ELEMENTARY LOGIC.

By Rev. JOHN LIGHTFOOT, M.A., D.Sc.

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No.
Let there be light!

ELEMENTARY LOGIC

FOR

UNIVERSITY AND CERTIFICATE STUDENTS.

(An Enlarged and Revised Edition of "Logic and Education.")

AN ELEMENTARY TEXT-BOOK OF
DEDUCTIVE AND INDUCTIVE LOGIC.

BY

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PREFACE.

This book is intended for students commencing the study of Logic. It is an elementary exposition of the subject on traditional lines. Modern logicians have added many refinements to the analysis which has been generally received since the days of Aristotle. But the more subtle questions of criticism can only be appreciated intelligently after the elements of the subject have been mastered. A brief account of the more important reforms in logical doctrine is given in the Appendix now added. The treatment will be found sufficiently full for the preliminary examinations at the Universities, and for the purposes of students preparing for any of the various examinations for a Teacher's Certificate. In the present edition the exposition of Inductive Logic has been considerably extended, and will now be found ample for all elementary purposes.

J. LIGHTFOOT.

Cross Stone Vicarage.

AUTHOR'S NOTE.

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Students making their first acquaintance with the subject are advised to adopt the following plan on their first reading of the book:—

Omit Chapter I. Read Chapter II., omitting the section "Divisions of the Subject." Omit Chapter III. Chapter IV. should be thoroughly mastered. Read Chapter V., omitting the "Predicables." Omit Chapter VI. Chapter VII. is very important, and the exercises on page 50 should be carefully worked. Read Chapter VIII., but "Obversion" and "Contraposition" may be omitted on first reading. Chapters IX. and X. are very important. Chapters XI. and XII. may be neglected on first reading. Chapter XIII. is important (to bottom of page 84). Chapter XIV. is easy and interesting. Pay special attention to the "petitio principii" and "ignoratio elenchi" (pages 92-94). Chapter XV. to end of book is very important. The Appendix may be omitted on first reading.

On second reading **no part** of the book should be neglected. Notice that the exercises contain only few questions involving mere reproduction of the text. It has been assumed that students can and will construct such questions for themselves.

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CHAPTER I.

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The Relation of Logic to other Branches of Philosophy.

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Logic is usually considered the proper introduction to the study of Philosophy. It is well, therefore, to get a preliminary view of the subjects which are included under the general term "Philosophy."

There are certain questions which must always be of great importance to those whose profession it is to train the minds of children. For instance, the question "What am *I*?" is obviously as important as the question "What is the sun?" Now, there are many similar questions to "What am I?" that will suggest themselves as of great importance in this respect; *e.g.*:—

> How did I become what I am? What are the fixed rules, or laws, which govern the development of mind in man? To what laws must all my conscious thinking conform, so that error and self-contradiction may be avoided?

Philosophy is the name given to that branch of study which attempts to give an answer to these and similar questions.

A course in Philosophy is usually divided into three sections :—**Logic, psychology, metaphysics**; and the object of

these three departments may be thus briefly stated :—

1. **Logic.**—Here we investigate the laws to which all our "thinking" must conform, in order that we may avoid error and self-contradiction in our thinking process.

2. **Psychology.**—Here is considered our *power* of thinking, its growth, and the laws by which "mind" in the individual is governed.

3. **Metaphysics.**—Under this name we discuss a number of very difficult and speculative questions about the ultimate grounds of our beliefs and opinions.

This very brief statement must, for the sake of clearness, be considered more in detail.

Philosophy, we have said, exists in three forms, and first we have :—

I. PHILOSOPHY IN THE FORM OF LOGIC.

Knowledge in its simplest form and from our earliest days, comes to us through sensation. But this elementary knowledge is from the first extended by reflection (*i.e.*, by thinking) and by reasoning. The earliest efforts of a teacher are devoted to making the scholar reason correctly. And this effort is continued throughout the pupil's school-life. Evidently, then, it is of first importance that all who teach should themselves clearly recognise the laws to which "correct reasoning" must conform.

Putting the matter in its briefest form, we may say that "correct reasoning" must have two qualities, *viz.* :—

(*a*) Self-consistency ;

(*b*) Consistency with known facts.

If our "reasoning" is wanting in either of these qualities, it is incorrect, or fallacious.

In our efforts, then, to extend and to improve our knowledge, Logic comes to our aid by showing how fallacies of

thought and fallacies of expression may be avoided. Logic is thus:—

A systematised body of tests and rules, by the aid of which we may determine whether

(*a*) Our thinking is correct thinking (*i.e.*, in accordance with the laws of thought); and

(*b*) Whether our thinking is in agreement with facts, and with the known laws of nature.

Logic and Rhetoric both aim at the formation of conclusions.
The *Logician* seeks to *convince* that the conclusion *must* be;
The *Rhetorician* seeks to *persuade* that the conclusion *ought* to be.

II. PHILOSOPHY IN THE FORM OF PSYCHOLOGY.

The simplest reflection suggests to us that all our knowing, reasoning and believing presupposes that we have a "mind" which knows, reasons and believes. What the "mind" really is we do not know. But we do know how the mind manifests itself. If we do not know the mind itself, we know its phenomena. Sensation, knowledge, memory, imagination, reasoning, all these are phenomena of the mind. All these, too, have their laws of growth and development, and Psychology is the orderly investigation of the phenomena of mind and the laws by which they are governed.

Hereafter it will be seen that all man's conscious thinking manifests itself in mental judgments. Logic investigates the laws which these judgments, when formed, must obey. Thus we see the relation between Logic and Psychology. Psychology investigates the process by which the mind forms judgments. Logic studies the result, *i.e.*, the judgments when formed.

III. PHILOSOPHY IN THE FORM OF METAPHYSICS.

We have remarked that we do not know what "mind" really is. But all the same, men have felt themselves bound to hold some theory about it. So, too, we do not know precisely what "matter" really is, but chemists and others have

a theory about it. So too in every department of human study there are certain things which have to be taken for granted; things which cannot be proved, but which we are compelled to assume. Now, Philosophy in the form of Metaphysics attempts to give some account of these subjects. It attempts to explain the ultimate nature of mind and matter, and to give a reasoned account of those truths which ordinary science takes for granted.

> The word "Metaphysic" suggests the subjects with which its study is concerned. The word means that which is "after," or beyond physical or ordinary scientific investigation.

It only remains in this general sketch of the province of Philosophy that we should see precisely the place of Moral Philosophy, or Ethics. Ethical study is the consideration of a certain definite set of facts and opinions which regulate the behaviour of men as individuals, and as members of the community. It is because of the great practical importance of these that they are reserved for special and separate treatment. In Ethics, or Moral Philosophy, we learn how the knowledge of moral distinctions (*i.e.*, of right and wrong) are obtained, and we investigate the laws which govern the moral life. But it must be observed that when we are studying the growth of the knowledge of moral distinctions we are really engaged in the study of Ethical Psychology. So, too, when we further consider questions like the ultimate nature and destiny of the soul, and its relation to the Supreme Being, we are then in the province of Ethical Metaphysics.

Having now got a general view of the whole province of Philosophy, the student will appreciate the reason why Logic is always prescribed as the proper introduction to this department of study; and, further, it will be clear that no course of study in the theory and practice of education can be considered at all complete which has not embraced the treatment of

Elementary Logic. At the Universities this subject always finds a place in the general schemes of instruction, and the tendency is to make it a compulsory one for all degree examinations. The student who has mastered the treatment in this work will be sufficiently well prepared for the preliminary and intermediate examinations at the Universities, as well as for the questions set by the Board of Education in its certificate examinations.

EXERCISES ON CHAPTER I.

1. Should Logic precede Psychology as a subject of study? Give reasons for your answer.

2. State briefly the fundamental relation in which Logic stands to Psychology and Metaphysics.

CHAPTER II.

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Definition of Logic and Divisions of the Subject.

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Logic is usually defined as "The Science of Reasoning or Inference." This definition, though sufficient for general purposes, is not sufficiently precise. The object of Logic is to unfold to us the ideal, or perfect conditions to which all our thinking must conform in order to be correct thinking. Its object is to show us how we must arrange the matter about which we think, in order that our thought shall be coherent and non-contradictory. A better definition, therefore, will be :—

Logic is the science of the laws which regulate valid thought.

It is of the utmost importance to fix clearly the meaning attached to each word in this definition.

(*a*) A **Science** is a systematised body of knowledge about some particular subject-matter.

(*b*) A **Law** is a statement of a general truth, *i.e.*, a truth which holds good generally in human experience.

(*c*) The word "**Thought**" is used both for the *process* of thinking, and for the *product* of thinking.

N.B.—Knowledge of isolated facts is not Science; nor is a truth which holds good only in certain instances a "Law."

In Logic we consider every simple complete thought as an assertion or a denial.

Every assertion or denial we call a "**judgment.**"

Every judgment when expressed in words we call a "**proposition.**"

Every assertion or denial which it is possible to make is of such a character that when it is made certain other assertions or denials follow from it as a necessary consequence. These latter are called "**inferences.**"

An inference is thus a judgment which follows as a necessary consequence from some previous assertion, and one which the mind is obliged to make on pain of self-contradiction. If I assert the general fact that "All men are mortal," I am obliged, on pain of self-contradiction, to infer that this or that particular man is mortal.

A child has had its attention called to the fact that a piece of cork thrown into the water always floats, and is also taught how to recognise a piece of cork. The child is now on any occasion able to make two assertions :—

(*a*) Cork always floats;

(*b*) This is a piece of cork;

and from these two assertions, the further one that we call an "inference" follows, and must follow, *viz.*, this piece of cork thrown into the water will float.

From our earliest days, our conscious life is largely occupied in drawing inferences. This will appear if we reflect how in teaching and in ordinary conversation we are constantly using such words as *therefore*, *for*, *because*, *since*, etc. Every such word marks the drawing of some inference. **Education at school, and experience in after life is really little else than the development of the inferential connections between propositions.**

Now this work of drawing inferences has certain definite rules and laws which must be observed. And it is by these laws that all our inferences may be tested. If the laws have been broken then the inference drawn is an invalid one. An

invalid inference we call a fallacy. No one knows better than a teacher how prone children are to draw wrong inferences. And indeed all through life men are liable to draw inferences which are fallacious, the main object of Logic being to show how these wrong conclusions may be avoided. It does this by educating man's power of distinguishing the *consistent* and the *conclusive* from that which is *inconsistent* and *inconclusive*.

The principles of Logic find their application in every walk of life, and in every branch of science. This is recognised by the names given to the various sciences. Thus in the name "Geology" the last four letters are only another form of the word "logic," and the term "geology" means "Logic applied to explain the crust of the earth." So, too, theology means "Logic applied to explain Divine matters," and so on. Since then the rules of Logic find their application in the processes of every special science, Logic has been very properly called the *Science of Sciences*.

> Some writers have considered it needful to discuss at great length whether Logic should be called a Science or an Art. As a matter of fact it may be considered as either or both. Science is sound knowledge, an Art is the instrument by which science works. In studying a science we are gathering knowledge, in learning an art we are preparing to do something.
>
> **Logic is a Science in so far as it unfolds the conditions of valid thought.**
>
> **Logic is an Art in so far as it devises rules for enabling men to apply their thought to things consistently and coherently.**

Of course it is not implied that a man is unable to think or reason correctly unless he has learnt Logic. Plenty of people speak correct English who have never learnt the rules of Grammar. From our earliest childhood we have been accustomed to draw conclusions, and no doubt we have generally obeyed the laws of Logic in doing so, without being in the least aware what those laws were. In such cases we were thinking logically without being conscious of the logical

principles which our thinking exemplified. On the other hand, a course of logical study must bring into prominence the laws which constitute valid, consistent thought. The student who has patiently worked through a course of Logic, is much more likely hereafter to think consistently and coherently than one who is ignorant of Logic. And this will be found to be especially true in those cases where, owing to the complexity of thought and the ambiguity of language, there is serious danger of fallacy even to the cleverest intellects.

DIVISIONS OF THE SUBJECT.

There are two main divisions of Logic, *viz.*:—

(*a*) **Deductive** and (*b*) **Inductive Logic.**

We must get a preliminary view of the scope of these:—

(*a*) **Deductive Logic** (sometimes called pure or Formal Logic). This division of the subject is the orderly, scientific unfolding of those forms and conditions to which our "thinking" must conform in order to be valid thinking.

Our "thinking" manifests itself in what for convenience may be called *three* stages, not that as a matter of fact they are separate and distinct. Each of these ideal stages has its peculiar product or result:—

PROCESS.	RESULT.
STAGE 1.	
Formation of ideas or concepts of things.	**Signifying these concepts or ideas by "Names."**
STAGE 2.	
Forming judgments about these ideas—*i.e.*, making mental assertions or denials about them.	**Expressing these judgments in "Propositions."**
STAGE 3.	
Drawing mental inferences from these judgments.	**Signifying this mental process by the "Syllogism."**

Stage 1, is often described as "simple apprehension," and stands for the action of the mind in being aware of anything, having an idea,

or concept of it. The psychological analysis of "simple apprehension," however, shows it to be a complex and not a simple process.

N.B.—Simple apprehension, judgment and **reasoning** (inference) are the psychological processes. **Names** (terms), **propositions** and **syllogisms** are the corresponding results which are the subject matter of Logic.

Consequently Deductive Logic has three sub-divisions answering to these three stages.

The laws of thought:—

1. Concerning our Concepts of things;
2. Concerning our Judgment of things;
3. Concerning our Reasoning about things.

And since our thinking is liable to error, we shall require to supplement the above by an account of the **Fallacies** to which we are liable if the laws of pure logic are violated.

(*b*) **Inductive Logic** (sometimes called Applied or Mixed Logic). The scope of this second main division is best seen from an account of its sub-divisions.

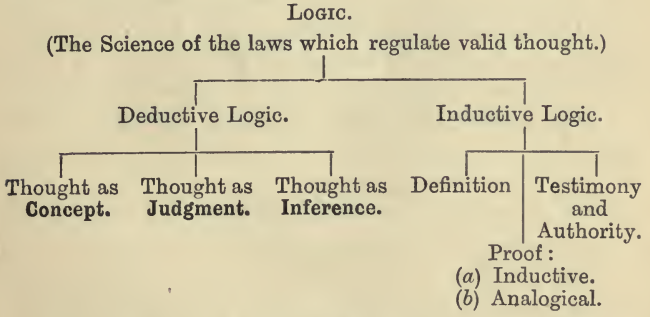
1. DEFINITION.—In all our thinking about nature we are obliged to use language. But language, at the best, is only an imperfect instrument for expressing all that is in the thinker's mind. The Logical doctrine of definition aims at improving the relations of language to thought, and especially to man's thought about things.

2. INDUCTIVE AND ANALOGICAL PROOF.—This section first investigates the meaning and value of the central presupposition of all science, *viz:* "The uniformity of nature." It reveals the fact that nature is not a chaos, but an orderly coherent system of cause and effect. Next it deals with the obvious fact that untrained minds are liable to confuse mere accidental coincidence with true consequence and real scientific connection. To guard against this, Logic provides certain canons or rules, and by these unfolds the standard of scientific proof. It thus enables men to distinguish between evidence which is properly

(*a*) Inductive and therefore reliable, and evidence which is only (*b*) Analogical or probable. This section is thus really the logic of all the physical sciences.

3. HISTORICAL PROOF.—If the language of nature is liable to be misunderstood, much more so is the spoken and written language of men. Therefore Logic has certain rules to lay down respecting human testimony which form the criteria of Historical Proof.

These divisions and sub-divisions of the whole domain of Logic may be shown thus:—



Note that LOGIC is concerned primarily with **Thought**, GRAMMAR with **Language**. Logic considers Language only as the instrument of Thought. The "Parts of Speech" which are the main feature in the grammatical analysis of language are not recognised by Logic. Only those words which can properly express a concept are within its range. Words that can express a concept are grouped together as **Terms**, and it is quite immaterial whether, in grammatical language, they are nouns, pronouns, adjectives, or verbs. In Grammar and Logic the simplest expression of a complete thought is a Simple Sentence. But the logical analysis of a sentence differs from the grammatical. The predicate of a logical sentence is always the complete assertion made of the subject.

EXERCISES ON CHAPTER II.

1. Define "Science" and "Art." Discuss the question whether Logic is a Science or an Art.

2. What is Logic? What are the chief uses of its study? Why should teachers especially make a study of it?

3. Many people think quite correctly who have never studied Logic; why, then, waste time in studying it?

4. Logic has been defined as "the science of the laws of thought." In this definition what is meant by the terms "Science," "Law," and "Thought"?

5. Discuss the relation of Logic to Grammar and Rhetoric.

6. What practical value may be attributed to Logic (a) *in the detection of error,* (b) *in the discovery of truth?*

7. Explain the logical words term, proposition and syllogism, and give the psychological words for the corresponding mental act of each.

CHAPTER III.

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The Axioms of Logic.

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Logic has thus far been shown to be the practical science which unfolds to us the ideal of self-consistent thought. In its later sections it supplies the student with certain canons or rules for applying our thought to things. Now, there are certain principles or axioms which form the essence of self-consistency. When these are drawn out they will appear to the student as self-evident truths. All the same they require consideration, and after we have examined them they must be regarded as axioms.

Speaking quite generally we may say that these several axioms imply one general truth, *viz.*:—

"*Thought which is evidently self-contradictory is impossible.*"

Every description of fallacy is really a thought that is self-contradictory. But we may so express ourselves that the contradiction is not obvious to those to whom we are speaking, nor even to ourselves. Thus, in such an argument as the following:—

"He who is most hungry eats most;
He who eats least is most hungry:
Therefore, he who eats least eats most,"

we feel there is contradiction somewhere, but it would require some consideration to point out where the contradiction really lay.

It might be better to say that a fallacy is really the absence of thought, *i.e.*, of logical thought; the absence usually being concealed under a veil of words.

The general truth stated above—*viz*, "thought which is evidently self-contradictory is impossible"—is expressed by logicians in three different ways. These three different forms are the three fundamental laws of which every valid thought is the exemplification. In other words, they are the essence of "self-consistent thought."

The three laws are known as :—

1. The Law of Identity.
2. The Law of Non-contradiction.
3. The Law of the Excluded Middle.

1. The Law of Identity.—This principle asserts that if any proposition is true, then any other proposition which is either (*a*) identical with it, or (*b*) logically included in it, must also be true.

A is (*i.e.*, must be) A.

Whatever is, is.

Everything is what it is.

If every A is B, then this A is B.

2. The Law of Non-contradiction.—This principle asserts the necessary logical disagreement of assertions with their contradictory denials. In other words, two contradictory assertions cannot both be true.

A is not non-A.

Nothing can both be and not be.

The same attribute cannot be at the same time affirmed and denied of the same subject.

3. The Law of the Excluded Middle.—This principle asserts that of two downright contradictory statements, either the one or the other must be true; no third or intermediate assertion is possible.

A either is or is not B.

Every assertion must be true or not true.

Besides these three generally accepted principles, there is another one which is less universally adopted. It is known as

The Law of Sufficient Reason, and the law may be expressed thus:—*Nothing happens without a reason why it should be so, rather than otherwise.* For instance:—If two forces are in exact equilibrium, there is no reason why the body on which they act should move in the direction of either force. If a body is acted on by two unequal forces in exactly opposite directions, it will move in the direction of the greater force. *If* the "reason why" is granted you must allow the consequence.

The student will notice that these laws are self-evident truths, they cannot be proved by reference to anything simpler. They neither require proof nor are capable of it.

Until the student has made some progress in the study of Logic, he is liable to confuse opposite terms with contradictory ones. Thus in considering the law of the excluded middle, he might fancy it possible to make intermediate assertions which the law says are impossible. Thus, if we say "every substance is either hard or not hard," the reply might be made that some substances are neither hard nor soft, but of medium quality. But Logic has nothing to do with degrees of hardness. It assumes that the word "hard" has a definite meaning, and all things which do not exactly agree with this meaning are "not hard." Even concerning things of which "hardness" could not properly be predicted, *e.g.*, heat, colour, taste, etc., it is still possible to say "heat is either hard or not hard."

EXERCISES ON CHAPTER III.

1. "*Things which are equal to the same thing are equal to each other.*" *Show that this is only another form of the Law of Identity.*

2. What are the laws of thought? State clearly what you understand by a law of thought.

3. Euclid in comparing things shows that they are either greater, equal to, or less than each other. How would these three alternatives be expressed in Logic?

CHAPTER IV.

Terms: their Definition and Classification.

THE simplest and most elementary manifestation of thought is an assertion or denial. Every assertion or denial that we can make is called in Logic a **judgment.** Now in every assertion or denial we affirm or deny something of something else.

When the judgment is expressed in words we call it a **proposition.** Every proposition must therefore contain two names:—

(*a*) The name of the thing about which the assertion is made; (*b*) The assertion itself, *e.g.*,—

(*a*) Gold (*b*) is a metal.

These two names are called the **"terms"** of the proposition, because they are the boundaries (terminals) of the proposition.

DEFINITION OF A TERM.—**A Term is a word or a combination of words, which can properly stand as the subject or predicate of a proposition.**

Every proposition must of course have a subject and a predicate either expressed or implied. Even in an exclamation such as "Fire!" the word is the predicate of a proposition the subject of which is implied, thus:—"*This house (etc.) is on fire.*"

From the definition given it follows that all **words** may be divided into two classes:—

(*a*) Those which can be used as Terms

(*b*) Those which cannot.

Words belonging to the former class are called **categorematic.** Those belonging to the latter class are called **syncategorematic.**

A categorematic word is one which can by itself be used as a term, *i.e.*, which can stand alone as the subject or predicate of a proposition. (In grammar such words are distinguished as nouns, pronouns, adjective, participles, but in Logic they form one class.)

A syncategorematic word is one which cannot by itself be used as a term, but only in combination with one or more other words. (Adverbs, prepositions, conjunctions and interjections.)

The student must carefully avoid speaking of syncategorematic terms. The contradiction is obvious.

Terms (categorematic words) are classified in *five* groups as follows :—

1. The **Common** term (or, as it is often called, the **General** term), as contrasted with the **Singular** (or **Proper**); and the **Collective** term.
2. **Concrete** and **Abstract** terms.
3. **Positive** and **Negative** terms.
4. **Connotative** and **Non-connotative** terms.
5. **Absolute** and **Relative** terms.

1. (*a*) The Common or General Term.—To logicians this is by far the most important of all. It is a term which can be affirmed or denied in the same sense of more things than one : as book, dog, man.

(*b*) The Singular or Proper Term is one which can be affirmed, in the same sense, of only one single thing. N.B.—A common term may of course be transformed into a singular term by means of some individualising prefix. Thus "man" is a common term, but "the first man" is a singular term.

(*c*) The Collective Term is one which can be affirmed or denied of two or more things taken together, **but** which cannot, like a common term, be affirmed or denied of each one of these when taken separately: as army, flock, library, etc.

> Notice that in words like "library" the sense in which the word is being used must be taken into account. Library, a collection of books, is a collective term. Library (*i.e.*, any library) is a common term.

2. (*a*) A Concrete Term is the name of an object; it stands for some individual thing, or a collection of individual things.

   (*b*) An Abstract Term represents an attribute or attributes, considered apart from the individual object of which it may be the attribute. Thus "man" is concrete, "humanity" is abstract; "living being" is concrete, "life" is abstract; "generous" is concrete, "generosity" is abstract.

3. (*a*) A Positive Term implies the presence of some attribute or group of attributes.

   (*b*) A Negative Term implies the absence of the attributes included in the corresponding positive term. Thus metallic, compound, light, are examples of positive terms; of which the corresponding negative terms are non-metallic, element, darkness.

4. (*a*) A Connotative Term is one which represents an individual thing, or group of individual things, *together with* one or more of their attributes. "Animal" is a connotative term, as it implies the attribute "animality"; so also is "mountain," which implies the attributes "height," etc.

   (*b*) A Non-Connotative Term signifies an individual thing only, and does not imply any attribute. Thus: Whiteness, London, are examples of non-connotative terms.

The student must carefully consider this distinction of terms. Think, for example, why "mountain" is called a connotative term, but Snowdon, the name of a particular mountain is not. Now the name "Snowdon" might suggest to anyone with sufficient geographical knowledge, all the attributes implied in the term "mountain." But a word is not connotative because it may *suggest* facts or attributes which are *otherwise known*, but only when it actually *implies* them. Many logicians have overlooked this, and have considered proper names connotative. In answering a question in an examination it would be wise to give your reason for considering a proper name as "non-connotative."

5. (*a*) An Absolute Term is a name which is complete in itself, *i.e.*, which in its meaning implies no reference to anything else; as gas, sound, tree, etc.

(*b*) A Relative Term is a name which not only denotes some object, but also implies in its signification the existence of some other object called the correlative. Thus when we use the term friend or father for some man, we imply the existence of some other person or persons to which the man stands in the relation of friendship or fatherhood.

These definitions of the various kinds of terms must be thoroughly understood, and the student must be well exercised in the classification of terms. When the appended examples are attempted it will be found a more difficult task than might be supposed. The main difficulty will be found in deciding whether an abstract term is general or singular. Some logicians argue that all abstract names are singular. Thus the adjective "red" is the name of red objects, but it implies the possession by them of the quality "redness," and this quality has one single meaning. It is much simpler, however, to consider some abstracts general on the ground that they are names of attributes of which there are various kinds or subdivisions; *e.g.*, the word colour which is a name common to whiteness, redness, etc., or the term whiteness in respect of the various shades of whiteness to which it is applied in common. But

just because the point is a disputed one, you should give your reason for classifying abstract terms as general or singular.

A further difficulty arises in dealing with terms that are equivocal, *i.e.*, capable of being used in several senses. Indeed, some writers make a further classification of terms, as UNIVOCAL (terms which can only suggest one meaning) and EQUIVOCAL or AMBIGUOUS (terms which may have two or more meanings). An equivocal term is really two or more terms with identical spelling, and should be so treated. Thus the term "force" is equivocal, as it might mean an army or that which causes motion, etc., and each meaning demands a distinct classification of the word. It is better, therefore, to say at once if a term is equivocal or univocal, and then proceed.

EXERCISES ON CHAPTER IV.

1. Discuss the grammatical parts of speech from a logical point of view.

2. May terms be classified as categorematic and syncategorematic? Give reasons for your answer.

3. Describe a "collective term." Illustrate the difficulty of distinguishing these from general or abstract terms.

4. Classify the following terms: donkey, reagent, red, redness, London, sugar, Mikado of Japan, intensity, also, vexation, blind, emotion, darkness, foot, Westminster Abbey, uncle.

5. Point out the ambiguity, if any, of the following terms: vice, hydrogen, peer, paper, sense, minister, tea-cup, interest.

6. Distinguish between the meaning of the terms abstract and concrete, and show the applicability of these terms (1) *to parts of speech, and* (2) *to arithmetic. Say what is the use of the distinction.*

CHAPTER V.

The Denotation and Connotation of Terms.

If the question were asked "What is an animal?" we can imagine two forms of answer being given: (*a*) an exact definition of the term; (*b*) an enumeration of the various classes of animals. The first answer might be expressed thus:—"An animal is a sentient, organised being." This definition tells us what **must** be the attributes of anything in the universe to which the name "animal" can be rightly applied. Such a definition is said to mark the **connotation** of the term. On the other hand the latter definition which proceeds to enumerate all the different classes of animals is said to mark the **denotation** of the term.

A Term, therefore, in Logic is considered to discharge a double function:—

1. Connoting the attributes of things.
2. Denoting individual things.

Notice that a term is a word which signifies a mental idea or concept. But in Logic we do not speak of the connotation or denotation of a concept. When speaking of concepts we use the words intension and extension.*

The intension of a concept corresponds to the connotation of the term signifying the concept.

The extension of a concept corresponds to the denotation of its related term.

The connotation of a term (or the intension of its corresponding concept) signifies the attributes implied in the meaning of the term.

* Some writers, however, speak of the intension and extension of *terms*, and even the denotation and connotation of *concepts*.

The denotation of a term (or the extension of its corresponding concept) signifies the number of individual things to which the term is applicable in the same sense.

The student is invited to reflect upon these definitions. It will then be seen that an important logical truth is involved. Every common term like man, bird, etc., stands for a **number** of individual things (different individual men or birds), **and a quantity** of attributes (rational being, feathered biped, etc.) Thought as expressed by "terms" is thus **a kind of quantity,** and all our affirmations and assertions about terms are really a comparison of quantities. If I say "men are animals," I mean that "men" are a quantity of things contained in a greater quantity of things called "animals."

Now the two particular kinds of quantity we are considering (connotation and denotation) have a mutual relation. For a moment's reflection will show that the wider or greater the *denotation* of a term becomes, the narrower or smaller must be its connotation. Thus compare the two terms "animal" and "man." The term animal embraces far more individual things under it than the term man, therefore its denotation is greater. But the term man implies a larger number of attributes than the term animal. For everything that you can say of animal you must say of man, but you also say of man certain things which you cannot say of all animals. Therefore the connotation of the term man is greater than that of the term animal.

As a fairly correct general rule it may be said that as the denotation of a term is increased, the connotation is diminished, and vice versa.

In other words the greater the number of individual things included under a common term, the fewer will be the number of attributes which can be predicated of the whole of them.

This is expressed by the Logical rule that the connotation and denotation of a term (or the intensive and extensive quantity of a concept) are in inverse ratio. The greater

the denotation, applicability or extent of a term—the less must be its connotation or comprehensive quantity. The maximum of the one must in all cases be the minimum of the other, and vice versa.*

Now observe when two common terms are so related that the whole connotation of the one is included within the greater connotation of the other—**the term which has the greater connotation is called the "Species," and the one which has the smaller, or included connotation, is called the "Genus."** Thus taking the two related terms "man" and "animal," the term "man" implies all the attributes that the term "animal" implies, as well as some further ones peculiar to itself. "Man" has the larger connotation, therefore "man" is a species of the genus "animal."

In the proposition "Man is an animal" we assert that "man" the species is included in "animal" the genus. Every affirmative proposition makes some such assertion respecting the subject of the proposition. The following problem, therefore, arises: "*Can the predicates of all propositions be classified in relation to their subjects under certain definite heads?*" Logic attempts this by the **Doctrine of the Predicables.**

The predicables, then, are a classification of all the possible relations of the predicate to the subject of a logical proposition. The following is the usual form of this classification:

Predicables are either { 1. Genus; 2. Species; 3. Differentia; 4. Proprium (property); 5. Accidens (accident) } of the subject.

These five heads of the predicate require consideration.

* This doctrine of "Connotation and Denotation being in inverse ratio" is given in accordance with traditional logic. It is open to much criticism, and is only a "FAIRLY correct general rule."

1. GENUS is a common term, signifying a wider class which is made up of other narrower classes, *e.g.*, animal, triangle.

2. SPECIES is the name given to the narrower classes, included in a genus, *e.g.*, Vertebrates, Invertebrates; equilateral triangle, etc.

Genus and species, then, are relative terms, and must be considered together. A genus would be meaningless apart from two or more species into which it is divided. A species would be equally meaningless apart from the genus in which it is contained.

The student will notice that the same term may be at the same time a **species** of the next more general class, and a **genus** to the less general classes included under it. Thus take the term "triangle." Triangle is a **species** of the genus "rectilinear figures," whilst at the same time it is a **genus** of the different kinds of triangles: equilateral, isosocles, etc.

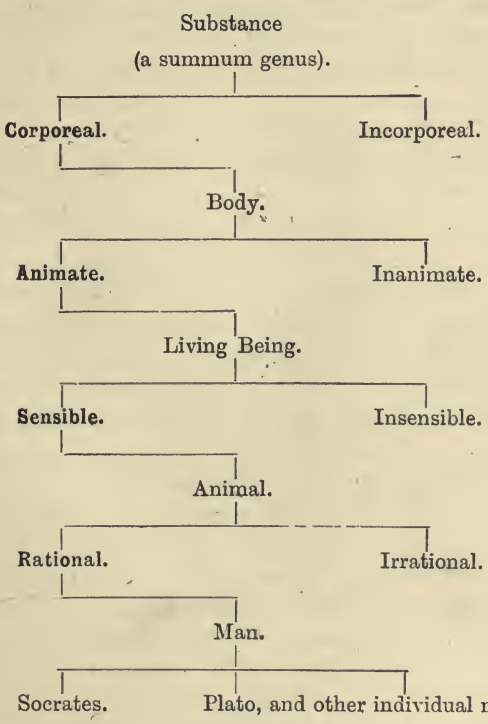
From this it follows that every term may be both a genus and a species. But the technical language of Logic implies, however, that this is not universally the case. It implies that there is a genus which is not a species of any higher genus; and that there is a species which is not a genus to any lower species. For Logic speaks of:—

1. **The highest genus;**
2. Intermediate genera or species;
3. **The lowest species.**

The highest or most general genus, *i.e.*, which can have none above it, is such a one as "Being." This is called **"the summum genus."** The lowest relative species, which can have none below it, is the name of any individual thing. This is called **"the infima species."**

Any highest genus broken up into its component species, and these component species in turn regarded as genera again broken up into their component species, and the process

repeated until you cannot proceed further, (*i.e.*, when an infima species is reached) is called a "Predicamental Line." A process such as this is illustrated by the ancient "Tree of Porphyry":



Here Substance is the Summum Genus and Man is the Infima Species (i.e., man cannot be divided into any smaller species, but only into individual men).

Each of the intermediate genera down the middle line (Body, Living Being, Animal), is called a subaltern genus or species, and the nearest genus to every term of which that term is itself a species, is called the proximum genus.

3. Differentia.—It has already being seen that a species has a larger connotation than its corresponding genus (*i.e.*, the species implies more attributes). Now take any term used as a species and compare it with its next, or proximate, genus. **The excess of the connotation of the species over the connotation of the genus is called the "Differentia" of the species.** Thus:—

Genus + Differentia = Species.

Referring to the Tree of Porphyry, "Living being" is a species of the genus "body." "Animate" is the attribute which forms the differentia of the species "living body," thus:—

Body + Animate = Living body
(genus) (differentia) (species).

4. Property (Proprium).—By property is meant any attribute which is common to **every** individual in a given class, but which is not necessary for distinguishing that class. This will be clear from the following illustration. Take the term "triangle." A triangle is a figure bounded by three straight lines. "Three-sided" is the differentia of a triangle. But triangles have many other **properties,** *e.g.*, "three-angled," "all their angles equal to two right angles," etc.

5. Accident (Accidens).—An accident is an attribute which has no **necessary** connection with the term to which it belongs. Thus the size of a triangle—*i.e.*, big or little—is an accident. Size does not at all affect what Euclid proves concerning triangles.

Accidents are usually divided into

Separable accidents—*e.g.*, how a man is dressed;

Inseparable accidents—*e.g.*, the colour of his hair.

EXERCISES ON CHAPTER V.

1. Define differentia, property, and inseparable accident, giving examples. How far may these distinctions be interchanged.

2. To which of the predicables would you refer the predicates in the following propositions, and why :—

(a) All men are animals.

(b) Mr. Gladstone was a great statesman.

(c) The three angles of a triangle are together equal to two right angles.

(d) All ducks are web-footed.

(e) John ruled badly.

(f) Alkalies by their union with acids form salts.

3. Explain clearly the connotation and the denotation of a term. What determines the connotation and denotation of terms? Have all terms a denotation and connotation?

4. Arrange the following terms in their order of extension :— Vertebrate, human, substance, child, organism, schoolboy.

5. Explain the terms intension and extension as applied to terms in Logic, and distinguish genus and species, illustrating your explanation by the terms cart, eagle and man.

6. Distinguish between denotation and connotation, and show the importance of the distinction in teaching.

7. Give the genus, the differentia, a proprium and an accident of silver, Darwinian, square, house.

8. "A generic term denotes a larger number of objects than a specific term; but it connotes a smaller number of attributes." Explain this statement and illustrate it by examples.

CHAPTER VI.

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Definition and Division of Terms.

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The definition of a term is the explicit statement of the connotation of the term.

Since every definition of a term must take the form of a proposition, it would be more convenient to have considered the logical doctrine of definition when we are discussing propositions. But it is usual to consider the subject at this stage of our study.

In a definition that which is defined is always the subject of a proposition. The predicate must declare with sufficient precision what the subject means. In other words, the predicate must show forth the attributes which separate the subject in question from all other subjects.

> All definitions are propositions, but all propositions are not definitions. Only those propositions are definitions in which the predicate so makes clear the attributes of the subject, as to separate it from all other subjects with which it might be confounded.

The subject and predicate of a definition are, therefore, exactly *co-extensive*. The difference between them is this:—what was latent—wrapped up, as it were, in the subject—is fully unfolded or analysed in the predicate. Logic asserts that this result is achieved when the predicate of the defining proposition exposes the proximate genus *and* the differentia of

a term. For the **genus** implies all the attributes of the term considered as a species of the genus; whilst the **differentia** displays those attributes which distinguish the term as a species. In Logic, then,

The definition of a term = proximate genus + differentia.

Notice that there are some terms which are incapable of logical definition, *e.g.*, a summum genus, all proper names, etc. The former has no proximate genus, the latter have such a multiplicity of attributes that we can only mention a number of them sufficient for the practical purpose of recognition. This enumeration, however, is "**description**" *not* **definition.**

The student must not confound logical definition with "**dictionary definition.**" In the latter all that is done is to substitute one word for another, assumed to have a similar connotation, on the ground that the new word is more familiar or intelligible than the one for which it is substituted.

There are certain simple rules which Logic lays down to which propositions must conform to entitle them to be regarded as **good logical definitions.**

1. The definition must bring into view the essential, distinguishing attributes (differentia) of what is defined.*

2. The definition must be adequate, and applicable exclusively to what is defined.

3. We must not define by negations.

4. The definition must be expressed in unambiguous, intelligible language.

Definition is a **most important** subject. Avoid confusing the definition of names with the definition of things. The definition of a *name* is the settlement of what the name shall be, by which a thing or a concept shall be designated. Any man is entitled to determine this as he pleases, so long as he adheres consistently to the name he has connected with the

* Obviously, to merely name properties or accidents can never be a logical definition.

concept or thing. Sounds or signs on paper, are in themselves indifferent to meaning. Each or any may be used to express any meaning that has been agreed upon by those who use the word. Definition of a *thing*, is not thus arbitrary. These definitions depend on what is involved in the essential nature of the thing defined. Men are apt to confound definitions of names with definitions of things, and to confuse both with that full analysis of the attributes implied in our concepts which it is the province of logical definition to bring into light. We are frequently asked to accept definitions of names as if they were the true definitions of things. Because we agree to employ a certain sound to express some meaning, it does not follow that the meaning so expressed corresponds to the essential attributes of the things signified by the sign.

LOGICAL DIVISION. — **Division is the analysis of the denotation of a term.**

It is always expressed in the form of a proposition, the term divided being the subject, and the exposition being the predicate.

There are other familiar kinds of division with which logical division must not be confounded, *e.g.*,

(*a*) Partition, which is the act of dividing some physical whole into its constituent parts, *e.g.*, ship=hull, mast, sails, etc.; man=head, trunk, limbs, etc.

(*b*) Distinction of ambiguous or equivocal terms, *e.g.*, Humanity=(1) human nature, or (2) the human race collectively; Vice=(1) a moral fault, or (2) a mechanical tool.

(*c*) Enumeration of individuals, *e.g.*, naming all the books in a library.

Logical division expounds the denotation of a term not by enumerating individuals. This would in most cases be impossible. No one could enumerate all the different men

included under the term "man." It proceeds by mentioning only the smaller **groups** denoted by the term.

Collective and singular terms cannot be divided into smaller groups, and, therefore, cannot be logically divided.

> A collective term can be transformed into a common term, and so become capable of logical division. Thus "the fourteenth regiment" may be transformed into "soldiers of the fourteenth regiment," and in this form may be divided into officers, privates, etc.

When we proceed to divide a term into terms expressive of smaller groups, we seek some attribute which may be predicated of certain members of the group, but which cannot be predicated of the rest. This attribute is called the **basis of division** (*fundamentum divisionis*). Of course, the same genus may be variously divided by adopting different bases of division. Thus in dividing the genus "triangles" we may adopt the relative length of their sides as our basis, and so divide triangles into equilateral, isosceles, and scalene. Or we might adopt the size of their angles as the basis, and so divide triangles into right-angled, acute-angled, and obtuse-angled. BUT two or more bases of division must never be confused together in the same division, or we fall into the error called in Logic "Cross division." It would, *e.g.*, be cross division to divide triangles into isosceles, right-angled, and scalene.

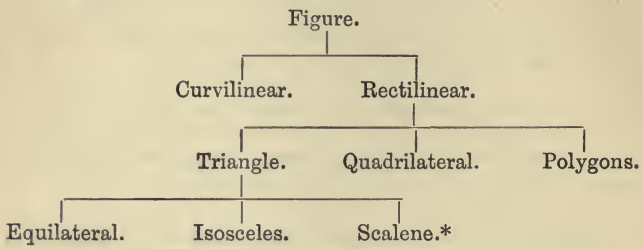
There are certain rules to which a logical division must conform, *viz.* :—

1. Each act of division must have one and only one basis of division, or cross division will ensue.

2. The division must be exhaustive, *i.e.*, the dividing members when taken together must be co-extensive with the divided whole.

3. If the division is a continued one (*i.e.*, embraces more than one step), each step should, as far as possible, be a

proximate one—in other words "proceed step by step." *e.g.* :—



When we turn from the division of our concepts as expressed in terms, and proceed to consider material things the logical doctrine of division becomes a theory of logical scientific CLASSIFICATION. The object of classification is to so arrange the facts with which we may be dealing that we can acquire the greatest command over them, and convey the greatest amount of information about them in a few words.

> Classification is really a branch of Inductive Logic. It is one of the important processes subsidiary to the application of the inductive canons. By its use we obtain a greater command over the knowledge we possess, and are put in the right avenue for obtaining additional information. It provides that our knowledge of things shall be so arranged that the facts may be more easily remembered, and that we may more readily perceive the laws by which they are governed.

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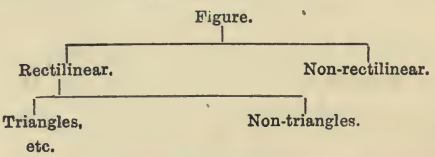
*There is a further method of division in which each step is a division into corresponding positive and negative terms, *e.g.* :—



This is called division by Dichotomy. It is extremely cumbersome and of small importance.

Logic considers all attempts at classification as either natural or artificial. By a natural classification is meant the grouping of facts in accordance with real natural distinctions. Thus an actual scientific knowledge of facts is a pre-supposed requisite for a natural classification. Different branches of science have different objects in view, and accordingly they often adopt a special basis for classification. The practical farmer divides plants into those which are useful, and those which are weeds. Whilst the botanist adopts the division into monocotyledons and dicotyledons as his basis. The student who has an elementary knowledge of Geology and Zoology will remember how differently fossils are classified in the two Sciences.

An artificial classification selects some point of resemblance amongst objects, and one which is easy to identify, and proceeds to classify related objects upon this basis. The Linnæan system of classification in Botany, which takes for its basis the number of stamens and pistils in a flowering plant, is a good illustration of an artificial system. In Zoology, where the primary basis of classification is into vertebrates and invertebrates, we have an example of a natural classification.

EXERCISES ON CHAPTER VI.

1. *Criticise the following definitions:—*
   (a) Ignorance is a blind guide.
   (b) The cat is a domestic animal.
   (c) Enjoyment means pleasure.
   (d) Tranquillity is the absence of unrest.
   (e) Alcohol is a kind of medicine.

2. *Define the terms gold, coal, legal nuisance, civilization, Cleopatra's Needle, bread, anger, Snowdon.*

3. *What do you understand by a perfect definition; and what processes of thought are employed in arriving at one? Give two or three examples which err by being either too wide or too narrow.*

4. *What is the difference between (a) a description, (b) a definition, (c) an explanation?*

5. *Explain what is meant by logical division, and briefly state its rules. Give instances which observe, and instances which violate the rules.*

6. Comment on the following as logical divisions :—

(a) Pens into quill pens and steel pens.

(b) Ireland into Ulster, Munster, Leinster and Connaught.

(c) Animals into vertebrate and invertebrate.

(d) Colour into whiteness, blackness and blueness.

(e) Lights into artificial, blue and red lights and moonlight.

(f) Vice into an immoral act and a mechanical tool.

(g) Englishmen into rich and poor, consumptive and bilious.

7. Show the relation between Definition, Division and Classification.

CHAPTER VII.

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Propositions.

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HAVING completed our investigation of the logical doctrine of "terms," we now proceed to consider the teaching of Logic with regard to "propositions." Just as a "term" is the outward expression for the inward (psychological) fact, which is called a "concept," so a "proposition" is the translation into language of the inward mental act, which is called "judgment." Now, it has already been shown that a judgment is the simplest and most elementary manifestation of a complete thought. Every assertion or denial that we can frame in our minds is a judgment. When this mental act is expressed in language, we have what is called in Logic a proposition.

A proposition, therefore, may be defined as **The verbal expression of a truth or falsity,** or **A sentence making an affirmation or denial.**

Propositions which make simple assertions or denials, without any condition attached, are called CATEGORICAL.

A Categorical Proposition is one which simply asserts or denies some fact, *e.g.*,

All men are mortal.
No men are infallible.

Notice that in a categorical proposition we bring together two terms, and connect them by the copula. *For logical purposes this copula is always the present tense of the verb "to be," with or without the negative particle "not."*

In ordinary language, of course, our categorical judgments are expressed in various ways. But Logic considers that every simple assertion or denial can be expressed in one general form, and, for logical purposes, the assertion or denial must be reduced to this form. Hence the student must become accustomed to expressing the ordinary forms of simple assertions and denials in the precise form required by Logic. There is no doubt that the logical form of an assertion will often appear awkward and "wordy," compared with ordinary conventional modes of expression, but the advantage gained by the precise exposition of our assertions is of the highest logical importance. Take as an illustration the assertion, "John was the brother of Richard." In order to get the present tense of the verb "to be" as the copula of this sentence, it must be expressed in some such form as:—"John is a person who was the brother of Richard." This transformation sometimes causes a little perplexity. Take, for example, the following sentences:—

(1) The bell will toll to-morrow.
(2) None but the brave deserve the fair.
(3) It does not rain.
(4) Fire!

These ordinary conventional sentences, when transformed into simple categorical propositions for logical purposes, become—

	Subject.	Copula	Predicate.
(1)	The tolling of the bell	is	an event which will happen to-morrow.
(2)	No not-brave persons	are	deserving of the fair.
(3)	Rain	is not	falling.
(4)	This property	is	on fire.

Observe, that when a sentence is being thus transformed for logical purposes, and divided into its logical elements (subject, copula, predicate), if any one of the elements has been omitted in the conventional form, it must be supplied in the precise logical form. Thus the exclamation "Fire!" is sufficient, for practical purposes, to convey definite information, but until its subject and copula have been supplied, it is useless for logical purposes.

A categorical proposition, then, is one which makes an unconditional assertion or denial. When the assertion is expressed as a proposition displaying its logical elements, the copula is in all cases the peremptory "is" or "is not." But many of the assertions or denials that we are making constantly are of such a nature as to forbid the employment of the unconditional "is" or "is not." To a large proportion of our judgments some condition or other is attached. Now, Logic draws a sharp distinction between judgments which are unconditional and those to which some condition is attached. **The former are categorical, the latter conditional.** We shall be chiefly concerned with categorical propositions, but it is needful to mention the two kinds of conditional propositions which are most common.

Conditional propositions are usually distinguished as HYPOTHETICAL and DISJUNCTIVE.

1. *Hypothetical propositions* have a conjunctive condition The following are examples :—

(*a*) If A is B, then also C is D.

(*b*) If Logic exercises the intellect, it ought to be studied.

(*c*) Where ignorance is bliss, 'tis folly to be wise.

Example (*a*) and similar examples, where symbols (A B, etc.) are used, are called abstract examples; (*b*) and (*c*) are called concrete examples.

2. *Disjunctive propositions* have an alternative condition, *e.g.*:—

(*a*) A is either B or C.

(*b*) He is either a knave or a fool.

(*c*) All men are either good or bad.

Sometimes we find propositions conditioned, at once conjunctively and disjunctively, *e.g.*:—

If A is B, then C is either D or E.

If a man becomes a soldier, then he must serve either at home or abroad.

Besides this obvious division of propositions into categorical (unconditional) and conditional, Logic further distinguishes them by their **quality** and their **quantity.**

The **quality** of a proposition is determined by the copula. The copula may be either "is" or "is not."

In the former case the proposition is AFFIRMATIVE, in the latter it is NEGATIVE.

A is B (affirmative) (1)

A is not B (negative) (2)

But we may also assert—

All A is B,

or only, Some A is B.

The distinction of propositions, according as the affirmation or denial is made of the *whole* or only a *part* of the subject, is what is meant by determining the **quantity** of a proposition.

Propositions, in which the assertion or denial is made of the whole of the subject, are called UNIVERSAL propositions. Propositions, in which only part of the subject is affected are called PARTICULAR propositions.

Notice carefully, that in universal propositions, the subject of the proposition is *distributed*, *i.e.*, taken in its full denotation.

In particular propositions the subject of the proposition is *undistributed*, *i.e.*, the extent of its denotation is indefinite.

PARTICULAR propositions are usually expressed in the form

Some A's are B.

Some A's are not B.

The word "some" is absolutely indefinite; it may mean "few" or "many," or indeed "all." In Logic it is the equivalent of "one at least."

The student should also carefully note that in UNIVERSAL propositions the subject may be either :—

(*a*) An undivided, whole class, of every member of which the predication is made, *e.g.*, "Men are mortal"; *i.e.*, All men and every individual man; *or*

(*b*) An indivisible individual, indicated by a proper name; *e.g.*, "John is mortal."

Propositions, which have a proper name for their subject, are sometimes called Singular Propositions. In most cases they may be considered only a sub-class of Universals. But instances arise which may cause perplexity. Thus: "John is sometimes eloquent," might be considered as universal with a somewhat complex predicate. (The student will have found, ere this, that in expressing propositions in logical form, the predicate is often very complex). The proposition in its full logical form would be: "John is a speaker who is sometimes eloquent." This is a true universal. On the other hand the proposition might be rendered: "Some of John's speeches are eloquent," in which case the subject is particular, not universal.

These various ways of dividing propositions may now be collected, thus :—

Propositions are divided

1. On the basis of their **quality** into (*a*) affirmative, (*b*) negative.
2. On the basis of their **quantity** into (*a*) universal, (*b*) particular.

The distinctions of quality and quantity are considered as applying only to categorical propositions. To some extent the same distinctions can be applied to conditional propositions.

But to attempt this would be quite beyond the scope of this elementary treatise.

From this we gather that all categorical assertions or denials may be grouped under four general forms. For, when our assertions are expressed in logical form, we *affirm* that the subject is, either

(1) In its whole logical extent, *or*

(2) In part of its logical extent,

contained under the logical extent of the predicate; or, on the other hand, the proposition *excludes* either

(3) The whole logical extent of its subject, *or*

(4) Part of the logical extent of its subject,

from the logical extent of its predicate.

This fourfold division answers to a combination of the divisions of propositions on the two bases of quality and quantity.

Every categorical proposition, true or false, that can be made on any subject whatever must find its place under one of the following heads :—

1. Universal affirmative, usually denoted by the symbol A.
2. Universal negative, ,, ,, ,, ,, E.
3. Particular affirmative, ,, ,, ,, ,, I.
4. Particular negative, ,, ,, ,, ,, O.

The symbols A, E, I, O, are taken from the Latin words *affirmo* and *nego*. A and I are the first two vowels of the former word, E and O the vowels of the latter word.

The student should carefully consider the following simple examples of the four forms of which, in each case, an abstract example, a concrete example, and a diagrammatic illustration are given. Notice the meaning of "is" in the propositional forms. "Is" means "is contained in"; "is not" means "is not contained in." "All X is Y" thus means "All X is contained in Y."

FORM A.—UNIVERSAL AFFIRMATIVE.

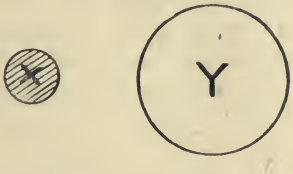
All X is Y.

All gold is yellow.

FORM E.—UNIVERSAL NEGATIVE.

No X is Y.

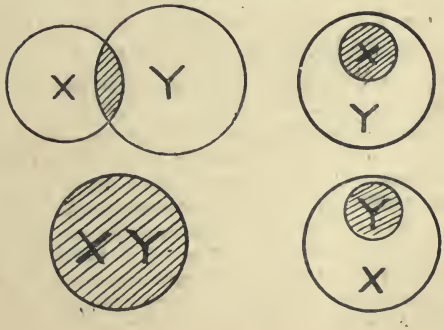
No man is infallible.



FORM I.—PARTICULAR AFFIRMATIVE.

Some X is Y.

Some men are wise.



FORM O.—PARTICULAR NEGATIVE.*

Some X is not Y.

Some men are not wise.

The following observations on this fourfold form of Propositions are of the utmost importance :—

FORM A.—**The subject is distributed,** *i.e.*, taken in its full extension: the predicate is not distributed. When we assert that "*all* gold is yellow," we mean that gold, at all times and in all forms, is yellow; therefore, the term "gold" is fully distributed. But the predicate is not distributed. For the proposition asserts only that amongst an indefinite number of yellow things, gold is always one.

FORM E.—**Both the subject and the predicate are distributed.** When we assert that "no man is infallible," we mean that the two terms "man" and "infallibility" are mutually exclusive. The attribute of infallibility cannot be predicated of any man in the whole universe.

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*In the diagrammatic illustrations the shaded parts always represent the subject of the proposition. The student must note that the proposition *only contains information about the part shaded.* Thus in the diagram representing the proposition "Some X is Y," our information is confined to the shaded part of X entirely We could not assume therefrom that some X is not Y. The proposition only asserts that some portion of X is included within Y. As a matter of fact all might be, but the proposition does not say so. These diagrammatic representations of propositions are called Euler's Circles, and are open to much criticism.

FORM I.—**Neither the subject nor the predicate is distributed.** When we assert that "some men are wise," we mean that amongst men there is an indefinite number, forming an equally indefinite proportion of those beings of whom the attribute of wisdom may be predicated.

FORM O.—**The predicate only is distributed.** When we assert that "some men are not wise," we mean that an indefinite number of men are excluded from the whole definite class of beings, of whom the attribute of wisdom may be predicated.

These observations may be summarised:—

Form A distributes its subject only.
,, E distributes both its subject *and* its predicate.
,, I distributes neither its subject nor its predicate.
,, O distributes its predicate only.

The student will notice that "this," "each," "every," "all," "no," and "some" are the only signs of quantity recognised by Logic. In ordinary speech many others are used, but they must be reduced to one of the signs given above before they can be considered in a logical reference.

Note particularly that expressions like "few," "many," or such fractional terms as "three-fourths" are all considered equivalent to "some." In short, "some" really stands for "some at least"; and beyond that, the word is altogether indefinite. "Any" and similar expressions must be considered as equivalent to "every."

Cases will sometimes arise in which it is a matter of uncertainty whether a given expression is intended to be taken as a universal or a particular. This is especially so in current sayings and proverbs, *e.g.*, "Knowledge is power," "Haste makes waste." Such cases can only be determined by a careful survey of the facts the expressions are supposed to summarise.

EXERCISES ON CHAPTER VII.

1. Define a logical proposition; and enumerate with examples, the various kinds of propositions.

2. What do you understand as the exact meaning of the logical copula?

3. What are the signs of quantity recognised by Logic? How do they compare with those used in grammar?

4. Give the logical equivalent of each of the following expressions: "All are not"; "Only these are"; "All except one"; "Scarcely any"; "Few are not."

5. Reduce each of the following to strict logical form, and indicate whether the proposition is A, E, I, or O:—

(a) All birds have two wings.
(b) All his shots except two hit the mark.
(c) The more the merrier.
(d) There's not a joy the world can give like that it takes away.
(e) All that glitters is not gold.
(f) He jests at scars who never felt a wound.
(g) None fail to remain poor who are both ignorant and lazy.

6. The following sentences are somewhat ambiguous. Make at least two logical propositions of each:—

(a) All are not clever who read much.
(b) Some of the guests behaved disgracefully.
(c) All the books cost a sovereign.

7. What logical proposition is implied in each case, when the following are declared to be false:—

(a) Honesty is the best policy.
(b) All men are liars.
(c) Some horse dealers are honest.

8. Express in the simplest logical form you can the sense of the following passages:—

(a) It never rains but it pours.
(b) You cannot have your cake and eat it.
(c) Unless help arrives we are beaten.
(d) Many are called, but few are chosen.

9. Say whether the following is a categorical or hypothetical proposition, and why:—Trespassers will be prosecuted.

CHAPTER VIII.

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Immediate Inference.

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THE whole of our study thus far has been a preparation for the investigation of inference or reasoning. Inference, in its wider meaning, is the derivation of one proposition from one other proposition or from two other propositions. Those cases in which *a conclusion is evolved from some one proposition, without the help of any other, are called* IMMEDIATE INFERENCES. Thus, when we say "All animals are organised beings," we are able to infer directly from this that any particular animal is an organised being, and, again, that "no unorganised beings are animals." Every single assertion or denial that can be made will yield quite a number of other propositions, which differ from the original proposition in logical quantity or quality, or both.

An **Immediate Inference,** then, is the inferential derivation of a new proposition from some *one* given proposition.

The number and variety of conclusions which can be immediately derived from any single proposition, will be quite surprising to one who is not familiar with this kind of exercise. Take, for example, the following A (universal affirmative) proposition:—"All X is Y."

What inferences can be immediately derived from this? Proceed thus: All X is Y; No X is not-Y; Some X is Y; Some X is not not-Y; No not-Y is X; All not-Y is not-X; Some not-X is not-Y; Some not-X is not Y.

This will be clearer if a concrete example is given:—"All men are mortal." From this we may infer: "No men are not-mortal"; "Some men are mortal"; "Some mortal beings are not not-men"; "No not-mortal beings are men," etc.

Now, without considering whether the examples just given are exhaustive, or whether all the conclusions are of practical importance, we will proceed to discuss the more important forms of Immediate Inference under the following heads:—

I. IMMEDIATE INFERENCES OF OPPOSITION.
II. ,, ,, ,, CONVERSION.
III. ,, ,, ,, PERMUTATION.

I. Inferences of Opposition.—Propositions are said to be *opposed* to each other when they have the some subject and predicate respectively, but differ in quantity or quality, or both.

Of the several kinds of opposition, that known as CONTRADICTORY OPPOSITION is the most perfect and of the greatest logical value. This kind of opposition is an application of the "law of the excluded middle," *viz.*, that, of two contradictory propositions, one must be true and the other false. This occurs when an A proposition is contradicted by an O proposition; or an E proposition is contradicted by an I proposition.

A.—All X is Y.
Contradictory=O.—Some X is not Y.

E.—No M is N.
Contradictory=I.—Some M is N.

Taking either of these pairs of propositions, we see at once that both cannot be true and that they cannot both be false. Therefore, if either of the two propositions is affirmed to be true, we *immediately infer* the falsity of the other.

CONTRARY OPPOSITION is that which exists between an A and an E proposition, having the same subject and predicate. In this case, both propositions may be false, but both cannot be true, *e.g.*:—

A.—All men are good.

Contrary=E.—No men are good.

This kind of opposition is of much less logical value. If we know that one proposition is true, we may immediately infer the falsity of the contrary. But if we know that one proposition is false, we **cannot** infer the truth of its contrary.

SUB-CONTRARY OPPOSITION is that which exists between an I and an O proposition, which both have the same subject and predicate :—

I.—Some men are wise.

Sub-contrary O.—Some men are not wise.

In this case both of the propositions may be true, but both cannot be false. If we know that one of them is false, we can immediately infer the truth of its sub-contrary.

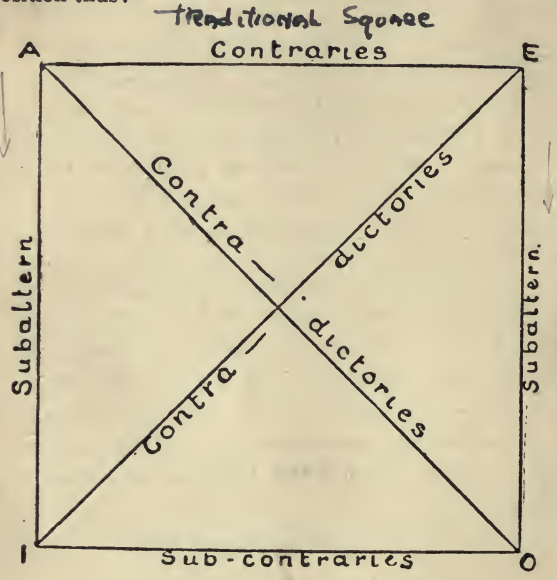
SUBALTERN OPPOSITION is that which exists between a universal and a particular proposition, *i.e.*, propositions which both have the same subject and predicate, but differ in *quantity*:—

A.—All men are mortal.

Subaltern I.—Some men are mortal.

From any universal proposition we can immediately infer the truth of any particular proposition of the same quality (an I from an A, or an O from an E), but **not vice versa.**

An ancient square sets forth these various relations of opposition thus:—



N.B.—Propositions must always have the same subject and predicate before we can place them in opposition.

There should now be found no difficulty in determining what inferences can be immediately drawn from the known truth or falsity of any one of the four ordinary propositional forms.

For convenience the student is advised to commit the following to memory:—

Contradictories cannot both be true, nor can they both be false.

Contraries may both be false, but both cannot be true.

Sub-contraries may both be true, but cannot both be false.

Subalterns may both be true and both false. If the universal is true so is the particular; but the truth of the particular does not imply the truth of the universal.

II. Inferences of Conversion.—By conversion is meant the immediate inferring of a new proposition from a given proposition, in which the subject of the given proposition forms the predicate of the new proposition, and the predicate the subject. Thus from "No stones are organised beings"—is obtained by conversion, "No organised beings are stones."

The remarks in Chapter VII. on the distribution of the subject and predicate in the four propositional forms, are of great consequence here. For, in converting a proposition, care must be taken that the two terms are used in precisely the same extent in the new (or inferred) proposition as they were in the original proposition. Now, in converting an E or an I proposition, no difficulty arises. "No X is Y" distributes both its subject and its predicate. Hence, we may at once say "No Y is X." So, also, the I proposition "Some M is N" distributes neither its subject nor its predicate. Thus, we can immediately say "Some N is M." But in the A proposition "All S is P," the subject S is distributed, but the predicate P is undistributed. If we converted this into "All P is S" we should distribute P in the new proposition, whereas it was not distributed in the original proposition. This we may not do. From "All S is P" we can only infer "Some P is S." Hence we say that A propositions can only be converted "by limitation" (*per accidens*).

Summarizing these points we learn that:—

From an A proposition we can infer an I proposition by "conversion by limitation."

From an E proposition we can obtain another E proposition by simple conversion.

From an I proposition we can infer another I proposition by simple conversion.

Lastly, we have to consider the case of O (particular negative) propositions. Can these be converted? Take, for instance, "Some X is not Y." Here X, the subject, is not

distributed. If we convert the proposition and say "Some Y is not X," we distribute X in the new proposition. But, in conversion, we may never distribute a term in the new proposition, which is undistributed in the original proposition. Hence, we conclude that O propositions cannot be converted.

Practice in drawing immediate inferences by the conversion of given propositions is a most valuable test of the student's progress in logical study. Both in ordinary discourse and in examinations most ludicrous results follow from not observing the rules of legitimate conversion. One examiner says that when he has asked for the converse of the proposition "None but the brave deserve the fair," students have said with perfect seriousness: "The fair deserve none but the brave," or "No one ugly deserves the brave." The error in such cases arises from the fact that the student has omitted to put the given sentence into exact propositional form, as logic requires. If this were done the sentence would become :—"No one who is not-brave is deserving of the fair," and this is a simple E proposition, and may therefore be converted simply into "No one deserving of the fair is not-brave," or, expressed more conventionally, "No one deserving of the fair is a coward."

III. **Inferences of Permutation.**—Of this kind of immediate inference there are several forms :—

(*a*) By Obversion.—Here we infer a new proposition, having for its predicate the contradictory of the predicate, *e.g.* :—

Original proposition.—All X is Y.
Inference by Obversion.—No X is not-Y.

We may always obvert a proposition, if at the same time we change its quality. The rule of obversion is usually given thus: Substitute for the predicate term its contrapositive, and change the quality of the proposition.

Contrapositive is a mediæval word for the opposite of a term. Thus "not-A" is the contrapositive of "A." It is convenient to use this word so that "contradictory" may be used exclusively of propositions.

Thus, All X is Y	yields	No X is not-Y.
No X is Y	,,	All X is not-Y.
Some X is Y	,,	Some X is not not-Y.
Some X is not Y	,,	Some X is not-Y.

(*b*) By Contraposition.—In this case we infer a new proposition which has the contrapositive of the original predicate for its subject, and the original subject for its predicate, *e.g.* :—

Original proposition.—All X is Y.

Contrapositive.—No not-Y is X.

Immediate inference by contraposition is sometimes called the converse by contraposition.

From A, E and O of the propositional forms we may infer a contrapositive, but not from I.

	Original Proposition.	Contrapositive.
A	All X is Y.	No not-Y is X.
E	No X is Y.	Some not-Y is X.
O	Some X is not Y.	Some not-Y is X.

In drawing immediate inferences accuracy is all important. The exposition in this chapter has been illustrated by symbols, but if the principles have been duly grasped it will not be difficult to apply them to concrete examples. In doing so the student must always reduce the sentences given as examples to strict logical form, if they are not already in that condition. The great importance of this subject makes it advisable that several worked examples should be presented for the reader's consideration.

1. *What immediate inferences are derivable from the proposition "All really happy men are virtuous"?*

(*a*) The Truth of the Subaltern: "Some really happy men are virtuous."

(*b*) The Falsity of the Contradictory: "Some really happy men are not virtuous."

(*c*) The Falsity of the Contrary: "No really happy men are virtuous."

(*d*) By Conversion: "Some virtuous men are really happy."

(*e*) By Obversion: "No really happy men are not virtuous."

(*f*) By Contraposition: "No not-virtuous men are really happy."

2. *Give the Converse, the Obverse and the Contrapositive of the following propositions*:—(*a*) *The longest road comes to an end;* (*b*) *Unasked advice is seldom acceptable.* (Each of these propositions must first be reduced to logical form.)

(*a*) This sentence="The longest road is limited." This is a universal affirmative.

Its Converse is: "Some (one) limited thing is the longest road."

Its Obverse is: "The longest road is not unlimited."

Its Contrapositive is: "No unlimited thing is the longest road."

(*b*) This sentence="Some unasked advice is unacceptable." This is a particular affirmative proposition.

Its Converse is: "Amongst (some) unacceptable things is unasked advice."

Its Obverse is: "Some unasked advice is not acceptable."

The sentence being an I proposition it has no contrapositive.

3. *Convert and contraposit the proposition,* "*For every wrong there is a legal remedy.*"

The proposition reduced to logical form is: "Every wrong is capable of a legal remedy."

Its converse is: "Some things capable of legal remedy are wrongs."

Its contrapositive is: "Nothing incapable of legal remedy is a wrong."

4. *What deductions are possible from the proposition,* "*Amethysts are precious stones*"? (*N.B.*—"*Eduction*" *is a term frequently used for* "*Immediate inference.*")

The given proposition is a universal affirmative, "All amethysts are precious stones," and may be treated as the proposition in the first-worked example.

EXERCISES ON CHAPTER VIII.

1. Explain and illustrate by examples the difference between the converse and the contradictory of a proposition; and say when and under what conditions the converse of a proposition is or is not necessarily true.

2. Explain with illustrations, the difference between the contrary and the contradictory of a proposition.

3. Explain why a universal negative proposition admits of the conversion of its terms. "All equilateral triangles are equiangular." Say whether the terms of this proposition are convertible. If not, why not?

4. Give the converse, the contradictory and contrary of "All A is B"; "Some men are wise."

5. Give the contradictory and the converse of:—

(a) Two blacks don't make a white.

(b) James struck John.

(c) Three-fourths of the candidates passed.

6. Assign the logical relation between each of the following propositions with the proposition "All crystals are solids":—

(a) *Some crystals are solids.*

(b) *No crystals are not solids.*

(c) *Some solids are crystals.*

7. What is mediate inference? Give where possible the converse, the obverse and the contrapositive of:—

(a) (said Hudibras): *"I smell a rat."*

(b) *Where no oxen are, the crib is clean.*

(c) *Only protestant princes can occupy the English throne.*

8. What is opposition? Which of the forms of opposition has the greatest value and why?

CHAPTER IX.

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Mediate Inference.—The Syllogism.

—o—

Immediate inference is the derivation of a new proposition from some given proposition. However useful this exercise may be, the new proposition is always recognised as only a different way of expressing the original proposition. Mediate inference professes to give a conclusion of a much more fruitful kind. In every example of a mediate inference, two propositions, and two only, are implied. In these two propositions the conclusion to be drawn is potentially contained, and out of these two propositions the conclusion is actually drawn by reasoning. The two propositions given are called the *premisses of the conclusion.*

It will be seen afterwards that in ordinary discourse the two premisses and the conclusion are seldom fully expressed. One of the premisses is generally left to be understood, but, in spite of this, it is implied in the reasoning. When, however, the two premisses and the derived conclusion are fully and formally stated, the expression is called a *Syllogism.* Formal Logic assumes that, in every instance in which we draw a new and fruitful conclusion, the reasoning when fully expressed must take the form of a Syllogism.

A Syllogism, then, is a conclusion expressly evolved from two propositions called its premisses.

Each of the premisses of a syllogism must once have been a conclusion from two other more remote premisses, unless one of the premisses is the statement of a truth which is axiomatic in its nature. All that we know, inferentially, about the universe, is known in the form of a vast number of conclusions drawn from other premisses. Knowledge is thus a net-work of conclusions, suspended ultimately upon a few axiomatic or self-evident truths. All arguing implies that there are certain remote premisses or assumptions, bearing logically on all questions, and about which the disputants must be agreed.

It is worthy of observation that some persons, who are not acute reasoners, are yet able to see truth at a glance. Others are subtle and ready reasoners whose natural intuition (insight) is small. Argument and insight are often found in inverse ratio. It has been remarked that, generally, women are more strongly endowed with insight, and men with reasoning power.

The following is a simple form of a Syllogism:—

All men may be educated.
Savages are men. } (PREMISSES.)

Therefore, Savages may be educated. (CONCLUSION.)

Notice that in this example there are three propositions. Of these the first two are the premisses, and the last the conclusion. There are also three terms: "men," "savages" and "educated," and the last two of these appear in the conclusion. The term which forms the *predicate* of the conclusion ("educated") is called the **major term,** and the term which forms the *subject* of the conclusion ("savages") is called the **minor term.** The term which appears in both the premisses, but which does not appear in the conclusion ("men") is called the **middle term.** Further, the premiss which contains the major term ("All men may be *educated*") is called the **major premiss;** and the premiss which contains the minor term ("All *savages* are men") is called the **minor premiss.** These general definitions hold good for all kinds of syllogisms.

From these definitions it will be easy to see that a syllogism is the logical comparison of the two terms which appear in the conclusion, by means of a third, or middle, term.

Logic lays down three fundamental rules which apply to every variety of syllogism.

1. Each syllogism must have three, and only three, terms; it must have three, and only three, propositions.

2. Of the three terms thus involved in every syllogism, the middle term (*i.e.*, the term common to both premisses) must be taken universally (*i.e.*, it must be distributed), at least in one of the premisses; and neither of the other terms, *i.e.*, the major or the minor, can be taken universally in the conclusion, *unless* it was taken universally in the premiss in which it occurred.

3. No conclusion can legitimately be drawn if both the premisses are negative; or if both are particular; and, if *one* of the premisses is particular, the conclusion must be particular; or, if one of the premisses is negative, the conclusion must be negative.

Notes on the Canons, or Rules of the Syllogism.

Rule I. – We require to add that the terms must be used throughout in exactly the same sense. Owing to the ambiguity of words it sometimes happens that a syllogism will seem only to contain three terms when in reality there are four, *i.e.*, one of the terms has been used in two distinct senses. In the fallacy quoted in the early part of this work we have an example of this:

He who is most hungry eats most,
He who eats least is most hungry,
Therefore he who eats least eats most.

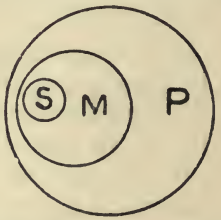
In this example a little reflection will show that terms are not being used throughout in the same sense, and that there are in reality more than three terms involved.

Rule 2.—The middle term must be once distributed, otherwise it cannot be a medium for comparing the other two terms. It must be either wholly in, or wholly out of one of the other terms before it can be the means of establishing a connection between them.

If we use a diagrammatic illustration of the Syllogism, the necessity of the distribution of the middle term is obvious.

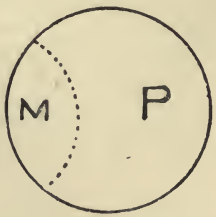
Thus let the Syllogism be

All M is P.
All S is M.
∴ All S is P.



Here M is the middle term, and it is shown to be wholly in P.

If, however, M were not wholly distributed we should have to represent it partially within P. Consequently, we should not be able to say whether S was contained in the part of M within P or in that part of M which is without P and of which we are not supposed to know anything.



A syllogism with an undistributed middle term is the most common form of erroneous reasoning.

A term must not be distributed in the conclusion that was not distributed in the premisses. Obviously, if an assertion is not made about the whole of a term in the premisses, we cannot make it of the whole of the term in the conclusion without going beyond what has been given. When this rule is broken in the case of the major term, it is called the *Illicit process of the Major*; and in the case of the minor term, *Illicit process of the Minor*. If we were to admit that a term might be taken universally in the conclusion, which was not so taken in the premisses, we should be admitting that the "part is greater than the whole."

Rule 3.—Two negatives cannot yield a conclusion. For two negative propositions are really a declaration that no connection exists between the major and minor term and the term by which they were to be compared—in other words there is no middle term, and no Syllogism can be formed with two negative premisses. That two particulars cannot give a valid conclusion, and that the conclusion follows the weakest premiss are corollaries from the previous rules.

The general rules of the Syllogism depend upon one great canon, *viz.*: "Two terms that logically agree with the same third term, must logically agree with each other; and two terms, one of which agrees while the other disagrees with the same third term must logically disagree with each other."

> The ultimate principle of reasoning thus defined, is expressed in its most general forms in the "*Dictum de omni et nullo*" of Aristotle. "Whatever is predicated affirmatively or negatively of any class, must, on pain of involving inconsistent (contradictory) thought, be predicated of whatever is contained under that class." Aristotle regarded this as the axiom on which all syllogistic inference is based.

Every conclusion drawn in a syllogism, where the above general rules have been observed, is an affirmative or negative proposition deduced by means of a minor (or applying) premiss

from a more general proposition that is assumed to be true, and in which the conclusion was virtually contained.

It can also be shown that there must be four, and need not be more than four syllogistic forms. For, the general (major) proposition, which virtually contains the conclusion must be universal (either A or E), and the applying (minor) premiss must bring either the logical whole, or a part only, of its subject into comparison with the middle term. The minor premiss, therefore, will be either A or I. The general rules of the syllogism decide the conclusion. Hence, we may say that every reasoning may be exhibited by one or other of the following combinations of the four propositional forms :—

A A A. A I I. E A E. E I O.

These letters, of course, tell us the quantity and quality of the two premisses, and the conclusion of the syllogism which each triplet forms. The arranging of the symbolic letters in different ways is called the *Mood* of the syllogism. Thus, A A A represents a syllogistic mood in which both the premisses and the conclusion are universal affirmatives. E I O represents a syllogistic mood in which the major premiss is a universal negative, the minor premiss a particular affirmative, and the conclusion a particular negative. The following are examples of four forms of syllogism :—

Mood A A A.

All men may be educated.

All savages are men.

∴ All savages may be educated.

All M is P. All S is M. ∴ All S is P.

[N.B.—This alone of all forms of syllogism gives a universal affirmative conclusion, and is, therefore, the one most convenient for expressing scientific reasonings with their universal affirmative conclusions.]

Mood A I I.

All educating influences are good.
Some difficulties are educating influences,
∴ Some difficulties are good.
All M is P. Some S is M. ∴ Some S is P.

Mood E A E.

No Europeans are cannibals,
All Englishmen are Europeans,
∴ No Englishmen are cannibals.
No M is P. All S is M. ∴ No S is P.

Mood E I O.

Whatever is followed by remorse is not desirable,
Some pleasures are followed by remorse,
∴ Some pleasures are not desirable.
No M is P. Some S is M. ∴ Some S is not P.

The student is advised at this point to transform some simple arguments from the form in which they are ordinarily used, into precise syllogistic form. Consider for example the following:—

1. "*There are no foreigners amongst the wounded, so no Frenchman received a wound.*"

Here we have given a major premiss and a conclusion. In order to express the statement in syllogistic form we must supply the minor premiss. The passage may then be written as a Syllogism in E A E:—

No foreigners are wounded
(All Frenchmen are foreigners)
∴ No Frenchmen are wounded.

2. "*No war is long popular; for every war increases taxation: and the popularity of anything that touches the pocket is short lived.*"

This may be written as a Syllogism in E A E thus:—

Nothing that increases taxation is long popular.
Every war increases taxation.
∴ No war is long popular.

3. "*For some wars there has been no justification; for they have been harmfully aggressive, and such aggression is without excuse.*"

This may be expressed as a Syllogism in E I O, thus:—

No harmful aggression is justifiable,
Some wars are harmfully aggressive,
∴ Some wars are not justifiable.

EXERCISES ON CHAPTER IX.

1. What is understood by a proposition, a premiss, a conclusion, and a syllogism? Give an example of each.

2. "From negative premisses you can infer nothing." Explain and illustrate this statement.

3. Show how logical form as displayed in the syllogism tends to clearness of thought.

4. Give a clear explanation of the rule concerning the middle term of a syllogism.

5. Enumerate the cases in which no valid conclusion can be drawn from two premisses.

6. Supply a premiss that will make the following reasoning correct: "There is no Englishman among the wounded, so no officer can have received a wound."

7. Put the following argument into syllogistic form:—"How can anyone maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good?"

CHAPTER X.

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The Figures of the Syllogism.

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In one or other of these four syllogistic forms all our reasonings might be expressed, just as all our judgments could be expressed in one or other of the propositional forms. But Logic takes cognisance of many other syllogistic forms besides these four. The question may suggest itself—why should we add to these four forms, if they are sufficient for the unabridged expression of all sorts of reasonings? The answer is that the addition is one of practical convenience. It will be found that many of the concrete reasonings of ordinary life, though capable of being expressed in one of the four syllogistic forms, yet find a more convenient and natural expression in one or other of the additional forms. The way in which the additional syllogistic forms are obtained is by *varying the position of the terms in the premisses*. The four syllogistic forms already considered have certain features in common. Thus, in each case the middle term is the subject of the major premiss and the predicate of the minor premiss. Also, the middle term is distributed in the major premiss but not in the minor premiss. On account of this similarity the four syllogistic forms already given are classed together, and constitute what is known as Figure I. of the syllogism.

But we can frame a series of syllogisms which violate none of the general rules of the syllogisms, in which the relations of Figure I. are varied. Thus we may have *the middle term as the predicate of each proposition.* In these cases the middle term will always be of greater logical extent than either of the other two. The syllogisms which exhibit these characteristics are classed together as FIGURE II.

Just as there were four valid moods under Figure I., so there are four valid moods under Figure II. The student should construct concrete illustrations by reference to the following abstract examples of each of the moods of Figure II.:—

A E E.	A O O.	E A E.	E I O.
All P is M.	All P is M.	No P is M.	No P is M.
No S is M.	Some S is not M.	All S is M.	Some S is M.
No S is P.	Some S is not P.	No S is P.	Some S is not P.

Notice that in Figure II. *the conclusion in each mood is a negative one.* Hence, this figure is the most convenient for expressing argumentative objections and refutations.

When *the middle term is made the subject* of each premiss, and is, therefore, of less logical extent than the other two terms, we get a series of syllogisms which are grouped together as forming FIGURE III. But for reasons that will afterwards appear, we can form six valid moods of this figure. Thus:—

A A I.	A I I.	E A O
All M is P.	All M is P.	No M is P.
All M is S.	Some M is S.	All M is S.
Some S is P.	Some S is P.	Some S is not P.

E I O.	I A I.	O A O.
No M is P.	Some M is P.	Some M is not P.
Some M is S.	All M is S.	All M is S.
Some S is not P.	Some S is P.	Some S is not P.

Notice *that a particular conclusion only is obtained in each mood of Figure III.* Hence this mood is well fitted for propounding examples argumentatively, or for establishing some particular or indefinite conclusion.

There is yet a further group of syllogisms, known as FIGURE IV, in which the middle term is the predicate of the major premiss, and the subject of the minor. This figure has five moods, *viz* :—

A A I.	A E E.	E A O.
All P is M.	All P is M.	No P is M.
All M is S.	No M is S.	All M is S.
Some S is P.	No S is P.	Some S is not P.

I A I.	E I O.
Some P is M.	No P is M.
All M is S.	Some M is S.
Some S is P.	Some S is not P.

The fourth figure is *clumsy and unnatural, and is omitted altogether by many logicians.* It is worth while to notice the following results, obtained from a comparison of the conclusions in the various moods of the four figures :—

A (universal affirmative) conclusions can only be obtained in one figure and in one mood of that figure.

E (universal negative) conclusions can be obtained in three figures or four moods.

I (particular affirmative) conclusions can be obtained in three figures or in six moods.

O (particular negative) conclusions can be obtained in each of the four figures or in eight moods.

From this it follows that A conclusions are the most difficult to establish, and the easiest to overthrow. O conclusions, on the other hand, are the easiest to argue for, but the hardest to

disprove. Or, more generally, universal and definite conclusions are most easily overthrown, and particular and indefinite conclusions are most easily maintained.

Special Rules of the Figures of the Syllogisms.

In addition to the general rules to which all syllogisms must conform, logicians have *deduced* certain simple rules applicable to the different Figures.

In the First Figure,—

(*a*) The major premiss must be universal.

(*b*) The minor premiss must be affirmative.

In the Second Figure,—

(*a*) The major premiss must be universal.

(*b*) One premiss and the conclusion must be negative.

In the Third Figure,—

(*a*) The minor premiss must be affirmative.

(*b*) The conclusion must be particular.

In the Fourth Figure,—

(*a*) When the major premiss is affirmative, the minor premiss must be universal.

(*b*) When the minor premiss is affirmative, the conclusion must be particular.

(*c*) In negative moods, the major premiss must be universal.

These special rules of the figures do not introduce new material, they are only a concise statement deduced from results previously obtained.

EXERCISES ON CHAPTER X.

1. What are the figures of the Syllogism? Examine whether I A I, E I O are valid or invalid in each of the figures.

2. Which figure is most convenient (1) for overthrowing an adversary's conclusion; (2) for establishing a negative conclusion; (3) for proving a universal truth.

3. Give the special rules of the Figures.

4. Express the following argument by a Syllogism of the third figure:—Some things which have a practical worth are also of theoretical value: for every science has a theoretical as well as a practical value.

5. What moods are good in the first figure and faulty in the second, and vice versâ? Why are they excluded in one figure and not in the other?

6. From which syllogisms can you infer universal, particular, negative inferences, or none at all?

7. Enumerate briefly the conditions of a valid deduction.

8. Construct a Syllogism in I A I to prove that some taxation is necessary.

CHAPTER XI.

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The Reduction of Syllogisms.

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It has already been observed that all reasonings may find their expression in one of the four moods of Figure I. The other figures are often convenient for special purposes, but inasmuch as the first figure is considered the most direct and perfect mode of expressing our reasoning, Logic shows how any syllogism of Figures II., III., and IV. (called the indirect figures) may be transformed into one of the moods of the first figure. *This process is called the Reduction of Syllogisms.* There are fifteen moods altogether in the three indirect figures, and thirteen of them may be reduced: (1) by the conversion of one or more of the three propositions in the syllogism to be reduced, (2) by the transposition of the premisses, or (3) by both of these processes.

[A O O, Figure II. and O A O, Figure III. are exceptions and will be considered separately.]

The reduction of the syllogisms of the indirect figures into direct or first figure syllogisms is one of the most profitable exercises in formal Logic. The process is not nearly as easy as might appear. To ensure accuracy and rapidity in the process an ingenious mnemonic has been used by logicians for more than 500 years. This mnemonic has been called "the magic verse of Logic," and certainly the words of which it is

composed are more full of meaning than any similar combination ever made. The usual form of the mnemonic, which must be learnt by heart, is as follows :—

Barbara, *Celarent*, *Darii*, *Ferio*que, prioris,
Cesare, *Camestres*, *Festino*, *Baroko*, secundae,
Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
Bokardo, *Ferison*, habet, Quarta insuper addit,
Bramantip, *Camenes* *Dimaris*, *Fesapo*, *Fresison*.

[The words in italics are the significant words, the others being only connectives.]

The following is the key to this famous mnemonic. Every mood in each of the four figures is represented by a different word. In the case of the indirect figures (II., III. and IV.), the mnemonic tells us to what mood of the first figure the various moods of these indirect figures are to be reduced. It gives us, also, full information as to *how* the reduction is to be performed.

1. The **vowels** in each word give the quantity and quality of the syllogism which the word represents. Thus B*a*rb*a*r*a*=a syllogism of Figure I., mood A A A. C*e*s*a*r*e*=a syllogism of Figure II., mood E A E.

2. The **initial** letters of the words in Figures II., III. and IV. tell us that a syllogism, represented by a word with that initial letter, may be reduced to the syllogism of the first figure, which is represented by a word having the same initial letter. Thus, the syllogism of the fourth figure, represented by the word *C*amenes, may be reduced to the syllogism of the first figure, represented by the word *C*elarent.

3. The **letter "s"** occurring in a word performs a **double function.** If it occurs in the *middle* of a word as in Cesare it means that, in the process of reduction, the proposition represented by the previous vowel is to be simply con-

verted. Thus, in reducing Cesare (Figure II.) to Celarent (Figure I.) the major premiss must be simply converted:—

$$\text{Cesare}\left\{\begin{array}{l}\text{No P is M}\\ \text{All S is M}\\ \text{No S is P}\end{array}\right\}=\text{Celarent}\left\{\begin{array}{l}\text{No M is P.}\\ \text{All S is M.}\\ \text{No S is P.}\end{array}\right.$$

When "s" occurs at the *end* of a word, it tells that the conclusion of the NEW syllogism requires to be converted in order to get the conclusion in the form given in the original syllogism.

4. When the letter "p" occurs in the MIDDLE of a word, it tells us that in the process of reduction the preceding proposition is to be converted *per accidens* (limitation). Thus, in reducing Darapti of Figure III. to Darii, Figure I., the minor premiss must be converted "per limitation."

$$\text{Darapti}\left\{\begin{array}{l}\text{All M is P}\\ \text{All M is S}\\ \text{Some S is P}\end{array}\right\}=\text{Darii}\left\{\begin{array}{l}\text{All M is P.}\\ \text{Some S is M.}\\ \text{Some S is P.}\end{array}\right.$$

When "p" occurs at the END of a word it signifies that the conclusion of the *new* syllogism must be converted per limitation in order that the new conclusion may appear in the same form as the conclusion in the original syllogism.

5. When the letter "m" occurs in a word it tells us that the premisses will require transposition in the process of reduction, *i.e.*, the minor will become the major. Thus, in reducing Ca*m*estres (Figure II.) to Celarent (Figure I.) the major and minor premisses exchange places:—

$$\text{Camestres}\left\{\begin{array}{l}\text{All P is M.*}\\ \text{No S is M.†}\\ \text{No S is P.}\end{array}\right\}=\text{Celarent}\left\{\begin{array}{l}\text{†No M is S.}\\ \text{*All P is M.}\\ \text{No P is S.}\end{array}\right.$$

This reduction also illustrates the use of the final "s" in Camestres. For in Celarent we have "No P is S" as the conclusion, and by applying the meaning of the final "s," we convert the conclusion and so obtain "No S is

P," which is the form of the conclusion in the original Camestres proposition.

6. There is still the significant letter "k" to be considered. It occurs in Baroko and Bokardo. It will be remembered that the moods A O O (Figure II.) and O A O (Figure III.) were reserved for exceptional treatment. At the time when these mnemonic lines were constructed, contrapositives were not recognised. In consequence of this, a somewhat roundabout method had to be employed in reducing syllogisms in Baroko and Bokardo. The process is known as *Reductio ad Absurdum*—a process quite familiar to students of Euclid. Suppose an argument in Baroko is proceeding. The two disputants agree about the premisses A and O in Figure II., *i.e.*:—

All P is M,
Some S is not M,

but one of the disputants will not accept the conclusion, Some S is not P. How, then, shall we show that the conclusion is the only valid one? We may say that if the conclusion, "Some S is not P," is incorrect, then its contradictory assertion must be correct, *viz.*, All S is P. We will assume, for the sake of argument, that All S is P. We had previously agreed that All P is M. Combining therefore, these two,

All P is M,
All S is P,

we draw the conclusion that All S is M.

But we agreed in our original premisses that Some S is not M. Therefore, the conclusion reached is absurd and impossible. When we convince an opponent in this way, by showing he cannot admit the premisses and deny the conclusion without contradicting himself, we are said to use the *Reductio ad Absurdum*.

The letter "k," therefore, tells us when this method is to be used. The position of the letter indicates that in

this process of *Reductio ad Absurdum*, the first step is to omit the premiss preceding it, and substitute in its place the contradictory of the conclusion. We then obtain two premisses in the corresponding mood of Figure I., which yield a conclusion contradicting the premiss omitted. But, since two contradictories cannot both be true, and since the truth of the original minor was granted, we reject the new conclusion and infer the truth of the original conclusion.

It may seem that this method of Reduction has no connection with those methods which converted syllogisms of the indirect figures to corresponding syllogisms in the first figure. The aim is, however, the same. The reason for reducing syllogisms to the first figure is, that the reasoning may be the more clearly seen, and that the conclusion may be vindicated. In the Reductio ad Absurdum the aim is also to vindicate the conclusion, but in a different manner. Although, in the mnemonic lines, this indirect method of vindication is contemplated only in the case of A O O and O A O propositions, it can be used in others if desired.

EXERCISES ON CHAPTER XI.

1. What is reduction? Say briefly what purpose the process is supposed to serve.

2. Construct an argument in Fresison and reduce it to Figure I.

3. In what moods and figures are the following syllogisms? Reduce them.

(*a*) *The nervous fluid will not travel along a tied nerve;*
Electricity will travel along a tied nerve;
Therefore electricity is not the nervous fluid.

(*b*) *No men are birds;*
All birds are animals;
Therefore some animals are not men.

4. Vindicate the truth of the following argument in A O O by Reductio ad Absurdum. "*Some successful persons are not industrious thinkers; for every industrious thinker is educated, but some successful persons are not educated.*

CHAPTER XII.

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Irregular and Compound Syllogisms.

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In ordinary arguments it is seldom necessary, for practical purposes, that both the premisses should be expressly stated. One or other of the premisses is frequently suppressed. The following, for example, is a method of argument often used:—

"That man is contemptible, for he is a coward."

In this case the major premiss, "All cowards are contemptible," has been taken for granted.

An **Enthymene** is a syllogism imcompletely stated, *i.e.*, one of the three propositions forming the syllogism is taken for granted, but not expressed. If the major premiss is omitted, the enthymene is said to be of the *first order*; if the minor premiss is omitted, it is said to be of the *second order*; and if the conclusion is omitted, it is said to be of the *third order*.

"That man is unhappy, for he is a miser." (*First order.*)

"All misers are unhappy, therefore that man is unhappy.' (*Second order.*)

"All misers are unhappy, and that man is a miser." (*Third order.*)

The above are examples of enthymenes of the three orders, the full syllogism being

All misers are unhappy,
That man is a miser,
∴ That man is unhappy.

An ENTHYMENE frequently occurs in a very terse form. Thus, "He must be mad to attempt that," is an enthymene. The premisses are "All who attempt that are mad; He is one who attempts that; Therefore, he is mad."

It has been remarked that all our thought consists of a chain or net work of premisses and conclusions, each premiss being really a conclusion drawn from previous premisses. Logic provides a nomenclature for the chains of reasoning of which our thought consists. Thus, a **Prosyllogism** is a syllogism the conclusion of which is used as a premiss in a succeeding syllogism. An **Episyllogism** is a syllogism of which one or both of the premisses are conclusions from preceding syllogisms.

The union of a prosyllogism with an episyllogism is called a **Polysyllogism.** Thus:—

Prosyllogism. { All M is P.
All S is M.
∴ All S is P. }
But All X is S. } **Episyllogism.***
∴ All X is P.

An **Epicheirema** is a polysyllogism in which the prosyllogism is only briefly stated, after the manner of an enthymene. Thus:—

All S is P, *because it is M.*
All X is S.
∴ All X is P.

A **Sorites** is a series of propositions, inferentially connected, in which the predicate of each is the subject of the next, and so on indefinitely. The conclusion is formed of the first subject and the last predicate. Thus:—All A is B, All B is C, All C is D, All D is E; therefore, All A is E.

In a Sorites there are really as many syllogisms as there are intermediate propositions between the first premiss and the conclusion. The example just given may be exhibited as the combination of three simple syllogisms. Thus:—

All A is B.	All A is C.	All A is D.
All B is C.	All C is D.	All D is E.
∴ All A is C.	∴ All A is D.	∴ All A is E.

* Of course, the chain may be continued indefinitely

In political speeches the Sorites is a frequent mode of argument. Take the following extract: "Free-trade is a great boon to the working man, for it increases trade and thus cheapens articles of ordinary consumption; this gives a greater purchasing power to money, which is equivalent to a rise in real wages; and any rise in real wages is a boon to the working man." This can be exhibited as a Sorites, but, of course, each of its general propositions must first be expressed in precise logical form. The passage will then be found to consist of the following propositions, joined together after the manner of a Sorites:—Free-trade is trade-increasing; every increase of trade is price-lowering; every fall in prices is money-value-raising; every rise of money-value is real-wage-raising; every rise in real wages is advantageous to working men; therefore, free-trade is advantageous to working-men.

There are two special rules of the valid **Sorites**:—

1. Only one premiss may be negative; and, if one premiss is negative, it must be the last one.

2. Only one premiss may be particular; and, if one premiss is particular, it must be the first one.

In the following argument in the form of a Sorites, there is a breach of the second rule, and consequently an invalid inference. "All thieves are dishonest; all dishonest persons are immoral; and some immoral persons go unpunished; therefore, some thieves go unpunished."

EXERCISES ON CHAPTER XII.

1. What kind of argument is the following:—

"Those who have shall not receive; those who do not receive do not want." Is the argument valid?

2. Define Prosyllogism and Episyllogism; and say of what genus of reasoning they are species.

3. Construct a valid Sorites argument with a negative premiss.

CHAPTER XIII.

Conditional Syllogisms.

In all the various kinds of argument that have so far been considered, the propositions employed have been categorical or unconditional. But in many of the concrete reasonings of our ordinary intellectual life, we are obliged to use general statements, to which some condition is attached. In some connection or other we are constantly using conjunctions, such as "if," "either," "whenever," etc., and this frequent use testifies to the number of conditional propositions and arguments in daily use. When a statement to which a condition is attached enters into an argument, the reasoning seems to turn on the condition. Hence, an amount of complexity is introduced into the argument. Logic recognises the fact that conditional arguments must have a place in our inferential thought, and exhibits the inner relation of such by the forms of conditional syllogism.

A Conditional Syllogism is one in which the major premiss, and that only, is a conditional proposition, and in which, accordingly, the reasoning seems to turn on the condition. We shall notice three forms of the conditional syllogism, *viz.*, (I.) the Conjunctive (or Hypothetical), (II.) the Disjunctive (or Alternative), and (III.) the Dilemma.

I. The Conjunctive or Hypothetical Syllogism.—

If A is B, C is D.
A is B.
∴ C is D. } *Modus ponens.*

If A is B, C is D.
C is not D.
∴ A is not B. } *Modus tollens.*

The first proposition in these examples is a complex proposition formed of two propositions, related in such a way that the truth of the one follows necessarily from the truth of the other. When two propositions are related in this manner, they are technically known as "the ANTECEDENT" and "the CONSEQUENT."

When two propositions are related as antecedent and consequent, the truth of the consequent follows from the truth of the antecedent; whilst the denial of the consequent is virtually the denial of the antecedent. *This is known as the Law of Antecedent and Consequent.*

From the statement of this law we may deduce TWO MOST IMPORTANT COROLLARIES, *viz.*:—

1. The affirmation of the consequent does not justify the affirmation of the antecedent. Granted that If A is B, C is D, we may not argue that because C is D, therefore A is B.

2. The denial of the antecedent does not justify the denial of the consequent. Granted that If A is B, C is D, we may not argue that because A is not B, therefore C is not D.

Applying these observations to the hypothetical proposition, "If rain has fallen, the grass is wet," consider what inference could be drawn (*a*) from the affirmation of the antecedent, (*b*) from the affirmation of the consequent, (*c*) from

the denial of the antecedent, (*d*) from the denial of the consequent.

(*a*) Affirmation of the antecedent, "Rain has fallen," yields a valid conclusion, "The grass is wet."

(*b*) Affirmation of the consequent, "The grass is wet," yields no conclusion.

Corollary (1) forbids us to conclude, "Rain has fallen."

(*c*) Denial of antecedent, "Rain has not fallen," yields no conclusion.

Corollary (2) forbids us to conclude, "The grass is not wet."

(*d*) Denial of the consequent, "The grass is not wet," yields a valid conclusion, "Rain has not fallen."

II. **The Disjunctive Syllogism** is one in which the major premiss, and that only, is a disjunctive proposition.

A is either B or C.
A is not B.
∴ A is C.

The principle which governs the reasoning here is that of the excluded middle. If we can assume that the disjunctive in the reasoning is exhaustive, *i.e.*, A is either B or C and cannot be anything else, then we may vary the general form of the disjunctive syllogism, thus:—

A is either B or C.
A is B.
∴ A is not C.

But great care must be taken in concrete reasoning that the assumption here involved is warranted. Consider the following example:—

Either the witness tells a lie or the prisoner is guilty.
The witness tells a lie.
∴ The prisoner is not guilty.

This is not a valid argument, for the disjunction in the major premiss is not exhaustive. For there are other alternatives—the witness may tell a lie, and the prisoner be guilty all the same.

III. The Dilemma.—The dilemma is a mode of reasoning designed to show the absurdity of the logical position of an opponent. It is a syllogism which has for its major premiss a hypothetical conjunctive proposition having more than one antecedent. For its minor premiss it has a disjunctive proposition. It thus offers an opponent a choice of alternatives, and the choice of either alternative leads to a conclusion which the opponent does not like. The dilemma is expressed in three principal forms: (1) SIMPLE CONSTRUCTIVE, (2) COMPLEX CONSTRUCTIVE, (3) DESTUCTIVE; of which forms the following are examples:—

1. SIMPLE CONSTRUCTIVE:—

If A is B, or if E is F, then C is D.
But either A is B or E is F:
Therefore C is D.

As a concrete example we may imagine the inhabitants of a town, against which a hostile army is approaching, arguing as follows:—"If we are bombarded we shall suffer loss, and if we surrender we shall suffer loss: but we must either surrender or be bombarded; therefore, in any case, we shall suffer loss."

2. COMPLEX CONSTRUCTIVE.—

If A is B, C is D; and, if E is F, G is H.
But either A is B or E is F:
Therefore, C is D or G is H.

A man in an upper room of a burning house, when the staircase has been destroyed, might use this form of argument in reasoning: "If I jump through the window I shall break my neck; if I remain here I shall be burnt to death; but I must do one or the other; therefore, in either case I must die.

3. Destructive Dilemma. — This dilemma in its commoner forms proceeds upon the denial of the consequent, as involving the denial of the antecedent. In the major proposition we obtain the admission that, if a certain thing holds good, it must be followed by one or other consequence. In the minor proposition we show that neither of these consequences follows, and so conclude that the antecedent is false, *e.g.* :—

If A is B, either C is D or E is F.
But neither C is D nor E is F:
Therefore, A is not B.

The dilemma has been known from time immemorial as the "horned syllogism," because in the major proposition the alternatives *assumed** to be exhaustive are opposed like "horns" to the opponent's position. The opponent's assertion is, in the minor proposition, thrown off each "horn," and finally rejected in the conclusion.

* The student should notice the expression "*assumed to be exhaustive.*" Dilemmatic arguments are often fallacious because all possible alternatives have not been exhausted. The fallacy in this is exposed by the construction of another dilemma equally to the point, but which gives an opposite conclusion. This method is called "rebutting a dilemma." Thus the complex constructive dilemma as given above might be rebutted thus:—

If A is B then C is not D or if E is F then G is not H.
But either A is B or E is F.
Therefore either C is not D or G is not H.

Compare this with the following dilemma and its corresponding rebutting dilemma. A mother is advising her son not to enter public life and argues thus:—"If you act justly men will hate you, if you act unjustly the Gods will hate you; but you must either act justly or unjustly; therefore public life must lead to your being hated." The son replies by a rebutting dilemma:—"If I act justly the Gods will love me, and if I act unjustly men will love me; therefore in either case I shall be beloved."

EXERCISES ON CHAPTER XIII.

1. Assuming the truth of the statement that, (*a*) *if A is B, C is D, say what inference, if any, can be drawn from each of the following further statements:* (*b*) *but A is B;* (*c*) *but A is not B;* (*d*) *but C is D;* (*e*) *but C is not D. If, in any of the cases, no inference can be drawn, give the reasons. Illustrate your answer by examples.*

2. What is meant by a disjunctive syllogism, and what conclusion does such a syllogism yield?

3. Express the following in the form of a dilemma:—Examinations are either needless or useless: for, if students are industrious, they are needless; and, if students are idle, they are useless.

4. Show that denying the antecedent or granting the consequent of a condition involves logical fault, if the argument be expressed in syllogistic form.

5. Examine the following dictum of the Caliph Omar, addressed to the custodians of the Alexandrian Library in 640 A.D.:—

> *"If your books are in conformity with the Koran, they are superfluous; if they are at variance with it they are pernicious. But, they must either be in conformity with the Koran, or at variance with it. Therefore, they are either superfluous or pernicious."*

CHAPTER XIV.

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Fallacies of Deduction.

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The study of logical forms, besides being a useful mental discipline, supplies a ready test for the detection of fallacies. Indeed, formal logic may be said to exist as a practical study for this purpose. The syllogistic forms may be regarded as a framework in which all our concrete reasonings may be unfolded or displayed, and one by which their weak points may be more readily discovered.

A Fallacy is a reasoning apparently correct, which, nevertheless, involves inconsistency in inferential thought. The conclusion appears to follow from the premisses, but in reality it does not. We do not class palpable, downright blunders as fallacies. A fallacy is an error so wrapped up in words that the mistake is not at once perceived, and thus tends to produce conviction. Hence, the work of defining and exemplifying the different kinds of fallacies is in one respect the chief end of the science. But fallacious reasoning is so diverse that it is impossible to exemplify every variety of it. Nor is it possible, sometimes, to decide to what class a given fallacy ought to be referred. For fallacies, like consistent reasonings, are mostly expressed elliptically, and it is not always clear what the unabridged reasoning is supposed to be. Thus, when a person argues "that a country is ill-governed, because misery prevails there," the unabridged syllogism may take two different forms

neither of which is correct. (1) It may have for its omitted premiss, "All miserable countries are ill-governed," which no reasonable opponent would admit; or, (2) the omitted premiss may be "Every ill-governed country is miserable," in which case the conclusion is invalid, for the middle term has not been distributed in either premiss. Again we do not consider wilful attempts to deceive as fallacies. To such attempts we apply a stronger term. When people who know the truth but suppress it by suggesting a wrong explanation (*suppressio veri et suggestio falsi*), this is moral not logical error. In short, a dishonest intention will evade all rules of Logic.

Ordinary common sense is competent to expose most fallacious reasonings by its own sagacity. But it not infrequently happens that common sense is aware of a fallacy in the course of argumentation, without being able to say exactly what is wrong. Arguments are felt to be wrong, but those unskilled in logical science are puzzled how to demonstrate the error or how to refute the fallacy. Logic supplies the needful help to enable students to localise and expose the error. It makes the student familiar with the common form of unsound inference. It keeps the attention fixed on the essential steps of all valid reasoning. It accustoms the student to mark accurately the exact meaning of terms used, and the relation of these terms to one another. And it shows the necessity of defining with precision the question in dispute. After a course of discipline like this, the mind forms a spontaneous habit of accurate judgment and self-consistent thought and reasoning.

Fallacies are usually divided into two classes:—

1. INTERNAL FALLACIES, where the unsound element appears in the mode of expression. These are called fallacies "*in dictione.*"

All internal fallacies may be detected even by those who are ignorant of the *matter* to which the reasoning relates.

Internal fallacies are subdivided into

(i.) **Purely formal fallacies,** which are a breach of one or other of the rules of Logic.

(ii.) **Verbal fallacies,** in which the error lies in some ambiguity in the words used.

(i.) **Purely formal fallacies** are breaches of one or other of the rules governing mediate and immediate inference. All that is needed here is to remind the student of the most obvious pitfalls, *viz.*:—

(*a*) Confusion of contradictory with contrary opposition of propositions.

(*b*) Simple conversion of *A* propositions.

(*c*) Syllogisms with an undistributed middle.

(*d*) Illicit process of the major or minor (see page 65).

(*e*) Arguing from two negative or two particular premisses.

(*f*) Neglect of the rules governing conditional syllogisms.

(ii.) **Verbal fallacies.** These are often mere quibbles. The following are the chief varieties of verbal fallacies:—

(*a*) *Ambiguity of a word* (*equivocation*).—A word is sometimes used in a different sense in the two propositions of a syllogism in which it occurs, *e.g.*, "Light is always cheering; some afflictions are light; therefore some afflictions are cheering." Obviously, the middle term "light" is used in a double sense, and there are four terms used instead of three.

(*b*) *Ambiguity in the grammatical structure of a sentence (amphibology)*, *e.g.*, "Twice two and three." This is ambiguous, for the answer may be either seven or ten. "What he was beaten with was what I saw him beaten with. I saw him beaten with my eye. Therefore he was beaten with my eye."

(*c*) *Composition.* This is the confusion of a universal with a collective term. When we assert something of

each and every member of a class, we may infer the same of the whole class. When we say that all the angles of a triangle are less than two right-angles, we use the word "all" distributively; but, when "all" is used collectively the sentence is incorrect. We could not say that "all the angles of a triangle *taken together* are less than two right-angles.

(*d*) *Division.* This fallacy is the converse of the fallacy of composition. What is said collectively may not be said of the various individuals included in the collective term. All the angles of a triangle *taken together*, are equal to two right-angles, but no individual angle of a triangle is equal to two right-angles.

(*e*) *Fallacy of accent.* This arises from the accent or emphasis being thrown on the wrong word in a sentence, *e.g.*, "And he said 'saddle me the ass'; and they saddled *him*."

2. External Fallacies.—The error here can only be recognised by those who are conversant with the matter about which the statement is made. These are said to be fallacies "*extra dictionem.*" It is not easy to give simple examples of them. When the wrongful argument is stated in simple language, the error is easily seen. But, when the error is diluted over a speech of an hour's length, it is more difficult to detect it. The following are the chief varieties of external fallacies :—

i. **Many Questions** (*plurium interrogationum*). This fallacy is committed when several questions are so combined into one, that, if you answer "yes" or "no," you are committed to something more than your real meaning. A man asks: "Have you ceased ill using your mother?" You would not care to answer "yes" or "no." Sometimes in a court of law, questions of this kind are asked, and a plain answer "yes" or "no" demanded. Such

questions should be at once broken up into their several parts and each part answered singly.

ii. **Fallacy of the Consequent,** better known by the familiar phrase "*non sequitur.*" This is the general name given to loose and pretended arguments, where there is no connection between the premisses and the conclusion.

iii. **The Fallacy of Accident** (*A dicto simpliciter ad dictum secundum quid*) is committed when we argue from a particular case. Thus, "To take interest upon a loan is just, therefore I do right to exact it from my own father in distress." The answer obviously is, "Circumstances alter cases." The converse of this fallacy is called "*A dicto secundum quid ad dictum simpliciter.*" In this case a statement is made in a certain sense, and then used in quite another. Thus, "I eat to-day what I bought yesterday. I bought raw meat yesterday, therefore raw meat is eaten to-day." Here the accidental qualification of "rawness" is added, whereas in the original premise the assertion is made without regard to any such accidental qualification.

iv. **The Fallacy of False Cause** (*post hoc, propter hoc*), where it is assumed that because one event follows another, the former event is the cause of the later. This is a purely inductive fallacy, and will be considered later.

v. **Irrelevant Conclusion** (*ignoratio elenchi*). This is a most important type of deductive fallacy. The name covers all those cases in which a conclusion is proved, which is really *not* the point in dispute, but which sufficiently resembles what was required to be proved, to be often mistaken for it. Scarcely any fallacy is so common or so dangerous as this. Arguing beside the point, distracting attention by irrelevant considerations, is as frequent as it is misleading. The incoherence of the *ignoratio elenchi* lies between the conclusion offered *and* the proper answer

to the question, but involves no breach of the rules of the syllogism. There are four varieties of this fallacy which should be noticed.

(*a*) The *argumentum ad hominem*. This is confusion as to what the point at issue really is. Thus, if a new law is proposed, it is no proper argument to urge that the proposer is not the right person to bring the question forward. When we have advice given to us, it is not logic to retort that the preacher should practise what he preaches. If a man is accused of a crime, it is not relevant to assert that the accuser is just as bad. In all such cases, the argument proceeds *not* upon the merits of the case, but upon the character of the persons engaged in it.

(*b*) *Fallacies of objections*.—We commit this fallacy when we argue that a proposal should be rejected because it is open to objections. Such an argument is always a fallacy, if the alternative can be shown to be open to greater objections or difficulties.

(*c*) *Argumentum ad verecundiam*.—This is an appeal to our respect for ancient or established authority. The fallacy lies in the assumption that whatever is old or well-established must *ipso facto* be good.

(*d*) *The argument in support of a change* is the opposite of (*c*). The fallacy here is the implication that all change is progress, whereas the contemplated change may occasion more or greater evils than would follow if no change were made.

vi. **The Surreptitious Assumption** (*Petitio principii*).—Every example of deductive reasoning starts from some general principle (major premiss) about which the disputants are assumed to be agreed. If one disputant adopts as his premiss a statement which the other disputant does not accept, the question at issue remains unsettled and no conclusion can be drawn between them. "Begging the

question" and "arguing in a circle" are familiar forms of the *petitio principii*. He who argues in a circle assumes the truth of his major premiss and by means of it reaches a conclusion, which he afterwards uses to establish the major premiss with which he started. Thus, an illogical divine might argue: "We know that there is a God, because the Bible tells us so; and we know that the Bible is true, because it is the Word of God." People are especially liable to fall into this fallacy when they use a mixture of English and classical words in the same reasoning. For they often seem to be proving one question by another which is identical with it, only expressed in words derived from another language; *e.g.*:—

> "Consciousness is the immediate knowledge of an object; for I cannot be said to know a thing unless my mind has been affected by the thing itself."

The detection of fallacies is such an important branch of logical study that a few typical fallacies are appended, with hints as to their solution.

1. *Examine the following:*—"*Every bird comes from an egg; every egg comes from a bird; therefore, every egg comes from an egg.*"

The premisses written in logical form are: "Every bird is an egg-product; every egg is a bird-product," *i.e.*, there are four terms, whereas a correct syllogism can have only three.

For exercise, test the following in the manner above indicated: "Knowledge is power; consequently, since power is desirable, knowledge is desirable."

2. *Is the following a valid argument?*—"*To assault another is wrong; consequently, a soldier who assaults another does wrong.*"

This is the **fallacy of accidents.** A soldier is a man with an accidental qualification, and we cannot argue from a general to a special in such a case.

Examine in the same way: "Intoxicants act as a poison to a drunkard, and everyone should avoid poison."

3. *Examine the following:*—"*He who is most hungry eats most; he who eats least is most hungry; therefore, he who eats least eats most.*"

This is the **fallacy of accidents**; "eats most" is taken generally in the conclusion, but specifically in the premiss.

4. *Examine:*—"*If Jack is a good boy he will do as he is told; he is a good boy (for, if he will do as he is told, he is a good boy); therefore, he will do as he is told.*"

A **petitio principii**—arguing in a circle.

5. *Examine:*—"*The sea was the place where the incidents of my tale happened; there is the sea; therefore, my story is true.*"

An **ignoratio elenchi.**

6. *Examine:*—"*A dog chases a tortoise: the tortoise has a hundred yards start, but the dog runs ten yards to every one run by the tortoise. When the dog has run a hundred yards the tortoise will be ten yards ahead; when the dog has covered these ten yards, the tortoise will be one yard ahead; when the dog has covered this one yard, the tortoise will be* $\frac{1}{10}$*th of a yard ahead, and so on. The tortoise will be always ahead and the dog will never overtake it.*"

This is an ancient specimen of an **ignoratio elenchi.** The argument pretends to prove that the dog will never overtake the tortoise; it really proves that the dog passes the tortoise between the 111th and 112th yards.

7. *Examine:*—"*If I am to pass this examination I shall pass it, whether I answer correctly or not; if I am not to pass it, I shall fail whether I answer correctly or not; therefore, it does not matter how I answer the questions.*"

Here it is tacitly assumed that "whether I answer correctly or not" is not a link in the fated chain of events. It is assumed that fate does not work through correct answering of questions, and the conclusion is merely a repetition of this assumption. It is the veneration of "Fate" that draws away our attention from the error of this delightful **petitio principii.**

EXERCISES ON CHAPTER XIV.

1. Describe any three fallacies in Logic, giving an example in each case.

2. Point out some of the ordinary forms of fallacy employed to mislead in argument or in oratory, and illustrate the forms named.

3. Explain and illustrate the terms "reductio ad absurdum" and "begging the question."

4. Examine the following:

(*a*) *You are not what I am; I am a man: therefore you are not a man.*

(*b*) *A fish is a cold-blooded animal and breathes by gills; neither of these things is true of a whale; therefore, it is not a fish.*

5. Explain exactly the nature of the fallacies called "accident," "argumentum ad hominem," and "argumentum ad verecundiam."

6. Show how a logical training enables a student to detect fallacies.

7. Explain and illustrate by examples the following terms:— Ambiguity; fallacy; premiss; suppressio veri et suggestio falsi.

CHAPTER XV.

Inductive Logic.

The fundamental lesson of Deductive Logic has been that no conclusion may ever contain more than was contained in the premisses from which it was drawn. Particular premisses, we saw, could not yield a general conclusion. The definition of Inductive Logic, therefore, will seem at the outset a paradox. For **Inductive Logic may be defined as the Inference from particulars to the general, or from the known to the unknown.** It is the establishing of general laws or principles from observed particular facts or instances. John, Thomas, etc., individual men, are mortal, from which the general inference is drawn that "All men are mortal." Here we have a general conclusion about *all* men, derived from an indefinite number of particular instances. At first it seems as if our conclusion was overdrawn. The conclusion contains more than is given in the premisses; it seems like a leap in the dark. Modern science consists throughout of such general conclusions, based on particular facts. Because certain things resemble each other in certain observed ways, we assume that they will resemble each other in certain previously unobserved ways. But is this general assumption warranted? Do any number of observed facts warrant a general universal conclusion. If the observed facts be two, or two hundred particular cases, are we warranted in making any assertion beyond the number observed? What right have we to *add* to

what we actually observe, as we certainly do, whenever we conclude, from seeing a number of particular events occur, that they will always occur? We say "all animals die," but we have not seen all animals die. Similarly, "all bodies gravitate," but our experience does not extend beyond particular instances of gravitation. Yet we are certain that these inferences are legitimate. What, then, is the ground of this certainty? Why are we able to conclude that "All must be so-and-so," because we have observed that "Some are so-and-so"; that "*all*" bodies gravitate because "*some*" have been observed to gravitate? This is the problem of induction. For, we must observe that this process of induction is attended with some perplexity. In some cases one single observation is enough to warrant a general conclusion. whilst in other cases we hesitate to draw a general conclusion from hundreds of observed instances. Euclid takes a single triangle and shows that its three angles are together equal to two right-angles. We therefore accept this as a general truth applying to triangles of every kind and everywhere. One single instance is sufficient to establish a general rule. On the other hand, though every crow I have seen is a black one, I should have no hesitation in believing someone who told me that he had seen a grey one. Whence come the certainty in the one case and the uncertainty in the other.

Induction is based upon one great axiom, *viz.*: "THE COURSE OF NATURE IS UNIFORM." In other words nature is not a chaos, it is an orderly system. Any event does not follow any other event in a haphazard way. The relation of things to each other is governed by what we call "law." This truth is axiomatic in its nature. It is the assumption of all Induction and of all science. It is not a truth which we can prove, nor does it need proof. If any one cares to deny it, we can offer no demonstration of it, beyond showing the denier that he himself acts upon the assumption every hour of the day.

This axiom is practically the assertion that things are related to each other by law, and the one general law, everywhere observable, is that of **cause and effect.** The truth, that every fact which has a beginning has its cause, is a truth coextensive with human experience. It is needful, then, to have a definite notion of what is meant by cause. We may define it as follows :—

A **cause** IS THAT WHICH IMMEDIATELY PRECEDES ANY CHANGE, AND WHICH, EXISTING AT ANY TIME AND IN SIMILAR CIRCUMSTANCES HAS BEEN ALWAYS AND WILL BE ALWAYS FOLLOWED BY A SIMILAR CHANGE.

In this sense of the word cause is synonymous with power, property, or quality. Thus: "Water has the power, property, or quality of melting salt" is the equivalent of "Water is the cause of the melting of salt." Each statement means that when water is poured upon salt, the solid is transformed into liquid. Two parts of a sequence are thus before our minds (*a*) the addition of water to salt, (*b*) the transformation of a crystalline solid into a liquid. These are respectively cause and effect. The powers, properties, qualities, or causes of things are not to be regarded as anything superadded to the thing. These are not the *things* plus their *powers*, but things alone. Things are the invariable antecedents of changes in similar circumstances. The changes occur in an order or with a uniformity, which we believe to be regular. It is this general fact which enables us to reason about nature and to draw general inferences. If the changes which we see continually happening were chaotic, without uniformity, there could be no reasoning about them, either inductive or deductive. Now, in our actual experience, causal connections are mixed up with casual or merely accidental coincidences. Even the unscientific man remarks that some sequences repeat themselves, whilst others do not. He watches the sequences and the coincidences happening within his range of observation. He

makes experiments. In this way he comes, ere long, to separate causal or constant sequences from those which are only casual and occasional. There are many practical difficulties to be overcome, arising from the fact that the same effect may be produced from several causes, and that effects are often produced partly from one cause and partly from another. Gradually, he learns the efficacy of experiment for helping him over these difficulties. Conjectures and hypotheses suggest further experiments. "Vary the circumstances," is a rule which commends itself more and more as he proceeds. Thus he puts aside immaterial circumstances which he finds to be casually and not causally connected with the phenomena he is investigating. Gradually, these casual antecedents and consequents drop off, and the true causal antecedents and consequents reveal themselves. Then, at last, the enquirer has discovered truth. He is no longer an enquirer or conjecturer, but he may claim to have established a general law or a scientific induction. The various methods which this sketch has suggested and which are exemplified in the various sciences, are known as the Canons or Rules of Valid Induction. Before these are examined in detail, it is needful to consider in detail some of those processes which are preliminary to induction, such as observation, experiment and conjecture.

(*For general exercises on Part II see Page 111.*)

CHAPTER XVI.

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The Preliminaries to Induction.

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OBSERVATION, EXPERIMENT, CONJECTURE.
(HYPOTHESIS.)

IN the closing sentences of the preceding chapter we have assumed a distinction between observation and experiment. Observations in the wide sense of the term are either simple or artificial, *i.e.*, they are conducted either with or without interference on the part of the observer. In simple observation the facts are taken just as they offer themselves. In artificial observation or experiment the spontaneous state or occurrence of things is modified by the observer's will. The phenomena to be observed, the effects and causes which are to be investigated, are put in such new circumstances as are most suitable for the detection of their causes and effects.

In OBSERVATION PROPER, we *watch* nature's experiments; in EXPERIMENTAL OBSERVATION we *interfere* with nature's experiments, in order to make others of our own.

Experimental observation is obviously a powerful auxiliary in the search for causes and effects in nature. It is simply the outcome of the old rule, "vary the circumstances," *i.e.*, vary the circumstances which surround the object whose causes or effects you wish to ascertain. Each fresh experimental

variation is a new opportunity for getting rid of the casual or companion circumstances, and for recognising the really constant or causal ones. Contrasting observation and experiment, the latter seems by far the more potent instrument. But certain things must be borne in mind. When we are endeavouring to ascertain what the effects of a given cause are, we may use experiment as freely as we choose. But, in the reverse process, *i.e.*, *in ascertaining what is the cause of a given effect, experiment is not always a safe guide.* We may take any given cause and see what effect it will produce, but we cannot always take an effect and try experimentally what will produce that effect. If we do so, we can only conclude that what we discover is one way of producing the effect out of many possible ways. So, too, in some cases we are shut out from experimental methods. In many applications of the science of medicine we have to rely upon observation entirely.

Conjecture or Hypothesis.—This is an important auxiliary in the search for the causes and effects of things. **An hypothesis is a provisional supposition about the true relation of things.** But we may not suggest hypotheses at random. There are certain reasonable conditions to which hypotheses must conform in order to be entitled to rank even as conditional explanations. An hypothesis is in itself a provisional conjecture, insufficiently supported by evidence. It is legitimately made in order that we may compare with the actual facts of the case, what would be the facts if it were well-founded; and, in proportion as it yields a reasonable result or the contrary, we may accept or reject it.

A **dogmatic** hypothesis is a conjecture not sufficiently supported by evidence, which we are asked to receive as an established truth. A **suggested** hypothesis is a provisional conjecture not sufficiently supported by evidence, which we are asked to try or test by means of the evidence.

The conditions of a legitimate hypothesis are:—

1. It must not be already known to be, or even strongly suspected of being, untrue.

2. It must be of a nature to admit of proof or disproof, for verification.

3. It must be adequate to explain all the phenomena of which it is offered as the explanation.

Hypotheses cannot be regarded as established general truths about nature, until they have conformed to the requirements of one or other of the Inductive Canons.

CHAPTER XVII.

The Inductive Canons.

THE general rules or conditions of successful search for causes and effects are the essence of Inductive Logic. They are to this branch of the subject what the syllogism is to Deductive Logic. They were first laid down in Bacon's *Novum Organum*. For the purposes of this elementary treatise it will be sufficient if we discuss the two fundamental canons known as (1) the Method of Agreement and (2) the Method of Difference.

> 1. **The Canon or Method of Agreement.**—"When all the antecedents of an effect *except one* can be absent without the disappearance of the effect, that *one* is causally connected with the effect, due precautions having been taken that no other circumstances have been present besides those taken account of."

The principle involved here is obvious. Whatever can be excluded from a sequence without affecting the phenomenon whose causes or effects we wish to ascertain, *cannot* be causally connected with it. Let a, b, c, d, e and f be circumstances observed on some particular occasion to attend some event which we will call x. We wish to find the cause of x. And, to do this, we must ascertain whether any of the given circumstances, a, b, c, etc., are causally connected with x. To settle this point we watch the occurrence of the phenomenon x again and again in a well selected variety of circumstances. In the first case we noticed that a, b, c, d, e and f were all

present. In the next case, perhaps, *a* was absent but all the rest were present. In another case *b* was absent but all the rest were present, and so on. But after many variations of the circumstances in all of which *x* occurs, we may find that one, say *f*, is a never-failing antecedent. All or any of the others may be absent, but whenever *x* is observed we find *f*, and *f* seems the only material circumstance. Hence we conclude that *f* is causally, and not merely casually, connected with *x*. whose cause we were seeking. Let us illustrate this by a concrete example: There has been an outburst of typhoid fever, and we wish to ascertain its cause. We take as many cases of the occurrence of the fever as possible. We notice the following points:—

(*a*) The cases occurred in different streets of the town.

(*b*) The ages of the victims differed.

(*c*) Their occupations differed, and so on.

But amongst all the circumstances there was *one* which was common to every case, *viz.*, all the patients had drunk milk coming from one farm. Comparison of the different circumstances enables us to say that (*a*) the locality, (*b*) the age of the sufferers, (*c*) their occupation, are not causally connected with the outbreak; but that the milk, being the only circumstance common to all, most probably *is* the cause.

Observe that we say "*most probably*." To be absolutely certain we should have to know that the given circumstance was the *only* material one in which all the cases agree. Although in many cases we may be certain enough for all practical purposes, we may not be logically certain. The Canon of Agreement, therefore, only enables us to clear away an indefinite number of casual, immaterial companion circumstances. But, it only more or less probably assures us that the residue is alone causal. We have to allow for the plurality of causes, *i.e.*, several causes producing or combining to produce the same effect.

The Method of Agreement is mainly, though not exclusively, one of observation rather than of experiment. It is applied more frequently and successfully to enquire into the causes of given effects, than into the effects of given causes. As we have before remarked, to find the effect of a given cause, experiment is better than observation; but to find the causes of a given effect, observation and its Method of Agreement are usually safer guides than experiment.

> 2. **The Canon or Method of Difference.**—"When the addition of an agent is followed by the appearance of a certain effect, or when the subtraction of an agent is followed by the disappearance of a certain effect, no other material circumstance having been added or subtracted at the same time or in the meanwhile, and no change having occurred among the original circumstances, that agent is a cause of the effect."

We have here two sets of circumstances, and we *know* that they differ from one another in one, and only one essential particular. What this essential particular is, we also know. Now, whatever happens in the set of circumstances, in which the given particular occurs, and which does not happen in the set of circumstances where the given particular is absent, *must* be due to the given particular. In other words it is the cause of the effect observed. To take an illustration with which the student will probably be familiar: a feather and a coin are suspended in the receiver of an air-pump, from which all air has been exhausted. They fall to the bottom at the same moment. Air is introduced into the receiver: the feather flutters to the bottom at some interval after the coin. Here the phenomenon under consideration is the retardation of the feather. This is an observed effect of which we desire to find the cause. The exhausted receiver is one set of circumstances; the receiver with air introduced is the other. The presence or

absence of air in the receiver is the only particular in which the circumstances differ. The phenomenon of retardation, occuring in one case but not in the other, is at once described as the effect of which "air" is in some way the cause.

It is quite possible to express the reasoning of the two canons in the form of a syllogism. The illustration given might be expressed in general terms as follows:—

"*All cases of observed or experimentally produced sequence which fulfil the conditions of the canon of difference must be cases of constant or causal sequence and not mere coincidences* (Major Premiss).

These cases are cases in which the conditions required by the canon of difference are realised (Minor Premiss).

Therefore, these cases are cases of causal connection (Conclusion).

In these days every teacher is obliged to pursue some amount of experimental work, and so becomes familiar with the logical principles involved in reasonings which obey the conditions of the canons of induction. In the constant search for the causes of effects, and the effects of causes, cases arise to which the two great canons, which we have already considered, are not directly applicable. Hence other canons have been formulated which must now be discussed.

3. **The Canon of the Joint Method of Agreement and Difference:** or, as it is more accurately called,

The Canon of the Joint Method of Agreement in Presence and in Absence.—"If two or more instances in which the phenomenon occurs, have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect or the cause or an indispensable part of the cause of the phenomenon."

This is the form in which the canon is usually given, but it will become somewhat more intelligible if stated a little more fully.

> **If** two or more instances in which a phenomenon occurs have only one other circumstance (either antecedent or consequent) in common:
>
> **whilst** two or more instances in which it does not occur (though in some important points they resemble the former set of instances) have nothing in common save the absence of that circumstance:
>
> **then** the circumstance in which alone the two sets of instances differ throughout (*i.e.*, being present in the first set and absent in the second set) is the effect or the cause, or an indispensable part of the cause of the phenomenon.

Although this canon reads somewhat complicated and difficult, it is as a matter of fact, only the precise statement of a form of reasoning which is in constant use. This will appear from a simple illustration. A man observes that whenever he eats cucumber he suffers from indigestion afterwards. Now by the method of agreement he might infer that the cucumber was the cause of his discomfort. But perhaps he is specially fond of cucumber, in which case he may endeavour to lay the fault of his sickness upon the salmon or the cheese, or something else that he had eaten along with the cucumber. But if he is a wise man he makes for himself a fresh set of instances where he has eaten cheese, salmon, etc., but no cucumber, and if he finds that on these occasions he has not suffered from indigestion, then he is bound to conclude that it was the cucumber that alone was responsible for the indigestion.

If the canon of the Joint Method is now read along with this simple illustration, the student will have no difficulty in grasping the steps in the reasoning.

It should be noticed that the Joint Method like the Method of Agreement rests mainly on observation, and a high degree of probability is the utmost that can be generally inferred by its use. It has, however, this special advantage over the Method of Agreement, that if the second set of instances, in which the phenomenon and its supposed antecedent (the indigestion and the cucumber in our illustration) are both absent, can be made exhaustive, then any hypothesis of a plurality of causes is precluded.

The principle of the Joint Method may be summed up in two propositions worth remembering:—

(*a*) That which is not followed by a given event is not the cause;

(*b*) That which cannot be left out without impairing a phenomenon is a condition of it.

4. **The Canon or Method of Concomitant Variations.**— "Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation."

This method is in reality a special case of the previous canons for use in cases to which they cannot be applied in their entirety. There are, for example, certain forces which can never be entirely eliminated, and consequently it is impossible to obtain negative instances. Thus we cannot entirely deprive a body of the whole of its heat. We can therefore only reason about the effects of heat by making changes in the amount of heat in a given body. If when the quantity of heat is varied we ascertain that there are concomitant changes in the accompanying circumstances, then we are able to establish the relation of cause and effect between the varying amount of heat and the attendant circumstances.

Thus, if on the occurrence of friction we find the temperature of a body increased, say ten degrees, and ascertain that besides the friction no other circumstance affecting the body has changed, then we are justified in concluding that the friction has been the cause of the rise in temperature. The law of the expansion of bodies of heat was ascertained by this method of reasoning.

The method of concomitant variations may now be illustrated quite generally :—

Let A and *a* be the two phenomena under consideration, and let A′, *a′*, and A″, *a″* represent corresponding alterations (whether of increase or decrease) of the phenomena. Now suppose we have three sets of circumstances, A B C, A′ D E, A″ F G, with corresponding phenomena *a b c*, *a′ d e*, *a″ f*, *g*. Now the one thing only in which the two sets of circumstances agree throughout, is that any alteration in A (*i.e.*, A′ or A″) is followed or accompanied by a corresponding alteration in *a* (*a′* or *a″*). From this we infer that most probably A is the cause of *a*. We say "most probably" for in the second case D E might be the cause of the alteration of *a* to *a′*, and in the third case F G might be the cause of the alteration of *a* to *a″*. But if after many trials it is found that A and *a* always vary together, then the probability of their causal connection becomes more and more a certainty. Yet just because it is nearly always possible that some unobserved cause is the real determinant of both A, *a*, and also their concomitant variations, absolute certainty can never, theoretically, be attained by this method.

These four methods are the reasonings which are employed when in the course of investigations we attempt to eliminate mere casual connections present with those which are related as cause and effect. After definite progress has been made by the use of these methods of elimination, further problems are greatly simplified by subtracting from any complex sequence

what has already been found to be the influence of ascertained causes. This process of subtraction or simplification is known as the method of residues.

> 5. **The Canon or Method of Residues.**—"Subtract from any phenomenon such part as previous induction has shown to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents."

The reasoning here is quite simple. Suppose that the antecedents A, B, C, D, are followed by the consequents *a*, *b*, *c*, *d*, and that by previous inductions it has been ascertained that B is the cause of *b*, C of *c*, D of *d*. Then by subtraction we infer that A is the cause of *a*.

It has been by this method that many of the elements of Chemistry have been discovered. Quite recently it was observed that nitrogen obtained from the atmosphere was slightly heavier than that obtained by ordinary chemical manipulation. This excess in weight was an instance of a residual phenomenon for which the cause must be sought in some peculiarity of the atmosphere. Lord Rayleigh investigated this and so discovered argon, a new element in the air, which had been present along with the nitrogen obtained from the atmosphere, and which accounted for the difference in weight.

Such, then, are the five Canons of Induction, or the logical conditions which regulate inferences about the laws of nature, and in particular the law of causation. They are calculated contrivances for finding out when causal connection really exists, and all scientific work exemplifies these rules. The exact words of each canon should be committed to memory, and the student should be prepared to give one or more concrete illustrations of each method.

It remains now to consider briefly the two conditions which, separately or together, tend to frustrate the methods in their practical application to the phenomena of nature. These are :—

1. *The fact that only comparatively few effects invariably follow one set of antecedents alone;* and

2. *It is only in comparatively few instances that a single effect can be kept apart and distinguishable.*

In short, there is a **Plurality of Causes** on the one hand, and an **Intermixture of Effects** possible on the other.

1. **The Plurality of Causes.**—This we have already seen is the special weakness of the Method of Agreement. Thus, for example, if the phenomenon of heat in a given body were under observation we could not with certainty infer that the particular amount of heat under observation was due to one cause (say friction) because, by the Method of Agreement, it had been established that friction was always accompanied by an increase of heat. For in this particular case the definite quantity of heat might have been the result of combustion, the solar ray, electricity, etc., or it might have been the result of several of these combined. The remedy for this inherent weakness of the Method of Agreement is to multiply instances as much as possible, and if practicable to apply the Joint Method. The multiplication of instances enables us to ascertain, possibly, *all* the causes which produce the effect, and then it is easier to say which of these could have been present in any special case and which of those actually present were free to operate to produce the effect. If, then, we could further apply the Joint Method and discover cases in *absence*, the conclusion would be decisive.

2. **The Intermixture of Effects.**—In nature the effects of various causes seldom remain separate and distinguishable.

More frequently is it that the effects of various causes unite in a single homogeneous total. For example, a good crop is a single effect, but the causes which have united to produce it are very numerous. Each cause has had its own effect, but the separate effects are united to form one single result. In such cases the Method of Concomitant Variations has a peculiar advantage. For when, amid a variety of causes, one cause happens to vary alone, we know that its effect will vary alone also. When this has been sufficiently observed, then cause and effect may frequently be singled out under circumstances of great complication.

CHAPTER XVIII.

Arguments Similar to Induction.

I. **Analogy.**—In ordinary life we are often as much obliged to act upon what is probably true as upon what we know is certainly true. Analogy is a form of reasoning which aims only at giving more or less probable certainty. *If we find two things closely resembling each other in certain observed ways, we argue that they will probably resemble each other in ways which we have not observed.* This is the formula of analogy. Induction argues: "These sequences have been found in some instances, therefore they will be found in *all* instances." Analogy argues: "These two things resemble each other in certain qualities, therefore they probably resemble each other in other qualities. Some of the planets are known to resemble the earth in certain respects, therefore they probably resemble the earth in being inhabited." Butler's great work on the "Analogy of Religion" argues that, because nature and revealed religion have many resemblances, therefore it is probable that they have a common Author.

Logic lays down the following rules for good analogical reasoning:—

1. The ratio or proportion in number of resemblances must be contrasted with the number of known differences. If the former are many and the latter few the

analogical conclusion is increased in probability, and *vice versâ*. N.B.—If one of the things about which we are arguing is only little known, the unknown points must be added to the points of difference in contrast with the resemblances. Thus, the argument about the planets being inhabited is weakened by the fact that we know very little about them.

2. The kind of resembling and differentiating circumstances must be carefully considered, and the general result compared with what we know of the laws of the universe. Thus, in the case of the planets, we know that life as it exists on the earth can only exist within certain definite limits of temperature and in connection with atmospheric air. Mercury is too hot, Saturn is too cold, whilst the moon has no atmosphere. All the resemblances, therefore, count for nothing when we consider the *kind* of differences that exist.

II. **Inductio per enumerationem simplicem.**—This argument is a kind of inductive fallacy. It argues that, because a case happens to be true in every instance in our experience, therefore it is a general law or truth. Before we can assume that a thing is universally true, *because* we have never known an instance to the contrary, we must have reason to suppose that, if there had been instances to the contrary, we should have heard of them.

III. **Post hoc, ergo propter hoc.**—This is *the* fallacy of Induction. It is the confusion of casual with causal connection, against which the Inductive Canons are designed to guard us. Thus, we have a National Debt and we have national prosperity. We are arguing *post hoc ergo propter hoc*, if we ascribe the prosperity to the Debt. "After, therefore because of" is the generic name for imperfect proof of causation from observed facts of succession.

IV. **Perfect Induction** is the name given to the conclusion, when all possible cases have been duly examined, and we have summarised the result in a general proposition. If, however, Induction is defined as an inference from the known to the unknown, it is obvious that "perfect induction" is really no induction at all.

Relation of Deduction to Induction.

Having thus briefly considered the aim and scope of Deductive and Inductive Logic, it only remains to get into clear perspective the relation between the two. Some modern logicians, seeing the vast practical importance of Inductive Logic as the logic of the physical sciences, have been led to doubt the value of Deductive Logic altogether. They argue that the syllogism is only a *petitio principii.* When we argue that, because all men are mortal, Socrates, being a man, is mortal, the conclusion was "begged" in the general proposition. But the number and variety of fallacies of deduction which abound in ordinary life, are a sufficient warrant to ensure the study of Deductive Logic a permanent and important place in a liberal education. The simplest way of expressing the relation between the two branches is to consider Inductive Logic as the orderly statement of those laws by which we arrive at general conclusions. The general conclusions have then validity, based securely on the principle of the uniformity of nature and the all-pervading law of universal causation. The general conclusions become a sort of memoranda in which our conclusions are expressed. But these memoranda require to be correctly interpreted and reasonably applied to particular cases. *This* is the proper work of Deductive Logic. *In short, the one is the counterpart of the other.*

EXERCISES ON INDUCTIVE LOGIC.

———o———

1. Explain and illustrate the difference between the Inductive and Deductive methods of arriving at truth. What are the chief dangers in reasoning from analogy? Explain and illustrate the terms "reductio ad absurdum" and "begging the question."

2. Illustrate the statement that in all discoveries of natural science, the processes of induction and deduction follow each other before a complete verification of a law can be obtained.

3. What is the exact difference between inductive and deductive reasoning? Give a simple example of each process in connection with some subject of instruction in an elementary school course.

4. Give some familiar examples of false induction, and say what school exercises are best calculated to encourage a habit of making a true use of the inductive process.

5. Distinguish between analogy and induction, hypothesis and theory. What is needed besides induction for ascertaining scientific truth?

6. By what processes of reasoning would you prove that the earth is round, or that the room in which you are is not empty, but filled with something; or, by examining a bird that it was an animal made to live in the air? What name would you give to the process in the last case?

7. Distinguish between generalisation and reasoning from analogy, and give an instance of each.

8. Distinguish between observation and experiment, and show how we may learn by experiment what we could not learn merely from observation.

9. "Induction is really the inverse process of Deduction.' Explain this.

10. What is meant by Inductio per enumeration simplicem?

11. Why is so called "perfect induction" not considered a really inductive process?

12. State exactly what you understand by the terms "Cause and Effect," and "the Plurality of Causes."

13. What is the meaning and significance of the principle known as "the Uniformity of Nature"?

14. Explain the principle of the Method of Agreement and the Method of Difference respectively, and say to what uses the two methods are appropriate.

Appendix.

—o—

MORE RECENT DEVELOPMENTS OF THE SYLLOGISM.

—o—

Since the time of Bacon and Locke, it has been largely the fashion to consider the syllogism as a worthless instrument for the discovery of new truths. Observation and experiment, conducted in accordance with the Canons of Induction, has alone been considered by some as worthy of serious attention. But in recent years the syllogism has again attracted to itself many sympathetic students who have exercised their ingenuity in extending and reconstructing its traditional forms. These newer developments are not noticed in the body of the present work, but some brief account of the three most important of them is now added here.

1. The Intensive or Comprehensive Interpretation of the Syllogism.—If the student will refer to page 46 of the present work, it will be seen that every proposition was regarded as an assertion respecting the logical extent of the subject and predicate of the proposition. But the terms which form the subject and predicate have connotation (intension) as well as denotation (extension). Consequently a proposition may be regarded as an assertion respecting the logical intension or comprehension of the subject and predicate. When extension alone is in question, the assertion is made on the relation of classes to classes. When intension is the point of view adopted, the assertion is grounded on the relation of attributes to attributes. An attribute contained in the predicate of an affirmative proposition must also be contained in the subject. From one point of view every genus is

seen to contain its species; at the other point of view every species is seen to contain its genus. The former is the point of view of the ordinary syllogism, *i.e.*, the syllogism in extension; the latter is the point of view of the syllogism in comprehension. Any syllogism may be interpreted in either way. In order to bring a syllogism out of extension into comprehension, the rule is:

Reverse the premises, and then read or interpret each proposition intensively.

Thus the following syllogism in extension—

"All M is under P
All S is under M
∴ All S is under P"

becomes, by the application of the rule, a syllogism in intension—

"All S contains M
All M contains P
∴ All S contains P."

It is not at all likely that a student of elementary logic will be questioned on this matter, but it is quite worth while to remember that every reasoning in the syllogistic form may be read either way, and that the student should understand how to translate a syllogism in extension to one in comprehension.

2. **The New Analytic.**—A much more important event in the history of the syllogism was the revolution suggested by Sir W. Hamilton, and known as "the new analytic of logical forms." When we use, *e.g.*, an A proposition, such as "All S is P," we mean that, amongst an **indefinite** number of things represented by P, all things represented by S are included. Hamilton considered that, in the precise language of logic, every proposition ought to say clearly all that it is meant to express. If this were done, then the ordinary A proposition

would be written "All S is **some** P." Thus the foundation of Hamilton's endeavour was the express and independent recognition of extensive quantity in the *predicates* as well as the subjects of propositions. Hence the system was described briefly as "the quantification of the predicate." Now, if this is carried out in each of the recognised propositional forms A, E, I, O, we shall obtain four entirely new ones, as shown by the following table :—

USUAL FORMS WITH PREDICATE QUANTIFIED.	NEW CORRESPONDING FORMS.
A. All S is some P	U. All S is **all** P
E. No S is any P	η. No S is some P
I. Some S is some P	Y. Some S is all P
O. Some S is not any P	ω. Some S is not some P

The four new forms have the symbols U η Y ω, corresponding to the traditional A E I O.

Considering briefly the four new forms, it may be remarked that two of them, U and Y, are in frequent use in ordinary language. Thus every definition is practically a U proposition; *e.g.*, "Europe, Asia, Africa, Australia, and America are all the continents"; "Common salt is the same as sodium chloride." Again, any exclusive assertion is an example of a Y proposition; *e.g.*, "Graduates only are eligible for the appointment," or "Some passengers are the only survivors." Since, therefore, U and Y propositions are in ordinary use, it would seem a valid contention that they should receive recognition in Logic. Of course, it is quite possible to express a U proposition in the older forms. Thus the proposition "All S is all P" may be resolved into two A propositions, "All S is P" and "All P is S," which, taken together, are equivalent to it.

Whilst there seem practical reasons for the recognition of U and Y propositions, it must be said that the same reason does not hold good for the new propositions η and ω. They

are theoretically possible, but they are seldom if ever found in ordinary use.

If the principle of the quantification of the predicate is adopted, many remarkable results follow. Every logical proposition becomes an equation between the two quantified terms which it contains. The relation between the terms of a proposition thus becomes one of co-extension, and this implies that each of the eight kinds of propositions may be converted simply. Again, when propositions with a quantified predicate are combined to form syllogisms, it is possible to express the whole syllogism equationally and without figure. Thus the ordinary syllogism—

"All patriots are brave,
Some persecuted persons are patriots,
∴ Some persecuted persons are brave,"

may be written as an equated syllogism as follows:—

"All patriots = some brave men,
Some persecuted persons = some patriots,
∴ Some persecuted persons = some brave men."

This equational theory of reasoning can also be developed in moods and figures as well as in unfigured syllogism. It is quite beyond the scope of an Elementary Logic to work out such development. But it may be interesting to note that in this way we obtain 3 valid figures and 108 valid moods, instead of the 4 figures and 19 moods recognised by traditional logic. The advantages claimed for the new analytic are:—

1. The special rules of each figure are abrogated, and their violation ceases to be illogical.

2. Reduction of syllogisms, like the conversion of propositions, ceases to be necessary.

3. Each figure is alike capable of expressing the relation of the terms in the reasoning, whilst each figure discharges a function specially its own.

Granting all these advantages, it still remains to be considered whether the forms of the new analytic are more convenient and useful as a framework for the unabridged expression of our assertions and reasonings, and also as an aid in the detection of fallacies. And further, are the advantages—scientific and practical—great enough to counterbalance the inconvenience of substituting it for that analysis that has been generally received since Aristotle? The general answer to these questions given by the great authorities on Logic is in the negative.

3. **The Numerically Definite Syllogism.**—Some logicians contend that, besides the definite "All" and the indefinite "Some," Logic ought to recognise definite arithmetical quantity. They would consider the following examples quite legitimate logical reasoning:—

"Two-thirds M is P
Two-thirds M is S
∴ Some S is P."

"Seventy per cent. of M are P
Sixty per cent. of M are S
∴ At least thirty per cent. are both S and P."

In neither of these cases is the middle term distributed in the premises, but the conclusion is correctly drawn according to the rules of arithmetic. De Morgan contends that in such cases it is permissible to mingle the relations of self-consistency with the principles of arithmetic, and no doubt, if this is admitted, the variety and complication of the syllogistic forms will be immensely increased. But it is properly urged, on the other hand, that where numerical evidence of the kind contained in the propositions which form the two illustrations above is obtainable, then the comparatively indefinite arguments of logic are needless and out of place.

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