	76	78	79	82	85
		14 000	56	59	61	63	66	67	68	70	72	73	76	79
0000	.528	Sea Level	104	111	115	119	123	125	128	132	136	139	144	150
		1000	101	107	111	115	119	121	124	128	132	134	139	145
		2000	97	103	107	111	115	117	120	123	127	130	134	140
		4000	89	95	99	102	106	107	110	113	117	119	124	129
		6000	83	88	92	95	98	100	102	105	108	111	115	120
		8000	77	81	85	88	91	92	95	97	100	103	106	110
		10,000	71	76	79	81	84	86	88	91	93	95	99	103
		12 000	66	70	73	75	78	79	81	84	86	88	91	95
		14 000	61	65	67	70	72	73	75	77	80	82	84	88

B & S No. and Circular Mils	Diameter in Inches	Elevation in Feet	Limit in Kilovolts between Conductors 3 Phase for Various Spacings X											
			3 Ft.	4 Ft.	5 Ft.	6 Ft.	7 Ft.	8 Ft.	9 Ft.	11 Ft.	13 Ft.	15 Ft.	19 Ft.	25 Ft.
250,000	.575	Sea Level	112	118	124	128	131	134	137	142	145	149	155	160
		1000	109	114	120	124	127	130	132	137	140	144	150	155
		2000	104	110	116	119	122	125	128	132	135	139	145	149
		4000	96	102	107	110	113	115	118	122	125	128	133	138
		6000	89	94	99	102	105	107	110	113	116	119	124	128
		8000	83	87	91	94	96	98	101	105	107	110	114	118
		10,000	77	81	85	88	90	92	94	97	99	102	106	110
		12 000	71	75	79	81	83	85	87	90	92	94	98	102
		14 000	66	69	73	75	77	79	80	83	85	87	91	94
300 000	.630	Sea Level	120	128	132	138	142	145	148	153	158	161	167	174
		1000	116	124	128	134	138	140	143	148	153	156	162	168
		2000	112	119	123	129	132	135	138	143	148	150	156	162
		4000	103	110	113	119	122	125	127	132	136	139	143	150
		6000	96	102	105	110	114	116	118	122	126	129	133	139
		8000	88	94	97	102	105	107	109	113	116	118	123	128
		10 000	82	88	90	95	97	99	102	105	108	110	114	119
		12 000	76	81	84	88	90	92	94	97	100	102	106	110
		14 000	70	75	77	81	83	85	87	90	93	95	98	102
350 000	.681	Sea Level	127	135	141	146	151	155	158	163	167	171	178	186
		1000	123	131	136	141	146	150	153	158	162	165	172	180
		2000	119	126	131	136	141	145	148	152	156	160	166	174
		4000	109	116	121	125	130	133	136	140	144	147	153	160
		6000	101	108	113	117	121	124	126	130	134	137	142	148
		8000	93	100	104	107	111	114	117	120	123	126	131	137
		10 000	87	92	96	100	103	106	109	112	115	117	122	127
		12 000	81	86	89	93	96	98	100	103	106	109	113	118
		14 000	75	79	82	86	89	91	93	96	98	101	105	109
400 000	.728	Sea Level	135	193	149	155	160	163	166	172	177	182	189	197
		1000	131	139	144	150	155	158	161	166	171	176	183	191
		2000	126	133	139	145	149	152	155	161	165	170	176	184
		4000	116	123	128	133	138	140	143	148	152	157	163	169
		6000	108	114	119	124	128	130	133	138	142	146	151	157
		8000	99	105	110	114	118	120	122	127	130	134	139	145
		10 000	92	98	102	106	110	112	114	118	122	125	129	135
		12 000	86	91	95	98	102	103	106	109	112	116	120	125
		14 000	79	84	87	91	94	96	97	101	104	107	111	116
460 000	.772	Sea Level	141	150	157	162	167	171	175	181	187	191	198	207
		1000	136	145	152	157	162	166	169	175	181	185	192	200
		2000	132	140	146	151	156	160	163	169	174	178	185	193
		4000	121	129	135	139	143	147	151	156	161	164	170	178
		6000	112	120	125	129	133	137	140	145	149	153	158	165
		8000	104	111	116	119	123	126	129	133	138	141	146	153
		10 000	97	103	108	111	115	117	120	124	128	131	136	142
		12 000	89	95	100	103	106	109	111	115	119	121	126	132
		14 000	83	88	92	95	98	101	103	106	110	112	116	121
500 000	.815	Sea Level	146	156	163	170	174	179	183	189	195	199	207	216
		1000	141	151	158	164	169	173	177	183	189	193	200	209
		2000	136	146	152	157	163	167	171	176	182	186	193	202
		4000	125	134	140	146	150	154	158	162	168	171	178	186
		6000	117	125	130	136	139	143	146	151	156	159	166	173
		8000	108	115	120	125	128	132	135	139	144	147	153	159
		10 000	100	107	112	117	119	123	126	129	134	137	142	148
		12 000	93	99	103	108	110	114	116	120	124	126	131	137
		14 000	85	91	96	100	102	105	107	111	114	117	121	127
750 000	.998	Sea Level	172	183	192	200	205	211	216	224	230	236	246	257
		1000	166	177	186	193	198	204	209	216	223	228	238	248
		2000	160	171	180	187	191	197	202	209	215	220	230	240
		4000	148	157	165	172	176	181	186	193	198	203	212	221
		6000	137	146	153	160	164	168	173	179	184	188	196	206
		8000	127	135	141	148	151	155	160	165	170	174	181	190
		10 000	118	126	132	137	141	145	148	154	158	162	169	176
		12 000	109	116	122	127	130	134	137	142	146	150	156	163
		14 000	101	107	113	117	120	124	127	131	135	139	145	150
1 000 000	1.152	Sea Level	192	205	216	224	231	237	243	252	260	267	277	290
		1000	186	199	209	217	223	229	235	244	252	258	268	280
		2000	179	191	202	209	215	221	227	235	242	249	258	271
		4000	165	176	185	193	198	204	209	217	224	230	238	250
		6000	153	164	173	179	185	190	194	202	208	214	222	232
		8000	142	151	159	165	170	175	180	186	192	147	205	214
		10 000	132	141	148	154	159	163	167	173	178	183	190	199
		12 000	123	130	137	142	147	151	154	160	165	169	176	185
		14 000	113	120	127	132	136	139	143	148	153	157	163	170

SOLID COPPER CONDUCTORS

B & S No.	Diameter in Inches	Elevation in Feet	3 Ft.	4 Ft.	5 Ft.	6 Ft.	7 Ft.	8 Ft.	9 Ft.	11 Ft.	13 Ft.	15 Ft.	19 Ft.	25 Ft.
4'	.204	Sea Level	51	54	56	58	59	60	61	63	64	65	68	70
		1000	49	52	54	56	57	58	59	61	62	63	66	68
		2000	47	50	52	54	55	56	57	59	60	61	63	65
		4000	44	46	48	50	51	51	52	54	55	56	58	60
		6000	41	43	45	46	47	48	49	50	51	52	54	56
		8000	37	40	41	42	43	44	45	46	47	48	50	51
		10,000	35	37	38	40	40	41	42	43	44	44	46	48
		12 000	32	34	35	37	37	38	39	40	40	41	43	44
		14 000	30	32	33	34	35	35	36	37	37	38	40	41
3	.229	Sea Level	57	60	63	64	65	67	68	70	71	73	75	78
		1000	55	58	61	62	63	65	66	68	69	71	73	75
		2000	53	56	59	60	61	62	63	65	66	68	70	73
		4000	49	51	54	55	56	57	58	60	61	63	64	67
		6000	45	48	50	51	52	53	54	56	57	58	60	62
		8000	42	44	46	47	48	49	50	51	52	54	55	57
		10 000	39	41	43	44	44	46	46	48	49	50	51	53
		12 000	36	38	40	40	41	42	43	44	45	46	47	49
		14 000	33	35	37	37	38	39	40	41	42	43	44	46
2	.258	Sea Level	62	66	68	70	72	73	75	77	79	80	83	86
		1000	60	64	66	68	70	71	73	75	76	77	80	83
		2000	58	61	63	65	67	68	70	72	74	75	78	80
		4000	53	57	58	60	62	63	64	66	68	69	71	74
		6000	49	52	54	56	57	58	60	61	63	64	66	69
		8000	45	48	50	51	53	54	55	57	58	59	61	63
		10 000	42	45	46	48	49	50	51	53	54	55	57	59
		12 000	39	42	43	44	45	46	47	49	50	51	53	54
		14 000	36	39	40	41	42	43	44	45	46	47	49	50
1	.289	Sea Level	69	72	75	77	79	81	83	85	87	89	92	95
		1000	66	69	72	74	76	78	80	82	84	86	89	92
		2000	64	67	70	72	74	76	77	79	81	83	86	89
		4000	59	62	64	66	68	69	71	73	75	76	79	81
		6000	55	57	60	61	63	65	66	68	69	71	73	76
		8000	51	53	55	57	58	60	61	63	64	65	68	70
		10 000	47	49	51	53	54	55	57	58	60	61	63	65
		12 000	44	45	47	49	50	51	52	54	55	56	58	60
		14 000	40	42	44	45	46	47	49	50	51	52	54	56

B & S No.	Diameter in Inches	Elevation in Feet	3 Ft.	4 Ft.	5 Ft.	6 Ft.	7 Ft.	8 Ft.	9 Ft.	11 Ft.	13 Ft.	15 Ft.	19 Ft.	25 Ft.
0	.325	Sea Level	75	79	82	85	87	89	91	94	96	98	102	105
		1000	72	76	79	82	84	86	88	91	93	95	98	102
		2000	70	74	76	79	81	83	85	88	90	92	95	98
		4000	64	68	70	73	75	76	78	81	82	84	88	90
		6000	60	63	65	68	69	71	73	75	76	78	82	84
		8000	55	58	60	62	64	66	67	69	71	72	75	77
		10 000	51	54	56	58	60	61	62	64	66	67	69	72
		12 000	47	50	52	54	55	56	58	60	61	62	65	67
		14 000	44	46	48	50	51	52	53	55	56	58	60	61
00	.365	Sea Level	83	88	91	94	96	98	100	104	106	109	112	116
		1000	80	85	88	91	93	94	97	100	103	105	108	112
		2000	77	82	85	88	90	92	93	97	99	102	104	108
		4000	71	75	78	81	82	84	86	89	91	94	96	100
		6000	66	70	73	75	77	78	80	83	85	87	89	92
		8000	61	65	67	69	71	72	74	76	78	80	82	85
		10 000	57	60	62	64	66	67	68	71	72	75	77	79
		12 000	53	56	58	59	61	62	63	66	67	69	71	73
		14 000	48	51	53	55	56	57	59	61	62	64	66	68
000	.410	Sea Level	91	96	100	103	106	109	111	114	117	120	124	128
		1000	88	93	97	100	102	105	107	110	113	116	120	124
		2000	85	90	93	96	99	102	103	106	109	112	116	120
		4000	78	82	86	88	91	93	95	98	100	103	107	110
		6000	73	77	80	82	84	87	89	91	93	96	99	102
		8000	67	71	74	76	78	80	82	84	86	88	91	94
		10 000	62	66	69	71	73	75	76	78	80	82	85	88
		12 000	58	61	63	65	67	69	70	72	74	76	79	81
		14 000	53	56	59	61	62	64	65	67	68	70	73	75
0000	.460	Sea Level	100	106	110	114	117	120	122	126	129	132	137	142
		1000	97	102	106	110	113	116	118	122	125	127	132	137
		2000	93	99	103	106	109	112	114	118	120	123	128	133
		4000	86	91	95	98	100	103	105	108	111	113	118	122
		6000	80	85	88	91	93	96	97	101	103	105	109	113
		8000	74	78	81	84	86	88	90	93	95	97	101	105
		10 000	69	72	75	78	80	82	84	86	88	91	94	97
		12 000	63	67	70	72	74	76	77	80	82	84	87	90
		14 000	59	62	64	67	69	70	71	74	76	77	80	83

x For single phase or 2 phase multiply the 3 phase values by 1.16. The above are the disruptive critical voltage values for fair weather based upon a temperature of 25° C. (77° F.) and values for M_0 of 0.87 for stranded and 0.93 for solid conductors. Derived by Peek's formula: Kilovolts to neutral $= 2.302\ M_0\ G_0\ \delta\ R\ \text{Log}_{10} \frac{S}{R}$, where $G_0 = 53.6$ Kilovolts per inch; S = Spacing in inches; R = Radius of conductor in inches; $\delta = \frac{17.9\ B}{459+T}$; T = Temperature in degrees F.; B = Barometer pressure in inches.

to operate the No. 0 copper conductors at as high a line voltage as 100 000 volts. If, however, for other reasons, 120 000 is selected as the desirable operating voltage, then either a large diameter copper conductor or an aluminum conductor having a greater diameter but an equivalent conductivity to that of the No. 0 copper conductor should be selected.

TABLE E—COMPARISON OF CORONA LOSS

For No. 0 Stranded Copper Conductors 105 560 cir. mil (diameter 0.373 in.) and equivalent Aluminum Conductors 167 800 cir. mil (diameter 0.501 in.)
Conductor Spacing (s) Delta = 144 in. Altitude 1000 feet—Barometer 28.9 inches. Calculated from formula (22)

Kilovolts		Corona Loss in Kw. per Mile for Three Conductors at 60 Cycles											
		Fair Weather—(Formula 22)						Stormy Weather—(Formula 22-A)					
		No. 0 Copper Radius 0.186 in.			Aluminum Radius 0.25 in.			No. 0 Copper Radius 0.186 in.			Alumioum Radius 0.25 in.		
Between Conductors	To Neutral	0° C 32° F $\delta=1.05$ $e_0=60.5$	25° C 77° F $\delta=0.967$ $e_0=55.7$	50° C 122° F $\delta=0.892$ $e_0=51.3$	0° C 32° F $\delta=1.05$ $e_0=77.5$	25° C 77° F $\delta=0.967$ $e_0=71.5$	50° C 122° F $\delta=0.892$ $e_0=66.0$	0° C 32° F $\delta=1.05$ $e_0=48.4$	25° C 77° F $\delta=0.967$ $e_0=44.5$	50° C 122° F $\delta=0.892$ $e_0=41.0$	0° C 32° F $\delta=1.05$ $e_0=62.$	25° C 77° F $\delta=0.967$ $e_0=57.2$	50° C 122° F $\delta=0.892$ $e_0=52.7$
100	57.8	0.0	0.1	0.2	0	0	0	0.3	6.5	11.3	0	0	1.1
110	63.5	0.3	2.3	6.0	0	0	0	7.8	13.3	20.3	0	1.7	4.6
120	69.2	2.6	6.7	12.8	0	0	0.4	14.8	22.6	32.0	2.0	6.2	12.6
130	75.1	7.25	13.9	22.6	0.0	0.5	3.8	24.4	34.6	46.5	6.7	13.7	23.2
140	80.8	13.8	23.3	34.8	0.3	3.7	10.1	35.8	48.7	63.7	13.9	23.8	36.4
150	86.7	22.4	35.5	50.2	3.3	9.9	19.7	50.2	66.	84.	24.	37.2	53.3
160	92.4	35.0	49.8	67.7	8.7	18.7	32.2	66.	85.	106	36.	53.	73.
180	104.8	66.0	89.0	115.0	29.3	47.3	69.5	108.	135.	163.	72.	96.	125.

Note: At 25 cycles the losses would be $\frac{f_1 + 25}{f + 25} = \frac{25 + 25}{60 + 25} = \frac{50}{85}$ times the above table values. For conductors in a row (flat spacing) the corona loss would be reduced below the values for delta or triangular arrangement. For the higher voltages in the above table the conductor spacings would, in an actual installation, be greater than 144 in. (upon which basis the table values are given) thus giving actually less corona loss for the higher voltages than indicated by the table values.

The accompanying photograph illustrating corona on an experimental line is published with the kind permission of F. W. Peek, Jr.

Since the formulas pertaining to corona effect are to some extent worked up from test data they may be slightly changed from time to time. In case the problem at hand seems vitally near the critical point it will be well to consult the latest literature at that time as an additional check on the work.

CORONA AT 230 KV. 1.19 CM. DIAMETER, 0.47" CABLE. 310 CM. 10 FEET SPACING.

CHAPTER V

SPEED OF ELECTRIC PROPOGATION—RESONANCE PARALLELING TRANSMISSION CIRCUITS HEATING OF BARE CONDUCTORS

SPEED OF ELECTRIC PROPAGATION

ASTRONOMERS and investigators by various methods of determination have arrived at slightly different values for the speed of light. The Smithsonian Physical Tables give 186 347 miles per second as a close average estimate. In electrical engineering, the speed of light is usually stated as approximately 3×10^{10} centimeters per second. This is the equivalent of 186 451 miles per second. The speed of electrical propogation (assuming zero losses) is the same as that of light.

ELECTRIC WAVE LENGTH

Suppose a frequency of 60 cycles per second is impressed upon a circuit of infinite length. At the end of one sixtieth of a second the first impulse (neglecting retardation due to losses) will have traversed a distance of 186 347 ÷ 60 or 3106 miles. A section of such a circuit 3106 miles long would be designated as having a full wave length for a frequency of 60 cycles per second.

In Fig. 14, the dotted line or one cycle wave is shown as extending over a circuit 3106 miles long. In this case, when the first part of the wave arrives at a point 3106 miles distant, the end of the same wave is at the beginning of the circuit. For each half wave length the current is of equal value but flowing in opposite directions in the conductor. Such a circuit is designated as of full wave length. Since the velocity of the electric propagation is slightly less than that of light, being slightly retarded due to resistance and leakage losses, the actual wave length will be slightly less than 3106 miles. Thus for a 300 mile, 60 cycle, three-phase circuit consisting of No. 000 copper conductors having 10 ft. flat spacing, the wave length is calculated to be 2959 miles. The wave length of such a circuit is indicated by the heavy line on the accompanying sketch. In the case of this particular circuit the electric field has been retarded approximately five percent, due to the losses of the circuit, as indicated by the displacement of the dotted and full line curves.

QUARTER WAVE RESONANCE

If the end of a long trough filled with water is struck by a hammer, the impact will cause a wave in the water to start in front of the point of impact and travel to the far end of the tank. When this wave reaches the far end of the tank it will be reflected, traveling back toward the point of origin, but on account of resistance encountered it will be of diminishing height or amplitude. If, at the instant it gets back to the point of origin, the end of the tank is again struck by the hammer, the resulting impulse will be that due to the second hammer blow plus that remaining from the first blow. The result will be that the second wave from the near end of the tank will be of greater amplitude than the first wave. If when the second wave arrives back at the near end, the end of the tank is struck again with the hammer the resulting third impulse will be of greater amplitude than the second impulse. If at the instant of the return of each succeeding impulse the end of the tank is struck, the result will be cumulative and each succeeding wave will be of greater magnitude than the one preceeding until the point is reached where the losses due to resistance become sufficient to prevent a further increase in amplitude of the wave.

Under certain conditions a similar phenomenon may occur in electric circuits and this is known as "quarter wave resonance". If an electric impulse* is sent into a

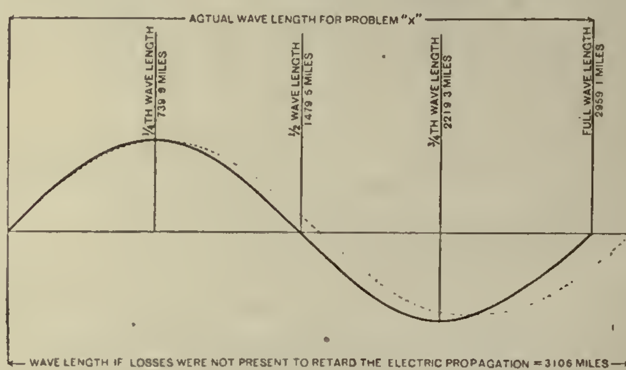


FIG. 14—WAVE LENGTH OF 60 CYCLE CIRCUIT

conductor, such as a transmission circuit, this impulse travels along the conductor at the velocity of light. If the circuit is open at the other end, the impulse is there reflected and returns at the same velocity. If at the moment when the impulse arrives at the starting point a second impulse is sent into the circuit, the returned first impulse adds itself to, and so increases the second impulse; the return of this second impulse adds itself to the third impulse, and so on; that is, if alternating impulses succeed each other at intervals equal to the time required by an impulse to travel over the circuit and back, the effects of successive impulses add themselves, and large currents and high e.m.f.'s may be produced by small impulses. This condition is known as quarter wave electric resonance. To produce this condition, it is necessary that the alternating impulses occur at time intervals equal to the time required for the impulses to travel the length of the line and back. For example, the time of one half wave or cycle of impressed e.m.f.

*For a complete study of this subject see "Transient Electric Phenomena and Oscillations" by C. P. Steinmetz, from which the above description of quarter wave resonance has largely been taken.

is the time required by light to travel twice the length of the line, or the time of one complete cycle is the time light requires to travel four times the length of the line. Stated another way, the number of cycles or frequency of the impressed alternating e.m.f.'s in resonance condition, is the velocity of light divided by four times the length of the line; or to have free oscillation or resonance condition, the length of the line is one quarter wave length of light. The cycles at which this condition is reached (if there were no losses present) would be determined as follows:—

$$\text{Frequency} = \frac{46587}{\text{Length in miles}} \quad (23)$$

or

$$\text{Length in miles} = \frac{46587}{\text{Frequency}} \quad (24)$$

RESONANCE LENGTHS OF CIRCUITS

Commercial frequencies are so low that to reach a quarter wave resonance condition with them the circuit would have to be of great length. The following values, for the sake of simplicity, are based upon the assumption that there are no losses in the circuit.

Fundamental Frequency	Resonance Length	Wave Length
15 cycles	3106 miles	12434 miles
25 cycles	1863 miles	7452 miles
40 cycles	1165 miles	4660 miles
60 cycles	776 miles	3106 miles

The above lengths are based upon the impressed or fundamental frequencies. If these impressed frequencies contain appreciable higher harmonics, some of the latter may approach resonance frequency and, if of sufficient magnitude, may cause trouble. Thus the length of circuit corresponding to resonance conditions of various harmonics of the fundamental is given below.

Cycles	Harmonics		
	3rd.	5th.	7th.
15	1035 miles	631 miles	444 miles
25	621 miles	372 miles	266 miles
40	388 miles	233 miles	166 miles
60	258 miles	155 miles	111 miles

Thus an impressed frequency of 60 cycles will not produce quarter wave electric resonance unless the circuit be approximately 776 miles long. If a third harmonic, however, is present in the impressed wave, this harmonic will develop quarter wave resonance in a circuit approximately 258 miles long, a 5th harmonic in a circuit approximately 155 miles long, and a 7th harmonic in a circuit approximately 111 miles long.

The above values are based upon no losses being encountered in transmission. Obviously this is an incorrect assumption, as electric propagation is always accompanied by more or less loss, depending upon the fundamental constants (resistance and leakage) of the circuit. The effect of such losses is to retard the velocity of the electric propagation, usually by an amount of five to ten percent below that of light. The above values of circuit lengths representing a condition for resonance may therefore be as much as ten percent above the actual lengths.

An investigation of the effects of higher harmonics of the impressed wave is of importance in connection with very long distance transmission systems.

PARALLELING TRANSMISSION CIRCUITS

Transmission lines are frequently constructed with duplicate circuits which are normally operated in parallel. In other cases two circuits may lead from the generating station in divergent directions and at some distant point come together and be connected in parallel.

If the two circuits are fed from different generators, or sources of supply, the only condition necessary for paralleling the circuits is that the phase rotation of the two circuits be the same and that the regulation in speed of the prime movers of the generators feeding the two systems can be adjusted so as to bring the phases of the two circuits together for paralleling.

If, however, the two circuits which are to be connected in parallel are fed from the same source of supply, the case may become involved. There will be no trouble in obtaining the correct phase rotation, for should the circuits not rotate alike, it is only necessary to transpose any two of the connections of either of the circuits (assuming that the circuits are three-phase). The other condition to be met is that the phases of both circuits to be paralleled are the same, i. e., the voltages in the phases to be paralleled must pass through their zero and maximum values at the same instant.

If neither circuit has transformers between the points where they are to be connected in parallel, their phases will coincide and there will be no trouble about connecting them in parallel. If one circuit has no transformers and the other has transformers, the phase relations of the two circuits will depend upon the kind of transformer connections employed. Suppose it is assumed that the raising transformers are connected delta to star and the lowering transformers are connected delta to delta. With these connections the phases of the two circuits will be 30 electrical degrees apart and it will be impossible to parallel the circuits. In other words one delta-star or star-delta transformer connection produces a phase displacement of 30 degrees. It will be obvious that a second delta-star or star-delta connection will restore the original phase relation. A delta-delta connection or a star-star connection does not affect the phase relations. If both circuits have an even number of star and even number of delta windings, the equivalent resultant will be the same as if all the connections were either delta-delta or star-star; hence, there will be no resultant change in phase relations and the two circuits can be paralleled with each other or with a circuit having no transformations. If, however, both circuits have an odd number of delta and an odd number of star windings, any attempt to resolve them into the equivalent number of delta-delta and star-star connections will leave one star and one delta; the effect is the same as if there was one star-delta connection in the circuits. This will twist the phase relations of the terminals 30 degrees out of phase from the generators. Since both circuits will have an equivalent phase displacement, they can be paralleled with one another, but since both are 30 degrees out of phase with the generators, they cannot be paralleled with a line having no transformations; nor with a line having an even number of star and delta connections.

When the phase angles of the two transmission circuits (receiving their power from a common source) are known to be such as to permit of parallel operation it is then necessary to phase them out before connecting the circuits together. The phase rotation can be checked most readily by means of a polyphase motor connected first to one circuit and then to the other, being careful to connect the leads in the same order in each case. If the motor runs in the same direction from both circuits, the phase rotation of the circuits will be the same. The phase angle can be readily tested by means of a single-phase synchroscope*. In case a polyphase motor and synchroscope are not available, the phasing out of the circuits may be accomplished by the use of a voltmeter and transformer.** As an illustration, assume that from a 4400 volt bus in a generating station a 4400 volt transmission circuit extends for some distance from the station. A second transmission circuit fed from the same bus but containing both raising and lowering transformers is to be paralleled at the farther end with the 4400 volt circuit which contains no transformers. The phase angles of the lines are assumed to be such as to permit paralleling the two circuits, with proper connections.

One of the transmission circuits is connected to one side of the paralleling switch as in Fig. 15 and the other circuit to the other side of the same switch. The three terminals on one side of the switch may be tagged *1-2-3*. Likewise the three terminals on the other side of the switch may be tagged *4-5-6*. Connect any two terminals together (*1* and *4* in this case) by a jumper. Take voltage readings across the corresponding terminals *2* to *5*, *3* to *6*, and *3* to *5*, *2* to *6*. From these voltage readings it is a simple matter to indicate by a vector diagram the relative phase relations at the switch contacts of the two circuits to be paralleled. In the case illustrated, the readings indicate that the relative voltage relations on the two sides of the paralleling switches are as indicated by the full line delta *1-2-3*, and the broken line delta *4-5-6*. It will be seen that phase *1-3* will parallel with phase *4-5*, that phase *1-2* will parallel with phase *6-5* and phase *2-3* will parallel with phase *4-6*. In order to bring about this phase relation it will be necessary to change the transformer connections on the low-tension side of the lowering transformers, inside of the delta. That is the *6* end of the transformer windings *5-6* will be connected to the *4* end of transformer *4-5*. The *4* end of transformer *4-6* will be connected to the *5* end of transformer, *5-6* and the *6* end of transformer *4-6* will be connected to the *5* end of transformer *4-5*. These changes will shift the position of the delta *4-5-6* so that it will coincide with delta *1-2-3*. A further test of voltage between switch terminals *2* to *5* and *3* to *6* should indicate zero voltage across the switch terminals to be connected together, in which case the paralleling switches may be closed. In order to measure the voltage across the paralleling switch contacts it will usually be necessary to employ a potential transformer. This transformer and voltmeter should be capable of withstanding 1.73 times the voltage of the circuit for, with the connections given in Fig. 15, one reading gave 7610 volts, whereas the voltage of the circuit was only 4400 volts.

In case there is a ground on both systems, the placing of a jumper across two of the switch contacts would result in a short-circuit. This jumper should not be placed across the switch until it has been shown by connecting a transformer across these two contacts that no potential exists between them.

*These tests are described in an article on "Phasing Out High Tension Lines" by E. C. Stone in the JOURNAL for Nov. 1917, p. 448.

**This method is described in an article on "Determination of Polarity of Transformers for Parallel Operation" by W. M. McConahey, in the JOURNAL for July 1912, p. 613. See also article on "Polarity of Transformers" by W. M. Dann in the JOURNAL for July 1916, p. 350.

HEATING OF BARE CONDUCTORS IN AIR

If the circuit is long, the voltage will probably be high and consequently the current to be transmitted

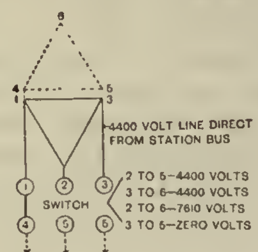


FIG. 15—TEST FOR PHASE SEQUENCE

small. In this case, the heating effect of the current will be small and unimportant. If, however, the circuit is short and an unusually large amount of power is to be transmitted, the current will be large. Since the I^2R loss varies as the square of the current and directly as the resistance, the heat generated, if the current is large, may be sufficient to overheat or anneal the material of the conductors. In some cases of unusually large amounts of power being transmitted short distances, the heating effect of the currents resulting may be sufficient to limit the amount of power that can be transmitted at a given voltage.

Table XXIII should be consulted in cases where the circuit is short and the amount of power to be transmitted large. In this table are columns containing current values which have been calculated corresponding to 10, 25 and 40 degrees C. rise in temperature for various sizes of bare copper conductors suspended in still air at a temperature of 25 degrees C. In other words these current values are based upon absolute temperatures of 35, 50 and 65 degrees C. The current values corresponding to a temperature rise of 40 degrees C.

ERRATUM

The formula used in calculating the values for table XXIII, page 43, embodied the only available information on this subject at the time the values were calculated. Recent exhaustive and carefully conducted tests, made by George E. Luke, indicate a wide difference in results from the table values, especially in the larger size conductors. The table values corresponding to 40° C rise should not, therefore, be used.

In the April, 1923 issue of the Electric Journal, page 127, appears an article entitled "Current Capacity of Wires and Coils" in which Mr. Luke gives the results of his tests and the empirical formula he developed as a result of the test.

TABLE XXIII—HEATING CAPACITY FOR 40° C. RISE

OF BARE COPPER CONDUCTORS SUSPENDED OUT OF DOORS

CONDUCTORS: Type	B & S No.	Area in Circular Mils	Diameter in Inches	Amperes—Bare Conductors in Still Air for Temperature Rises Stated: For 10°C Rise	For 25°C Rise	For 40°C Rise	Approximate carrying capacity in K.V.A. 3 phase corresponding to a temperature rise of 40°C (based upon amperes in column marked "For 40°C Rise") for bare copper conductors suspended in still air out of doors. 220 Volts K.V.A.	440 Volts K.V.A.	550 Volts K.V.A.	1100 Volts K.V.A.	2200 Volts K.V.A.	4000 Volts K.V.A.	4400 Volts K.V.A.	6000 Volts K.V.A.	6600 Volts K.V.A.	6900 Volts K.V.A.
Stranded		2 000 000	1.631	2140	3280	4050	1540	3080	3850	7700	15 400	27 900	30 800	42 000	46 200	48 300
		1 800 000	1.548	1980	3020	3760	1430	2860	3580	7150	14 300	26 000	28 600	39 100	42 800	44 800
		1 700 000	1.504	1890	2920	3600	1370	2740	3420	6850	13 700	24 800	27 400	37 400	40 000	43 000
		1 600 000	1.459	1810	2780	3440	1310	2620	3270	6550	13 100	23 800	26 200	35 800	39 200	41 000
		1 500 000	1.412	1720	2640	3300	1250	2500	3140	6280	12 560	22 800	25 120	34 300	37 600	39 200
		1 400 000	1.364	1635	2520	3100	1180	2360	2950	5900	11 800	21 400	23 600	32 200	35 400	37 000
		1 200 000	1.263	1460	2230	2760	1050	2100	2620	5250	10 500	19 100	21 000	28 700	31 500	33 000
		1 100 000	1.209	1360	2100	2580	980	1960	2460	4920	9840	17 800	19 680	26 800	29 400	30 800
		1 000 000	1.152	1270	1950	2420	920	1840	2300	4600	9200	16 700	18 400	25 100	27 600	28 800
		950 000	1.123	1215	1870	2320	880	1760	2200	4400	8800	16 000	17 600	24 100	26 400	27 700
		900 000	1.093	1175	1800	2220	840	1680	2110	4220	8440	15 300	16 880	23 000	25 300	26 500
		850 000	1.062	1120	1720	2130	810	1620	2030	4050	8100	14 700	16 200	22 100	24 300	25 400
		800 000	1.031	1075	1640	2030	770	1540	1930	3870	7740	14 000	15 480	21 100	23 200	24 200
		750 000	.998	1025	1580	1940	740	1480	1840	3690	7380	13 400	14 760	20 200	22 100	23 200
		700 000	.964	980	1490	1830	695	1390	1740	3480	6960	12 600	13 920	19 000	20 900	21 800
		650 000	.929	920	1410	1740	660	1320	1660	3320	6640	12 000	13 280	18 100	19 800	20 800
		600 000	.893	870	1330	1630	625	1250	1560	3120	6240	11 300	12 480	17 000	18 700	19 600
		550 000	.855	810	1250	1530	580	1160	1450	2900	5800	10 600	11 600	15 900	17 500	18 300
		500 000	.815	755	1160	1430	545	1090	1360	2720	5440	9860	10 880	14 900	16 300	17 100
		450 000	.772	700	1060	1320	500	1000	1250	2500	5000	9100	10 000	13 700	15 100	15 700
		400 000	.728	640	980	1210	460	920	1150	2300	4600	8350	9200	12 600	13 800	14 400
		350 000	.681	575	885	1090	415	830	1040	2080	4160	7500	8320	11 300	12 400	13 000
		300 000	.630	515	785	970	370	740	920	1840	3680	6700	7360	10 100	11 100	11 600
		250 000	.575	450	685	840	320	640	800	1600	3200	5800	6400	8700	9550	10 000
	0000	211 600	.528	385	605	750	285	570	715	1430	2860	5170	5720	7800	8550	8950
	000	167 772	.470	330	508	625	238	475	595	1190	2380	4320	4760	6500	7130	7450
	00	133 079	.418	280	425	527	200	400	500	1000	2000	3640	4000	5470	6000	6280
	0	105 560	.373	235	360	444	170	336	423	846	1692	3060	3384	4600	5050	5300
	1	83 694	.332	195	300	370	141	282	352	704	1408	2550	2816	3840	4220	4400
	2	66 358	.292	162	250	307	116	233	292	584	1168	2120	2336	3180	3500	3660
	3	52 624	.260	136	210	258	98	196	245	490	980	1770	1960	2680	2940	3080
	4	41 738	.232	114	176	235	89	178	224	448	896	1620	1792	2440	2680	2800
	5	33 088	.206	96	147	182	69	138	173	346	692	1260	1384	1890	2080	2180
Solid	0000	211 600	.460	370	565	728	275	550	690	1380	2760	5030	5520	7550	8300	8700
	000	167 772	.410	310	475	588	224	448	560	1120	2240	4060	4480	6100	6700	7000
	00	133 079	.365	258	420	495	188	376	470	940	1880	3420	3760	5150	5650	5900
	0	105 560	.325	218	335	415	158	316	395	790	1580	2860	3160	4300	4730	4950
	1	83 694	.289	182	280	348	132	264	330	660	1320	2400	2640	3620	3970	4150
	2	66 358	.258	154	235	295	112	224	280	560	1120	2040	2240	3060	3370	3520
	3	52 624	.229	128	200	245	93	186	233	465	930	1690	1860	2540	2790	2920
	4	41 738	.204	108	167	207	78	158	197	394	788	1430	1576	2150	2360	2470
	5	33 088	.182	90	140	174	66	132	165	330	660	1200	1320	1810	1980	2080

Type	B & S No.	Area in Circular Mils	Diameter in Inches	For 10°C Rise	For 25°C Rise	For 40°C Rise	10 000 Volts K.V.A.	11 000 Volts K.V.A.	13200 Volts K.V.A.	15000 Volts K.V.A.	20000 Volts K.V.A.	22000 Volts K.V.A.	30 000 Volts K.V.A.	33 000 Volts K.V.A.	50 000 Volts K.V.A.	60 000 Volts K.V.A.
Stranded		2 000 000	1.631	2140	3280	4050	70 000	77 000	92 200							
		1 800 000	1.548	1980	3020	3760	65 000	71 500	85 500	97 500						
		1 700 000	1.504	1890	2920	3600	62 300	68 500	82 000	93 500						
		1 600 000	1.459	1810	2780	3440	59 500	65 500	78 500	89 200						
		1 500 000	1.412	1720	2640	3300	57 000	63 000	75 300	85 500						
		1 400 000	1.364	1635	2520	3100	53 600	59 000	70 600	80 500						
		1 200 000	1.263	1460	2230	2760	47 700	52 500	63 000	71 600	95 400					
		1 100 000	1.209	1360	2100	2580	44 600	49 000	58 700	67 000	89 200	98 000				
		1 000 000	1.152	1270	1950	2420	41 800	46 000	55 200	63 000	83 600	92 000				
		950 000	1.123	1215	1870	2320	40 000	44 200	53 000	60 200	80 000	88 400				
		900 000	1.093	1175	1800	2220	38 400	42 200	50 500	57 500	77 800	84 400				
		850 000	1.062	1120	1720	2130	36 800	40 500	48 600	55 200	73 600	81 000				
		800 000	1.031	1075	1640	2030	35 100	38 600	46 300	52 600	70 200	77 200				
		750 000	.998	1025	1580	1940	33 600	36 900	44 200	50 500	67 200	73 800				
		700 000	.964	980	1490	1830	31 600	34 800	41 600	47 500	63 200	69 600	95 000			
		650 000	.929	920	1410	1740	30 100	33 000	39 700	45 000	60 200	66 000	90 000	99 000		
		600 000	.893	870	1330	1640	28 400	31 200	37 400	42 500	56 800	62 400	85 000	93 500		
		550 000	.855	810	1250	1530	26 400	29 000	34 900	39 700	52 800	58 000	79 400	87 000		
		500 000	.815	755	1160	1430	24 800	27 200	32 600	37 200	49 600	54 400	74 400	81 500		
		450 000	.772	700	1060	1320	22 800	25 000	30 100	34 200	45 600	50 000	68 400	75 000		
		400 000	.728	640	980	1210	21 000	23 000	27 600	31 400	42 000	46 000	62 800	69 000		
		350 000	.681	575	885	1090	18 900	20 800	24 800	28 300	37 800	41 600	56 600	62 000	94 000	
		300 000	.630	515	785	970	16 800	18 400	22 100	25 200	33 600	36 800	50 400	55 200	83 500	
		250 000	.575	450	685	840	14 500	16 000	19 100	21 800	29 000	32 000	43 600	47 800	72 500	
	0000	211 600	.528	385	605	750	13 000	14 300	17 100	19 500	26 000	28 600	39 000	42 700	65 000	78 000
	000	167 772	.470	330	508	625	10 800	11 900	14 200	16 200	21 600	23 800	32 400	35 600	54 000	64 800
	00	133 079	.418	280	425	527	9100	10 000	12 000	13 700	18 200	20 000	27 400	30 000	45 500	54 800
	0	105 560	.373	235	360	444	7700	8450	10 100	11 500	15 400	16 900	23 000	25 400	38 500	46 000
	1	83 694	.332	195	300	370	6400	7050	8420	9600	12 800	14 100	19 200	21 100	32 000	38 400
	2	66 358	.292	162	250	307	5300	5850	7000	7950	10 600	11 700	15 900	17 500	26 500	3[illegible] 800
	3	52 624	.260	136	210	258	4460	4900	5880	6700	8920	9800	13 400	14 700	22 400	26 800
	4	41 738	.232	114	176	232	4000	4420	5300	6000	8000	8840	12 000	13 200	20 000	24 000
	5	33 088	.206	96	147	182	3150	3470	4150	4700	6300	6940	9400	10 400	15 800	18 800
Solid	0000	211 600	.460	370	565	728	12 600	13 800	16 600	18 900	25 200	27 600	37 800	41 500	63 000	75 600
	000	167 772	.410	310	475	588	10 200	11 200	13 400	15 300	20 400	22 400	30 600	33 500	50 800	61 200
	00	133 079	.365	258	420	495	8550	9400	11 300	12 900	17 100	18 800	25 800	28 200	42 600	51 600
	0	105 560	.325	218	335	415	7180	7900	9450	10 800	14 360	15 800	21 600	23 600	36 000	43 200
	1	83 694	.289	182	280	348	6020	6620	7950	9050	12 040	13 600	18 100	19 800	30 000	36 400
	2	66 358	.258	154	238	295	5100	5600	6750	7650	10 200	11 200	15 300	16 800	25 500	30 600
	3	52 624	.229	128	200	245	4250	4660	5600	6350	8500	9320	12 700	14 000	21 200	25 400
	4	41 738	.204	108	167	207	3580	3940	4700	5370	7160	7880	10 740	11 800	17 800	21 480
	5	33 088	.182	90	140	174	3020	3320	3970	4500	6040	6640	9000	9900	15 000	18 000

Formula (from Foster's Hand Book) $Amperes = 1100\sqrt{\frac{TD^3}{R}}$ for stranded conductors, and $Amperes = 1250\sqrt{\frac{TD^3}{R}}$ for solid conductors, where T = temperature rise is degrees C., D = diameter of conductor in inches, and R = resistance in ohms per mil foot at the final temperature. Based on an air temperature of 25 degrees C.

(absolute temperature of 65 degrees C.) have also been expressed in the form of k.v.a., three-phase values corresponding to various transmission voltages. Thus No. 0000 stranded bare copper conductors suspended in still air out doors at 25 degrees C. will carry 750 amperes with a temperature rise of 40 degrees C. (absolute temperature 65 degrees C.). If the transmission voltage is 220 volts, the corresponding k.v.a. value will be 285 k.v.a. three-phase and if the transmission voltage is 10 000 volts, 13 000 k.v.a. may be transmitted with the same temperature rise.

As indicated by foot notes the values of the table were calculated by formulas from Foster's Handbook as follows:—

$$Amperes = 1100\sqrt{\frac{TD^3}{R}} \text{ for stranded conductor} \quad (25)$$

$$Amperes = 1250\sqrt{\frac{TD^3}{R}} \text{ for solid conductor} \quad (26)$$

Where

T = Temperature rise in degrees C.
D = Diameter of conductors in inches.
R = Resistance of conductors in ohms per mil-foot at final temperature.

CHAPTER VI

DETERMINATION OF FREQUENCY & VOLTAGE

FREQUENCY DETERMINATION

Cost of Transformers—Sixty cycle transformers cost approximately 30 to 40 percent less than 25 cycle transformers; or stated another way, 25 cycle transformers cost approximately 40 to 66 percent more than 60 cycle transformers. The saving in first cost may vary between $1.50 and $2.50 per kv-a. in favor of 60 cycles. Assuming that the total kv-a. of transformer capacity connected to a transmission circuit is 2.5 times the kv-a. transmitted over the circuit, the saving in favor of 60 cycle transformers would be $3.75 to $6.25 or an average of $5.00 per kv-a. transmitted. Assuming 20 000 kv-a. to be transmitted, the saving in cost at $5.00 per kv-a. will be $100 000 in favor of 60 cycle transformers. The actual difference in cost will depend upon the type of the transformers, that is, whether water or self-cooled and also upon their average capacity. The difference in cost will be greater for the self-cooled type and for the smaller capacities.

Weight and Space of Transformers—The less weight of 60 cycle transformers makes them easier to handle and they require less space for installation.

Higher Reactance—Inductive reactance at 60 cycles is 2.4 times its value at 25 cycles. This tends to produce poorer voltage regulation of the circuit. Higher reactance has one advantage for the larger systems in that it tends to limit short-circuit currents and thus assists the circuit opening devices to function properly. By virtue of the higher reactance it might be possible in some cases to obtain sufficient reactance in the transformers without the addition of current limiting reactance coils.

Efficiency—The efficiency of 60 cycle transformers is usually 0.25 to 0.50 percent higher than for 25 cycle transformers.

Charging Current—At 25 cycles both the charging current and the reactance are approximately 42 percent of their values for 60 cycles. This tends to give better regulation and usually higher efficiency in transmission. On the other hand, the higher transmission efficiency may be offset by the slightly lower efficiency of 25 cycle transformers. In cases of very long circuits (particularly if the circuits are in duplicate and both in service) or of transmission systems embracing many miles of high tension mains and feeders, the charging currents may be so great as to limit the choice in transmission voltage. On the other hand large charging currents may be permitted, provided under excited synchronous motors are used at various parts of the transmission system for partially neutralizing this charging current and for maintaining constant voltage.

Inductive Disturbances—Lightning, switching and other phenomena cause disturbances on conductors of transmission circuits. The frequency of these disturbances is independent of that impressed on the system. After the removal of the disturbing influence they oscillate with the natural frequency of the line.

The natural frequency of the line is far above commercial frequencies but, if the transmission line is long, there may be some odd harmonic present in the fundamental impressed frequency which corresponds with the natural period of the line. This might tend to produce an unstable condition or resonance. This condition is somewhat less likely to occur at 25 cycles.

Summary—Although there are a number of large 25 cycle transmission systems in operation, they were mostly installed before the design of 60 cycle converting apparatus and electric light systems had reached their present state of perfection. Unless it is desirable to parallel with an existing 25 cycle system located in adjoining territory without the introduction of frequency changers, it is now quite general practice to choose the frequency of 60 cycles.*

VOLTAGE DETERMINATION

From a purely economic consideration of the conductors themselves, Kelvin's law for determining the most economical size of conductors would apply. Kelvin's law may be expressed as follows:—

"The most economical section of a conductor is that which makes the annual cost of the I^2R losses equal to the annual interest on the capital cost of the conducting material, plus the necessary annual allowance for depreciation". That is, the economical size of conductor for a given transmission will depend upon the cost of the conducting material and the cost of power wasted in transmission losses. The law of maximum economy may be stated as follows:—"The annual cost of the energy wasted per mile of the transmission circuit added to the annual allowance per mile for depreciation and interest on first cost, shall be a minimum".

Attempts have been made to determine by mathematical expression the most economical transmission voltage, all factors having been taken into account. There are so many diverse factors entering into such a

*For a complete discussion of this subject see a paper by D. B. Rushmore before the Schenectady section A. I. E. E., May 17, 1912, on "Frequency" and an article by B. G. Lamme on "The Technical Story of the Frequencies" in the JOURNAL for June, 1918, p. 230.

treatment as to make such an expression complicated, difficult and unsatisfactory. There are many points requiring careful investigation, not embraced by Kelvin's law, before the proper transmission voltage can be determined. Some of these points are given below.

Cost of Conductors—For a given percentage energy loss in transmission, the cross-section and consequently the weight of conductors required by the lower and medium voltage lines (up to approximately 30 000 volts) to transmit a given block of power varies inversely as the square of the transmission voltage. Thus if this voltage is doubled, the weight of the conductors will be reduced to one fourth with approximately a corresponding reduction in their cost. This saving in conducting material for a given energy loss in transmission becomes less as the higher voltages are reached, becoming increasingly less as voltages go higher. This is for the reason that for the higher voltages at least two other sources of losses, leakage over insulators and the escape of energy through the air between the conductors (known as "corona") appear. In addition to these two losses, the charging current, which increases as the transmission voltage goes higher, may either increase or decrease the current in the circuit depending upon the power-factor of the load current and the relative amount of the leading and lagging components of the current in the circuit. Any change in the current of the circuit will consequently be accompanied by a corresponding change in the I^2R loss. In fact, these sources of additional losses may, in some cases of long circuits or extensive systems, materially contribute toward limiting the transmission voltage. The weight of copper conductors, from which their cost may readily be calculated, is given in Table E-1. As an insurance against breakdown, important lines frequently are built with circuits in duplicate. In such cases the cost of conductors for two circuits should not be overlooked.

TABLE E 1—WEIGHT OF BARE COPPER CONDUCTORS

B & S NO.	AREA IN CIRCULAR MILS	WEIGHT IN POUNDS — PER 1000 FEET OF CIRCUIT — NUMBER OF CONDUCTORS			WEIGHT IN POUNDS — PER MILE OF CIRCUIT — NUMBER OF CONDUCTORS		
		ONE	TWO	THREE	ONE	TWO	THREE
	2 000 000	6 180	12 360	18 540	32 630	65 260	97 890
	1 900 000	5 870	11 740	17 610	30 994	61 988	92 982
	1 800 000	5 560	11 120	16 680	29 357	58 714	88 071
	1 700 000	5 250	10 500	15 750	27 720	55 440	83 160
	1 600 000	4 940	9 880	14 820	26 083	52 166	78 249
	1 500 000	4 630	9 260	13 890	24 446	48 892	73 338
	1 400 000	4 320	8 640	12 960	22 810	45 620	68 430
	1 300 000	4 010	8 020	12 030	21 173	42 346	63 519
	1 200 000	3 710	7 420	11 130	19 589	39 178	58 767
	1 100 000	3 400	6 800	10 200	17 952	35 904	53 856
	1 000 000	3 090	6 180	9 270	16 315	32 630	48 945
	950 000	2 930	5 860	8 790	15 470	30 940	46 410
	900 000	2 780	5 560	8 340	14 678	29 356	44 034
	850 000	2 620	5 240	7 860	13 834	27 668	41 502
	800 000	2 470	4 940	7 410	13 042	26 084	39 126
	750 000	2 320	4 640	6 960	12 250	24 500	36 750
	700 000	2 160	4 320	6 480	11 405	22 810	34 215
	650 000	2 010	4 020	6 030	10 613	21 226	31 839
	600 000	1 850	3 700	5 550	9 768	19 536	29 304
	550 000	1 700	3 400	5 100	8 976	17 952	26 928
	500 000	1 540	3 080	4 620	8 131	16 262	24 393
	450 000	1 390	2 780	4 170	7 339	14 678	22 017
	400 000	1 240	2 480	3 720	6 547	13 094	19 641
	350 000	1 080	2 160	3 240	5 702	11 404	17 106
	300 000	926	1 852	2 778	4 889	9 778	14 667
	250 000	772	1 544	2 316	4 076	8 152	12 228
0000	212 000	653	1 306	1 959	3 448	6 896	10 344
000	168 000	518	1 036	1 554	2 735	5 470	8 205
00	133 000	411	822	1 233	2 170	4 340	6 510
0	106 000	326	652	978	1 721	3 442	5 163
1	83 700	258	516	774	1 362	2 724	4 086
2	66 400	205	410	615	1 082	2 164	3 246
3	52 600	163	326	489	861	1 722	2 583
4	41 700	129	258	387	681	1 362	2 043
5	33 100	102	204	306	539	1 078	1 617
6	26 300	81	162	243	428	856	1 284
7	20 800	64	128	192	338	676	1 014
8	16 500	51	102	153	269	538	807

Table E-1 contains the weights of bare stranded copper cables per 1000 feet of circuit, also per mile of circuit. For the purpose of facilitating rapid calculation for any given case, the weights are given corresponding to one, two and three conductors for these two lengths of circuit.

Reduced Electric Surges—The better insulation necessitated by higher transmission voltages tends to make the circuit more secure against ordinary disturbances. Also the smaller currents resulting with the higher voltages cause less disturbance in the circuit in the case of grounds, short-circuits, switchings, lightning and other disturbances.

Less Reactance Volts Drop—Since the current corresponding to higher transmission voltages goes down as the voltage goes up, the voltage necessary to overcome the reactance of the circuit will be less, and the percentage reactance volts much less for higher voltages. Thus, if the transmission voltage is doubled, the current will be halved and for the same spacing of conductors the reactance volts drop will be one half, resulting in one fourth the percentage of the reactance-volts drop.

TABLE F—PRESENT RELATIVE COSTS OF HIGH TENSION APPARATUS

Expressed in Percent (6600 Volt Costs Taken as 100%)

	6600 Volts	11000 Volts	13200 Volts	16500 Volts	22000 Volts	33000 Volts	44000 Volts	66000 Volts	88000 Volts	110000 Volts	120000 Volts
Transformers	100	102	104	106	108	115	125	150	175	200	225
Switches	100	100	100	100	100	110	115	155	255	420	
Electrolytic Arresters	100	151	160	195	205	320	430	640	1600	1900	2400
Insulators	100	135	185	365	430	650	1250	3500	5500	6500	7700

Cost of Transformers—If the transmission voltage exceeds 13 200 volts, banks of step-up transformers will be required of sufficient capacity to transform all of the kv-a. to be transmitted. A still greater capacity of step down transformers will be required to reduce the voltage to that suitable for operating motors and lights. In some cases two reductions from the transmission circuit voltage may be required, the first usually reducing to 22 000, 11 000 or 6600 volts for general distribution and the second reducing from the general distribution voltage to the proper voltage for motors and lights. The net result is that the total capacity in transformers connected to a transmission system employing both step up and step down transformers may vary from a minimum of two to a maximum of about four times the kv-a. transmitted over the high-tension circuits. The average condition we will assume as 2.5 times the kv-a. to be transmitted.

The cost of power transformers at the present time for 66 000 volts service will vary between $1.25 to $3.00 for 60 cycle and $2 to $5 per kv-a. for 25 cycle service, depending upon their type and capacity. The total cost per kv-a. of transformers on a system would therefore be represented by approximately 2.5 times the above costs. The present relative costs of transformers for different voltages are given in Table F. For instance if the transmission voltage is increased from 33 000 to 66 000 volts the transformers will cost in the neighborhood of 150 ÷ 115 or 31 percent more than they would cost for 33 000 volts. Knowing the amount of power to be transmitted, an approximate estimate may be made as to the additional cost of the necessary transformers for a higher voltage.

Cost of Insulators—Table F values indicate a wide difference in the cost of insulators for the higher voltages: thus the increased cost of 66 000 volt insulators above the cost of 33 000 volt insulators is stated as 3500 ÷ 650 or 540 percent.

Cost of Other Apparatus—The cost of lightning arresters, high-tension circuit breakers and general insulation increase with the voltage. The increased cost of these items, however, may not have sufficient weight to materially influence the selection of the transmission voltage.

Cost of Buildings — Lower voltage transformers, switching equipment and lightning arresters require less space for insulation. If this apparatus is to be placed indoors, the cost of necessary buildings may be less. The amount of real estate required may also be less in case of the lower voltage.

Relative Cost Values — Table F contains relative cost values for different transmission voltages. They indicate approximately the variation, at the present time, in cost of the principal material which is affected by a change in transmission voltage. Cost values are very unstable at present but the table will serve in a general way to indicate comparative costs.

Efficiency—The efficiency of transformers will be slightly higher for the lower voltages.

Small Customers — The furnishing of power to small customers at points along the transmission circuits should receive careful consideration. The cost of switching apparatus, lightning arresters and transformers required to permit service being given to such customers will be less for the lower voltage.

Charging Current — The amount of current required to charge the transmission circuits varies approximately as the transmission voltage. Therefore the charging current, expressed in kv-a. varies approximately as the square of the voltage. Thus the charging current required for a 33 000 volt circuit is approximately one half and the charging kv-a. one fourth that of a 66 000 volt circuit.

TABLE G—FORM OF TABULATION FOR DETERMINING VOLTAGES AND CONDUCTORS

BASED ON THE TRANSMISSION OF 10 000 KV-A. FOR TEN MILES AT 80 PERCENT POWER-FACTOR LAGGING, 60 CYCLES, THREE PHASE

VOLTAGE: BETWEEN CONDUCTORS	VOLTAGE: TO NEUTRAL	AMPERES FOR 10,000 KVA	CONDUCTORS: B & S OR CIRCULAR MILS	CONDUCTORS: TOTAL WEIGHT IN POUNDS	CONDUCTORS: RESISTANCE OHMS	TOTAL I^2R LOSS, 10,000 KVA: KW FOR 10 HRS	TOTAL I^2R LOSS, 10,000 KVA: LOSS IN %	TOTAL I^2R LOSS, 2500 KVA: KW FOR 14 HRS	TOTAL LOSS PER YEAR IN KW—HOURS	VOLTAGE DROP AT FULL LOAD: RESISTANCE IR IN %	VOLTAGE DROP: REACTANCE IX IN %	VOLTAGE DROP: VOLTS DROP IN %	FIRST COST: CONDUCTORS AT 25 CTS. PER POUND	FIRST COST: TRANSFORMERS 25,000 KVA	FIRST COST: HIGH TENSION SWITCHES	FIRST COST: LIGHTNING ARRESTERS	FIRST COST: INSULATORS	FIRST COST: TOTAL	ANNUAL OPERATING COST: INTEREST ON FIRST COST AT 6 %	ANNUAL OPERATING COST: DEPRECIATION ON FIRST COST AT 10 %	ANNUAL OPERATING COST: I^2R LOSSES AT 1 CT. PER KW—HOUR	ANNUAL OPERATING COST: TOTAL
16,500	9 526	350	500 000	243 930	1.17	430	5.3	27	1 707 470	4.3	21.7	17.5	$60 982	$75 000	$3 000	1 000	$900	$140 882	$8 453	$14 088	$17 075	39616
			300 000	146 670	1.96	720	9.0	45	2 857 950	7.2	22.7	20	36 670	75 000	3000	1000	900	116 570	6 994	11 657	28 580	47 231
			#000	82 050	3.50	1286	16.1	80	5 102 700	12.9	24.2	25	20 512	75 000	3 000	1000	900	100 412	6 025	10 041	51 027	67093
22,000	12 702	262	300 000	146 670	1.96	403	5.0	25	1 598 700	4.0	12.8	11	36 670	76 500	3 000	1 050	1 200	118 420	7 105	11 842	15 987	34 934
			#000	82 050	3.50	720	9.0	45	2 857 950	7.2	13.6	14	20 512	76 500	3 000	1 050	1 200	102 262	6 136	10 226	28 580	44942
			#0	51 630	5.55	1143	14.3	71	4 534 760	11.5	14.1	17.5	12 910	76 500	3 000	1 050	1 200	94 660	5 680	9466	45 348	60 494
33 000	19 053	175	#00	65 100	4.42	406	5.1	25	1 609 650	4.0	6.5	7.0	16 275	82 500	3 300	1 600	1 980	105 655	6 340	10 565	16 097	33002
			#2	32 460	8.83	811	10.1	50	3 215 650	8.0	6.8	10.5	8 117	82 500	3 300	1 600	1 980	97 497	5 850	9 749	32 156	47 755
			#4	20 430	14.1	1295	16.2	81	5 140 660	12.9	7.1	14.5	5 107	82 500	3 300	1 600	1 980	94 487	5 670	9 448	51 407	66 525
44 000	25 404	131	#2	32 460	8.83	454	5.7	29	1 805 290	4.6	3.9	6.0	8 117	90 000	3 450	2 200	3960	107 727	6 463	10 772	18 053	35 288
			#5	16 170	17.8	916	11.4	58	3 639 780	9.1	4.0	9.5	4 040	90 000	3 450	2 200	3960	103 650	6 219	10 365	36 398	52982

Summary — In deciding upon the transmission voltage, careful and full consideration should be given to the present (or probable future) voltage of any neighboring or adjacent systems. There is an increasing tendency to combine generating and transmission systems for purposes of economy, and insurance against breakdown in service. If a possible future consolidation is not kept in mind when selecting the transmission voltage, a voltage may be decided upon which would render it impossible to parallel with a neighboring system, except through connecting transformers. In this case the transformers of the two systems would probably not be interchangeable for service on either system.

If the contemplated transmission system is remote from any existing system, a study of the initial and operating costs should be made corresponding to various sizes of conductors and to various assumed transmission voltages. A suggested tabulation for such comparisons is shown in Table G. In this table, it is assumed that 10 000 kv-a. (8000 kw at 80 percent power-factor lagging), is to be transmitted a distance of ten miles at 60 cycles, three-phase for ten hours, followed by

2500 kv-a. (2000 kw at 80 percent power-factor lagging) for 14 hours. Delta spacing is assumed of three feet for the lower two and four feet for the higher two voltages. Raising and lowering transformers will be required of an assumed total capacity of 2.5 × 10 000 or 25 000 kv-a. Conductors of hard drawn stranded copper are employed, the resistance of the conductors being taken at a temperature of 25 degrees C. from Table II.

The cost of the pole or tower line, the right of way, buildings and real estate for buildings is not included in this tabulation. Neither is the difference in transformer efficiencies taken into account. The difference in these items will not be sufficient in this case greatly to influence the choice of the transmission voltage, because all of the voltages compared are relatively low. Because of the large amount of power to be transmitted a comparatively short distance, the approximate rule of 1000 volts per mile for short lines does not hold true for this problem.

Assuming for the sake of argument that the price values given in this form of tabulation are approximately correct for this problem and that there are no neighboring transmission systems, then the problem reduces to cost economics.

Since both the first and operating costs in Table G are higher for 16 500 volts than they are for 22 000 volts, it is evident that 16 500 volts is economically too low a voltage.

In the consideration of 22 000 volts it will be seen that, of the three sizes of conductors, the largest size (300 000 circ. mil.) will be the cheaper in the end. Thus, if No. 000 were selected, the first cost would be $16 159 less than for 30 000 circ. mil conductors, but the operating cost (due to greater loss in transmission) will be approximately $10 000 a year more. For a similar reason No. 0 conductors will be disqualified.

In the consideration of 33 000 volts, No. 00 conductors will be the choice and in the consideration of 44 000 volts, No. 2 conductors will be the choice. The choice then comes down to the following:—

Voltage Transmission	Conductors	Total Cost First	Annual Operating Cost
22 000	300 000 circ. mils	$118 420	$34 934
33 000	No. 00	105 655	33 002
44 000	No. 2	107 727	35 288

It will thus be seen that a voltage of 33 000 volts and No. 00 conductors are the most economical of those tabulated. The transmission loss will be 5.1 percent, the reactance 6.5 percent and the voltage drop seven percent at full load. The value assigned as the cost per kw-hour for power lost in transmission will obviously have great influence in determining the proper economic size of conductors for any given transmission voltage. The cost of the copper will have a relatively greater importance on longer lines. As a matter of fact, a larger size than any of the conductors listed in Table G would be still more economical, under the conditions given. There have been numerous mistakes made in under-estimating the ultimate demand for electrical power and consequently adopting too low a transmission voltage. When in doubt the higher voltage will, in the course of time, most likely justify its adoption by reason of future growth not apparent at the time the choice is made.

The design and construction of transformers, circuit breakers, lightning arresters, etc. for a multiplicity of high-tension voltages is expensive. The manufacturers of such apparatus are endeavoring to standardize transmission voltages for the purpose of minimizing the number of designs of high-tension apparatus. This point could with mutual profit be taken up with the manufacturers before any particular voltage is decided upon.

TABLE H—COMMON TRANSMISSION VOLTAGES

Length of Line	Voltages
1 to 3 miles	550 or 2200 volts
3 to 5 miles	2200 or 6600 volts
5 to 10 miles	6600 or 13 200 volts
10 to 15 miles	13 200 or 22 000 volts
15 to 20 miles	22 000 or 33 000 volts
20 to 30 miles	33 000 or 44 000 volts
30 to 50 miles	44 000 or 66 000 volts
50 to 75 miles	66 000 or 88 000 volts
75 to 100 miles	88 000 or 110 000 volts
100 to 150 miles	110 000 or 132 000 volts
150 to 250 miles	132 000 or 154 000 volts
250 to 350 miles	154 000 or 220 000 volts

The amount and cost of power to be transmitted is a very important factor in determining the economic transmission voltage. For average conditions isolated from existing transmission lines the voltages shown in Table H have been quite generally used. For exceptional cases, exceptional values will be used. For example if 40 000 kv-a. is to be transmitted 20 miles, 66 000 volts or higher might be used. On the other hand if a very small amount of power is to be transmitted, lower voltages would probably be selected.

At the present time the prospects seem bright for the standardization of the following "normal" system voltages.

44 000	132 000
66 000	154 000
88 000	*187 000
110 000	220 000

*The use of 187 000 volts is likely to occur only in case it is found necessary to have a voltage between 154 000 and 220 000 volts.

CHAPTER VII

PERFORMANCE OF SHORT TRANSMISSION LINES

(EFFECT OF CAPACITANCE NOT TAKEN INTO ACCOUNT)

THE PROBLEMS which come under the general heading of short transmission lines are those in which the capacitance of the circuit is so small that its effect upon the performance of the circuit may, for all practical purposes, be ignored. The effect of capacitance is to produce a current in leading quadrature with the voltage, usually designated as charging current. This leading component of current in the conductor does not appear in the load current at the receiving end of the circuit. It is zero at the receiving end of the circuit but increases at nearly a uniform rate as the sending end of the circuit is approached, at which point it ordinarily becomes a maximum.

The effect of this charging current flowing through the inductance of the circuit is to increase the receiving-end voltage and therefore to decrease the voltage drop under load. Since the charging current is 2.4 times greater for a frequency of 60 cycles than it is for a frequency of 25 cycles, its effect upon the voltage regulation will be considerably greater at 60 cycles than at 25 cycles. The effect of charging current upon the voltage regulation will also increase as the distance of transmission is increased.

If the circuit were without capacitance, there would be no charging current and consequently the mathematical and the two graphical solutions (impedance methods) which follow under the general heading of "short transmission lines" would all produce accurate results. All circuits, however, have some capacitance, and as the length or the frequency of the circuit increases, these three methods will therefore yield results of increasing inaccuracy. Some engineers consider these impedance methods sufficiently accurate for circuits 20 to 30 miles long while others use them for still longer circuits. To act as a guide, Table J indicates the error in the supply voltage as determined by these impedance methods, for circuits of different lengths corresponding to both 25 and 60 cycle frequencies. These three impedance methods produce practically the same results, and the sending end voltage, as determined by any of these methods, is always slightly high. In other words the effect of the charging current is to reduce the voltage necessary at the sending end, for maintaining a certain voltage at the receiving end of the circuit. The error referred to below for the three methods is expressed in percentage of the receiving end voltage. Thus, for a 30 mile, 25 cycle circuit, the error is 0.04 percent, and for a 30 mile, 60 cycle circuit the error is 0.2 percent. If an error of 0.5 percent is considered permissible, then the Dwight or the Mershon Chart methods, or the corresponding mathematical solution, may be used for 25 cycle circuits up to approximately 125 miles, and for 60 cycles circuits up to approximately 50 miles. Of course these impedance methods may be used for still longer circuits by making proper allowance to compensate for the fundamental error.

DIAGRAM ILLUSTRATING A SHORT TRANSMISSION CIRCUIT

Fig. 16 illustrates the relation between the various elements in short transmission circuits, when the effect of capacitance and leakage is not taken into account. The current flowing in such a circuit meets two opposing e.m.f's.; i.e. of resistance in phase with the current and reactance in lagging quadrature with the current.

The upper part of Fig. 16 illustrates such a circuit schematically and the lower part vectorially. The voltage component required at the sending end to overcome the resistance IR of the circuit is indicated in the vector diagram by a short line parallel with the base line I, representing the phase of the current. These lines are drawn parallel, since the resistance voltage drop is in phase with the current. The voltage component required at the sending end to overcome the reactance IX of the circuit is indicated by a line in quadrature or at right angles, to the phase of the current. The reactance is in quadrature with the current for the reason that the rate of change in the magnetic field (consequently the e.m.f. of self-induction or reactance) surrounding the conductor is greatest when the current is passing through zero. The hypotenuse IZ of this small right angle impedance triangle represents the impedance voltage of the circuit. It represents the direction and value of the resulting voltage necessary to overcome the combined effect of the resistance and the reactance of the circuit.

The relative values and phases of the receiving and

TABLE J

Length of Circuit (Miles)	Error in Percentage of Receiver Voltage	
	25 cycles	60 cycles
20	+0.02	+0.10
30	+0.04	+0.2
50	+0.1	+0.5
100	+0.4	+1.9
200	+1.4	+8.0
300	+3.3	+18.0

sending end voltages, and their phase relations with the current I, are also indicated on the vector diagram. This diagram is plotted for a receiving end load based upon 80 percent power-factor lagging. E_s represents the value of the voltage required at the sending end of the circuit to maintain the voltage E_r at the receiving end, when the impedance of the circuit is IZ and the receiving end power-factor is 80 percent lagging. The phase angle θ_s indicates the amount by which the current lags behind the voltage at the sending end; cos θ_s being the power-factor of the load as measured at the sending end. Likewise cos θ_r is the power-factor of the load at the receiving end.

TAPS TAKEN OFF CIRCUIT

Usually the main transmission circuit is tapped and power taken off at one or more points along the circuit. The performance of such a circuit must be calculated by steps thus:—Assume a circuit 200 miles long with 10 000 kw taken off at the middle and 10 000 kw at the receiving end. From the conditions known or assumed at the receiving end, calculate the corresponding send-

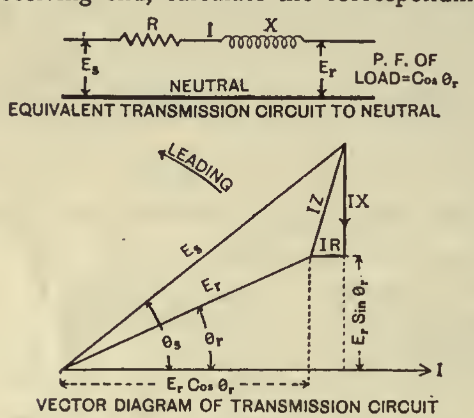


FIG. 16—DIAGRAMS FOR SHORT TRANSMISSION LINES
Impedance method, capacitance effect not taken into account.

ing end conditions, that is the voltage, power and power-factor at the substation in the middle of the circuit. To the calculated value of the actual power in kilowatts add the losses at the substation in the middle of the circuit. Any leading or lagging component in the substation load current must also be added algebraically, in order to determine the power-factor at the sending side of the substation. This will then be the receiving end conditions at the substation in the middle of the circuit, from which the corresponding conditions at the sending end of the circuit may be calculated. If the sending end conditions are fixed, and the receiving end conditions are to be determined, the substation losses will in such case be subtracted in place of added.

CABLE AND AERIAL LINES IN SERIES—COMPOSITE LINES

In some cases it is necessary to place part of a transmission circuit underground, and in other cases it may be desirable to use two or more sizes of conductors in series. The result will be that the circuit constants will be different for the various sections. If the effect of capacitance be neglected, the combined circuit may be treated as a single circuit having a certain total resistance R and a total reactance X.

PROBLEMS

Later a table will be presented listing a large number of transmission circuits from 20 to 500 miles long, at both 25 and 60 cycles operating at from 10 000 to 200 000 volts. These problems are numbered from *1* to *64*. When a reference is made in the following to some problem number it will refer to one of this list of problems.

SYMBOLS

The symbols which will be employed in the following treatment are given below:—

FOR LOAD CONDITIONS

$Kv\text{-}a_r$ = (total) at receiving end.
$Kv\text{-}a_{rn}$ = (one conductor to neutral) at receiving end.
$Kv\text{-}a_s$ = (total) at sending end.
$Kv\text{-}a_{sn}$ = (one conductor to neutral) at sending end.
Kw_r = Kw (total) at receiving end.
Kw_{rn} = Kw (one conductor to neutral) at receiving end.
Kw_s = Kw (total) at sending end.
Kw_{sn} = Kw (one conductor to neutral) at sending end.
E_r = Voltage between conductors at receiving end.
E_{rn} = Voltage from conductors to neutral at receiving end.
E_s = Voltage between conductors at sending end.
E_{sn} = Voltage from conductors to neutral at sending end.
I_r = Current in amperes per conductor at receiving end.
I_s = Current in amperes per conductor at sending end.
$Cos\ \theta_r$ = Power-factor at receiving end.
$Cos\ \theta_s$ = Power-factor at sending end.

FOR ZERO LOAD CONDITIONS

The symbols corresponding to zero load conditions are as indicated above for load conditions with the addition of a sub zero.

THE FUNDAMENTAL OR LINEAR CONSTANTS

The fundamental, or "linear constants" of the circuit for each conductor per unit length are represented as follows:—

r = Linear resistance in ohms per conductor mile (taken from Table II)
x = Linear reactance in ohms per conductor mile (taken from Table IV or V)
b = Linear capacitance susceptance to neutral in mhos per conductor mile (taken from Table IX or X)
g = Linear leakage conductance to neutral in mhos per conductor mile. (This represents the direct escape of active power through the air between conductors and of active power leakage over the insulators. These losses must be estimated for conditions similar to these of the circuit under consideration. For all lines except those of great length and high voltage it is common practice to disregard the effects of leakage or corona loss and to take g as equal to zero.
z = Linear impedance $= \sqrt{r^2 + x^2}$
y = Linear admittance $= \sqrt{g^2 + b^2}$

If the length of each conductor of the circuit in unit length is designated as l we have

rl = Total resistance in ohms per conductor $= R$
xl = Total reactance in ohms per conductor $= X$
bl = Total susceptance in mhos per conductor to neutral $= B$
gl = Total conductance in mhos per conductor to neutral $= G$

then,

$Z = \sqrt{R^2 + X^2}$ ohms

and, $Y = \sqrt{G^2 + B^2}$ mhos

IR = Voltage necessary to overcome the resistance.

IX = Voltage necessary to overcome the reactance.

IZ = Voltage necessary to overcome the impedance.

METHODS FOR DETERMINING THE CONSTANTS OF THE CIRCUIT

Several different methods for determining the fundamental constants of the circuit are in use. These methods are illustrated below.

Problem—Find the resistance volts IR and the reactance volts IX in percent of delivered volts E_r for the following conditions:—100 kw active power to be delivered at 1000 volts, three-phase, 60 cycles, over three No. 0000 stranded, hard drawn, copper conductors, circuit one mile long, with a symmetrical delta arrangement of conductors, two foot spacing, the temperature being taken as 25 degrees C.

Resistance of one mile of single conductor = 0.277 ohm (from Table II)

Reactance of one mile of single conductor = 0.595 ohm (from Table V)

Method No. 1—When three-phase circuits first came into use, it was customary (and correct), in determining the loss and voltage regulation, to consider them equivalent to two single-phase circuits, each single-phase circuit transmitting one-half the power of the three-phase system. This practice is still followed by some engineers; thus:—

$\frac{50\,000}{1000} = 50$ *amp. per conductor for each single-phase circuit.*

$\frac{0.277 \times 2 \times 50}{1000} \times 100 = 2.77\%$ *resistance volts drop of single-phase circuit.*

$\frac{0.595 \times 2 \times 50}{1000} \times 100 = 5.95\%$ *reactance volts drop of single-phase circuit.*

Method No. 2 consists of treating the case as a straight three-phase problem. Thus:

$\frac{100\,000}{1000 \times 1.732} = 57.73$ *amperes per conductor of three-phase circuit.*

$\frac{0.277 \times 1.732 \times 57.73}{1000} \times 100 = 2.77\%$ *resistance volts drop of three-phase circuit.*

$\frac{0.595 \times 1.732 \times 57.73}{1000} \times 100 = 5.95\%$ *reactance volts drop of three-phase circuit.*

Method No. 3 consists in assuming one-third the total power transmitted over one conductor with neutral or ground return (resistance and reactance of return being taken as zero). Such an equivalent circuit is shown by diagram in the upper part of Fig. 16. Thus the circuit constants for the above problem would be determined as follows:—

Watts per phase $= \frac{100\,000}{3} = 33\,333$ *watts.*

Volts to neutral = 1000 × 0.5774 *or* 577.4 *volts.*

$\frac{33\,333}{577.4} = 57.74$ *amperes per conductor; (same as for method No. 2)*

$\frac{0.277 \times 57.74}{577.4} \times 100 = 2.77\%$ *resistance volts drop of three-phase circuit.*

$\frac{0.595 \times 57.74}{577.4} \times 100 = 5.95\%$ *reactance volts drop of three-phase circuit.*

It will be seen that all three methods produce the same results. *Method No. 3* seems the most readily adaptable to various kinds of transmission systems and will be used exclusively in the treatment of the problems which will follow.

APPLICATION OF THE TABLES

Numerous tables of constants, charts, etc., have been presented, and a few more will follow. Chart II plainly indicates the application of these tables, etc. to the calculation of transmission circuits and the sequence in which they should be consulted.

GRAPHICAL VS. MATHEMATICAL SOLUTIONS

At the time of the design of a transmission circuit the actual maximum load or power-factor of the load that the circuit will be called upon to transmit is seldom known. An unforseen development leading to an increased demand for electrical energy may result in a greatly increased load to be transmitted. The actual length of a circuit (especially when located in a hilly or rolling country) is never known with mathematical accuracy. Moreover, the actual resistance of the conductors varies to a large extent with temperature variations along the circuit.

When it is considered that there are so many indeterminate variables which vitally affect the performance of a transmission circuit, it would seem that a comparatively long and highly mathematical solution for determining the exact performance, necessarily based upon rigid assumptions, is hardly justified. In many cases the economic loss in transmission will determine the size of conductors and, if the circuit is very long, synchronous machinery is likely to be employed for controlling the voltage.

Mathematical solutions have one very important virtue, in that they provide an entirely different but parallel route in the solution of such problems, and therefore are valuable as a check against serious errors in the results obtained by the more simple graphical solutions.

In the following treatment, simple but highly accurate graphical solutions will be first presented, for determining the performance not only of short transmission lines, but also for long lines. For short lines the Dwight and the Mershon charts will be used. For long lines, where the effect of capacitance must be accurately accounted for, the Wilkinson Charts, supplemented with vector diagrams will be used. These three forms of graphical solutions will, when correctly applied to any power transmission problem, produce results in which the error will be much less than that due to irregularities in line construction and inaccurate assumptions of circuit constants. These three graphical solutions will in each case be followed by mathematical solutions. In the case of short lines the usual formulas employing trigonometric functions will be employed, and in the case of long lines the convergent series, and two different forms of hyperbolic solutions will be employed.

GRAPHICAL SOLUTION

When the receiving end load conditions, that is, the voltage, the load and the power-factor are known, the *IR* volts required to overcome the resistance and the *IX* volts required to overcome the reactance of the circuit, may be readily calculated.

On a piece of plain paper or cross-section paper divided into tenths, a vector diagram of the current and of the various voltage drops of the circuit may be laid out to a convenient scale. Whichever kind of paper is used, the procedure will be as in the following example.

Single-Phase Problem—Find the voltage at the sending end of a single-phase circuit 16 miles long, consisting of two stranded, hard drawn No. 0000 copper conductors spaced three feet apart. Temperatures taken as 25 degrees C. Load conditions at receiving end assumed as 4000 kv-a (3200 kw at 80 percent power-factor lagging) 20 000 volts, single-phase, 60 cycles.

$$Kv\text{-}a_{rn} = \frac{4000}{2} = 2000 \text{ kv-a to neutral.}$$

$$E_{rn} = \frac{20\,000}{2} = 10\,000 \text{ volts to neutral.}$$

$$I_r = \frac{2\,000\,000}{10\,000} = 200 \text{ amperes per conductor.}$$

The fundamental constants per conductor are:—

$$R = 16 \times 0.277 \text{ (from Table II)} = 4.432 \text{ ohms}$$

$$X = 16 \times 0.644 \text{ (from Table V)} = 10.304 \text{ ohms}$$

and $IR = 200 \times 4.432 = 886$ *volts resistance drop*

$$= \frac{886}{10\,000} \times 100 = 8.86 \text{ percent}$$

$$IX = 200 \times 10.304 = 2061 \text{ volts reactance drop}$$

$$= \frac{2061}{10\,000} \times 100 = 20.61 \text{ percent}$$

Having determined the above values a vector diagram may be made as follows:—

Draw an arc quadrant having a radius of 10 000 (the receiving end voltage to neutral) to some convenient scale, as shown in Fig. 17. The radius which represents the base, or horizontal line will be assumed as representing the phase of the current at the receiving end of the circuit. Divide this base line into ten equal parts. These ten divisions will then correspond to loads of corresponding power-factors. Since a load has been assumed having a power-factor of 80 percent lagging, draw a vertical line from the 0.8 division on the base line, until it intersects the arc of the circle. From this point of intersection draw a line to the right and parallel with the base line. To the same scale as that plotted for the receiver voltage (10 000) measure off to the right 886 volts to *D*. This is the voltage which, as determined above is required to overcome the resistance of one conductor of the circuit. It is sometimes stated as the voltage consumed by the line resistance. It will be noted that this voltage drop is in phase with the current at the receiving end. From this point lay off vertically, and to the same scale, 2061 volts which is, as determined above, the volts necessary to overcome the reactance of one conductor of the circuit. This is sometimes stated as the voltage consumed by the line reactance. Connect this last point by a straight

CHART II.—APPLICATION OF TABLES TO SHORT TRANSMISSION LINES

(EFFECT OF CAPACITANCE NOT TAKEN INTO ACCOUNT) OVER HEAD BARE CONDUCTORS

Starting with the kv-a., voltage and power-factor at the receiving end known.

QUICK ESTIMATING TABLES XII TO XXI INC.

From the quick estimating table corresponding to the voltage to be delivered, determine the size of the conductors corresponding to the permissible transmission loss.

HEATING LIMITATION—TABLE XXIII

If the distance of transmission is short and the amount of power transmitted very large there is a possibility of overheating the conductors—to guard against such overheating the carrying capacity of the conductors contemplated should be checked by this table.

CORONA LIMITATION—TABLE XXII

If the transmission is at 30 000 volts, or higher, this table should be consulted to avoid the employment of conductors having diameters so small as to result in excessive corona loss.

RESISTANCE—TABLES I AND II

From one of these tables obtain the resistance per unit length of single conductor corresponding to the maximum operating temperature—calculate the total resistance for one conductor of the circuit—if the conductor is large (250 000 circ. mils or more) the increase in resistance due to skin effect should be added.

I^2R TRANSMISSION LOSS

Calculate the I^2R loss of one conductor by multiplying its total resistance by the square of the current—to obtain the total loss multiply this result by the number of conductors of the circuit.

REACTANCE—TABLES IV AND V

From one of these tables obtain the reactance per unit length of single conductor. Calculate the total reactance for one conductor of the circuit. If the reactance is excessive (20 to 30 percent reactance volts will in many cases be considered excessive) consult Table VI or VII. Having decided upon the maximum permissible reactance the corresponding resistance may be found by dividing this reactance by the ratio value in Table VI or VII. When the reactance is excessive, it may be reduced by installing two or more circuits and connecting them in parallel, or by the employment of three conductor cables. Using larger conductors will not materially reduce the reactance. The substitution of a higher transmission voltage, with its correspondingly less current, will also result in less reactance.

GRAPHICAL SOLUTION

A simple graphical solution, as described in the text, may be made by which the kv-a, the voltage and the power-factor at the sending end of the circuit may be determined graphically. Or the voltage at the sending end may be determined graphically by the use of either the Dwight or the Mershon chart. With the Mershon chart the power-factor at the sending end may be read directly from the chart.

MATHEMATICAL SOLUTION

As a precaution against errors the results obtained graphically should be checked by a mathematical solution, in cases where accuracy is essential.

line with the center E of the arc. The length of this line ES represents the voltage to neutral at the sending end which, for this problem, is 11 998 volts. The distance this line extends beyond the arc represents the drop in voltage for one conductor of the circuit. The voltage drop for this problem is $\frac{1998}{10\,000} \times 100 = 19.98$ percent of the receiving end voltage.

The phase difference between the current and the voltage at the receiver end is $\theta_r = 36° 52'$. This is the angle whose cosine is 0.8 corresponding to a power-factor at the receiving end of 80 percent. Likewise the phase difference between the receiving end current and the sending end voltage is $\theta_s = 42° 13'$ corresponding to a power-factor at the supply end of 74.06 percent. The difference in these two phase angles (5° 21′) represents the difference in the phase of the voltages at the sending and receiving ends of the circuit. The power-

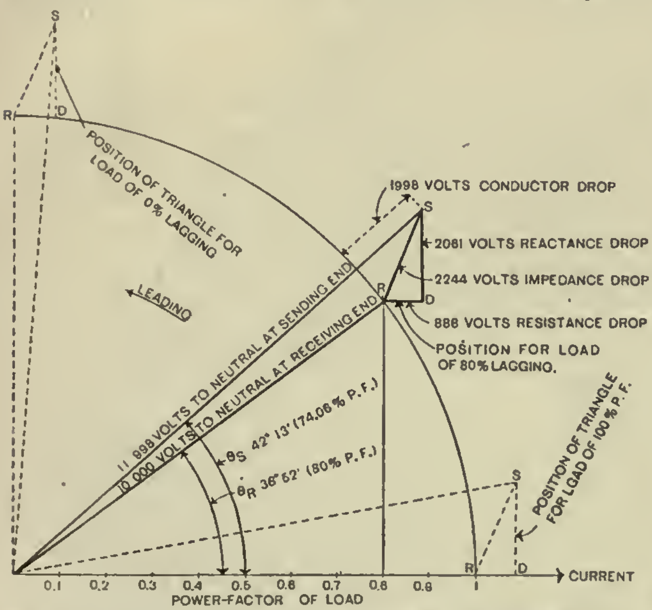


FIG. 17—GRAPHICAL SOLUTION FOR A SHORT TRANSMISSION LINE
Capacitance effect not taken into account.

factor at the sending end of the circuit may be readily obtained by dropping a vertical line down from the point where the line representing the sending end voltage ES intersects the arc of the circle, to the base line representing the phase of the receiving end current. Such a line will correspond to a power-factor of 74.06 percent. This assumption that the vector representing the direction of the receiving end current also represents the direction of the sending end current is upon the basis that the circuit is without capacitance. It, therefore, is permissible only with short lines.

In Fig. 17 the location of the impedance triangle is also indicated (by broken lines) in positions corresponding to a receiving end load of 100 percent power-factor; and also for a receiving end load of zero lagging power-factor. It is interesting to note that in the case of 100 percent power-factor the resistance drop (at right angle to the arc) has a maximum effect upon the voltage drop; whereas the reactance drop (nearly parallel with the arc) has a minimum effect upon the voltage drop. At zero lagging power-factor load just the reverse is true; namely the resistance drop is nearly parallel with the arc and causes a minimum voltage drop, while the reactance is at right angles and produces a maximum effect upon the voltage drop.

VOLTAGE AT SENDING END AND LOAD AT RECEIVING END FIXED

In cases of feeders to be tapped into main transmission circuits, the voltage at the sending end is usually fixed. It may be desired to determine what the voltage will be at the receiving end corresponding to a given load. This may be obtained graphically as follows:—

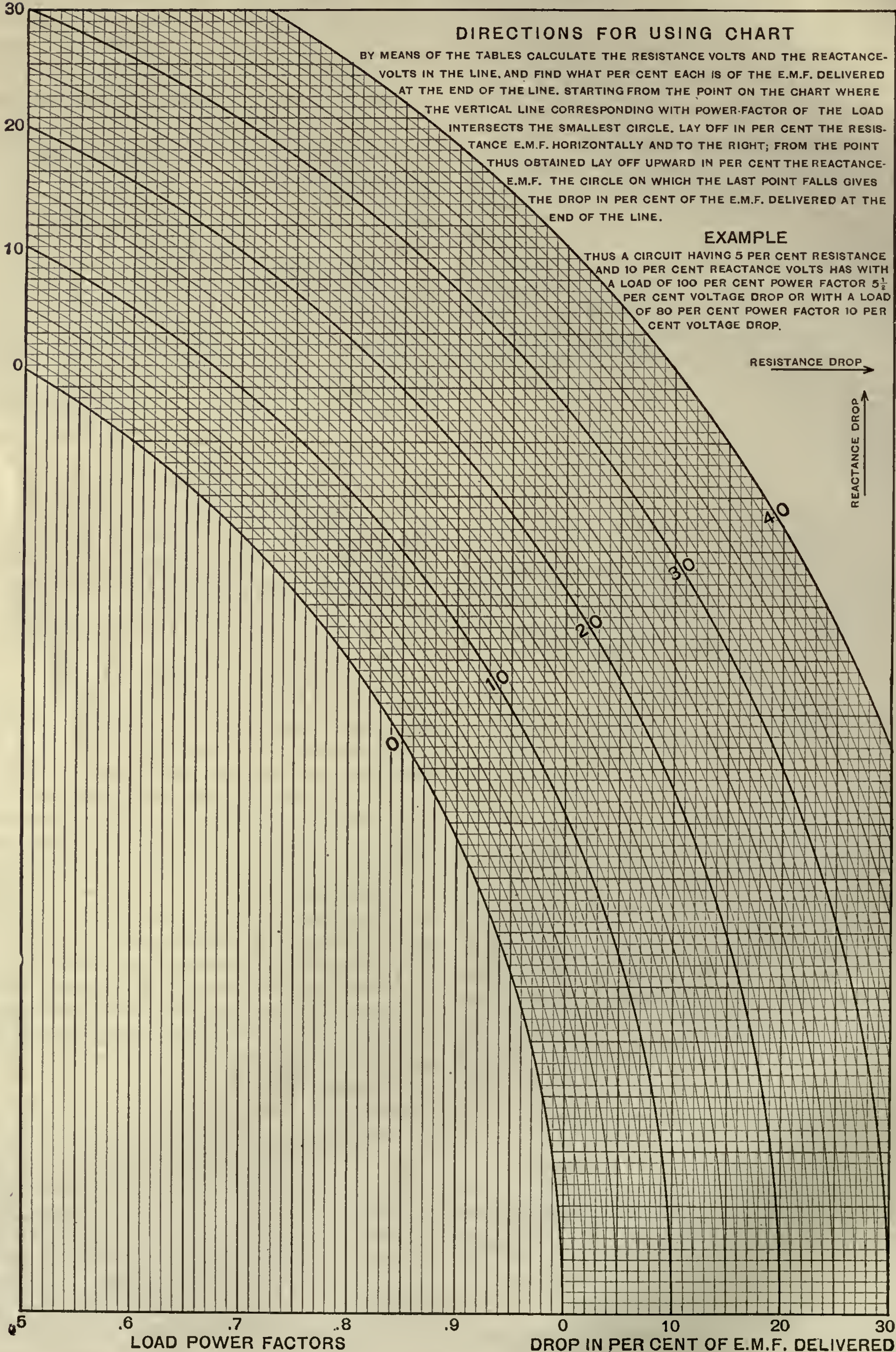
Draw a horizontal line which will be assumed to represent the phase of the current. (Fig. 17) Since the power-factor of the load at the receiving end is known, the angle whose cosine corresponds may be obtained from Table *K*. This angle represents the phase relation between the current and the voltage at the receiving end of the circuit. For the problem illustrated by Fig. 17 this angle is 36° 52′, corresponding to a power-factor of 80 percent. Having determined this angle, draw a second radial line intersecting the current vector at the angle corresponding to the receiving end load power-factor. This second line will then represent the direction of the voltage at the receiving end of the circuit. If the load power-factor is lagging, this line will be in the forward direction, and if the load power-factor is leading it will be in the backward direction from the current vector. Now with the intersection of the current and voltage vectors as a center, draw an arc of a circle to some suitable scale, representing the voltage at the sending end. Calculate the voltage necessary to overcome the resistance, and also that necessary to overcome the reactance of the circuit.

Draw a right angle impedance triangle to the same scale, using the resistance volts as a base. Cut out the impedance triangle to its exact size. Keeping the base of the triangle (resistance voltage) in a horizontal position (parallel with the current vector) move the triangle over the diagram in such a manner that its apex follows the arc of the circle representing the numerical value of the voltage at the sending end. Move the triangle up or down until a position is found where it makes connection with the vector representing the voltage at the receiving end. This is then the correct position for the impedance triangle, and the receiving end voltage may be scaled off.

GRAPHICAL SOLUTION BY THE MERSHON CHART

The above graphical solution is that employed in the well known chart which Mr. Ralph D. Mershon early presented to the electrical profession, and which is reproduced as Chart III. The Mershon Chart is simply a diagram on cross-section paper with vertical and horizontal subdivisions each representing one percent of receiving end voltage. On this chart a number of concentric arcs are drawn, representing voltage drops up to 40 percent. After the reactance and the resistance volts have been calculated and expressed in per-

CHART III—MERSHON CHART



cent of E_r the impedance triangle is traced upon the chart and the voltage drop in percentage of E_r is read directly as indicated by the directions. All values on the chart are expressed in percent of the receiving end voltage.

Single-Phase Problem—Taking the resistance voltage as 8.86 percent and the reactance voltages 20.61 percent of the receiving end voltage, for the above single-phase problem, (Fig. 17) and tracing these values upon the Mershon Chart for a receiving end load of 80 percent power-factor lagging, the voltage drop is determined as 19.9 percent. The calculated value being 19.98 percent, the error by the chart is seen to be negligible.

WHEN THE SENDING END CONDITIONS ARE FIXED

When the conditions at the sending end are fixed and those at the receiving end are to be determined, the solving of the problem by the Mershon Chart is more complicated. In such cases, it is usual to estimate what the probable receiving end condition will be. From these estimated receiving end conditions, determine by the chart the corresponding sending end conditions. If the conditions as determined by this assumption are materially different from the known conditions, another assumption should be made. The corresponding sending end conditions should then be checked with the known conditions. Several such trials will usually be necessary to solve such problems.

GRAPHICAL SOLUTION BY THE DWIGHT CHART

Mr. H. B. Dwight has worked up a straight line chart, shown as Chart IV, in which the resistance and the reactance of the circuit have been taken into account through the medium of spacing lines marked for various sizes of conductors.* The use of this chart does not, therefore, require the calculation of the resistance and reactance or the use of tables of such constants. The Dwight Chart is also constructed so as to be applicable to loads of leading as well as to loads of lagging power-factors, whereas the Mershon chart, as generally constructed, is applicable to loads of lagging power-factor only. However the Mershon Chart can be made applicable for the solving of problems of leading as well as lagging power-factor loads by extending it through the lower right-hand quadrant. The application of synchronous condensers frequently gives rise to loads of leading power-factor. The Dwight Chart is well adapted to the solution of such circuits. Still another feature of this chart is that formulas are given which take capacitance effect into account with sufficient accuracy for circuits with a length up to approximately 100 miles.

Single-Phase Problem—Find the voltage at the sending end of a single-phase circuit 16 miles long, consisting of two stranded, hard-drawn, No. 0000 copper conductors, spaced three feet apart. Temperature taken as 25 degrees C. Load condition at receiving end assumed as 4000 kv-a (3200 kw at 80 percent power-factor lagging) 20 000 volts single-phase, 60 cycles.

From Table II the resistance of No. 0000 stranded, hard-drawn, copper conductors at 25 degrees C. is found to be 0.277 ohm per wire per mile. Lay a straight edge across the Dwight Chart from the resistance value per mile 0.277 (as read on the lower half of the vertical line to the extreme right) to the spacing of three feet for copper conductors and 60 cycles at the extreme left. Along this straight edge read factor V = 0.62, corresponding to a lagging power-factor of 80 percent. This factor V is equivalent to the change in receiving end voltage per total ampere per mile of circuit, due to the line impedance.

It will be noted that opposite the resistance values (extreme right vertical line) is placed the corresponding sizes of copper and aluminum conductors on the basis of a temperature of 20 degrees C. If the temperature is assumed to be 20 degrees C. it will not be necessary to consult a table of resistance values. In such a case, the straight edge would simply be placed over the division of the vertical resistance line corresponding to the size and material of conductors. Marking a resistance value on this vertical line makes the chart adaptable to resistance values corresponding to conductors at any temperature. Had the power factor been leading, in place of lagging, the corresponding resistance point would have been located on the upper half of the vertical resistance line.

Continuing following the directions on the chart for short lines, we obtain the following. Since the circuit is single-phase, use $2\,V = 1.24$

$$\textit{Voltage drop in percent of } E_r = \frac{100\,000 \times 4000 \times 16 \times 1.24}{20\,000^2} = 19.84 \textit{ percent}$$

The voltage drop, as calculated mathematically, is 19.98 percent representing an error of 0.14 percent by the chart.

Three-Phase Problem (No. 33)—Find the voltage at the sending end of a three-phase circuit, 20 miles long, consisting of three No. 0000 stranded, hard-drawn, copper conductors, spaced three feet apart in a delta arrangement. Temperature taken as 25 degrees C. Load conditions at receiving end assumed as 1300 kv-a (1040 kw at 80 percent power-factor lagging) 10 000 volts, three-phase, 60 cycles.

From Table II, the resistance per wire per mile is again found to be 0.277 ohm and since the spacing and frequency are both the same as in the case of the above single-phase problem, we again obtain V = *0.62*. The voltage drop in percent of E_r is therefore

$$\frac{100\,000 \times 1300 \times 20 \times 0.62}{10\,000^2} = 16.12 \textit{ percent}$$

The voltage drop as calculated mathematically is 16.16 percent, representing an error of 0.04 percent.

CAPACITANCE

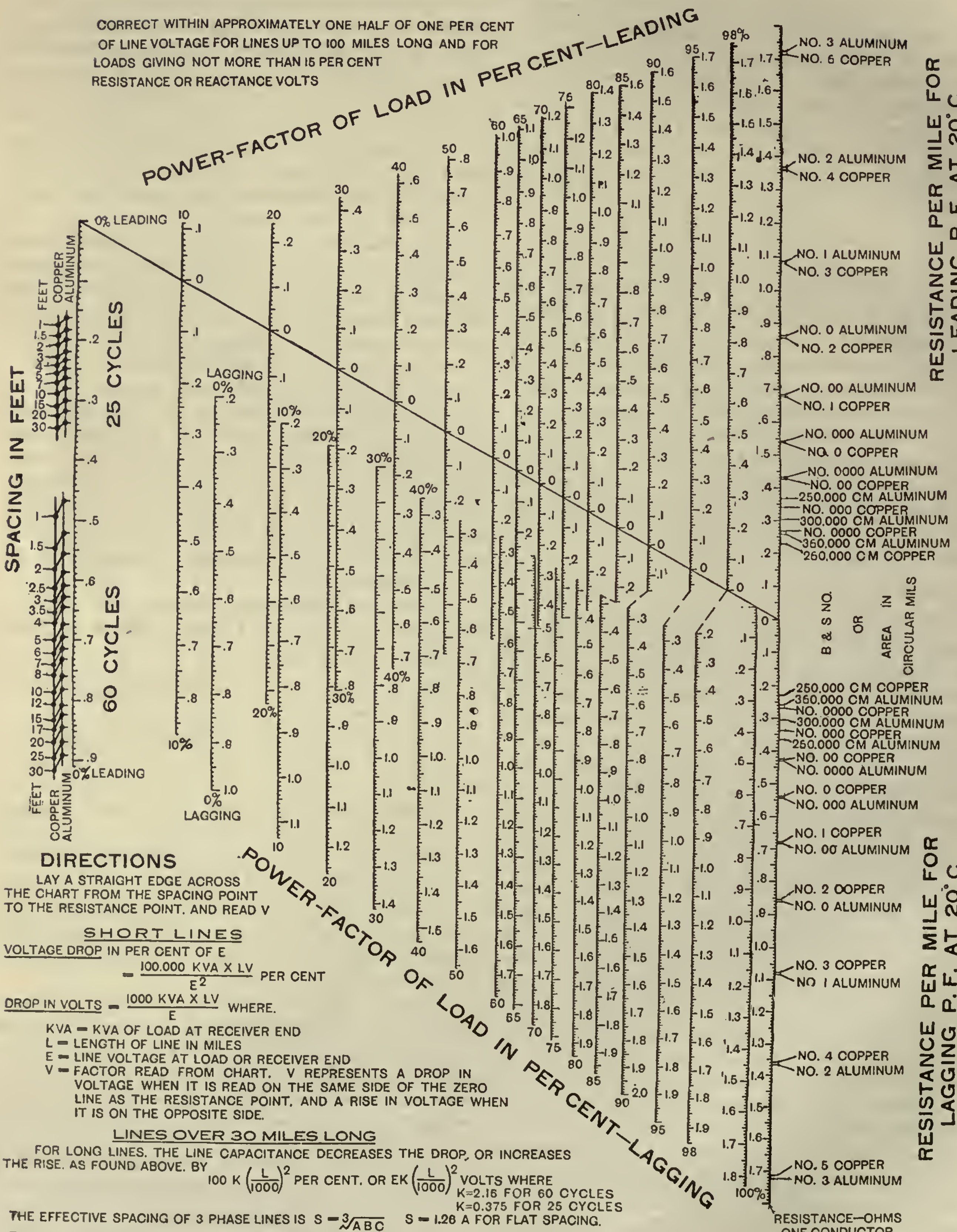
In long circuits the effect of capitance is to decrease the voltage drop, or increase the voltage rise, as

*The basis of the construction of this chart is described in the JOURNAL for July, 1915, p. 306.

CHART—IV DWIGHT CHART

FOR DETERMINING THE VOLTAGE REGULATION OF TRANSMISSION CIRCUITS CONTAINING CAPACITANCE

CORRECT WITHIN APPROXIMATELY ONE HALF OF ONE PER CENT OF LINE VOLTAGE FOR LINES UP TO 100 MILES LONG AND FOR LOADS GIVING NOT MORE THAN 15 PER CENT RESISTANCE OR REACTANCE VOLTS



DIRECTIONS

LAY A STRAIGHT EDGE ACROSS THE CHART FROM THE SPACING POINT TO THE RESISTANCE POINT, AND READ V

SHORT LINES

VOLTAGE DROP IN PER CENT OF E

$$= \frac{100.000 \text{ KVA X LV}}{E^2} \text{ PER CENT}$$

DROP IN VOLTS $= \frac{1000 \text{ KVA X LV}}{E}$ WHERE.

KVA = KVA OF LOAD AT RECEIVER END
L = LENGTH OF LINE IN MILES
E = LINE VOLTAGE AT LOAD OR RECEIVER END
V = FACTOR READ FROM CHART. V REPRESENTS A DROP IN VOLTAGE WHEN IT IS READ ON THE SAME SIDE OF THE ZERO LINE AS THE RESISTANCE POINT, AND A RISE IN VOLTAGE WHEN IT IS ON THE OPPOSITE SIDE.

LINES OVER 30 MILES LONG

FOR LONG LINES. THE LINE CAPACITANCE DECREASES THE DROP, OR INCREASES THE RISE. AS FOUND ABOVE. BY

$100 \text{ K} \left(\frac{L}{1000}\right)^2$ PER CENT. OR $EK \left(\frac{L}{1000}\right)^2$ VOLTS WHERE
K=2.16 FOR 60 CYCLES
K=0.375 FOR 25 CYCLES

THE EFFECTIVE SPACING OF 3 PHASE LINES IS $S = \sqrt[3]{ABC}$ S = 1.26 A FOR FLAT SPACING.

THE ABOVE EQUATIONS ARE FOR 2 AND 3 PHASE LINES.
FOR SINGLE PHASE USE 2 V IN PLACE OF V

will be explained later. The Dwight and Mershon charts do not recognize the effect which capacitance has upon the voltage drop. In the lower left hand corner of the Dwight Chart, however, there is placed a formula by which a correction may be applied to the voltage drop as given by the chart. This correction accounts for the effect of the charging current (resulting from capacitance) quite accurately, provided the circuit is not too long or the frequency too high. The application of this corrective factor will be evident from the following problem.

TABLE K—COSINES, SINES AND TANGENTS

ANGLE	COS θ (P F)	SIN θ	TAN θ
0° 00′	1.000	0.0000	0.0000
8° 06′	0.990	0.1409	0.1423
11° 28′	0.980	0.1988	0.2028
14° 04′	0.970	0.2430	0.2506
16° 15′	0.960	0.2798	0.2915
18° 11′	0.950	0.3120	0.3285
19° 56′	0.940	0.3410	0.3627
21° 33′	0.930	0.3673	0.3949
23° 04′	0.920	0.3918	0.4258
24° 29′	0.910	0.4144	0.4554
25° 50′	0.900	0.4357	0.4841
27° 07′	0.890	0.4558	0.5121
28° 21′	0.880	0.4748	0.5396
29° 32′	0.870	0.4929	0.5665
30° 41′	0.860	0.5103	0.5934
31° 47′	0.850	0.5267	0.6196
32° 51′	0.840	0.5424	0.6457
33° 54′	0.830	0.5577	0.6720
34° 54′	0.820	0.5721	0.6976
35° 54′	0.810	0.5864	0.7239
36° 52′	0.800	0.6000	0.7499
37° 48′	0.790	0.6129	0.7757
38° 44′	0.780	0.6257	0.8021
39° 38′	0.770	0.6379	0.8283
40° 32′	0.760	0.6499	0.8551
41° 24′	0.750	0.6613	0.8816
42° 16′	0.740	0.6726	0.9089
43° 06′	0.730	0.6833	0.9358
43° 56′	0.720	0.6938	0.9634
44° 45′	0.710	0.7040	0.9913
45° 34′	0.700	0.7141	1.0199
46° 22′	0.690	0.7238	1.0489
47° 09′	0.680	0.7331	1.0780
47° 55′	0.670	0.7422	1.1074
48° 42′	0.660	0.7513	1.1383
49° 27′	0.650	0.7598	1.1688
50° 12′	0.640	0.7683	1.2002
50° 57′	0.630	0.7766	1.2327
51° 41′	0.620	0.7846	1.2655
52° 24′	0.610	0.7923	1.2985
53° 07′	0.600	0.8000	1.3327
53° 50′	0.590	0.8073	1.3680
54° 32′	0.580	0.8145	1.4037
55° 14′	0.570	0.8215	1.4406
55° 56′	0.560	0.8284	1.4788
56° 37′	0.550	0.8350	1.5175
57° 18′	0.540	0.8415	1.5577
57° 59′	0.530	0.8479	1.5993
58° 40′	0.520	0.8542	1.6426
59° 20′	0.510	0.8601	1.6864
60° 00′	0.500	0.8660	1.7320
60° 39′	0.490	0.8716	1.7783
61° 18′	0.480	0.8771	1.8265
61° 57′	0.470	0.8825	1.8768

Three-Phase Problem (*No. 45*)—Find the voltage at the sending end of a three-phase circuit, 100 miles long, consisting of three No. 0000, stranded, hard-drawn copper conductors, spaced nine feet apart in a delta arrangement. Temperature assumed as 25 degrees C. Load conditions at receiving end assumed as 22 000 kv-a, 80 percent power-factor lagging, 88 000 volts, 60 cycles.

From Table II the resistance is found to be 0.277 ohm per mile. From Dwight Chart read V = *0.70*. Then, the voltage drop in percent of E_r, if the line were short, would be,

$$\frac{100\,000 \times 22\,000 \times 100 \times 0.70}{88\,000^2} = 19.89 \textit{ percent}$$

From directions on the Dwight chart for circuits over 30 miles long, the charging current of this circuit is found to be such as to decrease the voltage drop under load conditions or to increase the voltage at zero load by the amount of $100 \times 2.16\left(\frac{100}{1000}\right)^2 = 2.16$ percent. Hence the voltage at the sending end, under load conditions, will be *19.89* — *2.16* = *17.73* percent. The actual result as calculated rigorously is *17.94* percent. Thus the error by the Dwight graphical solution is approximately *0.21* percent.

If the power-factor of the load is assumed as 100 percent (problem 46) in place of 80 percent lagging, we get $V = 0.33$ and find the error for the Dwight graphical solution of this 100 mile, 60 cycle circuit to be approximately *0.75* percent. It should be noted, however, that the reactance volts are in this case 22 percent of the receiving end voltage.

SENDING END CONDITIONS FIXED

When the sending end conditions are fixed, a different form of solution must be employed to determine the size of conductors corresponding to a given voltage drop. In such cases, the Dwight Chart is particularly applicable. To use the chart for the solution of such problems proceed as follows. First V is calculated by means of the formulas on the chart, and then a straight edge is placed through V (on the line corresponding to the power-factor of the load) and the point for the spacing and frequency to be used, and the required size of conductor can be seen at a glance on the resistance scale at the right. To make this application of the chart clear, the following is given,—

$$\textit{Voltage drop in percent of } E_r = \frac{100\,000\ Kv\text{-}a \times L\ V}{E_r^2} \quad (28)$$

Hence

$$V = \frac{\textit{Voltage drop in percent of } E_r \times E_r^2}{100\,000\ Kv\text{-}a \times L} \quad \ldots\ldots\ldots\ldots (29)$$

Applying (29) to the above problem No. 33 we get

$$V = \frac{16.12 \times 10\,000^2}{100\,000 \times 1300 \times 20} = 0.62$$

Following the above directions, the resistance per mile is found to be 0.277 ohm and the corresponding size of conductor No. 0000 copper.

MATHEMATICAL SOLUTION

In order to check any one, or all of the above described graphical methods, a complete mathematical solution may be made by applying the various trigonometrical formulas, Fig. 18, to the values of the problem under consideration. These formulas have been arranged to meet the conditions of loads of either lagging or leading power-factors, and for conditions fixed at either the receiving or the sending ends.

There are numerous problems requiring a solution where the voltage at the sending end, and the kilowatts and the power-factor of the load at the receiving end are fixed. In such cases it is required to determine the corresponding receiving end voltage. This determination can be made mathematically, but such a solution is tedious, since the formulas applying to such cases are cumbersome. Formulas are given at the bottom of Fig. 18 which may be applied to such problems. Time and labor may, however, be saved in solving such problems by the employment of a cut-and-try method usually used in such cases, as follows:—

Assume what the voltage drop will be, corresponding to the size of conductors likely to be used. On the basis of this assumption the receiving end voltage is fixed; thus, all of the receiving end conditions are assumed to be fixed. The corresponding sending end voltage is then readily determined by one of the graphical methods described. If the sending end voltage thus determined is found to be materially different from the fixed sending end voltage, another trial, based upon a different receiving end voltage, will probably suffice.

Single-Phase Problem—Find the characteristics of the load at the sending end of a single-phase circuit, 16 miles long, consisting of two stranded, hard drawn, copper conductors, spaced three feet apart; temperature taken as 25 degrees C.; load conditions at receiving end assumed as 4000 kv-a (3200 kw at 80 percent power-factor lagging) 20 000 volts, 60 cycles; transmission loss to be approximately ten percent.

Following the procedure given in Chart II, consult Quick Estimating Table XVII for a delivered voltage of 20 000. Since the conditions of the above problem are a power-factor of 80 percent, and a temperature 25 degrees C, the corresponding kv-a values are as indicated at the head of the table on the basis of 10.8 percent loss in transmission for a three-phase circuit. For a single-phase circuit the corresponding values will be one-half the table values. Thus the 4 000 kv-a single phase circuit of the problem is equivalent to 8000 kv-a, three-phase on the table. From the table, it is seen that for a distance of 16 miles 7810 kv-a, three-phase can be transmitted over No. 0000 conductors with a loss of 10.8 percent. 7810 kv-a is near enough to 8000 kv-a, and the loss of 10.8 percent is near enough to an assumed loss of ten percent, so we decide that No. 0000 copper conductors come nearest to the proper size to meet the conditions of the problem. The loss with No. 0000 conductors will be $\frac{8000}{7810} \times 10.8 = 11.06$ percent, as will be shown later.

Table XXIII indicates that there will be no overheating of this size of conductor.

Table XXII indicates that 20 000 volts is too low to result in corona loss with No. 0000 conductors, at any reasonable altitude. Then,—

$$Kv\text{-}a_{rn} = \frac{4000}{2} = 2000 \textit{ kv-a to neutral.}$$

$$Kw_{rn} = \frac{3200}{2} = 1600 \textit{ kw to neutral.}$$

$$E_{rn} = \frac{20\,000}{2} = 10\,000 \textit{ volts to neutral.}$$

$$I_r = \frac{2\,000\,000}{10\,000} = 200 \textit{ amperes per conductor.}$$

The resistance per conductor is

$$R = 16 \times 0.277 \text{ (from Table II)} = 4.432 \textit{ ohms.}$$

The reactance per conductor is

$$X = 16 \times 0.644 \text{ (from Table V)} = 10.304 \textit{ ohms.}$$

and $IR = 200 \times 4.432 = 866$ *volts, resistance drop*

$$= \frac{886}{10\,000} \times 100 = 8.86 \textit{ percent}$$

$$IX = 200 \times 10.304 = 2061 \textit{ volts, reactance drop}$$

$$= \frac{2061}{10\,000} \times 100 = 20.61 \textit{ percent}$$

$$E_{sn} = \sqrt{(10\,000 \times 0.8 + 866)^2 + (10\,000 \times 0.6 + 2061)^2}$$
$$= 11\,998 \textit{ volts to neutral} \quad \ldots\ldots\ldots\ldots \quad (30)$$

$$\theta_s = tan^{-1}\left(\frac{(10\,000 \times 0.6) + 2061}{(10\,000 \times 0.8) + 886}\right) = 42^\circ\ 13' \quad \ldots\ldots \quad (31)$$

$$\textit{Percent } PF_s = (Cos.\ 42^\circ\ 13') \times 100 = 74.06 \textit{ percent} \ldots\ldots (32)$$

$$Kv\text{-}a_{sn} = \frac{200 \times 11\,998}{1000} = 2399.6 \textit{ kv-a per conductor} \ldots\ldots (33)$$

$$Kw_{sn} = 2399.6 \times 0.7406 = 1777.1 \text{ kw per conductor} \ldots\ldots (34)$$

$$\textit{Percent voltage drop} = \frac{11\,998 - 10\,000}{10\,000} \times 100 = 19.98 \textit{ percent} \quad \ldots\ldots (46)$$

$$\textit{Transmission loss} = \frac{(200)^2 \times 4.432}{1000} = 177.28 \textit{ kw per conductor} \quad \ldots\ldots (47)$$

$$\textit{Percent transmission loss} = \frac{177.28 \times 2}{3200} \times 100 = 11.08 \textit{ percent} \quad \ldots\ldots (48)$$

Three-Phase Problem (No. 33)—Find the characteristics of the load at the sending end of a three-phase circuit 20 miles long, consisting of three stranded, hard-drawn, copper conductors, spaced in a three foot delta. Temperature taken as 25 degrees C. Load conditions at receiving end assumed as 1300 kv-a. (1040 kw at 80 percent power-factor lagging) 10 000 volts, 60 cycles; transmission loss not to exceed ten percent.

Following the procedure given in Chart II, the following results are obtained:—

Consult Table XV for a delivered voltage of 10 000 volts. Since the conditions of the above problems are, power-factor of load 80 percent, temperature 25 degrees C. the corresponding three-phase kv-a values of the table are on the basis of 10.8 percent loss in transmission. From Table XV it is seen that 1240 kv-a, three-phase can be transmitted over No. 000 conductors, or 1560 kv-a., three-phase over No. 0000 conductors at 10.8 percent loss. Since the loss for the problem is not to exceed ten percent and 1300 kv-a is to be transmitted, we will select No. 0000 conductors. The loss for these conductors will therefore be $\frac{1300}{1560}$ of 10.8, or nine percent as will be shown later.

Table XXIII indicates that there will be no overheating of this size of conductor when carrying 1300 kv-a, three-phase.

Table XXII indicates that 10 000 volts is too low to result in corona loss with No. 0000 conductors at any reasonable altitude. Then:—

$$Kv\text{-}a_{rn} = \frac{1300}{3} = 433.33 \textit{ kv-a to neutral.}$$

$$Kw_{rn} = \frac{1040}{3} = 346.6 \textit{ kw to neutral.}$$

$E_{rn} = \frac{10\,000}{1.732} = 5774$ *volts to neutral.*

$I_r = \frac{433\,333}{5774} = 75.05$ *amperes per conductor.*

The resistance per conductor is,—

$R = 20 \times 0.277$ (*from Table II*) $= 5.54$ *ohms.*

The reactance per conductor is,—

$X = 20 \times 0.644$ (*from Table V*) $= 12.88$ *ohms.*

and

$IR = 75.05 \times 5.54 = 415.8$ *volts, resistance drop.*

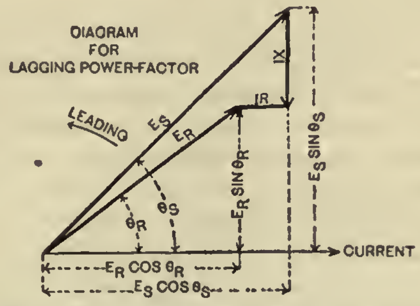
$= \frac{415.8}{5774} \times 100 = 7.20$ *percent.*

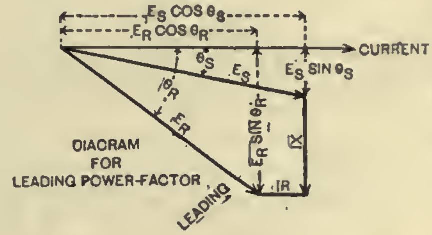
Transmission loss $= \frac{(75.05^2) \times 5.54}{1000} = 31.20$ *kw per conductor* . . . . . (*47*)

Percent transmission loss $= \frac{31.20 \times 3}{1040} \times 100 = 9.00$ *percent*

MIXED SENDING AND RECEIVING END CONDITIONS FIXED

Branch circuits are frequently run from the main transmission trunk circuit to the center of some local distribution. In such cases the voltage at the sending end and the current or the power and power-factor at





LOADS OF LAGGING POWER-FACTOR

WHEN RECEIVING-END CONDITIONS ARE FIXED

$$E_S = \sqrt{(E_R \cos\theta_R + IR)^2 + (E_R \sin\theta_R + IX)^2} \quad (30)$$

$$\theta_S = \tan^{-1}\frac{(E_R \sin\theta_R) + IX}{(E_R \cos\theta_R) + IR} \quad (31)$$

$$\%\,PF_S = \cos\theta_S \times 100 \quad (32)$$

$$KV\text{-}A_{SN} = \frac{I \times E_{SN}}{1000} \text{ PER CONDUCTOR} \quad (33)$$

$$KW_{SN} = KV\text{-}A_{SN} \times \cos\theta_S \text{ PER CONDUCTOR} \quad (34)$$

WHEN SENDING-END CONDITIONS ARE FIXED

$$E_R = \sqrt{(E_S \cos\theta_S - IR)^2 + (E_S \sin\theta_S - IX)^2} \quad (35)$$

$$\theta_R = \tan^{-1}\frac{(E_S \sin\theta_S) - IX}{(E_S \cos\theta_S) - IR} \quad (36)$$

$$\%\,PF_R = \cos\theta_R \times 100 \quad (37)$$

$$KV\text{-}A_{RN} = \frac{I \times E_{RN}}{1000} \text{ PER CONDUCTOR} \quad (38)$$

$$KW_{RN} = KV\text{-}A_{RN} \times \cos\theta_R \text{ PER CONDUCTOR} \quad (39)$$

LOADS OF LEADING POWER-FACTOR

WHEN RECEIVING-END CONDITIONS ARE FIXED

$$E_S = \sqrt{(E_R \cos\theta_R + IR)^2 + (E_R \sin\theta_R - IX)^2} \quad (40)$$

$$\theta_S = \tan^{-1}\frac{(E_R \sin\theta_R) - IX}{(E_R \cos\theta_R) + IR} \quad (41)$$

$$\%\,PF_S = \cos\theta_S \times 100 \quad (32)$$

$$KV\text{-}A_{SN} = \frac{I \times E_{SN}}{1000} \text{ PER CONDUCTOR} \quad (33)$$

$$KW_{SN} = KV\text{-}A_{SN} \times \cos\theta_S \text{ PER CONDUCTOR} \quad (34)$$

WHEN SENDING-END CONDITIONS ARE FIXED

$$E_R = \sqrt{(E_S \cos\theta_S - IR)^2 + (E_S \sin\theta_S + IX)^2} \quad (42)$$

$$\theta_R = \tan^{-1}\frac{(E_S \sin\theta_S) + IX}{(E_S \cos\theta_S) - IR} \quad (43)$$

$$\%\,PF_R = \cos\theta_R \times 100 \quad (37)$$

$$KV\text{-}A_{RN} = \frac{I \times E_{RN}}{1000} \text{ PER CONDUCTOR} \quad (36)$$

$$KW_{RN} = KV\text{-}A_{RN} \times \cos\theta_R \text{ PER CONDUCTOR} \quad (39)$$

GENERAL FORMULAS

WHEN THE VOLTAGE AT SENDING END AND THE AMPERES AND POWER-FACTOR AT RECEIVING END ARE FIXED

$$E_R = -I(R\cos\theta_R \pm X\sin\theta_R) + \sqrt{E_S^2 - I^2(R^2\sin^2\theta_R + X^2\cos^2\theta_R) \pm 2I^2 RX\cos\theta_R\sin\theta_R} \quad (44)$$

★ USE + WHEN THE POWER-FACTOR OF THE LOAD IS LAGGING AND − WHEN THE POWER-FACTOR IS LEADING.

WHEN THE VOLTAGE AT SENDING END AND THE POWER AND POWER-FACTOR AT RECEIVING END ARE FIXED

(POWER FACTOR LAGGING)

$$E_{RN} = A\sqrt{1 \pm \sqrt{1 - \frac{(R^2 + X^2)\,KW_{RN}^2 \times 10^6}{A^4\cos^2\theta_R}}} \quad \text{WHERE } A = E_{SN}\sqrt{\frac{1}{2} - \frac{1000\,KW_{RN}(R\cos\theta_R + X\sin\theta_R)}{E_{SN}^2\cos\theta_R}} \quad (45)$$

$$\%\text{ VOLTAGE DROP} = \frac{E_S - E_R}{E_R} \times 100 \quad (46)$$

$$\text{TRANSMISSION LOSS} = \frac{I^2R}{1000} \text{ KW PER CONDUCTOR} \quad (47)$$

$$\%\text{ TRANSMISSION LOSS} = \frac{\text{TOTAL } I^2R \text{ (IN KW)}}{\text{TOTAL } KW_R} \times 100 \quad (48)$$

FIG. 18—TRIGONOMETRICAL FORMULAS FOR SHORT TRANSMISSION LINES
Capacitance effect not taken into account.

$IX = 75.05 \times 12.88 = 966.6$ *volts, reactance drop.*

$= \frac{966.6}{5774} \times 100 = 16.74$ *percent.*

$E_{sn} = \sqrt{(5774 \times 0.8 + 415.8)^2 + (5744 \times 0.6 + 966.6)^2} = 6707$ *volts to neutral* . . . . . (*30*)

$\theta_s = \tan^{-1}\left(\frac{5774 \times 0.6 + 966.6}{5774 \times 0.8 + 415.8}\right) = 41°\ 22'$ . . . . . (*31*)

$PF_s = (\cos 41°\ 22') \times 100 = 75.05$ *percent* . . . . . (*32*)

$Kv\text{-}a_{sn} = \frac{75.05 \times 6707}{1000} = 503.4$ *kv-a per conductor.* . . . . (*33*)

$Kw_{sn} = 503.4 \times 0.7505 = 377.8$ *kw per conductor* . . . . . (*34*)

Percent voltage drop $= \frac{6707 - 5774}{5774} \times 100 = 16.16$ *percent* . . . . . (*46*)

the receiving end are approximately fixed. In such cases the calculation for the voltage at the receiving end requires more arithmetical work than is required when all the conditions at one end of the circuit are fixed. Such problems can be more readily solved graphically, as previously explained, but may be solved mathematically by applying formula (44) or (45), Fig. 18.

To illustrate the application of formula (44) we will apply the values of Problem *33* to formula (44) and calculate the receiving end voltage. Thus we have as fixed conditions:—

$E_{sn} = 6707$ *volts*
$I_r = 75.05$ *amperes*
$Cos\ \theta_r = 0.8$
$Sin\ \theta_r = 0.6$
$R = 5.54$ *ohms*
$X = 12.88$ *ohms*
$IR = 415.8$ *volts*

Then

$$E_r = -75.05\ (5.54 \times 0.8 + 12.88 \times 0.6) + \sqrt{6707^2 - 75.05^2\ (5.54^2 \times 0.6^2 + 12.88^2 \times 0.8^2) + 2 \times 75.05^2 \times 5.54 \times 12.88 \times 0.8 \times 0.6} \quad \ldots\ldots (44)$$

$$= -913 + \sqrt{44983849 - 660242 + 385831}$$

$$= -913 + 6637 = 5774 \text{ volts.}$$

To illustrate the application of formula (45) we will apply the values of Problem *33* to formula (45)

TABLE L
ILLUSTRATING VARIATION IN REACTANCE
Resulting from Changes in the Conductors and Transmission Voltages

CONDUCTORS	Total I^2R Loss (KW)	IR		IX		Approximate Voltage Regulation at	
		Volts	Per Cent.	Volts	Per Cent.	100 Per Cent. Power Factor	80 Per Cent. Power Factor (Lag.)
RECEIVING END VOLTAGE — 6600							
Single Circuit of three 500,000 circ. mil bare overhead conductors	129	123	3.22	622	16.32	4.5	12.8
Two circuits each of three 250,000 circ. mil bare overhead conductors.	129	123	3.22	333	8.73	3.6	7.7
One Circuit of 500,000 circ. mil three-conductor cable. Insulation thickness 13/64 by 13/64 inches.	129	1.23	3.22	172	4.52	3.2	5.0
RECEIVING END VOLTAGE — 13 200							
Single circuit of three 125,000 circ. mil bare overhead conductors.	129	247	3.22	354	4.64	3.2	5.1

and calculate the receiving end voltage. Thus we have as fixed conditions:—

$E_{sn} = 6707$ *volts*
$Kw_{rn} = 346.6$ *kw*
$R = 5.54$ *ohms*
$X = 12.88$ *ohms*
$Cos\ \theta_r = 0.8$
$Sin\ \theta_r = 0.6$

$$A = 6707\sqrt{0.5 - \frac{1000 \times 346.6\ (5.54 \times 0.8 + 12.88 \times 0.6)}{6707^2 \times 0.8}}$$

then

$$E_{rn} = A\sqrt{1 + \sqrt{1 - \frac{(5.54^2 + 12.88^2)\ 346.6^2 \times 10^6}{A^4 \times 0.8^2}}} \quad \ldots (45)$$

$$A = 6707\sqrt{0.5 - 0.1172} = 4152$$

$$E_{rn} = 4152\sqrt{1 + 0.936} = 5774 \text{ volts}$$

Alternative to (44) and (45)—The following formulas have been proposed by Mr. H. B. Dwight to meet the mixed conditions referred to,—

$E_{sn} = 6707$ *volts*
$1000 \times Kw_{rn} = 346\,600$ *watts*
$1000 \times$ *reactive* $Kv\text{-}a_{rn} = 346\,600 \times \frac{0.6}{0.8} = 260\,000$ *v-a*
$R = 5.54$ *ohms*
$X = 12.88$ *ohms*
$L = 346\,600 \times 5.54 + 260\,000 \times 12.88 = 5\,270\,000$
$M = 346\,600 \times 12.88 - 260\,000 \times 5.54 = 3\,025\,000$

$$E^2 = 0.5\ E_s^2 - L + 0.5\sqrt{E_s^4 - 4\ E_s^2\ L - 4\ M^2}$$

$E = 5774$ *volts*

or

$$E = E_s - \frac{L}{E_s} - \frac{L^2}{E_s^3} - \frac{M^2}{2\ E_s^3} - \frac{2L^3}{E_s^5} - \frac{3}{2}\frac{LM^2}{E_s^5} - \frac{5L^4}{E_s^7} - \frac{5\ L^2 M^2}{E_s^7} - \frac{5}{8}\frac{M^4}{E_s^7}\ *$$

$E = 5779$ *volts*

CIRCUITS OF EXCESSIVE REACTANCE

If a large amount of power is to be transmitted at comparatively low voltage, particularly if the frequency is high, the reactance of the circuit will be high compared with its resistance. If the reactance is excessive (20 to 30 percent reactance volts may in some cases be considered excessive), the voltage regulation of the circuit may be seriously impaired.

As will be seen by consulting Tables VI and VII, there is a fixed relation between the resistance and the reactance of a circuit for a given frequency, size and spacing of conductors. This ratio is 2.4 times greater for 60 cycle than it is for 25 cycle circuits. For a given size of conductor the reactance can be varied only slightly by changing the spacing of overhead bare conductors. Substituting a larger or smaller conductor may change the resistance materially, but this will have little effect upon the reactance.

The reactance may be reduced by either or all of the following methods. The circuit may be split up into two or more circuits employing smaller conductors and these circuits connected in parallel. The voltage may be raised, if the installation is new, and smaller conductors employed; or the overhead conductors may be replaced by three conductor cables. To illustrate the above methods, the following problem has been assumed and the results tabulated.

A HIGH REACTANCE PROBLEM

Table L refers to the following problem—4000 kv-a, three-phase, 60 cycles, is to be delivered a distance of three miles over hard-drawn, stranded copper conductors. The I^2R loss is to remain at 129 kw. The spacing of the overhead conductors assumed as 3 by 3 by 3 ft. Temperature 25 degrees C.

It is evident from Table L that if two three-phase circuits, each consisting of three 250 000 circ. mil. conductors are installed in place of one three-phase circuit, consisting of three 500 000 circ. mil. conductors, the reactance will be reduced by nearly one half, and a corresponding improvement in the voltage drop or regulation will occur, particularly if the load power-factor is 80 percent lagging. A further improvement along this line will be obtained if a single three-conductor cable is employed. Doubling the voltage for the overhead circuit and employing three 125 000 circ. mil. conductors results in practically as good performance in voltage regulation as for the 6600 volt three-conductor cable.

*See article by Mr. H. B. Dwight on "Effect of a Tie Line between Two Substations" in the *Electrical Review,* Dec. 21, 1918, p. 966. The formulas given in this article make complete allowance for the effect of capacitance and are very similar to the above.

CHAPTER VIII
PERFORMANCE OF LONG TRANSMISSION LINES
(GRAPHICAL SOLUTION)

THE E.M.F. of self-induction in a transmission circuit may either add to or subtract from the impressed voltage at the sending end, depending upon the relative phase relations between the current and the voltage at the receiving end of the circuit. This is illustrated by means of voltage vectors in Fig. 20, in which the phase of the current is assumed to be constant in the horizontal direction indicated by the arrow on the end of the current vector. The voltage at the receiving end is also assumed as constant at 100 volts. The vector representing the receiving end voltage ($E_r = 100$ volts) is shown in two positions corresponding to leading current, two positions corresponding to lagging current and in one position corresponding to unity power-factor. The components IR and IX of the supply voltage necessary to overcome the resistance R and the reactance X (e.m.f. of self-induction) of the circuit are assumed to be 10 volts and 20 volts respectively. Since the current is assumed as constant, IX and IR are also constant. The impedance triangle of the voltage components required to overcome the combined effect of the resistance and the reactance of this circuit is therefore constant. It is shown in five different positions about the semicircle, corresponding to five different load power-factors. The voltage E_s at the sending-end required to maintain 100 volts at the receiving-end is indicated for each of the five positions of the impedance triangle.

Counter-clockwise rotation of the vectors will be considered as positive. This means that when the current is lagging behind the impressed e.m.f., the voltage vector will be in the forward or leading direction from the current vector as indicated by the arrow. When the current leads the impressed voltage, the voltage vector will be in the opposite, or clockwise direction from the current vector. In other words, assuming the vectors all rotating at the same speed about the point O in a counter-clockwise direction, the current vector will be behind the voltage vector when the current is lagging and ahead of it when the current is leading.

The alternating magnetic flux surrounding the conductors, resulting from current flowing through them, generates in them a counter e.m.f. of self-induction. This e.m.f. of self-induction has its maximum value when the current is passing through zero and is therefore in lagging quadrature with the current. On the diagrams an arrow in the line IX, indicates the direction of the e.m.f. of self-induction. It will be seen that since the direction of the current is assumed constant, the e.m.f. of self-induction acts downward in all five impedance diagrams. The sending-end voltage is therefore opposed or favored by this self-induced voltage (see arrows) to a greater or less extent depending upon the power-factor of the load. Thus at lagging loads of high power-factor, the self-induced voltage acts approximately at right angles to the sending-end voltage, and therefore requires a small component of the sending-end voltage to balance or neutralize its effect. As the power-factor of the receiving-end load decreases in the lagging direction (upper quadrant of diagram) the sending-end voltage swings around more nearly in line with the direction of the induced voltage, thus requiring a greater component of the sending-end voltage to counter-balance its effect. At zero power-factor lagging, the direction of the sending-end voltage and that of the induced e.m.f. are practically in opposition, (as indicated by the arrows), so that the component of the sending-end voltage required to overcome the induced voltage is a maximum, or nearly as much as the e.m.f. of self-induction. It is interesting to note that at zero lagging power-factor, when the effect of self-induction on line voltage drop reaches a maximum, the sending-end voltage component IR necessary to overcome the resistance of the circuit, (now nearly at right angles to the supply voltage), is a minimum. The reverse of these conditions is true for receiving-end loads of power-factors near unity.

Now consider receiving-end loads of leading power-factors, (lower quadrant of diagram). It will be seen that the e.m.f. of self-induction does not now oppose the sending-end voltage (indicated by direction of the arrows) but has a direction more or less parallel to that of the sending-end voltage. At high leading power-factors, the e.m.f. of self-induction has little effect on the sending-end voltage, but as zero leading power-factor is approached these two e.m.f.'s more nearly come in phase with each other. At zero power-factor leading, the e.m.f. of self-induction adds almost directly to the sending-end voltage.

It will be seen, therefore, that for receiving-end loads of lagging power-factor, the sending-end voltage is greater than the receiving-end voltage, by an amount necessary to overcome the resistance and self-induction of the circuit. For receiving-end loads of leading power-factor, the sending-end voltage is less than the receiving-end voltage, for the reason that the e.m.f. of self-induction is in such a position as to assist the sending-end voltage.

The following values from Fig. 20 illustrate these conditions:

Power-Factor of Receiving End Load	Supply Voltage
0 percent lagging	120.4
80 percent lagging	120.4
100 percent	111.8
80 percent leading	98.5
0 percent leading	80.6

The condition of leading power-factor at the receiving-end would be unusual in practice, since the power-factor of receiving-end loads is usually lagging. In cases, however, where condensers are used for voltage or power-factor control, the power-factor at the receiving-end may be leading. If the circuit were without inductance, there could be no rise in voltage at the

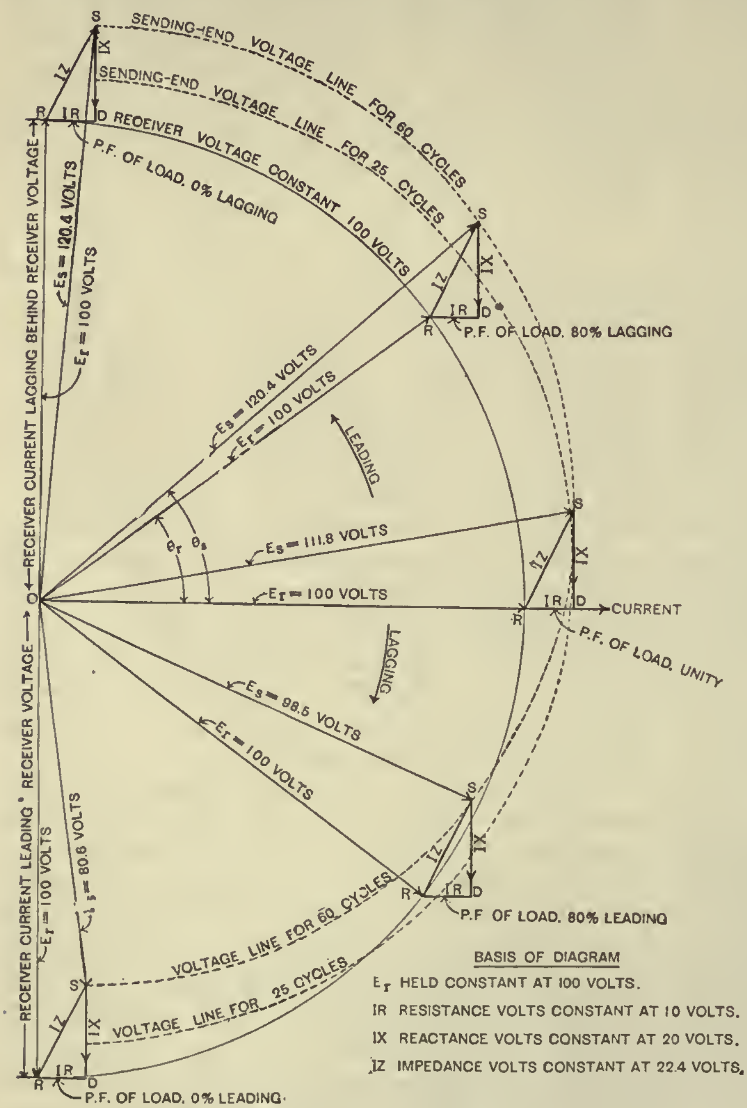


FIG. 20—EFFECT OF SELF INDUCTION ON REGULATION

receiving-end, for in such a case, *IX* of the diagram would disappear, and the voltage drop would be the same as with direct current. All alternating-current circuits are inductive, and the greater their inductance, the greater will be the voltage drop, or the voltage rise along the circuit.

Any alternating-current circuit may be looked upon as containing three active e.m.f.'s out of phase with each other. In addition to the impressed e.m.f. at the sending-end, there are two e.m.f 's of self-induction, one as the result of the receiving-end current and lagging 90 degrees behind it and the other as the result of the line charging current and lagging 90 degrees behind it. These two combine at an angle, with each other and with the impressed e.m.f. at the sending-end.

CHARGING CURRENT

Conductors of a circuit, being separated by a dielectric (such as air, in overhead circuits, or insulation in cables), form a condenser. When alternating-current flows through such a circuit, current (known as charging current) virtually passes from one conductor through the dielectric to the other conductors, which are at a different potential. This current is in shunt with the circuit, and differs from the current which passes between conductors over the insulators etc. (leakage current) or through the air (corona effect) only in that the charging current leads the voltage by 90 degrees, whereas the leakage current is in phase with the voltage.

For a given spacing of conductors, the charging current increases with the voltage, the frequency and the length of the circuit. For long high-voltage circuits, particularly at 60 cycles per second, the charging current may be as much as the full-load current of the circuit, or more. In some cases of long 60 cycle circuits, where a comparatively small amount of power is to be transmitted, it is necessary to limit the voltage of transmission, in order that the charging current may not be so great as to overload the generators. This charging current, being in leading quadrature with the voltage, represents nearly all reactive power, but it is just as effective in heating the generator windings as if it represented active power. On the other hand, it combines with the receiving-end current at an angle (depending upon the power-factor of the receiver load) in such a manner that the addition of the full-load receiving-end current, in extreme cases, may not greatly increase the sending-end current. In other words (if the charging current is near full-load current) the current at the generator end may not increase much when full load at the receiver end is added, over what it is when no load is taken off at the receiving-end.

Since the e.m.f. of self-induction due to the charging component is proportional to the charging current, its effect upon the voltage regulation of the circuit will also be proportional to the charging current. For a short low-voltage circuit, the charging current is so small that its effect on voltage regulation may be ignored. On the longer circuits, especially long 60 cycle circuits, such as will be considered later, its effect must be given careful consideration.

VARIATION IN CURRENT AND VOLTAGE ALONG THE CIRCUIT

It was explained above and illustrated in Fig. 20 that with a receiving-end load of leading power-factor, the voltage at the sending-end of the circuit might be less than that at the receiving-end. It was shown that the e.m.f. of self-induction, resulting from the leading current, tends to raise the voltage along the circuit. This boosting effect of the voltage is entirely due to the leading component of the load current.

If, now, it is assumed that the power-factor of the receiving-end load is 100 percent, there will be no leading component in the load current, and therefore there can be no boosting of the voltage due to the load current. Since, however, all circuits have capacitance, and since the current is alternating, charging current will flow into the line and this being a leading current, the same tendency to raise the voltage along the circuit will take place as is illustrated by Fig. 20.

The upper part of Fig. 21 is intended to give a physical conception of what takes place in an alternating-current circuit. As the load current starts out from the sending-end, and travels along the conductor, it meets with ohmic resistance. This is represented by r in Fig. 21. It also meets with reactance in quadrature to the current. This is represented by jx in the diagram. Superimposed upon this load current is a current flowing from one conductor to the others, in phase with the voltage at that point and representing true power. This current is the result of leakage over insulators and of corona effect between the conductors. It is represented by the letter g in the diagrams. Then there is the charging current in leading quadrature with the voltage. This current does not consume any active power except that necessary to overcome the resistance to its flow.

In Fig. 21 the four linear constants of the alternating-current circuit, r representing the resistance, jx representing the reactance, g representing the leakage and b representing the susceptance, are shown as located, or lumped, at six different points along the circuit. This is as they would appear in an artificial circuit divided into six units. In any actual line, these four constants are distributed quite evenly throughout the length of the circuit.

VOLTAGE AND CURRENT DISTRIBUTION FOR PROBLEM X

The effect of the charging current flowing through the inductance of the circuit gives rise to a very interesting phenomenon. In order to illustrate this effect, the current and voltage distribution for a 60 cycle, 1000 volt, three-phase circuit, 300 miles long, is plotted in Fig. 21. This circuit will be referred to as problem X. In such a long 60 cycle circuit, this phenomenon is quite pronounced; so that such a problem serves well as an illustration. The voltage and the current have been determined for points 50 miles apart along the circuit. Values for both the current and the voltage under zero load, also under load conditions have been plotted. The load conditions refer to a receiving-end load of 18 000 kv-a, at 90 percent power-factor, lagging, 60 cycle three-phase. The voltage is assumed as being held constant 104 000 volts at the receiving-end, for both zero and full-load conditions.

Zero-Load Conditions—Without any load being taken from the circuit, it will be seen that the charging current at the sending-end approaches in value that established when under full load; i.e., 94.75 amperes. The charging current drops down to approximately 50 amperes at the middle, and to zero at the receiving-end of the unloaded circuit. The lower full line curve shows how this current is distributed along the circuit. Starting at zero, at the receiving-end of the circuit, it increases as the sending-end of the circuit is approached, at which point it reaches its maximum value of 87.89 amperes. The voltage distribution under zero-load conditions is some-what opposite to that of the current distribution. That is the voltage (104 000 volts at the receiving-end) keeps falling lower until it reached a value of 84 676 at the sending-end. It should be noted that the voltage curve for zero load condition drops down rapidly as the sending-end is approached. The reason for this is the large charging current flowing through the inductance of the circuit at this end of the circuit. The larger the charging current the greater the resultant boosting of the receiving-end voltage.

Load Conditions—When 16 000 kv-a at 90 percent power-factor lagging is taken from the circuit at the receiving-end, the current at this end goes up to 99.92 amperes. As the supply end is approached the current becomes less, reaching its lowest value (approximately 83 amperes) in the middle of the circuit. At the supply end it is 94.75 amperes, which is less than it is at the receiver end. Thus the full line representing the current in amperes along the circuit assumes the form of an arc, bending downward in the middle of the circuit. The shape of this current curve is dependent upon the relative values of the leading and lagging components of the current at points along the circuit. The reason that the current is a minimum near the middle of the circuit, is because this is the point where the lagging current of the load and the leading charging current of the circuit balance or neutralize each other, and the power-factor is therefore unity. Starting at the receiving-end, the power-factor is 90 percent lagging. As the middle of the circuit is approached, the increasing charging current neutralizes an increasing portion of the lagging component of the load current. Near the middle of the circuit, this lagging component is entirely neutralized, and the power-factor therefore rises to unity. Passing the middle and approaching the sending-end there is no more lagging component to be neutralized, and the increasing charging current causes a decreasing leading power-factor which, when the sending-end is reached, becomes 93.42 percent leading. It will, therefore, be seen that the power-factor as well as the current and voltage varies throughout the length of the circuit.

The voltage distribution under load condition is indicated by the top broken line. In order that the receiving-end voltage may be maintained constant at 104 000 volts, the voltage at the sending-end will vary from 84 676 volts at zero load to 122 370 volts at the assumed load.

THE AUXILIARY CONSTANTS

With the impedance methods considered under the general heading of "Short Transmission Lines" the current was considered as of the same value throughout the circuit, and the voltage drop along the circuit was considered as proportional to the distance. These assumptions, which are permissible in case of short lines, are satisfied by simple trigonometric formulas.

The rigorous solution for circuits of great electrical length accurately takes into account the effect produced by the non-uniform distribution of the current and the voltage throughout the length of the circuit. This effect will hereafter be referred to as the *distribution effect* of the circuit, and may be taken into account

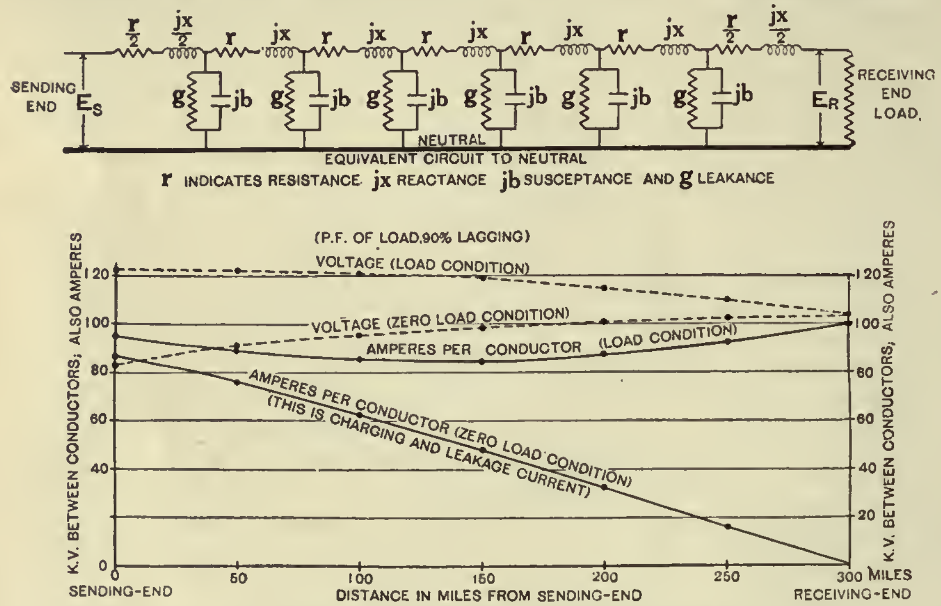


FIG. 21—DIAGRAMS OF TRANSMISSION CIRCUIT—PROBLEM *X*

300 miles long, 104 000 volts delivered, 60 cycle. The upper diagram gives a physical conception of the conditions along the line. The curves show the variation in current and voltage along the circuit.

through the application of the so called auxiliary constants of the circuit.

The auxiliary constants *A*, *B* and *C* of the circuit are functions of its physical properties, and of the frequency only. They are entirely independent of the voltage or current of the circuit. The various solutions for long transmission circuits are in effect schemes for determining the values of these three auxiliary constants. Mathematically they may be calculated, by hyperbolic functions or by their equivalent convergent series. Graphically they may be obtained to a high degree of accuracy from the accompanying Wilkinson Charts for overhead circuits not exceeding 300 miles in length. Having determined the values for these three constants for a given circuit, the remainder of the solution is just as simple as for short lines. It is only necessary to apply any desired load conditions to these constants and plot the results by vector diagrams.

DIAGRAM OF THE AUXILIARY CONSTANTS

In Fig. 22 are shown voltage and current diagrams representing the application of the auxiliary constants to the solution of transmission circuit problems. To construct the voltage vector diagram, the two auxiliary constants *A* and *B* are required, and to construct the current vector diagram, constants *A* and *C* are required.

Since these diagrams are based upon one volt and one ampere at the receiving-end, it is necessary to multiply the values of the auxiliary constants by the volts or the amperes at the receiving-end, in order to apply the auxiliary constants to a specific problem. Since the diagrams are shown corresponding to unity power-factor, it will also be necessary to change the position of the impedance and charging current triangles in case the power-factor differs from unity. This will be explained later.

Constants a_1 and a_2—Referring to the voltage diagram, Fig. 22, if the line is electrically short the charging current, and consequently its effect upon the voltage regulation is small. In such a case the auxiliary constant a_1 would be unity, and the auxiliary constant a_2 would be zero. In other words, the impedance diagram would (for a power-factor of 100 percent) be built upon the end of the vector *ER*, the point *O* coinciding with the point *R*. In such a case, the voltage at the sending end, at zero load, would be the same as that at the receiving-end. If the circuit contains appreciable capacitance, the e.m.f. of self-induction, resulting from the charging currents which will flow, will result in a lower voltage at zero load at the sending-end than at the receiving-end of the line, as previously explained. Obviously, the load impedance triangle must be attached to the end of the vector representing the voltage at the sending-end of the circuit at zero load. This is the vector *EO* of the voltage diagram, Fig. 22. This voltage diagram corresponds to that of a 60 cycle circuit, 300 miles in length. In such a circuit, the effect of the charging current is sufficiently great to cause the shifting of the point *O* from *R* (in a short line) to the position shown in Fig. 22. In other words, the voltage at zero load at the sending-end has shifted from *ER* for circuits of short electrical length, to *EO* for this long 60 cycle circuit. The auxiliary constants a_1 and a_2, therefore, determine the length and position of the vector representing the sending-end voltage at zero load. Actually, the constant a_2 represents the volts resistance drop due to the charging current, for each volt at the receiving-end of the circuit. That is, the line *OF* equals approximately one-half the charging current times the resistance R, taking into account, of course, the distributed nature of the circuit. If the circuit is short, it would be sufficiently accurate to assume that the total charging current flows through one-half of the resistance of the circuit. To make this clear, it will be shown later that, for problem *X*, the resistance per conductor $R = 105$ ohms and the auxiliary constant $C_2 = 0.001463$. Thus, this line will take 0.001463 ampere charging current, at zero load, for each volt maintained at the receiving-end, and since OF = approximately $I_c \times \frac{R}{2}$ we have $OF\ (a_2) = 0.001463 \times \frac{105}{2} = 0.0768075$. The exact value of a_2 as calculated rigorously, taking into account the distributed nature of the circuit, is 0.076831. Since the charging current is in

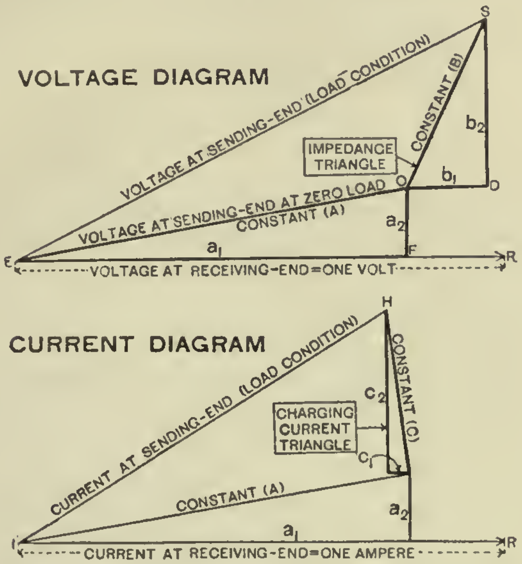


FIG. 22—DIAGRAMMATIC REPRESENTATION OF AUXILIARY CONSTANTS OF A TRANSMISSION CIRCUIT

The vectors are based upon one volt and one ampere being delivered to the receiving end at unity power-factor. These diagrams correspond to those of a long circuit.

leading quadrature with the voltage *ER*, the resistance drop *OF* due to the charging current is also at right angles to *ER*, as in Fig. 22.

The length of the line *FR* or $(1 - a_1)$, represents the voltage consumed by the charging current flowing through the inductance of the circuit. This may also be expressed with small error if the circuit is not of great electrical length as $I_c \times \frac{X}{2}$. The reactance per conductor for problem *X* is 249 ohms. Therefore $FR = 0.001463 \times \frac{249}{2} = 0.182143$ and $a_1 = 1.000000 - 0.182143 = 0.817857$. The exact value for a_1 as calculated rigorously, taking into account the distributed nature of the circuit, is 0.810558. The vector *FR*, representing the voltage consumed by the charging current flowing through the inductance, is naturally in quadrature with the vector *OF*, representing the voltage consumed by the charging current flowing through the resistance of the circuit.

Constants b_1 *and* b_2 represent respectively the resistance and the reactance in ohms, as modified by the distributed nature of the circuit. The values for these constants, multiplied by the current in amperes at the receiver-end of the circuit, give the *IR* and *IX* volts

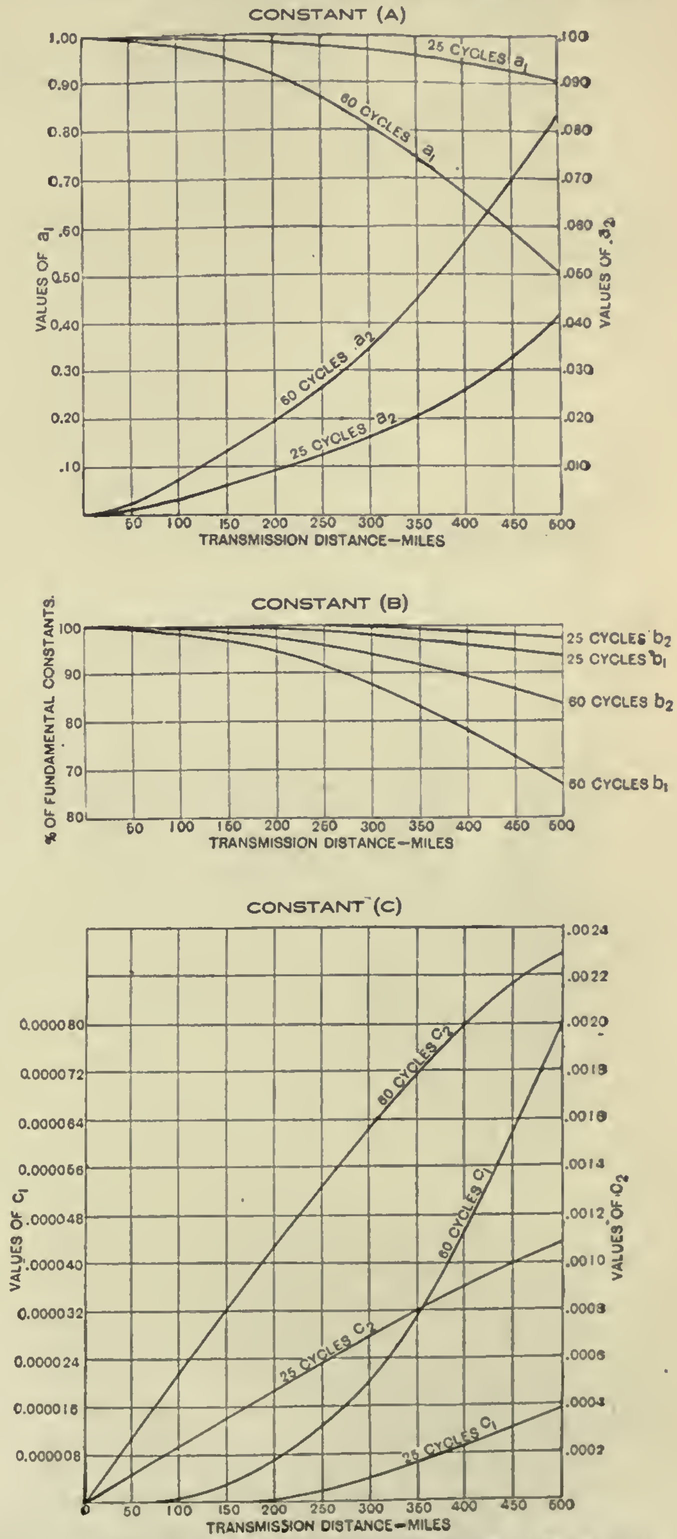


FIG. 23—VARIATION OF THE AUXILIARY CONSTANTS FOR CIRCUITS OF DIFFERENT LENGTHS

drop consumed respectively by the resistance and the reactance of the circuit. To illustrate this, the values of *R* and *X* for problem *X* are $R = 105$ ohms and $X = 249$ ohms per conductor. The distribution effect of the circuit modifies these linear values of *R* and *X* so that their effective values are $b_1 = 91.7486$ and $b_2 = 235.868$ ohms. The impedance triangle, as modified so as to take into exact account the distributed nature of the circuit, is therefore smaller than it would be if the circuit were without capacitance.

Constants c_1 and c_2 represent respectively conductance and susceptance in mhos as modified by the distributed nature of the circuit. The values for these constants, multiplied by the volts at the receiving-end of the circuit, give the current consumed respectively by the conductance and the susceptance of the circuit. To illustrate, the value of B for problem X is 0.001563 mho per conductor. The distribution effect of the circuit modifies this fundamental value so that its effective value $c_2 = 0.001463$. The value of c_1 is so small that its effect is negligible for all except very long circuits. For power circuits it will usually be sufficiently accurate to neglect c_1. The value c_2 will in such cases represent the charging current at zero load per volt at the receiving-end. Thus c_2, multiplied by the receiving-end voltage, gives the charging current at zero load for the circuit. For problem X, $c_2 = 0.001463$, and this, multiplied by the receiving-end voltage to neutral $60\,044 = 87.85$ amperes charging current per conductor.

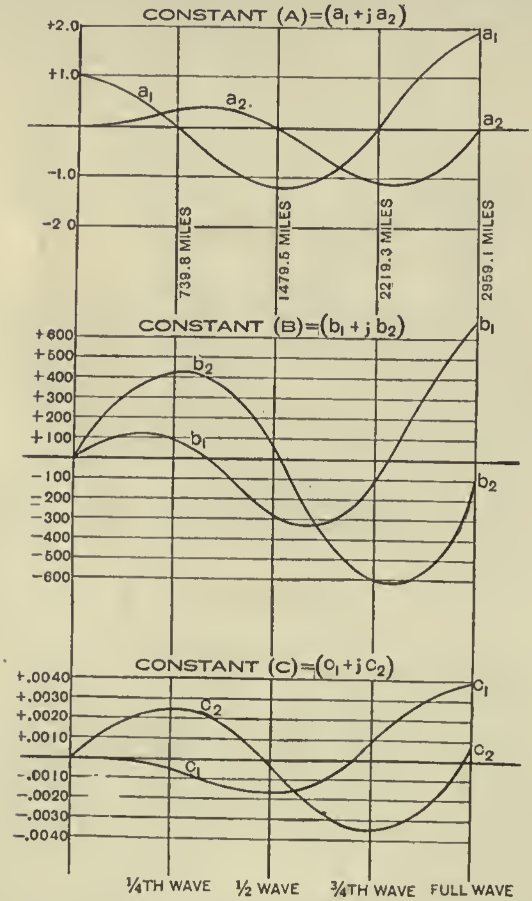


FIG. 24—VARIATION OF THE AUXILIARY CONSTANTS
For a 60 cycle circuit (problem X) up to full wave length.

VARIATION IN THE AUXILIARY CONSTANTS

The curves, Fig. 23, will serve to illustrate in a general way how the auxiliary constants vary for both 25 and 60 cycle circuits for lengths up to and including 500 miles. In other words these curves have been plotted from calculated values for these constants for certain circuits.

When the circuit is short, these constants do not vary materially from the linear constants of the circuit, but when the circuit becomes long, they depart rapidly, particularly if the frequency is high.

AUXILIARY CONSTANTS	WAVE LENGTH OF THE CIRCUIT AND TRANSMISSION DISTANCE—MILES							
	⅛TH	¼	⅜TH	½	⅝TH	¾	⅞TH	FULL
	369.9 MILES	739.8 MILES	1109.7 MILES	1479.5 MILES	1849.4 MILES	2219.3 MILES	2589.2 MILES	2959.1 MILES
a_1	+.716	0	−.789	−1.209	−.942	0	+1.191	+1.922
a_2	+.113	+.323	+.350	0	−.622	−1.104	−.958	0
b_1	+105	+87	−77.5	−276	−330	−122	+292	+670
b_2	+281	+428	+350	+55.5	−330	−605	−560	−135
c_1	−.000075	−.00050	−.0012	−.0016	−.00101	+.00071	+.0028	+.0039
c_2	+.00174	+.00247	+.00169	−.00032	−.00250	−.0035	−.00233	+.00078
(A)	.725 /8° 58′	.323 /90° 00′	.863 /156° 05′	1.209 /180° 00′	1.129 /213° 26′	1.104 /270° 00′	1.528 /321° 11′	1.922 /360° 00′
(B)	301.4 /69° 37′	437 /78° 34′	358.8 /102° 29′	282.3 /168° 34′	469.5 /225° 0′	619.3 /258° 34′	635.7 /297° 23′	682.4 /348° 34′
(C)	.001743 /92° 29′	.002527 /101° 26′	.002075 /125° 21′	.001633 /191° 26′	.002715 /247° 59′	.003582 /281° 26′	.003677 /320° 15′	.003947 /371° 26′

FIG. 25—VARIATION OF THE AUXILIARY CONSTANTS
For problem X up to full wave length.

The auxiliary constants have been calculated for problem X up to and including a full wave length, namely 2959 miles. Calculations were made only for distances representing each *1/8*th wave, that is each 370 miles. The results are tabulated in Fig. 25, and are plotted graphically in Fig. 24. It is interesting to note how these auxiliary constants vary with increasing negative and positive values as the circuit increases in length. A polar diagram is plotted in Fig. 26, indicating the manner in which the auxiliary constant A and its rectangular co-ordinates vary. Although these extreme variations are instructive and interesting, they are not encountered in power transmission circuits, although they will be in long distance telephone practice.

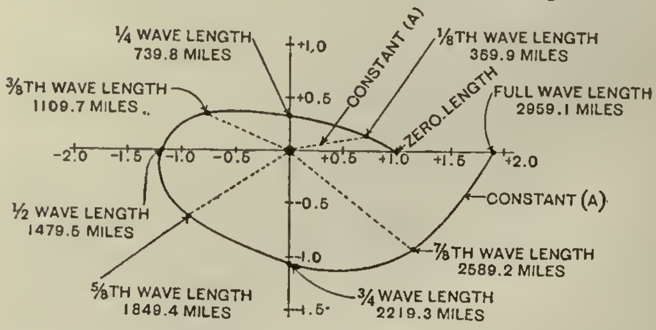


FIG. 26—POLAR DIAGRAM
Showing the variation of the auxiliary constant A for problem X, up to full wave length.

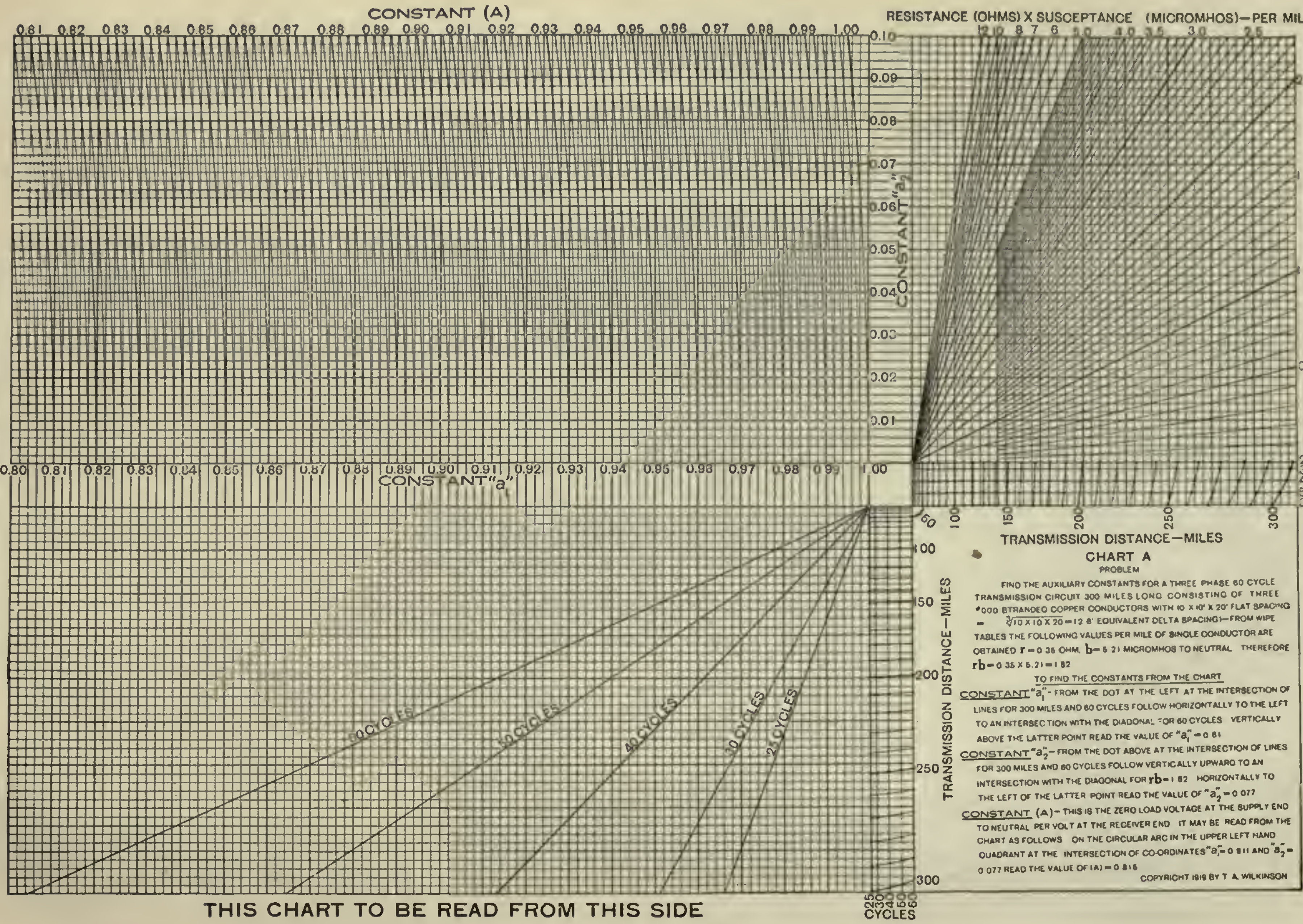
THE WILKINSON CHARTS

Mr. T. A. Wilkinson has prepared charts from which the auxiliary constants may be read directly, thus abridging a great amount of tedious mathematical calculation. These charts, are plotted for circuits of lengths up to and including 300 miles.*

*Similar Charts by Mr. Wilkinson were published in the *Electrical World* for Mar. 16, 1918.

CHART V—WILKINSON CHART A

(FOR DETERMINING AUXILIARY CONSTANTS—ZERO LOAD VOLTAGE)



their effective values are $b_1 = 91.7486$ and $b_2 = 235.868$ ohms. The impedance triangle, as modified so as to take into exact account the distributed nature of the circuit, is therefore smaller than it would be if the circuit were without capacitance.

Constants c_1 *and* c_2 represent respectively conductance and susceptance in mhos as modified by the distributed nature of the circuit. The values for these constants, multiplied by the volts at the receiving-end of the circuit, give the current consumed respectively by the conductance and the susceptance of the circuit. To illustrate, the value of B for problem X is 0.001563 mho per conductor. The distribution effect of the circuit modifies this fundamental value so that its effec-

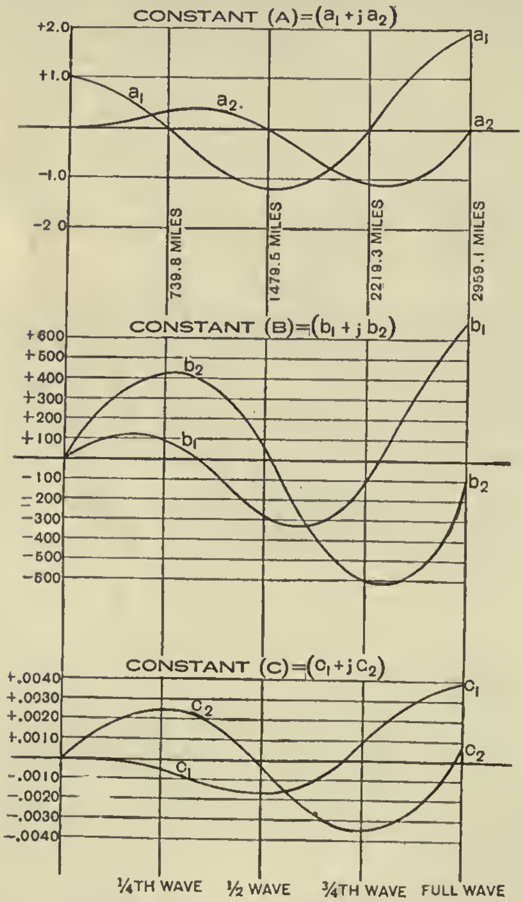


FIG. 24—VARIATION OF THE AUXILIARY CONSTANTS
For a 60 cycle circuit (problem X) up to full wave length.

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VARIATION IN THE AUXILIARY CONSTANTS

The curves, Fig. 23, will serve to illustrate in a general way how the auxiliary constants vary for both 25 and 60 cycle circuits for lengths up to and including 500 miles. In other words these curves have been plotted from calculated values for these constants for certain circuits.

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(B)	301.4 ∠69° 37′	437 ∠78° 34′	358.8 ∠102° 29′	282.3 ∠168° 34′	469.5 ∠225° 07′	619.3 ∠258° 34′	635.7 ∠297° 23′	682.4 ∠348° 34′
(C)	.001743 ∠92° 29′	.002527 ∠101° 26′	.002075 ∠125° 21′	.001633 ∠191° 26′	.002715 ∠247° 59′	.003582 ∠281° 26′	.003677 ∠320° 15′	.003947 ∠371° 26′

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For problem X up to full wave length.

The auxiliary constants have been calculated for problem X up to and including a full wave length, namely 2959 miles. Calculations were made only for distances representing each *1/8*th wave, that is each 370 miles. The results are tabulated in Fig. 25, and are plotted graphically in Fig. 24. It is interesting to note how these auxiliary constants vary with increasing negative and positive values as the circuit increases in length. A polar diagram is plotted in Fig. 26, indicating the manner in which the auxiliary constant A and its rectangular co-ordinates vary. Although these extreme variations are instructive and interesting, they are not encountered in power transmission circuits, although they will be in long distance telephone practice.

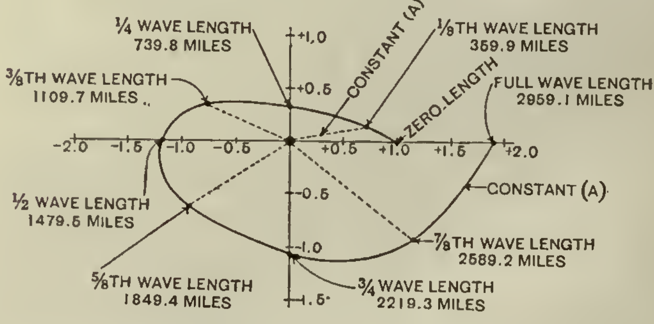

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CHART V—WILKINSON CHART A

(FOR DETERMINING AUXILIARY CONSTANTS—ZERO LOAD VOLTAGE)

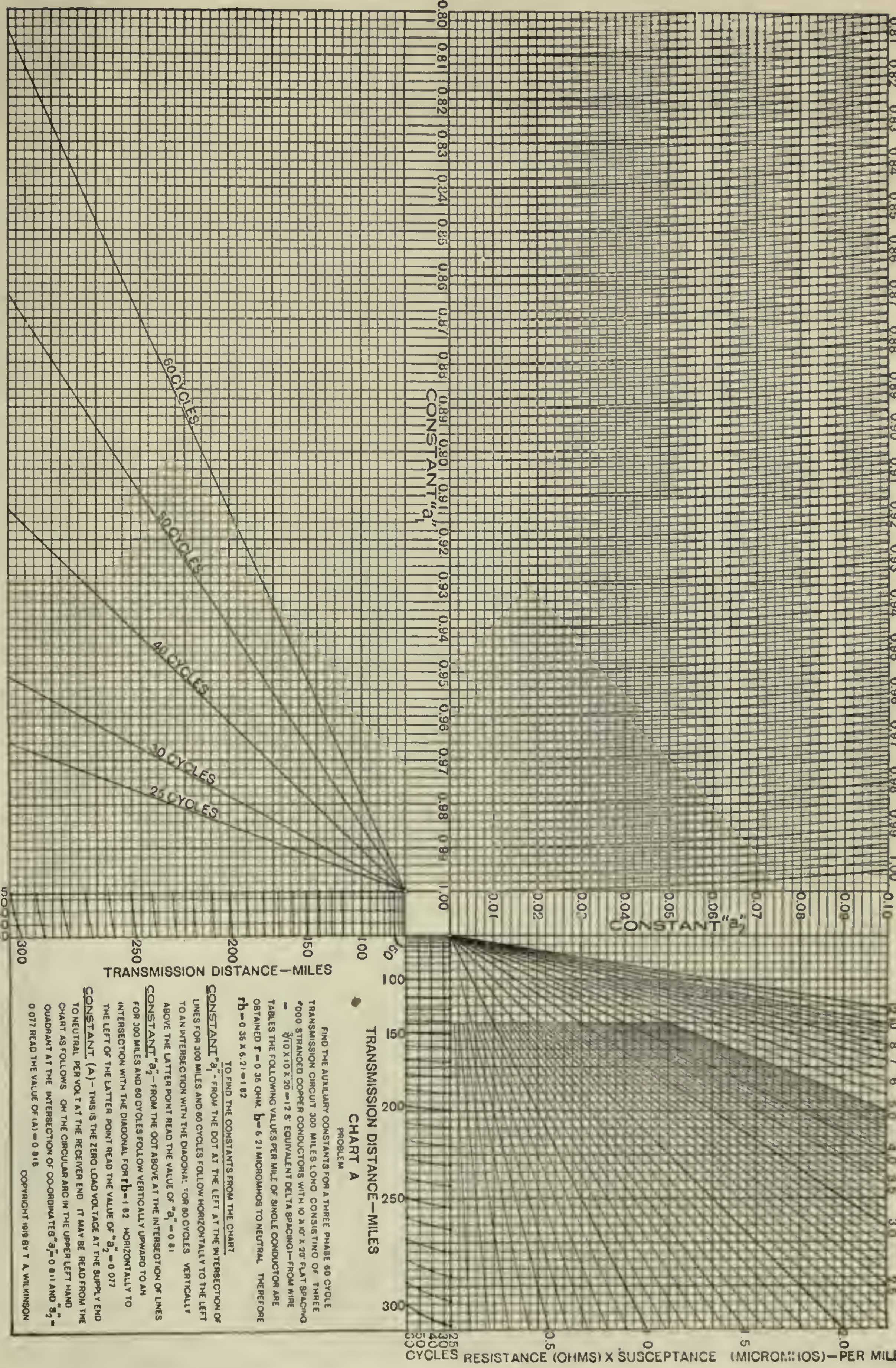


CHART VI—WILKINSON CHART B

(FOR DETERMINING AUXILIARY CONSTANTS—LINE IMPEDANCE)

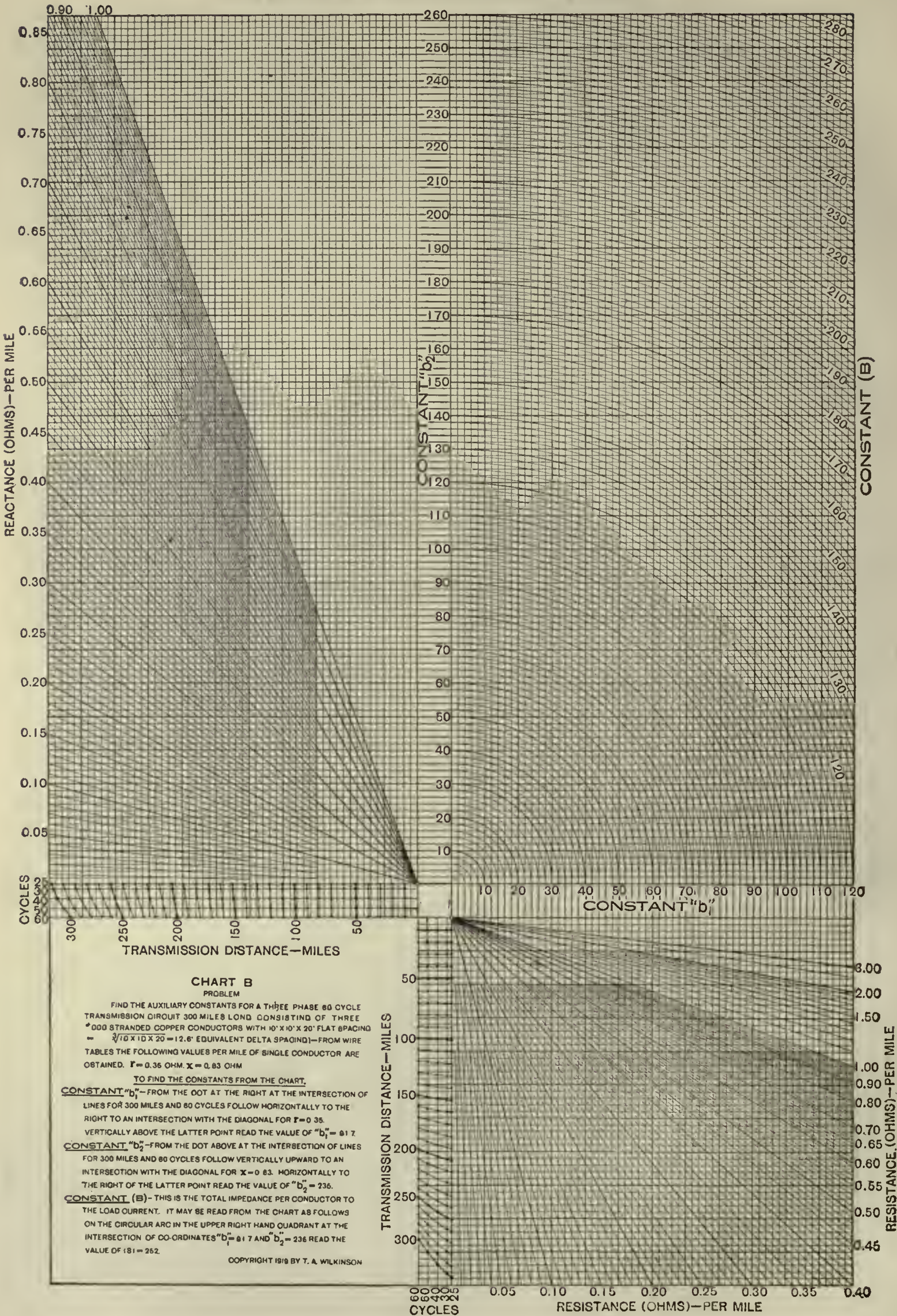
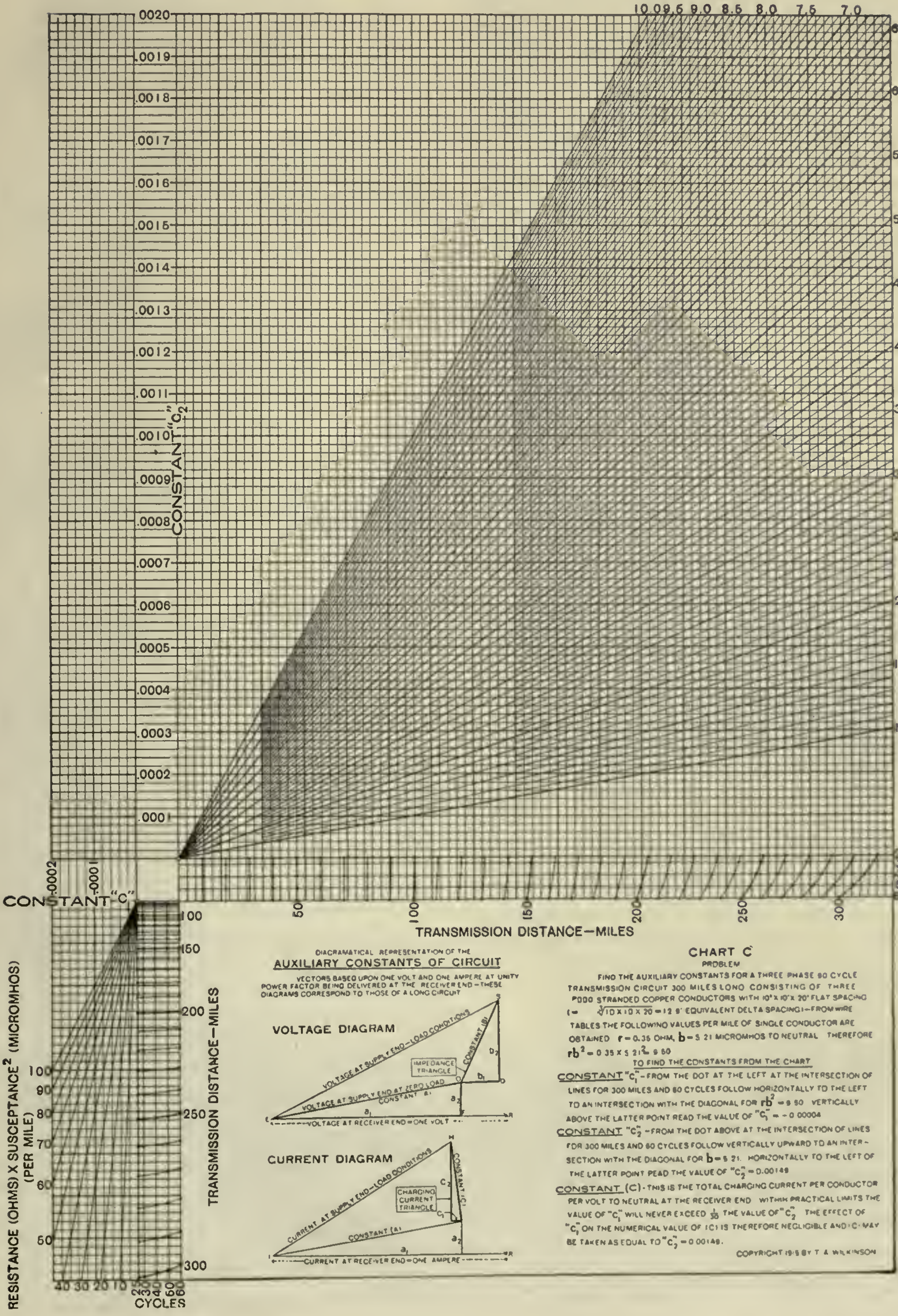


CHART VII—WILKINSON CHART C

(FOR DETERMINING AUXILIARY CONSTANTS—CHARGING CURRENT)



The reading of these charts is simplified by reason of the fact that all three charts are somewhat similar. In following any of them, the start is made from the intersection of the short arc representing length of circuit and the straight line representing the frequency. From this intersection a straight line is followed to a diagonal line and thence at right angles to the constant required. Thus in a few minutes the auxiliary constants of the circuit may be obtained directly from the chart, whereas by a mathematical solution from 15 minutes to an hour might be consumed in obtaining them. It is not, however, the time saved in obtaining these constants which is most important. The greatest advantage in this graphical solution for the auxiliary constants is that it not only abridges the use of a form of mathematics which the average engineer is inefficient in using, but it tends to prevent serious mistakes being made. In calculating these auxiliary constants by either convergent series or hyperbolic methods, an incorrect algebraic sign assigned to a number may cause a very serious error. Errors of magnitude are less likely to occur when using a comparatively simple graphical solution.

In order to determine the accuracy obtainable by a complete graphical solution, using the Wilkinson Charts for obtaining the auxiliary constants and vector diagrams for the remainder of the solutions, 48 problems were solved both graphically and mathematically. These problems consisted of circuits varying between 20 and 300 miles in length, and voltages varying between 10 000 and 200 000 volts. Twenty-four problems were for 25 cycle, and the same number for 60 cycle circuits. The maximum error in supply end voltage by the graphical solution employing a four times magnifying glass was one-fourth of one percent. A tabulation of the results as determined by various methods for these circuits will follow later.

APPLICATION OF TABLES

The application of the tables to long transmission lines follows, in general, the same plan as for short lines, published as Chart II, with such modifications as are produced by the effects of distributed capacitance and reactance. The procedure best suited for long transmission lines is shown in Chart VIII.

GRAPHICAL SOLUTION OF PROBLEM *X*

Problem X—Length of circuit 300 miles, conductors three No. 000 stranded copper spaced 10 by 10 by 20 feet (equivalent delta 12.6 feet) Temperature taken as 25 degrees C. Load conditions at receiving-end 18 000 kv-a, (16 200 kw at 90 percent power-factor lagging) 104 000 volts, three-phase, 60 cycles.

$$E_{rn} = \frac{104\,000}{1732} = 60\,046 \textit{ volts.}$$

$$I_r = \frac{6000 \times 1000}{60\,046} = 99.92 \textit{ amperes.}$$

CHART VIII.—APPLICATION OF TABLES TO LONG TRANSMISSION LINES

(EFFECT OF DISTRIBUTED CAPACITANCE TAKEN INTO ACCOUNT) OVERHEAD BARE CONDUCTORS

Starting with the kv-a., voltage and power-factor at the receiving end known.

QUICK ESTIMATING TABLES XII TO XXI INC.

From the quick estimating table corresponding to the voltage to be delivered, determine the size of the conductors corresponding to the permissible transmission loss.

CORONA LIMITATION—TABLE XXII

If the transmission is at 30 000 volts, or higher, this table should be consulted to avoid the employment of conductors having diameters so small as to result in excessive corona loss.

RESISTANCE—TABLE II

From this table obtain the resistance per unit length of single conductor corresponding to the maximum operating temperature—calculate the total resistance for one conductor of the circuit—if the conductor is large (250 000 circ. mils or more) the increase in resistance due to skin effect should be added.

REACTANCE—TABLES IV AND V

From one of these tables obtain the reactance per unit length of single conductor. Calculate the total reactance for one conductor of the circuit. If the reactance is excessive (20 to 30 percent reactance volts will in many cases be considered excessive) consult Table VI or VII. Having decided upon the maximum permissible reactance the corresponding resistance may be found by dividing this reactance by the ratio value in Table VI or VII. When the reactance is excessive, it may be reduced by installing two or more circuits and connecting them in parallel, or by the employment of three conductor cables. Using larger conductors will not materially reduce the reactance. The substitution of a higher transmission voltage, with its correspondingly less current, will also result in less reactance.

CAPACITANCE SUSCEPTANCE—TABLES IX AND X

From one of these tables obtain the capacitance susceptance to neutral, per unit length of single conductor. Calculate the total susceptance for one conductor of the circuit to neutral.

GRAPHICAL SOLUTION

From the Wilkinson charts obtain the auxiliary constants. Applying these auxiliary constants to the load conditions of the problems, make a complete graphical solution as explained in the text. Vector diagrams of the voltage and the current at both ends of the circuit are then constructed, from which the complete performance can be readily obtained graphically.

MATHEMATICAL SOLUTION

As a precaution against errors in those cases where accuracy is essential, the result obtained graphically should be checked by the convergent series or the hyperbolic method.

From tables the following linear constants per mile are determined.—

r = 0.35 *ohm* (*Table No. II*)
x = 0.83 *ohm* (*Table No. V by interpolation*)
b = 5.21 *micromhos* (*Table No. X by interpolation*)
g = (*in this case taken as zero*)

therefore,

$rb = 0.35 \times 5.21 = 1.82$

and,

$rb^2 = 0.35 \times 5.21^2 = 9.50$

The auxiliary constants of the above circuits are now taken directly from the Wilkinson Charts. This problem is stated on the Wilkinson chart. Following

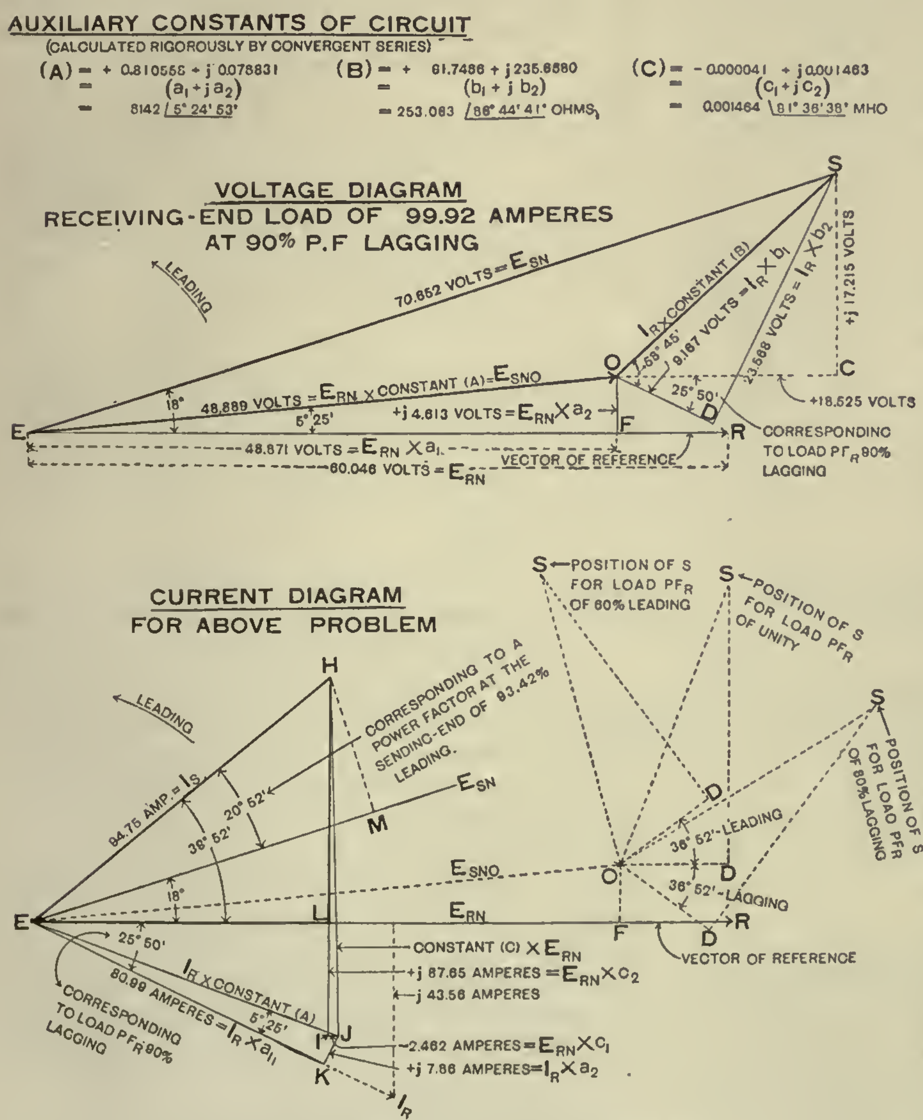


FIG. 27—GRAPHIC SOLUTION OF PROBLEM X

the directions printed on the charts, we obtain for this circuit the following values for the auxiliary constants.

$a_1 = 0.81 \quad b_1 = 91.7 \quad c_1 = 0.00004$
$a_2 = 0.077 \quad b_2 = 235 \quad c_2 = 0.00146$

From this point on, the solution is made graphically as indicated in Fig. 27. It should be noted here that the auxiliary constants obtained from the Wilkinson Charts are practically the same as those stated at the top of Fig. 27, which values were calculated rigorously by convergent series. We will employ the rigorous values in plotting the diagram so that the values on the diagram will agree with the values of voltage and current calculated rigorously which will appear in a later section. The j terms preceding some of the numerical values in Fig. 27 apply to the mathematical treatment, and have no significance in connection with the graphical solution.

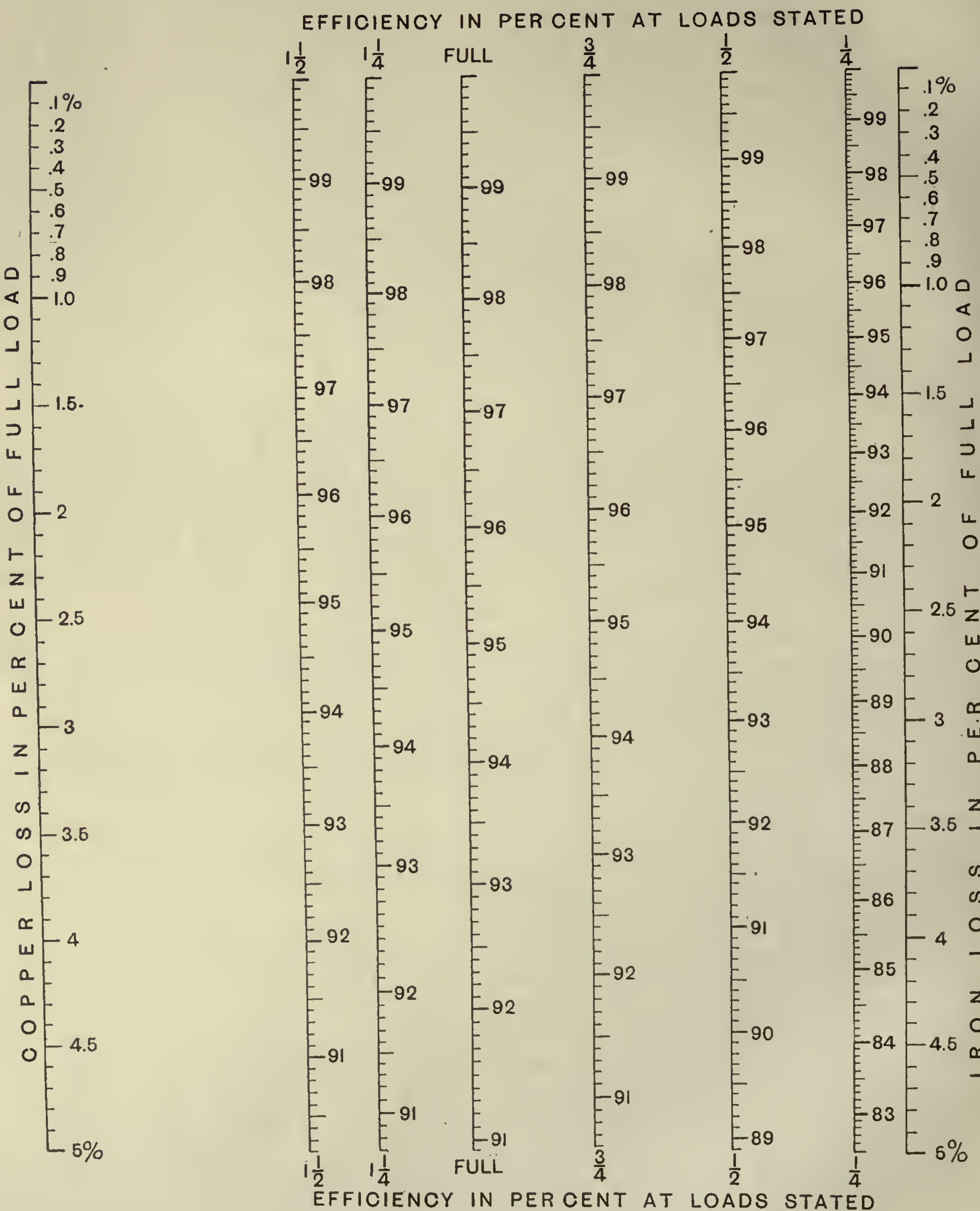
VOLTAGE DIAGRAM

The vector ER, representing the constant voltage at the receiving-end (for all loads) is first laid off to some convenient scale. Along this vector, starting from E, lay off a distance equal to the receiving-end voltage multiplied by the constant a_1 (60 046 × 0.810558 = 48,671 volts). This is EF of Fig. 27. From F lay off vertically (to the same scale) the line FO equal to the receiving-end voltage multiplied by the constant a_2 (60 046 × 0.076831 = 4613 volts). Connect the points O and E by a line. This line EO represents the voltage at the sending-end at zero load. This voltage vector may, if desired, be located by polar co-ordinates in place of rectangular co-ordinates. If it is desired to work with polar co-ordinates lay off the line EO at an angle of 5° 25′ in the forward direction from the receiving-end voltage vector ER. (For the graphical solution it is not necessary to take account of seconds in angles) The length of the vector EO will be found by multiplying the constant A by the receiving-end voltage (0.8142 × 60 044 = 48 889 volts).

Having located the point O, the impedance triangle is built upon it in the following manner. Since the power-factor of the load is 90 percent lagging, determine from a table of cosines what the angle is whose cosine is 0.9. This is found (from Table K) to be 25 degrees, 50 minutes. Lay off the line OD at an angle with the vector of reference ER of 25° 50′ in the lagging direction. The length of OD will be determined by multiplying the current in amperes per conductor by the auxiliary constant b_1 (99.92 × 91.7486 = 9167 volts). This represents the resistance drop per conductor. From the point D thus found draw a line DS at right angles with OD. This line DS represents the reactance volts per conductor; its length is found by multiplying the current in amperes per conductor by the auxiliary constant b_2 (99.92 × 235.868 = 23 568 volts). Connect the point S with E, the length of which represents the voltage (70 652 volts) at the sending-end for

CHART IX—PETER'S EFFICIENCY CHART

FOR DETERMINING TRANSFORMER LOSSES AND EFFICIENCIES

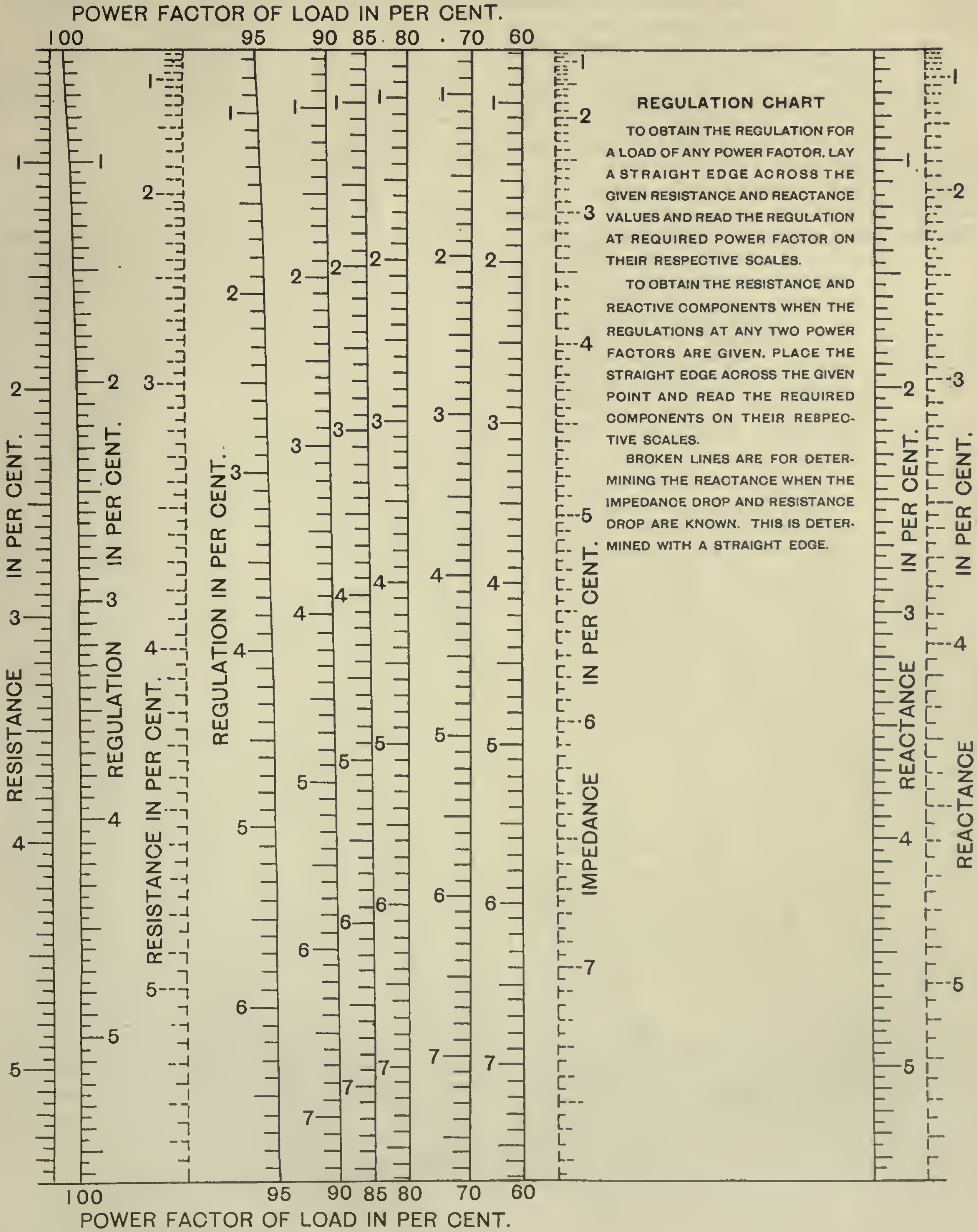


TO OBTAIN EFFICIENCY AT ANY LOAD LAY STRAIGHT EDGE AT GIVEN IRON AND COPPER LOSS POINTS AND READ THE EFFICIENCY AT REQUIRED LOAD ON THEIR RESPECTIVE SCALES WHERE STRAIGHT EDGE CROSSES THEM.

VICE VERSA, TO OBTAIN LOSSES, PLACE STRAIGHT EDGE ACROSS ANY TWO GIVEN EFFICIENCY POINTS AND READ PER CENT IRON AND COPPER LOSS ON THEIR RESPECTIVE SCALES.

CHART X--PETER'S REGULATION CHART

FOR DETERMINING TRANSFORMER REGULATION



the resistance per conductor including an equivalent value to correspond to the resistance in the high and low tension windings of two transformers will be,—

$$R + R_t = 105 + 6.25 + 6.25 = 117.5 \text{ ohms.}$$

The percent reactance volts of a transformer having 3.74 percent regulation at 80 percent lagging power-factor and 1.04 percent resistance volts may be read directly from Peter's Regulation Chart (Chart X) by laying a straight edge along the points corresponding to 1.04 percent resistance and 3.74 on the 80 percent power-factor line. The intersection of the straight edge with the last solid line at the right will give the percent reactance, = 4.85 percent.

The percent reactance volts can also be read directly from the Mershon Chart. To do this, follow upward the vertical line in the Mershon Chart corresponding to 80 percent power-factor until it intersects the first arc. From this point of intersection follow the horizontal line to the right a distance corresponding to 1.04 percent resistance volts. From this point thus obtained follow the vertical line until the arc representing 3.74 percent voltage drops is reached. The length of this vertical line will be the percentage reactance volts of the transformer, in this case 4.8 percent. Of course the reactance may, if desired, be calculated by following the general construction traced out as above described upon the Mershon chart, but the chart will give sufficiently accurate values for practical purposes.

TABLE M—APPROXIMATION OF RESISTANCE AND REACTANCE VOLTS FOR TRANSFORMERS OF VARIOUS CAPACITIES

Transformer Capacity in Kv-a	Voltage Drop in Percent			
	Resistance		Reactance	
	25 cycles	60 cycles	25 cycles	60 cycles
300	2.15	1.3	4.0	5.6
500	1.4	1.2	4.1	6.0
750	1.2	1.1	4.2	6.3
1000	1.7	1.1	6.0	6.5
1500	1.4	0.9	6.2	7.0
2000	1.3	0.8	6.4	7.0
3000	1.2	0.75	6.8	7.0
5000	1.1	0.65	7.2	7.0
7500	1.0	0.6	7.8	8.0
10 000	1.0	0.6	8.0	8.0
15 000	0.95	0.55	8.0	8.5
25 000	0.9	0.5	8.0	9.0

The volts necessary to overcome the reactance of the windings of one of these transformers is therefore found to be 60 046 × 0.048 = 2882 volts to neutral. The ohms reactance will therefore be $\frac{2882}{99.92} = 28.84$ ohms to neutral for each transformer. Since the reactance of each line conductor is 249 ohms, the reactance per conductor, including an equivalent value to correspond to the reactance in the high and low tension windings of two transformers will be,—

$$X + X_t = 249 + 28.84 + 28.84 = 306.68 \text{ ohms.}$$

The impedance of one conductor of the circuit of problem X including the raising and lowering transformers will be,—

$$Z = 117.5 + j\,306.68 \text{ ohms}$$

and Y = (*assumed to be the same as without the transformers*).

With the assumed values for the impedance, the performance of the combined circuit may be calculated as though there were no transformers in the circuit.

VOLTAGE AND CURRENT AT INTERMEDIATE POINTS ALONG THE CIRCUIT

Thus far we have considered the electrical condition at the two ends of a transmission circuit only. Occasionally it may be desired to determine the voltage or the current at a point, or at various points along the circuit. In Fig. 21, graphs of the voltage and of the current are shown for points between the terminals of a circuit corresponding to the condition of zero load, and also of rated load. The graphs were plotted by determining graphically the voltage and the current for points at 50 mile intervals along this 300 mile circuit, as follows:—

To determine the conditions 250 miles from the sending-end, (50 miles from the receiving-end) the three auxiliary constants were obtained from the Wilkinson charts corresponding to a circuit 50 miles long. In other words, it was assumed that the circuit was only 50 miles long. By multiplying these auxiliary constants by the known voltage and current at the receiving-end of the circuit, voltage and current diagrams were constructed as in Fig. 27 and on these, the corresponding values of voltage and current at the sending-end of the 50 mile section were scaled off. This gives the conditions, for the load assumed, at a point 250 miles from the sending-end. In a similar manner the voltage and current at this point, corresponding to zero load at the receiving-end, may be obtained. A similar precedure will determine the electrical conditions for a point 100 miles from the receiving-end (200 miles from the sending-end). The auxiliary constants will this time be read from the charts, corresponding to a 100 mile circuit, but the same receiving-end conditions will be used, as before. The electrical condition for any intermediate points along any smooth line, may thus be readily determined.

CHAPTER IX
PERFORMANCE OF LONG TRANSMISSION LINES
(RIGOROUS CONVERGENT SERIES SOLUTION)

THE APPROXIMATE electrical performance of overhead circuits having a length not exceeding 300 miles, may readily be determined by the use of the Wilkinson Charts for determining the values of the auxiliary constants, supplemented by vector diagrams representing the current and voltages of the circuits. In important cases, as a final check upon the values obtained by the simple graphical solution, a mathematical solution yielding rigorous results should be made. If the circuit is more than 300 miles long, a mathematical solution yielding rigorous values will be required for determining the correct values of at least the auxiliary constants.

FORMS OF RIGOROUS SOLUTIONS

The most direct method for determining mathematically the exact performance of circuits of great electrical length is by the employment of hyperbolic functions, and the fundamental equations are usually expressed in such terms. Many engineers have a general aversion to the use of mathematical expressions employing hyperbolic functions. One reason for this is that the older engineers attended college before the hyperbolic theory as applied to transmission circuits had been developed, and tables of such functions were not at that time available.

In 1893 Dr. A. E. Kennelly introduced vector arithmetic into alternating-current computation for the first time.* Although real hyperbolic functions had well recognized uses in applied science, it was in 1894** that he, for the first time, suggested and illustrated the application of vector hyperbolic functions to the determinations of the electrical performance of transmission circuits. Since that time Dr. Kennelly has been a most persistent advocate of the employment of these functions in electrical engineering problems. To advance their use, he has calculated and published numerous tables and charts of such functions. Such tables were, until recently, incomplete and the result was that it was necessary, in using these tables, to interpolate values, thus introducing complications and inaccuracies into the calculations.

Tables of hyperbolic functions and charts are now sufficiently extensive and complete for accurate work. The universities quite generally are encouraging instruction of students in the hyperbolic theory. It is therefore to be expected that, in the future, the employment of hyperbolic functions for the solution of long transmission lines will come into general use.

The fundamental hyperbolic equations expressing the electrical behavior of transmission circuits may be expressed in the form of convergent series and, in such form have, in some cases, certain advantages over the hyperbolic form. The convergent series form of solution does not require the employment of tables or charts of hyperbolic functions, whereas hyperbolic forms of solutions do require such tables or charts. If, therefore, such tables or charts are not available, hyperbolic solutions cannot be employed.

While the amount of arithmetical work involved is considerable, any degree of accuracy may readily be obtained by the convergent series solution by working out the terms for the auxiliary constants until they become too small to have any effect upon the results. This can also be done with hyperbolic functions, but exact interpolation of such functions from tabular values, may be considered more difficult than the working out of an extra term or two in the convergent series form of solution. The above remarks apply to cases where an unusual degree of accuracy is required. Later will be included a tabulation of the performance of 64 different electrical circuits, as determined by a rigorous, and also by eight different approximate methods of calculation. As the rigorous values are taken as 100 percent correct, in determining the percent error by the approximate methods, it was important that the so called "rigorous" values be exact. To make them so, it was found convenient to employ the convergent series form of solution for these particular problems, covering circuits up to 500 miles long and potentials up to 200 000 volts. For the calculation of the performance of practical power transmission circuits, tables of hyperbolic functions are now sufficiently complete to yield results well within the errors due to variation in the assumed linear constants of the circuits from their actual values.

The employment of convergent series requires a working knowledge of complex quantities only, whereas the employment of hyperbolic functions in addition leads into hyperbolic trigonometry. As literature pertaining to the hyperbolic theory becomes more generally available, and as the younger engineers take up active engineering work, the hyperbolic theory will become more generally used.

For the purpose of providing a choice of rigorous methods, both convergent series and two forms of hy-

*_Trans. Am. Inst. Elec. Engrs._, Vol. X, page 175 "Impedance."

**"_Electrical World_", Vol. XXIII, No. 1, page 17, January 1894, "The Fall of Pressure in Long-Distance Alternating-Current Conductors."

perbolic solutions are given. The numerical values employed in these solutions have been carried to what may appear as an unnecessary degree of precision. The reason for this is to demonstrate the fact that all of these rigorous solutions yield the same results. For practical problems less accuracy would be essential, thus reducing the amount of arithmetical work.

Before taking up the rigorous solutions, it has been thought desirable to review the rules regarding the use of complex quantities and vector operations.

COMPLEX QUANTITIES

The calculation of the auxiliary constants of the circuit by convergent series, and the further calculation of the electrical performance of the circuit, involve the use of complex numbers, that is, numbers containing j terms. Thus $A = a_1 + ja_2$ is a complex quantity. To the beginner, expressions containing j terms may seem difficult to understand. It cannot be made too emphatic that the rules governing the use of such terms are so simple (embodying only the simple rules of algebra) that the beginner will shortly be surprised with the ease at which complex quantities are handled.

j Terms—In the complex notation $Z = X + jY$, the prefix j indicates that the value Y is measured along the axis perpendicular to that of X, or what is called the imaginary axis. There need be no significance attached to the symbol j other than that of a mere distinguishing mark, to designate a distance above or below the reference axis in the vector diagram. However, great use is made of a further assigned significance. It has a numerical significance in the form of $j = \sqrt{-1}$ which enables all formal algebraic operations, multiplication, addition, extraction of roots, etc. incident to computation involving complex quantities, to be carried out rigorously. This numerical designation for j does not prevent its use as a designating symbol for the vertical direction in the vector diagram.*

PLANE VECTORS

Alternating voltages and currents which vary according to the sine or cosine law, may be represented graphically by directed straight lines, called plane vectors. The length of the vector represents the effective value of the alternating quantity, while the position of the vector with respect to a selected reference vector, base or axis, gives the phase displacement. The line OP, of Fig. 29, represents a plane vector inclined at an angle of 33° 41′ with the base OS (the axis of reference). The length of the line OP is a measure, to some assumed scale, of the effective value of the voltage or current, while the angle SOP gives the phase displacement.

Counter-clockwise rotation is considered positive. Thus, in Fig. 29, if the line OS represents the instantaneous direction of the current and the line OP that of the voltage at the same instant, the current is represented as lagging behind the voltage by the angle 33° 41′. By means of vectors the relative phase position and value of either currents or e.m.f.'s can be represented in the same manner as forces in mechanics.

The position of P, with respect to O, is usually defined in terms of rectangular or polar co-ordinates. In rectangular co-ordinates there are two fixed mutually perpendicular axes, $-XOX$ and $-YOY$ (Fig. 31) in the plane of reference. The former, $-XOX$, is called the real axis, or axis of real quantities. The latter, —YOY, is called the imaginary axis, or axis of imaginary quantities. The qualifying adjective "imaginary" does not mean that there is anything indeterminate or fictitious about this axis. The perpendicular projections of P-1 (Fig. 31) on the X and Y axes are respectively the real component X, and the imaginary component Y.

The magnitude and sign of the rectangular components X and Y completely determine the position of the vector OP. Positive is indicated to right and upward, negative to the left and downward as indicated in Fig. 30. Thus, if X and Y are both positive, OP lies in the first quadrant. If X and Y are both negative, OP lies in the third quadrant. If X is — and Y is +, OP lies in the second quadrant. If X is + and Y is —, OP lies in the fourth quadrant. Any plane vector may be completely specified by its real and imaginary components X and Y. Thus, beneath Fig. 31, is a table in which the point P is located in the plane by co-ordinates for all quadrants.

From Fig. 30 it is evident that, mathematically, the quadrature numbers are just as real as the others. The quadrature numbers represent the vertical, and the ordinary numbers the horizontal directions.

VECTOR OPERATIONS

In general, in the handling of complex numbers involving j terms, the simple rules of algebra are followed. In Fig. 32 two vector quantities are shown. Vector A has a magnitude of 5 units and is inclined in the positive or leading direction at an angle of 36° 52′ with the horizontal reference vector, and vector B has a magnitude of 4.47 units, and is inclined in the positive or leading direction at an angle of 63° 26′ with the reference vector. These vector quantities are expressed in rectangular co-ordinate as $\underset{.}{A} = +4 + j3$, $\underset{.}{B} = +2 + j4$ or in polar co-ordinates as $\underset{.}{A} = 5 \underline{/36° \ 52'}$, $\underset{.}{B} = 4.47 \underline{/63° \ 26'}$. The prefix j simply means that the number following it is measured along the vertical or Y axis. The dot under the vector designation indicates that A is expressed as a complex number, so that the absolute value of A would be $\sqrt{(4)^2 + (3)^2} = 5$ and of $B = \sqrt{(2)^2 + (4)^2} = 4.47$. The absolute value of a complex number is called its "size"; while the angle is called its "slope".

In order to illustrate the handling of complex quantities, the various operations of addition, subtraction, multiplication, division, evolution and involution of the vectors $\underset{.}{A}$ and $\underset{.}{B}$ in Fig. 32, will be performed.

*For an extended explanation of j terms, reference is made to Dr. Charles P. Steinmetz's "Enginering Mathematics", and Dr. A. E. Kennelly's "Artificial Electric Lines."

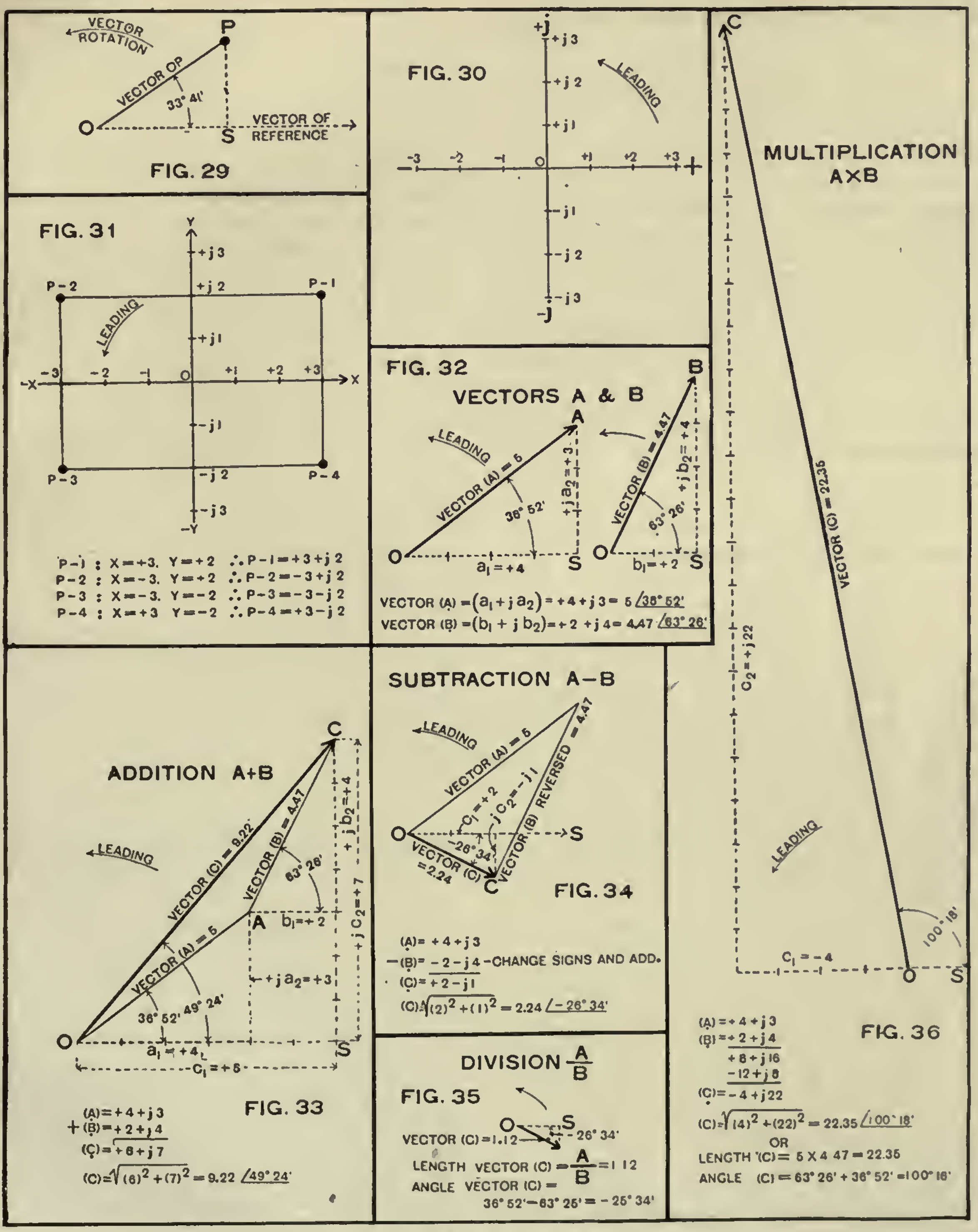
Addition—Fig. 33 illustrates the addition of these vectors expressed in rectangular co-ordinates. The resulting vector will have as its real component, the algebraic sum of the reals, and as its imaginary component, the algebraic sum of the imaginaries. Thus:

$$\dot{A} = +4 + j3$$
$$+\dot{B} = +2 + j4$$
$$\dot{A} + \dot{B} = \dot{C} = +6 + j7$$
$$C = \sqrt{(6)^2 + (7)^2} = 9.22 \textit{ absolute.}$$

The resulting vector has, therefore, a size of 9.22 units and is inclined in the forward direction at a slope of 49° 24′ with reference to the initial line, *OS*.

Subtraction—Fig. 34 illustrates the subtraction $\dot{A} - \dot{B}$. This is simply addition after the signs of both of the components of the vector to be subtracted have been reversed. Thus,—

$$\dot{A} = +4 + j3$$
$$-\dot{B} = -2 - j4$$
$$\dot{A} - \dot{B} = \dot{C} = +2 - j1$$
$$C = \sqrt{(2)^2 + (1)^2} = 2.24 \textit{ absolute.}$$



FIGS. 29 TO 36—EXAMPLES OF VECTOR SOLUTIONS

The resulting vector C has therefore a size of 2.24 units and a slope of — 26° 34′. In polar co-ordinates, $C = 2.24 \diagdown 26° 34'$.

Division—To divide one plane vector by another, divide their sizes and subtract their slopes, Fig. 35. Thus,—

$$\text{Absolute value of } C = \frac{5}{4.47} = 1.12$$

Angle of inclination of $C = 36° 52' - 63° 26' = -26° 34'$ in the negative direction. In polar co-ordinates $C = 1.12 \diagdown 26° 34'$.

Multiplication—Fig. 36 illustrates the multiplication of the vectors A and B. Here the rules of algebra also apply, except that when two j terms are multiplied signs are assigned opposite to those which would be used in the ordinary solution of an algebraic problem. This is for the reason that,—

$$j = \sqrt{-1}$$
$$\text{hence, } j^2 = -1$$

Hence where j^2 occurs it is replaced by its value -1 and therefore,—

$$-j \times j = +1$$
$$j^3 = -j$$
$$j^4 = +1$$
$$j^5 = +j, \text{ etc.}$$

Thus, to get the product of A and B:—

$$A = +4 + j\,3$$
$$B = +2 + j\,4$$
$$+8 + j\,6$$
$$-12 + j\,16$$
$$A \times B = C = -4 + j\,22 = 22.35 \text{ absolute}$$

The resulting vector C has therefore a size of 22.35 units and is inclined in the positive direction at an angle of 100° 18′ to the vector of reference. The polar expression is $C = 22.35 \diagdown 100° 18'$

The magnitude and position of the product may be also determined by multiplying the sizes of the vectors and adding their slopes. Thus:—

Size of $C = 5 \times 4.47 = 22.35$ (as above)
Slope of $C = 63° 26' + 36° 52' = 100° 18'$.

Involution—Involution is multiple multiplication. To obtain the power of a plane vector, find the power of the polar value and multiply the angle by the power to which the vector is to be raised. Thus,—vector $A = 5 \,/36° 52'$; and $(5 \,/36° 52')^2 = 5^2 \,/73° 44' = 25 \,/73° 44'$.

Evolution—To find the root of a polar plane vector, find the root of the polar value and then divide the slope by the root desired. Thus vector $A = 5 \,/36° 52'$; and $\sqrt{5 \,/36° 52'} = 2.236 \,/18° 26'$.

SOLUTION BY CONVERGENT SERIES

The hyperbolic formula for determining the operating characteristics of a transmission circuit in which exact account is taken of all the electric properties of the circuit is frequently expressed in the following form,—

$$E_s = E_r \cosh \sqrt{ZY} + I_r \sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} \ldots\ldots (51)$$

$$I_s = I_r \cosh \sqrt{ZY} + E_r \frac{1}{\sqrt{\frac{Z}{Y}}} \sinh \sqrt{ZY} \ldots\ldots (52)$$

Since $\sqrt{ZY}$ is complex, the hyperbolic functions of complex quantities are required in solving these equations.

In above formula, expressed in hyperbolic language, the three auxiliary constants A, B and C which take into account the "distributed" nature of the circuit are represented by the quantities—

$$A = \text{Cosh} \sqrt{ZY} \ldots\ldots (53)$$

$$B = \sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} \ldots\ldots (54)$$

$$C = \frac{1}{\sqrt{\frac{Z}{Y}}} \sinh \sqrt{ZY} \ldots\ldots (55)$$

Equations (51) and (52) above may therefore be expressed in terms of the auxiliary constants, A, B and C, as follows:—

$$E_s = E_r A + I_r B \ldots\ldots (56)$$
$$I_s = I_r A + E_r C \ldots\ldots (57)$$
$$\text{or } E_r = E_s A - I_s B \ldots\ldots (58)$$
$$I_r = I_s A - E_s C \ldots\ldots (59)$$

These three auxiliary constants may be calculated by convergent series as follows:—

$$A = \left[1 + \frac{YZ}{2} + \frac{Y^2 Z^2}{24} + \frac{Y^3 Z^3}{720} + \frac{Y^4 Z^4}{40320} + \text{etc.}\right] \ldots\ldots (60)$$

$$B = Z\left[1 + \frac{YZ}{6} + \frac{Y^2 Z^2}{120} + \frac{Y^3 Z^3}{5040} + \frac{Y^4 Z^4}{362880} + \text{etc.}\right] \ldots (61)$$

$$C = Y\left[1 + \frac{YZ}{6} + \frac{Y^2 Z^2}{120} + \frac{Y^3 Z^3}{5040} + \frac{Y^4 Z^4}{362880} + \text{etc.}\right] \ldots (62)$$

The above series are simply expressions for the auxiliary constants as previously stated. These constants are functions of the physical properties of the circuit and of the frequency only, and not of the voltage or the current. After the values for the auxiliary constants have been calculated for a given circuit and frequency their numerical values may be applied directly to any numerical values of E and I for which a solution is desired. From this point on, the performance of the circuit may be determined either by the graphical method previously described or by mathematical calculation.

Any degree of accuracy may be obtained by the use of convergent series for determining the auxiliary constants, by simply using a sufficient number of terms in the series. The rapidity of convergence of these series is dependent upon the value of the argument ZY and thus upon the square of the length of the circuit and frequency, and also, to a lesser extent upon the product of total circuit conductance and total circuit resistance.

As far as calculations based upon the more or less uncertain values of the fundamental constants of the circuit are concerned, the use of three terms in the series expression yields results in a 300 mile circuit which are sufficiently close to the exact values as given by the use of hyperbolic functions (infinite number of terms). In the case of shorter circuits two terms will give a high degree of accuracy. The number of terms necessary will be determined while doing the work, for it is usual to figure out the terms of the series until they become too small to be considered when added to $\frac{YZ}{2}$ or $\frac{YZ}{6}$.

In Table *N* are given values for the auxiliary constants (expressed in rectangular co-ordinates) illustrating the convergence of the series for a 300 mile, 60 cycle circuit (Problem X), the complete calculation of which will follow.

Table *N* shows that even for a 60 cycle, 300 mile circuit, three terms give sufficiently accurate results for determining constant *A*, whereas two terms are sufficient for determining constants *B* and *C*. This is on account of the slower convergence of the hyperbolic cosine series.

TABLE N—CONVERGENT SERIES TERMS FOR PROBLEM X.

No. of Terms	Constant A	Constant B	Constant C
1	1.000000 + j 0.000000	105 + j 249	0 + j 0.001563
2	+ 0.805407 + j 0.082057	+ 91.3788 + j 235.7211	— 0.000043 + j 0.001462
3	+ 0.810596 + j 0.076735	+ 91.7527 + j 235.8678	— 0.000041 + j 0.001463
4	+ 0.810558 + j 0.076832	+ 91.7486 + j 235.8680	— 0.000041 + j 0.001463
Infinite	+ 0.810558 + j 0.076831	+ 91.7486 + j 235.8680	— 0.000041 + j 0.001463

CALCULATION FOR THE AUXILIARY CONSTANTS BY CONVERGENT SERIES

The form of solution and procedure indicated in Chart XI for the calculation of the auxiliary constants by convergent series is suggested as being complete and easy to follow.

First the physical characteristics of the circuit and the frequency are stated. These are the only features having any bearing upon the value of the auxiliary constants for a given circuit. The voltage and current to be transmitted do not affect these constants. The resistance, reactance, conductance, and susceptance to neutral per mile are ascertained from the tables for one conductor of the circuit. These values are then multiplied by the length of the circuit in miles and set down as total per conductor.

The values of *Y* and *Z* must now be set down for the problem in the form of complex quantities. Thus $Z = R + jX = 105 + j249$ and $Y = G + jB = O + j0.001563$ since zero leakage conductance has been assumed for this case. Conductance *G* represents the true power loss in the form of leakage over insulators and of corona loss through the air between conductors. Corona loss corresponding to the assumed atmospheric conditions may be estimated by applying Peek's formula (See Chapter IV on Corona). Insulator leakage may be approximated from the most suitable test data available. It is general practice in the solution of all but the very longest high-voltage circuits to ignore the effect of the losses due to leakage and corona effect. These losses will be ignored in this case, so that *G* becomes zero. After *Z* and *Y* have been written down in the form of complex quantities the product *YZ* should be found as previously described for the multiplication of complex quantities. The second, third and fourth power of *YZ* may then be found, if desired. Chart XI shows the fourth power, but on all but the longest circuits a total of four terms will be sufficient, and for most problems three terms will give sufficient accuracy. The range of accuracy has been previously indicated for a 300 mile circuit on the basis of any number of terms being used up to and including infinity. The values in Chart XI are carried out to six decimal places whereas four places will usually give sufficient accuracy for calculating the values of the constants *A* and *B*. The smallness of the value of constant *C* may make six places desirable when calculating its value.

After the values of YZ, $Y^2 Z^2$, $Y^3 Z^3$ etc., have been calculated they are divided by 2, 24, 720 etc., respectively, set down and added to 1. This gives the value of the auxiliary constant *A*, as $+ 0.810558 + j\ 0.076831$ which is also referred to as $a_1 + ja_2$. The absolute value of the constant $A = 0.8142$ is simply the square root of the sum of the square of a_1 and a_2. The polar value of *A* is thus $0.8142 \underline{/5° 24' 53''}$.

The solution for the constant *B* is of the same general form as the solution for the constant *A*, except that the values of YZ, $Y^2 \cdot Z^2$, and $Y^3 Z^3$ etc., are divided by *6*, *120* and *5040* respectively. After these results are added to *1* they are multiplied by *Z*, the product being the value of the auxiliary constant *B* or $b_1 + jb_2$. The absolute value of *B* is obtained in the same manner as the absolute value of *A*.

The solution for *C* is the same as for *B* except that in place of the constant *B* series being multiplied by *Z* it is multiplied by *Y* and the values of *C* or $c_1 + jc_2$ obtained.

AUXILIARY CONSTANTS OF VARIOUS CIRCUITS

In Chart XII are tabulated exact values for the auxiliary constants for the 64 problems to which frequent reference will be made. These auxiliary constants have been calculated by convergent series, the results having been checked through the medium of three separate calculations made at different times. They are therefore believed exact to at least five significant digits. The results have been expressed in both rectangular and polar co-ordinates.

CALCULATIONS OF PERFORMANCE

In Chart XIII is given the complete calculation of the electrical performance for problem *X*, starting with

CHART XI—EXAMPLE ILLUSTRATING RIGOROUS SOLUTION FOR THE AUXILIARY CONSTANTS BY CONVERGENT SERIES FOR PROBLEM X.

PHYSICAL CHARACTERISTICS OF CIRCUIT — FREQUENCY

LENGTH, 300 MILES. CYCLES, 60.
CONDUCTORS — #000 STRANDED COPPER.
SPACING OF CONDUCTORS 10 X 10 X 20 FEET.
EQUIVALENT DELTA SPACING = 12.6 FEET.

LINEAR LINE CONSTANTS

FROM TABLES — PER MILE

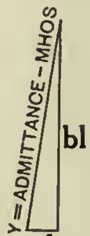
TABLE NO. 2, $r = .360$ OHM AT 26° C.
TABLE NO. 5, $x = .830$ OHM (BY INTERPOLATION).
TABLE NO. 10 $b = 5.21 \times 10^{-6}$ MHO (BY INTERPOLATION).
$g =$ (IN THIS CASE TAKEN AS ZERO).

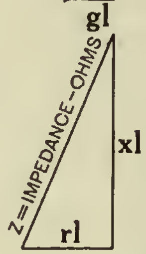
TOTAL PER CONDUCTOR

$R = rl = .360 \times 300 = 105$ OHMS TOTAL RESISTANCE.
$X = xl = .830 \times 300 = 249$ OHMS TOTAL REACTANCE.
$B = bl = 5.21 \times 300 \times 10^{-6} = .001563$ MHO TOTAL SUSCEPTANCE.
$G = gl = 0 \times 300 = 0$ MHO TOTAL CONDUCTANCE.

MULTIPLICATION OF YZ

$Y =$	$0 + j\,.001563$
$Z =$	$106 + j\,249$
	$0 \quad 0$
	$-\,.389187 + j\,.164116$
$YZ =$	$-\,.389187 + j\,.164116$
$YZ =$	$-\,.389187 + j\,.164115$
	$+\,.151466 - j\,.063871$
	$-\,.026934 - j\,.063871$
$Y^2Z^2 =$	$+\,.124532 - j\,.127742$
$YZ =$	$-\,.389187 + j\,.164115$
	$-\,.048466 + j\,.020437$
	$+\,.020964 + j\,.049715$
$Y^3Z^3 =$	$-\,.027502 + j\,.070152$
$YZ =$	$-\,.389187 + j\,.164115$
	$+\,.010703 - j\,.004513$
	$-\,.011513 - j\,.027302$
$Y^4Z^4 =$	$.000810 - j\,.031815$





NOTE

THE AUXILIARY CONSTANTS OF THE CIRCUIT (A) (B) & (C) MAY BE OBTAINED GRAPHICALLY FROM THE WILKINSON CHARTS

SOLUTION FOR (A)

$$(A) = \left[1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \frac{Y^3Z^3}{720} + \frac{Y^4Z^4}{40{,}320} + \right].$$

	$1.000\,000$
$\frac{YZ}{2} =$	$-\,.194593 + j\,.082057$
$\frac{Y^2Z^2}{24} =$	$+\,.005189 - j\,.005322$
$\frac{Y^3Z^3}{720} =$	$-\,.000038 + j\,.000097$
$\frac{Y^4Z^4}{40{,}320} =$	$-\,.000000 - j\,.000001$
$(A) =$	$+\,.810568 + j\,.076831$
$=$	$(a_1 + j a_2)$
$=$	$0.8142 \angle 5^\circ 24' 53''$

SOLUTION FOR (B)

$$(B) = Z\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \frac{Y^3Z^3}{5{,}040} + \frac{Y^4Z^4}{362{,}880} + \right].$$

	$1.000\,000$
$\frac{YZ}{6} =$	$-\,.064883 + j\,.027352$
$\frac{Y^2Z^2}{120} =$	$+\,.001038 - j\,.001064$
$\frac{Y^3Z^3}{5{,}040} =$	$-\,.000005 + j\,.000014$
$\frac{Y^4Z^4}{362{,}880} =$	$-\,.000000 - j\,.000000$
$(B) =$	$Z(+\,.93617 + j\,.026302)$
$Z =$	$106 + j\,249$
	$+\,98.2978 + j\,233.1063$
	$-\,6.6492 + j\,2.7617$
$(B) =$	$+\,91.7486 + j\,235.8680$
$=$	$(b_1 + j\,b_2)$
$=$	$253.083 \angle 68^\circ 44' 41''$ OHMS

SOLUTION FOR (C)

$$(C) = Y\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \frac{Y^3Z^3}{5.040} + \frac{Y^4Z^4}{362.880} + \right]$$

$(C) =$	$Y(+\;93617 + j\;026302)$
$Y =$	$0 + j\;001563$
$(C) =$	$-\,.000041 + j\;001463$
$=$	$(c_1 + j\,c_2)$
$=$	$.001464 \;\backslash 91^\circ 36' 18''$ MHO

the values for the auxiliary constants and the receiving end load conditions known. The calculations are carried out by the employment of complex numbers, the complete performance being calculated for both load and zero load conditions. In order to give a more clear understanding of these mathematical operations the reader is referred to the vector diagrams of Fig. 37.

In Chart XIII are given the formulas for determining the E_s and I_s values under load conditions. On Fig. 37 these two same formulas are given, but in the form of vector diagrams, upon which vectors the numerical values corresponding to problem *X* are stated. With the numerical values of the vectors and angles stated, it should be a comparatively simple manner to follow graphically (Fig. 37) the mathematical calculations shown in Chart XIII.

The formulas for E_s and I_s which are stated in Chart XIII and in Fig. 37 contain a complex number ($\text{Cos}\ \theta_r \pm j \sin \theta_r$) not previously stated in connection with the fundamental hyperbolic formulas for long circuits. The formulas previously given were based upon unity power-factor. The introduction of this new complex number is made necessary in order that the effect of the power-factor of the load current may be included in the calculations. The function of this new complex number is to rotate the current vector through an angle corresponding to the power-factor of the load current. It will be referred to as the rotating triangle. If the

CHART XII—AUXILIARY CONSTANTS OF VARIOUS CIRCUITS

PROBLEM NO.	LENGTH OF CIRCUIT—(MILES)	CONDUCTORS	SPACING—DELTA	LINEAR CONSTANTS TOTAL PER CONDUCTOR ★ rl FROM TABLE NO. 2	xl FROM TABLE NOS 4 & 5	bl FROM TABLE NOS 9 & 10	gl	AUXILIARY CONSTANTS OF CIRCUIT (THESE AUXILIARY CONSTANTS TAKE INTO ACCOUNT THE EFFECT OF DISTRIBUTED CAPACITANCE. THEY HAVE BEEN CALCULATED RIGOROUSLY BY CONVERGENT SERIES) CONSTANT (A) a_1 a_2	CONSTANT (B) b_1 b_2	CONSTANT (C) c_1 c_2
								2 5 C Y C L E S		
1	20	0000 COPPER	3	5.54	5.36	57.2	0	.999847 + j.000158	5.5394 + j 5.3600	0 + j.000057
2	"	"	"	"	"	"	0	= .999847 ∠0°0′32″	= 7.7081 ∠44°3′27″	= .000057 ∠90°0′0″
3	"	"	3	5.54	5.36	57.2	0	.999847 + j.000158	5.5394 + j 5.3600	0 + j.000057
4	"	"	"	"	"	"	0	= .999847 ∠0°0′32″	= 7.7081 ∠44°3′27″	= .000057 ∠90°0′0″
5	30	0000 COPPER	4	8.31	8.5	81.0	0	.999656 + j.000336	8.3082 + j 8.4999	0 + j.000081
6	"	"	"	"	"	"	0	= .999656 ∠0°1′10″	= 11.886 ∠45°39′12″	= .000081 ∠90°0′0″
7	"	"	4	8.31	8.5	81.0	0	.999656 + j.000336	8.3082 + j 8.4999	0 + j.000081
8	"	"	"	"	"	"	0	= .999656 ∠0°1′10″	= 11.886 ∠45°39′12″	= .000081 ∠90°0′0″
9	50	0000 COPPER	4	13.85	14.1	135	0	.999048 + j.000935	13.841 + j 14.0996	0 + j.000135
10	"	"	"	"	"	"	0	= .999048 ∠0°3′12″	= 19.757 ∠45°31′44″	= .000135 ∠90°0′0″
11	"	"	6	13.85	15.1	125	0	.999056 + j.000866	13.8413 + j 15.0991	0 + j.000125
12	"	"	"	"	"	"	0	= .999056 ∠0°2′58″	= 20.4833 ∠47°29′20″	= .000125 ∠90°0′0″
13	100	0000 COPPER	9	27.7	32.2	233	0	.996248 + j.003224	27.6307 + j 32.1894	0 + j.000233
14	"	"	"	"	"	"	0	= .996253 ∠0°11′7″	= 42.4218 ∠49°21′28″	= .000233 ∠90°0′0″
15	"	"	11	27.7	33.2	226	0	.996249 + j.003126	27.6308 + j 33.1874	0 + j.000226
16	"	"	"	"	"	"	0	= .996254 ∠0°10′47″	= 43.1841 ∠50°13′13″	= .000226 ∠90°0′0″
17	200	300M COPPER	11	39.2	64.8	464	0	.984991 + j.009049	38.808 + j 64.594	−.000001 + j.000462
18	"	"	"	"	"	"	0	= .985033 ∠0°31′35″	= 75.356 ∠59°0′10″	= .000462 ∠90°7′27″
19	"	"	17	39.2	69.2	434	0	.985009 + j.008464	38.8084 + j 68.965	−.000001 + j.000432
20	"	"	"	"	"	"	0	= .985050 ∠0°29′31″	= 79.134 ∠60°37′58″	= .000432 ∠90°7′54″
21	300	636M ALUM.	11	44.1	91.2	747	0	.966085 + j.016285	43.1033 + j 90.408	−.000004 + j.000739
22	"	"	"	"	"	"	0	= .966222 ∠0°57′1″	= 100.157 ∠64°30′36″	= .000739 ∠90°17′10″
23	"	"	21	44.1	101	672	0	.966219 + j.014650	43.1070 + j 100.077	−.000003 + j.000664
24	"	"	"	"	"	"	0	= .966330 ∠0°52′6″	= 108.966 ∠66°41′48″	= .000664 ∠90°15′24″
25	400	636M ALUM.	17	58.8	130	928	0.	.940161 + j.026738	56.4555 + j 127.927	−.000008 + j.000909
26	"	"	"	"	"	"	0	= .940541 ∠1°37′45″	= 139.83 ∠66°11′16″	= .000909 ∠90°30′14″
27	"	"	21	58.8	134	896	0	.940452 + j.025819	56.4664 + j 131.842	−.000008 + j.000878
28	"	"	"	"	"	"	0	= .940801 ∠1°34′20″	= 143.425 ∠66°48′54″	= .000878 ∠90°31′18″
29	500	636M ALUM.	17	73.5	163	1160	0	.906642 + j.041299	68.928 + j 158.928	−.000016 + j.001124
30	"	"	"	"	"	"	0	= .907583 ∠2°36′14″	= 173.23 ∠66°33′13″	= .001124 ∠90°48′56″
31	"	"	21	73.5	168	1120	0	.907109 + j.039880	68.9507 + j 163.76	−.000015 + j.001085
32	"	"	"	"	"	"	0	= .907985 ∠2°31′2″	= 177.684 ∠67°10′0″	= .001085 ∠90°47′33″
								6 0 C Y C L E S		
33	20	0000 COPPER	3	5.54	12.88	137	0	.999118 + j.000379	5.53675 + j 12.8769	0 + j.000137
34	"	"	"	"	"	"	0	= .999118 ∠0°1′18″	= 14.0167 ∠66°44′0″	= .000137 ∠90°0′0″
35	"	"	3	5.54	12.88	137	0	.999118 + j.000379	5.53675 + j 12.8769	0 + j.000137
36	"	"	"	"	"	"	0	= .999118 ∠0°1′18″	= 14.0167 ∠66°44′0″	= .000137 ∠90°0′0″
37	30	0000 COPPER	4	8.31	20.4	195	0	.998011 + j.00081	8.299 + j 20.3887	0 + j.000195
38	"	"	"	"	"	"	0	= .998011 ∠0°2′47″	= 22.014 ∠67°51′6″	= .000195 ∠90°0′0″
39	"	"	4	8.31	20.4	195	0	.998011 + j.00081	8.299 + j 20.3887	0 + j.000195
40	"	"	"	"	"	"	0	= .998011 ∠0°2′47″	= 22.014 ∠67°51′6″	= .000195 ∠90°0′0″
41	50	0000 COPPER	4	13.85	34.0	324	0	.994496 + j.002239	13.7992 + j 33.9479	0 + j.000323
42	"	"	"	"	"	"	0	= .994498 ∠0°7′33″	= 36.645 ∠67°52′45″	= .000323 ∠90°0′0″
43	"	"	6	13.85	36.4	301	0	.994526 + j.002081	13.7994 + j 36.3432	0 + j.000300
44	"	"	"	"	"	"	0	= .994528 ∠0°7′14″	= 38.874 ∠69°12′30″	= .000300 ∠90°0′0″
45	100	0000 COPPER	9	27.7	77.4	562	0	.97832 + j.007728	27.2996 + j 76.9116	−.000001 + j.000558
46	"	"	"	"	"	"	0	= .97835 ∠0°27′10″	= 81.6129 ∠70°27′36″	= .000558 ∠90°6′11″
47	"	"	11	27.7	79.7	542	0	.97847 + j.007452	27.302 + j 79.1963	−.000001 + j.000538
48	"	"	"	"	"	"	0	= .978498 ∠0°26′14″	= 83.77 ∠70°58′30″	= .000538 ∠90°6′27″
49	200	300M COPPER	11	39.2	156	1116	0	.914128 + j.021243	36.9541 + j 151.791	−.000008 + j.001084
50	"	"	"	"	"	"	0	= .914375 ∠1°19′31″	= 156.224 ∠76°19′2″	= .001084 ∠90°25′23″
51	"	"	17	39.2	166	1044	0	.914524 + j.019876	36.9641 + j 161.507	−.000007 + j.001014
52	"	"	"	"	"	"	0	= .914740 ∠1°14′40″	= 165.69 ∠77°6′31″	= .001014 ∠90°23′43″
53	300	636M ALUM.	11	44.1	220	1794	0	.808816 + j.037006	38.4655 + j 206.359	−.000023 + j.001678
54	"	"	"	"	"	"	0	= .809662 ∠2°37′0″	= 209.913 ∠79°26′28″	= .001678 ∠90°47′8″
55	"	"	21	44.1	243	1614	0	.810022 + j.033307	38.5002 + j 227.918	−.000018 + j.001510
56	"	"	"	"	"	"	0	= .810701 ∠2°21′14″	= 231.147 ∠80°24′43″	= .001510 ∠90°4′6″
57	400	636M ALUM.	17	58.8	314	2212	0	.671701 + j.057759	45.8726 + j 280.04	−.000044 + j.001958
58	"	"	"	"	"	"	0	= .674179 ∠4°54′54″	= 283.77 ∠80°41′50″	= .001959 ∠91°18′0″
59	"	"	21	58.8	322	2152	0	.672455 + j.056208	45.9013 + j 287.194	−.000042 + j.001912
60	"	"	"	"	"	"	0	= .674800 ∠4°46′39″	= 290.839 ∠80°55′10″	= .001913 ∠91°15′21″
61	500	636M ALUM.	17	73.5	390	2785	0	.502772 + j.084790	48.9614 + j 325.247	−.000085 + j.002307
62	"	"	"	"	"	"	0	= .509871 ∠9°34′20″	= 328.912 ∠81°26′21″	= .002309 ∠92°6′32″
63	"	"	21	73.5	402	2690	0	.504852 + j.081969	49.061 + j 335.414	−.000079 + j.002230
64	"	"	"	"	"	"	0	= .511463 ∠9°13′12″	= 338.98 ∠81°40′43″	= .002232 ∠92°1′45″

*rl is the resistance in ohms at 25° C (77° F), xl the reactance in ohms, bl the susceptance in micromhos to neutral (multiply by 10^{-6} to convert to mhos). The x and b values for the 636 000 circ. mil aluminum cable were taken as those of 700 000 circ. mil copper on the assumption that these two conductors would have approximately the same diameter. gl, the loss resulting from leakage over insulators and from corona has, for simplicity, been assumed as zero.

CHART XIII—RIGOROUS CALBULATION OF PERFORMANCE WHEN RECEIVING END CONDITIONS ARE FIXED

$KV\text{-}A_R = 18\,000.$ $KW_R = 16\,200.$ $E_R = 104\,000$ VOLTS 3 PHASE. $PF_R = 90.00\%$ LAGGING.

PER PHASE TO NEUTRAL

$KV\text{-}A_{RN} = \frac{18\,000}{3} = 6\,000.$ $KW_{RN} = \frac{18\,200}{3} = 5\,400.$ $E_{RN} = \frac{104\,000}{1.732} = 60\,046.$ $I_R = \frac{6\,000 \times 1000}{60\,046} = 99.92$ AMPERES.

AUXILIARY CONSTANTS OF CIRCUIT

$(A) = +.810558 + j.076831 = (a_1 + j a_2) = .8142 \angle 5°\,24'\,53''$

$(B) = +91.7486 + j\,235.868 = (b_1 + j\,b_2) = 253.083 \angle 68°\,44'\,41''$ OHMS

$(C) = -.000041 + j.001463 = (c_1 + j\,c_2) = .001464 \angle 91°\,36'\,18''$ MHO

SOLUTION FOR E_S — **LOAD CONDITIONS** — **SOLUTION FOR I_S**

$E_S = E_R(a_1 + j a_2) + I_R(\cos\theta_R \pm j \sin\theta_R)(b_1 + j b_2)$ ★

$I_S = I_R(\cos\theta_R \pm j \sin\theta_R)(a_1 + j a_2) + E_R(c_1 + j c_2)$ ★

★ ± THIS SIGN IS MINUS WHEN THE P. F. IS LAGGING AND PLUS WHEN THE P. F. IS LEADING

$(a_1 + j a_2)$	=	+.810558 + j.076831
$\times E_{RN}$	=	60046
$E_{RN}(a_1 + j a_2)$	=	+ 48671 + j 4613
$(\cos\theta_R - j \sin\theta_R)$	=	+ .9 − j .438
$\times I_R$	=	99.92
$I_R(\cos\theta_R - j \sin\theta_R)$	=	+ 89.93 − j 43.56
$\times (b_1 + j b_2)$	=	+ 91.75 + j 235.87
		+ 8251 + j 21212
		+ 10274 − j 3997
$I_R(\cos\theta_R - j \sin\theta_R)(b_1 + j b_2)$	=	+ 18525 + j 17215
$+ E_{RN}(a_1 + j a_2)$	=	+ 48671 + j 4613
E_{SN}	=	+ 67196 + j 21828
	=	$\sqrt{(67198)^2 + (21828)^2}$
E_{SN}	=	70 652 VOLTS TO NEUTRAL.

$I_R(\cos\theta_R - j \sin\theta_R)$	=	+ 89.93 − j 43.56
$\times (a_1 + j a_2)$	=	+ .810558 + j .076831
		+ 72.893 + j 6.909
		+ 3.347 − j 35.308
$I_R(\cos\theta_R - j \sin\theta_R)(a_1 + j a_2)$	=	+ 76 240 − j 28 399
$(c_1 + j c_2)$	=	−.000041 + j.001463
$\times E_{RN}$	=	60046
$E_{RN}(c_1 + j c_2)$	=	− 2.462 + j 87.85
$+ I_R(\cos\theta_R - j \sin\theta_R)(a_1 + j a_2)$	=	+ 76.240 − j 28.399
I_S	=	+ 73.778 + j 59.451
	=	$\sqrt{(73.778)^2 + (59.451)^2}$
I_S	=	94.75 AMPERES.

$KW_{SN} = (67.196 \times 73.778) + (21.828 \times 69.461) = 6.256$ KW PER PHASE.

$KV\text{-}A_{SN} = (70.662 \times 94.76) = 6\,694$ KV-A PER PHASE.

$LOSS_N = 6265 - 5400 = 855$ KW PER PHASE.

EFFICIENCY $= \frac{6\,400 \times 100}{6\,255} = 86.33\%$.

$PF_S = \frac{6\,255 \times 100}{6\,694} = 93.42\%$ LEADING.

PHASE ANGLES— AT FULL LOAD THE VOLTAGE AT THE SENDING END LEADS THE VOLTAGE AT THE RECEIVER END BY THE ANGLE $\tan^{-1}\frac{21\,828}{67196} = \tan^{-1}.326 = 18°\,00'$. AND THE CURRENT AT THE SENDING-END LEADS THE VOLTAGE AT THE RECEIVING-END BY THE ANGLE $\tan^{-1}\frac{69.451}{73.778} = \tan^{-1}.806 = 38°\,52'$. HENCE THE CURRENT AT THE SENDING-END LEADS THE VOLTAGE AT THE SENDING-END BY THE ANGLE $38°\,52' - \text{ANGLE } 18°\,00' = 20°\,52'$. THE POWER-FACTOR AT THE SENDING-END IS THEREFORE $\cos 20°\,52' = 93.42\%$ LEADING AT LOAD SPECIFIED.

ZERO LOAD CONDITIONS

$E_{SNO} = 48671 + j\,4813 = \sqrt{(48671)^2 + (4613)^2}$

$E_{SNO} = 48\,889$ VOLTS.

$I_{SO} = -2.462 + j\,87.85 = \sqrt{(-2.462)^2 + 87.85)^2}$

$I_{SO} = 87.89$ AMPERES.

$KW_{SNO} = (48.671 \times -2.462) + 14.613 \times 87.85) = 285.43$ KW PER PHASE.

$KV\text{-}A_{SNO} = 48.889 \times 87.89 = 4\,297$ KV A PER PHASE.

$PF_{SO} = \frac{285.43 \times 100}{4.297} = 6.64\%$ LEADING.

REGULATION

A RISE IN VOLTAGE AT THE SENDING-END OCCURS OF 70 652 − 48 889 = 21 763 VOLTS TO NEUTRAL WHEN THE LOAD IS INCREASED FROM ZERO TO 99.92 AMPERES AT 90% POWER FACTOR LAGGING AT THE RECEIVER END WITH CONSTANT VOLTAGE AT THE RECEIVING END.

PHASE ANGLES

AT ZERO LOAD THE VOLTAGE AT THE SENDING-END LEADS THE VOLTAGE AT THE RECEIVER END BY THE ANGLE $\tan^{-1}\frac{4\,613}{48\,671} = \tan^{-1}.0947 = 6°\,26'$ AND THE CURRENT AT THE SUPPLY END LEADS THE VOLTAGE AT THE RECEIVER END BY THE ANGLE $\tan^{-1}\frac{87.86}{-2\,462} = \tan(-35.7) = 91°\,36'$—HENCE THE CURRENT AT THE SUPPLY END LEADS THE VOLTAGE AT THE SUPPLY END BY THE ANGLE $(91°\,36') - (5°\,25') = 86°\,11'$. THE POWER FACTOR AT THE SENDING-END IS THEREFORE $\cos 86°\,11' = 6.64\%$ LEADING AT ZERO LOAD.

load power-factor is 100 percent, this rotating triangle will equal *1 ± j 0*, hence it has no effect or power to rotate. If the power-factor of the load is 80 percent the rotating triangle would have a numerical value of *0.8 ± j 0.6*.

The various phase angles given in Chart XIII show whether the power-factor at the supply end is leading or lagging. These various phase angles are given to make the discussion complete. Actually, in order to determine whether the power-factor at the supply end is leading or lagging, it is only necessary to note if the supply end current vector leads or lags behind the supply end voltage vector. At the lower end of Fig. 37 combined current and voltage vectors are shown for this problem, corresponding to both load and zero load conditions.

In Chart XIV is given a complete calculation of the electrical performance of problem *X*, starting with the values for the auxiliary constants and the sending end load condition known. In other words the supply end conditions which were derived by calculation in Chart XIII have in this case been assumed as fixed, and the receiver end conditions calculated. The reason that

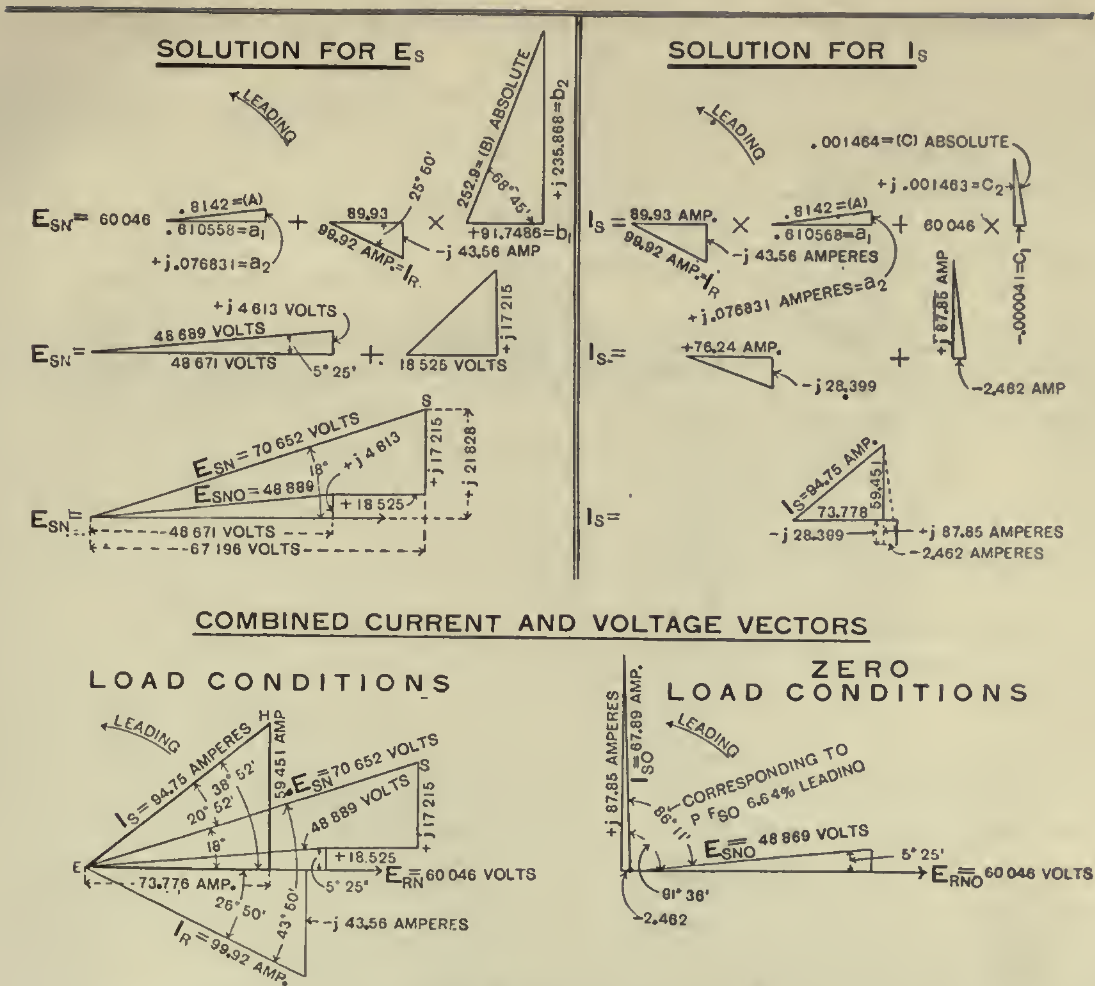


FIG. 37—GRAPHIC REPRESENTATION OF PROBLEM X

Illustrating rigorous calculations of performance when receiving end conditions are fixed.

there is a slight difference between the receiving end conditions as calculated on Chart XIV and the known receiving end conditions is that the value for the sine in the rotating triangle (0.436) in chart XIII was carried out to only three places, whereas in Chart XIV it was carried out to four places. If the values for the rotating triangles had been carried out to five or six places in the calculations in both charts, the receiving end conditions would have checked exactly.

TERMINAL VOLTAGES AT ZERO LOAD

For a given circuit and frequency, the relation of the voltage at the two ends of the circuit is fixed. The ratio of sending end to the receiving end voltage is expressed by the constant A. The ratio of receiving to sending end voltage is expressed by $\frac{I}{A}$. For example, problem X, the sending end voltage under load is 70 652 volts. If the load is thrown off, and this sending end voltage is maintained constant at 70 652 volts, the receiving end voltage will rise to a value of $\frac{70\,652}{0.8142}$ = 86 775 volts to neutral. The rise in percent of sending end voltage is therefore $\frac{100 \times 86\,775 - 70\,652}{70\,652}$ = 22.82 percent.

PERFORMANCE OF VARIOUS CIRCUITS

In Chart XV is tabulated the complete performance of the 64 problems for which the auxiliary constants are tabulated in Chart XII. The auxiliary constants in Chart XII were applied to the fixed load conditions as stated in Chart XV for the receiving end, and both load and zero load conditions at the sending end were calculated and tabulated.

The object of calculating and tabulating the values for the 64 problems was two fold. First to obtain data on 25 and 60 cycle problems covering a wide range which would provide a basis for constructing curves, illustrating the effect that distance in transmission has upon the performance of circuits and upon the auxiliary constants of the circuit. Second, to give the student a wide range of problems from which he could choose, and from which he could start with the tabulated values as fixed at either end and calculate the conditions at the other end. It is believed that such problems will furnish very profitable practice for the student and will also serve as a general guide when making calculations on problems of similar length and fundamental or lineal constants. It is not intended that the figures given for longer circuits, included in these tabulations, shall coincide with ordinary conditions encountered in practice.

CHART XIV—RIGOROUS CALCULATION OF PERFORMANCE WHEN SENDING END CONDITIONS ARE FIXED

$KV\text{-}A_S = 20\ 082.$ $KW_S = 18\ 785.$ $E_S = 122\ 369$ VOLTS 3 PHASE. $PF_S = 93.42\%$ LEADING.

PER PHASE TO NEUTRAL

$KV\text{-}A_{SN} = \frac{20\ 082}{3} = 6\ 694.$ $KW_{SN} = \frac{18\ 765}{3} = 6\ 265.$ $E_{SN} = \frac{122\ 369}{1.732} = 70\ 852.$ $I_S = \frac{8\ 694 \times 1000}{70\ 652} = 94.75$ AMPERES.

AUXILIARY CONSTANTS OF CIRCUIT

$(A) = +.810556 + j.076831 = (a_1 + j a_2) = .8142 \angle 5°\ 24'\ 53''$

$(B) = +91.7486 + j\ 236.868 = (b_1 + j\ b_2) = 253.083 \angle 68°\ 44'\ 41''$ OHMS

$(C) = -.000041 + j.001463 = (c_1 + j\ c_2) = .001464 \angle 91°\ 36'\ 18''$ MHO

LOAD CONDITIONS

SOLUTION FOR E_R

$$E_R = E_S(a_1 + j a_2) - I_S(\cos\theta_S \pm j \sin\theta_S)(b_1 + j b_2) \bigstar$$

$(a_1 + j a_2)$	=	$+.810558 + j.076831$
$\times E_{SN}$	=	$+70652$
$E_{SN}(a_1 + j a_2)$	=	$+67268 + j\ 5428$
$(\cos\theta_S + j \sin\theta_S)$	=	$+.9342 + j.3567$
$\times I_S$	=	$+94.75$
$I_S(\cos\theta_S + j \sin\theta_S)$	=	$+88.62 + j\ 33.8$
$\times (b_1 + j b_2)$	=	$+91.75 + j\ 235.9$
		$+8122 + 20882$
		$-7973 + j\ 3101$
$I_S(\cos\theta_S + j \sin\theta_S)(b_1 + j b_2)$	=	$+149 + j\ 23983$
$E_{SN}(a_1 + j a_2)$	=	$+57268 + j\ 5428$
$-I_S(\cos\theta_S + j \sin\theta_S)(b_1 + j b_2)$		$-149 - j\ 23983$ ← CHANGE SIGNS AND ADD
E_{RN}	=	$+67119 - j\ 18555$
	=	$\sqrt{(57119)^2 + (18555)^2}$

$E_{RN} = 80\ 067$ VOLTS TO NEUTRAL.

SOLUTION FOR I_R

$$I_R = I_S(\cos\theta_S \pm j \sin\theta_S)(a_1 + j a_2) - E_S(c_1 + j c_2) \bigstar$$

★ ± THIS SIGN IS MINUS WHEN THE P. F. IS LAGGING AND PLUS WHEN THE P. F. IS LEADING

$I_S(\cos\theta_S + j \sin\theta_S)$	=	$+88.52 + j\ 33.8$
$\times (a_1 + j a_2)$	=	$+.810558 + j.076831$
		$+71.751 + j\ 6.801$
		$-2.597 + j\ 27.397$
$I_S(\cos\theta_S + j \sin\theta_S)(a_1 + j a_2)$	=	$+69.154 + j\ 34.198$
$(c_1 + j c_2)$	=	$-.000041 + j.001483$
$\times E_{SN}$	=	70652
$E_{SN}(c_1 + j c_2)$	=	$-2.897 + j\ 103.36$
$I_S(\cos\theta_S + j \sin\theta_S)(a_1 + j a_2)$	=	$+69.154 + j\ 34.20$
CHANGE SIGNS AND ADD → $-E_{SN}(c_1 + j c_2)$	=	$+2.897 - j\ 103.36$
I_R	=	$72.051 - j\ 69.16$
	=	$\sqrt{(72.051)^2 + (69.16)^2}$

$I_R = 99.87$ AMPERES.

$KW_{RN} = (57.119 \times 72.051) + (18.565 \times 69.16) = 6\ 399$ KW PER PHASE.

$PF_R = \frac{6\ 399 \times 100}{6\ 998} = 90.01\%$ LAGGING.

$KV\text{-}A_{RN} = (60.057 \times 99.87) = 6\ 998$ KV-A PER PHASE.

$EFFICIENCY = \frac{6\ 399 \times 100}{6\ 265} = 88.32\%.$

$LOSS_N = 6\ 255 - 5\ 399 = 858$ KW PER PHASE.

PHASE ANGLES AT FULL LOAD THE VOLTAGE AT THE RECEIVER END LAGS BEHIND THE VOLTAGE AT THE SENDING-END BY THE ANGLE $\tan^{-1}\frac{18\ 555}{67\ 119} = \tan^{-1}.325 = 18°\ 0'$; AND THE CURRENT AT THE RECEIVER END LAGS BEHIND THE VOLTAGE AT THE SENDING-END BY THE ANGLE $\tan^{-1}\frac{69.16}{72.051} = \tan^{-1}.959 = 43°50'$. HENCE THE CURRENT AT THE RECEIVER END LAGS BEHIND THE VOLTAGE AT THE RECEIVER END BY THE ANGLE 43° 50' − ANGLE 18° 0' = 25° 50'. THE POWER-FACTOR AT THE RECEIVER END IS THEREFORE COS 25° 50' = 90% LAGGING.

ZERO LOAD CONDITIONS

$$E_{RNO} = \frac{E_{SNO}(a_1 - j a_2)}{(a_1^2 + a_2^2)} = \frac{48\ 898(.81058 - j.076831)}{(.81056)^2 + (.076831)^2} = \frac{39635 - j\ 3757}{.6629} = 59\ 790 - j\ 5667 = 60\ 058 \text{ VOLTS.}$$

$$I_{SO} = E_{SNO}\frac{(c_1 a_1 + c_2 a_2) + j(c_2 a_1 - c_1 a_2)}{(a_1^2 + a_2^2)} = 48\ 898\ \frac{((-.000041 \times .81056) + (.001463 \times .076831)) + j((.001463 \times .81056) - (-.000041 \times .076831))}{.6629}$$

$$I_{SO} = 48\ 898\ \frac{(+.0000792 + j.001189)}{.6629} = 48\ 898(.000119 + j.001794) = 48\ 898 \times .001798 = 87.92 \text{ AMPERES.}$$

REGULATION

A RISE IN VOLTAGE AT THE SENDING-END OCCURS OF 70 652 − 48 898 = 21 754 VOLTS TO NEUTRAL WHEN THE LOAD IS INCREASED FROM ZERO TO 99.87 AMPERES AT 90.01% POWER FACTOR LAGGING AT THE RECEIVER END WITH CONSTANT VOLTAGE AT THE RECEIVING END.

PHASE ANGLES

AT ZERO LOAD THE VOLTAGE AT THE RECEIVER END LAGS BEHIND THE VOLTAGE AT THE SENDING-END BY THE ANGLE $\tan^{-1}\frac{5\ 667}{69\ 790} = \tan^{-1}.0948 = 5°25'$; AND THE CURRENT AT THE SENDING-END LEADS THE VOLTAGE AT THE SENDING-END BY THE ANGLE $\tan^{-1}\frac{.001794}{.000119} = \tan^{-1}15.08 = 86°11'$. THE POWER-FACTOR AT THE SENDING-END IS THEREFORE COS 86° 11' = 6.64% LEADING AT ZERO LOAD.

CHART XV—CALCULATED PERFORMANCE OF VARIOUS CIRCUITS

PROBLEM	RECEIVING-END CONDITIONS FIXED — LOAD CONDITIONS		TO NEUTRAL					SENDING-END CONDITIONS—CALCULATED ★ — LOAD CONDITIONS — TO NEUTRAL							ZERO LOAD — TO NEUTRAL				
	$KV\text{-}A_R$	E_R 3 PHASE	$KV\text{-}A_{RN}$	KW_{RN}	E_{RN}	I_R	PF_R %	$KV\text{-}A_{SN}$	KW_{SN}	E_{SN}	I_S	★★ PF_S %	LINE DROP IN % OF E_{RN}	LINE LOSS IN % OF KW_R	$KV\text{-}A_{SNO}$	KW_{SNO}	E_{SNO}	I_{SO}	PF_{SO} %
25 CYCLES																			
1	1300	10 000	433.3	346.6	5 774	75	80 LAG	474.63	377.52	6 347	74.78	79.53	−9.92	8.42	1.963		5 773	.34	
2	"	"	"	433.3	"	"	100	465.09	464.21	6 202	74.99	99.81	−7.41	7.13	"		"	"	
3	5000	20 000	1666.6	1333.3	11 550	144.4	80 LAG	1821.9	1449.5	12 653	143.99	79.56	−9.55	8.71	7.622		11 548	.66	
4	"	"	"	1666.6	"	"	100	1 786.3	1 783.33	12 372	144.38	99.80	−7.12	7.00	"		"	"	
5	3500	20 000	1167	933	11 550	101	80 LAG	1 278.6	1017.45	12 733	100.42	79.58	−10.24	9.05	10.85		11 546	.94	
6	"	"	"	1167	"	"	100	1 253.5	1 254.22	12 415	100.97	99.82	−7.49	7.22	"		"	"	
7	8000	30 000	2667	2 133	17 320	154	80 LAG	2 928.2	2 329.8	19 125	153.11	79.56	−10.42	9.23	24.29		17 313	1.403	
8	"	"	"	2 667	"	"	100	2 869.8	2 860.5	18 640	153.96	99.68	−7.62	7.26	"		"	"	
9	5000	30 000	1667	1 333	17 320	96.2	80 LAG	1 817.3	1 459.2	19 184	94.73	80.29	−10.76	9.47	40.32		17 304	2.33	
10	"	"	"	1 667	"	"	100	1 796.4	1 794.2	18 685	96.14	99.88	−7.89	7.63	"		"	"	
11	20 000	60 000	6667	5 333	34 640	192.5	80 LAG	7 303.8	5 841.0	38 490	189.36	79.97	−11.11	9.53	149.8	.13	34 607	4.33	.09
12	"	"	"	6 667	"	"	100	7 192.1	7 181.2	37 387	192.37	99.85	−7.93	7.71	"	"	"	"	"
13	22 000	88 000	7333	5 867	50 810	144.4	80 LAG	7 762.5	6 419.6	56 619	137.1	82.70	−11.43	9.42	599.3	1.94	50 620	11.84	.32
14	"	"	"	7 333	"	"	100	7 915.4	7 915.2	54 820	144.39	100.00	−7.89	7.94	"	"	"	"	"
15	40 000	120 000	13 333	10 667	69 290	192.5	80 LAG	14 106	11 648	77 147	182.85	82.58	−11.34	9.19	1081	3.39	69 030	15.66	.31
16	"	"	"	13 333	"	"	100	14 366	14 366	74 642	192.47	100.00	−7.73	7.75	"	"	"	"	"
17	25 000	120 000	8 333	6 667	69 290	120.3	80 LAG	7 886.5	7 156.1	76 754	102.75	90.74	−10.77	7.34	2185	15.29	68 253	32.01	.70
18	"	"	"	8 333	"	"	100	9 025.4	8 913.0	73 401	122.96	98.75*	−5.93	6.96	"	"	"	"	"
19	40 000	140 000	13 333	10 667	80 830	165	80 LAG	13 270	11 610	91 761	144.52	87.49	−13.52	8.84	2780	17.44	79 622	34.92	.63
20	"	"	"	13 333	"	"	100	14 459	14 412	86 863	166.46	99.68*	−7.46	8.09	"	"	"	"	"
21	20 000	120 000	6 667	5 333	69 290	96.2	80 LAG	5 683.2	5 642.9	75 682	75.08	99.29	−9.22	5.81	3428	39.22	66 950	51.21	1.14
22	"	"	"	6 667	"	"	100	7 652.7	7 105.4	71 762	106.64	92.85*	−3.57	6.57	"	"	"	"	"
23	60 000	200 000	20 000	16 000	115 500	173.2	80 LAG	17 576	17 048	128 450	136.83	96.99	−11.21	6.55	8559	91.03	111 611	76.69	1.06
24	"	"	"	20 000	"	"	100	22 287	21 381	120 574	184.84	95.98*	−4.39	6.90	"	"	"	"	"
25	20 000	140 000	6 667	5 333	80 830	82.5	80 LAG	5 959.3	5 621.1	86 404	68.97	94.33*	−6.89	5.40	5585	109.6	76 024	73.47	1.96
26	"	"	"	6 667	"	"	100	8 808.9	7 165.1	81 647	107.89	81.34*	−1.01	7.47	"	"	"	"	"
27	50 000	200 000	16 667	13 333	115 500	144.4	80 LAG	14 295	14 222	127 267	112.32	99.49*	−10.19	6.67	11 018	202.04	108 663	101.4	1.84
28	"	"	"	16 667	"	"	100	20 322	18 066	118 833	171.01	88.90*	−2.89	8.40	"	"	"	"	"
29	15 000	140 000	5000	4 000	80 830	61.86	80 LAG	6 183.5	4 237	83 045	74.46	68.52*	−2.74	5.92	6 665	208.54	73 360	90.85	3.13
30	"	"	"	5 000	"	"	100	8 518.7	5 479	78 658	108.30	64.32*	+2.69	9.58	"	"	"	"	"
31	40 000	200 000	13 333	10 667	115 500	115.5	80 LAG	13 277	11 383	123 401	107.59	85.74*	−6.85	6.71	13 140	395.8	104 878	125.3	3.01
32	"	"		13333	"	"	100	19 096	14 672	115 162	165.82	76.83*	+0.29	10.05	"	"	"	"	"
60 CYCLES																			
33	1 300	10000	433.3	346.6	5 774	75	80 LAG	499.03	377.44	6702	74.46	75.63	−16.07	8.90	4.558		5 769	.79	
34	"	"	"	433.3	"	"	100	469.05	464.18	6 259	74.94	98.96	−8.40	7.13	"		"	"	
35	5 000	20 000	1 667	1 333	11 550	144.4	80 LAG	1 911.02	1448.95	13 333	143.33	75.82	−15.44	8.70	18.23		11 540	1.58	
36	"	"	"	1 667	"	"	100	1 800.6	1 783.3	12 480	144.28	99.04	−8.05	6.98	"		"	"	
37	3 500	20 000	1 167	933	11 550	101	80 LAG	1 341.0	1 016.8	13 482	99.47	75.82	−16.73	8.98	25.93		11 527	2.25	
38	"	"	"	1 167	"	"	100	1 264.0	1 251.2	12 537	100.82	98.99	−8.55	7.22	"		"	"	
39	8 000	30 000	2 667	2 133	17 320	154	80 LAG	3 073.6	2 327.9	20 268	151.65	75.74	−17.02	9.13	58.43		17 286	3.38	
40	"	"	"	2 667	"	"	100	2 894.7	2 864.1	18 830	153.73	98.94	−8.72	7.39	"		"	"	
41	5 000	30 000	1 667	1 333	17 320	96.2	80 LAG	1 879.2	1 456.2	20 331	92.43	77.40	−17.38	9.24	96.29	.22	17 225	5.59	.22
42	"	"	"	1 667	"	"	100	1 806.1	1 794.1	18 845	95.84	99.33	−8.80	7.62	"	"	"	"	"
43	20 000	60 000	6 667	5 333	34 640	192.5	80 LAG	7 597.8	5 830.1	40 976	185.42	76.73	−18.29	9.32	357.9	.75	34 450	10.39	.21
44	"	"	"	6 667	"	"	100	7 243.0	7 180.2	37 773	191.75	99.13	−9.05	7.70	"	"	"	"	"
45	22 000	88 000	7 333	5 867	50 810	144.4	80 LAG	7 578.7	6 380.0	59 925	126.47	84.18	−17.94	8.74	1 409	8.62	49 710	28.35	.61
46	"	"	"	7 333	"	"	100	7 915.5	7 915.3	54 869	144.26	100.00	−7.99	7.94	"	"	"	"	"
47	44 000	120 000	13 333	10 667	69 290	192.5	80 LAG	13 796	11 579	81 710	168.84	83.93	−17.92	8.55	2 528	14.49	67 800	37.28	.57
48	"	"	"	13 333	"	"	100	14 366	14 365	74 735	192.22	100.00	−7.86	7.74	"	"	"	"	"
49	25 000	120 000	8 333	6 667	69 290	120.3	80 LAG	7 082.3	7 075.1	79 000	89.65	99.89	−14.01	6.12	4 759	75.47	63 357	75.11	1.59
50	"	"	"	8 333	"	"	100	9 473.0	8 949.6	70 599	134.18	94.47*	−1.89	7.40	"	"	"	"	"
51	40 000	140 000	13 333	10 667	80 830	165	80 LAG	11 827	11 461	96 727	122.27	96.90	−19.67	7.44	6 060	89.78	73 938	81.96	1.48
52	"	"	"	13 333	"	"	100	14 666	14 438	84 862	172.82	98.44	−4.99	8.29	"	"	"	"	"
53	20 000	120 000	6 667	5 333	69 290	96.2	80 LAG	6 972.8	5 626.6	72 747	95.85	80.69*	−4.99	5.50	6 523	208.8	56 101	116.27	3.20
54	"	"	"	6 667	"	"	100	9 061.7	7 239.4	63 810	142.01	79.89*	+7.91	8.59	"	"	"	"	"
55	60 000	200 000	20 000	16 000	115 500	173.2	80 LAG	18 728	16 908	126 541	148.00	90.28*	−9.56	5.68	16 330	476.4	93 636	174.4	2.92
56	"	"	"	20 000	"	"	100	24 796	21 658	109 189	227.09	87.34*	+5.47	8.29	"	"	"	"	"
57	20 000	140 000	6 667	5 333	80 830	82.5	80 LAG	10 089	5 796.2	74 182	136.01	57.45*	+8.22	8.69	8 626	545.8	54 494	158.3	6.33
58	"	"	"	6 667	"	"	100	11 014	7 539.6	64 377	171.08	68.45*	+20.35	13.09	"	"	"	"	"
59	50 000	200 000	16 667	13 333	115 500	144.4	80 LAG	21 139	14 343	113 606	186.07	67.85*	+1.64	7.58	17 217	1 057	77 939	220.9	6.14
60	"	"	"	16 667	"	"	100	23 946	18 757	96 987	246.84	78.35*	+16.03	12.54	"	"	"	"	"
61	15 000	140 000	5 000	4 000	80 830	61.86	80 LAG	10 233	4 801.2	59 046	173.30	46.92*	+26.95	20.03	7 690	998.8	41 213	186.6	12.99
62	"	"	"	5 000	"	"	100	9 918.4	6 230.9	51 327	193.24	62.81*	+36.50	24.60	"	"	"	"	"
63	40 000	200 000	13 333	10 667	115 500	115.5	80 LAG	21 948	12 248	93 725	234.17	55.80	+18.81	14.82	15 223	1 907	59 074	257.7	12.52
64	"	"	"	13 333	"	"	100	21 750	16 020	80 106	271.52	73.66	+30.64	20.15	"	"	"	"	"

The above performances are based upon values for the auxiliary constants as given on Chart XII.

CHAPTER X

HYPERBOLIC FUNCTIONS

In the consideration of the hyperbolic theory as applied to transmission circuits, the writer desires to express his high appreciation of the excellent literature already existing. Dr. A. E. Kennelly's pioneer work and advocacy of the application of hyperbolic functions to the solution of transmission circuits has been too extensive and well known to warrant a complete list of his contributions. His most important treatises are "Hyperbolic Functions Applied to Electrical Engineering", 1916; "Tables of Complex Hyperbolic and Circular Functions", 1914; "Chart Atlas of Hyperbolic Functions", 1914, which provides a ready means of obtaining values for complex functions, thus materially shortening and simplifying calculations, and "Artificial Electric Lines", 1917.

"Electrical Phenomena in Parallel Conductors" by Dr. Frederick Eugene Pernot, 1918, is an excellent treatise on the subject and contains valuable tables of logarithms of real hyperbolic functions from $x = 0$ to $x = 2.00$ in steps of 0.001.

An article "Long-Line Phenomena and Vector Locus Diagrams" in the *Electrical World* of Feb. 1, 1919, p. 212, by Prof. Edy Velander is an excellent and valuable contribution on the subject, because of its simplicity in explaining complicated phenomena.

To employ hyperbolic functions successfully in the solution of transmission circuits it is not necessary for the worker to have a thorough understanding of how they have been derived. On the other hand it is quite desirable to understand the basis upon which they have been computed. A brief review of hyperbolic trigonometry is therefore given before taking up the solution of circuits.

CIRCULAR angles derive their name from the fact that they are functions of the circle, whose equation is $x^2 + y^2 = 1$. Tabulated values of such functions are based upon a radius of unit length. The geometrical construction illustrating three of the functions, the sine, cosine and tangent of circular angles is indicated in Fig. 38. The angle *AOP*, indicated by full lines in the positive or counter-clockwise direction, has been drawn to correspond to one radian. The radian is an angular unit of such magnitude that the length of the arc which subtends the radian is numerically equal to that of the radius of the circle. Thus, the number of radians in a complete circle is 2π. Expressed in degrees the radian is equal approximately to 57° 17′ 44.8″. The segment *AOP* of any angle *AOP* of one radian has an area equal to one-half the area of a unit square. Therefore the angle may be expressed in radians as,—

$$\frac{Length\ of\ arc}{radius} \quad or \quad \frac{2 \times area}{(radius)^2}$$

Circular functions are obtained as follows,—

$$Circular\ angle = \frac{2 \times area}{(radius)^2}\ radians$$

$$Sine\ \theta = \frac{Y}{R}$$

$$Cosine\ \theta = \frac{X}{R}$$

$$Tangent\ \theta = \frac{Y}{X}$$

The variations in the circular functions, sine, cosine and tangent are indicated graphically in Fig. 39 for a complete revolution of 360 degrees. Since for the second and each succeeding revolution these graphs would simply be repeated, circular functions are said to have a period equal to 2π radians. In other words, adding 2π to a circular angle expressed in radians does not change the value of a circular function.

REAL HYPERBOLIC ANGLES

Real hyperbolic angles derive their name because they are functions of an equilateral hyperbola. A hyperbola is a plane curve, such that the difference between the distances from any point on the curve to two fixed points called the foci is constant. In an equilateral hyperbola, Fig. 40, the asymptotes *OS* and *OS′* are straight lines at right angles to each other and make equal angles with the X-axis. The hyperbola continually approaches the asymptotes, and meets them at infinity. The equation of such a hyperbola is $x^2 - y^2 = 1$.

The hyperbolic angle *AOP* of Fig. 40, called for convenience θ*, has been drawn so as to correspond to an angle of one hyperbolic radian, or one "hyp" as it is usually designated. Hyperbolic angles are determined by the area of the sector they enclose. Thus the hyperbolic angle of one hyp *AOF*, encloses an area *AOP* of one-half, or the same as the area *AOP* of the corresponding circular angle of Fig. 38. It should be observed here that although one circular radian subtends an angle *AOP* of 57° 17′ 44.8″, one hyperbolic radian subtends a circular angle *AOP* of 37° 17′ 33.67″ (0.65087 circular radian).

In the same way as for the circle the hyperbolic angle may be expressed in radians as,—

$$\frac{Length\ of\ arc}{\rho} \quad or \quad \frac{2 \times area}{(radius)^2}$$

where ρ = the integrated mean radius from *O* to *AP*. As an illustration, the length of the arc *AP*, Fig. 40

*A "hyperbolic angle", in the sense above described, is not the opening between two lines intersecting in a plane, but a quantity otherwise analogous to a circular angle and the argument x of the function *sinh x, cosh x, tanh x*, etc. The use of the term hyperbolic angle can only be justified by its convenience of anology.

is 1.3167 and the mean integrated radius to arc AP is 1.3167.

Hyperbolic functions, distinguished from circular functions by the letter h affixed, are obtained as follows:—

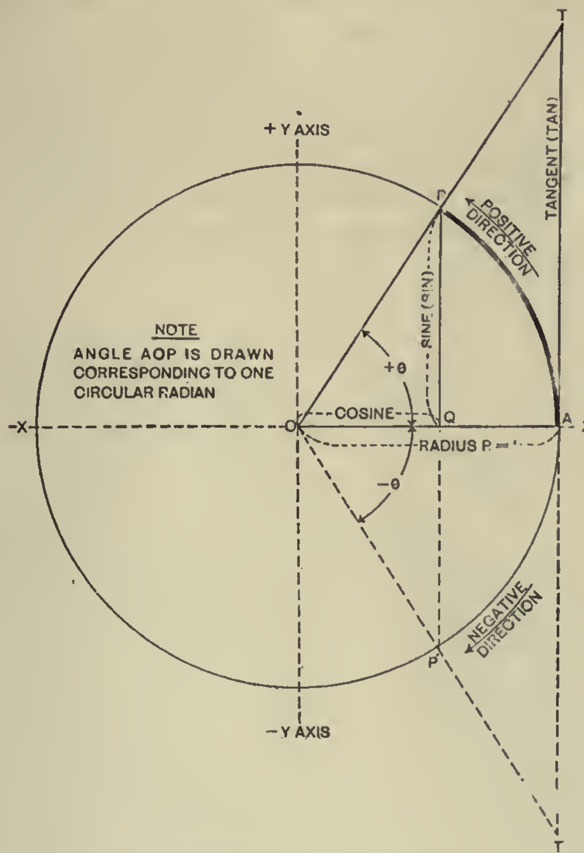


FIG. 38—REAL CIRCULAR ANGLES
$X^2 + Y^2 = 1$

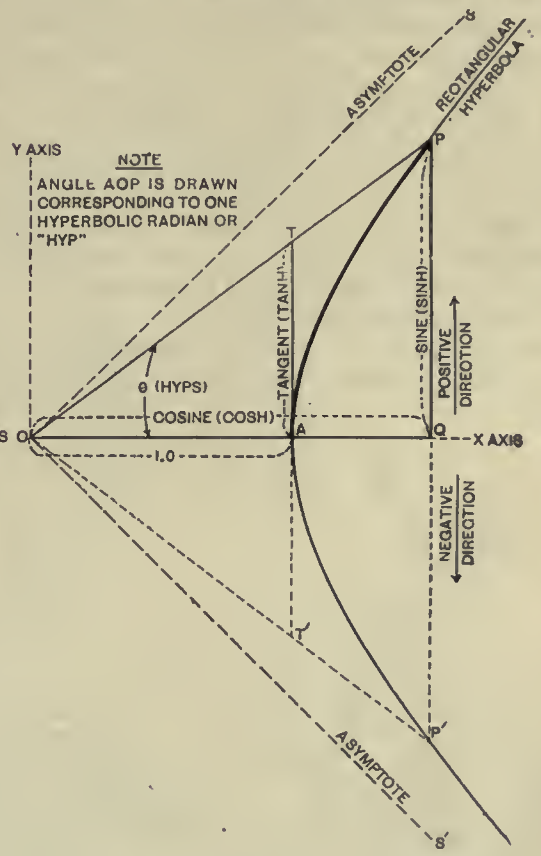


FIG. 40—REAL HYPERBOLIC ANGLES
$X^2 - Y^2 = 1$

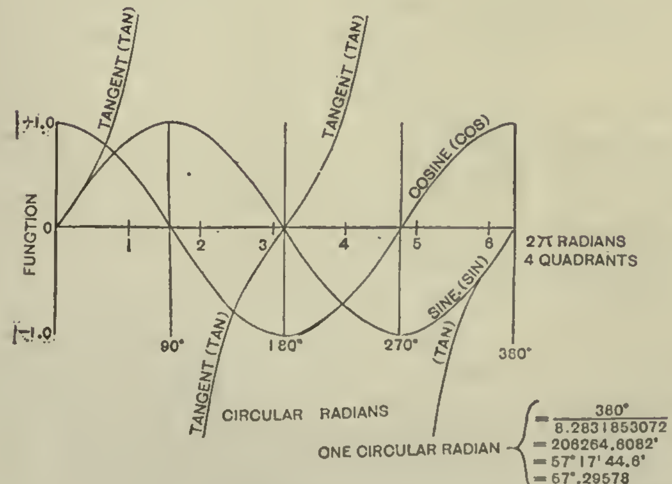


FIG. 39—GRAPHS OF CIRCULAR FUNCTIONS

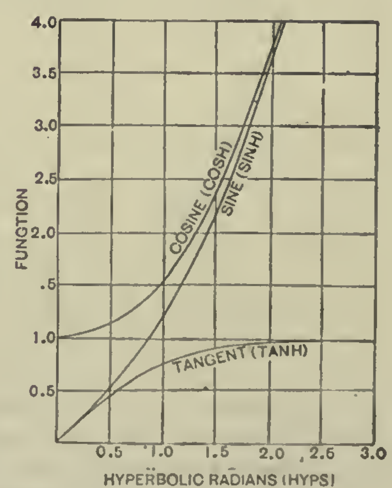


FIG. 41—GRAPHS OF HYPERBOLIC FUNCTIONS

$$\text{Hyperbolic angle } \theta = \frac{\text{Length of arc } AP}{\text{Length of mean radius}} \text{ radians.}$$

$$\text{Cosh } \theta = \frac{X}{OA}$$

$$\text{Sinh } \theta = \frac{Y}{OA}$$

$$\text{Tanh } \theta = \frac{Y}{X}$$

The variations in hyperbolic functions are indicated graphically in Fig. 41 for hyperbolic angles up to approximately 2.0 hyps for the sine and cosine and up to 3.0 hyps for the tangent.

Hyperbolic functions have no true period, but adding a $2\pi j$ to the hyperbolic angle does not change the values of the functions, hence these functions have an imaginary period of $2\pi j$.

Circular functions can be used to express the phase relations of current and voltage, but not the magnitude, or size, whereas hyperbolic functions, continually increasing or decreasing, can be used to express the magnitude of current in a long circuit.

In Fig. 42 is shown a circular angle corresponding to one circular radian divided into five equal parts, each of 0.2 radian. Assuming unity radius, each of the arcs will have a constant length of 0.2 and a constant mean radius of 1.0. In Fig. 42 is shown a hyperbolic angle corresponding to one hyperbolic radian divided into five equal hyperbolic angles each of 0.2 hyperbolic radian. In this case the length of the arcs corresponding to each subdivision increases as the hyperbolic angle increases. The lengths of the corresponding integrated mean radii vectors also increase with the angle. By dividing the length of the arc of any of the five subdivisions by the length of the mean radius for that subdivision it will be seen that each subdivision represents 0.2 hyps.

From the above it will be evident that in radian measure, the magnitudes of circular and hyperbolic

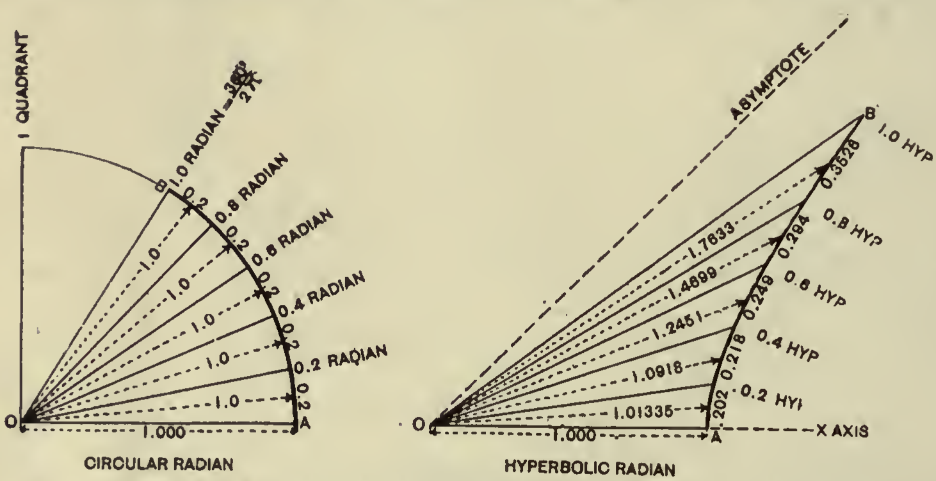


FIG. 42—SUBDIVISION OF A CIRCULAR AND A HYPERBOLIC RADIAN INTO FIVE SECTORS OF 0.2 RADIAN EACH

angles are similarly defined with reference to the area of circular and hyperbolic sectors.

COMPLEX ANGLES AND THEIR FUNCTIONS

A complex angle is one which is associated with both a hyperbolic and a circular sector. If the complex angle is hyperbolic, its real part relates to a hyperbolic and its imaginary to a circular sector. On the other hand, if the complex angle is circular, its real part relates to a circular and its imaginary part to a hyperbolic sector. Complex hyperbolic trigonometry and complex circular trigonometry thus unite in a common geometrical relationship.

In the following treatment for the solution of transmission circuits by hyperbolic functions, only hyperbolic complex angles will enter into the solution. Such a complex angle will then consist of a combination of a "real" hyperbolic sector and a so-called "imaginary" or circular sector. The circular sector will occupy a plane inclined at an angle to the plane of the hyperbolic sector. In other words, the complex angle will be of the three-dimensional order. The construction of such a complex angle may be difficult to follow if viewed only from one direction. In order to illustrate the form that a complex angle takes, the construction for the cosine of a hyperbolic complex angle is illustrated by Fig. 43.

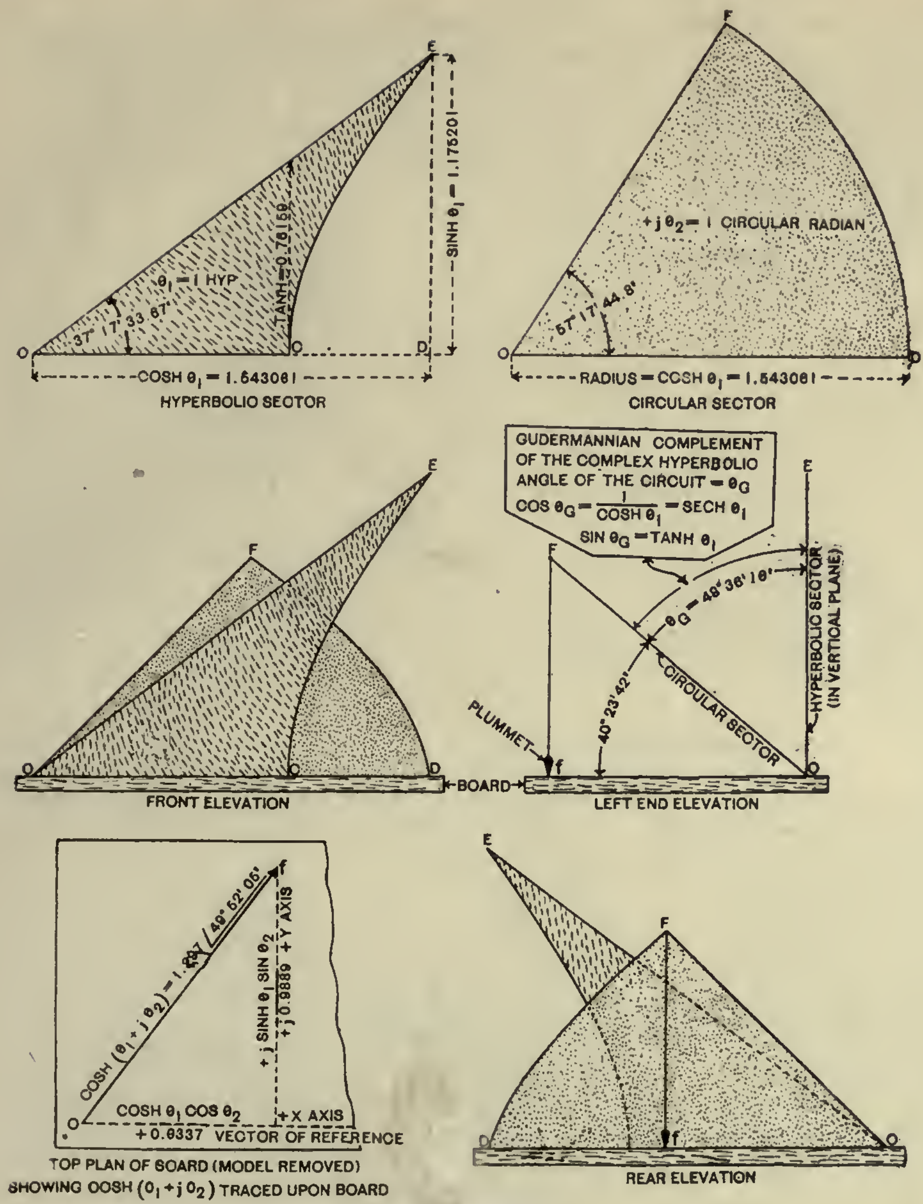
CONSTRUCTION FOR COSH θ

The construction, Fig. 43, assumes that the real part, that is the hyperbolic sector subtends an angle of one hyperbolic radian and the imaginary part, that is the circular sector, subtends an angle of one circular radian. This hyperbolic complex angle has therefore a numerical value of $1 + j\,1$ hyperbolic radian. These numerical values embrace sectors sufficiently large for the purpose of clear illustration. The actual construction for obtaining the complex function cosh $(\theta_1 + j\,\theta_2)$ = cosh ($1 + j\,1$ hyperbolic radians) may be carried out as follows:—

On a piece of stiff card board lay out to a suitable scale the hyperbolic sector $\theta_1 = EOC$, equal to one hyp as shown in the upper left hand corner of Fig. 43. This may readily be plotted by the aid of a table of real hyperbolic functions for say each one tenth of a hyp up to and including one hyp. These are then plotted on the cardboard and joined with a curved line thus forming the arc *EC* of Fig. 43. The ends of the arc are then joined with *O* by straight lines. The real part of this hyperbolic complex angle is then cut out of the cardboard.

The circular part $j\,\theta_2$ of this complex angle is traced upon the cardboard as follows:—With radius equal to cosh θ_1 (to the same scale as used when tracing the hyperbolic sector θ_1) draw the arc *DOF* of a length such that the angle *DOF* is 57° 17′ 44.8″ (one circular radian). Join the ends of the arc to *O* with straight lines. The circular part $j\,\theta_2$ of this complex angle is now cut out of the piece of cardboard. This gives models of the two parts of the complex angle which may be arranged to form the complex angle $1 + j\,1$ hyps. These two models are shown at the top of Fig. 43.

The two parts of the complex angle are arranged as follows:—Upon a drawing board or any flat surface occupying a horizontal plane, place the hyperbolic sector θ_1 in a vertical position. The plane of this hyperbolic sector will then be at right angles to the plane of the drawing board. The circular sector $j\,\theta_2$ is now placed in a vertical position just back of the hyperbolic sector. The toes *O* of each sector will then coincide, as well as the line *OD* of the circular sector with the line *OC* of the hyperbolic sector. The top of the circular sector is now turned back so that the plane of the circular sector lies at an angle with the vertical plane occupied by the hyperbolic sector. This displacement angle between the planes of the two sectors is



MATHEMATICAL SOLUTION

$\cosh(\theta_1 + j\theta_2) = (\cosh\theta_1 \cos\theta_2 + j \sinh\theta_1 \sin\theta_2)$

LOG COSH θ_1 = 0.188,386	LOG SINH θ_1 = 0.070,112
LOG COS θ_2 = $\bar{1}$.732,636	LOG SIN θ_2 = $\bar{1}$.925,039
$\bar{1}$.921,025	$\bar{1}$.995,151

$\cosh(\theta_1 + j\theta_2) = 0.8337 + j\,0.9889$

$= 1.297 \,/49° 52' 05''$

FIG. 43—GRAPHICAL CONSTRUCTION FOR THE HYPERBOLIC COSINE OF THE COMPLEX ANGLE $\theta_1 + j\theta_2 = 1 + j1$ Hyperbolic Radians.

known as the "gudermannian complement" of the hyperbolic angle θ. It will be referred to as θ_G. The front elevation of Fig. 43 illustrates how these two sectors would appear when viewed from the front. To the right of this illustration is shown how these two sectors would appear when viewed from the left hand end of the model. The displacement angle θ_G has a value for this particular complex angle of 49° 36′ 18″. This numerical value is determined by virtue of the fact that this displacement angle has a cosine of

$\frac{1}{\cosh\theta_1} = \frac{1}{1.543081} = 0.64805$ or cosine of $\theta_g = \text{sech}\,\theta_1 = 0.64805$. It has a sine of $\tanh\theta_1 = 0.76159$.

The angle whose cosine is 0.64805 and whose sine is 0.76159 is 49° 36′ 18″. Thus the top part of the circular sector of this complex angle is moved in the forward direction through an angle of 49° 36′ 18″ so that the plane of the circular sector assumes an angle of 90° 00′ 00″—49° 36′ 18″ = 40° 23′ 42″ with the horizontal plane of the drawing board. From the end of the circular sector (point F) thus inclined, a plummet may be suspended until it meets the horizontal plane of the drawing board at the point f of the illustration. In other words, the point F is projected orthogonally onto the horizontal plane of the drawing board.

A top view of the drawing board, with the model removed, is illustrated in the lower left hand corner of Fig. 43. The line OF (1.297 /49° 52′ 05″) traced upon the horizontal drawing board, is a vector representing the complex cosine of the complex angle $\theta_1 + j\,\theta_2 = 1 + j\,1$ hyperbolic radians. This complex cosine has rectangular coordinates of + 0.8337 and + j 0.9889.

At the bottom of Fig. 43 is given the mathematical expression for the exact solution for the cosine of a complex hyperbolic angle following the construction illustrated. There are numerous other mathematical equations with their equivalent geometrical constructions which will produce the same values for the cosine, but the above is probably as easy to follow as any, and will therefore be used exclusively hereafter.

CONSTRUCTION FOR SINH θ

The construction for the sine of the complex hyperbolic angle $1 + j\,1$ is indicated in Fig. 44. In this case the same construction may be used for obtaining the sinh as for determining the cosh of the complex angle with the following two exceptions.

The circular sector is made one quadrant (90°) larger. In other words the angle DOF' is 90° + 57° 17′ 44.8″ or 147° 17′ 44.8″ as indicated by Fig. 44. It occupies the same plane as when determining the cosh of the angle but is simply extended in the forward direction through one quadrant, as indicated by the dotted lines of Fig. 44. The plummet is again suspended, this time from point F' upon the horizontal board, which it meets at point f'. The other difference is that the sine OF' is read off from the Y axis as the vector of reference in place of the X axis as in the case of the cosine. Thus the circular sector has been carried forward through an angle of 90 degrees in the circular angle plane and the vector of reference has been advanced 90 degrees in the horizontal plane of reference. The sine of this angle is $1.446 \angle 63° 56' 37''$ and has rectangular components of $0.6349 + j1.2985$. The mathematical

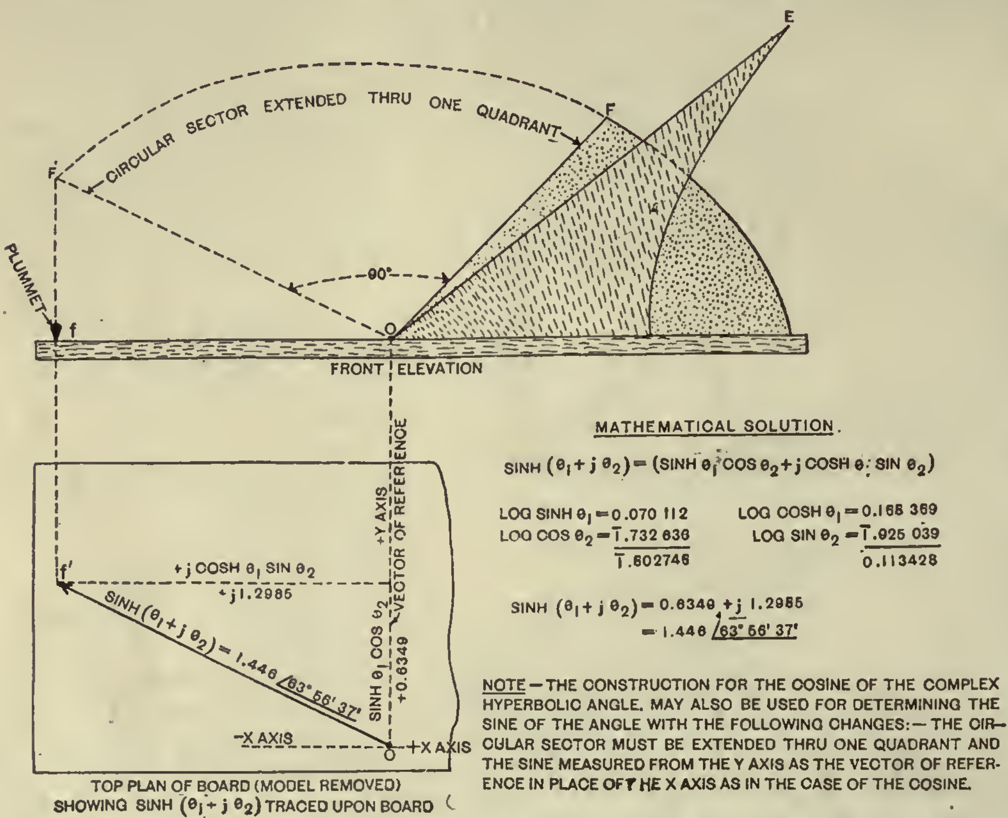


FIG. 44—GRAPHICAL CONSTRUCTION FOR THE HYPERBOLIC SINE OF THE COMPLEX ANGLE $\theta_1 + j\theta_2 = 1 + j1$ hyperbolic radians.

expression for exact solution for the sine of a complex angle likewise accompanies the illustrated geometrical construction.

MODEL FOR ILLUSTRATING THE FUNCTIONS OF A COMPLEX ANGLE

Dr. Kennelly has recently constructed a model* for illustrating complex angles and for obtaining approximate values for the functions of such angles. Drawings made from photographs of this model are shown in Figs. 45, 46 and 47. The construction of a complex angle as above described is that employed by Dr. Kennelly in building his model. Since the model is applicable to tracing out numerous complex angles, it may seem a little difficult at the start. It was therefore thought desirable to precede the description of the model which is applicable to the solution of so many angles with a similar solution of a single definite complex angle. With the procedure for the solution, as given above, for cosh and sinh of $1 + j\,1$ hyperbolic radians in mind, it is believed that Dr. Kennelly's description of the model and its application in determining the cosh and sinh of complex angles may be followed as given in the following paragraphs.

DESCRIPTION OF MODEL

In this model, the cosine or sine of a complex angle, either hyperbolic or circular, can be produced, by two successive orthogonal projections onto the XY plane, one projection being made from a rectangular hyperbola, and the other projection being then made from a particular circle definitely selected from among a theoretically infinite number of such circles, all concentric at the origin O, which circles, however, are not coplanar. The selection of the particular circle is determined by the foot of the projection from the hyperbola. This effects a geometrical process which is easily apprehended and visualized; so that once it has been realized by the student, the three-dimensional artifice is rendered superfluous, and he can roughly trace out a complex sine or cosine on an imaginary drawing board, with his eyes closed. The model, however, possesses certain interesting geometrical properties as a three-dimensional structure.

A drawing made from a photograph of the model is shown in Fig. 45. On an ordinary horizontal drawing board 53.5 by 31.8 cm., is a horizontal rod AB, which merely serves to support the various brass-wire semicircles, and a semihyperbola, in their proper positions. The axis of AB in the XY plane, on the upper surface of the board, is a line of symmetry for the structure, which, if completed, would be formed by full circles and a complete hyperbola. For convenience, however, only the half of the structure above the XY plane is presented, the omission of the lower half being readily compensated for in the imagination.

The eight wire semicircles are formed with the following respective radii, in decimeters: 1.0, 1.020..., 1.081..., 1.185..., 1.337..., 1.543..., 1.810..., and 2.150..., which are the respective cosines of 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, and 1.4 hyperbolic radians, according to ordinary tables of real hyperbolic functions. These successive semi-circles therefore have radii equal to the cosines of successively increasing real hyperbolic angles θ_1, by steps of 0.2, from 0 to 1.4 hyperbolic radians, inclusive. All of these semicircles have their common center at the origin O, in the plane $X\ O\ Y$, of the drawing board. The planes of the semicircles are, however, displaced. The smallest circle of unit radius (1 decimeter), occupies the vertical plane $X\ O\ Z$,

FIG. 45—DRAWING FROM A PHOTOGRAPH OF A GEOMETRICAL MODEL
For the orthogonal projection of the sines and cosines of complex angles. This model was developed by A. E. Kennelly.

*This model was described in a paper read by him at a meeting of the American Academy of Arts and Sciences in April 1919.

in which also lies the rectangular semi-hyperbola $X\ O\ H$. Angular distances corresponding to 0.2, 0.4, ... 1.4 hyperbolic radians, are marked off along this hyperbola at successive corresponding intervals of 0.2. The cosines of these angles, as obtainable projectively on the $O\ X$ axis are marked off between C and B along the brass supporting bar, and at each mark, a semicircle rises from the $X\ Y$ plane, at a certain angle θ_G with the vertical $X\ O\ Z$ plane. This displacement angle is determined by the relation,—

$$\cos \theta_G = \frac{1}{\cosh \theta_1} = \operatorname{sech} \theta_1$$

Where θ_1 is the particular hyperbolic angle selected. This means, as is well known, that the displacement angle θ_G between the plane of any semicircle and the vertical plane $Z\ O\ X$ is equal to the gudermannian of the hyperbolic angle θ_1.

The model is, of course, only a skeleton structure of eight stages. If it could be completely developed, the number of semicircles would become infinite, and they would form a smooth continuous surface in three dimensions. Along the midplane $Z\ O\ Y$, all of these circles would have the same level, raised one decimeter above the horizontal drawing board plane of reference $X\ O\ Y$. The circles would increase in radius without limit, and would cover the entire $X\ O\ Y$ plane to infinity, the hyperbola extending likewise to infinity towards its asymptote $O\ S$, in the $X\ O\ Z$ plane. The actual model is thus the skeleton of the upper central sheet of the entire theoretical surface, near the origin.

The semicircles are also marked off in uniform steps of circular angle. Each step is taken, for convenience, as nine degrees, or one tenth of a quadrant. Corresponding angular steps on all of the eight semicircles are connected by thin wires, as shown in the illustrations.

A front elevation of the model, taken from a point on the $O\ Y$ axis—15 units from O, is given in Fig. 46. It will be seen that any tie wire, connecting corresponding circular angular

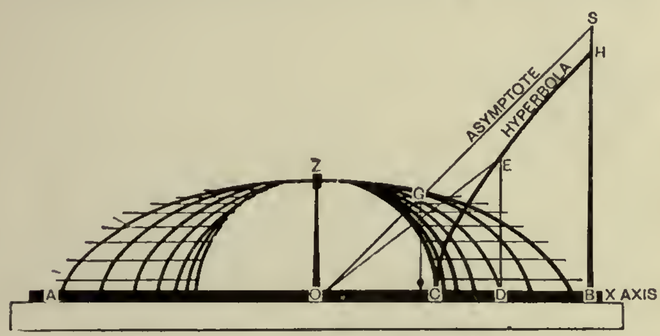


FIG. 46—FRONT ELEVATION OF MODEL
From a point on the $O\ Y$ axis, — 15 units from O.

points on the semicircles, is level, and lies at a constant height $\sin \theta_2$ decimeters above the drawing board. That is, the tie wire that connects all points of circular angle θ_2, measured from $O\ X$ positively towards OY, lies at the uniform height $\sin \theta_2$ decimeters above the drawing board.

A plan view of the model, taken from a point on the $O\ Z$ axis, + 15 units above O, is given in Fig. 47. It will be seen that each semicircle forms an ellipse, when projected on the base plane $X\ O\ Y$. The semi-major axis of this ellipse has length $\cosh \theta_1$, where θ_1 is the hyperbolic angle corresponding to that semicircle. The semi-minor axis is,—

$$\cosh \theta_1 \sin \theta_G = \cosh \theta_1 \tanh \theta_1 = \sinh \theta_1$$

from the well known relation that exists between a hyperbolic angle and its gudermannian circular angle; namely,—

$$\sin \theta_G = \tanh \theta_1$$

All of these ellipses have the same center of reference O. Any such system, having semi-major axes $\cosh \theta_1$, and semi-minor axes $\sinh \theta_1$, are well known to be confocal, and the foci must lie at the points +1 and —1 in the $X\ O\ Y$ plane, or the points in which the innermost circle cuts that plane.

PROCEDURE FOR PROJECTING COSH $(\pm \theta_1 \pm j\theta_2)$

Thus premised, the process of finding the cosine of a complex hyperbolic angle $\theta_1 + j\theta_2$; that is, the process of finding $\cosh(\theta_1 + j\theta_2)$ is as follows:

Find the arc $C\ E$, Fig. 45, from $C = +1$ along the rectangular hyperbola $C\ E\ H$, which subtends θ_1 radians. The hyperbolic sector comprised between the radius, $O\ C$, the hyperbolic arc, and the radius vector $O\ E$, on this arc from the origin O, will then include $\frac{\theta_1}{2}$ sq. dm. of area. Drop a vertical perpendicular from E onto $O\ X$. It will mark off a horizontal distance $O\ D$ equal to $\cosh \theta_1$. Proceed along the circle which rises at D, in a positive or counterclockwise direction, through θ_2 circular radians, thus reaching on that circle a point G whose elevation above the drawing board is $\sin \theta_2$ decimeters. The area enclosed by a radius vector from the origin O on the circle, followed between the axis $O\ C$ and the circular curve, will be $\frac{\theta_2}{2} \cosh^2\theta_1$ sq. dms.

From G, drop a vertical plummet, as in Fig. 46, on to the drawing board. In other words, project G orthogonally on the plane $X\ O\ Y$. Let g be the point on the drawing board at which the plummet from G touches the surface. Then it is easily seen that Og on the drawing board is the required magnitude and direction of $\cosh(\theta_1 + j\theta_2)$, in decimeters, with reference to $O\ X$ as the initial line in the plane $X\ O\ Y$. It may be read off either in rectangular coordinates along axes $O\ X$ and $O\ Y$ on a tracing cloth surface as shown in Fig. 47, or in polar coordinates printed on a sheet seen through the tracing cloth.

If the circular angle θ_2, i. e., the imaginary hyperbolic angle $j\theta_2$, lies between π and 2π radians, (in quadrants 3 and 4), the point G will lie on the under side of the plane $X\ O\ Y$, and the projection onto g in that plane must be made upwards, instead of downwards.

If the hyperbolic angle whose cosine is required has a negative imaginary component, according to the expression $\cosh(\theta_1 - j\theta_2)$, then starting from the projected point D, we must trace out the circular angle in the negative or clockwise direction, as viewed from the front of the model.

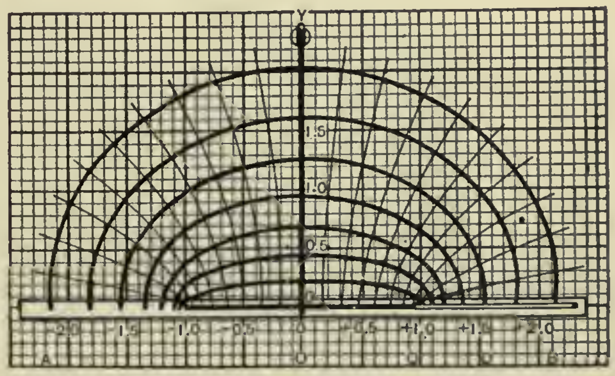


FIG. 47—PLAN VIEW OF MODEL
From a point on the $O\ Z$ axis, 15 units from O.

If the real part of the hyperbolic angle is negative, according to the expression $\cosh(-\theta_1 \pm j\theta_2)$; then since $\cosh -(\theta_1 \mp j\theta_2) = \cosh(\theta_1 \mp j\theta_2)$, we proceed as in the case of a positive real component, but with a change in the sign of the imaginary component.

The operation of tracing $\cosh(\pm \theta_1 \pm j\theta_2)$ on the $X\ Y$ plane, thus calls for two successive orthogonal projections onto that plane; namely (1) the projection corresponding to $\cosh(\pm \theta_1)$ as though $j\theta_2$ did not exist, and then (2), the projection corresponding to $\cosh j\theta_2 = \cos \theta_2$ independently of θ_1, except that the radius of the circle, and its plane, are both conditioned by the magnitude of θ_1.

If we trace the locus of $\cosh(\theta_1 \pm j\theta_2)$, where θ_1 is held constant, it is evident from Fig. 47 that we shall remain on one circle, which projects into the same corresponding ellipse on the $X\ Y$ plane. That is, the locus of $\cosh(\theta_1 \pm j\theta_2)$ with θ_1 held constant, is an ellipse, whose semi major and minor diameters are $\cosh \theta_1$ and $\sinh \theta_1$ respectively. If, on the other hand, we trace $\cosh(\pm \theta_1 + j\theta_2)$ with θ_2 held constant, we shall run over a certain tie wire bridging all the circles in the model, which tie wire is $\sin \theta_2$ dm. above the board, and its projection on the board, in the plane $X\ Y$ of projection, is part of a hyperbola.

PROCEDURE FOR SINH $(\theta_1 + j\theta_2)$

It would be readily possible to produce a modification of this model here described, which would enable the sine of a complex angle to be projected on the $X\ Y$ plane following constructions already referred to. The transition to a new model for sines is, however, unnecessary. It suffices to use the cosine model here described in a slightly different way. One has only to recall that

$$\sinh \theta = -j \cosh \left(\theta + j\frac{\pi}{2}\right)$$

or

$$\sinh (\theta_1 + j\theta_2) = -j \cosh \left[\theta_1 + j\left(\theta_2 + \frac{\pi}{2}\right)\right]$$

Consequently, in order to find the sine of a complex hyperbolic angle, we proceed on the model as though we sought the cosine of the same angle, increased by $\frac{\pi}{2}$ radians or one quadrant, in the imaginary or circular component. We then operate with $-j$ on the plane vector so obtained; i. e., we rotate it through one quadrant in the $X\ Y$ plane and in the clockwise direction. An equivalent step is, however, to rotate the X and Y axes of reference in that plane through one quadrant in the reverse or positive direction. That is, we may omit the $-j$ operation, if, in dealing with sine projections, we treat $O\ Y$ as an $O\ X$ axis, and $-O\ X$ as an $O\ Y$ axis, or read off the projections on the $X\ Y$ plane to the $-Y\ O\ Y$ axis as initial line.

The only difference, therefore, between projecting the cosine and the sine of a complex hyperbolic angle in the model, is that in the latter case the circular component is increased by one quadrant and the projected plane vector is read off to the $O\ Y$ reference axis as initial line. The model thus gives the projection of either cosh $(\pm\theta_1 \pm j\theta_2)$ or sinh $(\pm\theta_1 \pm j\theta_2)$ within the limits of $+1.4$ and -1.4 for θ_1, and for θ_2 between the limits $+\infty$ and $-\infty$. For accurate numerical work, reference would, of course, be made to the charts and tables of such functions already published, and which enable such functions to be obtained either directly or by interpolation, for all ordinary values of θ_1 and θ_2.

CHAPTER XI

PERFORMANCE OF LONG TRANSMISSION LINES

(RIGOROUS SOLUTION BY HYPERBOLIC FUNCTIONS)

AS STATED in the discussion of the convergent series solution, the performance of an electric circuit is completely determined by its physical characteristics;—resistance, reactance, conductance and capacitance and the impressed frequency. These five quantities are accurately and fully accounted for in the two complex quantities.

Impedance $Z = R + jX$
Admittance $Y = G + jB$

Having determined the numerical values for these two complex quantities, no further consideration need be given to the physical quantities of the circuit or to the frequency.

In the hyperbolic theory the circuit is said to subtend a certain complex angle, $\theta = \sqrt{ZY}$. This quantity represents in a sense the electrical length of the circuit. The numerical value of this angle θ is expressed in hyperbolic radians. If the circuit is very long electrically the numerical value of the angle will be comparatively large. Conversely, if the circuit is electrically short, it will be comparatively small. The numerical value of the angle θ is, therefore, a measure of the electrical length of the circuit and an indication of how much distortion in the distribution of voltage and current is to be expected as an effect of the capacitance and leakance of the circuit.

In order to give an idea of the extent of the variation in the complex θ and its functions cosh θ and sinh θ for power transmission circuits of various lengths corresponding to 25 and 60 cycle frequencies approximate values have been calculated, as shown in Table O.

This tabulation indicates that for circuits of from 100 to 500 miles in length, operated at frequencies of 25 and 60 cycles, the complex hyperbolic angle of the circuit (which is a plane-vector quantity) has a maximum modulus, or size of 0.41 for 25 cycles and of 1.05 for 60 cycles. It has an argument, or slope, lying between 70 and 78 degrees for 25 cycles and between 80 and 85 degrees for 60 cycles.

In the convergent series solution, the three so-called auxiliary constants *A*, *B* and *C* determine the performance of the circuit. These three auxiliary constants are simply expressions for certain hyperbolic functions of the complex hyperbolic angle θ of the circuit.

Thus

$$A = \cosh \theta$$

$$B = \sinh \theta \sqrt{\frac{Z}{Y}} = Z \frac{\sinh \theta}{\theta} = Z'$$

$$C = \sinh \theta \frac{1}{\sqrt{\frac{Z}{Y}}} = Y \frac{\sinh \theta}{\theta}$$

ADDITIONAL SYMBOLS

In addition to the symbols previously listed, the following will be employed in the hyperbolic treatment.

$\alpha =$ Linear hyperbolic angle expressed in hyps per mile. It is a complex quantity consisting of a real component α_1 and an imaginary component α_2. It is also known as the attenuation constant or the propagation constant of the circuit.

$\alpha_1 =$ The real component of the linear hyperbolic angle α, expressed in hyps. It is a measure of the shrinkage or loss in amplitude of the traveling wave, per unit length of line traversed.

$\alpha_2 =$ The imaginary component of the linear hyperbolic angle α, expressed in circular radians. It is a measure of the loss in phase angle of the traveling wave, per unit length of line traversed.

$\theta =$ The complex hyperbolic angle subtended by the entire circuit, expressed in hyps. It differs from α in that it embraces the entire circuit, whereas α embraces unit length of circuit (in this case one mile), $\theta = \alpha \times L$, where L is the length of the circuit expressed in miles.

$\theta_1 =$ The real component of the complex hyperbolic angle of the circuit expressed in hyps, and defines the shrinkage or loss in amplitude or size of a traveling wave, in traversing the whole length of the line.

$\theta_2 =$ The imaginary component of the complex hyperbolic angle of the circuit expressed in circular radians, expressing the loss in phase angle or slope of the traveling wave, in traversing the whole length of line.

$\epsilon =$ 2.7182818 which is the base of the Napierian system of logarithms. $\text{Log}_{10} \epsilon = 0.4342945$.

$\theta_s =$ Position angle at sending end.

$\theta_r =$ Position angle at receiving end.

$\theta_p =$ Position angle at point P on a circuit.

$\delta =$ Impedance load to ground or zero potential at receiving end line, in ohms at an angle.

$z_0 = \sqrt{\frac{z}{y}} =$ Surge impedance of a conductor in ohms at an angle.

$y_0 = \frac{1}{z_0} =$ Surge admittance of a conductor in mhos at an angle.

TABLE O—GENERAL EFFECT OF DISTANCE AND FREQUENCY UPON THE COMPLEX HYPERBOLIC ANGLE AND ITS FUNCTIONS

LENGTH OF CIRCUIT (MILES)	Z	Y	ZY	$\theta = \sqrt{ZY}$	COSH θ	SINH θ
25 CYCLES						
100	$43.3 \angle 50^\circ$	$0.000\,230 \angle 90^\circ$	$0.00996 \angle 140^\circ$	$0.10 \angle 70^\circ$	$0.99 \angle 0.2^\circ$	$0.10 \angle 70^\circ$
200	$80.6 \angle 60^\circ$	$0.000\,430 \angle 90^\circ$	$0.03466 \angle 150^\circ$	$0.19 \angle 75^\circ$	$0.98 \angle 0.5^\circ$	$0.19 \angle 75^\circ$
300	$109 \angle 65^\circ$	$0.000\,670 \angle 90^\circ$	$0.07303 \angle 155^\circ$	$0.27 \angle 77^\circ$	$0.96 \angle 0.9^\circ$	$0.27 \angle 77^\circ$
400	$143 \angle 66^\circ$	$0.000\,900 \angle 90^\circ$	$0.12870 \angle 156^\circ$	$0.36 \angle 78^\circ$	$0.94 \angle 1.5^\circ$	$0.35 \angle 78^\circ$
500	$156 \angle 67^\circ$	$0.001\,100 \angle 90^\circ$	$0.17160 \angle 157^\circ$	$0.41 \angle 78^\circ$	$0.91 \angle 2.5^\circ$	$0.39 \angle 79^\circ$
60 CYCLES						
100	$84.8 \angle 71^\circ$	$0.000\,560 \angle 90^\circ$	$0.04749 \angle 161^\circ$	$0.22 \angle 80^\circ$	$0.98 \angle 0.5^\circ$	$0.22 \angle 80^\circ$
200	$165 \angle 76^\circ$	$0.001\,050 \angle 90^\circ$	$0.17325 \angle 166^\circ$	$0.42 \angle 83^\circ$	$0.91 \angle 1.3^\circ$	$0.41 \angle 83^\circ$
300	$244 \angle 79^\circ$	$0.001\,600 \angle 90^\circ$	$0.39040 \angle 169^\circ$	$0.63 \angle 84^\circ$	$0.81 \angle 3.3^\circ$	$0.60 \angle 85^\circ$
400	$326 \angle 80^\circ$	$0.002\,150 \angle 90^\circ$	$0.70090 \angle 170^\circ$	$0.84 \angle 85^\circ$	$0.67 \angle 5.6^\circ$	$0.74 \angle 86^\circ$
500	$407 \angle 80^\circ$	$0.002\,700 \angle 90^\circ$	$1.09890 \angle 170^\circ$	$1.05 \angle 85^\circ$	$0.51 \angle 9.1^\circ$	$0.87 \angle 87^\circ$

These values are but roughly approximate to illustrate the general effect for certain circuits.

DETERMINATION OF THE AUXILIARY CONSTANTS

It was shown in Chart XI how values for the auxiliary constants *A*, *B* and *C* may be determined mathematically by convergent series form of solution, using problem *X* as an example. Chart XVI gives information as to how these same auxiliary constants may be determined by the use of real hyperbolic functions.

The solution for the auxiliary constants by real hyperbolic functions is given completely for problem *X* in Chart XVI. Vector diagrams are given to assist in following the solution. In the solution for the auxiliary constants by convergent series, the operations were carried out by aid of rectangular co-ordinates of the complex, or vector quantities. In Chart XVI, the operations are to a large extent carried out by the aid of polar co-ordinates. In the case of convergent series, most of the operations consist of adding the various terms of the series together. As addition and subtraction of complex quantities can be most readily carried out when expressed in rectangular co-ordinates, this form of expression is used for the convergent-series solution. On the other hand, powers and roots of complex quantities are most readily obtained by polar co-ordinate expression. In the solution by real hyperbolic functions Chart XVI, operations for powers and roots predominate, and for this reason polar expressions have been quite generally employed. The solution by real hyperbolic functions is briefly this:—

The impedance *Z* and the admittance *Y* are first set down in complex form and their product obtained. square root of this product gives the complex angle $\theta = \sqrt{ZY}$ of the circuit. This angle is then expressed in rectangular co-ordintes as $\theta_1 + j\,\theta_2$ for the purpose of determining the numerical value of its real part θ_1 (expressed in hyps) and its imaginary or circular part θ_2 expressable in circular radians. This circular part θ_2

CHART XVI—RIGOROUS SOLUTION FOR AUXILIARY CONSTANTS OF PROBLEM X BY REAL HYPERBOLIC FUNCTIONS

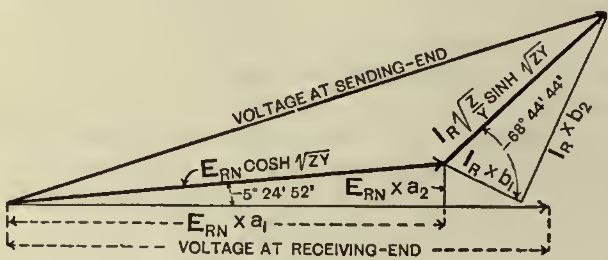
CHARACTERISTICS OF CIRCUIT

LENGTH 300 MILES. CYCLES 60.
CONDUCTORS—3 # 000 STRANDED COPPER.
SPACING OF CONDUCTORS 10 X 10 X 20 FEET.
EQUIVALENT DELTA SPACING = 12.6 FT.

LINEAR CONSTANTS OF CIRCUIT

TOTAL PER CONDUCTOR

R = 0.350 X 300 = 105 OHMS TOTAL RESISTANCE AT 25° C.
X = 0.830 X 300 = 249 OHMS TOTAL REACTANCE.
B = 5.21 X 300 X 10^{-6} = .001563 MHO TOTAL SUSCEPTANCE.
G = 0 X 300 = 0 MHO TOTAL CONDUCTANCE.
g = (IN THIS CASE TAKEN AS ZERO).



VOLTAGE DIAGRAM

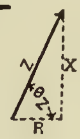
SOLUTION FOR $\theta = \sqrt{ZY}$

$\theta_Z = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{249}{105} = 67°\ 8'\ 8''$

$Z = R + jX = 105 + j\,249$

$= \sqrt{105^2 + 249^2} = 270.233$

$= 270.233\ \angle 67°\ 6'\ 6''$



$\theta_Y = \tan^{-1}\frac{B}{G} = \tan^{-1}\frac{0.001563}{0} = 90°$

$Y = G + jB = 0 + j\,0.001563$

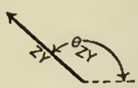
$= \sqrt{0^2 + 0.001563^2} = 0.001563$

$= 0.001563\ \angle 90°$

$\theta_{ZY} = \theta_Z + \theta_Y = 67°\ 8'\ 8'' + 90° = 157°\ 8'\ 8''$

$ZY = 270.233 \times 0.001563 = 0.4223745$

$= 0.4223745\ \backslash 157°\ 8'\ 8''$



$\theta = \sqrt{ZY} = 0.6499035\ \angle 78°\ 34'\ 4''$ HYP.

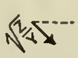
$= 0.128817 + j\,0.637009$ HYP.

$= \angle(\alpha_1 + j\,\alpha_2)$

$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{270.233\ \angle 67°\ 8'\ 8''}{0.001563\ \angle 90°}}$

$= \sqrt{172893\ \backslash 22°\ 51'\ 52''}$

$= 415.805\ \backslash 11°\ 25'\ 56''$



WAVE LENGTH

$\theta_2 = +j\,0.637009$ HYP.

$\alpha_2 = \frac{0.637009}{300} = 0.00212336$

WAVE LENGTH $= \frac{2\pi}{\alpha_2} = \frac{6.2831853072}{0.00212336} = 2959$ MILES

SOLUTION FOR (A)

(A) = COSH $\sqrt{ZY}$ = (COSH θ_1 COS θ_2 + j SINH θ_1 SIN θ_2)

$\theta_1 = 0.128817$ HYP $\qquad \theta_2 = \frac{360°}{2\pi} \times 0.637009 = 36°\ 29'\ 52''$

LOG COSH $\theta_1 = 0.003594$ $\qquad$ LOG SINH $\theta_1 = \bar{1}.111172$

LOG COS $\theta_2 = \bar{1}.905194$ $\qquad$ LOG SIN $\theta_2 = \bar{1}.774359$

LOG $a_1 = \bar{1}.908788$ $\qquad$ LOG $a_2 = \bar{1}.885531$

$a_1 = 0.81056$ $\qquad$ $a_2 = 0.07683$

$0.81056 + j\,0.07683$

(A) = $0.8142\ \angle 5°\ 24'\ 52''$

SOLUTION FOR (B)

(B) = $\sqrt{\frac{Z}{Y}}$ SINH $\sqrt{ZY} = \sqrt{\frac{Z}{Y}}$ (SINH θ_1 COS θ_2 + j COSH θ_1 SIN θ_2)

LOG SINH $\theta_1 = \bar{1}.11172$ $\qquad$ LOG COSH $\theta_1 = 0.003594$

LOG COS $\theta_2 = \bar{1}.905194$ $\qquad$ LOG SIN $\theta_2 = \bar{1}.774359$

$\bar{1}.016366$ $\qquad$ $\bar{1}.777953$

SINH θ_1 COS $\theta_2 = 0.10383$ $\qquad$ COSH θ_1 SIN $\theta_2 = 0.59973$

$\tan^{-1}\frac{0.59973}{0.10383} = \angle 80°\ 10'\ 40''$

SINH θ_1 COS θ_2 + j COSH θ_1 SIN θ_2 =

$= 0.10383 + j\,0.59973$

$= 0.60865\ \angle 80°\ 10'\ 40''$

$\sqrt{\frac{Z}{Y}} = 415.805\ \backslash 11°\ 25'\ 56''$

$= 415.805\ \backslash 11°\ 26'\ 55'' \times 0.60865\ \angle 80°\ 10'\ 40''$

(B) = $253.08\ \angle 68°\ 44'\ 44''$

SOLUTION FOR (C)

(C) = $\frac{1}{\sqrt{\frac{Z}{Y}}}$ SINH $\sqrt{ZY}$

$= \frac{1}{415.605\ \backslash 11°\ 25'\ 56''} \times 0.60855\ \angle 80°\ 10'\ 40''$

$= 0.002405\ \angle 11°\ 25'\ 56'' \times 0.60865\ \angle 80°\ 10'\ 40''$

(C) = $0.001464\ \backslash 91°\ 36'\ 35''$

As a check against possible serious errors in the calculations, the calculated values may be compared with values read from the Wilkinson Charts. The above results check exactly with those obtained by convergent series. (See Chart XI).

is converted to degrees by multiplying by 57° .29578. The hyperbolic cosine and sine of this complex angle are next obtained by the aid of logarithms of the functions of the component parts of the hyperbolic complex angle θ. The equation for cosh θ and sinh θ is given just above the solution. With a view of eliminating the necessity of calculation for each complex angle, cosh θ and sinh θ, Dr. Kennelly has prepared tables and charts from which these two functions (and others) may be obtained directly, thus very materially shortening the solution by hyperbolic functions. Since complex angles have two variable components $(\theta_1 + j\,\theta_2)$ tables of functions of such angles would have to be quite extensive in order that the steps for which values for the functions are given be not excessive. Although tables of functions of complex angles are not as complete as is desired they are a great help in the solution of ordinary power circuits. Functions corresponding to angles lying between the values for angles in these tables may readily be approximated by simple proportion, giving values sufficiently accurate for ordinary power transmission circuits. They have been calculated in Chart XVI for the purpose of illustrating such procedure and also as a high degree of accuracy was here desired for the purpose of illustrating the agreement of the results as obtained by different rigorous methods. Ordinarily these values would be taken from tables.

SOLUTION BY NOMINAL π METHOD

By this method, in place of considering the admittance of the circuit as being distributed (as it is in the actual circuit) it is based upon the assumption that the total conductor admittance may be lumped at two points, one half being placed at each end of the circuit. Such an artificial circuit is known as a "nominal π" circuit since the nominal values of impedance and admittance are ascribed to this circuit. On the above assumption, the current per conductor is the vector sum of the receiving end load and the receiving end condenser currents. The sending end current is the vector sum of the conductor and the sending end condenser currents. The performance of such a circuit may be determined either graphically or mathematically.

If the circuit is not of great electrical length, (say not over 100 miles at 60 cycles or 200 miles at 25 cycles) the performance of the corresponding nominal π circuit will not be materially different from that of the actual circuit having distributed constants which it imitates. If, however, the circuit is of great electrical length the performance of the nominal π circuit no longer closely imitates the performance of the actual circuit which it represents, owing to an error due to the lumpiness of the artificial circuit. Dr. Kennelly has shown that by making certain modifications in the linear or fundamental constants for the impedance and admittance of the nominal π circuit, the lumpiness error will vanish, so that the artificial circuit will then truly represent at the terminals the behavior under steady state operation, taking distributed admittance into account. Such a corrected artificial circuit is known as the "equivalent" π circuit, because it then becomes externally equivalent to the actual circuit, having distributed constants, in every respect.

The complex numbers which must be applied to the impedance, Z and the admittances, $\frac{Y}{2}$ and $\frac{Y'}{2}$ of the nominal π circuit in order to correct these nominal values into the equivalent circuit are called the correcting factors of the nominal π circuit. The nominal values of the impedance Z and the admittances $\frac{Y}{2}$ of the circuit must be multiplied by these vector correcting factors in order to convert them into the "equivalent" values; thus:—

$$Z' = Z\frac{\sinh\theta}{\theta}$$

$$\frac{Y'}{2} = \frac{Y}{2}\;\frac{\tanh\theta/2}{\theta/2}$$

Where $\theta = \sqrt{ZY}$ is the hyperbolic complex angle subtended by the circuit.

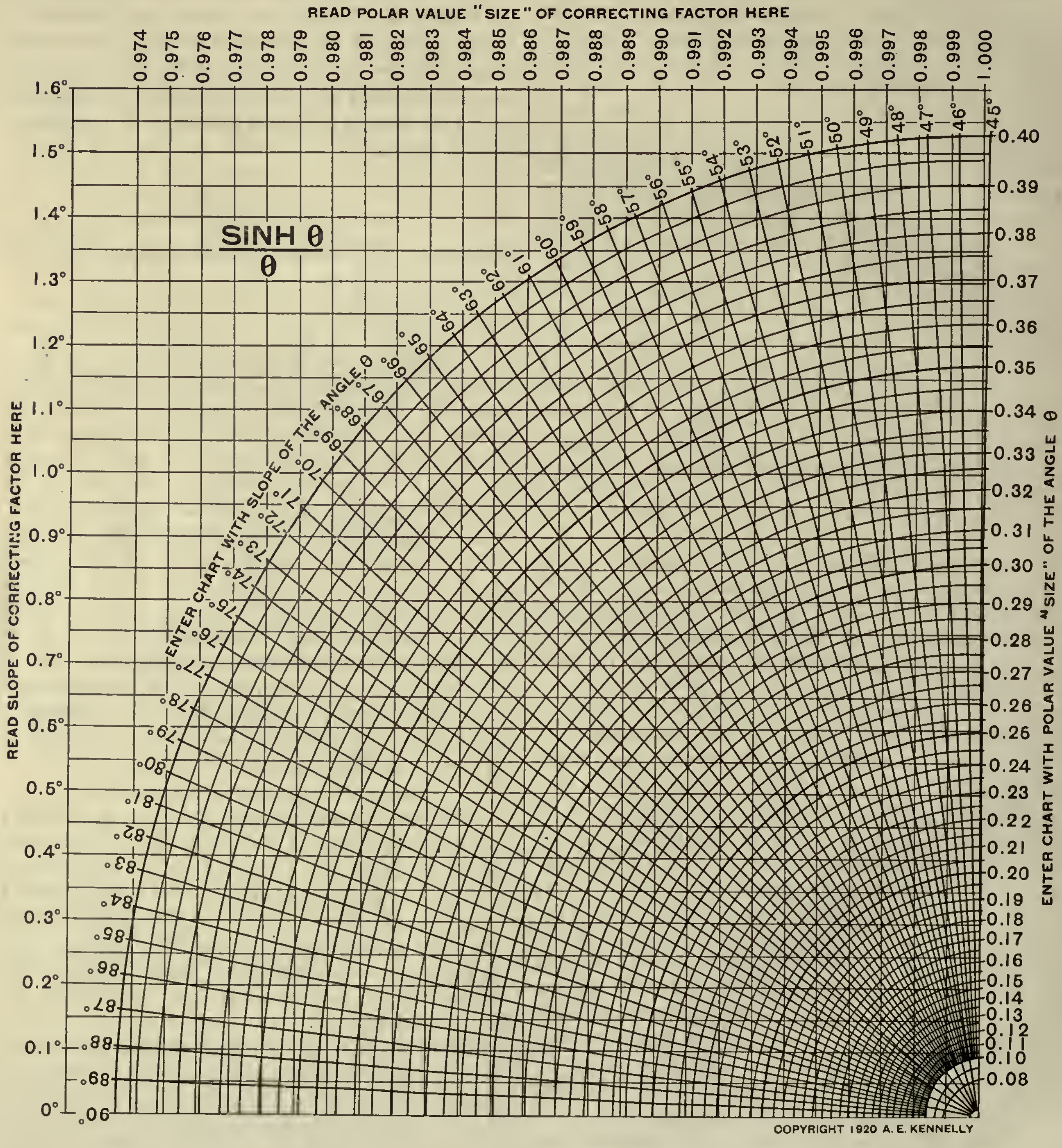
Complete tables of hyperbolic functions are not always available; then again, many engineers have a natural aversion to the use of such functions. In order to avoid these objections as well as to simplify calculations, Dr. Kennelly has charted these "correcting factors" for hyperbolic complex angles up to $\theta = 1.0$ radian in steps of 0.01 in size and 1 degree in slope. The writer is particularly indebted to Dr. Kennelly for these charts, which are reproduced herewith for the first time, as Charts XVIII, XIX, XX and XXI. It is believed that the use of these charts will greatly simplify the calculation of the performance of electric power transmission circuits by hyperbolic functions. They enable the vector values of these ratios to be read to at least three decimal places in sizes and to two decimal places in slope, and their availability makes the use of tables of hyperbolic functions unnecessary. The corrected conductor impedance Z' is the same as the familiar auxiliary constant B.

EQUIVALENT π SOLUTION FOR PROBLEM X

The solution for problem X by the equivalent π method is given in Chart XVII. At the top of the sheet are two diagrams, one a diagram for one conductor of the circuit of problem X and the other a corresponding vector diagram of the currents and the voltages at both ends. The numerical values of the angles and the quantities pertaining to problem X are placed upon the two diagrams for the purpose of assisting in following the mathematical solution.

The physical properties of the circuit are first set down, its linear constants obtained from the tables of constants and multiplied by the length of the circuit to obtain the total values per conductor. The next procedure is to calculate the hyperbolic angle θ of the circuit. To do this the impedance and the admittance of the circuit are set down as complex quantities in the form of polar co-ordinates and multiplied together by multiplying their slopes and adding their angles. The square root of the resulting vector is obtained by tak-

CHART XVIII
KENNELLY CHART FOR IMPEDANCE CORRECTING FACTOR
(FOR ANGLES HAVING POLAR VALUES BETWEEN 0 AND 0.40)



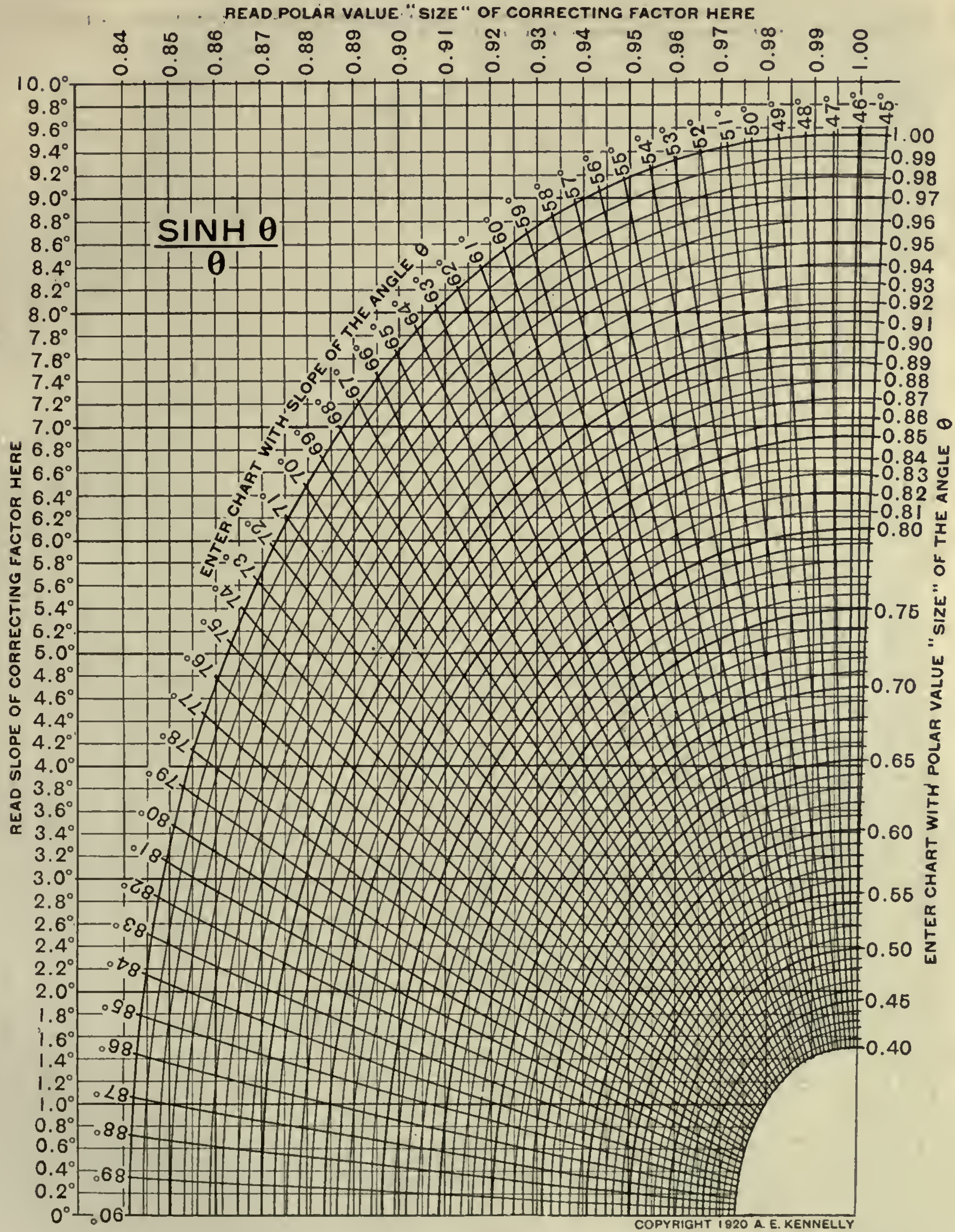
To find the vector "correcting factor" corresponding to any complex line angle θ, of a circuit, the angle θ is expressed in polar form with the slope in fractional degrees. The correcting factor as read from the chart will be in polar form with its slope in fractional degrees. Consult Table P for rapid conversion to minutes and seconds. For example:—

$\theta = 0.3 \angle 68^\circ$, correcting factor $= 0.9893 \angle 0^\circ.60 = 0.9893 \angle 0^\circ 36' 00''$

$\theta = 0.215 \angle 80^\circ.5$, correcting factor $= 0.9927 \angle 0^\circ.149 = 0.9927 \angle 0^\circ 08' 56''$

CHART XIX
KENNELLY CHART FOR IMPEDANCE CORRECTING FACTOR

(FOR ANGLES HAVING POLAR VALUES BETWEEN 0.40 AND 1.0)



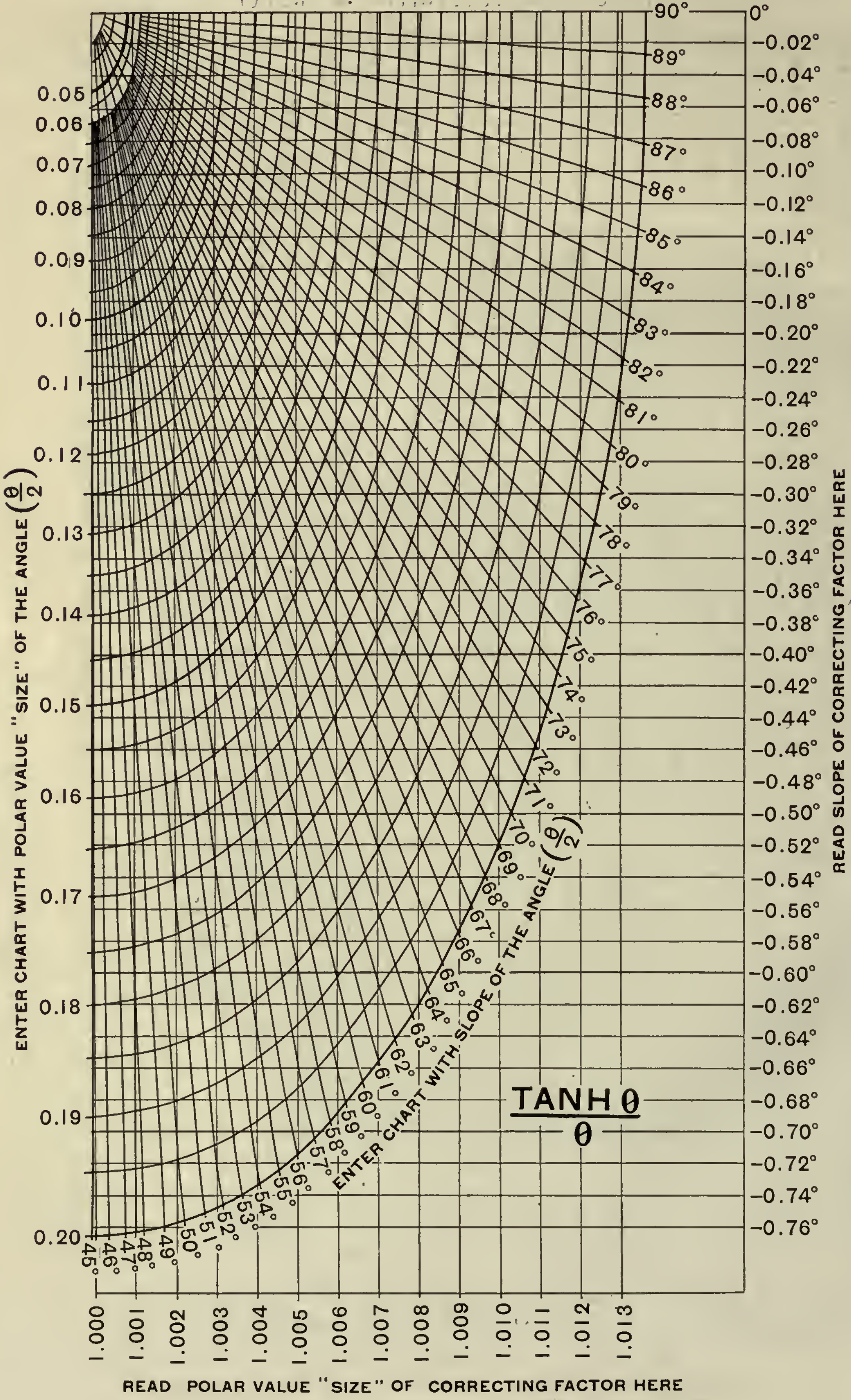
To find the vector "correcting factor" corresponding to any complex line angle θ, of a circuit, the angle θ is expressed in polar form with the slope in fractional degrees. The correcting factor as read from the chart will be in polar form with its slope in fractional degrees. Consult Table P for rapid conversion to minutes and seconds. For example:—

$\theta = 0.8 \angle 62^\circ$, correcting factor $= 0.943 \angle 5^\circ.19 = 0.943 \angle 5^\circ 11' 24''$

$\theta = 0.6499 \angle 78^\circ.57$, correcting factor $= 0.9365 \angle 1^\circ.61 = 0.9365 \angle 1^\circ 36' 36''$

CHART XX KENNELLY CHART FOR ADMITTANCE CORRECTING FACTOR

(FOR ANGLES HAVING POLAR VALUES BETWEEN 0 AND 0.20)



To find the vector correcting factor corresponding to any complex line angle θ of a circuit, the angle θ is expressed in polar form with the slope in fractional degrees. The angle of the line, θ, is then divided by $2\left(\frac{\theta}{2}\right)$ as it is necessary to enter the admittance charts with half the angle θ. The correcting factor as read from the chart will be in polar form with its slope in fractional degrees.

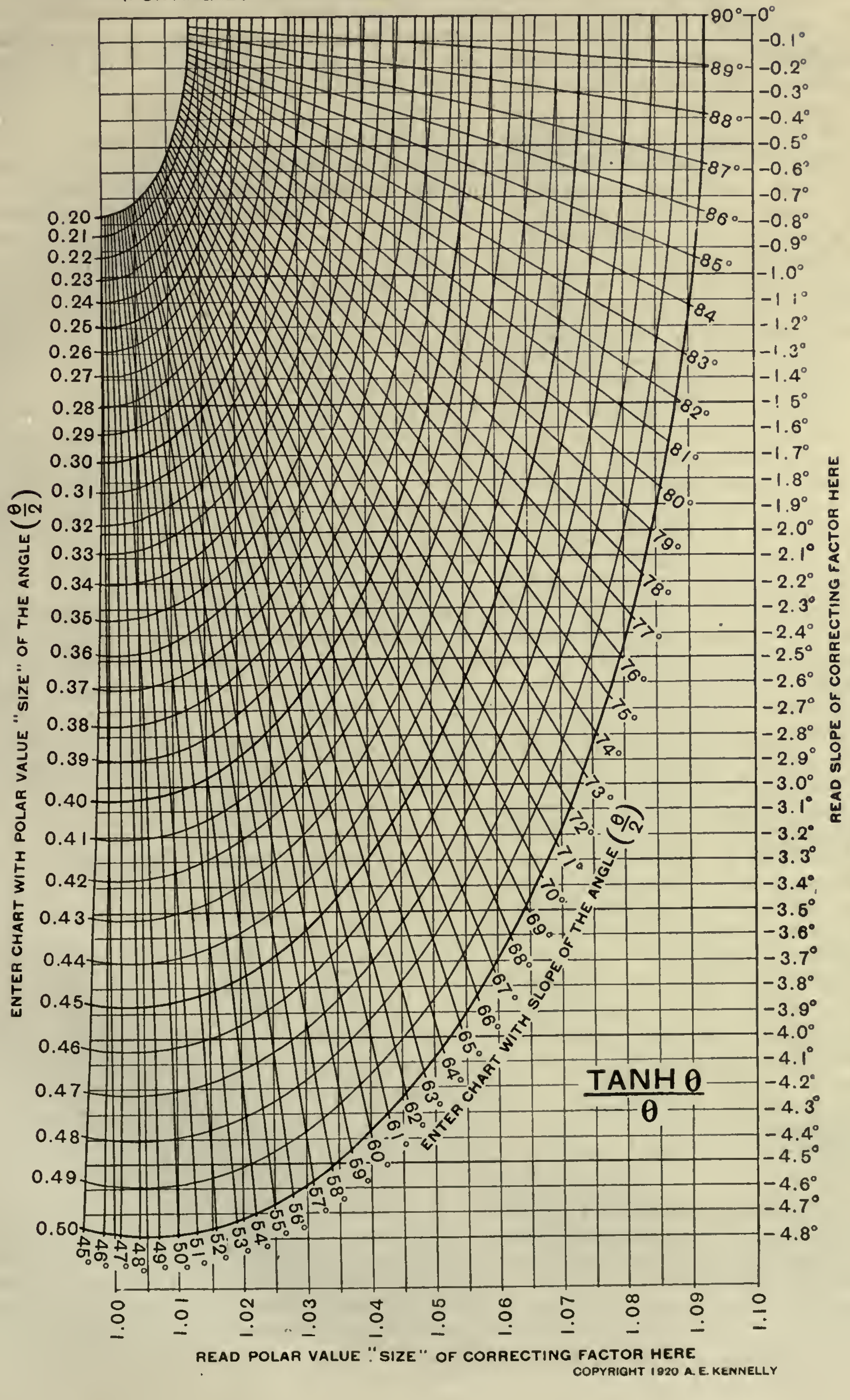
Consult Table P for rapid conversion to minutes and seconds. For example:—

$\theta = 0.4 \angle 61°$, $\left(\frac{\theta}{2}\right) = 0.2 \angle 61°$, correcting factor $= 1.007 \angle 0°.655$
$= 1.007 \angle 0°39'18''$

$\theta = 0.326 \angle 75°.5$, $\left(\frac{\theta}{2}\right) = 0.163 \angle 75°.5$, correcting factor $= 10078$
$\angle 0°.25 = 10078 \angle 0°15'00''$

CHART XXI KENNELLY CHART FOR ADMITTANCE CORRECTING FACTOR

(FOR ANGLES HAVING POLAR VALUES BETWEEN 0.20 AND 0.50)



To find the vector correcting factor corresponding to any complex line angle θ, of a circuit, the angle θ is expressed in polar form with the slope in fractional degrees. The angle of the line θ is then divided by 2 $\left(\frac{\theta}{2}\right)$ as it is necessary to enter the admittance charts with half the angle θ. The correcting factor as read from the chart will be in polar form with its slope in fractional degrees. Consult Table P for rapid conversion to minutes and seconds. For example:—

ing the square root of the slope and halving the angle. The result is the hyperbolic angle θ of the circuit expressed in hyps.

The ratio charts XIX and XXI are next consulted and the correcting values $\frac{\sinh \theta}{\theta}$ and $\frac{\tanh \theta/2}{\theta/2}$ corresponding to the hyperbolic angle of the circuit read off. Having thus obtained the correcting factors corresponding to this circuit, the linear impedance Z and linear admittance Y per conductor are multiplied respectively by the sinh and the tanh correcting factors.

If the circuit under consideration is electrically short the effect of these correcting factors upon the linear constants will be small and possibly negligible but, as the circuit becomes longer, their effect becomes increasingly greater. The effect of the correcting factors for problem X is to change the linear impedance Z from 270.233 $\underline{/67^\circ\ 08'\ 08''}$ to Z' = 253.083 $\underline{/68^\circ\ 44'\ 41''}$ and to change the linear admittance Y from 0.001563 $\underline{/90^\circ}$ to Y' = 0.001615512 $\underline{/89^\circ\ 10'\ 45''}$. In other words this circuit will behave in the steady state at 60 cycles as though its conductor resistance were reduced from 105 to 91.7486 ohms and its inductive reactance reduced from 249 to 235.866 ohms. Similarly it will behave as though a non-inductive leak of 11.571 micromhos, has been applied to each condenser in shunt.

In order to illustrate the exact agreement in the results as obtained by the equivalent π method with those obtained by either the convergent series or pure hyperbolic solution, the ratio values used for this problem were calculated and not obtained graphically. The accuracy in the performance resulting from the use of ratio values taken from the charts is well within the requirements of practical power circuits. The mathematical solution for these factors is given in Fig. 48.

Having determined the corrected values for the impedance Z' and the admittance Y' which will produce exact results, the remainder of the solution may be carried out graphically as indicated by the vector diagram in the upper right hand part of Chart XVII or mathematically as indicated under this vector diagram.

EQUIVALENT T SOLUTION

Dr. Kennelly has shown that the correcting factors which convert the nominal π into the equivalent π of the conjugate smooth line, are the same as those which convert the nominal T into the equivalent T, but in inverse order;—that is the correcting factors for the nominal T line are

$$Z' = Z\frac{\tanh \theta/2}{\theta/2}$$

$$Y' = Y\frac{\sinh \theta}{\theta}$$

Either the equivalent π or the equivalent T solution may be used by applying the two correcting factors properly. Usually less arithematical work will be required for the equivalent π solution.

ELECTRICAL CONDITIONS AT INTERMEDIATE POINTS

In the foregoing, the behavior of circuits at their terminals has been considered. In some cases it may be desirable to predetermine the voltage and the current at points along the circuit between the terminals. This may be particularly desirable in case of circuits of great electrical length and consequently having a pronounced bend or hump in the voltage graphs representing the voltage at points along the circuit. In Fig. 21 voltage and current graphs were shown for the circuit of problem X corresponding to zero load; also load conditions. Accompanying this stated was the step-by-step method by which the current and voltage at these intermediate points had been determined. In a corresponding manner the intermediate electrical conditions may be determined by the employment of hyperbolic functions. It is usual, however, when employing hyperbolic functions for determining the voltage or the current at points along a smooth circuit, in the steady state, to take advantage of the following facts relative to the variation in current and potential from point to point in such a circuit.

The potentials of any and all points of such a circuit are as the sines and the currents as the cosines of the corresponding position angles. This means that if the position angles corresponding to two points of a smooth circuit in the steady state are known, and the voltage or the current at one of these points is also known, then the voltage or current at any other point will be directly proportional to the sine or the cosine respectively of the corresponding position angles. In a similar manner, the impedance follows the tangents, the admittance the contagents and the volt-amperes the sines of twice the angles. Herein lies the beauty of the application of hyperbolic functions of complex angles for determining the electrical performance of electric circuits. The relationship expressed above (taken from Dr. Kennelly's "Artificial Electric Lines") are given in equation form below for ready reference:—

$$\frac{E_p}{E_c} = \frac{\sinh \theta_p}{\sinh \theta_c} \text{ numeric } \angle$$

$$\frac{I_p}{I_c} = \frac{\cosh \theta_p}{\cosh \theta_c} \text{ numeric } \angle$$

$$\frac{Z_p}{Z_c} = \frac{\tanh \theta_p}{\tanh \theta_c} \text{ numeric } \angle$$

$$\frac{Y_p}{Y_c} = \frac{\coth \theta_p}{\coth \theta_c} \text{ numeric } \angle$$

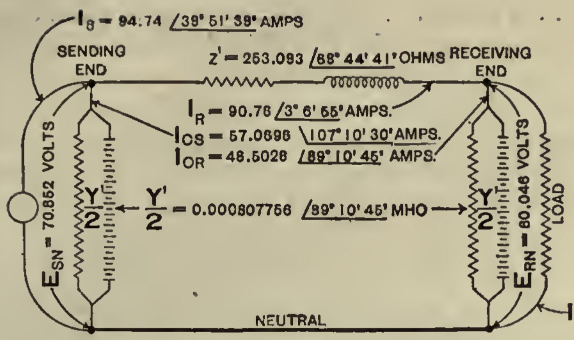
$$\left|\frac{Kv\text{-}a_p}{Kv\text{-}a_c}\right| = \left|\frac{\sinh 2\theta_p}{\sinh 2\theta_c}\right| \text{ numeric } \angle$$

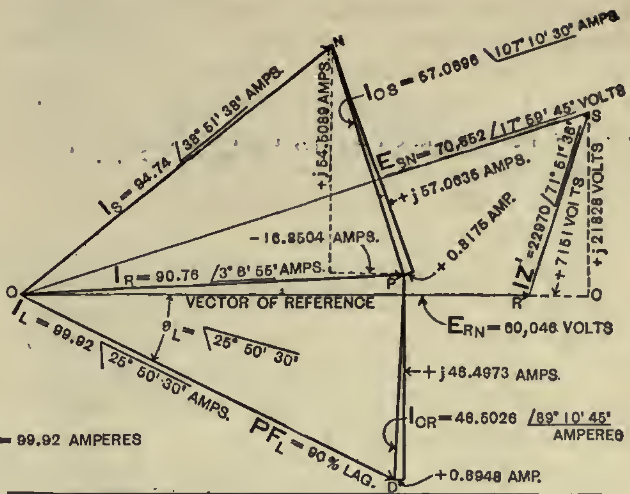
Where p and c are points along the circuit, c being some point where the electrical conditions are known, and p the point for which they are to be computed. The vertical lines enclosing the two parts of the last equation are for the purpose of indicating that the "size" of these complex quantities are referred to in this equation.

POSITION ANGLES

Reference has been made to the line as subtending a certain complex hyperbolic angle θ. Since the circuit through the load also encounters resistance and reactance, the load may be said to subtend also a certain complex hyperbolic angle, so that the receiving end of the circuit occupies an angular position θ_r. The total

CHART XVII—RIGOROUS EQUIVALENT π SOLUTION OF PROBLEM X





CHARACTERISTICS OF CIRCUIT

LENGTH, 300 MILES. CYCLES, 60.
CONDUCTORS—3 # 000 STRANDED COPPER.
SPACING OF CONDUCTORS 10 X 10 X 20 FEET.
EQUAVALENT DELTA SPACING = 12.6 FT.

LINEAR CONSTANTS OF CIRCUIT

FROM TABLES PER MILE

TABLE NO. 2, r = 0.350 OHM AT 25° C.
TABLE NO. 5, x = 0.830 OHM (BY INTERPOLATION).
TABLE NO. 10, $b = 5.21 \times 10^{-6}$ MHO (BY INTERPOLATION)
g = (IN THIS CASE TAKEN AS ZERO).

TOTAL PER CONDUCTOR

R = 0.350 X 300 = 105 OHMS TOTAL RESISTANCE.
X = 0.830 X 300 = 249 OHMS TOTAL REACTANCE.
$B = 5.21 \times 300 \times 10^{-6}$ = .001563 MHO TOTAL SUSCEPTANCE.
G = 0 X 300 = 0 MHO TOTAL CONDUCTANCE.

SOLUTION FOR HYPERBOLIC ANGLE $\theta = \sqrt{ZY}$

Z = 105 + j 249 Y = 0 + j 0.001563
= 270.233 /67° 8' 8" = 0.001563 /90°
$\theta = \sqrt{270.233\ /67° 8' 8" \times 0.001563\ /90°}$
$= \sqrt{0.4223745\ /157° 8' 8"}$
= 0.6499035 /78° 34' 4" HYP.
= 0.6499035 /78°.5678 HYP.
= 0.1289166 + j 0.6370092 HYP.

FROM DR. KENNELLY'S CHARTS

CHART XIX $\frac{\text{SINH } \theta}{\theta}$ = 0.9365365 /1°.6094 = 0.9365365 /1° 36' 33"
CHART XXI $\frac{\text{TANH } \theta/2}{\theta/2}$ = 1.033599 \0°.8208 = 1.033599 \0° 49' 15"

★ THESE VALUES WERE CALCULATED IN ORDER TO OBTAIN A HIGH DEGREE OF ACCURACY FOR THE PURPOSE OF DEMONSTRATING THE FUNDAMENTAL ACCURACY OF THIS METHOD.

CORRECTION OF LINEAR CONSTANTS

Z' = 270.233 /67° 8' 8" X 0.9365365 /1° 36' 33"
= 253.093 /68° 44' 41" (WHICH IS AUXILIARY CONSTANT (B))
= 91.7486 + j 235.986 OHMS
Y' = 0.001563 /90° X 1.033599 \0° 49' 15"
= 0.001615512 /89° 10' 45" MHO
$\frac{Y'}{2}$ = 0.000807756 /89° 10' 45"
= 0.000011571 + j 0.00080767
= 1238 \89° 10' 45" OHMS REACTANCE.

CALCULATION OF PERFORMANCE ★

PER PHASE TO NEUTRAL

$KV\text{-}A_{RN} = \frac{18,000}{3}$ = 6,000. $KW_{RN} = \frac{16,200}{.3}$ = 5,400.
$E_{RN} = \frac{104,000}{1.732}$ = 60,046. $I_R = \frac{6,000 \times 1000}{60,046}$ = 99.92.
PF_R = 90% LAGGING.

RECEIVING-END CONDITIONS

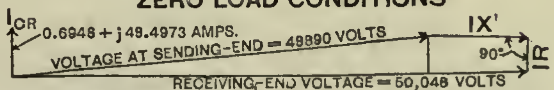
I_{CR} = 60,046 X 0.000807756 /89° 10' 45" = 48.5026 /89° 10' 45"
= 0.6948 + j 48.4973 AMP.
I_R = 99.92 (0.90 − j 0.436) + 0.6948 + j 48.4973
= 90.623 + j 4.9322 AMPS.
= 90.76 /3° 6' 55" AMPS.
PF_R = COS 3° 6' 55" = 99.85% LEADING.
KW_{CR} = 60,046 (0.6948 + j 48.4973)
= 41.72 + j 2912.069
KW_{RN} = 6000 (0.90 − j 0.436) + 41.72 + j 2912.07
= 5441.72 + j 296.07
$I_R Z'$ = 90.76 /3° 6' 55" X 253.093 /68° 44' 41"
= 22970 /71° 51' 36" VOLTS
= 7151 + j 21,828 VOLTS

SENDING-END CONDITIONS

E_{SN} = 60,046 + 7151 + j 21828
= 67,197 + j 21,828 VOLTS
= 70,652 /17° 59' 45" VOLTS
I_{CS} = 70,652 X 0.000807756 /89° 10' 45"
= 0.9175 + j 57.0635 (TO SUPPLY END VOLTAGE)
= 57.0696 \107° 10' 30" TO VECTOR OF REFERENCE
= − 16.8504 + j 54.5089
I_S = (90.623 + j 4.9322) + (− 16.8504 + j 54.5089)
= 73.77 + 59.44 AMPS.
= 94.74 /39° 51' 39" AMPS.

PF_S = COS 39° 51' 39" − 17° 59' 45" = 93.44 % LEADING
KW_{SN} = 70.652 X 94.74 X .9344 = 6255 KW PER PHASE
LOSS = 6255 − 5400 = 855 KW PER PHASE
EFF = $\frac{5400 \times 100}{6255}$ = 86.33 %

ZERO LOAD CONDITIONS



IR' = (0.6948 + j 48.4973) X 91.7486 = 64 + j 4449 VOLTS
IX' = (− 48.4973 + j 0.6948) X 235.986 = − 11438 + j164
$IR' + IX'$ = − 11374 + 4613
E_{SNO} = (60,046 − 11373) + 4613 = 48890 VOLTS /5° 24' 51"

*The above results check with those obtained by convergent series. (See Chart XIII).

angle of the circuit (line and load) will be $\theta_r + \theta = \theta_s$. By similar reasoning all points lying between the receiving and sending ends of a line will occupy or assume an angular position θ_p. If that part of the linear angle θ of the line between the receiving end and the point p be designated as θ_{pr}, then the angular position of the point p will be $\theta_p = \theta_r + \theta_{pr}$. Thus, at a point in the middle of the line, the position angle will be $\theta_p = \theta_r + \theta_{pr} = \theta_r + \theta/2$.

If the line is grounded or short-circuited at the receiving end, there will be no load containing resistance and reactance, and consequently no load angle. In such case $\theta_r = 0$ and the distribution of position angles along the line will be purely a linear function of the total line angle θ. In such a case $\theta_s = \theta$.

Load Conditions — In Fig. 49 the procedure is shown which may be followed for determining by complex functions of position angles the current and the voltage vectors at points 25 miles apart along problem *X* circuit, under load conditions.

The procedure is first to determine the complex angle θ_r, at the receiving end resulting from the load. The mathematical determination of this load angle is tedious. Such determination is given for problem *X* circuit under stated load in Fig. 49. This complex angle θ_t of the load (that is the position angle at the receiving end) is such that its complex tangent equals the impedance load δ to ground, or zero potential, at the receiving end of line (ohms $\angle$) divided by the surge impedance Z_0 of a conductor (ohms $\angle$). That is,—

$$\tanh \theta_r = \frac{\delta}{Z_0}$$

Since we are here interested only in the ratio between the load impedance and the surge impedance, the values may be taken either per unit length or total per conductor. Although tanh θ_r is readily calculated, as may be seen by consulting Fig. 49, the subsequent calculation for the corresponding angle θ_r is tedious. After having calculated the tanh θ_r, the corresponding angle θ_r may be obtained with sufficient accuracy from a table of tangents of complex angles or, more readily still, from a chart of such functions.* After having determined the angle θ_r by consulting a chart of tangents of complex angles, or by mathematical calculation, as in Fig. 49, the position angles at points along the circuit may easily and readily be determined as follows:

The change in the position angle from point to point along the circuit, due to the line impedance and the line admittance is purely a linear function of the line angle θ. This is the case whether the line is grounded, loaded or free at the receiving end.

Referring to Fig. 49, the angular position of the receiving end, due to the load conditions assumed, was calculated to be 0.48047 + j 1.06354. It is therefore necessary to add this angle to each of the linear line angles of the various points along the line in order to obtain the position angles of the points in question. Thus the linear line angle of the middle point of the circuit is 0.0644084 + j 0.3185046 and adding to this the load angle 0.48047 + j 1.06354 gives 0.544874 + j 1.3820446, which corresponds with the entry in the tabulation of Fig. 51 for the position angle at the middle of the circuit. In a similar manner position angles for the load assumed are readily determined for points 25 miles apart. Having determined the position angles for the various points along the circuit, the sines and the cosines corresponding to these position angles may be approximated closely from tables or charts of such complex functions, or may be calculated accurately by following the equations at the lower left hand corner of Fig. 51. Since the receiving end voltage and current are known to be 60 046 volts and 99.92 amperes respectively, the voltage and currents at all other points of this circuit will be as the sines and cosines of the corresponding position angles. From the vector quantities that have been assigned to the voltage and current at the points along the circuit, the power-factors at these points are readily determined.

The current and voltage graphs at the bottom of Fig. 51 were plotted from values as determined by the use of functions of position angles. These check exactly with similar graphs as determined by the Wilkinson charts and step-by-tep process (See Fig. 21).

Zero Load Condition—The procedure which may be followed for determining the position angles under zero load, their functions and the corresponding current and voltage distribution is the same as given above for load conditions and is shown in Fig. 50. In this case, however, there is no load and consequently no real part to the load angle. On the other hand the impedance of the load is infinite, that is $\delta = \infty$ so that $\theta_r = \tanh^{-1}\frac{\infty}{Z_0} = j\frac{\pi}{2}$. The effect of this supersurge impedance load at the receiving end at zero load is to cause a phase rotation of 90 degrees or one quadrant, $j\frac{\pi}{2} = 1.57080$ circular radians. Thus, at zero load, $\theta_{ro} = \left(0 + j\frac{\pi}{2}\right) = 0 + j\,1.57080$ and this angle must be added to each of the linear position angles of the points along the line. With the position angles corresponding to zero load thus obtained, and assigned to the points along the circuit, the voltage will be found to follow the sines, and the current the cosines, etc. of these position angles.

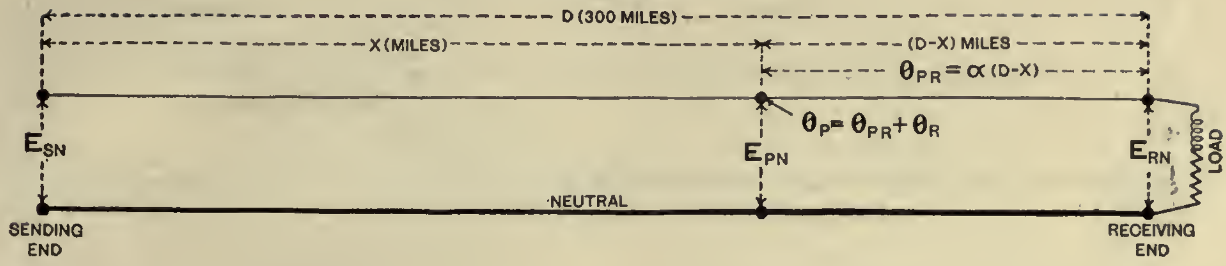
POLAR DIAGRAM OF CURRENT VOLTAGE

In Fig. 52 are shown the polar graphs of the voltage and the current for problem *X*, corresponding to load, and also to zero load conditions. These polar graphs were plotted from the vector values for current and voltage as tabulated in Figs. 49 and 50 for each 25 miles of circuit.

*Such as that worked out by Dr. Kennelly and published by the Harvard University Press. The chart atlas referred to contains graphs of complex tangents of complex angles, and by following the chart in the reverse from the usual direction the complex angle corresponding to any complex tangent may be read off directly.

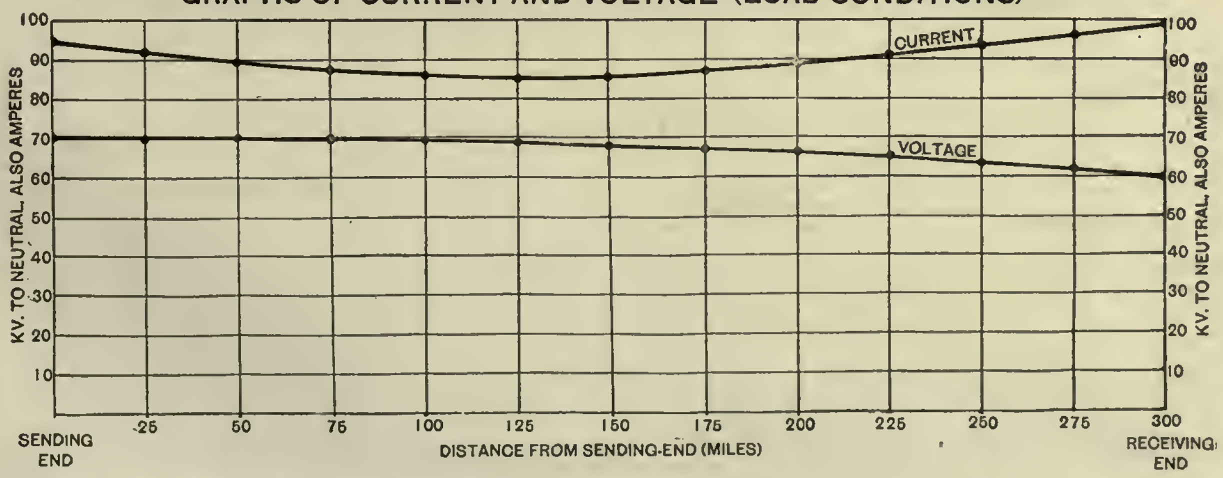
CURRENT & VOLTAGE DISTRIBUTION

(LOAD CONDITIONS)



(D-X) MILES	X MILES	POSITION ANGLE $\theta_P = \theta_{PR} + \theta_R$	SINH θ_P (THE VOLTAGE FOLLOWS THIS COMPLEX FUNCTION)	E_{PN} VOLTS ∠	COSH θ_P (THE CURRENT FOLLOWS THIS COMPLEX FUNCTION)	I_P AMPERES ∠	PF_P %
0	300	0.48047+j1.06354 θ_2= 60° 56′11″	0.24249+j0.97693 =1.00657 ∠76° 03′35″	60046 ∠0° 0′0″	0.54294+j0.43632 =0.69654 ∠38°47′10″	99.92 \25° 50′31″	−90.00
25	275	0.49120+j1.11662 θ_2= 63°58′40″	0.22426+j1.0092 =1.0338 ∠77° 28′16″	61670 ∠1°24′41″	0.49272+j0.45937 =0.67364 ∠42°59′38″	96.64 \21°38′03″	−93.83
50	250	0.50194+j1.16971 θ_2= 67° 01′10″	0.20430+j1.0391 =1.0590 ∠78°52′36″	63173 ∠2°49′01″	0.44064+j0.48176 =0.65288 ∠47°33′08″	93.66 \17°04′33″	−94.04
75	225	0.51267+j1.22279 θ_2= 70°03′39″	0.18259+j1.0663 =1.0819 ∠80°16′59″	64540 ∠4° 13′24″	0.38682+j0.50333 =0.63480 ∠52°27′25″	91.06 \12° 10′20″	−95.94
100	200	0.52341+j1.27587 θ_2= 73° 06′09″	0.15917+j1.0909 =1.1025 ∠81°41′55″	65770 ∠5°38′20″	0.33139+j0.52399 =0.61999 ∠57°41′22″	88.94 \6° 56′19″	−97.60
125	175	0.53414+j1.32895 θ_2= 76° 08′38″	0.13409+j1.1127 =1.1207 ∠83°07′43″	66854 ∠7°04′08″	0.27447+j0.54361 =0.60815 ∠63°12′39″	87.24 \1°25′02″	−98.90
150	150	0.54488+j1.38204 θ_2= 79°11′07″	0.10735+j1.1317 =1.1368 ∠84°34′52″	67815 ∠8°31′17″	0.21618+j0.56197 =0.60211 ∠68°57′32″	86.37 ∠4° 19′51″	−99.73
175	125	0.55561+j1.43512 θ_2= 82°13′36″	0.07908+j1.1477 =1.1504 ∠86° 03′30″	68626 ∠9°59′55″	0.15667+j0.57927 =0.60080 ∠74° 51′57″	86.34 ∠10° 14′16″	+99.99
200	100	0.56635+j1.48821 θ_2= 85°16′05″	0.04926+j1.1607 =1.1618 ∠87°34′11″	69306 ∠11°30′36″	0.09608+j0.59508 =0.60279 ∠80°49′42″	86.47 ∠16° 12′01″	+99.66
225	75	0.57708+j1.54129 θ_2= 88°18′35″	0.01798+j1.1707 =1.1708 ∠89°07′13″	69843 ∠13°03′38″	0.03455+j0.60939 =0.60962 ∠86°45′18″	87.45 ∠22° 07′39″	+98.75
250	50	0.58782+j1.59438 θ_2= 91°21′04″	−0.01471+j1.1775 =1.1775 ∠90°42′57″	70243 ∠14°39′22″	−0.02784+j0.62207 =0.62270 ∠92°33′44″	89.33 ∠27°56′03″	+97.32
275	25	0.59855+j1.64746 θ_2= 94°23′34″	−0.04863+j1.1811 =1.1821 ∠92°21′28″	70517 ∠16°17′53″	−0.09073+j0.63306 =0.63953 ∠98°09′22″	91.74 ∠34° 11′41″	+95.17
300	0	0.60929+j1.70055 θ_2= 97°26′03″	−0.08381+j1.1814 =1.1844 ∠94° 03′28″	70652 ∠17°59′53″	−0.15416+j0.64226 =0.66050 ∠103°29′45″	94.75 ∠38° 52′04″	+93.43

GRAPHS OF CURRENT AND VOLTAGE (LOAD CONDITIONS)



SINH $(\theta_1 + j\,\theta_2)$ = (SINH θ_1 COS θ_2 + j COSH θ_1 SIN θ_2).

COSH $(\theta_1 + j\,\theta_2)$ = (COSH θ_1 COS θ_2 + j SINH θ_1 SIN θ_2).

ANGLE AT RECEIVING END θ_R = 0.48047+ 1.06354

ANGLE OF LINE θ = 0.12682+ 0.63701

$\theta_S = \theta + \theta_R$ = 0.60929+ 1.70055

ONE QUADRANT = 1.57079632 CIRCULAR RADIANS.

ONE CIRCULAR RADIAN = 206264.8062″ = 57° 17′ 44.8″.

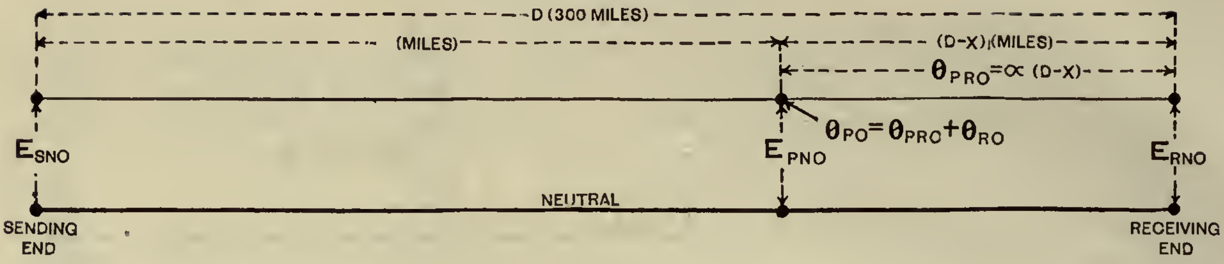
α = 0.00042939+j 0.00212336

FIG. 51—CURRENT AND VOLTAGE DISTRIBUTION

For problem *X* by position angles (load conditions).

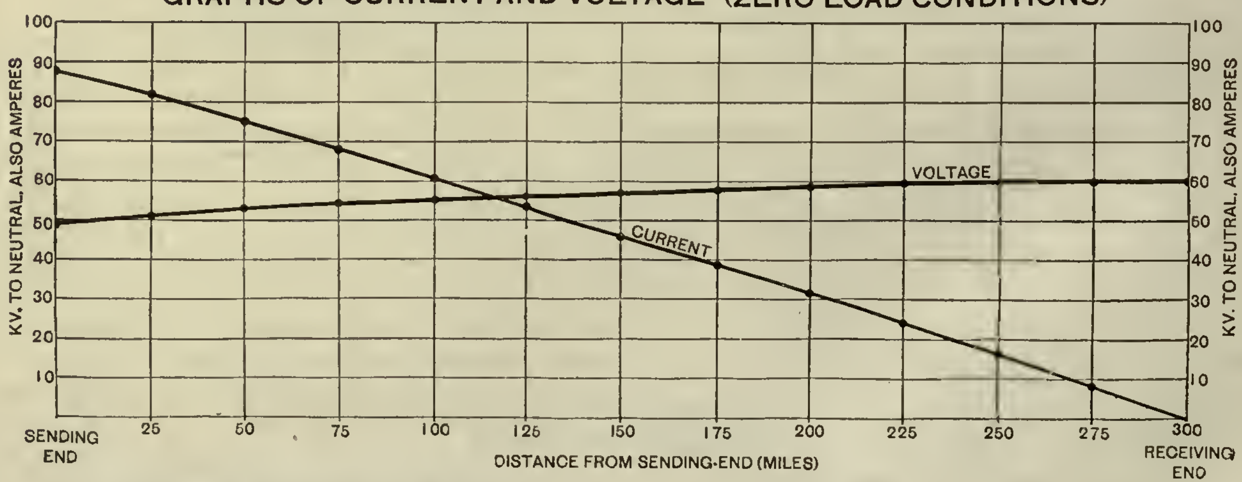
CURRENT AND VOLTAGE DISTRIBUTION

(ZERO LOAD CONDITION)



(D-X) MILES	X MILES	POSITION ANGLE $\theta_{PO}=\theta_{PRO}+\theta_{RO}$	SINH θ_{PO} (THE VOLTAGE FOLLOWS THIS COMPLEX FUNCTION)	E_{PNO} VOLTS ∠	COSH θ_{PO} (THE CURRENT FOLLOWS THIS COMPLEX FUNCTION)	I_{PO} AMPERES ∠	PF_{PO} %
0	300	0 +j1.57080 θ_2= 90°00′00″	0 +j1.00000 =1.00000 ∠90°	60046 ∠0°	0 0 ∠90°	0 ∠11°25′56″	
25	275	0.01073 +j1.62388 θ_2= 93°02′29″	0.00057+j0.99865 =0.99865 ∠89°58′03″	59965 ∠0°01′57″	0.05307+j0.01070 =0.05414 ∠11°23′58″	7.82 \90°01′58″	0
50	250	0.02146 +j1.67696 θ_2= 96°04′58″	0.00227+j0.99460 =0.99460 ∠89°52′09″	59803 ∠0°07′51″	0.10598+j0.02133 =0.10816 ∠11°22′48″	15.62 \90°03′08″	+00.12
75	225	0.03220+j1.73004 θ_2= 99°07′28″	0.00511 +j0.98785 =0.98786 ∠89°42′13″	59317 ∠0°17′47″	0.15866+j0.03179 =0.16181 ∠11°19′48″	23.37 \90°06′08″	+00.32
100	200	0.04294 +j1.78313 θ_2= 102°09′57″	0.00905+j0.97844 =0.97847 ∠89°28′12″	58753 ∠0°31′48″	0.21090+j0.04199 = 0.21503 ∠11°15′35″	31.05 \90°10′21″	+00.61
125	175	0.05367+j1.83621 θ_2= 105°12′26″	0.01409+j0.96638 =0.96648 ∠89°09′50″	58033 ∠0°50′10″	0.26269+j0.051842 =0.26776 ∠11°09′50″	38.66 \90°16′06″	+00.99
150	150	0.06441 +j1.88930 θ_2= 108°14′56″	0.02018+j0.95168 =0.95188 ∠88°47′07″	57156 ∠1°12′53″	0.31380 +j0.06120 =0.31970 ∠11°02′10″	46.17 \90°23′46″	+1.42
175	125	0.07514 +j1.94238 θ_2= 111°17′25″	0.02731 +j0.93436 =0.93476 ∠88°19′33″	56129 ∠1°40′27″	0.36417+j0.07006 =0.37085 ∠10°53′22″	53.55 \90°32′33″	+1.98
200	100	0.08588+j1.99546 θ_2= 114°19′54″	0.03543+j0.91452 =0.91522 ∠87°46′53″	54955 ∠2°13′07″	0.41354+j0.07835 =0.42090 ∠10°43′41″	60.77 \90°42′15″	+2.65
225	75	0.09661 +j2.04854 θ_2= 117°22′24″	0.04449+j0.89218 =0.89328 ∠87°08′43″	53638 ∠2°51′17″	0.46194+j0.08593 =0.46986 ∠10°32′16″	67.85 \90°53′40″	+3.40
250	50	0.10735 +j2.10164 θ_2= 120°24′53″	0.05445+j0.86735 =0.86905 ∠86°24′28″	52183 ∠3°35′32″	0.50917+j0.09275 =0.51755 ∠10°19′26″	74.73 \91°06′30″	+4.33
275	25	0.11808 +j2.15473 θ_2= 123°27′22″	0.06525+j0.84014 =0.84267 ∠85°33′33″	50599 ∠4°26′27″	0.55514+j0.09874 =0.56385 ∠10°05′07″	81.42 \91°20′49″	+5.41
300	0	0.12882 +j2.20781 θ_2= 126°29′52″	0.07683+j0.81056 = 0.81420 ∠84°35′08″	48889 ∠5°24′52″	0.59973+j0.10384 =0.60865 ∠9°49′22″	87.89 \91°36′34″	+6.64

GRAPHS OF CURRENT AND VOLTAGE (ZERO LOAD CONDITIONS)



SINH $(\theta_1+j\theta_2)=(\text{SINH}\,\theta_1\ \text{COS}\,\theta_2+j\,\text{COSH}\,\theta_1\ \text{SIN}\,\theta_2)$.
COSH $(\theta_1+j\theta_2)=(\text{COSH}\,\theta_1\ \text{COS}\,\theta_2+j\,\text{SINH}\,\theta_1\ \text{SIN}\,\theta_2)$.

ANGLE AT RECEIVING END θ_{RO} = 0 +j 1.57080
ANGLE OF LINE θ =0.12882+j 0.63701
$\theta_{SO}=\theta+\theta_{RO}$ =0.12882+j 2.20781

ONE QUADRANT=1.57079632 CIRCULAR RADIANS
ONE CIRCULAR RADIAN=206264,8082″=57° 17′ 44.8″

α =0.00042939+j 0.00212336

FIG. 50—CURRENT AND VOLTAGE DISTRIBUTION
For problem *X* by position angles (zero load conditions).

PROBLEM "X"

$$R = 105 \text{ OHMS.} \qquad X = 249 \text{ OHMS.}$$
$$G = 0 \text{ MHO.} \qquad Y = 0.001563 \text{ MHO.}$$
$$Z = 270.233 \angle 67^\circ 08' 08''$$
$$Y = 0.001563 \angle 90^\circ$$
$$\theta = \sqrt{ZY} = \sqrt{0.422\,374\,5 \angle 157^\circ 08' 08''}$$
$$= 0.649\,903\,5 \angle 78^\circ 34' 04''$$
$$= 0.1288168 + j0.6370092$$

CALCULATION FOR $\frac{\text{SINH } \theta}{\theta}$

$$\text{SINH } \theta = 0.103\,8393 + j0.599735$$
$$= 0.6086583 \angle 80^\circ 10' 38'' \quad *$$
$$\frac{\text{SINH}\theta}{\theta} = \frac{0.6086583 \angle 80^\circ 10' 37''}{0.6499035 \angle 78^\circ 34' 04''}$$
$$= 0.9365365 \angle 1^\circ 36' 33''$$
$$= \text{IMPEDANCE CORRECTING FACTOR.}$$

CALCULATION FOR $\frac{\text{TANH}(\theta/2)}{\theta/2}$

$$\frac{\theta}{2} = 0.324\,951\,8 \angle 78^\circ 34' 04''$$
$$= 0.0644084 + j0.318\,504\,6$$
$$\text{SINH}\frac{\theta}{2} = 0.0612112\,2 + j0.313\,796\,3$$
$$= 0.319\,710\,7 \angle 78^\circ 57' 43''$$
$$\text{COSH}\frac{\theta}{2} = 0.951\,6754 + j0.020\,183\,2$$
$$= 0.951\,889\,4 \angle 1^\circ 12' 54''$$
$$\text{TANH}\frac{\theta}{2} = \frac{\text{SINH}(\theta/2)}{\text{COSH}(\theta/2)} = \frac{0.319\,710\,7 \angle 78^\circ 57' 43''}{0.951\,889\,4 \angle 1^\circ 12' 54''}$$
$$= 0.335\,869\,6 \angle 77^\circ 44' 49''$$
$$\frac{\text{TANH}(\theta/2)}{\theta/2} = \frac{0.335\,869\,6 \angle 77^\circ 44' 49''}{0.324\,951\,8 \angle 78^\circ 34' 04''}$$
$$= 1.033\,598 \,\overline{\backslash 0^\circ 49' 15''}$$
$$= \text{ADMITTANCE CORRECTING FACTOR.}$$

$$\underline{\text{CHECK}},\ \text{SINH } \theta = 2 \text{ SINH}\frac{\theta}{2} \text{ COSH}\frac{\theta}{2} = 2 \times .0319\,7107 \angle 78^\circ 57' 44'' \times 0.951\,8894 \angle 1^\circ 12' 54''$$
$$= 0.608\,6584 \angle 80^\circ 10' 38'' \text{ (WHICH CHECKS WITH *).}$$

FIG. 48—MATHEMATICAL DETERMINATION OF CORRECTING FACTORS FOR EQUIVALENT π SOLUTION

PROBLEM "X"

$$Z = 105 + j249 = 270.233 \angle 67^\circ 08' 08'' \text{ OHMS.}$$
$$Y = 0 + j0.001563 = 0.001563 \angle 90^\circ \text{ MHO.}$$
$$\theta = \sqrt{ZY} = 0.1288168 + j0.6370092 \text{ HYP.}$$
$$\text{KV-A}_{RN} = 6000\,000 \backslash 25^\circ 50' 31'' \text{ WATTS.}$$
$$= 5\,400\,000 - j2\,615\,340.$$
$$E_{RN} = 60\,044.4 \text{ VOLTS TO NEUTRAL.}$$
$$I_R = 99.92605 \backslash 25^\circ 50' 31''$$

SOLUTION FOR TANH θ_R

$$\delta = \frac{E_{RN}}{I_R} = 600.888 \angle 25^\circ 50' 31'' \text{ OHMS.}$$
$$Z_0 = \sqrt{\frac{Z}{Y}} = 415.805 \backslash 11^\circ 25' 56'' \text{ OHMS.}$$
$$\text{TANH } \theta_R = \frac{600.888 \angle 25^\circ 50' 31''}{415.805 \backslash 11^\circ 25' 56''}$$
$$= 1.44512 \angle 37^\circ 16' 27''$$
$$= 1.14995 + j0.875209 \quad *$$
$$= (\theta_1 + j\theta_2)$$

SOLUTION FOR ANGLE θ_R

$$\text{TANH}^{-1}(\theta_1 \pm j\theta_2) = \frac{1}{2} \text{ LOGH} \sqrt{\frac{(1+\theta_1)^2 + \theta_2^2}{(1-\theta_1)^2 + \theta_2^2}} + j \left[\frac{\pi - \text{TAN}^{-1}\left(\frac{\theta_1 + 1}{\pm \theta_2}\right) + \text{TAN}^{-1}\left(\frac{\theta_1 - 1}{\pm \theta_2}\right)}{2} \right]$$

$$\theta_R = \frac{1}{2} \text{ LOGH} \sqrt{\frac{(1+1.14995)^2 + 0.875209^2}{(1-1.14995)^2 + 0.875209^2}} + j \frac{\left(180^\circ - \text{TAN}^{-1}\frac{2.14995}{0.875209} + \text{TAN}^{-1}\frac{0.14995}{0.875209}\right)}{2}$$

$$= \frac{1}{2} \text{ LOGH} \sqrt{\frac{4.62229 + 0.76599}{0.022485 + 0.76599}} + j \frac{(180^\circ - 67^\circ 50' 58'' + 9^\circ 43' 20'')}{2}$$

$$= \frac{1}{2}(\text{LOGH } 2.61415) + j\,60^\circ 56' 11''$$
$$= \frac{1}{2}(0.960939) + j1.0635397$$

$$\theta_R = 0.4804695 + j1.0635397$$
$$\theta = \underline{0.1288168 + j0.6370092}$$
$$\theta_S = 0.6092863 + j1.7005489$$

CHECK

$$\text{SINH } \theta_R = 1.006572 \angle 76^\circ 03' 36''$$
$$\text{COSH } \theta_R = 0.69653 \angle 38^\circ 47' 09''$$
$$\text{TANH } \theta_R = \frac{\text{SINH } \theta_R}{\text{COSH } \theta_R} = \frac{1.006572 \angle 76^\circ 03' 36''}{0.69653 \angle 38^\circ 47' 09''}$$
$$= 1.14995 + j0.875209 \text{ (WHICH CHECKS WITH *).}$$

FIG. 49—POSITION ANGLE θ_R AT RECEIVING-END

Mathematical determination at load conditions.

CHOICE OF VARIOUS METHODS

Two graphical and two mathematical forms of solution for circuits of long electrical length have been described thus far. These four methods have been given for the purpose of providing a choice of procedure for the beginner. Graphical solutions are more simple and more readily performed than mathematical solutions and, if used correctly and made to a large scale, will yield results well within the limits of permissible error for power transmission circuits. There is always a possibility of error with any method, even though the solution is carefully checked. For this reason it is desirable that errors be guarded against by the use of two different forms of solution. For instance the first solution could be made by making use of the Wilkinson charts followed by its accompanying graphical solution. The second solution could then be made by means of Dr. Kennelly's ratio charts XVIII to XXI, followed by its accompanying graphical solution. These two methods would then yield results obtained by two entirely different routes and methods of procedure. The use of two such methods would constitute check against errors being made in either solution.

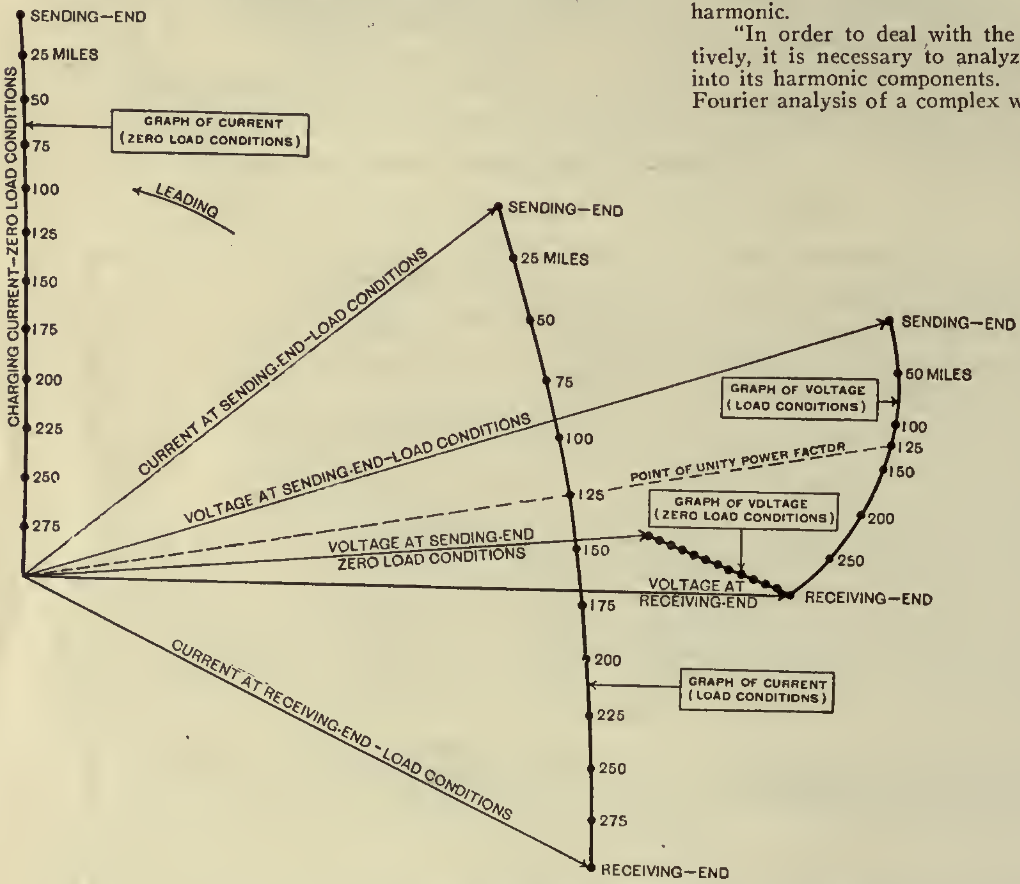


FIG. 52—POLAR DIAGRAM OF CURRENT AND VOLTAGE DISTRIBUTION FOR PROBLEM X

EFFECT OF HARMONIC CURRENTS AND VOLTAGES

The foregoing discussion is based upon the assumption that the fundamental wave is of sine shape and consequently free from harmonics. If harmonics of considerable magnitude are present in the fundamental wave, then it will be necessary to take their effect into account, if high accuracy is essential. In such a case there is an independent solution required of potential and current for each single frequency in turn, as though the others did not exist, and then the r.m.s. value at any point on the line is the perpendicular sum of the separate frequency values.

A detail discussion of the manner of including the effect of harmonic components in the current and voltage waves is quoted below from Dr. Kennelly's "Artificial Electric Lines."

"The ordinary complex harmonic impressed e.m.f. contains a fundamental frequency associated with multiple frequency harmonics. The *n*th multiple of the frequency is called the *n*th harmonic. The fundamental may thus be included as the first harmonic.

"In order to deal with the plural-frequency case quantitatively, it is necessary to analyze the impressed potential wave into its harmonic components. As is well known, the complete Fourier analysis of a complex wave may be written

$$V_0 + V'_1 \sin \omega t + V'_2 \sin 2\omega t + V'_3 \sin 3\omega t + V'_4 \sin 4\omega t + \ldots\ldots + V''_1 \cos \omega t + V''_2 \cos 2\omega t + V''_3 \cos 3\omega t + V''_4 \cos 4\omega t + \ldots\ldots \text{ volts} \quad (1)$$

where V_0 is a continuous potential, such as might be developed by a storage battery, ordinarily absent in an a. c. generator wave, V'_1, V''_1, V'_2, V''_2, etc., maximum cyclic amplitudes of the various sine and cosine components. The even harmonics are ordinarily negligible in an a. c. generator wave; so that V'_2, V''_2, V'_4, V''_4, etc., are ordinarily all zeros. If we count time from some moment when the fundamental component passes through zero in the positive direction, $V''_1 = 0$ and the series becomes

$$V'_1 \sin \omega t + V'_3 \sin 3\omega t + V'_5 \sin 5\omega t + \ldots\ldots + V''_3 \cos 3\omega t + V''_5 \cos 5\omega t + \ldots\ldots \text{ volts} \quad (2)$$

Compounding sine and cosine harmonic components into resultant harmonics of displaced phase, this may be expressed as

$$V_{r1} \sin \omega t + V_{r3} \sin (3\omega t + \beta_3{}^\circ) + V_{r5} \sin (5\omega t + \beta_5{}^\circ) + \ldots\ldots \text{ volts} \quad (3)$$

where $$V_{rn} = \sqrt{V'^2_n + V''^2_n} \quad \text{volts} \quad (4)$$

and $$\tan \beta_n{}^\circ = \frac{V''_n}{V'_n} \quad \text{numeric} \quad (5)$$

Formulas (1) and (2) give the wave analysis in *sine* and *cosine harmonics*, while (3) gives it in *resultant sine harmonics*.

"When considering a plural-frequency alternating-current line, we require to know the harmonic analysis of the impressed potential, either in sine and cosine harmonics, or in resultant harmonics, the latter analysis is preferable, as being shorter and containing fewer terms. A decision must be made as to the number of frequencies or upper harmonics which must be taken into account.

"Ordinarily, the sizes of the harmonics diminish as their order increases; but there are numerous exceptions to this rule, as when some particular tooth frequency in the alternating-current generator establishes a prominent size for that harmonic. Care must therefore be exercised not to exclude any important harmonics. On the other hand, the fewer the harmonics to be dealt with, the better, because the labor involved in correctly solving the problem increases in nearly the same ratio as the number of harmonics retained.

"The rule is to work out the position angle, r.m.s. potential and r.m.s. current distributions, over the artificial or conjugate smooth line, for each harmonic component in turn, as though it existed alone, and then to combine them, at each position, in the well-known way for root mean squares.

"*Combination of Components of Different Frequencies into a R.m.s. Resultant.*—Let the r.m.s. value of each alternating-current harmonic component be obtained by dividing its amplitude with $\sqrt{2}$ in the usual way, and let

$$V_n = \frac{V_{rn}}{\sqrt{2}} = \sqrt{\frac{V'^2_n + V''^2_n}{2}} \quad \text{r.m.s. volts} \quad (6)$$

be the r.m.s. value of the *n*th harmonic. Then the r.m.s. value of all the harmonics together, over any considerable number of cycles, will be

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2 + \ldots\ldots\ldots} \quad \text{r.m.s. volts} \quad (7)$$

or, as is well known, the joint r.m.s. value of a plurality of r.m.s. values of different frequency, is the square root of the sum of their squares. If a continuous potential V_0 be present, this may be regarded as a r.m.s. harmonic of zero frequency, and be included thus:

$$V = \sqrt{V_0^2 + V_1^2 + V_2^2 + V_3^2 + \ldots} \quad \text{r.m.s. volts} \quad (8)$$

Moreover, from (4), it is evident that the squares of the r.m.s. values of the sine and cosine terms of any harmonic may be

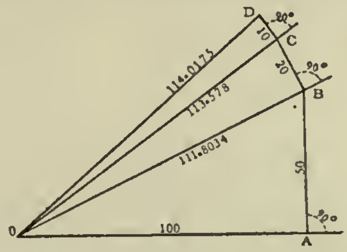


FIG. 53—GEOMETRICAL REPRESENTATION OF A JOINT R.M.S. VALUE OF PLURAL-FREQUENCY COMPONENTS BY PERPENDICULAR SUMMATION OR "CRAB ADDITION"

substituted for the square of their resultant; or that, in this respect, the sine and cosine terms may be treated as though they were components of different frequencies.

"The same procedure applies to plural-frequency currents. Find the r.m.s. resultant harmonics. The r.m.s. value of all together will be the square root of the sum of their squares. A continuous current, if present, may be included, as the r.m.s. value of an alternating current of zero frequency.

"*Graphical Representation of R.m.s. Plural-frequency Combination.*—The process represented algebraically in (7) or (8) may be represented graphically by the process of successive perpendicular summation, or "crab addition." An example will suffice to make this clear. A fundamental alternating current of 100 amp. r.m.s., is associated with a continuous current of 50 amp., and with two other alternating currents of other frequencies of 20 and 10 amp. r.m.s., respectively. What will be the joint r.m.s. current? Here by (8),

$$I = \sqrt{100^2 + 50^2 + 20^2 + 10^2} = \sqrt{10\,000 + 2500 + 400 + 100}$$
$$= \sqrt{13\,000} = 114.0175 \text{ amp. r.m.s.}$$

"In Fig. 53, *OA* represents the fundamental r.m.s. current. *AB*, added perpendicularly to *OA* represents the continuous current, or current of 50 r.m.s. amp. at zero frequency. The perpendicular sum of *OA* and *AB* is *OB* = 111.8034 amp. Adding similarly the other frequency components *BC* and *CD*, the total perpendicular sum is *OD* = 114.0175 amp. The order in which the components are added manifestly does not affect the final result, and it is a matter of insignificance whether the various frequencies coacting are "harmonic," *i. e.*, are integral multiples of a fundamental, or not, so long as they are different.

"The complete solution of an alternating-current line with complex harmonic potentials and currents thus requires an independent solution of potential and current for each single frequency in turn, as though the others were non-existent, and then the r.m.s. value at any point on the line is the perpendicular sum of the separate frequency values. The powers and energies of the different frequencies are independent of each other, and the total transmitted energy is the sum of the energies transmitted at the separate component frequencies."

BIBLIOGRAPHY

In order to give due prominence to some of the valuable contributions on the subject of performance of electrical circuits and as an acknowledgment to their authors of the assistance received from a study of them, the following publications are suggested as representing a very helpful and valuable addition to the library of the transmission engineer. They are given in the approximate order of their publication:—

Calculation of the High Tension Line and Output and Regulation in Long Distance Lines by Percy H. Thomas. (Published in *A. I. of E. E. Trans.* Vol. XXVIII, Part, 1, 1909). The former paper introduces a so-called "wave formula" for determining the performance of long lines having considerable capacity which embodies the use of algebra only. The second paper suggests the use of split conductors in order to adjust the ratio of the capacity and inductance of the line so that the leading and lagging components more nearly neutralize each other.

Formulae, Constants and Hyperbolic Constants by W. E. Miller. (Published in *G. E. Review,* supplement dated May 1910). This is a treatise upon the subject wherein hyperbolic functions of complex angles are tabulated for sinh and cosh (x + jy) up to x = 1, y= 1 in steps of 0.02.

Transmission Line Formulas by H. B. Dwight. (Published by John Wiley & Sons, Inc.). This book introduces what are known as "Dwight's '*K*' formulas," which permit the solution of transmission problems without the use of mathematics higher than arithmetic. It also contains working formulas based upon convergent series and the solution of many problems both by the *K* formulas and by convergent series.

Tables of Complex Hyperbolic and Circular Functions by Dr. A. E. Kennelly. (Published by the Harvard University Press). This book gives functions of complex angles for polar values up to 3.0 by steps of 0.1 and for angles from 45° to 90° by steps of one degree; also functions in terms of reactangular coordinates x + jy to x = 10 by steps of 0.05 and of y virtually to infinity by steps of 0.05.

Chart Atlas of Complex Hyperbolic and Circular Functions by Dr. A. E. Kennelly. (Published by Harvard University Press in large charts, 48 by 48 cm.) Presenting curves for all the tables published in above referred to "Tables of Complex Hyperbolic and Circular Functions" for rapid graphical interpolation.

Constant Voltage Transmission by H. B. Dwight. (Published by John Wiley & Son, Inc.). Embraces a very complete study of the use of over-excited synchronous motors for controlling the voltage of transmission.

The Application of Hyperbolic Functions to Electrical Engineering Problems by Dr. A. E. Kennelly. (Published by the McGraw-Hill Book Company). Every student should have a copy of this book because of its simplicity and completeness in explaining the application of hyperbolic functions to transmission circuit problems. It also contains a very complete bibliography of publications upon this general subject.

Artificial Electric Lines by Dr. A. E. Kennelly. (Published by McGraw-Hill Book Co.). This is a valuable treatise in which the subject is treated in accordance with the hyperbolic theory.

Electrical Phenomena in Parallel Conductors by Dr. F. E. Pernot. (Published by John Wiley & Son, Inc.). Being a very recent treatise, this book contains much practical and many readily understandable explanations for both the beginner and those further advanced in the study of this subject. It contains a six-place table of logarithms of real hyperbolic functions for values of *x* from 0.000 to 2.000 for intervals of 0.001 in the argument. This is the most complete table of real hyperbolic functions which the author has seen.

TABLE P—SUBDIVISIONS OF A DEGREE

SECONDS TO DEGREES		MINUTES TO DEGREES		DEGREES TO MINUTES AND SECONDS					
″=	°	′=	°	°=	′	″	°=	′	″
01	0.0003	01	0.0167	0.001	00	03.6	0.006	00	21.6
02	0.0006	02	0.0333	0.002	00	07.2	0.007	00	25.2
03	0.0008	03	0.0500	0.003	00	10.8	0.008	00	28.8
04	0.0011	04	0.0667	0.004	00	14.4	0.009	00	32.4
05	0.0014	05	0.0833	0.005	00	18.0	0.010	00	36.0
06	0.0017	06	0.1000						
07	0.0019	07	0.1167						
08	0.0022	08	0.1333						
09	0.0025	09	0.1500						
10	0.0028	10	0.1667						
11	0.0031	11	0.1833	0.01	00	36	0.51	30	36
12	0.0033	12	0.2000	0.02	01	12	0.52	31	12
13	0.0036	13	0.2167	0.03	01	48	0.53	31	48
14	0.0039	14	0.2333	0.04	02	24	0.54	32	24
15	0.0042	15	0.2500	0.05	03	00	0.55	33	00
16	0.0044	16	0.2667	0.06	03	36	0.56	33	36
17	0.0047	17	0.2833	0.07	04	12	0.57	34	12
18	0.0050	18	0.3000	0.08	04	48	0.58	34	48
19	0.0053	19	0.3167	0.09	05	24	0.59	35	24
20	0.0055	20	0.3333	0.10	06	00	0.60	36	00
21	0.0058	21	0.3500	0.11	06	36	0.61	36	36
22	0.0061	22	0.3667	0.12	07	12	0.62	37	12
23	0.0064	23	0.3833	0.13	07	48	0.63	37	48
24	0.0067	24	0.4000	0.14	08	24	0.64	38	24
25	0.0069	25	0.4167	0.15	09	00	0.65	39	00
26	0.0072	26	0.4333	0.16	09	36	0.66	39	36
27	0.0075	27	0.4500	0.17	10	12	0.67	40	12
28	0.0078	28	0.4667	0.18	10	48	0.68	40	48
29	0.0081	29	0.4833	0.19	11	24	0.69	41	24
30	0.0083	30	0.5000	0.20	12	00	0.70	42	00
31	0.0086	31	0.5167	0.21	12	36	0.71	42	36
32	0.0089	32	0.5333	0.22	13	12	0.72	43	12
33	0.0092	33	0.5500	0.23	13	48	0.73	43	48
34	0.0094	34	0.5667	0.24	14	24	0.74	44	24
35	0.0097	35	0.5833	0.25	15	00	0.75	45	00
36	0.0100	36	0.6000	0.26	15	36	0.76	45	36
37	0.0103	37	0.6167	0.27	16	12	0.77	46	12
38	0.0106	38	0.6333	0.28	16	48	0.78	46	48
39	0.0108	39	0.6500	0.29	17	24	0.79	47	24
40	0.0111	40	0.6667	0.30	18	00	0.80	48	00
41	0.0114	41	0.6833	0.31	18	36	0.81	48	36
42	0.0117	42	0.7000	0.32	19	12	0.82	49	12
43	0.0119	43	0.7167	0.33	19	48	0.83	49	48
44	0.0122	44	0.7333	0.34	20	24	0.84	50	24
45	0.0125	45	0.7500	0.35	21	00	0.85	51	00
46	0.0128	46	0.7667	0.36	21	36	0.86	51	36
47	0.0130	47	0.7833	0.37	22	12	0.87	52	12
48	0.0133	48	0.8000	0.38	22	48	0.88	52	48
49	0.0136	49	0.8167	0.39	23	24	0.89	53	24
50	0.0139	50	0.8333	0.40	24	00	0.90	54	00
51	0.0141	51	0.8500	0.41	24	36	0.91	54	36
52	0.0144	52	0.8667	0.42	25	12	0.92	55	12
53	0.0147	53	0.8833	0.43	25	48	0.93	55	48
54	0.0150	54	0.9000	0.44	26	24	0.94	56	24
55	0.0153	55	0.9167	0.45	27	00	0.95	57	00
56	0.0156	56	0.9333	0.46	27	36	0.96	57	36
57	0.0159	57	0.9500	0.47	28	12	0.97	58	12
58	0.0162	58	0.9667	0.48	28	48	0.98	58	48
59	0.0164	59	0.9833	0.49	29	24	0.99	59	24
60	0.0167	60	1.0000	0.50	30	00	1.00	60	00

EXAMPLES

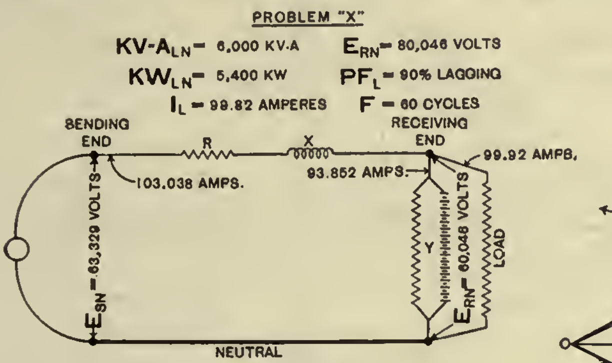
$0^\circ.41 = 0^\circ\,24'\,36''$ $\qquad 0^\circ 41'\,00'' = 0^\circ.6833.$

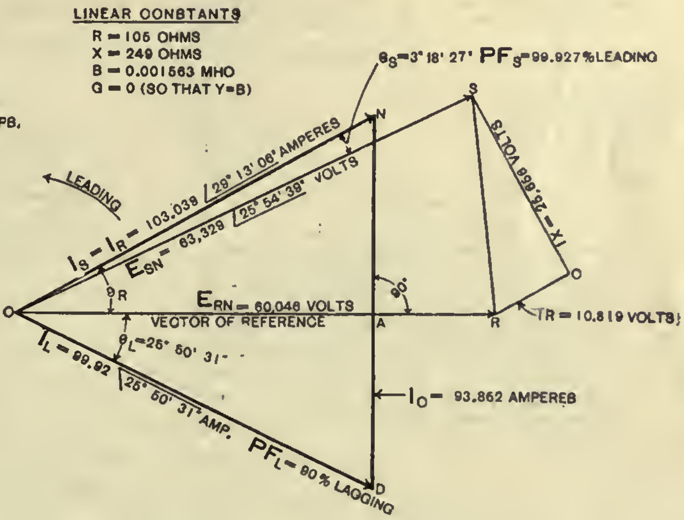
$0^\circ.005 = 0^\circ\,00'\,18''$ $\qquad 0^\circ 00'\,46'' = 0^\circ.0128.$

CHAPTER XII
COMPARISON OF VARIOUS METHODS

The "localized capacitance" or "localized admittance" methods are discussed below for the two following reasons. A discussion of them is of academic interest and a tabulation of the magnitude of the errors in the results as obtained by these approximate methods when applied to circuits of different lengths and frequencies should be helpful. These methods may be carried out either graphically or mathematically, but since they are only approximate the simpler graphical solution should suffice. Their principle virtue is the fact that they simplify the determination of performance, but this is obtained at the expense of accuracy. The more accurate of these methods is somewhat tedious to carry out. The graphical solution previously described in connection with the Wilkinson charts will be generally more accurate and shorter than these localized capacitance methods.

THE LOCALIZED CAPACITANCE methods are:—the single end condenser method; the middle condenser or T method; the split condenser or nominal π method and Dr. Steinmetz three condenser method. These four lumped capacitance methods assume the total capacitance of the circuit as being divided up and "lumped" in the form of condensers shunted across the circuit at one or more points.





RESULTS CALCULATED BY

RIGOROUS SOLUTION	SINGLE END CONDENSER METHOD	% ERROR
E_{SN} = 70,662 VOLTS	63,329 VOLTS	− 10.37%
I_S = 94.75 AMPS.	103.038 AMPS	+ 8.8%
PF_S = + 93.42%	+ 99.927 %	+ 7.0%
$LOSS_N$ = 855 KW	1120 KW	+ 30.9%

FIG. 54—SINGLE END CONDENSER METHOD
Problem X.

The single condenser method assumes the total capacitance as being lumped or shunted across the circuit at the receiving-end. On this assumption the total charging current for the circuit would flow over the entire circuit. Actually the charging current is distributed along the circuit so that the entire charging current does not flow over the entire circuit. Obviously the assumption of the total capacitance being lumped at the receiving-end will therefore give over compensation for the effect of the charging current upon the voltage regulation of the circuit. This method of solution yields a voltage too low at the sending end by nearly the same amount that the straight impedance method gives it too high. By averaging the values, as obtained by the impedance and single end condenser methods, usually an approximation to the true value may be obtained.

The middle condenser or T method assumes that the total capacitance may be shunted across the circuit at the middle point. On this assumption the total charging current will flow over one half the length of the circuit. This method is therefore more nearly accurate than the single-condenser method.

The split condenser or π method assumes one half the capacitance being shunted across the circuit at each end. In this case one-half of the charging current flows over the entire circuit. This assumed distribution of the charging current also more nearly represents the actual distribution than the single-condenser method.

Dr. Steinmetz has proposed a method assuming three condensers shunted across the circuit. One in the middle, of two-thirds, and one at each end, each of one sixth the total capacitance of the circuit. This method is equivalent to assuming that the electrical quantities are distributed along the circuit in a way representing an arc of a parabola. This method assumes one-sixth the charging current flowing over one half the entire circuit and five sixth the charging current flowing over the other half of the circuit. This method gives quite accurate results unless the circuit is very long and the frequency high.

Figs. 54-57 show leaky condensers placed at different points of the circuits, that is they indicate that there is a leak G, as well as a susceptance B. For simplicity pure condensers have been assumed in the accompanying calculations; that is we have assumed $G=0$. This is the usual assumption in such cases, for the reason that G is usually very small, and localized capacitance methods are approximations at best. In the equivalent π solution previously given, we have indicated the treatment when the condensers have a leak. In such case, however, the equivalent π method produces exact results, and the nature of such solution may demand a condenser having a material leak.

AUXILIARY CONSTANTS

Mr. T. A. Wilkinson and Dr. Kennelly have worked out the algebraic expressions for the auxiliary constants corresponding to these four circuits of localized capacitance. These are given in Table Q. It may be interesting to observe to what extent each of the four localized capacitance methods takes account of the three linear line constants R, X and B. The rigorous or exact expression for the auxiliary constants is given under Table Q for comparison with the values corresponding to the localized condenser methods. The numerals under the algebraic expressions correspond to problem X; that is, to a certain 60 cycle circuit, 300 miles long. They are given to illustrate for a long circuit, the account taken of the fundamental constants for each of the five methods listed. These numerals may be compared with the rigorous or exact values as given under the rigorous expressions at the bottom of the table.

SINGLE END CONDENSER METHOD

This method assumes that the total capacitance of the circuit may be concentrated across the circuit at the receiving-end. In such case the entire charging current would flow over the total length of the circuit.

Solution by Impedance Method—The diagrams of connections and corresponding graphical vector solution for problem X by the single end condenser method is indicated by Fig. 54. The current DN consumed by the condenser (zero leakage assumed) leads the receiving-end voltage OR by 90 degrees and is,—

$$I_c = 0.001563 \times 60.046 = 93.852 \textit{ amperes.}$$

The load current of 99.92 amperes, lagging 25° 50′ 30″ (90% power-factor) has a component OA of $99.92 \times 0.90 = 89.928$ *amperes* in phase with the receiving-end voltage and a component AD of $99.92 \times 0.4359 = 43.555$ amperes in lagging quadrature with the receiving-end voltage. This lagging component is therefore in opposite direction to the charging current, the effect of which is to neutralize an equivalent amount of charging current. The remaining current AN in leading quadrature with the receiving-end voltage is $93.852 - 43.555 = 50.297$ amperes. The current ON in the conductor is therefore:—

$$I_r = \sqrt{(89.928^2 + (50.297)^2} = 103.038 \textit{ amperes.}$$

The current at the sending-end leads the voltage at the receiving-end by the angle θ_R whose tangent is,—

$$\frac{50.297}{89.928} = 29^\circ\ 13'\ 06''$$

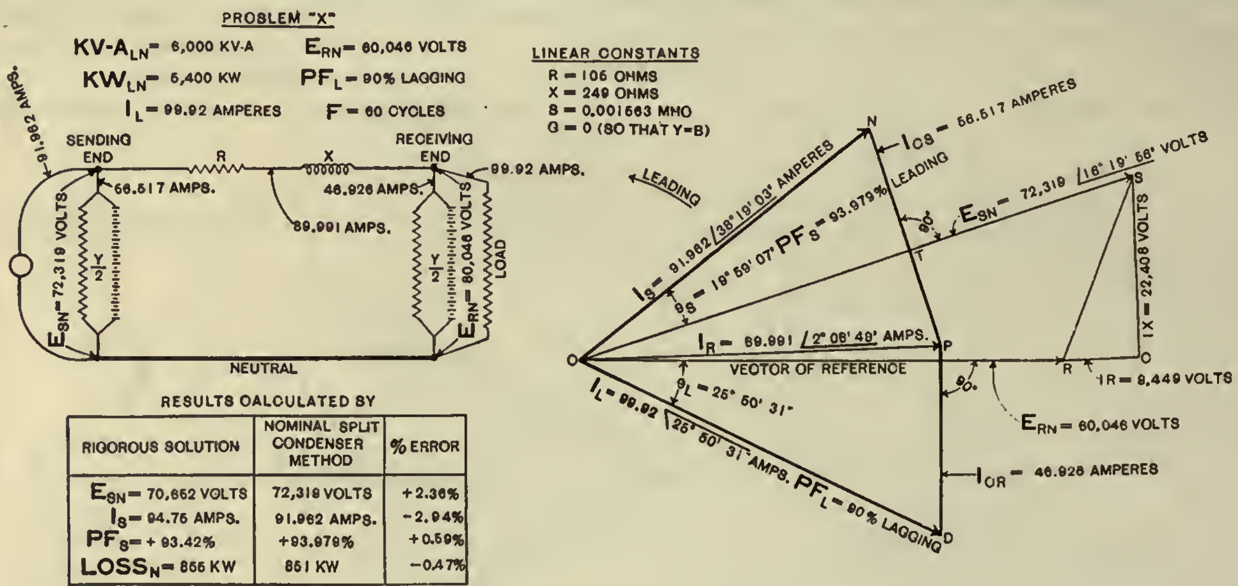
The voltage consumed by the resistance, and the reactance of each conductor is,—

$$IR = 103.038 \times 105 = 10\,819 \textit{ Volts (resistance drop)}$$
$$IX = 103.038 \times 249 = 25\,656 \textit{ Volts (reactance drop)}$$

The receiving-end conditions are thus,—

$$I_R = 103.038 \textit{ amperes}$$
$$\theta_R = 29^\circ\ 13'\ 06''$$
$$Cos\ \theta_R = 0.8772$$
$$Sin\ \theta_R = 0.4881$$

and from (40)



RESULTS CALCULATED BY

RIGOROUS SOLUTION	NOMINAL SPLIT CONDENSER METHOD	% ERROR
E_{SN} = 70,662 VOLTS	72,318 VOLTS	+2.36%
I_S = 94.75 AMPS.	91.962 AMPS.	−2.94%
PF_S = +93.42%	+93.979%	+0.59%
$LOSS_N$ = 855 KW	851 KW	−0.47%

FIG. 55—NOMINAL π OR SPLIT CONDENSER METHOD
Problem X.

$E_{sn} =$
$\sqrt{(60\,046 \times 0.8727 + 10\,819)^2 + (60\,046 \times 0.4881 - 25\,656)^2}$
$= 63\,329 \; \overline{\backslash 3° \; 18' \; 27''}$ *volts to vector ON*
$= 63\,329 \; \underline{/25° \; 54' \; 39''}$ *volts to vector of reference.*
$PF_s = Cos \; \underline{/3° \; 18' \; 27''} = 99.927$ *percent leading.*
$KV\text{-}A_{sn} = 103.038 \times 63.329 = 6525$ *kv-a.*
$KW_{sn} = 6525 \times 0.99927 = 6520$ *kw.*
$Loss_n = 6520 - 5400 = 1120$ *kw.*

Solution by Complex Quantities—From Table Q the auxiliary constants corresponding to the single end condenser method are found as follows:—

$a_1 = 1 - XB = 0.610813$
$a_2 = RB = 0.164115$
$b_1 = R = 105$ *ohms.*
$b_2 = X = 249$ *ohms.*
$c_1 = O$
$c_2 = B = 0.001563$ *mho.*

The voltage at the sending end is determined as follows:—

$$\begin{aligned} I_L \,(Cos\,\theta_L - j\,Sin\,\theta_L) &= 89.928 - j\,43.555 \\ \times\,(b_1 + j\,b_2) &= 20\,286 + j\,17\,819 \\ +E_{rn}\,(a_1 + j\,a_2) &= 36\,677 + j\,\;\,9854 \\ \hline E_{sn} &= 56\,963 + j\,27\,673 \\ &= 63\,329 \; \underline{/25° \; 54' \; 39''} \; volts. \end{aligned}$$

which checks exactly with the results as obtained previously by the impedance method.

The current at the sending end may be determined as follows:—

$$\begin{aligned} I_L\,(Cos\,\theta_L - j\,Sin\,\theta_L) &= 89.928 - j\,43.555 \\ +\,E_{rn}\,(c_2 + j\,c_1) &= \;\;\;\;\;\;\;\;0 + j\,93.852 \\ \hline I_s &= 89.928 + j\,50.297 \\ &= 103.038 \; \underline{/29° \; 13' \; 06''} \; amperes. \end{aligned}$$

which also checks exactly with the result as previously determined by the impedance method.

It should be noted here that in determining the sending-end current, the auxiliary constant $(a + j\,a_2$ did not enter into the calculation as it does in the rigorous solution; this is owing to the inherent dissymmetry of the single-end condenser. This is the only case in which the capacitance is applied dissymmetrically, consequently the current entering the line at the sending-end is completely determined by the load current at the receiving-end and the vector addition thereto of the current supplied at that end to the condenser under receiving-end voltage. For determining the sending-end voltage $A'_V = 1 + YZ$ and $B'_V = Z$; but for determining the sending-end current $A'_I = 1$ and $C'_I = Y$. If the condenser were applied symmetrically A'_V *and* A'_I would be identical.

SPLIT CONDENSER OR NOMINAL π SOLUTION

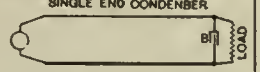
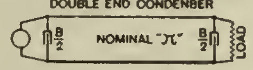
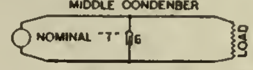
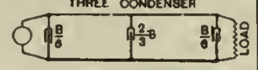
This method assumes that the total capacitance of the circuit may be concentrated at the two ends, one-half being placed across the circuit at either end. In this case one-half the charging current flows over the entire circuit. The total resistance and the total reactance of one conductor is placed between the two terminal condensers.

With this assumption the current consumed by the condenser across the receiving-end of the circuit is added vectorially to the load current and the power-factor of the combined currents calculated. With these new load conditions determined the conditions at the sending-end are calculated by the impedance method. This is the only calculation required when employing the nominal π method for determining the sending-end voltage. The voltage at the sending-end is therefore more readily calculated by this method than by the T method which requires the calculation of the two separate halves of the circuit. If, however, the current, power-factor and kw input are required, a second calculation must be made to determine them. In such cases the current consumed by the condenser at the sending-end must be added vectorially to that of the line conductors.

Solution by Impedance Method—The diagrams of connections and corresponding graphical vector solutions for problem X by the nominal π method is indicated in Fig. 55. The charging current consumed by the condenser (zero leakage assumed) at the receiving-

TABLE Q—AUXILIARY CONSTANTS

CORRESPONDING TO CIRCUITS OF LOCALIZED CAPACITANCE

METHOD	a_1	a_2	b_1	b_2	c_1	c_2	EQUIVALENT CONVERGENT SERIES FORM OF EXPRESSION ★
IMPEDANCE	1	0	R =105	X =+j249	0	0	$A'=1$ $B'=Z$ $C'=0$
SINGLE END CONDENSER 	$1-XB$ =0.610613	RB =+j0.184115	R =106	X =+j249	0	B =+j0.001663	$A'=1+YZ$ $B'=Z$ $C'=Y$
DOUBLE END CONDENSER 	$1-\frac{XB}{2}$ =0.6064066	$\frac{RB}{2}$ =+j0.082066	R =106	X =+j249	$-\frac{B^2R}{4}$ =−0.000 0641	$B-\frac{B^2X}{4}$ =+j0.001411	$A'=(1+\frac{YZ}{2})$ $B'=Z$ $C'=Y(1+\frac{YZ}{4})$
MIDDLE CONDENSER 	$1-\frac{XB}{2}$ =0.6064066	$\frac{RB}{2}$ =+j0.082068	$R-\frac{RXB}{2}$ =84.6677	$X-\frac{B}{4}(X^2-R^2)$ =+j229.081	0	B =+j0.001663	$A'=(1+\frac{YZ}{2})$ $B'=Z(1+\frac{YZ}{4})$ $C'=Y$
THREE CONDENSER 	$1-\frac{XB}{2}+\frac{B^2}{36}(X^2-R^2)$ =0.606866	$\frac{RB}{2}-\frac{RXB^2}{18}$ =+j0.0766091	$R-\frac{RXB}{3}$ =91.3766	$X-\frac{B}{6}(X^2-R^2)$ =+j236.721	$-\frac{6RB^2}{36}+\frac{RXB^3}{108}$ =−0.000 0347	$B-\frac{5XB^2}{36}+\frac{B^3}{216}(X^2-R^2)$ +j0.0014794	$A'=(1+\frac{YZ}{2}+\frac{Y^2Z^2}{36})$ $B'=Z(1+\frac{YZ}{6})$ $C'=Y(1+\frac{6YZ}{36}+\frac{Y^2Z^2}{216})$

★ THE EXACT OR RIGOROUS EXPRESSIONS FOR THE AUXILIARY CONSTANTS ARE GIVEN BELOW THE NUMERICAL FIGURES CORRESPOND TO PROBLEM "X"

$A = (1+\frac{YZ}{2}+\frac{Y^2Z^2}{24}+\frac{Y^3Z^3}{720}+\frac{Y^4Z^4}{40\,320}+)$ $= COSH\,\theta = 0.61066 + j0\,07693$

$B = Z(1+\frac{YZ}{6}+\frac{Y^2Z^2}{120}+\frac{Y^3Z^3}{5{,}040}+\frac{Y^4Z^4}{362{,}880}+)$ $= Z_o\,SINH\,\theta = 91\,7466 + j236.866$

$C = Y(1+\frac{YZ}{6}+\frac{Y^2Z^2}{120}+\frac{Y^3Z^3}{5{,}040}+\frac{Y^4Z^4}{362{,}880}+)$ $= Y_o\,SINH\,\theta = -0.000\,0411 + j\,0.0014634$

end of the circuit leads the receiving-end voltage by 90 degrees and is,—

$$I_{cr} = \frac{0.001563}{2} \times 60\,046 = 46.926 \text{ amperes.}$$

The current I_r in each conductor is the vector sum of the load and condenser currents and may be determined as follows:—

$$I_r = \sqrt{(99.92 \times 0.90)^2 + (I_{cr} + 99.92 \times -0.4359)^2}$$
$$= 89.991 \angle 2^\circ\ 08'\ 48'' \text{ amperes.}$$
$$PF_r = Cos\ 2^\circ\ 08'\ 48'' = 99.33 \text{ percent leading.}$$

The voltage consumed by the resistance, and the reactance of each conductor is,—

$$IR = 89.991 \times 105 = 9449 \text{ volts (resistance drop)}$$
$$IX = 89.991 \times 249 = 22\,408 \text{ volts (reactance drop)}$$

and from (40),—

$$E_{sn} = \sqrt{(60\,046 \times 0.9933 + 9449)^2 + (60\,046 \times 0.037458 - 22\,408)^2}$$
$$= 72\,319 \angle 16^\circ\ 11'\ 08'' \text{ volts to current vector } OP.$$
$$= 72\,319 \angle 18^\circ\ 19'\ 56'' \text{ volts to vector of reference } OR.$$

The charging current consumed by the condenser at the sending-end (zero leakage assumed) leads the voltage at the sending-end by 90° and is,—

$$I_{cs} = \frac{0.001563}{2} \times 72\,319 = 56.517 \text{ amperes.}$$

The current at the sending-end is the vector sum of the current in the conductor and the current consumed by the condenser at the sending-end. It may be calculated as follows:—

$$OT = 89.991\ (Cos\ 16^\circ\ 11'\ 08'') = 86.424 \text{ amperes.}$$
$$TP = 89.991\ (Sin\ 16^\circ\ 11'\ 08'') = 25.085 \text{ amperes.}$$
$$TN = 56.517 - 25.085 = 31.432 \text{ amperes.}$$

therefore,—

$$I_s = \sqrt{86.424^2 + 31.432^2}$$
$$= 91.962 \angle 19^\circ\ 59'\ 07'' \text{ amperes to vector } OS.$$
$$= 91.962 \angle 38^\circ\ 19'\ 03'' \text{ to vector of reference } OR.$$
$$PF_s = Cos\ 19^\circ\ 59'\ 07'' = 93.979 \text{ percent leading.}$$
$$KV\text{-}A_{sn} = 91.962 \times 72.319 = 6651 \text{ kv-a.}$$
$$KW_{sn} = 6651 \times 0.93979 = 6251 \text{ kw.}$$
$$Loss_s = 6251 - 5400 = 851 \text{ kw.}$$
$$Eff = \frac{5400 \times 100}{6251} = 86.37 \text{ percent.}$$

Solution by Complex Quantities—From Table *Q* the auxiliary constants corresponding to the nominal π method of solution are found as follows:—

$$a_1 = 1 - \frac{XB}{2} = 0.8054065.$$
$$a_2 = \frac{RB}{2} = 0.0820575.$$
$$b_1 = R = 105 \text{ ohms.}$$
$$b_2 = X = +j\ 249 \text{ ohms.}$$
$$c_1 = -\frac{B^2R}{4} = -0.0000641 \text{ mho.}$$
$$c_2 = B - \frac{B^2X}{4} = 0.001411 \text{ mho.}$$

The voltage at the sending-end is determined as follows:—

$$I_L\ (Cos\ \theta_L - j\ sin\ \theta_L) = 89.928 - j43.555.$$
$$\times\ (b_1 + jb_2) = 20\,286 + j17\,819 \text{ volts.}$$
$$+\ E_{rn}\ (a_1 + ja_2) = 48\,361 + j\ 4927 \text{ volts.}$$
$$E_{sn} = 68\,647 + j22\,746.$$
$$= 72\,319 \angle 18^\circ\ 19'\ 56'' \text{ volts.}$$

The current at the sending-end may be determined as follows:—

$$I_L\ (Cos\ \theta_L - j\ sin\ \theta_L) = 89.928 - j43.555.$$
$$\times\ (a_1 + ja_2) = +76.003 - j27.700 \text{ amperes.}$$
$$+\ E_{rn}\ (C_1 + jC_2) = -3.849 + j84.718 \text{ amperes.}$$
$$I_s = 72.154 + j57.018.$$
$$= 91.962 \angle 38^\circ\ 19'\ 03'' \text{ amperes.}$$

The above results check exactly with those previously obtained by impedance calculations. This agreement indicates that the nominal π solution may, if desired, be used with complex quantities, assuming values for the auxiliary constants as indicated in Table *Q*.

Convergent Series Expression—Table *Q* indicates that the nominal π solution is equivalent to using the following values for the auxiliary constants in the convergent series form of solution,—

$$A' = \left(1 + \frac{YZ}{2}\right), \quad B' = Z, \quad C' = Y\left(1 + \frac{YZ}{4}\right)$$

We will now show that the above expressions yield the same values for the auxiliary constants as given in Table *Q*. From chart *XI* the following values corresponding to problem *X* are taken.

$$ZY = -0.389187 + j\,0.164115$$

therefore,

$$A' = 1.0000000$$
$$-0.1945935 + j\,0.0820575$$
$$A' = 0.8054065 + j\,0.0820575$$
$$B' = 105 + j\,249$$
$$C' = 1.000000$$
$$-0.0972967 + j\,0.0410287$$
$$= Y\ (0.9027033 + j\,0.0410287)$$
$$C' = -0.0000641 + j\,0.001411$$

Thus the values for the auxiliary constants as determined by the above incomplete convergent series expression check with those as determined above from the equations in Table *Q*.

MIDDLE CONDENSER OR NOMINAL T METHOD

THIS METHOD assumes that the total capacitance of the circuit may be concentrated at its middle point. In such a case the entire charging current would flow over half of the circuit. The resistance and the reactance on each side of the capacitance or condenser is equal respectively to half the total conductor resistance and conductor reactance.

From an inspection of the diagram of such a circuit, Fig. 56, it is evident that two calculations will be required. Starting with the known receiving-end conditions, the conditions at the middle of the circuit are first calculated by the simple impedance method. To these calculated results the current consumed by the condenser shunted across the middle of the circuit must be vectorially added. This will give the load condition at the middle of the circuit from which the sending-end conditions may be calculated.

Solution by Impedance Method—The diagram of connections and the corresponding graphical vector solution for problem *X* by the nominal T method is indicated by Fig. 56. The electrical conditions at the middle of the circuit may be determined as follows:—

$$I_R \frac{R}{2} = 99.92 \times 52.5 = 5246 \textit{ volts (resistance drop)}$$

$$I_R \frac{X}{2} = 99.92 \times 124.5 = 12\,440 \textit{ volts (reactance drop)}$$

$$E_{mn} = \sqrt{(60\,046 \times 0.9 + 5246)^2 + (60\,046 \times 0.4359 + 12\,440)^2}$$
$$= 70\,753 \; \underline{/33^\circ\ 04'\ 36''} \textit{ to current vector } OD$$
$$= 70\,753 \; \underline{/7^\circ\ 14'\ 05''} \textit{ to vector of reference } OR$$

The current consumed by the condenser (zero leakage assumed) leads the voltage *OM* at the middle of the circuit by 90 degrees and is:—

$$I_c = 0.001\,563 \times 70\,753 = 110.587 \textit{ amperes}$$

The voltage consumed by the condenser current flowing back to the sending-end is:—

$$I_c \frac{R}{2} = 110.587 \times 52.5 = 5806 \textit{ volts (resistance drop)} = FC$$

$$I_c \frac{X}{2} = 110.587 \times 124.5 = 13\,768 \textit{ volts (reactance drop)} = FM$$

The voltage vector *OC* upon which the impedance triangle corresponding to the receiving-end load current $I_R = I_L$ flowing over the sending-end half of the circuit is constructed, may be found as follows:—

$$OC = \sqrt{(70\,753 - 13\,768)^2 + 5806^2}$$
$$= 57\,280 \; \underline{/5^\circ\ 49'\ 03''} \textit{ volts to vector } OM$$
$$= 57\,280 \; \underline{/13^\circ\ 03'\ 08''} \textit{ volts to vector of reference } OR$$

The voltage *OC* leads the receiving-end current *OD* by the angle 33° 04′ 36″ + 5° 49′ 03″ = 38° 53′ 39″ which angle corresponds to a power-factor of 77.831 percent. The voltage at the sending-end will therefore be:—

$$E_{sn} = \sqrt{(57\,280 \times 0.77831 + 5246)^2 + (57\,280 \times 0.62788 + 12\,440)^2}$$
$$= 69\,467 \; \underline{/44^\circ\ 10'\ 14''} \textit{ volts to vector } OD$$
$$= 69\,467 \; \underline{/18^\circ\ 19'\ 43''} \textit{ volts to vector of reference } OR$$

If desired, the receiving-end current and the condenser current may be combined and the corresponding impedance triangle for the sending-end half of the circuit constructed on the end of vector *OM* as indicated by the dotted lines.

The current at the sending-end may be determined as follows:—

$$OB = 99.92 \cos 33^\circ\ 04'\ 36'' = 83.727 \textit{ amperes.}$$
$$BD = 99.92 \sin 33^\circ\ 04'\ 36'' = 54.532 \textit{ amperes.}$$
$$BN = 110.587 - 54.532 = 56.055 \textit{ amperes.}$$
$$I_s = ON = \sqrt{(83.727)^2 + (56.055)^2}$$
$$= 100.76 \; \underline{/33^\circ\ 48'\ 06''} \textit{ amperes to vector } OB.$$
$$= 100.76 \; \underline{/41^\circ\ 02'\ 11''} \textit{ amperes to vector of reference } OR.$$

The current at the sending-end leads the voltage at the sending-end by the angle 41° 02′ 11″ — 18° 19′ 43″ = 22° 42′ 28″, which corresponds to a power-factor at the sending-end of 92.25 percent leading.

The power at the sending-end is:—

$$Kv\text{-}a_{sn} = 100.76 \times 69\,467 = 7000 \textit{ kv-a.}$$
$$Kw_{sn} = 7000 \times 0.9225 = 6457 \textit{ kw.}$$
$$Loss_n = 6457 - 5400 = 1057 \textit{ kw.}$$

Solution by Complex Quantities—From table Q the auxiliary constants corresponding to the nominal T method of solution are found as follows:

$$a_1 = 1 - \frac{XB}{2} = 0.805\,406\,5$$
$$a_2 = \frac{RB}{2} = 0.082\,057\,5$$
$$b_1 = R - \frac{RXB}{2} = 84.5677$$
$$b_2 = X - \frac{B}{4}(X^2 - R^2) = 229.081$$
$$c_1 = O$$
$$c_2 = B = 0.001\,563$$

The voltage at the sending-end is obtained as follows:—

$$I_R (\cos \theta_R - j \sin \theta_R) = 89.928 - j\,43.554$$
$$\times\ (b_1 + j\,b_2) = 17\,582 + j\,16\,918$$
$$+ E_{rL}\ (a_1 + j\,a_2) = 48\,361 + j\,4927$$
$$E_{sn} = 65\,943 + j\,21\,845$$
$$= 69\,467 \; \underline{/18^\circ\ 19'\ 43''}$$

The current at the sending-end may be calculated as follows:—

$$I_R (\cos \theta_R - j \sin \theta_R) = 89.928 - j\,43.554$$
$$\times\ (a_1 + j\,a_2) = 76.0026 - j\,27.6994$$
$$+ E_{rn}\ (c_1 + j\,c_2) = 0 + j\,93.8519$$
$$I_s = 76.0026 + j\,66.1525$$
$$= 100.76 \; \underline{/41^\circ\ 02'\ 11''} \textit{ amperes}$$

The above results check with those previously obtained by impedance calculations. This agreement indicates that the nominal T solution may, if desired, be made by complex quantities, assuming values for the auxiliary constants as indicated in Table Q.

Convergent Series Expression—Table Q indicates that the nominal T solution is equivalent to using the following values for the auxiliary constants in the convergent series form of solution:—

$$A' = \left(1 + \frac{ZY}{2}\right)$$

$$B' = Z\left(1 + \frac{ZY}{4}\right)$$

$$C' = Y$$

Comparing the above expressions for the auxiliary constants with the complete expression yielding rigorous values the following difference may be noted.

For auxiliary constant A' the first two terms in the complete series for the hyperbolic cosine are used and all terms beyond omitted. For auxiliary constant B' the first two terms of the complete series are also used except that the coefficient of the second term is given as ¼, whereas in the complete series it is 1/6. Auxiliary constant C' is equivalent to the first term only of the complete expression.

We will now show that the above expressions yield the same values for the auxiliary constants as given in Table Q. From Chart XI the following values corresponding to problem X are taken:—

$$Z = 105 + j\,249$$
$$ZY = -\,0.389\,187 + j\,0.164\,115$$
$$\text{Therefore } A' = 1.000\,000 - 0.1\,945\,935 + j\,0.0\,820\,575$$
$$A' = +\,0.8\,054\,065 + j\,0.0\,820\,575$$
$$B' = 1.000\,000 - 0.09\,729\,675 + j\,0.04\,102\,875$$
$$= Z\,(0.90\,270\,325 + j\,0.04\,102\,875)$$
$$B' = 84.5677 + j\,229.081$$
$$C' = 0 + j\,0.001\,563$$

Thus the values for the auxiliary constants as determined by the above incomplete convergent series expressions check exactly with those as determined above from the equations in Table Q.

THREE CONDENSER METHOD

This method (proposed by Dr. Chas. P. Steinmetz) assumes that the admittance of the circuit may be lumped or concentrated across the circuit at three points, one-sixth being localized at each end and two-thirds at the middle of the circuit. This is equivalent to assuming that the electrical quantities are distributed along the circuit in a manner represented by the arc of a parabola. It is evident that this method more nearly approaches the actual distribution of the impedance and the admittance of the circuit than any of the three previously described localized admittance methods, and therefore yields more accurate results.

From an inspection of the diagram of such a circuit, Fig. 57, it will be evident that it is necessary to calculate the performance of the two halves of the circuit in order to arrive at the sending-end voltage and an additional calculation will be required to determine the sending-end current, power and power-factor.

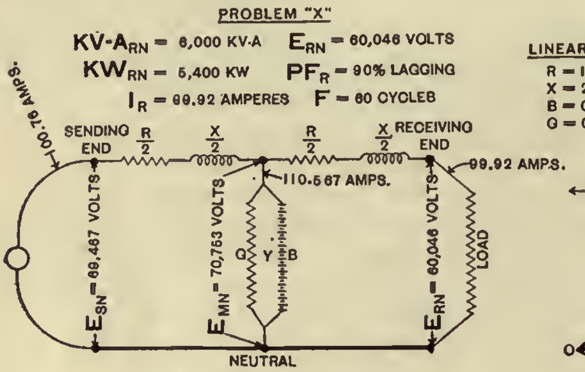
Solution by Impedance Method—The diagram of connections and corresponding graphical vector solution for problem X by the three condenser method is indicated by Fig. 57. The charging current consumed by the condenser (zero leakage assumed) at the receiving-end leads the receiving-end voltage by 90 degrees and is:—

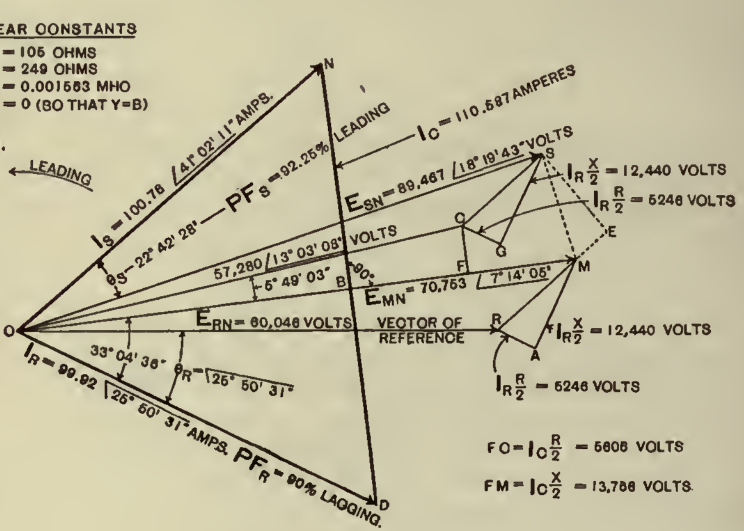
$$I_{cr} = \frac{0.001\,563}{6} \times 60\,046 = 15.642 \textit{ amperes.}$$

The current per conductor for the receiving-end half of the circuit is:—

$$I_r = \sqrt{(99.92 \times 0.9)^2 + (99.92 \times 0.4359 - 15.642)^2}$$
$$= 94.16 \angle 17°\ 14'\ 38'' \textit{ amperes}$$
$$PF_r = \cos \angle 17°\ 14'\ 38'' = 95.505 \textit{ lagging}$$

The voltage consumed by the resistance and the reactance per conductor between the receiving-end and the middle of the circuit is:—





RESULTS CALCULATED BY

RIGOROUS SOLUTION	MIDDLE CONDENSER METHOD	% ERROR
E_{SN} = 70,652 VOLTS	69,467 VOLTS	− 1.68%
I_S = 94.75 AMPS.	100.76 AMPS.	+ 6.34%
PF_S = + 93.42%	+ 92.25%	− 1.25%
$LOSS_N$ = 855 KW	1057 KW	+ 23.63%

FIG. 56—NOMINAL T OR MIDDLE CONDENSER METHOD

$$I_r \frac{R}{2} = 94.16 \times 52.5 = 4943.4 \textit{ Volts (resistance drop)}$$

$$I_r \frac{X}{2} = 94.16 \times 124.5 = 11\,723 \textit{ Volts (reactance drop)}$$

The voltage at the middle of the circuit is from (30):—

$$E_{mo} = \sqrt{(60\,046 \times 0.95\,505 + 4943.4)^2 + (60\,046 \times 0.29\,644 + 11\,723)^2}$$
$$= 68\,933 \ \angle 25^\circ\ 21'\ 33'' \textit{ volts to current vector OP}$$
$$= 68\,933 \ \angle 8^\circ\ 06'\ 55'' \textit{ volts to vector of reference OR}$$

The charging current consumed by the condenser (zero leakage assumed) at the middle of the circuit leads the voltage at the middle of the circuit by 90 degrees and is:—

$$I_{cm} = \frac{0.001\,563}{1.5} \times 68\,933 = 71.828 \textit{ amperes.}$$

The current per conductor for the sending-end half of the circuit may be determined as follows:—

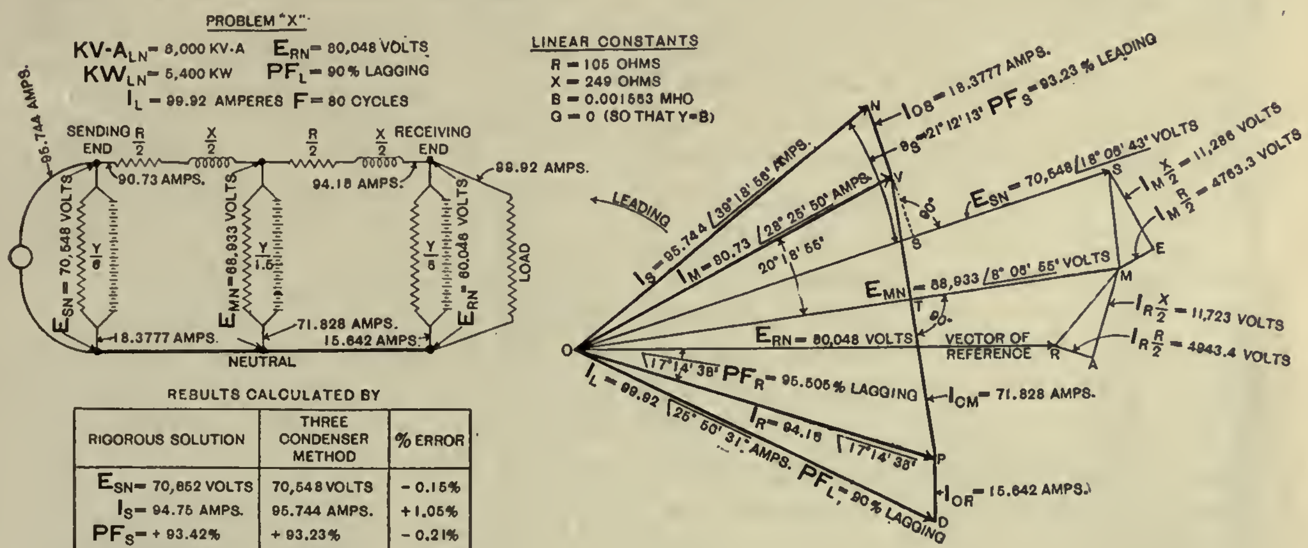
$$OT = Cos\ 25^\circ\ 21'\ 33'' \times 94.16 = 85.0867 \textit{ amperes.}$$

The current at the sending-end of the circuit may be determined as follows:—

$$OS = Cos\ 10^\circ\ 19'\ 07'' \times 90.73 = 89.2624$$
$$VS = Sin\ 10^\circ\ 19'\ 07'' \times 90.73 = 16.2516$$
$$NS = 16.2516 + 18.3777 = 34.6293 \textit{ amperes.}$$
$$I_s = \sqrt{89.2624^2 + 34.6293^2}$$
$$= 95.744 \ \angle 21^\circ\ 12'\ 13'' \textit{ to voltage vector OS.}$$
$$= 95.744 \ \angle 39^\circ\ 18'\ 56'' \textit{ to vector of reference OR.}$$
$$Kv\text{-}a_{sn} = 95.744 \times 70.548 = 6755 \textit{ kv-a}$$
$$PF_s = Cos\ (39^\circ\ 18'\ 56'' - 18^\circ\ 06'\ 43'')$$
$$= Cos\ 21^\circ\ 12'\ 13'' = 93.23 \textit{ percent leading}$$
$$Kw_{sn} = 6755 \times 0.9323 = 6298 \textit{ kw}$$
$$Loss_n = 6298 - 5400 = 898 \textit{ kw}$$
$$Eff. = \frac{5400 \times 100}{6298} = 85.75 \textit{ percent.}$$

Solution by Complex Quantities—From Table Q the auxiliary constants corresponding to the three condenser method of solution are found to be:—

$$a_1 = 1 - \frac{XB}{2} + \frac{B^2}{36}(X^2 - R^2) = 0.808866$$



RESULTS CALCULATED BY		
RIGOROUS SOLUTION	THREE CONDENSER METHOD	% ERROR
E_{SN} = 70,652 VOLTS	70,548 VOLTS	− 0.15%
I_S = 94.75 AMPS.	95.744 AMPS.	+ 1.05%
PF_S = + 93.42%	+ 93.23%	− 0.21%
$LOSS_N$ = 855 KW	898 KW	+ 5.03%

FIG. 57—DR. CHAS. P. STEINMETZ'S THREE CONDENSER METHOD

$$TP = Sin\ 25^\circ\ 21'\ 33'' \times 94.16 = 40.3278 \textit{ amperes.}$$
$$TV = 71.828 - 40.3278 = 31.5002 \textit{ amperes.}$$
$$I_m = \sqrt{85.0867^2 + 31.5002^2}$$
$$= 90.73 \ \angle 20^\circ\ 18'\ 55'' \textit{ amperes to voltage vector OM at middle.}$$
$$= 90.73 \ \angle 28^\circ\ 25'\ 50'' \textit{ to vector of reference OR}$$

The voltage consumed by the resistance and the reactance per conductor between the middle and sending-end of the circuit is:—

$$I_m \frac{R}{2} \times 90.73 \times 52.5 = 4763.3 \textit{ volts (resistance drop)}$$

$$I_m \frac{X}{2} \times 90.73 \times 124.5 = 11\,296 \textit{ volts (reactance drop)}$$

The voltage at the sending-end from (40) is:—

$$E_{sn} = \sqrt{(68\,933 \times 0.93779 + 4763.3)^2 + (68\,933 \times 0.34719 - 11\,296)^2}$$
$$= 70\,548 \ \diagdown 10^\circ\ 19'\ 07'' \textit{ volts to current vector OV}$$
$$= 70\,548 \ \angle 18^\circ\ 06'\ 43'' \textit{ volts to vector of reference OR}$$

The charging current consumed by the condenser (zero leakage assumed) at the sending-end of the circuit leads the voltage at the sending-end by 90 degrees and is:—

$$I_{cs} = \frac{0.001563}{6} \times 70\,548 = 18.3777 \textit{ amperes.}$$

$$a_2 = \frac{RB}{2} - \frac{RXB^2}{18} = 0.0785091$$

$$b_1 = R - \frac{RXB}{3} = 91.3785$$

$$b_2 = X - \frac{B}{6}(X^2 - R^2) = 235.7208$$

$$c_1 = -\frac{5RB^2}{36} + \frac{RXB^2}{108} = -0.0000347$$

$$c_2 = B - \frac{5XB^2}{36} + \frac{B^3}{216}(X^2 - R^2) = +0.0014794$$

These values for the auxiliary constants are in close agreement with the rigorous values.

$$I_L\ (Cos\ \theta_L - j\ Sin\ \theta_L) \times (b_1 + jb_2)$$
$$= 18\,484 + j\ 17\,218$$
$$E_{ro}\ (a_1 + j\ a_2) = 48\,569 + j\ 4714$$

$$E_{so} = 67\,053 + j\ 21\,932$$
$$= 70\,548 \ \angle 18^\circ\ 06'\ 43'' \textit{ volts}$$

The current at the sending-end is:—

$$I_L\ (Cos\ \theta_L - j\ Sin\ \theta_L) \times (a_1 + j\ a_2)$$
$$= 76.159 - j\ 28.170$$
$$E_{ro}\ (C_1 + j\ C_2) = -2.084 + j\ 88.832$$

$$I_s = 74.075 + j\ 60.662$$
$$= 95.744 \ \angle 39^\circ\ 18'\ 56'' \textit{ amperes}$$

By comparing these results with those obtained

CHART XXII—COMPARISON OF RESULTS BY VARIOUS METHODS

Problem No.	Length of Circuit—Miles	Conductors	Spacing in Feet—Delta	Linear Constants R	X	B × 10^{-6}	G	Load at Receiving-End KV-A ★	E_R	E_{RN}	PF %	I_R	% Error in Receiving-End Voltage as Determined by: Rigorous Solution (these values are exact) E_{SN}	Semi-Graphical Method % Error	Complete Graphical Method % Error	H. B. Dwight's "K" Formulas % Error	Dr. Chas. P. Steinmetz's Three Condenser Method % Error	Nominal "T" or Middle Condenser Method % Error	Nominal "π" or Split Condenser Method % Error	Single End Condenser Method % Error	Impedance Method % Error
25 CYCLES																					
1	20	#0000	3	5.54	5.36	57.2	0	1,300	10,000	5,774	80	75	6,347	0	+.05	0				-.01	0
2	"	COPPER	"	"	"	"	0	"	"	"	100	"	6,202	0	+.03	0				-.01	0
3	20	#0000	3	5.54	5.36	57.2	0	5,000	20,000	11,550	80	144.4	12,653	-.04	+.06	-.01				-.02	+.02
4	"	COPPER	"	"	"	"	0	"	"	"	100	"	12,372	-.05	+.02	-.01				-.02	+.02
5	30	#0000	4	8.31	8.5	81	0	3,500	20,000	11,550	80	101	12,733	+.02	+.06	-.02				-.03	+.04
6	"	COPPER	"	"	"	"	0	"	"	"	100	"	12,415	+.08	+.04	-.02				-.03	+.04
7	30	#0000	4	8.31	8.5	81	0	8,000	30,000	17,320	80	154	19,125	+.05	0	-.01				-.03	+.03
8	"	COPPER	"	"	"	"	0	"	"	"	100	"	18,640	+.08	+.05	+.01				-.02	+.03
9	50	#0000	4	13.85	14.1	135	0	5,000	30,000	17,320	80	96.2	19,184	0	-.07	+.03				-.08	+0.1
10	"	COPPER	"	"	"	"	0	"	"	"	100	"	18,685	-.05	-.03	+.02				-.05	+0.1
11	50	#0000	6	13.85	15.1	125	0	20,000	60,000	34,640	80	192.5	38,490	-.03	-.04	+.01			0	-.08	+0.1
12	"	COPPER	"	"	"	"	0	"	"	"	100	"	37,387	-.02	-.03	+.01			0	-.07	+0.1
13	100	#0000	9	27.7	32.2	233	0	22,000	88,000	50,810	80	144.4	56,619	-.07	-.03	0		-.01	+.01	-.31	+0.3
14	"	COPPER	"	"	"	"	0	"	"	"	100	"	54,820	-.02	-.04	0		-.01	+.01	-.30	+0.3
15	100	#0000	11	27.7	33.2	226	0	40,000	120,000	69,290	80	192.5	77,147	-.02	-.06	0		-.01	+.02	-.30	+0.4
16	"	COPPER	"	"	"	"	0	"	"	"	100	"	74,642	-.02	-.03	0		-.01	+.02	-.31	+0.4
17	200	300,000 C.M.	11	39.2	64.8	464	0	25,000	120,000	69,290	80	120.3	76,754	+.06	-.01	0		-.04	+.06	-1.24	+1.4
18	"	COPPER	"	"	"	"	0	"	"	"	100	"	73,401	+.05	-.06	0		-.04	+.06	-1.24	+1.4
19	200	300,000 C.M.	17	39.2	69.2	434	0	40,000	140,000	80,830	80	165	91,761	-.02	+.05	-.08		-.05	+.06	-1.19	+1.4
20	"	COPPER	"	"	"	"	0	"	"	"	100	"	86,863	-.02	-.01	+.01		-.04	+.05	-1.19	+1.4
21	300	636,000 C.M.	11	44.1	91.2	747	0	20,000	120,000	69,290	80	96.2	75,682	0	+.02	+.05		-.09	+.15	-2.83	+3.2
22	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	71,762	+.08	-.02	+.02		-.08	+.13	-2.88	+3.2
23	300	636,000 C.M.	21	44.1	101	672	0	60,000	200,000	115,500	80	173.2	128,450	-.04	+.04	+.03		-.11	+.17	-2.74	+3.4
24	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	120,574	0	-.06	+.01		-.11	+.15	-2.82	+3.3
25	400	636,000 C.M.	17	58.8	130	928	0	20,000	140,000	80,830	80	82.5	86,404	-.05	—	+.11	0	-.19	+.26	-5.08	+5.7
26	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	81,647	-.06	—	+.05	0	-.18	+.23	-5.30	+5.7
27	400	636,000 C.M.	21	58.8	134	896	0	50,000	200,000	115,500	80	144.4	127,267	-.05	—	+.09	0	-.22	+.32	-4.80	+5.6
28	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	118,833	-.03	—	0	0	-.21	+.29	-4.89	+5.6
29	500	636,000 C.M.	17	73.5	163	1160	0	15,000	140,000	80,830	80	61.86	83,045	-.06	—	+.06	-0.04	-.27	+.36	-8.22	+9.2
30	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	78,658	+.05	—	+.01	-0.04	-.23	+.31	-8.32	+9.3
31	500	636,000 C.M.	21	73.5	168	1120	0	40,000	200,000	115,500	80	115.5	123,401	+.08	—	+.02	-0.04	-.40	+.50	-7.65	+9.0
32	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	115,162	+.08	—	-.02	-0.04	-.37	+.44	-5.99	+9.0
60 CYCLES																					
33	20	#0000	3	5.54	12.88	137	0	1,300	10,000	5,774	80	75	6,702	0	-.15	0				-.07	+.07
34	"	COPPER	"	"	"	"	0	"	"	"	100	"	6,259	+.06	+.02	+.02				-.06	+0.1
35	20	#0000	3	5.54	12.88	137	0	5,000	20,000	11,550	80	144.4	13,333	+.02	-.10	0				-.07	+0.1
36	"	COPPER	"	"	"	"	0	"	"	"	100	"	12,480	+.02	+.04	0				-.07	+0.1
37	30	#0000	4	8.31	20.4	195	0	3,500	20,000	11,550	80	101	13,482	+.07	+.06	0				-.15	+0.2
38	"	COPPER	"	"	"	"	0	"	"	"	100	"	12,537	+.05	+.06	0				-.16	+0.2
39	30	#0000	4	8.31	20.4	195	0	8,000	30,000	17,320	80	154	20,268	+.03	+.06	+.01		0	0	-.15	+0.2
40	"	COPPER	"	"	"	"	0	"	"	"	100	"	18,830	-.03	+.05	+.02		0	0	-.16	+0.2
41	50	#0000	4	13.85	34	324	0	5,000	30,000	17,320	80	96.2	20,331	0	+.04	+.03		-.02	+.04	-.42	+0.5
42	"	COPPER	"	"	"	"	0	"	"	"	100	"	18,845	+.08	-.03	-.02		-.02	+.03	-.44	+0.5
43	50	#0000	6	13.85	36.4	301	0	20,000	60,000	34,640	80	192.5	40,976	+.03	-.01	+.02		-.02	+.03	-.41	+0.5
44	"	COPPER	"	"	"	"	0	"	"	"	100	"	37,773	+.04	-.01	+.03		-.02	+.04	-.42	+0.5
45	100	#0000	9	27.7	77.4	562	0	22,000	88,000	50,810	80	144.4	59,925	+.03	-.09	+.08	0	-.08	+.13	-1.61	+1.91
46	"	COPPER	"	"	"	"	0	"	"	"	100	"	54,869	0	-.13	+.05	0	-.07	+.13	-1.69	+1.96
47	100	#0000	11	27.7	79.7	542	0	40,000	120,000	69,290	80	192.5	81,710	-.08	-.06	-.02	0	-.08	+.14	-1.60	+1.84
48	"	COPPER	"	"	"	"	0	"	"	"	100	"	74,735	-.05	-.11	+.01	0	-.07	+.12	-1.68	+1.92
49	200	300,000 C.M.	11	39.2	156	1116	0	25,000	120,000	69,290	80	120.3	79,000	+.03	+.14	+.18	-0.04	-.46	+.61	-6.53	+7.8
50	"	COPPER	"	"	"	"	0	"	"	"	100	"	70,599	+.08	+.07	+.14	-0.04	-.39	+.47	-6.99	+8.1
51	200	300,000 C.M.	17	39.2	166	1044	0	40,000	140,000	80,830	80	165	96,727	-.07	+.14	+.22	-0.04	-.51	+.73	-5.96	+7.5
52	"	COPPER	"	"	"	"	0	"	"	"	100	"	84,862	+.07	+.02	+.25	-0.04	-.45	+.62	-6.37	+7.8
53	300	636,000 C.M.	11	44.1	220	1794	0	20,000	120,000	69,290	80	96.2	72,747	-.04	+.05	+.46	-0.21	-1.51	+1.34	-15.68	+19
54	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	63,810	-.02	+.21	-.25	-0.21	-1.34	+1.01	-16.43	+20
55	300	636,000 C.M.	21	44.1	243	1614	0	60,000	200,000	115,500	80	173.2	126,541	-.05	+.25	+.46	-0.21	-1.60	+1.60	-14.50	+18
56	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	109,189	-.07	+.15	+.51	-0.21	-1.44	+1.29	-15.12	+19
57	400	636,000 C.M.	17	58.8	314	2212	0	20,000	140,000	80,830	80	82.5	74,182	+.01	—	-.08	-0.71	-3.92	+2.15	-29.34	+37
58	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	64,377	+.03	—	.0	-0.71	-3.65	+1.74	-27.16	+39
59	400	636,000 C.M.	21	58.8	322	2152	0	50,000	200,000	115,500	80	144.4	113,606	-.08	—	-.49	-0.75	-4.11	+2.99	-25.82	+35
60	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	96,987	+.01	—	+1.04	-0.81	-3.92	+2.84	-22.38	+36
61	500	636,000 C.M.	17	73.5	390	2785	0	15,000	140,000	80,830	80	61.86	59,046	+.06	—	-1.47	-1.97	-9.18	+2.28	-40.82	+70
62	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	51,327	+.06	—	+.25	-1.89	-8.64	+2.54	-20.62	+73
63	500	636,000 C.M.	21	73.5	402	2690	0	40,000	200,000	115,500	80	115.5	93,725	+.06	—	-.28	-1.84	-9.32	+4.44	-35.54	+64
64	"	ALUMINUM	"	"	"	"	0	"	"	"	100	"	80,106	+.06	—	+3.8	-1.72	-8.84	+5.43	-13.53	+65

*It would be commercially impractical to transmit such small amounts of power some of the extreme distances indicated by the tabulation. The problems are stated simply for the purpose of illustrating in an approximate manner the effect distance of transmission has upon the voltage drop as calculated by various methods.

by the impedance method of procedure, it will be seen that they are in exact agreement.

Convergent Series Expression—Dr. F. E. Pernot in "Electrical Phenomena in Parallel Conductors," Vol. 1, shows that the above described three condenser solution is equivalent to using the following values for the auxiliary constants in the convergent series form of solution:—

$$A' = \left(1 + \frac{ZY}{2} + \frac{Z^2\,Y^2}{36}\right)$$

$$B' = Z\left(1 + \frac{ZY}{6}\right)$$

$$C' = Y\left(1 + \frac{5\,ZY}{36} + \frac{Z^2\,Y^2}{216}\right)$$

Comparing the above expressions for the auxiliary constants with the complete expressions yielding rigorous values, the following differences may be noted. For constant A' the first two terms are the same as in the complete series, but the third term is less than in the complete series, and all terms beyond the third are omitted. For constant B' the first two terms are the same as in the complete series, but all terms beyond the second are omitted. For constant C' both the ZY and the $Z^2\ Y^2$ terms are smaller than in the complete series and all terms beyond the third are omitted.

The above expressions yield the same values for the auxiliary constants as given in Table Q. Thus from chart XI, the following values corresponding to problem X are taken:—

$$ZY = -0.389187 + j\,0.164115$$
$$Z^2\,Y^2 = +0.124532 - j\,0.127742$$

Therefore

$$\begin{aligned} A' = {} & 1.000000 \\ & -0.194593 + j\,0.0820575 \\ & 0.003459 - j\,0.0035484 \\ \hline A' = {} & 0.808866 + j\,0.0785091 \\ B' = {} & 1.000000 \\ & -0.0648645 + j\,0.0273525 \\ \hline & Z\,(0.9351355 + j\,0.0273525) \\ B' = {} & 91.3785 \quad + j\,235.7208 \\ C' = {} & 1.000000 \\ & -0.0540538 + j\,0.0227938 \\ & +0.0005765 - j\,0.0005914 \\ \hline & Y\,(0.9465227 + j\,0.0222024) \\ C' = {} & -0.0000347 + j\,0.0014794 \end{aligned}$$

It will be seen that the above convergent series expression for the auxiliary constants check exactly with those as determined by the equations in Table Q.

COMPARATIVE ACCURACY OF VARIOUS METHODS

In order to determine the inherent error in various methods of solution, when applied to circuits of increasing length; also for frequencies of both 25 and 60 cycles, 64 problems were solved. These problems embrace thirty-two 25 cycle circuits, varying in length from 20 to 500 miles and in voltage from 10 000 to 200 000 volts. Fixed receiving-end load conditions were assumed for unity, and also for 80 percent power-factor lagging. These same problems were also solved for a frequency of 60 cycles.

These 64 problems with corresponding linear constants and assumed load conditions are stated on Chart XXII. This is followed by columns in which have been tabulated the error in voltage at the sending-end of these circuits as determined by nine different methods. The errors are expressed in percent of receiving-end voltage. Obviously the inherent error corresponding to various methods will vary widely for conductors of various resistances and to some extent for different receiving-end loads. The tabulated values should therefore be looked upon as comparative rather than absolute for all conditions.

Rigorous Solution—The column headed "Rigorous Solution" contains values for the sending-end voltage which are believed to be exact. These values were obtained by calculating values for the auxiliary constants by means of convergent series and then calculating the performance mathematically. The calculations were carried out to include the sixth place and terms in convergent series were used out to the point where they did not influence the results.

The first values calculated were checked by a second set of values calculated independently at another time and where differences were found the correct values were determined and substituted. This corrected list of values was again checked by a third independent calculation. It is therefore believed that the values contained in this column are exact, representing 100 percent.

Semi-Graphical Solution—The next column contains the error in the results as derived by the combination of an exact mathematical solution for the auxiliary constants and a graphical solution from there on. This combination gave results in which the maximum error does not exceed eight one hundredths of one percent of receiving-end voltage for either frequency. In other words, since the values for the auxiliary constants used in this method were exact, the maximum error of eight one hundredths of one percent occurs in the construction and reading of the graphical constructions.

Complete Graphical Solution—This solution employs Wilkinson's charts for obtaining graphically the auxiliary constants, the remainder of the solution being also made graphically as previously described. It will be seen that the maximum error as obtained by this complete graphical solution is seven hundredths of one percent for the 25 cycle and twenty-five hundredths of one percent for the 60 cycle circuits. These errors represent the combined result of various errors. First there is a slight fundamental error in the basis upon which the Wilkinson Charts are constructed when used for circuits employing conductors of various sizes and spacings, the introduction of this error making possible the simplification attained. Then there is the inherent limitation of precision obtainable in the construction and reading of the charts and vector diagrams.

These results show that the inherent accuracy of this simplified, all graphical solution is sufficiently accurate for all practical power circuits up to 300 miles long.

Dwight's "K" Formulas—The high degree of accuracy resulting by the use of H. B. Dwight's "K" formulas should be noted. This error is a maximum of eleven hundredths of one percent for these 32 twenty-five cycle problems. The statement is therefore justified that these "K" formulas are sufficiently accurate for all 25 cycle power circuits.

For the 60 cycle problems the maximum error by the "K" formulas for problems up to and including 200 miles is one-fourth of one percent of receiving-end voltage. For 300 mile circuits this error is one-half of one percent and increases rapidly as the circuit exceeds 300 miles in length. The accuracy of the "K" formulas for 60 cycle circuits is therefore well within that of the assumed values of the linear constants for circuits up to approximately 300 miles in length.

The "K" formulas are based upon the hyperbolic formula expressed in the form of convergent series. In the development of these formulas, use was made of the fact that the capacitance multiplied by the reactance of non-magnetic transmission conductors is a constant quantity to a fairly close approximation. This assumption has enabled the "K" formulas to be expressed in comparatively simple algebraic form without the use of complex numbers. To those not familiar or not in position to make themselves familiar with the operation of complex numbers, such as is used in the convergent series or hyperbolic treatments, the availability of the Dwight "K" formulas will be apparent.*

Localized Capacitance Methods—The next four columns contain values indicating the error in results as determined by the four different localized capacitance methods previously described in detail. It is interesting to note the high degree of accuracy inherent in Dr. Steinmetz's three condenser method. It is also interesting to note that three of these methods over compensate (that is, give receiving-end voltages too low) and one (the split condenser method) gives under compensation.

Impedance Method—The values of the sending-end voltage as obtained by the impedance method (which takes no account of capacitance) are always too high when applied to circuits containing capacitance. The results by this method are included here simply to serve as an indication of how great is the error for this method when applied to circuits of various lengths and frequencies of 25 and 60 cycles. Some engineers prefer to use this method for circuits of fair length and allow for the error. These tabulations will give an approximation of the necessary allowance to be made.

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*These have been included with much other valuable material in "Transmission Line Formulas" by H. B. Dwight, published by D. Van Nostrand Co. of New York City.

CHAPTER XIII
CABLE CHARACTERISTICS
Heating Limits for Cables

THE MAXIMUM safe-limiting temperatures in degrees C at the surface of conductors in cables is given in the Standardization Rules of the A. I. E. E. (1918) as follows:—

For impregnated paper insulation (85—E)
For varnished cambric (75—E)
For rubber insulation (60—0.25 E)

Where E represents the effective operating e.m.f. in kilovolts between conductors and the numerals represent temperature in degrees C. Thus, at a working pressure of 5 kv, the maximum safe limiting temperature at the surface of the conductors in a cable would be:—

For impregnated paper insulation (80 degrees C)
For varnished cambric insulation (70 degrees C)
For rubber compound insulation (58.75 degrees C)

The actual maximum safe continuous current load for any given cable is determined primarily by the temperature of the surrounding medium and the rate of radiation. This current value is greater with direct than with alternating current and decreases with increasing frequency, being less for a 60 cycles than for 25 cycles. The carrying capacity of cables will therefore be less in hot climates than in cooler climates and will be considerably increased during the winter.

Cables immersed in water, carry at least 50 percent more than when installed in a four-duct line, and when buried in the earth 15 to 30 percent more than in a duct line, depending upon the character of soil moisture, etc. Circulating air or water through conduits containing lead covered cables will increase their capacity. From the above it is evident that no general rule relative to carrying capacity can be formulated to apply in all cases, and it is necessary, therefore, to consider carefully the surroundings when determining the size of cables to be used.

The practicability of tables which specify carrying capacity for cables installed in ducts will generally be questioned, for the reason that operating conditions are frequently more severe than those upon which table values are based. A duct line may operate at a safe temperature throughout its entire length, except at one isolated point adjacent to a steam pipe or excessive local temperatures due to some other cause. If larger cables are not employed at this point, burnouts may occur here when the remainder of the cable line is operating well within the limits of safe operating temperature. The danger in using table values for carrying capacity without carefully considering the condition of earth temperatures throughout the entire duct length is thus evident.

HEATING OF CABLES—TABLE XXIV

The basis upon which the data in Table XXIV has been calculated is covered by foot notes below the table. The kv-a values are determined from the current in amperes and are based upon 30 degree C rise and a maximum of 3000 volts.* Expressing the carrying capacity of cables in terms of kv-a (corrected for the varying thickness of insulation required for various voltages) may be found more convenient than the usual manner of expressing it in amperes. It will be noted that the kv-a values of the table are on the basis of a four-duct line and that for more than four ducts in the line the table kv-a values will be reduced to the following:—

For a 4 duct line—100 percent.
For a 6 duct line— 88 percent.
For an 8 duct line— 79 percent.
For a 10 duct line— 71 percent.
For a 12 duct line— 63 percent.
For a 16 duct line— 60 percent.

When applied to all sizes of cables, the above values are only approximate. The reduction of carrying capacity caused by the presence of many cables is more for large cables than for small ones. Also, where load factors are small, the reduction due to the presence of many cables is less than the value assigned, although the carrying capacity of a small number of cables is only slightly affected.

REACTANCE OF THREE-CONDUCTOR CABLES

Tables XXV and XXVI contain values for the inductance, reactance and impedance of round three-conductor cables of various sizes and for the thicknesses of insulation indicated.All values in the tables are on the basis of one conductor of the cable one mile long.

The table values were calculated from the fundamental equation (4),

$$L = 0.08047 + 0.741 \log_{10}\frac{D}{R}$$

where L = the inductance in millihenries per mile of each conductor, R the actual radius of the conductor and D the distance between conductor centers expressed in the same units as R. As indicated in Section I, under Inductance,** this formula has been derived on the basis of solid conductors. In the case of cables, the effective radius is actually slightly less than that of the stranded conductor. The values for

*These current values are taken from General Electric Bulletin No. 49302 dated March 1917. They are in general slightly higher than those published by the Standard Underground Cable Company in their Hand Book dated 1906.

**Chapter I.

TABLE XXIV—CARRYING CAPACITY OF INSULATED COPPER CONDUCTORS

The following values for carrying capacity must not be assumed unless it is positively known that the conditions upon which they are based will not be exceeded in service.

THREE CONDUCTOR CABLES

K.V A. WHICH MAY BE TRANSMITTED AT THREE PHASE AND THE FOLLOWING VOLTAGES OVER PAPER INSULATED LEAD COVERED CABLES INSTALLED IN A FOUR DUCT LINE WITH 30° C RISE IN TEMPERATURE BASED UPON THE ASSUMPTION THAT ALL DUCTS CARRY LOADED CABLES AND UPON A NORMAL EARTH TEMPERATURE OF 20° C

FOR A 6 DUCT LINE THESE K.V.A. VALUES WOULD BE REDUCED TO APPROXIMATELY 88 PER CENT FOR AN 8 DUCT LINE TO 79 PER CENT. FOR A 10 DUCT LINE TO 71 PER CENT FOR A 12 DUCT LINE TO 63 PER CENT AND FOR A 18 DUCT LINE (4 WIDE AND 4 HIGH) TO 60 PER CENT OF THE TABLE VALUES. XXXX.

B & S NO. AREA IN CIRCULAR MILS	XX CARRYING CAPACITY IN AMPERES DIRECT CURRENT BASED UPON 30° C RISE AND A MAXIMUM OF 3000 VOLTS, PAPER INSULATION	220 VOLTS	440 VOLTS	650 VOLTS	1100 VOLTS	2200 VOLTS	3300 VOLTS	4000 VOLTS	6000 VOLTS	6600 VOLTS	10000 VOLTS	11000 VOLTS	12000 VOLTS	13200 VOLTS	15000 VOLTS	20000 VOLTS	22000 VOLTS	25000 VOLTS
14	18	7	14	17	34	68	103	124	184	202	300	328	356	390	438	570	620	693
12	22	8	17	21	42	84	125	152	225	247	367	400	435	477	536	695	757	847
10	30	11	23	28	57	114	171	206	307	336	500	547	595	650	730	950	1035	1155
8	40	15	30	38	76	152	228	275	410	450	667	730	792	867	975	1265	1380	1540
6	55	21	42	52	104	209	314	378	562	616	917	1000	1090	1190	1340	1740	1895	2120
4	70	26	53	66	133	266	400	482	715	785	1170	1275	1385	1520	1710	2215	2410	2700
2	95	36	72	90	180	361	543	655	970	1065	1585	1730	1880	2060	2310	3000	3270	3650
1	110	42	84	105	209	418	628	755	1125	1230	1840	2050	2180	2390	2680	3480	3780	4240
0	130	49	99	123	247	495	740	895	1330	1460	2170	2370	2580	2820	3170	4120	4470	5000
00	150	57	114	143	285	570	855	1030	1535	1710	2500	2740	2970	3260	3660	4750	5170	5770
000	170	65	130	162	323	647	970	1170	1740	1940	2840	3100	3370	3690	4150	5375	5850	6550
0000	200	76	152	190	380	760	1140	1375	2040	2240	3340	3650	3960	4340	4870	6325	6900	7700
250 000	225	86	172	214	428	857	1285	1550	2300	2520	3750	4100	4450	4880	5470	7125	7750	8650
300 000	250	95	190	238	475	950	1430	1720	2560	2800	4170	4550	4950	5425	6100	7900	8600	9625
350 000	280	106	213	266	532	1065	1600	1925	2860	3140	4670	5100	5550	6070	6825	8850	9650	10800
400 000	310	118	236	295	590	1180	1770	2130	3170	3480	5170	5650	6150	6720	7550	9800	10700	11950
450 000	340	129	258	323	647	1295	1940	2340	3480	3810	5670	6200	6750	7370	8300	10750	11700	13100
500 000	360	137	274	342	685	1370	2050	2480	3680	4040	6000	6550	7130	7800	8770	11400	12400	13850
600 000	400	152	304	380	760	1520	2280	2760	4100	4500	6680	7300	7920	8680	9750	12650	13800	15400
700 000	470	179	358	447	895	1790	2680	3240	4800	5270	7820	8570	9300	10200	11450	14850	16200	18100

SINGLE CONDUCTOR CABLES

K.V A. WHICH MAY BE TRANSMITTED AT THREE PHASE AND THE FOLLOWING VOLTAGES OVER THREE PAPER INSULATED LEAD COVERED CABLES INSTALLED IN A FOUR DUCT LINE WITH 30° C RISE IN TEMPERATURE BASED UPON THE ASSUMPTION THAT ALL DUCTS CARRY LOADED CABLES AND UPON A NORMAL EARTH TEMPERATURE OF 20° C

FOR A 6 DUCT LINE THESE K.V.A. VALUES WOULD BE REDUCED TO APPROXIMATELY 88 PER CENT FOR AN 8 DUCT LINE TO 79 PER CENT FOR A 10 DUCT LINE TO 71 PER CENT; FOR A 12 DUCT LINE TO 63 PER CENT AND FOR A 18 DUCT LINE (4 WIDE AND 4 HIGH) TO 60 PER CENT OF THE TABLE VALUES XXXX

B & S NO AREA IN CIRCULAR MILS	CARRYING CAPACITY IN AMPERES DIRECT CURRENT: N. E. CODE – INTERIOR CONDUCTORS: X TABLE A RUBBER INSULATION	X TABLE B OTHER INSULATION	XXX BASED UPON 30° C RISE & A MAXIMUM OF 3000 VOLTS PAPER INSULATION	220 VOLTS	440 VOLTS	660 VOLTS	1100 VOLTS	2200 VOLTS	3300 VOLTS	4000 VOLTS	6000 VOLTS	6600 VOLTS	10000 VOLTS	11000 VOLTS	12000 VOLTS	13200 VOLTS	15000 VOLTS	20000 VOLTS	22000 VOLTS	25000 VOLTS
14	15	20	24	9	18	23	46	92	137	165	245	270	400	437	475	520	585	758	826	925
12	20	25	30	11	23	28	57	114	171	206	306	337	500	547	595	650	732	948	1033	1155
10	25	30	40	15	30	38	76	152	228	275	408	450	667	730	792	868	975	1265	1378	1540
8	35	50	55	21	42	52	104	209	314	378	560	617	917	1010	1090	1195	1340	1740	1895	2120
7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6	50	70	75	28	57	71	142	284	428	515	765	842	1250	1270	1485	1630	1830	2370	2580	2890
5	55	80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
4	70	90	95	36	72	90	181	362	540	653	970	1065	1585	1730	1880	2060	2320	3000	3270	3660
3	80	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	90	125	125	47	95	119	237	475	712	860	1275	1405	2085	2280	2480	2710	3050	3950	4300	4820
1	100	150	150	57	114	143	285	570	855	1030	1530	1685	2500	2715	2970	3260	3660	4740	5170	5780
0	125	200	170	65	130	162	323	647	970	1170	1740	1910	2830	3100	3370	3690	4150	5380	5850	6550
00	150	225	200	76	152	190	380	760	1140	1380	2040	2280	3330	3650	3960	4350	4880	6320	6880	7700
000	175	275	230	87	175	219	437	875	1310	1580	2350	2620	3840	4200	4550	5000	5600	7270	7920	8860
0000	225	325	270	103	205	256	512	1025	1540	1860	2760	3030	4500	4920	5350	5850	6580	8530	9300	10400
250 000	—	—	300	114	228	285	570	1140	1710	2060	3070	3370	5000	5470	5950	6500	7320	9500	10350	11550
300 000	275	400	340	129	259	324	647	1295	1940	2340	3470	3820	5670	6200	6730	7380	8300	10750	11700	13200
350 000	—	—	380	144	289	362	723	1445	2170	2620	3880	4270	6330	6920	7520	8250	9260	12000	13100	14650
400 000	325	500	410	156	312	390	780	1560	2340	2820	4180	4600	6850	7480	8120	8900	10000	12950	14150	15800
450 000			450	172	343	428	855	1710	2560	3100	4600	5050	7500	8200	8900	9770	10900	14200	15500	17350
500 000	400	600	480	183	365	456	912	1825	2740	3300	4900	5400	8000	8750	9500	10400	11700	15200	16550	18500
600 000	450	680	550	209	418	522	1045	2090	3140	3790	5620	6180	9170	10000	10900	11950	13400	17400	18950	21200
700 000	500	760	610	232	464	580	1160	2320	3480	4200	6220	6850	10150	11100	12100	13250	14850	19300	21000	23500
800 000	550	840	670	255	510	638	1275	2550	3820	4600	6850	7520	11150	12200	13250	14550	16350	21200	23100	25800
900 000	600	920	720	274	548	685	1370	2740	4100	4950	7350	8100	12000	13150	14250	15650	17550	22700	24800	27700
1 000 000	650	1000	780	297	594	742	1480	2960	4450	5370	7950	8750	13000	14200	15450	16950	19000	24650	26800	30100
1 100 000	690	1080	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1 200 000	730	1150	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1 250 000	—	—	900	342	685	855	1710	3420	5130	6200	9200	10100	15000	16400	17800	19550	21900	28400	31000	34700
1 300 000	770	1220	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1 400 000	810	1290	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1 500 000	850	1360	1030	392	785	980	1960	3920	5870	7100	10500	11550	17200	18800	20400	22400	25100	32600	35500	39700
1 600 000	890	1430	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1 750 000	—	—	1130	430	860	1075	2150	4300	6450	7770	11550	12700	18800	20600	22400	24500	27500	35700	39000	43600
1 900 000	1010	1610	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2 000 000	1050	1670	1260	480	960	1200	2400	4800	7200	8670	12850	14150	21000	23000	25000	27300	30700	39800	43300	48600

X For purposes of comparison these values are given for interior conductors.

XX For four conductor cables these ampere values would be reduced by 12.5 percent.

XXX For solid conductors these ampere ratings would be reduced by seven percent. For two conductor cables made up either round or flat, they would be reduced by 15 percent. For two conductor concentric cables they would be reduced by 25 percent. They will also be reduced in the case of the larger conductors when used on alternating-current circuits on account of skin effect, unless special cables having non-conducting cores are used. These special cables should be used for 700 000 circ. mils and larger for 60 cycle and 1 000 000 circ. mils and larger for 25 cycle service.

XXXX For the higher voltage cables the kv-a values of the table have been reduced by one percent for each 2000 volts that the working pressure exceeds 3000 volts, that is by 11 percent for a 25 000 volt cable. For insulated aluminum conductors the safe carrying capacity (based upon 61 percent conductivity) is 79.3 percent of the above table values with the same kind of insulation. These kv-a values are based upon the current in columns headed by XX and XXX.

TABLE XXV—INDUCTANCE, REACTANCE AND IMPEDANCE, AT 25 CYCLES, PER MILE OF SINGLE CONDUCTOR FOR THREE CONDUCTOR CABLES

INSULATION THICKNESS IN 64THS OF AN INCH ★★

AREA IN CIRCULAR MILS B & S NO.	DIAMETER IN INCHES	RESISTANCE PER MILE IN OHMS ★	3/64 BY 3/64			4/64 BY 4/64			5/64 BY 5/64			6/64 BY 6/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.338	.0530	.128	.349	.0547	.129	.360	.0565	.129	.370	.0580	.130
450 000	.772	.129	.340	.0534	.140	.351	.0552	.140	.362	.0568	.141	.373	.0585	.142
400 000	.728	.145	.343	.0537	.155	.354	.0554	.155	.367	.0576	.155	.377	.0592	.157
350 000	.681	.166	.346	.0542	.175	.357	.0560	.176	.370	.0581	.176	.380	.0596	.177
300 000	.630	.194	.349	.0547	.204	.361	.0567	.204	.374	.0587	.205	.386	.0605	.207
250 000	.575	.233	.353	.0554	.240	.366	.0575	.240	.381	.0597	.240	.394	.0619	.242
0000	.528	.275	.357	.0560	.281	.372	.0585	.281	.387	.0607	.282	.403	.0633	.282
000	.470	.346	.362	.0567	.352	.379	.0595	.352	.397	.0623	.352	.411	.0645	.353
00	.418	.437	.369	.0579	.441	.388	.0609	.442	.406	.0637	.442	.423	.0665	.442
0	.373	.550	.377	.0592	.552	.398	.0625	.554	.417	.0653	.554	.432	.0677	.554
1	.332	.695	.384	.0603	.697	.405	.0635	.698	.429	.0673	.698	.447	.0700	.699
2	.292	.879	.393	.0617	.882	.417	.0655	.882	.441	.0691	.882	.463	.0727	.882
3	.260	1.11	.403	.0633	1.11	.431	.0675	1.11	.454	.0712	1.11	.476	.0746	1.11
4	.232	1.40	.413	.0648	1.40	.442	.0695	1.40	.469	.0736	1.40	.494	.0775	1.40
6	.184	2.21	.437	.0685	2.21	.470	.0737	2.21	.501	.0785	2.21	.529	.0830	2.21

			7/64 BY 7/64			8/64 BY 8/64			9/64 BY 9/64			10/64 BY 10/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.379	.0595	.130	.389	.0610	.131	.398	.0625	.132	.407	.0640	.133
450 000	.772	.129	.384	.0602	.143	.393	.0617	.143	.403	.0634	.144	.411	.0645	.145
400 000	.728	.145	.389	.0610	.158	.396	.0622	.158	.409	.0642	.159	.417	.0655	.160
350 000	.681	.166	.395	.0620	.177	.402	.0630	.178	.415	.0652	.178	.423	.0664	.179
300 000	.630	.194	.399	.0626	.207	.409	.0642	.205	.421	.0660	.205	.431	.0675	.206
250 000	.575	.233	.409	.0642	.242	.419	.0658	.242	.430	.0675	.242	.442	.0693	.243
0000	.528	.275	.415	.0652	.283	.427	.0673	.283	.441	.0690	.284	.452	.0708	.285
000	.470	.346	.429	.0673	.353	.440	.0690	.353	.455	.0714	.354	.466	.0730	.355
00	.418	.437	.439	.0690	.443	.455	.0714	.443	.469	.0735	.443	.483	.0758	.444
0	.373	.550	.453	.0712	.554	.466	.0731	.554	.485	.0760	.555	.498	.0780	.556
1	.332	.695	.466	.0732	.698	.483	.0757	.697	.501	.0785	.699	.516	.0810	.700
2	.292	.879	.483	.0758	.882	.502	.0787	.882	.521	.0816	.833	.537	.0843	.883
3	.260	1.11	.499	.0782	1.11	.519	.0814	1.11	.538	.0845	1.11	.558	.0875	1.11
4	.232	1.40	.518	.0813	1.40	.538	.0845	1.40	.558	.0875	1.40	.577	.0905	1.40
6	.184	2.21	.557	.0873	2.21	.580	.0910	2.21	.601	.0943	2.21	.622	.0975	2.21

			11/64 BY 11/64			12/64 BY 12/64			13/64 BY 13/64			14/64 BY 14/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.417	.0655	.133	.427	.0670	.133	.434	.0681	.134	.441	.0691	.135
450 000	.772	.129	.423	.0665	.145	.431	.0675	.145	.439	.0690	.146	.449	.0705	.147
400 000	.728	.145	.429	.0673	.160	.436	.0683	.160	.446	.0700	.161	.457	.0717	.162
350 000	.681	.166	.436	.0685	.180	.446	.0700	.180	.453	.0710	.180	.464	.0729	.181
300 000	.630	.194	.444	.0697	.206	.456	.0715	.206	.461	.0722	.207	.473	.0742	.208
250 000	.575	.233	.454	.0712	.244	.465	.0730	.244	.475	.0745	.245	.486	.0762	.245
0000	.528	.275	.465	.0730	.285	.476	.0745	.285	.486	.0760	.286	.498	.0782	.287
000	.470	.346	.481	.0755	.355	.493	.0775	.355	.503	.0790	.355	.516	.0810	.356
00	.418	.437	.498	.0780	.445	.510	.0800	.445	.521	.0816	.445	.535	.0840	.446
0	.373	.550	.514	.0805	.556	.528	.0828	.556	.539	.0845	.556	.554	.0870	.557
1	.332	.695	.531	.0830	.700	.546	.0855	.700	.559	.0877	.700	.573	.0900	.700
2	.292	.879	.554	.0870	.882	.570	.0895	.882	.583	.0915	.883	.598	.0938	.884
3	.260	1.11	.574	.0900	1.11	.591	.0927	1.11	.606	.0950	1.11	.618	.0970	1.11
4	.232	1.40	.596	.0935	1.40	.613	.0962	1.40	.627	.0983	1.40	.643	.1010	1.41
6	.184	2.21	.643	.1010	2.21	.661	.1037	2.21	.678	.1063	2.21	.696	.1090	2.21

			16/64 BY 16/64			18/64 BY 18/64			20/64 BY 20/64			22/64 BY 22/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.457	.0717	.136	.474	.0744	.138	.487	.0764	.140	.501	.0785	.141
450 000	.772	.129	.462	.0725	.148	.481	.0754	.150	.496	.0778	.151	.509	.0800	.152
400 000	.728	.145	.471	.0738	.163	.487	.0764	.164	.505	.0792	.165	.519	.0815	.166
350 000	.681	.166	.480	.0753	.182	.496	.0778	.183	.513	.0805	.185	.529	.0830	.186
300 000	.630	.194	.491	.0770	.208	.511	.0802	.210	.526	.0825	.211	.541	.0848	.212
250 000	.575	.233	.505	.0792	.246	.524	.0822	.247	.541	.0848	.248	.557	.0875	.249
0000	.528	.275	.517	.0810	.287	.536	.0840	.288	.556	.0870	.289	.573	.0900	.290
000	.470	.346	.536	.0840	.357	.556	.0870	.357	.575	.0905	.358	.592	.0930	.360
00	.418	.437	.552	.0865	.446	.578	.0907	.446	.599	.0940	.447	.618	.0970	.448
0	.373	.550	.575	.0902	.558	.601	.0942	.558	.621	.0972	.558	.641	.1005	.559
1	.332	.695	.598	.0938	.700	.623	.0980	.700	.645	.1010	.701	.666	.1045	.702
2	.292	.879	.623	.0978	.884	.649	.1017	.884	.674	.1060	.885	.693	.1085	.886
3	.260	1.11	.649	.1018	1.11	.674	.1060	1.11	.698	.1095	1.11	.721	.1130	1.12
4	.232	1.40	.673	.1055	1.41	.701	.1100	1.41	.725	.1138	1.41	.746	.1170	1.41
6	.184	2.21	.725	.1135	2.22	.754	.1180	2.22	.780	.1225	2.22	.809	.1270	2.22

*Resistance based upon 100 percent conductivity at 25 degrees C (77 degrees F), including two percent allowance for spiral of strands and two percent allowance for spiral of conductors. For a temperature of 65 degrees C (149 degrees F) these resistance values would be increased 15 percent.

**The inductance is in millihenries; the reactance and the impedance are in ohms.

The table values were derived from the equation $L = 0.08047 + 0.741 \, Log_{10} \frac{D}{R}$ where R is the radius of conductor, D the distance between centers of conductors expressed in the same terms as R, and L the inductance in millihenries per mile of each conductor. All values in the table are single-phase and based upon a single conductor one mile long.

TABLE XXVI—INDUCTANCE, REACTANCE AND IMPEDANCE, AT 60 CYCLES, PER MILE OF SINGLE CONDUCTOR FOR THREE CONDUCTOR CABLES

AREA IN CIRCULAR MILS B & S NO.	DIAMETER IN INCHES	RESISTANCE PER MILE IN OHMS ★	INSULATION THICKNESS IN 64THS OF AN INCH ★★											
			3/64 BY 3/64			4/64 BY 4/64			6/64 BY 5/64			6/64 BY 6/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.338	.127	.172	.349	.131	.175	.360	.136	.178	.370	.140	.182
450 000	.772	.129	.340	.128	.181	.351	.132	.184	.362	.137	.189	.373	.141	.191
400 000	.728	.145	.343	.129	.195	.354	.134	.197	.367	.138	.201	.377	.142	.204
350 000	.681	.166	.346	.130	.211	.357	.135	.214	.370	.140	.217	.380	.143	.220
300 000	.630	.194	.349	.132	.235	.361	.136	.237	.374	.141	.240	.386	.145	.244
250 000	.575	.233	.353	.133	.268	.366	.138	.271	.381	.144	.274	.394	.149	.277
0000	.528	.275	.357	.135	.308	.372	.140	.309	.387	.146	.313	.403	.152	.316
000	.470	.346	.362	.136	.373	.379	.143	.375	.397	.150	.378	.411	.155	.381
00	.418	.437	.369	.139	.460	.388	.146	.461	.406	.153	.464	.423	.160	.466
0	.373	.550	.377	.142	.569	.398	.150	.571	.417	.157	.572	.432	.163	.573
1	.332	.695	.384	.145	.711	.405	.152	.713	.429	.162	.715	.447	.168	.716
2	.292	.879	.393	.148	.893	.417	.157	.894	.441	.166	.896	.463	.174	.896
3	.260	1.11	.403	.152	1.12	.431	.162	1.12	.454	.171	1.12	.476	.180	1.12
4	.232	1.40	.413	.156	1.41	.442	.167	1.41	.469	.177	1.41	.494	.186	1.41
6	.184	2.21	.437	.165	2.22	.470	.177	2.22	.501	.189	2.22	.529	.200	2.22
			7/64 BY 7/64			8/64 BY 8/64			9/64 BY 9/64			10/64 BY 10/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP OHMS
500 000	.814	.116	.379	.143	.184	.389	.146	.186	.398	.150	.190	.407	.153	.192
450 000	.772	.129	.384	.145	.194	.393	.148	.195	.403	.152	.200	.411	.155	.202
400 000	.728	.145	.389	.147	.206	.396	.149	.208	.409	.154	.212	.417	.157	.230
350 000	.681	.166	.395	.149	.222	.402	.151	.224	.415	.157	.229	.423	.160	.231
300 000	.630	.194	.399	.150	.245	.409	.154	.246	.421	.158	.251	.431	.162	.254
250 000	.575	.233	.409	.154	.279	.419	.158	.282	.430	.162	.285	.442	.166	.286
0000	.528	.275	.415	.157	.318	.427	.161	.320	.441	.166	.323	.452	.170	.323
000	.470	.346	.429	.162	.383	.440	.166	.385	.455	.172	.388	.466	.176	.389
00	.418	.437	.439	.166	.467	.455	.171	.471	.469	.177	.473	.483	.182	.474
0	.373	.550	.453	.171	.578	.466	.176	.578	.485	.183	.580	.498	.188	.582
1	.332	.695	.466	.176	.718	.483	.182	.697	.501	.189	.720	.516	.195	.721
2	.292	.879	.483	.182	.900	.502	.189	.900	.521	.196	.902	.537	.202	.902
3	.260	1.11	.499	.188	1.13	.519	.195	1.13	.538	.203	1.13	.558	.211	1.13
4	.232	1.40	.518	.195	1.41	.538	.203	1.41	.558	.210	1.42	.577	.218	1.42
6	.184	2.21	.557	.210	2.22	.580	.219	2.22	.601	.226	2.22	.622	.234	2.22
			11/64 BY 11/64			12/64 BY 12/64			13/64 BY 13/64			14/64 BY 14/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.417	.157	.195	.427	.161	.198	.434	.164	.202	.441	.166	.202
450 000	.772	.129	.423	.160	.204	.431	.162	.208	.439	.165	.211	.449	.170	.211
400 000	.728	.145	.429	.161	.216	.436	.164	.220	.446	.168	.222	.457	.172	.224
350 000	.681	.166	.436	.164	.235	.446	.168	.237	.453	.171	.240	.464	.175	.240
300 000	.630	.194	.444	.167	.256	.456	.172	.260	.461	.174	.262	.473	.178	.264
250 000	.575	.233	.454	.171	.289	.465	.175	.292	.475	.179	.295	.486	.183	.296
0000	.528	.275	.465	.175	.328	.476	.180	.330	.486	.183	.332	.498	.188	.334
000	.470	.346	.481	.181	.392	.493	.186	.395	.503	.190	.396	.516	.194	.398
00	.418	.437	.498	.188	.476	.510	.192	.479	.521	.196	.480	.535	.202	.482
0	.373	.550	.514	.194	.584	.528	.199	.586	.539	.203	.589	.554	.209	.590
1	.332	.695	.531	.200	.724	.546	.206	.725	.559	.211	.726	.573	.216	.728
2	.292	.879	.554	.209	.905	.570	.215	.906	.583	.220	.908	.598	.225	.910
3	.260	1.11	.574	.216	1.13	.591	.222	1.13	.606	.228	1.13	.618	.233	1.14
4	.232	1.40	.596	.224	1.42	.613	.231	1.42	.627	.236	1.42	.643	.242	1.42
6	.184	2.21	.643	.242	2.22	.661	.249	2.22	.678	.256	2.22	.696	.262	2.23
			16/64 BY 16/64			18/64 BY 16/64			20/64 BY 20/64			22/64 BY 22/64		
			IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS	IND. M.H.	REAC. OHMS	IMP. OHMS
500 000	.814	.116	.457	.172	.208	.474	.179	.212	.487	.183	.217	.501	.189	.222
450 000	.772	.129	.462	.174	.218	.481	.181	.224	.496	.187	.228	.509	.192	.232
400 000	.728	.145	.471	.178	.230	.487	.183	.235	.505	.190	.240	.519	.196	.244
350 000	.681	.166	.480	.181	.246	.496	.187	.252	.513	.193	.254	.529	.200	.260
300 000	.630	.194	.491	.185	.270	.511	.192	.274	.526	.198	.279	.541	.204	.282
250 000	.575	.233	.505	.190	.302	.524	.197	.306	.541	.204	.311	.557	.210	.314
0000	.528	.275	.517	.195	.338	.536	.202	.342	.556	.210	.348	.573	.216	.352
000	.470	.346	.536	.202	.403	.556	.209	.406	.575	.217	.410	.592	.223	.415
00	.418	.437	.552	.208	.486	.578	.218	.490	.599	.226	.494	.618	.233	.496
0	.373	.550	.575	.217	.592	.601	.226	.596	.621	.234	.599	.641	.242	.602
1	.332	.695	.598	.225	.732	.623	.235	.734	.645	.243	.737	.666	.251	.740
2	.292	.879	.623	.235	.912	.649	.245	.914	.674	.254	.917	.693	.261	.920
3	.260	1.11	.649	.245	1.14	.674	.254	1.14	.698	.262	1.14	.721	.272	1.14
4	.232	1.40	.673	.254	1.42	.701	.264	1.43	.725	.273	1.43	.746	.281	1.43
6	.184	2.21	.725	.273	2.22	.754	.284	2.23	.780	.294	2.23	.809	.305	2.23

*Resistance based upon 100 percent conductivity at 25 degrees C (77 degrees F), including two percent allowance for spiral of strands and two percent allowance for spiral of conductors. For a temperature of 65 degrees C (149 degrees F) these resistance values would be increased 15 percent.

**The inductance is in millihenries; the reactance and the impedance are in ohms.

The table values were derived from the equation $L = 0.08047 + 0.741\ Log_{10}\frac{D}{R}$ where R is the radius of conductor, D the distance between centers of conductors expressed in the same terms as R, and L the inductance in millihenries per mile of each conductor. All values in the table are single-phase and based upon a single conductor one mile long.

inductance, as determined by the fundamental formula, would thus tend to give values several percent less than the actual when applied to three-conductor cable calculations. On the other hand spiraling the conductors of three conductor cables tends to increase their reactance by several percent. It may, therefore, be assumed that the use of the fundamental formula in the case of three-conductor cables give results approximately correct. Skin effect on the larger cables will, however, tend to decrease the reactance slightly, particularly at 60 cycles.

CAPACITANCE OF 3 CONDUCTOR CABLES

Formulas for determining the approximate capacitance of three-conductor cables are cumbersome. They give reasonably accurate results only in the case of a homogeneous dielectric and in cases where the conductors are small compared to the radius of the sheath. They give inaccurate results in cases of large conductors closely spaced. Fig. 58* illustrates the various capacitances of a three-conductor cable. Formulas taken from Russel's "Alternating Currents" have been combined and converted to common logarithms and are given below. They were derived by the method of images and on the assumption that the conductors are round and symmetrically spaced with respect to the axis of the sheath.

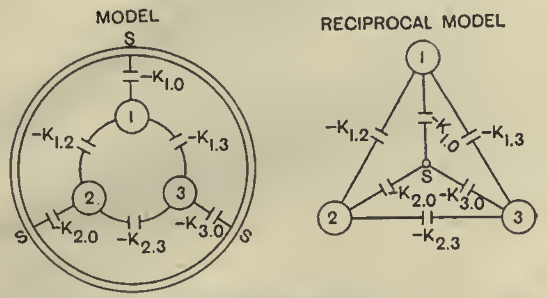


FIG. 58—REPRESENTATION OF CAPACITANCES OF A SYMMETRICAL THREE-PHASE CABLE

$$C_1 = \left[\frac{1}{13.82 \log_{10} \dfrac{R^4 - d^4}{3 R^2 d^2 r}} + \frac{1}{6.91 \log_{10} \left(\dfrac{1.73d}{r} \times \dfrac{R^2 - d^2}{(R^4 + R^2 d^2 + d^4)^{1/2}} \right)} \right] \times 0.179 \times K. \quad (70)$$

$$C_{12} = \left[\frac{1}{13.82 \log_{10} \dfrac{R^4 - d^4}{3 R^2 d^2 r}} - \frac{1}{13.82 \log_{10} \left(\dfrac{1.73d}{r} \times \dfrac{R^2 - d^2}{(R^4 + R^2 d^2 + d^4)^{1/2}} \right)} \right] \times 0.179 \times K \quad (71)$$

Where,—

- R = inside radius of sheath in centimeters (Fig. 59).
- r = radius of conductor in centimeters.
- d = distance between axis of conductor and axis of sheath in centimeters.
- K = the dielectric constant. For impregnated paper insulation it varies between 3 and 4; for varnished cambric insulation it varies between 4 and 6; for rubber insulation it varies between 4 and 9.
- C_1 = capacitance in microfarads per mile between one conductor and the other two conductors plus the sheath.
- C_{1-2} = mutual capacitance in microfarads per mile between any two conductors. The capacitance to neutral is twice this value.
- C_{12} is used in determining the capacitance for various combinations or arrangements as explained below.

CAPACITANCE AND SUSCEPTANCE—TABLE XXVII

Table XXVII contains values for capacitance and susceptance of three conductor paper insulated cable for the various sizes of conductors and thicknesses of insulation indicated. All values are based upon a value for *K of 3.5* and, as indicated, a thickness of insulation for the jacket the same as that surrounding each conductor. The values were calculated by equations (70) and (71).

The susceptance values given for 25 and 60 cycles are to neutral. In calculating the voltage regulation of circuits, it is general practice to calculate the regulation on the basis of one conductor to neutral. The susceptance between two of the conductors would be half the table values to neutral. The values for susceptance were calculated from the equation,—

Susceptance to neutral in micromhos $= 2 \pi f C$

Thus No. 0 three-conductor cable with 7/64 and 7/64 insulation has a capacitance between conductors of 0.195 microfarads (0.39 microfarads to neutral). The susceptance to neutral at 60 cycles therefore is,— $2 \pi 60 \times 0.39 = 147$ microfarads, as indicated by the table.

INTER-RELATION OF CAPACITANCE OF THREE-CONDUCTOR CABLES

The following equations for determining the effective capacitance for various arrangements of the three conductors and the sheath are given in Russell's "Alternating Currents."

Capacitance between 1 and 2 $= \frac{1}{2} (C_1 - C_{12})$ ...... (72)

Capacitance between 1 and 2, 3 $= \frac{2}{3} (C_1 - C_{12})$ .... (73)

Capacitance between 1 and S (2 and 3 insulated) $= \dfrac{(C_1 - C_{12})(C_1 + 2 C_{12})}{C_1 + C_{12}}$ .... (74)

Capacitance between 1 and S, 2 (3 insulated) $= \dfrac{(C_1 - C_{12})(C_1 + C_{12})}{C_1}$ .... (75)

Capacitance between 1 and S, 2, 3 $= C_1$ .... (76)

Capacitance between S and 1, 2, (3 insulated) $= \dfrac{2 (C_1 - C_{12})(C_1 + 2 C_{12})}{C_1}$ .... (77)

Capacitance between 1, S and 2, 3 $= 2 (C_1 + C_{12})$.... (78)

Capacitance between S and 1, 2, 3 $= 3 (C_1 + 2C_{12})$... (79)

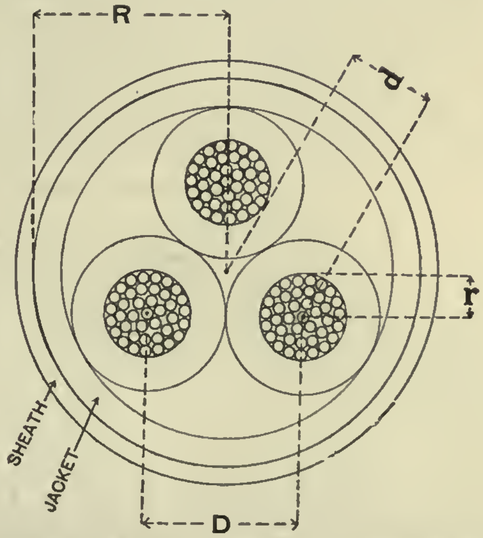


FIG. 59—DIMENSIONS OF A SYMMETRICAL THREE-PHASE CABLE

C_1 (76) may be measured in the ordinary way, by reading the throw of a mirror galvanometer and comparing with the throw given by a standard condenser. A further measurement of (78) or (79) will give a simple equation to find C_{12}. For instance, if measurements were taken of (78) and (79) and were found to be:—

*Reproduced from Alexander Russel's "Alternating Currents."

TABLE XXVII—CAPACITANCE AND SUSCEPTANCE PER MILE OF THREE CONDUCTOR PAPER INSULATED CABLES

INSULATION THICKNESS IN 64THS OF AN INCH

AREA IN CIRCULAR MILS / B & S NO.	3/64 BY 3/64					4/64 BY 4/64					5/64 BY 6/64					6/64 BY 6/64				
	CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL	
	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES
500 000	.680	-.217	.448	141	337	.613	-.175	.394	124	297	.555	-.154	.354	111	267	.505	-.137	.321	101	242
450 000	.667	-.197	.432	136	325	.590	-.169	.379	119	286	.538	-.149	.343	108	259	.488	-.130	.309	97	233
400 000	.657	-.194	.425	133	320	.570	-.159	.364	114	274	.517	-.142	.329	103	248	.475	-.125	.300	94	226
350 000	.640	-.189	.414	130	313	.560	-.158	.359	113	270	.506	-.138	.322	101	242	.460	-.119	.289	91	218
300 000	.606	-.176	.391	123	294	.545	-.153	.349	110	263	.490	-.131	.310	97	234	.446	-.116	.281	88	212
250 000	.590	-.171	.380	120	286	.518	-.142	.330	104	249	.468	-.125	.296	93	223	.427	-.109	.268	84	202
0000	.570	-.160	.365	111	265	.500	-.134	.317	100	239	.440	-.115	.280	88	211	.407	-.103	.255	80	192
000	.535	-.147	.341	107	257	.475	-.125	.300	94	226	.420	-.107	.262	82	198	.384	-.095	.239	75	180
00	.513	-.140	.327	103	246	.447	-.116	.281	88	212	.398	-.101	.249	78	187	.364	-.088	.226	71	170
0	.494	-.123	.308	97	232	.422	-.107	.264	83	199	.374	-.090	.232	73	175	.342	-.081	.211	66	159
1	.462	-.119	.290	91	219	.398	-.099	.248	78	187	.356	-.086	.221	69	167	.323	-.074	.198	62	149
2	.420	-.107	.263	83	198	.373	-.091	.232	73	175	.332	-.077	.203	64	153	.305	-.070	.187	59	141
3	.402	-.101	.251	79	189	.352	-.084	.218	69	165	.314	-.072	.193	61	145	.284	-.062	.173	54	131
4	.378	-.100	.239	75	180	.330	-.077	.203	64	153	.295	-.066	.180	57	136	.270	-.059	.164	52	124
6	.342	-.081	.211	66	159	.301	-.063	.182	57	137	.264	-.056	.160	50	121	.239	-.050	.144	45	108

AREA IN CIRCULAR MILS / B & S NO.	7/64 BY 7/64					8/64 BY 8/64					9/64 BY 9/64					10/64 BY 10/64				
	CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL	
	C_1	C_{12}	$C_{1\&2}$	26 CYCLES	80 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	80 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES
500 000	.468	-.124	.296	93	224	.435	-.115	.275	86	207	.410	-.104	.257	81	193	.392	-.097	.244	77	184
450 000	.454	-.119	.286	90	216	.427	-.107	.267	84	201	.405	-.103	.254	79	191	.380	-.093	.236	74	178
400 000	.442	-.116	.279	88	210	.415	-.105	.260	82	196	.392	-.099	.245	77	184	.368	-.090	.229	72	173
350 000	.426	-.108	.267	84	201	.398	-.099	.248	78	187	.380	-.093	.236	74	178	.358	-.087	.222	70	167
300 000	.415	-.105	.260	82	196	.390	-.096	.243	76	183	.365	-.089	.227	71	171	.348	-.081	.215	68	162
250 000	.400	-.101	.250	79	188	.370	-.089	.229	72	173	.352	-.087	.219	69	165	.332	-.078	.205	65	155
0000	.380	-.094	.237	75	178	.354	-.085	.220	69	166	.334	-.076	.205	64	155	.316	-.073	.194	61	146
000	.358	-.086	.222	70	168	.332	-.079	.205	64	155	.315	-.073	.194	61	146	.296	-.066	.181	57	136
00	.336	-.080	.208	65	157	.313	-.071	.192	60	145	.295	-.067	.181	57	136	.278	-.061	.169	53	127
0	.317	-.073	.195	61	147	.293	-.065	.179	56	135	.279	-.061	.170	54	128	.263	-.056	.159	50	120
1	.299	-.068	.183	58	138	.280	-.061	.170	54	128	.261	-.056	.158	50	119	.247	-.055	.151	47	114
2	.279	-.062	.170	54	128	.264	-.056	.160	50	121	.247	-.052	.150	47	113	.233	-.048	.140	44	106
3	.264	-.056	.160	50	121	.248	-.052	.150	47	113	.232	-.048	.140	44	106	.222	-.044	.133	42	100
4	.250	-.053	.151	47	114	.233	-.048	.140	44	106	.221	-.045	.133	42	100	.210	-.041	.125	39	94
6	.221	-.045	.133	42	100	.209	-.041	.125	39	94	.198	-.037	.117	37	88	.188	-.036	.112	35	85

AREA IN CIRCULAR MILS / B & S NO.	11/64 BY 11/64					12/64 BY 12/64					13/64 BY 13/64					14/64 BY 14/64				
	CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL	
	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES
500 000	.371	-.089	.230	72	173	.355	-.085	.220	69	166	.343	-.082	.212	67	160	.329	-.078	.203	64	153
450 000	.364	-.087	.225	71	170	.352	-.085	.218	68	164	.332	-.078	.205	64	155	.321	-.075	.198	62	149
400 000	.356	-.085	.220	69	166	.338	-.080	.209	66	157	.326	-.076	.201	63	152	.310	-.071	.190	60	143
350 000	.340	-.080	.210	66	158	.328	-.077	.202	63	152	.317	-.073	.195	61	147	.300	-.068	.184	58	139
300 000	.329	-.078	.203	64	153	.313	-.071	.192	60	145	.303	-.069	.186	59	140	.290	-.065	.177	56	133
250 000	.316	-.072	.199	63	150	. 98	-.068	.183	58	138	.288	-.064	.176	55	133	.276	-.061	.168	53	127
0000	.302	-.069	.185	58	140	.235	-.067	.176	55	133	.278	-.061	.169	53	127	.264	-.056	.160	50	121
000	.282	-.061	.171	54	129	.271	-.060	.165	52	124	.261	-.056	.158	50	119	.251	-.053	.152	48	115
00	.267	-.058	.162	51	122	.255	-.054	.154	48	116	.247	-.052	.150	47	113	.237	-.048	.142	45	107
0	.250	-.053	.151	48	114	.241	-.050	.145	46	109	.233	-.048	.140	44	106	.222	-.044	.133	42	100
1	.237	-.050	.143	45	108	.228	-.047	.137	43	103	.220	-.044	.132	42	100	.212	-.042	.127	40	96
2	.222	-.045	.133	42	100	.216	-.044	.130	41	98	.208	-.041	.124	39	94	.199	-.039	.118	37	89
3	.212	-.042	.127	40	96	.204	-.039	.121	38	91	.195	-.037	.116	36	88	.190	-.036	.113	36	85
4	.201	-.039	.120	38	91	.192	-.037	.114	36	86	.186	-.034	.110	35	83	.180	-.033	.106	33	80
6	.181	-.033	.107	34	81	.174	-.031	.102	32	77	.168	-.030	.099	31	75	.163	-.029	.096	30	73

AREA IN CIRCULAR MILS / B & S NO.	16/64 BY 16/64					18/64 BY 18/64					20/64 BY 20/64					22/64 BY 22/64				
	CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL		CAPACITANCE			SUSCEPTANCE TO NEUTRAL	
	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES	C_1	C_{12}	$C_{1\&2}$	25 CYCLES	60 CYCLES
500 000	.308	-.071	.189	59	143	.288	-.065	.176	55	133	.276	-.061	.168	53	127	.263	-.057	.160	50	121
450 000	.302	-.069	.185	58	140	.282	-.062	.172	54	130	.269	-.058	.163	51	123	.255	-.054	.154	48	116
400 000	.292	-.066	.179	56	135	.276	-.060	.168	53	127	.260	-.055	.157	50	118	.249	-.053	.151	47	113
350 000	.282	-.061	.171	54	129	.266	-.057	.161	51	121	.252	-.053	.152	48	115	.240	-.049	.144	45	108
300 000	.271	-.059	.165	52	125	.256	-.054	.155	49	117	.242	-.050	.146	46	110	.232	-.047	.139	44	105
250 000	.260	-.055	.157	49	118	.244	-.051	.147	46	111	.231	-.048	.139	44	105	.222	-.044	.133	42	100
0000	.248	-.052	.150	47	113	.233	-.049	.141	44	106	.222	-.044	.133	42	100	.211	-.041	.126	40	96
000	.234	-.048	.141	44	106	.221	-.044	.132	42	100	.211	-.041	.126	40	95	.200	-.038	.119	38	90
00	.222	-.044	.133	42	100	.209	-.041	.125	39	94	.199	-.038	.118	37	89	.191	-.036	.113	36	85
0	.210	-.041	.125	39	94	.199	-.038	.118	37	89	.188	-.036	.112	35	85	.180	-.030	.105	33	79
1	.198	-.038	.118	37	89	.188	-.035	.111	35	84	.181	-.034	.107	33	81	.172	-.030	.101	32	76
2	.190	-.036	.113	36	86	.179	-.032	.105	33	79	.170	-.030	.100	32	76	.163	-.029	.096	30	73
3	.179	-.034	.107	34	81	.171	-.031	.101	32	76	.162	-.028	.095	30	72	.155	-.026	.090	28	68
4	.170	-.030	.100	32	76	.162	-.027	.094	30	71	.154	-.026	.090	28	68	.148	-.025	.086	27	65
6	.154	-.025	.089	28	67	.147	-.024	.085	27	64	.141	-.023	.082	26	62	.136	-.021	.078	25	59

Capacitance—The values in table for capacitance were derived by formulas in Alexander Russel's "Alternating Currents." These values are as follows:—C_1 values are the capacitance in microfarads per mile between one conductor and the other two conductors plus sheath. $C_{1\text{-}2}$ values are the mutual capacitance in microfarads per mile between any two conductors. The capacitance to neutral is twice these values. C_{12} values per mile are used in the application of Russel's formulas for determining the capacitance corresponding to various arrangements of the three conductors and the sheath.

The Charging Current in amperes per mile for each conductor to neutral = susceptance in micromhos to neutral (taken from Table) $\times$ volts to neutral $\times$ 10^{-6}.

Dielectric Constant—All of the above table values are based upon a value for the dielectric constant K of 3.5. For all other values of K the table values will change in direct proportion. Values for K will usually be found between the following limits; for impregnated paper 3.0 to 4.0; for varnished cambric 4.0 to 6.0 and for rubber 4.0 to 9.0.

TABLE XXVIII—THREE-PHASE CHARGING KV-A PER MILE OF THREE-PHASE CIRCUIT OF THREE CONDUCTOR PAPER INSULATED CABLES

25 CYCLES

CHARGING KV-A PER MILE (EXPRESSED IN KV-A 3 PHASE) FOR PAPER INSULATED THREE CONDUCTOR CABLES BASED UPON A VALUE FOR 'K' OF 3.5 AND UPON A THICKNESS OF INSULATION SURROUNDING THE CONDUCTORS AND OF THE JACKET INDICATED.

AREA IN CIRCULAR MILS / B & S NO.	220 VOLTS 4/64	440 VOLTS 4/64	550 VOLTS 4/64	1100 VOLTS 5/64	2200 VOLTS 6/64	4400 VOLTS 8/64	6000 VOLTS 10/64	6000 VOLTS 14/64	6600 VOLTS 10/64	6600 VOLTS 14/64	6900 VOLTS 10/64
500 000	.00600	.0240	.0376	.134	.488	1.66	2.76	2.26	3.35	2.79	3.66
450 000	.00575	.0230	.0360	.131	.469	1.62	2.66	2.22	3.22	2.70	3.52
400 000	.00550	.0220	.0346	.125	.455	1.58	2.58	2.15	3.13	2.61	3.42
350 000	.00545	.0218	.0342	.122	.440	1.51	2.51	2.08	3.04	2.52	3.33
300 000	.00532	.0213	.0333	.117	.425	1.47	2.44	2.01	2.96	2.44	3.23
250 000	.00502	.0201	.0315	.113	.406	1.39	2.33	1.90	2.83	2.31	3.09
0000	.00483	.0193	.0303	.106	.387	1.33	2.19	1.79	2.65	2.18	2.90
000	.00454	.0182	.0285	.099	.363	1.24	2.04	1.72	2.48	2.09	2.71
00	.00424	.0170	.0266	.094	.343	1.16	1.90	1.61	2.31	1.96	2.52
0	.00400	.0160	.0250	.0883	.319	1.08	1.79	1.51	2.18	1.83	2.37
1	.00376	.0151	.0236	.0836	.300	1.04	1.68	1.43	2.05	1.74	2.23
2	.00352	.0141	.0221	.0775	.286	.965	1.58	1.33	1.92	1.61	2.09
3	.00333	.0133	.0209	.0740	.261	.908	1.51	1.29	1.83	1.57	2.00
4	.00309	.0124	.0194	.0690	.252	.850	1.40	1.18	1.70	1.44	1.85
6	.00275	.0110	.0173	.0605	.218	.755	1.26	1.08	1.52	1.31	1.66

25 CYCLES

B & S NO.	10.000 VOLTS 12/64	11000 VOLTS 12/64	11000 VOLTS 14/64	13.200 VOLTS 12/64	13.200 VOLTS 16/64	16.500 VOLTS 14/64	20.000 VOLTS 16/64	20.000 VOLTS 18/64	22,000 VOLTS 18/64	26000 VOLTS 18/64	26000 VOLTS 20/64
500 000	6.93	8.35	7.77	12.00	10.25	17.35	23.6	22.0	26.6	34.5	33.3
450 000	6.83	8.23	7.50	11.80	10.10	16.80	23.3	21.6	26.1	33.8	32.0
400 000	6.62	7.98	7.26	11.50	9.75	16.25	22.4	21.2	25.6	33.2	31.4
350 000	6.33	7.62	7.02	10.95	9.40	15.75	21.6	20.4	24.6	32.0	30.1
300 000	6.02	7.27	6.78	10.45	9.05	15.20	20.8	19.6	23.7	30.7	28.8
250 000	5.82	7.02	6.42	10.10	8.52	14.40	19.6	18.4	22.2	28.8	27.6
0000	5.53	6.66	6.05	9.56	8.17	13.55	18.8	17.6	21.3	27.6	26.4
000	5.23	6.30	5.82	9.05	7.65	13.05	17.6	16.8	20.3	26.3	25.1
00	4.82	5.80	5.45	8.36	7.30	12.25	16.8	15.6	18.8	24.4	23.2
0	4.62	5.57	5.09	8.00	6.78	11.40	15.6	14.8	17.8	23.2	22.0
1	4.32	5.21	4.84	7.48	6.43	10.85	14.8	14.0	16.9	21.9	20.7
2	4.12	4.97	4.48	7.13	6.26	10.05	14.4	13.2	15.9	20.7	20.1
3	3.82	4.60	4.36	6.60	5.92	9.78	13.6	12.8	15.5	20.1	18.8
4	3.62	4.36	4.00	6.27	5.57	8.95	12.8	12.0	14.5	18.8	17.6
6	3.22	3.87	3.63	5.57	4.87	8.15	11.2	10.8	13.1	16.9	16.3

60 CYCLES

B & S NO.	220 VOLTS 4/64	440 VOLTS 4/64	550 VOLTS 4/64	1100 VOLTS 5/64	2200 VOLTS 6/64	4400 VOLTS 8/64	6000 VOLTS 10/64	6000 VOLTS 14/64	6600 VOLTS 10/64	6600 VOLTS 14/64	6900 VOLTS 10/64
500 000	.0143	.0574	.0900	.323	1.17	4.00	6.58	5.49	8.00	6.65	8.73
450 000	.0138	.0554	.0858	.313	1.13	3.88	6.40	5.33	7.75	6.48	8.46
400 000	.0132	.0530	.0830	.301	1.09	3.79	6.20	5.13	7.53	6.22	8.22
350 000	.0131	.0523	.0818	.292	1.05	3.61	5.98	4.98	7.25	6.07	7.93
300 000	.0127	.0510	.0798	.283	1.02	3.54	5.80	4.78	7.05	5.80	7.70
250 000	.0120	.0483	.0755	.270	.975	3.35	5.55	4.56	6.75	5.52	7.37
0000	.0115	.0463	.0725	.255	.925	3.22	5.24	4.33	6.35	5.26	6.93
000	.0109	.0437	.0685	.240	.870	3.00	4.88	4.12	5.92	5.00	6.47
00	.0102	.0410	.0643	.226	.820	2.80	4.55	3.84	5.52	4.65	6.03
0	.0096	.0385	.0602	.212	.768	2.61	4.30	3.59	5.22	4.35	5.70
1	.0090	.0362	.0566	.202	.720	2.48	4.08	3.44	4.95	4.17	5.42
2	.0084	.0339	.0530	.185	.680	2.34	3.80	3.19	4.60	3.87	5.04
3	.0080	.0320	.0500	.176	.632	2.18	3.59	3.05	4.35	3.70	4.75
4	.0074	.0296	.0465	.165	.600	2.05	3.37	2.87	4.08	3.48	4.46
6	.0066	.0265	.0415	.147	.522	1.82	3.05	2.62	3.70	3.17	4.05

60 CYCLES

B & S NO.	10.000 VOLTS 12/64	11,000 VOLTS 12/64	11,000 VOLTS 14/64	13.200 VOLTS 12/64	13.200 VOLTS 16/64	18.600 VOLTS 14/64	20.000 VOLTS 16/64	20.000 VOLTS 18/64	22.000 VOLTS 18/64	25000 VOLTS 18/64	25000 VOLTS 20/64
500 000	16.7	20.1	18.5	28.9	24.9	41.5	57.3	53.3	64.2	83.3	79.5
450 000	16.5	19.8	18.1	28.5	24.4	40.3	56.0	52.0	62.7	81.5	77.0
400 000	15.8	19.0	17.3	27.2	23.5	38.8	54.0	50.9	61.3	79.5	74.0
350 000	15.3	18.4	16.8	26.4	22.4	37.6	51.6	48.5	58.3	75.8	72.2
300 000	14.6	17.6	16.1	25.2	21.8	36.0	50.0	46.8	56.5	73.3	69.0
250 000	13.9	16.7	15.4	24.0	20.5	34.5	47.3	44.3	53.6	69.5	65.7
0000	13.4	16.1	14.7	23.2	19.6	32.8	45.3	42.5	51.2	66.5	62.7
000	12.5	15.0	13.9	21.6	18.4	31.2	42.5	40.0	48.3	62.7	59.5
00	11.7	14.1	13.0	20.2	17.4	29.0	40.0	37.6	45.4	59.0	55.7
0	11.0	13.2	12.1	18.9	16.3	27.2	37.6	35.7	43.0	55.8	53.3
1	10.4	12.5	11.6	17.9	15.5	26.0	35.6	33.6	40.5	52.5	50.7
2	9.85	11.9	10.8	17.0	14.9	24.1	34.5	31.7	38.2	49.5	47.6
3	9.15	11.0	10.3	15.8	14.1	23.0	32.4	30.4	36.7	47.6	45.2
4	8.65	10.4	9.7	15.0	13.2	21.7	30.4	28.4	34.3	44.5	42.6
6	7.75	9.3	8.8	13.4	11.7	19.8	26.9	25.6	30.9	40.2	38.9

The values in Table XXVIII are based upon a value for the dielectric constant K of 3.5. For all other values of K the table values will change in direct proportion. Values for K will usually be found between the following limits; for impregnated paper 3.0 to 4.0; for varnished cambric 4.0 to 6.0 and for rubber 4.0 to 9.0.

$2C_1 + 2C_{12} = 0.410$ *mf. per mile* .... (78)

And $3C_1 + 6C_{12} = 0.450$ *mf. per mile* .... (79)

Therefore $C_1 = 0.26$ *mf. per mile*

$C_{12} = -0.055$ *mf per mile*

Numerical Examples—From Table XXVII for a 250 000 circ. mil., three-conductor cable having a band of insulation surrounding each conductor of 16/64 of an inch and an insulation jacket surorunding all three conductors of the same thickness, the following values are obtained:—

$C_1 = 0.260$ *mf. per mile.*

$C_{12} = -0.055$ *mf. per mile.*

Then, in the order in which the capacitance increases,—

Capacitance between 1 and 2 = 0.157 mf. per mile.... (72)

Capacitance between 1 and 2, 3 = 0.210 mf. per mile.. (73)

Capacitance between 1 and S (2 and 3 insulated) = 0.230 mf. per mile .... (74)

Capacitance between 1 and S, 2 (3 insulated) = 0.248 mf. per mile .... (75)

Capacitance between 1 and S, 2, 3 = 0.260 mf. per mile .... (76)

Capacitance between S and 1, 2 (3 insluated) = 0.363 mf. per mile .... (77)

Capacitance between 1, S and 2, 3 = 0.410 mf. per mile .... (78)

Capacitance between S and 1, 2, 3 = 0.450 mf. per mile .... (79)

COMPARISON OF CALCULATED CAPACITANCE WITH TEST RESULTS

The difference between measured results of capacitance and the results calculated by the above formulas are given in Fig. 60. It will be seen that in all cases these calculated results are less than the corresponding test results, the discrepancy being greater as the conductor becomes larger and the separation less. The differences vary from zero to as much as eleven percent for the largest cable, at the minimum spacing shown. The discrepancy is greatest with the minimum thickness of insulation. Since such cables would be used only for low-voltage service, the charging current would be small and consequently this error would probably be of little importance. For 6600 volt cables the results by the formula would seem to be approximately five percent too low.

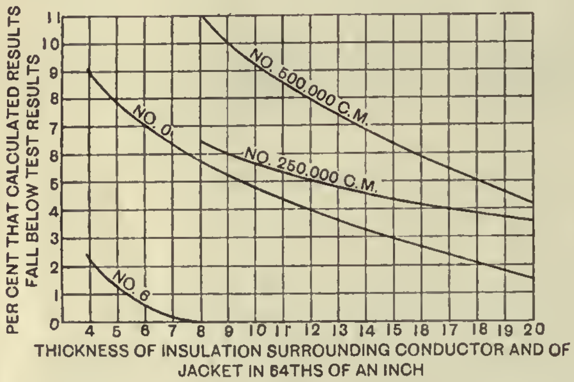


FIG. 60—COMPARISON OF CALCULATED AND MEASURED CAPACITANCES
Tests made on three conductor paper insulated cables, K = 3.5.

The cause of the discrepancy between the formula and test results is as follows:—In order to obtain a mathematical solution, Russell found it necessary to make certain approximations to the true physical conditions. Thus the resulting mathematical formula cannot give exact results. The approximation made by Russell is very close to the actual physical fact where the conductors are small compared with the insulation thickness, but it is not very close where the conductors are large compared with the insulation.

CHARGING KV-A—TABLE XXVIII

Table XXVIII contains values for charging current (expressed in kv-a, three-phase) for three-conductor paper insulated cables, both 25 and 60 cycles, based upon a value for K of 3.5. For other values of K, the table values would vary in proportion. For other thicknesses of insulation, the kv-a values would vary as the susceptance values corresponding to the thickness of insulation (See Table XXVII). In some cases, such for instance, as grounded neutral systems, the thickness of insulation of the jacket may be less than that surrounding the conductors. In such cases it might be desirable to calculate the susceptance and charging current, if accurate results were desired. The values for charging current corresponding to two thicknesses of insulation are included for some of the commonly employed transmission voltages.

These kv-a values were calculated by using the values for susceptance in Table XXVII which, in turn, were derived from the capacitance in the same table obtained by formulae (70) and (71). Thus a 350 000 circ. mil cable with 10/64 and 10/64 paper insulation has a 60 cycle susceptance to neutral of 167 micromhos per mile. Since the charging current in amperes to neutral equals the susceptance to neutral $\times$ volts to neutral $\times$ 10^{-6} and assuming 6600 volts, three-phase between conductors, we have:—

$$167 \times \frac{6600}{1.73} \times 10^{-6} = 0.637 \textit{ amperes to neutral.}$$

Charging kv-a = 0.637 × 3815 × 3 = 7.25 *kv-a*,
as indicated in Table XXVIII.

VALUES FOR K

The capacitance of any cable depends upon the dielectric constant of the insulating material and a dimension term or form factor. The dielectric constant should be determined from actual cables and not from samples of material. The usual range in value for K is given below.

	Value of K
Impregnated Paper	3.0 to 4.0
Varnish Cambric	4.0 to 6.0
Rubber	4.0 to 9.0

All values in Tables XXVII and XXVIII are based upon a value of K of 3.5. For all other values of K all table values will vary in the same proportion as their K values. The actual value of permittivity of most paper insulation runs about ten percent less than the value 3.5 which has been used in calculating the accompanying table values. The true alternating-current capacitance is always considerably lower than the capacitance measured with ballistic galvanometer.

REFERENCES

"Electric Power Conductors," by W. S. Del Mar.

"Electric Cables," by Coyle and Howe, London, England.

"The Heating of Cables with Current," by Melsom & Booth, *Journal I. E. E.*, Vol. 47—1911.

"Current and Rating of Electric Cables," by Atkinson & Fisher, *Trans. A. I. E. E.*, 1913, p. 325.

"The Heating of Cables Carrying Current," by Dushman, *Trans. A. I. E. E.*, 1913, p. 333.

"Effect of Moisture in the Earth on Temperature of Underground Cables," by Imlay, *Trans. A. I. E. E.*, 1915, Part I, p. 233.

"Temperature Rise of Insulated Lead Covered Cables," by Richard A. Powell, *Trans. A. I. E. E.*, 1916, Part II, p. 1017.

"The Restoration of Service After a Necessary Interruption," by Rickets, *Trans. A. I. E. E.*, 1916, Part II, p. 635.

"The Dielectric Field in an Electric Power Cable," by Atkinson, *Trans. A. I. E. E.*, June 1919.

"The Current Carrying Capacity of Lead Covered Cables," by Atkinson, *Journal A. I. E. E.*, Sept. 1920.

CHAPTER XIV

SYNCHRONOUS MOTORS AND CONDENSERS FOR POWER-FACTOR IMPROVEMENT

BEFORE discussing the employment of synchronous machinery for improving the power-factor of circuits, it may be desirable to review how a change in power-factor affects the generators supplying the current.

Fig. 61 shows the effect of in-phase, lagging and leading components of armature current upon the field strength of generators*. A single-coil armature is illustrated as revolving between the north and south poles of a bipolar alternator. The coil is shown in four positions 90 degrees apart, corresponding to one complete revolution of the armature coil. The direction of the field flux is assumed to be constant as indicated by the arrows on the field poles of each illustration. In addition to this field flux, when current flows through the armature coil another magnetic flux is set up, magnetizing the iron in the armature in a direction at right angles to the plane of the armature coil. This will be referred to as armature flux.

This armature flux varies with the armature current, being zero in a single-phase generator when no armature current flows, and reaching a maximum when full armature current flows. It changes in direction relative to the field flux as the phase angle of the armature current changes.

The revolving armature coil generates an alternating voltage the graph of which follows closely a sine wave, as shown in Fig. 61. When it occupies a vertical plane marked *start* no voltage is generated, for the reason that the instantaneous travel of the coil, is parallel with the field flux.** As the coil moves forward in a clockwise direction, the field enclosed by the armature coil decreases; at first slowly but then more rapidly until the rate of change of flux through the coil becomes a maximum when the coil has turned 90 degrees, at which instant the voltage generated becomes a maximum. As the horizontal position is passed the voltage decreases until it again reaches zero when the coil has traveled 180 degrees or occupies again a vertical plane. As the travel continues the voltage again starts to increase but since the motion of the coil relative to the fixed magnetic field is reversed the voltage in the coil builds up in the reverse direction during the second half of the revolution. When the coil has reached the two 270 degree position the voltage has again become maximum but in the opposite direction to that when the coil occupied the position of 90 degrees. When the coil returns to its original position at the start the voltage has again dropped to zero, thus completing one cycle.

If the current flowing through this armature coil is in phase with the voltage, it will produce cross magnetization in the armature core, in a vertical direction, as indicated by the arrows at the 90 and 270 degree positions. The cross magnetization neither opposes nor adds to the field flux at low loads and therefore has comparatively little influence on the field flux. At heavy loads, however, this cross magnetization has considerable demagnetizing effect, due to the shift in rotor position resulting from the shifting of the field flux at heavy loads.

If the armature is carrying lagging current, this current will tend to magnetize the armature core in such a direction as to oppose the field flux. This action is shown by the middle row of illustrations of Fig. 61. Under these illustrations is shown a current wave lagging 90 degrees representing the component of current required to magnetize transformers, induction motors, etc. When the lagging component of current reaches its maximum value the armature coil will occupy a vertical position (position marked start, 180 degrees and 360 degrees) and in this position the armature flux will directly oppose the field flux, as indicated by the arrows. The result is to reduce the flux threading the armature coil and thus cause a lowering of the voltage. This lagging current encounters resistance and a relatively much greater reactance, each of which consumes a component of the induced voltage, as shown in Fig. 62. When the armature current is lagging, the voltage induced by armature inductance is in such a direction as to subtract from the induced voltage, and thus the voltage is still further lowered, as a result of the armature self induction. In order to bring the voltage back to its normal value it will be necessary to increase the field flux by increasing the field current. Generators are now usually designed of sufficient field capacity to compensate for lagging loads of 80 per cent power-factor.

If the armature is carrying a leading current this leading component will tend to magnetize the armature core in such a direction as to add to the field flux.

*For a more detailed discussion of this subject the reader is referred to excellent articles by F. D. Newbury in the ELECTRIC JOURNAL of April 1918, "Armature Reaction of Polyphase Alternators"; and of July 1918, "Variation of Alternator Excitation with Load".

**For the sake of simplicity this and the following statements are based upon the assumption that armature reaction does not shift the position of the field flux. Actually, under load, the armature reaction causes the position of the field flux to be shifted toward one of the pole tips, so that the position of the armature coil is not quite vertical at the instant of zero voltage in the coil.

This action is shown by the bottom row of illustrations of Fig. 61. Under these illustrations is shown a current wave leading the voltage wave by 90 degrees. When the leading component of current reaches its maximum values, the armature coil will again occupy vertical positions, but the armature flux will add to that of the field flux, as indicated by the arrow. The resulting flux threading the armature coil is thus increased causing a rise in voltage. This leading current flowing through the generator armature encounters resistance and a relatively much greater reactance, each of which consumes a component of the induced voltage, as shown in Fig. 62. When the armature current is lead-

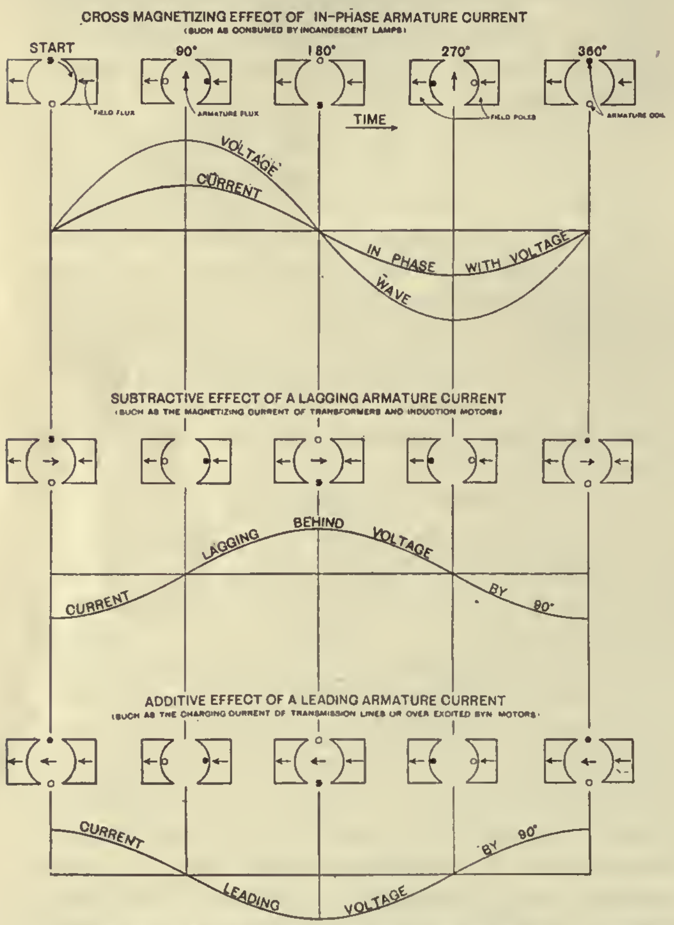


FIG. 61—EFFECT OF ARMATURE CURRENT UPON FIELD EXCITATION OF ALTERNATING-CURRENT GENERATORS

ing, the voltage induced by armature inductance is in such a direction as to add to the induced voltage and thus the voltage at the alternator terminals is still further increased as the result of armature self-induction. In order to reduce the voltage to its normal value it is necessary to decrease the field flux by decreasing the field current.

With alternators of high reaction the magnetizing or de-magnetizing effect of leading or lagging current will be greater than in cases where the armature reaction is low. For instance if the alternator is so designed that the ampere turns of the armature at full armature current are small compared to its field ampere turns, the voltage of such a machine would be less disturbed with a change in power-factor of the armature current than in an alternator having armature ampere turns large compared with its field ampere turns.

Modern alternators are of such design that when carrying rated lagging current at zero power-factor they require approximately 200 to 250 percent of their no-load field-current and when carrying rated leading current at zero power-factor they require approximately —15 to +15 percent of their no-load field current. Thus with lagging armature current the iron will be worked at a considerable higher point on the saturation curve and the heating of the field coils will increase because of the greater field current required.

The voltage diagrams of Fig. 62 are intended to show only the effect of armature resistance and armature reactance upon voltage variation. Voltage regu-

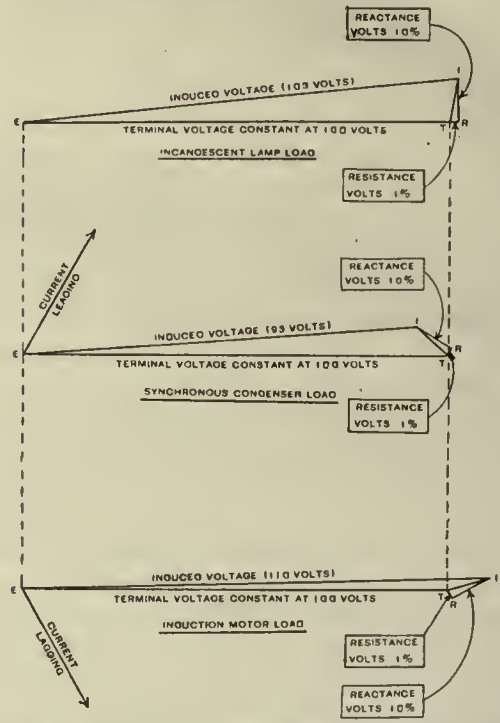


FIG. 62—VECTORS ILLUSTRATING THE EFFECT OF ARMATURE REACTANCE AND RESISTANCE UPON THE TERMINAL VOLTAGE FOR IN-PHASE, LEADING AND LAGGING CURRENTS

lation is the combined effect of armature impedance and armature reaction. Turbogenerators have, for instance, very low armature reactance but their armature reaction is higher, so that the resulting voltage regulation may not be materially different from that of a machine with double the armature reactance. Under normal operation armature reaction is a more potent factor in determining the characteristics of a generator than armature reactance. In the case of a generator with a short circuit ratio of unity, this total reactive effect may be due, 15 percent to armature reactance and 85 percent to armature reaction.

For the case illustrated by Fig. 62 the field flux corresponds to the induced voltage indicated, but the field current does not. The field current corresponds to a value obtained by substituting the full synchronous impedance drop for that indicated.

SYNCHRONOUS CONDENSERS AND PHASE MODIFIERS

The term "synchronous condenser" applies to a synchronous machine for raising the power-factor of circuits. It is simply floated on the circuit with its fields over excited so as to introduce into the circuit a leading current. Such machines are usually not intended to carry a mechanical load. When this double duty is required they are referred to as synchronous motors for operation at leading power-factor. On long transmission circuits, where synchronous condensers are used in parallel with the load for varying the power-factor, thereby controlling the transmission voltage, it is sometimes necessary to operate them with under excited fields at periods of lightloads. They are then no longer synchronous condensers but strictly speaking become synchronous reactors.

Whether synchronous motors for operation at leading power-factor, synchronous condensers or synchronous reactors be used they virtually do the same thing, that is; their function is to change the power-factor of the load by changing the phase angle between the armature current and the terminal voltage. They

TABLE R—SYNCHRONOUS CONDENSER LOSSES

Kv-a	Loss (Kw)	Kv-a	Loss (Kw)
100	12	3500	180
200	18	5000	220
300	22	7500	320
500	32	10000	420
750	47	15000	620
1000	55	20000	820
1500	70	25000	1000
2000	120	35000	1400
2500	130	50000	2000

are, therefore, sometimes referred to as "phase modifiers." This latter name seems more appropriate when the machine is to be operated both leading and lagging, as when used for voltage control of long transmission lines.

Rating — Synchronous condensers as regularly built may be operated at from 30 to 40 percent of their rating lagging, depending upon the individual design. Larger lagging loads result in unstable operation on account of the weakened field. Phase modifiers can be designed to operate at full rating, both leading and lagging, but they are larger, require larger exciters, have a greater loss and cost 15 to 20 percent more than standard condensers.

Starting—Condensers are furnished with squirrel-cage damper windings, to prevent hunting, which also provides a starting torque of approximately 30 percent of normal running torque. They have a pull-in torque of around 15 percent of running torque. The line current at starting varies from 50 to 100 percent of normal. The larger units are sometimes equipped for forced oil lubrication, which raises the rotor sufficiently to permit of oil entering the bearing, thus reducing the starting current.

Mechanical Load—Synchronous condensers are generally built for high speeds and equipped with shafts of small diameter. If they are to be used to transmit some mechanical power it may be necessary to equip them with larger shafts and bearings, particularly if belted rather than direct connected. If a phase modifier is to furnish mechanical energy and at the same time to operate lagging at times of light load for the purpose of holding down the voltage on an unloaded transmission line there may be danger of the machine falling out of step, if a heavy mechanical load occurs when the machine is operating with a weak field.

Losses—At rated full load leading power-factor the total losses, including those of the exciter, will vary from approximately 12 percent for the smallest capacity to approximately four percent for the larger capacity 60 cycle synchronous condensers. The approximate

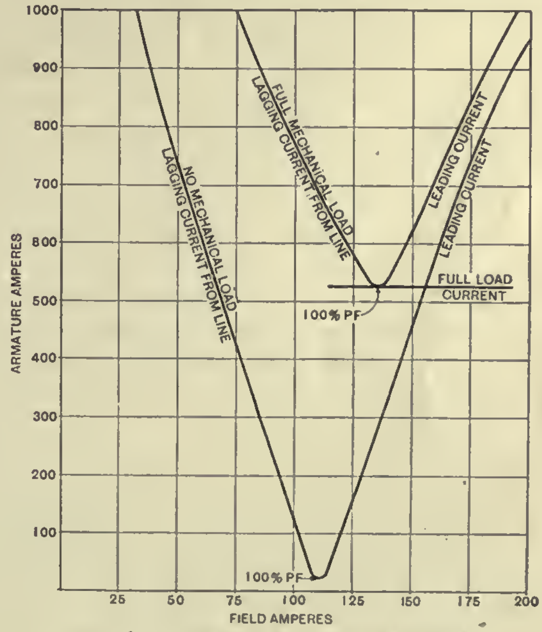


FIG. 63—V-CURVES OF A PHASE MODIFIER

values given in Table R may be of service for preliminary purposes.

"V" Curves—The familiar V curves shown in Fig. 63 serve to give some idea of the variation in field current for a certain phase modifier when operating between full load lagging and full load leading kv-a.* For this particular machine the excitation must be increased from 112 amperes at no load minimum input or unity power-factor to 155 amperes at full kv-a output leading or a range of 1.4 to 1 in. field excitation. For operation between full lagging and full leading, with no mechanical work done, the range of excitation is from 67 to 155 or 2.3 to 1.

Generators as Condensers—Ordinary alternators may be employed as synchronous condensers or synchronous motors by making proper changes in their field poles and windings to render them self-starting

*These curves have been reproduced from H. B. Dwight's book "Constant Voltage Transmission".

and safely insulated against voltages induced in the field when starting.

Where transmission lines feed into a city net work and a steam turbine generator station is available these generating units can serve as synchronous condensers by supplying just enough steam to supply their losses and keep the turbine cool. When operated in this way they make a reliable standby to take the important load quickly in case of trouble on a transmission line.

Location for Condensers—The nearer the center of load that the improvement in power-factor is made the better, as thereby the greatest gain in regulation, greatest saving in conductors and apparatus are made since distribution lines, transformers, transmission lines and generators will all be benefited.

How High to Raise the Power-Factor—Theoretically for most efficient results the system power factor should approach unity. The cost of synchronous apparatus having sufficient leading current capacity to raise the power-factor to unity increases so rapidly as unity is approached, as to make it uneconomical to carry the power-factor correction too high. Not only the cost but also the power loss chargeable to power-factor improvement mounts rapidly as higher power-factors are reached. This is for the reason that the reactive kv-a in the load corresponding to each percent change in power-factor is a maximum for power-factors near unity. It usually works out that it doesn't pay to raise the power factor above 90 to 95 percent, except in cases where the condenser is used for voltage control, rather than power-factor improvement.

DETERMINING THE CAPACITY OF SYNCHRONOUS MOTORS AND CONDENSERS FOR POWER-FACTOR IMPROVEMENT

A very simple and practical method for determining the capacity of synchronous condensers to improve the power-factor is by aid of cross section paper. A very desirable paper is ruled in inch squares, sub-ruled into 10 equal divisions. With such paper, no other equipment is required.

With a vector diagram it is astonishing how easy it is to demonstrate on cross section paper, the effect of any change in the circuit. A few typical cases are indicated in Fig. 64. These diagrams are all based upon an original circuit of 3000 kv-a at 70 percent power-factor lagging, shown by (1). It is laid off on the cross section paper as follows. The power of the circuit is 70 percent of 3000 or 2100 kw, which is laid off on line *AB*, by counting 21 sub-divisions, making each sub-division represent 100 kw or 100 kv-a. Now lay a strip of blank paper over the cross section paper and make two marks on one edge spaced 30 sub-divisions apart. This will then be the length of the line *AC*. This blank sheet is now laid over the cross section paper with one of the marks at the edge held at the point *A*. The other end of the paper is moved downward until the second mark falls directly below the point *B* thus locating point *C*. The length of the line *BC* represents the lagging reactive kv-a in the circuit, in this case 2140 kv-a.

Diagram (2) shows the effect of adding a 1500 kv-a synchronous condenser to the original circuit. The full load loss of this condenser is assumed as 70 kw. The resulting kv-a and power-factor are determined as follows: Starting from the point *C* trace to the right a line 0.7 of a division long. This is parallel to the line *AB* for the reason that it is true power, so that there is now 2170 kw true energy. The black triangle represents the condenser, the line *CD*, 15 divisions long, representing the rating of the condenser. In this case, however, the vertical line is traced upward in place of downward, because the condenser kv-a is leading. This condenser results in decreasing the load from 3000 kv-a at 70 percent power-factor to 2275 kv-a at 95.4 percent power-factor. The line *AD* represents in magnitude and direction, the resulting kv-a in this circuit. The power-factor of the resulting circuit is the ratio of the true energy in kw to the kv-a or 95.4 percent, in this case. Since the line *AD* lays below the line *AB*, that is in the lagging direction, the power-factor is lagging.

Diagram (3) is the same as (2) except that the condenser is larger, being just large enough to neutralize all of the lagging component of the load, resulting in a final load of 2215 kw at 100 percent power-factor. Diagram (4) is similar to (3) except that a still larger condenser is shown. This condenser not only neutralizes all of the lagging kv-a of the load but in addition introduces sufficient leading kv-a into the circuit to give a leading resultant power-factor of 94 percent with an increase in kv-a of the resulting circuit from 2215 of (3) to 2400 kv-a of (4).

Diagram (5) illustrates the addition to the original circuit of a 100 percent power-factor synchronous motor of 600 hp. rating As this motor has no leading or lagging component, there is no vertical projection. The power-factor of the circuit is raised from 70 to 77 percent as the result of the addition of 500 kw true power (load plus loss in motor) to the circuit. A resistance load would have this same effect.

Diagram (6) shows a 450 kw (600 hp.) synchronous motor of 625 kv-a input at 80 percent leading power-factor added to the original circuit. The input to this motor (including losses) is assumed to be 500 kw. The resulting load for the circuit is 3150 kv-a at 82.5 percent lagging power-factor.

The Diagram (7) shows an 850 kw, (1140 hp.) synchronous motor generator of 1666 kv-a input at 60 percent power-factor leading added to the original circuit. This gives a resulting load of 3200 kv-a at 96.9 percent lagging power-factor.

Diagram (8) shows the addition to the original circuit of the following loads, including losses.

A 550 kw synchronous converter at 100 percent power-factor.
A 650 kw induction motor at 70 percent lagging power-factor.
A 500 kw synchronous motor.

The resultant load of this circuit is 3800 kw, and if a power-factor of 95 percent lagging is desired the total kv-a will be 4000. The line *AD* may be located by a piece of marked paper and the capacity of the necessary synchronous motor scaled off. This is found to be 1650 kv-a at 30.3 percent power-factor.

The Circle Diagram—The circle diagram in Fig. 65 shows the fundamental relations between true kw, reactive kv-a and apparent kv-a corresponding to different power-factors, the values upon the chart being read to any desired scale to suit the numerical values of the problem under consideration. This diagram is suffi-

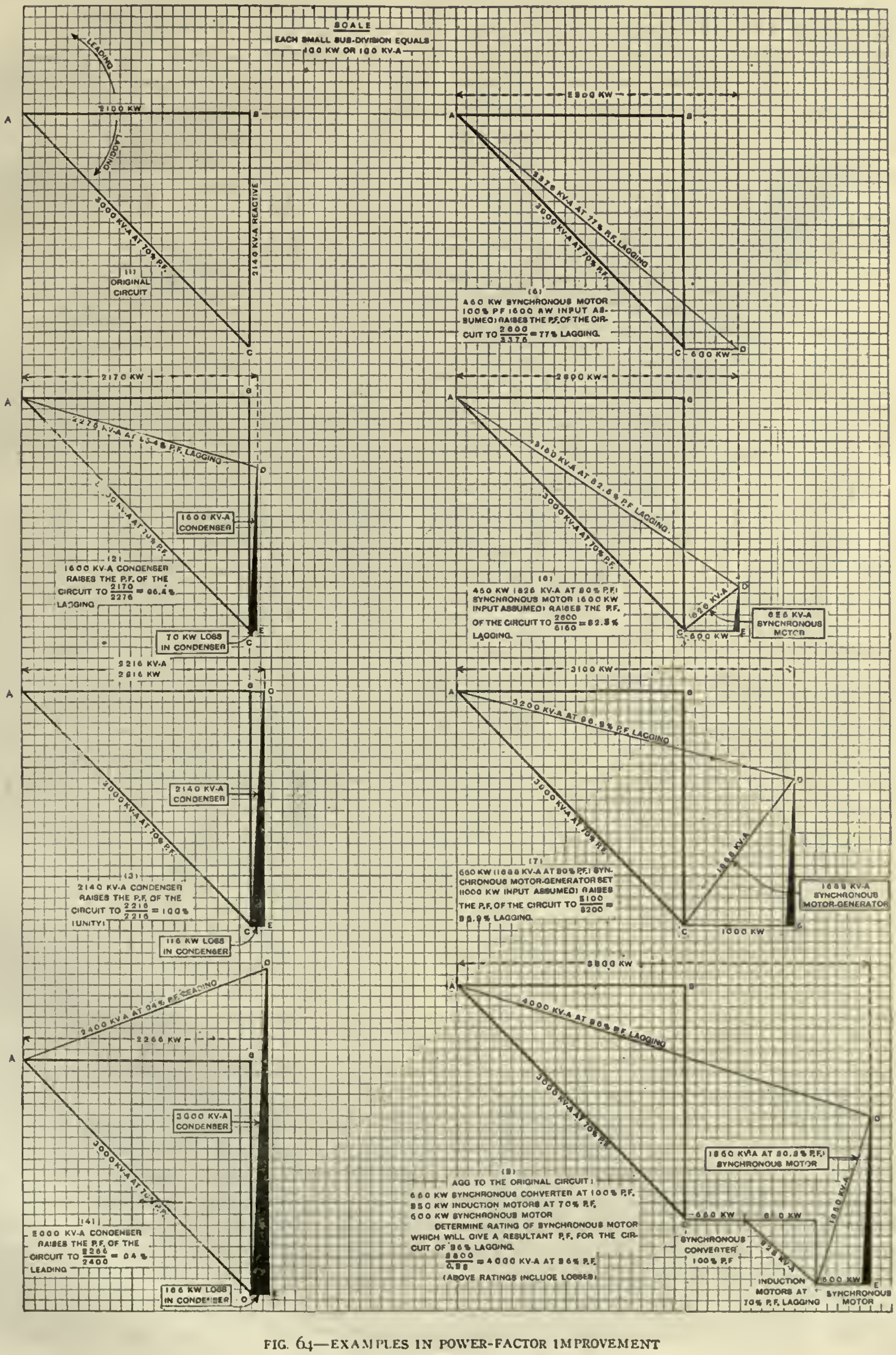


FIG. 64—EXAMPLES IN POWER-FACTOR IMPROVEMENT

ciently accurate for ordinary power-factor problems. In place of drawing out the vector diagrams as just explained they are traced out with a pin point on the circle diagram.

Assume again a load of 2100 kw at 70 percent power-factor lagging, and that the power-factor is to be raised to 95.4 percent as in (2) of Fig. 64, and that the loss in the condenser necessary to accomplish this is again taken as 70 kw. The capacity of the synchronous condenser may be traced on the circle diagram as follows: From the true power load of 2100 kw (top horizontal line) follow vertically downward until the diagonal line representing 70 percent power-factor is reached. This is opposite 2140 kv-a reactive component. From the point thus obtained, go horizontally to the right a distance representing 70 kw power. From this point go vertically upward until the diagonal line representing 95.4 percent power-factor is reached. Then read the amount of reactive kv-a (640) corresponding to this last point. The original lagging component of 2140—640=1500 kv-a which is approximately the capacity of the condenser necessary to accomplish the above results. Actually the rating of the condenser would be the hypotenuse rather than the vertical projection. The error in assuming the vertical projection as the rating of the condenser is negligible unless the condenser furnishes mechanical power, in which case the hypotenuse should be marked on a separate strip of paper and its length determined from the kv-a scale.

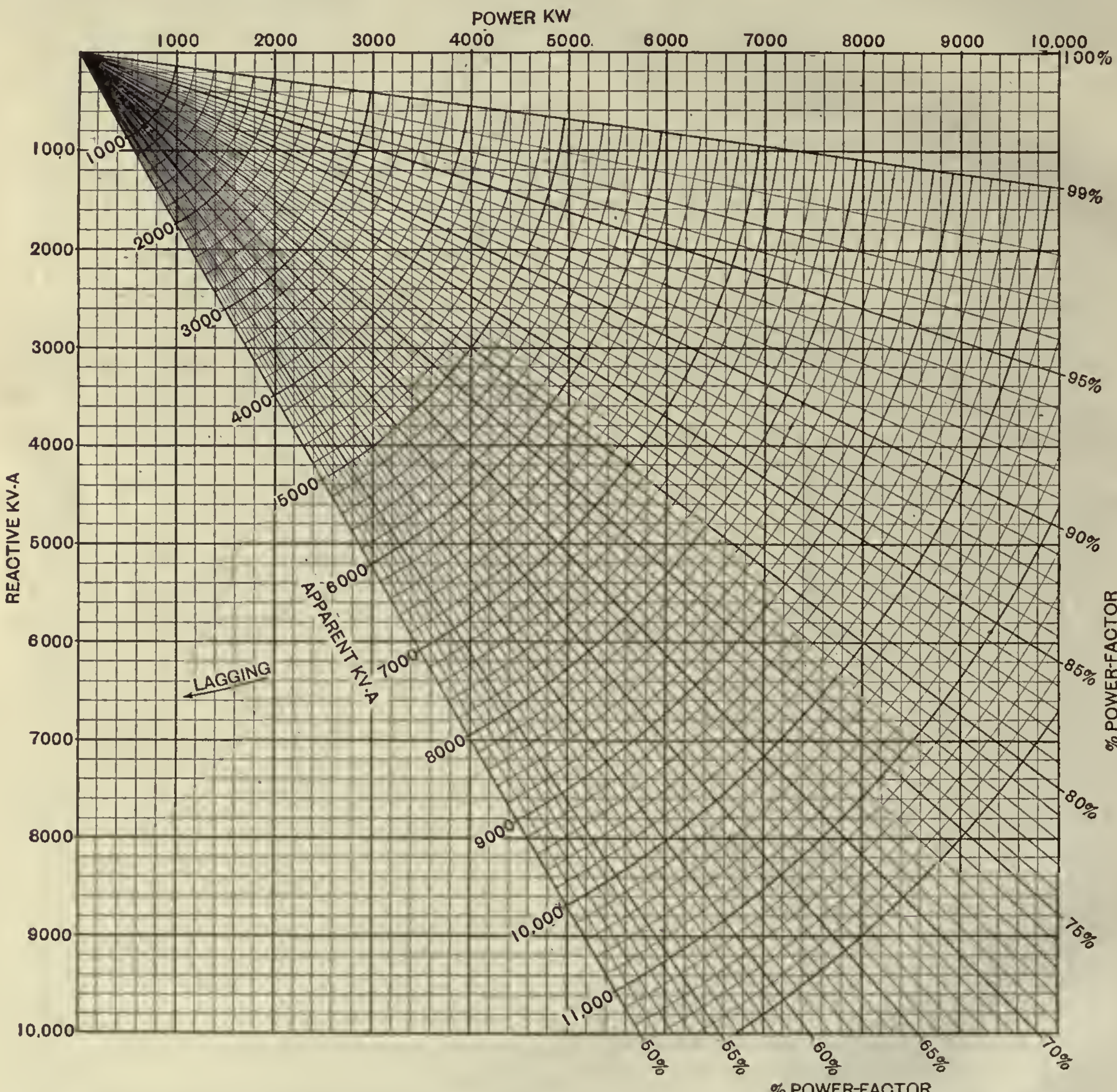


FIG. 65—RELATION BETWEEN ENERGY LOAD, APPARENT LOAD AND REACTIVE KV-A FOR DIFFERENT POWER FACTORS

ADVANTAGE OF HIGH POWER-FACTOR

Less Capacity Installed—Low power-factors demand larger generators, exciters, transformers, switching equipment and conductors. Loads of 70 percent power-factor demand equipment of 28 percent greater capacity than would be required if the power-factor were 90 percent. The cost of apparatus for operation at 70 percent power-factor would be approximately 15 percent greater than the cost of similar apparatus for 90 percent power-factor operation, since the capacity of apparatus to supply a certain amount of energy is inversely proportional to the power-factor.

Higher Efficiency—Assume that the power-factor of a 1000 kv-a (700 kw at 70 percent power-factor) transmission circuit is raised to 90 percent. As the copper loss varies as the square of the current, raising the power-factor reduces the copper loss approximately 40 percent. If we assume an efficiency for the generator of 93 percent (one percent copper loss); for combined raising and lowering transformers 94 percent (three percent copper loss) and for the transmission line 92 percent, the saving in copper loss corresponding to 90 percent power-factor operation would be as follows:

Generators .... 0.4 percent
Transformers .... 1.2 percent
Transmission line .... 3.2 percent
Total .... 4.8 percent or approximately 33 kw.

To raise the power-factor to 90 percent would require a synchronous condenser of 375 kv-a capacity. This size condenser would have a total loss of about 30 kw, resulting in a net gain in loss reduction of three kw. Against this gain would be chargeable, the interest and depreciation of the condenser cost with its accessories, also any cost of attendance which there might be in connection with its operation. It is evident that in this case it would not pay to install a condenser if increased efficiency were the only motive.

TABLE S—COST OF POWER-FACTOR CORRECTION WITH SYNCHRONOUS MOTORS

Syn. Motor Kv-a	Motor Will Furnish		Chargeable to Power-Factor Correction	
	Mech. Kw	Leading Kv-a	Loss Kw	Difference in Price
140	100	100	1.6	$500.00
280	200	200	2.5	500.00
420	300	300	5.0	500.00
700	500	500	8.0	800.00
1050	750	750	9.0	1000.00
1400	1000	1000	14.0	1200.00

The improvement in power-factor can be more cheaply and efficiently obtained by the installation of one or more synchronous motors designed for operation at leading power-factor. Sufficient capacity of these will give, in addition to mechanical load, sufficient leading current to raise the power-factor to 90 percent. The extra expense and increased loss of synchronous motors enough larger to furnish the necessary leading component for power-factor correction is very small. Table S gives in a very approximate way, some idea of the amount of loss and proportional cost of synchronous motors chargeable to power-factor improvement when delivering both mechanical power and leading current.

Thus if a synchronous condenser is used on the above circuit there is a loss of 30 kw, chargeable to power-factor improvement, whereas if a synchronous motor of sufficient capacity (530 kv-a) to give 375 kw mechanical work and at the same time the necessary 375 kv-a leading current for power-factor improvement, the extra loss chargeable to power-factor improvement would be something like six kw. The increased cost of a synchronous motor to furnish 375 kv-a leading current in addition to 375 kw power would be about $600 whereas the cost of a 375 kv-a condenser would be in the neighborhood of $4000. Varying costs and designs make cost and loss values unreliable. They are given here only to illustrate the points which should be considered when considering synchronous motors vs synchronous condensers.

Improved Voltage Regulation—The voltage drop under load for generators, transformers and transmission lines rapidly increases as the power-factor goes down. Table T gives an idea of the variation in voltage drop corresponding to various power-factors at 60 cycles.

Automatic voltage regulation may be used to hold the voltage constant at the generators or at some other point, but it cannot prevent voltage changes at all points of the system.

Increased Plant Capacity—The earlier alternators were designed for operation at 100 percent power-factor with prime movers, boilers, etc. installed on the same basis. Increasing induction motor loads have resulted in power-factors of 70 and 80 percent. As a result, some of the older generating stations are being operated with prime movers, boilers etc. underloaded because the 100 percent power-factor generators which

TABLE T—EFFECT OF POWER-FACTOR ON VOLTAGE DROP

Percent Power-Factor.	100	90	80	70
Generators *(older design)	8.0	-	25.0	-
Transformers	1.2	4.1	4.9	5.5
Transmission line	7.9	13.0	14.2	15.2

they drive limit the amount of power that can be generated without endangering the generator windings. This condition some times makes it necessary to operate three units, where two might be sufficient to carry the load at unity power-factor. The shutting down of a unit would result in a considerable saving in steam consumption. A recent case came up of a transmission line 30 miles long, fed at each end by a small generating station. On account of heavy line drop it was necessary to operate both stations to furnish the comparatively light night load. Investigation developed that by installing a synchronous condenser at one of these terminal stations for reducing the voltage drop in the line, one generating station could be shut down during the night, thereby resulting in a very large annual saving in coal and labor bills.

A station may have some generating units designed for 100 percent power-factor and other units designed for 80 percent power-factor; or again, where two generating stations feed into the same transmission system, one may have 100 percent power-factor generating units and the other 80 percent power-factor

*The present-day design of maximum rated generators with a short-circuit ratio of about unity will barely circulate full-load current with normal no-load excitation. Under such conditions the terminal voltage would be practically zero regardless of the power-factor.

generating units. In such cases, the field strength of the generators may be so adjusted as to cause the 80 percent power-factor units to take all the lagging current, thus permitting the 100 percent power-factor units to be loaded to their full kw rating.

BEHAVIOR OF A. C. GENERATORS WHEN CHARGING A TRANSMISSION LINE*

It has been shown above how leading armature current, by increasing the field strength, causes an increase in the voltage induced in the armature of an alternator and consequently an increase in its terminal voltage. It was also shown that the terminal voltage is further increased as result of the voltage due to self induction adding vectorially to the voltage induced in the armature.

If an alternator with its fields open is switched onto a dead transmission line having certain electrical characteristics, it will become self exciting, provided there is sufficient residual magnetism present to start the phenomenon. In such case, the residual magnetism in the fields of the generator will cause a low voltage to be generated which will cause a leading line charging current to flow through the armature. This leading current will increase the field flux which in turn will increase the voltage, causing still more charging current to flow, which in turn will still further increase the line voltage. This building up will continue until stopped by saturation of the generator fields. This is the point of stable operation. Whether or not a particular generator becomes self exciting when placed upon a dead transmission line depends upon the relative slope of the generator and line characteristics.

In Fig. 66 are shown two curves for a single 45 000 kv-a, 11 000 volt generator, the charging current of the transmission line being plotted against generator terminal voltage. One curve corresponds to zero excitation, the other curve to 26.6 percent of normal excitation. A similar pair of curves correspond to two duplicate generators in parallel**. The straight line representing the volt-ampere characteristics of the transmission line fed by these generators corresponds to a 220 kv, 60 cycle, three-phase transmission circuit, 225 miles long, requiring 69 000 kv-a to charge it with the line open at the receiving end.

The volt-ampere charging characteristic of a transmission line is a straight line, that is, the charging current is directly proportional to the line voltage. On the other hand the exciting volt-ampere characteristic for the armature has the general slope of an ordinary saturation curve.

*For a more detailed discussion of this subject see the following articles:—"Characteristics of Alternators when Excited by Armature Currents" by F. T. Hague, in the JOURNAL for Aug. 1915; "The Behavior of Alternators with Zero Power-Factor Leading Current" by F. D. Newbury, in the JOURNAL for Sept. 1918; "The Behavior of A. C. Generators when Charging a Transmission Line" by W. O. Morris, in the *General Electric Review* for Feb. 1920.

**It is assumed that with the assumed field current such generators can be synchronized and held together during the process of charging the line.

If the alternator characteristic lie above the line characteristic at a point corresponding to a certain charging current the leading charging current will cause a higher armature terminal voltage than is required to produce that current on the line. As a result the current and voltage will continue to rise until, on account of saturation, the alternator characteristic falls until it crosses the line characteristic. At this point the voltage of the generator and that of the line are the same for the corresponding current. If on the other hand the alternator characteristic falls below the line characteristic the alternator will not build up without permanent excitation.

As stated previously, whether or not a generator becomes self-exciting when connected to a dead transmission line depends upon the relative slopes of generator and transmission line characteristics. The relative slopes of these curves depend upon:—

a—The magnitude of the line charging current.
b—The rating of the generators compared to the full voltage charging kv-a of the line.
c—The armature reaction. High armature reaction, (that is low short-circuit ratio) favors self-excitation of the generators.
d—The armature reactance. High armature reactance also favors self-excitation of the generators.

Methods of Exciting Transmission Lines—If the relative characteristics of an alternator and line are such as to cause the alternator to be self-exciting, this condition may be overcome by employing two or more

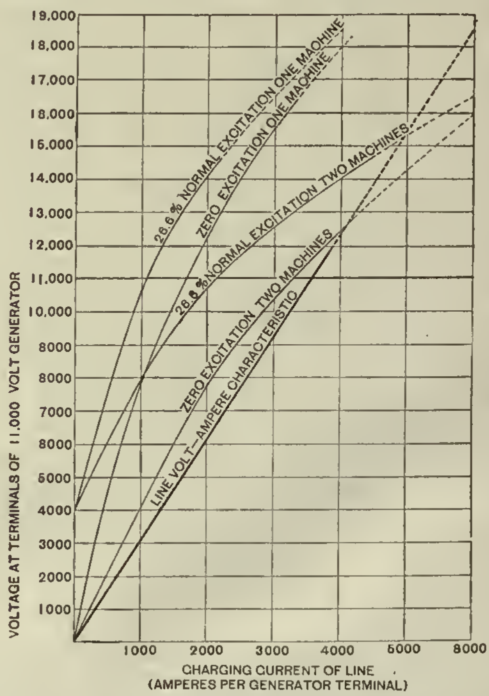


FIG. 66—VOLT AMPERE CHARACTERISTICS OF ONE 45 000 KV-A, 11 000 VOLT GENERATOR; TWO DUPLICATE 45 000 KV-A GENERATORS; AND A THREE-PHASE, SINGLE-CIRCUIT, 220 KV TRANSMISSION LINE

alternators (provided they are available for this purpose) to charge the transmission line. The combined characteristics of two or more alternators may be such as to fall under the line characteristic, in which case the alternator will not be self-exciting. In such case, the alternators could be brought up to normal speed, and given sufficient field charge to enable them to be synchronized and held in step, after which they could be connected to the dead transmission line and their voltage raised to normal.

Generators as normally designed will carry approximately 40 percent of their rated current at zero leading power-factor. If more than this current is demanded of them they are likely to become unstable in operation. By modifying the design of normal alternators so as to give low armature reaction, they may be made to carry a greater percentage of leading current. If the special design is such that with zero voltage field excitation when carrying half the line charging kv-a, the armature voltage will not exceed 70 percent of normal, this reduced voltage will result in a line charging kv-a of half of normal value. Specially designed alternators usually result in larger and more costly machines and the gain resulting in the special design is usually not sufficient to warrant the extra cost.

If a single generator with its field circuit open were connected to a dead transmission circuit such as the one whose volt-ampere characteristics are shown in Fig. 66, and there were sufficient residual magnetism to start the phenomenon, the generator voltage would rise to approximately double normal value before the point of staple operation is reached. If, however, two generators having 26.6 percent of normal excitation were paralleled and connected to this circuit, a point of staple operation would be reached at a terminal voltage of approximately 15 500 volts. Actually stable operation would be reached at a somewhat less terminal voltage for the reason that the line would probably not be open at the receiving end, but would probably have the lowering transformers connected to it. In such case the magnetizing current required for lowering transformers would lower the receiving end voltage, resulting in less line charging current.

In either case the curves of Fig. 66 show that either more than two generators will be required to charge the line when unloaded, or some other method of charging must be resorted to. Reactance coils could be used at the receiving end to furnish lagging current for neutralizing some of the line charging current, but there might be difficulty in removing these from the circuit when the line is fully charged At the present time it is expected that the problem of charging long transmission lines may usually be solved by starting one or more generators with sufficient field strength to permit them to be synchronized and held in step. One or more phase modifiers with under-excited fields may then be connected to the line at the receiving end and brought up to normal speed with the generators. Such a method of solving this problem has been employed by the Southern California Edison Company.

TABLE U—INSTALLATIONS OF LARGE PHASE MODIFIERS (1921)

By American Manufacturers

Kv-a	R.P.M.	Volts	Cycles	No. of Units	Date of Order	NAME AND LOCATION
30 000	600	6600	50	1	1919	So. Cal. Ed. Co., Los Angeles, Cal.
20 000	600	11 000	60	2	1921	Pacific Gas & Elec.
15 000	375	6600	50		1912	Southern Cal. Ed. Co., Los Ang., Cal.
15 000	375	6600	50	1	1912	Pacific Lt. & Pr. Co.
12 500	500	22 000	50	2	1918	Andhra Valley, India
7500	400	6600	60	2	1913	Utah Pr. & Lt. Co., Salt Lake, Utah
7500	400	6600	60	2	1916	Canton El. Co., Canton, Ohio
7500	600	13 800	60	1	1917	Blackstone Valley Gas & Elec. Co., Pawtucket, R. I.
7500	600	13 800	60	1	1917	New England Pr. Co., Worcester, Mass.
7500	720	13 800	60	1	1918	New England Pr. Co., Fitchburg, Mass.
7500	800	11 500	40	1	1918	Adirondack El. Pwr. Corp., Watervliet, New York
7500	750	11 000	50	1	1919	Energia Electrica de Cataluna, Barcelona, Spain
7500	600	11 000	60	1	1920	Duquesne Light Co.
7500	600	1200	60	2	1918	J. G. White, Engineers
7500	600	11 000	60	1	1918	Duquesne Light Co.
7500	600	11 000	60	1	1916	Duquesne Light Co.
7500	600	11 000	60	2	1917	Duquesne Light Co.
6500	750	2200	50	1	1917	Shanghai Municipal Council, Shanghai, China
6000	500	16 500	50	1	1914	So. Cal. Ed. Co., Los Angeles, Cal.
5000	600	7200	60	1	1916	Pac. Pwr. & Lt., Kennewick, Wash.
5000	500	6600	50	2	1915	Tata Hydro El. Pr. & S. Co., India
5000	750	6600	50	3	1917	Ebro Irrigation & Pr. Co., Barcelona, Spain
5000	750	11 500	50	1	1919	Societa Lombarda Distribuziona Energia Elettrica, Italy
5000	600	2300	60	1	1918	Turnbull Steel Co., Warren, Ohio
5000	720	2300/ 4000	60		1921	Public Service of N. Ill.
5000	720	11 000	60	1	1921	Takata & Co., Japan.
5000	600	13 200	60	1	1919	Conn. Lt. & Pr. Co.

CHAPTER XV
PHASE MODIFIERS FOR VOLTAGE CONTROL

WITH alternating-current transmission there is a voltage drop resulting from the resistance of the conductors, which is in phase with the current. In addition there is a reactance voltage drop; that is a voltage of self-induction generated within the conductors which varies with and is proportional to the current, and may add to or decrease the line voltage. If the line is long, the frequency high or the amount of power transmitted large, this induced voltage will be large, influencing greatly the line drop. By employment of phase modifiers the phase or direction of this induced voltage may be controlled so that it will be exerted in a direction that will result in the desired sending end voltage.

A certain amount of self-induction in a transmission circuit is an advantage, allowing the voltage at the receiving end to be held constant under changes in load by means of phase modifiers. It may even be made to reduce the line voltage drop to zero, so that the voltage at the two ends of the line is the same for all loads. Self-induction also reduces the amount of current which can flow in case of short-circuits, thus tending to reduce mechanical strains on the generator and transformer windings, and making it easier for circuit breaking devices to function successfully. On the other hand, high self-induction reduces the amount of power which may be transmitted over a line and may, in case of lines of extreme length, make it necessary to adopt a lower frequency. It also increases the capacity of phase modifiers necessary for voltage control. High reactance also increases the surge overvoltage that a given disturbance will set up in the system.

On the long lines, the effect of the distributed leading charging current flowing back through the line inductance is to cause, at light loads, a rise in voltage from generating to receiving end. At heavy loads, the lagging component in the load is usually sufficient to reverse the low-load condition; so that a drop in voltage occurs from generating to receiving end. The charging current of the line is, to a considerable extent, an advantage; for it partially neutralizes the lagging component in the load, thus raising the power-factor of the system and reducing the capacity of synchronous condensers necessary for voltage control.

The voltage at the receiving end of the line should be held constant under all loads. To partially meet this condition, the voltage of the generators could be varied to a small extent. On the longer lines, however, the voltage range required of the generators would be too great to permit regulation in this manner. In such cases, phase modifiers operating in parallel with the load are employed. The function of phase modifiers is to rotate the phase of the current at the receiving end of the line so that the self-induced voltage of the line (always displaced 90 degrees from the current) swings around in the direction which will result in the desired line drop. In some cases a phase modifier is employed which has sufficient capacity not only to neutralize the lagging component at full load, but, in addition, to draw sufficient leading current from the circuit to compensate entirely for the ohmic and reactance voltage drops of the circuit. In this case, the voltage at the two ends of the line may be held the same for all loads. This is usually accomplished by employing an automatic voltage regulator which operates on the exciter fields of the phase modifier. The voltage regulator may, if desired, be arranged to compound the substation bus voltage with increasing load.

CHECKING THE WORK

A most desirable method of determining line performance is by means of a drawing board and an engineer's scale. A vector diagram of the circuit under investigation, with all quantities drawn to scale, greatly simplifies the problem. Each quantity is thus represented in its true relative proportion, so that the result of a change in magnitude of any of the quantities may readily be visualized. Graphical solutions are more readily performed, and with less likelihood of serious error than are mathematical solutions. The accuracy attainable when vector diagrams are drawn 20 to 25 inches long and accurate triangles, T squares, straight edges and protractors are employed is well within practical requirements. Even the so-termed "complete solution" may be performed, graphically with ease and accuracy. A very desirable virtue of the graphical solution which follows is that it exactly parallels the fundamental, mathematical solution. For this reason this graphical solution is most helpful even when the fundamental mathematical solution is used, for it furnishes a simple check against serious errors. The result may be checked graphically after each individual mathematical operation by drawing a vector in the diagram paralleling the mathematical operation. Thus, any serious error in the mathematical solution may be detected as soon as made.*

*A method of checking arithmetical operations which requires little time and is an almost sure preventative of errors is that known as "casting out the nines." This method is given in most older arithmetics but has been dropped from many of the modern ones. A complete discussion is given in Robinson's "New Practical Arithmetic" published by The American Book Company.

When converting a complex quantity mathematically from polar to rectangular co-ordinates, or vice versa, the results may readily be checked by tracing the complex quantity on cross-section paper and measuring the ordinates and polar angle, or for approximate work the conversion may be made graphically to a large scale. For instance, in using hyperbolic functions, polar values will be required for obtaining powers and roots of the complex quantity. For approximate work much time will be saved by obtaining the polar values graphically.

In the graphical solution of line performance it will usually be desirable to check the line loss by a mathematical solution in cases which require exact loss values. Since the line loss may be five percent or less of the energy transmitted, a small error in the overall results might correspond to a large error in the value of the line loss.

EFFECT OF TRANSFORMERS IN THE CIRCUIT

Usually long transmission circuits have transformers installed at both ends of the circuit and one or more phase modifiers in parallel with the load. Such a transmission circuit must transmit the power loss of the phase modifiers and of the receiver transformers. In addition to this power loss, a lagging reactive current is required to magnetize the transformer iron. A complete solution of such a composite circuit (generator to load) requires that the losses of the phase modifiers and transformers be added vectorially to the load at the point where they occur so that their complete effect may be included in the calculation of the performance of the circuit. A complete solution also requires that three separate solutions be made for such a circuit.* First with the known or assumed conditions at the load side of the lowering transformers the corresponding electrical conditions at the high voltage side of the transformers is determined by the usual short line impedance methods. With the electrical conditions at the receiving end of the high-tension line thus determined, the electrical conditions at the sending end of the line are determined by one of the various methods which take into account the distributed quantities of the circuit. With the electrical condition at the sending end thus determined the electrical conditions at the generating side of the raising transformers are determined. The above complete method of procedure, is tedious if carried out mathematically, but if carried out graphically is comparatively simple.

It is the general practice to neglect the effect of condenser and lowering transformer loss in traveling over the line, but to add this loss to the loss in the high-tension line after the performance has been calculated. If the loss in condensers and lowering transformers is five percent of the power transmitted the error in the calculated results would probably be less than 0.5 percent, a rather small amount.

In order to simplify calculations, it is the general practice to consider the lumped transformer impedance as though it were distributed line impedance by adding it to the linear constants of the line and then proceeding with the calculations as though there were no transformers in the circuit. This simplifies the solution but at the expense of accuracy, particularly if the line is very long, the frequency high or the ratio transformer to line impedance high. This simplified solution introduces maximum errors of less than two percent in the results for a 225 mile, 60-cycle line.

It has been quite general practice to disregard the effect of the magnetizing current consumed by transformers. The magnetizing current required to excite transformers containing the older transformer iron was about two percent and therefore its effect could generally be ignored. Later designs of transformers employ silicon steel, and their exciting current varies from about 20 percent for the smaller of distribution type transformers, to about 12 percent on transformers of 100 kv-a capacity and about five percent for the very largest capacity transformers. The average magnetizing current for power transformers is between six and eight percent. This magnetizing current is important for the reason that it is practically in opposition to the current of over-excited phase modifiers used to vary the power-factor. If in a line having 100 000 kv-a transformer capacity at the receiving end, the magnetizing current is five percent, there will be a 5000 kv-a lagging component. If the capacity of phase modifiers required to maintain the proper voltage drop under this load is 50 000 kv-a the lagging magnetizing component of 5000 kv-a will subtract this amount from the effective rating of the phase modifiers, with a resulting error of ten percent in the capacity of the phase modifiers required.

In the diagrams and calculations which follow, the transformer leakage, consisting of an in-phase component of current (iron loss) and a reactive lagging component of current (magnetizing current), is considered as taking place at the low-tension side of the transformers. A more nearly correct location would be to consider the leak as at the middle of the transformer, that is, to place half the transformer impedance on each side of the leak. To solve such a solution it would be necessary to solve two complete impedance diagrams for the transformers at each end of the circuit. The gain in accuracy of results would not, for power transmission lines, warrant the increased arithmetical work and complication necessary.

In the case of lowering transformers, it would seem that the magnetizing current would be supplied principally from synchronous machines connected to the load. If phase modifiers are located near the lowering transformers, the transformers would probably draw most of their magnetizing current from

*A method for calculating a transmission line with transformers at each end in one solution is given in the articles by Messers. Evans and Sels in the JOURNAL for July, August, September, *et seq.* 1921.

them rather than from the generators at the distant end of the line. Partly for this reason, but more particularly for simplicity, the leak of the lowering transformers will be considered as taking place at the load side of the transformers. On this basis we first current also from the low side; that is from the generators. Both the complete and the approximate methods of solving long line problems which follow, include the effect of not only the magnetizing current consumed by the transformers, but also the losses in

TABLE V—COMPARISON OF RESULTS AS OBTAINED BY FIVE DIFFERENT METHODS OF CALCULATIONS

75,000 KW (88,235 KV-A AT 85% PF) 3 PHASE. 60 CYCLES RECEIVER VOLTAGE HELD CONSTANT AT 220 KV. 50,000 KV-A CONDENSER AT RECEIVING END

LENGTH OF TRANSMISSION 225 MILES ALL TABULATED VALUES REFERRED TO NEUTRAL

AREA IN CIRCULAR MILS	* METHOD	RECEIVING END TO NEUTRAL: LOW TENSION SIDE OF TRANSFORMERS. VOLTS E_{LN}	AMPS I_L+I_C	PF_L LEAD	HIGH TENSION SIDE OF TRANSFORMERS. VOLTS E_{RN}	AMPS I_R	PF_R LAG	SENDING END TO NEUTRAL: HIGH TENSION SIDE OF TRANSFORMERS. VOLTAGE E_{SN}	%	CURRENT I_S	%	PF_S LEAD	LOW TENSION SIDE OF TRANSFORMERS. VOLTAGE E_{GEN-N}	%	CURRENT I_{GEN}	%	PF_{GEN} LEAD	LOSSES IN KW TO NEUTRAL: LOWERING TRANSFORMERS IRON	COPPER	CONDENSER	HIGH TENSION LINE KW_N	LOSS IN % OF KW_L	RAISING TRANSFORMERS IRON	COPPER	TOTAL LOSS KW_N	LOSS IN % OF KW_L
605 000	A	127020	202.3	99.90	127 556	204.9	9963	129 090	100	227.8	100	9377	126 920	100	2261	100	97.49	235	130	666	1542	6 16	235	165	2973	11 89
"	B	"	"	"				124 247	963	2365	1039	9335						"	"	"	1634	6 53	"	178	3078	12 31
"	C	"	"	"									124 657	98.2	232 3	1028	95 14	"	"	"	1583	6 33	"	172	3021	12 08
"	D	"	"	"				126783	98.4	2287	1004	9432						"	"	"	1553	6 21	"	166	2985	1194
"	E	"	"	"									127537	100.5	224.6	993	95.87	"	"	"	1510	6.04	"	160	2936	1174
715 500	A	"	"	"	127 556	2049	9963	127911	100	2287	100	9350	125 668	100	226 6	100	9736	"	"	"	1320	528	"	166	2752	1101
"	B	"	"	"				123 041	962	237.5	103.9	9309						"	"	"	1408	563	"	180	2854	11.42
"	C	"	"	"									123 409	982	233 1	102.8	94.93	"	"	"	1338	535	"	173	2777	11.11
"	D	"	"	"				125 576	98 2	2297	100.4	94.09						"	"	"	1349	5 40	"	168	2783	11.13
"	E	"	"	"									126 292	100.5	225.4	99.5	95 67	"	"	"	1273	5.09	"	162	2701	10.80
795 000	A	"	"	"	127 556	204.9	9963	127 196	100	229.3	100	93.33	124 909	100	2274	100	97 24	"	"	"	1192	4 77	"	167	2625	10.50
"	B	"	"	"				122 313	962	2380	103 8	9294						"	"	"	1260	5 04	"	181	2707	10 83
"	C	"	"	"									122 648	98.2	233.5	102.7	94.80	"	"	"	1177	4.71	"	174	2617	10 47
"	D	"	"	"				124 846	981	2302	100.4	93.94						"	"	"	1198	4.79	"	169	2633	10.53
"	E	"	"	"									125 532	100.5	225.8	99.3	95.55	"	"	"	1126	4.50	"	162	2554	10 22
954 000	A	"	"	"	127 556	204.9	9963	126 132	100	2304	100	9293	123 740	100	228.4	100	9699	"	"	"	976	390	"	169	2411	964
"	B	"	"	"				121 212	961	239.4	1039	9255						"	"	"	1059	4 23	"	183	2508	10.03
"	C	"	"	"									121 488	98 2	2351	1029	94 51	"	"	"	1020	4 08	"	177	2463	9 85
"	D	"	"	"				123 737	98.1	231 5	100 5	93 58						"	"	"	1014	4 05	"	170	2450	9 80
"	E	"	"	"									124 368	100.5	227 3	99.5	95.31	"	"	"	984	3 93	"	165	2415	966

**A*—Transformer impedances treated as lumped at the ends of the line. This is the most nearly accurate of the five methods. It is referred to in the text as the complete solution.

B—This assumes the impedance of the lowering transformers as line impedance. It takes no account of the leakage of the lowering transformers.

C—This assumes the impedance of both lowering and raising transformers as line impedance—It takes no account of the leakage of the lowering and raising transformers.

D—This is the same as *B* except that the leakage of the lowering transformers has been added to the load—It is referred to in the text as the approximate solution.

E—This is the same as *C* except that the leakage of the lowering transformers has been added to the load.

have a load current expressed in rectangular coordinates with the load voltage as a temporary vector of reference. To this we add algebraically a phase modifier current (loss + j or leading) and to this we add the transformer leakage (loss — j or lagging). In other words, these three components of current at the receiving end of the line add up algebraically upon a transformers and phase modifiers flowing over the line.

For the purpose of determining the magnitude of errors in the calculated results corresponding to simplified methods of calculation where transformers are required at both ends of the line, the calculations shown in Table V were made. Five methods of calculations were made for each of four sizes of cable. A con-

TABLE W—PERCENTAGE ERRORS IN RESULTS, AS DETERMINED BY VARIOUS METHODS OF CALCULATION.

These methods do not take complete account of the effects of the transformers in the circuit

Method	At Generator Percent Error E_{gen}	I_{gen}	PF_{gen}	At Sending End Percent Error E_s	I_s	PF_s	Line Loss Percent Error	Transformer Account
A	0	0	0	0	0	0	0	Complete method—Assumed for comparison as resulting in 100 percent values.
B	...	...	...	−3.7	+3.9	−0.42	+0.37	Leak of lowering transformers ignored. Impedance of lowering transformers assumed as line impedance.
C	−1.8	+2.8	−2.35	...	...	...	+0.17	Leaks of raising and lowering transformers ignored. Impedance of raising and lowering transformer assumed as line impedance.
D	...	...	...	−1.6	+0.4	+0.55	+0.05	Same as *B* except that the transformer leak has been added to the load.
E	+0.5	−0.7	−1.62	...	...	...	−0.12	Same as *C* except that the transformer leak has been added to the load.

common vector of reference, thus making it very easy to obtain the resulting load at the receiving end of the line.

The transformers at the sending end of the line have been considered as receiving their magnetizing stant load, load voltage and condenser capacity were assumed for all calculations and the results of these calculations are tabulated in Table W. Thus method *B* which does not take any account of the lowering transformer magnetizing current and assumes the transformer impedance as line impedance, gives the sending end voltage too low by 3.7 percent and the current too high by 3.9 percent.

Table X contains approximate data upon transformers of various capacities 25 and 60 cycles. Since such data will vary greatly for different voltages it must be considered as very approximate but may be found useful in the absence of specific data for the problem at hand.

Fig. 67 shows complete current and voltage diagrams for both short and long lines. The diagram illustrating short lines is based upon the current having the same value and direction at all points of the circuit. On this basis the IR drops of the line and of the raising and lowering transformers will be in the same direction. Likewise their individual IX drops will also be in the same direction. It is evident, therefore, that, for short lines where the capacitance is neglible, the transformer impedance may be added directly to the line impedance, provided the electrical characteristics on the high-tension side of the transformers are not required.

As the line becomes longer, the current changes in both amount and direction from point to point, as a result of the superimposed distributed charging current of the line. The result of this is that the impedance triangles of the line and of lowering and raising transformers change in both size and relative position; so that their individual impedances can no longer be added together and considered as all line impedance, without accepting an error in the results thus obtained. The complete diagram for long lines shown by Fig. 67 will be considered later.

TRANSFORMER IMPEDANCE TO NEUTRAL*

Transformer constants are referred to the high voltage circuit in order to combine properly with the linear constants of the line. Although all calculations are made in terms of the high-voltage circuit the results may, if desired, be converted to terms of the low voltage circuit, by applying the ratio of transformation.

The transformer impedance to neutral is one-third the equivalent single-phase value. The reason for this is that the I^2R and I^2X for one phase is identical whether to neutral or between phases. Since the current between phases is equal to the current to neutral divided by $\sqrt{3}$, the square of the phase current would be one-third the square of the current to neutral; therefore, R and X to neutral will be one-third the phase values. Another way of looking at this is that the resistance and reactance ohms vary with the square of the voltage, and since the phase voltage is $\sqrt{3}$ times the voltage to neutral, the phase resistance and phase reactance would be three times that to neutral. In calculating the impedance to neutral, the results will be the same whether star or delta connection is used.

Even if the transformers at both ends of the transmission line are duplicates their impedance will not be the same if operated on different taps of the windings to accommodate different voltages. In such cases, their impedances will vary as the square of the voltages. For instance, if they are operated at 220 and 230 kv at the receiving and sending end respectively, then their impedances will have the relation of $\frac{220^2}{230^2} = 0.915$. In other words, if the resistance and reactance of the receiving end transformers is 3.185 and 39.82 ohms respectively, the sending end transformers will have resistances and reactances of 3.481 and 43.52 ohms respectively; provided transformer taps corresponding to this higher voltage are used.

The impedance in ohms of an 18 000 kv-a, three-phase, or of three 6000 kv-a single-phase transformers, connected in a bank, may be determined as fol-

TABLE X—APPROXIMATION OF RESISTANCE AND REACTANCE VOLTS, OF IRON AND COPPER LOSSES AND OF MAGNETIZING CURRENT FOR TRANSFORMERS OF VARIOUS CAPACITIES

Capacity of Transformer KV-A	60 Cycles per Second					25 Cycles per Second				
	Percent *Resistance	Percent *Reactance	Percent Loss		Percent Magnetizing Current	Percent Resistance	Percent Reactance	Percent Loss		Percent Magnetizing Current
			Iron	Copper				Iron	Copper	
200	1.5	5.5	1.4	1.5	10	2.6	4.0	1.1	2.6	10
300	1.3	5.6	1.3	1.3	9	2.15	4.0	1.0	2.15	10
500	1.2	6.0	1.2	1.2	8	1.85	4.1	1.0	1.85	9
750	1.1	6.3	1.0	1.1	8	1.65	4.2	0.9	1.65	9
1000	1.1	6.5	0.9	1.1	7	1.55	6.0	0.8	1.55	8
1500	0.9	7.0	0.8	0.9	6	1.4	6.2	0.8	1.4	8
2000	0.8	7.0	0.7	0.8	6	1.3	6.4	0.7	1.3	8
3000	0.75	7.0	0.7	0.75	6	1.2	6.8	0.6	1.2	7
5000	0.65	7.0	0.6	0.65	6	1.1	7.2	0.5	1.1	7
7500	0.6	3.0	0.6	0.6	5	1.0	7.8	0.5	1.0	7
10000	0.6	8.0	0.5	0.6	5	1.0	8.0	0.5	1.0	6
15000	0.55	8.5	0.5	0.55	5	0.95	8.0	0.6	0.95	6
25000	0.5	9.0	0.6	0.5	5	0.9	8.0	0.6	0.9	6
35000	0.5	9.5	0.6	0.5	5	0.9	9.0	0.6	0.9	6
50000	0.5	10.0	0.6	0.5	5	0.9	9.0	0.6	0.9	6

*The actual ohms resistance and ohms reactance will vary as the square of the voltage. The values in above table must be considered as only roughly approximate. They will vary materially with transformers wound for different voltages

*The writer desires to express his appreciation of helpful assistance and useful data on transformer characteristics received from Mr. J. F. Peters.

lows. Assume that they are operated at 104 000 volts between conductors (60 046 to neutral) and that the resistance voltage is 1.04 percent and reactance voltage is 4.80 percent.

The single-phase values are:—

$$\frac{6\,000\,000}{104\,000} = 57.7 \text{ amperes}$$

$$R_t = \frac{104\,000 \times 0.0104}{57.7} = 18.75 \text{ ohms resistance}$$

$$X_t = \frac{104\,000 \times 0.048}{57.7} = 86.52 \text{ ohms reactanee}$$

The values to neutral are, as stated above, one-third of the above; but, for the sake of uniformity in determining values to neutral, should preferably be determined as follows:—

$$\frac{6\,000\,000}{60\,046} = 99.92 \text{ amperes to neutral}$$

$$R_{tn} = \frac{60\,046 \times 0.0104}{99.92} = 6.25 \text{ ohms resistance to neutral}$$

$$X_{tn} = \frac{60\,046 \times 0.0480}{99.92} = 28.84 \text{ ohms reactance to neutral}$$

If two or more banks operate in parallel, the resulting impedance Z_r can be obtained by taking the reciprocal of the sum of the reciprocals of the individual impedance. Thus:—

$$Z_r = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

In the above example $Z_t = \sqrt{1.04^2 + 4.8^2} = 4.91$ percent.

To parallel two banks containing transformers duplicates of the above, we get, by the above rule, the following resultant impedance:—

$$Z_r = \frac{4.91 \times 4.91}{4.91 + 4.91} = 2.45 \text{ percent}$$

Which is just half the impedance of a single bank, as is evident without applying the rule.

Where two or more banks are to be operated in parallel consisting of transformers not duplicates, then the above rule must be applied to determine the resultant impedance. If the impedances are expressed in percent, as is usual, then they must be both referred to the same kv-a base. For instance, if a 6000 kv-a and a 3000 kv-a transformer each have a resistance of 1.04 percent and a reactance of 4.8 percent, their impedance is 4.91 percent. Before combining the impedances, that of the 3000 kv-a unit should be put in terms of the 6000 kv-a, and the resultant would be:—

$$Z_r = \frac{4.91 \times 9.82}{4.91 + 9.82} = 3.27 \text{ percent at 6000 kva.}$$
$$= 0.69 \text{ percent resistance volts at 6000 kva.}$$
$$= 3.19 \text{ percent reactance volts at 6000 kva.}$$

If the impedance triangles of the two banks to be paralleled are considerably different (that is their ratio of resistance to reactance) it will be necessary to express the impedances in complex form. We have assumed above that the triangles are proportional, otherwise they would not divide the load evenly at all power-factors. Solving the preceding problem for the resultant impedance by complex notation, we get:

$$Z_r = \frac{(1.04 + j4.8) \times (2.08 + j9.6)}{(1.04 + j4.8) + (2.08 + j9.6)}$$
$$= \frac{-43.917 + j19.968}{3.12 + j14.4}$$
$$= \frac{48.25 \diagdown 155^\circ 32' 58''}{14.734 \angle 77^\circ 46' 29''}$$
$$= 3.27 \angle 77^\circ 46' 29'' \text{ ohms}$$
$$= 0.69 + j\,3.19 \text{ ohms}$$

Which checks with the results determined above on the percentage basis.

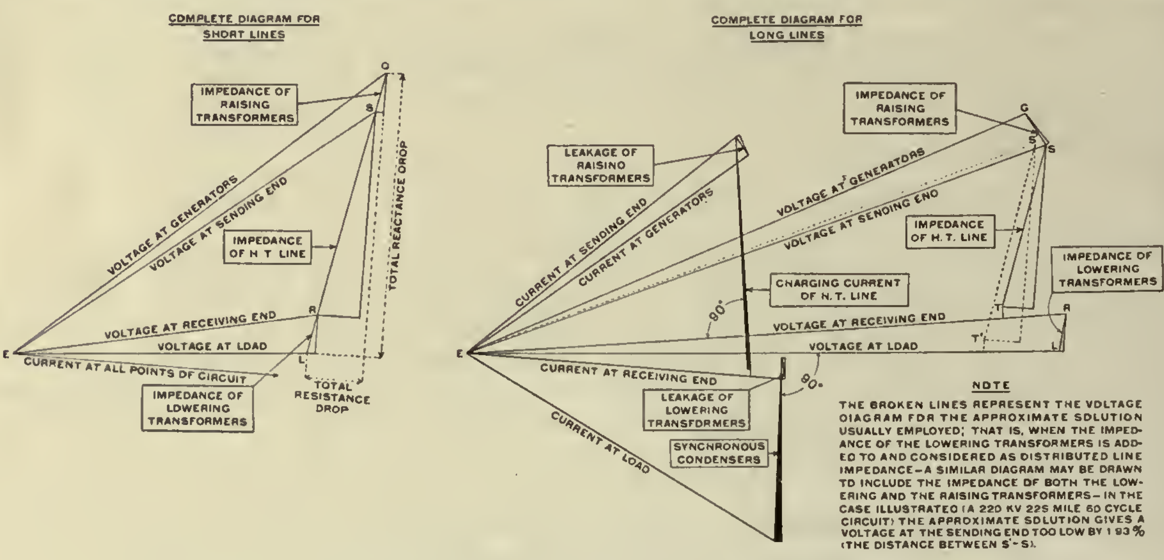


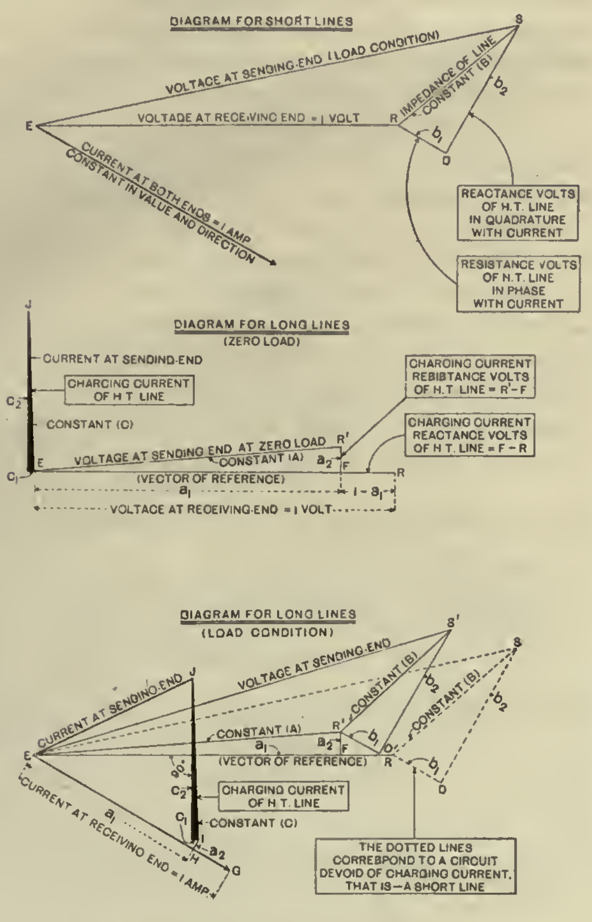
FIG. 67—VECTOR DIAGRAMS FOR SHORT AND LONG LINES

THE AUXILIARY CONSTANTS

The graphical construction for short lines represented typically by the Mershon Chart is so generally known and understood that a similar construction modified to take into accurate account the distribution effect of long lines will readily be followed. Both the short and the long line diagrams are reproduced in Fig. 68. From these diagrams it will be seen how the three auxiliary constants correct or modify the short line diagram adapting it to long line problems. The two mathematical and three graphical methods of obtaining the auxiliary constants are indicated at the bottom of this figure. Since the auxiliary constants are functions of the physical properties of the circuit and of the frequency only, they are entirely independent of the voltage or the current. Having determined

VECTORS BASED UPON ONE VOLT AND ONE AMPERE AT 85% POWER FACTOR BEING DELIVERED AT THE RECEIVING END—THE DIAGRAMS CORRESPOND TO A LONG LINE

$$E_S = E_R(a_1 + j a_2) + I_R(b_1 + j b_2)$$
$$I_S = I_R(a_1 + j a_2) + E_R(c_1 + j c_2)$$



HOW THE AUXILIARY CONSTANTS MAY BE OBTAINED

$(A) = (a_1 + j a_2) = \left[1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \frac{Y^3Z^3}{720} + \frac{Y^4Z^4}{40,320} + \text{ETC.}\right]$ (BY CONVERGENT SERIES—SEE CHART XI)

$= \cosh\theta$ (BY REAL HYPERBOLIC FUNCTIONS—SEE CHART XVI)

$= \frac{\sinh\theta/\theta}{\tanh\theta/\theta}$ (GRAPHICAL—SEE KENNELLY CORRECTING FACTOR CHARTS XVIII-XIX-XX-XXI)

$= \cosh\theta$ (GRAPHICAL—SEE KENNELLY'S CHART ATLAS, HARVARD PRESS)

$= \cosh\theta$ (ALL GRAPHICAL FROM WILKINSON'S CHART "A"—SEE CHART V)

$(B) = (b_1 + j b_2) = Z\left[1 + \frac{YZ}{8} + \frac{Y^2Z^2}{120} + \frac{Y^3Z^3}{5,040} + \frac{Y^4Z^4}{362,660} + \text{ETC.}\right]$ (BY CONVERGENT SERIES—SEE CHART XI)

$= \sqrt{\frac{Z}{Y}}\sinh\theta$ (BY REAL HYPERBOLIC FUNCTIONS—SEE CHART XVI)

$= Z\frac{\sinh\theta}{\theta}$ (GRAPHICAL—SEE KENNELLY'S CORRECTING FACTOR CHARTS XVIII - XIX)

$= \sqrt{\frac{Z}{Y}}\sinh\theta$ (GRAPHICAL—SEE KENNELLY'S CHART ATLAS, HARVARD PRESS)

$= \sqrt{\frac{Z}{Y}}\sinh\theta$ (ALL GRAPHICAL FROM WILKINSON'S CHART "B"—SEE CHART VI)

$(C) = (c_1 + j c_2) = Y\left[1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \frac{Y^3Z^3}{5,040} + \frac{Y^4Z^4}{362,880} + \text{ETC.}\right]$ (BY CONVERGENT SERIES—SEE CHART XI)

$= \frac{1}{\sqrt{\frac{Z}{Y}}}\sinh\theta$ (BY REAL HYPERBOLIC FUNCTIONS—SEE CHART XVI)

$= Y\frac{\sinh\theta}{\theta}$ (GRAPHICAL—SEE KENNELLY'S CORRECTING FACTOR CHARTS XVIII - XIX)

$= \frac{1}{\sqrt{\frac{Z}{Y}}}\sinh\theta$ (GRAPHICAL—SEE KENNELLY'S CHART ATLAS, HARVARD PRESS)

$= \frac{1}{\sqrt{\frac{Z}{Y}}}\sinh\theta$ (ALL GRAPHICAL FROM WILKINSON'S CHART "C"—SEE CHART VII)

WHERE $\theta = \sqrt{ZY}$

FIG. 68—HOW THE AUXILIARY CONSTANTS MODIFY SHORT LINE DIAGRAMS ADAPTING THEM TO LONG LINE PROBLEMS

by any of the five methods referred to, the value for the auxiliary constants corresponding to a given circuit, the remainder of the solution for any receiving end current or voltage is readily performed graphically.

Constants a_1 and a_2—If the line is short electrically the charging current, and consequently its effect upon the voltage regulation is small. In such a case constant a_1 would be unity and constant a_2 would be zero, and the line impedance triangle would be attached to the end of the vector *ER* representing the receiving end voltage, since this vector also represents the sending end voltage at zero load.

If, however, the circuit contains appreciable capacitance, the e.m.f. of self-induction resulting from the charging current will result in a lower voltage at zero load at the sending end than at the receiving end of the line. Obviously, the load impedance triangle must be attached to the end of the vector representing the voltage at the sending end of the circuit at zero load. This is the vector *ER′* of the long line diagrams of Fig. 68. In such a circuit the effect of the charging current is sufficiently great to cause the shifting of the point *R* for a short line to the position *R′* for the long line. The constants a_1 and a_2 therefore, determine the length and position of the vector representing the sending end voltage at zero load. Actually the constant a_2 represents the volts resistance drop due to the charging current for each volt at the receiving end of the circuit. That is, the line *FR′* equals approximately one-half the charging current times the resistance *R*, taking into account, of course, the distributed nature of the circuit. For a short line, it would be sufficiently accurate to assume that the total charging current flows through one-half the resistance of the circuit. To make this clear, it will be shown later that, for a 220 kv problem, the resistance per conductor is $R = 34.65$ ohms and the auxiliary constant $C_2 =$ 0.001211 mho. Thus, this line will take 0.001211 ampere charging current, at zero load, for each volt maintained at the receiving end, and since *FR′* = approximately $I_{cc} \times \frac{R}{2}$ we have *FR′* or $a_2 = 0.001211 \times \frac{34.65}{2} = 0.020980$. The exact value of a_2 as calculated by hyperbolic functions, taking into account the distributed nature of the circuit is 0.020234. Since the charging current is in leading quadrature with the voltage *ER*, the resistance drop *FR′* due to the charging current is also at right angles to *ER*.

The length of the line *FR* or (one-a_1), represents the voltage consmed by the charging current flowing through the inductance of the circuit. This may also be expressed with small error if the circuit is not of great electrical length as $I_{cc} \times \frac{X}{2}$. The reactance per conductor for the 220 kv problem is 178.2 ohms. Therefore, $FR = 0.001211 \times \frac{178.2}{2} = 0.107900$ and $a_1 = 1 - 0.107900 = 0.892100$. The exact value of a_1 as calculated rigorously, is 0.893955.

Constants b_1 and b_2—These constants represent respectively the resistance and the reactance in ohms, as modified by the distributed nature of the circuit. The values for these constants, multiplied by the current in amperes at the receiving end of the circuit, give the IR and IX volts drop consumed respectively by the resistance and the reactance of the circuit. To illustrate this, the values of R and X for the 220 kv problem are 34.65 ohms and 178.2 ohms per conductor. The distributed effect of the circuit modifies these linear values of R and X so that their effective values are $b_1 = 32.198$ and $b_2 = 172.094$ ohms. The line impedance triangle, as modified to take into exact account the distributed nature of the circuit, is therefore smaller than it would be if the circuit were without capacitance.

Constants c_1 and c_2—These constants represent respectively the conductance and susceptance in mhos as modified by the distributed nature of the circuit. The values for these constants, multiplied by the volts at the receiving end of the circuit, give the current consumed respectively by the conductance and the susceptance of the circuit. To illustrate, the linear value of c_2 for the 220 kv problem is 0.001211 mho. The distribution effect of the circuit modifies this linear value so that its effective value $c_2 = 0.001168$. The value of c_1 is so small that its effect is negligible for all except for long circuits. An exception to this statement would be that if the line loss is very small compared to the amount of power transmitted the percent error in the value of line loss may be considerably increased if the effect of c_1 is not included in the solution. If c_1 is ignored, c_2 will represent the charging current at zero load per volt at the receiving end. Thus c_2 multiplied by the receiving end voltage, gives the charging current at zero load for the circuit. For the 220 kv problem $c_2 \doteq 0.001168$ and this multiplied by 127 020, the receiving end voltage to neutral, gives 148.36 amperes charging current per conductor.

Referring to the formulas at the top of Fig. 68, $E_r\ (a_1 + j\ a_2)$ is that part of E_s which would have to be impressed at the sending end if $I_r = 0$, or the line was freed at the receiving end with E_r steadily maintained there. It may be called "free" component of E_s*. Again $I_r\ (b_1 + j\ b_2)$ is that other part of E_s which would have to be impressed at the sending end, if $E_r = 0$, or the line was short-circuited at the receiving end, with I_r steadily maintained there. It may be called the "short" component of E_s.

Similarly, the term $I_r\ (a_1 + j\ a_2)$ is the component of I_s necessary to maintain I_r at the receiving end without any voltage there ($E_r = 0$); while $E_r\ (c_1 + j\ c_2)$ is the component of I_s necessary to maintain E_r at the receiving end without any current there ($I_r = 0$). The reason that c_1 is likely to be negative in ordinary power lines is because the complex hyperbolic angle of any good power transmission line has a large slope, being usually near 88 degrees. The sinh of such an angle, within the range of line lengths and sizes of 6 ordinarily present, is also near 90 degrees in slope. The surge impedance $Z_0 = \sqrt{\frac{Z}{Y}}$ of such a line is not far from being reactanceless; but it usually develops a small negative or condensive slope. This means that the surge admittance $Y_0 = \frac{1}{Z_0}$ usually develops a small positive slope. Consequently, C or the product $E_r\ (c_1 + j\ c_2)$ usually slightly exceeds 90 degrees in slope; or c_1 becomes a small negative rectilinear component.

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*See paper by Houston and Kennelly on "Resonance in A. C. Lines" in Trans. A. I. E. E. April, 1895

CHAPTER XVI
A TYPICAL 220 KV PROBLEM

TO illustrate the method of determining the performance of long lines requiring phase modifiers for voltage control, the following 220 kv problem will be considered, which is typical of many likely to be considered in the near future. A line necessitating such large expediture would warrant a thorough investigation before determining the final design. The conclusions are given only for the purpose of illustrating the procedure.

The Problem—It is assumed that 300 000 kw at 85 percent lagging power-factor is to be delivered a distance of 225 miles, at 220 kv, three-phase, 60 cycles. Two lines will be required, so that in case one is under repair, the other will transmit the entire 300 000 kw load. Since the self-induced voltage would be excessive if the 300 000 kw were transmitted in emergency over a single-circuit tower line, we will assume that each tower line will support two three-phase circuits. The cost of two three-phase circuits per tower line will not be greatly in excess of a single circuit tower line employing conductors of double the cross-section. On this basis each of the four three-phase circuits will normally transmit 75 000 kw and, under emergency condition, each of the two circuits on one tower line will transmit 150 000 kw. Such a transmission is illustrated by Fig. 69*

Economic Size of Conductors—For a fixed transmission voltage and material of conductors, the most economic size of conductor will be found by applying Kelvin's law extended to include, in addition to the cost of conductors, that part of the cost of towers, insulators, line construction, phase modifiers, etc. which increases directly with the cost of conductors. Kelvin's law is as follows:—

"The most economical section of a conductor is that which makes the annual cost of the I^2R losses equal to the annual interest on the capital cost of the conducting material plus the necessary annual allowance for depreciation". Stated another way, "The annual cost of the energy wasted, added to the annual allowance for depreciation and interest on first cost shall be a minimum".

In Table Y is shown a comparison of values of capitalized losses vs. first costs of conductors for four sizes of aluminum-steel cables considered in connection with this 220 kw problem**. The cost of power losses is based upon rates of 0.3, 0.4 and 0.5 cents per kw hour, an average load corresponding to 80 percent of the full load loss and a capitalization of these losses at 15 percent. The cost of the cables is based upon 29 cents per pound for the complete cable (aluminum plus the steel). All tabulated data is based upon fotir three-phase circuits. The losses include those in the high voltage line only. If the capacity of transformers or phase modifiers varies materially for different conductors, the difference in their losses should be included.

If the base load power generated in such a large amount by water power costs 0.3 of a cent per kw-hr.,

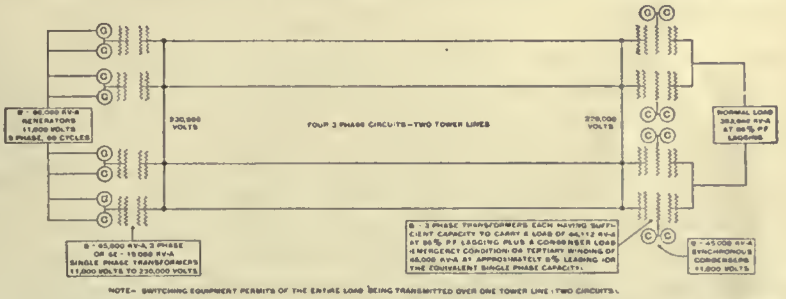

FIG. 69—THE TRANSFORMER AND CONDENSER ARRANGEMENT UPON WHICH THE CALCULATIONS FOR THE 220 KV PROBLEM HAVE BEEN BASED.

It is not intended that this arrangement would, upon a complete study of the problem, be found to be the most desirable. If single-phase transformers were selected, possibly three banks for each double circuit would be found more desirable than four banks, as indicated above.

the values in Table Y show that the smallest size cable, 605 000 circ. mil. will be the cheapest to install. At 0.3 cents per kw-hr. the power loss for this cable, capitalized at 15 percent, represents the equivalent of an investment of $2 593 000 for the four three-phase circuits, whereas the cost of the conductors is $3 224 000. If the cost of power loss is taken as 0.4 cents per kw-hr., the next larger cable will be the most economical size to use, provided that there is no increased cost of towers, insulators, etc. If the losses in transformers or condensers vary for the different sizes of cables compared such losses should be included with the conductor losses.

There is always a question as to what price should be charged in Kelvin's equation in estimating the cost of power loss. If all power saved could be promptly sold, the cost to allow might be considered the cost at the consumers meter. If, on the contrary, none of the power saved can be sold under any circumstances,

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*The calculations and the illustrations in this article were made in such a way as to be equally suited for the series of articles on "Electrical Characteristics of Transmission Circuits" and the Superpower Survey. Figs. 69, 70, 72 and 75 and Charts XXIII, XXV and XXVII appear also in the report of the latter, which is printed as *Professional Paper* 123 by the United States Geological Survey. Similarly, Charts XXIV, XXVI and XXVIII appear in the Paper by L. E. Imlay in the *Journal of the A. I. E. E.* for June, 1921. (Ed.)

**An interesting graphic presentation of Kelvin's Law is given in the article by Mr. L. J. Moore in the *Electrical World* for Sept. 24, 1921, p. 612.

then the cost to allow is the cost at the generating switchboard. Intermediate cases may occur.

The conductor losses of Table Y were taken from the calculated values by the complete method *A* listed in Table V*. It is usually sufficient to calculate the

TABLE Y—APPLICATION OF KELVIN'S LAW

Conductors Circ. Mill.	Total Loss in 12 Conductors Kw	Cost of Power lost In 12 Conductors, Capitalized at 15%			Cost of 12 Conductors at 29c per Lb.
		At 0.3c per Kw-hr.	At 0.4c per Kw-hr.	At 0.5c per Kw-hr.	
*605 000	18 504	$2 593 000	$3 458 000	$4 322 000	$3 224 000
715 500	15 840	$2 220 000	$2 960 000	$3 700 000	$3 837 000
795 000	14 304	$2 040 000	$2 673 000	$3 341 000	$4 244 000
954 000	11 712	$1 641 000	$2 188 000	$2 736 000	$5 011 000

*This is the smallest conductor which is, in this case, permissible on account of corona limitations. These tabulations are total for four three-phase circuits. It will usually be sufficiently accurate to calculate the conductor I^2R loss for one size of conductor and assume that the loss for other sizes will be proportional to their resistances. This assumes that the distribution of current throughout the length of circuit will be approximately the same for the different sizes of conductors compared. The above data is based upon 75 000 kw at 85 percent power-factor, three-phase, 60 cycles, delivered over each of the four circuits a distance of 225 miles at 220 kv with a 50 000 kv-a condenser in parallel with the load on each of the four circuits and an average load equivalent to 80 per cent of full load. It should be noted that the third, fourth and fifth columns do not give the actual cost of the power lost, but give instead the values at which these losses are capitalized.

loss in the conductors for one size of cable and to estimate it for other sizes of cable, assuming that this loss varies as the resistance of the conductors, that is, for a given line, frequency, load, delivery voltage and condenser capacity the current distribution in the line is approximately the same for various sizes of conductors likely to be considered. Since the conductor loss varies as the square of the current and directly as the resistance, it will be sufficient to estimate the loss for other conductors as being inversely proportional to their resistance.

The various constants corresponding to the four sizes of conductors considered are listed in Table Z. It may be interesting to note the variation in these constants corresponding to the different sizes of cable for the high-tension line alone, and also when the transformer impedances are included with the line impedance.

SOLUTION OF THE 220 KV PROBLEM

Assuming that 605 000 circ. mil. aluminum-steel cables work out as the most economical size, the next step is the determination of the auxiliary constants A, B, and C for this size of conductor, spacing and 60 cycles. (These constants would have previously been determined when determining the most economical size). Mathematically these constants may be calculated by real hyperbolic functions (Chart XVI) or by convergent series (Chart XI). Graphically, they may be obtained from Wilkinson's charts (Charts V, VI and VII) or through the medium of Dr. Kennelly's charts (Charts XVIII, XIX, XX and XXI). When using charts it is desirable to read the results from them at two different times as a check against errors in reading, or the constants may be read from both the Wilkinson and Kennelly charts and the results compared. From Table V we find $r = 0.154$ ohms, so that $R = 0.154 \times 225 = 34.65$ ohms and $x = 0.792$ so that $X = 0.792 \times 225 = 178.2$ ohms. From Table X we obtain $b = 5.38 \times 10^{-6}$ so that $B = 5.38 \times 225 \times 10^{-6} = 0.001211$ mho. G is assumed here as zero.

From Wilkinson Charts—

$$
\begin{aligned}
a_1 &= 0.892\\
\text{and since } rb &= 0.828\\
a_2 &= 0.020\\
b_1 &= 32.2 \text{ ohms}\\
b_2 &= 173.5 \text{ ohms}\\
\text{and since } rb^2 &= 4.457\\
c_1 &= \text{(too small to read)}\\
c_2 &= 0.001175
\end{aligned}
$$

From Dr. Kennelly's Charts—We must first obtain the hyperbolic complex angle of the circuit as follows:—

$$
\begin{aligned}
Z &= 34.65 + j\,178.2\\
&= 181.54 \;\underline{/78^\circ 59' 46''}\\
Y &= 0 + j\,0.001211\\
&= 0.001211 \;\underline{/90^\circ}\\
ZY &= 0.21984 \;\underline{/168^\circ 59' 46''}\\
\theta = \sqrt{ZY} &= 0.4689 \;\underline{/84^\circ 29' 53''}
\end{aligned}
$$

$$\text{From Chart XIX, } \frac{Sinh\,\theta}{\theta} = 0.964 \;\underline{/0.4^\circ} = 0.964 \;\underline{/0^\circ 24' 00''}$$

$$\text{From Chart XXI, } \frac{Tanh\,\theta}{\theta} = 1.0785 \;\overline{\backslash 0.88^\circ} = 1.0785 \;\overline{\backslash 0^\circ 52' 48''}$$

TABLE Z—CABLE AND CIRCUIT CONSTANTS CORRESPONDING TO A THREE-PHASE, 60 CYCLE CIRCUIT, 225 MILES LONG CONSISTING OF FOUR SIZES OF ALUMINUM CABLES OF AN ARRANGEMENT EQUIVALENT TO 21 FEET DELTA

AREA OF CONDUCTORS (C M) ALUM	STEEL	TOTAL	COPPER EQUIV	DIA. OF ALUM COND.	STRANDS AL.	ST	LINEAR CONSTANTS OF LINE TO NEUTRAL r	x	g	$b \times 10^{-6}$	R	X	G	$B \times 10^{-6}$	IMPEDANCE TO NEUTRAL OF 50 000 KV A BANK OF TRANSFORMERS ★ R_{TN}	X_{TN}	G_{TN}	$B_{TN} \times 10^{-6}$
605,000	78,000	683,500	380,400 EQUIV	0.952	54	7	0.154	0.792	0	5.38	34.65	178.2	0	1211	6.37	79.64	0	0
715,500	92,900	808,900	450,000 EQUIV	1.036	54	7	0.131	0.782	0	5.45	29.48	175.9	0	1226	6.37	79.64	0	0
795,000	103,100	898,100	500,000 EQUIV	1.092	54	7	0.117	0.775	0	5.49	26.33	174.4	0	1235	6.37	79.64	0	0
954,000	123,700	1,077,700	600,000 EQUIV	1.196	54	7	0.0978	0.764	0	5.58	22.00	171.9	0	1256	6.37	79.64	0	0

	LINEAR CONSTANTS R	X	G	B ×10⁻⁶	HYPERBOLIC QUANTITIES $\theta = \sqrt{ZY}$	$Z_0 = \sqrt{\frac{Z}{Y}}$	AUXILIARY CONSTANTS OF CIRCUIT (A)	(B)	(C)
HIGH TENSION LINE (TO NEUTRAL)									
605,000	3465	1782	0	1211	.04496 + j.466738 = .4689 /84°29′53″	387.2 \5°30′07″	.893955 + j.020234 = .89418 /1°17′47″	32.198 + j 172.094 = 175.1 /79°24′10″	−.000 008 + j.001168 = .001168 \90°24′24″
715,500	2948	1759	0	1226	.03879 + j.46599 = .4676 /85°14′29″	381.4 \4°45′25″	.894044 + j.017429 = .8942 /1°07′01″	27.394 + j 169.818 = 172.01 /80°50′11″	−.000 007 + j.001182 = .001182 \90°21′01″
795,000	2633	1744	0	1235	.034933 + j.46538 = .4667 /85°42′26″	377.9 \4°17′33″	.894183 + j.015681 = .8944 /1°0′16″	24.47 + j 168.344 = 170.11 /81°43′47″	−.000 006 + j.001191 = .001191 \90°18′54″
954,000	2200	1719	0	1256	.029684 + j.46576 = .4667 /86°21′12″	371.6 \3°38′47″	.893944 + j.013326 = .89404 /0°51′14″	20.44 + j 165.88 = 167.1 /82°58′29″	−.000 006 + j.001211 = .001211 \90°16′04″
HIGH TENSION LINE PLUS IMPEDANCE OF LOWERING TRANSFORMERS (TO NEUTRAL)									
605,000	3783	212.02	0	1211	.044417 + j.515789 = .5177 /85°04′41″	427.47 \4°55′18″	.870783 + j.021911 = .87106 /1°26′29″	34.5653 + j 208.83 = 211.67 /80°36′06″	−.000 009 + j.001158 = .001158 \90°26′43″
715,500	3266	215.72	0	1226	.03882 + j.515741 = .5172 /85°41′43″	421.85 \4°18′16″	.870587 + j.019150 = .87079 /1°15′36″	29.837 + j 206.545 = 208.69 /81°46′49″	−.000 007 + j.001173 = .001173 \90°23′22″
795,000	2951	214.29	0	1235	.035347 + j.51559 = .5168 /86°04′41″	418.44 \3°55′18″	.870556 + j.017429 = .87073 /1°08′49″	26.9585 + j 205.0706 = 206.83 /82°30′39″	−.000 007 + j.001181 = .001181 \90°21′15″
954,000	2518	211.72	0	1256	.030613 + j.51658 = .5175 /86°36′31″	412.01 \3°23′28″	.869919 + j.015121 = .87005 /0°59′45″	22.992 + j 202.588 = 203.89 /83°31′30″	−.000 006 + j.001201 = .001201 \90°18′23″
HIGH TENSION LINE PLUS IMPEDANCE OF BOTH LOWERING AND RAISING TRANSFORMERS (TO NEUTRAL)									
605,000	41.02	257.8	0	1211	.04431 + j.56045 = .5622 /85°28′47″	469.3 \4°31′13″	.84782 + j.023566 = .8481 /1°35′31″	36.86 + j 244.92 = 247.67 /81°26′37″	−.000 01 + j.001149 = .001149 \90°29′03″
715,500	35.85	258.5	0	1226	.03917 + j.56103 = .5624 /86°00′24″	458.7 \3°59′37″	.84735 + j.020846 = .8476 /1°24′33″	32.196 + j 242.62 = 244.8 /82°26′28″	−.000 01 + j.001163 = .001163 \90°25′43″
795,000	32.70	254	0	1235	.03598 + j.561247 = .5624 /86°19′55″	455.4 \3°40′04″	.84715 + j.019155 = .8474 /1°17′43″	29.36 + j 244.1 = 242.9 /83°03′25″	−.000 01 + j.001171 = .001171 \90°23′39″
954,000	28.37	251.5	0	1256	.03165 + j.562922 = .5638 /86°46′55″	448.98 \3°13′02″	.84612 + j.016893 = .8463 /1°08′38″	25.452 + j 238.628 = 240 /83°54′43″	−.000 007 + j.001191 = .001191 \90°20′52″

*Since two 50 000 kv-a banks of transformers will be required at each end the corresponding values for impedance will be half these amounts.

$$A = \frac{Sinh\,\theta/\theta}{Tanh\,\theta/\theta} = \frac{0.964 \;\underline{/0^\circ 24' 00''}}{1.0785 \;\overline{\backslash 0^\circ 52' 48''}} = 0.8939 \;\underline{/1^\circ 16' 48''}$$

$$a_1 = 0.8937$$
$$a_2 = 0.01996$$

*In the JOURNAL for Dec. 1921, p. 544.

$$B = Z\frac{\text{Sinh}\,\theta}{\theta} = 181.54\ \underline{/78^\circ 59' 46''} \times 0.964\ \underline{/0^\circ 24' 00''}$$
$$= 175.0\ \underline{/79^\circ 23' 46''} \text{ ohms}$$
$$b_1 = 32.2 \text{ ohms}$$
$$b_2 = 172 \text{ ohms}$$
$$C = Y\frac{\text{Sinh}\,\theta}{\theta} = 0.001211\ \underline{|90^\circ} \times 0.964\ \underline{/0.24' 00''}$$
$$= 0.001167\ \underline{\backslash 90^\circ 24' 00''} \text{ mho}$$
$$c_1 = -0.000\,008 \text{ mho}$$
$$c_2 = 0.001167 \text{ mho}$$

The auxiliary constants as obtained graphically and by exact mathematical solution, are given in Table ZZ. It is thus seen that the Kennelly charts, although primarily intended for correcting the linear impedance and the linear admittance of circuits for the equivalent π solution, are highly adaptable to determining the values of the auxiliary constants to a very close degree of accuracy. The use of these charts for obtaining auxiliary constants requires more arithmetical work than the use of the Wilkinson charts. For instance the hybolic angle, $\theta = \sqrt{ZY}$ of the circuit must first be calculated before the charts can be employed. The results, read from charts, must then be multiplied by the impedance and the admittance of the circuit for obtaining auxiliary constants B and C. Auxiliary constant A cannot be taken directly from a single Kennelly chart. To obtain this auxiliary constant from these charts it is necessary to divide the values read from two of these charts since $A = \dfrac{\sinh\ \theta/\theta}{\tanh\ \theta/\theta}$. Chart tanh θ/θ is constructed for angles up to and including 0.50 polar values. This makes it adapted to angles up to 1.0 polar value when used for determining correcting factors for the equivalent π solution. This is for the reason that for obtaining such correcting factors we enter this chart with $\theta/2$. However for obtaining auxiliary constant A by means of values read from these charts we must enter this chart with θ in place of $\theta/2$. This limits the use of the Kennelly charts for obtaining auxiliary constant A to circuit angles not exceeding 0.5 polar values. In case the circuit angle has a polar value greater than 0.5, Wilkinson chart A may be used provided the line is not over 300 miles long. If the circuit is over 300 miles long the auxiliary constants should be determined by mathematical calculation.

In the following discussion the calculated values for the auxiliary constants will be used, since exact results are required for the purpose of comparing the results with those obtained by the approximate method, a description of which follows the complete solution.

NORMAL LOAD—COMPLETE SOLUTION

The complete solution for normal load is given by Chart XXIII. At the top is illustrated the circuit diagramatically. Underneath this is stated the load conditions, linear and the auxiliary constants for this circuit. The transformer data and method of determining the amperes iron loss, magnetizing current and impedance to the neutral of the lowering transformer is also shown. Actually the impedance of raising and lowering transformers, even when duplicates, is slightly different when the connections are not made to similar taps. This difference is so slight (and so far as the raising transformer is concerned so unimportant) that for simplicity, we are assuming that both raising and lowering transformers have the same impedance. This comprises all the data required for a complete mathematical or graphical solution of this circuit.

Following the data is a complete graphical vector solution of this circuit with symbols placed on all vectors indicating the manner of obtaining their values. At the lower left hand corner is placed a complete mathematical solution of the problem, which parallels the graphical solution (one method of solution checking the other). In the calculations of the high-voltage circuit the current, in order to include the power-factor, must always be expressed in complex form referred to the vector of reference, as indicated by a dot under the symbol I.

At the lower right hand corner a method is indicated of determining the transmission loss from the calculated quantities. The loss in the high-tension line can be determined graphically by scaling off the voltage and the current at each end of the high-tension line and measuring the angle between the vectors representing the current and the voltage. The current times the voltage times the cosine of this angle will give the power at the point considered and the difference between the power as so determined at the two ends of the high-tension line is the line loss. The losses in transformers and condensers are known and stated at the top of the chart.

TABLE ZZ—AUXILIARY CONSTANTS FOR 220 KV PROBLEM APPROXIMATE SOLUTION

	Calculated	From Wilkinson Chart	From Kennelly Chart
a_1	0.893955 = 100%	0.892 = 99.78%	0.8937 = 99.97%
a_2	0.020234 = 100%	0.020 = 98.85%	0.01996 = 98.65%
b_1	32.198 = 100%	32.2 = 100%	32.2 = 100%
b_2	172.094 = 100%	173.5 = 100.82%	172 = 99.95%
c_1	-0.000008 = 100%	can't read	-0.000008 = 100%
c_2	0.001168 = 100%	0.001175 = 100.6%	0.001167 = 99.91%

The complete vector diagram is constructed as follows: First draw the horizontal line representing E_{LN}, the voltage at the load to neutral. This should be drawn to as large a scale as possible. All other voltage vectors will of course be drawn to the same scale. The vector I_L representing the load current is now drawn to as large a scale as can be used without mixing the current vectors with the voltage vectors. This is drawn at an angle of 31° 47′ from E_{LN} in the lagging direction, corresponding to a lagging load of 85 percent power-factor. It usually works out that for normal load the power-factor at the receiving end should be slightly lagging and at the sending end slightly leading so that the average power-factor of the line will be close to unity. This will necessitate a phase modifier in parallel with the load, having approximately the capacity of the lagging kv-a in the load.

The lagging kv-a in the load is equal to the kv-a of the load times the sine of the angle of the load. In this case it is 88 235 $\times$ sin 31° 47′ = 46 500 kv-a. The vector diagram is constructed on the basis of a 45 000 kv-a condenser in parallel with the load. This condenser has a power loss of 4.72 amperes to neutral and since this is in phase with the load voltage, we trace from the end of the load current vector horizontally to the right a distance representing 4.72 amperes by the current scale. The current per terminal for the condenser is 118.09 amperes so that the leading component of the current input of the condenser is 118.00 amperes. Since this is leading it is drawn vertically upward from the last point determined. Actually we will not need to determine the 118 amperes leading component, but will complete the solid black condenser triangle, since the length of the input line is 118.09 amperes. To the vector sum of load and condenser currents thus determined we now add the leakage current of the lowering transformers, the lagging component of which materially effects the capacity of the phase modifiers required because of its nearly direct opposition to it under load. We have assumed that the leakage current required by the lowering transformers will be supplied by the phase modifier on account of its close electrical proximity to the lowering transformers. On this assumption the triangle representing this transformer leakage will be located as indicated. There is a loss current of 1.85 amperes in phase with the load voltage and a magnetizing current of 13.9 amperes in lagging quadrature with the load voltage. We thus find that the current I_R at the receiving end of the line is 204.17 amperes, lagging 5° 1′ 16″ behind the load voltage. In this case the magnetizing current of the lowering transformer reduces the effective capacity of the phase modifier by an amount of 13.9 amperes; that is by 5.3 percent of the total capacity of the lowering transformers.

We next determine the voltage at the high-voltage side of the lowering transformers; that is the voltage E_{RN} at the receiving end of the transmission line. Knowing the resistance and reactance of the lowering transformer banks to neutral and the curent I_R, the transformer resistance voltage drop is plotted in phase with the current I_R and the reactance voltage drop in quadrature with the resistance drop as indicated. The voltage at the sending end E_{SN} of the transmission line is next determined by applying auxiliary constants A and B to the voltage and current respectively of the receiving end.

The base of the impedance triangle for the high-tension line $I_R \times b_1$ represents the resistance drop of the high-tension line in phase with the receiving end current. In quadrature to this is the reactance volts drop of the line $I_R \times b_2$. The voltage at the sending end is thus determined to be 131 858 volts which corresponds to slightly less than 230 000 volts between conductors. An arc of a circle corresponding to the voltage to be maintained at the sending end will serve as a guide in determining the proper capacity condenser necessary to maintain this sending end voltage. An increase in condenser capacity rotates the vector I_R in a counter-clockwise direction, swinging the line impedance triangle also in a counter-clockwise direction thus decreasing the voltage E_{SN} and reducing the line drop. A decrease in condenser capacity rotates the vector I_R in a clockwise direction, swinging the line impedance triangle also in a clockwise direction, thus increasing the voltage E_{SN} and increasing the line drop. Thus the effect upon line voltage drop may be readily determined for condensers of various capacities.

The next step is to determine the current at the sending end. This is done by applying auxiliary contants A and C to the current and voltage respectively of the receiving end. It will be noted that the charging current is drawn as leading by 90 degrees the high-tension voltage at the receiving end, which voltage is taken as the vector of reference as in previous discussions. The current at the sending end is thus determined to be 220.34 amperes leading the vector of reference by 35° 12′. The impedance triangle for the raising transformers may now be drawn in, the resistance drop of same being drawn parallel with I_S. This then gives the voltage at the generators. The current at the generators is determined by adding vectorally to I_S the leakage of the raising transformers. It is assumed that the raising transformers will receive their excitation from the generators, in which case the leakage triangle will occupy the position shown, resulting in a current at the generators of 218.88 amperes.

NORMAL LOAD—APPROXIMATE SOLUTION

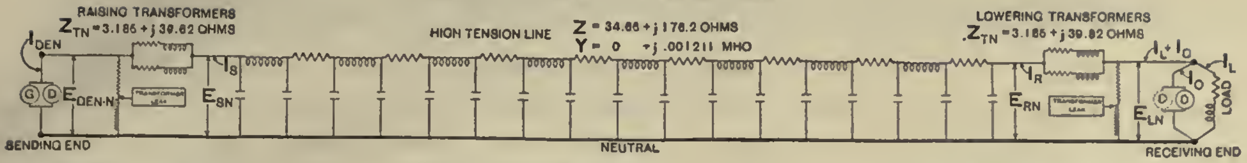
The approximate solution for normal load is given in Chart XXIV. It differs from the complete solution in that the impedance of the lowering transformers is added to and considered as a part of the line impedance so that there are no transformer impedance triangles to construct. It differs also in that, in the case illustrated, the conditions at the sending end only are obtained, whereas in the complete diagram the conditions at both sending end and generators were determined. If the condition at the generators in place of at the sending end is required, the impedance of the raising transformers would also be added to that of the line, the general construction of the diagram remaining the same as for the complete solution.

If it is not necessary to know conditions at both sides of the raising and lowering transformer banks, then it will be seen from a comparison of the two diagrams that the approximate solution will be simpler, although the results will be somewhat incorrect. For instance, for the 220 kv problem illustrated, the errors in the results will, according to tabulations in the lower right hand corner, vary from 0.88 to 2.38 percent. If the losses in condensers and transformers were not added to the load (as they are in both these complete and approximate methods) and the transformer mag-

CHART XXIII—220 KV PROBLEM—NORMAL LOAD

(COMPLETE SOLUTION)

*(LOW TENSION VALUES REFERRED TO THE HIGH TENSION CIRCUIT)



NORMAL LOAD

PER 3 PHASE CIRCUIT	PER PHASE TO NEUTRAL
$KV\text{-}A_L$ = 86.236	$KV\text{-}A_{LN}$ = 29.412
KW_l = 75.000	KW_{LN} = 25.000
PF_L = 85% LAG.	PF_{LN} = 85% LAG.
E_L = 220.000	E_{LN} = 127.020
I_L = 231 65	I_{LN} = 231 65

60 CYCLES

CONDENSER

(ONE REQUIRED)

3 PHASE	TO NEUTRAL
$KV\text{-}A_O$ = 45.000	$KV\text{-}A_{ON}$ = 15.000
E_C = 220.000	E_{CN} = 127.020
I_C = 118.09	I_{CN} = 118.09
$KW_{C\text{-}LOSS}$ = 1 800	$KW_{C\text{-}LOSS\text{-}N}$ = 600
$I_{C\text{-}LOSS}$ = 4.72	$I_{C\text{-}LOSS\text{-}N}$ = 4.72

NOTE – THE CONDENSER INDICATED BY BROKEN LINE CIRCLE SERVES AS A SPARE DURING NORMAL OPERATION BUT IS REQUIRED FOR THE EMERGENCY CONDITION.

LINEAR CONSTANTS

Z = 34.65 + j 176.2 OHMS
Y = 0 + j .001211 MHO

AUXILIARY CONSTANTS

$(A) = (a_1 + j\,a_2)$ = COSH θ = .893.655 + j .020.234 = .89410 /1° 17′ 47″

$(B) = (b_1 + j\,b_2) = Z\,\frac{\text{SINH}\,\theta}{\theta} = \sqrt{\frac{Z}{Y}}\,\text{SINH}\,\theta$ = 32.196 + j 172.084 OHMS

$(C) = (c_1 + j\,c_2) = Y\,\frac{\text{SINH}\,\theta}{\theta} = \frac{1}{\sqrt{\frac{Z}{Y}}}\,\text{SINH}\,\theta$ = −.000.006 + j .001.158 MHO

WHERE $\theta = \sqrt{ZY}$

TRANSFORMERS

(TWO BANKS IN PARALLEL AT EACH END OF THE LINE)

ON BASIS OF 100.000 KV-A RATING FOR TWO BANKS:

RESISTANCE VOLTS	= 0.666 %
REACTANCE VOLTS	= 6.225 %
MAGNETIZING CURRENT	= 5.300 %
IRON LOSS	= 0.706 %

VALUES TO NEUTRAL

$KV\text{-}A_{TN}$ = 33.333. E_{TN} = 127.020. I_{TN} = 262.4

$R_{TN} = \frac{.00668 \times 127.020}{262.4}$ = 3.186 OHMS RESISTANCE

$X_{TN} = \frac{.08225 \times 127.020}{262.4}$ = 39.82 OHMS REACTANCE

MAGNETIZING CURRENT = $\frac{.0530 \times 33.333.333}{127.020}$ = 13.9 AMPS AT 127.020 VOLTS

IRON LOSS = $\frac{.00706 \times 33.333.333}{127.020}$ = 1.85 AMPS AT 127.020 VOLTS

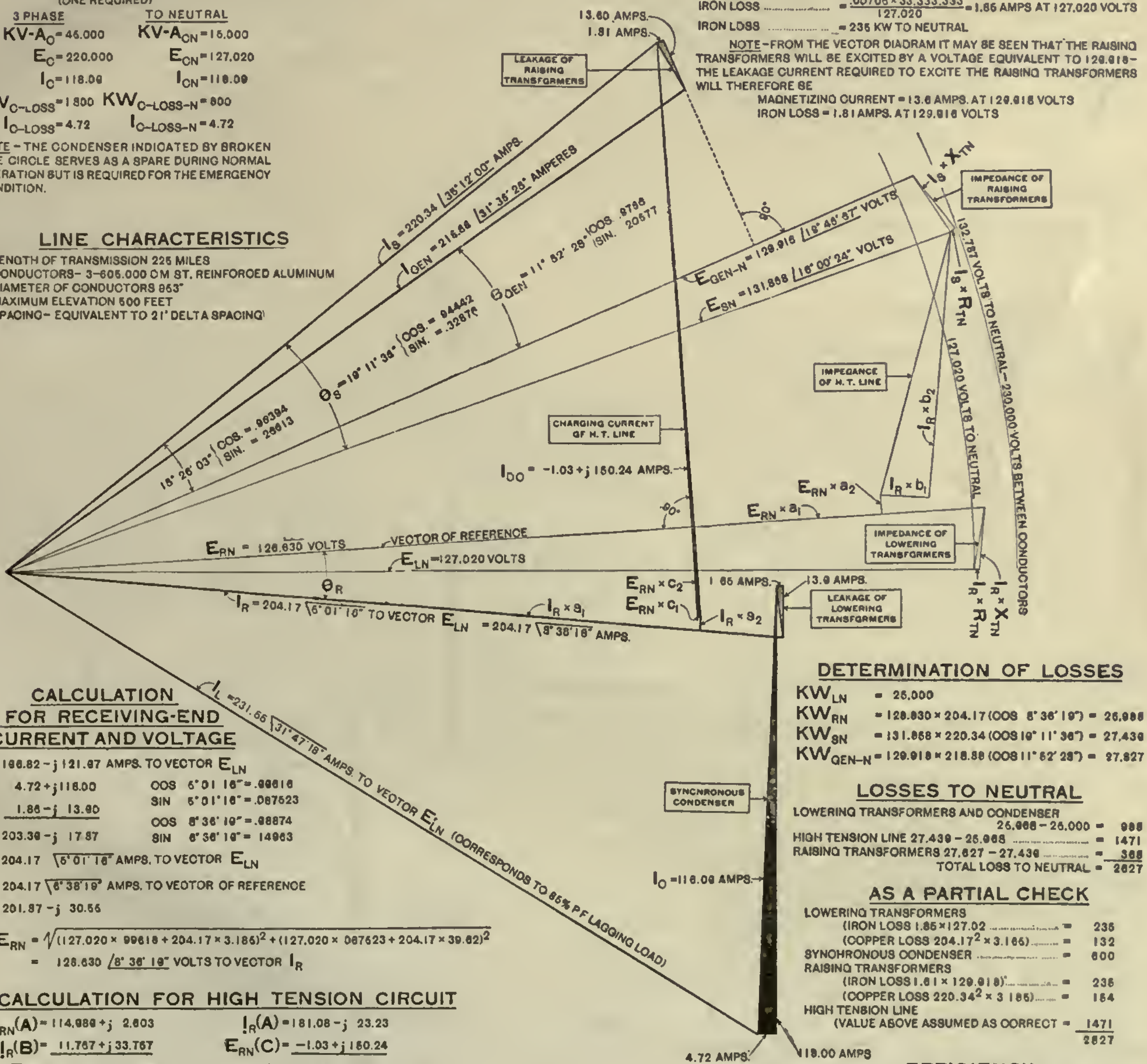
IRON LOSS = 235 KW TO NEUTRAL

NOTE—FROM THE VECTOR DIAGRAM IT MAY BE SEEN THAT THE RAISING TRANSFORMERS WILL BE EXCITED BY A VOLTAGE EQUIVALENT TO 129.916—THE LEAKAGE CURRENT REQUIRED TO EXCITE THE RAISING TRANSFORMERS WILL THEREFORE BE

MAGNETIZING CURRENT = 13.6 AMPS. AT 129.916 VOLTS
IRON LOSS = 1.81 AMPS. AT 129.916 VOLTS

LINE CHARACTERISTICS

LENGTH OF TRANSMISSION 225 MILES
CONDUCTORS– 3–605.000 CM ST. REINFORCED ALUMINUM
DIAMETER OF CONDUCTORS 953″
MAXIMUM ELEVATION 500 FEET
SPACING– EQUIVALENT TO 21′ DELTA SPACING



CALCULATION FOR RECEIVING-END CURRENT AND VOLTAGE

I_L = 196.82 − j 121.97 AMPS. TO VECTOR E_{LN}
I_C = 4.72 + j 118.00
I_T = 1.86 − j 13.90
I_R = 203.39 − j 17.87

COS 6° 01 16″ = .99616
SIN 6° 01′ 16″ = .087523
COS 8° 38′ 19″ = .98874
SIN 8° 38′ 19″ = 14963

= 204.17 \6° 01′ 16″ AMPS. TO VECTOR E_{LN}

= 204.17 \6° 38′ 19″ AMPS. TO VECTOR OF REFERENCE

= 201.87 − j 30.56

$E_{RN} = \sqrt{(127.020 \times 99616 + 204.17 \times 3.186)^2 + (127.020 \times 087523 + 204.17 \times 39.82)^2}$

= 126.630 /8° 36′ 19″ VOLTS TO VECTOR I_R

CALCULATION FOR HIGH TENSION CIRCUIT

$E_{RN}(A)$ = 114.989 + j 2.603	$I_R(A)$ = 181.08 − j 23.23
$I_R(B)$ = 11.757 + j 33.757	$E_{RN}(C)$ = −1.03 + j 160.24
E_{SN} = 126.746 + j 36.360	I_S = 180.05 + j 127.01
= 131.668 /16° 00′ 24″ VOLTS	= 220.34 /36° 12′ 00″ AMPS.

CALCULATION FOR GENERATOR VOLTAGE AND CURRENT

$E_{GEN\text{-}N} = \sqrt{(131.656 \times 94442 + 220.34 \times 3.186)^2 + (131\ 856 \times .32876 - 220.34 \times 39.82)^2}$

= 129.916 /15° 26′ 03″ VOLTS TO VECTOR I_S

= 129.916 /19° 45′ 57″ VOLTS

220.34 (.96394 + j .26613)
= 212.39 + j 58.84
= 1.81 − j 13.80 (LEAKAGE OF RAISING TRANSFORMERS)

$I_{GEN\text{-}N}$ = 214.20 + j 45.04

= 218.88 /11° 52′ 25″ AMPERES TO VECTOR E_{GEN}

= 218.88 /31° 36′ 25″ AMPERES

$KV\text{-}A_S$ = 131.869 × 220.34 × 3 = 87.163 PER 3 PHASE CIRCUIT
$KV\text{-}A_{GEN}$ = 129.916 × 218.88 × 3 = 85.306 PER 3 PHASE CIRCUIT
$KV\text{-}A_{CO}$ = 131 866 × 160.24 × 3 = 58.431 PER 3 PHASE CIRCUIT

DETERMINATION OF LOSSES

KW_{LN} = 25.000
KW_{RN} = 126.630 × 204.17 (COS 8° 36′ 19″) = 25.966
KW_{SN} = 131.868 × 220.34 (COS 19° 11′ 36″) = 27.439
$KW_{GEN\text{-}N}$ = 129.916 × 218.88 (COS 11° 52′ 28″) = 27.827

LOSSES TO NEUTRAL

LOWERING TRANSFORMERS AND CONDENSER 25.966 − 25.000 =	966
HIGH TENSION LINE 27.439 − 25.968 =	1471
RAISING TRANSFORMERS 27.827 − 27.439 =	388
TOTAL LOSS TO NEUTRAL =	2827

AS A PARTIAL CHECK

LOWERING TRANSFORMERS	
(IRON LOSS 1.85 × 127.02 =	235
(COPPER LOSS 204.17^2 × 3.166) =	132
SYNCHRONOUS CONDENSER =	600
RAISING TRANSFORMERS	
(IRON LOSS 1.81 × 129.918) =	235
(COPPER LOSS 220.34^2 × 3 186) =	154
HIGH TENSION LINE	
(VALUE ABOVE ASSUMED AS CORRECT =	1471
	2827

EFFICIENCY

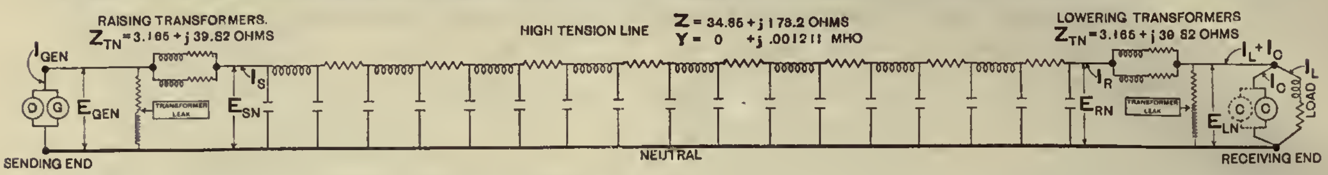
EFFICIENCY. (HIGH TENSION LINE) = $\frac{25.966}{27.439}$ = 94.64 %

EFFICIENCY. (GENERATORS TO LOAD) = $\frac{25.000}{27.827}$ = 89.84 %

CHART XXIV—220 KV PROBLEM—NORMAL LOAD

(APPROXIMATE SOLUTION)

THIS APPROXIMATE SOLUTION ASSUMES THAT THE IMPEDANCE OF THE LOWERING TRANSFORMERS MAY BE ADDED TO THE LINE IMPEDANCE AND TREATED AS THOUGH IT WERE DISTRIBUTED LINE IMPEDANCE—THIS ASSUMPTION SIMPLIFIES THE SOLUTION AT THE EXPENSE OF ACCURACY (SEE LOWER RIGHT HAND CORNER OF PAGE; ALSO TEXT).—THE SOLUTION BELOW IS BASED UPON THE VOLTAGE BEING HELD CONSTANT AT THE LOAD SIDE OF THE LOWERING TRANSFORMERS AND AT THE HIGH TENSION SIDE OF THE RAISING TRANSFORMERS.—IF THE VOLTAGE IS TO BE HELD CONSTANT AT THE GENERATOR BUS, THE IMPEDANCE OF THE RAISING TRANSFORMERS MUST ALSO BE ADDED TO THAT OF THE LINE—ALL LOW TENSION VALUES ARE REFERRED TO THE HIGH TENSION CIRCUIT.



NORMAL LOAD

PER 3 PHASE CIRCUIT	PER PHASE TO NEUTRAL
$KV\text{-}A_L$ = 88,235	$KV\text{-}A_{LN}$ = 29,412
KW_L = 75,000	KW_{LN} = 25,000
PF_L = 85% LAG.	PF_{LN} = 85% LAG.
E_L = 220,000	E_{LN} = 127,020
I_L = 231.55	I_{LN} = 231.55

60 CYCLES

CONDENSER

(ONE REQUIRED)

3 PHASE	TO NEUTRAL
$KV\text{-}A_C$ = 45,000	$KV\text{-}A_{CN}$ = 15,000
E_C = 220,000	E_{CN} = 127,020
I_C = 118.09	I_{CN} = 118.09
$KW_{C\text{-}LOSS}$ = 1800	$KW_{C\text{-}LOSS\text{-}N}$ = 600
$I_{C\text{-}LOSS}$ = 4.72	$I_{C\text{-}LOSS\text{-}N}$ = 4.72

NOTE – THE CONDENSER INDICATED BY BROKEN LINE CIRCLE SERVES AS A SPARE DURING NORMAL OPERATION BUT IS REQUIRED FOR THE EMERGENCY CONDITION.

LINEAR CONSTANTS

Z = 37.835 + j 218.02 OHMS ★
Y = 0 + j .001211 MHO

★ THIS INCLUDES IMPEDANCE OF LOWERING TRANSFORMERS

AUXILIARY CONSTANTS

$(A) = (a_1 + j a_2) = \cosh \theta = .870783 + j .021911$

$(B) = (b_1 + j b_2) = Z \frac{\sinh \theta}{\theta} = \sqrt{\frac{Z}{Y}} \sinh \theta = 34.5853 + j 208.83$ OHMS

$(C) = (c_1 + j c_2) = Y \frac{\sinh \theta}{\theta} = \frac{1}{\sqrt{\frac{Z}{Y}}} \sinh \theta = -.000,009 + j .001158$ MHO.

WHERE $\theta = \sqrt{ZY}$

TRANSFORMERS

(TWO BANKS IN PARALLEL AT EACH END OF THE LINE)

ON BASIS OF 100,000 KV-A RATING FOR TWO BANKS:

- RESISTANCE VOLTS = 0.858 %
- REACTANCE VOLTS = 8.225 %
- MAGNETIZING CURRENT = 5.300 %
- IRON LOSS = 0.705 %

VALUES TO NEUTRAL

$KV\text{-}A_{TN}$ = 33,333. E_{TN} = 127,020. I_{TN} = 262.4

$R_{TN} = \frac{.00858 \times 127,020}{262.4} = 3.165$ OHMS RESISTANCE

$X_{TN} = \frac{.08225 \times 127,020}{262.4} = 39.82$ OHMS REACTANCE

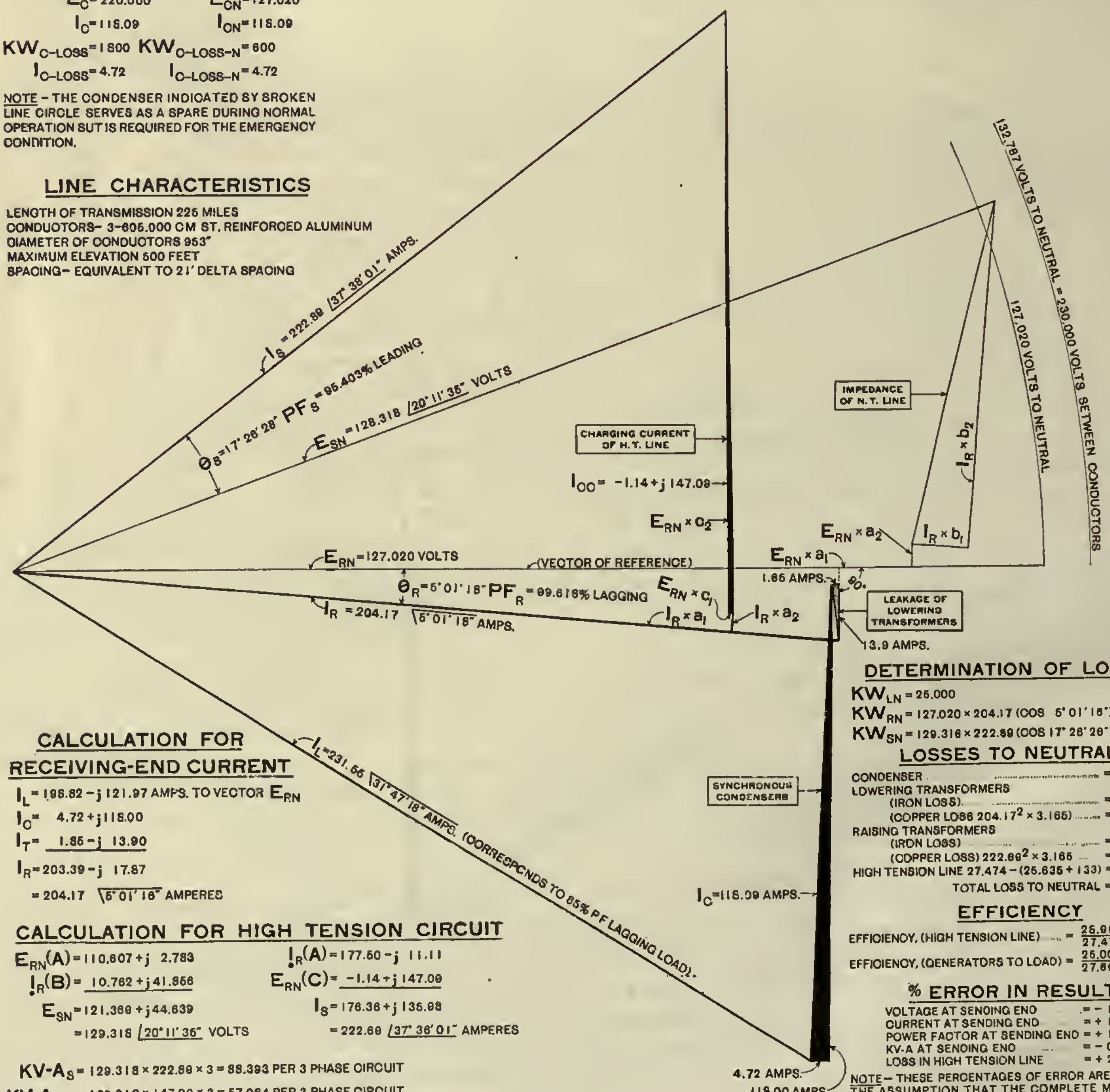
MAGNETIZING CURRENT $= \frac{.0530 \times 33,333,333}{127,020} = 13.9$ AMPS AT 127,020 VOLTS

IRON LOSS $= \frac{.00705 \times 33,333,333}{127,020} = 1.85$ AMPS AT 127,020 VOLTS

IRON LOSS = 235 KW TO NEUTRAL

LINE CHARACTERISTICS

LENGTH OF TRANSMISSION 225 MILES
CONDUCTORS– 3–605,000 CM ST. REINFORCED ALUMINUM
DIAMETER OF CONDUCTORS 953"
MAXIMUM ELEVATION 500 FEET
SPACING– EQUIVALENT TO 21' DELTA SPACING



CALCULATION FOR RECEIVING-END CURRENT

I_L = 195.82 – j 121.97 AMPS. TO VECTOR E_{RN}
I_C = 4.72 + j 118.00
I_T = 1.85 – j 13.90
I_R = 203.39 – j 17.87
= 204.17 \5° 01' 16" AMPERES

CALCULATION FOR HIGH TENSION CIRCUIT

$E_{RN}(A)$ = 110,607 + j 2,783	$I_R(A)$ = 177.60 – j 11.11
$I_R(B)$ = 10,762 + j 41,856	$E_{RN}(C)$ = –1.14 + j 147.09
E_{SN} = 121,369 + j 44,639	I_S = 176.36 + j 135.98
= 129,318 /20° 11' 35" VOLTS	= 222.89 /37° 38' 01" AMPERES

$KV\text{-}A_S$ = 129.318 × 222.89 × 3 = 86,393 PER 3 PHASE CIRCUIT

$KV\text{-}A_{CC}$ = 129.318 × 147.09 × 3 = 57,064 PER 3 PHASE CIRCUIT

DETERMINATION OF LOSSES

KW_{LN} = 25,000
KW_{RN} = 127.020 × 204.17 (COS 5° 01' 18") = 25,835
KW_{SN} = 129.318 × 222.89 (COS 17° 26' 28") = 27,474

LOSSES TO NEUTRAL

CONDENSER	= 600
LOWERING TRANSFORMERS	
(IRON LOSS)	= 235
(COPPER LOSS 204.17² × 3.165)	= 133
RAISING TRANSFORMERS	
(IRON LOSS)	= 235
(COPPER LOSS) 222.89² × 3.165	= 158
HIGH TENSION LINE 27,474 – (25,835 + 133)	= 1506
TOTAL LOSS TO NEUTRAL	= 2867

EFFICIENCY

EFFICIENCY, (HIGH TENSION LINE) $= \frac{25,968}{27,474} = 94.52$

EFFICIENCY, (GENERATORS TO LOAD) $= \frac{25,000}{27,867} = 89.71.$

% ERROR IN RESULTS

VOLTAGE AT SENDING END	= – 1.63 %
CURRENT AT SENDING END	= + 1.05 %
POWER FACTOR AT SENDING END	= + 1.02 %
KV-A AT SENDING END	= – 0.66 %
LOSS IN HIGH TENSION LINE	= + 2.36 %

NOTE— THESE PERCENTAGES OF ERROR ARE BASED UPON THE ASSUMPTION THAT THE COMPLETE METHOD PRODUCES 100 % VALUES.—THE MINUS SIGNS SIGNIFY RESULTS TOO LOW; THE PLUS SIGNS RESULTS TOO HIGH.

netizing current were not taken into account, (as it also is in both these methods) the error resulting from the use of the approximate method would be considerably greater than the above values.

The simplified graphical approximate solution illustrated by Chart XXIV will yield results sufficiently accurate for preliminary work, although for final results it should be supplemented by a mathematical solution and, in cases of very long lines, a complete mathematical solution might be desirable. A complete solution as given by Chart XXIII may be followed as a guide in such cases.

The method of obtaining the auxiliary constants corresponding to the approximate solution is given below. The linear constants of the circuit including transformer impedance are determined as follows:—

	Resistance (*Ohms*)	Reactance (*Ohms*)
Line	34.650	178.20
Transformers	3.185	39.82
Total	37.835	218.02

Dividing these total values by 225 we obtain the following as the impedance per mile of the combined circuit.

$$r = 0.1681 \text{ ohms}$$
$$x = 0.969 \text{ ohms}$$

TABLE ZZZ—AUXILIARY CONSTANTS FOR 220 KV PROBLEM, APPROXIMATE SOLUTION

Calculated	From Wilkinson Chart	From Kennelly Chart
$a_1 = 0.870783 = 100\%$	$0.892 = 102.44\%$ $0.868 = 99.68\%$ (corrected)	$0.8713 = 100.05\%$
$a_2 = 0.021911 = 100\%$	$0.0221 = 100.86\%$	$0.02206 = 100.68\%$
$b_1 = 34.5653 = 100\%$	$34.3 = 99.23\%$	$34.561 = 99.99\%$
$b_2 = 208.83 = 100\%$	$211.2 = 101.14\%$	$208.92 = 100.04\%$
$c_1 = -0.000009 = 100\%$	$-0.00001 = 111.11\%$	$-0.000009 = 100\%$
$c_2 = 0.001158 = 100\%$	$0.001163 = 100.43\%$	$0.001159 = 100.09\%$

The admittance per mile is assumed the same as before namely:—

$$b = 5.38 \times 10^{-6} \text{ mho}$$
$$g = 0$$

From Wilkinson's Charts

$$a_1 = 0.892$$
$$\text{and since } rb = 0.904$$
$$a_2 = 0.221$$
$$b_1 = 34.3 \text{ ohms}$$
$$b_2 = 211.2 \text{ ohms}$$
$$\text{and since } rb^2 = 4.865$$
$$c_1 = -0.000010$$
$$c_2 = 0.001163$$

From Dr. Kennelly's Charts

$$Z = 37.835 + j\,218.02$$
$$= 221.28 \ \underline{|80^\circ 09' 23''}$$
$$Y = 0 + j\,0.001\,211$$
$$= 0.001\,211 \ \underline{|90^\circ}$$
$$ZY = 0.26797 \ \overline{\backslash 170^\circ 09' 23''}$$
$$\theta = \sqrt{ZY} = 0.5177 \ \underline{|85^\circ 04' 41''}$$
$$\text{from Chart XIX } \frac{Sinh\,\theta}{\theta} = 0.957 \ \underline{/0.45^\circ}$$
$$= 0.957 \ \underline{/0^\circ 27' 00''}$$
$$\text{from Chart XXI } \frac{Tanh\,\theta}{\theta} = 1.098 \ \overline{\backslash 1^\circ 00' 00''}\text{*}$$

*This was interpolated since this angle lies beyond the range of this chart.

$$A = \frac{Sinh\,\theta/\theta}{Tanh\,\theta/\theta} = \frac{0.957 \ \underline{/0^\circ 27' 00''}}{1.098 \ \overline{\backslash 1^\circ 00' 00''}}$$
$$= 0.8716 \ \underline{/1^\circ 27' 00''}$$
$$a_1 = 0.8713$$
$$a_2 = 0.02206$$
$$B = Z\frac{Sinh\,\theta}{\theta} = 221.28 \ \underline{|80^\circ 09' 23''} \times 0.957 \ \underline{/0^\circ 27' 00''}$$
$$= 211.76 \ \underline{/80^\circ 36' 23''} \text{ ohms}$$
$$b_1 = 34.561 \text{ ohms}$$
$$b_2 = 208.92 \text{ ohms}$$
$$C = Y\frac{Sinh\,\theta}{\theta} = 0.001211 \ \underline{|90^\circ} \times 0.957 \ \underline{/0^\circ 27' 00''}$$
$$= 0.0011589 \ \underline{|90^\circ 27' 00''}$$
$$c_1 = -0.000\,009$$
$$c_2 = 0.001\,159$$

The auxiliary constants as obtained graphically and by exact mathematical results are given in Table ZZZ.

The same remarks in regard to use of the Kennelly charts for obtaining the auxiliary constants as given under the complete solution also apply when the approximate solution is used. Wilkinson chart *A*, if used when transformer impedance is added to the line impedance, as in the approximate method, requires a correction to constant a_1. Constant a_2 as read from this chart will be correct but constant a_1 as read from the chart will be too high for the following reason. Constant c_1 accounts for the rise in voltage along the line at zero load due to the charging current flowing through the line inductance adding directly to the sending end voltage. The section of Wilkinson chart *A* applying to constant a_1 is based upon dis tance and frequency only, so that values read from this section would be the same for a given distance and frequency regardless of whether or not transformer impedance is included with the line constants. This section of chart *A* therefore takes acount only of the voltage lowering effect of the charging current flowing through the line inductance. In addition to this, it flows also through the transformer inductance, which further lowers the value of a_1. The value of a_1 read from the chart must therefore be reduced. From the chart, $a_1 = 0.892$ volt corresponding to a voltage rise of 0.108 volt which results from a linear conductance reactance of 178.02 ohms. Actually the reactance of the circuit including lowering transformers is 218.02 ohms or 22.5 percent greater. Increasing 0.108 volt by 22.5 percent we get 0.132 volt rise, so that a_1 becomes 1.000 — 0.132 = 0.868, which is 99.68 percent of the calculated results.

In the following solutions calculated values for the auxiliary constants are used since exact results are required for the purpose of comparing the results with those previously obtained by the complete solution.

EMERGENCY LOAD—COMPLETE SOLUTION

The complete solution for emergency load conditions shown by Chart XXV follows the same construction as covered by Chart XXIII for normal load. The difference being that the load is doubled and the condenser capacity for a circuit increased nearly four times. Thus to force double the amount of power through the line and transformer impedance, with the same voltage drop, it is necessary in this case, nearly to quadruple the condenser capacity per circuit. Thus to meet the emergency condition nearly double the total condenser capacity will be required. This large increase in condenser capacity necessitated drawing the current vectors to one half the scale used for current vectors in the normal load diagram.

EMERGENCY LOAD—APPROXIMATE SOLUTION

The approximate solution for emergency load shown by Chart XXVI follows the same construction as in Chart XXIV for normal load with the exception of increased load and condenser capacity.

ZERO LOAD—COMPLETE SOLUTION

The complete solution for zero load is shown by Chart XXVII. In this case the load is made up of a lagging phase modifier load and the leakage of the lowering transformers. The same constructions are used as for the other complete solutions.

ZERO LOAD—APPROXIMATE SOLUTION

The approximate solution for zero load is shown by Chart XXVIII. It may be seen from the tabulated errors that this approximate method produces at zero load larger errors than the corresponding errors for loaded conditions. This is usually of little importance, however, as the light load conditions are generally not important.

PHASE MODIFIER CURVES

Frequently the normal and maximum amount of power to be transmitted is known; that is the transmission line, condensers and transformers are designed for a certain maximum load and it is of little importance what condenser capacity would be required for other loads or for various sending end voltages. At other times, especially in preliminary surveys, such data may be very necessary.

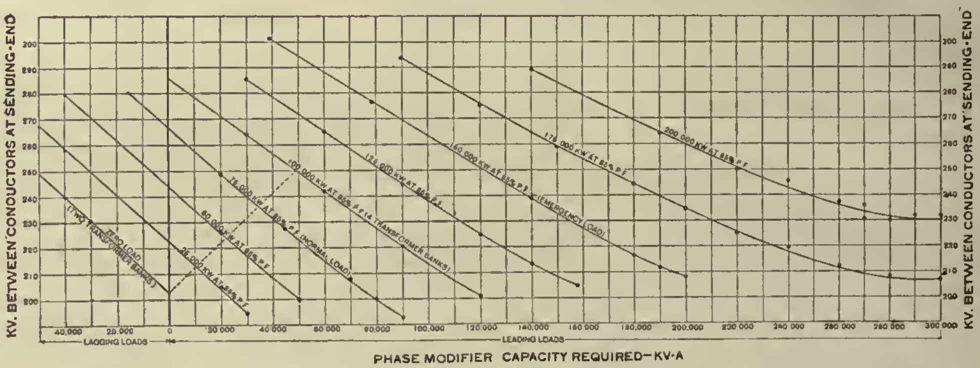


FIG. 70—PHASE MODIFIER CAPACITY REQUIRED TO MAINTAIN CONSTANT RECEIVER VOLTAGE.

These curves indicate for a constant load power-factor of 85 percent lagging and constant load voltage of 220 kv, the amount of energy which may be delivered to the load over one 225 mile, 60 cycle, three-phase circuit consisting of three 605 000 circ. mil aluminum-steel conductors corresponding to various voltages between conductors at the high-tension side of the raising transformers. The values by which these curves were drawn were determined graphically. For 230 kv at the sending end the maximum amount of power which can be transmitted is approximately 200 000 kw and to force this amount of power through the line impedance will require approximately 300 000 kv-a capacity in phase modifiers.

In Fig. 70 are plotted curves* showing the phase modifier capacity required to produce certain voltages at the sending end corresponding to various receiving-end loads at 85 percent power-factor and 220 kv. At 85 percent power-factor and 220 kv 200 000 kw is approximately the maximum amount of power which may be transmitted through the lowering transformers and over this line of three 605 000 circ. mil. cables if the sending end voltage is not permitted to exceed 230 kv. This is indicated by the fact that the curve corresponding to this load becomes flat when it reaches the 230 kv horizontal line. To deliver this maximum load at 220 kv through the impedance of this line will require a total condenser capacity of about 300 000 kv-a. The economic capacity of the line is reached at loads very much below the maximum theoretical limit of 200 000 kw.

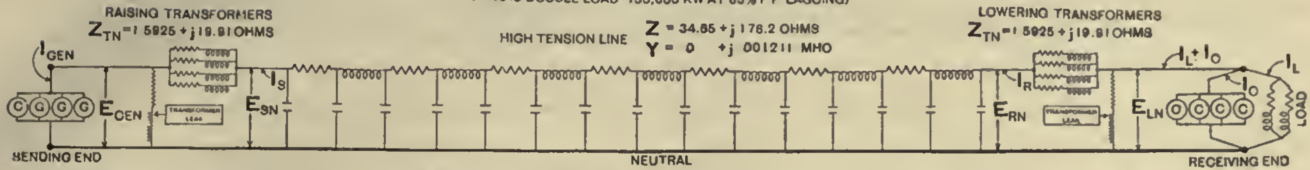
The sending end voltages corresponding to various capacities of phase modifiers in parallel with different receiving end loads for drawing curves such as shown by Fig. 70 are most readily obtained by the following graphical procedure. After auxiliary constants A and B for the circuit under investigation have been determined (preferably through the medium of both the Wilkinson and Kennelly charts) a tabulation of the current to neutral corresponding to each load for which curves are desired is made. A further tabulation of current to neutral for condensers of various capacities is made. The current to neutral which represents the loss in the various condensers, is also tabulated. The resist-

*Such curves were suggested by Mr. F. W. Peek, Jr. in an article on "Practical Calculations of Long Distance Transmission Line Characterictics" in the *General Electrical Review* for June, 1913, p. 430.

CHART XXV—220 KV PROBLEM—EMERGENCY LOAD

(COMPLETE SOLUTION)

(THIS IS DOUBLE LOAD–150,000 KW AT 85% P F LAGGING)



EMERGENCY LOAD

PER 3 PHASE CIRCUIT	PER PHASE TO NEUTRAL
$KV\text{-}A_L$ = 176.470	$KV\text{-}A_{LN}$ = 58.824
KW_L = 150.000	KW_{LN} = 50 000
PF_L = 85% LAG.	PF_{LN} = 85% LAG.
E_L = 220.000	E_{LN} = 127.020
I_L = 463.1	I_{LN} = 463.1

60 CYCLES

TRANSFORMERS

(FOUR BANKS IN PARALLEL AT EACH END OF THE LINE)

R_{TN} = 1.5925 OHMS RESISTANCE X_{TN} = 19.91 OHMS REACTANCE

MAGNETIZING CURRENT = 27 8 AMPS. AT 127 020 VOLTS
IRON LOSS .... = 3.70 AMPS. AT 127.020 VOLTS
IRON LOSS .... = 470 KW TO NEUTRAL

MAGNETIZING CURRENT = 28.46 AMPS. AT 133.552 VOLTS
IRON LOSS .... = 3.52 AMPS. AT 133.552 VOLTS

LINEAR CONSTANTS

Z = 34.65 + j 176.2 OHMS
Y = 0 + j .001211 MHO

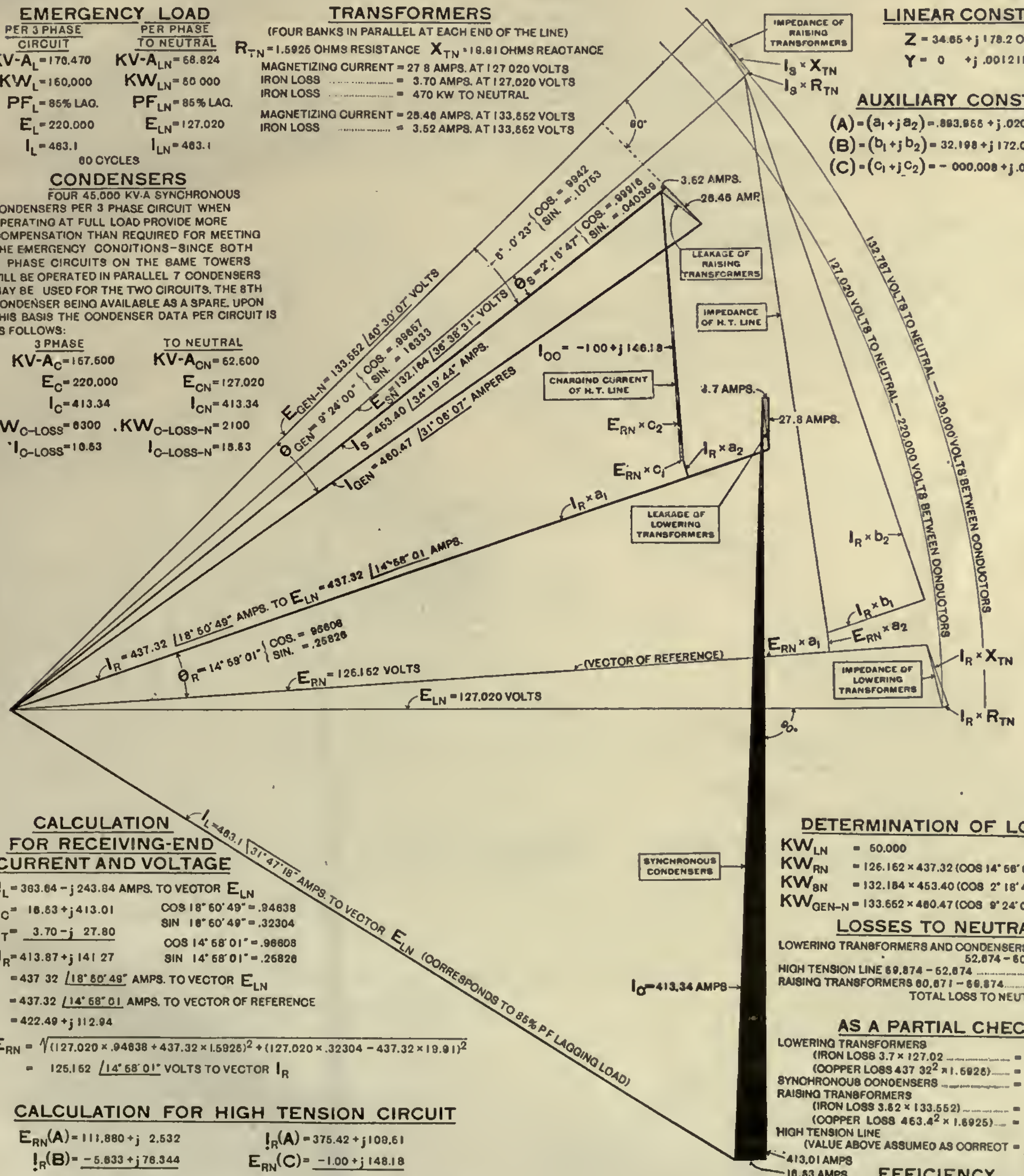
AUXILIARY CONSTANTS

$(A) = (a_1 + j a_2)$ = .893.955 + j .020.234
$(B) = (b_1 + j b_2)$ = 32.198 + j 172.094 OHMS
$(C) = (c_1 + j c_2)$ = − 000.008 + j .001.168 MHO

CONDENSERS

FOUR 45.000 KV-A SYNCHRONOUS CONDENSERS PER 3 PHASE CIRCUIT WHEN OPERATING AT FULL LOAD PROVIDE MORE COMPENSATION THAN REQUIRED FOR MEETING THE EMERGENCY CONDITIONS–SINCE BOTH 3 PHASE CIRCUITS ON THE SAME TOWERS WILL BE OPERATED IN PARALLEL 7 CONDENSERS MAY BE USED FOR THE TWO CIRCUITS. THE 8TH CONDENSER BEING AVAILABLE AS A SPARE. UPON THIS BASIS THE CONDENSER DATA PER CIRCUIT IS AS FOLLOWS:

3 PHASE	TO NEUTRAL
$KV\text{-}A_C$ = 157.600	$KV\text{-}A_{CN}$ = 52.600
E_C = 220.000	E_{CN} = 127.020
I_C = 413.34	I_{CN} = 413.34
$KW_{C\text{-}LOSS}$ = 6300	$KW_{C\text{-}LOSS\text{-}N}$ = 2100
$I_{C\text{-}LOSS}$ = 16.53	$I_{C\text{-}LOSS\text{-}N}$ = 16.53



CALCULATION FOR RECEIVING-END CURRENT AND VOLTAGE

I_L = 393.64 − j 243.94 AMPS. TO VECTOR E_{LN}
I_C = 16.53 + j 413.01
I_T = 3.70 − j 27.80
I_R = 413.87 + j 141 27
= 437 32 /18° 50′ 49″ AMPS. TO VECTOR E_{LN}
= 437.32 /14° 58′ 01 AMPS. TO VECTOR OF REFERENCE
= 422.49 + j 112.94

COS 18° 50′ 49″ = .94638
SIN 18° 50′ 49″ = .32304
COS 14° 58′ 01″ = .96608
SIN 14° 58′ 01″ = .25826

$E_{RN} = \sqrt{(127.020 \times .94638 + 437.32 \times 1.5925)^2 + (127.020 \times .32304 - 437.32 \times 19.91)^2}$
= 126.152 /14° 58′ 01″ VOLTS TO VECTOR I_R

CALCULATION FOR HIGH TENSION CIRCUIT

$E_{RN}(A)$ = 111.880 + j 2.532
$I_R(B)$ = −5.833 + j 76.344
E_{SN} = 106.047 + j 78.876
= 132.164 /36° 38′ 31″ VOLTS

$I_R(A)$ = 375.42 + j 109.51
$E_{RN}(C)$ = −1.00 + j 146.18
I_S = 374.42 + j 255.69
= 453.40 /34° 19′ 44″ AMPERES

DETERMINATION OF LOSSES

KW_{LN} = 50.000
KW_{RN} = 126.152 × 437.32 (COS 14° 58′ 01″) = 52.674
KW_{SN} = 132.164 × 453.40 (COS 2° 18′ 47″) = 59.874
$KW_{GEN\text{-}N}$ = 133.552 × 460.47 (COS 9° 24′ 00″) = 60.671

LOSSES TO NEUTRAL

LOWERING TRANSFORMERS AND CONDENSERS 52.674 − 50.000	= 2674
HIGH TENSION LINE 59.874 − 52.674	= 7000
RAISING TRANSFORMERS 60.671 − 59.874	= 797
TOTAL LOSS TO NEUTRAL	= 10.671

AS A PARTIAL CHECK

LOWERING TRANSFORMERS	
(IRON LOSS 3.7 × 127.02	= 470
(COPPER LOSS 437 32² × 1.5925)	= 304
SYNCHRONOUS CONDENSERS	= 2100
RAISING TRANSFORMERS	
(IRON LOSS 3.52 × 133.552)	= 470
(COPPER LOSS 463.4² × 1.5925)	= 327
HIGH TENSION LINE (VALUE ABOVE ASSUMED AS CORRECT	= 7000
	10.671

EFFICIENCY

EFFICIENCY (HIGH TENSION LINE) = $\frac{52.674}{59.874}$ = 88.31%

EFFICIENCY. (GENERATORS TO LOAD) = $\frac{50.000}{60.671}$ = 82.41%

CALCULATION FOR GENERATOR VOLTAGE AND CURRENT

$E_{GEN\text{-}N} = \sqrt{(132.164 \times .99918 + 453.4 \times 1.5925)^2 + (132.164 \times .040369 + 453.4 \times 19.91)^2}$
= 133.552 /8° 10′ 23″ VOLTS TO VECTOR I_S
= 133.552 /40° 30′ 07″ VOLTS

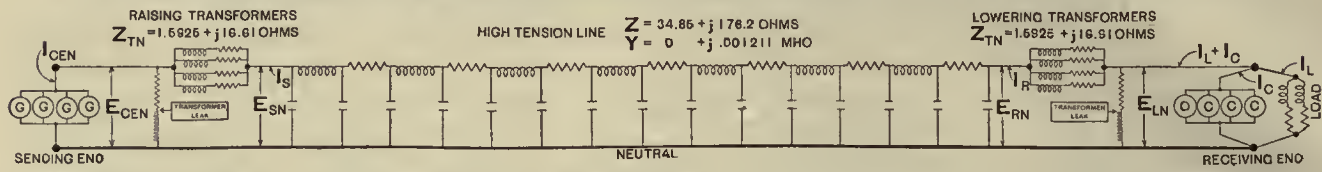
$KV\text{-}A_S$ = 132.164 × 453.40 × 3 = 179.769 PER 3 PHASE CIRCUIT
$KV\text{-}A_{GEN}$ = 133.552 × 460.47 × 3 = 184.490 PER 3 PHASE CIRCUIT
$KV\text{-}A_{CC}$ = 132.164 × 146.18 × 3 = 57.959 PER 3 PHASE CIRCUIT

453.4 (.9942 − j .10753)
= 450.77 − j 48.75
= 3.52 − j 28.46 (LEAKAGE OF RAISING TRANSFORMERS)
$I_{GEN\text{-}N}$ = 454.29 − j 75.21
= 460.47 \9° 24′ 00″ AMPERES TO VECTOR E_{GEN}
= 460.47 /31° 06′ 07″ AMPERES

CHART XXVI—220 KV PROBLEM—EMERGENCY LOAD

(APPROXIMATE SOLUTION)

THIS APPROXIMATE SOLUTION ASSUMES THAT THE IMPEDANCE OF THE LOWERING TRANSFORMERS MAY BE ADDED TO THE LINE IMPEDANCE AND TREATED AS THOUGH IT WERE DISTRIBUTED LINE IMPEDANCE--THIS ASSUMPTION SIMPLIFIES THE SOLUTION AT THE EXPENSE OF ACCURACY (SEE LOWER RIGHT HAND CORNER OF PAGE; ALSO TEXT).--THE SOLUTION BELOW IS BASED UPON THE VOLTAGE BEING HELD CONSTANT AT THE LOAD SIDE OF THE LOWERING TRANSFORMERS AND AT THE HIGH TENSION SIDE OF THE RAISING TRANSFORMERS.--IF THE VOLTAGE IS TO BE HELD CONSTANT AT THE GENERATOR BUS, THE IMPEDANCE OF THE RAISING TRANSFORMERS MUST ALSO BE ADDED TO THAT OF THE LINE--ALL LOW TENSION VALUES ARE REFERRED TO THE HIGH TENSION CIRCUIT.



EMERGENCY LOAD

PER 3 PHASE CIRCUIT	PER PHASE TO NEUTRAL
$KV\text{-}A_L$ = 176.470	$KV\text{-}A_{LN}$ = 58.824
KW_L = 150.000	KW_{LN} = 50.000
PF_L = 85% LAG.	PF_{LN} = 85% LAG.
E_L = 220.000	E_{LN} = 127.020
I_L = 463.1	I_{LN} = 463.1

60 CYCLES

CONDENSERS

FOUR 45.000 KV-A SYNCHRONOUS CONDENSERS PER 3 PHASE CIRCUIT WHEN OPERATING AT FULL LOAD PROVIDE MORE COMPENSATION THAN REQUIRED FOR MEETING THE EMERGENCY CONDITIONS—SINCE BOTH 3 PHASE CIRCUITS ON THE SAME TOWERS WILL BE OPERATED IN PARALLEL 7 CONDENSERS MAY BE USED FOR THE TWO CIRCUITS. THE 8TH CONDENSER BEING AVAILABLE AS A SPARE. UPON THIS BASIS THE CONDENSER DATA PER CIRCUIT IS AS FOLLOWS:

3 PHASE	TO NEUTRAL
$KV\text{-}A_C$ = 157.500	$KV\text{-}A_{CN}$ = 52.500
E_C = 220.000	E_{CN} = 127.020
I_C = 413.34	I_{CN} = 413.34
$KW_{C\text{-}LOSS}$ = 6300	$KW_{C\text{-}LOSS\text{-}N}$ = 2100
$I_{C\text{-}LOSS}$ = 16.53	$I_{C\text{-}LOSS\text{-}N}$ = 16.53

LINEAR CONSTANTS

Z = 37.635 + j 218.02 OHMS ★
Y = 0 + j .001211 MHO

★ THIS INCLUDES IMPEDANCE OF LOWERING TRANSFORMERS

AUXILIARY CONSTANTS

$(A) = (a_1 + j a_2) = \cosh\theta = .670783 + j .021611$

$(B) = (b_1 + j b_2) = Z \frac{\sinh\theta}{\theta} = \sqrt{\frac{Z}{Y}} \sinh\theta = 34.5653 + j\,208.83$ OHMS

$(C) = (c_1 + j c_2) = Y \frac{\sinh\theta}{\theta} = \frac{1}{\sqrt{\frac{Z}{Y}}} \sinh\theta = -.000.009 + j .001158$ MHO.

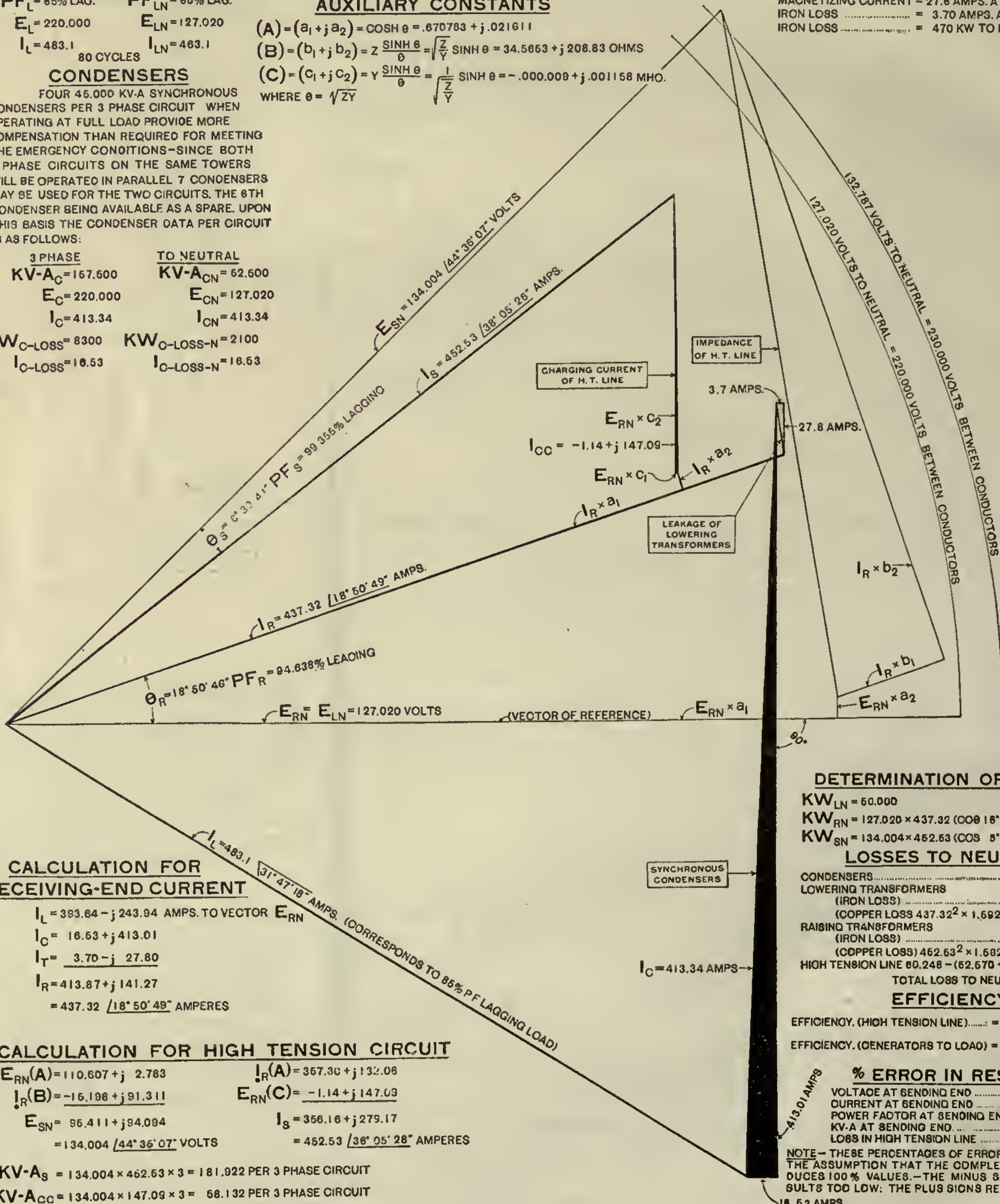
WHERE $\theta = \sqrt{ZY}$

TRANSFORMERS

(FOUR BANKS IN PARALLEL AT EACH END OF THE LINE)

R_{TN} = 1.5925 OHMS RESISTANCE
X_{TN} = 16.91 OHMS REACTANCE

MAGNETIZING CURRENT = 27.8 AMPS. AT 127.020 VOLTS
IRON LOSS .... = 3.70 AMPS. AT 127.020 VOLTS
IRON LOSS .... = 470 KW TO NEUTRAL



CALCULATION FOR RECEIVING-END CURRENT

I_L = 393.64 − j 243.94 AMPS. TO VECTOR E_{RN}
I_C = 16.53 + j 413.01
I_T = 3.70 − j 27.80
I_R = 413.87 + j 141.27
= 437.32 /18° 50' 49" AMPERES

CALCULATION FOR HIGH TENSION CIRCUIT

$E_{RN}(A)$ = 110.607 + j 2.783	$I_R(A)$ = 357.30 + j 132.08
$I_R(B)$ = −16.196 + j 91.311	$E_{RN}(C)$ = −1.14 + j 147.09
E_{SN} = 95.411 + j 94.094	I_S = 356.16 + j 279.17
= 134.004 /44° 36' 07" VOLTS	= 452.53 /36° 05' 28" AMPERES

$KV\text{-}A_S$ = 134.004 × 452.53 × 3 = 181.922 PER 3 PHASE CIRCUIT
$KV\text{-}A_{CC}$ = 134.004 × 147.09 × 3 = 59.132 PER 3 PHASE CIRCUIT

DETERMINATION OF LOSSES

KW_{LN} = 50.000
KW_{RN} = 127.020 × 437.32 (COS 18° 50' 49") = 52.570
KW_{SN} = 134.004 × 452.53 (COS 5° 30' 41") = 60.248

LOSSES TO NEUTRAL

CONDENSERS	= 2100
LOWERING TRANSFORMERS	
(IRON LOSS)	= 470
(COPPER LOSS $437.32^2 \times 1.5925$)	= 306
RAISING TRANSFORMERS	
(IRON LOSS)	= 470
(COPPER LOSS) $452.53^2 \times 1.5925$	= 326
HIGH TENSION LINE 60.248 − (52.570 + 306)	= 7373
TOTAL LOSS TO NEUTRAL	= 11,044

EFFICIENCY

EFFICIENCY. (HIGH TENSION LINE) .... = $\frac{52,876}{60,248}$ = 87.76%

EFFICIENCY. (GENERATORS TO LOAD) = $\frac{50,000}{61,044}$ = 81.91%

% ERROR IN RESULTS

VOLTAGE AT SENDING END	= + 1.36%
CURRENT AT SENDING END	= − 0.20%
POWER FACTOR AT SENDING END	= − 0.56%
KV-A AT SENDING END	= + 1.16%
LOSS IN HIGH TENSION LINE	= + 5.33%

NOTE—THESE PERCENTAGES OF ERROR ARE BASED UPON THE ASSUMPTION THAT THE COMPLETE METHOD PRODUCES 100% VALUES.—THE MINUS SIGNS SIGNIFY RESULTS TOO LOW: THE PLUS SIGNS RESULTS TOO HIGH.

NOTE:—Linear constant Z, as used in this chart, incorrectly includes impedance of two banks, whereas it should have included four banks of transformers. This error will not, however, materially affect the result.

ance, reactance, iron loss and magnetizing currents of the transformer banks to neutral should also be determined for all capacity transformer banks required. With the above data tabulated any draughtsman can be instructed how to draw vector diagrams of the circuit to determine the sending end voltages corresponding to ficient to locate the curve, although more points were calculated for drawing the curves of Fig. 70. This method of obtaining condenser capacities corresponding to sending end voltages is a cut and try method. It has one important advantage in its favor. That is, the results check each other, so that an error in one

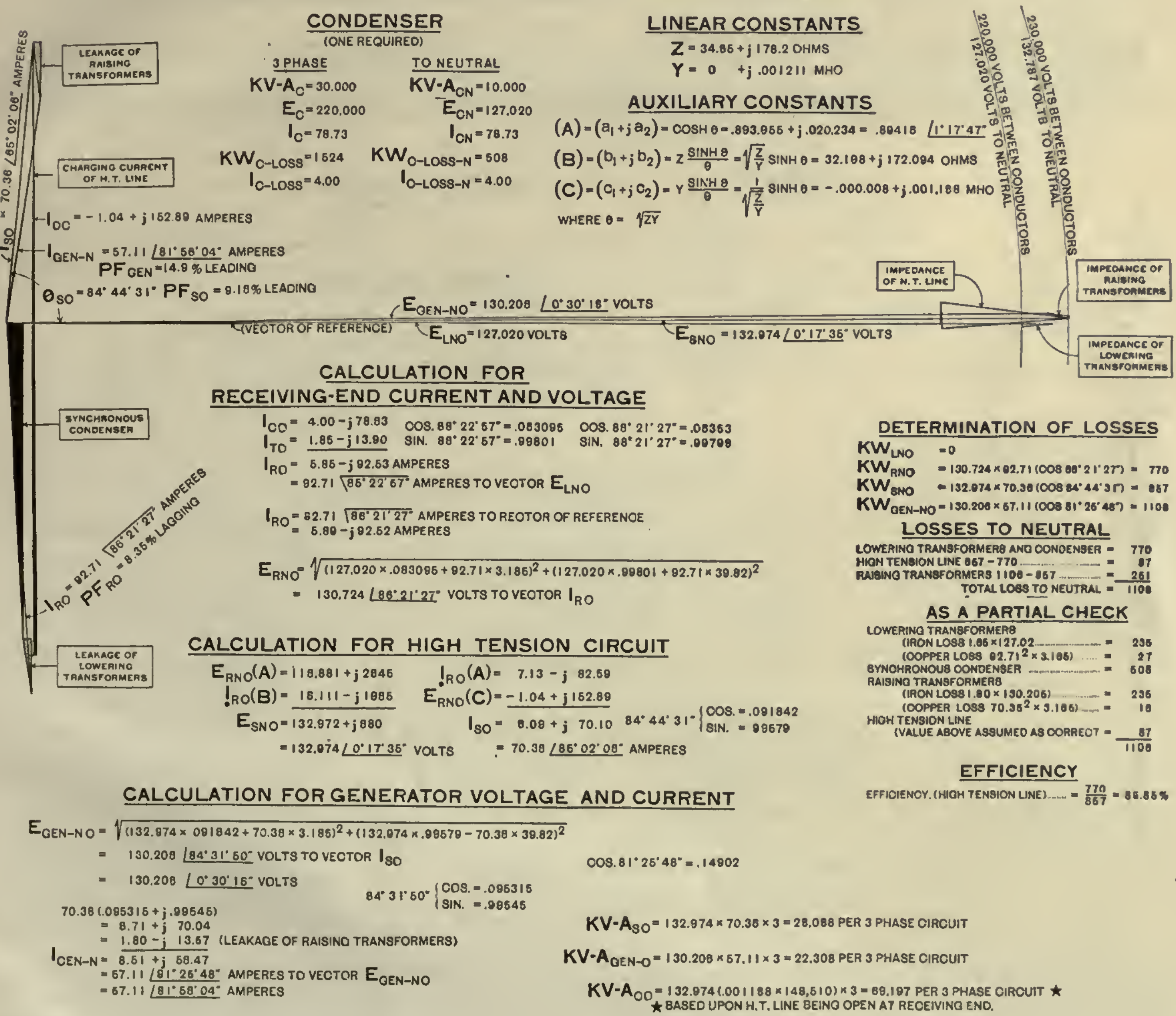
CHART XXVII—220 KV PROBLEM—ZERO LOAD

(COMPLETE SOLUTION)

(THIS CORRESPONDS TO NORMAL LOAD CONNECTIONS)

AT ZERO LOAD, WITH 230,000 VOLTS MAINTAINED BETWEEN CONDUCTORS (132,787 VOLTS TO NEUTRAL), AT THE HIGH TENSION SIDE OF THE RAISING TRANSFORMERS THE VOLTAGE AT THE HIGH TENSION SIDE OF THE LOWERING TRANSFORMERS (NEGLECTING THE EFFECT OF THE LAGGING MAGNETIZING CURRENT OF THE LOWERING TRANSFORMERS) WILL RISE TO 230,000 DIVIDED BY (A)=230,000 DIVIDED BY .89418 = 257,219 VOLTS BETWEEN CONDUCTORS (148,810 VOLTS TO NEUTRAL). ACTUALLY THE GREATLY INCREASED LAGGING MAGNETIZING CURRENT OF THE LOWERING TRANSFORMERS WHEN EXCITED BY ABNORMALLY HIGH VOLTAGE WILL NOT PERMIT OF THE RECEIVING END VOLTAGE REACHING SUCH A HIGH VOLTAGE UNLESS THE GENERATOR VOLTAGE RAISES MOMENTARILY TO A VALUE GREATLY IN EXCESS OF 230,000 VOLTS. IF HOWEVER THE LOWERING TRANSFORMERS ARE DISCONNECTED FROM THE CIRCUIT, THE INCREASED LEADING CHARGING CURRENT OF THE LINE, REACTING UPON THE GENERATOR FIELDS, COMBINED WITH A MOMENTARY OVER SPEED OF THE GENERATORS MAY CAUSE THE RECEIVING END VOLTAGE TO GREATLY EXCEED THE ABOVE VALUE.

IN ORDER TO HOLD THE VOLTAGE AT THE RECEIVING END CONSTANT AT 220,000 VOLTS BETWEEN CONDUCTORS (127,020 VOLTS TO NEUTRAL) AT ZERO LOAD IT WILL BE NECESSARY TO PLACE AN ARTIFICIAL LAGGING LOAD AT THE LOAD END OF THE LINE--THIS IS ACCOMPLISHED BY OPERATING ONE OF THE SYNCHRONOUS CONDENSERS WITH ITS FIELDS UNDER EXCITED.--BY CONSTRUCTING SEVERAL VECTOR DIAGRAMS FOR THIS CIRCUIT EACH BASED UPON DIFFERENT VALUES OF REACTOR LOAD, A CURVE MAY BE DRAWN BY PLOTTING THE REACTOR LOADS AGAINST THE CORRESPONDING SENDING END VOLTAGES.--FROM THIS CURVE THE REACTOR CAPACITY CORRESPONDING TO 230,000 VOLTS BETWEEN CONDUCTORS AT THE SENDING END WILL BE SEEN TO BE APPROXIMATELY 30,000 KV-A



the various receiving end loads and different phase modifier capacities.

The graphical method used in determining the values to plot the curves of Fig. 70, is illustrated by Fig. 71. Three solutions are illustrated, two with condensers of different size and one without condensers. Three such solutions for each load will usually be sufficient to locate the curve, although more points were calculated for drawing the curves of Fig. 70.

of the graphical constructions corresponding to a given load will be detected, since the point will not lay in the curve and an error in a curve corresponding to a given load will be detected by the curves of Fig. 72.

CAPACITY OF PHASE MODIFIERS

The curves of Fig. 70 show that, for a constant delivered load, power-factor and voltage, the leading capacity of phase modifiers required goes down as the line drop increases. For instance 75 000 kw at 85 per-cent power-factor and 220 kv can be delivered over this line with 230 kv sending end voltage, if 43 000 kv-a condenser capacity is placed in parallel with the load. If, however, a line drop of 20 kv is selected in place of 10 kv, the sending end voltage will be 240 kv and the corresponding condenser load will be reduced to approximately 30 000 kv-a. On the other hand this increased line drop will necessitate a greater capacity The dotted line in Fig. 70 is simply the zero load line thrown over to the leading load side to facilitate study in phase modifier capacity. For instance, projection from the points where the dotted line intersects a load curve will give the minimum capacity of phase modifier on the bottom scale and the corresponding sending end voltage on the vertical scale to the left. Thus with a load of 75 000 kw, intersection of the dotted line with this load curve indicates that 33 000 kv-a phase modifier capacity will be required both at this load and at zero

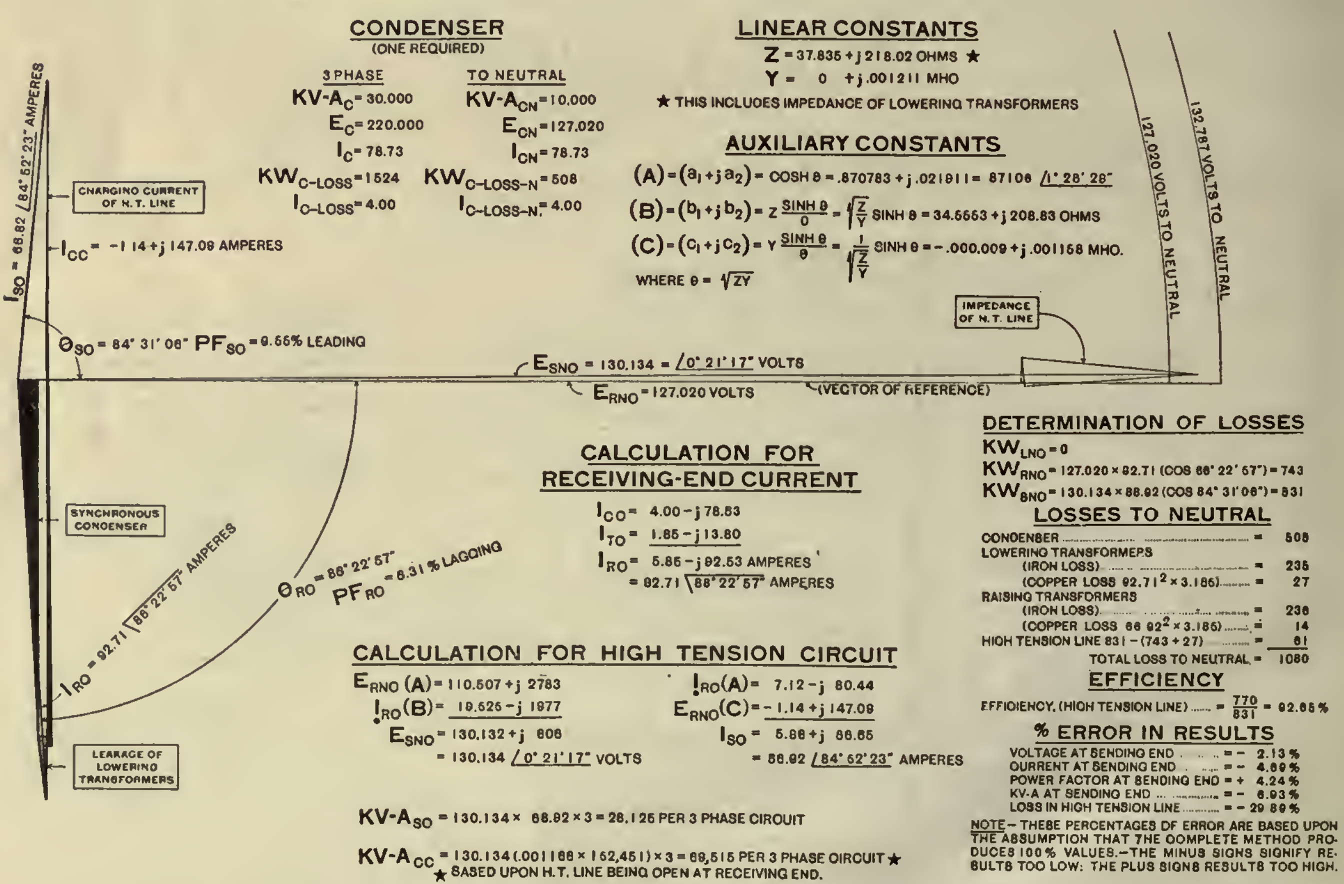
CHART XXVIII—220 KV PROBLEM—ZERO LOAD
(APPROXIMATE SOLUTION)

(THIS CORRESPONDS TO THE NORMAL LOAD CONNECTIONS)

THIS APPROXIMATE SOLUTION ASSUMES THAT THE IMPEDANCE OF THE LOWERING TRANSFORMERS MAY BE ADDED TO THE LINE IMPEDANCE AND TREATED AS THOUGH IT WERE DISTRIBUTED LINE IMPEDANCE--THIS ASSUMPTION SIMPLIFIES THE SOLUTION AT THE EXPENSE OF ACCURACY (SEE LOWER RIGHT HAND CORNER OF PAGE; ALSO TEXT).--THE SOLUTION BELOW IS BASED UPON THE VOLTAGE BEING HELD CONSTANT AT THE LOAD SIDE OF THE LOWERING TRANSFORMERS AND AT THE HIGH TENSION SIDE OF THE RAISING TRANSFORMERS.--IF THE VOLTAGE IS TO BE HELD CONSTANT AT THE GENERATOR BUS, THE IMPEDANCE OF THE RAISING TRANSFORMERS MUST ALSO BE ADDED TO THAT OF THE LINE--ALL LOW TENSION VALUES ARE REFERRED TO THE HIGH TENSION CIRCUIT.

AT ZERO LOAD, WITH 230,000 VOLTS MAINTAINED BETWEEN CONDUCTORS (132,787 VOLTS TO NEUTRAL), AT THE HIGH TENSION SIDE OF THE RAISING TRANSFORMERS THE VOLTAGE AT THE HIGH TENSION SIDE OF THE LOWERING TRANSFORMERS (NEGLECTING THE EFFECT OF THE LAGGING MAGNETIZING CURRENT OF THE LOWERING TRANSFORMERS) WILL RISE TO 230,000 DIVIDED BY (A)=230,000 DIVIDED BY .87106 =264,046 VOLTS BETWEEN CONDUCTORS (152 461 VOLTS TO NEUTRAL). ACTUALLY THE GREATLY INCREASED LAGGING MAGNETIZING CURRENT OF THE LOWERING TRANSFORMERS WHEN EXCITED BY ABNORMALLY HIGH VOLTAGE WILL NOT PERMIT OF THE RECEIVING END VOLTAGE REACHING SUCH A HIGH VOLTAGE UNLESS THE GENERATOR VOLTAGE RAISES MOMENTARILY TO A VALUE GREATLY IN EXCESS OF 230,000 VOLTS. IF HOWEVER THE LOWERING TRANSFORMERS ARE DISCONNECTED FROM THE CIRCUIT, THE INCREASED LEADING CHARGING CURRENT OF THE LINE, REACTING UPON THE GENERATOR FIELDS, COMBINED WITH A MOMENTARY OVER SPEED OF THE GENERATORS MAY CAUSE THE RECEIVING END VOLTAGE TO GREATLY EXCEED THE ABOVE VALUE.

IN ORDER TO HOLD THE VOLTAGE AT THE RECEIVING END CONSTANT AT 220,000 VOLTS BETWEEN CONDUCTORS (127,020 VOLTS TO NEUTRAL) AT ZERO LOAD IT WILL BE NECESSARY TO PLACE AN ARTIFICIAL LAGGING LOAD AT THE LOAD END OF THE LINE--THIS IS ACCOMPLISHED BY OPERATING ONE OF THE SYNCHRONOUS CONDENSERS WITH ITS FIELDS UNDER EXCITED.--BY CONSTRUCTING SEVERAL VECTOR DIAGRAMS FOR THIS CIRCUIT EACH BASED UPON DIFFERENT VALUES OF REACTOR LOAD, A CURVE MAY BE DRAWN BY PLOTTING THE REACTOR LOADS AGAINST THE CORRESPONDING SENDING END VOLTAGES.--FROM THIS CURVE THE REACTOR CAPACITY CORRESPONDING TO 230,000 VOLTS BETWEEN CONDUCTORS AT THE SENDING END WILL BE SEEN TO BE APPROXIMATELY 30,000 KV-A



at zero load in order to maintain 240 kw constant at the sending end. Thus with 230 kv at the sending end, about 30 000 kv-a reactor load will be required at zero load, whereas with 240 kv at the sending end, about 40 000 kv-a reactor load will be required at zero load.

Obviously the smallest phase modifier capacity possible to maintain regulation is one in which full capacity leading will be required under maximum load and full capacity lagging under zero load. At half load such a phase modifier would operate at near zero kv-a.

load and that the corresponding sending end voltage will be approximately 236 kv. At 100 000 kw load, nearly 50 000 kv-a phase modifier capacity will be required, and the corresponding sending end voltage would be 250 kv.

As previously stated, phase modifiers which may be operated at rated load both lagging and leading are special, and cost more than standard phase modifiers. On account of unstable operation due to weakened field, standard condensers usually cannot be operated at lagging loads above approximately 70 percent of their full load leading rating. To deliver 75 000 kv-a at 85 percent power-factor requires approximately 42 000 kv-a in phase modifier capacity with 230 kv at the sending end. To maintain the sending end voltage of 230 kv at zero load requires approximately 30 000 kv-a lagging. This is 70 percent of the capacity leading, thus permitting of employing a standard 43 000 kv-a condenser. To provide margin a 45 000 kv-a standard condenser might be selected for this normal load condition.

Under emergency conditions (that is, double or 150 000 kw load at 85 percent power-factor) 157 000 kv-a phase modifier capacity will be required if 230 kv is not to be exceeded at the sending end. If the generator can be operated during the emergency condition at increased voltage of, for instance, 240 kv, the phase modifier capacity could be reduced to approximately 140 000 kv-a. However, too much liberty in variation of generator operating voltage should not be taken. If the voltage is held constant at the high-voltage side of the raising transformers, the generator operating voltage will have to be varied to compensate for the regulation of the sending end transformers, and to provide a still greater range in generator operating voltage might impose a hardship on the generator designers. The voltage drop through the transformers is small under load conditions, since the power-factor will be near unity, but under zero load condition the drop will be considerable, due to the low power-factor, especially if a large phase modifier load is required at zero load. It will be seen that it is the emergency condition which determines the total capacity of phase modifiers, for the 220 kv problem. For instance at normal load, 43 000 kv-a in capacity is required, whereas for the double or emergency load 157 000 kv-a capacity (nearly four times) is required This large increase is due to the fact that the line charging current (which tends to reduce phase modifier capacity under load) has not changed, and that the line impedance volts has become twice as much, making it necessary to turn the line impedance triangle through a large angle in the counter-clockwise direction in order that the sending end voltage be not increased.

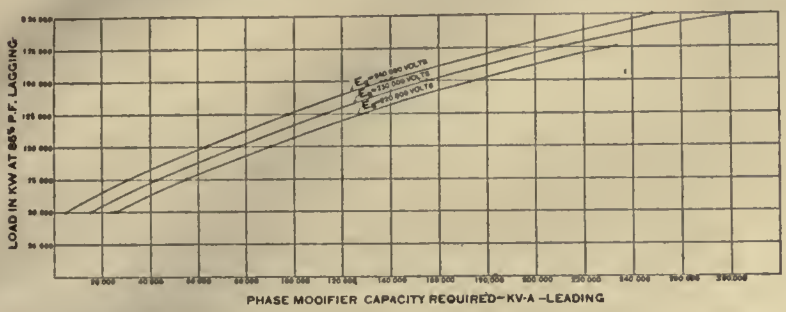


FIG. 72—PHASE MODIFIER CAPACITY REQUIRED FOR THE VARIOUS LOADS

These curves are plotted from values read from the curves of Fig. 70 and are on the basis of a constant load voltage of 220 kv.

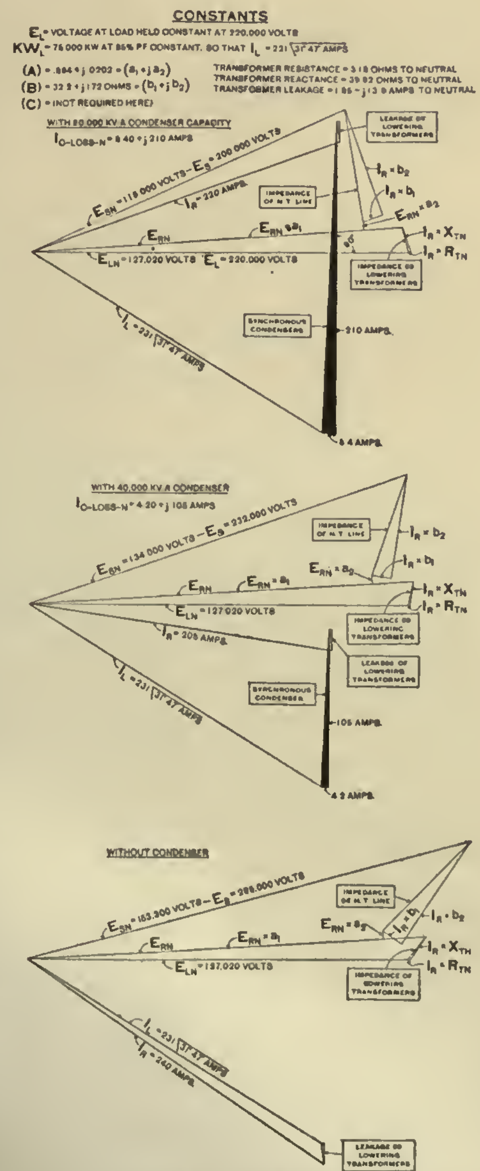


FIG. 71—GRAPHIC METHOD FOR DETERMINING THE VOLTAGE AT THE SENDING END.

Corresponding to different condenser loads in parallel with a constant power load of 75 000 kw at 85 percent power-factor and 220 kv. The results as plotted in Fig. 70 were obtained by similar constructions.

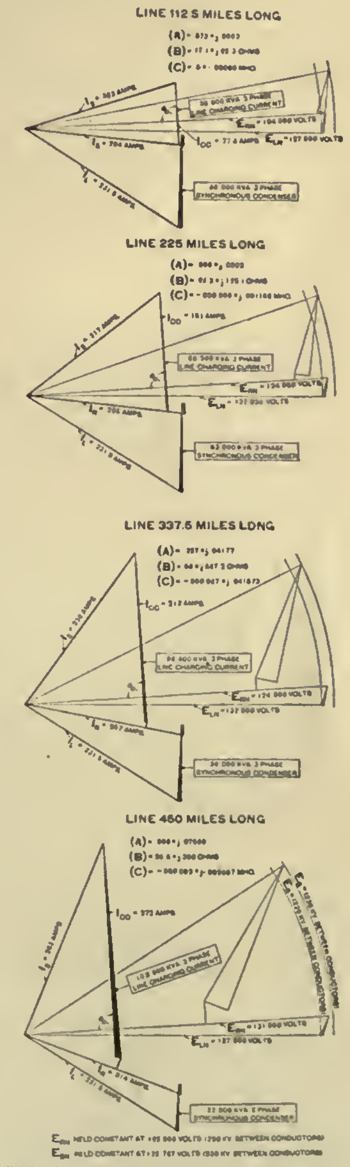


FIG. 73—VECTOR DIAGRAMS SHOWING THE EFFECT OF THE LENGTH OF THE LINE ON THE PHASE MODIFIER CAPACITY REQUIRED

The diagrams represent a three-phase, 60 cycle circuit, consisting of three 605 000 circ. mil aluminum steel reinforced conductors, when delivering 75 000 kw at 85 percent lagging power-factor at a load voltage of 220 kv with a sending end voltage of 230 kv.

The zero load curve on Fig. 70 is drawn for the normal load connection; that is, for two 50 000 kv-a transformer banks in parallel. For the emergency load four transformer banks in parallel will be required. The result of the increased magnetizing current consumed by four in place of two transformer banks will be to reduce the capacity of phase modifiers required under zero load. A second zero load line could be added, covering four transformer banks. Such a line would lie directly above the one for two transformer banks but would not materially affect the results. For load conditions of 100 000 kw at 85 percent power-factor and above, the points for the curves were determined on the basis of four transformer banks.

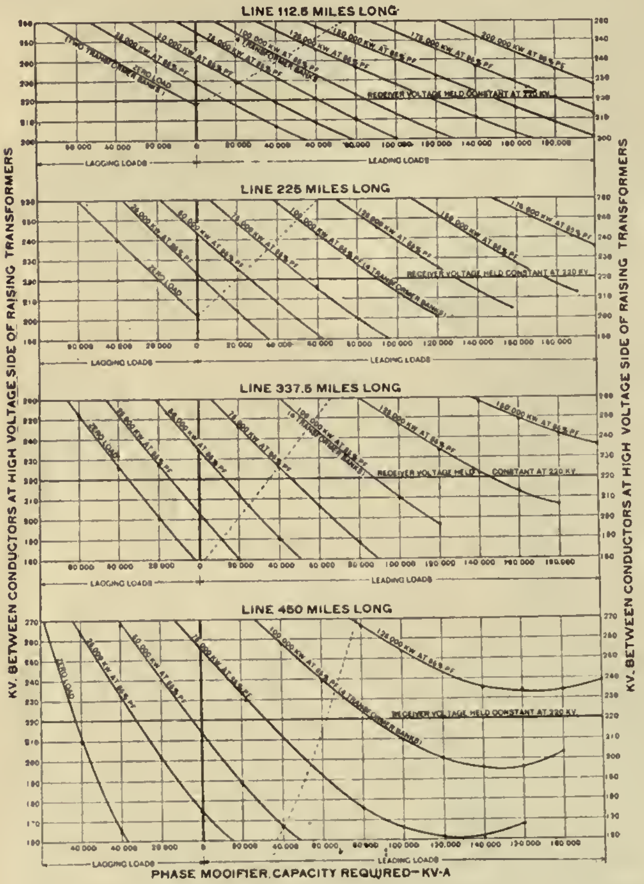


FIG. 74—CURVE SHOWING THE RELATION BETWEEN PHASE MODIFIER CAPACITY AND SENDING END VOLTAGE

For various receiving end loads of 85 percent lagging power-factor and a constant load voltage of 220 kv. These curves apply to a three phase, 60 cycle circuit consisting of three 605 000 circ. mil aluminum steel conductors. The vector construction of these four lines is shown in Fig. 73.

In the above it was assumed that the power-factor of the load would be 85 percent lagging. A long line such as this would probably feed into an extended distribution net work, having numerous load centers. At these load centers synchronous condensers would probably be located for the purpose of holding the voltage constant. This would necessitate operating the condenser leading at heavy loads thus raising the power-factor of the entire system under load, and in effect reducing the capacity of phase modifiers required for voltage control at the receiving end of the line. This point should be investigated where a long line such as this feeds a net work on which condensers are required for voltage control.

It may be desired to investigate the effect of line charging current on phase modifier capacity for lines of different lengths. For this purpose the vector diagrams Fig. 73, and the phase modifier curves, Fig. 74, were prepared. These vector diagrams and curves are based upon a constant load of 75 000 kw at 85 percent power-factor delivered at 220 kv and a line drop of 10 kv. In other words the only variable for the four different lines is the length and this varies in equal increments.

The vector diagrams of Fig. 73 show the influence of line charging current upon condenser capacity. As the length of the line increases, the influence of the increased line charging current is toward a reduction in condenser capacity; that is the line itself furnishes a large part of the leading current necessary to maintain the proper line voltage drop. If this line were longer than 450 miles, the line charging current at a certain length would be sufficient in itself to maintain the desired voltages at the two ends of the line without the aid of condensers. In such a case, however, a large reactor capacity would be required at zero and low loads to hold the receiving end voltage at a constant value.

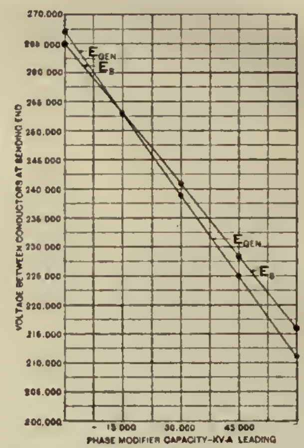


FIG. 75—CURVES SHOWING THE VOLTAGE ON EACH SIDE OF THE RAISING TRANSFORMERS

Corresponding to condenser loads of various capacities in parallel with a constant load of 75 000 kw at 85 percent power factor lagging and 220 kv. The vertical distance between the two voltage lines is the voltage drop or voltage rise through the raising transformers. For condenser loads up to 15 000 kv-a there is a drop in voltage through the raising transformers. For condenser loads above 15 000 kv-a there is a rise in voltage through the raising transformers.

The reason that a short line may necessitate more condenser capacities for voltage control than a long line is simple. For the 112.5 mile line the charging current will be about one half as much as for a 225 mile line. Since the line is only half as long this smaller charging curent will flow through only half the inductance so that the net result of half the line charging current and half the inductance will be about one fourth the voltage boosting effect due to line charging current. On the other hand the line impedance will be only half as great, but the net result will be more condenser capacity for the short line. A large part of the condenser capacity is required for neutralizing the lagging reactor component of the load.

Auxiliary constant *A*, as previously explained, accounts for the effect of the line charging curent flowing through the impedance of the circuit; that is, the voltage boosting effect of the charging current. Thus for the 112.5 mile line (Fig. 73) a_1 which accounts for the line charging current flowing through the inductance of the circuit is near unity and a_2 near zero, but for the 450 mile line a_1 drops to 0.594 and a_2 increases to 0.07508. As the length of line increases, constant *A* moves the line impedance triangle to the left and raises its toe somewhat. The increased line impedance and slightly increased current at the receiving end increases the size of the line impedance triangle.

The curves of Fig. 74 show the relation between phase modifier capacity and sending end voltage for different receiving end loads of 85 percent lagging power-factor and a constant load voltage of 220 kv. It is interesting to note the effect of distance for fixed size conductors upon the maximum amount of power which can be transmitted over a circuit, as evidenced by the load curves bending upward as the line length increase. It is also interesting to note the decrease in phase modifiers leading capacity and increase in phase modifier lagging capacity as the line becomes larger, as evidenced of the load curves shifting to the right. The curves, Fig. 75, show the voltage at each side of the raising transformer, corresponding to various condenser capacities in parallel with a constant load of 75 000 kw at 85 percent lagging power-factor and 220 kw.

H. B. DWIGHT'S METHOD.

In the various methods for determining the performance of transmission lines which are described above, current and voltage vectors or corresponding vector quantities have been employed throughout. It was believed that solutions embodying the use of current and voltage vectors would be the more easily followed by the young engineer, for the assistance of whom this book has been primarily written.

H. B. Dwight worked out and published in book form formulas for determining the complete performance of circuits by the employment of quantities not generally employed in the methods described above. These quantities require a new set of symbols applicable to his method. Partly to prevent confusion in symbols but principally because his method has been so completely and clearly set forth and illustrated with numerous examples worked out in the two books referred to his method has not been detailed in this book. To include it here would simply be a duplication of what is already available in very complete form.

THE CIRCLE DIAGRAM

Various forms of circle diagrams as an aid in determining the performance of *short* transmission lines have been frequently described by writers, notably by R. A. Philip 'thru the medium of the A. I. E. E. transactions of February 1911. Following this H. B. Dwight worked out a solution and construction for a circle diagram which accurately takes into account the effect of capacitance in transmission lines that is, a circle diagram for *long* high voltage lines. This circle diagram consists of curves which indicate the phase modifier capacity (leading or lagging) required to maintain a certain reciving end voltage corresponding to all values of delivered load up to the maximum capacity of the line. In other words it gives data such as is given by the curves of Fig. No. 70.

The next step in the development of the circle diagram was to so alter the constants upon which it is constructed that it will take accurately into account the localized impedance and loss in raising or lowering transformers or in both. Of course the transformer impedance may be added to the line impedance as is frequently done and considered as distributed line impedance. Such procedure, will, however, in the case of the circle diagram for the line alone result in objectionable errors in the results. In order to correctly apply the circle diagram to *long* lines so as to accurately *include* the effect of transformers in the circuit it is necessary to develop new formulas for obtaining values for the constants by which the circle diagram is constructed. See articles on transmission line constants by R. D. Evans and H. K. Sels in the Electric Journal, page 306 July 1921, page 356 August 1921 and page 530 December 1921.

To the expert who spends much time investigating transmission problems the general use of the circle diagram should be of great assistance. It indicates performance at all loads, which with other methods would have to be obtained by a separate calculation or vector diagram construction for each load.

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*Transmission Line Formulas, 1913, D. Van Nostrand Co., New York City and Constant-voltage Transmission, 1915, John Wiley & Sons Inc., New York City.

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