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Elements of Descriptive Geometry

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PREFACE

This book, like its companion volume, "Elements of Drawing" by the same authors, is based on the work of this character required of all first-year students in Sibley College, Cornell University. It is the outgrowth of an effort to modify and shorten somewhat the scope and method of presentation of this subject, from the lengthy discussion often presented, to one more in keeping with the relative importance of the subject in an engineering curriculum.

This work was undertaken for me some years ago by Professor Blessing, and the book is the outgrowth of a series of lectures and an accompanying drawing-room course first given by him and later continued by Professor Darling; both being at the time members of the instructing staff of the Department of Machine Design and Construction of Sibley College. The work of putting this material into the form of a book was undertaken by the authors at the writer's request, the object in view being twofold, namely: to obtain a book exactly suited to the needs of the Department, which we had hitherto been unable to do, and also to put into permanent shape the methods and principles used in this work, thus forming one of a series of correlated textbooks which eventually it is expected will cover the entire work of the Department.

The authors brought to the task a full knowledge of the more advanced work of the Department, having had experience in teaching the advanced subjects in design, which with their experience elsewhere both in practical and teaching positions was of great aid in improving and simplifying this more elementary work. For five years this material has been used in mimeographed form in this and other institutions with such unqualified success as to warrant the belief that the methods of presentation are sound and the scope sufficient to give the student all the training and information necessary to pursue advanced work in drawing and design, without omitting any essential details necessary in training engineering students. The following suggestions by the authors as to the use of the book may make their point of view clearer:

"*The presentation of an experiment as a means of bringing out fundamental principles* has been found to appeal to the student and it has enabled many to grasp the subject with comparative ease who had considerable difficulty by other methods in 'visualizing' these simple problems. For the average student the mere reading of the experiment is sufficient but others may require the actual 'building up' of the experiment to fully grasp the principles involved. The analysis and solution of each problem should be thoroughly understood and, as a test of the thoroughness and accuracy, the solution of the 'check' will serve.

"A somewhat unusual feature of the book is the use of the *first angle* for the solution of many problems relating to the point, line and plane, while the *third angle* is used exclusively for problems relating to solids. It is believed that the fundamental principles of the subject are more easily grasped by beginners if they are presented in the first angle on account of the ease with which the problem can be built up in this angle when necessary and if these principles are once thoroughly understood the student will find no difficulty in applying them to any other angle. Experience in the use of this method of presentation justifies the belief that this is a good way of approaching the subject. The third angle is made use of only after the student has gained some proficiency, and is used for all work involving solids in order to conform to modern practice of making drawings in the third angle.

"The drawing-room course of problems in Chapter X can be varied and others introduced by the instructor as occasion demands. It is believed that the method of definite measurements used in this drawing-board work is fully justified by the training it gives the student in method and accuracy, although admittedly a slightly greater number of problems can be solved in a given time by placing them on the sheet in a haphazard way. This systematic and definite arrangement of problems will be found helpful to the instructor where the work of a large number of students must be handled."

There is nothing of an experimental nature in the book, as the material it contains has been used in Sibley College for several years with increasing success; and it is hoped that it may be found helpful elsewhere.

DEXTER S. KIMBALL,
Professor of Machine Design and Construction,
Sibley College, Cornell University.

ITHACA, N. Y.
July 29, 1912.

AUTHORS' NOTE

While teaching this work and collecting data for this book the authors have studied carefully all available literature on the subject. When a suggestion, method of presentation or problem was found especially good it was adopted. So much similarity exists in these books that individual credit cannot be given, but the following books were found especially useful:

Elementary Descriptive Geometry by C. H. McLeod; Descriptive Geometry by A. E. Church; Essentials of Descriptive Geometry by G. H. Follows; Notes on Descriptive Geometry by C. L. Adams; Descriptive Geometry by V. T. Wilson; Descriptive Geometry for Engineering Students by J. A. Moyer; Elements of Descriptive Geometry by C. E. Ferris.

Grateful acknowledgment is also made to the following persons for assistance and helpful criticism:

Prof. Dexter S. Kimball and Prof. John T. Williams, Sibley College, Cornell University; Prof. James G. White, State University of Kentucky; and Prof. George W. Lewis, Swarthmore College.

CONTENTS

CHAPTER III

Point and Line Problems

CHAPTER IV

LINE AND PLANE PROBLEMS

CHAPTER V

SOLIDS

SECTION PAGE

CHAPTER VI

TANGENT PLANES AND DOUBLE-CURVED SURFACES OF REVOLUTION

CHAPTER VII

SECTIONS

SECTION PAGE

CHAPTER VIII

INTERSECTIONS AND DEVELOPMENTS

SECTION PAGE

CHAPTER IX

Isometric Projections

CHAPTER X

Set of Drawing Exercises in Descriptive Geometry

APPENDIX A

Drawing-room System

APPENDIX B

ELEMENTS OF DESCRIPTIVE GEOMETRY

CHAPTER I

INTRODUCTION

1. **Descriptive Geometry** is that branch of geometry which deals with the solution of problems involving points, lines, planes, and solids; also the methods of *representing* these problems and their solutions upon a sheet of drawing paper in such a way that the *position* of points, the *position and length* of lines, the *position, length, and breadth* of planes, and the *position, length, breadth, and thickness* of solids can be determined with unfailing accuracy.

2. **The practical value of Descriptive Geometry** lies in the knowledge gained in solving graphical problems which arise in engineering and architecture, and in making and reading working drawings; also, in the sense of accuracy acquired, and in the training of the mind to analyze, and the imagination to visualize, graphical problems. **The power to visualize a problem** consists in being able to call up a *mental picture* that will enable the draftsman to see all the parts of the problem in their true geometrical relations *without* the aid of a model. This power is characteristic of all successful inventors, designers, and constructive engineers, and the student should endeavor to develop along this line by *visualizing* each problem presented.

3. The latter part of the definition of Descriptive Geometry (see § 1) is very important, since it indicates the **properties of the data** dealt with in the subject: that is, the only geometric property of a *point* is its position, and therefore it is entirely defined when its position in space is known; a *line* has not only

position, but also length; a *plane* has position, length, and breadth; while a *solid* has position, length, breadth, and width.

4. **Position** indicates the *exact* location of an object in space, and the simplest way to give the position of an object is to state the *distance and direction* it is from a second object, whose position is *always* known.

5. **EXPERIMENT I.** Cut and letter a piece of drawing paper as shown in Fig. 1.

Crease the paper along the lines G–L and G_1–L_1 and glue the sides with gummed stickers to form the "solid angle" shown in Fig. 2. Secure a fine plumb line with a small "bob"; tie a knot D in the line so that it will be *exactly* three inches from the point of the bob, *after* the knot E has been tied. Other knots are then tied to correspond to A, B, and C.

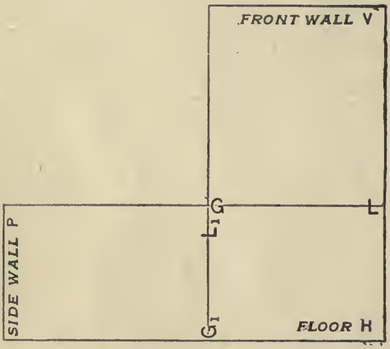


Fig. 1. — Paper Cut and Marked for Experiment 1.

Hold the solid angle formed by the paper so that the "front wall" and "side wall" are vertical and the "floor" is horizontal, or in the position of an ordinary floor. Then, *facing* the *front* wall (this places the side wall to the left), hold the plumb line so that it is located according to the dimensions shown in Fig. 2. If instructions have been accurately followed the plumb line will be *perpendicular* to and the point of the bob will just touch the "floor." Mark this point d where the bob touches the "floor" and note that it marks the *foot of a perpendicular* (the plumb line) to a horizontal plane (the "floor") and through the point D in space. The point d on the "floor" is called the **horizontal projection** of the point *D in space.*

Next, with the plumb line still in position, pass the wire No. 1 through the point D (the knot) in such a manner that it will just touch the "front-wall" and be *perpendicular* to it. This point where the wire touches the " front-wall " is marked d', and it is termed the **vertical projection** of the point D in space. In a similar manner, with wire No. 2, locate and mark the point d_1' on the side wall. The point d_1' is also termed a vertical projection, because it, too, marks the foot of a perpendicular to a vertical plane and through the point D in space. At this stage of the experiment note that **the number of vertical projections** a point in space may have depends *only* upon the *number of vertical planes* considered. If only the front plane is considered, the point being projected has one vertical and one horizontal projection. When the " side wall " is also considered the point has two vertical projections (d' and d_1') but only one horizontal (d) projection. A third vertical wall might have existed on the right, in which case a third vertical projection could have been located on this wall and so on for *any number* of vertical planes. Note especially that the vertical projection shows *how far* the point in space is above the horizontal plane (the floor). Hence, all *vertical projections* of the *same point* on different vertical planes are the *same distance* from their respective *G–L* lines, because these lines lie in both the vertical and horizontal planes.

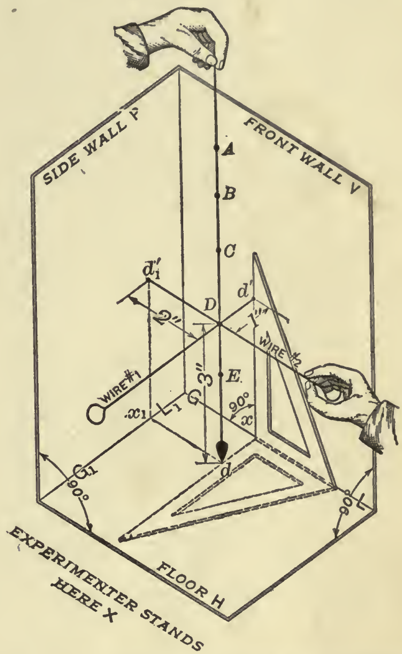


Fig. 2. — To Illustrate the Principles of Orthographic Projection.

With wires No. 1 and No. 2 still in position (that is, through the point D in space and *perpendicular* respectively to the front and side walls) observe that *three* measurements must be made to *definitely* locate the point D in the paper angle.

For example, suppose the point D to be an electric light bulb which is to occupy some *exact* position over a desk in a room, it will be necessary to instruct the workman *exactly* how far the light is to be from the side wall, the front wall, and the floor. These measurements are always understood to be the shortest distances, i.e., the *perpendicular* distances, and *all three* must be known in order to fix the point where the light is to hang. To illustrate, unless the distance *D–d* is fixed, the bulb could occupy *any* position on the plumb line such as *A*, *B*, *C*, or *E*, and its distance from the floor would *not* be fixed, although it would be the proper distance from both vertical walls. Similarly, unless the distance of the bulb from the front wall is fixed it might be the proper distance from the side wall and floor, but could occupy *any* position whatever along the wire No. 1. Similar reasoning applies to wire No. 2.

Next observe that the following principle is true, namely the horizontal and *one* vertical *projection* of the point *D* would fix *definitely* its position in the paper angle; this is due to the fact that the horizontal projection shows how far the point is from *both vertical* planes (the front and side walls) and *either* one of the vertical projections shows how far the point *D* is from the horizontal plane.

Again referring to Fig. 2, place a triangle against the front wall as shown and draw a dashed line *d′–x* from the point *d′* to the floor; then rotate the triangle to the "floor" so it lies in the dotted position shown and draw the dashed line *d–x* from the point *d* to the front wall. If this has been properly done the dashed lines *d′–x* and *d–x* meet at the point *x* on *G–L* and each is perpendicular to *G–L*. Also the dashed line *d′–x* drawn on the front wall is *exactly equal and parallel to the line D–d* in space, and the line *d–x* drawn on the floor is exactly equal and parallel to the part *D–d′* of the wire No. 1. The dashed lines d_1'–x_1 and *d*–x_1 are similarly drawn and are equal and parallel to their corresponding line in space.

Next remove the plumb line and wires, open the paper angle, spreading it out flat on the drawing board as it was originally in Fig. 1. The result is shown in Fig. 3 which is a graphic record or *Descriptive Geometry drawing* of all that has taken place. If the drawing as shown in Fig. 3 had been made *first*, the position of the point *D* in space could have been fixed by reversing the process. The operation by which the vertical and horizontal projections of the point *D* were found and the drawing produced is termed **Orthographic** (or **perpendicular**) **projection**, and is the method employed by the draftsman in making mechanical drawings.

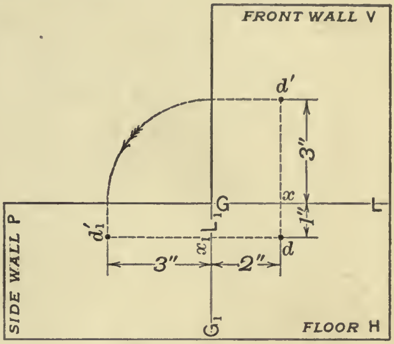


Fig. 3. — Descriptive Geometry Drawing of Data of Fig. 2.

In making a Descriptive Geometry drawing of this data in Fig. 2 the rectangle to the left of G_1–L_1 made to represent the side wall, in Fig. 3, would be omitted. *The left-hand border line of the drawing paper would serve as the line G_1–L_1 for measuring off the distance 2 inches shown in Fig. 2* and the vertical projection d_1' gives no information *not* contained on the front wall. That is, both vertical projections simply tell that the point in space is 3 inches above the floor.

6. The expression "in space" is constantly used in Descriptive Geometry and in general means that the object (spoken of as being "in space") does not lie *on or coincide with* the planes of projection (i.e., the "walls" or "floor"). Thus, the point *D* in Fig. 2 is "in space" but the projections d_1' and *d* are in the vertical and horizontal *planes* of projection. Any *material object* such as the plumb line and the wire projectors used in the experiment are considered as existing "in space," but the dashed lines *d–x*, *d′–x*, etc., *ruled on the paper*, are spoken of as being *in* the planes of projection.

7. The expression "projecting a point in space" means that a *shadow of the point* has been thrown forward upon the plane of projection by a single ray of light from the eye, somewhat after the manner that a lantern projects the picture from a slide upon a screen. Note especially that *points*, not objects, are projected by a *single* ray of light and that this ray must always be

perpendicular to the plane upon which the point is to be projected. Figure 4 illustrates how a line $A-B$ might be projected from the slide to a screen by a lantern, and Fig. 5 (a) illustrates how it would be *orthographically* projected by the eye, upon the vertical plane of projection. In Fig. 4 the rays of light *diverge* and form a triangular sheet of light with its base on the screen, but in Fig. 5 (a) a *single* ray of light first projects the point A to a' and the *position* of the eye must then be changed so as to project B to b' and the straight line connecting a' and b' is the *projection* of the line $A-B$ on the vertical plane.

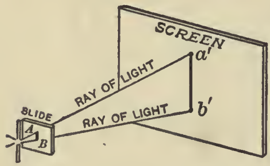
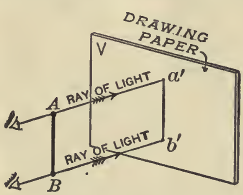


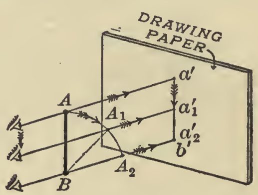
Fig. 4. — Projecting a Line by Divergent Rays.

Assume that in Fig. 5 the points in space (A and B) are the same distance from the vertical plane V, then the line $A-B$ in space is *parallel* to V, the ray of light $A-a'$ is equal to $B-b'$, and the two rays are also parallel because both are perpendicular to V.

Hence $a'-b'$ is *equal* to $A-B$ or this truth has been established: namely, **if a line in space is parallel to a plane of projection its projection on that plane is equal in length to the line itself.**



(a) Projecting a Line by Parallel Rays.



(b) How the Projection of a Line is affected by Revolution.

Fig. 5.

In Fig. 5 (b) assume that $A-B$ is the radius of a circle, B being the center. Revolve A about B and in the plane $A-a'-b'-B$ until the radius $A-B$ occupies the position A_2-B, that is, until it is perpendicular to the V plane, and note the effect. When the point A in space assumes such a position as A_1 the eye would be in the second position shown and the vertical projection of the line A_1-B would be $a_1'-b'$. When the point A reached A_2 both A_2 and B would be projected by the same ray of light, and the line A_2-B would be projected as a point (marked $a_2'-b'$

for convenience) on the V plane. This serves to illustrate the principle that **the maximum projection of a line in space is its own length,** and occurs when the line is *parallel* to the plane of projection. **The minimum projection that a line in space can have is a point,** and occurs when the line is *perpendicular* to the plane on which it is projected.

CHAPTER II

DEFINITIONS, NOTATION, AND FIRST PRINCIPLES, PROBLEMS ON THE POINT

8. **Principal Planes of Projection.** As stated in § 4, page 2, the position of any object can be fixed by giving the distance and direction it is from some *other* object the position of which is always known.

The *standard* objects from which positions (that is, distances and directions) are given in Descriptive Geometry, consist of two planes at right angles to each other. These are termed the planes of projection and they always occupy a *fixed* position in space.

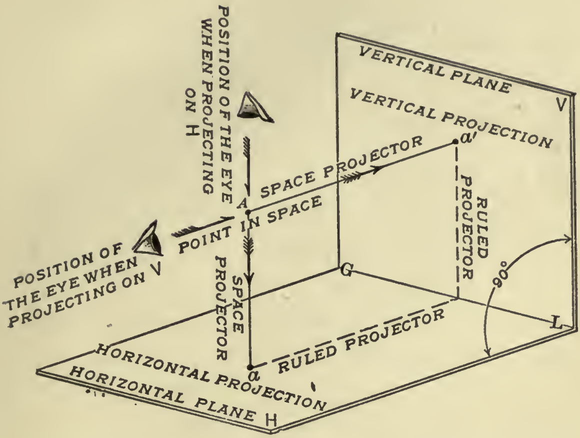
9. One of these planes is termed the **horizontal plane** of projection, because its position in space is always horizontal and for brevity it is designated as H.

10. The other plane is at right angles to H and is termed the **vertical plane** of projection because its position in space (being at right angles to H) is always vertical. This plane is designated as V.

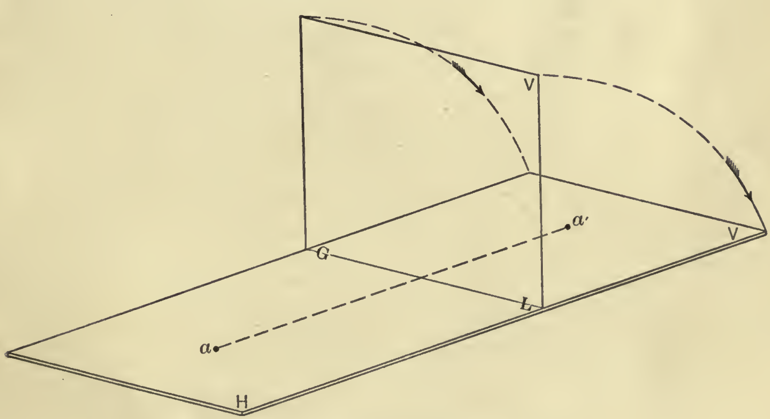
11. The line of intersection of these two planes (V and H) is termed the **ground line** and is designated as *G–L.*

12. A line [real, as the wire in Fig. 2; or imaginary, as the ray of light in Fig. 6 (a)] passing through a point in space and perpendicular to a plane of projection is termed a **space projector.**

13. The point where a space projector *pierces* V is termed the **vertical projection** of all points in space, through which the space projector passes. The vertical projection shows *how far* the point in space is from H [see Fig. 6 (a)]. Thus the point a', where the ray of light from the eye (perpendicular to V) pierces V, is the vertical projection of the point *A* which is in space.



(a) To Project a Point which is in Space.



(b) The Projection of the Point recorded on Paper.

Fig. 6. — Method of Projecting a Point.

14. The point where a space projector *pierces* H is termed the **horizontal projection** of all points in space through which the space projector passes. The horizontal projection shows *how far* the point in space is from V [see Fig. 6 (a)].

The point *a*, where a space projector (perpendicular to H) pierces H, is the horizontal projection of *A*.

15. A straight dashed line ruled on the drawing paper perpendicular to *G–L* and containing the vertical and the horizontal projections of a point is termed a **ruled projector.** If the paper is creased along *G–L* and held in the position of V and H [see Fig. 6 (a)], *each ruled projector is equal and parallel to a space projector which is directly opposite* and for this reason can *represent* space projectors on the drawing.

16. **Orthographic projection** is a method of projecting points in space upon planes of projection (V and H), by lines (space projectors) perpendicular to these planes. By projecting a sufficient number of points of an object an accurate drawing or *projection* of the object can be made. Projecting by the use of *space projectors* is usually an imaginary operation [see Fig. 6 (a)], and the real process [see Fig. 6 (b)] consists in dividing the drawing paper into two parts, by a straight line to represent *G–L*; all the area of the paper on one side of *G–L* being considered as if in V and all on the other side as if in H, and the V and H projections of the points are located in these areas respectively and on *ruled* projectors drawn perpendicular to *G–L.*

17. **THEOREM I. The vertical and the horizontal projection of a point determines its position in space with reference to V and H.**

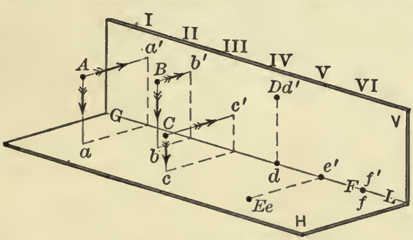
Proof. — If a perpendicular to V is passed through the vertical projection of the point it will pass through the point in space. If a perpendicular to H is passed through the horizontal projection of the point, it, also, will pass through the point in space.

Hence, as the point in space is common to *both* perpendiculars it must be at their intersection. These perpendiculars coincide with the space projectors and the point is fully determined.

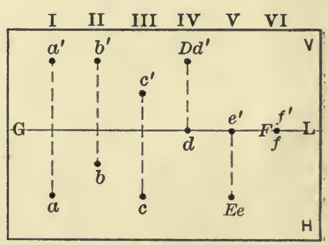
18. Corollary I. The vertical and horizontal projections of a point are on one and the same straight line (the ruled projector), perpendicular to the ground line (see Fig. 6 (b), page 9), because the ruled projector from the vertical projection is perpendicular to H and therefore to *G–L*; the ruled projector from the horizontal projection is perpendicular to V and therefore to *G–L*. The ruled projectors are parallel and equal to the space projectors, and these two sets of lines form a rectangle, and hence the ruled projections must meet at a point. The only point possible is on *G–L* and when V and H are folded into one plane [see Fig. 6 (b)], the ruled projectors drawn on V and H fall 180 degrees apart, hence lie in the same straight line.

19. Corollary II. A point situated upon either plane of projection is its own projection on that plane, and its other projection is in *G–L* [see Fig. 7, IV, V, and VI, this page].

20. A point in space may occupy any one of six different positions with reference to V and H [see Fig. 7 (a)].



(a) The Point Itself.



(b) Projections.

Fig. 7.—Positions which a Point may Occupy.

(1) It may be the same distance from both V and H [see Fig. 7 (a)–I].

(2) It may be nearer to V than H [see Fig. 7 (a)–II].

(3) It may be nearer to H than V [see Fig. 7 (a)–III].

(4) It may lie in V [see Fig. 7 (a)–IV].

(5) It may lie in H [see Fig. 7 (a)–V].

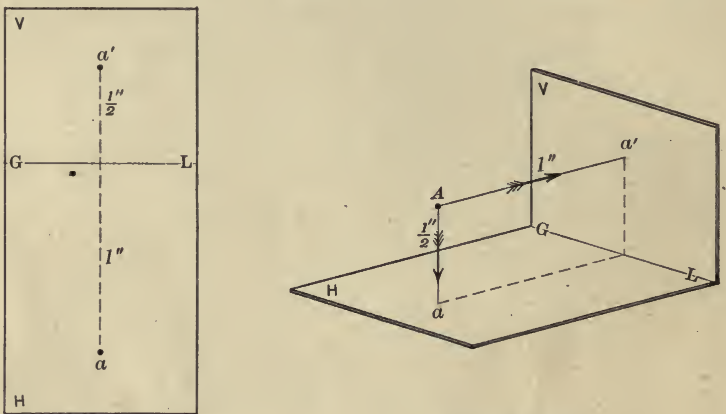
(6) It may lie in V and H (therefore in *G–L*) [see Fig. 7 (a)–VI.]

Figure 7 (b) illustrates the method of representing each of the six cases on the drawing paper.

21. PROBLEM 1. Given the position of a point in space to determine its projections.

Let the point be $\frac{1}{2}$ inch from H and 1 inch from V [see Fig. 8 (b)].

Analysis. The vertical projection shows the perpendicular distance that the point in space is from H; the horizontal pro-



(a) Projections. **(b) The Point Itself.**

Fig. 8. — To determine a Point by its Projections.

jection shows the perpendicular distance it is from V, and both projections must lie on the same straight line perpendicular to *G–L*.

Construction. See Fig. 8 (a). Draw an indefinite horizontal line to represent *G–L*; draw an indefinite line perpendicular to *G–L* (ordinate) and lay off a point $\frac{1}{2}$ inch above *G–L* for the vertical projection and a point 1 inch below *G–L* for the horizontal projection.

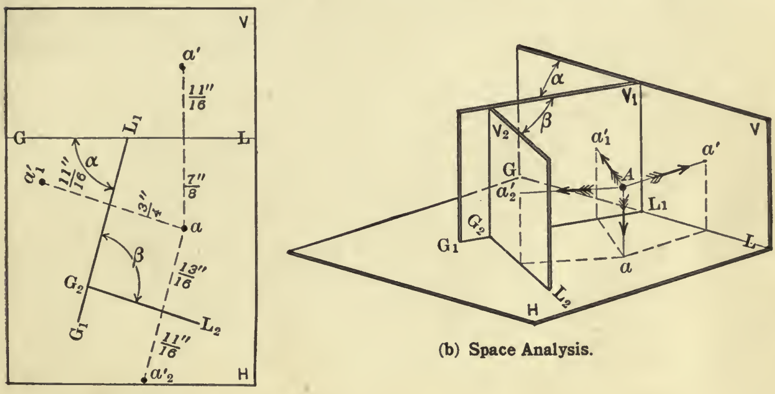
Line Conventions. The *ground line* (*G–L*) is a light unbroken line. The *ruled projector* connecting the vertical and horizontal projections is composed of a series of dashes $\frac{1}{16}$ inch long, separated by $\frac{1}{32}$ inch spaces and is light "weight."

Notation. The ground line has *G* written at one extremity and *L* at the other.

The point *in space* is designated by a capital letter (A in this case); its vertical projection by a' and its horizontal projection by a.

22. PROBLEM 2. Given the projections of a point on V and H to find its new vertical projection on any new vertical plane.

Analysis. The same horizontal projection serves for any number of new vertical planes (Experiment I, page 2). By § 18, page 11, the horizontal projection and the new vertical projection must lie on the *same* straight line, perpendicular to the new ground line G_1–L_1. By Experiment I, also Fig. 9 (b) the



(a) Projections.

(b) Space Analysis.

Fig. 9. — To determine New Vertical Projections on a New Vertical Plane.

new vertical projection on any new vertical plane, such as V_1 or V_2, will be the same distance from the new ground lines G_1–L_1 and G_2–L_2 as the original vertical projection was from the original G–L.

Construction. See Fig. 9 (a). Draw the indefinite straight line G–L to divide the drawing paper into two parts, the area above G–L to represent V revolved back into the plane of the drawing board, and the area below G–L to represent H. Draw an indefinite straight line perpendicular to G–L to serve as the ruled projector for the projections of the point on V and H. Assume that the point A in space is $\frac{11}{16}$ inch from H and $\frac{7}{8}$ inch from V. The vertical projection is then found by measuring $\frac{11}{16}$ inch *above* G–L on the ruled projector. The horizontal projection is found by measuring $\frac{7}{8}$ inch *below* G–L on the *same* ruled projector.

To draw G_1–L_1 first determine the point L_1 (this would be given in the problem), where the new vertical plane V_1 meets V and then draw G_1–L_1 so that it makes the same angle (α) with G–L that the plane V_1 makes with V. From a, the horizontal projection of A, draw a line of indefinite length and perpendicular to G_1–L_1. This is the ruled projector containing the vertical projection of A on V_1. Measure off $\frac{11}{16}$ inch (the distance the point A is from H) from G_1–L_1 on this projector and a_1' is the *new* vertical projection sought. The vertical projection on the plane V_2 is found in a similar manner.

NOTE. Observe that the distance ($\frac{11}{16}$ inch) which the vertical projections are above their respective ground lines is the same in each case, because the same H serves for V, V_1, and V_2. The distances, however, from the horizontal projection a to G–L, G_1–L_1, and G_2–L_2 are different because the perpendicular distance from the point A in space to the planes V, V_1, and V_2 is different. Let no confusion arise from the fact that a_1' and a_2' are located on a portion of the paper that was *originally* set aside as H. When the plane V_1 was folded *away* from the point A in space to bring V_1 into the plane of the paper, a_1' fell at the place indicated, then all the surface of the paper on the left of G_1–L_1 became vertical plane V_1 area *when working with* G_1–L_1; but it represents H when working with G–L. When V_2 was folded *away* from the point A in space into the plane, of the paper, it also fell on the *original* H and the surface *below* G_2–L_2 became vertical plane area when working with V_2 and G_2–L_2. *Do not pass over this note without understanding it thoroughly, as the principles involved are of constant recurrence.*

Notation. The *original* or principal vertical plane is designated as V, the *first new* vertical plane as V_1, the second V_2, etc. The lines marking the intersection of these new planes with H are notated as G_1–L_1, G_2–L_2, etc., and the new vertical projections on these planes are primed as on V and the sub-number of the plane (1, 2, etc.) is added. Thus a_1' is the vertical projection of A on V_1 and a_2' is the vertical projection of the same point on V_2, etc. Any number of new vertical planes can thus be used.

23. The data for problems in Descriptive Geometry consist in the location of isolated points in space and of points which limit the dimensions of lines, planes, and solids. As a **point** has no dimensions it is fully specified by its position. **A line** is fully specified by its position and length; **a plane** by its position, length, and width; and **a solid** by its position, length, width, and height.

Notice very carefully that the *position* and *length* of a straight line are determined by the position of its two ends; that the *position* of a plane is determined by a straight line and a point (*or by three points only*) which lie in the plane, and its length and width are determined by several limiting lines; that the position and dimensions of solids are determined by the position and dimensions of the planes outlining their form. Hence it is seen that *all the data* used in Descriptive Geometry may be stated in terms of the position of *points*, and whether thus stated or not it will be finally used in this form and *every principle involved in the Descriptive Geometry of the point must be understood thoroughly.*

24. The Four Dihedral Angles. Thus far all points and lines in space have been made to occupy a position in *front* of V and

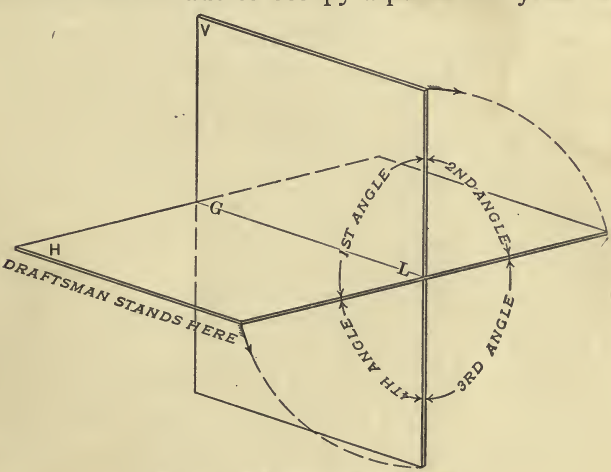


Fig. 10. — The Four Dihedral Angles.

above H, and no mention has been made of points lying behind V and below H. In order to have the methods of Descriptive

Geometry general, they must deal with data which may involve *any* position whatever in space, and this is accomplished by considering V to extend below H, and H to extend beyond V, thus forming four *equal* dihedral angles (see Fig. 10, page 15) and numbered 1st, 2d, 3d, 4th, in the order shown.

The draftsman always views an object from the first angle (that is, the eye is always above H and in front of V) regardless of the angle in which that object is located.

The 1st angle is *above* the horizontal, and *in front* of the vertical plane.

The 2d angle is *above* the horizontal, and *behind* the vertical plane.

The 3d angle is *below* the horizontal, and *behind* the vertical plane.

The 4th angle is *below* the horizontal, and *in front* of the vertical plane.

Figure 11 shows dimensioned planes which can be cut out of cardboard, or very thin celluloid, and used as a model in studying

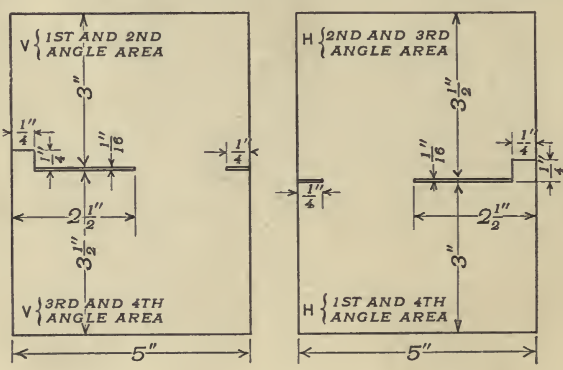


Fig. 11. — Layout for "Planes of Reference."

problems involving several angles. Figure 12 shows this assembled model viewed *edgewise* with points *A*, *B*, *C*, *D*, located in the 1st, 2d, 3d, and 4th angles respectively. As stated above **for all angles the position of the eye** (as indicated by the arrows) is always *in front* of V in determining *vertical* projections, and *above* H in determining *horizontal* projections. Therefore for each point projected the ray of light (or space projector) passes from the eye *through* V for vertical projections in the second angle, *through* H for horizontal projections in the fourth angle, and through *both* V and H for the projections in the third angle. In order to represent the vertical and horizontal projections on the same sheet of paper the planes that stand at right angles to one another in space must be brought into a single plane. This is accomplished by *opening out* the first angle as shown in Fig. 6 (b), page 9, and this would open the third angle and close the second and fourth, so that in the second angle V and H both fall *above* *G–L* and in the fourth both fall *below* *G–L*. The plane H falls above *G–L* in the third angle while V falls below *G–L*.

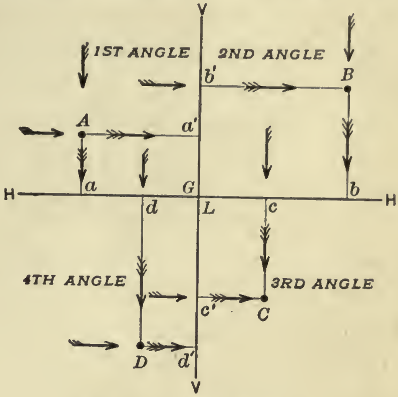


Fig. 12. — Viewing the Point.

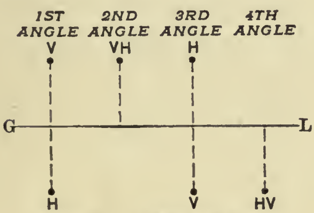


Fig. 12. — Viewing the Point.

The positions of the vertical and horizontal projections relative to *G–L* for the different angles are as shown in the accompanying figure. Also the points *A*, *B*, *C*, and *D* of Fig. 12 are shown in projection in Fig. 13.

With the *single exception* that the vertical and horizontal projections may be located, on *different* sides of *G–L* for the various angles, *all principles established for the first angle apply to all other angles.*

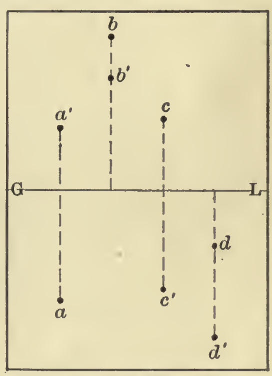


Fig. 13. — Projection of a Point which lies in Each Angle.

25. General Notation. The following notation is *general* and special notation will be given as required.

Points in space are designated by *capital letters*, beginning with *A* and continuing through the first part of the alphabet, thus *A*, *B*, *C*, *D*, etc.

Horizontal projections of points are designated by the *small letters* corresponding to the capital letters used to designate the points being projected, thus, *a*, *b*, *c*, *d*, etc.

Vertical projections of points are designated in the same manner as horizontal projections *except* the letters are primed, thus, *a′*, *b′*, *c′*, *d′*, etc.

Lines in space are designated, in general, by the two points which mark their extremities and the two letters designating the line are separated by a dash, thus, *A–B*, *B–C*, *C–A*, etc.

The horizontal projections of lines are designated by the *small letters* corresponding to the capital letters designating the lines in space and are separated by a dash thus, *a–b*, *b–c*, *c–a*, etc.

The vertical projections of lines are designated in the same manner as horizontal projections *except* the letters are primed, thus, *a′–b′*, *b′–c′*, *c′–a′*, etc.

26. Line Conventions. The following line conventions cover the *ordinary* cases and special conventions will be given as required.

The ground line is an *unbroken* line of *light weight* with the capital G printed at one extremity and the capital L at the other, thus: G————————————————L

A ruled projector is a *broken* line of *light weight* and is made up of a series of dashes $\frac{1}{16}$ inch long separated by spaces $\frac{1}{32}$ inch long, thus: - - - - - - - - - - - - - - - - - -

A visible line, given or required, is an *unbroken* line of *medium weight,* thus: ————————————

Construction lines used in the solution of a problem are drawn *broken* and of *light weight* and are of a long and short dash construction, the long dash being $\frac{1}{4}$ inch, the short dash being $\frac{1}{16}$ inch, and all spaces being $\frac{1}{16}$ inch, thus:

—— - —— - —— - —— - ——

All invisible lines are *broken* and are of *medium* weight. They are made up of $\frac{1}{8}$ inch dashes separated by $\frac{1}{16}$ inch, thus:

The path traced by a point is indicated by a broken line of *light weight.* The line is composed of dashes $\frac{1}{4}$ inch long separated by spaces $\frac{1}{16}$ inch long, and a *half* arrow shows the direction of motion, thus:

The direction of sight is indicated by a *full* arrow thus:

CHAPTER III

POINT AND LINE PROBLEMS

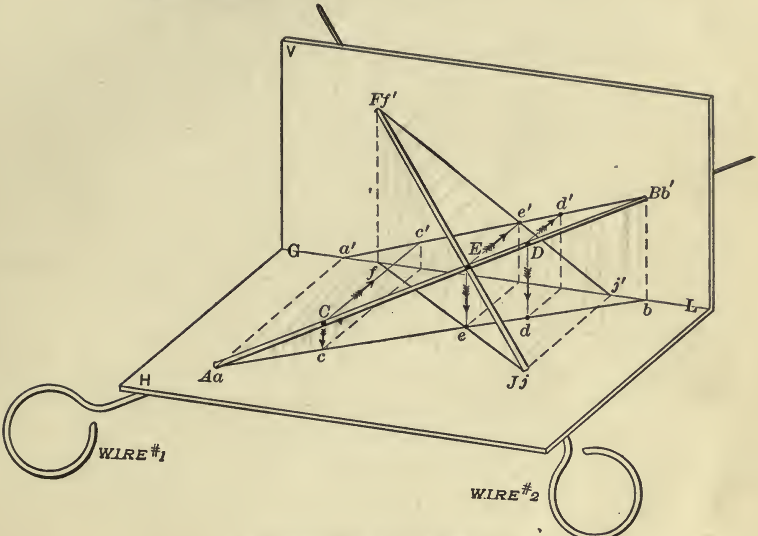
27. **EXPERIMENT II.** Crease, letter, and fold a piece of drawing paper to represent V and H in space. Thrust wire No. 1 through V and H so that it lies across the angle as shown in Fig. 14 (a), call the point where the wire *pierces* H the **horizontal piercing point** of the wire and mark it *Aa*. The vertical projection *a′* of this horizontal piercing point *Aa* is necessarily *G–L*. Call the point where the wire *pierces* V the **vertical piercing point** and mark it *Bb′*. The horizontal projection *b* of this vertical piercing point *Bb′* is in *G–L*. The line connecting *Aa* with *b* is the **horizontal projection** of the wire and the line connecting *a′* and *Bb′* is the **vertical projection** of the wire.

Next thrust wire No. 2 through H at about *Jj* and sliding it along wire No. 1 so that the two wires intersect (or touch) at about *E* thrust the wire to pierce V, and mark this piercing point *Ff′*. Draw, as in the case of wire No. 1, the vertical and horizontal projections of wire No. 2. Determine and mark the projections of the point of intersection *E* of the two wires also, determine the projections of any other two points, as *C* and *D*, on wire No. 1.

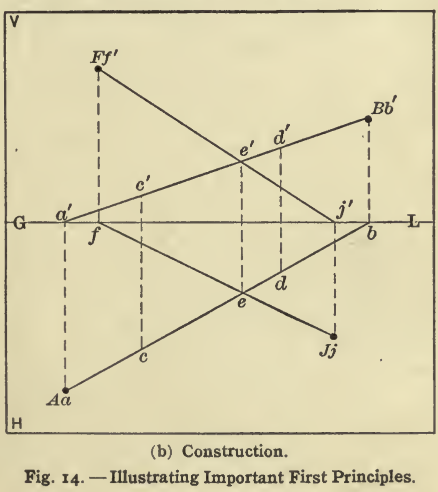
Remove the wires, fold V backward into the plane of the drawing board (that is, into the plane of H) and if instructions have been accurately followed the result will be as shown in Fig. 14 (b). By referring to Figs. 14 (a) and (b), the following **important principles** are evident.

1. Lines in space can be represented on a drawing by their projections.
2. A single line in space such as *C–D* requires two projections, *c′–d′* and *c–d* (the vertical and horizontal), to define its position.
3. Since a straight line in space is determined by two of its points, so the projections of such a line are determined by the projections of two points of the line.

Thus the line *C–D* in space is fully determined by the points *C* and *D* and the projections of the line are determined by the projections *c*, *c′* and *d*, *d′* of these points.



(a) Perspective of Experiment II.



(b) Construction.

Fig. 14. —Illustrating Important First Principles.

4. Since two points determine a straight line *in space*, a line may be determined by its intersections with the planes of projection, that is, by determining its hori-

zontal and vertical piercing points. A line can have only two piercing points [see the line *A–B*, Fig. 14 (a)]; but if parallel to *one* plane of projection it will have but one piercing point, and if parallel to *G–L* will have none.

5. If a point such as *C* or *D* is on a line such as *A–B* the projections *c* and *c′* or *d* and *d′* will be on the projections *Aa–b* and *a′–Bb′* of the line.
6. If any two lines, as *A–B* and *J–F*, intersect at some point in space, their vertical projections will intersect at *e′*, and their horizontal projections at *e*, as these are the vertical and horizontal projections of the *same* point *E* in space, and the ruled projector joining *e′* and *e* must be perpendicular to *G–L*.
7. The horizontal piercing point *Aa*, *a′* of any line *A–B* is found by producing its vertical projection *Bb′–a′* until it meets *G–L*. At this junction *a′* draw a perpendicular *a′–aA* (to *G–L*) which will cut the horizontal projection *b–Aa* of the line at the horizontal piercing point.
8. The vertical piercing point *Bb′*, *b* of any line *A–B* is found by producing its horizontal projection *Aa–b* until it cuts *G–L* at *b*. From *b* draw a perpendicular *b–Bb′* (to *G–L*) which will cut the vertical projection *a′–Bb′* of the line at the vertical piercing point.
9. The point of intersection *E*, of two lines *A–B* and *F–J* in space, has its projections *e* and *e′* at the intersection of the projections of the line.
10. The piercing point *Aa*, *a′* or *Bb′*, *b* (or the trace) of a line is the point of intersection of the line *A–B* in space with a plane such as V or H. (See problem 3, page 26, for further discussion of piercing point.)

28. Possible Positions which a Line may have with reference to V and H. A straight line may occupy *any one* of the following general positions in space [see Fig. 15].

(1) Parallel to both V and H. [Figs. 15 (a) and (b)–I.]

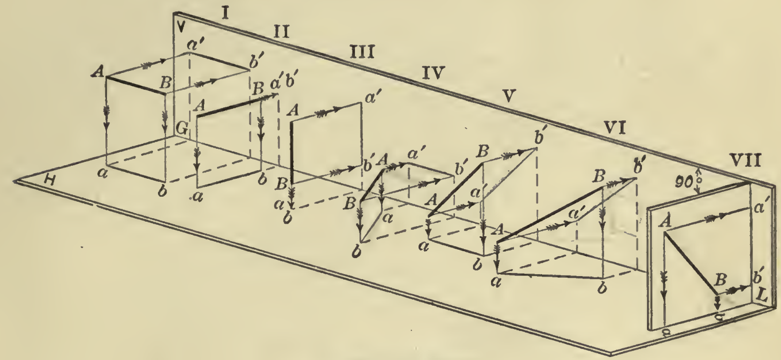
(2) Perpendicular to V (hence parallel to H). [Figs. 15 (a) and (b)–II.]

(3) Perpendicular to H (hence parallel to V). [Figs. 15 (a) and (b)–III.]

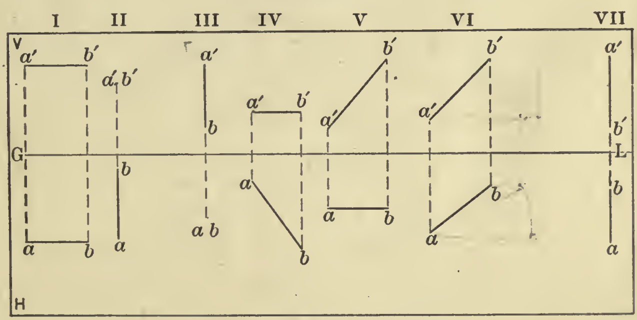
(4) Parallel to H and inclined to V. [Figs. 15 (a) and (b)–IV.]

(5) Parallel to V and inclined to H. [Figs. 15 (a) and (b)–V.]

(6) Inclined to V and H. [Figs. 15 (a) and (b)–VI.]



(a) The Line Itself.



(b) Projections of the Line.

Fig. 15.—Positions which a Line in Space may Occupy.

(7) In a plane perpendicular to V and H (that is, in a plane perpendicular to *G–L*). [Figs. 15 (a) and (b)–VII.]

The lines *in space* are shown in Fig. 15 (a), and Fig. 15 (b) illustrates the method of representing, *on a drawing*, the line in each of the above positions.

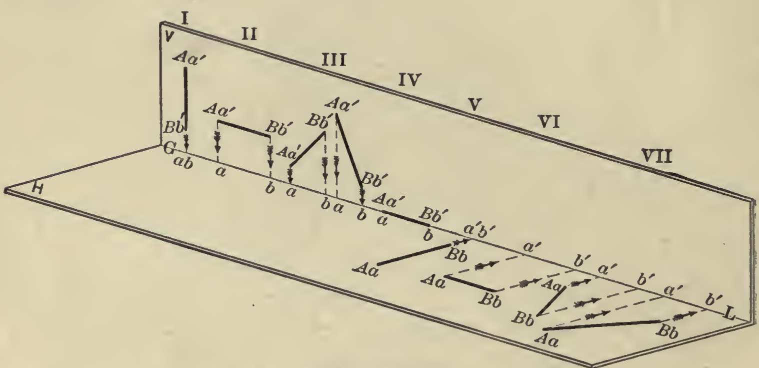
If a line does not occupy one of the above seven *general* positions *in space*, it may lie in a plane of projection as follows:

(1) It may lie in V and be perpendicular to H. [Figs. 16 (a) and (b)–I.]

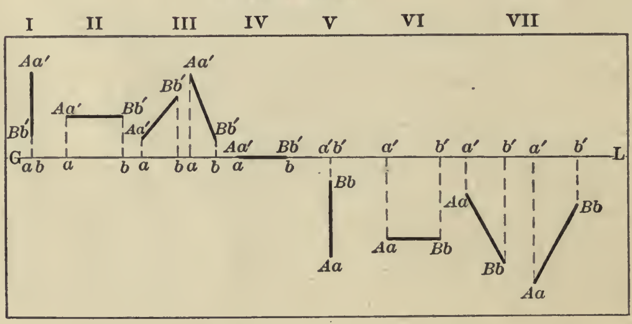
(2) It may lie in V and be parallel to H. [Figs. 16 (a) and (b)–II.]

(3) It may lie in V and be inclined to H (either direction). [Figs. 16 (a) and (b)–III.]

(4) It may lie in V and in H, that is, in *G–L*. [Figs. 16 (a) and (b)–IV.]



(a) The Line Itself.



(b) Projections of the Line.

Fig. 16.—Positions which a Line which lies on a Projection Plane may Occupy.

(5) It may lie in H and be perpendicular to V. [Figs. 16 (a) and (b)–V.]

(6) It may lie in H and be parallel to V. [Figs. 16 (a) and (b)–VI.]

(7) It may lie in H and be inclined to V (either direction). [Figs. 16 (a) and (b)–VII.]

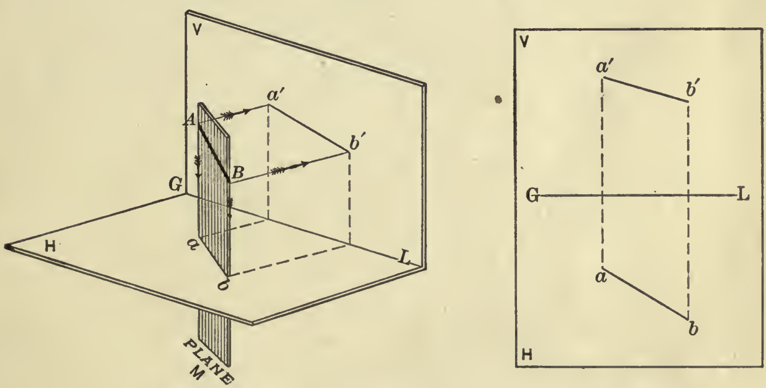
The lines *in space* are shown in Fig. 16 (a). In Fig. 16 (b) is illustrated the method of representing, *on a drawing*, the line in each of the above positions.

29. Principle. Lines in space are represented on a drawing by their projections. As both the vertical and horizontal projections of a point represent but a single point in space, so a *single* line in space requires *two* projections to define its position.

30. Principle. Two points are always necessary to *fully determine* a straight line (that is, define its length as well as its directions), but one point and its direction may *locate* the line.

31. Principle. Since a straight line in space is determined by two of its points, similarly the projections of a line are *fully determined* by the projections of two of its points, but the projection of one point and the direction may *locate* the projection of a line.

32. THEOREM II. Every point on a line in space has its horizontal projection on the horizontal projection of the line, and its vertical projection on the vertical projection of the line.



(a) Perspective. (b) Construction.

Fig. 17. — Projection of the Points of a Line.

Proof. — Let A and B [Figs. 17 (a) and (b)] determine any line in space. Pass the plane M through A–B and *perpendicular* to H. Drop the space projectors from A and B to H and determine the horizontal projections a and b. Connect a and b with a straight line.

The space projector A–a (also B–b) must lie in M because the plane M is *perpendicular* to H and contains every perpendicular that can be drawn from the line A–B to H. The projections a and b therefore lie in both M and H as also does the line a–b joining them; hence a–b is the **line of intersection** of M and H, and must contain every point common to these planes, therefore it contains the foot of every perpendicular line from A–B to H and is the horizontal projection of A–B. In the same manner a'–b' can be shown to contain the ruled projector from *every* point of the line A–B.

33. **Principle.** If two lines intersect in space, their *projections* must also intersect, and the *ruled* projector joining the points in which the two projections intersect must be perpendicular to G–L; for the intersection of two lines must be a point (common to both lines) whose projections must be on the horizontal and vertical projections of each of the lines, hence at their intersections respectively. See Figs. 18 (a) and (b), also 19 (a) and (b).

34. **PROBLEM 3. To find the piercing points (or traces) of a line.**

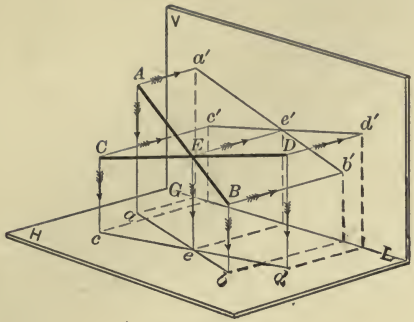
Fundamentals. The trace of a line is the point in which the line pierces V or H.

Since a trace lies in one of the planes of projection it must have one projection in G–L [see Figs. 14 (a) and (b), page 21].

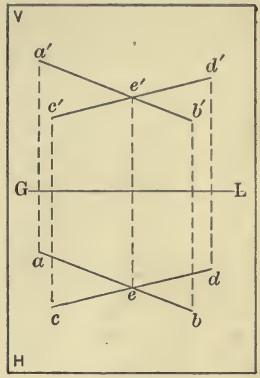
Since the projections of the traces must be on the projections of the line (see Theorem II, page 25), one projection of the trace must lie where a projection of the line cuts G–L, and the other projection, at the intersection of the ruled projector and the *other* projection of the line.

(1) *To determine where a line pierces V.*

Analysis. Move the point of a pencil along the line from A toward B [see Fig. 20 (a)], then the horizontal projection of the pencil point follows along a–b; also, the perpendicular distance from any point on the projection a–b to G–L would measure the distance the pencil point on A–B (in space) is from V when at that position. Hence when the horizontal projection of the point reaches G–L the point in space (the pencil point) is on V. And since the vertical and horizontal projection of the point must be on the *same* ruled projector, the point Cc' where the

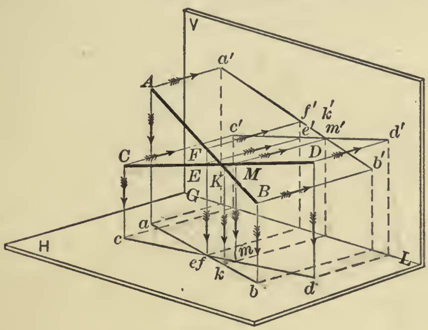


(a) Perspective.

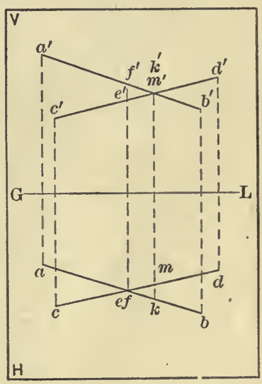


(b) Construction.

Fig. 18. — Projection of a Point Common to Two Lines.

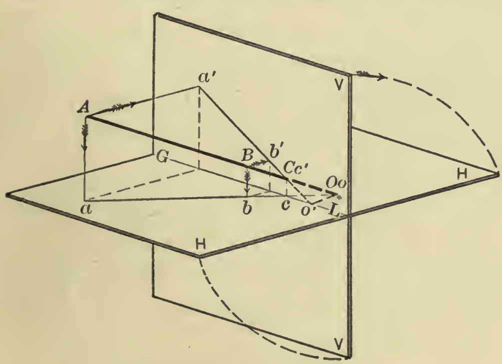


(a) Perspective.

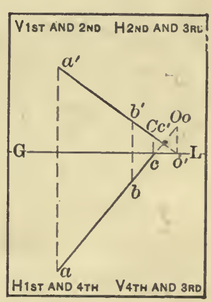


(b) Construction.

Fig. 19. — Intersecting Lines have One Point Projected in Common.



(a) Perspective.



(b) Construction.

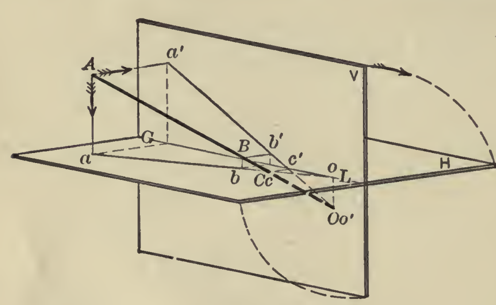
Fig. 20. — The Vertical Trace of a Line.

perpendicular from this projection *c* cuts the vertical projection *a′–b′* of the line (produced if necessary) determines the vertical trace of the line.

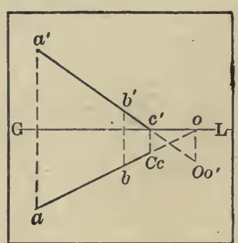
Construction. Extend *a–b* [see Fig. 20 (b)] until it cuts *G–L* at *c* and draw an indefinite perpendicular from *c* upward. Extend *a′–b′* to intersect this perpendicular at *Cc′*, the required point.

(2) *To determine where a line pierces H.*

Analysis. See Fig. 21 (a) and (b). The vertical projection *a′–b′* of the line must cut *G–L* in order that the line shall touch H.



(a) Perspective.



(b) Analysis.

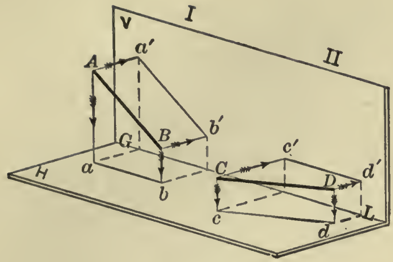
Fig. 21. — The Horizontal Trace of a Line.

Draw an indefinite perpendicular from this point *c′* and extend the horizontal projection *a–b* of the line until it intersects this ruled projector. This point of intersection *Cc* establishes the horizontal trace of the line.

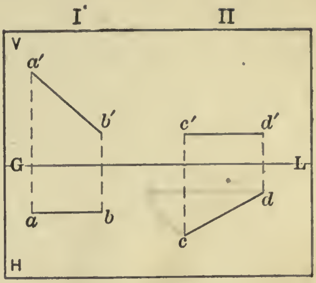
35. **THEOREM III. If a line in space is parallel to a plane of projection its projection on that plane shows the true length of the line.**

Proof. — Let *A–B* [Fig. 22 (a) and (b)–I] be parallel to V, then every point on *A–B* is the same distance from V; hence *A–a′* is equal to *B–b′*. Also as *A–a′* and *B–b′* are both space projectors to V, they must both be perpendicular to V; therefore *A–a′–b′–B* is a rectangle and *a′–b′* is equal to *A–B*.

Similarly *c–d* can be proven equal to *C–D*. [Figs. 22 (a) and (b)–II.]



(a) Perspective.



(b) Construction.

Fig. 22. — A Line Parallel to a Plane of Projection.

36. **Corollary I.** The true distance between two points in space can be found by determining the length of the line joining them.

37. **Corollary II.** The angle a true length projection makes with *G–L* is the same as the angle that the line in space makes with its *other* projection; hence the same angle as the line in space makes with the plane containing the other projection.

38. **Corollary III.** The true angle between two intersecting lines can be found by bringing both lines parallel to the same plane of projection and determining the projections of the lines.

39. **Corollary IV.** To measure off a true distance on a line, measure it off on the projection *which is parallel* to a plane of projection.

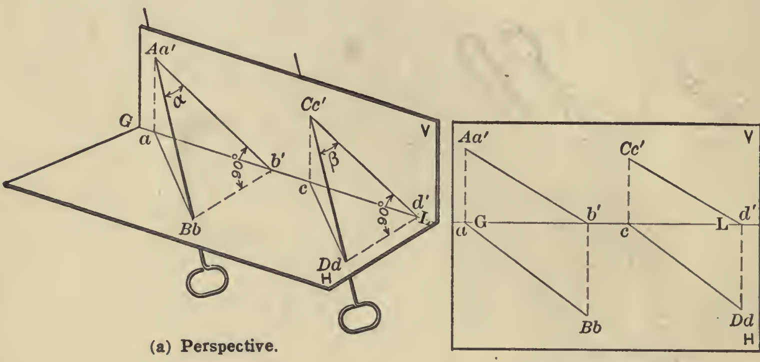
40. **THEOREM IV. If two lines in space are parallel, their vertical projections will be parallel, also their horizontal projections will be parallel.**

Proof. — Let *A–B* and *C–D* [Figs. 23 (a) and (b)] be any two lines in space which are parallel.

Determine their piercing points on V and H.

Then, by hypothesis *Aa′–Bb* [Fig. 23 (a)] is parallel to *Cc′–Dd* and by construction *Bb–b′* is parallel to *Dd–d′*; hence the angle α is equal to the angle β. By construction *Bb–b′–Aa′* = 90° = *Dd–d′–Cc′*.

Therefore the triangle *Bb–b′–Aa′* is similar to the triangle *Dd–d′–Cc′* and the side *Aa′–b′* is parallel to *Cc′–d′*. Similarly the horizontal projection *a–Bb* can be proven parallel to the horizontal projection *c–Dd*.

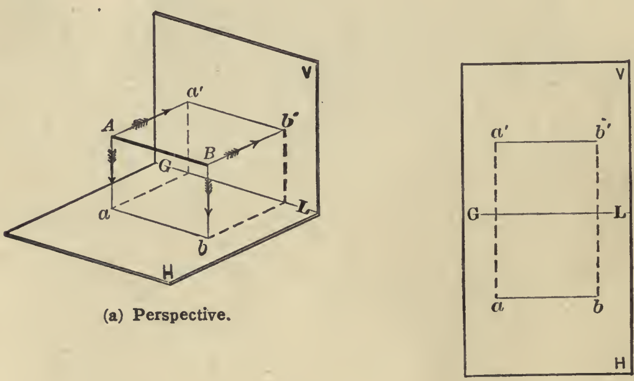


(a) Perspective.

(b) Construction.

Fig. 23. — Projection of Parallel Lines.

41. Corollary I. If a line *A–B* [Fig. 24 (a)] in space is parallel to both V and H, then both of its projections [Fig. 24 (b)] will be parallel to *G–L*.



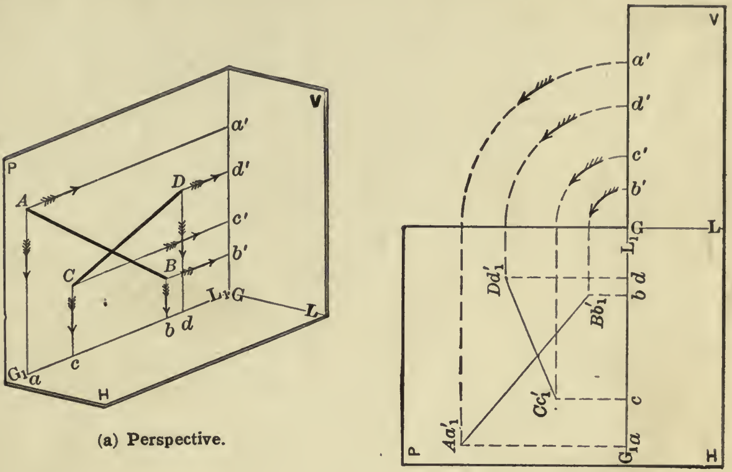
(a) Perspective.

(b) Construction.

Fig. 24. — A Line Parallel to Both V and H.

42. Corollary II. If a line in space is parallel to a plane of projection, its projection on that plane is the true length of the line in space, and its projection on the other plane will show parallel to *G–L* [see Figs. 22 (a) and (b), page 29].

NOTE. The converse of Theorem IV, namely, *two lines in space are parallel if both the vertical and horizontal projections are*



(a) Perspective.

(b) Construction.

Fig. 25. — Projection of a Line lying in a Profile Plane.

parallel, is true provided the lines do *not* lie in a **profile plane** [see Figs. 25 (a) and (b)], that is a plane perpendicular to *G–L.*

43. PROBLEM 4. To pass a line through a given point and parallel to a given line.

Analysis. The vertical projection of the line must pass through the vertical projection of the point, and the horizontal projection of the line through the horizontal projection of the point. The vertical projections of the given and required lines must be parallel, also their horizontal projections must be parallel.

Construction 1. *When both projections are inclined to V and H.* Let *e–e'* (Fig. 26) be the projections of the point in space and *a–b*, *a'–b'* be the projections of the given line. Through *e* draw *c–d*, parallel to *a–b*. Through *e'* draw *c'–d'* parallel to *a'–b'*. *c–d* and *c'–d'* are the projections of the required line.

Construction 2. *When the line lies in a plane perpendicular to G–L.* Assume a new V (shown at G_1–L_1, Fig. 27), and determine the new vertical projections e_1' of the given point E and a_1'–b_1' of the given line A–B. Through e_1' draw c_1'–d_1' and determine the position of e' on c'–d'.

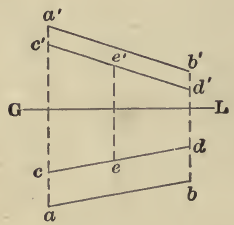


Fig. 26. — A Line passed through a Given Point and Parallel to a Given Line.

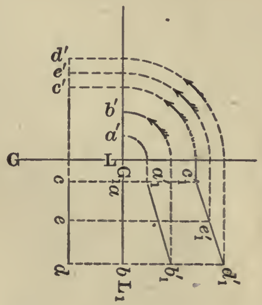
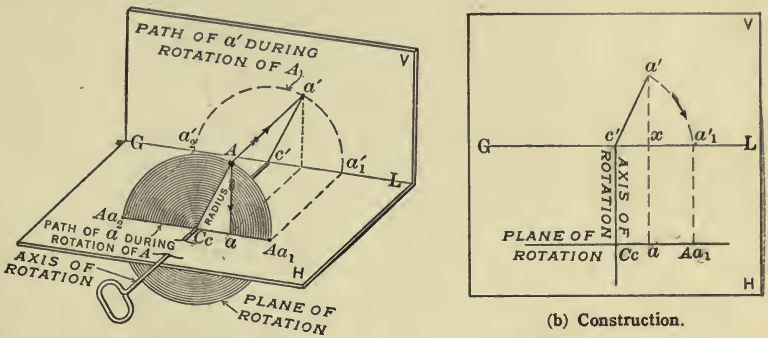


Fig. 27. — A Line passed through any Given Point and Parallel to a Given Line in a Profile Plane.

44. **EXPERIMENT III.** Crease, letter, and fold a piece of drawing paper to represent V and H in space. Cut out a circular disc of cardboard (about 2 inches in diameter) and mark a point A on its circumference. Cut a slit in H, parallel to G–L, and 2 inches long. Place the disc in the slit and with the wire through its center and perpendicular to the plane of the disc, as shown in Fig. 28 (a). The wire must now lie on H, and the disc must be parallel to V.

Place A in about the position shown and mark its vertical projection a' on V and its horizontal projection a on H; also mark the vertical and horizontal projections c' and Cc of the center point of the disc. Keeping the disc in a plane perpendicular to the wire (that is, parallel to V), revolve the disc about the wire as an axis until A coincides with H at Aa_1; during this operation the vertical projection of A moves along the arc of a circle a'–a_1' to the ground line. The horizontal projection moved along a straight line (the slit) — that is perpendicular to the axis (the wire) — and to the new position Aa_1.

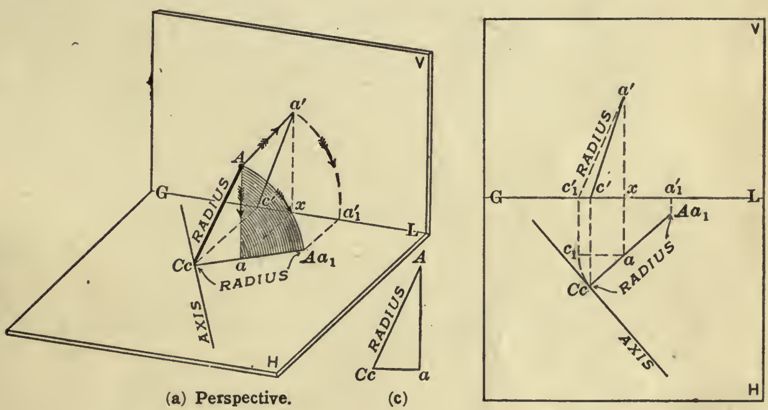
Note that the point A is always a *fixed* distance Cc–A from the wire during its revolution, and that in every position for A *this radius is always the hypothenuse of a right triangle* the two sides (Cc–a and A–a) of which are *constantly* changing. Hence, to revolve any point in space about an axis which lies in or parallel to H until the point also lies in or is parallel to this plane, it is only necessary to draw the projections of the axis [see Fig. 28 (b)],



(a) Perspective. (b) Construction.

Fig. 28. —Rotation of a Point about a Line in H and Perpendicular to V.

and through the horizontal projection a of the point draw a line a–Aa_1 perpendicular to the horizontal projection of the axis; this determines the center Cc about which A is to revolve and the horizontal projection Aa_1 (the revolved position of A) must



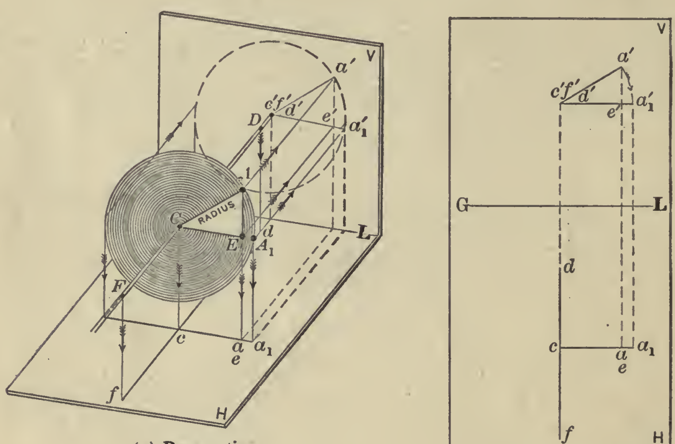
(a) Perspective. (c) (b) Construction.

Fig. 29. —Rotation of a Point about a Line in H and Inclined to V.

be on this line; the distance from Cc to Aa_1 will be the hypothenuse of the right triangle A–Cc–a (*in space*).

When the axis of rotation is perpendicular to V, as shown in Figs. 28 (a) and (b), this hypothenuse is projected in its true length on V; and hence it is only necessary to swing a' to a_1' about c' and draw its ruled projector to Aa_1 as shown in Fig. 28 (b). When the axis lies *in* H but is *not* perpendicular to V as shown in Fig. 29 (a), the hypothenuse *Cc–A* is *not* parallel to V and consequently is *not* shown in true length in c'–a'. It therefore becomes necessary to construct a right triangle [Fig. 29 (c)] with *Cc–a* as the base, and with an altitude equal to *x–a'*. In Fig. 29 (b), an equivalent of this construction has been carried out by revolving *Cc–a* parallel to *G–L*; but this triangle might have been constructed at any convenient position as its only use is to determine the **radius of rotation** which is then marked off from *Cc* to determine Aa_1.

Figure 30 shows the method of revolving the point when the axis is *parallel* to but not *in* the plane H. Note especially that



(a) Perspective.

(c) Construction.

Fig. 30. — Rotation of a Point about a Line in Space which is Perpendicular to One of the Planes of Projection.

in this case the *altitude A–E* of the triangle *C–A–E* becomes the *difference* between the *original* distance *A–a* which the point *A* in space is from H, and the distance A_1–a_1 after the radius has been brought *parallel* to H.

45. **Axiom.** By referring to Experiment III, page 32, it is seen that the radius *C–A* (which is perpendicular to the wire) projects

on H as a straight line perpendicular to the projection of the wire, in all positions except when it is perpendicular to this plane H. Hence the important truth is evident that *if one of two lines forming a right angle is parallel to a plane of projection, the projection of the right angle will be a right angle unless the second line projects as a point.*

46. The Revolution of Points, Lines, Planes, and Solids. It frequently happens that the solution of a problem can be simplified by the revolution of some point, line, plane, or solid involved in the data.

The method of **revolving a point** has been illustrated in Experiment III, page 32, and **the revolution of a line, plane, or solid** consists in revolving a *sufficient* number of points to establish the new position and dimensions of such data.

Referring to Experiment III, it will be seen that the revolution of a point involves the following essentials:

(1) *An axis* (the wire) of revolution, around which the point revolves.

(2) *A radius of revolution,* which is the *unchanging* distance that the point in space is from the axis during the *entire* revolution.

(3) *A center of revolution,* which is the point about which the revolving point is to move; it marks the *intersection of* the axis and radius.

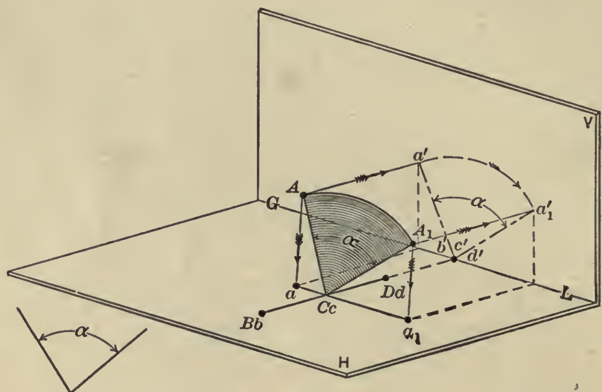
(4) *An arc or circle of revolution* which represents the *path* of the revolving point and measures the angle through which it moves.

(5) *The plane of revolution* which is *always* perpendicular to the axis and contains the arc or circle of revolution.

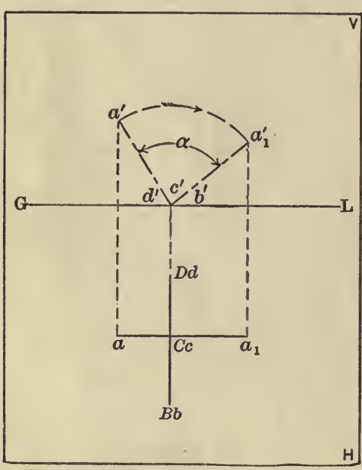
47. PROBLEM 5. To revolve a point through a given angle and around an axis which is perpendicular to V.

Analysis. Let *B–D* [see Fig. 31 (a)] be the axis, *A* the point, and α the given angle. The point *A* will move in the arc A–A_1 of a circle about the center *C* and with a radius *C–A*. The path of the point during revolution will be in the plane of revolution (which is perpendicular to *B–D*), and therefore parallel to V.

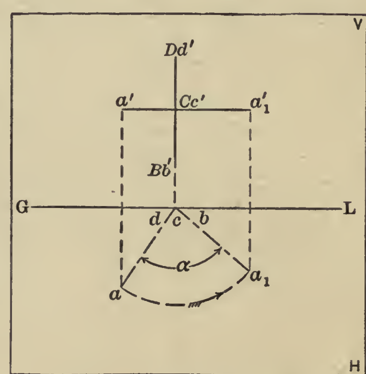
Hence the radius, the path, and the angle moved through by the point must be shown in their true size on V.



(a) Perspective.



(b) Construction with Axis in H.

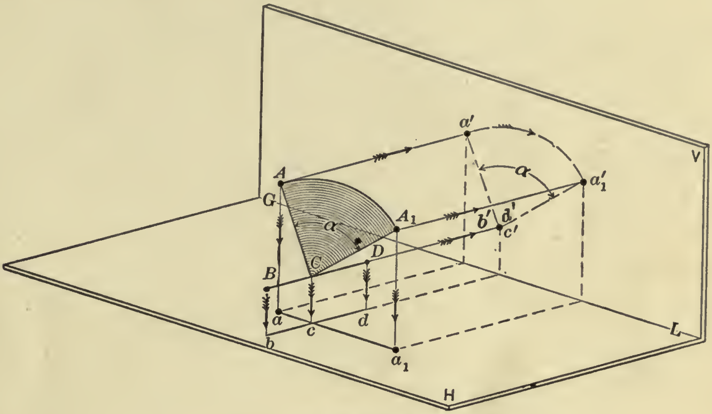


(c) Construction with Axis in V.

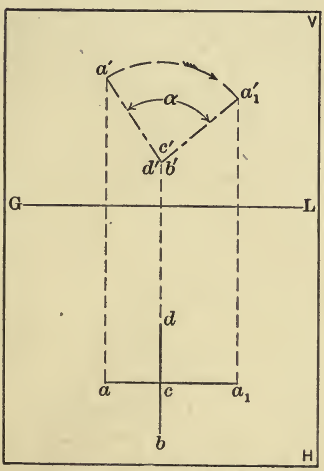
Fig. 31. — Revolution of a Point through a Given Angle around an Axis in One Plane and Perpendicular to the Other Plane of Projection.

Construction. Draw the projections *a*, *a′* of the point *A*, and *Bb–Dd*, *b′–d′* of the axis *B–D*. Figure 31 (b) shows the axis lying in H.

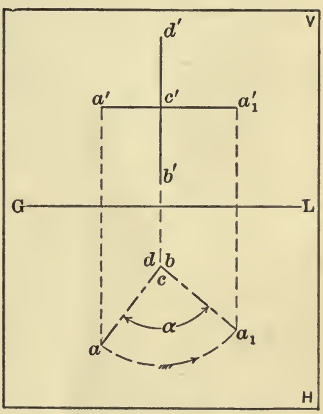
Through *a* draw the line *a–Cc* perpendicular to the horizontal projection Bb–Dd of the axis. Draw the vertical projection c'–a' of the radius of revolution and swing a' through the angle α to a_1'. Draw the ruled projector from a_1' to meet a–Cc produced.



(a) Perspective.



(b) Construction with Axis about H.



(c) Construction with Axis in front of V.

Fig. 32.—Revolution of a Point through a Given Angle about a Line Perpendicular to a Plane of Projection.

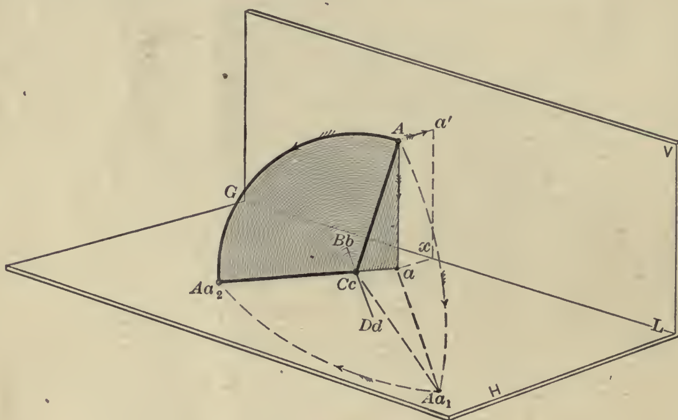
Then a_1 is the horizontal projection and a_1' the vertical projection of the point A in its *revolved* position.

Figure 31 (c) shows a solution when the axis lies in V and is perpendicular to H.

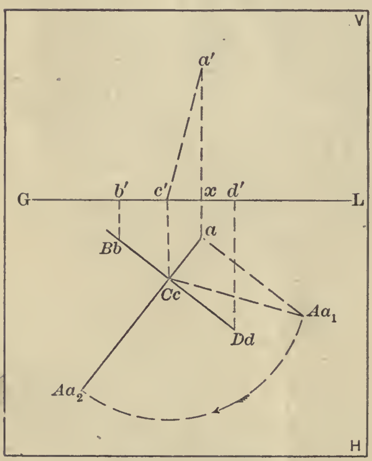
Figures 32 (a) and (b) show the solution when the axis is perpendicular to V but above H.

Figure 32 (c) shows the solution when the axis is perpendicular to H and in front of V.

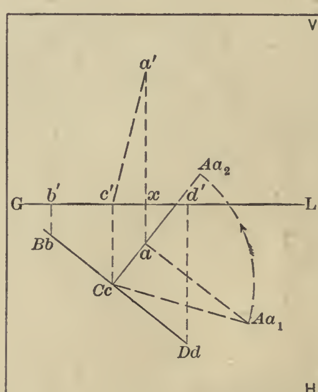
The student should be required to analyze, construct, and visualize problems similar to these in the *third* angle.



(a) Perspective.



(b) Construction I.



(c) Construction II.

Fig. 33.—Revolution of a Point about Any Line lying in a Plane of Projection.

48. PROBLEM 6. To revolve a point in space into H about an axis which lies in H and is oblique to V.

Analysis. Let *B–D* be the axis and *A* the point [see Fig. 33 (a)]. The point *A* will move in the arc $A–a_2$ of a circle, with center at *Cc* and having a radius of *A–C*. This radius is the hypothenuse of the triangle *A–a–Cc* which has a right angle at *a*. The altitude of this triangle is the distance from the point *A* to H, and the base of the triangle is the *perpendicular* distance from the axis to the projection *a*. As the radius *A–Cc* is always at right angles to the axis *B–D*, and the center of revolution, *Cc* does not change and the point *A* must fall upon a line in H drawn through *Cc* and perpendicular to *B–D*. If *A* is moved toward the left it falls at the point Aa_2 so that the distance $Cc–Aa_2$ is equal to the radius of revolution *Cc–A*.

Construction. In Fig. 33 (b) let *a*, *a′* be the projections of the given point *A*, and *Bb–Dd* be the horizontal projection of the axis. From *a* draw a perpendicular $a–Aa_2$ to the axis *Bb–Dd* and the intersection *Cc* of these lines is the center of revolution. Then *a–Cc* is the base of the triangle and *x–a′* is the altitude. The hypothenuse (the radius of rotation) can now be found by constructing the triangle. For convenience draw from *a* the perpendicular $a–Aa_1$ and mark off the distance $Aa_1–a$ equal to *x–a′*; with *Cc* as a center describe an arc with radius $Cc–Aa_1$ to cut *a–Cc* (extended) at Aa_2 which is the revolved position of the point *A*. Figure 33 (c) shows the solution when *A* is revolved *toward* V.

NOTE. If the point *A* is directly above the axis, the base of the triangle becomes zero and the radius of revolution is equal to the distance of the point above H.

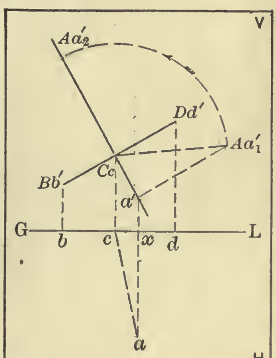


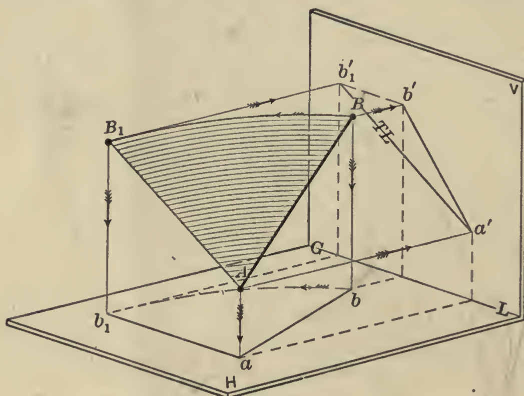
Fig. 34.—Revolution of a Point about an Oblique Axis in V.

49. PROBLEM 7. To revolve a point in space into V about an axis which lies in V and is oblique to H.

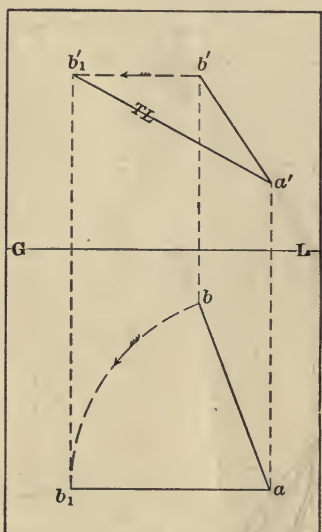
Analysis. In this case the altitude of the right triangle whose hypothenuse is the radius of revolution becomes the distance of the point *A* from V; the base of the triangle is the perpendicular distance of *a′* from the axis; and the construction is made in V (see Fig. 34).

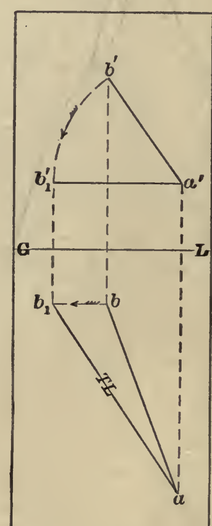
50. PROBLEM 8. To determine the distance between two points in space.

Fundamentals. The distance between two points will be measured by the straight line joining them. This line will be



(a) Perspective.





(b) Construction. (c) Construction.

Fig. 35.—To determine the Distance between Any Two Points in Space.

projected in its true length on that plane of projection to which it is parallel, or, on which it lies.

Analysis 1. See Fig. 35 (a). Revolve the line A–B joining the points, about the space projector (which can be used as an axis) of any one of its points, say A, until the line is parallel to V.

Its new projection on V (the plane to which it is parallel) is its true length a'–b_1'. Similarly it could have been revolved parallel to H.

Construction 1. See Fig. 35 (b). Let the projections of the given points be a, a' and b, b'. Revolve the projection a–b about a until it is parallel to G–L. Since the point B in space must not change its distance from H, the new vertical projection b_1' will take a position along the line b'–b_1', which is parallel to G–L. The length of the horizontal projection a–b does not change, therefore b moves in the arc of a circle to b_1 until b_1 and a are the same distance from G–L. The projection b_1' must lie on the ruled projector from b_1, hence at b_1'. Similarly the line could have been revolved parallel to H [see Fig. 35 (c)].

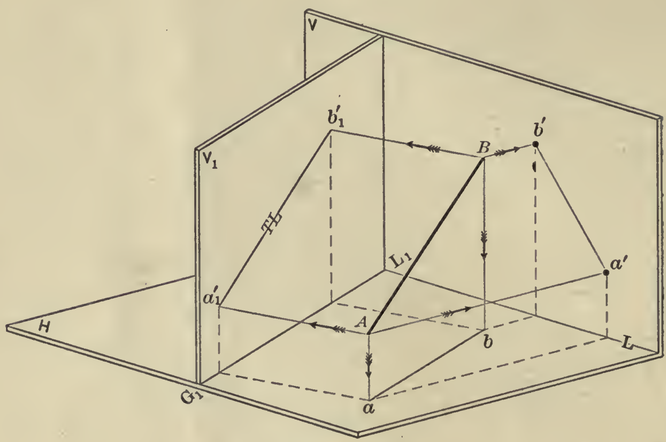
Analysis 2. See Fig. 36 (a). If a new vertical plane V_1 is assumed parallel to the line A–B in space, the new vertical projection a_1'–b_1' of the line on this plane will be the true length of the line.

Construction 2. See Fig. 36 (b). The new G_1–L_1 must be parallel to the horizontal projection a–b of the line. Ruled projectors are drawn from a and b perpendicular to G_1–L_1 and a_1' and b_1' are the same distance from G_1–L_1 that a' and b' are from G–L. Join a_1' and b_1' with a straight line and mark it *T. L.* (true length).

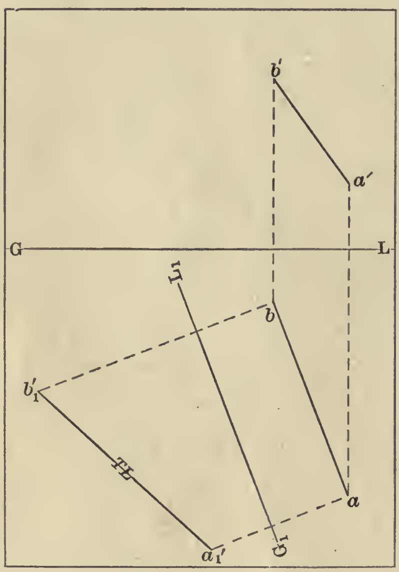
NOTE. In this method the new V is often made to pass through the line in space, hence G_1–L_1 will coincide with a–b, and the distances on the ruled projector from b and a to G_1–L_1 become zero. The distance from G_1–L_1 to b_1' and a_1' remains unchanged [see Fig. 36 (c)].

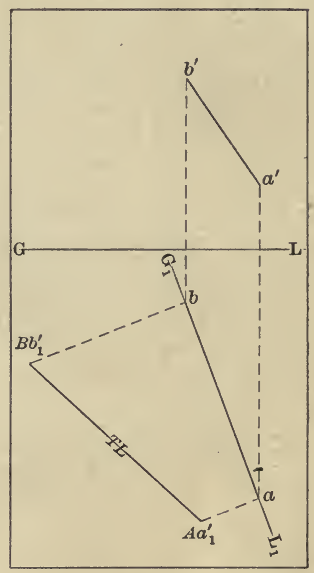
Notation. When the solution of a problem requires that the true length of a line be found, the line is marked *T. L.* as in Fig. 36 and the projection is an *unbroken* line of *medium* weight.

51. PROBLEM 9. To determine the true size of the angle between two intersecting lines.



(a) Perspective.

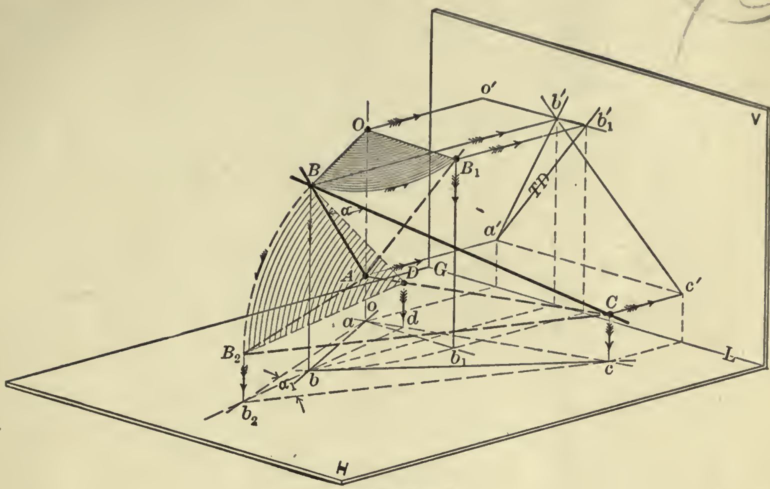




(b) Construction. (c) Construction.

Fig. 36. — To determine the Distance between Two Points in Space.

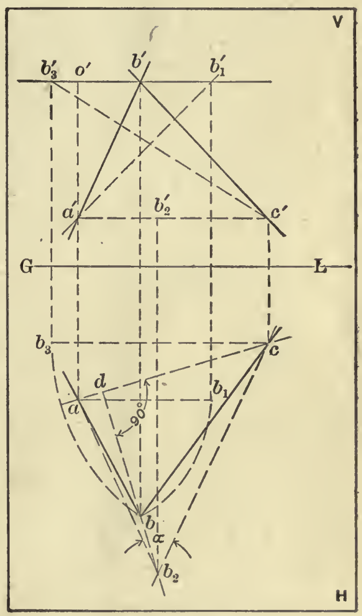
Analysis. The two lines must be brought either into or parallel to a plane of projection without changing the angle between them. When in this position the projections of the lines on the plane to which they are parallel represent the true



(a) Perspective.

length of the lines in space and the angle between these projections is the same as the angle between the lines in space.

If A–B and B–C [Fig. 37 (a)] are the two lines in space and α is the angle, the problem consists in determining a point (A and C) on the lines respectively which is the same distance a–A = C–c from H, and then revolving the vertex B of the angle about the axis A–C and in the plane of the sector B–D–B_2 until the distances B_2–b_2 = A–a = C–c, that is B is brought to the same distance as is A and C from H. The lines A–B and B–C are then parallel to H and α is shown on H as α_1 in its true size.



(b) Construction.

Fig. 37.—To determine the True Angle between Two Intersecting Lines.

A simple method, however, of performing this operation on the drawing consists in keeping A stationary and revolving B about a vertical axis O–o and in the plane of the sector B–O–B_1 (which is parallel to H) until A–B_1 is parallel to V [see Fig. 37 (a)]. The true distance from A to B is then shown by the vertical projection a'–b_1' marked TD.

The true distance from B to C is found in a similar manner [not shown in Fig. 37 (a)] and the triangle a–b_2–c is constructed on the true distance a–c and this determines the required angle α_1.

Construction. Let a–b, a'–b' and b–c, b'–c' [Fig. 37 (b)], represent the projections of the two intersecting lines A–B and B–C.

To determine the points A and C the line a'–c' is drawn parallel to G–L and to intersect the vertical projections of the lines. Since a'–c' is parallel to G–L, the horizontal projection shows the true distance between the points A and C in space. Then a and c are determined by ruled projectors from a' and c'. A–B and B–C will be shown in their true length by the projections a–b_2 and b_2–c when these lines in space are brought parallel to H. Hence find the true length of A–B and B–C (by revolving each of them parallel to V) and with the true lengths as radii and a and c as centers describe arcs of circles intersecting at b_2. Then a–b_2–c is the required angle.

Check. The perpendicular b_2–d to the line a–c and through b_2 must pass through b.

52. PROBLEM 10. To measure off a given distance on a given line.

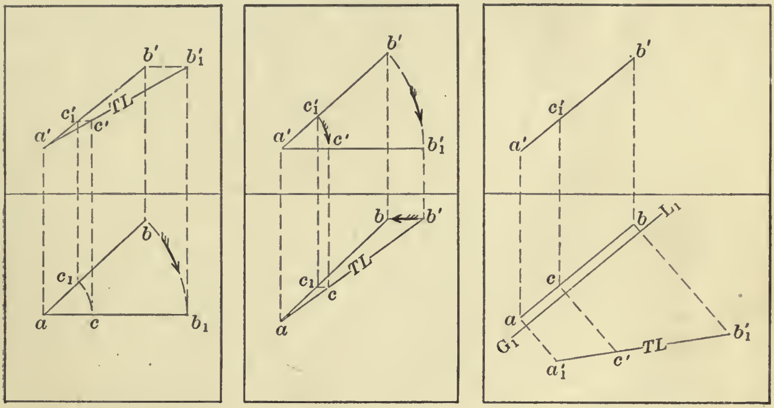
Analysis. The line must be parallel, or brought parallel, to a plane of projection and the given distance measured on the projection which shows the true length of the line.

The projections of the point limiting the distance can then be found on the original projections of the given line by counter revolution, that is, by revolving the line back to its original position.

Construction 1. Let A–B represent the given line and A–C the given distance which is to be measured from A. Swing the line A–B about the space projector (from A to a) until it is parallel to V [see Fig. 38 (a)]. The new projections of the line are a–b_1 and a'–b_1'. From a' mark off the length a'–c' equal to A–C, the given distance. By drawing the ruled projector determine c. When the line is counter revolved c swings to c_1 and c_1' is the corresponding vertical projection and c_1 and c_1' are the projections of the required point in its required position.

Construction 2. Similarly, swing the line $A-B$ about the space projector $A-a'$ until the line is parallel to H [see Fig. 38 (b)].

Construction 3. Assume a new V_1 shown by G_1-L_1 [Fig. 38 (c)] at any convenient point and parallel to the line $A-B$.



(a) Construction I. **(b) Construction II.** **(c) Construction III.**

Fig. 38.—To measure a Given Length on a Given Line.

Find the new vertical projection (true length) $a_1'-b_1'$ and measure off the required distance $A-C$ from a_1' to c'. Determine the horizontal projection c and the required vertical projection c_1' by ruled projectors.

NOTE. The plane could have been passed through the line perpendicular to H.

53. **Definitions.** An **element of a cone** is any *straight* line drawn on its surface from the vertex to the base.

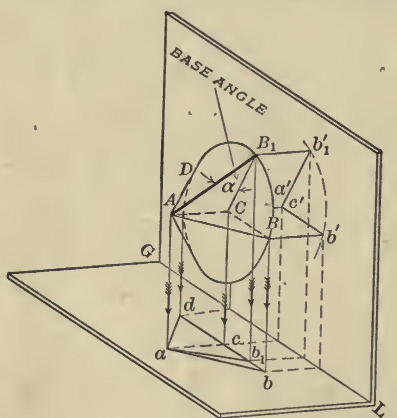
If the base of a cone is a circle the cone is said to be circular, and a line passing through the vertex and the center of the base is called the **axis of the cone.** If this axis is perpendicular to the base, the cone is called a **right circular cone.**

A right circular cone is also called a **cone of revolution** because it can be generated by the revolution of a right-angled triangle about one of its *shorter* sides.

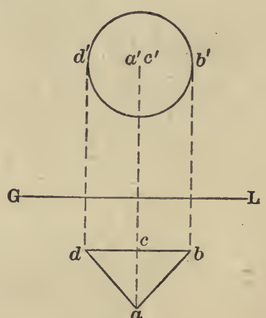
The **base angle** of a right circular cone is the angle made by an element and a line drawn from the center of the cone base to the point of intersection of the element with the base [see Fig. 39 (a)].

54. PROBLEM 11. To draw the projections of a line which makes a given angle with V and H.

Analysis. If a line *A–B* in space [see Fig. 39 (a)] is revolved about a second line *A–C*, which intersects it and is perpendicular to V, a cone of revolution is generated. This cone projects as



(a) Perspective.

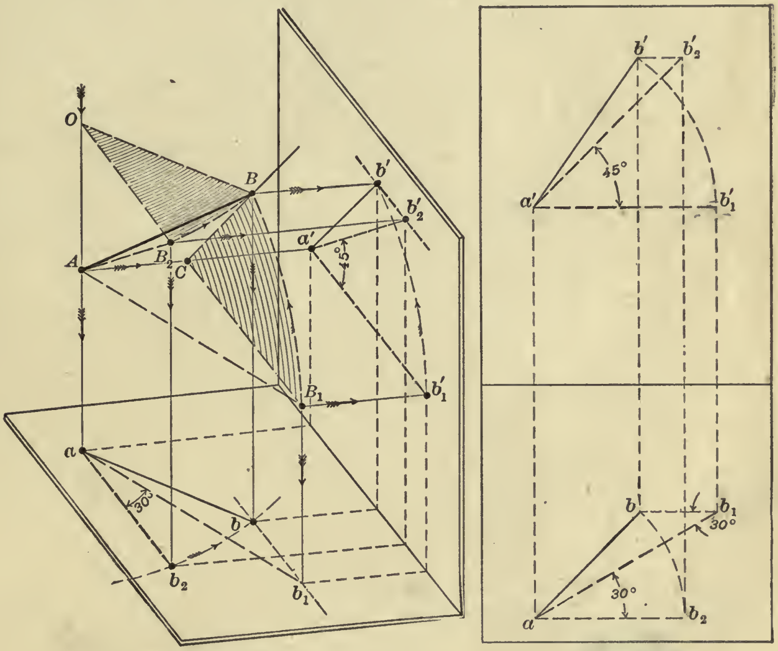


(b) Layout.

Fig. 39. — To determine the Projection of a Line which makes a Given Angle with a Plane of Projection.

a triangle on H [see Fig. 39 (b)]. The altitude *a–c* of the triangle will be the same as the altitude *A–C* [see Fig. 39 (a)] of the cone; the base *d–b* of the triangle will be the same as the diameter *D–B* of the cone, and each slant side of the triangle will be the same as the line *A–B* of the cone. The cone projects on V as a circle which is equal to the base of the cone since it is parallel to V. As the line *A–B* revolves to generate the cone, its projection on H changes from its true length (when parallel to H) to a length equal to the altitude of the cone when it is directly above the axis. The vertical projection of *A–B*, however, does not change its length in changing its position. See *a–b*, $a–b_1$ and *a′–b′*, $a'–b_1'$, Fig. 39 (a), for the illustration of these facts.

Construction. Revolve the line $A-B$ until it is parallel to H [see $a'-b_1'$, $a-b_1$, Figs. 40 (a) and (b)] and at the required angle (say 30 degrees) to V. The vertical projection $a'-b_1'$ of this line gives the *radius* of the circle which represents the vertical *projection* of the cone base. The horizontal projection $a-b_1$ gives the *slant height* of the triangle that represents the horizontal *projection* of the cone. Since the base of the cone is projected on H as a straight line parallel to $G-L$, the complete projection of the cone can be drawn [see Fig. 40 (c), page 48].



(a) Perspective. (b) Construction.

Fig. 40. — To determine the Projections of a Line that makes a Given Angle with a Plane.

Place a line $A-B_2$ of the same length as $A-B$ (with one end at A) and parallel to V [see $a-b_2$, $a'-b_2'$, Figs. 40 (a) and (b)] and at the required angle (say 45 degrees) with H. The horizontal projection $a-b_2$ of this line gives the radius of the circle that represents the horizontal *projection* of a *second* cone base. The triangle representing the vertical projection is found by having the slant height $a'-b_2'$ and the base $a-b_2$.

The vertical projections of the two cones intersect at b', and a' is common to both. Therefore a'–b' and a–b represent the projections of an element common to both cones. It therefore makes the required angles with both V and H.

Check. The ruled projector b'–b should come perpendicular to G–L.

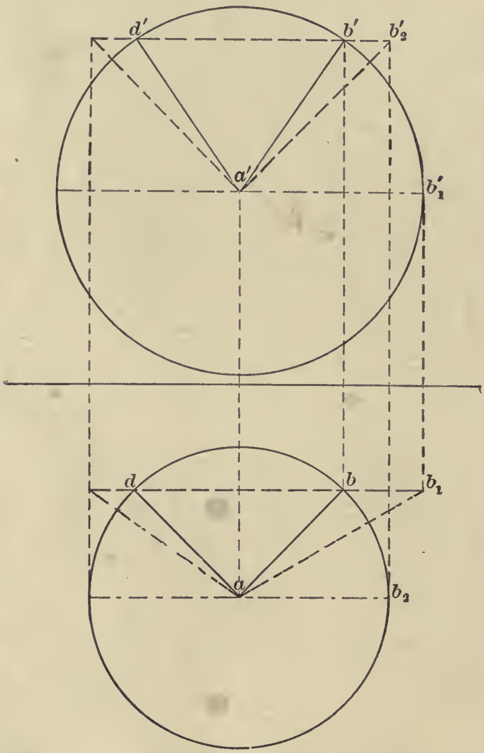


Fig. 40. — (c) Layout.

A second line A–D in the opposite direction [see Fig. 40 (c)] would fulfill the requirements of this problem.

NOTE. If the cones are tangent, the line in space is in a plane perpendicular to G–L and the sum of the two angles made with V and H is 90 degrees.

CHAPTER IV

LINE AND PLANE PROBLEMS

55. **Definitions and Principles.** The position of any plane with reference to the co-ordinate planes (that is V and H) is determined by the lines of intersection of the plane with V and H.

These lines of intersection are termed the **traces** of the plane. If the trace lies in V it is termed the **vertical trace**; if the trace lies in H it is termed the **horizontal trace.**

The vertical and horizontal traces of a plane must intersect on *G–L*, because a plane can intersect a line only in *one* point.

Since the traces of a plane are lines *in* either V or H they are their own projections on the plane *containing* them and their *other* projection lies in *G–L.*

56. **Notation and Line Convention. Planes are designated** by *capital letters*, the last letters of the alphabet being used, thus, P, Q, R, S, T, U.

The horizontal trace of a plane is designated by the capital letter of the plane to which the capital *H* has been prefixed, thus: *HP*, *HQ*, *HR*, etc.

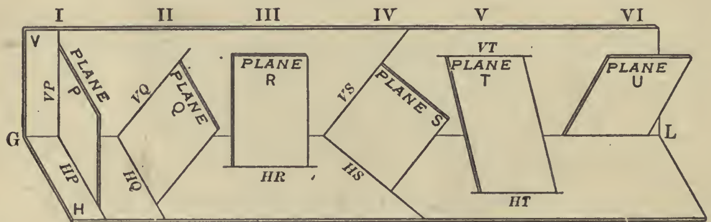
The vertical trace of a plane is designated by the capital letter of the plane to which the capital *V* has been prefixed, thus: *VP*, *VQ*, *VR*, etc.

Traces of planes unless used in construction are *unbroken* lines of *medium* weight and the designating letters are printed close to the line. See Fig. 41, page 50.

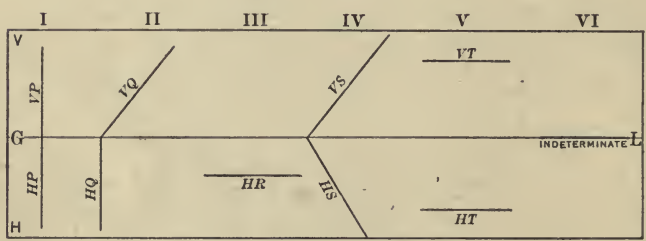
Traces of planes used in construction or the solution of a problem are *broken* lines of *light weight* and are drawn the same as construction lines (see page 18, § 26) and in addition the traces are designated by letter in the same manner as the traces of given or required planes.

57. Positions which a plane may have with reference to V and H. A plane may have any one of the following positions with reference to V and H.

(1) Perpendicular to both V and H, hence perpendicular to *G–L*; the traces of such a plane are represented by a straight line perpendicular to *G–L* [see Figs. 41 (a) and (b)–I].



(a) Perspective.



(b) Construction.

Fig. 41. — Positions a Plane may occupy.

(2) Perpendicular to one plane (either V or H) and inclined to the other. One trace of this plane will be perpendicular to *G–L* and the other will be inclined to it [see Figs. 41 (a) and (b)–II].

(3) Perpendicular to one plane (either V or H) and parallel to the other. This plane is represented by a single trace, which is parallel to *G–L* and the trace is located on the co-ordinate plane to which the plane is *not* parallel [see Figs. 41 (a) and (b)–III].

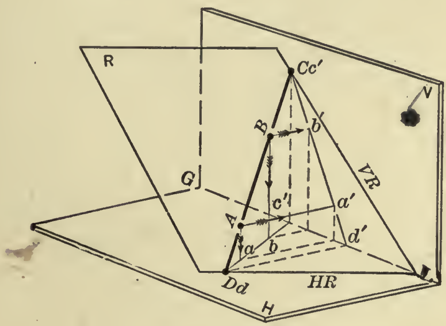
(4) Inclined to both V and H hence to *G–L*. Both traces will incline to *G–L* [see Figs. 41 (a) and (b)–IV].

(5) Inclined to both V and H but paralled to *G–L*. Both traces are parallel to *G–L* [see Figs. 41 (a) and (b)–V].

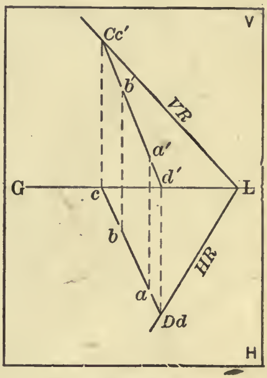
(6) Passed through *G–L* giving no trace either on V or H [see Figs. 41 (a) and (b)–VI].

58. THEOREM V. If a line lies in a plane its vertical piercing point (or vertical trace) must lie in the vertical trace of the plane, and its horizontal piercing point in the horizontal trace of the plane.

Proof. If a line *A–B* [Fig. 42 (a) and 42 (b)] lies in a plane R, every point of the line lies in the plane, and the line intersects



(a) Perspective.



(b) Construction.

Fig. 42. — The Traces of a Line lie in the Traces of a Plane containing it.

every other line in the plane to which it is not parallel. Extend the line *A–B* until it pierces V. The vertical piercing point *Cc'* of *A–B* is a point common to both the given plane R and to V. Also, the vertical trace *VR* of the plane contains every point common to the given plane R and V, hence the vertical piercing point *Cc'* of the line *A–B* must lie in the vertical trace *VR* of the plane.

Likewise the horizontal piercing point *Dd* of the line must lie in the horizontal trace *HR* of the plane.

59. PROBLEM 12. To assume a line in a given oblique plane.

Analysis 1. Let R in Fig. 43 (a) be the given plane. From Theorem V, this page, any point *Aa'* in the vertical trace *VR* may be assumed to be the vertical piercing point of some line in R. Also, any point *Bb* in the horizontal trace *HR* may be assumed to be the horizontal piercing point of some line in R. Therefore, since two points determine a straight line, *A–B* is the required line.

Construction 1. Draw *VR* and *HR* [Fig. 43 (b)] to represent the vertical and horizontal traces respectively of the plane

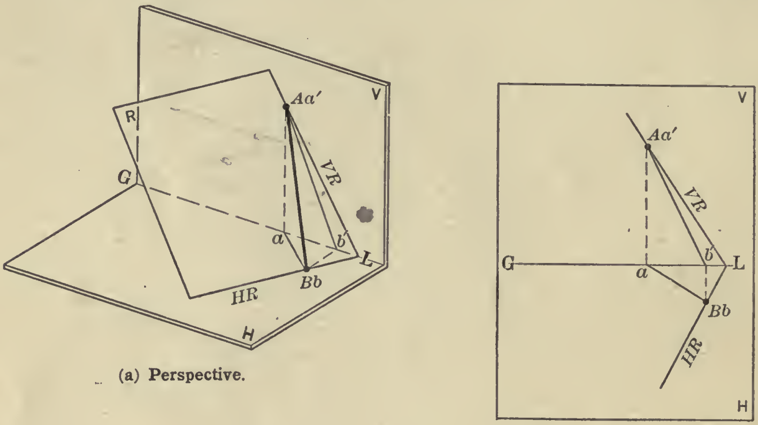
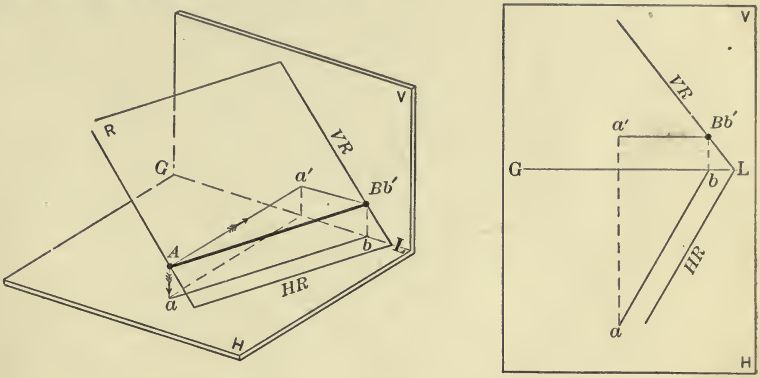


Fig. 43.—To determine the Projections of Any Line assumed in Any Given Plane.

R. Assume any point *Aa′* on *VR*. Since *Aa′* is on V it will be horizontally projected in the ground line at *a*. Similarly, the second assumed point *Bb* on *HR* is vertically projected at *b′*. By joining the vertical projections *Aa′* and *b′* and next the horizontal projections *a* and *Bb*, the vertical and horizontal projections of the line are determined.

Analysis 2. Let R in Fig. 44 (a) be the given plane. Draw a line *A–B* in the plane R and parallel to H. This line will be parallel to the horizontal trace *HR* of the plane, and have its vertical piercing point *Bb′* in the vertical trace *VR* of the plane.

Construction 2. Draw *VR* and *HR* [Fig. 44 (b)] to represent the traces of the given plane. As the line *A–B* is parallel to *HR* the horizontal projection *a–b* must be parallel to *HR*, and the vertical projection *a′–Bb′* must be parallel to *G–L* (since the vertical projection of *HR* is in *G–L*). The point *b* where the horizontal projection *a–b* cuts *G–L* marks the horizontal projection of the point where the line *A–B* pierces V, hence draw the ruled projector from *b* to *VR* to determine the vertical projection *Bb′*. The vertical projection of the required line is *Bb′–a′* and is parallel to *G–L*.



(a) Perspective. (b) Construction.

Fig. 44. — To determine the Projections of a Line which is Parallel to a Plane of Projection and lies in Any Given Plane.

A line in the plane R and parallel to the vertical trace *VR* could be similarly assumed.

NOTE. **To assume a point in an oblique plane** it is only necessary to assume one projection of the point, pass any line *of*

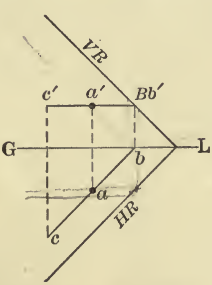


Fig. 45. — Determining a Point in Any Plane by Means of a Horizontal Line of the Plane passed through the Point.

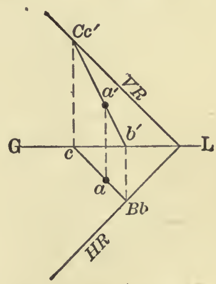
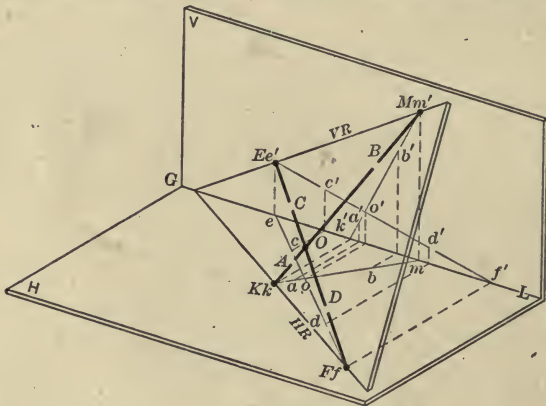


Fig. 46. — Determining a Point in Any Plane by Means of Any Line of the Plane passed through the Point.

the plane through this projection and the other projection of the point will be found in the other projection of the line (see Figs. 45 and 46).

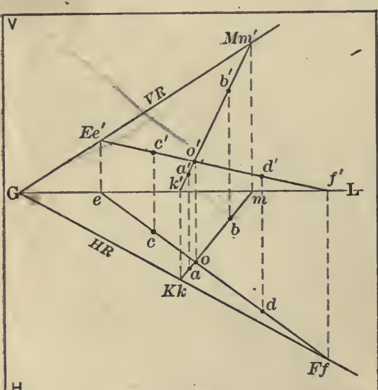
60. PROBLEM 13. To pass a plane through two intersecting lines.

Analysis. Let *A–B* and *C–D* intersecting at *O* [see Fig. 47 (a)] be the given lines, then since the lines lie in the required plane



(a) Perspective.

R, they must pierce H and V in the traces of this plane, and since two points are sufficient to determine a straight line; the vertical trace of the plane is determined by the vertical piercing points *E* and *M* of the two lines. The horizontal trace is determined by the horizontal piercing points F and K of the lines.



(b) Construction.

Fig. 47. — To pass a Plane through Any Two Lines which intersect.

Construction. See Fig. 47 (b). Extend the horizontal projection *a–b* of *A–B* and *c–d* of *C–D* to intersect *G–L* and find the vertical projections *Ee′* and *Mm′* of the vertical piercing points (see page 51, § 58). Through these projections *Ee′* and *Mm′* draw the vertical trace *VR* of the plane R. Similarly, find the horizontal projections of the horizontal piercing points, and through these projections *Ff* and *Kk* draw the horizontal trace *HR* of the required plane R.

Check. If the traces of the plane R intersect *G–L* at the same point, the work is correct.

NOTE. **If one trace of a plane is given,** the other trace can be determined if a single point lying in it is known, for this trace must intersect *G–L* at the same point as the given trace or it must be parallel to *G–L*.

61. PROBLEM 14. To pass a plane through three given points.

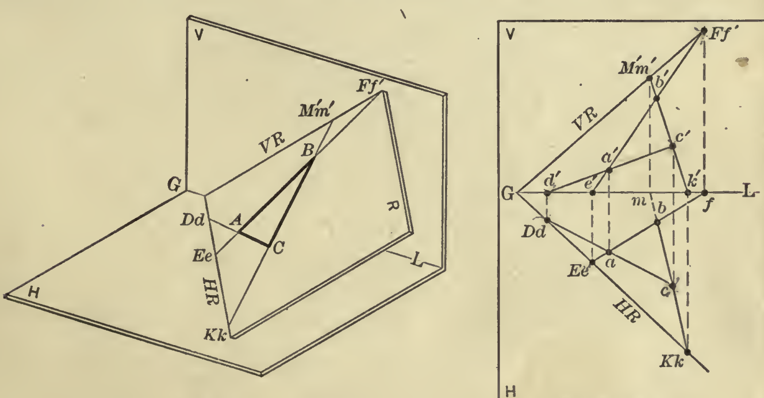
It is only necessary to draw lines through the points and proceed as in Problem 13, page 54.

62. PROBLEM 15. To pass a plane through a point and a line.

Draw a line through the point intersecting the given line, and proceed as in Problem 13, page 54.

63. PROBLEM 16. To determine the traces of a plane to contain any given plane figure.

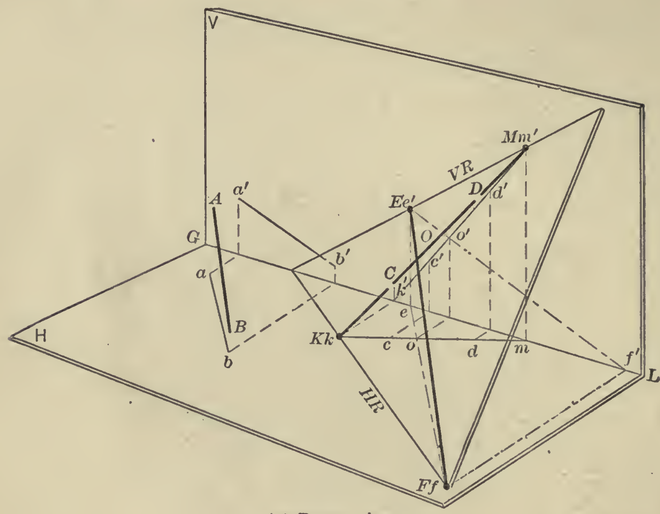
It is only necessary to extend two sides of the figure until they pierce V and H and draw the traces through these piercing points [see Figs. 48 (a) and 48 (b)]. See Problem 13, page 54.



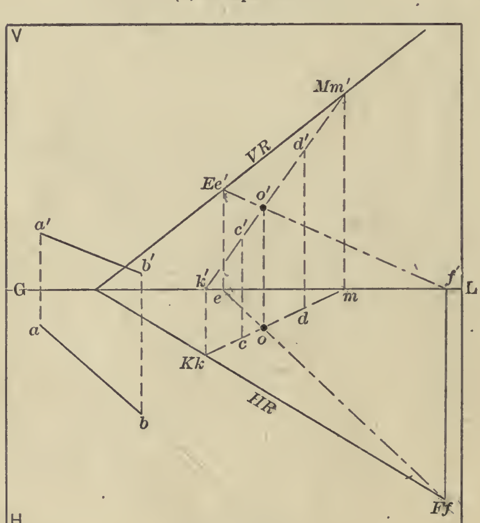
(a) Perspective. (b) Construction.

Fig. 48.—To pass a Plane through Any Given Plane Figure.

64. THEOREM VI. If two planes in space are parallel, their vertical traces are parallel, also their horizontal traces are parallel.



(a) Perspective.



(b) Construction.

Fig. 49. — To pass a Plane through a Given Straight Line and Parallel to Another Given Straight Line.

As the vertical traces of the planes are their intersections with V and their horizontal traces are their intersections with H, the proof of this theorem follows directly from a theorem in Solid Geometry which establishes the truth that, if two parallel planes are cut by a third plane, the intersections are parallel.

65. Principles. A theorem in Solid Geometry proves that through any given straight line, a plane can be passed parallel to any other straight line. Also, a plane is parallel to a given straight line when it contains any straight line parallel to the given line.

66. PROBLEM 17. To pass a plane through a given straight line and parallel to another given straight line. See §65, this page.

Analysis. In Fig. 49 (a) let it be required to pass a plane R through the line *C–D* and parallel to the line *A–B*. Through any convenient point *O* on the given line draw a straight line *E–F* parallel to *A–B*. The plane containing these two intersecting lines is the required plane.

Construction. In Fig. 49 (b) draw the projections *Ee′–f′* and *e–Ff* of the line *E–F* through the projections *o′* and *o*, and parallel to the projections *a′–b′* and *a–b*. Find the piercing points of the lines *E–F* and *C–D* on V and H. Draw *VR* and *HR* through the vertical and horizontal piercing points respectively.

Check. *VR* and *HR* must meet on *G–L*, and any other straight line drawn through any other point on *C–D* and parallel to *A–B* must pierce V and H in *VR* and *HR*.

67. PROBLEM 18. To pass a plane through a given point and parallel to two given straight lines.

Analysis. Let the two lines *A–B* and *C–D* and the point *O* be given by their projections. If two lines are drawn through the point *O* parallel to the given lines *A–B* and *C–D* they will lie in the plane R and the traces of R will pass through the piercing points of these lines.

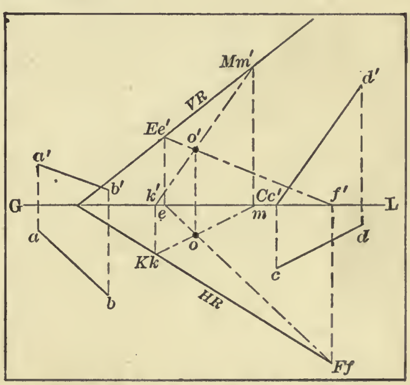


Fig. 50. — A Plane passed Parallel to Two Given Straight Lines and through a Given Point.

Construction. Draw the projections of the *construction* lines (see page 18, § 26) *E–F* and *K–M* through the projections of the point *O*, and parallel respectively to the projections of the given lines *A–B* and

C–D (see Fig. 50). Determine the piercing points of these lines *E–F* and *K–M* on V and H and draw the traces *VR* and *HR* of the required plane containing them.

Check. The traces of the plane must meet on *G–L* and a straight line drawn through the point *O* parallel to *VR* must pierce H in *HR* or if drawn through *O* parallel to *HR* must pierce V in *VR*.

68. PROBLEM 19. To determine a plane which shall be parallel to a given plane and contain a given point.

Analysis. Since the planes are parallel, their corresponding traces will be parallel and one point on each trace will determine the required plane. A straight line through the given point and parallel to either trace of the given plane will pierce V or H in the corresponding trace of the required plane. The intersection of this trace with *G–L* is a point on the other trace and its direction is known. Hence the trace can be drawn.

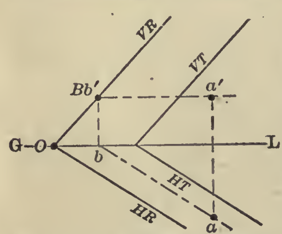


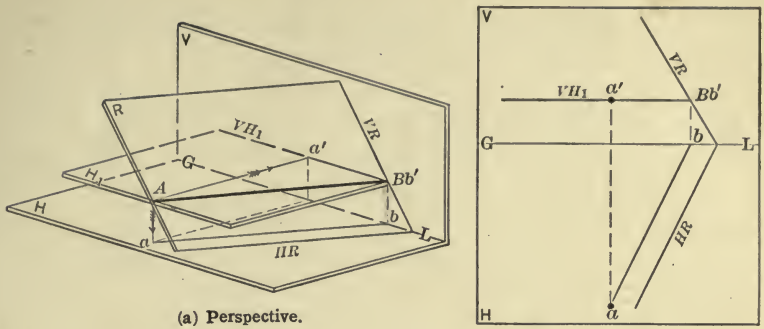
Fig. 51. — To determine a Plane to contain a Given Point and to be Parallel to a Given Plane.

Construction. (See Fig. 51.) Through *a* draw *a–b* parallel to *HT*. Through *a′* draw *a′–Bb′* parallel to *G–L*. The point *Bb′* is on the vertical trace of the required plane, hence through *Bb′* draw *VR* parallel to *VT*. The trace *HR* must intersect *VR* on *G–L*; therefore *HR* is drawn through *O* parallel to *HT*.

Check. See Problem 18, page 57.

69. THEOREM VII. If a plane in space is intersected by a plane parallel to H, the horizontal projection of the line of intersection will be parallel to the horizontal trace of the plane and the vertical projection will be parallel to G–L.

Proof. See Figs. 52 (a) and (b). Let R be the given plane cut by H_1 parallel to H. The line of intersection *A–B* is parallel to *HR* because *HR* is the line of intersection of R with a plane H parallel to H_1 (see Analysis 2, page 52). If *A–B* and *HR* are parallel their projections must be parallel. Hence, as *HR* is its own horizontal projection, it is parallel to *a–b*, and as the vertical projection of *HR* is in *G–L*, the vertical projection *a′–Bb′* must be parallel to *G–L*.



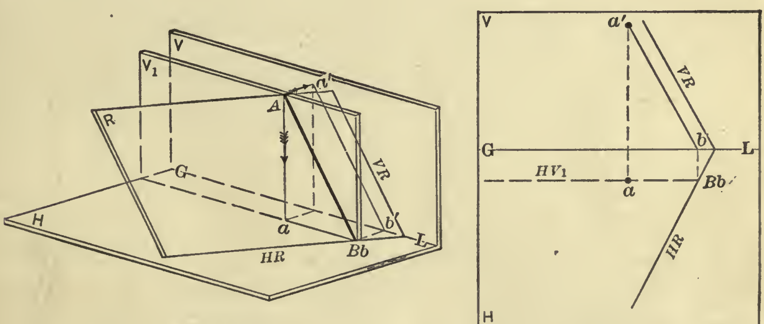
(a) Perspective.

(b) Construction.

Fig. 52. — A Plane in Space intersected by a Plane Parallel to H.

70. Corollary. Any line which lies in a given plane and which is parallel to H, will be parallel to the horizontal trace of that plane. The vertical projection of this line will be parallel to *G–L*, and the horizontal projection will be parallel to the horizontal trace of the plane.

See Figs. 53 (a) and (b) for similar proof when the assumed plane is parallel to V



(a) Perspective.

(b) Construction.

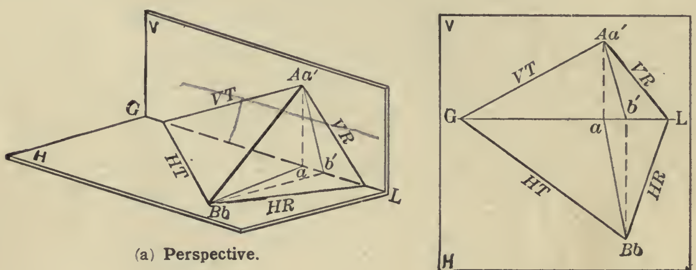
Fig. 53. — A Given Line in a Given Plane which is Parallel to V.

71. Axiom. The line of intersection of two planes is a straight line common to both planes. Hence, to locate this line of intersection the projections of two points common to both planes must be determined and the like projections of the points connected for the projections of the line.

72. PROBLEM 20. To determine the line of intersection of two planes.

Analysis. The vertical traces *VT* and *VR* [see Fig. 54 (a)] intersect at *Aa′* and thus determine the point *A* which lies on V and is common to both the planes T and R. Since *A* is on V, its horizontal projection *a* must be on *G–L*. The intersection of the horizontal traces *HT* and *HR* determines a second point *B* that is common to both planes. Since *B* is on H, its vertical projection *b′* must be on *G–L*.

A line drawn to join the vertical projections *Aa′* and *b′* of the points *A* and *B* determines the vertical projection of the line



(a) Perspective.

(b) Construction.

Fig. 54. — To determine the Line of Intersection between Two Planes.

of intersection of the two planes. The horizontal projection of the line is determined by joining *a* and *Bb*, the horizontal projections of these points.

Construction. See Fig. 54 (b). Let *VT*, *HT* and *VR*, *HR* represent the two given planes. The vertical traces intersect at *Aa′*, horizontally projected at *a*. The horizontal traces intersect at *Bb*, vertically projected at *b′*. Join *Aa′* and *b′* for the vertical projection of the line of intersection and *a* and *Bb* for the horizontal projection.

Check. Draw any line to cut the line of intersection *A–B* and the vertical trace of the plane T; this line lies in T and therefore when extended must pierce H on *HT*. Apply the same test to R.

Special Cases. Solutions for the special cases where the traces intersect *within* the limits of the drawing are given in Figs. 55 to 61 inclusive.

In each case the analysis given for Problem 20 (page 60) applies, but the solutions depend upon the relative positions of the traces. The student is expected to analyze, "build up," and explain in detail each of the special cases shown in Figs. 55 to 61 inclusive.

Fig. 55 represents the solution when the traces meet on *G–L*. Fig. 56 represents the solution when all the traces are parallel to *G–L*. Both of these solutions require the use of a profile plane. Fig. 57 represents a solution by the use of a plane parallel to H when the traces meet on *G–L*, as in the case of Fig. 55. Fig. 58 represents the solution when one plane is perpendicular to H and inclined to V and the other is perpendicular to V and inclined to H. Fig. 59 represents the solution when one plane is inclined to V and H while the other plane is perpendicular to H but inclined to V. Fig. 60 represents the solution when one plane is inclined to V and H while the other plane is perpendicular to V but inclined to H. Fig. 61 represents the solution when the line of intersection lies in a plane perpendicular to *G–L*.

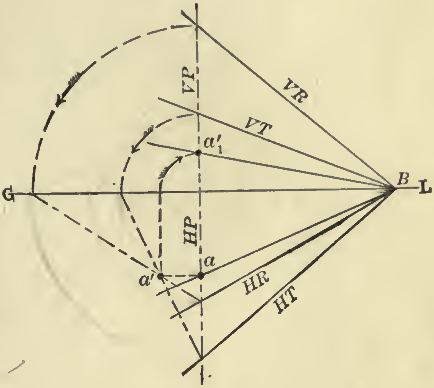


Fig. 55. — Traces intersect on G–L (Construction I).

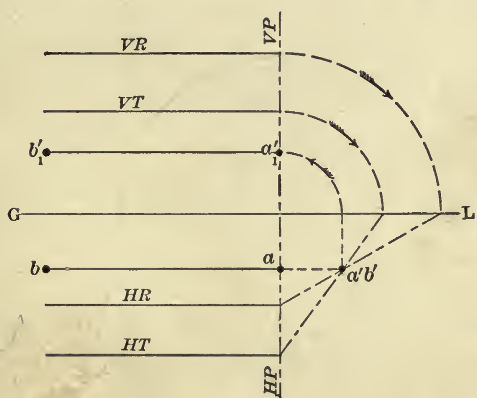


Fig. 56. — All Traces Parallel to G–L.

73. Principle. In plane geometry the principle is established that parallel planes intersect a third plane in parallel lines.

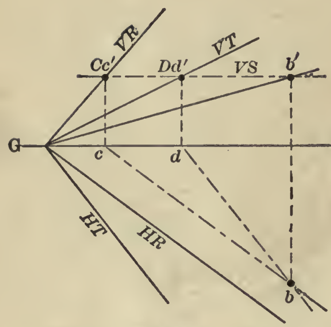


Fig. 57. — Traces Meet on G-L. (Construction II.)

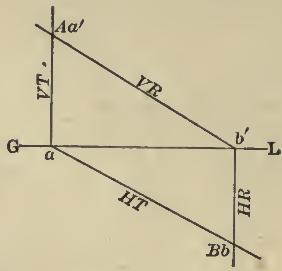


Fig. 58. — Planes Perpendicular to V and H respectively but Inclined to H and V respectively.

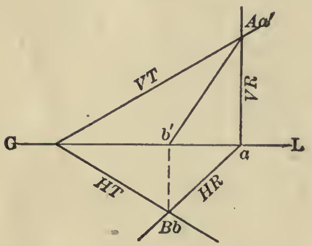


Fig. 59. — One Plane Perpendicular to H and Inclined to V, the other Inclined to H and V.

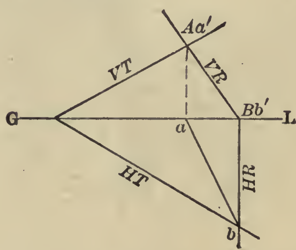


Fig. 60. — One Plane Perpendicular to V and Inclined to H, the other Inclined to V and H.

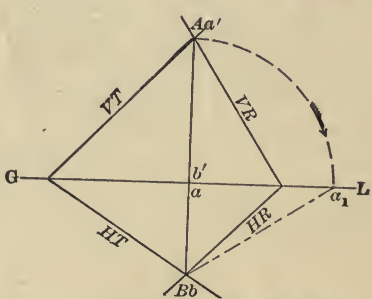
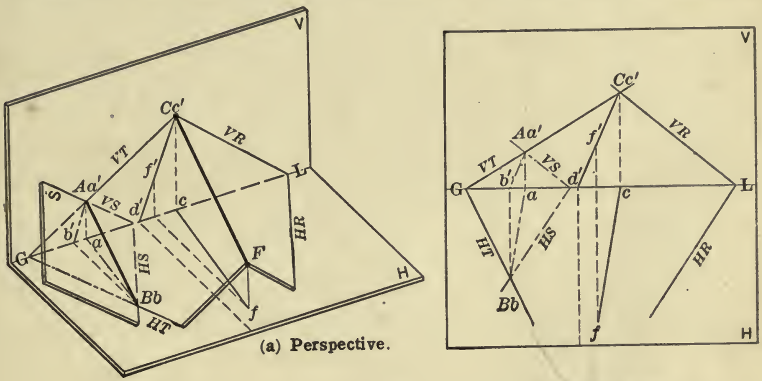


Fig. 61. — Both Planes Inclined to V and H.

74. PROBLEM 21. To determine the line of intersection of two planes when the horizontal traces do not intersect within the limits of the drawing.

Analysis. Let T and R [Fig. 62 (a)], be the given planes. Since the vertical traces intersect at *Cc'*, horizontally projected at *c*, the point *Cc'* is one point on the required line of intersection.

If an auxiliary plane S, parallel to R, is passed to intersect T at such a position that *VS* intersects *VT* and *HS* intersects *HT*, the line of intersection of the plane S with T can be found. Then, as the plane S is parallel to the plane R, the line of intersection of S with T must be parallel to the line of intersection



(a) Perspective.

(b) Construction.

Fig. 62.—Intersection of Two Planes when only One Set of Traces Intersect within the Limits of the Drawing.

of R with T. Since one point and the direction determines the position of a straight line, the line *Cc'–d'* parallel to *Aa'–b'* is the vertical projection, and *c–f* parallel to *a–Bb* is the horizontal projection of the required line of intersection.

Construction. See Fig. 62 (*b*). Let *HT* and *HR*, which do not intersect, represent the horizontal traces of the planes T and R; and *VT* and *VR* represent the vertical traces which do intersect at the point *Cc'*. Assume *VS* and *HS* parallel respectively to *VR* and *HR* and find the projections of the line of intersection of S with T. Next draw *Cc'–d'* parallel to *Aa'–b'* and *c–f* parallel to *a–Bb*. Then *Cc'–d'* is the vertical projection, and *c–f* is the horizontal projection of the line *C–D* in space which is the required line of intersection of the plane T with R.

Check. Assume a new horizontal plane (i.e. parallel to H) and if the lines cut from T and R intersect *C–D* the construction is accurate.

NOTE. As the horizontal traces *HT* and *HR* do not intersect within the limits of the drawing, *d* cannot be determined and *F* is the last point on the line of intersection that can be shown in horizontal projection. A similar solution applies if the horizontal traces of the plane intersect within the limits of the drawing and the vertical traces do not.

75. PROBLEM 22. To determine the line of intersection of two planes when neither horizontal nor vertical traces intersect within the limits of the drawing.

Analysis. Cut the two given planes T and R by a third plane S parallel to H. The plane S will cut from T and R straight lines (which will be parallel to H) and which will intersect in a point common to both T and R, hence, in their line of intersection. Having determined one point on the line of intersection, its direction can be found, and the projections drawn as in Problem 21, page 62.

Construction. See Fig. 63. Let *VT*, *HT* and *VR*, *HR* represent the traces of the given planes. Draw *VS* the vertical trace of the horizontal plane S. This plane S cuts the plane T along the line *E–C* which is vertically projected at *Ee′–c′* and horizontally projected at *e–c*; it cuts the plane R along the line *C–F* which is vertically projected at *c′–Ff′* and horizontally projected at *c–f*. Hence the point *C* is common to both planes and is therefore one point on their line of intersection. Next determine the line of intersection *A–B* of the planes T and the auxiliary plane Q (which is passed parallel to R). Through *c′* draw *c′–d′* parallel to *Aa′–b′* and through *c* draw *c–k* parallel to *Bb–a*. The line *C–D in space*, represented by these projections, is the required line of intersection of the two given planes.

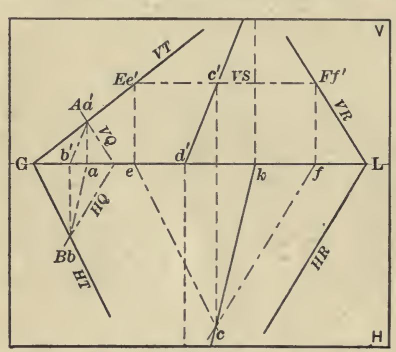


Fig. 63. — To Determine Line of Intersection when the Traces do not Intersect within Drawing Limits.

Check. Same as for Problem 21, page 63.

76. PROBLEM 23. To determine the point in which a given straight line pierces a given plane.

Analysis. In Fig. 64 (a) *A–B* is the given line and T the given plane. If a plane R is passed through the line *A–B* so as to intersect the plane T, the line of intersection *Cc–Dd′* of these planes contains all the points common to both R and T. As the piercing point must be in the line *A–B* and on *Cc–Dd′* it must be at *O* their point of intersection.

Construction. Any number of planes could be passed through *A–B*, but the simplest one to use in the solution of the problem is a plane perpendicular to H [see Fig. 64 (b)]. The trace *HR* must therefore pass through the horizontal projection *a–b* of the given line, and *VR* must be perpendicular to *G–L*. *Cc–d* is the horizontal projection, and *c′–Dd′* the vertical projection of *C–D*, the line of intersection of the planes. The horizontal projections of *A–B* and *C–D* fall along the same straight line, but their vertical projections intersect at *o′*. Hence the line *A–B* pierces the plane T at the point *O* in space.

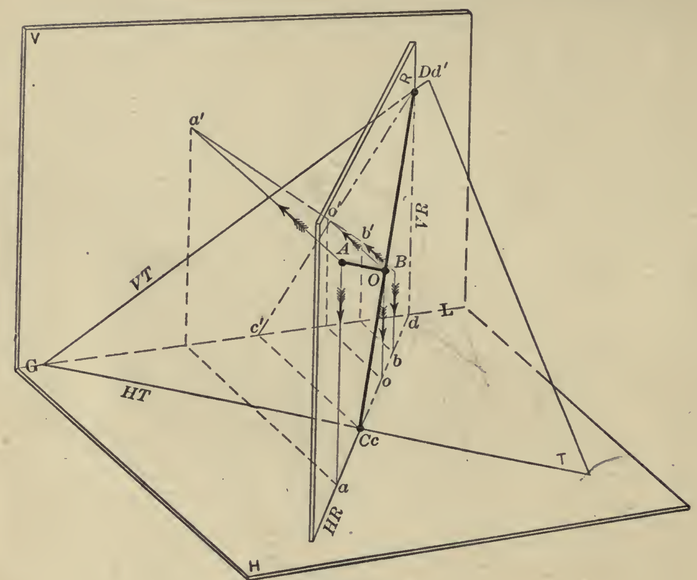
Check. Pass a plane through *A–B* and perpendicular to V, and find the piercing point *O*.

77. **Definitions. A dihedral angle** is the angular *space* included between two planes which intersect, and its measure is the angle formed by drawing a line on each plane from the same point on their line of intersection and perpendicular to it (see Fig. 69, page 72).

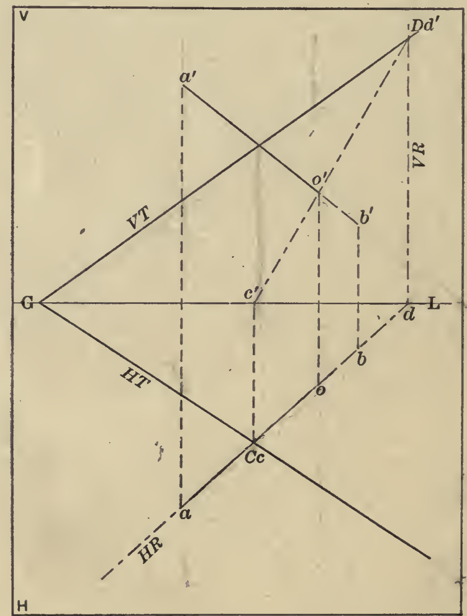
If one plane meets another in such a way as to make the adjacent dihedral angles equal, the planes are said to be perpendicular to each other.

78. **Principle.** A theorem in Solid Geometry proves that if each of two intersecting planes are perpendicular to a third plane, their line of intersection is perpendicular to that plane. Also a scholium in Solid Geometry states that a perpendicular to a plane is perpendicular to every straight line drawn in the plane through the foot of this perpendicular.

79. **THEOREM VIII. If a line is perpendicular to a plane, the vertical projection of the line is perpendicular to the vertical trace of the plane, and the horizontal projection of the line is perpendicular to the horizontal trace of the plane.**

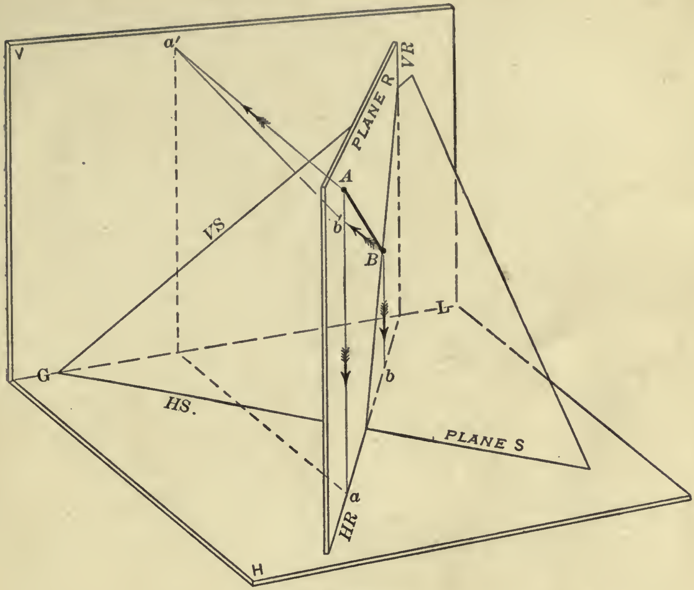


(a) Perspective.

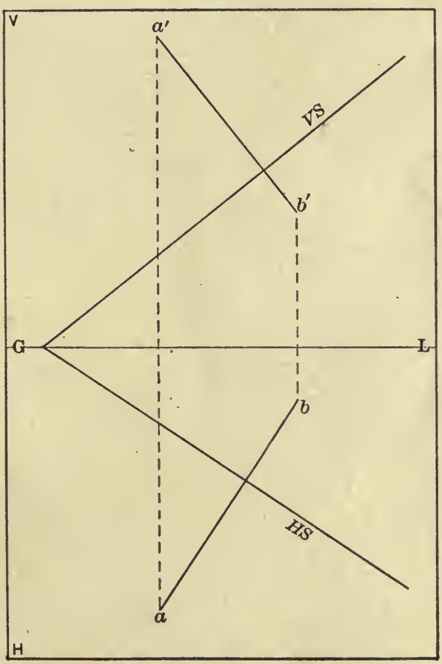


(b) Construction.

Fig. 64. — To determine the Piercing Point of any Line on any Plane.



(a) Perspective.



(b) Construction.

Fig. 65. — The Projections of a Line Perpendicular to a Plane are respectively Perpendicular to the Traces of that Plane.

Proof. (See Fig. 65.) If an auxiliary plane R is passed through the line *A–B* and perpendicular to H, the horizontal trace *HR* of this plane must coincide with the horizontal projection of the line. The plane R will also be perpendicular to the plane S since it was passed through the line *A–B* which is perpendicular to S. The traces *HR* and *HS* are perpendicular to each other, being the traces of two intersecting planes which are at right angles, and which two planes are cut by a third plane H. Hence the horizontal trace of the given plane S is perpendicular to the horizontal projection *a–b* of the given line. Similarly, it can be proven that the vertical projection of the line is perpendicular to the vertical trace of the plane.

80. PROBLEM 24. To draw through a given point a straight line perpendicular to a given plane, and to determine the distance from the point to the plane.

Analysis. Through the given point pass a perpendicular to the given plane. Find the point in which the perpendicular pierces the plane. Determine the true distance between this piercing point and the given point.

Construction. Let *VT* and *HT* (Fig. 66) represent the traces of the plane T and *a′* and *a* the projections of the point. Through *a′* draw *a′–b′* perpendicular to *VT*, and through *a* draw *a–b* perpendicular to *HT*. To find where *A–B* pierces T pass the plane R through *A–B* and perpendicular to H. Find the line of intersection *C–D* of R and T, and the point *B* where *A–B* crosses this line of intersection *C–D* is a point common to the line *A–B* and the plane T, hence is the piercing point of the line on the plane. Revolve the line *A–B* so that it is parallel to V and the true distance from the point to the plane is determined since the line then projects on *V* in its true length.

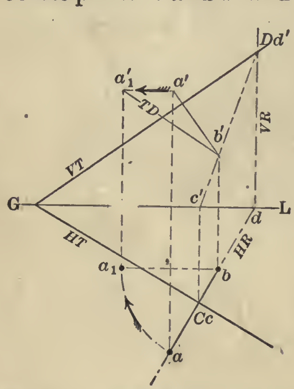


Fig. 66. — To Draw through a Point a Line Perpendicular to any Plane.

Check. Any line in the plane T drawn through B must be perpendicular to A–B. Also to test the distance of the point from the plane, revolve the line A–B into H.

81. Notation. A projected point on a plane other than V or H takes the subscript of the plane, thus the point A projected on the plane T is marked A_T and the projection of A_T on V is a'_T and on H is a_T.

82. PROBLEM 25. To project a given straight line on a given oblique plane.

Analysis. If the line is *not* perpendicular to the plane its projection on the plane will be a straight line.

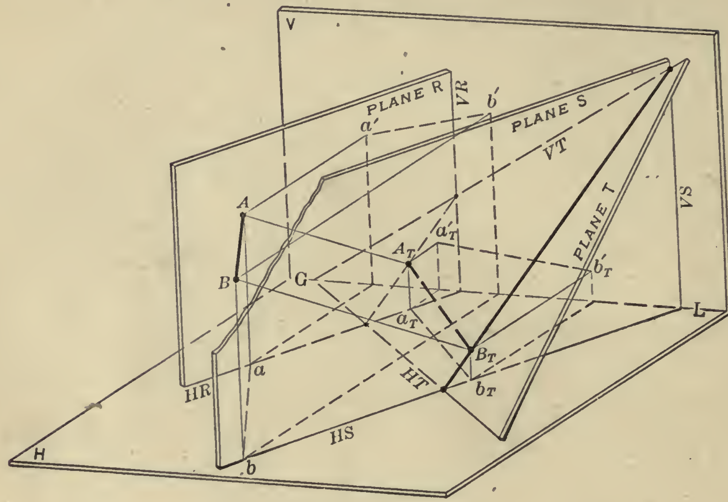
Let A–B [Fig. 67 (a)] be the given line in space, and its projections on V and H are a'–b' and a–b. The projection of A–B on the given plane T, however, is A_T–B_T, and the required projections (i.e. on V and H) are a'_T–b'_T and a_T–b_T.

Through the points A and B of the line in space, pass perpendiculars to the given plane T. Through these perpendiculars to T, pass the planes R and S perpendicular to H, and determine the point in which the perpendiculars pierce T. The straight line joining the two vertical projections (on V) of these piercing points on T is the vertical projection of the required line, and the straight line joining the horizontal projections of the two piercing points of the perpendiculars to T is the horizontal projection of the required line.

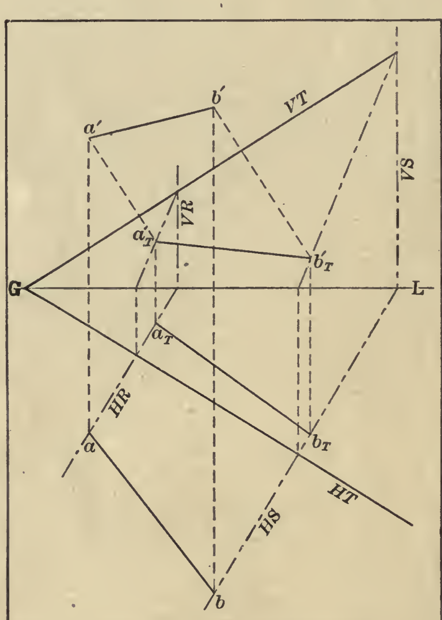
Construction. Let a'–b' and a–b represent the given line and VT, HT [Fig. 67 (b)] the given plane. From A and B draw perpendiculars and determine the projections a'_T, a_T and b'_T, b_T, of the points where these perpendiculars pierce the plane T. The straight line a'_T–b'_T is the vertical projection and a_T–b_T is the horizontal projection of the required *projection* A_T–B_T.

Check. Any point on the given line other than A or B when projected on T must have its projections on a'_T–b'_T and a_T–b_T.

83. PROBLEM 26. To determine a plane which shall pass through a given point and be perpendicular to a given straight line.

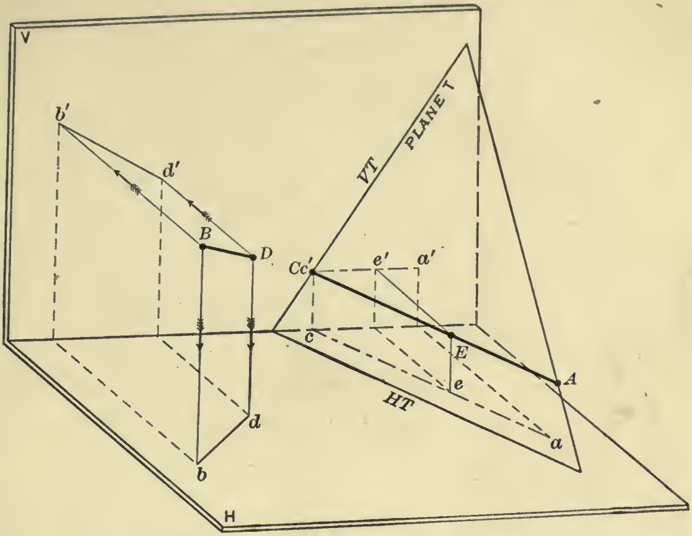


(a) Perspective.

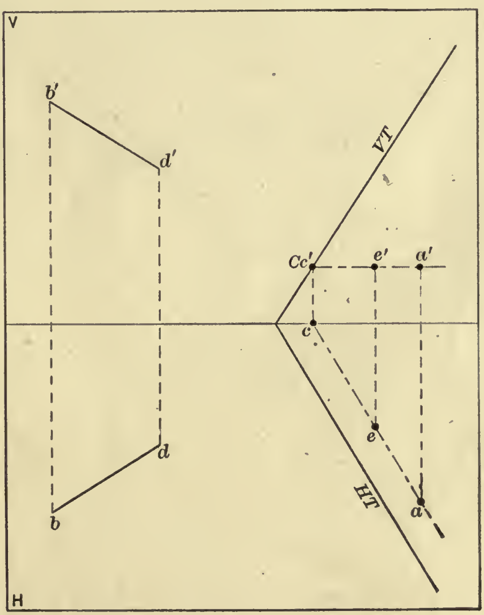


(b) Construction.

Fig. 67. — To Project a Given Straight Line in a Given Oblique Plane.



(a) Perspective.



(b) Construction.

Fig. 68. — To Pass a Plane through a Given Point and Perpendicular to a Given Straight Line.

Analysis. See Fig. 68 (a). Since the plane T is to be perpendicular to the line *B–D*, its vertical trace *VT* must be perpendicular to the vertical projection *b′–d′* of the line, and its horizontal trace *HT* must be perpendicular to the horizontal projection *b–d* of the line. If, therefore, a line *A–C* be drawn through the point *E* and perpendicular to the line *B–D*, also parallel to the horizontal trace of this plane, it will be a line of the required plane. The vertical piercing point *Cc′* of the line *A–C* is a point on the vertical trace *VT* of the plane, and since the direction of the trace is known, one point is sufficient to determine it.

Construction. Let *b′–d′* and *b–d* in Fig. 68 (b) be the vertical and horizontal projections of the given line. Let *e′* and *e* be the projections of the given point. Through *e* draw *e–c* perpendicular to *b–d* and determine the piercing point *Cc′*. Through *Cc′* draw *VT* perpendicular to *b′–d′*. Draw *HT* parallel to *a–c* and to meet *VT* on *G–L*.

Check. Pass a plane through *B–D* and perpendicular to H. Revolve the line of intersection of this plane with the required plane into V, carrying the line *B–D* also into V. If the line *B–D* and the line of intersection of the planes are perpendicular when revolved into V the construction is correct.

84. Principle. The angle α between any two planes such as R and T (see Fig. 69) must be measured in a plane P, perpendicular to both R and T, hence it is the angle made by the lines of intersection of P with R and T.

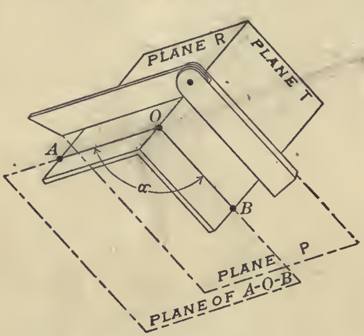
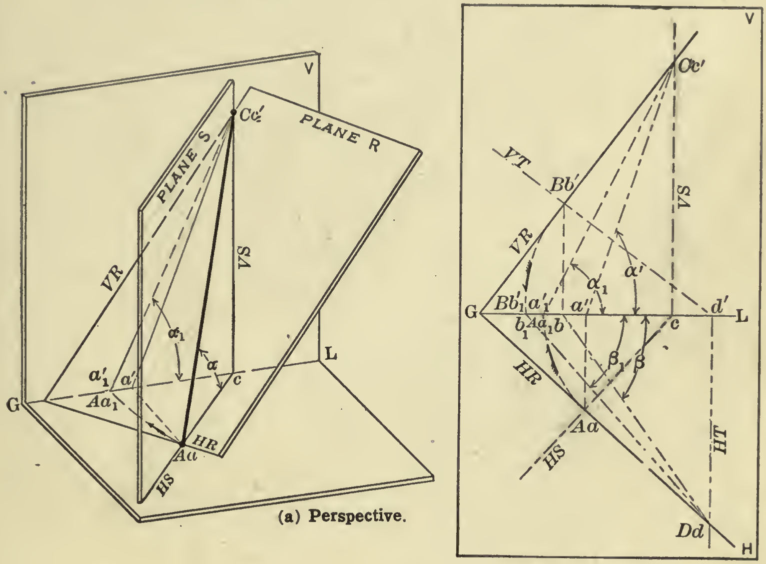


Fig. 69.—The Angle between Two Intersecting Planes.

85. PROBLEM 27. To determine the angle that a given plane makes with V and H.

Analysis. A plane S [Fig. 70 (a)] passed perpendicular to the horizontal trace *HR* of the given plane R will cut a line from R and also one from H. The angle between these two intersecting lines is the measure of the angle the plane R makes with H.

Construction. Let the given plane be represented by VR and HR [Fig. 70 (b)]. Through any convenient point Aa on HR draw HS, the horizontal trace of a plane perpendicular to HR; the vertical trace of the plane S must be perpendicular to G–L at c. The plane S cuts from R the line A–C given by its pro-



(a) Perspective.

(b) Construction.

Fig. 70. — To Determine the Angle a Given Plane makes with V or H.

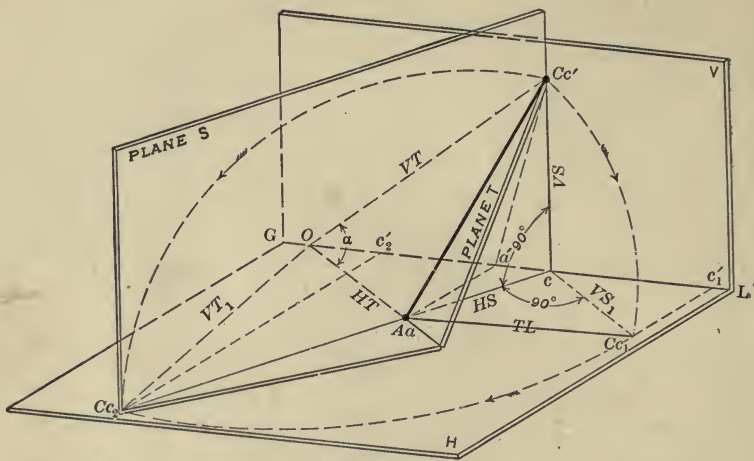
jections a'–Cc' and Aa–c. Revolve A–C into V and α_1 is the required angle that R makes with H.

In a similar way by taking an auxiliary plane perpendicular to the vertical trace of the given plane, the angle β that the given plane makes with V is determined. See Fig. 70 (b).

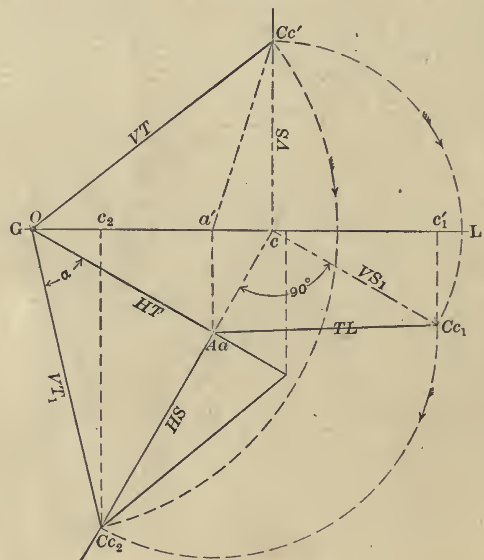
Check. Revolve the plane S about HS into H and determine the angle α.

86. PROBLEM 28. To determine the true angle between the traces of a plane.

Analysis. This is *not* the angle the traces make with G–L nor the angle between these traces when V and H are folded into the plane of the paper, but it is the angle α [see Fig. 71 (a)] between the traces measured on the plane *itself*. To find this angle, a



(a) Perspective.



(b) Construction.

Fig. 71. — To Find the True Angle between the Traces of a Plane.

plane S perpendicular to H and the trace *HT* is passed to intersect the given plane T, and the line of intersection *Aa–Cc′* of these two planes and the trace *VS* are revolved about *HS* into H.

Construction. The given plane is represented by *VT* and *HT* in Fig. 71 (b). Pass the plane S to intersect T also perpendicular to H and to T; find the projections of the line of intersection *A–C*. When *C* is revolved about *HT* into H it will fall along the line *HS* and at a distance from *Aa* equal to its true length. Swing *VS* into H as shown at VS_1, to find the true length of *A–C*. Swing Cc_1 to its proper position at Cc_2. Then the angle between *HT* and VT_1 is the one required.

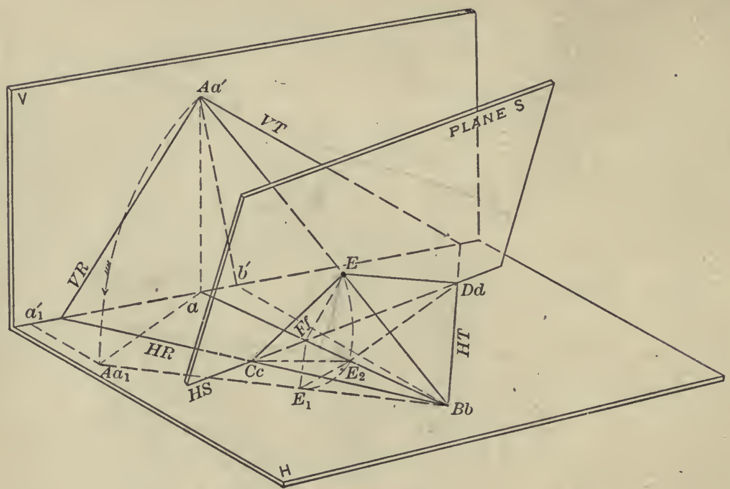
Check. Since the point *Cc′* must fall on HS when revolved into H, and as *O–Cc′* is a true length (being on V) the arc from *O* with the radius *O–Cc′* must cut *HS* at Cc_2.

87. PROBLEM 29. To determine the angle between two intersecting planes.

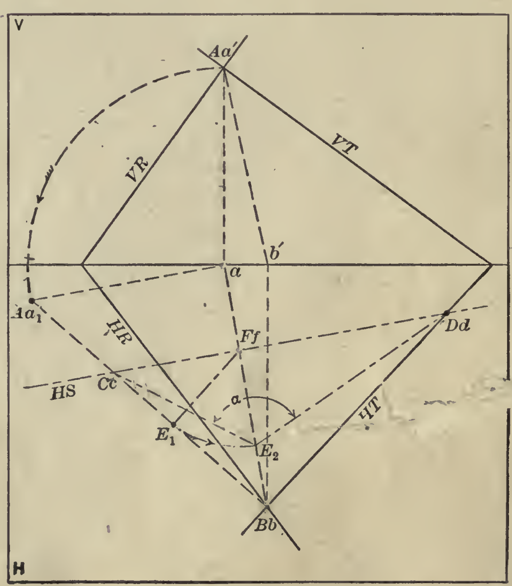
Analysis. See Fig. 72 (a). Pass a plane S perpendicular to the line of intersection *A–B* of the two planes, and it will cut from the plane R the straight line *C–E* and from the plane T the straight line *E–D*. Each of these lines is perpendicular to the line of intersection *A–B* at a common point *E*. The angle between the lines *C–E* and *E–D* will be the measure of the required angle, and may be found by revolving the plane S containing the desired angle into H.

Construction. Draw the traces *HR*, *VR*, and *HT*, *VT* of the planes R and T [see Fig. 72 (b)] and determine the line of intersection *A–B*. The horizontal *trace* of the plane S is perpendicular to the horizontal *projection* of the line *A–B*. The line of intersection of S with R and T is found by revolving the line *A–B* into H at Aa_1–*Bb* carrying with it the point E (which is the vertex of the required angle) to E_1. If, however, the plane S is revolved into H about *HS*, the point *E* would fall in the perpendicular *a–Bb* to *HS* and at a distance from *Ff* equal to *Ff*–E_1, hence at E_2. As *Cc* and *Dd* are both on *HS* their positions remain fixed when the plane S is revolved into H and *Cc*–E_2–*Dd* is the required angle. The vertical projection of the angle has not been shown, as it is not necessary to the solution of the problem.

Check. Take another plane perpendicular to *A–B*. The intersection of this plane with R, P, and H forms a triangle whose base lies in H; find the true length of the other two sides and on this base construct the triangle in H. The angle at the vertex of the triangle so found is the required angle.



(a) Perspective.



(b) Construction.

Fig. 72. To Determine the Angle between Two Intersecting Planes.

88. Principle. If a line in space is perpendicular to a plane in space, the line and the plane make complemental angles with the plane of projection. That is, if as shown in Fig. 73 the plane T makes α degrees with H then O–B which is perpendicular to T must make 90–α degrees, in order that the sum of the angles of the triangle C–O–B equal 180 degrees.

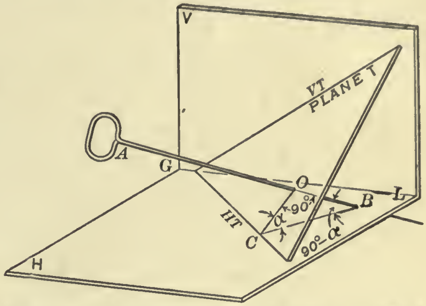


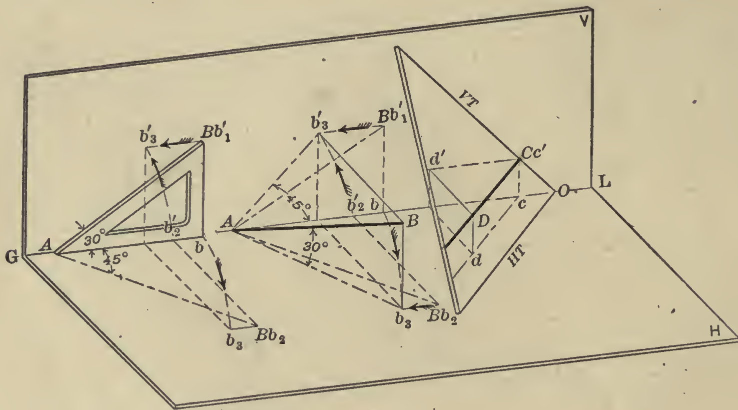
Fig. 73.—The Angle a Line Perpendicular to One of Two Intersecting Planes makes with the Other Plane is the Complement of the Angle between the Two Planes.

89. PROBLEM 30. To determine the traces of the plane that contains a given point and makes given angles with V and H.

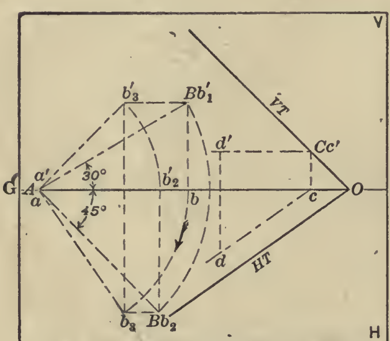
Analysis. If in Fig. 74 (a) the line A–B in space is brought to a position where it makes the complemental angles with V and H that the plane is required to make, then the plane T perpendicular to A–B and containing the given point D fulfills the requirements of the problem.

Construction. Assume that the required plane is to make 60 degrees with H and 45 degrees with V. Draw a construction line, A–Bb_1', [see Fig. 74 (b)] of any convenient length, and in V so that it makes $90° - 60° = 30°$ with G–L. A–b therefore represents the length of the horizontal projection of a line, whose real length in space is A–Bb_1', when it makes 30 degrees with H. Let the point A remain stationary and swing B out from V. If the angle Bb_1'–A–b is kept constant b must move along the arc b–b_3 and Bb_1' moves along the straight line Bb_1'–b'_3 parallel to G–L. Similarly, draw a line of the same length as A–B in H so that it makes $90° - 45° = 45°$ with G–L. This determines the length of the vertical projection of the original line when it makes the proper angle (90°–45°) with V. Hence the intersection of an arc of a radius A–b_2' (having center at A) with the straight line Bb_1'–b_3' determines b_3'. Similarly, b_3 is determined and the check on the accuracy of the work is that b_3' and b_3 will fall on the same line perpendicular to G–L. Draw a line

D–C through the given point *D* so that *D–C* is parallel to H and its horizontal projection *d–c* is perpendicular to $A–b_3$. This



(a) Perspective.



(b) Construction.

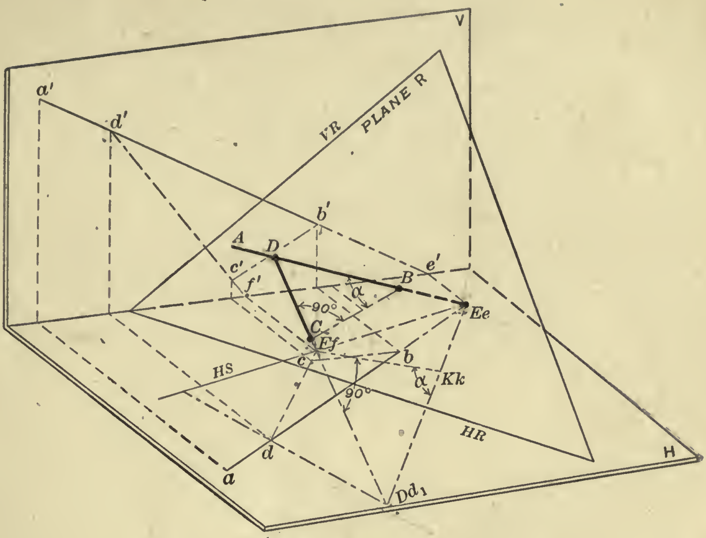
Fig. 74.—To Determine the Plane that will contain a Given Point and make Given Angles with V and H.

line will be a line in the required plane and parallel to its horizontal trace. Determine the piercing point *Cc′* of this line. Draw *VT* through *Cc′* perpendicular to $A–b_3'$; draw *HT* through *O* and parallel to *d–c*.

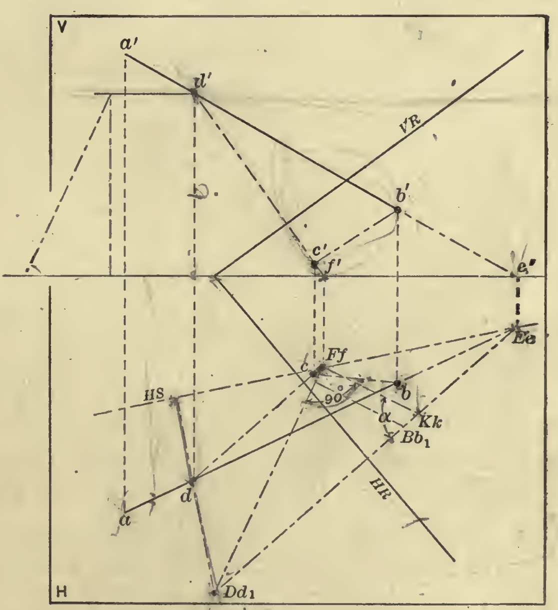
Check. See Problem 27, page 72.

90. **Principle.** The angle which a straight line makes with a plane is the angle which the line makes with its projection on the plane. Also, this angle is the complement of the angle formed by the line in space, and a perpendicular to the plane from some point on the line. Therefore, if this last angle is determined, and subtracted from 90 degrees, the result will be the angle required.

91. **PROBLEM 31. To determine the angle which a given straight line makes with a given plane.**



(a) Perspective.



(b) Construction.

Fig. 75. — To Determine the Angle between a Given Line and a Given Plane.

Analysis. See Fig. 75 (a). From any convenient point D on the given line $A-B$ draw a perpendicular $D-C$ to the given plane R. The projections of $D-C$ will pass through the projections of the assumed point and be respectively perpendicular to the traces of the plane. Find where the line $A-B$ and the perpendicular $D-C$ each pierce H and a line $Ff-Ee$ drawn through these points will be the horizontal trace of a plane S containing the line $A-B$ and the perpendicular $D-C$. Revolve the plane S into H about its horizontal trace and the true measure of the complementary angle is found; subtracting this angle from 90 degrees gives the required angle.

Construction. In Fig. 75 (b) draw HR and VR to represent the plane R, also $a'-b'$ and $a-b$ to represent the line $A-B$. From d and d' draw the projections $d-f$ and $d'-f'$ perpendicular respectively to HR and VR. The line $D-F$ pierces H at Ff and $A-B$ pierces H at Ee. Through Ff and Ee draw HS the horizontal trace of the plane containing the *complementary* angle. Revolve S into H. The points Ff and Ee remain fixed and D falls at Dd_1. $Ff-Dd_1-Ee$ is the angle included between the lines. From Ff draw a perpendicular to $Ff-Dd_1$ to meet Dd_1-Ee and α is the angle the line $A-B$ makes with R.

Check. Project the line $A-B$ upon the plane R and this projection will be the base of a right angle triangle, in which the line itself is the hypotenuse, and the perpendicular to the plane the altitude. Revolve this triangle into V or H.

92. PROBLEM 32. To draw a straight line through a given point to intersect a given line at a given angle.

Analysis. Pass a plane through the given point and given line; revolve this plane parallel to V or H, and when in this position draw the required line from the point to intersect the given line at the given angle. Counter-rotate the plane and find the new projections of the required line.

Construction. Through the given point D (Fig. 76) draw a line $D-F$ to intersect the given line $A-B$ at any convenient point F. Determine the traces of the plane T which contains the lines $A-B$ and $D-F$. Revolve the plane T about HT as an axis into H. From Dd_1 the revolved position of the point D, draw the line D_1–C_1 to intersect A–B_1 at the required angle α. Counter-rotate C_1, and D–C_1 shown by its projections d–c and d'–c' is the required line.

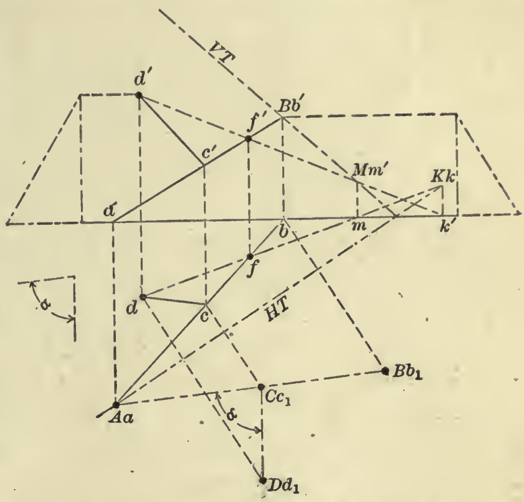


Fig. 76. — A Straight Line drawn through a Given Point to Intersect a Given Line at a Given Angle.

93. **PROBLEM 33. To determine the true form of any plane figure given by its projections.**

Analysis 1. Let A–B–C (Fig. 77) be the figure whose true form is required. If the true length of each side is found by revolving each of these sides in turn parallel to H or V, the triangle can be constructed. If the figure is other than a triangle it can be divided into several triangles by diagonal lines.

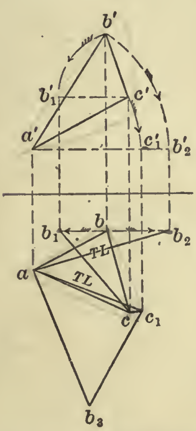


Fig. 77. — To Determine the True Form of a Plane Figure. (Construction I.)

Construction 1. Swing each line of the figure parallel to H and thus determine the true lengths in H. On the most convenient true-length, a–c, construct the triangle.

Analysis 2. Determine the traces of the plane which contains the figure and revolve each vertex of the figure into V or H and about one of the traces.

Construction 2. See Fig. 78. Extend the sides of the figure until they pierce V and H. Two such piercing points Ee', Ff' and Kk, Mm in each plane of projection determines the vertical trace VR and the horizontal trace HR of the plane containing the figure. The radius of rotation for the point A will be the true distance from this point in space to the point y on HR. This may be found by constructing the right triangle a–A–y at any convenient place. A–a is equal to the perpendicular distance from the point A in space to a, that is, the same as from A to H, hence is equal to a'–x, and a–y is equal to the perpendicular distance from a to HR, hence is equal to a–y.

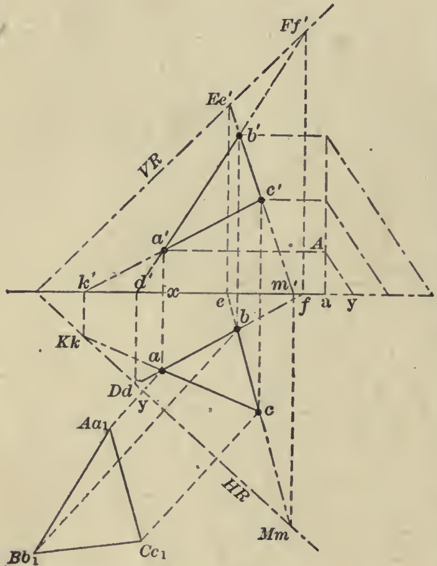


Fig. 78. — To Determine the True Form of a Plane Figure. (Construction II.)

Swing the figure about HR into H. The horizontal projection of the path traced by each point as it swings about HR is a straight line perpendicular to HR. Hence through a draw a perpendicular to HR, and lay off the distance y–Aa_1 equal to y–a. Similarly, construct the other triangles and determine the revolved position of B and C. The figure Aa_1–Bb_1–Cc_1 is the one required.

Analysis 3. Determine as before the traces of the plane which contains the figure, and revolve this plane into H or V about its horizontal or vertical trace.

Construction 3. (See Fig. 79.) When the plane R is revolved about HR into H, the distance O–Ff' must equal O–Ff_1' because both are true lengths. Also Ff' when revolved must fall on the perpendicular to HR through f, hence at Ff_1 and as Dd is on HR it does not change position and the revolved position of F–D is determined.

Similarly, determine the revolved position of E–M. The point Aa_1 falls on the intersection of the line Ff_1–Dd and the perpendicular to HR through a.

The points B and C in their revolved position are similarly determined.

Check. As a check that the plane contains the figure, draw two intersecting lines through the figure and if these lines pierce V and H in the traces of the plane the work is accurate. Any one of the above methods may be used as a check on another to determine the true shape of the figure.

94. **Axiom.** The above method can be used for **finding the angle between two intersecting lines** such as *A–B* and *B–C*. **When the angle** is shown in its true size it **can be bisected or otherwise divided** and the **dividing line can then be revolved back to the original position with the plane R, and its projections found.**

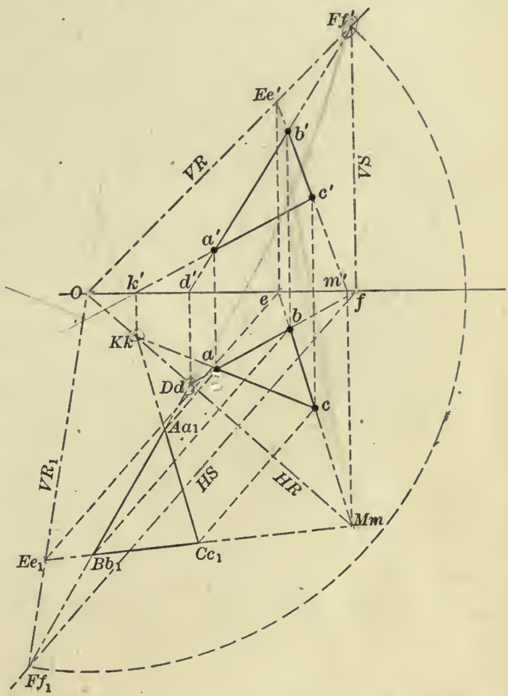
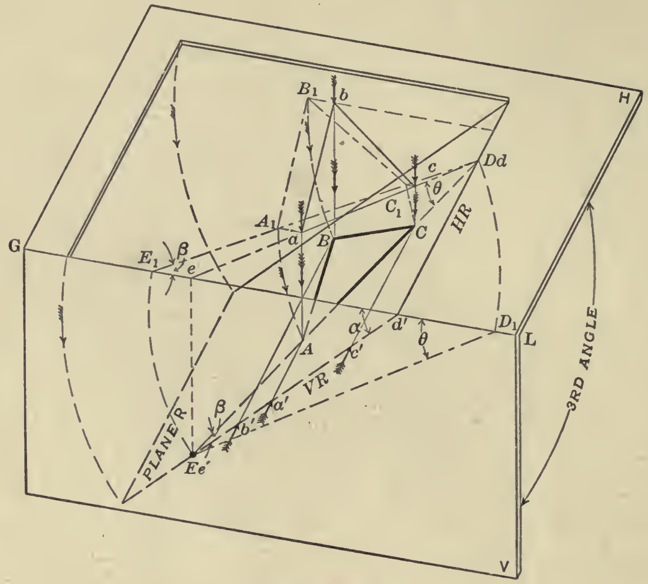


Fig. 79. — To Determine the True Form of a Plane Figure. (Construction III.)

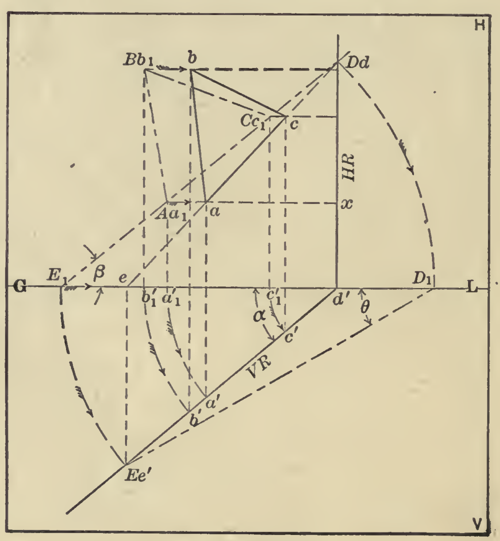
95. PROBLEM 34. To determine the third angle plan and elevation* of any plane figure when one edge of the figure is inclined at any given angle to V and the plane of the figure is perpendicular to V and inclined at any given angle to H. Also to determine the angle the given edge makes with H.

Analysis. Fold the paper to represent the *third* angle [see Fig. 80 (a)]. Draw *HR* and *VR* to represent the traces of the plane that will contain the required figure. *HR* will be perpendicular to *G–L* and *VR* will make the angle α with *G–L* that the plane of the figure *A–B–C* makes with H. Draw E_1Dd

* The vertical projection is technically known as an *elevation* and the horizontal projection as a *plan* view.



(a) Perspective.



(b) Construction.

Fig. 80. — To Determine the Third Angle Projection of a Plane Figure to Lie in a Definite Position.

on H so that it makes with $G-L$ the given angle β that the edge of the figure is to make with V. Construct the required figure $A_1-B_1-C_1$ on the line E_1-Dd. Cut H along the double line as shown in the figure and swing it about HR until the edge cut along the $G-L$ coincides with VR. The figure $A_1-B_1-C_1$ then occupies the required position shown by $A-B-C$, and its plan on H would be represented by $a-b-c$ found by the space projectors shown. The elevation would necessarily be in VR and at $a'-b'-c'$.

Construction. [See Fig. 80 (b)]. Draw HR perpendicular to $G-L$ and VR at the required angle α that the plane of the figure must make with H. Draw E_1-Dd to make the angle β with $G-L$ that the edge of the figure is required to make with V. On E_1-Dd construct the required figure $Aa_1-Bb_1-Cc_1$. Swing $Aa_1-Bb_1-Cc_1$ around HR into the plane R. Thus a_1' moves in the arc of the circle to a' and Aa_1 moves along the perpendicular Aa_1-x to the horizontal projection a, determined by the ruled projector from a'.

Check. Keeping E_1 against V, swing the line E_1-Dd into the plane R; $e-Dd$ must then pass through $a-c$.

96. NOTE. To determine the angle that the edge $A-C$ makes with H, swing the line $E-D$ about E into V. If the edge $A-C$ is required to make the angle θ with H and the angle α is the angle the plane R makes with H and which is to be determined, assume the point Ee' and draw $Ee'-D_1$ making the angle θ with H. Swing D_1 to Dd and then swing $E-D$ into H. On E_1-Dd construct the figure and swing it into the plane R.

97. **Principle.** The projection of a circle is determined by finding the projections of a number of points in its circumference and drawing an ellipse through the points thus projected. In general the projections of circles are ellipses, but if the plane of the circle is perpendicular to the plane of projection the projection of the circle becomes a straight line, having a length equal to the diameter of the circle.

98. **PROBLEM 35. To draw the third angle projections of a circle in a given plane.**

Analysis. Let it be required to find the plan and elevation of a circle which lies in the plane R, represented by its traces *HR* and *VR* (see Fig. 81).

The plane R which contains the circle is revolved about its vertical trace *VR* into V and the circle is drawn in its true size.

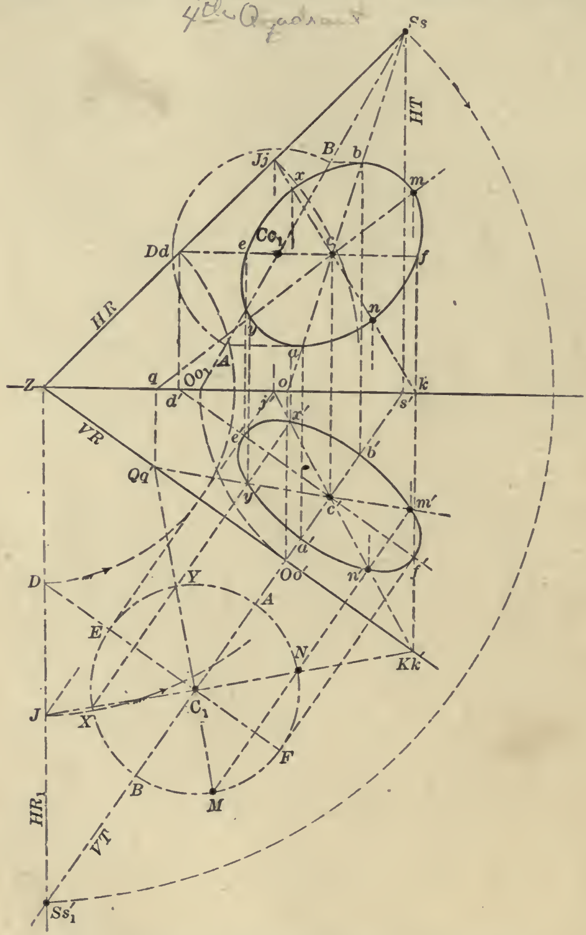


Fig. 81. — Third Angle Projection of a Circle in a Given Plane.

The circle is divided into eight (or any number) equal parts by the points *B–M–F–N*, etc., marked on its circumference. By counter-revolution these points are brought into their true position (in the plane R) and the ellipses drawn through the projections of these points give the required plan and elevation of the circle.

Construction. Determine the traces *VR* and *HR* of the plane R so that this plane contains the point *C* and makes the required angles with V and H (see Problem 30, page 77). Revolve the plane R and with it the point *C* about *VR* into V; to do this, pass the plane T through the point *C* and perpendicular to *VR*. The revolved position of *Ss* must be on the trace *VT* and at a distance *Z–Ss* from *Z*, hence it falls at the position Ss_1'. The true distance of C_1 from *Oo′* is Oo_1–Cc_1, and is found by revolving the line of intersection *O–S* of the planes R and T into H. On c_1 as a center describe the circle of the given size. Divide the circumference into eight equal parts as shown. As the points *Qq′* and *Kk′* are on *VR* which is the axis of rotation, and the projections of *C* are already determined, the projections of the lines *K–C–J* and *Q–Y–C* can be drawn through the respective projections of the point *C* and these lines will then lie in the plane R.

Since the points *X*, *Y*, and *M*, *N* must move (in projection) during counter-revolution along perpendiculars to *VR*, the projections *x′*, *y′*, *m′*, and *n′* are determined by the intersections of these perpendiculars and the vertical projections *Kk′–j* and *Qq′–m′* of the lines; also *y*, *x*, *n*, and *m* must be on the horizontal projections of the lines *K–J* and *Q–M* on which the points *N*, *X*, *Y*, and *M* are located, and on the ruled projectors from *x′*, *y′*, *n′*, and *m′*, hence at the points shown. The line *D–C–F* is drawn parallel to *VR*, hence when counter-revolved it will pass through *c′* and fall parallel to *VR*, therefore its projections are *d′–c′–f′* and *Dd–c–f* and the projections of *E* and *F* fall as shown.

To determine *A* and *B* in projection lay off the diameter of the circle on Oo_1–*Ss* and revolve the points to *o–Ss* as shown. Then *a′* and *b′* are at the intersection of the ruled projectors from *a* and *b* and the vertical projection of *O–S*.

Check. Pass any line through the circle parallel to *HR*, and see if its projections cut the projections of the circle on the same ruled projectors or, in other words, pass horizontals or verticals through the circle.

99. PROBLEM 36. To determine the third angle plan and elevation of a plane figure inclined to both V and H.

Analysis. Construct the figure *Aa′–Bb′–Cc′–Dd′* in V, drawing it in its true size and shape; revolve it about an axis *VT* lying in V and perpendicular to H, until its horizontal projection makes the given angle, say 45 degrees, with *G–L*; revolve the vertical projection about an axis through *A* and perpendicular to H, until an edge of the figure makes the given angle, say 60 degrees, with *G–L*.

Construction. If *Aa′–Bb′–Cc′–Dd′* (Fig. 82) represents the true shape and size of the plane figure, its horizontal projection *a–b–c–d* must be in *G–L*. Draw *VT* through *Aa′* and perpendicular to *G–L*. Then draw *HT* to make the required angle, say 45 degrees, with *G–L* that the figure in space is to make with H. When the figure is revolved about *VT* as an axis, the projections *b*, *c*, *d* move in the arcs of circles to b_1–c_1–d_1 on *HT*. As the distance from H of each point of the figure does not change, the new vertical projections fall at the intersections of the ruled projectors and the construction lines *E–Bb′*, etc., drawn parallel to *G–L*. If next the horizontal projection is to be determined when the side Aa'–d_1' makes, say 60 degrees, with *G–L*, transfer (for clearness) and revolve the vertical projection by means of the base line Aa_1'–x_1–y_1–z_1, and the ordinate b_1'–x_1, etc., as shown. In this transfer and revolution, the distance of the points from V do not change; hence the distances from *G–L* of the new horizontal projections are determined by projecting across from a–b_1–c_1–d_1 and up from Aa_2'–b_2'–c_2'–d_2'.

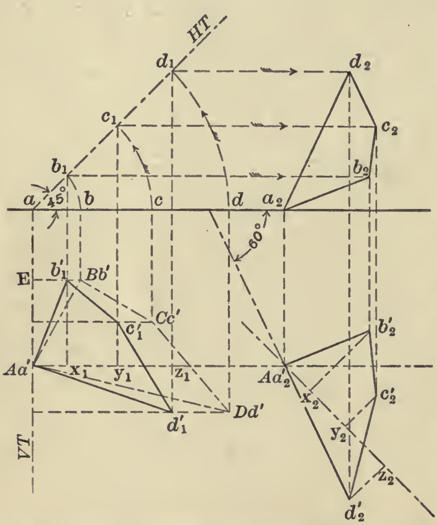


Fig. 82.—The Third Angle Projection of a Plane Figure in a Plane Inclined to V and H.

NOTE. Had the plane figure in the preceding problem been a circle it would have been projected by means of assumed points on the original circle (see Fig. 83).

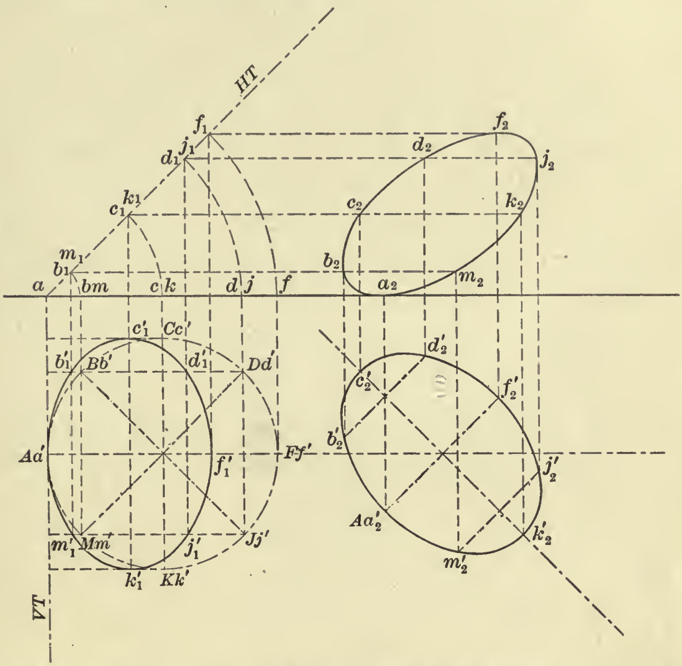


Fig. 83.—Third Angle Projection of a Circle lying in a Plane Inclined to V and H.

CHAPTER V

SOLIDS

100. A solid has length, breadth, and thickness and is completely bounded by surfaces, which may be plane or curved. **To project a solid** it is necessary to project the surfaces which bound its exterior form. These surfaces are represented by lines which would generate them if moved according to some fixed law. Since lines are fixed in length and direction by two points, one at each extremity, the projection of solids really consists of projecting a series of points.

101. PROBLEM 37. To draw the third angle plan and elevation of a cube when one face is in H and a vertical face makes a given angle with V.

Analysis. The face in H (i.e., the plan) will show as a square, the true size of one face of the cube. The side of this square nearest to V will make the same angle with *G–L* that the corresponding face of the cube makes with V. As four edges of the cube are perpendicular to H (i.e., parallel to V) they will appear in their true length in the elevation.

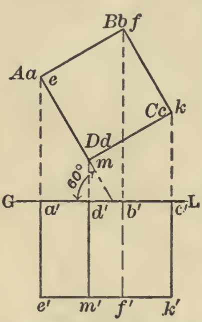


Fig. 84. — Third Angle Projection of a Cube.

Construction. (See Fig. 84.) Draw the side *Aa–Dd* to make the required angle, say 60°, with *G–L* that the face *A–D–M–E* of the cube makes with V. On the side *Aa–Dd* construct the square to represent the true size of the face in H. The *elevation* of this square is in *G–L* and at *a′–b′–c′–d′*. From these points draw perpendiculars *a′–e′*, *d′–m′*, *b′–f′*, and *c′–k′* each equal in length to an edge of the cube. Draw *e′–m′–f′–k′* parallel to *a′–b′–c′–d′* to represent the lower face of the cube. As the cube is regarded as opaque the edge *b′–f′* must be of a hidden-line construction, that is, of the dash-space construction, to show that it is invisible to the draftsman (see § 26, page 18).

Check. To test the accuracy of construction, measure each edge and check to see that the edges parallel in space are parallel in projection.

102. PROBLEM 38. To draw the third angle plan and elevation of a cube when one edge lies in H and makes 30° or any given angle with V and two parallel faces make 60° or any given angle with H.

Analysis I. Place the cube with one edge *B–F* in H and perpendicular to V. Swing the cube around this edge until the required parallel faces *A–B–F–E* and *C–D–M–K* make 60° with H. Maintaining this angle with H, swing the edge, *F–B*, until it makes 30° with V. The projections in this position are the required plan and elevation.

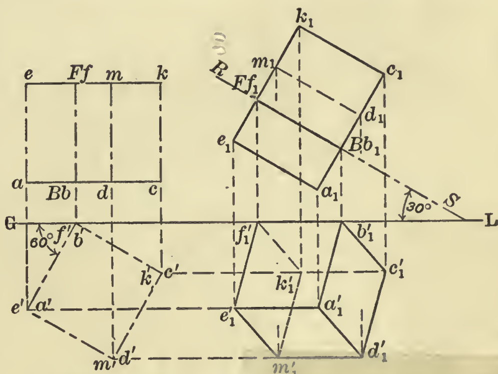


Fig. 85. — Projection of a Cube Inclined to V and H.

Construction. (See Fig. 85.) Draw the line *a′–b′* to make the required angle (60°) that the parallel faces of the cube make with H. On this line construct the square *a′–b′–c′–d′* to represent the elevation of the cube when the parallel faces make the required angle with H and are perpendicular to V. If the angle of 60° is maintained with H and the edge *Bb–Ff* is kept in H, this edge can take any angle to V without changing the *shape* of the plan. The elevation, however, changes shape.

Hence, at some convenient point, draw the line *R–S* to make the required angle (30°) that the edge in H is to make with V, and on this line lay off the edge Ff_1–Bb_1 and construct the original plan. Since the angle of the faces (60°) does not change, neither do the distances that the points *a′*, *e′*, *m′*, *d′*, *k′*, and *c′*

are from G–L. That is, they move parallel to G–L and are determined in their new positions a_1', e_1', m_1', d_1', k_1' and c_1' by the ruled projectors from a_1, e_1, m_1, d_1, k_1, c_1, etc.

Analysis II. Instead of revolving the plan as in Fig. 85 to make the given angle (30°) with V, take a new V (see Fig. 86) at 30° with the edge B–F. Project the plan upon this new G_1–L_1. In drawing the new elevation a_1'–Bb_1'–c_1'–d_1', etc., it must be noted that the distances on the ruled projectors from a_1', Bb_1', c_1', d_1', etc., to G_1–L_1 are the same as the distances from a', Bb', c', d', etc., to G–L (see § 22, page 13).

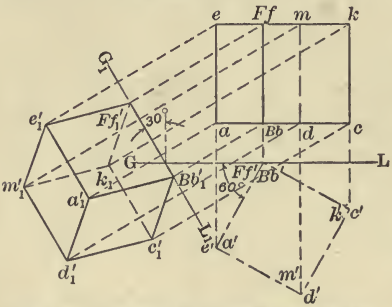


Fig. 86. — Projecting a Cube by use of a new V Plane.

NOTE. This method saves one drawing of the plan.

Check. As in Problem 37, page 91.

103. PROBLEM 39. To draw the third angle plan and elevation of a cube when one edge is inclined at 45° or any given angle to H and 30° or any given angle to V, and the diagonal of one face is perpendicular to V.

Analysis I. This problem consists of four distinct operations:

1st. Represent the cube with one face A–B–C–D in H and the *diagonal* B–D perpendicular to V. Draw the plan and elevation in this position (see Fig. 87 I).

2nd. Without changing the distance that the points E, A, C, K on the cube are from V incline the cube by swinging the point E about A until the given edge E–A makes the required angle (45°) with H. This changes the position of the elevation but not its shape or size, and all points move in planes parallel to G–H–L (see Fig. 87 II).

3rd. Revolve the given edge A–E until it makes the given angles with V and H (see Fig. 87 III, also § 54, page 46).

4th. Without changing the distance of any point on the cube from H, revolve the cube about the point A until the edge A_5–E_5 is parallel to the revolved position of the line A_4–E_4 shown in

Fig. 87 III. This revolution changes the position of the plan but not its shape or size, and all points in elevation move parallel to G–L (see Fig. 87 IV).

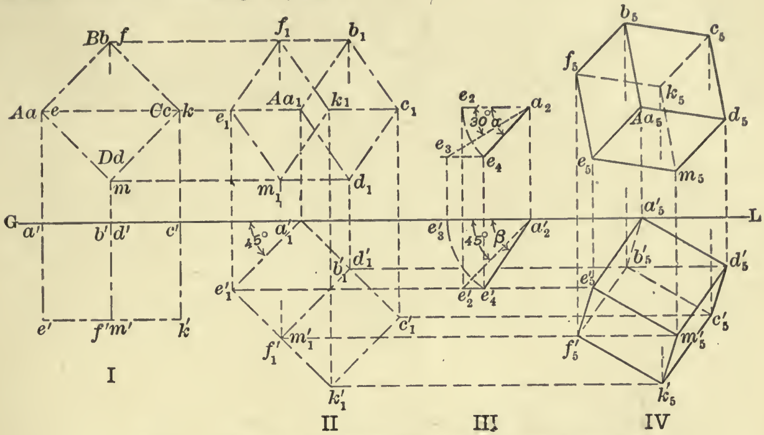


Fig. 87. — Projecting a Cube which is Inclined to both V and H.

Construction I. As the base of the cube is in H its plan will be the square *Aa–Bb–Cc–Dd*. Since the diagonal *B–D* makes 90° with V, *Bb–Dd* is drawn perpendicular to *G–L*, as shown in Fig. 87 I. The elevation of the base is in *G–L*. At a', b', c',

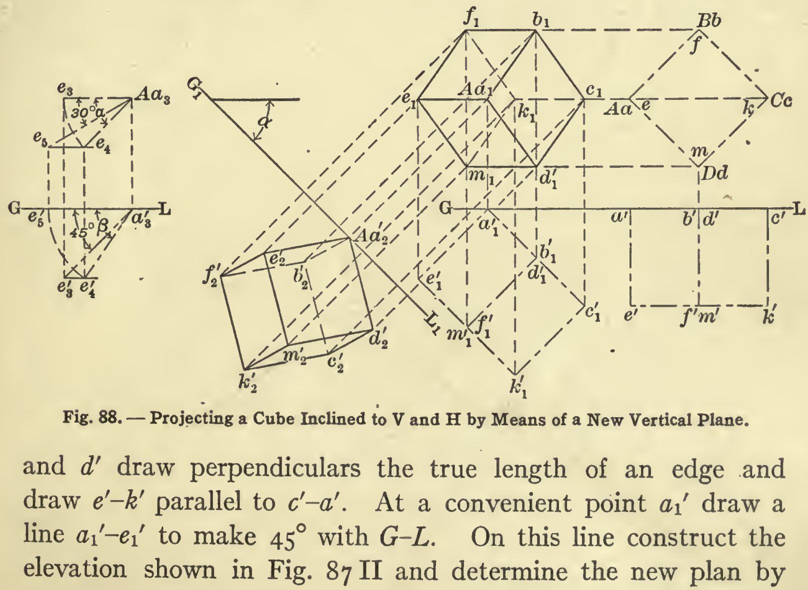


Fig. 88. — Projecting a Cube Inclined to V and H by Means of a New Vertical Plane.

and d' draw perpendiculars the true length of an edge and draw e'–k' parallel to c'–a'. At a convenient point a_1' draw a line a_1'–e_1' to make 45° with G–L. On this line construct the elevation shown in Fig. 87 II and determine the new plan by projecting across from the plan 87 I to meet the ruled projectors from the elevation in 87 II.

Take a line E_4–A_2 equal and parallel to the edge E–A and at some convenient point, and revolve it until it makes 45° with H and 30° with V (see Fig. 87 III). At a convenient point draw the line E_5–A_5 (shown by projections e_5'–a_5' and e_5–Aa_5) parallel to E_4–A_2 and on e_5–Aa_5 construct the plan drawn in Fig. 87 II, and determine the new elevation by projecting across from the elevation in Fig. 87 III and the ruled projectors from Fig. 87 IV.

Visualize the cube to determine which are visible edges.

Analysis II. (See Fig. 88.) Take a new G_1–L_1 making the angle α with the edge e_1–Aa_1 and determine the new elevation as in the second method of Problem 38, page 92.

Check. Parallel edges of the cube must be parallel in projection.

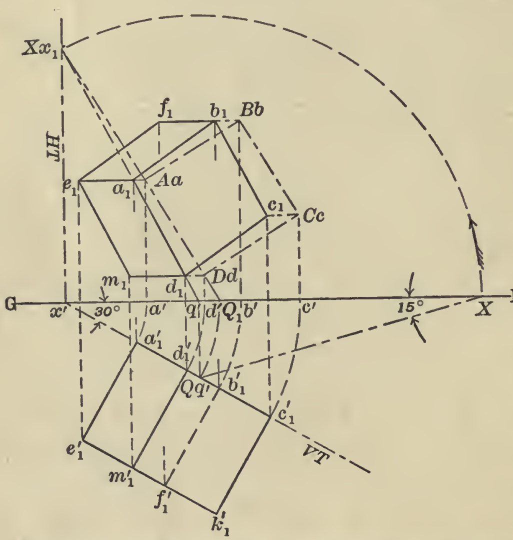


Fig. 89. — Projection of a Cube with One Face and One Edge definitely Inclined to H.

104. PROBLEM 40. To draw the third angle plan and elevation of a cube when one face is inclined to H at a given angle and an edge of that face also makes a given angle with H.

Analysis. Determine the traces of the plane which contains the given face of the cube. Determine a line in this plane which makes with H the angle of the given edge. Upon this line (as the given edge) construct the plan of the face and complete the elevation of the cube upon the elevation of this face.

Construction. Let the given face A–B–C–D make 30° with H; also let the given edge A–D make 15° with H. Draw HT

and VT (see Fig. 89) of the plane T which makes 30° with G–L.

Assume any point Oq' in VT and draw $Qq'-X$ to make 15° with H. Swing the line Q–X into the plane T by revolving X into HT at Xx_1. Swing Xx_1–Q into H and upon Xx_1–Q_1 construct the face of the cube. Revolve this face into T and upon its elevation erect perpendiculars a_1'–e_1', d_1'–m_1', b_1'–f_1', and c_1'–k_1' to the plane T and equal in length to the edges of the cube. These are the elevations of the edges and e_1'–f_1'–k_1'–m_1' is the elevation of the face parallel to the face A–D–B–C. The plan is completed from the elevation.

Visualize the cube to determine the invisible edges and check the parallelism of the edges.

105. Definitions. **A pyramid** is a solid bounded by a polygon, called the base, and a series of triangles having a common vertex. The *common* vertices of the triangular faces is called the **apex** of the pyramid.

The **altitude** is the perpendicular distance from the apex to the plane of the base.

The **axis** of a pyramid is the line joining its apex to the *center* of its base.

A right pyramid is one with its axis perpendicular to its base.

An oblique pyramid is one with its axis oblique to its base.

106. PROBLEM 41. To draw the third angle plan and elevation of a right hexagonal pyramid when its axis is inclined at any given angle β to H and α to V.

Analysis I. This problem can best be analyzed by steps as follows:

(1) Draw the plan and elevation of the pyramid when it is in its simplest position, that is, with its axis perpendicular to H (see Fig. 90 I).

(2) Move the pyramid to the right (see Fig. 90 II) and swing it around an axis X–Y drawn through the point D and perpendicular to V, until the axis O–Q assumes the given angle with H. During this rotation every point of the solid moves in the arc of a circle (equivalent to 90°–β) and parallel to V.

Hence the elevation moves through the angle $90^\circ-\beta$ but does not change shape or size.

Draw $a_1'-d_1'$ equal to $a'-d'$ and incline to $G-L$ at the angle $90^\circ-\beta$. Lay off the distances $a_1'-b_1'$, $b_1'-c_1'$, etc., equal to $a'-b'$, $b'-c'$, etc. From o_1' draw the vertical projection of the axis $o_1'-q_1'$ making 90° with the base, and complete the elevation by connecting each of the points a_1', b_1', c_1', etc., with the vertical projection of the apex q_1'.

Draw the ruled projectors upward from the new vertical projection and draw the transfer lines parallel to $G-L$ and toward

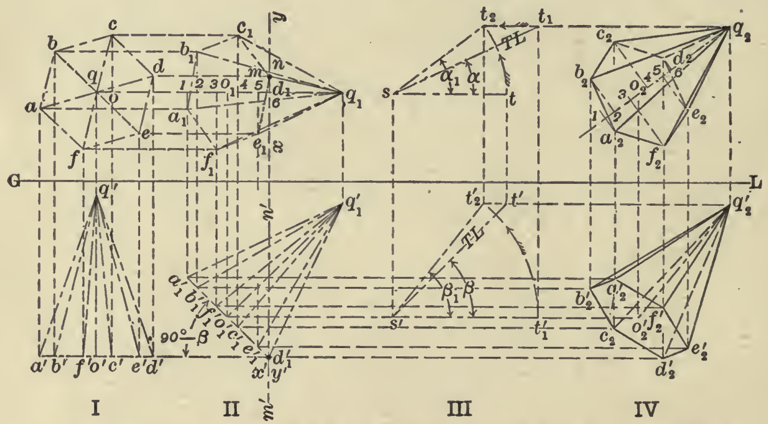


Fig. 90. — Third Angle Projection of a Right Pyramid whose Axis is Inclined to V and H.

the right from the original plan. The intersection of the ruled projectors and the transfer lines determine the new plan.

(3) Draw the projections of a line $S-T$ at the same angles (β with H and α with V) that the axis of the pyramid is required to make with the projection planes (see Fig. 90 III).

(4) Move the pyramid as before (Fig. 90 II) and change its inclination to V by swinging it round an axis $M-N$ which passes through D and is perpendicular to H. This will not change the distance of the pyramid from H and hence the plan will not change shape or size, but only its position to $G-L$. Place the horizontal projection o_2-q_2 of the axis parallel to the horizontal projection of a line $S-T$ (Fig. 90 III), and transfer the plan in this position. The plan is most easily transferred by means of

the base line *1–2–3–4–5–6* and the perpendiculars from a_1, b_1, c_1, etc. Project down and across for the new elevation (Fig. 90 IV).

Visualize the pyramid in its different positions in space and note which edges are visible.

Analysis II. Instead of revolving the plan take $G_1–L_1$ (see Fig. 91) to make the angle α_1 with the axis and determine the new elevation as in the second method of Problem 38, page 92.

Check. The accuracy of construction may be ascertained by noting the parallelism of the projections of lines which are parallel in space.

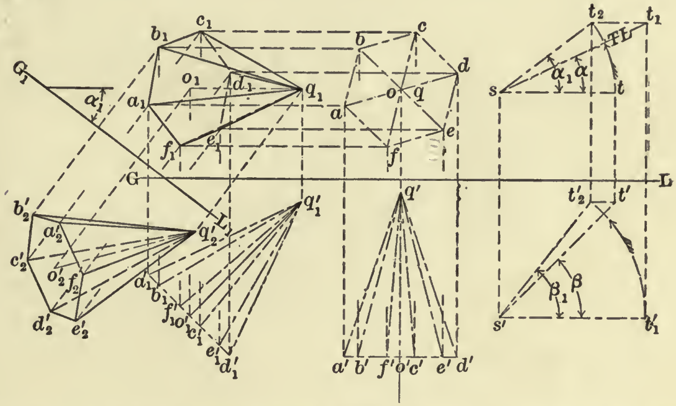


Fig. 91. — **Projection of a Pyramid whose Axis is Inclined to V and H by use of a New V Plane.**

107. PROBLEM 42. To draw the third angle plan and elevation of a right pyramid of given base and altitude and with its base in a given oblique plane.

Analysis. This problem can best be analyzed by steps as follows:

(1) Revolve the center *O* of the base into V about *VT* as an axis. On the revolved position O_1 draw the base in its true shape and size (see Fig. 92). Revolve the base *A–B–C–D–E–F* into the given plane T by any convenient method (see Fig. 78, page 82, Fig. 79, page 83, and Fig. 81, page 86).

(2) Pass a plane R through *O*, perpendicular to T and V. Revolve the line of intersection *I–J* of these planes R and T into H. This carries *O* into H. Erect the perpendicular $Oo_1–Q$ equal in length to the true altitude of the pyramid. Counter-

rotate Oo_1–Q into its true position shown by the projections o'–q' and o–q. The projections of the apex of the pyramid is then q,q', when it occupies the required position in space.

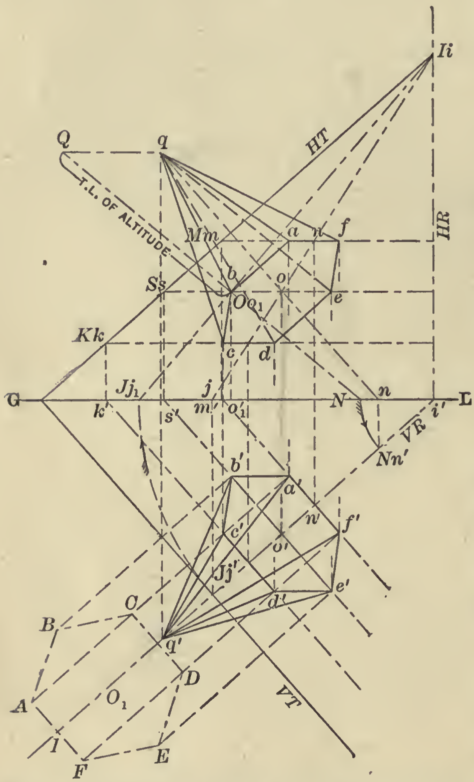


Fig. 92.—**Projection of a Right Pyramid whose base is in an Oblique Plane.**

(3) Connect the projections of the apex with the projections of the corners of the base to complete the figure.

(4) Visualize the pyramid to determine the invisible edges.

108. PROBLEM 43. To draw the third angle plan and elevation of an oblique hexagonal pyramid when its axis is inclined to both V and H (see Fig 93).

The solution of this problem is similar to that of Problem 41, page 95.

109. PROBLEM 44. To draw the third angle plan and elevation of a right pentagonal (five-angled) pyramid when one slant face *C–Q–D* is parallel to H and the horizontal edge *C–D* of the base is inclined at any angle to *G–L*.

Analysis. This problem can best be analyzed by steps as follows:

(1) Draw the plan and elevation of the pyramid in its simplest position. This will be with a base edge *C–D* perpendicular to V and the axis perpendicular to H (see Fig. 94 I).

(2) Swing the pyramid around *C–D* as an axis until the face *C–Q–D* is parallel to H (see Fig. 94 II). Every point of the

pyramid revolves in the arc of a circle which in each case is parallel to V, hence the new elevation is precisely the same as in

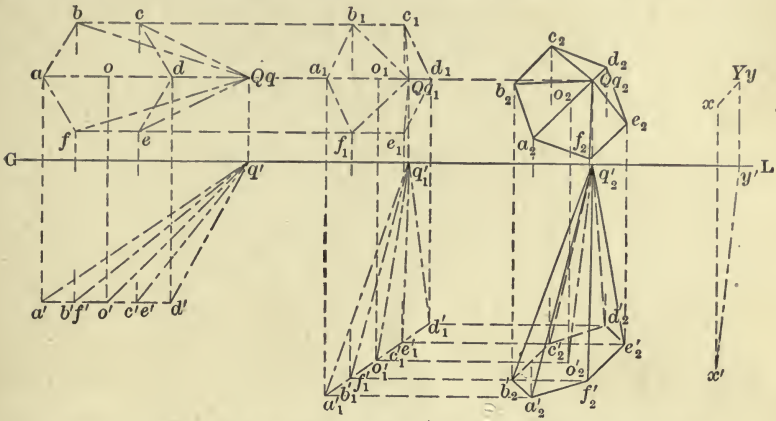


Fig. 93. — Projection of an Oblique Pyramid whose Axis is Inclined to V and H.

Fig. 94 I, but its position is changed relative to *G–L*. The plan changes shape and position.

(3) Move the pyramid to the right (see Fig. 94 III) and with-

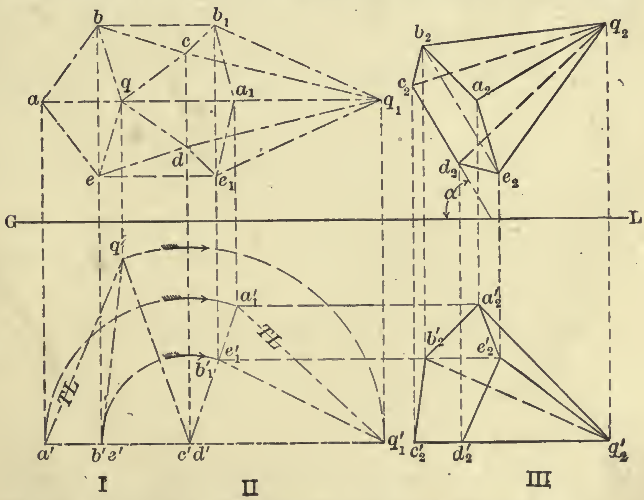


Fig. 94. — Projection of a Pentagonal Pyramid with Axis Inclined to V and H.

out changing the distance of its face *C–Q–D* from H swing it until the given edge *C–D* makes the required angle α with *G–L*. The new plan will be precisely the same as in Fig. 94 II, but its position relative to *G–L* is changed. Hence draw the line c_2–d_2 at the proper angle to *G–L* and upon this line construct the plan exactly as in Fig. 94 II. Project down and across to determine the new elevation.

Check. The accuracy of the construction may be tested by finding the true lengths of the edges and comparing them with the true length of *A–Q* of the original figure shown in Fig. 94 II.

110. **Definitions.** **A conical surface** is a curved surface generated by a line moving so that it touches a given circular or elliptical curve and passes through a fixed point not in the plane of the curve.

The moving line in any of its positions is called **an element.** **A cone** is a solid bounded by a conical surface and a plane that cuts all of its elements. The plane is the **base** of the cone. The **altitude** of a cone is the *perpendicular* distance from the apex to the plane of the base. **A circular cone** is one whose base is a circle. The **axis** of a circular cone is a line from the center of the base circle to the apex.

A right circular cone is a circular cone in which the axis is perpendicular to the base.

An oblique circular cone is a circular cone in which the axis is *not* perpendicular to the base.

111. **PROBLEM 45. To draw a cone in any position similar to the pyramids of Problems 41, 42, 43, and 44.**

Project a sufficient number of points on the base to establish the ellipse which represents the projection of the base, and complete as in the above problems.

For example see Figs. 95 and 96.

112. **Definitions.** **A prism** is a solid having equal and parallel faces for bases, and parallelograms for its other sides.

The altitude of a prism is the *perpendicular* distance between the planes of its bases.

A right prism is one whose lateral faces are perpendicular to its bases. A regular prism is also termed a right prism.

An **oblique prism** is one whose lateral edges are *not* perpendicular to its bases.

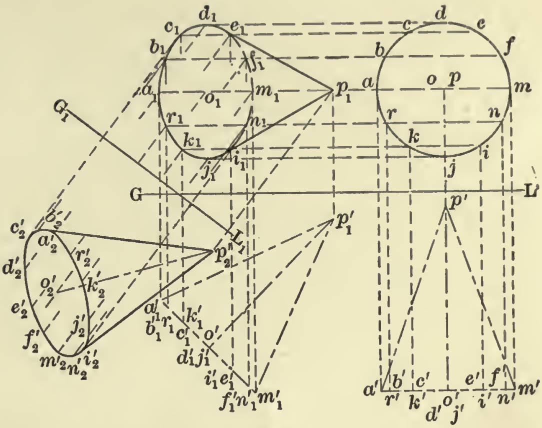


Fig. 95.—Projection of a Right Cone whose Axis is Inclined to V and H by use of a New V-Plane.

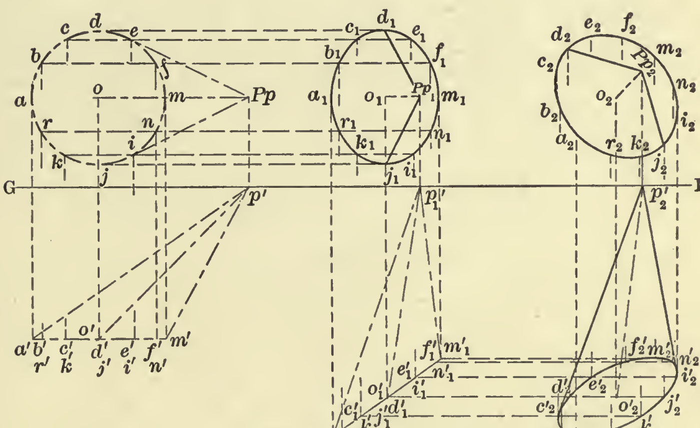


Fig. 96.—Projection of an Oblique Cone whose Axis is Inclined to the Planes of Projection.

113. **PROBLEM 46.** To draw the third angle plan and elevation of a hexagonal prism when the axis is parallel to V, but inclined to H at any given angle, and an edge of one end is parallel to H and perpendicular to V.

Analysis. This problem can best be analyzed by steps as follows:

(1) Draw the plan and elevation in its simplest position, that is, with two parallel faces perpendicular to V (see Fig. 97 I).

(2) Move the prism to the right (for convenience) and swing it about the line *B–C* as an axis until the face *C–B–F–E* makes the given angle α with *G–L*. In this rotation all points on the prism move in the arc of circles parallel to V, that is, no point on the prism changes its distance from V. Hence, the new elevation (Fig. 97 II) is precisely the same in shape and size as at Fig. 97 I, only its position to *G–L* has changed. Project up and across for the new plan (see Fig. 97 II).

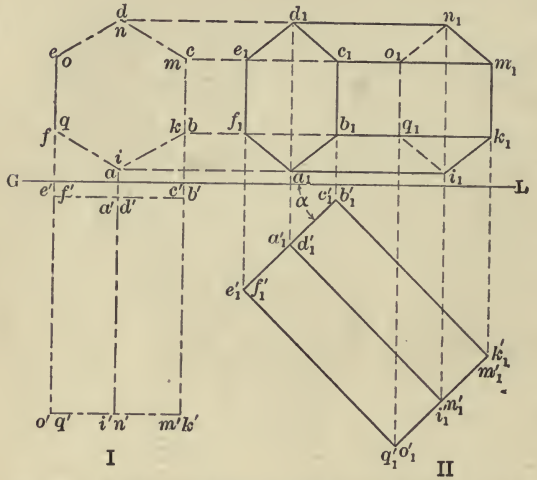


Fig. 97.—Projection of a Prism whose Axis is parallel to V and Inclined to H.

114. PROBLEM 47. To draw the third angle plan and elevation of a hexagonal prism, when one edge of an end is in H and inclined at any given angle to V and the axis is inclined at any given angle to H.

Analysis. This problem can best be analyzed by steps as follows:

(1) Determine, as in the problem 46, page 101, the plan and elevation when the axis is parallel to V but inclined at the given angle α with H (see Fig. 98 I).

(2) Draw G_1–L_1 at any convenient point and at the angle β with Bb_1–Cc_1 that the edge *B–C* is to make with V. The elevation on the new vertical plane is obtained by drawing the perpendiculars to G_1–L_1 from the points on the plan at Fig. 98 II; measuring on these perpendiculars from G_1–L_1, the corresponding distances from *G–L* on the elevation Fig. 98 II.

Check. The accuracy of the construction may be checked by comparing the true lengths of several edges shown in the final plan and elevation with those shown at Fig. 98 I.

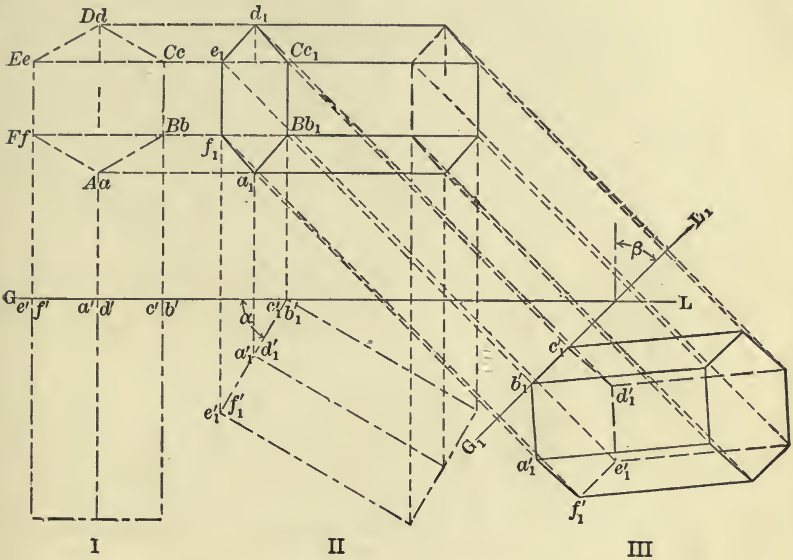


Fig. 98. — Projection of a Prism whose Axis is Inclined to V and H.

115. PROBLEM 48. To draw the third angle plan and elevation of a right cylinder when its axis is parallel to V but inclined to H at any given angle.

Analysis. This problem can best be analyzed by steps as follows:

(1) Draw the plan and elevation of the cylinder in its simplest position, that is, when one end or face is parallel to H. The plan will be a circle with a diameter equal to the diameter of the cylinder base; the elevation will be a rectangle of length equal to the length of the cylinder and width equal to the diameter of the cylinder (see Fig. 99 I).

(2) Divide the cylinder into twelve (or any convenient number) equal parts by lines parallel to the axis. (See elements through *A*, *B*, *C*, *D*, *E*, etc., in Fig. 99 I.) Move the elevation to the right and incline its base at 90° minus the given angle α. Determine the horizontal projection of the points on the circum-

ference exactly as was done in finding the plan of end of prism (see Problem 47, page 102) and join these points by a curved line, in place of straight lines as in the previous problems.

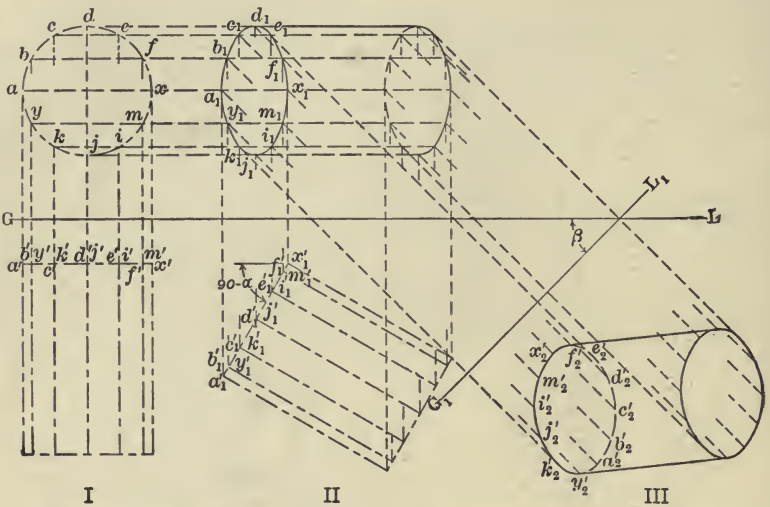


Fig. 99.—Third Angle Projection of a Right Cylinder whose Axis is Parallel to V and Inclined to H.

116. PROBLEM 49. To draw the third angle plan and elevation of a right cylinder when its axis is inclined to both V and H.

Analysis. Assume that the axis makes the angle α with H and determine the new elevation on a new vertical plane which is inclined to the original V at an angle of β. See Fig. 99.

(1) Proceed as in Problem 48, page 103.

(2) Draw G_1–L_1 to make the angle β with G–L that the new vertical plane V_1 makes with V and determine the new vertical projection as in Fig. 95, page 101, and Fig. 98, page 103.

117. Definition. A helix is a curve generated by a point which moves along the surface of a cylinder in such a way that a constant ratio is maintained between its travel *around* the cylinder and *along* its length.

Thus in Fig. 100 let the moving point start from A and travel around the cylinder and at the same time along its length. Divide the circumference of the base into twelve (any number) equal parts, and divide the length into the *same* number of equal parts, and through these divisions draw horizontal lines.

Assume the point starting at a moves to b in plan, it must then move to b' in elevation, or when it has moved through one-twelfth of a revolution, it has also moved through one-twelfth the distance a'–r'. When it has moved through one-half revolution, to i, it has moved through one-half of a'–r' or to i'. When it has moved through a complete revolution in plan, it has moved in elevation to r'. The distance a'–r' through which the moving point travels along the axis while it describes one complete revolution is termed the *pitch* of the helix.

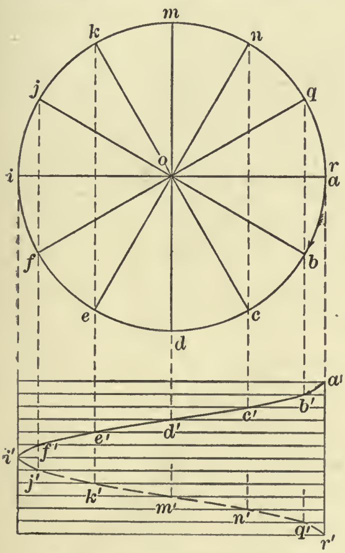


Fig. 100.—Construction of a Helix.

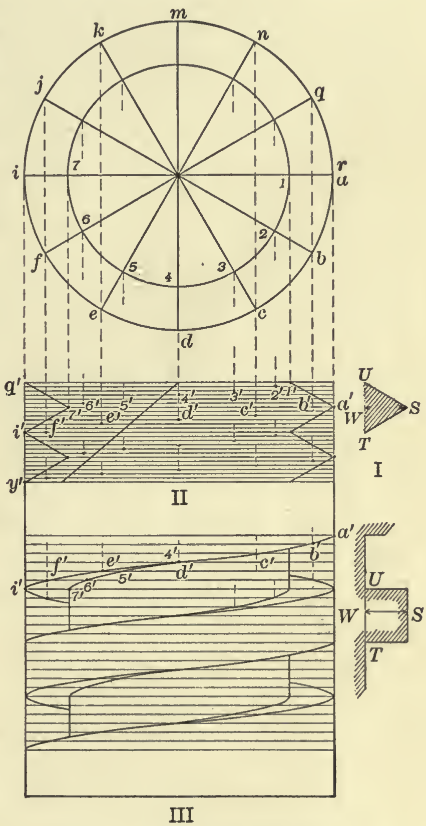


Fig. 101.—Projection of Screw Thread.

A helicoid is a surface generated by a straight line which glides along a helix and maintains an invariable position with relation to the axis of the curve. Its most common use is in screw threads.

118. PROBLEM 50. To project a V screw thread.

Analysis. Let the section of the thread be the triangle *U–S–T* shown at Fig. 101 I. The side *U–T* is the pitch of the thread. The bottom and top edges of the thread are each a helix. The helix for the top of the thread is generated on a cylinder having a diameter *a–i*, while the one to represent the bottom of the thread is on a cylinder having a diameter 1–7. The difference between these two diameters is equal to twice the perpendicular distance *S–W*, or twice the depth of the thread.

Construction. The plan of the thread will be the plan of the two cylinders on which the helices that are to represent the top and bottom of the thread are generated.

The elevation is obtained by marking off lengths *q′–i′* and *i′–y′* each equal to the pitch, and dividing these distances into the same number of equal parts as the circumference. Next draw horizontal lines through the points of division, and determine the helicoidal lines on the outer and inner cylinders. (See § 117, page 104.) Finish by properly joining the bottoms of the threads to the tops (see Fig. 101 II).

The square thread is similarly drawn (see Fig. 101 III).

CHAPTER VI

TANGENT PLANES AND DOUBLE-CURVED SURFACES OF REVOLUTION

119. Definitions and General Considerations. Let *A–F–E–B* be any plane curve in space (see Fig. 102). If the point *E* of the secant *F–E* remains fixed at *E* and the *point F* of this secant is moved along the curve until it coincides with *E*, then the line *F–E* coincides with *C–E–D* and becomes **a tangent** to the curve at the point *E*. By further reference to this figure it is seen that if a straight line *C–D* is tangent to a plane curve *A–E–B* then the tangent will lie in the plane of the curve, because the secant *F–E* moved in the plane of the curve as it changed its position to *C–E–D* where it *became* the tangent.

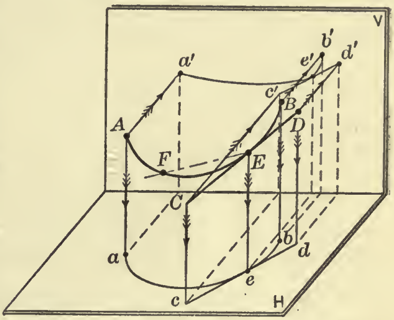


Fig. 102.—Projection of a Plane Curve and a Line Tangent to the Curve.

As the **point of tangency** *E* is a point, and the only point, common to *both* the curve and the tangent, the projections of the curve and of the tangent on V are tangent, as also are their projections on H. In this connection it is self evident that two *straight* lines tangent to each other will coincide.

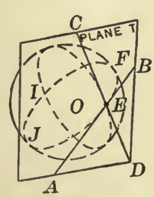


Fig. 103.—A Line, also a Plane, shown Tangent to a Surface.

A straight line is **tangent to a surface** at a given point when it is tangent to a line of the surface at that point. Thus, the line *A–B* (Fig. 103) is tangent to the sphere *O* at the point *E*, because it is tangent to the great circle *E–F–I–J* at the point *E*.

A plane is tangent to a surface at a given point when it contains all the straight lines tangent to the surface at that point. The tangent plane has one point in common with the surface to which it is tangent, which is the point of tangency. Thus, the plane T is tangent to the sphere *O*, at the point *E* because it contains all the straight lines tangent to the sphere at this point. The point *E* is common to the sphere and the plane.

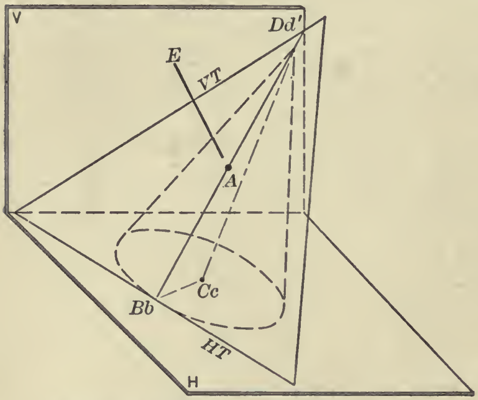


Fig. 104.—A Plane which is Tangent to a Cone.

If a surface has rectilinear elements, the tangent plane must contain the element passing through the point of tangency. This element is called the **element of tangency.** In Fig. 104 the plane T is tangent to the cone at the point *A*, hence tangent to the cone along the element of tangency *B–D*. Figure 105 shows the plane T tangent to the cylinder at the point *A*, hence along the element of tangency *B–D* through the point *A*. **A straight line is normal to a surface at a given point when it is perpendicular to the plane which is tangent to the surface at that point.** Thus, the line *E–A* (Fig. 104) is perpendicular to the plane T at the point *A*, hence it is normal to the cone at the point *A*.

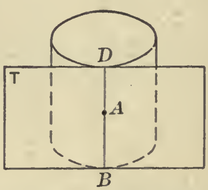


Fig. 105.—A Plane which is Tangent to a Cylinder.

120. PROBLEM 51. To pass a plane tangent to a cone through a given point on the surface.

Analysis. An element of the cone through the given point will be a line in the tangent plane and will pierce H in a point on the *horizontal trace* of the required plane (see Fig. 106). Also since this trace and the cone base are both in H they will be tangent at the point where the element of tangency pierces H. The vertical piercing point of the element of tangency (when produced) gives a point in the vertical trace of the required plane.

Construction. Let the cone be given in the third angle by its projections as in Fig. 106, and let a, a' be the projections of the given point on the surface through which the tangent plane T is to pass. Through a, a' and the apex of the cone b, b' draw the element of tangency, which is a line of the required plane. This element $A-B$ pierces H at D, and the tangent HT to the base (drawn perpendicular to the radius $C–D$) at this point is the horizontal trace of the required plane. Produce the element of tangency until it pierces V at E, which point is therefore on the vertical trace of the required plane; hence VT drawn through E and O is the required vertical trace.

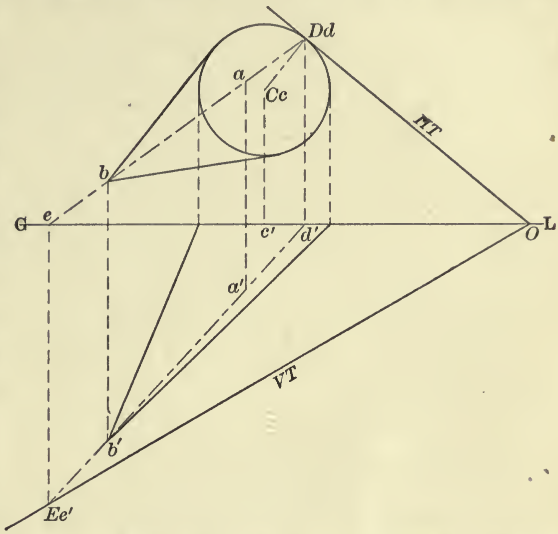


Fig. 106.—A Plane drawn Tangent to a Cone through a Given Point on an Element of the Cone.

Check. Any line drawn through any point on the element of tangency (other than A) and parallel to the vertical trace VT will be a line in the required plane and hence must pierce H on HT.

121. PROBLEM 52. To pass a plane tangent to a cone through a point without the surface.

Analysis. A line drawn through the given point and the apex of the cone must be a line of the required plane; hence it will pierce V and H in the vertical and horizontal traces of the required plane. The horizontal trace of the required plane will be tangent to the horizontal projection of the base of the cone and will pass through the horizontal piercing point of the line.

Construction. Let the cone be given by its projections as in Fig. 107, and let *a*, *a′* be the given point through which the tangent plane must pass. Draw the line *A–E* through this given point and the apex *E* of the cone; determine the piercing points *C* and *B* of the line *A–E* extended. The horizontal trace *HT* is drawn through *Cc* and tangent to the projection of the base of the cone. The vertical trace is drawn through *Bb* and intersects *HT* on *G–L*.

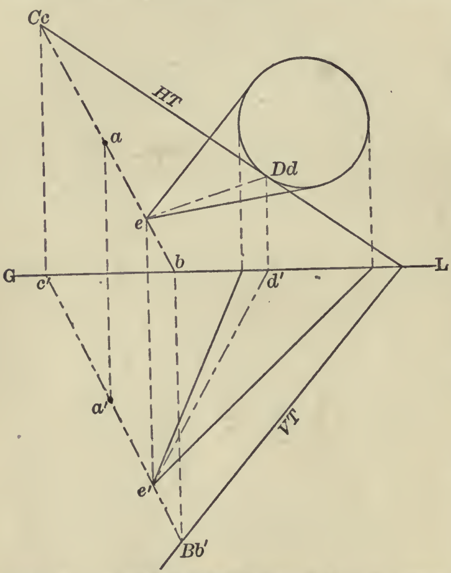


Fig. 107.— To pass a Plane Tangent to a Given Cone and through a Given Point without its Surface.

NOTE. The plane T is tangent to the cone on the *near* side, and a second plane could be determined tangent on the far side of the cone.

Check. Pass a line through *A* parallel to *HT*. This line must pierce V on *VT* and intersect the element of tangency.

122. **PROBLEM 53. To determine the traces of a plane which shall contain a given line and make a given angle with H.**

Analysis. Such a plane must contain the given line and be tangent to a right cone, whose base is in H and whose elements make the given angle with the base.

Construction. Let *A–B* (Fig. 108) represent the given line, and *Aa* and *Bb′* its piercing points. From any convenient point *C* on this line draw the line *C–D* making the given angle α with *G–L*. This line *C–D* represents an element of a right cone which is parallel to V. Draw the horizontal projection of the cone and *HT* will be tangent to the horizontal projection of the base outline and will pass through *A*. The trace *VT* will meet *HT* on *G–L* and will pass through *B*.

Check. The plane T must contain the given line since its traces are passed through the piercing points of the line and so as to meet on *G–L*. Determine the angle the plane makes with H (see Problem 27, page 72).

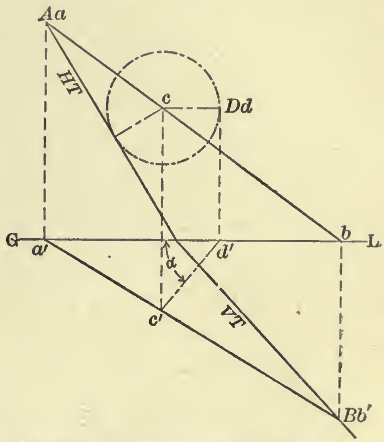


Fig. 108.—To determine a Plane which will make a Given Angle with H and contain a Given Line.

123. PROBLEM 54. To pass a plane tangent to a cone and parallel to a given straight line.

Analysis. Since the required tangent plane must contain the vertex of the cone, a straight line through the vertex and parallel to the given line will be a line of the required plane. The horizontal trace of the required plane must pass through the horizontal piercing point of this line and be tangent to the cone base.

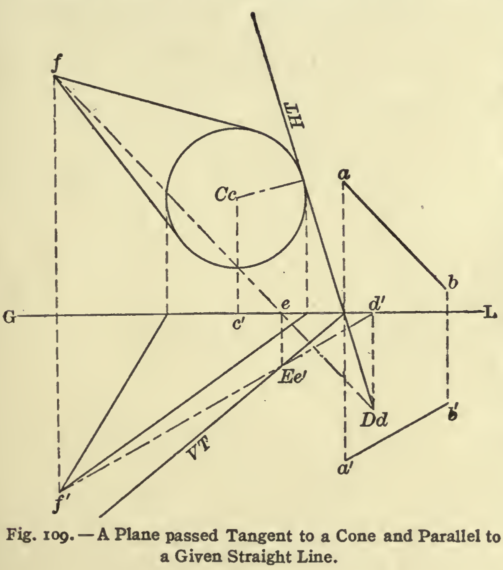


Fig. 109.—A Plane passed Tangent to a Cone and Parallel to a Given Straight Line.

Construction. Let the cone and the given line *A–B* be represented by their projections in Fig. 109. Draw *F–E* through the vertex of the cone and parallel to *A–B*. Through *Dd* the horizontal piercing point of *F–E* draw the horizontal trace *HT* of the required plane tangent to the cone base. The vertical trace *VT* is drawn through the vertical piercing point *Ee'* of the line and to meet *HT* on *G–L*.

Check. The element of tangency must pierce V in *VT*.

124. PROBLEM 55. To pass a plane tangent to a cylinder through a given point on the cylinder.

Analysis. Since the plane is tangent to the cylinder at a point, it is tangent all along the element through this point. Hence the element through the given point is a line of the tangent plane and will pierce V and H in the traces of this plane. If the base of the cylinder is on H, the horizontal trace of the tangent plane must be tangent to the base and pass through the horizontal piercing point of the element of tangency.

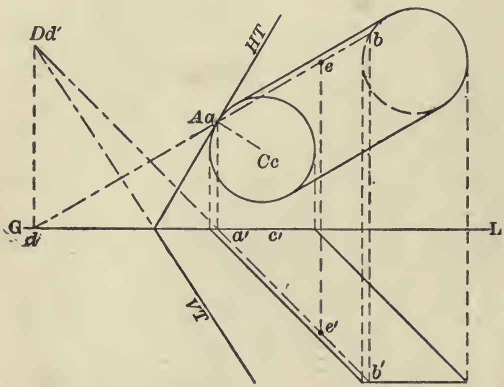


Fig. 110.—A Plane passed through a Given Point on a Cylinder and Tangent to it.

Construction. Let the cylinder be given by its projections in Fig. 110, and let E be the given point on its surface. Through the projections e, e', of the point E, draw the projections of the element A–B and determine its piercing points Aa and Dd'. The horizontal trace of the tangent plane will be perpendicular to the radius A–C, that is, tangent to the base at the point Aa. The vertical trace of the plane will pass through the vertical piercing point Dd' of the element of tangency A–B and meet the horizontal trace on G–L.

Check. Any line intersecting the element of tangency and parallel to HT must pierce V in VT.

125. PROBLEM 56. To pass a plane tangent to a cylinder and through a given point without the surface.

Analysis I. Since a plane which is tangent to a cylinder is tangent all along an element, the problem is solved by passing a plane through the element of tangency and the given point.

Construction I. Let the cylinder be given by its projections in Fig. 111 and let A be the given point. Draw the line A–D

through the given point *A* and parallel to the elements of the cylinder; extend *A–D* to pierce H at *Dd*. A line through *Dd* tangent to the base is the horizontal trace of the required plane. Through *A* draw the line *A–E* parallel to *HT* and it will pierce V at *E*, which is a point on *VT*. The element of tangency is *B–F*.

Check. The element of tangency *B–F* must pierce V in *VT*.

SPECIAL CASE. When the cylinder is one of revolution and its axis is parallel to *G–L*.

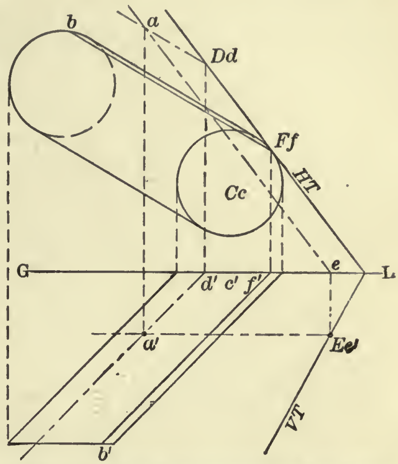


Fig. 111.—A Line drawn through a Point and Tangent to a Cylinder.

Construction II. Let *A–B* in Fig. 112 be the axis of the cylinder and *J* the given point. A straight line through *J* and parallel to *A–B* must be a line of the required plane, both traces of which must be parallel to *G–L*.

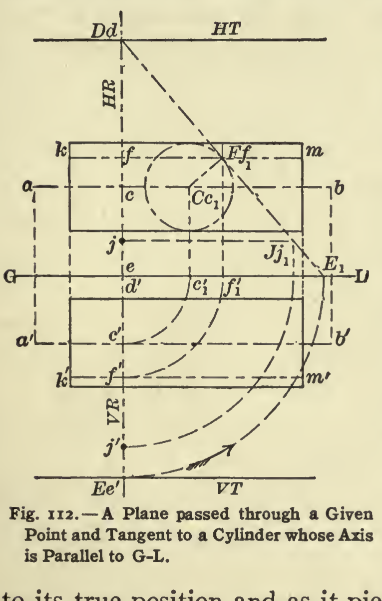


Fig. 112.—A Plane passed through a Given Point and Tangent to a Cylinder whose Axis is Parallel to G-L.

Pass a **profile plane** R through the point *J* and it will cut a circle, with center at *C*, from the cylinder. Revolve the plane R around its horizontal trace into H. The center of the circle will fall at Cc_1, and the point *J* at Jj_1. Next draw the circle in its true size about the center Cc_1 and the line $Dd–E_1$ drawn tangent to this circle is a line of the required plane revolved into H. Revolve $Dd–E_1$ to its true position and as it pierces H at *Dd*, and V at *Ee'*, the line *HT* through *Dd* and parallel to *G–L* is the horizontal trace and VT through Ee' and parallel to G–L is the vertical trace of the required plane.

To determine the element of tangency revolve the point of tangency Ff_1, f_1' to its true position and through it draw the projections of K–M parallel to G–L.

126. PROBLEM 57. To pass a plane tangent to a sphere at a given point on its surface.

Analysis. A sphere shows in projection on V and H as a circle with a diameter equal to the diameter of the sphere. The radius of the sphere drawn to the point of contact of the tangent plane is perpendicular to this plane. Hence, if a plane is passed through the given point and perpendicular to the radius of the sphere drawn to the tangent point, it will be the required plane.

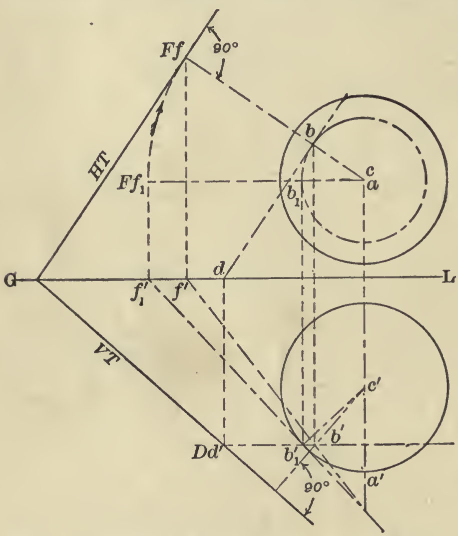


Fig. 113.—To pass a Plane Tangent to any Given Point on the Surface of a Sphere.

Construction. Let the sphere be given by its projections shown in Fig. 113, and let b, b' be the given point on the sphere. Through the point B draw B–D perpendicular to the radius B–C and parallel to H. The vertical projection of B–D will then be parallel to G–L and its horizontal projection will be parallel to the required horizontal trace HT. Through Dd' draw VT perpendicular to c'–b'. Through the intersection of VT with G–L draw HT parallel to d–b.

Check. Swing the sphere about a vertical axis C–A until the radius B–C is parallel to V. Draw a line tangent to the great circle of the sphere at b_1, b_1', and determine its horizontal piercing point Ff_1. When the line B–F is revolved to its true position the point F must fall on HT.

127. PROBLEM 58. To pass a plane tangent to a sphere and through a given straight line without the sphere.

Analysis. A plane passed through the center of the sphere and perpendicular to the given line will be pierced by this line (pro-

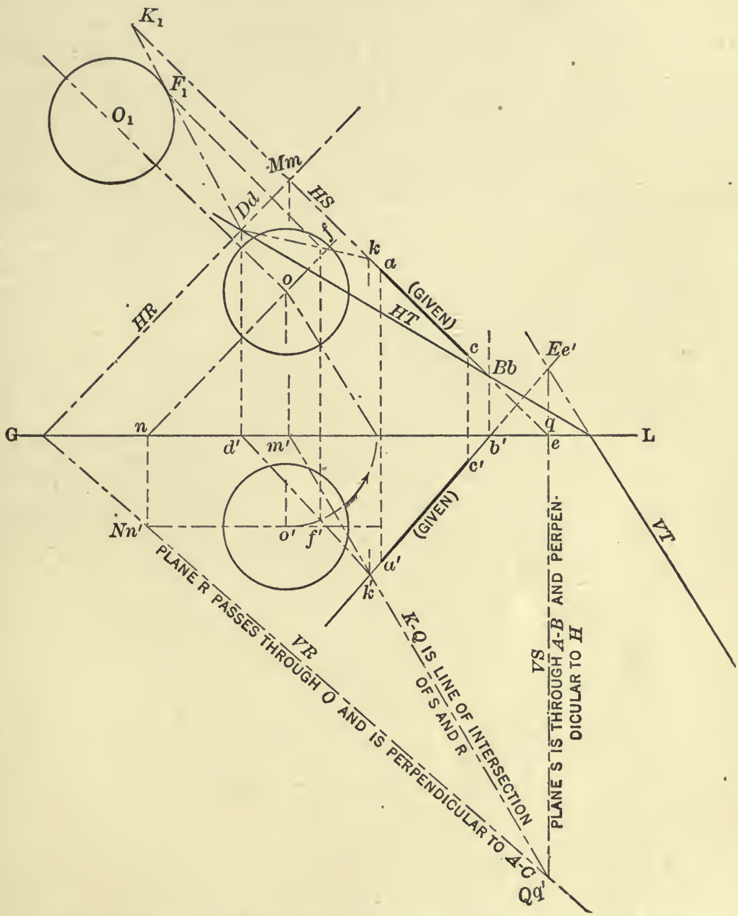


Fig. 114.—To pass a Plane through a Given Straight Line and Tangent to a Sphere.

duced if necessary) and it will cut a great circle from the sphere. An auxiliary line drawn from the point in which the given line pierces this plane, and tangent to the circle cut from the sphere by the auxiliary plane, will be a line of the required plane. This line with the given line will determine the required plane.

Construction. Let the sphere be given by its projections in Fig. 114, and let *a–c*, *a′–c′* be the projections of the given line. Pass the plane R through the center of the sphere *O*, and perpendicular to the given line *A–C* (see Problem 26, page 69). By use of the plane S, determine the piercing point *K* of the line *A–C* on the plane R (see Problem 23, page 64). Revolve the plane R, and with it the center *O* of the sphere, around *HR* as an axis into H. Draw a great circle of the sphere about O_1 (the revolved position of O) and through the revolved position K_1 of K draw K_1–Dd tangent at F_1 to the great circle. The projection *Dd* must be a point on *HT*. Determine the horizontal piercing point *B* of the given line *A–C* and draw the required horizontal trace *HT* through *Dd* and *Bb*. Determine *Ee′* the vertical piercing point of *A–C* and draw *VT* through *Ee′* and the point of intersection of *HT* with *G–L*.

The plane T touches the sphere at *F* and is perpendicular to *A–C*.

128. Definitions. An **hyperboloid of revolution** is a warped surface of revolution generated by revolving a straight line around an axis at a constant distance from and kept at a constant angle with the axis.

An **hyperboloid of revolution** of two nappes is a double curved surface of revolution generated by revolving an hyperbola about its *transverse* axis.

A **torus** is a double curved surface of revolution generated by revolving a circumference about a line which is in the plane of and outside the circumference.

129. PROBLEM 59. To draw the third angle plan and elevation of an hyperboloid of revolution with its axis perpendicular to H and to assume a point on its surface.

Analysis. In order to draw the hyperboloid, the distance of the generating line from the axis must be known as well as its inclination to a plane perpendicular to the axis. As the generating line revolves, the path of the horizontal projection of each point is a circle and the vertical projection is a straight line parallel to *G–L*. Hence, by finding the projections of points on the generating line in the proper position, the projections of the hyperboloid can be determined. **To assume a point on the surface** draw the two projections of any generating line; assume a point on one projection of the line and draw the ruled projector to the other projection of the line.

Construction. In Fig. 115 let *a–b*, *a′–b′* represent the given axis. Let *m–k*, *m′–k′* represent the generating line, at the proper distance from *A–B*, and making the correct angle with H. Mark off any number of convenient points such as *C*, *D*, *E*, *F*, and *M*. As these points revolve about *A–B* their horizontal projections move in circles and their vertical projections in straight line parallel to *G–L*. Revolve all points until they are in a plane parallel to V as at $C_1D_1E_1F_1$ and M_1 and then draw in the curve.

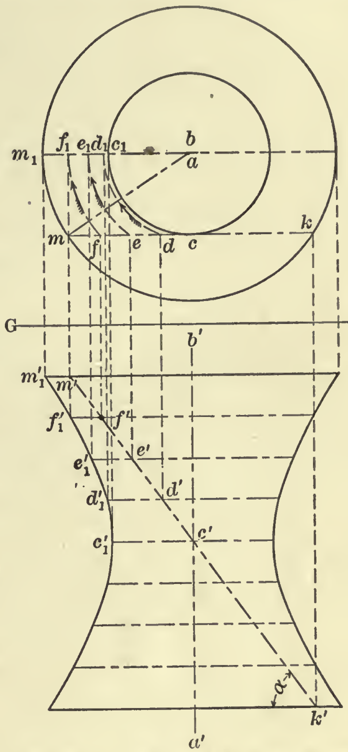


Fig. 115.—Projection of an Hyperboloid of Revolutions.

130. PROBLEM 60. To draw the third angle plan and elevation of a torus which is inclined to both V and H.

Analysis I. The projection of the torus consists of projections of lines tangent to the generating circle.

Construction I. Draw the torus in its simplest position, that is, with its axis perpendicular to H [see Fig. 116(a)]. Several positions of the generating circle are shown, the center being at C, C_1, C_2, etc. Fig. 116(b) shows the construction when the axis is inclined to H and parallel to V. Fig. 116 (c) shows the construction when the axis is inclined to both H and V.

Construction II. The method described above makes it necessary to use a generating circle, which in most positions projects as an ellipse. A simpler construction results by assuming the torus to be generated by a sphere instead of a circle. This generating sphere (center at S) projects in all positions as

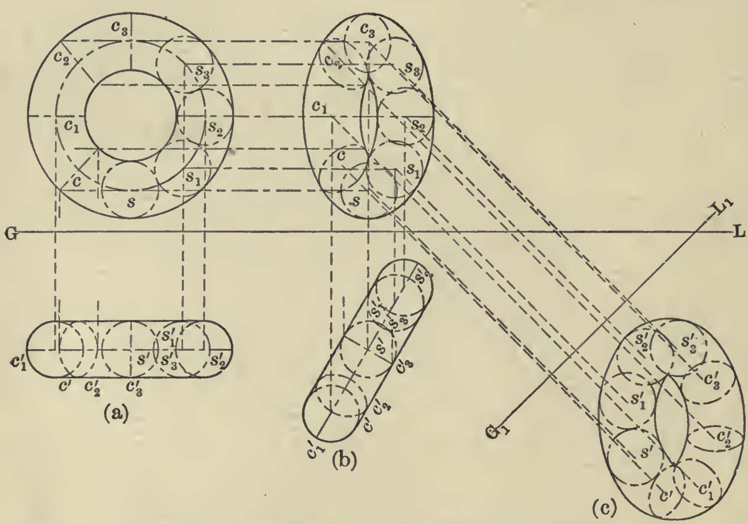


Fig. 116.—Projection of a Torus Inclined to V and H.

a circle, hence it is only necessary to determine the projections s, s', s_1, s_1', of several positions of the center S of the sphere, and then describe circles about these projections and draw the projections of the torus tangent to these circles. See Fig. 116.

CHAPTER VII

SECTIONS

131. Definitions and General Considerations. A section is a plane figure cut from an object by a plane passing through it (see Figs. 117, 118, and 119). Such a plane as T or R is termed a **cutting plane. The section** illustrates the appearance of the cut surface of the object with one of the "cut parts" removed.

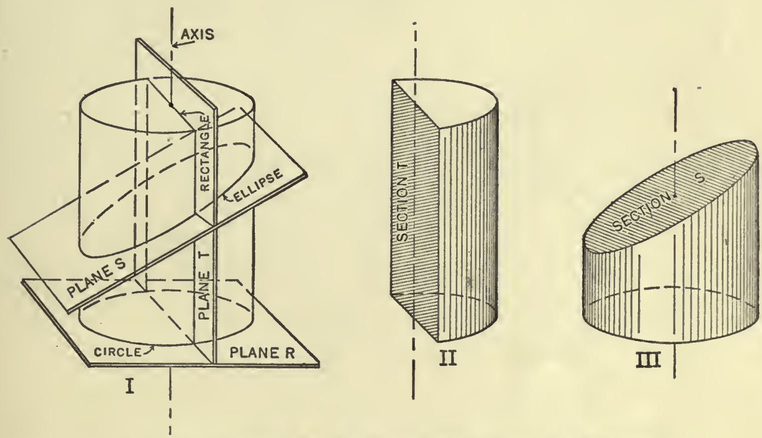


Fig. 117. — Sections that can be cut from a Cylinder by a Plane.

The section is usually shown in its true form, but it may be represented by its plan and elevation. The line in which the cutting plane intersects the object is termed the **line of intersection,** and is common to both the solid and the cutting plane; hence, whatever the form of the object, the outline of intersection must be a plane line, and the section a flat surface (see Figs. 117 II and III). The lines outlining the form of a section may be straight, curved, or a combination of these two, depending upon the *form* of the solid cut and the *position* of the cutting plane. Thus in Fig. 117 I the right cylinder can be cut in any one of three general positions, each position giving a definite form of section. The section cut by a plane T, through or parallel to the axis, is termed a **longitudinal section** and is a **rectangle** in form. The section cut by a plane R, perpendicular to the axis, is termed a **transverse section** and in form is **a circle.** The section cut by a plane S, inclined to the axis, is termed an **oblique section** and in form is **an ellipse.**

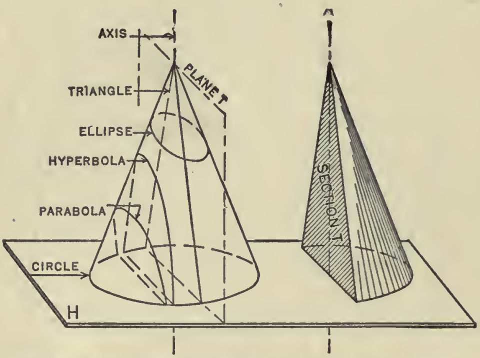


Fig. 118. — Sections that can be cut from a Cone by a Plane.

Five sections may be cut from a cone (see Fig. 118); **a circle,** if the cutting plane is *perpendicular* to the axis; **a triangle,** if the cutting plane *contains* the vertex; **an ellipse,** if the cutting plane makes an angle with the axis of the cone *greater* than that of an element; **an hyperbola,** if the angle the cutting plane makes with the axis is *less* than that of the element; and **a parabola,** if the angle the cutting plane makes with the axis is *equal* to that of the element.

But *one* form of **section** can be cut **from a sphere,** namely, a circle (see Fig. 119). **The diameter** of this circle depends upon the distance the cutting plane is from the center of the sphere. When the cutting plane T passes through the center of the sphere, the section is a *great circle* of the sphere. That is, the diameter of the circle is the same as that of the sphere. When the cutting plane H is tangent to the sphere the section has diminished to a point.

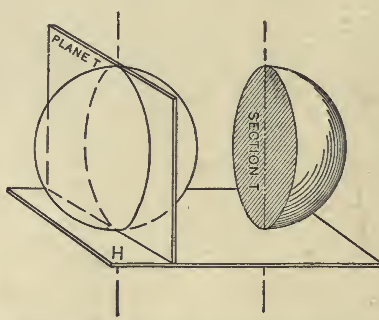


Fig. 119. — The Section cut from a Sphere by a Plane.

132. PROBLEM 61. To determine any section of a right square prism.

Analysis. Place the prism with a base against H and a diagonal of the base perpendicular to V. Take a cutting plane perpendicular to V and inclined to H. The plane will cut the edges of the prism in points the projections of which are known. By connecting these points with straight lines, the line of intersection of the cutting plane with the prism is determined. By revolving this outline of intersection into V or H the true form of the section is determined.

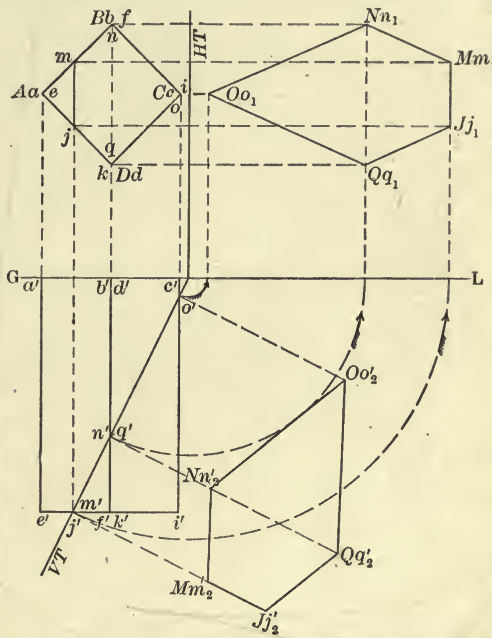


Fig. 120. — **Section cut from a Right Prism by a Plane Perpendicular to V and Inclined to H.**

Construction. Let the prism be shown by its projections in Fig. 120, and let *VT* and *HT* represent the cutting plane. The plane T cuts the edge *B–F* of the prism in the point *N*, the vertical projection *n'* of which is determined by the intersection of the vertical trace *VT* with the vertical projection *b'–f'* of the edge. Since *B–F* is perpendicular to H the horizontal projection of all its points must be in a single point, hence the horizontal projection *n* of *N* is the same point as the horizontal projections *b* of the point *B* and *f* of *F*. Similarly, the points *Q* and *O* are found. The point *J* must have its horizontal projection on the ruled projector from *j'* and on the edge *E–K*, hence at *j*. Similarly, *M* must be on the edge *E–F*.

Therefore *j–m–n–o–q* must represent the plan of the section and *j'–m'–n'–o'–q'* its elevation. To determine the true form of the section revolve the polygon *j–m–n–o–q* about *HT* into H as shown in Oo_1–Nn_1–Mm_1–Jj_1–Qq_1.

Check. Revolve the section into V about VT as an axis, and the resulting polygon $Oo_2'-Nn_2'-Mm_2'-Jj_2'-Qq_2'$ will check with $Oo_1-Nn_1-Mm_1-Jj_1-Qq_1$ if the construction is accurate.

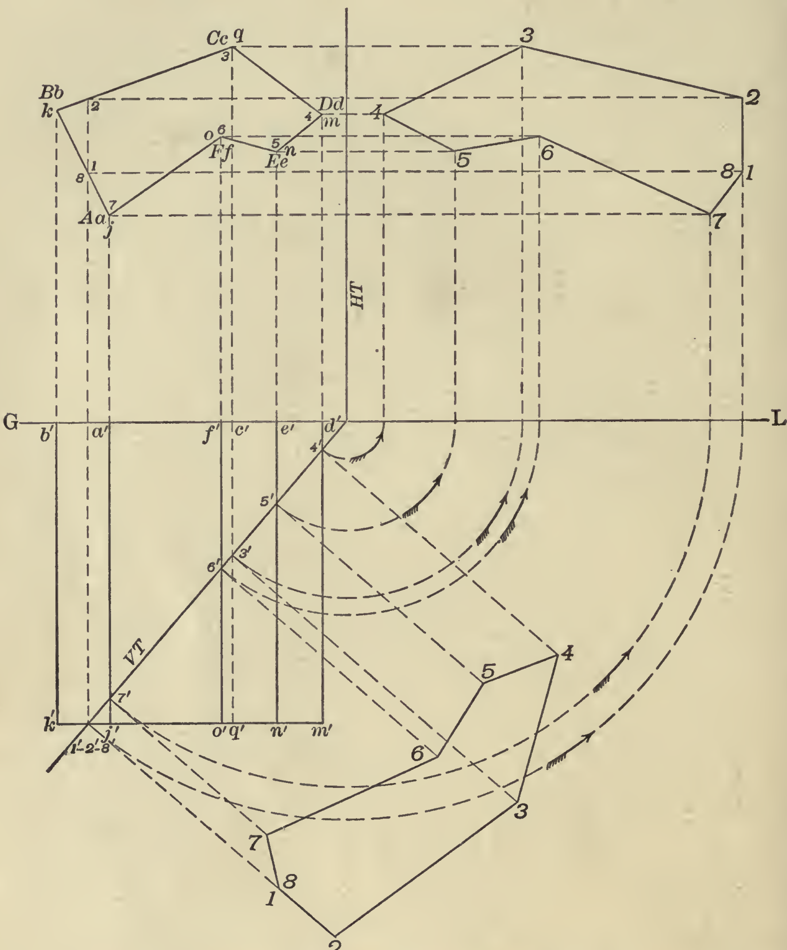


Fig. 121. — To determine the Section cut from any Right Prism by a Plane.

133. PROBLEM 62. To determine any section of any right prism. Proceed as in Problem 61, page 120. See Fig. 121.

134. PROBLEM 63. To determine the section cut from a right cylinder by a plane making a given angle with its axis.

Analysis. Divide the circumference of the cylinder into any convenient number of parts; draw elements through these points;

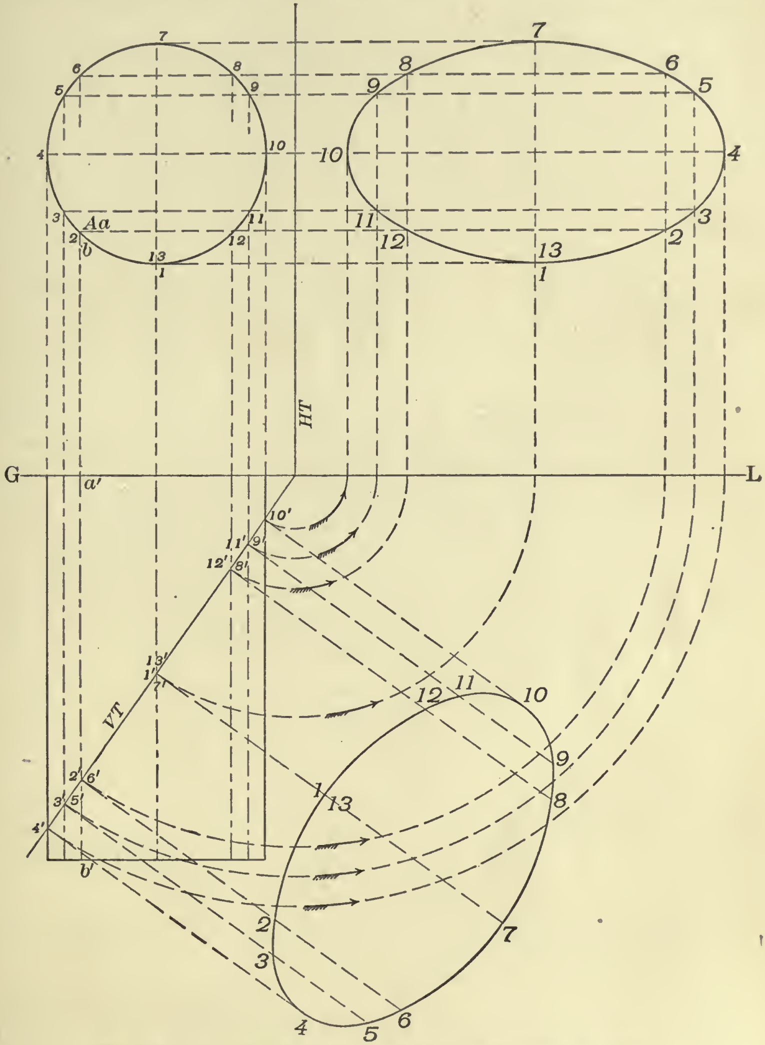


Fig. 122.—To determine the Section cut from a Cylinder by a Plane.

determine the intersection of the elements and the cutting plane and revolve the section so found into V or H.

Construction. Let the cylinder be given as in Fig. 122, and let *VT* and *HT* be the traces of the cutting plane. Divide the circumference of the base of the cylinder into any convenient number of parts (1–12 as shown in Fig. 122) and draw the projections of elements of the cylinder through these points. Thus *A–B* is one such element. The others are drawn, but, to avoid confusion, are not lettered. The plan of the section will fall in the plan of the cylinder and the elevation will fall in *VT*. Revolve the points 1 to 13 into H about *HT* and join these points (in revolved position) with a curved line.

Check. Revolve into V about *VT*.

135. PROBLEM 64. To determine the section of a pyramid when cut by a plane making a given angle with its axis.

Analysis. Determine the intersection or piercing point of the edges of the pyramid with the cutting plane. Connect these points to form a polygon and revolve the polygon into V or H.

Construction. Let the pyramid be given as in Fig. 123, and let *VT* and *HT* represent the cutting plane.

VT cuts the vertical projection of the edges in the points i', k', j', m', n', r', s', and the corresponding horizontal projections of all points, except the point *J* on the edge *B–Q*, can be determined by the ruled projectors. The edge *B–Q* coincides with the ruled projector, hence *B–Q* can be revolved parallel to V; the point J_1 determined, and then counter-revolved to its correct position. The polygon *i–k–j–m–n–r–s* is the plan of the section and Ii_1–Kk_1–Jj_1–Mm_1–Nn_1–Rr_1–Ss_1 is its true form and size found by revolving the section into H around HT.

Check. The polygon Ii_2'–Kk_2'–Jj_2'–Mm_2'–Nn_2'–Rr_2'–Ss_2' is the section revolved into V and should check with the section in H.

136. PROBLEM 65. To determine the section of a right cone when cut by a plane making a given angle with its axis.

Analysis I. Divide the base of the cone into any convenient number of parts and draw elements of the cone through these points. Determine the points of intersection of these elements with the cutting plane. Draw a curved line through these points of intersection and revolve this figure into V or H.

Construction I. Let the cone be given as in Fig. 124 and let *HT* and *VT* represent the cutting plane. Let *a*, a' and *c*, c' be

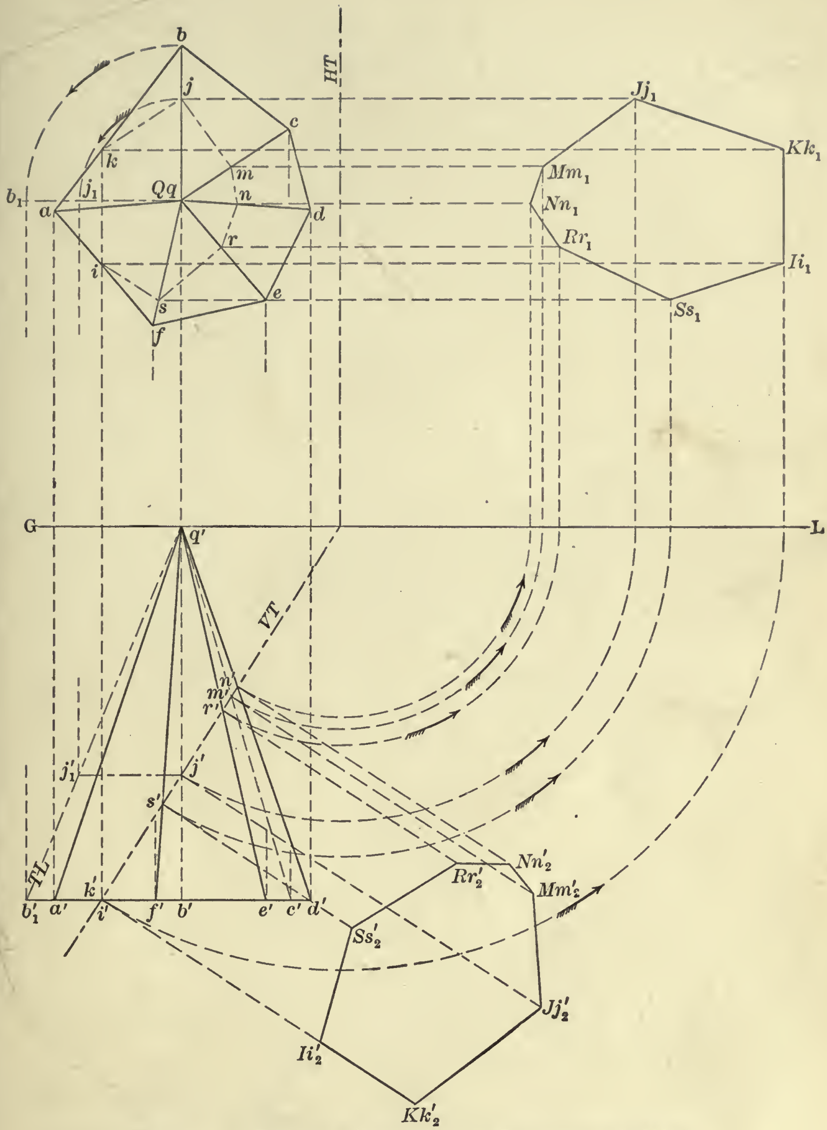


Fig. 123.—To determine the Section cut from a Pyramid by a Plane.

the projection of any two points of division on the base of the cone, and let *a–q*, *a′–q′* and *c–q*, *c′–q′* represent the elements through these points. The vertical trace *VT* cuts *a′–q′* at *b′*; then *b* is found by drawing the ruled projector through *b′*

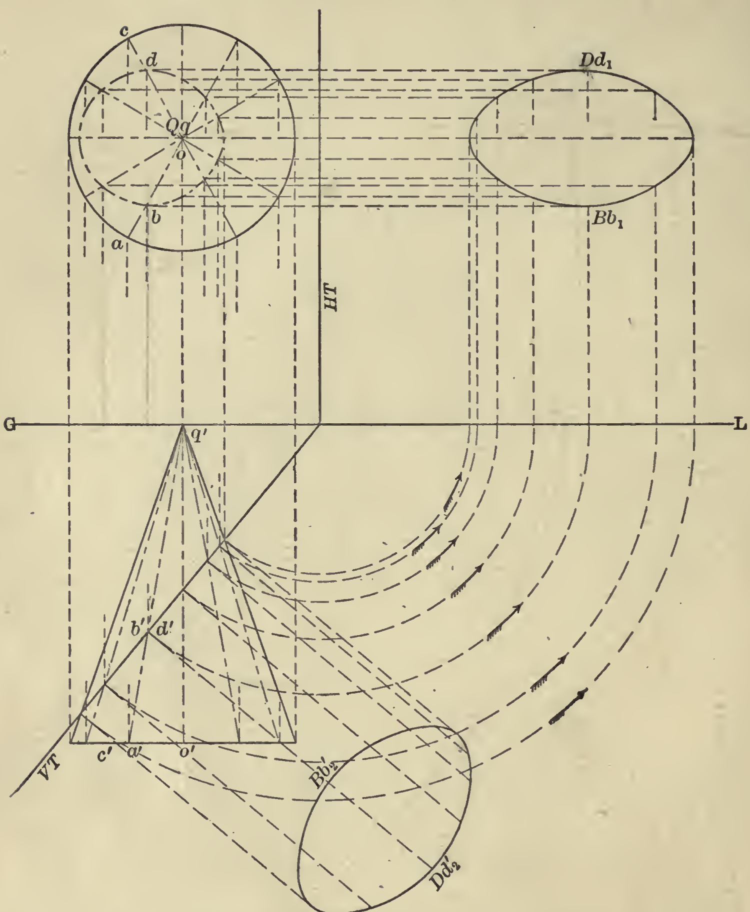


Fig. 124.—Section cut from a Right Cone by a Plane Inclined to its Axis.

to cut a–q, hence is at b. Similarly, determine other points and draw the curve as shown. Revolve this figure, which is an ellipse, into H at Bb_1–Dd_1 or revolve into V at Bb_2'–Dd_2'.

Analysis II. Pass a series of horizontal planes through the cone. Such planes will cut circles from the cone, which are shown in plan as circles and in elevation as straight lines parallel to G–L. The points of intersection of these circles with the cutting plane marks points on the required section.

Construction II. Let the cone be given as in Fig. 125, and let the plane cut the cone parallel to a single element, so that the section cut is a parabola. *HT* and *VT* represent the cutting plane. Take any horizontal plane S and it will cut from the cone a circle of radius *O–C* horizontally projected in the circle *c–b–a* and vertically projected in the trace *VS*. The horizontal projections *a* and *b* must be on the horizontal projection *a–b–c* of the circle, and on the ruled projector from *a′*, *b′*, *c′*, etc., hence at

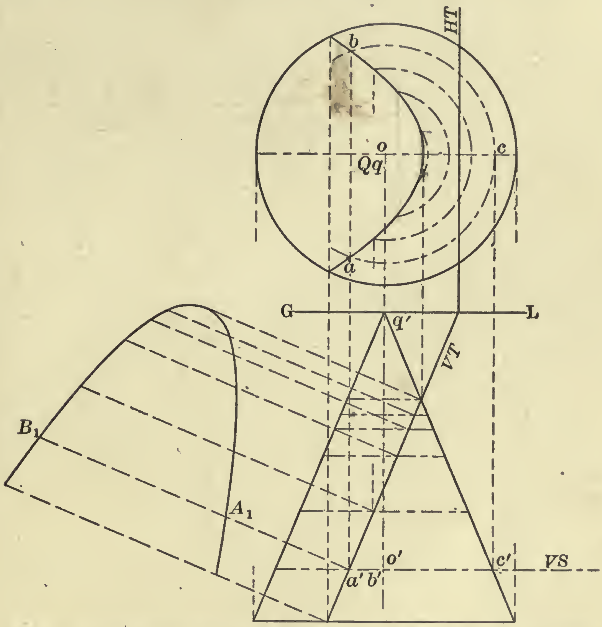


Fig. 125.—Section of a Right Cone cut by a Plane Parallel to an Element.

a, *b* and *c*. Other points are determined in the same manner and all these points are revolved into either V or H to determine the true shape of the section.

137. PROBLEM 66. To determine the section of an annular torus when cut by a plane making a given angle with H.

Construction. See Fig. 126. Cut the torus by planes parallel to H, such as S. This plane S cuts two circles from the torus, one of radius *O–C*, shown in vertical projection in the trace VS and in horizontal projection as the circle *a–c–b*, the other of radius *O–D*, shown also in the vertical projection VS and in the horizontal projection *e–d–f*. The cutting plane T intersects each of these circles in two points; the circle $A–C–B$ in the points A and B and the circle $E–D–F$ in the points E and F, and the four points are located in the required section. In a similar

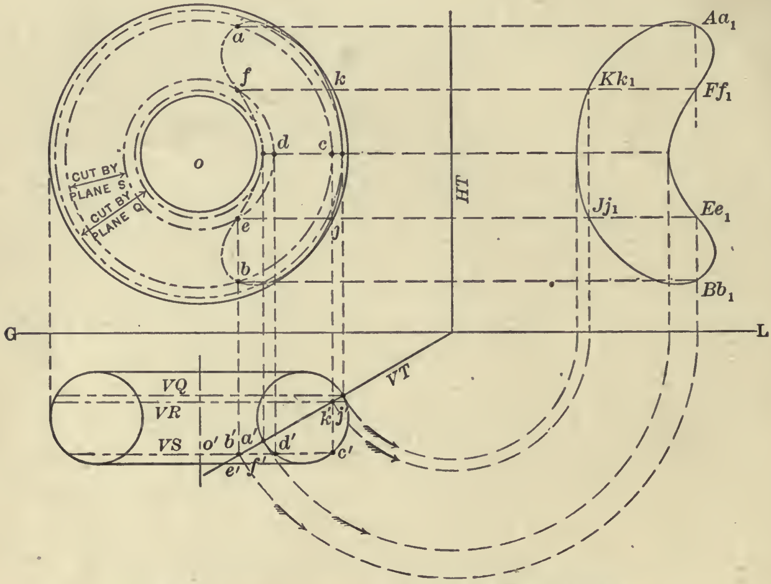


Fig. 126. — Section cut from a Torus by a Plane Inclined to its Axis.

manner a sufficient number of points can be found so as to determine the projections of the section, and then by revolving the section about HT into T or about VT into V it can be fully determined.

CHAPTER VIII

INTERSECTIONS AND DEVELOPMENTS

138. Definitions and General Considerations. When two surfaces meet or penetrate, the line along which they meet is termed their line of intersection. **The line of intersection** is therefore a line common to both intersecting surfaces, and its *form* is determined by the character of the surfaces and their relative size and position. It is necessary to determine the projections of the line of intersection in order to represent the surfaces, and the true form of the line of intersection is necessary in order to develop the surfaces. By **the development of a surface** is meant that it is laid out *flat* without any part of it being "stretched."

When the intersecting surfaces are solids, the **general method of determining the line of intersection** is to pass planes through the solids which will cut intersecting sections. The boundaries of these sections intersect in points which are *common* to both solids, hence are points on their line of intersection.

The development of a solid is effected when every line of its surface has been brought into a *single* plane and it is shown in its *true* size and shape. The development can then be cut out of cardboard or sheet metal and when properly folded or shaped it will make a surface exactly like the surface of the solid illustrated. Only those bodies which are bounded by planes, such as cubes, prisms, and pyramids, and single curved surfaces such as cylinders and cones, can be developed. **Double curved surfaces** such as spheres and ellipsoids can *not* be developed. The **cube, prism, and pyramid are developed** by placing one face in contact with a plane, and then turning the solid about its edges to bring all of the remaining faces of the solid in contact with the plane so that their true size and shape can be outlined. It is such an outline of all the faces of the solid shown in true size and shape which constitutes the development of the solid.

To develop a cone or cylinder, one element is placed in contact with a plane and then the solid is rolled on this plane until every other element has touched the plane.

That portion of the plane covered by the solid in its revolution represents the development of the surface of the solid.

139. PROBLEM 67. To determine the true form of the line of intersection of an hexagonal prism when cut by a plane meeting its axis at any given angle, and to develop the surface of the prism.

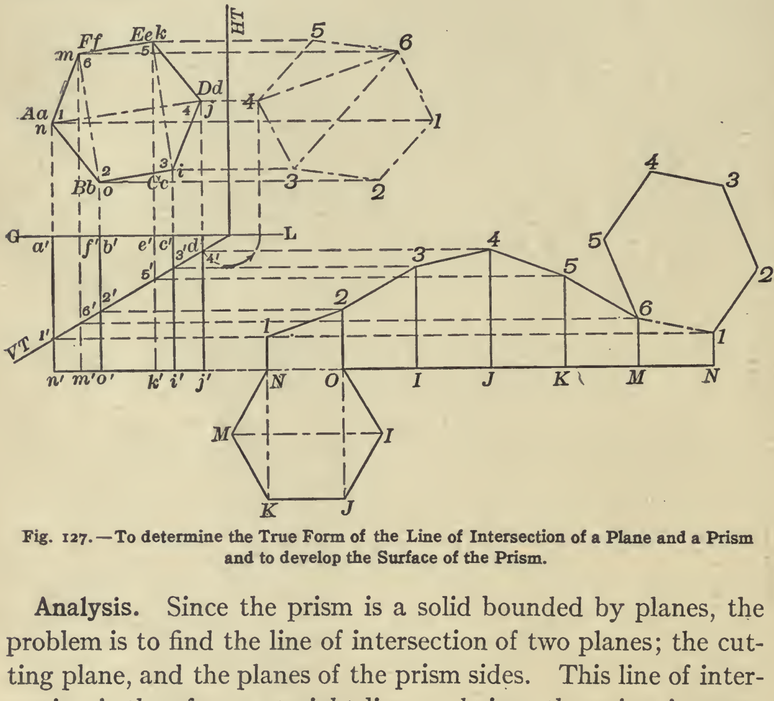


Fig. 127.—To determine the True Form of the Line of Intersection of a Plane and a Prism and to develop the Surface of the Prism.

Analysis. Since the prism is a solid bounded by planes, the problem is to find the line of intersection of two planes; the cutting plane, and the planes of the prism sides. This line of intersection is therefore a straight line, and since the prism is represented by its edges, the problem is really the determination of the points where the edges of the prism pierce the cutting plane. Such points are common to both the prism and the cutting plane, hence, when connected in the proper order by straight lines, these points determine the line of intersection.

Construction. Draw the plan and elevation of the prism and the traces of the plane as shown in Fig. 127. It is evident that the edges of the prism pierce the cutting plane T in the points, *1*, *2*, *3*, *4*, *5*, and *6*. Lay off on *G–L* produced (or any convenient straight line) the distance *N–O* = *n–o*; *O–I* = *o–i*; etc. Draw the perpendiculars *N–1*, *O–2*, etc. (equal in length to *n′*–1′ and *o′–2′* respectively), to represent the edges of the prisms in development.

Project the points *1′* to *1*, *2′* to *2*, etc., to represent the position of *1*, *2*, *3*, etc., in development. Connect the points *1–2*, *2–3*, etc., to get the *true form* of the line of intersection.

To draw the development of the section, revolve the plane T into H about the trace *HT*. Transfer this figure *1–2–3–4–5–6* to the development by building it up on the side *6–1*, making it exactly the same size and shape as the section would be if revolved into H. Since the plan of the prism shows the base in its true size and shape, the developed end or base *N–O–I–J–K–M* can be laid off on the side *N–O*, and the development is complete.

Check. The top portion of the prism can be developed and the true size and shape of the section can be determined by revolving T into V about *VT*. The sum of the developed segments of any given edge should equal the height of the prism.

140. PROBLEM 68. To determine the true form of the line of intersection of a right cylinder cut by a plane meeting its axis at any given angle, and to develop the surface of the cylinder.

Analysis. Let the cylinder and plane be given as in Fig. 128. Divide the base of the cylinder into a sufficient number of equal parts (say twelve), and through these points of division draw elements of the cylinder. Determine the piercing points of these elements on the cutting plane, exactly as if they were the edges of a prism. Revolve the piercing points into H about *HT* and connect them with a smooth curve. Develop the surface as in Problem 67, page 130.

Construction. Draw a straight line *B–D–F–B* equal in length to the circumference of the cylinder. Divide this line into the same number of equal parts as the base of the cylinder. Erect perpendiculars at the points of division, determine the points

1-2-3, etc., as in Problem 67, and through these points draw a *smooth* curve. Complete and check as in Problem 67.

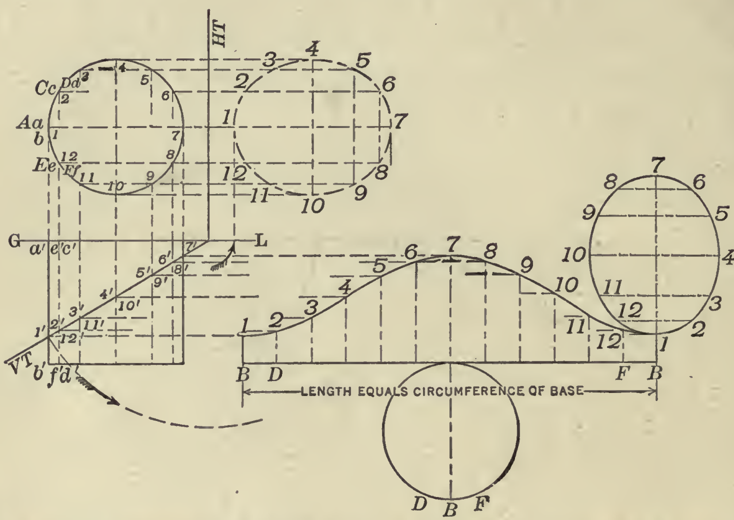


Fig 128.—To determine the True Form of the Line of Intersection of a Plane and a Cylinder and to Develop the Surface of the Cylinder.

141. PROBLEM 69. To determine the true form of the line of intersection of a pyramid cut by a plane which meets the axis of the pyramid at any given angle, and to develop the surface of the pyramid.

Analysis. The vertical projections of the points where the edges pierce the cutting plane are found by inspection; the horizontal projections are determined by drawing ruled projectors from the vertical projection of the point, to the horizontal projection of the edge, upon which the point is located. Connecting the horizontal projections of the points in proper order determines the horizontal projection of the section cut by the plane. The true shape of this section is determined by revolving it into H as in the preceding problems.

To develop the surface of a right pyramid, draw an arc, having as its radius the true length of an edge. On this arc lay off chords equal in length and equal in number to the base edges of the pyramid. At the extremities of each of these chords draw straight lines to the center of the circular arc. These lines represent the developed edges of the pyramid. **On each** developed edge mark off the true distance from the base to the piercing point on that edge. Connect these points in proper order for the true development of the pyramid surface. The development of the base and of the section are brought into proper relation as in problem 67, page 130.

Construction. See Fig. 129. Determine the projections, *1′*, *1*; *2′*, *2*; etc., of the piercing points and revolve the plane T into H

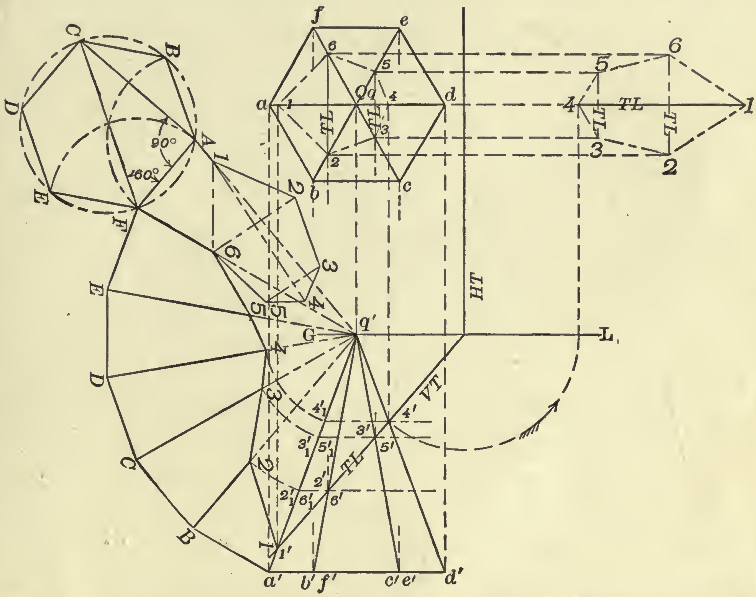


Fig. 129.—To determine the True Form of the Line of Intersection of a Plane and a Pyramid and to develop the Surface of the Pyramid.

about *HT* to get the true section *1–2–3*, etc. Swing the inclined edges of the pyramid parallel to V to determine the true distances *a′* to *2′*, *a′* to *3′*, etc., *a′–q′* being a true length projection; draw an arc with *q′* as a center and *q′–a′* as a radius. Step off *a′–B = a–b*, *B–C = b–c*, etc., until the complete base line is stepped off, and connect these points in successive order. From these points draw *B–q′*, *C–q′*, etc., and measure off the distances $B\text{–}2 = a'\text{–}2_1'$; $C\text{–}3 = a'\text{–}3_1'$, etc., to get the true distances from the base along the edges to the piercing points. Connect the points *1* to *2*, *2* to *3*, etc., and on the edge *1–6* construct the section *1–2–3–4–5–6* making it the same in all respects as the section which was revolved into H about HT. Also construct the base $A–B–C–D–E–F$ the same in all respects as $a–b–c–d–e–f$.

Check. The portion of the pyramid from the apex to the plane could be developed and should check up with the developed lower portion.

142. **PROBLEM 70. To determine the true form of the line of intersection of a right cone when cut by a plane meeting the cone axis at any given angle, and to develop the surface of the cone.**

Analysis. Divide the base line into a sufficient number of equal parts (say twelve). Through these points of division draw elements of the cone. Proceed as in Problem 69, page 132, treating the elements as if they were edges of a pyramid, with the *single exception* that the points are connected by a *smooth* curved line.

Construction. Divide the base circle into twelve equal divisions, $A–B$, $B–C$, etc., as shown in Fig. 130. Draw the elements $A–Q$, $B–Q$, etc., projected at $a–q$, $a'–q'$, $b–q$, $b'–q'$, etc. Proceed as explained in Problem 69, page 132.

Check. Same general method as in Problem 69.

NOTE. The horizontal projections of the points *4* and *10* (see Fig. 130) cannot be determined by ruled projectors, since they lie in a plane perpendicular to $G–L$. To determine these projections, pass a plane R through the cone at these points (*2* and *10*), and at right angles to the cone axis. This plane will cut a circle from the cone, and this circle will pierce the plane T in the required points, because those points are common to both the cone and the plane T, hence on their line of intersection; thus the points are completely determined.

Check. The horizontal projections of the points *1*; *2*, *12*; *3*, *11*, etc., can also be determined in the above way.

143. **PROBLEM 71. To determine the line of intersection of a prism when cut by a plane inclined to V and H. Also to develop the surface and determine the true form of the line of intersection.**

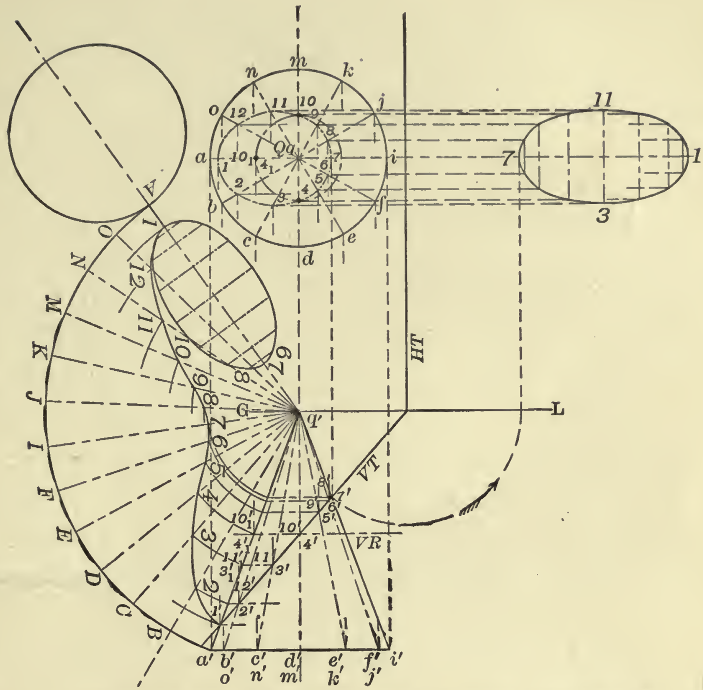


Fig. 130.—To determine the True Form of the Line of Intersection of a Plane and a Cone and to develop the Surface of the Cone.

Analysis. Pass a series of planes through the edges of the prism so as to intersect the given plane. Each of these planes will cut a line from the given plane, which line intersects the edge or edges of the prism through which the auxiliary plane was passed. The projections of the points of intersection are the projections of the piercing point of the edges of the prism on the given plane, since they represent points which are common to the cutting plane and the edge of the prism. The development is made as in Problems 67 to 70 inclusive, page 130.

Construction. Assume one base of the prism in H and let the plane T be given by its traces *HT* and *VT* (see Fig. 131). Pass a series of planes (such as R) through the vertical edges (such as *M–N*), cutting out the lines of intersection (such as *K–I*) on the plane T. The vertical projection *m′–n′* of the edge intersects the vertical projection *K′–Ii′* at the point *a′*, which is the vertical projection of the piercing point of the edge $M-N$ on the plane T. In like manner, the projection of all the piercing points are determined and joined in proper order for the line of intersection of the given plane and prism. The

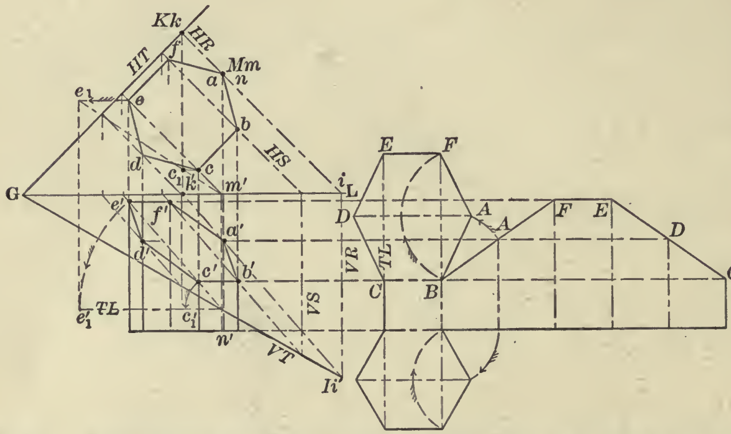


Fig. 131.—To determine the Line of Intersection cut by a Plane from a Prism and to develop the Surface of the Prism.

development is found and checked as in Problems 67 to 70 inclusive, page 130.

144. PROBLEM 72. To determine the line of intersection of a cylinder cut by a plane inclined to V and H. Also to develop the surface and determine the true form of the line of intersection.

Analysis. Divide the base of the cylinder into any number of equal parts, and through the points of division draw elements of the cylinder. Treat these elements of the cylinder exactly as if they were edges of a prism, and determine points of the line of intersection as in Problem 71, page 134; these points are connected with a smooth curve.

Construction. See Fig. 132. Pass several auxiliary planes R, S, etc., through the cylinder and perpendicular to H. Determine the projections of the line of intersection as in Problem 71, page 134.

Revolve $X-Y$ until parallel to V, and draw lines through the projections, *1*, *2*, *3*, *4*, etc., each parallel to $Xx-x_1$, to get the true distances $Y-1$, $Y-2$, etc. Knowing the true length of the major axis (Xx–y being perpendicular to HT) and the true length and location of ordinates to this axis, the true section Xx–A–Y–B can be constructed.

Check. The section can be checked by revolving it about HT into H. The surface is developed similarly to that in Problem 70, page 134.

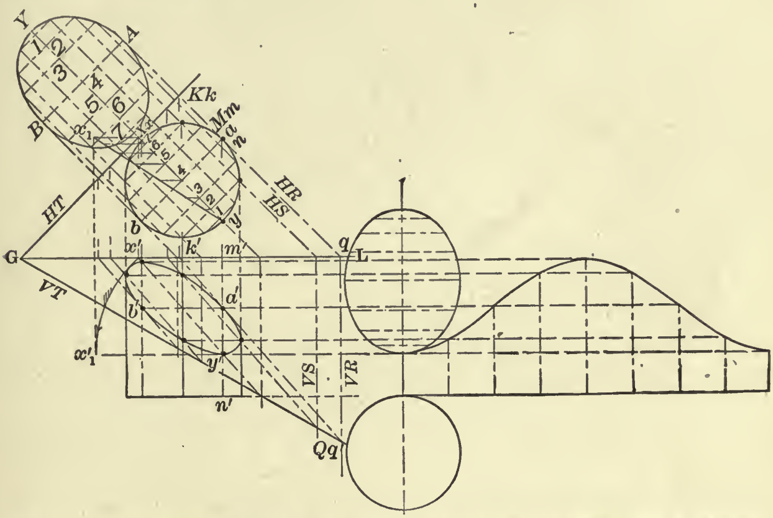


Fig. 132.—To determine the Line of Intersection of a Cylinder and a Plane and to develop the Surface of the Cylinder.

145. PROBLEM 73. To determine the line of intersection of a pyramid cut by a plane which is inclined to V and H. Also to develop the surface and determine the true form of the line of intersection.

Analysis. Pass auxiliary planes through the edge of the pyramid (these planes will be perpendicular to H), and determine the line of intersection as in Problem 72, page 136. Develop the surface as in Problems 69 and 70, page 132.

Construction. See Fig. 133. Pass the auxiliary plane S through the edges as shown and determine the projection of the two points D and E where the edges X–Y and X–Z pierce the plane T. There are two points on the section.

Similarly determine all other points on the section.

Check. The lines of intersection of all auxiliary planes and T must pass through the point C.

Develop the surface of the pyramid as in Problem 69, page 132.

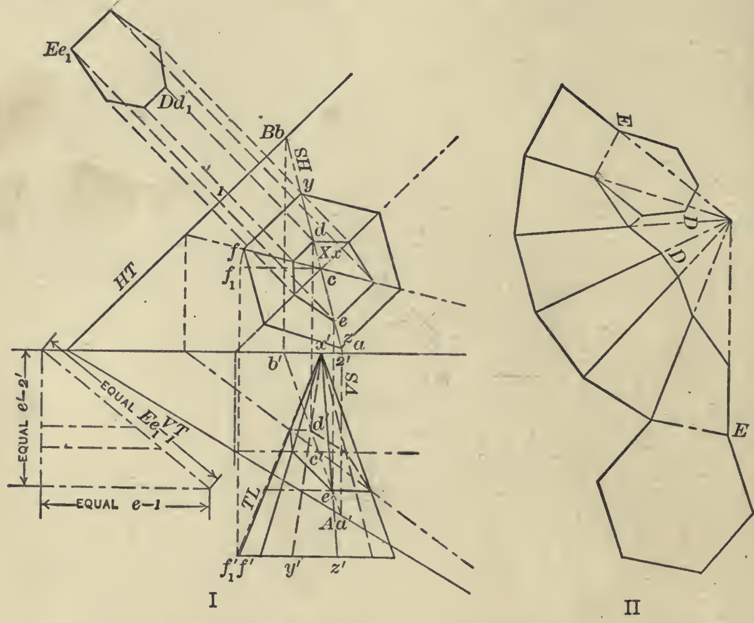


Fig. 133. — Intersection of a Plane and a Pyramid and Development of the Pyramid.

146. PROBELM 74. To determine the line of intersection of a cone cut by a plane inclined to V and H. Also to develop the surface of the cone and determine the true form of the line of intersection.

Analysis. Divide the base into a sufficient number of parts (say twelve), and through these points draw elements of the cone. Proceed as in Problems 70 and 73, page 134, with the *exception* that projections and developed points of the section are to be connected by a smooth curve.

Construction. See Fig. 134. The general construction is similar to that of Problem 73, page 137.

147. PROBLEM 75. To determine the true form of the line of intersection of two square prisms, meeting at any angle, and to develop the surfaces of the prisms.

Analysis. Since the faces of the prisms are planes, all lines

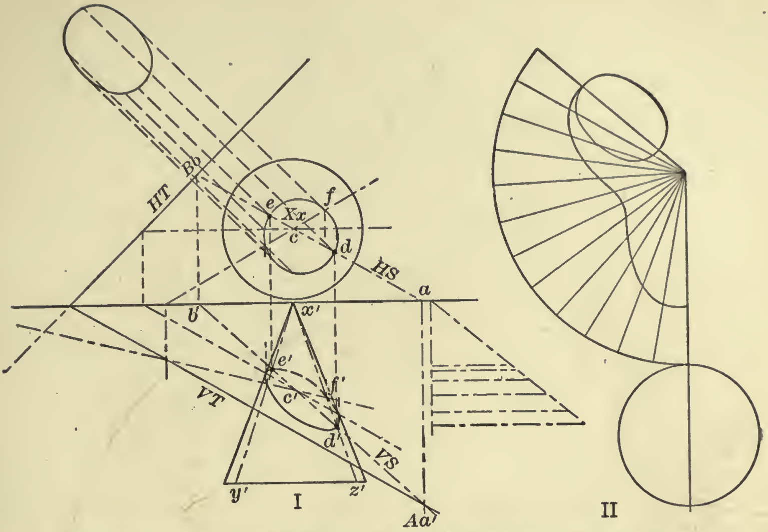


Fig. 134.—Intersection of a Plane and a Cone and Development of the Cone.

of intersection are straight lines, and these are determined by finding points common to both prisms and connecting these points by straight lines.

Construction. Let the principal prism be vertical, that is, let it have its base parallel to H, and let it be intersected first by a prism (on the left) which has its axis parallel to H, and next by a prism (on the right) which has its axis at any angle to H (see Fig. 135).

The vertical prism will show in plan as a square *a–b–c–d*, and in elevation as two tangent rectangles, *d′–j′–i′–c′* and *i′–c′–b′–f′*. To draw the vertical projection of the prism intersecting on the left, draw projections of the square *K–L–M–N* to represent the base of the prism in its proper vertical position when viewed in the direction of the arrow. Project the edges *m′–r′*; *k′–o′*; *l′–q′* and *n′–p′*, against the elevation of the vertical prism. The plan of the prism can be drawn in a similar manner. It is then evident that the projections *p*, *q*, *r*, and *o* are the horizontal projections of points common to both prisms. The vertical projections of these points are determined by the ruled projectors. Thus, *p′* must be at the intersection of the ruled projector from *p* and the vertical projection of the edge *n′–p′*, that is at *p′*.

Connect the points thus found to determine the line of intersection of the prisms. The lower prism *X–W–Y–Z* is projected by determining the position of its axis and making the construction as in the previous case. To develop the prism *A–B–C–D*, use *G–L* produced as a base line; lay off the dis-

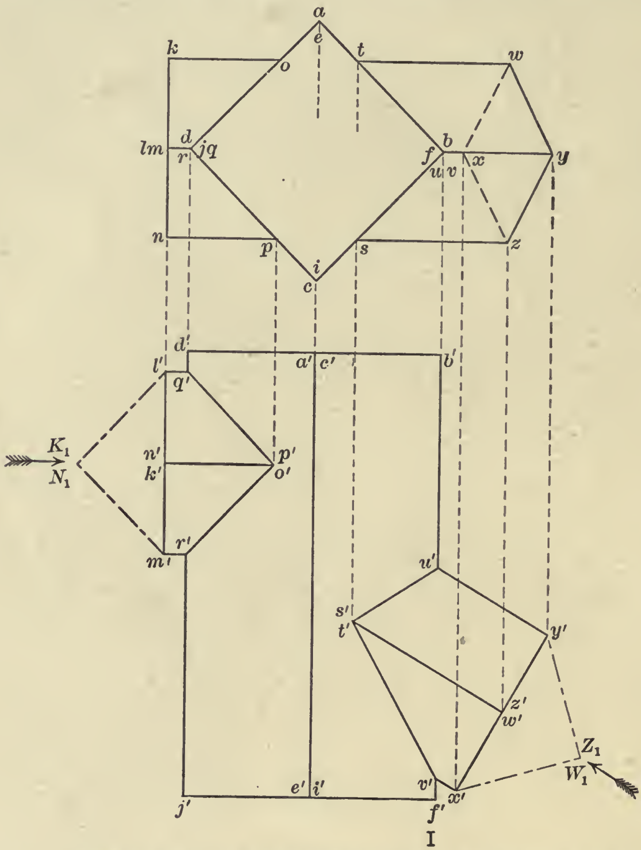


Fig. 135.—To determine the True Form of Intersection of Two Prisms and to develop the Surfaces of the Prisms.

tances C_1–B_1, B_1–A_1, A_1–D_1, D_1–C_1, each equal to a side of the prism. Draw the perpendiculars C_1–I_1, B_1–F_1, etc., making each equal to the length of the prism. These represent the edges of the prism in the development. As all vertical heights are shown true length in elevation, the points *q′* and *r′* can be projected directly to the edge D_1–J_1 upon which they are located. Thus the points Q_1 and R_1 are determined. The points O_1 and P_1 are determined by drawing the elements *1–2* and *3–4*, making E_1–*2*

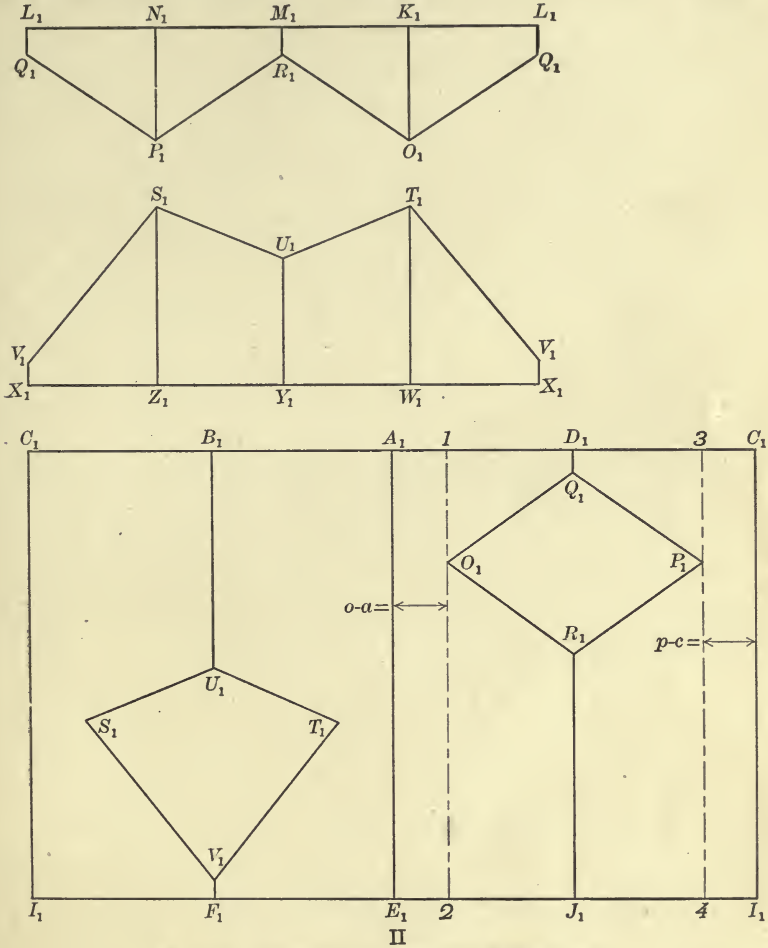


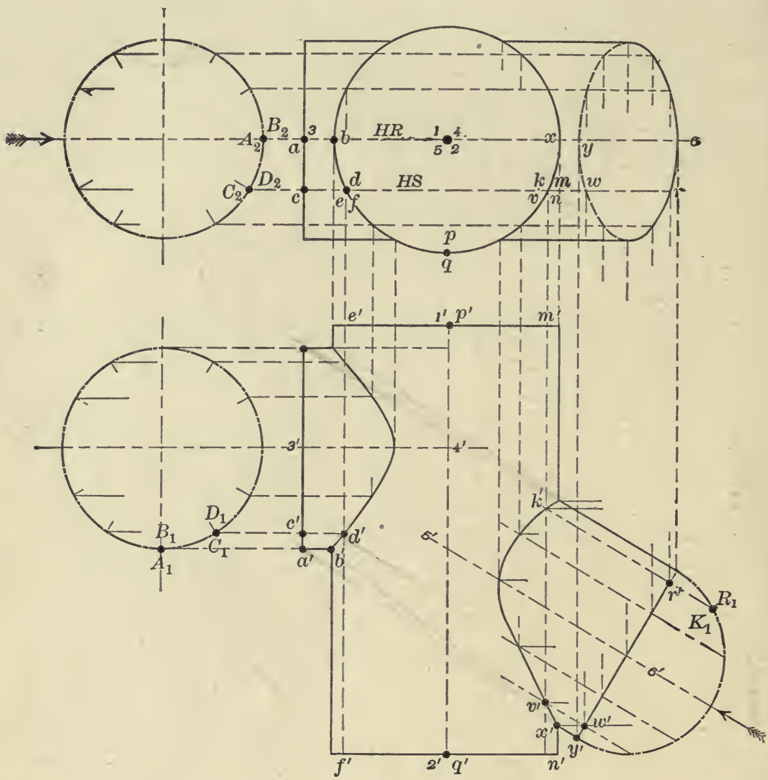
Fig. 135. — Continued. — Development of the Surfaces.

equal to *o–a*, and *4*–I_1 equal to *p–c*. Also, S_1–U_1–T_1–V_1 is found in the same manner.

The development of the prism *W–X–Y–Z* is determined by laying off X_1–Z_1, Z_1–Y_1, etc., each equal to a side of the prism base, and erecting the perpendicular X_1–V_1 equal to x'–v'; Z_1–S_1 equal to z'–s', etc.; x'–v', z'–s', etc., being true length projections of these edges.

148. PROBLEM 76. To draw the plan and elevation of two right cylinders, the axes of which meet at any given angle, and to develop the surfaces and determine the true form of the line of intersection.

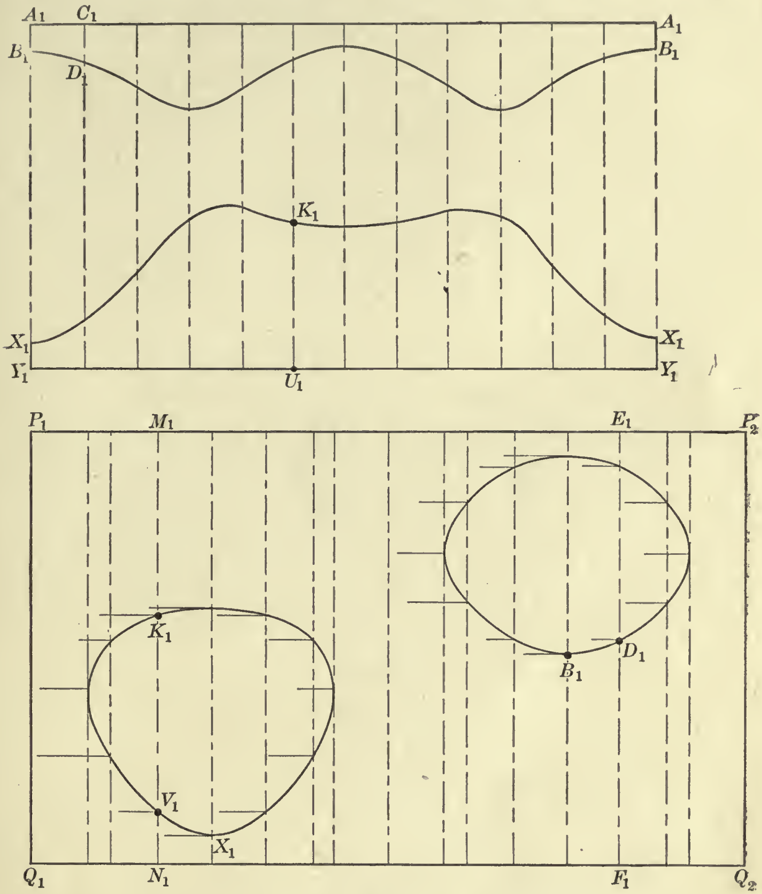
Analysis. Assume that the axes of the cylinders lie in a plane parallel to V. Also, that the axis of one cylinder is perpendicular



I

Fig. 136.—To determine the True Form of Intersection of Two Cylinders and to develop the Surfaces of the Cylinders.

to H, another is parallel to H, and a third makes 60 degrees with the first cylinder. To determine the lines of intersection, pass a series of planes through the cylinders parallel to V. Such planes cut elements from the cylinders, and the intersection of these elements determines points on the line of intersection of the cylinders. Develop the surfaces by laying off rectangles having a height equal to the height of the cylinder and a length equal to the circumference of the base of the cylinder; determine the position of elements of the cylinder in this



II

Fig. 136. — Continued. — Development of the Surfaces.

development and locate the points of intersection on these elements.

Construction. See Fig. 136. Draw the projections of the axes *1–2*, *3–4*, and *5–6*. All are parallel to V in plan, and in elevation *1′–2′* is perpendicular to *G–L*, *3′–4′* makes 90 degrees with *1′–2′*, and *5′–6′* makes 60 degrees with *1′–2′*. The cylinder *1–2* will show in plan as a circle having a diameter equal to the diameter of the cylinder, and in elevation as a rectangle having a width equal the diameter of the cylinder and a length equal the length of the cylinder. To find the projections of the cylinder *3–4*, draw its base on *3–4* produced as shown. This represents the base when looking in the direction of the arrow. Similarly draw the base in elevation. Divide this base into any number of equal parts (say twelve) and through these points of division pass planes R, S, etc., parallel to V. These planes cut from the cylinders elements which intersect at points common to both cylinders, hence on the line of intersection. Thus, the plane S cuts the elements $E–F$ and $M–N$ from the cylinder *1–2* and $V–W$ and $K–R$ from the cylinder *5–6*. The elements $M–N$ and $K–R$ intersect at K, therefore the point K is a point on the line of intersection. Similarly determine all other points and draw a smooth curve through them. The line of intersection of the cylinders *3–4* and *1–2* is similarly obtained.

To develop the cylinder *1–2*, use $G–L$ produced as a base line. Draw a vertical line at $Q_1–P_1$ to represent the element $Q–P$. Lay off the length $Q_1–N_1–F_1–Q_2$ equal to the circumference of the cylinder and draw the element $Q_2–P_2$, which coincides with $Q_1–P_1$ when the development is rolled together to form the cylinder.

Next locate on the development the element $M_1–N_1$, etc., of the cylinder which passes through the points K, V, etc., of the line of intersection by laying off the distance $Q_1–N_1$ equal to the arc $q–n$, etc. The altitudes of the points K, V, etc., can be transferred with dividers or in this special case projected directly across by the horizontal lines $k'–K_1$, $v'–V_1$, etc.

To obtain the development of the cylinders *3–4* and *5–6*, draw at any convenient position a base line equal in length to the circumference of the cylinder and proceed as in the case of the prisms of Problem 75, page 138.

149. PROBLEM 77. To draw the plan and elevation of a pyramid and prism which intersect; to develop the surfaces and determine the true form of the line of intersection.

Analysis. Assume the axis of the pyramid perpendicular to H and the axis of the prism parallel to H. Pass auxiliary planes through the pyramid so as to be perpendicular to V and to contain the edges of the prism and pyramid, and determine the points in which the boundaries of the sections intersect. Connect these points in proper order for the line of intersection. Develop the surfaces as in Problem 69, page 132, and Problem 75, page 138.

Construction. Let the pyramid and prism be given as in Fig. 137 I. Pass the horizontal plane S through the edge *E–F* of the prism. The plane S must cut a square section from the pyramid. This square must have one corner at the point *P* where *VS* intersects the edge *O–M*, since *P* is common to both the plane S and the edge *O–M*.

This square section is shown in plan at *p–1–2–3*, and the projection *k* where *e–f* intersects *p–1* determines the horizontal projection of a point which is common to both pyramid and prism, hence a point of their line of intersection.

The vertical projection *k′* of this point is determined by the ruled projector *k–k′*. Since the point *K* is located in the face *M–O–N*, it shows that the edge *E–F* pierces the pyramid in this face. In a similar manner, pass the plane T through the edge *C–D* and determine the point *I*. Also pass the plane U through the edge *A–B* and determine the point *X*. The points *K* and *X* are both in the face *M–O–N*, and hence can be connected. The point *I* cannot be connected to either *K* or *X*, as it lies on the opposite side of the edge *O–M* and in a different face of the prism. To determine the points where the edge *O–M* pierces the prism, pass the plane R through the edge *O–M* and perpendicular to V. The plane R cuts the edges of the prism at points vertically projected at *5′–6′–7′*, hence *5–6–7* is the horizontal projection of the section cut from the prism by this plane. Also *y* and *z* are both horizontal projections of points common to the prism and pyramid; their vertical projections are determined by ruled projectors. The point *Y*, being on the edge *O–M*, can be connected to both *X* and *I*, and the point *Z* can be connected to both *K* and *I*. This determines the line of intersection on the right, and that on the left is determined in a similar manner.

Check. Produce the edge 8–9 of the square section cut from the pyramid by the plane T until it intersects the edge *C–D* of the prism. The lines *8–9* and *C–D* must intersect, as both lie in the plane T and are not parallel. Also, as the line *8–9* lies

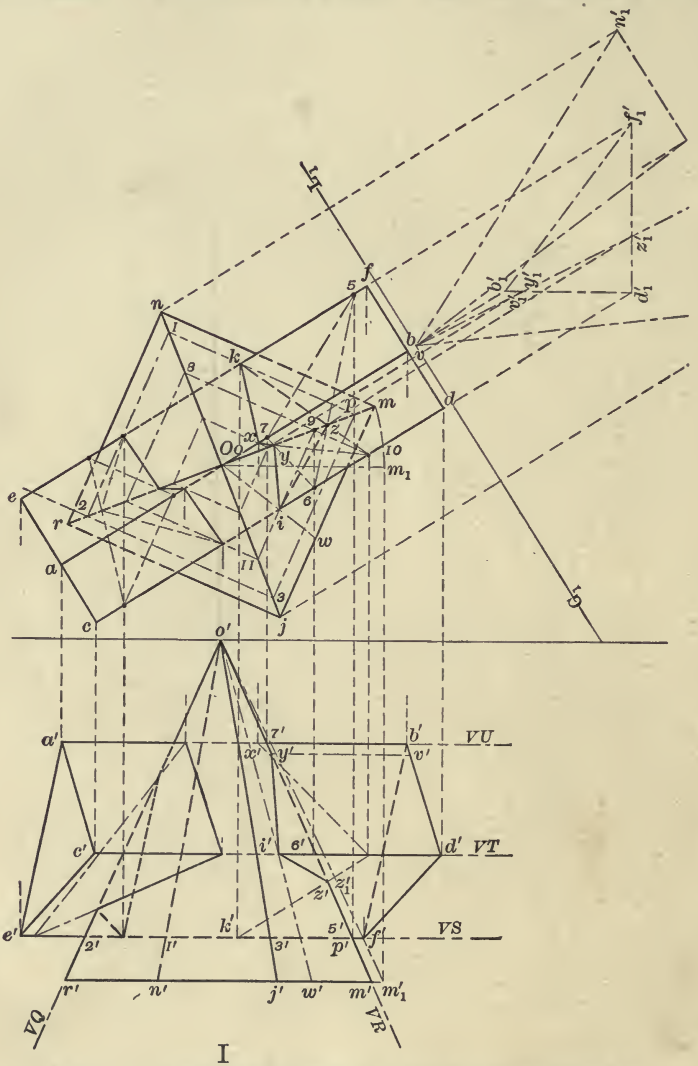
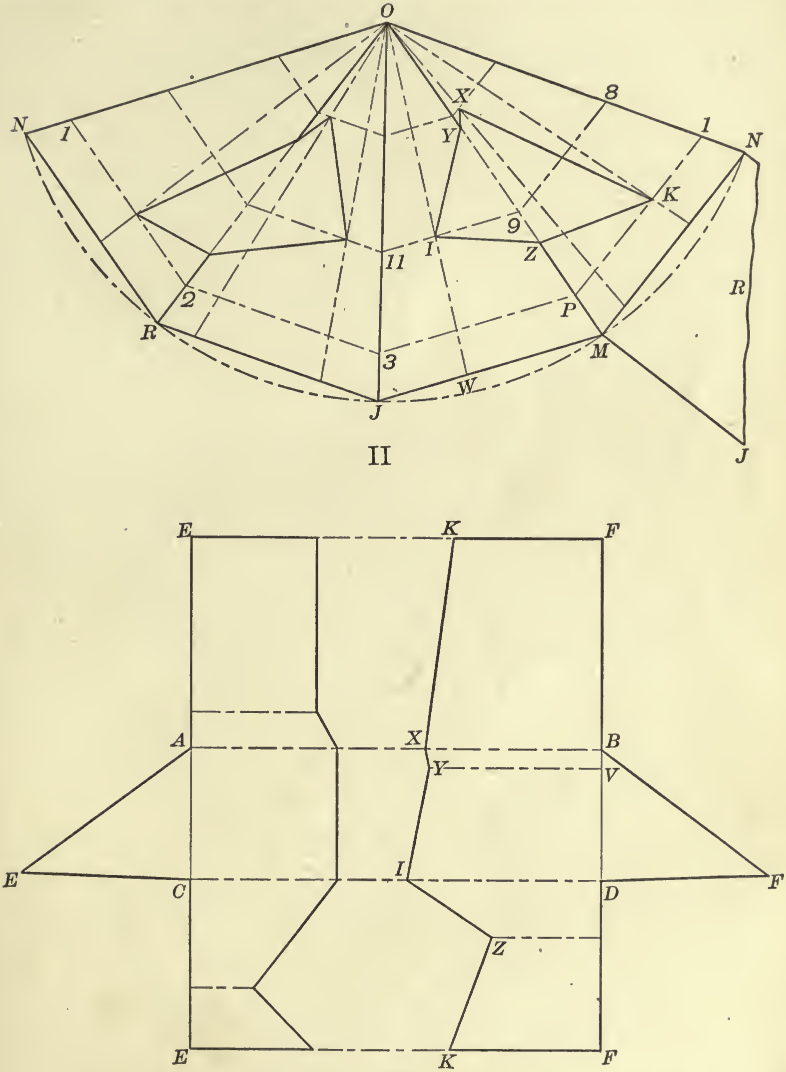


Fig. 137.—To determine the Projection of the Line of Intersection of a Pyramid and Prism.

in the plane of the pyramid face *N–O–M*, it must intersect *C–D* in the plane of this face or in the plane produced; hence at the point *10*. As the points *10* and *K* both lie in the plane of *N–O–M* they can be connected, and if the line *K–10* intersects the edge *O–M* at *Z*, this part of the construction is correct.



III

Fig. 137. — Continued. — To develop the Surfaces of the Pyramid and Prism.

Also, as the point *X* lies in the plane of *N–O–M*, the point *Y* can be similarly checked.

Development. Lay out the surface of the pyramid as in Problem 69, page 132, and determine the location of the square sections *1–P–3*; *8–9*, etc.

To determine the true form of the line of intersection, locate the point Z on the edge $O–M$ by swinging $O–M$ parallel to V, when z' will fall at z_1', and $m_1'–z_1'$ is the true distance that the point Z is from M (see Fig. 137 I). Similarly locate the point Y on the edge $O–M$ in development (see Fig. 137 II). To determine the point I, draw the element $O–W$ which will contain the point I. As the edge $J–M$ of the pyramid is parallel to H, the distance $j–w$ is a true length, and hence $J–W$ in the development is known. Join $W–O$ in the development (Fig. 137 II), and where this line intersects the edge *11–9* the point I is located. All other points are similarly determined and connected in proper order.

To develop the prism, draw $G_1–L_1$ (Fig. 137 I) parallel to the prism base. Determine the new vertical projection of the prism and $d_1'–b_1'–f_1'$ shows each edge in its true length. Hence draw $F–B$, Fig. 137 III, equal to $f_1'–b_1'$ of Fig. 137 I, etc. The edge $F–K$ is shown in true length in $f–k$, Fig. 137 I, hence can be transferred at once to the development. Similarly determine $B–X$. To determine the point Y of the development, draw the projections of the prism element $Y–V$ (see Fig. 137 I). The length $b_1'–v_1'$ is a true length and determines $B–V$ of the development. The element $V–Y$ is parallel to $B–X$, and hence can be drawn. The projection $v–y$ is a true length, hence the length $V–Y$ is known.

Similarly, all other points are determined and connected in proper order for the true form of the line of intersection.

Check. Similar lines in Figs. 137 II and 137 III must be equal. That is, $K–X$ of Fig. 137 II must equal $K–X$ of Fig. 137 III, etc. as these lines coincide on the objects which intersect.

150. PROBLEM 78. To draw the plan and elevation of a cone and cylinder which intersect and to develop the surfaces and determine the true form of the curve of intersection.

Analysis I. Assume the axis of the cone to be perpendicular to H and the axis of the cylinder parallel to V and H. Pass a series of auxiliary horizontal planes through the cone and cylinder. These planes cut circular sections from the cone and rectangular sections from the cylinder. The points of intersection of these circles and rectangles which are cut by the same plane are points common to both solids, and hence are

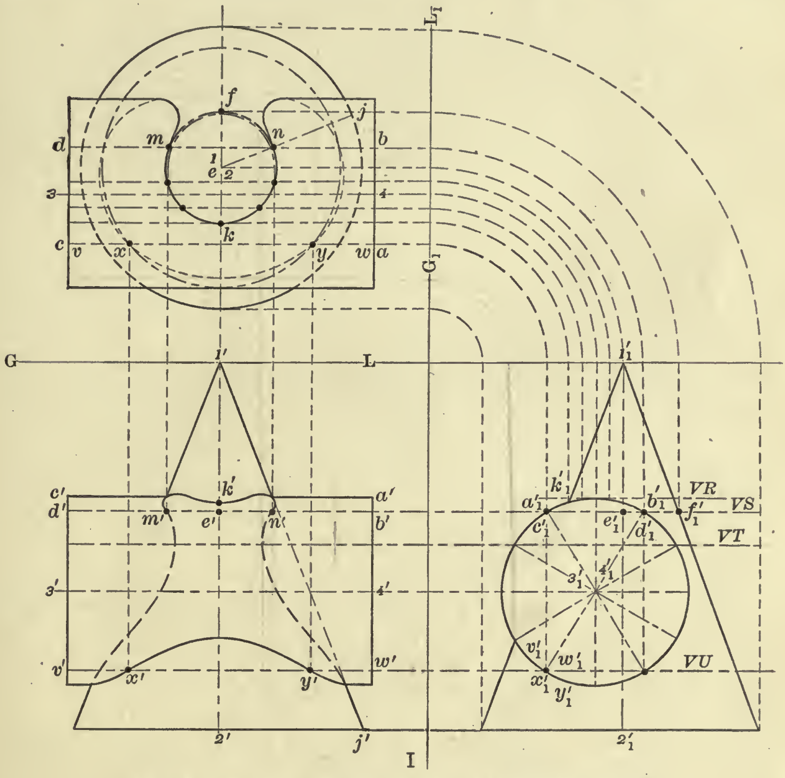
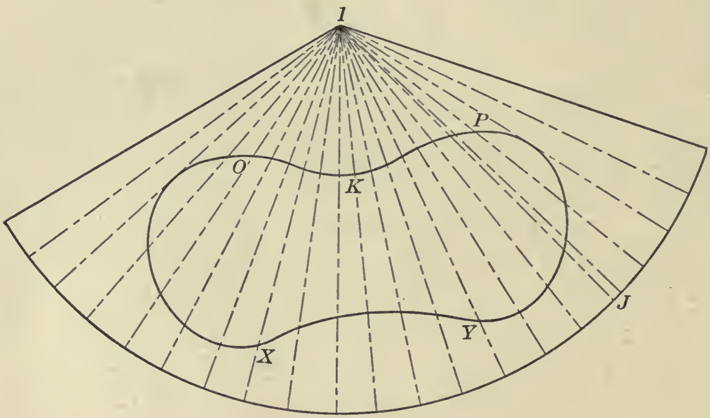


Fig. 138. — To determine the Intersection of a Cone and Cylinder.

points on their curve of intersection. Develop the surfaces as in Problem 70, page 134, and Problem 68, page 131.

Construction. Draw 1–2, $1'$–$2'$ and 3–4, $3'$–$4'$ (see Fig. 138 I) to represent the projections of the axes of the cone and cylinder respectively. Upon these axes draw the plan and elevation of the solids. Assume a new vertical plane (shown by G_1–L_1, Fig. 138 I) parallel to the base of the cylinder and draw in the new elevation on $1_1'$–$2_1'$ and $3_1'$–$4_1'$. Pass the horizontal plane

S through the solids. This plane cuts the cylinder along the elements *A–C* and *B–D*, and from the cone it cuts a circle of radius e_1'–f_1'. This circle, shown in plan at *f–m–n*, cuts the horizontal projection of the element *B–D* at *m* and *n*. Hence these



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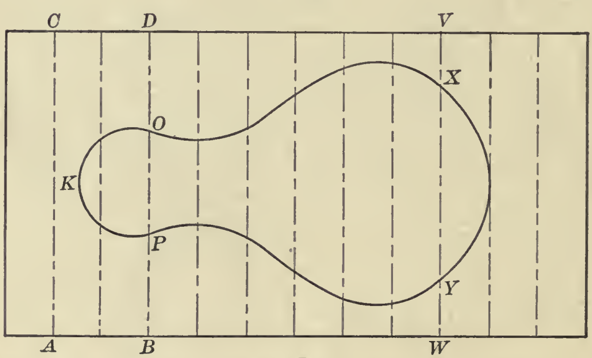

III

Fig. 138. — Continued. — To develop the Surfaces of the Cone and Cylinder which Intersect.

are horizontal projections of two points on the curve of intersection. Their vertical projections are determined by ruled projectors. The projection *f–m–n* of the circle does not reach the horizontal projection of the element *A–C*, therefore the cone does not intersect the cylinder on this element. This is also seen by inspection, as k_1' marks the last point on top-and-near-side of the curve of intersection. The points *x* and *y* are not on the element $A-C$, but are on $V-W$, an element cut by the plane U. This plane also cuts from the cone the circle drawn in plan through x and y. Similarly determine all points necessary and draw the curve of intersection.

To develop the surface of the cone, proceed as in Problem 70, page 134. To develop the cylinder, proceed as with the prism in Problem 68, page 131, using a sufficient number of elements of the cylinder in the same manner as the edges of the prism were used in problem 67. For developments see Figs. 138 I and 138 II. The bases of cone and cylinder have been omitted in this development.

Check. The curve of intersection $O-K-P-Y-X$ on the development of the cone, Fig. 138 II, must be equal in length and of the same size and shape as $O-K-P-Y-X$ on the development of the cylinder, Fig. 138 III.

151. PROBLEM 79. To determine the curve of intersection of two double-curved surfaces of revolution whose axes intersect.

Analysis. Let the two surfaces be those of an ellipsoid and a sphere given by their projections in Fig. 139. Intersect the surfaces of the ellipsoid and sphere by a series of auxiliary spheres whose centers are at some point O on the major axis of the ellipsoid. The section line between such an auxiliary sphere and the original sphere, as well as between the auxiliary sphere and ellipsoid, are both circles. The intersection of two such circular sections determines points on the line of intersection of the surfaces.

Construction. Let the axes of both the ellipsoid and sphere intersect at O and be parallel to V. Let $c'-e'-d'-f'$ represent the vertical projection of an auxiliary sphere having its center at O; $c'-d'$ is the vertical projection of the circle in which the auxiliary sphere meets the given sphere, and $e'-f'$ is the vertical projection in which the auxiliary sphere meets the ellipsoid. These two projections intersect at Q and N. Draw the horizontal projection $e-f$ of the circle also the ruled projector from q' and n' to intersect the horizontal projection of this circle at two points q and n, hence these are the horizontal projections of two points on the required curve of intersection. The horizontal projections of the auxiliary sphere and the circular section cut

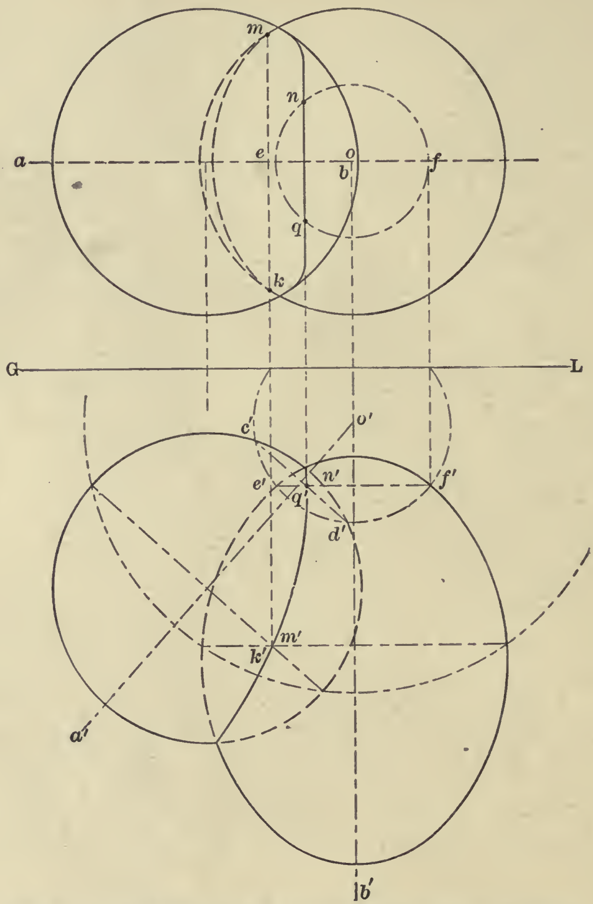

Fig. 139.—To determine the Intersection of an Ellipsoid and a Sphere.

from the sphere *O–A* are not necessary in the construction and hence are omitted. By the above method any number of points such as *k*, *k'* and *m*, *m'* can be determined and the projections of the curve of intersection can then be drawn.

CHAPTER IX

ISOMETRIC PROJECTION

152. Introductory. Isometric drawings can frequently be used to give those *not* skilled in the interpretation of *mechanical* drawings a clear idea of the construction and appearance of a structure or machine. Also, it frequently happens that peculiarities of construction can be more clearly presented by this method than by the use of the ordinary plan, elevation, and sections. Isometric drawings *resemble* perspective drawings in that they show the three principal dimensions — length, width, and height — of an object in a *single* view. A perspective drawing, however, cannot be completely dimensioned, whereas an isometric drawing is almost as easily dimensioned as is a mechanical drawing.

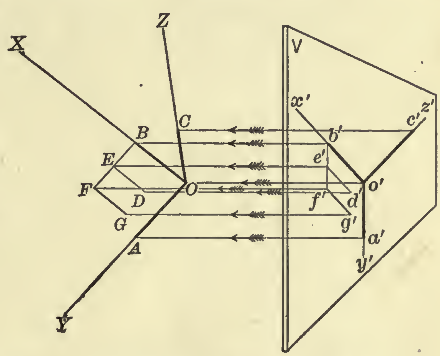

Fig. 140. — Fundamental Principle of Isometric Projection, Illustrated.

The method by which isometric drawings are made is termed **isometric projection,** and this consists in projecting an object upon a *vertical* plane, to which its three principal dimension lines are *equally* inclined. For example, let O–X, O–Y, and O–Z, in Fig. 140, represent the three edges of a right trihedral angle in space. Let O–Y be in a vertical plane that is in a plane perpendicular to V, and let each of the other two edges make the same angle with V.

When occupying this position, if the three edges are projected upon V, their projections o'–x', o'–y', and o'–z' will radiate from o' and form three *equal* angles of 120 degrees each.

The projection o'–y' will be vertical, while o'–x' and o'–z' will each make 30 degrees, in opposite directions, with a horizontal line through o'.

The lines in space, O–X, O–Y, and O–Z, are termed **coördinate axes,** and the planes determined by the coördinate axes are termed **coördinate planes.**

The **isometric origin** is o', and o'–x', o'–y', and o'–z' are the **isometric axes.**

If O–A, O–B, and O–C coincide with O–Y, O–X, and O–Z respectively, and *represent* the height, width, and length of a cube itself, the three faces bounded by these lines can be shown in projection on V, and the *projections* will be bounded by o'–a', o'–b', and o'–c'.

Every point located on one of the faces of the cube would have its projection *in the angle* formed by the projections of the edges which terminate or bound that face. Thus, the point D is on the face A–O–B–E, and its isometric projection is in the angle a'–o'–b'. If a second point E on this same plane had been in such a position that the line D–E joining the points were parallel to O–X, then the isometric projection d'–e' of the line will be parallel to the isometric axes o'–x'. Furthermore, if two lines in space, D–E and F–G, are parallel, their isometric projections will be parallel. It should here be observed that any solid whatever could be placed in the right trihedral angle so that its three principal lines of dimensions will either coincide or be parallel to as many of the three axes O–Y, O–X, and O–Z as possible, and when in this position it can be isometrically projected on V. Theoretically, it is necessary to use a *special* scale, called an **isometric scale,** to measure off distances on the plane V. This is because the projection of a line when "inclined" is *always* shorter than the line itself. In the case of isometric projections, the three axes O–X, O–Y, and O–Z are each inclined to V at an angle of 35° 16′ and hence an inch length on one of these axes will be represented by the product resulting from multiplying 1 inch by the natural cosine of this angle, which is 0.816. That is, 1 inch actual measurement on the object *projects* only 0.816 of an inch, and a scale constructed upon this basis could be used to measure all distances on V *parallel* to one of the isometric axes. The isometric scale is mentioned here as a matter of theoretical interest, since such scales are now seldom used in practice.

Isometric drawings are usually made with the same scales as are used in the making of mechanical drawings, in order to facilitate scaling and dimensioning.

153. **Fundamental Principles.** Based on the discussion in §152, page 153, the cube shown in Fig. 141 has been drawn. By reference to this figure the following five **fundamental principles** will be understood.

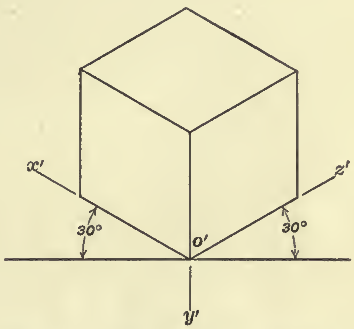

Fig. 141.—Isometric Projection Illustrated.

(1) There are three isometric axes which radiate from a common point termed the origin. One axis is drawn vertical, one is drawn at 30 degrees toward the right, and one at 30 degrees toward the left.

(2) The isometric axes represent lines mutually *perpendicular* to each other *in space*. They correspond to the three dimensions,—height, length, and width,—and on the drawing measurements can only be laid off on or parallel to one of these axes.

(3) Lines vertical on the object are vertical on the drawing.

(4) Lines parallel on the object are parallel on the drawing.

(5) Right angles on the object are usually either 60 degrees or 120 degrees on the drawing.

154. **PROBLEM 80. To determine the isometric projection of a point which lies in one coördinate plane and at a given distance from the other two coördinate planes.**

Analysis. Since the point lies in one of the coördinate planes, the isometric projection of the point must be at the intersection of isometric ruled projectors which lie in this same plane.

From the origin measure off distances along the proper coördinate axes to locate the foot of the isometric ruled projectors.

Construction. See Fig. 142. Let the point lie in the plane $X\text{–}O\text{–}Z$, it being $\frac{5}{8}$ inch from the plane $Y\text{–}O\text{–}Z$ and $\frac{7}{8}$ inch from the plane $Y\text{–}O\text{–}X$. Let $o'\text{–}x'$, $o'\text{–}y'$, and $o'\text{–}z'$ be the projections of the isometric axes. Measure off o'–a_z' equal to $\frac{7}{8}$ inch (the distance the point is from the coördinate plane X–O–Y), and through a_z' draw the ruled projector a_z'–a' parallel to o'–x'.

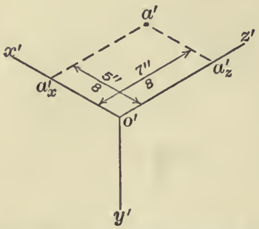

Fig. 142. — To Determine the Isometric Projection of a Point on a Coördinate Plane.

Make o'–a_x' equal to $\frac{5}{8}$ inch (the distance the point is from the coördinate plane Z–O–Y), and through a_x' draw the ruled projector a_x'–a' parallel to o'–z'.

The lines a_z'–a' and a_x'–a' are the *isometric* ruled projectors for the point A in space, and a' is the required *isometric projection* of the point.

155. PROBLEM 81. To determine the isometric projection of any point in space when its position relative to the coördinate planes is known.

Construction. See Fig. 143. Let the point A be located as in Problem 80, page 155, with reference to the planes X–O–Y and Y–O–Z but let it be $\frac{1}{2}$ inch *below* the plane X–O–Z. Its position relative to the plane X–O–Z is determined as in Problem 80. From o' measure o'–a_y' off equal to $\frac{1}{2}$ inch along o'–y'. At a_y' draw a_y'–3 parallel to o'–z', and lay off the distance a_y'–3 equal to $\frac{7}{8}$ inch. Similarly, from a_y' lay off a_y'–1 equal to $\frac{5}{8}$ inch and parallel to o'–x'.

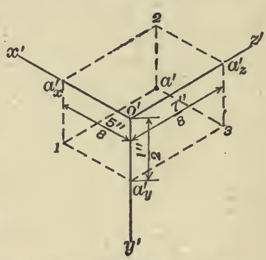

Fig. 143. — Isometric Projection of Any Point in Space.

This method of constructing an "offset" parallelogram 1–a_y'–3–a' illustrates one way of determining the projection a'.

From o' proper distances could be measured off along o'–z' and o'–x' to locate 2 from which a' could be located.

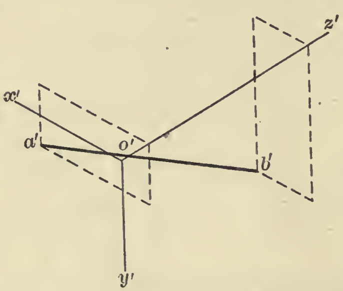

Fig. 144. — To Determine the Isometric Projection of Any Straight Line.

156. PROBLEM 82. To determine the isometric projection of any straight line when the position of any two of its points is known with reference to the coördinate planes.

Analysis and Construction. See Fig. 144. Determine the projections *a′* and *b′* of the two given points *A* and *B*, as in Problem 81, page 156, and the line connecting the projections of the points is the required *projection* of the line.

157. PROBLEM 83. To draw the isometric projection of a square figure of given dimensions which lies in one coördinate plane and at known distances from the other two.

Analysis and Construction. Let the figure be 1 inch square. Assume that it lies in *X–O–Z*, with one side ¼ inch from and parallel to *O–Z*, and another side 1 inch from and parallel to *O–X*, that is the sides are respectively parallel to these two axes.

Determine the projection *d′* (see Fig. 145) of one corner, which is 1 inch from *o′–x′* and ¼ inch from *o′–z′* (see Problem 80). Draw *d′–c′* parallel to *o′–z′* and measure off along this line 1 inch to determine *c′*. Draw *d′–a′* parallel to *o′–x′* and measure off along this line 1 inch to determine *a′*. Draw *c′–b′* parallel to *o′–x′*, and *a′–b′* parallel to *o′–z′*. This locates *b′*, and *a′–b′–c′–d′* is the projection required.

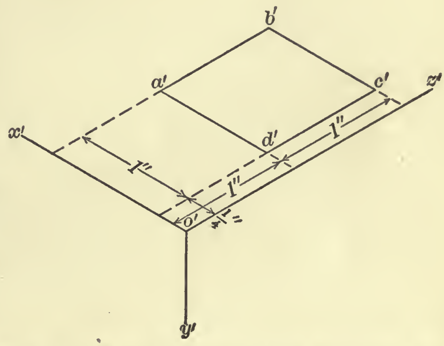

Fig. 145.—To Draw the Isometric Projection of a Square Which Lies in a Definite Position in One of the Coördinate Planes.

158. PROBLEM 84. To draw the isometric projection of a cube.

Analysis. Consider a vertex of the cube as coinciding with the origin of coördinates, *O*, and three adjacent edges coinciding with the coördinate axes. In this position these three edges of the cube will be isometrically projected on the isometric axes, and the remaining edges will be projected in lines parallel to these axes.

Construction. Lay off *o′–a′* and *o′–c′* (see Fig. 146), each equal to an edge of the cube. Draw *a′–b′* parallel to *o′–z′*, and *b′–c′* parallel to *o′–x′*. The intersection *b′* of these two lines determines another vertex of the cube. Lay off $o'-g'$ equal to an edge of the cube and draw $g'-d'$ and $g'-f'$ parallel to $o'-x'$ and $o'-z'$ respectively. From a' and c' draw lines parallel to $o'-y'$. Then $a'-b'-c'-o'$ represents the top, $d'-a'-o'-g'$ the left-hand front face, and $g'-o'-c'-f'$ the right-hand front face of the cube.

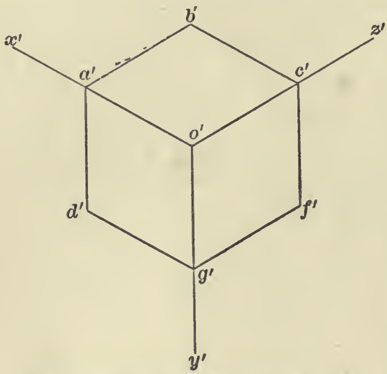

Fig. 146.—Isometric Projection of a Cube.

159. PROBLEM 85. To draw the isometric projection of an hexagonal pyramid.

Analysis and Construction. Let the plan of the pyramid be shown in Fig. 147 II. Assume the pyramid to be circumscribed by a rectangular prism, the base of which circumscribes the base of the pyramid, and the altitude of which equals the altitude of the pyramid. Draw the isometric projection of the right prism as in Problem 84. Draw the diagonals $1'-3'$ and $2'-4'$ to determine the center q' of the top face of the prism. The point Q' is the apex of the pyramid. Determine the vertices $a'-b'-c'-d'-e'-f'$ of the base as in Problem 84.

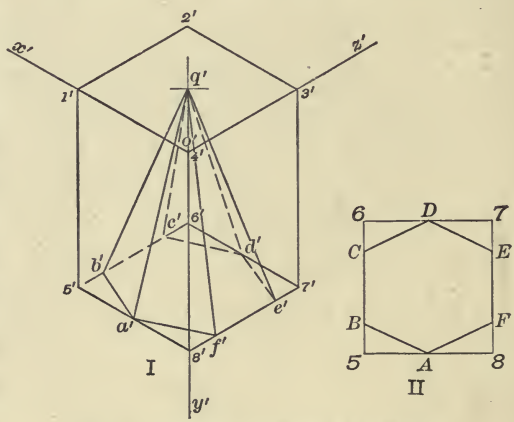

Fig. 147.—Isometric Projection of a Pyramid.

To draw the projection of the base of the pyramid, make $8'-a'$ in Fig. 147 I equal to $8-A$ in Fig. 147 II and $a'-f'$ equal to $A-F$. Make $8'-e'$ equal to $8-E$ and connect f' and e'. The projections of other vertices of the base can be similarly determined.

Connecting each vertex of the base with the apex q' determines the projections of the edges of the pyramid. The edges that are invisible can be determined by inspection.

160. PROBLEM 86. To make an isometric drawing of a circle.

Analysis and Construction. Circumscribe a square about the circle and locate any number of points as shown in Fig. 148 II. The distances 1′–7′, 7′–6′, 6′–5′, etc., in Fig. 148 I, are respectively equal to 1–7, 7–6, 6–5, etc., in Fig. 148 II, etc.

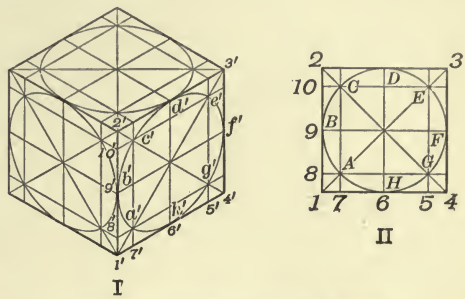

Fig. 148.—Isometric Drawing of a Circle.

161. PROBLEM 87. To make an isometric drawing of any plane curve.

Fig. 149 illustrates clearly how a plane curved figure can be shown in isometric.

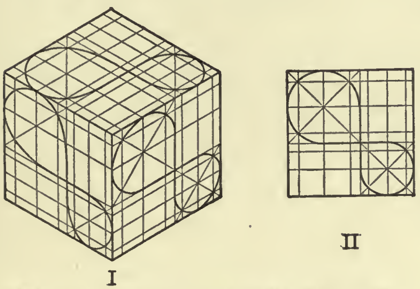

Fig. 149.—Isometric Drawing of a Plane Curve.

162. PROBLEM 88. To make an isometric drawing of a mortise and tenon.

Fig. 150 clearly shows how this can be done.

163. PROBLEM 89. To make an isometric drawing of a piping system.

Fig. 151 clearly shows how this can be done.

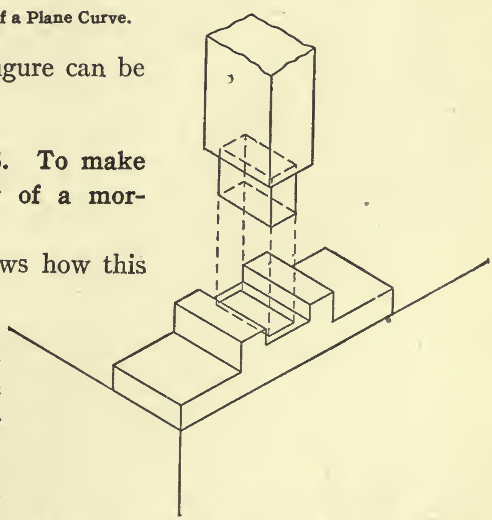
Fig. 150.—Isometric Drawing of a Mortise and Tenon.

164. Cavalier or Cabinet Projection. Sketches made in **cavalier or cabinet projection** are sometimes preferred to those made in isometric projection (see §152, page 153). This system is similar to the isometric in that it makes use of three axes along or parallel to which measurements corresponding to the three space dimensions — length, width, and height — can be made. The axes, however, do *not* occupy the same position as in isometric projection, one axis being vertical, one horizontal, and one 45 degrees to the horizontal (see Fig. 152). Measurements are made *true size* on and parallel to the vertical and horizontal axes, and *half-size* on and parallel to the 45-degree axis. This foreshortens the depth of the drawing and gives the sketch a perspective effect. The front face and all others parallel to it are drawn in their true shape and size as in a mechanical drawing.

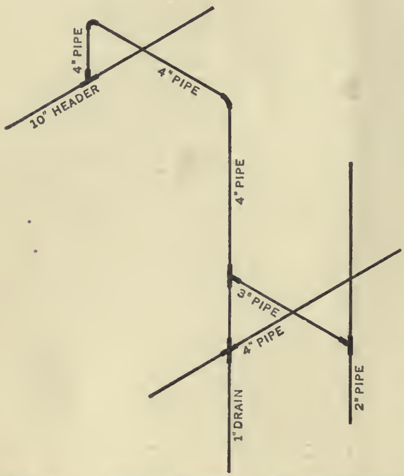

Fig. 151. — Isometric Drawing of a Piping Layout.

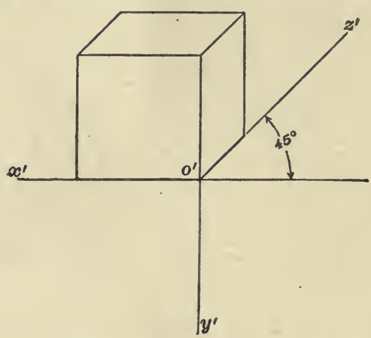

Fig. 152. — Cavalier or Cabinet Projection of a Cube.

165. Pseudoperspective. This method is in principle a modification of cabinet projection (see § 164). Both vertical and horizontal axes are used the same as in cabinet projection, but the third axis is drawn at a *greater* angle than 45 degrees. In this system it is necessary that measurements made in the plane of the paper or parallel to the paper be made in their

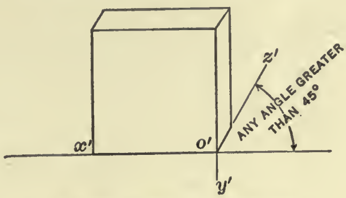

Fig. 153. — Pseudoperspective.

true size, while those representing lines perpendicular to the plane of the paper are reduced to some assumed ratio of their actual length the exact ratio in each case being that which gives the best results. See Fig. 153. This method gives the drawing a sense of depth similar to a perspective and can be used to good advantage in making illustrations and sketches.

CHAPTER X

SET OF DRAWING EXERCISES IN DESCRIPTIVE GEOMETRY

166. Introductory. It is intended that the following exercises be solved in the drawing room, immediately after the articles indicated have been taken up in the drawing or classroom.

It is expected that in solving *each* drawing board exercise, the student will study the general text matter covering the principles involved. The proper text to study can be easily found by use of the INDEX (page 201) *which has been made unusually complete to facilitate quick reference.*

The exercises are designed to be solved within border lines measuring 11 × 16 inches (see Fig. 167, page 196). For full instructions as to working up the drawing board exercises see page 195, Appendix A.

167. PLATE I. Exercises 1 to 25 inclusive. This plate requires a working knowledge of § 1 to § 40 inclusive.

G–L, $2\frac{1}{2}''$ *and* $7\frac{5}{8}''$.

Draw the projections for the point *A* in the following positions given in exercises 1 to 6 inclusive.

1. — [$\frac{1}{2}$] $1\frac{1}{4}''$ in front of V and an equal distance from H.

2. — Move *A* $\frac{1}{2}''$ to the right and $\frac{1}{4}''$ nearer V.

3. — Move *A* from the last position $\frac{1}{2}''$ to the right and $\frac{1}{2}''$ nearer H.

4. — Move *A* from the last position $\frac{1}{2}''$ to the right and into V.

5. — Indicate [$2\frac{1}{2}$] *B* in H and $\frac{3}{4}''$ from V.

6. — Indicate [3] *C* in both V and H.

7. — Determine the projections of a line A–B; [$3\frac{1}{2}$] A, $\frac{1}{2}''$ in front of V and $\frac{1}{2}''$ above H; b' is $1\frac{1}{2}''$ above G–L and the line is perpendicular to H.

8. — Determine the projections of a line C–D; [4] C, $\frac{1}{4}''$ in front of V and $1''$ above H. The line is perpendicular to V and is $1''$ long.

9. — Indicate the line E–F; [$4\frac{1}{2}$] E, $\frac{1}{2}''$ above H and $1''$ in front of V; the line is $1''$ long and is parallel to V and H.

10. — Move the line E–F to the right $1\frac{1}{2}''$ and without changing its distance from V draw its projections when it is equally distant from V and H and parallel to G–L.

11. — Indicate the line A–B; [$7\frac{1}{2}$] A, [$8\frac{1}{4}$] B, when the line lies in G–L. Keeping the line in V, raise it until A and B are each $1\frac{1}{4}''$ from H and draw its projections. Next swing b' about a' as a center and toward G–L. Determine its projections when the vertical projections make 45 degrees and 90 degrees with G–L. What is the position of the line in space in each case?

12. — Indicate the line which is terminated by the points [$9\frac{1}{4}$] B, $1''$ above H and $\frac{1}{4}''$ in front of V; [$10\frac{1}{4}$] C, $\frac{3}{8}''$ above H and $\frac{3}{4}''$ in front of V. Determine the points E and F where the line first pierces V and H.

13. — Draw G_1–L_1 perpendicular to G–L at [11] and determine the new vertical projections of the line E–F and of the points B and C on this line. Swing the horizontal projection of the horizontal piercing point of the line about the horizontal projection of the vertical piercing point until F lies in G–L. Where does the line now lie in space? What is the true distance in space from the point E (on V) to F (on H)? Also between B and C. Mark the true angle (TA) that the line E–F makes with H, and the projection of this angle mark PA.

14. — Determine the projections of the line *A–B*. [13½] *A* is $\frac{3}{8}''$ above H and $\frac{3}{8}''$ in front of V; [14¾] *B* is $1\frac{3}{8}''$ above H and $1\frac{3}{8}''$ in front of V. Revolve b' about a' as a center until b_1' is $\frac{3}{8}''$ from *G–L* and determine b_1. The point *B* is not to change its distance from *V*.

15. — State where the line *A–B* (of Exercise 14) lies in space in its original and revolved positions. Determine its piercing points in each position.

16. — The point *P* bisects the line *A–B*; draw its projections for each position of the line.

17. — Swing b_1 about *a* until b_2 is $\frac{3}{8}''$ above *G–L* and determine b'. Where does the line now lie in space and where are its piercing points?

18. — The line *C–D* passes through [½] *C*, $1\frac{3}{8}''$ in front of V and $1\frac{1}{4}''$ above H; [1½] *D*, $\frac{1}{2}''$ in front of V and $\frac{1}{4}''$ above H. Determine the horizontal piercing point of the line. Draw $G_1–L_1$ through [2½] and parallel to *c–d*. Determine the new vertical projection of the line on V_1. The point *F* is on *C–D* and is $\frac{1}{2}''$ from *C*. Determine its projections.

19. — Indicate [3] *A*, $\frac{3}{4}''$ below H and $1\frac{1}{4}''$ back of V.

20. — The line *A–B* is in the *third* angle. [3½] *A* is $1\frac{3}{8}''$ below H and $1\frac{1}{8}''$ back of V; [4½] *B* is $\frac{1}{4}''$ below H and $\frac{1}{2}''$ back of V. Determine where *A–B* pierces V and H. Take $G_1–L_1$ through [5½] and parallel to *a–b* and determine the new vertical projection of the line.

21. — Indicate the point [6] *A*, $\frac{1}{2}''$ back of V and $1''$ above H.

22. — Indicate the point [6½] *B*, $\frac{1}{2}''$ in front of V and $1''$ below H.

23. — Indicate the line *A–B*; [7] *A* on V and $1''$ below H; [8½] *B* on H and $1''$ in front of V.

24. — Solve this exercise in the *first* angle:
Pass a line through the points [9] A, $1\frac{1}{2}''$ from V and $1\frac{1}{2}''$ from H; [$10\frac{1}{2}$] B, $\frac{3}{8}''$ from V and $\frac{1}{4}''$ from H. Determine where A–B pierces V and H. Through the point [$9\frac{1}{2}$] E, $1\frac{1}{2}''$ from Hand $\frac{1}{2}''$ from V, pass a second line E–F parallel to A–B and determine its piercing points. Through [12] pass a new vertical plane parallel to the lines and determine the **true length** of the portions of each line that lies only in the first angle.

25. — [$12\frac{1}{2}$] A, [14] B, [13] E. Otherwise data same as in Exercise 24, but this exercise to be solved in the *third* angle. Take G_1–L_1 through [$15\frac{3}{8}$].

168. PLATE II. Exercises 26 to 35 inclusive. This plate requires a working knowledge of the text through § 50.

G–L, $3\frac{1}{4}''$ *and* $7\frac{3}{4}''$.

26. — Indicate the line A–B; [$\frac{1}{2}$] A, $1\frac{3}{4}''$ above H and $2''$ in front of V; [$2\frac{1}{2}$] B, $\frac{1}{4}''$ above H and $\frac{1}{16}''$ in front of V. The point [$1\frac{1}{2}$] E is on A–B. Indicate the line C–D to intersect A–B at E. [$\frac{1}{2}$] C is $\frac{1}{2}''$ above H and $\frac{1}{4}''$ in front of V. From [$\frac{3}{4}$] F on A–B draw F–L to intersect C–D on [$2\frac{1}{4}$]. See § 27, p. 20; § 33, p. 26.

27. — The line [$3\frac{1}{2}$] A–B touches H and is also perpendicular to H. It is $1''$ from V and is $1\frac{1}{4}''$ long. The point [3] D is $1''$ above H and $\frac{1}{2}''$ in front of V. The point [$4\frac{1}{4}$] E is $1''$ above H and $\frac{1}{4}''$ in front of V. Revolve each point "clockwise" through 45 degrees about A–B as an axis. See § 44, p. 32; § 46, p. 35.

28. — The line A–B is $1\frac{1}{4}''$ long, lies in the *first* angle, and is perpendicular to V. [6] A is $1\frac{1}{4}''$ above H and $\frac{1}{4}''$ in front of V. The line C–D makes 60 degrees with A–B, it intersects A–B at D $1\frac{1}{4}''$ from V, and C is approximately $\frac{11}{16}''$ from V. Revolve C–D

clockwise about A–B as an axis through 360 degrees and determine projections for its position at each 45 degrees of the line's revolution. See § 47, p. 35.

29. — The point [9] A is on V and $1\frac{1}{2}''$ from H, and is to be revolved into H. Draw an axis of rotation O–C in H, through [$10\frac{1}{4}$] O, and inclined to the left, making 30 degrees with G–L. Revolve the radius of rotation Aa'–Cc into V. Keeping A in V, revolve the radius of rotation A until it lies in H (that is, in G–L). Next revolve A in H until it occupies the true position Aa_2 that it would take if revolved from its original position in V about C–O as an axis and into H. Revolve Aa' about a as a center until A lies in H at Aa_3. Check the solution by noting if Cc–Aa_3 is equal to Cc–Aa_2. Mark the radius of rotation in each revolution. See § 48, p. 38.

30. — The point [$13\frac{1}{8}$] A lies $\frac{3}{8}''$ in front of V and $1\frac{1}{2}''$ above H. Draw an axis of rotation C–O through [$14\frac{3}{4}$] O and revolve A as in Exercise 29. Also determine Aa_5, which is the position of A when revolved about C–O into the *second* angle.

31. — [$\frac{1}{2}$] A is $\frac{1}{2}''$ above H and $\frac{7}{8}''$ in front of V. [2] B is on V and $\frac{1}{2}''$ above H. [$1\frac{3}{4}$] D is $2''$ above H and $1\frac{1}{4}''$ in front of V. Revolve D about A–B until it is $\frac{1}{2}''$ above H.

32. — [3] A is $1\frac{3}{4}''$ above H and $1''$ in front of V. [$4\frac{1}{4}$] B is $\frac{3}{4}''$ above H and $\frac{1}{4}''$ in front of V. Determine the true length of the line A–B by revolving B away from V until is occupies its proper position. Check construction by the use of a *new* V taken through [5].

33. — The line A–B is in the *first* angle and perpendicular to H. [$6\frac{1}{2}$] A is $1\frac{1}{4}''$ from V and $\frac{3}{8}''$ from H. B is $2\frac{1}{4}''$ from H. [$5\frac{1}{2}$] D is $1\frac{1}{4}''$ in front of V and $1\frac{3}{4}''$ above H. [7] E is $\frac{7}{8}''$ above H and $\frac{1}{2}''$ in

front of V. First, revolve the line *D–E* *clockwise* about *A–B* as an axis, through 60 degrees, and then continue the rotation until the line is 90° from its *original* position.

34. — Indicate the line *A–B*; [8] *A*, $1\frac{1}{2}''$ above H and $\frac{3}{4}''$ in front of V, [$10\frac{1}{2}$] *B*, $1\frac{1}{2}''$ above H and 2″ in front of V. Indicate the line *D–E*; [$8\frac{1}{8}$] *D*, $2\frac{1}{4}''$ above H and $1\frac{3}{8}''$ in front of V; [$10\frac{1}{4}$] *E* is in front of V and *D–E* intersects *A–B* at [9] *C*. Revolve *D–E* about *A–B* as an axis until *D* is as close to V as possible (that is, until *D–E* is parallel to H). Show the projections of the *shortest* line possible (that is, the perpendicular distance) from the point *B* to the line *D–E*.

35. — Indicate the line *A–B*; [$12\frac{1}{2}$] *A*, $\frac{5}{8}''$ above H, $1\frac{1}{2}''$ in front of V; [14] *B*, 2″ above H and $\frac{1}{2}''$ in front of V. Determine *C*, the horizontal piercing point of the line. Determine the true length of the line *A–B* first, by revolving it about *A* parallel to H next, by revolving it about *C* into H and finally by revolving *A* and *C* about *B* parallel to H.

169. PLATE III. Exercises 36 to 45 inclusive. This plate requires a working knowledge of the text through Chapter III.

G–L, $2\frac{3}{4}''$ *and* 8″.

36. — Indicate the line *A–B* in the *third* angle; [$\frac{1}{2}$] *A*, 1″ from V and $\frac{3}{8}''$ from H; [$2\frac{1}{2}$] *B*, $1\frac{3}{8}''$ from H and $\frac{1}{2}''$ from V. Draw a line through [$2\frac{1}{4}$] *C*, $1\frac{1}{4}''$ from V and $\frac{1}{2}''$ from H to intersect the line *A–B*, $\frac{3}{4}''$ from V. What is the distance from this point to H?

37. — Indicate the line *A–B*; [$3\frac{1}{4}$] *A*, $1\frac{1}{8}''$ above H and $1\frac{3}{8}''$ in front of V; [$5\frac{1}{8}$] *B*, $1\frac{1}{8}''$ above H and $\frac{1}{2}''$ in front of V. The line *C–D*; [$3\frac{3}{8}$] *C*, $1\frac{1}{2}''$ above H and $\frac{5}{16}''$ in front of V; [5] *D*, $\frac{1}{4}''$ above H and $1\frac{1}{4}''$ in front of H. Keeping *A–B* parallel to H swing it about *B* until it intersects *C–D*. Keeping *A–B* par-

allel to H and at the *same* angle to V as in its original position, show the distance it would have to be lowered to intersect *C–D*.

38. — Solve this exercise in the *first* angle:
The vertices of a triangle are projected at [$5\frac{3}{4}$] *A*, $\frac{7}{8}''$ from H and $\frac{3}{4}''$ from V; [$6\frac{3}{4}$] *B*, $1\frac{3}{8}''$ from H and from V; [$7\frac{1}{8}$] *C*, $\frac{1}{8}''$ from H and $\frac{1}{4}''$ from V. Take a *new* V through the line *C–B* and determine the true length of *C–B* on this plane. On this true length construct the true shape of the triangle *A–B–C*. Make construction to the right of the horizontal projection.

39. — Construct the same figure as in Exercise 38 on ruled projectors through [$9\frac{1}{2}$], [$10\frac{1}{2}$], and [$10\frac{7}{8}$], and make the construction in V by taking G_1–L_1 through *c′–b′*. Make construction to the right of *a′–b′–c′*.

40. — The lines *A–B* and *C–D*, in the *first* angle, intersect at *E*; the point [$12\frac{1}{4}$] *A*, $\frac{1}{4}''$ from H and $\frac{3}{8}''$ from V; [14] *E*, $1\frac{1}{2}''$ from H and $1\frac{3}{4}''$ from V; [$15\frac{1}{4}$] *D*, $1\frac{1}{4}''$ from H and $1\frac{3}{8}''$ from V. Determine the true angle between the lines. Draw the true angle in H. See § 51, p. 41.

41. — Indicate the line *A–C*; [$\frac{1}{2}$] *A*, $1\frac{1}{2}''$ above H and $1''$ in front of V; [$3\frac{1}{2}$] *C*, $1''$ back of V and $1\frac{1}{2}''$ below H. Revolve the part of the line included in the *first* angle about *A* parallel to V and determine its true length. Likewise determine, by revolution about *C*, the portion of the line that lies in the *third* angle.

42. — Repeat Exercise 41 by revolving in each case until parallel to H.

43. — Indicate the line *A–C* as in Exercise 41 with [4] *A* and [7] *B*. Swing the entire line about *A* until parallel to H to determine its true length, the length of the line in each angle.

44. — Repeat Exercise 43 by revolving the line *A–C* about *A* until parallel to V, and next lay off a point on the line *A–C* that is $1\frac{1}{4}''$ from *A* and determine the projections of the point in each position of the line.

45. — *A–B–C* are the three corners of a triangular piece of cardboard lying in the *third* angle. *A–B* is $2\frac{3}{16}''$ long; *B–C* is $2\frac{13}{16}''$ long; *C–A* is $2\frac{5}{16}''$ long. The edge *C–A* rests in V; [$11\frac{1}{2}$] *C* is $1\frac{3}{4}''$ from H; [$13\frac{3}{8}$] *A*, $\frac{3}{8}''$ from H. Determine the vertical and horizontal projections of the cardboard figure when *B* rests against H. Take G_1–L_1 at [$13\frac{5}{8}$] and determine the new vertical projection. Draw a line from *A* to bisect the line *B–C*, and show its projections in each view.

170. PLATE IV. Exercises 46 to 58 inclusive.

NOTE. At this point there should be a review and examination of Chapters I to III inclusive and of all exercises and lectures given up to the time of completing Plate IV.

This plate requires a working knowledge through Chapter III.

G–L, $2\frac{1}{2}''$ and to be drawn only $9''$ long.

46. — Indicate the point [$\frac{1}{2}$] *A* in the second angle $1\frac{1}{4}''$ from H and $\frac{3}{4}''$ from V.

47. — Same as Exercise 46, except [1] *B*, $1''$ from H and on V.

48. — Same as Exercise 46, except [$1\frac{1}{2}$] *C*, $1\frac{1}{4}''$ from V and on H.

49. — Same as Exercise 46, except the line *A–B*, [2] *A*, $\frac{3}{4}''$ from H and $\frac{1}{4}''$ from V; *B*, $1\frac{1}{2}''$ from H and the line is perpendicular to H.

50. — Same as Exercise 46, except the line *C–D* is perpendicular to V. The line is $1''$ long and [$2\frac{1}{2}$] *C* is $\frac{1}{2}''$ from V and $1''$ from H.

51. — The point [3] *A* is $\frac{1}{2}''$ back of V and in H, [3] *B* is $1\frac{1}{4}''$ above H and in V. Show the **true distance** (*T. D.*) between the points in space.

52. — Indicate the line *A–B* parallel to *G–L.* [4] *A* is $\frac{1}{2}''$ above H and $1''$ back of V. [5] *B.* Keeping the line parallel to *G–L* raise it $\frac{1}{4}''$ and then move $\frac{1}{4}''$ toward V and show its projections.

53. — Indicate the line *A–B* in the *second* angle; [$5\frac{1}{2}$] *A*, $1\frac{1}{4}''$ from V and $\frac{1}{2}''$ from H; [8] *B*, $1\frac{1}{2}''$ from H and $\frac{1}{4}''$ from V. Continue *A–B* and determine its vertical piercing point. Determine the projections of the point *D* on *A–B* and $\frac{3}{4}''$ from H. Draw the projections of a line from the point [$6\frac{3}{4}$] *E*, $1\frac{1}{2}''$ from both V and H to intersect the line *A–B*, $\frac{1}{2}''$ from V.

Second G–L, $4\frac{3}{4}''$.

54. — Indicate the line *A–B* in the *fourth* angle; [$\frac{1}{2}$] *A*, $\frac{1}{4}''$ from H and $1\frac{3}{4}''$ from V; [$2\frac{1}{2}$] *B*, $\frac{1}{4}''$ from V and $2''$ from H. *C–D* is $1\frac{1}{4}''$ from V and parallel to V. [$\frac{1}{2}$] *C* is $\frac{1}{2}''$ from H. The lines *A–B* and *C–D* intersect $1\frac{1}{4}''$ from V. Determine the projections of *C–D* and indicate the **point of intersection** of the two lines. Swing *A–B* about *A* parallel to H and determine where *A–B* pierces V when in this position.

55. — *A*, *B*, and *C* are the corners of a triangular figure in the *fourth* angle. [$4\frac{1}{4}$] *A* is $\frac{1}{4}''$ from H and $\frac{1}{2}''$ from V; [$5\frac{1}{4}$] *B* is $1\frac{1}{2}''$ from both V and H. [$6\frac{1}{2}$] *C* is $\frac{3}{4}''$ from H and $1\frac{1}{4}''$ from V. From the point [$4\frac{1}{4}$] *D*, $1''$ from V and $1\frac{1}{4}''$ from H, draw a line *D–E* to intersect the edge *A–C* at $\frac{3}{8}''$ from H. Does this line *D–E* lie in the plane of the triangle? Determine the vertical piercing point of *D–E*.

Third G–L, $8\frac{3}{4}''$ *and drawn* $6\frac{1}{2}''$ *long.*

56. — The line *A–B* is in the *third* angle. [$\frac{1}{2}$] *A* is $1\frac{3}{4}''$ from V and $\frac{1}{4}''$ from H. The line *C–D* is in the same

angle and is parallel to V. [$\frac{1}{2}$] C is $1\frac{1}{4}''$ from V and $\frac{1}{2}''$ from H. [$2\frac{1}{2}$] D is $1\frac{1}{2}''$ from H. The two lines intersect at [$1\frac{1}{4}$] E, $\frac{7}{8}''$ from H. How long is A–B and what angle does it make with V?

57. — A triangular figure A–B–C is in the *third* angle. [4] A is $\frac{1}{2}''$ from V and $\frac{1}{4}''$ from H. [5] B is $1\frac{1}{2}''$ from both V and H. [$6\frac{1}{4}$] C is $\frac{3}{4}''$ from H and $1\frac{1}{4}''$ from V. Draw the projections of the figure. Draw a line in the plane of the figure to intersect A–C, $\frac{3}{8}''$ from H, and to intersect A–B, $\frac{3}{4}''$ from H. Where does this line pierce H and V?

Fourth G–L, $4\frac{3}{4}''$.

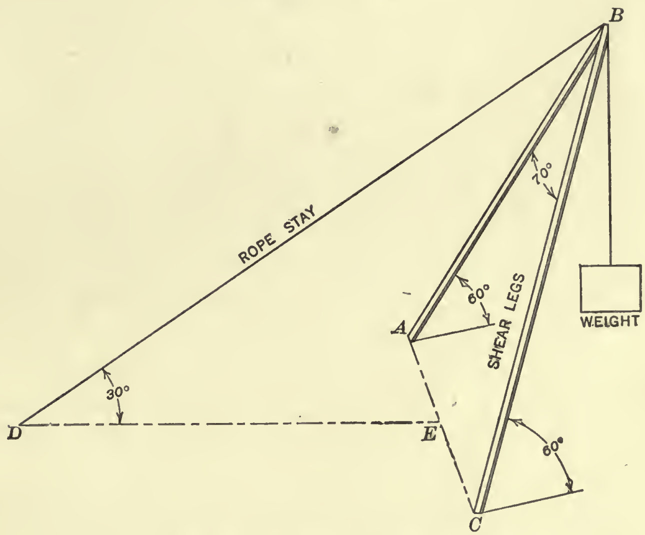

Fig. 154. — Perspective of Shear Legs.

58. — Fig. 154 shows a perspective of a pair of shear legs. The legs A–B and B–C are each 18 feet long and A and C are 12 feet apart. The shears rest on a flat plane and are turned so that the line D–E makes 45 degrees with V. Locate [$11\frac{1}{8}$] E, $2\frac{1}{2}''$ in front of V and in H. With B to the right and D to the left, make a *first-angle* drawing of the complete shear legs. Do *not* attempt to show thickness of legs and ropes but make a "center line" drawing.

Suggestion. Draw the shear legs first with *B–D* parallel to V and the legs in H. Then swing the legs about *A* and *C* to make 60 degrees with H and complete the drawing in this position. Next swing the horizontal projection about *E* until *D–E* makes 45 degrees with V and complete the drawing. Scale $\frac{1}{4}''$ = 1′ – 0″.

171. PLATE V. Exercises 59 to 71 inclusive. This plate requires a working knowledge of the text through § 71.

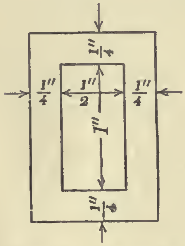

Fig. 155.—Layout of Cardboard Figure.

G–L, 3″ and 8″.

59. — [$\frac{1}{2}$] *A*. Draw the plan and elevation of the cardboard figure (see Fig. 155) when it stands $\frac{3}{8}''$ above H and $\frac{1}{2}''$ in front of V and is perpendicular to both V and H. Draw *A–B* vertical and nearest V.

NOTE. Cut out of cardboard a figure as shown and place it in the various positions called for as an aid in solving the exercises.

60. — [1]. Move the cardboard of Exercise 59, $\frac{1}{2}''$ to the right and swing *C–D anticlockwise* about *A–B* until the cardboard makes 45 degrees with V and draw its plan and elevation in this position.

61. — [$2\frac{1}{4}$]. Again move the cardboard of Exercise 59 to the right until *A–B* is at [$1\frac{1}{2}$], and continue the revolution until the cardboard is parallel to V and draw its plan and elevation.

62. — Place the cardboard of Exercise 59 so the corner [$3\frac{3}{4}$] *B* is $\frac{1}{2}''$ above H and $\frac{3}{4}''$ in front of V, with *B–D* parallel to H. Tilt the cardboard until *A–C* is $\frac{1}{4}''$ from V and draw its plan and elevation.

63. — Draw the projections of the cardboard of Exercise 59 in H with [$6\frac{1}{4}$] *A*, $\frac{1}{4}''$ in front of V and *B* on the

ruled projector [6]. Draw a line G_1–L_1 through *Aa* and [$8\frac{1}{2}$] on *G–L*. Without changing the angle the edge *A–C* makes with G_1–L_1 revolve the cardboard about this line as an axis until *A–C* and *B–D* are both parallel to V, and then draw the plan and elevation.

Suggestion. Draw *Aa*–c_1 parallel to *G–L*. Then determine the vertical projection of C_1 by constructing the right triangle at [$7\frac{1}{4}$]. The distance from c_1 to the axis of rotation being known as well as the true length of the radius, c_1 can be determined. The projection c_1'–d_1' will be perpendicular to a'–c_1', hence its direction is known. The projection c_1'–d_1' will pierce H on the axis of rotation, and this determines the direction of d_1–c_1 and the point d_1. The projection d_1' is then determined by the ruled projector. Check the construction by means of other triangles similar to the one for D_1.

64. — *VS* and *HS* are the traces of a triangular cardboard figure in the *first* angle; they meet at [$8\frac{1}{2}$] and are each $2\frac{1}{2}''$ long. *VS* makes 60 degrees and *HS* makes 30 degrees with *G–L*. Draw the projections of the third side of the figure and determine its true size and shape by revolving it about *HS* into H.

NOTE. **In designating the location of points** in some of the following exercises, distances above H will be + and below H will be −; distances in front of V will be + and behind V will be −.

All **traces of planes are to be drawn to the right** unless stated thus; (left). Make all traces of planes $2\frac{1}{2}''$ long unless the conditions of the problem require that they be otherwise.

65. — Determine the traces of the plane which contains the points whose projections are [$13\frac{1}{8}$] $a' + \frac{5}{8}''$; $a + \frac{1}{2}''$; [$13\frac{3}{4}$] $b' + \frac{1}{4}''$; $b + 1\frac{1}{4}''$; [$14\frac{3}{4}$] $c' + 2''$; $c + \frac{1}{8}''$. See § 63, p. 55; § 62, p. 55.

66. — The horizontal trace of the plane R meets *G–L* at [$\frac{1}{2}$] and at an angle of 30 degrees. The point whose projections are [$1\frac{3}{4}$] $a' + \frac{1}{2}''$ and $a + \frac{5}{8}''$ is in the plane. Determine the vertical trace of the plane.

67. — *VR* and *HR* (each $2\frac{1}{2}''$ long) meet at [$3\frac{1}{8}$]. *VR* makes 60 degrees and *HR* makes 30 degrees with *G–L*. What is the *vertical* distance from the point whose projections are [$4\frac{1}{2}$] $a' + 2''$ and $a + \frac{1}{2}''$ to the plane R?

68. — Pass a plane R through the points whose projections are [$7\frac{3}{4}$] $a' + \frac{1}{2}''$; $a + \frac{7}{8}''$. [$7\frac{7}{8}$] $b' + 1\frac{1}{4}''$; $b + \frac{1}{2}''$, such that *VR* makes 45 degrees and *HR* makes 30 degrees with *G–L*. See § 59, p. 53; § 68, p. 58.

69. — The traces of the plane S meet at [$8\frac{3}{8}$]. *VS* makes 45 degrees and *HS* makes 30 degrees with *G–L*. Pass a plane T parallel to S and through the point whose projections are [10] $a' + \frac{3}{4}''$; $a + \frac{1}{4}''$. See § 58, p. 58.

70. — What is the distance measured perpendicularly to V from a point [10] *B* which is $1\frac{1}{4}''$ in front of V, to the plane T in Exercise 69. See § 76, p. 64.

71. — Indicate the line *A–B*. [$11\frac{3}{8}$] $a' + \frac{1}{4}''$; $a + \frac{3}{4}''$. [$12\frac{3}{8}$] $b' + 1\frac{1}{2}''$; $b + \frac{1}{4}''$. Also indicate the line *C–D* [15] $c' + 1''$; $c + \frac{1}{8}''$. [$15\frac{1}{2}$] $d' + \frac{1}{2}''$; $d + \frac{3}{4}$ and the point [14] $f' + \frac{3}{4}''$; $f + \frac{1}{2}''$. Pass a plane R through the point *F* and parallel to the two lines *A–B* and *C–D*. See § 67, p. 57.

172. PLATE VI. Exercises 72 to 82 inclusive. This plate requires a working knowledge of the text through § 83.

G–L $3''$ *and* $8''$. *Exercises in first angle.*

72. — [$\frac{1}{2}$] *VR* makes 45 degrees and *HR* makes 30 degrees with *G–L*. [$5\frac{1}{8}$] *VS* makes 30 degrees (left) and *HS* makes 45 degrees (left) with *G–L*. Determine the line of intersection of the planes R and S. See § 72, p. 60.

73. — Draw a line on each plane, parallel to and $\frac{1}{2}''$ from V.

74. — [$5\frac{5}{8}$] *VR* makes 60 degrees and *HR* makes 75 degrees with *G–L*. [$7\frac{1}{8}$] *VS* makes 75 degrees and *HS* makes 90 degrees with *G–L*. Determine the line of intersection of the planes.

75. — [9] [$11\frac{1}{2}$]. The traces of the planes R and S are all parallel to *G–L*. The trace *VR* is 2″ above and *HR* is 1″ below *G–L*. The trace *VS* is $1\frac{1}{4}$″ above and *HS* is $1\frac{1}{2}$″ below *G–L*. Determine the line of intersection of the planes. Take a profile plane through [$9\frac{1}{4}$]. See § 72, p. 61.

76. — [12] *VS* makes 30 degrees and *HS* makes 45 degrees with *G–L*. [12] *VR* makes 45 degrees and *HR* makes 30 degrees with *G–L*. Determine the line of intersection of the planes by use of a profile plane through [$13\frac{3}{4}$]. See § 72, p. 61.

77. — Pass a line through the points whose projections are [$1\frac{1}{2}$] $a' + \frac{5}{8}$″; $a + 1\frac{3}{8}$″; and [$2\frac{7}{8}$] $b' + 1\frac{1}{2}$″; $b + \frac{3}{8}$″. Also, pass a line through the point whose projections are [$1\frac{7}{8}$] $c' + \frac{11}{16}$″; $c + \frac{3}{4}$″; and a point *D* on *A–B* and $\frac{1}{2}$″ from V. Determine the traces of the plane R containing the lines. See § 33, p. 26; § 60, p. 54.

78. — [$4\frac{7}{8}$] *VS* makes 60 degrees (left) with *G–L*. *HS* is a continuation of *VS*. Determine the intersection of the plane R and the plane S.

79. — [$7\frac{7}{8}$] *VR* makes 45 degrees (left) and *HS* makes 30 degrees (left) with *G–L*. Indicate the point whose projections are [$6\frac{3}{8}$] $b' + 2\frac{1}{8}$″; $b + 1\frac{3}{8}$″. Determine the perpendicular distance from the point *B* to the plane R. See § 80, p. 68.

80. — [$8\frac{3}{8}$] *VR* makes 45 degrees and *HR* makes 30 degrees with *G–L*. The projections of the line *A–B* are [$8\frac{5}{8}$] $a' + 1\frac{1}{8}$″; $a + 1\frac{1}{4}$″, and [$9\frac{5}{8}$] $b' + 1\frac{3}{4}$″; $b + 1\frac{1}{2}$″. Project the line *A–B* upon the plane R. See § 81, p. 69.

81. — The line A–B is projected at [$11\frac{1}{2}$] $a' + 2''$; $a + 2''$, and [$12\frac{1}{2}$] $b' + 1''$; $b + 1\frac{1}{4}''$. The point C is projected at [13] $c' + \frac{1}{2}''$; $c + 1\frac{1}{2}''$. Pass a plane R through the point C and perpendicular to the line A–B. See § 83, p. 69.

82. — Through [$13\frac{1}{8}$] draw the traces of a plane S parallel to R and determine the perpendicular distance between the planes.

173. PLATE VII. Exercises 83 to 91 inclusive. This plate requires a working knowledge of the text through § 94.

G–L, $2\frac{1}{2}''$ and $7\frac{1}{2}''$. Exercises in first angle.

83. — The triangular figure A–B–C is projected at [$1\frac{3}{8}$] $a' + \frac{1}{4}''$; $a + \frac{1}{2}''$. [2] $b' + 1\frac{5}{8}''$; $b + \frac{1}{4}''$. [$2\frac{3}{8}$] $c' + \frac{5}{8}''$; $c + 1''$. Determine the true size of the figure by revolving it into H about the horizontal trace of the plane R in which it lies. Check Exercise 83 by revolving the triangle about VR and into V. See § 93, p. 81.

84. — [$3\frac{1}{4}$] HR makes 30 degrees with G–L. VR passes through [$5\frac{1}{2}$] $d' + 1\frac{15}{16}''$. Determine the angle that the plane R makes with H. Take the auxiliary plane through [$5\frac{1}{2}$]. See § 85, p. 72.

85. — [$8\frac{1}{4}$] VR makes 45 degrees (left) with G–L. HR passes through the projection [6] $d + 1\frac{5}{8}''$. Determine the angle that the plane R makes with V. Take the auxiliary plane through [6]. See § 89, p. 77.

86. — Pass a plane R through the point C whose projections are [$10\frac{1}{2}$] $c' + \frac{7}{16}''$; $c + \frac{3}{4}''$ such that it will make 45 degrees with H and 60 degrees with V. Begin construction at [$8\frac{3}{4}$]. See § 89, p. 77.

87. — Pass a plane S through A projected at [$13\frac{5}{8}$] $a' + \frac{1}{2}''$; $a + 1\frac{3}{16}''$ parallel to the plane R of Exercise 86. By

passing auxiliary planes T and U through A, prove the accuracy of the construction of Exercise 86. See § 85, p. 72.

88. — [$\frac{1}{2}$] VR makes 30 degrees and HR 45 degrees with G–L. The line A–B is projected at [$1\frac{3}{8}$] $a' + 2''$; $a + 1\frac{3}{4}''$ and [$2\frac{5}{8}$] $b' + \frac{3}{4}''$; $b + 1\frac{1}{4}''$. Determine the true angle that the line A–B makes with the plane R. See § 91, p. 78.

89. — [$4\frac{3}{8}$] VR makes 45 degrees and [$6\frac{7}{8}$] VS makes 60 degrees (left) with G–L. HR and HS are parallel and make 75 degrees (left) with G–L. Determine the true angle between the planes R and S. See § 87, p. 75.

90. — [$7\frac{3}{8}$] VR makes 45 degrees and HR makes 30 degrees with G–L. [$10\frac{7}{8}$] VS makes 60 degrees (left) and HS makes 75 degrees (right) with G–L. Determine the angle between the planes R and S. See § 72, p. 60.

91. — The planes R and S are parallel to G–L. Each trace passes through [12] and [$13\frac{1}{2}$]. VR is $\frac{3}{4}''$ and VS is $2''$ above G–L. HR is $2''$ and HS is $1''$ below G–L. Determine the angle between the planes. Take the auxiliary plane P through [$13\frac{1}{2}$] and revolve it into H and to the right. See § 72, p. 60.

174. PLATE VIII. Roof and Stack Problem. This plate requires a working knowledge of the text through Chapter IV. For layout see Fig. 156.

NOTE. At this point there should be a review and examination of exercises covered by plates V to VIII inclusive and of the text matter and lectures given up to the time of completing plate VIII.

92. — The **rafters** of a boiler-house roof make 30 degrees with the **joist,** and the **eave** of the roof makes 45 degrees with a **front vertical wall.** A **smokestack** of

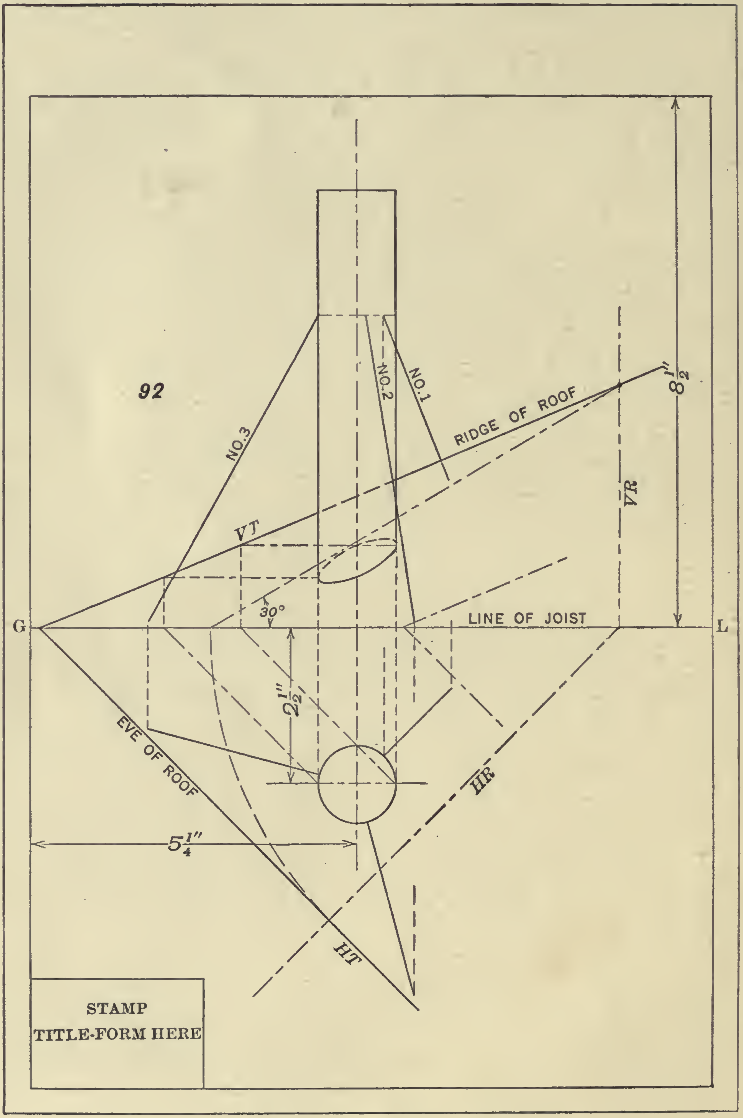

Fig. 156. — Layout for Plate VIII.

30-inch diameter passes through the roof (center at the point "*C*") and extends 14 feet above the line of the joist. Four feet from the top of the stack 3 guy wires 120 degrees apart are attached and extend down to the roof to act as braces. The wire No. 1 is to have a direct pull against the roof (that is, it will be perpendicular to the roof). The other two wires are to make *the same angle with the stack* that No. 1 makes. A mechanical drawing is to be made of the above in the *first* angle, using a scale of $\frac{1}{2}''$ = 1 foot. Also determine the length of the guy wires and the angle each makes with the roof.

Suggestions. The roof will be referred to as the plane T, the front wall as V, and the plane of the joist as H.

Draw *G–L* and *HT* (the eave of the roof). *VT* is the ridge of the roof and is found by passing an auxiliary plane R perpendicular to *HT* and $9\frac{3}{8}''$ from where *HT* cuts *G–L*; swinging R into V determines where the line of intersection of R and T cuts *VR*. This gives one point on *VT* and a second point is where *HT* cuts *G–L*. Locate the position of the stack and draw it in *very lightly*. By using *auxiliary horizontal* planes, find where 9 elements of the stack pierce the roof and draw in the curve of intersection. Draw this curve by using irregular curves, —*free-hand work will not be accepted.* Draw the horizontal projection of wire No. 1, and locate the vertical projection of the point where it is attached to the stack. Draw the vertical projection of the wire and find where it pierces the roof. The plane containing the wire and perpendicular to V will not intersect T within the limits of the drawing; take the parallel plane through [6]. Revolve the wire parallel to V and find the angle it makes with the stack. Draw the horizontal projection of the other wires. Pass a plane through wire No. 2 perpendicular to H, and revolve the plane about its vertical trace until its line of intersection with the roof and its line of intersection with the stack are in V; draw the wire and counter-rotate. Draw wire No. 3 and determine the angle it makes with the roof.

175. PLATE IX. Exercises 93 to 102 inclusive. This plate requires a working knowledge of the text through § 103.

G–L, $3\frac{3}{8}''$ *and* $8\frac{1}{2}''$.

93. — *This exercise is in the first angle.* *A–B–C–D* outlines a figure lying in H. [$\frac{1}{2}$] $a + 1''$; [$1\frac{7}{8}$] $b + \frac{3}{8}''$; [$2\frac{3}{4}$] $c + \frac{3}{4}''$; [$1\frac{3}{8}$] $d + 1\frac{3}{4}''$. Raise this figure about *A* as a center until the plane of the figure makes 45 degrees with H and draw its *plan* and *elevation* in this position. Move *a* to [$3\frac{1}{4}$] $a_2 + 1\frac{1}{4}''$; $a_2' + 0''$; revolve the plan a_1–b_1–c_1–d about a_2 and through 30 degrees toward V and determine the elevation of the figure.

94. — *This exercise is in the third angle.* [$7\frac{5}{8}$] *HR* make 45 degrees (left) and *VR* makes 30 degrees (left) with *G–L*. The point [$5\frac{5}{8}$] *A* lies in the plane R, $\frac{7}{8}''$ from V, and is the center of a circle of $1\frac{1}{2}''$ diameter which lies in R. Determine the projections of the circle. Locate the right triangle required for this construction with the vertex of the right angle $6\frac{1}{4}''$ from left border line and $5\frac{5}{8}''$ from the top border line. See § 99, p. 88.

95. — *This exercise is in the first angle.* [$8\frac{1}{8}$] *VR* makes 45 degrees and *HR* makes 90 degrees with *G–L*. The plane R contains an equilateral triangle having sides $1\frac{1}{2}''$ long. The side *A–B* makes 45 degrees with V. *A* is $\frac{3}{8}''$ from *HR* and $1\frac{1}{2}''$ from V. Determine the projections of the triangle.

96. — Solve Exercise 95 in the *third angle* with the traces of the plane meeting at [$10\frac{3}{4}$].

97. — Repeat Exercise 96 with the traces meeting at [$13\frac{3}{8}$]. Upon the triangle as a base construct a prism of $\frac{3}{8}''$ altitude and determine its plan and elevation. Make a third-angle *mechanical* drawing of the figure.

98. — Make a third angle mechanical drawing of a hollow cube (see Fig. 157), taking [$1\frac{1}{4}$] C, $1\frac{1}{8}''$ from V and on H and a side parallel to V. See § 103, p. 92.

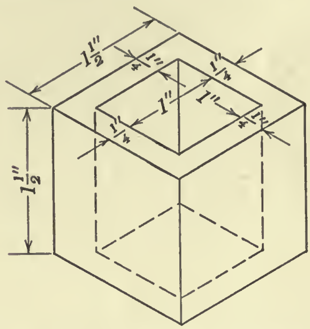

Fig. 157. — Isometry Drawing of a Hollow Cube.

99. — Make a third angle mechanical drawing of the cube of exercise 98, taking [$3\frac{5}{8}$] C, $1\frac{1}{8}''$ from V and on H, and with the parallel side of Exercise 98 revolved *clockwise* about C and through 30 degrees.

100. — With the cube in the position of Exercise 99, move it to the right until the corner A is at [7] a–$1\frac{3}{8}''$; keeping the corner at 7″ on H, drop the left-hand corner until the top face of the cube makes 30 degrees with H and draw the plan and elevation.

101. — Determine a new elevation of the cube of Exercise 100 upon a profile plane through [8].

102. — Make a *third-angle* drawing of a cone with base diameter of $1\frac{3}{4}''$ and altitude of $1\frac{1}{2}''$, when the vertex [$11\frac{1}{2}$] A is on H and $1\frac{1}{4}''$ from V, and the base is parallel to H. Also determine a new elevation when G_1–L_1 is tangent to the original plan of the base and cuts G–L at $13\frac{1}{4}''$.

176. PLATE X. Exercises 103 to 107 inclusive. This plate requires a working knowledge of § 116. See Fig. 158 for general layout.

G–L, $2\frac{1}{4}''$.

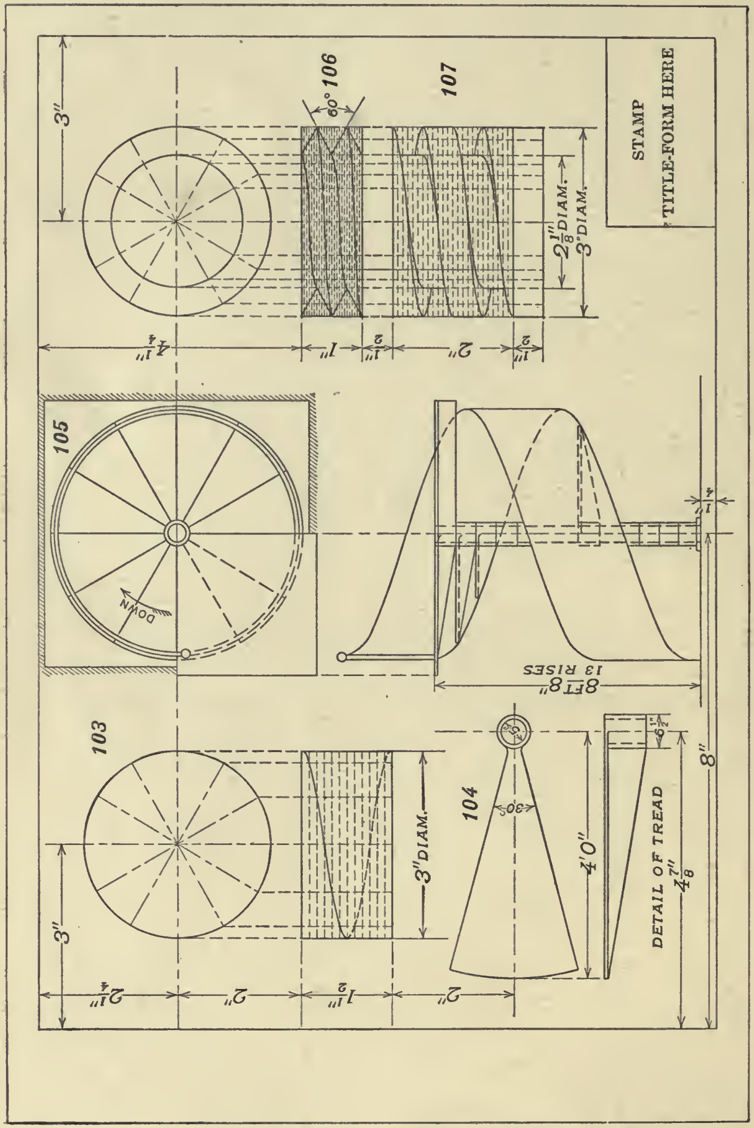

Fig. 158. — Layout for Plate X.

103. — Construct **right-hand helix** of radius $1\frac{1}{2}''$ and pitch $1\frac{1}{2}''$.

104. — Make a detail drawing of **tread** for spiral stairway (see dimensions in data for Exercise 105).

105. — Draw the plan and elevation of a **spiral stair** of dimensions as follows:

Total rise, 8 Ft. 8″. Single rise, 8″.
Radius of **tread,** 4 Ft. **Thickness of metal,** 1″.
Center-post, $4\frac{7}{8}''$ diam. Solid **balustrade,** 3 Ft. high.

Draw to a scale of $\frac{1}{2}'' = 1$ foot.

106. — Make a drawing of a right-hand **V-threaded screw,** having a pitch of $\frac{1}{2}''$ and sides at an angle of 60-degrees.

NOTE. Use twelve points on the circumferences for Exercises 106 and 107. The $\frac{1}{12}''$ and $\frac{1}{24}''$ spaces will be found on the scale of $1'' = 1$ foot.

107. — Make a drawing of a right-hand **square-threaded screw** having pitch of 1″ and depth of thread the same as the V thread of Exercise 106.

177. PLATE XI. Exercises 108 to 114 inclusive. This plate requires a working knowledge of the text through Chapter VI.

G–L, 3″ and 8″.

108. — The base of an **oblique cone** rests on H in the *third* angle; it is $1\frac{1}{4}''$ in diameter and has its center [$2\frac{3}{8}$] *A*, 1″ back of V. The altitude of the cone is $1\frac{3}{4}''$ and its vertex is at [$1\frac{1}{8}$] *B*, $\frac{3}{8}''$ back of V. Determine the traces of a plane R passing through the point [$\frac{7}{8}$] *C*, 1″ back of V and $\frac{7}{8}''$ below H and tangent to the cone.

109. — The line *A–C* passes through [$4\frac{1}{4}$] *A*, $1\frac{15}{16}''$ back of V and $\frac{7}{16}''$ below H. [$5\frac{1}{4}$] *C* is $1\frac{1}{4}''$ back of V and 1″ below H. Determine the traces of a plane R passing through *A–B* and making 45 degrees with H.

Suggestion. Take axis of cone through *C*.

110. — The base of an **oblique cone** is $1\frac{1}{4}''$ diameter and rests against H in the *third* angle. The center of the base is at [10] *C*, $\frac{7}{8}''$ back of V. The horizontal projection of the cylinder axis makes 30 degrees (right) and the vertical projection 45 degrees (right) with *G–L*. The altitude of the cylinder is $1\frac{1}{2}''$. Determine the traces of the plane R tangent to the cylinder at the point [$10\frac{3}{4}$] *A*, $1\frac{7}{8}''$ *a*.

111. — [$12\frac{3}{4}$] and [$15\frac{1}{2}$] limit the end planes of a cylinder in the third angle. The cylinder is $1\frac{1}{4}''$ in diameter and is tangent to both V and H. Determine the traces of the plane R tangent to the cylinder along an element which is $1''$ from V.

112. — [$4\frac{1}{8}$] $1''$ back of V and $\frac{7}{8}''$ below H represents the center of a **sphere** of $1\frac{1}{2}''$ diameter. [$3\frac{3}{4}$] *B*, $1\frac{3}{8}''$ back of V is a point on the sphere. Determine the traces of the plane R tangent to the sphere at *B*.

113. — [$6\frac{3}{4}$] *C* is on H and $1\frac{1}{2}''$ in front of V and is the center of an oblique cylinder resting on H. The diameter of the base of the cylinder is $1''$ and the altitude is $\frac{7}{8}''$. The vertical projection of the cylinder axis makes 60 degrees (left) with *G–L* and the horizontal projection is parallel to the vertical. [7] *A* is $\frac{1}{4}''$ above H. Determine the traces of a plane R tangent to the cylinder at the point *A*.

114. — [$10\frac{5}{8}$] *A* is $2\frac{1}{8}''$ above H and $1\frac{1}{16}''$ in front of V. [$11\frac{5}{8}$] *B* is $\frac{3}{8}''$ above H and $1\frac{1}{8}''$ in front of V. [$10\frac{5}{8}$] *C* is $\frac{5}{8}''$ above H and $1''$ in front of V, and is the center of a sphere of $1''$ diameter. Determine the traces of a plane R containing the line *A–B* and tangent to the sphere.

178. PLATE XII. Exercises 115 to 122 inclusive. This plate requires a working knowledge of the text through Chapter VII.

NOTE. At this point there should be a review and examination of exercises covered by plates IX to XII inclusive and

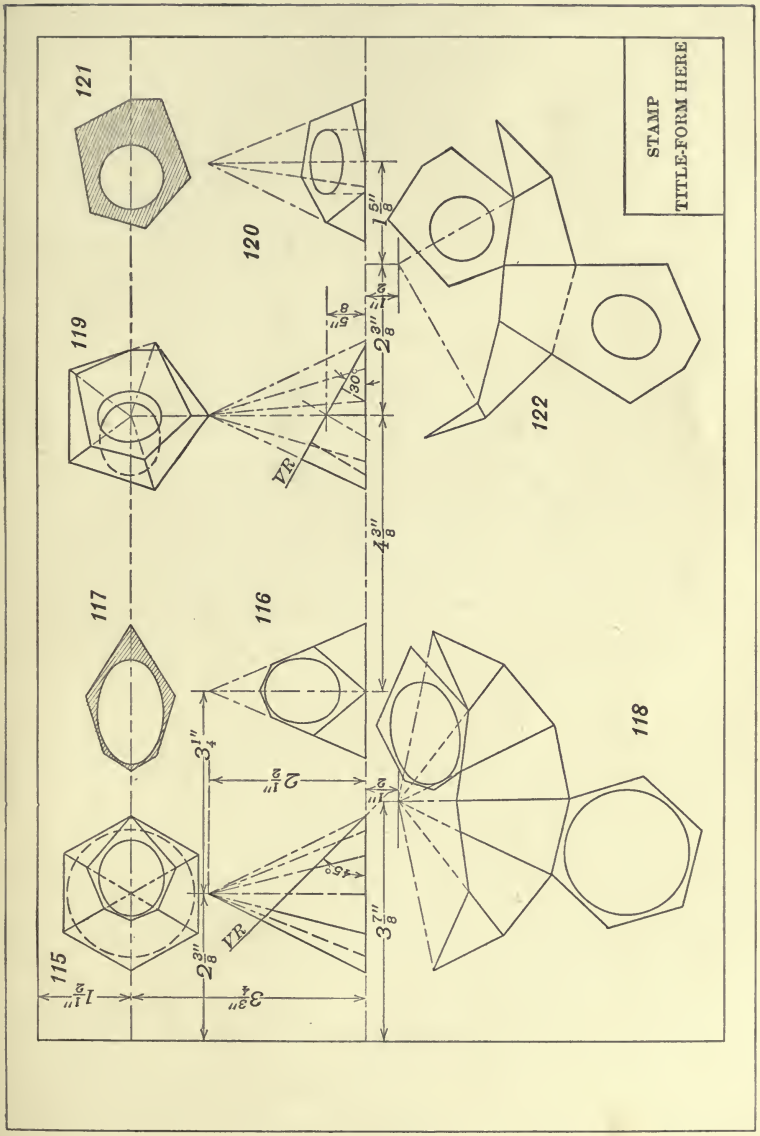

Fig. 159. — Layout for Plate XII.

of the text matter and lectures given up to the time of completing plate XII.

For location of G–L and layout of plate, see Fig. 159.

115. — A **hexagonal pyramid** having an altitude of $2\frac{1}{2}''$ and the sides $1\frac{1}{4}''$ at the base has a conical hole cut through its central portion. The diameter of the hole at the base of the pyramid is $2''$, and at the vertex is $0''$. Cut the pyramid by a plane R as shown in layout and draw its plan and elevation.

116. — Draw the side elevation of the pyramid of Exercise 115.

117. — Show the true shape of the section cut from the figure by the plane in Exercise 115.

118. — Develop the surface of the figure shown in Exercise 115.

119. — A **pentagonal pyramid** of $2\frac{1}{2}''$ altitude and having a $2\frac{1}{2}''$ base diameter is cut by a plane as shown in layout. A hole of $1''$ diameter is drilled as shown (that is perpendicular to the plane and through the axis of the pyramid).

120. — Draw the side elevation of the pyramid of Exercise 119.

121. — Show the true shape of the section cut from the figure by the plane in Exercise 119.

122. — Develop the surface of the figure shown in Exercise 119.

179. PLATE XIII. Exercises 123 to 128 inclusive. This plate requires a working knowledge of the text through Chapter VII.

G–L. See layout of sheet, Fig. 160.

NOTE. Length of *vertical* prism $2\frac{1}{2}''$. Length of *inclined* prism $2\frac{3}{4}''$.

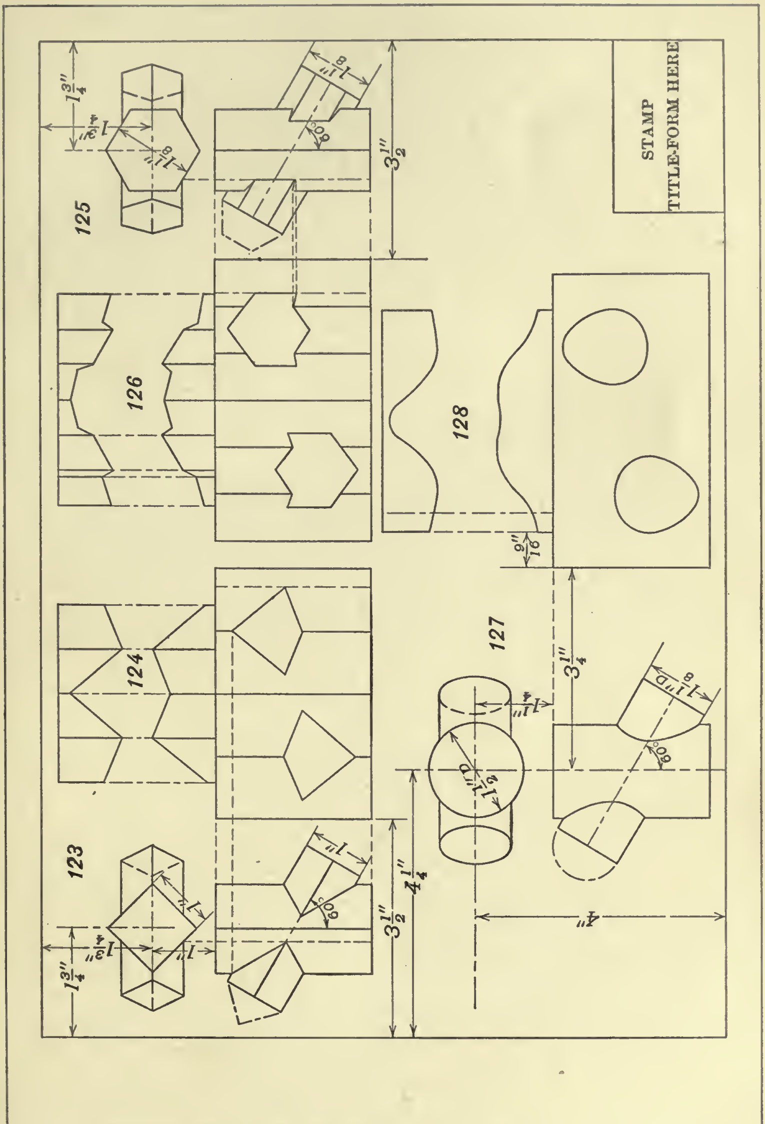

Fig. 160.—Layout for Plate XIII.

123. — Draw complete plan and elevation of the intersecting prisms showing lines of intersection.

124. — Develop the surfaces of Exercise 123.

125. — Draw complete plan and elevation of the intersecting prisms showing lines of intersection.

126. — Develop the surfaces of Exercise 125.

127. — Draw complete plan and elevation of the intersecting cylinders showing their line of intersection. Use 12 elements in the solution of this exercise.

128. — Develop the surfaces of Exercise 127.

NOTE. This plate may be varied by changing the angle of intersection of the axes. Also, the axes may be taken so as not to intersect.

180. PLATE XIV. Exercises 129 and 130. This plate requires a working knowledge of the text through Chapter VIII. See Fig. 161 for layout of this plate.

129. — The center of a circle of 1″ diameter is $\frac{5}{8}$″ from V and $\frac{3}{4}$″ from H. Determine the projections of the circle such that it shall lie in a plane whose vertical and horizontal trace each makes 45 degrees with *G–L*.

130. — Make a *third-angle* mechanical drawing from the sketch and the following data. The **pulleys** are 7 Ft. – 3″ from center to center; they are on **shafts** at right angles to each other, and located as shown in Fig. 161. A **guide pulley** is to be introduced between pulley No. 1 and No. 2 in such a manner as to change the direction of the belt at a point 3 Ft. – 0$\frac{1}{2}$″ from the center of pulley No. 1. Diameter of guide pulley = 24″; diameter of pulley No. 1 = 30″; diameter of pulley No. 2 = 42″; **face** of all pulleys = 8″; thickness of **rim** = 1$\frac{1}{2}$″; length of **hubs** = 11″; diameter of hubs = 8″; width of **belt** = 7″. Show thickness of belt $\frac{1}{2}$″; thickness of web 2″; diameter of **shaft** 4″. Use a scale 1″ = 1 Ft. – 0″.

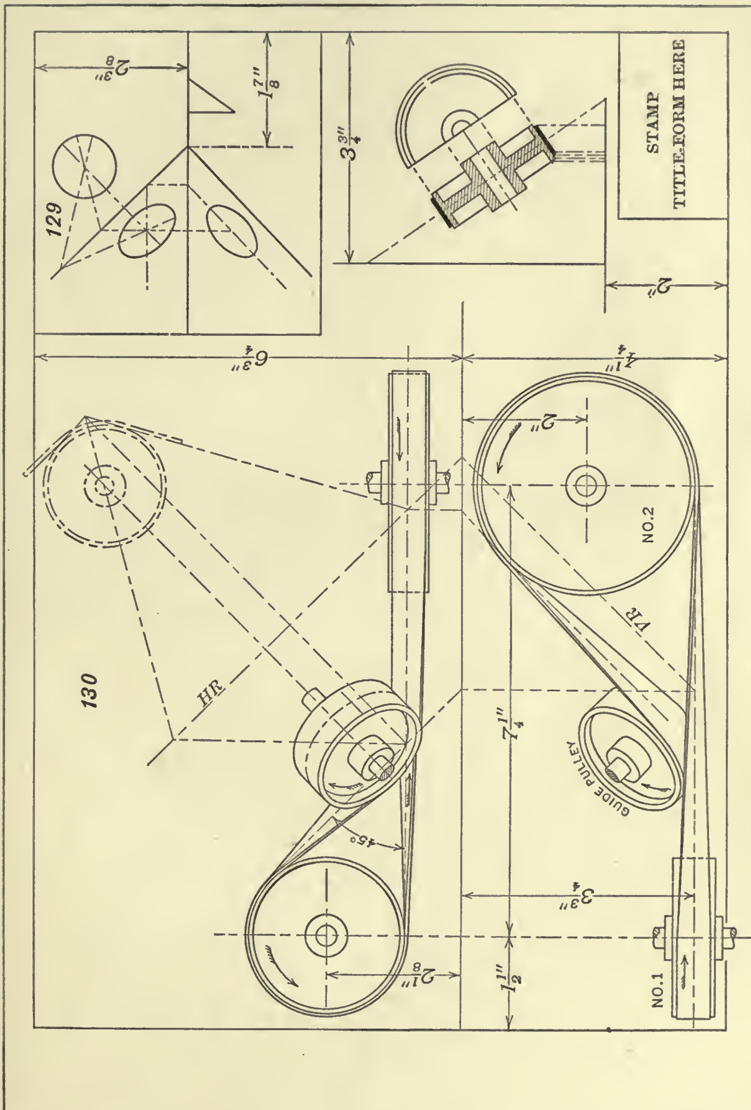

Fig. 161. — Layout for Plate XIV.

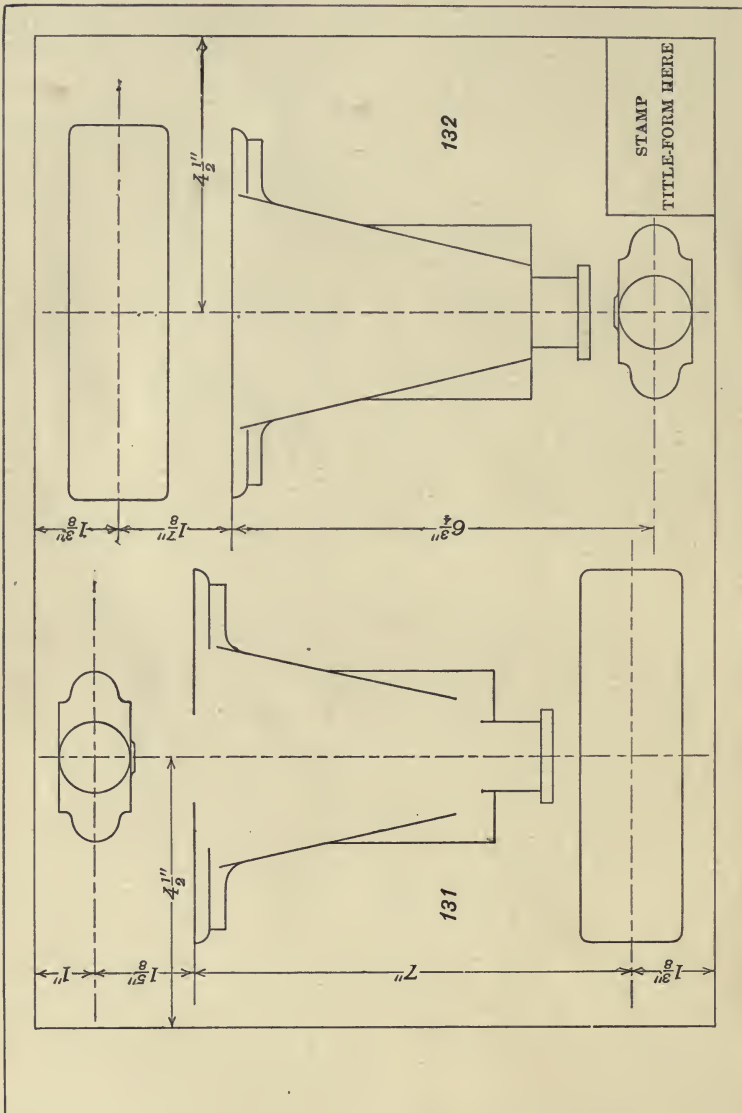

Fig. 162. —Layout for Plate XV.

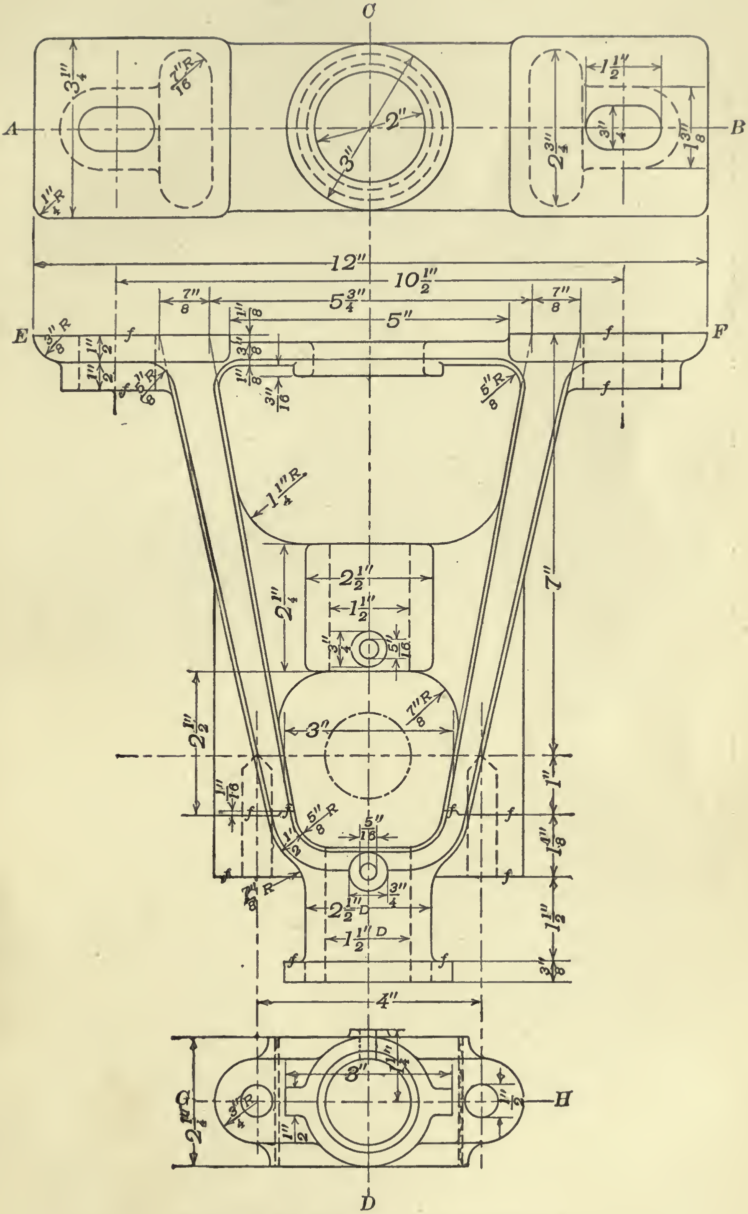

Fig. 163.—Mechanical Drawing of Shaft-hanger.

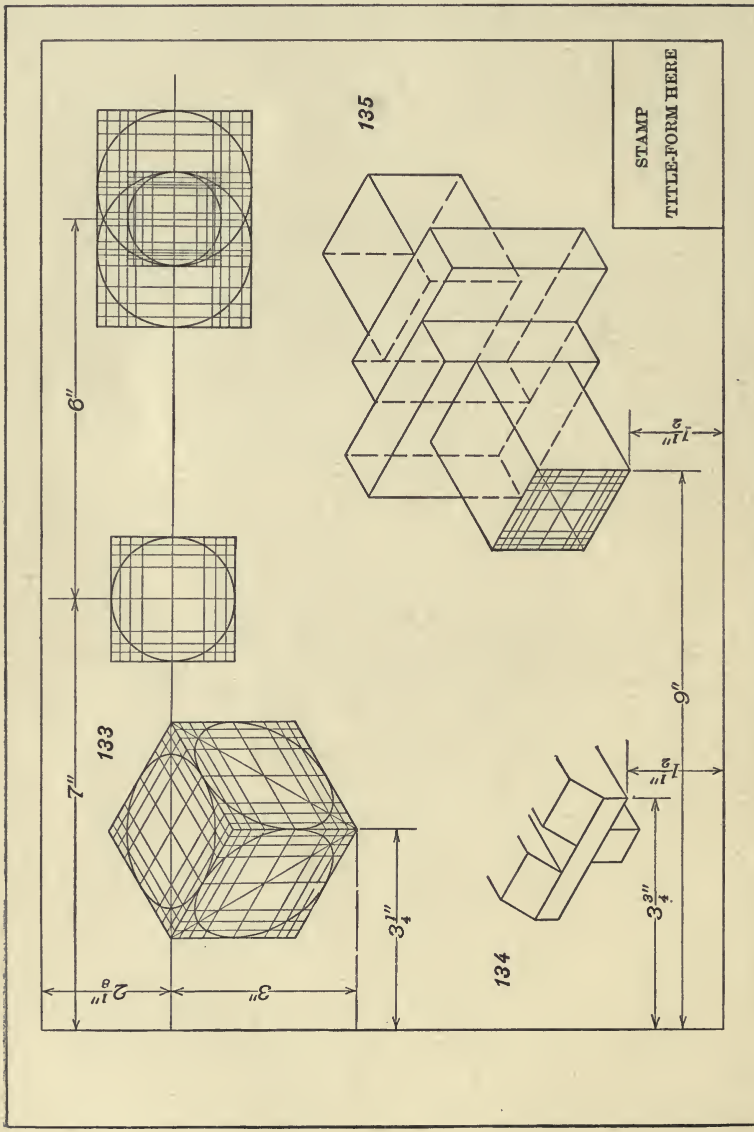

Fig. 164. — Layout for Plate XVI.

181. PLATE XV. Exercises 131 and 132. A practical plate from which the advantages of using the "third angle" in making drawings can be studied. See Fig. 162 for layout of this plate.

> 131. — Make a mechanical drawing in the first angle of the **shaft-hanger** shown in Fig. 163.

> 132. — Make a mechanical drawing in the third angle of the **shaft hanger** shown in Fig. 163.

182. PLATE XVI. Exercises 133, 134 and 135. This plate requires a working knowledge of the text through Chapter IX. See Fig. 164 for layout of sheet.

NOTE. At this point there should be a review and examination of exercises covered by plates XIII to XVI inclusive and of the text matter and lectures given up to the time of completing XVI.

> 133. — Make an *isometric* projection of a 2″ cube having a 2″ diameter circle on each face.

> 134. — Make an *isometric* projection of Fig. 165.

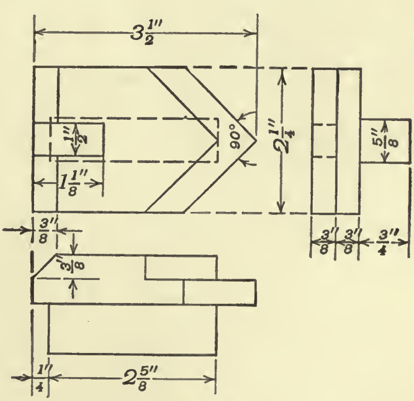

Fig. 165.

135. — Make an *isometric* projection of Fig. 166.

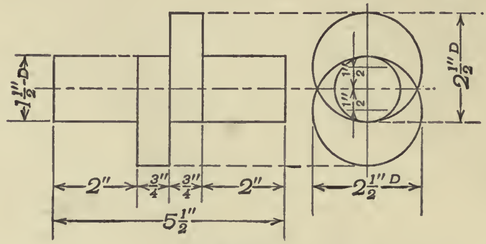

Fig. 166.

183. **Final Examination.** The determination of the students' knowledge and proficiency in this subject gained by a study of the text and lectures and by the solution of drawing-board exercises will be determined in accordance with the methods used in the institution where the work is taken.

APPENDIX A

DRAWING-ROOM SYSTEM

184. **Introductory.** A set of drawing-board exercises in Descriptive Geometry is presented in Chapter X.

It is expected that these exercises will be worked up as presented unless modified or substituted for by the instructor. The general subject matter in the first nine chapters of this book is to be studied *as needed in solving the drawing-board exercises*,* and the student must be prepared for examination at any time on a drawing plate and the general text matter assigned; also on lectures given in connection with the working-up of that plate. The exercises presented in Chapter X are designed to be worked up by mechanical and *not* free-hand drawing. Therefore a knowledge of *elementary* mechanical drawing is necessary to properly solve these exercises.

This work can be much more satisfactorily done under an efficient system, and it is expected that either the system outlined below or a system provided by the instructor will be followed.

(*a*) **General Method of Procedure.** Having secured *instruments* and *supplies* (see Appendix B, page 199) proceed with the drawing-board exercises in Descriptive Geometry given in Chapter X, page 162.

In working up the exercises on each plate proceed in the following manner:

1. Read all the text matter applicable and be prepared for examination on this text and on any lectures given.

2. Tack down the drawing paper and draw in the border line (see Fig. 167, page 196).

3. Stamp in the title form (see Fig. 168, page 197) in the lower right-hand corner just within the border line, unless otherwise

* General reference is given for each plate but special text reference needed in solving each exercise can be found by use of the *index*, which has been made unusually complete for that purpose.

instructed. Write in *with ink* the wording of the title form as far as possible, at the start.

4. Proceed with the solution of the exercises. All drawing is to be done mechanically unless other instructions are given.

The first step in the solution of an exercise is to gain some understanding of *how the data would appear in space* and in the beginning this idea may be obtained by " building up " the exercise. The art of " building up " data so the student may

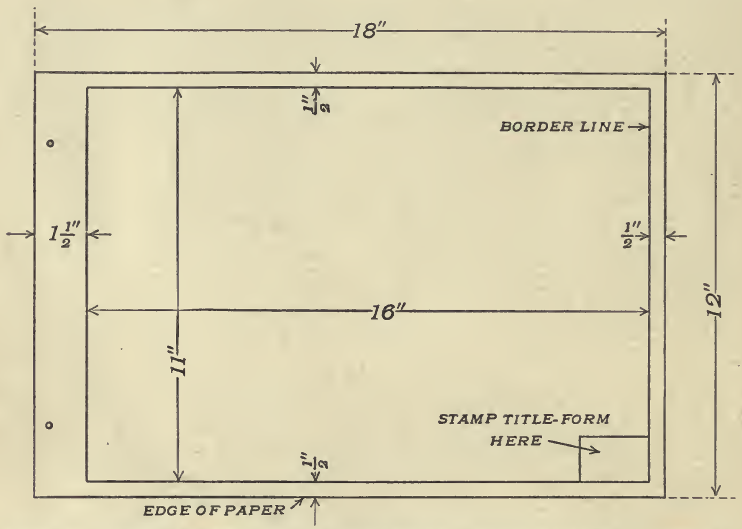

Fig. 167.—Layout of "Plate" for the Drawing-board Exercises as given in Chapter X, page 162.

actually see *in space* what he is to represent *on the drawing paper* is very important and for this reason the instructor should occasionally require the student to " build up " an exercise as a part of his examination.

In building up exercises for the *first* angle a piece of Bristol board $7\frac{1}{2}$ by 10 inches, folded as shown in Fig. 8, page 12, will be used for " planes of projection."

In building up exercises for a general space solution cut out " planes of projection," as shown in Fig. 11, page 16, and assemble them as shown in Fig. 10, page 15.

NOTE. Each exercise must be *carefully* drawn and checked before the instructor's criticism is asked. When the exercises on a drawing plate have been brought to the highest standard of perfection possible the instructor will carefully inspect the work and examine the student. If the instructor is satisfied with the quality of the work done and the student's knowledge of the exercises on the plate he will permit the student to complete the filling in of the title form and to submit the plate for final criticism or acceptance.

(*b*) **Position of Plate on the Drawing Board.** All plates are to be tacked down on the drawing board with the *short* dimensions to the sides and with the punch holes to the left, unless other instructions are given.

(*c*) **Size and Layout of Plates.** All plates are 12 by 18 inches and are laid out as shown in Fig. 167, page 196, unless otherwise noted.

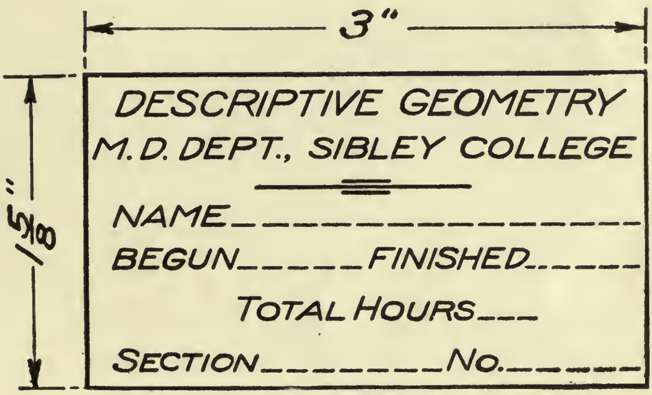

Fig. 168. — Title Form to be used in this Work.

(*d*) **Title Form.** Locate the title form in the lower right-hand corner, just within the border lines, unless otherwise instructed. A rubber stamp of the title-form shown* in Fig. 168 should be provided for the use of the students.

(*e*) **Numbering of Plates.** The plates are to be numbered by use of *Roman* numerals beginning with I.

(*f*) **Locations of Ground Lines** on a plate are stated thus: *G–L*, $2\frac{1}{2}$ and 8 inches, which indicates that there are *two* ground lines on the plate in question. The first ground line is $2\frac{1}{2}$ inches

* The wording of the title form is to be appropriately modified to suit the institution.

from and parallel to the *top border line,* and the second is 8 inches from and parallel to the *top border line.*

(*g*) **Location of Points.** In the set of exercises, before giving the location of a point, a number is always given in brackets; thus, [$7\frac{1}{2}$] A, which indicates that the *ruled* projector for the point A is $7\frac{1}{2}$ inches from the *left-hand border line.* Wherever a dimension is given in brackets, it is to be understood that this is a measurement *along the ground line* and from the *left-hand border line.* The projections are then measured off on the ruled projector. All dimensions are in inches.

(*h*) **Style of Letters and Numbers.** The *free-hand Slant Gothic Alphabet* is to be used. The 4 H drawing pencil is to be used for lettering and numbering, and the pencil point is to be kept well sharpened to a *cone* point.

(*i*) **Numbering of Exercises.** All exercises are to be neatly numbered. Each exercise number is to be conveniently located near the exercise to which it applies.

(*j*) **Notation.** The notation as given in § 25, page 18, and elsewhere in the text, is to be used, and in this connection it must be remembered that *the correct notation is as much a part of each exercise as is the correct graphical solution.*

(*k*) **Line Conventions.** The line conventions as given in § 26, page 18, and elsewhere in the text, are to be used, and in this connection it must be remembered that *the correct line convention is as much a part of each exercise as is the correct graphical solution.*

(*l*) **The Mechanical Penciled Line.** All mechanical lines are to be " clear cut " and of the correct *construction and weight.* A 6 H pencil is to be used and must be kept well sharpened to a cone point and *not* to a chisel edge.

(*m*) **Inking of Exercises.** No instructions are given for the inking of exercises, as it is believed that the time which would be spent in inking can be better spent in solving additional problems. Where, however, instructors do not agree with this view, they can introduce their own color scheme for ink notation.

APPENDIX B

LIST OF INSTRUMENTS AND SUPPLIES

The following instruments and supplies are needed in this work:—

Drawing Paper. 20 sheets of drawing paper accurately cut to 12 by 18 inches and punched. A *good quality* of "detail" paper is required.

Drawing Board. One pine drawing board about 19 by 26 inches.

Thumb Tacks. One dozen thumb tacks with small "round top" heads of about $\frac{5}{16}$-inch diameter.

T-square. One T-square with solid head and polished surfaces. Blade about 26 inches long.

Triangles. One 7-inch transparent triangle 45° and one 10-inch transparent triangle, 30–60°.

Irregular Curve. One transparent irregular curve. Similar to K. and E., No. 21 or Dietzgen No. 20.

Pencil Pointer. One medium cut file (with handle) or a small pad of fine sandpaper.

Pencils. One 4 H and one 6 H drawing pencil, each of hexagon cross section.

Erasers. One block of "Artgum" or a sponge rubber. One Faber's (or equivalent) soft green "Emerald" eraser.

Erasing Shield. One metal or celluloid erasing shield.

Drawing Instruments. One set of drawing instruments of *good* design and quality consisting of at least the following:

(a) Compass, 5 inch, pivot-jointed instrument with handle, lengthening bar, and detachable pencil leg.

(b) Dividers, 5-inch instrument with hair-spring adjustment.

(c) Bow dividers having a maximum capacity of $1\frac{1}{4}$-inch radius.

(d) Bow pencil having a maximum capacity of $1\frac{1}{4}$-inch radius.

Scale. One architects' 12-inch scale with the ordinary graduations.

Protractor. Not absolutely necessary in this course.

Instrument Cleaner. One piece of cotton cloth about 18 by 18 inches for brushing off drawing and cleaning instrument.

Paper Fasteners. Four No. 3 brass paper fasteners with washers.

Bristol Board. One-half sheet of Patent Office Bristol Board, size $7\frac{1}{2}$ by 10 inches.

Sheet Celluloid. One sheet ivory celluloid, polished on one side only, size 10 by 10 inches.

INDEX

A.

B.

C.

G.

H.

I.

L.

M.

N.

O.

P.

R.

U.

V.