The
ELEMENTARY
Arithmetic,
ORAL AND WRITTEN.

BY ALBERT N. RAUB A. M.
PRINCIPAL OF THE   PENNSYLVANIA
STATE NORMAL SCHOOL.

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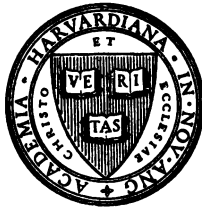
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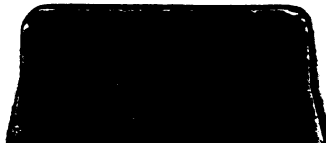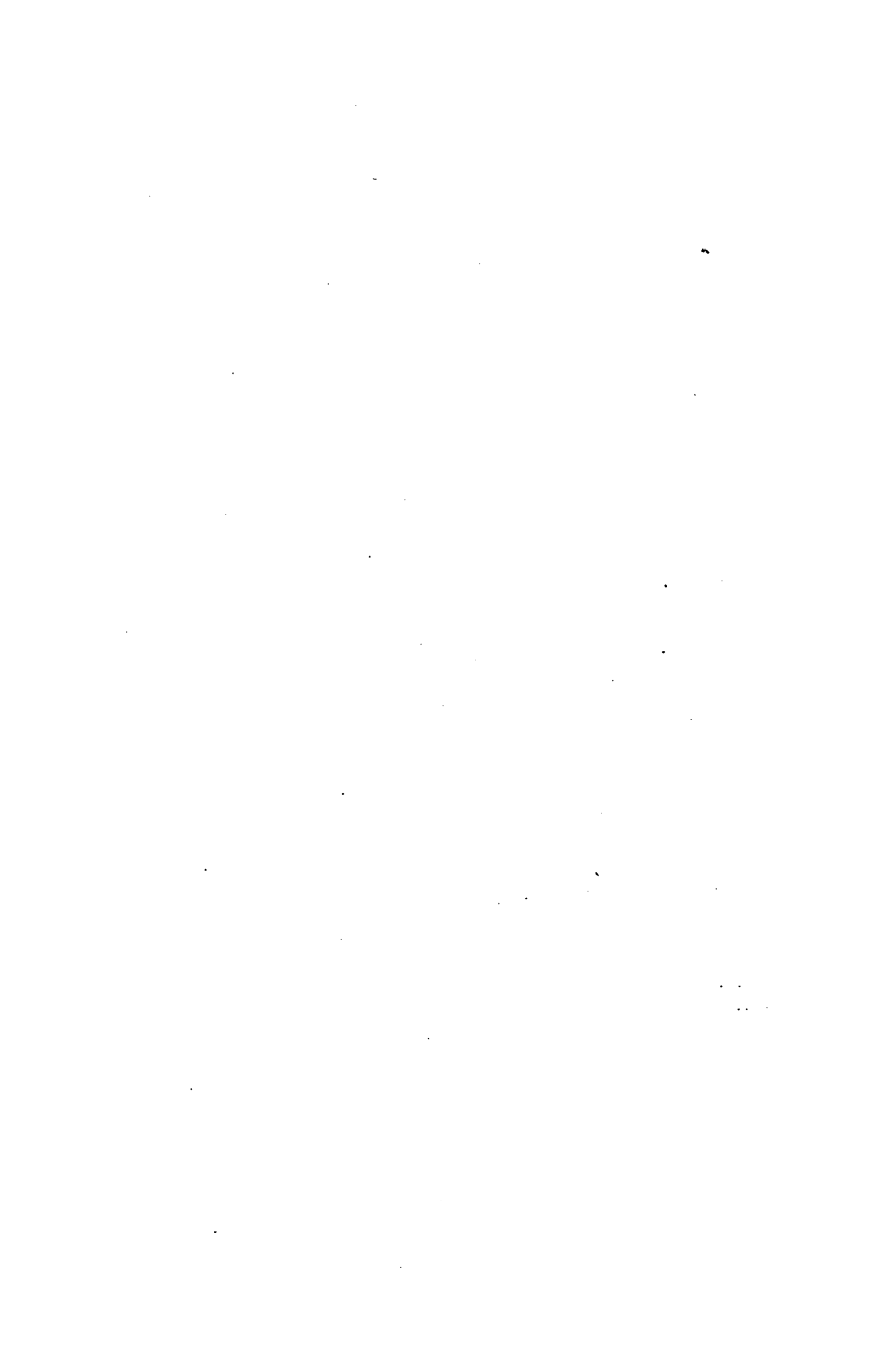
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PREFACE.

BELIEVING that Oral and Written Arithmetic form properly but a single study, and that they are best taught together, the author has in this, as in his Complete Arithmetic, combined both oral and written exercises in the same book. He claims that by this arrangement much valuable time may be saved to both teacher and pupil, and the science of Arithmetic be even better taught than by the plan of teaching the two from separate books.

In the composition of this book two main objects have been steadily kept in view: first, that of teaching the elements of Arithmetic thoroughly; and, secondly, that of training the pupil to think for himself and apply his knowledge of the science as he acquires it to the actual business operations of life. To accomplish the first of these objects, the author has adopted a natural and systematic gradation, not only of subjects, but also the divisions of these subjects and the various operations under these divisions. To this end also the problems have been systematically graded throughout the book.

To accomplish the second object, simple, concise solutions have been given as a basis where it was thought they could be of advantage; but in many cases the pupil is left to depend upon his own knowledge, gained from his previous study of the book, to frame his own solutions. He is required to give his explanation of many of the written solutions of the problems as given by the author.

It is believed that this will, under the guidance of a ju-
dicious teacher, give him valuable training, not only in
reasoning, but also in the use of language.   He is re-
quired also, in many cases, to derive his own rules from
the preceding solutions and principles.

In order that he may understand the practical appli-
cation of Arithmetic, and at the same time acquire a
knowledge of the business use of the science as he pro-
ceeds, the problems given are largely, almost wholly,
drawn from the actual business operations of life.

The fact that a large number of pupils never study
Arithmetic for any great length of time, and the neces-
sity of their studying that only which will be of practical
utility to them in after life, has been kept constantly in
view in adapting the book to their wants.

Extended exercises are given in the application of the
fundamental rules, because these are the basis of all the
operations of Arithmetic.

The definitions given are brief and accurate, and it is
thought they are all clothed in such language as the
pupils can readily comprehend.

Every effort has been put forth to make the book a
desirable one, not only for the pupil in learning the ele-
ments of Arithmetic, but also for the teacher in present-
ing the principles to the mind of the learner.   It is
believed, and confidently hoped, that progressive teachers
everywhere will find it well adapted to their wants.

<div align="right">A. N. R.</div>

CENTRAL STATE NORMAL SCHOOL,
  LOCK HAVEN, PA., July 19, 1877.

SUGGESTIONS TO TEACHERS.

THE author neither desires nor aims to give here a detailed method of teaching the whole subject of Arithmetic, but rather to suggest a few hints which he hopes may be of practical service to those who use the book.

OBJECTS.—Since children acquire knowledge most readily through their perceptive faculties, the teacher should illustrate largely by the use of objects. This means of illustration should be used from the beginning, for the reason that the child thinks concretely—that is, in connection with objects. The idea of ten ones, or a *ten*, can be best taught by putting together ten objects and calling the collection a *ten*; in a similar manner *ten* tens, forming a *hundred*, and so on. So also the idea of a fraction and of fractional parts can be taught best in connection with objects. Objects themselves are much superior in this respect to pictures.

In the selection of objects care should be taken, especially in teaching the idea of a fraction, that such objects be used as cannot be divided without destroying their unity. Thus, an apple is much better for illustration than a stick or a line; for if an apple be divided into any number of equal parts, none of these parts are units; but a stick may be divided into other sticks, or a line into other lines, each of which is still a unit.

NUMERAL FRAME.—Teachers will find the numeral frame a great aid in teaching all the fundamental rules. It has the advantage of being compact and convenient, and is always at hand. It is among the most serviceable articles of school furniture.

1*

COUNTING.—Give much attention to the exercises in counting following each of the tables. It is believed that if these exercises are judiciously used, pupils will perform the operations of the fundamental rules with much greater rapidity and exactness. Don't continue these exercises, however, till they become tedious or monotonous.

TESTS.—Whenever possible give your pupils experience in the actual application of measurements. Thus, let them in the absence of a foot-rule or a yardstick mark a lath into feet and inches, and by the use of this determine the height and length of their desks, the blackboard, the recitation seats, the doors, etc., and of the schoolroom itself. If no set of weights can be secured, let paper or muslin bags filled with sand or corn, representing the ounce, the pound, the quarter-pound, etc., be used. The pupil is thus more readily taught, because he gains his knowledge largely by observation, and the judgment is trained and cultivated while the knowledge is being acquired.

SOLUTIONS.—The solutions given in this book are simple and concise. Teachers are not expected, however, to confine themselves strictly to those given by the author. Many problems will admit of several solutions. It will be well, therefore, for the teacher to encourage the pupils to give solutions of their own. He should be careful to see, however, that no unnecessary words are used, and that the solutions are given in good language. This will train the pupils to think and reason for themselves. In the written work everything should be done neatly, whether on the slate or on the blackboard. Pupils should be taught how to make figures. They frequently do not make neat figures, because they have never been taught.

THOROUGHNESS.—Do not hurry. Let your pupils mas-

ter everything thoroughly as they pass over it. It is believed that the exercises in this book are sufficiently extended and well graded to enable the learner to master the elements of the different subjects presented. But should the teacher deem it necessary to have more extended exercises, both teacher and pupils will find it advantageous often to originate problems for themselves. The teacher should be sure that the pupil understands the principles as he proceeds. It is too often the case that he is hurried along from one subject to another without having a clear comprehension of each, and therefore spends years in doing the work of as many months.

HELP.—Let the teacher give no help where the pupil can overcome any difficulty by his own efforts. Every obstacle the learner surmounts unaided, and every difficulty he overcomes, makes him stronger and better prepared to contend successfully with the next. If help is needed, let it come indirectly in the shape of hints or suggestions. A single suggestion will often put a pupil on the right track or start a train of thought which will lead to success, and the knowledge thus gained without actual aid will be doubly valuable to him.

ORAL ARITHMETIC.—The exercises in Oral Arithmetic are not designed to be comprehensive. In many cases the teacher will find it advantageous to have his pupils solve some of the written problems orally. A portion of the class may be solving their problems on the blackboard while another part of the class may be solving by the oral process. Thus much valuable time may be economized, and the solutions and principles, being discussed by both methods in the same recitation, will be more fully comprehended and more definitely understood.

CONTENTS.

ELEMENTARY ARITHMETIC.

CHAPTER I.

INTEGERS.

SECTION I.

NOTATION AND NUMERATION.

1. IN writing numbers, ten figures are used. They are—

$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$$

called *naught, one, two, three, four, five, six, seven, eight, nine.*

Naught, 0, is sometimes called *zero*, or a *cipher*. The other nine figures are sometimes called *digits*.

In writing numbers greater than nine, two or more of these figures are taken together.

Repeating or naming the numbers in order is called **Counting** ; as, *one, two, three, four, five*, etc.

NUMBERS TO 100.

One	1	Fourteen	14	Twenty-seven	27
Two	2	Fifteen	15	Twenty-eight	28
Three	3	Sixteen	16	Twenty-nine	29
Four	4	Seventeen	17	Thirty	30
Five	5	Eighteen	18	Thirty-one	31
Six	6	Nineteen	19	Thirty-two	32
Seven	7	Twenty	20	Thirty-three	33
Eight	8	Twenty-one	21	Thirty-four	34
Nine	9	Twenty-two	22	Thirty-five	35
Ten	10	Twenty-three	23	Thirty-six	36
Eleven	11	Twenty-four	24	Thirty-seven	37
Twelve	12	Twenty-five	25	Thirty-eight	38
Thirteen	13	Twenty-six	26	Thirty-nine	39

Forty 40
Forty-one 41
Forty-two 42
Forty-three 43
Forty-four 44
Forty-five 45
Forty-six 46
Forty-seven 47
Forty-eight 48
Forty-nine 49
Fifty 50
Fifty-one 51
Fifty-two 52
Fifty-three 53
Fifty-four 54
Fifty-five 55
Fifty-six 56
Fifty-seven 57
Fifty-eight 58
Fifty-nine 59
Sixty 60

Sixty-one 61
Sixty-two 62
Sixty-three 63
Sixty-four 64
Sixty-five 65
Sixty-six 66
Sixty-seven 67
Sixty-eight 68
Sixty-nine 69
Seventy 70
Seventy-one 71
Seventy-two 72
Seventy-three 73
Seventy-four 74
Seventy-five 75
Seventy-six 76
Seventy-seven 77
Seventy-eight 78
Seventy-nine 79
Eighty............. 80

Eighty-one 81
Eighty-two 82
Eighty-three...... 83
Eighty-four 84
Eighty-five 85
Eighty-six 86
Eighty-seven 87
Eighty-eight 88
Eighty-nine 89
Ninety............ 90
Ninety-one....... 91
Ninety-two....... 92
Ninety-three..... 93
Ninety-four...... 94
Ninety-five....... 95
Ninety-six....... 96
Ninety-seven 97
Ninety-eight 98
Ninety-nine 99
One hundred..... 100

EXERCISE IN COUNTING.

1. Count from 1 to 10.
2. Count from 10 to 30.
3. Count from 30 to 60.

4. Count from 60 to 100.
5. Count from 1 to 99.
6. Count from 99 to 1.

With what figure does 17 begin? 18? 35? 39? 47? 46? 43? All the forties? All the sixties? All the eighties?

2. When we write numbers, *ten* ones are called a *ten*, which is written............................ 10

Two tens, or *twenty*, is written................. 20

Three tens, or *thirty*, "   "   30

Four tens, or *forty*,   "   "   40

Five tens, or *fifty*,   "   "   50

Six tens, or *sixty*,   "   "   60

Seven tens, or *seventy*, "   "   70

Eight tens, or *eighty*, "   "   80

Nine tens, or *ninety*, "   "   90

One ten and one is called *eleven*, and is written... 11
One ten and two, or *twelve*,   "   "   ... 12
One ten and three, or *thirteen*,   "   "   ... 13
One ten and four, or *fourteen*,   "   "   ... 14
One ten and five, or *fifteen*,   "   "   ... 15
One ten and six, or *sixteen*,   "   "   ... 16
One ten and seven, or *seventeen*,   "   "   ... 17
One ten and eight, or *eighteen*,   "   "   ... 18
One ten and nine, or *nineteen*,   "   "   ... 19
Two tens and one, or *twenty-one*,   "   "   ... 21
Two tens and two, or *twenty-two*,   "   "   ... 22
Three tens and four, or *thirty-four*,   "   "   ... 34
Five tens and seven, or *fifty-seven*,   "   "   ... 57
Six tens and two, or *sixty-two*,   "   "   ... 62
Eight tens and five, or *eighty-five*,   "   "   ... 85
Nine tens and nine, or *ninety-nine*,   "   "   ... 99

EXERCISE.

1. Write in figures thirteen, nineteen, fourteen, twenty-four, thirty-six, forty-three, fifty-two, sixty-one, seventy-seven, eighty-five, ninety-eight.

2. Write in words 12, 18, 27, 32, 86, 72, 64, 92, 26, 75.

3. Write in figures seventeen, twenty-nine, thirty, forty-eight, sixty-five, eighty-one, forty-five, seventy-eight, eighty-three.

4. Write in words 39, 96, 62, 74, 87, 34, 72, 27, 98.

5. Write in figures thirty-one, twenty-three, fifty-eight, sixty, seventy-nine, forty-four, thirty-three.

6. Write in words 22, 54, 27, 99, 23, 26, 68, 98, 77.

3. Ten *tens* taken together are called a *hundred*.

One hundred is written........................ 100
Two hundred   "   "   200
Three hundred "   "   300
Four hundred "   "   400
Five hundred "   "   500
Six hundred   "   "   600
Seven hundred "   "   700
Eight hundred "   "   800
Nine hundred "   "   900

4. When numbers are expressed by figures, the first place on the right is called *ones* or *units;* the second place, *tens;* the third place, *hundreds.* Thus, 326 is three hundreds, two tens and six ones, and is read three hundred and twenty-six.

EXERCISE.

1. Write in figures one hundred and twenty-five, two hundred and seventy-three, three hundred and seventy-two, four hundred and twenty-six, eight hundred and ninety-three, three hundred and ninety-eight, nine hundred and eighty-three.

2. Write in words 126, 472, 264, 746, 827, 722, 468.

3. Write in figures eight hundred and twenty-one, three hundred and forty-three, nine hundred and seventy-four, two hundred and seventy-six, three hundred and forty-five, five hundred and forty-three, four hundred and thirty-five.

4. Write in words 263, 348, 482, 576, 647, 446, 644.

5. Write in figures five hundred and forty-one, one hundred and fourteen, one hundred and nineteen, two hundred and one, seven hundred and twenty-three, eight hundred and four, six hundred and five.

6. Write in words 468, 144, 987, 207, 763, 875, 445.

7. Write in figures four hundred and fifty-two, eight hundred and thirteen, nine hundred and eleven, seven hundred and ninety-eight, two hundred and twenty-one, eight hundred and eighty-seven, four hundred and sixty.

8. Write in words 567, 348, 843, 654, 388, 777.

9. Write in figures seven hundred and twenty-six, three hundred and ninety-nine, eight hundred and seventy-two, three hundred and eighty-seven, six hundred and twenty-three, seven hundred and eighty-nine, seven hundred and sixty-four.

10. Write in words 111, 406, 460, 723, 327, 273, 732.

5. Ten *hundreds* taken together are called a *thousand.*

One thousand is written............................	1000
Twenty hundreds, or *two thousand,* is written...	2000
Thirty hundreds, or *three thousand,* " " ...	3000
Forty hundreds, or *four thousand,* " " ...	4000
Fifty hundreds, or *five thousand,* " " ...	5000
Sixty hundreds, or *six thousand,* " " ...	6000
Seventy hundreds, or *seven thousand,* " " ...	7000
Eighty hundreds, or *eight thousand,* " " ...	8000
Ninety hundreds, or *nine thousand,* " " ...	9000

6. When numbers of more than three figures are written, the fourth place to the right is called *thousands.* Thus, 6723 is read six thousand seven hundred and twenty-three.

7. The fifth place to the right is called *ten-thousands.* Thus, 86423 is 8 ten-thousands, 6 thousands, 4 hundreds, 2 tens and 3 ones, or eighty-six thousand four hundred and twenty-three.

8. The sixth place to the right is called *hundred-thousands.* Thus, 768357 is 7 hundred-thousands, 6 ten-thousands, 8 thousands, 3 hundreds, 5 tens and 7 ones, or seven

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hundred and sixty-eight thousand three hundred and fifty-seven.

9. In numbers of more than three figures, every three figures counting from the right are called a *Period.* These periods are separated from each other by commas.

10. The first period at the right consists of ones, tens and hundreds. The second consists of thousands, ten-thousands and hundred-thousands. Thus, in 845,675, the first right-hand period consists of 5 ones, 7 tens and 6 hundreds; and the second period, of 5 thousands, 4 ten-thousands and 8 hundred-thousands.

11. The first right-hand period is also called the *units'* period, and the second to the right the *thousands'* period. Thus, 643,876 is 643 thousands and 876 units or ones. It is read six hundred and forty-three thousand eight hundred and seventy-six.

12. If there are no figures for any of the places, naughts must be supplied. Thus, write—

Eighteen thousand........ 18,000
Six hundred and three thousand.............. 603,000
Four hundred and eighty thousand........... 480,000
Seven thousand five hundred.................... 7,500
Twenty thousand nine hundred 20,900
Sixteen thousand seven hundred and eight.. 16,708
Five hundred and seven thousand two hun-
    dred 507,200
Five hundred and seven thousand two hun-
    dred and six.................................... 507,206
Sixty thousand and thirteen..................... 60,013
One hundred and seventy-three thousand
    and six.......... 173,006

EXERCISE.

1. Write in figures seven hundred and eighty-four, six thousand, eight thousand, five thousand five hundred.

2. Write in figures one hundred and seventeen thousand, one hundred and seventy thousand, eight hundred and fifty thousand, one hundred and seven thousand.

3. Write in words 1193; 12,700; 97,800; 20,600.

4. Write in words 21,500; 180,000; 180,700; 60,500.

5. Write in figures fifteen thousand four hundred and sixty, twenty-seven thousand six hundred and four, twenty thousand one hundred and seventeen, ninety-five thousand one hundred and ninety-five.

6. Write in figures eighty thousand six hundred and seventy, eighty-five thousand and sixty-seven, seventy-two thousand and five, sixty thousand and seventy-six.

7. Write in words 16,750; 25,702; 11,720; 19,559.

8. Write in words 86,700; 56,708; 52,007; 70,067.

9. Write in figures one hundred and seventeen thousand six hundred and fifty, one hundred and forty thousand and forty, eight hundred and five thousand five hundred and five, six hundred and seven thousand seven hundred and seventy.

10. Write in figures five hundred and forty-three thousand two hundred and two, one hundred and ninety-six thousand six hundred and nine, five hundred thousand and sixty-seven, four hundred thousand and six.

11. Write in words 711,560; 440,010; 550,805; 770,076.

12. Write in words 647,032; 546,607; 607,006; 600,003.

MILLIONS.

13. The third period of figures is called *millions*.

14. A *million* is a thousand thousand.

15. The third period consists of millions, ten-millions and hundred-millions. Thus we write—

3 million 600 thousand and 65........	3,600,065
17 million 420 thousand 642......	17,420,642
45 million 316 thousand 718.............	45,316,718
264 million 376 thousand 945............	264,376,945
740 million 360 thousand and 80........	740,360,080
700 million 300 thousand and 6.........	700,300,006

EXERCISE.

1. Write in figures one hundred and seventy-eight thousand nine hundred and sixteen, one hundred and eighty-seven thousand five hundred and sixty.

2. Write in figures six million seven hundred and sixty-three thousand four hundred and twenty-one, eighteen million seven hundred and sixteen thousand seven hundred and fourteen.

3. Write in words 967,478 ; 875,642.

4. Write in words 7,843,617 ; 15,673,424.

5. Write in figures one hundred and eighty-seven million six hundred and forty-five thousand six hundred and six, seven hundred and forty-three million eight hundred thousand six hundred and five.

6. Write in figures six hundred million seven hundred and fifteen thousand and fifteen, eight hundred and seventy-seven million four hundred and three thousand six hundred and six.

7. Write in words 244,707,644; 873,600,707.

8. Write in words 705,604,403; 700,600,006.

9. Name the periods in order from the right.

10. Name the places in order from the right.

TABLE.

PERIODS,	Millions.			Thousands.			Units.	
	7	4	5,	6	8	3,	7	2	6

PLACES.	Hundred-millions,	Ten-millions,	Millions,	Hundred-thousands,	Ten-thousands,	Thousands,	Hundreds,	Tens,	Units.

10 units.................. are 1 ten.
10 tens .... " 1 hundred.
10 hundreds............ " 1 thousand.
10 thousands............ " 1 ten-thousand.
10 ten-thousands........ " 1 hundred-thousand.
10 hundred-thousands. " 1 million.
10 millions.............. " 1 ten-million.
10 ten-millions......... " 1 hundred-million.

16. A **Unit** is one or a single thing.

17. A **Number** is one or more units.

18. A whole number—as, 1, 3, 4, etc.—is called an **Integer.**

19. Arithmetic is the science of numbers and the art of computing by them.

20. Notation is the art of writing or expressing numbers by figures or letters.

21. Numeration is the art of reading numbers.

22. Figures have two values, *simple* and *local.*

23. The *simple* value of a figure is the value when it stands alone.

24. The *local* value of a figure is the value given it by the place it occupies.

Thus, 7, 6 and 4, when taken alone, mean 7 ones, 6 ones and 4 ones; but when combined, as in 764, the value of 7 is 7 hundreds; of 6, 6 tens; and of 4, 4 ones.

REVIEW EXERCISE.

1. Express by figures seven hundred and nine, seven thousand six hundred and nine, forty-three thousand two hundred and seventy-five, sixteen thousand and six.

2. Express by figures one hundred and fifteen thousand and fifteen, five hundred thousand and five, eighteen thousand and six, eighteen thousand and eighteen.

3. Express by figures six hundred and fourteen million seven hundred and eighty-three thousand and sixty-three, one hundred and forty-nine million six hundred thousand, eighteen million, one hundred and seventy million and five.

4. Express by figures nine hundred million and nine, seven hundred and forty-three million seven hundred and forty-three, seven hundred and forty million seven thousand four hundred and seventy-four, two hundred and four million six hundred and seventy thousand and sixty-seven.

5. Express by figures seventeen million and seventeen, nine hundred and sixty-two million nine hundred and sixty-two thousand nine hundred and sixty-two, four hundred and twenty-eight million four hundred and twenty-eight thousand, nine hundred and eight million seventy thousand six hundred and five.

6. Express by words 1,865; 6,006; 18,014; 73,642.

7. Express by words 114,000; 14,006; 128,128; 900,019.

8. Express by words 462,352; 109,109,109; 16,000,000; 160,000,000.

9. Express by words 1,600,000; 160,000; 4,004,004; 45,045,045.

10. Express by words 87,500,875; 731,640,005; 62,062,062; 405,060,708.

SECTION II.

ADDITION.

1. A boy has one right hand and one left hand: how many hands has he?

2. A horse has two front feet and two hind feet: how many feet has he?

3. A girl has two apples in one hand and one in the other: how many apples has she?

4. John had three cents, and his mother gave him two more: how many had he then?

5. Mary has three cakes and George has three: how many have they both?

25. Uniting two or more numbers of the same kind, so as to find how much they all equal, is called **Addition**.

26. The number found by adding two or more numbers together is called the *Sum*.

27. The *sign* of addition, +, is called *plus*, and when placed between two numbers shows that they are to be added.

28. The following sign, =, is called the *sign of equality*, and when placed between two numbers shows that they are equal. Thus, $2 + 3 = 5$, and is read 2 plus 3 equals 5.

PRINCIPLE. — Only similar numbers can be added. Thus, 3 boys and 2 boys, 4 cents and 7 cents.

ADDITION TABLE.

	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10
2	0	1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2	2	2
	2	3	4	5	6	7	8	9	10	11
3	0	1	2	3	4	5	6	7	8	9
	3	3	3	3	3	3	3	3	3	3
	3	4	5	6	7	8	9	10	11	12
4	0	1	2	3	4	5	6	7	8	9
	4	4	4	4	4	4	4	4	4	4
	4	5	6	7	8	9	10	11	12	13
5	0	1	2	3	4	5	6	7	8	9
	5	5	5	5	5	5	5	5	5	5
	5	6	7	8	9	10	11	12	13	14
6	0	1	2	3	4	5	6	7	8	9
	6	6	6	6	6	6	6	6	6	6
	6	7	8	9	10	11	12	13	14	15
7	0	1	2	3	4	5	6	7	8	9
	7	7	7	7	7	7	7	7	7	7
	7	8	9	10	11	12	13	14	15	16
8	0	1	2	3	4	5	6	7	8	9
	8	8	8	8	8	8	8	8	8	8
	8	9	10	11	12	13	14	15	16	17
9	0	1	2	3	4	5	6	7	8	9
	9	9	9	9	9	9	9	9	9	9
	9	10	11	12	13	14	15	16	17	18
10	0	1	2	3	4	5	6	7	8	9
	10	10	10	10	10	10	10	10	10	10
	10	11	12	13	14	15	16	17	18	19

ORAL EXERCISE.

1. Count by 1's from 1 to 100; thus, 1 and 1 are 2, 2 and 1 are 3, 3 and 1 are 4, and so on.

2. Count by 2's from 2 to 100; thus, 2 and 2 are 4, 4 and 2 are 6, and so on.

Count by 2's from 4 to 60.

3. Count—
   By 3's from 3 to 60.
   By 3's from 4 to 100.

4. Count—
   By 4's from 1 to 49.
   By 4's from 2 to 74.
   By 4's from 3 to 99.

5. Count—
   By 5's from 1 to 51.
   By 5's from 2 to 77.
   By 5's from 3 to 93.
   By 5's from 4 to 119.

6. Count—
   By 6's from 1 to 73.
   By 6's from 2 to 98.
   By 6's from 3 to 117.
   By 6's from 5 to 155.
   By 6's from 0 to 180.

7. Count—
   By 7's from 2 to 51.
   By 7's from 4 to 95.
   By 7's from 5 to 110.
   By 7's from 0 to 133.

8. Count—
   By 8's from 3 to 83.
   By 8's from 5 to 125.
   By 8's from 7 to 119.
   By 8's from 1 to 145.

9. Count—
   By 9's from 1 to 82.
   By 9's from 2 to 128.
   By 9's from 8 to 188.

10. Count—
   By 10's from 2 to 102.
   By 10's from 5 to 135.
   By 10's from 9 to 199.

CASE I.

To Add any Column of Figures whose Sum does not exceed Nine.

ORAL EXERCISE.

1. How many are 3 pins and 4 pins?
2. How many are 6 pins and 2 pins?
3. How many are 4 boys and 5 boys?
4. How many are 5 boys and 4 boys?
5. How many are 7 men and 2 men?

6. How many are 6 men and 2 men?

7. How many are 4 cups and 4 cups?

8. How many are 3 spoons and 5 spoons?

9. How many are 3 apples, 2 apples and 1 apple?

10. How many are 2 cows, 3 cows and 4 cows?

WRITTEN EXERCISE.

Ex. How many are—

3 horses,   *Explanation.*—We write the numbers under one an-
2 horses,   other, and add thus: 3 and 2 are 5, and 3 are 8. Hence,
3 horses?   the sum is 8 horses.
8 horses.

Find the sum of—

(1.)	(2.)	(3.)	(4.)
6 books,	6 men,	3 pens,	4 boys,
3 "	2 "	4 "	2 "
		2 "	2 "

(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
6	7	3	4	2	5	2	4	6
0	0	4	2	1	2	1	1	2
1	2	2	3	6	2	5	1	1

14. How many are 21 cents, 15 cents and 12 cents?

SOLUTION.   *Explanation.*—We write the numbers so that units
21 cents,   stand under units and tens under tens, and begin
15 cents,   at the right to add. Thus, 2 and 5 are 7, and 1 is 8,
12 cents.   which we write in the place of units; adding the tens,
48 cents.   we have 1 and 1 are 2, and 2 are 4, which we write
in the tens' place.

Find the sum of—

(15.)	(16.)	(17.)	(18.)	(19.)
16 horses,	18 men,	12 books,	10 cents,	71 boxes,
21 "	20 "	14 "	22 "	6 "
10 "	60 "	13 "	34 "	12 "

(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
23	12	20	18	23	123
44	14	30	20	42	322
22	13	46	40	20	400

(26.)	(27.)	(28.)	(29.)	(30.)
422	221	165	141	6
320	143	200	23	21
150	20	30	32	432

31. A man travelled 21 miles on Monday and 16 on Tuesday: how far did he travel in both days?

32. There are 33 boys and 25 girls in school: how many pupils are there in the school?

33. A horse cost 105 dollars, a cow 43 dollars and a sheep 10 dollars: how much did they all cost?

34. A boy bought two hens for 50 cents, a duck for 25 cents and some feed for 12 cents: how much did he pay for all?

35. If I pay 122 dollars for a buggy, 45 dollars for harness and 200 dollars for a horse, how much do I pay for all?

How much is—

36. $6+2+1$?

37. $4+12+3$?

38. $21+22+43$?

39. $10+25+60$?

40. $23+42+122$?

41. $130+200+304$?

42. $125+320+201$?

43. $172+205+602$?

44. $232+300+107$?

45. $455+203+340$?

CASE II.

To Add when the Sum of a Column exceeds Nine Units of that place.

ORAL EXERCISE.

1. How many are 5 apples and 5 apples? 3 pens and 8 pens?

2. How many are 4 dollars and 6 dollars?

3. A boy has 5 fingers on each hand: how many has he on both?

4. How many are 5 cents, 6 cents and 7 cents?

5. A hog cost 10 dollars, a sheep 5 dollars and some hens 7 dollars: how much did they all cost?

6. A pair of boots cost 12 dollars, a hat 7 dollars and some socks 2 dollars: how much did they all cost?

7. How many are 7 books, 9 books and 22 books?

8. Mary has 10 cents, John has 12 cents and James has 15 cents: how many cents have they together?

9. A boy has 12 hens, 6 turkeys and 15 ducks: how many fowls has he?

10. A boy learns 6 letters one day, 5 letters the next, 4 letters the next, and 2 letters the next: how many does he learn in the four days?

WRITTEN EXERCISE.

Ex. How many are 6 horses, 8 horses, 4 horses and 5 horses?

SOLUTION.
6 horses.
8   "
4   "
5   "
—
23 horses.

Explanation.—We write the numbers under one another, and add, beginning at the bottom: 5 and 4 are 9, and 8 are 17, and 6 are 23, which is 2 tens and 3 ones. We write the 3 in the units' place, and the 2 in the tens' place.

Add—

(1.)	(2.)	(3.)	(4.)	(5.)
5 cows.	8 sheep.	6 boys.	2 cents.	6 dollars.
6 "	4 "	7 "	6 "	7 "
4 "	3 "	4 "	9 "	6 "
3 "	5 "	2 "	4 "	9 "

(6.)	(7.)	(8.)	(9.)	(10.)
6	7	6	8	8
4	8	4	7	9
5	3	7	5	7
9	2	3	6	2

(11.)	(12.)	(13.)	(14.)	(15.)
5	7	6	5	9
6	8	7	5	7
8	5	8	6	6
3	4	3	4	8
2	2	4	5	3

ADD.
423
347
849
1619

Explanation.—We write the numbers so that units are under units, tens under tens, and hundreds under hundreds, and begin to add at the right: 9 and 7 are 16, and 3 are 19 ones, or 1 ten and 9 ones. We write the 9 in the units' place, and add the 1 ten to the tens: 1 and 4 are 5, and 4 are 9, and 2 are 11 tens, or 1 hundred and 1 ten. We write the 1 ten in the tens' place, and add the 1 hundred to the hundreds: 1 and 8 are 9, and 3 are 12, and 4 are 16 hundreds, or 1 thousand and 6 hundreds. We write the 6 hundreds in the hundreds' place and the 1 thousand in the thousands' place. The result is, therefore, 1619.

Add—

(1.)	(2.)	(3.)	(4.)
42 dollars.	18 cents.	55 ducks.	48 books.
28 "	16 "	13 "	25 "
43 "	44 "	84 "	72 "

(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
45	84	16	46	84	95
69	72	61	64	46	50
32	91	85	51	87	68

3

(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
642	272	615	465	956	925
347	447	421	641	508	575
872	638	879	848	467	259

(17.)	(18.)	(19.)	(20.)	(21.)	(22.)
752	342	253	897	156	851
423	426	541	111	481	318
709	151	422	343	423	805
820	737	735	625	782	167

(23.)	(24.)	(25.)	(26.)	(27.)	(28.)
4813	1122	2291	3574	4449	1357
5914	7914	5723	3333	2575	2468
6115	1234	2102	4680	4404	5555
7036	8024	6838	3391	3685	6666

(29.)	(30.)	(31.)	(32.)	(33.)	(34.)
5788	3455	2729	4044	3282	1185
2693	6521	8272	5260	6341	5073
1112	6817	3228	3788	3161	9962
6762	7773	9561	5473	2827	9467
8104	6839	5587	2667	7214	3478

(35.)	(36.)	(37.)	(38.)
43474	73422	77823	13536
38242	75638	21684	71882
67891	18208	18516	81385
84870	32378	33902	80246
22171	27225	14656	91257

(39.)	(40.)
433827	28513534
563725	47224456
434958	31821745
367624	18714924
233647	73584627

Find the sum—

41. Of 6472+8733+4633+4854.
42. Of 2762+8756+9783+4578.
43. Of 1617+8743+7284+9621.
44. Of 2650+4062+8705+9030.
45. Of 5005+6007+7583+4783.
46. Of 27845+67832+74281+68432.
47. Of 47823+68421+70070+60504.
48. Of 127+6434+7805+60007.
49. Of 10+8756+405+66782.
50. Of 7560+804+7854+87400.
51. Of 1525+960+820+16+37800.

PROBLEMS.

1. A merchant sells 44 yd. of cloth on Monday, 62 yd. on Tuesday and 30 yd. on Wednesday: how much does he sell in the three days?   *Ans.* 136 yd.

2. A farmer raises 650 bu. of wheat in one field, 420 bu. in another and 725 bu. in another: how many bushels does he raise in the three fields?   *Ans.* 1795 bu.

3. A grocer sold 850 lb. of sugar in March, 927 lb. in April and 1640 lb. in May: how many pounds did he sell in the three months?   *Ans.* 3417 lb.

4. A knife cost 62 cts., a slate 44 cts., a reader 95 cts. and a grammar 65 cts.: how much did they all cost?
    Ans. 266 cts.

5. A gentleman owns a farm worth 16000 dollars, a store worth 12500 dollars, and a house worth 25000 dollars: how much are they all worth? *Ans.* 53500 dollars.

6. If a horse cost 157 dollars, a carriage 225 dollars and a set of harness 120 dollars, what do they all cost?
Ans. 502 dollars.

7. In my orchard I have 124 apple trees, 63 peach trees and 27 plum trees: how many trees are there in the orchard?   *Ans.* 214.

8. A merchant has three pieces of carpet; the first contains 127 yd., the second 145 yd., and the third 162 yd.: how many yards do they all contain?   *Ans.* 434 yd.

9. A man pays 1200 dollars for rent, 2750 dollars for clerk-hire and 175 dollars for insurance: how much does he pay altogether?   *Ans.* 4125 dollars.

10. A man was born in 1813, and died when he was 62 years old: when did he die?   *Ans.* In 1875.

11. A man receives as rent from five buildings the following amounts: from the first 125 dollars a year, from the second 320 dollars a year, from the third 650 dollars a year, from the fourth 240 dollars a year, and from the fifth 175 dollars a year: how much does he receive from all?   *Ans.* 1510 dollars.

12. A merchant has two pieces of muslin of 45 yd. each, and three of 46 yd. each: how much muslin has he?
Ans. 228 yd.

13. What is the cost of three horses at 125 dollars each, and four cows at 43 dollars each?   *Ans.* 547 dollars.

14. A drover bought 120 sheep for 625 dollars, 243 sheep for 763 dollars and 571 sheep for 2855 dollars: how many sheep did he buy, and how much did they cost him?
Ans. 934 sheep for 4243 dollars.

15. A farmer has four hogs which weigh as follows:

327 lb., 405 lb., 377 lb. and 396 lb.: how much do they all weigh?   *Ans.* 1505 lb.

16. The three stories of a building rent as follows: 1st story, 1560 dollars; 2d, 750 dollars; and 3d, 425 dollars: what amount of rent was received for the building?
                                                        Ans. 2735 dollars.

17. Five timber-rafts contain the following amounts, respectively: 1260 ft., 1340 ft., 973 ft., 1122 ft. and 1065 ft.: how many feet do they all contain?   *Ans.* 5760 ft.

18. A builder receives 1800 dollars for building a house, 743 dollars for building a barn and 2255 dollars for building a store property: how much does he receive for building all?   *Ans.* 4798 dollars.

19. Independence was declared in 1776, and the Constitution was adopted 11 yr. later: in what year was the Constitution adopted?   *Ans.* In 1787.

20. America was discovered in 1492, and the Puritans settled at Plymouth 128 yr. after: when did the Puritans settle at Plymouth?   *Ans.* In 1620.

21. New York was settled by the Dutch in 1613, and Pennsylvania was settled 69 yr. later: when was Pennsylvania settled?   *Ans.* In 1682.

22. The Puritans landed at Plymouth in 1620, and the battle of Lexington was fought 155 yr. later: when was the battle of Lexington fought?   *Ans.* In 1775.

23. Washington was inaugurated as the first President in 1789, and gold was discovered in California 59 yr. later: when was gold discovered in California?
                                                        Ans. In 1848.

24. I bought a house for 2750 dollars, and sold it so as to gain 1225 dollars: how much did I get for it?
                                                        Ans. 3975 dollars.

25. I paid 175 dollars for a horse, 150 dollars for a

buggy and 35 dollars for harness, and sold all at a gain of 13 dollars: how much did I get? *Ans.* 373 dollars.

26. A merchant's cash sales were as follows: Monday, 127 dollars; Tuesday, 67 dollars; Wednesday, 173 dollars; Thursday, 187 dollars; Friday, 25 dollars; and Saturday, 316 dollars: what was the amount of his sales for the week? *Ans.* 895 dollars.

27. The distance from Buffalo to Erie is 88 mi., from Erie to Toledo 207 mi., and from Toledo to Chicago 243 mi.: how far is it from Buffalo to Chicago?

Ans. 538 mi.

(28.)	(29.)	(30.)
27506	453	32556
3741	117642	897634
82	75	7347
640	7065	86424
8714	82436	17
618	4782	1700
44106	954	476
27	8	16964275
8742	12645	8740
196	7253	60006
94372		

SECTION III.

SUBTRACTION.

1. A boy had 2 cents, and lost 1: how many cents had he then?

2. A girl had 3 cakes, and gave her sister 1: how many had she then?

3. Four birds sat on a bush; 2 flew away: how many remained?

4. John had 5 cents, and bought an orange for 2 cents: how many cents had he left?

5. How many would he have left if he had paid 3 cents for his orange?

6. Mary bought 6 cups, but broke 3: how many had she remaining?

7. How many are 6 cups less 3 cups?

8. How many are 6 cups less 2 cups?

9. How many are 6 cups less 4 cups?

10. Charles had 8 ducks, but 3 died: how many had he then?

11. How many are 8 less 3? 8 less 4?

12. How many are 8 less 2? 8 less 5?

29. Finding the difference between two numbers is called **Subtraction.**

30. The number found by taking one number from another is called the *Difference.*

31. The number from which the other is taken is called the *Minuend.*

32. That which is taken from the minuend is called the *Subtrahend.*

33. The sign of subtraction, $-$, is called *minus,* and when placed between two numbers shows that the one on the right of the sign is to be taken from the one on the left of it. Thus, $6 - 2$ is read 6 minus 2, and means that 2 is to be taken from 6.

PRINCIPLE.—Only similar numbers can be subtracted; thus, 3 boys from 6 boys, 5 cents from 7 cents, etc.

SUBTRACTION TABLE.

	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10	11
2	2	2	2	2	2	2	2	2	2	2
	0	1	2	3	4	5	6	7	8	9
	3	4	5	6	7	8	9	10	11	12
3	3	3	3	3	3	3	3	3	3	3
	0	1	2	3	4	5	6	7	8	9
	4	5	6	7	8	9	10	11	12	13
4	4	4	4	4	4	4	4	4	4	4
	0	1	2	3	4	5	6	7	8	9
	5	6	7	8	9	10	11	12	13	14
5	5	5	5	5	5	5	5	5	5	5
	0	1	2	3	4	5	6	7	8	9
	6	7	8	9	10	11	12	13	14	15
6	6	6	6	6	6	6	6	6	6	6
	0	1	2	3	4	5	6	7	8	9
	7	8	9	10	11	12	13	14	15	16
7	7	7	7	7	.7	7	7	7	7	7
	0	1	2	3	4	5	6	7	8	9
	8	9	10	11	12	13	14	15	16	17
8	8	8	8	8	8	8	8	8	8	8
	0	1	2	3	4	5	6	7	8	9
	9	10	11	12	13	14	15	16	17	18
9	9	9	9	9	9	9	9	9	9	9
	0	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15	16	17	18	19
10	10	10	10	10	10	10	10	10	10	10
	0	1	2	3	4	5	6	7	8	9

ORAL EXERCISE.

1. Subtract by 1's from 100 to 1; thus, 1 from 100 leaves 99, 1 from 99 leaves 98, and so on.

2. Subtract by 2's from 100 to 2; thus, 2 from 100 eaves 98, 2 from 98 leaves 96, and so on.

3. Subtract by 2's from 95 to 1; thus, 2 from 95 leaves 3, 2 from 93 leaves 91, and so on.

- Subtract—
    By 3's from 100 to 1.
    By 3's from 99 to 0.
    By 3's from 98 to 2.
- Subtract—
    By 4's from 100 to 0.
    By 4's from 99 to 3.
    By 4's from 98 to 2.
    By 4's from 97 to 1.
5. Subtract—
    By 5's from 100 to 0.
    By 5's from 99 to 4.
    By 5's from 98 to 3.
    By 5's from 97 to 2.
    By 5's from 96 to 1.
7. Subtract—
    By 6's from 100 to 4.
    By 6's from 99 to 3.
    By 6's from 98 to 2.
    By 6's from 97 to 1.
    By 6's from 96 to 0.
    By 6's from 95 to 5.
8. Subtract—
    By 7's from 100 to 2.
    By 7's from 99 to 1.

By 7's from 98 to 0.
By 7's from 97 to 6.
By 7's from 96 to 5.
By 7's from 95 to 4.
By 7's from 94 to 3.

9. Subtract—
    By 8's from 100 to 4.
    By 8's from 99 to 3.
    By 8's from 98 to 2.
    By 8's from 97 to 1.
    By 8's from 96 to 0.
    By 8's from 95 to 7.
    By 8's from 94 to 6.
    By 8's from 93 to 5.
10. Subtract—
    By 9's from 100 to 1.
    By 9's from 99 to 0.
    By 9's from 98 to 8.
    By 9's from 97 to 7.
    By 9's from 96 to 6.
    By 9's from 95 to 5.
    By 9's from 94 to 4.
    By 9's from 93 to 3.
    By 9's from 92 to 2.

11. Count by 4's from 3 to 39, and back again to 19.
12. Count by 5's from 6 to 66, and back again to 26.
13. Count by 7's from 18 to 53, and back again to 11.
14. Count by 8's from 25 to 65, and back again to 1.

C

CASE I.

To Subtract where no Figure of the Subtrahend is greater than the corresponding Figure of the Minuend.

WRITTEN EXERCISE.

Ex. 1. Subtract 3 from 8.

PROCESS.
```
  8
  3
  5
```
Explanation.—We write 3 under 8, and say 3 ones from 8 ones leaves 5 ones.   Or, 3 from 8 leaves 5.

EXAMPLES FOR PRACTICE.

	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
From	6 pens,	7 cows,	6 cents,	8	7	8
Take	2 "	3 "	4 "	5	5	4

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
9	8	9	8	3	9
4	2	7	6	2	3

14. Subtract 4 from 8 ; 6 from 9.
15. Subtract 5 from 11 ; 6 from 15.
16. Subtract 4 from 13 ; 9 from 17.
17. Subtract 35 cents from 69 cents.

PROCESS.
```
69 cents.
35   "
34 cents.
```
Explanation.—We write the numbers so that units stand under units, and tens under tens, and begin to subtract at the right: 5 units from 9 units leaves 4 units ; 3 tens from 6 tens leaves 3 tens.   The remainder is 3 tens and 4 units, or 34.

(18.)	(19.)	(20.)	(21.)	(22.)	(23.)
From 65	74	49	54	65	46
Take 22	33	36	31	43	25

(24.)	(25.)	(26.)	(27.)	(28.)	(29.)
673	478	645	897	745	683
542	135	325	783	444	560

(30.)	(31.)	(32.)	(33.)	(34.)	(35.)
6434	7859	4765	8493	4682	1378
3233	6537	3542	4272	2460	1164

(36.)	(37.)	(38.)	(39.)	(40.)
87643	· 97485	68432	22486	85434
35223	66283	46231	11375	74321

(41.)	(42.)	(43.)
847562	8764397	64863387
224530	4231264	53621105

44. A farmer had 75 sheep, and sold 32 of them: how many had he remaining? *Ans.* 43.

45. A merchant had a piece of muslin containing 47 yards; he sold 26 yards of it: how much remained?
Ans. 21 yd.

46. A man earns 878 dollars in a year, and spends 642 dollars: how much does he save? *Ans.* 236 dollars.

47. I buy a horse for 175 dollars, and sell him for 162 dollars: how much do I lose? *Ans.* 13 dollars.

48. A farmer had 6742 bu. of oats, and sold 1430 bu.: how much had he remaining? *Ans.* 5312 bu.

49. A man paid 6750 dollars for a farm, and sold it for 6620 dollars: how much did he lose? ·
Ans. 130 dollars.

50. A lady bought a farm for 6320 dollars, and sold it for 7460 dollars: how much did she make?
Ans. 1140 dollars.

51. A carpenter bought 7500 shingles to put on a roof, and had 1200 remaining: how many did he put on the roof? *Ans.* 6300.

52. A school had 85 pupils, but 23 left: how many remained? *Ans.* 62.

CASE II.

To Subtract when a Figure of the Subtrahend is greater, or expresses more, than the corresponding Figure of the Minuend.

WRITTEN EXERCISES.

Ex. 1. Subtract 8 from 17.

PROCESS.
```
17
 8
——
 9
```
Explanation.—Writing the 8 under the 17, we say 8 from 17 leaves 9.

Ex. 2. Subtract 29 from 85.

PROCESS.
```
85
29
——
56
```
Explanation.—We write the numbers so that units stand under units and tens under tens. Since we cannot take 9 units from 5 units, we take 1 ten of the 8 tens, which equals 10 units, and unite it with the 5 units, making 15 units; 9 units from 15 units leaves 6 units, which we write in the units' place.

Since we have used one of the 8 tens, we now have 7 tens : 2 tens from 7 tens leaves 5 tens, which we write in the tens' place.

	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
From	16	14	15	23	45	63
Take	9	7	8	14	27	29

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
26	75	83	72	65	84
19	46	64	48	37	26

Find the value—

15. Of 75 − 48.
16. Of 64 − 57.
17. Of 84 − 26.

18. Of 96 − 77.
19. Of 85 − 49.
20. Of 73 − 19.

21. In a school of 65 pupils, 27 left: how many remained ?   *Ans.* 38.

22. A boy had 75 cents, and spent 39 of them for a book : how much had he left ?   *Ans.* 36.

23. A box of soap contained 60 lb., but 47 lb. of it have been used: how much remains? *Ans.* 13 lb.

24. Mary had 72 pins, but lost 16: how many has she now? *Ans.* 56.

25. Charles bought a cow for 69 dollars, and sold her for 95 dollars: how much did he make? *Ans.* 26 dols.

26. Subtract 1657 from 8265.

SOLUTION.   *Explanation.*—Writing the numbers so that units
8265   stand under units, tens under tens, and so on, we sub-
1657   tract, beginning at the right.
6608   Since 7 units cannot be taken from 5 units, we take
one of the 6 tens, which is equal to 10 units, and unite it with the
5 units, making 15 units; subtracting 7 units from 15 units, we have
8 units.

Since we have taken one of the 6 tens, there are but 5 tens; 5 tens from 5 tens leaves nothing.

Since we cannot take 6 hundreds from 2 hundreds, we take 1 thousand of the 8 thousands, which is equal to 10 hundreds, and uniting this with the 2 hundreds, we have 12 hundreds; subtracting 6 hundreds from 12 hundreds, we have 6 hundreds.

Since we have taken one of the 8 thousands, there are but 7 thousands remaining; subtracting 1 thousand from 7 thousands, we have 6 thousands. Hence, the result is 6 thousands 6 hundreds and 8 units, or 6608.

	(27.)	(28.)	(29.)	(30.)	(31.)	(32.)
From	643	724	642	486	645	863
Take	428	617	139	395	596	469

(33.)	(34.)	(35.)	(36.)	(37.)	(38.)
749	472	566	435	673	462
376	318	197	264	647	378

(39.)	(40.)	(41.)	(42.)	(43.)	(44.)
450	507	570	607	725	742
375	263	387	469	678	387

4

(45.)	(46.)	(47.)	(48.)	(49.)	(50.)
6437	5347	8546	8432	8450	9720
2863	3726	7615	6547	6375	6483

(51.)	(52.)	(53.)	(54.)	(55.)	(56.)
6248	5337	8756	8645	8643	9674
3673	4276	4165	3427	5750	2803

(57.)	(58.)	(59.)	(60.)	(61.)	(62.)
86523	42347	83742	73388	61345	52384
63748	26353	35635	37756	22378	41307

(63.)	(64.)	(65.)	(66.)
74332	634725	7346000	1000006
38856	378462	4683745	346087

Find the value—

67. Of 6700 – 1864.	*Ans.* 4836.
68. Of 7820 – 6437.	*Ans.* 1383.
69. Of 1877 – 1798.	*Ans.* 79.
70. Of 6342 – 6007.	*Ans.* 335.
71. Of 3600 – 2225.	*Ans.* 1375.
72. Of 72500 -- 63497.	*Ans.* 9003.
73. Of 82000 – 18640.	*Ans.* 63360.
74. Of 60000 – 17400.	*Ans.* 42600.
75. Of 62000 – 18649.	*Ans.* 43351.
76. Of 56009 – 27346.	*Ans.* 28663.

PROBLEMS.

1. A horse was bought for 125 dollars, and sold for 117 dollars : how much was lost by the sale? *Ans.* 8 dollars.

2. A farmer took 620 bu. of potatoes to market, and sold 455 bu. : how many had he remaining?

Ans. 165 bu.

3. Washington was born in 1732, and died in 1799: how old was he? *Ans.* 67 yr.

4. A roll of carpet contained 156 yd., but 79 yd. have been sold from it: how much remains? *Ans.* 77 yd.

5. The battle of New Orleans was fought in 1815; America was discovered in 1492: how long was America discovered before the battle of New Orleans was fought?
Ans. 323 yr.

6. A house cost 5440 dollars, and was sold for 6000 dollars: how much was the gain? *Ans.* 560 dollars.

7. From a flock containing 820 sheep 417 were sold: how many remain? *Ans.* 403.

8. A merchant began business with 20000 dollars, and lost 1463 dollars: how much had he remaining?
Ans. 18537 dollars.

9. A man having 1600 dollars in bank, withdrew 977 dollars: how much remained? *Ans.* 623 dollars.

10. How many dollars are 15000 dollars minus 6743 dollars? *Ans.* 8257 dollars.

11. I bought a lot for 450 dollars, and sold it for 525 dollars: how much did I gain? *Ans.* 75 dollars.

12. At an election one candidate received 23204 votes, and his opponent 18675 votes: what was the majority?
Ans. 4529.

13. A farmer had 240 acres of land; he sold 27 acres to one man and 148 acres to another: how much remained? *Ans.* 65 acres.

14. A man died in 1877 at the age of 75 years: when was he born? *Ans.* In 1802.

COMBINATION PBOBLEMS.

15. A merchant received cash 1125 dollars, and paid rent 215 dollars, and for clerk-hire 567 dollars: how much cash had he remaining? *Ans.* 343 dollars.

16. Three horses cost as follows: 175 dollars, 116 dollars and 95 dollars; the three were sold for 300 dollars: how much was the loss?   *Ans.* 86 dollars.

17. A horse cost 125 dollars, a carriage 173 dollars and some harness 62 dollars: if the owner sell all for 400 dollars, how much will he gain?   *Ans.* 40 dollars.

18. Mr Miller owed a man 1500 dollars; he has paid him 650 dollars, 325 dollars and 92 dollars: how much does he still owe him?   *Ans.* 433 dollars.

19. How long is it since the discovery of America?

20. A farmer raised 320 bu. of corn in one field, 290 bu. in another and 700 bu. in another; he sold 1269 bu.: how much has he remaining?   *Ans.* 41 bu.

21. A boy had 120 apples; he found 72 more, and then gave away 59: how many had he remaining?

Ans. 133.

22. A gentleman paid 6000 dollars for his farm; he built a house for 3976 dollars, and then sold both for 12000: how much did he gain?   *Ans.* 2024 dollars.

23. A merchant paid 6000 dollars for his store; he sold the goods for 8642 dollars, and paid his clerks 1640 dollars: how much did he gain?   *Ans.* 1002 dollars.

24. Two men bought store goods to the amount of 1644 dollars, and sold them so that each made 386 dollars: how much did they receive for the goods?

Ans. 2416 dollars.

25. In a regiment of 900 men, 127 were killed and 345 were wounded: how many escaped unhurt?

Ans. 428.

26. What is the value of $1862 + 1744 - 673$?

Ans. 2933.

27. What is the value of $1683 - 420 + 9684 - 6472$?

Ans. 4475.

28. What is the value of 89644 + 7842 − 6845 + 67340?

Ans. 157981.

29. What is the value of 6400 + 7856 + 6834 − 20465 ?

Ans. 625.

30. What is the value of 6000 − 1463 + 8674 − 6340 + 1009 − 1001 ?

Ans. 6879.

SECTION IV.

MULTIPLICATION.

1. One boy has 2 eyes: how many eyes have 2 boys ?

SOLUTION.—2 boys have 2 eyes and 2 eyes, or 2 times 2 eyes, which are 4 eyes.

2. If 1 apple cost 2 cents, how much will 3 apples cost?

SOLUTION.—3 apples will cost 2 cents and 2 cents and 2 cents, or 3 times 2 cents, which are 6 cents.

3. A chair has 4 legs: how many legs have 2 chairs ? How many legs have 3 chairs ?

4. A girl has 5 fingers on one hand : how many has she on both ?

5. If 1 orange cost 5 cents, how much will 4 oranges cost ?

6. If 1 hat cost 5 dollars, how much will 3 hats cost?

7. One cat has 4 feet : how many feet have 5 cats?

8. How many feet have 4 cats ?

9. How many are 5 times 6 ?

SOLUTION.—5 times 6 are 6 + 6 + 6 + 6 + 6, or 30.

34. The process of taking one of two numbers as often as there are units in the other is called **Multiplication.**

35. The number to be multiplied or repeated is called the *Multiplicand.*

4 *

36. The number showing how often the multiplicand is repeated is called the *Multiplier*.

37. The result obtained by the process of multiplying is called the *Product*.

38. The sign of multiplication, ×, is read *times* or *multiplied by;* 5×6 is read 5 times 6, or 5 multiplied by 6.

MULTIPLICATION TABLE.

ONCE		TWICE		3 TIMES		4 TIMES		5 TIMES		6 TIMES	
1 is	1	1 are	2	1 are	3	1 are	4	1 are	5	1 are	6
2 "	2	2 "	4	2 "	6	2 "	8	2 "	10	2 "	12
3 "	3	3 "	6	3 "	9	3 "	12	3 "	15	3 "	18
4 "	4	4 "	8	4 "	12	4 "	16	4 "	20	4 "	24
5 "	5	5 "	10	5 "	15	5 "	20	5 "	25	5 "	30
6 "	6	6 "	12	6 "	18	6 "	24	6 "	30	6 "	36
7 "	7	7 "	14	7 "	21	7 "	28	7 "	35	7 "	42
8 "	8	8 "	16	8 "	24	8 "	32	8 "	40	8 "	48
9 "	9	9 "	18	9 "	27	9 "	36	9 "	45	9 "	54
10 "	10	10 "	20	10 "	30	10 "	40	10 "	50	10 "	60
11 "	11	11 "	22	11 "	33	11 "	44	11 "	55	11 "	66
12 "	12	12 "	24	12 "	36	12 "	48	12 "	60	12 "	72

7 TIMES		8 TIMES		9 TIMES		10 TIMES		11 TIMES		12 TIMES	
1 are	7	1 are	8	1 are	9	1 are	10	1 are	11	1 are	12
2 "	14	2 "	16	2 "	18	2 "	20	2 "	22	2 "	24
3 "	21	3 "	24	3 "	27	3 "	30	3 "	33	3 "	36
4 "	28	4 "	32	4 "	36	4 "	40	4 "	44	4 "	48
5 "	35	5 "	40	5 "	45	5 "	50	5 "	55	5 "	60
6 "	42	6 "	48	6 "	54	6 "	60	6 "	66	6 "	72
7 "	49	7 "	56	7 "	63	7 "	70	7 "	77	7 "	84
8 "	56	8 "	64	8 "	72	8 "	80	8 "	88	8 "	96
9 "	63	9 "	72	9 "	81	9 "	90	9 "	99	9 "	108
10 "	70	10 "	80	10 "	90	10 "	100	10 "	110	10 "	120
11 "	77	11 "	88	11 "	99	11 "	110	11 "	121	11 "	132
12 "	84	12 "	96	12 "	108	12 "	120	12 "	132	12 "	144

ORAL EXERCISE.

1. Multiply by 2 from 1 to 12; thus, 2 times 1 are 2, 2 times 2 are 4, and so on.

2. Multiply by 3 from 1 to 6.

3. Multiply by 3 from 12 to 6 ; thus, 3 times 12 are 36, 3 times 11 are 33, and so on.

4. Multiply—
    By 4 from 3 to 9.
    By 4 from 12 to 6.
5. Multiply—
    By 5 from 2 to 7.
    By 5 from 12 to 4.
6. Multiply—
    By 6 from 3 to 10.
    By 6 from 12 to 4.
7. Multiply—
    By 7 from 2 to 8.
    By 7 from 12 to 5.
8. Multiply—
    By 8 from 3 to 10.
    By 8 from 12 to 2.

9. Multiply—
    By 9 from 1 to 11.
    By 9 from 12 to 3.
10. Multiply—
    By 10 from 3 to 8.
    By 10 from 12 to 2.
11. Multiply—
    By 11 from 4 to 9.
    By 11 from 12 to 3.
    By 11 from 10 to 2.
12. Multiply—
    By 12 from 1 to 7.
    By 12 from 12 to 5.
    By 12 from 2 to 9.
    By 12 from 10 to 3.

PRINCIPLES.—1. When two numbers are multiplied, either one may be taken as the multiplier. Thus, $4 \times 5 = 20$, or $5 \times 4 = 20$.

2. The product is the same kind as the multiplicand. Thus, 3×3 cents are 9 cents; 2×5 boys are 10 boys.

ORAL EXERCISE.

1. What will 5 hats cost at 7 dollars each?

SOLUTION.—If 1 hat cost 7 dollars, 5 hats will cost 5 times 7 dollars, or 35 dollars.

2. What will 4 pairs of boots cost at 6 dollars a pair?

3. A sheep cost 7 dollars: how much will 6 sheep cost at the same rate?

4. If a lemon cost 3 cents, how much will 11 lemons cost?

5. John is 8 years old : how much is 4 times his age?

6. A boat cost 11 dollars : how much would 5 boats cost at the same rate?

7. If a coat cost 10 dollars, how much will 8 coats cost?

8. There are seven days in a week : how many days are there in 8 weeks?

9. How much will 12 tons of coal cost at 6 dollars a ton?

10. A man earns 4 dollars a day : how much can he earn in 9 days?

11. At the rate of 8 marbles for a cent, how many can be bought for 12 cents?

12. If a cow eat 12 pounds of hay, how much will 10 cows eat?

13. What will 7 lead-pencils cost at 7 cents apiece?

14. If a chicken has 8 toes, how many toes have 9 chickens?

15. John is 7 years old; his father is 9 times as old : how old is his father?

WRITTEN EXERCISE.
CASE I.
When the Multiplier is a Single Figure.

Ex. Multiply 355 by 5.

Addition.	*Multiplication.*	*Explanation.*—We write the multi-
355	355	plier under the multiplicand, and be-
355	5	gin at the right to multiply : 5 times 5
355	1775	units are 25 units, or 2 tens and 5
355		units. Write the 5 units in the units'
355		place, and add the 2 tens to the product
1775		of tens : 5 times 5 tens are 25 tens, and

2 tens added are 27 tens, or 2 hundreds and 7 tens. Write the 7 tens in the tens' column, and add the 2 hundreds to the next product:

5 times 3 hundreds are 15 hundreds, and 2 hundreds added are 17 hundreds, which write·in its proper place.

The following explanation is shorter than the preceding:

Explanation 2.—5 times 5 are 25; write the 5 and add the 2 to the next product.

5 times 5 are 25, and 2 are 27; write the 7 and add the 2 to the next product.

5 times 3 are 15, and 2 are 17. Hence, the product is 1775.

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
Multiply	26	34	42	63	72	81
By	2	3	3	4	4	4

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
25	18	24	35	54	63
5	5	6	6	7	8

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
43	64	75	87	95	19
7	7	6	8	9	5

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)
324	645	732	841	681	375
7	8	9	5	7	6

(25.)	(26.)	(27.)	(28.)	(29.)	(30.)
463	572	986	785	487	604
8	7	6	9	5	8

Multiply—

31. 315 by 6.	41. 6742 by 8.
32. 480 by 7.	42. 6040 by 9.
33. 614 by 5.	43. 61783 by 7.
34. 7842 by 3.	44. 60784 by 6.
35. 6843 by 7.	45. 85643 by 5.
36. 8742 by 5.	46. 170604 by 6.
37. 9764 by 8.	47. 683471 by 5.
38. 8973 by 6.	48. 863478 by 7.
39. 14068 by 5.	49. 785473 by 8.
40. 18007 by 4.	50. 246853 by 9.

CASE II.

When the Multiplier consists of Two or more Figures.

Ex. Multiply 642 by 57.

PROCESS.   *Explanation.*—We write the multiplier under the
642   multiplicand, as in the previous case, and begin to mul-
 57   tiply at the right. By Case I., multiplying 642 by 7
4494   gives 4494 ones. Multiplying 642 by 5 gives 3210, and
3210   since the multiplier is 5 *tens*, the result is 3210 *tens*,
36594   which, added to the previous product, 4494 ones, gives
the correct product, 36594.

Multiply—

1. 32 by 43.	*Ans.* 1376.	11. 723 by 88.	*Ans.* 63624.
2. 46 by 65.	*Ans.* 2990.	12. 647 by 77.	*Ans.* 49819.
3. 67 by 35.	*Ans.* 2345.	13. 493 by 82.	*Ans.* 40426.
4. 73 by 87.	*Ans.* 6351.	14. 761 by 41.	*Ans.* 31201.
5. 122 by 73.	*Ans.* 8906.	15. 875 by 65.	*Ans.* 56875.
6. 144 by 96.	*Ans.* 13824.	16. 944 by 66.	*Ans.* 62304.
7. 347 by 52.	*Ans.* 18044.	17. 871 by 73.	*Ans.* 63583.
8. 954 by 63.	*Ans.* 60102.	18. 945 by 78.	*Ans.* 73710.
9. 725 by 75.	*Ans.* 54375.	19. 674 by 65.	*Ans.* 43810.
10. 864 by 64.	*Ans.* 55296.	20. 777 by 77.	*Ans.* 59829.

21. How many yards of muslin in 24 pieces of 43 yd each? *Ans.* 1032.

22. If a cow cost 35 dollars, how much will 29 cow cost? *Ans.* 1015 dollars.

23. A railway train runs 38 mi. an hour: how far doe it run in 75 hr.? *Ans.* 2850 mi.

24. How much will a farm of 46 acres cost at 95 dol lars an acre? *Ans.* 4370 dollars.

25. What is cost of 27 bu. of oats at 63 cts. a bushel ? *Ans.* 1701 cts.

26. What is the cost of 64 doz. of eggs at 27 cts. dozen? *Ans.* 1728 cts.

27. There are 12 things in a dozen: how many ar there in 64 doz.? *Ans.* 768.

28. A drover had some horses worth 125 dollar each: what are 23 of these horses worth?
  Ans. 2875 dollars.

29. What is the value of 45 A. of land at 164 dollar an acre? *Ans.* 7380 dollars.

30. What will 55 mules cost at 185 dollars apiece?
  Ans. 10175 dollars.

31. A clerk earns 755 dollars in a year: how muc at the same rate can he earn in 15 yr.?
  Ans. 11325 dollars.

32. There are 63 gal. in a hogshead: how many gallon are there in 27 hhd.? *Ans.* 1701.

33. If 1 boy solve 227 problems in a week, how man at the same rate can 53 boys solve? *Ans.* 12031.

34. If a man earn 27 dollars a week, how much can h earn in 52 wk., or 1 yr.? *Ans.* 1404 dollars.

35. A merchant has 47 pieces of calico, each containin 43 yd.: how many yards has he? *Ans.* 2021.

Ex. Multiply 6741 by 475.

PROCESS.

        6741   *Explanation.*—Multiplying 6741 by 5 units, we have
         475   33705 units; multiplying 6741 by 7 tens, we have 47187
        33705   tens; multiplying 6741 by 4 hundreds, we have 26964
        47187   hundreds; writing these products in their proper places
        26964   and adding them, we have the true product, 3201975.
       3201975

Multiply—

36. 744 by 635.   46. 6484 by 6372.
37. 895 by 336.   47. 7856 by 3475.
38. 972 by 243.   48. 6748 by 6334.
39. 825 by 682.   49. 4878 by 3437.
40. 973 by 745.   50. 8547 by 7733.
41. 8462 by 781.   51. 85474 by 2547.
42. 9643 by 683.   52. 46887 by 3489.
43. 8532 by 763.   53. 56184 by 5474.
44. 8984 by 133.   54. 56664 by 4871.
45. 4659 by 886.   55. 25473 by 4487.

56. 64374 by 78561.   *Ans.* 5057285814.
57. 648487 by 678432.   *Ans.* 439954332384.
58. 7846825 by 397.   *Ans.* 3115189525.
59. 75456593 by 6471.   *Ans.* 488279613303.
60. 96458 by 7354.   *Ans.* 709352132.

61. What will 655 A. of land cost at 164 dollars an acre?   *Ans.* 107420 dollars.

62. If a train go 597 mi. a day, how far will it go in 313 d.?   *Ans.* 186861 mi.

63. How much will 864 horses cost, if 1 cost 95 dollars?   *Ans.* 82080 dollars.

64. If it cost 125 dollars a year to board 1 student, how much will it cost to board 453 students?
                                        Ans. 56625 dollars.

65. A drover sold 197 horses at 168 dollars each : how much did he receive for them ? *Ans.* 33096 dollars.

66. If 1 log cut 475 feet of lumber, how much will 374 such logs cut ? *Ans.* 177650 ft.

67. A man had 4623 tons of iron, which he sold at 73 dollars a ton : how much did he get for it ?
Ans. 337479 dollars.

68. A teamster hauls 675 bricks in one load : how many does he haul in 688 loads ? *Ans.* 464400 bricks.

69. If a book has 723 pages, how many pages have 642 similar books ? *Ans.* 464166 pages.

70. If 1 cotton-bale weigh 397 lb., how much at the same rate will 256 bales weigh ? *Ans.* 101632 lb.

CASE III.

To Multiply when there are Naughts at the Right of either the Multiplicand or the Multiplier, or both.

Ex. 1. Multiply 643 by 700.

PROCESS.　　*Explanation.*—Multiplying 643 by 7 hundreds gives
643　　4501 hundreds, or 450100.
　700　　　This result is the same as is obtained by multiply-
450100　ing by 7, and annexing as many naughts on the right
as there are naughts at the right of the 7.

Ex. 2. Multiply 614000 by 600.

PROCESS.　　*Explanation.*—Multiplying the multiplicand by 6
614000　　hundreds, gives 3684000 hundreds, or 368400000.
　　600　　　This result is the same as that obtained by multi-
368400000　plying 614 by 6, and annexing to the right five
naughts, which is the number of naughts to the right of both the multiplier, 6, and the multiplicand, 614.

Hence, *When there are naughts to the right of either multiplier or multiplicand, multiply the other figures and annex as many naughts as are at the right of both numbers.*

Find the value—

1. Of 743 × 600.
2. Of 847 × 700.
3. Of 9642 × 6300.
4. Of 1875 × 6340.
5. Of 27 × 9000.
6. Of 6000 × 43.

7. Of 18000 × 623.
8. Of 6400 × 640.
9. Of 650 × 650.
10. Of 83600 × 7500.
11. Of 9230 × 7000.
12. Of 8000 × 61000.

13. There are 2000 lb. in a ton: how many pounds are there in 75 T.?   *Ans.* 150000.

14. What is the weight of 20 loads of coal, if each load weigh 1800 lb.?   *Ans.* 36000.

15. A man sold 500 cows at 40 dollars apiece: how much did he get for them?   *Ans.* 20000 dollars.

16. If a teacher receive 600 dollars a year salary, how much would 35 teachers receive at the same rate?
                         Ans. 21000 dollars.

17. What are 160 A. of land worth at 150 dollars an acre?   *Ans.* 24000 dollars.

18. What are 600 horses worth at 200 dollars each?
                         Ans. 120000 dollars.

CASE IV.

To Multiply when there are Naughts in the Multiplier.

Ex. Multiply 6043 by 7006.

PROCESS.
6043
7006
───────
36258
42301
───────
42337258

Explanation.—Multiplying 6043 by 6 units equals 36258 units. Multiplying 6043 by 7 thousands equals 42301 thousands, which we write in its proper place. Adding these products, we have the true product, 42337258.

NOTE.—Since 0 times any number is 0, it is not necessary to multiply by the naught.

In multiplying by any number of thousands, etc., the first figure of the result should be placed in the same column as the multiplier.

Thus, in multiplying above by 7 thousands, we say 7 times 3 are 21, and write the 1 in the thousands' column, adding the 2 to the next product.

Find the value—

1. Of 806 × 307.	6. Of 4600 × 4006.
2. Of 7500 × 1406.	7. Of 8740 × 6040.
3. Of 1800 × 709.	8. Of 9000 × 70500.
4. Of 6742 × 604.	9. Of 6004 × 6004.
5. Of 12000 × 709.	10. Of 86457 × 300017.

11. A lot cost 420 dollars: how much will 105 lots cost at the same rate?   *Ans.* 44100 dollars.

12. A drover has 406 cows worth 30 dollars each: how much are they all worth?   *Ans.* 12180 dollars.

13. How much will it cost to build 307 miles of railroad at 4060 dollars a mile?   *Ans.* 1246420 dollars.

14. A contractor built 604 miles of railroad at 6500 dollars a mile: how much did he get for it?
                   Ans. 3926000 dollars.

COMBINATION PROBLEMS.

1. A merchant has 26 pieces of cloth of 54 yd. each, which he sells at 65 cts. a yard: how much does he receive for all of it?   *Ans.* 91260 cts.

2. A merchant sells 12 boxes of starch, each containing 60 lb., at 11 cts. a pound: how much does he receive for it?   *Ans.* 7920 cts.

3. A man earns 25 dollars a week, and pays 5 dollars a week for board: how much does he save in 6 wk.?
                   Ans. 120 dollars.

4. A boy bought 3 ducks at 25 cts. each, and 4 hens at 30 cts. each: how much did he pay for all?
                   Ans. 195 cts.

5. What is the value of 7 horses at 125 dollars and 14 cows at 27 dollars each?   *Ans.* 1253 doll

6. A farmer has the following stock: 6 horses 140 dollars each, 13 cows worth 32 dollars each, ɛ hogs worth 16 dollars each: what is his stock wortl
                                        Ans. 1512 dol

7. A builder hired 12 men at 2 dollars a day eac 5 boys at 1 dollar a day each: how much did he p in 27 d.?   *Ans.* 783 dol

8. A farmer took 10 bu. of potatoes worth 75 bushel to market, and traded them for 12 yd. oɪ worth 60 cts. a yard, the balance to be paid in cash much cash did he get?   *Ans.* 30

9. A merchant bought 27 lb. of butter at 33 pound, and gave in exchange 64 lb. of sugar at 1ɛ pound, and the balance in cash: how much did he cash?   *Ans.* 59

10. If a train travel 28 mi. an hour, how far wᴄ go in 6 d. of 24 hr. each?   *Ans.* 4032

11. What is the cost of 40 horses at 106 dollarɛ and 60 cows at 65 dollars each?   *Ans.* 8140 dol

12. What is the value of 165+178+347−612?
                                        Ans

13. What is the value of 17×643, 27×647 an 745?   *Ans.* 36

14. What is the value of 18×24 and 16×2' 30×28?   *Ans*

15. A man earns 427 dollars a month, and spen dollars a month: how much does he save in 16 mo
                                        Ans. 4032 dol

16. A farmer bought 17 cows at 45 dollars eaᴄ 53 hogs at 12 dollars each: which cost the most, aɪ much?   *Ans.* The cows, 129 dol

17. A farmer took to market 12 hens at 33 cts. each, and 15 bu. of potatoes at 75 cts. a bushel; he bought 14 books at 95 cts. each: how much money had he remaining?

Ans. 191 cts.

18. A merchant buys 12 geese at 85 cts. each, and 16 turkeys at 94 cts. each; he gives in exchange 27 yd. of calico at 9 cts. a yard, and 25 yd. of delaine at 34 cts. a yard: how much money does he have to pay?

Ans. 1431 cts.

19. A man paid 1600 dollars for a house, and paid a man for 15 days' work at 2 dollars a day for fencing: how much did the property cost him?

Ans. 1630 dollars.

20. A lady sold 27 tubs of butter, each weighing 43 lb., at 27 cts. a pound, and bought 175 yd. of carpet at 90 cts. a yard: how much money had she left?

Ans. 15597 cts.

21. A man had 465 dollars; he earned 750 dollars, and then spent 540 dollars and lost 27 dollars: how much had he remaining? *Ans.* 648 dollars.

SECTION V.

DIVISION.

1. Two boys have 4 eyes: how many eyes has 1 boy?

2. How many times 2 eyes are 4 eyes?

3. Three dogs have 12 feet: how many feet has 1 dog?

4. How many times 4 feet are 12 feet?

5. A bush has 8 roses: how many times 2 roses has it? How many times 4 roses?

6. A house has 12 doors: how many times 3 doors has it? How many times 4 doors has it?

5*

7. An orchard has 20 trees: how many times 5 trees has it? How many times 4 trees? How many times 10 trees?

8. How many times 6 is 24? How many times 5 is 25?

9. How many times is 5 contained in 15?

SOLUTION.—Since 3 times 5 are 15, 5 is contained 3 times in 15.

10. How many times is 4 contained in 20?

11. How many times is 6 contained in 30?

12. How many times is 5 contained in 30?

13. How many times is 3 contained in 18?

14. How many times is 6 contained in 18?

15. If a boy earn 24 dollars, how many times 4 dollars does he earn? How many times 6 dollars? How many times 8 dollars?

16. If a man has 30 cts., how many times 10 cts. has he? How many times 6 cts.? How many times 5 cts.? How many times 3 cts.?

17. How many times 6 boys are 30 boys?

18. How many times 7 horses are 21 horses?

19. How many times is 7 contained in 28?

39. The process of finding how often one number is contained in another is called **Division.**

40. The number to be divided is called the *Dividend.*

41. The number which is contained in the other is called the *Divisor.*

42. The result obtained by the division is called the *Quotient.*

43. The *sign* of division, ÷, is read *divided by,* and when placed between two numbers shows that the first is to be divided by the second. Thus, 165 ÷ 15 is read 165 divided by 15.

DIVISION TABLE.

1 in		2 in		3 in		4 in	
1,	1 time.	2,	1 time.	3,	1 time.	4,	1 time.
2,	2 times.	4,	2 times.	6,	2 times.	8,	2 times.
3,	3 "	6,	3 "	9,	3 "	12,	3 "
4,	4 "	8,	4 "	12,	4 "	16,	4 "
5,	5 "	10,	5 "	15,	5 "	20,	5 "
6,	6 "	12,	6 "	18,	6 "	24,	6 "
7,	7 "	14,	7 "	21,	7 "	28,	7 "
8,	8 "	16,	8 "	24,	8 "	32,	8 "
9,	9 "	18,	9 "	27,	9 "	36,	9 "
10,	10 "	20,	10 "	30,	10 "	40,	10 "
11,	11 "	22,	11 "	33,	11 "	44,	11 "
12,	12 "	24,	12 "	36,	12 "	48,	12 "

5 in		6 in		7 in		8 in	
5,	1 time.	6,	1 time.	7,	1 time.	8,	1 time.
10,	2 times.	12,	2 times.	14,	2 times.	16,	2 times.
15,	3 "	18,	3 "	21,	3 "	24,	3 "
20,	4 "	24,	4 "	28,	4 "	32,	4 "
25,	5 "	30,	5 "	35,	5 "	40,	5 "
30,	6 "	36,	6 "	42,	6 "	48,	6 "
35,	7 "	42,	7 "	49,	7 "	56,	7 "
40,	8 "	48,	8 "	56,	8 "	64,	8 "
45,	9 "	54,	9 "	63,	9 "	72,	9 "
50,	10 "	60,	10 "	70,	10 "	80,	10 "
55,	11 "	66,	11 "	77,	11 "	88,	11 "
60,	12 "	72,	12 "	84,	12 "	96,	12 "

9 in		10 in		11 in		12 in	
9,	1 time.	10,	1 time.	11,	1 time.	12,	1 time.
18,	2 times.	20,	2 times.	22,	2 times.	24,	2 times.
27,	3 "	30,	3 "	33,	3 "	36,	3 "
36,	4 "	40,	4 "	44,	4 "	48,	4 "
45,	5 "	50,	5 "	55,	5 "	60,	5 "
54,	6 "	60,	6 "	66,	6 "	72,	6 "
63,	7 "	70,	7 "	77,	7 "	84,	7 "
72,	8 "	80,	8 "	88,	8 "	96,	8 "
81,	9 "	90,	9 "	99,	9 "	108,	9 "
90,	10 "	100,	10 "	110,	10 "	120,	10 "
99,	11 "	110,	11 "	121,	11 "	132,	11 "
108,	12 "	120,	12 "	132,	12 "	144,	12 "

ORAL EXERCISE.

1. Divide by 2's from 2 to 12; thus, 2 in 2, 1 time; 2 in 4, 2 times, and so on.

2. Divide by 2's from 12 to 2; thus, 2 in 12, 6 times; 2 in 10, 5 times, and so on.

3. Divide—
   By 3's from 3 to 24.
   By 3's from 36 to 12.

4. Divide—
   By 4's from 4 to 28.
   By 4's from 48 to 24.

5. Divide—
   By 5's from 5 to 40.
   By 5's from 60 to 20.

6. Divide—
   By 6's from 6 to 42.
   By 6's from 72 to 24.

7. Divide—
   By 7's from 7 to 63.
   By 7's from 84 to 28.

8. Divide—
   By 8's from 8 to 56.
   By 8's from 96 to 32.

9. Divide—
   By 9's from 9 to 81.
   By 9's from 108 to 27.

10. Divide—
   By 10's from 10 to 80.
   By 10's from 120 to 40.

11. Divide—
   By 11's from 11 to 132.
   By 11's from 132 to 11.

12. Divide—
   By 12's from 12 to 96.
   By 12's from 144 to 48.

1. How many oranges at 5 cents each can I buy for 30 cents?

SOLUTION 1.—If 1 orange cost 5 cents, for 30 cents I can buy 6 oranges, because 6 times 5 are 30.

SOLUTION 2.—If 1 orange cost 5 cents, for 30 cents I can buy as many oranges as 5 is contained times in 30, or 6 oranges.

2. If a hat cost 4 dollars, how many hats can be bought for 20 dollars?

3. There are 4 pecks in a bushel: how many bushels are there in 24 pecks?

4. There are 7 days in 1 week: how many weeks are there in 42 days?

5. There are 8 quarts in a peck: how many pecks in 56 quarts?

6. How many coats can be bought for 63 dollars if 1 coat cost 9 dollars?

7. If 1 quart of milk cost 10 cents, how many quarts can be bought for 80 cents?

8. How many pair of boots at 7 dollars a pair can I buy for 77 dollars?

9. There are 12 eggs in a dozen: how many dozen are there in 108 eggs?

10. If 12 yards of calico make a dress, how many dresses can be made from 84 yards?

11. If a sheep cost 6 dollars, how many can be bought for 54 dollars?

12. If a man earn 8 dollars a week, how long will it take to earn 80 dollars?

13. A pair of shoes cost 3 dollars: how many pairs can be bought for 36 dollars?

14. How many horses will eat 60 bushels of corn if 1 horse eat 5 bushels?

15. If a cord of wood is worth 6 dollars, how many cords can be bought for 54 dollars?

CASE I.

The Divisor One Figure.

Ex. Divide 984 by 4.

PROCESS.
4)984
246

Explanation.—We write the divisor at the left of the dividend, separating them by a curved line, and draw a line under the dividend. 4 is contained in 9 hundred 2 hundreds times, with 1 hundred, equal to 10 tens, remaining; adding this remainder to 8 tens, the next number in the dividend, we have 18 tens; 4 is contained in 18 tens, 4 tens times, with a remainder of 2 tens, or 20 units; adding this remainder to 4

units, the next number of the dividend, we have 24 units; 4 is contained in 24 units 6 units times.

NOTE.—In practice we shorten the operation and say thus: 4 is contained in 9 twice, and 1 remaining; 4 is contained in 18, 4 times and 2 remaining; 4 is contained in 24, 6 times.

Divide as follows—

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
2)6	3)9	4)12	4)20	5)25	5)30

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
4)28	5)35	3)36	6)12	6)36	7)42

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
6)54	7)56	6)60	9)63	5)45	6)24

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)
7)49	8)32	8)56	8)72	8)96	8)80

(25.)	(26.)	(27.)	(28.)	(29.)	(30.)
6)18	7)70	9)72	9)45	9)81	9)99

(31.)	(32.)	(33.)	(34.)	(35.)	(36.)
4)48	5)50	6)66	4)84	3)963	2)864

(37.)	(38.)	(39.)	(40.)	(41.)	(42.)
5)70	6)96	8)96	7)147	6)126	5)735

(43.)	(44.)	(45.)	(46.)	(47.)	(48.)
6)738	7)483	8)624	5)720	8)944	4)6832

(49.)	(50.)	(51.)	(52.)
5)68405	6)68436	9)78462	6)46824

(53.)	(54.)	(55.)	(56.)
9)468972	7)468342	8)847632	6)345678

NOTE.—If the divisor is not contained in the dividend a whole number of times, write the remainder at the right, with the plus-sign between the answer and the remainder. Thus,

$$3)\underline{644}$$
$$214+2 \text{ rem.}$$

57. If a ton of coal cost 4 dollars, how many tons can be bought for 464 dollars? *Ans.* 116.

58. There are 7 days in a week: how many weeks in 364 days? *Ans.* 52.

59. A pair of boots cost 9 dollars: how many pairs at the same rate can be bought for 225 dollars? *Ans.* 25.

60. If 9 horses are worth 1215 dollars, how much is 1 horse worth? *Ans.* 135 dollars.

61. If a farmer can raise 1504 bushels of potatoes on 8 acres, how many bushels can he raise on 1 acre?
Ans. 188.

62. If a man can earn 6 dollars in one week, how many weeks will it take to earn 690 dollars? *Ans.* 115.

63. In an orchard of 240 trees there are 8 trees in a row: how many rows are there? *Ans.* 30.

64. A train of cars runs 280 miles in 8 hours: how far does it move in an hour? *Ans.* 35 miles.

65. If 7 horses cost 980 dollars, how much does 1 horse cost? *Ans.* 140 dollars.

44. The mode of dividing in the foregoing problems is called **Short Division.**

The divisor, the dividend and the quotient only are written in Short Division.

45. When also the different steps of the solution are written, the process is called **Long Division.**

Ex. Divide 744 by 3.

PROCESS 1.

3)744|248
6̲
14
1̲2̲
24
24

Explanation 1.—3 is contained in 7 hundreds 2 hundred times; 2 hundred times 3 are 6 hundreds; 6 hundreds from 7 hundreds, leaves 1 hundred. 1 hundred and 4 tens are 14 tens. 3 is contained in 14 tens 4 tens times; 4 tens times 3 is 12 tens, which subtracted from 14 tens leaves 2 tens. 2 tens and 4 units are 24 units. 3 is contained in 24 units 8 units times; 8 times 3 are 24, which subtracted from 24 leaves nothing. Hence, 744 divided by 3 equals 248.

PROCESS 2.

3)744|248
6̲
14
12̲
24
24

Explanation 2.—3 is contained in 7, 2 times; 2 times 3 are 6; 6 from 7 leaves 1; bringing down the 4, we have 14. 3 is contained in 14, 4 times; 4 times 3 are 12, which subtracted from 14 leaves 2; bringing down 4, the next figure of the dividend, we have 24. 3 is contained in 24, 8 times; 8 times 3 are 24, which subtracted from 24 leaves nothing. Hence, the quotient is 248.

WRITTEN EXERCISES.

1. Divide 32 by 2.
2. Divide 48 by 3.
3. Divide 60 by 4.
4. Divide 75 by 5.
5. Divide 96 by 6.
6. Divide 112 by 8.
7. Divide 324 by 3.
8. Divide 316 by 4.
9. Divide 432 by 3.
10. Divide 612 by 3.
11. Divide 846 by 3.
12. Divide 780 by 4.
13. Divide 840 by 4.
14. Divide 932 by 4.
15. Divide 825 by 5.
16. Divide 960 by 5.

17. Divide 800 by 5.
18. Divide 744 by 6.
19. Divide 894 by 6.
20. Divide 366 by 6.
21. Divide 761 by 7.
22. Divide 875 by 7.
23. Divide 672 by 7.
24. Divide 768 by 8.
25. Divide 960 by 8.
26. Divide 936 by 8.
27. Divide 999 by 9.
28. Divide 657 by 7.
29. Divide 675 by 9.
30. Divide 348 by 2.
31. Divide 4264 by 4.
32. Divide 6243 by 3.

CASE II.

To Divide when the Divisor consists of Two or more Figures.

Ex. Divide 4550 by 14.

PROCESS.

```
14)4550|325
   42
   ──
   35
   28
   ──
   70
   70
```

Explanation.—14 is contained in 45 hundreds 3 hundreds times; 3 hundreds times 14 are 42 hundreds; 42 hundreds from 45 hundreds leaves 3 hundreds, or 30 tens; bringing down the 5 tens, the dividend is 35 tens. 14 is contained in 35 tens 2 tens times; 2 tens times 14 are 28 tens; 28 tens from 35 tens leaves 7 tens, or 70 units, which is the dividend. 14 is contained in 70 units 5 units times; 5 times 14 equals 70, which being subtracted from 70 leaves nothing. Hence, the quotient is 325.

Divide—

1. 385 by 11.	20. 6095 by 23.
2. 312 by 12.	21. 8760 by 24. *Ans.* 365.
3. 660 by 15.	22. 4125 by 33. *Ans.* 125.
4. 923 by 13.	23. 14770 by 35. *Ans.* 422.
5. 756 by 14.	24. 14625 by 45. *Ans.* 325.
6. 520 by 20.	25. 27398 by 38. *Ans.* 721.
7. 735 by 21.	26. 17316 by 39. *Ans.* 444.
8. 608 by 16.	27. 32928 by 42. *Ans.* 784.
9. 324 by 18.	28. 26136 by 44. *Ans.* 594.
10. 1064 by 19.	29. 37904 by 46. *Ans.* 824.
11. 1650 by 22.	30. 35280 by 48. *Ans.* 735.
12. 1600 by 25.	31. 34048 by 56. *Ans.* 608.
13. 432 by 24.	32. 27720 by 72. *Ans.* 385.
14. 1680 by 30.	33. 33280 by 64. *Ans.* 520.
15. 1134 by 27.	34. 35406 by 63. *Ans.* 562.
16. 765 by 17.	35. 21489 by 87. *Ans.* 247.
17. 3191 by 28.	36. 62167 by 107. *Ans.* 581.
18. 6110 by 26.	37. 79005 by 115. *Ans.* 687.
19. 5175 by 15.	38. 53125 by 125. *Ans.* 425.

6

39. 1509534 by 234.   *Ans.* 6451.

40. 3160542 by 462.   *Ans.* 6841.

41. There are 24 hr. in a day : how many days are there in 1032 hr.?   *Ans.* 43.

42. If a cow cost 32 dollars, how many at that rate can be bought for 2080 dollars? *Ans.* 65.

43. If some pigs are worth 12 dollars apiece, how many can be bought for 180 dollars? *Ans.* 15.

44. If a man walk 25 mi. in a day, how long will it take him to walk 950 mi.? *Ans.* 38 d.

45. If a train run 33 mi. an hour, how long will it be in running 1386 mi.?   *Ans.* 42.

46. There are 60 min. in an hour : how many hours in 3900 min.?   *Ans.* 65.

47. At 55 cts. a pair, how many pairs of chickens can be bought for 1210 cts.?   *Ans.* 22.

48. There are 16 oz. in a pound : how many pounds are there in 1968 oz.?   *Ans.* 123.

49. Bought mules at 115 dollars each : how many did I get for 2760 dollars?   *Ans.* 24.

50. A received 5250 dollars for horses at 125 dollars each : how many did he sell? *Ans.* 42.

51. How many pineapples at 16 cts. each may be bought for 400 cts.?   *Ans.* 25.

52. How many pounds of beef at 18 cts. a pound can be bought for 540 cts.?   *Ans.* 30.

53. There are 32 qt. in a bushel : how many bushels in 1536 qt.?   *Ans.* 48.

54. There are 64 pt. in a bushel : how many bushels in 2688 pt.?   *Ans.* 42.

55. There are 24 sheets in a quire : how many quires in 864 sheets?   *Ans.* 36.

CASE III.

To Divide when there are Ciphers at the Right of the Divisor.

Ex. Divide 6783 by 600.

PROCESS.

6|00)67|83
11+183 rem.

Explanation.—600 is contained in 67 hundreds 11 times, with a remainder of 1 hundred. 600 is not contained in 83; hence, the entire remainder is 183.

NOTE.—When the divisor with the ciphers cut off is greater than 12, divide by Long Division.

Find the value—

1. Of 725 ÷ 30.
2. Of 864 ÷ 70.
3. Of 892 ÷ 80.
4. Of 7642 ÷ 60.
5. Of 6484 ÷ 200.
6. Of 8645 ÷ 500.
7. Of 7887 ÷ 700.
8. Of 9484 ÷ 600.
9. Of 8642 ÷ 1200.
10. Of 54224 ÷ 1500.

11. Of 3786 + 1700.
12. Of 25761 + 2100.
13. Of 46483 + 2500.
14. Of 31400 + 1600.
15. Of 61380 ÷ 3300.
16. Of 75611 + 4000.
17. Of 21500 ÷ 3600.
18. Of 75643 + 4500.
19. Of 45742 + 6000.
20. Of 378751 + 12300.

CASE IV.

To find the Equal Parts of a Number.

Ex. A man bought 15 cows for 495 dollars: how much did 1 cost?

SOLUTION.

15)495|33
45
45
45

Explanation.—If 15 cows cost 495 dollars, 1 cow cost as many dollars as 15 is contained times in 495, or 33 dollars.

WRITTEN PROBLEMS.

1. If 16 hens lay 848 eggs in a summer, how many eggs will 1 hen lay at the same rate?   *Ans.* 53.

2. If 15 horses cost 1740 dollars, how much will 1 horse cost? *Ans.* 116 dollars.

3. If 23 tons of hay are sold for 552 dollars, how much is 1 ton worth? *Ans.* 24 dollars.

4. What can 1 man earn in a month if 24 men earn 1344 dollars? *Ans.* 56 dollars.

5. If 42 men lay 14700 bricks in a half day, how many can 1 man lay at the same rate? *Ans.* 350.

6. A drover sells 64 cows for 2880 dollars: how much is that for each cow? *Ans.* 45 dollars.

7. A drover sells 344 hogs for 4816 dollars: how much does he get apiece? *Ans.* 14 dollars.

8. If 43 ducks lay 5332 eggs in a season, how many eggs does 1 duck lay? *Ans.* 124.

9. A farm of 118 A. was sold for 17110 dollars: how much was that per acre? *Ans.* 145 dollars.

10. A farmer raises 7656 bu. of potatoes on 29 A: how many bushels does he raise on 1 A.? *Ans.* 264.

COMBINATION PROBLEMS.

1. What is the value of $644 + 584 - 500, \div 104$? *Ans.* 7.

2. What is the value of $1182 + 4208 - 4030, \div 85$?
Ans. 16.

3. What is the value of $74400 + 63300, \div 324$?
Ans. 425.

4. From 6492 subtract 3468, and divide the remainder by 27. *Ans.* 112.

5. A man bought 16 horses at 120 dollars each, and sold them all for 2000 dollars: what was the gain on 1 horse? *Ans.* 5 dollars.

6. A drover bought 25 cows for 1000 dollars, and sold them for 1200 dollars: how much did he gain on each cow? *Ans.* 8 dollars.

7. A man earns 25 dollars a week, and spends 12 dollars a week ; he saves 195 dollars : how many weeks does he work ? *Ans.* 15.

8. If a farmer buy 16 horses at 110 dollars each, and gain 240 dollars on the lot, at what price each does he sell the horses? *Ans.* 125 dollars.

9. I traded 16 hens at 45 cts. each for ducks at 40 cts. each : how many ducks did I get ? *Ans.* 18.

10. Sold 6 lb. of butter at 35 cts. a pound, and 4 chickens at 30 cts. apiece, and took in exchange muslin at 15 cts. a yard : how many yards did I get ? *Ans.* 22.

11. A laborer worked 16 d. at 80 cts. a day, and took his pay in potatoes at 40 cts. a bushel : how many bushels of potatoes did he get ? *Ans.* 32.

12. I sell to a merchant 3 bu. of potatoes at 80 cts. a bushel, and 15 lb. of butter at 25 cts. a pound ; he pays me cash 75 cts., and the rest in coffee at 30 cts. a pound : how many pounds of coffee do I get ? *Ans.* 18.

13. A boy who wishes to buy some books worth 75 dollars, saves 7 dollars a week for 9 wk. : how much does he still need ? *Ans.* 12 dollars.

14. If two men have 15 horses worth 90 dollars each and 10 cows worth 40 dollars each, what is the value of each one's share of the stock ? *Ans.* 875 dollars.

15. A man earns 544 dollars in 16 wk., but spends 3 dollars a week of this amount : how much does he save each week ? *Ans.* 31 dollars.

16. A farmer has 24 cows and 93 sheep, worth 1521 dollars : if the sheep are worth 5 dollars each, how much is each cow worth ? *Ans.* 44 dollars.

17. A drover bought 16 horses at 120 dollars each and 8 horses at 150 dollars each : what was the average price ? *Ans.* 130 dollars.

6 *   E

18. If a carpenter charge 18 dollars a week for 9 wk. in building a barn, and 420 dollars for his lumber, what does the barn cost?   *Ans.* 582 dollars.

19. A farmer gave his farm of 160 acres for a store worth 12000 dollars: what was the land worth an acre?
                                        Ans. 75 dollars.

20. If a clerk has a salary of 2500 dollars a year, and spends 6 dollars a day for 365 days, how much has he left at the close of the year?   *Ans.* 310 dollars.

CHAPTER II.

UNITED STATES MONEY.

SECTION I.

DEFINITIONS AND PRINCIPLES.

46. United States Money, sometimes called **Federal Money**, consists of dollars, cents and mills.

TABLE.

10 mills (m.)	= 1 cent, c.
10 cents	= 1 dime, d.
10 dimes, or 100 cents	= 1 dollar, $.
10 dollars	= 1 eagle, E.

47. In business, dollars, cents and mills only are used. A *quarter-dollar* is 25 cents, a *half-dollar* is 50 cents.

The *dollar* is denoted by the following sign, $, called the dollar-sign.

48. Dollars are separated from cents, in writing, by a

point, called a *separatrix*. Thus, 3 dollars and 25 cents is written $3.25; four dollars and 5 cents is written $4.05.

Cents occupy the second place at the right of the point; the first place is occupied by dimes.

When there are no dimes or no cents the vacant places are filled with naughts.

49. United States money may be either *paper money* or *coins.* Coin is sometimes called specie; and paper money, paper currency.

ORAL EXERCISE.

1. How many cents in 3 dimes?
2. How many cents in 2 dollars?
3. How many cents in 3 dollars and 16 cents?
4. How many cents are equal to a five-dollar bill?
5. How many cents are equal to a dollar bill and 25 cents?
6. How many cents in a half-dollar and a quarter-dollar?
7. How many cents in 1 dollar and a half?
8. How many dimes in 4 dollars?
9. How many cents are equal to 2 five-dollar bills?
10. How many dollars in 3 eagles?

Read the following:

$1.15.	$14.25.	$21.50.	$.243.
$3.24.	$6.00.	$107.16.	$.803.
$11.17.	$18.05.	$107.60.	$8.03.
$19.30.	$25.07.	$100.70.	$6.003.

Write—

1. One dollar and twelve cents.
2. Eight dollars and twenty-five cents.
3. Three hundred and fourteen dollars and forty-two cents.

4. Ten dollars and nine cents.
5. Six cents and six mills.
6. One dollar and a half.
7. Eighty dollars and eighty cents.
8. Twenty dollars two cents and two mills.
9. Three hundred dollars and thirty cents.
10. Thirty dollars and three mills.

SECTION II.

REDUCTION OF UNITED STATES MONEY.

NOTE.—Since 1 cent equals 10 mills, and $1 equals 100 cents, or 1000 mills, we have the following

RULES.

1. *To reduce cents to mills, multiply by* 10, *or annex a cipher.*

2. *To reduce dollars to cents, multiply by* 100, *or annex two ciphers.*

3. *To reduce dollars to mills, multiply by* 1000, *or annex three ciphers.*

4. *To reduce dollars and cents to cents, or dollars, cents and mills to mills, remove the dollar-sign and the separatrix.*

5. *To reduce mills to cents, divide by* 10 ; *cents to dollars, divide by* 100 ; *and mills to dollars, divide by* 1000, *and write the sign of the denomination required.*

WRITTEN EXERCISE.

1. How many mills in 3 cents? In 45 cents?

2. How many cents in 5 dollars? In 40 dollars? In $87.

3. How many mills in 42 cents?   In $4.18?
4. How many cents in $41.16?   In $84.12?
5. How many mills in 4 cents and 4 mills?
6. How many mills in $6.066?
7. Reduce 647 mills to cents and mills.
8. Reduce 847 cents to dollars.
9. Reduce $8.475 to mills.
10. Reduce $400.03 to cents.

SECTION III.

ADDITION OF UNITED STATES MONEY.

ORAL EXERCISE.

1. A hen cost 40 cents, and a duck 30 cents: how much did both cost?

SOLUTION.—If a hen cost 40 cents, and a duck 30 cents, the two cost 40 cents plus 30 cents, or 70 cents.

2. A book cost $1.25, and a slate 50 cents: how much did they both cost?

3. A pair of shoes cost $2.50, and a hat $2.25: how much did they both cost?

4. If I pay $1.20 for a turkey, $1.15 for a goose, and 60 cents for some butter, how much do I pay for all?

5. If a cow cost $25.50, and a sheep $7.75, how much will both cost?

6. If I earn 20 cents on Monday, and 75 cents on Tuesday, how much do I earn in both days?

7. If I pay $7.50 for a coat, and $2.25 for a pair of pants, how much do both cost?

8. A book cost 90 cents, a pen-holder 10 cents, and a slate 35 cents: how much did they all cost?

WRITTEN PROBLEMS.

NOTE.—In solving problems in Addition of United States Money, write the numbers, placing cents under cents and dollars under dollars, and add as in simple addition. Separate dollars and cents by a point and prefix the dollar-sign. ⌐

Ex. Add $18.30, $16.25, $18.40 and $13.21.

PROCESS.   *Explanation.*—1 and 5 are 6 cents; 2 and 4 are 6, and
$18.30   2 are 8, and 3 are 11 dimes, or 1 dollar and 1 dime; 1
  16.25   and 3 are 4, and 8 are 12, and 6 are 18, and 8 are 26
  18.40   dollars; write the 6 dollars, and add the 2 ten-dollars to
  13.21   the next column. 2 and 1 are 3, and 1 are 4, and 1 are
$66.16   5, and 1 are 6 tens of dollars. Hence, the sum is $66.16.

Add the following:

(1.)	(2.)	(3.)	(4.)
$18.25	$54.34	$57.60	$105.20
17.24	71.56	75.20	110.00
121.43	84.93	18.00	409.05
67.44	62.71	10.00	1000.65

5. I sold 1 horse for $87.50, and another for $94.75: how much did I get for both?   *Ans.* $182.25.

6. I bought sugar for $4.24, coffee for $1.25 and rice for 63 cts.: how much did they all cost me? *Ans.* $6.12.

7. A merchant's receipts for 4 d. were $16.20, $13.40, $27.42 and $19.75: how much were his receipts?
Ans. $76.77.

8. In building a barn I paid the carpenter $97.40, the mason $25, and for lumber $367.45: how much did my barn cost?   *Ans.* $489.85.

9. I paid $9.75 for a coat, $1.50 for a vest, $3.50 for a hat and $3.75 for a pair of pants: what was the cost of all?   *Ans.* $18.50.

10. A man's coal-bill for the year was $50.50, his rent $225, his groceries $460.73 and his other expenses $630.73: what were his expenses for the year? *Ans.* $1366.96.

SECTION IV.

SUBTRACTION OF U. S. MONEY.

ORAL EXERCISE.

1. If a knife cost \$1.25, and it is sold for \$1.75, what is the gain?

SOLUTION.—If a knife cost \$1.25, and it is sold for \$1.75, the gain is the difference between \$1.75 and \$1.25, which is 50 cents.

2. A merchant bought some goods at \$2.62 a yard, and sold them at \$3.89 a yard: what was the gain per yard?

3. A huckster bought turkeys at \$1.20 apiece, paid 10 cts. apiece for their feed, and sold them at \$1.75 apiece: what was the gain?

4. Bought some rice for 60 cts., some sugar for 45 cts. and some tea for \$1: how much change should I get from a five-dollar bill?

5. Bought a horse for \$120, a saddle for \$15, and sold both for \$150: what was my gain?

WRITTEN PROBLEMS.

NOTE.—In solving problems in Subtraction of United States Money, write the numbers, placing cents under cents and dollars under dollars, and proceed as in simple subtraction. Separate the dollars from the cents in the result and prefix the dollar-sign.

Ex. Subtract \$14.26 from \$19.41.

PROCESS.   *Explanation.*—We cannot subtract 6 cts. from 1 ct.;
\$19.41   hence we take 1 dime, or 10 cts., which with 1 ct. equals
14.26   11 cts.; 6 cts. from 11 cts. leaves 5 cts.; 2 dimes from 3
\$5.15   dimes leaves 1 dime; 4 dollars from 9 dollars leaves 5
dollars. Hence, the difference is \$5.15.

	(1.)	(2.)	(3.)	(4.)
From	\$6.24	\$27.62	\$75.90	\$60.50
Take	3.12	19.45	27.54	18.46

5. What is the difference between $18 and $9.75?

<div align="right">*Ans.* $8.25.</div>

6. If a horse was bought for $175, and sold for $181.25, what was the gain?   *Ans.* $6.25.

7. A carpenter's whole expense in building a house was $1562.35; he received $1700: how much did he make?   *Ans.* $137.65.

8. Bought a house for $1875 and a lot for $525, and sold both for $2350: how much did I lose? *Ans.* $50.

9. My salary is $1000 a year; I pay for rent $150, for groceries $325.40, for butter $60.30, for dry goods $127.63, and for other expenses $75.60: how much do I save?

<div align="right">*Ans.* $261.07.</div>

10. A house and lot cost me $1927.50; I paid for repairs $127.67, and sold the property for $2500: what was my gain?   *Ans.* $444.83.

SECTION V.

MULTIPLICATION OF U. S. MONEY.

ORAL EXERCISE.

1. What will 5 pencils cost at 6 cents each?

SOLUTION.—At 6 cents each 5 pencils will cost 5 times 6 cents, or 30 cents.

2. What will 3 books cost at 90 cents each?

3. If 1 yard of muslin is worth 15 cents, how much are 40 yards worth?

4. If a pair of shoes cost $2.50, how much will 12 pairs cost?

5. What is the cost of 2 horses at 120 dollars each, and 3 cows at 40 dollars each?

6. If I buy 2 hats at $1.20 each, and a book for $2, how much change do I get from a five-dollar bill?

7. A man bought 3 bushels of wheat at $1.25, and sold the flour he made from it for $4.50: what did he gain?

WRITTEN PROBLEMS.

Note.—Multiply in United States Money as in simple numbers, separate dollars from cents, and prefix the dollar-sign.

Ex. Multiply $16.25 by 7.

Process.   *Explanation.*—7 times 5 cents are 35 cents, or 3 dimes
$16.25   and 5 cents; 7 times 2 dimes are 14 dimes, + 3 dimes are
7   17 dimes, or 1 dollar and 7 dimes; 7 times 6 dollars are
$113.75   42 dollars, + 1 dollar are 43 dollars; we write the 3 dollars, and add the 4 to the next product; 7 times 1 ten-dollar are 7 ten-dollars, + 4 ten-dollars are 11 ten-dollars. Hence, the product is $113.75.

1. Multiply $27 by 3.
2. Multiply $121 by 16.
3. Multiply $8.17 by 4.
4. Multiply $20.96 by 14.
5. Multiply $70.15 by 21.
6. Multiply $18.27 by 84.
7. Multiply $107.27 by 46.
8. Multiply $150.05 by 705.

9. What are 19 hogs worth at $13.75 each?
Ans. $261.25.

10. What will 43 yards of carpet cost at $1.45 a yard?
Ans. $62.35.

11. What is the cost of 93 cords of wood at $3.75 a cord?
Ans. $348.75.

12. What is the value of 12 books at $1.50 each, and 25 books at $1.75 each?
Ans. $61.75.

13. A farmer sells 117 bushels of wheat at $1.35 a bushel, and 40 bushels of oats at 62 cents a bushel: how much does he get for both?
Ans. $182.75.

14. A merchant sells 19 yards of cloth at $2.25 a yard,

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and 27 yards of carpet at 95 cents a yard: how much does he get for both?   *Ans.* $68.40.

15. A lady goes to market with 10 dollars; she buys 6 dozen of eggs at 27 cents, 7 pounds of meat at 16 cents, and 3 bushels of potatoes at $1.25: how much money has she remaining?   *Ans.* $3.51.

16. A drover bought 95 cows at $37.25 each, and sold them at $40 each; how much did he make?

Ans. $261.25.

SECTION VI.

DIVISION OF UNITED STATES MONEY.

ORAL EXERCISE.

1. If a melon cost 25 cents, how many can be bought for $1.50.

SOLUTION.—If a melon cost 25 cents, for $1.50 there can be as many bought as 25 cents is contained times in $1.50, or 150 cents, which is 6.

2. If 6 sheep cost $42, how much will 1 sheep cost?

3. At 10 cents apiece, how many copy-books can be bought for $2.50?

4. If 9 cords of wood cost $36, how much will 1 cord cost?

5. At 20 cents apiece, how many pineapples can I buy for $1.80?

6. At the rate of 25 cents a dozen, how many dozen buttons can be bought for $3?

7. If a man earn $40 in 8 d., how much can he earn in 1 d.?

8. If I buy 17 lb. of sugar at 10 cts. a pound, how many oranges at 5 cts. each can I get for the change due me from a five-dollar bill ?

9. A yard of calico is worth 9 cts.: how many yards can I get for 10 doz. of eggs, worth 18 cts. a dozen ?

10. If I trade 6 lb. of butter at 20 cts. a pound, and 10 lb. of lard at 12 cts. a pound, for sugar at 12 cts. a pound, how many pounds of sugar do I get ?

CASE. I.

To find how often One Sum of Money is contained in Another.

WRITTEN PROBLEMS.

NOTE.—When finding how often one sum of money is contained in another, reduce both sums to the same denomination and divide as in simple numbers.

Ex. How often is 14 cts. contained in $3.22 ?

PROCESS.

14)322|23
  28
   42
   42

Explanation.—$3.22 is equal to 322 cts.; 14 cts. is contained in 322 cts. 23 times.

1. Divide $6000 by $12.

2. Divide $3200 by $160.

3. Divide $56 by 80 cts.

4. At $16 each, how many coats can be bought for $384 ?   *Ans.* 24.

5. At $45 an acre, how many acres of land can be bought for $3330 ?   *Ans.* 74.

6. If hats cost $2.25 apiece, how many can be bought for $56.25 ?   *Ans.* 25.

7. How many bushels of oats at 55 cts. a bushel can I buy for $58.85 ?   *Ans.* 107.

8. How many cows at $24 each can be bought for $1512?   *Ans.* 63.

9. At 27 cts. a yard, how many yards of dress goods can I get for $14.31 ?   *Ans.* 53.

10. At $2.25 a yard, how many yards of cloth can be bought for $36 ?   *Ans.* 16.

CASE II.
To Divide a Number into Equal Parts.

NOTE.—To find the equal parts of a number, divide as in simple numbers, and separate dollars from cents with the separatrix.

Ex. If 5 cows cost $125.75, how much will 1 cow cost?

SOLUTION.   *Explanation.*—If 5 cows cost $125.75, or 12575 cts.,
5)125.75   1 cow will cost as many cents as 5 is contained in
   25.15   12575, or 2515 cts., equal to $25.15.

1. Divide $9.50 into 5 equal parts.

2. Divide $5.10 into 6 equal parts.

3. If 6 books cost $6.90, how much will 1 book cost?
                                               Ans. $1.15.

4. If 8 inkstands cost $6, how much will 1 inkstand cost?   *Ans.* $.75.

5. If 11 T. of coal cost $63.25, how much will 1 T. cost?   *Ans.* $5.75.

6. If 12 bu. of oats cost $3.60, how much will 5 bu. cost?   *Ans.* $1.50.

7. If a man earn $7.50 in 5 d., how much can he earn in 6 d.?   *Ans.* $9.

8. If 9 lb. of butter cost $2.43, how much will 50 lb. cost?   *Ans.* $13.50.

9. If 42 A. of land cost $1890, how much will 65 A. cost?   *Ans.* $2925.

10. If 23 yd. of carpet cost $28.75, how much will 40 yd. of the same carpet cost?   *Ans.* $50.

COMBINATION PROBLEMS.

1. A farmer brings to a grocer 10 doz. of eggs at 18 cts. a dozen, 26 lb. of lard at 15 cts. a pound, and takes in exchange 10 lb. of coffee at 23 cts. a pound, and a set of dishes worth $2: how much is due him in cash? *Ans.* $1.40.

2. A farmer gave a horse worth 175 dollars and 3 cows worth 40 dollars each for some sheep worth $5 each: how many sheep did he get? *Ans.* 59.

3. A farmer exchanged 60 bu. of potatoes at 45 cts. a bushel, 3 bbls. of flour at $6.50, and 50 bu. of wheat at $1.20 a bushel, for carpet worth $1.50 a yard: how many yards did he get? *Ans.* 71.

4. A merchant sells the following goods: 10 yd. of calico at 9 cts. a yard, 21 yd. of muslin at 13 cts. a yard, 18 yd. of delaine at 35 cts. a·yard, and takes in exchange 15 bu. of corn at 62 cts. a bushel, and the remainder in cash: how much cash did he get? *Ans.* $.63.

5. A man buys a farm for $16000; he buys 6 horses at $130 each, 15 cows at $32 each, 1 pair of oxen for $96, 15 pigs at $4.25 each, and 70 sheep at $3.75 each, and sells all for $18000: how much does he gain?

Ans. $317.75.

BILLS.

50. A **Bill** of goods is a written statement of the goods sold, giving quantity and price of each article and total cost, also the date of the sale, with the names of the buyer and the seller.

51. The party who owes is called a *Debtor*, and the party to whom a debt is owed is called a *Creditor*.

NOTE.—In mercantile and other business transactions if the fractional part of a cent is less than one-half, it is not counted; if it is equal to a half cent or greater, it is counted a cent.

7 *

Make out the following bills:

(1.)

LOCK HAVEN, PA., *July* 3, 1877.

MR. WILSON KISTLER,

Bought of JACOB BROWN.

15 lb. Coffee at 32¢	$4.80
16 " Lard at 15¢	2.40
25 " Sugar at 13¢	3.25
16 " Ham at 16¢	2.56
	Amt. $13.01

(2.)

READING, PA., *Aug.* 7, 1877.

DR. J. H. BARTON,

Bought of W. W. RANKIN & CO.

3 tons Coal at $5.75
16 " Coal at 5.25
4 " Coal at 5.00

Amt. $121.25.

(3.)

LANCASTER, PA., *June* 6, 1876.

MR. SAMUEL CHRIST,

Bought of S. D. BALL.

6 tons Hay   at $18
16 bushels Rye at 1.25
27 " Corn at .65

Amt. $145.55.

(4.)

BUFFALO, N. Y., *Jan.* 1, 1877.

HON. J. W. SMITH,

Bought of ABRAHAM BEST.

11 bbl. Flour at $6.75
32 bu. Wheat at 1.63
97 lb. Beef at .09
17 bu. Corn at .45

Amt. $142.79.

(5.)

BROOKLYN, N. Y., *May* 1, 1875.

MR. M. W. HERR,

To JOHN A. ROBB, *Dr.*

To 27 yd. Muslin at 10¢
" 16 pr. Shoes at $1.25
" 25 yd. Carpet at 1.15
" 27 yd. Silk at 85

Or.

By 46 bu. Potatoes at $.75
" 33 lb. Butter at .25
" 16 doz. Eggs at .18

Bal. due, $28.77.

6. Jan 13, 1877, L. A. Thompson bought of A. C. Harrison of Harrisburg, Pa., 32 lb. sugar, at 13¢; 11 lb. coffee, at 27¢; 26 lb. soap, at 9¢; 14 lb. rice, at 9¢; 127 lb. fish, at 13¢, and 18 lb. crackers, at 12¢.
Make out a bill.   *Ans.* $29.40.

CHAPTER III.

PROPERTIES OF NUMBERS.

SECTION I.

FACTORS AND MULTIPLES.

52. A **Divisor** of a number is any number that will exactly divide it.

53. A **Factor** of a number is one of its exact divisors.

54. A **Prime Number** is one that has no factors, and therefore no exact divisor.

55. A Composite Number is one that may be divided, and always is the product of two or more factors.

2, 3, 5, 7, 11, etc. are *prime* numbers.

4, 6, 8, 9, 10, etc. are *composite* numbers.

56. A Prime Factor is a factor that cannot be divided.

57. When any number is a factor of two or more numbers, it is called their *common* factor. Thus, 2 is a common factor of 6 and 8.

58. An Even Number is one whose right-hand figure is 0, 2, 4, 6 or 8. All even numbers are divisible by 2.

59. An Odd Number is one whose right-hand figure is 1, 3, 5, 7 or 9.

60. Factoring is the process of finding the factors of a composite number.

61. A factor is said to be common to two or more numbers when it is found in each of them.

ORAL EXERCISE.

1. What are the factors of 10?

SOLUTION.—The factors of 10 are 2 and 5, because 10 is divisible by 2 or 5.

2. What are the factors of 6? Of 8? Of 12?

3. What are the factors of 12? Of 15? Of 20?

4. What are the factors of 18? Of 24? Of 30?

5. What are the factors of 33? Of 49? Of 50?

6. What are the prime factors of 30?

SOLUTION.—The prime factors of 30 are 3, 2 and 5, because 3, 2 and 5 are the only prime numbers by which 30 is divisible.

7. What are the prime factors of 12? Of 8?

8. What are the prime factors of 15? Of 20? Of 22?

9. What are the prime factors of 25? Of 27? Of 36?

10. What are the prime factors of 33? Of 44? Of 60?

11. What are the prime factors of 75? Of 100? Of 80?

12. What prime factors are found in 4 and 6? In 6 and 9?

13. What prime factor is common to 10 and 12? 8 and 12?

14. What prime factor is common to 15 and 20? 18 and 20? 16 and 20? 18 and 24? 30 and 50?

15. Name the prime numbers from 1 to 50.

16. Name the prime numbers from 50 to 100.

17. Name the composite numbers from 1 to 100.

18. Name the factors of the composite numbers from 1 to 50.

WRITTEN EXERCISE.

Ex. What are the prime factors of 360?

SOLUTION.   *Explanation.*—Dividing 360 by the prime factor 3,
  3)360   the result is 120; dividing 120 by the prime factor 3,
  3)120   the result is 40; dividing 40 by the prime factor 2,
  2)40   the result is 20; dividing 20 by the prime factor 2,
  2)20   the result is 10; dividing 10 by the prime factor 2,
  2)10   the result is 5, which is also prime. Hence, the prime
    5   divisors or prime factors of 360 are 3, 3, 2, 2, 2, 5.

Find the prime factors—

1. Of 30.   *Ans.* 2, 3, 5.   9. Of 420.
2. Of 60.   *Ans.* 2, 2, 3, 5.   10. Of 270.
3. Of 48. *Ans.* 2, 2, 2, 2, 3.   11. Of 475.
4. Of 125.   *Ans.* 5, 5, 5.   12. Of 800.
5. Of 120. *Ans.* 3, 2, 2, 2, 5.   13. Of 1200.
6. Of 144.   14. Of 1875.
7. Of 175.   15. Of 4620.
8. Of 180.   16. Of 1440.

F

What prime factors are common—

17. To 10 and 20 ?	22. To 25 and 125 ?
18. To 12 and 15 ?	23. To 27 and 81 ?
19. To 18 and 24 ?	24. To 120 and 600 ?
20. To 30 and 40 ? .	25. To 500 and 700 ?
21. To 60 and 70 ?	26. To 144 and 180 ?

CANCELLATION.

62. Cancellation is the process of shortening operations in Division by rejecting or cancelling equal factors common to both dividend and divisor.

Rejecting the same factor from both dividend and divisor does not affect the value of the quotient.

Ex. Divide 4×60 by 3×8.

PROCESS.

$$\frac{\cancel{4} \times \overset{\overset{10}{\cancel{2\cancel{0}}}}{\cancel{6\cancel{0}}}}{\cancel{3} \times \underset{\cancel{2}}{\cancel{8}}} = 10$$

Explanation.—Since dividing both dividend and divisor does not alter the value of the quotient, dividing both by 3, 4 and 2 by cancelling, the quotient is 10.

EXAMPLES FOR PRACTICE.

1. Find the value of $\dfrac{3 \times 4 \times 5 \times 6}{4 \times 3 \times 6}$.

2. Divide $16 \times 4 \times 5$ by $8 \times 2 \times 10$.   *Ans. 2.*

3. Divide $7 \times 16 \times 6$ by $14 \times 3 \times 8$.   *Ans. 2.*

4. Divide $108 \times 10 \times 12$ by $6 \times 9 \times 20$.   *Ans. 12.*

5. Divide $9 \times 7 \times 16 \times 16$ by $21 \times 32 \times 2$.   *Ans. 12.*

6. Divide 108 by 27.

SOLUTION. $\dfrac{108}{27} = \dfrac{2 \times 2 \times \cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3}} = 4.$

7. Divide 96 by 32 ; 96 by 24.   *Ans. 3 ; 4.*

8. Divide 225 by 15 ; 256 by 32.   *Ans. 15 ; 8.*

9. Divide 288 by 3×48 ; 500 by 5×20.   *Ans. 2 ; 5.*

10. Divide $54 \times 12 \times 28$ by $18 \times 7 \times 4$.   *Ans.* 36.

11. Divide $18 \times 45 \times 280$ by $30 \times 35 \times 24$.   *Ans.* 9.

12. Divide $4 \times 50 \times 12$ by 5×30.   *Ans.* 16.

13. How many hens at 30 cents each can be bought for 2 bushels of corn at 75 cents a bushel ?   *Ans.* 5.

14. How many cows at $25 each will cost as much as 12 horses at $75 each ?   *Ans.* 36.

15. Three pieces of cloth containing 30 yards each, worth $5 a yard, were exchanged for 5 pieces of cloth containing 45 yards each: what was the second kind worth per yard ?   *Ans.* $2.

16. If a farmer exchange 25 bushels of wheat at $1.20 a bushel for delaine at 40 cents a yard, how many yards does he get ?   *Ans.* 75.

SECTION II.

GREATEST COMMON DIVISOR.

63. A **Common Divisor,** or common factor of two or more numbers, is a number that will exactly divide each of them.   Thus, 3 is a common divisor of 6, 9 and 12.

64. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.   Thus, 6 is the greatest common divisor of 12 and 18.

ORAL EXERCISE.

Name a common divisor—

1. Of 6 and 9.   Of 12 and 10.
2. Of 4 and 6.   Of 15 and 18.
3. Of 16 and 20.   Of 18 and 21.
4. Of 25 and 30.   Of 20 and 30.
5. Of 16 and 40.   Of 30 and 50.

Name the greatest common divisor—

      6. Of 6 and 9.   Of 12 and 10.

      7. Of 15 and 18.   Of 12 and 24.

      8. Of 20 and 15.   Of 20 and 30.

      9. Of 25 and 50.   Of 25 and 40.

     10. Of 24 and 48.   Of 48 and 72.

PRINCIPLE.—The product of the common factors of two or more numbers is the greatest common divisor of those numbers.

WRITTEN PROBLEMS.

Ex. Find the greatest common divisor of 15, 30 and 60.

SOLUTION 1.

$15 = 3 \times 5$
$30 = 3 \times 5 \times 2$
$60 = 3 \times 5 \times 2 \times 2$

$3 \times 5 = 15 = $ G. C. D.

Explanation 1.—By examining the solution we find the only prime factors common to 15, 30 and 60 are 3 and 5; hence the greatest common divisor of these numbers is 3×5, or 15.

SOLUTION 2.

$5) \underline{15, \quad 30, \quad 60}$
$3) \underline{3, \quad 6, \quad 12}$
$\quad 1, \quad 2, \quad 4$

$5 \times 3 = 15 = $ G. C. D.

Explanation 2.—5, being contained in all the numbers, is a common factor; 3, being contained in all the numbers, is also a common factor, and since the numbers have no other common factors, the greatest common divisor is 3×5, or 15.

From these solutions we derive the rule for finding the greatest common divisor of two or more numbers:

RULE.

Find the factors common to all the numbers, and take the product of these factors.

Find the greatest common divisor—

      1. Of 15, 20, 30.   *Ans.* 5.

      2. Of 16, 20, 24.   *Ans.* 4.

      3. Of 25, 50, 100.   *Ans.* 25.

4. Of 18, 36, 72.   *Ans.* 18.

5. Of 28, 56, 42.   *Ans.* 7.

6. Of 120, 240, 72.   *Ans.* 24.

7. Of 210, 180, 150.   *Ans.* 30.

8. Of 44, 110, 77.   *Ans.* 11.

9. Of 210 miles, 90 miles, 75 miles.   *Ans.* 15 miles.

10. A man has 2 logs which he wishes to cut into boards of equal length; one is 24 feet, and the other 16 feet long : how long can he cut the boards?   *Ans.* 8 ft.

11. What is the greatest common divisor of $27, $36 and $72 ?   *Ans.* $9.

12. What is the greatest equal lengths into which two trees can be cut, one being 105 feet in length and the other 84 feet?   *Ans.* 21 feet.

SECTION III.

LEAST COMMON MULTIPLE.

65. A **Multiple** of a number is a number which is exactly divisible by that number.

66. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each. Thus, 18 is a common multiple of 9 and 6, because it is divisible by each of them.

67. The **Least Common Multiple** of two or more numbers is the least number exactly divisible by each of them. Thus, 30 is the least common multiple of 10 and 15, because it is the least number exactly divisible by each of them.

ORAL EXERCISES.

What is a multiple—

1. Of 3 ? Of 4 ? Of 5 ? Of 10 ? Of 6 ? Of 20 ?

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What is a common multiple—

  2. Of 3 and 4 ?   Of 4 and 5 ?   Of 5 and 10 ?

  3. Of 6 and 12 ?   Of 5 and 15 ?   Of 5 and 6 ?

What is the least common multiple—

  4. Of 2 and 3 ?   Of 3 and 4 ?   Of 4 and 5 ?

  5. Of 2 and 5 ?   Of 3 and 5 ?   Of 5 and 6 ?

  6. Of 2 and 7 ?   Of 3 and 6 ?   Of 4 and 9 ?

  7. Of 5 and 7 ?   Of 6 and 9 ?   Of 10 and 12 ?

PRINCIPLES.—1. Every multiple of a number contains the prime factors of that number.

2. The least common multiple of two or more numbers contains all the prime factors of each of the numbers, and no other factors.

WRITTEN PROBLEMS.

Ex. Find the least common multiple of 10, 15 and 20.

SOLUTION.

$10 = 2 \times 5$
$15 = 3 \times 5$
$20 = 2 \times 2 \times 5$
L. C. M. $= 2 \times 5 \times 3 \times 2 = 60$

Explanation.—The least common multiple of the given numbers must contain the factors 2 and 5 to be divisible by 10; it must contain the factors 3 and 5 to be divisible by 15; it must contain the factors 2, 2 and 5 to be divisible by 20. Since the number 60 contains all these factors and no others, it is the least common multiple of 10, 15, 20.

From the foregoing we derive the rule for finding the least common multiple of two or more numbers:

RULE.

Find the prime factors of the numbers, and take the product of these factors, using each the greatest number of times it occurs in any of the given numbers.

What is the least common multiple—

  1. Of 15, 10 and 5 ?   *Ans.* 30.

  2. Of 20, 10 and 30 ?   *Ans.* 60.

3. Of 9, 12 and 18 ?   *Ans.* 36.

4. Of 2, 3, 4 and 5 ?   *Ans.* 60.

5. Of 3, 6 and 18 ?   *Ans.* 18.

6. Of 10, 25 and 30 ?   *Ans.* 150.

7. Of 15 and 24 ?   *Ans.* 120.

8. Of 24, 30 and 36 ?   *Ans.* 360.

9. Of $20, $30 and $40 ?   *Ans.* $120.

10. Of 18 men, 16 men and 12 men ? *Ans.* 144 men.

11. What is the smallest tract of land that may be cut into 6-acre, 5-acre or 4-acre lots ?   *Ans.* 60 acres.

12. What is the smallest sum for which I can hire workmen at 6, 8 or 9 dollars a week ?   *Ans.* $72.

13. What is the least common multiple of 10, 15, 20, 25 and 24 ?   *Ans.* 600.

CHAPTER IV.

FRACTIONS.

SECTION I.

DEFINITIONS.

IF a unit be divided into *two* equal parts, each of these parts is called *one-half;* if into *three* equal parts, each part is called *one-third;* if into *four* equal parts, each part is called *one-fourth;* *two* of the four equal parts are called *two-fourths,* and so on.

These parts of a unit—one-half, one-third, one-fourth, two-fourths, three-fourths, etc.—are called *fractions.*

68. A **Fraction** is one or more of the equal parts of a unit.

69. A fraction is usually expressed by writing the numbers one above the other with a short horizontal line between them. Thus, three-fourths is written $\frac{3}{4}$; two-thirds, $\frac{2}{3}$, etc.

70. The number below the line is called the *Denominator*, and shows into how many equal parts the unit is divided.

71. The number above the line is called the *Numerator*, and shows how many of the equal parts are taken. Thus, in $\frac{4}{5}$ the denominator, 5, shows that the unit is divided into 5 equal parts; and 4, the numerator, shows that 4 of these parts are taken.

72. The numerator and the denominator are called the *Terms* of the fraction.

EXERCISE.

Read—$\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{2}{8}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{4}{7}$, $\frac{7}{8}$, $\frac{9}{10}$.

Write one-third, two-fourths, three-ninths, six-sevenths, five-sevenths, three-eighths, five-ninths, seven-tenths, thirteen-fifteenths, sixteen-fortieths.

73. A **Common Fraction** is one in which the numerator and the denominator are both expressed by figures.

74. Common fractions are either *proper* or *improper*.

75. A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{2}{3}$, $\frac{3}{4}$, etc.

76. An **Improper Fraction** is one whose numerator is equal to or greater than its denominator; as, $\frac{3}{3}$, $\frac{5}{4}$, $\frac{8}{5}$, etc.

77. A **Mixed Number** is one consisting of a whole number and a fraction.

SECTION II.

REDUCTION OF FRACTIONS.

CASE I.

To Reduce Whole or Mixed Numbers to Fractions.

ORAL EXERCISE.

1. How many halves in 1 apple? How many thirds?
2. How many fourths in an orange? How many fifths?
3. How many halves in 3 apples?

SOLUTION.—In 1 apple there are 2 halves, and in 3 apples 3 times 2 halves, or 6 halves.

4. How many halves in 4? In 5? In 6? In 8?
5. How many thirds in 2? In 4? In 5? In 10?
6. How many fourths in 3? In 6? In 9? In 12?
7. How many fourths in $5\frac{2}{4}$?

SOLUTION.—In 1 there are $\frac{4}{4}$; in 5 there are 5 times $\frac{4}{4}$, or $\frac{20}{4}$; $\frac{20}{4}$ and $\frac{2}{4}$ are $\frac{22}{4}$. Hence, in $5\frac{2}{4}$ there are $\frac{22}{4}$.

8. How many fifths in 3? In 4? In $5\frac{3}{5}$?
9. How many sixths in 5? In 6? In $7\frac{5}{6}$?
10. How many eighths in 4? In 10? In $6\frac{7}{8}$?
11. How many thirds in $7\frac{2}{3}$? In $8\frac{1}{3}$? In $12\frac{2}{3}$?
12. How many tenths in $6\frac{7}{10}$? In $5\frac{8}{10}$? In $12\frac{2}{10}$?

WRITTEN PROBLEMS.

Ex. Reduce $17\frac{1}{3}$ to an improper fraction.

SOLUTION.
$17\frac{1}{3}$
3

$\overline{52 \text{ thirds}} = \frac{52}{3}$.

Explanation.—In 1 there are $\frac{3}{3}$, and in 17 there are 17 times $\frac{3}{3}$, or $\frac{51}{3}$; $\frac{51}{3} + \frac{1}{3} = \frac{52}{3}$.

From the foregoing we derive the rule for reducing mixed numbers to improper fractions:

8 *

RULE.

Multiply the whole number by the denominator, and to this add the numerator, writing the product over the given denominator.

Reduce to improper fractions—

1. $4\frac{3}{4}$.	*Ans.* $\frac{19}{4}$.	7. $10\frac{6}{7}$.	*Ans.* $\frac{76}{7}$.
2. $5\frac{1}{6}$.	*Ans.* $\frac{31}{6}$.	8. $25\frac{3}{8}$.	*Ans.* $\frac{228}{8}$.
3. $6\frac{1}{4}$.	*Ans.* $\frac{25}{4}$.	9. $30\frac{4}{5}$.	*Ans.* $\frac{154}{5}$.
4. $7\frac{1}{2}$.	*Ans.* $\frac{15}{2}$.	10. $27\frac{3}{8}$.	*Ans.* $\frac{219}{8}$.
5. $8\frac{3}{6}$.	*Ans.* $\frac{53}{6}$.	11. $26\frac{15}{16}$.	*Ans.* $\frac{431}{16}$.
6. $9\frac{3}{4}$.	*Ans.* $\frac{39}{4}$.	12. $24\frac{11}{12}$.	*Ans.* $\frac{299}{12}$.

CASE II.

To Reduce Improper Fractions to Whole or Mixed Numbers.

ORAL EXERCISE.

1. How many ones in $\frac{15}{5}$?

SOLUTION.—Since one equals $\frac{5}{5}$, in $\frac{15}{5}$ there are as many ones as $\frac{5}{5}$ is contained times in $\frac{15}{5}$, or 3.

2. How many ones in $\frac{6}{2}$?   In $\frac{6}{3}$?   In $\frac{8}{8}$?
3. How many ones in $\frac{12}{3}$?   In $\frac{12}{4}$?   In $\frac{16}{4}$?
4. How many ones in $\frac{20}{4}$?   In $\frac{20}{5}$?   In $\frac{80}{5}$?
5. How many ones in $\frac{18}{6}$?   In $\frac{25}{5}$?   In $\frac{35}{7}$?
6. Reduce $\frac{9}{4}$ to a mixed number.

SOLUTION.—Since one equals $\frac{4}{4}$, in $\frac{9}{4}$ there are as many ones as $\frac{4}{4}$ is contained times in $\frac{9}{4}$, which is 2 times and $\frac{1}{4}$ remaining, or $2\frac{1}{4}$.

7. Reduce $\frac{9}{4}$ to a mixed number.
8. Reduce $\frac{27}{5}$ to a mixed number.
9. Reduce $\frac{45}{8}$ to a mixed number.
10. Reduce $\frac{81}{8}$ to a mixed number.

WRITTEN PROBLEMS.

Ex. Reduce $\frac{63}{8}$ to a mixed number.

SOLUTION. *Explanation.*—Since there are $\frac{8}{8}$ in 1, in $\frac{63}{8}$ there
8)63   are as many ones as $\frac{8}{8}$ is contained times in $\frac{63}{8}$, which
  7$\frac{7}{8}$ is 7, and $\frac{7}{8}$ remaining, or 7$\frac{7}{8}$.

From the foregoing we derive the

RULE.

*Divide the numerator of the improper fraction by the
denominator ; the quotient is the whole or mixed number.*

Reduce to whole or mixed numbers—

1. $\frac{21}{6}$.   *Ans.* 3$\frac{3}{8}$. | 6. $\frac{35}{10}$.   *Ans.* 3$\frac{5}{10}$.
2. $\frac{25}{4}$.   *Ans.* 6$\frac{1}{4}$. | 7. $\frac{75}{8}$.   *Ans.* 9$\frac{3}{8}$.
3. $\frac{64}{11}$.   *Ans.* 5$\frac{7}{11}$. | 8. $\frac{80}{6}$.   *Ans.* 13$\frac{2}{6}$.
4. $\frac{63}{5}$.   *Ans.* 12$\frac{3}{5}$. | 9. $\frac{78}{9}$.   *Ans.* 8$\frac{6}{9}$.
5. $\frac{29}{12}$.   *Ans.* 2$\frac{5}{12}$. | 10. $\frac{127}{12}$.   *Ans.* 10$\frac{7}{12}$.

CASE III.

**To Reduce Fractions to Equivalent Fractions having
Higher Terms.**

ORAL EXERCISE.

1. How many sixths are there in $\frac{1}{3}$?

SOLUTION.—Since there are $\frac{6}{6}$ in 1, in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{6}{6}$, or $\frac{2}{6}$.

2. How many fourths in $\frac{1}{2}$?
3. How many sixths in $\frac{1}{2}$?   In $\frac{1}{3}$?
4. How many eighths in $\frac{1}{2}$?   In $\frac{1}{4}$?
5. How many tenths in $\frac{1}{2}$?   In $\frac{1}{5}$?
6. How many twelfths in $\frac{1}{3}$?   In $\frac{1}{4}$?   In $\frac{1}{6}$?
7. How many twelfths in $\frac{3}{4}$?

SOLUTION.—In $\frac{1}{4}$ there are $\frac{3}{12}$, and in $\frac{3}{4}$ there are 3 times $\frac{3}{12}$, or $\frac{9}{12}$.

8. How many twelfths in $\frac{2}{3}$?   In $\frac{3}{4}$?

9. How many fifteenths in $\frac{2}{5}$? In $\frac{3}{5}$? In $\frac{2}{3}$?

10. How many twentieths in $\frac{3}{4}$? In $\frac{3}{5}$? In $\frac{7}{10}$?

PRINCIPLE.—Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction. Thus, $\frac{1}{2}$ equals $\frac{3}{6}$, which is the same as multiplying $\frac{1}{2}$ by $\frac{3}{3}$.

WRITTEN PROBLEMS.

1. Reduce $\frac{3}{4}$ to twentieths.

SOLUTION.   *Explanation.*—To reduce fourths to twentieths it is
$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$ necessary to multiply the denominator by 5; but to
preserve the value of the fraction the numerator must
also be multiplied by the same number. Hence, $\frac{3}{4} = \frac{15}{20}$.

From the foregoing we derive the rule for reduction of fractions to higher terms.

RULE.

Multiply both numerator and denominator by that number which will produce the required denominator.

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to 12ths.   *Ans.* $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

3. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ to 30ths.   *Ans.* $\frac{15}{30}$, $\frac{10}{30}$, $\frac{6}{30}$.

4. Reduce $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{15}$ to 30ths.   *Ans.* $\frac{24}{30}$, $\frac{25}{30}$, $\frac{14}{30}$.

5. Reduce $\frac{8}{9}$, $\frac{5}{12}$, $\frac{3}{4}$ to 36ths.   *Ans.* $\frac{32}{36}$, $\frac{15}{36}$, $\frac{27}{36}$.

CASE IV.

To Reduce Fractions to their Lowest Terms.

ORAL EXERCISE.

1. How many thirds in $\frac{4}{6}$?

SOLUTION.—Since $\frac{1}{3} = \frac{2}{6}$, there are as many thirds in $\frac{4}{6}$ as $\frac{2}{6}$ is contained times in $\frac{4}{6}$, or $\frac{2}{3}$. Hence, in $\frac{4}{6}$ there are $\frac{2}{3}$.

2. How many halves in $\frac{4}{8}$, $\frac{12}{8}$, $\frac{8}{6}$?

3. How many thirds in $\frac{6}{9}$, $\frac{4}{6}$, $\frac{12}{9}$?

4. How many fourths in $\frac{2}{8}$, $\frac{6}{8}$, $\frac{9}{12}$?

5. How many fifths in $\frac{6}{10}$, $\frac{12}{10}$, $\frac{9}{15}$?

6. How many sixths in $\frac{4}{12}$, $\frac{8}{12}$, $\frac{9}{18}$?

7. Reduce $\frac{9}{12}$ to fourths ; $\frac{8}{12}$ to thirds.

8. Reduce $\frac{10}{12}$ to sixths ; $\frac{9}{15}$ to fifths.

9. Reduce $\frac{6}{15}$ to fifths ; $\frac{10}{15}$ to thirds.

10. Reduce $\frac{12}{24}$ to halves ; $\frac{18}{24}$ to fourths.

11. Reduce $\frac{12}{24}$, $\frac{18}{24}$ to their lowest terms.

NOTE.—When a fraction cannot be reduced to one having a less denominator, it is said to be in its lowest terms.

12. Reduce $\frac{8}{12}$, $\frac{10}{16}$, $\frac{20}{24}$ to their lowest terms.

13. Reduce $\frac{18}{36}$, $\frac{30}{36}$, $\frac{20}{48}$ to their lowest terms.

PRINCIPLE.—The division of both terms of a fraction by the same number does not change its value. Thus, if both terms of the fraction $\frac{6}{12}$, equal to $\frac{1}{2}$, be divided by 6, the quotient is $\frac{1}{2}$. The value is not changed.

WRITTEN PROBLEMS.

Ex. Reduce $\frac{120}{144}$ to its lowest terms.

SOLUTION. *Explanation.*—Dividing both terms of $\frac{120}{144}$ by 12,
$\frac{120}{144} = \frac{10}{12} = \frac{5}{6}$ it reduces to $\frac{10}{12}$. Dividing both terms of $\frac{10}{12}$ by 2,
Or, it reduces to $\frac{5}{6}$. Since 5 and 6 do not have a com-
$\frac{120}{144} \div \frac{12}{12} = \frac{10}{12}$ mon divisor, the lowest terms of the fraction $\frac{120}{144}$
$\frac{10}{12} \div \frac{2}{2} = \frac{5}{6}$ is $\frac{5}{6}$.

From the solution we derive the following

RULE.

To reduce a fraction to its lowest terms, divide both terms of the fraction by a common divisor, and this result again by a common divisor, and so on till the terms have no common divisor.

Reduce—

1. $\frac{8}{12}$ to thirds.   *Ans.* $\frac{2}{3}$. | 4. $\frac{3}{4}\frac{6}{8}$ to twelfths.   *Ans.* $\frac{9}{12}$.
2. $\frac{9}{12}$ to fourths.   *Ans.* $\frac{3}{4}$. | 5. $\frac{4}{6}\frac{0}{0}$ to fifteenths. *Ans.* $\frac{10}{15}$.
3. $\frac{12}{20}$ to fifths.   *Ans.* $\frac{3}{5}$. | 6. $\frac{2}{4}\frac{5}{0}$ to sixths.   *Ans.* $\frac{5}{8}$.

Reduce to their lowest terms—

7. $\frac{24}{44}$.   *Ans.* $\frac{3}{4}$. | 11. $\frac{60}{125}$.   *Ans.* $\frac{12}{25}$.
8. $\frac{100}{120}$.   *Ans.* $\frac{5}{6}$. | 12. $\frac{70}{240}$.   *Ans.* $\frac{7}{24}$.
9. $\frac{60}{108}$.   *Ans.* $\frac{5}{9}$. | 13. $\frac{132}{441}$.   *Ans.* $\frac{15}{18}$.
10. $\frac{77}{132}$.   *Ans.* $\frac{7}{12}$. | 14. $\frac{88}{187}$.   *Ans.* $\frac{8}{17}$.

CASE V.

To Reduce Compound Fractions to Simple Ones.

78. A Compound Fraction is a fraction of a fraction, as $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{2}{5}$ of $2\frac{1}{2}$.

ORAL EXERCISE.

1. What is $\frac{1}{2}$ of $\frac{1}{8}$?

     SOLUTION.—Since $\frac{1}{8}$ equals $\frac{2}{8}$, $\frac{1}{2}$ of $\frac{1}{8}$ is $\frac{1}{2}$ of $\frac{2}{8}$, or $\frac{1}{8}$.

2. What is $\frac{1}{8}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{7}$?
3. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{6}$?
4. What is $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{5}$?

5. If I have $\frac{1}{2}$ a dollar, and give $\frac{1}{2}$ of it away, how much do I give away?

6. If a man own $\frac{1}{3}$ of a store, and sell $\frac{1}{4}$ of his share, how much does he sell?

7. A boy had $\frac{1}{4}$ of a dollar, and lost $\frac{1}{4}$ of it: what part of a dollar did he lose?

8. Bought $\frac{1}{4}$ of a farm, and sold $\frac{1}{5}$ of my share: what part of the farm did I sell?

9. How much is $\frac{2}{3}$ of $\frac{2}{5}$?

     SOLUTION.—$\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$; $\frac{1}{3}$ of $\frac{2}{5}$ is twice as much, or $\frac{2}{15}$; and $\frac{2}{3}$ of $\frac{2}{5}$ is 2 times $\frac{2}{15}$, or $\frac{4}{15}$. Hence, $\frac{2}{3}$ of $\frac{2}{5}$ is $\frac{4}{15}$.

10. What is $\frac{2}{3}$ of $\frac{4}{5}$? | 15. What is $\frac{1}{4}$ of $\frac{5}{6}$?
11. What is $\frac{3}{4}$ of $\frac{2}{3}$? | 16. What is $\frac{5}{4}$ of $\frac{3}{8}$?
12. What is $\frac{3}{4}$ of $\frac{3}{5}$? | 17. What is $\frac{5}{8}$ of $\frac{7}{8}$?
13. What is $\frac{4}{5}$ of $\frac{3}{4}$? | 18. What is $\frac{5}{6}$ of $\frac{5}{7}$?
14. What is $\frac{5}{8}$ of $\frac{2}{3}$? | 19. What is $\frac{5}{8}$ of $\frac{7}{10}$?

20. If Charles has $\frac{2}{3}$ of a dollar, and loses $\frac{3}{4}$ of it, what part of a dollar does he lose?

21. A boy has $\frac{3}{4}$ of a pound of candy, and eats $\frac{2}{3}$ of it: how much does he eat?

11. If I have $\frac{3}{4}$ of a bushel of peas, and give away $\frac{2}{3}$ of it, what part of a bushel do I give away?

WRITTEN PROBLEMS.

Ex. What is $\frac{3}{5}$ of $\frac{3}{4}$?

SOLUTION.	ANALYSIS.
$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$	$\frac{1}{5}$ of $\frac{1}{4} = \frac{1}{20}$;
	$\frac{1}{5}$ of $\frac{3}{4} = 3 \times \frac{1}{20} = \frac{3}{20}$;
	$\frac{3}{5}$ of $\frac{3}{4} = 3 \times \frac{3}{20} = \frac{9}{20}$.

What is—

1. $\frac{1}{5}$ of $\frac{3}{4}$? $Ans.$ $\frac{3}{20}$. | 4. $\frac{3}{5}$ of $\frac{7}{9}$? $Ans.$ $\frac{21}{45}$.
2. $\frac{2}{3}$ of $\frac{7}{9}$? $Ans.$ $\frac{14}{27}$. | 5. $\frac{9}{7}$ of $\frac{5}{6}$? $Ans.$ $\frac{15}{42}$.
3. $\frac{3}{4}$ of $\frac{5}{6}$? $Ans.$ $\frac{15}{24}$. | 6. $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{3}{4}$? $Ans.$ $\frac{6}{36}$, or $\frac{1}{6}$.

Reduce to simple fractions—

7. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. $Ans.$ $\frac{2}{5}$.
8. $\frac{1}{5}$ of $\frac{2}{4}$ of $\frac{5}{6}$. $Ans.$ $\frac{1}{6}$.
9. $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{3}{4}$. $Ans.$ $\frac{1}{10}$.
10. $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$. $Ans.$ $\frac{1}{3}$.
11. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{10}$. $Ans.$ $\frac{9}{16}$.
12. $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{9}{9}$. $Ans.$ $\frac{3}{5}$.
13. $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{7}{9}$ of $\frac{8}{10}$. $Ans.$ $\frac{7}{54}$.

COMMON DENOMINATOR.

ORAL EXERCISE.

1. Reduce $\frac{2}{3}$ and $\frac{2}{5}$ to fifteenths.

SOLUTION.—Since there are $1\frac{2}{3}$ in 1, in $\frac{1}{3}$ there are $\frac{1}{3}$ of $1\frac{2}{3}$, and in $\frac{2}{3}$, twice $\frac{5}{15}$, or $1\frac{0}{15}$. In $\frac{1}{5}$ there are $\frac{1}{5}$ of $1\frac{2}{3}$, or $\frac{3}{15}$, an twice $\frac{3}{15}$, or $\frac{6}{15}$. Hence in $\frac{2}{3}$ and $\frac{2}{5}$ there are $1\frac{0}{15}$ and $\frac{6}{15}$.

2. How many twelfths in $\frac{2}{3}$ and $\frac{3}{4}$?

3. How many tenths in $\frac{2}{5}$ and $\frac{1}{2}$?

4. How many twentieths in $\frac{2}{5}$, $\frac{1}{4}$ and $\frac{3}{4}$?

5. How many thirtieths in $\frac{3}{5}$, $\frac{2}{3}$ and $\frac{5}{6}$?

6. Reduce $\frac{1}{3}$ and $\frac{1}{4}$ to fractions having a commo nominator.

NOTE.—When fractions have the same denominator they a to have a *common denominator*.

7. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to fractions having a commo nominator.

8. Reduce $\frac{4}{5}$ and $\frac{5}{6}$ to fractions having a commo nominator.

9. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to fractions having a commo nominator.

10. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $2\frac{1}{4}$ to fractions having a con denominator.

WRITTEN PROBLEMS.

1. Reduce $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$ to fractions having a common de inator.

SOLUTION 1.

$\frac{3}{4} \times \dfrac{3 \times 2}{3 \times 2} = \frac{18}{24}$

$\frac{2}{3} \times \dfrac{4 \times 2}{4 \times 2} = \frac{16}{24}$

$\frac{1}{2} \times \dfrac{3 \times 4}{3 \times 4} = \frac{12}{24}$

Explanation 1.—Multiplying any of the deno tors by the others, we find the common denom to be 24. Multiplying both terms of $\frac{3}{4}$ by t nominators 3 and 2, it equals $\frac{18}{24}$; multiplyin terms of $\frac{2}{3}$ by the denominators 4 and 2, it $\frac{16}{24}$; multiplying both terms of $\frac{1}{2}$ by the deno tors 3 and 4, it equals $\frac{12}{24}$.

SOLUTION 2.

Denominator $= 4 \times 3 \times 2 = 24$

$\frac{3}{4} = \frac{18}{24}$

$\frac{2}{3} = \frac{16}{24}$

$\frac{1}{2} = \frac{12}{24}$

Explanation 2.—We find the common denominator, 24, by multiplying together the given denominators 4, 3 and 2, and reduce each fraction to 24ths.

From these solutions we derive the following rule for reducing fractions to a common denominator:

RULE.

Multiply both terms of each fraction by all the denominators except its own.

Reduce to fractions having a common denominator—

2. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}.$ *Ans.* $\frac{12}{24}, \frac{8}{24}, \frac{6}{24}.$

3. $\frac{2}{3}, \frac{3}{4}, \frac{1}{5}.$ *Ans.* $\frac{40}{60}, \frac{45}{60}, \frac{12}{60}.$

4. $\frac{3}{4}, \frac{5}{6}, \frac{3}{8}.$ *Ans.* $\frac{144}{192}, \frac{160}{192}, \frac{72}{192}.$

5. $\frac{4}{5}, \frac{6}{7}, \frac{2}{3}.$ *Ans.* $\frac{84}{105}, \frac{90}{105}, \frac{70}{105}.$

6. $\frac{2}{9}, \frac{3}{10}, \frac{4}{5}.$ *Ans.* $\frac{100}{450}, \frac{135}{450}, \frac{360}{450}.$

7. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}.$ *Ans.* $\frac{60}{120}, \frac{40}{120}, \frac{30}{120}, \frac{24}{120}.$

8. $\frac{5}{8}, \frac{8}{9}, \frac{7}{10}.$ *Ans.* $\frac{450}{720}, \frac{640}{720}, \frac{504}{720}.$

79. The **Least Common Denominator** of two or more fractions is the least denominator to which they can all be reduced.

It is also the least common multiple of their denominators.

WRITTEN PROBLEMS.

Ex. Reduce $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$ to equivalent fractions having their least common denominator.

SOLUTION.

2)4, 6, 8

2)2, 3, 4

1, 3, 2

$2 \times 2 \times 3 \times 2 = 24$, L. C. D.

$\frac{3}{4} = \frac{18}{24}$

$\frac{5}{6} = \frac{20}{24}$

$\frac{7}{8} = \frac{21}{24}$

Explanation. — The least common multiple of 4, 6 and 8 is 24, which is therefore the least common denominator. By analysis, $\frac{1}{4} = \frac{6}{24}$, and $\frac{3}{4} = 3 \times \frac{6}{24}$, or $\frac{18}{24}$; $\frac{1}{6} = \frac{4}{24}$, and $\frac{5}{6} = 5 \times \frac{4}{24} = \frac{20}{24}$; $\frac{1}{8} = \frac{3}{24}$, and $\frac{7}{8} = 7 \times \frac{3}{24} = \frac{21}{24}$.

9   G

The following is the rule for reducing fractions to eq**l**alent ones having their least common denominator:

RULE.

Find the least common multiple of the denominators the least common denominator; divide this common den inator by each denominator, and multiply both terms by quotient.

NOTE.—Reduce mixed numbers and compound fractions to ple ones, and these to their lowest terms, before proceeding to the least common denominator.

Reduce the following fractions to their least denomina1

1. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{3}{8}$.   *Ans.* $\frac{20}{24}$, $\frac{18}{24}$, $\frac{9}{24}$

2. $\frac{3}{4}$, $\frac{9}{10}$, $\frac{7}{12}$.   *Ans.* $\frac{45}{60}$, $\frac{54}{60}$, $\frac{35}{60}$

3. $\frac{4}{5}$, $\frac{7}{9}$, $\frac{11}{12}$.   *Ans.* $\frac{144}{180}$, $\frac{140}{180}$, $\frac{165}{180}$

4. $\frac{7}{8}$, $\frac{11}{14}$, $\frac{2}{10}$.   *Ans.* $\frac{245}{280}$, $\frac{220}{280}$, $\frac{56}{280}$

5. $\frac{8}{15}$, $\frac{3}{20}$, $\frac{2}{3}$.   *Ans.* $\frac{32}{60}$, $\frac{9}{60}$, $\frac{41}{60}$

6. $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{9}{10}$ of $\frac{7}{8}$.   *Ans.* $\frac{30}{80}$, $\frac{63}{80}$

SECTION III.

ADDITION OF FRACTIONS.

ORAL EXERCISE.

1. How many are $\frac{1}{6}$, $\frac{2}{6}$ and $\frac{4}{6}$?
2. How many are $\frac{2}{8}$, $\frac{4}{8}$ and $\frac{7}{8}$?
3. How many are $\frac{9}{7}$, $\frac{3}{7}$ and $\frac{4}{7}$?
4. What is the sum of $\frac{3}{8}$, $\frac{2}{8}$, $\frac{7}{8}$?
5. What is the sum of $\frac{1}{3}$ and $\frac{1}{4}$?

SOLUTION.—$\frac{1}{3}$ equals $\frac{4}{12}$, and $\frac{1}{4}$ equals $\frac{3}{12}$; $\frac{4}{12}$ and $\frac{3}{12}$ are Hence, the sum of $\frac{1}{3}$ and $\frac{1}{4}$ is $\frac{7}{12}$.

What is the sum—

6. Of $\frac{1}{2}$ and $\frac{1}{3}$?

7. Of $\frac{1}{6}$ and $\frac{1}{2}$?

8. Of $\frac{1}{4}$ and $\frac{1}{3}$?

9. Of $\frac{1}{2}$ and $\frac{2}{3}$?

10. Of $\frac{2}{8}$ and $\frac{3}{4}$?

11. Of $\frac{3}{4}$ and $\frac{5}{6}$?

12. Of $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$?

13. Of $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{4}$?

14. Of $2\frac{1}{6}$ and $3\frac{1}{4}$?

NOTE.—Questions of this kind may also be solved by adding the whole numbers and the fractions separately; thus, 2 and 3 are 5; $\frac{1}{3}$ and $\frac{1}{4}$ equal $\frac{4}{12}$ and $\frac{3}{12}$, or $\frac{7}{12}$. Hence, $2\frac{1}{3}$ and $3\frac{1}{4}$ equal $5\frac{7}{12}$.

15. Of $2\frac{1}{2}$ and $3\frac{1}{4}$?

16. Of $3\frac{1}{4}$ and $2\frac{1}{3}$?

17. Of $3\frac{1}{8}$ and $3\frac{1}{4}$?

18. Of $2\frac{2}{3}$ and $3\frac{1}{2}$?

19. Of $4\frac{1}{6}$ and $2\frac{3}{4}$?

20. Of $3\frac{3}{5}$ and $4\frac{2}{3}$?

21. Of $4\frac{1}{8}$ and $2\frac{5}{6}$?

22. Of $6\frac{1}{4}$ and $5\frac{3}{7}$?

23. Of $4\frac{1}{7}$ and $3\frac{5}{6}$?

24. Of $7\frac{3}{4}$ and $6\frac{7}{8}$?

25. If I pay $\frac{3}{4}$ of a dollar for a turkey, and $\frac{1}{2}$ a dollar for a goose, what do I pay for both?

26. If I pay $\frac{1}{4}$ of a dollar for butter, $\frac{2}{3}$ of a dollar for eggs, and $\frac{1}{2}$ of a dollar for cheese, how much do I pay for all?

27. A pair of boots cost \$5$\frac{1}{2}$, a hat \$3$\frac{3}{4}$, and a vest \$2$\frac{1}{8}$: how much did they all cost?

80. Addition of Fractions is the process of finding the sum of two or more fractions.

81. *Similar fractional units* are those having the same name.

PRINCIPLES.—1. Only similar fractional units can be added. Thus, thirds can be added to thirds, but not to fifths or other fractional units.

2. Fractions to be added must have a common denominator.

WRITTEN PROBLEMS.

Ex. Find the sum of $\frac{3}{4}$ and $\frac{2}{5}$.

SOLUTION.

$\frac{3}{4} + \frac{2}{5}$

$= \frac{15}{20} + \frac{8}{20}$

$= \frac{23}{20}$, or $1\frac{3}{20}$

Analysis.—The common denominator of $\frac{3}{4}$ and $\frac{2}{5}$ is 20.

$\frac{3}{4} = \frac{15}{20}$ and $\frac{2}{5} = \frac{8}{20}$; $\frac{15}{20}$ and $\frac{8}{20}$ are $\frac{23}{20}$, or $1\frac{3}{20}$.

From the foregoing principles and solution we derive the following rules for Addition of Fractions:

RULES.

1. *Reduce the fractions to a common denominator, add the numerators and write the result over the common denominator.*

2. *To add mixed numbers, add the whole numbers and the fractions separately, and then add the results.*

NOTE.—1. Reduce each fraction to its lowest terms before reducing to a common denominator.

2. Reduce fractional results to their lowest terms, and improper fractions to whole or mixed numbers.

Find the sum—

1. Of $\frac{5}{6}$ and $\frac{3}{4}$.　　　　　　*Ans.* $1\frac{7}{12}$.

2. Of $\frac{5}{6}$ and $\frac{4}{5}$.　　　　　　*Ans.* $1\frac{5}{12}$.

3. Of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.　　　　　*Ans.* $2\frac{1}{24}$.

4. Of $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$.　　　　*Ans.* $1\frac{13}{16}$.

5. Of $\frac{5}{6}$, $\frac{5}{8}$, $\frac{3}{4}$.　　　　*Ans.* $2\frac{5}{36}$.

6. Of $\frac{4}{7}$, $\frac{3}{8}$, $\frac{2}{5}$.　　　　*Ans.* $1\frac{77}{280}$.

7. Of $\frac{1}{4}$, $\frac{3}{10}$, $\frac{1}{4}$.　　　　*Ans.* $\frac{4}{5}$.

8. Of $\frac{2}{5}$, $\frac{7}{10}$, $\frac{11}{12}$.　　　*Ans.* $2\frac{1}{6}$.

9. Of $3\frac{1}{2}$ and $5\frac{3}{5}$.

PROCESS.

$3\frac{1}{2} = 3\frac{5}{10}$

$5\frac{3}{5} = 5\frac{6}{10}$

$8\frac{11}{10} = 9\frac{1}{10}$

10. Of 2¼ and 3½.   *Ans.* 5¾.

11. Of 3⅔, 6⅘, 3¾.   *Ans.* 14⅟₆⅟₅.

12. Of 7⅓, 9⟋₁₀, 2¼.   *Ans.* 18⅟₆⅟₅.

13. Of 6⅝ and 9⅞.   *Ans.* 16½⅟₄.

14. Of ½ of ⅛ and ⅛ of ¼.   *Ans.* ₃⟋₁₂.

15. Of 2⟋₇ and ⅔ of ⅘.   *Ans.* 2⅘⅘.

16. What is the sum of $19¼, $16½, $27⅘?
                 Ans. $63½⅙.

17. What is the sum of $23¾, $19¾, $16⅝?
                 Ans. $60⟋₂₄.

18. What is the sum of 9⅞ yd., 10¾ yd., 18⅝ yd.?
                 Ans. 39¼ yd.

SECTION IV.

SUBTRACTION OF FRACTIONS.

ORAL EXERCISE.

1. What is the difference between ⅝ and ¾?

2. How much is ¾ less ¼?   ⟋₇ less ²⟋₇?

3. How much is ₇⟋₁₂ less ₅⟋₁₂?   ⅜ less ⅔?

4. How much is ¾ − ½?

SOLUTION.—¾ equals ⅜, and ½ equals ⅜; ⅜ less ⅜ is ⅛; hence ¾ −
½ is ¼.

Subtract—

5. ¼ from ½.	10. ⅝ from ⅞.
6. ⅔ from ¾.	11. ⅝ from ⅔.
7. ⅔ from ⅘.	12. ⅚ from ⅞.
8. ¾ from ⅚.	13. ⟋₇ from ¾.
9. ⅘ from ⅚.	14. ⅔ from ⅚.

15. What is the difference between 2⅛ and 1½?

SUGGESTION.—Reduce the mixed numbers to improper fractions
before subtracting.

9*

16. Between $3\frac{1}{8}$ and $2\frac{3}{4}$?   18. Between $6\frac{1}{4}$ and $5\frac{3}{8}$?
17. Between $4\frac{1}{4}$ and $5\frac{1}{3}$?   19. Between $10\frac{1}{3}$ and $7\frac{1}{4}$?

What is the value—

20. Of $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$?   23. Of $\frac{3}{4} + \frac{2}{3} - \frac{5}{8}$?
21. Of $\frac{1}{2} + \frac{2}{3} - \frac{1}{4}$?   24. Of $\frac{5}{6} + \frac{3}{4} - \frac{2}{3}$?
22. Of $\frac{3}{8} + \frac{1}{4} - \frac{1}{2}$?   25. Of $3\frac{1}{4} + 2\frac{1}{3} - 3\frac{1}{2}$?

26. If a boy has $\frac{1}{2}$ a dollar, and spends $\frac{1}{4}$ a dollar, how much has he remaining ?

27. I have 14 dollars, and owe $10\frac{1}{4}$ dollars: if I pay what I owe, how much have I remaining ?

28. John is $7\frac{1}{2}$ years old, and Henry is $9\frac{1}{4}$ yr. old: how much older is Henry than John ?

29. A merchant has 10 yd. of cloth, from which he cuts $2\frac{3}{4}$ yd.: how much remains ?

30. A farmer sold $\frac{1}{2}$ his corn in the fall, and fed $\frac{1}{4}$; he has 60 bu. remaining: how many bushels had he ?

82. Subtraction of Fractions is the process of finding the difference between two fractions.

PRINCIPLES.—1. Two fractions can be subtracted only when their fractional units are similar.

2. Fractions must have a common denominator before their difference can be found.

NOTE.—Reduce compound fractions to simple ones, and each fraction, as well as the resulting difference, to its lowest terms.

WRITTEN PROBLEMS.

1. Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

SOLUTION.

$\frac{5}{6} - \frac{2}{3}$
$= \frac{25}{30} - \frac{18}{30} = \frac{7}{30}$

Explanation.—$\frac{5}{6}$ equal $\frac{25}{30}$, and $\frac{2}{3}$ equal $\frac{18}{30}$; $\frac{25}{30}$ less $\frac{18}{30}$ are $\frac{7}{30}$.

The following is the rule for subtraction of fractions:

RULE.

Reduce the fractions to a common denominator, subtract the numerators and write the result over the common denominator.

How much is—

1. $\frac{8}{5} - \frac{2}{7}$?   Ans. $\frac{6}{85}$.
2. $\frac{3}{4} - \frac{3}{10}$?   Ans. $\frac{9}{20}$.
3. $\frac{5}{6} - \frac{4}{5}$?   Ans. $\frac{1}{30}$.
4. $\frac{6}{8} - \frac{2}{7}$?   Ans. $1\frac{2}{7}$.
5. $1\frac{1}{12} - \frac{9}{10}$?   Ans. $\frac{1}{60}$.

6. $\frac{9}{11} - \frac{9}{12}$?   Ans. $\frac{3}{44}$.
7. $\frac{1}{2} + \frac{1}{3} - \frac{1}{6}$?   Ans. $\frac{2}{3}$.
8. $\frac{3}{4} + \frac{7}{8} - \frac{5}{8}$?   Ans. $1\frac{2}{24}$.
9. $\frac{5}{6} - \frac{7}{12}$?   Ans. $\frac{1}{4}$.
10. $\frac{9}{11} - \frac{13}{17}$?   Ans. $\frac{10}{187}$.

11. From $6\frac{2}{3}$ take $3\frac{3}{5}$.

SOLUTION 1.

$6\frac{2}{3} - 3\frac{3}{5}$
$= \frac{20}{3} - \frac{18}{5}$
$= \frac{100}{15} - \frac{54}{15} = \frac{46}{15}$, or $3\frac{1}{15}$

ANALYSIS.

$6\frac{2}{3} = \frac{20}{3}$, or $\frac{100}{15}$;
$3\frac{3}{5} = \frac{18}{5}$, or $\frac{54}{15}$;
$\frac{100}{15} - \frac{54}{15} = \frac{46}{15}$, or $3\frac{1}{15}$.

SOLUTION 2.

$6\frac{2}{3} = 6\frac{10}{15}$
$3\frac{3}{5} = 3\frac{9}{15}$
$\overline{\hphantom{xx}3\frac{1}{15}}$

NOTE.—Let the pupil explain.

12. From $6\frac{1}{4}$ take $3\frac{1}{5}$.   Ans. $3\frac{1}{20}$.
13. From $6\frac{3}{4}$ take $5\frac{2}{3}$.   Ans. $1\frac{1}{12}$.
14. From $5\frac{1}{6}$ take $3\frac{1}{4}$.   Ans. $1\frac{11}{12}$.
15. From $7\frac{2}{3}$ take $2\frac{1}{4}$.   Ans. $5\frac{1}{24}$.
16. From $7\frac{1}{2}$ take $4\frac{3}{5}$.   Ans. $2\frac{9}{10}$.
17. From $15\frac{1}{3}$ take $10\frac{3}{4}$.   Ans. $4\frac{7}{12}$.
18. From $17\frac{1}{2}$ take $12\frac{5}{8}$.   Ans. $4\frac{7}{8}$.
19. From $18\frac{1}{3}$ take $10\frac{3}{4}$.   Ans. $7\frac{1}{8}$.
20. From $27\frac{4}{5}$ take $19\frac{3}{4}$.   Ans. $8\frac{1}{20}$.
21. From $27\frac{1}{2}$ take $18\frac{5}{8}$.   Ans. $8\frac{7}{8}$.

What is the value—

22. Of $\frac{3}{4}$ of $\frac{2}{3} - \frac{1}{2}$ of $\frac{3}{5}$?   Ans. $\frac{1}{5}$.

23. Of $6\frac{1}{3} - \frac{3}{4}$ of $2\frac{1}{2}$?   *Ans.* $4\frac{11}{12}$.

24. Of $\frac{4}{5}$ of $7\frac{1}{2} - \frac{6}{7}$ of $2\frac{1}{3}$?   *Ans.* 4.

25. Of $\frac{3}{4} + \frac{2}{3} + \frac{1}{2} - \frac{3}{7}$ of $1\frac{2}{5}$?   *Ans.* $1\frac{19}{60}$.

26. A farmer buys sheep at $5\frac{1}{4}$ dollars apiece, and sells them at 8 dollars each: how much does he gain?

                                Ans. $\$2\frac{3}{4}$.

27. If I buy groceries amounting to $\$3\frac{1}{3}$, and give the merchant a twenty-dollar bill, how much change do I get?

                                Ans. $\$16\frac{2}{3}$.

28. From a farm of $120\frac{3}{4}$ acres there were sold $19\frac{15}{16}$ acres: how many acres remained?   *Ans.* $100\frac{13}{16}$ acres.

29. A merchant has one piece of muslin containing $45\frac{3}{4}$ yards, and another of $43\frac{1}{2}$ yards; he sells $50\frac{3}{8}$ yards: how much remains?   *Ans.* $38\frac{7}{8}$ yards.

30. Having $6\frac{3}{4}$ tons of hay, I sold $2\frac{4}{5}$ tons: how much remains?   *Ans.* $3\frac{19}{20}$ tons.

SECTION V.

MULTIPLICATION OF FRACTIONS.

CASE I.

To Multiply a Fraction by an Integer.

ORAL EXERCISE.

1. How much will 5 pairs of ducks cost at $\frac{3}{4}$ of a dollar a pair.

SOLUTION.—If 1 pair of ducks cost $\frac{3}{4}$ of a dollar, 5 pairs will cost 5 times $\frac{3}{4}$ of a dollar, or $\frac{15}{4}$ dollars, equal to $3\frac{3}{4}$ dollars.

2. At $\frac{1}{4}$ of a dollar each, how much will 6 melons cost?

3. At $\frac{2}{5}$ of a dollar each, what must I pay for 10 hens?

4. If a horse can eat $\frac{1}{2}$ of a bushel of oats in a day, how much can 12 horses eat in the same time?

5. If a hat cost $\frac{4}{5}$ of a dollar, how much will 10 hats cost at the same rate?

How many are—

6. 3 times $\frac{6}{10}$? 4 times $\frac{4}{5}$? 6 times $\frac{2}{8}$?
7. 5 times $\frac{4}{7}$? 8 times $\frac{3}{4}$? 8 times $\frac{3}{8}$?
8. 6 times $\frac{3}{8}$? 4 times $\frac{5}{9}$? 5 times $\frac{7}{8}$?
9. 3 times $\frac{7}{8}$? 6 times $\frac{7}{8}$? 8 times $\frac{4}{5}$?
10. 10 times $\frac{3}{7}$? 9 times $\frac{5}{8}$? 10 times $\frac{6}{13}$?

Since $2 \times \frac{1}{5} = \frac{2}{5}$, $3 \times \frac{3}{4} = \frac{9}{4}$, etc., we have the following

PRINCIPLE.—A fraction may be multiplied by multiplying its numerator.

Since $2 \times \frac{4}{8} = \frac{4}{8}$, or $\frac{1}{2}$, $3 \times \frac{2}{9} = \frac{6}{9}$, or $\frac{2}{3}$, etc., we have the following

PRINCIPLE.—A fraction may be multiplied by dividing its denominator.

WRITTEN PROBLEMS.

1. Multiply $\frac{9}{20}$ by 4.

SOLUTION 1. $\frac{9}{20} \times 4 = \frac{36}{20} = \frac{9}{5}$, or $1\frac{4}{5}$. Let the pupil explain.

SOLUTION 2. $\frac{9}{\underset{5}{20}} \times 4 = \frac{9}{5}$, or $1\frac{4}{5}$. *Explanation.*—Since dividing the denominator of a fraction multiplies the value, $4 \times \frac{9}{20}$ equals $\frac{9}{5}$, or $1\frac{4}{5}$.

RULES.

1. *To multiply a fraction by an integer, multiply the numerator or divide the denominator.*

2. *To multiply a mixed number by an integer, multiply the whole number and the fraction separately, and add the results.*

Multiply—

2. $\frac{9}{10}$ by 4.	*Ans.* 3$\frac{3}{5}$.	7. $\frac{8}{12}$ by 9.	*Ans.* 6.
3. $\frac{6}{8}$ by 8.	*Ans.* 6$\frac{3}{4}$.	8. $3\frac{1}{3}$ by 6.	*Ans.* 20.
4. $\frac{7}{8}$ by 10.	*Ans.* 8$\frac{3}{4}$.	9. $8\frac{3}{4}$ by 8.	*Ans.* 70.
5. $\frac{7}{9}$ by 12.	*Ans.* 9$\frac{1}{3}$.	10. $7\frac{5}{6}$ by 12.	*Ans.* 94.
6. $\frac{14}{15}$ by 6.	*Ans.* 5$\frac{3}{5}$.	11. $18\frac{1}{4}$ by 25.	*Ans.* 456$\frac{1}{4}$.

CASE II.

To Multiply an Integer by a Fraction.

ORAL EXERCISE.

1. If 1 yard of muslin cost 12 cents, how much will $\frac{1}{3}$ of a yard cost?

SOLUTION.—If 1 yard cost 12 cents, $\frac{1}{3}$ of a yard will cost $\frac{1}{3}$ of 12 cents, or 4 cents.

2. A boy had 25 cents, and lost $\frac{1}{5}$ of it: how much did he lose?

3. If a man earn $60 a month, how much will he earn in $\frac{1}{4}$ of a month?

4. A cow cost $40, and a sheep $\frac{1}{8}$ as much: how much did the sheep cost?

5. A horse cost $120, and a pig $\frac{1}{12}$ as much: how much did the pig cost?

6. If a ton of hay cost $25, how much will $\frac{4}{5}$ of a ton cost?

SOLUTION.—If 1 ton of hay cost $25, $\frac{1}{5}$ of a ton will cost $\frac{1}{5}$ of $25, which is $5, and $\frac{4}{5}$ of a ton will cost 4 times $5, or $20.

7. John has 30 cents, and James has $\frac{2}{5}$ as many: how many has James?

8. A house cost $800, and a barn $\frac{3}{4}$ as much: how much did the barn cost?

9. $\frac{3}{7}$ of 14 yards of calico cost 60 cents: what is 1 yard worth?

10. If 2 hens cost 50 cents, how much will $\frac{2}{5}$ of 30 hens cost?

11. $\frac{4}{5}$ of \$50 is 8 times the cost of a shawl: what was the cost of it?

12. $\frac{2}{3}$ of 15 are how many times $\frac{1}{3}$ of 6?

13. $\frac{3}{4}$ of 16 are how many times $\frac{1}{5}$ of 20?

14. $\frac{5}{6}$ of 24 are how many times $\frac{1}{6}$ of 30?

15. What is $\frac{1}{5}$ of 6?

SOLUTION.—$\frac{1}{5}$ of 1 is $\frac{1}{5}$, and $\frac{1}{5}$ of 6 is 6 times $\frac{1}{5}$, or $\frac{6}{5}$. Hence, $\frac{1}{5}$ of 6 is $\frac{6}{5}$.

16. What is $\frac{1}{5}$ of 7? $\frac{1}{4}$ of 6?

17. What is $\frac{1}{4}$ of 8? $\frac{1}{6}$ of 9?

18. What is $\frac{1}{8}$ of 12? $\frac{1}{10}$ of 16?

19. What is $\frac{3}{4}$ of 9?

SOLUTION.—$\frac{1}{4}$ of 9 is $\frac{9}{4}$, and $\frac{3}{4}$ of 9 is $3 \times \frac{9}{4}$, or $\frac{27}{4}$. Hence, $\frac{3}{4}$ of 9 is $\frac{27}{4}$.

20. What is $\frac{2}{3}$ of 10? $\frac{3}{5}$ of 10?

21. What is $\frac{3}{8}$ of 8? $\frac{4}{5}$ of 8?

22. What is $\frac{5}{6}$ of 4? $\frac{4}{7}$ of 5?

23. What is $\frac{3}{5}$ of 7? $\frac{2}{7}$ of 8?

24. What is $\frac{5}{6}$ of 4? $\frac{3}{10}$ of 16?

25. What is $\frac{3}{4}$ of \$26? $\frac{2}{3}$ of 17 miles?

PRINCIPLE.—The product of an integer by a fraction equals the fraction of the integer.

WRITTEN PROBLEMS.

1. Multiply 45 by $\frac{3}{4}$.

SOLUTION.

$45 \times \frac{3}{4}$

$= \frac{3}{4} \times 45 = \frac{135}{4}$, or $33\frac{3}{4}$

Explanation.—45 times $\frac{3}{4}$ is the same as $\frac{3}{4}$ of 45, which is $\frac{135}{4}$, or $33\frac{3}{4}$.

Multiply—.

2. 27 by $\frac{2}{3}$.	*Ans.* 18.	5. 29 by $\frac{3}{7}$.	*Ans.* 12$\frac{3}{7}$.
3. 36 by $\frac{2}{3}$.	*Ans.* 14$\frac{2}{5}$.	6. 54 by $\frac{3}{8}$.	*Ans.* 20$\frac{1}{4}$.
4. 60 by $\frac{3}{8}$.	*Ans.* 22$\frac{1}{2}$.	7. 75 by $\frac{1}{3}$.	*Ans.* 25

8. Multiply 65 by 4$\frac{2}{3}$.

SOLUTION.

```
  65
  4⅔       Explanation.—4 times 65 are 260, and ⅔ of 65 are
 260        ₁⅓⁰, or 43⅓.  Adding the two results, we have 303⅓.
 43⅓
303⅓
```

The following are the rules for the multiplication of integers by fractions:

RULES.

1. *Multiply the integer by the numerator, and divide the result by the denominator. Or, Divide the integer by the denominator, and multiply the result by the numerator.*

2. *When the multiplier is a mixed number, multiply the integer and the fraction separately, and add the results.*

Multiply—

9. 25 by 3$\frac{3}{4}$.	*Ans.* 93$\frac{3}{4}$.	13. 60 by 15$\frac{2}{3}$.	*Ans.* 925$\frac{2}{3}$.
10. 20 by 6$\frac{1}{5}$.	*Ans.* 124.	14. 12 by $\frac{3}{4}$ of $\frac{5}{6}$.	*Ans.* 7$\frac{1}{2}$.
11. 35 by 10$\frac{3}{4}$.	*Ans.* 376$\frac{1}{4}$.	15. 42 by $\frac{1}{2}$ of 7$\frac{5}{6}$.	*Ans.* 163$\frac{1}{4}$.
12. 48 by 8$\frac{3}{5}$.	*Ans.* 412$\frac{4}{5}$.	16. 125 by 18$\frac{3}{4}$.	*Ans.* 2343$\frac{3}{4}$.

CASE III.

To Multiply a Fraction by a Fraction.

ORAL EXERCISE.

1. What is $\frac{3}{4}$ of $\frac{2}{3}$?

SOLUTION.—$\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$; $\frac{1}{4}$ of $\frac{2}{3}$ is 2 times $\frac{1}{12}$, or $\frac{2}{12}$; and $\frac{3}{4}$ of $\frac{2}{3}$ is 3 times $\frac{2}{12}$, or $\frac{6}{12}$, equal to $\frac{1}{2}$. Hence, $\frac{3}{4}$ of $\frac{2}{3}$ is $\frac{1}{2}$.

2. What is $\frac{3}{5}$ of $\frac{3}{4}$? | 5. What is $\frac{5}{6}$ of $\frac{8}{8}$?
3. What is $\frac{3}{4}$ of $\frac{5}{8}$? | 6. What is $\frac{3}{4}$ of $3\frac{1}{2}$?
4. What is $\frac{4}{5}$ of $\frac{9}{7}$? | 7. What is $\frac{5}{6}$ of $4\frac{3}{4}$?

8. How much will $\frac{3}{4}$ of a bushel of potatoes cost at $\frac{4}{5}$ of a dollar a bushel?

9. What is the cost of $\frac{5}{8}$ of a peck of beans at $\frac{4}{10}$ of a dollar a peck?

10. What cost $1\frac{1}{2}$ doz. of eggs at $12\frac{1}{2}$ cts. a dozen?

11. A hat cost $\frac{1}{2}$ of $\frac{3}{4}$ of $16\frac{1}{2}$ dollars: how much did it cost?

12. If 4 pairs of shoes cost $9, how much will 2 pairs cost?

PRINCIPLE.—The product of a fraction by a fraction equals the fraction of that fraction.

Remark.—The process of multiplying fractions may be shortened by cancellation.

WRITTEN PROBLEMS.
1. What is the product of $\frac{5}{6}$ of $5\frac{1}{4}$ by $\frac{3}{7}$?

SOLUTION 1. *Explanation.*—Since $5\frac{1}{4}$ equals $\frac{21}{4}$, $\frac{5}{6}$ of
$\frac{5}{6}$ of $5\frac{1}{4} \times \frac{3}{7}$ $5\frac{1}{4} \times \frac{3}{7}$ equals $\frac{5}{6}$ of $\frac{21}{4} \times \frac{3}{7}$, which is $\frac{315}{168}$,
$= \frac{5}{6} \times \frac{21}{4} \times \frac{3}{7} = \frac{315}{168}$, or $1\frac{7}{8}$ equal to $\frac{15}{8}$, or $1\frac{7}{8}$.

SOLUTION 2.
$\frac{5}{6}$ of $5\frac{1}{4} \times \frac{3}{7}$
$= \frac{5}{6} \times \frac{21}{4} \times \frac{3}{7} = \frac{15}{8}$, or $1\frac{7}{8}$.

The following is the rule for multiplying fractions by fractions:

RULE.

Multiply the numerators together for the numerator, and the denominators for the denominator, of the product.

NOTE 1.—Reduce mixed numbers first to improper fractions.
2. Cancel common factors in the numerator and the denominator.

10

Remark.—This case is practically the same as finding the fractional part of a fraction.

Multiply—

2. $\frac{9}{10}$ by $\frac{3}{8}$.   *Ans.* $\frac{27}{80}$.

3. $\frac{7}{8}$ by $\frac{5}{6}$.   *Ans.* $\frac{35}{48}$.

4. $2\frac{1}{4}$ by $\frac{3}{8}$.   *Ans.* $\frac{7}{8}$.

5. $\frac{3}{4}$ of $\frac{9}{7}$ by $\frac{5}{4}$.   *Ans.* $1\frac{13}{35}$.

6. $\frac{15}{20}$ by $\frac{2}{5}$ of $\frac{5}{6}$.   *Ans.* $\frac{9}{20}$.

7. $\frac{5}{6}$ of $\frac{7}{8}$ by $\frac{2}{3}$ of $\frac{4}{5}$.   *Ans.* $\frac{7}{18}$.

8. $\frac{5}{6}$ of 7 by $2\frac{1}{4}$.   *Ans.* $13\frac{1}{8}$.

9. $8\frac{1}{5}$ by $\frac{2}{3}$ of $7\frac{1}{4}$. *Ans.* $23\frac{33}{45}$.

SECTION VI.

DIVISION OF FRACTIONS.

CASE I.

To Divide a Fraction by an Integer.

ORAL EXERCISE.

1. If 3 ducks cost $\frac{6}{10}$ of a dollar, how much will 1 duck cost?

SOLUTION.—If 3 ducks cost $\frac{6}{10}$ of a dollar, 1 duck, which is $\frac{1}{3}$ of 3 ducks, will cost $\frac{1}{3}$ of $\frac{6}{10}$ of a dollar, which is $\frac{2}{10}$, or $\frac{1}{5}$ of a dollar.

2. If 3 caps cost $\frac{9}{10}$ of a dollar, how much will 1 cap cost?

3. If 5 melons cost $\frac{7}{10}$ of a dollar, how much will 1 melon cost?

SOLUTION.—If 5 melons cost $\frac{7}{10}$ of a dollar, 1 melon will cost $\frac{1}{5}$ of $\frac{7}{10}$ of a dollar, which is $\frac{7}{50}$ of a dollar.

4. How much will 1 boy earn in a day if 3 boys earn $\frac{7}{8}$ of a dollar?

5. If 6 men can do $\frac{3}{4}$ of a piece of work, what part of the work can 1 man do?

6. If 6 cows eat $\frac{5}{6}$ of a ton of hay in a certain time, how much will 1 cow eat in the same time?

7. If $\frac{5}{6}$ of a farm be divided into 8 equal parts, how much will each part be?

8. 5 persons own $\frac{2}{3}$ of a ship: what is each one's share?

9. 6 boys pick $10\frac{1}{2}$ quarts of berries: how much does each boy pick?

10. What is $\frac{1}{5}$ of $5\frac{1}{4}$? $\frac{1}{4}$ of $3\frac{7}{10}$? $\frac{1}{7}$ of $10\frac{1}{10}$?

Since $\frac{4}{5}$ divided by 2 is $\frac{2}{5}$, $\frac{6}{7}$ divided by 3 is $\frac{2}{7}$, etc., we have the following

PRINCIPLE.—A fraction may be divided by dividing its numerator.

Since $\frac{1}{2}$ divided by 2 is $\frac{1}{4}$, $\frac{1}{3}$ divided by 4 is $\frac{1}{12}$, etc., we have the following

PRINCIPLE.—A fraction may be divided by multiplying its denominator.

WRITTEN PROBLEMS.

1. Divide $\frac{12}{14}$ by 3.

SOLUTION. *Explanation.*—$\frac{12}{14}$ divided by 1 is $\frac{12}{14}$; hence, $\frac{12}{14}$ di-
$\frac{12}{14} \div 3 = \frac{4}{14}$ vided by 3 equals $\frac{1}{3}$ of $\frac{12}{14}$, or $\frac{4}{14}$.

2. Divide $\frac{11}{12}$ by 6.

SOLUTION. *Explanation.*—$\frac{11}{12}$ divided by 1 equals $\frac{11}{12}$; hence,
$\frac{11}{12} \div 6$ $\frac{11}{12}$ divided by 6 equals $\frac{1}{6}$ of $\frac{11}{12}$, or $\frac{11}{72}$. Therefore,
$= \frac{1}{6} \times \frac{11}{12} = \frac{11}{72}$ $\frac{11}{12}$ divided by 6 is $\frac{11}{72}$.

Divide—

3. $\frac{12}{15}$ by 6.	*Ans.* $\frac{2}{15}$.	7. $\frac{9}{10}$ by 7.	*Ans.* $\frac{9}{70}$.
4. $\frac{18}{19}$ by 9.	*Ans.* $\frac{2}{17}$.	8. $\frac{15}{16}$ by 10.	*Ans.* $\frac{15}{160}$.
5. $\frac{45}{46}$ by 5.	*Ans.* $\frac{9}{80}$.	9. $3\frac{1}{4}$ by 5.	*Ans.* $\frac{2}{3}$.
6. $\frac{3}{4}$ by 6.	*Ans.* $\frac{1}{8}$.	10. $7\frac{1}{5}$ by 7.	*Ans.* $\frac{36}{35}$.

CASE II.

To Divide an Integer by a Fraction.

ORAL EXERCISE.

1. At $\frac{3}{4}$ of a dollar apiece, how many turkeys can I buy for 3 dollars ?

SOLUTION.—If 1 turkey cost $\frac{3}{4}$ of a dollar, for 3 dollars, or $\frac{12}{4}$ dollars, as many turkeys can be bought as $\frac{3}{4}$ is contained times in $\frac{12}{4}$, or 4 turkeys. Hence, if 1 turkey cost $\frac{3}{4}$ of a dollar, for 3 dollars 4 turkeys can be bought.

2. At $\frac{2}{3}$ of a dollar each, how many hats can I buy for 6 dollars?

3. If 1 hen cost $\frac{1}{4}$ of a dollar, how many hens can I buy for 2 dollars ?

4. How many baskets holding $\frac{2}{4}$ of a peck each will it take to hold 6 pk. ?

5. If 1 doz. of fish cost $\frac{1}{6}$ of a dollar, how many dozen can I buy for 3 dollars ?

6. If $\frac{3}{5}$ of a dollar buy 1 yd. of cloth, how many yards can I buy for 9 dollars ?

7. How many pairs of gloves at $\frac{4}{5}$ of a dollar a pair can I buy for 4 dollars?

8. How many times is $\frac{1}{2}$ contained in 2? In 3? In 4?

9. How many times is $\frac{2}{3}$ contained in 2? In 4? In 6?

10. How many times is $\frac{3}{4}$ contained in 6? In 9? In 12?

WRITTEN PROBLEMS.

1. Divide 9 by $\frac{3}{4}$.

SOLUTION.   *Explanation.*—Since 9 divided by 1 equals 9, 9
$9 \div \frac{3}{4} =$ divided by $\frac{1}{4}$ equals 4×9, and divided by $\frac{3}{4}$ it equals
$9 \times \frac{4}{3} = \frac{36}{3} = 12$ $\frac{1}{3}$ of 4×9, or $\frac{4}{3}$ of 9, which is equal to $\frac{36}{3}$, or 12.

The following is the rule for dividing an integer by a fraction :

RULE.

Multiply the integer by the denominator, and divide the result by the numerator of the given fraction. Or, Multiply the integer by the divisor inverted.

Divide—

2. 12 by $\frac{3}{4}$.	*Ars.* 16.	7. 30 by $\frac{7}{8}$.	*Ans.* $34\frac{2}{7}$.
3. 15 by $\frac{5}{6}$.	*Ans.* 18.	8. 60 by $\frac{5}{9}$.	*Ans.* 108.
4. 20 by $\frac{4}{5}$.	*Ans.* 25.	9. 40 by $3\frac{1}{2}$.	*Ans.* $11\frac{3}{7}$.
5. 18 by $\frac{6}{7}$.	*Ans.* 21.	10. 55 by $5\frac{1}{2}$.	*Ans.* 10.
6. 25 by $\frac{3}{5}$.	*Ans.* $41\frac{2}{3}$.	11. 75 by $6\frac{1}{2}$.	*Ans.* $11\frac{7}{13}$.

CASE III.

To Divide a Fraction by a Fraction.

ORAL EXERCISE.

1. At $\frac{2}{10}$ of a dollar each, how many caps can I buy for $\frac{4}{5}$ of a dollar?

SOLUTION.—If 1 cap cost $\frac{2}{10}$ of a dollar, for $\frac{4}{5}$ of a dollar, which is equal to $\frac{8}{10}$ of a dollar, I can buy as many caps as $\frac{2}{10}$ is contained times in $\frac{8}{10}$, or 4 caps.   Hence, at $\frac{2}{10}$ of a dollar each, I can buy 4 caps for $\frac{4}{5}$ of a dollar.

2. How many pens at $\frac{1}{4}$ of a cent each can I get for $2\frac{1}{4}$ cents?

3. How many slates at $\frac{1}{4}$ of a dollar each can I get for $\$3\frac{1}{2}$?

4. How many oranges at $\frac{2}{5}$ of a dime apiece can I buy for $4\frac{4}{5}$ dimes?

5 If silk cost $\frac{3}{4}$ of a dollar a yard, how many yards can I buy for $\$5\frac{1}{2}$?

6. If 6 hens cost one dollar and a half, how many can I buy for $\$4\frac{1}{2}$?

7. Three hats are sold for $\$4\frac{1}{2}$: how many can be bought for $\$13\frac{1}{2}$?

8. Divide $\frac{8}{8}$ by $\frac{2}{10}$.

SOLUTION.—Since 1 is contained in $\frac{8}{8}$, $\frac{8}{8}$ times, $\frac{1}{10}$ is contained in $\frac{8}{8}$ 10 times $\frac{8}{8}$ times, or $\frac{80}{8}$ times; and $\frac{2}{10}$ is contained in $\frac{8}{8}$, $\frac{1}{2}$ of $\frac{80}{8}$ times, or $\frac{40}{8}$ times, equal to 8. Hence, $\frac{8}{8}$ divided by $\frac{2}{10}$ equals 8.

9. Divide $\frac{3}{4}$ by $\frac{8}{12}$.
10. Divide $\frac{2}{3}$ by $\frac{1}{6}$.
11. Divide $3\frac{1}{2}$ by $\frac{3}{8}$.
12. Divide $3\frac{1}{2}$ by $\frac{5}{8}$.
13. Divide $\frac{3}{4}$ by $\frac{2}{3}$.
14. Divide $\frac{7}{12}$ by $\frac{7}{24}$.

15. Divide $1\frac{5}{2}$ by $\frac{3}{4}$.
16. Divide $2\frac{2}{5}$ by $\frac{1}{10}$.
17. Divide $2\frac{2}{8}$ by $\frac{5}{6}$.
18. Divide $9\frac{1}{2}$ by $\frac{1}{8}$.
19. Divide $9\frac{1}{2}$ by $\frac{2}{3}$.
20. Divide $9\frac{1}{2}$ by $2\frac{1}{4}$.

WRITTEN PROBLEMS.

1. Divide $\frac{3}{4}$ by $\frac{5}{6}$.

SOLUTION.   Analysis.—1 is contained in $\frac{3}{4}$, $\frac{3}{4}$ times; $\frac{1}{6}$ is
$\frac{3}{4} \div \frac{5}{6}$ contained in $\frac{3}{4}$, $6 \times \frac{3}{4}$ times, and $\frac{5}{6}$ is contained
$= \frac{3}{4} \times \frac{6}{5} = 1\frac{8}{20}$, or $\frac{9}{10}$ $\frac{1}{5}$ of $6 \times \frac{3}{4}$ times, or $\frac{6}{5} \times \frac{3}{4}$, equal to $1\frac{8}{20}$, or $\frac{9}{10}$.
Hence, $\frac{3}{4}$ divided by $\frac{5}{6}$ is $\frac{9}{10}$.

The following are the rules for the division of fractions by fractions:

RULES.

1. *Divide the dividend by the numerator of the divisor, and multiply by its denominator.*

2. *Invert the terms of the divisor, and multiply the numerators together, also the denominators.*

3. *Reduce the fractions to a common denominator, and divide the numerator of the dividend by the numerator of the divisor.*

Divide—

2. $\frac{3}{4}$ by $\frac{4}{5}$.   *Ans.* $1\frac{5}{16}$.
3. $\frac{5}{6}$ by $\frac{2}{3}$.   *Ans.* $1\frac{1}{4}$.
4. $\frac{2}{5}$ by $\frac{5}{6}$.   *Ans.* $1\frac{18}{25}$.
5. $\frac{5}{6}$ by $\frac{3}{8}$.   *Ans.* $\frac{25}{27}$.
6. $\frac{4}{5}$ by $\frac{7}{9}$.   *Ans.* $\frac{31}{35}$.
7. $\frac{3}{15}$ by $\frac{1}{5}$.   *Ans.* 1.

8. $3\frac{1}{2}$ by $\frac{1}{3}$.   *Ans.* $10\frac{1}{2}$.
9. $3\frac{3}{4}$ by $\frac{2}{5}$.   *Ans.* $9\frac{3}{8}$.
10. $4\frac{1}{2}$ by $\frac{7}{8}$.   *Ans.* $5\frac{1}{7}$.
11. $3\frac{1}{2}$ by $2\frac{1}{3}$.   *Ans.* $1\frac{1}{2}$.
12. $6\frac{1}{4}$ by $3\frac{1}{7}$.   *Ans.* $1\frac{87}{88}$.
13. $\frac{1}{2}$ of $17\frac{1}{2}$ by $3\frac{1}{4}$. *Ans.* $2\frac{9}{18}$.

CASE IV.

Complex Fractions.

NOTE.—**Complex Fractions** may be regarded as being one form of *Division of Fractions.* The number or fraction in the numerator may be regarded as the dividend, and the number or fraction in the denominator as the divisor. Thus, in the fraction $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{2}{3}$ is to be divided by $\frac{4}{5}$.

WRITTEN PROBLEMS.

1. Reduce $\frac{\frac{2}{3}}{\frac{5}{6}}$ to its simplest form.

SOLUTION.

$\frac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} \div \frac{5}{6}$ *Explanation.*—$\frac{2}{3}$ divided by $\frac{5}{6}$ equals $\frac{2}{3}$ multiplied by $\frac{6}{5}$, which equals $\frac{12}{15}$, or $\frac{4}{5}$.

$= \frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$, or $\frac{4}{5}$.

Reduce the following fractions to their simplest forms:

2. $\frac{\frac{3}{4}}{\frac{4}{5}}$.   *Ans.* $1\frac{5}{16}$.   7. $\frac{\frac{8}{4}}{9}$.   *Ans.* $\frac{1}{12}$.

3. $\frac{\frac{2}{3}}{\frac{1}{2}}$.   *Ans.* $\frac{4}{5}$.   8. $\frac{10}{\frac{3}{5}}$.   *Ans.* $16\frac{2}{3}$.

4. $\frac{\frac{5}{6}}{\frac{7}{8}}$.   *Ans.* $\frac{20}{21}$.   9. $\frac{6\frac{3}{4}}{7\frac{1}{2}}$.   *Ans.* $\frac{9}{10}$.

5. $\frac{\frac{6}{7}}{\frac{3}{4}}$.   *Ans.* $1\frac{1}{7}$.   10. $\frac{\frac{2}{5} \text{ of } 6\frac{1}{4}}{6\frac{1}{3}}$.   *Ans.* $\frac{15}{76}$.

6. $\frac{\frac{7}{8}}{6}$.   *Ans.* $\frac{7}{48}$.   11. $\frac{\frac{2}{3} + 4\frac{1}{2}}{3\frac{1}{3} - 2\frac{1}{4}}$.   *Ans.* $4\frac{18}{13}$.

REVIEW.

WRITTEN PROBLEMS.

1. Reduce $21\frac{5}{7}$ to an improper fraction.
2. Reduce $87\frac{1}{14}$ to an improper fraction.
3. Reduce $\frac{179}{16}$ to a mixed number.

4. Reduce $\frac{1800}{21}$ to a mixed number.

5. Reduce $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ to a simple fraction.

6. Reduce $\frac{1}{4}$ of $\frac{15}{2}$ of $6\frac{3}{4}$ to a simple fraction.

7. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{7}{12}$ to their least common denominator.

8. Reduce $\frac{5}{6}$, $\frac{7}{8}$, $\frac{13}{4}$ to their least common denominator.

9. Add $4\frac{1}{2}$, $3\frac{2}{3}$ and $6\frac{1}{4}$.   *Ans.* $14\frac{1}{24}$.

10. Add $\frac{1}{2}$ of $\frac{4}{5}$, $3\frac{1}{2}$ and $\frac{2}{3}$ of $6\frac{1}{4}$.   *Ans.* $7\frac{4}{5}$.

11. From $44\frac{1}{4}$ take $16\frac{5}{6}$.   *Ans.* $27\frac{2}{3}$.

12. From $18\frac{1}{4}+16\frac{1}{4}$ take $\frac{1}{2}$ of 27.   *Ans.* $20\frac{12}{25}$.

13. Multiply $12\frac{1}{2}$ by $12\frac{1}{2}$.   *Ans.* $156\frac{1}{4}$.

14. Multiply $\frac{2}{3}$ of $18\frac{3}{4}$ by $\frac{4}{5}$ of $23\frac{1}{4}$.   *Ans.* $231\frac{3}{4}$.

15. Divide $18\frac{1}{2}$ by $6\frac{1}{4}$.   *Ans.* $2\frac{24}{25}$.

16. Divide $\frac{5}{6}$ of $16\frac{1}{4}$ by $\frac{4}{5}$ of 16.   *Ans.* $\frac{27}{64}$.

17. Reduce $\dfrac{\frac{5}{6} \text{ of } \frac{3}{4}}{\frac{2}{3} \text{ of } \frac{5}{8}}$ to a simple fraction.   *Ans.* $1\frac{11}{14}$.

18. Reduce $\dfrac{1\frac{1}{2}+6\frac{3}{5}}{\frac{1}{2} \text{ of } 7+8\frac{1}{4}}$ to a simple fraction. *Ans.* $\frac{47}{70}\frac{7}{8}$.

19. A man earns $3\frac{2}{3}$ dollars one day, $6\frac{1}{4}$ dollars the next, and $4\frac{1}{2}$ the next: how much does he earn in three days?   *Ans.* $\$14\frac{5}{12}$.

20. A boy has $6\frac{2}{3}$ dozen hens, $3\frac{1}{4}$ dozen ducks and $1\frac{7}{12}$ dozen turkeys: how many fowls has he? *Ans.* $11\frac{1}{2}$ doz.

21. A farmer has $26\frac{2}{3}$ acres of woodland, and sells $16\frac{1}{4}$ acres: how much has he remaining? *Ans.* $10\frac{11}{12}$ acres.

22. If a man have $27\frac{2}{3}$ dollars, and earn $9\frac{8}{25}$ dollars more, how much will he have if he lose $16\frac{1}{4}$ dollars?

   Ans. $20\frac{67}{100}$ dollars.

23. One hat costs $\$3\frac{3}{4}$: how much will 14 hats cost at the same rate?   *Ans.* $\$52\frac{1}{2}$.

24. George earned $\$17\frac{1}{4}$ one week, and 4 times as much during the month, less $\$3\frac{3}{4}$: how much did he earn in the month?   *Ans.* $\$65\frac{1}{10}$.

25. Twenty-seven carpenters earn in one day $87¾ : how much does each earn in a day ?   *Ans.* $3¼.

26. If a store is worth $26246⅔, and is bought by a company of 16 men, how much does each one pay ?

Ans. $1640⅔.

27. A man owning a 40-acre farm planted ⅕ in potatoes, ⅔ in wheat and the remainder in corn : how many acres did he plant in each ?

Ans. 8 A. in potatoes, 26⅔ A. in wheat, 5⅓ A. in corn.

28. A farmer owned 127½ A. ; he sold 16¼ A. to one man and 45⅔ to another : how much has he remaining ?

Ans. 65¹³⁄₁₈ A.

29. If 15 bu. of wheat cost $18¾, how much will 16 bu. cost ?   *Ans.* $20.

30. A boy who is 14 years old is ₇⁄₂₀ as old as his father and ₇⁄₃₀ as old as his grandmother : how old is each ?

Ans. Father, 40 years ; grandmother, 60 years.

31. If a man sleep 7⅙ hr. a day, how many hours does he sleep in 365 d., or a year ?   *Ans.* 2615⅚ hr.

32. A man bought a barrel of sugar ; he sold ⅓, ¼ and ⅛ of it : what part of the barrel remains ?   *Ans.* ₇⁄₂₄.

33. What is the cost of 13⅛ ℔. of meat at 15 cts. a pound ?   *Ans.* $1.96⅞.

34. A man earns $2¾ a day, and spends $5½ a week for board : how much does he save in a week ?   *Ans.* $11.

35. A farmer sells ⅔ of his land, and finds that 12½ A. is ⅛ of the remainder : how much had he at first ?

Ans. 93¾ A.

36. What is the value of a turkey weighing 14⅜ ℔., at 12½ cts. a pound ?   *Ans.* $1.79¹¹⁄₁₆.

CHAPTER V.

DECIMAL FRACTIONS.

IF a unit be divided into ten equal parts, each of the parts is called a *tenth*.

If each tenth be divided into ten equal parts, each of the parts is called a *hundredth*.

In the same manner, each of the ten equal parts of a hundredth is called a *thousandth*.

83. A **Decimal Fraction** is a number of *tenths, hundredths, thousandths,* etc.—that is, a number of the decimal divisions of a unit.

SECTION I.

NOTATION AND NUMERATION.

84. A decimal fraction is usually written without its denominator; when so written, a point called the *decimal point* or *separatrix* is placed before the numerator. Thus, $\frac{3}{10}$ and $\frac{27}{100}$, when written decimally, are written .3 and .27.

85. When decimal fractions are written without their denominator, they are called *Decimals.*

86. The divisions, beginning at the decimal point and reading to the right, are named as follows: *tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths,* etc.

The following table shows the relative place of decimals and whole numbers:

Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.
4	4	4	4	4	4.	4	4	.	4	4	4	4	4	4	4	4

$\underbrace{\qquad\qquad}_{\text{Integers.}}$ $\underbrace{\qquad\qquad}_{\text{Decimals.}}$

The decimal .175 is read thus: 1 tenth, 7 hundredths and 5 thousandths, or 175 thousandths.

87. Decimals are read by naming the numerator as given, and the denominator, which is 1, with as many ciphers annexed as there are figures in the given numerator.

Read the following:

.8	.64	.125	.3142	6.23
.6	.72	.107	.0641	7.8
.4	.83	.604	.0037	19.303
.3	.27	.718	.0005	6.006

NOTE.—When the fraction is expressed decimally, the units' figure of the numerator is placed in the order indicated by the decimal. Thus, in $\frac{129}{10000}$ the 9 is placed in the ten-thousandths' place, and the other figures in their proper connection with it. It is written .0129, a cipher being used to fill the vacant space.

Write the following decimally:

$\frac{3}{10}$	$\frac{16}{100}$	$\frac{117}{1000}$	$6\frac{3}{10}$
$\frac{2}{10}$	$\frac{25}{100}$	$\frac{205}{1000}$	$\frac{137}{10000}$
$\frac{9}{10}$	$\frac{8}{100}$	$\frac{14}{1000}$	$\frac{6}{10000}$

142 thousandths. ·	16 hundredths.
·165 thousandths.	19 millionths.
63 hundredths.	14 .thousandths.
4 hundredths.	106 ten-thousandths.
107 thousandths.	6 hundred-thousandths.
707 ten-thousandths.	76 hundredths.
15 ten-thousandths.	1 thousandth.
5 tenths.	32 millionths.
12 thousandths.	16 ten-millionths.
104 thousandths.	106 hundred-thousandths.

88. An integer and a fraction may be written as one number.   Thus, $17\frac{3}{1000}$ is written decimally 17.003.

Write the following:

9 and 7 tenths.	603 and 19 ten-thousandths.
16 and 3 hundredths.	721 and 17 thousandths.
14 and 6 thousandths.	6 and 6 ten-thousandths.
16 and 18 hundredths.	3 and 13 millionths.
18 and 9 tenths.	13 and 3 millionths.
15 and 18 thousandths.	16 and 16 ten-thousandths.

PRINCIPLES.—1. The denominator of a decimal fraction is 1, with as many ciphers annexed as there are figures in the numerator.

2. Moving the decimal point to the right increases the decimal tenfold for every removal of one space.

3. Moving the decimal point to the left decreases the value of the decimal tenfold for every removal of one space.

4. Annexing a cipher to the right of a decimal, or removing a cipher from the right of a decimal, does not alter the value, as it simply multiplies or divides the numerator and the denominator by the same number.

SECTION II.

REDUCTION OF DECIMALS.

CASE I.

Reduction to Higher or Lower Terms.

ORAL EXERCISE.

1. How many tenths in 3 ones? In 8 ones?
2. How many hundreds in 6 tenths? In 9 tenths?
3. How many thousandths in 8 tenths? In 6 hundredths? In .7?
4. How many tenths in .60? In .500?

WRITTEN PROBLEMS.

1. Reduce .16 to thousandths.

PROCESS.   *Explanation.*—Since there are 10 thousandths in 1
.16 = .160   hundredth, in 16 hundredths there are 16 times 10
        thousandths, or 160 thousandths.

2. Reduce .13 to thousandths.
3. Reduce .6 to thousandths.
4. Reduce .17 to hundred-thousandths.
5. Reduce .14 to thousandths.
6. Reduce .600 to tenths.
7. Reduce .0700 to hundredths.
8. Reduce .6000 to tenths.

CASE II.

Reduction of Decimals to Common Fractions.

ORAL EXERCISE.

1. In .4 how many fifths?
2. How many fourths in $\frac{25}{100}$? In .75? In .500?
3. How many fifths in .20? In .60? In .0500?
4. How many twentieths in .5? In .40? In .3000?

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WRITTEN PROBLEMS.

1. Reduce .16 to a common fraction.

PROCESS.   Let the pupil explain.

$.16 = \frac{16}{100} = \frac{4}{25}$

NOTE.—In reducing a decimal to a common fraction, omit the decimal point and write the denominator.

Reduce the following decimals to common fractions :

2. .4.	*Ans.* $\frac{2}{5}$.	9. .125.	*Ans.* $\frac{1}{8}$.
3. .15.	*Ans.* $\frac{3}{20}$.	10. .625.	*Ans.* $\frac{5}{8}$.
4. .25.	*Ans.* $\frac{1}{4}$.	11. .0625.	*Ans.* $\frac{1}{16}$.
5. .60.	*Ans.* $\frac{3}{5}$.	12. 4.0375.	*Ans.* $4\frac{3}{80}$.
6. .08.	*Ans.* $\frac{2}{25}$.	13. 4.375.	*Ans.* $4\frac{3}{8}$.
7. .09.	*Ans.* $\frac{9}{100}$.	14. .035.	*Ans.* $\frac{7}{200}$.
8. .36.	*Ans.* $\frac{9}{25}$.	15. .1875.	*Ans.* $\frac{3}{16}$.

CASE III.

Reduction of Common Fractions to Decimals.

ORAL EXERCISE.

1. How many tenths in $\frac{1}{2}$; $\frac{1}{4}$; $\frac{2}{5}$?
2. How many hundredths in $\frac{1}{2}$; $\frac{1}{10}$; $\frac{2}{5}$?
3. How many thousandths in $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{20}$; $\frac{6}{20}$?

WRITTEN PROBLEMS.

1. Reduce $\frac{5}{40}$ to a decimal.

PROCESS.

40)5.000|.125
  40
 ───
 100
  80
 ───
 200
 200

Explanation.—$\frac{5}{40}$ equals $\frac{1}{40}$ of 5 ; 5 equals $\frac{50}{10}$, or $\frac{500}{100}$, or $\frac{5000}{1000}$; $\frac{1}{40}$ of $\frac{5000}{1000}$ is $\frac{125}{1000}$, or .125.

NOTE.—To reduce common fractions to decimals, annex ciphers to the numerator and divide by the denominator, and point off as many places in the quotient as there are ciphers annexed.

Reduce the following to decimals:

2. $\frac{1}{2}$.	*Ans.* .5.	12. $\frac{5}{8}$.	*Ans.* .625.
3. $\frac{1}{4}$.	*Ans.* .25.	13. $6\frac{1}{4}$.	*Ans.* 6.25.
4. $\frac{1}{8}$.	*Ans.* .125.	14. $\frac{4}{15}$.	*Ans.* .26$\frac{2}{3}$.
5. $\frac{2}{5}$.	*Ans.* .4.	15. $18\frac{3}{4}$.	*Ans.* 18.75.
6. $\frac{3}{4}$.	*Ans.* .75.	16. $56\frac{1}{4}$.	*Ans.* 56.25.
7. $\frac{3}{20}$.	*Ans.* .15.	17. $12\frac{1}{2}$	*Ans.* 12.5.
8. $\frac{7}{40}$.	*Ans.* .175.	18. $16\frac{3}{5}$.	*Ans.* 16.6.
9. $\frac{6}{80}$.	*Ans.* .075.	19. $\frac{5}{16}$.	*Ans.* .3125.
10. $1\frac{2}{8}$.	*Ans.* .2.	20. $7\frac{3}{4}$.	*Ans.* 7.6$\frac{3}{4}$.
11. $\frac{7}{16}$.	*Ans.* .4375.	21. $8\frac{3}{32}$.	*Ans.* 8.09375.

NOTES.—1. When the denominator of the fraction contains other prime factors than 2 or 5, the division will not terminate.

2. When a sufficient number of decimal places is obtained, the remainder may be expressed by a common fraction.

SECTION III.

ADDITION OF DECIMALS.

WRITTEN PROBLEMS.

1. Add 3.7, 18.24, 163.017 and 614.16.

PROCESS.

$$\begin{array}{r} 3.7 \\ 18.24 \\ 163.017 \\ \underline{614.16} \\ 799.117 \end{array}$$

Explanation.—The decimals are so written that units of the same order stand in the same perpendicular column, and the addition is then performed the same as in simple numbers.

RULE.

Write the numbers so that the decimal points shall stand in the same perpendicular column, and add as in simple numbers.

2. Add 16.15, 18.17, 9.16 and 20.13. *Ans.* 63.61.

3. Add 15.04, 7.7, 18.05 and 16.43.   *Ans.* 57.22.
4. Add 3.64, .75, 127.5 and 27.003.   *Ans.* 158.893.
5. Add .718, 72.5, 5.316, 7.29.   *Ans.* 85.824.
6. Add 18.43, 17.03, 10.894, 1.707.   *Ans.* 48.061.
7. Add 33.624, .006, 600, 800.08.   *Ans.* 1433.71.
8. Add 21.05, 26.005, 18.0005, 20.5. *Ans.* 85.5555.
9. Add 14.003, 1.275, 1.324, 16.5.   *Ans.* 33.102.
10. Add 80.07, 645.3, 3.003, 64.00016.

Ans. 792.37316.

SECTION IV.

SUBTRACTION OF DECIMALS.

WRITTEN PROBLEMS.

1. From 65.13 take 18.344.

PROCESS.   *Explanation.*—Since annexing a cipher to the right
65.130   of a decimal does not alter its value, 65.13 may be
18.344   written 65.130.   Writing the number so that the deci-
46.786   mal points are immediately above each other, the sub-
traction is performed as in simple numbers.

RULE.

*Write the numbers one above the other, so that the decimal
point of the minuend shall be immediately above the decimal
point in the subtrahend, and subtract as in simple numbers.*

Find the value—

2. Of 87.15 – 63.24.   *Ans.* 23.91.
3. Of 119.3 – 65.784.   *Ans.* 53.516.
4. Of 107.07 – 6.45.   *Ans.* 100.62.
5. Of 615.34 – 7.183.   *Ans.* 608.157.
6. Of 7.004 – 4.7.   *Ans.* 2.304.
7. Of 14 – .14.   *Ans.* 13.86.
8. Of 3.3. – .033.   *Ans.* 3.267.

9. Of 21.5 – 16¼.   *Ans.* 5.25.

10. Of 307¾ – 194.785.   *Ans.* 112.965.

11. Of 12½ – 6¾.   *Ans.* 5.75.

12. From 9 and 7 tenths take 3 and 45 hundredths.
                               Ans. 6.25.

13. From 8 and 3 hundredths take 83 millionths.
                          Ans. 8.029917.

SECTION V.

MULTIPLICATION OF DECIMALS.

WRITTEN PROBLEMS.

1. Multiply 3.12 by 5.25.

PROCESS.

  3.12
  5.25   *Explanation.*—3.12, or $\frac{312}{100}$, multiplied by 5.25, or
 1560 $\frac{525}{100}$, equals $\frac{163800}{10000}$, or 16.38.
  624
1560
16.3800

From the foregoing we have the rule for multiplication of decimals:

RULE.

Multiply decimals as simple numbers, and point off from the right as many decimal places as there are in the multiplier and the multiplicand.

NOTE. —If the number of figures in the product is less than the number in the two factors, prefix as many ciphers as may be necessary to make the number of decimal places in the product equal the number in both factors.

2. Multiply 16.14 by .6.   *Ans.* 9.684.

3. Multiply 17.21 by .15.   *Ans.* 2.5815.

4. Multiply 27.48 by .07.   *Ans.* 1.9236.

   11 *

5. Multiply 18.23 by 1.17.   *Ans.* 21.3291.

6. Multiply 27.007 by 1.006.   *Ans.* 27.169042.

7. Multiply 20.02 by 4.8.   *Ans.* 96.096.

8. Multiply 372.006 by 4.09.   *Ans.* 1521.50454.

9. Multiply 264.0078 by 5.19.   *Ans.* 1370.200482.

10. Multiply 8.0008 by 6 tenths and 7 thousandths.
                                    Ans. 4.8564856.

11. Multiply 96 and 374 thousandths by 81.18.
                                    Ans. 7823.64132.

SECTION VI.

DIVISION OF DECIMALS.

WRITTEN PROBLEMS.

1. Divide 29.295 by 2.17.

PROCESS.

2.17)29.295|13.5
  217
  ——
  759
  651
  ——
  1085
  1085

Explanation.—29.295 is equal to $\frac{29295}{1000}$; 2.17 is equal to $\frac{217}{100}$; $\frac{29295}{1000}$ divided by $\frac{217}{100}$ equals $\frac{135}{10}$, or 13.5.

NOTE.—Since the quotient multiplied by the divisor equals the dividend, it is evident, according to the preceding section, that the number of decimal places in the divisor plus the number in the quotient equals the number in the dividend; hence, the number in the dividend less the number in the divisor equals the number in the quotient. Hence the following rule for division of decimals:

RULE.

Divide as in simple numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number of those in the divisor.

NOTE.—If the number of decimal places in the divisor exceeds the number in the dividend, first annex enough ciphers to the divi-

dend to make the number of places equal to the number in the divisor.

   2. Divide 123.39 by 4.5.   *Ans.* 27.42.
   3. Divide 77.935 by 1.09.   *Ans.* 71.5.
   4. Divide 205.2608 by 24.32.   *Ans.* 8.44.
   5. Divide 2.7306 by 15.17.   *Ans.* .18.
   6. Divide 16644.3728 by 23.08. *Ans.* 721.16.
   7. Divide 6.54 by 26.16.   *Ans.* .25.
   8. Divide 13.68 by 18.24.   *Ans.* .75.
   9. Divide 164.835 by 20.25.   *Ans.* 8.14.
10. Divide .0121344 by .016.   *Ans.* .7584.

REVIEW PROBLEMS.

   1. A horse cost $125.5, a cow $15.75 and some sheep $124.63: how much did they all cost? *Ans.* $265.88.

   2. A farm consists of 23.385 A. of meadow, 16.315 A. of woodland, and 45.3 A. of tillable land: how many acres are in the farm?   *Ans.* 85 A.

   3. If a man have 27.75 dollars, and spend 18.125 dollars, how much money has he remaining?
                               Ans. 9.625 dollars.

   4. If a house cost $1162.5, a store $3146.1875, and both were sold for $4400, how much was gained by the sale?   *Ans.* $91.3125.

   5. A merchant has 650.5 lb. of sugar; he buys 425¼ lb. more, and sells 824.125 lb.: how much remains?
                              Ans. 251.625 lb.

   6. If a man travel 3.125 mi. an hour, how far will he travel in 6.2 hr.?   *Ans.* 19.375 mi.

   7. How much will 7.125 lb. of meat cost at 12.5 cts. a pound?   *Ans.* 89.0625 cts.

   8. What is the cost of 93.5 lb. of lard at 12.5 cts. a pound?   *Ans.* $11.6875.

9. What is the cost of 16$\frac{8}{50}$ A. of land at $125.5 an acre?   *Ans.* $2012.70625.

10. If a man travel at the rate of 31.15 mi. a day, how far will he travel in 14.375 days?

Ans. 447.78125 mi.

11. If a man travel at the rate of 31.5 mi. a day, in how many days can he travel 204.75 mi.?   *Ans.* 6.5 d.

12. If 18.75 lb. of meat cost $2.34375, how much will 1 lb. cost.?   *Ans.* 12.5 cts.

13. If I trade 15$\frac{1}{2}$ lb. of butter at 37.5 cts. a pound for coffee at 18.75 cts. a pound: how many pounds of coffee do I get?   *Ans.* 31 lb.

14. A man bought a farm of 45.375 A. for $7260: how much did it cost him per acre?   *Ans.* $160.

15. Exchanged 3.5 T. of coal at $4.80 a ton for lath at 30 cts. a hundred: how many lath did I get?

Ans. 5600.

CHAPTER VI.

DENOMINATE NUMBERS.

SECTION I.

DEFINITIONS.

89. A **Concrete Number** is one which is applied to a particular unit; as, 3 sheep, 2 pounds, etc.

90. A **Denominate Number** is one whose unit is named; as, 6 gallons, 3 inches, etc.

Denominate numbers are always concrete.

91. Numbers are of the same denomination when they have the same unit; as, 2 pints, 7 pints, etc.

92. A Compound Number is one consisting of several denominate numbers, but of the same measure; as, 6 feet 3 inches.

SECTION II.

TABLES AND MEASURES.

VALUE.

Note.—**United States Money** has been considered on page 66 and the pages following.

ENGLISH CURRENCY.

93. English Currency is the money used in Great Britain and Ireland.

TABLE.

4 farthings (far.)	= 1 penny, d.
12 pence	= 1 shilling, s.
20 shillings	= 1 pound, £.
21 shillings	= 1 guinea, G.

1 £ = 20 s. = 240 d. = 960 far.

ORAL EXERCISES.

1. How many farthings in 3 d. ?   In 5 d. ?
2. How many pence in 3 s. ?   In 12 s. ?
3. How many shillings in 6 £?   In 9 £?
4. How many shillings in 60 d. ?   In 120 d. ?
5. How many pence in 12 s. 6 d. ?   In $6\frac{1}{2}$ s. ?
6. How many £ in 480 d. ?   In 80 s. ?

I

WRITTEN PROBLEMS.

1. Reduce 2 £ 3 s. to shillings.

SOLUTION.

2 £ 3 s.
20

40
3

43 s.

Explanation.—Since 1 £ equals 20 s., 2 £ equal 2 times 20 s., or 40 s.; 40 s. plus 3 s. are 43.

2. Reduce 975 d. to £, etc.

SOLUTION.

12)975 d.

20)81 s. 3 d.

4 £ 1 s.

Explanation.—Since 12 d. equal 1 s., in 975 d. there are as many shillings as 12 is contained times in 975, or 81 s. 3 d.

Since 20 s. equal 1 £, there are as many £ in 81 s. as 20 is contained times in 81, or 4 £ 1 s.

3. Reduce 5 £ 6 s. to shillings.   *Ans.* 106 s.
4. Reduce 1 £ 5 s. 6 d. to pence.   *Ans.* 306 d.
5. Reduce 1680 d. to £, s. and d.   *Ans.* 7 £.
6. Reduce 16740 far. to £, etc.   *Ans.* 17 £ 8 s. 9 d.

WEIGHT.

AVOIRDUPOIS WEIGHT.

94. Avoirdupois Weight is used in weighing produce, coal, iron, groceries etc.

TABLE.

16 drams (dr.)	= 1 ounce,	oz.
16 ounces	= 1 pound,	lb.
25 pounds	= 1 quarter,	qr.
4 quarters	= 1 hundred-weight,	cwt.
20 hundred-weight	= 1 ton,	T.

1 ton = 20 cwt. = 80 qr. = 2000 lb = 32000 oz.

1 lb. avoirdupois equals 7000 grains.

The *long ton*, equal to 2240 lb., is used in collecting

duties at the United States custom-houses and in selling
coal at wholesale.

The following are also used:

32 lb. of oats,		
45 " of timothy seed,		
48 " of barley,	= 1 bushel.	
56 " of rye or Indian corn,		
60 " of wheat, potatoes or clover seed,		
56 " of butter	= 1 firkin.	
196 " of flour,		
200 " of beef or pork,	= 1 barrel.	
100 " of dry fish	= 1 quintal.	
100 " of nails	= 1 keg.	

ORAL EXERCISE.

1. How many ounces in 3 lb.? In 5½ lb.?
2. How many pounds in 64 oz.? In 240 oz.?
3. How many pounds in 4 T.? In 2 T. 6 cwt.?
4. I exchange 2 cwt. 3 qr. of flour at 2 cts. a pound for
muslin at 10 cts. a yard: how many yards do I get?

WRITTEN PROBLEMS.

1. Reduce 6 T. 1 cwt. 3 qr. to quarters.

SOLUTION.

```
6 T. 1 cwt. 3 qr.
   20
  120
    1
  121 cwt.
    4
  484
    3
  487 qr.
```

Hence, 6 T. 1 cwt. 3 qr. = 487 qr.

2. Reduce 2750 lb. to tons, etc.

SOLUTION.

```
25)2750 lb.
  4)110 qr.
 20)27 cwt. 2 qr.
     1 T. 7 cwt.
```

Hence, 2750 lb. = 1 T. 7 cwt. 2 qr.

Let the pupil explain.

3. Reduce 6 T. 3 cwt. to hundred-weight. *Ans.* 123 cwt.

4. Reduce 18 T. 1 cwt. 22 lb. to pounds.

Ans. 36122 lb.

5. Reduce 2300 lb. to tons, etc.   *Ans.* 1 T. 3 cwt.

6. What is the value of 1 T. 4 cwt. of flour at 2½ cts. a pound?   *Ans.* $60.

7. What will 16 bbl. of flour cost at 2¾ cts. a pound?

·*Ans.* $86.24.

8. How much will 3 T. of wheat cost at $1.10 a bushel?

Ans. $110.

9. A farmer sold 60 bags of wheat, each weighing 183 lb., at $1.15 a bushel : how much did he get for his wheat?

Ans. $210.45.

10. What is the value of 40 bu. of clover-seed at 13½ cts. a pound?   *Ans.* $324.

TROY WEIGHT.

95. Troy Weight is used in weighing gold, silver, gems and jewels.

TABLE.

24 grains (gr.)   = 1 pennyweight, dwt.
20 pennyweights = 1 ounce,   oz.
12 ounces   = 1 pound,   lb.

1 lb. Troy = 12 oz. = 240 dwt. = 5760 gr.

APOTHECARIES' WEIGHT.

96. Apothecaries' Weight differs from Troy weight in the division of the ounce.   It is used in weighing medicines.

TABLE.

20 grains (gr.) = 1 scruple, Ͽ.
3 scruples   = 1 dram,   Ʒ.
8 drams   = 1 ounce,   ℥.
12 ounces   = 1 pound,   lb.

1 lb. = 12 ʒ = 96 ʒ = 288℈ = 5760 gr.

1 lb. avoirdupois = 7000 gr.; 1 oz. avoirdupois = 437½ gr.
1 lb. Troy   = 5760 gr.; 1 oz. Troy   = 480 gr.
1 lb. apothecaries' = 5760 gr.; 1 oz. apothecaries' = 480 gr.

ORAL EXERCISE.

1. How many ounces in 6 lb. Troy? In 4½ lb.?

2. How many ounces in 3 lb. 11 oz. Troy? In 10 lb. 14 oz.?

3. How many pounds in 49 oz.? In 59 oz.? In 63 oz.?

4. How many pennyweights in 4 oz.? In 5 lb.? In 2 lb. 6 oz.?

5. How many ounces in 60 dwt.? In 960 gr.? In 2½ lb.?

6. How many scruples in 3 ʒ 2 ʒ? In 1 lb. 6 ʒ 5ʒ?

7. What is the difference in weight between an ounce of gold and an ounce of iron?

WRITTEN PROBLEMS.

1. Reduce 8 lb. 3 oz. 4 dwt. to pennyweights.

SOLUTION.

```
  8 lb. 3 oz. 4 dwt.
 12
 96
  3
 99 oz.
 20
1980
   4
1984 dwt.
```

Hence, 8 lb. 3 oz. 4 dwt. = 1984 dwt.

2. Reduce 8600 gr. to pounds.

SOLUTION.

```
24)8600 gr.
  20)358 dwt. 8 gr.
    12)17 oz. 18 dwt.
        1 lb. 5 oz.
```

Hence, 8600 gr. = 1 lb. 5 oz. 18 dwt. 8 gr.

Let the pupil explain.

12

3. Reduce 6 lb. 7 oz. Troy to ounces. *Ans.* 79.

4. Reduce 7 lb. 8 oz. 4 dwt. to pennyweights.

Ans. 1844 dwt.

5. Reduce 1600 dwt. to pounds, etc. *Ans.* 6 lb. 8 oz.

6. Reduce 9000 gr. to pounds, etc.

Ans. 1 lb. 6 oz. 15 dwt.

7. How much is a gold chain weighing 1 oz. 3 dwt. worth, at 90 cts. a pennyweight? *Ans.* $20.70.

8. How many grains in 6½ lb. Troy? *Ans.* 37440.

9. How many grains in 6½ lb. avoirdupois?

Ans. 45500.

EXTENSION.

97. Extension is that which has one or more of the dimensions—length, breadth and thickness.

98. A *Line* has length only.

99. A *Surface* has length and breadth.

100. A *Volume* has length, breadth and thickness.

LONG MEASURE.

101. Long Measure is used in measuring length and distance.

TABLE.

12 inches (in.)	= 1 foot,	ft.
3 feet	= 1 yard,	yd.
5½ yards, or 16½ feet	= 1 rod,	rd.
40 rods	= 1 furlong,	fur.
8 furlongs	= 1 mile,	mi.
3 miles	= 1 league,	lea.

1 mi. = 8 fur. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

The inch is usually divided into halves, quarters, etc.

ORAL EXERCISE.

1. How many inches in 3 ft.?   In 5 ft.?   In 10 ft.?
2. How many feet in 24 in.?   In 60 in.?   In 84 in.?
3. How many feet in 6 yd.?   In 40 yd.?   In 3 yd. 2 ft.?
4. How many yards in 18 ft.?   In 36 ft.?   In 72 in.?
5. How many yards in 3 rd.?   In 4½ rd.?
6. How many rods in 22 yd.?   In 49½ ft.?
7. How many inches in 6 ft. 3 in.?   In 3 yd. 2 ft. 9 in.?
8. At 40 cents a foot, how much will 20 rd. of fence cost?

WRITTEN PROBLEMS.

1. Reduce 7 yd. 2 ft. 3 in. to inches.

SOLUTION.

```
   7 yd. 2 ft. 3 in.
   3
  ――
  21
   2
  ――
  23 ft.
  12
 ―――
 276
   3
 ―――
 279 in.
```

Hence, 7 yd. 2 ft. 3 in. = 279 in.

2. Reduce 940 in. to rods.

SOLUTION.

```
12)940 in.
   3)78 ft. 4 in.
  5½)26 yd.
   2    2
  ――  ――
  11 )52
        4 rd. ⅔ yd., or 4 yd.
```

NOTE.—5½ = ¹¹⁄₂, and 26 = ⁵²⁄₂.

Hence, 940 in. = 4 rd. 4 yd. 4 in.

3. Reduce 16 ft. 5 in. to inches.   *Ans.* 197 in.
4. Reduce 5 yd. 2 ft. 10 in. to inches.   *Ans.* 214 in.
5. Reduce 10 rd. to feet; to inches.   *Ans.* 165 ft.
6. Reduce 620 in. to yards, etc.   *Ans.* 17 yd. 8 in.
7. Reduce 720 ft. to rods, etc. *Ans.* 43 rd. 3 yd. 1 ft. 6 in.
8. Reduce 800 in. to rods, etc.   *Ans.* 4 rd. 8 in.

SURFACE OR SQUARE MEASURE.

**102. Square Measure** is used in measuring areas or surfaces.

103. A *Surface* has length and breadth only.

104. A *Rectangle* is a fig-
ure having four sides, each
of which is perpendicular to
two of the others. A slate,
a book and a sheet of paper
are examples of rectangles.

105. A *Square* is a rectangle whose
four sides are equal.

A square inch is a square whose
sides are each one inch long.
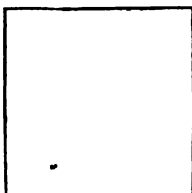
106. The *Area* of a surface is equal
to the product of the two numbers rep-
resenting the two dimensions. Thus,
in the rectangle A B C D,
5 ft. long and 3 ft. wide, the
unit, which is 1 sq. ft., is
contained 3×5 times, or 15
times. This may be found
also by actual count of the
small squares found in the
rectangle.
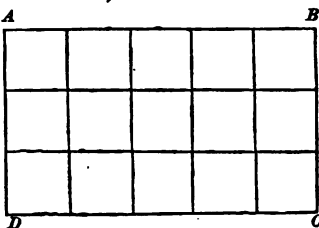
From this we have the rule to find the area of a rect-
angle or similar surface:

RULE.

Multiply the length by the breadth.

TABLE.

144 square inches (sq. in.)	= 1 square foot, sq. ft.
9 square feet	= 1 square yard, sq. yd.
$30\frac{1}{4}$ square yards,	
$272\frac{1}{4}$ square feet }	= 1 square rod, sq. rd.
160 square rods	= 1 acre, A.
640 acres	= 1 square mile, sq. mi.

1 A. = 160 sq. rd. = 4840 sq. yd. = 43560 sq. ft. = 6272640 sq. in.

ORAL EXERCISE.

1. How many square inches in 2 sq. ft.?   In 1 sq. yd.?
2. How many square feet in 3 sq. yd.?   In 5 sq. yd.?
3. How many square feet in 3 sq. rd.?
4. How many square yards in 5 sq. rd.?   In 90 sq. ft.?
5. How much will 1 sq. mi. of land cost at $50 an acre?

WRITTEN PROBLEMS.

1. How many square rods in 2 A. 4 sq. rd.? *Ans.* 324.
2. Reduce 435600 sq. ft. to acres?   *Ans.* 10 A.
3. How many acres in 3 sq. mi.?   *Ans.* 1920 A.
4. A town-lot is 160 ft. long and 60 ft. wide : how many square feet does it contain ?   *Ans.* 9600.
5. In a rectangular piece of land 480 ft. long and 320 ft. wide, how many acres are there?

   Ans. 3 A. 84 sq. rd. 51 sq. ft.

6. If I buy 10 A. of land at $150 an acre, and sell it at 2 cts. a square foot, how much do I gain? *Ans.* $7212.
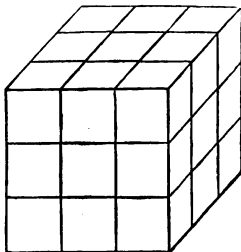
MEASURES OF VOLUME.

107. Cubic Measure is used in computing the contents of volumes or solids.

108. A *Volume* or *Solid* has three dimensions—length, breadth and thickness.

109. A *Cube* is a volume bounded by six equal squares, which are called its *faces.* The three dimensions of a cube are equal.

A cubic inch is a cube each side of which is one inch in length.

12 *

110. The *Volume* of a body is expressed by the product of the numbers representing its three dimensions.

TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot, cu. ft.
27 cubic feet   = 1 cubic yard, cu yd.
1 cu. yd. = 27 cu. ft. = 46656 cu. in.

WOOD MEASURE.

TABLE.

16 cubic feet (cu. ft.) = 1 cord foot, cd. ft.

8 cord feet, ⎫
or 128 cubic feet ⎬ = 1 cord,   cd.

A *cord* of wood is a pile 8 ft. long, 4 ft. high and 4 ft. wide.

24¾ cubic feet = 1 perch of stone or masonry, pch.

ORAL EXERCISE.

1. How many cubic feet in 3 cu. yd. ?   In 3456 cu. in.?

2. How many cubic feet in 3 cords ?   In 2 cords?   In 1½ cords?

3. How many cubic feet in a block 3 ft. long, 2 ft. thick and 2 ft. high ?

4. How many cubic feet in 2 pch. of stone?   In 4 pch.?

WRITTEN PROBLEMS.

1. How many cubic feet in 6 pch. of stone?
Ans. 148½.

2. How many cubic inches in 10 cu. ft ?   *Ans.* 17280.

3. In ¼ of a cord of wood how many cubic feet ?
Ans. 32.

4. How many cords in a pile of wood 60 ft. long, 4 ft. wide and 6 ft. high?   *Ans.* 11¼.

5. How many perch of stone in a wall 200 ft. long, 1½ ft. thick and 6 ft. high?   *Ans.* 72$\frac{6}{11}$.

6. How many cubic inches in a tank 6 ft. long, 4 ft. wide and 3 ft. deep?   *Ans.* 124416.

MEASURES OF CAPACITY.

LIQUID MEASURE.

111. Liquid Measure is used in measuring liquids.

TABLE.

4 gills (gi.) = 1 pint,   pt.
2 pints   = 1 quart, qt.
4 quarts   = 1 gallon, gal. = 231 cu. in.

1 gal. = 4 qt. = 8 pt. = 32 gi. = 231 cu. in.

112. The ale gallon contains 282 cu. in., but it is now rarely used.

113. The barrel and the hogshead are no longer fixed measures. Their contents are estimated by gallons.

114. In estimating the contents of cisterns, reservoirs, etc.,

1 barrel (bbl.)   = 31½ gallons.
1 hogshead (hhd.) = 63 gallons, or 2 barrels.

ORAL EXERCISE.

1. How many gills in 3 pt.? In 3 qt.?
2. How many pints in 40 gi.? In 3 gal.?
3. How many quarts in 20 gal.? In 40 gi.?
4. How many pints in 3 gal. 2 qt.?
5. If a gill of oil cost 3 cents, how much will a gallon cost?

6. If 2 gal. of wine cost $5.12, how much will a gil' cost?

WRITTEN PROBLEMS.

1. Reduce 16 gal. 3 qt. to quarts.   *Ans.* 67 qt.
2. Reduce 73 gal. 2 qt. 1 pt. to pints.   *Ans.* 589 pt.
3. Reduce 1 gal. 3 qt. to gills.   *Ans.* 56 gi.
4. Reduce 600 gi. to gallons, etc. *Ans.* 18 gal. 3 qt.
5. Reduce 900 pt. to gallons, etc. *Ans.* 112 gal. 2 qt.
6. How many cubic inches in a barrel of $31\frac{1}{2}$ gal.
                    Ans. $7276\frac{1}{2}$ cu. in.

DRY MEASURE.

115. Dry Measure is used in measuring grain, fruit, salt, soft coal, etc.

TABLE.

    2 pints (pt.) = 1 quart, qt.
    8 quarts,   = 1 peck, pk.
    4 pecks   = 1 bushel, bu. = 2150.42 cu. in.

  1 bus. = 4 pk. = 32 qt. = 64 pt. = 2150.42 cu. in.

1 pint dry measure equals nearly $1\frac{1}{6}$ pints liquid measure.

116. In measuring grain, beans and other small seeds the measure must be *even* full. But in measuring beets, potatoes and other coarse vegetables it must be *heaped* measure. Four *heaped* measures are equal to 5 *even* measures. A heaped bushel equals $\frac{5}{4}$ of an even bushel.

ORAL EXERCISE.

1. How many pints in 6 qt.? In 3 pk.?
2. How many quarts in 16 pt.? In 7 pk.?
3. How many quarts in 2 bu.? In 1 bu. 3 pk.?
4. How many pecks in 80 qt.? In 4 bu. 3 pk.?
5. How many pints in 1 bu.? In 3 bu. 2 pk.?

WRITTEN PROBLEMS.

1. How many quarts in 3 bu. 2 pk.? *Ans.* 112.
2. How many bushels in 634 qt. of beans?
Ans. 19 bu. 3 pk. 2 qt.
3. How many pints in 17 bu. 3 pk. 6 qt.? *Ans.* 1148 pt.
4. If I buy 2 bu. of walnuts at $1.50 a bushel, and sell them at 3 cts. a pint, how much do I gain? *Ans.* 84 cts.

APOTHECARIES' FLUID MEASURE.

117. This measure is used in measuring liquids in preparing medicines.

TABLE.

60 minims, or drops (m.) = 1 fluid drachm, f℥.
8 fluid drachms   = 1 fluid ounce, f℥.
16 fluid ounces   = 1 pint,   O.
8 pints   = 1 gallon,   Cong.

1 gal. = 8 pt. = 128 f℥ = 1024 f℥ = 61440 m.

ORAL EXERCISE.

1. How many ounces in 3 pt.?
2. How many f℥ in 3 f℥?
3. How many f℥ in 3 Cong.?
4. How many minims in ½ f℥?
5. How many f℥ in 3 O. 5 f℥.

TIME.

118. Time is a measured part of duration.

TABLE.

60 seconds (sec.) = 1 minute, min.
60 minutes   = 1 hour,   hr.
24 hours   = 1 day,   d.
365 days   = 1 year,   yr.
366 days   = 1 leap year.
100 years   = 1 century, cen.

Also,

7 days = 1 week, wk.
4 weeks = 1 month, mo.
52 weeks = 1 year, yr.

119. In business transactions 30 days is considered a month, and 12 months a year.

. The months of April, June, September and November have 30 days each.

February has 28 d., except in leap year, when it has 29.

The remaining months, January, March, May, July, August, October and December, have 31 days each.

ORAL EXERCISE.

1. How many seconds in 3 min. ? In $\frac{1}{2}$ min. ?
2. How many minutes in 120 sec. ? In 4 hr. ? In 1½ hr. ?
3. How many hours in 240 min. ? In 4 d. ?
4. How many hours in 5 d. ? In 6 d. ?
5. How many weeks in 35 d. ? In 60 d. ?
6. How many days in 6 wk. ? In 5 wk. 3 d. ?
7. What part of a day is 12 hr. ? 4 hr. ?

WRITTEN PROBLEMS.

1. Reduce 6000 sec. to hours, etc. *Ans.* 1 hr. 40 min.
2. How many minutes in 2 d. ?   *Ans.* 2880.
3. How many hours from 3 o'clock in the morning till 9 o'clock at night?   *Ans.* 18 h.
4. How many minutes from 6 o'clock A. M. to 4 o'clock P. M. ?   *Ans.* 600 min.

NOTE.—A. M. means before noon ; M., noon ; and P. M., after-noon.

5. How much time passes from 10 minutes before 7 in the morning to 8 o'clock 25 min. P. M.?

Ans. 13 hr. 35 min.

CIRCULAR MEASURE.

120. Circular or **Angular Measure** is used in measuring angles and arcs of circles. Also in determining the latitude and longitude of places.

121. The measuring unit is called a *Degree.* It is $\frac{1}{360}$ part of the circumference of a circle.

122. A *Circle* is a plane figure bounded by a curved line, every part of which is equally distant from a point within called the *Centre.*
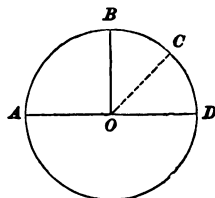
123. The bounding line is called the *Circumference.* The space within the circumference is the circle.

124. An *Arc* is any part of the circumference, as A B or A C.

125. An *Angle* is the difference in the direction of two lines meeting at a common point called the *Vertex.* Thus, B O C and A O C are angles.

126. A *Right Angle* is one in which the two lines are perpendicular to each other. Thus, A O B and B O D are right angles.

127. An angle is measured by the arc of the circle included between its two sides. Thus, the measure of the angle B O C is the arc B C.

128. Any straight line extending from the centre to the circumference, as O B, is called a *Radius,* and any straight line passing through the centre and limited by the circumference, as A D, is called a *Diameter.*

129. Every circumference is divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* and each minute into 60 equal parts, called *seconds.*

TABLE.

60 seconds (″)	= 1 minute,	(′).
60 minutes	= 1 degree,	(°).
30 degrees	= 1 sign,	s.
12 signs or 360°	= 1 circumference, c.	
90°	= 1 quadrant, or right angle.	

ORAL EXERCISE.

1. How many degrees in 120′ ?
2. How many minutes in 5° ?   In 240″ ?
3. How many minutes in 4° 30′ ?
4. How many degrees in 7200″ ?

WRITTEN PROBLEMS.

1. Reduce 18900″ to degrees, etc.   *Ans.* 5° 15′.
2. Reduce 19° 18′ to minutes.   *Ans.* 1158′.
3. Reduce 1850′ to degrees, etc.   *Ans.* 30° 50′.
4. Reduce 4° 6′ 30″ to seconds.   *Ans.* 14790″.
5. How many degrees, etc., in 27604″ ? *Ans.* 7° 40′ 4″.

COUNTING.

130. The following table is used in counting buttons, screws, etc.

TABLE.

12 things	= 1 dozen,	doz.
12 dozen	= 1 gross,	gro.
12 gross	= 1 great-gross,	gr.-gro.
20 units	= 1 score,	sc.

1 gr.-gro. = 12 gro. = 144 doz. = 1728 units.

PAPER.

24 sheets = 1 quire,   qr.
20 quires = 1 ream,   rm.
2 reams   = 1 bundle, bun.
5 bundles = 1 bale,   B.

1 B. = 5 bun. = 10 rm. = 200 qr. = 4800 sheets.

ORAL EXERCISE.

1. How many are $1\frac{1}{2}$ doz.? $4\frac{1}{4}$ doz?
2. How many dozen in 3 gro.?   In 2 gr.-gro.?
3. How many buttons in 5 gro.?
4. How many gross in 432 screws?
5. How many sheets in 6 qr. of paper?
6. How many sheets of paper in $\frac{1}{2}$ rm.?
7. How many quires in 3 rm.?   In 240 sheets?
8. How many sheets in $3\frac{1}{2}$ rm.?   How many quires?

WRITTEN PROBLEMS.

1. How many things in 3 gr.-gro.?   *Ans.* 5184.
2. How many things in $6\frac{1}{4}$ gro.?   *Ans.* 900.
3. How many matches in 5 gr.-gro.?   *Ans.* 8640.
4. How many gross in 1500?   *Ans.* 10 gro. 5 doz.
5. How many quires in 1700 sheets of paper?
                                Ans. 70 qr. 20 sh.
6. How many sheets of paper in 11 qr. 11 sh.?
                                Ans. 275 sheets.
7. How many sheets in 3 bales of paper? *Ans.* 14400.
8. How many reams, etc. in 2500 sh. of paper?
                                Ans. 5 rm. 4 qr. 4 sh.
9. If a man use 12 sheets of paper in a day, how much does he use in 150 days?   *Ans.* 3 rm. 15 qr.
10. If 1 desk require 6 doz. and 2 screws, how many screws do 20 desks require?   *Ans.* 123 doz. 4 screws.

13   K

COMPOUND NUMBERS.

1. ADDITION OF COMPOUND NUMBERS.

131. Addition of Compound Numbers is the process of finding the sum of two or more similar compound numbers.

WRITTEN PROBLEMS.

1. Find the sum of—

lb.	oz.	dwt.	gr.
6	5	6	0
18	3	9	4
26	0	2	16
0	3	4	18
51	0	2	14

Explanation.—We write the numbers in vertical columns, so that units of the same denomination stand in the same column. The sum of the first right-hand column is 38 gr., or 1 dwt. 14 gr. Writing the 14 gr. in the column of grains and adding the 1 dwt. to the column of pennyweights, the sum of the second column is 22 dwt., or 1 oz. 2 dwt. Writing the 2 dwt. in the proper column, and adding the 1 oz. to the column of ounces, the sum of the third column is 12 oz., or 1 lb., which we add to the column of pounds. The sum of the fourth column is 51 lb. Hence, the sum of the given numbers is 51 lb. 2 dwt. 14 gr.

From this and similar solutions we derive the following rules for the addition of compound numbers:

RULES.

1. *Write the numbers so that units of the same denomination shall stand in the same column.*

2. *Beginning with the lowest denomination, add as in simple numbers, and reduce the sum to the next higher denomination, writing the remainder, if any, under the column added, and adding the quotient obtained by the reduction to the next column.*

3. *Proceed in the same manner with all the columns to the last.*

NOTE.—If any places are wanting, supply ciphers.

What is the sum of the following:

(2.)			(3.)			(4.)		
£	s.	d.	pk.	qt.	pt.	gal.	qt.	pt.
6	3	4	3	2	1	3	1	1
8	9	6	6	4	0	4	3	1
3	12	3	3	6	1	7	0	1

(5.)			(6.)			(7.)			
yd.	ft.	in.	d.	hr.	min.	lb.	oz.	dwt.	gr.
6	2	7	7	3	15	3	4	18	6
3	0	8	6	4	12	9	6	12	18
2	1	6	18	9	20	13	4	0	12
8	2	0	40	3	30	18	8	6	6

(8.)				(9.)			
℥	ℨ	Ə	gr.	cwt.	qr.	lb.	oz.
4	3	2	8	6	2	15	8
6	6	1	12	18	3	20	12
6	4	0	13	15	2	6	5
8	7	2	15	16	0	22	15

10. Add 6 mi. 3 fur. 16 rd., 18 mi. 14 rd., 16 mi. 7 fur. and 20 mi. 6 fur. 35 rd. *Ans.* 62 mi. 1 fur. 25 rd.

11. Add 12 bu. 2 pk. 3 qt. 1 pt., 17 bu. 3 pk., 14 bu. 1 pk. 1 pt., 8 bu. 6 qt. 1 pt. *Ans.* 52 bu. 3 pk. 2 qt. 1 pt.

12. A farmer raises in four years the following quantities of corn: 106 bu. 3 pk. 1 qt., 125 bu. 1 pk. 1 qt. 1 pt., 205 bu. 3 pk. 5 qt., 250 bu. 2 pk. 7 qt. 1 pt.: how much did he raise? *Ans.* 688 bu. 2 pk. 7 qt.

13. How long is it from 2 hr. 6 min. 40 sec. before noon to 15½ min. past 4 o'clock in the afternoon?

Ans. 6 hr. 22 min.

2. SUBTRACTION OF COMPOUND NUMBERS.

132. Subtraction of Compound Numbers is the process of finding the difference between two similar compound numbers.

WRITTEN PROBLEMS.

(1.) PROCESS.

	lb.	oz.	dwt.	gr.
From	63	4	18	13
Take	27 "	6 "	4 "	16 "
	35 lb.	10 oz.	13 dwt.	21 gr.

Explanation. — We write the numbers as in addition of compound numbers, and begin to subtract at the lowest denomination. We cannot take 16 gr. from 13 gr.; we therefore take 1 dwt., equal to 24 gr., which added to 13 gr. equals 37 gr.; 16 gr. from 37 gr. leaves 21 gr.; 4 dwt. from 17 dwt. leaves 13 dwt. We cannot take 6 oz. from 4 oz.; we therefore take 1 lb., equal to 12 oz., which added to 4 oz. equals 16 oz.; 6 oz. from 16 oz. leaves 10 oz.; 27 lb. from 62 lb. leaves 35 lb.

NOTE.—Let the pupil derive the rule.

(2.)

	lb.	oz.	dwt.
From	27	6	15
Take	14	7	8

(3.)

	£	s.	d.
From	25	6	3
Take	18	4	11

(4.)

	yr.	mo.	wk.	d.
From	115	7	6	2
Take	97	8	3	12

(5.)

	mi.	fur.	rd.
From	18	4	27
Take	12	7	36

(6.)

	bu.	pk.	qt.	pt.
From	140	3	2	1
Take	60	2	5	0

(7.)

	T.	cwt.	lb.	oz.
From	18	7	20	6
Take	7	4	26	15

(8.)

	yr.	mo.	d.
From	1867	6	13
Take	1855	3	26

(9.)

	yr.	mo.	d.
From	1706	1	6
Take	1608	12	9

10. From a barrel containing 31 gal. 2 qt. there leaked out 6 gal. 3 qt. 1 pt.: how much remained?

Ans. 24 gal. 2 qt. 1 pt.

11. A farmer had 126 bu. 3 pk. of seed-wheat, and sold 111 bu. 2 pk. 5 qt. 1 pt.: how much remained?

Ans. 15 bu. 2 qt. 1 pt.

12. A pile of wood contained 7 cd. ; a teamster hauled away 4 cd. 6 cu. ft.: how much remained?

Ans. 2 cd. 122 cu. ft.

13. How old was a man who was born Jan. 3, 1832, and died July 1, 1877?   *Ans.* 45 yr. 5 mo. 28 d.

14. A lady was born Nov. 22, 1837: how old was she Sept. 2, 1876?   *Ans.* 38 yr. 9 mo. 10 d.

15. A girl was born Jan. 13, 1862: how old was she Sept. 2, 1877?   *Ans.* 15 yr. 7 mo. 19 d.

16. What is the time from 15 min. past 4 P. M. to 3 min. before 10 P. M.   *Ans.* 5 hr. 42 min.

17. A note was dated Jan. 15, 1877, and was to be paid Aug. 10, 1877: how long from the date of it to the time it was paid?   *Ans.* 6 mo. 25 d.

3. MULTIPLICATION OF COMPOUND NUMBERS.

133. Multiplication of Compound Numbers is the process of finding the product of two numbers, one of which is compound.

WRITTEN PROBLEMS.

1. Multiply 12 £ 6 s. 9 d. by 7.

PROCESS.

12 £ 6 s. 9 d.
   7
―――――――――
86 £ 7 s. 3 d.

Explanation.—7 times 9 d. are 63 d., or 5 s. 3 d.; 7 times 6 s. are 42 s.; 42 s. and 5 s. are 47 s., or 2 £ 7 s.   7 times 12 £ are 84 £; 84 £ and 2 £ are 86 £. Hence, 12 £ 6 s. 9 d. multiplied by 7 equals 86 £ 7 s. 3 d.

NOTE.—Let the pupil derive the rule.

13 *

	(2.)	
6 cwt.	3 qr.	7 lb.
Multiplied by	5	
34 cwt.	0 qr.	10 lb.

	(3.)	
18 mi.	3 fur.	16 rd.
		7
128 mi.	7 fur.	32 rd.

	(4.)	
14 gal.	3 qt.	1 pt.
		8

	(5.)	
15 yr.	3 mo.	3 wk.
		12

	(6.)	
3 d.	6 hr.	15 min.
		10

	(7.)	
3 £	6 s.	4 d.	1 far.
			14

8. Multiply 4 gal. 3 qt. 1 pt. by 20. *Ans.* 97 gal. 2 qt.

9. If a farmer can raise 16 bu. 3 pk. 4 qt. of wheat on an acre, how much can he raise on 40 acres?

Ans. 675 bu.

10. If a horse can haul 17 cwt. 3 qr. 15 lb. 4 oz. of coal in a load, how much can he haul in 16 loads?

Ans. 14 T. 6 cwt. 1 qr. 19 lb.

4. DIVISION OF COMPOUND NUMBERS.

134. Division of Compound Numbers is the process of dividing one number by another, the dividend being compound.

WRITTEN PROBLEMS.

1. Divide 40 bu. 3 pk. 5 qt. by 7.

PROCESS.   *Explanation.*—⅐ of 40 bu. is 5 bu., and 5
7)40 bu. 3 pk. 5 qt. bu., or 20 pk., remaining. 20 pk. and 3 pk.
  5 bu. 3 pk. 3 qt.   are 23 pk.; ⅐ of 23 pk. is 3 pk., and 2 pk., or
                16 qt., remaining; 16 qt. and 5 qt. are 21 qt.;
⅐ of 21 qt. is 3 qt.

NOTE.—Let the pupil derive a rule.

Perform the following divisions:

(2.)	(3.)
8)25 £ 15 s. 4 d.	9)80 bu. 1 pk. 6 qt.
3 £ 4 s. 5 d.	8 bu. 3 pk. 6 qt.

(4.)	(5.)
12)58 gal. 2 qt.	15)244 mi. 2 fur. 20 rd.

(6.)	(7.)
20)125 yr. 10 mo.	24)244 £ 4 s.

8. If 8 hens weigh 42 lb., how much does each weigh?
Ans. 5 lb. 4 oz.

9. If 12 bags contain 31 bu. 2 pk. of oats, how much does 1 contain?   *Ans.* 2 bu. 2 pk. 4 qt.

NOTE.—When both divisor and dividend are concrete numbers, reduce both to the lowest denomination in either, and divide as in simple division.

10. Divide 40 £ 4 s. by 2 £ 10 s. 3 d.

PROCESS.

$$\frac{40 £ \quad 4 s.}{2 £ \; 10 s. \; 3 d.} = \frac{9648 \text{ d.}}{603 \text{ d.}} = 16.$$

11. Divide 78 lb. 12 oz. avoirdupois by 3 lb. 15 oz.
Ans. 20.

12. A man travels 410 mi. 6 fur. 10 rd. by travelling 16 mi. 3 fur. 18 rd. a day: how many days does he travel?   *Ans.* 25.

13. How many loads of coal, each weighing 1 T. 3 cwt. 27 lb., will weigh 46 T. 10 cwt. 80 lb.?   *Ans.* 40.

14. A number of boys gather 10 bu. 3 pk. 1 qt. of chestnuts; they divide them equally, and each receives 3 pk. 4 qt. 1½ pt.: how many boys were there? *Ans.* 12.

CHAPTER VII.

PERCENTAGE.

SECTION I.

DEFINITIONS AND PRINCIPLES.

135. Percentage is the name applied to computations in which 100 is the unit or measure.

136. *Per cent.* is an abbreviation of the Latin phrase *per centum,* meaning *by the hundred.*

ORAL EXERCISE.

1. What is $\frac{1}{100}$ of 100? $\frac{2}{100}$ of 100? $\frac{5}{100}$ of 100?
2. What is $\frac{1}{100}$ of 200? $\frac{1}{100}$ of 400? $\frac{3}{100}$ of 600?
3. How many hundredths of $100 are $3? $5? $10?

NOTE.—$\frac{1}{100}$ of any number is 1 per cent. of that number; $\frac{2}{100}$ is 2 per cent. of it; $\frac{3}{100}$ is 3 per cent. of it; $\frac{15}{100}$ is 15 per cent. of it, and so on.

4. How many hundredths of a number is 5 per cent. of it? 8 per cent.? 10 per cent.? 20 per cent.?
5. How many hundredths of a number is 15 per cent. of it? 75 per cent.? 120 per cent.? 100 per cent.?
6. What per cent. of a number is $\frac{3}{100}$ of it? $\frac{8}{100}$ of it? $\frac{15}{100}$ of it? $\frac{25}{100}$ of it?
7. What per cent. of a number is .14 of it? .27 of it? .12$\frac{1}{2}$ of it? 1.19 of it?

Per cent. is usually written $\%$; thus, 25 per cent. is written 25$\%$.

8. How many hundredths is 5$\%$? 15$\%$? 20$\%$? 40$\%$? 75$\%$? 6$\frac{1}{4}\%$? 12$\frac{1}{2}\%$? 135$\%$?

9. What per cent. of a number is $\frac{1}{5}$ of it?

SOLUTION.—$\frac{1}{5}$ equals $\frac{20}{100}$, or 20%. Hence, $\frac{1}{5}$ of a number is 20% of it.

10. What per cent. of a number is $\frac{1}{2}$ of it? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{1}{20}$? $\frac{2}{5}$? $\frac{3}{4}$? $\frac{7}{10}$? $\frac{6}{20}$? $\frac{1}{8}$? $\frac{2}{8}$? $\frac{1}{8}$? $\frac{3}{8}$?

11. What fractional part of a number is 5% of it?

SOLUTION.—$5\% = \frac{5}{100}$, or $\frac{1}{20}$. Hence, 5% of a number is $\frac{1}{20}$ of it.

12. What fractional part of a number is 10% of it? 20%? 25%? 50%? 30%? 60%? 24%? 48%? 65%? 92%? 87%?

13. What part of a number is $\frac{1}{4}\%$ of it?

ANALYSIS.—$\frac{1}{4}\% = \frac{1}{4}$ of $\frac{1}{100} = \frac{1}{400}$.

14. What part of a number is $\frac{1}{2}\%$ of it? $\frac{1}{5}\%$ of it? $\frac{3}{4}\%$ of it? $\frac{7}{10}\%$ of it?

WRITTEN PROBLEMS.

Express decimally the following:

1. 20%; 25%; 18%; 30%; 40%; 60%.

2. 75%; 4%; 8%; $12\frac{1}{2}\%$; $93\frac{3}{4}\%$; 175%.

Write the following, with the *per cent.* sign ($\%$):

3. $.25$; $.18$; $.42$; $.12\frac{1}{4}$; $.18\frac{3}{10}$; $.16\frac{2}{3}$.

4. 1.24; 8.75; $\frac{1}{4}$; $\frac{3}{8}$; $\frac{1}{8}$; $\frac{5}{8}$.

137. The number of hundreds is called the *Rate.* Thus, in 5%, or $\frac{5}{100}$, 5 is the rate.

138. The *Rate per cent.* is the fraction which denotes how many hundredths are taken.

Every number is $\frac{100}{100}$, or 100 per cent. of itself.

139. The number of which the per cent. is taken is called the *Base.*

140. The result of taking the per cent. of the base is called the *Percentage.*

141. The base added to the percentage is called the *Amount.*

142. The base less the percentage is called the *Difference.*

CASE I.

The Base and the Rate Per Cent. being given, to find the Percentage.

ORAL EXERCISE.

1. What is 30% of 120?

SOLUTION.—30% = $\frac{30}{100}$, or $\frac{3}{10}$; $\frac{3}{10}$ of 120 is 36.

2. What is 5% of 40?	5. What is 40% of 75?
3. What is 10% of 70?	6. What is 65% of 39?
4. What is 15% of 50?	7. What is 90% of 30?

8. What is 18% of $50?

9. What is 32% of 75 cows? -

10. What is 45% of 120 cents?

11. A man's wages were $1.20 a day, but they were reduced 10%: how much did he then get?

12. A clerk's salary was $70 a month, but his employer reduced the salary 10%: how much did the clerk then get?

13. A merchant who sells some goods at 40 cts. a yard, reduces the price 20%: what does he sell at then?

14. A boy had 84 chickens, but 25% of them died: how many had he remaining?

15. A horse cost me $140; if I sell him so as to gain 15%, what do I get for him?

WRITTEN PROBLEMS.

1. What is 37% of 120 A. of land?

SOLUTION.

120 A.
.37   *Explanation*—Since 37% equals .37, the required
840   percentage is .37 of 120 A., or 44.4 A.
360
44.40

From the foregoing we derive the

RULE.

Multiply the base by the rate per cent.; the result is the percentage.

2. What is 21% of $170?

3. What is 15% of 40 cows?

4. What is 25% of $18?

5. What is 12% of 50 chickens?   ➡

6. What is 9% of 200 mi.?

7. What is 43% of $72?

8. I bought a horse for $95, and sold him at a gain of 26%: what was my gain?   *Ans.* $24.70.

9. If my salary is $75 a month, and it is reduced 5%, how much do I get a month?   *Ans.* $71.25.

10. A boy has a flock of 60 hens, and they increase in one year 33⅓%: how many has he at the end of the year?   *Ans.* 80.

11. If a man receive $1200 a year salary, and spend 15% of it for board and 6% of it for books, how much has he remaining?   *Ans.* $948.

12. A boy whose knife cost him $2.50 sold it at a gain of 34%: how much did he get for it?
          Ans. $3.35.

CASE II.

The Base and the Percentage being given, to find the Rate Per Cent.

ORAL EXERCISE.

1. What per cent. of 35 is 7 ?

ANALYSIS.—7 is $\frac{1}{5}$ of 35; it is therefore $\frac{1}{5}$ of 100% of 35, or 20% of 35.

2. What per cent. of 20 is 10 ?

What per cent.—

3. Of 40 boys is 8 boys ?
4. Of 60 ducks is 15 ducks ?
5. Of $90 is $18 ?

6. Of $55 is $11 ?
7. Of 40 is 5 ?   Is 8 ?
8. Of $200 is $40 ?

9. 16 is what per cent. of 200 ?

10. 36 is what per cent. of 108 ?

11. 14 is what per cent. of 35 ?

12. $35 is what per cent. of $140 ?

13. A boy had 60 marbles, but lost 15 : what per cent. of his number did he lose ?

14. If I have a hat worth $5, and sell it for $4, what per cent. do I lose ?

15. If from a barrel of vinegar containing 40 gal. I sell 15 gal., what per cent. do I sell, and what per cent. remains ?

WRITTEN PROBLEMS.

1. What per cent. of 150 is 21 ?

SOLUTION.

$$150)\overline{21.00}\,|.14$$
$$\underline{150}$$
$$\overline{600}$$
$$\underline{600}$$

Explanation—21 is $\frac{21}{150}$ of 150, or $\frac{21}{150}$ of 100% of 150, which is $\frac{2100}{150}$ %, equal to 14%.

RULE.

To find the rate per cent., divide the percentage by the base.

2. What per cent. of 600 is 150?

3. What per cent. of 80 is 12?

4. What per cent. of $6.40 is 16 cts.?   *Ans.* 2½.

5. What per cent. of 20 A. is 7 A.?   *Ans.* 35.

6. What per cent. of 3 bu. is 2 pk.?   *Ans.* 16⅔.

7. What per cent. of 5 gal. is 3 qt.?   *Ans.* 15.

8. A man wishing to purchase a house for $1000 had only $800 in money: what per cent. could he pay cash?
                            Ans. 80.

9. If I buy muslin at 8 cts. a yard and sell it at 11 cts., what per cent. do I make?   *Ans.* 37½.

10. If a book cost me 90 cts., and I sell it at $2.25, what per cent. do I make?   *Ans.* 150.

CASE III.

The Rate Per Cent. and the Percentage being given, to find the Base.

ORAL EXERCISE.

1. 40 is 20% of what number?

ANALYSIS.—20% is $\frac{20}{100}$, or ⅕; since 40 is ⅕ of a number, ⅖, or the number, is 5 × 40, or 200.

2. 20 is 10% of what number?

3. 10 is 20% of what number?

4. 24 is 6% of what number?

5. 25 is 5% of what number?

6. $40 is 20% of what a horse cost: how much did he cost?

7. $36 is 9% of the cost of a town-lot: how much did it cost?

8. If 12% of the cost of my coat is $3, how much did it cost?

9. If 28% of my money is $14, how much money have I?

14

10. If 15% of the cost of a knife is 45 cts., how much did it cost?

WRITTEN PROBLEMS.

1. 120 is 30% of what number?

SOLUTION 1.
30% = 120
1% = $\frac{1}{30}$ × 120 = 4   Let the pupil explain.
100% = 100 × 4 = 400

SOLUTION 2.   *Explanation.*—If .30 of a number is
120 ÷ .30 = $\frac{12000}{30}$ = 400   120, the number is as many as .30 is con-
tained times in 120, or 400.

RULE.

To find the base when the rate per cent. and the percentage are given, divide the percentage by the rate per cent.

2. $260 is 13% of what number?   *Ans.* $2000.

3. If I receive $210 rent for a house, which is 7% of the value, what is the value?   *Ans.* $3000.

4. If 20 bu. of potatoes is 5% of my whole number, how many bushels have I?   *Ans.* 400.

5. A merchant sold muslin at 15 cts. a yard, which was 25% more than he paid for it: what did it cost him?   *Ans.* 12 cts. a yd.

6. A clerk spent 20% of his salary for board and 30% for clothing and books; he saved $400: what was his salary?   *Ans.* $800.

7. If $175 is 14% of my money, how much money have I?   *Ans.* $1250.

8. If a farmer sell 60% of his corn, and feed 25% of it, how much was the crop if he has 105 bu. remaining?   *Ans.* 700 bu.

SECTION II.

INTEREST.

143. Interest is a certain percentage paid for the use of money. It is reckoned at a certain rate per cent. for each year.

144. The sum on which the interest is paid is called the *Principal.*

145. The per cent. paid per annum is called the *Rate.*

146. The sum of the principal and the interest is called the *Amount.*

147. Simple Interest is that which is reckoned on the principal alone.

SIMPLE INTEREST.

1. GENERAL METHOD.

ORAL EXERCISE.

1. What is the interest of $100 for 2 yr., at 6% ?

SOLUTION.—At 6%, $\frac{6}{100}$ of the principal equals the interest for 1 year, and for 2 years 2 times $\frac{6}{100}$, or $\frac{12}{100}$; $\frac{12}{100}$ of $100 equals $12. Hence, the interest of $100 for 2 yr. at 6% is $12.

2. What is the interest of $200 for 1 yr., at 5% ?
3. What is the interest of $400 for 2 yr., at 5% ?
4. What is the interest of $300 for 2 yr., at 6% ?
5. What is the interest of $300 for 2½ yr., at 6% ?
6. What is the interest of $140 for 2½ yr., at 6% ?
7. What is the interest of $500 for 3 yr. 6 mo., at 6% ?
8. What is the interest of $120 for 5 yr., at 6% ?
9. What is the interest of $25 for 6 yr. 3 mo., at 8%.
10. What is the interest of $60 for 8 yr. 4 mo., at 5% ?

WRITTEN PROBLEMS.

1. What is the interest of $360 for 3 yr. 8 mo., at 6% ?

SOLUTION.

$360
.06

21.60
      3⅔ yr. = 3 yr. 8 mo.

6480
1440

$79.20

Explanation.—6%, or .06, of $360 is $21.60, or the interest of $360 for 1 yr., at 6%, and for 3 yr. 8 mo., or 3⅔ yr., the interest is 3⅔ × $21.60, or $79.20.

2. What is the interest of $500 for 7 yr. 2 mo., at 6% ?
Ans. $215.

3. What is the interest of $350 for 7 yr., at 6% ?
Ans. $147.

4. What is the interest of $200 for 6 yr. 6 mo., at 6% ?
Ans. $78.

5. What is the interest of $75 for 3 yr. 8 mo., at 5% ?
Ans. $13.75.

6. What is the interest of $12 for 2 yr. 9 mo., at 10% ?
Ans. $3.30.

7. What is the interest of $600 for 7 yr. 3 mo., at 8% ?
Ans. $348.

8. What is the interest of $140 for 8 yr. 4 mo., at 3% ?
Ans. $35.

9. What is the amount of $120 for 6 yr. 5 mo., at 6% ?
Ans. $166.20.

NOTE.—Add the interest to the principal to find the amount.

10. What is the amount of $740.50 for 7 yr. 3 mo., at 7% ?
Ans. $1116.30.

11. What is the amount of $180 for 2 yr. 6 mo. 12 d., at 5% ?
Ans. $202.80.

12. What is the amount of $160 for 1 yr. 3 mo. 18 d., at 6% ?
Ans. $172.48.

2. DECIMAL METHOD.

NOTE.—By this method the years, months and days are reduced to the decimal part of a year.

WRITTEN PROBLEMS.

1. What is the interest of $130 for 6 yr. 7 mo. 24 d., at 6% ?

SOLUTION.

```
30)24 d.            $130
12)7.8 mo.           .06
    6.65 yr.        ─────
                    7.80
                    6.65
                   ─────
                    3900
                    4680
                    4680
                   ───────
                  $51.8700
```

Explanation. — 6 yr. 7 mo. 24 d. equals 6.65 yr. The interest for 1 yr. at 6% is $7.80, and for 6.65 yr. it is 6.65 × $7.80, or $51.87.

2. What is the interest of $150 for 3 yr. 4 mo., at 5% ?
Ans. $25.

3. What is the interest of $450 for 6 yr. 3 mo. 24 d., at 6% ? *Ans.* $170.55.

4. What is the interest of $190 for 2 yr. 4 mo. 18 d., at 7% ? *Ans.* $31.70.

5. What is the interest of $225 for 1 yr. 9 mo. 18 d., at 7% ? *Ans.* $28.35.

6. What is the interest of $240.50 for 7 yr. 5 mo. 6 d., at 6% ? *Ans.* $107.26+.

7. What is the interest of $920.14 for 5 yr. 8 mo., 12 d., at 7% ? *Ans.* $367.14−.

8. What is the interest of $742.10 for 6 yr. 5 mo. 12 d., at 6% ? *Ans.* $287.19+.

9. What is the amount of $225 for 3 yr. 2 mo. 12 d., at 6% ? *Ans.* $268.20.

10. What is the amount of $220.40 for 1 yr. 7 mo. 21 d., at 5% ? *Ans.* $238.49+.

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11. What is the amount of $40.50 for 6 yr. 11 mo. 12 d., at 4%? *Ans.* $51.76 −.

12. What is the amount of $320.50 for 7 yr. 3 mo., 16 d., at 6%? *Ans.* $460.77 +.

REVIEW PROBLEMS.

1. If a man earn $20 a week and pay $6 a week board, how much can he save in 15 wk.? *Ans.* $210.

2. How long will it take a boy to save $105, if his wages are $12 a month and he spends $5 a month?

Ans. 15 mo.

3. What is the cost of 6 cows at $27.50 each, and 4 horses at $115 each? *Ans.* $625.

4. A boy had 80 cts., but lost $\frac{2}{5}$ of his money: how much has he remaining? *Ans.* 48 cts.

5. If 49 years is $\frac{1}{2}$ of $2\frac{1}{3}$ times my age, how old am I? *Ans.* 42 yr.

6. How many sheets of paper in $3\frac{1}{4}$ reams? *Ans.* 1560.

7. A man who was worth $25000 willed $1600 to the poor, and the remainder to his six children, to be divided equally: how much did each one get? *Ans.* $3900.

8. How much will 13 lb. 6 oz. of butter cost at 18 cts. a pound? *Ans.* $2.41 −.

9. If I trade 16 chickens worth 30 cts. each for 20 yd. of gingham at 22 cts. a yard, how much do I lose?

Ans. 40 cts.

10. If I trade 15 hens for 30 yd. of calico at 12 cts. a yard, how much do I get for my hens each? *Ans.* 24 cts.

11. If I take to a store 6 lb. 4 oz. of butter at 20 cts. a pound, and 16 lb. 6 oz. of lard at 16 cts. a pound, and get in exchange sugar at 9 cts. a pound, how much sugar do I get? *Ans.* 43 lb.

12. What is the value of 6 bu. 2¼ pk. of onions at $1.60 a bushel?   *Ans.* $10.50.

13. How many dozen of eggs at 12½ cts. a dozen can I buy for 3 bu. potatoes worth 75 cts. a bushel?
                    Ans. 18 doz.

14. How many yards of carpet 1 yd. wide will it take to carpet a room 15 ft. long and 12 ft. wide? *Ans.* 20 yd.

15. A farmer sells 18 bags of wheat, each containing 2½ bu., at $1.50 per bushel: how much does he get for his wheat?   *Ans.* $67.50.

16. How much will it cost to paint a floor 18 ft. long and 16½ ft. wide, at 5 cts. a square foot?   *Ans.* $14.85.

17. How much will it cost to dig a cellar 40 ft. long, 30 ft. wide and 5 ft. deep, at ¾ ct. a cubic foot? *Ans.* $45.

18. A merchant had a piece of cloth containing 42½ yd., from which he cut 12 coats of 2¾ yd. each: how much remains?   *Ans.* 9½ yd.

19. If a farmer buy 26 sheep at $5 each, but 6 of them die, how much should he receive for the others that he may not lose?   *Ans.* $6.50 each.

20. A party of 6 go on a fishing excursion; their carfare is $1.20 each; their meals $10.20 for all of them, and they pay $1.20 expressage on their baggage: what is each one's share of the bill?   *Ans.* $3.10.

21. If a merchant buy goods at 40 cts., how must he sell them to make 15%?   *Ans.* 46 cts.

22. If I borrow $120 June 17, 1876, and pay it back July 5, 1877, how much interest do I pay at 6%?
                    Ans. $7.56.

23. What is the interest on $140.50 from May 10, 1876, to Aug. 1, 1877, at 7%?   *Ans.* $12.05.

24. What is your age in years, months and days?

25. If one boy travels 16.5 mi. a day, and another

travels 20.25 mi. a day, how far apart will they be in 12.6 days if they travel the same direction? *Ans.* 47.25 mi.

26. If the two boys mentioned in the preceding problem travel in opposite directions, how far apart will they be at the end of 6.4 days? *Ans.* 235.2 mi.

27. How many acres in a street 60 ft. wide and 1½ mi. long? *Ans.* 10 A. 145 sq. rd. 13.75 sq. yd.

28. A marketman buys a turkey weighing 16 lb. at 8 cts. a pound, live weight, and sells it at 12½ cts. a pound, dressed: how much does he make if the turkey loses 30% in dressing? *Ans.* 12 cts.

29. A miller sold 4.5 tons of flour at $7.25 a barrel: how much did he get for the flour? *Ans.* $332.91.

30. What is the value of 2740 lb. of hay at $12.50 a ton? *Ans.* $17.12½.

31. A boy sells 2 pk. 3 qt. of chestnuts at $4 a bushel: how much does he get for them? *Ans.* $2.37½.

32. A farmer plants 6 bu. potatoes worth $2 a bushel on some ground; he pays a man for cultivating them 90 cts. a day for 12 d., and raises 120 bu. which he sells at 80 cts. a bushel: how much does he make? *Ans.* $73.20.

33. Into how many building lots, 140 ft. long and 20 ft. wide, can a piece of ground containing 10 A. 1200 sq. ft. be cut? *Ans.* 156.

THE END.

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