ELEMENTARY SURVEYING

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PREFACE

Now that Elementary Surveying is regarded as something more than a mere adjunct to mathematics and geography, it appeared to the writer that there might be a place for a little book which aims at opening a vista of the educational and professional possibilities of the subject, presenting it as the application of a few general geometrical principles rather than something akin to a handicraft with each operation an entity. It is hoped this book will stimulate enthusiasm among those who contemplate entering one of the professions implied in the Introduction or, otherwise, create an interest in the other man's job.

The text is based largely upon the syllabus in Elementary Surveying in the General School Examination of the University of London, and matter outside this curriculum is indicated with an asterisk, suggesting the introduction to an intermediate course in the subject. Also many of the questions are taken from papers set by the writer in this particular examination; and he takes this opportunity of expressing his indebtedness to the Senate of that University for their courtesy in permitting him to reproduce this material.

In addition to the theoretical exercises, a number of field exercises are added, and these no doubt will suggest lines upon which others can be devised in keeping with what may be (conveniently) styled "local" conditions. These examples are short, and anticipate the adoption of parties of three (four at most) pupils, this organisation, in the writer's opinion, being the only rational way of handling the subject. Parts of larger surveys or schemes can be allocated to these parties, who retain their identity as far as is practicable. Prior to going into the field the routine should be outlined so as to reduce supervision to a minimum, and, better still, to leave the parties to their own devices.

The writer takes this opportunity of expressing his indebtedness to Mr. A. N. Utting, of the Cambridge University Engineering Laboratory, for preparing the drawings from which the figures are reproduced, also his thanks to Mr. S. G. Soal, M.A., of Queen Mary College, for his kindness in reading the proofs.

In conclusion the writer acknowledges the agency of his wife, whose influence really led him to undertake this short but pleasant enterprise.

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c/o King's College,
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ARTHUR LOVAT HIGGINS

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INTRODUCTION

Surveying may be described as the art of making measurements upon the earth's surface for the purpose of producing a map, plan, or estimate of an area. Levelling is combined with surveying when the project requires that the variations in the surface surveyed shall be delineated by contour lines, or shown in a vertical section, or used in the calculation of a volume content.

Surveying may thus be defined as making measurements in the horizontal plane, and levelling as taking measurements in the vertical plane.

The converse operation to surveying is setting-out work, or field engineering, as when a constructional project, such as a railway, highway, or reservoir, is pegged out on the ground. Hence it is obvious that whatever possibilities the future may hold for aerial methods of surveying, the lowlier methods of ground survey will always be utilised in the setting-out of works for the use and convenience of man.

Surveying is divided primarily into (1) *Geodetical Surveying* and (2) *Plane Surveying*. In geodesy, the earth is considered a sphere, and in plane surveying a plane, the approximation being within the permissible limits of error for areas up to about 100 square miles. The former involves a knowledge of spherical trigonometry, and the latter of plane trigonometry.

Mathematics. The mention of trigonometry introduces aptly the question as to the extent of mathematical knowledge necessary in the various professions in which surveying plays an important part. In applied science, mathematics is a good servant but a bad master, and philosophic doubts often overcome enterprise. This suggestion of more advanced mathematics may cast a shadow over the aspirations of the reader; but let him be comforted in the thought that few boys are gifted with real mathematical ability, and not infrequently this is at the expense of vision and initiative, by one of those balancing feats of nature, which always settles its account with the least effort. Normally mathematical knowledge is a slow growth in hard-worked ground, and many brilliant scientists and engineers would admit that their knowledge in this connection has grown with mental development arising from other interests, the complex filling the voids in a wide, open structure of essential principles.

It is unfortunate that the syllabuses of certain examinations do not insist upon an elementary knowledge of plane trigonometry. In fact, a degree may be taken in geography, evading trigonometry by cumbersome artifices in map projections, while, at the bench the workman can often use the tables with facility as merely a part of a day's work. Therefore get into touch with your trigonometical tables. Four-figure tables will suffice when angles are only required to degrees, five figures when minutes of arc occur, and seven figures whenever seconds are involved.

In ordinary surveying, such as occurs in connection with estate management, valuation, building, municipal engineering, town-planning, and quantity surveying, a knowledge up to and including the solution of plane triangles is necessary; and the subject is subordinate to mensuration, the application of which demands speed, accuracy, and orderliness. In civil engineering a knowledge of spherical trigonometry and the calculus will be desirable, as also is the case in cartography and hydrography, while geodetic surveying will demand still more advanced mathematics, particularly knowledge of the theory of errors.

Errors! What are errors? They are as natural to surveying as colds and measles are to the young. Scientifically, they are not "mistakes," and you make no apology for making them, though you do your utmost to keep them in their place. The true error in a measured quantity is never known, simply because the really true measurement of that quantity is not known. But this is a very advanced argument. All you know is the "discrepancy" between successive measurements of the same quantity, all of which may contain error; though, of course, comparison with a precise standard will convince you whether the error is great or small. Though you may never aspire to a knowledge of the theory of errors, you must learn to control and adjust your errors, always avoiding mistakes with professional contempt by never dropping a chaining arrow (or an odd ten in calculations) or reading a foot out on the levelling staff. But this digression is looking years ahead. You want to know something about the scope of the subject, which is shown in the following list, where the relative degrees of accuracy are given in descending order, the demands of accuracy gradually giving way to the exigencies of speed and time.

TRIGONOMETRICAL SURVEYING, for the preparation of maps of large extents of territory.

LAND SURVEYING, ranging from the Land Division System of the United States and extensive topographical surveys and work for boundary commissions to small areas, such as farms and estates.

HYDROGRAPHICAL SURVEYING, ranging from coast surveys to plans for harbour works.

ENGINEERING LOCATION SURVEYING, for the construction of highways, railways, and various public works. Mine surveys are to be included in this category.

PRELIMINARY AND PARLIAMENTARY SURVEYS, in connection with a projected scheme, such as the construction of a railway or a waterworks.

PIONEER AND EXPLORATORY SURVEYING, for geological, engineering, and mining enterprises, also work in connection with archeological expeditions.

MILITARY SURVEYING, ranging from reconnaissance to maps by aerial photographic methods. In war, these are carried out in dangerous situations, and accuracy must be subordinated to speed.

Some writers subdivide the subject in accordance with the instrument used; e.g., *The Chain*, *The Theodolite*, *The Compass*, etc., and others by the methods, as *Photographic Surveying*, *Tacheometrical Surveying*, *Plane Tabling*, etc.

ORDNANCE SURVEY MAPS. Most countries issue a series of maps for the various subdivision of their states and departments, further sheets showing municipalities, etc., based upon these. In the United Kingdom, this is done by the Ordnance Survey Department. The best known of the Ordnance sheets are the Six-Inch, or "County" maps, on a scale of 6 inches to the mile or a representative fraction of 1 : 10,560, which is used largely in connection with parliamentary plans; the Twenty-five Inch, approximately 25 inches to the mile, or exactly 1 : 2,500, as used for certain constructional surveys, and (double scale) in land valuation; and the One-Inch, or 1 : 63,360, either plain or coloured, contoured and hill-shaded. Various other maps are obtainable, formerly the 1 : 500 "Town" map for certain districts, down to the latest series for the Land Utilisation Survey.

The commoner Ordnance sheets should be carefully examined, and notes made as to the conventional signs used to represent such features as county, borough, and parish boundaries, roads, marshes, canal locks, tunnels, etc., etc. Levels are marked on these maps, and, in addition, the 25-inch gives the areas of enclosures, the well-known bond indicating that a detached area is included in a given acreage.

CHAPTER I

FUNDAMENTAL PRINCIPLES

In introducing the First Five Principles of Surveying, it may be advisable for us to recall our acquaintanceship with Co-ordinates, or "graphs," as you doubtless call them. In Fig. 1 you will recognise the axes of rectangular (or Cartesian) co-ordinates, with the X and Y axes corresponding to *abscissae* x and *ordinates* y, the origin being at O.

"Positive north and positive east,
Negative south and negative west."

Rectangular co-ordinates are also used in plotting surveys by the Method of Latitudes and Departures, the four quadrants representing the four quarters of the compass, as indicated by the letters, N.E., N.W., S.W., S.E.

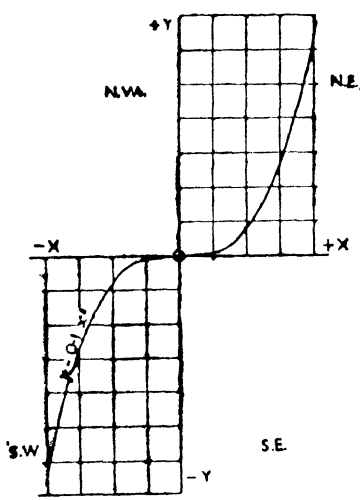

FIG. 1

Possibly you have also met the *cubic parabola*, $y = 0.1x^3$, as plotted with respect to the axes in Fig. 1. It is not altogether an intruder here, being a member of the same family as $y = cx^3$, which is the transition curve the railway surveyor sets out, in order to ease the passage of a train from the tangent-straight to the circular curve against the effects of centrifugal force on the train's motion.

Other forms of co-ordinates are used in surveying; in particular, *Polar Co-ordinates*, in which the point P is fixed with respect to the axes by the distance OP and the bearing or angle β. But there are endless applications of our mathematical principles in applied science, and each is not a stranger living in the same house.

I. FIRST FIVE PRINCIPLES

In the introduction it was stated that Surveying consisted in making measurements in the horizontal plane, and Levelling taking measurements in the vertical plane. Actually, in surveying the measurements consist in fixing the positions of points in the horizontal plane; two points fix a straight line, and three or more straight lines determine the plan of a plane figure. If the actual position of a point P is also found in a vertical plane, vertically above its plan p, the point P is fixed in space; and this is the basis of topographical surveying, which leads to a map in which the surface features are delineated, and usually represented by contour lines.

All surveying operations are based upon these principles, as will appear in the summaries appended to the following methods.

FIRST METHOD *Rectangular Co-ordinates*

Here the point p is fixed with respect to the survey line AB by the distance qp measured at right angles to AB from the point q (Fig. 2).

Uses. (1) Auxiliary, as in taking right angle offsets to the boundaries from the skeleton outline of a survey.

(2) Setting out buildings and certain engineering works.

(3) Fundamental in important operations, such as the U.S.A. Lands Survey. Here the X co-ordinates are really parallels of latitude, and the Y co-ordinates meridians, guide and principal; and as the area surveyed becomes extensive account has to be taken of the fact that on a spherical earth the meridians must *converge* in order to pass through the poles.

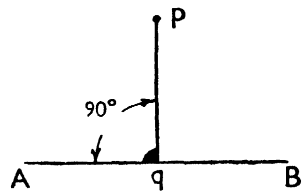

FIG. 2

Thus in a few lines our little mathematics has carried us from mechanics to geography.

SECOND METHOD *Focal Co-ordinates*

Here the point p is "tied" by the distances ap, bp, which are measured respectively from a and b, known points in the survey line AB (Fig. 3).

Uses. (1) Auxiliary, as in surveying boundaries with long offsets, particularly in surveying frontage lines in town surveying.

(2) Basis of all chain surveying, whether "chain triangulation" or traversing.

(3) Method of referencing survey stations on the completion of the field work.

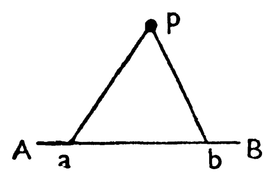

FIG. 3

THIRD METHOD *Angular Co-ordinates*

Here the point p is fixed with respect to the line AB by the intersection of two visual lines, ap, bp, which at known points a and b make observed angles θ and φ respectively with AB (Fig. 4).

This method is peculiarly applicable to the locating of inaccessible points and objects, such as mountain peaks, sounding boats, and through the medium of electrical communication, the position of aeroplanes in flight.

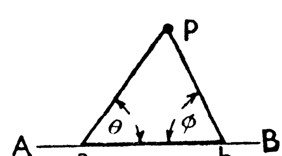

FIG. 4

Uses. (1) Basis of the method of "intersections" with the plane table and compass, also the kindred process in ordinary and stereoscopic photographic surveying.

(2) Method embodied in range-finders and telemeters, the base *ab* being near the observer; and conversely, the principle employed in tacheometry, the optical measurement of distances, the base being at the distant point observed.

(3) Basis of all *pure* triangulation, which may range from a simple net of triangles to a major and minor system, or even a primary, a secondary, and a tertiary net, as in the Ordnance Survey of the United Kingdom.

FOURTH METHOD *Polar Co-ordinates*

Here the point *p* is fixed with reference to the survey line *AB* by the distance *ap* measured from a known point *a* in *AB* at a known angle β from that line.

Uses. (1) Method of locating details by "angles and distances."

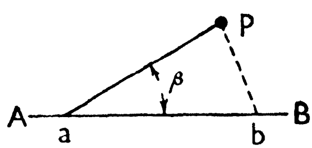

FIG. 5

(2) Method of "radiation" and "progression" in plane tabling, where the angles are measured goniographically; i.e. constructed without account of their magnitudes.

(3) Basis of traversing with the compass or theodolite, *AB* being a reference meridian or N. and S. line.

Inverse polar co-ordinates occur in certain operations, the (dotted) distance *bp* being measured instead of *ap*.

FIFTH METHOD *Trilinear Co-ordinates*

Here the point *p* is fixed by θ and φ, the angles subtended at *p* by three visible and mapped points, *A*, *B*, and *C* (Fig. 6).

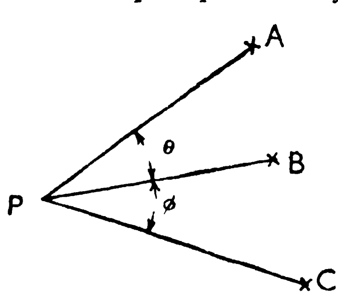

FIG. 6

Uses. (1) The "three-point problem" in resection with the plane table, also with the compass and the theodolite.

(2) Important method in marine surveying, *P* being the sounding boat and *A*, *B*, *C*, three points plotted on the chart.

(3) Method embodied in resection in space in stereoscopic methods of surveying.

(4) Locating positions by wireless signals from three known transmitting stations.

Now the mere knowledge of these principles is not the sole qualification of a surveyor. There is the art or technique of the subject, which alone is acquired by practice and experience. Primarily, this consists in judiciously selecting methods and instruments to suit the objects and nature of the survey. It is not acquired by making a crazy-patchwork map merely to show that you have used every instrument at your disposal, though of course contingencies may arise in which it is expedient to depart from the one prevailing method of the survey. Secondly, the art requires that you shall make all your measurements with uniform accuracy, never mixing the crude and precise promiscuously. Unfortunately there are many obsessed with the idea that rough measurements will accommodate themselves, not only obligingly, but correctly between points surveyed with great precision as a basic framework. Thirdly, simplicity and economy are to be considered with due regard to the strength or rigidity of the basic figure or scheme.

TRIANGULATION AND TRAVERSING. There are two primary methods of making a survey: (1) *Triangulation* and (2) *Traversing.*

In triangulation the area is covered as nearly as may be with a scheme of triangles, and in traversing, by a polygonal outline, also approximating to the boundary or fences (Figs. 7, 8), the latter being more applicable to areas devoid of interior detail.

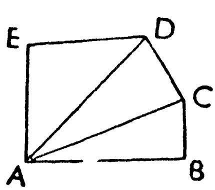

FIG. 7

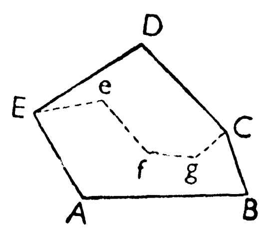

FIG. 8

Traverses may be *closed*, as *ABCDEA*, or *open*, as indicated by the dotted lines *EefgC*, which actually makes a *compound* traverse with the boundary survey, which is the case in certain town and park surveys.

In triangulation surveys, only one side, the base, may be measured (*Pure triangulation*); or certain sides and angles (*Mixed triangulation*); or only the sides are measured, as in chain surveys (*Chain triangulation*).

The strongest figure is that with the fewest sides, hence the triangle; and the skeleton of a traverse becomes weaker as the number of sides increases, so that it may be necessary to brace it up with triangles.

STATIONS. The angular points of a triangulation net or traverse skeleton are called stations, and are usually indicated thus in chain surveys ⊙, and ⚠ whenever angular observations are involved. Commonly stations are referenced with capital letters, A, B, C, etc.; and if subsidiary stations occur the small letters, a, b, c, etc., are requisitioned. In extensive surveys it is advisable to retain the letters for main stations, and to utilise numerals, 1, 2, 3, etc., for the sub-stations. Otherwise the large and small letters are soon exhausted, and applying dashes (or primes) leads to confusion in the field notes. The use of the Greek alphabet is not usually successful, as the surveyor's classical knowledge seldom gets beyond Epsilon, ϵ.

Stations are established in the ground in a manner consistent with the duration of the field-work. In small surveys, such as you will undertake, $1\frac{1}{2}$-in. square pegs will suffice, the error of planting the picket or flagpole beside it introducing no serious error in chain surveys. In practice, however, where the work is likely to last weeks, a metal socket is let into concrete, the flagpole being inserted in the socket when the station is not occupied. As the survey grows and lines are thought of in miles, not chains, the stations become more permanent, and when the distances reach up to 60 or even 100 miles scaffolds are erected with signals for day and night observations.

Fig. 9

II. FIELD-WORK

The first item in the surveyors' outfit consists of ranging rods, or poles, commonly in the 6-ft. length, known as pickets, which, like the 8-ft. and 10-ft. poles, are painted in one-foot lengths, alternately red, white, and black, and are shod with a steel point.

A bundle of six pickets forms a convenient *set* for a party. The longer sizes are more convenient in larger surveys. Flags of red and white fabric are desirable when visibility is impaired by distance or background. Pickets can be supplemented by builders' laths in "ranging out" or "boning out" long lines so as to guide the chainmen as they follow the ups and downs of the ground.

CHAINS. The standard chain length of 66-ft. or 4 poles, was introduced by the celebrated mathematician, Edmund Gunter, in 1620. The length is not only convenient to handle, but is such that ten square chains are comprised in an English acre. Wherefore Americans style it the "surveyors' " chain in distinction with the 100-ft. unit, or "engineers' " chain, which is now used extensively in this country in engineering surveys.

Both chains are made up of one hundred long pieces of steel or iron wire, each bent at the ends into a ring, and connected with the ring of the next piece by two or three oval rings, which afford flexibility to the whole and render the chain less likely to become entangled or kinked. Two or more swivels are inserted in the chain so that it may be turned without twisting.

The entire length of the chain is 66 ft. (or 100 ft.) outside the handles, and the hundredth part of the whole is a link (or a foot), this decimal division allowing lengths to be written as 8·21 lks. (or ft.). Each link, with the exception of those at the handles, is 7·92 in.=0·66 ft. (or 1 ft.), as measured from the middle ring of the three connecting rings to the corresponding ring in the next length. The handle and short link at each end constitute the first and hundredth link (or foot). Every tenth link is marked with a brass tag or teller in a system that allows either end to be used as zero, as indicated in Fig. 10, where the one finger tag can be 10 or 90, and the four-finger tag 40 or 60 lks., or ft., as the case may be.

The Gunter chain is much more easily manipulated than the 100-ft. length, and on this account some surveyors insist upon 6-in. links in order to increase the flexibility. Sometimes a 50-ft. chain is used for offsets, or where traffic exists, but the reversion of few tellers makes it inconvenient to read.

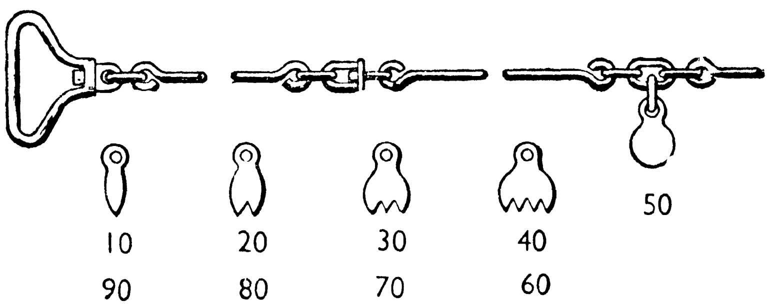

Fig. 10

Steel wire chains are lighter and more easily manipulated than those of iron wire, and the three-ring pattern in No. 12 gauge wire by Chesterman is recommended. Iron wire chains are still used in rough farm and estate work, and though more cumbersome, are more easily corrected and repaired than steel chains. A 100-ft. chain in iron wire would be a tough proposition for a young surveyor; and in the language of Huckleberry Finn, "A body would need the fist o' Goliar to cast it."

Chaining arrows are used to mark the ends of the chain lengths. These are preferably made of steel wire, so as to allow the use of a lighter gauge. A common length is 1 ft. in the pin, but 18 in. or even 2 ft. may be necessary in long grass or stubble. Arrows should be made conspicuous by tying strips of wide red tape to the rings. They should be carried on a steel snap ring or in a quiver slung across the shoulders. Ten arrows comprise a set, and the number should be checked from time to time.

Tapes and Bands. In better class work the surveyor often uses a blue steel band fitted with handles, and wound on a windlass when not in use. The links or feet are marked with brass studs, a small plate denoting the tens. Extreme care should be exercised in the use of these, particularly in the narrow or lighter patterns. Steel bands are elastic, and it is quite easy to pass the elastic limit and so produce a permanent set. For this reason, though more especially to keep the length constant, some surveyors use a tension handle or a spring balance, in order to apply a constant pull, which may vary from 5 lb.

to 20 lb., according to the cross-sectional area of the steel. Also, the tapes are easily snapped, and are liable to corrosion, and need wiping and oiling to preserve them.

The Linen tape is indispensable to the surveyor, civil engineer, and valuer. These tapes are usually wound into round leather cases, and are obtainable in the 50-ft., 66-ft., and 100-ft. lengths, showing feet and inches, and, desirably, links on the reverse side. Similar patterns are made in bright steel with etched divisions, but these are expensive, and their use should be restricted to high-class work and experienced hands. Linen tapes should be dry when wound into their cases; if dirty, they should be washed, wiped carefully, and allowed to dry.

CHAINING. Happily the following instructions are being given to young and enthusiastic surveyors who do not regard it *infra dig.* to get down (literally, and on one knee) to a job of the first importance. Good chaining is a great accomplishment, which can be appreciated only by those who have had good, bad, and indifferent chainmen. Some surveyors are fortunate enough to have trained chainmen, whereas the resident engineer is sometimes at the mercy of a contractor's foreman, who in his wisdom lends him the two men whom he regards as surplus to requirements.

Let it be assumed that you have ranged out a line between two station poles, *A* and *B*, by standing behind one pole *A*, sighting the other *B*, and directing by hand signals the interpolating of pickets and laths at intermediate points in the line. Presumably you have agreed who shall be Leader (L) and who Follower (F) in chaining the line.

First of all, the chain must be cast in the following manner: Remove the strap from the chain, and unfold five links from each handle, then, holding both handles in the left hand, throw the chain well forward, retaining hold of the handles. If the chain has been done up correctly, no tangles will occur.

Leader (L), on receiving ten arrows, counts them, and drags the chain forward along the line *AB*. As he approaches a chain's length from the Follower (F), he moves slowly, and on receiving the order, "Halt," turns and faces F with an arrow gripped against the handle. He bends down in readiness for further instructions. Follower (F), bending down, holds his handle against the starting-point *A*. He then jerks the chain to expel kinks, directs L to tighten or ease his pull, lines L in with the forward station *B*, and finally, with hand signals, directs L to fix Arrow No. 1. Meanwhile, L, holding the chain clear of his person (and preferably facing F), responds to the orders from F, and on receiving the final "stick," fixes an arrow firmly as No. 1. L now takes up the remaining nine arrows, and drags the chain forward for the second length, which is measured in the same manner, except that F holds his handle against Arrow No. 1. On receiving the signal "stick," L fixes Arrow No. 2 and goes forward, dragging the chain. Meanwhile F takes up Arrow No. 1 and carries it to No. 2. The process is repeated, L inserting arrows, and F collecting them duly. If the line is long, the leader L calls out "Ten" on fixing his last arrow, No. 10. The best practice now is to proceed to measure the eleventh length, the Leader having no arrows. When the eleventh length has been laid down, L stands on his end of the chain until F comes up with ten arrows, which he hands to L, who sticks one (11 chains) before dragging the chain forward for the twelfth length.

Folding the chain properly means the saving of much annoyance when next it is used. Take it up by the middle (50) teller and shake it out so that it drags evenly on each side of that teller. Transfer it to the left hand, and place the first five links on each side of the 50-teller, side by side, two at a time together, turning the links in the palm of the hand. Now invert the folded portion in the left hand so that the 50-teller hangs down, and, turning it slowly in the palm of the hand, fold links equidistant from the middle across it, two at a time, not straight, as at first, but sloping obliquely to the left at the top. Continue this oblique folding until the handles are reached, and secure it by means of the strap in this form, which is that of a *hyperboloid of revolution.*

Testing Chains. The limits of this book preclude various hints as to the care, correction, and repair of chains. Nevertheless, these should be tested from time to time. Students in their enthusiasm may unwittingly provoke chaining into a tug-of-war, and even the rings of steel chains will open under the strain. Although this will not occur as far as you are concerned, it is essential that a *Standard Length* be laid down carefully with a steel tape on stone flags or a concrete surface, the ends being marked with cuts into the surface, or, better, by inserting metal plugs cut with a fine cross and filled with solder. Sometimes startling disclosures are made in checking a chain against the standard length. Quite easily a chain may be forgotten during a break for lunch, and an inoffensive ploughman may run it down, and be little alarmed at the repair he has made with not more than three links missing.

Then there is always the danger that a chain of correct length l_0 has attained an incorrect length l after protracted use. Hence lines of correct length L_0 are measured as L, and consequently true areas A_0 are computed as A; but if the chain has been tested and the incorrect length l observed, the correct values can easily be reduced by the following relations:

$$L_0 = \frac{l}{l_0} \cdot L; \qquad\qquad A_0 = \left(\frac{l}{l_0}\right)^2 A.$$

Offsets. Offsets are measurements made from the outer survey lines of a triangulation or traverse skeleton to the boundary of a property, the root of a hedge, a fence, or a wall, as the case may be. Usually these are taken at right angles to the survey lines, and their length is limited roughly to 50 links, though some latitude is allowed in certain circumstances. Whenever necessarily long, they should also be "tied" from another point in the chain as it lies along the survey line. Offsets are usually measured with the linen tape, though formerly the offset staff was used. The right angle is estimated, but when the offset is long, this is best done by "swinging" the tape in the following manner: *A* directs *B* to hold the ring (*O*) end of the tape at a point in the boundary or detail, and, pulling the tape out gently, *A* swings it over the chain and notes the lowest readings both on the chain and tape as the respective chainage and length of the offset.

Objects—buildings in particular—are located by finding points on the chain which are in line with the end walls of houses, as judged by sighting along these while standing on the chain as it lies on the ground in the survey line. Fig. 11 (left) shows how a building is fixed by rectangular offsets, the diagonals *bc* and *ad* being sometimes measured as checks. The line *cd* being thus fixed, the position of the building may be plotted, and since it is rectangular, it may be constructed on the side *cd* when the remaining sides have been measured up with the linen tape. Fig. 11 (right) shows a common method of "tying in" a building which lies obliquely to the survey line. Here the points a' and c' are selected so that they are in line with the respective sides $f'g'$, $e'h'$, of the building and the corners f' and e' are tied with the lines of $a'f'$, $b'f'$, and $c'e'$, $d'e'$, respectively, the readings a', c', b', and d' being suitably recorded in the field notes.

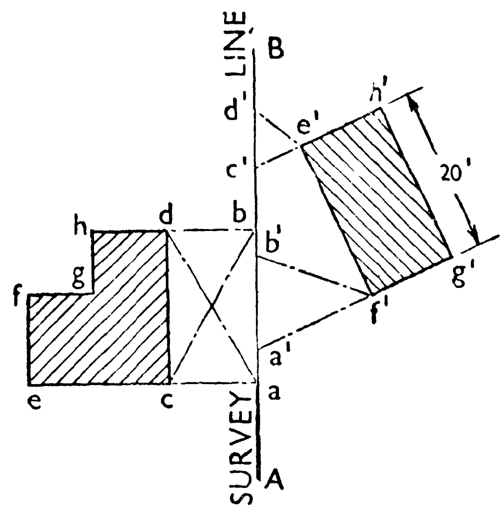

FIG. 11

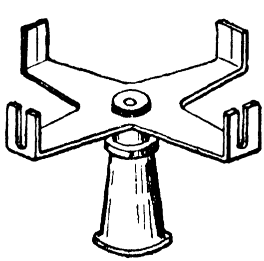

FIG. 12

At this stage we may consider two simple instruments which are used to set out the right angles of long perpendiculars, the geometrical construction of which will be dealt with in Chapter IV.

CROSS STAFF. The *cross-head* is best known in the open form, shown in Fig. 12, the more complicated patterns possessing little to qualify their use. This pattern consists of four metal arms, turned up at the ends, and cut with vertical sighting slits at right angles. The head is attached to an iron-shod staff, which is planted at the point at which it is desired to set out the right angle. Two slits are sighted along the survey line, and the right angle is set out by sighting in a picket through the other pair of slits. The chief difficulty is that of planting the staff (or Jacob) truly vertical, but this can be facilitated by the simple artifice hereafter described.

Cross heads can be constructed in the manual training classes, and even if metal is not available, quite useful instruments can be made from hard wood. The best way of ensuring that the staff is vertical is to use a *ring-plummet*, which may be improvised as follows. Drill $\frac{1}{16}$-in. holes near the alternate corners of the hexagonal face of a backnut for $1\frac{1}{2}$-in. gas-pipe, and drill three corresponding holes in the rim of socket in which the staff is inserted. Suspend the nut by three threads from the socket; then, when the staff is vertical, it will appear centrally in the hole of the backnut.

**Optical Square.* The *optical square* belongs to a class of *reflective* instruments of which the *Sextant* is the representative instrument in modern surveying. The best-known form consists of a circular box about 2 in. diameter and $\frac{5}{8}$ in. deep. The lid, though attached, can be slid round so as to cover the sight-holes and thus protect the mirrors when out of use. Fig. 13 shows a plan of the square when the lid, or cover, is removed; *h* is the half-silvered *horizon glass*, rigidly attached in a frame to the sole plate, and *i* the wholly silvered *index glass*, which in some patterns is adjustable by means of a key. The three openings required for sighting are cut alike in the rims of the case and cover: a square hole *Q* for the *Horizon* sight, a similar one *O* for the *Index sight*, and a pin-hole *e* for the *Eye sight*.

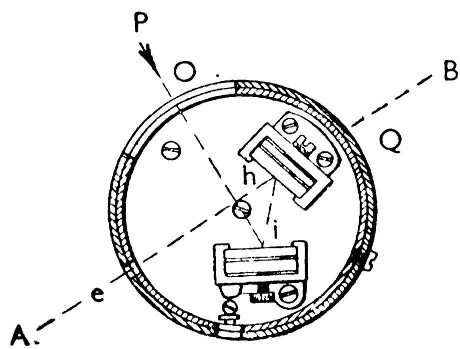

FIG. 13

The index glass *i* is set at an angle of 105° to the index sight line *Oi*, and, since the angle between the planes of the mirrors is 45°, the rays coming from a pole *P* fixed at right angles to the survey line *AB* will be finally reflected to the eye along the eye-horizon line *he*, which is perpendicular to *Oi* by the optical fact that the angles of *incidence* and *reflection* are equal. Prisms are sometimes used in optical squares, and a pair of 45° prisms are embodied in the Line Ranger, a device for interpolating points in survey lines.

A perpendicular is erected in the following manner, the optical square being inverted if the right angle is to be erected on the left of a line, *AB*, as indicated by pickets.

Place the square on the top of a short pole interpolated in AB at the point at which the perpendicular is required. Send out an assistant to the required side of the line AB, estimating the right angle, as well as you can. Then, sighting B through the eye-horizon, direct the assistant to move until you see his picket by reflection vertically above B, as viewed directly, raising the eye momentarily in obtaining the coincidence.

III. SLOPING DISTANCES

Already, doubtless, you have been wondering how hills, valleys, and undulations will affect your measurements. Over two thousand years ago Government officials were worried about the matter, and quite possibly at this moment some contractor has a headache about it. Polybius told those in authority that no more houses could be built upon a hillside than within the same limits on level ground. Other *economic* arguments are that the majority of plants shoot up vertically, and no more trees or crops can be grown on a hill than on its *productive base*, as the horizontal equivalent is called. Exception, however, occurs in the case of certain creeping plants. There is also the *geometrical* argument which contends that a map must represent areas of any surface on a plane sheet. For instance, a triangle can be plotted with any three distances, and so the four-sided skeleton of an irregular field which slopes steeply from one corner will plot as two triangles on a diagonal as a common base, even though all the measurements are made on the actual ground surface; but if the other diagonal be measured likewise, its length will not check with the resulting figure, being too long or too short, to an extent dependent upon its own slope and the distortion induced by the other irregularly measured lengths.

Hence, all measurements must be reduced to a common basis, which for general convenience is the *horizontal plane.*

Wherefore, an "area" is the superficial content of a horizontal plane surface of definite extent, and this definition is understood in the valuation of land. No account is taken of the nature or relief of the surface, which theoretically is thus "reduced to horizon," or in other words, projected on to a horizontal plane.

On the other hand, certain exceptions must be admitted, and these refer to the work of the labourer, which consists of lineal or superficial measurements, such as mowing, hedging, and ditching.

Let us hope by this time that our contractor has fathomed the reason why more concrete will be required in constructing the road up Hilly Rise than the amount he estimated by scaling from the map.

Slopes are expressed either (*a*) by the vertical angle α the surface makes with the horizontal, or (*b*) by the ratio of the vertical rise in the corresponding horizontal distance, 1 in x, say. If the actual sloping distance is l, the vertical rise d is $l \tan \alpha$, and not $l \sin \alpha$, as used in certain connections; that is, the gradient on a road or railway is the tangent of the angle of slope, expressed as a fraction; 1 in 12, or 1 in 75, as the case may be.

It is very difficult to assess slopes by eye, and the limit at which they should be taken into account depends upon the accuracy required in the work, angles up to 3° or 5° being neglected in ordinary work.

In Fig. 14, it is evident that the horizontal distance b corresponding to the sloping length l is

$$b = l.\cos\alpha \tag{1}$$

Corrections are sometimes given in reduction tables, or are engraved as such on clinometers, being differences

$$c = l(1 - \cos\alpha) \tag{2}$$

which are subtracted from the measurements made on the incline.

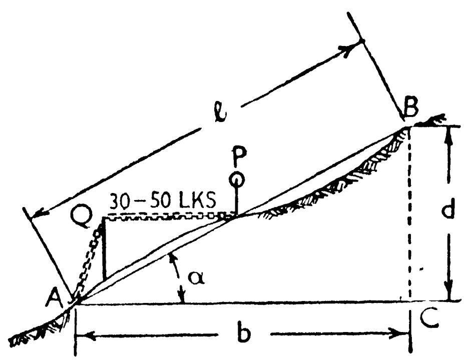

Fig. 14

Now $\cos\alpha = \sqrt{1 - \sin^2\alpha}$; and if α is very small, $\cos\alpha = 1 - \frac{1}{2}\sin^2\alpha$ where $\sin\alpha = d/l$. Hence

$$c = \frac{d^2}{2l} \tag{3}$$

the rule used when pegs are driven on steep slopes and their differences d found by levelling.

Rule (3) shows that if we ignore a difference in height (or in alignment) of 1·42 ft. = 17 in., in a length of 100 ft., the error in length will not exceed $\frac{1}{8}$ in., or 0·01 ft.

Also, in surveying, a correction is of the same magnitude but opposite sign to the corresponding error. Hence, if we prescribe a *ratio of precision* to our chaining, such as 1 : r, it is possible to determine the slope at which it is necessary to apply a correction.

Thus in rule (2), if the ratio c/l must not exceed $1/r$, cos. $\alpha = 1 - 1/r$. Hence if we are to chain to 1 in 1,000 or 1 in 5,000, the angles of slope must not exceed 2° 34′ and 1° 08′ respectively, even assuming that error does not arise from other sources.

There are two general methods of determining horizontal distances when measuring slopes:

(1) *Stepping*, by taking such precautions as will ensure that the chain or tape is stretched out horizontally.

(2) *Observing Slopes* when taking hypotenusal measurements or chaining along the actual slope, the angle α or the gradient 1 in x being observed, frequently with the clinometer.

(1) STEPPING. In this method it is usual to employ short portions of the chain, lengths varying from 20 to 50 links, in accordance with the steepness of the slope and the weight of the chain. In the latter respect the sagging effect of the chain may be so serious that the tape must be used in accurate work. Some surveyors insert arrows slantwise when they require the slope to be taken into account, and sticking arrows in this way facilitates the use of a plumb-bob, which is far better than "drop arrows," loaded with a lead plummet, to ensure a vertical fall.

Let us proceed to measure *down* the slope from B to A with P and Q as chainmen, R going outwards to the side of the line with a straight rod (or picket) in his hand (Fig. 14).

P, at the starting-point B, puts Q into line, holding his handle of the chain on the ground. Q, gripping a plumb-line at (say) the 40-teller, exerts a pull, almost invariably holding his end too low. (In fact, the sense of looking horizontally is badly impaired when working on slopes.) Hence the advisability of the services of R, whose duty it is to see that the chain is horizontal. R, standing some distance to the side of the line, looks for telegraph wires or ridges of roofs, in order to direct Q in raising or lowering his end of the length PQ. When no horizontal object can be viewed, R extends his right hand and balances the rod on his forefinger, and uses this artifice in judging the horizontal. When "All right" is signalled, Q fixes an arrow and proceeds for the next length.

Stepping uphill is more difficult, as it requires that both Q and P must move their ends of the length used, or that P also must be provided with a plumb-bob.

Stepping has the advantage that it is quick and does not necessitate any alteration in the field notes, but its use is limited to lines that involve few or no offsets. When there is much detail, as in surveying streets or crooked fences, the following method must be used, since the chain will of necessity lie on the ground for some time.

(2) OBSERVING SLOPES. The instrument most commonly used in this operation is known as a clinometer, an instrument made in more forms and types than any other surveying instrument, the compass included.

At present we need only examine it in its simplest and improvised form. Take a 5-in. or 6-in. celluloid protractor, insert a stout pin at its centre *o*, tie a thread to the pin, and attach a light weight, say a bunch of keys, at the other end of the thread. Appoint somebody of your own height to proceed up the slope, directing him into the line *AB*. Now sight along the straight edge of the protractor which should be held with its plane vertical, and move it until you see the eyes of your helpmate; then grip the thread and the protractor at the edge near the point *g*, and, bringing it down thus, read the angle, which will be the complement $90° - \alpha$ in observing *angles* of *elevation* (up the slope) and/or *angles* of *depression* (down the slope).

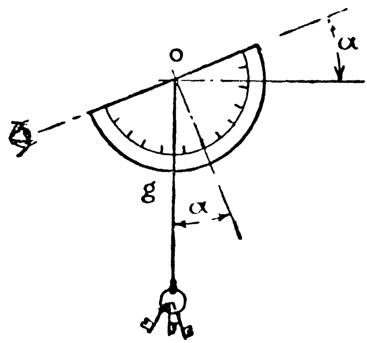

FIG. 15

Obviously the foregoing process requires some practice, but it suggests, failing a proprietary instrument, the lines of constructing a good substitute. Attach a piece of three-ply, 6 in. $\times$ 6 in. to a piece of hard wood, ¾ in. square, and attach the protractor to the plywood, keeping its zero line parallel to the upper surface of the wood. Take two brass strips, ½ in. wide, drill a pin-hole sight in one, and cut a ⅜ in. square hole in the other. Bend the strips at right angles, ¾ in. from the pin-hole and the bottom of the square hole respectively, and attach these sights to the upper face of the wood with brass screws. Insert a tiny picture-ring in the centre of the protractor, so that a plummet with a hook attachment can be readily suspended. Finally make a wooden handle and fix it to the back of the baseboard. Figure around the outside of the protractor the even slope ratios, tan α, as 1 : 1, 1 : 5; 1 : 12, etc. As a further refinement, the corrections to surface measurements can be inscribed in accordance with Rule 2, preferably from the tables in a surveying manual. Such a device can be used in many connections.

The chief difficulty is knowing when and where to take the slopes, since these often vary on a hillside or consist of featureless undulations. What is big in the field is small on a map; and the sense of appreciating the general trend must be cultivated.

Apart from injudicious selection of slope limits, the chief drawback to this method is the fact that the field notes must show the angles of slope or their ratios together with the limits of each different slope. In practice the distances along the survey lines must be duly amended before plotting, preferably as red ink corrections. Only measurements along survey lines will be affected; not offsets normally.

LINEAR MEASUREMENT. Since one aim of this little book is to give a broad view of the subject, a summary of the different methods of measuring lines will not be out of place, particularly if some idea of the relative degrees of accuracy are shown. In surface measurements the precision is influenced mainly by the nature of the ground and the precautions that are taken at the expense of speed. The ratios for ordinary chaining are 1 : 750 to 1; 1,500, with a fair average of 1 : 1,000 for careful work on good ground. A limit of 1 : 50,000 seems reasonable for surface measurement with steel tapes and every precaution. In optical and other measurements, instruments and atmospheric conditions are the controlling factors; and the ratios given are representative of average practice.

(*a*) *Pacing*, after training (1 : 75 to 1 : 150); lower value in route surveys.

(*b*) *By Perambulator*, in road measurement and exploratory work (1 : 150 to 1 : 300).

(*c*) *By river launch*, towing the patent log (1 : 500 to 1 : 900).

(*d*) *By optical measurement*, by tacheometer or range-finder (1 : 300 to 1 : 650).

(*e*) *By sound signals*, guns being fired alternately between ships or the shore and a ship (1 : 500 to 1 : 2,000).

(*f*) *By aeroplane*, in controlled flight over ground stations (1 : 1,000 to 1 : 3,000).

(*g*) By base tapes and compensated bars (1 : 300,000 to 1 : 1,000,000).

Practised pacing is a great asset to the surveyor, and is particularly useful in reconnaissance, route, and military surveying; but the difficulty of counting is a great handicap, even if stones are transferred from pocket to pocket at every hundred paces. The passometer, or pace-counter, is a useful investment, and is to be preferred to the pedometer, which gives distances, and suffers from the refinements necessary to setting it to the individual step. Both instruments are similar and like watches in appearance, the mechanism being operated by a delicate pendulum device. They should be carried vertically above the waist; in the vest pocket or clipped inside the collar opening of the waistcoat. If carried in the trousers pockets, they usually count only half-paces. They respond to well-defined paces, and not to the shuffling gait of a celebrated comedian of the silent films: a fact that may be useful when the user does not want counts to be recorded.

IV. FIELD-CODE

In the writer's youth the text-books gave much sound personal advice to the surveyor, even on matters of dress and deportment. Doubtless this would appear superfluous in a modern text-book, even though sound sense and good taste are not experience, the "obvious" being evident only after the event. Possibly the line of approach should be through the medium of a code, which at least has an official air.

(1) Surveying equipment is expensive, and if damaged or neglected is likely to impair some other fellow's work. Sheep and cattle are naturally inquisitive, and range-poles are easily snapped. Horses masticate flags (and lunch haversacks), cows chew tripods, and two lambs can overturn an expensive level in two to four minutes.

Wherefore, instruments should never be left unguarded, and, during recesses, should be left in enclosures, tripods firmly planted, and staves and poles left on the ground, and never leant against trees or walls.

(2) Instruments should be securely attached to tripods, security in this respect being checked from time to time. When necessarily exposed to rain, levels, compasses, and theodolites should be protected with a waterproof cover, the tennis racket case serving this purpose well. Wet instruments should be carefully dried. Tripods and poles should not be shouldered in streets or through doorways, and levels, etc., should be carried under the arm, the instrument forward, except in the open.

(3) Permission should always be asked before entering any field, yard, or forecourt. Every respect should be given to property. Chaining or walking through crops of all kinds may lead to a claim for damage. Hedges must never be opened or cut in order to make stations intervisible. Fences should never be climbed in order to shorten journeys; barbed wire is no respecter of clothing, and the proper way is the shortest.

(4) Gates should be properly closed and fastened, even for temporary egress. An open gate may lead to straying cattle, with consequent damage and expense; and neglect in this respect may lead to the withdrawal of your permit.

(5) Chaining on paths and highways should be carried out with extreme caution, and always under supervision. Pedestrians and cyclists are easily tripped, and a stretched chain may lead to a motor accident. When only municipal parks are available, special attention should be given to the conditions of the permit. It should always be remembered that these are places of recreation; and that undue interest by the public will soon subside if you work silently and show no resentment.

(6) Field notes should be legible, explicit, and easily interpreted by a surveyor who has never seen the area. They should be complete before leaving the field. It may be impossible to supply an omitted measurement, and the entire work may be rendered invalid.

(7) Stock should be taken of the equipment before leaving the field. Chains, range-poles, and arrows are easily forgotten when clearing the ground. Station pegs should be removed. If driven where they are likely to cause accidents, they should be removed nightly, and the position of the station carefully referenced.

(8) Shouting instructions is bad taste. In public spaces it provokes ridicule, and in private lands annoyance or curiosity.

Surveying affords excellent opportunities of trying out the semaphore code. But a simpler system is desirable: something like the following, which is suggestive rather than standard.

SIGNALS

(*a*) "*Halt.*" Raise the right arm full length vertically above the head, the right hand extended.

Directing staffmen and chainmen, but obviated by "fix picket" in boning-in.

(*b*) "*Fix.*" Extend the forearms forward horizontally, and depress the hands briskly.

Ranging out lines and establishing stations. "Fix arrow" is indicated by depressing the right hand sharply, the sign implying "All right" in short distances.

(*c*) "*Stay There.*" Raise both arms full length vertically above the head, the hands extended.

Directing staffmen to remain while a reading is taken, and generally to await further instructions.

(*d*) "*Go Ahead.*" Extend the right arm full length above the head, and wave it between this and a position horizontally in front of the body, graduating the motion to the desired forward movement, and bringing the arm full length to the *halt* position.

Directing staffmen in levelling and chainmen in fixing stations.

(*e*) "*Right*" *or* "*Left.*" Extend the *right* or *left* elbow in the required direction, and graduate the motion of the forearm to suit the lateral movement required.

When it is desired to bring staffmen or chainmen round through a considerable distance from their present positions, emphasise the signal by swinging the arm and body in the required direction, periodically indicating the required spot with the arm extended.

(*f*) "*Come Nearer.*" Circle the right arm over the head, slackening the motion as the required position is approached, and finally bringing the arm to the *halt* position.

"Come here" or "Come in" is indicated by bringing the hand to the crown of the head after every few turns.

(*g*) "*Plumb Staff.*" "To your Right." Extend the right arm upwards slightly to the right, and swing the entire body to the right, checking further movement by thrusting out the left arm. Vice versa in plumbing to the staffman's *left.*

Plumbing the staff in levelling and adjusting station poles.

(*h*) "*Higher.*" Hold the left hand, palm downwards, in front of the body, and raise the right hand briskly above it; repeat after momentary pauses, emphasising the motion by raising the body until the signal is interpreted and obeyed.

The signal implies "Too Low," and instructs the staffman to extend a telescopic staff or to move to higher ground. The signal may be reversed to suggest movement to lower ground.

(*i*) "*All Right.*" Swing both arms from the sides simultaneously, bringing the hands together above the head several times.

For great distances where the less-emphatic "Fix" would not be recognised.

American surveyors signalled "O.K." for "All Right" fully forty years before we took it into our vernacular.

CLASS EXERCISES

1 (*a*). Describe with reference to neat sketches, the following methods of measuring sloping distances with the chain:

(*a*) *Stepping;* (*b*) *Observing slopes.*

1 (*b*). In chaining you are instructed to take into account the slope of the ground when it gives rise to an error of measurement of 1 in 1,000 in Land Surveys and 1 in 3,000 in Town Surveys.

Express as angles or otherwise the slopes corresponding to these errors.

(2° 34′ or 1 : 22 and 1° 29′ or 1 : 39.) (G.S.)

1 (*c*). Describe how you would "reference" a survey station so that you could re-establish its position if required.

1 (*d*). A purchaser disputed the area of a field which was stated to be 54 a. 3 r. 24 p., the sale price being £300 per acre. It was proved, however, that the Gunter chain used was 0·4 link too short; and the court decided that the excess payment should be refunded to the purchaser.

Calculate the amount of the refund. (G.S.)

(Excess, 0·438 acres; Refund, £131 8*s*. 0*d*.)

1 (*e*). Describe the optical square, indicating its principles on a neat sketch.

FIELD EXERCISES

Problem 1 (*a*). Examine and test the assigned chains against a standard length.

Equipment: Chains, scriber or chalk, rule, and in the absence of a permanent standard, an accurate steel tape or band.

Problem 1 (*b*). Investigate the accuracy of chaining by measuring the line *AB* . . . times.

Equipment: Chain, arrows, rule and a set of pickets.

Problem 1 (*c*). Ascertain the average length of the natural pace and assess the accuracy of careful pacing.

Equipment: Chain, arrows and set of pickets, and desirably a passometer.

Problem 1 (*d*). Measure up the specified portion of the . . . Building.

Equipment: Set of pickets, chain, arrows, and a linen tape.

Problem 1 (*e*). Set out one of the following in the playing-field:

(*a*) Tennis court; (*b*) Hockey ground.

Equipment: Set of pickets, chain (50 ft.), arrows, tape, and cross staff.

(On hard surfaces improvised tripods may be used, the feet tied to prevent opening out. A picket (or plumb-bob) may be inserted in the junction-piece to which the legs are hinged.)

ORIGINAL PROBLEMS

. . . .

. . . .

(e.g. Use of a cycle wheel as a road-measuring *perambulator*, the strikes on a gong serving as an improvised *trocheameter*.)

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CHAPTER II

CHAIN SURVEYING

Not the least of the educational values of surveying is the fact that the introduction to the art is through the medium of the simple chain survey; something utilitarian as well as instructive, and something that merges into the complex naturally and unobtrusively.

The execution of an extensive chain survey is the finest training for the surveyor; though the field of imagination, effort, and resource has been impaired by the premature inception of the theodolite, which is often introduced inexpediently. There is a place for everything in the field; but a proper place. In chain surveys the selection of stations can be truly pioneer work, since all lengths must be measured, and reconnaissance in order to obtain inter-visibility becomes a matter of greatest importance. But the labour is usually rewarded by satisfaction in the results, which in no small way arises from the fact that all measurements are of the same *order*, often the same precision, and not mixed, as in accurate angles and rough chaining. Sanction to purchase a theodolite may sound important in the council chamber, but disillusionment is often the lot of the surveyor.

The writer recalls some notes he encountered thirty-five years ago; the records of a very extensive chain survey carried out in the 'sixties. A classical piece of work, but a monumental piece of plotting, particularly in view of the fact that page 22 of the duplicated sheets was missing. But let us proceed, in order that you may foster your own reminiscences.

Equipment. The usual outfit in work of the present nature will consist of one or two sets of range-poles or pickets (flags), chain, arrows, linen tape (cross staff or optical square), pocket compass, and, above all, the field-book.

(*By the way, see that the linen tape is also figured in links when measurements are to be made in Gunter chains.*)

FIELD NOTES. The field notes are entered in a book with stiff covers, about $7\frac{1}{2}$ in. by $4\frac{1}{2}$ in., containing plain leaves, opening lengthwise, and secured with an elastic band. Usually two red lines, about $\frac{5}{8}$ in. apart, are ruled centrally down the middle of the page to represent the survey line, and the notes are recorded *up* the page, as in looking forward along the chain to the next forward station. This method of upward booking should be characteristic of all forms of *line* notes. In keeping field notes, scale is relatively unimportant compared with neatness and clarity of interpretation, particularly in regard to offset detail. Some of the notes recorded seventy years ago emphasise a marked decline in handwriting and lettering and general presentation, which to-day is not infrequently loose and half legible. Few surveyors record their notes in precisely the same way, but vary their conventional signs, though, of course, these follow a more or less general scheme.

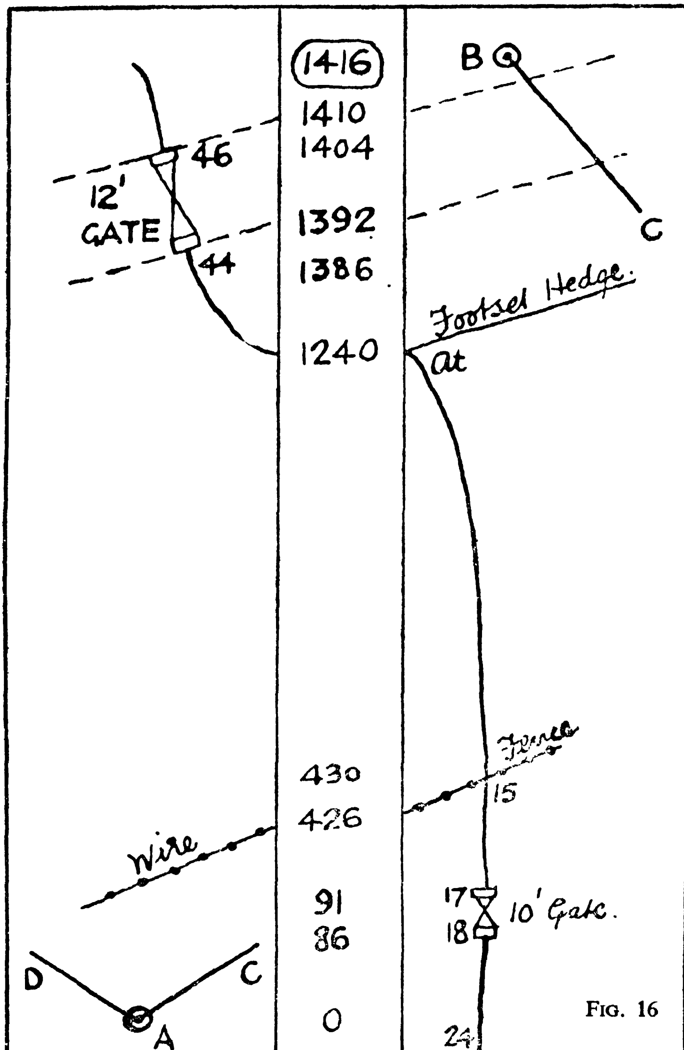

FIG. 16

(It is now suggested that the reader study the survey of "Conventional Farm," page 37, and obtain some idea of representing detail and objects, improvising clear abbreviations.)

Fig. 16 shows a specimen page of the notes of a chain survey, various symbols being introduced. Space will not allow discussion of the various points of contention, such as the merits of using a single red line instead of a pair, whether lines should be numbered or not, etc. Usually a page is allocated to a line regardless of its length, though obviously very long lines or lines with much detail will require two or more pages. Also, two strokes are drawn to denote the end of a line, even if this is not stated in words. Some insert direction marks at the stations, without further remark, or with arrow-heads and letters indicating the directions of the adjacent stations concerned. Frequently the station chainages are inserted in rings. The crossing of a road or fence requires that the double lines be imagined as a single line, by breaking the road or fence; while contact with a fence corner necessitates contact at a red line with the zero offset distance "*O*" suggested by the word "At." Right-angle offsets are generally understood, but when these are long, necessitating "tying," dotted lines are inserted with the tape measurements figured along them.

Keeping the Field Notes. The keeping, or better, the custody of the field notes affords no difficulty in actual practice, but is a matter of serious importance in instructional classes, booking being a substantial part of the training. A class under instruction may appear like a rush of reporters in an American gangster film, overwhelming the story or the instrument in their enterprise. On the other hand, the lone keeper of records may be a well-meaning but irresponsible student, who fails to produce the evidence when required, and often loses it—and the labours of his fellows. A middle course must be found by deputing a trustworthy student to be responsible for the "party copy," and at least one other student of that party should transcribe the notes before leaving the field. Often the "class copy" must inevitably be the work of several hands, often inadept, and the leader must keep an eye to the book from time to time. Some object to the indoor transcription of notes, and even like to see the marks of the field (which need little cultivation); but a copy is a copy wherever made, and whether in pencil or waterproof ink. Indelible pencils may serve in official capacities, but there is no place for them in plotting or surveying, except for marking stakes.

I. CHAIN TRIANGULATION

Let us consider Fig. 17, bearing in mind the following rules:

(*a*) As long lines as possible, consistent with short offsets, which latter should be restricted to 50 links, though even a chain may be permissible if it obviates a subsidiary triangle in an unimportant gap.

(*b*) As few main triangles as possible, consistent with covering the area without a number of subsidiary triangles for outlying boundaries, inlying details, etc.

(*c*) As well-formed triangles as possible, with no angle under 30° or greater than 120°, in the main, but with reasonable latitude in subsidiary triangles.

(*d*) As strong check lines as possible, in order to verify all main triangles with an additional measurement, unless these are otherwise checked by interior fence or road lines. Small or isolated subsidiary triangles need not be checked.

The diagonal AC in Fig. 17 is selected as the basis of the work, and is frequently styled the base line, quite without qualification. A line alongside the approach road usually assumes this capacity in plotting. On AC are built the two triangles ABC and ADC, which together comprehend the area without requiring long offsets to the boundaries, this difficulty being obviated by inserting the triangle efg in the gap.

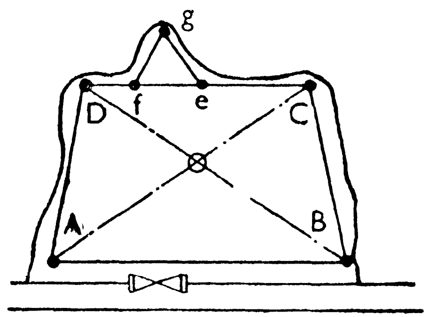

FIG. 17

Now any three lengths will form a triangle, and if a chain length is "overlooked" in measuring a line, a plan will certainly follow, but one of sorts. Hence it is essential to measure a *check line*, such as BD, and, in doing this, a pole O should be interpolated so that it is both in AC and BD, its position being recorded in both these lines: thus: AB, 6·64 chs. with O at 3·28 chs., and BD, 6·72 chs. with O at 2·04 chs., no offsets being taken from these lines. In the case of farms, etc., where several fields are included, it is seldom necessary to think about checks, as these will arise from lines along farm roads and fences—often a check too many in slipshod work.

The field-work may be detailed concisely as follows:

(1) Reconnoitre the ground and select suitable points for the stations A, B, C, D, etc., consulting existing maps, if available, in the case of large surveys. Select the stations in accordance with the foregoing rules, aiming at simplicity and strength, and never sacrificing a strong triangle in order to avoid a difficulty. Establish the stations suitably with pegs, and if necessary fix flags to the station poles.

(2) Sketch an "index map" on the first page of the field-book, and insert the survey lines. This item often permits simplification in the field-book. In large surveys an index to the lines is desirable, so that in plotting, the lines can be readily found from the numbered pages of the book.

(3) Proceed to measure the lines and the offsets to the adjacent boundaries, selecting the order most suitable to prevailing conditions. Thus in the afternoons, in winter, visibility along the long lines may become very poor, and these should be measured first. Normally, read the chain to the nearest link, since in plotting this will introduce an error of less than 1/200 inch on a scale as large as 1 chain to 1 inch. In certain connections, particularly with the 100-ft. chain, it is necessary

to work more accurately, as calculations may be involved, or certain portions may be required on a very large scale.

(4) Concurrently with the measurement of the lines, take offsets from the chain as it lies on the ground, sending out the ring end of the tape to the roots of hedges, fences, walls, etc., and swinging the tape as already described in estimating the right angles. Widths of gateways should be figured in addition to the offsets to the posts, and particular note should be made whenever the chain line crosses fences, roads, footpaths, and ditches. When a subsidiary triangle is set out, such as *efg*, offsets should be taken from *eg* and *fg*, though occasionally this may be done after closing the line on the end station. At least two corners, fixing the faces of buildings should be located, and these, like all important measurements, should be "tied" even though they may be squared off from the survey line. Trees need little attention when they grow along boundaries, but otherwise their positions should be found, particularly if isolated or if they are planted along avenues. Clusters of trees may often be surveyed from a line between them, and often the general limits and a mere count as to their number is sufficient. On large scale plans, it is often desirable to represent trees by a conventional circle rather than to obscure the ground with artistic matter. Offsets should be taken where there is a distinct change in the direction of a boundary, remembering what is large to the eye is often undistinguishable on the map. Two (three at most) are necessary in the case of straight fences.

Never take offsets at regular intervals merely in order to use Simpson's Rule, which should be restricted to distances scaled from the plan.

(5) Continue the work on these lines until all the lines are measured or surveyed, taking care that no important triangle is unchecked and that no important detail is omitted. Reference two stations of an important line in order to facilitate re-survey, by tying the stations with two tape measurements from trees, gate-posts, or prominent points on buildings. Finally remove all station pegs, poles, and laths.

The plotting of the survey will be discussed in a later chapter.

Hedge and Ditch. There are very few cases in which the surveyor can tell by mere inspection the precise position of a legal boundary line between properties. In the case of brick and stone walls, the centre sometimes forms the division line, in which case it is known as a *party wall*, while in other cases the wall is built entirely on one property and the boundary line is then the outer face. Frequently, local inquiries have to be made as to the positions of stones and marks on parish boundaries. Also the boundary between properties and parishes may be the centre of a brook or a stream. When a hedge has a ditch on either side of it, or none at all, the root is the boundary line if it divides the property of two different owners *A* and *B*. But when a hedge has a ditch (or the remains of one) the hedge and the ditch usually belong to the same property, the clear side or brow forming the boundary line. Thus, in Fig. 18, the boundary of Mr. A.'s property is the line XY, while Mr. B.'s property includes both the hedge and the ditch. There are exceptions to this rule. Usually the owner's side is denoted by a "T" when a hedge is represented by a mere line on a map.

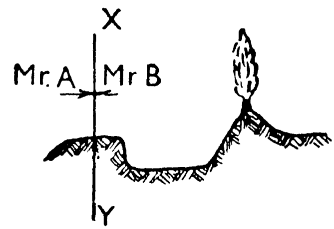

FIG. 18

Commonly all measurements are taken to the root of the hedge, the following allowances being made: 5 or 6 to 7 links according as adjacent fields belong to the same or different owners, and 7 to 10 links when abutting on public lands. Further discussion may get us entangled in the Law of Property, and that is best left to lawyers or chartered surveyors.

II. CHAIN TRAVERSES

Traversing denotes the running of consecutive survey lines more or less in conformity with the configuration of a wood, pond, or plantation, or a route, road, river, or stream, the two categories representing the primary classes of traverse surveys:

(*a*) *Open Traverses*, and (*b*) *Closed Traverses.*

What in themselves are open traverses may occur between triangulation stations, or between the stations of closed traverses, placing the latter in the category of *Compound Traverses*.

Strictly, the chain alone is not the ideal method of dealing with a traverse, which is best surveyed with the compass and chain, or, better, the theodolite and chain. Needless to say, it would be incongruous to run a closed traverse around a wood, and then introduce one of these instruments in order to survey an interior road. Strange things like this happen in surveying when a proper examination of the ground is not made.

(*a*) *Open Traverses.* In chain traverses it is necessary to fix the relative positions of the lines *AB*, *BC*, *CD*, etc., by means of ties *ab*, *cd*, etc., whereas otherwise the directions would be determined by the angles or bearings at *A*, *B*, and *C*. Usually the bearing of the first line, *AB*, is taken with a pocket compass, as in the case of chain triangulation, so that a magnetic meridian, or N. and S. line, can be drawn on the plan.

The tendency is to use ties far too short, or otherwise giving too acute or oblique intersections, so that the directions of the main traverse lines may be in error. In the case of roads through woods it is often extremely difficult to get in ties at all.

Fig. 19 shows the main traverse lines and ties with reference to a portion of a stream, which by a stretch of imagination may be a road, or even a contour line. The main stations *A*, *B*, *C*, and *D* are selected so as to render the offsets short, and the tie stations, *a*, *b*, *c*, and *d*, to fix the angles rigidly, incidentally serving for offset measurements when close to the stream. The routine differs little from that detailed for triangulation surveys. The notes, however, should not terminate with the end station, *B*, say, but should include the end *b* of the tie on the next line, so as to retain continuity and avoid omissions. There is certainly much to be said for the use of the single red line instead of the pair in work of this nature.

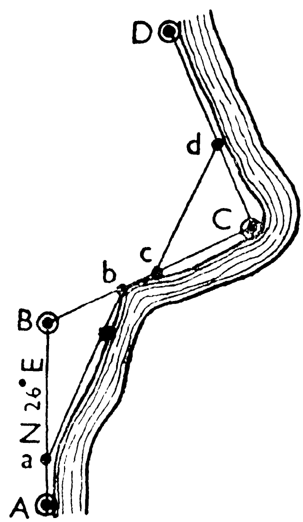

FIG. 19

When the traverse is run between stations more rigidly located by chain or other triangulation, the traverse lines can be adjusted to fit between the main stations by the methods described in Chapter VII.

(*b*) *Closed Traverses.* Fig. 20 shows the foregoing method applied to the case of a pond. Sometimes certain of the ties afford a convenient basis for offset measurements, as, for instance, the line *de*. Each main line requires an angle tie, and not infrequently several main lines are laid down whereas few would suffice. This is evident in the triangle *bcd*, which not only replaces two main lines, but doubtless affords a better basis for offset measurements.

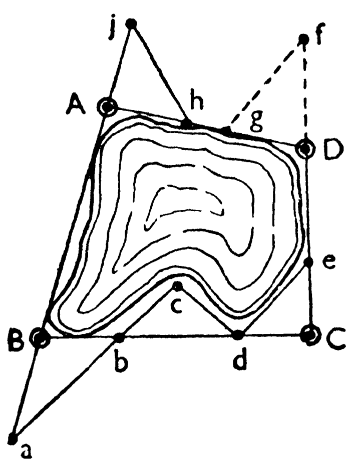

FIG. 20

The area in Fig. 20 is shown traversed in the counter-clockwise direction simply because when a theodolite is used, "back angles" will also be the interior angles of the polygonal skeleton. The principles and methods of chain triangulation are also employed in mixed triangulation surveys with the theodolite, which in the case of Fig. 17 might obviate the chaining of the diagonals by the observation of four angles; and this would be an extravagant innovation unless great accuracy is required or obstructions impede the measurement of *AB* and *CD*. Also, the principles could be extended to compound chain surveys, such as those of farms and estates. Some idea of surveying Conventional Farm might be obtained by inspection of Plate I. Generally, however, the sketching of lines on diagrammatic surveys is of little value unless contours and other information are supplied. Examine an area and you will discover that this is something more than a diagram. This comment does not apply to plotting from unseen field notes, such as are given in text-books. Often a field class has to be abandoned on account of the weather, limitations of time, or an omission on the part of a member of the party. Hence, extract notes from text-books must be resorted to. Nevertheless, nothing is so good as notes brought in from the field.

CLASS EXERCISES

2 (*a*). Sketch the plan of a farm which consists of six adjacent fields and a building, the whole area approximating to a rectangle with a road running along the south boundary. Assuming that the interior fences are low and the ground fairly level, indicate clearly how you would survey the farm with the chain, measuring tape, and range-poles only.

2 (*b*). Draw up a page of a field-book, and insert the imaginary notes of an important line in the survey in Qu. 2 (*a*).

2 (*c*). In measuring a survey line *BC*, chaining was done on the surface of the ground, and the slopes taken with the clinometer at the sections indicated.

	1 : 12		1 : 10			1 : 8		
(*B*) 0	50	245	360	510	720	824	960	1128 (*C*)

Enter these on a page of field notes and make the necessary corrections for sloping ground.

2 (*d*). Sketch an isolated wood of irregular shape, containing a road leading to a quarry; and indicate how you would survey this with the chain, tape, and poles only.

2 (*e*). Sketch the plan of a street you know, and indicate how you would survey the frontage lines of buildings and fix other details from a survey line which runs down the centre of the carriage-way.

FIELD EXERCISES

Problem 2 (*a*). Survey the (specified) field by chain methods only.

Equipment (*which is also the same in the following problems*): *Chain, arrows, set of pickets, pocket compass, and linen tape,*

Problem 2 (*b*). Survey the (specified) pond (wood or plantation) with the chain, poles and tape only.

Problem 2 (*c*). Survey the (specified) road between the range-poles marked *A* and *B*.

Problem 2 (*d*). Survey the (specified) cottage (gate lodge) and garden.

Problem 2 (*e*). Survey the (specified) farmyard, and measure up the buildings.

ORIGINAL PROBLEMS

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CHAPTER III

PLOTTING PLANS AND MAPS

It is but natural that the young surveyor is eager to see how his own efforts show up on paper; and, in deference to his wishes, the present chapter is inserted somewhat prematurely, possibly overlooking various difficulties he has encountered. On the other hand, it is desirable to proceed slowly, in order to take a wider view of the subject of plotting rather than to distribute it throughout the book, though matters not of immediate interest may be revised at a second reading. Anyway, the uses and construction of scales is a matter of primary importance.

I. SCALES

A scale is used to measure straight lines on plans or maps in certain conventional ratios to the actual lengths of the corresponding lines in space. Scales may be expressed in the following three ways:

(*a*) *By a Statement*, such as 1 *Chain to* 1 *Inch*, 6 *Inches to* 1 *Mile*, etc.

(*b*) *By a Representative Fraction* (R.F.), such as 1 : R, the denominator being the number of units in space represented by one scale unit; the in. or cm., as the case may be. Thus the R.F. of the scales stated above are respectively 1 : 792 and 1 : 10,560. The method is universal, applying to all systems of measurement; and most Continental maps are characterised by even ratios, such as our 25-in. Ordnance sheet, which is 1 : 2,500, and not, therefore, precisely 25 inches to the mile. The R.F. is absolutely necessary when two unrelated systems of units are involved.

(*c*) *By a Divided Line*, or map scale, which is usually "open" divided. Usually (*a*) and (*c*) are combined to express the scale, and all three modes are used on the Ordnance maps.

Scales occur in two forms, which are *Open Divided* or *Close Divided*, according as only the first main division or all the main divisions are subdivided.

(1) As refined or improvised drawing instruments for plotting maps and plans, and (2) as an important feature of the plan or map for convenience in scaling measurements and distances.

(1) *Office Scales*. Office scales are constructed of boxwood, celluloid, or ivory, the flat section bearing two scales, being better than those of triangular section which carry six scales. Oval section scales carry four different sets of divisions, and are usually (open) divided for engineering and architectural plans. Surveying plotting scales are close divided, and are sometimes provided with a short length of the same dividing known as an offset scale.

High-class scales are expensive, and, failing access to these, the students must content himself with a good 12 in. boxwood rule, improvising wherever necessary special divisions on strips of drawing-paper. A useful and inexpensive item is the protractor scale, 6 in. long, and similar to the so-called military protractor. The boxwood pattern is the best. One form show inches with eights and tenths, centimetres, a diagonal scale, giving hundredths of an inch, and ¼, ½, ¾ (inch) to 1 ft. (1 ch., or 100 ft.), also a scale of chords, three edges being divided for the construction and measurement of angles. It is exceedingly useful also in the field, though its principal use is giving fine measurements through the medium of the dividers when constructing scales by the methods hereafter described.

(2) *Map Scales*. Scales of this category are open divided, and are drawn on the map to facilitate measurement with a paper strip or a pair of dividers, and to provide against the shrinkage of the paper over the lapse of years. A "shrunk" scale is made when the surveyor has omitted to insert a scale on his map, and the paper shows evidence of shrinkage. It is then necessary to find two prominent points on the map which still exist in the area; to measure carefully the distance between them, and then to construct a true scale so that it can be used in the future, although it carries the statement of the exact scale on which the survey was plotted.

Scales should be drawn with extreme care, never unduly short or long, and preferably with a single line. A double line with alternate primary divisions blacked in is often used. Here, unfortunately, the artist covers up his inaccuracies, so that often the scale is of little use, except to the eye of the beholder. Students have a habit at first of setting off primary divisions, and figuring these with fractional values and their multiples. This must never be done. The primary divisions must show integral values of the units, however fractional the actual lengths may be, in inches, etc.

Among the various kinds of scales that may have to be constructed are *Comparative Scales* and *Time Scales*. Comparative scales show two different systems, such as feet and metres on the same representative fraction; and time scales show time intervals instead of yards or metres for a given statement or representative fraction, being used for pacing, trotting, etc., in military surveying and exploratory mapping.

Constructing Scales. When the division introduces fractions, it is usual to resort to construction by diagonal division, as shown in Fig. 21.

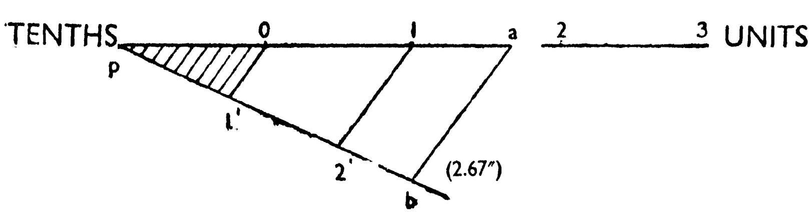


Fig. 21

A horizontal line *pa* is drawn, and at any convenient acute angle to it a line *pb*. When a convenient length has been marked off, as *pa*, say 4 in., a drawing scale is placed along *pb*, and the division is chosen so that it represents conveniently the number of units (usually fractional and integral) represented by the 4-in. length of *pa*. Next, *b* and *a* are joined, and parallels to *ba* are drawn through the even points of division, 2′ and 1′, the subdivisions of *p*1′ giving likewise the subdivisions of *p*0.

The following examples introduce the types of problems that commonly arise:

(i) Construct a scale showing chains and tenths, given the statement, say, 10 ft. to 1 mile.

Find the number of chains that are represented by a convenient length, 4 inches, say. Here 4 in.$=$176 ft.$=$2·667 chs. Take the decimally-divided scale and measure off $pb=2{\cdot}67$ in. Join *ab* and draw parallels through 2′ and 1′, the primary divisions on *pb* which give on *pa* divisions each corresponding to one chain. Write "0" at the end of the first division and subdivide the division on the left into 10 parts, as indicated. Extend the scale a convenient length to the right.

(ii) Construct a scale showing chains and tenths, given the *R. F,* 1 : 528. Here 1 in.$=$528 in.$=$44 ft.; and 4 in.$=$2·667 chs., which is the scale of the preceding example.

(iii) Construct comparative scales of 1 : 500 showing yards and metres. Here 1 in.$=$500 in.$=$13·9 yds., while alternatively 1 cm.$=$5 metres. If *pa* is still 4 in., *pb* would have to represent 55·6 yds., and *b* could conveniently be 5·56 in., so that tens of yards would appear as primary divisions, with single yards on the left. In the metric system, the scale would be constructed by merely setting off 2 cm. primary divisions, each to represent 10 metres.

(iv) Construct on the scale of 6 in. to the mile a scale for marching at 100 paces per minute with an average length of 27 in.

Here 100 paces will cover $\dfrac{27\times 100}{36}=75$ yds. per min., or 375 yds. in 5 min. while on the given scale 1 in.$=29\dot{3}$ yds. or 1·28 in.$=$375 yds. Hence a suitable scale would be 6 in. to 7½ in. in length, the primary divisions being 5 minute intervals, and the close divisions 1 minute intervals.

(v) Construct the scale stated as 2 chains to 1 inch, omitted from an old shrunk map, given that a line scaling 5·80 in. was found to be 11·85 chs. on recent re-measurement.

Here 2 chs. are actually represented by $\dfrac{5{\cdot}80\times 2}{11{\cdot}85}=0{\cdot}98$ in. Otherwise the 4 in. length of *pa* in Fig. 21 will represent $\dfrac{4\times 11{\cdot}85}{5{\cdot}8}=8{\cdot}17$ chs., and the true scale can be constructed by joining *b* at 8·17 chs. to *a*, and drawing parallels 8′7, 7′6, 1′0, 0 being the zero of the open divisions.

Mapping Requisites. Apart from the drawing-board, T-square, set-square, compasses, and dividers, all of which are too well known to require description, there are certain items which must be discussed at length.

(1) First a good drawing-pen is necessary for drawing lines in ink, the common mapping-pen serving for lettering and inserting details.

Waterproof Indian ink should be used, particularly whenever a colour wash is to be applied. Ordinary writing-ink should never be used on plans, nor crayons, which emphasise only bad taste.

(2) A clinograph is preferable to the lever types of parallel rules for transferring parallels to oblique lines, as in plotting bearings. An adjustable T-square will also serve the purpose; and often one can be improvised from a broken T-square, the stock being secured to the head with an adjustable thumb nut.

(3) Good quality drawing-paper should be used; never the soft surface material which becomes ragged along inked lines. A sample of the paper should be tested as to how it will take ink, stand erasures, even with sand-paper in the event of accidents, and, possibly, how it will react to water-colours. The sizes that will be used in the present connection are the *Half Imperial* (23 in. × 16 in.) and *Imperial Sheets* (30 in. × 22 in.).

Always use a hard pencil, HH or HHH, chisel-pointed, and a round-pointed H or HH for lettering, etc. A *pricker* is recommended for marking off scale distances on survey lines.

(4) Finally, the chief item is the beam compass, since the lengthening bar will extend the use of ordinary compasses only to relatively short lines. A good quality beam compass should be available, though a few additional ones could be improvised in the workshop with $\frac{5}{8}$-in. or $\frac{3}{4}$-in. square mahogany rods, 18 in. to 24 in. long, by making adjustable clips and attaching these to the points and pencil-holders of old compasses. In an emergency strips of paper, 15 in. × $1\frac{1}{2}$ in., might be used, a stout pin serving as the centre.

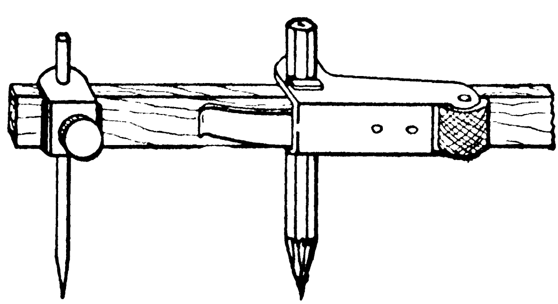


FIG. 22
BEAM COMPASS

The beam compass is not only used in plotting chain surveys, but often in laying down accurate triangulation nets, the sides of the triangles having been calculated by the Sine Rule from observed angles and the one measured side, the base.

II. PLOTTING THE SURVEY

There are four principal steps in the routine of plotting maps and plans: (1) *Selecting the Scale;* (2) *Placing the Survey;* (3) *Constructing the Triangulation or Skeleton; and* (4) *Inserting the Detail.* Finishing the map will be discussed later.

(1) In selecting the scale the objects of the survey and the extent of surface to be represented must be borne in mind. Incidentally, centimetres and chains or feet must never be mixed, as in 1 cm. to 1 ch., even if this would be geometrically convenient. All "irregularities" must be avoided; such as 64 ft. to 1 in., simply because a scale reading to $\frac{1}{16}$ in. is available; or 132 ft. to 1 in., when the scale is definitely 2 chs. to 1 in. If distances are to be scaled to the nearest foot, the scale should not be less than 50 ft. to the inch. The tendency is to use too large a scale, leaving very little margin, while, within reason, a fairly wide margin is effective.

(2) Only in maps of extensive areas is it desirable that the true north should be at the top of the sheet, the side border lines being true north and south lines.

The area should be viewed from the local aspect, and thus the approaches to the property should appear at the bottom, with roads approximately parallel to the bottom edge of the paper, regardless of the position of the meridian needle, true or magnetic. Considerable thought may be involved in placing a survey on the paper, so as to be pleasing to the eye, and easily evident to the least intelligent.

(3) In plotting the triangles, a survey line (*AB* in Fig. 17) is selected as the base, and this is drawn to scale in the best position on the paper as can be judged, often from a trial plotting. The beam compass is then set to the respective scale lengths of the sides, *AC*, *BC*, adjacent to the base, and, with *A* and *B* as centres, arcs are swung accordingly, intersecting at the apex *C* of the triangle. On this triangle another, *ACD*, is constructed likewise; and the process is continued until the entire framework is completed. Subsidiary triangles, such as *efg*, in Fig. 17, can be inserted with the ordinary compasses. Stations should be indicated by small circles, appropriately lettered *A*, *B*, *C*, *D*, etc.

(4) Details of surveys of the present class will be inserted by offsets, mostly rectangular, long offsets being tied by means of ordinary compasses. The most rapid method of inserting right-angle offsets is by the conjoint use of a close divided surveying scale and an offset scale, the main scale being held in position by a pair of shoe-shaped weights. Few students will have these latter at hand, and the T-square and set-square must be brought into service. (A straight edge is better than a T-square, since it is more easily manipulated.) The plotting scale, zero at the beginning station of the line, is placed carefully along the pencilled line, and the points at which offsets were taken are pricked off; then with the aid of the set-square, short perpendiculars are erected

for right-angle offsets, the positions of tie line offsets being indicated by a short stroke across the survey line. The scale is then applied to the offset measurements, the ends carefully pricked or pencilled, and the fences, etc., etc., are inserted with the aid of the set-square, or, occasionally, a French curve. Likewise, the nearest walls of buildings are inserted, the corners usually being fixed by ties described with ordinary compasses.

Finishing Survey Maps. Inserting details is a step closely related to the finishing of survey maps, and at this stage the imaginary survey of Conventional Farm should be consulted (page 37). All details should be carefully outlined in pencil, reducing the use of erasers to a minimum. In instructional surveys it is usual to insert the survey lines in very fine red ink lines, red circles being drawn for the stations of chain surveys and triangles wherever angles are observed. The station letters should also be inserted in red, but not too conspicuously. This retention of the skeleton occurs in practice only where constructional work is likely to follow, as in mine surveys particularly. When a number of subsidiary traverses occur in instructional mapping, the lines are sometimes shown in another colour: green or blue ink.

Handwriting should be restricted to one detail: the signature. Free-hand lettering cannot be taught by a text-book, being normally the outcome of practice and training.

Many have improved themselves in free-hand lettering by practising to some scheme, such as the following. Write a sentence three times on the top three lines of a sheet of ruled foolscap; and hence find the natural slope of the downstrokes. Next, with the aid of the squares, cover the remainder of the sheet with a series of strokes, ruled parallel, about ¼ in. apart, and then rule guide-lines parallel to the paper ruling, giving the height of the body of the letters. Finally, reproduce the written sentences by changing script into hand-lettering.

Half an hour's practice a day for a week often has a revolutionary result. Erect letters emphasise defects far more than slanting, though an experienced draughtsman can work to any slope with facility.

Titles are best inserted by outlining them first between guide-lines with a fairly soft pencil, so as to obtain uniform heights and spacing. Then the squares are used, finally giving the outline in block or other letters, which are inked with the drawing-pen and the mapping-pen, and filled in or finished in black ink. The celluloid open stencils are exceedingly useful in obtaining well-proportioned outlines in pencil.

Unfortunately craftsmanship is at a discount to-day, and the will seems to be taken for the deed; but the fact remains that draughtsmanship is a great accomplishment, and the master-touch can give an atmosphere even to a prosaic survey map.

Finally, always test your pen from time to time on a piece of the same paper as that on the board; and keep a cloth at hand to wipe the pen when out of temporary use. Never let the ink coagulate in the pen. Also, seize the opportunity of instruction in sharpening a drawing-pen. Use soft rubbers generally, and clear the rubbings from the board before inking or colouring.

The General Requirements of a Map or Plan are: (*a*) The *Title*, (*b*) the *Scale*, (*c*) the *Meridian Needle*, and (*d*) the *Border Lines*; also, possibly, (*e*) an *Explanation* or *Legend*, as to the symbols employed, and (*f*) a *Terrier*, showing the acreage held by various owners. The surveyor's signature and the date should always be given. Special requirements include (*g*) Contours, hachures, or spot-levels, and (*h*) *Constructional* lines and symbols for building and engineering works.

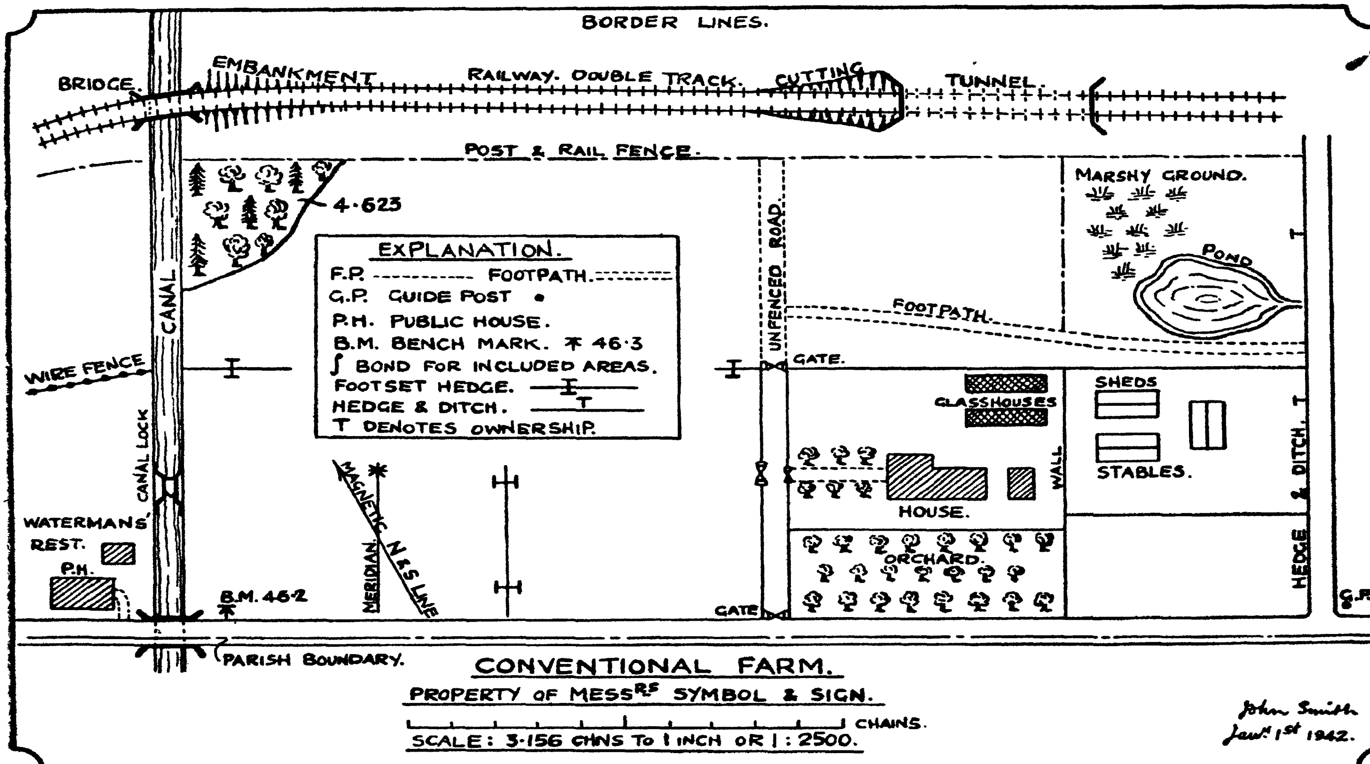
Simplicity is the keynote of modern mapping, and this involves great skill, as the artist cannot hide defects with trimmings. Early cartographic art was characterised by wonderful embellishments, often in rich mezzo-tint, and far more entertaining than the prosaic land and sea: birds, beasts, and equally gigantic men everywhere, even leviathans basking where America is now known to be. Gradually these have vanished like prehistoric monsters, giving place to accuracy in the terrain; in fact, the artistic touch has almost disappeared during the past fifty years. One of the remains is the touch applied at the bottom right corners of tree-trunks, indicating the shadows cast by conventional light coming from the top left-hand corner of the plan. This convention is sometimes used in connection with objects, the lower and right-hand edges being outlined boldly.

Conventional signs are used to indicate features, such as boundaries, roads, buildings, lakes and constructional objects, and, obviously, there is a host of these, certain symbols being varied in accordance with the scale of the map. Many of those in everyday use are shown in the Survey of Conventional Farm, where the ownership suggests that the names are the property of Symbol & Sign (Plate I). The artist never writes "Tree," "Horse," or "Inn" on a painting, though there would certainly be some justification for doing this in certain specimens of modern art. The noun must be used only when duly qualified; as, for example, "R. Medway," "G. Junction Canal," "G.W.R.," "Beverley Brook," etc., etc.

Signs are sometimes modified to resemble the object more closely in large-scale maps, where they become relatively important in the small area portrayed. Thus, walls become double lines, and hedges shoot from the root line. Trees should never be drawn taller than $\frac{1}{4}$ in. or $\frac{3}{8}$ in. When, as in the case of a house and garden, these are actually large to scale, it is better to represent them with small circles, with added verdure, if desirable, but never so as to obscure the ground below.

The title should be printed neatly and compactly in what appears to be the best position. In certain plans there is a more or less fixed position for the title. The divided scale line should always be inserted as well as the statement of the scale for the reasons already mentioned.

PLATE I


BORDER LINES.
BRIDGE.
EMBANKMENT.
RAILWAY. DOUBLE TRACK.
CUTTING.
TUNNEL.
POST & RAIL FENCE.
CANAL
4·623
EXPLANATION.
F.P. FOOTPATH.
G.P. GUIDE POST
P.H. PUBLIC HOUSE.
B.M. BENCH MARK. 46·3
BOND FOR INCLUDED AREAS.
FOOTSET HEDGE.
HEDGE & DITCH.
T DENOTES OWNERSHIP.
UNFENCED ROAD.
FOOTPATH.
MARSHY GROUND.
POND
GATE.
WIRE FENCE
CANAL LOCK
GLASSHOUSES
SHEDS
STABLES.
WALL
HOUSE.
HEDGE & DITCH.
ORCHARD.
WATERMANS' REST.
P.H.
MAGNETIC N & S LINE
MERIDIAN
B.M. 46·2
GATE
G.P.
PARISH BOUNDARY.
CONVENTIONAL FARM.
PROPERTY OF MESSRS SYMBOL & SIGN.
CHAINS.
SCALE: 3·156 CHNS TO 1 INCH OR 1 : 2500.
John Smith
Jany 1st 1942.

The meridian needle is best drawn with a star for the true north, and an arrow-head for the magnetic north. If, as it should be, the map is dated, the magnetic declination can be found from a book, and the true north indicated, thus admitting of additions after the lapse of years. Contour lines are either indicated by alternate dots and dashes, or are traced in sepia, the contour heights being figured on the high sides, or in gaps in the contours.

A border line with an appropriate margin gives a finish to the map; but it is often a finish indeed when a passing student collides with the head or stock of the T-square. A neat margin usually requires double lines about $\frac{1}{16}$ in. apart, and the effort demands great courage when a border-pen is not at hand. A break in the border for outlying details looks far better than one very close to the edges of the paper. Also, rounded corners are often preferred to plain right angles.

Stencil plates are convenient for pencilling the outlines of various features, but, again, these should be small, and letter stencils never of the size used in directing boxes for passage by rail.

COLOURING. This is a subject that must be introduced diffidently, since it may lead to the ruination of a nicely plotted plan. Happily, however, some maps drawn on good paper in waterproof ink have been resuscitated after a necessary immersion in a tank or bath of clean water. Students are exceedingly liberal with colour washes: pastures, green indeed, and soil exceedingly rich, even if the roads are veritable quagmires. The secret is to apply only the faintest suggestion of colour. "Use sparingly," in the words on the labels of certain proprietary articles. Above all, practise on a piece of similar paper first, and always colour before inking when waterproof ink is not available. Incline the drawing-board towards you, inserting wood blocks beneath it. Transfer a pool of colour to the top of the portion to be tinted, and wash the area over rapidly, lightly sweeping the surface, and, above all, avoid brushwork, as in painting domestic objects; garage doors, for instance. Perhaps it is providential that the demand for colouring is declining in modern practice.

The best water-colours are sold in cakes, which are rubbed down in saucers and mixed to give any desired shades, always excluding dirt, dust, and treacherous particles of colour. Numerous conventions are in use, the following being fairly common:

Water. Prussian blue, toned from deep at the banks to faint at the centres of *rivers*, *lakes*, etc. A touch is also applied to the conventional sign for marshes.

Land. Arable, burnt umber or sepia; *Pasture*, Hooker's green, preferably varied in adjoining fields; *Trees*, *Hedges*, etc., in green, but darker.

Buildings. Brick, crimson lake; *Timber*, India yellow.

Roads. Roman ochre.

Property surrounding the portion for which the survey was made is not coloured, but all the conventional signs in black ink are usually inserted.

Since the present chapter deals mainly with Office Work, opportunity will be taken to include certain relevant operations.

III. CONSTRUCTING ANGLES

The most obvious method of constructing angles is by means of the protractor; but it must be borne in mind that the ordinary pattern, say in the 6-in. size, is not sufficiently accurate except for inserting details, and never for plotting the skeletons of theodolite traverses. It would be impossible to plot to nearer than 10 minutes of arc on the 3-in. radius, and when this is extended as a survey line to 12 in., say, the error would be considerable, though admissible in rough compass surveys. Accurate celluloid protractors are the best of this category, as far as constructing angles is concerned. Silver-plated types reflect light, and are particularly disconcerting in examinations, where the protractor is usually allowed for plotting compass traverses. There are, of course, elaborate forms with vernier arms; and there is the cheaper form of cardboard protractor with an 18-in. open circle, as used for plotting bearings. Also there is the scale of chords, which is no more accurate than a small protractor, even though it introduces a highly important method of constructing and measuring angles through the medium of the tables headed "Chord."

There are also the trigonometrical tables, preferably those giving minutes in four- or five-figure trigonometrical ratios. Two methods of constructing or measuring angles must be considered, for they are not only useful in plotting angles, but in constructing and measuring angles in the field when the theodolite is not at hand.

*(1) *Chord Method.* The following method is actually that which would result if a table of chords were included in the tables, as they are in the more precise, such as Chambers' Seven Figure Mathematical Tables, where the values refer to a unit chord. But the unit may be conveniently 10 in. in plotting and 100 ft. in field construction, which merely means that the decimal point is moved respectively *one* or *two* places to the right. Of course centimetres could be used consistently, but centimetres and inches must never be mixed.

Anyway, the sine of an angle is always at hand, and the sine is a kind of half-brother to the chord, as will be seen in Fig. 23, where an angle θ is to be set out at a station A, being measured from Ab.

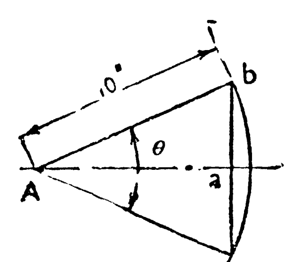


FIG. 23

(1) Set the beam compass at 10 in. exactly, and with A as centre, swing

an arc of convenient length *bc*. (2) Find in the table of chords the unit value for the angle θ, and multiply this by 10 for the length of the chord *bc* in inches. Otherwise, look up the sine of the half-angle, $\frac{1}{2}\theta$, and find the value for the hypotenuse *Ab*, which, being 10 in., gives the chord *bc* as 20 times the tabular value of the sine in inches. (3) Swing an arc with the chord *bc* as radius about *b*, cutting the arc *bc* at *c*. Join *Ac* for the required angle $bAc=\theta$.

The alternative method in (2) follows from the fact that $bc=2Ab.\sin\frac{1}{2}\theta$.

Thus, for $\theta=44° 20'$, $\frac{1}{2}\theta=22° 10'$, and $\sin.\frac{1}{2}\theta=0{\cdot}3773$; whence for a 10-in. (cm.) radius, the chord $bc=2\times3{\cdot}773=7{\cdot}55$ in. (cm.).

In the field, the angle θ could be measured by inserting arrows at *b* and *c* by swinging a 100-ft. radius, measuring the chord *bc*, halving *bc* at *d*, and finding $\frac{1}{2}\theta$ from $bd=Ab.\sin\frac{1}{2}\theta$.

*(2) *Tangent Method.* Although applicable to angles, the principle is *par excellence* in plotting the bearings in traverse surveys, a subject that will be treated with reference to the compass in due course. The method involves the table of tangents, and is applied with a base of 10, the base now taking the place of the hypotenuse. Consider the method with reference to the closed traverse *ABCD* of Fig. 24.

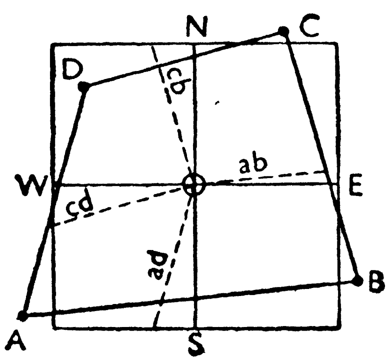


FIG. 24

(1) Through a point *O* in the centre of the paper, rule a vertical line and a horizontal line, N.S. and W.E. respectively; and make $ON=OS=OW=OE=10$ in., and draw parallels so as to form an outer square of 20 in side exactly.

Obviously, on a half-imperial sheet, centimetres must supersede inches unless some larger convenient decimal scale is at hand. Assume the central vertical to be a north and south line, the four interior squares representing the four quadrants: N.E., S.E., S.W., N.W.

Draw up a table showing the *Traverse* Lines, their *Lengths* and *Bearings*, also the *Tangents* of bearings under 45° and the *Co-tangents* of bearings over 45°.

Incidentally, a *bearing* is under 90°, and is measured from the N. or the S. point, being defined by N. or S. in front of the magnitude and E. or W. following, as described on page 90. Also the tangent of 45° is unity, so that when a bearing exceeds this value, the complement, the co-tangent, must be introduced: Cot. $\beta=\tan(90°-\beta)$.

(3) Plot the direction line for each traverse line in its proper quadrant, measuring ten times the tabular value of the tangent or co-tangent along the outer side of a small square, and joining the point thus found

to the centre of the large square. Tangent distances are scaled outwards from the north and south points on the upper and lower sides respectively, while co-tangent distances are scaled from the east and west points, upwards or downwards, accordingly.

(4) Draw the parallel to each direction line in its correct position on the paper, using the clinograph or adjustable T-square.

Whole circle bearings (or azimuths), styled bearings in military surveying and applied geography, are plotted in a similar manner, the lines falling within quadrants which exhibit the angles as bearings proper, values under 90° being expressed by β, over 90° as $180°-\beta$, over 180° as $\beta-180°$, and over 270° as $360°-\beta$, corresponding respectively with N.E., S.E., S.W., and N.W. bearings.

IV. ENLARGING AND REDUCING MAPS

Frequently enlarged or reduced copies of maps and plans are required. In practice, this is usually done mechanically, with the aid of the *pantagraph*, or *photographically*, as in photo-engraving. When the necessary instruments are not available, recourse must be made to *Graphical Methods*, the best known of which is that of (1) *Proportional Squares*, though often the method of (2) *Angles and Distances* serves as an excellent substitute.

(1) *Proportional Squares*. This method consists in covering the original map with a network of squares (otherwise called a grid or graticules), either actually on the map or on a superimposed sheet of tracing-paper. These squares are then reproduced proportionally larger or smaller on a clean sheet of paper, and the lines of the survey are inserted with reference to the sides of the squares by plotting distances in the proportions they bear to the sides of the squares of the original. In a great many cases of enlargement and reduction the scale of the copy is either a simple multiple or sub-multiple of the scale of the original, and the squares of the original can be made a convenient mapping unit, 1 in., say, while those of the copy will be simply so many inches, or so much of an inch; say, 2 in. and ½ in. respectively. But cases constantly arise in which the given statements or representative fractions, or both, and the squares of the copy involve complex fractions of the mapping unit. The same inconvenience arises when squares with sides representing chains, hundreds of feet, or other even units of measurement, are used instead of convenient mapping units, inches, centimetres, etc. It is always advisable to ascertain if one or other of these units, a mapping or a field unit, will lead to simple square dimensions in both the original and the copy; for various simple relationships, not evident at sight, are often discovered by such procedure. As a rule, field units are to be preferred when mapping units introduce squares of equally inconvenient dimensions; but even they cannot be considered unless suitable direct relationship to inches, or

simple parts thereof, exists in one or other of the given statements or representative fractions, as the case may be.

Let us consider the matter with reference to the three cases that may arise:

(*a*) *When Statements are Given.* Suppose it be required to enlarge a map from 1 chain to 1 in. to 40 ft. to 1 in. Here 1-in. squares on the original will require 66/40, or 1·65-in. squares on the enlargement. Thus, both convenient mapping and field units are inherent in the smaller scale. But if reduction from 5 ft. to the mile to 4 ch. to 1 in. is required, 2-in. squares, representing 2·66 chs. on the original will necessitate 0·66-in. squares on the copy, while 2 chs. represented by 1½ in. on the original, will merely require ½-in. squares on the copy.

(*b*) *When Representative Fractions are Given.* Suppose it be required to reduce a portion of the Ordnance 1 : 1,056 sheet to the engineering scale of 1 : 1,200. Here feet are the units in view, although the original scale is directly related to chain units, 1 in. representing 88 ft., or ¾ in. one chain. But all relations between the given scales will introduce fractional dimensions in the squares of either the original or the copy, and, in general, squares representing 100 ft. on the copy would be preferred to 1-in. squares on the original.

(*c*) *When Representative Fractions and Statements are Given.* Let it be required to enlarge the 25-in. Ordnance sheet to a scale of 100 ft. to 1 in. Here 1 : 2,500 corresponds to a scale of 25·344 in. to the mile, or 1 in. to 208·296 ft., and since one of the given scales is simply connected with hundreds of feet, the enlargement can be made with equal facility with either field or mapping units. Thus with 1-in. squares

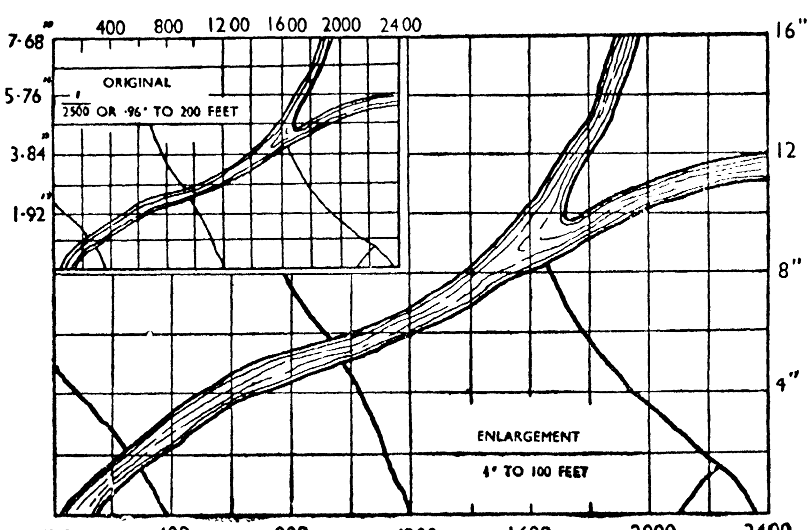


Fig. 25

or the original, squares of 2·08 in. side will be required on the copy, while 2-in. squares representing 200 ft. on the copy will require squares of 0·96 in. side on the original. The latter case is illustrated in Fig. 25.

Once the original map and the copy sheet have been covered with suitable squares, the plotting is quite simple, the intersections of fences, etc., with the sides of the squares, and the positions of points, etc., in the squares, being judged by eye with reference to the corners. As in all graphical methods, the use of proportional compasses facilitates plotting and raises the accuracy of the work.

One of the slotted limbs of these double-pointed compasses is graduated for a series of proportions between opposite pairs of points, the compasses being set for the desired proportion by changing the sliding block so that the index line coincides with the mark figured with that proportion on the graduated limb.

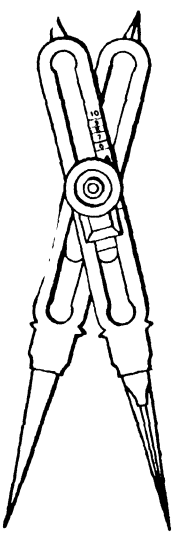


FIG. 26

(2) *Angles and Distances.* The following method is particularly suitable for areas in which important detail is sparse or localised, and the accuracy of reproduction is highly important.

Describe a circle of any convenient size in the centre of the area to be enlarged or reduced, and through its centre *o* draw a reference meridian *ns*, and rays to important points, such as *p*, *q*, and *r*. If necessary, produce these rays to cut the circle in *a*, *b*, and *c* respectively.

Describe a circle of the same radius on a clean sheet of paper, and insert the meridian.

The copy is assumed to be superposed over the original in Fig. 27, capitals superseding the small letters.

Measure the chord distances *na*, *nb*, *nc*, etc., on the original, and set them off as *NA*, *NB*, *NC*, etc., on the circle of the copy. Draw rays through *A*, *B*, *C*, etc., in the latter, and along these rays set off the computed distances of the selected points *P*, *Q*, and *R* in the proportion that the scale of the copy is greater or less than the scale of the original. Having thus fixed the *ruling points*, fill in the intervening detail by eye.

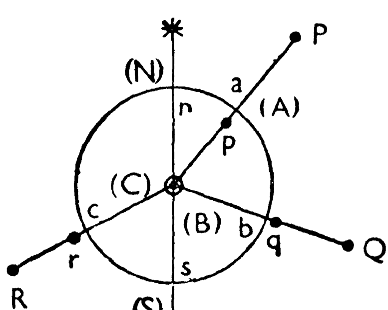


FIG. 27

The use of this method is not advised, unless a simple proportion, or one readily obtained with proportional compasses, exists between the scale of the original and the copy.

CLASS EXERCISES

3 (*a*). The 1 : 10,000 Service Map of France is to be used in the following connections:

(*a*) Laying Decauville track with measurements both in metres and feet.

(*b*) Reconnaissance, with pacing at the rate of 100 paces of 30 inches per minute.

Construct the respective "comparative" and "time" scales.

3 (*b*). Construct the following scales to the representative fraction of 1: 1250:

(*a*) Reading to 10 ft., with main divisions of 100 ft.

(*b*) Reading to single metres, with main divisions of 10 metres. (G.S.)

(0·96 in. to 100 ft.; 0·8 cm. to 10 m.)

3 (*c*). A survey map dated 1860 is stated to be on a scale of 4 chains to 1 inch, although no scale is drawn. Believing that the paper had shrunk considerably, a surveyor found two prominent points on the map that are still existing: he measured the distance between these and found it to be 15·39 chains whereas it scaled only 15·20 chains on the map.

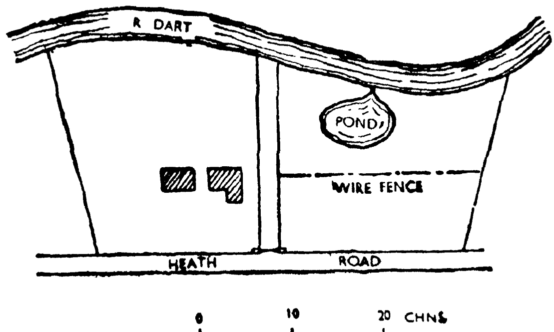
Construct a scale for the old map, suitable for measuring lengths up to 20 chains. (G.S.)

(20 ch. represented by 4·94 in.)

3 (*d*). The scale of an old French map is 1,000 toises to 1 French inch. You wish to copy the map on the scale of 1 mile to 1 inch by the method of squares. If you draw ¼-in. squares on the old map, what size must they be on the new one, given that 1 toise was 72 French inches. Draw a scale of yards for the new map (0·284 inches.)

3 (*e*). Enlarge the plan shown in Fig. 27 (*e*) to a scale twice the size of that of the figure.

FIG. 27 (*e*)



OFFICE EXERCISES

Problem 3 (*a*). Plot the survey from the notes given on Plate II. (G.S.)

Problem 3 (*b*). Plot the survey from the notes given on Plate III. (G.S.)

Problem 3 (*c*). ditto Plate IV. (G.S.)

Problem 3 (*d*). Plot your survey of (specified area) and finish it in the prescribed manner.

Problem 3 (*e*). Enlarge the specified portion of the assigned map to a scale of . . .

ORIGINAL PROBLEMS

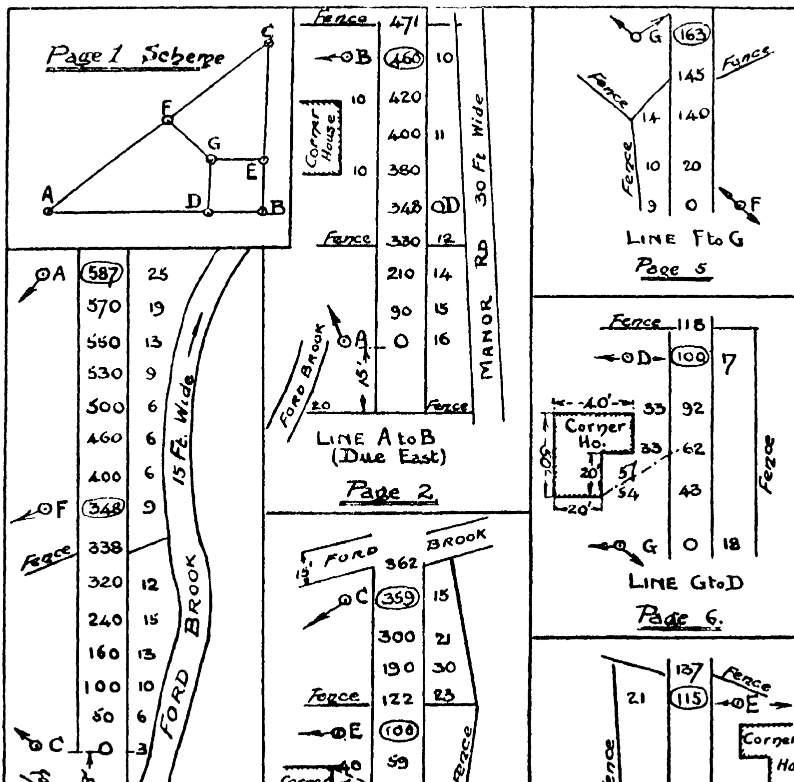
. . . .

. . . .

. . . .

. . . .

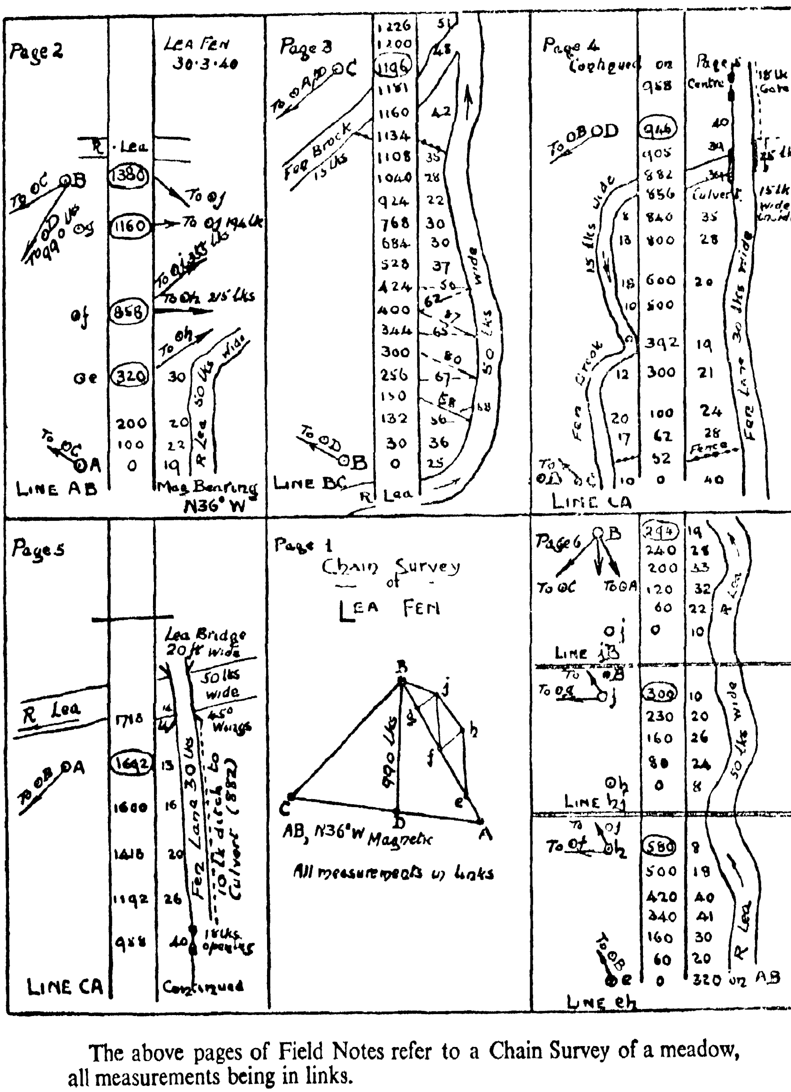
PLATE II



The above pages of Field Notes refer to a survey in which only the chain, tape, and range-poles were used, *all* measurements being in feet.

Plot the survey on a scale of 50 ft. to 1 inch.

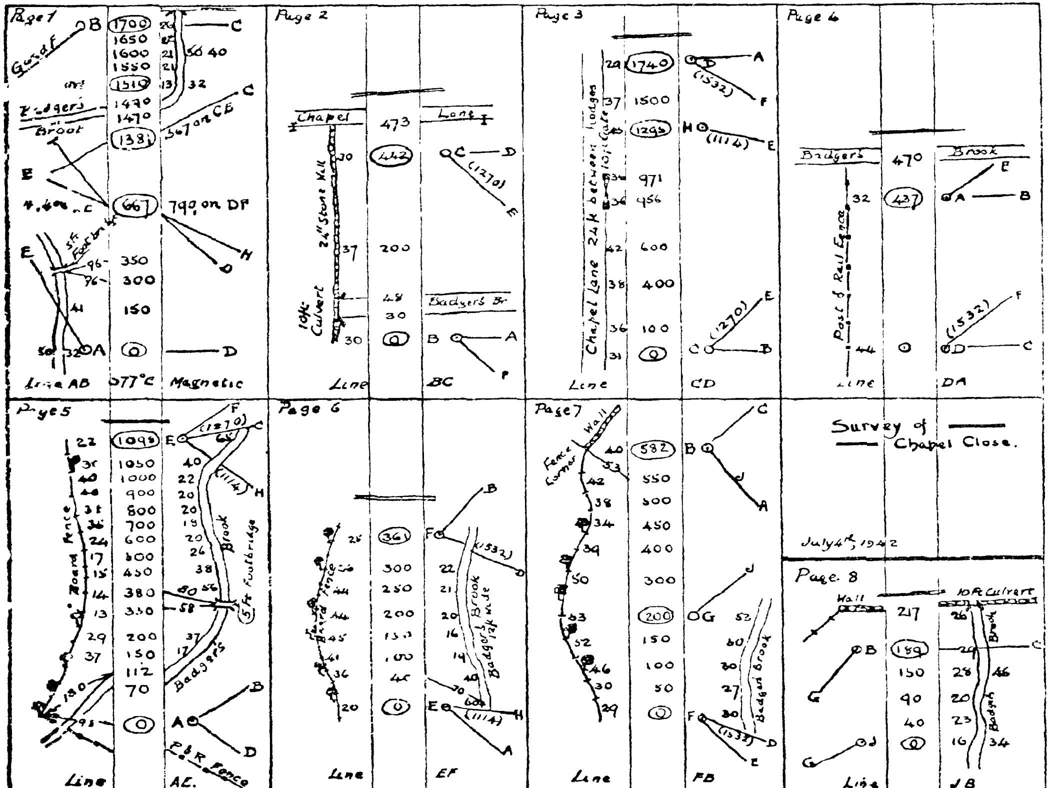
PLATE III



The above pages of Field Notes refer to a Chain Survey of a meadow, all measurements being in links.

Plot the survey on a scale of 1 chain to 1 inch, placing the Magnetic N. and S. line parallel to the short edges of the paper with A $2\frac{1}{2}$ inches from the lower and right-hand edges.

PLATE IV



The following notes refer to a chain survey of Chapel Close, the northern boundary of which is outside the detour of a brook, 12 feet in width, *the inner side* of the detour being exceedingly marshy.

Plot the survey on a scale of 1 chain to 1 inch, placing the *true* north at the top of the sheet, so that the lane runs along the bottom. Finish with title, scale, meridian needle and border. All measurements are in links, and the magnetic declination was 130° W.

N.B. Since a well-finished plan is required, the scheme of the survey lines should be sketched before the plotting is begun.

CHAPTER IV

FIELD GEOMETRY

It may be well at this stage to consider a number of problems, some of which you may have already encountered in the field; and the title of the present chapter must be understood to include also the applications of geometrical principles in dealing with obstructed survey lines, more commonly, however, when only the chain, tapes, and poles are at your disposal.

The subject of ranging out survey lines strictly should have preceded their measurement, though, on the other hand, many cases of ranging out lines become matters of obstructed distances. There is no geometry in the following artifice.

Reciprocal Ranging. It often happens that a hill or high ground intervenes so that the end stations, A and B, are not visible from each other; and it is necessary to interpolate pickets in the line AB in order to guide the chainmen. Reciprocal ranging is also useful in interpolating additional stations in a survey line without going to the end stations, A or B, in order to direct the boning-out. It is also convenient on level ground in foggy weather when the station poles can be seen for about only five-eighths of the distance AB.

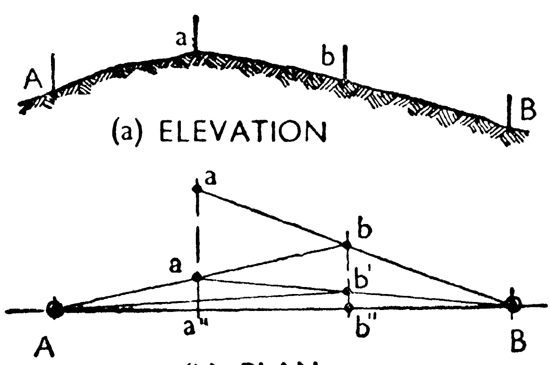


FIG. 28

Observers a and b, each holding a picket, place themselves on the ridge of the hill, in the line between A and B as nearly as they can guess, and so that one can see the other and the station beyond him. Observer a looks to b, and by signals, puts b's picket in line with B. Observer b then looks to A, and put a's picket in line at a'. Observer a repeats his operation from a', and is then himself moved by b to a_1 (not shown). In this manner the two alternately line in each other, gradually approaching the straight line between the stations A, B, till at last they find themselves exactly in it at a'' and b'', as shown in Fig. 28 (b).

FIELD GEOMETRY. The primary operations in field geometry consist in (i) *Laying down Perpendiculars*, and (ii) *Running in Parallels*. The work presents little difficulty when a theodolite is available, though there are instances when even this would be of little use, and the surveyor must resort to purely linear methods. Underlying the use of linear methods is the fact that right angles must be reduced to a minimum, since these can never be set out precisely with auxiliary instruments, such as the optical square or the cross staff, while the construction of right angles is a tedious matter with the chain alone, particularly when these are but part of a method. These facts should be borne in mind also when measuring obstructed distances.

I. PERPENDICULARS

Perpendiculars may have to be (*a*) *Erected at given points in Survey Lines*, or (*b*) *Let fall from given external points to Survey Lines*.

These cases will be considered separately, p and p' denoting respectively *internal* and *external* points with respect to a survey line AB.

(1) The 3 : 4 : 5 Method. It is almost a matter of propriety that the subject should be introduced by the application of Pythagoras' Theorem, introducing the fact that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. The longest perpendicular is usually required, and this can be laid down with the following combinations of the available unit lengths: (15—20—25) ft. with the 50-ft. chain; and (33—44—55) ft./lk. with the 100 ft./lk. chain.

Commonly, however, the method is applied in the following manner with the Gunter chain (Fig. 29):

(*a*) *Internal Points*. (1) Measure from the given point p a length of 30 lks. (ft.) along the survey line AB to a point q. (2) Hold one handle of the chain at p and at q its 90-lk. (ft.) ring. (3) Pull out the chain evenly by the teller 40 lks. (ft.) from p until it takes up the form shown in Fig. 29. (4) Fix an arrow at p', a point in the required perpendicular.

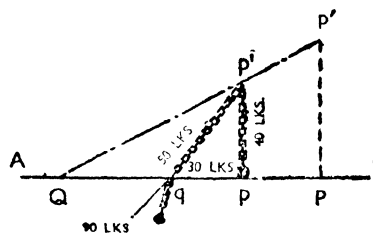


FIG. 29

(*b*) *External Points*. The method is not directly applicable to this case, and would involve calculation by similar triangles, as in the following cumbersome construction:

(i) Erect a perpendicular pp', as above, at any convenient point p in AB. (2) Line in a point Q in AB with p' and the given external point P'. (3) Measure the sides pQ, $p'Q$, and $P'Q$, and calculate the position of P, the foot of the required perpendicular from $QP=\dfrac{P'Q\times pQ}{p'Q}$

The following methods, though detailed with reference to the chain and its limitations, have a wide range of application when long lengths of cord or wire are used.

As in the foregoing method, the steel tape cannot be subjected to sharp bends, and on this account two steel tapes would need to be tied together. Also wires or cords could not be used in the 3 : 4 : 5 : method, since figured lengths are required. But in the following, light steel wires or strong cord (such as sea-fishing line) or (in an emergency) the linen tape, can all be used to advantage, provided a safe and uniform pull is applied. In fact, a small spring balance is desirable in precise work, care always being that the elastic limit is not passed. The chief advantage is that lengths may be bisected by merely doubling the cord or wire back upon itself. Various methods must be improvised for marking lengths. For instance, an electric wiring connector may be taken from its porcelain container, sawn in two, and each half secured in its temporary position on the wire by means of a pocket screwdriver. One index secured in the middle of the whole length, and an adjustable one on each side are desirable. Small pipe clips, as used in chemistry, are convenient in the case of cord lines. Loops should be made at the ends, and the total length made a convenient multiple of a standard length, carefully tested. The erecting of a long perpendicular at a point in a straight wall becomes a simple matter, and lengths up to 300 ft. can be laid down expeditiously and accurately; but in all accurate work arrows should be lined in, either by eye or with the aid of the stretched cord or wire.

(2) *Chord-Bisection Method.* In general, this is the best linear method of (*a*) erecting perpendiculars at given points in survey lines.

(*a*) *Internal Points.* (1) Hold or secure the ends of the chain in the survey line *AB* at *a* and *b*, points 15 to 40 ks. (ft.) equidistant from the given point *p*. (2) Pull out the chain evenly, and fix an arrow at the 50-teller to indicate *p′*, a point on the required perpendicular.

When long perpendiculars are required, it is advisable to lay down a second triangle, such as *acb* or *ac′b*, so as to obtain a second point on the perpendicular (Fig. 30). This, of course, applies more particularly when working with 200 ft., etc., lengths of wire or cord, and in this case an outer index should be set at a convenient distance for the loop from the lengths *ap*, *bp*.

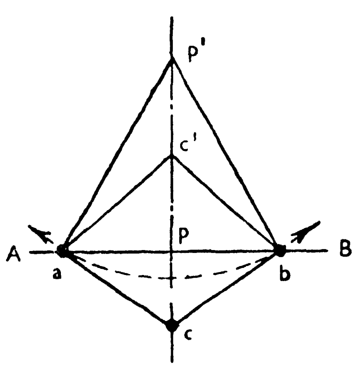


Fig. 30

(*b*) *External Points.* (1) With one end of the chain at the external point *p′*, swing an arc about *p′*, cutting the survey line *AB* in the points *a* and *b*. (2) Measure the length *ab*, and bisect it at *p*. Otherwise obviate measurement by laying down a second triangle (by the preceding method), preferably on the opposite side of *AB*, as *acb*, and lining an arrow at *p* with those at *p′* and *c* (Fig. 30)

This method is in effect, the process of swinging the tape "to and fro" in measuring short offsets, the lowest readings on tape and survey line being at the point of tangency to the arc. Incidentally, the length of the chain or tape should at least be 5 per cent greater than the length of the required perpendicular.

(3) *Semicircle Method.* In general this is the best linear method of (*b*) laying down perpendiculars from given external points, and though it might be extended to Case (*a*), there is little to commend such practice under ordinary circumstances.

The construction is based upon the theorem that the angle in a semicircle is a right angle.

(*b*) *External Points.* (1) Select a point *a* in the survey line *AB* so that the line is at an angle between 30° and 60° with the direction of the given external point *p'*. (2) Measure the distance *ap'*, and bisect it at the centre *o*. (3) With one end of the chain held or secured at *o*, swing an arc of radius *op'*, cutting the survey line at *p*, the foot of the required perpendicular (Fig. 31).

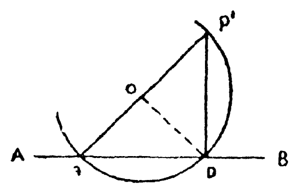


FIG. 31

II. PARALLELS

A parallel may be required (*a*) *through a given external point*, or (*b*) *at a given distance from the survey line AB.*

The second case reduces to the first by setting off the given perpendicular distance *pp'* from *AB*, preferably as a part of the construction.

(1) *By Right-angle Offsets.* (*a*) *Through Given Points.* The following method would be inapplicable to the present case with a theodolite, since it involves the dropping of a perpendicular from a given point; and the method of alternate angles would be used, as with all angle-measuring instruments. In lower grade work it is easily effected by trial with the optical square or the cross staff, or the chain might be used alone, though the setting-out of two right angles should be the limit in this, as in other constructions.

(1) Let fall a perpendicular from the given point *p'* on to the survey line *AB*. (2) At any convenient point *q* in *AB*, erect a perpendicular *qq'*, making *qq'*=*pp'*. The line through the points *p'* and *q'* is the required parallel. (3) Test the accuracy of the construction by the equality of the diagonals *qp'*, *pq'* (Fig. 32).

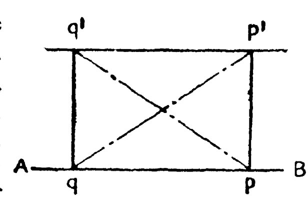


FIG. 32

(*b*) *At Given Distances.* The following construction is expressly suited to the use of the theodolite, sextant, etc., and the cross staff and optical square may be used likewise, except in accurate work. Occasionally it might be the best method when only the chain and arrows are at hand, but it has the defect of requiring two right angles.

(1) Erect perpendiculars at convenient points, p, q, in the survey line, and along each of these set out the given distance pp' and qq' of the required parallel. (2) Test the accuracy of the construction by erecting a third perpendicular of the same length, and noting the alignment of its extremity with the points p' and q'.

With the theodolite, only the perpendicular pp' might be erected and a perpendicular to pp' set out at p', thus making the alternate angles each 90° in the following method.

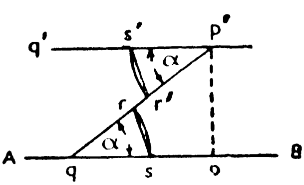


FIG. 33

(2) *By Alternate Angles.* This purely angular method should supersede all other methods in the case of (*a*) *parallels through given points* when an angle measuring instrument is at hand. A convenient point q is selected in AB, and the angle $pqp' = \alpha$ is measured. Then at p', the angle $q'p'q$ is set out equal to α. (*b*) *At a given distance* pp', a right angle to AB is set at p, and at p' another right angle, $pp'q'$, would be measured off, giving the parallel $p'q'$ (Fig. 33).

The method might be effected by purely linear means by swinging arcs about q and p' with 50-ft. or 100-ft. radii, and then measuring the chord $r's'$ equal to the chord rs, thus tying the alternate angles α. It would seldom be, if ever, used in laying down a parallel with the chain or tape at a given distance from AB.

(3) *By Similar Triangles.* The use of similar triangles is recommended when only the chain and poles are available.

(*a*) *Through Given Points.* (1) From a convenient point q in the survey line AB, measure the distance to the given point p', and bisect qp' at o. (2) From another convenient point r in AB, run in a line through o to s', making os' equal to ro. The line through p' and s' is the required parallel (Fig. 34). If it is inconvenient to bisect qp' at o, select o appropriately and measure qo and op', also ro, and then make os' equal to $\frac{or \times p'o}{qo}$. This also applies to the case when the point p' is not very distant from the survey line AB. In this case the point o is selected outside the parallels, and is the common apex of similar triangles, whose bases are on the survey line and the parallel. Again, it is convenient to swing an arc about o so that the triangles are isosceles (Fig. 35).

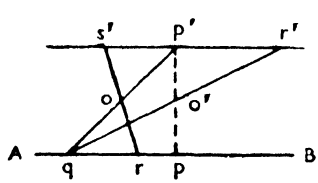


FIG. 34

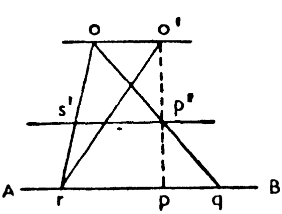


FIG. 35

(*b*) *At Given Distances.* (1) At any given point p in AB, erect a perpendicular pp'

equal in length to the given distance, thus fixing the point p'. (2) Then proceed as before, running the line from q through o' to r', making $o'r'$ equal to qo'. The line through p' and r' is the required parallel. The use of an external apex o' is dealt with on similar lines.

III. MEASURING ANGLES BY LINEAR METHODS

Contingencies often arise when it is necessary to construct or measure angles in the field; and in the absence of a theodolite, this may often be done with sufficient accuracy by means of the tape, with the aid of a table of sines or chords, as described on page 39. Imagine yourself in charge of setting out certain engineering works, and the theodolite has not arrived, no tables are at hand, and the foreman is none too amiable because his men are held up for the want of an angle to fix a direction. The science of surveying is to know fundamental principles, and the art, to apply them with dexterity and accuracy in any emergency. You know that a *radian* is the angle subtended at the centre of a circle by an *arc* equal in length to the radius of the circle, the angle thus being 57·3°. Hence, if a radius of 57·3 ft. (lks.) is described about the angular point A, the length around the arc will be the magnitude of the angle in degrees and decimals. Suppose that the foreman requires a direction fixed by an angle $\theta = 36° \ 15'$. Well, merely centre the ring end of the tape on A, and swing an arc from r with a radius of 57·3 ft. (lks.), at the same time inserting arrows (or clean twigs) around the arc rs; then measure carefully round the arrows from r with the tape, and fix s at a distance of 36 ft. 3 in. (36¼ lks.) so as to give the required direction AC.

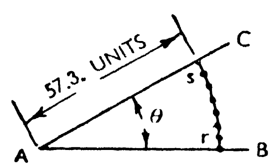


FIG. 36

IV. OBSTRUCTED DISTANCES

Although the stations of a survey should be selected so as to avoid obstacles as far as possible, a survey line of great importance, even if it be obstructed in some way, must not be discarded for others less suitable to the general scheme. In chain surveys, particularly, it often happens on hilly ground that the end stations of a line are readily intervisible, but when the chainmen work into a hollow they find themselves confronted by a pond or even a building. Also an essential extension of a survey line may introduce similar difficulties.

Now obstacles may (i) impede the chaining of a survey line only or, in addition, (ii) prevent the alignment or prolongation of the line.

Impeded chaining means that a geometrical construction must be resorted to in order that the distance may be determined, while, in addition, *broken alignment* will require additional construction in order that the direction of the line may be re-established after the obstacle is passed. *Broken alignment* requires that two points shall be established beyond the obstacle, or one point and an angle shall be likewise fixed, the angle often being a right angle.

Detached, or Isolated Obstacles, are of two classes: (*a*) those which impede chaining only, such as ponds, lakes, and low plantations, and (*b*) those which impede both measurement and alignment, such as buildings and woods.

Continuous Obstacles likewise fall into two classes: (*c*) those which impeded chaining only, such as rivers and canals, and (*d*) those which impede both measurement and alignment, such as high boundary walls and blocks of buildings.

The first two classes (*a*) and (*b*) introduce the same basic constructions, and may therefore be treated together, keeping in view the fact that (*b*) will require one extra point or an angle, in order that the line may be continued on the far side of the obstacle. Classes (*c*) and (*d*) differ essentially, and must therefore be considered separately.

The best-known methods will be considered with reference to an obstructed survey line *AB*, *A* being on the "working" side of the obstacle and *B* on the "distant" side, as suggested by progress from *A* to *B*. Also right angles will be blacked in, or otherwise indicated, in the diagrams.

Obstacles of Classes (*a*) *and* (*b*). (1) *By Right-angle Offsets.* This method is best adapted to close sites, one side of the obstacle being impassable. Its use at once suggests the theodolite, particularly in precise work, since a number of right angles are involved, and these cannot be set out very accurately with the cross staff or optical square.

(*a*) Erect *ab*, *dc*, equal perpendiculars to the survey line *AB*, at points *a* and *d* in that line, on opposite sides of the obstacle. Measure *bc*, which should be equal in length to the obstructed distance *ad*. If necessarily carried out with the chain and range-poles only, some check will be desirable; say, for instance, prolonging *bc* to *e*, and comparing the lengths of the diagonals *ed* and *cf* after having measured *df* equal to *ce* (Fig. 37).

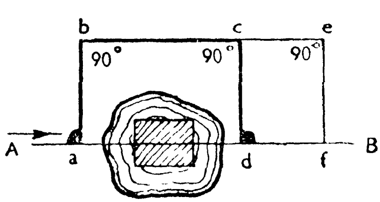


FIG. 37

(*b*) At *a* erect *ab*, a perpendicular to the survey line *AB*; at *b*, set out *bc*, a perpendicular to *ab*; at *c* erect *cd*, a perpendicular to *bc*, measuring *cd* equal to *ab*; and finally re-establish the alignment at *d* by setting out *dB* perpendicular to *cd*. Otherwise, or as a check, obtain a second point *f* on the required prolongation by producing *bc* to *e*, erecting *ef* perpendicular to *ce*, and measuring *ef* equal to *cd*.

Since it is never advisable to set out two consecutive right angles by linear methods, this construction should not be attempted with the chain and poles alone.

(2) *By One Random Line.* Though best modified to the interpolation of points in woods, etc., this method can be used in fairly close sites when the necessary deviation from the survey line AB is not great. The necessary right angles should never be set out with the chain alone, though in average work the optical square and cross staff may be used. It is modified to a purely linear method by running a second random line, which requires that the obstacle shall be passable on both sides.

(*a*) Select a point e as the apex of a right angle included between two lines that meet the survey line AB, one on each side of the obstacle. Sighting from e at a point a in AB, set out a right angle, thus fixing the point d on the distant side of the obstacle. Measure the lengths ae, ed, and calculate the obstructed distance from $ad = \sqrt{(ae)^2+(ed)^2}$. (Fig. 38.)

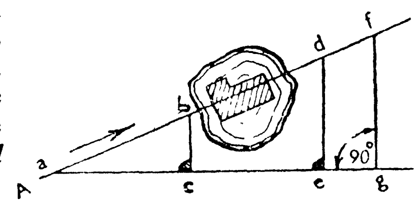


FIG. 38

(*b*) From a, a point on the working side, run a random line ag; and at c, a point in it, erect a perpendicular to meet the survey line at b, a point also on the working side of the obstacle. Measure ab, bc, and ac, and calculate the values of ab/ac and bc/ac. At e and g, points in the random line, ae and ag units respectively from a, erect perpendiculars ed, gf, having calculated their respective lengths from $ae.cb/ac$ and $ag.cb/ac$. The points d and f thus determined are on the required prolongation of the survey line, and the impeded distance, ad or af, is calculated from $ad=\frac{ab}{ac}.ae$ or $af=\frac{ab}{ac}.ag$ accordingly.

(3) *By Equilateral Triangles.* This purely linear method commends itself by its simplicity, though it is somewhat extravagant of space and time, as regards obstacles of Class (*a*). Also, it demands extreme care in prolonging the angular ties when applied to obstacles of Class (*b*), where otherwise it is a most useful method (Fig. 39).

(*a*) On the working side of the survey line AB, lay down the side ac of an equilateral triangle abc of side L, L being 50 ft., 66 ft., or any convenient unit. Produce ab, one of its sides, to d, a point conveniently clear of the obstruction, and on de lay down a tie triangle, also of side L, and produce the side df to meet the survey line AB on the distant side of the obstacle at a point h. Then $ad=ah$.

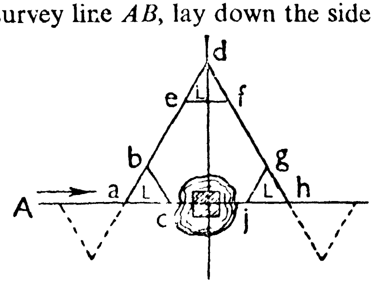


FIG. 39

(*b*) Proceed in the above manner, and produce df to h, making dh equal to ad. Then on gh as base construct an equilateral triangle ghj

of side *L* (if possible) in order to obtain a second point *j* on the distant side of the obstacle. Re-establish the direction of the line by sighting through *j* and *h* in the direction thus given to *B*.

The small triangles may be laid down on the other side of *AB*, if more convenient, as indicated by the dotted lines in Fig. 39.

(4) *By Two Random Lines.* Although more complicated than the preceding method, the following construction, also wholly linear, gives a stronger figure, but involves calculations and also requires that the obstacle be passable on both sides (Fig. 40).

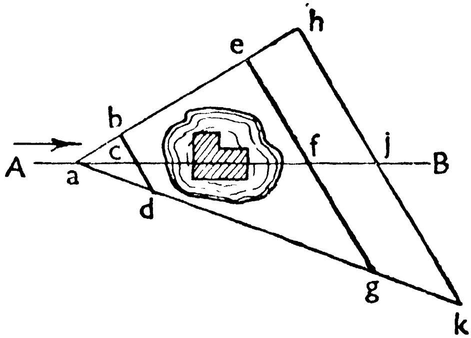


FIG. 40

(*a*) From a point *a* in the working side of the survey line *AB*, measure the lines *ae*, *ag*, one on either flank of the obstacle, and conveniently beyond it. Line in with the points *e* and *g* the point *f* in *AB* on the distant side of the obstacle. From a point *b* in *ae* on the working side, lay down a parallel to *eg*, calculating the position of *d* in *ag* from $ab/ae = ad/ag$. Measure along *AB* the length *ac*, and calculate the impeded distance *af* from either $(ae/ab)ac$ or $(ag/ad)ac$.

(*b*) Run in the lines *ae*, *ag*, as in the preceding case, and produce them conveniently. From a point *b* on the working side of the obstacle, measure any convenient line across to *d* in *ag*, noting the reading where the survey line is crossed at *c*. Measure also *ab*, *ac*, and *ad*; and calculate the value of ab/ad and either bc/ab or cd/ad. Measure from *a* along *ag* to any two convenient points *g* and *k*; calculate $ag(ab/ad)$ and $ak(ab/ad)$, and measure these distances from *a* along *ae* to the points *e* and *h* respectively. Determine *f* and *j*, two points on the required prolongation, by measuring either from *e* and *h* along *eg* and *hk*

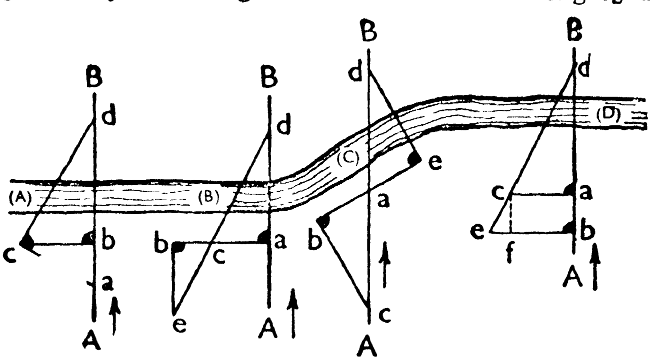


FIG. 41

distances respectively equal to $ae\,(bc/ab)$ and $ah\,(bc/ab)$, or in the opposite direction, distances equal to $ag\,(cd/ad)$ and $ak\,(cd/ad)$ respectively. The obstructed distance af is equal to either $\frac{ac}{ab}.ae$ or $\frac{ac}{ad}.ag$.

Obstacles of Class (*c*). The four constructions shown in Fig. 41 are based upon the relations of similar triangles, and are expressly applicable to continuous obstacles of the present class. Each requires two right angles, which in ordinary work might be set out with the optical square or the cross staff or by means of the chain.

Method (*A*). At a point *b*, erect a perpendicular *bc* to the survey line *AB*. At *c*, lay down a perpendicular to the visual line *cd* to meet the survey line at a point *a*. Measure *ab* and *ac*, and calculate the impeded distance from $ad=\frac{(ac)^2}{ab}$ (Fig. 41A).

Method (*B*). At a point *a* erect *ab*, a perpendicular to *AB*, and in it determine a point *c*, visually in line with the point *e*, the mid point of *ab*, and a point *d* on the distant side of the obstruction. Measure *be*, a length equal to the impeded distance *ad* (Fig. 41B).

Method (*C*). If the survey line *AB* is at an angle to the river, lay down *ab* at a convenient angle and produce it backwards, making *ae* equal to *ab*. Erect a perpendicular to this line at each of its extremities *b* and *e*, and determine where each of these lines intersects the survey line; namely, the points *c* and *d*. Measure *ac*, a length which is equal to the impeded distance *ad*. (Fig. 41C).

Method (*D*). Erect a perpendicular of length *ac* at a convenient point *a* in the survey line *AB*. Erect a perpendicular at *b*, another point in *AB*, and in this perpendicular find a point *e* in line with *c* and a point *d* in *AB* on the distant side of the obstacle. Measure *ab*, *ac* and *be*. Then

$$ad=\frac{ac\times ab}{be-ac} \text{ (Fig. 41D).}$$

Instead of perpendiculars at *a* and *b*, these may be parallels at any convenient angle, the same expression holding for the length *ad*.

Obstacles of Class (*d*). This class includes the most difficult cases that arise in land surveying: obstacles that in many cases may be essayed with the theodolite or compass, though not always expediently, and, failing these, must be negotiated by some artifice especially adapted to the circumstances.

(1) *High Boundary Walls*. Obtain a piece of ¾-in. board, about 9 in. wide and at least 4 in. longer than the thickness of the wall. Fix two 2-in. × 1-in. battens on the underside, close along the short edges, and along the centre line, parallel to the long edges, drive 4½-in. wire nails straight up through each batten to serve as a pair of sights.

Using a ladder, place the board on the coping course with the wall between the battens and the nail points uppermost. From the distant side of the wall, sight the pickets in the direction of *A*, and shift the board until the sights are both exactly in line with the pickets. Secure the board in this position by means of wood wedges. Now with the ladder on the working side of the wall, instruct the chainmen to fix pickets on the distant side, lining them in towards *B* by means of the nail sights.

The objection to the method is that the line is prolonged through the medium of plain sights seldom more than 15 in. apart.

A more accurate, though laborious, method would be to procure a straight scaffold pole, 20 ft. long, and, with the aid of a cord stretched centrally down the length of the pole, to insert four picture rings; one near each end and one about 7 ft. from each end. Balance the pole across the wall with the rings downwards, and suspend a plumb-bob from each of the rings, *a*, *b*, *c*, *d*, say. In windy weather it will be necessary to damp the vibrations of the bobs by immersing them in buckets of water. Next, standing back to the wall on the working side, look towards *A*, and instruct the chainmen to move the pole until two adjacent plumb-lines, *b*, *a*, come exactly into line with the pickets already inserted. Finally, go to the distant side of the wall, and with the back to it sight through the other plumb-lines, *c*, *d*, and direct the chainmen to fix pickets in the direction of *B*.

(ii) *Two High Walls enclosing Roads, etc.* When two walls of about the same height, and no great distance apart, cross the survey line, it is often possible to bone out by "mutual ranging." Observer on wall *M* puts his pole and that of an observer on the wall *N* in line with the pickets interpolated on the working side from *A*. Observer on *N* then sights in poles towards *B* on the side beyond *M* in line with his pole and that held by the observer on *M*.

Much art has been lost, not in surveying alone, by evading obstacles.

CLASS EXERCISES

4 (*a*). Show with reference to neat sketches what you consider to be the best method of dealing with each of the following obstructions in a survey line, when only the chain and poles are at your disposal:

(*a*) *Pond, passable on one side only.*
(*b*) *Isolated building.* (G.S.)

4 (*b*) (*a*). You are surveying in foggy weather, and it is possible only to see the pole at *B* for five-eighths of the distance *AB*. Describe, with reference to a sketch, how you would proceed to measure the line *AB*.

(*b*) In the same survey the line *FG* must cross a river 35 yds. in width. Describe, with reference to a sketch, how you would measure the length *FG* with only the chain and range-poles at your disposal. (G.S.)

*4 (*c*). Describe, with neat sketches, how you would overcome the following difficulties when only the chain tape, and range-poles are at your disposal:

(*a*) Chaining between stations when the line is obstructed by a building passable on one side only.

(*b*) Measuring a line between stations when the line is crossed by a road which is fronted by boundary walls 12 ft. in height.

(*c*) Interpolating a subsidiary station in a survey line without ranging from the end stations, which are 24 chains apart. (U.L.)

4 (*d*). Describe with sketches how you would overcome the following difficulties in chain surveying:

(*a*) Ranging a line over a hill between stations when the latter are not mutually intervisible, but are both visible for a considerable distance on the hill itself.

(*b*) Chaining between stations when the line is obstructed by a detached building, passable on both sides.

(*c*) Measuring a line between stations when a boundary wall 12 ft. in height crosses the line. (U.L.)

4 (*e*). Describe two methods of measuring angles by means of a tape.

FIELD EXERCISES

Problem 4 (*a*). The range-poles *A* and *B* are the end stations of a survey line which is obstructed by the (specified) building. Determine the length of *AB*.

Equipment: Chain, arrows, and set of pickets.

Problem 4 (*b*). The pickets *A*,*B*, and *C*,*D*, represent stations on the opposite banks of an imaginary river too wide to be chained across. Using two different methods, find the lengths of *CD* and *AB*.

Equipment, as in 4 (*a*).

*Problem 4 (*c*). Find the error that would result in measuring with the tape the three angles of the triangle, as indicated by the pickets *A*, *B*, and *C*. Check the work by the "radian" method.

Equipment: Tape (steel or linen), arrows, and table of Sines or Chords.

*Problem 4 (*d*). Determine the perpendicular distance of the (specified) inaccessible point 0 from the survey line indicated by the range poles *A* and *B*.

Equipment: Chain, arrows (cross staff), and set of pickets.

The selection of points is indicated by the numbers on Fig. 42 (*d*), the lines actually measured being crossed (//) and chained in the order suggested by the numbers at the ends of the lines.

*Problem 4 (*e*). Run a line through the given point 0 parallel to the inaccessible survey line indicated by the range-poles *A* and *B*.

Equipment, as in 4 (*d*).

The selection of stations is indicated by the numbers on Fig. 42 (*e*), which also suggest the order in which the auxiliary parallels (shown) thick are run in obtaining the required parallel 04.

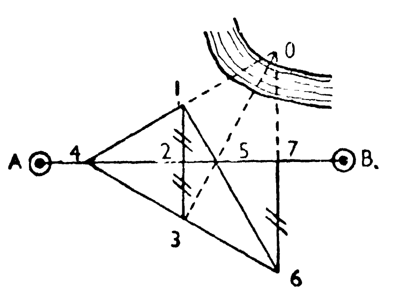


FIG. 42 (*d*)

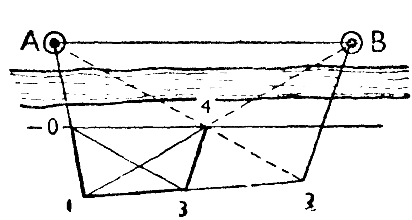


FIG. 42 (*e*)

CHAPTER V

LEVELLING

Levelling is the art of determining the differences in elevation of points on the earth's surface for the purposes of (*a*) tracing contour lines, (*b*) plotting vertical sections to represent the nature of that surface, and (*c*) establishing points at given elevations in constructional projects.

The methods of levelling may be divided into the following categories:

(1) *Gravitational Levelling*, (2) *Angular Levelling*, and (3) *Hypsometrical Levelling.*

Gravitational methods include Spirit Levelling, as usually understood in practice; Angular methods, the application of trigonometry or tacheometry, and hypsometry, those methods which depend upon variations of the pressure of the atmosphere, as utilised in the barometer, the boiling-point thermometer, and the altimeter as used in finding heights in aircraft navigation. The three systems in the general sense represent three degrees of accuracy in descending order: precise to accurate, accurate to moderate, and moderate to approximate. At the same time they represent in ascending order their applications to small, medium, and great differences of elevation, which the writer prefers to designate "Reduced Levels," "Elevations," and "Altitudes" accordingly.

I. PRINCIPLES OF LEVELLING

Levelling is surveying in the vertical plane, and the systems of vertical co-ordinates involved are respectively: (1) Rectangular Co-ordinates, (2) Inverse Polar Co-ordinates, while (3) has obviously no geometrical basis (see page 82).

Fundamentally, all levelling is based upon gravitation since the ruling levels of all methods are based upon spirit levelling.

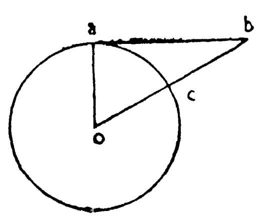


FIG. 43

In practice all elevations are referred to some "datum." This may be some assumed level plane, known as a local datum, or it may be some level spherical surface, such as that of the Ordnance Survey, which is the "approximate mean water at Liverpool." Points of reference to the datum are known as bench marks, which are figured on maps conveniently with the elevation above datum.

Now a *Level Line* is strictly a line concentric with the earth's mean figure as given by mean sea level, *acd* in Fig. 43. Since a plumb-line is a vertical line, always tending to point to the centre of the earth, a *Horizontal Line* is a tangent to the earth's curve, as *ab*. We must see horizontally, and a levelling instrument constrains us to look horizontally. Hence a surveyor at *a* sights along *ab*, and the distance *bc* is the earth's curvature, which, being 8 inches per (mile)^2, would not be detected by ordinary instruments.

Only gravitational levelling will be treated in the present chapter, and some applications of angular levelling will be given in the following chapter, where reference to hypsometry will also be made.

HISTORICAL DEVELOPMENT. The basis of gravitational levelling is the plumb-line, or plummet, and, by a stretch of the imagination, the bubble of a spirit level is a plumb-bob with an exceeding long line, making it so accurate and sensitive that its vibrations could never be nulled. In fact, a way of specifying the accuracy or sensitiveness of a bubble tube is by its "equivalent" plumb-line, which may be 300 ft. in length or more.

Plumb-line Level. Let us consider the primitive instrument shown in Fig. 44. Here a builders' square is attached to the top of a vertical stake *B*, which is driven into the ground, the stock being adjustable and secured in a horizontal position by means of a thumb nut *c*. In the head of *A*, at *d* is a hook, at *e* a ring, and from *d* a plumb-bob *P* is suspended. On top of the stock are fitted two sights *h*, *h*, of equal height. Now if the stock is so adjusted and clamped at *c* so that the plumb-line passes centrally through the ring, the sights *h*, *h*, will be truly horizontal, and the eye at *E* will be constrained to look horizontally, which is the basis of levelling. If, then, a vertical staff, divided into feet and tenths, were moved in the direction in which the stock points the readings taken from *E* on the staff would show the relative heights of the ground at the different staff stations, and, by subtraction, the differences of elevation.

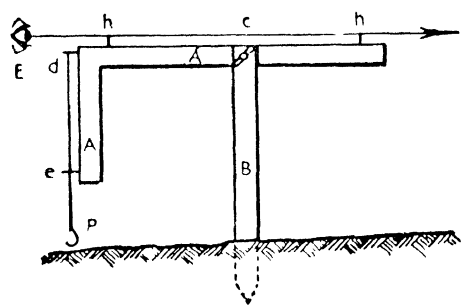


FIG. 44

Another way of introducing the principle would be to take a mirror, about 1¼-in. square, remove the silver above a diagonal, and mount the mirror in a metal frame so that the line of demarcation (say *h*, *h*) is horizontal. Then if the frame were suspended from the uppermost of its corners (*v*, say) and a heavy weight were suspended from the lowest corner, the plane of the mirror would be truly vertical. Hence if the instrument were held near the eye, the pupil would be seen by reflection, and above the silver edge the vision would be horizontal, so that a sight on a staff could be taken. Such an instrument could be constructed in the metal-work classes.

The principle is employed in two well-known instruments: the *Reflecting Level* and *De Lisle's Clinometer.*

Water Level. Suppose a $\frac{3}{4}$-in. glass tube were turned up at the ends and fitted by means of a knuckle joint to the top of a stand or tripod, the tube being almost full of water, as shown in Fig. 45. Then if the eye E is placed near one end, looking over the menisci, a horizontal sight is obtained, and a vertical levelling staff could be read as before. Again this is the principle of gravitation, in that water at rest has found its own level. The principle survives in the water level, a simple instrument used for transferring levels in spaces so confined that the use of any other instrument would be impossible.

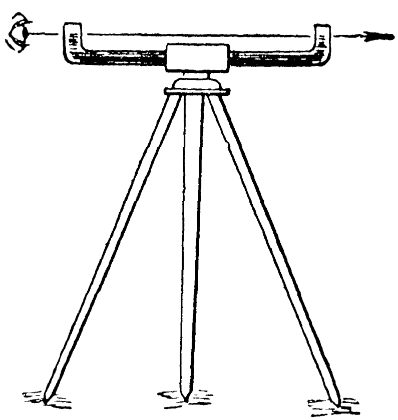


FIG. 45

This instrument consists of a pair of glass tubes, like test tubes, but with a short open glass pipe sealed into the bottom, fine lines being etched on the tubes. Attached to the pipes are the ends of a length of rubber tubing, the whole being nearly filled with water. The open ends are plugged until the tubes approximate to the same height, so that the levels can be transferred when the water reaches the etched marks.

Telescope. Doubtless a water level was attached to a metal sighting tube provided with a pinhole eye-sight and a horizontal hair line at the open end, the whole being mounted on a tripod. Still there was the limitation that the naked eye could not estimate to a tenth of a foot on a staff at distances exceeding about 150 ft. But as soon as a water-level tube was mounted upon a telescope, the range of sighting would be increased by 20 times, or 1/100 ft. could be read at distances up to 300 ft.

Fig. 46 is a longitudinal section of an ordinary, or draw tube, telescope, the type still found in the majority of surveying instruments. A is the outer tube and B the inner draw tube, which is moved by means of the rack and pinion R by an external focusing screw at the side of the body, but here hidden from view. O is the double-convex *object glass*, which throws an inverted image of the levelling staff on the plane of the *diaphragm D*. The diaphragm, which is supported by the screws d, d, consists of one horizontal line and two vertical lines, either etched finely on glass or actually spider webs. E is the *eyepiece*, which magnifies the image through the medium of two plano-convex lenses, giving a magnifying power in the ratio of the focal length f of the object glass to the focal length f_0 of the eyepiece. Thus the staff is seen between the two vertical lines of the diaphragm, and the reading is taken at the horizontal line (or crosswire), the numbers on the staff

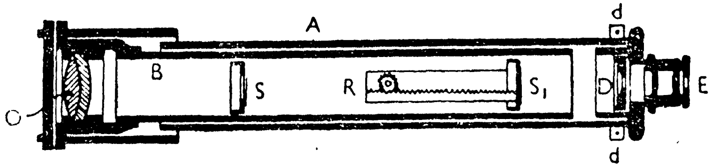


FIG. 46

being also seen inverted. Incidentally the landscape is also seen inverted, as is also the case with most surveying instruments. Rarely an *inverting eyepiece* is used: a tube fitted with four lenses instead of two. At first sight an inverting eyepiece appears an investment; though, on the other hand, the surveyor would think something was radically wrong if he lost his *habit* of seeing things upside down. Also there is an adage, "More glass, less light."

Since the end of the Great War of 1914-18, the internal focusing telescope has superseded the foregoing pattern in instruments of recent manufacture. In this type, the distance between the objective *O* and the diaphragm is fixed, and instead of the focusing screw moving the draw tube it moves a double concave (or negative) focusing lens. This leads to a more compact telescope, and one less susceptible to constructional defects, being on the whole an improvement, though, on the other hand, many surveyors of great field experience are inveterately conservative, and repeat among other things the slogan, "More glass, less light."

Bubble Tubes. A water-level tube on the top of the telescope leaves much to be desired, apart from sensitiveness. Hence it is superseded by the bubble tube, or phial, which is usually filled with pure alcohol so that a bubble of vapour is contained when the ends of the tube are sealed. Bubble tubes must be curved, either bent bodily or ground internally to a curve, or they would represent plumb-lines of infinite length, far too sensitive for mundane matters. Cheap bubble tubes, such as those fitted in carpenters' levels, are usually bent; but in all proper surveying instruments they are ground, often with such precision that each of the small division marks could be used for measuring small vertical angles, even as small as 5-seconds of arc. All good bubble tubes are costly and demand respect, if not for the skill in producing them, for the cost in replacing them when broken.

The idea is that the vapour bubble, being lighter than its spirit, rises to the uppermost point of the curve of the tube. Hence, if marks fixing its length are etched on the tube, the tube can be mounted or inset in a block of metal or wood, which, tried on a truly horizontal surface, will register the deviation from the horizontal when laid on any other plane surface.

Now the radius of the curve to which the bubble tube is bent or ground is the length of the equivalent plumb-line. Suppose that a vertical staff is sighted at a horizontal distance D from a levelling instrument with the bubble out of the centre and nearer the eyepiece. Then if the bubble be moved an equal distance from its central position towards the objective end, the staff reading will alter accordingly, and the difference of the staff readings will be the intercept s. Hence, if R is the radius of the bubble tube and n the number of divisions of length v through which the bubble has travelled, it follows that

$$R = \frac{nv}{s} D.$$

Measure six divisions of a bubble tube with a diagonal scale and find the length *in feet* of a single division, v. The rest is simple. Feet must go with feet, even though it is a privilege of youth to mix units indiscriminately.

In recent years great improvements have been made in the production of bubble tubes; in particular, the type in which the bubble has a constant length, even in tropical climates.

Thus the modern spirit levelling instrument is evolved. To-day it is made in three predominant forms, all of which embody the same essential principles, differing only in manipulation and adjustment. For nearly 100 years the *Dumpy Level* has characterised British practice, and must therefore be our representative instrument.

THE DUMPY LEVEL. The Gravatt level was called "dumpy," because it was more compact than its immediate predecessor, the "Y" level, so called because the telescope rested in crutches of this form.

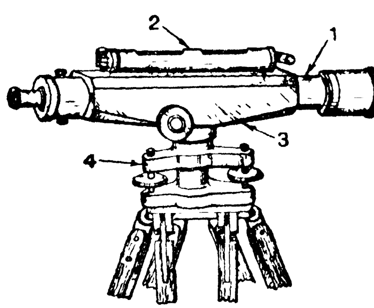


FIG. 47

The dumpy level, like most levels, consists of four primary parts: (1) the *Telescope*; (2) the *Level Tube*; (3) the *Limb*, and (4) the *Levelling Head*.

In the instrument shown in Fig. 47, the limb is really a casing around the telescope, and terminating in a vertical conical spindle, which rotates in the levelling head. The objective end of the telescope is covered with a ray shade, or sun cap, which is used for cutting out the glare of the sun. Underneath the telescope will be seen a *clamp* and its *slow motion screw*, not always fitted (or wanted) in levels, but provided here in order that the telescope may be moved gradually round about the vertical axis of rotation, particularly when the diaphragm is "webbed" with fine metal points. instead of spider lines or lines on glass (page 62). The diaphragm webbing is variously styled the crosswires, the cross hairs, or lines.

The levelling head of the model shown is of the *Tribrach*, or Three Screw pattern. Earlier instruments were provided with Four Screw *Parallel Plates*. These plates had an awkward habit of "locking," but virtue was found in this vice when the instrument was in skilful hands. The lower *tribrach sprang* or parallel plate is bored internally and threaded so that the instrument can be mounted upon its stand, or tripod, which may be of the solid round form, or framed, more like that of a photographic camera.

The sight line is known as (i) the "*line of collimation*," and is the line between the centres of the object-glass and the horizontal cross wire; also (ii) the *bubble line* is an imaginary line tangential (or axial) to an undistorted bubble, being horizontal when the bubble is in the middle of its run.

(i) *The one condition essential to accurate levelling is that the line of collimation shall be parallel to the bubble line, so that when the bubble is centralised by means of the foot or plate screws, the line of collimation will be horizontal.*

(ii) Another condition that has somewhat fallen into abeyance by the restoration of the old principle of the tilting screw in the modern level is that the bubble should "traverse," that is, remain in the middle of its run for all directions in which the telescope may point. Traversing is not a necessity, but a great convenience, particularly in trial and error work, such as contouring. In fact few levels traversed perfectly, but a slight touch to the foot screw soon put matters right. Certainly the tilting screw does the same thing, but there is a difference, apart from the prejudice of experience.

The levelling up of the instrument and the focusing of the telescope preparatory to taking observations with any mounted instrument are known as *Temporary Adjustments*, as distinct from *Permanent Adjustments*, which are those by which the correct relations are made between the *Fundamental Lines*, as (i) and (ii) are called in the present connection.

Caution. Nothing is so disconcerting to a student as a proper level out of adjustment; a fact that is evinced in closing a circuit of levels upon the starting-point. The causes and effects of maladjustments should be kept in mind. Apart from accidents, instruments may be thrown out of adjustment by numerous causes, seen and unseen, such as forcing them into their cases, staffmen sitting on the cases, careless transport, storage under extreme conditions of temperature, etc., even undue admiration in a pawnbroker's shop in the interim between pledging and redemption.

The essential relation between the bubble line and the line of collimation is usually restored by means of the antagonistic pair of capstan-headed diaphragm screws, which must be treated with care; and one with less than the engineering degree standard of training should not undertake the making of permanent adjustments—apart, of course, from men of practical experience. Less than forty years ago it was regarded as the next offence to capital crime for a capable assistant engineer or pupil to attempt to adjust even a dumpy level. Even this was considered "a matter for the maker" by men who have left very substantial monuments behind them. Happily, to-day, the outlook has changed, and there may be a surveyor in the locality who will undertake the adjustment with a little persuasion, for as you will doubtless find, "traditions die hard." The maker will certainly adjust the instrument most satisfactorily, but this involves the trouble of careful packing and certain risks of transport, though the motor car has removed the possibility of the case being thrown out of the luggage van.

In this connection, it must be known if it is the level that is really at fault.

There is little difficulty in testing the adjustment by the "two peg" method, which is the only absolute field test as to the accuracy of *any* levelling instrument. The various ways of applying this test are described in most text-books on surveying, and are detailed at length in the writer's *Field Manual.* In the present connection the best plan would be to introduce the method of "*Reciprocal Levelling*," the process used to eliminate instrumental defects and the errors of curvature and refraction in the very long but necessary sights, such as occur in levelling across a wide valley or river.

(1) Select two points, A and B, on a fairly level piece of ground at an estimated distance of about 4 chains or 300 feet apart; and at these points drive pegs firmly in grassland or chalk crosses on concrete or asphalt surfaces. (2) Set up the level near one point, A, say, so that when the staff is held vertically on that peg or point it will be possible to measure directly up to the eyepiece a staff reading a_1. (3) Sight through the telescope, and read at the horizontal wire the (same) staff held vertically on the peg or mark B, noting the reading b_1 when the bubble is central. (4) Now set the level up likewise near B, and measure the staff reading b_2 up to the centre of the eye-piece. (5) Sight through the telescope and observe at the horizontal wire the reading a_2 on the staff now held on the peg or point A when the bubble is central. (6) Find the differences (a_1-b_1) and (a_2-b_2), and if these are equal the level is in adjustment; but if this is not the case, the error $E=\frac{1}{2}((a_1-b_1)-(a_2-b_2))$, which is corrected by means of the diaphragm screws, gently slackening one screw and taking up the slackness more gently with the other in moving the diaphragm over an image, which is also *real.*

LEVELLING STAVES. Levelling staves are made in two forms: *telescopic* and *folding.* Telescopic staves have the advantage that they are heaviest at the bottom and are not top-heavy like the folding patterns, but this embodies the disadvantage that the uppermost, the third length, is very narrow and therefore more difficult to read. The former are made commonly in 14-ft. lengths, though 16-ft. and even 18-ft. are obtainable, while the latter are usually 10-ft. or 12-ft. when extended fully in all cases.

The type of staff used mainly in this country is the *Self-reading* staff, so called because it is read from the telescope; but in America another form is also used, the *Target Rod* (the Boston, the Philadelphia, and the New York patterns), a target being set by the staffman under the directions of the surveyor at the instrument. Target staves are sometimes used in precise work in this country, and apparently they were seen at the time *Alice in Wonderland* was written.

Although numerous "readings," or modes of division, have been designed, the prevalent one is the Sopwith "ladder," shown in Fig. 48. This shows (*a*) Primary divisions in feet, the numerals of which are shown in *red* on the left of the staff; (*b*) Secondary divisions of tenths of $\frac{1}{10}$ foot, which are indicated alternately in *black* figures of that height on the right of the staff; and (*c*) *Steps*, or blocks; subdivisions alternately black and white, each one-hundredth of a foot in height. Feet are read at the tops of the red figures in line with the wider black spaces, which here, as at all tenths, denote the pointings of the secondary portions. The tops and bottoms of the alternate black figures are also in line with the tops of these wider black spaces. Sometimes a black diamond and a dot are placed at the bottom of the middle shorter black space to denote each half of a tenth of a foot reading. Also small red numerals are painted at intervals along the staff to provide against the event that a large red numeral does not appear in the field of view of the telescope.

FIG. 48

A real levelling instrument must be available if only for demonstration purposes. An older model can be purchased at a reasonable figure. Military instruments will not serve the purpose of an engineer's level satisfactorily, even though they may prove excellent substitutes for theodolites.

Improvised Levels. A number of sighted levels and staves reading to tenths might be constructed if necessary. For instance, light brass tubing, about $1\frac{1}{4}$-in. diameter, could be cut into 10-in. lengths; a circular disc with a pin-hole centre could be soldered in as an eyesight at one end, and at the other end a horse hair could be stretched across from small holes in the horizontal diameter, with two similar hairs vertically, so that the three represent diaphragm webbing. A rectangular plate soldered at this end with its upper edge across the horizontal diameter, would serve the purpose of the horizontal web, as in the case of Abney levels. A spirit level, about 4 in. or 5 in. long, in a metal container, could be attached to the top of the sighting tube by means of adjustable clips, so that the bubble could be set central when the line of sight is established truly horizontally in the manner described in the Two Peg test.

The chief difficulty, however, is the means of attaching the level to the tripod in such a way that the bubble can be adjusted. This may mean a piece of work for a fitter, though much can be done with thumbscrews and $\frac{1}{4}$-in. plate, if a drill and screw taps are available. Otherwise some stiff form of ball joint could be improvised. Light frame tripods are easily constructed from 1-in. square ash or pine, six 5-ft. lengths being required. Pairs are screwed together and pointed at the toes (desirably shod), while the tops are opened out to fit on $\frac{1}{2}$-in. bolts through lugs projecting from a triangular plate, similar to that into which the levelling thumbscrews are threaded.

Improvised Staves. Folding staves can be constructed from two 5-ft. lengths of well-seasoned pine, both 3 in. wide, the lower being $1\frac{1}{2}$-in. thick and the upper 1-in., so as to avoid top heaviness. These should be given two coats of white paint, and then painted to show alternate tenth-of-a-foot black blocks across, black numerals, also one-tenth, being inserted at the edges of the white spaces. The lengths should be secured together with a strong brass butt hinge, and a bolt should be fitted at the back to retain the upper length in position when extended. The bolt should shoot from the upper length into the loop on the lower, otherwise the staff-man's fingers are exposed to great risks.

Otherwise, and particularly when proper levels are used, *staff* papers for renovating old staves could be purchased from any of the surveying instrument makers, who supply these, plain or varnished, with full instructions. Since these are usually divided for telescopic staves, the lower 5-ft. length should be planed out with a rectangular channel, so that a narrower upper length will lie in it when folded, thus protecting the divisions in transport. It is desirable that the backs and sides of the wood should be lightly stained and varnished, and especially that a brass sole plate should be fitted; 10-ft. readings, painted as above on fabric, could be used, but these are best attached to a staff—with the zero exactly at the foot.

TEMPORARY ADJUSTMENTS. Although these are a part of the field routine with both the level and the theodolite, they will be detailed here to avoid interruptions in the procedure of the Practice of Levelling, the important subject of the following subsection. A note will be added in order to avoid repetition when the subject arises in connection with the theodolite.

Consider the three (and four) dots, lettered *A*, (*A*) and *B*, *B*, in Fig. 49 to represent respectively the plan(s) of the tribrach and plate screws of a surveying instrument. For brevity, these will be called "foot-screws," and the remarks relative to the four plate screws will be enclosed in brackets. The small *o* in the centre is the plan of the vertical axis about which the telescope or instrument rotates. In the following instructions it must be remembered that aptitude in levelling up an instrument cannot be acquired from mere words: there is that little something else which practice alone gives.

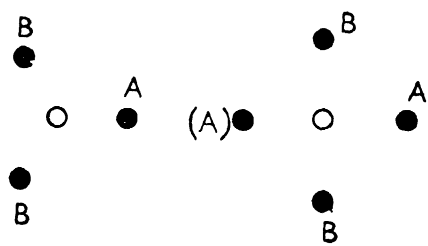


FIG. 49

(*a*) *Setting up the Instrument.* (1) Plant the tripod firmly with the telescope at a convenient height for sighting, and press the toes of the legs into soft ground, or place them in crevices in hard surfaces, always so that the lower sprang (or parallel plate) is fairly horizontal. (2) Turn the telescope so that it lies with its eyepiece over the screw *A*; then, by means of this screw *A* (and (*A*)) bring the bubble to the middle of its run (in the case of a pair of screws *A*, (*A*), working these equally in opposite directions). The bubble will move towards the screw that is worked in the clockwise direction as viewed from above. (3) Turn the telescope through a right angle so that it lies parallel to (or over) the other screws, and by means of these screws *B*, *B*, bring the bubble to the middle of its run, working the screws equally in opposite directions.

(4)* Return the eyepiece over the screw A, and by means of this screw (and its opposite fellow (A)) restore the bubble to the middle of its run if necessary.

If the level tube is in adjustment, the bubble will remain central for a complete rotation of the telescope, or the deviation will be so small that a mere touch to the foot screw nearest to the eyepiece will set matters right, for the bubble must always be central when reading the staff.

***When a bubble departs considerably from its mid-position on repeating the foregoing routine, it suggests that adjustment of the level tube is necessary; but this must not be attempted, since in most patterns of dumpy levels this would derange the all-important parallelism of the bubble line and line of collimation.**

After all, a "traversing" bubble is a convenience, not a necessity. Some modern levels are levelled approximately on similar lines, though often with the aid of an auxiliary circular bubble, which is brought to the centre of a circle etched on the glass cover. The main bubble is then set to its mid-position for every sight by means of a tilting screw.

*Usually, in the case of theodolites, two small *plate levels* are fitted at right angles to each other, and these can be set parallel to the lines B, B, and A, (A), thus avoiding the necessity of turning the telescope through a right angle in the second step.

(*b*) *Focusing the Telescope.* The foci of the object-glass and eyepiece must both be in the plane of the cross wires; otherwise the accuracy of the reading will be impaired by "visual parallax." Parallax can be detected by moving the eye up and down when sighting the staff or a station and noting if the cross hair appears fixed to the (inverted) image or if it moves relative to that image. The latter condition denotes parallax, which in many cases is due to incorrect focusing of the object-glass with the focusing screw, and not to the oft-innocent eyepiece. Usually it is better first to point the telescope to the clear sky with the focusing tube well in, and then move the eyepiece with a screwing motion until the cross wires are seen clearly and sharply. But our instrument is levelled up. Hence we had better look at a sheet of white paper held obliquely in front of the telescope and set the eyepiece when sighting this. Now direct the telescope towards the levelling staff, and by means of the focusing screw obtain a clear image of the staff. Test for parallax, but try refocusing with the screw before moving the eyepiece to eliminate parallax.

A perennial source of annoyance in an instructional class is the focusing of the telescope to suit the real and unreal idiosyncrasies of many eyes, and the fellow with spectacles might often oblige by removing them. Inexperienced surveyors are always tampering with the eyepiece, and in a class seventy per cent of the eyepiece adjustments are unnecessary, leading to wear if not damage to the instrument.

II. PRACTICE OF LEVELLING

Even now we cannot proceed until we acquaint ourselves with a few more terms and definitions. On page 60 we saw that the *Datum* is the plane or surface to which elevations are referred, and that the "reduced level" is the elevation of a point above this datum surface (or below in the case of soundings at sea). Reduced levels are connected with the datum through the medium of "benchmarks," which may be *official* or *local* according as the Ordnance datum is adopted, or any convenient horizontal plane of reference is assumed, the latter serving in the case of small or instructional surveys. Ordnance benchmarks are indicated thus, *B.M.* ⩚ 62·3, on the official maps, and are likewise indicated by the symbol alone cut into the walls of buildings, etc., the centre of the horizontal bar being the reference line. (By the way, this symbol was taken from the armorial bearings of an early chief of the Ordnance Department, and it has no connection whatever with Dartmoor.) An interesting excursion after studying an Ordnance sheet would be a search for the benchmarks indicated in a given area. When the motor hunts of twenty years ago were the thrill of "the bright young people," it was observed that a benchmark hunt in Richmond Park would be equally exciting. Benchmarks on posts and boundary stones have a ghostlike habit of disappearing and reappearing.

Benchmarks improvised in small jobs or on the cessation of a day's levelling are known as *Temporary Benchmarks* (*T.B.M.*).

All levelling operations must begin at a benchmark, which may be temporary with an assumed value (50·0 or 100·0) if an Ordnance *B.M.* is not in the immediate neighbourhood; and *all levelling operations must close on a benchmark*, even for the day. The state recognition of a benchmark may give a sense of dignity, but this has no effect upon the work, except that legal requirements may demand due respect for the Ordnance datum.

Finally, levelling is peculiar in that the point at which the staff is held is the station, and not the position of the instrument, which may be anywhere within sight and reason.

Backsight. A backsight (*B.S.*) is a reading taken on a staff held at a point of *known* elevation. It is the first reading taken on setting up the level *anywhere*, and is taken on a benchmark at the beginning of all levelling operations.

Backsighting is equivalent to measuring up from the datum, for if the reduced level of the staff station be known, say, 50·0, and the observed backsight reading is 4·24, the height of the horizontal plane in which the line of collimation revolves is 54·24, the contraction for feet being understood (and, therefore, unprofessional). Hence the rule: Add the backsight to the reduced level for the *Height of Plane of Collimation* (*H.P.C.*), or "height of Instrument" (*H.I.*) or even *Collimation*, " as it is variously styled.

Foresight. A foresight (*F.S.*) is a reading taken on the staff held on a point of *unknown* elevation in order to ascertain what distance that point is below the plane of collimation, and thus to determine the reduced level of the ground at the foot of the staff. It is the last reading taken before removing the level *anywhere*, and is taken on a benchmark at the close of a day's operations.

Foresighting is equivalent to measuring down from the horizontal plane of collimation, for if the reduced level of the plane of collimation is 54·24, and the foresight reading is 5·26, the reduced level of the foot of the staff is $54{\cdot}24-5{\cdot}26=48{\cdot}98$. Hence the rule: Subtract the foresight from the height of collimation for the reduced level of the staff station.

Now these two terms in no way denote direction, for often a backsight and a foresight are taken in the same direction.

Incidentally, the original method of reducing levels ignores the plane of collimation, and merely conceives the difference of the back and foresight readings as a *Rise* or a *Fall*, the difference $5{\cdot}26-4{\cdot}24=1{\cdot}02$ denoting that the ground has fallen from 50·00 to 48·98.

Backsights and foresights are taken on firm ground, embedded stones, or even footplates, since both the continuity and the accuracy of the work depends upon these.

Intermediate Sights. An intermediate sight (*Int.S.*) is virtually a foresight taken solely in order to ascertain the reduced level of a point or to establish a point thereat to a given reduced level. It has the algebraical sense of a foresight, but not the importance of one, being often booked to the nearest tenth, especially on rough ground. All readings between the backsights and the foresights are "intermediates."

Change Points. A *change point*, or *turning point*, is a staff station on which two staff readings are taken; a foresight prior to removing the level and a backsight in order to fix the new collimation height on again setting up the level. Occasionally, the term "Shift" is used colloquially, but this involves risk, since the cold (or weary) staffman may misinterpret "That's a Shift" as a welcome command. The importance of hard points for shifts is again emphasised.

A change point is characterised by two features: (i) that two staff readings are taken at it, and (ii) that these readings must appear in the *same* line in the level book, simply because they refer to the *same* point.

It is unnecessary to say that it is bad form to note a change point in the level book when it is thus evident.

Level Books. There are two methods of booking level notes: (1) *The Rise and Fall System*, and (2) *The Collimation System*. These will be considered together as we run our first line of levels. Common to both books are columns for *B.S.*, *Int.S.*, *F.S.*, and "Remarks," a column for distances being provided when measurements are made between staff stations, as in running vertical sections along a line. All level notes should read *down* the page, the notable exception occurring in the American method of contouring and cross sectioning on railways.

Level books should be simple and adapted to the immediate demands of the work in hand and not complex or wasteful so as to comprehend all the various work that may arise.

But let us draw up a page for each system, adding *two* columns for *Rise* and *Fall* in the former, and one for *Collimation* in the latter. Now let us move on to the benchmark on the wall of the "Spotted Dog" (*B.M.* 50·0). The staffman already stands there with the foot of the staff held exactly at the centre of the cross-bar, which is about a couple of feet above the ground. He will find it easier in the open when he stands behind the staff with the foot between his feet, holding it vertically with a hand on each side, never covering the divisions with his fingers.

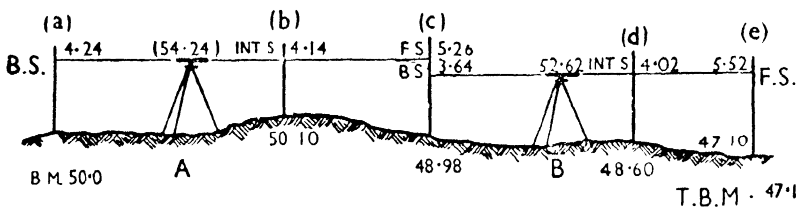


FIG. 50

Fig. 50 shows the instrument levelled up at *A*, a convenient distance to the east of the licensed premises. From here a reading 4·24 is taken on the staff (*a*) still painfully held on *B.M.* 50·0. This is entered as a backsight in both books, while the staffman moves to (*b*). The reduced level of the *B.M.* is also recorded and a note as to its location in the *Remarks* column.

(Added to 50·0 the *B.S.* of 4·24 gives the height of collimation (54·24) shown above the level, and also booked in the Collimation column of System (2).) The reading taken on the staff held at (*b*) is an intermediate sight of 4·14, which is entered as such in the proper columns. The readings at (*a*) and (*b*) suggest that the ground has risen 4·24−4·14, and 0·10 rise is entered in the appropriate column of System (1), where, added to 50·0, the reduced level of the point (*a*) it gives 50·10 as the reduced level of (*b*), which is entered in the column provided. (In System (2), the intermediate sight is merely subtracted from the collimation height of 54·24, giving the value 50·10, which is booked in the column for reduced levels.) The staffman is waiting at (*c*). Tell him that is a change point so that he can make a firm footing for the staff. This is a foresight of 5·26, which is duly recorded. In System (1), a fall of 5·26−4·14=1·12 is observed and recorded, and subtracted from 50·10 to give the recorded reduced level of 48·98. (In System (2) the reduced level of (*c*) is found by merely subtracting

5·26 from the collimation height of 54·24 for the recorded reduced level of 48·98 of this change point.)

Instruct the staffman to turn the face of the staff towards the position you indicate as the second position *B* of the level. Set up the instrument at *B* and level it carefully.

Now take a backsight on (*c*), and check it before booking it as 3·64; and above all enter it in the same line as 5·26 and 48·98. (In System (2) a new collimation is established, and the backsight must be added to the reduced level of (*c*) for the new collimation height, which is booked as 52·62 in the column provided.) Direct the staffman to hold the staff on the point (*d*). This is certainly an intermediate sight of 4·02, and is entered as such in both books. In System (1) a fall from (*c*) to (*d*) of 4·02−3·64=0·38 is recorded and subtracted from 49·88 for the reduced level of 48·60. (In System (2) the reading 4·02 is subtracted from the new collimation height of 52·62 for this reduced level, which is recorded as 48·60.) Direct the staffman to go to that mark on the step at the church gate, as indicated by (*e*). It was a temporary *B.M.* of 47·12, interpolated during the main drainage scheme. Record this foresight of 5·52 in both books. In System (1) this shows a fall from (*d*) to (*e*) of 1·50 which, subtracted from 48·60, gives the reduced level on the *T.B.M.* of 47·10 against 47·12. Excellent work for a first effort! In System (2) the reading 5·52 is subtracted from 52·62 for the reduced level 47·10.

The error of 0·02 would represent fair work with an engineers' level, but an error of 0·10 to 0·20 ft. might be expected with a sighting tube level—tenths of feet only being read on the staff.

The notes of the line of levels run as in Fig. 50 are recorded on the appended forms:—

(1) *Rise and Fall System*

B.S.	*Int. S.*	*F.S.*	*Rise*	*Fall*	*Reduced Level*	*Remarks*
4·24					50·00	*B.M.* "Spotted Dog" P.H.
	4·14		0·10		50·10	
3·64		5·26		1·12	48·98	
	4·02			0·38	48·60	
		5·52		1·50	47·10	*T.B.M.* 47·12, Church Gate
7·88		10·78	0·10	3·00	50·00	
		7·88		0·10	47·10	
	Fall	2·90	=	2·90	=2·90	

(2) *Collimation System*

B.S.	*Collimation*	*Int.S.*	*F.S.*	*Reduced Level*	*Remarks*
4·24	54·24			50·00	*B.M.* "Spotted Dog" P.H.
		4·14		50·10	
3·64	52·62		5·26	48·98	
		4·02		48·60	
			5·52	47·10	*T.B.M.* 47·12, Church Gate
7·88			10·78	50·00	
			7·88	47·10	
		Fall	2·90 =	2·90	

Checking the Book. The figures at the bottoms of the columns are the checks; two common to both systems and a third in the Rise and Fall System:

(1) Diffs. of sums of *B.S.*'s and *F.S.*'s
=(2) Diff. of first and last reduced levels
=(3) Diffs. of sums of Rises and Falls.

These are merely checks on the arithmetic, and never on the levelling work, though they have often raised the surveyor's spirits until he discovers that a drastic mistake has occurred outside. On the other hand, more than one line of levels has been run again unnecessarily when the arithmetical check would have shown what a simple slip in addition can do.

Choice of Systems. In the following points of comparison it must be remembered that these refer to the booking and not to levelling operations, which are identical:

(1) In the Rise and Fall System the remainder of the reduced levels may depend upon the reduction of a single intermediate sight. But there is a check upon the intermediates, whereas in the Collimation system any intermediate sight may be wrongly reduced without affecting the remainder of the levels. Age and habit are apt to exaggerate the merits of the Rise and Fall System; and it is a matter of visualisation as to whether rises and falls are more readily evident in a figure than in the field.

(2) Also in the Rise and Fall System there is either one more addition or subtraction in each reduction whenever intermediate sights are taken, and thus there is a considerable saving in bookwork in the Collimation system when numerous spot levels occur.

(3) Then the second decimal place from backsights and foresights

must be carried through the intermediates in the Rise and Fall System unless direct subtraction between backsights and foresights are made. Whereas in the Collimation system the intermediates can be taken only to the nearest tenth when desired, without giving thought to the backsights and foresights, which are necessarily read to the hundredth of a foot.

(4) Finally, the Collimation System has the indisputable merit of emphasising the relatively greater importance of backsights and foresights in the field, but whether this system is more scientific, being closely related to fundamental principles, is again a matter of opinion.

Levelling Operations. It would be unfair to dismiss the subject without a word as to what all the fuss is about. Hence the following summary of the applications of spirit levelling.

(1) *Check Levels.* If a sewage disposal scheme, or other works, is under construction, it would be necessary to have numerous temporary benchmarks, based upon the Ordnance datum, at convenient points throughout the area. A main circuit is established and levels are run carefully round, checking on the starting-point; cross lines are run through the benchmarks in the middle of the area, closing on the outer benchmarks of the system. Usually this is carried out by accurate or precise spirit levelling.

(2) *Flying Levels.* Suppose that there had been no *T.B.M.* at the Church Gate in Fig. 50, it would be necessary either to run levels forward to the next Ordnance *B.M.*, or back to the *B.M.* on the wall of the Spotted Dog. Flying levels consist only of backsights and foresights and are run solely to check the accuracy of the work.

(3) *Section Levels.* When a highway, railway, or other scheme is projected, it is necessary to run levels along straight lines or around curves for the purposes of preparing a *longitudinal section* from which the gradients and earthwork volumes can be estimated. *Cross sections* are also run in connection with roads and railways at right angles to the longitudinal sections, and, similarly in connection with surveys for reservoirs, etc. Sections require that the distance between the staff stations shall be measured. These distances are best recorded in a "Distance" column rather than in the remarks, though this is often done in cross-sectioning.

(4) *Spot Levels.* Spot levels are intermediates taken in areas reserved for building or the construction of public works. Sometimes contour lines are interpolated between spot levels. In these, as in much highway and railway surveying, the surface levels are taken to the nearest tenth of a foot.

LEVELLING DIFFICULTIES. The length of sight with a telescopic levelling instrument should not exceed 5 chs. or 350 ft., and as far as possible the lengths of the foresights should equal the lengths of the backsights, either individually or in sum, in order that the small errors of adjustment may not affect the accuracy of the work. When exceedingly long sights are necessary, as in sighting across a wide river, the method of Reciprocal Levelling, as suggested on page 66, shoula be resorted to, but preferably with the use of a target staff. The averages of the differences of level as observed in each direction is taken as the true difference of level, since this average eliminates instrumental errors and the effects of the earth's curvature and atmospheric refraction. The effect of curvature c is indicated by bc in Fig. 43, where it is evident that it increases the staff reading and thus makes very distant points appear too low. The effect c is 8 inches per mile, varying as the square of the distance. It is therefore about 0·01 ft. in 10 chs., a distance at which no ordinary staff could be read directly. Refraction reduces the effect of curvature, bending ab so that b is depressed $r = \frac{1}{7}(bc)$ towards c. The value of r is really uncertain, as refraction becomes very capricious near the horizontal. Anyway, the matter is largely academic in spirit levelling, and the net correction $c-r$ is taken at $\frac{4}{7}(D)^2$ ft., the distance D being in statute miles. Refraction is a very important correction in trigonometrical levelling and astronomy.

In conclusion, there is bound to be something omitted, possibly a difficulty that will be encountered the first time the level is taken into the field. But the difficulties that arise in levelling are legion, and could not be summarised in a book of this nature. Nevertheless a few hints may be given among others.

(1) When reading near the top of the staff, ensure that it is truly vertical by instructing the staffman to wave it gently to and fro towards you so that you can record the lowest reading.

(2) When working up and down a steep hill, avoid very short telescopic sights by setting up the level to the sides of the line and zigzag thus so as to obtain as nearly as possible a balance of the total lengths of backsights and foresights.

(3) When sighting the staff very near to a telescopic level, instruct the staffman to hold a piece of paper against the staff as a target from which the reading can be taken directly. A target improvised in this way is necessary in taking very long sights, also in testing the adjustments of the level.

(4) When a benchmark is considerably above the level, as under an arch, invert the staff (foot on the *B.M.*) and record this (and regard it) as a negative backsight or foresight, as the case may be.

(5) When a board fence crosses the line, drive a spike through to support the staff on each side and regard the spike as a change point. Also a lake of still water too wide to be sighted across can be regarded as a single change point if pegs are driven flush with the water surface. (Incidentally, this suggests a method of checking the accuracy of the collimation adjustment of a level.)

(6) When a wall is encountered, drive pegs in the line on either side and measure with the staff to the top of the wall, which is regarded a change point.

CLASS EXERCISES

5 (*a*). Draw up the headings of a specimen page of the following level books:

(a) *Rise and Fall System;* (b) *Collimation System.*

The following readings were recorded in running a line of levels, the nearest tenth of a foot being taken in the case of intermediate sights:

(B.M. 62·4) (B.M. 63·3)
3·12, 2·4, 1·8, 0·94; 2·84, 3·1, 3·6, 4·12

Reduce these in the system you prefer, stating the reasons for your choice.
(No error.) (G.S.)

5 (*b*). In taking the following readings with a dumpy level, the surveyor started at a benchmark and returned to it, in order to check his work. He took staff readings on *A* and *B* as points for temporary benchmarks in both the outward and homeward directions.

Record and reduce the levels on a page of a level book, and indicate where a mistake was made in reading the staff.

B.S.	*Int.S.*	*F.S.*	*Reduced Level*	*Remarks*
2·34			76·42	B.M.
	5·24			(Outwards)
4·62		9·63		
	8·52			Point *A*
7·64		5·88		
	7·24			Point *B*
4·26		4·32		
8·82		0·38		(Homewards)
5·44		10·89		
6·88		10·17		Point *B*
12·66		7·92		Point *A*
		1·47	76·42	B.M.

(Read 9·92 for last *F.S.* on *A*.) (G.S.)

5 (*c*). The following levels were taken along the bed of a water course. Reduce the levels and find the rates of inclination along the bed of the water course.

B.S.	*Int.S.*	*F.S.*	*Reduced Level*	*Distance (ft.)*	*Remarks*
2·43			37·43	—	B.M. (Mill Ho.)
	8·86			0	Section A
	7·64			180	
6·46		7·22		230	
	4·70			420	
	3·34			540	Section B
		0·82	38·28	—	B.M. (Culvert)

(1 in 148; 1 in 119; 1 in 108; 1 in 88.) (G.S.)

5 (*d*). The following staff readings were taken in levelling down a hill between benchmarks 76·4 and 43·8:

(B.M. 76·4) 3·44, 6·78, 12·44; 2·06, 5·66, 11·74;
1·04, 3·68, 7·22, 9·16, 12·88 (B.M. 43·8.)

Interpret these notes in a level book of the "Collimation" System. (U.L.)

5 (*e*). A surveyor runs flying levels down a hill from a temporary benchmark (162·40) to an Ordnance B.M. (123·4), recording his staff readings as follows:

1·62	11·44	12·68	12·80	8·64
	2·86	0·82	1·24	

Prepare a page of a level book, and on it record and reduce the above readings. (G.S.)

(O.B.M. reduces to 123·38.)

FIELD EXERCISES

Problem 5 (*a*). The points marked *A*, *B*, *C*, *D*, etc., around the (specified) building are selected as temporary benchmarks, the assumed reduced level of *A* being 100·0. Determine the reduced levels of these and find the error in closing the circuit on *A*.

Equipment: Level on tripod, staff and chalk.

Problem 5 (*b*). Run the levels for a longitudinal section between the stations indicated by the range-poles *A* and *B*.

A convenient B.M. ( ) is . . .

Equipment: Level on tripod, staff, chain, and arrows.

Problem 5 (*c*). The pickets *A* and *B* indicate the direction for a proposed drain, and surface levels at 50 ft. intervals are required. Submit these on an appropriate form, and check the book.

Equipment: as in 5 (*b*).

Problem 5 (*d*). Find the reduced levels of the survey which is being made by Group ( ).

Equipment: as in 5 (*a*).

Problem 5 (*e*). Test the accuracy of adjustment of the assigned level by the Reciprocal Method.

Equipment: Level on tripod, staff, two pegs, and a mallet.

ORIGINAL PROBLEMS

. . . .

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CHAPTER VI

ANGULAR LEVELLING

In the preamble to Chapter V it was stated that the methods of angular levelling are based on Inverse Polar Co-ordinates, though fundamentally they are dependent upon a horizontal line, such as AB, which is determined by gravitational methods in which the plumb-line may take the form of a weighted sector or the guise of a spirit level.

The term inverse polar co-ordinates is coined somewhat loosely, for both in trigonometrical and tacheometrical levelling, the relation between the *vertical height H* and the *horizontal distance D* is simply

$$H = D.\tan.\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where α is the vertical angle of *elevation* if above the horizontal sight line AB, or of *depression* if below AB. "Acclivity" and "declivity" are terms used synonymously with these. (Fig. 51.)

In angular levelling the horizontal distance AB is determined by triangulation, being found graphically by the intersection of rays on the plane table or by photo-intersections in photogrammetry. In tacheometry, the height H is found from the intercept observed on a staff at B, but directly or indirectly the horizontal distance D is involved.

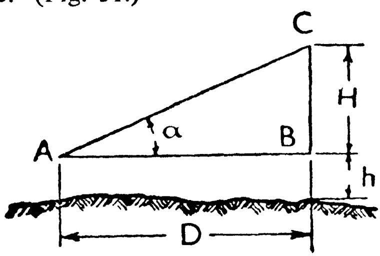

FIG. 51

(*a*) *Base known or accessible.* Fig. 51 shows the case of an accessible base, and Fig. 52 the case of triangulation, the right-hand lines corresponding with those in Fig. 51.

It is evident that the method requires some instrument for measuring vertical angles, and this may be one of the numerous forms of clinometers, the sextant, or the theodolite, the accuracy thus increasing from low to high.

If α is 45° in (1), $H=D$, since $\tan 45°=1$.

This fixed relation is embodied in the Apecometer, which is a simple instrument for measuring heights of trees and buildings, the bases of which are accessible to direct linear measurement. This little instrument is essentially an optical square which reflects at 45° instead of at 90°, being held edgewise in sighting. The observer sights a point near the foot of the object and moves along AB until he finds a point X from which the top C can also be seen. Then $XB=H$, the height h of the eye being afterwards added.

The Brandis "Hypsometer" is really a clinometer for finding heights generally, various reducing data being inscribed on the instrument.

The "Dendrometer" is another form of instrument used in connection with a 10-ft. rod. Some of these devices are exceedingly handy in forestry and preliminary survey. Road tracers are clinometers on stands used in connection with sighting targets, and very large clinometers are mounted on tripods, various scales being engraved on the plumbing sector.

Also various improvised forms could be suggested, as, for example, the principle of Fig. 24. Here the N.E. quadrant could be a frame, levelled with a bubble, and a sighting-arm could be pivoted at the centre *o*, so that vertical angles could be read as such, or their slope ratios, on the outer edges of the square. This is the . . . After all, we are not finding the height of the Tower of Babel.

Failing a better instrument, we have our improvised clinometer (page 17).

Fig. 52 illustrates the case of triangulation.

With the theodolite, the horizontal distance *AB* (or *DB*) is calculated from the observed angles δ and β, and the base *AD* by (Angular Co-ordinates). In the case of the plane table *B* is fixed by intersecting rays drawn first from *A* and then from *B*, the end stations of a measured base. Since the horizontal distance *AB* (or *DB*) is known or plotted, as the case may be, the height *H* can be found as above, graphically or by calculation, even though *B* is inaccessible as it so often is. The height is calculated in the case of the theodolite, and for great distances curvature and refraction are important considerations. *H* may also be calculated in plane tabling; but since this method is graphical, the height may be found by setting off a right angle at *B*, and constructing the angle θ at *A*, so as to determine the point *C*, *BC* being the height *H* to scale. In Practical Geometry, this is known as rabatting the triangle *ABC* into the horizontal plane or, in other words, Fig. 52 is seen, not as an elevation, but as a plan, with *AB* the horizontal projection of *AC*. Laussedat, the pioneer of photogrammetry (1854), introduced this graphical process in determining the heights of points from pairs of photographs taken from the ends of bases, such as *AD*. Our few principles go a long way.

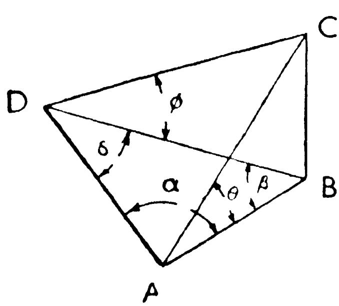

Fig. 52

Mention must be made of the India Pattern Clinometer, which is specially adapted to work with the plane table, the board of which serves as a base for the instrument. A pin-hole sight is used in conjunction with a sighting index, which can be set to the observed vertical angles or their tangents, sometimes by means of a rack and pinion movement.

(*b*) ***Base Inaccessible.*** Frequently it is necessary to determine the elevation of a point, the base of which is inaccessible, and it would be inexpedient to resort to triangulation, as in Fig. 52. In this case it is necessary to measure a base AD of length L in the vertical plane of the elevated point C and the instrument stations A and D from which the two angles θ and φ are observed. Consider Fig. 53, the general

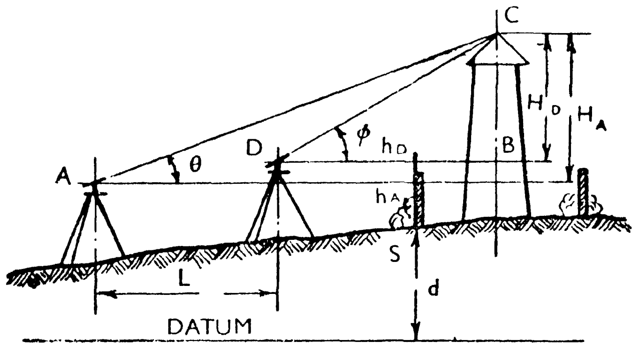

FIG. 53

case in which the slope of the ground is appreciable, giving instrumental heights h_A and h_D on a staff held as near as possible to the base of the object, with $h = h_A - h_D$ algebraically.

Then for the height of the point C above the instrument at A:

$$AB = H_A \cot \theta.$$

$$DB = H_D \cot \varphi.$$

$$L = AB - BD = H_A \cot \theta - H_D \cot \varphi.$$

But

$$H_A = H_D - h, \text{ and}$$

$$H_A = \frac{L - h \cot \varphi}{\cot \theta - \cot \varphi} \tag{2}$$

If the ground is level, or nearly so, $h = O$, and $H_A = \dfrac{L}{\cot \theta - \cot \varphi}$ (2a)

and if the instrument is divided with *slope ratios*, r horizontally to 1 vertically, these are co-tangents, and (2a) becomes $H_A = \dfrac{L}{r_A - r_B}$ where r_A and r_B are the ratios for the angles observed at A and B in Fig. 53. Often, however, the tangents of the angles of slope are shown; otherwise *gradients*, 1 vertically in r horizontally. Also, if the point S, the staff station, is of known elevation above datum, the reduced level will be $H_A + d$.

Abney Level. Mention must be made of what is possibly the best known of all clinometers, the Abney level; an instrument which extends the principle of the reflecting spirit level to the measurement of vertical angles, the primary function of all clinometers.

The sighting tube in Fig. 54 is square in section, and is provided with a pin-hole sight on the right, and, axial with this, the edge of a

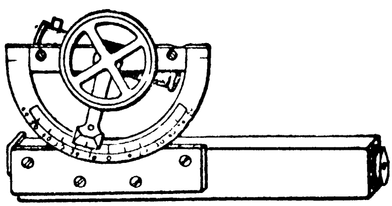

FIG. 54

sighting plate at the object end. Some patterns are telescopic, but the innovation is questionable, as with any hand instrument, except the sextant. Inside the tube is a mirror, the upper edge of which leans towards the object end, the mirror being half-silvered, with the line of division either vertical, or coincident with the line of vision on the horizontal edge of the silver. In the reflecting spirit level, the bubble tube is fixed on the top of the sighting tube, being embedded in an open recess, so that the bubble can be read by reflection, its image in contact with the horizontal sight on a levelling staff. This hand level is used extensively in route contouring in America, being strapped, or otherwise attached, to the top of a 5-ft. staff, known as a "Jacob." In the Abney level, however, the bubble tube is carried on an axis which forms the centre of a graduated arc fixed to the sighting tube. An index arm is also fixed to this axis, and the bubble tube and arm are turned by the little wheel in the front of the figure. Thus, for any inclination of the line of sight, the bubble is moved so as to give the reflected coincidence that corresponds to its middle position, and the vertical angle of the observed point is read on the vernier of the index arm. The arc is also provided with graduations giving the tangents of these angles, or gradients, or the corrections to be made in chaining slopes. In the latter connection, it is usually sufficient to sight the eyes of an assistant of one's own stature. Abney levels can be used as they lie on the boards of plane tables, also in similar connections in various mechanical experiments.

An objection to clinometers and other hand instruments is the difficulty of keeping them steady when taking observations. In this respect, it is well to note that a "bipod" of surprising steadiness can be improvised by inserting the knob end of a walking-stick in the left-hand jacket pocket, and gripping the stick at the height of the eye, with the right hand, the thumb and one or two fingers supporting the instrument.

* BAROMETRY. Although barometrical levelling is outside the scope of Elementary Surveying, this chapter affords the temptation of introducing the third mode of levelling to those who may proceed further in the subject, with a view to engineering, geography, or aerial navigation. In 1647, Pascal demonstrated that the variation in the density of the atmosphere with changes in altitude might be applied to the determination of heights; and this was made possible by Torricelli's invention of the mercurial barometer, the readings of which are found to decrease in geometrical progression as the altitudes increase in arithmetical progression. Thus, the barometer and the boiling-point thermometer (also *alias* the *Hypsometer*) are strictly the

preserves of Physics; and more than one experienced surveyor considers this to be the proper place. Eminent physicists prepared tables with different initial assumptions, and the surveyors were not infrequently bewildered with apparently confusing data and corrections. Often the wrong tables with the right instrument, on top of no knowledge of physics. The barometer as a meteorological instrument is not the barometer as a surveying instrument.

The portable form of barometer is known as the *aneroid*, which merely signifies "no liquid."

Possibly you may have heard the story of the fair young examinee, who was asked how she would find the height of a tower if she had a pocket aneroid. Her answer was to the effect that she would unpick her jumper, let down the "thing like a watch," and then measure the length of wool paid out with her ruler. Whatever the examiner thought, she was a born surveyor, for she was evidently aware that the wool would have broken had the instrument been sufficiently large and sensitive to respond to a difference in elevation of (say) 180 ft. Also, she may have seen a similar method used in transferring levels down the shaft of a mine, where tenths of feet matter, even if they do not in travellers' stories.

Incidentally, the *altimeter* used in connection with air survey cameras is a form of aneroid, but, being small, will not give absolute heights to within 200 ft. Usually a *statoscope*, or differential aneroid, is used in addition, so that the variations can be more accurately determined.

The surveying aneroid in itself is an ingenious piece of work; and its idiosyncrasies are no fault of the maker. Household barometers are meteorological instruments, and often an excellent solution to the problems of presents, or prizes at sports meetings. But in the field a surveying model, never less than 4 in., should be used, and always with respect for the instructions supplied by the maker. For instance, the working range should always be taken to about 2,000 ft. less than the limit engraved on the fixed altitude scale. The aneroid is indispensable in exploratory and pioneer work, and good results will follow if the instrument is used with care and understanding.

The principle of the instrument, as given in respect to the diagram of Fig. 55, is exceedingly simple.

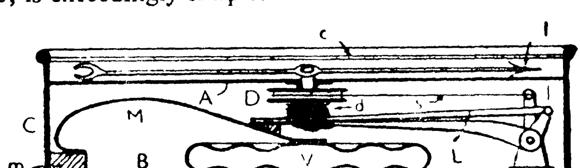

FIG. 55

The aneroid consists of a circular metal case C with a glass cover c, the base plate carrying the entire mechanism and the cover the dial. Fixed to the base plate B is the all-important vacuum chamber V, which is circular and corrugated, and constructed of German silver. The walls of this chamber are under 10 to 15 lb. per sq. in. of *suction*, and would immediately collapse under the outside pressure except for the material support of the mainspring *M*, which is fixed to the bridge-piece *m*. Now variations in the outside atmospheric pressure are as tiny weights in the pans of a delicate balance, and these induce pulsations in the vacuum, which are accompanied by movements of the mainspring. These movements are transmitted and magnified by means of the compensated lever *L*, which transmits them to the crank system *l*. A second crank of this system *l* transmits them to the chain *s*, which turns the drum *D* and the indicator *I*, the motion being resisted by the hairspring *d*, keeping the chain *s* taut. The pulsations are thus finally read as altitudes (ft.) and pressures (in.) on the dial *A*.

The only correction that has to be considered with the aneroid is for the temperature of the intermediate air. Compensation for temperature refers to the instrument and not to this correction. Care must be taken to ascertain the initial temperature for which the instrument is divided; say, 32° F. or 50° F.

The peculiarities of the instrument should be studied, preferably by comparison with a standard mercurial pattern; also the results should be compared with those of a boiling-point thermometer for absolute altitudes.

Possibly you have observed the tapping of the glass of the barometer in the vestibule of an hotel. This is not a religious rite, if carried out with the solemnity of one. It is merely to eliminate "stiction," which is statical friction with a following—here the mechanism ending in the spindle of the indicating needle.

In addition, there is a "lag" effect, analogous to that which occurs in other connections, the instrument being sluggish in responding to a descent after an ascent. Hence, when a series of journeys is made uphill and downhill, the greater importance should be attached to the mean value reduced from the ascents. Surveyors often work to the height (of barometer) in inches, as though it were the mercurial form, reducing the altitudes either by formulae or by means of tables. On returning from the peak station *B* to the base station *A*, they can deduce the probable height in inches at *A* at the instant the reading was taken at *B*. It is possible to make an approximate correction by comparison of the height and altitude scales.

CLASS EXERCISES

6 (*a*). You are required to find the height of the bottom of a tank on a water tower, which is surrounded by a high hedge about 25 ft. from the tower. The tank is fitted with a gauge and the zero (0) of this is level with the bottom of the tank.

The following vertical angles were read with a clinometer at *A* and *B* respectively, *A*, *B*, and 0 being in the same vertical plane:

13° 20°

In each case the eye was 5 ft. above A and B, between which the ground was level. AB measured 185 ft. and the reduced level of A was 64·6.

Determine the height of the bottom of the tank above Ordnance datum. (186·41 ft.) (G.S.)

*6 (*b*). During a plane table survey, sights were taken to points A, B, C, D, and E with a clinometer, which was 4·5 ft. above the table station O, a peg at a reduced level of 155·5. The horizontal distances scaled from O to the observed stations were as follows:

A	B	C	D	E
760′	420′	315′	880′	1260′
$+7°$	$+14\frac{1}{2}°$	$+16°$	$-10°$	$+12\frac{1}{2}°$

Determine graphically or otherwise the reduced levels of A, B, C, D, and E. (249·0; 264·3; 245·9; 0·6; 336·5 ft.).

*6 (*c*). Outline three essentially different modes of levelling, one applicable to each of the following:

(*a*) Small differences of reduced level;
(*b*) Medium „ „ elevation;
(*c*) Great „ „ altitude.

*6 (*d*). How can an explorer in unknown country obtain rough determinations of absolute heights? Explain fully, showing the limitations of the methods you suggest.

*6 (*e*). Draw a sectional view of an aneroid barometer, explaining carefully how the instrument functions in determining altitudes.

FIELD EXERCISES

Problem 6 (*a*). Determine the heights of the accessible points indicated on the (specified) building.

Equipment: Clinometer, chain, and arrows.

Problem 6 (*b*). Determine the height of the spire on the (specified) building.

Equipment: Clinometer, chain, arrows, and set of pickets.

Problem 6 (*c*). Determine the difference in elevation of the two (specified) points, P and Q.

Equipment: as in 6 (*b*).

Problem 6 (*d*). Supply Group . . . with the elevations of the stations of the plane table survey they have in hand.

Equipment: Clinometer, preferably India pattern or Abney level.

*Problem 6 (*e*). Determine the height of (specified) hill with the aneroid. (during excursion or field class in the country).

ORIGINAL PROBLEMS

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CHAPTER VII

THE COMPASS

The compass may be defined as an instrument in which a magnetic needle assumes a more or less definite line of reference from which angular direction lines known as bearings can be measured.

The origin of the compass is lost in antiquity, to adopt the common phrase. All that is known is that the mariner's compass was used by the Italians or the Portuguese in the twelfth century A.D., and that there are indications that it was known in China in the eighth century B.C.

Compasses are made in at least fifty-seven varieties, ranging from the tiny charms on watch-chains to the most elaborate forms of mining dials, excluding, of course, the types used in navigation.

As surveying instruments, compasses are made in three forms at the present time: (1) *Occasional Compasses*, as found in pocket dials, trough compasses, etc., incidental to the plane table, theodolite, and even old pattern levels. (2) *Reconnaissance Compasses*, including the service forms of luminous compasses, and the prismatic non-luminous form, as used extensively on preliminary and route surveys. (3) *Surveying Compasses*, usually mounted, and fitted with a pair of vertical sights. In America, the last class stands as designated. These instruments were used extensively on land surveys, and had the merit that their accuracy was consistent with that of chaining. Harmony between measurements, both angular and linear, is essential in surveying; but the compass brought harmony of another kind, giving us "Dixie," when those two English surveyors, Mason and *Dixon* ran the disputed boundary between Maryland and Pennsylvania in 1767. In England the surveying compass faded out as the "compass circumferentor," on the advent of the small theodolite, but reappeared with some slight alterations as the "mining dial," the compass needle still holding a prominent place.

The prismatic compass is the type best known to British surveyors, and this will be described in detail in order to emphasise the points essential to a good compass.

Prismatic Compass. This instrument was invented in 1814 by Captain Kater, whose famous pendulum is a source of anxiety in most physical laboratories. It is the most convenient instrument for rapid traverses, particularly in dense forests and jungles. The characteristic feature is the prism reading, which enables the surveyor to observe bearings without resting his compass on the ground or a wall, or deputising someone to read the divisions for him.

The prismatic compass consists of four main parts: (1) *The Compass Box*, (2) *the Dial*, (3) *the Prism*, and (4) *the Window*.

(1) The *box*, a metal case, 2½ in. to 6 in. diameter, carries the needle pivot or bearing at the centre of its base. On the rim is fitted the prism *P* and, diametrically opposite, the vertical sighting window *V*. Under the window at *C*, a pin is inserted to actuate a light check spring *B′*, which, touching the dial, damps its oscillations (or fixes the reading with a very doubtful degree of accuracy). A glass cover is fitted to the box, and a metal lid is provided to protect this glass when the instrument is out of use.

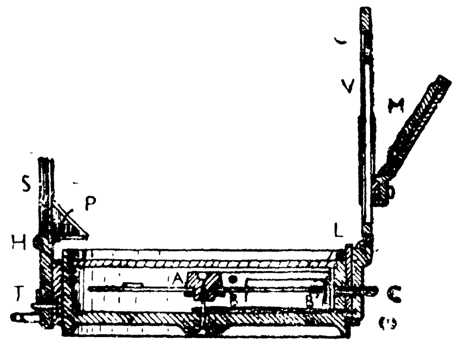


Fig. 56

(By the way, "Boxing the Compass" does not refer to the last step; but means, in nautical language, calling the "32 points," or rhumbs, in order from north by way of east.)

(2) The *dial*, which is carried by the magnetic needle *A*, is made of card in small instruments and of aluminium in larger patterns. It is figured from 0° to 360° in the clockwise direction, but has its numbers *reversed* (as seen in a looking-glass) and *advanced* 180° so that bearings can be read directly through the prism *P*, as though they appeared at the *forward* or window end of the dial. (Remember forward end of the needle for forward bearings.) The needle is usually *mounted* with a bearing centre *B* of agate or chrysolite. (This can wear the pivot and impair accuracy if the needle is not raised by its *Lift* during transport.)

(3) The *prism* is cut to 45° on one face, and to 90° on the other two faces, which are worked to a convex surface so as also to give magnification of the numbers on the dial. The prism box is provided with a sight slit *S*, and is hinged to a projection, which, for focusing, can be slid up and down by means of a *thumb-nail* stud *T*. The hinge *H* is fitted so that the prism box can be folded back for compactness when out of use. Sometimes a ring is fitted under the prism box for attaching the instrument to the person as a precaution against accidents.

(4) The *window* consists of an open frame fitted with a central vertical hair *V*, which, in sighting, is used in conjunction with the sight slit, *S*. The window-frame is hinged, and, when turned down for compactness, lifts the needle *A* from its pivot, the base pressing down the outer end of the lifting lever *L*.

Additional parts may include (*a*) the *mirror M* for sighting points considerably above or below the horizon; (*b*) *sunshades*, which may be placed in front of the mirror in solar observations; and (*c*) a *tripod*, which is desirable with the heavier patterns.

Use. (1) Remove the cover and open out the prism and window, and, holding the compass as level as possible, focus the prism by raising or lowering its case until the divisions appear sharp and clear. If necessary, lower the needle on to its pivot. (2) Holding the compass box with the thumb under the prism at T and the forefinger near the stud C, sight through the slit S and the hair-line V at the object or station, lowering the eye to read the required bearing as soon as the dial comes to rest naturally (or by cautiously damping its swings by pressing the stud C).

The bearing read will be a "forward" bearing and normally a "whole circle" bearing, a clockwise angle between 0° and 360°.

Military Compasses. Although these are made in various forms, the service patterns are usually of the prismatic class, the box being about 2 in. diameter, provided with a finger-ring under the prism. The more conspicuous differences from the larger pattern just described are as follows: (1) An external ring, divided 0° to 360° in the counter-clockwise direction, is fitted around the circumference of the box. (2) A movable glass cover, provided with a luminous patch (a), is fitted in a milled ring over the box and secured by means of a clamping screw. (3) Two sets of dial divisions, both figured 0° to 360°, the inner set being normal for direct readings; i.e. without the use of the prism. On top of the dial is a luminous pointer (b), which is used in connection with the patch (a). (4) The lid carries a circular window, placed eccentrically. Down this window is scribed a vertical line which serves as a sighting vane in daylight operations. At the extremities of this line are two patches (c), which are also used in connection with (a) and (c) in night operations.

The walking-stick "bipod" described on page 82 (Chapter VI) is also useful with this instrument.

Now if a compass were supported in gimbals and constantly in view, as on a ship, it would be possible for the observer to keep on a given bearing from A to B. But on foot or horseback this is impossible, and if B is not a visible landmark or station, the observer will soon find himself moving in a direction parallel to AB. Hence, in marching on merely a given bearing at night it is often necessary to work to a selected series of stars, or, failing this, to work in conjunction with two men by mutual alignment in order to maintain the direction. Some knowledge as to the identity of conspicuous stars and their apparent positions and movement is obviously necessary.

Some advice might be found in a story of the last war, when an old lady was informed that the officers were not holding an egg-and-spoon race, as she surmised, but were undergoing instruction in marching with the compass.

The advantages of the compass as a surveying instrument may be summarised generally, its simplicity and portability being recognised.

(1) Running rapid traverses without regard to preceding lines, a more or less fixed line of reference for forward bearings existing at

all stations. (2) Running lines through forests where obstructions impeding the line of sight are more easily overcome than with other instruments. (3) Facility of fixing positions by resection on two or three points, already mapped. (4) Retracing lines which were run with the compass before the introduction of the theodolite; an application of the surveying compass in the U.S.A. (5) Facility with which bearings lend themselves to the use of latitudes and departures, particularly if the circle is divided in the "quadrant system."

Its disadvantages are (1) that lines of great length cannot be run with great accuracy unaided by a telescope; (2) that at best the method is not precise, since at best bearings cannot be read within five to ten minutes (of arc) under the most favourable conditions; and (3) that the needle is unreliable, and that local attraction may render rapid work impracticable or impossible.

II. BEARINGS

The axis of the compass needle serves as a reference line known as a magnetic meridian, or *n* and *s* line. This line differs from the *true* meridian, as would be given by a line between the observer *O* and the north pole, by a horizontal angle known as the *magnetic declination*, of which more will be said later.

The true meridian *NS* is shown faintly and the magnetic meridian boldly as *ns* in Fig. 57, where the angle *NOn* is the declination, being to the west, as it is in this country to-day.

But the Pole Star actually rotates about the north pole, making an angle with the earth's centre of about 1° at the present time, so that its direction fixes the true north only when it is vertically above or below the pole, or at upper or lower transit, in astronomical language.

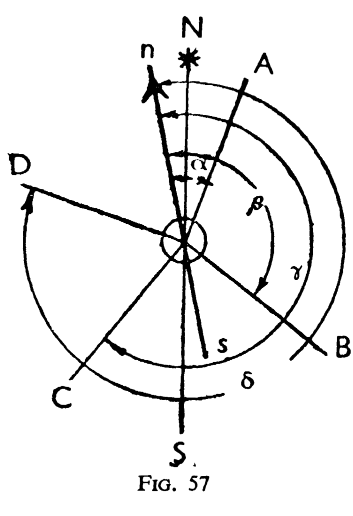


FIG. 57

In general, bearings are horizontal angles measured from the north and south points of reference meridians, and may be *true* or *magnetic* bearings accordingly.

Now we encounter the true British profusion of terminology; but once and for all let us classify the two modes of observing bearings as "Azimuths" and "Bearings," but keeping in mind the synonymous uses of the terms.

(*a*) *Whole Circle Bearings* (W.C.B.). or simply *Azimuths*, are angles measured clockwise from the north point from 0° to 360°. Most geographical and army and air force text-books define these simply as bearings, but not altogether without reason. The graduated circles of British theodolites and compasses are divided in the *Whole Circle* System, 0° to 360° clockwise, which are read directly, but have to be reduced to the angles we have adopted as bearings.

(The true meridian *NS* may be forgotten a while, since we are working with the compass at present, and booking magnetic bearings.)

Thus the azimuth of *OA* is α; of *OB*, β; of *OC*, γ; of *OD*, δ; as simply read on the circle, being respectively 30°, 140°, 230°, and 300° in Fig. 57.

(*b*) *Reduced Bearings* (R.B.), nautical bearings, or simply *Bearings*, are horizontal angles measured from the north and south point, in either direction from 0° to 90°, the angular value being preceded by the initial letter *N* or *S*, and followed by the terminal letter *E* or *W*.

They are read directly on circles divided in the *Quadrant System*, 0° to 90° to 0° to 90° to 0°, which obviates reduction in later methods.

Otherwise they are readily reduced from observed whole circle bearings as follows:

(*a*)	α	β	γ	δ
(e.g.)	30°	140°	230°	300°
(*b*)	N.(α)E.	S.(180° − β)E.	S.(γ − 180°)W.	N.(360° − δ)W.
(e.g.)	N.30°E.	S.40°E.	S.50°W.	N.60°W.

(2) Although azimuths are read directly, it is now desirable to work with bearings, not only because back bearings are readily evident, but that azimuths have little future in front of them until they are converted to bearings.

Every line has two bearings: a *forward* bearing and a *backward* bearing, the forward bearing being understood in plotting. Thus the forward bearing of a line *AB*, as suggested by progress from *A* to *B*, is merely β or N. β E., and the back bearing, as suggested by sighting from *B* to *A*, is $180°+\beta$, or S. β W. Hence, with azimuths we merely add or subtract angles exceeding 90° to or from 180° (or 360°), while with bearings we merely interchange the initial letters (N. and S.) and the terminal letters (E. and W.).

Thus, in Fig. 58, the forward bearing of *AB* is N. β E., and its back bearing S. β W.; the forward bearing of *BC* is S. γ E., and its back bearing N. γ W. . . . with the forward bearing of *DA*, N. α W. and its back bearing S. α E.

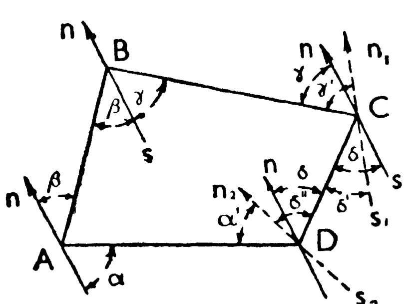


FIG. 58

Possibly you will appreciate the process better if you write 30° for β, 40° for γ, 50° for δ, and 60° for α.

Also, the forward azimuth of *AB* is simply β and its back bearing $180°+\beta$; the forward azimuth of *BC* $(180°-\gamma)$ and its back azimuth $(360°-\gamma)$ . . .

with the forward azimuth of *DA* ($360°-\alpha$) and its back azimuth ($180°-\alpha$), though these would be read merely as 30° and 210°, 140° and 320° . . . and 300° and 120° on ordinary whole circle divisions. The conjoint use of forward and backward bearings is an important artifice known as working "fixed needle" in the presence of magnetic disturbances.

(3) MAGNETIC DECLINATION. The magnetic declination, the horizontal angle between the magnetic and the true meridian at any place, varies at different times and at different points on the earth's surface. In physics, lines of equal declination on maps are known as *Isogonic Lines*, the lines of zero declination being *Agonic Lines*. The term expresses the fact that magnetic and true meridians are not coincident, and imply some magnitude at any given date; but in military surveying and mathematical geography, the term "variation" is often used exclusively and synonymously with the term declination. This is incorrect, since the magnitude at any given place and date is itself subject to variations, or changes, which are of the following three kinds: (*a*) *Secular*, (*b*) *Annual*, and (*c*) *Diurnal*.

(*a*) Secular variations occur with the lapse of years, centuries revealing that the motion is periodic in character, something like that of a pendulum impelled to oscillate in a complex vibration. Thus, at *Greenwich*, the declination was 11° 36′ *East* in 1580; in 1663 (three years before the *Annus Mirabilis*) it was *zero*; and in 1818 it reached its most westerly value of 25° 41′. Since then it has decreased steadily with an increasing annual movement, and is now 10° 45′ W. (1940). Its value for any year can be found from some reference work, such as *Whitaker's Almanack*.

(*b*) Annual variations are cyclical changes in which the year is the period, the variation being greatest at springtime, decreasing to midsummer, and then increasing during the following nine months. At most places it amounts to less than a minute, and is therefore of secondary importance. Annual variation is totally different from the progressive angular change due to secular variation.

(*c*) Diurnal variations are the more or less regular changes in the needle from hour to hour, leading to a total difference of ten minutes in any one day near London. This variation differs for different localities and for different seasons of the year, being less in winter than in summer, when it may amount to 15′ at places. The cause is attributed to the influence of sunlight.

There are also irregular fluctuations which seem to coincide with the appearance of the aurora borealis, earthquakes, and volcanic eruptions, the needle becoming extremely capricious. Hence the mine surveyor relies upon the notices issued as to magnetic storms by the appropriate department of the Royal Observatory.

(4) LOCAL ATTRACTION. There are also disturbances of the magnetic needle which can be attributed to *Acts of Man*. Local Attraction

(facetiously known as the feminine of magnetic interference) denotes the influence which renders compass bearings inaccurate in the neighbourhood of certain bodies, particularly iron, steel, and certain iron ores, or even nickle, chromium, and manganese.

Thus disguised steel spectacles, keys (cigarette cases), and knives may cause trouble, and the unassuming chain and arrows are not always above suspicion. Also, steel helmets and box-respirators have been known to have been overlooked; and, by the way, a well-meant cleaning of the glass cover may electrify it so as to attract the needle.

Outside these *avoidable sources* there are enough *fixed sources* to fill a catalogue. Fences, manhole covers, railway metals, trolley wires, steel structures, etc., in view; and, unseen, underground pipes, ironwork, etc., etc.

By this time you have doubtless agreed that the compass is best suited to exploratory work, and the safest place for it is a forest, desert, or jungle. On the contrary, local attraction can be "bypassed," and this a mere detail of the mine surveyor's work.

Suppose we return to Station A of Fig. 58. Here we observe the forward bearing β of AB as N. 30° E. (30°). Next we proceed to B. Here we find the back bearing, BA, is also β, as S. 30° W. (210°). Hence it is fairly safe to assume that neither A nor B is under magnetic influence, and with confidence we take the forward bearing γ of BC, reading S. 40° E. (or 140°). Then we proceed to C, and, lo! the back bearing of CB, is γ', not N. 40° W. (320°), but N. 45°W. (315°). Knowing that B was immune, C is suspected, local attraction causing the needle to assume the dotted position n_1s_1.

Anyway, let us take the forward bearing δ' of CD (which we believe should be δ) as S. 50° W. (230°), even though we record S. 45° W. (225°). Also, let us proceed to D. Here we observe a back bearing, δ'', as N. 55° E. (55°), and suspect that D is also unduly influenced. But here we are near to our starting-point, A, which we know to be immune. Hence we take a forward bearing, DA, of α', and this is N. 55°W. (305°). Therefore we hurry to A, and observe α, the back bearing AD, as S. 60° E. (120°).

Now we review our notes and see that our false reference meridian *lies* 5° to the west at D, and 5° to the east at C.

Also it is evident that the exterior angle at $C=$

$$180°+\gamma+\delta=180°+\gamma'+\delta'$$

and that the interior angle at C is this value subtracted from 360°, or $360°-(180°+40°+50°)=360°-(180°+45°+45°)=90°$. But the interior angle at D is simply $(60°+50°)=(55°+55°)=110°$.

In fact, it would not have mattered if magnetic interference had existed at all the stations. The record of forward and back bearings would have enabled us to find the true interior angles of the polygon with just one geometrical defect. The polygon would be *orthomorphic* (correct shape), but it would be displaced on the drawing-paper by the error in the magnetic meridian we assume for our first station. Hence, it is desirable that one station should be unaffected, and this can be ascertained by taking a bearing at an intermediate point in a line and noting if this agrees with one of the end bearings.

This process is known as working "fixed needle." Free needle traversing is the normal method by forward bearings. No regard is thus taken of back bearings, so that each line is independent; but if local attraction exists, the configuration of the traverse will be incorrect.

Finally, the sum of the interior angles should be equal to twice the number N of right angles as the figure has sides, less four right angles, or, algebraically, $(2N—4)90°$. Naturally you will seldom obtain this sum, for, apart from local attraction, there are those natural ailments, errors of observation; and the *error of closure*, as it is called, may be from $(\frac{1}{4}° \text{ to } 1°)\sqrt{N}$. degrees difference from $(2N—4)90°$, according to the size and quality of the compass.

A simple polygon should be traversed with a compass influenced by suspended keys, or, better, the shadow of a steel helmet, if privileged to wear one.

III. TRAVERSING

A good compass would have been exceedingly welcome when we were running the open traverse of the stream in Fig. 19, or fixing the traverse angles around the pond in Fig. 20. This would have obviated the use of those terrible ties, essential to fixing directions when only the chain is at hand. Also the field notes would have been simplified, since the mere entry "N. 12° E.," etc., would have superseded the entries relative to the lengths and positions of the angle ties. At the same time, it is always desirable in all traverses to keep a tabular record, showing *Line*, *Length*, *Bearing*, with further columns for future calculations if likely to be required.

However, an illustrative example of the application of *Polar Co-ordinates*, the *fourth* principle of surveying, is desirable.

Now traverse surveys of quiet country lanes are very interesting, particularly if they are run between definite landmarks shown on, say, the 25-in. Ordnance sheets.

Starting at a point, or station A on the roadside, a sight is taken on a distant point B, a heap of stones, or a mark, and the forward bearing of AB is observed. Then the distance AB is carefully paced, aided desirably by that useful present, the passometer. (Notes as to buildings, width of road, etc., are jotted down.) Also, if there is a prominent landmark, say, a church spire, a bearing should be taken to this, in order to serve as a check on the work.

From B, another station C is seen, possibly at a definite point near the bend of the road. The bearing of BC is observed, and the distance BC is paced, notes with estimated distances to objects being recorded incidentally. Possibly from B, the church spire may again be visible,

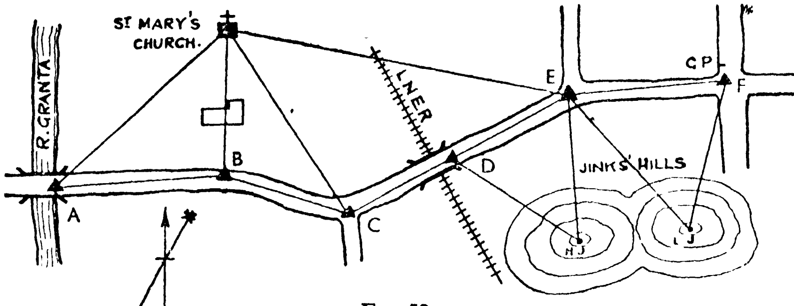


FIG. 59

but, if not, it may be sighted from *C*. So the open traverse is made, until the end *F* is reached, which might be a guide-post on the grass verge near the cross-roads.

En route, various inaccessible objects, such as hill-tops, might be located by "intersections," or *Angular Co-ordinates*, the *third* principle of surveying. If a clinometer were at hand, the heights of conspicuous points might be found by the method outlined in Chapter V.

A pocket prismatic compass has been in view in this traverse. If a larger pattern were available the distances might be measured with a convenient length of R.E. tracing tape. This is fairly safe in traffic. But, of course, chaining thus makes it a two-man job.

Now the explorer actually does the same thing, measuring distances by time on foot, or horseback, etc. His distances will be very much greater, and the scale of plotting very much smaller; but the principles are the same. In fact, Tom Sawyer would see our prosaic country lane as a rough mountain valley, the little river, the Mississippi, the church, the Grand Canyon, and the twin hills as Council Bluffs.

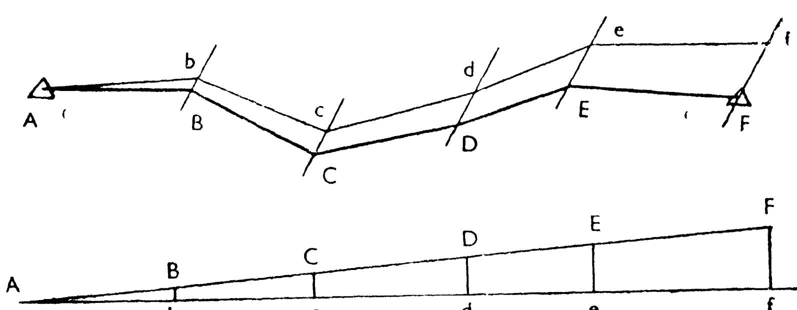
But we are anxious to see how our first effort works out on paper, plotting it, say, on the scale of 1 : 2,500. The work will be simplified if we summarise our notes in tabular form, as follows:

Line	*Length ft.*	*Compass Bearing*	*Station*	*Notes*
AB	930	N. 85° E.	*A*	St. Mary Ch., N. 52° E.
BC	755	S. 66° E.	*B*	
CD	750	N. 64° E.	*C*	St. Mary Ch., N. 27° W.
DE	623	N. 62° E.	*D*	High Jinks, S. 54° E.
EF	1052	N. 83° E.	*E*	H.J., S. 19° E.; L.J., S. 43° E.
			F	Low Jinks, S. 14° W.

Adjustment of Traverse Surveys. Although the following process more particularly applies to closed traverse polygons, it is also used in correcting open traverses, when these begin and end at points accurately surveyed, as in the case of the triangulation stations of a main survey, or definite points on the 6-in. or other Ordnance sheets. Rough and medium grade compass traverses are adjusted graphically, but those made with accurate surveying compass are adjusted arithmetically, as is usually the case with the theodolite, the correction being carried out through the rectangular co-ordinates, the latitudes, and departures of the lines.

There is only one rational method of graphical adjustment, and this is based upon the method devised by the celebrated mathematician, Nathaniel Bowditch (1807). This requires that the correction to each traverse line shall be in the proportion that the line bears to the total length of the lines, or the perimeter of the traverse. This process affects both bearings and lengths alike, and was devised for the compass, the theodolite, as we understand it, being beyond the dreams of the land surveyor at that time. Yet to-day many surveyors use the method implicitly in the arithmetical adjustment of theodolite traverses; and often wisdom would be folly when this blissful ignorance achieves its end.

The graphical process is as follows when applied to the traverse of Fig. 59, where *A* and *F* are the points or stations on the Ordnance sheet, from which a tracing has been made for the sake of econony.



FIGS. 60 and 61

Let *Abcdef* be the traverse as plotted to the given scale with a good protractor, showing *fF* as the error of closure on the point *F* on the Ordnance sheet.

Draw parallels to the direction *fF* through *b*, *c*, *d*, and *e*, in Fig. 60. Set off on the same scale (or some convenient fraction thereof) the consecutive lengths of the traverse lines along the horizontal base *Af* in Fig. 61; and at *f* erect a perpendicular to *Af* equal to the error of closure *fF*. Join *AF*, and erect perpendiculars to the base *Af*, giving *bB*, *cC*, *dD*, *eE*, the corrections to be made at the stations *b*, *c*, *d*, and *e*.

Obviously these corrections are in the required ratios of the lengths of the sides of the sum of these lengths, and they would be the same if *Af* were one-half, one-quarter, etc., the scale value of the total length *A* to *f* in Fig. 60.

Finally, set off the corrections *Bb*, *Cc*, *Dd*, and *Ee* along the parallels at *b*, *c*, *d*, and *e*, so as to obtain the adjusted traverse *ABCDEF*, as indicated in thick lines in Fig. 60.

The method is applied in a similar manner to a closed traverse polygon. Copy the outline of the traverse *ABCD* in Fig. 20, but with *A'*, say ¼ in. above and to the left of *A*, giving an error of closure *AA'*. Then proceed as above, but with the horizontal base divided only into four lengths, *AB*, *BC*, *CD*, and *DA'*. No! Run round an irregular pentagon with the compass and chain, and then see what you have to say about it.

IV. COMPASS RESECTION

Although the method of *Trilinear Co-ordinates*, as understood in the "three-point problem," is usually associated with the plane table, it has numerous applications with the compass in exploratory and preliminary surveys.

In theory a point *P* can be fixed by the bearings observed from *P* to any two visible and mapped stations or points, *A*, *B*, the magnetic north serving as the third point. But, apart from the sluggishness or other defects of the compass, the magnetic declination might vary considerably over the area, and the direction assumed for the magnetic meridian might be true only for a part of the area if the latter were very extensive. Hence, it is advisable to observe three points, *A*, *B*, and *C*, and by subtracting their bearings from *P*, to find the angles θ and φ as subtended by *AB* and *BC* at *P*, θ being $\beta-\alpha$ and φ, $\gamma-\beta$, as shown in Fig. 61. The direction of the magnetic meridian *np* thus becomes of little importance, since θ and φ would be the same whatever the extent of local attraction. The rays from *P* to *A*, *B*, and *C* are reproduced on a piece of tracing-paper with the aid of a protractor; and the tracing is shifted over the map of the area until the rays pass through *A*, *B*, and *C* at the same time. Then if *P* on the tracing is pricked through to the map the required position is fixed.

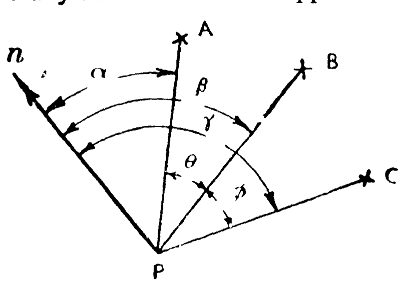


FIG. 62

The solution will fail if *A*, *B*, *C*, and *P* are all on the circumference of a circle, *P* thus having an indefinite number of positions.

The method is particularly useful in fixing positions at which observations for altitudes have been carried out with the aneroid or the boiling-point thermometer; and in selecting positions for stations in an extensive scheme of triangulation.

Here is an idea for an adventure story in your magazine, with mystery introduced through the medium of a complex code, which gives clues both to points and bearings in finding the hidden secret.

Incidentally, this *fifth* principle of surveying is also the basis of fixing the position P of a wireless receiving station with reference to three transmitting stations of known wave-length, A, B, and C, the positions of which are shown on a map. A directional frame aerial at P is fitted with a horizontal circle, so that the direction of the vertical plane of the aerial can be determined when it is turned edgewise towards the transmitting stations so as to receive the signals at maximum strength. From the divided circle the angles θ and φ are found, and P is determined in a manner similar to that described for compass re-section.

In practice this is not so simple as it appears to be, for there is "local attraction" above and below, and all along the paths of the wireless waves.

OFFICE AND CLASS EXERCISES

7 (A). Plot the survey from the notes given on Plate IV. (G.S.)

7 (B). Plot the survey from the notes given on Plate V. (G.S.)

7 (a). The following bearings were taken with a prismatic compass in an open traverse $ABCDE$ through an area in which magnetic interference was suspected:

AB, 39°; BA, 219°; BC, 84°; CB, 267°; CD, 122°; DC, 294°; DE, 129°; ED, 314°.

State the values of the corrected magnetic bearings with which you would plot the survey.

(C and D affected —3° and + 5°; CD, 119°; DC, 299°; DE, 134°) (G.S.)

7 (b). Draw an equilateral triangle of 6 in. side to represent a triangle ABC, with A, B, and C running in clockwise order, CB being horizontal and 2 in. above the bottom of the page.

A represents a spire; B, a coastguard station; and C, a castle.

A smuggler hurriedly buries some treasure at a point O, and observes the following compass bearings from O to A, B, and C respectively:

36° 135° 230°

Show how he would find the position of O with a compass bearing and distance, both from B, the scale of the map being 1 in 1,000.

(Tracing paper will be supplied if required.) (G.S.)

(4·85 in., or 405 ft. and 315° from B.)

7 (c). Describe any form of prismatic compass, giving (if possible) a sectional view.

State concisely what you know about the following:

(a) Magnetic declination and variation.

(b) Magnetic interference in surveying. (G.S.)

7 (d). The following compass traverse was run between two stations, A and B, which were fixed by triangulation, B being respectively 475 ft. and 1,200 ft. due N. and due E. of A with reference to the true meridian.

Both forward and backward bearings were taken as local attraction was suspected.

Line	Length (ft.)	Magnetic bearings	
		Observed	Corrected
Ab	510	N. 34° E.	
bA	510	S. 34° W.	
bc	195	N. 79° E.	
cb	195	S. 84° W.	
cd	540	S. 70° E.	
dc	540	N. 75° W.	
dB	370	S. 58° E.	
Bd	370	N. 58° W.	

Plot the traverse with the corrected true bearings, taking the magnetic declination to be 13° W., and using a scale of 1 in 2,400. If necessary, adjust the traverse lines to fit between the main stations *A* and *B*.

(Place the true north parallel to the short edges of the Answer Book and assume *A* about 2¾ in. from the S.W. corner of the page.)

(Station *c* affected +5°. Closing error from 60 to 75 ft. reasonable.)

*7 (*e*). Three wireless transmitting stations *A*, *B*, and *C*, are situated in clockwise order at the vertices of an equilateral triangle of 6 in. side on a map plotted to a scale of 20 miles to the inch.

An explorer has mapped three stations as *P*, *Q*, and *R*, with the following magnetic bearings and distances:

PQ, 43·8 mls., 62°; *QR*, 37·8 mls., 30½°, *P* being 34·8 mls. on a line 60° N.E. of *C*.

He then uses his wireless receiver to check his positions at *P*, *Q*, and *R*, and by means of a directional aerial he ascertains the magnetic bearings of three transmitting stations he identifies as *A*, *B*, and *C*.

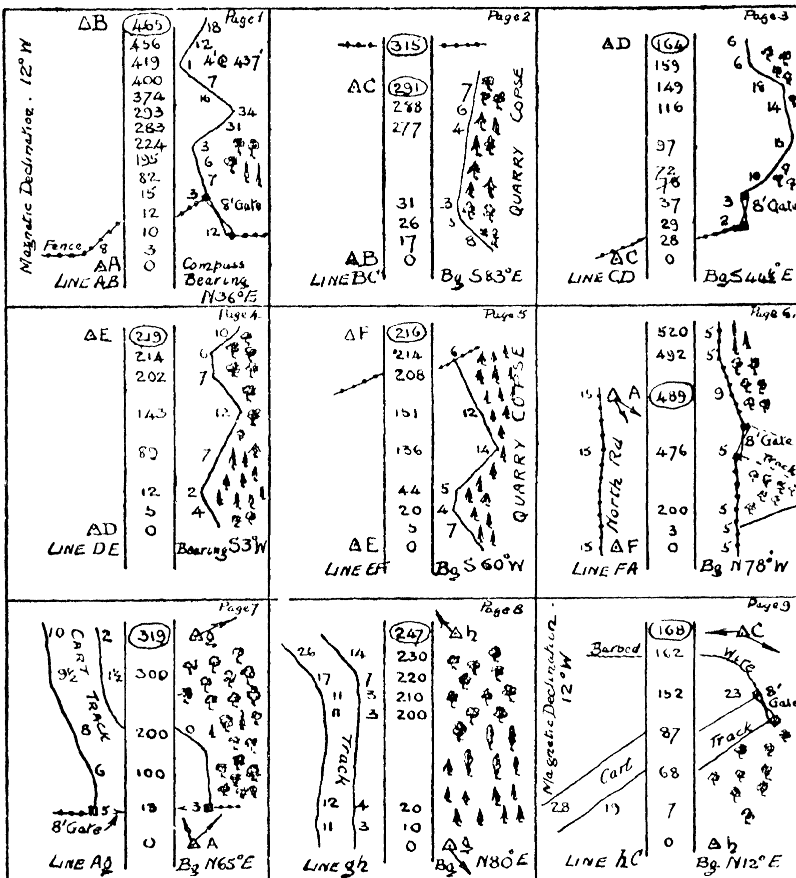
Observer's Station	Magnetic Bearings to Transmitting Stations		
	A	*B*	*C*
P	20½°	103°	237°
Q	354°	125½°	243°
R	314½°	157°	228½°

(*a*) Plot the traverse on the scale stated from the given distances and bearings, of *P*, *Q*, *R*.

(*b*) Plot the positions of *P*, *Q*, and *R*, as determined by the wireless signals, using the tracing paper supplied.

(Stations *P*, *Q*, and *R*, located by wireless signals approximately 2·4 mls. N., 3·2 mls. S., and 5·9 mls. N. of respective survey traverse stations.)

PLATE V



The above pages of Field Notes refer to a Compass-Chain survey of a Copse, *all* measurements being in feet.

Plot the survey on a scale of 50 ft. to 1 inch, placing the true N. and S. parallel to the short edges of the paper, with *A* about 4 inches from the lower and left-hand edges.

PLATE VI

The following notes refer to the survey of a meadow which was mainly under water except alongside a stream running through the area. In consequence a straight backbone *ABCD* was run near the *southern* bank of the stream up to Station *C*, where the stream bears N.E., entering the river near Station *h*. From the stations *A*, *B*, *C*, *D*, compass bearings were taken in order to fix the boundaries, which were straight, except along the bank of the river on the north of the area.

Plot the triangulation network for the survey on a scale of 50 feet to 1 inch, using a protractor and scale. In doing this, place the Magnetic North parallel to the short edges of the paper with *A* 1½ in. from the left-hand short edge and 6½ in. above the bottom edge of the paper. Add as much of the detail as possible, following the notes given on the right of the notes.

N.B. The bearings are measured E. and W. from the N. and S. points.

(G.S.)

COMPASS SURVEY OF BRAY'S PIECE

Station	*Sighting to*	*Magnetic Bearing*	*Length (ft.)*	*Notes*
A	*e*	N. 2° 30′ E.	*AB*	*l*, *A* and *e*, 12 ft. E. of fence to
	f	N. 48° 15′ E.	=	Bray's Lane; 10 ft. footpath on
	B	N. 90° 00′ E.	410	E. side and 30 ft. carriage way.
	k	S. 64° 30′ E.		*A*, *B*, *C*, *h*, 15 ft. from S. bank of
	l	S. 2° 30′ W.		Mill Brook, 12 ft. wide.
B	*e*	N. 55° 30′ W.	*BC*	*e*, *f*, *g*, *h* along S. bank of River
	f	N. 9° 00′ W.	=	Dee; 30 ft. wide.
	g	N. 52° 15′ E.	271	
	C	N. 90° 00′ E.		15 ft. S. of Mill Brook/Bears N.E.
	j	S. 0° 00′ E.		Wall corner, Grove Mill.
	k	S. 0° 00′ E.		Fence corner, Grove Mill.
	l	S. 65° 30′ W.		*l*, *k*, Iron fence, Grove Mill.
C	*g*	N. 24° 00′ W.	*CD*	S. bank of R. Dee.
	h	N. 37° 30′ E.	=	S. bank of R. Dee/Also at Crown
	D	N. 90° 00′ E.	237	Inn wall. *D* also at this wall.
	i	S. 63° 30′ E.		*hDi*, straight wall, Crown Inn.
	j	S. 44° 00′ W.		*ji*, straight wall, Grove Mill.
	k	S. 56° 00′ W.		*kj*, short wall, Grove Mill.
D	*h*	N. 22° 00′ W.		Straight wall, Crown Inn.
	i	S. 22° 00′ E.		Straight wall, Crown Inn.

Details: Centre of culvert, 210 ft. from *l* (21 ft. from *A*): diameters, 12 ft. inside; 15 ft. outside. Footbridge, 4 ft. wide near B. Bridge over R. Dee: 35 ft. clear span: 40 ft. between walls.

FIELD EXERCISES

Problem 7 (*a*). Survey the (specified) pond (or wood) by means of the chain and compass.

Equipment: Compass, chain, compass, arrows, tape, and a set of pickets.

Problem 7 (*b*). Run an open traverse of the (specified) road (or stream).

Equipment: As in 7 (a).

Problem 7 (*c*). Make a compass-pacing traverse between . . . (two specified places).

Equipment: Small prismatic compass (clinometer and passometer).

Problem 7 (*d*). Determine the error in the sum of the interior angles of the polygon indicated by the range poles *A*, *B*, *C*, *D*, and *E*.

Equipment: Compass.

Problem 7 (*e*). Determine the distance and height of the (specified inaccessible points) from and above the station indicated at the range pole *A*.

Equipment: Compass, two pickets, chain, arrows, and clinometer.

ORIGINAL PROBLEMS

. . . .

. . . .

(e.g. Measuring the interior angles of a polygon when the compass needle is influenced by an attached key).

. . . .

. . . .

(e.g. Finding the treasure buried at *P* from the bearings of *A*, *B*, and *C*, as obtained from Group . . . (the smuggling party).)

CHAPTER VIII

PLANE TABLING

Although it scarcely needs any introduction to-day, the plane table may be described as a drawing-board mounted upon a tripod to form a table upon which surveys can be plotted concurrently with the field work through the medium of a combined sighting device and plotting scale.

Hence angles are not observed in magnitude, as in the case of any *goniometer*, or angle measurer, such as the compass, sextant, and theodolite, but instead are constructed directly, so that the instrument is a *goniograph*, or angle plotter.

Suppose you insert two pins vertically at the ends of a ruler, and place this on a table at a point O; then, using these pins as sights, you glance through them first to one corner A of the room, then to the other, B, the angle AOB could be constructed if lines were drawn along the edge of the ruler. That is the sole geometrical principle of the plane table.

The *Table* is made in numerous forms and sizes, ranging from small, light patterns with a simple board and thumb-nut attachment to the tripod, to elaborate boards with every refinement for levelling the drawing surface and rotating the board, even to carrying a continuous roll of drawing-paper.

The sighting device may also range from a simple *Sight Rule* with vertical eye-sights to a *Telescopic Alidade*, which may be simple or the upper part of a complete transit theodolite mounted upon a rule.

Sight rules are often engraved with a scale on each edge, but the base rules of alidades are seldom divided, thus necessitating the use of independent plotting scales.

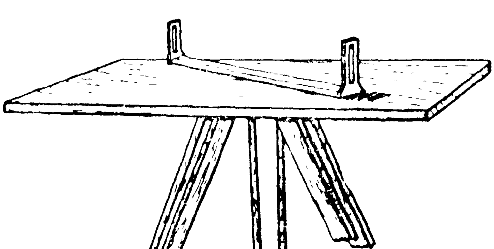


FIG. 63

A simple plane table is shown in Fig. 63.

Common to all patterns are the spirit levels and compass. In simple patterns the spirit level for levelling the board is sometimes fitted as a cross bubble to the sight rule, a separate trough compass being supplied. In more expensive patterns a large circular compass sometimes carries two large bubbles at right angles to each other. Simple tables are levelled up solely by manipulation of the legs of the tripod, but high-class models are fitted with a tribrach levelling base. Also the clamping of the board in an important position is effected solely with that unsatisfactory device, the thumb-nut, on the one hand, while on the other, a refined clamp and slow motion is provided. In the writer's opinion, no board and tripod can be too good in practice, and a telescopic alidade is essential, though the tendency is to make this unduly elaborate.

In larger models, a *Plumbing Bar* is supplied, so that a mark on a station peg can be transferred up to the board through the medium of a plumb-bob attached to the under-arm of the plumbing bar.

Useful plane tables are readily constructed from half-imperial drawing boards, battened rather than framed patterns. A hard wood or metal plate is secured to the centre of the under face of the board, and a long screw with a thumb-nut is inserted in this, exactly at the centre of the board. The thumb-nut serves to secure the board to the top of the tripod and to clamp it in any desired position in the field. When it is really necessary to construct tripods, these should be of the framed pattern, as suggested on page 67. The triangular plate, or headpiece, to which the halves of the tripod legs are secured, should be drilled to take the screw which protrudes from the under-face of the board. Wooden camera tripods, though they possess the merit of telescopic portability, are seldom rigid enough; and size should not be sacrificed in order to lighten the board. A 3-in. or 4-in. bubble in a metal case will serve for levelling purposes, and the compass should be of a type that the N. and S. line of the case may be easily transferred to the plotting paper. Incidentally a waterproof satchel should be made, or improvised from American cloth. Sight rules should be purchased. Boxwood patterns in the 10-in. or 12-in. size should be selected, with one of the scales showing inches, tenths, etc., however it may be specified: 10, or 100, as the case may be. Some of the scales engraved on sight rules are of little use in classes, the work being kept normally to 50-ft. (lks.), or 100-ft. (lks.) to the inch. The 1 : 2,500 means access to the 25-in. Ordnance sheets, and these are very expensive in classes, while the 6 in. to 1 mile would suggest a man's job on a half imperial drawing board. Plausible as these scales may sound, their use impedes the work in instructional classes.

Terms and Definitions. On small scales, or in lower grade work, the board is regarded a point; and only on very large scales is the plumbing bar justified. Scale plays an important part in plane tabling. It is a kind of denominator which fixes the speed of the work; and, in a way, speed multiplied by accuracy equals a constant; "more accuracy less speed." Scale also suggests two terms we must know before going into the field: (*a*) *Orienting the Board*, and (*b*) *Setting the Board.*

(*a*) Orienting the board means turning it on the top of the tripod so that plotted lines are parallel to (or coincident with) the corresponding lines in the field.

(6) Setting the board means orienting it roughly by placing the north end of the compass box over the north end of a magnetic needle drawn on the board, and turning the board until the compass needle comes to rest in the common magnetic meridian.

Military sketch-boards are set in this manner, as also are certain rough plane table maps. On small scales, the error from orientation will be very small, but on large and even medium scales the defects of the compass will soon be evident.

Perhaps you will understand the distinction in the following emergency You are stranded at the junction *J* of five roads, where the guide-post has been removed as a war-time precaution. But you have a map of the locality; say 1 in 10,000, or the 6 in. to 1 mile. You find your position on this without difficulty, and you also see that church *C* in the distance. Then, if you spread the map flat, and turn it while looking along the line between *J* and *C* on the map until the church comes into view, the map will be oriented, strictly, if not accurately.

But if the country around is thickly wooded, you will need a compass in addition. You lay your map on the ground, and place the compass upon it with the north end mark on the compass box exactly over the north end of the magnetic meridian of the map. Then you turn the map slowly until the needle comes to rest in the common magnetic meridian. If only the true meridian is shown on the map, a magnetic meridian must be pencilled across it at the declination for the date and place. This is "setting" strictly, though both processes are referred to as such in military surveying.

Notation. The edge along which rays are drawn on the paper in plane tabling is known as the "fiducial edge of the alidade," which we will contract to "ruling edge." "Centring the alidade" (or sight rule) at a point or station on the plan means placing the ruling edge over the plotted position of that point. "Centring the sight rule" is facilitated by inserting a bead-headed pin at the point, and keeping the edge in contact with this pin.

As far as possible capital letters will denote stations in the field, such as *A*, *B*, *C*, etc., and the corresponding small letters will indicate the corresponding points on the board, as *a*, *b*, *c*, etc.

METHODS OF PLANE TABLING. It is not without reason that the plane table is considered the simplest and best instrument for demonstrating the principles of surveying, though, even in this capacity, it is seldom treated as a versatile demonstrator. A student from the Orient is said to have observed that the plane table is the best of all surveying instruments because there are only two things to be remembered about it. If he meant, as he presumably did, the processes of intersection and three-point resection, he had the academic outlook fairly well assessed. Apart from economic and climatic considerations, there is a place for all the five principles in practice, which is not all solving the three-point problem for resection's sake.

Now there are three primary methods of surveying with the plane table:

(1) *Radiation*; (2) *Intersection*; and (3) *Progression*, or Traversing.

Although seldom used in entire surveys, they are commonly used in filling in the details of triangles and polygons surveyed by more accurate methods.

(1) *Radiation.* (*a*) Reconnoitre the ground, making an index sketch, or adding notes to one copied from an existing map. Select as the station *O* a point from which all points to be surveyed are visible, say, *A*, *B*, *C*, and *D*, as in Fig. 64.

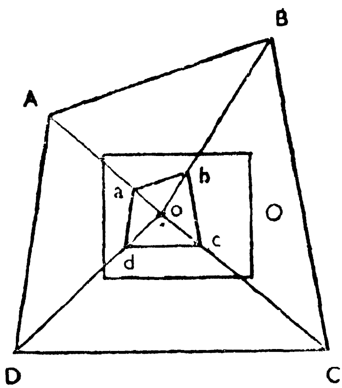


FIG. 64

(*b*) Level the table over the station *O*, and, referring to the index sketch, clamp the board in the best position for *placing* the survey on the paper. Fix a pin in the board at *o*, to represent *O*, selecting this point so that the entire survey can be plotted on the proposed scale. (Using the compass, insert a magnetic N. and S. line in a convenient place to serve as a dated meridian, and in large surveys in roughly orienting the board.)

(*c*) Centre the sight rule against the pin, and, sighting the stations *A*, *B*, *C*, etc., in order, draw rays towards them, but only round the margins, and not in the body of the paper, which is the place for the map. Reference these *A*, *B*, *C*, etc., in the margins.

(*d*) Chain the radial distances from *O* to *A*, *B*, *C*, etc., and set them off to scale as *oa*, *ob*, *oc*, etc. Connect *ab*, *bc*, *cd*, etc., with firm lines, if these are actually straight boundaries.

In practice, however, this method is used for details, such as inserting contours, where the radial distances may be found also by means of the tacheometer, or on small scales, even by pacing. In simple surveys it is sometimes used as an auxiliary method to progression.

(2) *Intersection.* (*a*) Prepare an index sketch, as stated above, incidentally obtaining some idea as to the distances and lengths involved.

(*b*) Select a suitable situation for the base line *PQ*, observing that all points to be plotted must be visible from both ends of the base. Chain the *one* direct linear measurement, the base *PQ* with great care, using a steel tape if one is available. Fix a pole at *Q*.

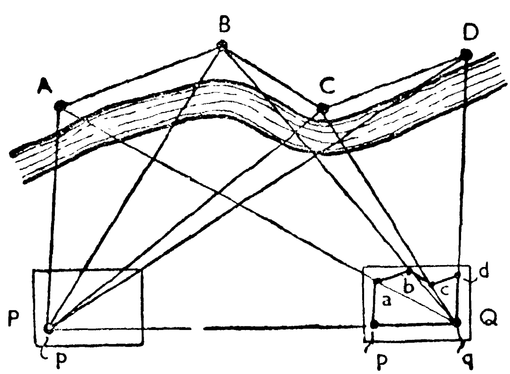


FIG. 65

(*c*) Set up the table, centring and levelling the board over one end of the base *P*. Clamp the board in the most convenient position for placing the survey, and, having carefully selected the position for the base *pq*, fix a pin at *p*, to represent *P*, the station occupied. (Insert a magnetic meridian by means of the compass.)

(*d*) Centre the sight rule against the pin at *p*, and sight at *A*, *B*, *C*, etc., salient points in the survey, drawing rays near the margins and referencing them accordingly *A*, *B*, *C*, etc. Sight the pole at *Q* with the ruling edge still centred against the pin at *p*. Draw a ray towards *Q*, and along it set off *pq* to the scale adopted to represent the measured base *PQ*. Fix a pole at *P* on vacating the station.

(*e*) Set up the table, and centre and level the board over the other end of the base, *Q*. Fix a pin at *q* to represent *Q*. Orient the board by sighting with the ruling edge along *qp* to the pole at *P*, and clamp the board thus. Centre the sight rule against the pin at *q*, and, sighting the points *A*, *B*, *C*, etc., draw rays towards them to intersect the corresponding rays from *P* in *a*, *b*, *c*, etc. Avoid intersections that are very oblique, or very acute, bearing in mind the rule for all triangulation -30° to 120° at the point intersected.

The chief objection to intersections as a sole method of surveying is the difficulty of selecting a base so proportioned that definite intersections will result, and of plotting that base with respect to both scale and position so that the resulting map is neither absurdly small nor so unduly large that certain intersections fall outside the limits of the paper. The method has been used with some measure of success by fixing stations by intersections around the boundaries, and then measuring between them in order to take offsets. Too often, however, the more accurate chain measurements will not agree with the intersected positions of the stations, which must be adjusted. Here we have fair linear measurements not mixing with poor angular measurements; and, as hinted before, the surveyor's headaches are not all due to eye-strain. In general, the method of intersections is best used as an auxiliary to some other method, particularly for locating inaccessible objects, such as mountain peaks, points across rivers, etc., etc., also outlying and broken boundaries.

It is particularly interesting to note that plane tabling by intersections is analogous to ground photographic surveying, particularly as regards determining elevations, the clinometer being used in conjunction with sight rules and the vertical arc with telescopic alidades. The India pattern clinometer is especially suited to determining elevations in plane table surveys, the tangents of the vertical angles being read directly. Here the elevations above the table are found from $V=D \tan \alpha$, as described in Chapter VI, D being the horizontal distance to the observed point as scaled on the board.

But there is this great difference between plane tabling and photographic surveying. Plane table surveys are plotted almost entirely in the field, and the field work is protracted at the saving of office work, whereas the field work in photographic surveying is brief, but at the expense of protracted office work. Thus photographic surveying is especially adapted to observations in exposed or dangerous situations. In fact, a photographic survey was being made at Sedan at the time the city capitulated in 1871. While the topic is still before us, it might be noted that the plane table was the forerunner of the elaborate plotting machines now used in connection with aerial surveys.

(3) *Progression.* (*a*) Prepare an index sketch of the area, as described with reference to radiation, incidentally considering the first traverse line on the proposed scale. No difficulty arises in this respect when filling in the details of a previously surveyed polygon by more accurate methods.

(*b*) Select and establish the stations, *A*, *B*, *C*, etc., bearing in mind that if the boundaries are straight, these may be more distant from the boundaries, but if the fences are undulating, short offsets must be used, as in ordinary land surveying.

(*c*) Set up the table over one of the stations *A*, and ascertain from the index sketch, the best position for the board and a point indicating the station *A*. As before, fix a pin at *a*, and on vacating *A*, remove the pin to the next forward station, *b*, etc. (Using the compass, draw a line in the magnetic meridian. Here this will merely serve as a magnetic meridian for the finished survey; but, in general, it assists in orienting when it is necessary to resort to resection.)

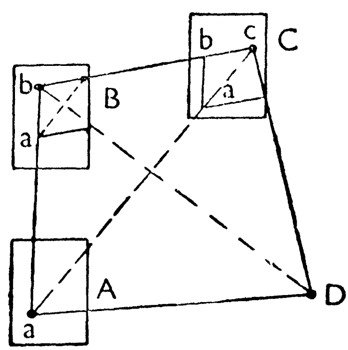


FIG. 66

(*d*) Centre and level up the table over station *A*, and clamp the board in the most suitable position. Sight back on the rear station *D* with the ruling edge centred against the pin at *a*. Draw a line towards *D*. Sight at the forward station *B*, still keeping the sight rule centred on *A*. Draw a line towards *B*.

(While the rule is still centred on *a*, draw a fine or dotted line towards *C* as a check sight. This will intersect later with the line drawn from *B* towards *C*, fixing *c* as a check. But checks must take second place to the main measurements, though they are very helpful in checking unseen movements of the board.)

(*e*) Locate detail near *A* by radial distances, using the sight rule, if the fences are straight, but if the fences are crooked, measure offsets in the usual way while chaining from *A* to *B*. (Radial distances are measured just as though station *A* were station *O* in the first method.) Measure *AB*, and plot it to scale as *ab* on the board. (Offset detail may be plotted either in the field or the office, and time and weather are the deciding factors.) Fix a pole at *A* and proceed to station *B*.

(*f*) Centre and level up the board over the next forward station *B*. Orient the plan by sighting back at *A* with the ruling edge along *ba*. Clamp the board thus. Sight at the next forward station *C* with the sight rule centred on *b*, and draw a line. (Note that this line will intersect the dotted line from *a*, giving a check. Incidentally draw a dotted fine line towards *D* for a similar check on *d*; but regard this as a check, never letting it supersede a chained measurement except when a serious movement has occurred.) Locate fence corners, buildings, etc., near *B* by radial distances, if otherwise offsets have not been taken. Fix a pole at *B* on vacating this station.

(*g*), (*h*) Occupy in order the stations *C*, *D*, etc., in order, levelling and orienting the board and chaining and plotting the traverse lines *BC*, *CD*, etc., as detailed in (*f*).

Progression is the best method of making purely plane table surveys, but it is seen at its best in traverses of roads and rivers, particularly in exploratory work, where intersections are invaluable in fixing lateral detail, mountain peaks, and the like. The value of radiation can only be assessed by surveying contour points and other features from a table set up at stations previously traversed by means of the theodolite and chain. Combined with tacheometric measurement the method still has a well-deserved place in topography.

Resection. The characteristic feature of resection is that the point *p* plotted is the station *P* occupied by the table. Strictly there are two general cases of plotting *p* from not less than two visible and plotted points, called *known* points: (*a*) when the line through *P* and *A*, one of the known points, is drawn, and (*b*) when *P* is no way connected with any known point, *A*, *B*, or *C*. The former is simple resection, and *p* is plotted by orienting the board by sighting along the line drawn through *a* towards *P* and fixing *p* by a sight through *b* to *B*. The second introduces the well-known "three-point" problem, which is often regarded as resection proper.

Now it often happens that a point of excellent command and general usefulness to the survey as a whole is not a station, and this can be occupied and plotted at once if three known points are visible. Simple resection would possibly involve a return journey to *A*, a great distance often, in order to draw a ray through *A* towards *P*. There are many occasions when resort to the method is expedient. But the three-point problem should never be resorted to for resection's sake; for, after all, it is merely incidental in actual work, even though it may be made a matter of great academical moment.

The three-point problem can be solved (1) *By trial*, (2) *Mechanically*, (3) *Graphically*, and (4) *Analytically*, the last applying more particularly to the theodolite.

Actually there is little to commend trial methods, beyond that an expert can readily eliminate a small "triangle of error" which may result from the mechanical method. The paper is the property of the map and not the place for a confusion of efforts at trial solutions. It is difficult enough to keep the paper clean without cultivating dirt from erasures of unnecessary lines. Graphical methods may be good in expert hands, but the following mechanical artifice will meet the demands of most cases.

Mechanical Solution. (Place the compass with its box N. and S. line along the meridian drawn on the map, and turn the board until the needle comes to rest in the common meridian. Otherwise set the board by eye.)

(1) Fasten a piece of tracing-paper on the board, and, as near its correct position as can be estimated, assume a point p' to represent the station occupied by the table.

(2) Centring the sight rule on this point p', sight successively at the three points A, B, and C, and draw rays accordingly along the ruling edge. Unfasten the tracing-paper.

(3) Move the tracing-paper about on the board until the three rays pass through a, b, and c, the plotted positions of A, B, and C, then prick through the point p', obtaining its true position on the map, say p.

(4) Orient the board by sighting through p and a to A, and check by sighting through b and c to B and C respectively.

If the check rays do not pass exactly through p, they will form a "triangle of error." The marine surveyor performs this operation with a three-armed protractor known as a "station pointer."

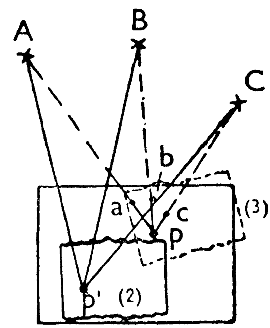


FIG. 67

The value of the plane table is often wrongly assessed in practice. It is frequently compared with combinations of other instruments rather than judged by its own peculiar merits.

(*a*) The plane table dispenses with field notes, and the survey is plotted concurrently with the field work, thus obviating mistakes in plotting recorded measurements. Also the area is in view, and measurements which might otherwise be overlooked are at once detected. In contour work it is superb under good climatic conditions, and features that mean little in field notes are seen as they really are. Even the finest photographic methods are impaired by shadows and other defects in the negatives.

But field plotting is disagreeable, if not impossible, under certain conditions of weather and climate, and the observer's position is cramped and tiring, being exceedingly trying in the heat of the sun. Also no notes are available for precise calculations of areas and the like.

(*b*) Little knowledge is required to use the plane table, but to manipulate it correctly and effectively demands considerable skill. It is cumbersome and awkward to carry and requires several accessories.

(*c*) The chief use of the instrument is the filling in of details of surveys where the skeleton has been surveyed with the theodolite. Used discriminately in this way, it is unsurpassed in certain topographical work. It is rapid, covering more ground in a given time than any other instrument, when plotting is also taken into account. Inaccessible points can be plotted without trigonometrical calculations, and elevations are readily found from the graphical construction of the tangents of vertical angles observed with a clinometer or the vertical arc of a complete alidade. Also the facility of three-point resection permits the occupation of unknown points which give excellent command without working through obstacles or areas devoid of detail.

Withal it is not intended for extremely accurate work, yet exceptionally good results can be obtained if due care and understanding are exercised.

In conclusion, a few practical hints must suffice, though pages could be devoted to the technique of plane tabling from the surveyors' point of view.

(1) *Paper.* Good quality paper should be used for surveys proper, the cheap grades being admissible only to rough exercise work Faintly tinted papers are best in intense sunlight They relieve eye-strain and obviate the necessity of coloured spectacles in bright sunlight. The paper should be fastened to the board so that neither the wind nor the movements of the alidade can disturb it As few drawing-pins as possible should be used on the drawing surface of the board The best plan is to cut the sheets barely the *width* of the board, turn under the excess *length*, and fix pins either in the edges or underneath, never using more than two pins on the upper surface. A waterproof cover is desirable in the case of sudden showers, and if this is not at hand the board must be removed and turned upside down.

(2) *Plotting.* Either a HH or HHH pencil should be used, one end being sharpened to a chisel point for ruling lines, and the other to a round point for indexing. Both points should be kept sharp, and for this purpose a sandpaper block should be suspended from the top of a leg of the tripod. Lines should be few, fine, and short, and unnecessary lines, or parts of lines, should be avoided. There is never any need for continuous lines, except for clearness in the case of main survey lines. A line, half an inch or so in length, at the station and a similar one near the estimated position of the observed point, will suffice in both radiation and intersection. Some simple system of referencing and indexing the lines should be adopted, and notes should never be written in the vicinity of points. A very good plan is to produce lines, not actually drawing them, except at the margins, where

half-inch lengths can be referenced without the possibility of confusion. Cleanliness is of highest importance, and the paper can be messed up with the slightest provocation. Pencils should never be sharpened over the board, erasures should be as few as possible, the cleanings flicked off the paper; the base of the rule should be kept clean, particularly if metal, and heavy alidades should be placed in position, never slid.

(3) *Manipulation.* See that the legs of telescopic or folding tripods are secure, and press these into the ground lengthwise, never crosswise. Aim at getting a level board, oriented and centred over the station, with the board a little below the bent elbow. Avoid unnecessary scruples in centring over a station when working to medium scales. Remember that eccentricity between point and station varies inversely as the observed distance, and that 1 inch means 1 minute in error in 280 ft., and 1 minute represents the highest grade work. Level up with the spirit level central in two positions at right angles near the centre of the board, and also test near the edges. Avoid undue pressure or leaning on the board and keep all accessories off the table when not in immediate use.

(4) *Sighting.* Fix a pin in the board at stations and keep the ruling edge against it when sighting. Fine bead-headed pins are the best. Always draw a magnetic meridian with the compass in the top left-hand corner when levelled up and oriented at the first station. It may prove useful besides giving a necessary detail in the finish of a plan. Always sight at the lowest possible points of station poles, pickets, etc. When vertical angles are observed, ensure that the board is level, and place table pattern clinometers as nearly as may be at the centre of the board.

Brief as these words of advice may be, they imply that you will aim at making a proper plan, rather than thinking you know how it is done.

Finally, do not mind if you are corrected for using the original term "oriented," which is now being ousted by the affectation "orientated."

CLASS EXERCISES

8 (*a*). An unfinished map is fixed to the board of a plane table at a station *A*. The map contains a magnetic north and south line, but only one plotted station, *B*, is visible.

Describe how you would (*a*) *Set* the map by means of the compass, and (*b*) *Orient* it by use of the sight rule, or alidade.

State clearly, giving reasons, which of these two methods you would use. (G.S.)

8 (*b*). Describe with reference to neat sketches the use of the India pattern clinometer in connection with the plane table.

8 (*c*). Describe how you would carry out the following surveys with the plane table, a chain, tape, and pickets being included in your outfit.

(a) *A large isolated wood;* (b) *a flat open field with straight fences;* (c) *a crooked boundary with a stream running along the inner side.*

8 (*d*). In a plane table survey it is essential to occupy as a station a point *P*

from which three stations, A, B, and C, are visible and are already plotted as a, b, and c respectively.

Describe with reference to a sketch how you would plot the position of P and orient the board at the point for further plotting. Under what conditions would your method fail.

8 (*e*). Describe how you would make a rapid survey of a mountain valley, determining the heights and positions of mountain peaks *en route*.

Your outfit consists of a light plane table with sight rule and compass, clinometer, and a passometer. The proposed scale is 1 in 25,000.

FIELD EXERCISES

Problem 8 (*a*). Survey the (specified) wood by means of the plane table.

Equipment: Plane table with sight rule and compass, chain arrows, tape, and a set of pickets.

Problem 8 (*b*). Survey the (specified) pond (or lake) by means of the plane table.

Equipment: As in 8 (*a*).

Problem 8 (*c*). Survey (specified owner's) field with the plane table.

Equipment: As in 8 (*a*).

Problem 8 (*d*). Make a rapid plane table survey of the (specified) lane between (named points).

Equipment: Plane table with sight rule and compass (passometer), and three pickets.

Problem 8 (*e*). The points indicated, A, B, and C, have been plotted as a, b, and c, on the board of the assigned plane table, which now stands at an unknown station P. Determine and plot the position of P with the aid of the tracing-paper supplied.

ORIGINAL PROBLEMS

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. . . .

Survey a part of the (specified) building by means of the plane table.

. . . .

. . . .

Select three prominent points on the 6-inch Ordnance sheet attached to the board of the assigned plane table. Find the horizontal distances and heights of these with respect to the station at which the table now stands.

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CHAPTER IX

CONTOURING

A contour is a line drawn through points of the same elevation on any portion of the earth's surface as represented on a map.

Contour lines are figured with that elevation above datum as an integral or whole value, and successive contour lines are inserted at regular increments from that value, such as 5 ft., 50 ft., or 10 metres.

The difference in elevation, or reduced level, of successive contour lines is known as the *contour interval,* or *vertical interval* (V.I.) in geographical and military surveying, where the corresponding distance in plan is called the horizontal equivalent (H.I.), leading to the relation (H.I.)=(V.I.) cot. α, with α the angle of slope between successive contours.

Contour intervals vary from 1 ft. to 10 ft. in engineering work, 5 ft. being the usual interval in English-speaking countries; from 10 ft. to 50 ft. in preliminary and pioneer surveys; and 100 ft. and upwards in exploratory surveys.

There would have been no reason for the inclusion of this chapter if, during the Great Flood, the waters had receded from the peak of Mount Ararat with solar regularity, and at noon each day had left a permanent watermark on the face of the earth, at intervals of 4 cubits, which approximate to our fathom—units so easily conceived by the mind of mankind, if not by that of the scientist.

In solid geometry these watermarks would be defined as the traces of horizontal section planes, and the ground plane, or horizontal plane of reference, would be the sea-level datum to which the water ultimately receded.

Uses of Contours. The uses to which contours are put may be summarised concisely as follows:

(1) Giving general information as to the surface characteristics of the country and showing if points are intervisible, as in military surveying.

(2) Giving data for drawing trial vertical and oblique sections for the construction of roads, railways, etc., and the layout of engineering schemes.

(3) Giving data for the calculation of earthwork volumes *indirectly,* as in the case of cuttings and embankments, and *directly,* as in the case of impounding reservoirs.

Characteristics. Among the various characteristics of contour lines the following should be noted:

(*a*) Contour lines close upon themselves somewhere, each to its own elevation, if not within the limits of the map.

(*b*) Contour lines cannot intersect one another, whether they be of the same elevation or not.

(*c*) Contour lines on the tops of ridges and in the bottoms of valleys either close or run in pairs within the limits of the map; and no single line can ever run between two of higher or lower elevation.

(*d*) Contour lines indicate *uniform* slopes when they are *equally* spaced; *convex* slopes when becoming farther apart with increasing elevations; and *concave* slopes when becoming closer together with increasing elevations.

METHODS. Contouring involves both surveying and levelling; in fact all the first five principles are employed in surveying, or *Horizontal Control*, as it may be called, and the two geometrical principles in levelling, or in *Vertical Control*, though the use of hypsometrical levelling is resorted to in the case of great intervals.

Contouring is the prime feature of topographical surveying, and there is no branch of surveying in which so many combinations of instruments and methods have been employed.

There are two general methods of Contour Location: (1) *Direct*, and (2) *Indirect*.

(1) *Direct Contouring*. As the term should imply, points on the actual contour lines are found on the surface by spirit levelling which, with one possible exception, is the sole practical method of vertical control. These *contour points* are then surveyed in the horizontal plane, and any of the methods of horizontal control are at the surveyor's disposal, though only one is selected primarily, another being in reserve for parts where the primary method would prove inexpedient.

At first sight, the procedure in vertical control appears to be tedious (as it really is at the outset), but on ground of definite surface character it is often best for intervals of 5 ft., though it may prove tedious on intervals of 2 ft., and exhausting in body to the staffman (and in soul for the levelman) on intervals of 10 ft. The levelman improvises targets from paper, straps, pocket handkerchiefs, etc., attaches these to the levelling staff; two and sometimes three. He then directs the staffman by signals until a point is found at which the (level) line of collimation strikes the target, which may be as wide as 3 inches in locating 5 ft. intervals. Unfortunately, among students, signals are soon evolved into acrobatics and clamours, the latter of the nature: "Go back," "Come nearer," "Up a bit," "Up yards," and (that gesture of comfort) "Down a millimetre." Then a lull. The staffman thinks he is forgotten until the levelman discovers that the line of sight is really "yards" above the top of the staff on account of a forgotten bubble. (It might be noted here that a traversing bubble is really necessary, since the modern tilting level is of little use in trial and error work.)

But after a few points have been found, the clamours will subside and the staffman will begin to sense the trend of the contours. And when direct contouring is done, it is done; which is to say, that hours of monotonous office work will not follow, as in the case of indirect methods.

In small parties the levelman signals to the surveying party that the staffman awaits them at a contour point, which may be located immediately with some instruments, such as the plane table with a stadia alidade, or the theodolite provided with stadia lines in the telescope. The work is then in *Dual Control.* Otherwise, or in extensive surveys, the staffman inserts a short length of coloured lath, selected from a haversack, which carries white laths for the 50 contour, red for the 55, black for the 60, green for the 65, blue for the 70, etc., all prepared by dipping the tops into paint pots. These coloured whites can then be located in *Detached Control,* the surveying party working at their own convenience and collecting the sticks as the points are located. A convenient plan of booking contour points is to enclose the interval number in a circle, thus 60; and in general, tabular notes are best, with a page devoted only to one station.

Something sensational, though analogous to the above process, happens when photographic plate pairs are inserted in a *Stereo-comparator,* where a single plastic, or relief model, is seen with a wandering mark moving robot-like along the contours like a willing (and silent) staffman.

(2) *Indirect Contouring.* In this method salient points in the area are selected as ruling points of elevation representative of the general surface character; the elevations of these are found and the points are conveniently recorded by a cross with the reduced level; thus, × (57·6). All the methods of vertical control are used in indirect contour location. Also, again all the methods of horizontal control are at the surveyor's disposal, but he chooses one primarily and never makes his notes an encyclopaedia of methods. More often than not dual control is kept, though occasions arise when detachment is advisable for practical reasons.

Contours are inserted between these ruling points (×) by interpolation, at best a very monotonous undertaking which is usually carried out in the office, though occasionally in the field on the plane table.

Indirect location is the only method that can reasonably be considered for intervals over 10 ft. to 20 ft. Singularly the method is also best for very small intervals, 1 ft. or 2 ft., or even 5 ft., when the ground is devoid of surface character. The question arises, "What is surface character?" The flattest area certainly has character; *flatness* which is nothing in topography; undulatory ridges and furrows, even on hillsides are of indefinite character; but hill and valley features are definitely character, if pronounced in the immediate landscape.

Fieldwork. A description of one combination of each of the general methods will be given with an outline as to how these are varied with other instruments in particular cases.

(1) *Plane Table (H.C.) and Dumpy Level (V.C.).* Let *A*, *B*, and *C* in Fig. 68 represent a traverse which may have been run solely with the

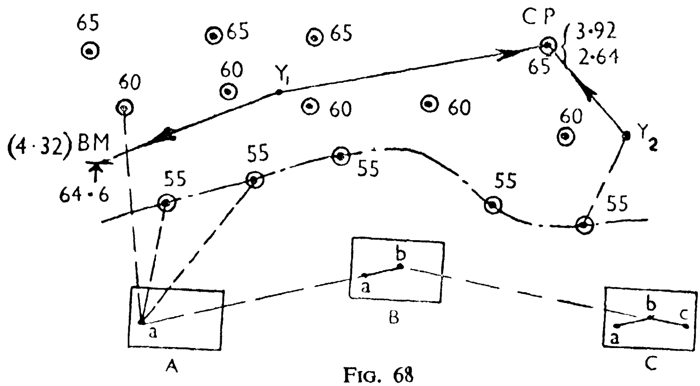

FIG. 68

plane table and chain or by means of the theodolite and chain, and afterwards plotted on the board of the plane table. Also let Y_1, Y_2, etc., represent positions of the level, ⊙ points on the contours, and the rectangles the board at the stations.

Control. It will be best to work in detached control, since it is assumed that only a sight rule or simple alidade is at our disposal; for, obviously, chaining (or even pacing in rougher work) would hold up the levelling party. Intersections certainly may be used in horizontal control wherever possible, though as a rule the use of these in proper contouring is more restricted than what it may appear. Hence sticks of conventional colours should be fixed at the contour points, or, failing paint, cleft twigs may be used with coloured tickets.

(V.C.). A backsight of 4·32 is taken with the dumpy level on a staff held on B.M. 64·6, giving a height of collimation at Y_1 of 68·92. Hence for a reading of 8·92, the foot of the staff will be on the 60 contour, while readings of 13·92 and 3·92 will likewise give the 55 and 65 contours respectively.

The staffman is ready to move in search of contour points as soon as he has attached paper targets to the staff at these readings. Assuming that working uphill is the more convenient, the points on the 55 contour are found first, then points on the 60 contour, and then on the 65 contour, the sights being up to 500 ft., which length is permissible in work of this nature.

When the contours are nearly straight or are flat curves, the contour points may be from 100 ft. to 200 ft. apart, but on sharp curves they may be as close as 20 ft. or even less.

After a while it will be necessary to move eastwards to locate the contour points in the vicinity of C. A foresight (of 3·92) is taken on the contour point (65 *C.P.*), and with this as a change point the level is set up at Y_2, whence a backsight of 2·64 is read, giving a new collimation height of 67·64. The paper targets will now be shifted to 12·64, 7·64, and 2·64 for the 55, 60, and 65 contour points respectively, which will be found in the manner described.

(H.C.). Meanwhile the plane tabler draws rays towards the different contour points from A, represented by a pin at a, and directs the chainmen to measure the radial distances rapidly to the nearest 2-ft., working in a manner to reduce walking to a minimum. These distances are then scaled off along the corresponding rays, a circle and dot is inserted to represent each contour point, and the contour is inserted, advantage being taken of the fact that the ground is in view. After all the points have been surveyed in this manner, the plane table is set up at B, and is duly oriented by sighting back along ba to A. When the points commanded from B are surveyed, the table is moved to C, and so on till the work is completed.

The ideal method of horizontal control is to use a stadia alidade so that the radial distances can be found by the length of staff seen intercepted between the stadia lines of the telescope, or, better, a tacheometer, introducing the same principle, could be stationed beside the table. Dual control is then possible in small surveys, the staffman turning the face of the staff towards the plane table as soon as the levelman has signalled that the staff is on a contour point.

Similarly the tacheometer, or a theodolite with a stadia telescope, could replace the plane table and the contour points could be fixed by back angles or azimuths; but of course all the plotting would be done indoors.

Dual control is sometimes possible by using the stadia lines in the telescope of the dumpy level at Y_1 Y_2, etc., thus obtaining from the staff the distances from Y_1, Y_2, etc., to the staff. The positions of Y_1 and Y_2 are plotted by radial distances or by intersections from A to B. Then the rays drawn from a, b, etc., are intersected with arcs centred on Y_1, Y_2, etc., the radii being the stadia distances from these positions of the level.

Sometimes compass-chain traverses are run through the contour points, and the latter are fixed by offsets, as in the case of boundaries in land surveying. Occasionally straight lines can be run likewise, particularly in areas with a general slope in a definite direction. A special application of direct location is the American method of contouring the proposed routes of railways and highways. A reflecting hand level is strapped to the top of a 5 ft. staff (called a Jacob), and points on the contours on each side of the centre line are fixed with great rapidity, the distances right and left being measured with the tape.

(2) (*a*) *Grid Squares* (*H.C.*) *and Dumpy Level* (*V.C.*). This is one of the most effective methods on ground that displays little or no surface character, and it is applicable to intervals up to 10 ft., but its more immediate use is for intervals of 1 ft. or 2 ft. in connection with building or constructional sites, sports grounds, etc. In the latter connection it

also provides a ready means of calculating earthwork excavation from the truncated prisms of which the unit squares are the plans. As the term implies, the horizontal control consists merely in covering the area with a network of squares of 50 ft., 66 ft., or 100 ft. side, basing these on the most convenient side of the survey skeleton, as surveyed with the chain only, or the theodolite or compass and chain.

(H.C.). In Fig. 69, *ABCD* is the skeleton of a chain survey. Along *AC* at distances of (say) 50 ft., sticks are inserted, and at each of these points perpendiculars are erected by means of the cross staff or the optical square, the theodolite being used in highest class work. At 50 ft. distances along each of the perpendiculars, sticks are inserted right and left of *AC*, so that in effect the entire area is covered with a grid. It is convenient to number the perpendiculars to *AC* as "Line 1," "Line 2," etc., with this and "O" along *AC*, and the points to the right and left "Line 2, 100′ L," "Line 6, 250′ R," meaning the corners of the squares 100 ft. to the left of *AC* on Line 2, and 250 ft. to the right of *AC* on Line 6. Cards inserted in the clefts of the sticks are used.

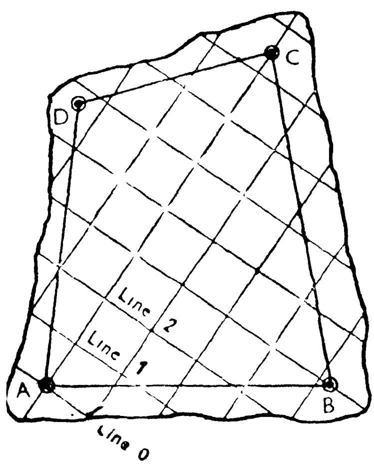

FIG. 69

(V.C.). Starting at a benchmark, the levelling party take the levels at the corners of the squares, the staffman removing the sticks as soon as the levelman has recorded the staff reading and its position in his notes. A great deal of thought is necessary in devising a form of notes from which the corner levels can be found at a glance.

A good plan is to bring the table only of the plane table into the field and appoint a topographer who will reduce the staff readings to corner elevations as soon as they are brought to him by someone in the humbler (though none the less useful) capacity of "runner." Meanwhile the topographer can interpolate the contours in the manner described at the close of the present chapter.

(2) (*b*) *Plane Table* (*H.C.*) *and Dumpy Level* (*V.C.*). Now Method 1 (*a*) can be used if the signs ⊙ used for contour points are replaced by crosses × denoting points figured with irregular reduced levels, not 55, 60, and 65, but values such as 47·6, 58·4, 66·6, etc., taken along bottoms of valleys, tops of ridges, or at definite changes in the surface character. The field work will be more expeditious than in the direct method, and very often dual control is possible, though in every other respect there is little difference in the field work, beyond the knowledge

that the laborious process of interpolating contours is yet to come. This combination is best adapted to intervals up to 10 ft., beyond which ordinary spirit levelling ceases to be economical. This, the representative method of low interval contouring, is best carried out with a tacheometer in both horizontal and vertical control for intervals of 10 ft. to 20 ft. Also the work with the plane table would be greatly expedited by the use of a stadia alidade or an independent tacheometer beside the plane table.

(2) (*c*) *Plane Table* (*H.C.*). When the contour interval exceeds 20 ft., the India pattern clinometer is exceedingly useful and the work becomes more of the nature of an exploratory survey, such as would be carried out in a valley with eminences of considerable height on either side of the traverse. The ruling points for elevations would be conspicuous points, which would of necessity be fixed by intersections in horizontal control. The elevations of the ruling points would be found from the tangents of the vertical angles α observed with the clinometer, with $V = D \tan \alpha$, where $\tan \alpha$ would be read directly and D the horizontal distance scaled from the map. A similar process is used in ordinary photographic surveying where, as here, the elevations above the camera may be found graphically, as described in Chapter VI. In reconnaissance work the compass may supersede the plane table, the ruling points being fixed by bearings observed from the ends of the traverse lines.

(2) (*d*) *Compass* (*H.C.*) *and Clinometer* (*V.C.*). In reconnaissance and pioneer work it is sometimes possible to run "direction" lines which radiate from the stations of a compass traverse, the distances between the traverse stations A, B, C, and D being found by pacing, riding, or by range-finder. The direction lines, which are fixed by compass bearings, are chosen along lines in which the ground surface has a fairly uniform slope; and the slopes are observed with the clinometer, as angles 4°, 6°, etc., or, preferably, as cotangents of α.

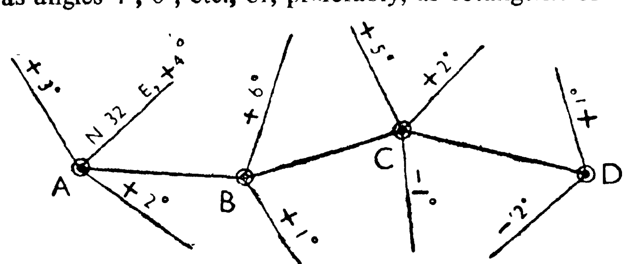

FIG. 70

If the reduced levels of A, B, C, etc., are known, it is possible to interpolate contours in accordance with the relation, $D = V \cot. \alpha$, where D is the distance between the contours on the direction lines and V the interval, which should not be under 20 ft. in the best applications.

For example, if A in Fig. 70 is 404 ft. above datum, and the contour interval is 20 ft., then along the N. 32° E. direction line, the horizontal distance from A for the 420 contour would be 16 cot. $4° = 16 \times 14{\cdot}3 = 229$ ft., after which the 440, 460, etc., contours would follow at even horizontal spacings of 286 ft., also to the scale of the map.

Finally, as an idea of the scope of indirect contouring, the use of the compass and aneroid may be mentioned in regard to intervals of 100 ft. and upwards. The elevations of salient points are found with the aneroid, and are then fixed in horizontal control either by compass three-point resection by a lone observer, or by compass or plane table intersections, made by other observers.

Interpolating Contours. Now that accurate transparent papers, ruled accurately in tenth-inch or millimetre squares, are readily obtainable, there is only one method really worth considering; and the work is very different from the days when the surveyor was compelled to rule tracing cloth, and, to preserve this, fixed a strip of paper on the left from time to time for jotting down the values he assigned to the thicker rulings.

In general, strips of transparent squared paper are cut in widths from 1 in. to 3 in., 4 in. to 6 in. long, or widths 2 cm. to 6 cm., 10 to 15 cm. long. A system of diagonal decimal division is thus at hand and the $\frac{1}{10}$-in. spaces, or 1 mm. spaces, may represent 0·5 ft., 1 ft., or even 10 ft., according to the range of elevations and the scale of the map. In Fig. 71 the values 0, 1, 2, and 3, are merely shown for illustrative purposes, and the elevations actually assigned to the main rulings may

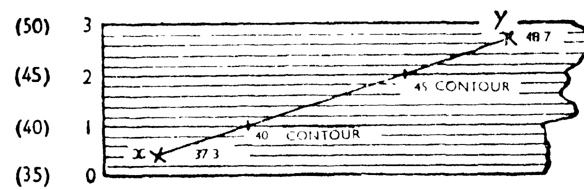

FIG. 71

have any temporary values, as indicated by the bracketed figures jotted down on the left.

Let it be required to interpolate 5 ft. contours between two points x and y of respective elevations 37·3 and 48·7 by means of a strip of 1 inch divided transparent squared paper. (*Fifths only are shown for clearness in Fig. 71.*) Assume the zero ruling to represent an elevation of 35 ft., the next main ruling 40 ft., the next 45 ft., and so on. Each of the (*nine*) intermediate lines will then represent 0·5 ft., and it is possible to estimate here to 0·1 ft., which is the lowest reading usually observed in contouring.

Place the strip so that the cross x is between the 4th and 5th lines from 0, being 0·6 of a small spacing above the 4th for 37·3 ft. Prick through x with a needle point, and, with this as a pivot, turn the strip until the cross y is seen between the 7th and 8th lines above the main ruling 2, being 0·4 of a small space above the 7th line for 48·7 ft. The main readings 1 and 2 will intersect the line between x and y for points respectively on the 40 and 45 contours.

The process is simplified in connection with unit squares, and in many cases the crossings of contours between x and y are estimated, sometimes with the aid of a scale.

CLASS EXERCISES

9 (*a*). As a surveyor with a trained assistant and two men, you are required to insert the 5-ft. contours on a plane table survey of an area in which it is advisable to trace the actual contours on the ground.

Describe concisely, giving sketches, how you would carry out this work with the following equipment at your disposal: Plane table with sight rule (or alidade), chain and arrows, tape, dumpy level and staff, range-poles, and a bundle of laths. (G.S.)

9 (*b*). The scale of an old map is unknown, but at a place where there is a regular slope the map shows 5-ft. contours spaced exactly 0·9 inch apart. The slope of the ground was found by means of a dumpy level, and a fall of 2·5 ft. was observed in a horizontal distance of 45 ft. Draw a scale for the map. (100 ft. to 1 inch.)

9 (*c*). You are required to make a survey of a small lake to show under-water contours as well as the plan of the lake.

Describe your procedure with the aid of sketches.

*9 (*d*). The following notes were recorded in a reconnaissance survey in mountainous country, altitudes being determined with the aneroid barometer and the positions of stations fixed by compass bearings on two known points P and Q. The magnetic bearing of the line PQ was 82° and its length 5,500 ft.

Using a scale of 1 inch to 1,000 ft., insert the spot levels and, as far as possible, interpolate approximate contours at 100-ft. intervals.

Observer's Station	*Bearings to*		*Altitude Ft.*
	P	*Q*	
A	12°	58°	1,540
B	350°	32°	2,200
C	314°	44°	1,160
D	320°	15°	1,735
E	280°	25°	1,010
F	310°	350°	1,950

*9 (*e*). *A*, *B*, *C*, *D*, are four points on a straight line in a valley, *AB* being 1,530 ft., *BC*, 1,650 ft., and *CD*, 1,840 ft., and the line has a true bearing of N. 45° E. The four points are used as stations in determining the angles of uniform ground slope in the area by means of a clinometer and the bearings of these direction lines by means of the compass. The notes are tabulated below, the ***plus*** and ***minus*** signs indicating respectively angles of elevation and depression along the direction lines.

Plot the survey on a scale of 1 inch to 1,000 ft. and insert the 300, 350, and 400-ft. contours.

Station	*Elevation of Station (ft.)*	*Direction Lines*	
		Angle of Slope	*True Bearing*
A	260	$+3°$ $+4°$	355° 160°
B	290	$+2\frac{1}{2}°$ $+2°$	5° 140°
C	312	$+2°$ $+2°$	10° 85°
D	348	$+1\frac{1}{2}°$ $-1°$	30° 220°

FIELD EXERCISES

Problem 9 (*a*). Trace the 5-ft. contours within the triangle indicated by the pickets *A*, *B*, and *C*, and plotted as *a*, *b*, and *c* on the board of the assigned plane table.

Equipment: Plane table, sight rule, dumpy level, levelling staff, chain, arrows, and laths.

Problem 9 (*b*). The coloured laths inserted by Group . . . are at points on the . . . ft., . . . ft., and . . . ft. contours.

Survey the positions of these by means of the compass, chain, and tape, with reference to the stations *A* and *B*, as indicated by flag-poles.

Equipment: Compass, chain, tape, arrows, and a set of pickets.

Problem 9 (*c*). The flag-poles *A* and *C* are at the end stations of the diagonal of (specified) field, as surveyed by Group . . . .

Obtain the data for interpolating contours at an interval of 5 ft. by means of grid squares and spirit levelling.

Equipment: Chain, arrows, cross staff (or optical square), set of pickets, laths, dumpy level and levelling staff.

Problem 9 (*d*). The points indicated (on rough ground) are plotted as a triangle *pqr* on the board of the assigned plane table. Working in conjunction with Group II, plot sufficient ruling points in order that 10-ft. contours may be interpolated within the area *pqr*.

Equipment: Group I, Plane table with sight rule, chain, arrows; Group II, dumpy level, levelling staff, and bundle of laths.

(B.M. to be indicated.)

Problem 9 (*e*). Survey the (specified inaccessible) portion of . . . Hills by means of the plane table, and make sufficient observations of prominent points so that contours at 20 (50) ft. intervals can be interpolated.

Equipment: Plane table, clinometer, chain, arrows, and set of pickets.

ORIGINAL PROBLEMS

Excursion with compass, aneroid, and 6 inch Ordnance map in very hilly country.

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CHAPTER X

AREAS AND VOLUMES

In a way this chapter brings us back to the seeming drudgery of arithmetic, and therefore to the things that really count. Those prisms, pyramids, and cylinders may be something more than geometrical solids, and little is ever lost of what has been learnt in mensuration.

The calculations that arise from surveying notes require three things: (*a*) *System* in setting out the data simply and methodically, avoiding unnecessary repetitions and cumbersome arithmetical processes; (*b*) *Soundness*, selecting the method to meet the particular requirements of the work in hand; and (*c*) *Certainty* in arithmetic, subjecting the results to checks, preferably by simple processes; for mistakes, as distinct from errors, can always intrude, and, in a practical world, a single arithmetical slip can lead to a very material loss, possibly thousands of pounds (sterling) in a contract. Much more could be said, but the present would be untimely.

I. AREAS

The British and American unit of square measure in land valuation is the *acre*, but the square yard and square foot are used in constructional projects.

1 acre = 4 roods = 160 perches = 10 square chains = 43,560 square feet.
1 square mile = 640 acres.

The acre was the estimated amount of land that could be ploughed by a horse in one day; "*by the rod make one rood.*" It was generally regarded as an area 10 chains in length, which is 1 furlong, or "*furrow long*," with a breadth of 1 chain, which was divided into 72 furrows of eleven inches.

Simple Plane Figures. The rules for the areas A of simple plane figures will be summarised only, since the computation of these is common mensuration. Wherefore, plane rectilineal figures will be considered with the letters A, B, C, (D) at the angular points in counter-clockwise order, A being uppermost and also to the left in the case of quadrilaterals. The altitude, or height above a horizontal plane or base line, will be denoted by h, and radii by r, the letter R indicating an outer or larger concentric radius.

Triangles. In the following rules the angles will be expressed in magnitude by A, B, and C, and the opposite sides by a, b, and c respectively, the semi-sum of the sides being $s=\frac{1}{2}(a+b+c)$.

Altitude h and base a. $A=\frac{1}{2}hb=\frac{1}{2}ab \sin. C=\frac{1}{2}ac. \sin. B=\frac{1}{2}bc \sin. A$. Also the formula attributed to Hero of Alexandria (120 B.C.):

$$A=\sqrt{s(s-a)(s-b)(s-c)}.$$

Apart from the square and rectangle, the other (1) *Parallelograms* include (2) the *Rhombus*, with all four sides equal, and *Quadrilaterals* including

(3) the *Trapezoid*, with two sides parallel, and (4) the *Trapezium*, the general case of a quadrilateral, no sides being parallel. The Euclidian definition given is not rigidly adhered to, and often the names of the trapezoid and trapezium are interchanged.

(1) *Parallelogram:* sides a, b, altitude h. $A=bh=ab$ sin. B.

(2) *Rhombus:* side a, diagonals AC, BD. $A=\frac{1}{2}(AC\times BD)=a^2$ sin. B.

(3) *Trapezoid:* parallel sides BC, AD, separated by perpendicular distance h. $A=\frac{1}{2}h(BC+AD)$.

(4) *Trapezium,* with perpendiculars h_1, h_2, let fall from A and C on the diagonal BD. $A=\frac{1}{2}BD(h_1+h_2)$.

Circle. Sector of radius r, subtending angle of θ radians at the centre, $A=\frac{1}{2}r^2\theta$.

When $\theta=2\pi$, for the entire circle, $A=\pi r^2$.

Annulus with outer and inner radii R and r, $A=\pi(R^2-r^2)$.

Approximation to area of a segment intercepted between a chord of length C and the circle, the perpendicular distance being h at the middle of the chord. $A=2/3\ hC$.

Ellipse, with semi-major and semi-minor axes a and b respectively, $A=\pi ab$.

Sphere: radius r. $A=4\pi r^2$.

Zone intercepted between two parallel section planes, distance h apart, $A=2\pi rh$. When $h=2r$, $A=4\pi r^2$.

METHODS. The areas of surveys may be determined (1) *Arithmetically*, (2) *Graphically*, and (3) *Mechanically*.

(1) *Arithmetical Methods.* Occasionally areas are calculated directly from the field notes, usually as (*a*) *areas of skeletons*, and (*b*) *outlying areas at boundaries*. The areas of skeletons are readily found by the above trigonometrical rules, or by co-ordinates, but the outlying strips involve tedious calculation by trapezoids between offsets, which also can be facilitated by a co-ordinate method.

(2) *Graphical Methods.* Sometimes the area is calculated from (i) Partial areas, (*a*) and (*b*) as above, and sometimes as (ii) Entire areas.

(i) *Partial areas.* (*a*) The area of the skeleton is taken off by scale measurements, usually of the altitudes and bases of the constituent triangles, which is more accurate and expeditious than the artifice of reducing polygons to triangles of equal areas.

(*b*) Although the actual offsets introduced in plotting might be used as in the foregoing method, the usual plan is to erect false offset ordinates at regular distances along the survey line.

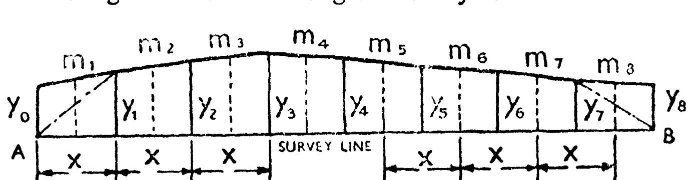

FIG. 72

The areas of the strips may then be calculated by *Trapezoids*, or by *Simpson's Rule*, or by *Mid-ordinates*, as indicated on the right of Fig. 72.

(1) *By Trapezoids.* Let y_0 and y_n be the end ordinates, y_1, y_2, y_3, etc., the intermediate ones, x the common distance between the ordinates, and Y the sum of the intermediate ordinates. Then the area A by the trapezoidal rule:

$$A=\tfrac{1}{2}x\,(y_0+2Y+y_n).$$

If the ends converge, as shown dotted at A and B, the terms y_0 and y_n disappear, and $A=xY$.

(2) *By Mid-ordinates.* A common method, particularly with computing strips, is to insert ordinates midway between the false offset ordinates. In this case the trapezoidal rule becomes:

$$A=x\Sigma m,$$

where Σm is the sum of the ordinates m_1, m_2, m_3, etc. (Fig. 72.)

Both the foregoing methods are based upon the assumption that the several offset figures are trapezoids, and this leads to results that are sufficiently accurate for most purposes. If, however, the boundaries are really curved to such an extent that appreciable error is likely to be introduced, the areas should be calculated by Simpson's parabolic rule, sometimes called Simpson's *First Rule.*

(3) *By Simpson's Rule.* In applying this, the better known of the two rules, it is necessary to divide the area into an even number of strips of the same width x, the odd number of ordinates again being the several false offset distances to the boundary. If, as before, y_0 and y_n be the end ordinates, y_1, y_2, y_3, etc., the intermediate ordinates, and x the common distance between them, then

$$A=x/3\,(y_0+y_n+2\,(y_2+y_4+y_3,\text{ etc.})+4\,(y_1+y_3+y_5,\text{ etc.})$$

$$\frac{\text{Width}}{3}\left\{\frac{\text{Sums of Ordinates}}{\text{Once End}+\text{Twice Even}+\text{Four Odd}}\right\}$$

(ii) *Entire Areas.* The chief methods applied to whole areas are (1) *By Division into Triangles* and (2) *By Division into Trapezoids,* introducing the computing scale and the use of Simpson's rule.

(1) *By Division into Triangles.* In this method the resulting outlying sides of triangles are not wholly inside the boundaries or identical with those of the survey skeleton, but are such that they balance out the inequalities of the boundaries by serving as "give and take" lines. Fig. 73 shows a survey with irregular boundaries, pencilled into triangles for treatment by this method. The resulting triangles *ABC*, *CDA*, *DEA*, are inserted so that their outlying sides *AB*, *BC*, *CD*, *DE*, and *EA* each takes into its own area portions equal to those which it gives

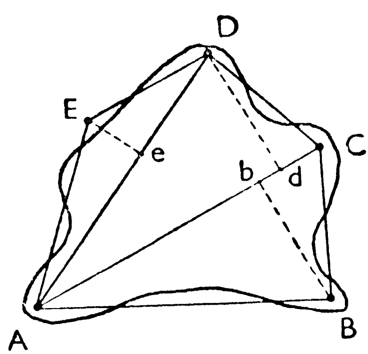

FIG. 73

outside. These outlying sides are found by stretching a fine thread along the boundaries, or, better still, by using a couple of set squares. After a little adjustment, the lines are drawn, resolving the area into triangles, the areas of which are found by multiplying half the respective altitudes by the corresponding bases; thus:

$$\tfrac{1}{2}dD \cdot AC;\ \tfrac{1}{2}bB \cdot AC, \text{ and } \tfrac{1}{2}eE \cdot AD.$$

The method is far more accurate than it first appears to be, since the portions equalised are small in comparison with the areas of the corresponding triangles.

(2) *By Division into Trapezoids.* In principle the area is divided into a number of parallel strips of the same width x, not by ruling equidistant parallels across the plan, but preferably on a sheet of tracing-paper. This tracing-paper is placed over the plan, and is shifted about so that the area is exactly enclosed between extreme parallels, thus avoiding an odd area at one extremity.

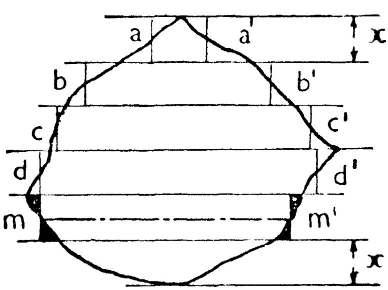

FIG. 74

The process of taking out an area consists in finding the area of every constituent strip of the figure. This is done by measuring the mean lengths of the strips, as indicated by mm' in Fig. 74, where the dotted line reduces the length to that of an equivalent rectangle, the area of which is equal to the width x of the strip multiplied by the length mm'. Thus the area of the survey,

$$A = x\ (aa' + bb' + cc', \text{ etc.}),$$

indirectly in square inches or *directly* in acres, according to the width x employed.

Square Inches. Commonly strips of convenient width, 1 inch, say, are used, and the map area is taken out in square inches, which are afterwards reduced to acres.

This method has the advantage that transparent squared paper can be used, the small squares serving in obtaining the lengths aa', bb', etc. In fact, large areas can be dealt with on small sheets of squared paper if the survey is appropriately divided into parts; four, for instance.

Acres. Even if only an ordinary decimally-divided inch scale is available, it is possible to rule a sheet of tracing-paper so that the acreage can be found directly for any given scale. This is done by making the common width x between the parallels that value in inches which would be expressed by 10 divided by the square of the number n of chains to the inch in the scale of the survey. Every inch length of the strips will then represent an acre and every tenth of an inch one square chain.

Thus, for 2 chains to an inch, the distance between the parallels will be $10/4=2\frac{1}{2}$ in.; for 3 chains to an inch, $10/9=1\cdot11$ in.; and for 4 chains to an inch, $10/16=\frac{5}{8}$ in.

Because on a scale of n chs. to 1 in., the width x will be $\frac{10}{n^2}$, and this will represent $\frac{10}{n^2}.n=\frac{10}{n}$ chains on the map, and since an inch length of the strip represents n chs., the product of these measurements will represent $\frac{10}{n}n=10$ sq. chs. $=1$ acre.

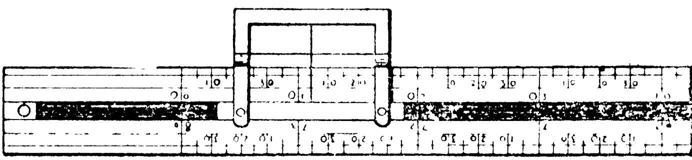

FIG. 75

*The best-known device for computing areas in this way is the computing scale, one form of which is shown in Fig. 75. These instruments can be obtained in various divisions, ranging from two ordinary scales to universal patterns with six chain scales and two Ordnance Survey scales. Some patterns are divided for use with $\frac{1}{4}$-inch strips and others for strips representing one chain widths; and therefore each scale can be applied directly only to maps on the scale for which it is divided.

The use of the computing scale needs little explanation, once the survey is enclosed between parallels on tracing-paper appropriately divided. The indicator of the sliding frame is set to zero on the scale, and the scale is placed parallel to the rulings, with the wire cutting the beginning of the first strip, "squaring the boundary," as at a in Fig. 74. The frame is then slid until the wire cuts the end of the strip, squaring the boundary, as at a', the scale being held firmly in position. The scale is then lifted and placed parallel for the second strip by moving it bodily until the wire cuts the beginning of the second strip, as at b. Then, as before, the scale is held firmly in position while the frame is moved until the end of the groove in which the frame moves is reached. A mark is now made under the wire and the scale is inverted and placed with the wire at the mark, after which the frame is moved as before, summing up the strips until the other end of the groove is reached. A mark is then made at the wire, and against it the acreage of the first double travel is recorded. Next the scale is set right way up again, and the process is repeated until all the strips have been measured. The acreage is then cast up for the number of double travels noted plus the final reading of the scale.

**Simpson's Rule.* The method described on page 125 is sometimes used in computing entire areas from figures divided into an even number of strips at regular intervals x inches apart, preferably on a sheet of tracing-paper. Often this is shifted so as to enclose the area exactly between parallels, and this means that the sums of the first and last ordinates is zero. Since an open frame is unnecessary when linear ordinates are measured, a brass frame, or cursor, can be fitted to an ordinary decimally-divided scale, a pointer at the upper edge serving in summing up the sets of ordinates, odd and even, as the case may be.

Obviously the area may also be determined by covering the survey, or a portion of it, with a sheet of transparent paper, or paper may be ruled so that each square contains so many square chains. Usually it is quicker to work in square inches on prepared paper and afterwards reduce the acreage.

*(3) *Mechanical Methods.* The most popular mechanical method is by means of an instrument known as the planimeter, which is used less in surveying than in other connections. This instrument, in its best-known form, consists of two arms jointed together so as to move relatively to each other with perfect freedom. Near the joint is the *rolling wheel,* and at the extremity of one arm is the *fixed* pole P, while at the extremity of the other arm is the tracing point T, a tiny handle being provided for guiding it over the plan. Connected with a gear to the rolling wheel is the *index wheel,* or *dial,* which shows the number of units of area encompassed, fractional parts being read with a vernier at the edge of the rolling wheel.

In simple patterns one revolution of the indicator corresponds to ten revolutions of the rolling wheel, which is divided into 100 divisions, a tenth of each division being read by means of the vernier.

The theory of the planimeter is beyond the scope of this book, since it involves a knowledge of integration. Also the instrument is made in many patterns, though the original Amsler instrument had arms of fixed length, giving areas in square inches or square centimetres. In later patterns the tracing arm was made adjustable, being divided to correspond with official scales, giving areas directly, some designs even allowing for shrinkage of maps.

Therefore it is always desirable to test an instrument by running it round a square or circle of known area.

In use the pole P is set outside the area, if possible, the point being pressed into the drawing board. If P is inside the area, a correction, as stamped on a weight which fits over P, must be added. The index wheel is then set to zero by rotating the rolling wheel, stopping with the zero of this wheel at the vernier index. (Otherwise the initial reading must be subtracted from the final reading of the instrument.) The tracing point T is then guided round the area in the clockwise direction, following the boundary lightly and carefully, and stopping at the starting point. The nearest lower value on the index wheel is recorded, and to this is added the fractional part as read on the rolling wheel and the vernier. Also the constant value must be added if the point P was necessarily inside the area during the operation.

Thus with a simple instrument divided for square inches, if 9 is the reading on the index wheel, and 72 on the rolling wheel, with coincidence at 3 on the vernier, the area is 9·723 sq. in.

It is interesting to note that a planimeter has been improvised with a jack knife, though with indifferent success on the part of many.

In emergencies areas have been cut out of cardboard of uniform thickness and compared in weight with a square of the same material after careful weighings on a chemical balance.

II. VOLUMES

The unit of cubic content, or volume, in earthwork estimates is the yard cube, which was regarded as the amount that could be hauled in a one-horse cart. It is also used in the measurement of concrete and brickwork. In related connections, the cubic foot and the bushel may be used, while brickwork may be measured in rods and timber in cubic feet or standards.

The following summary shows the volume content V of certain simple solids, A being the base area, r the radius generally, and h the altitude as measured perpendicularly to the base.

Prisms, right or oblique. $V=A.h$; square rectangular, trapezoidal, etc.

Cylinders, right or oblique. $V=A.h=\pi r^2.h$. Hollow $V=\pi(R^2-r^2)h$.

Frustrum of right cylinder. $V=\frac{1}{2}\pi r^2(h_1+h_2)$, where h_1 and h_2 are the greatest and least heights.

Cones, right or oblique. $V=\frac{1}{3}A.h=\frac{1}{3}\pi r^2.h.$

Frustrum of cone or pyramid, $\frac{1}{3}h(A+\sqrt{Aa}+a)$, with h between sectional areas A and a.

Pyramid, right or oblique. $V=\frac{1}{3}A.h.$

Spheres. $V=\frac{4}{3}\pi r^3=\frac{1}{3}$ (surface) $\times$ (radius).

The solids most commonly associated with earthwork calculations may be defined as (1) *Section Prismoids*; (2) *Truncated Prisms*, and (3) *Contour Prisms*.

(1) *Section Prismoids.* A vertical section of the earth's surface as found by levelling along the centre line of a projected railway or reservoir is known as a *longitudinal* section and vertical sections at right angles to these are known as *cross sections*, the shapes of which are also determined by levelling.

(Although the subject of sections is dealt with at the end of this chapter, it might be consulted during the reading of the present section.)

The solids in the present category are derived from the irregular cross sections of cuttings and embankments in the construction of railways and highways. The method consists in finding the areas A_1, A_2, A_3, etc., of successive cross sections, usually 1 ch. (or 100 ft.) apart, and using these in the trapezoidal or prismoidal rules for volume content. The solids approximate to irregular truncated pyramids, and may be considered prismoids, which are solids having for their ends any dissimilar parallel plane figures of the same number of sides and all faces plane figures.

Three standard types of cross sections will be considered. In these d will be the centre line (C.L.) depth of cutting or banking, w the half formation width, s the side slope ratio, s horizontally to 1 vertically, and r the crosswise or lateral slope of the ground, r : 1 likewise. Thus s and r are the co-tangents of the angles which the side slopes or the ground surface make with the horizontal, but s is commonly 1, $1\frac{1}{2}$, or 2, whereas r can have a wide range of values.

In Cases (*a*) and (*b*) of the following treatment, the imaginary formation triangle OPQ, which is neither excavated nor made, will be incorporated in order to simplify the formulae. Its area a is consistently w^2/s and therefore its volume v is $w^2/s \times l$ in a length l, and, being a right triangular prism, this value will follow from *all* rules. Hence, *whole areas* A' will be calculated from *whole depths* $D=d+w/s$, and the true areas A will be $A'-a$.

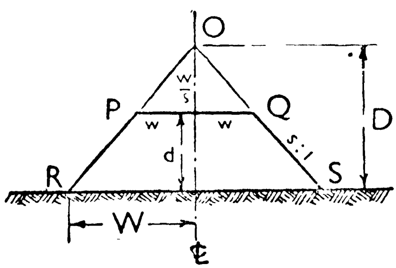

FIG. 76

Case (*a*) *Ground Level Across* (Fig. 76). Here the *side widths* are

$$W=sD=s(d+w/s)=w_2+sd;$$

the whole area $A'=sD^2=s(d+w/s)^2=(2w+sd)d+w^2/s$ . . . . (1)

But since the area of the formation triangle $a=w^2/s$, the true area $A=(2w+sd)d$: a very inconvenient expression.

The values would be precisely the same for a cutting.

Thus for a cutting in which the formation width $2w$ is 20 ft., and the centre line depth of cutting d is 10 ft., and the side slope ratio 1 : 1, the whole depth $D=d+w/s=20$ ft., the whole area $A'=sD^2=1\times(20)^2=400$ sq. ft., and the true area $A'=A'-w^2/s=400-100=300$ sq. ft.

Case (*b*) *Ground Sloping Across* ($d>w/r$) (Fig. 77). Here the side widths are different, being W_l and W_r on the left and right respectively:

$$W_l=CT=W-R'T=W-s(RT)=W-sW_l \tan \alpha =W-W_l s/r.$$

$$\text{Or } W_l=\frac{W}{1+s/r}=\frac{sD}{1+s/r}.$$

$$\text{Also } W_r=CT'=W+S'T'=W+s(ST')=W+sW_r\tan=W+W_r s/r$$

$$\text{Or } W_r=\frac{W}{1-s/r}=\frac{sD}{1-s/r}.$$

The whole area $A'=\frac{1}{2}(W_l+W_r)D=\dfrac{sD^2}{1-s^2/r^2}$, ... (2)

the latter expression being that for ground level across in (1) divided by $1-s^2/r^2$, Also the former expression in (2) frequently occurs in "three level" sections in the American method of cross-sectioning.

Similar expressions would follow if an embankment were considered with the higher ground on the left.

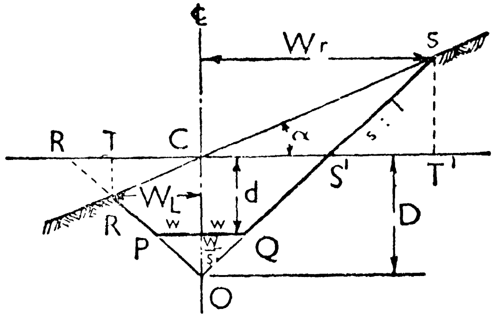

FIG. 77

Thus for an embankment in which the formation width is 20 ft., the centre line height of bank 10 ft., the side slope ratio $1\frac{1}{2}:1$ and the sidelong ground slope $r:1=9:1$, the whole depth $D=10+10/1\frac{1}{2}=16{\cdot}7$ ft., and the whole area

$$A'=\frac{1\frac{1}{2}(16{\cdot}7)^2}{1-\frac{1}{36}}=430{\cdot}2 \text{ sq. ft.}$$

Also the true area $A=363{\cdot}5$ sq. ft., since the area of the formation triangle $w^2/s=66{\cdot}7$ sq. ft.

Case (*c*) *Hillside Sections* ($d<w/r$) (Fig. 78). Cross sections of this nature are best treated right away as true areas since one portion is in 'cut" and the other in "fill." Also the side slope s' is necessarily flatter for the banking than for the cutting, "made ground" being less stable than that under excavated earth.

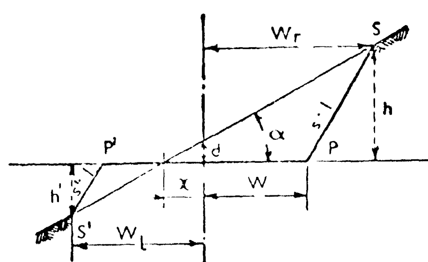

FIG. 78

Here $h=(w+x+sh)1/r$ and $h'=(w-x+s'h')1/r$

$$=\frac{w+x}{r-s} \qquad =\frac{w-x}{r-s'}$$

$$\text{True area } Right=\frac{(w+x)^2}{2(r-s)};\quad \text{true area } Left=\frac{(w-x)^2}{2(r-s')} \quad . \quad . \quad . \quad (3)$$

If these areas are equal, $\dfrac{w+x}{w-x}=\sqrt{\dfrac{r-s}{r-s'}}=k;$

$$\text{and } x=w.\frac{k-1}{k+1}.$$

The side widths $W_l=h'\cot.\alpha-x=h'r-x$

$$W_r=h\cot.\alpha+x=hr+x$$

Two rules are used in the calculation of volume content from cross sectional areas A_1, A_2, A_3, etc., l units apart, l being usually 66 ft. and 100 ft., convenient submultiples being introduced.

(1) *Trapezoidal Rule.* In this, the *average end area rule,* the volume is calculated from the mean of the areas at the ends of horizontal lengths l along the centre line, A_1 and A_2 being true or whole areas.

$$\text{Thus } V=\tfrac{1}{2}l(A_1+A_2).$$

Hence for a series of areas A_1, A_2, A_3 . . . A_n, all l ft. apart, the total volume will be

$$V=\tfrac{1}{2}l(A_1+2A_2+2A_3+2A_4+\ \ldots\ A_n) \text{ cu. ft.}$$

The well-known earthwork tables of Bidder were based upon this rule, the centre line distances being in Gunter chains. The trapezoidal rule is commonly used in preliminary estimates.

(2) *Prismoidal Rule.* In this rule, which was the basis of Sir John MacNeill's tables, it is assumed that the surface of the ground between any two vertical cross sections is such that the volume content is a prismoid, the end areas not necessarily being similar, but of any shape whatever, provided the surfaces between their perimeters can be regarded plane.

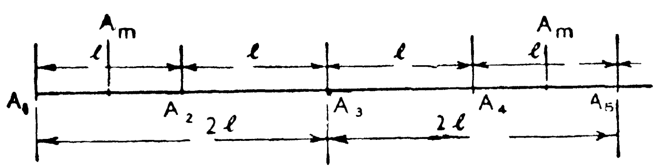

FIG. 79

According to this rule,

$$V=\frac{1l}{6}\cdot(A_1+4A_m+A_2),$$

where A_m is the area midway between sections proper, such as A_1 and A_2. Now A_m is not the mean or average of the areas A_1 and A_2, and in complex sections it should be determined. If the surface slope is the same as at A_1 and A_2, the area A_m may be calculated from the mean depth $\tfrac{1}{2}(d_1+d_2)$ or $\tfrac{1}{2}(D_1+D_2)$, as the rule will apply to both whole and true areas. But if the end sections have different lateral slopes, the central depth will still be the mean of the end depths, but the lateral slope will be the harmonic mean of the end slopes. The prismoidal rule is used in final estimates, sometimes by applying "prismoidal corrections" to the trapezoidal rule. It is not strictly confined in practice to solids with plane faces, but has been used in calculating curved volumes.

Viewed from this standpoint the prismoidal rule becomes applicable when the solid is not strictly a prismoid.

Now if the even numbered sections are used as the middle sections,

l becomes $2l$ as indicated in Fig. 79, and the rule reduces to that of Simpson, A_1 corresponding to y_0 in Fig. 72:

$$V=\frac{2l}{6}(A_1+4A_2+2A_3+4A_4+2A_5, \text{ etc.})$$

$$=\frac{l}{3}\text{ (End Sum}+4\text{ Even Sum}+2\text{ Odd Sum)}$$

As in the case of areas, it is inadvisable to say dogmatically that this method overestimates or that underestimates the content. Such statements usually refer to areas with straight boundaries between ordinates or plane surfaces between sections, and not as would really be given by field measurements or levels. Also the comparisons are not always consistent, one often being in fact an approximation.

Example. The following notes refer to actual sections taken on level across ground at 50 ft. intervals for an embankment with a formation width of 20 ft. and side slopes 2 horizontally to 1 vertically. In order to show the discrepancies that can arise from calculations alone, the volumes are calculated by the *Trapezoidal Rule* (*a*) for sections 100 ft. apart, (*b*) for the 50 ft. sections, and by the *Prismoidal Rule* applied as Simpson's rule, (*c*) with extreme sections 200 ft. apart and (*d*) with these 100 ft. apart, the 50 ft. sections serving as the middle areas.

Distance	0	50	100	150	200 ft.
Centre line depth d	6·2	6·9	8·4	10·0	9·8 ft.
Whole depth $D=d+w/s$	11·2	11·9	13·4	15·0	14·8 ft.
Whole area $A'=sD^2$	250·9	283·2	359·1	450·0	438·1 sq. ft.

The volume of the formation prism, 200 ft. in length, =10,000 sq. ft. =370 cu. yds. is to be deducted.

Trapezoidal Volumes:

(*a*) $\frac{100}{2}(250·9+2(359·1)+438·1)=70{,}360$ cu. ft.$=2{,}606$ cu. yds.
Net. 2,236 cu. yds.

(*b*) $\frac{50}{2}(250·9+2(283·2)+2(359·1+2(450)+438·1)=71{,}840$ cu. ft.$=$ 2,661 cu. yds.
Net 2,291 cu. yds.

Prismoidal Volumes:

(*c*) $\frac{200}{6}(250·9+4(359·1)+438·1)=70{,}847$ cu. ft.$=2{,}661$ cu. yds.
Net 2,254 cu. yds.

(*d*) $\frac{100}{6}(250·9+4(283·2)+2(359·1)+4(450)+438·1)=72{,}333$ cu. ft.$=$ 2,679 cu. yds.
Net 2,309 cu. yds.

Since both (*b*) and (*d*) introduce two more measurements, they will more closely represent the true content, which possibly lies between the given values, since (*d*) is not strictly, if practically, applied to the prismoid, though its use is justified by the assumption common to Simpson's rule for volumes. Although a single comparison is poor evidence, it shows that a small discrepancy of 0·8 per cent occurs between (*b*) and (*d*) against 2·4 per cent between the two applications of the prismoidal rule in (*c*) and (*d*).

(2) *Truncated Prisms.* Whenever a considerable width of surface is to be excavated, as in the case or rectangular reservoirs, building, or other sites, the most accurate method is that of taking out volumes from a series of vertical truncated prisms, squares being laid out and levels taken at the corners, as described in Chapter IX with reference

to Fig. 69 in the method of contouring with grid squares in horizontal control.

Fig. 80 shows a portion of an area marked out in unit squares of (say) 50 ft. or 100 ft. side, the corner reduced levels having been taken by means of a dumpy level. Now if the reduced level of the formation, or finished surface, is fixed, the difference of reduced level will be the *cut* or *fill* at each corner, and the total volume of excavation or filling will be the sum of the volumes of the constituent truncated prisms, the right section of which is a unit square. Also the volume of any right truncated prism is the area of its right section multiplied by the distance between the centres of its bases, and, in Fig. 80, is the area of a unit square multiplied by the mean difference of reduced level of the four corners, which for a level formation, is the average reduced level of the corners less the reduced level of formation.

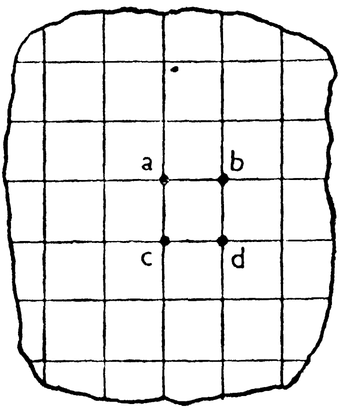

Fig. 80

Hence, if the corners *abcd* of the squares indicated are 51·8 ft., 53·9 ft., 52·7 ft., and 54·8 ft. above datum respectively, and the formation level is 40 ft., the mean height of the truncated prism will be $\frac{1}{4}(11{\cdot}8+13{\cdot}9+12{\cdot}7+14{\cdot}8)=13{\cdot}3$, while if the square is of 50-ft. side, the volume of the prism will be $13{\cdot}3 \times 50 \times 50 = 33{,}250$ cu. ft. $= 1{,}321$ cu. yds.

If the finished surface is to be inclined or graded, the calculations will be the same, but the mean height is taken as the differences between the reduced levels of the corners in the surface and in the formation.

Obviously the area to be levelled or graded may not be exactly rectangular, and in this case, a number of irregular solids will occur at the boundaries. These will be mainly trapezoidal in section, triangles occurring now and then; but the same rule applies as to the height of the truncated prisms, being the average difference of the reduced levels at the corners, while the areas will be merely those of trapezoids or triangles.

In practice, there will usually be parts of the area which are to be excavated and parts which are to be banked. Hence it is convenient to prefix the values at the corners + or – , signifying *cuts* and *fills* respectively. The cuts will be separated from the fills at formation level by irregular lines which are actually contours in the case of level formations. Sometimes these areas are given a light wash of colour to distinguish them from each other, areas at formation level being left white.

After taking out the content by the foregoing rules, account must be taken of the fact that earth expands on excavation and shrinks to some extent after being placed as filling, the allowance varying with different earths and materials.

The foregoing methods are mainly arithmetical since the calculations are made directly from the field notes.

Graphical Methods. Sometimes, however, wide vertical cross-sections are plotted, their areas found graphically or by means of the planimeter, and the corresponding volumes are calculated by the average end area rule, or even the prismoidal method.

(3) *Contour Prisms.* Also, estimates are sometimes taken from the *horizontal* sections given by contour lines, as in the case of the water content of the impounding reservoir shown in Fig. 81. Here a dam AB with a vertical water face is shown, the top water level (T.W.L.) being 80 ft., as indicated near the contour. The successive areas at the different contour elevations are found graphically or mechanically as A_{80}, A_{70}, A_{60}, and A_{50}. (Fig. 81.)

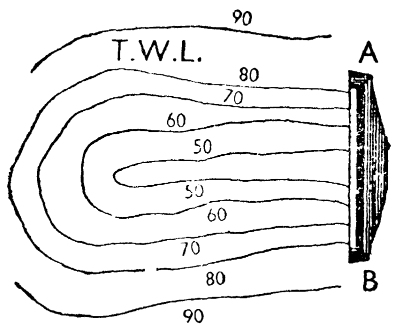

FIG. 81

The volume is then calculated for the several layers, or laminae, 10 ft. in depth from $V=10(\frac{1}{2}A_{80}+A_{70}+A_{60}+\frac{1}{2}A_{50})$ cu. ft., which may be expressed in millions of gallons, with 6·24 gallons to the cubic foot.

Another of the various methods consists in covering the contour area with a grid, and reversing the process by finding the corner elevations by interpolating between the contours on the map.

III. LONGITUDINAL SECTIONS

Longitudinal sections, called "profiles" in the U.S.A., are an important feature in engineering plans. They are *false* sections because the vertical scale which shows reduced levels is larger than the horizontal section which shows the corresponding horizontal distances along the centre line of the proposed railway, road, or sewer. The vertical scale is roughly 8 to 10 times that of the horizontal scale taken to the nearest convenient figure. Thus, with a horizontal scale of 200 ft. to 1 in., the vertical scale could be 20 ft. to 1 in.; with a horizontal scale of 50 ft. to 1 in., 5 ft. to 1 in. If the horizontal base were the actual datum of the survey this would often lead to an unsightly section, and waste of paper. Hence, in order to obtain a neat section, it is usual to raise the datum, stating the fact thus along the base A_0B_0, "50 ft. above datum," as in Fig. 82. Also, the sections are opened out like a screen unfolded, so that points in elevation are not vertically above the same points in plan, as in geometrical projection. The fact that the section is longer than the plan is evident in Fig. 82, which shows the traverse and section of a portion of a proposed railway, the distances "running through" in chains continuously from the beginning of the line. The section shows the reduced levels at 1 chain intervals along the centre line, with additional values at the points at which the direction of the traverse changes. These points and the beginning and the end of the section are shown with thicker lines than the rest of the ordinates, and it is a rule in plotting sections to join the tops of the ordinates with straight lines, never with a free curve as in the case of a graph.

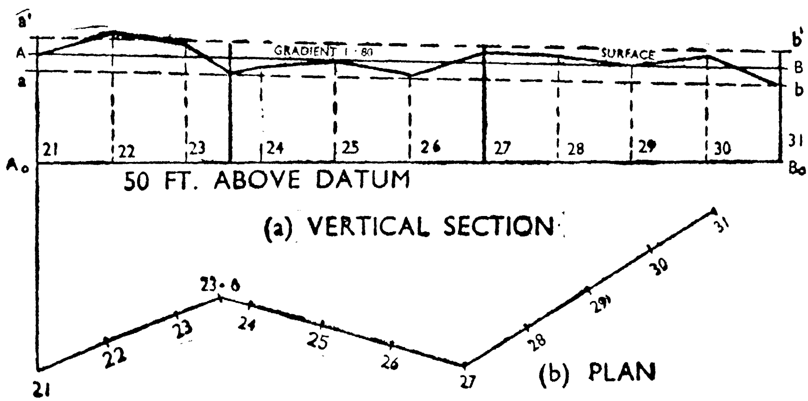

FIG. 82

The straight line AB in the section is drawn at formation level, and is known as the *gradient*, which is expressed by the tangent of the vertical angle, as 1 in x horizontally, or 1/80, or as a percentage, 1·25 per cent, being sometimes prefixed with the *plus* or *minus* sign, according as it is rising (upgrade) or falling (downgrade).

Incidentally, the gradient of pipes is taken at the "invert," which is the lowest point on the interior.

In Fig. 82 (*a*) it will be seen that a cutting will occur between 21·0 and 23·3 chs., a bank between 23·3 chs. and 26·6 chs., and a cutting between 26·6 and 30·4 chs. Also, the ordinates above or below AB are the centre line cuts and fills, which in the days of more elaborate plans were often tinted red and blue respectively, a neat array of information being tabulated along the base A_0B_0. Now if the gradient is settled upon from the section, a rapid means of estimating the earthwork volume is at the surveyor's disposal. Thus, if lines ab and $a'b'$ are drawn parallel to AB at depths w/s above or below formation, these will be whole depths D, and if the ground surface is level across the cross-sectional areas will be sD^2, while for ground with a lateral slope of r to 1, they will have this value divided by $1-\frac{s^2}{r^2}$ as explained in (2) of page 130.

Cross-sections are *true* sections, and when drawn, are plotted on a common horizontal and vertical scale.

It is a difficult matter to write a conclusion to a chapter of this character, since old heads cannot be put on young shoulders, and experience is something that cannot be imparted by words. In the preamble to this chapter, the term "correctness" was used to imply arithmetic devoid of mistakes, since the word "accuracy" alone might suggest the use of an approximation that would fully satisfy practical requirements. Briefly, when information is required the methods should be adequately accurate and the calculations arithmetically correct. Now there are not only arithmetical approximations in calculations, but also visual approximations in the field, in that a feature which appears even marked to the eye may be trifling as a part of the whole. Thus, level ground should not suggest a bowling green, but anything up to a general slope of 3°, or a surface warped to a series of slight irregularities. Thus, it often happens that elaborate rules are really ineffective, and this is frequently the case in ascertaining the cross-sectional areas of rivers. Of course, there may be some satisfaction in using the complex. Earlier engineers and surveyors held the mathematician in awe, and misapplied his teachings, reverently, at least, little realising that the natural errors of their work overwhelmed any refinements these rules might otherwise have introduced. Simplicity is the surest path until experience proves its limitations. Apart from these there are economic factors that demand rapid or good estimates, each of which has its place; the former utilising graphs, charts, and other artifices, and the latter discreet and careful calculations with appropriate checking. It has been said that there are computations, estimates, guesses, and back answers, the last suggesting an absurd response to a ridiculous request for a statement in an unreasonable period of time. But this presupposes that the reader will proceed further with the subject and will learn that much truth is said in jest. Hence, the work should be kept to the first two categories; and this means no juggling with rules or scrupling with trivialities, but getting ahead confidently, obtaining the right data and applying it with reason.

CLASS EXERCISES—AREAS

10 (*a*). Draw an irregular figure about $4\frac{1}{2}$ in. × 3 in. to represent a survey on a scale of 5 chains to 1 inch, and describe with reference to this area two ways in which you would determine its acreage. (G.S.)

10 (*b*). Draw an irregular closed figure about $3\frac{1}{2}$ in. × $2\frac{1}{4}$ in. to represent a pond on a scale of 1 in 2,500, and describe with reference to this figure *two* methods of determining its area other than by the use of squared paper

10 (*c*). A race track is to consist of two straight portions and two semicircular ends, the width of the track being 29 ft. and the length $\frac{1}{4}$ mile, measured around the inner edge of the track.

A rectangular plot which exactly encloses the track is to be purchased for the purpose at £400 per acre.

The committee suggest (*a*) that the straight portions should be equal in length to the outer diameters of the ends, while the surveyor recommends (*b*) that the outer radii should be 110 ft.

Calculate the saving that would result by taking the surveyor's advice.

(G.S.)

(£245 6*s*. 8*d*.)

10 (*d*). Sketch an irregular figure approximating to a rhombus of about 4 in. side to represent an area on a scale of 2 chains to 1 inch.

Determine its area by the following methods:

(*a*) Give and take lines; (*b*) Division into trapezoids; (*c*) Simpson's rule.

10 (*e*). You have a computing scale divided into inches and decimals, and you are required to find acreages directly on the following scales:

4 chs. to 1 in.; 5 chs. to 1 in.; 1 in 2,500.

State the spacings of the parallel rulings on tracing-paper for use with the scale. (0·625 in., 0·40 in., 1·03 in.)

CLASS EXERCISES—VOLUMES

10 (*A*). A straight and level roadway, 20 ft. wide, is being cut through a plane hillside which slopes 1 vertically in 9 horizontally at right angles to the road although it is level in the direction of the road.

The side slopes of the cutting will be 1 vertically in 1 horizontally and the depth of the cutting will be 10 ft. on the centre line of the road.

Calculate the volume of excavation in a horizontal length of 500 ft. (G.S.)

(5,648 cu. yds.)

10 (*B*). A reservoir is to be constructed with a flat rectangular bottom in which the length is $1\frac{1}{2}$ times the breadth. It is to hold one million gallons of water with a depth of 15 ft. Calculate the dimensions of the surface and bottom rectangles, given that the side slopes are 3 horizontally to 1 vertically. 1 cu. ft. of water $= 6\frac{1}{4}$ gallons.

(95·44′. × 63·63′., 185·44′. × 153·63′.)

10 (*C*). The following distances and reduced levels were taken in connection with a drain:

Distance	0	25	50	75	100	125	150	175	200 ft.
Red. level	91·8	92·0	92·4	93·6	94·2	95·2	96·4	95·7	94·1

The invert level of the drain is 88·6 at the beginning and falls 1 in 100. If the trench is rectangular, 2 ft. 6 in. wide, calculate the cost of excavation at 1*s*. 1*d*. per cu. yd.

Plot a section of the ground surface and the bottom of the trench on a horizontal scale of 25 ft. to 1 inch and a vertical scale of 10 ft. to 1 inch.

(£4 11*s*. 4*d*.)

10 (*D*). The following sectional areas were taken at 50 ft. intervals in a straight trench:

32·5 33·0 35·0 36·0 38·0 sq. ft.

In calculating, the prismoidal rule was used with only the end and middle areas. Determine the error in cu. yds. due to this misapplication of the rule.

(G.S.)

(75 cu. ft. overestimate. 7,016·7; 6,491·7.)

10 (*E*). The following heights of embankment were reduced at 100 ft. sections on a proposed railway, the ground being level across. The formation width is to be 30 ft. and the side slopes 2 horizontally to 1 vertically. Calculate the volume of the embankment by Simpson's rule.

0 7 12 14 13 9 8 4 0 ft. (12,720 cu. yds.)

FIELD AND PLOTTING EXERCISES

10 (*F*). *Morning:*

(*A*) The range-poles indicate a line *AB* which is to be levelled with staff readings at intervals of 50 ft., starting from an imaginary benchmark 50·0 (chalked $\overline{\wedge}$). Take staff readings at the 50 ft. points and reduce the level on a form you have prepared in the Answer Book.

Afternoon:

(*B*) Using your level notes, plot the corresponding vertical section with a horizontal scale of 25 ft. to 1 in. and a vertical scale of 20 ft. to 1 in. Finish the section neatly in pencil and insert the horizontal scale.

Imagine that a trench for a drain is to be dug with its bottom 2 ft. below ground level at the lower point *A*, and 3 ft. below ground level at *B*.

(*a*) Insert the line of the bottom of the trench on your section and find its gradient.

(*b*) Calculate the volume of excavation in cubic yards, given that the trench is uniformly 3 ft. wide. (G.S.)

ORIGINAL PROBLEMS

Calculate the areas of the surveys in Problems . . .

Find the subaqueous contours of a pond, and from these and the survey estimate the water content.

Calculate the volume of earth in a knoll from contours directly located.

Estimate the earthwork in levelling a plot for a tennis-court with 15 ft. level margins around.

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CHAPTER XI

*THEODOLITE SURVEYING

It would seem unfair to the reader if his curiosity were not appeased by some mention of that instrument which has come to be regarded as the embodiment of surveying: the *Theodolite*, the most perfect of all *goniometers*, or angle-measuring instruments. Thus, opportunely, this chapter may ease the passage to the more advanced branches of surveying.

The first mention of the rudimentary form of the instrument in English literature concerns the "theodolitus" of Thomas Digge's "*Pantometria*," 1571. The name, derived from *theodicoea*, was, in that old writer's sense of *perfection*, the most perfect of known surveying instruments. Also, there are grounds to believe that an equivalent Arabic root has given us the word "Alidade," which is associated with the plane table or the upper part of a modern theodolite.

The theodolitus consisted merely of a horizontal circle divided and figured up to 360°, and fitted with a centred, sighted alidade, the entire instrument being mounted upon a stand. The nearest instrument of this form is the almost-extinct "circumferentor" of about seventy years ago, some patterns of which very closely resembled the American Surveyors' Compass. It was not until the close of the eighteenth century that the theodolite assumed its present form, largely at the hands of Jonathan Sissons, the inventor of the Y-level; and in the early years of the nineteenth century Ramsden added substantial improvements; in particular, the transit principle, by which the telescope could be rotated in the vertical plane. Quite a romance could be written about the evolution of the theodolite, introducing its various forms, ranging from the Great Theodolite of the Ordnance Survey and Borda's Repeating Circle to the modern geodetic and engineering models. It is gratifying to know that English makers have been foremost in the design and construction of surveying instruments; and some of the pioneers of American instrument-making received their early training in this country.

As may have already been concluded, the primary function of the theodolite is the accurate measurement of horizontal and vertical angles, i.e. angles respectively in the horizontal and vertical planes.

Apart from special designs, the modern theodolite is made in sizes ranging from 3 in. to 12 in., the *size* being specified by the diameter of the horizontal graduated circle.

In an elementary text-book it is impossible to describe theodolites in general, though it is desirable that any description should refer to an actual instrument rather than to an improvised model. For this reason, a vernier pattern of a general purpose transit theodolite will be considered as the representative instrument. This is shown dissected in Fig. 83, in order that the essentials of theodolites may be explained.

The theodolite consists of the following four primary portions, which are shown separated, the reference letters corresponding to those on the diagram.

I. *The Vertical*; II. *The Plate-Standards*; III. *The Limb*; and IV. *The Levelling Head.* I and II together form the "Alidade" of the instrument.

I. *Vertical.* This comprises (1) the telescope with its eyepiece *E* and ray shade *R*, the azimuthal level *B*, and the horizontal or transverse axis *o*; (2) the vertical circle and (behind) its two verniers *M*, the magnifiers *m* being omitted; (3) the clipping frame with its clipping screws *jj*, and (behind) the clamp *V* and tangent screw or slow-motion *v* to the vertical circle; hereafter called the "*Vertical Motion.*"

Sometimes the level *B* is fitted as an altitude level on the top of the clipping or vernier frame.

Frequently the verniers *M* of the vertical circle are stamped *C* and *D*, but usually the former is understood as the vernier.

The horizontal axis fits into the bearings at the tops of the standards (4) and is secured with little straps and a screw.

II. *Plate-Standards.* These consist of (4) the standards (here *A* frames) which at the tops provide a trunnion bearing *O* for the horizontal axis *o*, and also carry the plate levels, *p*, for levelling the instrument; (5) the upper horizontal plate, which carries the two verniers *N* (their magnifiers *n* being omitted), and the clamp *U* to the upper plate with its tangent screw or slow motion *u*, hereafter called the "*Upper Motion.*"

Frequently the verniers *N* are stamped *A* and *B*, the former being understood unless qualified.

Centrally at the bottom of the upper plate is the solid inner spindle (ii) which fits into the outer hollow spindle (iii) of the limb.

III. *Limb.* This simple component consists of the horizontal circle divided and figured in degrees on silver, and the outer hollow spindle (iii) which fits into the bearing afforded inside the levelling head.

IV. *Levelling Head.* Here the older pattern four-screw device is shown, the lower parallel plate being bored and threaded in order that the entire instrument may be screwed to the top of its tripod. *F, F* are the plate screws with which the instrument is levelled after the tripod has been planted, the levelling being regulated by the position of the bubbles of the plate levels *p, p*, as described on page 68. A small hook is inserted in the nut which secures the spindles (ii) and (iii) in position in the levelling head, and from this hook a plumb-bob is suspended. Some levelling heads are fitted with a centring stage or shifting plates so that the plumb-bob can be set exactly to a cross on a peg at the station beneath the instrument.

In the model shown the levelling head carries the clamp *L* of the

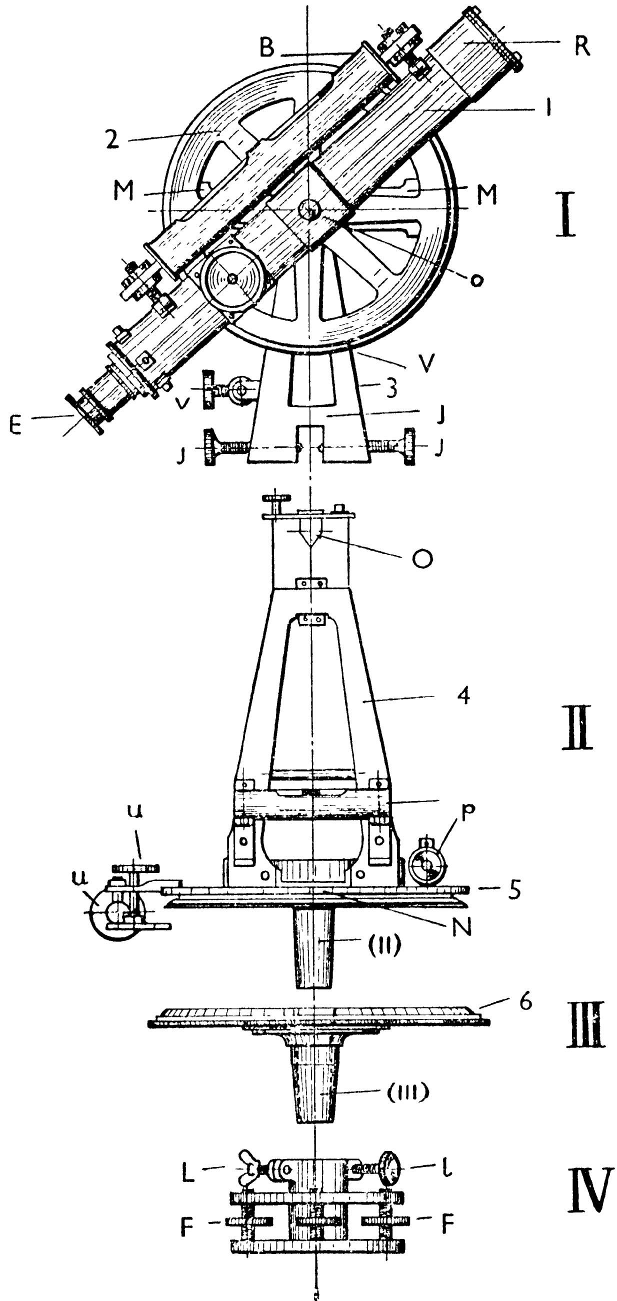

Fig. 83

THE THEODOLITE

limb and its tangent screw, or slow motion l, hereafter called the "*Lower Motion.*"

Manipulation. When the instrument has been re-assembled and levelled up at a station O, say, the outer spindle can be clamped to the levelling head by means of L, while if the inner spindle is unclamped, U being slack, the vernier N can be moved relatively to the divisions on the horizontal circle, or limb. Hence, in setting the A vernier to zero (i.e. 360°), the upper plate and superstructure are turned until the vernier index is at 360°, as nearly as may be; the upper plate is then clamped by means of U, and the index is set exactly at 360° by means of the tangent screw u, the vernier being viewed through its magnifier n. With the upper motion thus clamped, a station P, say (normally the one to the left) can be sighted by slackening the clamp L, and turning the entire superstructure about the outer spindle, as an axis, until the foot of the station pole is seen inverted near the intersection of the cross-wires of the telescope; the lower motion is then clamped by means of L, and the image of the foot of the pole is exactly bisected by the vertical wire by turning the tangent screw l. If now the upper motion is unclamped by slackening U, the telescope can be directed towards the station Q, say (normally to the right of P), the inner spindle moving inside the clamped outer spindle; and after a near sight at the foot of the station pole, the upper plate can be clamped by means of U, and the inverted image of the foot of the pole exactly bisected by the vertical wire by turning the tangent screw u, the upper plate thus moving relatively to the horizontal circle. The magnitude of the angle POQ is then read on the A vernier (Fig. 85).

Circles and Verniers. Circles of British and American instruments are divided into the *Sexagesimal* division of 1 degree (1°)=60 minutes (60′); 1′=60 seconds (60″). This system actually follows from the ancient nomenclature of the first and second subdivisions of the degree: "*pars minuta prima*" and "*pars minuta seconda,*" although Ptolemy (A.D. 85-165) actually worked in arcs, not angles, dividing the circumference of the circle into 360 equal arcs. The Continental angular measure is the circle of 400 grades, 100g. being equal to 90°. *Whole Circle Clockwise* (*0° to 360°*) is the division of horizontal circles used exclusively in this country; and the most rational system for vertical circles is the *Quadrant*, or Quarter circle division (0°-90°-0°-90°-0°), the zeros being in a horizontal line. This quadrant division is favoured by surveyors who prefer to observe bearings directly, particularly in North America, where the *Half Circle* (0° to 180° in both directions) is also used, in each case subsidiary to the whole circle division.

Simple as it sounds, some confusion usually arises as to what the whole circle division should read when the upper, or vernier, plate is turned in the counter-clockwise direction. There seems no better answer to this than to say that if a clock stops at 20 minutes to 5, then, on re-winding at 11.20, it will read 20 minutes past 11 whether the hands be turned backwards or forwards on re-winding, apart, of course, from the fact that it would be indiscreet to turn the hands of a striking movement in the retrograde direction.

Most of the smaller patterns of theodolites are fitted with verniers, the simplest and most reliable mechanical contrivance for reading exact subdivisions of a main division; 1°, $\frac{1}{2}$°, or $\frac{1}{3}$°, in the case of the circles of theodolites. Named after its inventor, the vernier is a small sliding scale on which n divisions of length v are equal to $n-1$ scale divisions of length c. Thus if the scale divisions c are $\frac{1}{10}$ in. and 9 of these are equal 10 divisions v on the vernier, then $c-v=\frac{1}{10}-\frac{9}{10}(\frac{1}{10})$ in. $=\frac{1}{100}$ in. which is the *least count*, signifying that the vernier will read to $\frac{1}{100}$ in., which would require a diagonal scale 1 in. in width.

No difficulty need ever arise with surveying instruments. Merely divide the angular value of c, the smallest division on the circle, by the number n of corresponding divisions on the vernier. Now n is not necessarily the number stamped on the vernier, this often corresponding to even minutes only.

This simple rule merely follows from:

$$(n-1)c=nv;\ v=\left(\frac{n-1}{n}\right)c;\ \text{and}\ x=c-v=c-\left(\frac{n-1}{n}\right)c=\frac{c}{n}.$$

Thus, if $c=\frac{1}{2}°$ on the circle and $n=30$ on the vernier, then $x=\frac{1}{60}°=1'$.

Verniers of vertical circles are often read upwards and downwards, and are frequently figured in both directions. Much trouble would be saved if these were marked plus and minus. Anyway, always read the vernier with its numbers counted in the *same* direction as the figures on the circle.

Since the graduations are finely etched on the circles, it is necessary to take the readings of the verniers through a magnifier or reader, attached near the vernier. This device must not be confused with the micrometer microscopes which are fitted instead of verniers on the more elaborate instruments. In more accurate work it is usual to read and take the mean value from both verniers; the A and the B on the horizontal circle and the C and D on the vertical circle. This is a precaution against "eccentricity," which is seldom encountered to any appreciable extent except in old or damaged instruments.

MEASUREMENT OF ANGLES. Let us assume that the tripod has been firmly planted at the station O with the telescope at a convenient height for sighting, the lower plate of the levelling head being fairly horizontal. The instrument must now be levelled up in the manner described on page 68, the bubbles of the plate levels being central. Next the telescope must be focused, eliminating parallax, in the manner also described. The cross-wires will appear as in Fig. 84, and the images of the station poles will appear at these, finally with that of the pole or point exactly bisected by the vertical wire.

(Some diaphragms will also be webbed or etched with the *stadia lines* shown dotted in Fig. 84. The object of these is that of determining horizontal distances D from the amount of vertical staff seen intercepted between them, D being $100s$, but always subject to corrections for vertical angles above 5°, known as Reductions to Horizontal.)

Horizontal Angles. (1) Clamp the lower motion by means of the clamp L. Unclamp the upper motion, and set the A vernier at zero; clamp U, and finally set the vernier index at 360° by means of the tangent screw u.

(2) Unclamp the lower motion and sight the lowest point of the pole at the left-hand station P; clamp L, and obtain an exact bisection of the image of P by means of the tangent screw l.

(3) Unclamp the upper motion, and sight the pole at the right-hand station Q; clamp U, and obtain an exact bisection of the image of Q by means of the tangent screw u.

(4) Read the A vernier and record this reading as the magnitude of the angle POQ (Fig. 85).

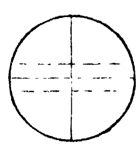

FIG. 84

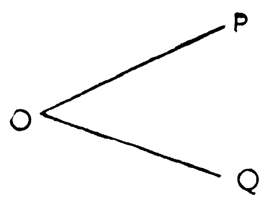

FIG. 85

If it is impossible to sight the lowest points of station poles, these should be carefully "plumbed."

If the magnetic bearing of a line is required, the A vernier should be set at zero by means of the upper motion, and the lower motion should be unclamped and the alidade turned until the magnetic needle comes exactly into its meridian, clamping L, and obtaining exact coincidence by use of the tangent screw l. Then the station P (or Q) should be sighted by means of the upper motion, clamping U and obtaining an exact bisection of the image of the station by means of the tangent screw u. The bearing of OP (or OQ) is then read on the A vernier.

Vertical Angles. When vertical angles are observed, greater accuracy will result if the azimuthal or altitude level B is utilised in a more exact levelling-up of the instrument.

(1) Set the C vernier to zero by means of the vertical motion, clamping at 0° by means of V, and setting the vernier index precisely by means of the tangent screw v.

(2) Set the bubble of the level B central by means of the clipping screws jj, the process being the same whether the level is on the clipping frame or on the telescope (Fig. 83).

(3) Unclamp the vertical motion, and sight the elevated point; clamp V, and obtain exact coincidence of the intersection of the cross-wires and the image by means of the tangent screw v.

Read the magnitude of the vertical angle on the C vernier, taking care that the vernier is counted in the proper direction.

Face Left and Right. In the case of transit theodolites, it is possible to "transit," or rotate the telescope about its horizontal axis o, which means that the vertical circle may be either on the right or the left of the observer's eye. These are known as the Face Left (F.L.) and Face Right (F.R.) positions; one of which is retained in ordinary usage, this "normal" position being *Face Left* preferably. When angles are observed with both faces thus, the mean horizontal angle will be free from instrumental errors of adjustment, but this is never the case with vertical angles. Both faces are used thus when great accuracy is required, as in triangulation surveys.

Back Angles and Bearings. In British practice, horizontal angles are usually measured directly, as above, or as *Back Angles*, which are the angles measured clockwise from a zero reading on the preceding rear station, a practice commonly followed in town surveying. Thus, if the pond in Fig. 20 is traversed with the lines, AB, BC, CD, and DA running in the counter-clockwise direction, these back angles will be the interior angles of the skeleton; and this is convenient in applying the check of the angular sum; $(2N—4)$ 90°, where N is the number of sides or angles. Any error in the observed sum of the angles may then be divided equally among the angles, and each part applied appropriately as a *correction* to the observed angles, provided each angle is measured with equal accuracy, or equal weight, as it is called. If reduced bearings are required for plotting by co-ordinates, as described hereafter, these must be calculated with the bearing of one side of the traverse, observed with reference to the magnetic or the true meridian or assumed with reference to any convenient so-called north and south line. Most theodolites are provided with a magnetic compass, sometimes in the trough or the telescopic form, and sometimes in the form of a dial. If then the bearing of one side, AB, say, is observed, or if AB is assumed to have some bearing, conveniently N.0° 0′ E., then the bearings of the remaining sides can be reduced from the observed interior angles. The characteristic of direct angular measurement is that all angles are measured separately, and errors are not carried through to succeeding lines. Its advantage is that angles may be repeated with alternate faces of the instrument, thus eliminating the effects of instrument errors.

In North America, azimuths and bearings are observed directly by sighting on the preceding rear station *face right*, transitting the telescope, and then sighting forward *face left* consistently. This expedites the work and gives a direct reading of the total angular error on the horizontal circle, but it confines the work to one vernier, angles to one measurement, and also exaggerates the effects of errors of instrumental adjustment. If the bearing of the first line is observed the compass need not be consulted again, for, in fact, the survey will be run "fixed needle."

The angular measurements of the surveys shown in both Figs. 19 and 20 will be more precise than when the compass was used (page 93), and, strictly, the accuracy of the chaining should be raised, or the results may appear disappointing, simply because the crude and precise cannot mix.

II. LATITUDES AND DEPARTURES

Like as the traverse of a polygon should close upon the first station, so is it fit and proper that this little book should return to the co-ordinates of the opening chapter.

Latitudes and departures are nothing more or less than Cartesian co-ordinates, more commonly known as "graphs." The Y-axis of Fig. 1 merely becomes the N.S. axis of Latitude and the X-axis the W.E. axis of Departure, the origin still remaining at O.

Now if s_1 be the length of a survey line OA and $N\beta E$ its bearing; then its latitude will be the projection on the N.S. axis, which is $\lambda_1 = s_1 \cos \beta_1$.

Also, its departure will be the projection δ_1 on the W.E. axis, which is $\delta_1 = s_1 \sin \beta_1$. Hence, $\tan \beta_1 = \delta_1/\lambda_1$.

Likewise for the line OC, of length s_3 and bearing $S\beta_3 W$, the latitude and departure will be respectively,

$$\lambda_3 = s_3 \cos \beta_3;\ \delta_3 = s_3 \sin \beta_3;$$
$$\tan \beta_3 = \delta_3/\lambda_3.$$

Again the opening rhyme may be repeated:

> "*Positive north and positive east,*
> *Negative south and negative west.*"

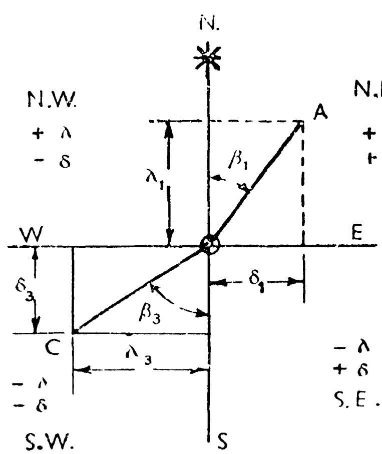

FIG. 86

Howare these signs determined?

Simply from the initial and final letters of the bearings; N. and S. and E. and W. respectively, as given in the rhyme.

North bearings give plus latitudes, or "Northings"; south bearings give minus latitudes, or "Southings"; east bearings give plus departures, or "Eastings"; and west bearings give minus departures, or "Westings."

Thus, λ_1 and δ_1 are both plus, while λ_3 and δ_3 are both minus, lines in other quadrants having signs prefixed to them as in the four quadrants of Fig. 86. Algebraical signs are of utmost importance in all problems which introduce latitudes and departures.

Now latitudes and departures are used in two forms, which, to avoid confusion, may be styled (*a*) *Individual Co-ordinates*, and (*b*) *Total Co-ordinates*. At all stations the existence of the co-ordinate axes must be imagined when thinking of individual latitudes and departures, while in working with total co-ordinates these axes exist in fact, as with graphs, with the origin at the most westerly station of the survey; and the total co-ordinates of any point are the individual latitudes and departures summed algebraically from this origin.

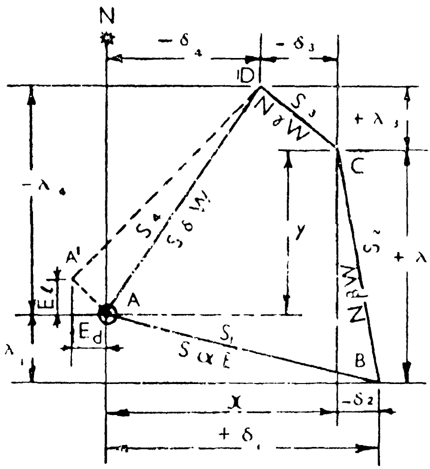

FIG. 87

Consider Fig. 87, which is a quadrilateral traversed in the counter-clockwise direction, so that the forward reduced bearing of *AB* is S.E.; of *BC*, N.W.; of *CD*, N.W.; and of *DA*, S.W. The origin *O* is taken at *A*, which is the most westerly station, and the individual latitudes and departures λ and δ are written appropriately on the diagram.

Now at *C*, the total plus departure, or easting, is $x=+(\delta_1-\delta_2)$, and the total plus latitude, or northing, is $y=+(\lambda_2-\lambda_1)$.

On returning to *A* by way of *CD* and *DA*, the total latitude *y* will be *O*, since $+(\lambda_2-\lambda_1)-(\lambda_4-\lambda_3)=O$, while the total departure x will also be *O*, since $+(\delta_1-\delta_2)-(\delta_3+\delta_4)=O$.

This introduces a very important principle, which is the basis of adjusting traverse surveys arithmetically.

Adjusting Traverses. Now it seldom happens that either the algebraical sum of the latitudes or of the departures is exactly zero, but will be small values which are the total errors in latitude and departure, E_l and E_d respectively, as indicated by the dotted line *AA'* in Fig. 87. The true error of closure of the traverse is linear, and is $E=\sqrt{E_l^2+E_d^2}$, while the angular error of closure α is found from the difference of the observed sum of the interior angles of the figure and the geometrical sum, as found from $2(N-4)90°$, where *N* is the number of sides.

Bowditch's method is easily applied by finding the ratios:

$m=\dfrac{E_l}{\Sigma s}$ and $n=\dfrac{E_d}{\Sigma s}$, where $\Sigma s=s_1+s_2+s_3$, etc., or the perimeter of the traverse.

The corrections in latitude and departure will be l_1, l_2, etc., d_1, d_2, etc., to the sides s_1, s_2, etc., accordingly:

$$l_1 = \pm ms_1;\ l_2 = \pm ms_2;\ \text{etc.; and } d_1 = \pm ns_1;\ d_2 = \pm ns_2,\ \text{etc.,}$$

which are prefixed with the sign of the corresponding total error in latitude and departure. These corrections are then subtracted algebraically from the corresponding calculated values for the corrected latitudes and departures to be used in plotting the survey. Many practical men do not worry about signs. When, for instance, they sum up the latitudes and find that E_l is negative, they say they have too much negative latitude, and merely increase the plus latitudes and decrease the minus latitudes by the values of the corrections l_1, l_2, etc. Likewise for the departures.

Plotting Surveys. The method of total co-ordinates provides possibly the best and most accurate method of plotting surveys. But before the latitudes and departures are calculated, it is advisable to consider how the survey is to be "placed" on the drawing-sheet; as, for example, with the approach road along the bottom of the sheet, which usually will mean that the meridian will not run parallel to the vertical edges. Consequently, it is advisable to plot the traverse roughly with the aid of a protractor. This will not only reveal which is the most westerly station, but will indicate the angle θ through which the entire survey must be twisted so that a meridian will run parallel with the vertical edges of the sheet. Some even value of θ is then subtracted from all the bearings, and the latitudes and departures are calculated with reference to the resulting "false" meridian. They are then duly corrected, as explained in the preceding paragraph. Otherwise it might be necessary to re-calculate the entire set of latitudes and departures; and this is no small undertaking without traverse tables, since each pair of values requires four to five minutes in reducing with five-figure mathematical tables. It might happen that an extra-outsize sheet of paper, known as "antiquarian," might be found, and from this the modest "imperial" sheet could be cut out after plotting.

The individual latitudes and departures are added algebraically from the most westerly station adopted as the fixed origin, the total latitude of a station being either plus or minus, and the total departure always plus. On reaching the origin again the sum will be zero. The values thus tabulated are the co-ordinates with which the stations of the survey are plotted with reference to the origin.

In general, it is best to draw a reference rectangle which will exactly enclose the skeleton to scale, the most westerly station being on the left-hand side, while the upper, lower, and right-hand sides pass through the most northerly, southerly, and easterly stations respectively. The vertical dimension of this rectangle is given by the arithmetical sum of the greatest total northerly and southerly latitudes, and the horizontal dimension is merely the greatest total easterly departure.

The work may be expedited in surveys with much detail by covering the rectangle with a graticule or grid of unit squares, each side of which shows on the scale of the plan a convenient unit of latitude and departure; 1 chain, 100 ft., etc. Otherwise the stations would have to be plotted with their total scale distances, north and south and east of the origin *O*. It is, of course, possible to plot from the two nearest sides of the rectangle by subtracting the tabular distances of the stations or points from the lengths of the sides of the rectangle.

The foregoing are only two of the uses of the method of latitudes and departures. The principles are also used in (*c*) calculating areas, (*d*) supplying omitted measurements, (*e*) parting land and rectifying boundaries, and (*f*) overcoming obstructions where no other method would be effective.

* EXAMPLE. The foregoing methods may be illustrated through the medium of the following closed theodolite and chain traverse, in which back angles were observed, the magnetic bearing of *AB* being S. 64° 36′ E.

Reducing Bearings:

Line	*AB*	*BC*	*CD*	*DE*	*EA*
Length (chs.)	23·53	7·20	10·66	8·89	22·45

	EAB	*ABC*	*BCD*	*CDE*	*DEA*
Back angle	58° 42′	101° 06′	143° 30′	131° 18′	105° 24′

It will be seen that the back angles sum up exactly to 540° 00′, which frequently happens in careful work with theodolites reading to single minutes.

The bearings may now be reduced, and since this survey may be plotted with the magnetic north at the top of the sheet, the latitudes and departures may be calculated and tabulated also.

Calculating Latitudes and Departures:

Line	*Length (lks.)*	*Bearing*	*Latitude (lks.)*	*Departure (lks.)*
AB	2353	S. 64 36′ E.	−1009·3	+2125·6
BC	720	N. 36 30′ E.	+ 578·8	+ 428·3
CD	1066	N. 0 00′ E.	+1066·0	00·0
DE	889	N. 48 42′ W.	+ 586·7	− 667·9
EA	2245	S. 56 42′ W.	−1231·8	−1875·6

$E_l - 9{\cdot}6$; $E_d + 10{\cdot}4$

Adjusting the Traverse. It will be seen that there is an excess of minus latitude of 9·6 lks. and of plus departure of 10·4 lks. in a perimeter of 7273 lks.

The correction factors *m* and *n* can now be calculated, although the fractions are more conveniently run off on a slide rule:

$$m = -\frac{E_l}{\Sigma s} = -\frac{9{\cdot}6}{7273}; \; n = +\frac{E_d}{\Sigma s} = +\frac{10{\cdot}4}{7273}.$$

The corrections to the latitude and the departure, l and d, are calculated by multiplying respectively m and n by the lengths of the sides thus:

	AB	*BC*	*CD*	*DE*	*EA*		
l	−3·1	−1·0	−1·4	−1·2	−3·0;	sum	−9·7
d	+3·4	+1·0	+1·5	+1·3	+3·2	„	+10·4

These corrections are now subtracted algebraically from the observed latitudes and departures, giving the corrected values in the following table. In practice, a Traverse Sheet is drawn up with sufficient columns for the entire notes; but this would require a folding sheet, which is not desirable in a book of this nature. Hence the tables are separated, and so curtailed in width that it is impossible to show + and − latitudes in separate columns as northings and southings respectively, and + and − departures as eastings and westings respectively—a great convenience in summing algebraically.

Plotting the Traverse. Now A also happens to be the most westerly station of the traverse, and the following total co-ordinates are summed algebraically from that station.

Station	*Line*	*Corrected*		*Total*	
		Latitude (lks.)	*Departure (lks.)*	*Latitude (lks.)*	*Departure (lks.)*
	AB	−1006·2	+2122·2		
B				−1006·2	+2122·2
	BC	+ 579·8	+ 427·3		
C				− 426·4	+2549·5
	CD	+1067·4	− 1·5		
D				+ 641·0	+2548·0
	DE	+ 587·9	− 669·2		
E				+ 1228·9	+1878·8
	EA	−1228·8	−1878·8		
A				+ 0·1	0·0

E_l +0·1 E_d 0·0

If a boundary rectangle is used in plotting, its horizontal length will be 2549·5 lks. to scale, and its vertical width will be (1067·4+1228·8)=2296·2 lks. to scale. On a scale of 1 chain to 1 inch the dimensions would thus be 25·50″×22·96″.

CLASS EXERCISES

11 (*a*). The following readings were obtained in a triangle *ABC*, the mean reading of the two verniers being given in each case. Tabulate the mean observed value of each angle, and state the corrected values, assuming that the total error is to be distributed equally among the angles.

Station	Point Observed	Face Left	Face Right
		° ′ ″	° ′ ″
A	*B*	0 00 00	229 10 30
	C	49 10 30	278 20 50
B	*C*	91 39 00	350 53 30
	A	170 43 30	69 58 10
C	*A*	195 56 20	67 13 50
	B	247 41 10	118 58 20

	° ′ ″		° ′ ″
BAC	49 10 15		49 10 25
CBA	79 04 35		79 04 45
ACB	51 44 40		51 44 50
Observed	179 59 30	Corrected	180 00 00

11 (*b*). Discuss the measurement of a horizontal angle with the theodolite when great accuracy is required. State what errors will be eliminated by the various steps of your procedure.

11 (*c*). The following back angles were observed in a traverse survey, the area being traversed in the counter-clockwise direction: *ABC*, 172° 48′; *BCD*, 96° 50′; *CDE*, 148° 42′; *DEF*, 128° 43′; *EFG*, 70° 40′; and *FGA*, 102° 17′. Reduce these to magnetic bearings, given that the line *AB* had a forward bearing of N. 24° 12′ E.

(N. 24° 12′; N. 17° 00′ E.; N. 66° 10′ W.; S. 82° 32′ W.; S. 31° 15′ W.; S. 78° 05′ E.; N. 24° 12′ E. (check)).

11 (*d*). The following notes show the co-ordinates of a closed traverse with straight boundaries, the stations running in counter-clockwise order.

Line	Latitude (*lks.*)	Departure (*lks.*)
AB	00·00	+ 1133·90
BC	+404·00	− 188·80
CD	+437·50	− 269·10
DE	−445·40	− 526·50
EA	−396·10	− 149·50

Plot the survey and calculate the area from the co-ordinates.

(563,126 sq. lks. = 5·63126 acres).

11 (*e*). The following notes were recorded in a theodolite and chain traverse in which the length and bearing of the closing line *EA* were omitted:

AB: 2342 ft., S. 84° 21′ E.; *BC*: 782 ft., N. 14° 44′ E.; *CD* 1510 ft., S. 88° 32′ W.; *DE*: 462 ft., S. 38° 24′ W.

Calculate the latitudes and departures, and hence determine the length and bearing of the missing closing line, assuming that all the measurements were made with uniform accuracy.

($EA = \sqrt{(\lambda^2)+(\delta^2)} = 744$ ft., where λ and δ are the latitude and the departure that make the entire sums both algebraically zero.

Tan Bearing $EA = \dfrac{\delta}{\lambda}$, the signs indicating the quadrant, S. 80° 20′ W.)

FIELD EXERCISES

11 (*A*). Measure the angles of the triangle *ABC*, using both faces of the instrument and taking the mean of both verniers. Record the results in appropriate form, and adjust the triangle to close.

Equipment: Transit theodolite, and three range-poles.

11 (*B*). Determine the error in the sum of the interior angles of the polygon *ABCDE* by observing the back angles with a theodolite. Observe the magnetic bearing of *AB*; reduce the corrected bearings, and record these on an appropriate note form.

Equipment: Theodolite and five range-poles.

11 (*C*). Determine the height of the finial on (specified) Tower above the ordnance datum, given that the reduced level of the peg *A* is . . .

Equipment: Theodolite, two pickets, chain or band, and levelling staff.

11 (*D*). Make a theodolite and chain traverse of the (specified) field, wood, or pond. Plot the skeleton of the survey by latitudes and departures.

Equipment: Theodolite, chain, arrows, tape, and set of pickets.

11 (*E*). Make a theodolite and chain traverse of (specified) road between . . . and . . . (or . . . brook, between . . . and . . .).

Equipment: as in 11 (*D*).

ORIGINAL PROBLEMS

Determine the height and distance of . . . (specified point) on . . . Hill (inaccessible). . . . .

. . . .

Determine the true north from the mean horizontal angle when observing a circumpolar star at equal altitudes on each side of the pole. . . . .

. . . .

. . . .

. . . .

. . . .

TRIGONOMETRICAL TABLE

Deg.	Chord	Sine	Tangent	Cotangent	Cosine		Deg.
0°	0	0	0	∞	1	1·414	90°
1	·017	·0175	0175	57 2900	·9998	1 402	89
2	·035	·0349	0349	28 6363	·9994	1 389	88
3	·052	0523	·0524	19 0811	·9986	1·377	87
4	·070	·0698	·0699	14 3006	·9976	1·364	86
5	·087	·0872	·0875	11 4301	·9962	1 351	85
6	105	·1045	·1051	9 5144	·9945	1·338	84
7	·122	·1219	·1228	8 1443	·9925	1·325	83
8	·139	·1392	·1405	7 1154	·9903	1·312	82
9	·157	·1564	·1584	6 3138	·9877	1·299	81
10	·174	·1736	·1763	5 6713	·9848	1·286	80
11	·192	·1908	·1944	5 1446	·9816	1·272	79
12	·209	·2079	·2126	4 7046	·9781	1·259	78
13	·226	·2250	·2309	4·3315	·9744	1 245	77
14	·244	·2419	·2493	4 0108	·9703	1·231	76
15	·261	·2588	·2679	3·7321	·9659	1·217	75
16	·278	·2756	·2867	3 4874	·9613	1 204	74
17	·296	·2924	·3057	3 2709	·9563	1 190	73
18	·313	·3090	·3249	3 0777	·9511	1 176	72
19	·330	·3256	·3443	2 9042	·9455	1 161	71
20	·347	·3420	·3640	2 7475	9397	1·147	70
21	·364	·3584	·3839	2 6051	·9336	1 133	69
22	·382	·3746	·4040	2 4751	·9272	1 118	68
23	·399	·3907	·4245	2 3559	·9205	1 104	67
24	·416	·4067	·4452	2 2460	·9135	1 089	66
25	·433	·4226	·4663	2 1445	·9063	1 075	65
26	·450	·4384	·4877	2 0503	8988	1 060	64
27	·467	4540	·5095	1 9626	8910	1 045	63
28	484	·4695	·5317	1 8807	·8829	1·030	62
29	·501	·4848	·5543	1 8040	8746	1 015	61
30	·518	·5000	·5774	1 7321	·8660	1·000	60
31	·534	·5150	·6009	1 6643	·8572	·985	59
32	·551	·5299	·6240	1 6003	·8480	·970	58
33	·568	·5446	·6494	1 5399	·8387	·954	57
34	·585	·5592	·6745	1·4826	·8290	·939	56
35	·601	·5736	·7002	1·4281	·8192	·923	55
36	·618	·5878	·7265	1 3764	·8090	·908	54
37	·635	·6018	·7536	1 3270	·7986	·892	53
38	·651	·6157	·7813	1 2799	·7880	·877	52
39	·668	·6293	·8098	1 2349	·7771	·861	51
40	·684	·6428	8391	1·1918	·7660	·845	50
41	·700	·6561	·8693	1 1504	·7547	·829	49
42	·717	·6691	·9004	1 1106	·7431	813	48
43	·733	·6820	.9325	1 0724	·7314	·797	47
44	·749	·6947	.9657	1 0355	7193	·781	46
45	·765	·7071	1 0000	1 0000	·7071	·765	45
Deg.		Cosine	Cotangent	Tangent	Sine	Chord	Deg.

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