ELECTRIC POWER CONDUCTORS

BY

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NEW YORK:
D. VAN NOSTRAND COMPANY
23 MURRAY AND 27 WARREN STS.
LONDON:
CROSBY LOCKWOOD & SON
7 STATIONERS' HALL COURT, LUDGATE HILL
1909

The Scientific Press
Robert Drummond and Company
New York

PREFACE

THE purpose of this book is to present, for the benefit of the users of power conductors, a clear account of all the engineering considerations which affect the purchase and use of such conductors.

The book will be found practical and up to date; being based upon notes prepared by the author for his own use, and there is nothing in the book which has been copied from any published data without having been thoroughly studied and found reliable.

The arrangement of the book follows the rational order of the series of engineering considerations which affect the purchase of conductors, namely, the determination of material, insulation, and size, the specifications, test, and installation.

The text is made as brief as possible, and where explanation or theoretical discussion is advisable, the text is supplemented by appendices.

The sections on Alternating Current Feeder Calculations and Stress in Spans, were written by Dr. Harold Pender, who also suggested the method of calculation given in the sections on Skin Effect and Kelvin's Law.

Dr. Pender's method of calculations are distinguished for their thorough adaptability to practical work with the minimum amount of labor and for their careful scientific foundation. The author, therefore, has pleasure in expressing his indebtedness to Dr. Pender for his valuable contributions.

The author also acknowledges the courtesy of Mr. W. W. Weaver of the *Electrical World* and of Mr. J. H. Smith of the *Electrical Age* in permitting the use of material from their respective journals.

W. A. Del Mar

New York, June, 1909

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ERRATA.

Page 1. 2d line of table, 3d column, change ".995" to "3.31."

Page 4. 22d line, change "4×6^6" to 4×10^6."

Page 14. 3d line, change $\left(\frac{D}{P}\right)^2$ to $\left(\frac{\pi D}{P}\right)^2$.

Page 27. 4th line from bottom, change "9.516" to "9.5916."

Page 49. To the note at the bottom of Table B, add "except for smaller sizes than No. 0, B. & S., where the divergence between experiments is greater."

Page 125. Cancel entire paragraph following words "Appendix IV," from "If" to "practice" inclusive.

Page 284. Line after first formula "8π" should be "0.008π."

Page 292. Table I, column headed "Error Thickness," read in reverse order, i.e., 5/64 in. heading the table and 3/128ths in. ending it.

Page 296. 13th line, between "4000" and "respectively," add "millions."

Page 298. Top of last column, the number "0.315625," change to "0.515625."

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ELECTRIC POWER CONDUCTORS

CHAPTER I

MATERIALS AND GAUGES

1. MATERIALS

COMPARISON OF ALUMINUM AND COPPER

General Properties.

	Aluminum.	Copper (Hard Drawn).	Copper, Soft Drawn.
Specific gravity.	2.68	8.93	8.89
Relative specific gravity.	1.00	3.33	0.995
Conductivity (Matthiessen's Standard).	61 to 63	96 to 99	99 to 102
Elastic limit, solid wire (lbs. per sq.in.).	14,000	35,000 to 40,000	3000 to 5000
Coefficient of expansion per degree F.	0.000,012,8	0.000,009,6	0.000,009,6
Modulus of elasticity, solid wire	7.5 to 9×10^6	8 to 16×10^6	. . . .
Melting point (about).	1200° F.	2000° F.	2000° F.
Lbs. per cu.in.	0.097	0.32	0.32
Tensile strength, solid wire, lbs. per sq.in.	20,000 to 35,000	45,000 to 68,000	25,000 to 45,000

Comparison of Aluminum and Copper of Equal Length and Conductance.

s = specific gravity of aluminum;
S = specific gravity of copper;
c = conductivity of aluminum;
C = conductivity of copper;
t = tensile strength of aluminum, lbs. per sq.in.;
T = tensile strength of copper, lbs. per sq.in.;
p = price of aluminum, per lb.
P = price of copper, per lb.

Then to compare a given aluminum wire with a copper wire of equal length and conductance,

Relative cost, $\frac{\text{Aluminum}}{\text{Copper}} = \frac{spC}{SPc}$

Relative cross-section, $\frac{\text{Aluminum}}{\text{Copper}} = \frac{C}{c}$

Relative diameter, $\frac{\text{Aluminum}}{\text{Copper}} = \sqrt{\frac{C}{c}}$

Relative weight, $\frac{\text{Aluminum}}{\text{Copper}} = \frac{sC}{Sc}$

Relative breaking strength, $\frac{\text{Aluminum}}{\text{Copper}} = \frac{tC}{Tc}$

Relative current carrying capacity, $\frac{\text{Aluminum}}{\text{Copper}} = \sqrt[4]{\frac{C}{c}}$

The following table is calculated for $s = 2.68$, $S = 8.93$, $t = 25{,}000$, and $T = 55{,}000$:

Conductivity (Matthiessen's Standard).	Copper.	Aluminum			
	98	63	62	61	60
Relative cost	1.00	$0.467p$	$0.474p$	$0.482p$	$0.489p$
Relative cross-section	1.00	1.556	1.581	1.606	1.633
Relative diameter	1.00	1.247	1.258	1.268	1.278
Relative weight	1.00	0.467	0.474	0.482	0.489
Relative breaking strength	1.00	0.708	0.719	0.731	0.743
Relative current carrying capacity *	1.00	1.117	1.121	1.126	1.130

* For wires of the same diameter aluminum will carry only 80% of the current carried by copper.

Advantages of Aluminum Compared with Copper.

(1) For equal conductance aluminum is cheaper. In the United States the price is held about 10% less than that of copper.

(2) For equal conductance aluminum is lighter and therefore easier to string.

(3) Sleet does not adhere so readily as to copper.

Disadvantages of Aluminum Compared with Copper.

(1) Aluminum wire must be strung with a greater sag than copper wire of equal conductance due to its lower tensile strength and greater surface exposed to wind and sleet. For long spans higher towers are therefore required.

(2) Low melting-point makes wire more liable to break off under influence of an arc either at the insu-

lators or when foreign wires fall on the line. Wires must therefore be placed further apart, necessitating the use of longer cross arms.

(3) Scrap value very small on account of artificial price of new product.

(4) Aluminum is much softer than copper; greater care must therefore be observed in stringing to avoid denting or abrasion.

TENSILE STRENGTH AND ELASTIC PROPERTIES OF COPPER

The properties of commercial hard-drawn copper seldom resemble those given in the old text-books, as the commercial article used for aerial power wires is much softer than that usually described as hard-drawn copper. The modulus of elasticity instead of being 16×10^6 (in lb.-in. units) varies from 8×10^6 to 16×10^6; the tensile strength instead of being over 60,000 lbs. per sq.in., varies from 45,000 to 68,000. The point where the strain ceases to be proportional to the stress, called the elastic limit, varies from 35,000 to 45,000 lbs. per sq.in., 38,000 being a value easy to obtain. These values apply to solid wire; for stranded cables the modulus of elasticity varies from 4×10^6 to 12×10^6, the tensile strength from 45,000 to 60,000 lbs. per sq.in.; the elastic limit from 25,000 to 35,000 lbs. per sq.in.

If the elastic limit is considerably exceeded, the wire becomes so attenuated that the actual stress, i.e., the force per sq.in. of actual section gradually increases, and ultimately teaches a value sufficient to break the wire. Therefore a stress considerably under the nominal breaking stress will break a wire if continued for a sufficient length of time. Working a wire having 60,000 lbs. per sq.in. ultimate strength, at a stress of 10,000 lbs. per sq.in., therefore gives an actual safety factor of less than six instead of six, as is usually computed.

The hardness of copper depends upon the amount of drawing it has been subjected to, and all degrees of hardness are obtainable from soft annealed copper to the hard material used for telephone wires. Telephone wires can be made very hard because they are drawn to such a small size. It is therefore important to take into account the size of wire in specifying its degree of hardness and the various mechanical properties dependent thereon. This is well illustrated by the curves of Fig. 1.

Curve *A* is what is usually called half-hard drawn and curve *D* is a very hard-drawn telephone wire of 1/10 inch diameter, having an elastic limit of 50,000 lbs. and an ultimate strength of 69,000 lbs. per sq.in. with an elongation of 1%.

It should be noted that in hard-drawn copper of various degrees of hardness, the elongation at the elastic limit is usually about ½%, whatever the modulus of elasticity.

Soft-drawn copper cannot be used alone in tension on account of its low elastic limit, about 3000 to 5000 lbs. per sq.in. It is used with hard-drawn copper wires for the cores of concentric cables, where a knowledge of its stresses under various elongations is essen-

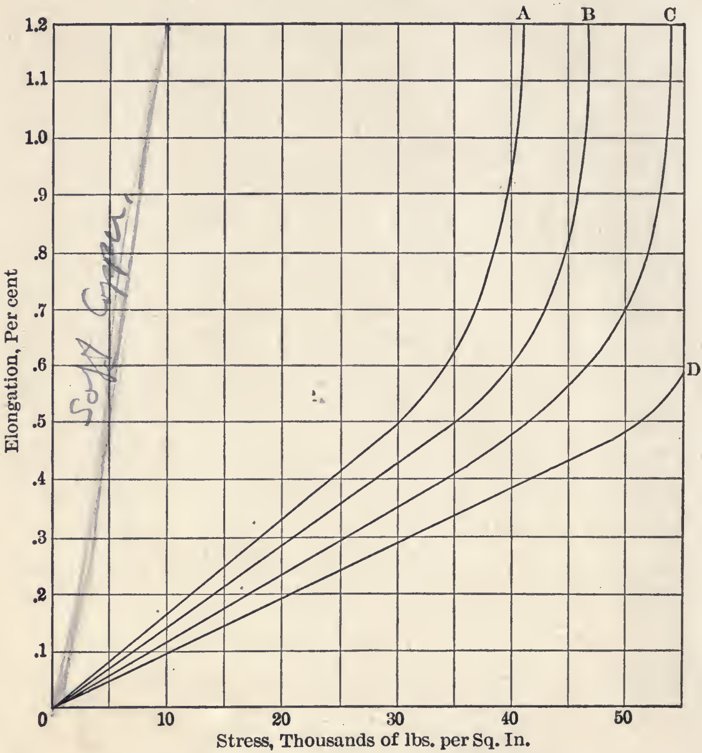


FIG. 1.—Typical Stress-Strain Diagrams, Hard Drawn Copper Wire.

tial for the calculation of the strength of the cable. Fig. 2 is a typical stress strain diagram for commercial soft-drawn copper, and is based on the following table:

	Lbs. per Sq.in. of Original Area Elastic Limit.	Elongation Per Cent of Original Length.
	3,000	.2
	5,000	.4
	10,000	1.1
	15,000	2.1
	20,000	3.5
	25,000	5.0
	30,000	6.7
	35,000	9.0
	40,000	12.5
	41,000	13.6
	41,500	15.0
Ultimate strength	42,000	45.0

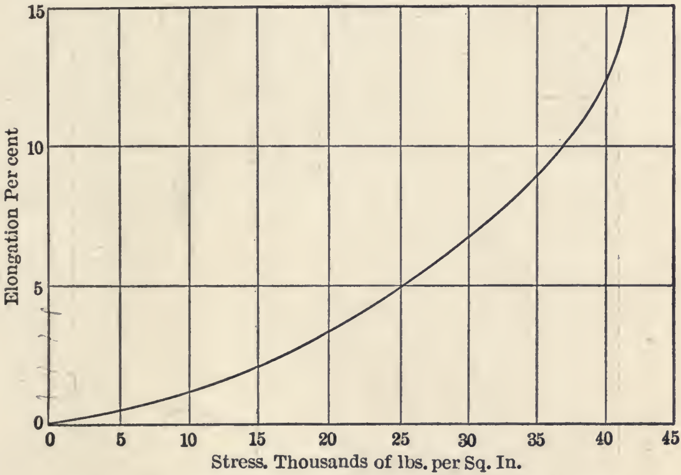


FIG. 2.—Typical Stress-Strain Diagram, Soft Drawn Copper.

The ultimate strength of soft-drawn copper is of no practical importance as, when the elastic limit is somewhat exceeded and the load maintained, the wire stretches until it breaks. The ultimate strength varies from 25,000 to 45,000 lbs. per sq.in. with an elongation of from 25% to 45%.

Wire used for the core of hard-drawn cables frequently has an ultimate strength of about 45,000 lbs. per sq.in., with an elongation of 8% to 10%. The elastic limit of such wire is about 20,000 lbs. per sq.in. and the modulus 8 to 10 millions.

2. SOLID WIRES.

RATING OF WIRES

American or Brown and Sharpe Gauge

A.W.G. B. & S.	Diameter. Inches.	Area.		Copper.		Aluminum.	
		Circular Mils.	Square Mils.	Lbs. per Foot.	Feet per Lb.	Lbs. per Foot.	Feet per Lb.
0000	0.460	211,600	166,190	0.6405	1.561	0.1929	5.185
000	0.4096	167,800	131,790	0.5080	1.969	0.1529	6.539
00	0.3648	133,100	104,518	0.4028	2.482	0.1213	8.246
0	0.3249	105,500	82,887	0.3195	3.130	0.09618	10.40
1	0.2893	83,690	65,732	0.2533	3.947	0.07629	13.11
2	0.2576	66,370	52,128	0.2009	4.977	0.06050	16.53
3	0.2294	52,630	41,339	0.1593	6.276	0.04797	20.85
4	0.2043	41,740	32,784	0.1264	7.914	0.03805	26.28
5	0.1819	33,100	25,999	0.1002	9.980	0.03017	33.15
6	0.1620	26,250	20,618	0.07946	12.58	0.02393	41.79
7	0.1443	20,820	16,351	0.06302	15.87	0.01898	52.69
8	0.1285	16,510	12,967	0.04998	20.01	0.01505	66.44
9	0.1144	13,090	10,283	0.03963	25.23	0.01193	83.82
10	0.1019	10,380	8,155	0.03143	31.82	0.009462	105.7
11	0.09074	8,234	6,467	0.02493	40.12	0.007505	133.2
12	0.08081	6,530	5,129	0.01977	50.59	0.005952	168.0
13	0.07196	5,178	4,067	0.01568	63.79	0.004720	211.9
14	0.06408	4,107	3,225	0.01243	80.44	0.003743	267.2
15	0.05707	3,257	2,558	0.009858	101.4	0.002968	336.9
16	0.05082	2,583	2,029	0.007818	127.9	0.002354	424.8
17	0.04526	2,048	1,609	0.006200	161.3	0.001867	535.6
18	0.04030	1,624	1,276	0.004917	203.4	0.001480	675.7
19	0.03589	1,288	1,012	0.003899	256.5	0.001174	851.8
20	0.03196	1,022	802	0.003092	323.4	0.000931	1074.1

COMBINATION OF WIRES APPROXIMATELY EQUIVALENT TO ONE WIRE

(Based upon approximate equivalence of $\sqrt[6]{2}$ and $\sqrt[39]{92}$.)

B. & S. No.	2 of B. & S. No.	4 of B. & S. No.	8 of B. & S. No.	16 of B. & S. No.	32 of B. & S. No.	64 of B. & S. No.	One Each of B. & S. Nos.
0000	0	3	6	9	12	15	
000	1	4	7	10	13	16	
00	2	5	8	11	14	17	1 and 3
0	3	6	9	12	15	18	2 " 4
1	4	7	10	13	16	....	3 " 5
2	5	8	11	14	17	....	4 " 6
3	6	9	12	15	18	....	5 " 7
4	7	10	13	16	....	....	6 " 8
5	8	11	14	17	....	....	7 " 9
6	9	12	15	18	....	....	8 " 10
7	10	13	16	....	....	....	9 " 11
8	11	14	17	....	....	....	10 " 12
9	12	15	18	....	....	....	11 " 13
10	13	16	....	....	....	....	12 " 14
11	14	17	....	....	....	....	13 " 15
12	15	18	....	....	....	....	14 " 16
13	16	....	....	....	....	....	15 " 17
14	17	....	....	....	....	....	16 " 18
15	18						

Circular Mils. A circular mil is the area of a circle of 1 mil (thousandth of an inch) diameter. The area of any conductor in circular mils is equal to the square of its diameter in mils, or one million times the square of its diameter in inches.

$$\frac{\text{one square mil}}{\text{one circular mil}} = \frac{4}{\pi} = 1.27.$$

BIRMINGHAM OR STUBB'S WIRE GAUGE

B. W G. Stubb's.	Diameter. Inches.	Area.		Lbs. per Foot. Copper.
		Circular Mils.	Sq. Mils.	
0000	0.454	206,100	161,883	0.6239
000	0.425	180,600	141,863	0.5468
00	0.380	144,400	113,411	0.4371
0	0.340	115,600	90,792	0.3499
1	0.3000	90,000	70,686	0.2724
2	0.2840	80,660	63,347	0.2441
3	0.2590	67,080	52,685	0.2031
4	0.2380	56,640	44,488	0.1715
5	0.2200	48,400	38,013	0.1465
6	0.2030	41,210	32,365	0.1247
7	0.1800	32,400	25,447	0.09808
8	0.1650	27,230	21,382	0.08241
9	0.1480	21,900	17,203	0.06630
10	0.1340	17,960	14,103	0.05435
11	0.1200	14,400	11,310	0.04359
12	0.1090	11,880	9,331	0.03596
13	0.0950	9,025	7,088	0.02732
14	0.08300	6,889	5,411	0.02085
15	0.07200	5,184	4,072	0.01569
16	0.06500	4,225	3.318	0.01279
17	0.0580	3,364	2,642	0.01018
18	0.04900	2,401	1,886	0.007268
19	0.04200	1,764	1,385	0.005340
20	0.03500	1,225	962	0.003708

TABLE OF COMPARATIVE SIZES OF WIRE GAUGES, IN DECIMALS OF AN INCH

No. of Wire Gauge.	Brown & Sharpe.	American Steel & Wire Co. or Washburn & Moen.	Birmingham or Stubb's.	English Legal Standard.	Old English or London.
0000000	....	0.4900	.....	0.500	.....
000000	0.58000	0.4615	.....	0.464	.....
00000	0.51650	0.4305	0.500	0.432	.....
0000	0.46000	0.3938	0.454	0.400	0.454
000	0.40964	0.3625	0.425	0.372	0.425
00	0.36480	0.3310	0.380	0.348	0.380
0	0.32495	0.3065	0.340	0.324	0.340
1	0.28930	0.2830	0.300	0.300	0.300
2	0.25763	0.2625	0.284	0.276	0.284
3	0.22942	0.2437	0.259	0.252	0.259
4	0.20431	0.2253	0.238	0.232	0.238
5	0.18194	0.2070	0.220	0.212	0.220
6	0.16202	0.1920	0.203	0.192	0.203
7	0.14428	0.1770	0.180	0.176	0.180
8	0.12849	0.1620	0.165	0.160	0.165
9	0.11443	0.1483	0.148	0.144	0.148
10	0.10189	0.1350	0.134	0.128	0.134
11	0.09074	0.1205	0.120	0.116	0.120
12	0.08081	0.1055	0.109	0.104	0.109
13	0.07196	0.0915	0.095	0.092	0.095
14	0.06408	0.0800	0.083	0.080	0.083
15	0.05706	0.0720	0.072	0.072	0.072
16	0.05082	0.0625	0.065	0.064	0.065
17	0.04525	0.0540	0.058	0.056	0.058
18	0.04030	0.0475	0.049	0.048	0.049
19	0.03589	0.0410	0.042	0.040	0.040
20	0.03196	0.0348	0.035	0.036	0.035

TABLE OF COMPARATIVE SIZES WIRE GAUGE, IN DECIMALS OF AN INCH—*Continued.*

No. of Wire Gauge.	Brown & Sharpe.	American Steel & Wire Co. or Washburn & Moen.	Birmingham or Stubb's.	English Legal Standard.	Old English or London.
21	0.02846	0.03175	0.032	0.032	0.0315
22	0.02535	0.0286	0.028	0.028	0.0295
23	0.02257	0.0258	0.025	0.024	0.0270
24	0.02010	0.0230	0.022	0.022	0.0250
25	0.01790	0.0204	0.020	0.020	0.0230
26	0.01594	0.0181	0.018	0.018	0.0205
27	0.01420	0.0173	0.016	0.0164	0.01875
28	0.01264	0.0162	0.014	0.0148	0.01650
29	0.01126	0.0150	0.013	0.0136	.001550
30	0.01003	0.0140	0.012	0.0124	0.01375
31	0.00893	0.0132	0.010	0.0116	0.01225
32	0.00795	0.0128	0.009	0.0108	0.01125
33	0.00708	0.0118	0.008	0.0100	0.01025
34	0.00630	0.0104	0.007	0.0092	0.00950
35	0.00561	0.0095	0.005	0.0084	0.00900
36	0.00500	0.0090	0.004	0.0076	0.00750
37	0.00445	0.0085	.....	0.0068	0.00650
38	0.00396	0.0080	.....	0.0060	0.00575
39	0.00353	0.0075	.....	0.0052	0.00500
40	0.00314	0.0070	.....	0.0048	0.00450

The Edison Gauge is the area in circular mils divided by one thousand.

3. MECHANICAL PROPERTIES OF CABLES

The terminology relating to electric cables having evolved out of that used for ages in connection with ordinary rope, is unsatisfactory and indefinite. It is therefore necessary to define our terms before considering the properties of electric cables. The following definitions are based principally on common usage.

Cable. A conductor composed of a number of wires twisted together.

Strand. A conductor composed of a straight central wire surrounded by one or more layers of spirally laid wires. This construction is frequently called a *concentric strand.*

Rope Strand. A conductor composed of a straight central strand surrounded by one or more layers of spirally laid strands. For example, a rope-stranded cable may be built up of seven strands, each strand of nineteen wires, such a cable is briefly described as a 19×7 cable.

Stranding or Laying. The process or method of twisting the wires or strands into a cable.

Pitch. The length, measured along the cable axis, of a complete turn of a strand of cable. The word lay is sometimes used instead of pitch. Thus the standard pitch recommended by the British Institution of Electrical Engineers is defined as "a lay of twenty times the pitch diameter."

Pitch Diameter. The diameter of the spiral made by the axis of a wire or strand.

The following definition is suggested by the author:

Pitch Factor. The ratio of the length of a wire to the corresponding axial length of the cable.

If D = pitch diameter;

P = pitch; then,

$$\text{Pitch Factor} = \sqrt{1 + \left(\frac{D}{P}\right)^2}.$$

Number of Wires in Cables. Concentric-strand cables are made up of 7, 19, 37, 61, 91, 127, etc., wires, the numbers being obtainable by the following formula, in which n is the number of layers over the core.

The number of wires $= 3(n^2 + n) + 1$.

The number of wires per layer increases by six for each successive layer; thus, the first layer has six, the second twelve, the third eighteen, and so on.

Rope-strand cables are usually made of strands composed of seven wires each. The number of strands in such a cable follows the same law as the number of wires in a concentric-strand cable. The total number of wires therefore follows the following law:

Core $= 7$.
1 layer $= 7 \times 7$.
2 layers $= 7 \times 19$.
3 " $= 7 \times 37$, etc.
n " $= 7 \times [3(n^2 + n) + 1]$.

If the rope is made up of strands composed of any other number of wires, that number should be used in the place of the seven in the above formula.

WIRES IN CONCENTRIC CABLES

Number of Layers over Core.	Core of One Wire.		Core of Two Wires.		Core of Three Wires.		Core of Four Wires.	
	Per Layer.	Total.	Per Layer.	Total.	Per Layer.	Total.	Per Layer.	Total.
1	6	7	8	10	9	12	10	14
2	12	19	14	24	15	27	16	30
3	18	37	20	44	21	48	22	52
4	24	61	26	70	27	75	28	80
5	30	91	32	102	33	108	34	114
6	36	127	38	140	39	147	40	154

Cables having more than one wire in the core are seldom used.

WIRES IN ROPE CABLES

Number of Layers over Core.	Number of Strands.	Total Number of Wires.		
		7 Wires per Strand.	19 Wires per Strand.	37 Wires per Strand.
1	7	49	133	259
2	19	133	361	703
3	37	259	703	1369
4	61	427	1159	2257
5	91	637	1729	3367
6	127	889	2413	4699

Diameter of Cables. It being standard practice to run alternate layers in opposite directions, the wires cannot fit into the grooves between the other wires, and the total diameter is therefore

$$d(1+2n),$$

where d is the diameter of each wire or strand, and n the number of layers over the core.

Thus a 5 layer 91 wire cable composed of wires of 0.1048 in. diameter, has a total diameter of

$$0.1048(1+10)=1.1528 \text{ in.}$$

WEIGHT OF CABLES

Number of Layers.	Wires or Strands in Cable.	Weight of Cable, Lbs. per Foot.
1	7	$w(1+6p_6)$
2	19	$w(1+6p_6+12p_{12})$
n	$3(n^2+n)+1$	$w(1+6p_6+12p_{12}+$, etc.)

w = weight of each wire or strand, lbs. per foot;
p_6 = pitch factor of first or 6 wire layer;
p_{12} = pitch factor of second or 12 wire layer, etc.
(Definition of pitch factor on page 13.)

Pitch. The British standard pitch is twenty times the pitch diameter, and is the only standard pitch agreed upon by any large body of manufacturers. In America there is no standard pitch, this being usually left to the manufacturers.

The cable user is interested in obtaining the largest pitch with which the wires will hold together and that obviously depends upon the size and number of wires and upon their stiffness. The longer the pitch the greater the conductance and tensile strength.

The cable manufacturers, on the other hand, generally prefer a short pitch. The pitch to be used should therefore be agreed upon by manufacturers and buyers when specifications are to be prepared. For cables of hard-drawn copper for aerial lines, a pitch of from twenty to thirty-five times the pitch diameter is usual practice.

Minimum Pitch. The minimum pitch or lay with which n wires of diameter d can be coiled spirally on a pitch diameter D, is

$$\frac{\pi D . nd}{\sqrt{(\pi D)^2 - (nd)^2}}.$$

In the case of regular concentric cables in which successive layers have 6, 12, 18, etc., wires, the minimum pitch is 10.1 times the pitch diameter if all the wires are of equal size. The constant 10.1 equals $\frac{3\pi}{\sqrt{(\pi+3)(\pi-3)}}$.

Ultimate Strength of a Seven-Wire Strand with Soft Core.

Let p = pitch factor of six-wire layer;

d = diameter of each wire (in.);

t = tensile strength of outer wires, lbs. per sq. in.;

e = elongation, per cent, at which outer wires break;

s = stress in lbs. per sq.in. in core with elongation e (see Fig. 2, p. 7, for soft-drawn copper).

$$\text{Ultimate strength (lbs.)} = \frac{\pi}{4} d^2 \left(s + \frac{6t}{p} \right).$$

Ultimate Strength of a Nineteen-Wire Strand with Soft Core.

Let p_6 = pitch factor of six-wire layer;

p_{12} = pitch factor of twelve-wire layer;

d = diameter of each wire (in.);

t = tensile strength of outer wires, lbs. per sq.in.;

e = elongation, per cent, at which outer wires break;

s = stress, lbs. per sq.in. in core with elongation e (Fig. 2, p. 7, for soft-drawn copper).

$$\text{Ultimate strength (lbs.)} = \frac{\pi}{4} d^2 \left(s + \frac{6t}{p_6} + \frac{12t}{p_{12}} \right).$$

With a 37-wire strand, the bracketed expression should have a term for the 18-wire layer, namely, $\frac{18t}{p_{18}}$, and so on, for all sizes.

Space Wasted in Concentric-Strand Cables.

n = number of concentric layers around one central wire;

R = ratio of copper area to area of circle circumscribing the outside of cable;

$$R = \frac{3(n^2 + n) + 1}{(2n + 1)^2}.$$

This neglects the increase of ratio due to wires being arranged in spiral form.

Number of Layers.	Number of Wires.	*R.*
–	1	1.000
1	7	0.778
2	19	0.760
3	37	0.755
4	61	0.753
5	91	0.752

RESISTANCE AND WEIGHT OF STANDARD BRITISH CABLES

Wires in Cable.	Ratio of Resistance of Cable, to Resistance of One Wire.	Ratio of Weight of Cable, to Weight of One Wire.
7	0.14436	7.0736
19	0.05324	19.2207
37	0.02735	37.4414
61	0.01659	61.7356
91	0.01112	92.1034

Based upon the British Institution of Electrical Engineers' Standard of a lay or pitch of twenty times the pitch diameter which corresponds to a pitch factor of 1.0122. Both the weight and resistance of the strand are about one per cent higher than for a solid wire of same cross section.

DIAMETER OF WIRES IN STRANDS

Size of Cable.	Number of Wires in Strand.					
	7.	19.	37.	61.	91.	127.
Circ. Mils.						
2,000,000	0.5345	0.3244	0.2324	0.1811	0.1482	0.1255
1,750,000	0.5000	0.3035	0.2175	0.1694	0.1387	0.1174
1,500,000	0.4629	0.2810	0.2013	0.1568	0.1284	0.1087
1,250,000	0.4226	0.2565	0.1838	0.1431	0.1173	0.0992
1,000,000	0.3779	0.2294	0.1644	0.1281	0.1048	0.0887
750,000	0.3273	0.1986	0.1428	0.1109	1.0908	0.0769
500,000	0.2673	0.1622	0.1162	0.0906	0.0661	0.0628
250,000	0.1889	0.1147	0.0822	0.0640	0.0524	
B. & S.						
0000	0.1739	0.1055	0.07563	0.0589		
000	0.1548	0.09398	0.0674	0.0525		
00	0.1379	0.08369	0.060			
0	0.1228	0.07453				
1	0.1094	0.06637				
2	0.0974	0.05911				
3	0.0867					
4	0.0772					

DIMENSIONS AND WEIGHTS OF CABLES

Copper and Aluminum

Size.	Number of Wires in Strand.	Diameter of Individual Wires in Inches.	Diameter of Bare Cables in Inches.	Approximate Weight of Copper per 1000 Ft. in Lbs.	Approximate Weight of Aluminum per 1000 Ft. in Lbs.
B. & S.					
14	7	0.0243	0.0729	13	3.87
12	7	0.0306	0.0918	20	5.95
10	7	0.0386	0.1158	32	9.54
8	7	0.0485	0.1455	51	15.2
6	7	0.0613	0.1839	81	24.1
5	7	0.0688	0.2064	101	30.2
4	7	0.0773	0.2319	128	38.5
3	7	0.0867	0.2604	161	48.5
2	7	0.0974	0.2922	203	61
1	19	0.0664	0.3320	256	77
0	19	0.0745	0.3750	323	97
00	19	0.0837	0.4190	408	123
000	19	0.094	0.4700	514	155
0000	19	0.1055	0.5280	647	195
CM.					
250,000	37	0.0822	0.5754	765	239
300,000	37	0.0906	0.6342	919	276
350,000	37	0.0974	0.6818	1070	322
400,000	37	0.104	0.7280	1220	368
450,000	37	0.111	0.7770	1380	414
500,000	61	0.0906	0.8154	1530	460
550,000	61	0.095	0.8550	1680	506
600,000	61	0.0992	0.8928	1840	552
650,000	61	0.1033	0.9297	1990	597
700,000	61	0.1072	0.9648	2140	643
750,000	61	0.1109	0.9990	2300	690
800,000	61	0.1146	1.031	2450	735
900,000	61	0.1216	1.094	2750	834
1,000,000	61	0.1281	1.153	3060	920
1,000,000	91	0.1048	1.153	3030	924
1,250,000	91	0.1173	1.290	3830	1150
1,500,000	91	0.1284	1.412	4590	1380
1,750,000	127	0.1174	1.526	5360	1610
2,000,000	127	0.1255	1.631	6120	1840
2,000,000*	133	0.1226	1.84	6220	1850

* Rope.

The above figures should be regarded as approximate only, as the cable diameters and weights depend upon the pitch of the spirals.

An allowance of 1% is made for increase of weight due to spiralling.

The size of area is based upon the united areas of the individual wires cut at right angles to their axes and laid out straight.

CHAPTER II

ELECTRICAL PROPERTIES OF CONDUCTORS

1. RESISTANCE OF WIRES AND CABLES

MATTHIESSEN'S STANDARD

The recognized standard of conductivity of copper wire is that established by Matthiessen, from experiments on pure copper. Matthiessen's standard for soft-drawn copper is that a wire one meter long, of uniform cross-section, weighing one gram, has a resistance of 0.141729 ohm at 0° C.

While Matthiessen's standard is often reached and even exceeded in commercial copper, it is usual to accept soft-drawn copper having 98% and hard-drawn copper having 97% of the above standard conductivity.

Matthiessen's special standard for hard-drawn copper is not used in America.

The conductivity of aluminum is from 55% to 63% of Matthiessen's standard for copper, the usual commercial figure being 62%, which is equivalent to 15.47 ohms per mil-foot at 0° C.

The variation of resistance with temperature, both for copper and aluminum, is about 0.42% per degree Centigrade or 0.23% per degree Fahrenheit.

RESISTANCE OF A MIL-FOOT OF COPPER, OHMS

(One circular mil area, 1 foot long.)

Temperature Degrees.		Per Cent Conductivity—Matthiessen.				
Cent.	Fahr.	100.	99.	98.	97.	96.
0	32	9.59	9.69	9.79	9.89	9.99
10	50	9.99	10.1	10.2	10.3	10.4
15.5	60	10.2	10.3	10.4	10.5	10.6
20	68	10.4	10.5	10.6	10.7	10.8
24	75.2	10.6	10.7	10.8	10 9	11.0
30	86	10.8	10.9	11.0	11.1	11.2
40	104	11.2	11.3	11.4	11.5	11.7
50	122	11.6	11.7	11.8	12.0	12.1
60	140	12.0	12.1	12.2	12.4	12.5
70	158	12.4	12.5	12.7	12.8	12.9
80	176	12.8	12.9	13.1	13.2	13.3
90	194	13.2	13.4	13.5	13.6	13.8
100	212	13.6	13.7	13.9	14.0	14.2

Based on Matthiessen's Standard, 9.5916 ohms per mil-foot at 0° C. and the A.A.I.E E. temperature coefficient, 0.0042 from 0° C.

For any other percentage conductivity divide the number in the column headed 100 by the conductivity expressed as a decimal fraction. For example, the ohms per mil-foot for aluminum of 62% conductivity at 70° C. is $\frac{12.4}{0.62} = 20.0$.

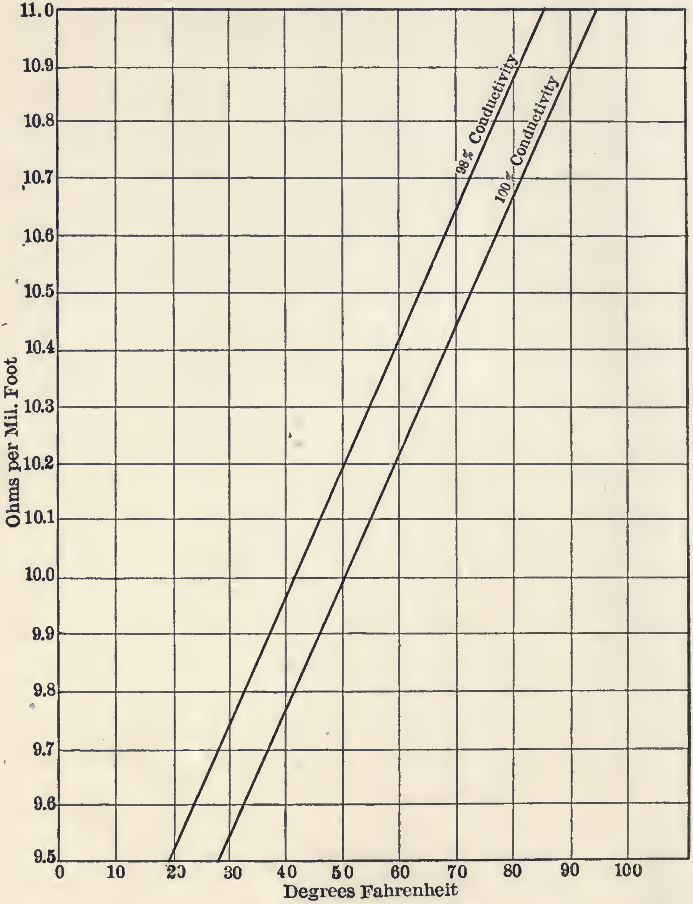


FIG. 3.—Resistance of Copper. Based on Standards adopted by A.I.E.E.

RESISTANCE OF SOLID COPPER WIRE

CONDUCTIVITY 100 PER CENT—MATTHIESSEN'S STANDARD

Ohms per 1000 Feet.

Size.	0° C. 32° F.	10° C. 50° F.	20° C. 68° F.	50° C. 122° F.
Millions of C.M.				
5	0.001918	0.001999	0.002079	0.002321
4	0.002398	0.002499	0.002599	0.002901
3	0.003197	0.003331	0.003466	0.003869
2	0.004796	0.004997	0.005199	0.005803
1¾	0.005481	0.005711	0.005941	0.006632
1½	0.006394	0.006663	0.006932	0.007737
1¼	c.007673	0.007996	0.008318	0.009285
1	0.009592	0.009994	0.01040	0.01161
¾	0.01279	0.01333	0.01386	0.01547
½	0.01918	0.01999	0.02079	0.02321
¼	0.03837	0.03998	0.04159	0.04642
B. & S.				
0000	0.04528	0.04718	0.04909	0.05479
000	0.05716	0.05956	0.06196	0.06916
00	0.07207	0.07510	0.07813	0.08721
0	0.09089	0.09470	0.09852	0.1100
1	0.1146	0.1194	0.1242	0.1387
2	0.1445	0.1506	0.1566	0.1749
3	0.1822	0.1899	0.1975	0.2205
4	0.2298	0.2394	0.2491	0.2780
5	0.2898	0.3019	0.3141	0.3506
6	0.3654	0.3807	0.3961	0.4421
7	0.4608	0.4801	0.4995	0.5575
8	0.5810	0.6054	0.6297	0.7029
9	0.7325	0.7633	0.7941	0.8863
10	0.9239	0.9627	1.001	1.118
11	1.165	1.214	1.263	1.410
12	1.469	1.531	1.592	1.777
13	1.852	1.930	2.008	2.241
14	2.335	2.434	2.532	2.826
15	2.945	3.069	3.192	3.563
16	3.713	3.869	4.025	4.493
17	4.683	4.880	5.077	5.667
18	5.906	6.154	6.402	7.146

Based upon Matthiessen's Standard of 9.5916 ohms per mil-foot at 0° C. and the A.I.E.E. temperature coefficient of 0.0042 per degree Centigrade temperature rise above 0° C.

Resistance at t° C. is equal to that at zero multiplied by $(1 + 0.0042t)$.

RESISTANCE OF SOLID COPPER WIRE

Conductivity 98 per cent—Matthiessen's Standard

Ohms per 1000 Feet.

Size.	0° C. 32° F.	10° C. 50° F.	20° C. 68° F.	50° C. 122° F.
Millions of C.M				
5	0.001957	0.002040	0.002122	0.002369
4	0.002447	0.002550	0.002652	0.002961
3	0.003262	0.003400	0.003536	0.003948
2	0.004894	0.005099	0.005305	0.005921
1¾	0.005593	0.005828	0.006063	0.006767
1½	0.006525	0.006799	0.007073	0.007895
1¼	0.007830	0.008159	0.008488	0.009474
1	0.009787	0.01020	0.01061	0.01184
¾	0.01305	0.01360	0.01415	0.01579
½	0.01957	0.02040	0.02122	0.02369
¼	0.03915	0.04079	0.04244	0.04737
B. & S.				
0000	0.04621	0.04820	0.05009	0.05597
000	0.05833	0.06078	0.06323	0.07057
00	0.07355	0.07663	0.07972	0.08899
0	0.09274	0.09664	0.1005	0.1122
1	0.1169	0.1219	0.1268	0.1415
2	0.1475	0.1537	0.1598	0.1784
3	0.1860	0.1938	0.2016	0.2250
4	0.2345	0.2443	0.2542	0.2837
5	0.2957	0.3081	0.3205	0.3578
6	0.3728	0.3885	0.4042	0.4511
7	0.4702	0.4899	0.5097	0.5689
8	0.5928	0.6177	0.6426	0.7173
9	0.7475	0.7789	0.8103	0.9044
10	0.9427	0.9823	1.022	1.141
11	1.189	1.238	1.288	1.438
12	1.499	1.562	1.625	1.814
13	1.890	1.970	2.049	2.287
14	2.383	2.483	2.583	2.884
15	3.005	3.131	3.257	3.636
16	3.789	3.948	4.107	4.585
17	4.779	4.980	5.180	5.783
18	6.027	6.280	6.533	7.292

Based upon Matthiessen's Standard of 9.5916 ohms per mil-foot at 0° C. and the A.I.E.E. temperature coefficient of 0.0042 per degree Centigrade temperature rise above 0° C.

Resistance at t° C. is equal to that at zero multiplied by $(1 + 0.0042t)$.

RESISTANCE OF ALUMINUM WIRE

Conductivity 62 per cent—Matthiessen's Standard

Ohms per 1000 Feet.

Size.	0° C. 32° F.	10° C. 50° F.	20° C. 68° F.	50° C. 122° F.
Millions of C.M.				
5	.003094	.003224	.003354	.003744
4	.003868	.004030	.004192	.004680
3	.005157	.005373	.005590	.006239
2	.007735	.008060	.008385	.009360
1¾	.008840	.009211	.009583	.01070
1½	.01031	.01075	.01118	.01248
1¼	.01238	.01290	.01342	.01497
1	.01547	.01612	.01677	.01872
¾	.02063	.02149	.02236	.02496
½	.03094	.03224	.03354	.03744
¼	.06188	.06448	.06708	.07488
B & S.				
0000	.07304	.07610	.07917	.08837
000	.09219	.09606	.09994	.1116
00	.1162	.1211	.1260	.1407
0	.1466	.1527	.1589	.1774
1	.1848	.1926	.2004	.2237
2	.2331	.2429	.2527	.2820
3	.2939	.3063	.3186	.3556
4	.3706	.3862	.4017	.4484
5	.4674	.4870	.5066	.5655
6	.5893	.6141	.6388	.7131
7	.7432	.7744	.8056	.8992
8	.9370	.9764	1.016	1.134
9	1.181	1.231	1.281	1.430
10	1.490	1.553	1.615	1.803
11	1.879	1.958	2.037	2.273
12	2.369	2.469	2.568	2.867
13	2.988	3.113	3.239	3.615
14	3.767	3.925	4.083	4.558
15	4.750	4.949	5.149	5.747
16	5.989	6.241	6.492	7.247
17	7.554	7.871	8.188	9.140
18	9.526	9.926	10.33	11.53

Based upon Mathiessen's Standard of 9.516 ohms per mil-foot at 0° C. and the temperature coefficient of 0.0042 per degree Centigrade temperature rise above 0° C.

Resistance at t° C. is equal to that at zero multiplied by $(1 + .0042t)$.

The following rules, which are easily remembered, enable one to determine approximately the constants of any size of copper or aluminum wire on the B. & S. gauge without reference to a wire table.

1. A No. 10 copper wire has a resistance of approximately one ohm per 1000 feet, a cross section of 10,000 C.M. and weight of 32 lbs. per 1000 ft.

2. A No. 10 aluminum wire has a resistance of approximately 1.6 ohms per 1000 feet, a cross section of 10,000 C.M. and weights 9.5 per 1000 feet.

3. An increase of one in the number of a wire increases the resistance 25 per cent; an increase of two in the number increases the resistance 60 per cent; an increase of three in the number doubles the resistance an increase of ten in the number increases the resistance ten times.

4. The cross section and weight of a wire varies inversely as the resistance; the diameter in mils is equal to the square root of the cross section in circular mils (a stranded wire has a diameter about 15 per cent greater).

Examples: The resistance of a number 18 copper wire is $4 \times 1.60 = 6.4$ ohms per thousand feet; the cross section is $\frac{10,000}{6.4} = 1560$ C.M.; the diameter is $\sqrt{1560} = 39.5$ mils; the weight $\frac{32}{6.4} = 5.00$ lbs. per 1000 feet. The resistance of a number 00 stranded aluminum wire is $\frac{1.6}{10 \times 1.25} = 0.128$ ohms per 1000 feet; the cross section $\frac{1.6}{0.128} \times 10{,}000 = 125{,}000$ C.M.; the diameter $1.15\sqrt{125{,}000} = 406$ mils; the weight $\frac{1.6}{0.128} \times 9.5 = 119$ lbs. per 1000 feet.

Increase of Resistance Due to Spiralling. The area of a cable for electrical purposes is taken to be the sum of the areas of the wires when laid out straight and measured in a plane at right angles to their axes. Hence, calculating the resistance of a cable accurately we must take into account the increase in effective length due to spiralling.

Let a = area of each wire in circular mils.

k = resistivity of the wires in ohms per mil-foot.

p_6 = pitch factor of layer of 6 wires.

p_{12} = pitch factor of layer of 12 wires, etc.

The resistance of a seven-wire cable equals

$$\frac{k}{a} \cdot \frac{p_6}{6 + p_6} \text{ ohms per foot.}$$

The actual path of the current is along the spiral, a very small proportion passing from wire to wire.

Formulæ for larger cables are cumbersome, but calculations may be made by considering the layers individually and grouping them in multiple. The proper value of p for each layer being assumed, we have the following resistances.

Wires in Layer.	Resistance of Each Layer, Ohms.
1	$\frac{k}{a}$
6	$\frac{k}{6a} \cdot p_6$
12	$\frac{k}{12a} \cdot p_{12}$
18	$\frac{k}{18a} \cdot p_{18}$
etc.	etc.
n	$\frac{k}{na} \cdot p_n$

See p. 19 for Resistance of Standard British Cables, for which the pitch is twenty times the diameter.

VARIATION OF RESISTANCE WITH TEMPERATURE

All materials suffer a slight increase of resistance with rise of temperature. For all pure metals except iron and nickel, this amounts to about two-fifths of one per cent per degree Centigrade. Iron and nickel show an increase of .005 and .007 respectively.

The law of increase of resistance, although for most purposes proportional, is not always exactly so, and depends not only on the metal but also on the physical condition of the sample experimented on.

Measurements by Kennelly and Fessenden appear to show that the resistance of commercial copper follows a straight-line law, that is, the equation connecting resistance and temperature is of the form,

$$R = r(1 + at),$$

where R = resistance at $t°$ Cent.;

r = resistance at 0° Cent.

The coefficient a appears to depend on the quality of the sample. The following values are used:

Authority.	Coefficient a.
American Institute of Electrical Engineers Standardization Report, value used in U. S. A. and accepted by American Authorities as correct.	.0042
British Engineering Standards Committee.	.00428
German	.0040

Matthiessen (Phil. Transac. 1862) gave the following formula, which was used in making up the American Institute of Electrical Engineers' Wire Table:

C_t = conductivity at $t°$ C.

C_0 = conductivity at 0° C.;

$$C_t = C_0(1 - .003{,}890{,}1t + .000{,}009{,}009t^2).$$

The second significant figure being doubtful, the absurdity of having five is apparent. The reciprocal formula is in the form of a convergent series and is unwieldy. The following widely published formula is obtained by omitting the terms containing the higher powers of t than t^2:

$$R = r(1 + .00387t + .000{,}005{,}968t^2).$$

It was pointed out by F. B. Crocker (*Elect. World*, Feb. 23, 1907), that the higher terms are not negligible and that an error of over 1.7% is obtained at 100° C. The following approximation is more nearly correct:

$$R = r(1 + .004t + .000,002,4t^2).$$

The error at 100° C. is only 1/10 of 1% compared with Matthiessen's formula. Professor Crocker, in the article above referred to, says that "the formula adopted in the A. I. E. E. Standardization Report is probably as nearly correct as any general expression can be made."

The author's concurrence with this statement led him to calculate new wire tables to supersede that of the A. I. E. E., these tables being given on pages 25 and 26.

The temperature coefficient of aluminum is practically the same as that of copper, but is sometimes given as .00423 per degree Centigrade.

Temperature - Resistance Calculations for Copper. Slide-rule Method. The following method is of great value on account of its simplicity, but requires a slide rule marked as described below.

Mark *slide* (lower scale) as follows:

Slide Rule Number.	Marking of New Scale.
238	0
248	10
258	20
268	30
278	40
288	50
298	60
308	70
318	80
328	90
338	100
etc.	etc.

Example showing how to use temperature scale: Suppose a copper wire to have a resistance of 300 ohms at 13° C., what will be its resistance at 100° C.?

Set 13 on the new slide scale opposite 300 on the lower scale and read on the lower scale the desired resistance 404 opposite 100 on the new scale.

This method is based on the coefficient 0.0042 adopted by the American Institute of Electrical Engineers, using the formula

$$R = \frac{238 + t}{238 + t_1}.$$

(H. Pender, *Elect. World*, New York, April 13, 1907).

2. RESISTANCE OF NETWORKS OF CONDUCTORS

KIRSCHOFF'S LAWS

(1) In any branching network of wires, the algebraic sum of the currents in all the wires that meet in any point, is zero.

(2) When there are several electromotive forces acting at different points of a circuit, the total electromotive force around the circuit is equal to the sum of the resistances of its separate parts multiplied each into the strength of the current that flows through it.

Maxwell's Imaginary Currents. In any network of conductors it is permissible, for purposes of calculation, to replace the actual currents through the network, by imaginary currents flowing in the closed cricuits formed by each mesh. These imaginary currents are taken as circulating in one direction, say the clockwise direction, and are all given the same sign, say the positive. Should it be convenient, for any reason, to take a current flowing in the opposite direction, it should be given a negative sign. In each mesh the sum of the IR drops equals the E.M.F. in the mesh, this being zero unless there is a generator. If the generator E.M.F. is in the same direction as the current, the E.M.F. is positive; if it opposes the imaginary current, its sign is negative.

Example. Let a, b, c, d, e, f, g, h, and i be the resistances of the various branches of the network represented in Fig. 4, and w, x, y, and z the imaginary

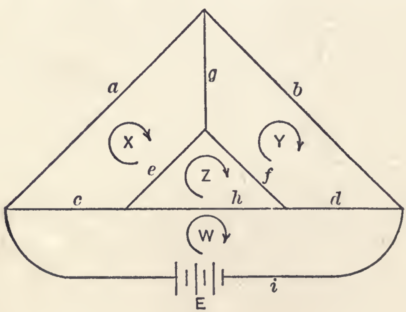


FIG. 4.

currents in the various meshes as shown, the direction of each current being assumed to be clockwise. The only E.M.F. in the system is E, produced by a battery in the branch i. Then,

$$
\begin{aligned}
&X(a+g+e+c)-Yg-Ze-Wc=0;\\
&Y(g+b+d+f)-Xg-Zf-Wd=0;\\
&Z(e+f+h)-Xe-Yf-Wh \quad =0;\\
&W(i+c+h+d)-Xc-Yd-Zh=E.
\end{aligned}
$$

Rearrange the equations so as to make them all of the same form; thus,

$$
\begin{array}{llr}
-Wc+X(a+g+e+c)-Yg-Ze & & =0;\\
Wd-Xg & +Y(g+b+d+f)-Zc & =0;\\
-Wh-Xe & -Yf+Z(e+f+h) & =0;\\
W(i+c+h+d)-Xc-Yd-Zh & & =E.
\end{array}
$$

These equations may be solved in the ordinary way or by *determinants*, as described below.

SOLUTION OF EQUATIONS BY DETERMINANTS

In order to solve such a series of equations the following pair of " determinants " are written out:

$$
W=\frac{\begin{vmatrix}
0 & (a+g+e+c) & -g & -e\\
0 & -g & (g+b+d+f) & -c\\
0 & -e & -f & (e+f+h)\\
E & -c & -g & -h
\end{vmatrix}}{\begin{vmatrix}
-c & (a+g+e+c) & -g & -e\\
-d & -g & (g+b+d+f) & -c\\
-h & -e & -f & (e+f+h)\\
(i+c+h+d) & -c & -g & -h
\end{vmatrix}}
$$

In the above equation it should be noted that the denominator consists of the terms of the four equations with the W, X, Y and Z omitted. The numerator differs from the denominator only in that the column of W terms is replaced by the terms on the right-hand side of the four equations. Were X the unknown, the second column of the numerator would be replaced by the terms, 0, 0, 0, and E.

The numerator and denominator of the above equation, each constitute what is called a determinant, and are simplified by the following rules. When the value of W has been found, the resistance of the circuit external to the generator is $\frac{E}{W}$.

Rules. (1) If a determinant has two equal rows or columns, it is equal to zero.

(2) To any row or column it is possible to add or subtract any number of times any other row or column without altering the value of the determinant.

(3) To multiply any row or column by a number is equivalent to multiplying the whole determinant by that number.

(4) If all the terms in a row or column except one are zero, the determinant reduces to one of a lower order which may be obtained by striking out the row and column which intersect at the term in question, and multiplying the whole by that term, the sign of the determinant being settled in the following way:

The line of terms beginning at the upper left-hand corner and ending at the lower right-hand corner, is called the principal diagonal of the determinant. If the uncancelled term in the line of zeros is on the principal diagonal or is removed from it by an even number of terms, the term by which the determinant is multiplied in lowering its order, is positive. If, however, this term is removed from the principal diagonal by an odd number of terms, the multiplying terms is negative. Thus,

$$\begin{vmatrix} 1 & 5 & 6 & 3 \\ 2 & 1 & 1 & 5 \\ 4 & 3 & 2 & 1 \\ 0 & 2 & 0 & 0 \end{vmatrix} = 2 \begin{vmatrix} 1 & 6 & 3 \\ 2 & 1 & 5 \\ 4 & 2 & 1 \end{vmatrix}$$

and

$$\begin{vmatrix} 1 & 5 & 6 & 3 \\ 2 & 1 & 1 & 5 \\ 4 & 3 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{vmatrix} = -2 \begin{vmatrix} 1 & 5 & 3 \\ 2 & 1 & 5 \\ 4 & 3 & 1 \end{vmatrix}$$

the principal diagonal being that with the figures 1, 2, and 0. It is immaterial whether the distance from the diagonal is counted along a row or a column.

(5) A determinant of the second order is expanded in the following way

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1.$$

The reduction of determinants is effected by altering the terms according to the above rules until a row or column is obtained in which all terms but one are zero. This enables a reduction or order to be effected in accordance with rule 4. Reductions are continued until one of the second order is obtained.

Example 1.

$$x + y + z = 6;$$
$$x + 2y + z = 8;$$
$$x + y + 2z = 9.$$

Then

$$x = \frac{\begin{vmatrix} 6 & 1 & 1 \\ 8 & 2 & 1 \\ 9 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}} = \frac{\begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ -3 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} 2 & 1 \\ -3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{1}{1} = 1$$

In the numerator, the following steps were taken. Six times the last column was subtracted from the first, and the last column was subtracted from the second. In the denominator, the first column was subtracted from the last. The determinants were then reduced to the second order by rule 4, and expanded by rule 5.

Similarly,

$$y = \frac{\begin{vmatrix} 1 & 6 & 1 \\ 1 & 8 & 1 \\ 1 & 9 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}} = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = 2$$

Hence $z=3$, by subtraction.

Actual cases are usually worked out without copying the various steps of the determinant, the changes being made with pencil and eraser.

Example 2. Reduce the following determinant.

$$\begin{vmatrix} 2 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 8 & 9 \end{vmatrix}$$

Subtract twice the first column from the second, and $\frac{7}{2}$ of the first column from the third.

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 3 & 2 & -\frac{3}{2} \end{vmatrix}$$

Reducing to the second order—

$$2 \begin{vmatrix} 1 & 1 \\ 2 & -\frac{3}{2} \end{vmatrix}$$

Expanding—

$$2\left[-\tfrac{3}{2} - 2\right] = -7.$$

3. RESISTANCE TO ALTERNATING CURRENTS OR SKIN EFFECT

NATURE OF SKIN EFFECT

The current induced in a conductor begins at the surface and rapidly diffuses inward. When an alternating E.M.F. is applied, the current started by a positive impulse has only time to diffuse a short distance from the surface before the succeeding impulse starts an opposite current from the surface. The effect is that the current never attains its full value. A conductor therefore offers greater resistance to alternating than to direct current.

Calculation of Skin Effect for a Cylindrical Wire. Let

R = ratio of alternating current resistance to direct current resistance.

M = area of conductor, circular mils.

N = cycles per second.

μ = permeability of conductor.

k = resistance of a mil-foot of the conductor at the temperature under consideration.

$$Z=\sqrt{\frac{MN\mu}{10^6 k}}.$$

The relation between R and Z is given by the curve of Fig. 5, and by the following table.

TABLE I

APPROXIMATE VALUE OF R

Z less than 1.4	$R=1$
Z between 1.4 and 4.0	R is as given in Table II
Z is greater than 4.0	$R=0.314Z+0.24$

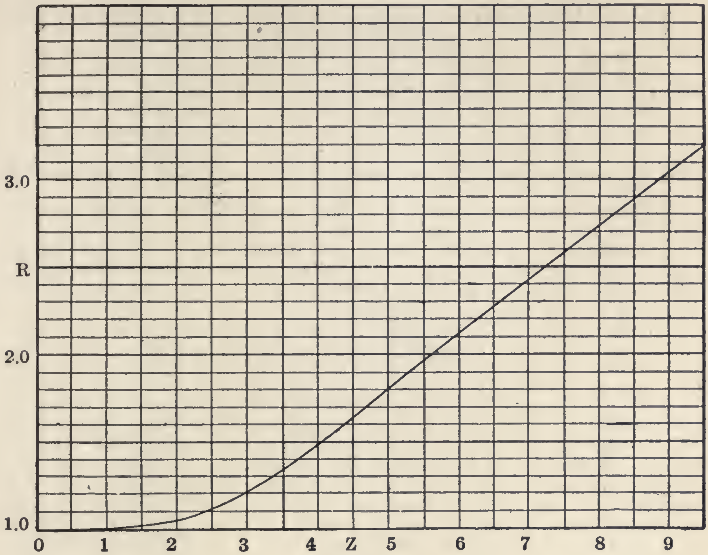


FIG. 5.

TABLE II

Z.	R.	Z.	R.
1.48	1.01	2.80	1.16
1.64	1.02	2.84	1.17
1.78	1.03	2.89	1.18
1.90	1.04	2.94	1.19
2.00	1.05	2.99	1.20
2.11	1.06	3.03	1.21
2.20	1.07	3.08	1.22
2.28	1.08	3.12	1.23
2.36	1.09	3.14	1.24
2.43	1.10	3.20	1.25
2.50	1.11	3.24	1.26
2.57	1.12	3.27	1.27
2.63	1.13	3.31	1.28
2.68	1.14	3.34	1.29
2.74	1.15	3.38	1.30

The calculation of skin-effect in copper and other non-magnetic conductors presents no difficulties because μ is unity. In the case of iron and other magnetic metals, calculation is rendered difficult by the necessity of using the proper value of μ which depends on the current. The following table gives the results of tests by L. Lichenstein. (*Electrician*, London, Aug. 23, 1907.)

TEST ON RAIL

Cycles per Second.	Amperes.	A.C. Resistance. / D.C. Resistance.	Equivalent μ.	
58.5	49	4.34	8.0	Rail bonded with copper
48.7	153.8	5.55	7.2	
28.2	62.5	2.85	14.8	
25.4	108.4	3.76	15.0	
19.4	36.4	2.5	16.0	
17.3	123.2	2.93	19.3	
58.6	35	2.68	9.6	Rail not bonded
48.6	152	3.42	8.6	
28.4	46.2	1.94	11.0	
25.7	169	2.2	14.4	

Area of rail, 5160 sq.mm. = 8 sq.in.

LARGE CABLES ON A.C. CIRCUITS

Owing to the fact that alternating current flowing in large cables has greater density on the surface of the conductor than in the center (so-called skin effect), an ordinary cable will not carry as much alternating current with the same temperature rise as direct current. In order to overcome this it is advisable on single conductor cables, 700,000 cm. and larger, for 60 cycle circuits and 1,250,000 cm. and larger for 25 cycle circuits, to make up the cable with a fibre core and the copper stranded around it. The weight of copper in this type of cable is the same per foot as in an ordinary cable, but owing to its annular cross section the cable is much more efficient in carrying alternating current, and also has a somewhat greater current carrying capacity due to the larger radiating surface.

Size.	Diameter Fiber Core in Inches.	Number of Wires in Strand.	Size Wire in Strand.	Overall Diameter Copper Core.	Ampere Capacity.	
					30° C.	60° C.
2,000,000	7/8	210	0.099	2.065	1400	1750
1,750,000	25/32	210	0.091	1.870	1300	1625
1,500,000	11/16	162	0.091	1.780	1200	1500
1,250,000	9/16	148	0.086	1.590	1150	1400
1,000,000	15/32	98	0.102	1.280	900	1150
800,000	11/32	51	0.125	1.100	775	925
700,000	9/32	51	0.117	0.990	700	830

(G. E. Co. Bulletin.)

4. CARRYING CAPACITY

In the following table the lower limit is specified for rubber-covered wires to prevent gradual deterioration of the insulation by the heat of the wires, not from fear of igniting the insulation.

The carrying capacity of Nos. 16 and 18 B. & S. gauge wire is given, but no smaller than No. 14 is used, except for fixture work and flexible cord.

TABLE OF CARRYING CAPACITY OF COPPER WIRES AND CABLES—INTERIOR WIRING

(National Electric Code.)

B. & S. Gauge.	Table A. Rubber Insulation. Amperes.	Table B. Other Insulations Amperes.	Circular Mils.
18	3	5	1,624
16	6	8	2,583
14	12	16	4,107
12	17	23	6,530
10	24	32	10,380
8	33	46	16,510
6	46	65	26,250
5	54	77	33,100
4	65	92	41,740
3	76	110	52,630
2	90	131	66,370
1	107	156	63,690
0	127	185	105,500
00	150	220	133,100
000	177	262	167,800
0000	210	312	211,600
	200	300	200,000
	270	400	300,000
	330	500	400,000
	390	590	500,000
	450	680	600,000
	500	760	700,000
	550	840	800,000
	600	920	900,000
	650	1000	1,000,000
	690	1080	1,100,000
	730	1150	1,200,000
	770	1220	1,300,000
	810	1290	1,400,000
	850	1360	1,500,000
	890	1430	1,600,000
	930	1490	1,700,000
	970	1550	1,800,000
	1010	1610	1,900,000
	1050	1670	2,000,000

For insulated aluminum wire the safe carrying capacity is 84% of that given above for copper wire with the same kind of insulation. (Nat. Elec. Code.)

CURRENT CARRYING CAPACITY OF INSULATED LEAD COVERED COPPER CABLES IN DUCTS *

Initial Temperature, 20° C.

(G. E. Bulletin 4591.)

Size of Cable in Circular Mils.	National Electric Code, 1907, Rubber.	Low Tension Cable, Single Conductor.		High Tension Cable, Three Conductor.
		Rubber 30° C. Rise. Amperes.	Var. Cam. or Paper 60° C. Rise. Amperes.	Rubber and Var. Cam. 30° C. Rise Paper, 35° C. Rise Amperes on Each Conductor.
2,000,000	1050	1400	1750	
1,500,000	850	1200	1500	
1,000,000	650	900	1150	
750,000	525	750	900	
500,000	390	550	660	440
400,000	330	460	560	360
300,000	270	370	450	290
250,000	235	230	390	250
200,000	200	270	310	210
150,000	160	220	260	175
125,000	140	180	210	140
100,000	120	160	190	125
80,000	104	140	165	110
60,000	82	110	130	85
40,000	63	75	90	60
6 B. & S. solid	46	50	60	40
8 B. & S. solid	33	30	36	24
10 B. & S. solid	24	20	24	16

* The table gives the maximum continuous load in amperes for high and low tension cables with rubber and varnished cambric or paper insulation, the ultimate rise in temperature being marked at the head of each column. For high tension single conductor, use figures given for single conductor rubber.

Experience has shown that the maximum temperature which cables should be permitted to attain is 50° C. for rubber and 80° C. for varnished cambric and paper insulated. (From G. E. Co. Bulletins.)

GENERAL FORMULA FOR THE CARRYING CAPACITY OF COPPER WIRES AND CABLES:

I = Current, amperes;

T = Temperature rise, deg. Cent.;

$$I = ABCD\sqrt{\frac{T}{k}},$$

where

k is given by Table		I
A	"	" II
B	"	" III
C	"	" IV
D	"	" V

For multiple conductor cables, the value of I is for one conductor. The carrying capacity of *Aluminum* of 62% conductivity is 80% that of copper.

When I is known, and T is required, use the following formula

$$T = \frac{1 + 0.0042 T_0}{N - 0.0042},$$

where

T_0 = initial temperature in deg. Cent.;

$$N = \frac{1}{k_0}\left(\frac{ABCD}{I}\right)^2;$$

k_0 = value of k at 0 C., as given by Table I.

For *Aluminum* of 62% conductivity $k_0 = 15.5$.

Table I

VALUES OF *k*

Resistance (Ohms) of a Mil-foot of Copper

Use of Table

Find the temperature corresponding to the rise T by adding the initial temperature thereto. Then take the value of k corresponding to that temperature, from the table.

Temperature.		Values of *k*.		Temperature.		Values of *k*.	
° C.	° F.	98% Conductivity.	99% Conductivity.	° C.	° F.	98% Conductivity.	99% Conductivity.
0	32	9.79	9.69	55	131	12.0	11.9
5	41	10.0	9.90	60	140	12.2	12.1
10	50	10.2	10.1	65	149	12.4	12.3
15.5	60	10.4	10.3	70	158	12.7	12.5
20	68	10.6	10.5	75	157	12.9	12.7
24	75.2	10.8	10.7	80	176	13.1	12.9
30	86	11.0	10.9	85	185	13.3	13.1
35	95	11.2	11.1	90	194	13.5	13.4
40	104	11.4	11.3	95	203	13.7	13.6
45	113	11.6	11.5	100	212	13.9	13.7
50	122	11.8	11.7				

Based on Matthiessen's Standard and the A.I.E.E. temperature coefficient.

Table II

VALUES OF *A*

$$A=\sqrt{12\pi d^3},$$

where d = diameter of a solid wire of the size given, inches

Size of Conductor.	A.	Size of Conductor.	A.
Millions of C.M.		No. B. & S.	
2	10.32	0000	1.91
1.9	9.93	000	1.61
1.8	9.54	00	1.35
1.7	9.14	0	1.14
1.6	8.74	1	0.955
1.5	8.32	2	0.802
1.4	7.90	3	0.675
1.3	7.46	4	0.566
1.2	7.03	5	0.476
1.1	6.59	6	0.400
1.0	6.15	7	0.337
0.9	5.67	8	0.282
0.8	5.19	9	0.237
0.75	4.95	10	0.200
0.7	4.70	12	0.147
0.6	4.18	14	0.0995
0.5	3.65	16	0.0703
0.4	3.08	18	0.0496
0.35	2.79	20	0.0351
0.3	2.49	22	0.0248
0.25	2.17		

TABLE III

VALUE OF B

$$B = 1000\sqrt{W}$$

W = watts dissipated per sq.in. of single conductor cable per deg. Cent. Temperature rise.

Where Installed.	Type of Cable.					
	Bare.		Rubber Covered.		Paper or Cloth and Lead.	
	Solid.	Stranded.	Solid.	Stranded.	Solid.	Stranded.
Open air	160	175	105	110	105	110
Still air	130	143	94	100	89	93
Wooden moulding..	...	...	140	150	...	...
$3\frac{1}{2}''$ tile duct;						
No. 0000 B. & S..	...	...	89	93	86	90
$\frac{3}{4}$M.	...	...	78	82	75	79
1M.	...	...	75	79	72	75
Under water, leaded and armored......	...	...	110	115	100	105

The values given in the above table are averages based on experimental data from various sources; the maximum variation from the average in the values thus found was about 5%.

TABLE IV

VALUE OF C

(Standard Underground Cable Co. Handbook.)

Type of Cable.	C.
Single conductor..................	1
Two conductor, flat or round......	0.87
Two conductor, concentric.........	0.79
Three conductor, triplex..........	0.75
Three conductor, concentric	0.60

Table V

VALUES OF *D*

Number of Similarly Loaded Cables in Group of Ducts.	*D*.
1	1.0
2	0.92
3	0.86
4	0.79
5	0.75
6	0.70
7	0.66
8	0.63
9	0.59
10	0.56
11	0.53
12	0.50

The thickness of insulation probably has a considerable effect upon the radiation, but experimental data on this point are not available. The above table is based principally upon tests of low voltage cables.

Carrying Capacity of Wires of Various Metals. The carrying capacity or current causing a given temperature rise is inversely proportional to the square root of the specific resistance of the metal, and directly proportional to the square root of the heat radiation per unit area. Assuming the latter to be the same for all wires, the relative carrying capacities of wires of different metals, referred to Matthiessen's annealed copper as unity are given in the following table:

Metal.	Relative Carrying Capacity.
Silver, annealed.	1.04
Copper, annealed.	0.99 to 1.01
Copper, annealed, 100% cond.	1
Copper or silver, hard drawn.	0.98 to 1.0
Gold, hard drawn.	0.87
Aluminum, annealed.	0.74
Aluminum wire, 62% cond.	0.79
Zinc, pressed.	0.53
Phosphor bronze.	0.45
Platinum, annealed.	0.42
Iron, annealed.	0.40
Nickel, annealed.	0.36
Tin, pressed.	0.35
Lead, pressed.	0.29
German silver, from.	0.28
to.	0.23
Platinoid.	0.22
Antimony, pressed.	0.21
Manganin.	0.19
Krupp metal.	0.14
Mercury.	0.13
Bismuth, pressed.	0.12

Knowing Heating with One Current, to Find Heating with Another Current. The chart (Fig. 6) is used as follows:

Suppose a switch or cable has a rise of 20° C. with 200 amperes, what will the rise be with 300 amperes? Referring to the curve, the vertical line 200 is followed upward until it intersects the diagonal which starts at 20°. This diagonal is followed upward until it intersects a vertical line at 300 amperes. The horizontal line intersecting the vertical line at this point gives the rise in degrees, namely, 45.

As noted on the diagram the current scale is correct for amperes, milli-amperes, or any other unit,

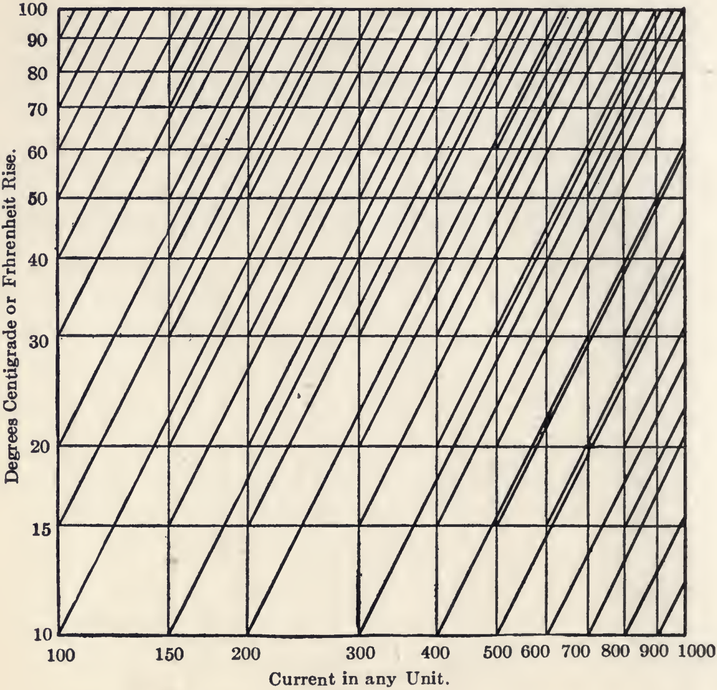


Fig. 6.

and the temperature scale is correct for either Centigrade or Fahrenheit. (Based on article by C. C. Badeau, *Elec. World*, Jan. 11, 1908.)

INTERMITTENT CARRYING CAPACITY

Let P = time of full period (minutes) assuming the current periodically on and off;

a = portion of full period (minutes) that current is on;

T = time (minutes) in which temperature rise becomes 0.633 times maximum temperature rise. This depends on size and type of cable and is given in the following table:

C = maximum permissible constant current;

pC = maximum permissible intermittent current.

To find pC:

Find T for size of cable under consideration.

Thence calculate $\frac{a}{T}$ and $\frac{a}{P}$ and from the table find the corresponding value of p.

VALUES OF T

CABLE INSULATED FOR 700 VOLTS

Sq.Mm.	Value of T.	
	Single Cond.	Triplex.
50	14	21
100	21	32
150	28	42
200	32	50
300	38	63
400	41	70
500	42	..
600	44	..
700	46	..
800	48	..
900	49	..
1000	50	..

VALUES OF p

$\frac{a}{T}$	$\frac{a}{P}$									
	0.1.	0.15.	0.2.	0.3.	0.4.	0.5.	0.6.	0.7.	0.8.	0.9
0.0	3.15	2.65	2.25	1.8	1.55	1.45	1.3	1.2	1.1	1.05
0.1	2.55	2.35	2.2	1.7	1.5	1.4	1.25	.....	.....	1.05
0.2	2.2	2.05	1.9	1.6	1.45	1.35	1.25	.....	.....	1.05
0.3	1.9	1.85	1.7	1.55	1.4	1.3	.....	.....	.....	1.05
0.4	1.7	1.7	1.6	1.5	1.35	1.3	.....	.....	.....	1.05
0.5	1.6	1.55	1.55	1.45	1.3	1.3	1.2	1.15	.....	1.05
1.0	1.25	1.25	1.25	1.25	1.2	1.2	1.15	1.1	1.05	1.05
2.0	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
∞	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

THE SHORT-PERIOD CARRYING CAPACITY OF CABLES*

The formula should not be used without understanding the assumptions made in deriving it; for although they are quite reasonable under ordinary conditions, they do not necessarily hold under certain extreme conditions. These assumptions are as follows:

(1) That the heat dissipated by the cable is directly proportional to the temperature rise.

(2) That the specific heat of the conductor and insulation does not vary greatly over the temperature range considered, an average value being assumed.

(3) That the cable insulation is raised to the same temperature as the conductor. This assumption is approximately correct for thin insulation on large

* *Electrical World*, Dec. 12, 1908.

cables; the assumption is not true for cables smaller than No. 00 B. & S., or for cables insulated for over 1000 volts. This restriction is of little moment, however, as the important use of the formula is in connection with large power cables, a knowledge of the carrying capacity of which may lead to considerable economy of copper.

The following formula gives the time (t = minutes) during which a cable will carry I amperes with a temperature rise of D deg. Fahr.:

$$t = 40.5\,PAKGZ,$$

P, A, K, and G are constants of the cable and are defined as follows:

P = [(specific heat of conductor × weight in pounds per foot) + (specific heat of insulation × weight in pounds per foot)];

A = cross-sectional area of cable in circ. mils;

K = average of the reciprocals of the ohms per mil-foot over the range of temperature considered. For practical purposes AK is the reciprocal of the resistance per foot of the cable at the temperature midway between the initial and final temperatures assumed;

$G = \frac{F}{I^2}$ where F is the final temperature rise which would occur with I amperes applied steadily.

It is a constant for every cable under given conditions of thermal exposure, and may be obtained from any pair of values of F and I.

Z is a function of $\frac{D}{GI^2}$, which in turn equals $\frac{D}{F}$ and may be taken from Table I or calculated by the formula,

$$Z = -\log\left(1 - \frac{D}{GI^2}\right),$$

the logarithm being to the base 10.

The product, 40.5 $PAKG$, which is a constant for a given cable under given conditions, is the time in minutes required to raise the temperature of the cable to 90% of its final temperature rise.

Table I

VALUES OF Z

$\frac{D}{GI^2}=\frac{D}{F}$	Z	$\frac{D}{GI^2}=\frac{D}{F}$	Z	$\frac{D}{GI^2}=\frac{D}{F}$	$Z.$
0.005	0.00218				
0.01	0.00436	0.31	0.161	0.61	0.409
0.02	0.00877	0.32	0.167	0.62	0.420
0.03	0.0132	0.33	0.174	0.63	0.432
0.04	0.0177	0.34	0.180	0.64	0.444
0.05	0.0223	0.35	0.187	0.65	0.456
0.06	0.0269	0.36	0.194	0.66	0.469
0.07	0.0315	0.37	0.201	0.67	0.481
0.08	0.0362	0.38	0.208	0.68	0.495
0.09	0.0410	0.39	0.215	0.69	0.509
0.10	0.0458	0.40	0.222	0.70	0.523
0.11	0.0506	0.41	0.229	0.71	0.538
0.12	0.0555	0.42	0.237	0.72	0.553
0.13	0.0605	0.43	0.244	0.73	0.569
0.14	0.0655	0.44	0.252	0.74	0.585
0.15	0.0706	0.45	0.260	0.75	0.602
0.16	0.0757	0.46	0.268	0.76	0.620
0.17	0.0809	0.47	0.276	0.77	0.638
0.18	0.0861	0.48	0.284	0.78	0.658
0.19	0.0915	0.49	0.292	0.79	0.678
0.20	0.0969	0.50	0.301	0.80	0.699
0.21	0.102	0.51	0.310	0.81	0.721
0.22	0.108	0.52	0.319	0.82	0.745
0.23	0.113	0.53	0.328	0.83	0.770
0.24	0.119	0.54	0.337	0.84	0.795
0.25	0.125	0.55	0.347	0.85	0.824
0.26	0.131	0.56	0.357	0.86	0.854
0.27	0.137	0.57	0.367	0.87	0.886
0.28	0.142	0.58	0.377	0.88	0.921
0.29	0.149	0.59	0.387	0.89	0.959
0.30	0.155	0.60	0.398	0.90	1.000

TABLE II

VALUES OF *P* FOR BARE COPPER CABLES

Specific heat = 0.093.

Size.	No. of Strands.	*P*.
2 million C.M.	127	0.558
1½	91	0.425
1¼	91	0.354
1	61	0.284
¾	61	0.212
½	61	0.142
¼	37	0.0709
0000 B. & S.	19	0.0600
000 B. & S.	19	0.0476

For aluminum of the same resistance, increase *P* by 11 per cent.

TABLE III

RECIPROCALS OF OHMS PER MIL-FOOT

Deg. F.	Reciprocal of Ohms per Mil.-Foot.	Deg. F.	Reciprocal of Ohms per Mil. Foot.
50	0.0980	100	0.0883
55	0.0970	105	0.0874
60	0.0960	110	0.0866
65	0.0950	115	0.0858
70	0.0940	120	0.0850
75	0.0930	125	0.0842
80	0 0920	130	0.0833
85	0.0910	135	0.0826
90	0.0900	140	0.0818
95	0.0892	145	0.0811
100	0.0883	150	0.0804

The above table is based on 98% conductivity.

CHAPTER III

INSULATION AND INSULATED CONDUCTORS

1. INSULATION

THE principal materials used for insulating power cables are:

(1) Paper saturated with oil.

(2) Varnished muslin, variously known as varnished cambric or varnished cloth.

(3) Compounds containing rubber.

The first two, being made of staple commercial materials, are generally reliable, but compounds containing rubber vary from the cheap material used for insulating "code wire" to the high-grade compound required by the U. S. Navy.

NECESSITY OF UNIFORM STRUCTURE

If two conducting plates are arranged at such a distance apart that the air is just able to withstand for an indefinite time, say, 10,000 volts maintained by a transformer, and then a strip of glass is introduced between them, the insulation will break down, although the glass has greater dielectric strength than air. The explanation is quite simple: the fall of volts per centimeter in the air before the glass is inserted is the highest the air can withstand; as glass has a higher specific inductive capacity the potential gradient in the glass is less steep than in the air, and the consequent increased steepness in the air due to the insertion of the glass, causes the air to break down. This experiment shows the necessity of having the insulation free from air spaces or weak spots, and in the case of high tension cables, of having no air spaces between insulation and sheath; it also explains why an insulated cable without a sheath should not be supported directly on metal brackets. However, by adapting the specific inductive capacity of the insulations to the potential gradient, an increased total dielectric strength may be obtained as in "graded" cables; that is, those in which the small area in contact with the conductor is made of greater specific inductive capacity than the peripheral areas, in order to decrease the potential gradient in the insulation adjacent to the conductor.

RUBBER INSULATION

Rubber insulation, so-called, is a compound of various substances in which rubber seldom predominates. It is therefore not surprising to find the properties of rubber compounds varying between very wide limits according to the nature of the substances of which they are composed, and according to the process of compounding.

The qualities which a rubber compound should possess, in order to fulfil all requirements as cable insulation, are as follows:

(1) High dielectric strength.

(2) High mechanical strength.

(3) Fair elasticity.

(4) Fair specific resistance.

(5) Permanence or long life.

The first four qualities are not difficult to obtain. and it is easy to test a compound for their presence. The fifth quality, permanence, depends upon two conditions. The first of these is chemical equilibrium, *i.e.*, the rubber and substances associated with it must have no affinity for one another, for the conductor, for air or for moisture. The second condition is that the compound shall contain no substance tending to change its physical state, as for example, a volatile, photo-sensitive or crystallizable substance.

Within wide limits compounds of various compositions can be made balanced, and therefore permanent, provided that conditions inconsistent with the condition of balance are not specified. There are no known tests which will infallibly distinguish between a balanced and an unbalanced compound. A short discussion of the tests and restrictions which have been suggested for this purpose is given below.

RUBBER GUM

Rubber is a gum extracted from a tree which grows in the tropical countries of Africa and South America. The quality of this gum varies in many ways, but the characteristic which most affects its commercial value is the amount of resinous extract which it contains. The amount of extract is usually estimated by digesting the gum in acetone for several hours, and thereby dissolving out the extract. The proportion of acetone extract in different grades of gum varies from less than 1 per cent to over 20 per cent, the grades having the smaller proportion of extract being generally from South America.

The best grade of South American rubber is known as fine Para, and is the most desirable kind to use in insulating compounds. While it is usual to specify that compounds shall contain only the finest dry Para rubber, there is no practical way to ascertain whether the rubber did actually come from Para. Furthermore, it is of no practical import whence the rubber is from, provided that the percentage of extract does not exceed, say, 3 per cent. A greater percentage of extract indicates a cheap grade of rubber, which it is difficult to manufacture into a balanced compound.

VULCANIZATION

Rubber gum, in its native state, is of little use for insulating purposes, owing to its property of absorbing water and oxidizing. When mixed with sulphur and heated to a temperature of from 248° to 302° Fahr., a combination takes place which renders the rubber more stable and at the same time increases its mechanical and electrical strength. This process is known as vulcanization.

COMPOUNDING

It has been found by experience that 60 to 70 per cent of adulterant may be added to rubber gum without destroying its useful qualities after vulcanization. Above this percentage, the qualities of the rubber cease to predominate, and the compound partakes markedly of the characteristics of the adulterant. It is for this reason that 30 per cent pure rubber is generally adopted as the standard proportion, and that 40 per cent pure rubber is required for shipboard work in the navy, the larger proportion being adopted as a special precaution on account of the necessity of absolute reliability.

TENSILE STRENGTH

A good 30% Para compound, properly vulcanized, should show a tensile strength of at least 800 pounds per square inch. This figure is agreed to by practically every manufacturer of rubber compound in the United States, but the proportion of compounds which actually show this tensile strength is small.

A sample should be cut so that the ends gripped shall be considerably larger than the center, where the break should occur. The sample should be bent slightly, in every direction, before testing, in order to magnify and reveal any surface incisions which might reduce the total cross-section.

SET AFTER STRETCHING

When stretched three times its original length, a sample should show a set not greater than $18\frac{3}{4}\%$ after a stated time has elapsed. Although the time is a matter of controversy, this percentage set is agreed to by all the leading manufacturers.

Nevertheless, it is well known that certain excellent compounds entirely fail to meet the regular stretch tests. It is, perhaps, better to lose the use of this class of compounds and take advantage of the selective action of the stretch test; and if this is done it should be specified that the test may be performed by the purchaser at any temperature between 50° and 100° Fahr. It should also be specified that the sample tested shall not have been submitted to any previous stretching, because a sample with a permanent set will not show much additional set when further stretched. Stretching should be steadv and release instantaneous.

SPECIFIC RESISTANCE

The specific resistance of insulation sold as 30% Para compound varies between the enormously wide limits of 150 millions of megohms per inch cube and 4000 millions of megohms per inch cube.

From the standpoint of leakage a mere fraction of the smaller value would be sufficient. It is, therefore, only as a test of quality that high megohms may be demanded, and the value of such test is open to doubt.

A minimum of 750 millions of megohms per inch cube is conservative, and there is certainly nothing to be gained by specifying over 1200 millions of megohms per inch cube.

Much more important is the permanence of the insulation resistance. A good compound should show little decrease of insulation resistance after 100 hours of test with current applied continuously.

TEMPERATURE COEFFICIENT OF RESISTANCE

The rate of change of resistance with regard to temperature should not exceed 2.6% per degree Fahr. This is in agreement with the tables used by the most reputable manufacturers. The object of specifying this quantity is twofold: First, to prevent the manufacturer using any temperature correction factor which will give a figure which complies with the specifications; second, as a measure of quality of the compound as pointed out by H. G. Stott, Proc. Am. Inst. Elec. Eng., 1906.

The author's experience confirms Mr. Stott's opinion of the value of this test.

"HYSTERESIS TEST"

If extensions and contractions are plotted on a base of load, a "hysteresis" loop is obtained, as shown in Fig. 7. The area of this loop should

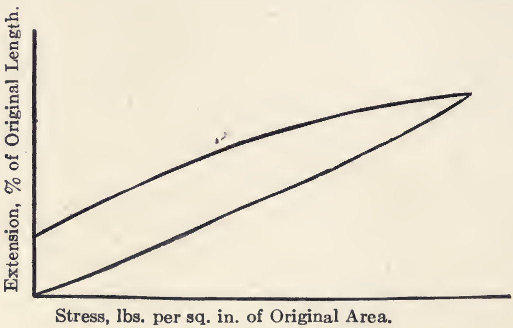


FIG. 7.

generally be small in good compound; there are, however, exceptions to this rule.

SULPHUR

Sulphur in rubber compound may be in three conditions:

(1) Free;

(2) Combined with rubber;

(3) In barium sulphate, etc.

Poor quality rubber requires a great deal of sulphur to vulcanize it, and is, therefore, often revealed by the large amount of combined sulphur.

Excess of free sulphur, say over 1%, usually indicates an unstable compound, as the sulphur is liable to combine with the copper or tin coating over the copper.

PERCENTAGE OF RESINOUS MATTER

Brand of Rubber.	Resin in Washed Rubber, Per Cent.	Resin in Vulcanized Rubber, Per Cent.
Para, fine	1.2	4.04
Ceara	2.1	5.12
Upper Congo	3.7	7.60
Lagos	4.5	7.13
Sierra Leone	6.1	9.97
Borneo	10.3	14.44

C. O. Weber, "Chemistry of India Rubber."

EFFECT OF TEMPERATURE ON RUBBER

Rubber insulation begins to deteriorate at ordinary air temperatures; the deterioration is rapidly accelerated when exposed to temperatures in excess of 50° C. The following are the effects noted.

(1) *Loss of Strength or Cohesion.* Rubber with a low coefficient of vulcanization is liable to develop this defect, particularly if the time for vulcanization has been short.

(2) *Hardening with Brittleness.* Rubber may contain white substitutes (chlorosulphides), but more

commonly is due to the presence of a considerable amount of free sulphur.

(3) *Stickiness and Darkening in Color.* Rubber containing mineral oils, large quantities of recovered rubber, or large proportions of sulphide substitutes.

Change in State of Para Rubber with Temperature

Temperature Deg. Cent	
90–100	Slightly sticky.
145	Sticky, but slightly elastic.
150–160	Surface melts and rubber darkens.
170–190	Gradually melts.
240	Can be mixed up and thermometer easily pushed into the mass.
255	Appearance of decomposition and boiling.
340	Gas evolved, which burns with a luminous flame.

The liquid obtained on heating becomes viscid on cooling, but it does not again solidify.

TENACITY AND TEMPERATURE

Temperature in Deg. F.	Loss of Tenacity, Per Cent.
68	2
138	5
248	10
328	15
418	20
438	22
488	25

A. Schwartz, Journal Inst. E. E., 1907.

RUBBER INSULATION UNDER WATER

Rubber insulation will last indefinitely under fresh or salt water if the compound is balanced and if the water is not contaminated with sewage, etc. Where the insulation is intended for this service, the manufacturer should be so advised.

EFFECT OF OVER-MASTICATION OF RUBBER

Rubber overworked in the masticator oxidizes very rapidly, yielding a much greater amount of extract than before mastication. (C. O. Weber, *Journal of Society of Chemical Industry*, 1903, p. 875 and p. 103.)

EFFECT OF LIGHT ON RUBBER

The action of light on rubber, whether vulcanized or unvulcanized, is an oxidizing action, but the oxidation is faster the lower the degree of vulcanization. (C. O. Weber, *Journal of Society of Chemical Industry*, 1903, p. 875.)

The significance of this statement has been overlooked by the majority of manufacturers and users of rubber. If a number of samples of rubber insulation of different makes are subjected for a long period to the action of light, those compounds which are *black* will almost invariably remain unchanged, while those which are white or of light shade, will become stiff and brittle. This fact has been made use of by certain manufacturers of black compounds, who claim that their product has a longer life than others, because when subjected to the open air "weathering test" it outlasts nearly all others. This claim is unjustifiable because the test is really a photo-chemical one and has nothing to do with weathering. If protected from light, the white compounds last as well as the black ones, and are therefore just as good if used under black braiding or lead sheathing.

The explanation of these facts is that rubber is normally translucent and unless rendered quite opaque by the presence of black matter is affected photo-chemically throughout its mass. Black compounds, on the other hand, are only affected superficially by light, becoming coated with a powdery, white film, which is readily brushed off.

DETERIORATION OF CONGO RUBBER

The deterioration of Congo rubber is due to the presence of albuminous substances primarily. Coagulated albumin is not removed by washing, causing finished goods to be more or less brittle, according to the amount of albumin present. (C. O. Weber, *Journal of Society of Chemical Industry*, 1902, p. 712.)

EXCESS OF LITHARGE

Certain varieties of rubber do not become properly vulcanized when treated with sulphur only, but do so readily if a considerable proportion of litharge is present during the process. The effect of litharge, however, is to make the rubber brittle. (C. O. Weber, *Journal of Society of Chemical Industry*, 1903, p. 103.)

AVERAGE DIELECTRIC STRENGTH OF RUBBER INSULATION

128 kilovolts per in., conservative testing stress.
56 kilovolts per in., conservative working stress.
400 kilovolts per in., breakdown stress (approx.).

PAPER INSULATION

Paper ribbon is wound spirally around the conductor in numerous layers, until the desired thickness is obtained. The cable is then immersed in a bath of oily insulating compound, until saturated. The whole is then enclosed in a lead sheath, which not only serves to retain the compound, but also to exclude moisture.

This type of cable is cheaper than varnished cambric or good quality rubber, and is almost universally used for voltages from 5000 up. It is also very largely used for lower voltages.

Owing to the hygroscopic qualities of paper insulation, it should not be used where the cable is exposed to the direct action of water, as, for example, in submarine work, or in badly drained splicing chambers. For this service, rubber or varnished cambric insulation is to be preferred, as, in the event of a burn-out, the insulation will not be spoiled, except at the actual point of trouble.

Dr. Jona (Int. Elec. Congress, 1904), says that paper subjected to dielectric strain for an hour, with progressively increasing voltage, will stand from eight to ten kilovolts per millimeter. These numbers represent good commercial averages, but it is not unusual to find paper with 20 or 30 per cent greater dielectric strength.

FACTORS FOR CORRECTION OF INSULATION RESISTANCE TO 15.5° C.

Temperature, Deg. C.	Factor for High-Grade Paper.
30	5.38
29	5.20
28	4.82
27	4.45
26	4.09
25	3.71
24	3.32
23	2.97
22	2.61
21	2.24
20	2.00
19	1.78
18	1.57
17	1.36
16	1.14
15.5 (60° F.)	1.00
15	0.92
14	0.78

The resistance at 15.5° C. is found by multiplying the observed resistance by the factor corresponding to the temperature at which the resistance is measured.

VARIATION OF INSULATION RESISTANCE WITH TIME OF ELECTRIFICATION (PAPER INSULATION)

Time of Electrification, Minutes.	Relative Insulation Resistance, Referred to Value after One Minute Electrification.
0	....
0½	0.824
1	1.00
1½	1.09
2	1.16
2½	1.21
3	1.24
3½	1.28
4	1.31
4½	1.33
5	1.35

This test represents average results, but must *not* be taken as correct for any particular cable.

VARNISHED CAMBRIC

Prepared cotton fabric is coated on both sides with multiple films of insulating varnish. The coated cloth is cut into strips and wound spirally on the copper core, with films of non-drying viscous adhesive compound between the layers. A separator is sometimes applied between the copper core and the taping, in order to prevent any possible action of the varnished films on the copper.

This insulation, unlike paper, does not absorb moisture and may be used for indoor work without a lead sheath. It is suitable for high-tension cables, especially where a lead sheath cannot be used, as, for example, when subjected to vibration. In such cases it is usual to protect the insulation by a spiral galvanized steel tape; this construction is not suitable however for single conductor cables carrying alternating currents.

Cambric insulation is considerably more flexible than paper, it being possible to bend cables to a radius of six times their diameter, without injury. Unlike rubber-insulated cables, the insulation remains concentric with the core.

Other advantages of varnished cambric are that splices are simple, and that mineral oils have no effect upon it.

Varnished cambric is the best insulation for high tension station wiring, as it can be installed without the metallic sheath and end bells required for paper cable, while it stands heat, static discharges, and overloads much better than the best grade of rubber insulation.

Oil of the variety generally used in switches and in the lubrication of generators does not injure varnished cambric. The cables can, therefore, be run directly into oil switches and oil-filled transformers, or can be used as leads to generators. Rubber cables when used in similar circumstances are ruined in a very short time.

FACTOR FOR CORRECTION OF INSULATION RESISTANCE TO 15.5° C.

Temperature, Deg. C.	Factor for Varnished Cambric.
30	13.00
29	11.00
28	9.35
27	8.07
26	6.76
25	5.92
24	5.00
23	4.06
22	3.30
21	2.76
20	2.32
19	2.00
18	1.70
17	1.40
16	1.16
15.5 (60° F.)	1.00
15	0.88

The resistance at 15.5° C. is found by multiplying the observed resistance by the factor corresponding to the temperature at which the resistance is measured.

2. INSULATED CABLES

Underground. Paper being the cheapest kind of insulation which is permanent, is more extensively used than any other kind for underground work. As long as the sheath is intact, it is as good as any other material. If the sheath is punctured either mechanically or electrolytically or any other way, moisture penetrates the paper and grounds the conductor. If the cable is under water, the zone affected by water will probably extend in both directions from the puncture and may necessitate the removal of the cable length from splicing chamber to splicing chamber. In such situations, varnished cambric or rubber should be used, preferably the former if the voltage is high. Another application of varnished cambric or rubber is to direct current railway feeders and third rail jumpers where, as explained below, a lead sheath is undesirable.

The cables which give the greatest trouble in underground conduit lines are direct-current railway feeders of large carrying capacity. The reasons for this are:

(1) When they are punctured the cable is short-circuited to the sheath, and the current is so great that the sheath is melted, often for a length of several hundred feet.

(2) If the direct-current cable sheath is in metallic

connection with other sheaths, the short-circuit current will distribute itself among these sheaths, and may melt them in the same way as the sheath of the original cable.

(3) When cable sheaths are melted or burned, not only are the cables put out of use; but it is often impossible to withdraw them from the ducts, which therefore have to be broken into and replaced. If this does not occur, the lining of the ducts may be so roughened as to render new ones necessary.

(4) The arc established at the point of short circuit is so intense as to be a source of danger to linemen, to other cables, and to the structure of the splicing chamber itself.

(5) If the short circuit occurs far from the station-bus, the resistance of the line may be sufficient to keep the value of the short-circuit current below that at which the circuit breakers are set to open. Such short circuits are particularly dangerous because there is no way to distinguish them on the station meters from a regular load.

With these facts in view, the following precautions should be adopted with large direct-current cables:

(1) Where possible, keep the direct-current cables out of the duct lines which carry the alternating-current cables. It may be advisable to put the required conductivity in the third rails in order to avoid the necessity of auxiliary copper in duct lines along the track.

(2) If it is necessary to put direct- and alternating-current cables in the same duct line it is well to isolate the direct-current cables as much as possible in the splicing chambers. This may be effected by running them in open-face ducts, or by protecting them with split ducts put around the cables and held together by clay. Such ducts may be supported on one or two light angle irons extending longitudinally through the chamber.

(3) The racks on which direct-current cables are supported in splicing chambers should not be in metallic connection with other racks. If, however, this is unavoidable, the cables should not lay directly on the racks, but on insulating pads or blocks.

(4) The electrostatic charges on the sheaths of low-tension cables are insignificant, and none can be derived from the high-tension cables if these are properly grounded. It is therefore not necessary to ground the sheaths of direct-current cables of an insulated system. With a grounded return system, however, the case is different, for however well the cable sheath is insulated in a duct line, in the case of a short circuit the current will find its way through tne most unexpected paths, thereby creating wide-spread damage. As grounded return systems are used principally for railways, it is usual to ground the direct-current cable sheaths directly to the track rails through stout wires.

Where this is done, high tension cables in the same subways should not be grounded to the track rails. Railroad tracks having insulated sections for automatic block signals cannot be used in this way, as the sections would be electrically connected through the cable sheaths. In such cases, feeders should either be kept out of the duct line or protected by short circuit indicator wires, as described on page 93.

If retaining walls or tunnel walls are available, it is easy to support weatherproof cable on large porcelain clamp insulators attached to the walls. In the open, however, it is usually necessary to support the cables on insulators placed in a wooden or concrete trough, which is filled with viscous insulating and waterproof compound. This is known as the "solid system."

Arcs produced by the rupture of alternating-current tables are less intense than those produced by direct-current cables, for the following reasons:

(1) The periodic reversal of the current tends to extinguish the arc twice in every cycle.

(2) The amperes per kilowatt transmitted are less than with direct-currents, owing to the high voltages used in alternating-current transmission systems.

(3) The use of two or three conductor cables helps to make a clean short circuit, which will operate the power-house relays at once and thus open the circuit. It also does away with the tendency to follow

any roundabout path to ground, as with direct-current cables.

(4) The carrying capacity of the lead sheath of a high-tension cable is usually great enough to take without injury sufficient current to operate the relays in the power station, especially where resistance is used in the grounded neutral of the generators.

There is, however, a danger inherent to high-tension cables which must be carefully guarded against, namely, electrostatic induction.

When an electrically charged body is introduced, without touching, into a cylindrical conductor, a charge is induced on the inner surface of the cylinder, which is equal in magnitude but opposite in sign to the charge on the electrified body. If the cylinder is insulated from the earth there is also induced on its outer surface a charge of equal magnitude and similar sign to that of the electrified body. The difference of potentials between the charged body and cylinder depends on their dimensions and relative locations, and may be very considerable. If, however, the cylindrical conductor is connected to the ground by a metallic wire, it will be maintained at ground potential.

A high-tension cable in a lead sheath acts precisely like the charged body in a cylindrical conductor described above, inducing a charge on the sheath which may raise the latter to a dangerously high potential.

It is therefore necessary to ground the sheaths of cables at intervals, in order to carry off their "static," as the induced charge is commonly called.

Under Water. Rubber is almost invariably used for submarine power work on account of its absolute waterproofness and inertness with respect to salt water.

While it is usual to enclose the rubber in a lead sheath protected by steel armor, the lead sheath may be dispensed with unless there is sewage or other injurious impurities in the water.

Owing to their inaccessibility for repairs, submarine cables should be free from all defects which might give rise to trouble in the event of excessive current or voltages occurring in them. Such defects are splices in the conductors, faulty patches in the insulation, faulty patches in the sheathing, injury to sheathing by tight armor, etc. These defects should be guarded against by careful inspection at the factory.

The author has seen a length of 11,000 volt triplex submarine cable supplied by one of the best-known manufacturers in the country in which were discovered a group of wire splices which had become loose in service, an unvulcanized patch in the insulation, and two improperly repaired splits in the lead sheath. The cable broke down in service and was a total loss.

On Walls, etc., in the Open. Varnished cambric exposed to the heat of the sun deteriorates owing to the softening of the compound and its consequent settlement from the upper part to the lower part of the cable. This phenomenon occurs where cables are laid either horizontally or vertically. In consequence of this, rubber is less liable to give trouble in exposed locations. Paper is but slightly affected by the softening and flow of compound where the cables are horizontal, but where cables are vertical it is likely to give trouble.

In House Conduits. For house wiring, rubber insulation covered with tape and braid is almost invariably used, and except for the larger sizes it is practically alone in the field. Beginning with No. 6 B. & S. varnished cambric is a rival to the rubber if the wire does not have to be pulled around sharp bends.

"Code" insulation is a cheap rubber compound or substitute for rubber which is very largely used for house wiring and its use is probably responsible for the large number of fires due to defective insulation.

Thickness of Insulation. The thickness of insulation which should be used for a given voltage and size, is determined largely by experience. The dielectric strength of impregnated cambric is so great that a very thin film of this material, under laboratory conditions, will suffice for most voltages in practical use. An enormous factor of safety, however, is necessary in order to compensate for inevitable defects in manufacture and to allow for injury in handling, especially in bending. Rubber, on the other hand, is comparatively weak dielectrically, and superior mechanically, making the calculation of thickness a possibility. Table I gives the proper thickness calculated according to the theory given in Appendix 3. Tables II was prepared by the engineer of an important manufacturing firm. Tables III and IV have been adopted as standards by the Rubber Covered Wire Engineers Association (1907). Table V and its accompanying data were prepared by Mr. H. G. Stott, whose experience with paper insulated cables is probably unequalled.

Table VI is from a G. E. Co. bulletin, and represents the best practice with varnished cambric.

The thickness of insulation on cables for very high voltages can be considerably reduced by grading. The conductor is first insulated with rubber and the cambric is then applied to secure the insulating wall necessary for the required test.

The width of tape over the insulation is usually equal to twice the square root of the cable diameter over the insulation.

Table I

THICKNESS OF RUBBER INSULATION, 64THS INCH

See Appendix III

Size.	Number of Strands.	Single Phase Volts between Conductor and Sheath.			
		710.	2300.	3900.	6700.
		Direct Current.	Three Phase Volts between Conductors. Thickness of Insulation around each Conductor.		
		Up to 1000V	4000.	6750.	11500.
B. & S.					
14	1	3 or 4	10		
12	1	3 or 4	8		
10	1	3 or 4	7		
8	7	4	7	14	
6	7	4	7	12	
4	7	4	7	11	25
2	19	4	7	11	22
1	19	5	7	11	20
0	19	5	7	10	19
00	19	5	7	10	18
000	19	5	7	10	18
0000	19	6	7	10	18
Millions of Circ. Mils.					
0.25	37	6	8	10	17
0.35	61	7	8	11	17
0.5	61	7	8	11	17
0.75	91	8			
1.0	91	9			
1.25	127	10			
1.50	127	10			
1.75	127	11			
2.00	133	12			

Stranding, concentric, except for 2,000,000 C.M., which is rope.

Table II

RUBBER INSULATION

Puncture Tests (30% Para Compound

Low Potential, 600 Volts

B. & S. Gauge.	Wall.	Voltage Test for 1 Minute.
Nos. 14 to 8	3/64 in.	1,000
" 6 to 2	4/64 "	1,000
" 1 to 4/0	5/64 "	1,000
250,000 to 500,000 cir. mils	6/64 "	1,000
550,000 to 1,000,000 "	7/64 "	1,000
Medium Potential, 3500 Volts		
Nos. 14 to 8	3/32 in.	5,000
" 6 to 2	3/32 "	5,000
" 1 to 4/0	3/32 "	5,000
250,000 to 500,000 cir. mils	3/32 "	5,000
550,000 to 1,000,000 "	4/32 "	5,000
5000 Volts Working Pressure		
Nos. 4 to 4/0	6/32 in.	10,000
250,000 to 500,000 cir. mils	6/32 "	10,000
550,000 to 1,000,000 "	6/32 "	10,000
11,000 Volts Working Pressure		
Nos. 4 to 4/0	9/32 in.	15,000
250,000 to 500,000 cir. mils	9/32 "	15,000
550,000 to 1,000,000 "	9/32 "	15,000
Nos. 4 to 4/0	10/32 in.	20,000
250,000 to 500,000 cir. mils	10/32 "	20,000
550,000 to 1,000,000 "	10/32 "	20,000
Nos. 4 to 4/0	12/32 in.	20,000
250,000 to 500,000 cir. mils	12/32 "	20,000
550,000 to 1,000,000 "	12/32 "	20,000

J. Langan, Am. Inst. Elect. Eng., 1906.

Table III

RUBBER INSULATION

Megohms per Mile. 60° F. One Minute Electrification

	Thickness of insulation in inches.									
	3/64.	2/32.	5/64.	3/32.	7/64.	4/32.	5/32.	6/32.	7/32.	8/32.
C.M.										
1,000,000	.....	.....	.....	.....	300	340	420	490	560	630
900,000	.....	.....	.....	.....	320	360	440	510	590	660
800,000	.....	.....	.....	.....	330	380	460	540	610	690
700,000	.....	.....	.....	.....	350	400	490	570	650	730
600,000	.....	.....	.....	.....	380	430	520	610	690	770
500,000	.....	.....	.....	360	410	460	570	660	750	830
400,000	.....	.....	.....	400	450	510	620	720	820	910
300,000	.....	.....	.....	450	520	580	700	810	910	1010
250,000	.....	.....	.....	490	560	630	750	870	980	1090
4/0 Strand	.....	.....	450	530	610	680	820	940	1060	1170
3/0 Strand	.....	.....	500	590	670	740	890	1020	1150	1270
2/0 Strand	.....	.....	560	650	740	820	980	1130	1260	1380
1/0 Strand	.....	.....	600	710	800	890	1060	1210	1350	1470
1 Solid	.....	.....	750	870	970	1080	1270	1440	1600	1740
2 Solid	.....	680	820	950	1070	1170	1380	1560	1720	1870
3 Solid	.....	750	900	1040	1160	1280	1490	1680	1850	2000
4 Solid	.....	820	980	1130	1260	1380	1610	1800	1980	2140
5 Solid	.....	910	1070	1230	1370	1500	1740	1940	2130	2290
6 Solid	.....	990	1160	1330	1480	1610	1860	2070	2260	2430
8 Solid	950	1170	1370	1560	1720	1870	2140	2360	2570	2750
9 Solid	1040	1280	1490	1680	1850	2000	2280	2520	2730	2910
10 Solid	1130	1390	1610	1810	1990	2150	2440	2680	2890	3000
12 Solid	1340	1620	1860	2080	2270	2440	2750	3000	3220	3420
14 Solid	1550	1860	2120	2360	2560	2740	3060	3320	3550	3750

Rubber Covered Wire Engineers Association (1907).

Table IV

VOLTAGE TEST FOR FIVE MINUTES

For 30 Minutes Test, take 80% of These Figures

Size.	Thickness of Insulation in Inches.									
	3/64.	2/32.	5/64.	3/32.	7/64.	4/32.	5/32.	6/32.	7/32.	8/32.
1,000,000 to 550,000	...	.....	.....	.....	6000	8000	12000	16000	19000	22000
500,000 to 250,000	...	.....	.....	5000	7000	9000	13000	16000	19000	22000
4/0 to 1	.....	.....	4000	6000	8000	10000	13000	16000	19000	22000
2 to 7	.....	3000	5000	7000	9000	11000	14000	16000	18000	20000
8 to 14	3000	4500	6000	7500	9000	10000	11000	12000		

Rubber Covered Wire Engineers Association (1907).

Thickness of Insulation (Paper). "As the result of some fifteen years of experience with underground cables, the following table, giving thickness of insulation and lead sheath for various sizes of conductors and working pressures, is submitted as representing conservative practice:

Table V

PAPER INSULATION

Standard Working Pressure of 4000 Volts

Size of Conductors.	Thickness of Insulation.	Thickness of Lead.	
		Single Cond.	Three Cond.
Nos. 6 to 2 B. & S.	5/32 in.	5/64 in.	3/32 in.
" 1 to 00 "	5/32 "	3/32 "	7/64 "
No. 000 to 300,000 cm.	6/32 "	7/64 "	9/64 "
400,000 to 750,000 cm......	6/32 "	7/64 "	
800,000 to 1,000,000 "	6/32 "	4/32 "	
1,250,000 to 2,000,000 "	7/32 "	9/64 "	

"For each 1000 volts increase of pressure above 4000 add 1/32-in. insulation to the wall until 11,000 volts is reached, and after that add 1/64 in. for each 1000 volts. For example, the insulation required on a No. 0 B. & S. 25,000-volt cable would be 19-32 in. If 35% Para rubber compound or varnished cambric is used for insulation the above empirical rule may be changed to read: for each 1000 volts increase above 3000, add 1/64 in. insulation to the thickness of wall until 25,000 volts is reached. For the insulation of low-potential cables, 4/32 in. paper should be used on all sizes up to 1,000,000 cm., and from 1,250,000 to 2,000,000 cm., 5/32 in. should be used.

"From a purely electrical point of view, one-half of this insulation would be ample to withstand 650 volts working pressure, but the mechanical effects of reeling and unreeling the cable and pulling it into ducts and bending around the manholes, are to practically destroy the insulating qualities of the layer of paper next the lead, so that we really start in with a cable having approximately 1/32 in. of its insulation destroyed before it is put into commission; this mechanical destruction of insulation is especially marked in cold weather, as the oils used with the paper tend to congeal when subjected to a temperature below 32° F. The cable manufacturers have met this difficulty by using more fluid oil, with the result that the insulation resistance of the cable may not be more than 50 megohms at 60° F., but by the use of this very soft insulation they have produced a cable giving a very low insulation, but a high puncture test, and at the same time have met, to a great extent, the difficulty of handling paper cable in cold weather. It is always advisable, however, if a cable is to be used in a temperature below 32° F., to keep it in a warm place, such as a boiler-room, for at least twelve hours before drawing it in. The cable may then be used in the coldest weather, as it gives up its heat very slowly." (*H. G. Stott, Am. Street and Interurban Ry. Assoc., Oct., 1906.*)

The working voltages in Table VI are based on all conductors of the circuit being insulated. For direct-current 600-volt railway single conductor, leaded cables, use 2000-volt class. For three-phase "Y" connected circuits with grounded neutral with three conductor cables, thickness of insulation between conductors and ground need only be $^7/_{10}$ of that between conductors. Tests on such cable in proportion to thickness of insulation: Example, three-phase, 12,000-volt circuit "Y," neutral grounded, insulation on each conductor $^6/_{32}$ in. (total between conductors $^{12}/_{32}$ in.), outer belt $^3/_{32}$ in. (total $^9/_{32}$ in.); test pressure at factory for five minutes between conductors 30,000 volts, each conductor to earth 22,500 volts. For mechanical reasons, thickness of insulation on individual conductors of three-conductor cables 3000 volts and less is made somewhat greater than required by working pressure on some sizes.

Table VI

WORKING AND TEST VOLTAGES

Varnished Cambric

Kilo Volts Working Pressure.	Sizes.	Thickness Insulation.	Test in Kilo Volts: At Factory. 5 min.	30 min.	60 min.	After Installation. 5 min.	30 min	60 min.
1	6–2	1/16	2.5	2	1.6	2	1.6	1.3
1	1–0000	5/64	2.5	2	1.6	2	1.6	1.3
1	250,000–500,000	3/32	2.5	2	1.6	2	1.6	1.3
1	550,000–1,000,000	7/64	2.5	2	1.6	2	1.6	1.3
1	1,100,000 and over	4/32	2.5	2	1.6	2	1.6	1.3
2	6–0000	3/32	5.	4	3.2	4	3.2	2.6
2	250,000–500,000	7/64	5.	4	3.2	4	3.2	2.6
2	550,000–2,000,000	4/32	5.	4	3.2	4	3.2	2.6
3	All sizes	9/64	7.5	6	4.2	6	4.8	3.8
4	"	5/32	10.	8	6.4	8	6.4	5.1
5	"	6/32	12.5	10	8.	10	8.	6.4
6	"	7/32	15.	12	9.6	12	9.6	7.7
7	"	8/32	17.5	14	11.2	14	11.2	9.0
8	"	17/64	20.	16	12.8	16	12.8	10.2
9	"	9/32	22.5	18	14.4	18	14.4	11.5
10	"	10/32	25.	20	16.	20	16.	12.8
11	"	11/32	27.5	22	17.6	22	17.6	14.1
12	"	12/32	30.	24	19.2	24	19.2	15.4
13	"	12/32	32.5	26	20.8	26	20.8	16.6
14	"	13/32	35.	28	22.4	28	22.4	17.9
15	"	13/32	37.5	30	24.	30	24.0	19.2
16	"	14/32	40.	32	25.6	32	25.6	20.5
17	"	14/32	42.5	34	27.2	34	27.2	21.7
18	"	15/32	45.	36	28.8	36	28.8	23.0
19	"	15/32	47.5	38	30.4	38	30.4	24.3
20	"	16/32	50.	40	32.	40	32.	25.5
21	"	16/32	52.5	42	33.6	42	33.6	26.8
22	"	17/32	55.	44	35.2	44	35.2	28.1
23	"	17/32	57.	46	36.8	46	36.8	29.4
24	"	18/32	60.	48	38.4	48	38.4	30.7
25	"	18/32	62.5	50	40.	50	40.	31.9

G. E. Bulletin 4591.

Belted and Unbelted Triplex Cable. In a three-conductor cable for, say, 11,000 volts, the insulation can be most advantageously disposed if each condutor is insulated for half of 11,000, i.e., 5500 volts, and the group insulated by a belt good for 900 volts, this being the difference between 5500 and 6400, the voltage from conductor to ground. A triplex cable built on this plan, i.e., with an exterior belt, is therefore dielectrically the strongest as long as the belt is intact. For this reason paper insulated cables are almost invariably of the belted type.

Rubber cables differ from paper in not necessarily breaking down when the sheath is punctured. It is therefore desirable to design such cables so that they will not be put out of service in the event of water getting at the insulation. When a triplex cable of the belt type is punctured so as to admit water under the belt, the whole surface under the belt and between conductors becomes filled with water for a considerable distance on each side of the puncture, perhaps even for the whole length of the cable. The result of such a puncture is to put a stress of 6400 volts on the 5500 volt insulation. The puncturing of a sheath of an unbelted triplex cable is attended with no such injurious result, and if the insulation of only one conductor is injured, the other two are intact. The former may be used if supplemented by a new single conductor cable or by a similar uninjured wire from another injured cable.

The processes of manufacture of belted triplex cables with rubber insulation also place this type at a disadvantage compared with the unbelted type. The insulation on the individual conductors being vulcanized and tested before the conductors are assembled is subjected to an additional cooking when the belt is vulcanized. This is liable to alter its electrical and mechanical characteristics after test, which is very undesirable.

Diameter of a Triplex Cable.

Let d = diameter of each conductor;
t = thickness of insulation around each conductor;
T = sum of thickness of sheath and outer belt of insulation, if any.

Diameter $= 2.15d + 4.3t + 2T$.

Thickness of Sheath. The following sheath thicknesses are recommended as representing the best practice for cables in tile ducts:

Size.	Thickness of Sheath. Inches.
14–8 B. & S.	3/64
6–1 B. & S.	4/64
0 B. & S. to 250,000 cm.	5/64
500,000 to 750,000 cm.	6/64
1,000,000 cm.	7/64
1,250,000–2,000,000 cm.	8/64
Triplex–000 B. & S.	8/64
Triplex–0000 B. & S.	9/64

Short Circuit Indicator. Direct-current feeders fed through circuit breakers set for large currents may be protected against the effects of short circuits by means of the following device:

The automatic relay feature of the circuit breaker is connected to a small wire or a pair of wires clipped or taped to the feeder cable along its entire length in such a way that a short circuit will burn these wires and thereby open the relay circuit. The relay is of the low voltage release type, so that the interruption of its circuit has the effect of promptly opening the circuit breaker. A diagram of connections is shown in Fig. 8.

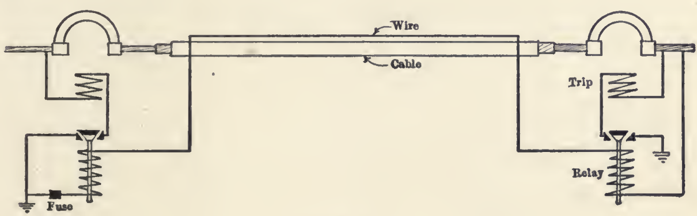


FIG. 8.

A No. 12 B. & S. wire with $\frac{1}{16}$ in. 30% Para rubber compound taped and braided is usually suitable for this service, but the correct size should be worked out for each installation, taking into account both the carrying capacity and potential drop. The fuse on the negative side comes into service in case the short circuit melts the indicator wire into contact with the main feeder metal thereby maintaining the continuity of the circuit. In such a case, the rush of current to ground blows the fuse and interrupts the relay circuit. This system has been in successful operation on the New York Central R. R. to protect feeders along the Park Avenue viaduct and tunnel. It was devised by the author early in 1906, and is unpatented.

3. INSULATORS, PINS, ETC.

REQUIREMENTS OF A GOOD INSULATOR

1. Dielectric strength.
2. Resistance to surface arcing.
3. Mechanical strength.
4. Ease of erection.
5. Facility of cleaning.
6. Negligible electrostatic capacity, this being, however, the least important qualification.

Dielectric Strength. This quality is affected by dielectric strength of material, by thickness of material, and by freedom from flaws.

Porcelain and glass are the only materials used extensively, although there are several compositions which have had success particularly for low tension work. Porcelain is almost universally used for high tension work, notable exceptions, however, being the use of glass for 57,000 volts by the Missouri River Power Company, and for 40,000 volts by the Madison River Power Company, Butte, Montana.

A thick head adds to the dielectric strength but reduces the mechanical strength. The "Italian" type is solid and is provided with a wide petticoat at each end and two small intermediate petticoats. The usual American practice for high tension work is to make the insulator in two or more pieces, each individually tested and assembled with litharge and glycerine cement. This construction adds considerably to the dielectric strength.

Porcelain which absorbs water should be avoided, although it is not uncommon to find an absorption of 1% or 2% in commercial porcelain.

Resistance to Surface Arcing. This quality is affected by material, texture of surface, and shape of insulator.

With regard to material, porcelain is universally conceded to be superior to glass on account of its less hygroscopic nature. The surface should be very smooth and uniform.

The shape is a matter of great importance, and there is a division of opinion as to the relative merits of many petticoats or a wide umbrella or bell combined with a long pin shield. Petticoats give long leakage surface but shorter arcing distance, and are more difficult to manufacture.

Mechanical Strength. Mechanical strength depends upon strength of material, thickness of material, and judicious design. Porcelain is superior to glass mechanically, and glass is more subject to internal stresses developed in manufacture. Glass, however, being transparent, has the advantage of enabling flaws to be readily detected.

Facility of Cleaning. Facility of cleaning depends upon the size of spaces between petticoats. The bell and shield type is decidedly superior to the petticoat type in this characteristic. Glass in some cases has the advantage of permitting inspection more readily on account of its transparency. The transparency has the further advantage of preventing insects from building cocoons under the petticoats.

Electrostatic Capacity. An insulator in service acts as the dielectric of a condenser, the two conductors of which are the wire and pin. The capacity of the insulator should be as low as possible to minimize operating troubles. This can be accomplished by having a considerable thickness of insulation between line and pin, precaution being taken to distribute the potential so as to make each shell carry its share of the potential stress. In fact, a multipart insulator acts as several condensers in series, the voltage stress in the different shells being dependent upon the relative capacities of the several condensers.

Shape. In a severe rainstorm the wind and spattering from the top surfaces of shells are liable to wet practically all of the insulator surfaces, except possibly the under surface of the inner shell. In order to keep this inner surface dry, the insulator must be carefully mounted with respect to the cross-arm. The ideal multipart insulator of the umbrella type should therefore have its inside shell so designed that alone it can carry the full line potential without puncture or arcing. This condition usually obtains on low voltage insulators but seldom on those for 60,000 volts or more.

With a given diameter and height, maximum sparking distance between adjacent rim and shell can be obtained by using the curved type of shell, but there is a point where this advantage is counterbalanced by the increased risk of spattering from the other shells. The flare of the shell is often determined by a radius taken about the rim of the upper shell as center, the curve beginning at the hypothetical dry line, assuming that the rain falls at an angle of 30° from the horizontal.

TEST VOLTAGE FOR INSULATORS

Dry Test, insulator assembled on metal pin; fifteen minutes at three times line voltage.

Wet Test, precipitation $\frac{1}{4}$ in. per minute, 45° angle spray nozzle; fifteen minutes at $1\frac{1}{2}$ times line voltage.

Puncture Test, for each shell; fifteen minutes at from $\frac{3}{4}$ to $1\frac{1}{2}$ times line voltage, the former figure for high voltages and the latter for low voltage.

By line voltage is meant the normal voltage between line and ground.

INSULATION FACTORS

Ratio of Arcing Distance $\frac{\text{Wet}}{\text{Dry}}$. This ratio varies from 0.3 to 0.9, averaging between 0.6 and 0.7

Ratio of Dry Creeping Surface with 45° Rain and Dry. This ratio varies from 0.5 to 0.85 and averages between 0.75 and 0.7.

Working Volts per Inch Thickness of Insulation. Above 10,000 volts this varies between 20,000 and 60,000, averaging between 30,000 and 40,000. Below 10,000 volts mechanical considerations settle the thickness.

Factor of Safety (ratio of breakdown to working volts). Above 20,000 volts the factor of safety varies between 2½ and 3 dry, and between 1½ and 2½ wet. At voltages around 10,000 the factor is usually between 6 and 8 dry and between 3 and 6 wet.

Puncturing Voltage of Porcelain. C. J. Greene (*Elec. Rev.*, Lond., Apr. 24, 1908) says that the average puncturing voltage of porcelain tested by him is approximately 100 kv. per inch.

LINK INSULATORS

The insulator consists of a solid porcelain piece having a flanged rim which affords a long creepage surface between live parts and insures some portion of the surface sheltered from rain. There are two interlinked holes in the center (Fig. 9) through which the cables or guy wires are threaded, thereby bringing a compressive strain on the porcelain.

An insulator of 10 in. diameter is suitable for 25,000 volt service and a 6½ in. insulator for 12,000 volts. For higher voltages, several disks are used in series spaced at a distance approximately equal to their diameter.

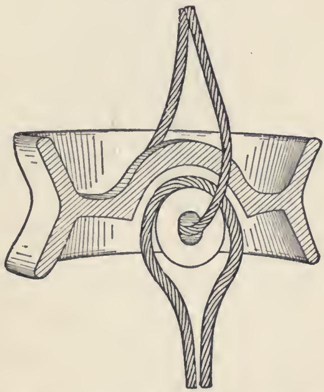


FIG. 9.—Cross Section of Link Strain Insulator.

The advantages of this tvpe of insulator are as follows:

(1) The material is subjected on.y to compress.ve strains.

(2) By the use of the proper number of insulators in series practically any line voltage can be used.

(3) High factor of safety both electrically and mechanically.

(4) Less likelihood of torsional strains in cross arms in the event of a wire breaking.

The chief disadvantages are:

(1) Increased height of poles or towers.

(2) Necessity of frequent anchoring of the line wire.

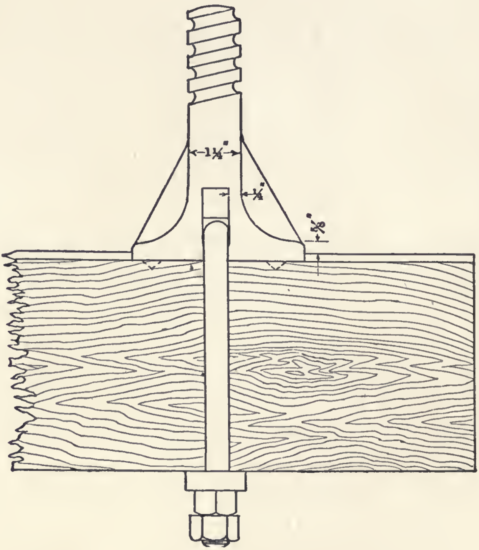
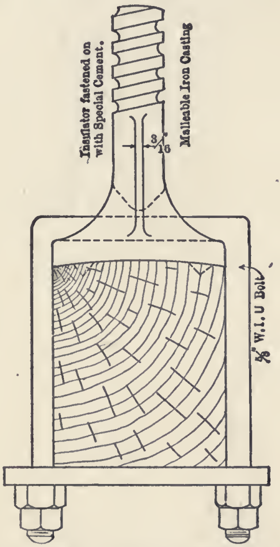


FIG. 10.

Where several discs are used in series, they should be linked together by hard drawn copper cable held fast by bolted clamps. Brass wire has been tried and found unsuitable on account of its uneven structure, and galvanized steel has been found to deteriorate rapidly.

PINS

Wooden pins are largely used for low-tension work, but are now considered risky for high-tension lines. The most approved type of pin is that of the Long Island R. R., a malleable cast-iron pin which it attached to the cross arm by a U bolt passing around the cross arm, as shown, in Figs. 10 and 11.



Insulator Pin for H. T. Lines. Long Island R. R. Type.
Scale ½ Full Size. Dimensions approx. only.

FIG. 11.

This construction obviates the drilling of holes in the cross arms. The advantages of metallic pins are long life, and if grounded, rapid and clean short circuit in the event of an insulator failing, thereby preventing protracted arcing and operating circuit breakers with certainty

Locust and eucalyptus are the most approved kinds of wood for insulator pins.

PROPOSED STANDARD PINS

(See Fig. 12.)

	A.	*B.*	*C* Nominal.	*C* Actual.	*D.*	*E.*	*F.*	*G.*	*H.*	*I.*
5	$4\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{15}{32}$	$1\frac{7}{16}$	$1\frac{1}{4}$	1	$\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{2}$
7	$6\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{3}{4}$	$1\frac{23}{32}$	$1\frac{11}{16}$	Same for all sizes.	Same for all sizes.	Same for all sizes.	$2\frac{1}{8}$	Same for all sizes.
9	$8\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{7}{8}$	$1\frac{27}{32}$	$1\frac{13}{16}$				$2\frac{1}{4}$	
11	$10\frac{3}{4}$	$4\frac{3}{4}$	2	$1\frac{31}{32}$	$1\frac{15}{16}$				$2\frac{3}{8}$	
13	$12\frac{3}{4}$	$4\frac{3}{4}$	$2\frac{1}{8}$	$2\frac{3}{32}$	$2\frac{1}{16}$				$2\frac{1}{2}$	
15	$14\frac{3}{4}$	$4\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{7}{32}$	$2\frac{3}{16}$				$2\frac{5}{8}$	
17	$16\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{3}{8}$	$2\frac{11}{32}$	$2\frac{5}{16}$				$2\frac{3}{4}$	
19	$18\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{15}{32}$	$2\frac{7}{16}$				$2\frac{7}{8}$	

Trans. Am. Inst. E. E., vol. XX, p. 415.

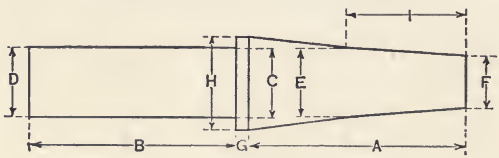


FIG. 12.

CONDUCTIVITY OF ATMOSPHERE AT HIGH VOLTAGES

(From Amer. Inst. Elec. Eng., 1904, H. J. Ryan.)

E = maximum value of voltage curve (to obtain R.M.S. value divide by $\sqrt{2}$);

r = radius of conductor, inches;

s = distance between conductors, center to center;

D = strength of electrostatic field, coulombs per sq.in., causing atmospheric rupture.

d = distance from the surface of the conductor at which atmospheric rupture is initially caused.

$$E = 2055 \log_{10} \cdot \left(\frac{s}{r}\right) D(r+d) \times 10^{10}.$$

The following table gives the relation between d, D', and r at a pressure of 29.5 in. of mercury and a temperature of 70° F.:

B. & S. Gauge.	$2r$.	d.	D.
20	0.03196	0.0050	350×10^{-10}
15	0.05706	0.0100	300 "
10	0.10189	0.0180	275 "
8	0.12849	0.0220	258 "
6	0.16202	0.0350	200 "
4	0.20431	0.0700	171 "
2	0.25763	0.0700	170 "
up to .625 in. diam.	....	0.0700	170 "

Amended to allow for barometric pressure and temperature, the above formula reduces to the following, in which b = barometric pressure in inches of mercury, and t = temperature, F. deg.,

$$E = \frac{17.94b}{459+t} \times 350{,}000 \log_{10}\left(\frac{s}{r}\right)(r+0.07).$$

If the surface of the wire is rough, the voltage at which it glows is less than given above.

Experiments of R. D. Mershon at Niagara give the critical voltage approximately 40% less than the values calculated from Ryan's formula. (See Proc. Am. Inst. Elect. Eng. June 30, 1908.)

CHAPTER IV

DETERMINATION OF SIZE OF CONDUCTORS

1. VOLTAGE AND SYSTEMS OF DISTRIBUTION

GENERAL IMPORTANCE OF HIGH VOLTAGE

The amount of copper required to transmit a given amount of power at a given loss over a given distance, other things being equal, is inversely proportional to the square of the potential used, whatever the system of distribution.

Comparison of the different systems, such as two-wire single phase, three-wire three-phase, and quarter-phase is given below on the basis of equality of power delivered, loss and potential.

In low-potential circuits, as secondary networks, where the potential is not limited by the insulation strain in the transmission system but by the potential of the apparatus connected into the system, as, for example, incandescent lamps, the proper basis of comparison is equality of the potential per branch of the system, or per phase.

On the other hand, in long distance transmission where the potential is not restricted by any consideration of apparatus suitable for a certain maximum potential only, but where the limitation of potential depends upon the proper insulation of the conductors against disruptive discharge, the correct comparison is on the basis of equal maximum dielectric strain on the insulation; for overhead lines this means equality of potential to ground as it is between ground and wire that the insulation (other than air) has to be provided.

COMPARISON OF SYSTEMS WITH EQUAL EFFECTIVE DIFFERENCE OF POTENTIAL ACROSS BRANCH OR PHASE OF LOWEST DIFFERENCE OF POTENTIAL

No. of Wires	System.	Relative Amount of Copper.
2	Continuous current	100
2	Single-phase	100
3	Edison three-wire, d. c. or single-phase, neutral full section	37.5
3	Edison three-wire, d. c. or single-phase, neutral half section	31.25
3	Inverted three-phase (derived from two branches of a 3-phase system by transformation by means of two transformers, whose secondaries are connected in opposite direction with respect to their primaries)	56.25
3	Quarter-phase with common return	72.9
3	Three-phase	75.0
4	Three-phase with neutral wire, full section	33.3
4	Three-phase with neutral wire, half section	29.17
4	Independent quarter-phase	100
5	Edison five-wire, d. c. or single-phase, full neutral	15.625
5	Edison five-wire d. c. or single phase, half neutral	10.93
5	Four wire, quarter phase, with common neutral, full section	31.25
5	Four wire, quarter-phase, with common neutral, half section	28.125

We see herefrom that in distribution for lighting, with the same minimum potential and with the same number of wires, the single phase system is superior to any polyphase system.

COMPARISON OF SYSTEMS WITH EQUAL MAXIMUM POTENTIAL TO GROUND

No. of Wires	System.	Relative Amount of Copper.
2	Single-phase, either without ground * or with one wire grounded.	100
2	Single-phase, center grounded.	25
2	Continuous current, either without ground* or with one wire grounded.	50
2	Continuous current, center grounded.	12.5
3	Three-phase, either without ground* or with one wire grounded.	75
3	Three-phase, neutral grounded.	25
3	Quarter-phase with common return, without ground or with either outer grounded.	145.7
3	Quarter-phase with grounded common return.	72.9
4	Independent quarter-phase, either without ground * or with one wire grounded.	100

* Even when no part of the system is grounded each wire has to be insulated from ground for a difference of potential equal to that between wires, since the difference of potential between any wire and ground may be anything from zero to full potential between wires.

Since the comparison is made on the basis of equal maximum potential and the maximum potential of an alternating system is $\sqrt{2}$ times that of a continuous-current circuit of equal effective potential, the alternating circuit of effective potential e compares with the continuous-current circuit of potential $e\sqrt{2}$, which latter requires only half the copper of the alternating system.

(The author is indebted to C. P. Steinmetz, "Alternating Current Phenomena," for much of the above data.)

Standard Transmission Line Voltages. The following three-phase voltages have been adopted by the General Electric Company as standard for railway work:

11,000 volts with delta connected transformers.
19,000 volts with delta connected transformers.
33,000 volts "Y" or delta connected transformers.
57,000 volts "Y" connected transformers.

These voltages step up in the ratio of the square root of three to one, allowing the voltage of any system to be raised in case of extensions from one standard to the next higher, by changing the transformer primary connections from delta to "Y." The lowest voltage (11,000), is the only one suited for direct generation without step-up transformers, and is generally so installed. Such systems are not readily changed over, for which reason 19,100 volt transformers are delta connected only. On account of the prevailing use of 13,200 volts, transformers and switching apparatus can be supplied for this voltage also. (*G. E. Review*, May, 1908.)

2. LAMP WIRING CALCULATIONS

PRELIMINARY

The following data are necessary for the wiring calculations.

(1) Length of feeder from bus to branches. Use length of wire, which is usually twice the distance.

(2) Number of branches.

(3) Length of wire and current taken by each branch.

(4) Permissible volts drop from bus to branches, in both wires.

(5) Permissible volts drop in each branch. Usually the same for all branches.

Calculation of Wire for Branches. Construct a table as shown below, giving for each branch the permissible drop, the length of wire, and the current. Then by the formula $\text{C.M.} = \frac{10.8 \times \text{ampere-feet}}{\text{Volts drop}}$, the size of the wire is calculated.

TABLE FOR CALCULATION OF BRANCH WIRES

Branch Number.	Permissible Drop of Volts in Branch $= v$.	Length of Wire in Branch, Feet $= F$.	Amperes taken by Branch $= A$.	$\frac{10.8AF}{v} =$ Circular Mils.	Size B. & S.
1					
2					
3					
etc.					

Calculation of Wire for Feeder or Main.

$$\text{C.M.} = \frac{10.8 \times \text{total current} \times \text{total length}}{\text{Permissible volts drop}}.$$

While the above form is the most usual, the formula may also be written as follows:

$$\text{C.M.} = \frac{1080 \times \text{total ampere-feet}}{p \times V.},$$

where V = volts delivered,

p = drop in mains in per cent of volts delivered.

Slide Rule for Wiring. A simple slide rule for wiring calculations devised by E. P. Roberts, is made by constructing a table as shown below, and cutting along the line between the first and second columns.

Size of Wire.	Thousands of Ampere Feet. Thousands of Circular Mils. Volts Loss.
500	500
400	400
320	320
250	250
0000	200
000	160
00	125
0	100
1	80
2	64
3	50
4	40
5	32
6	25
7	20
8	16
9	12
10→	10
11	8
12	6
13	5
14	4
15	3
16	2
17	2

Then to use the rule all that is required is to put the arrow-head opposite the figures in the second column representing volts loss allowable, and opposite thousands of ampere-feet, read in the second column, will be found in the first column the size of the wire required.

The action of the rule is based upon the fact that No. 10 wire has a resistance of practically one ohm per 1000 feet, and therefore with No. 10 wire 10,000 ampere-feet would give 10 volts loss. Also No. 10 wire has practically 10,000 circular mils cross-section, and the size of the wire doubles for each third size larger.

Three-Wire System. The outside wires are calculated by the above rules, ignoring the center or neutral wire, and treating two lamps in series as one lamp of double voltage.

The neutral wire of a branch is usually made the same size as the outers, although in most cases a smaller size would be possible.

Alternating Currents. The inductance of house wiring, where the two wires of a circuit are run in the same pipe or moulding, is negligible.

3. CONTINUOUS-CURRENT RAILWAY FEEDER CALCULATIONS

PERMISSIBLE POTENTIAL DROP

The total drop of potential in the positive and negative conductors is governed by four conditions, namely: the possibility of starting the cars, the brilliancy of the lights, the limiting of drop in the grounded conductors and the relative economy of low first cost compared with low energy loss. With regard to the question of starting the cars, the voltage required may be derived from a study of the motor curves.

With the multiple-unit system of control, the limiting voltage is usually that at which the contactors will operate satisfactorily, this being about one-half the normal running voltage. The voltage at which the car lights become too dim is about 90% of the rated voltage of the group of lamps. However, by using lamps, rated considerably below the normal bus voltage, it is permissible to let the voltage drop more than 10% without affecting the lights too seriously; although lamps thus used get an over-voltage when the load is light, causing a shortening of their life.

The drop in grounded conductors is usually covered by city ordinances, which require it not to exceed a specified amount.

The investment in a system of conductors may be expressed as an initial cost or as an annual interest thereon. The value of the kilowatt-hours of energy lost in these conductors is most conveniently expressed as an annual expense. The sum of these two annual items is the total annual expense of the feeders, which it is desirable to make as small as possible.

AUXILIARY FEEDERS

Any direct-current feeder system consists of two distinct parts, the conductors which supply current from the power-house to the line and the contact conductors which yield their current directly to the cars. In many cases the contact conductors will be sufficiently large to fulfil both functions, but more often they are supplemented by auxiliary copper feeders. The various steps at which auxiliary copper may have to be added are given below in the order in which they usually have to be treated.

I. If the drop in the grounded conductors exceeds the legal limit or the limit prescribed by danger of electrolysis, copper will have to be added to these conductors.

II. If with this additional copper the total drop in the positive and negative feeders is still too great to enable the cars to start, additional copper must again be resorted to, but this time it may be added to either the positive or negative system. Whether it will be more economical to add it to the positives or negatives will have to be worked out for each case, although an indication is given by the fact that if the unit price of conductors installed is the same for both, it is more economical to distribute the copper so as to make the resistance of the two systems equal.

III. Having provided copper to maintain the voltage high enough to start the cars, it remains to determine whether it is also high enough to keep the lamps bright. If not, more copper must be added in the way described above.

IV. The feeder system having been made of ample dimensions to meet all the conditions of the service it remains to determine whether the annual loss in the conductors is great enough to justify the addition of more copper in order to keep down the operating expenses. If the conductivity is sufficient, there is nothing to be done; but if the considerations of operating economy call for more copper, the engineer is justified in recommending it.

In order to determine the most economical copper investment, it is convenient to compile a table showing the following six quantities: (1) Value of proposed additional conductors. (2) Total annual energy loss (kilowatt-hours) in the entire positive and negative system, including the proposed additional conductors. (3) Value of this lost energy. (4) Value of the additional conductors. (5) Annual interest on value of additional conductors. (6) Sum of value of total annual energy loss and the interest on proposed additional conductors. When selecting the figures for the first column two values should be assumed initially and all the other columns worked out for them, in order to give an indication of the range of values which is most convenient to work with.

An abbreviation of this calculation is given under Kapp's and Pender's modifications of Kelvin's law.

V. If after these conditions are satisfied, the carrying capacity is insufficient, more copper must be added.

DISTRIBUTION OF CURRENT

A certain current passing from the positive to the negative system at the end of the line farthest from the power station being assumed to cause a total drop of V volts, the same total current taken from from the line in n equal amounts at n equidistant points along the line will produce a total drop of $\left(1+\frac{1}{n}\right)\frac{V}{2}$ volts. If n is infinite, that is, if the drain of current is uniform along the line, the drop will be $\frac{V}{2}$. If, however, n is not infinite, the drop will be greater than $\frac{V}{2}$ by $\frac{100}{n}$ per cent, a quantity which is quite small when n is considerable. It is therefore usual to assume a uniform drain of current, a procedure which is further justified by the continuous motion of the load which causes it to act as if more distributed. Such an assumption, however, is by no means justifiable on interurban or trunk line railroads, as in such cases the trains are usually far apart. This case is treated separately below.

DISTRIBUTION OF COPPER

The drop of potential depends largely on how the copper is distributed along the line. It is therefore important to secure the most economical distribution of copper which will give the required drop. The auxiliary copper may be connected to the contact conductor at such frequent intervals that it virtually forms a part of it; it may, on the other hand, be connected at one end only, or it may be connected at such distances as not to be covered by either of the above cases. Each of these schemes requires separate consideration, a general method of treatment being given for each, which covers the addition of copper to either the positive or negative system, as the case may require.

AUXILIARY COPPER FREQUENTLY CONNECTED

The diagram in Fig. 13 shows the most economical way of distributing the feeder metal; the formulæ for circular mils, volume of copper, watts lost and potential drop are also given.* The following symbols are used in both Figs. 13 and 14.

C.M. = Area in Circular mils, where one C.M. is the area of a circle of $^1/_{1000}$ inch diameter.

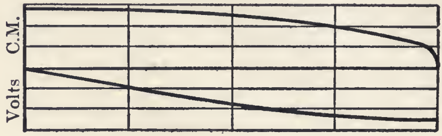


FIG. 13.

C.M.-Ft. = Volume in Circular mil-feet, where one C.M.-Ft. is the volume of a cylinder of one c.m. area and one foot long. A volume of copper in c.m.-ft. divided by any number of c.m. gives the number of feet of cable of that area required to make up the given volume of copper.

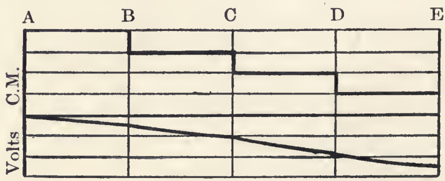


FIG. 14.

r = the resistance of a c.m.-ft. of copper, measured along its length, at about 60° F.

r = 10.2 for copper of 100% cond

10.3 for copper of 99% cond.

10.4 for copper of 98% cond.

10.5 for copper of 97% cond.

* See Appendix 4.

If the conductors are partly of iron, as with a third rail, it is usual to reduce the area of iron to its equivalent area of copper.

V = drop of potential from the station bus to the end of the line in either the positive or negative conductors, as the case may be.

A = total current delivered from the station bus to the section under consideration.

L = length of the section, feet.

x = distance (feet) of any point from the end of the line farthest from the station.

$$\text{C.M.} = \frac{2}{3} r \cdot \frac{A}{V} \cdot \sqrt{L} \cdot \sqrt{x},$$

$$\text{C.M.} - \text{feet} = \frac{4}{9} \cdot r \cdot \frac{A}{V} \cdot L^2.$$

$$\text{Drop} = \frac{V}{\sqrt{L^3}} \cdot \sqrt{x^3},$$

$$\text{Watts lost} = \frac{3}{5} A V.$$

It is, of course, impossible to exactly realize the most economical distribution in practice, so that a series of steps, as shown in the second diagram, should be arranged so as to approximate as closely as possible to the theoretical curve. It should be remembered that the curve of most economical distribution shows the total feeder metal, including the contact conductors.

The approximation to the most economical distribution is calculated in the following way. Referring to Fig. 14:

X_1 = distance ED, and Y_1 = c.m. of copper in ED.
X_2 = distance EC, and Y_2 = c.m. of copper in DC.
X_3 = distance EB, and Y_3 = c.m. of copper in CB.
X_4 = distance EA, and Y_4 = c.m. of copper in BA.

$$k = \tfrac{1}{2} r \frac{A}{L}.$$

$$\text{Drop in } DE = k\frac{X^2{}_1}{Y_1}$$

$$\text{“} \quad CD = k\frac{X^2{}_2 - X^2{}_1}{Y_2}$$

$$\text{“} \quad BC = k\frac{X^2{}_3 - X^2{}_2}{Y_3}$$

$$\text{“} \quad AB = k\frac{X^2{}_4 - X^2{}_3}{Y_4}$$

Total watts lost =

$$\frac{1}{3} \cdot r \cdot \frac{A^2}{L^2} \Sigma \frac{1}{Y_2} (X^3{}_2 - X^3{}_1)$$

The drop given by the above formula is from the far end of the line. The drop from the station end may be obtained by subracting this value from V.

AUXILIARY COPPER CONNECTED AT END

The auxiliary feeder, in this case being merely a uniform conductor with the same current along its entire length, may be treated by Ohm's law in its simplest form. Auxiliary conductors of this sort are useful in connection with grounded returns in which it is desired to minimize the drop. Two or more insulated conductors, connected to the line at various points will each take off its proportion of the current without making the entire current accumulate near the station, as would be the case with a single connection direct from the bus. This gives rise to a series of rises and falls of potential along the line, but there will be no serious drop in the grounded conductors, irrespective of what the drop may be in the insulated feeders connected thereto.

FEEDERS INFREQUENTLY CONNECTED

This condition occurs where a feeder cable runs parallel to the line and is tapped in at intervals through circuit breakers or switches. The expense of the breakers renders it necessary to have as few such connections as possible. Fig. 15 shows an example of such a system, comprising four conductors, some of which may be contact conductors and others, auxiliary feeders. Fig. 16 shows this scheme

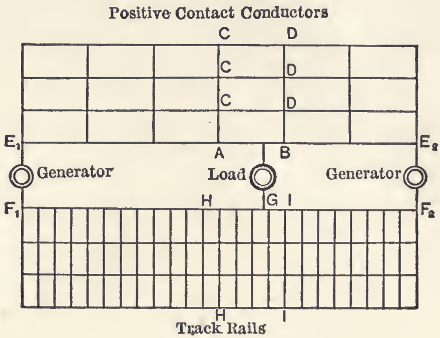


FIG. 15.

in diagrammatic form with corresponding points indicated by identical letters.

The resistance of this system may be calculated in

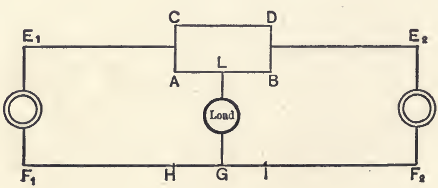


FIG. 16.

two ways, the first of which is simpler, but the second more complete, as it gives the point of maximum resistance.

First Method. Referring to Figs. 15 and 16, the resistances of the various sections are designated as follows:

Points.	Conductors.	Resistance.
E_1 to A	All tracks	c
E_2 to B	All tracks	d
C to D	All feeders but one	e
A to L	One track	a
B to L	One track	b
F_1 to G	All tracks*	m
F_2 to G	All tracks*	n

* Including negative feeders.

Resistance from load to both substations equals

$$\frac{AF-\dfrac{Ab^2+Fa^2}{B}}{A+F-\dfrac{(b+a)^2}{B}},$$

where

$$A=c+a+m;$$
$$F=d+b+n;$$
$$B=a+b+e.$$

(Derivation of above formula given in Appendix IV.)

Second Method. Where the maximum resistance is required, the following formulæ may be used. The resistances and lengths are as follows:

R = resistance of third rail per 1000 ft.;
r = resistance of all track rails per 1000 ft.
l = length AB in thousands of feet;
x = distance from A to point of maximum resistance, thousands of feet;

c = resistance from E_1 to A, all tracks;
d = resistance from E_2 to B, all tracks;
k = resistance from F_1 to H, all tracks;
y = resistance from F_2 to I, all tracks;
$Q = R + r$;
$v = (d+y) + (c+k) = H + D$, say;
$w = (d+y) - (c+k) = H - D$;

$$P = Q^2 + \frac{R^2 v + RrlQ}{e};$$

$$S = P + w\left(\frac{Q}{l} + \frac{Rr}{e}\right);$$

$$T = \frac{P}{l};$$

$$U = \left\{R(H+e) + \frac{He}{l} + (Rl+e)r\right\}\frac{D}{e};$$

$$V = (v + lr)\frac{R}{e} + Q + \frac{v}{l}.$$

Then

$$\text{Resistance} = \frac{-Tx^2 + Sx + U}{V}$$

and resistance is a maximum where

$$x = \frac{S}{2T}.$$

(Derivation of above formula given in Appendix IV.)

Third Method. Unlike the two previous methods, this is intended to be used where there is only one substation feeding the section, as shown in Fig. 17.

Let R = resistance in ohms per thousand feet of single contact conductor;

r = resistance in ohms per thousand feet of combined track rails;

e = multiple resistance of all conductors between A and B except the loaded one;

$A = R + r$;

$B = \frac{R^2}{Rl + e}$;

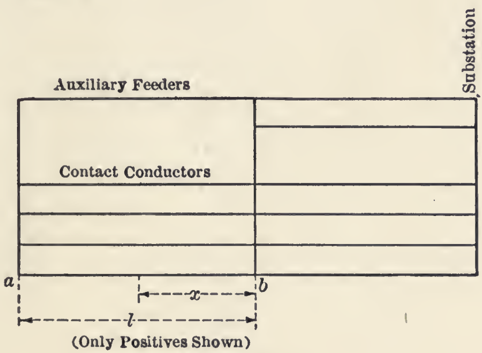


FIG. 17.

D = resistance from b to c. all conductors in multiple;

$E = r \times$ length bc in thousands of feet;

x = distance from b to point of maximum resistance from substation;

$= \frac{A}{2B}$.

The resistance from substation to point of maximum resistance from substation,

$$= Ax + D + E - Bx^2.$$

This may be applied to the section *bc* as well as to the section *ab*.

Fourth Method. The circuit, shown in Fig. 18, is that of a feeder system in which both positive and negative feeders are infrequently cross-bonded.

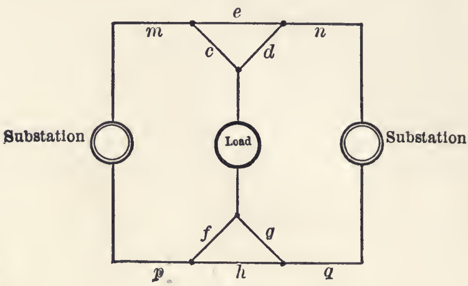


FIG. 18.

The resistances are designated by letters on the diagram and by the following:

$$A = m + c + f + p;$$

$$F = n + d + g + q;$$

$$B = c + d + e;$$

$$D = f + g + h;$$

$$P = \left(\frac{d^2}{B} + \frac{g^2}{D}\right);$$

$$Q = \left(\frac{c^2}{B} + \frac{f^2}{D}\right).$$

The total resistance from the load to both substations in multiple is given by the following expression:

$$\frac{AF-AP-FQ-2\left(\frac{fg}{D}\times\frac{cd}{B}\right)+\frac{f^2d^2+c^2g^2}{DB}}{A+F-P-Q-2\left(\frac{fg}{D}+\frac{cd}{B}\right)}.$$

(The derivation of the above formula is given in Appendix IV.)

If there are several trains between the two substations, the maximum drop in the section will be the sum of the drops computed for each train as if it were the only one on the line, and the trains should be distributed so as to give the worst condition that would arise in practice.

MISCELLANEOUS FORMULÆ

The potential drop in any uniform conductor in which the current varies along its length, is given by

Volts = ohms per ft. × area of current curve in ampere-feet.

The watts lost in any conductor along which there is a uniform drain of current are given by

Watts lost = amperes per ft. × area of drop curve in volt-feet.

If a curve of potential drop in any feeder system be plotted for one load, the drop curve for any other load similarly distributed may be derived from it by merely changing the ordinates in the ratio of the two loads in question.

VALUE OF CURRENT USED IN CALCULATIONS

Purpose.	Current.
Electrolysis.......	Depends upon local ordinances.
Car starting.......	Average current during half minute of maximum load.
Car lighting.......	If cars are closely spaced, the R.M.S. current during hour of maximum load. If cars are infrequent, it is better to use various unit train loads and estimate whether their effect upon the candle-power is excessive, when concentrated at various points.
Copper economy...	R.M.S. current of whole year. If the trains are too infrequent to permit the assumption of uniform current drain, the best approximation is to assume the R.M.S. current for the year, concentrated at the point of average resistance.
Heating of cables..	R.M.S. current taken over several periods of maximum load.

Let i_1, i_2, i_3, etc., be the currents flowing for t_1, t_2, t_3, etc., minutes respectively, and let T be the minutes in the total interval considered. Then the R.M.S. current $= \frac{1}{T}\sqrt{i_1^2 t_1 + i_2^2 t_2 + i_3^2 t_3 + \ldots}$

COST OF ENERGY

The cost of producing energy may be divided into two items.

Fixed Charges, which are independent of variations of output, and

Operating Expenses, which are practically proportional to the output. Fuel, water, and oil are included in this item.

In feeder calculations only the operating expenses should be used because the fixed charges exist independent of any saving in line losses.

4. NEGATIVE BOOSTER CALCULATIONS

In railway feeder work it is usual to assume the load to be uniformly distributed along the line, so that going towards the power station the current

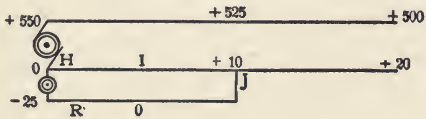


FIG. 19.

in the negative feeders, including the return rails, uniformly increases. The current flowing in the feeders to the bus bars will then be represented by a straight line diagram, provided that all the feeders are connected together so as to virtually form one conductor. When, however, a booster cable is connected to the negative feeders, as shown in Fig. 19, the booster cable being insulated from the other feeders, except at one point, the current will be drawn from the line into the booster cables and the current diagram will take one of the forms shown in

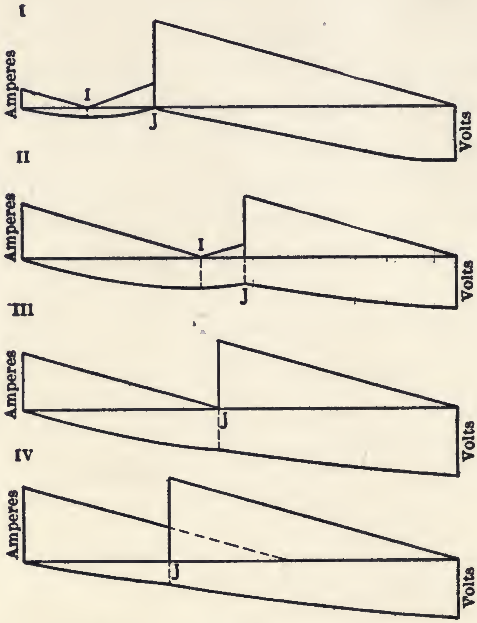


FIG. 20.

Fig. 20, these four forms, however, being treated in exactly the same way in the voltage calculations described below. Case I shows a booster which entirely neutralizes the drop in the booster cables and reduces the point of connection, *J*, to the same potential as the bus bar. In this case current is drawn into the booster cables from both sides of the point of connection, the current dividing at a point I, from which the resistance to the bus equals the resistance to the point J. In Case II, the booster only partially neutralizes the drop in its cables, but draws current from both sides of the point of connection. Case III shows a booster drawing current only from beyond the point of connection, the whole of the current on the other side returning to the bus by the line feeders. In Case IV, the booster draws only part of the current from beyond the point of connection, the remainder returning to the bus through the line feeders with the current from between the station and that point. A fifth case might be added to these, which is only useful when the permissible drop is very small. In this case the point of connection, I, is maintained at a lower potential than the negative bus itself.

The relation between line drop, booster E.M.F. and current may be found either by calculation or graphically. Considering the former method first, let

a = amperes entering negative feeder system per foot of line;

r = resistance of negative feeder system per foot;

i = total amperes entering negative feeder system;

i_0 = total amperes taken off by booster;

$l = HI = \frac{i - i_0}{a}$ = distance from H to the point at which the current in the negative feeders is zero. (Fig. 21);

$l_1 = IJ$;

$l_2 = JD$.

The volts drop in the various sections of the negative feeder is

$$\text{From } H \text{ to } I: \quad D = \tfrac{1}{2}arl^2;$$
$$I \text{ to } J: \quad D_1 = \tfrac{1}{2}arl_1^2;$$
$$J \text{ to } D: \quad D_2 = \tfrac{1}{2}arl_2^2.$$

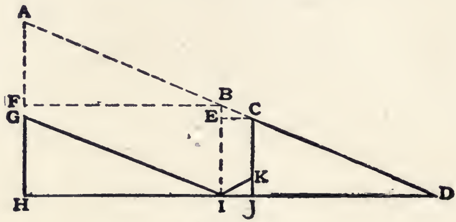


FIG. 21.

These drops can be read directly from a curve plotted from the equation

$$D = \tfrac{1}{2}arl^2.$$

The drop to the point J is $(D - D_1)$ and the total drop is $(D - D_1 + D_2)$.

The booster voltage is

$$e = Ri_0 - D + D_1,$$

where R is the resistance of the booster cable.

In case $I_0 < \tfrac{1}{2}I$ the current curve takes the form shown in Case IV, Fig. 20.

In this case there is no point in the negative feeders at which the current is zero. Mathematically, however, we still define the distance HI by the formula $e\frac{i-i_0}{a}$. The length l_1 is then negative, but since the lengths are squared in the above formula for drop, these formulas also hold in this case.

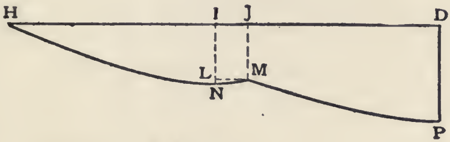


FIG. 22.

The voltage curves shown in Figs. 20 and 22 are composed of parts of a general voltage curve, the equation of which is

$$V = \tfrac{1}{2}arD^2,$$

where

V = voltage rise from where the current is zero, to a point D feet away.

a = current increment in amperes per foot, i.e., total load on section divided by length of section.

r = resistance of return conductors per foot of line.

Therefore, if one such curve be drawn with its corresponding current diagram over it, as shown in Fig. 23, the voltage curve for any of the schemes shown in Fig. 20 may be traced from it.

Thus to obtain the voltage curve shown in Fig. 22, set off *HD* and *DP* on tracing paper to the same scale as the general voltage curve, and select any point *J* for the booster feed point. Put *P* over the point *O* on the general curve, make *HD* parallel to *XX*, and trace the voltage curve to *M*, where it intersects the perpendicular through *J*. Then, still keeping *HD* parallel to *XX*, run *M* along the general voltage curve until *H* lies on that curve.

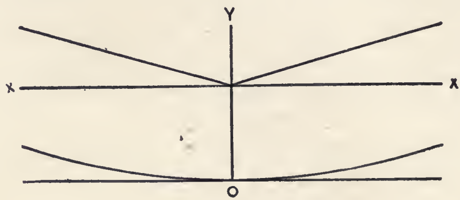


FIG. 23.

The intersection of *OY* and *HD* is the point *I* where the current divides. This, having been marked, avoid shifting the papers and trace the remainder of the voltage curve, i.e., *HNM*.

Knowing *I*, draw the current diagrams *HGI* and *JKI* (Fig. 21). The current in the booster and its cable, will be the sum of *JK* and *JC*. The booster voltage will be the sum of the drop in the booster cables, and (*DP*−*MP*), Fig. 22. This should be tried for various positions of *J*, and the best selected.

5. ALTERNATING-CURRENT TRANSMISSION LINE CALCULATIONS *

(From an article by H. Pender, also published in part in the *Electrical World.*)

Let E = pressure between adjacent wires at receiving end in kilovolts (thousands of volts);

V = pressure between adjacent wires at the generating end in kilovolts (thousands of volts);

W = power delivered in megawatts (thousands of kilowatts);

k = power factor of the load expressed as a decimal fraction;

t = tangent corresponding to $k = \cos \alpha$ (Table III);

k_0 = power factor at the generating end, expressed as a decimal fraction;

L = in case of a three-phase system, the length of each wire in miles; in the case of a single-phase system, the total length of both wires in miles;

r = resistance of each conductor per mile;

$x = x_1 + x_2$ = reactance of each conductor per mile, where x_1 is the reactance per mile of a number 0000 B. and S. wire (Table I), and x_2 the difference in the reactance per mile of a No. 0000 wire and that of the wire actually used (Table II);

* See Appendix IV for derivation of formulæ.

Q = power lost in transmission as a fraction of the delivered power;

P = pressure drop as a fraction of the delivered pressure.

$R = \frac{(kE)^2}{LW}$ = equivalent resistance of receiver per mile of line.

$T = t + \frac{x}{R}$.

TABLE I

REACTANCE PER MILE OF A No. 0000 B. & S. WIRE = x_1

Distance Apart of Wires in Feet.	15 Cycles.	25 Cycles.	40 Cycles.	60 Cycles.	125 Cycles
1	0.128	0.213	0.340	0.510	1.063
2	0.149	0.248	0.396	0.594	1.238
3	0.161	0.268	0.429	0.644	1.341
4	0.170	0.283	0.452	0.678	1.413
5	0.176	0.294	0.470	0.705	1.470
6	0.182	0.303	0.485	0.728	1.516
7	0.187	0.311	0.498	0.746	1.555
8	0.191	0.318	0.508	0.763	1.589
9	0.194	0.324	0.518	0.777	1.618
10	0.197	0.329	0.526	0.790	1.645
15	0.210	0.350	0.559	0.839	1.748
20	0.218	0.364	0.582	0.874	1.820
25	0.225	0.375	0.601	0.901	1.877

Case I. Given the delivered pressure E, the power delivered W, the power factor of the load k, the length of the line L, the frequency, the size, and spacing of the wires. The following are exact expressions * for the quantities to be determined.

Power loss $Q=\frac{r}{R}$.

Pressure drop $P=k\sqrt{(1+Q)^2+T^2}-1$.

Power factor at generating end $k_0=\frac{1+Q}{1+P}k$.

Case II. Given the delivered pressure E, the power delivered W, the power factor of the load k, the length of the line L, the frequency and the allowable power loss Q. The size wire to use is determined by the following exact formula:

Resistance of each wire per mile, $r=RQ$,

the corresponding size of wire being given in Table II. The pressure drop and power factor at the generating end can then be determined by the formulæ given in Case I.

Case III. Given the delivered pressure E, the power delivered W, the power factor of the load k, the length of the line L, the frequency and spacing of the wires, and the allowable pressure drop P.

An exact determination of the size of wire to use in this case cannot be made directly, since this would

* These formulæ can also be used to determine the overall efficiency, regulation and power factor of any number of circuits in series (e.g. line and transformers) if we let r and x represent the sum of the component resistances and reactances respectively and R the total equivalent resistance of the receiver.

require the solution of a logarithmic equation. However, since the reactance of commercial sizes of wire for a given frequency and spacing differ but slightly from one another, a close approximation to the exact size of wire to use can be obtained by assuming that the reactance, for a given frequency and spacing, for any size between 1,000,000 circular mils and a No. 6 B. and S. wire is equal to that of a No. 0000 wire. It will be found that except when the line reactance is large compared to the line resistance, the error due to this assumption will not cause a change in the size of wire; that is, the error will be less than half the percentage difference (26%) between successive sizes on the B. and S. gauge. On the other hand a large error in the approximate formula for the size of wire, indicates immediately that the drop is due chiefly to the line reactance, and that by allowing a very small increase in the permissible drop, or by employing two separate circuits instead of one, a very considerable saving in copper can be effected.

Put

$$T_1 = t + \frac{x_1}{R},$$

where x_1 is the reactance of a No. 0000 wire. Then to a close approximation, resistance of each wire per mile

$$r_1 = R\left[\sqrt{\left(\frac{1+P}{k}\right)^2 - T_1^2} - 1\right]$$

the corresponding size of wire being given in Table II, as well as the difference x_2 between the reactance corresponding to this size and the reactance of a No. 0000 wire.

Table II

RESISTANCE * PER MILE OF COPPER AND ALUMINUM CABLES AND REACTANCE INCREMENT x_2.

Size C.M. and B. & S.	Ohms per Mile at 20° C.		Difference in Reactance per Mile of any Size Wire and that of No. 0000 B. & S. Wire $=x_2$.†				
	Copper.	Aluminum.	15 Cycles.	25 Cycles.	40 Cycles.	60 Cycles.	125 Cycles.
1,000,000	0.0566	0.0894	−0.024	−0.039	−0.063	−0.094	−0.196
900,000	0.0629	0.0993	−0.022	−0.037	−0.059	−0.088	−0.183
800,000	0.0707	0.1118	−0.020	−0.034	−0.054	−0.081	−0.168
700,000	0.0808	0.1278	−0.018	−0.030	−0.048	−0.073	−0.152
600,000	0.0943	0.1490	−0.016	−0.026	−0.042	−0.063	−0.132
500,000	0.1131	0.1788	−0.013	−0.022	−0.035	−0.052	−0.109
450,000	0.1257	0.1987	−0.011	−0.019	−0.031	−0.046	−0.095
400,000	0.1414	0.224	−0.005	−0.009	−0.014	−0.021	−0.044
350,000	0.1616	0.255	−0.008	−0.013	−0.020	−0.031	−0.064
300,000	0.1886	0.298	−0.005	−0.009	−0.014	−0.021	−0.044
250,000	0.226	0.358	−0.003	−0.004	−0.007	−0.010	−0.021
0000	0.267	0.423					
000	0.337	0.533	+0.004	+0.006	+0.009	+0.014	+0.029
00	0.425	0.672	+0.007	+0.012	+0.019	+0.028	+0.059
0	0.536	0.848	+0.011	+0.018	+0.028	+0.042	+0.088
1	0.676	1.068	+0.014	+0.023	+0.038	+0.056	+0.117
2	0.852	1.347	+0.109	+0.029	+0.047	+0.070	+0.147
4	1.355	2.14	+0.025	+0.041	+0.066	+0.098	+0.205
6	2.15	3.41	+0.032	+0.053	+0.084	+0.127	+0.264

* Stranded wire, copper 98%, aluminum 62% conductivity, resistance increased 1% on account of stranding, temperature coefficient 0.42% per degree C.

† The total reactance of a wire for any spacing and frequency is $x=x_1+x_2$ where x_1 is the reactance of a No. 0000 wire under the same conditions.

By substituting for T_1 in the above formula the value $T=T_1+\frac{x_2}{R}$ the error in the value of r caused by neglecting x_2 can be readily found. As stated above, in any practical case this will, as a rule, be negligible, but should the error in the particular problem in hand be sufficient to give a new value for r, for which the corresponding value for x_2 differs appreciably from the first value found, r should be again calculated, using this second value for x_2, and so on, until the difference in x_2 for two successive values of r, as thus determined, becomes negligible. In this way an exact determination of the size corresponding to the given drop can be readily made, although, as stated above, a large error in the first approximation immediately indicates that the feasibility of increasing the permissible drop, or of dividing the circuit, should be investigated. If the formula gives a negative value of r, it is impossible, with any amount of copper, to transmit the assumed amount of power with the drop and inductance assumed.

Case IV. Given the pressure at the generating end V, the power delivered W, the power factor of the load k, the length of the line L, the frequency the size and spacing of the wires.

In this case R, the equivalent resistance of the receiver per mile of line, can be expressed in terms of the pressure at the generating end V.

Put $$M=\frac{(kV)^2}{2LW}-(r+xt)k^2$$

$$e=\frac{k^2(r^2+x^2)}{M^2}$$

Then $$R=M[1+\sqrt{1-e}].$$

Using this value for R, the exact formulæ given under Case I become immediately applicable.

TABLE III

VALUES OF t=tan CORRESPONDING TO k=cos α

k.	0.00.	0.01.	0.02.	0.03.	0.04.	0.05.	0.06.	0.07.	0.08.	0.09.
0.5	1.732	1.687	1.643	1.600	1.559	1.519	1.479	1.442	1.404	1.368
0.6	1.333	1.299	1.265	1.233	1.201	1.169	1.138	1.108	1.078	1.049
0.7	1.020	0.992	0.964	0.936	0.909	0.882	0.855	0.829	0.802	0.776
0.8	0.750	0.724	0.698	0.672	0.646	0.620	0.593	0.567	0.540	0.512
0.9	0.489	0.456	0.426	0.395	0.363	0.329	0.292	0.251	0.203	0.143

Effect of Line Capacity.—A complete and accurate treatment of transmission lines, taking into account the capacity and leakage, is given below. In most practical cases, however, the leakage is negligible and the effect of line capacity can be determined with sufficient accuracy by assuming that this effect is the same as would be produced by two condensers, each having a capacity equal to half that of the line, shunted across the line at the receiving and sending ends respectively. The effect of the condenser at the receiving end is to increase both the equivalent resistance of the load and also the load power factor; the condenser at the sending end has no effect on the power loss and line drop, but merely increases the resultant power factor at the generating end.

Table IV

SIZE AND WEIGHT OF STRANDED COPPER AND ALUMINUM WIRES

Size B. & S.	Circ. Mils.	Diameter, Ins.	Lbs. per Mile.*	
			Copper.	Aluminum.
	1,000,000	1.152	16,140	4,870
	900,000	1.092	14,530	4,380
	800,000	1.035	12,910	3,890
	700,000	0.963	11,300	3,410
	600,000	0.891	9,690	2,920
	500,000	0.819	8,070	2,430
	450,000	0.770	7,260	2,190
	400,000	0.728	6,460	1,947
	350,000	0.679	5,650	1,703
	300,000	0.630	4,840	1,460
	250,000	0.590	4,040	1,217
0000	211,600	0.530	3,420	1,030
000	167,800	0.470	2,710	817
00	133,100	0.420	2,150	648
0	105,500	0.375	1,703	513
1	83,690	0.330	1,351	407
2	66,370	0.291	1,071	323
4	41,740	0.231	674	203
6	26,250	0.183	420	128

* Increased 1% over weight of solid wire on account of stranding.

In addition to the above symbols let

b = capacity susceptance* per mile of two parallel wires for a frequency of one cycle per second (Table V);

$B = nbL$ for a *three-phase* line or $\frac{nbL}{4}$ for a *single-phase* line, where n is the number of cycles per second, and L as defined above is the length in miles of *each* wire for a *three-phase* line or the length of both wires for a *single-phase* line.

Then the equivalent power factor at the receiving end is the cosine k' corresponding to the tangent t' where

$$t' = t - \frac{BE^2}{W}.$$

The above formulæ for power loss and pressure drop (Case I) are then immediately applicable, substituting for k and t the values k' and t'; the formula for predetermining the size of wire in terms of the pressure drop (Case III) may also be applied, assuming the capacity susceptance equal to that of a No. 0000 wire, an assumption which will introduce but a slight error, since the capacity susceptance varies but slightly with the size of wire. The power factor formula $k_0 = \frac{1+Q}{1+P} k'$, given under Case I, is

* $b = 2\pi C$ where C is the capacity per mile in farads of the condenser found by each pair of wires.

the power factor at the generating end *excluding* the second condenser, the actual power factor at the generator is the cosine k'_0 corresponding to t'_0 where

$$t'_0 = t_0 - \frac{BV^2}{W_0},$$

where $W_0 = (1 + Q)W$, the total power supplied at the generating end.

Table V

CAPACITY SUSCEPTANCE PER MILE OF TWO PARALLEL STRANDED WIRES FOR FREQUENCY OF ONE CYCLE PER SECOND

Size C.M. and B. & S.	Distance Apart of Wires in Feet.				
	1.	2.	3.	6.	10.
1,000,000	9.3×10^{-8}	7.5×10^{-8}	6.8×10^{-8}	5.8×10^{-8}	5.3×10^{-8}
500,000	8.3×10^{-8}	6.9×10^{-8}	6.3×10^{-8}	$5.4\times10^{8-}$	4.9×10^{-8}
250,000	7.6×10^{-8}	6.4×10^{-8}	5.8×10^{-8}	5.1×10^{-8}	4.7×10^{-8}
0000	7.4×10^{-8}	6.2×10^{-8}	5.7×10^{-8}	5.0×10^{-8}	4.6×10^{-8}
1	6.6×10^{-8}	5.7×10^{-8}	5.2×10^{-8}	4.6×10^{-8}	4.3×10^{-8}
6	5.8×10^{-8}	5.0×10^{-8}	4.7×10^{-8}	4.2×10^{-8}	3.9×10^{-8}

NOTE.—The charging current per mile of *single-phase line* (2 miles of wire) is equal to $10^3 \times bnE$; for a *three-phase line* the charging current per wire per mile of line (3 miles of wire) is equal to $1.16 \times 10^3 nbE$, where n is the cycles per second, b the capacity susceptance given in the table, and E the kilovolts between wires.

A. C. Trolley. The resistance and reactance of various combinations of overhead trolleys and 100-lb. return rails are given in the Table VI. This table is based on extended tests made by A. W. Copley on the New York, New Haven and Hartford Railroad and other single-phase roads, the results of which were published in the Proceedings of the American Institute of Electrical Engineers for December, 1908. Unfortunately there is no reliable data on rails of smaller section, but as the greater part of the resistance is in the trolley, and only a small percentage of the total reactance is due to the magnetic field in the rail, the values given for the combined resistance and reactance respectively may also be used with but slight error in case the rail is of smaller size. It should be noted that the reactance for the three sizes of wire given are constant to within 5% for any height from 15 to 30 feet above the track.

The figures showing the division of current between the track and the earth refer to intermediate portions of long sections (over three miles); a greater portion of the current flows in the track near the load and the power house. It will be noted that if we let p' be the percentage current in each trolley and p'' the percentage current in each rail, and the respective resistances r' and r'', the total resistance of any combination of trolleys and rails, as measured by Mr. Copley, is approximately $p'r' + p''r''$; similarly the total reactance is $p'x' + p''x''$, where x' and x'' are the reactances of a single trolley and rail respectively; using these formulæ, a closely approximate value for the equivalent resistance and reactance for any other combination of trolleys and rails for any division of current between the rails and the earth can be ob-

Table VI

RESISTANCE AND REACTANCE OF SINGLE-PHASE TROLLEY WITH 100-LB. RAIL-RETURN

No. of Tracks.	No. of Trolley Wires.	No. of Return Rails.	Percentage of Current Returning this Rail.	Resistance Ohms per Mile.			
				0000 Trolley		000 Trolley	
				25 Cycles.	15 Cycles.	25 Cycles.	12 Cycles.
	1	..	...	0.26*	0.26	0.33	0.33
	..	1	100	0.16*	0.13	0.16	0.13
	1	1	25	0.30	0.29	0.37	0.36
1	Track 1	2	40*	0.29*	0.28*	0.36	0.35
2	" 2	4	58*	0.155*	0.15*	0.20	0.19
4	" 4	8	75*	0.086*	0.082*	0.11	0.10

No. of Tracks.	No. of Trolley Wires.	No. of Return Rails.	Percentage of Current Returning this Rail.	Resistance Ohms per Mile.		Reactance Ohms per Mile.	
				00 Trolley.		No. 0000, No. 000 or No. 00 Trolley.	
				25 Cycles.	12 Cycles.	25 Cycles.	12 Cycles.
	1	..	...	0.42	0.42	0.38	0.23
	..	1	100	0.16	0.13	0.44	0.26
	1	1	25	0.46	0.45	0.49	0.30
1	Track 1	2	40*	0.45	0.44	0.47*	0.282*
2	" 2	4	58*	0.24	0.23	0.269*	0.161*
4	" 4	8	75*	0.13	0.12	0.168*	0.101*

Note.—The figures marked thus (*) are taken directly from Mr. Copley's paper; the others are derived from these. At the point where the current enters the rail Mr. Copley found that 70% of the current starts toward the power house on a singe track road and similarly 87% on a four track road, in each case the rail currents falling to the values given in the table in a distance of about three miles and from that point on remaining practically constant until near the power house.

tained. In case of a catenary suspension a certain percentage of the overhead current is carried by the messenger cable, but on account of the high effective resistance of a steel cable to alternating currents, this current will be quite small. (In a $\frac{7}{16}''$ messenger cable carrying a No. 0000 wire Mr. Copley gives the messenger current as but 3.5% of the total.)

To determine the power loss, pressure drop, etc., for a single-phase trolley system, the formulæ given above under Case I are directly applicable, putting L equal to the distance in miles of the load from the power house (or substation) and r and x equal respectively to the combined resistance and reactance of trolley and track per mile, as given in Table VI. Similarly, the proper size of trolley for any given set of conditions can be determined by the formulæ given under Case III, taking from Table VI the reactance per mile (which is constant for the three sizes of trolley given and likely to be used in good practice), and selecting the size of trolleys from Table VI corresponding to the value of the resistance per mile r_1, given by the formula

$$r = R\left[\sqrt{\left(\frac{1+P}{k}\right)^2 - T^2} - 1\right].$$

NUMERICAL EXAMPLES

Case I. A load of 5000 kilowatts at 80% factor is to be delivered at 40,000 volts over a three-phase line of No. 2 B. and S. copper wire 30 miles long, frequency 25 cycles per second, wires spaced 4 feet apart. To find the power loss, pressure drop, and power factor at generating end we have

$$E=40;$$

$$W=5;$$

$$k=0.8;$$

$$t=0.75;$$

$$L=30;$$

$$r=0.852;$$

$$x=0.283+0.029=0.312:$$

$$R=\frac{(0.8\times 40)^2}{30\times 5}=6.83;$$

$$T=0.75+\frac{0.312}{6.83}=0.796.$$

Then

Power loss $$Q=\frac{0.852}{6.83}=0.125.$$

Pressure drop $P = 0.8\sqrt{(1.125)^2 + (0.796)^2} - 1 = 0.102.$
Generator power factor

$$k_0 = \frac{1.125}{1.102} \div 0.8 = 0.817.$$

Case II. A load of 5000 kilowatts at 80% power factor is to be delivered at 40,000 volts over a three-phase line of copper wire 30 miles long, allowable power loss 12.5%. To find the size wire to use, we have

$$E = 40;$$

$$W = 5;$$

$$k = 0.8;$$

$$L = 30;$$

$$Q = 0.125;$$

$$R = \frac{(0.8 \times 40)^2}{30 \times 5} = 6.83.$$

Then, using the formula $r = RQ$,

$$\text{Resistance per mile } r = 0.125 \times 6.83 = 0.854,$$

whence from Table II we find that the proper size is No. 2 B. and S.

Case III. A load of 5000 kilowatts at 80% power factor is to be delivered at 40,000 volts over a three-phase line of copper wire, 30 miles long, frequency

25 cycles per second, wires spaced 4 feet apart, allowable pressure drop 10.2%. To find the size wire to use we have

$$E=40;$$

$$W=5;$$

$$k=0.8;$$

$$t=0.75;$$

$$L=30;$$

$$P=0.102;$$

$$x_1=0.283;$$

$$R=\frac{(0.8\times 40)^2}{30\times 5}=6.83;$$

$$T_1=0.75+\frac{0.283}{6.83}=0.791.$$

Then

Resistance per mile

$$r_1=6.83\left[\sqrt{\left(\frac{1.102}{0.8}\right)^2-(0.791)^2-1}\right]=0.880,$$

whence from Table II we find the nearest size wire is No. 2 B. and S. The value of x_2 corresponding to $r_1=0.880$ is 0.030, which makes $T=0.796$ and gives 0.854 as the corresponding value for r, showing that the error in the first approximation for r is only 3%.

The above example may also be used to illustrate an extreme case, in which the first approximation for r may be entirely erroneous, but by successive applications of the above formula a correct solution can be obtained.

Keeping the other conditions the same, suppose we change the frequency to 125 cycles per second. Then

$$x_1 = 1.413,$$

$$T_1 = 0.75 + \frac{1.413}{6.83} = 0.957.$$

First approximation

$$r_1 = 6.83\left[\sqrt{\left(\frac{1.102}{0.8}\right)^2 - (0.957)^2} - 1\right] = -0.060,$$

which shows that it is impossible to deliver power under the conditions stated over a line having a reactance as great as that of a No. 0000 wire.

As a second approximation assume a reactance equal to that of a 500,000 c.m. wire. The resistance per mile then works out 0.041 ohm, which is again too small a value, because the reactance corresponding to a wire having this resistance is less than that of a 500,000 c.m. wire.

As a third approximation assume a reactance equal to that of a 700,000 c.m. wire. The resistance per mile then works out 0.079, the reactance of which is about 1% less than that of the 700,000 c.m. assumed.

The nearest commercial size of wire corresponding to a drop of 10.2% is 700,000 circ. mil.

As a matter of fact, however, the drop for any size between a No. 0000 wire and a 1,000,000 c.m. wire would be substantially the same, as will be readily seen by calculating the drop for these two sizes by the exact formula of Case I, which gives a drop of 9.6% for a 1,000,000 c.m. wire and 13.0% for a No. 0000 wire, as against the 10.2% specified. Therefore, by increasing the drop to 13.0%, say, a saving of 70% in copper can be effected. (Were the drops proportional to the resistance the saving in the copper for the same increase in drop would be only 21.5%.) Again, the use of two circuits of No. 2 wire each would give a drop of but 9.5%, and would effect a saving of 81.0% in copper.

Case IV. Take the example given under Case I, but assume the pressure at the generator 44,080 volts, the receiver pressure E being unknown. Then

$$M=\frac{(0.8\times 44.08)^2}{2\times 30\times 5}-(0.852+0.312\times 0.75)^2(0.8)^2=3.45;$$

$$e=\frac{(0.8)^2(\overline{0.852}^2+\overline{0.312}^2)}{(3.46)^2}=0.044,$$

whence

$$R=3.45[1+\sqrt{1-0.044}]=6.83,$$

which agrees with the value found in Case I. The power loss, pressure drop (as a fraction of the delivered pressure), and power factor at generating end then work out the same as in Case I. The pressure drop as a fraction of the pressure at the generating end is

$$\frac{P}{1+P}=\frac{0.102}{1.102}=0.0926.$$

Effect of Capacity. Take the example given under Case I. From Table 5

$$b=5.0\times10^{-8},$$

whence $B=25\times30\times5.0\times10^{-8}=3.8\times10^{-5}$,

and $t'=0.75-\dfrac{3.8\times10^{-5}\times(40)^2}{5}=0.738.$

$$k'=0.805;\quad \text{(Table III.)}$$

$$R=\frac{(0.805\times40)^2}{30\times5}=6.91;$$

$$T=0.738+\frac{0.312}{6.91}=0.783.$$

Then

Power loss $Q=\dfrac{0.852}{6.91}=0.123.$

Pressure drop $P=0.805\sqrt{(1.123)^2+(0.783)^2}-1=0.102.$

$$k_0=\frac{1.123}{1.096}\times0.805=0.821;$$

$$t_0=0.683\quad \text{(Table III)};$$

$$t'_0=0.683-\frac{3.8\times10^{-5}\times(44.08)^2}{5.62}=0.670;$$

Generator power factor $k'_0=0.831$ (Table III.)

A. C. Trolley. 2000 kilowatts are to be supplied to a locomotive at 90% power factor, and 10,000 volts at a distance of twenty miles from the power house (or substation); No. 0000 trolley, return circuit two 100 lb. running rails, frequency 25 cycles. To find the power loss, pressure drop and power factor at power house, we have, assuming the division of current between rails and earth, as given in Table VI,

$$E=10$$

$$W=2$$

$$L=20$$

$$k=0.9$$

$$t=0.489$$

$$r=0.29$$

$$x=0.47$$

$$R=\frac{(0.9\times 10)^2}{20\times 2}=2.05$$

$$T=0.489+\frac{0.47}{2.05}=0.718$$

Then

Power loss $Q=\dfrac{0.29}{2.05}=0.141.$

Pressure drop $P=0.9\sqrt{(1.141)^2+(0.718)^2}-1=0.213.$

Power factor at power house

$$k_0=\frac{1.141}{1.213}\times 0.9=0.847.$$

Taking the reverse problem, suppose that we wish to determine the size of trolley to use for a drop of 21.3% between power house and locomotive, the other conditions being the same as given in the preceding example. We then have by formula under Case III,

$$r = 2.05\left[\sqrt{\left(\frac{1.213}{0.9}\right)^2 - (0.718)^2} - 1\right] = 0.29,$$

whence, from Table IV, the proper size of trolley is a No. 0000.

Transmission Line with Resistance, Reactance, Leakage, and Capacity. The following is a complete solution involving no approximations. The only assumptions made are that the resistance, reactance, leakage, and capacity are true constants and that sufficient time has elapsed for steady conditions to have become established.

Let E = volts between each wire and neutral at generator end;

I = amperes per wire at generator end;

$\cos\phi$ = power factor at generator end;

$W = EI\cos\phi$ = total watts deliver edto line per wire.

These same symbols with the subscript "o" refer to the receiver.

$Y_0 = \frac{I_0}{E_0} = \frac{W_0}{E_0^2 \cos\phi_0}$ = equivalent admittance of the receiver;

r = resistance of each wire per unit length;

x = reactance of each wire per unit length;

$z=\sqrt{r^2+x^2}$ = impedance of each wire per unit length;

$\cos \varepsilon=\frac{r}{z}$ = power factor of the line $\left(\sin \varepsilon=\frac{x}{z}\right)$;

g = leakage conductance between each wire and neutral per unit length;

b = leakage susceptance * between each wire and neutral per unit length;

$y=\sqrt{g^2+b^2}$ = leakage admittance per unit length;

$\cos \eta=\frac{g}{y}$ = power factor of leakage circuit $\left(\sin \eta=\frac{b}{y}\right)$;

L = length of line in any unit.

Calculate the following quantities: †

$$a=\sqrt{\frac{z}{y}};$$

$$\beta=\frac{\eta+\varepsilon}{2} \quad \text{and} \quad \gamma=\frac{\eta-\varepsilon}{2};$$

$$\log m=0.4343ayL \cos \beta;$$

$$\mu=114.6ayL \sin \beta;$$

$$\sigma_0=\phi_0+\gamma;$$

$$U_0=aY_0;$$

* $b=2\pi fC$ where f is the frequency in cycles per second and C the capacity between each wire and the neutral per unit length.

† Greek letters are used to represent angles in degrees. The logarithm is to the base ten.

$$p = \frac{m}{2}\sqrt{1 + U_0^2 + 2U_0 \cos \sigma_0};$$

$$q = \frac{1}{2m}\sqrt{1 + U_0^2 - 2U_0 \cos \sigma_0}.$$

$$\cos \delta_0 = \frac{1 - U_0^2}{4q}.$$

δ_0 has the same sign as σ_0.

$$\delta = \delta_0 - \mu;$$

$$D = \sqrt{p^2 + q^2 + 2pq \cos\delta};$$

$$Q = \sqrt{p^2 + q^2 - 2pq \cos\delta};$$

$$\cos \theta = \frac{p^2 - q^2}{DQ}.$$

θ has the same sign as δ. Then

Volts at generator end, $E = DE_0$

Amperes at generator end, $I = \dfrac{QE_0}{a}$

Power factor at generator end, $\cos \phi = \cos (\theta - \gamma)$

Total watts delivered to line, $W = EI \cos \phi$

(End of H. Pender's article.)

Voltage Drop and Synchronous Apparatus. An excessive ohmic drop in the transmission lines is liable to cause hunting of rotary converters or synchronous motors. The exact amount permissible depends upon the design of the rotary converter, those designed for normal A.C. starting requiring less drop than those designed for D.C. starting. In the latter type of machine an ohmic drop of 20% is generally permissible whether or not a simultaneous reactive drop exists. Converters of the A.C. starting type, do not, as a rule, operate satisfactorily if the ohmic drop is so high.

6. ECONOMICAL SIZE OF CONDUCTORS. (Kelvin's Law)

The total expenditure on a transmission system is made up of the initial cost plus the annual expenses. The most economical system to install for permanent use is that in which the sum of these items is a minimum.

The annual expenses consist of maintenance, depreciation, and power lost due to resistance.

It is usual to reduce the initial cost to a yearly basis for purposes of comparison, this yearly basis being the interest which must be paid for the use of the money, or which is lost by withdrawing the money from a profitable investment and putting it in feeder metal.

As a rule, it is necessary to work out the sum of the expenses for various sizes of wires and select that size which gives the minimum total cost.

When, however, the capital outlay is proportional to the amount of copper in the system, the following law, given by Lord Kelvin, is of use.

"The most economical area of conductor will be that for which the annual interest on capital outlay equals the annual cost of energy wasted."

One side of this equation would be the interest, depreciation, maintenance, and repairs; the other, the cost of producing energy at the station bus, including interest, depreciation, and operating expenses.

Kapp has made Kelvin's law of more universal application by changing it to the following form:

"The most economical area of conductor is that for which the annual cost of energy wasted is equal to the annual interest on that portion of the capital outlay which can be considered proportional to the weight of metal used."

The simplest way of applying Kelvin's law is that due to Dr. Pender. The most economical current density per million circular mils is

$$A\sqrt{\frac{L}{C}},$$

where L = increase in annual charges on transmission line resulting from increasing the weight of feeders one ton (2000 lbs.), and C = increase in annual operating and capital charges on the power station resulting from increasing the output one kilowatt. A is a constant whose value is

$$2170\sqrt{\frac{\text{Weight of conductors, lbs. per cu.in.}}{\text{Specific resistance, ohms per mil-foot}}}.$$

For copper, $A = 380$
Aluminum, $A = 165$

Calculations of this kind are often rendered useless by the following circumstances:

1. The rate of interest on the capital outlay is difficult to estimate exactly.

The discount of bonds depending on the value below par at which they are sold cannot be predicted for the future.

2. The life of insulation is difficult to estimate.

3. The cost of copper, lead, and insulation constantly fluctuates. It makes a material difference in the depreciation whether the price of copper and lead is assumed to rise or fall during the period it is in use.

4. There is not always a market for power that can be saved by additional feeder metal.

Owing to the inaccuracy of these premises, it is advisable to make two calculations, using for one the maximum possible value of L and the minimum possible value of C, and in the other the minimum value of L and the maximum value of C.

The economical current density will then be between the extremes thus obtained.

It is thus obvious that the size of conductors to be used is more a matter of judgment than of mathematics.

CHAPTER V

DETERMINATION OF SIZE FOR GIVEN STRESS IN SPAN

ALGEBRAIC METHOD

(Abstracted by permission from article in *Electrical World*, N. Y. Jan. 12, 1907, by H. Pender, Ph.D.)

Formulæ are closely approximate:*

a = coefficient of expansion of wire per degree Fahrenheit;

D = deflection of wire at center of span in feet, in the direction of the resultant force at temperature t;

ι = length of span in feet;

M = modulus of elasticity (pounds, square inches);

m = weight of wire per cubic inch in lbs.;

ρ = ratio of the resultant of weight of wire and sleet and wind pressure to the weight of wire, at temperature t;

ρ_0 = corresponding ratio at temperature t_0;

T = tension at center of span in thousands of lbs. per sq.in. at temperature t;

* See Appendix V.

T_0 = tension at center of span in thousands of lbs. per sq.in. at temperature t_0;

$$K = \frac{\rho l}{T};$$

$$K_0 = \frac{\rho_0 l}{T_0}$$

t and t_0 described above under D, K, K_0, T, and T_0, and are in degrees Fahrenheit.

General Formulæ for Points of Support on the same Level

$$t - t_0 = \frac{10^{-6}}{a}\left[6m^2(K^2 - K^2_0) + \frac{10^9}{M}(T_0 - T)\right],$$

$$D \doteq 0.0015mlK.$$

Copper wire: *

$$t - t_0 = 0.0644[(K^2 - K^2_0) + 135(T_0 - T)],$$

$$D = 0.00048lK.$$

Aluminum wire: †

$$t - t_0 = 0.00441[(K^2 - K^2_0) + 1965(T_0 - T)].$$

$$D = 0.000145lK.$$

Making numerical calculations, choose various values for T and plot the corresponding values of t in the form of a curve, from which the value of the tension for the temperature in question can be taken.

* For Copper for which $m = 0.321$, $a = 9.6 \times 10^{-6}$, $M = 12 \times 10^6$.

† For Aluminum for which $m = 0.0967$, $a = 12.8 \times 10^{-6}$, $M = 9 \times 10^6$.

The value of K is obtained from this value of T and used in the formula for D.

GRAPHICAL METHOD

Instead of the trial method above outlined, a graphical method giving a direct answer was outlined by Dr. Pender in the *Electrical World*, Sept. 28, 1907.

The two charts, Figs. 24 and 25, are the essential parts of this method. (See p. 172 for method of constructing these charts.)

Calculation of Tension and Sag

Given: A span of length l and the points of support on the same level; tension T_1; ratio of resultant force to weight of wire, ρ_1. To find the tension T when the temperature rises t degrees and the ratio of resultant force to weight of wire changes to ρ (for example, sleet melts off).

1. On the line corresponding to l find the point 3 having the abscissa t on the temperature scale.

2. On the curve corresponding to ρ_1 find the point having the abscissa T_1 and at this point lay off the length of the ordinate of point 3, upward if t is positive or downward if t is negative.

3. Through the point 2 thus obtained draw a line parallel to the line l.

4. The abscissa of the point 4 where this line cuts

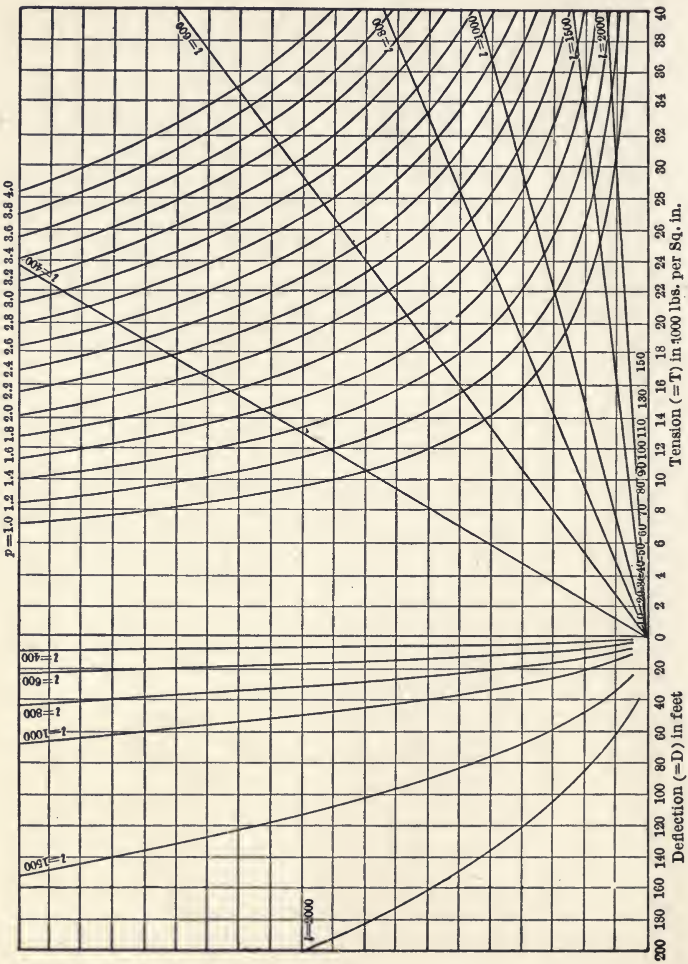

p=1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0
l=400
l=600
l=800
l=1000
l=1500
l=2000
Tension (=T) in 1000 lbs. per Sq. in.
Deflection (=D) in feet
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
20 40 60 80 100 120 140 160 180 200
10 20 30 40 50 60 70 80 90 100 110 130 150

FIG. 24.

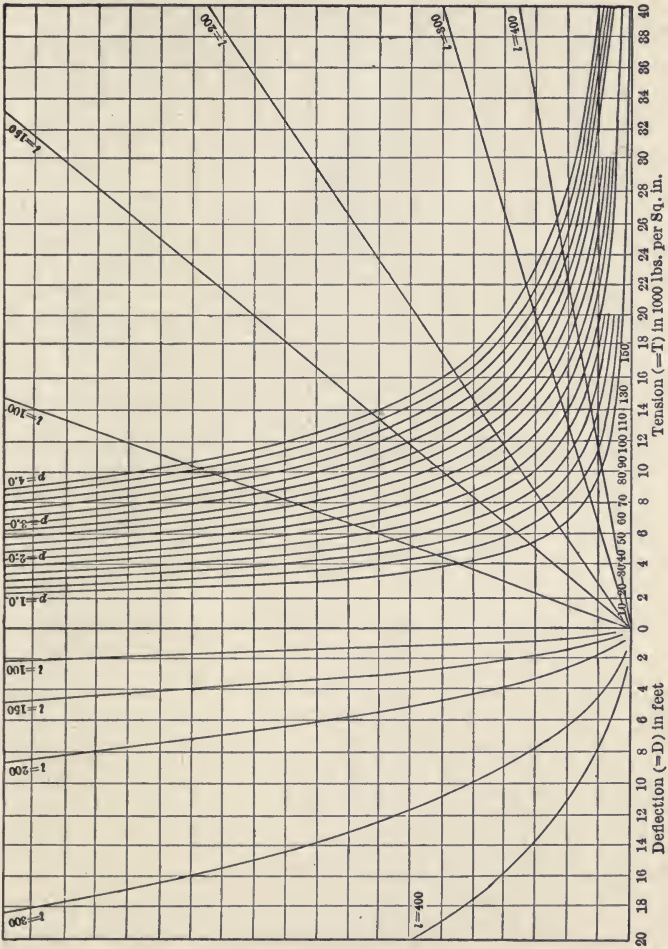

Tension (=T) in 1000 lbs. per Sq. in.
Deflection (=D) in feet
l=100
l=150
l=200
l=300
l=400
p=1.0
p=2.0
p=3.0
p=4.0
10 20 30 40 50 60 70 80 90 100 110 130 150

FIG. 25.

the curve corresponding to ρ is the tension T at the new temperature when the ratio of the resultant force to weight of wire is ρ.

5. The abscissa of the point 5 where the horizontal line through 4 cuts the parabolic curve corresponding to l gives the corresponding deflection D at the center of the span in feet.

Instead of actually drawing the straight line 2–4 a pair of compasses may be used; i.e., lay off the distance 1–2, then open the compasses until the lower

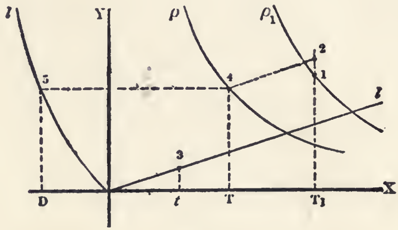


FIG. 26.

point touches the straight line l; then keeping the compasses vertical, slide the lower point along l until the upper point intersects the curve corresponding to ρ. If t is negative, i.e., if the temperature decreases, lay off 1–2 in the opposite direction.

The deflection under any conditions can also be calculated from the formula

$$D = 0.0015 m l^2 \frac{\rho}{T},$$

when T is known.

Calculation of ρ

Let ω = weight of wire in pounds per foot.

The weight of sleet (and hemp core, if any) in pounds per foot of wire is

$$\omega_1 = 0.312(d^2{}_1 - d^2) + 0.32d^2{}_0,$$

where d is the diameter of the wire, and d_1 the diameter over sleet and d_0 the diameter of the core, all in inches.

The wind pressure in pounds per foot of wire is*

$$\omega_2 = 0.00021\,V^2 d_1,$$

where V is the actual wind velocity in miles per hour; $d_1 = d$ in case of no sleet. The relation between indicated wind velocity (as given by U. S. Weather Reports) and actual velocity is as follows:

Indicated Velocity.	Actual Velocity.	$0.00021\,V^2$.
10	9.6	0.0194
20	17.8	0.0667
30	25.7	0.139
40	33.3	0.233
50	40.8	0.350
60	48.0	0.485
70	55.2	0.640
80	62.2	0.812
90	69.2	1.01
100	76.2	1.22

* H. W. Buck, in Transactions International Electric Congress, 1904.

The ratio ρ, when the wind is horizontal, is then

$$\rho = \sqrt{\left(1+\frac{\omega_1}{\omega}\right)^2+\left(\frac{\omega_2}{\omega}\right)^2}.$$

When the wind is acting vertically downward,

$$\rho = 1+\frac{\omega_1+\omega_2}{\omega}.$$

Calculation of Sag with Wind Blowing. In case of no wind, or the wind blowing vertically downward, the vertical sag S will be the same as the deflection D. A horizontal wind gives a horizontal component to the resultant force, so that the vertical sag when the wind is blowing horizontally is

$$S = \frac{D}{\sqrt{1+\left(\frac{\omega_2}{\omega+\omega_1}\right)^2}}.$$

Example: A No. 00 stranded copper cable is to be strung in still air at 70° F. between two points on the same level 800 ft. apart, so that at a temperature of 0° F., with a coating of sleet ½ in. thick all around, and wind blowing horizontally directly across the span at 65 miles an hour (actual velocity), the tension in the cable will be 30,000 lbs. per sq.in.; (1) at what tension must the cable be strung, and (2) what will be the vertical sag at stringing temperature, i.e., 70°, also (3) what will be the sag at zero temperature when the cable is coated with ½ in. of sleet and wind is blowing with a velocity of 65 miles an hour, and (4) what will be the sag as a temperature of 150° in the still air?

We have

$$\omega = 0.406$$

$$\omega_1 = 0.312(\overline{1.418}^2 - \overline{0.418}^2) = 0.574$$

$$\omega_2 = 0.00021 \times \overline{65}^2 \times 1.419 = 1.26.$$

Therefore, at 0° with wind and sleet

$$\rho_0 = \sqrt{\left(1 + \frac{0.574}{0.406}\right)^2 + \left(\frac{1.26}{0.406}\right)^2} = 3.93.$$

(1) Measure off with compasses on Chart No. 1 the vertical distance from $t = 70$ on X axis to the straight line corresponding to $l = 800$. Lay this distance off vertically above the point on the curve corresponding to $\rho = 3.93$ having the abscissa $T = 30$. Keep the upper point fixed, open the compasses until the lower point touches the line $l = 800$; then, keeping the compasses vertical, slide the lower point along the line $l = 800$, until the upper point intersects the curve $\rho = 1$ at $T = 8.35$; the cable must therefore be strung at a tension of 8350 lbs. per sq.in. This value of T is readily checked by finding, by the alge-

braic method given in the preceding section, the temperature rise corresponding to $T=8.35$. Thus,

$$K_0=\frac{3.93\times 800}{30}=104.8;$$

$$K=\frac{800}{8.35}=95.8;$$

$$K^2-K^2_0=95.8^2-104.8^2=-1837;$$

$$135(T_0-T)=135(30-8.35)=2922;$$

$$t-t_0=0.0644(-1837+2923)=70°,$$

which is the temperature rise given. (2) The abscissa of the point on the parabolic curve $l=800$, having the same ordinate as the point corresponding to $\rho=1$ and $T=8.35$ is $D=36.9$ ft., which is the vertical sag S in still air at 70° F.

(3) The deflection at 0° with sleet and wind is the abscissa of the point on the parabolic curve $l=800$ having the same ordinate as the point corresponding to $\rho_0=3.93$ and $T_0=30$, i.e., $D_0=40.4$ ft.

The vertical sag is

$$S=\frac{40.4}{\sqrt{1+\left(\frac{1.26}{.980}\right)^2}}=24.8 \text{ ft.}$$

(4) To find the sag at 150° proceed as under (1) and (2) taking $t=150$. The sag will be found to be $S=39.2$ ft.

Wire Suspended from Points Not on the Same Level. The charts also apply directly to the determination of the change in tension in spans when the points of support are at different heights. In this case, however, the vertical sag S_1 (=deflection in case of no wind) below the higher point of support, is given by the formula

$$S_1 = S\left(1 + \frac{h}{4S}\right)^2,$$

where h is the difference in height of the two points of support and S is the vertical sag for a span of *equal* length but points of support on the *same* level; S is calculated by the formula given above, i.e.,

$$S = \frac{D}{\sqrt{1 + \left(\frac{w^2}{w + w_1}\right)^2}}.$$

D being the deflection, taken directly from the chart, for a span of *equal* length but points of support on the *same* level; in case of no wind $S = D$.

The distance of the point of maximum sag from the lower point of support is

$$\frac{l}{2}\left(1 - \frac{h}{4S}\right).$$

When h is greater than $4S$ the lower point of support is the point of maximum sag, i.e., the lowest point in the span.

Consider three consecutive poles A, B, and C. Let l, h, and S refer to the span of AB, and l', h', and S' refer to the span BC, where S and S' are the sags for spans of length l and l' but with points of support on the *same level* and h, is the height of the point of support at A above the point of support at B, and h' is the height of the point of support at B above the point of support at C; if B is below C, h' is to be taken as negative. Then the total vertically downward pull on the insulator B due to the span on the two sides is

$$\frac{W}{2}\left[l\left(1-\frac{h}{4S}\right)+l'\left(1+\frac{h'}{4S'}\right)\right],$$

where W is the weight of wire, sleet, and vertically downward wind per lineal foot.

If the quantity in the bracket is negative there will be an upward lift on the insulator. It is necessary to apply this criterion only for those spans for which h is greater than $4S$; if h is less than $4S$ for the span on each side of the pole B, both the terms $\left(1-\frac{h}{4S'}\right)$ and $\left(1-\frac{h'}{4S'}\right)$ in the above expression are positive. It should be noted that the sag S will be a minimum when the temperature is at its lowest value, but no sleet on the wire, and the wind is blow-directly across the span at maximum velocity; the above criterion should therefore be applied for these conditions. (See p. 176 for graphical method.)

Example. In the example given above, suppose the difference in height of the points of support is 20 ft. Then (1) the tension at 70° will still be 8350 lbs. per sq.in. (2) The corresponding vertical sag at 70° in still air for points of support at same level is 36.9 ft., therefore, for the span under consideration the vertical sag from the highest point of support is

$$36.9\left(1+\frac{20}{4\times 36.9}\right)^2=47.6 \text{ ft.}$$

(3) The vertical sag at 0° with sleet and wind for points of support on the same level is 24.8 ft.; therefore, for a 20-ft. difference in the height of points of support the vertical sag from the highest point of support is

$$24.8\left(1+\frac{20}{4\times 24.8}\right)^2=35.9 \text{ ft.}$$

(4) The vertical sag at a temperature of 150° for points of support on the same level is 39.2 ft.; therefore, for a 20-ft. difference in height of the points of support the vertical sag from the highest point of support is

$$39.2\left(1+\frac{20}{4\times 39.2}\right)^2=49.8 \text{ ft.}$$

The diagrams, Figs. 24 and 25, are reduced to such a small scale that they are of little use for actual calculations. It is therefore necessary, in practical work, to draw a series of curves of suitable scale. These curves are plotted from the following equations:

Inclined straight lines: $y = \frac{10^9}{6Mm^2l^2}T$.

Parabolic curves on left-hand side: $D = 0.0015ml^2\sqrt{y}$. (D is measured to the left from the origin.)

Hyperbolic curves on the right-hand side: $y = \left(\frac{\rho}{T}\right)^2$.

Temperature scale * on the axis of T: $x = 10^{-3}\,Mat$.

Symbols used in formulæ above:

M = modulus of elasticity of wire, lbs. per sq.in.;
m = weight of wire per cubic inch, lbs.;
l = length of span, feet;
T = tension at center of span, thousands of lbs. per sq.in.;
D = deflection at center of span, feet;
a = coefficient of expansion of wire per degree F.;
t = temperature rise, degrees F.;
ρ = ratio of the resultant of the weight of wire, the weight of sleet, and the wind pressure to the weight of wire;
y = an arbitrary quantity, the physical meaning of which does not appear in the calculations, as all the quantities entering the problem are given as abscissæ; y being merely a common ordinate for all the curves.

* x is the distance on the scale of T corresponding to the temperature t.

TABLE GIVING THE VALUE OF T FOR VARIOUS VALUES OF ρ AND $y = \left(\frac{\rho}{T}\right)^2$.

Values of $y=\left(\frac{\rho}{T}\right)^2$	Values of ρ.							
	1.0.	1.2.	1.4.	1.6.	1.8.	2.0.	2.2.	2.4.
0.2	2.24	2.68	3.13	3.58	4.02	4.47	4.92	5.37
0.17	2.43	2.91	3.40	3.88	4.37	4.85	5.34	5.82
0.13	2.77	3.33	3.88	4.44	4.99	5.55	6.10	6.66
0.10	3.16	3.79	4.43	5.06	5.69	6.32	6.96	7.59
0.07	3.78	4.54	5.29	6.05	6.80	7.56	8.32	9.07
0.04	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00
0.03	5.77	6.92	8.08	9.24	10.39	11.55	12.70	13.86
0.02	7.07	8.49	9.90	11.31	12.73	14.14	15.56	16.97
0.017	7.67	9.20	10.74	12.27	13.81	15.34	16.87	18.41
0.014	8.45	10.14	11.83	13.52	15.21	16.90	18.59	20.3
0.012	9.13	10.95	12.78	14.61	16.43	18.26	20.1	21.9
0.010	10.00	12.00	14.00	16.00	18.00	20.00	22.0	24.0
0.008	11.18	13.42	15.65	17.89	20.1	22.4	24.6	26.8
0.006	12.91	15.49	18.07	20.7	23.2	25.8	28.4	31.0
0.005	14.14	16.97	19.80	22.6	25.5	28.3	31.1	33.9
0.004	15.81	18.97	22.1	25.3	28.5	31.6	34.8	37.9
0.0035	16.90	20.3	23.7	27.0	30.4	33.8	37.2	40.6
0.0030	18.26	21.9	25.6	29.2	32.9	36.5	40.2	43.8
0.0025	20.00	24.0	28.0	32.0	36.0	40.0	44.0	48.0
0.0020	22.4	26.8	31.3	35.8	40.2	44.7	49.2	53.7
0.0015	25.8	31.0	36.1	41.3	46.5	51.6	56.8	61.9
0.0010	31.6	37.9	44.3	50.6	56.9	63.2	69.6	75.9
0.0005	44.7	53.7	62.6	71.6	80.5	89.4	98.4	107.3

TABLE GIVING THE VALUE OF T FOR VARIOUS VALUES OF ρ AND $y=\left(\frac{\rho}{T}\right)^2$—(*Continued*)

Values of $y=\left(\frac{\rho}{T}\right)^2$	Values of ρ.							
	2.6.	2.8.	3.0.	3.2.	3.4.	3.6.	3.8.	4.0.
0.2	5.81	6.26	6.71	7.16	7.60	8.05	8.50	8.94
0.17	6.31	6.79	7.28	7.76	8.25	8.73	9.22	9.70
0.13	7.21	7.77	8.32	8.87	9.43	9.98	10.54	11.09
0.10	8.22	8.85	9.49	10.12	10.75	11.38	12.02	12.65
0.07	9.83	10.58	11.34	12.10	12.85	13.61	14.36	15.12
0.04	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.0
0.03	15.01	16.16	17.32	18.47	19.63	20.8	21.9	23.1
0.02	18.38	19.80	21.21	22.63	24.0	25.5	26.9	28.3
0.017	19.94	21.5	23.0	24.5	26.1	27.6	29.1	30.7
0.014	22.0	23.7	25.4	27.0	28.7	30.4	32.1	33.8
0.012	23.7	25.6	27.4	29.2	31.0	32.9	34.7	36.5
0.010	26.0	28.0	30.0	32.0	34.0	36.0	38.0	40.0
0.008	29.1	31.3	33.5	35.8	38.0	40.2	42.5	44.7
0.006	33.6	36.1	38.7	41.3	43.9	46.5	49.1	51.6
0.005	36.8	39.6	42.4	45.2	48.1	50.9	53.7	56.6
0.004	41.1	44.3	47.4	50.6	53.8	56.9	60.1	63.2
0.0035	43.9	47.3	50.7	54.1	57.5	60.8	64.2	67.6
0.0030	47.5	51.1	54.8	58.4	62.1	65.7	69.4	73.0
0.0025	52.0	56.0	60.0	64.0	68.0	72.0	76.0	80.9
0.0020	58.1	62.6	67.1	71.6	76.0	80.5	85.0	89.4
0.0015	67.1	72.4	77.4	82.6	87.8	92.9	98.1	103.2
0.0010	82.2	88.5	94.9	101.2	107.5	113.8	120.2	126.5
0.0005	116.3	125.0	134.2	143.1	152.0	161.0	170.0	178.9

Relation between Sag and Length. [Approximate method based on parabolic equation, and applicable only when tension (lbs.) at center is very great compared with weight of wire (lbs. per foot)].

Let s = length of wire, support to support;

l = horizontal distance between supports;

D = deflection (in same units).

Then,
$$s = l + \frac{8}{3}\frac{D^2}{l}.$$

[The exact method is based on the catenary equations and involves hyperbolic functions.]

Equations of Elastic Catenary

Let D = deflection of wire at center of span in feet;

l = distance between supports, feet;

m = weight of wire per cu.in.;

ρ = ratio of the resultant of weight of wire and sleet and wind pressure to the weight of wire;

T = tension at center of span in lbs. per sq.in.;

s = length of wire, support to support;

$k = \frac{T}{12\rho m}$.

Then the exact formulæ for D and s are

$$D = k\left[\left(\cosh \frac{l}{2k}\right) - 1\right],$$

$$s = 2k\left(\sinh \frac{l}{2k}\right).$$

Tables of hyperbolic functions are published in the Smithsonian Tables.

Vertical Stresses on Poles. Every span from pole to pole is part of a large imaginary span, as shown

Fig. 27.

in Fig. 27. If there is an intermediate pole of such height as to just touch the imaginary span, the outside poles will carry the entire load, leaving the

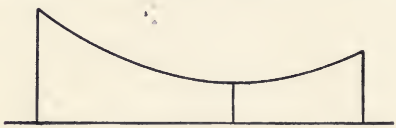


Fig. 28.

intermediate pole unloaded. This condition is shown in Fig. 28. If the intermediate pole is of such height as to bring the wire above the imaginary span, the

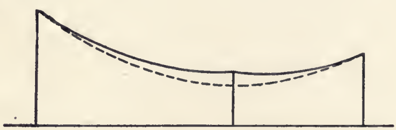


Fig. 29.

intermediate pole will carry part of the load, a condition illustrated in Fig. 29. If, on the other hand, the intermediate pole brings the wire below the imaginary span, there will be an uplift at the intermediate pole, tending to detach the wire from its insulator, a condition illustrated in Fig. 30. In practice, the imaginary span is plotted on tracing cloth to the same scales as the pole line plans, and laid upon the latter so as to determine whether there are any places where there is an uplift on the poles. If any such places are discovered, the pole heights should be altered.

FIG. 30.

The greatest care should be taken to hold the curve with its base line horizontal. The test curve should be plotted from the following equation:

$$d = \frac{Wx^2}{2F};$$

d = rise, feet, measured upward from lowest point;
W = weight of wire, pounds per foot;
F = tension in wire, pounds, corresponding to lowest temperature and wind blowing directly across the span with maximum velocity (the uplifting tendency is greatest for this condition);
x = distance from center of span, feet.

While the above curve is only approximate, it is so close to the exact catenary that for ordinary working tensions it cannot be distinguished from the latter.

The numerical value of the upward pull on the insulator may be calculated as follows, referring to Fig. 31:

If the pole C were removed, the span LC would support the span CO, O being the lowest point of the curve. Since, however, the span CO does not exist, pole C takes the force exerted by the span LC, which is equal to the weight of wire from C to O.

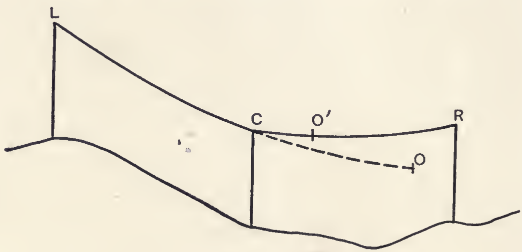


FIG. 31.

Pole C also supports the length CO', found by sliding the test curve along until it touches the tops of C and R; the weight of this length CO' is accordingly subtracted from the weight CO in order to obtain the total uplift at C.

The exact equation for the curve of sag is

$$d = k\left[\left(\cosh \frac{x}{k}\right) - 1\right],$$

where $k = \frac{F}{W}$, and the hyperbolic cosines, written "cosh," may be obtained from the Smithsonian Physical Tables.

CHAPTER VI

SPECIFICATIONS

I. CABLES FOR AERIAL LINES

In writing specifications for bare wire cables for aerial lines the following points should be noted:*

(1) Service to be used for.

(2) Conductors.

  (*a*) Solid or stranded.

  (*b*) Number of wires in strand.

  (*c*) Material of wires, aluminum, hard-drawn or soft-annealed copper.

  (It is usual in copper cables for aerial use to have the central wire medium soft, and the outers hard; sometimes a core of hemp is used in place of the central wire.)

  (*d*) Combined area of wires when laid out straight and measured at right angles to their axes.

  (*e*) Pitch of each layer of wires.

  (*f*) Conductivity in terms of Matthiessen's Standard for soft-annealed copper as

* See Appendix VI.

given in the A.I.E.E. Standardization Report.

(3) Strength.

(*a*) Hard-drawn copper or aluminum.

The tensile strength shall be not less than . . . . lbs. per sq.in. Elastic limit shall not be less than . . . lbs. . . . . . . lbs. per sq.in., with an elongation of not less than . . . %.

(*b*) Annealed copper.

Tensile strength shall be not less than . . . lbs. per sq.in. with an elongation of *not less* than . . . % in a five-foot length.

Elastic limit shall be not less than . . . lbs. per sq.in.

It is usual to have a "flexibility" test, such as a requirement that each of the wires composing the strand shall be capable of being wrapped around a wire of its own diameter in a spiral of six turns, without surface injury or cracking. This test should be performed at 32° F.

The finished cable shall be sufficiently flexible between the temperatures of 0° F. and 150° F., so that it may be bent around a cable of its own diameter without injury to the cable so bent.

The necessary apparatus for making all tests shall be furnished by the contractor.

(4) Length of cable to be supplied on each reel.

(5) The length of cable on each reel shall be continuous; there shall be no joints or splices either in the cable as a whole or in the individual wires of the strand.

(6) Aluminum cables shall be without dents or scratches which might impair their strength.

(7) The cross-section of the cable shall not exceed the specified amount by over 2%.

2. INSULATED CABLE

TITLE

Issue No...... Date............

General. This cable will be used

(1) In tile ducts;
(2) Buried in the ground;
(3) Under water;
(4) In iron or fiber pipes;
(5) In iron or fiber pipes subjected to vibration;
(6) On poles and supported by messenger wire;
(7) In the open air.

} to carry

(1) Direct;
(2) ....phase, ...cycle alternating

} current

at a normal working voltage of .... volts.

All workmanship and materials shall be first-class and shall be in entire accord with the best engineering practice.

Form of Cable. (In the case of single conductor cable). The cable shall consist of

{ (1) The specified number of / (2) (State number) } { (1) hard / (2) soft } drawn copper wires

{ (1) Stranded concentrically / (2) Rope laid in . . . strands of . . . wires each } and insulated with

{ (1) Paper / (2) Rubber / (3) Varnished cloth }

(In case of two conductor cables.)

The cable shall be of oval form and shall consist of two insulated conductors and tarred jute laterals bound together with cotton tape thoroughly saturated with rubber compound.

Each conductor shall consist of . . . . . . wires stranded into cable.

(In case of multiple conductor cables.)

The cable shall consist of . . . . (state number) conductors insulated from one another with

{ (1) Paper; / (2) Rubber; / (3) Varnished cloth. } and stranded into cable with jute laterals.

Each conductor shall consist of . . . . (state number) wires stranded into a cable.

(In the case of cables having several layers, as for example, in control cables, add following clauses.)

The outer layer of conductors shall have a covering of tarred rope. Adjacent layers shall be wound in opposite direction, and around each layer there shall be a spiral of insulating tape. Each layer shall include one conductor differently colored from the others.

The group of conductors shall have a belt of (state material) insulation.

Conductors. The conductors shall have a minimum conductivity of ninety-eight (98) per cent, Matthiessen's standard, as given in the A. I. E. E. Standardization Report, for soft drawn copper wire, and (if rubber or cambric insulated) shall be provided with a heavy uniform coating of tin without projections.

The combined area of the wires, composing each conductor, when laid out straight and measured at right angles to their axes, shall be not less than ...... circular mils (No. .... B. and S.).

Insulation. The insulation (around each conductor) shall be not less than ...... of an inch thick, and shall consist of

{ (1) Paper;
(2) Rubber;
(3) Varnished cloth, } conforming with the accompanying insulation specification.

The insulating belt shall be not less than .... of an inch thick, and shall consist of

{ (1) Paper; (2) Rubber; (3) Varnished cloth, } conforming with the accompanying insulation specification.

Taping and Braiding. The cable shall be taped and braided in accordance with the following table (give table stating number of layers of taping and braiding).

The tape shall be of closely woven cotton filled with rubber compound, and shall lap at least one-third its width, making a smooth surface. Layers in double taping to be wound in opposite directions.

(If cable is to be braided use the following):

(Optional.) Eight-ply jute thoroughly tarred shall be applied spirally over the taping.

A braiding of { (1) Double cotton (2) Six-lea hemp (3) ... inch asbestos } saturated with high flash-point coal tar-compound shall be applied over the { (1) Taping (2) Jute }. The compound shall neither be injuriously affected by nor shall have injurious effect upon the materials of the cable at any temperature below 200° F.

Sheath. A sheath consisting of an alloy of tin and lead containing not less than ninety-eight (98) per cent pure lead and from one (1) to two (2) per cent tin shall be applied uniformly over the insulation. The thickness of the sheath shall be

(1) .... of an inch.
(2) According to the following table.
(Give table of sheath thickness for different sizes of cable.)

Armor. (For rubber or varnished cloth without sheath.)

The above cable shall be protected by a double taping of galvanized steel .... inch wide and ..... of an inch thick applied in such a manner that the spiral space between adjacent turns shall be not more than one-eighth ($\frac{1}{8}$) of an inch; the outer taping to entirely cover the spiral space left between the turns of the inner taping.

(For lead-sheathed cables):

The sheath shall be protected by two layers of asphalted or tarred jute having a combined thickness of three-sixteenths ($^3/_{16}$) of an inch, and shall be armored with galvanized mild steel wire not smaller than No....... B. and S. Over the armor there shall be a similar covering of jute, the layers of which shall be wound in opposite directions.

Tests. Factory tests to be made in the presence of the company's inspector.

(Rubber or varnished cloth insulation):

The electrical tests shall be made upon the cable after twenty-four (24) hours' immersion in water before the lead sheath or braiding is applied.

(Paper insulation):

The electrical tests shall be made upon the cable after it has been (1) passed through a bath of water not less than six feet in length and of sufficient depth to submerge the cable; (2) immersed in water for......hours.

(Paper or varnished cloth):

A high potential test of .... volts alternating shall be applied for a period of .... minutes between

{ (1) Conductor and sheath;
(2) Conductors and between conductors and sheath (the latter for multiple conductor cables). }

(Where cable has *braid* or steel *armor* instead of sheath, substitute *braid* or *armor* for *sheath*.)

The insulation resistance after one minute's electrification with a battery of not less than one hundred (100) or more than five hundred (500) volts shall be measured and reduced to sixty (60) degrees F. and shall not be less than

{ (1) Is required for successful operation.
(To be used for varnished cloth.)
(2) Specified in insulation specification No.......
(3) ......megohms per mile. }

Each finished cable shall be sufficiently flexible between the temperatures of zero and one hundred degrees F. so that it may be bent to a radius of

{ (1) ......times its diameter
(2) ......inches } without injury.

(The apparatus required for making all tests shall be furnished by the contractor.)

(Field tests to be made in the presence of the contractor's inspector.)

Capacity Guarantee. The contractor shall state in his proposal the guaranteed electrostatic capacity of the cables at 60°, 100°, and 150° F. under the following tests: (1) Each conductor against the other(s) and lead sheath; (2) Between (any) two conductors; (3) Between any conductor and lead sheath. (If braided or taped instead of sheathed, use corresponding word instead of "lead sheath" above.)

Inspection. Cables furnished under this specification shall be available for inspection during the process of manufacture except..........................

The contractor shall notify the company when the manufacture of cable is to begin in order that inspection may be arranged for.

Data. The manufacturer shall supply the following data:

(*a*) Weight per foot.
(*b*) Diameter of wire.
(*c*) Diameter of cable over all.
(*d*) Diameter of splice.
(*e*) Length of splice.

Installation in Ducts:

(*a*) By whom cable is to be installed.

If installed by cable contractor the following clauses are necessary:

(*b*) By whom cable lengths will be determined.
(*c*) By whom ducts will be rodded and wired.
(*d*) By whom terminal end bells, clamps, etc., will be supplied and erected.

(*e*) Type of joint.
  (1) Sleeve or interlaced strands.
  (2) Compound to be used.
  (3) Material of taping, paper, cambric, or rubber.

(*f*) By whom cables will be racked in splicing chambers.

(*g*) Limit to number of splices per cable permissible in one splicing chamber; usually one.

(*h*) How and by whom cable sheaths will be grounded. Description of ground connections.

(*i*) How and by whom cables will be wrapped or otherwise protected in splicing chambers.

(*j*) Contractor shall furnish and install bushings or cushions for the cables where they leave the ducts.

(*k*) Contractor shall tag every cable in splicing chamber and station with a brass tag having the cable number stamped on it.

(*l*) Contractor shall refill sleeves after .... months if any perceptible settlement has taken place.

(*m*) Tests. A high potential test of .... volts alternating shall be applied for a period of ...... minutes between conductors and sheath.

The insulation resistance after one minute's electrification with a battery of not less than one hundred (100) or more than five hundred (500) volts, shall be measured and reduced to sixty (60°) F., and (1) shall be not less than ...... megohms per mile, (2) shall decrease at a rate not exceeding ...... per cent per annum.

3. THIRTY PER CENT PARA RUBBER INSULATION

The following specification is offered by the author as more suitable for obtaining a high grade insulation than that of the Rubber Covered Wire Engineers Association given below.

Description of Insulation. Insulation supplied under this specification shall contain not less than thirty (30) per cent and not more than thirty-three (33) per cent by weight of rubber. All the rubber shall be the finest dry Para gum, which has not previously been used in rubber compound.* The gum itself shall not contain more than three (3) per cent of acetone extract. The compound shall be properly vulcanized, and after vulcanization shall contain not more than two (2) per cent by weight of acetone extract which is volatile below 212° F.; and not more than one (1) per cent of free sulphur. The insulation must be tough, elastic, and homogeneous, and placed concentrically about

* This may be objected to by certain manufacturers who claim the use of shoddy to be beneficial.

the cable. (The thickness specified in the tables below means the minimum thickness at any point.*)

If exigencies of manufacture require repairs or joints in the insulation, the entire material of such joints shall conform with the specification, except that over thirty-three (33) per cent of rubber may be used and the work shall be done in such a way as to leave the repaired part or joint as strong and durable electrically and mechanically as the remainder of the insulation.

Tests. The electrical tests shall be made upon the cable after twenty-four hours' immersion in water and before the braid or sheath is applied. The high potential test voltages shall be applied for a period of one minute. The insulation resistance shall be measured following a one-minute electrification with a battery of not less than 100 and not more than 500 volts, and the results corrected to the standard temperature of 60° F. In the case of cables made up of separately insulated wires or cables, the insulation resistance test shall be made before assembling the wires; the high potential test shall be made both before assembling and after the cable is complete. The insulation resistance shall be not less than that given in the accompanying tables. (Tables should be prepared as under "Rubber Insulation," p. 84.)

The change in insulation resistance with tempera-

* Optional.

ture shall be at a rate in accordance with the table herewith, which is based on a rate of two and six-tenths (2.6) per cent per degree F. between the limits of forty (40) and seventy-five (75) degrees F.

A sample of the insulation of the thickness of the insulating wall, without taping, shall be taken from the cable and cut to such a width as to give a cross-sectional area of about one-thirty second ($^1/_{32}$) sq.in. If the total cross section of insulation is less than this, the whole of the insulation shall be used. Marks shall be placed two (2) ins. apart on the sample, which shall then be stretched until the marks are six (6) ins. apart, and one end immediately released; five seconds after release the marks shall be not over two and three-eighths ($2\frac{3}{8}$) ins. apart, except as noted below. The sample shall then be stretched until the marks are eight (8) ins. apart, and one end immediately released; five seconds after release, the marks shall be not over two and five-eighths ($2\frac{5}{8}$) ins. apart, except as noted below. Should the cable be insulated with over twelve sixty-fourths ($^{12}/_{64}$) ins. of rubber, the stretching may be measured thirty seconds after release.* Any sample may be stretched until the marks are nine (9) ins. apart, before breaking. The tensile strength of the compound shall be not less than eight hundred (800) lbs. per sq.in.

* The stretch test may be made less severe if the insulation is intended for high voltages.

These conditions shall be satisfied at any temperature between fifty (50) and one hundred (100) degrees F., and none of the samples shall have been stretched at all prior to the commencement of these tests.

The necessary apparatus for making all tests shall be furnished by the contractor.

Inspection. The contractor shall afford every facility for the engineer to assure himself that the specified proportion and quality of rubber is put into the compound.

TEMPERATURE COEFFICIENT OF RESISTANCE

30% PARA RUBBER COMPOUND

Temperature, Degrees F.	Coefficient shall be not greater than	Temperature, Degrees F.	Coefficient shall be not less than
40	1.68	60	1.000
41	1.64	61	0.974
42	1.60	62	0.949
43	1.56	63	0.925
44	1.52	64	0.901
45	1.48	65	0.878
46	1.44	66	0.855
47	1.41	67	0.833
48	1.37	68	0.812
49	1.34	69	0.791
50	1.30	70	0.771
51	1.27	71	0.751
52	1.24	72	0.732
53	1.20	73	0.713
54	1.17	74	0.695
55	1.14	75	0.677
56	1.11		
57	1.09		
58	1.06		
59	1.03		
60	1.00		

The insulation resistance (megohms) at a given temperature may be reduced to that at 60° Fahr. by dividing by the coefficient corresponding to that temperature.

4. RUBBER COVERED WIRE ENGINEERS' ASSOCIATION

SPECIFICATIONS FOR THIRTY PER CENT RUBBER INSULATING COMPOUND

The compound shall contain not less than 30% by weight of fine dry Para rubber which has not previously been used in rubber compounds. The composition of the remaining 70% shall be left to the discretion of the manufacturer.

Chemical. The vulcanized rubber compound shall contain not more than 6% by weight of acetone extract. For this determination the acetone extraction shall be carried on for five hours in a Soxhlet extractor, as improved by Dr. C. O. Weber.

Mechanical. The rubber insulation shall be homogeneous in character, shall be placed concentrically about the conductor, and shall have a tensile strength of not less than 800 pounds per square inch.

From any wire on which the wall of insulation does not exceed $^4/_{32}$ inch, a sample of vulcanized rubber compound not less than 4 inches in length shall be cut with a sharp knife held tangent to the copper. Marks should be placed on the sample 2 inches apart. The sample shall be stretched until the marks are 6 inches apart and then immediately released; one minute after such release the marks shall not be over $2\frac{3}{8}$ inches apart. The sample shall then be stretched until the marks are 9 inches apart before breaking.

In case the wall of insulation exceeds $^4/_{32}$ inch, the return required shall be $2\frac{1}{2}$ inches instead of $2\frac{3}{8}$ inches and the stretch before breaking shall be 8 inches instead of 9 inches.

For the purpose of these tests, care must be used in cutting to obtain a proper sample, and the manufacturer shall not be responsible for results obtained from samples imperfectly cut.

These tests shall be made at a temperature not less than 50° F.

For high tension service, it is recommended that the above mechanical requirements of the rubber be eliminated.

Electrical. Each and every length of conductor shall comply with the requirements given in the following table. The tests shall be made at the works of the manufacturer when the conductor is covered with vulcanized rubber and before the application of other covering than tape or braid.

Tests shall be made after at least twelve hours' submersion in water and while still immersed. The voltage specified shall be applied for five minutes. The insulation test shall follow the voltage test, shall be made with a battery of not less than 100 nor more than 500 volts, and the reading shall be taken after one minute's electrification. Where tests for acceptance are made by the purchaser on his own premises, such tests shall be made within ten days of receipt of wire or cable by purchaser.

Inspection. The purchaser may send to the works of the manufacturer a representative who shall be afforded all necessary facilities to make the above specified electrical and mechanical tests, and also to assure himself that the 30% of the rubber above specified is actually put into the compound, but he shall not be privileged to inquire what ingredients are used to make up the remaining 70% of the compound.

For insulation thickness and test voltages recommended by the Rubber Covered Wire Engineers Association, see p. 86.

5. VARNISHED CAMBRIC INSULATION

The insulation shall consist of closely-woven cotton tape filled and uniformly coated with a solid film of insulating compound. The tape shall be applied spirally with turns overlapping and successive layers staggered. Groups of layers may be wound in opposite directions. Between the layers of tape there shall be a film of waterproof, viscous, slow-drying and adhesive compound. Should the contact of copper with this insulation give rise to any injurious reaction, a separator shall be applied between insulation and copper. The insulation shall not deteriorate at a constant temperature of 150° F.

6. PAPER INSULATION

Shall consist of the best grade of Manila paper, containing no particles of iron, wood pulp, or any trace of alkali or acid, and shall not be injured by a continued temperature of 130° F. The paper shall be applied spirally with turns overlapping and successive layers staggered and shall be saturated with an insulating compound.

Splicing sleeves shall be filled with........compound. (For example, paraffin wax, Voltax, G. E. No. 67, ozite, etc. The object of specifying this is to secure uniformity.) *

7. RAIL BONDS

(1) *Style of Adhesion:*

- (*a*) Expanded terminal.
- (*b*) Compressed terminal.
- (*c*) Soldered.
- (*d*) Brazed.
- (*e*) Amalgam, or plastic.

(2) *Location to be Applied:*

- (*a*) Exposed.
- (*b*) Concealed.
- (*c*) Head.
- (*d*) Web.
- (*e*) Foot.

* This clause omitted unless cable contractor is to install the cable.

(3) *Type of Conductor:*

(*a*) Ribbon.

(*b*) Cable.

(*c*) Solid.

(4) *Size:*

(*a*) Cross-sectional area measured at right angles to axes of individual strands.

(*b*) Formed length between centers of terminals, or end to end.

(*c*) Contact area of stud or other contact surface.

(5) *Material.* The bond shall be of copper having a conductivity of 98%, Mathiessen's standard, for soft-drawn copper wire, A. I. E. E. Standardization Report.

8. HIGH TENSION LINE INSULATOR

(1) *Service to be used for.*

(*a*) Voltage.

(*b*) A. C. or D. C.

(*c*) Size of cable to be carried.

(2) *Number of pieces.*

(3) *General dimensions* according to accompanying drawings. Permissible variation from dimensions on drawings.

(4) *Color* (usually) white or chocolate brown.

(5) *Quality of material.* The insulator shall be of porcelain (or other specified material) free from

pits, cracks, and other imperfections, and the material shall be sound throughout. The rest marks (on which the insulator is supported in glazing) shall be not larger than .... by .... inch.

(6) *Tests*. The contractor shall furnish for test .... per cent of the insulators from each furnace charge (if porcelain).

(*a*) Absorption Test.

The test insulators shall be broken and the exposed surfaces moistened with red ink. If the ink spreads or is absorbed the insulators will be rejected.

(*b*) Structure.

The fracture shall exhibit surfaces free from cracks, blow holes, etc., and having a close uniform grain.

The following tests shall be made on *all* insulators:

(*c*) The insulator (complete and assembled) shall be inverted, immersed in water up to the center of the side wire groove, and the pin-hole filled with water to the top of the thread. With ...... kilovolts (A. C.) applied for one minute, there shall be no indication of breakdown, leakage, or excessive brush discharge.

(*d*) Mounted on an upright metal pin and subjected to a precipitation of ...... in. (say $\frac{3}{4}$ in.) fresh water per minute, the insulators shall not break down or arc over at less than ...... kilovolts between pin and side wire groove.

Tests may also be required for the individual shells of which the insulator is built up.

If the insulator is of the Hewlett type, use the following clauses instead of (*c*) and (*d*).

(*e*) The insulator (complete and assembled shall be subjected to a potential of ...... kilovolts A. C. applied between opposite wire holes, the upper one being filled with water. After one minute exposure to this voltage, there shall be no indication of breakdown, leakage, or excessive brush discharge.

(*f*) The insulator (complete and assembled), subjected to a precipitation of .... in. (say $\frac{3}{4}$ in.) fresh water per minute, shall not break down or arc when exposed to a potential difference of ...... kilovolts between opposite holes.

CHAPTER VII

TESTING OF WIRES AND CABLES

Wheatstone's Bridge.
- Resistance.
- Inductances.

Resistance by Ammeter and Voltmeter.
Resistance by Differential Galvanometer.
Resistance by Substitution.
Standard Resistances.
Plug Resistance Box.
Reichsanstalt Resistances.
Shunts.
Capacity by Direct Discharge.
Insulation Resistance by Direct Deflection.
Voltmeter Test of Insulation Resistances.
Locating Faults.
- Murray Loop Test.
- Fisher Loop Test.
- Varley Loop Test.
- Point by Point Method.

WHEATSTONE'S BRIDGE OR CHRISTIE'S BRIDGE

Resistance Measurements. Two conducting branches (Fig. 32) *PSQ*, *PTQ*, are joined in parallel, and a current sent through the arrangement, as indicated by the arrows, then in passing from *P* to *Q*, either along the conductor PSQ, or along the conductor PTQ, there are points having all potentials between the potential of P and that of Q, therefore it follows that for every point in the conductor PSQ there must be a point in the conductor PTQ having the same potential. Let S and T be two such points; then, if they were joined with the terminals of a galvanometer, there would be no deflection. Given one point S, the corresponding point T can therefore be experimentally

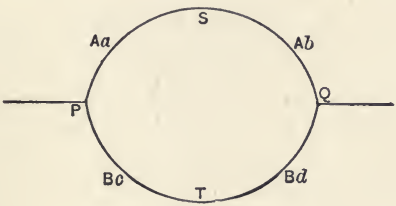


FIG. 32.

found by joining one terminal of a galvanometer to S, and touching the other conductor PTQ at different points with a wire attached to the other terminal of the galvanometer, until a point T is found for which there is no deflection.

Let A be the current flowing in PSQ, B the current flowing in PTQ, and a, b, c, d the resistances respectively of PS, SQ, PT, TQ; then, since the potential difference between P and S is the same as between P and T,

$$Aa = Bc.$$

Similarly since the potential difference between S and Q is the same as between T and Q,

$$Ab = Bd.$$

Hence,

$\frac{a}{b} = \frac{c}{d}$ (Adapted from W. E. Ayrton's "Practical Electricity.")

Inductance Measurements. Place the inductance to be measured in one arm of a Wheatstone's bridge and balance the bridge with a steady current. Then replace the simple galvanometer by a ballistic one and place a key in the battery circuit. Upon depressing the key the galvanometer needle will swing θ degrees. Now destroy the balance of the bridge by inserting a resistance r in the same arm as the inductance, and note the permanent deflection ϕ. Then

$$L = \frac{r}{\tan \phi} \cdot \frac{T}{\pi} \sin \tfrac{1}{2}\theta,$$

where

T = time of natural swing of galvanometer needle.

RESISTANCE BY AMMETER AND VOLTMETER

This method is generally used for resistances from .001 ohm to .01 ohm, using a milli-voltmeter and an ammeter. By Ohm's law,

$$\text{Ohms} = \frac{\text{Millivolts}}{1000 \times \text{amperes}}.$$

For low-resistance work care should be exercised that the voltmeter reading is taken across the resistance to be measured, not including the ammeter.

RESISTANCE MEASUREMENT BY DIFFERENTIAL GALVANOMETER

The arrangement of circuits, as shown in Fig. 33, is the simplest. The adjustable resistance R is

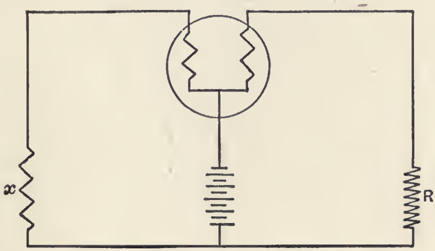


FIG 33.

varied until the galvanometer shows no deflection, when R is equal to x.

If the resistance to be measured is small in comparison with that of the galvanometer, a reversing switch should be used, enabling readings to be taken with the current in either direction through the galvanometer coils. The true resistance will be the mean of the resistances found before and after reversal of current. The connections for this test are shown in Fig. 34.

RESISTANCE MEASUREMENT BY SUBSTITUTION

A battery giving a constant E.M.F., a galvanometer, an adjustable resistance, and the resistance to be measured are all connected in series. The adjustable resistance is short circuited and the deflection of the galvanometer noted. The unknown resistance is then short circuited and the adjustable

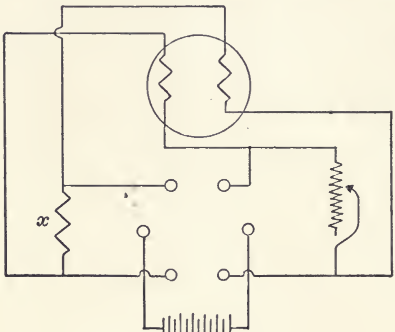


FIG. 34.

resistance cut in until the same deflection is obtained. The resistance thus cut in equals the unknown resistance. The measured resistance should be large in comparison with the total resistance of the circuit; this method is therefore suitable for measuring insulation resistance of machines and cables.*

* See p. 210.

ACCURACY OF RESISTANCE MEASUREMENTS

(W. E. Ayrton, *Electrician*, London, 1907).

By comparison with a standard ohm, a resistance of one ohm may be obtained accurately to $^1/_{100}$ of 1%, whereas a resistance of $^1/_{10,000}$ ohm could be obtained accurately to only about 1%.

STANDARD RESISTANCES

The standard ohm represented by a column of mercury is replaced in practice by coils of high resistance wire with a small temperature coefficient such as manganin, platinoid, or German silver.

PLUG TYPE RESISTANCE BOX (Fig. 35)

A resistance box of the plug type contains coils of wire C_1, C_2, etc., wound on insulating bobbins. The ends of these coils are soldered to stiff wires, w, which are fastened to the brass pieces, b_1, b_2, b_3, which are screwed to the insulating top of the box. When a plug P is inserted tightly between the contact pieces b_1 and b_2, the coil c_1 is short-circuited and practically all the current takes the short path through the plug. If, however, a plug is withdrawn, as at N, all the current passes through the coil C_2. Hence in a box containing coils of various resistance, by taking out one or more plugs, the resistance in the circuit may be varied at will. The brass contact pieces are shaped to form a space *S*, in order to render the surface of the box accessible for cleaning.

The number near each plug indicates the resistance which is put *in* circuit when the plug is taken *out*.

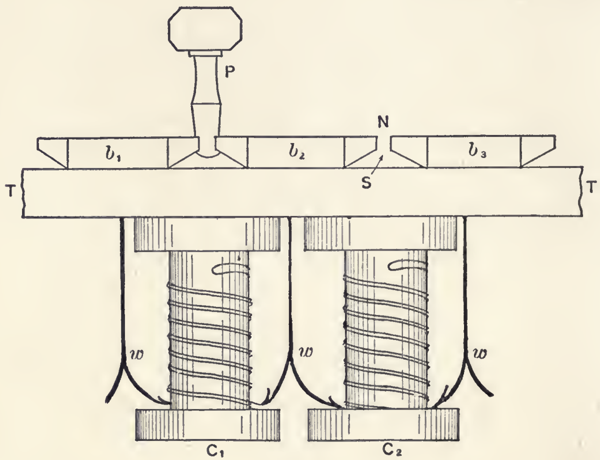


FIG. 35.

When put in the hole, a plug should be given a slight downward screwing motion, which, if the plug is properly made, should make it hold firmly.

Coils are wound double so as to form a loop at one end and two free ends at the other. This makes the current flow around the bobbin an equal number of times in each direction and nullifies the magnetic effect, thereby rendering the coil practically *non-*

inductive and preventing any magnetic action on neighboring instruments.

REICHSANSTALT RESISTANCES

Standard resistances of very low value, say less than one-tenth ohm, cannot be satisfactorily made in the ordinary form, owing to the errors introduced by the contact devices and leads. Instead, the resistance between two points on a conductor is used, and the conductor made suitable for large currents. Current is sent through a pair of large terminals and the drop between the two small terminals taken.

SHUNTS

Resistance boxes are sometimes made up as adjustable shunts for decreasing the current in a galvanometer in a known ratio.

Let C_1 = current in unshunted galvanometer;
C_2 = current in shunted galvanometer;
g = resistance of galvanometer;
s = shunt resistance;
m = resistance of remainder of circuit (see Fig. 36).

Then

$$\frac{C_2}{C_1}=\frac{s(m+g)}{m(g+s)+gs}.$$

With the arrangement shown in Fig. 36, calculations have to be made for every combination of galvanometer and resistance, as the effect of the shunt depends on both the galvanometer and outside resistance. *The Ayrton & Mather Universal Shunt*, which is shown diagramatically in Fig. 37, can be used with any galvanometer, and circuit

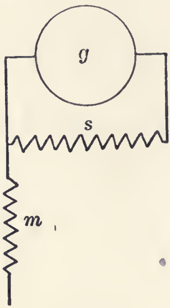


FIG. 36.

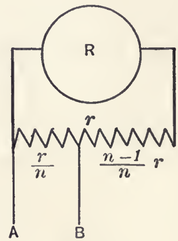


FIG. 37.

without calculation. In order to obtain $^1/_{10}$, $^1/_{100}$, or $^1/_{1000}$ of the current when unshunted, it is only necessary to pull out the plug corresponding to that fraction. Referring to Fig. 37,

R = resistance of galvanometer

r = resistance of shunt coil, which is connected permanently across the galvanometer during the tests.

I_1 = current in A if opposite main is connected to the other end of r;

I_2 = current in mains A and B;

B is any variable point which divides r into two parts having resistances $\frac{r}{n}$ and $\frac{n-1}{n}r$, respectively.

C_1 = galvanometer current with mains connected across r;

C_2 = galvanometer current with mains connected to A and B.

Then,

$$\frac{I_1}{I_2}=\frac{C_1}{C_2}.$$

The use of the universal shunt produces less change in the resistance of the circuit from its original value than the employment of the ordinary shunt, provided that r is less than

$$g(n+\sqrt{n^2+n}).$$

TESTING CAPACITY BY DIRECT DISCHARGE

Apparatus. Ballistic galvanometer and standard condenser.

Method. Obtain galvanometer constant by noting deflection d, due to the discharge of the standard condenser after a charge of say ten seconds from a given E.M.F. Then discharge the unknown capacity through the galvanometer after ten seconds' charge and note the deflection d'. Then,

$$\text{Capacity} = C\frac{d'}{d},$$

C being the capacity of the standard condenser.

INSULATION RESISTANCE BY DIRECT DEFLECTION

Referring to Fig. 38,

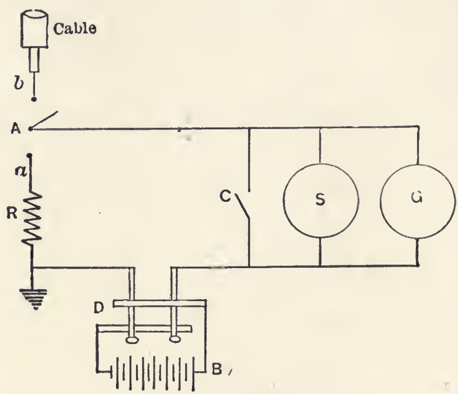


FIG. 38.

G is a mirror galvanometer,
S is a shunt for the above,
B is a battery giving between 100 and 500 volts,
R is a resistance box of 100,000 ohms,
D is a battery reversing key,
C is a short circuit key for galvanometer.

(1) Put the switch A to the lower point, and using $\frac{1}{n}$ of the number of cells to be used in the cable test, obtain the galvanometer deflection.

$$\text{Galvanometer const.} = \frac{G \text{ deflection} \times s \times r \times n}{1,000,000},$$

where r is the resistance unplugged in the resistance box,

s is the multiplying value of the shunt.

The 1,000,000 is to reduce megohms.

(2) Put the switch A to the upper point and disconnect the lead b from the cable. Upon depressing the key D, the insulation resistance of the lead b may be determined from the deflection of the galvanometer. The deflection should be zero, but if not, it should be deducted from the deflection obtained when testing the cable.

(3) Close switch C and connect b to cable.

(4) Open C carefully to see if there are any earth currents. If any, note deflection due to them, and deduct from battery reading if in the same direction, or add to it if in opposite direction.

(5) Close C, depress one knob of D, using, say, the $^1/_{100}$ shunt. After a few seconds open C; if the spot goes off the scale, use a higher shunt; if the deflection is low, use a lower shunt. After one minute's electrification, note the deflection.

The insulation resistance in megohms,

$$= \frac{\text{constant}}{\text{deflection} \times \text{shunt}}.$$

(6) It is desirable to take readings at the end of two, three, or even four and five minutes. The deflection should gradually decrease.

(7) It is desirable to repeat operations with the battery reversed; if there are no earth currents the readings with opposite poles of battery to the cable should not differ appreciably.

The last two and the fourth operations are unnecessary for tests on cables in tanks.

This method is the one universally used for power cable testing.

VOLTMETER TEST FOR INSULATION RESISTANCE

Connect one voltmeter between bus and cable sheath, and a second voltmeter between bus and conductor.

Let V_1 = volts between bus and cable sheath.
V_2 = volts between bus and conductor.
r = resistance of voltmeter on which V_2 is read, ohms.
R = megohms between conductor and sheath.
L = Length of cable, miles,

$$R = \frac{r}{10^6}\left(\frac{V_1}{V_2} - 1\right).$$

"Megohms per mile" $= R \times L$.

LOCATING FAULTS

Murray Loop Test. This test is applicable when a good wire is available, having practically the same resistance as the faulty wire, a condition which occurs when the fault is on one wire of a duplex, triplex, or other multiple conductor cable.

The apparatus being connected, as shown in Fig. 39, the resistance R is varied until the galvanometer is not deflected in either direction.

L = combined length of good and bad wire (equal to twice the length of cable in case of multiple conductor cable).

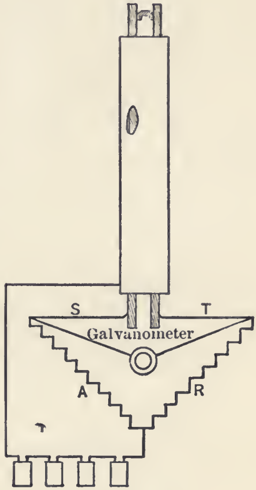


FIG. 39.

$$\text{Distance to fault} = \frac{AL}{A+R},$$

if the leading wires are very short. If the leading wires are of the same size as the conductors in the cable,

$$\text{Distance to fault} = \frac{A}{A+R}(L+S+T)-S.$$

If the leading wires are different in size from the cable wires, for S and T must be substituted the length of a wire of the same size as that of the cable wire which will have a resistance equal to that of S and T respectively.

Fisher Loop Test. This test is applicable when there are two good conductors parallel to the faulty one. The resistance of all three conductors can differ without affecting the test.

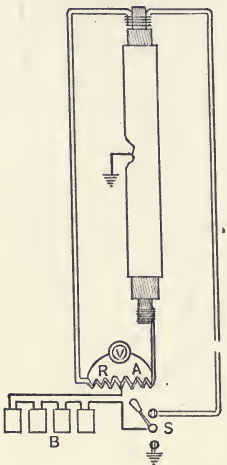


FIG. 40.

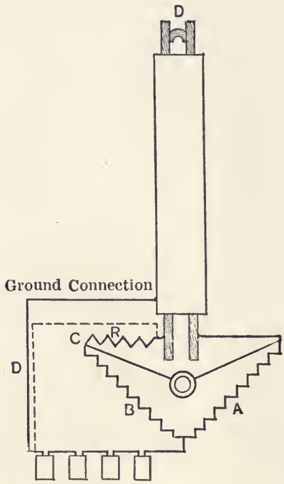


FIG. 41.

The apparatus being connected as shown in Fig. 40, with the switch S thrown down, the resistance R is varied until the galvanometer shows no deflection. Then switch S is then thrown up and the value of R_1 found in like manner.

Then the distance to the fault $= \dfrac{A(A_1+R_1)}{A_1(A+R)}L$, where

L = length of the faulty conductor, and A_1 the value of A when the variable resistance is R_1.

This method is applicable to the locating of crosses if the lower terminal of the switch, instead of being grounded, is connected to the wire crossed with the one used in the test.

Varley Loop Test. The apparatus being connected as shown in Fig. 41, where the faulty conductor is shown on the left-hand side, the resistance R is adjusted so that the galvanometer is not deflected, and a record made of the respective values of A, B, and R. Then measure the combined resistance of the two conductors of the cable and of a and b, the leads from the testing resistances R and A respectively, to the cable. Let this be r, and let x be the resistance of the conductor as far as the fault. Then

$$x = \frac{Br - AR}{A + B} - a.$$

The resistances a and b are obtained by the use of the Wheatstone's bridge, the dotted line battery connection being used and the ground connection taken off the battery.

The above method is also applicable for locating a cross between two wires, if a connection to the wire which is crossed be substituted for the ground connection.

LOCATING FAULTS BY POINT-BY-POINT METHOD

The cable fault-locating outfit of the New York Interborough Rapid Transit Company consists essentially of a Brush D. C. arc generator fitted with a current reversing device, this being located in the power-station, and of a current detector used outside along the cable line.

One terminal of the generator is connected to the cable to be tested and the other terminal grounded to the sheath of the cable. The slowly alternating current from the reversing device is revealed by the current detector if the latter is placed near the cable between the power-station and the fault; beyond the fault the current indicator should show no signs of the slowly alternating current.

The Brush generator is a 9.6 ampere machine giving a maximum of 4000 volts. It is direct driven by a 50 H.P. three-phase induction motor.

The reversing device, from the mechanical standpoint, consists of a wooden cylinder, about 8 inches diameter and 18 inches long, mounted on a horizontal shaft set in brass bearings and fitted with a gear wheel operated by a worm on the shaft of a small induction motor. The entire cylinder and appurtenances are immersed in oil and contained in a closed cast iron box. The general appearance is shown by Fig. 42.

The electrical features consist of three brass rings each $\frac{3}{4}$ inch wide by $\frac{1}{4}$ inch deep, screwed to the cylin-

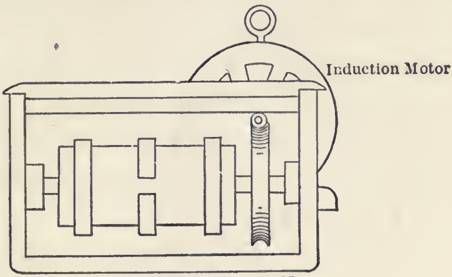


FIG. 42.

der at equal distances along the axis. The center ring is split into two equal nearly semicircular arcs, the gap between parts being about $\frac{3}{4}$ inch. The rings

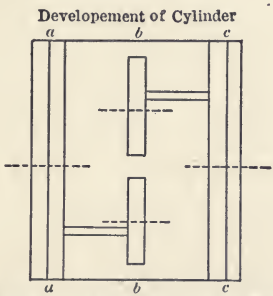


FIG. 43.

are connected as shown in the development of the cylinder, Fig. 43. A copper brush makes contact with each of the outside rings and a pair of brushes

180° apart make contact with the two halves of the split ring. The cylinder makes one revolution in ten seconds, this being therefore the period of a complete reversal or cycle.

The current reverser is connected to the generator and cable, as shown in the diagram, Fig. 44.

The generator, before commencing a test, is short-circuited through a knife switch. When the cable

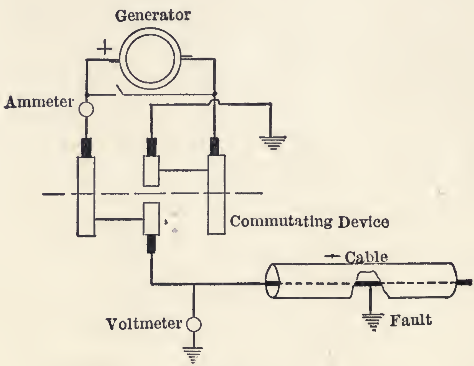


FIG. 44.

is connected, the switch is opened and the machine allowed to build up. The automatic features of the machine keep the current down to less than 10 amperes, the voltage rising in proportion to the resistance of the fault.

The detectors used outside are of two kinds, a compass and a "listening coil." The compass is used wherever possible by laying it on the cable sheath and looking for a periodic swing. Where live D. C.

feeders are close by, the compass needle is too much disturbed to be reliable, and a "listening coil," consisting of a loop of wire connected to a telephone receiver, is used instead.

When a cable breaks down it is tested both by the Varley loop method and by the above described method.

The use of a mercury arc rectifier instead of the arc generator has been suggested and promises satisfactory results.

CHAPTER VIII

INSTALLATION OF CABLES

1. INSTALLATION OF UNDERGROUND CABLES

THE duct having been rodded and wired, is ready at any time to receive a cable. When this is to be done, a drawing rope, usually a Manila rope of from $\frac{3}{4}$ inch to $1\frac{1}{2}$ inches in diameter, is attached to the wire and the wire pulled at the opposite end until the rope is pulled in, leaving sufficient slack at each end.

The cable reel is placed on the ground near the manhole over the duct into which the cable is to be pulled and in such a position that the cable by a slight straightening will unwind from the top of the reel into the manhole and thence into the duct. The free end of the cable must first be fastened in some way to the draw rope.

1. It is usual to grip the cable by means of a pair of iron wires wound around it, as shown in Fig. 45, the end loops being hooked to the pulling rope. Ready-made woven wire cable grips are now largely used. These grips consist of loosely woven wires,

and are placed over the end of the cable. When stretched longitudinally they shrink laterally round the cable and grip it firmly in the same way as the improvised grip above described. As the wires are liable to cut into the lead sheath and pull it off, it is not unusual with large rubber and cambric cables to bare the conductors and fasten the pulling rope to them, thus relieving the sheath of the severe tension caused by the wires. This, however, should never be done if there is any moisture in the ducts, as water is liable to get into the cable.

Fig. 45.

2. Cables may be pulled in any one of three ways:

(*a*) Direct pulling by a gang of men at the rope. This is used only for small cables or very short lengths.

(*b*) Pulling by block and tackle. This is the process most commonly used, although difficult for large cables.

(*c*) Pulling by capstan or winch. This process is only used for long sections of large cable. The winch or capstan is sometimes operated by gasoline or electricity, but such devices have not been universally successful and are not economical unless a large amount of cable pulling is to be done.

When pulling is done by block and tackle, the pulling rope must be successively gripped at points nearer the cable as the cable is pulled along; it is therefore necessary to attach it to the pulling device in a simple way and so that it can be readily removed.

Such a way is shown in Fig. 46, in which it will be noted that the upper part of the rope presses upon the under part *B* when the cable is pulled, thus holding it fast, in spite of the absence of knots or other permanent fastenings.

When the cable has been pulled as far as the tackle will permit, the rope is loosened, the tackle pulled back and the hook attached to a new part of the

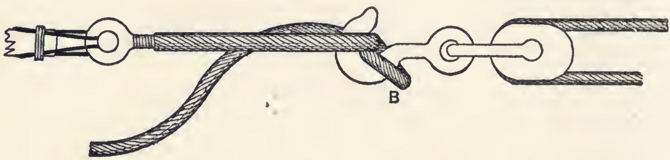


FIG. 46.

pulling rope. This process is repeated until the cable is in place.

A capstan device fitting in a splicing chamber is shown in Fig. 47 and the method of using the rope, in Fig. 48, the end of the rope which reels off, being held tight against the drum by hand and the slack allowed to coil up.

Street capstans are also used, but the difficulty of fastening them securely is a serious objection to their use.

Before pulling cable the edges of the duct should be covered with a piece of lead, such as a piece of scrap cable sheathing, in order to protect the cable from abrasion during drawing. A man should be stationed in the chamber to superintend the feeding-in

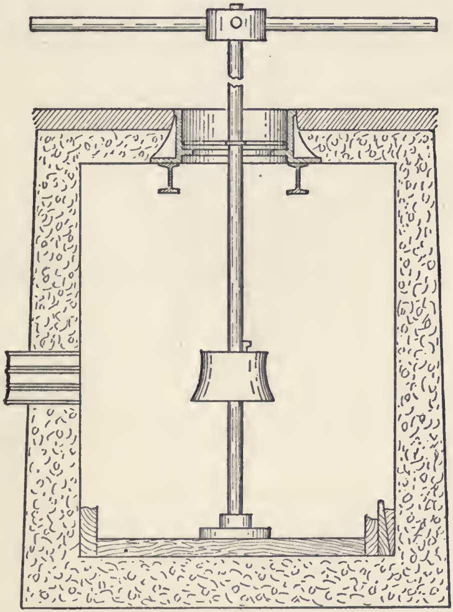


FIG. 47.

of the cable, taking care that it runs tangentially into the duct without injury to its surface. In the event of the feeding-in not progressing properly, this man should notify the men above to signal the pullers to stop or proceed with caution. There should also be a man in the chamber at the pulling end to notify the pullers of anything amiss and to signal stop when the cable has been pulled sufficiently. When a chamber capstan is used, it is often necessary to move the draw rope from the end of the cable grip to a point at the side of the cable in order to pull the cable end beyond the capstan, so that it will not be necessary to use the part injured by the grip.

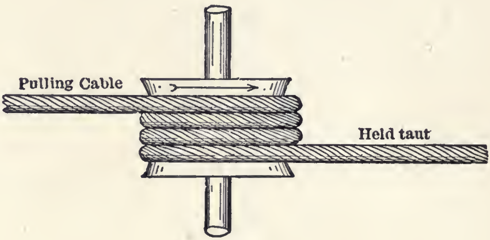


FIG. 48.

2. INSTALLATION OF OVERHEAD WIRES

The erection of overhead wires is performed in various ways, depending upon local conditions and upon the preferences of the engineer. There are, however, two entirely different styles of construction to be considered, namely, the simple span and the messenger wire.

The former is used where the conductors have sufficient tensile strength to support the stresses due to their own weight and the weight of wind and ice; the latter is used where the conductors are unable to support these stresses, as, for example, in the case of insulated cables.

Simple Spans. Starting at an anchored pole, a rope is placed over the cross arm and the wire pulled over the latter and drawn to the next pole, where it is again pulled up by means of the rope and so drawn along from pole to pole until the reel, which remains at the starting point, is exhausted. The pulling may be done by a gang of men, by horses, or by a locomotive if the pole line parallels a railroad. Care must be taken, as the end of the reel is approached, that the wire does not slip away and fall over the first pole.

The next step is to place the cable on the insulators. This may be accomplished by means of a block and tackle if there is a cross arm above, but unless the wire is very large there is no difficulty in doing it by hand. Where the cable is very heavy and there is no cross arm above, the best procedure is to rig up a temporary cross arm or boom projecting from the pole.

The wire, being set upon the insulators, must be drawn up to the required tension. Starting at the first pole after the anchorage, the wire is gripped by a clamp attached to a rope and the rope pulled until the wire is drawn up to the required sag. The wire is then firmly attached to the insulator and the process repeated at the other poles.

The foreman should be provided with a table or curve showing the proper sag at different temperatures and spans. The desired sag is obtained by sighting from pole to pole by means of devices attached to the cross arm or wire, the wire being drawn up until the point of lowest sag is tangential to the sight line.

Messenger Construction. The messenger wire, usually a steel cable, having been erected as described above, a "leading-up" wire is stretched from an anchorage to the messenger wire on the starting pole. A rope is fastened to the end of the cable to be suspended and carried along the messenger wire over the first two poles. The cable is then slowly drawn up the inclined wire, under the cross arm, and along the messenger wire, carriers being attached to the cable as it is paid out from the reel. Men stationed on each pole remove the carriers from the messenger, pass them under the cross arm, and replace them on the other side. The cable is pulled along, in this way, until the reel is exhausted. A common type of carrier for this purpose is shown in Fig. 49, but wire hooks are sometimes used instead.

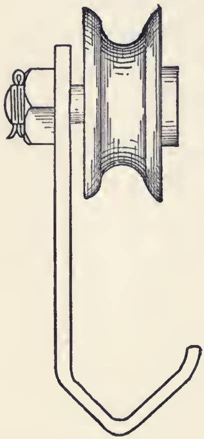


FIG. 49.

When the whole length of cable is suspended, a lineman rides along the messenger wire in a "carriage" or trolley-seat, and replaces the carriers by permanent clips which firmly fasten the cable to the messenger wire.

3. SPLICING

JOINING BARE WIRES

Copper. Solid wires up to No. 00 B. and S. are almost invariably joined by the Western Union method. To make such a joint, bring the two ends of the wire together so that they lap from 3 to 8 ins. Then beginning midway between the two ends wind each overlapping end spirally around the adjacent wire, as illustrated in Fig. 50. With hard drawn copper it is important to avoid giving the wire too much twist. This is accomplished by making the first turn at a small angle and then gradually bringing successive turns nearer to a right angle until they form a close spiral.


Fig. 50.

Cables are generally joined by unstranding them for three or four feet, dovetailing the wires together and wrapping them one by one round the unopened part of the cable. Solder should not be used on overhead wires lest the tensile strength be reduced by overheating.

Numerous mechanical connectors have met with varying success, but do not enjoy the vogue of the ordinary line splice described above.

Aluminum. Aluminum cables are joined mechanically without the use of solder.

Splices between wires of an area equal to No. oooo B. and S. gauge, or anything smaller, are best made by twisting. The two ends to be joined are inserted, side by side, into a piece of flattened aluminum tubing, after which the ends of the tubing are gripped by a pair of connectors having a groove of the same shape as the tube, and from two and one-half to four complete twists given to the tube with its contained wires.

Larger conductors than No. oooo B. and S. may be joined by special connectors supplied by the cable manufacturers or firms dealing in such specialties. A representative joint of this type is made by inserting the ends of the cable into a cast aluminum sleeve. The sleeve is then inserted between dies in a hydraulic jack and pressure applied to the dies until the metal of the sleeve and of the cable flow together into a solid homogeneous mass. A modified form of this joint has the sleeve made in two parts, which are pressed on the cable at the factory. These terminals are provided with internally threaded ends, one right-handed and the other left-handed, and cables are joined by screwing a right- and left-hand threaded stud into the terminals. Such joints, however, are not as popular as the ordinary cable splice, which is made by unstranding the cable for three or four feet, dovetailing the wires together and wrapping them one at a time round the unopened part of the cable.

JOINING INSULATED CABLES

Preliminary. (1) Inspect cable from edge of duct to end, looking for mechanical injury.

(2) Be certain to select the corresponding incoming and outgoing sections.

(3) Place bushings in the mouths of ducts.

(4) Bend cables neatly, taking care to avoid sharp curves, until the ends meet properly at the point of designated for the joint. The completed joint should lie between supports in such a way that there will be no strain on the joint itself. In single conductor cables, where a butt joint is used, the cables should overlap very slightly, but in multiple conductor cables, where the wire joints must be staggered, the cables should overlap sufficiently to allow for the proper distribution of wire splices.

Drying Ends of Cable. The ends of the cable should be carefully examined for moisture, and if any is discovered, the cable should be cut back until all evidence of moisture disappears, care being taken not to cut back so far as to render it too short to make the joint. If moisture is still evident, apply heat to the lead cover of the cable, beginning near the duct and very slowly approaching the open end. This heating may be effected either by pouring on very hot insulating compound and catching it in a vessel held underneath, or by means of a gasoline torch. If the dryness of the cable remains doubtful, an insulation test should be made before jointing, and if the insulation is abnormally low, the cable section should be replaced. Never cut off the end of one section until sure there is no moisture in the other section, as it may be possible to change the location of the splice in case the other end is defective.

Removing the Lead. (1) Mark the lead at the point it is to be removed and make a deep cut around the sheath, gradually increasing the depth of the cut until the lead is cut through, taking care not to cut the insulation in the slightest degree. A chipping knife and hammer or a special tool may be used for this purpose.

(2) Cut the lead lengthwise from the circular cut to the end, taking the precaution to hold the knife tangent to the insulation so that it will pass between the insulation and the sheath.

(3) Pull off the lead with a pair of pliers.

(4) When the lead is removed examine all parts of the bared insulation and remove all loose and projecting particles of lead, especially at the edge of the circular cut.

Preparing Cable Sleeves. (1) Scrape the ends of the sleeve for a length of about 2 inches along the outside, using a knife or a shave-hook and smear the cleaned surfaces with tallow.

(2) Slip the sleeve over the more convenient end of the cable and push it out of the way.

Removing Insulation. Cut back the insulation of each section for a length equal to half the length of the connector plus from $\frac{1}{4}$ to $\frac{1}{2}$ inch, depending on the size of the cable.

With multiple conductor cables having an outer insulating belt it is necessary to cut the outer insulation further back than the inner insulation. In doing this is it essential to avoid cutting the inner insulation in the slightest degree.

Tinning the Copper. Pour molten solder over the copper, using a tallow candle as flux.

Joining Copper by a Connector. The usual way to join the cable ends is to use a copper sleeve, having a cross-sectional area at least equal to that of the cable itself. This condition is obtained by making the outside diameter of the connector about $1\frac{1}{2}$ times that of the wire.

The usual length of sleeve is shown in Table II below:

(1) Put the connector over one cable end and then slip the other cable end in until the two ends butt in the center of the connector.

(2) Sweat on the connector by pouring on solder from a ladle, catching the surplus solder in a pot below.

(3) When thoroughly saturated with molten solder, wipe the joint with a wiping cloth, taking care to

leave no projecting points or sharp edges. This is extremely important in high-tension cables, as sharp points or edges greatly increase the dielectric stress

TABLE I

DATA ABOUT CABLE SLEEVES

(Standard Underground Cable Co.)

	Outside Diameter of Cable, Mils.	Inside Diameter of Sleeve, Inches.	Length of Sleeve, Inches.	Gallons of Compound per Joint.	Wiping Solder per Joint, Lbs.
Single Conductor, light and power, up to 6600 volts	Up to 550	1	8	0.05	0.9
	551–950	1½	10	0.1	1.7
	951–1350	2	12	0.2	2.8
	1351–1750	2½	12	0.3	4.2
	1751–2150	3	14	0.5	5.5
	2151–2550	3½	14	0.6	6.8
Single conductor, light and power, above 6600 volts	Up to 550	1	10	0.05	0.9
	551– 950	1½	12	0.1	1.7
	951–1350	2	14	0.2	2.8
	1351–1750	2½	16	0.4	4.2
	1751–2150	3	18	0.6	5.5
	2151–2550	3½	18	0.8	6.8
Multiconductor, light and power, all voltages	Up to 800	1½	14	0.2	1.5
	801–1200	2	16	0.25	2.5
	1201–1600	2½	16	0.35	3.7
	1601–2000	3	18	0.6	5.0
	2001–2400	3½	18	0.8	6.3
	2401–2800	4	18	1.0	7.6
	2801–3200	4½	20	1.4	8.3

Joining Copper without a Connector. (1) Cut the wires alternately short and long, so that when the two ends are butted, the long wires of one cable will fit against the shortened wires of the other cable and the two cables will be interlaced.

TABLE II

SIZE OF COPPER CONNECTORS

Size of Cable.	Length of Connector.
0 B. & S. to 000 B. & S.	1 in. to 2 in.
0000 B. & S. to 1,000,000 c.m.	2½ in. to 4 in.
1,250,000 to 2,000,000 c.m.	5 in. to 6 in.

(2) Bind the joint with binding wire.

(3) Sweat the cables together by pouring on molten solder.

This type of joint is superior to the connector joint for cables larger than, say, one-half million c.m., because there is less danger of the cables being pulled apart.

Insulating the Joint with Tape. (1) If the cable insulation is thicker than the connector, taper it gradually with a sharp knife.

(2) Then wind on insulating tape of the same material as the cable insulation until a thickness somewhat greater than that of the cable insulation is obtained. The tape should be wound tightly and evenly, running up the tapered part of the cable insulation until well attached to it.

(3) "Boil out" the insulation by pouring over it hot compound. The compound should be hot enough to throw off moisture readily without being hot enough to ignite a piece of paper dipped into it. The surplus compound should be caught in a pan, and when heated may be used again.

The jointer should not take a pot of insulation into a splicing chamber until he has taken off the lid and assured himself that it is at the proper temperature. Many accidents to men and cables are caused by neglect of this precaution.

Insulating the Joint with Sleeves. Instead of winding on insulating tape, an insulating sleeve may be slipped over the wires before soldering and put in place when the wires are joined. The internal diameter of the sleeve must be great enough to permit it to slip easily over the insulation.

(1) After the wires are joined, wind cotton tape tightly over them until entirely covered up to the level of the original insulation.

(2) Slip the insulating tube over the taped joint and fasten it in place with a layer of cotton tape.

(3) " Boil out " the joint by pouring on insulation.

With multiple conductor cables having an outside belt it is necessary to slip a large tube over the belt before splicing the wires.

Tubes may be of prepared paper, varnished cloth, or micanite.

Wiping on the Sleeve. (1) Bring the lead sleeve into position so as to extend equally over the lead on each cable end, and dress down the ends close to the lead of the cable, taking care to make the sleeve concentric with the cable.

(2) Join the sleeve and sheath by means of a wiped solder joint. That is to say, solder is poured

on with a ladle and as quickly wiped with a cloth. This is continued until an absolute air-tight joint is obtained. The joint should be carefully inspected, a small mirror being used to examine the under

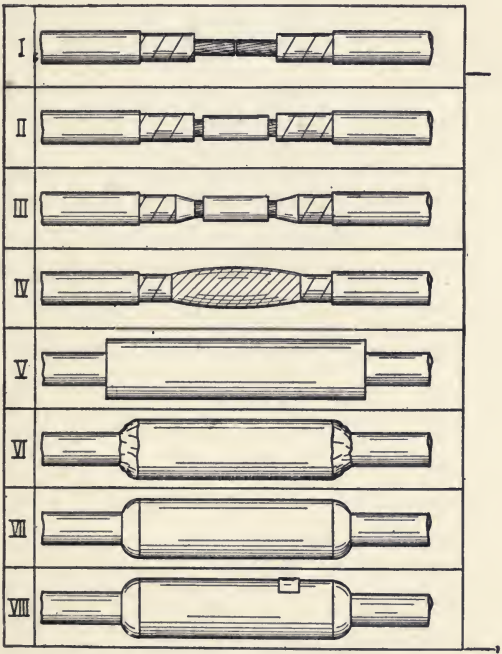


FIG. 51.

side, and if any roughness or weakness is discovered, should be worked over. A small blow-hole undetected at this stage will give great trouble later.

Filling the Sleeve. (1) When the sleeve is well wiped on, make two small holes in the top of the sleeve and pour hot insulation in one hole until it appears at the other, and then in each hole alternately until the sleeve is filled. If any frothing appears on the insulation, continue pouring it in one hole while it escapes out the other, until the frothing stops.

(2) Leave the joint to cool for say an hour, and then add compound to compensate for settling

(3) Put a small piece of lead over the holes and solder it on.

(4) Allow the joint to thoroughly cool and solidify and then put it in its permanent place.

The following compounds are used for filling sleeves.

Paraffin wax;
Ozite;
G. E. Co. No. 67 compound;
Voltax, etc.

Key to Fig. 51. The various stages for a typical joint in a single conductor cable are shown in Fig. 51.

I shows the lead stripped and the wires ready to be joined.

II shows the wires joined by a copper connector.

III shows the insulation tapered to receive the tape.

IV shows the joint insulated with tape.

V shows the lead sleeve slipped in position.

VI shows the ends of the lead sleeve hammered down preparatory to wiping.

VII shows the lead sleeve wiped on.

VIII shows the lead sleeve filled and the holes in it closed by a sheet of lead.

CHAPTER IX

DEPRECIATION AND DETERIORATION

1. DEPRECIATION

DEPRECIATION is a "lessening of value" which may be brought about by the following causes.

1. Deterioration due to the ravages of time and the effects of the elements.

2. Wear and tear incident to use.

3. Displacement by reason of obsolescence or supersession, resulting from developments of the art.

The natural life of cables in ducts is estimated by R. Hammond as thirty years.

Value of a Cable after Installation. Calling the original cost 100, let

V = value of cable immediately after installation expressed as percentage of the original cost.

L = life of cable, years;

S = scrap value at end of L, years, expressed as per cent of original cost;

x = value after being installed y years, expressed as per cent of original cost. Then if the cable is assumed to depreciate by a constant amount every year,

$$x = V - y\frac{V-S}{L}.$$

If, however, the cable is assumed to depreciate at a constant rate per annum,

$$x = V\left(\frac{S}{V}\right)^{\frac{y}{L}}.$$

V is less than the original cost for the following reasons:

(1) Price at which cable was bought may be artificially controlled so as to be above a free market price.

(2) The cable lengths will probably be unsuitable for other installations and will have to be reduced, thereby wasting some cable.

(3) Cable is injured to some extent during installation.

It is important to distinguish between the value of a cable if removed and its value as an integral part of a transmission system, this latter depending upon its efficacy as a revenue producer as well as upon its cost and age.

Life and Depreciation. Equipment worth p per cent of its original cost after y years, is said to depreciate at the rate of P per cent, where

$$P = \frac{(100-p)}{y}.$$

If C = original cost, depreciation is offset by an annuity to redeem $\frac{pC}{100}$ in y years.

Depreciation Calculations. The effects of depreciation may be offset by putting aside a depreciation fund which, added to the scrap value of the old cables, will enable new ones to be purchased.

The payment p to be made at the end of each year, in order to possess the sum s at the end of n years, is as follows.

$$p = s\frac{r}{(1+r)^n - 1}.$$

The payment p to be made at the beginning of each year, in order to possess the sum s at the end of n years, is as follows:

$$p = s\frac{r}{(1+r)^{n-1} - (1+r)}.$$

2. DETERIORATION BY ELECTROLYSIS AND MISCELLANEOUS CAUSES

Principles of Electrolysis Protection. Where a current passes through an electrolyte, the latter is decomposed, hydrogen, metals, and alkaline bases appearing at the cathode or negative electrode and oxygen and acids at the anode or positive electrode. In other words, the corrosive agents, oxygen and acids, travel against the current, and it is therefore only at the anodes or places where the current leaves the metal to enter the electrolyte that the electrolytic corrosion occurs.

The important condition for electrolysis prevention is therefore to keep current from flowing *from* any underground metal work to earth or water in contact with it. The current flowing from the underground metal work can be kept a minimum in three ways in grounded return railway systems.

First. Keeping the potential difference between metal and ground very low, or in other words, by keeping down the drop in the grounded return circuit.

(*a*) By thorough bonding.

(*b*) By frequent cross bonding between tracks.

(*c*) By using negative feeders to reduce the drop in the grounded system.

(*d*) By using insulated negative feeders taking current from the rails at numerous points.

(*e*) By negative boosters on the track rails.

Second. Keeping the metal electronegative to the earth.

(*a*) By connecting to the station negative bus.

(*b*) By means of negative boosters connected to the metal work.

Third. Insulating the metal work.

(*a*) In concrete.

(*b*) By paint.

Those methods requiring special comment are more fully described below.

Insulated Negatives. The drop in the grounded rails may be diminished by taking the current off by numerous insulated cables connected to the track rails. The drop in these cables may be of any magnitude without affecting electrolytic conditions. This system is in use in the New York subways.

Negative Boosters Connected to Tracks. This subject is treated under negative boosters.

Connecting Metal to Station Bus. Pipes, columns, etc., connected by an insulated cable to the negative bus are immune from electrolysis. This method of protection is especially applicable in connection with the insulated feeder system described above, as where that is used the main insulated negative feeders are available for this purpose and special grounding cables are unnecessary.

Negative Boosters connected to Metal. Important iron work, such as iron tunnels and pipes under water, may be protected from electrolysis by using a booster to render them negative to the surrounding water. Such boosters are usually motor driven, and have their negative terminal connected at intervals to the iron work. The positive terminal should be connected to a cable paralleling the tunnel and connected at intervals to graphite anodes. Where conditions permit, a single anode may be sufficiently effective. The voltage must be sufficient to supply a slight current after polarization has been established

Electrolysis of Concrete Encased Steel. Concrete being porous, when saturated with water permits the passage of current. Hence if a current is established through concrete electrolysis can take place through it.

There is, however, a marked increase of resistance following the application of current and consequent tendency of the corrosive action to cease. This increase of resistance may be from ten to fifteen fold before it becomes constant.

Insulation of Metal Work by Paint and Asphalt. Metal work *perfectly* covered with non-conducting paint is impervious to electrolytic corrosion. Unfortunately a slight flaw in the paint will often suffice to start trouble. Several coats of paint are therefore essential for proper protection. Asphalt paint and others of similar nature are generally used as common paints are acted upon by the damp ground especially if alkalis are present.

Alternating-Current Electrolysis. (J. L. R. Hayden, Proc. Am. Inst. Elec. Eng., 1907.) Alternating current electrolysis is not a phenomenon like direct current electrolysis on which quantitative general laws can be formulated; but it is of the character of a secondary effect; that is, the action of the positive half wave is not quite reversed by the action of the negative half wave leaving a small difference rarely exceeding $\frac{1}{2}\%$ of the electrolytic action of an equal direct current.

A direct current about 1.5% of the alternating current is a perfect protection against 25-cycle current. The corrosion increases with decrease of frequency.

Deterioration from Miscellaneous Causes. Cable sheaths are generally somewhat injured, during installation, by projecting points on the surface of the ducts. When it is remembered that a great length of cable is pulled over each projection of this sort, the possible extent of the damage is seen to be very great and the importance of thorough and conscientious examination of ducts realized. Duct inspection is often performed by incompetent people or in a perfunctory manner, which is encouraged by duct manufacturers. This arises from the fact that the process of glazing usually develops a very high percentage of defective ducts which the manufacturer is anxious to dispose of, bids being usually made on the assumption that the customer will be lenient in the inspection. Engineers should remember that electrolysis is a gentle agency of destruction compared with the ripping action of a projection in a duct.

In warm climates, lead sheathing is attacked by beetles, caterpillars, and even wasps. The Home Telephone Co. of Santa Barbara, Cal., has been troubled by insect holes of an eighth of an inch diameter in their cable sheaths.

CHAPTER X

THIRD-RAIL CIRCUITS

The design of railway feeders is so much influenced by the systems of contact conductors to which they are connected, that a study of the general characteristics of such systems constitutes an important phase of the feeder problem.

The first principle in the design of contact conductor circuits is that, when the contact conductor becomes grounded on account of any kind of accident, this grounding shall not be the cause of dangerous or expensive damage of any kind. Such damage may involve material on the right of way, rolling stock, *feeder conductors*, power and control equipment, and may seriously derange the schedule by delaying trains on one or more tracks. It is therefore essential to sectionalize the third rail or trolley wire in such a way as to localize this damage as much as possible. One way of doing this immediately suggests itself, namely, the use of automatic circuit breakers which will open when a ground

* Abstracted from an article by the author in the *Electrical World*, April 22, 1905.

occurs. This method, however, is not as simple as it seems, for, although it is easy to get a circuit breaker that will open with a certain current, it is impossible to get one that has the power of distinguishing between a ground and an abnormal load. This is the principal difficulty encountered when designing a system of third-rail sectionalizing devices. A very destructive short circuit may take even less current than a normal load, and it will, therefore, not open a circuit breaker set to open at an

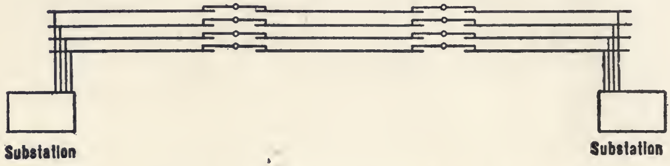


FIG. 52.—Third Rails not Interconnected but Sectionalized.

abnormal current. When, however, the circuit breaker does open, its contacts may be so damaged that it cannot be put back in circuit. For these reasons it is obvious that the promiscuous use of circuit breakers is not desirable, and that none should be installed without a very thorough consideration of the advantages and disadvantages which may arise from local conditions in each case.

The designer of a sytem of third rails should remember that it is far more important to have a reliable system of *connection* between the bus and the cars than the most complete system of automatic or other *interrupting* devices. It is obvious that whereas the interruption of current is an incidental and unusual requirement, certainty of supply is the requirement of fundamental importance. Hence, certainty of supply must not in any way be sacrificed to certainty of non-interruption. Judging from some complicated and expensive systems now in use, it would seem that this fundamental proposition is not universally appreciated. One corollary to be drawn from this is that it is not desirable to

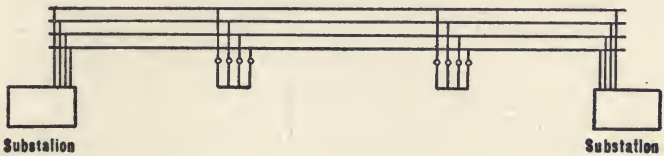


FIG. 53.—Third Rails Interconnected but not Sectionalized.

have circuit-breakers between the load and the source of current, unless they are under constant supervision. As a rule, this means that there should be no circuit-breakers in series with the line except those in the power house or substation.

It is desirable that an accident to the third rail of one track should not in any way interfere with the traffic on the other tracks. For this reason, each track should be separately fed from the bus without any other connections. Unfortunately this system of separate feeding is very uneconomical, as it does not utilize all the available feeder metal to carry the current, unless all the tracks are always equally loaded. In order to obtain the advantages of separately fed tracks, and to secure maximum feeder economy, the method of connecting together all the tracks through circuit-breakers immediately suggests itself. Damage to such circuit breakers is not liable to cause serious trouble, as they do not interrupt the current along each third rail, and they are therefore not essential in the scheme of supply. Whether the tracks are to be

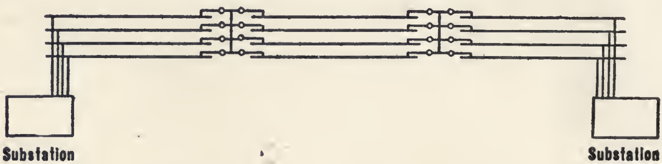


FIG. 54.—Third Rails Interconnected and Sectionalized.

permanently connected or connected through switches or circuit-breakers, or not connected at all, will depend upon local conditions as viewed by the engineer.

In order to confine the effects of a short-circuit to a limited portion of the track on which it occurs, it is desirable to divide the third rail into a number of sections. It is, however, not advantageous to carry this division very far, as an accident at any point on a track will affect the traffic a long way behind. As a rule it is sufficient to break the rail in front of the substations and at *cross-overs*. Breaks at cross-overs are essential in order that a train may go around a dead section of rail by crossing to another track. Breaks in front of the substations are convenient because it is possible to break the rail there without having to install switches or circuit-breakers on the line. Breaks in the rail at cross-overs distant from the substations involve the use of circuit-breakers or switches to interrupt the conductor which joins the sections. As circuit-breakers in series with the line are undesirable, it only remains

FIG. 55.—Knife Switches at Cross-overs.

to recommend the use of switches for this service. It is desirable, however, to use a type of switch which can be opened under load. It is often desirable to locate section breaks at passenger stations on the "far side," in order to enable trains to reach a station in spite of trouble ahead.

The third rail may be sectionalized for another purpose besides confinement of accidents. It sometimes occurs that the current normally carried by the substation circuit-breakers is of such unusual magnitude that the circuit-breakers are materially damaged whenever they operate. It is therefore necessary to divide the third rail into two or more sections, each of which is directly fed from the substation by feeders, thereby dividing the current between two or more circuit-breakers. The breaks at substations are useful in effecting the same purpose.

A weak point in the ordinary feeder system is found in the cable which connects the bus to the third rail. Should a ground occur in this cable, it will not suffice to open the breaker between it and the bus, for the ground will be fed through the third rail from the other substations. It is therefore desirable to have a switch at the third rail between the third rail and its feeder. It should be remembered that a ground of this kind will necessitate the interruption of current from all sources and may, therefore, seriously delay traffic.

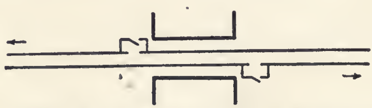


FIG. 56.—Third Rails Sectionalized at Passenger Station.

With separately fed third rails, auxiliary copper may have to be provided for each rail, whereas with rails connected together, auxiliary copper may not be required, but if it is, it will serve to feed all the rails and may be connected to them with the same system of switches or breaks as are used to connect the rails.

A much-discussed subject is the advisability of using short *isolated sections* of third rail at gaps between separately fed sections. The object of these is to prevent a car or train from spanning across a gap between a live and a grounded rail. With the simple multiple-unit system—that is, where only the control wiring runs from car to car—an isolated section may be used with advantage. It must be so proportioned as to render it impossible for one or more cars to span both gaps which isolate the section, and the section on each track must be fed through a separate circuit-breaker. When a bus line connects the main wiring of all the cars a short section of about a car length is quite useless. In this case the section has to be of about a train length, and in order to avoid burning out the train bus line, the isolated section may be protected by a circuit-breaker arranged to open when either of the main third-rail circuit-breakers is open. A train length section is in use on the New York Central R. R., where it has been of considerable service during alterations and repairs to the third rails. An alternative scheme which has been found satisfactory in the I. R. T. subway, New York, is a system of signals at the gaps arranged to show danger when the rail on either side is dead.

CHAPTER XI

RAIL BONDS

CLASSIFICATION

Table I

RAIL BONDS

Classified according to Method of Adhesion

- Chemical Adhesion
  - Soldered Bond (may be applied to Head, Web or Foot).
  - Amalgamated or Plastic Bond.
  - Brazed and Welded Bond.
- Mechanical Adhesion (Adhesion is obtained by expanding terminal into hole in the rail. Expansion effected by pressure applied in following ways.)
  - Pressure from inside. Pin Expanded Bond for Web or Foot.
  - Pressure from one end. Compressed Head Bond.
  - Pressure from both ends. Compressed Web or Foot Bond.

TABLE II

RAIL BONDS

CLASSIFIED ACCORDING TO TYPE OF CONDUCTOR

- Exposed
  - Solid
  - Cable
  - Ribbon
- Concealed
  - Solid
  - Cable
  - Ribbon

Classification. Rail bonds differ in the form of conductors, and in the methods of securing the terminals of the conductor to the rails. Table I shows the classification according to the method of securing adhesion between terminals and rail, and Table II the classification according to type of conductor. Each of the classes mentioned in these tables is commented on below.

Soldered Bond (Figs. 57, 58, and 59). Soldered bonds are very easy to apply, but do not always last

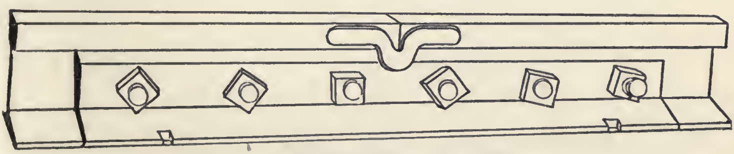


FIG. 57.—Soldered Bond Head Type.

well. Good performance for several months should not be taken as a guarantee of excellence, because failures only begin to occur after several months' use. Under conditions of light service soldered bonds are quite serviceable.

When soldered bonds become loose or are taken

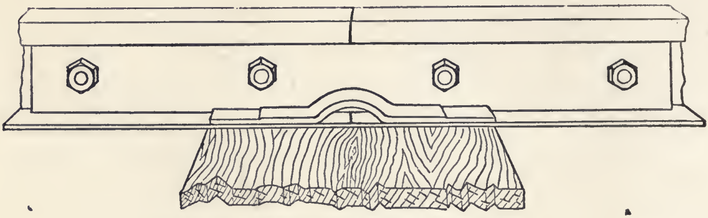


FIG. 58.—Soldered Ribbon Foot Bond.

off for rail repairs or renewals, they can be used again.

In order to apply a soldered bond, the rail surface is made bright with an emery or carborundum wheel,

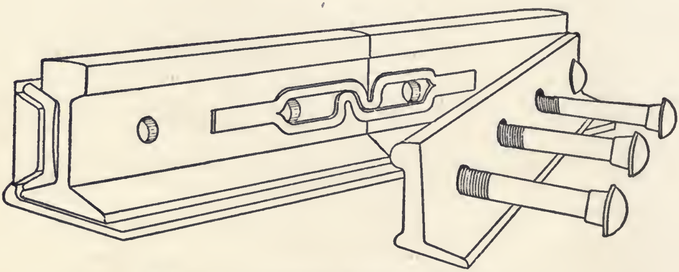


FIG. 59.—Concealed Soldered Web Bond.

and further cleaned with hydrochloric acid before the solder is put on. The soldering is done with a blow torch, the bond being held in place with clamps.

Soldered bonds being short and requiring no drilling, are considerably cheaper than most other kinds.

Brazed Bond. Similar to soldered bond except that brass is used instead of solder.

Welded Bond (Copper Welding). A mould is set around the bond terminal and back along the rail a little distance, and then some copper is brought to a red heat in a crucible placed in a small furnace using hard coal or coke and served with an air blast. A portion of the melted copper is poured through a small opening in the mould where the point of contact is desired; sufficient is poured in to bring the strands of the bond and the steel to the welding point, the mould being provided with an overflow opening for superfluous copper. When it has solidified the mould is taken off and the overflow knocked off with a hammer to be used again.

The heating may also be done electrically by a process similar to that described below for moulding rail joints, the flat copper bond head being welded to the rail.

Plastic Bonds. The conductivity of the fish plate is made use of by interposing between it and the rail a copper bond brought into intimate contact with the iron with the aid of a soft mercury amalgam.

Another type has a copper plate which makes electrical contact with the rail by means of a plastic amalgam, the plate itself being held in position by the reaction of a spring pressing against the fish plate.

The latest type (Fig. 60) consists of a copper plug surrounded with amalgam, placed in a hole drilled through the flange of a girder rail and into the fish plate.

Bonds with Mechanical Adhesion, General. These bonds are more generally used than any other type owing to their greater durability. When once removed they are scrap metal, the life of the bonds

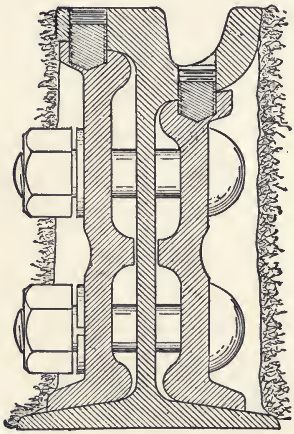


Fig. 60.—Plastic Bond Plug Type.

being therefore limited to the life of the rails on which they are installed.

There are a great many different types on the market differing principally in the method of applying the terminal to the rail. There is, however, little ground for discrimination between types.

Drilling of the rail must be accomplished without the use of oil, the permissible lubricants being soapy water or caustic soda solution.

Pin-expanded Bonds. The pin-expanded terminal has a conical hole into which a steel pin is pressed by screw or hydraulic pressure. This presses the copper outward into firm contact with the rail and leaves a head on the outside of the terminal which, acting like a rivet head, helps to hold the bond firmly in place.

The steel-core type resembles the ordinary pin-expanded type in many respects, but the steel pin

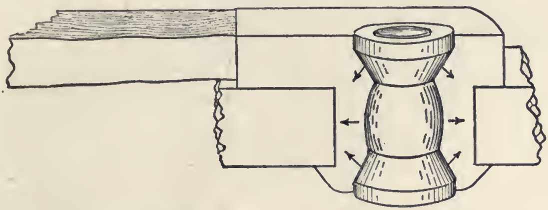


FIG. 61.—Pin Expanded Bond. **G. E. Co. Type with Steel Core.**

is retained in the terminal after it is installed. The core is similar to a double-headed rivet which, when upset by longitudinal compression, expands radially, forcing the walls of the rail hole in the directions shown by the arrows in Fig. 61.

Compressed Head Bonds. One or more holes are drilled in the side of the rail head and the bond terminal pressed firmly into the hole until expanded sufficiently to hold tight. Reaming the sides of the hole so as to produce cavities to catch the bond head does not add much to the security of this type of bond.

If constructed so that rail motion will not tend to rotate the bond terminals in their rail holes, this type of bond is very satisfactory.

Compressed Web or Foot Bond. The bond terminal is put in a hole drilled *through* the rail web or foot,

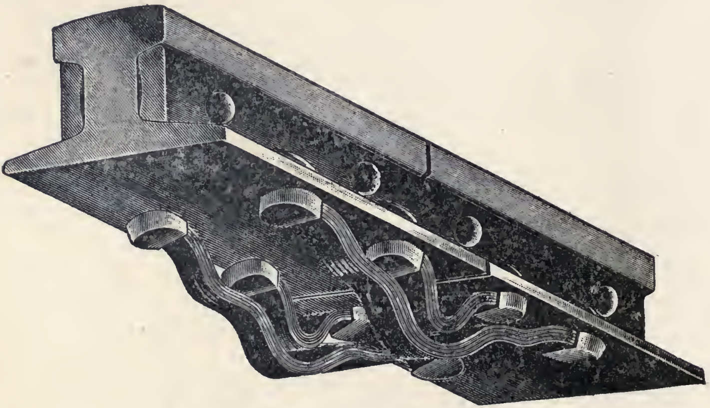


FIG. 62.—Compressed Bond. Foot Type.

and pressure applied at both ends until the copper terminal is squeezed into the shape of a rivet, its ends being spread out to form the rivet heads (Figs. 62, 63, 64, and 65).

Exposed and Concealed Bonds. Exposed bonds are desirable on account of the facility of inspecting, where there is little danger of theft or external injury.

Concealed bonds, i.e., bonds under the fish plate, are necessary where there is danger of theft or external injury. Concealed soldered bonds are not favored for heavy work because soldered bonds require constant inspection and repairs.

Head, Web and Foot Bonds. Open bonds may be applied to the head, web, or foot of the rail.

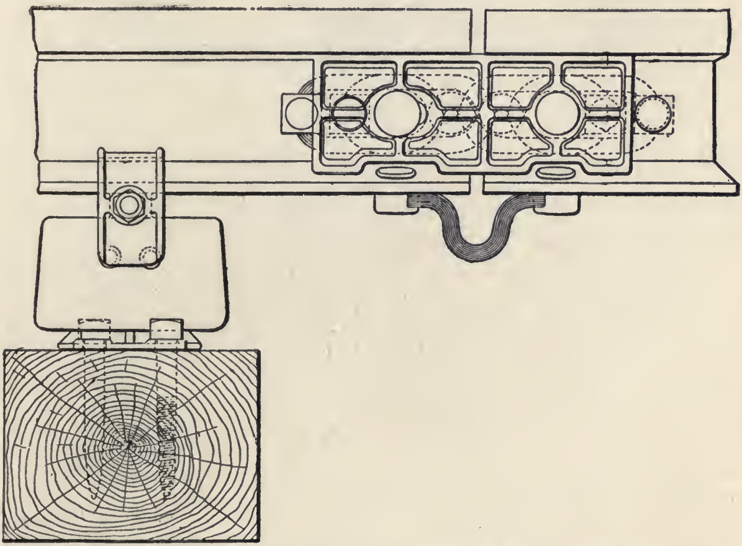


Fig. 63.—Compressed Foot Bond and Compressed Concealed Web Bond.

The only advantage of head bonds is the lower resistance due to the fact that most of the current in a rail is carried in the head. This type of bond is practical for third rails only, on account of the wear on the heads of track rails.

Web bonds are commonly used because concealed bonds are necessarily of that type and expanded terminal bonds are most conveniently applied to the web.

Foot bonds are little used except for third-rail work.

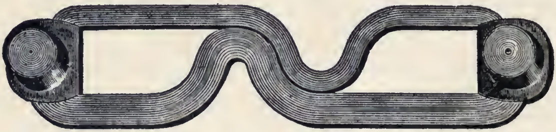


Fig. 64.—Protected Ribbon Bond with Compression Terminals.

Soldered bonds are most easily applied to the upper surfaces of the foot, while compressed terminal bonds are more generally applied underneath.

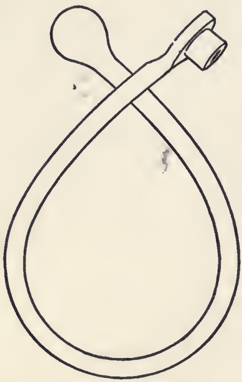


Fig. 65.—Solid Wire Bond.

Solid, Cable, and Ribbon Bonds. Bonds of all classes are made either of solid copper, stranded cable, or multiple ribbons of copper.

Solid bonds, unless of great length and small cross-section, are too stiff for traction work, but are largely used for signal and telegraph circuits.

Exposed bonds are usually of wire cable, as on account of its flexibility in all directions this material is well adapted to withstand vibration.

Ribbon bonds are usually used under fish plates on account of their compactness and the ease with which they lend themselves to tucking about the fish-plate bolts.

Efficiency of Bonding. The efficiency of a rail bond is the ratio of the conductivity of the bonded joint to the conductivity of an equivalent length of continuous rail. If a rail of length L has a sectional area equivalent to A c.m. of copper, and a bonded joint of length l has a section equivalent to a c.m., the efficiency of the bonded joint neglecting contact resistance is $\frac{a}{A} = J$.

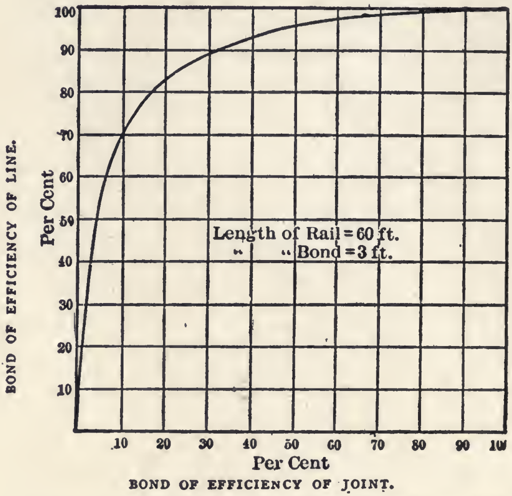
The efficiency of the bonding of a line of rail is the ratio of the conductivity of the bonded line to the conductivity of the line, supposing the rail to be continuous.

The relation between the efficiency of the bonding of a line and the efficiency of a bonded joint is given by the equation,

$$\text{Efficiency of the bonding of line} = \frac{L}{L + \frac{l}{J}(1 - J)}.$$

Fig. 66 shows this equation plotted as a curve for

$L=60$ and $l=3$. It will be noted that the bond efficiency may be very low without materially reducing the efficiency of the bonded line. It therefore appears that the size of bond to be adopted depends



RELATION BETWEEN BOND EFFICIENCY OF JOINT AND OF LINE.

FIG. 66.

more upon the carrying capacity than upon the conductivity.

Carrying Capacity of Bonds. The carrying capacity of a bond cannot be calculated by the ordinary rules for wires or ribbons, on account of the great cooling effect of the rails. A soldered bond will become loose on account of the fusion of the solder without the copper being in any way injured.

Thus a No. 0000 soldered bond will melt off in five or ten minutes at 10,000 amperes.

It should be noted that short bonds have far greater carrying capacity than long bonds on account

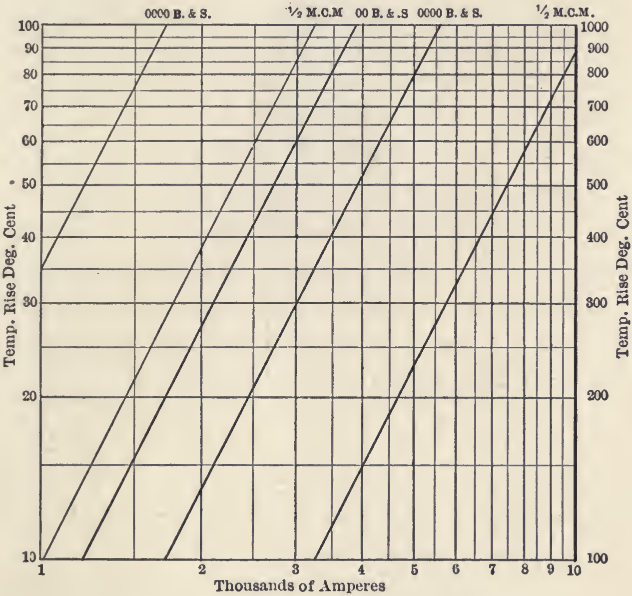


FIG. 67.—9-in. Bond with Mechanical Adhesion.

of the proportionately greater cooling effect of the rails.

There is, at the present time, little reliable data on the carrying capacity of the various types of bonds.

The diagram (Fig. 67) refers to a 9-in. exposed bond with mechanical adhesion or welded. The heavy lines should be used in connection with the right-hand temperature scale, and the light lines with the left-hand temperature scale.

Importance of Cleanliness in Bonding. In order to secure good bonding it is essential to guard against dirty bonds, and bond holes, rough and irregular bond holes insufficient pressure on compressed terminals, unclean rails, and insufficient heat on soldered bonds. The average track construction gang, if entrusted with bonding, even under the eyes of a vigilant inspector, usually makes joints which, while mechanically good, are electrically imperfect. For this reason many companies now have special bonding forces under a foreman with sufficient electrical training to understand the importance of good electrical contact.

TABLE III

CIRCULAR MILS OF COPPER EQUIVALENT TO VARIOUS WEIGHTS OF RAIL

Weight of Rails, Lbs. per Yard.	Ratio of Resistance of Steel to Resistance of Copper.						
	6.	7.	8.	9.	10.	11.	12.
50	1,061,030	909,455	795,773	707,354	636,618	578,743	530,515
60	1,273,236	1,091,346	954,928	848,825	763,942	694,491	636,618
70	1,485,442	1,273,237	1,114,083	990,296	891,266	810,239	742,721
75	1,591,545	1,364,183	1,193,660	1,061,031	954,927	868,115	795,773
80	1,697,648	1,455,127	1,273,238	1,131,766	1,018,589	925,989	848,825
90	1,909,854	1,637,018	1,432,393	1,273,237	1,145,913	1,041,735	954,928
100	2,122,060	1,818,910	1,591,546	1,414,708	1,273,236	1,157,486	1,061,030

Single and Double Bonding. Single bonding has the advantage of being more likely to be in good repair, as a defective bond soon reveals itself. Double bonding affords a factor of safety very important on busy roads.

Welded Rail Joints. Both bonding and the mechanical connection of rails are replaced by various types of welded joints, although some companies use the welded joint for its mechanical features only, preferring to use copper bonds to maintain electrical continuity.

CAST WELDING

A mould is placed around the rail joint and molten iron poured into it.

There are various ways of effecting this, differing in the type of mould and method of applying the iron, but in all of them thorough cleansing of the rails at the joints and protection of the rail top from molten metal are of prime importance.

It is claimed by some that cast welding changes the character of the steel at the joints so that the joints do not wear the same as the rest of the track, and will in time hammer down. This is apparently due to defects in workmanship, as this trouble is not experienced by all users of cast welded joints.

It is important to use plenty of metal in order that it may not be too rapidly chilled.

THERMIT WELDING

A mould is placed around the rail joint, and molten iron poured into it. The process differs from the ordinary cast weld in the method of preparing the molten iron.

Preparing the Rails. The rails having been aligned properly, the ends are thoroughly cleaned with a sand blast or wire brush a few inches each side of the joint. The rails are then heated by a gasoline or oil blow torch, to expel all moisture. Some advise heating to a dull red heat.

The Moulds. The moulds consist of iron frames lined with a mixture of sand and 10% cheap rye flour. This mixture is slightly moistened, so as to retain its form when pressed in the hands, and in this condition placed in the iron frames and baked at about the same temperature as bread. By adding a teaspoonful of turpentine to each pair of moulds, the material is hardened. This, however, is unnecessary except for special work.

The mould frames are securely clamped to the rails, one on each side, the interstices between moulds and rails luted with clay about the consistency of putty, and common earth heaped around the frames.

The rail head is then painted with a watery paste of common red clay, which the heated rail immediately dries to a thin coating. This is to prevent the molten steel uniting with or burning the rail head.

The moulds and rails are then given a final warming with the torch.

The Crucible and its Use. The crucible on its tripod is placed with its pouring hole directly over and about two inches above the gate in the mould. After placing the topping pin, iron disk, asbestos disk, and refractory sand in the bottom of the crucible to act as a plug for the opening, the thermit compound is poured in and in the center of the top is placed about one-third of a teaspoonful of ignition powder, which is set off with a match.

The compound is composed of a mixture of iron oxide and aluminum, both in granular or flake form. The ignition powder is composed of barium peroxide and aluminum in fine powder. When the match is applied to the ignition powder, the aluminum ignites, drawing the necessary oxygen from the barium peroxide. The heat thus developed ignites the aluminum of the thermit compound, which draws the oxygen from the iron oxide and liberates the iron. The latter settles immediately to the bottom of the crucible. While this is going on, the contents of the crucible form a glowing, seething mass, and in about thirty seconds the action is completed.

The crucible is tapped by striking the tapping pin with a special iron spade, and the incandescent steel runs smoothly into the mould, the slag following. In five minutes the mould can be removed to permit the passage of cars.

The mould must be of generous proportions, otherwire the rail will chill the iron and the latter will not adhere.

It is found that if thermit welding is performed when the temperature is rising, the expansion of the rails is apt to cause a hump at the joints.

For this reason it is better to work on cool days or when the temperature is falling.

ELECTRIC WELDING

An iron bar is fitted against the web of the rail and welded thereto by heating both the bar and the rail to a white heat by means of an electric current.

Preparing the Rails. The rails having been aligned properly, the ends are thoroughly cleaned with a sand blast or wire brush a few inches along both sides of the web. The iron bars are applied one on each side of the web and clamped to one rail.

Source and Application of Current. A small motor generator set on a wagon is operated by power taken from the trolley, and supplies alternating current to a step-down transformer. The secondary of this transformer supplies current at very low voltage but enormous amperage which, when applied to the clamps which hold the bars to the rails, brings both bars and rails to a white heat and welds them into one.

While still hot, the bars are clamped to the other rail and the current applied until the welding is effected. As the bars cool, they contract and draw the rails firmly together.

In order to obtain good results the rails must be well abutted before welding.

Table IV

BONDING AREAS

Internal Contact Area of Hole in Rail

Diameter, Inches.	Length 1 Inch	Length $\frac{9}{16}$ Inch.	Length $\frac{5}{8}$ Inch.	Length $2\frac{1}{4}$ Inch.	Length $2\frac{1}{2}$ Inch.
	Sq. In.	Sq. In.	Sq. In.	Sq. In.	Sq. In.
$\frac{5}{8}$	1.964	1.105	1.232	4.419	4.918
$\frac{3}{4}$	2.356	1.324	1.471	5.298	5.890
$\frac{7}{8}$	2.749	1.545	1.721	6.185	6.871
1	3.142	1.767	1.962	7.068	7.854
$1\frac{1}{16}$	3.338	1.875	2.085	7.509	8.345
$1\frac{1}{4}$	3.927	2.206	2.452	7.825	9.818
$1\frac{1}{2}$	4.712	2.648	2.941	10.602	11.780
$1\frac{3}{4}$	5.498	3.088	3.436	12.371	13.745
2	6.283	3.528	3.925	14.137	15.708
$2\frac{1}{4}$	7.069	3.974	4.418	15.905	17.673
$2\frac{1}{2}$	7.854	4.418	4.913	17.672	19.635

Cross-section of Bonds in C.M. and Sq. In.

C.M.	Sq. In.	C.M.	Sq. In.	C.M.	Sq. In.
1,000,000	0.785	400,000	0.314	200,000	0.157
900,000	0.707	350,000	0.275	000	0.132
800,000	0.628	300,000	0.236	00	0.104
750,000	0.489	250,000	0.196	125,000	0.098
600,000	0.472	225,000	0.177	0	0.083
500,000	0.392	0000 *	0.166	100,000	0.079
450,000	0.354				

* B. & S.

CHAPTER XII

INDUCTANCE, REACTANCE, AND CAPACITY

1. TABLES OF INDUCTANCE AND REACTANCE OF PARALLEL WIRES *

Inductance of Single Phase Lines. *To find the inductance in millihenrys per mile of each of two parallel non-magnetic wires, find* A *corresponding to the distance apart of the wires, and* B *corresponding to the size of wire, and add together* A *and* B. *The sum will be the required inductance.*

Thus the inductance of a 1,000,000 circular mil cable, distant 50 feet from a similar cable, will be

$$2.724 - .363 = 2.36 \text{ millihenrys per mile.}$$

The inductance of a No. 36 wire, distant 10 inches from a similar wire, will be

$$1.407 + 1.338 = 2.745.$$

Reactance. Express L in millihenrys. Then if f = cycles per second,

$$\text{Reactance} = 2 \times 10^{-3} \pi f L.$$

* See Appendix VII.

To find the reactance in ohms per mile of each of two parallel wires, find a *corresponding to the distance apart of the wires, and* b *corresponding to the size of wire, and add together* a *and* b. *The sum will be the reactance at* 100 *cycles. At other frequencies the reactance will be in proportion to the frequency.*

Table I

SINGLE PHASE

Values of *A*

d, Distance between Centers of Wires, Ins.	*A*.	*d*. Distance between Centers of Wires, Ins.	*A*.	*d*, Distance between Centers of Wires, Ins.	*A*.
1	0.6654	21	1.645	41	1.861
2	0.8886	22	1.660	42	1.868
3	1.019	23	1.675	43	1.876
4	1.112	24	1.688	44	1.883
5	1.183	25	1.701	45	1.891
6	1.242	26	1.714	46	1.898
7	1.292	27	1.726	47	1.905
8	1.335	28	1.738	48	1.911
9	1.373	29	1.749	49	1.918
10	1.407	30	1.760	50	1.925
11	1.437	31	1.771	51	1.931
12	1.465	32	1.781	52	1.937
13	1.491	33	1.791	53	1.943
14	1.515	34	1.800	54	1.949
15	1.537	35	1.810	55	1.955
16	1.558	36	1.819	56	1.961
17	1.577	37	1.828	57	1.967
18	1.596	38	1.836	58	1.972
19	1.613	39	1.845	59	1.978
20	1.630	40	1.853	60	1.983

Table II

SINGLE PHASE

Values of A

d, Feet.	A.	d, Feet.	A.	d, Feet.	A.
1	1.465	15	2.336	29	2.549
2	1.688	16	2.357	30	2.560
3	1.819	17	2.368	35	2.609
4	1.911	18	2.395	40	2.652
5	1.983	19	2.413	45	2.690
6	2.042	20	2.428	50	2.724
7	2.091	21	2.445	60	2.783
8	2.134	22	2.460	70	2.832
9	2.172	23	2.474	80	2.875
10	2.206	24	2.488	90	2.913
11	2.237	25	2.501	100	2.947
12	2.265	26	2.513	500	3.465
13	2.290	27	2.525	1000	3.688
14	2.314	28	2.537		

Table III

SINGLE PHASE

Values of B

Size of Wire No. B. & S.	B.	Size of Wire, No. B. & S	B.	Size of Wire, No. B. & S.	B.
0000	−0.112	11	0.411	24	0.896
000	−0.075	12	0.448	25	0.933
00	−0.037	13	0.485	26	0.970
0	0	14	0.522	27	1.008
1	0.037	15	0.560	28	1.044
2	0.075	16	0.597	29	1.082
3	0.112	17	0.634	30	1.120
4	0.149	18	0.672	31	1.157
5	0.187	19	0.709	32	1.194
6	0.224	20	0.746	33	1.232
7	0.261	21	0.784	34	1.269
8	0.298	22	0.822	35	1.306
9	0.336	23	0.859	36	1.344
10	0.373				

TABLE IV

SINGLE PHASE

VALUES OF a

($a = 0.46565 \log d + 0.41811$)

Distance between Centers of Wires, Inches = d.	a.	Distance between Centers of Wires, Inches = d.	a.
1	0.4181	21	1.0338
2	0.5626	22	1.0432
3	0.6413	23	1.0522
4	0.7071	24	1.0608
5	0.7436	25	1.0691
6	0.7805	26	1.0770
7	0.8116	27	1.0846
8	0.8386	28	1.0920
9	0.8625	29	1.0991
10	0.8838	30	1.1059
11	0.9030	36	1.1428
12	0.9206	42	1.1740
13	0.9368	48	1.2010
14	0.9518	54	1.2248
15	0.9658	60	1.2461
16	0.9788	66	1.2654
17	0.9911	72	1.2830
18	1.0026	78	1.2992
19	1.0136	84	1.3142
20	1.0239	90	1.3281
		96	1.3412

Thus the reactance at 25 cycles of a mile of No. 0000 B. and S. 36 in. between wires, is as follows:

$$a = \quad 1.1428$$
$$b = - \quad .0703$$
$$1.0725$$

dividing by $\dfrac{100}{25} = 4$

.2681 ohm.

TABLE V

SINGLE PHASE

VALUES OF b

($b=0.023443n$)

Size of Wire.	b.	Size of Wire.	b.
1,000,000 C.M.	−0.2272	7	0.1641
750,000 C.M.	−0.1982	8	0.1876
500,000 C.M.	−0.1572	9	0.2110
250,000 C.M.	−0.0872	10	0.2344
0000 B. & S.	−0.0703	11	0.2579
000	−0.0469	12	0.2813
00	−0.0235	13	0.3048
0	0	14	0.3282
1	0.0235	15	0.3517
2	0.0469	16	0.3751
3	0.0703	17	0.3986
4	0.0938	18	0.4220
5	0.1172	19	0.4454
6	0.1407	20	0.4689

n is the number of the wire on the B. & S. gauge.

Impedance.

$$\sqrt{\text{Resistance}^2+\text{reactance}^2}=\text{impedance}.$$

In a three phase line with wires symmetrically arranged the reactive drop in the *loop* formed by any two wires is $\sqrt{3}\times$reactance of each wire$\times$current in the wire.

Inductance for Parallel Iron Wires (approximate).

d = distance apart, center to center, of wires.

r = radius of wires.

L = inductance of each wire in millihenrys.

Formulæ,

$$L=\left[75+\left(2\log\frac{d}{r}\right)\right]10^{-6}, \text{ per centimeter.}$$

$$L \text{ per centimeter} = .000{,}075 + .000{,}004{,}6 \log \frac{d}{r}.$$

$$L \text{ per inch} = .000{,}191 + .000{,}011{,}68 \log \frac{d}{r}.$$

$$L \text{ per foot} = .002{,}286 + .000{,}14 \log \frac{d}{r}.$$

$$L \text{ per 1000 feet} = 2.286 + .14 \log \frac{d}{r}.$$

$$L \text{ per mile} = 12.070 + .741 \log \frac{d}{r}.$$

(Permeability assumed to be 150.)

Table VI

APPROXIMATE OHMIC RESISTANCE AND IMPEDANCE OF THREE CONDUCTOR CABLES

Size.	Resistance, Ohms per Mile.	Impedance Ohms per Mile. Working Voltage.					
		3000	5000	7000	10000	15000	20000
2	0.850	0.858	0.859	0.863	0.867	0.872	0.884
1	0.674	0.692	0.696	0.700	0.706	0.712	0.724
0	0.535	0.545	0.547	0.552	0.558	0.565	0.580
00	0.424	0.436	0.439	0.444	0.452	0.460	0.478
000	0.336	0.352	0.352	0.357	0.365	0.374	0.396
0000	0.267	0.280	0.283	0.288	0.296	0.306	0.332
250,000	0.227	0.245	0.245	0.252	0.261	0.272	0.299
300,000	0.188	0.210	0.210	0.217	0.227	0.241	0.270
350,000	0.161	0.187	0.187	0.194	0.204	0.217	0.250
400,000	0.141	0.166	0.166	0.174	0.185	0.199	0.234
450.000	0.127	0.148	0.148	0.156	0.167	0.182	0.221
500,000	0.113	0 137.	0.137	0.144	0.156	0.172	0.212

Based on pure copper at 75° F. with an allowance of 3% for spiral path of conductors, 60 cycles per second, and standard thickness of varnished cambric insulation.

Values are practically the same for other types of insulation.

NOTE.—*These figures are approximately correct for 98% conductivity copper at 65° F.*—G. E. Co. Bulletin.

Overcoming Effects of Mutual Induction. Neighboring circuits having currents of the same frequency affect each other so that the inductive drop in one circuit is increased, and in the other decreased. If the currents differ in frequency, the potential will rise in one circuit when the waves come in step, and will fall in the other circuit.

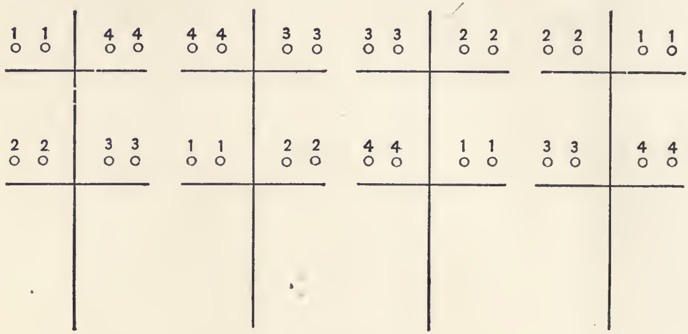


FIG. 68.

The simplest cure for this evil is to put the wires of a circuit close together compared with their distance from the other circuit. Another way is to transpose the wires so that the induction along one-half the line will neutralize the induction in the other half. This is illustrated in Fig. 68, in which each diagram shows how the wires should be arranged for one-quarter of the entire length.

2. CAPACITY

General. The capacity of a transmission line is distributed over the whole length of the conductor, so that the circuit can be considered as shunted by an infinite number of infinitely small condensers scattered along its entire length. Where the capacity of the line is small, however, it may, with sufficient approximation, be represented by one condenser of the same capacity as the line, shunted across the line, either at the generator end, the receiver end, or at the middle.

The best approximation is to consider the line as shunted at the generator and at the receiver end, by two condensers of one-sixth the line capacity each, and in the middle by a condenser of two-thirds the line capacity. This approximation, based on Simpson's rule, assumes the variation of the electric quantities in the line as parabolic.

(Abstracted from "Alternating Current Phenomena," C. P. Steinmetz.)

Injurious Effects of Capacity. The principal objection to high capacity in a line is the large charging current which necessitates a greater generating and transforming equipment. The current, being wattless, does not give rise to much energy loss.

In case the line is supplying a low power factor load, a high capacity in the line may be a distinct advantage, as it improves the power factor at the generating station by neutralizing the lagging current taken by the load.

Two Parallel Wires (Bare). The capacity given by the following formulæ are for the pair of wires, such a pair forming with the air between them, the equivalent of a condenser.

$$\text{Microfarads per mile,} \quad \frac{.038,83}{\log \frac{D}{r}},$$

$$\text{Microfarads per 1000 ft.,} \quad \frac{.007,361}{\log \frac{D}{r}},$$

$$\text{Microfarads per meter,} \quad \frac{.000,02415}{\log \frac{D}{r}},$$

where

r = radius of wire;

D = distance apart, center to center.

The logarithms are to the base 10.

In the above formulæ it is assumed that the disturbing effect of the earth and other neighboring conductors, is negligible.

Charging Current.

E = potential difference between wires, volts;

K = capacity in microfarads of the condenser formed by any two line wires;

f = frequency in cycles per second;

I = charging current, amperes per wire;

For a *single* phase line,

$$I = 2\pi f K E \div 10^6.$$

For a *three* phase line,

$$I = \frac{4\pi f K E}{\sqrt{3} \times 10^6}.$$

Single Overhead Wire with Earth Return.

h = height of wire above ground,
r = radius of wire.

(These to be given in the same units.)

The capacity of such a wire is equal to that of a pair situated a distance $2h$ apart. In other words, the capacity which such a wire forms with the earth is equal to that which it forms with its reflected image in the earth, assuming the earth to be a perfect conductor.

Microfarads per mile, $\frac{.03883}{\log \frac{2h}{r}}$;

Microfarads per 1000 ft., $\frac{.007361}{\log \frac{2h}{r}}$;

Microfarads per meter, $\frac{.000,02415}{\log \frac{2h}{r}}$;

Single-Phase Two Conductor Cable.

Let a = capacity between one conductor and the other in parallel with the sheathing;
$b = -(a - \frac{1}{2}$ capacity between the two conductors in parallel and the sheathing).

These two are readily measurable quantities. Then the capacity between the two conductors equals $\frac{1}{2}(a-b)$.

Three-Phase Cable with Neutral Grounded.

Let a = capacity between one wire and the other two in parallel with the sheathing,
$b = -(a - \frac{1}{2}$ capacity between two wires in multiple and one wire and sheathing in multiple).

These two are readily measurable quantities. Then

Capacity between one wire and other two in parallel with the sheathing $= a$;
Capacity between two wires in multiple and one wire and sheathing in multiple $= 2(a+b)$;
Capacity between three wires in multiple and the sheathing $= 3(a+2b)$
Capacity between two wires $= \frac{1}{2}(a-b)$;
Capacity between one wire and two in multiple $= \frac{2}{3}(a-b)$

(Alex. Russel, *Journal Inst. Elect. Eng.*, London, 1901.)

APPENDIX I

BASIS OF AMERICAN OR BROWN AND SHARPE GAUGE

The American or Brown and Sharpe gauge is based upon a geometrical progression beginning with a wire of five mils diameter, called No. 36, and ending with a wire of 460 mils diameter, called No. 0000. These numbers and sizes were selected in order to make the gauge conform approximately with existing systems.

The ratio of this geometrical progression is

$$R = \sqrt[39]{\frac{460}{5}} = \sqrt[39]{92} = 1.122932$$

and its common logarithm is

$$.050,353,53.$$

The diameter of a No. n wire is that of a No. 0 wire divided by R^n, and as the diameter of a No. 0

wire is that of a No. 0000, divided by R^3, the diameter of a No. n wire in mils is equal to

$$\frac{460}{R^{n+3}}, \text{ exactly,}$$

or,
$$\frac{32,486}{1.1229^n}, \text{ approximately.}$$

The area in circular mils being equal to the square of the diameter, is equal to

$$\frac{211,600}{R^{2n+6}}, \text{ exactly,}$$

or
$$\frac{105,534}{1.2605^n}, \text{ approximately.}$$

The number on the B. and S. gauge of a conductor of A circular mils area is given by the following equation, which is derived from the above equation for area,

$$n=\left(\frac{39}{2 \log 92} \cdot \log \frac{211,600}{A}\right)-3,$$

or
$$n=\left(9.92978 \log \frac{211,600}{A}\right)-3.$$

Numbers of conductors larger than No. 0 are given as negative quantities. Thus

B. and S. No.	n
0	0
00	−1
000	−2
0000	−3

etc.

The ratio R is approximately equal to the sixth root of 2, which is 1.12246. This fact makes it possible to have a group of wires having approximately the same area as any single wire, all being regular sizes on the B. and S. gauge. This approximation gives rise to the following formulæ:

$$\text{Diameter, mils} = \frac{3249}{2^{\frac{n}{6}}};$$

$$\text{Area, circ. mils.} = \frac{105{,}500}{2^{\frac{n}{3}}}$$

$$\text{Ohms, per 1000 ft.} = \frac{2^{\frac{n}{3}}}{10};$$

$$\text{Pounds per 1000 ft.} = \frac{320}{2^{\frac{n}{3}}}.$$

APPENDIX II

BASIS OF SKIN EFFECT AND CARRYING-CAPACITY FORMULÆ

SKIN EFFECT

USING the same symbols as on p. 40, the exact expression for R is as follows:

$$R = \tfrac{1}{2}p\frac{\text{ber.}\ p \cdot \text{bei}'.\ p - \text{bei.}\ p \cdot \text{ber.}'\ p}{(\text{ber.}'\ p)^2 + (\text{bei.}'\ p)^2},$$

where $p = 0.875\ Z$, and 0.875 is the square root of 8π times the number of centimeters in one foot.

Bessel's functions may be avoided by substituting a series, but for all practical purposes the approximation given is sufficient.

CARRYING CAPACITY

A conductor heated by a current assumes a steady temperature when the power generated in it equals the power dissipated from it. The rate of generation of heat is given by the well known equation

$$\text{Watts} = \frac{I^2 k}{a},$$

where I = amperes;

k = specific resistance of conductor in ohms per circular mil-foot at the temperature corresponding to the rise T;

a = cross-sectional area of conductor, circ. mils.

The rate of dissipation of heat cannot be expressed by any exact equation because heat is dissipated by conduction, convection and radiation, and these methods of heat dissipation are not susceptible of exact expression.

It is usual to assume the dissipation of heat to be entirely effected by one method, either radiation or conduction, the former being nearly correct for bare conductors and the latter, for insulated cables in ducts.

Assuming heat dissipation by radiation and using Newton's law of cooling,

$$I = K_1 \sqrt{T},$$

where K_1 is a constant depending upon the size and style of conductor.

Assuming heat dissipation by conduction,

$$I = K_2 \sqrt{\frac{T}{1 + aT}},$$

where K_2 is a constant, and

$$a = \frac{1}{238.1 + t},$$

t being the initial temperature. This is based upon the assumption that the thermal resistivity and outside temperature of the heat insulator surrounding the conductor are constant.

The best experimental data available is that of Fisher, Ferguson, and Kennelly, but their results do not exactly agree with any formula available. The author has therefore adopted the simplest formula, namely, that based upon dissipation proportional to the temperature rise, and has derived his constants so as to include all the best experimental data within his knowledge.

The formula is as follows:

Let

A = cross-sectional area of conductor, sq.in.;

I = amperes;

C = circumference of conductor, inches;

W = watts dissipated per sq.in. of surface per degree C. temperature rise;

T = temperature rise, degree C.

r = specific resistance of conductor, ohms per inch cube at the temperature corresponding to the rise T.

Then in an inch of conductor,

$$\text{Watts generated} = I^2\frac{r}{A},$$

$$\text{Watts dissipated} = CWT.$$

Hence,

$$I^2\frac{r}{A}=CWT.$$

or

$$I=\sqrt{\frac{ACTW}{r}}=\sqrt{A}\,\sqrt{C}\,\sqrt{W}\,\sqrt{\frac{T}{r}}.$$

By changing the constants to a more practical form, the formula of p 46 is obtained.

The Short-Period Carrying Capacity of Cables. The watts generated in a cable on account of its ohmic resistance are partly absorbed by the cable and partly dissipated from it.

The joules absorbed when the temperature is raised $D°$ F., equal pD, where $p=1055$ [(specific heat of conductor $\times$ its weight in pounds per foot) + (specific heat of insulation $\times$ its weight in pounds per foot)]. Hence, the watts absorbed equal

$$p\frac{dD}{dt},$$

t being the time in seconds.

The watts dissipated per foot of cable when the temperature rise is $D°$ F., are equal to

$$qD,$$

q being the watts dissipated per foot per degree temperature rise.

Then, if W = watts generated per foot of cable,

$$W = \text{watts absorbed} + \text{watts dissipated}$$

$$= p\frac{dD}{dt} + qD. \qquad (1)$$

The temperature of the cable rises until the watts dissipated equal the watts generated, so that when

$D = F$, the final temperature rise,

$$qD = W$$

or

$$F = \frac{W}{q}.$$

Equation (1) may then be written

$$\frac{W}{q} = \frac{p}{q}\frac{dD}{dt} + D = F,$$

whence,

$$\frac{dt}{dD} = \frac{p}{q} \cdot \frac{1}{F - D},$$

$$\therefore \quad dt = \frac{p}{q} \cdot \frac{dD}{F - D},$$

$$\therefore \quad t = \frac{p}{q} \log_e \frac{F}{F - D}. \qquad (2)$$

The F in the numerator is the constant of integration and is determined from the condition that when $D = 0$, t must be zero. As $F = \frac{W}{q}$ and $W = I^2 r$, where r is the resistance of the cable in ohms per foot,

$$F = \frac{I^2 r}{q},$$

and

$$\frac{F}{I^2} = \frac{r}{q},$$

which is a constant and may be called G.

Substituting $\frac{G}{r}$ for $\frac{1}{q}$, equation (2) becomes

$$t = \frac{pG}{r} \log_e \left(1 - \frac{GI^2}{D}\right). \quad . \quad . \quad . \quad . \quad (3)$$

Reducing to minutes, replacing p by P, which is $\frac{p}{1055}$, and substituting common logarithms for the Naperian,

$$t = 40.5 \frac{GP}{r} \log \left(1 - \frac{GI^2}{D}\right). \quad . \quad . \quad . \quad (4)$$

Writing Z for the logarithm, the equation becomes,

$$t = 40.5 \frac{GP}{r} Z. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The above deduction is based on the assumption that r is constant. As, however, r varies with the temperature, the time t will be proportional to the mean value of $\frac{1}{r}$. Hence, we write,

$$\frac{1}{r} = AK,$$

where A is the cross-sectional area in circ. mils and K is the mean of the reciprocal of the ohms per mil-foot over the temperature range considered. Hence, equation (5) reduces to

$$t = 40.5\ PAKGZ. \qquad (6)$$

The equation considered above connects the variables t and I, D being constant. The same equation, however, may be used to express the relation between t and D, I being maintained constant, and for this purpose is most conveniently written

$$t = 40.5\ PAK\frac{F}{I^2}Z,$$

where $Z = \log_{10}\left(1 - \frac{F}{D}\right)$ and F is the final temperature rise with I amperes applied indefinitely.

Experiments on the time element of fuses by Schwartz and James, detailed in the Journal of the Institution of Electrical Engineers, July, 1908, p. 71, may be accurately represented by this equation, although a range of 180° F. is covered. Thus, with an enclosed fuse consisting of a No. 27 S. W. G. copper wire surrounded by Calais sand in a ½-inch fiber tube, the temperature rise is represented by

$$t = 20\ Z,$$

the current being 20 amperes.

(From an article by the author in the *Electrical World*, 1908.)

APPENDIX III

THICKNESS OF RUBBER INSULATION

The thickness of insulation to be placed on a wire is governed by three features:

1. Errors in size of wire, eccentric situation of wire in the insulation, and similar irregularities.

2. Insulation not to be strained by application of test voltage.

3. Insulation to be thick enough to have mechanical strength.

ERROR THICKNESS

The thickness of insulation required to make up for errors and irregularities of manufacture may be termed the "*Error Thickness*." This quantity can be determined in only one way and that is by observation.

The Rubber-Covered Wire Engineers' Association have adopted the following values in which the error thickness depends only upon the size of wire and not upon the thickness of insulation.

The error thickness used in the N. Y. C. R. R. specification are based partly upon the Rubber-

Covered Wire Engineers' Association values and partly upon a series of measurements, a curve being plotted through the mean of the numerous points obtained.

Table I

ERROR THICKNESS USED IN SPECIFICATIONS OF RUBBER-COVERED WIRE ENGINEERS' ASS'N AND OKONITE CO.

Size of Conductor.	Error Thickness.
1,000,000 to 550,000 C.M.	3/128 in.
500,000 to 250,000 C.M.	2/64 in.
4/0 to 1 B. & S.	3/64 in.
2 to 7	4/64 in.
8 to 14	5/64 in.

Table II

ERROR THICKNESS USED IN N. Y. C. & H. R. R. SPECIFICATIONS

Size of Conductor.	Error Thickness, Inches.	Size of Conductor.	Error Thickness, Inches.
14 B. & S.	0.018	250,000 C.M.	0.053
12	0.020	500,000	0.063
10	0.022	750,000	0.070
8	0.025	1,000,000	0.075
6	0.028	1,250,000	0.080
4	0.032	1,500,000	0.083
2	0.036	1,750,000	0.086
1	0.038	2,000,000 conc.	0.089
0	0.040	2,000,000 rope	0.095
00	0.042		
000	0.045		
0000	0.047		

Some engineers believe that the error thickness depends upon the thickness of insulation, being greater for heavily insulated cables than for those lightly insulated. A series of measurements to elucidate this point gave uncertain results.

DIELECTRIC STRESS

When a high potential is established across the insulation of a cable, the insulation is subjected to a strain which depends upon the degree of concentration of electric force. When this concentration reaches a certain value, the insulation will no longer be able to stand the strain and will break down. It will not necessarily be punctured, but will be disintegrated only where the concentration of electrical force has been excessive. For purposes of analysis, it is usual to represent the intensity of electric force by the density of imaginary lines of force stretching radially from wire to sheath.

Let F = dielectric stress in kilovolts per inch;
V = test potential, kilovolts;
t = thickness of insulation, inches, over error thickness.

Then

$$F = \frac{V}{t}, \text{ for a uniform static field of force.}$$

The field of force around a cylindrical wire, however, is not uniform, the lines extending radially from the wire to the outside of the insulation. The density of the force lines is therefore greater at the surface of the wire than at the outside of the insulation. This explains the well-known fact that small wires insulated for high potentials often show a disintegration of the inner layers of insulation without any visible defect on the outside. In this case

$$F = \frac{.434\,V}{r \log \frac{(t+r)}{r}},$$

where r is the radius of the wire, inches, and the logarithm is to the base 10.

This gives

$$V = 2.3026\,Fr \log \frac{t+r}{r}.$$

This is not strictly true for stranded cables, the dielectric stress being from 1.23 to 1.46 times the value given by the above formula.

The smaller value holds for thick insulation and the latter for very thin insulation.

(The exact formula for stranded cables, according to Professor Levi-Civita, is given by E. Jona in the Transactions of the International Electrical Congress at St. Louis, 1904.)

For stranded cables, therefore,

$$F = 1.345 \times \frac{.434\ V}{r \log \frac{(t+r)}{r}} = \frac{.585\ V}{r \log \frac{(t+r)}{r}}.$$

(The figure 1.345 is the mean of 1.3 and 1.46.)

MECHANICAL THICKNESS

The error thickness and the electrical thickness of insulation are often insufficient for mechanical reasons. Table III shows the minimum thickness of insulation which is permitted by mechanical considerations. The thickness of the insulation on a cable should never be less than the value given in this table, irrespective of what voltage it is designed for. This table, while based on average practice, may not meet the requirements of some engineers, and should, therefore, be carefully examined before it is used.

TABLE III

Diameter of Conductor, Inches.	Mechanical Thickness, 64th Inch.	Diameter of Conductor, Inches.	Mechanical Thickness, 64th Inch.
.0	3	1.2	9
.2	4	1.4	10
.4	5	1.6	11
.6	6	1.8	12
.8	7	2.0	13
1.0	8		

INSULATION RESISTANCE

The insulation resistance of a cable is derivable from the following formula:

$$M = 58 \times 10^{-7} \times S \times \log \frac{T+r}{r},$$

where M = megohms per mile;

S = specific resistance in megohms per inch cube;

T = thickness of insulation, inches;

r = radius of wire, inches;

logarithm is to base 10.

This formula is sometimes written

$$M = K \log \frac{T+r}{r},$$

where $$K = 58 \times 10^{-7} \times S.$$

The value of K varies from 870 to 23,200 for $S = 150$ and $S = 4000$, respectively. The use of K instead of S has the advantage of brevity and is endorsed by the manufacturers.

In calculating insulation resistance, the total thickness of insulation should be used.

EXAMPLE OF CALCULATION

It is desired to find the thickness of insulation for a cable to be tested for 15 kilovolts (using a stress of 127 kilovolts per inch), the size being No. 4–0 B. and S. stranded.

Using the formula

$$F = \frac{.585\ V}{r \log \frac{t+r}{r}},$$

we obtain

$$\log (t+r) = \frac{.585\ V}{Fr} + \log r.$$

Inserting the figures,

$$\log (t+r) = \frac{.585 \times 15}{127 \times .23} + \log .23 = \bar{1}.662.$$

$$\therefore \quad t = .229$$

The error thickness from Table III is .047; hence the total thickness of insulation is

$$.229 + .047 = .276 = \frac{18}{64} \text{ in.}$$

In the above case the thickness is well above the amount required for mechanical strength, which would be about $^6/_{64}$ inch. If the thickness had worked out to an amount less than is required for mechanical strength, the proper thickness would have to be taken from Table III.

In such cases the error thickness has to be calculated and subtracted from T in order to obtain t, for which the test voltage is calculated.

The table on p. 84 is calculated by the above method, using a dielectric stress of 57 kilovolts per inch for the working voltage.

The cables, therefore; normally operate with a factor of safety of 7, assuming a breakdown stress of 400 kv. per in. The actual factor of safety is liable to be much below this, as some brands of rubber compound have a very low dielectric strength.

The megohms per mile, assuming $K = 4000$, are

$$M = 4000 \log \frac{\cdot 377 + \cdot 23}{\cdot 23} = 4000 \times .421 = 1684.$$

ONE INCH IN FRACTIONS AND DECIMALS

64th.	32nds.	16th.	8ths.	4ths.	Decimal.	64th.	32nds.	16th.	8ths.	4ths.	Decimal.
1	....	....	....	....	0.015625	33	16½	....	....	....	0.315625
2	1	....	....	....	0.031250	34	17	....	....	....	0.531250
3	1½	....	....	....	0.046875	35	17½	....	....	....	0.546875
4	2	1	....	....	0.062500	36	18	9	....	....	0.562500
5	2½	....	....	....	0.078125	37	18½	....	....	....	0.578125
6	3	....	....	....	0.093750	38	19	....	....	....	0.593750
7	3½	....	....	....	0.109375	39	19½	....	....	....	0.609375
8	4	2	1	....	0.125000	40	20	10	5	....	0.625000
9	4½	....	....	....	0.140625	41	20½	....	....	....	0.640625
10	5	....	....	....	0.156250	42	21	....	....	....	0.656250
11	5½	....	....	....	0.171875	43	21½	....	....	....	0.671875
12	6	3	....	....	0.187500	44	22	11	....	....	0.687500
13	6½	....	....	....	0.203125	45	22½	....	....	....	0.703125
14	7	....	....	....	0.218750	46	23	....	....	....	0.718750
15	7½	....	....	....	0.234375	47	23½	....	....	....	0.734375
16	8	4	2	1	0.250000	48	24	12	6	3	0.750000
17	8½	....	....	....	0.265625	49	24½	....	....	....	0.765625
18	9	....	....	....	0.281250	50	25	....	....	....	0.781250
19	9½	....	....	....	0.296875	51	25½	....	....	....	0.796875
20	10	5	....	....	0.312500	52	26	13	....	....	0.812500
21	10½	....	....	....	0.328125	53	26½	....	....	....	0.828125
22	11	....	....	....	0.343750	54	27	....	....	....	0.843750
23	11½	....	....	....	0.359375	55	27½	....	....	....	0.859375
24	12	6	3	....	0.375000	56	28	14	7	....	0.875000
25	12½	....	....	....	0.390625	57	28½	....	....	....	0.890625
26	13	....	....	....	0.406250	58	29	....	....	....	0.906250
27	13½	....	....	....	0.421875	59	29½	....	....	....	0.921875
28	14	7	....	....	0.437500	60	30	15	....	....	0.937500
29	14½	....	....	....	0.453125	61	30½	....	....	....	0.953125
30	15	....	....	....	0.468750	62	31	....	....	....	0.968750
31	15½	....	....	....	0.484375	63	31½	....	....	....	0.984375
32	16	8	4	2	0.500000	64	32	16	8	4	1.000000

APPENDIX IV

BASIS OF DIRECT AND ALTERNATING CURRENT TRANSMISSION FORMULÆ

BASIS OF DIRECT-CURRENT FORMULÆ

Most Economical Distribution of Copper. The formula for the most economical distribution of copper is derived as follows:

The current decreases uniformly from the station to the end of the line, where a drop of V volts is to be allowed. Required to find the arrangement which will give this drop with the minimum amount of copper:

(1) Divide the line into a number of short pieces of length l.

The current in the first section from the far end $=al$, in the second section $2al$, in the third, $3al$, and so on, where $a=$amperes taken from the line per foot of length.

The volume of copper in the first section may be called y_1l, in the second y_2l, in the third y_3l, and so on, these quantities being c.m.-ft.

The resistance of the first section is $\frac{10.5}{y_1} \cdot l$, of the second $\frac{10.5}{y_2} \cdot l$, of the third $\frac{10.5}{y_3} \cdot l$, etc., where 10.5 is the ohms per mil-foot.

The drop in the first section is $10.5al^2 \frac{1}{y_1}$, in the second $10.5al^2 \frac{2}{y_2}$, in the third, $10.5al^2 \frac{3}{y_3}$, etc.

The total copper $= l(y_1 + y_2 + y_3 +$, etc.).

Total drop $= 10.5al^2\left(\frac{1}{y_1} + \frac{2}{y_2} + \frac{3}{y_3} +, \text{etc.}\right)$.

It is required to make $l(y_1 + y_2 + y_3 +$, etc.) a minimum subject to the condition that $\left(\frac{1}{y_1} + \frac{2}{y_2} + \frac{3}{y_3} +, \text{etc.}\right)$ shall be a constant.

Multiply the latter series by a constant P having the dimensions of a length to the fourth power and add the two series. The following one is obtained:

$$\left(\frac{P}{y_1} + y_1\right) + \left(\frac{2P}{y_2} + y_2\right) + \left(\frac{3P}{y_3} + y_3\right) +, \text{etc.}$$

The series $l(y_1 + y_2 + y_3 +$, etc.), is a minimum when the above series is a minimum. This occurs when the differential coefficient of each term with regard to its y is zero. Hence differentiating and equating to zero,

$$\left(1-\frac{P}{y_1^2}\right),\quad \left(1-\frac{2P}{y_2^2}\right),\quad \text{etc.}=0.$$

$$\therefore\quad \frac{nP}{y_n^2}=1.$$

Now, nl is x, the distance from the far end, and l is a constant.

Hence, $y=\sqrt{\frac{P}{l}}\sqrt{x}$.

The value of the constant $\sqrt{\frac{P}{l}}$ must be found. The drop in dx, which may be called dv, equals current at distance x from the end multiplied by

$$\frac{10.5\cdot dx}{\text{c.m. at that distance}}.$$

Hence,

$$dv=ax\cdot\frac{10.5}{y}dx=10.5a\sqrt{\frac{l}{P}}\frac{x}{\sqrt{x}}\cdot dx=10.5a\sqrt{\frac{l}{P}}\sqrt{x}\cdot dx,$$

$$\therefore\quad V=10.5a\sqrt{\frac{l}{P}}\int_0^L\sqrt{x}\cdot dx=10.5a\sqrt{\frac{l}{P}}\frac{2}{3}\cdot L^{\frac{3}{2}},$$

$$\therefore\quad \sqrt{\frac{P}{l}}=\frac{2}{3}\times 10.5\frac{A}{V}\sqrt{L},$$

$$\therefore\quad y=\frac{2}{3}\times 10.5\frac{A}{V}\sqrt{L}\cdot\sqrt{x}.$$

This equation gives a parabola of circular mils that represents the most economical distribution of copper with uniform drain of current.

This deduction is based upon a modification of the "Method of Undetermined Multipliers" given in Chapter XI of Williamson's "Differential Calculus."

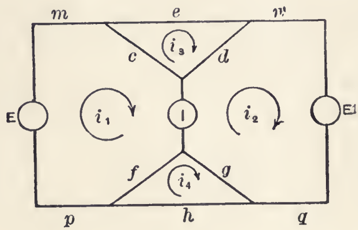


FIG. 69.

Resistance with Infrequent Cross-Bonding. Referring to Fig. 69, the following additional notation is used:

R = equivalent resistance of load;

$A = m + c + f + p;$

$F = n + d + g + q;$

$B = c + d + e;$

$D = f + g + h.$

Using Maxwell's method of imaginary currents the following four equations are obtained:

$$
\begin{array}{llllll}
Ai_1 & & -ci_3 & -fi_4 & +IR_1 & = E \\
 & Fi_2 & -di_3 & -gi_4 & -IR & = -E \\
-ci_1 & -di_2 & +Bi_3 & & & = 0 \\
-fi_1 & -gi_2 & & +Di_4 & & = 0 \\
i_1 & -i_2 & & & & = I
\end{array}
$$

Then, using determinants

$$R = \frac{\begin{vmatrix} A & 0 & -c & -f & E \\ 0 & F & -d & -g & -E \\ -c & -d & B & 0 & 0 \\ -f & -g & 0 & D & 0 \\ 1 & -1 & 0 & 0 & I \end{vmatrix}}{\begin{vmatrix} A & 0 & -c & -f & I \\ 0 & F & -d & -g & -I \\ -c & -d & B & 0 & 0 \\ -f & -g & 0 & D & 0 \\ 1 & -1 & 0 & 0 & 0 \end{vmatrix}}$$

R having been obtained by solving the above determinant, is used in the following formula, in which x is the resistance from the load to the two stations in multiple:

$$x = \frac{E}{I} - R.$$

As the expression for R contains $\frac{E}{I}$, this quantity cancels out, leaving

$$x = \frac{AF - AP - FQ - 2\left(\frac{fg}{D} \cdot \frac{cd}{B}\right) + \frac{f^2d^2 + c^2g^2}{DB}}{A + F - P - Q - 2\left(\frac{fg}{D} + \frac{cd}{B}\right)}.$$

By making $f=0$, $g=0$, and $h=\infty$,

$$x=\frac{AF-\frac{1}{B}(Ad^2+Fc^2)}{A+F-\frac{1}{B}(d+c)^2},$$

which is the formula given on p. 121 with somewhat different notation.

The formulæ for x and R given on p. 122 are obtained by differentiating R with respect to x, equating to zero and rearranging the terms.

BASIS OF ALTERNATING-CURRENT FORMULÆ

With the exception of the problem of determining the size of wire to use for a given pressure drop, the solution in each case is given directly by means of a comparatively simple formula; in the particular case of determining the size of a wire for a given drop, an approximation is first obtained and then the error involved in the approximation determined, which error, however, will be found negligable in most practical cases. Moreover, the use of this particular approximate formula, followed by a determination of the error involved, has a distinct advantage, since a large error immediately indicates that the drop for any size of wire within a wide range will differ only slightly from the permissible drop given, and that therefore, either by allowing a slight increase in the drop, or, if this is not feasible, by employing two separate circuits instead of one, a very considerable saving in copper can be effected.

The formulæ given are all readily derived from the usual diagram of two impedances in series, namely, the impedance of the load, and the impedance of the line, remembering that the ratio of power lost to power delivered is equal to the ratio of line resistance to load resistance, and that the ratio of the pressure at the generating end to the pressure delivered is equal to the ratio of the total impedance to the load impedance.

The reactance tables are based upon the fact that the reactance of a wire for a given frequency can be considered as the sum of two quantities, one varying only with the spacing of the wires and the other only with the size. The resistances have been calculated for copper of 98% conductivity and for aluminum of 62% conductivity (Matthiessen's standard, i.e., one meter-gram of soft-drawn copper = 0.141729 international ohm at 0° C.), both at 20° C., plus an increase of 1% on account of stranding, temperature coefficient 0.42% per degree C. The weights given are the weights of solid wire of equal cross-section increased 1% on account of stranding.

BASIS OF FORMULÆ FOR TRANSMISSION LINE WITH RESISTANCE, REACTANCE, LEAKAGE, AND CAPACITY

(H. Pender.)

Let i = instantaneous value of current at time t;
V = instantaneous value of difference of potentials between wire and neutral at time t;
l = distance from load to the point where current and voltage are being considered;
C = capacity of each line wire to neutral;
L = inductance of each line wire;
g = leakage susceptance per line wire;
r = resistance per line wire.

The formulæ are derived from the following differential equations:

$$\frac{di}{dl} = gV + C\frac{dV}{dt},$$

$$\frac{dV}{dl} = ri + L\frac{di}{dt},$$

from which can be derived by differentiation the following differential equations of the second order:

$$\frac{d^2i}{dl^2} = gri + (Cr + Lg)\frac{di}{dt} + CL\frac{d^2i}{dt^2},$$

$$\frac{d^2V}{dl^2} = grV + (Cr + Lg)\frac{dV}{dt} + CL\frac{d^2V}{dt^2}.$$

These equations are of the form

$$\frac{d^2y}{dl^2} = Ax + G\frac{dx}{dt} + H\frac{d^2x}{dt^2},$$

and are satisfied by the integral relation

$$y = Ae^{kx} \cos (\omega t + hx - \alpha).$$

The various constants can be found by substituting this integral in the differential equations.

APPENDIX V

BASIS OF FORMULÆ FOR STRESSES IN SPANS

THE approximate equations for a wire suspended between two points are

$$D = \frac{3\rho ml^2}{2T} = \frac{3mlK}{2},$$

$$L = l\left[1 + \frac{8}{3}\left(\frac{D}{l}\right)^2\right] = l[1 + 6m^2K^2],$$

where D = deflection of wire at center of span in feet in the direction of the resultant force at temperature t;

L = length of wire at temperature t under tension T;

ρ = ratio of the resultant of weight of wire and sleet and the wind pressure to weight of wire;

m = weight of conductor per cubic inch;

l = length of span, feet;

$$K = \frac{\rho l}{T}.$$

Letters with subscript zero refer to corresponding quantities at temperature t_0 and tension T_0. Hence,

$$L_0 = l\,[1 + 6m^2K_0^2].$$

The relation between the length L of wire at temperature t under tension T to length L' at zero temperature and unstressed, is given by the equation,

$$L = L'\left(1 + \frac{T}{M}\right)(1 + \alpha t).$$

Similarly,

$$L_0 = L'\left(1 + \frac{T_0}{M}\right)(1 + \alpha t_0),$$

where α is the coefficient of expansion per degree.

Combining the last four equations and neglecting cross products of the term $6m^2K^2$, $\frac{T}{M}$, and αt, since these quantities are of the order of 10^{-3} or less in any practical case, we get the following expression,

$$t - t_0 = \frac{1}{\alpha}\left[6m^2(K^2 - K_0^2) + \frac{1}{M}(T_0 - T)\right].$$

The graphical method is based upon the above formulæ, the equations of the curves being given in Chapter IV.

APPENDIX VI

EXPLANATION OF SPECIFICATIONS

1. CABLES FOR AERIAL LINES

SOLID conductors are only used for the smaller sizes, say up to No. 0 B. and S., seven strands being used up to 250,000 c.m. and a larger number for sizes above that.

The total effective area of copper is that of the sum of the individual wires laid out straight and measured at right angles to their axes, because the current follows the spiral of the cable without appreciably passing from one strand to another.

The pitch is important on account of its effect upon the tensile strength of the cable (see p. 17).

Pounds per square inch at the elastic limit divided by the elongation expressed as a decimal fraction gives the modulus of elasticity.

The object of the " flexibility " test is to assure the possibility of making Western Union joints with solid conductors and to assure the absence of undue stresses in strands. Theoretically, the wrapping test should be performed at the lowest temperature to which the wire will be exposed in practice, but the lowest temperature conveniently attainable is 32° F., which is accordingly specified.

The permissible excess of area is limited in order to prevent the manufacturer obtaining the specified conductivity and strength by using more metal. This is often done where, as is usually the case, the cable is sold by the pound, and should be avoided, not only on account of the extra expense, but also on account of the decreased strength of the wire per square inch.

2. INSULATED CABLE

General. It is advisable to state the conditions under which the cable is to be used in order that the manufacturer may run no chance of misunderstanding any part of the specifications, thereby producing a cable unsuitable for the purpose for which it is intended to be used. Furthermore, it gives the manufacturer an opportunity to judge which, of several products fulfilling the specification, is best suited to the conditions.

Form of Cable. Soft-drawn copper is almost universally used for insulated conductors in preference to the hard-drawn product, on account of its comparative cheapness and its superior flexibility and conductivity. Hard-drawn copper is, however, used for special work, such as long spans of insulated wire.

Solid wire may be used where flexibility is of little importance but for larger sizes than No. 10 B. and S. stranded conductors are desirable if they have to be drawn into conduits. Conductors of 2,000,000 c.m. area and over are inconveniently stiff even in the form of concentric cables and are therefore often rope-laid.

Two conductor cables of oval form contain less lead and filling than round ones, and are therefore preferred on account of their cheapness.

The lateral fillings not only serve the purpose of making the cable mechanically solid, but also to prevent static discharges between the insulation and the lead; such discharges arising from the steep potential gradient in the air spaces due to the low specific inductive capacity of air compared with that of the insulating compound.

Multiple conductor cables being generally composed of small wires furnished with sufficient insulation for their individual mechanical protection, require some further protection on account of their greater size and consequent liability of injury in handling. For this reason a covering of tarred rope is advised.

The object of one conductor differently colored from the others is to facilitate the identification of wires at the opposite ends, care being taken in splicing to first join the ends of the marked wires, and then join the others in their natural order.

A final insulating belt over the rope serves principally to hold the wires and ropes together and to give a smooth surface to the lead or braid covering. This is very important with lead, as a projection on the inner surface of the sheath greatly reduces the dielectric strength of the cable.

Conductors. While soft drawn copper of over 100% Matthiessen's standard is obtainable, the manufacturers have difficulty in producing it steadily, and therefore charge an abnormal price for it; 98% conductivity is about the best commercially obtainable.

Rubber insulation, owing to its sulphur, attacks copper, which must therefore be protected by a coating of metal not affected by sulphur. Varnished cambric also affects copper when certain chemicals are used in the preparation of the oils, and therefore requires a separator like rubber. Either tin or unvulcanized rubber containing no sulphur is used for this purpose.

In stranded conductors the major part of the current follows the spirals of the strands. The increase of copper area due to spiralling, therefore, has no effect in reducing the resistance, and the effective area of copper is the combined area of the strands when laid out straight and measured at right angles to their axes.

Insulation. Many engineers leave the thickness of insulation to be determined by the manufacturers from the specified tests. This practice has the disadvantage of permitting the various competing manufacturers to present bids based on different factors of safety with the results that all the manufacturers will use as little insulation as possible and that the lowest bidder will probably be the one who is using the lowest safety factor. If, on the other hand, the insulation thickness is specified, the manufacturer who produces a compound of higher dielectric strength than his competitors is reduced to an equality with them, and the buyer loses an opportunity of obtaining the cheapest product. This objection, however, is of little weight at the present time, as little difference exists in the dielectric strength of different makes of paper and cambric insulation, and rubber is seldom used under high dielectric stress.

Taping and Braiding. Rubber insulation cannot be properly vulcanized without a covering of tape. The majority of manufacturers vulcanize in a tape which becomes a permanent part of the insulation, but some vulcanize in a temporary tape of tin-foil or other non-adhesive material and put the permanent tape on the cold, vulcanized insulation. In either case, the tape serves as a mechanical protection by giving a hard surface to the insulation, but its principal function in lead-sheathed cables is to protect the surface of the insulation from being burned in the lead press.

Successive turns of the tape should overlap, but the overlap should be less than half the width of the tape, in order to avoid ridges where turns would be superimposed. On the other hand, the overlap should be sufficient to insure protection when the cable is bent to a sharp radius.

Braiding is simply a cheap sheathing for cables to be used in dry places or where, for any other reason, lead cannot be used. Six-lea hemp is hemp yarn having six times 300 yards to the pound, a lea of hemp being 300 yards.

Sheath. Pure lead is too soft for sheathing, but alloyed with a small quantity of tin it has excellent mechanical properties. Two per cent of tin is found to be ample for this purpose, a greater quantity having the effect of rendering the metal liable to crystallize.

Armor. Armor is used either as a substitute or as a protection for sheathing.

When used as a substitute it is usually in the form of a galvanized steel tape. It is used where cables are exposed to vibration which would crystallize the sheath metal.

Armor is used as a protection for sheathing on submarine cables, and on cables intended to be laid in the ground without ducts. For these purposes galvanized wire is preferable to steel tape owing to the possibility of putting on a greater thickness without making the cable too stiff.

Tests. Cable should be immersed for a sufficient time to enable the water to penetrate anywhere it could penetrate after the cable is installed. In the case of rubber or varnished cambric insulation this requires from twelve to twenty-four hours, but a very short period is sufficient for paper insulation as it is very hygroscopic.

The conditions prescribed for the megohms test constitute a convenient standard, which is universally accepted.

Capacity Guarantee. Cables of high electrostatic capacity should be avoided for high tension work on account of the large charging current they take. The proposals should therefore be scanned with the view of eliminating cables of undesirable capacity.

It is seldom necessary to initially specify the capacity, as the standard products of the manufacturers are satisfactory in that respect.

Installation. The responsibility for correct cable lengths should be placed on the contractor whenever possible, in order to avoid troubles arising from errors in measurement. Lengths should never be estimated from subway plans, as splicing chambers can seldom be built exactly according to plan.

It is advisable to specify the compound to be used in the sleeves in order to avoid the use of more than one kind of compound, plurality of compounds giving rise to trouble in maintenance and repair work.

3. THIRTY PER CENT PARA RUBBER COMPOUND

Description of Insulation. The object of specifying that not more than 33% of rubber, is to be assured that only Para rubber is used. If an inferior grade of rubber is used the compound will have to contain more than 33% rubber to meet the test requirements. As the permanence of these inferior grades is doubtful their use should be guarded against. Furthermore, in the presence of low grade rubber, it is practically impossible to determine how much high grade rubber is in the compound.

The small amount of extract in the gum is the essential quality which differentiates the finest dry Para rubber from other kinds. The small amount of volatile extract specified for the complete compound is to assure the absence of an excess of volatile matter which would evaporate and leave the insulation dry and also to prevent the over-mastication of rubber during manufacture.

The amount of sulphur is limited in order to protect the conductors from corrosion.

Tests. There is some question about the proper electrical properties which rubber insulation should possess. From the operating standpoint a very low insulation resistance should suffice, but it appears that a high insulation resistance is some indication of sound homogeneous structure. High insulation resistance may be secured, however, by artificial means, such as by the use of paraffine wax, and is therefore not a reliable indication of quality.

High dielectric strength is very desirable but it is often obtained at the cost of permanence, it being possible to greatly increase the dielectric strength by putting more or less volatile oils in the compound.

High insulation resistance and high dielectric strength are each strongly recommended by different manufacturers, but their reasons for doing so are more commercial than technical.

The remarks under the heading of tests in specification No. 2 apply equally to this specification. The object of making the megohms test of multiplex cables before assembling, is to have test figures which can be checked by theory, there being no way of calculating the insulation resistance of a multiplex cable. The high voltage test is made before assembling in order to eliminate faulty pieces and after assembling in order to detect faults which may have arisen during assembling.

The temperature coefficient of insulation resistance is specified for two reasons: first, in order to prevent the manufacturer using a coefficient which will make any test results agree with the specifications; and second, because it has been found that compounds of high temperature coefficient (i.e., over 3% per deg. F.) generally do not contain 30% Para rubber.

The stretch tests are somewhat arbitrary, being founded partly upon manufacturers' recommendations and partly upon experience with various grades of rubber. While many excellent compounds entirely fail to meet this test, it cannot be questioned that, combined with the restriction in the quantity of rubber, it practically bars objectionable compounds.

The paragraph containing temperature limits is intended to prevent the heating and stretching of rubber prior to tests, a little judicious handling often having the effect of making a doubtful sample pass.

4. RUBBER-COVERED WIRE ENGINEERS' ASSOCIATION SPECIFICATIONS FOR THIRTY PER CENT RUBBER COMPOUND

This specification is a compromise agreed upon by the principal manufacturers, but while doubtless prepared in good faith, the number of different compounds which it is intended to cover is so great that it will practically pass anything. In other words, this specification contains no requirement which cannot be met by all the manufacturers, and this comprehensiveness is obtained at the sacrifice of that severity which makes a specification really useful.

5. and 6. VARNISHED CAMBRIC AND PAPER INSULATION

These specifications need little explanation beyond the statement that cambric and paper being staple articles of manufacture of undoubted permanence and excellent electrical qualities, they need no further specification than a general description. The insulation resistance may be left to the manufacturer, provided that it is sufficiently high for successful operation, but the voltage test should be severe.

APPENDIX VII

BASIS OF TABLES GIVING SELF-INDUCTION OF PARALLEL WIRES

It is surprising to note the errors made by technical writers in their attempts to express the inductance of a pair of parallel wires, especially since a very simple and accurate formula has been available in most of the standard mathematical treatises on electricity from J. Clerk-Maxwell to Alex. Russel.

The inductance of a circuit is a measure of the magnetic energy associated with the current in it and is defined by the well known equation

$$E = Li^2,$$

where E is the energy in the magnetic field interlinked with a circuit of inductance L, carrying an unvarying current i.

This definition gives rise to the following equation:

$$L = \left[.5 + 2 \log_e \frac{d}{r} \right] 10^{-6},$$

where d = distance apart of wires, center to center;
r = radius of wires in same unit;
L = inductance of each wire in millihenrys.

The formulæ given in Chapter XII are based upon the above equation.

In the case of a circuit composed of two parallel wires the size of which is negligible in comparison with their distance apart, the inductance is approximately equal to the total flux embraced by the circuit due to the unit current therein.

This definition, although based upon an approximation, is often assumed to be exact and used as the basis of various self-induction formulæ.

The flux around a wire is plotted from the well-known equations

$$B=\frac{2i}{r} \text{ outside the wire,}$$

and

$$B=\frac{2ir}{R^2} \text{ inside the wire,}$$

where B is the flux density, lines per sq.cm. at distance; r cms. from the center of a long straight wire of radius R cms. carrying a current of i absolute units.

When two conductors carrying currents in opposite directions are brought into proximity, the magnetic whirls around the conductors are squeezed together and the axes of the two whirls are pushed away from the axes of the conductors.

If the integration is taken between the centers of wires, a formula containing the term $\log \frac{d-r}{r}$ instead of $\log \frac{d}{r}$ will be obtained; if taken between the axes of the whirls a very long and complicated formula is obtained.

One of these incorrect formulæ is often given in text-books as exact, and the exact formula derived from it as an approximation, the authors of these books neglecting the fact that their original definition involved an approximation.

It should be noted that where only a part of a circuit is involved, there may be some magnetic energy interlinked with it, but originated by the current in some other part of the circuit. Such extraneous magnetism adds to the "flux due to unit current," but not to the magnetic energy associated with the current in that part of the circuit under consideration.

The tables given in Chapter XII are based upon the fact that the equation for inductance may be resolved into a sum of two quantities, one of which depends upon the size of wire and the other upon the distance apart of the wires, a simple fact first utilized by H. Pender and published in Foster's "Electrical Handbook."

The fundamental formula given above may be resolved into the various forms given below.

Let d = distance apart of wires, center to center;
r = radius of wires in same unit;
L = self-induction of each wire in millihenrys, or thousandths of a henry.

The logarithms are common, i.e., to the base 10.

$$L \text{ per cm.} = .000{,}000{,}5 + .000{,}004{,}605 \log \frac{d}{r}.$$

$$L \text{ per in.} = .000{,}001{,}27 + .000{,}011{,}68 \log \frac{d}{r}.$$

$$L \text{ per ft.} = .000{,}015{,}24 + .000{,}140{,}3 \log \frac{d}{r}.$$

$$L \text{ per 1000 ft.} = .015{,}24 + .140{,}3 \log \frac{d}{r}.$$

$$L \text{ per mile} = .080{,}47 + .74111 \log \frac{d}{r}.$$

$$L \text{ per kilometer} = .05 + .460{,}5 \log \frac{d}{r}.$$

For magnetic wires the first constant in each of the above formulæ should be multiplied by permeability of the wire. An average value of the permeability for high grade iron telegraph wire is 150, which value has been used in the formulæ given on p. 275.

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