ELECTRIC LIGHTING

AND MISCELLANEOUS APPLICATIONS
OF ELECTRICITY

·The MM Co·

ELECTRIC LIGHTING

AND MISCELLANEOUS APPLICATIONS OF ELECTRICITY

A TEXT BOOK FOR
TECHNICAL SCHOOLS AND COLLEGES

BY

WILLIAM SUDDARDS FRANKLIN

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., Ltd.
1912

Set up and electrotyped. Published May, 1912

PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.

PREFACE.

This book is intended to be a companion volume to *Dynamos and Motors*.

In the preparation of these two volumes especial attention has been given to the physical principles which underlie operating engineering, and but little attention has been given to the principles of design.

April 22, 1912.

TABLE OF CONTENTS.

IMPORTANT INDEXES AND TABLES AND SPECIAL PUBLICATIONS.

Engineering Index Annual, published by The Engineering Magazine, London.

Science Abstracts, published by E. & F. N. Spon, London, Series *A*, Physics; Series *B*, Electrical Engineering.

Science Abstracts is issued by the *Institution of Electrical Engineers* of Great Britain in association with the *Physical Society* of London, and with the coöperation of the *American Physical Society*, the *American Institute of Electrical Engineers*, and the *Associazione Elettrotecnica Italiana.*

Physico-Chemical Tables, by John Castell-Evans, Chas. Griffen & Co., London.

Physikalisch-Chemische Tabellen, by Landolt & Börnstein, Julius Springer, Berlin.

Manufacturers' Bulletins and Circulars contain a great deal of important information.

ELECTRIC LIGHTING.

CHAPTER I.

INSTALLATION AND OPERATION COSTS AND THE SELLING PRICE OF ELECTRICAL ENERGY.

1. General statement.—From one point of view engineering is a composite of all the physical sciences, and from another point of view it is a branch of the science of economics. Any elementary treatise on engineering should, however, be chiefly devoted to purely physical problems inasmuch as the student must become familiar with engineering as a branch of physical science before he can possibly undertake as a practising engineer to choose that particular physical solution of an engineering problem which will best meet the requirements of economy.

The economic problem is in every case to produce satisfactory results at a minimum of ultimate cost, and this is always a very complicated problem inasmuch as the cost of erection and maintenance of engineering works and the value of the service rendered thereby are both dependent upon minute variations of local conditions, they both fluctuate from year to year with the varying stress of business activity and they both change with every improvement in industrial processes.

The first estimate of the cost of any engineering undertaking is usually based upon general statistics of the cost of similar undertakings, and the final cost is determined by the bids of contractors who have had some experience in the particular line of work and whose margin of profit must in general be great enough to cover the many uncertain items that always appear in any new undertaking.

The following data on the cost of installing and operating steam and electric plants are averages based upon the record of a large number of actual cases, and the cost of a given station may depart considerably from these figures on account of peculiar local and temporary conditions.

THE STUDENT IS WARNED THAT AVERAGES ARE ALWAYS MISLEADING IN COMPLICATED MATTERS LIKE COSTS, BECAUSE TO TAKE AN AVERAGE IS TO ELIMINATE EVERY ELEMENT OF DIFFERENCE BETWEEN INDIVIDUAL CASES, AND THESE DIFFERENCES ARE VERY GREAT.* THE GENERAL ESTIMATES WHICH ARE GIVEN IN THIS CHAPTER WILL BE WORSE THAN USELESS IF THE ONE WHO USES THEM DOES NOT CONSIDER THE PECULIAR FEATURES OF THE PARTICULAR CASE UNDER CONSIDERATION.

2. **Cost of steam power.**†—The ordinates of the curve A in

* In this connection the student should read R. S. Hale's article "The Real Theory of Real Rates" in the *General Electric Review* for April, 1911, Vol. XIV, pages 157–169.

† The data from which the curves in Figs. 1 and 2 are plotted are taken from a paper by William O. Weber, *The Engineer* (Cleveland), Vol. XI, page 145, February 2, 1903.

Important data on the cost of steam power and on the cost of electrical equipment are given by C. E. Emery, *Transactions American Institute of Electrical Engineers*, Vol. XII, pages 358–389, 1895; by H. A. Foster, *Transactions American Institute of Electrical Engineers*, Vol. XIV, pages 385–421, 1897; and by R. C. Carpenter, *Electrical World*, Vol. XLIII, page 1016, May 28, 1904.

An important paper on the cost of power in very large plants is given by H. G. Stott, *Transactions American Institute of Electrical Engineers*, Vol. XXVIII, pages 1479–1502, December, 1909. This paper gives data as to cost of plants and as to fuel and operation costs, and it includes plants driven by reciprocating steam engines, by steam turbines and by gas engines.

The cost of power in industrial plants where exhaust steam may be used for heating is discussed by John C. Parker, *Proceedings American Institute of Electrical Engineers*, March, 1911, pages 467–483; and by A. E. Hibner, *Proceedings American Institute of Electrical Engineers*, March, 1911, pages 485–503.

The economics of a small producer-gas plant are discussed by F. C. Tyron, *Power* (New York), Vol. XXVIII, pages 619–620, April 21, 1908. The economics of a suction-gas plant are discussed by P. W. Robinson, *Electrician* (London), Vol. LXI, pages 898–901, September 25, 1908. A description of the equipment of a gas-engine electric plant together with data as to operation costs of same is given by E. B. Latta, Jr., *Proceedings American Institute of Electrical Engineers*, Vol. XXIX, pages 475–507, April, 1910.

Fig. 1 give the approximate total cost per horse-power capacity, of boiler and engine plants of various sizes, including buildings, chimneys and all accessories, and the ordinates of curve *B* give the approximate coal consumption in pounds per horse-power-hour.*

The ordinates of the curves *C*, *D*, *E*, *F* and *G* in Fig. 2 give the costs per horse-power-year (308 days of 10 hours each) of the following items:

Curve *C*. Fixed charges per horse-power-year. This item is reckoned at 14 per cent. of the total investment, and it includes interest at six per cent., depreciation† at 4 per cent., repairs at 2 per cent., insurance and taxes at two per cent.

Curve *D*. Cost of coal per horse-power-year at $4.00 per ton of 2,000 pounds.

Curve *E*. Wages per horse-power-year.

Curve *F*. Cost of oil and supplies per horse-power-year.

Curve *G*. Total cost per horse-power-year.

These curves are based on the assumption that engines of the

The cost of water power is discussed in the *Mechanical Engineer* (Manchester, England), Vol. XII, page 694, November 21, 1903. The comparative cost of steam and water power is discussed by H. von Schon, *Engineering Magazine* (London), Vol. XXX, pages 35–48, 184–194, 353–380, and 611–622, May to July, 1907. See also W. O. Webber, *Engineering Magazine*, Vol. XXX, pages 889–893, September, 1907. A paper on hydro-electric plants by H. L. Doherty together with a very complete discussion of the subject is given in *Transactions American Institute of Electrical Engineers*, Vol, XXVIII. pages 1361–1478, December, 1909.

* The coal consumption in a large steam-turbine power plant is considerably less than the coal consumption given by the ordinates of curve *B* in Fig. 1, and the coal consumption in a gas-engine plant is less than the coal consumption in a steam-turbine plant. See paper by H. G. Stott, *Transactions American Institute of Electrical Engineers*, Vol. XXVIII, pages 1479–1502, December, 1909; and a paper by H. G. Stott and J. S. Pigott, *Proceedings American Institute of Electrical Engineers*, Vol. XXIX, pages 1455–1501, September, 1910.

† Depreciation is a very uncertain item. The estimate for the whole plant of 4 per cent. per year is perhaps too low. The depreciation of buildings is less than 4 per cent. and the depreciation of electrical machinery is perhaps more than 4 per cent., but the depreciation of boilers and engines is certainly considerably greater than 4 per cent. per year.

See paper by Henry Floy for a detailed discussion of depreciation: *Proceedings American Institute of Electrical Engineers*, Vol. XXX, pages 1143–1185, June, 1911.

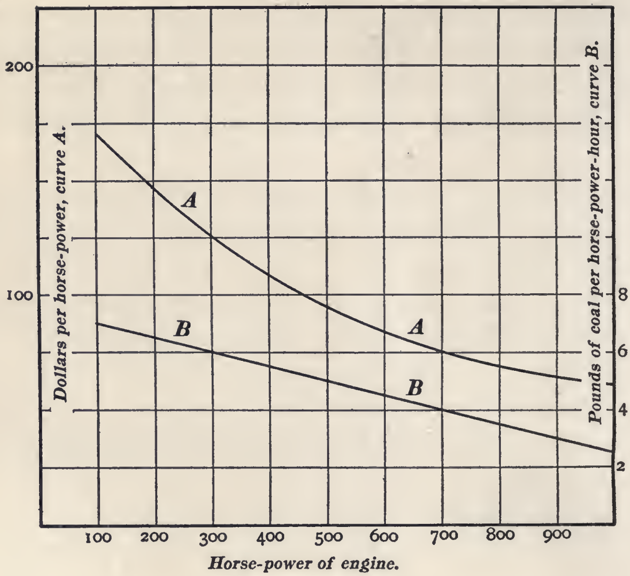

200
100
Dollars per horse-power, curve A.
Pounds of coal per horse-power-hour, curve B.
8
6
4
2
A
A
B
B
100
200
300
400
500
600
700
800
900
Horse-power of engine.

Fig. 1.

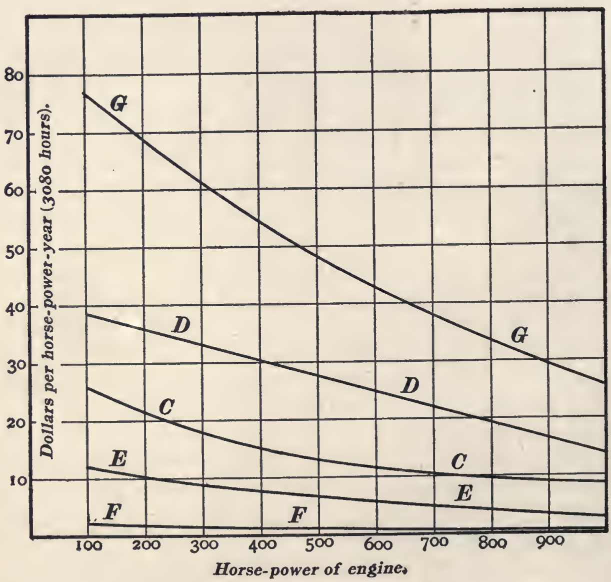

80
70
60
50
40
30
20
10
Dollars per horse-power-year (3080 hours).
G
G
D
D
C
C
E
E
F
F
100
200
300
400
500
600
700
800
900
Horse-power of engine.

Fig. 2.

smaller sizes are simple non-condensing engines of the Corliss type, that engines of the medium and larger sizes are simple condensing engines, and that the engines of the largest sizes are compound condensing engines.

The wages in a 100-horse-power plant are taken to be as follows: An engineer at $2.50 per day and a fireman at $1.50 per day, which would amount to about $1,200 per year. In a 1,000-horse-power plant the wages are taken to be as follows: One engineer at $3.00 per day, one assistant engineer at $2.00 per day and three men in the boiler room during the day and one night watchman at $1.50 per day each, and the total amount would be about $3,500 per year.

The cost of coal per horse-power-year at any given price per ton may be easily determined from curve *D*, Fig. 2, the ordinates of which represent the cost of coal* per horse-power-year at $4.00 per ton, and the total cost per horse-power-year, as given by curve *G*, may be corrected accordingly.

It is important to remember that the cost of power as given by curve *G*, Fig. 2, is based on the assumption that the engines are working at full load for 10 hours per day, 308 days in the year. When a plant operates at a fraction of its full-load capacity the cost per horse-power-hour is increased as explained in Art. 4.

3. Cost of electrical power at the switch board.—The ordinates of the curve *H*, Fig. 3, give the capacities in kilowatts of electrical equipments corresponding to various horse-power capacities of engine plants. This curve is not exactly a straight line inasmuch as the efficiency of a large electric generator is greater than the efficiency of a small electric generator of the same design.

The following estimate of the cost of electrical power at the switch board is based upon the cost of steam power as given in Figs. 1 and 2 and upon the assumption that the plant operates at full load, 10 hours per day for 308 days per year. The ap-

* A good quality of coal giving 14,500 B.T.U. per pound is here referred to.

proximate cost of the electrical part* of the equipment in dollars per kilowatt of station capacity may be obtained by dividing the ordinate of curve *J*, Fig. 4, by 0.14. This cost ranges from about $45.00 per kilowatt for a 100-kilowatt station to about $35.00 per kilowatt for a 700-kilowatt station. The cost of

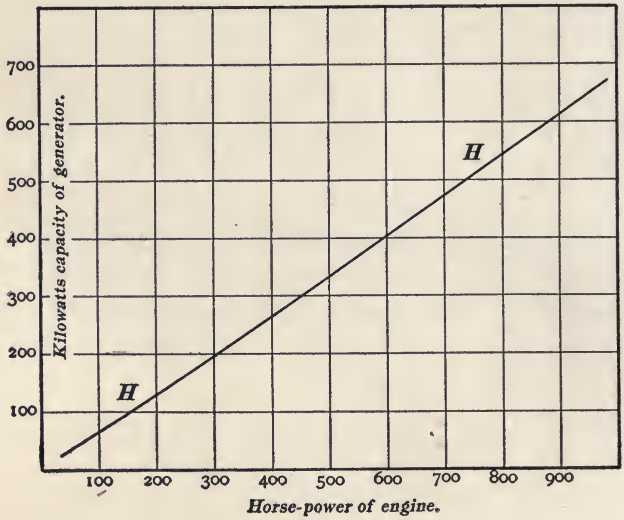


Fig. 3.

additional labor required by the electrical part of the plant is about 25 per cent. of the cost of labor for the steam plant alone, and the additional cost of oil and supplies is about 40 per cent. of the cost of this item for the steam plant alone.

The curves in Fig. 4 show an approximate analysis of the cost of electrical power as follows:

Curve *I* shows the cost of steam power per kilowatt-year of electrical output (determined from curve *G* of Fig. 2, using Fig. 3).

Curve *J* shows the fixed charges on the electrical equipment per kilowatt-year. This item is reckoned at 14 per cent. of the cost of the electrical equipment, and it includes interest at 6

* Not including the distributing system. See Art. 5.

per cent., depreciation 4 per cent., repairs 2 per cent., insurance 1 per cent., and taxes 1 per cent.

Curve *K* shows the cost per kilowatt-year for attendance (labor) of electrical equipment in addition to attendance of steam plant.

Curve *L* shows the cost per kilowatt-year of oil and supplies for electrical equipment.

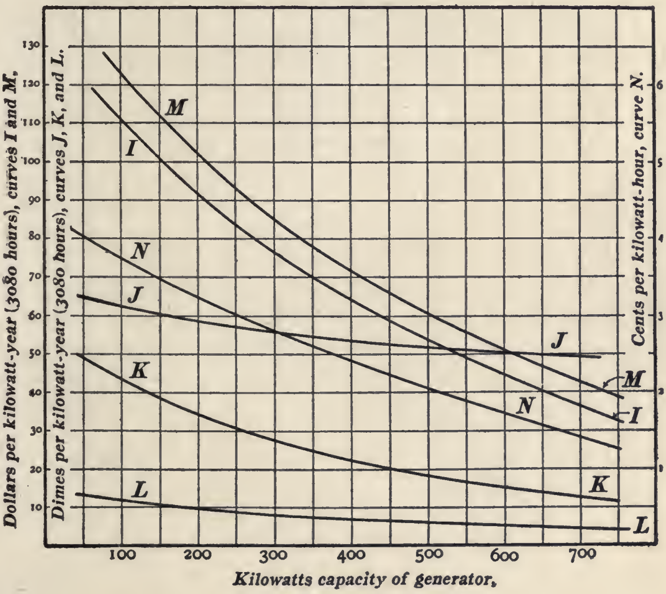


Fig. 4.

Curve *M* shows total cost per kilowatt-year of electrical output.

Curve *N* shows the total cost per kilowatt-hour of electrical output.

4. **Load factor and its influence on the cost of power.***—The discussion of cost of power which is given in the foregoing articles

* A very complete table of costs of electric power at the switchboard showing a wide range of load factors is given by R. W. Conant, *Electrical World*, Vol. XXXII, pages 313–319, September 24, 1898. See also *Street Railway Journal*, Vol. XVIII, page 827, December 7, 1901, and Vol. XXV, pages 126–128, January 21, 1905; and *Street Railway Review*, Vol. XIII, pages 185–198, April, 1903.

is based upon the assumption of continuous operation of a power plant at full load. As a matter of fact, however, the demand for power always fluctuates greatly so that a power plant which is designed to deliver the maximum amount of power demanded must be operated for a large portion of the time at a fraction of its rated capacity.

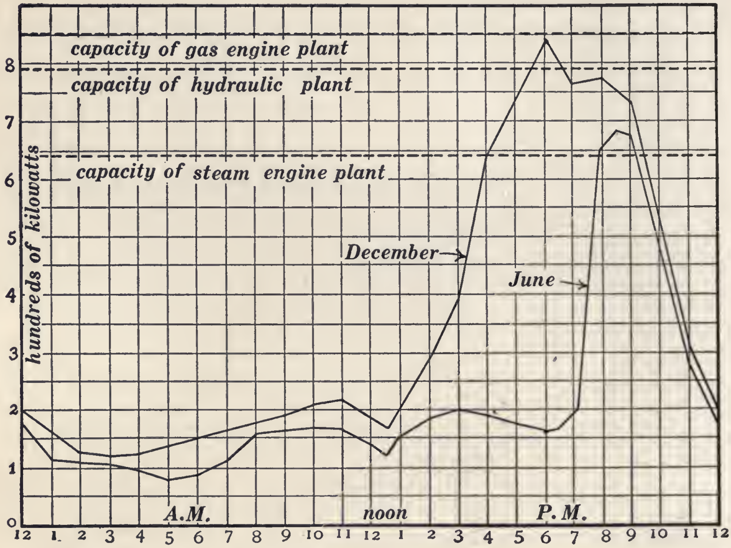


Fig. 5. Winter and summer load curves of lighting plant.

The average load on a plant (in kilowatts) divided by the capacity of the plant in kilowatts is called the *load factor*. For example, a 465-kilowatt plant* operating 14 hours per day 365 days per year delivers a total of 464,000 kilowatt-hours, so that the average load is $\frac{464,000}{14 \times 365} = 90.8$ kilowatts, and the load

The effect of load factor on the cost of power is discussed by W. M. Archibald, *Electrical World*, Vol. XLV, pages 303–305, February, 1905.

A paper by H. G. Stott, *Transactions American Institute of Electrical Engineers*, Vol. XXVIII, pages 1479–1502, December, 1909, takes account of the influence of load factor on the cost of power. The curves given in Fig. 8 are taken from this paper.

* See page 32.

factor is $\frac{90.8}{465}$ or 0.195. In specifying a load factor it is important to give also the average daily run in hours.

Figure 5 shows typical summer and winter load-curves on a power plant supplying power for lighting; Fig. 6 shows a typical load-curve on a power plant supplying power for the operation

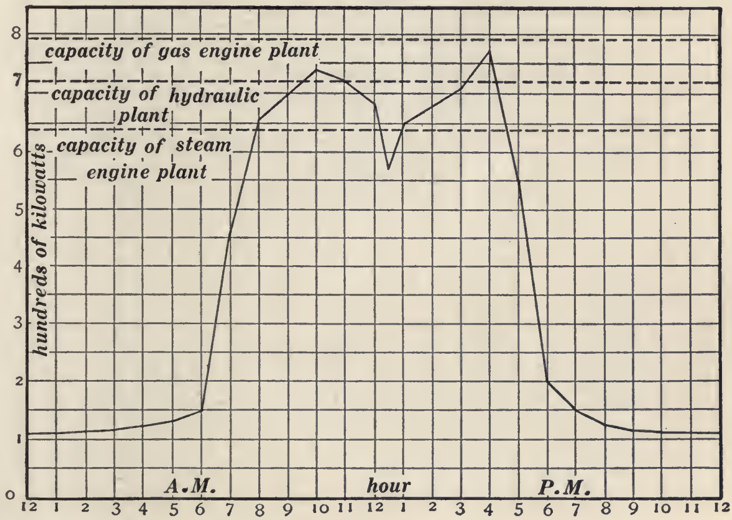


Fig. 6. Load curve of plant supplying motors in manufacturing district.

of motors in a manufacturing district; and Fig. 7 shows typical load-curves on a power plant supplying power to a large city electric railway system.

The horizontal dotted lines in Figs. 5, 6 and 7 show suitable power ratings of plants for the respective load curves. A properly designed steam plant has a very large overload capacity, a hydraulic plant has a small overload capacity, and a gas-engine plant has practically no overload capacity. Therefore the "peak of the load" (maximum load) may be 25 or 30 per cent. in excess of the rated capacity of a steam plant, not more than 5 or 10 per cent. in excess of the rated capacity of a hydraulic plant, and not at all in excess of the rated capacity of a gas-engine plant. A further consideration which bears upon the proper rating of a power plant is the probability of excessive demand for power on special occasions. Thus an excessive demand for power is likely to occur in a street railway system, and it is for this reason that the rated plant capacities in Fig. 7 are

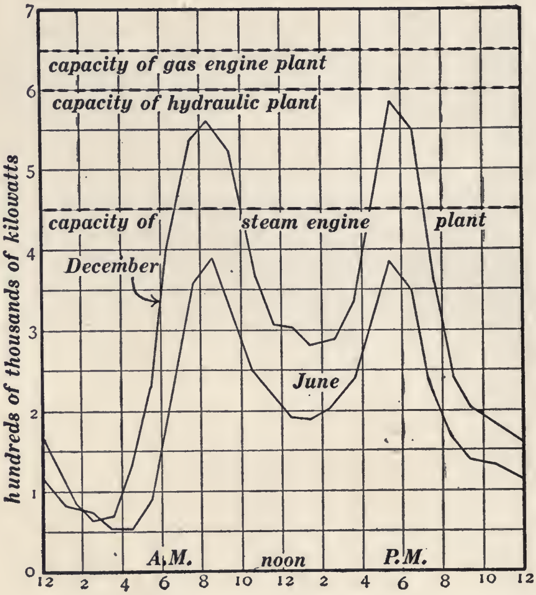


Fig. 7. Winter and summer load curves of electric railway plant.

larger as compared with the normal peak of the load than in Figs. 5 and 6.

The load factor on an ordinary electric lighting station may be as low as 0.15 for a small plant in a small town where there is but little lighting during the day. If the plant supplies current for motors during the day to any considerable extent the load factor may be as large as 0.40. The load factor of a plant which supplies current for electric railways may be as low as 0.20 or 0.30 when there are but few cars in operation, or as high as 0.40 or 0.50 when many cars are operated. When a single large power plant supplies power for lighting, for manufacturing and for street railways, the combined demand for power is much more nearly uniform than for either kind of service alone, and the load factor in this case is sometimes as large as 0.60 or even larger.

The influence of load factor on the cost of power is illustrated by the following example: Consider a 1,000 horse-power electric light station representing a total investment of $100,000, not including the distributing system. The fixed charge (interest, depreciation, repairs, insurance and taxes) is, say, 14 per cent. on the total investment, or $14,000 per year. If the station were run at full load day and night the year round, the running expenses would be approximately as follows: (*a*) wages of one chief engineer and superintendent at $5.00 per day, two assistant engineers at $2.50 per day, three helpers in the engine room at $1.50 per day and six men in the boiler room at $1.50 per day, making a total of $8,600 per year; (*b*) coal at 2.5 pounds per horse-power-hour at $4.00 per ton would amount to about $43,700 per year; and (*c*) petty stores would amount to about $6,500 per year. The total annual expense of $72,800 would be the cost of approximately 6,000,000 kilowatt-hours, which would be at the rate of 1.22 cents per kilowatt-hour delivered at the switchboard.

If the station were run night and day the year around at an average load equal to 0.25 of its full-load capacity, the running expenses would be approximately as follows: (*a*) wages, one engine room helper and three firemen less than before, would amount to $5,100 per year; (*b*) the coal consumption of a 1,000-horse-power engine running at one quarter load would be about 6 pounds per horse-power-hour, therefore, the cost of coal would be about $26,200 per year; and (*c*) petty stores would be about $5,500 per year. The total annual expense of $51,400 would be the cost of 1,500,000 kilowatt-hours which would be at the rate of 3.43 cents per kilowatt-hour delivered at the switchboard.

This example illustrates the great importance of load factor as a condition affecting the cost of power. Certain items of expense are nearly constant whatever the load on the station may be. The sum of these items (interest, depreciation, taxes and administrative expenses) is called the *fixed charge*. When a station is operated at light load the item of wages may be somewhat reduced especially if the station is shut down during certain hours each day, also the item of fuel is reduced; but neither is reduced in proportion to the reduction of load. Therefore the cost per kilowatt-hour increases greatly with decrease of load on the station. The following table shows the increasing cost

TABLE SHOWING EFFECT OF LOAD FACTOR ON COST OF POWER.

Costs per kilowatt-hour in cents * (150-kilowatt steam plant).

Load-factor.	0.2.	0.4.	0.6.	0.8.
Interest on investment	1.70	0.85	0.57	0.42
Depreciation	1.20	0.60	0.44	0.33
Management and taxes	1.50	0.75	0.50	0.40
Repairs	1.00	0.55	0.40	0.30
Fuel	1.50	1.40	1.30	1.20
Labor	1.50	1.20	1.00	0.90
Petty stores	0.20	0.15	0.10	0.08
Total cost per kilowatt-hour in cents.	8.60	5.56	4.31	3.63

per kilowatt-hour (delivered to the consumer, see Art. 5) with decrease of load factor as estimated by B. J. Arnold.† These figures refer to a power station having a full-load capacity of about 150 kilowatts. Figure 8 shows the influence of load factor (per cent. load) on the cost of electrical energy as estimated by H. G. Stott ‡ for a large steam-turbine plant costing $75 per kilowatt of rated capacity. The ordinates of curve *A* show the cost of coal [high grade coal (14,500 B.T.U. per pound) at $3.00 per ton] and water per kilowatt-hour, the ordinates of curve *B* show the total operation cost (coal and water, boiler and engine room labor, coal and ash handling, oil and engine room supplies)

* Costs in this table include interest, depreciation, repairs, etc., on the distributing system. See Art. 5.

† *Electrical World*, Vol. XXIV, pages 104–107 and page 120, August 4 and 11, 1894.

‡ *Transactions American Institute of Electrical Engineers*, Vol. XXVIII, page 1482, December, 1909.

per kilowatt-hour, the ordinates of curve *C* show the fixed charge (interest at 5 per cent., taxes and administrative expenses 1 per cent., and depreciation 5 per cent.) per kilowatt-hour, and the ordinates of curve *D* show the total cost per kilowatt-hour. Thus at 100 per cent. load factor the cost is

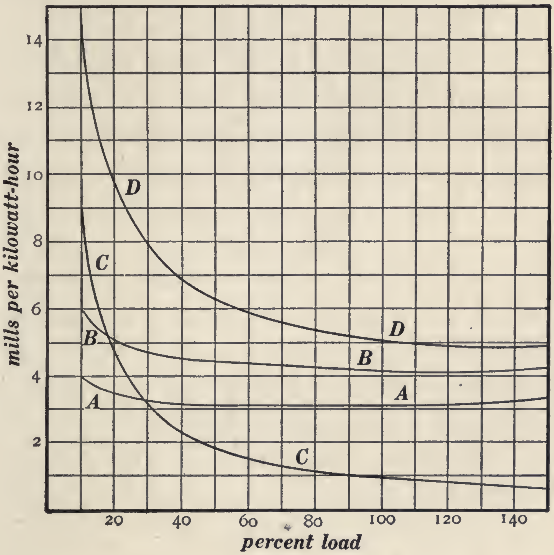


Fig. 8.

half-a-cent per kilowatt-hour, and at 20 per cent. load factor the cost is 0.97 cent per kilowatt-hour at the switchboard.

Multiplying the tabulated costs in the above table by the corresponding values of the load factor, one can see the extent to which the various items of total cost change with the load on the station, as estimated by B. J. Arnold. Similarly the ordinates of the various curves in Fig. 8 may be multiplied by "per cent. load" to give the various items of total cost at different station loads as estimated by H. G. Stott.

5. Cost of electrical energy delivered to the consumer.—The foregoing articles discuss the cost of electrical energy at the switchboard. The cost of electrical energy delivered to the consumer is greater than the cost at the switchboard by an amount sufficient to cover the following items: (*a*) The interest on the cost, and depreciation of the distributing system; (*b*) the energy lost in the distributing system; (*c*) the interest on the cost of meters, and depreciation and repair of the meters; and (*d*) the cost of reading meters, making out bills and collecting same.

The cost of the distributing system varies so greatly with local conditions that it is impossible to give any general estimate. The cost is especially great where the consumers are scattered over a large district, or where costly underground distributing cables are used; and the cost varies greatly with the system of distribution employed. Thus high-voltage alternating-current distribution with step-down transformation is much cheaper than low-voltage direct-current distribution when the consumers are widely scattered.

Complete statistics of a small municipal electric lighting plant are given in Art. 15, and some idea of the difference between cost of electrical energy at switchboard and cost delivered to the consumer may be obtained by comparing these statistics with the cost statistics which are given in Art. 3.

6. Customer's load factors. The diversity factor.*—The average power delivered to a customer divided by his maximum demand is called his *load factor*. Thus the general average of 24,177 small living apartments in Chicago was 18.34 kilowatt-hours monthly consumption (representing an average continuous consumption of 25.5 watts per customer) and an average maximum demand of 370 watts per customer, so that the average load factor of these customers was 0.069.

If the load factor of an electric lighting station were as low as

* See a paper by H. G. Gear, *Transactions American Institute of Electrical Engineers*, Vol. XXIX, pages 375–384, March 1910. The examples of customer's load factors which are given in the text are taken from this paper.

See also a paper which was read by Mr. E. W. Lloyd before the Atlantic City Convention of the National Electric Light Association, June, 1909. The curves of Fig. 9 are taken from this paper.

0.069 the cost of electric lighting would be almost prohibitive, but the load factor of a station is always very much larger than the average load factor of the customers, because the maximum demands of the customers never come at the same time. This diversity among the customers is a very important element in the cost of electric lighting.

The combined actual maximum demand of a group of customers divided by the sum of their individual maximum demands is called their *diversity factor*. The diversity factor is here defined with respect to a group of customers but it can also be applied to a group of lamps or a group of feeders as shown in the following examples.

Diversity factor of a customer's group of lamps.—A customer has fifty 50-watt lamps and of course the sum of the individual maximum demands of the lamps is 2,500 watts ("connected load"). The customer's maximum demand, however, is 1.5 kilowatts. Therefore the diversity factor* of the customer's group of lamps is 0.60. The ordinates of the curves in Fig. 9

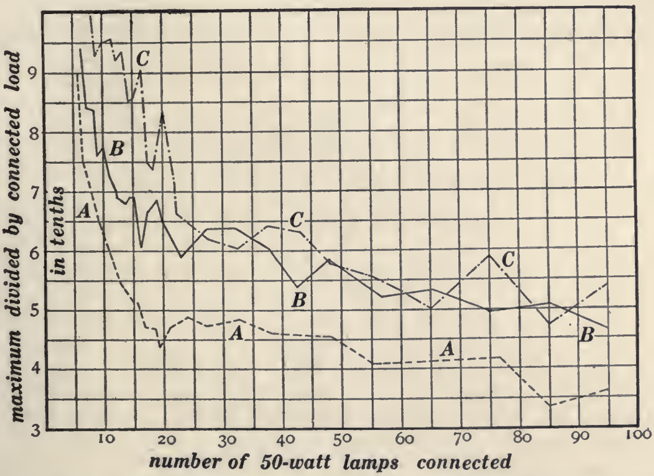


Fig. 9. Curve *A* average of 30,729 residences. Curve *B* average of 5,392 offices. Curve *C* average of 9,149 small stores.

* The diversity factor of a customer's group of lamps, namely, the ratio of maximum demand to connected load is usually called the *demand factor* of the customer.

show the ratio *maximum demand to connected load* for various kinds of electric lighting service in Chicago.

Diversity factor of a group of customers.—The sum of the individual maximum demands of the various customers in a densely populated residence block in Chicago was 63 kilowatts, whereas the maximum demand upon the service transformer which supplies the block was 18 kilowatts. Therefore the diversity factor of the group of customers was 0.286.

Diversity factor of a number of feeders.—The actual maximum demand upon a particular substation in Chicago was 1,000 kilowatts, and the sum of the maximum loads on the 10 sets of feeders which go out from the substation was 1,200 kilowatts. The diversity factor of the group of feeders was therefore 0.833.

7. Analysis of costs of electric service.*—There are three fairly distinct items of cost of a customer to a central station, and any equitable schedule of rates for electric service should be based on a careful consideration of these costs which are as follows:

(*a*) *Connection cost.*—A certain service is rendered to a customer in that power from a central station is available for his use at all times even if he does not actually make use of it. This service of the central station is usually called "readiness to serve" and its cost to the central station includes a small part of the total annual expense of the entire plant and distributing system. It includes a large part of the interest on the cost of the customer's wiring connections and meter; and it includes a large part of the cost of maintaining and reading

* This matter is discussed at some length in the following papers: "Central Station Charging Systems in Use in the United States," *Electrical World and Engineer*, Vol. XL, pages 361–366, September 6, 1902. "Methods of Charging for Electrical Energy," E. H. Crapper, *Electrician* (London), Vol. LII, pages 330–332, December 18, 1903. "Method of Charging for Electrical Energy," Schönborn, *Electrotechnische Zeitschrift*, Vol. XXV, pages 377–378, May 12, 1904. "Price of Electricity," by R. S. Hale, Superintendent Sales Department, Boston Edison Company. This paper was read before the New England Section of the National Electric Light Association on March 16, 1910.

his meter, and a portion of the cost of the station book-keeping. The cost to the central station of "readiness to serve" is called *connection cost.*

(*b*) *Demand cost.*—A certain service is rendered to a customer because of and in proportion to his maximum demand for power. The cost of this service to the central station is called the customer's *demand cost.* The size of a station (and therefore the total investment) is determined by the total maximum demand for power, and therefore interest on investment, taxes, and a portion of the depreciation should be distributed among the various customers as a demand charge; but a customer who never uses power at the time of heavy load (on the peak) may pay little or nothing for demand.

(*c*) *Consumption cost.*—A certain service is rendered to a customer because of and in proportion to his total energy consumption in kilowatt-hours, and the cost of this service to the central station is called the customer's *consumption cost.* The total consumption cost of all the customers of a station is approximately equal to the cost of coal plus the cost of labor and station supplies.

A careful analysis of service costs (electric lighting) has been made by Mr. S. E. Doane* upon the basis of very full statistics of 112 central stations in the United States, and the results are shown in the following table.

* "High Efficiency Lamps and their Effect on the Cost of Light to the Central Station," a paper read before the St. Louis convention of the National Electric Light Association, May, 1910.

A very important source of information in matters of operation costs and the fixing of rates for electrical service is the series of *Annual Reports* of the Massachusetts Commissioners of Gas and Electric Light.

The Railway Commission of Wisconsin has made a most exhaustive study of the costs of electric service in their handling of the celebrated case of The State Journal Printing Company *vs.* The Madison Gas and Electric Company. The decision of the Commission was rendered March 8, 1910, and copies of it may be obtained by addressing the commission. The decision is discussed in some detail by Mr. Percy. H. Thomas in the *Electrical Journal,* Vol. VII, pages 560–574, July, 1910.

	Average of 40 Small Stations in the West. Per Cent.	Average of 70 Small Stations in the East. Per Cent.	One Large Station Giving Free Lamp Renewals. Per Cent.	Another Large Station Giving Free Lamp Renewals. Per Cent.
Connection cost .	11.5	11.3	18.0	17.7
Demand cost....	59.6	50.8	58.5	55.1
Consumption cost	28.9	37.9	23.5	27.2
Total..........	100.0	100.0	100.0	100.0

8. The fixing of rates for electric service.*—Inasmuch as the cost of electrical service may be resolved into connection cost, demand cost, and consumption cost, therefore the rational method of charging for the service would be to separate the charge into three items, namely, (*a*) a fixed charge per year to cover connection cost, (*b*) a yearly charge per kilowatt of maximum demand, and (*c*) a charge of so much per kilowatt-hour of consumption. Each of these items should exceed the corresponding cost so as to give a fair margin of profit.

In establishing a schedule of rates, however, a number of things must be carefully considered, some of which are as follows: (1) There is a strong popular prejudice against a fixed charge (connection charge), and therefore this item is seldom or never included in price schedules; (2) the price schedule must be arranged to attract business by setting a low price (low margin of profit) on business that is open to sharp competition; and (3) a simple price schedule is a practical necessity. There are three more or less distinct price schedules used by every central station as follows:

(*a*) For small residence lighting. A certain fairly high rate, usually 11 to 15 cents per killowatt-hour, for energy consumption with a minimum charge which is usually $1.00 per month.

(*b*) For large consumers whose maximum demands are on the peak. A charge of, say, $60.00 per year for each kilowatt

* A very good discussion of rates is given by R. S. Hale in the *General Electric Review*, Vol. XIV, pages 157–169, April, 1911. The student is likely to get an altogether erroneous idea of rate making from the foregoing analysis of costs *because the employment of averages tends to obscure the fact that there is an endless variety of conditions to be met with in electric service.* The reading of Mr. Hale's article is strongly recommended.

of maximum demand, and a low rate, say 2 or 3 cents per kilowatt-hour, for energy consumption.

(*c*) For operating motors off the peak. A consumption charge of 3 to 8 cents per kilowatt-hour.

Equitable charges for electric service may be realized by resolving the service into its cost elements ("connection," "demand" and "consumption") and charging all classes of customers at the same rate for each element; or the customers may be carefully classified on the basis of their "connection," "demand" and "consumption" costs, and a certain equitable rate per kilowatt-hour of consumption may be set for each class. The following table illustrates the complete equivalence of the two systems of charging.

Class of Service.	Connected Load Expressed in 50-watt Units.	Connection Charge, Dollars per Month.	Maximum Demand in Kilowatts.
A. Domestic (small) . .	10	0.50	0.35 at peak
B. Domestic (medium)	20	0.75	0.50 at peak
C. Domestic (large) . .	50	1.25	1.00 at peak
D. Store lighting	80	1.50	4.00 at peak
E. Small factory lighting.	80	1.50	4.00 off peak
F. Motor driving.	100 (5 H.P.)	1.75	5.00 off peak

Class of Service.	Demand Charge, Dollars per Month, at $5 per Month per Kilowatt on Peak.	Yearly Average of Monthly Consumption in Kilowatt-hours.	Consumption Charge, Dollars per Month, at 3 Cents per Kilowatt-hour.	Total Monthly Charge in Dollars.	Equivalent Class Rate in Cents per Kilowatt-hour.
A.	1.65	15	0.45	2.60	17.3
B.	2.50	35	1.05	4.30	12.3
C.	5.00	80	2.40	8.65	10.8
D.	20.00	360	10.80	22.30	6.2
E.	——	240	7.20	8.70	3.6
F.	——	900	27.00	28.75	3.2

An objection to the charging for electrical service by the kilowatt-hour of consumption (using classified rates) is that it places a false premium on the economical use of current by the customer. Thus a customer might reduce his monthly bill to one-half by reducing his killowatt-hour consumption to one-half, whereas his cost to the central station would be reduced but very little if his maximum demand is not reduced in proportion to his kilowatt-hour consumption. The time of day when a customer economizes is very important to the central station.

Another objection (which applies during a time when an improved high-efficiency lamp like the tungsten lamp is being introduced) is that a customer can reduce his kilowatt-hour consumption to, say, one-third for the same amount of light, and thus reduce his monthly bill to one-third; whereas his cost to the central station will not be reduced in the same proportion. In fact the advent of the tungsten lamp has brought the whole question of electric service costs into a prominence which it has not had for years.

9. The meter-rate system and the flat-rate system for selling electrical service.—A central power station supplies energy to its customers and it may, therefore, seem that the use of an energy meter (a watt-hour-meter) would, like the use of a gas meter, furnish a complete and equitable basis for charging customers, but it is not so. In supplying gas for lighting, the large storage reservoir makes the gas generating plant independent of the irregular consumption of the gas. In an electric plant, on the other hand, electrical energy must be generated as used, except in the few cases where storage batteries can be economically employed, and, therefore, the capacity of the electric plant must be sufficient to meet the maximum demand for power. This matter is discussed in detail in Articles 5 to 8. The selling of electrical energy by the kilowatt-hour as indicated by the watt-hour meter is called the *meter-rate system.*

In the early days of electric lighting the customer paid so much per month for each lamp installed. This method of selling electrical service is called the *flat-rate system.* The flat-rate system is more satisfactory than the meter-rate system because it simplifies the station book-keeping and avoids the cost of installing, maintaining and reading of meters, and it avoids the consumption of energy in the meter. On the other hand, the flat-rate system is unsatisfactory because under this system a wasteful customer pays no more than an economical one, and a dishonest customer can connect lamps in excess of the number he pays for. These two disadvantages of the flat-rate system may, however, be overcome to some extent as follows: In the first place if the customer uses high-efficiency tungsten lamps the cost of the lamps themselves is an incentive to short hours of use; and in the second place the so-called *excess indicator* can be used to limit the amount of power delivered to the customer. The excess indicator is a device essentially like an ordinary bell or buzzer; the current which is delivered to a customer flows through the winding of an electromagnet and when the customer turns on more than a prescribed number of lamps the armature of the electro-magnet is attracted and the circuit is interrupted repeatedly, causing the lights to flicker in a manner which makes them practically useless. The excess indicator is much cheaper than a watt-hour meter and it needs less attention. Without doubt the extension of electrical service to great numbers of very small customers is hampered by the prevailing idea that the meter-rate system must be used, and the cost of installing, maintaining and reading meters is probihitive in the case of a very small customer. The very small customer should be supplied on the flat-rate system buying his own tungsten lamps and using them under the control of an excess indicator.

10. **The watt-hour meter*** is a device for summing up the total amount of work or energy delivered to a circuit. This meter is sometimes used on a station switchboard to measure the energy output of the station. Thus the annual output of the Wallingford station (464,000 kilowatt-hours, see page 32) was

* A good description of the various types of watt-hour meter is given in *The Watt-hour Meter*, by Shepard and Jones, Technical Publishing Company, San Francisco, California, 1910. This book also contains information relating to the maintenance of the meter department of an electric power station.

Everyone who has to do with watt-hour meters should possess a copy of the *Meter Code* of the National Electric Light Association.

measured by a watt-hour meter in the station. The watt-hour meter is chiefly used, however, to measure the energy delivered to a customer.

In the following discussion of the two most important types of watt-hour meter (the *commutator-motor* type and the *induction-motor* type), it is shown that the speed of the motor spindle is proportional to the watts delivered to the receiving circuit; that is to say, *the rate at which the spindle turns is proportional to the rate at which energy is delivered to the receiving circuit;* therefore* the total angle (or number of revolutions) turned by the spindle is proportional to the total energy delivered, and a revolution counter may be arranged to indicate the delivered energy in kilowatt-hours.

11. **The Thomson watt-hour meter** is a small electric motor of the commutator type (without iron) driving a revolution counter. The field coils *BB*, Fig. 10, are made of coarse wire and they are connected in series with the receiving circuit or "load" so that the total current, i, which is delivered to the receiving circuit flows through the field coils The armature A is wound with fine wire and it is connected across the supply mains in series with a non-inductive resistance R so that a current equal to e/R flows through the armature, e being the voltage between the mains. The brushes *dd* are made of metal and they rub very lightly on a small commutator *e*. This commutator has silver bars so as to give good electrical connection with a minimum of friction. The armature is mounted on a vertical spindle which rests on a jewel so as to turn with as little friction as possible, and the opposition to rotation is due to the

* This simple case of integration occurs frequently in mathematical arguments, and it should be understood perfectly by the student. Thus the rate of change of one quantity y is proportional to the rate of change of another quantity x; that is to say, the rate of change of y is k times as great as the rate of change of x. Therefore, if y and x start from zero together, y must be always k times as large as x. If one person A saves money 10 times as fast as another person B, then A will always have ten times as much as B if A and B start from zero together.

electromagnetic drag of the permanent magnets MM upon the copper disk f which is attached to the spindle. The result is that the speed of the armature and spindle is almost exactly proportional to the driving torque which is exerted on the

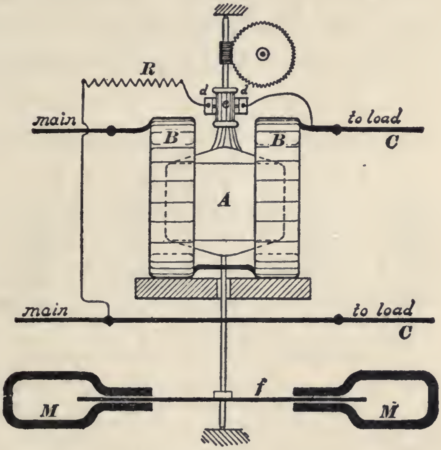


Fig. 10.

armature A by the field coils BB, and this driving torque is proportional to the product of field current i and armature current e/R. *Therefore the speed is proportional to* ei (*watts delivered to the receiving circuit*).

The commutator-motor type of watt-hour meter can be used on direct-current or alternating-current circuits.

The compensation for friction in the Thomson meter.—An auxiliary fine-wire field coil connected in series with the armature is always provided in the Thomson meter. The effect of this coil is to give a constant driving torque (supply voltage assumed to be constant) sufficient to overcome friction. This auxiliary field coil is called a *starting coil* and it gives a great increase of accuracy of the meter.

12. The induction watt-hour meter is a small induction motor driving a revolution counter, and it can be used only on alternating-current circuits. The essential features of the motor part of the meter are shown in Fig. 11. A thin disk DD of aluminum or copper is mounted on a spindle and rotates in front of the three lugs BAB of a laminated iron structure as shown. The lugs BB are wound with coarse wire and this winding (the *current winding*) is connected in series with the receiving circuit so that the current i which is delivered to the receiving circuit flows through the windings on BB. The magnetic field in front of the lugs BB is therefore proportional to i. The lug A is wound with fine copper wire and this winding (the *voltage winding*)

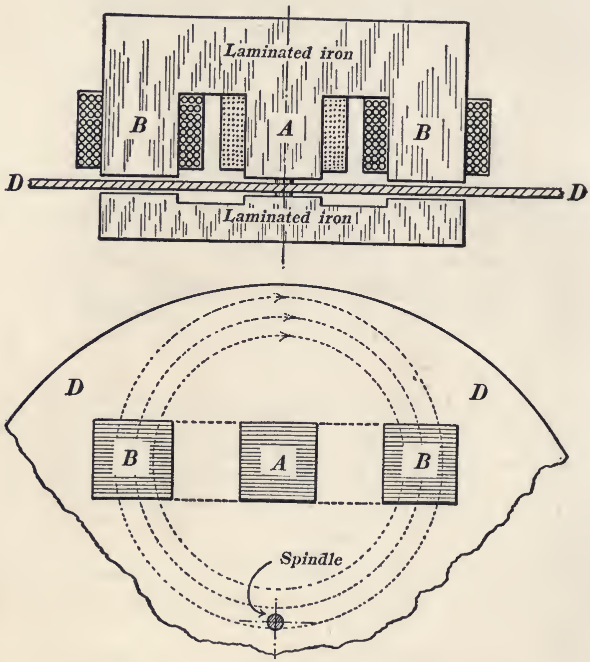


Fig. 11.

is connected directly across the supply mains. If the resistance of the winding on A is negligible,* then the alternating magnetic flux through the lug A induces a voltage in the winding A which balances (and is therefore equal to) the supply voltage e, and this alternating flux induces a proportional voltage (proportional

* Negligible in comparison with the very large reactance of the coil. It is not wholly negligible. See discussion of compensation for lag.

to e) along the dotted lines in the disk (around the bundle of flux which passes through the disk). This voltage in the disk produces a current in the disk which is proportional to it and to e (disk assumed to be non-inductive), and this current as it flows under the lugs BB is pushed sidewise by the flux with a force which is proportional to the current in the disk and to the flux density under the lugs BB. But the current in the disk is proportional to e, and the flux density under BB is proportional to i, therefore a driving torque proportional to ei is exerted on the disk.*

The meter spindle carries a copper disk† similar to f in Fig. 10, which rotates between the poles of permanent magnets like MM, Fig. 10, and *therefore the speed of the meter is proportional to* ei, *as in the Thomson meter.*

Compensation for friction and lag in the induction meter.‡—If the driving torque of the induction meter were strictly proportional to the delivered watts, the speed of the meter spindle would not be proportional to delivered watts because of friction. The effect of friction may be to a great extent eliminated, however, by means of a device whereby the voltage winding (on lug A, Fig. 11) alone produces an amount of torque sufficient to overcome friction. This is usually done by placing a flat short-circuited coil S between the lug A and the disk DD, as shown in Fig. 12, and adjusting this coil sidewise until the desired torque is produced by the voltage coil. The coil, S, is called a *shading coil.*

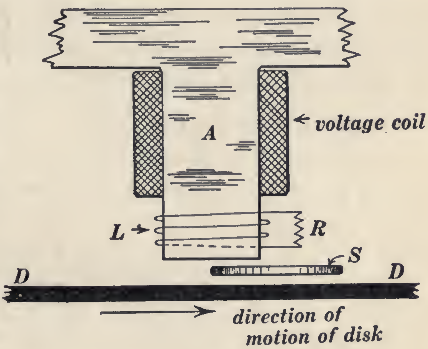


Fig. 12.

In order that the combined action of lugs A and B, Fig. 11, may produce a driving torque which is accurately proportional to the delivered watts, the voltage induced in the disk by the magnetic flux from lug A must produce a current in

* The flux from lugs B also induces currents in the disk, these currents flow across the face of lug A, and the flux under lug A pushes widewise on these currents. It can be easily shown that this torque is also proportional to ei.

† Usually the disk DD, Fig. 11, serves also as the damping disk.

‡ Full details of the compensation of the induction meter may be found on pages 28–38 of Shepard and Jones' *The Watt-hour Meter.*

the disk which is in phase with the voltage across the mains. In fact, however, the current in the disk is not in phase with the supply voltage for two reasons: (1) The voltage winding does not have a negligible resistance, and (2) The current paths in the disk are not non-inductive. The error due to this effect is especially noticeable when the meter is used to measure energy delivered to a receiving circuit of low power factor, and this error may be to a great extent eliminated by means of an auxiliary secondary coil L on lug A, as shown in Fig. 12, this coil being short circuited through an adjustable resistance R. This auxiliary coil constitutes what is called the *lag adjustment* of the meter.*

13. The two-rate meter.—One of the most satisfactory schemes for selling electrical service is to charge at a high rate per kilowatt-hour during the period of heavy station load (on the peak) and to charge at a low rate per kilowatt-hour during the period of light station load (off the peak). To carry this scheme into effect requires the use of the *two-rate meter* which is an ordinary watt-hour meter with two sets of dials and a clock which throws into gear one set of dials during the period of heavy station load (between 7 and 10 P.M., for example) and the other set of dials during the remainder of the 24 hours of each day. One set of dials thus registers the kilowatt-hours of consumption on the peak and the other set of dials registers the kilowatt-hours of consumption off the peak. The practical objection to the two-rate meter is that a high grade and expensive clock is required in order that the clock may run for a month without too great an error.

14. The maximum-demand meter.†—The system of selling electrical service in which a certain charge per year is made for each *kilowatt* of maximum demand and a certain charge per *kilowatt-hour* of consumption, requires the use of two distinct meters, namely, a maximum-demand meter and a watt-hour meter. The most extensively used maximum-demand meter is the Wright meter, which is essentially a large thermometer, the bulb of which is heated by a low-resistance strip of metal through which the current demanded by the customer flows. A mo-

* The theory of the lag adjustment of the induction watt-hour meter is fully explained on pages 28–33 of Shepard and Jones' *The Watt-hour Meter.*

† See Shepard and Jones' *The Watt-hour Meter*, pages 100–105.

mentary short circuit does not heat the bulb of the thermometer perceptibly, but a heavy demand lasting for five or six minutes heats the bulb up to a certain temperature, a portion of the liquid in the bulb flows over into a graduated trap, and the maximum demand for current is indicated by the amount of liquid in this trap. After reading, the instrument is tilted so as to cause the liquid in the trap to flow back into the bulb.

The maximum-demand meter is not as yet extensively used. A central station using a schedule of rates which involves a yearly charge for maximum demand* may estimate the maximum demands of its customers on the basis of statistics such as are represented in Fig. 9.

15. The Borough Electric Plant of Wallingford, Connecticut.†—The cost of erection of this plant was met by the proceeds ($56,500.00) from the sale of 20-year municipal bonds bearing interest at 3½ per cent., the face value of the bonds being $55,000.00. The plant began operating on December 23, 1899. The station building is of brick and steel construction, 45 feet × 104 feet. At the beginning the plant included the necessary boilers, a 150-horse-power eng ne belted to a 75-kilowatt alternator, a 225-horse-power engine belted to a 150-kilowatt alternator, the necessary switchboards and accessory apparatus, including three constant-current transformers for operating three arc-lamp circuits for street lighting, and a distributing system for commercial lighting.

A 450-horse-power engine and a 240-kilowatt, 2-phase alternator with additional boilers were installed in 1904. A small water-power plant with a generator capacity of 120 kilowatts was installed in 1907. The distributing system (pole lines and transformers) has been slowly extended year by year as roughly indicated by the yearly increase of the number of customers (see Fig. 14, curve *G*).

* See Art. 8.

† A series of detailed annual reports of this plant and the kind permission of the manager. Mr. A. L. Pierce, to use them here makes it possible to give a good example of the economies of a small lighting plant.

In 1906 about 45 kilowatts of power were expended for operating the street lamps (89 enclosed-arc lamps and eight 100-watt incandescent lamps). In 1907 fifty-eight incandescent lamps (carbon filament) and one enclosed-arc lamp were added to the street lighting system which was extended to an adjoining village, and the total power expended for street lighting was about 50

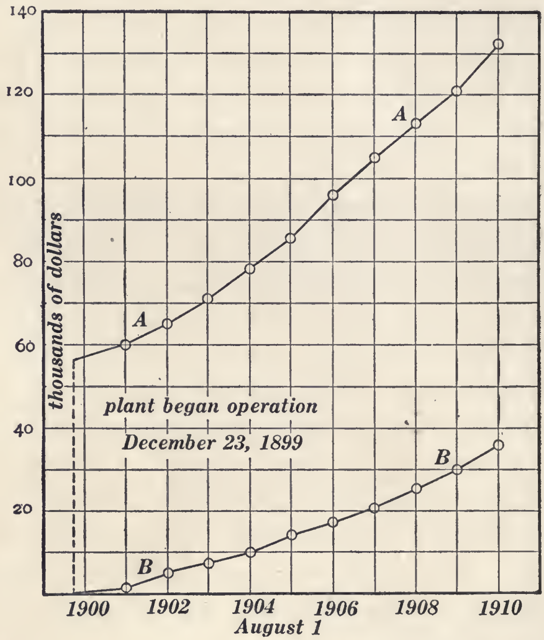


Fig. 13. Curve *A* assets. Curve *B* accumulated depreciation.

kilowatts. In 1910, all but 9 of the enclosed-arc lamps and all of the carbon-filament lamps were replaced by tungsten lamps, and the power consumed for lighting was about 40 kilowatts.

The plant was operated only at night (about 14 hours per day) until 1908, when it was operated about 18 hours per day and in 1909 a continuous day and night service was begun.

The population of Wallingford was about 7,000 in 1900, and about 5,000 inhabitants were included within the service area of the plant. In 1910 the distributing system had been extended to include the adjoining villages of Yalesville and Tracy and about 10,000 inhabitants were included in the service area of the plant.

The entire capital invested in the plant has grown out of the proceeds of the bonds above mentioned, and the net assets of

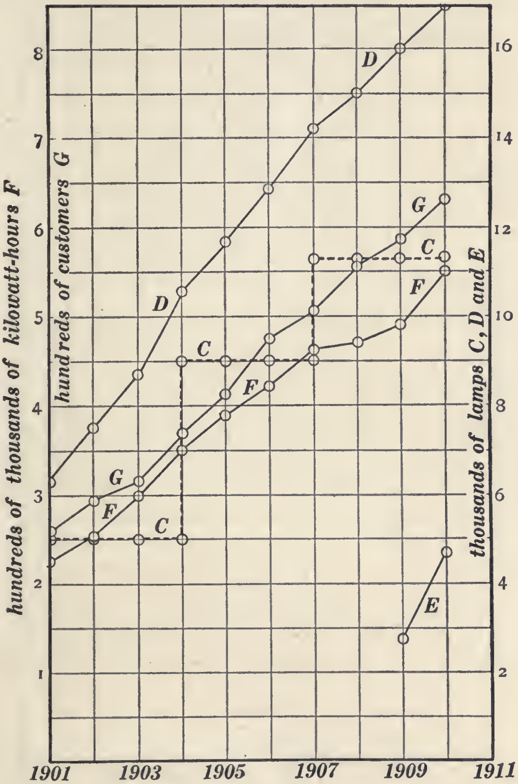


Fig. 14. Curve *C* capacity of plant. Curve *D* connected lighting load. Curve *E* connected load of motors and heating apparatus. Curve *F* annual output in kilowatt-hours. Curve *G* number of customers.

the plant have increased to $96,881.00* (on July 31, 1910) although the bond interest of $1,925 per year is paid out of the earnings of the plant. In judging the financial success of the

* $133,000 assets *minus* $36,119 of accumulated depreciation as carried forward on the books.

plant, however, one must remember that it pays no taxes. The ordinates of curve *A*, Fig. 13, show the growth of assets year by year. In reckoning these assets the plant equipment* is estimated at its first cost, and the assets include an increasing stock of electric lighting supplies (new), and interest-bearing loans and bonds. The latter amounted to $13,243.50 on August

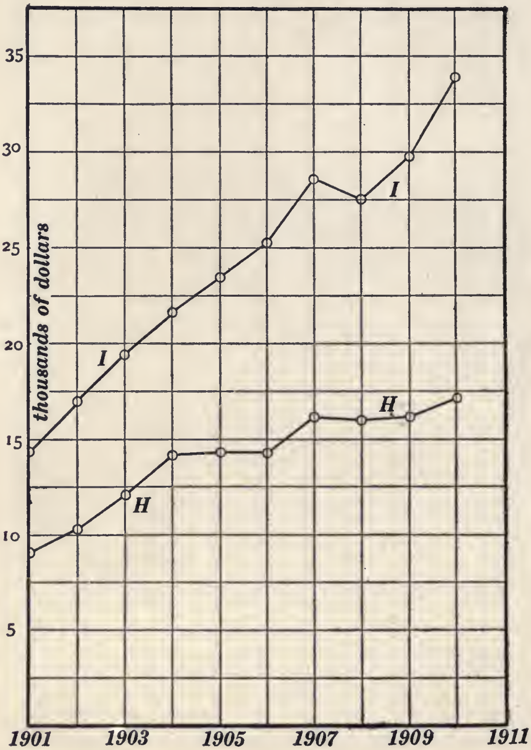


Fig. 15. Curve *H* yearly operating expenses. Curve *I* yearly income.

1, 1910. The ordinates of curve *B* represent the accumulated depreciation charge of 5 per cent. per year which a municipal plant in Connecticut is required by law to carry forward in its annual reports.

* Permanent equipment, only, is included. Lamps and other materials which are discarded after short periods of use are not included.

The ordinates of the curves in Fig. 14 show the year by year growth of the following items:

Curve *C* shows the plant capacity expressed in 50-watt units (lamps).

Curve *D* shows the connected lighting load expressed in 50-watt units (lamps).

Curve *E* shows the connected load of motors and heating apparatus expressed in 50-watt units.

Curve *F* shows the yearly kilowatt-hours output.

Curve *G* shows the number of customers.

Fig. 15 shows the year by year growth of the following items:

Curve *H* shows yearly operating expenses including the cost of maintaining the fire-alarm system.

Curve *I* shows yearly income including $500.00 per year paid by the Borough for the maintenance of the fire-alarm system.

INVESTMENT (1907).

Station building and real estate	$12,346.38
Steam equipment	21,435.65
Electric equipment	13,437.04
Line equipment, including street lighting circuits	25,068.29
Transformers	3,523.72
Incandescent lamps	3,520.16
Meters	473.73
Arc lamps and arc lamp supplies	1,008.72
Average amount of coal and supplies on hand	2,500.00
Total capital invested	$83,313.69

OPERATING EXPENSES (1907).

Maintenance of lamps*	$1,200.51
Trimming and maintaining arc lamps	682.82
Labor and superintending	4,896.93
Coal (at $4.00 per ton)	7,409.72
Oil and waste	147.97
Building and boiler insurance	374.13
Liability insurance	288.00
Office rent	30.00
Plant stationery and incidentals	1,622.13
Total expenses	$16,652.21

* Customers are supplied with new lamps as the old ones burn out.

Kilowatt-hours output during the year........................464,000

3½ per cent. interest on investment	$2,916.00	6 per cent. interest on investment	$4,999.00
4 per cent. depreciation on equipment	2,740.00	4 per cent. depreciation on equipment	2,740.00
Total cost per kilowatt-hour	0.048	1 per cent. taxes on $80,000	800.00
		Total cost per kilowatt-hour	0.0543

INCOME (1907).

For street lighting	$ 6,513.36
For commercial lighting	21,422.47
For maintenance of fire-alarm system	500.00

Profits in excess of 3½ per cent. interest and 4 per cent. depreciation	$6,118.00	Profits in excess of 6 per cent. interest and 4 per cent. depreciation and 1 per cent. taxes	$3,245.00

The foregoing tables show the details for the year ending July 31, 1907. In order to show the important relations between investment, operating expenses, and income, the newly purchased water-power plant (which was not operated during 1907) is not included as a part of the investment.

It may seem that the profits should be much greater than what is indicated in the table in view of the fact that the actual cost per kilowatt-hour is about 5 cents whereas the lighting rate is 11 cents per kilowatt-hour; but the sum of all customers' meter readings is less than the station output because of distribution losses, and off-peak service is sold at much less than 11 cents per kilowatt-hour.

The cost of maintaining the fire-alarm system cannot be clearly separated from other items of expense. In fact the charge of $500 is about equal to the cost, which includes care of fire-alarm apparatus and lines, and fuel and labor for keeping up steam for the whistle during the daytime.

The schedule of service rates in force in 1907 was a complicated mixture of flat rates and meter rates, and a knowledge of this schedule beyond the mere fact that it represented approximately a meter rate of 11 cents per kilowatt-hour is not necessary to an understanding of the economies of the plant during 1907.

MISCELLANEOUS ITEMS (1907).

Total station capacity expressed in 50-watt units (lamps)........	9,000
Total connected load (commercial) in 50-watt units............	13,316
Street-lighting load expressed in 50-watt units (lamps)........	1,000
3 street-lighting circuits, aggregate length of in miles.........	26.5
Length of street covered by 2-wire high-voltage line (primary circuits) in miles....................................	14.4
Length of street covered by 3-wire low-voltage line (secondary circuits) in miles....................................	6.8
Number of transformers in service..........................	58
Aggregate capacity of transformers in kilowatts..............	346
Rate charged per year for enclosed-arc street lamps in dollars..	69.58
Commercial lighting rate per kilowatt-hour in cents..........	11

Plant in operation about 14 hours per day on the average.

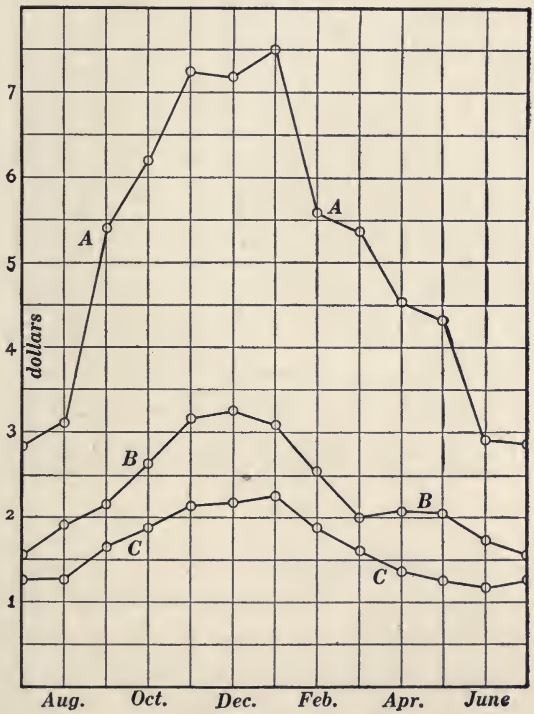


Fig. 16.

The ordinates of the curves in Fig. 16 show the average monthly bills of the customers of the Wallingford station after meters were installed and the service changed to a meter basis.

Curve A refers to a group of customers having an average of 50 connected lamps, curve B refers to a group of customers having an average of 30 connected lamps, and curve C refers to a group of customers having an average of 20 connected lamps. Individual monthly bills depart very widely from the averages shown

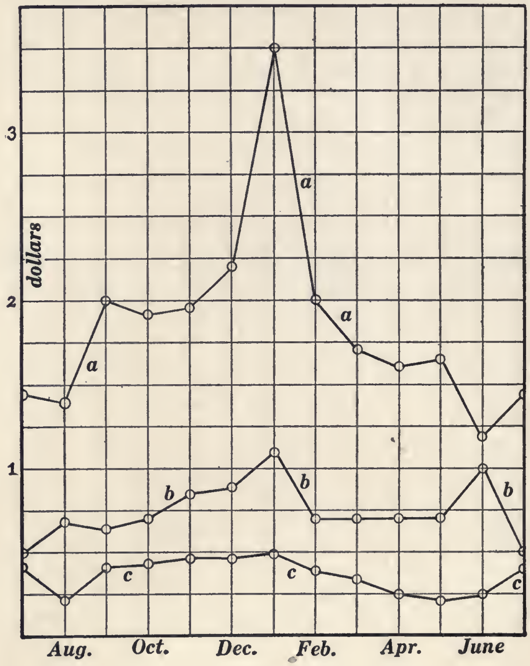


Fig. 17.

in Fig. 16, and the ordinates of the curves a, b and c in Fig. 17 show the probable departure* of an individual bill from the mean. For example, the ordinate of curve a for December is $2.21, which means that the December bill of any customer taken at random is as likely to depart *less* than $2.21 from the mean as it is to depart *more* than $2.21 from the mean for December ($7.18, see Fig. 16).

* Reckoned exactly as the "probable error of a single observation" is reckoned in the theory of least squares.

The bills of the 10-lamp customers are not shown in Figs. 16 and 17 because these bills come down very frequently to the fixed minimum of $1.00 per month. In fact the mean of the 10-lamp customers is about $1.10 per month in midsummer and about $1.96 per month in midwinter, and the "probable departure" of an individual's bill from the mean is about 5 cents in midsummer and about 18 cents in midwinter.

CHAPTER II.

ELECTRIC DISTRIBUTION AND WIRING.

16. Parallel and series systems of distribution.*—In practice a large number of receiving units (lamps or motors) are always operated from a single generator, and it is always desirable to put any lamp or motor into service or to take any lamp or motor out of service at will without affecting the other lamps or motors. The easiest method for doing this is to connect the lamps and motors in parallel with each other between supply mains and maintain a constant voltage between the mains. In this case the receiving units must all be of the same voltage rating, and a given unit is taken out of service by opening its circuit. This arrangement constitutes what is called the *constant-voltage system of distribution* and it is frequently called the *parallel system of distribution*. Another method is to connect the lamps or motors in series and to maintain a constant current through the circuit. In this case the receiving units must all be of the same current rating, and a given unit is taken out of service by short-circuiting it by means of a by-pass switch. This arrangement constitutes what is called the *constant-current system of distribution* and it is frequently called the *series system of distribution*.

The constant-voltage system is almost universally used nowadays both for direct current distribution and for alternating current distribution. This system is exemplified by great numbers of plants for supplying current to lamps and motors in our cities, by nearly every electric railway installation, and by numerous installations for the long distance transmission of power.

* A brief discussion of these two systems (with special reference to the characteristics of the generators) is given in *Dynamos and Motors*, The Macmillan Co.

The constant-current system of supply is advantageous when a fixed number of widely-distributed lamps, arc or incandescent, are to be operated; the lamps are connected in series and supplied with constant current. This system is exemplified by many street-lighting installations and by the Thury system* of power transmission.

Combinations of series and parallel connections. (*a*) *Connections of series-groups in parallel.*—When the voltage of supply in the constant-voltage method of distribution is greater than can be conveniently used for operating single lamps, the lamps are usually arranged in groups, each group consisting of a number of lamps connected in series, and these groups of lamps are connected in parallel with each other across the mains. This arrangement is exemplified in the lighting of electric cars where the standard supply voltage is 550 volts and where the lamps for lighting the cars are usually 110-volt lamps connected in series-groups of 5 lamps each, these groups being connected in parallel with each other between the trolley and the rail. A similar arrangement is frequently employed for tungsten lamps, especially in the case of the low-candle-power tungsten lamps which are extensively used for decorative and sign lighting. Thus five 22-volt tungsten lamps may be connected in a series-group, and such groups may be connected in parallel with each other across standard 110-volt mains.

(*b*) *Connections of parallel-groups of lamps in series.*—In the early days of electric lighting the constant-current method of supplying arc lamps for street lighting was quite common. Many towns which were provided with this series system of distribution were not provided with any other means for supplying incandescent lamps, and the only feasible method for operating incan-

* The Thury system (direct-current) is a constant-current system of distribution. It is exemplified by several large, long-distance, power-transmission plants in Europe. A description of the Thury system is given by Wieshofer in the *Zeitschrift für Electrotechnik* (Vienna), Vol. XVI, pages 5–10, 1898. See also a paper by Cuénod and Thury in the *Bulletin de la Societété Internationale des Electriciens*, Vol. XVII, pages 9–93, January, 1900.

descent lamps was to connect a group of such lamps in parallel and to connect this group in series in the arc lamp circuit. This arrangement is now seldom or never used.

Advantages and disadvantages of the connections of series-groups of lamps in parallel.—In order to understand the advantages of grouping electric lamps in series one must keep in mind the fact that the electric lamp is essentially a low-voltage device, so that if one wishes to use a high-voltage distributing line in order to reduce* the amount of line copper required, the lamps must be arranged in series groups in order to adapt them to the high-voltage supply. The disadvantage of the series grouping of lamps is that each group must be turned off and on as a unit unless a special device is used to connect an equivalent resistance in place of a lamp which is to be turned off.

The grouping of incandescent lamps in series is exemplified in the use of incandescent lamps for street lighting. When such a group is supplied with constant current (*i. e.*, with a current which is automatically kept at a constant value regardless of the number of lamps) then any given lamp is simply short-circuited by a by-pass connection when for any reason the lamp is taken out of service. When, however, such a group is connected to a constant voltage supply then when a lamp is broken or otherwise damaged a by-pass containing a similar auxiliary

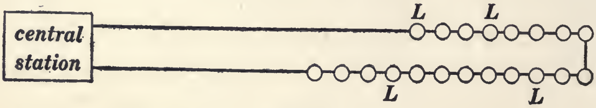


Fig. 18.

lamp must be provided. Thus Fig. 18 shows twenty 110-volt lamps connected in series and supplied from 2,200-volt mains in a central station, and Fig. 19 shows how each lamp in Fig. 18 is provided with a by-pass. Each lamp *L* of the series has a similar auxiliary lamp *A* connected in parallel with it, and the circuit of this auxiliary lamp is broken by a thin piece of

* See Art. 23.

paper *p* between two metal springs *SS*. If the lamp *L* happens to break the full voltage of supply (2,200 volts) is brought to bear upon the thin paper *p*, thus puncturing it and establishing a metal bridge from spring to spring. In this way the auxiliary lamp *A* is substituted for the lamp *L*. The inspector on his rounds seeing *A* burning knows that a new lamp is needed in place of *L*, and when a new lamp is installed a fresh bit of paper is placed between the springs *SS*. When a constant current is supplied to a series group of lamps, as shown in Fig. 18, then each lamp has a by-pass as shown in Fig. 19 except that the auxiliary lamp *A* is not used.

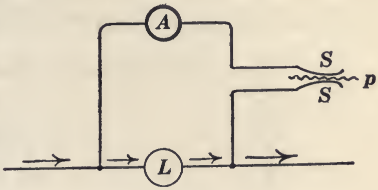


Fig. 19.

17. The Edison three-wire system of distribution.—Figure 20 shows a number of 110-volt lamps connected in series-groups of two lamps each to 220-volt mains and supplied with current from two 110-volt generators connected in series; and Fig. 21 shows an arrangement which is the same as Fig. 20 except

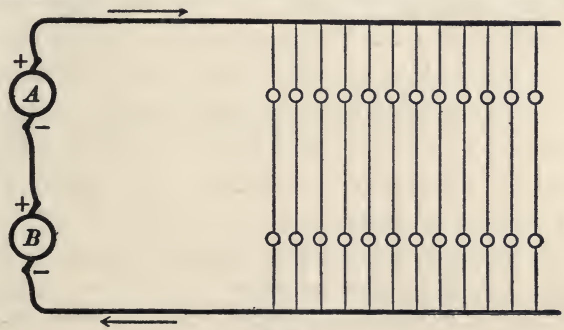


Fig. 20.

that a third main *CD* is added as shown. By placing some of the lamps of each customer in the *A*-set and some in the *B*-set (see Fig. 21), there will always be nearly the same number of lamps in each set even when each customer exercises entire freedom in the turning off and on of single lamps; under these conditions the middle main need never carry very much current, and therefore the middle main may be made of comparatively small wire. In fact the current in the middle main will be a small current coming into the station when the *A*-set contains a few more lamps than the *B*-set, or a small current going out

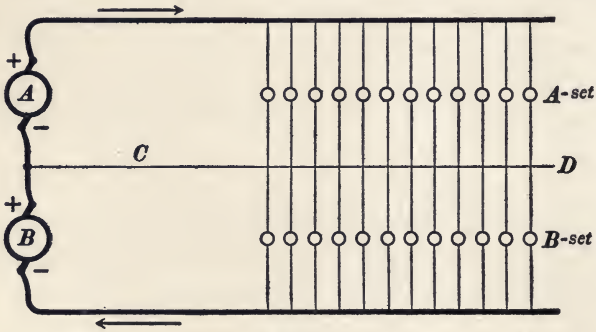


Fig. 21.

from the station when the *B*-set contains a few more lamps than the *A*-set. In this arrangement, therefore, most of the advantage (economy of line copper) due to the use of 220-volt distribution is realized although 110-volt lamps are used. The arrangement is called the *Edison three-wire system of distribution.* In practice the middle main is usually made of the same size of wire as each outside main but each outside main need be only one-quarter as heavy* as would be required to supply the same number of lamps in the simple parallel system using 110 volts. Therefore to supply a given number of 110-volt lamps in the Edison three-wire system requires only three eighths as much copper in the mains as would be required in the simple parallel system of distribution with the same percentage drop of voltage.

When the number of lamps in the *A*-set in Fig. 21 is different from the number of lamps in the *B*-set the system is said to be *unbalanced.* When the system is unbalanced the middle main

* See Art. 23, page 57.

carries current as above explained, and the voltage drop in the middle main tends to increase the voltage which acts on one set of lamps and to decrease the voltage which acts on the other set of lamps. These voltage relations are clearly represented

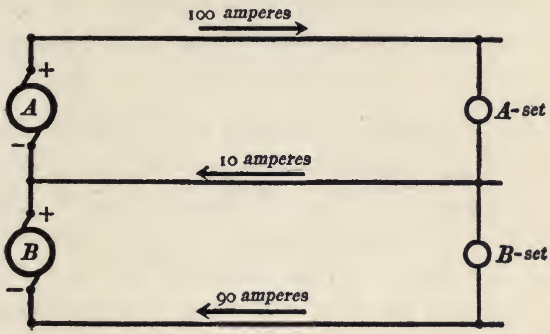


Fig. 22*a*.

for a particular case in Figs. 22*a* and 22*b*. These figures show the state of affairs when the *A*-set of lamps takes 100 amperes and the *B*-set takes 90 amperes, each main having 1/20 ohm

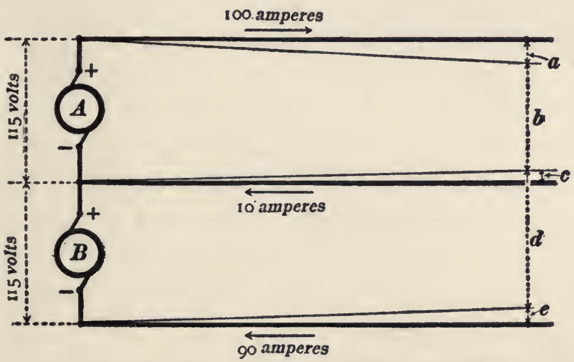


Fig. 22*b*.

resistance; the lamps being supposed to be bunched at the ends of the mains for the sake of simplicity. Electric current may always be considered as flowing downhill, as it were, so that the electric level (or potential) is to be thought of as falling off along each main in the direction of the current as indicated by the fine inclined lines in Fig. *22b*. The distances between the inclined lines at the ends of the mains represent the voltages acting on the two sets of lamps. Thus the voltage *b* acting on the *A*-set is 115 volts − 5 volts − ½ volt = 109.5 vo'ts, and the voltage *d* acting on the *B*-set is 115 volts − 4½ volts + ½ volt = 111 volts.

18. Special three-wire generators and three-wire balancers for direct-current systems.—The use of two generators as indicated in Figs. 21 and 22 involves an added expense for machinery in a generating station, and the extensive use of the Edison three-wire system has given rise to the so-called *three-wire generator* which can be used for supplying current to a three-wire system, and to the so-called *three-wire balancer* which enables a single 220-volt generator of the ordinary type to supply a 110-volt Edison three-wire system.

The double-current generator.—Several types of three-wire generators have been proposed and used to some extent* but the machine which is now almost universally used to deliver direct current to the Edison three-wire system is the synchronous converter (rotary converter). This machine when engine driven is called the *double-current generator.*† The arrangement of the three-ring double-current generator as a three-wire direct-current generator is shown in Fig. 23. Three similar choke coils (inductance coils) C', C'' and C''' are connected as shown to the middle main and the other ends *a*, *b* and *c* of the coils are connected to the alternating-current brushes, *a*, *b* and *c*, of the machine

The motor-generator balancer.—The two generators shown in Fig. 21 may be replaced by a single 220-volt generator of the ordinary type (having two brushes), and the current which

* The three-wire generator of Dettmar is described in *Electrotechnische Zeitschrift*, Vol. XVIII, pages 55 and 320, 1897.

† The structural features of the double-current generator (rotary converter) are described in Art. 145 of *Dynamos and Motors*.

flows into or out of the station on the middle main may be taken care of by a small motor-generator consisting of two simple shunt-wound dynamos P and Q with their armatures

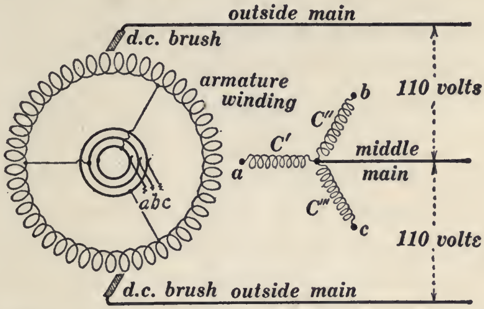


Fig. 23. Terminals a, b and c connected to brushes a, b and c respectively.

mounted on one shaft and connected electrically as shown in Fig. 24. Consider the particular case in which the upper main carries a large outward current of 100 amperes, the middle main a return current of 10 amperes, and the lower main a return current of 90 amperes as shown in the figure. Assuming the efficiency of the motor-generator to be 100 per cent. (for the

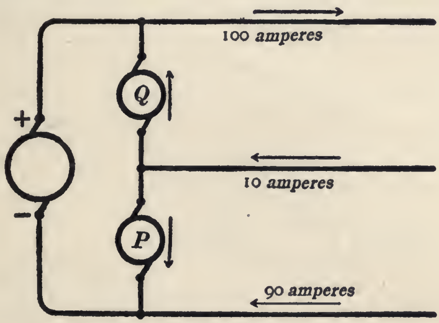


Fig. 24.

sake of simplicity of statement), the action would then be as follows: One-half of the current entering the station on the middle main would flow downhill, as it were, through P to the negative terminal of the large generator; P would therefore act as a motor, drive Q as a generator, and cause Q to pump the remainder of the current in the middle main uphill, as it were, to the positive terminal of the large generator. With outward current flowing in the middle main Q would operate as a motor and P would operate as a generator. In order to keep the potential of the middle main at the proper value so as to divide the electromotive force of the large generator into two equal parts, the machines P and Q must have carefully adjusted compound field windings, or the field rheostat of P or Q must be repeatedly adjusted as the current in the middle main changes in value.

In the use of a motor-generator balancer it is desirable to keep the system approximately balanced so as to reduce the duty of the motor-generator. By carefully grouping the consumers' lamps and motors, the unbalancing of a 3-wire system may be usually kept within 8 or 10 per cent. and under these conditions the rated output capacity of each of the dynamos P and Q in Fig. 24 would need to be only 8 or 10 per cent. of the rated output capacity of the main generator.

19. The supply of alternating current to an Edison three-wire system.—A transformer with a divided secondary coil can

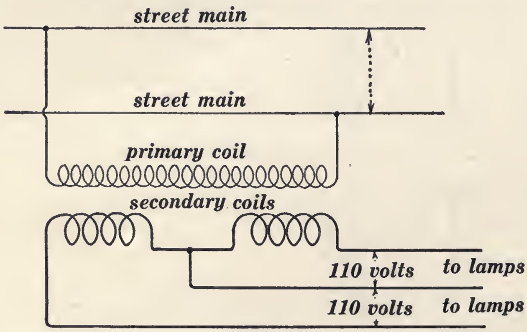


Fig. 25.

be used to supply alternating current to an Edison three-wire system by arranging connections as shown in Fig. 25; or two independent transformers can be used for the same purpose as shown in Fig. 26.

The Edison three-wire system must not be confused with the polyphase three-wire system.* The fundamental idea in the Edison three-wire system is to deliver current to two approximately similar groups of lamps in series, as explained in Art. 17; and the fundamental idea of the polyphase system is to deliver alternating currents from electrically separate alternators over separate lines to separate receivers.

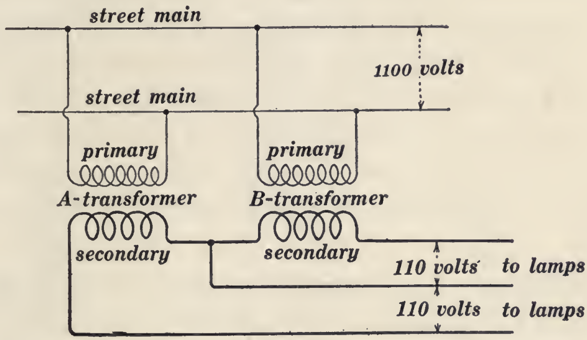


Fig. 26

20. Factors which determine the size of wires in practice.—There are five conditions which should be considered in selecting the sizes of wires for the distribution of electric current, namely, (*a*) the wire must have sufficient strength to withstand the mechanical stresses to which it may be subjected. This condition applies especially to wires strung on poles. (*b*) The wire must be large enough to carry the prescribed current without becoming so hot as to damage its insulation or ignite adjacent inflammable material. This condition applies especially to wires in a building. (*c*) The wire must be large enough to keep the variations of voltage at the lamps or other receiving units within certain limits. This condition applies only to the distributing wires of a "constant-voltage" system. See Art. 23. (*d*) The wire should be large enough so that the annual value of the RI^2 losses in the wire are not greater than the interest on the cost

* See Arts. 117–120 of *Dynamos and Motors*.

of the wire. See Art. 26. (*e*) In extreme cases the size of a wire may be determined by a consideration of the electric strength of the air or other insulating substance surrounding the wire because the ability of an insulating medium to withstand the electric stress between two wires due to a given voltage between the wires depends in part upon the size and shape of the wires. See Art. 27.

Whenever in a given case any one of these conditions demands a larger wire than would be required by any of the other conditions, the larger wire should be used. Frequently an engineer is guided by one only of the above conditions in laying out the preliminary plans for a distributing system. When this is the case the preliminary plans should be examined carefully to see that all of the conditions are satisfied before the plans are finally adopted.

21. Mechanical stresses in aerial wires and their supports.* **Stresses in the supports.**—The stresses in the insulator pins, cross-arms, and poles are: (*a*) The stresses due to the weight of the wire plus the weight of an occasional coating of ice; this weight is to be considered as resting directly upon the insulators and constituting a force acting vertically downwards.† (*b*) The stresses due to the unbalanced tensions‡ of the wire on the opposite sides of an insulator. The tensions of the wire on the opposite sides of an insulator are in nearly every case sensibly equal in value and unbalancing occurs only where the wire terminates or changes its direction. In the case of a straight pole-line on a slope the tension of the wire is generally greater on the down-hill side of the pole, but the unbalanced force is in this case a force acting vertically downwards, that is, a given insulator

* A discussion of the details of pole-line construction is beyond the scope of this text. Information concerning these details may be found in *Electrical Transmission of Energy*, A. V. Abbott, 1905 edition, Chapter III.

† This force is the sum of the vertical components of the tension of the wire on the two sides of an insulator.

‡ Horizontal components of the tensions, inasmuch as vertical components are considered under (*a*).

supports a large part of the weight of the lower span of wire and a correspondingly small part of the weight of the upper span of wire. (*c*) Stresses due to wind pressure.

(1) The weight of wire and ice produces, in the poles and pins, stresses of simple compression, which stresses may nearly always be neglected, inasmuch as poles and pins which are strong enough to withstand the bending stresses to which they are subjected are not perceptibly affected by these slight stresses of compression.

The weight of wire and ice produces a bending stress in the cross-arms, and the breadth, b, and depth, d, of the cross-arms must be sufficient to sustain this bending stress, the length of the cross-arms being determined by the number of wires and their required distance apart. The simplest case is that shown in Fig. 27, which

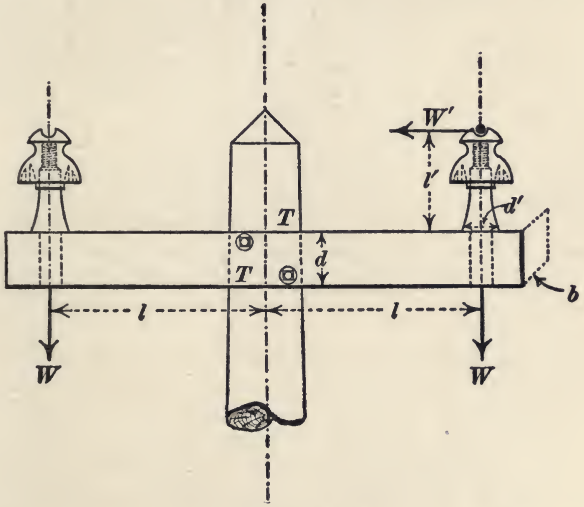


Fig. 27.

shows a cross-arm carrying two wires. In this case the dimensions, b, d, and l, as shown in the figure must satisfy the equation:

$$S = \frac{6Wl}{bd^2} \qquad (1)$$

in which S is the permissible fiber stress of the cross-arm material in pounds per square inch at the points, TT, Fig. 27, and W is the total weight in pounds resting on one pin. The dimensions, b, d, and l, are expressed in inches.

A coating of ice one eighth of an inch thick is seldom exceeded, and it is cheaper to repair the line after an excessively severe sleet storm than it is to make it strong enough to sustain much more than one eighth of an inch of ice on the wires.

The permissible values of S may be taken from the table of tensile strengths of timber.

(2) The stresses due to unbalanced tensions are the most important stresses to be considered in pins and poles. Having given the value of the tension and the angle turned at a corner, the side force, W', Fig. 27, is easily determined, and the dimensions, l' and d' (diameter of pin at base), Fig. 27, must satisfy the equation:

$$S' = \frac{32W'l'}{\pi d'^3} \qquad (2)$$

in which S' is the maximum permissible fiber stress in pounds per square inch, W' is the resultant horizontal force in pounds acting on the insulator, and l' and d' are expressed in inches.

The cross-arms on a corner pole are usually set so as to be parallel to the resultant force due to wire tensions, and hence, except at the end of a line, this resultant force does not produce a bending stress in the cross-arms.

The unbalanced tensions of the wires produce bending stresses in the poles; and the diameter, d', of the pole at the ground and the height, l', of the pole, both in inches, must satisfy equation (2), using for W' the resultant horizontal force due to all of the wires. In most cases a corner pole is guyed or braced so that the bending stress in the pole is to a great extent eliminated.

(3) Stresses due to wind pressure vary with the direction as well as the velocity of the wind. When the wind blows parallel to the line its effect is slight because the wires are parallel to the wind. It is considered sufficient in practice to provide the necessary strength to withstand a side wind giving a maximum pressure of from 20 to 30 pounds per square foot of surface, according to the degree of exposure of the line. In calculating the force of a side wind on a cylinder like a pole or wire, the effective exposed area is taken as two thirds of the product of the diameter of the cylinder times its length.

The effect of a side wind is to produce bending stresses in the insulator pins and in the poles, and the dimensions of the pin in inches, as shown in Fig. 27, must satisfy equation (2), where W' is the total force of the wind on the wire in pounds, and S' is the maximum permissible fiber stress in pounds per square inch. Also the height of l' of the pole and its diameter, d', at the ground, both in inches, must satisfy equation (2), in which case W' is the force of the wind on all the wires plus about half or two thirds of the force of the wind upon the pole and cross-arms.

In estimating the stresses on pin, cross-arm and pole, due to weight of wire and ice, or the stresses due to wind pressure on the wires, a length of wire equal to the distance between adjacent poles must be assumed to be supported by each insulator.

TENSILE STRENGTH OF TIMBER IN POUNDS PER SQUARE INCH.

Cedar (American)	11,000
Chestnut	7,000 to 13,000
Cypress	6,000
Elm	6,000 to 10,000
Oak	10,000
Pitch pine	7,600
Yellow pine	5,000 to 12,000

White pine	8,000
Red wood (California)	11,000
Spruce	5,000 to 10,000

The usual factor of safety being 4 to 6, the permissible fiber stress in pounds per square inch is from one sixth to one fourth of the values given in this table.

Stresses in the wire.—In stringing a wire on poles two things in particular should be provided for, namely, (*a*) an approximate equality of wire tension on the two sides of each insulator, and (*b*) a certain maximum tension in the wire when it is shortened by the coldest winter weather.

The first condition is desirable not only because it relieves the pins, cross-arms, and poles from unnecessary stress, but also because it is difficult to tie a line wire to an insulator so that it cannot slip lengthwise through the tie, unless the line wire is bent, which it should not be if it can be avoided. The horizontal components of the wire tension can always be made equal on the two sides of an insulator; but in the case of a pole line on a grade the vertical component of the wire tension will be somewhat greater on the down-hill side of an insulator when the horizontal components are equal.

The second condition is explained in the following discussion.

Pole line on a level.—The calculation of the tension in a span of wire in terms of length of span, vertical sag at the center of the span, and weight of the wire, or the calculation of the sag corresponding to a prescribed tension, is based upon the equation of the curve formed by the wire. When the sag is a small fraction of the length of the span, say one twentieth or less, the curve formed by the wire is sensibly a parabola and the working formulæ are:

$$T = \frac{l^2 w}{8h} \tag{3}$$

and

$$s = l + \frac{8h^2}{3l} \tag{4}$$

in which T is the tension of the wire in pounds, l is the length of the span in feet, h is the sag at the center of the span as shown in Fig. 28, s is the length in feet of the wire in a span, and w is the weight of the wire in pounds per foot. Equation (3) gives the tension of the wire at the center of the span. The tension at the ends of the span is wh pounds greater than at the center; but this difference amounts to only 2 per cent. when the sag is one twentieth of the length of the span, and it is always negligible. The important use of equation (4) is in making allowance for the effects of changes of temperature.

Equation (3) when solved for l gives:

$$l = \sqrt{\frac{8hT'}{w}}$$

where T' represents the maximum safe tension of the wire in pounds, which is equal to the breaking tension T_b in pounds divided by the factor of safety. See following tables.

The spacing of the poles is usually chosen tentatively as the first step in the design of a pole line. When a great deal depends upon the permanence of a line, as in a transmission line supplying power to many customers, the poles are placed close together in order to make the line substantial and in order that the sag may be small enough to avoid the possibility of the wires swaying into contact. Close spacing is especially necessary in the case of heavy wires so as to distribute the weight of the heavy wire over a large number of insulators, the insulator being one of the weakest elements in the construction. Poles are usually spaced as follows on straight-pole lines: (*a*) Heavy power transmission lines about 80 feet, which is the spacing on the Niagara-Buffalo transmission line; (*b*) ordinary electric-lighting circuits in city or suburban districts, from 100 to 125 feet; (*c*) telegraph and telephone lines 125 to 150 feet. In every case the poles should be placed near together where the pole line follows a curve, thus making the line turn a very obtuse corner at each pole, in order to avoid excessive stresses in the supporting structure due to unbalanced tensions of the wire. Furthermore pole spacing is often determined by surrounding local conditions such as the presence of obstacles or the recurrence of cross-streets in cities.

The amount of sag in a span of line wire should be small in order to prevent the swaying of the wire by the wind. This swaying is objectionable because it tends to break the wire where it is fastened to the insulators and because it is likely to bring adjacent wires into contact. Once the spacing of the poles is chosen, the minimum permissible sag is determined as explained in the next paragraph; although the amount of sag that may be allowed has a great deal to do with the choice of the pole spacing. Very long spans, such as spans across rivers, have a sag equal to one twentieth or one thirtieth of the span. In ordinary pole-lines the sag seldom exceeds one one-hundred-and-fiftieth of the length of span, in coldest weather.

Effects of temperature.—Wires are usually strung on poles during warm weather, the wire grows shorter as the temperature falls, and the tension of the wire is therefore greatly increased during cold winter weather. Hence, it is important to string a wire with sufficient sag (and a correspondingly low tension) so that the coldest weather may not increase the tension of the wire beyond the safe value, T'. Knowing the temperature, t, of the wire when it is strung, and the lowest winter temperature, t', the calculation of the necessary sag h, and tension, T, at temperature, t, is carried out as follows: Take the values of T' ($= T_b$ divided by the factor of safety) and w from the following tables, and from these, together with the chosen distance, l, between poles, calculate the winter sag, h', using equation (3), and calculate the corresponding length of wire, s', in a span using equation (4). Then calculate the length of the wire at summer temperature, t, by the equation

$$s = s' [1 + \beta (t - t')]$$

in which β is the coefficient of linear expansion of the wire as given in the following tables. From the value of s, so calculated, the value of the sag, h, at temperature, t, may be calculated from equation (4), and then finally the tension, T, at summer temperature, t, may be calculated from equation (3).

It is to be noted that as a line wire cools and shortens, its tension increases, so that its thermal contraction is accompanied by an elastic elongation due to the increase of tension; but this effect is generally neglected in practical line calculations, inasmuch as the error is always on the safe side, that is, the actual winter tension is less than that anticipated in the calculations.

It is to be remembered that equations (3) and (4) are approximate because equation (3) is based on the assumption that the suspended wire forms a parabolic curve, and equation (4) is based on the assumption that this parabola is sensibly a circle.

For very long spans or where the sag is greater than about 0.05 of the length of span more accurate equations are desirable.*

Pole line on a grade.—It is usual to make the horizontal component of the tension of the wire the same in value all along a pole line on a grade, so that the actual tension of the wire is slightly greater on the down-hill side than on the up-hill side of each pole. The problem of determining the sag corresponding to a given horizontal tension, and the problem of allowing for the effects of temperature are treated in the same way as in case of a pole line on a level except that the following equations are used instead of equations (3) and (4):

$$T = \frac{l^2 H w}{2d^2}\left(1 - \sqrt{1 - \frac{d}{H}}\right)^2 \tag{5}$$

$$s = l + \frac{2Hd}{3l\left(1 - \sqrt{1 - \frac{d}{H}}\right)} + \frac{2(H-d)^2 d}{3ld + 3lH\left(1 - \sqrt{1 - \frac{d}{H}}\right)} \tag{6}$$

in which T, l, w, and s represent the same quantities as in equations (3) and (4), d is the difference in level between the ends of the span, and H is the sag of the wire below the upper end of the span, as shown in Fig. 29.

TENSILE STRENGTHS, WEIGHTS AND COEFFICIENTS OF EXPANSION OF WIRES.

	Tensile Strength in Pounds per Circular Mil $= a$.	Density in Pounds per Mil-Foot $= b$.	β-Coefficient of Linear Expansion per Degree F.
Steel...............	0.0785	2.65×10^{-6}	0.0000064
Iron................	0.0417	2.65×10^{-6}	0.0000064
Hard-drawn copper..	0.0300	3.03×10^{-6}	0.0000094
Aluminum.........	0.0204	0.91×10^{-6}	0.0000128

* Charts for facilitating these accurate calculations of long spans have been constructed by Mr. Percy H. Thomas. See *Proceedings American Institute of Electrical Engineers*, Vol. XXX, pages 1131–1142, June, 1911.

Other important papers on long spans are: W. LeRoy Robertson, *Proceedings American Institute of Electrical Engineers*, Vol. XXX, pages 1111–1130, June, 1911; Pender and Thomson, *Proceedings American Institute of Electrical Engineers*, Usual factor of safety from 2 or 3 in warm climates to 6 or 7 in cold climates.

Factor of safety for aluminum must be larger than for other metals on account of low elastic limit of aluminum.

Breaking tension of wire $T_b = ad^2$ in pounds.

Weight of wire $w = bd^2$ in pounds per foot, where d is the diameter of the wire in mils.

Length at $t°$ F. = length at $t' \times [1 + \beta(t - t')]$.

For weight of galvanized iron or steel wire add about 6 per cent. to weight of plain wire.

Derivation of equations (3) to (6).—Consider a wire, Fig. 28, suspended between two points, p and p'. If the wire is nowhere greatly inclined the actual length of any element, ab, of the wire is very nearly equal to the horizontal projection, dx, of the element. Therefore the weight of the element is very nearly equal to

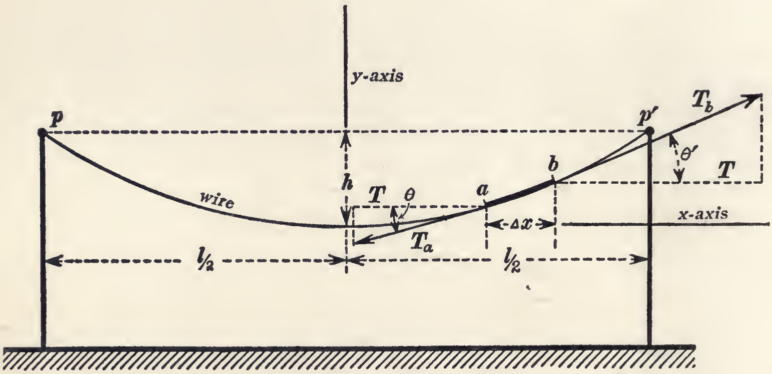


Fig. 28.

$w \cdot dx$, w being the weight of the wire per unit length. Furthermore, the horizontal component of the tension of the wire has necessarily the same value, T, all along the span of wire.

Consider the element, ab, of the wire of which the coördinates of the end, a, are x and y, and the coördinates of the end, b, are $x + dx$ and $y + dy$. Let dy/dx be the value of the first differential coefficient of y at the end, a, then $dy/dx + d^2y/dx^2 \cdot dx$ is its value at the end, b. The force, T_a, pulling at the end, a, of the element is the tension of the wire at a, its horizontal component is T, and its component vertically downwards is $T \tan \theta$ or $T dy/dx$. The force T_b pulling at the end, b, of the element is the tension of the wire at b, its horizontal component is T, and its component vertically upwards is $T \tan \theta'$ or $T(dy/dx + d^2y/dx^2 \cdot dx)$. Therefore the unbalanced force pulling upwards on the element, ab, is $T d^2y/dx^2 \cdot dx$,

Vol. XXX, pages 1379–1416, July, 1911; and F. O. Blackwell, *Transactions International Electrical Congress*, Vol. II, pages 331–347, St. Louis, 1904.

and this unbalanced force is equal to the weight of the element, $w \cdot dx$, so that·

$$T\frac{d^2y}{dx^2} = w \qquad \text{(i)}$$

whence

$$Ty = \tfrac{1}{2}wx^2 + cx + c'$$

but, since $y = 0$ and $dy/dx = 0$ when $x = 0$, the constants, c and c', must be each equal to zero, so that:

$$y = \frac{w}{2T} \cdot x^2 \qquad \text{(ii)}$$

From Fig. 28 it is evident that $y = h$ when $x = l/2$; therefore, substituting these values in equation (ii), we have equation (3).

The second member of equation (4) consists of the first two terms of the infinite series which expresses the length of the arc of a parabola in terms of its chord, l, and the distance, h, of the middle of the arc from the chord.

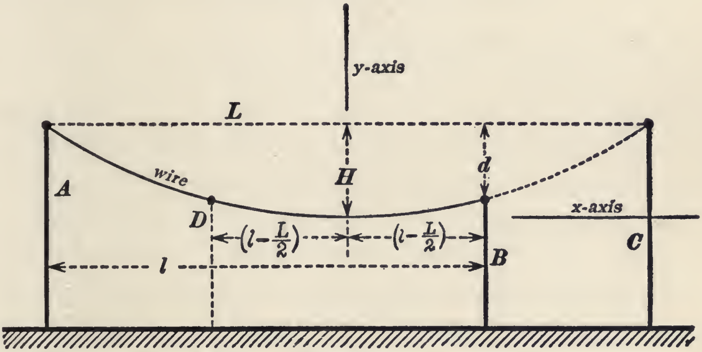


Fig. 29.

Equations (5) and (6) are derived from equations (3) and (4). Consider a given span of wire between two poles, A and B, Fig. 29, at a horizontal distance, l, from each other, d being the difference in level of the tops of the poles, and H the sag of the wire below the top of pole, A, as shown. The given span, AB, may be considered as part of a longer span, AC, of which the length is L, as shown in the figure; and the portion, BD, of the given span may be looked upon as a span also. Let S be the length of wire in the long span, AC, and P the length of wire in the short span, BD. Then the length of wire in the given span, AB, is,

$$s = \frac{S}{2} + \frac{P}{2} \qquad \text{(iii)}$$

Furthermore, applying equations (3) and (4) to the span, AC, we have

$$T = \frac{L^2w}{8H} \qquad \text{(iv)}$$

and

$$S = L + \frac{8H^2}{3L} \tag{v}$$

Applying equation (4) to the span, BD, gives:

$$P = (2l - L) + \frac{8(H - d)^2}{3(2l - L)} \tag{vi}$$

The equation of the parabolic curve formed by the wire is

$$y = \frac{4H}{L^2} \cdot x^2;$$

and at the top of the pole, B, $y = H - d$, and $x = l - L/2$, so that:

$$H - d = \frac{4H}{L^2}\left(l - \frac{L}{2}\right)^2 \tag{vii}$$

Equations (5) and (6) are obtained by eliminating L, P and S from (iii) and (iv) by means of equations (v), (vi) and (vii).

22. Safe carrying capacity.—An electric wire rises in temperature until it gives off heat to its surroundings as fast as heat is generated in it by the current. Therefore, the rise of temperature for a given current (or the current which corresponds to a prescribed rise of temperature) depends upon the facility with which the wire gives off heat and this facility varies greatly with the degree of ventilation in the region in which the wire is placed and with the nature of the adjacent materials. Thus a wire entirely covered with a wooden moulding gets hotter than a wire exposed to the open air, and a wire which lies against a wooden wall gets hotter than a wire which lies against a stone wall.

The opposite table gives the safe carrying capacity of wires according to the National Board of Fire Underwriters. This table refers to the most unfavorable cases, namely, when wires are covered with wooden moulding or enclosed in narrow air spaces inside the walls of building.

23. Voltage drop as a factor determining the size of wires.—The so called "constant-voltage" system of distribution is generally used in electric-light and power installations, and the sizes of distributing wires in such a system are usually determined from a *prescribed allowable voltage drop* (loss of voltage between the generator and the lamps). When more and more lamps are put into service as the darkness of evening comes on the current flowing over the distributing wires increases and a larger and larger loss of voltage takes place in the wires causing a decrease of voltage at the lamps even though the voltage at the generator be kept at a fixed value. Such variation of voltage at the lamps causes an undesirable variation of brightness, *the various lamps and motors are not independent of each other*, and therefore it is necessary to provide for small drop of voltage in the distributing wires so as to obviate large fluctuations of voltage at the lamps.

TABLE OF CARRYING CAPACITIES OF COPPER WIRES.

(*From National Electrical Code.*)

For insulated aluminum wire the safe carrying capacity is eighty-four per cent. of that given in the following tables for copper wire with the same kind of insulation.

Brown and Sharpe Gauge.	Sectional Area in Circular Mils.	Rubber Insulation. Amperes.	Other Insulation. Amperes.	Bare Wires in Still Air for 50° F. Rise of Temperature.
18	1,624	3	5	6.0
16	2,583	6	8	8.5
14	4,107	12	16	12.1
12	6,530	17	23	17.1
10	10,380	24	32	24.3
8	16,510	33	46	41.5
6	26,350	46	65	58.8
5	33,100	54	77	69.7
4	41,740	65	92	83.3
3	52,630	76	110	98.8
2	66,370	90	131	117.6
1	83,690	107	156	140.0
0	105,500	127	185	169.8
00	133,100	150	220	201.5
000	167,800	177	262	240.2
0000	211,600	210	312	286.0
——	400,000	330	500	463.0
——	600,000	450	680	631.0
——	1,000,000	650	1,000	922.0
——	1,500,000	850	1,360	1,250.0
——	2,000,000	1,050	1,670	1,550.0

The lower carrying capacities of rubber-covered wires is due to a tendency of rubber to deteriorate rapidly when warm.

The question of voltage drop is not considered in this table.

Figure 30 represents a central station with a pair of feeders delivering current to a *center of distribution C* from which pairs of *street mains* *m*, *m*, *m* radiate. The points *p*, *p*, *p* are here called *service points*, and the wires which lead from the service points into the houses are called *service wires*.

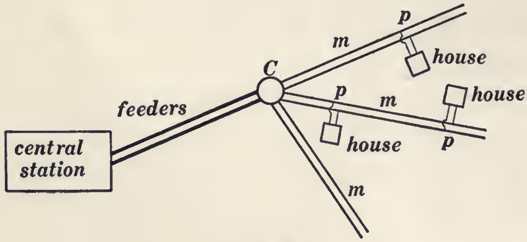


Fig. 30.

The loss of voltage in the feeders is usually compensated by what is called *feeder control* at the station. This is especially the case in alternating-current distribution because in this case feeder control is easily accomplished.* Thus we have constant voltage at the center of distribution *C*. The loss of voltage in the street mains and the loss of voltage in the service wires and house wires are not compensated, and these voltage losses must therefore be small because they affect the value of the voltage at the lamps. A total voltage drop of about five per cent. (two per cent. in the street mains and three per cent. in the service and house wires) is frequently allowed, although a greater or less drop may be advisable if the lamps are very far from or very close to the center of distribution.

There are two causes of voltage drop in distributing wires, namely, resistance and reactance.† Resistance drop of voltage occurs in direct-current and alternating-current distribution. Reactance drop of voltage occurs only in alternating-current distribution.

The voltage across a group of glow lamps is not appreciably affected by reactance drop in the service wires if the wires supply current to glow lamps only. The peculiarity of glow lamps is that they are non-inductive, and the *present discussion refers*

* See Art. 193, *Dynamos and Motors*.

† See Art. 112, *Dynamos and Motors*.

alike to alternating and direct currents when the receiver is non-inductive, that is, when the receiver has unity power factor.*

The resistance drop of voltage along a distributing line is equal to RI volts, where R is the resistance of the line (both wires) in ohms and I is the current in amperes which flows out in one wire and back in the other.

The resistance in ohms of the two wires (copper) of a line is given by the equation

$$R = 10.8 \frac{2l}{d^2} \tag{7}$$

in which l is the length of the line in feet ($2l$ is the length of the two wires), and d is the diameter of the wire in mils. One mil is equal to 0.001 inch.

The weight W in pounds of $2l$ feet of copper wire d mils in diameter is given by the equation

$$W = 0.00000303 \times 2ld^2 \tag{8}$$

Proposition.—The weight in pounds of the wire required to transmit a given amount of power with a given percentage loss of voltage is proportional to l^2/E^2, where l is the distance from the generator to the lamps and E is the voltage of the generator.

Proof.—Let P be the power in watts to be delivered by the generator and let p be the percentage loss of voltage (actual loss of voltage equals $pE/100$). Then P/E is the curent in the line in amperes, and $R \times P/E$ is the loss of voltage in the line. Therefore

$$\frac{pE}{100} = \frac{RP}{E} \tag{i}$$

Eliminating d^2 between equations (7) and (8) we get R in terms of W and l; then substituting this value of R in equation (i) and solving for W we have

$$W = 0.01308 \frac{Pl^2}{pE^2} \tag{9}$$

* See *Dynamos and Motors*, Art. 49.

Inasmuch as the weight of the copper is inversely proportional to E^2, according to equation (9), it is evident that a very great saving in copper may be effected by using high voltage. The permissible voltage at the lamps is limited, however, (*a*) by the fact that incandescent lamps cannot be made to operate satisfactorily at voltages higher than about 220 volts, and (*b*) by the danger that is involved in the use of high voltages.

The saving in copper by the use of high voltage, combined with the practical necessity of low-voltage delivery, has led to the use of the Edison three-wire system as explained in Art. 17. In the alternating-current system of distribution power can be transmitted at any desired high voltage and cheaply and efficiently transformed near the place of consumption to any desired low voltage. Therefore the alternating-current system permits of very great economy of copper in the transmission lines and does not involve any of the difficulties or dangers incident to the utilization of high voltages at lamps and motors.

When the voltage-drop in a transmission line is not limited by the necessity of maintaining an approximately constant voltage at the lamps, or other receiving units, the size of wire should be determined on the basis of economic considerations as explained in Art. 26; *and it is to be particularly noted that the weight of copper, demanded by economic considerations, for the delivery of a given amount of power is not proportional to* l^2/E^2 *but to* l/E.

24. Wiring calculations for a motor or for a concentrated group of lamps.—Two important cases arise in the laying out of wiring plans in a constant-voltage system, namely, (*a*) the case in which current is delivered at one point to a motor or to a group of lamps, constituting what is called a *concentrated load;* and (*b*) the case in which current is delivered to a scattered group of lamps or motors, constituting what is called a *distributed load.*

The problem of determining the size of wire required to deliver a specified amount of power P (in watts) to a **concentrated load** at a specified voltage E (at the lamps) with a specified drop of voltage D in the line is solved as follows:

(*a*) The current I is equal to P/E. Sometimes the current is given directly, as when a given number of half-ampere lamps, for example, are to be supplied.

(*b*) The given voltage drop D is equal to RI, so that, I being known, the resistance R of the line (both wires) may be determined.

(*c*) Knowing the length $2l$ of the wire and its resistance R, its diameter d in mils may be calculated with the help of equation (7).

The final result (current being given) is expressed by the equation

$$d^2 = \frac{21.6lI}{D} \qquad (10)$$

in which d^2 is the sectional area in circular mils of copper wires required to deliver I amperes to a group of lamps distant l feet from the center of distribution, D is the voltage drop, and E is the voltage at the lamps.

Note 1.—In laying out the wiring for a house much time is saved by using *wiring charts* which give at a glance the solution of equation (10) for any particular case.

Note 2.—Equation (10) is frequently used to determine approximately the size of wires required to deliver a specified current to a *distributed load*. In this case l is the distance from the center of distribution to the middle point of the distributed load and D is the allowable voltage drop. See rule 2 on page 62.

25. Wiring calculations for distributed loads.—When a group of widely distributed lamps is supplied with current by one pair of service wires, or when a group of widely distributed customers is supplied by one pair of street mains, we have what is called a *distributed load*. The problem of determining the size of street mains to supply a number of scattered customers is the same as the problem of determining the size of service wires to supply a number of scattered lamps. In the first case the voltage-drop between the center of distribution and the various service points is the important thing, and in the second case the voltage-drop between the service point and the individual lamps is the important thing.

In a distributed load two kinds of variation of voltage occur, namely, (*a*) the variation of voltage *from lamp to lamp* when the number of lamps in operation is fixed, and (*b*) the variation of voltage *at any given lamp* as the number of lamps in operation is increased or decreased.

Concerning the first type of variation it may be stated in general that the lamp voltage is less and less the more remote the lamp is from the service point, the most remote lamp having always the lowest voltage.

Concerning the second type of variation it may be stated in general that the voltage at every lamp falls off to some extent when additional lamps are turned on, and rises when lamps already in operation are turned off. The range of variation in voltage at a given lamp is from a lowest value, when all the lamps are in operation, to a value very nearly equal to the voltage at the service point, when the given lamp, only, is in operation. Therefore, the lamp that is most remote from the service point is subject to the greatest range of variation of voltage as other lamps are turned on and off.

There are two clearly defined cases that arise in the laying out of wires for distributed loads, namely, *Case I*, in which the lamps supplied by a given pair of service wires are turned on and off separately, and *Case II*, in which all of the lamps supplied by a given pair of service wires are turned on and off together. In the first case the wiring must be laid out so as to keep the voltage variations of both types (*a*) and (*b*) within certain limits; and in the second case the wiring may be laid out with reference to the limitation of voltage variations of the first type only, that is, variations of voltage from lamp to lamp, inasmuch as voltage variations of the second type (*b*) do not exist in Case II.

Case I.—An example of a distributed load is shown in Fig. 31. When all of the lamps are in operation the end lamp, *L*, has the lowest voltage of any lamp in the group, and the voltage at this lamp varies through the greatest range when other lamps are turned off and on. Therefore, if the voltage at the end lamp is

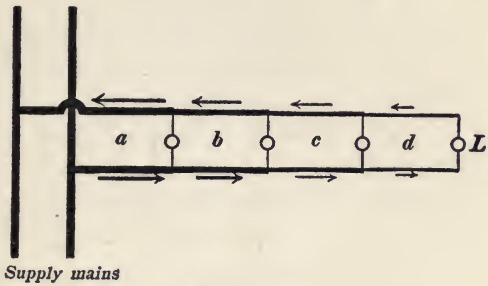


Fig. 31.

to be kept within, say, three volts of its normal value (which is the value when all the lamps are in operation) then the voltage-drop in the wires must not exceed three volts when all the lamps are in operation.

To secure a specified drop out to the end lamp, *L*, when all the lamps are in operation, with the minimum weight of copper in the wires, the sectional area of each portion, *a*, *b*, *c* and *d*, Fig. 31, of the wires must be* proportional to the square root of the current in that portion.† Thus, if each lamp in Fig. 31 takes the same amount of current, then the current values in the portions *a*, *b*, *c* and *d*, are as 4 : 3 : 2 : 1, and the sectional areas of the respective portions of the wires should be as $\sqrt{4} : \sqrt{3} : \sqrt{2} : \sqrt{1}$ in order to give a minimum voltage-drop at the end lamp, *L*, with a given amount of copper, or to give a

* It should be kept in mind that the fundamental condition here is a minimum amount of copper for a given voltage-drop. A minimum amount of copper for given watts lost in the line requires the sectional area of the wires to be proportional to the current at each point; that is, the number of circular mils per ampere must be the same throughout the system to give a minimum amount of copper for a given loss of power in watts. See Art. 26.

† The general proof of this proposition involves the highly elaborate methods of the calculus of variations and therefore the proof of the proposition is not given here.

minimum amount of copper for a specified voltage-drop at the end lamp, L.

In laying out street mains to supply a group of scattered customers it is generally advisable, on account of the large amount of copper involved, to taper the mains in steps in going farther and farther from the center of distribution; but, as a rule, the successive steps should be made longer than the distance between adjacent customers, in order to avoid an excessive number of joints in the mains.

In laying out service wires to supply current to a scattered group of lamps, it is generally not advisable to taper the wires in steps, because the amount of copper involved may not be large; whereas the expense of making many joints, together with the expense of inserting fusible cut-outs at each point where wires of unequal size are joined, as required by the insurance rules, may be considerable.

Rule 1.—When it is desired to reduce the size of a pair of street mains (or service wires) in steps so as to secure the greatest economy of copper, the size of each portion of the mains is determined as follows. Having given the total drop to be allowed out to the end of the line, calculate the factor s from the equation:

$$s = \frac{2 \times 10.8 \times (a\sqrt{i_1} + b\sqrt{i_2} + c\sqrt{i_3} + \cdots)}{\text{total drop in volts}}$$

in which $a, b, c \cdots$ are the lengths in feet of the respective portions of the pair of mains, and $i_1, i_2, i_3 \cdots$ are the currents in amperes in the respective portions. The sectional areas of the various portions of the mains in circular mils are then equal to $s\sqrt{i_1}$, $s\sqrt{i_2}$, $s\sqrt{i_3}$, $\cdots$ respectively.

Rule 2.—When service wires of uniform size are to be used for supplying current to a scattered group of lamps, the size of the wire to give a prescribed drop may be determined as follows: Estimate the distance, L, of the "center of gravity" of the group of lamps by the formula:

$$L = \frac{l' + l'' + l''' + \cdots}{n}$$

where l', l'', l''', etc., are the distances in feet of the individual lamps* from which the service point and n is the total number of lamps in the group. Then calculate the size of the wire that would be required to supply the n lamps as a concentrated group at the prescribed total drop and at the distance L from the service point.

Case II.—When the lamps of a group *are always turned on and off together* the variation of voltage from lamp to lamp can of course be kept within bounds by limiting the voltage-drop between the service point and the end lamp of the group as in Case I. In fact a group of lamps which is to be operated as a unit is generally wired according to rules 1 and 2 of Case I; but a special wiring scheme, called the *return-loop* scheme,† may be used to eliminate voltage variations of the first type (see page 60) in a group of lamps that is operated as a unit, whatever the total voltage-drop may be.

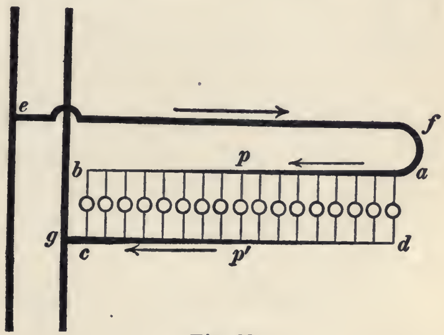


Fig. 32.

The fundamental idea of the return-loop scheme may be seen with the help of Fig. 32. The current in the wire, *ab*, at any point, *p*, is proportional to the distance, *pb*, the lamps being assumed to be uniformly distributed; and the current at any point, *p'*, in the wire, *cd*, is proportional to the distance *p'd*. If the wires, *ab* and *cd*, are tapered so as to have sectional areas proportional to the current at each point, then the value of Ri is the same in both wires between any pair of lamps; but Ri is a *drop* of voltage in a given direction along one wire and a

* If the lamps are arranged in subgroups it is easier to take l' as the product of the distance of the first subgroup times the number of lamps in that subgroup, l'' as the product of the distance of the second subgroup times the number of lamps in that subgroup, and so on.

† Sometimes called the *anti-parallel* scheme.

rise of voltage in the same direction along the other wire, therefore the lamp voltage is constant throughout the group of lamps, whatever the total voltage-drop between the service point and the lamps may be.

The use of tapered wires is of course impracticable and the return loop scheme is always carried out either with wires tapered in steps or with wires of uniform size, usually the latter. Under such conditions the voltage varies to some extent from lamp to lamp but the range of this variation is very much less than the total drop.

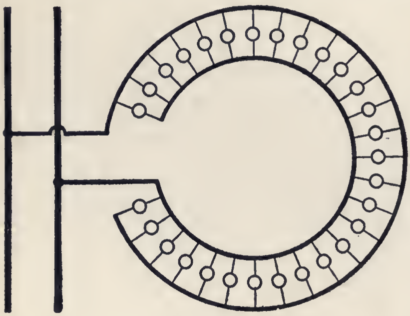


Fig. 33.

The return-loop scheme of wiring evidently requires three wires of a given length instead of two, and therefore it requires much more copper than the simple parallel wiring scheme for the same total voltage-drop. The advantage of the return loop scheme however is that a very large voltage-drop is permissible.* See page 63.

The return-loop scheme is usually employed in the wiring of churches, lecture halls and theaters, where the lamps are either all in use or all out of use, or where the lamps in certain groups are either all in use or all out of use.

In many cases the lamps in a group are arranged in a circular or reëntrant row. In such a case the return-loop scheme is carried out as shown in Fig. 33, or as shown in Fig. 34.

Return-loop scheme with wires of uniform size.—When the wires used in the return-loop scheme are of uniform size (not tapered) the middle lamp, *p*, Fig. 35, has the lowest voltage of any lamp in the group, and the size of the wires, *efab* and *gcd*, is usually determined with reference to the voltage-drop between

* The limit should be determined by Kelvin's law, as explained in Art. 26.

the service point, *eg*, and the middle lamp, *p*. Let I be the total current delivered to the group of lamps, and let the lamps be assumed to be uniformly distributed as shown in Fig. 35;

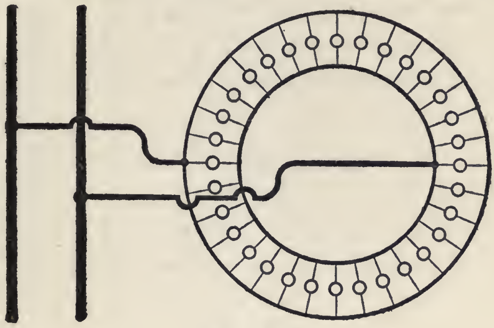


Fig. 34.

then the current in the element, Δx, is $I(X - x)/X$, and the voltage-drop in the element, Δx, is $\rho \cdot \Delta x$ times $I(X - x)/X$, where ρ is the resistance per unit length of the wire, *cd*. Therefore the voltage-drop along *cd* from *c* to the middle lamp is:

$$\frac{\rho I}{X}\int_{x=0}^{x=X/2}(X - x)dx = \tfrac{3}{8}\rho XI$$

Also the voltage-drop along *ab* from *a* to the middle lamp is $\tfrac{3}{8}\rho XI$, so that the total drop between the service point, *eg*, and

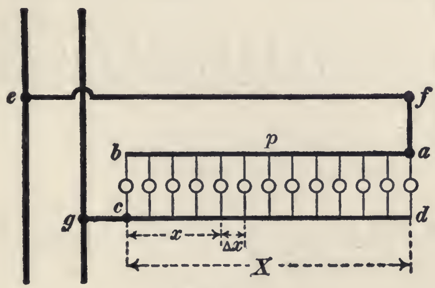


Fig. 35.

the middle lamp, *p*, is $I(r' + r'')$, where r' is the resistance of *gc* plus the resistance of *efa*, and r'' is equal to $\tfrac{3}{4}\rho X$, where ρX is the resistance of one of the wires, *ab* or *cd*, Fig. 35.

6

Rule 3.—To give a prescribed voltage-drop between the service point and the middle lamp of a row, which is connected according to the return-loop scheme, make the wire of such size that the total current delivered to the group of lamps would give the prescribed drop over a length $l' + l''$ of the wire, where l' is the sum of the distances, *gc* and *efa*, in Fig. 35, and l'' is three fourths of the distance *ab*.

Modifications of Cases I and II.—Every practical case of wiring in the constant-voltage system of distribution can be treated as a slight modification of Cases I and II above described. Thus Fig. 36 shows two groups of lamps each exactly like the

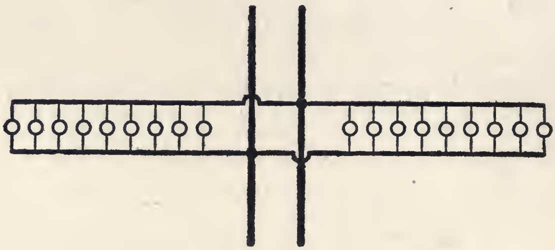


Fig. 36.

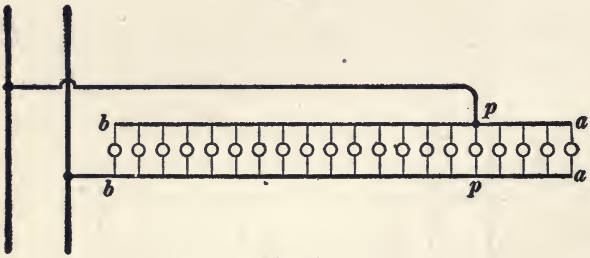


Fig. 37.

single group in Fig. 31; and Fig. 37 shows a combination of Figs. 31 and 32, that is, the portion, *pa*, of the group of lamps in Fig. 37 is arranged in conformity with Fig. 31 and the portion, *pb*, is arranged in accordance with the return loop scheme.

26. The economic balance between loss of power and the cost of copper in the distribution of electric current.—The original cost of erection of a distributing line consists of two nearly independent parts, namely (*a*) the cost of the copper and (*b*) the cost of poles, cross-arms, pins and insulators and the cost of erection. That is to say, even if one were to double the size of wires to be used the cost of item (*b*) would not be increased to any considerable extent. The disadvantage of using large wires lies, therefore, almost wholly in the annual "charge," including interest on the cost of the wire, depreciation of the wire, and taxes thereon. The advantage* of using large wires, on the other hand, lies in the decreased loss of power in the wires. Therefore, the most economical size of wire is that for which the additional annual "charge" on a larger wire would exceed the annual value of the power saved by the use of the larger wire, or, in other words, the most economical size of wire is that for which the sum of the annual "charge" on the total copper plus the annual value of the power lost in the wires is a minimum.

The economic balance between loss of power and cost of copper always leads to a definite number of circular mils of sectional area of wire per ampere of current, without regard to the voltage or to the distance of transmission.

Electric power is to be supplied for h hours each year to a customer. The cost of power at the switchboard is p dollars per kilowatt-hour, the cost of copper is c dollars per pound, and the interest charge on invested capital (including a small percentage to cover the depreciation of copper wires and taxes) is t per cent. per annum. It is required to find the sectional area of the copper wire in circular mils per ampere of current on the condition that any increase in the amount of copper would effect a saving of power of which the annual value would be less than the interest on the cost of the additional copper. Let $2l$ be the length of the wire in feet (equal to twice the length of the line), s its sectional area in circular mils, R its resistance in ohms, W its

* It is to be kept in mind that we are not here considering the fact that in the constant-voltage system the wires must be large enough to limit the voltage-drop, as explained on pages 54–66.

weight in pounds, and I the current in amperes. Then $R = 10.8 \times 2l/s$ so that the lost power in kilowatts is $\frac{21.6}{1,000} \cdot \frac{l}{s} I^2$, and the annual loss of energy is $\frac{21.6}{1,000} \cdot \frac{l}{s} I^2h$ kilowatt-hours, of which the value at p dollars per kilowatt-hour is $\frac{21.6}{1,000} \cdot \frac{l}{s} \cdot I^2ph$ dollars per year. On the other hand $W = 0.00000303 \times 2ls$ pounds, of which the cost is $0.00000606lsc$ dollars, the interest on this cost is $0.0000000606lsct$ dollars per year, and the quantity to be made a minimum by choosing s is $\frac{21.6l}{1,000s} I^2ph + 0.0000000606lsct$. Differentiating this expression with respect to s and placing the differential coefficient equal to zero gives:

$$-\frac{21.6l}{1,000s^2} \cdot I^2ph + 0.0000000606lct = 0$$

from which l cancels out, and we find:

$$\frac{s}{I} = \text{circular mils per ampere} = 597 \sqrt{\frac{ph}{ct}}$$

or

$$s = 597I \sqrt{\frac{ph}{ct}} \qquad (11)$$

The meanings of the symbols, s, I, h, p, c and t, are specified above. When the delivered current, I, is not constant, the average value of the current must not be used, but the square-root-of-the-average-value-of-the-square should be used in equation (11).

Example 1.—The cost of power at the switchboard of an electric-power station is 1.6 cents per kilowatt-hour ($p = 0.016$), the interest on invested capital is 5 per cent., and the annual depreciation and taxes is three per cent. ($t = 8$), the cost of copper wire is 16 cents per pound ($c = 0.16$), and a current of 200 amperes is delivered to a customer for 1,000 hours each year. Considerations of economy would lead, under these conditions, to the use of transmission wires 650 mils in diameter, whatever the distance of the customer from the station may be.

If the distance from the station to the consumer is 538 feet, then the total length of wire is 1,076 feet, its resistance is 0.0275 ohm, and the voltage-drop with 200 amperes is 5.5 volts. That is, if current is to be supplied to the customer at 110 volts, the size of wire required on the basis of a 5 per cent. drop of voltage is the same as the size of the wire required to give an economic balance between the loss of power and the cost of copper under the specified conditions. If the distance is greater than 538 feet, then considerations of economy would give a smaller wire than would be required by a 5 per cent. drop in voltage; and, if the distance is less than 538 feet, then considerations of economy would give a larger wire than would be required by a 5 per cent. drop in voltage.

Example 2.—Cost of power, rate of interest and cost of copper being the same as in example 1, it is required to find the most economical size of wire for carrying 100 amperes for 400 hours each year and 300 amperes for 600 hours. The average square of the current is

$$\frac{(100^2 \times 400) + (300^2 \times 600)}{400 + 600} = 58{,}000 \text{ amperes-squared}$$

and the square-root-of-average-square is 241 amperes. Therefore using $h = 400 + 600$ hours and $I = 241$ amperes in equation (11), we have $s = 509{,}200$ circular mils, or the diameter of the wire must be 713 mils.

Kelvin's law.—*Dependence of total weight of copper on voltage of delivery and distance of customer from station.*—A given amount of power, P, is to be delivered to a customer at a distance, l, from the station and at a voltage, E. The current is P/E so that from equation (10) we have:

$$s = 597\frac{P}{E}\sqrt{\frac{ph}{ct}}$$

which, substituted in the formula $W = 0.00000303 \times 2ls$, gives:

$$W = 0.003618P \frac{l}{E} \sqrt{\frac{ph}{ct}} \qquad (12)$$

which shows that the amount of copper required by economic considerations is proportional to the distance, l, and inversely proportional to the voltage of delivery.

It is important to note the difference between equations (9) and (12). To deliver a specified amount of power requires a weight of copper which is proportional to l^2/E^2 when the percentage voltage drop is fixed whereas it requires a weight of copper proportional to l/E if maximum economy is the ruling condition.

Limitations of Kelvin's law.—The economic balance between the loss of power in transmission wires and the cost of copper was first pointed put by Lord Kelvin, and the condition expressed by equation (12) is sometimes called Kelvin's law of economy.* In the derivation of equation (12) it was assumed, first, that the cost of poles, cross-arms and pins, and the cost of erection of the pole line are the same whatever the size of the wire may be, and second, that the cost of the wire is so much per pound irrespective of size. The first assumption is approximately true only for wires of moderate weight. For very heavy wires the supporting structure must be very strong and therefore expensive. The second assumption is approximately true only for bare wires. For insulated wires the cost per pound varies considerably with the size of the wire.

27. Corona formation as a factor determining the size of wires.—When the voltage† between two line wires is increased more and more a point is ultimately reached where the air in

* A very full discussion of Kelvin's law is given by Dr. F. A. C. Perrine in his book entitled *Conductors for Electric Distribution*, pages 161–178 (D. Van Nostrand, 1903).

† This article refers to alternating voltages. Direct voltages exceeding a few thousand volts never occur in ordinary engineering practice.

the neighborhood of the wires breaks down and we have what is called the *corona*, a faint bluish glow surrounding the wires. A sufficiently high voltage causes the complete break-down of the air insulation and the formation of an electric arc from wire to wire.

The corona may be easily shown as follows: Two very fine wires (line wires) are supported on glass rods at a distance of 6 or 8 inches apart in a very dark room, and the wires are connected to the secondary of a good-sized induction coil the primary of which is connected to alternating current supply mains. A much smaller induction coil will suffice if its primary is excited by direct current using a Wehnelt interrupter.

By using two pairs of "line wires," one pair much finer than the other, and connecting both pairs to the induction coil simultaneously it can be shown that the corona starts on the finer wires more easily (at a lower voltage) than on the coarser wires. Indeed the approximate voltage E required to start an electrical breakdown of the air between two line wires is:

$$E = 150d \cdot \log_{10}\left(\frac{2S}{d}\right) \tag{13}$$

in which d is the diameter of the line wires in mils, and S is the distance of the wires apart center to center in mils. Thus about 55,000 volts is required to start the corona on wires 125 mils in diameter and 4 feet apart center to center; and about 3,000 volts is required to start the corona on wires 6 mils in diameter and 6 inches apart center to center. In the case of very fine wires, however, the corona is limited to the region very near to the wires unless the voltage greatly exceeds the corona-starting voltage.

The only cases in practice where corona formation occurs is near the very high-voltage terminals of transformers and on very high-voltage transmission lines, and in such cases d and S are both made large enough to give a corona-starting voltage about two times as great as the voltage which is used.

Thus half-inch line wires would have to be 25 inches apart center to center to require a corona-starting voltage of 150,000 volts, and therefore half-inch wires 25 inches apart would be suitable for about 75,000 volts;* or if the distance apart of the wires is fixed at 25 inches it would not be allowable to use wires much smaller than 500 mils in diameter on a 75,000-volt transmission line.

A very full discussion of electrical stresses and a derivation of equation (13) are given in Appendix A.

28. Pole line insulation.—When the voltage between two line wires is distinctly less than that required to cause an electrical break-down of the air in the neighborhood of the wires, the leakage of current through the air from wire to wire is entirely negligible; the only appreciable leakage of current is at the supporting insulators and through branches of trees and other objects which happen to touch the wires.

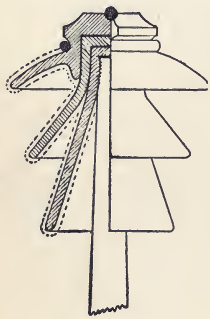


Fig. 38.

If the insulators are made of glass or thoroughly vitrified porcelain the leakage of the current through the material of the insulator is always negligible, unless the insulator is ruptured, but the leakage of current over the surface of the insulator may be considerable. This leakage over the surface of an insulator is reduced to a minimum by designing the insulator so that the leakage path measured along the surface (see dotted line in Fig. 38) may be as long as possible and so that a portion of the surface may be shielded from rain or mist. This is accomplished by making a series of deep grooves around the bottom of the insulator as shown by the dotted lines at the base of the insulator in Fig. 27. Figure 38 shows a typical high-voltage insulator. This type of insulator is deficient in

* An alternating voltage of 75,000 effective value has a maximum value of about 105,000 volts on the peak of the wave so that a corona-starting voltage of 150,000 volts on a 75,000-volt transmission line would mean a "factor of safety" of about 3/2.

mechanical strength and a more recent type called a strain insulator is shown in Fig. 39. The strain insulator is used as a link in a chain which supports the transmission wire as shown in Fig. 40.

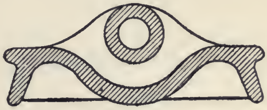


Fig. 39.

The strength of an insulator to withstand high voltage without rupture has nothing directly to do with its insulation resistance, in the same way, that the strength of a porous earthenware jar to withstand hydraulic pressure without bursting has nothing directly to do with the facility with which the porous walls of the jar permit the water to flow through them. Therefore the most important test of a high-voltage insulator is the break-down test. In this test the insulator pin and a wire tied around the insulator are connected to the secondary of a high-voltage testing transformer and the voltage is raised to the desired value. In testing a lot of insulators which are to be used the test voltage is run up to $1\frac{1}{2}$ or 2 times the voltage which the insulators are to stand in service.

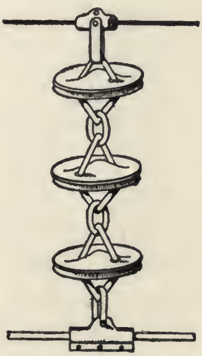


Fig. 40.

Wires on pole lines are provided with insulating covering only when it is desired to reduce the risk of accidental *momentary* contacts. Thus the moderately high-voltage lines (2,200 to 5,000 volts) which supply arc lamps on city streets usually have insulating covering. The wires of high-voltage transmission lines are always bare because any ordinary insulating covering would be entirely inadequate.

29. Insulation of underground, house and station wires.—In the installation of underground, house and station wires, two things are kept in view, namely, (*a*) the insulation of the wires and (*b*) the protection of the wires from mechanical injury. It is especially important to use an absolutely waterproof insulation such as rubber or lead encased fiber; and the most satisfactory mechanical protection is that which is afforded by an iron pipe or by a vitrified clay conduit laid in concrete or built in the floor and walls of a building.

30. National Electrical Code.—Rules governing the installation of electrical apparatus of all kinds have been formulated by a national conference* with the object of minimizing fire risks and risks of personal injury. These rules are published in convenient form and sold at a nominal price by the National Board of Fire Underwriters† under the title "The National Electrical Code."

The rules and requirements which constitute the National Electrical Code are classified in the 1911 edition as follows:

Class *A*. Rules applying to stations and dynamo rooms.

Class *B*. Rules applying to outside work.

Class *C*. Rules applying to inside work (wiring and lamp and motor installations).

Class *D*. Rules applying to fittings, materials and details of construction.

Class *E*. Miscellaneous. Rules applying to telephone and telegraph, fire and burglar alarms, etc.

Class *F*. Marine work.

Classes *A*, *B*, *C* and *E* are issued in one volume for general distribution, and Classes *D* and *F* are issued in another volume which may be obtained from the National Board by any one interested in the special rules contained therein. Contractors and station managers are specially interested in the list of

* The following is a list of the Associations composing this National Conference: American Institute of Architects, American Institute of Electrical Engineers, American Society of Mechanical Engineers, American Institute of Mining Engineers, American Street Railway Association, Associated Factory Mutual Fire Insurance Companies, Association of Edison Illuminating Companies, International Association of Fire Engineers, International Association of Municipal Electricians, National Board of Fire Underwriters, National Electric Light Association, National Electrical Contractors' Association and Underwriters' National Electric Association.

A brief history of the development of the National Electrical Code is given by C. E. Skinner, *Electric Journal*, Vol. II, pages 262–265, January, 1906.

† General agency, 34 Nassau St., New York City.

"approved" fittings which is issued semi-annually by the National Board for general distribution.

WEIGHTS AND RESISTANCES OF COPPER WIRE.

Brown and Sharpe Gauge.

Gauge Numbers.	Diameters in Mils = *d*.	Areas in Circular Mils = d^2.	Weights.		Resistances per 1,000 Feet in International Ohms.	
			Per 1,000 Feet.	Per Mile.	At 60° F.	At 75° F.
0000	460	211,600	641	3,382	.04811	.04966
000	410	168,100	509	2,687	.06056	.06251
00	365	133,225	403	2,129	.07642	.07887
0	325	105,625	320	1,688	.09639	.09948
1	289	83,521	253	1,335	.1219	.1258
2	258	66,564	202	1,064	.1529	.1579
3	229	52,441	159	838	.1941	.2004
4	204	41,616	126	665	.2446	.2525
5	182	33,124	100	529	.3074	.3172
6	162	26,244	79	419	.3879	.4004
7	144	20,736	63	331	.491	.5067
8	128	16,384	50	262	.6214	.6413
9	114	12,996	39	208	.7834	.8085
10	102	10,404	32	166	.9785	1.01
11	91	8,281	25	132	1.229	1.269
12	81	6,561	20	105	1.552	1.601
13	72	5,184	15.7	83	1.964	2.027
14	64	4,096	12.4	65	2.485	2.565
15	57	3,249	9.8	52	3.133	3.234
16	51	2,601	7.9	42	3.914	4.04
17	45	2,025	6.1	32	5.028	5.189
18	40	1,600	4.8	25.6	6.363	6.567
19	36	1,296	3.9	20.7	7.855	8.108
20	32	1,024	3.1	16.4	9.942	10.26
21	28.5	812.3	2.5	13	12.53	12.94
22	25.3	640.1	1.9	10.2	15.9	16.41
23	22.6	510.8	1.5	8.2	19.93	20.57
24	20.1	404	1.2	6.5	25.2	26.01
25	17.9	320.4	.97	5.1	31.77	32.79
26	15.9	252.8	.77	4	40.27	41.56
27	14.2	201.6	.61	3.2	50.49	52.11
28	12.6	158.8	.48	2.5	64.13	66.18
29	11.3	127.7	.39	2	79.73	82.29
30	10	100	.3	1.6	101.8	105.1
31	8.9	79.2	.24	1.27	128.5	132.7
32	8	64	.19	1.02	159.1	164.2
33	7.1	50.4	.15	.81	202	208.4
34	6.3	39.7	.12	.63	256.5	264.7
35	5.6	31.4	.095	.5	324.6	335.1
36	5	25	.076	.4	407.2	420.3

This table is based on a resistance of 10.51 ohms per mil-foot at 75° F., a temperature coefficient of resistance of 0.0022 per degree Fahrenheit, and a density of 3.03×10^{-6} pound per mil-foot, or 555 pounds per cubic foot.

According to the Report of the Standardization Committee of the American Institute of Electrical Engineers* it is commercially feasible to supply annealed copper for electric wires and cables having a resistance not greater than 102 per cent. of the "annealed copper standard," and hard-drawn copper having a resistance not exceeding 105 per cent. of the "annealed copper standard." The annealed copper standard is 10.36 ohms per mil-foot at 20° C.

Aluminum.—Resistances in above table are to be multiplied by 1.61 for aluminum wire.

Weights in above table are to be multiplied by 0.30 for aluminum wire.

Iron and Steel.—Resistances in above table are to be multiplied by about 7 for iron and steel wires.

Weights in above table are to be multiplied by 0.876 for iron and steel wires.

IMPORTANT BOOKS ON WIRING.

National Electrical Code, National Board of Fire Underwriters.

Standard Tables for Electric Wiremen, C. P. Poole, McGraw-Hill Book Co.

Electric Light Wiring, C. E. Knox, McGraw-Hill Book Co.

Conductors for Electrical Distribution, F. A. C. Perrine.

Electrical Transmission of Energy, A. V. Abbott, Van Nostrand. This book contains a very full discussion of pole lines and of underground conduits and cables.

* Paragraph 260 of Rules as adopted on June 27, 1911. See *Proceedings of Institute* for August, 1911, page 1942.

CHAPTER III.

ALTERNATING-CURRENT LINES.

31. Direct-current and alternating-current calculations compared.—Kelvin's law of economy (see Art. 26) applies without distinction to direct current and to alternating current *when the value of the current is given.* If an amount of power P is to be delivered at voltage E, the current is equal to P/E in the case of direct-current transmission, but the current is equal to $P/(pE)$ in the case of alternating-current transmission, where p is the power factor of the receiver.

When the size of wires is to be determined on the basis of a prescribed voltage drop (see Arts. 23–25) then alternating-current calculations differ from direct-current calculations, although *the direct-current method (which refers only to resistance drop of voltage) is approximately correct for alterating-current wiring when the power factor of the receiver is nearly unity.*

This chapter refers to alternating-current wiring calculations on the basis of a prescribed voltage drop but when the power factor of the receiver is not equal to unity.

The methods of this chapter give very accurate results for transmission distances up to ten or twenty miles at the usual alternating-current frequencies, but they give only approximate results for very long transmission lines.

On a very long transmission line the effects of line capacity must be taken into account in precise calculations.* The effect of line capacity is primarily to cause a difference between the current which flows into the line at the generator end and the current which flows out of the line at the receiver end. This

* The simplest rigorous discussion of the alternating-current transmission-line problem is that which is given in Chapter V (pages 116–153) of *Electric Waves*, by W. S. Franklin, The Macmillan Co., 1909. This discussion is easily followed because the geometric and physical features of the problem are kept in view, and

difference is called the *charging current of the line.* The effects of capacity are ignored in this chapter.

32. Resistance drop and reactance drop on a line.*—Figure 41 is a clock diagram in which the line OI represents the current flowing through the transmission line and through the receiving circuit, and the line E_1 represents the voltage across the receiving circuit, θ being the phase difference between E_1 and I as

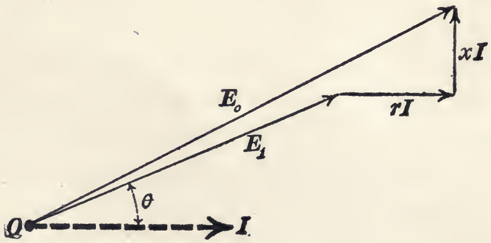


Fig. 41.

shown. Let r be the resistance and x the reactance of the line. Then rI is the *resistance drop* on the line and it is in phase with I (parallel to I in the clock diagram), and xI is the reactance drop on the line and it is 90° ahead of I in phase. The generator voltage E_0 is equal to the vector sum of E_1, rI and xI.

The *numerical difference* between E_0 and E_1 is usually called simply the *line drop*. It is evident from Fig. 41 that the line drop is a complicated function of I, θ, E_1, r and x.

33. Line resistance.—The resistance of a wire for alternating current is in nearly all practical cases sensibly equal to the resistance of the same wire for direct current. When the wire is very large, however, or when the frequency is very high, the alternating

because the solution is expressed in terms of familiar exponential functions. These exponential functions (as they appear in the solution of this problem) *are* the so-called hyperbolic sines and cosines; but it is distinctly misleading to call them such, because to do so is to convey the impression that the mathematics of the problem involves something which is entirely unfamiliar and new, which is distinctly not the case; and every table of logarithms is in effect a table of hyperbolic sines and cosines provided one ignores these names and looks at the simple facts.

* This matter is discussed in *Dynamos and Motors*, Art. 112. A general statement is repeated here for the sake of clearness.

current flows chiefly through the surface layers of the wire, and the resistance of the wire is very perceptibly larger for alternating current than for direct current. This effect is called the *skin effect.**

34. **Line reactance.**—The reactance of a transmission line (outgoing and returning wires side by side) depends upon the size of the wires and upon their distance apart center to center, and it is proportional to the length of the line and to the frequency.†

RESISTANCE AND REACTANCE OF ONE MILE OF WIRE (½ MILE OF TRANSMISSION LINE).

Size of Wire B. & S. Gauge.	Resistance in Ohms.	Reactance in Ohms.					
		At 60 Cycles per Sec.			At 125 Cycles per Sec.		
		Wires 12 Inches Apart.	Wires 18 Inches Apart.	Wires 24 Inches Apart.	Wires 12 Inches Apart.	Wires 18 Inches Apart.	Wires 24 Inches Apart.
0000	.259	.508	.557	.591	1.06	1.17	1.23
000	.324	.523	.573	.607	1.09	1.20	1.26
00	.412	.534	.588	.618	1.12	1.23	1.29
0	.519	.550	.603	.633	1.15	1.26	1.32
1	.655	.565	.614	.648	1.18	1.28	1.35
2	.826	.580	.629	.663	1.21	1.31	1.38
3	1.041	.591	.644	.674	1.24	1.34	1.41
4	1.313	.606	.656	.690	1.26	1.37	1.44
5	1.656	.620	.670	.704	1.30	1.40	1.47
6	2.088	.633	.685	.720	1.32	1.43	1.49
7	2.633	.647	.700	.730	1.35	1.46	1.52
8	3.320	.662	.712	.742	1.38	1.48	1.55
9	4.186	.677	.727	.761	1.41	1.51	1.58
10	5.280	.688	.742	.776	1.44	1.54	1.62

The following table gives the resistance and reactance per half mile of transmission line, using copper wires.

35. **Calculation of a single-phase transmission line to give a specified line drop.**‡—A single-phase transmission line is to

* See Ernest Merritt, *Physical Review*, Vol. V, pages 47–60, July, 1897.

† Derivations of the formulas for line reactance and line capacity are given in *Electric Waves*, by W. S. Franklin, Appendix A.

‡ A good discussion of the exact theory of line drop on a long alternating-current transmission line is given on pages 141–153 of Franklin's *Electric Waves*. The Macmillan Company, 1909.

A series of papers on the exact calculation of alternating-current transmission line problems, by W. F. Miller, is given in the *General Electric Review*, Vol. XIII, deliver a prescribed amount of power P at a prescribed electromotive force E_1 to a receiving circuit of which the power factor, $\cos\theta$, is given; the line drop, frequency, length of time, and distance apart of wires being given.

The generator voltage E_0 is equal to the sum (numerical sum) of E_1 and line drop.

The full load current I is found from the relation $E_1 I \cos\theta$ equals P.

The component of E_1 parallel to I is $E_1 \cos\theta$, and the component of E_1 perpendicular to I is $E_1 \sin\theta$.

By treating the problem first as a direct-current problem, the approximate resistance r' of the line is found from the relation $r'I$ equals line drop. From this approximate resistance and the known length of the line, the approximate size of the wire and the line reactance x may be found from the table; and since the line reactance varies but little with the size of the wire, the value of x need not be further approximated.

The component of E_0 parallel to I is $E_1 \cos\theta + rI$, where r is the true resistance of the line, and the component of E_0 perpendicular to I is $E_1 \sin\theta + xI$. Therefore

$$E_0^2 = (E_1 \cos\theta + rI)^2 + (E_1 \sin\theta + xI)^2$$

or

$$r = \frac{\sqrt{E_0^2 - (E_1 \sin\theta + xI)^2} - E_1 \cos\theta}{I} \qquad \text{(i)}$$

From this equation the true line resistance r may be found and thence the correct size of wire.

Example:

E_1 = 20,000 volts.

P = 1,000 kilowatts.

$\cos\theta$ = 0.85 = power factor of receiving circuit.

E_0 = 23,000 volts, or line drop = 3,000 volts.

frequency = 60 cycles per second.

pages 177–181, 220–227, 264–267, and 326–330, April to July, 1910. These papers refer especially to three-phase lines. A table of hyperbolic functions is published in a supplement to the *General Electric Review* for May, 1910.

distance = 30 miles.

distance apart of wires = 18 inches.

From these data we find:

$I = 58.8$ amperes.

$r' = 51$ ohms.

Therefore, from the table we find that, approximately, a No. 2 B. & S. wire is required so that $x = 37.7$ ohms.

Furthermore,

$$E_1 \cos\theta = 17{,}000 \text{ volts}$$

$$E_1 \sin\theta + xI = 12{,}700 \text{ volts}$$

and from equation (i) we find

$$r = 37.3 \text{ ohms}$$

from which the correct size of wire is found to be, approximately, a No. 1 B. & S.

36. Calculation of double line for two-phase transmission (*four wires*).—In this case each line is calculated to deliver half the prescribed power. Thus, if it is desired to deliver 1,000 kilowatts at 20,000 volts two-phase, at a frequency of 60, line drop of 3,000 volts, etc., then each line is calculated as a single-phase line to deliver 500 kilowatts at 3,000 volts line drop.

37. Calculation of a three-wire transmission line for three-phase currents.—The calculation will be carried out for the case in which both the generator and the receiver are Y-connected as shown in Fig. 42. If it is desired to state the problem by specifying *the voltage between mains* at generator and at receiver,

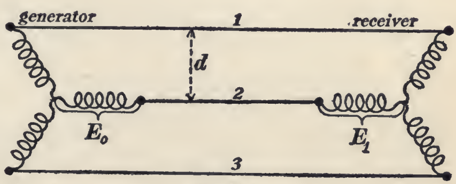


Fig. 42.

and *current in each main*, the specified voltage between mains may be divided by $\sqrt{3}$ to give the values of E_0 and E_1 in Fig. 42.

Let $\cos\theta$ be the power factor of each receiving circuit, P the total power to be delivered, E_1 the electromotive force between the terminals of each receiving circuit, and E_0 the electromotive force of each armature winding on the generator; all prescribed (see Fig. 42). Then

$$P = 3E_1 I \cos \theta$$

from which the full-load line current I may be calculated.

The numerical difference $E_0 - E_1$ is the electromotive force drop in one wire. Therefore, looking upon the problem as one in direct currents, we have $E_0 - E_1 = r'I$, where r' is the approximate resistance of *one wire.* From this the approximate size of the wire may be found from the table.

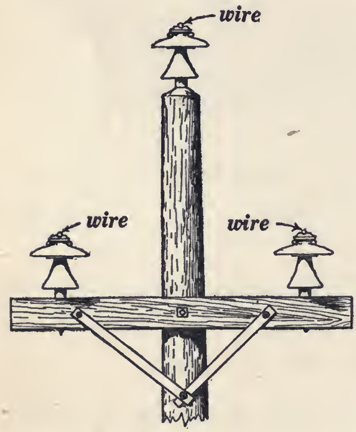


Fig. 43.

Consider one of the wires, say wire number 2. The other two wires together constitute the return circuit for this wire, and, the three wires being arranged as indicated in Fig. 43, the distance from wire number 2 to each of the other wires is equal to d, which is given. Find the reactance x of a *pair* of wires at the prescribed distance apart center to center, from the above table.

The component of E_1 parallel to I is $E_1 \cos \theta$, and the component of E_1 perpendicular to I is $E_1 \sin \theta$.

The resistance drop in one main is rI and the reactance drop in one main is $\frac{1}{2}xI$, the former being parallel to I and the latter being perpendicular to I. Then the components of E_0 are $E_1 \cos \theta + rI$, and $E_1 \sin \theta + \frac{1}{2}xI$, respectively, so that

$$E_0^2 = (E_1 \cos \theta + rI)^2 + (E_1 \sin \theta + \tfrac{1}{2}xI)^2$$

whence

$$r = \frac{\sqrt{E_0^2 - (E_1 \sin \theta + \frac{1}{2}xI)^2} - E_1 \cos \theta}{I} \qquad \text{(i)}$$

which gives the true resistance r *of one wire* from which the correct size of wire is easily found.

Example.—The electromotive force between mains at the receiving station is to be 20,000 volts. Therefore, the electromotive force between terminals of Y-connected receiving circuits would be

$$E_1 = 20{,}000 \div \sqrt{3} = 11{,}550 \text{ volts (see Fig. 42)}$$

The electromotive force between mains at the generating station is to be 23,000 volts. Therefore,

$$E_0 = 23{,}000 \text{ volts} \div \sqrt{3} = 13{,}280 \text{ volts (see Fig. 42)}$$

Further specifications: $P = 1{,}000$ kilowatts, $\cos\theta = 0.85$, frequency = 60 cycles per second, distance = 30 miles, distance apart of wires = 21 inches.

From these data we find $I = 34$ amperes, and $r' = 50.9$ ohms. Therefore, approximately, a number 5 wire is required. The reactance x of a 30-mile double line of number 5 wires, 21 inches apart center to center, at 60 cycles per second, is

$$x = 41.2 \text{ ohms}$$

which substituted in equation (i) gives

$$r = 46.5 \text{ ohms}$$

so that a wire between number 4 and number 5 would give the prescribed line drop.

38. Line interference.—An alternating-current transmission line induces an alternating current in any adjacent line; this is called *line interference.* Line interference depends upon three distinct effects as follows: (*a*) *Magnetic induction.* The alternating-current line acts like the primary of an induction coil and an adjacent telephone line, for example, like the secondary of the induction coil, or in other words, the magnetic action of the alternating current induces an alternating electromotive force in an adjacent line. (*b*) *Electrostatic induction.* The alternating-current transmission wires are repeatedly charged first in one sense and then in the opposite sense with the reversals of the alternating electromotive force, a repeatedly reversed charge is produced on the wires of an adjacent line by influence and the flow of this charge into and out of the adjacent line produces an alternating current in the adjacent line. (*c*) *Leakage.* If the alternating-current line is not thoroughly insulated from the

Fig. 44.

adjacent line more or less current leaks across from one to the other.

Line interference is seldom a serious matter, except in the case of a telephone line exposed to the influence of an alternating-current transmission line, and in such a case the interference may be obviated (*a*) by what is called *transposition of wires* of one of the lines thus tending to eliminate magnetic and electro-

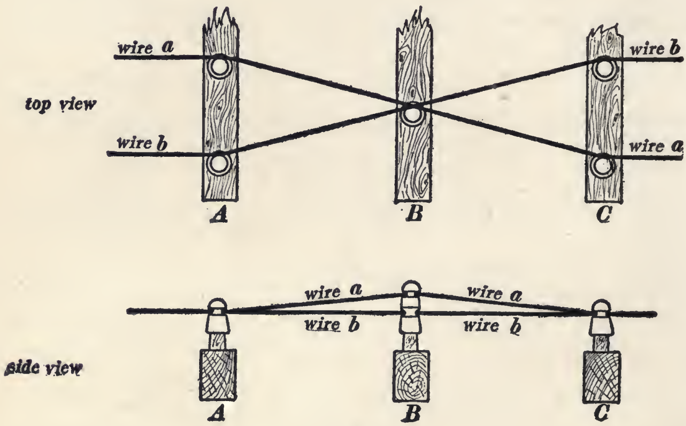


Fig. 45.

static induction, and (*b*) by thorough insulation, thus tending to eliminate leakage.*

* See a paper by P. M. Lincoln, *Transactions of the American Institute of Electrical Engineers*, Vol. XXI, pages 245–251, and a paper by F. F. Fowle, *Transactions of the American Institute of Electrical Engineers*, Vol. XXIII, pages 659–689. The important part of this latter paper is included in pages 674–687.

Figure 44 shows the essential features of a transposed single-phase (two-wire) transmission line. Any two successive sections or loops of the line may be looked upon as complete circuits around which the current flows in opposite directions and across which the voltage is reversed so that the magnetic actions of two successive sections of the transposed line on an adjacent (untransposed) line are opposite to each other, and the electrostatic actions of two successive sections of the transposed line on an adjacent line are opposite to each other.

Line interference between an alternating current line and a telephone line may be eliminated by the transposition of wires of the transmission line or by the transposition of wires of the telephone line. It is more usual to transpose the wires of the

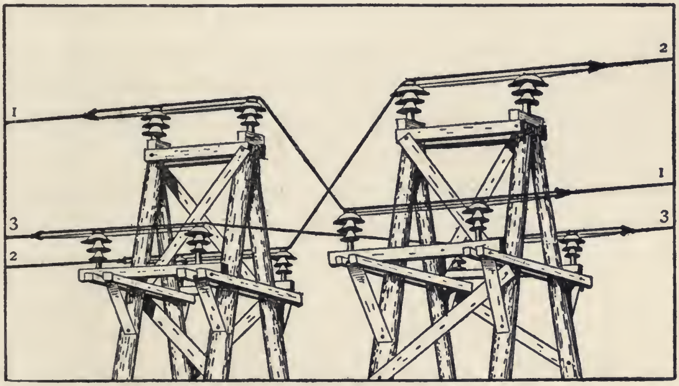


Fig. 46.

telephone line. Figure 45 shows the method of transposing a telephone line. Three cross-arms are attached to the pole at which the transposition is to be made, and upon the middle arm a "two-story" insulator is placed so as to bring one of the wires above the other at the crossing point as indicated in the figure.

Long distance alternating-current transmission lines are always transposed so as to minimize interference with adjacent lines of all kinds. Thus Fig. 46 shows a cross-over or transposition on a three-wire three-phase line.

CHAPTER IV.

PHOTOMETRY AND ILLUMINATION.

39. Radiant heat. Light.—The radiation from a hot body may be resolved into simple component parts, each of which is a train of ether waves of definite wave-length. All of these component parts of the total radiation have one common property, namely, they generate heat in a body which absorbs them. Therefore every portion of the radiation from a hot body is properly called *radiant heat*. The intensity of a beam of radiant heat is measured by the heat it delivers per second to an absorbing body. Thus, the radiant heat emitted by a standard candle represents a flow of about 450 ergs of energy per second across one square centimeter of area at a distance of one meter from the candle.

Radiant heat of which the wave-length lies between 39 and 75 millionths of a centimeter affects the optic nerves and gives rise to sensations of light. Therefore radiant heat of which the wave-length lies between these limits is called *light*. These limits, which are called the limits of the visible spectrum, are not sharply defined, but vary considerably with the intensity of the radiation and with the degree of fatigue of the optic nerves, and they vary greatly with different persons.

40. The physical intensity of a beam of light is measured by its perfectly definite thermal effect, that is, by the heat energy it delivers per second to an absorbing body. Thus, those parts of the radiation of a standard candle which lie within the visible spectrum represent the flow of about 9.3 ergs per second across an area of one square centimeter at a distance of one meter from the candle. Comparing this with the flow of energy which is represented by the total radiation from a standard candle (450 ergs per second across an area of one square centimeter at a distance of one meter from the candle), it follows that only about two per cent. of the energy radiated by the standard candle lies within the visible spectrum, that is, only about two per cent. of the radiation from the standard candle is light. Full sunlight represents a flow of about two million ergs (or 0.2 of a watt) per second across one square centimeter. About one third or one half of this energy is absorbed by the atmosphere. The luminous part of the sun's rays represents about four hundred thousand ergs per second, or 0.04 of a watt per square centimeter.

41. **The luminous intensity of a beam of light** is *presumably* measured by the intensity of the light sensation it can produce, but the intensity of the light sensation which is produced by a given beam of light is extremely indefinite. A given beam of light entering the eye may produce a strong or weak sensation, depending upon various individual peculiarities of the person and on the degree of fatigue of the retina; and the vividness of the sensation depends upon the extent to which it is enhanced by attention. Our sensations are not quantitative in the physical meaning of that term; in fact, they enable us merely to distinguish objects, to judge whether things are alike or unlike, and the certainty and precision with which we can do this is exemplified in every outward aspect of our daily life. *The ratio of the lumionus intensities of two beams of light is measured by using a* **device** *to alter, in a known ratio, the* **physical intensity** *of one beam until it gives, as nearly as one can judge, a degree of illumination on a screen which is equal to (like) the illumination produced by the other beam. Such a device is called a* **photometer.*** The Bunsen photometer is described in Art. 57.

42. **Simple photometry and spectrophotometry.**—The measurement of the light emitted by a lamp is called *photometry*. This measurement is always made by comparing the beam of light

*A very complete and interesting discussion of Photometric Devices is given by C. H. Sharp, on pages 411–506 of Vol. I of the *Johns Hopkins University Lectures on Illuminating Engineering*, The Johns Hopkins Press, 1911.

from a given lamp with the beam of light from a standard lamp as explained in Article 41.

The comparison of the total light in a beam from a given lamp with the total light in a beam from a standard lamp is called *simple photometry;* whereas the comparison, wave-length by wave-length, throughout the spectrum, is called *spectrophotometry.**

The fundamental difficulty in simple photometry is that different lamps usually show differences of color, and these differences of color do not disappear when the attempt is made to adjust a photometer so that two lamps give equal (*like*) illumination on a screen. This difficulty is overcome to some extent by the use of the flicker photometer which is described in Art. 61.

43. Standard lamps. The fundamental light units.—The *British standard candle* is a sperm candle made according to exact specifications.† When this candle burns 120 grains of sperm per hour it is a standard candle, and the actual candle-power during a given test is taken to be $a/120$ where a is the number of grains of sperm actually burned per hour during the test.

The *Hefner lamp,*‡ so called from its inventor, is a lamp which burns pure amyl acetate; the wick and its containing tube are of prescribed dimensions, and the wick is turned up to give a flame of prescribed height.

The Vernon-Harcourt pentane lamp§ is a lamp which burns the vapor of pentane. A stream of air flows through a chamber containing pentane, the air becomes saturated with pentane vapor,

* Some examples of the results of spectrometrical measurements are given in Fig. 79.

† See *American Gas Light Journal,* Vol. LX, page 41, 1894.

‡ A full discussion of the Hefner lamp may be found in *Photometrical Measurements* by Wilbur M. Stine, The Macmillan Co., 1904. In particular, see the discussion of Influence of Atmospheric Moisture, Influence of Carbon Dioxide, Influence of Atmospheric Pressure, and Influence of Atmospheric Temperature on the brightness of the Hefner lamp on pages 153–157.

§ The pentane lamp is described on pages 132–134 of Wilbur M. Stine's *Photometrical Measurements.* Pentane is one of the more volatile constituents of gasolene.

and this mixture of pentane vapor and air is burned in an Argand* burner of prescribed dimensions.

The Carcel lamp is an Argand burner of prescribed dimensions burning rape-seed oil. This lamp has been extensively used as a standard lamp in France.

Light units.—The intensity of the horizontal beam of light from a Hefner lamp is called a *hefner-unit* or a *hefner*. If a lamp were to give one hefner-unit of light intensity in every direction, the *amount of light*, or the so-called *flux of light* emitted by the lamp would be what is called one *spherical-hefner*.

The *candle*, or *candle-unit*, or *candle-power*, as it is variously called, is a beam of light of which the intensity is 1.11 hefner-units; that is to say, a horizontal beam from a Hefner lamp has an intensity of 0.90 candle-power. This is the definition of the candle-power which is used by the United States Bureau of Standards, and the candle-power so defined is called the *international candle* to distinguish it from the old British Standard Candle which is now obsolete.

A lamp which would give one candle-unit of intensity in every direction would emit one *spherical-candle* of light flux.

For most photometric work nothing is better as a working standard than *a properly aged and standardized incandescent lamp*. Such lamps can be obtained from the United States Bureau of Standards with certificates specifying their candle-power in a prescribed direction when operated with a prescribed voltage between their terminals.

44. Conical intensity and sectional intensity of a beam of light.—The expression, *intensity of a beam of light*, which is used in the above definitions of the hefner-unit and candle-unit, refers to the amount of light in a unit-sized cone of rays. This *conical intensity*, which it may be called for brevity, is expressed in

* The Argand burner is a type of burner in which a supply of air is admitted to the interior of a flame, as in the familiar student lamp. See article *Argand burner* in any good encyclopædia.

hefners or candles; and it is independent of distance, since the light in a given cone of rays always remains in that cone.*

The intensity of a beam of light may also refer to the amount of light per unit sectional area of the beam. This *sectional intensity*, which it may be called for brevity, decreases as the square of the distance from the lamp increases, as explained in Art. 48.

45. Intrinsic brilliancy of a lamp.—The candle-power of a lamp in a given direction divided by the luminous area † of the lamp is called the *intrinsic brilliancy* of the lamp.

Examples.—The intrinsic brilliancy of the crater of a powerful carbon-arc lamp approaches two hundred thousand candles per square inch. The tungsten-filament lamp has an intrinsic brilliancy of about one thousand candles per square inch. The carbon-filament lamp has an intrinsic brilliancy of about three hundred candles per square inch. A kerosene lamp flame has an intrinsic brilliancy of from four to eight candles per square inch.

A lamp of great intrinsic brilliancy is very painful to look at, and such a lamp should always be surrounded by a diffusing globe or shade so as to hide the luminous surface of the lamp itself. Thus, an arc lamp when used indoors is always provided with a diffusing globe, and the tungsten lamp should always have a diffusing globe or shade when it is used for interior lighting.

46. Unit of spherical angle. Definition of the lumen.—To understand the relationship of the various light units one must understand what is called *solid* or *spherical* angle. Consider a cone and a sphere with its center at the apex of the cone. Let A be the area of the spherical surface which is inside the cone,

* It is assumed in these fundamental definitions that the light source is very small in size; it requires a very elaborate discussion to establish the fundamental ideas of photometry if this assumption is not made.

Some matters relating to light sources which are not negligibly small are discussed in Arts. 50 and 56. A good example of the application of the fundamental ideas of photometry to large luminous sources is the paper, "Geometrical Theory of Radiating Surfaces with Discussion of Light Tubes," by E. P. Hyde, *Bulletin of the Bureau of Standards*, Vol. III, pages 81–104, 1907.

† Projected area at right angles to the given direction.

and let r be the radius of the sphere. Then the ratio A/r^2 measures what is called the *spherical angle* of the cone.* Thus one unit of spherical angle is subtended by one square meter of the surface of a sphere of one meter radius, or by one square foot of a sphere of one foot radius and the complete surface of a sphere represents 4π units of spherical angle. In the following discussion one unit of spherical angle is called a *unit-cone.*

Imagine a lamp which gives an intensity of one candle-power in every direction. The amount of light, or light flux, passing out from such a lamp in one unit-cone is called the *lumen* of light flux. Such a lamp would emit one spherical-candle of light flux, inasmuch as the conical intensity is assumed to be one candle-power in every direction; but the whole spherical surface represents 4π unit-cones, each of which contains one lumen of light; therefore there are 4π lumens of light flux in one spherical-candle.

47. Sectional intensity of a beam of light. Definition of the foot-candle. Definition of the lux.—Imagine a lamp which gives out one candle-power in every direction, and consider a sphere of one foot radius with its center at the lamp. One square foot of the surface of this sphere is contained inside of a unit-cone, and such a unit-cone contains one lumen of light flux. Therefore one lumen of light flux passes through each square foot of the surface of the sphere; that is, the light which radiates from the given lamp has a sectional intensity of one lumen per square foot *at a distance of one foot from the lamp.* This sectional intensity is sometimes called the *foot-candle.* That is to say, the foot-candle is the sectional intensity of a one-candle-power beam at a distance of one foot from the lamp.

The *meter-candle* is the sectional intensity of a one-candle-power beam at a distance of one meter from the lamp. The meter-candle is one lumen per square meter and it is sometimes called the *lux.*

* To express the value of a spherical angle as the quotient of the spherical area divided by square of spherical radius is analogous to the method of expressing a plane angle as the quotient of the arc of a circle divided by the radius.

One foot-candle is equal to 10.75 meter-candles or luxes.

The relation between the various light units may be kept in mind most easily with the help of Fig. 47.

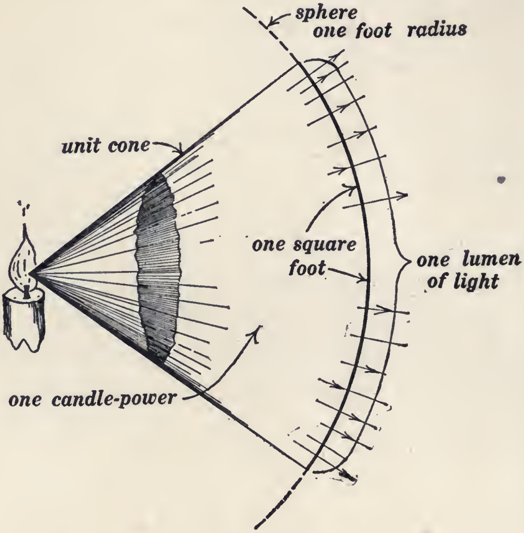


Fig. 47.

48. The law of inverse squares.—It is evident that the sectional intensity of a beam of light from a lamp decreases with increasing distance from the lamp. Indeed the sectional intensity of a beam from a lamp is given by the equation

$$I = \frac{C}{d^2} \tag{14}$$

in which C is the conical intensity of the beam in candle-power, and I is the sectional intensity of the beam in foot-candles at a distance of d feet from the lamp. This is evident when we consider that the amount of light in a cone of rays remains constant, whereas the sectional area of the cone increases as the square of the distance from the apex of the cone.

Equation (14) expresses what is called *the law of inverse squares*. This law applies strictly to the light which comes from *a very* *small portion of the luminous surface of a lamp*, a point source as it is called. The law is approximately true, however, for a whole lamp at distances which are large compared with the size of the luminous surface of the lamp. For example, consider the light which is given off by a brightly illuminated flat disk of paper. The sectional intensities of this light at points on the axis of the disk at different distances from the disk are exhibited in the accompanying table. The heavy-faced numbers show what the sectional intensities would be according to the law of inverse squares, and the light-faced figures show the actual intensities.

PAPER DISK 2 FEET IN DIAMETER.

Distances from Disk in Feet.	5	10	20	40	80	160
Sectional intensities according to the law of inverse squares	**1024**	**256**	**64**	**16**	**4**	**1**
Actual sectional intensities	984.6	253.5	63.84	15.99	3.9992	0.99996

The error of the law of inverse squares does not exceed two-tenths of one per cent. for distances exceeding ten times the maximum dimension of the luminous surface of the lamp. See Art. 50.

49. To find the relation between the intrinsic brilliancy and the lumens of light emitted per square foot of a flat diffusing surface of plaster or uncalendered paper.—Consider a small element ΔA of the plaster surface, and let C_n be the candle-power of the beam which is given off by the element normally. Then $C_n/\Delta A (= B)$ is the intrinsic brilliancy of the plaster surface according to Art. 45. The candle power of the normal beam is therefore $C_n = B \cdot \Delta A$ and the candle-power of the oblique beam b, Fig. 48, is equal to $C_n \cos\theta$ according to Lambert's cosine law (see Art. 53). Therefore we have:

$$C_b = B \cos\theta \cdot \Delta A \tag{i}$$

in which C_b is the candle-power of the oblique beam b in Fig. 48, B is the intrinsic brilliancy of the plaster surface, ΔA is the area of the plaster surface, and θ is the angle shown in the figure.

Consider the zone zz of the reference sphere, the zone extending entirely around the "pole" p. The area of this zone is $2\pi r \sin\theta \times r \cdot \Delta\theta$, the sectional intensity of the beam b at zz is equal to C_b/r^2 according to equation (14), and the amount of light passing through the zone zz is equal to the product of the area of zz and the sectional intensity of b. Therefore we have

$$\Delta L = 2\pi B . \Delta A \cdot \sin\theta \cos\theta \cdot \Delta\theta \qquad \text{(ii)}$$

in which ΔL is the amount of light in lumens passing through *zz*.

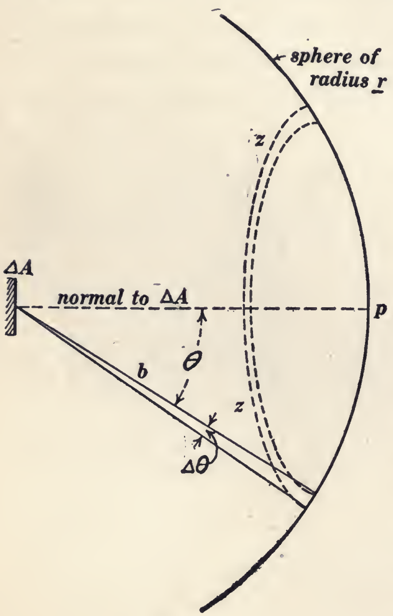


Fig. 48.

To find the total amount of light L emitted by the element of plaster surface, equation (ii) must be integrated between the limits $\theta = 0$ to $\theta = 90°$, which gives:

$$L = \pi B \cdot \Delta A \qquad \text{(iii)}$$

whence

$$B = \frac{L}{\pi \cdot \Delta A} \qquad \text{(iv)}$$

That is, the intrinsic brilliancy of a flat diffusing surface of plaster or uncalendered paper is equal to the lumens of light emitted per unit area $(L/\Delta A)$ divided by π.

50. Given a flat circular disk of plaster or uncalendered paper of which the intrinsic brilliancy is B, to find the amount of light falling on unit area at a point p in the axis of the disk as shown in Fig. 49.—Consider the beam of light which reaches p from a small element e of the circular strip *ss* of the plaster disk *DD*. The conical intensity of this beam is given by equation (i), where ΔA is the area of the element e; and the sectional intensity of the beam at p is found by dividing C_b by the square of the distance *ep* which is $(R^2 + \rho^2)$. Therefore, multiplying this sectional intensity by the projected value of the unit of area at p (namely unit of area $\times \cos\theta$), we find the amount of light falling upon the unit of area at p from the element e. But every element of the circular strip *ss* sends the same amount of light to the unit of area at p, and therefore the total amount of light ΔL received by the unit of area at p from the entire circular strip *ss* is

$$\Delta L = B \times 2\pi\rho \cdot \Delta\rho \times \frac{1}{R^2 + \rho^2} \times \cos^2\theta \qquad \text{(v)}$$

but

$$\cos\theta = \frac{R}{\sqrt{R^2 + \rho^2}},$$

so that equation (v) becomes:

$$\Delta L = 2\pi B R^2 \cdot \frac{\rho \cdot \Delta\rho}{(R^2 + \rho^2)^2} \qquad \text{(iv)}$$

and the total amount of light falling on unit of area at p from the entire disk *DD* is found by integrating equation (vi), from $\rho = 0$ to $\rho = r$, that is,

$$L = 2\pi BR^2 \int_{\rho=0}^{\rho=r} \frac{\rho \cdot d\rho}{(R^2 + \rho^2)^2} \qquad \text{(vii)}$$

or

$$L = \frac{\pi Br^2}{R^2 + r^2} \qquad \text{(viii)}$$

And of course the lumens of light received by the unit of area at p, in Fig. 49, is the intensity of illumination of the unit of area by the light from the plaster or paper disk. The actual sectional intensities which are given in the table in Art. 48 were calculated from equation (viii), using r = one foot, and using for R the series of values 5 feet, 10 feet, 20 feet, 40 feet, etc.

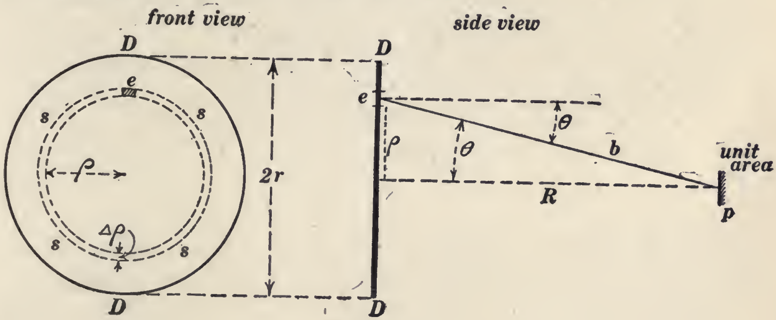


Fig. 49.

51. The intensity of illumination of a surface depends upon the amount of light falling upon one unit of area of the surface. Therefore, intensity of illumination is expressed in terms of the same unit as sectional intensity of a beam of light. Thus an intensity of illumination of one lux, or one *meter-candle*, is one lumen of light falling an each square meter; it is the intensity of illumination produced by a standard candle at a distance of one meter. The *foot-candle* is the intensity of illumination at a distance of one foot from a standard candle; it is equal to one lumen per square foot.

Examples.—The light from a 10-candle-power lamp falls perpendicularly upon a sheet of paper at a distance of two feet from the lamp. Substituting $C = 10$ and $d = 2$, in equation (14) we find the value of I to be 2.5 foot-candles.

52. Oblique illumination of a flat surface.—When a beam of light falls obliquely upon a flat surface as shown in Fig. 50, the light is spread over an area greater than the sectional area of the beam in the ratio *ac/ab*, and therefore the intensity of illumination of the surface is less than the sectional intensity of the beam in the ratio *ab/ac*. That is to say, the intensity of illumination of the surface is equal to $I \cos \theta$ where I is the sectional intensity of the beam and θ is the angle shown in Fig. 50.

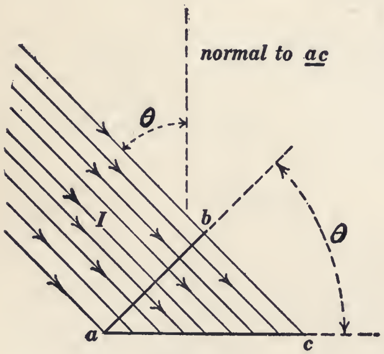


Fig. 50.

Let C be the conical intensity of the beam *bb* from a lamp as shown in Fig. 51, and let us consider the intensity of illumination I_n on a surface normal to the beam at *p*, the intensity of illumination I_h on a horizontal surface at *p*, and the intensity of illumination I_v on a vertical surface at *p*. The sectional intensity of the beam at *p* is C/d^2 according to equation (14),

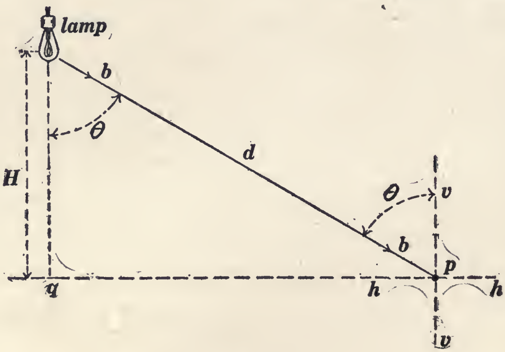


Fig. 51.

and, since the normal intensity of illumination produced by a beam is equal to the sectional intensity of the beam, therefore we have

$$I_n = \frac{C}{d^2} \tag{15a}$$

Furthermore from the above discussion of Fig. 50 we have

$$I_h = \frac{C}{d^2} \cdot \cos \theta \qquad (16a)$$

$$I_v = \frac{C}{d^2} \cdot \sin \theta \qquad (17a)$$

where θ is the angle shown in Figs. 50 and 51.

In many cases the height H of the lamp above the illuminated plane qp and the angle θ are given to find I_n, I_h and I_v. In such cases more convenient equations can be obtained by substituting $d = H/\cos \theta$ in the above equations giving

$$I_n = \frac{C}{H^2} \cdot \cos^2 \theta \qquad (15b)$$

$$I_h = \frac{C}{H^2} \cdot \cos^3 \theta \qquad (16b)$$

and

$$I_v = \frac{C}{H^2} \cdot \sin \theta \cos^2 \theta \qquad (17b)$$

53. Regular reflection and diffuse reflection. Lambert's cosine law.—The reflection from a polished surface like a mirror is called *regular reflection.* The reflection from a rough surface like plaster or uncalendered paper, is called *diffuse reflection.*

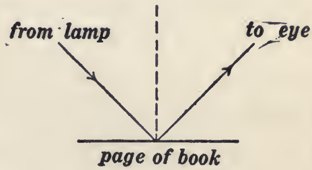


Fig. 52.

Most surfaces show both regular reflection and diffuse reflection. Thus everyone is familiar with the unpleasant shining reflection from the page of a book when it is viewed as shown in Fig. 52. A scraped plaster surface has no perceptible regular reflection. The following statements refer to such a surface.

Proposition (a).—The apparent brightness of an illuminated plaster surface is independent of its distance from the observer's *eye.* This may be understood from the following argument. If the distance of the illuminated surface from the eye is doubled then, according to the law of inverse squares,* one quarter as much light enters the eye from the surface (size of pupil remain-

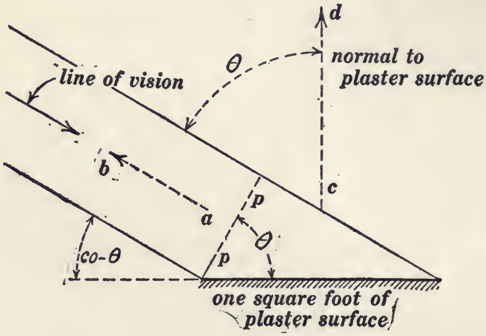


Fig. 53.

ing unchanged), but the image of the illuminated sufrace on the retina is reduced to one quarter of its original area, so that one quarter as much light falls on one quarter as large a spot on the retina, and therefore the intensity of illumination of the retina is unchanged.

Proposition (b).—Surfaces like rough plaster and uncalendered paper, which diffuse light very completely, have an apparent brightness which remains the same as they are viewed more and more obliquely. This is an observed fact, and an important consequence of it is as follows. Consider one square foot of an illuminated plaster surface which is viewed obliquely as shown in Fig. 53, so that the projected area *pp* of the plaster surface is reduced, say, to one half of a square foot. Then the area of the image on the retina is one half as great as it would be if the plaster surface were turned at right angles to the line of vision (the distance of the eye from plaster surface remaining unchanged). But to make the plaster surface appear of the same brightness in Fig. 53 with the image on the retina reduced to half-size (in area), one half as much light must enter the eye.

* The law of inverse squares is true for a small portion of an illuminated surface.

That is to say, when an illuminated diffusing surface is viewed obliquely so that the apparent area of the surface is reduced say to one half, then the amount of light which enters the eye from the surface is reduced to one half also. In general the apparent area of a flat surface is reduced in the ratio $1 : \cos\theta$, where θ is the angle shown in Fig. 53. Therefore the intensity* of the light given off in the direction *ab*, Fig. 53, is less than the intensity* of the light given off in the direction *cd* in the ratio $1 : \cos\theta$. This relation is called *Lambert's cosine law.*

Consider the light emitted by an element ΔA of an illuminated surface, the element being so small that it may be considered as a point source of light. Then the candle power of the light beam emitted normally to the element is $B \cdot \Delta A$, where B is the intrinsic brilliancy of the surface, and the candle power of the beam *bb*, in Fig. 54, is $B \cdot \cos\theta \cdot \Delta A$.

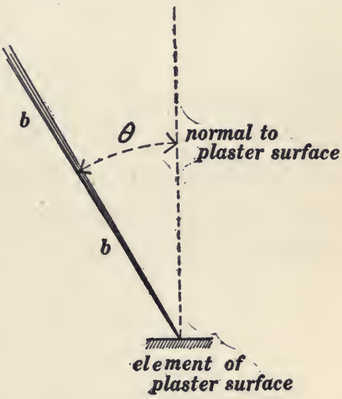


Fig. 54.

54. Absorption of light by illuminated surfaces. — An absolutely white surface would be a surface which would reflect (diffusely) all of the light falling upon it. Consider a surface which reflects, say, 0.6 of the light which falls upon it and which absorbs the remaining 0.4 of the light. The fraction 0.6 is called the *reflection coefficient* of the surface, and the fraction 0.4 is called the *absorption coefficient* of the surface. The reflection coefficient of a rough surface is sometimes called the *albedo* of the surface.

55. Change of conical intensity by lenses. When the light source is very small.—Light from a small source A passes through a lens and is brought to a focus at B, as shown in Fig.

* Conical intensity; or sectional intensity at a given distance from the plaster surface; see following paragraph.

COEFFICIENTS OF REFLECTION * OF WALL PAPERS.

Material.	Coefficients of Reflection.	Coefficients of Absorption.
White blotting paper	0.82	0.18
White cartridge paper	0.80	0.20
Ordinary foolscap paper	0.70	0.30
Chrome yellow paper	0.62	0.38
Orange paper	0.50	0.50
Yellow wall paper	0.40	0.60
Yellow painted wall	0.40	0.60
Light pink paper	0.36	0.64
Yellow cardboard	0.30	0.70
Light blue cardboard	0.25	0.75
Brown cardboard	0.20	0.80
Yellow painted wall (dirty)	0.20	0.80
Emerald green paper	0.18	0.82
Dark brown paper	0.13	0.87
Vermilion paper	0.12	0.88
Bluish green paper	0.12	0.88
Cobalt blue paper	0.12	0.88
Black paper	0.05	0.95
Ultamarine blue paper	0.035	0.965
Black velvet	0.004	0.996

55. Beyond the focus B the light spreads out again in a conical beam gg as shown in the figure. It is evident that the conical intensity of the beam gg is the same as the conical intensity of

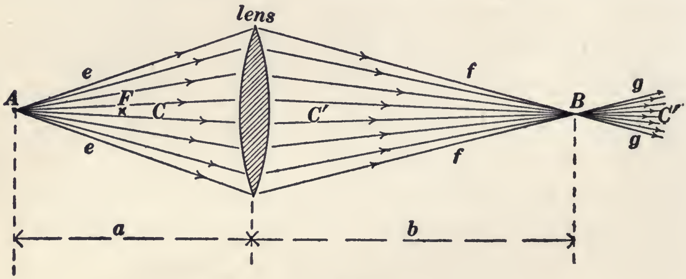


Fig. 55.

the beam ff because the total amount of light is the same in both beams and both cones are of the same size. Let C be the conical intensity of the beam ee, and C' the conical intensity of the beam ff (or of the beam gg). Then

* Taken from the *Standard Handbook for Electrical Engineers*, section 12, paragraph 37, by Louis Bell.

$$\frac{C}{C'} = \frac{a^2}{b^2} \qquad (18)$$

in which a and b are the distances indicated in the figure. This equation is based on the assumption that no light is lost in passing through the lens. Let S be the area of the lens. Imagine a sphere of radius a with its center at A, and a sphere of radius b with its center at B. The portions of these spherical surfaces which are included within the cones ee and ff are approximately equal in area to S. Therefore the spherical angle of the cone ee is equal to S/a^2 and the spherical angle of the cone ff is equal to S/b^2. Let L be the amount of light-flux in the cone ee, then the conical intensity of the beam ee is equal to L divided by S/a^2, and the conical intensity of the beam ff is equal to L divided by S/b^2. That is, $C = La^2/S$ and $C' = Lb^2/S$, whence equation (18) follows at once.

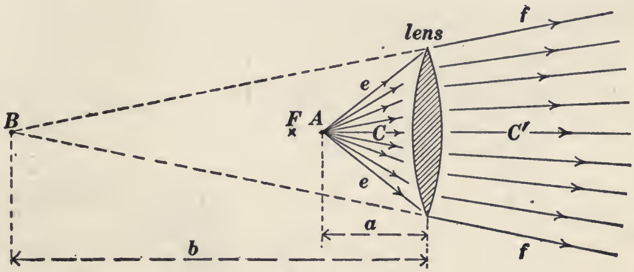


Fig. 56.

The action of a concave mirror is the same as the action of a converging lens in altering the conical intensity of a beam of light.

The above discussion applies also to Fig. 56 in which the light source A is inside of the principal focus F.

If the distance a in Fig. 55 is equal to the focal length of the lens, then the emergent beam ff is a beam of parallel rays, or, in other words, the distance b is infinity. Thus, Fig. 57 shows a very small light source placed at the principal focus of a converging lens. In this ideal case (ideal in that the source is assumed to be a point) the beam *ff* is a beam of parallel rays, and the conical intensity of the beam *ff* is infinitely greater than the conical intensity of the beam *ee*, according to equation (18).

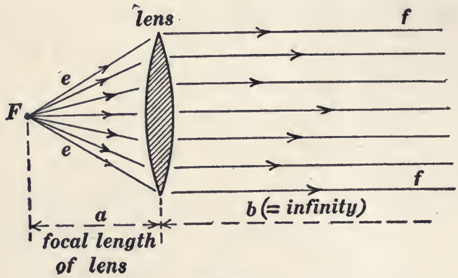


Fig. 57.

But there is no such thing physically as a point source, and when an actual light source is placed at the principal focus of a lens or concave mirror, the size of the light source must be taken into consideration in the discussion of the action of the lens or mirror, as in the following article.

56. The searchlight.—The searchlight consists of a lamp placed at the principal focus of a lens or concave mirror. The action

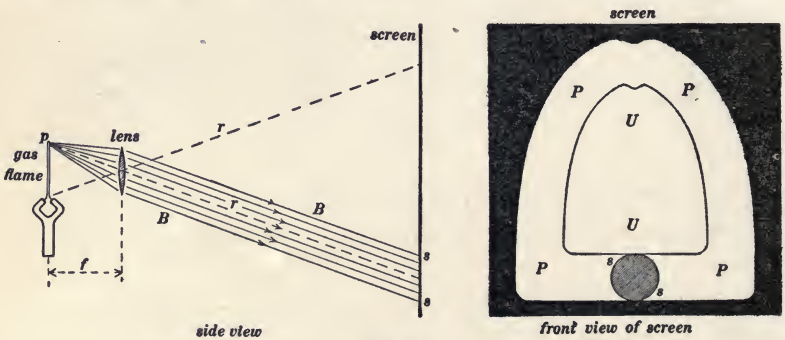


Fig. 58.

of the searchlight is shown in Fig. 58, in which the luminous source is a gas flame. *The light from each point p of the flame is converted into a beam of parallel rays by the lens as shown in the figure.* Therefore the light from each point of the source illuminates a circular spot ss on a distant screen, and the diameter of this spot is equal to the diameter of the lens. The illuminated field on the distant screen consists of a central portion UU which is uniformly illuminated (if the flame is uniformly bright), and a fringe PP which shades off gradually to complete darkness as shown in the figure. In fact the illuminated field produced by a searchlight is an inverted blurred image of the source, as may be understood by a careful study of Fig. 58. Thus, one sees a large inverted blurred image of the acetylene flame of an automobile searchlight when the searchlight beam falls on a flat surface like the side of a house.

When the light source is uniformly bright, then the beam from a searchlight has a definite conical intensity at a great distance from the lamp, as may be understood from the following considerations. If the screen in Fig. 58 is at a very great distance from a searchlight, the diameter of ss becomes negligible in comparison with the diameter of the illuminated field, or, in other words, the fringe of the illuminated field becomes negligible. That is to say, all of the light in the searchlight beam may be thought of as being contained within the cone rr, and consequently the searchlight beam may be thought of as having a definite conical intensity.

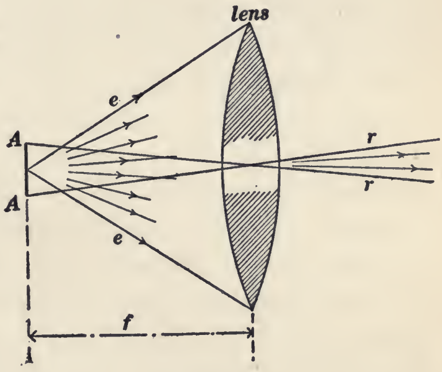


Fig. 59.

To determine the conical intensity of the beam of a searchlight, consider a source AA, Fig. 59, placed at the principal focus of a lens as shown. The light which falls upon the lens is contained within the cone ee, and the searchlight beam at a great distance from the lens is contained in the cone rr as above explained.

Therefore, assuming that no light is lost in the lens, the conical intensity of the searchlight beam is greater than the conical intensity of the beam from the lamp in the inverse ratio of the spherical angles of the two cones *ee* and *rr* in Fig. 59. Let s be the area of the luminous surface of the lamp AA in Fig. 59, and let S be the area of the lens. Then the spherical angle of the cone *rr* is sensibly equal to s/f^2, and the spherical angle of the cone *ee* is sensibly equal to S/f^2. Multiplying the conical intensity of the beam *ee* by the spherical angle of the cone *ee* gives the number of lumens of light that strike the lens. Ignoring loss of light, this is the amount of light in the cone *rr*, and it may be divided by the spherical angle of the cone *rr* to give the conical intensity of the searchlight beam.

In the above discussion the lens or mirror is supposed to be entirely free from the error called spherical aberration. This is never the case in practice, especially with a lens or mirror whose diameter is large as compared with its focal length. The effect of spherical aberration is to cause more blurring on a distant screen than is represented in Fig. 58 and to cause a slightly increased divergence of the searchlight beam.

As before stated, it is proper to speak of the candle-power (conical intensity) of a searchlight beam when the lamp presents a uniformly brilliant surface and when the searchlight beam is considered only at great distances from the search lamp. The effect of spherical aberration is to make it more generally permissible to speak of the candle-power of a searchlight beam, because the blurring due to spherical aberration tends to make the central portion of the field in Fig. 58 uniformly illuminated even when the lamp does not present a uniformly illuminated surface. This is especially the case in a searchlight using a closely coiled tungsten-filament lamp. The illuminated field on a distant screen is of course a blurred image of the filament, and it is not permissible to speak of the candle-power of the searchlight beam unless the blurring is sufficient to obliterate every trace of this image of the filament.

When a lamp having a small straight filament is used in a searchlight, then to confine the searchlight beam to the smallest possible cone, it is best to arrange the filament in the axis of the lens or mirror as shown in Fig. 60. Thus the light from the

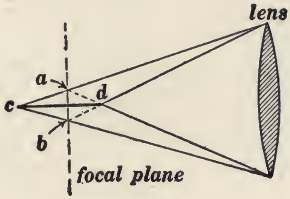


Fig. 60*a*.

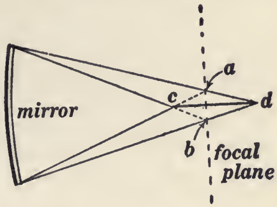


Fig. 60*b*.

small straight filament *cd*, in Fig. 60, may be thought of as coming from a circular spot *ab* on the focal plane, and *ab* is much shorter than *cd*, if the focal length of the lens is large as compared with its diameter.

When the diameter of the lens or mirror is large as compared with its focal length, as shown in Fig. 61, then the only feasible

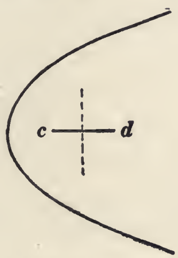


Fig. 61*a*

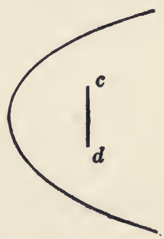


Fig. 61*b*.

method of finding the best position of the lamp filament is by trial; it is not feasible to calculate the relative advantages of the two positions shown in Fig. 61*a* and Fig. 61*b*.

57. The Bunsen photometer.—The most extensively used device for comparing the conical intensities of the light from two lamps is the *Bunsen photometer*. A given lamp and a standard lamp are placed at the ends of a horizontal bar, and a screen (see Fig. 62) of thin paper is moved along the bar until the two sides of the screen are equally illuminated by the two lamps. Equal intensities of illumination on the two sides of the screen indicate equal sectional intensities (at the screen) of the beams from the two lamps. Let C and C' be the conical intensities of the beams from the two lamps as shown in Fig. 62. Then the

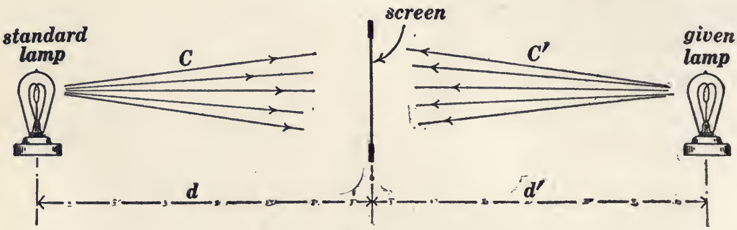


Fig. 62.

sectional intensity at the screen of the beam from the standard lamp is C/d^2, and the sectional intensity at the screen of the beam from the given lamp is C'/d'^2, according to equation (14), and since these sectional intensities are equal, as above explained, we have

$$\frac{C}{d^2} = \frac{C'}{d'^2}$$

or

$$\frac{C}{C'} = \left(\frac{d}{d'}\right)^2 \qquad (19)$$

from which the ratio of the conical intensities may be calculated when d and d' have been observed.

An irregular grease spot on the thin paper screen enables one to judge better when the illumination is the same on the two sides of the screen. This spot should be made with clean paraffine, and the excess of paraffine should be drawn out of the screen by placing it between folds of absorbent paper and applying a hot flat-iron. To facilitate the seeing of both faces of the screen simultaneously two mirrors are usually placed as shown in Fig. 63.

In judging the equality of illumination on the two sides of the

Bunsen photometer screen, one eye only should be used. In using both eyes, one unconsciously looks at one side of the screen with one eye and at the other side of the screen with the other eye, and the difference between the two eyes leads to a constant error of setting.

With extremely dim lamps (or with bright lamps at great distances from the screen of the Bunsen photometer) the accuracy

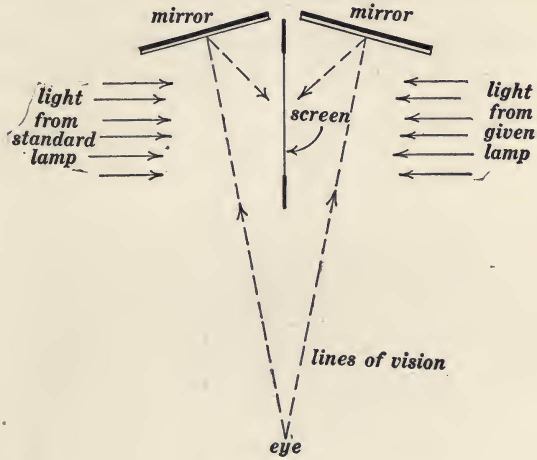


Fig. 63.

of setting is very low; a number of settings taken under such conditions may deviate from each other by several per cent. With increasing intensity of illumination on the screen the accuracy of setting increases and reaches about one half of one per cent. when the intensity of illumination on the screen is about one foot-candle or more.*

FECHNER'S RATIO.—Given a surface with intensity of illumination I and let ΔI be the added illumination which is barely perceptible to the eye. The ratio $\Delta I/I$ is called *Fechner's ratio.* Thus defined this ratio is approximately equal to the percentage accuracy of setting of a Bunsen photometer.

* The limit of accuracy of the setting of a Bunsen photometer is about two tenths of one per cent., and the error which is introduced by the inaccuracy of the law of inverse squares should be considerably less than two tenths of one per cent. Therefore the maximum dimension of the luminous surface of a lamp should not exceed about one twentieth of the distance of the lamp from the photometer screen. See Art. 48.

The Bunsen photometer screen, together with the two mirrors which are shown in Fig. 63 is usually mounted in a box which is called a *screen box* or *sight box*. An extensively used substitute for the Bunsen sight box is the Lummer-Brodhun sight box in which an optical device (a prism-set) is used for showing portions of the two sides of a white opaque screen side by side in the same field of view. Two forms of the Lummer-Brodhun prism-set are described in Arts. 63 and 65.

58. Distribution of light around a lamp.—In defining the spherical-candle the idea of uniformity of distribution of light

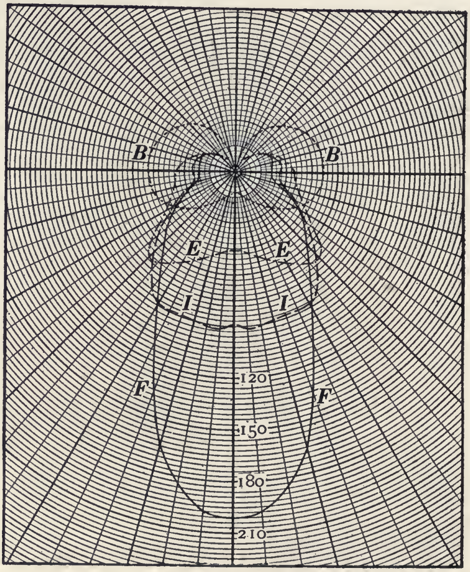


Fig. 64.

around a lamp was introduced for the sake of simplicity. In fact, however, no lamp gives complete uniformity of distribution of light, but the conical intensity (candle-power) is always greater in certain directions and less in other directions. Thus curve *B*, Fig. 64, shows the distribution of light around a tungsten lamp, and curves *E*, *I* and *F* show the distribution of candle-power around the same lamp when it is equipped with "extensive," "intensive" and "focusing" prismatic glass reflectors respectively.* Figure 65 shows the distribution of light around an ordinary carbon-filament lamp without a shade. In these figures the conical intensity (candle-power) in each direction is represented to scale by the length of the radius vector of the curve.

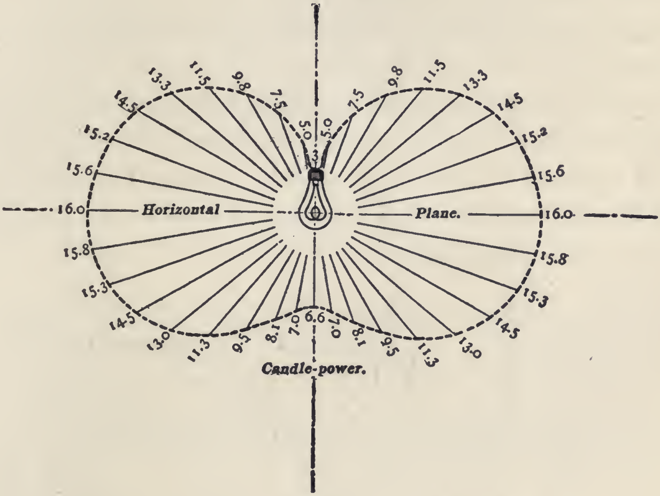


Fig. 65.

The distribution of candle-power about a lamp, which, like a carbon-filament lamp, can be held in any position, may be determined by mounting the lamp in a universal holder at one end of the photometer bar, turning it step by step in various positions and taking the photometer reading for each position.

In some cases a lamp is symmetrical with respect to an axis so that a complete knowledge of the distribution of candle-power about the lamp may be obtained by determining the candle-power in different directions in a single plane which contains the axis of symmetry of the lamp.

In many cases, a lamp is approximately symmetrical with respect to an axis, so that the slight variations of candle-power

* These reflectors are described in Articles 73 and 75.

around the axis of approximate symmetry are of no importance. In such a case the lack of symmetry may be averaged out, as it were, by rotating the lamp at a speed of three or four revolutions per second about its axis of approximate symmetry while the photometer readings are being taken. The data for Figs. 64 and 65 were obtained in this way. The vertical dotted line in Fig. 65 is the axis of approximate symmetry, and the lamp (with its shade) was rotated about this axis while the various readings were taken.

In the case of a lamp which must be held in a fixed position, such as an arc lamp, a gas lamp, or a kerosene lamp, one or more mirrors are used to reflect the different beams from the lamp along the photometer bar. Thus Fig. 66 shows three mirrors

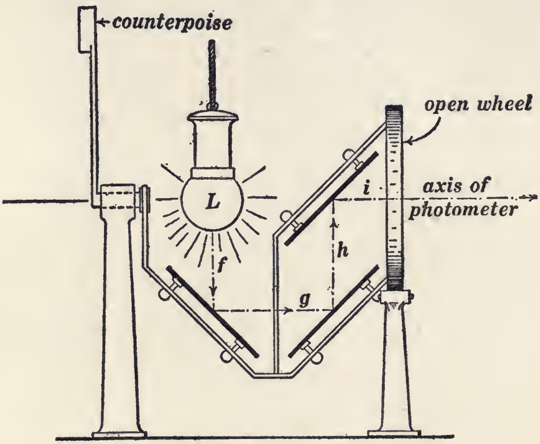


Fig. 66.

arranged to reflect the light from a fixed lamp along a photometer bar. The three mirrors are suspended in a rigid frame which may be rotated about the axis of the photometer as indicated in the figure. The figure shows the mirrors in the position to reflect the downward beam from the lamp along the photometer bar.

The mirrors shown in Fig. 66 must be large enough so that, with the eye placed at the photometer screen, one can see the entire luminous surface of the lamp including the globe or shade; and in using equation (19) the distance of the lamp from the photometer screen must be taken as the sum of the distances f, g, h and i in Fig. 66.

The mirrors in Fig. 66 reflect only a certain fractional part of the light from the lamp, and therefore the photometer reading must be multiplied by a correction factor when the mirrors are used. This correction factor may be found by observing the photometer readings corresponding to a certain beam from the lamp (*a*) *with the mirrors*, and (*b*) *without the mirrors*, making due allowance for the effective distance from the lamp to screen in each case.

If it is feasible the lamp should be rotated steadily about the vertical axis in Fig. 66 while the photometer readings are being taken.

Distribution of light around a very fine straight filament.—In the ordinary tungsten lamp the light is emitted by a number of nearly straight parallel tungsten wires, and the distribution of light is very nearly of the kind one would get from a single straight filament.

Let C be the candle-power of the equatorial beam as shown in Fig. 67. If the filament is very fine and if it radiates light like a perfectly diffusing surface (an illuminated surface of plaster for example), then, according to Lambert's cosine law, the candle-power of the beam b is:

$$b = C\cos\theta \qquad \text{(i)}$$

The sectional intensity of the beam b at the surface of the reference sphere is b/r^2, according to equation (14); the area of the zone zz of the reference sphere is $2\pi r\cos\theta\times r\cdot\Delta\theta$; and the amount of light in lumens which passes through the zone zz is the product of the sectional intensity b/r^2 and the area of the zone. Therefore

$$\Delta L = 2\pi C\cos^2\theta\cdot\Delta\theta \qquad \text{(ii)}$$

whence

$$L = 2\pi C\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos^2\theta\cdot d\theta = \pi^2 C \qquad \text{(iii)}$$

in which L is the total amount of light in lumens emitted by the filament.

To find the mean spherical candle-power of the filament, divide L by the spherical angle corresponding to the entire reference sphere, namely, 4π. This gives

$$\left.\begin{array}{r}\text{Mean spherical}\\ \text{candle-power of the filament}\end{array}\right\} = \frac{\pi}{4}\cdot C \qquad \text{(iv)}$$

Now the equatorial candle-power, C, corresponds to what is usually called the *mean horizontal candle-power*, and the factor by which the mean horizontal candle-power of a lamp must be multiplied to give the mean spherical candle-power is called the *spherical reduction factor* of the lamp. Therefore the spherical reduction factor of a lamp which has a fine straight filament is $\pi/4$ or 0.7854.

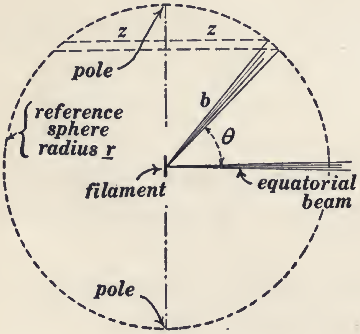


Fig. 67.

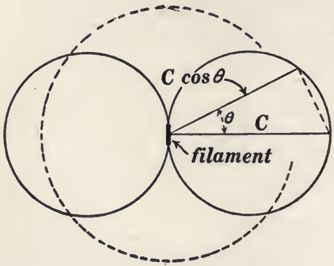


Fig. 68.

The candle-power distribution curve of a very fine straight filament is made up of two circles having the filament as their common tangent, as shown in Fig. 68, and the diameter of each circle is the equatorial candle-power, C. The radius of the dotted circle in Fig. 68 is equal to $\pi/4 \times C$ and it represents the mean spherical candle power of the filament.

The candle-power distribution curve of an ordinary tungsten lamp without a shade is shown by the curve BB in Fig. 64, and it is very similar to the two full-line circles in Fig. 68. The spherical reduction factor of an ordinary tungsten lamp should be nearly equal to $\pi/4$. In fact the spherical reduction factor of such a lamp is about 0.79.

59. Measurement of total light flux from a lamp.—If a lamp were to emit light of the same conical intensity (same candle-power) in all directions, then the candle-power in any direction as measured by a Bunsen photometer would be numerically equal to the total amount of light expressed in spherical-candles, and multiplying by 4π would reduce to lumens. In general, however, light is emitted by a lamp unequally in different directions, and the amount of light emitted by a lamp is determined by measuring the candle-power in every direction and taking the average. This average is the amount of light in spherical-candles; to reduce to lumens multiply by 4π.

If the average is to be calculated simply by adding and dividing then the directions in which the separate readings are taken must be distributed uniformly over the surface of a sphere with its center at the lamp. This sphere is called the *reference sphere* for the sake of brevity. If the readings are not so distributed, then *each reading must be multiplied by the spherical area which may be properly assigned to it, and the sum of such products must be divided by the total area of the sphere to give the correct average.*

When a lamp can be rotated at a speed of three or four revolutions per second about its axis of approximate symmetry, the total light flux from the lamp may be determined by taking readings of candle-power in different directions in *one plane only*, namely, a plane which includes the axis of rotation. Thus the lamp L in Fig. 69 would be rotated about the vertical axis PQ, and the conical intensities C_1, C_2, C_3, C_4, etc., at equal angular distances would be measured. On account of the rotation of the lamp each setting of the photometer gives the average candle-power along a parallel of latitude, as it were. Each of the readings C_1, C_2, C_3, C_4, etc., represents, therefore, the candle-power over a zone of the reference sphere. Consequently the readings must be multiplied by the areas of the respective zones, and the sum of these products must be divided by the total area of the reference sphere to get the correct average candle-power in all directions, the mean spherical candle-power as it is called.

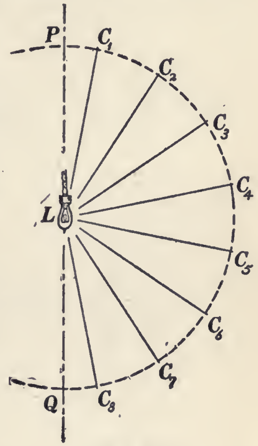


Fig. 69.

This calculation may be easily understood with the help of Fig. 70 in which *ccc* is a given candle-power curve, the curve shown in Fig. 65 for example. Consider the candle-power which is represented by the radius vector Oa. This candle-power may be thought of as referring to the zone zz of the reference sphere.

Taking the diameter DD of the reference sphere as representing the total area of the sphere then the height h of the zone zz represents the area of the zone.* Therefore the product $Oa \times h$ is the product, *candle-power* $Oa \times$ *area of the corresponding zone*, and the sum of all such products has to be divided by the diameter DD of the reference sphere to give the mean spherical candle-power of the lamp.

It is especially to be noted that the *area* enclosed by a candle-power distribution curve is *not* a measure of the total amount of

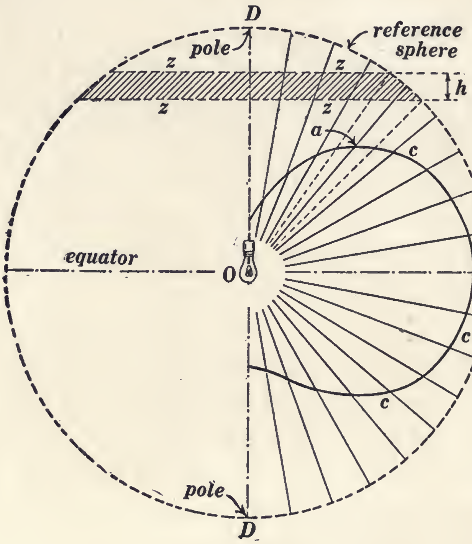


Fig. 70.

light emitted by a lamp. Thus each of the four candle-power distribution curves in Fig. 64 represents approximately the same total amount of light, indeed each of the curves EE, II and FF represents only about 88 per cent. as much light as curve BB.

60. Rousseau's diagram.—When the observed candle-powers of a lamp correspond to zones of the reference sphere as above

* That is, the rule for finding the area of a spherical zone is to multiply the entire area of the sphere $(4\pi r^2)$ by h/D, where h is the altitude of the zone and D is the diameter of the sphere.

explained, then the multiplication of each candle-power by the area of the corresponding zone, the adding of these products together, and the dividing of the total sum by the area of the reference sphere may be done graphically. The most familiar graphical method is due to Rousseau, and the drawing which is necessary to carry it out is called *Rousseau's diagram.*

The curve *ccc*, Fig. 71, is the given candle-power curve (like Fig. 65, for example). Construct another curve $c'c'c'$, Fig. 72, such that each abscissa $O'a'$ is equal to the corresponding radius

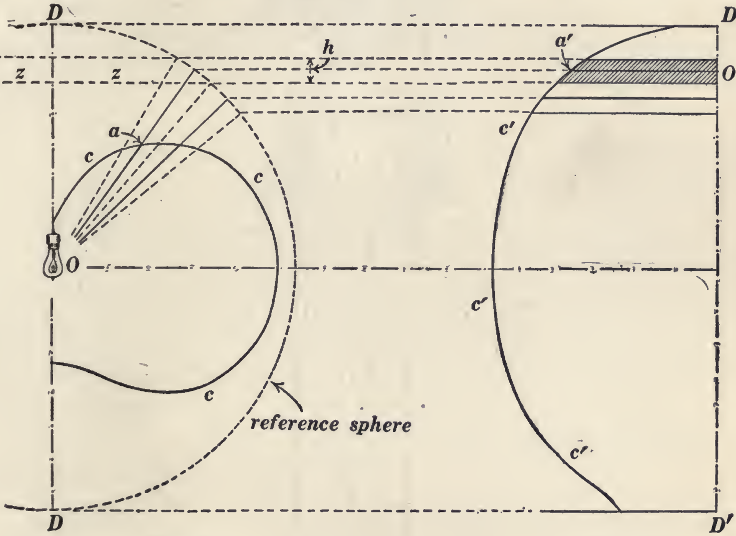


FIG. 71. Fig. 72.

vector Oa. The candle-power Oa multiplied by the area of the corresponding zone* is evidently the same thing as $O'a'$ multiplied by h, and this product is equal to the shaded area in Fig. 72. Consequently, the sum of all such products is equal to the total area between the curve $c'c'c'$ and the axis $D'D'$. Therefore this area can be measured by a planimeter and divided by the distance $D'D'$ (the diameter of the reference sphere) to give the true mean spherical candle-power of the lamp.

* See discussion of Fig. 70 in Art. 59.

Other graphical methods have been proposed for the determination of mean candle-power by Kennelly* and by Wohlauer.†

61. The flicker photometer.—The flicker photometer is a device for eliminating, to some extent, the error in the setting of a photometer which is due to differences in color of the lamps which are being compared. The following is the principle upon which the elimination of color error is based. When one looks at a thing such as a photometer screen one has a sensation of

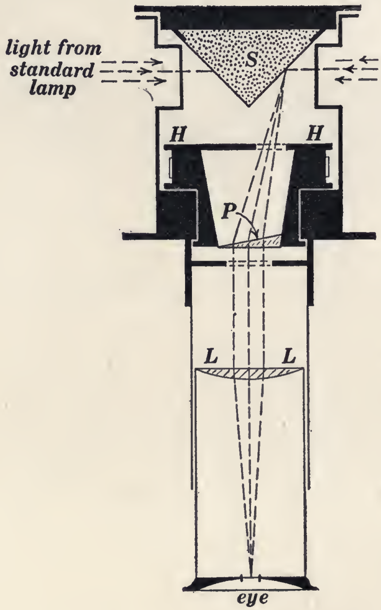


Fig. 73.

brightness and a sensation of *color*. Both of these sensations persist for an appreciable interval of time after stimulation ceases, but the sensation of color persists much longer than the sensation of brightness. Therefore if the two sides of the photometer screen are brought into the same field of view in rapid succession (with high frequency of interchange), the *color sensa-*

* See *Electrical World*, March 28, 1908.

† See *Illuminating Engineering*, Vol. III, page 655.

tion produced by the two sides of the screen and also the *brightness sensation* produced by the two sides of the screen will both become perfectly steady (devoid of flicker), whereas a much lower frequency of interchange will suffice to give a steady color sensation but leave a flickering sensation of brightness *unless the two sides of the screen have the same brightness.* Therefore if we use a frequency of interchange just sufficient to give a steady color sensation, the two sides of the photometer screen can be brought to equality of brightness by adjusting the photometer until the brightness-flicker disappears. The various flicker photometers differ in the arrangement used for bringing about the rapid interchange of the two sides of the photometer screen in the same field of view, and the most satisfactory device is perhaps that due to Marten.* The two faces of a prism of white plaster *S*, Fig. 73, are illuminated by the standard lamp and a given lamp respectively. The eye is focused upon one of the illuminated faces of the plaster prism by means of the lens *LL*, and the thin glass prism *P* which is carried in a rotating holder *HH* directs one's vision to one face and then to the other face of the plaster prism in rapid succession. This arrangement constitutes a sight box, and it is used on a photometer bar in the same way as the Bunsen sight box, as explained in Art. 57.

Another form of flicker sight-box which is quite satisfactory is that of Simmance and Abady.†

62. The complete photometer; laboratory type.—The photometer consists of a sight-box moving along a track or bar, and the lamps to be compared are placed at the ends of this bar. The sight-box may be of the Bunsen type as described in Art. 57, or of the Lummer-Brodhun type as described in Arts. 63 and 65, or a sight-box of the flicker-type may be used as described in Art. 61. Usually the photometer setting is made by moving the

* This flicker photometer is manufactured by Schmidt and Haensch, of Berlin.

† The Simmance-Abady flicker sight-box is described on pages 491–492 of Vol. I, *Johns Hopkins University Lectures on Illuminating Engineering*, Johns Hopkins Press, 1911.

sight-box along the photometer bar, but sometimes it is more convenient to make the setting by moving the standard lamp as in the Sharp-Millar portable photometer (see Art. 63).

The essential features of the laboratory type of photometer are shown in Fig. 74. Dead black surfaces *BB* are placed back

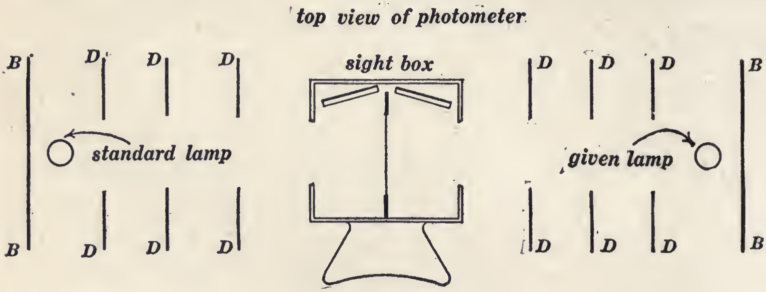


Fig. 74.

of each lamp, and a series of black diaphragms *DD* are placed so as to shield the photometer screen and the observer's eyes from all stray light. Freshly brushed black velvet is the best material to use for these diaphragms and for the back pieces *BB*.

The law of inverse squares is assumed in the use of a photometer, as explained in Art. 57, and therefore the distance from the photometer screen to either lamp should never be less than about 20 times the maximum dimension of the luminous surface of the lamp. A 10-foot photometer bar is therefore suitable for use with ordinary *bare* incandescent lamps, but when a lamp has a large shade the bar should be much longer because the shade is, in effect, the luminous source. The short distance between the standard lamp and the photometer screen in the Sharp-Millar photometer (see next article) is permissible because of the small size of the standard lamp.

It is best to set up the photometer in a dark room with dead black walls, but if the back curtains *BB* and the diaphragms *DD* are properly arranged the photometer room may be dimly lighted without interfering with accurate photometric work.

Tables should be arranged at the ends of the photometer bar for the gas or electric meters and other accessory apparatus, such as pressure regulators (for gas) and rheostats and switches (for electric current).

A single photometer setting is apt to be considerably in error, and therefore a number of settings is taken whenever possible. The taking of a set of readings is greatly facilitated by placing a strip of paper on the photometer bar and marking the successive positions of the photometer screen for a series of readings; this marking may be done very quickly by a pencil point which is carried by a flat spring attached to the sight-box of the photometer.

A photometer for incandescent lamp tests is usually provided with a universal rotating holder so that the lamp under test can be kept rotating and the axis of rotation can be given any desired position.

It is important to use a storage battery for supplying current to electric lamps under test because other sources of supply are usually subject to sudden changes of voltage. When a storage battery is not available the standard lamp and the lamp under test should both be operated from the same mains so that both lamps may be affected to approximately the same degree by voltage variation.

63. The portable photometer and illuminometer.—The most extensively used portable photometer is perhaps the *Weber photometer*.* The essential features of this instrument are embodied in the more recent and improved portable photometer of Sharp and Millar, and it is therefore sufficient to explain the construction and use of the Sharp-Millar instrument which can be used for measuring the candle-power of any lamp in service or for the measurement of the intensity of illumination at any point on a street or in a room. The essential features of the Sharp-Millar photometer are shown in Figs. 75 and 76.

The observer's eye is focused on the diagonal plane *ff* of a

* See *Industrial Photometry*, pages 85–91. Palaz (translated by G. W. and M. R. Patterson), Van Nostrand, 1894.

Lummer-Brodhun prism set *L-B*, and this diagonal plane is the field of view of the observer. Through the center of the field of view (where the prisms are in contact) the observer sees the diffusing plate *DD* in Fig. 75, or the translucent milk-glass plate

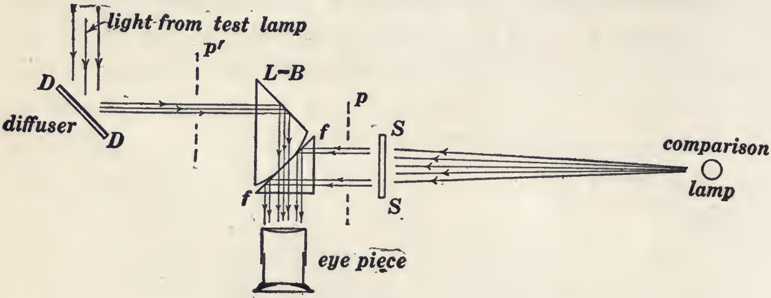


Fig. 75.

S′S′ in Fig. 76; and through the edge portions of the field of view (where the prisms are not in contact) the observer sees the translucent milk-glass plate *SS* which is illuminated by the comparison lamp (standard lamp) in the instrument.

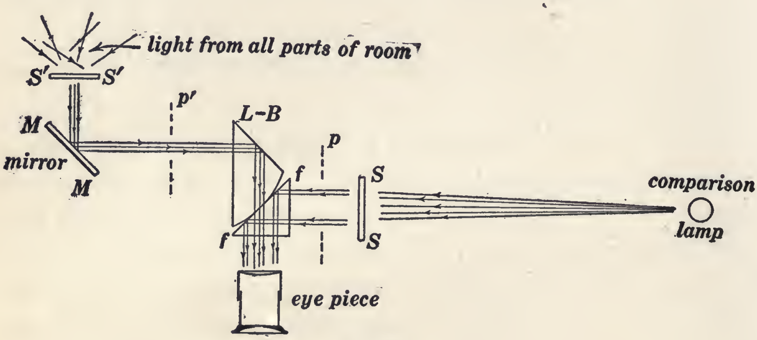


Fig. 76.

To set the photometer the comparison lamp is moved towards or away from *SS* until the central and edge portions of the field of view are equally bright, and the reading of the instrument is the distance of the comparison lamp from *SS*. To interpret the reading of the instrument calibration is necessary as follows:

(*a*) *Calibration for candle-power.*—A series of lamps of known candle-powers are placed in succession at a chosen distance d from DD (in place of the test lamp in Fig. 75), the corresponding readings of the photometer are taken, and a curve is plotted showing readings as abscissas and candle-powers as ordinates. Then if the lamp to be tested is at a distance d' from DD, the candle-power of the lamp is $(d'/d)^2$ times the candle-power which is given by the curve.

(*b*) *Calibration for intensities of illumination on $S'S'$.*—A lamp of known candle-power is placed at a series of measured distances from $S'S'$ in Fig. 76, thus producing known intensities of illumination upon $S'S'$; the corresponding readings of the photometer are taken; and a curve is plotted showing readings as abscissas and foot-candles as ordinates. Then to determine the intensity of illumination of, say, a horizontal surface at a given point in a room, the photometer is set up with the plate $S'S'$ at the given point and horizontal, the photometer reading is taken, and the desired value of the intensity of illumination is found from the curve.

The commercial form of the Sharp-Millar photometer is usually provided with a *direct-reading scale*, but it should be occasionally calibrated as above explained.

The comparison lamp is a very small tungsten lamp which has been thoroughly aged before it is mounted in the instrument, and the voltage between the terminals of the comparison lamp is kept at a prescribed value.

The Sharp-Millar photometer as furnished by the manufacturers* is provided with a smoke-glass plate which can be placed at p if very dim illumination is to be measured or at p' if very bright illumination is to be measured. This smoke-glass is usually adjusted by the manufacturers so that when used at p the photometer reading (in case the photometer is direct reading) must be divided by 10, and when used at p' the photometer reading must be multiplied by 10.

* Foote, Pierson & Co., of New York City.

Figure 77 shows the arrangement of the Sharp-Millar photometer. The containing box is blackened inside, and a series of diaphragms like *DD*, Fig. 74, are arranged between the comparison lamp *L* and the translucent screen *SS*. The elbow tube *T* can be turned so that its end *E* points in any desired direction. The translucent plate *S'S'* is placed over the end *E*, and *DM* is a plate of which one face is ground milk-glass for diffusion, and the other face is a thin plate of clear polished glass backed with silver (a mirror). The mirror face is used in Fig. 76 and the ground milk-glass face is used in Fig. 75.

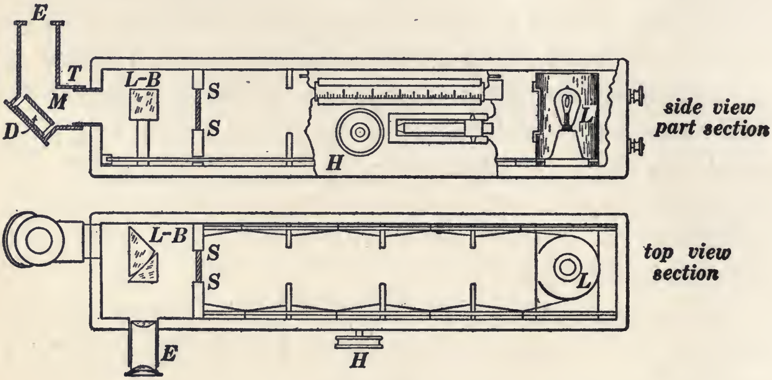


Fig. 77.

An interesting use of the Sharp-Millar photometer is to turn the mirror face of *DM* outward as in Fig. 76, remove the milk-glass plate *S'S'* so as to leave the end *E* of the elbow tube open, and direct the open end of the elbow tube towards a piece of uncalendered white paper which faces a lamp to be tested. The photometer reading is taken for a lamp of known candle-power at distance d from the paper screen, and then the candle-power of any given lamp is $(d'/d)^2$ times the photometer reading (photometer thought of as being direct reading for the sake of simplicity of statement), where d' is the distance of the given lamp from the paper screen. This result is independent of the distance of the piece of white paper from the instrument, and if the paper is free from gloss the result is independent of obliquity of vision of the white paper as seen from the end of the elbow tube of the instrument. The piece of white paper must face the lamp to be tested and it must be large enough to completely fill the central part of the observer's field of view (where the Lummer-Brodhun prisms are in contact in Fig. 76). This use of the Sharp-Millar photometer furnishes a striking illustration of the two propositions (*a*) and (*b*) in Art. 53.

64. The globe photometer of Ulbricht.*—Several types of photometer for determining the mean spherical candle-power of a lamp by a single setting of the photometer have been devised. The most notable are perhaps the integrating photometers of C. P. Matthews,† but the Ulbricht globe photometer is the most convenient in use. The arrangement of this photometer is shown in Fig. 78; *AA* and *BB* are two hemispheres on wheel-bases so that they can be easily moved apart. The interior of these hemispheres is painted with a dead-white paint (a white paint free from gloss).‡ The lamp to be tested and a "comparison lamp" of which the mean spherical candle-power has been previously determined are hung in the sphere as shown in the figure. A small milk-glass diffusing window *S′S′* is placed in one side of the sphere and

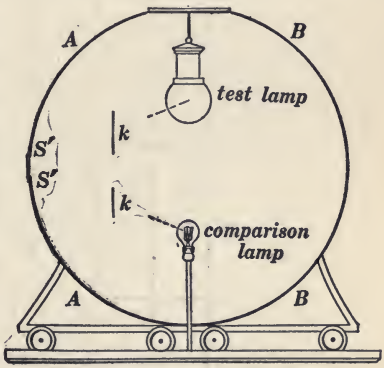


Fig. 78.

* The globe photometer is fully described and the results of a thorough investigation of its reliability are given by L. Bloch in *Electrotechnische Zeitschrift*, pages 1047–1052 and 1074–1078, November 16 and 23, 1905. See also a paper on the Ulbricht integrating sphere by C. H. Sharp, *Johns Hopkins University, Lectures on Illuminating Engineering*, Vol. I, pages 481–485, Baltimore, 1911.

† See *Transactions of the American Institue of Electrical Engineers*, Vol. XVIII, pages 677–697, 1901; and Vol. XX, pages 59–70, 1902.

‡ The most satisfactory paint is barium sulphate in zapon lacquer.

shaded from the direct light of the lamps by two white cardboard screens *kk*. The milk-glass diffusing window $S'S'$ is the illumination plate of a Sharp-Millar photometer (the same as $S'S'$ in Fig. 76). The photometer reading is taken with the "comparison lamp" in place. This lamp is then extinguished, the lamp to be tested is lighted, and the photometer reading is again taken. Multiplying the known mean spherical candle-

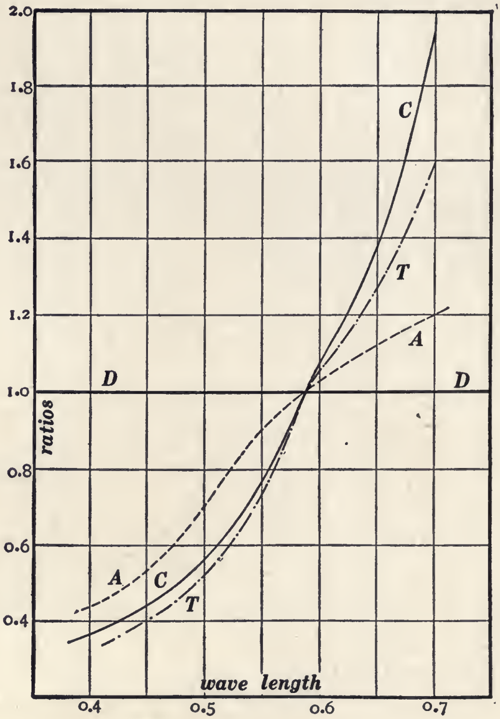


Fig. 79.

CC, carbon-filament lamp; *TT*, tungsten-filament lamp; *AA*, carbon-arc lamp; *DD*, daylight.

power of the "comparison lamp" by the ratio of the two photometer readings gives the mean spherical candle-power of the lamp under test. In this statement the photometer is assumed to be direct reading.

65. The spectrophotometer is a combination of a spectroscope and a photometer arranged for comparing the intensities of two beams of light, wave-length by wave-length. Figure 79 shows the results of a spectroscopic comparison of the light from a carbon-filament lamp (curve C), the light from a tungsten-filament lamp (curve T), and the light from the crater of a carbon-arc lamp (curve A), each with daylight (curve D). The meaning of these curves is as follows: *For the same intensity at the sodium line* (wave-length 0.589 millionth of a meter), the light from the carbon-filament glow lamp is 1.93 times as bright as daylight in the extreme red and 0.36 as bright as daylight in the extreme violet; *for the same brightness at the sodium line*, the light from the hot carbon tips of the carbon-arc lamp is 1.2 times as bright as daylight in the extreme red and 0.45 as bright as daylight in the extreme violet.

One of the best forms of spectrophotometer is the spectrophotometer of Lummer and Brodhun, the essential features of which are shown in Fig. 80, in which L-B is a Lummer-Brodhun prism-set. The observer's eye, placed at the narrow slit

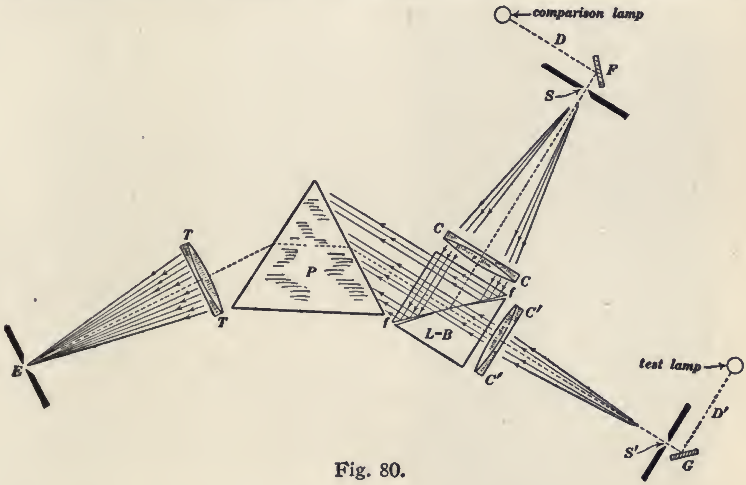


Fig. 80.

E, looks through the lens TT and the glass prism P and is focused on the diagonal face ff of the Lummer-Brodhun set so that this diagonal face is the observer's field of view. The observer sees the central portion of the field of view illuminated by light of one wave-length from the diffusing plate G, whereas the edge portions of the field of view are illuminated by light of the same wave-length from the diffusing plate F. The two diffusing plates F and G are illuminated by the comparison lamp and the test lamp, respectively, and the distances D and D' are adjusted until the observer's field of view is uniformly illuminated. Then the ratio of brightness of the two lamps for the given wave-length is equal to the ratio of the squares of the distances D and D'. In some forms of the Lummer-Brodhun spectrophotometer, the observer's field of view is brought to uniform intensity of illumination by adjusting the widths of the two slits S and S'.

CHAPTER V.

ELECTRIC LAMPS. LAMP SHADES AND REFLECTORS.

66. The electric arc. The arc lamp.—When two carbon or metal rods are connected to supply mains, brought into contact* and then separated, the current flows across the gap between the ends of the rods producing what is called an *electric arc*. Thus Fig. 81 shows the appearance of a direct-current arc between carbon rods. The direction of flow of the current is indicated by the arrow. The column of hot conducting vapor is called the *arc stream*, and the carbon or metal rods are called the *electrodes* (*anode* and *cathode*), as shown in Fig. 81.

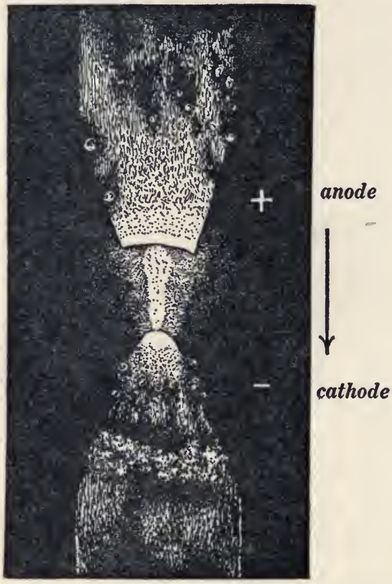


Fig. 81.

The arc between pure carbon electrodes is called the *carbon arc*, and an arc lamp in which pure carbon electrodes are used is called a *carbon-arc lamp*. Nearly all of the light of a carbon-arc lamp comes from the hot tips of the carbons, indeed the greater portion of the light comes from the slightly concave end of the anode carbon. The arc stream gives off a pale violet light.

The arc between metal electrodes or between electrodes containing metallic oxides or salts is called the *luminous arc*, because

* A rheostat must be included in the circuit.

the arc stream itself gives off a great deal of light. In this case the electrodes are not intensely heated and they do not give off any appreciable amount of light. An arc lamp in which a luminous arc is used is called a *luminous-arc lamp.* The luminous arc stream gives off a smoke or cloud of condensed metal oxide, and a flow of air must be maintained through the arc chamber of a luminous-arc lamp to carry away this oxide. Otherwise an

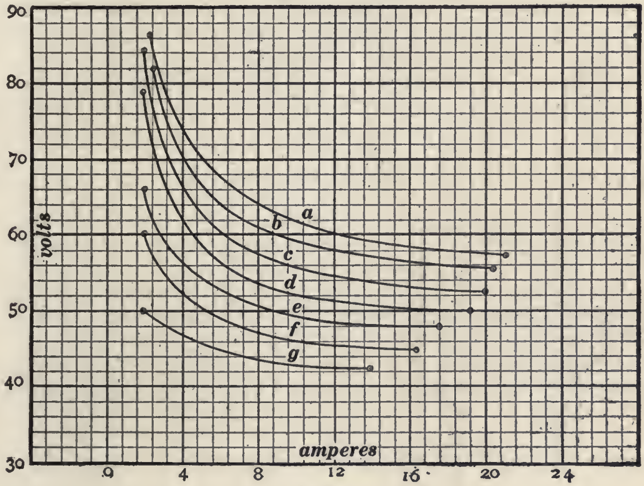


Fig. 82.

Curve *a* refers to arc 2.76 inches long.
" *b* " " " 2.36 " "
" *c* " " " 1.97 " "
" *d* " " " 1.58 " "
" *e* " " " 1.18 " "
" *f* " " " 0.79 " "
" *g* " " " 0.39 " "

opaque deposit would form on the inner walls of the enclosing glass globe.

The electric arc between pure carbon rods or between carbon rods impregnated with metallic salts can be maintained by direct current or by alternating current; but it is not practicable to maintain an alternating-current arc when either of the electrodes is of metal. Apparently the cathode of an arc must be at a temperature sufficiently high to vaporize the cathode material, and with a massive metal electrode the heat is taken away so rapidly that the necessary rise of temperature is not produced unless a very large current is used.*

An important property of the electric arc is that the voltage across the arc decreases with increasing current, as shown by the curves in Fig. 82. Consequently it is necessary to place resistance in series with an arc lamp which is connected across constant-voltage supply mains. Without this resistance the current would increase indefinitely and the arc would constitute a short circuit of the system. This resistance is called a *ballast resistance*, and the loss of energy in the ballast resistance is usually about 30 per cent. of the total energy delivered by the supply mains.

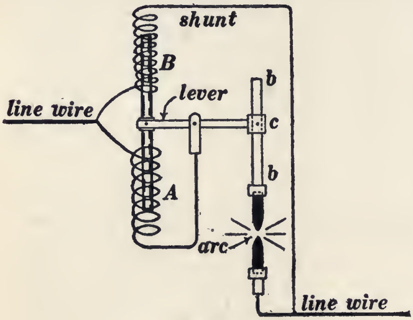


Fig. 83.

In an alternating-current arc lamp a choke coil (an inductance) can be used as a ballast. In a direct-current arc lamp a resistance must be used as ballast. When arc lamps are connected in series to a constant-current supply, no ballast is necessary.

An important part of an arc lamp is the mechanism for automatically moving the electrodes so as to keep the arc steady. Thus Fig. 83 shows the essential features of an arc-lamp mechanism for lamps which are to be connected in series. A very small portion of the current flows through a shunt coil B without passing through the arc, and the remainder of the current flows through the coil A and thence through the arc. An iron rod AB passing loosely into the coils A and B is attached to one end of

* For a discussion of the physics of the electric arc see C. P. Steinmetz, *Transactions of International Electrical Congress*, Vol. II, pages 710–730, St. Louis, 1904. Also see W. R. Whitney, *Transactions of American Electrochemical Society*, Vol. VII, pages 291–299, 1905.

a lever which is pivoted at its center, and the other end of the lever is provided with a clutch *c* through which a smooth brass rod *bb* passes. This brass rod supports one of the carbon electrodes, and the clutch is so constructed that it releases the rod *bb* when the iron rod *AB* is raised, thus allowing the carbons to come together. Each of the coils *A* and *B* acts to pull the rod *AB* into itself, and a spring which is attached to the lever is adjusted so that when the arc is burning properly the combined action of this spring and the two coils *A* and *B* holds the lever in such a position that the clutch clasps the brass rod *bb*. As the arc continues to burn the carbons are slowly consumed, causing the gap between the carbon tips to widen. This increases the voltage across the arc and causes a greater portion of the current to flow through the shunt coil *B*, which pulls up on the iron rod *AB*, moves the lever, lowers the carbon and ultimately releases the clutch so as to allow the rod *bb* to fall. The carbons usually come too near together when the clutch *c* is thus released, but the current in coil *B* is thereby greatly reduced so that the current in coil *A* quickly pulls the lever down and separates the carbons to the desired extent.

67. Carbon-arc lamps. The open-arc lamp and the enclosed-arc lamp.—In the oldest form of arc lamp the carbons are exposed to the open air or surrounded by a glass globe through which the air circulates freely. This type of lamp is called the *open-arc lamp*. The carbons in this type of lamp are consumed rapidly by the oxygen of the air, and the lamp must be trimmed, that is, the carbons must be renewed, about once in 12 hours.

In the *enclosed-arc lamp* the ends of the carbon rods project into a small glass bulb which is very nearly air-tight. In this type of lamp the carbons last about 150 hours.

The carbon-arc lamp operates satisfactorily on direct-current circuits or on alternating-current circuits. The mechanism is, however, slightly different in the two cases so that an arc lamp which has been designed especially for direct current will not operate satisfactorily with alternating current.

10

The carbon-arc lamp is rapidly going out of use. Tungsten lamps are much better and cheaper where small units are needed, and luminous-arc lamps and mercury-vapor lamps are much more efficient where large units are needed.

68. Luminous-arc lamps. The magnetite-arc lamp and the flame-arc lamp. — In the *magnetite-arc lamp* the anode is a short rod of copper as shown in Fig. 84,* and the cathode is composed of a mixture of iron oxide (magnetite), titanium oxide and chromium oxide. The copper anode remains relatively cool and wears away with extreme slowness. The end of the cathode is heated to a moderately high temperature during the operation of the lamp, and the oxides are slowly vaporized producing an intensely luminous arc. The copper anode lasts five thousand hours or more, and the rod of magnetite lasts about one hundred and fifty hours. The light emitted by the lamp is a brilliant white. It is not feasible to operate the magnetite-arc lamp by alternating current.

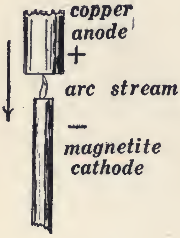


Fig. 84.

In the *flame-arc lamp* carbon rods are used as electrodes, and these carbon rods are impregnated with metallic salts. In the familiar flame-arc lamp, which gives an extremely brilliant yellow light, the carbons are impregnated with calcium fluoride

The flame arc gives off a cloud of oxide, and it is impracticable to enclose the flame arc in a small bulb such as is used in the enclosed carbon-arc lamp. Therefore when the flame-arc lamp was first brought out the arc was not enclosed and the carbons burned away rapidly. Consequently very long carbons were required for a 12-hour run, and the use of long carbons led to the arrangement shown in Fig. 85. This type is called the *short-*

* Figure 84 shows the arrangement of the electrodes in the General Electric Company's lamp. In the Westinghouse magnetite-arc lamp the magnetite cathode is above and the anode is below.

burning inclined-carbon flame-arc lamp. It is now practically obsolete. The objection to this type of flame-arc lamp is the cost of the frequent trimming.

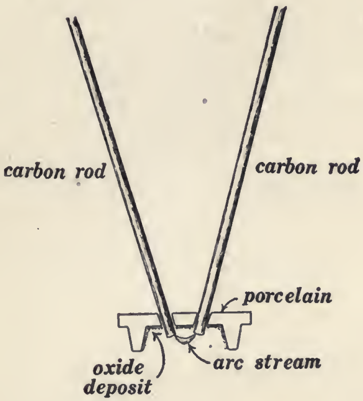


Fig. 85.

A satisfactory enclosed flame-arc lamp was placed on the market in 1911, the cloud of oxide from the arc stream being carried by a natural draft into a condensing chamber where the oxide is deposited, and the clear cooled air retuns to the arc chamber, but no fresh air has access to the arc chamber. In the enclosed flame-arc lamp the carbon electrodes are placed vertically one over the other as in the ordinary carbon-arc lamp, and the electrodes last about 100 hours.

The flame-arc lamp can be operated either by direct current or by alternating current.

69. The mercury-vapor lamp.—In 1860 it was known that a steady and brilliantly luminous effect could be obtained by passing an electric current through a glass tube containing mercury vapor, and in 1881 the proposal was made to utilize this effect in an electric lamp. In the early attempts to produce a practicable mercury-vapor lamp the necessary cooling of the vapor tube was accomplished by the circulation of water, and it was not until Peter Cooper Hewitt (about 1898) designed a vapor tube with enlarged condensing chambers (which, indeed, do not need to be very large on a long tube) that the mercury-vapor lamp became a success.

The essential features of the Cooper-Hewitt lamp are shown in Fig. 86. A long glass tube with a bulb on each end is provided

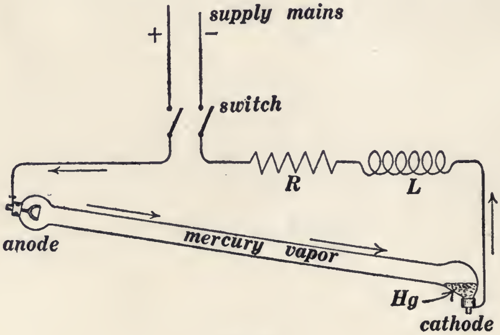


Fig. 86.

with sealed-in lead-wires one of which connects with an electrode (the anode) of iron or graphite and the other connects with a pool of mercury (the cathode). The air is removed from the tube by an air pump.

To start the lamp the tube is brought into a horizontal position so that a thread of mercury bridges across from electrode to electrode, the tube is then brought back to an inclined position and when the thread of mercury breaks the current continues to flow through the mercury vapor.* A ballast resistance R is necessary, and an inductance (a choke coil) L is used to prevent the stoppage of current by a momentary drop of the supply voltage.

The type of lamp which is started by tilting is provided with a mechanism in which an electromagnet automatically tilts

* A momentary high voltage produced by a spark-coil is sometimes used for starting the flow of current through the mercury-vapor lamp.

the tube and brings it back to the running position when the control switch is closed. A general view of a Cooper-Hewitt lamp of this type, with its inverted-trough reflector is shown in Fig. 87. The ballast and the tilting mechanism are contained in the sheet-metal case above the lamp tube.

Fig. 87.

The simple form of Cooper-Hewitt lamp cannot be operated by alternating current. The alternating-current lamp has two anodes and its connections are similar to the connections of the mercury-vapor rectifier.*

The light of the Cooper-Hewitt lamp is intensely green and it is entirely unsatisfactory where objects must be seen in their natural colors as by day-light. When color values are not important, for example in shops and draughting rooms, the lamp is quite satisfactory.

The *Quartz lamp* is a mercury-vapor lamp, with a containing tube made of fused silica or quartz. It differs from the Cooper-Hewitt lamp in that a greater amount of energy can be delivered to a small tube. The tube becomes red-hot during the operation of the lamp and the enclosed vapor gives an extremely brilliant light. The light from the quartz lamp contains a small amount of red, and the light is therefore of a more pleasing quality than the light of the Cooper-Hewitt lamp.

70. The glow lamp.—The most extensively used type of electric lamp is the familiar *incandescent lamp* or *glow lamp*, in which a fine filament of carbon or metal is heated to incandescence by the electric current. The filament is enclosed in a glass bulb from which the air is exhausted, the object being to protect the filament from the oxygen of the air and to elim-

* See *Dynamos and Motors*, Chapter XIII.

inate the great cooling effect which exists when the filament is surrounded by gas of any kind.

There are five important kinds of glow lamps as follows:

(*a*) The old style *carbon-filament lamp.*

(*b*) The "*metalized*" *carbon-filament lamp* which differs from the old style in that the carbon filament is heated to an extremely high temperature in an electric furnace before it is mounted in the lamp.

(*c*) The *tantalum lamp* in which the filament is a fine wire of metallic tantalum.

(*d*) The *tungsten lamp* in which the filament is a fine wire of metallic tungsten. The tungsten lamp is sometimes called the *mazda lamp.*

(*e*) The *Nernst lamp* in which the glower is a small rod of porcelain-like material.*

The carbon-filament lamp (old style and metalized) and the tungsten lamp are by far the most important.

The carbon-filament lamp stands rough handling without breakage of the filament and it is cheap; but it is not very efficient. The tungsten lamp on the other hand is more fragile

* An interesting article on carbon-filament lamp manufacture by M. K. Eyre is given in *The Electrical World*, pages 9–13, January 5, 1895.

The "metalizing" process of carbon-filament manufacture is described by J. W. Howell in the *Transactions of the American Institute of Electrical Engineers*, Vol. XXIV, pages 839–849, June, 1905.

A discussion of the tantalum-filament glow lamp is given by Bolton and Feuerlein, *Electrotechnische Zeitschrift*, Vol. XXVI, pages 105–108, January, 1905.

" New Types of Incandescent Lamps," a discussion of the early process of tungsten filament manufacture and a discussion of the characteristics of metal filament lamps, by C. H. Sharp, *Transactions of the American Institute of Electrical Engineers*, Vol. XXV, pages 815–864, November, 1906.

The manufacture of malleable tungsten which can be drawn into fine wire is described by W. C. Coolidge, *Transactions of the American Institute of Electrical Engineers*, Vol. XXIX, pages 961–965, May, 1910.

The Nernst lamp is described by A. J. Wurts in the *Transactions of the American Institute of Electrical Engineers*, Vol. XVIII, pages 545–587, 1901.

The process of manufacture of the Nernst lamp is described in *The Electrical World and Engineer*, Vol. XLIII, pages 981–985, May 21, 1904.

COST OF LIGHTING BY GLOW LAMPS

(Energy at 10 cents per kilowatt-hour.)

1	2	3	4	5	6	7	8	9	10	11
Lamp.	Actual Watts.	Watts per Candle.*	Candle-power.*	Hours Total Life.	Cost of Lamp in Cents.†	Energy Consumed During Life in Kilowatt-hours.	Candle-hours During Life.	Cost of Lamps for 1,000 Candle-hours in Cents.	Cost in Cents of Energy for 1,000 Candle-hours at 10 Cents per Kilo-watt-hour.	Cost of Energy and Lamp Renewals for 1,000 Candle-hours in Cents.
Metalized carbon	50	2.50	20.0	700	16.59	35	11,900	1.395	29.40	30.79
Tungsten.............	25	1.31	19.1	1,000	39.50	25	18,200	2.170	13.73	15.90
Tungsten.............	40	1.23	32.5	1,000	43.45	40	30,900	1.407	12.94	14.35
Tungsten.............	60	1.18	50.8	1,000	59.25	60	48,400	1.224	12.40	13.60
Tungsten.............	100	1.18	84.7	1,000	86.90	100	80,500	1.079	12.40	13.48
Tungsten.............	150	1.18	127.0	1,000	130.35	150	120,800	1.079	12.40	13.48
Tungsten.............	250	1.13	221.7	1,000	181.70	250	210,000	0.865	11.90	12.76

* The words *candle* and *candle-power* signify *mean horizontal candle-power.*

† Prices of lamps are net prices, March, 1912, on $300 contract.

Old style carbon lamps and tantalum lamps are practically obsolete and they are therefore omitted from this table.

Candle-hours during life is found by multiplying initial candle-power by hours total life by deterioration factor.

This table refers to what is called "top efficiency." See page 139.

and it is more expensive than the carbon-filament lamp, but it is much more efficient.

A comparison of costs of carbon, tantalum and tungsten lamps is shown in the accompanying table, energy cost being 10 cents per kilowatt-hour.

It is not strictly correct to compare a carbon lamp with a tungsten lamp on the basis of mean horizontal candle-power as is done in this table because the mean spherical candle-power of the regular type of carbon lamp is about 0.85 of its mean horizontal candle-power whereas the mean spherical candle power of a regular type of tungsten lamp is only about 0.79 of its mean horizontal candle-power. The correct basis of comparison is the *total amount of light emitted;* to reduce the values in columns 9, 10 and 11 to this correct basis the tungsten costs should be multiplied by 0.85/0.79.

An important matter which is shown very clearly in this table is that *more than nine-tenths of the cost of lighting by glow lamps is for energy, and less than one-tenth of the cost is for lamp renewals.* The users of electric light hesitate, however, to purchase a high-priced tungsten lamp when they can get a low-priced carbon lamp or when their contract with the lighting company calls for free renewals of their carbon lamps. This is a very short-sighted policy because the higher-priced tungsten lamp has a longer life than the carbon lamp and this longer life alone makes up for a large part of the greater cost, but especially because the saving in energy-cost which can be realized by the high-priced tungsten lamp may be ten or twenty times the cost of the lamp.

A given glow lamp is usually rated by the manufacturers for a specified supply voltage as explained below, but the lamp can, of course, be o erated at a higher or lower voltage. The effect of a higher voltage is to give a greatly increased candle-power, an increased power consumption (watts), a decreased watts-per-candle, and a shortened life. A low power consumption in watts per candle is, of course, desirable because it saves in the cost of energy, but a short life is undesirable because short life involves high renewal cost. It is therefore important to make a proper compromise between these two items of cost so as to obtain a minimum total cost. This matter is shown by the curves in Fig. 88. The pairs of curves *A*, *B*, *C*, *D* and *E* refer to energy costs of 2, 4, 6, 8 and 10 cents per kilowatt-hour respectively. These curves refer to an 80-candle-power tungsten lamp. The abscissas represent watts per candle, and the ordinates represent the costs for 1,000 hours' use of an 80-candle-power lamp of the following items: (*a*) energy cost, (*b*) renewal cost, and (*c*) total cost. The wavy line connects the minimum points of the total cost curves. Thus when energy costs 10 cents per kilowatt-hour a tungsten lamp of the size specified gives light at a minimum total cost when operated so as to consume about 1.1 watts per mean horizontal candle-power.

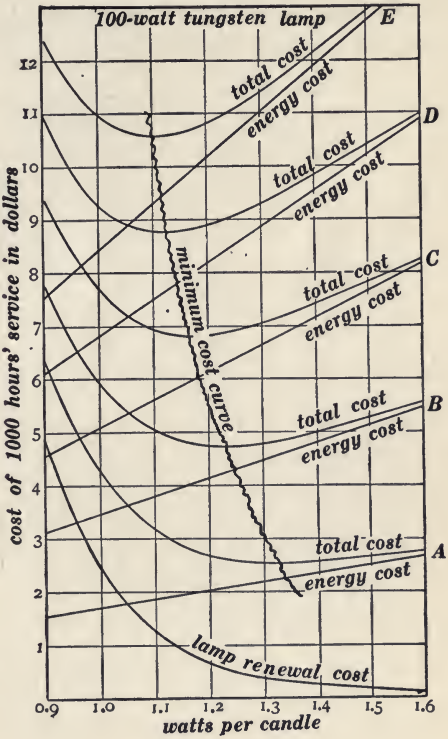


Fig. 88.

The cost of energy per kilowatt-hour in a large manufacturing plant is usually less than two cents and sometimes less than one cent. It is therefore important to consider total costs of lighting by glow lamps (lamp renewal cost plus energy cost) when the rate per kilowatt-hour is less than 2 cents. These costs are shown by the ordinates of the curves in Fig. 89. These curves show that the tungsten lamp is cheaper than the carbon-filament lamp down to an energy rate of about 0.2 cent per kilowatt-hour.

The carbon-filament lamp is properly used where it is subjected to rough handling or where it is used only a very small portion of the time. Thus the drop-lamp which a machinist uses about a lathe or boring-mill and the lamps one uses in a cellar or closet should be carbon-filament lamps.

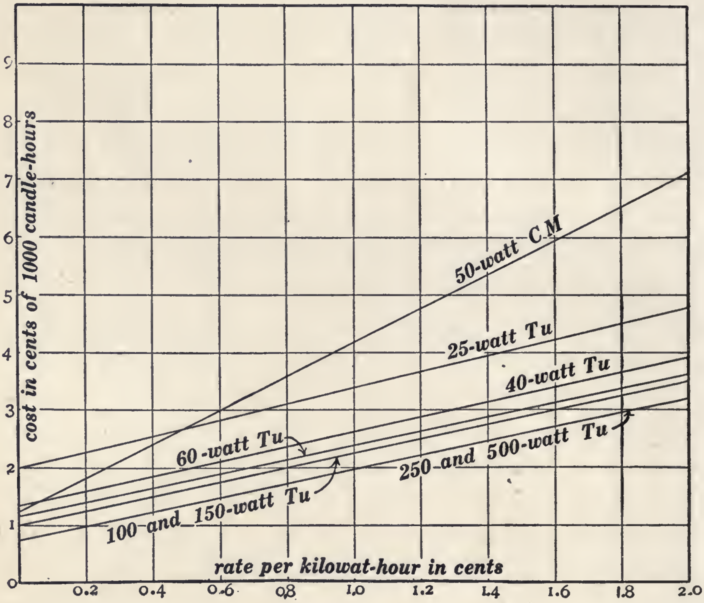


Fig. 89.

*Glow lamp ratings.**—To rate a thing like a glow lamp is to specify the conditions under which it is to be used, and the results to be obtained by its use. Manufacturers usually rate a glow lamp in terms of the voltage for which the lamp is to be used. Thus a 110-volt lamp is one that is designed to operate from 110-volt mains. In addition to its voltage rating, it is necessary to specify the approximate candle-power of the lamp

* See Circular No. 13 of the U. S. Bureau of Staudards for standard specifications for the purchase of incandescent electric lamps.

The Electrical Testing Laboratories, 80th Street and East End Avenue, N. Y. City, have unsurpassed facilities for testing lamps for purchasers.

or to specify the approximate power consumption of the lamp in watts at its rated voltage. Carbon-filament lamps are usually rated in candle-power,* and tungsten lamps are usually rated in terms of their power consumption. Thus we speak of a 16-candle-power carbon-filament lamp or of a 100-watt tungsten lamp.

The three-efficiency scheme of rating tungsten lamps.—From Fig. 88 it is evident that it is sometimes economical to burn tungsten lamps at low watts-per-candle and sometimes economical to burn tungsten lamps at high watts-per-candle. Therefore, manufacturers offer the purchaser a choice of watts-per-candle in lamps of any given voltage and power rating. Three efficiencies are offered in regular lamps; namely, "top efficiency," "middle efficiency" and "bottom efficiency" as shown in the following table. If a lamp user pays a high rate per kilowatt-hour for his energy, he should order "top efficiency" lamps, and if he pays a low rate per kilowatt-hour for his energy, he should order "bottom efficiency" lamps.

LIFE AND EFFICIENCY TABLE FOR REGULAR TUNGSTEN LAMPS.

Size of Lamp in Watts.	Top Efficiency.		Middle Efficiency.		Bottom Efficiency.	
	Watts per Candle.*	Life in Hours.	Watts per Candle.	Life in Hours.	Watts per Candle.	Life in Hours.
25	1.31	1,000	1.37	1,300	1.43	1,700
40	1.23	1,000	1.28	1,300	1.33	1,700
60	1.18	1,000	1.23	1,300	1.28	1,700
100	1.18	1,000	1.23	1,300	1.28	1,700
150	1.18	1,000	1.23	1,300	1.28	1,700
250	1.13	1,000	1.18	1,300	1.23	1,700

Candle-power ratings of glow lamps.—It is the universal practice to rate a glow lamp by giving its mean horizontal candle-power.

*Mean horizontal candle-power.

The mean horizontal candle-power, however, is not an exact measure of the amount of light emitted by a lamp; the amount of light emitted must be expressed in spherical-candles or in lumens. The factor by which the mean horizontal candle-power of a lamp must be multiplied to give its mean spherical candle-power is called the *spherical reduction factor* of the lamp. The spherical reduction factor of the regular carbon-filament lamp is about 0.85, and the spherical reduction factor of the regular tungsten lamp is about 0.79. To reduce mean spherical candle-power to lumens multiply by 4π.

Variations of candle-power and watts due to variation of voltage.—Glow lamps are generally supplied with current from "constant voltage" mains, but the supply voltage always varies irregularly

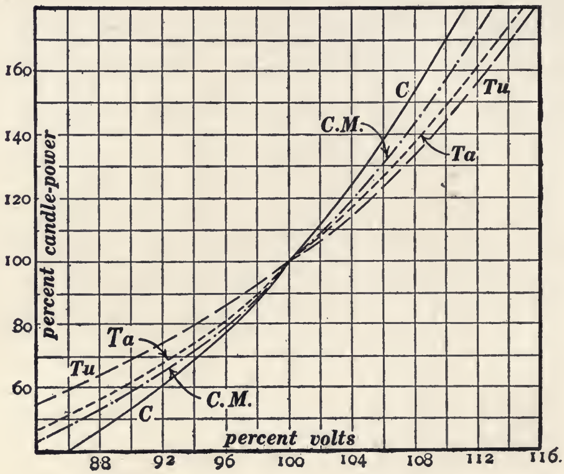


Fig. 90.

through a range of one or two per cent. (indeed the variation of voltage is much more than one or two per cent. when a central station is poorly designed or carelessly operated), and all of the connected lamps fluctuate in candle-power as the supply voltage rises and falls. This fluctuation of candle-power is very unpleasant, and metal-filament glow lamps have an advantage over carbon-filament lamps in that the variation of candle-power due to a given variation of voltage is less for metal-filament lamps than for carbon-filament lamps. The ordinates of the curves in Figs. 90 and 91 show *candle-powers* and *watts* for various

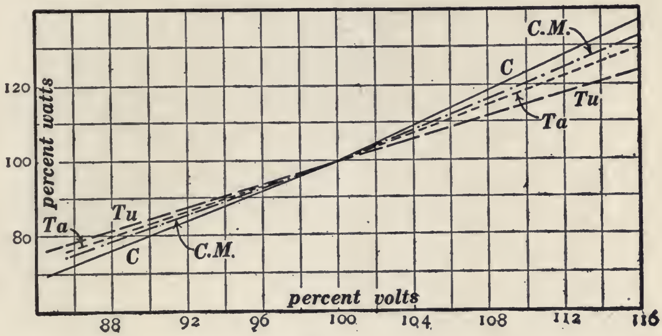


Fig. 91.

values of *voltage* (abscissas). The normal value of each item (voltage, candle-power and watts) is taken as 100 so that the departures from the normal values may be read off directly in per cent. Thus a two per cent. increase of voltage (100 to 102 in Fig. 90) causes a 12 per cent. increase of candle-power of an old style carbon-filament lamp (curve *C*), a 10 per cent. increase of candle-power of a metalized carbon-filament lamp (curve *CM*). and a 7 per cent. increase of candle-power of a tungsten lamp (curve *Tu*).

Slow deterioration of glow lamps in service.—A glow-lamp filament is always operated at a temperature which causes a slow change of the filament and an ultimate deterioration of the lamp. This deterioration is chiefly of two kinds, namely, (*a*) an increase of resistance of the filament which causes a decrease of power consumption and a very considerable decrease of candle-power, and (*b*) a blackening of the lamp bulb which involves a very great loss of light. If a lamp is used long enough the filament is weakened until it breaks.

It is usually advisable to use a tungsten lamp until the filament breaks. Occasionally, however, the bulb of a tungsten lamp blackens and such a lamp should be discarded when the blackening causes a serious loss of light.

A carbon-filament lamp should, as a rule, *not* be used until the filament breaks, it is cheaper to throw away an old carbon-filament lamp and buy a new one than it is to use the old lamp at a greatly decreased efficiency (increased watts-per-candle). A carbon-filament glow lamp is usually considered to have reached the end of its useful life when its candle-power falls to 80 per cent. of its initial value.

The curves in Fig. 92* show the change of candle-power of various kinds of glow lamps with age; *C*, *CM*, *Ta* and *Tu*

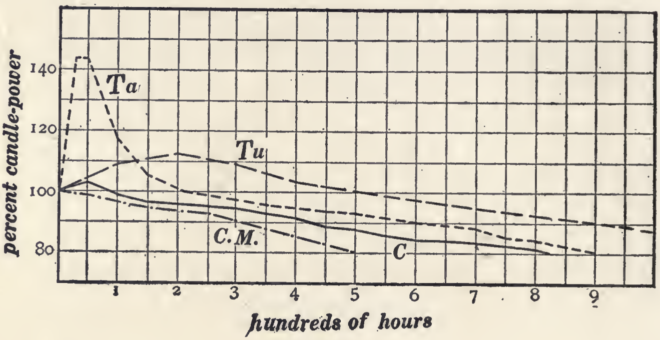


Fig. 92.

refer to carbon, "metalized" carbon, tantulum and tungsten, respectively.

In estimating the cost of lighting it is important to estimate the illuminating power of a lamp on the basis of its average candle-power during its life. Thus the average candle-power of a tungsten lamp during its life is from 0.90 to 0.95 of its candle-power when new. See page 149.

Regular lamps and special lamps.—Great numbers of glow lamps are used, in the ordinary constant-voltage system, for

* These curves are taken from Wickenden's *Illumination and Photometry*. The tungsten lamp curve refers, apparently, to an old-style tungsten lamp. More recent deterioration curves, and a discussion of the important subject of deterioration are given by Sydney W. Ashe, *Transactions of the Illuminating Engineering Society*, Vol. VI, pages 503–570, June, 1911.

lighting houses of all kinds, and these lamps are made of standard form so as to fit standard forms of sockets and shades. Such lamps are called *regular lamps.* Glow lamps for street lighting, sign lighting, and car lighting, are called *special lamps.* Very small lamps for batteries are called *miniature lamps.*

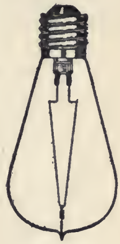


Fig. 93.

Figure 93 shows the special form of tungsten lamp which is used when a large number of lamps are connected in series for street lighting. These lamps have a heavy filament (low voltage) and they are made in a variety of sizes, from 25 to 350 candle-power with current ratings* from 3.50 to 7.5 amperes.

71. Comparison of electric lamps.—An important matter in connection with a lamp is the distribution of light around the lamp. Thus the candle-power distribution curves of carbon-filament and tungsten-filament glow lamps with and without

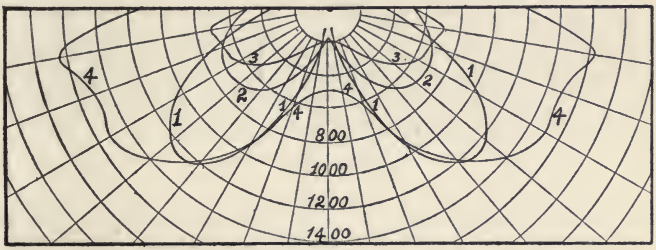


Fig. 94 *a.*

shades are shown in Figs. 64, 65 and 71. Candle-power distribution curves of the more important types of arc lamps are shown in Figs. 94 *a* and 94 *b.* The numbers of the curves correspond to the serial numbers in the following tables.† Curve 4 in Figs. 94 *a* and 94 *b* refers to the same lamp, namely, the 6.6 ampere magnetite-arc lamp equipped with an enamel reflector,

* Series tungsten lamps are rated on the basis of current and candle-power (mean horizontal candle-power).

† Candle-power distribution curves of mercury-vapor lamps are given by Sydney W. Ashe, *Transactions of the Illuminating Engineering Society,* Vol. VI, pages 513–516, June, 1911.

as furnished by General Electric and Westinghouse Companies for street lighting.

In laying out the lighting plans of a shop or factory one needs to consider the intensity of illumination on the working plane as explained in Art. 81. The following table* gives the values of

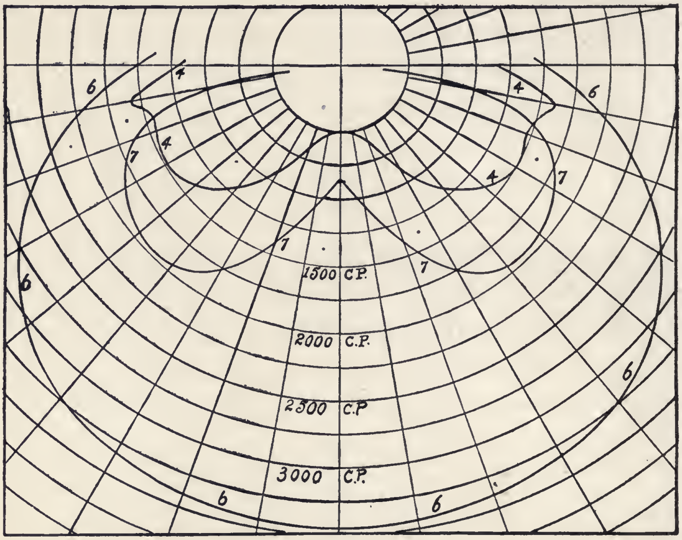


Fig. 94 *b*.

I_h (horizontal illumination) produced by various arc lamps. The height of the lamp above the working plane is in each case assumed to be 50 feet, and x is the horizontal distance from the lamp to the point on the working plane where I_h is reckoned.

By making use of the law of inverse squares (Art. 48) it is easy to use this table to find the intensity of illumination (horizontal) at a point at any given distance d horizontally from any one of the lamps placed at any given height H above the working plane. The rule is as follows: *Find from the table the value of* I_h *for the lamp* 50 *feet high, and for* $x = d \times 50/H$, *and multiply the value of* I_h *so found by* $(50/H)^2$.

* This table was calculated from the candle-power curves of Figs. 94*a* and 94*b*, using equation (16 *b*).

VALUES OF ILLUMINATION ON HORIZONTAL PLANE IN FOOT-CANDLES.

Height of lamps 50 *feet, horizontal distance from lamp* = x.

(Values of I_h.)

Serial Number.	Type of Lamp.	Values of x in Feet.									
		0	50	100	150	200	250	300	350	400	450
3	4-ampere magdetite-arc lamp	0.0720	0.0653	0.0216	0.00791	0.00374	0.00196	0.00125	0.00086	0.00062	0.00045
4	6.6-ampere magnetite-arc lamp	0.196	0.181	0.0537	0.0191	0.00895	0.00467	0.00298	0.00205	0.00146	0.00106
6	Inclined-carbon flame-arc lamp	1.36	0.453	0.102	0.0291	0.0120	0.0060	0.0032	0.0018	0.0013	0.00096
7	Vertical-carbon flame-arc lamp	0.349	0.288	0.069	0.0260	0.0074	0.0033	0.00174	0.00109	0.00075	0.00053

Example.—To find the horizontal illumination at a point 80 feet from an inclined-carbon flame-arc lamp 40 feet above the working plane. Multiply 80 by 50/40, giving $x = 100$ feet. The value of I_h in the table corresponding to this value of x is 0.102, which, multiplied by $(50/40)^2$ gives the desired result, namely, 0.127 foot-candle.

To find the distance d from a lamp H feet above the working plane to give a prescribed horizontal illumination, *multiply the prescribed horizontal illumination by* $(H/50)^2$ *to get* I_h **as per table,** *find the corresponding value of* x *from the table and multiply this value of* x *by* $(H/50)$ *to get the desired value of* d.

Example.—To find d for which a 6.6-ampere magnetite-arc lamp hung 30 feet high will give 0.5 foot-candle horizontal illumination; the value of $(H/50)^2 \times 0.5$ is 0.18 foot-candle, and the corresponding value of x is 50 feet, so that the desired value of d is 30 feet.

Comparative costs.—The following table* shows the approximate cost of producing 100,000 downward lumens 6 hours per day 300 days per year using different kinds of lamps. It must be remembered that this table refers to average† conditions, and to interpret the table properly the following matters must be taken into consideration.

(*a*) *Multiple arc lamps and series arc lamps.*—When few arc lamps are to be installed, it is always most convenient to supply them from existing constant-voltage mains. Each lamp is provided with a ballast, as explained on page 128, and the lamps are connected singly‡ across the constant voltage mains. Lamps designed to be connected in this way are called *multiple lamps.*

When many arc lamps are to be installed, they may be con-

* This table has been prepared from data collected from various sources. A discussion of costs of street lamps is given in *Bulletin No. 51* of the Illinois Engineering Experiment Station by J. M. Bryant and H. G. Hake, on Street Lighting.

† See statement concerning averages on page 2.

‡ Sometimes two lamps are connected in series. In case of a 500-volt supply six or seven lamps can be connected in series. Lamps designed to be connected in this way are called *series-multiple lamps.*

ANNUAL COST OF 100,000 DOWNWARD LUMENS.

(6 hours per day, 300 days per year.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Serial Number.	Type of Lamp.	Number of Lamps to Give Approximately 100,000 Downward Lumens.	Actual Downward Lumens.	Ampere Rating of Lamp.	Volts Across Lamp Terminals.	(*a*) Inner Globe. (*b*) Outer Globe. (*c*) Reflector.	Total Watts.	Annual Cost of Energy at 2 Cents per Kilowatt-hour.	Annual Cost of Maintenance.	First Cost of Lamps Ready for Service.	Interest at 6 Per Cent. on First Cost.	Depreciation at 10 Per Cent. on Non-renewable Parts.	Total Annual Cost.
1*a*	500-watt tungsten (multiple).......	29	100,000		110	Enamel reflector	14,500	$522	$227	$177	$11	$ 6	$ 766
1*b*	100-watt tungsten (multiple).......	186	99,600		110	Prismatic glass reflector	18,600	670	415	424	26	24	1,135
1*c*	350-c.p. tungsten (series).........	42	101,000	†		Enamel reflector	17,052	614	215	304	18	15	862
1*d*	60-c.p. tungsten (series).........	254	100,000	†		Enamel reflector	18,000	648	325	1117	67	89	1,129
2	Enclosed carbon-arc (multiple).......	49	99,000	5	110	(*a*) Light opal (*b*) None (*c*) Opal glass	27,000	972	163	784	47	78	1,260
3	4-amp. magnetite-arc (series)......	30	101,000	4	77.5	Enamel reflector inside of clear globe	9,300	335	64	675	41	68	508
4	6.6-amp. magnetite-arc (series)......	12	100,400	6.6	77.5	Enamel reflector inside of clear globe	6,120	220	37	300	18	30	305
6	Inclined-carbon flame-arc, short burning (*AC* series)..........	9	101,000	10	55	(*a*) None (*b*) Clear (*c*) See Fig. 85.	5,050	182	284	450	27	45	538

ANNUAL COST OF 100,000 DOWNWARD LUMENS.—(*Continued.*)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Serial Number.	Type of Lamp.	Number of Lamps to Give Approximately 100,000 Downward Lumens.	Actual Downward Lumens.	Ampere Rating of Lamp.	Volts Across Lamp Terminals.	(a) Inner Globe. (b) Outer Globe. (c) Reflector.	Total Watts.	Annual Cost of Energy at 2 Cents per Kilowatt-hour.	Annual Cost of Maintenance.	First Cost of Lamps Ready for Service.	Interest at 6 Per Cent. on First Cost.	Depreciation at 10 Per Cent. on Non-renewable Parts.	Total Annual Cost.
7	Vertical-carbon flame-arc, long burning (D.C. series)*	11	96,000	6.5	77	(a) Clear (b) Light opal	5,500	198	78	418	25	42	343
8	Cooper-Hewitt (multiple)	26	98,000	3.5	110	Enamel trough reflector	10,000	360	104	842	51	38	553
9	Quartz tube mercury-vapor (multiple)	6	98,000	3.5	220	Enamel plate reflector	4,620	166	68	437	26	21	281

* The data given for lamp No. 7 refer to a multiple lamp taking 6.5 amperes at 110 volts, and correction has been made for the loss of energy in the ballast so as to give a fair comparison with the other series arc lamps.

† Current rating is 1.75 to 7.5 amperes; see page 143.

This table must be used with due consideration of the qualifications mentioned in the text.

It is very important to make allowance for the deterioration of a lamp during its life. This can be done by dividing the values in column 14 by the deterioration factors of the respective lamps.

No central station can supply a small amount of power for a short period each day for less than 6 or 8 or 10 cents per kilowatt-hour. Column 9 can be easily recalculated for any rate per kilowatt-hour and column 14 corrected accordingly.

nected in series to a constant-current supply (see Art. 87). This arrangement reduces the cost of wiring to a minimum and it eliminates the waste of energy in lamp ballasts. Lamps designed to be connected in this way are called *series lamps.*

A multiple lamp is rated by specifying the voltage of supply for which the lamp is designed and the current which the lamp will take when it is properly adjusted. A series lamp is rated by specifying the current for which the lamp is designed.

All of the arc lamps referred to in the table are series lamps. Nearly every type of arc lamp, however, is offered for sale either as a series lamp or as a multiple lamp; a multiple lamp uses about 1.4 times as much power as a series lamp when the same amount of power is expended at the arc.

(*b*) *Cost of energy.**—As a rule series lamps are used under conditions which involve low cost of energy (long hours of service and large demand for power), whereas multiple lamps are, as a rule, used under conditions which involve high cost of energy (short hours of service and small demand for power). Thus a company might furnish power for operating the street lamps of a city (about 4,000 hours of service per year) at 2 or 3 cents per kilowatt-hour, and be fully justified in charging 8 or 10 cents per kilowatt-hour for power for operating two or three arc lamps in a store (about 1,000 hours service per year).

(*c*) *Deterioration factor.*—The number of lamps in column 3 is reckoned on the basis of *initial candle-power*, whereas a fair comparison of costs must be based upon *average candle-power during the life of a lamp.* The ratio, average candle-power during the life of a lamp divided by the initial candle-power of the lamp, is called the *deterioration factor* of the lamp. The annual costs in column 14 should be divided by the deterioration factors. The values of this factor are as follows:

* See Art. 87, page 183.

TABLE OF DETERIORATION FACTORS*

Tungsten.	Carbon-arc.	Magnetite-arc.	Enclosed Flame-arc.	Cooper-Hewitt.
0.90 to 0.95	0.80 to 0.85	0.90	0.85	0.65 to 0.75

(*d*) *A given number of downward lumens does not mean the possibility of illuminating a certain floor area.*—Thus the light from a few very high candle-power lamps cannot be satisfactorily distributed over a large floor space unless the lamps are placed at a sufficient height overhead and unless the space is free from obstructions such as beams and belts and shafting (see Art. 78). *It is misleading to compare the costs of small lamps and large lamps on the basis of quantity of light.*

Moderately small tungsten lamps give the cheapest satisfactory illumination when the height of the lamps is limited to 12 or 15 feet or where there are many obstructions. On the other hand quartz tube lamps are only suitable for illuminating large open spaces where the lamps can be placed 40 or 50 feet high. Between these two extremes there is a wide field of usefulness for high candle-power tungsten lamps, for Cooper-Hewitt lamps and for magnetite-arc and flame-arc lamps.

(*e*) *Maintenance.*—The maintenance cost as given in column 10 includes in every case the cost of regular inspection, the cost of cleaning globes and reflectors, the cost of material and labor for renewal of parts which wear out† or are occasionally broken, and the cost of material and labor for repairing lamp mechanisms.

The maintenance cost varies greatly: (*a*) because different kinds of service differ greatly in the required frequency of cleaning

* These results are based upon very few observations, and they are therefore not to be depended upon. This is especially true in view of the fact that frequency of cleaning of globes and reflectors has a very great deal to do with deterioration.

† Such as tungsten lamps, arc lamp electrodes, Cooper-Hewitt vapor tubes, globes, etc.

of globes and reflectors and in the amount of breakage, (*b*) because in some cases the lamps are difficult of access and the cost of trimming and cleaning is excessive, and (*c*) because the maintenance labor can be more efficiently organized when a great many lamps of a kind are to be cared for than when the number of lamps is small.

The maintenance cost in the table refers to two or three or four thousand hours use per year. When lamps are used only a few hundred hours per year the maintenance is more expensive per hour.

(*f*) *Color.*—When whiteness of light is a necessity, it is of course meaningless to compare the cost of carbon-arc lamps and tungsten lamps with the cost of flame-arc lamps and mercury-vapor lamps. See Art. 76.

72. Lamp globes, shades and reflectors.*—Let one consider the lamps which one sees everywhere, on city streets, in stores and public halls, and in residences, and one will realize that lamps are almost universally equipped with shades and reflectors. These shades and reflectors are used for three distinct purposes, namely, (*a*) to eliminate glare,† (*b*) to throw the light in a desired direction, and (*c*) for decoration.

The use of a shade for the elimination of glare is exemplified by the opal and ground glass globes which enclose indoor arc lamps and Welsbach gas lamps, and the use of a shade for throwing the light of a lamp in a desired direction is exemplified by the canopy reflectors commonly used on street lamps.

In most cases shades and reflectors are used for the double purpose of eliminating glare and directing the light from a lamp.

* See Cravath and Lansingh, *Practical Illumination*, pages 25–135, McGraw Publishing Company, 1907.

The results of an extensive series of experimental studies of globes and reflectors by R. B. Williamson and J. H. Klinck are given in *Journal of Franklin Institute*, Vol. CXLIX, page 66, 1900.

Also see papers by V. R. Lansingh, *Transactions of Illuminating Engineering Society*, Vol. II, pages 371–399, and Vol. V, pages 49–74.

† See Art. 77.

Thus the shades which are used on desk lamps eliminate glare by hiding the lamp itself from view, and they throw the light downwards upon the desk.

Lamp shades for indoor use should always be to some extent decorative. Decorative effects are, indeed, the primary consideration in the elaborate and deeply colored shades which are frequently used in parlors, and these shades are highly effective in the elimination of glare; but if a shade is to direct the light of a lamp where it is needed, the shade must not depart very widely in shape from a simple cone or bowl; and if excessive loss of light by absorption is to be avoided, the shade must not have colored parts which are intended to transmit light.

73. Extensive, intensive and focusing shades.—Lamp shades which are not primarily decorative may be classified according to their directing action on the light of a lamp. Thus we have the *focusing shade* which throws the light downwards in a very intense narrow beam, the *intensive shade* which throws the light downwards in a fairly narrow beam of moderate intensity, and the *extensive shade* which throws the light downwards in a wide beam. Thus curve B of Fig. 64 shows the distribution of candle-power* around a bare tungsten lamp, curve E shows the distribution of candle-power when the lamp is equipped with a typical† extensive reflector, curve I shows the distribution of candle-power when the lamp is equipped with a typical intensive reflector, and curve F shows the distribution of candle-power when the lamp is equipped with a typical focusing reflector.

Extensive shades or reflectors are the most generally used. They are adapted to residence lighting where small rooms with low ceilings are the rule. Intensive shades are suitable for large rooms with high ceilings where the lamps are distributed

* The radius vector of one of the curves in Fig. 64 represents the candle-power in that direction, and therefore it is strictly correct to speak of the curves as representing the distribution of candle-power. Indeed the expression *distribution of candle-power* is better than the more general expression *distribution of light.*

† In fact curves E, I and F refer to standard line prismatic glass reflectors of the Holophane Company.

uniformly over the room. Focusing shades are used for rooms with very high ceilings and for producing intense local illumination on a desk or draughting board or on a work bench.

To classify shades according to their directing action tends to take one's attention away from an equally important matter, the elimination of glare. This matter is best considered, however, in the following description of particular shades.

74. Metal and milk-glass reflectors.—A familiar type of reflector is the sheet metal cone, which is made deep for the focusing type, less deep for the intensive type, and nearly flat for the extensive type. In the cheaper grades, this shade is made of sheet tin painted white on the inside, but the better grades are made of pressed sheet steel with white porcelain enamel on the inside. Milk-glass is also extensively used for these cone reflectors.

When lamps which are used for illuminating tables and desks, are also depended upon to illuminate the upper part of the room, metal reflectors are not very satisfactory, especially intensive and focusing types of metal shades are not satisfactory under the stated conditions because metal shades allow no light to pass from the lamp to the upper part of the room. Milk-glass cones are, however, quite satisfactory in this respect as are also the prismatic reflectors which are described in Art. 75.

Porcelain enamel reflectors are used very extensively in street and mill lighting. As compared with prismatic glass, enamel reflectors are cheaper, they do not catch and hold the dust as badly, and when properly made they throw a greater portion of the light of a lamp downwards. For store and residence lighting, however, prismatic glass reflectors are more extensively used than enamel reflectors for reasons above stated and because prismatic glass is more decorative.

When the shades above described are not deep enough to hide the lamp from view, the lower portion of the lamp bulb should be frosted* to reduce the glare of the visible portion of the lamp

* Lamps so treated are said to be *bowl-frosted.*

filament. This is quite necessary when brilliant tungsten lamps are used, and it is advisable even when the lamps with their open shades are hung near the ceiling of any ordinary room because it is impossible in an ordinary room to place a lamp entirely outside of the field of vision.

75. **The prismatic glass reflector** is a cone of clear glass with vertical prismatic ribs on its outside surface. Its action may be

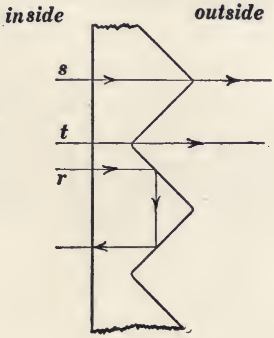


Fig. 95.

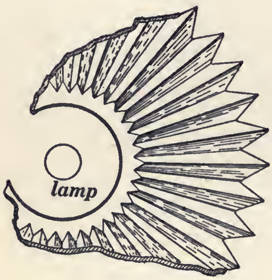


Fig. 96.

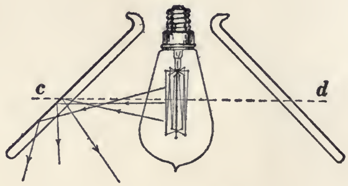


Fig. 97.

understood from Figs. 95, 96 and 97. Any ray like *r*, Fig. 95, is turned backwards (and downwards) by total* reflection, whereas such rays as *s* and *t* pass through the rounded edges of the prisms and the rounded bottoms of the intervening grooves.

* When light in a dense medium like glass strikes the surface obliquely it is totally reflected. This phenomenon is exemplified by the brilliant silvery appearance of the surface of the water in a tumbler when the surface is viewed obliquely from below.

Fig. 96 is a top view of a prismatic reflector showing a *section* of the reflector along the plane *cd* in Fig. 97. The small circle at the center in Fig. 96 represents the lamp. The rays of light from the lamp strike the faces of the prismatic ribs very obliquely and are totally reflected downwards as shown in Fig. 97.

The open cone-shaped prismatic reflectors are not usually deep enough to completely hide the lamp from view and the lamp should therefore be bowl-frosted to reduce the glare.

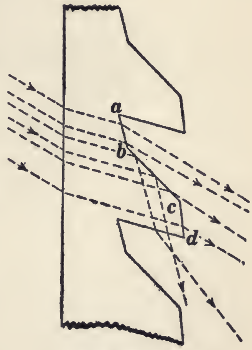


Fig. 98.

Another type of prismatic glass shade has horizontal prismatic ribs which act partly by refraction and partly by reflection as shown in Fig. 98 in which the dotted lines represent rays of light from the lamp. The faces *ab* and *cd* are refracting faces and the faces *bc* are total reflecting faces. This type of prismatic lamp shade was brought out in England in 1882 by Mr. A. P. Trotter.* It is now manufactured in a variety of rather ornamental forms by the Holophane Company. These ornamental prismatic shades are usually made in the form of complete spheres which enclose the lamp, and they eliminate glare almost completely.

* A very interesting account of the development of this shade is given in Trotter's *Illumination*, pages 263–274, Macmillan and Co., London, 1911.

CHAPTER VI.

INTERIOR ILLUMINATION.

76. The illumination of a room.*—A room may be said to be well lighted when the eye is easily able to distinguish the various objects in the room in minute detail of perception. This completeness of visual perception depends upon three conditions: namely, (*a*) a sufficient brightness of illumination, (*b*) a proper location of the light sources so as to bring out that combination of soft shadows which is so essential to the perception of form, and (*c*) a proper composition† of the light so as to bring out those physical differences in objects which the eye perceives as variations of color.

(*a*) The necessity of having a sufficient intensity of illumination is, of course, known to everyone. The ability to perceive fineness of detail (called *visual acuity*) depends chiefly upon intensity of illumination.

Visual acuity is always measured in an arbitrary way, for example, one may measure visual acuity as the distance from one's eye at which clear black print of a chosen size may be read, and the dependence of visual acuity upon intensity

* An extremely interesting discussion of the conditions which determine visual perception is given by Helmholtz in his popular lecture on *The Relation of Optics to Painting* which is translated (by E. Atkinson) in the second series of Helmholtz's *Popular Lectures*, Longmans, Green & Co., 1903. Everyone who is concerned with the practical problems of illumination should read this lecture. Helmholtz's *Popular Lectures* are published in German under the title *Vorträge und Reden*, 2 volumes, Braunschweig, Vieweg und Sohn, 1884.

Three lectures in Helmholtz's first series (translated by Dr. Pye-Smith; Longmans, Green & Co., 1873), *On the Theory of Vision*, also have a bearing upon the important practical subject of illumination.

See also the lectures by Percy W. Cobb and by Robt. M. Yerkes, pages 525–604, Vol. II, *Johns Hopkins University Lectures on Illuminating Engineering*, Baltimore, 1911.

† The composition of light refers to the relative intensities of the various wavelengths which are present in the light.

FIG. 99.

FIG. 100.

FIG. 101.

To face page 157.

of illumination may be determined by finding the distance at which the given type can be read for different intensities of illumination. In this way it is found that visual acuity is very low when the intensity of illumination is one or two tenths of a foot-candle. It increases rapidly up to one or two foot-candles and then it increases slowly and reaches a maximum at about eight or ten foot-candles.

(*b*) The importance of the second condition is illustrated by Figs. 99, 100 and 101, which show a face illuminated in three different ways. In Fig. 99 the face is illuminated by light from a single concentrated source (an electric arc), without any reflection from the walls of the room to soften the effect, and the shadows are extremely harsh; in Fig. 100 the face is illuminated by light from a broad source and largely from one side, and the shadows are soft; in Fig. 101 the face is illuminated by light coming equally from all directions and there are no shadows at all.

A room may be sufficiently illuminated by a single arc lamp but such illumination is unsatisfactory, even when the eye is shaded from the direct light of the lamp, because the excessive harshness of the shadows renders the perception of form almost impossible. The light from a single brilliant lamp is always softened, however, by the reflection from the walls and ceiling of a room.

The second condition is not important where purely flat-surface vision is required as in a draughting room, where indeed it is important to eliminate all shadows on the sheet of drawing paper.

(*c*) The importance of the third condition is evident when one attempts to distinguish delicate colors by ordinary lamp light. Thus the light of an ordinary kerosene lamp is very deficient in the short wave-lengths (blue and violet), and a deep blue or violet piece of cloth appears almost black by kerosene lamp light, False color values are produced in a very striking way by the light from a mercury-vapor lamp on account of the almost complete absence of the longer wave-lengths (red) in the light from this lamp. The most striking illustration of false color values, however, may be obtained by illuminating a batch of brilliantly colored worsteds by the light from a sodium flame in a room from which all white light is excluded. All differences of tint disappear under these conditions, and a given piece of worsted merely appears to be light or dark according as it is able or unable to reflect the yellow light of the sodium flame.

The carbon-arc lamp gives a nearer approach to daylight than any other commercial form of lamp.* The whiteness of the light from the carbon-arc lamp is spoiled, however, by the excess of violet light from the arc stream and by the excess of red and orange light from the moderately heated parts of the carbons. These defects are corrected to some extent in the *intensified carbon-arc lamp* which burns with a short arc thus reducing the violet light, and small carbons are used thus reducing the moderately heated areas near the ends of the carbons.

77. Glare.—The presence of excessively brilliant lamps or excessively brilliant patches of light in a field of vision greatly hinders visual perception. The eye adapts itself automatically to the brightest lights in the field of view, and all perception of detail in the shadows is lost. This effect is called *glare*, and it is especially marked when the field of vision includes a bright unshaded lamp.

The explanation of glare is as follows: In the first place a beam of light entering the eye from a bright source illuminates the whole interior of the eye just as a beam of sunlight entering a window illuminates a room. This diffused light in the eye illuminates and excites the entire retina, including those portions where the images of the deeper shadows fall, and thereby tends to obliterate all detail of perception. In the second place the portions of the retina upon which the brilliant light falls become greatly reduced in sensitiveness by fatigue, the continual wandering of the eye brings the image of a dark region upon this fatigued

* The Moore vacuum-tube lamp (with carbon dioxide) is better than the carbon-arc lamp. The use of bluish glass for absorbing the excess of red and yellow light from a tungsten or carbon-arc lamp is briefly discussed in the *General Electric Review* for December 1911.

portion of the retina, and the result is almost total blindness like that produced when one looks out of a window and then turns towards a dark corner of a room. In the third place the pupils of the eyes contract greatly when there is a bright light in the field of vision, and this contraction lessens the effective brightness not only of the bright portions of the field, but also of the deep shadows; but the deep shadows are already insufficiently illuminated and the contraction of the pupils of the eyes tends to make them (the shadows) appear like black patches entirely devoid of detail.

An interesting case of excessive contrast or glare is that in which a workman at a loom, for example, has his immediate work illuminated to a fair degree of brightness while the remainder of the room is left in darkness. If the workman could keep his eyes upon his work incessantly, it is conceivable that this kind of illumination might be satisfactory; but the eye moves about in spite of everything one can do, and, under the assumed conditions, the workman would be unable to see when he glanced about the room and he would be blinded when he glanced back at his work. To avoid this impracticable situation a general illumination of the room is necessary. If a brilliant light is needed upon one's work, the whole room must be fairly well lighted, and the necessary local illumination must be produced in addition thereto.

It is very important, in arranging for the illumination of a room, to place the lamps outside of the field of vision if possible, so that no light can enter the eye directly from the lamps and render the eye insensible to the delicate shading of surrounding objects. The excessive discomfort that is produced by the glare of improperly located lamps, such, for example, as the exposed footlights of a poorly arranged stage, is due not only to the physical pain that is associated with long-continued looking at a bright light but more especially to the incessant effort of trying to peer into the dark region beyond.

When a lamp cannot be removed from the field of vision the bad effects of glare may be greatly reduced by enlarging the luminous surface of the lamp by means of a translucent globe or shade.

78. Small lamps versus large lamps.—A small lamp, as the term is here used, is a lamp which gives a small amount of light; and a large lamp is a lamp which gives a large amount of light. A given amount of light can be produced more cheaply by large lamps than by small lamps because large lamps are, as a rule, more efficient (less watts per lumen) than small lamps and because a few large lamps are cheaper to install and cheaper to maintain than many small lamps. The use of large brilliant lamps is, however, limited by two conditions as follows:

(*a*) *Satisfactory distribution of light.*—Whenever it is necessary to use a large number of lamps in order to get a satisfactory distribution of light, small lamps are used because to use large lamps would give an unnecessarily large quantity of light. It is not desirable, however, to have light too uniformly distributed (by using a great number of small lamps) because the resulting illumination is flat, that is, devoid of satisfactory shadows.

(*b*) *Elimination of glare.*—A large lamp in one's field of vision produces a much more unpleasant glare than a small lamp, and therefore (even if a proper distribution could be secured) it is not advisable to use large lamps in rooms with low ceilings because with a low ceiling a lamp cannot be placed high enough to remove it entirely from one's field of vision. Large lamps must be placed high overhead. This is especially true of lamps which have a high intrinsic brilliancy; such lamps must not be placed in the field of vision unless they are shaded.

When the indirect system of lighting is employed, however, very large brilliant lamps can be used in small rooms.

79. The indirect system of lighting.*—A favorite, although somewhat extravagant, system of illumination is to place the

* A good discussion of indirect lighting is given by L. B. Marks, *Johns Hopkins University Lectures on Illuminating Engineering*, Vol. II, pages 691–702, Baltimore, 1911. See also an article by J. R. Cravath, *Transactions of the Illuminating Engineering Society*, Vol. IV, pages 290–306, 1909.

lamps entirely out of sight, and so that the light from the lamps may fall upon the ceiling of the room. Thus Fig. 102 shows the essential features of such an arrangement. The lamps are placed in a cove within a few feet of the ceiling and are hidden from view by a high moulding as indicated in the figure.

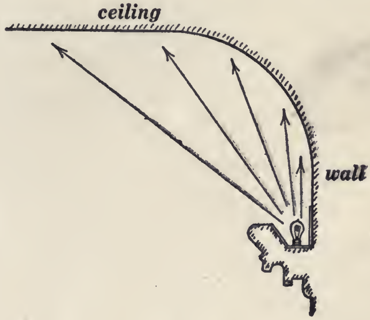


Fig. 102.

The indirect system of illumination gives an extremely diffused light which is itself beautiful and pleasing, but it does not give the shadows which are essential for the visual perception of form. This system is satisfactory for auditoriums and draughting rooms, but it is not satisfactory where the perception of form is of great importance. It should never be used where the walls and ceiling are liable to become even slightly discolored by dust or smoke.

A modification of the indirect system of lighting is to place arc lamps or very high-candle-power tungsten lamps with reflectors to throw the light upwards against the ceiling or against a white diffusing surface. Thus Fig. 103 shows an arc lamp with an opal reflector for throwing the light upwards against a corrugated *diffuser* having a white enamel surface. This type of lamp and diffuser is now practically obsolete; it is, like every other device for indirect lighting, too wasteful.

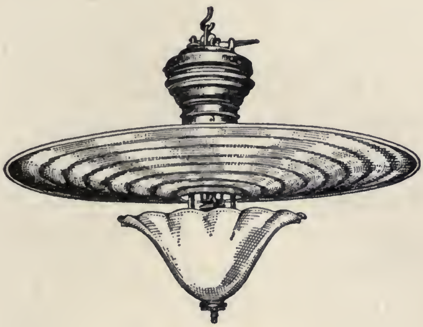


Fig. 103.

80. Influence of absorption on illumination.—Everyone is familiar with the fact that more lamps are required to illuminate a room with dark walls than are required to illuminate a similar room with light walls. This is because the useful light in a room comes, not only directly from the lamps, but is also reflected from the walls and objects in the room to the object which is in the field of vision. Indeed, if all the illuminated surfaces in a room could be made to reflect *all* the light which falls upon them, then the degree of illumination of the room would increase steadily after the turning on of a lamp; and the ultimate degree of illumination would be infinitely great. The walls and objects in a room, however, always absorb light, and after a lamp is turned on *the intensity of illumination in a room increases quickly (almost instantaneously) until the rate of absorption of light by the illuminated surfaces is equal to the rate of emission of light by the lamp.*

A given surface absorbs a definite fractional part of the light which falls upon it and this fraction is called the *coefficient of absorption* of the surface. Thus a surface which absorbs 0.4 of the light which falls upon it has a coefficient of absorption equal to 0.4.

Let I be the average intensity of illumination of the walls and objects in a room in foot-candles (lumens per square foot), let A be the area in square feet of the walls and objects in the room, and let k be the average coefficient of absorption of the illuminated surfaces. Then AI is the number of lumens of light falling upon the illuminated surfaces and kAI is the number of lumens absorbed. That is to say, the amount of light which is being continually absorbed is kAI lumens, and this must be equal to the total amount of light L (in lumens) which is being emitted by the lamps. Therefore we must have:

$$L = kAI \tag{20}$$

Example 1.—Consider one room B which is twice as long, twice as wide and twice as high as another room A, the character of all surfaces being the same in both rooms, and the furniture area being four times as great in the larger room. Under these conditions the total absorption area is four times as great in the larger room, and, since the coefficient of absorption is assumed to be the same, it will take exactly four times as much light to illuminate the larger room to the same intensity as the smaller room. *The amount of light required to illuminate a room to a given intensity of illumination is proportional to the total area of the illuminated surfaces, the coefficient of absorption being given.*

Example 2.—According to equation (20) the amount of light required to illuminate a room to a given intensity of illumination is proportional to the absorption coefficient (average) of the illuminated surfaces in the room. Thus ten lamps give a desired intensity of illumination in a given room when the walls are dark and have a coefficient of absorption equal to 0.80. The same intensity of illumination (average) will be produced in the room by one half as many lamps if the walls are given a light finish of which the coefficient of absorption is 0.40.

Example 3.—A room is 15 feet wide, 20 feet long and 12 feet high. The floor has an area of 300 square feet and the effect of furniture is to add, say, 150 square feet to this amount making a total area of 450 square feet of floor and furniture. The average coefficient of absorption of floor and furniture is 0.88. The coefficient of absorption of the ceiling and walls of the room is 0.40. Required, the number of lumens to give an average intensity of illumination in the room of one foot-candle (one lumen per square foot).

The average coefficient of absorption of the illuminated surfaces in the room is found approximately as follows:

$$
\begin{aligned}
450 \text{ square feet} \times 0.88 &= 396 \\
1{,}140 \text{ square feet} \times 0.40 &= \underline{456} \\
\text{Sum total} &= 852
\end{aligned}
$$

Dividing this sum total by the entire area of the exposed surfaces in the room (1,590 square feet) we have 0.54 as the average value of k. Therefore, substituting in equation (20) $A = 1{,}590$,

$k = 0.54$ and $I = 1.0$, we find $L = 852$ lumens or 67.7 spherical-candles.

The method of calculating the average coefficient of absorption of the illuminated surfaces of a room as shown in this example is not strictly correct,* but the result of the above calculation is sufficiently exact for most practical purposes.

81. Calculation of light flux (lumens) required to illuminate a room.—In planning for the illumination of a room equation (20) might be used as in example 3 of Art. 80, but in practice the following method is used.

INTENSITIES OF ILLUMINATION RECOMMENDED FOR VARIOUS CLASSES OF SERVICE.

(*Foot-candles.*)

Auditorium or ball room	2.0	Stores	
Café	2.5	Art	4.0
Car, passenger	2.0	Baker	3.0
Car, mail	7.0	Book	3.5
Church	2.0	Butcher	3.5
Desk	4.0	China	2.5
Draughting and engraving	7.0	Cigar	3.0
Factory and shop		Clothing	5.0
(*a*) General illumination	1.5	Cloak & Suit	5.0
(*b*) Bench and machine-tool illumination, local, in addition to *a*	4.0 to 5.0	Confectionery	3.0
		Decorator	3.0
		Department (see each department).	
(*c*) General illumination to take the place of *a* and *b*	3.0 to 5.0	Drug	3.0
Garage	2.0	Dry goods	4.0
Library		Florist	3.0
Stack room	1.5	Furniture	5.0
Reading room when local desk illumination is not supplied in addition	3.5	Furrier	5.0
		Grocery	3.0
		Haberdasher	3.5
Reading room when local desk illumination is supplied in addition	0.7	Hardware	4.5
		Hat	4.0
		Jewelry	3.5
Office	4.0	Lace	3.0

* The correct expression for the total amount of light absorbed is $\Sigma kI \cdot \Delta A$, where ΔA is an element of surface, k is its coefficient of absorption, and I is the intensity of illumination at ΔA. Therefore $L = \Sigma kI \cdot \Delta A$; but this formula would lead to extremely tedious calculations even if all the necessary data were known.

Reading (book print)	2.0
Reading (newspaper print)	2.5
Residence	
Porch	0.2
Porch (reading light)	1.0
Hall (entrance)	0.7
Reception room	1.5
Parlor	1.5
Sitting room	1.5
Library	2.0
Music room	2.0
Dining room	1.5
Pantry	2.0
Kitchen	2.0
Laundry	1.5
Hall (upstairs)	0.5
Bed room	1.5
Bath room	2.0
Furnace room	0.7
Store room	0.7
School room	2.5
Shop (see factory).	
Show window	
Light goods	8.0
Medium goods	16.0
Dark goods	20.0
Sign	8.0
Stable	1.0
Leather	3.5
Meat	3.5
Men's furnishings	3.5
Millinery	4.0
Music	3.0
Notions	3.0
Piano	4.0
Post cards	3.0
Show	3.5
Stationery	3.5
Tailor	4.0
Tobacco	3.0
Street	
Business streets (in addition to light from show windows and signs)	0.5
Residence streets	0.1
Prominent streets in residence districts	0.2
Country roads	0.05
Theater	
Lobby	3.0
Auditorium	2.0

(*a*) It is assumed that the surface to be illuminated is a plane at a height of 30 inches above the floor. This plane is called the *working plane*, and the problem is to determine the number of lamps required to give a specified average degree of illumination on this plane. The intensities of illumination required for various kinds of service as found from practice are given in the accompanying table. *Multiply the desired intensity of illumination in foot-candles (lumens per square foot) by the area of the working plane in square feet to get the required light flux in lumens.*

(*b*) A certain fraction, only, of the light emitted by a lamp reaches the working plane and the accompanying table gives the values of this fraction under various conditions. *Divide the total lumens required on the working plane by this fraction to get the lumens delivered by the lamps.*

(*c*) Glow lamps are always rated in mean horizontal candle-power, and to find the total lumens delivered by a given lamp one must know the factor by which mean horizontal candle-power must be multiplied to give mean spherical candle-power. The following table, however, gives the rating in lumens of the present regular types of lamps. *Divide the total lumens to be delivered by the lamps by the rating of the chosen type of lamp in lumens to get the number of lamps.*

PERCENTAGE OF LIGHT DELIVERED TO THE WORKING PLANE BY INCANDESCENT LAMPS.*

(*When globes and reflectors are clean.*)

Equipment of Lamp.	Ceiling.	Walls.	Percentage.
Clear holophane reflector..............	light	light	57
Clear holophane reflector..............	light	dark	45
Clear holophane reflector..............	dark	dark	38
No shade or reflector.................	light	light	39
No shade or reflector.................	light	dark	23
Opal reflector........................	light	light	50
Opal reflector........................	light	dark	41

See pages 147 and 148 for data concerning arc lamps.

Example.—It is proposed to install electric lights in a men's furnishing store 80 feet long, 25 feet wide and 12½ feet high. The ceiling is light, but the walls are covered with shelves containing dark goods, and therefore the walls cannot be counted on to reflect much light.

Referring to the table we find that an illumination of about 3.5 foot-candles is usual in stores of this kind, but the store under consideration is not in a metropolitan district where comparisons would be made, and therefore 3.0 foot-candles may be taken as a sufficient degree of illumination.

The chosen degree of illumination of 3.0 foot-candles (or 3 lumens per square foot) is to be produced over the "working

* From the publications of the Holophane Company. The percentage is less for a tungsten lamp than for a carbon-filament lamp especially when the lamps are bare. The table refers primarily to tungsten lamps and the values may be used for carbon lamps without serious error.

plane" which contains 2,000 square feet. Therefore 6,000 lumens of light are required on the working plane.

TOTAL LUMENS GIVEN BY REGULAR TYPE LAMPS.*

This table gives the total light emitted by regular type lamps when operated at "top efficiency." Light is expressed in lumens. For ratings of arc lamps (in downward lumens) see pages 147 and 148.

Rated Watts.	Tungsten.		Tantalum.		Metallized Carbon.	Carbon.	
	100–130 Volts.	200–260 Volts.	100–130 Volts.	200–260 Volts.	100–130 Volts.	100–130 Volts.	200–260 Volts.
10	—	—	—	—	—	21	—
15	110	—	—	—	—	—	—
20	150	—	—	—	—	50	—
25	185	—	125	—	—	84	—
30	—	—	—	—	105	96	—
35	—	—	—	—	—	—	84
40	320	300	220	—	160	—	—
50	—	—	275	250	205	175	—
60	500	455	—	—	250	210	170
80	—	—	445	400	335	—	—
100	830	760			420	350	—
120	—	—			—	420	340
125	—	—			525		
150	1,250	1,137			—		
187.5	—	—			785		
250	2,170	1,895			1,050		
400†	3,520	—					
500†	4,400	4,030					

It is intended to use tungsten lamps with holophane reflectors, and according to the above table such lamps so equipped deliver 45 per cent. of their light to the working plane in a room with light ceiling and dark walls. Therefore the total amount of light to be delivered by the lamps is 13,333 lumens.

Dividing the total lumens by the rating of the various sizes of tungsten lamps as given in the above table, we find that the required average degree of illumination of the working plane would be produced by any of the following:

* This table is taken from the publications of the General Electric Company and it is standard at this date, April 1, 1912. The table is reissued by the company whenever changes are made in the ratings of regular lamps.

† Round bulb lamps; all other lamps in the table have regular type bulbs.

	seventy-two	25-watt tungsten lamps,
	forty-two	40-watt tungsten lamps,
	twenty-seven	60-watt tungsten lamps,
	sixteen	100-watt tungsten lamps,
	eleven	150-watt tungsten lamps,
	six	250-watt tungsten lamps,
or	three	500-watt tungsten lamps,

The choice of size of lamp is considered in the following article, where this example is carried to a conclusion.

82. The choice of size of lamps.—Economy in first cost and economy in operation leads one to choose the largest size lamps that can be used in a given room, and the limit of size is determined by the necessity of producing a fairly uniform illumination of the working plane. In general the higher the ceiling the larger the lamps that can be used to give satisfactory illumination. The following rules relating to spacing and heights of lamps serve as a basis for choice of size of lamps. These rules have been formulated by the engineers of the Holophane Company.

Rule a.—In narrow stores a single row of lamps may be used. In this case the lamps should be equipped with extensive reflectors, the height H of the lamps above the working plane should be from four tenths to five tenths of the width of the room, and the distance apart s of the lamps should not exceed two times their height above the working plane.

Rule b.—In large rooms with unusually high ceilings, divide the space as nearly as possible into equal squares, place a lamp at the center of each square, use focusing reflectors, and make the height of lamps above the working plane equal to about one and one third times their distance apart.

Rule c.—In large rooms with ordinary ceiling heights or in stores in which it is intended to use two or more rows of lamps, place lamps according to rule *b*, but use intensive reflectors and make the height of lamps above the working plane equal to four fifths of their distance apart.

Where architectural restrictions make it impossible to follow this rule, the height of lamps above the working plane may be varied from 0.66 to 1.0 times their distance apart without seriously affecting the uniformity of illumination on the working plane.

Rule d.—Where outlets are located too far apart for rule *c*, or where it is found impracticable to supply as many outlets as rule *c* requires, divide the room into approximately equal squares, place a lamp at the center of each square, use extensive reflectors and make the height of the lamps above the working plane approximately one half their distance apart.

Example.—The example given in Art. 81 may now be carried to a conclusion. The given room is 12½ feet high, and the working plane is 2½ feet above the floor. It does not look well to place lamps close up against the ceiling; therefore let the lamps be placed one foot below the ceiling. This gives a height of 9 feet $(= H)$ above the working plane. The lamps with the chosen type of reflectors (intensive reflectors) should therefore be 11 feet apart $(= s = 5/4 H)$. Consequently the room should be divided into squares approximately 11 feet × 11 feet.

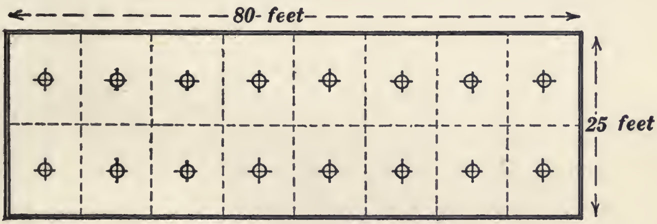


Fig. 104.

Therefore the room may be divided into 16 squares as shown in Fig. 104 each square being 10 feet × 12½ feet, and the desired quantity of light would be produced by placing a 100-watt tungsten lamp at the center of each square (see Art. 81), or the room may be divided into 12 squares each 13.3 feet × 12½ feet and the desired quantity of light would be produced by placing a 150-watt tungsten lamp at the center of each square. The sixteen 100-watt lamps would give a slightly more satisfactory illumination, but the twelve 150-watt lamps would be slightly cheaper to install. Either arrangement would be satisfactory.

CHAPTER VII.

STREET LIGHTING.

83. Detail vision and block vision.—Every one knows that to see the fine details of an object one must look directly at the object. Let the reader fix his eyes on this single letter *A* and consider how narrow his field of acute vision really is; it is scarcely possible to recognize any of the surrounding letters while looking directly at *A*. Everyone knows also how quickly one sees, for example, a hand which is moved up along side of one's head from behind, although one may have to turn and look directly at the hand before one recognizes that it is a hand.

That kind of vision where one looks directly at an object and sees minute detail may be called *detail vision*, and that kind of vision where one sees things without minute detail may be called *block vision*. Detail vision requires strong illumination and the field of detail vision is very narrow. Block vision does not require strong illumination and the field of block vision is very wide. In fact block vision is very good when the intensity of illumination is 0.02 of a foot-candle, which is the intensity of illumination in full moon light. An important characteristic of block vision in very dim light is that the eye must be *kept* in the dark; after a momentary exposure of the eye to bright light one can scarcely see at all in very dim light, and the blinding effect of the bright light lasts for five minutes or more. Block vision in very dim light is sometimes called *twilight vision*.

Two kinds of organs are recognized in the retina of the eye, namely, the *rods* which are distributed over the whole retina except a small central spot which is called the *fovea*, and the *cones* which are crowded together in great numbers in the fovea and which are distributed rather sparsely over the remainder of the retina. To see the fine details of an object the image of the object must fall on the fovea, The fovea is used for detail vision and the remainder of the retina is used for block vision; or the cones are used for detail vision and the rods are used for block vision.*

* This statement is perhaps not strictly correct. An interesting discussion of vision, including color vision, is given by Percy W. Cobb on pages 525–574, Vol. II, *Johns Hopkins University Lectures on Illuminating Engineering*, Baltimore, 1911.

Therefore detail vision is sometimes called *cone vision*, and block vision is sometimes called *rod vision*.

84. Intensity of illumination required for street illumination.* —The intensity of illumination required for interior lighting varies from one to five or six foot-candles (see table on page 164), and this brilliant illumination is required because it is detail vision that must be provided for; furthermore the perception of colors requires brilliant illumination, in very dim light all color differences disappear. It is out of the question to provide street illumination sufficiently bright to give complete detail vision and color vision, the cost would be prohibitive.

It is customary in the discussion of street lighting to specify intensity of illumination *normal to beam*. This quantity is represented by the symbol I_n; see Art. 52. Furthermore, *an intensity of, say, 0.05 foot-candle of normal illumination at a point midway between two street lamps is understood to mean* 0.05 *due to each lamp*. The intensities of illumination actually used for street lighting range from 0.01 foot-candle (average) to 0.1 foot-candle (average) in well lighted cities, as follows:

(*a*) Streets where the night traffic is heavy require good illumination, as do also certain streets where criminal disturbances are likely to occur. For such streets a minimum of 0.05 foot-candle (normal to the beam) and an average of 0.1 foot-candle is quite satisfactory.

(*b*) Ordinary residence streets and streets where the night traffic is light are satisfactorily lighted if an average of 0.05 foot-candle with a minimum of 0.025 foot-candle (normal to beam) is provided. Streets so lighted are as bright as full moonlight at the darkest places.

(*c*) Streets where the houses are scattering and where there is very little night traffic are satisfactorily lighted when a minimum of 0.01 foot-candle (normal to beam) is provided, and the

* A very good discussion of this matter is given by Louis Bell on pages 795–837. Vol. II, *Johns Hopkins University Lectures on Illuminating Engineering*, Baltimore, 1911.

effectiveness of this dim illumination is very greatly increased by placing the lamps (small lamps, of course) high overhead so as to avoid local regions of intense illumination which tend to destroy the extreme sensitiveness of the eye in very dim light as stated in Art. 83.

A condition which favors vision on a street at night is as follows: When one is at a dimly lighted part of the street one sees a dark object on the street projected against the brightly illuminated field near a distant lamp.

85. Arc lamps for street lighting.—Everyone is familiar with the street arc lamp. In the earlier days this was an open carbon-arc lamp operated by direct-current from a Brush or Thomson-Houston generator. Later the enclosed carbon-arc lamp was extensively used for street lighting, and in some cases these carbon-arc lamps (open and enclosed) were operated by alternating current. In new installations magnetite-arc lamps are most extensively used, and the long-burning flame-arc lamp (the enclosed flame-arc lamp) and the quartz tube mercury-vapor lamp are coming into use.

Arc lamps are suitable only for streets which are to be brightly lighted, and for ordinary first class street illumination (minimum 0.05 foot-candle, average 0.1 foot-candle normal to beam) the 4-ampere magnetite-arc lamp is perhaps the most economical.

Very high candle-power lamps are wasteful for street lighting (unless very brilliant illumination is desired) even when they are placed very high above the street, because the diameter of the circular field illuminated by such a lamp is usually three or four or five times the width of the street.

A street with many trees cannot be properly lighted by arc lamps at a reasonable cost; such streets should be lighted by many small lamps (tungsten lamps).

A street lamp should give the greater part of its light in a direction slightly below the horizontal. Thus the magnetite-arc lamp (curves 3 and 4, Fig. 94 *a*) and the vertical-carbon long-burning flame-arc lamp (curve 7, Fig. 94 *b*) are well adapted to street lighting.

Arc lamps are not, as a rule, hung sufficiently high on the streets of American cities, the usual practice being to hang the 4-ampere magnetite-arc lamp 20 or 25 feet above the street and the 6.6-ampere magnetite-arc lamp 25 or 30 feet above the street.

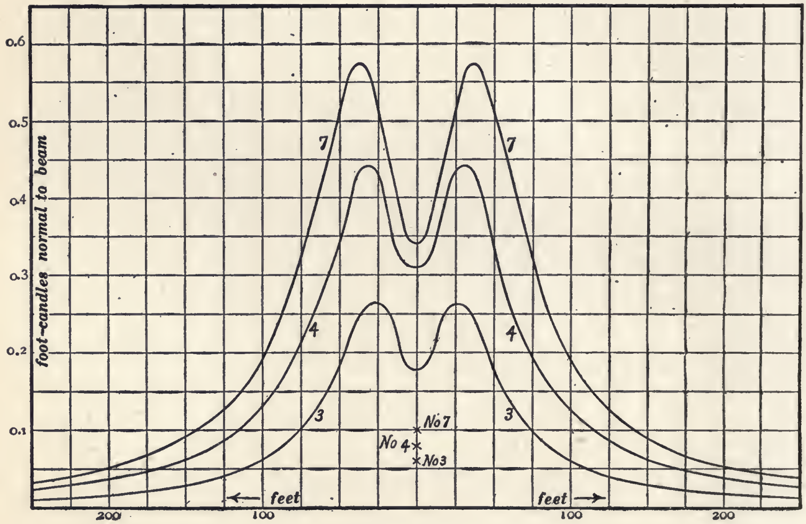


Fig. 105.

The ordinates of the curves 3, 4, and 7 in Fig. 105 show the values of I_n (intensity of illumination normal to beam) at various horizontal distances along a street from the various lamps, the heights of the lamps above the street being 30 feet, 40 feet, and 50 feet respectively, as represented by the small crosses in the figure. The values of I_n are reckoned at the surface of the street. The inclined-carbon flame-arc lamp is not adapted to ordinary street lighting.

The accompanying table gives the values of I_n at points on a street distant x feet horizontally from the various kinds of lamps, *the height of the lamp in each case being* 50 *feet.*

VALUES OF ILLUMINATION NORMAL TO BEAM NI FOOT-CANDLES.

Height of lamps 50 feet, horizontal distance from lamp = x.

(Values of I_n.)

Serial No.	Type of Lamp.	Values of x in Feet.									
		150	200	250	300	350	400	450	500	550	600
3	4.4-ampere magnetite-arc lamp	0.0250	0.0154	0.0100	0.0076	0.0061	0.0050	0.0041	0.0031	0.0022	0.0014
4	6.6-ampere magnetite-arc lamp	0.0605	0.0369	0.0238	0.0181	0.0145	0.0118	0.0096	0.0077	0.0054	0.0033
6	Inclined-carbon flame-arc lamp...	0.100	0.0496	0.0293	0.0204	0.0163	0.0134	0.0107	0.0086	0.0064	0.0043
7	Vertical-carbon flame-arc lamp ..	0.075	0.0310	0.0175	0.0105	0.0070	0.0055	0.0045	0.0035	0.0027	0.0017

By making use of the law of inverse squares (Art. 48) it is easy to use this table to find the intensity of illumination (normal to beam) at a point on the street at any given distance d horizontally from any one of the lamps placed at any given height H above the street. The rule is as follows: *Find from the table the value of* I_n *for the lamp* 50 *feet high and for* $x = d \times 50/H$, *and multiply the value of* I_n *so found by* $(50/H)^2$.

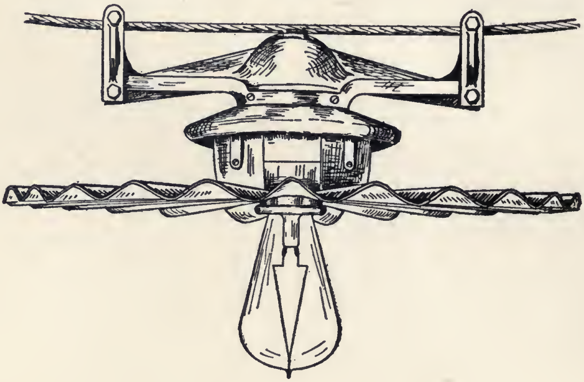


Fig. 106.

Also the table can be used to find the horizontal distance d from a lamp (H feet above the street) to the point on the street where the normal illumination has a prescribed value. The rule is as follows: *Multiply the prescribed normal illumination by* $(H/50)^2$ *to get* I_n **as per table,** *find the corresponding value of* x *from the table and multiply this value of* x *by* $(H/50)$ *to get the desired value of* d.

86. Glow lamps for street lighting.—The great advantage of glow lamps for street lighting is that a fairly uniform distribution of light along a street can be produced much cheaper by small lamps than by large lamps. Tungsten lamps, for example, equipped with suitable reflectors (see below) and properly spaced along a street give an illumination which is entirely satisfactory for residence and suburban districts, and the cost is less perhaps than any other kind of street lighting.

Figure 106 shows a common form of suspension fixture and reflector for a tungsten street lamp. The reflector is made of sheet metal with a white enamel surface, and it has radial flutings. Figure 107 shows an ornamental street lighting cluster of tungsten lamps mounted on top of a post and equipped with holophane street reflectors.

Fig. 107.

The dotted curve in Fig. 108 shows the distribution of candle-power around a 40-candle-power tungsten lamp without a shade, and the full-line curve shows the distribution of candle-power of the same lamp equipped as shown in Fig. 106.

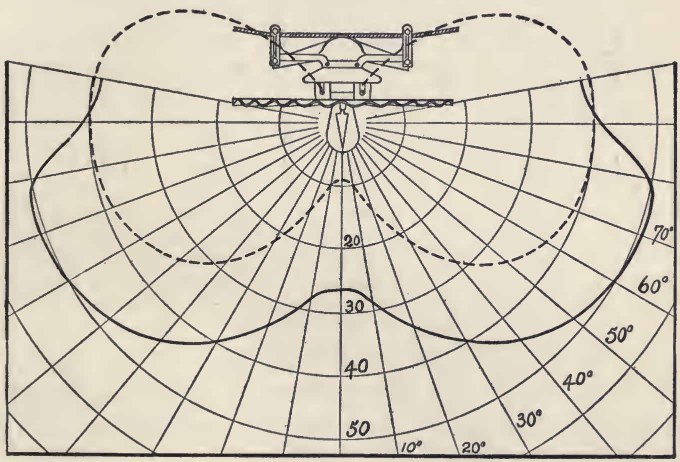


Fig. 108.

For sidewalk lighting 32-candle-power or 40-candle-power tungsten lamps are usually employed and hung 10 or 12 feet above the walk. For street lighting larger tungsten lamps are used and they are hung over the middle of the street at heights of from 18 to 24 feet above the street.

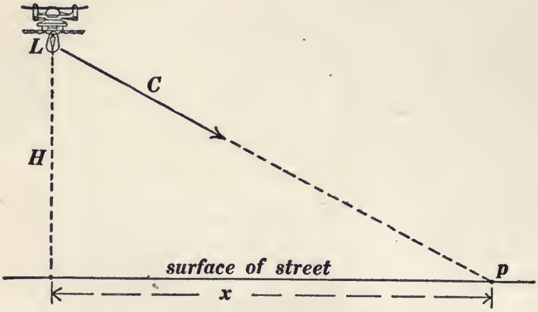


Fig. 109.

NORMAL ILLUMINATION IN FOOT-CANDLES DUE TO 40-CANDLE-POWER TUNGSTEN LAMP * WITH RADIAL REFLECTOR AS SHOWN IN FIG. 106 OR 108.

Distance x (see Fig. 109) in feet	Height of Lamp H. 10 Feet.	12 Feet.	15 Feet.	18 Feet.
0	0.260	0.181	0.115	0.0803
5	0.288	0.201	0.127	0.0860
10	0.222	0.174	0.123	0.0884
15	0.146	0.125	0.0985	0.0774
20	0.098	0.0882	0.0744	0.0622
25	0.0688	0.0637	0.0565	0.0496
35	0.0383	0.0368	0.0342	0.0316
50	0.0196	0.0193	0.0185	0.0177
75	0.00856	0.00875	0.00872	0.00858
100	0.00466	0.00483	0.00489	0.00492

From this table the normal illumination at any point on a street produced by a tungsten lamp of any candle-power C (equipped like Fig. 106) can be found by multiplying the tabulated value of I_n by $C/40$.

If it is desired to find the intensity of illumination (normal) produced at a distance d horizontally from a lamp hung H

* This table refers specifically to the special low-voltage tungsten lamp which is described on page 143 and shown in Fig. 93, the lamp being equipped with a "radial wave" reflector as shown in Fig. 106.

feet above the street, the method used in Art. 85 can be employed as shown by the following examples:

Example 1.—What is the normal illumination at a point on the street distant 150 feet from a 200-candle-power lamp like Fig. 106, the lamp being 24 feet above the street. Divide the given distance (150 feet) and the given height (24 feet) by some factor b to give a height, say, of 12 feet which appears in the table. In this case $b = 2$. Find I_n from the table for this reduced distance (75 feet) and reduced height (12 feet), and divide the value of I_n so found by b^2. This gives the normal illumination which would be produced by a 40-candle-power lamp at a distance of 150 feet, the height of the lamp being 24 feet; and if we multiply this by 200/40 we have the desired result, namely, 0.0109 foot-candle.

Example 2.—How far apart must 200-candle-power tungsten lamps (like Fig. 106) hung 24 feet above the street be placed to give a minimum of 0.025 foot-candle (normal) at a point on the street midway between two lamps. Let $2d$ be the desired spacing, then the problem is to find the distance d from a 200-candle-power tungsten lamp for which the normal illumination due to the lamp is 0.025 foot-candle. Divide the specified height (24 feet) and the distance d by a factor b which will reduce the height to say 12 feet which appears in the table. In this case the value of b is 2. Multiply the desired illumination, 0.025, by 40/200 to get the illumination which would be given by a 40-candle-power lamp. Multiply this result by b^2 $(= 4)$ to get the illumination (0.020) corresponding to reduced height (12 feet) and reduced distance $(d/2)$. Find the value of x from the table corresponding to a height of 12 feet and an illumination of 0.02. This value of x (which is found to be a little less than 50 feet) is equal to $d/2$. Therefore the desired spacing $(2d)$ is a little less than 200 feet.

87. Systems of street lighting.—The kinds of lamps used for street lighting are mentioned in Arts. 85 and 86. It remains to discuss the methods of connecting the lamps in groups and the mode of delivery of current to the groups of lamps.

Single arc lamps are always connected across constant-voltage supply mains. Such lamps are called *multiple lamps* as stated in Art. 71. Multiple lamps are seldom used for street lighting.

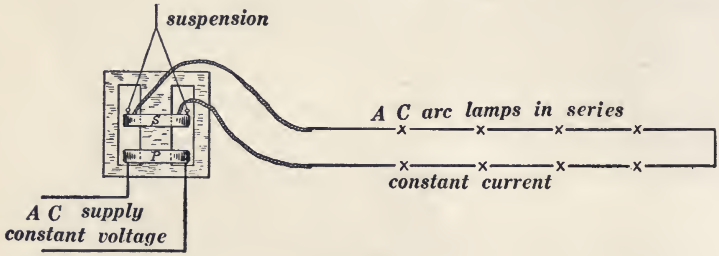


Fig. 110.

When a great number of arc lamps are used in one installation, the lamps are always designed to be operated in series; and a group of lamps in series must be operated by a constant current. In the early days of electric street lighting the constant current for operating arc lamps in series was supplied by a "constant-current" generator of the direct-current type, but this arrangement is now obsolete. New arc-lamp street lighting installations are now series magnetite-arc lamps operated by constant direct current derived from a constant-voltage alternating source by means of the *constant-current transformer* and the *mercury vapor rectifier*. Flame-arc lamps can be operated by direct current and these lamps (if they have the proper current rating) can be operated in series with magnetite-arc lamps. A large group of flame-arc lamps would be most conveniently operated by a constant alternating current derived from a constant-voltage supply by means of the constant-current transformer.

Figure 110 shows the scheme for supplying a group of series arc lamps with constant *alternating current* derived from a constant-voltage supply. The lamps used in this case cannot be magnetite-arc lamps because such lamps cannot be operated by alternating current. Figure 111 shows the scheme for supplying a group of series arc lamps with constant *direct current* derived from a constant-voltage alternator. The lamps used in this case may be magnetite-arc lamps or flame-arc lamps or both. A general view of a constant-current transformer is shown in Fig. 112. This transformer has a movable secondary coil which is delicately counterpoised. When one or more lamps are taken out of service (by short-circuiting, or by-pass switches) the tendency is for the current to increase but this tendency is counteracted by the movement of the secondary coil.

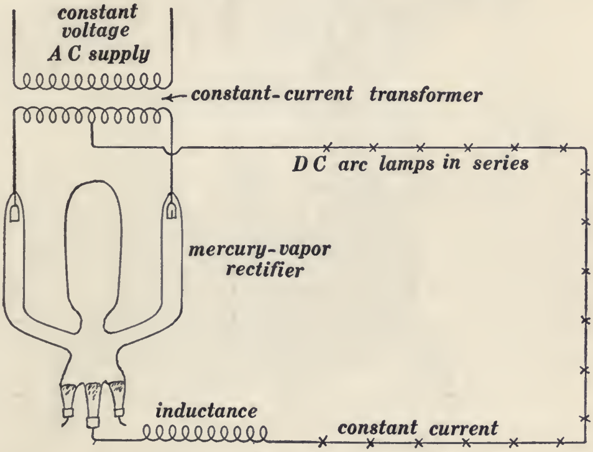


Fig. 111

Glow lamps for street lighting are sometimes connected in parallel to constant-voltage supply mains in the same way that house lamps are usually connected. In fact this arrangement is always used when a very few glow lamps are used at widely separated parts of a town or city. The lamps are connected across the low-voltage terminals of a transformer which supplies the neighboring houses.

When many glow lamps are used on the streets of a town or city the lamps are always connected in series, and two different schemes are employed to supply current to such series groups of lamps as follows:

(*a*) The series group may be connected directly to the *constant-voltage* bus bars in the station. Thus twenty 110-volt lamps may be connected in series to a 2,200-volt supply. In this case provision must be made to place an auxiliary lamp in circuit when one of the service lamps breaks, as explained in connection with Figs. 18 and 19 in Chapter II.

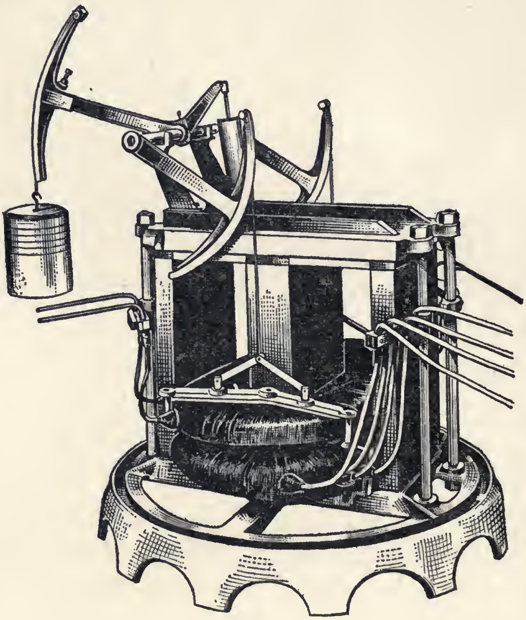


Fig. 112.

(*b*) Each series group of lamps may be supplied with a constant alternating current derived from a constant-voltage source by means of a constant-current transformer exactly as in the case of a series group of alternating-current arc lamps as represented in Fig. 110. In this case provision must be made to close a by-pass around a lamp which burns out or is broken. This is accomplished exactly as explained in connection with Figs. 18 and 19 in Chapter II, except that no auxiliary lamp is used.

Operation of lamps of different current ratings in a series circuit using alternating current.—Given a series group of arc lamps operated by, say, 12 amperes alternating current. A series group of lamps of any current rating may be operated in conjunction with the given group by connecting as shown in Fig. 113. A

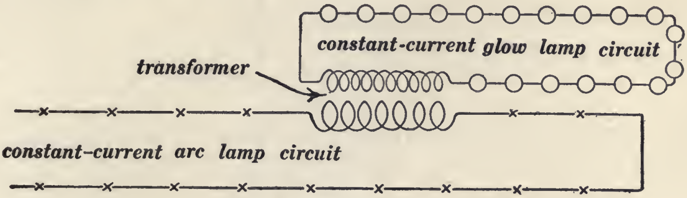


Fig. 113.

transformer* connected as shown in Fig. 113 gives a secondary current which is equal to Z'/Z'' times the primary current, where Z' and Z'' are the numbers of turns of wire in primary and secondary coils respectively.

Increased cost of power due to the use of constant-current transformer and rectifier.—A slight addition to the cost of power is involved in the use of the constant-current transformer and rectifier on account of the interest on the cost of the transformer and rectifier, depreciation of non-renewable parts, cost of renewals of rectifier bulbs, and loss of energy in transformer and rectifier.†

* It is distinctly misleading to call this transformer a "series" transformer. It is simply an ordinary transformer.

† See Bulletin No. 51 of the Illinois Engineering Experiment Station (Bryant and Hake), pages 46–47.

CHAPTER VIII.

ELECTROLYSIS AND BATTERIES.

88. Electrolysis.*—Two sheets of copper *A* and *C*, Fig. 114, dipping into a solution of copper sulphate are connected to direct-current supply mains as shown and the flow of electric current is indicated by the arrows. During the flow of current the copper plate *A* is slowly dissolved and metallic copper is slowly deposited upon the plate *C*.

The slow dissolving of the plate *A* and the slow deposition of metallic copper upon plate *C* are evidences of chemical action at *A* and *C*. Chemical action thus produced by the electric current is called *electrolysis*, the solution through which the current flows is called an *electrolyte*, the arrangement *VV* is

* The methods of electroplating and electrotyping are described by Samuel Field in *Principles of Electrodeposition*, Longmans, Green & Co., 1911. The most complete discussion of this subject is *Electrolytische Metallniederschläge*, by W. Pfanhauser, Jr., Berlin, 1910. See also *Handbuch der electrolytischen Metallniederschläge* by Georg Langbein, Leipzig, 1906.

The electrolytic refining of copper is a simple process of electroplating in which pure copper is deposited out of a solution while the impurities are left in the solution. See *Electrochemical and Metallurgical Industry* (now *Metallurgical and Chemical Engineering*), Vol. I, pages 561–562, December, 1903.

The extraction of aluminum from bauxite is described by Joseph W. Richards, *Electrochemical and Metallurgical Industry* (now *Metallurgical and Chemical Engineering*), Vol. I, pages 158–162, Jan., 1903.

The electrolytic manufacture of alkali and chlorine is described by L. E. Baekeland, *Metallurgical and Chemical Engineering*, Vol. V, pages 209–212, June, 1907.

The manufacture of chlorates by electrolysis is described by G. Rossert. *L'Eclairage Electrique*, beginning July 27, 1907.

The manufacture of metallic sodium and of sodium peroxide is described in the *Electrochemical and Metallurgical Industry* (now *Metallurgical and Chemical Engineering*), Vol. I, pages 11–22, Sept., 1903.

Electrolysis is used extensively for recovering tin from tinned iron scrap, for refining gold and silver, and for reducing lead ore. Most of these processes are described in the *Metallurgical and Chemical Engineering* called an *electrolytic cell* and the copper plates A and C are called *electrodes*. The electrode A at which the current enters the solution is called the *anode*, and the electrode C at which the current leaves the solution is called the *cathode*.

Solutions of acids and salts generally are electrolytes, also fused salts are electrolytes. That is to say, the flow of electric current through such a solution or through a fused salt produces chemical action at the point where the current enters the liquid (at the anode) and at the point where the current leaves the liquid (at the cathode).

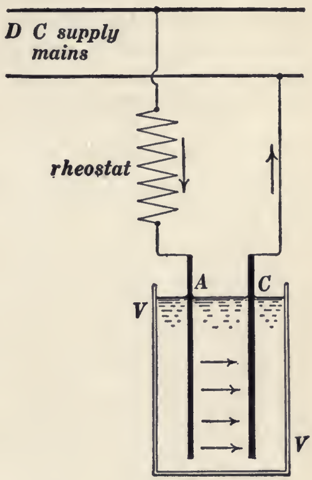


Fig. 114.

A good example of electrolysis is the electrolysis of a solution of hydrochloric acid (HCl) between electrodes of carbon. In this case hydrogen (H) is liberated at the cathode and chlorine (Cl) is liberated at the anode. In general the molecule of any dissolved salt or acid is separated into two parts by electrolysis; one part is liberated at the cathode and is called the *cathion*, and the other part is liberated at the anode and is called the *anion*. Thus hydrogen (H) is the cathion and chlorine (Cl) is the anion of hydrochloric acid. In all metallic salts the metal constitutes the cathion and the acid radical or halogen constitutes the anion. In acids the hydrogen constitutes the cathion and the acid radical or halogen constitutes the anion. Thus in the case of copper sulphate ($CuSO_4$) the cathion is copper (Cu), and the anion is the acid radical (SO_4).

*The dissociation theory of electrolysis.**—Many of the known facts of electrolysis

* The dissociation theory is used very extensively in advanced treatises in the correlation of experimental results. A good discussion of this subject is given in Chapter 7 of Nernst's *Theoretical Chemistry*, Macmillan and Co. See also Chapters 9–12 of Whetham's *Theory of Solution*, Cambridge University Press. A good discussion of electrolysis is also given in H. C. Jones' *Elements of Physical Chemistry*, The Macmillan Co.

may be clearly conceived in terms of the dissociation theory which is briefly as follows: Consider a solution of common salt (NaCl) for example. Every molecule of the salt in a dilute solution is supposed to be separated into *positively-charged-sodium-atoms* (+ Na) and *negatively-charged-chlorine-atoms* (− Cl) which are called *ions*. The positively charged sodium atoms are called *cathions*, and the negatively charged chlorine atoms are called *anions*. Ordinarily these ions wander about in the solution, but the application of an electromotive force to the electrodes produces an electric field throughout the solution, and the forces exerted on the ions by this electric field cause the cathions to drift towards the cathode and the anions to drift towards the anode. When the cathions reach the cathode they give up their positive charges and enter into chemical combination or are deposited as neutral metal or hydrogen; when the anions reach the anode they give up their electric charges in the same way.

In the electrolysis of hydrochloric acid the cathion material (H) is actually set free at the cathode and the anion material (Cl) is actually set free at the anode. In most cases, however, the cathion material is not actually set free at the cathode nor is the anion material actually set free at the anode. Thus in the electrolysis of a solution of sodium chloride (NaCl), the cathion material (Na) when it is "liberated" at the cathode immediately reacts with the water forming NaOH and free hydrogen; in the electrolysis of a solution of copper sulphate ($CuSO_4$) between copper electrodes, the anion material (SO_4) combines with the copper of the anode forming fresh $CuSO_4$ which goes into solution, or it is deposited as crystals on the anode if the solution is saturated; in the electrolysis of H_2SO_4 between inert electrodes, the hydrogen is set free at the cathode as a gas and the anion material (SO_4) reacts on the water according to the formula $SO_4 + H_2O = H_2SO_4 + O$, and the free oxygen escapes as a gas.

89. **Current density at the electrodes.**—Generally the *flow of current* from the anode plate into the electrolyte and from the electrolyte into the cathode plate is not uniformly distributed over the surfaces of the plates. There is always a tendency for the flow of current to be concentrated at the sharp edges of the plates and at any sharp projecting part of the plates. Thus Fig. 115 represents a top view of an electrolytic cell, *AA* being the

anode and CC being the cathode, and the fine curved lines represent the stream lines of the electric current through the electrolyte. The crowding together of the stream lines at the corners of the electrodes and at the sharp point on CC represents the concentration of the current-flow at these places. With electrodes placed as shown in Fig. 115 there is extremely little flow of current into or out of the back faces *bb bb* of the electrodes.

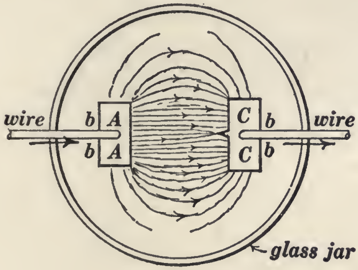


Fig. 115.

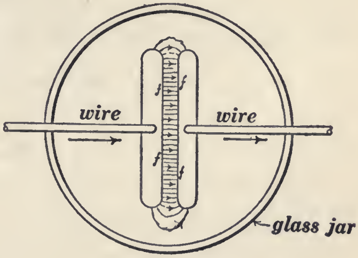


Fig. 116.

When the electrodes are parallel flat plates at a distance apart which is small as compared with the size of the plates, then the flow of current is almost wholly confined to the front faces *ff ff* of the electrodes as shown in Fig. 116, the flow of current, furthermore, is distributed over the faces *ff ff* with approximate uniformity, and the total current divided by the area of one of the front faces *ff* is called the *current density*.

The current density at an electrode has a great deal to do with the character of the chemical action at the electrode. Thus, if the electrolyte contains zinc sulphate and copper sulphate, copper only is deposited on the cathode if the current density is low, but both copper and zinc (and also some hydrogen) are deposited on the cathode if the current density is large. In the operation of silver plating or copper plating or nickel plating the deposited metal is soft and granular if the current density is too high.

90. Definition of electrochemical equivalent. Chemical calculations in electrolysis.—The amount of silver deposited per second in the operation of silver plating is proportional to the strength of the current in amperes, and the amount of silver deposited in one second by one ampere is called the *electrochemical equivalent* of silver; it is equal to 0.001118 gram per ampere-second or 4.025 grams per ampere-hour.

In the great majority of cases no material is actually deposited on either electrode in an electrolytic cell, but chemical action is always produced in the immediate neighborhood of the electrodes, and *it is important to consider the amount of chemical action which takes place in a given time due to the flow of a given current through the cell.* A general statement of this matter involves the use of a number of chemical terms. These terms are exhibited in the following schedules.

The *valencies* of various metals, acid radicals, etc., are shown by the numbers in the following exhibit which shows the chemical symbols of several common acids and salts.

EXHIBIT OF VALENCIES.

Name.	Hydrochloric Acid.		Silver Nitrate.		Nitric Acid.		Sulphuric Acid.	
Chemical symbol........ Valency....................	H 1	Cl 1	Ag 1	NO_3 1	H 1	NO_3 1	H_2 2	SO_4 2
Name.	**Cupric Sulphate.**		**Zinc Sulphate.**		**Aluminum Sulphate.**			
Chemical symbol........ Valency....................	Cu 2	SO_4 2	Zn 2	SO_4 2	Al_2 6	$(SO_4)_3$ 6		

The *chemical equivalents* of various metals, acid radicals, etc., are shown in the following exhibit. One chemical equivalent of a metal or acid radical is hereafter called a *gram-val* of the substance.

EXHIBIT OF CHEMICAL EQUIVALENTS IN GRAMS.

Symbol of Substance.	H	Ag	Cl	NO_3	SO_4	Cu	Zn	Al
Atomic or molecular weight..............	1.01	108	35.5	62	96	63.6	65.4	27.1
Valency..................	1	1	1	1	2	2	2	3
Chemical equivalent in grams. (The gram-val)....................	1.01	108	35.5	62	48	31.8	32.7	9.03

The number of grams of a substance in one gram-val of that substance is equal to the atomic or molecular weight of the substance divided by the valency of the substance. Thus one gram-val of silver is 108 grams of silver, and to deposit one gram-val of silver on the cathode in a silver plating cell requires 96,540 ampere-seconds or 26.82 ampere-hours. **In general, 96,540 ampere seconds "liberates" one gram-val of anion material at the anode and one gram-val of cathion material at the cathode whatever the particular electrolyte may be.** It is evident therefore that 96,540 ampere-seconds is an important unit of *current* × *time;* this unit is called the *faraday.* For example:

One faraday (96,540 ampere-seconds) "liberates"

at the anode	at the cathode
62 grams of NO_3 from nitric acid or any nitrate solution.	23 grams of Na from a solution of NaOH, or from a solution of any sodium salt.
48 grams of SO_4 from sulphuric acid or any sulphate solution.	31.8 grams of Cu from a solution of any cupric salt.
35.5 grams of Cl from hydrochloric acid or any chloride solution.	63.6 grams of Cu from a solution of any cuprous salt.*
16.01 grams of OH from a solution of caustic soda or potash, etc., etc.	9.03 grams of Al from a solution of any aluminum salt, etc., etc.

91. The electrochemical unit of work.—Consider an electrolytic cell through which a current of I amperes is flowing and across the terminals of which the electromotive force is E volts. Work is being delivered to the cell (done on the cell) at the rate EI watts, and in t seconds the amount of work done is EIt joules.†

It is important to consider the amount of work done by an electromotive force of one volt during the flow of one faraday (96,540 ampere-seconds) through a cell. This amount of work may be conveniently called a *volt-faraday*, it is evidently equal to 96,540 joules and it is therefore equivalent to 23,000 calories.

* Cupric copper has a valency of 2, cuprous copper has a valency of 1. Thus cupric chloride is $CuCl_2$ and cuprous chloride is CuCl.

† It takes 4.2 joules to raise the temperature of one gram of water 1° Centigrade. That is, one calorie is the heat equivalent of 4.2 joules.

The work done by E volts during the flow of one faraday is E volt-faradays, or $96{,}540E$ joules and it is equivalent to $23{,}000E$ calories.

92. Expenditure of work in electrolysis. Heat and chemical work.—A portion of the work done in forcing a current through an electrolytic cell is converted into heat. This is shown* by the fact that the temperature of an electrolytic cell rises during the flow of current. Also a portion of the work done in forcing a current through an electrolytic cell is left tied up, as it were, in the chemical changes which are produced by the current. This work which becomes tied up in chemical changes is called *chemical work.* Thus, when dilute sulphuric acid is electrolized between electrodes of carbon or platinum, hydrogen and oxygen gases are set free, and it is evident that energy is tied up in these free gases because if the gases are re-combined by burning, energy is obtained in the form of heat.

The amount of heat developed by the current in an electrolytic cell depends in part upon the resistance† of the electrolyte in ohms and it depends in part upon irreversible‡ actions at the electrodes; *when the current is very small the energy which is lost as heat is usually negligible in comparison with the energy which becomes tied up in the chemical changes which are brought about by the current.*

93. Decomposition voltage.—Consider the electrolysis of dilute sulphuric acid between inert electrodes of carbon or platinum, the current being so small that the loss of energy in the production of heat is negligible, and let E be the electromotive force across

* This argument is not strictly correct because the chemical work done in an electrolytic cell may be in some cases greater than the electrical work delivered to the cell, the cell being actually cooled in the process.

† The matter of electrolyte resistance is discussed at some length in Whetham's *Theory of Solution* and in H. C. Jones' *Physical Chemistry.*

‡ This is a thermodynamic term which the beginner is not expected to understand. See two papers on Reversible and Irreversible Polarization by W. S. Franklin and L. A. Freudenberger, *Transactions of the American Electrochemical Society*, Vol. VII, pages 33–49; and Vol. VIII, pages 227–237.

the terminals of the cell. *During the flow of one faraday through the cell E volt-faradays of work will be done on the cell*, and, heat losses being negligible, all of this energy will be tied up in the 1.01 grams of free hydrogen and the 8 grams of free oxygen produced.

Now the amount of energy tied up in these gases is approximately* equal to the heat of combustion of 1.01 grams of hydrogen, and the heat of combustion of hydrogen is 34,700 calories per gram. Therefore the setting free of 1.01 grams of H and 8 grams of O involves the tying up of the energy equivalent of 35,000 calories, which is 147,000 joules or 1.52 volt-faradays, and this must be equal to the work done on the cell, namely, E volt-faradays. Therefore we have

$$E \text{ volt-faradays} = 1.52 \text{ volt-faradays}$$

and *consequently E, which is the voltage required to send a very small current through the given electrolytic cell, is equal to* 1.52 *volts, and it is called the* **decomposition voltage** *of sulphuric acid.*

The decomposition voltages of some common acids and salts between inert electrodes are given in the following table:

DECOMPOSITION VOLTAGES.†

(For Normal Solutions.)

Sulphuric acid	1.67 volts.	Barium nitrate	2.25 volts.
Nitric acid	1.69 volts.	Sodium nitrate	2.15 volts.
Hydrochloric acid	1.31 volts.	Calcium chloride	1.89 volts.
Oxalic acid	0.75 volts.	Potassium chloride	1.96 volts.
Sodium hydroxide	1.69 volts.	Sodium chloride	1.98 volts.

Qualifications of Above Statements Concerning Decomposition Voltages.— The above statements concerning decomposition voltages are not in accord with experiment in the following particulars:

* The heat of combustion includes the heat of condensation of the water vapor produced by the combustion, but a small amount of available energy is represented by the placing of the condensed water back into the acid solution. This small amount of energy is neglected.

† As determined by observation by LeBlanc, *Zeitschrift für Physikalische Chemie*, Vol. VIII, page 299, 1891. One must not be surprised at the discrepancy between the value 1.52 volts above calculated and the value 1.67 volts as observed by LeBlanc for dilute sulphuric acid. See page 192.

(*a*) A very small current through sulphuric acid between inert electrodes *does not* set free hydrogen and oxygen gases. A trace of oxygen is dissolved in the solution and it combines with the hydrogen as it is "liberated" at the cathode, and the supply of dissolved oxygen is replenished by diffusion from the neighborhood of the anode. Also a trace of hydrogen is dissolved in the solution and it combines with the oxygen as it is "liberated" at the anode, and the supply of dissolved hydrogen is replenished by diffusion from the neighborhood of the cathode. Any voltage, however small, can maintain a very small current through sulphuric acid between inert electrodes, and the same is true of any electrolyte whatever. There is, however, a definite voltage below which no oxygen and hydrogen gases are set free, and this is the decomposition voltage above referred to. Similarly, there is a definite decomposition voltage for any electrolyte between inert electrodes.

(*b*) The setting free of oxygen and hydrogen gases at a rate which is actually perceptible, by electrolizing sulphuric acid between inert electrodes, requires about 1.67 volts. Therefore 1.67 volt-faradays of work is done on the cell during the flow of one faraday through the cell, 1.52 volt-faradays of this work is tied up in the free oxygen and hydrogen gases, and the remainder, namely, 0.15 volt-faraday, is converted into heat at the electrodes because of irreversible actions at the electrodes. That is to say, the heat generated in an electrolytic cell does not always become negligible as the current approaches zero. The heat generated in the body of the electrolyte because of the resistance of the electrolyte (in ohms) does become negligible.

(*c*) The application of thermodynamics to electrolysis shows that it is possible for the "chemical work" done in an electrolytic cell to exceed the electrical work done on the cell, the deficiency being made up by the cooling of the cell. In such a case the "decomposition voltage" would be less than what is calculated by the theory above outlined.

94. Electrode polarization.—When a metal plate is allowed to stand in an acid or salt solution a definite potential-difference or voltage comes into existence between the solution and the metal. This potential-difference or voltage measures what is called the *electrode polarization*. Thus, according to Neumann* the values of electrode polarization between ordinary metals and solutions of their salts are as follows, *the voltage being reckoned as positive when it is from the solution to the metal:*

ELECTRODE POLARIZATION VOLTAGES.

(Equilibrium Values in Volts.)

Metal.	Salt of Metal.		
	Sulphate.	Nitrate.	Chloride.
Zinc	−0.524	−0.473	−0.503
Iron	−0.093	——	−0.087
Lead	——	+0.115	+0.095
Copper	+0.515	+0.615	——

* *Zeitschrift für physikalische Chemie*, Vol. XIV, page 229, 1894.

The polarization voltages given in this table are equilibrium values, that is, the values which exist when no current flows from metal to electrolyte or from electrolyte to metal. When there is a flow of current the polarization voltage changes considerably. This change is due chiefly to the change of concentration of the electrolyte which always takes place near an electrode as explained in connection with the lead storage battery in Art. 105.*

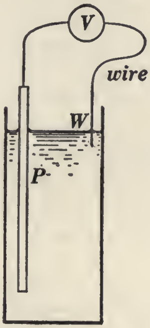


Fig. 117.

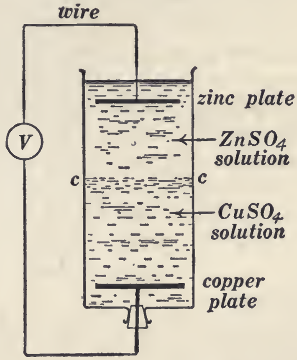


Fig. 118.

One might think that the polarization voltage between a solution and a metal plate P could be measured by connecting a voltmeter as shown in Fig. 117, but there is a polarization voltage between the solution and the wire W so that the voltmeter would measure the sum:

volts from wire W to solution + volts from solution to plate P,

or the difference

− volts from solution to wire W + volts from solution to plate P.

The measurement of the polarization voltage between the metal plate P and a solution depends upon the use at W of an electrode whose polarization voltage has been determined.† Such an electrode is called a *standard electrode*.

Example.—Consider the electrolytic cell which is shown in Fig. 118, in which the zinc sulphate solution floats on the heavier copper sulphate solution. Neglecting the voltage across the contact cc of the solutions, the voltage indicated by the voltmeter V is the sum:

volts from zinc to solution + volts from solution to copper,

* Another cause of change of polarization voltage with current is discussed in two papers on Reversible and Irreversible Polarization by W. S. Franklin and L. A. Freudenberger which are referred to on page 190.

† Two methods have been employed for measuring the polarization voltage of the standard electrode. This matter is fully discussed in Whetham's *Theory of Solution*, Chapter XI, Cambridge University Press, 1902.

or the difference

− volts from solution to zinc + volts from solution to copper.

According to the above table, voltage from solution to zinc is − 0.524, and voltage from solution to copper is + 0.515, so that the voltage indicated by the voltmeter would be 1.039. This voltage depends upon the degree of concentration of the solutions. With a concentrated solution of copper sulphate and a fairly dilute solution of zinc sulphate the voltmeter would indicate about 1.08 volts.

Calculation of electrode polarization.—The total voltage (the decomposition voltage) required to force an infinitesimal current through an electrolytic cell may be calculated from the chemical work as explained in Art. 93; and if one could determine the portion of the chemical work which is done at the anode and the portion which is done at the cathode then the total decomposition voltage could be divided into two known parts, and these parts would be the anode polarization and the cathode polarization respectively. There is, however, no chemical data in existence upon which such a calculation can be based, and even if such chemical data did exist the results would be subject to the qualifications outlined in the fine print in Art. 93.

95. The voltaic cell.*—The chemical action produced by the flow of current through an electrolytic cell is confined wholly to the immediate neighborhood of the electrodes, and this chemical action is usually forced, that is, *work has been done* to bring it about, or, in other words, an outside electromotive force is required to push the current through the cell. But in many cases the chemical action produced by the flow of current through an electrolytic cell is a *source* of energy. In such a case the electrolytic cell itself can maintain a current through the electrolyte from electrode to electrode and through an outside circuit of wire which connects the electrodes. Such an electrolytic cell is called a *voltaic cell* or *primary battery*.

Example.—When a strip of clean zinc and a strip of copper or carbon are dipped into dilute sulphuric acid, no chemical action takes place. But when the plates are connected together by a wire, a current immediately starts to flow through the circuit, leaving the cell at the copper or carbon electrode (the cathode) and entering the cell at the zinc electrode (the anode). This current decomposes the sulphuric acid (H_2SO_4), the hydrogen is

* A number of voltaic cells connected together constitute a voltaic battery. The word battery is, however, frequently applied to a single cell.

set free at the copper or carbon cathode and escapes from the cell as a gas, and the sulphuric acid radical (SO_4) which is "liberated" at the zinc anode combines with the zinc and forms zinc sulphate ($ZnSO_4$) which goes into solution. The combination of Zn and SO_4 develops more energy than is required for the decomposition of the H_2SO_4 so that the chemical action in this cell is a source of energy. The cell here described is called the *simple voltaic cell.*

96. Electromotive force of a voltaic cell.—A clear idea of the electromotive of a voltaic cell may be obtained by the following argument: (*a*) The amount of chemical action which takes place in a given voltaic cell is proportional to $I \times t$, where I is the current and t is the time during which the current continues to flow, as explained in Art. 90. Furthermore the energy evolved by the chemical action is proportional to the amount of chemical action; therefore the energy evolved by the chemical action is proportional to $I \times t$ and it can be expressed as eIt where e is a constant for a given type of voltaic cell.

(*b*) The energy delivered to an electric circuit is equal to RI^2t joules, where R is the resistance of the circuit in ohms, I is the current in amperes, and t is the elapsed time in seconds.

(*c*) *If all of the energy of the chemical action were available for the maintenance of the current* then eIt (the energy used to maintain the current) would be equal to RI^2t (the energy which appears in the electric circuit). That is:

$$eIt = RI^2t$$

or

$$I = \frac{e}{R}$$

Or, in words, the current produced by the given voltaic cell in a circuit of resistance I is equal to the factor e [as defined under (*a*)] divided by R. Now this relation is what is familiarly known as Ohm's law and the factor e is the *electromotive force* of the given voltaic cell.

The electromotive force of a voltaic cell is entirely independent of the size of the cell, it depends only on the character of the chemical action in the cell.

The electromotive force of a voltaic cell is most easily measured by connecting a voltmeter to the terminals of the cell.

The above argument may well be repeated as applied to a particular case. (*a*) Consider the energy evolved by the dissolution of the zinc in the simple voltaic cell which is described in Art. 95. The net result of the chemical action in this cell is represented by the formula

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

During the flow of one faraday through such a cell one gram-val (32.7 grams) of zinc would be dissolved and a corresponding amount of hydrogen would be set free. Now the dissolution of one gram of zinc in dilute sulphuric acid develops 578 calories of heat which is the equivalent of 2428 joules or 0.822 volt-faradays of work.

(*b*) Let E be the electromotive force of the cell. Then during the flow of one faraday the cell would deliver E volt-faradays of electrical work.

(*c*) If all the energy evolved by the dissolution of the zinc in the simple voltaic cell were available in the form of electrical-energy output then, from (*a*) and (*b*) we would have

$$E \text{ volt-faradays} = 0.822 \text{ volt-faradays}$$

or

$$E = 0.822 \text{ volts.}$$

(*d*) As a matter of fact the electromotive force of the simple voltaic cell under discussion is 1.03 volts on open circuit and it falls off very greatly when the cell delivers current.

97. Polarization of a voltaic cell.—The electromotive force between the terminals of a voltaic cell is always less when the cell is delivering current than it is when the cell is not delivering current. This difference is at first due chiefly to the loss of voltage due to the resistance of the cell. Thus a voltaic cell has an "open-circuit" electromotive force of 2 volts and an internal resistance of 0.05 ohm, and the electromotive force between the terminals of the cell drops *instantly* to 1.5 volts when the cell is called upon to deliver 10 amperes. The loss of voltage due to internal resistance is in this case 0.5 volt and it is equal to the current of 10 amperes multiplied by the internal resistance of 0.05 ohm.

When a voltaic cell continues to deliver current the electromotive force between the terminals of the cell falls off more and more. This falling off of the electromotive force of a voltaic cell is called *polarization*.* The chief cause of the polarization of a voltaic cell is as follows: The chemical action in the neighborhood of the electrodes brings about local changes of composition and of concentration of the electrolyte, less energy is evolved by the chemical action *at* the electrodes, and consequently the voltage of the cell decreases.

98. The bichromate cell.—*The available energy of the chemical action which takes place in the simple voltaic cell which is described as an example in Art. 95 may be greatly increased by providing an oxidizing agent in the neighborhood of the cathode so that the hydrogen may be oxidized at the moment of its "liberation" by the current.* The increase of available energy because of this oxidation gives a greatly increased voltage. An oxidizing agent used in this way is called a *depolarizer*.† The bichromate cell furnishes a good example of the use of an oxidizing agent in a voltaic cell.

The simplest form of bichromate cell, which is sometimes called the *Grenet cell* from its inventor, is shown in section in Fig. 119*a*. The electrodes are a carbon plate *C* and an amalgamated‡ zinc plate *Z*, and the electrolyte *e e e* is dilute

* This term must not be confused with the term electrode polarization as defined in Art. 94.

† In the whole history of physics words without meanings have been used for things not understood. Thus the word *polarize* does a great variety of questionable duty in the subject of electrolysis in the same way that the word *induce* does a great variety of questionable duty in electromagnetism! It would be a great help if all such words could be thrown away.

‡ Covered with a thin coating of mercury.

sulphuric acid in which potassium bichromate or chromic acid has been dissolved. Without the potassium bichromate the hydrogen escapes as a gas and the voltage of the cell is 1.03; with the bichromate the hydrogen is oxidized and the voltage of the cell is 1.90. There is a very rapid waste of zinc (and acid) in this cell and the cell is now seldom used.

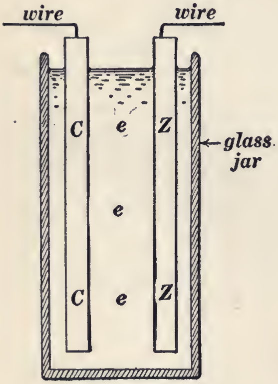


Fig. 119*a*.

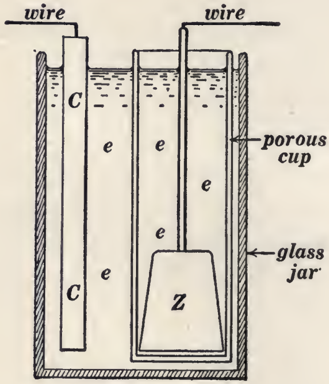


Fig. 119*b*.

A modified form of bichromate cell, known as the *Fuller cell*, is shown in section in Fig. 119*b*. In this cell the electrolyte is dilute sulphuric acid but the zinc anode is contained in a porous earthenware cup and the potassium bichromate or chromic acid is dissolved only in that portion of the electrolyte which surrounds the carbon cathode. There is not a rapid waste of zinc (and acid) in this cell and the cell is extensively used.

99. Voltaic action and local action.—Two kinds of chemical action are to be distinguished in a voltaic cell: (*a*) the chemical action which depends upon the flow of current, and does not take place when there is no current; and (*b*) the chemical action which is independent of the flow of current, and which does take place whether the current is flowing or not.

The chemical action which depends on the current is proportional to the current, it is essential to the operation of the voltaic cell as a generator of current, its energy is available for the maintenance of the current, and it is called *voltaic action*.

The chemical action in a voltaic cell which is independent of the flow of current does not help in any way to maintain the current, it represents absolute waste of materials, and it is called *local action*.

Local action takes place more or less in every type of voltaic cell and it is especially great in the Grenet cell where the oxidizing agent comes into contact with the zinc plate. Local action is greatly reduced by coating the zinc plate with a thin coating of metallic mercury and by using very pure zinc.

The amount of zinc usefully consumed by voltaic action while a voltaic cell in delivering a given current for a specified time, is equal to the amount of zinc that would be deposited by the given current during the specified time upon the cathode of an auxiliary electrolytic cell containing a solution of a zinc salt. For example five amperes flowing for one hour will deposit 6.1 grams of zinc, and therefore 6.1 grams of zinc (and a corresponding amount of acid and bichromate) are usefully consumed in a Grenet cell in the delivery of 5 amperes for one hour. In an actual test of a Grenet cell delivering 5 amperes for one hour the loss of zinc was 30 grams. Under the conditions of the test, therefore, 23.9 grams of zinc (and a corresponding amount of acid and bichromate) were wasted by local action and 6.1 grams of zinc (and a corresponding amount of acid and bichromate) were usefully consumed.

100. Open-circuit cells and closed-circuit cells.—A voltaic cell which can be left standing unused but in readiness at any time for the delivery of current when its circuit is closed is called an *open-circuit cell*. An open-circuit cell must, above all things, be nearly free from local action. The cell most extensively used for open-circuit service is the LeClanché cell which is described as the manganese-dioxide cell in Art. 103.

A voltaic cell which is suitable for delivering a current steadily is called a *closed-circuit cell*. A closed-circuit cell should be of a type of which the voltage does not fall off greatly with continued delivery of current, and, of course, local action should be eliminated if possible. The gravity Daniell cell, the Fuller cell, the copper oxide cell, and the lead storage cell are most extensively used for closed circuit work.

101. The gravity Daniell cell.—The essential features of the gravity Daniell cell are shown in Fig. 118. The usual form of the cell is shown in Fig. 120. During the operation of the cell metallic copper is deposited upon the copper cathode at the bottom of the cell, and the SO_4 combines with the zinc of the anode, forming $ZnSO_4$. The electromotive force of this cell varies from about 1.05 volts to 1.10 volts, depending upon the degree of concentration of the zinc sulphate and copper sulphate solutions.

Fig. 120.

This cell has a considerable amount of local action when it is allowed to stand unused because of the upward diffusion of the copper sulphate. It is used very extensively in telegraphy and for operating the "track circuit" relays in automatic railway signalling.

102. The copper-oxide cell.—In this type of cell the anode is zinc, the electrolyte is a solution of caustic soda, and the cathode is a highly compressed block of copper oxide. The flow of current through the cell "liberates" Na at the cathode and OH at the zinc anode and the following reactions take place:

At the cathode $\mathbf{Na} + \frac{1}{2}(CuO) + \frac{1}{2}(H_2O) = \frac{1}{2}Cu + NaOH$

↑ direction of flow of current through the cell.

At the anode $\mathbf{OH} + \frac{1}{2}Zn + NaOH = \frac{1}{2}(Na_2ZnO_2) + H_2O.$

The atomic weights (or molecular weights) of the various substances in this schedule, multiplied by the factor $\frac{1}{2}$ or not as the case may be, express the number of grams of the various substances involved in the chemical action produced by the flow of one faraday through the cell.*

The copper-oxide cell has but little local action and its electromotive force is about 0.6 volt. This cell is used extensively for operating railway signals; that is for operating the small motors which move the signal arms. The track relays are usually operated by gravity cells.

103. The manganese-dioxide cell.—In this cell the anode is zinc, the electrolyte is a solution of ammonium chloride (sal ammoniac) and the cathode is a plate of carbon with bits of coke and manganese dioxide packed closely around it. The earliest cell of this type is called the *LeClanche cell* from its inventor, and in this cell the manganese dioxide and coke are contained in a porous earthenware cup. The ordinary *dry cell*† is a manganese-dioxide cell in which the containing vessel is made of sheet zinc and it serves as the anode, the electrolyte is held in a porous mass of saw-dust or other absorbent material, and several thicknesses of unglazed paper keep the manganese dioxide and powdered coke away from the zinc.

The flow of current through the manganese-dioxide cell "liberates" chlorine at the zinc anode and NH_4 at the carbon cathode and the following reactions take place:

At the cathode $NH_4 + \frac{1}{2}(MnO_2) = NH_3 + \frac{1}{2}(H_2O) + \frac{1}{2}(MnO)$

↑ Direction of current through the cell.

At the anode $Cl + \frac{1}{2}Zn = \frac{1}{2}(ZnCl_2)$

There is but little local action in the manganese-dioxide cell if the zinc and ammonium chloride are pure,‡ but the cell polarizes rapidly when it delivers steady current. Reliable manufacturers

* This statement applies also to the reaction schedules given on pages 204, 205 and 206.

† The dry cell has been defined as a cell which being sealed is always wet, whereas the wet cell being open to the air often becomes dry.

‡ See important papers on the dry cell, *Transactions of the American Electrochemical Society*, Vol. XVI, pages 97 and 109; Vol. XVII, page 341; and Vol. XIX, page 31.

always stamp the date of manufacture on their dry cells, and the purchaser should not accept a cell which is more than two or three months old. The best test of a dry cell is to observe the short-circuit current by connecting the cell momentarily to an ammeter. A fresh cell $2\frac{1}{2}$ inches in diameter and 6 inches high should give at ordinary room temperature about 20 amperes short-circuit current. The electromotive force of the manganese-dioxide cell is about 1.6 volts.

104. Regeneration of a voltaic cell by a reversed current.— *An essential feature of voltaic action is that* **it is reversed** *if a current is forced backwards through a voltaic cell by an outside agent,* provided that no material that has played a part in the previous voltaic action has been allowed to escape from the cell. Thus, in the operation of the simple voltaic cell consisting of a zinc anode and carbon cathode in dilute sulphuric acid, the H_2SO_4 is decomposed, $ZnSO_4$ is formed at the anode, and hydrogen is liberated at the cathode. If the current is reversed so that the carbon plate becomes the anode and the zinc plate the cathode, then the $ZnSO_4$ previously formed will be decomposed, metallic zinc will be deposited upon the zinc cathode, and SO_4 will be liberated at the carbon anode where it will combine with the trace of hydrogen that is clinging to the carbon plate and form H_2SO_4. In this cell the greater part of the liberated hydrogen has of course escaped and the reversed chemical action, due to a reversed current cannot long continue. *Local action, on the other hand, being independent of current, is not affected by a reversal of the current.*

The storage cell.—*A voltaic cell which is free from local action and in which all of the materials which take part in the voltaic action are kept in the cell, may be* **regenerated** *after use by sending through it a reversed current.* This regeneration is due to the reversed chemical action that is produced by the reversed current, as explained above. A voltaic cell which can be thus regenerated is called a *storage cell.* The process of regeneration is called *charging* and the use of the cell as an electric generator is called *discharging.*

The capacity of a storage cell is expressed in ampere-hours. Thus a cell which can deliver 10 amperes for 8 hours is said to have a capacity of 80 ampere-hours.

The electrode *out of which current flows while a storage cell is discharging* is called the *positive electrode,* and the other electrode is called the *negative electrode.* This matter can be

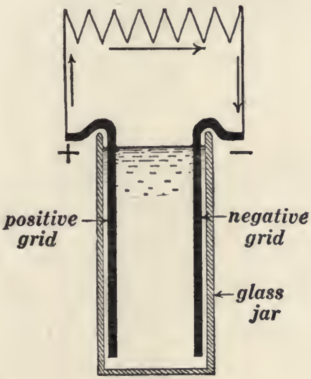


Fig. 121.

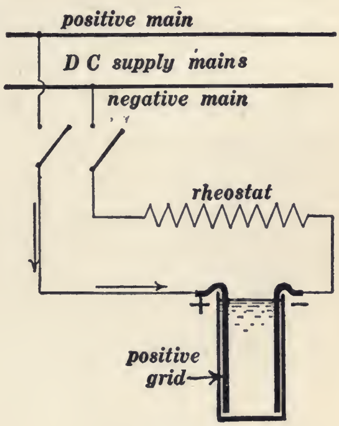


Fig. 122.

most easily remembered with the help of diagrams. Thus Fig. 121 shows a storage cell discharging, and Fig. 122 shows a storage cell being charged.

Almost any kind of voltaic cell can be used to some extent as a storage cell, that is to say, almost any kind of voltaic cell can be regenerated to some extent by forcing a reversed current through it, but a good storage cell must be free from local action, and the materials which take part in the voltaic action must be kept in the cell, as pointed out above; and the electrodes must not crumble to pieces with frequent charging and discharging of the cell. The only voltaic cells which, up to the present time, have been found to meet these requirements are the lead cell* and the nickel-iron cell.†

105. The lead storage cell.—When the *lead cell* is fully charged it has a positive electrode of lead peroxide (PbO_2), a negative electrode of spongy metallic lead (Pb), and an electrolyte of dilute sulphuric acid (H_2SO_4). When the cell discharges the lead peroxide is reduced to lead oxide (PbO) which absorbs sulphuric acid from the solution and is converted into insoluble lead sulphate ($PbSO_4$), and the spongy metallic lead is oxidized and similarly converted into insoluble lead sulphate.

The lead peroxide and the spongy metallic lead (or the insoluble lead sulphate into which these materials are converted by discharge) constitute the *active electrode materials* of the lead cell. These active materials are held in grooves or pockets, in plates or grids of solid metallic lead.

The chemical actions which take place in the charging and discharging of the lead storage cell are shown in the following schedule:

DISCHARGING.

Positive grid $H + \frac{1}{2}(PbO_2) + \frac{1}{2}(H_2SO_4) = H_2O + \frac{1}{2}(PbSO_4)$

↑ Direction of current through the cell

Negative grid $\frac{1}{2}(SO_4) + \frac{1}{2}Pb = \frac{1}{2}(PbSO_4)$

CHARGING.

Positive grid $\frac{1}{2}(SO_4) + \frac{1}{2}(PbSO_4) + H_2O = H_2SO_4 + \frac{1}{2}(PbO_2)$

↓ Direction of current through the cell.

Negative grid $H + \frac{1}{2}(PbSO_4) = \frac{1}{2}(H_2SO_4) + \frac{1}{2}Pb$

Effects of charge and discharge.—The conversion of the PbO_2 and the Pb into $PbSO_4$ on discharge causes a great decrease in the concentration of the electrolyte, especially in the pores of the active material, and an increase of volume of the active

* The details of construction of the lead storage cell are very fully described on pages 147–200 of Lyndon's *Storage Battery Engineering*, McGraw-Hill Book Co., 1911.

† Sometimes called the *Edison cell*. The earlier type of nickel-iron storage cell is described by A. E. Kennally, *Transactions of the American Institute of Electrical Engineers*, Vol. XVIII, pages 219–244, May, 1901.

electrode materials. The reconversion of the $PbSO_4$ into PbO_2 and Pb by charging causes a corresponding increase in the concentration of the electrolyte and a decrease of volume of the active electrode materials.

The changes of concentration of the electrolyte are the chief causes* of the decrease of voltage of the cell during discharge and of the increase of voltage of the cell while it is being charged (see Fig. 123).

The expansion and contraction of the active electrode materials is the chief cause of the disintegration of the electrodes, and it also tends to cause the plates or grids to buckle or warp, especially if the action is not the same on both sides of a plate. A well-designed storage battery grid allows the active electrode material to expand and contract with a minimum of mechanical damage to the grid.

106. The nickel-iron storage cell.—When the *nickel-iron cell* is fully charged it has a positive electrode of nickel peroxide (NiO_2), a negative electrode of spongy metallic iron (Fe), and the electrolyte is a solution of caustic potash (KOH). When the cell discharges, the nickel peroxide is reduced to nickel oxide and the spongy metallic iron is oxidized.

The nickel peroxide and the spongy metallic iron (or the nickel and iron oxides into which these materials are converted by discharge) constitute the *active electrode materials* of the nickel-iron cell. These active materials are held in grooves or pockets, in plates or grids of metallic nickel and steel.

The chemical actions which take place in the charging and discharging of the nickel-iron storage cell are shown in the following schedule:

DISCHARGING.

Positive grid $\mathbf{K} + \frac{1}{2}(NiO_2) + \frac{1}{2}(H_2O) = \frac{1}{2}(NiO) + KOH$

↑ Direction of current through the cell.

Negative grid $\mathbf{OH} + \frac{1}{2}Fe = \frac{1}{2}(FeO) + \frac{1}{2}(H_2O)$

* The increase and decrease of voltage during charging and discharging are also due in part to the resistance of the cell and in part to what is called *irreversible actions at the electrodes.*

CHARGING.

Positive grid $OH + \frac{1}{2}(NiO) = \frac{1}{2}(NiO_2) + \frac{1}{2}(H_2O)$

↓ Direction of current through the cell.

Negative grid $K + \frac{1}{2}(FeO) + \frac{1}{2}(H_2O) = \frac{1}{2}Fe + KOH$

Effects of charge and discharge.—The NiO_2 contracts and the Fe expands during discharge, and *vice versa*; but there is no general increase or decrease of concentration of the electrolyte due to charging and discharging. During discharge, however, there is an increase of concentration near the positive grid and a decrease of concentration near the negative grid and *vice versa.*

107. Charging and discharging curves. Comparison of the lead storage cell and the nickel-iron storage cell.—The ordinates of the curve in Fig. 123 show the change of voltage between the

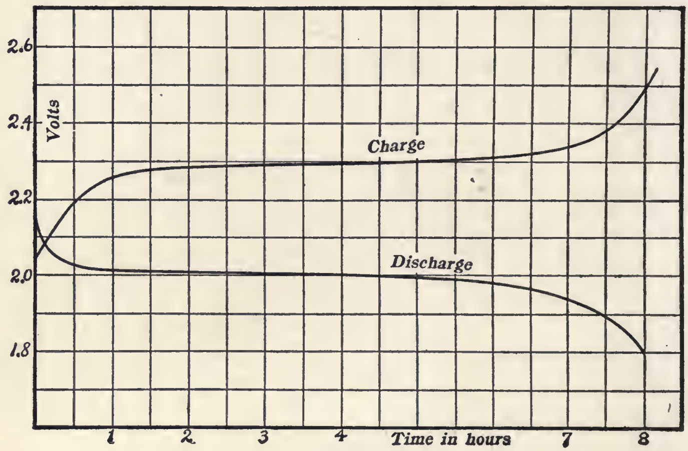


Fig. 123.

terminals of a "30-ampere" lead storage cell when it is charged for 8 hours and discharged for 8 hours, the current being 30 amperes in each case. The ordinates of the curve in Fig. 124 show the change of voltage between the terminals of a "30-ampere" nickel-iron storage cell when it is charged for 5.5 hours and discharged for 5 hours, the current being 30 amperes in each case.

From these figures it is evident that the voltage of a lead storage cell stands at a more nearly constant value during discharge than the voltage of a nickel-iron storage cell. The average voltage of a lead cell while discharging is about 81 per cent. of the average voltage while being discharged, whereas the average

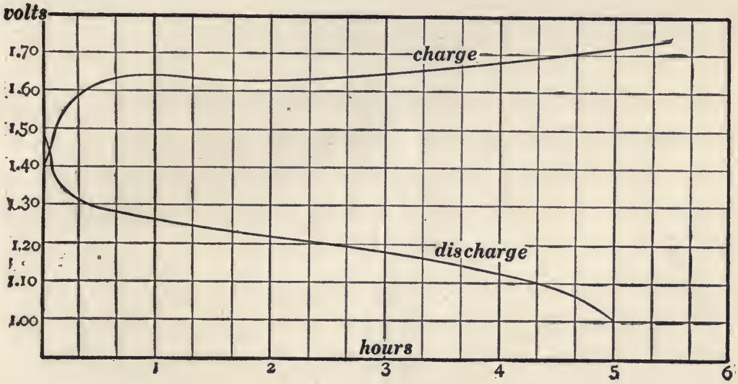


Fig. 124.

voltage of a nickel-iron cell while discharging is about 72 per cent. of the average voltage while being charged The energy efficiency of a lead storage cell when charged and discharged as represented by the curves in Fig. 123, is about 80 per cent.; and the energy efficiency of a nickel-iron cell when charged and discharged, as represented by the curves in Fig. 124, is about 60 per cent.

The nickel-iron storage cell seems to make a better portable cell than the lead storage cell because the nickel-iron cell is lighter than the lead cell and because the nickel-iron cell may be allowed to stand for a long time partially or wholly discharged. Also the nickel-iron cell is perhaps better than the lead storage cell for driving electric vehicles because the nickel-iron cell does not demand careful attention. For electric vehicle driving a constant-voltage supply of current is not necessary and therefore the great change of voltage of the nickel-iron cell is not especially objectionable for this kind of service.

The lead storage battery is better than the nickel-iron storage battery for stationary service of all kinds because of its lower first cost, because of its more nearly constant voltage, and because of its higher efficiency. The greater constancy of voltage is especially important when current is to be supplied to incandescent lamps; the voltage even of a lead storage battery must be regulated when it is used for this purpose.

108. Portable type and stationary type of storage cells. Costs and weights.—There are two more or less distinct types of lead storage cells, namely, the portable type which is made as light as possible by using thin grids and hard-rubber containing jars, and the stationary type which has very heavy grids and glass jars, or, in the case of very large cells, lead-lined wooden tanks.

The portable type of lead storage cell has about 10 watt-hours of storage capacity (a normal discharge rate of 1.25 watts) per pound of gross weight. The stationary type of lead storage cell has about 4 watt-hours of storage capacity (a normal discharge rate of 0.5 watt) per pound of gross weight. The nickel-iron storage cell has about 14 watt-hours (2.8 watts normal discharge rate) per pound of gross weight.

Some idea of the cost of storage cells is given by the following schedules:

LEAD STORAGE BATTERY, HEAVY STATIONARY TYPE, 50 CELLS, 400 AMPERE-HOURS.

To deliver 5 kilowatts for 8 hours.*

	Cost.		Weight.	
	Total.	Per Kilowatt-hour of capacity.	Total.	Per Kilowatt-hour of capacity.
In glass jars.............	$1,400	$35.00	10,300 lbs.	258 lbs.
In lead-lined wooden tanks.	1,650	41.25	14,600	365

Depreciation 6 or 7 per cent. per year when properly cared for.

* Or 7 kilowatts for 5 hours, or 10 kilowatts for 3 hours, or 20 kilowatts for 1 hour. See page 211.

LEAD STORAGE BATTERY, LIGHT PORTABLE TYPE, 50 CELLS, 200 AMPERE-HOURS.

To deliver 2.5 kilowatts for 8 hours.*

	Cost.		Weight.	
	Total.	Per Kilowatt-hour of capacity.	Total.	Per Kilowatt-hour of capacity.
In covered rubber jars.....	$800	$40	2,340 lbs.	117 lbs.

Depreciation 15 per cent. per year or more.

NICKEL-IRON BATTERY, 60 CELLS, 225 AMPERE-HOURS.

To deliver 3.24 kilowatts for 5 hours.

	Cost.		Weight.	
	Total.	Per Kilowatt-hour of Capacity.	Total.	Per Kilowatt-hour of Capacity.
In covered steel tanks......	$960	$59.25	1,200 lbs.	71 lbs.

Depreciation unknown.

109. Comparative costs of electrical energy from storage batteries and from ordinary primary batteries.—(*a*) A storage battery is essentially like any ordinary battery except that a storage battery can be regenerated by forcing a current through it backwards. A storage battery which has delivered one kilowatt-hour of electrical energy can be made as good as new by the expending of a little more than a kilowatt-hour in forcing current backwards through the battery at a cost of say 20 cents. Therefore, the output of a storage battery may cost as low as 20 cents per kilowatt-hour.

(*b*) A Grenet or Fuller cell can be regenerated after use by replacing the zinc and the electrolyte. Therefore the cost of the electrical energy delivered by a Grenet or Fuller cell is equal to the cost of the materials consumed plus a few cents for the labor of setting up the cell. The voltaic action corresponding to 100 ampere-hours represents the consumption of 0.26 pound of zinc at 15 cents per pound, 0.40 pound of potassium bichromate

* Or 3.5 kilowatts for 5 hours, or 5 kilowatts for 3 hours, or 10 kilowatts for 1 hour.

at 15 cents per pound and 0.6 pound of sulphuric acid at 2 cents per pound (the zinc is reckoned considerably higher in price than ingot zinc for several reasons, one of which is the cost of mercury for amalgamating the zinc). Therefore, counting five cents for the labor cost we have a total cost of 16.1 cents for 100 ampere-hours. The electromotive force of the bichromate cell is about 2 volts, therefore 100 ampere-hours represents 200 watt-hours, and 200 watt-hours at 16.1 cents gives a rate of 80 cents per kilowatt-hour. But the total consumption of materials in a Grenet cell is at least five times the consumption corresponding to the voltaic action alone, and the total consumption of materials in a Fuller cell is at least two times the consumption corresponding to the voltaic action alone. Therefore the output of a Grenet cell costs about $4.00 per kilowatt-hour and the output of a Fuller cell costs about $1.60 per kilowatt-hour.

(*c*) When an ordinary dry cell is discharged the entire cell is thrown away, and therefore, the first cost of the cell is the cost of its output of electrical energy. Thus a dry cell costing 25 cents has about 50 ampere-hours of discharge capacity at 1.6 volts* which is equivalent to 80 watt-hours, and 25 cents for 80 watt-hours is at the rate of $3.12 per kilowatt-hour.

Comparing (*a*), (*b*) and (*c*) it is evident that a storage battery is very much cheaper than an ordinary primary battery provided the storage battery can be charged without excessive loss of energy in a rheostat and without carrying the battery to a charging station. The following example shows the excessive cost of a storage battery where these two conditions are not realized. A three-cell storage battery having a capacity of 50 ampere-hours at 6 volts is delivered to the proper place for charging at a cost of 50 cents counting return delivery and the battery is charged from 110-volt direct-current mains so that 5,500 watt-hours† of energy is consumed in charging the battery.

* No allowance is here made for the fact that the terminal voltage of the cell may be considerably less than its open-circuit voltage.

† The battery would be connected to the supply mains in series with a rheostat and about 94 per cent. of the energy taken from the mains would be lost in this

At 15 cents per kilowatt-hour this energy amounts to 82 cents, and the total cost of $1.32 for charging the battery is the cost of the 300 watt-hours of battery output, which is at the rate of $4.40 per kilowatt-hour.

110. The management and care of a lead storage battery.*— The lead storage battery deteriorates rapidly in service when it is not properly cared for, and, the first cost of the storage battery being high, it is important that it should have proper care. The deterioration shows itself by a decrease of ampere-hour capacity, by a continued disintegration of the active materials of the electrodes, by a slow corrosion of the massive lead grids and by warping and buckling of the grids.

Setting up a storage battery.—Detailed directions for setting up a storage battery are always supplied by the manufacturer. A person who has not had experience in handling acids must exercise great care. The concentrated acid should be poured into the water in a thin stream and the water should be stirred with a wooden paddle, great care being taken to avoid splashing. The acid should be mixed in an earthenware jar or in a clean wooden tub. Metal must not be used.

The electrolyte.—The electrolyte is dilute sulphuric acid having a density of about 1.21 at 70° F. when the battery is charged. This acid must be quite pure (free from iron, hydrochloric acid or nitric acid), and pure water, preferably distilled water, must be used for mixing the electrolyte.

When the surface of the electrolyte falls because of evaporation, pure water must be added so as to keep the tops of the grids covered to a depth of about one-half inch.

Discharging.—The normal discharge current of a lead storage

rheostat. To avoid this loss of energy many storage cells must be connected in series for charging from 110-volt mains.

* See Chapters XXII and XXIII of Lyndon's *Storage Battery Engineering*, McGraw-Hill Book Company, 1911.

Manufacturers of storage batteries publish circulars giving instructions for setting up and operating, and the manufacturers are usually glad to send these circulars to any one who is interested in storage battery work.

cell (on the basis of an eight-hour discharge) is about 5 amperes per square foot of positive grid area, both sides of each positive grid being counted.

A current exceeding 4 or 5 times the normal discharge current should never be taken from a storage cell; if a greater current must be taken from a cell it should be for a few minutes only and the battery should be at full charge.

A lead storage battery should never be discharged so as to cause the voltage to drop below 1.75 volts per cell, while the normal discharge current is flowing.

A lead storage cell should never be allowed to stand discharged.

Charging.—Usually a lead storage battery is charged by a current equal to the normal discharge current, the time required for charging being eight or eight and one-half hours.

The charging current may be high, however, when the battery is nearly discharged, and it should be low when the battery approaches full charge, especially after evolution of gas begins. A good rule for rapid charging is to deliver 35 per cent. of the total ampere-hours during the first hour, 52 per cent. during the next two hours, and 14 per cent. during the fourth hour. Thus a 100-ampere-hour cell may be completely charged in four hours by using a charging current of 35 amperes during the first hour, 26 amperes during the second and third hours, and 14 amperes during the fourth hour.

Overcharging.—A battery should be overcharged once a week, or once every two weeks if it is not used daily. This overcharge is a prolongation of the regular charge (at the normal eight-hour rate) until the voltage across each cell reaches a maximum, that is, until five successive voltmeter readings, 15 minutes apart, show no further increase of voltage, or until gas is developed in all the cells freely.

Voltmeter test.—The voltage of each cell should be taken just before the end of the weekly overcharge *with current flowing at the normal eight-hour rate.* The normal voltage under these conditions is about 2.56 volts per cell. The voltage normally reached during the regular charging is about 2.45 volts per cell. An abnormally low voltage shows that a cell has been discharged by internal short circuit.

Inspection.—Just before the weekly overcharge, every cell should be inspected carefully, especial attention being given to those cells which have shown abnormally low voltage on previous tests. The object of the inspection is to see that no internal short circuits exist. Short circuits are to be removed by means of a thin strip of hard wood pushed down between the grids. Evidences of sulphatation should also be noted as explained under the heading sulphatation.

Treatment of cells which show abnormally low voltage.—If the voltage of a cell does not rise to the normal value during an overcharge, it must be cut out of circuit when the battery is discharged and cut in again just before beginning the next charging. If this does not bring it up to normal voltage the process must be repeated.

Sediment.—The accumulation of sediment in the bottom of the jars must be watched and the sediment must be removed before it reaches the bottom of the grids. In the case of small cells the grids may be lifted out after the battery has been fully charged, the electrolyte drawn off, and the sediment washed out of the jars. It is important to get the elements back and covered with electrolyte again as quickly as possible. Fresh electrolyte must be added to make up for the electrolyte lost with the sediment.

Sulphatation.—Sulphatation of the grids of a lead storage cell consists* of the conversion of portions of the active material *wholly* into lead sulphate. This pure sulphate is a very poor conductor and, once it is formed, it is difficult to make it act as anode or as cathode and thus reconvert it to lead peroxide or to spongy lead respectively. A layer of pure lead sulphate sometimes forms between the active material and the metallic lead

* It is claimed by some authorities that sulphatation consists in the formation of hydrated lead sulphate.

of the grid, and sometimes the external surface of the active material becomes covered with a crust of pure sulphate. Pure lead sulphate is white and whenever white spots appear on the grids of a lead storage cell, the cell should be subjected to a very long-continued over-charge in the attempt to reduce the pure lead sulphate into active material.

A very good method* for treating sulphated cells is as follows:

1. Remove the acid electrolyte and rinse with pure water.

2. Fill the cells with a solution of pure sodium sulphate using 200 grams of the crystallized salt per liter of water.

3. Charge the battery in the usual way at the 8-hour rate for 50 hours or more.

4. Remove the sodium sulphate solution and rinse with pure water.

5. Replace the acid electrolyte, using a little fresh acid to bring it up to correct strength.

6. Charge the battery

One change of water is sufficient for each rinsing. The cost of the entire treatment counting labor, materials and energy is about 21 cents per cell for cells rated at 60-ampere-hours.

Variation of capacity with discharge rate.—When a storage cell is discharged slowly, the discharge can be carried further than when the cell is discharged rapidly, that is to say, the ampere-hour capacity is greater with a small ampere discharge than with a large ampere discharge. The relation between ampere-hours of capacity and amperes of discharge rate for the stationary batteries of the Electric Storage Battery Company is as follows: A cell that can deliver 12.5 amperes for eight hours can deliver 17.5 amperes for five hours, 25 amperes for three hours or 50 amperes for one hour.

111. The use of storage batteries.†—Storage batteries are extensively used in the place of ordinary primary batteries in

* See a paper by C. W. Bennett and D. S. Cale; read before the Boston General Meeting of the American Electrochemical Society, April 18, 1912.

† Fundamentally, of course, a storage battery is used to store electrical energy at a given time and place in order that the energy may be used when and where it is needed.

telephone and telegraph work and in railway signalling.* Storage batteries are also used for driving electric vehicles and for railway car lighting, and very large storage batteries are used in central stations for one or more of the following purposes:

(*a*) *For supplying the station output during the hours of small demand.* In this case the battery is charged while the station is in operation, and discharged during the remainder of the day, thus obviating the expense of operating the station continuously.

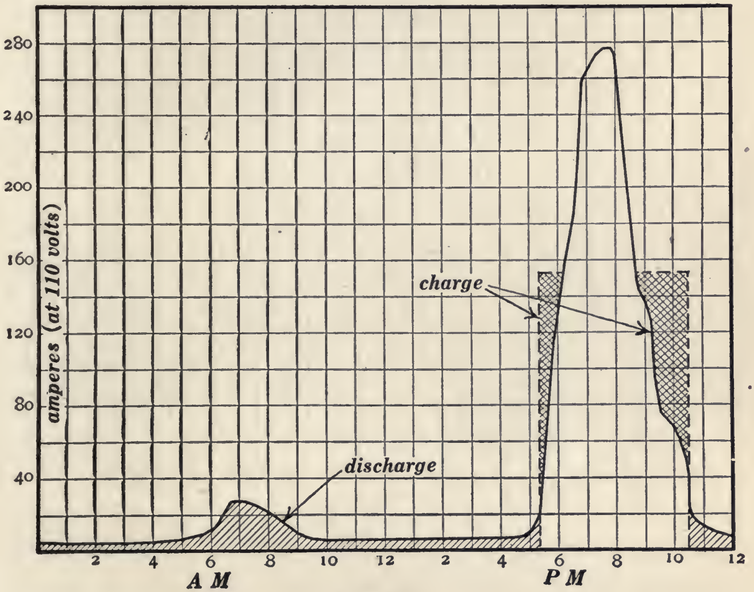


Fig. 125.

(*b*) *For equalizing a fluctuating station load.* In this case provision is made for the battery to charge while the station load is below the average and to discharge while the station load is above the average. This is an important use of large storage battery installations, and the cost of installing and maintaining the battery is set over against the saving in the first cost of the station and the saving in the cost of operating the station.

* In many cases a small motor-generator is used to take current from 110-volt mains (direct-current or alternating-current) and deliver *direct current* at low voltage for charging. In some cases direct current for charging is derived from alternating-current supply mains by using the mercury-vapor rectifier.

(*c*) *As a reserve.*—The primary object of a storage battery may be to supply the output of a station during certain hours of the day or to equalize a fluctuating load on a station. In both cases the battery will be valuable also as a reserve to supply the station load in case of a break-down.

Figure 125 illustrates the use of a storage battery for supplying the total station output during the hours of small demand. The

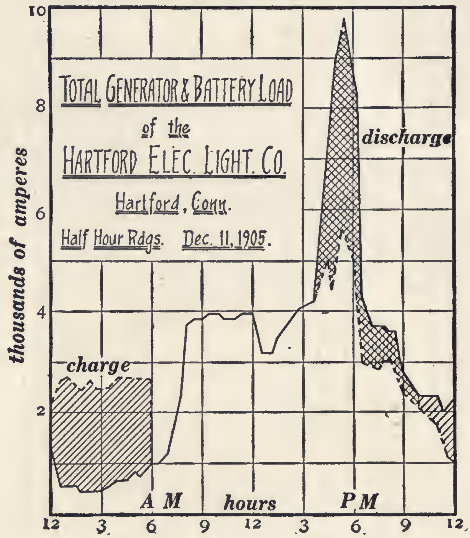


Fig. 126.

curve represents the operation of a small direct-current plant at Milan, Michigan, in 1902. The engine and generators were operated from 5:30 P. M. to 10:30 P. M. From 5:30 to about 6:20 P. M. the generators supply the station load (which is small) and charge the battery. Between 6:20 P. M. and 8:40 P. M. the generators supply the station load, only, and the battery is disconnected. Then from 8:40 P. M. until 10:30 P. M. the generators supply the station load (which is small) and charge the battery. The engine is shut down at 10:30 P. M. and the battery carries the entire station load until 5:30 the next evening.

The use of a storage battery for equalizing the load on a generator is shown in Fig. 126. This curve shows the operation of the central station of the Hartford Electric Light Company on December 11, 1905. The battery was charged from about 10:30 P. M. to 6 A. M., and discharged from about 4 P. M. to 10:30 P. M., during the peak of the station load.

The use of a storage battery for equalizing the rapidly fluctuating load of an electric railway station is shown in Fig. 127. The dotted curve in this figure shows the actual generator load and

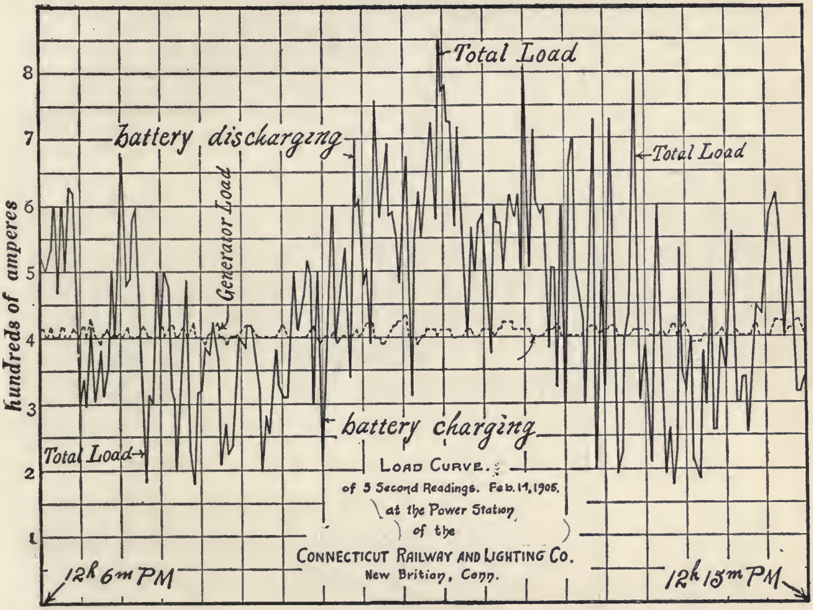


Fig. 127.

the full-line curve shows the rapidly fluctuating station load between 12:06 P. M. and 12:15 P. M. Readings were taken every five seconds.

The use of a storage battery as represented in Figs. 125, 126 and 127 depends upon the employment of controlling devices as described in the following articles.

112. The use of a storage battery for supplying the output of a station during the hours of small demand.—When a storage battery is used for this purpose it is nearly always desired to deliver current at constant voltage, and some device for controlling the voltage of the battery is necessary as explained below. A sufficient number of storage cells is used to give the required voltage when the battery is discharged and has 1.8 volts per cell, and the controlling device is arranged to take up the excess voltage when the battery voltage is higher than the desired value.

Control of voltage by rheostat.—The current, I, delivered by the battery flows through a resistance, R, Fig. 128, and this resistance is adjusted so that the excess of battery voltage may be used up as the voltage drop RI in this resistance. When the station output is constant this method of control is fairly satisfactory, for, in this case, the resistance has to be adjusted only occasionally as the battery voltage falls off. When the station output fluctuates, however, the rheostat, R, Fig. 128, requires constant attention because the voltage drop RI in the rheostat changes when the station output changes.

Control of voltage by counter-electromotive-force cells.—When current flows through a low-resistance electrolytic cell consisting of plain lead plates in dilute sulphuric acid, the voltage drop through the cell varies from about 2.3 to 2.5 volts according to the value of the current. The excess voltage of a discharging storage battery may be taken up by causing the current to flow through a number of such cells connected in series, the number being reduced as the battery voltage decreases. The advantage of this arrangement is that the voltage which is lost in these controlling cells does not vary greatly with the current. This method of voltage control is seldom used in practice. It has no advantage over the rheostat method when the load is constant, and the end-cell method is usually preferred when the load is variable.

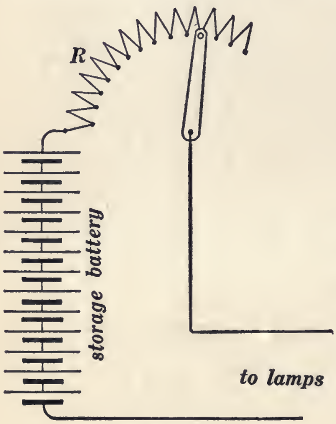


Fig. 128.

Control of voltage by end-cells.—This method of control will be explained by giving an actual example of a battery delivering current at 110 volts. The lowest permissible voltage at the end of the discharge is usually taken to be 1.8 volts per cell. Therefore the number of cells required to give a minimum of 110 volts is 110 ÷ 1.8, which is equal to 61. The highest voltage is about 2.15 volts per cell at the very beginning of the discharge (see Fig. 123), and 51 cells are therefore required at the very beginning of the discharge to give 110 volts. Therefore, the entire battery being fully charged, 51 cells are used at the beginning of the discharge, and as the voltage of the battery falls off the number of cells is increased, by connecting-in additional cells at one end of the set, until, when the battery reaches the limit of discharge, all of the 61 cells are in service. Under these conditions it is evident that the end-cells, which are in service only a portion of the time during the delivery of current by the battery, are not completely discharged Therefore, when the battery is recharged, the end-cells are placed in circuit at the start and cut out one by one as they become fully charged, as indicated, for example, by the copious evolution of gas.

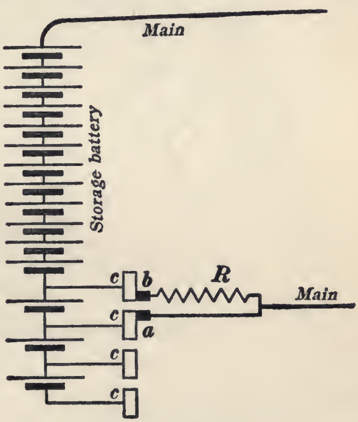


Fig. 129.

An important detail in the carrying out of the end-cell method of voltage control is the design of the switch for connecting and disconnecting the end-cells without interrupting the delivery of current, and without momentarily short-circuiting the individual cells. The essential features of this *end-cell switch** are shown in Fig. 129. The terminals of the end-cells are brought out to a series of contact blocks, *cccc*, which are

* Various types of end-cell switches are described on pages 285–324 of Lyndon's *Storage Battery Engineering.*

rather widely separated from each other. The movable contact arm of the connecting and disconnecting device has two fingers, *a* and *b*, which are far enough apart to bridge across between two of the blocks *cc* as shown in the figure. When the contact arm is moved it stands for a moment in the position shown in the figure and short-circuits one of the cells of the battery, but this short-circuit takes place through the resistance *R* so that no damage is done. The contact arm is arranged to move quickly past the position shown in the figure and stand with both fingers *a* and *b* in contact with one of the blocks *c*.

113. The booster.—Consider, for example, the use of a storage battery for supplying the output of a station during the hours of small demand, the voltage of the station being, say, 110 volts. A battery of 61 cells would be required as explained above, and to charge such a battery a voltage of about 150 volts would be

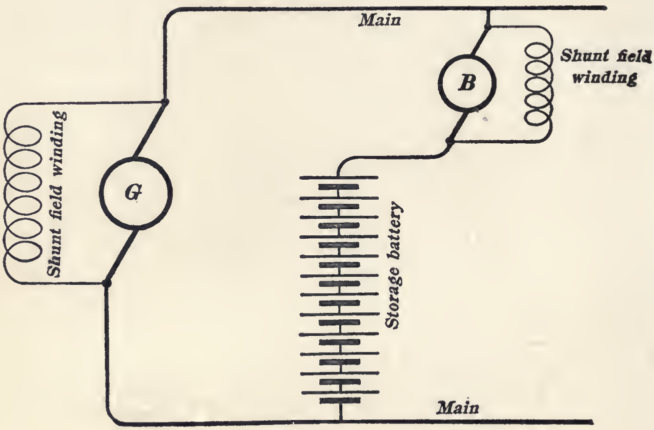


Fig. 130.

required towards the end of the charge (2.45 volts per cell). Now the battery is usually charged from the main generator of the station while the generator is supplying regular station output at 110 volts, and therefore the generator voltage is not sufficient to charge the battery. In practice a small auxiliary generator *B*, Fig. 130, is connected in series with the storage battery and this auxiliary generator helps to force the charging current through the battery. The auxiliary generator B is called a *booster.*

114. Automatic boosters.*—When the station load changes slowly, as is usually the case in an electric lighting station, there is ample time for an attendant to connect up a booster and charge a storage battery when the station load is small, and to disconnect the booster and make the necessary arrangements for discharging the battery when the peak of the load comes on. When, however, the station load fluctuates rapidly and irregularly as is usually the case in an electric railway power station, hand control of the storage battery is impossible. In such cases an automatic booster must be used.

The differential booster.—Fig. 131 shows an arrangement, due to Mailloux, in which a booster, B, is actuated by variations

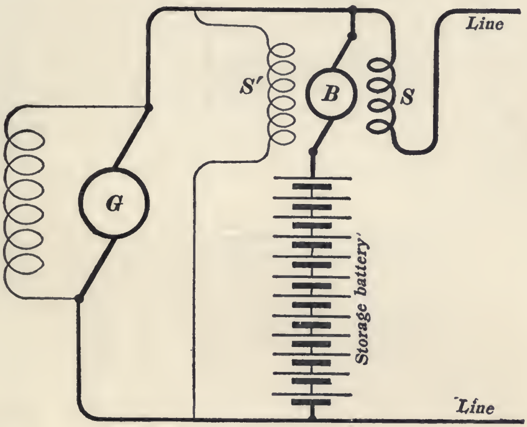


Fig. 131.

of line current. The booster has two opposing field windings, S and S'. When the demand for current is at its average value the windings, S and S', balance each other, the small generator, B, develops no electromotive force, and the battery neither charges nor discharges. When the line current is excessive the

* A very full discussion of boosters and booster systems is given on pages 325–470 of Lyndon's *Storage Battery Engineering.*

winding, *S*, predominates, and the voltage of *B* helps the battery to discharge; when, however, the line current is small the winding, *S′*, predominates and the reversed voltage of *B* helps the line voltage to charge the battery.

Booster with automatic carbon rheostat control.—Fig. 132 shows a carbon rheostat, *RR′*, connected across the terminals of the storage battery; and the field winding, *F*, of the booster, *B*, is connected from the middle of the rheostat to the middle of the battery. The solenoid *S* pulls on an iron plunger which is attached to one end of the lever *ll* and two lugs on this lever push on two piles of carbon plates *R* and *R′* which constitute the rheostat. A large line current gives a strong pull of the solenoid which compresses the pile *R* of carbon plates and greatly

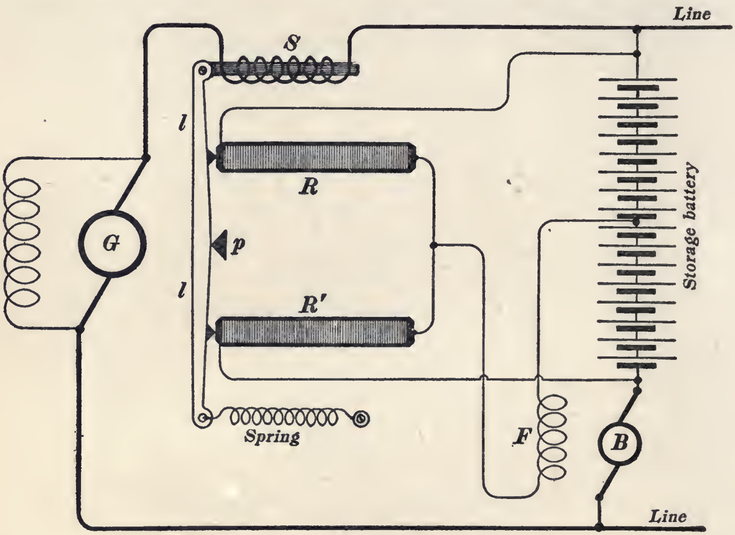


Fig. 132.

reduces its resistance. A small line current reduces the pull of the solenoid and the pull of the spring compresses the pile of *R′* of carbon plates thus greatly reducing its resistance. The current in the field winding *F* of the booster is zero when the resistances of *R* and *R′* are equal, and the current through the field winding is in one direction when R is greater than R' and in the opposite direction when R is less than R'. In this manner the field of the booster is so excited as to cause the booster to help the storage battery discharge when the station output is large and to help the station voltage to charge the battery when the station output is small.

The ordinates of the extremely irregular curve in Fig. 127 represent the fluctuating demand for current on a railway power station during a period of ten minutes. Without a storage battery the generators would have to meet this extremely irregular demand, varying from a minimum of about 180 amperes to a maximum of about 850 amperes. The ordinates of the slightly undulating dotted curve show the values of generator output when an adequate storage battery is installed and controlled by a booster as shown in Fig. 132 with the field excitation of the booster under the control of a carbon rheostat. When the total load curve is above the dotted curve (generator output) the battery discharges, and when the total load curve is below the dotted curve the battery charges.

The general average of the generator load must be slightly greater than the general average of the station output because some energy is lost in the battery, but the average generator load during a short period may be greater or much less than the average station output during that period. Thus the average station output during the ten minute period, which is shown in Fig. 127, was evidently greater than the average generator load during that period so that the battery was on the whole discharging.*

115. The floating battery.—The simplest arrangement for causing a storage battery to operate automatically and tend to equalize a station load, is that which is frequently employed in connection with long feeders over which a considerable drop of voltage takes place when a large current is delivered. This

* A good example of a large storage battery installation is described by Franklin E. Moore in the *Street Railway Journal* for September 21, 1911.

arrangement is shown in Fig. 133, in which *G* is the main generator and *B* is the storage battery. Any great demand for

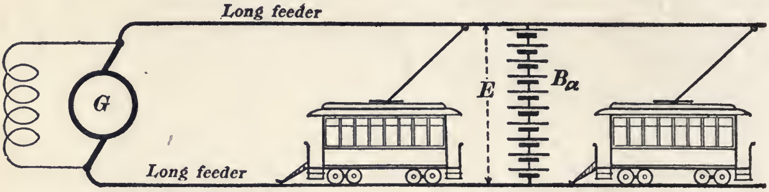


Fig. 133.

current causes the voltage, *E*, to decrease, so that the battery can discharge, and when the demand for current is small the voltage, *E*, rises and the battery is charged. A battery connected as shown in Fig. 133 is called a *floating battery*. Such a floating battery cannot completely equalize the demand on the station, inasmuch as the rise and fall of the voltage, *E*, depends upon some decrease and increase of the current flowing through the long feeders.

116. The negative booster.—An arrangement which produces an effect which is exactly equivalent to Fig. 133 is shown in Fig. 134. A generator *G* supplies current at constant voltage and

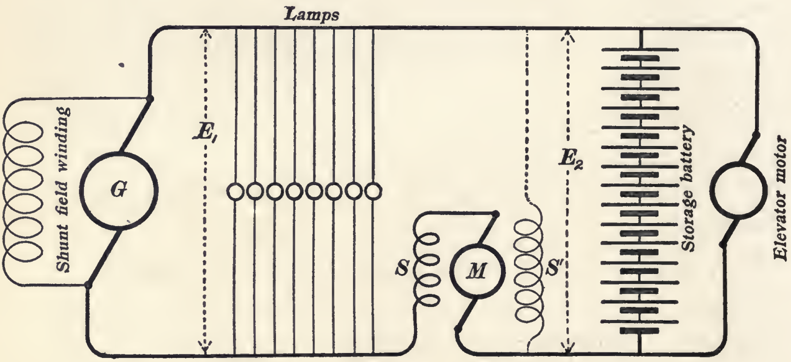


Fig. 134.

an elevator motor takes current from the constant voltage mains through a series motor *SM*. This series motor is belted to the main generator G so as to run at constant speed, and so that the power generated by SM may be belted back to the main generator. When the elevator motor takes large current the field of the series motor is strongly excited and its counter electromotive force is large so that the voltage E_2 is much less than E_1. Under these conditions the storage battery discharges. When the elevator motor takes small current the field magnet of the series motor is only weakly excited and its counter electromotive force is small so that E_2 is nearly as large as E_1 and the storage battery charges.

The series motor SM in Fig. 134 is called a *negative booster*, and the loss of voltage in the series motor due to its counter electromotive force is exactly analogous to the drop of voltage in the long feeders in Fig. 133. That is to say the battery in Fig. 134 acts exactly like the floating battery in Fig. 133.

CHAPTER IX.

MISCELLANEOUS APPLICATIONS.

117. The Morse telegraph.*—An insulated wire leads from one station to another and back again (the earth is generally used instead of a return wire). An electric current from a battery is sent intermittently through this circuit by operating at one station a key which makes and breaks the circuit. This current excites an electromagnet at the other station, and the armature of this electromagnet makes a record on a moving strip of paper or produces sound signals which are interpreted by the operator at the other station.

Relays and sounders.—A fairly strong electric current is required to operate the instrument which produces the signals at the receiving station, and it is not desirable to send so strong a current over a long line because of the great number of voltaic cells that would be required. This difficulty is obviated by the use of a *relay*. A small current flows over the line and through many turns of fine wire wound upon an electromagnet (of the relay) at the receiving station. This magnet actuates a very light lever, and this lever is arranged to open and close what is called a *local circuit* as it moves back and forth between stops. The local circuit which is opened and closed by the light lever of the relay contains a battery which supplies a moderately large current for the operation of the instrument which produces the sound signals. This instrument is called a *sounder*. It consists

* A good treatise on telegraphy is *American Telegraphy* by William Maver, Jr., New York, 1892.

It is not practicable to operate a very long telegraph line as one circuit for reasons which are explained in the article on submarine telegraphy. Long circuits are, therefore, broken up into sections. In the early days messages were repeated from one section to another by hand, but an automatic device called a *repeater* is now used.

of an electromagnet which is wound with moderately coarse wire and which actuates a massive lever, and the lever produces sharp clicks as it moves back and forth between stops.

Way stations.—An ordinary telegraph may include relays placed at points along the line, and a make and break key may be operated at any point along the line if all the other keys are

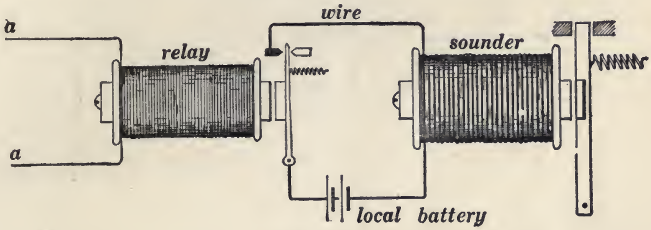


Fig. 135.

closed. When any key is thus operated all of the instruments on the line respond simultaneously. The simple railway telegraph is usually arranged in this manner. It is not unusual to have a single circuit one hundred and fifty miles or more in length containing fifteen or twenty way stations.

The arrangement of relay and sounder is shown in Fig. 135, *aa* being the wires which connect the fine-wire winding on the relay in

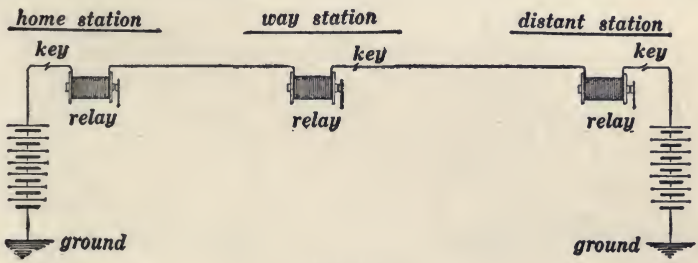


Fig. 136.

circuit with the telegraph line. Figure 136 shows a simple telegraph circuit with two end stations and one way station. The local circuits and sounders (one at each station) are omitted from this diagram for the sake of clearness. Of course all of the keys are normally closed, and when a telegram is to be sent from any station the key at that station is manipulated so as to make and break the circuit of the line.

118. Duplex telegraphy.—The sending of two messages (in opposite directions) over one line wire simultaneously is called

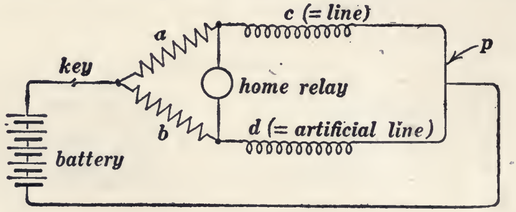


Fig. 137.

duplex telegraphy. There are two systems of duplex telegraphy, namely the *bridge duplex*, and the *differential duplex*. Way stations cannot be used in the duplex system.

Figure 137 shows the principle of the bridge duplex. The four resistances *a*, *b*, *c*, and *d* constitute a Wheatstone bridge arrangement, and no portion of the battery current flows through the home relay when the home key is closed. The resistance *c* represents the line and the apparatus at the distant station (which is interpolated at *p*). The actual arrangement of the bridge

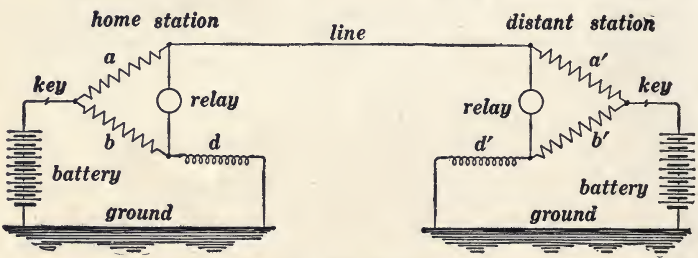


Fig. 138. Bridge duplex.

duplex is shown in Fig. 138. In Fig. 138 the distant relay responds to the home key and the home relay responds to the distant key. The local circuits and sounders are omitted for the sake of clearness.

The differential duplex makes use of the *differentially wound relay* or the *differential relay*, and the principle of the differential duplex is shown in Fig. 139. The battery current divides equally

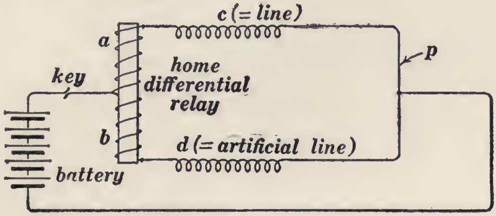


Fig. 139.

between the two similar branches *c* and *d*, and the two equal parts of the battery current circulate in opposite directions in the two windings *a* and *b* of the differential relay so that the iron core of the differential relay is not magnetized when the key is closed in Fig. 139. The resistance *c* represents the line and the apparatus at the distant station (which is interpolated at *p*). The actual arrangement of the differential duplex is shown in Fig. 140. In Fig. 140 the distant relay responds to home key and the home relay responds to the distant key. The local circuits and sounders are omitted for the sake of clearness.

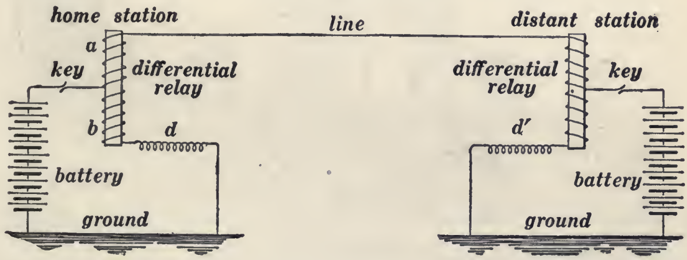


Fig. 140. Differential duplex.

The bridge duplex prevails in Europe and especially in England, and the differential duplex is almost universal in the United States. The bridge duplex requires the use of a larger battery than the differential duplex.

The artificial line.—An ordinary telegraph line has not only resistance but also a certain amount of inductance, and there is also a certain electrostatic capacity between the line and the ground. The artificial line *d* in Figs. 138 and 140 must duplicate the properties of the real line in every respect in order to completely eliminate the effect of the home battery on the home relay in Figs. 138 and 140. This artificial line is arranged as shown in Fig. 141 in which *r* and *r′* are adjustable resistances, *C* is a condenser and *L* is an inductance. This arrangement does not enable a very long line to be exactly matched.*

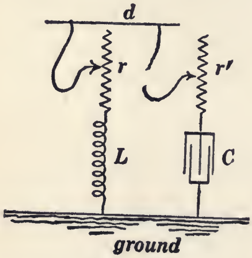


Fig. 141.

119. Diplex telegraphy.—The sending of two messages (in the same direction) over one line wire simultaneously is called *diplex telegraphy*. Diplex telegraphy depends upon the use of

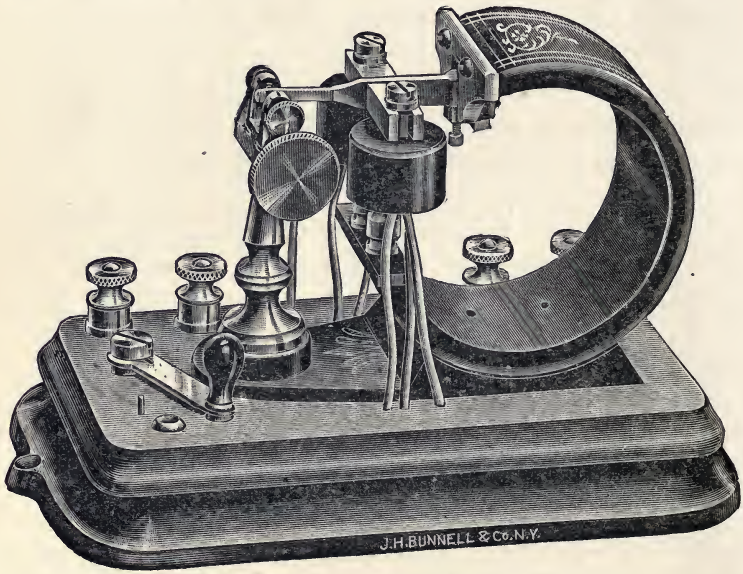


Fig. 142. Polarized relay.

* Artificial duplicates of very long lines and submarine cables are described in Maver's *American Telegraphy*, pages 276 to 280.

two kinds of relays, namely, (*a*) An *ordinary relay* with a fairly stiff spring so that the lever of the relay responds to an increase and decrease of current, the current being never reduced to zero; and (*b*) The so-called *polarized relay*, of which the lever responds to reversals of current. The ordinary relay is usually called the *neutral relay* to distinguish it from the polarized relay.

The polarized relay.—A general view of a polarized relay is shown in Fig. 142, and the essential features of the relay are

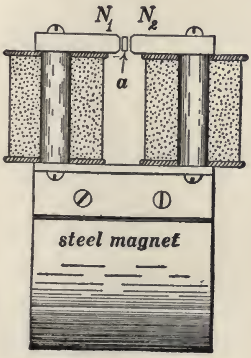


Fig. 143.

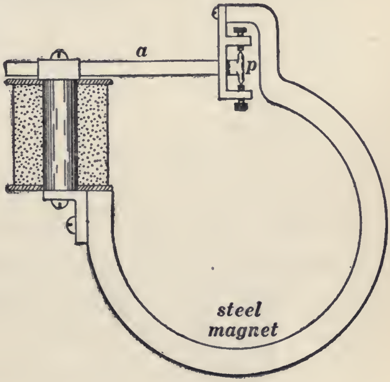


Fig. 144.

shown in Figs. 143 and 144. An ordinary electromagnet (with soft iron cores) is mounted on one pole of a U-shaped permanent magnet of steel, and a light iron lever, a pivoted at p, plays between the two poles N_1 and N_2 of the electromagnet.

When current flows in a certain direction through the coils of the electromagnet one of the poles, say N_1, is greatly strengthened and attracts the lever a. When the current is reversed the other pole N_2 is strengthened and attracts the lever a. Thus the lever a is pulled towards N_1 or N_2 according to the direction of the current, and therefore the lever a may be made to open and close a local circuit in response to reversals of the line current which flows through the windings on N_1 and N_2.

Diplex telegraphy.—Figure 145 shows the essential features of the arrangements for diplex telegraphy. The action is evident; the polarized relay responds to the reversing key *R*, and the neutral relay responds to the increase-and-decrease key, *I*. An

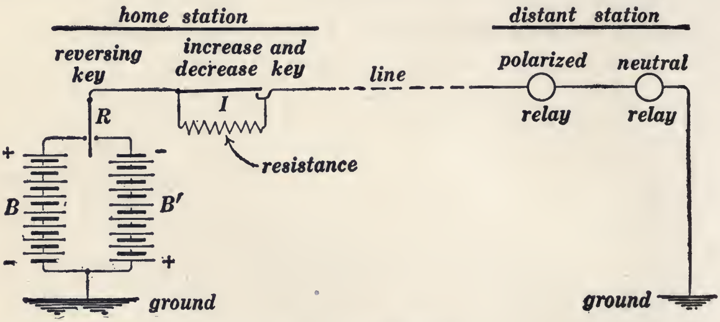


Fig. 145. Diplex telegraph.

important thing which is not shown in the figure is that the reversing key *R* must be arranged so that it is impossible for the operator to hold its lever midway between the contact points, because the reversal of current must take place as quickly as possible so that the lever of the neutral relay may not have time to respond. The lever of the reversing key is therefore usually actuated by an electromagnet, and the electromagnet is controlled by a hand-operated key which opens and closes the local circuit of the electromagnet. Also it is an advantage to connect condensers as shown in Fig. 146 so as to eliminate sparking, and to ensure the quickest possible reversal of current.*

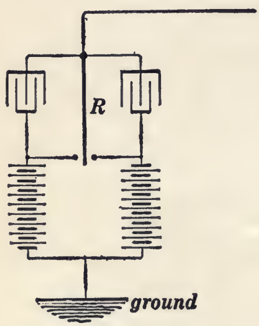


Fig. 146.

A neutral relay and a polarized relay might be placed in circuit with a diplex line at a way station so

* The action of a condenser in causing a quick reversal of current is explained in Franklin and MacNutt's *Electricity and Magnetism* (The Macmillan Co.); see index.

that the way station and the distant end station could both receive messages from the sending station. This arrangement, however, is never used. Indeed diplex telegraphy is used only in conjunction with duplex telegraphy to give quadruplex telegraphy as explained in the next article.

120. Quadruplex telegraphy.—The sending of four messages (two messages each way) over one line wire simultaneously is

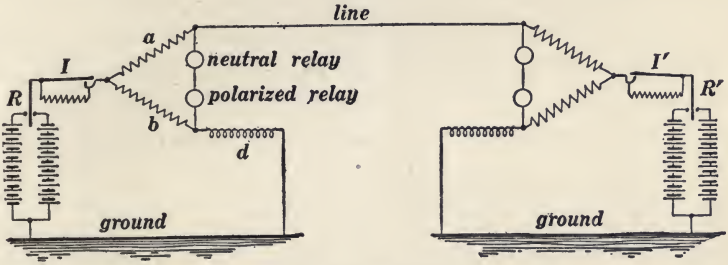


Fig. 147. Bridge Quadruplex.

called *quadruplex telegraphy*. This is accomplished by combining the arrangements for duplex (either bridge duplex or differential duplex) and diplex telegraphy. Thus a key arrangement like that shown in Fig. 145 may be installed at each station in Fig.

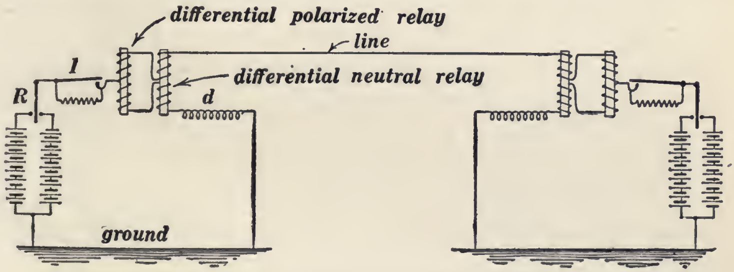


Fig. 148. Differential Quadruplex.

138, and the single relay at each station in Fig. 138 may be replaced by two relays, a neutral relay and a polarized relay, as shown in Fig. 147. The local circuits and sounders are omitted for the sake of clearness.

Figure 148 shows the combination of the diplex and the differential duplex. The differential duplex has an advantage over the bridge duplex in that the resistances a and b in Figs. 138 and 147 are not necessary in the differential system, and therefore it is possible to operate the differential system with less battery power.

In Figs. 147 and 148 the neutral relay at each station responds to the I key at the other station, and the polarized relay at each station responds to the R key at the other station.

The quadruplex system is very extensively used where the line wires are not used also for telephones as explained in Arts. 129–135.

Way stations cannot be served in the quadruplex system.

121. The printing telegraph* is an arrangement by means of which a simple form of typewriter is operated at a distant station from a key board at the sending station. The simplest form of printing telegraph is the well known ticker which prints in one line on a long strip of paper. The action of the ticker is as follows: Twenty-six equidistant pins are arranged in a helical row around a long metal cylinder. This cylinder is rotated by a small electric motor or by clock work, and above the cylinder is a bank of twenty-six lettered keys so arranged that when a key is depressed one of the pins comes against it and the cylinder is stopped in a certain position; the next key would stop the cylinder 1/26 of a revolution farther on, and so on. Attached to the rotating cylinder is a device for reversing an electric current

* The ticker as now generally used in American cities is somewhat different from the device here described. See Maver's *American Telegraphy*, pages 395–420.

When a person is thoroughly familiar with the elements which enter into the construction of a machine, that is, when a person is familiar with shafts and wheels and with simple devices like switches for opening and closing circuits and for reversing connections, a more easily intelligible description of a complicated machine can be made without illustrative diagrams and drawings than can be made with the help of diagrams and drawings. Indeed it would be confusing under the specified conditions to have recourse, even, to a working model of a complicated machine when the object in view is to impart a clear idea of its fundamental features. The only element of a ticker which may not be familiar to the student is the escapement device, the oscillations of which turn a toothed wheel notch by notch.

fifty-two times for each revolution of the cylinder. This repeatedly reversed electric current passes over the telegraph line and through two electromagnets at the receiving station. One of these electromagnets is like a neutral relay with a heavy lever, and the other is like a polarized relay with a light lever which oscillates with the rapid reversals of current and actuates an escapement which turns a type wheel with the twenty-six letters arranged around its periphery. This type wheel is thus turned step by step, keeping pace with the rotating cylinder at the sending station. When the cylinder at the sending station is stopped by depressing a key, the A-key for example, the current-reversing device stops also, a steady current flows through the line, the lever of the polarized relay stops oscillating, the type wheel stops, and the steady current excites the neutral relay, the lever of which pushes a strip of paper against the type wheel and prints the letter A. When the key at the sending station is raised the current reversals begin again, the type wheel at the receiving station starts, and at the same time the lever of the neutral relay falls back and actuates a device which moves the strip of paper a step forward for the printing of the next letter.

122. Submarine telegraphy.—Figure 149 shows a full size sectional view of a submarine telegraph cable. The conductor at the center consists of a number of strands of copper wire. Surrounding this is a layer of gutta percha, and the whole is protected by a covering of tarred hemp and steel wire.

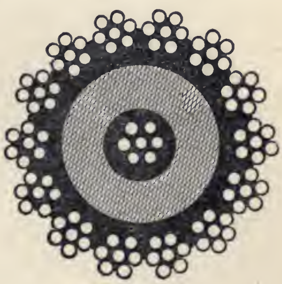


Fig. 149.

The conductor and metal sheath of the cable, together with the intervening insulating material, constitute a condenser of large electrostatic capacity. The effect* of this large electrostatic capacity is as follows: At the instant a battery is connected to a

* The effect which is here described is exaggerated by the action of the inductance of the cable.

cable a very large current begins to flow into the cable. Most of this current goes to charge the cable, and, as the cable becomes charged, the entering current falls off in value, settling finally to a steady value which is determined by the resistance of the copper wires of the cable. The ordinates of curve A, Fig. 150, show the successive values of current which enters a cable from a battery, the abscissas being time reckoned from the instant the battery is connected.

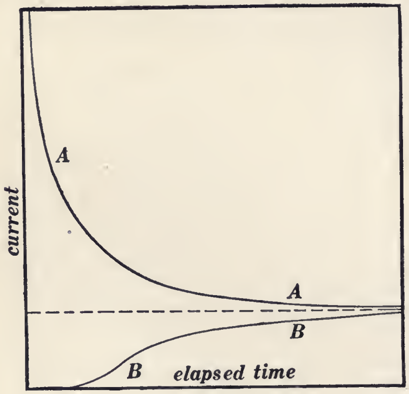


Fig. 150.

At the distant end of the cable an infinitesimal current begins to flow out of the cable almost at the instant the battery is connected to the cable at the sending station, and as the cable becomes charged this outflowing current rises in value until it reaches a steady value very nearly equal to the steady value of the entering current. The curve B, Fig. 150, shows the growth of current flowing out of the distant end of a cable after a battery is connected to the near end. When the battery is disconnected, the entering current ceases at once, but the outflowing current at the distant end of the cable drops slowly to zero as the accumulated charge flows out of the cable.

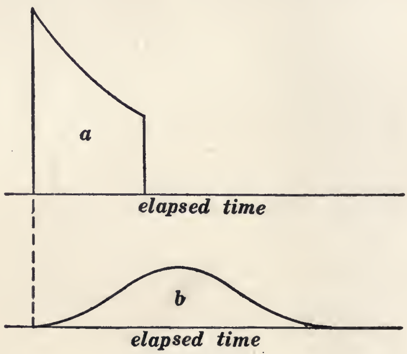


Fig. 151.

Distortion of current pulses by a cable.—The curve a, Fig. 151, shows the character of the current pulse which enters a cable when a battery is momentarily connected to the cable, and the curve, *b*, shows the character of the current pulse which flows out of the distant end of the cable. The action of a cable in thus altering the character of a current pulse is called *distortion.* Land lines distort current pulses to some extent, and it is for this reason that a very long telegraph line cannot be satisfactorily operated as a single circuit. Distortion very seriously impairs the distinctness of telephonic transmission in land lines four or five hundred miles long or more.*

The distortion of electric current pulses by a submarine cable is analogous to the distortion of water-current pulses by a long thin-walled rubber tube. If water is forced into one end of such a tube in sharply defined pulses, the water will flow out of the other end of the tube in one long continued pulse, and a succession of separate pulses of inflowing water would show themselves as slight variations of outflowing current.

The curves *aaaa*, Fig. 152, represent four short current pulses sent into a cable at one end, and the curve *b* represents the pulse of current which flows out of the cable at the other end. The four successive pulses of inflowing current show themselves as four slight humps on the curve *b* of outflowing current, and it is evident that these four successive pulses of inflowing current could not be detected by means of an ordinary relay and sounder at the distant end of the cable. The receiving instrument in submarine telegraphy is a galvanometer arranged to trace the curve of outflowing current at the receiving end of the cable, and the separate current pulses that are sent into the cable at the sending

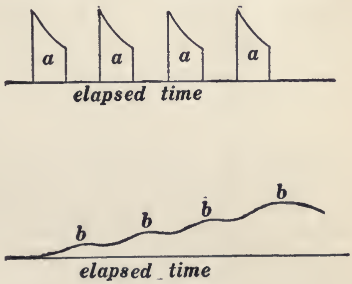


Fig. 152.

* Wave distortion is very fully and simply discussed on pages 29, 30, 108–115 of Franklin's *Electric Waves.*

end are inferred from the slight humps in the curve which is traced by the receiving instrument.

123. The syphon recorder.*—The receiving instrument commonly used in submarine telegraphy is called the *syphon recorder*. The current flowing out of the distant end of a cable passes through a D'Arsonval galvanometer, the moving coil of which produces sidewise motion of a pen which traces an ink line on a moving strip of paper; the pen thus traces a current curve like *b*, Fig. 152.

124. The telephone.—The telephone set includes a transmitter, a receiver, and an arrangement for calling. The *transmitter* is a device for producing over the line a current which is reversed with each to and fro movement of a diaphragm, the diaphragm being set into vibration by a speaker's voice; and the *receiver* is a device in which a diaphragm is set into vibration by these rapidly reversed currents (which come to it over the line from the transmitter) thus reproducing the original sound.

The transmitter.—A sectional view of a telephone transmitter is shown in Fig. 153, and the connections are shown in Fig. 155. An electric circuit contains a battery, the primary of a small transformer (induction coil), and a mass of granular carbon

* The syphon recorder was devised by Lord Kelvin, who contributed more, perhaps, to the development of transatlantic telegraphy than any other man. In an article by Professor W. E. Ayrton, which appeared in the London *Times* shortly after Lord Kelvin's death (reprinted in *Popular Science Monthly* for March, 1908), much interesting information is given concerning what Kelvin did for submarine telegraphy. "When signals through the 1858 Atlantic cable became weak, and a message from the President to our Queen took thirty hours in transmission although containing only 150 words, and which would need only three or four minutes to transmit through any one of our good Atlantic cables of to-day, the only remedy of those who looked down upon the theories of the young Glasglow professor was to use Whitehouse's "thunder pump," a magneto-electric machine which produced a sudden large electromotive force when the armature of a permanent magnet was jerked off the poles of the magnet. But these shocks only sent sparks through the gutta-percha insulating coating and hurried the poor cable to its doom, so that even the three words per minute which would have been the utmost limit of speed possible had this cable been entirely uninjured, were replaced by absolute silence."

between corrugated carbon blocks, all in series. The black patches in Fig. 153 represent the carbon blocks, one of which is supported rigidly, and the other of which is attached to the diaphragm *DD*. The speaker's voice causes the diaphragm to vibrate and the resistance of the granular carbon increases and decreases as the diaphragm moves to and fro. This variation of resistance causes the battery current to increase and decrease, and this increase and decrease of battery current in the primary of the small transformer produces in the secondary a current which flows in one direction and the other alternately as the diaphragm moves to and fro.

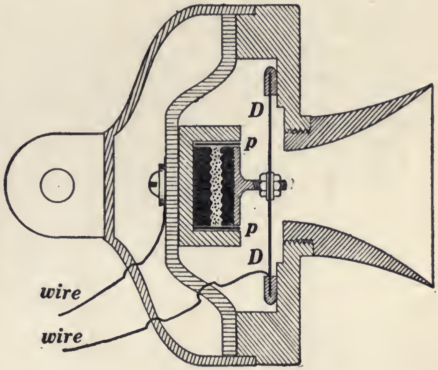


Fig. 153.

The receiver.—The simplest type of telephone receiver is shown in Fig. 154. A coil of very fine wire, *C*, is wound around one end of a permanent steel magnet *MM*, and the reversals of current from the distant transmitter in flowing through this coil strengthen and weaken the steel magnet alternately, and the thin iron diaphragm *dd* is moved to and fro by the variations of strength of the steel magnet, thus reproducing the original sound. The most approved form of telephone receiver has a bi-polar magnet.

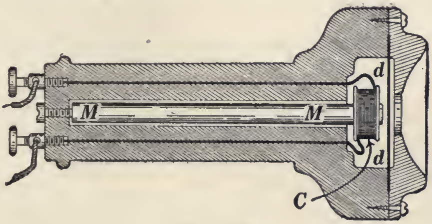


Fig. 154.

Two telephone stations all connected up for talking are shown in Fig. 155. In order to give a call at the distant station a small magneto generator is used to operate a bell,* and the change from connections required to operate the bell to connections required for the operation of transmitter and receiver is made by

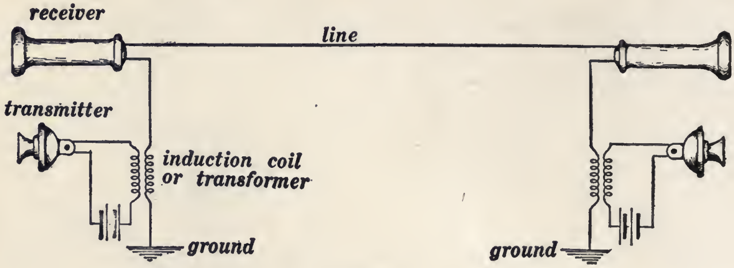


Fig. 155.

the movement of the hook when the telephone is taken from the hook. Figure 156 shows the hook down, and the connections, as indicated by the full lines (dotted lines are dead), are proper

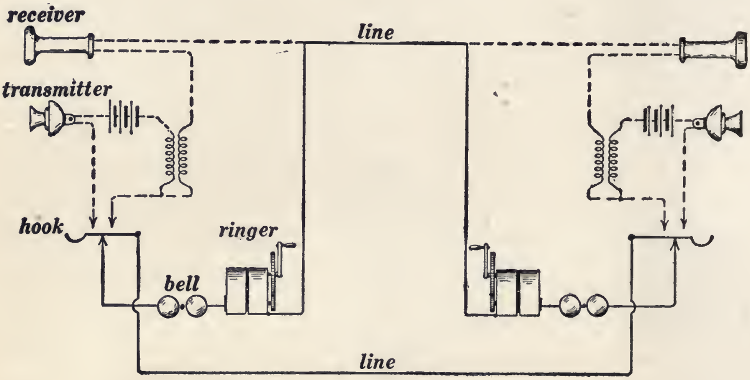


Fig. 156. Connections for ringing.

for operating the bell at the distant station. Figure 157 snows the hook up, and the connections are proper for operating the transmitters and receivers.

*Let it be understood that we are not discussing central exchange telephone systems, but simple two-station telephone lines such as are extensively used in railway work. The student is referred to Kempster B. Miller's *American Telephone Practice* for full information on telephone practice.

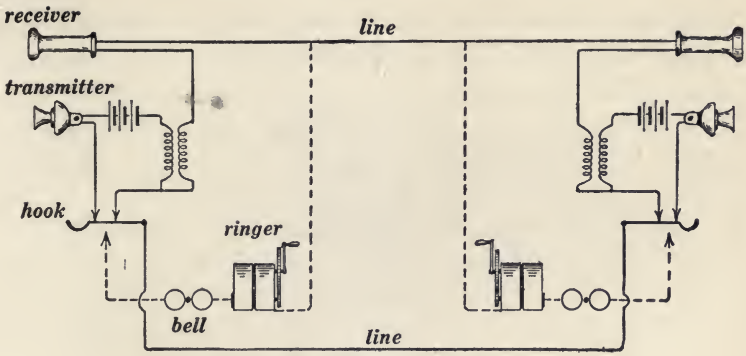


Fig. 157. Connections for talking.

125. The ground return versus the metallic circuit.—Many private and railway telephone lines use a single wire with ground return. Such lines are objectionable for two reasons, namely, (*a*) The atmospheric electricity gathered by such a line flows to ground through the telephone receivers and makes an almost incessant crackling sound which is very annoying; and (*b*) It is impossible in the case of such a line to eliminate the disturbances due to adjacent electric light and power lines. First class telephone service demands therefore a wire circuit (two wires), and such a telephone line is called a *metallic circuit.* The advantages of the metallic circuit are (*a*) That disturbances can be more completely eliminated when two wires are used, and (*b*) A two-wire line lends itself more readily than a single-wire line to combination uses as explained later in connection with simplex, composite and phantom circuits.

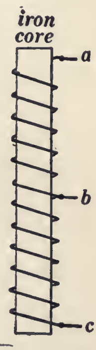


Fig. 158.

126. The use of divided choke-coils on metallic telephone circuits.—A divided choke-coil is a continuous winding of wire on an iron core with a lead wire brought out from the middle of the winding, as shown in Fig. 158. *There is great inductive opposition to the flow of alternating current through the entire coil, but equal alternating currents can enter at a and c and flow out at b without induc-*

tive opposition because the magnetizing action of one half of the winding is neutralized by the opposite magnetizing action of the other half of the winding.

Figure 159 shows a metallic telephone circuit (the telephone sets being like those shown in Figs. 156 and 157) with a divided

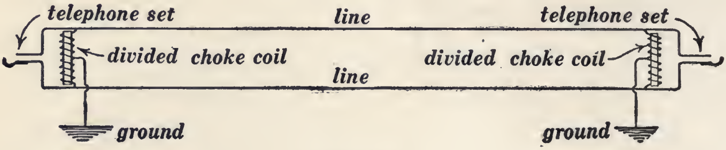


Fig. 159.

choke-coil connected between the line wires at each end of the line, the middle lead of each choke-coil being connected to earth. The high frequency telephone currents cannot flow through the choke-coils to any appreciable extent, *but any current (alternating or direct) which flows in the* **same direction** *in both line wires can flow without inductive opposition through the two halves of a choke-coil and to ground without affecting the telephones.*

Thus the atmospheric electricity which gathers equally on the two line wires has an easy path to earth without flowing through either telephone, and any current which is induced equally and in the same direction in the two line wires has an easy path to ground without flowing through either telephone.

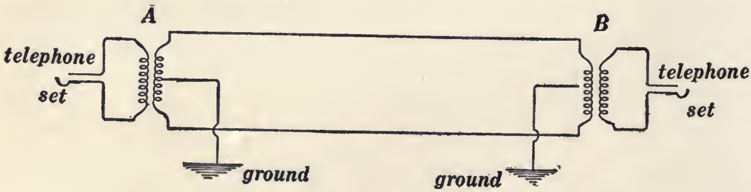


Fig. 160.

Figure 160 shows a metallic telephone circuit in which the high frequency telephone currents are delivered to the line (and from the line) by two small transformers *A* and *B*; and the transformer coils which are connected to the line have their middle points grounded. This arrangement is equivalent to the arrangement shown in Fig. 159. The small transformers A and B in Fig. 160 with the lead coming out of the middle of one of their coils are called *divided repeating coils*

127. The phantom telephone circuit.—Two metallic telephone circuits like Fig. 159 (or like Fig. 160) can be used as telephone circuits and at the same time a third telephone circuit can be established as indicated in Fig. 161, in which AA' is one set of telephones like Fig. 159, BB' is another set of telephones like

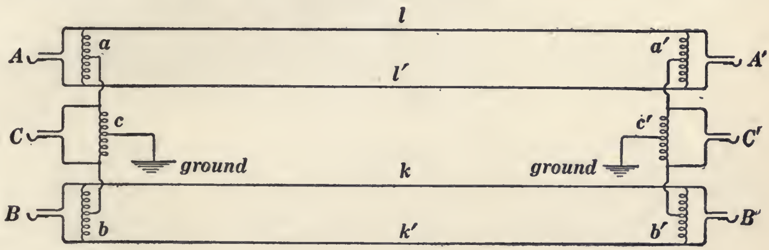


Fig. 161.

159, and CC' is a third set of telephones, all of which operate independently of each other. Telephone current (high frequency alternating) from C cannot flow across the choke-coil c but can enter the two wires l and l' through the two halves of choke-coil a, flow through the telephone set C' and return through the two wires k and k'. This circuit is called the *phantom circuit.*

128. The choke-coil and the condenser.—The simultaneous use of wires for Morse telegraph and for telephone depends in part upon the use of choke-coils and in part upon the use of condensers; and it is important to understand that high frequency alternating current cannot flow through a choke-coil but can flow freely through a condenser, whereas a very low frequency alternating current or a direct current can flow freely through a choke-coil but cannot flow through a condenser.

If the main rod in Fig. 162 oscillates back and forth at high frequency the heavy weight does not move perceptibly, but all of the motion of the main rod is accommodated by motion of the end *C* of the lever *CL*. If the main rod oscillates back and forth at very low frequency the end *C* of the lever does not move

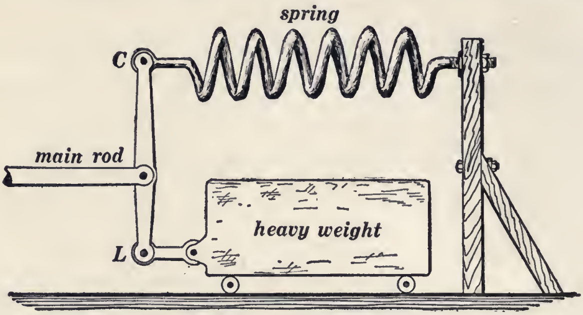


Fig. 162.

perceptibly, but all of the motion of the main rod is accommodated by motion of the end *L* of the lever. If any agent *A* causes the main rod to oscillate back and forth at high frequency and if another agent *B* causes the main rod to move back and forth at low frequency the two motions are added together so far as the main rod is concerned, that is the main rod performs both motions simultaneously, but the end *C* of the lever will move as if agent *A* were acting alone and the end *L* of the lever will move as if agent *B* were acting alone; indeed, end *C* of the lever will respond to agent *A* and end *L* of the lever will respond to agent *B*.

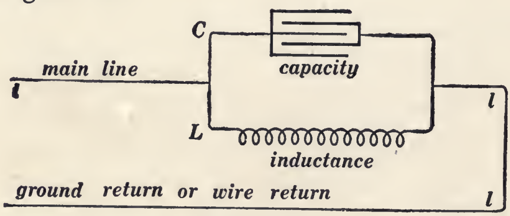


Fig. 163.

In action the mechanical arrangement in Fig. 162 is exactly analogous to the electrical arrangement in Fig. 163. If one agent *A* produces a high frequency alternating current through the circuit *llll* and if another agent *B* produces at the same time a low frequency alternating current through the circuit, then the high frequency current produced by *A* will flow through the condenser (capacity) and the low frequency current produced by *B* will flow through the inductance.

129. The railway composite.—The arrangement for using an ordinary ground-return telegraph line for telephone service at the same time that it is being used for telegraph service is called the *railway* or *one-wire composite.* Such an arrangement with a way station served both by telegraph and telephone is shown in Fig. 164.* The high frequency telephone currents and the low frequency telegraph currents flow together over the line and

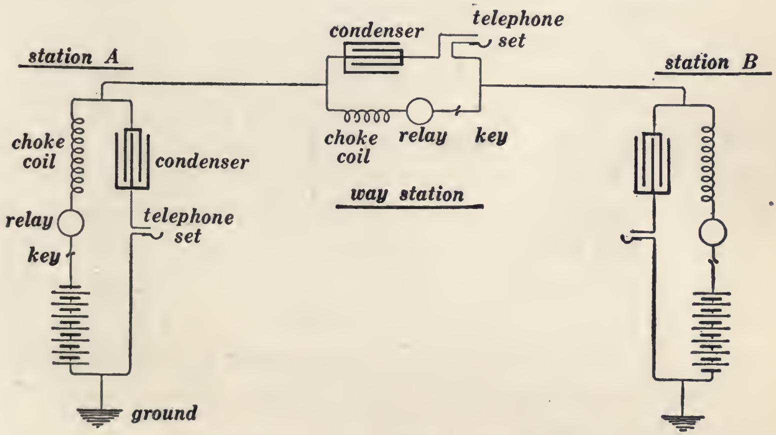


Fig. 164. The railway composite.

return through the ground, but the telephone currents, only, flow through the condensers and telephones, and the telegraph currents, only, flow through the choke-coils, relays and keys (which are of course all closed but one).

The arrangement shown in Fig. 164 is not very satisfactory because a number of telephone sets do not operate satisfactorily in series. It is better to connect the telephones all from line to

* The telephone calls are usually made by means of the telegraph. See Art. 137.

ground so that any transmitter supplies current to all of the receivers in parallel. This arrangement, which is shown in Fig. 165, allows all of the telegraph relays to be operated in series and at the same time it allows any telephone transmitter to deliver current at all of the telephone receivers in parallel.

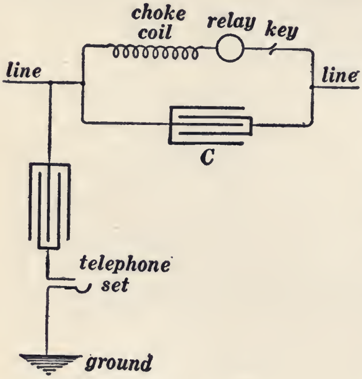


Fig. 165.

The excessively quick change of current due to the opening and closing of a telegraph key causes a momentary flow of current through the condensers and telephones so that the telegraph signals are audible in the telephones. This difficulty is to a great extent obviated by "bridging" a condenser across each key or across each key and relay taken together, as shown in Figs. 168 and 169. Each relay should also be shunted by a non-inductive high resistance so as to eliminate high frequency oscillations around the circuit which is formed at each station by the telephone and telegraph apparatus (at the way station by the telegraph apparatus and the condenser *C* in Fig. 165). Of course the relays are themselves choke-coils and in some cases therefore the additional choke-coils shown in Figs. 164 and 165 might be omitted.

130. The simplex circuit.—The railway composite is useful but it has all of the disadvantages of a ground-return telephone line (see Art. 125) and the telegraph signals cannot be entirely eliminated from the telephones. An arrangement in which these objectionable features are avoided is the simplex circuit which is shown in Fig. 166. The action of this arrangement may be understood with the help of the discussion of Arts. 126 and 127. The two wires of the metallic telephone circuit serve as one wire for a ground-return telegraph circuit, and, if there are no way stations, this telegraph circuit can be operated duplex or quadruplex by installing apparatus like Figs. 147 or 148 for the simple telegraph apparatus shown in Fig. 166.

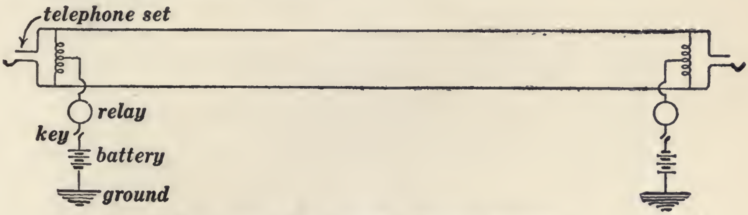


Fig. 166. The simplex telegraph circuit.

Figure 167 shows a simplex circuit with a telegraph set at a way station. The way station may also be served by a telephone, the telephone set* being connected across ("bridged" across) between the two wires on either side of the condensers in Fig. 167.

131. The simplex on phantom.—Telegraph apparatus can be inserted in the two ground leads in Fig. 161, thus giving three metallic telephone circuits and one ground-return telegraph circuit on four wires.

Duplex or quadruplex apparatus like Figs. 147 or 148 can be inserted in the ground leads in Fig. 161, thus giving three independent metallic telephone circuits and permitting the sending

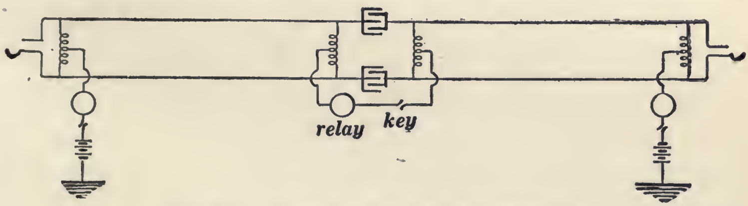


Fig. 167. Telegraph way station on simplex.

of four telegraph messages over the four wires of Fig. 161 all at the same time. The matter of duplex and quadruplex working on telephone lines is discussed briefly from a practical point of view in Art. 138.

* The magneto and bell are not shown in Figs. 166 and 167. See discussion of telephone calls in Art. 137.

132. Phantom on two simplex circuits.—A phantom telephone circuit may be superposed upon two simplex circuits like Fig. 166. This arrangement is shown in Fig. 168. Theoretically both of

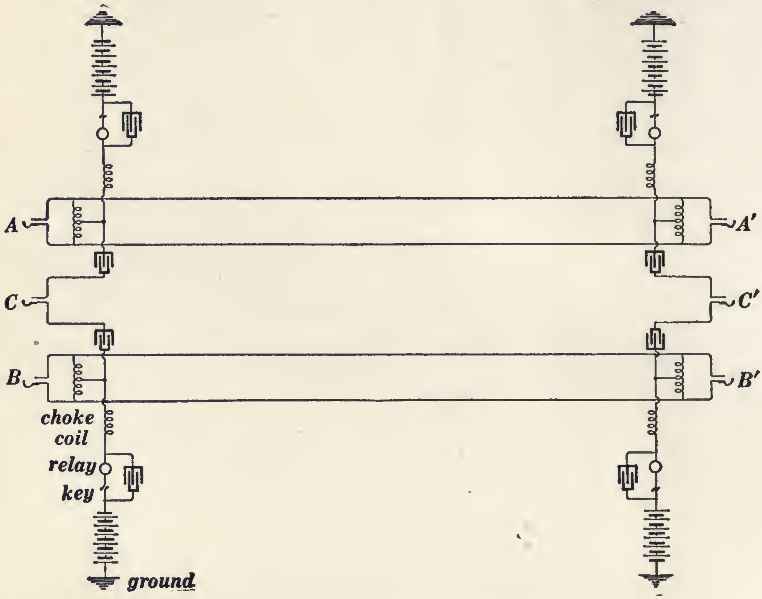


Fig. 168. Telephone phantom on two simplex circuits.

the telegraph circuits in Fig. 168 can be operated duplex or quadruplex. See Art. 138.

133. The two-wire composite circuit.—A metallic telephone circuit making use of two ordinary ground-return telegraph lines is called a *two-wire composite circuit.* Such an arrangement is shown in Fig. 169.

134. Phantom on two two-wire composites.—A phantom telephone circuit may be superposed upon two two-wire composite circuits as shown in Fig. 170.

135. Simplex blocks worked in series for telegraph.—A number of metallic telephone circuits such as are used for communicating between signal stations on a railway can be connected for use

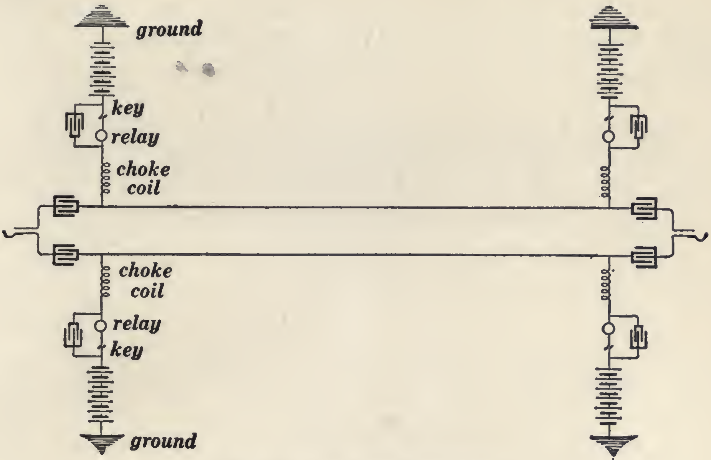


Fig. 169. Two-wire composite.

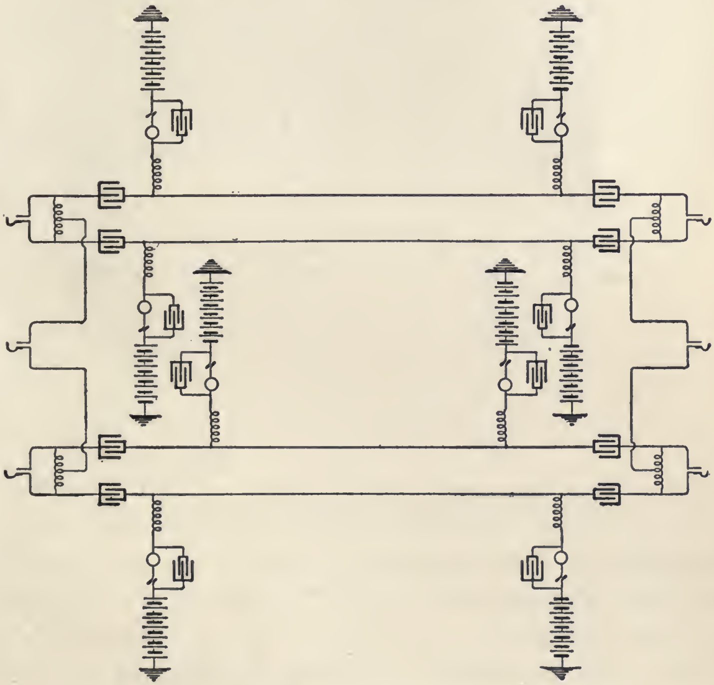


Fig. 170. Telephone phantom on two two-wire composites.

as a long telegraph line as shown in Fig. 171, and the telegraph circuit can be carried onwards as a single-wire line as shown in Fig. 172.

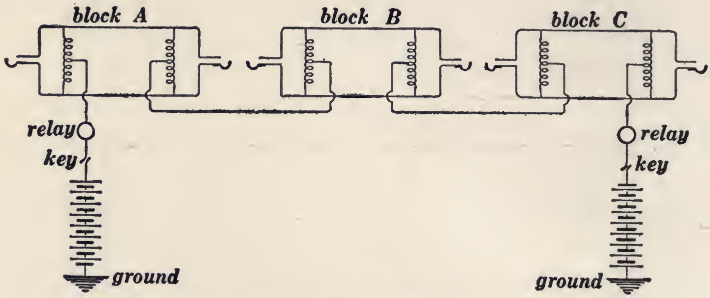


Fig. 171. Simplex blocks in series for telegraph.

136. Way stations in general.—In the simple Morse telegraph a number of way stations can be arranged, the relays and keys being all in series. Five or six way stations can also be arranged on a telephone line (metallic circuit or ground-return); in this case it is best to use telephone receivers and call bells wound with many turns of fine wire and to connect the telephone set at

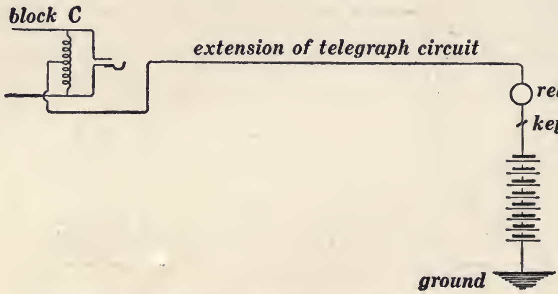


Fig. 172. Extension of telegraph circuit of Fig. 171.

each station between the two wires of the line or between the single line wire and the ground. That is, all of the telephone sets are in parallel.

Way stations cannot be arranged in duplex or quadruplex systems.

In simplex circuits and composite circuits telegraph way stations may be arranged by breaking the telegraph circuit* with a condenser and shunting the telegraph set and a choke-coil around the condenser.

In simplex circuits and composite circuits telephone way stations may be arranged either by shunting the telephone apparatus (with a condenser) around the telegraph apparatus (with a choke-coil), or by connecting the telephone apparatus (with a condenser) between the two wires or legs of the telephone circuit.

137. Telephone calls.—The ordinary magneto generator which is used for operating telephone call bells gives an alternating current of which the frequency is about 16 cycles per second, and a one or two microfarad condenser is a very great obstruction to the flow of such low-frequency alternating current. Therefore the ordinary magneto and bell cannot be used for making telephone calls on composite circuits such as are shown in Figs. 164, 169 and 170. Telephone calls are frequently made on composite circuits by a momentary use of the telegraph apparatus. Another arrangement is to use a small induction coil with a high-frequency interrupter for producing moderately high-frequency alternating current for calling. These high-frequency currents can flow through the condensers and operate a bell which is specially designed to be operated by high-frequency current, or the telephone receiver can be left permanently in circuit in which case the high-frequency current will produce a loud howling sound in the telephone receiver. This latter arrangement is called the *howler call*.

The divided choke-coils (or the divided repeating coils) which are used in the simplex circuit can be made large enough to obstruct the flow of the 16-cycle bell current without hindering the telegraph current and therefore the ordinary magneto and bell can be used on simplex circuits.

* If a pair of wires forms one side of the telegraph circuit both wires must be broken as stated. See Fig. 167.

138. Duplex and quadruplex on simplex and composite circuits.—There are two difficulties involved in the use of duplex and quadruplex telegraph apparatus or simplex and composite circuits, namely, (*a*) the complexity is such that the balanced condition between the "line" and the "artificial line" is difficult to maintain, even to a degree sufficient to keep the different telegraph signals separated, and (*b*) large voltages (sometimes 150 volts or more) and heavy currents are required, especially for quadruplex working, and the telegraph signals are usually heard in the telephones. *It is essential to keep the telegraph current as low as possible in simplex and composite circuits.* Therefore the quadruplex is practically abandoned on such circuits.

139. The automatic railway block signal.—When railway trains are run closely following each other it is quite necessary to install a system of signals to show to the engineer that he has a mile or two of clear track ahead, the signal system being so designed that the presence of a train ahead *or any chance derangement of the apparatus* will show a danger signal.

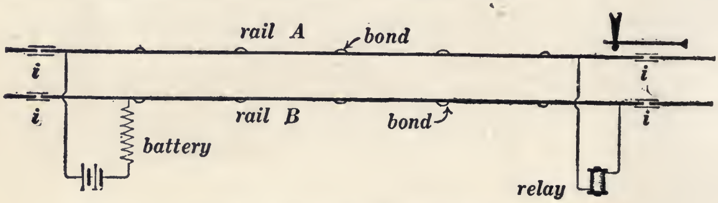


Fig. 173. Short railway block with single-arm semaphore.

The essential features of the block signal system are shown in Fig. 173. The rails of a block are separated from the rails of the adjoining blocks by insulating joints *iiii*, and the rails in the block are connected together by wire bonds so as to make each rail a continuous conductor. A battery with some resistance *r* in series with it* is connected between the rails at one end of the block, and the windings of a relay are connected between

* Gravity Daniell cells are generally used, and the internal resistance of such a battery is usually sufficient without the insertion of any additional resistance *r*.

the rails at the other end of the block. The battery current thus flows steadily through the relay, and the signal arms are held in the position which shows that the track is clear. When a train comes into the block the wheels and axles of the train make a short-circuit connection from rail to rail, and the voltage between the rails falls to zero so that no perceptible current flows through the relay. The relay lever is therefore released and allowed to fall back, a local circuit is opened and the signal arms move to the "danger" position. It is of the utmost importance that the danger signal be displayed when any derangement of the signal apparatus occurs; therefore the arms of the semaphore are weighted so as to fall to "danger" position when the local circuit is opened either by the track relay or by accident.

When the train moves out of the block, current again flows through the relay, the local circuit is closed and a small motor in the local circuit moves the signal arms to the "safe" position. Copper oxide cells are extensively used for the local circuits for operating the small motors for moving the signal arms.

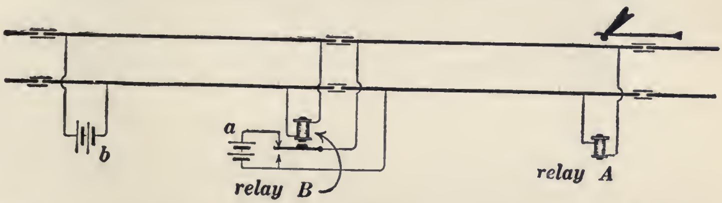


Fig. 174. Arrangement for working very long section of track as a single block.

There is always considerable leakage of current across from rail to rail in Fig. 173 through the ties and ballast, especially when the road bed is wet, and because of this leakage of current a relay connected as shown in Fig. 173 does not operate satisfactorily if the block is more than 6,000 or 7,000 feet in length. When the traffic on a railway is light it is desirable to have blocks two or three or four miles in length, and blocks of this length can be arranged as shown in Fig. 174. Relay A controls the motor which moves the signal arm, and relay B makes the two sections operate as one block. When both sections are clear, battery *b* energizes relay *B* and the lever of relay *B* connects battery *a* to the rails of section *A* so that relay *A* is energized and the signal arm is held at "safe" position. A train on either section causes the current to cease flowing through relay *A* and the signal arm falls to "danger."

140. The overlap track circuit.—A system of signals which is extensively used is as follows: A locomotive engineer brings a train up to the entrance to a new block, and if he sees both arms of a double semaphore at "safe" he knows the entire block is clear; if he sees one arm at "safe" and the other at "danger" he knows that a train is on the distant half of the block, and if

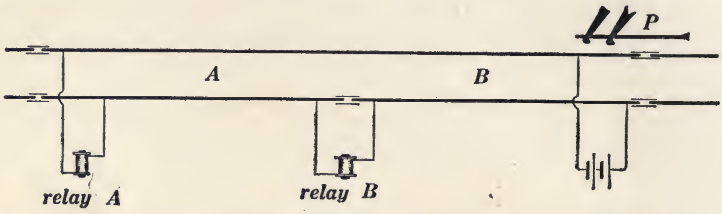


Fig. 175. Arrangement for operating double-arm semaphore.

he sees both arms at "danger" he knows that a train is on the adjacent half of the block. In the first case the engineer goes ahead at full speed, in the second case the engineer goes ahead cautiously at reduced speed, and in the third case the engineer stops the train until the signals show that the first half of the block is clear.

Figure 175 shows an arrangement for operating a double semaphore as above explained. When the train is on section *B* both relays are without current and both semaphore arms (at *P*) are at "danger." When the train moves into section *A*, relay *B* is energized and one semaphore arm (at *P*) is moved to "safe" position, and when the train moves out of section *A* both relays are energized and both semaphore arms (at *P*) move to "safe" position. The arrangement shown in Fig. 176 is called the overlap track circuit.*

REFERENCES.

Fire alarm and police telegraphy:

Maver's *American Telegraphy*, J. H. Bunnell & Co., 1892

Wireless telegraphy:

Poincaré-Vreeland, *Maxwell's Theory and Wireless Telegraphy*, McGraw-Hill Book Co., 1904.

J. A. Fleming, *The Principles of Wireless Telegraphy*, Longmans, Green & Co., 1908.

G. W. Pierce, *The Principles of Wireless Telegraphy*, McGraw-Hill Book Co., 1910.

Electric furnace work:

J. B. Kershaw, *The Electric Furnace in Iron and Steel Production*, Van Nostrand, 1907.

Rodenhauser & Schoenawa, *Electrische Ofen in der Eisenindustrie*, Leipzig, 1911.

J. Bronn, *Electrische Ofen im Dienste der keramischen Gewerbe und der Glas und Quarzglasererzeugung*, Halle, 1910.

Borchers-Solomon, *Electric Furnaces*, Longmans, Green & Co., 1908.

A. Neuburger, *Handbuch der praktischen Electrometallurgie*, Berlin, 1907.

J. B. Kershaw, *Electrometallurgy*, Van Nostrand, 1908.

M. deKay Thompson, *Applied Electrochemistry*, The Macmillan Co., 1911.

Protection of buildings from lightning:

Oliver J. Lodge, *Lightning Conductors and Lightning Guards*, London, 1892.

Electric welding:

R. N. Hart, *Welding*, McGraw-Hill Book Co., 1910.

Ore concentration:

C. G. Gunther, *Electromagnetic Ore Separation*, McGraw-Hill Book Co., 1909.

Ozone:

F. M. Perkin, *The Industrial Uses of Ozone, Nature*, February 22, 1912.

A good discussion of the manufacture and use of ozone is given in *Bulletin No.* 4912 of the General Electric Company.

M. W. Franklin, *Ozone; Its Properties and Commercial Production*, Proceedings American Institute of Electrical Engineers, May, 1812, pages 597–607.

* The student is referred to Maver's *American Telegraphy*, pages 494–508 and to *Railway Signaling* published by *The Electric Journal* of Pittsburgh, Pa. This is a reprint of a series of articles which appeared in *The Electric Journal* during 1907 and it gives a great deal of information on the subject. The Union Switch and Signal Company of Swissvale, Pa., and The Hall Signal Company of Elizabeth, N. J., will send descriptive circulars to engineering students who are interested in railway signaling.

Electric Railroads:

W. C. Gotshall, *Electric Railway Economics*, McGraw-Hill Book Co., 1903.

Ashe & Keily, *Electric Railways*, Van Nostrand, 1905.

Wilson & Lydall, *Electrical Traction*, Longmans, Green & Co., 1907.

Herrick & Boynton, *American Electrical Railway Practice*, McGraw-Hill Book Co., 1907.

Sheldon & Hausman, *Electric Traction and Transmission Engineering*, Van Nostrand, 1911.

C. F. Harding, *Electric Railway Engineering*, McGraw-Hill Book Co., 1911.

Section 13 of *The Standard Handbook for Electrical Engineers* on Traction, by A. H. Armstrong, McGraw-Hill Book Co.

APPENDIX A.

DIELECTRIC STRESSES.

The equation of the condenser is

$$Q = CE \tag{1}$$

in which E is the electromotive force in volts applied to the condenser plates, Q is the amount of charge in coulombs drawn out of one plate and forced into the other plate, and C is the capacity of the condenser in farads.*

The capacity in farads of a parallel plate condenser with air as the dielectric is

$$C = 884 \times 10^{-16} \frac{a}{x} \tag{2}$$

in which a is the area of one of the plates (sectional area of the dielectric) in square centimeters, and x is the thickness of the dielectric in centimeters.

To substitute oil or any other dielectric for the air increases the capacity of a condenser in a certain ratio k so that the capacity may then be expressed by the equation

$$C_{farads} = 884 \times 10^{-16} \frac{ka}{x}. \tag{3}$$

The factor k is called the *inductivity* of the dielectric (also sometimes called *specific capacity* of the dielectric). Thus the inductivity of kerosene is about 2, which means that the capacity of an air condenser is doubled if kerosene is substituted for the air dielectric.

In the following discussion the letter B is used to designate the factor 884×10^{-16}.

* An elementary discussion of the condenser is given on pages 162–173 of Franklin and MacNutt's *Elements of Electricity and Magnetism*, The Macmillan Co., 1908.

Gauss's theorem.—A theorem of fundamental importance in electrostatic theory is as follows: The total electric flux Φ emanating from a charged body is equal to $Q \div B$, or the total charge on a body is equal to $B\Phi$, where Q is the charge in coulombs and Φ is electric flux expressed in volt-centimeters in air. (An electric field intensity is expressed in volts per centimeter, and the product of this field intensity by an area in square centimeters gives electric flux in volt-centimeters, the area being perpendicular to the field.)

When applied to a parallel-plate air condenser, Gauss's theorem may be derived by multiplying both members of equation (2) by E (the electromotive force between the plates), giving

$$CE = Q_{coulombs} = Ba\frac{E}{x},$$

but $E \div x$ is the electric field intensity between the plates in volts per centimeter, so that $a \times E \div x$ is the electric flux from plate to plate in volt-centimeters and therefore, $a \times E \div x = \Phi$ whence we have

$$Q = B\phi. \qquad (4)$$

Gauss's theorem may be derived for a parallel plate condenser with any dielectric by multiplying both members of equation (3) by E (the electromotive force between the plates), giving

$$CE = Q = B \,.\, a \,.\, k\frac{E}{x},$$

but $E \div x$ is, as before, the electric field intensity between the plates and $k \times E \div x$ may be defined as the *electric flux density** in the dielectric, so that $a \times kE \div x$ is the total flux, and we thus arrive again at equation (4).

* The product kf which is here called electrical flux density or electrical strain, was called dielectric polarization by Maxwell. See Maxwell's Treatise.

Magnetic and Electric Parallel.

$\mathcal{B} = \mu\mathcal{H}$,

where $\mathcal{H}$ is intensity of magnetic field in gausses, μ is the permeability of the medium, and $\mathcal{B}$ is the magnetic flux density.

$F = kf$,

where f is intensity of electric field in volts per centimeter $(E \div x)$, k is the inductivity of the medium, and F is the electric flux density in volt-centimeters per square centimeter.

Electrical Stress and Mechanical Stress.

The stretching force per unit of sectional area of a rod is called the *stress* on the rod, and the elongation of unit length of the rod is called the *strain;* and the strain is proportional to the stress. That is

$$\begin{Bmatrix}\text{mechanical}\\ \text{strain}\end{Bmatrix} = n \times \begin{Bmatrix}\text{mechanical}\\ \text{stress}\end{Bmatrix},$$

where n is a constant for a given substance.

The intensity of an electric field in volts per centimeter (or the volts per centimeter in a layer of dielectric between metal plates) is frequently called *electrical stress*, and the electric flux density kf in the dielectric is frequently called the *electrical strain*. Therefore we have

$$\begin{Bmatrix}\text{electrical}\\ \text{strain}\end{Bmatrix} = k \times \begin{Bmatrix}\text{electrical}\\ \text{stress}\end{Bmatrix},$$

where k is a constant for a given dielectric.

The potential energy of mechanical strain per cubic centimeter of the strained substance is equal to one-half the product of stress and strain.

The potential energy of electrical strain per cubic centimeter of the dielectric is equal to one-half the product of electrical stress (f) and electrical strain (kf).

The elastic condition of a substance can be specified as follows:

(*a*) By giving the stress in pounds per square inch;

(*b*) By giving the strain, as for example the percentage elongation of a wire under tension; or

(*c*) By giving the potential energy per unit volume of the strained substance.

The electric condition of a dielectric can be specified as follows:

(*a*) By giving the electrical stress in volts per centimeter;

(*b*) By giving the electrical strain in volt-centimeters or in coulombs per square centimeter*; or

(*c*) By giving the potential energy per unit volume of the dielectric.

* Electric flux can be expressed in coulombs according to equations (2) and (3) and therefore electric flux density can be expressed in coulombs per square centimeter. If a metal ball has Q coulombs of charge per square centimeter of its surface, then Q/B volt-centimeters of electric flux emanate, from each square centimeter of the ball, according to equation (4), or, in other words, the electric flux density in the dielectric near the surface of the ball $(= kf)$ is Q/B volt-centimeters per square centimeter, and therefore the electric field intensity near the surface of the ball is $Q/B \div k$ volts per centimeter.

Electric stresses in plane layers of different dielectrics.—Consider two metal plates with air and glass between them as shown in Fig. 1. The thing which is constant throughout the region between the metal plates, that is, the thing which has the same value in glass and air is the electric flux density kf, because there are equal and opposite charges on two plates, and, therefore, according to Gauss's theorem the total flux passing out from the positively charged plate $(+Q)$ is equal to the total flux passing in towards the negatively charged plate $(-Q)$. Now since kf is the same in the glass and in the air, and since $k = 1$ for air and $k = 6$ for glass, therefore, the electric field intensity or stress in volts per centimeter (f) is six times as great in the air as in the glass.

Consider the special case in which the glass and air are of equal thickness as indicated in Fig. 1. Then six sevenths of the battery voltage is impressed on the air layer and one seventh on the glass layer. If glass and air are each 1 centimeter thick, and if the total voltage is 35,000 volts, then, assuming the air not to break down, the voltage across the air will be 30,000 volts, and the voltage across the glass will be 5,000 volts.

If the glass plate is removed, leaving 2 centimeters of air, then the electrical stress on the air will be 17,500 volts per centimeter. Therefore the electrical stress in the air between two plates 2 centimeters apart is increased from 17,500 volts per centimeter to 30,000 volts per centimeter by filling half of the space between the plates with glass of inductivity 6.

This effect can be shown in a very beautiful manner by connecting two metal plates to a high-voltage transformer and adjusting the plates to a distance such that the intervening air layer is barely sufficient to sustain the voltage. Then if a glass plate be introduced between the metal plates, the electrical stress in the remaining air will be increased sufficiently to break the air down at each reversal of the alternating voltage, as shown by the bluish luminosity of the air layer.

The above discussion of the stresses in layers of glass and air as based on Fig. 1 can be simplified as follows: Imagine a thin sheet of metal *mm* to be placed between the air and glass as shown in Fig. 2. We thus have two exactly similar condensers, C' and C, of glass and air, and the capacity of the glass

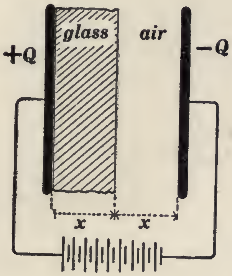


Fig. 1.

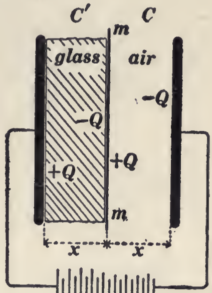


Fig. 2.

condenser is six times as great as the capacity of the air condenser, according to equations (2) and (3). But the charges on C' and C are the same because they have been charged in series. Therefore the voltage across the glass condenser is one-sixth of the voltage across the air condenser, according to equation (1).

The concentration of the greater part of the voltage upon the air layer in Figs. 1 and 2 is exactly analogous to the concentration of the greater part of the magnetomotive force of a dynamo field-winding upon the air gap in the magnetic circuit; only a small portion of the magnetomotive force is required to force the magnetic flux through the highly permeable iron, and a large portion of the magnetomotive force is required to force the magnetic flux through the less permeable air layer. A small portion of the battery voltage is required to force the electric flux through the highly inductive glass in Fig. 1 and a large portion of the voltage is required to force the electric flux through the less inductive air.

Mechanical analog of Fig. 1.—A difficulty in obtaining a simple mechanical idea of the concentration of the greater part of the battery voltage on the air layer in Fig. 1 arises from the following fact: the glass and the air are *in series* in Fig. 1 (and the electric flux density or *electric strain* or yield, in the two is the same), whereas two mechanical elements have the same *stress* when they are in series; to have the same strain or yield, two mechanical elements must be *in parallel.* Thus Fig. 3 shows a column of steel and a column of rubber equally compressed between two bars A and B (the steel and rubber columns are in parallel

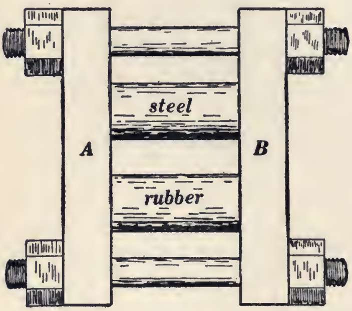


Fig. 3.

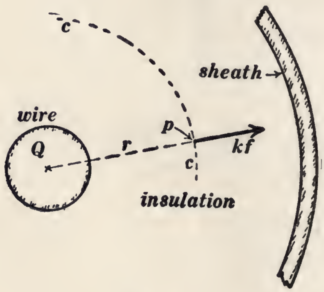


Fig. 4.

and they are equally shortened); but the easily yielding rubber (high inductivity) supports a small part of the compressing force, and the stiff steel (low inductivity) supports a large part of the compressing force.

Distribution of electrical stress in the region between core and sheath of an insulated cable, core and sheath being cylindrical and coaxial.—Consider unit length of the wire core of a cable and let Q be the amount of electric charge thereon. Let f be the electric field intensity in volts per centimeter at the point p in the insulation distant r from the axis of the cable and let k be the inductivity of the insulating material at p. Then kf (see Fig. 4) is the electric flux density at p, and $2\pi r \times kf$ is the flux across the cylindrical surface cc (of unit length), that is $2\pi rkf$ is the flux emanating from Q, and, therefore, according to equation (4), we have

$$Q = B \times 2\pi rkf$$

or

$$f = \frac{Q}{2\pi B} \cdot \frac{1}{k} \cdot \frac{1}{r}. \tag{5}$$

In interpreting this equation we will consider two cases, namely, (*a*) The case in which the cable insulation is all of one kind of material, and (*b*) The case in which the cable insulation is built around the core in layers of decreasing inductivity.

In the first case r is the only variable in equation (5) and the electrical stress at any point in the cable insulation is inversely proportional to the distance r from the axis of the cable. The electrical stress is greatest near the wire core and least near the sheath. This concentration of the stress near the wire core of a cable makes it impossible to utilize the full electrical strength of a cable insulation in case (*a*). In a somewhat similar manner the mechanical stress in the steel of a gun barrel is concentrated near the bore and consequently it is impossible to utilize the full mechanical strength of a solid forged gun barrel. The concentration of stress near the bore will start cracks in the walls while the outer portions of the gun barrel are very far indeed from being severely strained.

In the second case if we could select the materials for the successive layers of cable insulation so as to decrease k as r increases it would be possible to keep the product kr constant and then the electrical stress f in volts per centimeter would be the same in value throughout the insulating material. It is impracticable to accomplish this result completely, but high-voltage cable insulation is usually put on in two or three layers decreasing in inductivity outwards. Such a cable is said to have a *graded insulation*.

There is an interesting mechanical analogy to the graded cable insulation. If a thick-walled steel tube is subjected to internal pressure as in a cannon, the material next the bore is stretched to its stress-limit before the outer portions of the steel are brought into full action. If easily yielding (highly elastic like rubber) steel could be used for the inner portions of the gun tube, then the greater yield of the inner material would tend to bring all of the material of the tube up to the limiting stress simultaneously.* There is in fact but little variation in the elastic coefficient of various kinds of steel, and this method of gun construction is therefore impracticable. There are, however, great differences in the inductivities of different insulating materials, and therefore the grading of cable insulation is to some extent practicable.

Observable effects dependent upon variations of dielectric inductivity.—The greatest obstacle to a clear understanding of the theory of dielectric stress is that students and engineers are not familiar with the simple observable effects which involve inductivity. Indeed some of these effects are so simple that it is sufficient merely to describe them as follows:

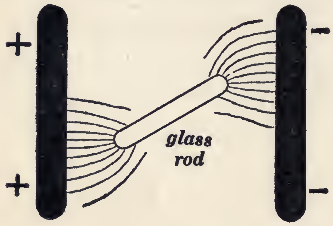


Fig. 5.

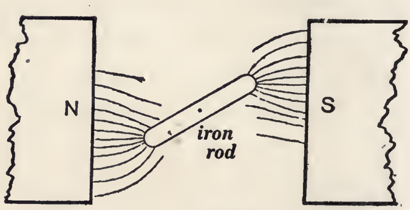


Fig. 6.

The lines of force in an electric field converge upon and pass through a glass rod (high inductivity), and the lines of force in a magnetic field converge upon and pass through an iron rod (high permeability). A glass rod suspended in an electric field oscillates to and fro through an equilibrium position parallel to the field in the same way that a suspended iron rod oscillates in a magnetic field. (Figs. 5 and 6.)

A glass plate is drawn into the intense electric field between positively charged metal plates in the same way that a piece of iron is drawn into the intense magnetic field between two opposite magnet poles as shown in Figs. 7 and 8. In the same way oil

* The steel tube of a cannon is strengthened in practice by shrinking a series of jackets over the tube.

and especially water is drawn into the most intense part of an electric field.

A thin glass cell partly filled with oil and provided with metal terminals A and B is placed in a lantern and the terminals A and B are connected to a Toepler-Holtz machine, as indicated

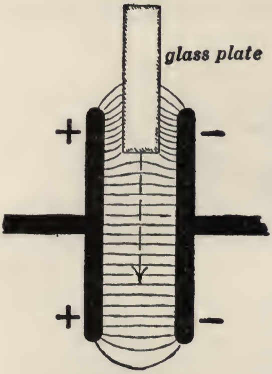


Fig. 7.

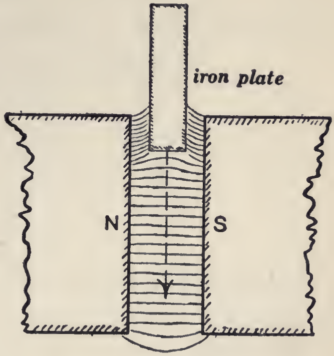


Fig. 8.

in Fig. 9. The oil (high inductivity) is drawn up as shown, and eventually a column of oil is formed reaching up to terminal A. In the same way a magnetic liquid (permeability greater than unity) would be drawn up to a magnet pole. Bubbles of air rising in oil in front of a pointed metal terminal (charged) are repelled.

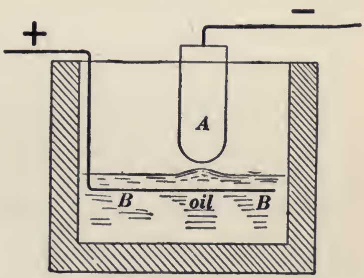


Fig. 9.

A charged gold-leaf electroscope is placed in a lantern with the plate of the electroscope connected by a fine wire to an insulated plate PP on the lecture table, as shown in Fig. 10. When a slab of paraffin wax W is placed in the region CC, the electroscope leaves fall slightly. The capacity of the condenser CC has been increased by the paraffin slab and a greater portion of the charge on the insulated system flows into PP thus decreasing the charge on the electroscope leaves. If one had two inflated rubber bags connected by a tube, and if one were to make the walls of one bag more yielding by dissolving off a portion of the rubber (if that were possible), then the weakened bag would swell and the other bag would shrink. The plate of paraffin makes the dielectric around *PP* more yielding and some charge flows from *EE* into *PP*.

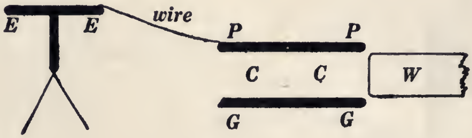


Fig. 10.

Dielectric hysteresis.—The most prominent kind of dielectric hysteresis is a kind which is closely analogous to what is technically called *elastic lag* in mechanics. Glass, for example, when subjected to a mechanical stress takes on a certain amount of strain (deformation) quickly, after which the strain slowly increases for a time; and when the stress is removed, a remnant of the strain persists for a time. This kind of hysteresis is sometimes called *viscous hysteresis*, and it is very different from the magnetic hysteresis in iron or steel, although a slight amount of viscous hysteresis does exist in very soft iron.

Dielectric hysteresis of the viscous type has long been known to exist, and it is the cause of the so-called "residual charge" which accumulates in a Leyden jar when the jar is highly charged and then completely discharged and allowed to stand.

A Leyden jar is charged. The coatings of the jar are then momentarily connected by wire, and then the jar is left standing on open circuit. After a time the coatings are again connected and a second slight discharge is obtained.

A rubber tube is stretched. This stretch corresponds to the electrical strain of the glass walls of the Leyden jar. The end of the tube is momentarily released, and the end is then clamped fast in what seems to be its equilibrium position. After a time the end is again released and a second slight "discharge" or movement takes place.

Dielectric strength.—The voltage required to puncture a layer of dielectric between flat metal plates is approximately proportional to the thickness of the dielectric layer. That is to say, a definite electrical stress in volts per centimeter is required to break down a dielectric, and this limiting electrical stress measures what is called the *dielectric strength.*

TABLE OF DIELECTRIC STRENGTHS.*

Substance.	Strength in Volts per Inch.	Substance.	Strength in Volts per Inch.
Oil of turpentine	235,000	Beeswaxed paper.........	1,350,000
Paraffine oil............	217,500	Air (thickness 5 cm.).....	59,500
Olive oil................	205,000	CO_2 (thickness 5 cm.)	56,750
Paraffine (melted)......	140,000	O (thickness 5 cm.)	55,500
Kerosene oil...........	125,000	H (thickness 5 cm.)	37,750
Paraffine (solid)........	325,000	Coal gas (thickness 5 cm.)	55,750
Paraffined paper........	900,000		

Concentration of electrical stresses by points.—The ease with which a bar of hard tool-steel can be broken when a sharp-bottomed nick is made in one side of the bar is well known. Fig. 11 shows the lines of stress passing around the bottom of a sharp groove in a bent bar. The stress is very greatly concentrated near the bottom of the groove, and the groove deepens by the formation of a crack. The stress is then concentrated at the edge of the crack, and the crack is extended farther and farther until the bar is broken in two.

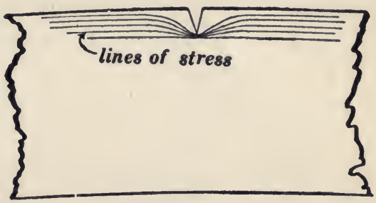


Fig. 11.

It is perhaps not generally known that the glass-cutting diamond does not make a scratch. Such a scratch would be a shallow flat-bottomed groove, and no very great concentration of stress would occur at the bottom of such a groove when the pane of glass is slightly bent. The end of a cutting diamond

* From the measurements of Macfarlane and Pierce, *Physical Review*, Vol. I, page 165, 1894.

is a perfectly rounded "corner" of a natural diamond crystal (the diamond is a crystal with curved faces), and when a cutting diamond is drawn properly across a pane of glass a minute crack is formed under the diamond on account of the excessive local compression. This crack causes a very great concentration of stress when the pane of glass is subjected to a very slight bending action, and the result is that the crack runs through the pane.

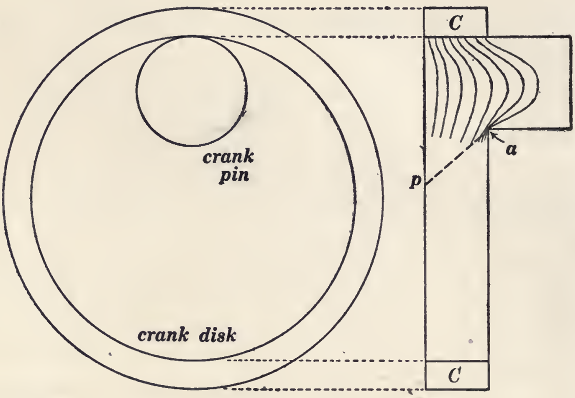


Fig. 12.

When a diamond is drawn heavily across a pane of glass a very considerable exertion is required to break the glass and the crack does not always follow the groove. When a diamond is drawn properly across a pane of glass a very slight bending effort is sufficient to break the glass, and the break nearly always follows the minute crack produced by the diamond.

A very interesting accident occurred at the Bethlehem Steel Works a number of years ago when an attempt was made to strengthen a crank-disk by shrinking a collar upon it. The disk had a crank-pin on one side, and the disk sheared off along the dotted line *ap* in Fig. 12 on account of the excessive concentration of stress at the reëntrant angle *a*. The fine curved lines show the approximate trend of the stress lines in the disk due to the collar *CC*.

An interesting experiment is to place a small piece of window glass on a flat plate of steel (or plate glass) and press a sharp-pointed file against it as shown in Fig. 13. The stresses in the window glass are very greatly concentrated at the sharp point of the file, and it takes but little force on the file to break the glass to pieces. If, however, a bit of soft copper is placed under the point of the file, one cannot push hard enough to break the glass; the copper yields (breaks down mechanically) and distributes the stress.

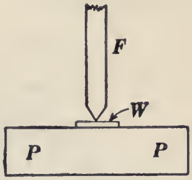


Fig. 13.

When a voltage is applied to the metal terminals *MM* in Fig. 14, the electric lines of force (the electrical stress lines) converge upon the sharp metal point, and the electrical stress is very greatly concentrated near the point. Indeed a comparatively low voltage will rupture the glass plate in Fig. 14 because of the starting of an electric rupture by the excessive concentration of the stress near the metal point. To produce this result, however, the region *rr* must be filled with a substance of great

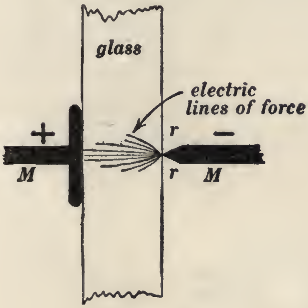


Fig. 14.

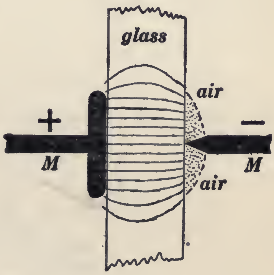


Fig. 15.

dielectric strength like turpentine or wax. If the region *rr* is filled with a substance of low dielectric strength like air, the portion in the immediate neighborhood of the metal point breaks down electrically and becomes a conductor, and the resultant distribution of electrical stress in the glass plate (which is shown in Fig. 15) is the same as if the glass plate were between two flat metal plates as shown in Fig. 16. Under these conditions the electrical stress in the glass is nearly uniform, and a very high voltage is required to puncture the glass plate, because there is no region of concentrated stress to start the electrical break- down.

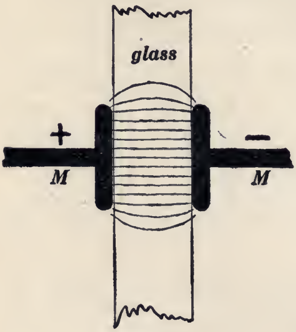


Fig. 16.

Having air around the metal point in Fig. 14 is like having a bed of soft copper around the point of the file in Fig. 13. The copper breaks down mechanically and distributes the stress, thus preventing excessive concentration of stress near the point of the file and the starting of a crack thereby. The air breaks down electrically and distributes the stress, thus preventing excessive concentration of stress near the metal point and the starting of an electric puncture thereby.

An electrical breakdown in a solid dielectric (and usually in liquid and gaseous dielectrics also) is always in the form of a puncture, that is the breakdown occurs along a line; and this line of breakdown is an electrical conductor. Therefore the electrical stresses in the dielectric are concentrated at the end of an incipient puncture, as at a metal point, and the puncture is thus

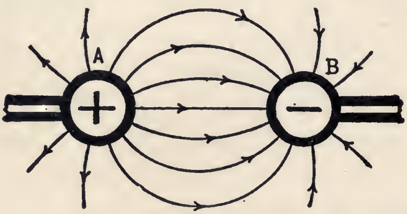


Fig. 17.

carried through the dielectric or into regions where the electrical stresses were far below the breakdown value before the puncture started. Thus Fig. 17 shows the electric lines of force between two metal balls, and Fig. 18 shows how the electric lines of force rearrange themselves when an electric puncture starts.

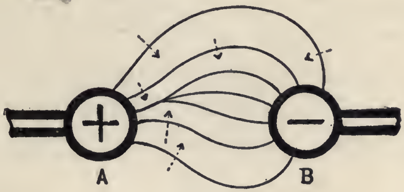


Fig. 18.

Maximum stress in homogeneous cable insulation for given voltage between core and sheath.—Equation (5) cannot be used to calculate the actual value of the electrical stress f at a given distance r from the axis of the cable because the value of Q is not known. The quantity which is always specified or which can be most easily observed is the voltage E between the core and the sheath, and, therefore, it is desirable to derive an equation which gives f in terms of E as follows:—Let R_1 be the radius of the wire core and let R_2 be the inside radius of the sheath, both expressed in centimeters. It is required to find the total voltage along the line *ab*, Fig. 19. Consider the element Δr of the

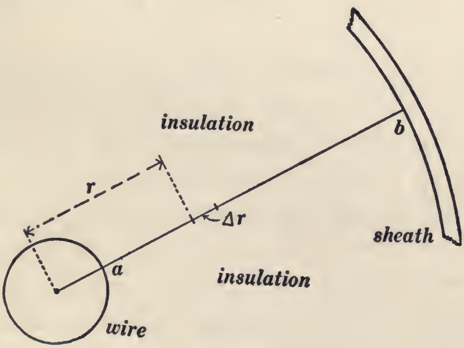


Fig. 19.

line *ab*. The voltage along this element is $f \cdot \Delta r$ because f is the volts per centimeter at Δr (compare Fig. 4). Therefore we have $\Delta E = f \cdot \Delta r$, or using the value of f from equation (5) we have

$$\Delta E = \frac{Q}{2\pi Bk} \cdot \frac{\Delta r}{r},$$

whence by integrating* between the limits $r = R_1$ to $r = R_2$ we have

$$E = \frac{Q}{2\pi Bk} \log_e \left(\frac{R_2}{R_1} \right). \tag{6}$$

Therefore the entire factor $Q/2\pi Bk$ in equation (5) is equal to $E/[\log_e(R_2/R_1)]$, and equation (5) may be written:

$$f = \frac{E}{\log_e \left(\frac{R_2}{R_1} \right)} \cdot \frac{1}{r} \tag{7}$$

or, reducing to common logarithms, we have

$$f = \frac{0.435 E}{\log \left(\frac{R_2}{R_1} \right)} \cdot \frac{1}{r}. \tag{8}$$

The greatest value of f occurs at the surface of the wire core where $r = R_1$ and therefore from equation (8) we have

$$f_{max} = \frac{0.435 E}{R_1 \log \left(\frac{R_2}{R_1} \right)}, \tag{9}$$

in which f_{max} is expressed in volts per centimeter if E is expressed in volts and R_1 and R_2 in centimeters, or in volts per inch if E is expressed in volts and R_1 and R_2 in inches. For example consider a cable having a quarter-inch wire core ($R_1 = 0.125$ inch) and one half-inch of insulation ($R_2 = 0.625$ inch) with 10,000 volts between wire and sheath. Then the maximum electrical stress is 49,700 volts per inch and the electrical stress near the sheath is 9,940 volts per inch. On the other hand, half an inch of insulation between *flat metal plates* would be under a *uniform* stress of 20,000 volts per inch with 10,000 volts between the plates.

* Of course k is assumed to be constant; that is the cable insulation is all one kind of material.

Maximum electrical stress between parallel wires.—Figure 20 is a sectional view of the two wires W' and W'' and it is desired to find the expression for the electrical stress f at any point p distant x centimeters from the axis of W' when there is $+Q$ coulombs of electric charge on each centimeter of W' and $-Q$ coulombs on each centimeter of W''. The wires are small in diameter as compared with their distance apart D, and therefore

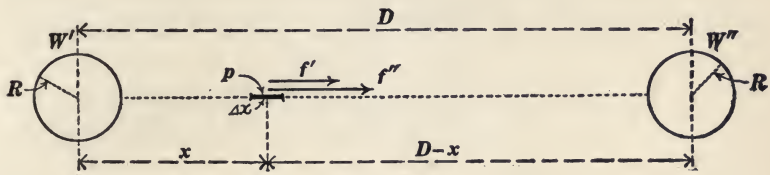


Fig. 20.

the charge may be assumed to be uniformly distributed around each wire, and *the electric field due to either wire alone* may therefore be assumed to radiate symmetrically around the wire exactly as if the wire were surrounded by a co-axial sheath.

Let f' be the electrical stress at p due to W' and let f'' be the electrical stress at p due to W''. Then according tc equation (5) we have:

$$f' = \frac{Q}{2\pi Bk} \cdot \frac{1}{x} \tag{i}$$

and

$$f'' = \frac{Q}{2\pi Bk} \cdot \frac{1}{(D-x)}. \tag{ii}$$

But the total electrical stress at p is $f' + f''$ and therefore we have

$$f = f' + f'' = \frac{Q}{2\pi Bk}\left(\frac{1}{x} + \frac{1}{D-x}\right) \tag{10}$$

and the voltage ΔE along the element Δx is equal to $f \cdot \Delta x$ so that

$$\Delta E = \frac{Q}{2\pi Bk}\left(\frac{1}{x} + \frac{1}{D-x}\right) \cdot \Delta x, \tag{iii}$$

whence, by integrating* between the limits $x = R$ and $x = D - R$ (from surface to surface of the wires in Fig. 20) we have:

$$E = \frac{2Q}{2\pi Bk} \cdot \log_e \left(\frac{D - R}{R} \right) \qquad \text{(iv)}$$

where R is the radius of each wire and D is the distance between centers of wires.

From equation (iv) we find the value of the factor $Q/(2\pi Bk)$ to be $E/\{2 \log_e [(D - R)/R]\}$ so that equation (10) becomes

$$f = \frac{E}{2 \log_e \left(\frac{D - R}{R} \right)} \cdot \left(\frac{1}{x} + \frac{1}{D - x} \right). \qquad \text{(v)}$$

The maximum value of f occurs at the surface of either wire where $x = R$ (or where $x = D - R$). Furthermore $1/R$ is quite large as compared with $1/(D - R)$. Therefore substituting R for x in equation (v) and discarding $1/(D - R)$ as negligible in comparison with $1/R$, we have

$$f_{\max} = \frac{E}{2R \log_e \left(\frac{D - R}{R} \right)}, \qquad \text{(vi)}$$

whence, using ordinary logarithms we have

$$f_{\max} = \frac{0.435 E}{2R \log \left(\frac{D - R}{R} \right)}. \qquad \text{(11)}$$

For most practical purposes $D - R$ is sensibly equal to D and therefore equation (11) may be written:

$$f_{\max} = \frac{0.435 E}{2 R \log \left(\frac{D}{R} \right)},$$

in which $f_{\max}$ is expressed in volts per centimeter if E is expressed in volts and D and R in centimeters, or in volts per

* Of course k is assumed to be constant; that is the wires are surrounded by insulating material all of one kind.

inch if E is expressed in volts and D and R in inches. For example consider a transmission line consisting of two quarter-inch wires ($R = 0.125$ inch) 24 inches apart center to center ($D = 24$ inches) with 50,000 volts between the wires. Then the maximum electrical stress is 38,100 volts per inch.

The equalization of the electrical stresses in the insulation of transformer terminals.—A certain amount of electric flux starts out from the wire core of a cable, and *this same amount of flux continues outwards unchanged in value*, and therefore the flux density decreases at increasing distances from the core. In the case of a long cable, the electrical stress in the insulation can be

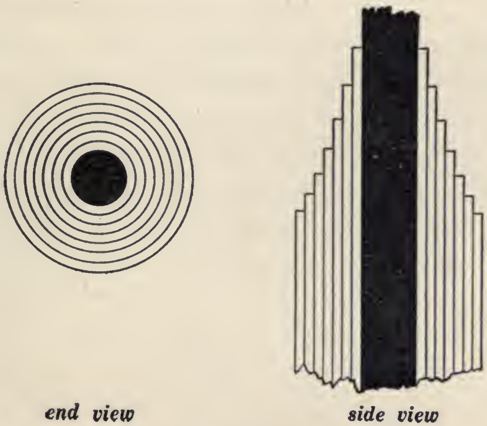
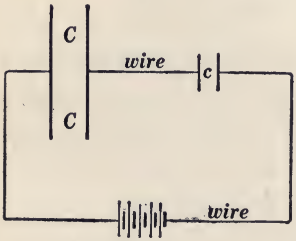


Fig. 21.

made uniform by the grading of the inductivity of the insulation as above explained. In the case of a short rod, the electric flux which emanates from the rod can be more and more *crowded together endwise* at increasing distances from the rod so as to compensate for the circumferential spreading and thereby give the same electric flux density and the same electrical stress throughout the insulation. This crowding together endwise of the electric flux is accomplished by dividing the insulation into layers of equal thickness which are separated by sheets of tin-foil as shown in

Fig 21. This arrangement of the insulation of a rod is called the *condenser type of insulation*.

The principle of the condenser type of insulation may be understood with the help of Fig. 22, which shows two condensers



F g. 22.

c and C connected in series to a battery. The two condensers receive the same amount of charge Q when arranged in this way, the voltage across c is Q/c and the voltage across C is Q/C, or, in other words, the voltages across the respective condensers are inversely as their capacities. Any layer of insulation together with the adjacent sheets of tin-foil in Fig. 21 constitutes a condenser, and the condensers formed by all the layers of insulation and sheets of tin-foil are in series. Therefore the voltage is the same across every layer if the various capacities are equal. To make the various capacities equal (with same thickness of dielectric in each case) the sectional areas of the various layers of dielectric must be the same, that is to say, the length parallel to the rod of each successive layer must be reduced in proportion to the increasing circumference around the rod.

APPENDIX B. PROBLEMS.

CHAPTER I.

COSTS.

1. Calculate the values of station load factor corresponding to each of the curves in Fig. 5, to the curve in Fig. 6, and to each of the curves in Fig. 7.

2. From the data given in the table on page 12 calculate each item of cost of operating the 150-kilowatt plant at 0.2 load factor and at 0.8 load factor.

3. From the data given in Fig. 8 calculate the following items of cost of operating a 10,000-kilowatt steam turbine plant at full load and at 30 per cent. load: (*a*) Coal and water [see page 12 for statement as to quality of coal and cost of coal per ton]; (*b*) boiler room and engine room labor, coal and ash handling, oil and engine room supplies; and (*c*) fixed charge.

4. A 150-kilowatt lighting plant operates night and day at 0.2 load factor and the schedule of operation costs are shown in the table on page 12. It is impossible to increase the lighting load because the peak of the load already reaches the limit of overload capacity of the plant. It is possible, however, to increase the output of the plant by supplying current for motors off the peak if the motor rate is made very low so as to attract customers. What would be the cost to the station per kilowatt-hour of additional output if the load factor of the station were raised to 0.4 by supplying motors off the peak?

5. Find from the curve *A* of Fig. 9 the probable peak load of a station supplying 20 residences in each of which 60 lamps are installed, 50 residences in each of which 40 lamps are installed, 100 residences in each of which 20 lamps are installed, and 200 residences in each of which 10 lamps are installed.

6. A certain customer might be asked to pay $2.00 per month "connection" charge, his maximum demand is 5 kilowatts on the peak for which he might be asked to pay at the rate of $60 per kilowatt per year, and his yearly consumption is 5,000 kilowatt-hours for which he might be charged at the rate of 3 cents per kilowatt-hour. What would be the cost of electrical energy to this customer per kilowatt-hour?

Note.—Compare table on page 19.

7. A small customer has ten 20-watt lamps. Asssuming 10 cents per month as a reasonable connection charge (without a meter), $60 per year per kilowatt of maximum demand, and 3 cents per kilowatt-hour of consumption what would be the equivalent flat rate for the 10 lamps, an excess indicator being installed to limit the maximum demand to 160 watts, the yearly consumption being 43 kilowatt-hours?

8. A Thomson watt-hour meter without a starting coil starts on a 75-watt load. The meter is adjusted to give a true watt-hour record when run on a 500-watt load. What will the instrument indicate after running for 4 hours on a constant load of 200 watts, running friction being assumed to be equal to half of starting friction. See note to problem 10. Ans. 702.8 watt-hours.

9. The watt-hour meter specified in problem 8 is provided with a starting coil so as to start, on 110-volt mains, when the power delivered to the receiving circuit is 40 watts. At what load will the meter start on 55-volt mains? See note to problem 10. Ans. 66.25 watts.

10. The watt-hour meter of problem 9 is adjusted to record a 500-watt load correctly on 110-volt mains. At what load will it record correctly on 55-volt main? Ans. 5,750 watts.

Note.—The driving torque, not counting that due to the starting coil, is proportional to the watts delivered to the receiving circuit, and it may be conveniently expressed in "watts." The driving torque due to the starting coil (with given voltage between the supply mains) may be expressed as the difference between the starting watts with and without the starting coil. The running friction (a torque) may be expressed as one half the starting load in watts without the starting coil. The speed of the meter may be conveniently expressed in "watt-hours recorded per hour."

Subtracting from the total driving torque (including the torque due to the starting coil) the running friction, gives the net torque used to overcome the retarding action of the damping magnets, and the speed of the meter is proportional to this net torque. The torque produced by the starting coil is proportional to the square of the voltage between the mains.

11. The net assets of the Wallingford plant year after year are represented by the differences between the ordinates of the curves A and B in Fig. 13, and the capitalization at the beginning was $55,000. Find the rate at compound interest which is represented by the growth of the net assets to $96,881 on July 31, 1910 ($10\frac{1}{2}$ years from the beginning). Ans. 5.53 per cent.

Note.—This rate of interest does not represent the real profits of the plant; to find the real profits one must take into consideration the annual payment of bond interest as explained in problem 12. If the Wallingford plant continues in the future to make the same profits it has made in the past then in 1920 the net assets will be $161,700 and there will remain $106,700 of net assets after the bonds are paid. Furthermore, the accumulated depreciation charge will in 1920 probably amount to more than the actual depreciation so that the $106,700 will be real tangible assets.

12. The Wallingford plant has been paying $1,925 annually as interest on its bonds, beginning on August 1, 1900, so that on August 1, 1910, eleven payments had been made. Find the accumulated cash value on August 1, 1910, of all these payments (the accumulation being reckoned at $3\frac{1}{2}$ per cent. compound interest), add this to the net assets on August 1, 1910, and find the rate at compound interest which is represented by the growth from $55,000 in $10\frac{1}{2}$ years. Ans. 7.9 per cent.

CHAPTER II.

ELECTRIC DISTRIBUTION AND WIRING.

13. A span, 150 feet long, of hard-drawn copper wire, No. 8 Brown and Sharpe gauge, is to be strung at a temperature of 75° F. at a place where the winter temperature sinks to —20° F. The maximum tension of the wire is to be 164 pounds. Find: (*a*) The sag at —20° F.; (*b*) the sag at 75° F., and (*c*) the tension at 75° F. Ans. (*a*) 0.86 foot; (*b*) 2.86 feet; (*c*) 49 pounds.

14. Five hundred glow lamps each taking one-half an ampere at 110 volts are supplied with current from a 115.5-volt generator at a distance of 1,000 feet from the lamps. Find: (*a*) The size of copper wire required, (*b*) the total weight of the wire, and (*c*) the total cost of the wire at 16 cents per pound. Ans. (*a*) 982,000 circular mils; (*b*) 5,950 pounds; (*c*) 953 dollars.

15. Five hundred glow lamps each taking one-half an ampere at 110 volts are supplied with current from a 231-volt generator at a distance of 1,000 feet from the lamps. The Edison three-wire system is used and the system is balanced. Find: (*a*) The size of copper wire required for the outside mains; (*b*) the total weight of all three mains, the middle main having one-half the sectional area of either outside main; and (*c*) the total cost of the three mains at 16 cents per pound. Ans. (*a*) 245,500 circular mils; (*b*) 1,860 pounds; (*c*) 298 dollars.

16. The three-wire system of problem 15 supplies 300 lamps (150 amperes) on one side and 200 lamps (100 amperes) on the other side, all at a distance of 1,000 feet from a 231-volt generator. A balancer is used in the station to take care of the current in the middle main and to keep the voltage between the middle main and each outside main equal to 115.5 volts. Find: (*a*) The voltage across the set of 300 lamps; and (*b*) the voltage across the set of 200 lamps. Ans. (*a*) 104.5 volts; (*b*) 115.5 volts.

Note.—If the lamps above specified are all exactly alike it is evident that the current in each lamp of the 300 set cannot be the same as the current in each lamp of the 200 set. It is usual, however, in wiring calculations to consider that lamps of a given size and type take a definite amount of current irrespective of the slight variations of voltage.

17. The three-wire system of problem 15 supplies 300 lamps (150 amperes) on one side and 100 lamps (50 amperes) on the other side, and the total voltage of 231 volts at the generator is equally divided by the balancer as explained in problem 16. Find: (*a*) The voltage across the set of 300 lamps; and (*b*) the voltage across the set of 100 lamps. Ans. (*a*) 100.1 volts; (*b*) 122.1 volts.

18. A group of ten lamps, each taking one-half an ampere, is ten feet distant from 115-volt mains. (*a*) Find the size of wire required in order to give a drop of 5 volts. (*b*) What size of wire (rubber insulation) would be required according to the table of safe carrying capacity given on page 55? Ans. (*a*) 14.7 mils diameter; (*b*) No. 16 Brown and Sharpe gauge (51 mils diameter).

Note.—It is evident that a wire 14.7 mils in diameter would be excessively heated by a current of 5 amperes. Usually the size of wire for supplying lamps near to the generator or center of distribution is determined by the table of safe carrying capacity.

The insurance rules forbid the use of wire smaller than No. 14 Brown and Sharpe gauge for house wiring.

19. A pair of street mains leading out from a central station delivers 50 amperes of current to a consumer at a distance of 200 feet from the station, 75 amperes to a second consumer at a distance of 350 feet from the station, and 40 amperes to a third consumer at a distance of 600 feet from the station. The station voltage is 115 volts and the voltage at the distant end of mains is 110 volts. Find: (*a*) The size, weight, and cost of mains of uniform size; and (*b*) the size of each section of the mains, and their total weight and cost, when the size is reduced in steps so as to give the specified voltage-drop with a minimum amount of copper. The cost of copper is to be taken at 16 cents per pound. Ans. (*a*) 260,300 circular mils, 946.5 pounds, 151.2 dollars, (*b*) first section 319,700 circular mils, second section 267,000 circular mils, third section 157,400 circular mils, 868 pounds, 139 dollars.

Note.—In estimating the distance, *L*, of the "center of gravity" of the consumers in the above problem, one may use one ampere as the unit instead of one lamp.

20. An electric railway 33,300 feet in length is divided into three sections of which the lengths, 9,000 feet, 10,800 feet and 13,500 feet, are proportional to the schedule speeds of cars on the respective sections so that a car running from end to end of the line traverses each section in 10 minutes. Four cars are always on the first section five minutes apart going each way, two cars are always on the second section ten minutes apart going each way, and a single car is always on the third section making the round trip in 20 minutes. Owing to frequent stops on the first section the cars taken an average current of 125 amperes each, on the second section the stops are less frequent and each car takes an average of 105 amperes, and on the third section the stops are least frequent and the car that is always on this section takes an average of 95 amperes.

The "center of gravity" of the four cars that are always on the first section is at the middle of the section, the "center of gravity" of the two cars that are always on the second section is at the middle of that section, and the most unfavorable position of the single car that is always on the third section is when it is at the extreme end of the line. Assume, therefore, that 500 amperes are delivered continuously at the middle of the first section (4,500 feet from the city end), that 210 amperes are delivered continuously at the middle of the second section (14,400 feet from the city end), and that 95 amperes are delivered continuously at the extreme end of the line. If the power house is located at the city end of the line, find: (*a*) The size of each section of the feeder to give a total drop of 75 volts at the extreme end of the line with a minimum amount of copper; and (*b*) the total cost of feeder copper at 16 cents per pound. Ans. (*a*) First 4,500 feet of feeder 1,980,000 circular mils, next 9,900 feet of feeder 1,219,000 circular mils, and remaining 18,900 feet of feeder 680,000 circular mils; (*b*) 16,400 dollars.

Note.—The resistance of the bonded track, which is used as a return feeder, is very uncertain and it is here to be assumed equal to zero for the sake of simplicity.

21. (*a*) Find the position in which the power house should be placed on the railway specified in problem 20 in order that the feeder copper may be reduced to a minimum; (*b*) find the size of each section of the feeder on the assumption that the two cars on the middle section are in the most unfavorable positions, namely, at the two ends of the section, and on the assumption that the car on the third section is at the extreme end of the section; and (*c*) find the total cost of the feeder copper at 16 cents per pound. Total drop to each end of the line to be 75 volts. Ans. (*a*) 10,480 feet from the city end of the railway; (*b*) first section 441,000 circular mils, city end of second section (1,480 feet) 485,200 circular mils, suburban end of second section (9,320 feet) 536,700 circular mils, and third section 369,900 circular mils; (*c*) 7,119 dollars.

Note.—The power house should be placed at the center of "gravity" of a system in the sense in which this term is defined on page 62 in order that the amount of feeder copper may be a minimum. Similarly, a center of distribution, from which electric lamps are to be supplied by street mains, should be located at the "center of gravity" of the consumers, each consumer being "weighted" in proportion to the current delivered to him.

The feeder on city section of 9,000 feet is assumed to be of uniform section of 441,000 circular mils throughout, but it would be advisable in fact to make the extreme city end of this section of the feeder much smaller than 441,000 circular mils.

22. A nearly reëntrant row of 100 lamps, each taking one-half an ampere, is to be wired in accordance with the return loop scheme, using wire of uniform size. The row is 200 feet long, one end of the row is 50 feet from the service point, and the other end of the row is 60 feet from the service point. The voltage at the service point is 115 volts. Find the size of wire to give 105 volts at the middle lamp of the row. Ans. 14,040 circular mils sectional area.

Note.—Such problems as this and problem 23 are most easily solved on the assumption that the given group of lamps is equivalent to a 50-ampere load distributed with ideal uniformity over the whole length of 200 feet.

23. Find the voltage at each end lamp in the row specified in problem 22. Ans. 106.93 volts.

Note.—The drop all the way along *ab* (or *cd*) of Fig. 35 is equal to

$$\frac{\rho I}{X}\int_{x=0}^{x=X}(X-x)dx$$

(as may be understood from the discussion on page 65) and this is equal to $\frac{1}{2}\rho XI$. The voltage across lamp at either end of row is the service voltage minus $\frac{1}{2}\rho XI$, minus the drop in *efa*, and minus the drop in *ge*.

24. All the lamps except the middle lamp in the row specified in problem 22 are turned off. Find the rise of voltage at the middle lamp. Ans. From 105 volts to 114.88 volts.

25. Find the size of wire required to supply the row of 100 lamps specified in problem 22 by the simple parallel scheme, both service wires being led from the service point to the nearer end of the row: (*a*) when the drop between the service point and the most remote lamp is 10 volts; (*b*) when the drop to the most remote lamp exceeds the drop to the nearest lamp by the amount (106.93 − 105) volts; and (*c*) when the drop to the most remote lamp is 5 volts. Ans. (*a*) 16,200 circular mils sectional area; (*b*) 56,000 circular mils; (*c*) 32,400 circular mils.

Note.—In case (*a*) we have the same total drop as in problem 22, but the voltage at the lamps ranges from 105 volts at the remote end to 111.67 volts at the near end of the row, that is, a range of 6.67 volts; whereas with the return loop scheme as specified in problem 22 the voltage at the lamps ranges from 105 at the middle lamp to a maximum of 106.93 volts, that is, a range of only 1.93 volts, and the wire in the return loop scheme is the smaller.

In case (*b*) the voltage at the lamps has the same range as in problem 22 and the lamps therefore would operate equally well as in the return loop scheme as specified in problem 22, provided the lamps are all in use or all out of use, but the wire in case (*b*) is nearly four times as heavy as in problem 22. This shows in a striking way the saving of copper by the return loop scheme, for the same range of voltage among the lamps, when the group of lamps forms a nearly reëntrant row and when the lamps are always either all on or all off.

On the other hand, the result of problem 24 shows that the return loop as specified in problem 22 is not at all suited to the case in which part of the lamps in the group are turned off, the effect being to cause a very considerable rise of voltage at the remaining lamp (or lamps).

26. A group of 50 lamps each taking 1.0 ampere is to be installed at a distance of one mile from a lighting station. It is understood that whenever any of the lamps are in use all are in use, so that the drop in the feeders which supply the lamps may have any value that economy demands. The lamps are to be operated for 300 hours each year. The cost of power at the station is 3.5 cents per kilowatt-hour, the cost of copper is 16 per cents per pound, the annual charge on the cost of the wire is 10 per cent. (interest 6 per cent., depreciation 3 per cent., and taxes 1 per cent.), and the station voltage is 125 volts. Find: (*a*) The size of the feeders to give a balance between loss of power and cost of copper, and (*b*) the voltage at the lamps. Ans. (*a*) 76,370 circular mils; (*b*) 50.45 volts.

Note 1.—The only objection to the application of the economic principle of the balance between loss of power and cost of copper to a case like the one here considered is that the voltage at the lamps may be very different from the voltage which prevails in the other parts of the lighting system, so that the station management would have to be careful to supply special lamps suited to the special voltage. It is evident that it is expensive at best to supply the fifty lamps at a distance of a mile, for, under the conditions of problem 26, it requires $391.40 worth of copper with a loss of $39.14 worth of power each year, and to transmit the required power (2.522 kilowatts, or 22.93 amperes with 110 volts at the lamps) with 15 volts drop would take $892 worth of copper with a loss of $3.61 worth of power each year.

Note 2.—It is instructive to solve problem 26 graphically as follows: Assume, say, 50,000, 60,000, 70,000, 80,000, 90,000, and 100,000 circular mils. Use these sectional areas as abscissas of two curves, *A* and *B*; the ordinates of curve *A* representing the values in dollars of the power lost each year, and the ordinates of curve *B*, representing the annual charge in dollars on the total cost of the copper. Then plot a third curve, *C*, of which each ordinate is the sum of the corresponding ordinates of curves *A* and *B*, and the abscissa corresponding to the minimum ordinate of this curve, *C*, is the required sectional area.

27. The customer mentioned in problem 26 is to pay at the rate of 11 cents per kilowatt-hour for all energy delivered to his special line in excess of the usual 5 per cent. line loss. At what rate per kilowatt-hour must this customer pay on the basis of his watt-hour meter?

28. A consumer pays 10 cents per kilowatt-hour not only for the energy he uses in his lamps but also for the energy that is lost in the wires that lead from the watt-hour meter to his lamps.

If the customer uses his lamps 2 hours per day the year round, find the size of wires he should use in his house, in circular mils per ampere, for greatest economy, the cost of copper being 16 cents per pound, and the interest and depreciation being 8 per cent. Ans. 4,507 circular mils per ampere.

29. The cost of power at the switch-board in an arc-lighting station is 2 cents per kilowatt-hour. The plant supplies a current of 6.6 amperes to a circuit of 50 arc lamps which are operated on a moonlight schedule for 2,160 hours each year. The cost of copper is 21 cents per pound, and the annual charge on the cost of the wire is 11 per cent. (interest 6 per cent., depreciation 3.5 per cent., and taxes 1.5 per cent.). The cost of the wire is high and its depreciation is large because the wire is insulated. The price of 21 cents per pound is on the net weight of copper in the wire and this price is intended to cover the cost of the insulation. Find the size of wire to give a balance between loss of power and cost of copper. Ans. 17,040 circular mils.

Note.—The falling of an arc light wire into the street would be very dangerous on account of the high voltage. Therefore, it is important that an arc-lamp circuit be very substantial. It is usually not considered allowable for this reason, to use wire smaller than No. 6 Brown and Sharpe gauge (26,000 circular mils) for an arc-lamp circuit.

30. Find the cost to the station owners of 16.5 kilowatts of power delivered for 1,200 hours each year at 220 volts over a special line to a single customer at a distance of one mile from the station, the wire being of such size as to give a balance between loss of power and cost of copper. Determine the size of wire on the basis of a 6 per cent. annual charge on the cost of the copper at 16 cents per pound, the cost of power at the switch-board being 2.5 cents per kilowatt-hour. Reckon the total cost of the line at 2.25 times the cost of the copper, and reckon the total annual cost of interest, depreciation, taxes, and maintenance of line at 15 per cent. of the total cost of the line. Ans. $1,176.80 per year, or about 6 cents per kilowatt-hour delivered.

31. Given ten groups of lamps, each group taking 10 amperes, the groups being 10 feet apart. The lamps are supplied with current from 115-volt mains according to the wiring scheme shown in Fig. 37. The end group, *bb* (see figure), is 10 feet from the mains, the point, *pp*, is 70 feet from the mains, and No. 2 Brown and Sharpe gauge copper wire is used throughout. Make a drawing like Fig. 22*b*, showing the voltage at every group of lamps. Sample answer: 1.964 volts drop to the group of lamps which is 50 feet from the service point.

Note.—It would be permissible for practical purposes to calculate drops to the various lamps in this problem on the assumption that the lamps constitute a load which is distributed with ideal uniformity. It is here intended, however, that the drops be calculated as they actually are and so represented in the drawing.

32. The accompanying figure, Fig. 32*p*, shows two motors, M and M', two groups of glow lamps, L and L', and a group of arc lamps, A, all supplied from the 115-volt service point, P. The distances are all so small that the sizes of all wires are to be determined from the table of safe carrying capacities. The motor, M, takes 35 amperes, the motor, M', takes 18 amperes, the group, L, contains 7 half-ampere lamps, the group, L', contains 3 half-ampere lamps, and each arc lamp takes 5 amperes.

(*a*) Make a sketch of Fig. 32*p* and indicate the values of the current at the points *a*, *b*, *c*, *d*, *e* and *f*; (*b*) indicate the size of

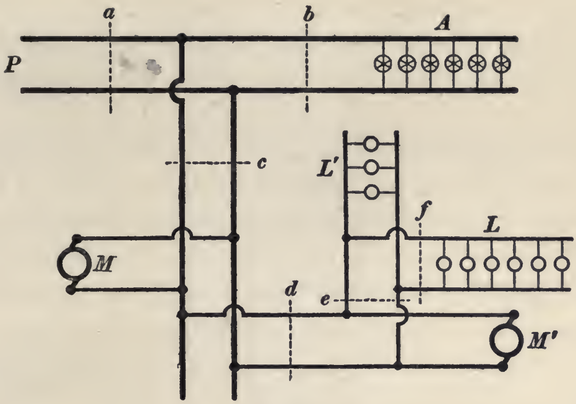


Fig. 32*p*.

each wire assuming rubber insulated wire to be used; (*c*) show the location and mark the current rating of every fusible cut-out and branch block required by the National Electrical Code.

CHAPTER IV.

PHOTOMETRY AND ILLUMINATION.

33. The conical intensity of a beam of light from a lamp is 50 candle-power. The lamp is 5 feet from the center of a hole in a wall, and the diameter of the hole is 3 feet. Find the amount of light passing through the hole in lumens.

Note.—The area of a spherical zone is equal to the area of the sphere multiplied by the altitude of the zone and divided by the diameter of the sphere.

34. A 50-candle-power lamp is at the center of a circular band which is 6 feet in diameter and one foot wide. Find the amount of light which falls on the band in lumens.

35. As seen from a given direction, the luminous area of a lamp (projected on a plane at right angles to the line of sight) is 0.75 square inch and the conical intensity of the light in the given direction is 150 candle-power. What is the intrinsic brightness of the lamp?

36. Find the sectional intensity of the light from a 50-candle-power lamp at a distance of 5 feet from the lamp, expressing the result in lumens per square foot and in spherical-candles per square foot.

37. The intensity of illumination at a distance of four feet from a 16-candle lamp is sufficient for easy reading of ordinary book type. (*a*) Find the distance from a 20-candle lamp at which the lamp gives the same intensity of illumination; (*b*) express this intensity of illumination in spherical-candles of light per square foot of illuminated surface; and (*c*) express this intensity of illumination in luxes. Ans. (*a*) 4.47 feet; (*b*) 0.0796 spherical-candle per square foot; (*c*) 12.21 luxes.

38. The glow lamp which is used as a standard in a Bunsen photometer has a candle-power of 16.8 candles in the direction towards the photometer screen. Another lamp B is placed at the other end of the photometer bar and when the screen is adjusted to equality of illumination on both sides, it is 2.61 meters from the lamp B, and 1.80 meters from the standard lamp. What is the candle-power of B in the direction towards the screen? Ans. 35.3 candle-power.

Note.—This problem is to be solved with the help of the law of inverse squares. Equality of illumination on the two sides of the screen means that the beams from the two lamps have the same sectional intensity at the screen.

39. Find the intensity of illumination in foot-candles at a point on a floor distant 6 feet horizontally from a lamp which is 8 feet above the floor, the candle-power of the lamp in the direction towards the specified spot being 65.

40. Let the brightness of daylight with the sun in the zenith be taken as unity. What is the brightness of daylight when the altitude of the sun is 75°, 60°, 45°, 30° and 15°, above the horizon respectively, ignoring increase of atmospheric absorption with increase of zenith distance of the sun?

Note.—The relative brightness of daylight is inversely proportional to the area of country covered by a beam of sunlight of, say, one square mile in sectional area, and this is inversely proportional to the sine of the sun's altitude above the horizon.

41. A certain photographic lens gives a good photograph with an exposure of 1/50 second when the sun is 75° above the horizon. What exposure would be required with the same lens when the sun is 5° above the horizon, ignoring increase of atmospheric absorption with increase of zenith distance of sun?

42. A direct-current arc lamp gives the following distribution of candle-power:

Angle from vertical.	10°	20°	30°	40°	50°	60°	70°	80°
Candle-power........	290	440	670	1,080	1,220	1,080	795	580

Calculate the intensities of illumination at points along a level open street distant h tan 10°, h tan 20°, h tan 30°, etc., horizontally from the lamp: (*a*) When the height h of the lamp above the street is 15 feet, and (*b*) when the height h of the lamp above the street is 50 feet. Express the intensities of illumination in foot-candles.

Plot two curves showing horizontal distances from the lamp as abscissas and intensities of illumination as ordinates.

43. A beam of light consisting of parallel rays has a sectional intensity of 100 foot-candles. Find the conical intensity of the beam after it passes through a lens of which the focal length is 18 inches.

44. An open-arc lamp is placed at a distance of five feet from a converging lens, and an image of the arc is formed at a distance of one foot beyond the lens. The light from the lamp has a conical intensity of 2,500 candles. Assuming that the luminous surface of the lamp is negligibly small, and ignoring loss of light at the lens by reflection and absorption, find the conical intensity of the beam beyond the image. Ans. 100 candles.

45. Two lamps A and B are placed at the ends of a Bunsen photometer bar, and the photometer screen is adjusted to give equality of illumination on its two sides. The screen is then one meter from lamp A and 3 meters from lamp B. A lens of which the focal length is 25 centimeters is placed 50 centimeters from lamp B, and the screen is left in its original position. Find how far lamp A must be placed from the screen to give equal illumination on the two sides of the screen, neglecting losses of light in the lens. Ans. 0.67 meter.

46. An acetylene flame is placed at the focal point of a lens which is 8 inches in diameter and the focal length of the lens is 25 inches. The luminous area of the flame is 0.7 square inch. Find the approximate candle-power of the light beyond the lens.

47. The powerful arc lamp of a searchlight emits a beam of which the conical intensity is 10,000 candle-power. The luminous area of the arc is 0.1 square inch, the diameter of the searchlight lens is 12 inches and the focal length of the searchlight lens is 25 inches. What is the approximate conical intensity of the searchlight beam?

48. A 35.3-candle-power lamp is placed at a distance of 35 inches from the center of a large mirror which reflects the light from the lamp along a photometer bar towards the photometer screen, and when the screen is adjusted to give equal illumination on its two sides it is 92.5 inches from the standard lamp and 91 inches from the center of the mirror. The candle-power of the standard lamp is 16.8 in the direction towards the photometer screen. Find the factor by which the apparent candle-power of any lamp, when measured by the light reflected from the above mirror must be multiplied in order to correct for the loss of light at the mirror. Ans. 1.13.

49. Calculate the mean spherical candle-power of the bare glow-lamp from data given in Fig. 65. Ans. 13.33 candle-power.

Note.—Solve this problem by the method explained at the top of page 114.

50. Construct a Rousseau diagram for the candle-power curve shown in Fig. 65, and, if a planimeter is available, measure the area of curve $c'c'c'$ (see Fig. 72) and calculate the mean spherical candle-power of the lamp.

CHAPTER V.

ELECTRIC LAMPS. LAMP SHADES AND REFLECTORS.

51. A closet or cellar lamp is used on the average one minute per day and energy costs 10 cents per kilowatt-hour. Compare the yearly cost (including interest on cost of lamp) of using a 20-candle-power (50-watt) carbon-filament lamp and a 60-watt tungsten-filament lamp. Assume that the tungsten lamp is broken once every two years by rough handling.

Note.—The table on page 135 gives all the data required for the solution of this problem.

52. From the tables on pages 135 and 167 find the annual cost of producing 13,333 lumens 6 hours per day for 300 days per year (see example on page 166) by 60-watt tungsten lamps and by 50-watt metalized carbon lamps, the cost of energy being 8 cents per kilowatt-hour. Include interest at 6 per cent. per year on cost of lamps and assume no breakage due to rough handling.

53. Find the annual cost 13,333 lumens produced by 60-watt tungsten lamps for 1,800 hours per year when energy costs 2 cents per kilowatt-hour and when 8 cents per kilowatt-hour—lamps to be burned at top-efficiency and at bottom-efficiency in each case. Include interest on first cost of lamps at 6 per cent. and neglect breakage.

Note.—See tables on pages 167, 139 and 135.

CHAPTER VI.

INTERIOR ILLUMINATION.

54. Half the light from a lamp is reflected from the walls of a room. When this reflected light again strikes the walls half of it is reflected and so on. How much light is there crossing and re-crossing the room expressed in terms of the amount of light coming directly from the lamp?

55. The mean distance across the room in problem 54 is 20 feet. What fraction of a second elapses after turning on the lamps until the light in the room has eached 1023/1024 of its final steady value?

CHAPTER VII.

STREET ILLUMINATION.

55. Using the table on page 175 find the distance apart of lamps No. 3, No. 4 and No. 7 to give 0.05 foot-candle normal to beam (due to each lamp) at a point midway between the lamps when the lamps are hung 24 feet above the street.

56. Using the table on page 178 find distance apart of 350-candle-power tungsten lamps (equipped like Fig. 106) to give 0.05 foot-candle normal to beam (due to each lamp) at a point midway between lamps when the lamps are hung 24 feet above the street.

57. The dimensions of a certain city block are shown in Fig. 57*p*. Find the minimum mean value of normal illumination

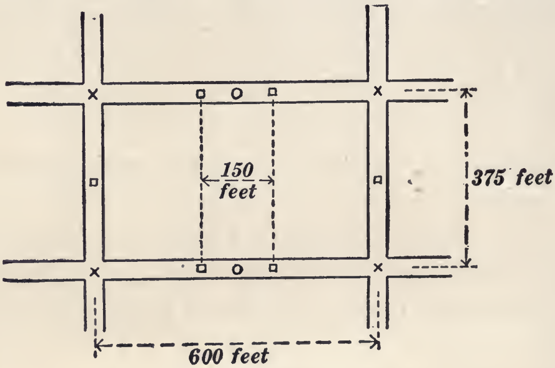


Fig. 57*p*.

(I_n) and find annual cost per mile of street with 4,000 hours of service per year and energy at 3 cents per kilowatt-hour:

(*a*) With 6.6 magnetite-arc lamps at crosses and 350-candle-power tungsten lamps at circles.

(*b*) With 4-ampere magnetite-arc lamps at crosses and 200-candle-power tungsten lamps at circles.

(*c*) With 200-candle-power tungsten lamps at crosses and 50-candle-power tungsten lamps at squares.

Note.—By mean value of I_n at a given point on the street it is here intended to refer to half the sum $I_n' + I_n''$, where I_n' and I_n'' refer to the two lamps between which the given point lies.

Assume all lamps to be 24 feet high and plot curve for I_n' and I_n'' with the help of the tables on pages 175 and 178. Then plot the curve of which the ordinates represent $\frac{1}{2}(I_n' + I_n'')$. Assume, in the absence of more exact data, that maintenance, first cost, interest and depreciation are the same for a 200-candle-power tungsten lamp as for a 350-candle-power lamp.

In reckoning the cost per mile of street note that four crosses and four circles (or four crosses and ten squares) represent 3,900 feet of street in Fig. 57*p*.

A serious objection to the use of many small lamps along a street is the cost of the lamp-supporting structure. The cost of this structure is not to be considered in this problem.

CHAPTER VIII.

ELECTROLYSIS AND BATTERIES.

58. The anode of an electrolytic cell is a copper rod 1 inch in diameter and the cathode is a hollow copper cylinder 6 inches inside diameter; the two electrodes are co-axial and they stand vertically in an electrolyte 8 inches deep. A current of 5 amperes flows through the cell. Find the current density at the cathode and the current density at the anode.

59. Calculate the number of cubic centimeters of oxygen and the number of cubic centimeters of hydrogen liberated in one hour by a current of one ampere; inert electrodes being used in dilute sulphuric acid or in a solution of potassium or sodium hydrate. The gases being reckoned dry at 0° C. and 760 millimeters pressure. Ans. 209 cubic centimeters of oxygen and 418 cubic centimeters of hydrogen.

Note.—The density of dry oxygen at 0° C. and 760 millimeters pressure is 0.00143 grams per cubic centimeter and the density of dry hydrogen at 0° C. and 760 millimeters prssure is 0.0000902 grams per cubic centimeter. The atomic weight of silver is 107.93 and the atomic weight of hydrogen is 1.01 (o = 16).

59. An electrolytic generator* for oxygen and hydrogen requires 3 volts per cell. Find the cost of one cubic foot of oxygen

* A good form of electrolytic generator for hydrogen and oxygen is described by W. S. Franklin, *Physical Review*, Vol. IV, pages 61–64, July, 1896. A large generator of this type *using cast iron frames and sodium hydrate solution* has been in use for about 13 years by the Nernst Lamp Company; and the depreciation and repairs have been negligibly small. The generator consists of 35 cells in series supplied with current from 110-volt mains. See *American Electrician*, Vol. XI, pages 526–527, November, 1899.

and two cubic feet of hydrogen when energy costs 2 cents per kilowatt-hour making no allowance for interest on cost of generators and no allowance for depreciation and repairs of generator.

60. The heat of combustion of one gram of hydrogen is 34,700 calories. What fraction of the energy delivered to the generator of problem 59 is represented by or tied up in the oxygen and hydrogen produced? Ans. 50.7 per cent.

61. The cost of gravity-cell zinc is, say, 6 cents per pound, and the cost of copper sulphate crystals ($CuSO_4 + 5H_2O$) is, say, 6.5 cents per pound. Half of the materials consumed in a gravity cell is wasted by local action and about one third of the zinc is left as scrap and is worth about 2 cents per pound. Furthermore the copper which is deposited on the cathode is worth, as scrap, about 10 cents per pound. The terminal electromotive force of the cell while it is delivering 0.16 ampere is about 0.72 volt. What is the cost per kilowatt-hour of the output of the cell making no allowance for cost of labor? Ans. $1.64.

62. The electromotive force of a gravity cell is 1.08 volts. Find the rise of temperature when 10 grams of finely divided zinc are stirred into 2,000 cubic centimeters (approximately 2,000 grams) of dilute copper sulphate solution.

Note.—The specific heat of dilute copper sulphate solution is sensibly the same as the specific heat of water. In a Daniell cell (gravity cell) arranged to have no local action the whole of the chemical energy is converted into electrical energy.

62. A lead storage cell delivers 10 amperes for 8 hours. Find the increase of weight of each electrode. Ans. The positive electrode gains 0.2105 pound and the negative electrode gains 0.3232 pound.

63. The storage cell specified in problem 62 contains 4,000 cubic centimeters of dilute sulphuric acid of which the density at 18° C. is 1.700 grams per cubic centimeter when the cell is fully charged. Find the density of the electrolyte (at 18° C.) after the cell has delivered 10 amperes for 8 hours. Ans. 1.1286 grams per cubic centimeter.

Note.—For the solution of this problem the following table is needed.

DENSITY OF DILUTE SULPHURIC ACID IN GRAMS PER CUBIC CENTIMETER AT 18° C.

Percentage Strength.	Density.
0	0.9986
10	1.0673
20	1.1414
30	1.221

Percentage strength in this table means the number of grams of H_2SO_4 in 100 grams of the solution.

To solve the problem find grams of H_2SO_4 and grams of H_2O in the solution at the beginning.

Then find grams H_2SO_4 taken from the solution and grams of H_2O given to the solution by the discharge.

Then find percentage strength of solution after discharge and find density from table.

INDEX.