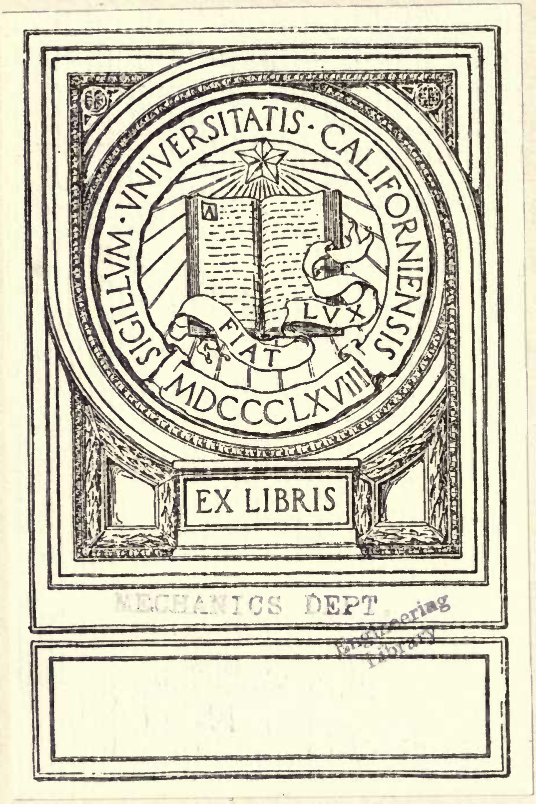

ELEMENTS OF

ENGINEERING THERMODYNAMICS

BY

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PREFACE

This treatise is an extension of a briefer work entitled "Engineering Thermodynamics" by James A. Moyer and James P. Calderwood. The additions and changes are made to suit the needs of those who had successful experience in using the original publication and found it desirable to add supplementary material to make it sufficiently inclusive for special institutional requirements. A great deal of the new material is supplied by A. A. Potter.

This book is intended to bring out the fundamental principles of Engineering Thermodynamics, and is particularly intended for use in technical colleges where it is possible to give special courses on the subjects of steam turbines, internal combustion engines, refrigeration and other applications of thermodynamics.

The new material includes the theory of the hot air engine and internal combustion engine cycles. The appendix includes logarithmic tables, the properties of gases, the properties of steam, ammonia, sulphur dioxide and carbon dioxide.

Every engineering student should become familiar with standard works on the subject of thermodynamics. This book should consequently be supplemented by references to standard works on this subject. A representative list of such reference books is given in a table of the appendix.

The authors are particularly indebted to the following professors and instructors in Mechanical Engineering Departments who gave valuable suggestions and criticisms: Edwin A. Fessenden, Pennsylvania State College; J. E. Emswiler, University of Michigan; G. L. Christensen, Michigan College of Mines; Roy

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SYMBOLS

A = area in square feet, also used to represent the reciprocal of the mechanical equivalent of heat, $\frac{1}{778}$.
B.t.u. = British thermal units (= 778 ft.-lbs.).
C_p = specific heat at constant pressure in B.t.u. per pound per degree.
C_v = specific heat at constant volume in B.t.u. per pound per degree.
C = a general constant in equations of perfect gases.
E = external work in B.t.u. per pound; also sometimes used to express efficiency, usually as a decimal.
E_a = available energy in B.t.u. per pound.
F = force in pounds.
H = heat per pound in B.t.u.*
$H_{\text{sup.}}$ = total heat of superheated steam, B.t.u. per pound.
I_H = total internal energy of steam (above 32° F.) in B.t.u. per pound, sometimes designated by E and i''.
I_L = internal energy of evaporation of steam in B.t.u. per pound, sometimes designated by ρ.
J = mechanical equivalent of heat = 778 (use becoming obsolete).
K = specific heat in foot-pound units.
L = latent heat of evaporation in B.t.u. per pound.
P = pressure in general or pressure in pounds per square foot.
Q = quantity of heat in B.t.u.
R = thermodynamic constant for gases; for air it is 53.3 (in foot-pound units per pound.)
T = absolute temperature, in Fahr. degrees = $460 + t$.
V = volume in cubic feet, also specific volume and velocity in feet per second.
W = work done in foot-pounds.
a = area in square inches.
c = constant of integration.
d = distance in feet.
e = subscript to represent base of natural logarithms.
g = acceleration due to gravity = 32.2 feet per second per second.
h = heat of the liquid per pound in B.t.u. (above 32° F.) also designated by q and i'.
i' = heat of liquid in B.t.u. per pound (above 32° F.).
i'' = total internal energy of steam.
k = a constant.
log = logarithm to base 10.
$\log_e$ = logarithm to natural base e (Naperian).

* In steam tables it is total heat above 32° F.

n = general exponent for V (volume) in equations of perfect gases, also sometimes used for entropy of the liquid in B.t.u. per degree of *absolute* temperature.
p = pressure in pounds per square inch.
q = sometimes used for heat of the liquid in B.t.u. per pound (above 32° F.).
r = ratio of expansion also sometimes used for latent heat of evaporation in B.t.u. per pound.
s' = entropy of the liquid.
s'' = total entropy of vapors.
t = temperature in ordinary Fahr. degrees.
u = difference between the specific volume of a vapor and that of the liquid from which it is formed.
v = specific volume, in cubic feet per pound (in some steam tables).
w = weight per cubic foot = density, also used to designate weight of a substance in pounds.
x = quality of steam expressed as a decimal.
γ = ratio of specific heats $\frac{C_p}{C_v}$.
ϕ = total entropy $\frac{dQ}{T}$, sometimes designated by S''.
θ = entropy of the liquid in B.t.u. per pound per degree of *absolute* temperature.
ρ = internal energy of evaporation, B.t.u. per pound.
σ = volume of water in a cubic foot of saturated steam.
$''$ = inches.

ELEMENTS OF
Engineering Thermodynamics

CHAPTER I

THERMODYNAMIC PRINCIPLES AND DEFINITIONS

Thermodynamics. Thermodynamics deals with the relation between heat and mechanical work. The object of the study of thermodynamics is to consider factors which influence the efficiency of heat power machinery.

Thermodynamics makes it possible to predict the performance of steam engines and steam turbines when operating under conditions of increased pressure, of higher vacuums and expansions, of higher superheats, of jacketing cylinders, of using the working medium in several cylinders one after the other, — that is, compounding instead of expanding the steam only in a single cylinder, — of inserting receivers between the cylinders and of reheating the working medium as it passes from one cylinder to the next.

By means of calculations based upon the study of thermodynamics it is possible to determine the effect of increasing the compression pressures of a gas engine mixture before it is ignited, the result of incomplete cooling upon the efficiency of an air compressor, and similar problems.

Another important service which the study of thermodynamics renders is that of showing what **maximum efficiency is attainable** for any heat engine operating under a given set of conditions. It often happens that tests indicate an efficiency very much better than is usually obtained with any of the present types of engines. In such cases thermodynamic calculations will show conclusively whether the results secured are possible. The ability to interpret correctly the results of experiments performed on all kinds of heat engines requires a knowledge of the basic principles of thermodynamics.

Heat. Heat is a form of energy and not a material substance. The heat of a body depends on the vibratory motion of the small particles or molecules of which the body is built up, the greater the velocity and amplitude of the vibration of these molecules, the higher is the temperature of the body.

Heat can be transferred by conduction, radiation and convection.

The transfer of heat between the different particles of the same body or between several bodies is called **conduction.**

In a heated body the particles are in violent agitation and as a result waves are formed and are emitted or are **radiated** through space to other bodies.

The transfer of heat by the motion of the heated particles is called **convection.** This phenomenon is exhibited in liquids and gases. Thus, when a hot body heats the air in contact with it, the currents of air which are produced by the process of convection ascend and are replaced by cooler air.

Temperature. Temperature is the indication of the sensible heat of a substance and can be measured by a thermometer. The temperature does not measure the quantity of heat energy in the substance, but indicates only the relative heat intensity, which can be revealed by the senses of the observer.

Thermometers. There are three thermometric scales:

The **Centigrade** or **Celsius** degree is $\frac{1}{100}$ of the temperature interval between the melting point of ice and the boiling point of water at atmospheric pressure, these two fixed points being denoted 0° C. and 100° C. respectively.

The **Fahrenheit** degree is $\frac{1}{180}$ of the temperature interval between these two fixed points, the melting point of ice being taken at 32° F. and the boiling point of water at 212° F.

The **Reaumur** scale has the melting point of ice at 0° R. and the boiling point of water at 80° R.

The thermometric scales generally used are the Centigrade and Fahrenheit, the relations between these scales being:

$$\text{Degrees C.} = \tfrac{5}{9}\,[\text{degrees F.} - 32]. \qquad (1)$$

$$\text{Degrees F.} = \tfrac{9}{5}\,\text{degrees C.} + 32. \qquad (2)$$

Mercury thermometers, as ordinarily constructed, have the space above the mercury under a vacuum. Such thermometers cannot be used for the measurement of temperatures exceeding 500° F., as the vacuum reduces the boiling point of mercury. The range of mercury thermometers can be increased to about 900° F. by filling the space above the mercury with some inert gas like nitrogen.

For the measurement of very high temperatures thermoelectric pyrometers are best suited.

Absolute Zero. In the graduation of liquid thermometers, the vaporization of the liquid at high temperatures and its freezing at low temperatures limits the thermometric range. The fundamental scale of temperature measurement is based on Thomson's absolute thermometric scale, which is independent of the nature of any thermometric substance. The zero of the absolute scale or the absolute zero is taken as a point which marks the absence of heat energy or of molecular vibrations of a body.

The absolute zero is 459.5° (practically 460°) below the zero on the Fahrenheit scale and 273.0° below the zero on the Centigrade scale.

Calling the absolute temperature T and the temperature as measured by a thermometer t,

$$\text{On the Fahrenheit scale } T = t + 460. \tag{3}$$

$$\text{On the Centigrade scale } T = t + 273. \tag{4}$$

Units of Heat. Heat is measured in heat units. A heat unit is the amount of heat required to raise the temperature of one unit weight of water one degree. In the English system of measures the heat unit is the British thermal unit (B.t.u.), which is defined as the amount of heat required to raise the temperature of one pound of water one degree on the Fahrenheit scale. The heat unit in the metric system is the calorie * which is defined as the heat required to raise the temperature of one kilogram of water one degree on the Centigrade scale.

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* Since one kilogram = 2.204 pounds and one degree C. = $\frac{9}{5}$° F., 1 calorie = $\frac{9}{5} \times 2.204 = 3.968$ B.t.u.

To correctly define the British thermal unit it is necessary to state at what temperature the rise of one degree on the Fahrenheit scale is to occur, because the specific heat of water is slightly variable. This heat unit (B.t.u.) is sometimes defined as the amount of heat required to raise the temperature of water one degree Fahrenheit at the condition of maximum density of water, that is, between 39 and 40 degrees Fahrenheit. Other definitions are based on the amount of heat required to raise the temperature of water one degree Fahrenheit between 60 to 61 degrees Fahrenheit. Still another definition, which is generally considered to be the most accurate, defines a British thermal unit as one one-hundred-and-eightieth ($\frac{1}{180}$) of the amount of heat required to raise the temperature of water from 32 to 212 degrees Fahrenheit. In other words, according to this last definition, the British thermal unit is the average value of the amount of heat required to raise the temperature of one pound of water one Fahrenheit degree between the conditions of freezing and boiling at atmospheric pressure.

Specific Heat. As the addition of the same quantity of heat will not, as a rule, produce the same temperature changes in equal weights of different substances, it is evident that the amount of heat in any substance will depend on the capacity of the substance for heat. For this reason it is necessary to allow for the relative heat capacity or the specific heat (C) of a substance. Specific heat can be defined as the ratio of the heat added to the temperature change produced in a unit weight of a substance. It can also be defined as the resistance which a substance offers to a change in its temperature, the resistance of water being taken as unity. In the English system the specific heat is the number of British thermal units (B.t.u.) required to raise the temperature of a pound of the substance one degree Fahrenheit.

Thus if Q is the quantity of heat added to one pound of a substance, the temperature change is,

$$t_2 - t_1 = \frac{Q}{C}, \tag{5}$$

or

$$Q = C\,(t_2 - t_1).$$

If the specific heat is a variable,

$$Q = \int_{t_1}^{t_2} C\,dt. \tag{6}$$

The following problem illustrates the application of equation (6):

The specific heat of a substance is expressed by the equation,

$$C = 0.24112 + 0.000009\,t.$$

What amount of heat is required to raise the temperature of one pound of the substance from 0° to 100° F.?

Solution. Since the specific heat is variable,

$$Q = \int_{t_1}^{t_2} C\,dt.$$

substituting the value of C and integrating,

$$\begin{aligned} Q &= \int_{t_1}^{t_2} (0.24112 + 0.000009\,t)\,dt \\ &= 0.24112\,\Big[t\Big]_{0^\circ}^{100^\circ} + 0.000009\left[\frac{t^2}{2}\right]_{0^\circ}^{100^\circ} \\ &= 0.24112\,(100) + 0.000009\,(5000) \\ &= 24.112 + 0.045 = 24.157 \text{ B.t.u.} \end{aligned}$$

The specific heat of gases and vapors changes considerably in value according to the conditions under which the heat is applied. If the heat is applied to a gas or a vapor held in a closed vessel, with no change in volume, no work is performed, and, therefore, all the heat added is used to increase the temperature. This is the condition in a boiler when no steam is being drawn off. In this case the symbol C_v represents the specific heat during heat application at **constant volume.** If, on the other hand, the heating is done while the pressure is kept constant and the volume is allowed to change permitting expansion and the performance of work, the symbol C_p is used and represents the specific heat during heat application at **constant pressure.**

When the problem deals with w pounds of a substance instead of a unit weight, equation (6) becomes,

$$Q = w\int_{t_1}^{t_2} C\,dt. \tag{7}$$

Pressure. Force per unit of area is called pressure. Thus, the pressure exerted by a gas or vapor is expressed in the English system in pounds per square inch, pounds per square foot, inches of mercury or atmospheres. In the metric system, pressure is expressed in kilograms per square centimeter or millimeters of mercury.

Gages always read pressures above atmospheric pressure or above vacuum. The absolute pressure is the sum of the gage and atmospheric pressures. Thus, if a gage reads 75 pounds pressure (per square inch), and the barometer is 29.65 inches of mercury, the atmospheric or barometric pressure is, in pounds per square inch,

$$29.65 \times 0.491 = 14.56.$$

(0.491 is the weight of a cubic inch of mercury at 70° F.)

The absolute pressure is:

$$75 + 14.56 = 89.56 \text{ pounds per square inch.}$$

In thermodynamic equations the unit of pressure is usually expressed in pounds per square foot.

Volume. By specific volume is meant the amount of space occupied by a unit weight of a substance, expressed in cubic feet or in cubic meters. Thus, the volume of one pound of steam at atmospheric pressure is its specific volume and is equal to 26.79 cubic feet.

Work. Work is done by a force during a given displacement and is independent of the time. The foot-pound is the unit of work in the English system. Thus, when a body weighing one pound is raised through a distance of one foot, the resulting work is a foot-pound. Similarly, the product of the pressure in pounds per square foot and the volume in cubic feet is equal to work in foot-pounds.

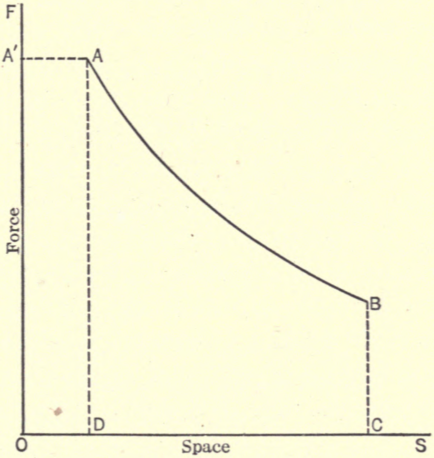

FIG. 1. — Work Diagram.

Work being the product of two dimensions, it may be represented graphically by the area of a closed figure, the coördinates of which are force and distance, or pressure and volume. Thus, *A* (Fig. 1) represents the position of a body between *F* (force) and *S* (space) coördinates, *O* being the origin or starting point. The distance *AD* from the *OS* line represents the force against which it is acting and *AA′*, measured from the *OF* line, is the distance it has moved from the starting point. If the body moves from *A* to *B* along the path *AB*, the area *ABCD* represents the work done.

If the equation of the path is known, the work is

$$W = \int_{S_A}^{S_B} F\,dS. \tag{8}$$

Power. Power is the rate of doing work, or the work done divided by the time required to do it. In the English system, the unit of power is the horse power. It is the power required to raise 550 pounds through a vertical distance of one foot in one second, or 33,000 pounds one foot in one minute. To obtain the horse power, the work in foot-pounds per minute must be obtained and the result divided by 33,000. In the metric system, the unit of power is the watt. One horse power is equal to 746 watts. The French horse power (cheval) is 542.5 foot-pounds per second.

Indicated Horse Power. The term indicated horse power is applied to the rate of doing work by a gas or a vapor in the cylinder of an engine. It is obtained by means of an engine indicator.

If P represents the mean effective pressure (average unbalanced pressure) in pounds per square inch, as shown by the indicator card, and A the effective area of the piston in square inches, the total pressure exerted on the piston is PA. If the piston has a stroke of L feet, the work per stroke is PAL and the work per minute is $PALN$, where N represents the number of revolutions per minute. The indicated horse power is

$$\text{I.h.p.} = \frac{\text{Work per minute}}{33{,}000} = \frac{PALN}{33{,}000}. \tag{9}$$

Brake Horse Power represents the actual power which an engine can deliver for the purposes of work. The difference between indicated and brake horse power of an engine represents the horse power lost in friction. The brake horse power can be measured by some form of friction brake, which absorbs the power measured, and is called an absorption dynamometer, or by a transmission dynamometer. In either type of dynamometer, if F is the effective pull in pounds, L the lever arm, in feet, through which the weight is exerted, and N the number of revolutions of the shaft per minute, the brake horse power is

$$\text{B.h.p.} = \frac{2\pi FLN}{33{,}000}. \tag{10}$$

Mechanical Equivalent of Heat. There is a definite quantitative relation between work expended and heat produced. This relation between heat and work is called the mechanical equivalent of heat and is designated by J. In the English system,

$$J = 778 \text{ foot-pounds}$$

or

$$1 \text{ B.t.u.} = 778 \text{ foot-pounds}.$$

In the metric system,

$$J = 427 \text{ kilogrammeters}$$

or

$$1 \text{ calorie} = 427 \text{ kilogrammeters.}$$

The reciprocal of J, or the heat equivalent of work, is designated by A, where $A = \frac{1}{778}$ in the English system or $\frac{1}{427}$ in the metric system.

First Law of Thermodynamics. The statement of the definite relation between heat and mechanical work constitutes what is known as the first law of thermodynamics. It is usually expressed:

"Heat and mechanical energy are mutually convertible and heat requires for its production and produces by its disappearance mechanical work in the ratio of the mechanical equivalent of heat." In other words, this law is a statement of the conservation of energy as regards the equivalence of mechanical work and heat.

Second Law of Thermodynamics. "In order to transform the heat of a body into work, heat must pass from that body into another at a lower temperature." Thus, there must be a difference of level in the transformation of heat energy into work and heat cannot be transformed from one body to another at a higher temperature, unless work is expended in order to produce such a transfer of heat.

This law states as regards heat engines the limits to their possible performance, which would be otherwise unlimited, if only the "first law" of thermodynamics is considered. It means, also, that no heat engine converts or can convert into work all of the heat supplied to it. A very large part of the heat supplied is necessarily rejected by the engine in the form of unused heat.

Effects of Heat Application. If a quantity of heat Q is imparted to a body, the following effects will be produced:

1. The temperature of the body will rise;
2. The volume of the body will increase;
3. The body will be capable of doing external work.

Representing the above effects by S, V and W,

$$Q = S + V + W, \tag{11}$$

in which S represents that quantity of heat, termed the sensible heat, which was utilized in raising the temperature of the body; V represents that quantity of heat which was absorbed by the body in increasing its store of internal energy other than that associated with the sensible heat; and W is the heat which was absorbed to perform the external work in increasing the volume of the body against the resistance offered by external substances. $S + V$ represents that portion of the heat that was chargeable to the increase of internal work. This may be termed the intrinsic energy increase and designated by I.

Thus, for the addition of an infinitesimal quantity of heat,

$$dQ = dI + dW$$

or

$$Q = \int_{I_1}^{I_2} dI + \int_{V_1}^{V_2} P\,dV. \tag{12}$$

In equation (12), which represents the effect of heat application, the intrinsic energy change $\int_{I_1}^{I_2} dI$ depends on the physical state of the body, that is, whether it is a solid, a liquid, a vapor, or a gas, while the external work $\int_{V_1}^{V_2} P\,dV$ depends on the character of the path in the work diagram.

The Heat Engine. The heat engine is a machine which converts the heat energy of solid, liquid or gaseous fuel into work. This conversion depends on the variation in the pressure, volume and temperature of a gas or a vapor and can be accomplished in two ways:

First, by "external combustion," in which case the fuel is burned outside of the engine cylinder; the heat developed by the combustion of the fuel is conducted to the working substance or heat medium through walls; this working substance does work on a piston in the case of a reciprocating engine or on a vane or

" blade " in a turbine. To this class belong steam engines of the reciprocating, turbine or rotary types and also external-combustion hot air engines. Thus, in the case of the steam engine, the fuel, which may be coal, wood, petroleum or gas, is burned outside of the boiler shell and the resulting heat is transmitted by conduction through the metal of the shell to the working substance, which is water. When enough heat has been added to the water to produce a change in its physical state, water vapor or steam is formed at the required pressure. This vapor, which may be dry, wet or superheated, if allowed to act on the piston of the engine, will do work.

Another method of converting heat into mechanical energy is by burning the fuel rapidly or slowly inside of an engine cylinder or in a communicating vessel, the products of combustion being allowed to act directly on the piston of the engine. To this class belong gas, petroleum and alcohol engines which are called " internal combustion " engines.

Thermal Efficiency of a Heat Engine. By thermal efficiency (E) is meant the ratio of the heat converted into work (AW) to the heat supplied by the engine (Q_1), or

$$E = \frac{AW}{Q_1}. \tag{13}$$

Since only a part of the heat supplied to an engine can be converted into work, the above ratio is a fraction always less than unity.

PROBLEMS

1. Give examples of the transfer of heat by conduction, radiation and convection.

2. Show that the kilogram calories per kilogram × 1.8 give B.t.u. per pound.

3. Prove that the product of a pressure in pounds per square foot and a volume in cubic feet results in foot-pounds of work.

4. If the specific heat of a substance is 0.65, how many B.t.u. are required to raise the temperature of 10 lbs. of the substance through 10° F.?

5. The specific heat of a substance is expressed by the equation,

$$C = 0.5 - 0.02\, t.$$

What heat is required to raise the temperature of 5 lbs. of the substance from 10° to 100° F.?

6. Convert − 40° C. into degrees Fahrenheit.

7. Change 350° F., 212° F. and 160° C. to absolute Fahrenheit temperatures.

8. Prove that the weight of 1 cu. in. of mercury is equal to 0.491 lb.

9. If the barometric reading is 29.2 ins., change 140 lbs. per sq. in. gage and also 27 ins. vacuum into pounds per square inch absolute pressure.

10. Calculate the indicated horse power of a 12″ × 13″ steam engine which operates at a speed of 265 r.p.m. The mean effective pressure of the head end is 27.5 lbs. per sq. in. and of the crank end 27.8 lbs. per sq. in. The diameter of the piston rod is $1\frac{5}{8}$ inches.

11. Calculate the brake horse power developed by an engine as measured by a Prony brake, the effective pull being 32 lbs. at 250 r.p.m. and the lever arm 32 inches long.

12. One pound of fuel has a heating value of 14,500 B.t.u. How many foot-pounds of work is it capable of producing, if all this heat is converted into work?

13. An engine developed 15,560 foot-pounds of work. How much heat in B.t.u. was theoretically required?

14. A heat engine receives 100,000 B.t.u. of heat in the form of fuel and during the same period 30,000 B.t.u. are converted into work. What percentage (thermal efficiency) of the heat received by the engine was converted into work?

15. A gas engine receives 20,000 B.t.u. of heat in the form of fuel and during the same period 3,112,000 foot-pounds of work are developed. What is the thermal efficiency of the engine?

16. It is claimed that a certain motor generates 300,000 foot-pounds of work per hour and during this period receives 400 B.t.u. of heat in the form of fuel. Are such results possible?

17. An oil engine uses 0.74 lb. of fuel per b.h.p. per hour. Calculate the thermal efficiency of this engine if the oil has a calorific value of 18,600 B.t.u. per lb.

18. An engine receives 200 B.t.u. of heat per minute and exhausts during the same period 100 B.t.u. If no losses of heat occur within the cylinder,

(a) How many B.t.u. of heat are being transformed into work?
(b) What number of foot-pounds does this heat produce?
(c) What horse power is being developed?
(d) What is the thermal efficiency of the engine?

19. In the manufacture of certain explosives, acids are mixed with an oxidizable substance. During the process the mixture must be constantly agitated by a stirring mechanism to maintain uniform conditions, and the

temperature of the mixture must be kept below a certain predetermined value to prevent explosion. If during the process of manufacture 5000 foot-pounds of work are delivered per minute to the agitator, and 1000 B.t.u. are generated during the same period by the chemical reaction, how much heat must be absorbed per hour to maintain the temperature of the mixture constant?

20. Prove that one horse power developed for one hour is equivalent to the consumption of 2545 B.t.u. of heat in the same period.

CHAPTER II

PROPERTIES OF PERFECT GASES

In the study of thermodynamics, the working substance, or heat medium, through which the heat engine converts heat into work, is in the condition of a perfect gas or vapor. The laws governing the action of the two classes of substances differ. For this reason the study of thermodynamics is divided into the thermodynamics of gases and of vapors.

When the term " gas " is used it refers to what is more properly called a perfect gas. A **perfect gas** may be defined as a fluid which remains in the gaseous state when subjected to changes in pressure or in temperature. Gases which are near their point of condensation are not perfect gases. Oxygen, hydrogen, nitrogen, air and carbon dioxide are practical examples of perfect gases.

Vapors are fluids which are readily transformed into liquids by a very moderate reduction in temperature or increase in pressure. Common examples of vapors are steam and ammonia.

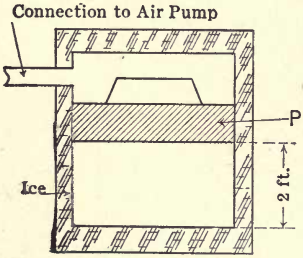

FIG. 2. — Constant Temperature Apparatus for Demonstrating Relation between Pressure and Volume of a Gas.

Relation between Pressure, Volume and Temperature of a Perfect Gas. In practically all heat engines, work is done by changes of volume of a fluid, and the amount of work performed depends only on the relation of pressure to volume during such change and not at all on the form of the vessel containing this fluid.

Figure 2 shows a vessel containing a perfect gas and surrounded by a jacket filled with cracked ice. Its temperature will, therefore, be at 32° F. This vessel has a tightly fitting piston P of which the lower flat side has an area of **one square foot.** In the position shown the piston is two feet from the bottom of the vessel, so that the volume between the piston and the bottom of the vessel is **two cubic feet.** The pressure on the gas is that due to the piston and the weights shown. Assume this total weight is 100 pounds and that the air pump connected to the top of the vessel maintains a vacuum above the piston. Then the pressure on the gas below the piston is 100 pounds per square foot. If now the weights are increased to make the pressure on the gas 200 pounds per square foot the piston will sink down until it is only one foot from the bottom of the vessel, provided the ice keeps a constant temperature. If the temperature is not maintained constant, because of the tendency of gases to expand with increase in temperature, it will be necessary to apply a total weight greater than 200 pounds to reduce the volume to one cubic foot. Similarly, if the weight on the gas were reduced to 50 pounds and the vessel were made high enough, the lower side of the piston would then be four feet from the bottom of the vessel.

Examination of the above figures shows that if the temperature is constant **the product of pressure and volume** is a constant, and in this particular case it is always equal to 200 foot-pounds. These facts are expressed by **Boyle's Law** which can be stated as follows:

Boyle's Law. If a unit weight of gas is compressed or expanded at constant temperature, the pressure **varies inversely as the volume,** or the product of pressure and volume remains a constant. Thus, if P_1, V_1 are the initial pressure and volume and P_2, V_2 the final pressure and volume,

$$P_1V_1 = P_2V_2. \tag{14}$$

The laws of thermodynamics dealing with volume and pressure changes corresponding to temperature variations may be stated as follows:

(1) **Under constant pressure the volume of a given mass of gas varies directly as the absolute temperature.**

(2) **Under constant volume the absolute pressure of a given mass of gas varies directly as the absolute temperature.**

These fundamental principles, often called Gay-Lussac's or **Charles' Laws,** may also be stated thus:

$$\text{With } \textbf{pressure constant,}\ \frac{V_1}{V_2} = \frac{T_1}{T_2}, \tag{15}$$

$$\text{With } \textbf{volume constant,}\ \frac{P_1}{P_2} = \frac{T_1}{T_2}, \tag{16}$$

where V_1 and V_2 are respectively the initial and final volumes, P_1 and P_2 are the initial and final absolute pressures, and T_1 and T_2 are the absolute temperatures corresponding to the pressures and volumes of the same subscripts.

The following problem shows applications of Charles' laws:

A gas has a volume of 2 cubic feet, a pressure of 14.7 pounds per square inch absolute and a temperature of 60° F.

(a) What will be the volume of this gas if the temperature is increased to 120° F., the pressure remaining constant?

(b) What will be the pressure if the temperature is increased as in (a) but the volume remains constant?

Solution. (a) Since the pressure remains constant and the substance is a gas, the volume varies directly as the absolute temperature.

Letting V_1 and T_1 be the initial conditions and V_2 and T_2 be the final conditions, then

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{2}{V_2} = \frac{60 + 460}{120 + 460},$$

$$V_2 = 2.23 \text{ cubic feet.}$$

(b) Since the volume remains constant,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{14.7}{P_2} = \frac{60 + 460}{120 + 460},$$

$$P_2 = 16.39 \text{ pounds per square inch absolute.}$$

Combination of Boyle's and Charles' Laws. Equations (14), (15), and (16) cannot often be used as they stand, because it does not often happen that any one of the three variables (P, V and T) remains constant. A more general law must be developed, therefore, allowing for variations in all of the terms P, V and T. This is accomplished by combining the above equations.

Assume a pound of gas of which the **initial** conditions of pressure, volume and temperature are represented by P_1, V_1 and T_1, while the corresponding **final** conditions are given by P_2, V_2 and T_2. The first step is in changing the volume from V_1 to V_2 and the pressure from P_1 to some intervening pressure P_2' while the temperature T_1 remains constant. This change can be expressed by Boyle's law (equation 14).

With constant temperature (T_1),

$$\frac{V_1}{V_2} = \frac{P_2'}{P_1}, \tag{17}$$

from which, by solving,

$$P_2' = \frac{P_1 V_1}{V_2}, \tag{18}$$

where P_2' is the resulting pressure of the gas when its volume is changed from V_1 to V_2, with the temperature remaining constant at T_1.

The second step is in the change in pressure from P_2' to P_2 and in temperature from T_1 to T_2, while the volume remains constant at V_2. This step is expressed as follows:

With constant volume (V_2),

$$\frac{P_2'}{P_2} = \frac{T_1}{T_2}, \tag{19}$$

which may be written

$$P_2 = \frac{P_2' T_2}{T_1}. \tag{20}$$

Substituting now the value of P_2' from (18) in (20), we have

$$P_2 = \frac{P_1 V_1 T_2}{V_2 T_1}, \tag{21}$$

which may be arranged to read,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}. \qquad (22)$$

The following problem shows the application of equation (22): A quantity of air at atmospheric pressure has a volume of 2000 cubic feet when the barometer reads 28.80 inches of mercury and the temperature is 40° C. What will be the volume of this air at a temperature of 0° C. when the barometer reads 29.96 inches of mercury?

Solution. Volume, pressure and temperature vary in this case as in the following equation,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}.$$

Letting $P_1, V_1, T_1 =$ initial conditions,

$P_2, V_2, T_2 =$ final conditions,

then

$$\frac{28.80 \times 2000}{40 + 273} = \frac{29.96 \times V_2}{0 + 273},$$

$$V_2 = 1676 \text{ cubic feet.}$$

Now, since P_2, V_2 and T_2 in equation (22) represent any **simultaneous condition** of the gas, we may also write the following more general relations:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} = \text{a constant}, \qquad (23)$$

and, therefore,

$$PV = RT, \qquad (24)$$

where R is the "**gas constant,**" and is equal to $\frac{PV}{T}$.

The Law of Perfect Gases. Equation (24) expresses the law connecting the relation between pressure, volume and temperature of a perfect gas. In this equation V is the specific volume, or the volume occupied by a unit weight of a gas at the absolute pressure P and absolute temperature T; R is the **gas constant**

in foot-pounds and depends on the density of the gas and on the units of measurement adopted.

By means of equation (24) if the pressure, volume and temperature of a gas for one given condition are known, the value of R can be determined.

Example 1. If the volume of air at freezing point and atmospheric pressure is 12.39 cubic feet per pound, calculate the value of R in the English units.

Solution. $R = \frac{P_0V_0}{T_0} = \frac{14.7 \times 144 \times 12.39}{32 + 460} = 53.3.$

Example 2. If one pound of air occupies 5 cubic feet at a temperature of 200° F., find the corresponding pressure.

Solution. Using the value of R for air as calculated above

$$P_1 = \frac{RT_1}{V_1} = \frac{53.3\,(460 + 200)}{5} = 7040 \text{ pounds}$$

per square foot absolute, which is the same as 34.2 pounds per square inch gage pressure.

In calculating R by equation (24) care must be taken not to confuse the units of measurement. It must also be remembered that the method, as illustrated in the above examples, gives the value of R for one pound of air, or for one unit weight of the gas in question; for w pounds, the value of the constant is wR, or:

$$PV = wRT, \tag{25}$$

where P = absolute pressure in pounds per square foot,
V = volume in cubic feet,
w = weight of gas in pounds,
R = the "gas constant" for one pound of gas in foot-pound units,
T = the absolute temperature in Fahrenheit degrees.

This equation is applicable to any perfect gas within the limits of pressure and temperature employed in common engineering practice. The "thermodynamic" state of a gas is known when its pressure, volume, temperature, weight and composition are known; when any four of these quantities are known the fifth can be found by equation (25).

Heat and Its Effect Upon a Gas. In equation (12) it was shown that in general the effect of adding heat upon a substance was to increase the intrinsic energy and to overcome the external resistances producing work. This law can be stated as follows:

Heat supplied = increase in intrinsic energy + the external work done.

The converse of this statement is equally true. The heat abstracted from a gas equals the decrease in intrinsic energy plus the negative work done.

External Work. The external work or the work done by a gas in its expansion is represented graphically by Fig. 3. The area under the expansion line *BC* is proportional to the work done in the expansion. If the initial condition of the gas at *B* as regards pressure and volume is represented by P_1 and V_1 and the final condition at *C* by P_1 and V_2 (expansion being at constant pressure), the force *P* moved through a distance of abscissas $(V_2 - V_1)$ is a measure of the work done. Obviously the area under the line *BC* divided by the horizontal length $(V_2 - V_1)$ is the average value of the force *P*. If, further, and in general, we represent the area under *BC* by the symbol *A*, then we can write,

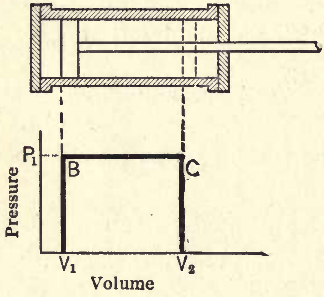

FIG. 3.—External Work of Expansion.

$$\frac{A}{V_2 - V_1} = \text{average value of } P,$$

whether or not *P* is constant. And also,

$$\text{Work done} = \frac{A}{V_2 - V_1} \times (V_2 - V_1) = A.$$

The same principle applies whether the line *BC* is a straight line, as shown in Fig. 3, or a very irregular curve, as will be shown later.

Internal Energy. The heat energy possessed by a gas or vapor, or the heat energy which is contained in a gas or vapor in a form similar to "**potential**" energy, is called its **internal energy.** It is also called intrinsic energy, since it may be said to "**reside**" within the substance and has not been transferred to any other substance. Thus, an amount of heat added to a substance when no work is performed is all added to the internal energy of that substance. On the other hand, when heat is added while work is being performed, the internal energy is increased only by the difference between the heat added and the work done.

Internal energy may also be defined as the energy which a gas or vapor possesses by virtue of its temperature, and for one pound of gas may be expressed as follows:

$$\text{Internal energy} = C_v\, dT \text{ (in B.t.u.)},$$

where T is the absolute temperature and C_v the specific heat at constant volume. From the paragraph on specific heat it will be remembered that C_v takes into account only that heat required to raise the temperature, since under constant volume conditions no external work is done; and, therefore, in dealing with internal energy, C_v is always used.

Increase in internal energy in B.t.u. (for one pound of a gas)

$$= C_v (T_2 - T_1). \qquad (26)$$

Joule's Law. In the case of ideally perfect gases, such as thermodynamic equations must deal with, it is assumed, when a gas expands without doing external work and without taking in or giving out heat (and, therefore, without changing its stock of internal energy), that its temperature does not change. It was for a long time supposed that when a gas expanded without doing work, and without taking in or giving out heat, that its temperature did not change. This fact was based on the famous experiments of Joule. Later investigations by Lord Kelvin and Linde have shown that this statement is not exactly correct as all known gases show a change in temperature under these conditions. This change in temperature is known as the "Joule-Thomson" effect.

Relation of Specific Heats and the Gas Constant. If heat is added at constant pressure, then

$$Q = wC_p (T_2 - T_1). \tag{27}$$

Also, by equation (26), the increase in internal energy when heat is added

$$= wC_v (T_2 - T_1). \tag{28}$$

$$\text{External work} = P (V_2 - V_1) \text{ foot-pounds}$$

$$= \frac{P (V_2 - V_1)}{778} \text{ B.t.u.} \tag{29}$$

Since the heat added = increase in internal energy + external work,

$$wC_p (T_2 - T_1) = wC_v (T_2 - T_1) + \frac{P (V_2 - V_1)}{778}. \tag{30}$$

By equation (25),

$$P_2 V_2 = wRT_2 \text{ and } P_1 V_1 = wRT_1.$$

Substituting these values in (30),

$$wC_p (T_2 - T_1) = wC_v (T_2 - T_1) + w\frac{R (T_2 - T_1)}{778}. \tag{31}$$

Simplifying,

$$C_p = C_v + \frac{R}{778}. \tag{32}$$

$$C_p - C_v = \frac{R}{778} = AR. \tag{33}$$

Equation (33) shows that the difference between the two specific heats is equal to the gas constant R, which when measured in foot-pounds represents the external work done by one pound of a gas when its temperature is increased by one degree under constant pressure.

Example. The specific heat of air at constant pressure (C_p) is 0.2375 B.t.u. and R = 53.3 foot-pounds. Calculate the specific heat at constant volume (C_v).

Solution.

$$C_v = C_p - AR = 0.2375 - \frac{53.3}{778} = 0.1690 \text{ B.t.u.}$$

Ratio of the Two Specific Heats $\left(\frac{C_p}{C_v}\right)$. The constant representing the ratio of the two specific heats of a perfect gas is represented by γ where

$$\gamma = \frac{C_p}{C_v} = \frac{C_p}{C_p - \frac{AR}{C_p}} = \frac{1}{1 - \frac{AR}{C_p}}. \tag{34}$$

Example. Calculate the value of γ for air. $C_p = 0.2375$ B.t.u. $R = 53.3$ foot-pounds.

Solution. $\gamma = \frac{1}{1 - \frac{AR}{C_p}} = \frac{1}{1 - \frac{53.3}{778 \times 0.2375}} = 1.405.$

The constant designating the relation between the specific heats of air is often designated by k instead of γ.

Values of the Specific Heats. Regnault after carrying on experiments on the specific heat of hydrogen, oxygen, air and carbon dioxide concluded that all gases have a constant specific heat under varying pressures and temperatures. Recent experiments tend to show that the specific heats of substances vary with the pressure and temperature. The variability in the values of the specific heats does not influence to any very great extent most thermodynamic computations.

In the application of thermodynamics to internal combustion engines the exact values of the specific heats are of considerable importance.

Tables 1 and 2 in the appendix give constants for various gases.

PROBLEMS

1. Air at constant pressure with an initial volume of 2 cu. ft. and temperature of 60° F. is heated until the volume is doubled. What is the resulting temperature in degrees Fahrenheit?

2. Air is cooled at constant volume. The initial pressure is 30 lbs. per

sq. in. absolute and the initial temperature is 101° F. The final condition has a temperature of 50° F. What is the final pressure?

3. One pound of hydrogen is cooled at constant pressure from a volume of 1 cu. ft. and temperature of 300° F. to a temperature of 60° F. What is the resulting volume?

4. A tank whose volume is 50 cu. ft. contains air at 105 lbs. per sq. in. absolute pressure and temperature of 80° F. How many pounds of air does the tank contain?

5. An automobile tire has a mean diameter of 34 inches and a width of 4 inches. It is pumped to 80 lbs. per sq. in. gage pressure at a temperature of 60° F.; atmospheric pressure 14.6 lbs. per sq. in. absolute.

(a) How many pounds of air does the tire contain?

(b) Assuming no change of volume, what would be the gage pressure of the tire if placed in the sun at 100° F.?

6. A gas tank is to be made to hold 0.25 lb. of acetylene when the pressure is 250 lbs. per sq. in. gage, atmospheric pressure 14.4 lbs. per sq. in. absolute, and the temperature of the gas 70° F. What will be its volume in cubic feet?

7. A quantity of air at a temperature of 70° F. and a pressure of 15 lbs. per sq. in. absolute has a volume of 5 cu. ft. What is the volume of the same air when the pressure is changed at constant temperature to 60 lbs. per sq. in. absolute?

8. How many pounds of air are required for the conditions in problem 7?

9. The volume of a quantity of air is 10 cu. ft. at a temperature of 60° F. when the pressure is 15 lbs. per sq. in. absolute. What is the pressure of this air when the volume becomes 60 cu. ft. and the temperature 60° F.?

10. How many pounds of air are required for the conditions in problem 9?

11. A tank contains 200 cu. ft. of air at a temperature of 60° F. and under a pressure of 200 lbs. per sq. in. absolute.

(a) What weight of air is present?

(b) How many cubic feet will this air occupy at 14.7 lbs. per sq. in. absolute and at a temperature of 100° F.?

12. The volume of a quantity of air at 70° F. and at a pressure of 14.2 lbs. per sq. in. absolute is 20 cu. ft. What is the temperature of this air when the volume becomes 5 cu. ft. and the pressure 80 lbs. per sq. in. absolute?

13. If the specific heat of carbon dioxide under constant pressure C_p is 0.2012 and the value of R is 35.10, find the value of the specific heat under constant volume C_v.

14. How many B.t.u. are required to double the volume of one pound of air at constant pressure from 50° F.

15. A tank filled with 200 cu. ft. of air at 15 lbs. per sq. in. absolute and 60° F. is heated to 150° F.

(a) What will be the resulting air pressure in the tank?

(b) How many B.t.u. will be required to heat the air?

16. A tank contains 200 cu. ft. of air at 60° F. and 40 lbs. per sq. in. absolute. If 500 B.t.u. of heat are added to it, what will be the resulting pressure and temperature?

CHAPTER III

EXPANSION AND COMPRESSION OF GASES

The equation of the perfect gas in the form $PV = wRT$ for the expansion or compression of gases has three related variables, (1) **pressure,** (2) **volume** and (3) **temperature.** For a given weight of gas with any two of these variables known the third is fixed. As regards the analysis of the action of heat engines, the pressure and volume relations are most important, and graphical diagrams, called pressure-volume or P-V diagrams, are frequently needed to assist in the analysis. The indicator diagram is a pressure-volume diagram drawn autographically by the mechanism of an engine indicator. A pressure-volume diagram is shown in **Fig. 4,** in which the vertical scale of coördinates represents **pressures** and the horizontal, **volumes.** Assume that the pressure and volume of a pound of a gas are given by the coördinates P and V_1, which are plotted in the middle of the diagram. It will be assumed further that the **pressure remains constant** in the changes to be indicated. Now if the gas is **expanded** until its volume becomes V_2, then its condition as regards pressure and volume would be represented by PV_2. If, on the other hand, the gas had been **compressed** while a constant pressure was maintained, its final condition would be represented by the point PV_3 to the left of PV_1. Similarly, any line whether straight or curved extending from the initial condition of the gas at PV_1 will represent an expansion when drawn in the direction away

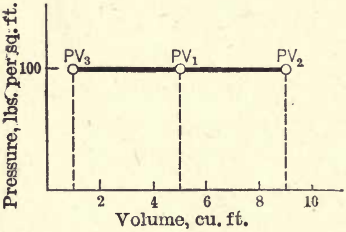

FIG. 4. — Diagram of Expansion and Compression at Constant Pressure.

from the zero of volumes and will represent a compression when tending toward the same zero.

It has been shown (page 20) that areas on such diagrams represent the product of pressure and volume, and, therefore, work or energy. Thus in **Fig. 4** the area under the curve PV_3 to PV_2 represents on the scales given 100 (pounds per square foot) × (9 − 1) cubic feet or 800 foot-pounds irrespective of whether it is an expansion or a compression from the initial condition.

Most of the lines to be studied in heat engine diagrams are either straight or else they can be exactly or approximately represented by an equation in the form

$$PV^n = \text{a constant,} \tag{35}$$

where the index n, as experimentally determined, has varying numerical values, but is constant for any one curve. When the lines of the diagram are straight the areas of simple rectangles and triangles need only be calculated to find the work done. The two most common forms of curves to be dealt with in expansions are (1) when there is expansion with addition of heat at such a rate as to maintain the temperature of the gas constant throughout the expansion. Such an expansion is called **isothermal.** The other important kind of expansion (2) occurs when work is done by the gas without the addition or abstraction of heat. To do this work some of the internal heat energy contained in the gas must be transformed in proportion to the amount of work done. Such an expansion is called **adiabatic.**

The following problems show the application of the foregoing principles to various types of expansions:

1. Expansion at Constant Pressure. One pound of air having an initial temperature of 60° F. is expanded to 100° F. under constant pressure. Find

(a) External work during expansion;

(b) Heat required to produce the expansion.

Solution. The heat added equals the increase in internal energy plus the external work done. In solving for the heat added or required during any expansion it is only necessary to find the external work (which is equal to the area under the expansion curve) and add to it the heat needed to increase the internal energy.

The external work $= W = P_1 (V_2 - V_1).$ (36)
Its equivalent is: $wR (T_2 - T_1).$ (37)
$W = 1 \times 53.3 [(100 + 460) - (60 + 460)] = 2132$ ft.-lbs.

The increase in internal energy:

$$I = wC_v (T_2 - T_1) \qquad (38)$$
$$I = 1 \times 0.169 [(100 + 460) - (60 + 460)]$$
$$= 6.76 \text{ B.t.u.}$$

$$\text{Heat required } (Q) = 6.75 + \frac{2132}{778} = 9.50 \text{ B.t.u.}$$

Another method of computing the heat required to produce the expansion is:

$$Q = wC_p (T_2 - T_1). \qquad (39)$$
$$Q = 1 \times 0.237 [(100 + 460) - (60 + 460)]$$
$$= 9.50 \text{ B.t.u. approximately.}$$

2. Expansion at Constant Volume. One pound of air having an initial temperature of 60° F. is heated at constant volume until the final temperature is 120° F. Find:

(a) External work;
(b) Heat required.

Solution.
Heat added (Q) = increase in internal energy + external work.
External work = 0.

Then

$$\begin{aligned}\text{Heat added} &= \text{increase in internal energy} + 0\\ &= wC_v (T_2 - T_1) + 0\\ &= 1 \times 0.169 [(100 + 460) - (60 + 460)] + 0\\ &= 6.76 \text{ B.t.u.}\end{aligned}$$

3. Expansion and Compression at Constant Temperature (Isothermal). In an isothermal expansion or compression the temperature of the working substance is kept constant throughout the process. The form of the isothermal curve on pressure-volume coördinates depends upon the substance. In the case of perfect gases Boyle's Law (equation 14) applies and we have:

$$PV = C = \text{a constant.} \tag{40}$$

Equation (40) is that of a rectangular hyperbola. It is the special case of the general equation $PV^n = \text{constant}$ (35), in which the index $n = 1$ and is represented by Fig. 5.

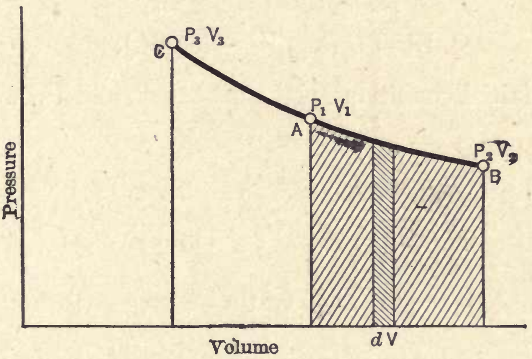

FIG. 5. — Work done during Isothermal Expansion and Compression.

The external work performed is shown graphically by the shaded area under the curve between **A** and **B** (**Fig. 5**). The two vertical lines close together in the figure are the limits of a narrow closely shaded area and indicate an infinitesimal volume change dV. Work done during this small change of volume is, the

$$dW = P\,dV,$$

and for a finite change of volume of any size as from V_1 to V_2 the work done, W (foot-pounds), is:

$$W = \int_{V_1}^{V_2} P\,dV. \tag{41}$$

For the integration of this form it is necessary to substitute P in terms of V. Assume that P and V are values of pressure and volume for any point on the curve of expansion of a gas of which the equation is

$$PV = C.$$

Then
$$P = \frac{C}{V}.$$

Substituting this value of P in equation (41),

$$W = \int_{V_1}^{V_2} \frac{C}{V} dV = C \int_{V_1}^{V_2} \frac{dV}{V}.$$

$$W = C (\log_e V_2 - \log_e V_1). \tag{42}$$

Since the initial conditions of the gas are P_1 and V_1, we have

$$PV = C = P_1V_1,$$

and substituting this value of C in equation (42), we obtain

$$W = P_1V_1 (\log_e V_2 - \log_e V_1)$$

or
$$W = P_1V_1 \log_e \frac{V_2}{V_1} \text{ (in foot-pounds).} \tag{43}$$

Units of weight do not enter in equations (42) and (43). For a certain weight of gas under the same conditions, since

$$P_1V_1 = wRT \text{ (in foot-pounds) and } \frac{V_2}{V_1} = \frac{P_1}{P_2},$$

then the work for w pounds is:

$$W = wRT \log_e \frac{V_2}{V_1} = wRT \log_e \frac{P_1}{P_2} \text{ (in foot-pounds).} \tag{44}$$

Often the ratio $\frac{V_2}{V_1}$ is called **the ratio of expansion** and is represented by r. Making this substitution we have,

$$W = wRT \log_e r. \tag{45}$$

Equations (42), (43), (44) and (45) refer to an expansion from P_1V_1 to P_2V_2. If, on the other hand, the work done is the result of a **compression** from P_1V_1 to P_3V_3 the curve of compression would be from A to C and the area under it would be its graphical representation. Equations (43), (44) and (45) would represent the work done for compression the same as for expansion, except that the expression would have a **negative** value; that is, work is to be done **upon** the gas to decrease its volume.

The isothermal expansion or compression of a perfect gas causes no change in its stock of internal energy since the temperature T is constant. During such an expansion the gas must **take in an amount of heat just equal to the work it does,** and conversely during an isothermal compression it must **reject an amount of heat just equal to the work spent upon it.** This quantity of heat Q (in B.t.u.) is, from equation (45),

$$Q = \frac{wRT}{778} \log_e \frac{V_2}{V_1}. \tag{46}$$

The following problem shows the application of the foregoing formulas to isothermal expansions:

Air having a pressure of 100 pounds per square inch absolute and a volume of 1 cubic foot expands isothermally to a volume of 4 cubic feet. Find

(a) External work of the expansion;
(b) Heat required to produce the expansion;
(c) Pressure at end of expansion.

Solution. (a) Since the expansion is isothermal,

External work,
$$\begin{aligned} W &= P_1V_1 \log_e{}^* \frac{V_2}{V_1} \\ &= 100 \times 144 \times 1 \times 1.3848 \\ &= 19{,}940 \text{ foot-pounds.} \end{aligned}$$

(b) Since the heat added equals increase in internal energy plus external work, and since the temperature remains constant (requiring therefore no heat to increase the internal energy), the internal energy equals zero and the heat added equals the work done.

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* 2.3 × log base 10 = log base e. Tables of natural logarithms are given in the appendix.

$$\text{Then:}\quad \text{Heat added} = \text{external work} = \frac{19{,}940}{778} = 25.6 \text{ B.t.u.}$$

(c) Since $P_1V_1 = P_2V_2,$

then $100 \times 1 = P_2 \times 4,$

$P_2 = 25$ pounds per square inch absolute.

If a gas expands and does external work without receiving a supply of heat from an external source, it must derive the amount of heat needed to do the work from its own stock of internal energy. This process is then necessarily accompanied by a lowering of **temperature** and the expansion obviously is not isothermal.

4. Adiabatic Expansion and Compression. In the adiabatic mode of expansion or compression the working substance neither receives nor rejects heat as it expands or is compressed. A curve which shows the relation of pressures to volumes in such a process is called an **adiabatic line** (see **Fig. 6**). In any adiabatic process the substance is neither gaining nor losing heat by **conduction, radiation** or internal chemical action. Hence the work which a gas does in such an expansion is all done at the expense of its stock of internal energy, and the work which is done upon a gas in such a compression all goes to increase its internal energy. Ideally adiabatic action could be secured by a gas expanding, or being compressed, in a cylinder which in all parts was a perfect non-conductor of heat. The compression of a gas in a cylinder is approximately adiabatic when the process is very rapidly performed, but when done so slowly that the heat

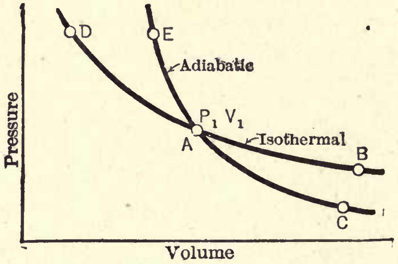

FIG. 6.— Isothermal and Adiabatic Expansion Lines.

has time to be dissipated by conduction the compression is more nearly isothermal. Fig. 6 shows on a pressure-volume diagram the relation between an isothermal and an adiabatic form of expansion or compression from an initial condition P_1V_1 at A to final conditions at B and C for expansions, and at D and E for compressions.

In order to derive the pressure-volume relation for a gas expanding adiabatically, consider the fundamental equation (page 20):

Heat added = increase in internal energy + external work, or:

$$Q = wK_v (T_2 - T_1) + P\,dV \text{ (foot-pounds),} \tag{47}$$

where K_v is the specific heat in foot-pound units, i.e., 778 C_v. In the adiabatic expansion no heat is added to or taken away from the gas by conduction or radiation, and, therefore, the left hand member of the above equation becomes zero. Furthermore, since equation (25) can always be applied to perfect gases, the following simultaneous equations may be written:

$$0 = wK_v\,dT + P\,dV, \tag{48}$$

$$PV = wRT. \tag{49}$$

When P, V and T vary, as they do in adiabatic expansion, equation (49) may be written as follows:

$$P\,dV + V\,dP = wR\,dT, \tag{50}$$

and

$$dT = \frac{P\,dV + V\,dP}{wR}.$$

Substituting the value of dT in (48),

$$P\,dV + wK_v \frac{P\,dV + V\,dP}{wR} = 0. \tag{51}$$

$$RP\,dV + K_vP\,dV + K_vV\,dP = 0.$$

To separate the variables divide by PV:

$$R\frac{dV}{V} + K_v\frac{dV}{V} + K_v\frac{dP}{P} = 0. \tag{52}$$

Collecting terms,

$$(R + K_v)\frac{dV}{V} + K_v\frac{dP}{P} = 0.$$

Integrating,

$$(R + K_v)\log_e V + K_v \log_e P = \text{a constant} = c. \tag{53}$$

$$\frac{R + K_v}{K_v}\log_e V + \log_e P = c.$$

$$\log_e PV^{\frac{R+K_v}{K_v}} = c, \tag{54}$$

since from equation (33),

$$R + K_v = K_p,$$

where K_p and K_v are respectively the specific heats at constant pressure and at constant volume **in foot-pound units,** then equation (54) becomes:

$$\log_e PV^{\frac{K_p}{K_v}} = c.$$

From equation (34),

$$\frac{C_p}{C_v} = \gamma, \quad \text{or} \quad \frac{K_p}{K_v} = \gamma,$$

therefore:

$$PV^{\gamma} = \text{a constant}. \tag{55}$$

Following the method used for obtaining an expression for the work done in isothermal expansion (equation 43), the work done, W (in foot-pounds), for a change of volume from V_1 to V_2,

$$W = \int_{V_2}^{V_1} P\,dV. \tag{56}$$

For purposes of integration, P can be substituted in terms of V as outlined below. In the general expression $PV^n = c$, a constant (see equation 35), where P and V are values of pressure and volume for any point on the curve of expansion of a gas of which the initial condition is given by the symbols P_1 and V_1, we can then write,

$$P = \frac{c}{V^n}. \tag{57}$$

And substituting (57) in (56),

$$W = \int_{V_1}^{V_2} \frac{c}{V^n}\,dV = c\int_{V_1}^{V_2} \frac{dV}{V^n}. \tag{58}$$

$$= c\left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$= c\left[\frac{V_2^{1-n} - V_1^{1-n}}{1-n}\right]. \tag{59}$$

Since $PV^n = c = P_1V_1^n = P_2V_2^n$, we can substitute for c in (59) the values corresponding to the subscripts of V as follows:

$$W = \frac{P_2V_2^nV_2^{1-n} - P_1V_1^nV_1^{1-n}}{1-n},$$

$$W = \frac{P_2V_2 - P_1V_1}{1-n},$$

or

$$W = \frac{P_1V_1 - P_2V_2}{n-1}\text{ (foot-pounds)}. \tag{60}$$

Since $PV = wRT$,

$$W = \frac{wR\,(T_1 - T_2)}{n-1}\text{ (foot-pounds)}. \tag{61}$$

Equations (60) and (61) apply to any gas undergoing expansion or compression according to PV^n = a constant. In the case of an adiabatic expansion of a perfect gas $n = \gamma$ (see equation 55).

Change of Internal Energy During Adiabatic Processes. Since in an adiabatic expansion no heat is conducted to or away from the gas, the work is done at the expense of the internal energy and, therefore, the latter decreases by an amount equivalent to the amount of work performed. This loss in internal energy is readily computed by equation (60) or (61). The result must be divided by 778 in order to be in B.t.u.

During an adiabatic **compression** the reverse occurs, i.e., there is a gain in internal energy and the same formulas apply, the result coming out negative, because work has been done on the gas.

Relation between Volume, Pressure and Temperature in Adiabatic Expansion of a Perfect Gas. Since P, V and T vary during an adiabatic expansion, it will be necessary to develop formulas for obtaining these various quantities. It will be remembered that equation (25), applies to perfect gases at all times. Therefore, in the case of an **adiabatic expansion or compression** we can write the two simultaneous equations:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}. \tag{A}$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}. \tag{B}$$

$\frac{P_1^{\frac{1}{\gamma}}}{P_2} = \left(\frac{v_2}{v_1}\right)^{\gamma}$

By means of these two equations we can find the final conditions of pressure, volume and temperature, having given two initial conditions and one final condition.

For instance, having given V_1, V_2 and T_1, to find T_2, divide (A) by (B), member for member. Then

$$\frac{V_1}{T_1V_1^{\gamma}} = \frac{V_2}{T_2V_2^{\gamma}},$$

$$\frac{T_2}{T_1} = \frac{V_2^{1-\gamma}}{V_1^{1-\gamma}},$$

$$T_2 = T_1\left(\frac{V_2}{V_1}\right)^{1-\gamma},$$

or

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}. \tag{62}$$

In like manner the following formulas can be obtained:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}, \tag{63}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}, \tag{64}$$

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}, \tag{65}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}, \tag{66}$$

$$V_2 = V_1 \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}. \tag{67}$$

It should be noted that the above formulas can be used for any expansion of a perfect gas following PV^n = a constant, provided γ in the formulas is replaced by n.

It is also to be noted that these equations can be used for any system of units so long as the same system of units is employed throughout an equation.

There are many cases of expansions which are neither adiabatic nor isothermal and which are not straight lines on P-V diagrams. It will be observed from the equations in the discussion of the internal work done by an expanding gas and for the change of internal energy, that if in the general equation PV^n = a constant the exponent or index n is less than γ, the work done is greater than the loss in internal energy. In other words for such a case, the expansion lies between an adiabatic and isothermal and the gas must be taking in heat as it expands. On the other hand, if n is greater than γ the work done is less than the loss of internal energy.

Example. Given a quantity of pure air in a cylinder at a temperature of 60° F. ($T_1 = 460 + 60 = 520$ degrees absolute) which is suddenly (adiabatically) compressed to half its original volume. Then $\frac{V_1}{V_2} = \frac{2}{1}$, and taking γ as 1.405, the temperature immediately after compression is completed, T_2, is calculated by equation (62) as follows:

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 520 \left(\frac{2}{1}\right)^{1.405-1} = 520 \times 2^{0.405} = 688° \text{ absolute,}$$

or t_2 in ordinary Fahrenheit is 688 − 460 or 228 degrees.

The work done in adiabatic compression of one **pound** of this air is calculated by equation (61):

$$W = \frac{wR\,(T_1 - T_2)}{\gamma - 1} = \frac{53.3\,(520 - 688)}{1.405 - 1} = \frac{53.3\,(-168)}{0.405} = -22{,}110$$

foot-pounds per **pound** of air compressed. The negative sign means that work has been done **on** the gas or the gas was compressed. If the sign had been positive it would have indicated an expansion.

As the result of this compression the internal energy of the gas has been increased by $\frac{22{,}110}{778}$ B.t.u., but if the cylinder is a conductor of heat, as in practice it always is, the whole of this heat will become dissipated in time by conduction to surrounding air and other bodies, and the internal energy will gradually return to its original value as the temperature of the gas comes back to the initial temperature of 60° F.

PROBLEMS

1. Calculate the heat required to produce a temperature change of 50° F. in three pounds of air at constant pressure.

2. How many foot-pounds of work are done by 2 lbs. of air in expanding to double its volume at a constant temperature of 100° F.?

3. Three pounds of air are to be compressed from a volume of 2 to 1 cu. ft. at a constant temperature of 60° F. How many B.t.u. of heat must be rejected from the air?

4. An air compressor has a cylinder volume of 2 cu. ft. If it takes air at 14.4 lbs. per sq. in. absolute and 70° F. and compresses it isothermally to 100 lbs. per sq. in. absolute, find

    (a) Pounds of air in cylinder at beginning of compression stroke.
    (b) The final volume of the compressed air.
    (c) The foot-pounds of work done upon the gas during compression.
    (d) The B.t.u. absorbed by the air in increasing the internal energy.
    (c) The B.t.u. to be abstracted from the cylinder.

5. A cubic foot of air at a pressure of 150 lbs. per sq. in. gage expands isothermally until its pressure is 50 lbs per sq. in. gage. Calculate the work done during this expansion.

6. Air at 100 lbs per sq. in. absolute and a volume of 2 cu. ft. expands along an $n = 1$ curve to 25 lbs. per sq. in. absolute pressure. Find

(a) Work done by the expansion.
(b) Heat to be supplied.

7. A quantity of air at 100 lbs. per sq. in. absolute pressure has a temperature of 80° F. It expands isothermally to a pressure of 25 lbs. per sq. in. absolute when it has a volume of 4 cu. ft. Find (1) the mass of air present, (2) Work of the expansion in foot-pounds, (3) Heat required in B.t.u.

8. Air at 100 lbs. per sq. in. absolute pressure and 2 cu. ft. expands to 25 lbs. per sq. in. absolute adiabatically. What is the final volume?

9. One cubic foot of air at 60° F. and a pressure of 15 lbs. per sq. in. absolute is compressed without loss or addition of heat to 100 lbs. per sq. in. absolute pressure. Find the final temperature and volume.

10. Two pounds of air are expanded from a temperature of 300° F. to 200° F. adiabatically. How many foot-pounds of work are developed?

11. A quantity of air having a volume of 1 cu. ft. at 60° F. under a pressure of 100 lbs. per sq. in. absolute is expanded to 5 cu. ft. adiabatically. Find the pounds of air present, the final temperature of the air and the work done during this expansion.

12. Data the same as in Problem 4 but the compression is to be adiabatic. Find

(a) The final volume of the compressed air.
(b) The final temperature of the compressed air.
(c) The foot-pounds of work required to compress this air.
(d) The B.t.u. absorbed by the air in increasing the internal energy.
(e) The B.t.u. to be abstracted from the gas.

13. A pound of air at 32° F. and atmospheric pressure is compressed to 4 atmospheres (absolute). What will be the final volume and the work of compression if the compression is (a) isothermal, (b) adiabatic?

14. Plot the curve $PV^n = C$, when $n = 1.35$. Initial pressure is 460 lbs. per sq. in. gage, initial volume 0.5 cu. ft., and final volume 8 cu. ft.

15. Prove that the work of an adiabatic expansion can be expressed by the formula:

$$W = \frac{P_1V_1}{\gamma - 1}\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right].$$

CHAPTER IV

CYCLES OF HEAT ENGINES USING GAS

The Heat Engine Cycle. In the heat engine, the working substance or heat medium undergoes changes in its physical properties converting heat into mechanical work. The series of such changes by the repetition of which the conversion of heat into work takes place forms the heat engine cycle.* The heat engine cycle usually consists of four events which are: heating, cooling, expansion, and compression.

1. THE CARNOT CYCLE

Very important conclusions regarding theoretically perfect heat engines are to be drawn from the consideration of the action of an ideal engine in which the working substance is a perfect gas which is made to go through a cycle of changes involving both isothermal and adiabatic expansions and compressions. This ideal cycle of operations was invented and first explained in 1824 by Carnot, a French engineer. This cycle gave the first theoretical basis for comparing heat engines with an ideally perfect engine. The ideal Carnot cycle requires an engine, illustrated in **Fig. 7,** which consists of the following parts:

(1) A piston and cylinder, as shown in **Fig. 7,** composed of perfectly non-conducting material, except the **cylinder-head** (left-hand end of the cylinder) which is a good conductor of heat. The space in the cylinder between the piston and the cylinder head is occupied by the working substance, which can be assumed to be a perfect gas.

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* A thermodynamic machine performing a cycle in which heat is changed into work is called a **heat** engine, and one performing a cycle in which heat is transferred from a medium at a low temperature to one at a higher temperature is **called a refrigerating machine.**

(2) A **hot body H** of unlimited heat capacity, always kept at a temperature T_1.

(3) A perfectly **non-conducting cover N.**

(4) A **refrigerating or cold body R** of unlimited heat receiving capacity, which is kept at a constant temperature T_2 (lower than T_1).

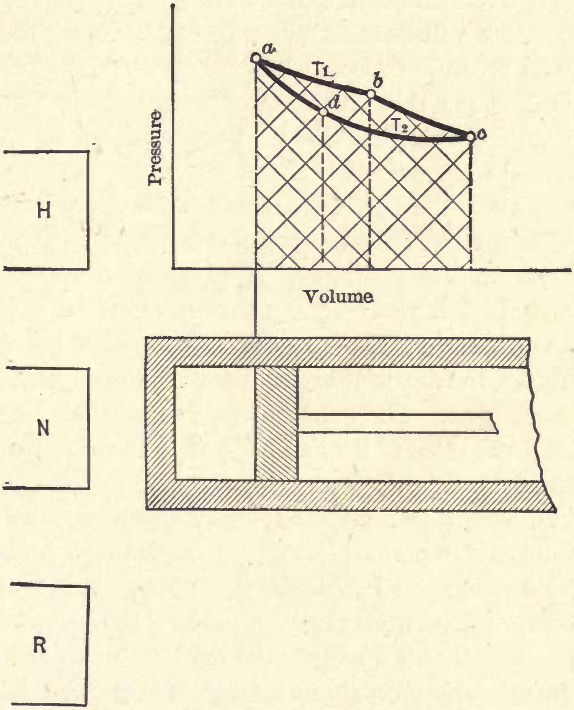

Fig. 7. — Apparatus and Diagram Illustrating a Reversible (Carnot) Cycle.

It is arranged that **H, N** or **R** can be applied, as required, to the cylinder head. Assume that there is a **charge** of one pound of gas in the cylinder between the piston and the cylinder head, which at the beginning of the cycle, with the piston in the position shown, is at the temperature T_1, has a volume V_a, and has a pressure P_a. The subscripts attached to the letters V and P refer to points on the pressure-volume diagram shown in Fig. 7.

This diagram shows, by curves connecting the points **a, b, c** and **d,** the four steps in the cycle.

The operation of this cycle will be described in four parts as follows:

(1) Apply the hot body or heater **H** to the cylinder head at the left-hand side of the figure. The addition of heat to the gas will cause it to expand **isothermally** along the curve **ab,** because the temperature will be maintained constant during the process at T_1. The pressure drops slightly to P_b when the volume becomes V_b. During this expansion external work has been done in advancing the piston and the heat equivalent of this work has been obtained from the hot body **H.**

(2) Take away the hot body **H** and at the same time attach to the cylinder head the **non-conducting cover N.** During this time the piston has continued to advance toward the right, doing work without receiving any heat from an external source, so that the expansion of the gas in this step has been done at the expense of the stock of internal energy in the gas along the adiabatic curve **bc.** The temperature has continued to drop in proportion to the loss of heat to the value T_2. Pressure is then P_c and the volume is V_c.

(3) Take away the non-conductor **N** and apply the refrigerator **R.** Then force the piston back into the cylinder. The gas will be compressed **isothermally** at the temperature T_2. In this compression, work is being done **on** the gas, and heat is developed, but all of it goes into the refrigerator **R,** in which the temperature is always maintained constant at T_2. This compression is continued up to a point **d** in the diagram, so selected that a further compression (adiabatic) in the next (fourth) stage will cause the volume, pressure and temperature to reach their initial values as at the beginning of the cycle.*

(4) Take away the refrigerator **R** and apply the non-conducting cover **N.** Then continue the compression of the gas without

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* Briefly the **third** stage of the cycle must be stopped when a point **d** is reached, so located that an adiabatic curve (PV^{γ} = constant) drawn from it will pass through the "initial" point **a.**

the addition of any heat. It will be the **adiabatic** curve **da.** The pressure and the temperature will rise and, if the point **d** has been properly selected, when the pressure has been brought back to its initial value P_a the temperature will also have risen to its initial value T_1. The cycle is thus finished and the gas is ready for a repetition of the same series of processes comprising the cycle.

To define the Carnot cycle completely we must determine how to locate algebraically the proper place to stop the third step (the location of **d**). During the second step (**adiabatic expansion** from **b** to **c**) by applying equation (62), the following temperature and volume relations exist:

$$\frac{T_1}{T_2} = \left[\frac{V_c}{V_b}\right]^{\gamma-1};$$

also for the **adiabatic compression** the **fourth** step can be similary stated,

$$\frac{T_1}{T_2} = \left[\frac{V_d}{V_a}\right]^{\gamma-1}.$$

Hence,

$$\left[\frac{V_c}{V_b}\right]^{\gamma-1} = \left[\frac{V_d}{V_a}\right]^{\gamma-1}.$$

Simplifying and transposing, we have

$$\frac{V_b}{V_a} = \frac{V_c}{V_d}. \tag{68}$$

$\frac{V_b}{V_a}$ is the ratio of expansion **r** for the **isothermal** expansion in the first step of the cycle. This has been shown (Equation 68) to be equal to $\frac{V_c}{V_d}$ in the isothermal compression in the third step in order that the adiabatic compression occurring in the fourth step shall complete the cycle.

A summary of the heat changes to and from the working gas (per pound) in the **four steps** of the Carnot cycle is as follows:

(ab). Heat **taken in** from hot body (by equation (45), in foot-pounds) is:

$$RT_1 \log_e \frac{V_b}{V_a}. \tag{69}$$

(bc). No heat taken in or rejected.

(cd). Heat **rejected** to refrigerator (by equation (45), in foot-pounds) is:

$$RT_2 \log_e \frac{V_d}{V_c} = -RT_2 \log_e \frac{V_c}{V_d}.^* \tag{70}$$

(da). No heat taken in or rejected.

Hence, **the net amount of work done, W,** by the gas in this cycle, being the mechanical equivalent (foot-pounds) of the **excess** of heat taken in over that rejected, is the **algebraic sum** of (69) and (70):

$$W = R\left(T_1 \log_e \frac{V_b}{V_a} - T_2 \log_e \frac{V_c}{V_d}\right) = R\,(T_1 - T_2) \log_e \frac{V_b}{V_a}. \tag{71}$$

The thermal or heat efficiency of a cycle is defined as the ratio of

$$\frac{\text{Heat equivalent of work done}}{\text{Heat taken in}}.$$

The heat equivalent of **work done** is, by equation (71),

$$R(T_1 - T_2) \log_e \frac{V_b}{V_a},$$

and the **heat taken in** is, by equation (69),

$$RT_1 \log_e \frac{V_b}{V_a}.$$

The ratio above representing the efficiency E is:

$$E = \frac{R\,(T_1 - T_2) \log_e \dfrac{V_b}{V_a}}{RT_1 \log_e \dfrac{V_b}{V_a}} = \frac{T_1 - T_2}{T_1}. \tag{72}$$

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* $\text{Log}\,\frac{V_d}{V_c} = -\log \frac{V_c}{V_d}.$

The efficiency of the Carnot cycle as expressed by equation (72) is the maximum possible theoretical efficiency which may be obtained with any heat engine working between the temperature limits T_1 and T_2. This equation can be used as a standard for the comparison of the efficiencies of actual heat engines working between two temperature limits.

Reversible Cycles. A heat engine which is capable of discharging to the "source of heat" when running in the reverse direction from that of its normal cycle the same quantity of heat that it would take from this source when it is running direct and doing work is said to operate with its cycle reversed, or, in other words, the engine is reversible. A reversible heat engine then is one which, if made to follow its indicator diagram in the reverse direction, will require the same horse power to drive it as a refrigerating machine as the engine will deliver when running direct, assuming that the quantity of heat used is the same in the two cases. An engine following Carnot's cycle is, for example, a reversible engine. The thermodynamic idea of **reversibility** in engines is of very great value because no heat engine can be more efficient than a reversible engine when both work between the same limits of temperature; that is, when both engines take in the same amount of heat at the same higher temperature and reject the same amount at the same lower temperature.

It was first proved conclusively by Carnot that no other heat engine can be more efficient than a reversible engine when both work between the same temperature limits. To illustrate this principle, assume that there are two engines **A** and **B**. Of these let us say **A is reversible and B is not.** In their operation both take heat from a hot body or heater **H** and reject heat to a refrigerator or cold body **R**. Let Q_H be the quantity of heat which the reversible engine **A** takes in from the hot body **H** for each unit of work performed, and let Q_R be the quantity of heat per unit of work which it discharges to the refrigerator **R**.

For the purpose of this discussion, assume that the non-reversible engine **B** is more efficient than the reversible engine **A**.

Under these circumstances it is obvious that the engine **B** will take in less heat than **A** and it will reject correspondingly less heat to **R** per unit of work performed. The heat taken in by the non-reversible engine **B** from the hot body **H** we shall designate then by a quantity less than Q_H, or $Q_H - X$, and the heat rejected by **B** to the refrigerator **R** by $Q_R - X$. Now if the non-reversible engine **B** is working direct (when converting heat into work) and is made to drive the reversible engine **A** according to its reverse cycle (when converting work into heat), then for every unit of work done by the engine **B** in driving the reversible engine **A**, the quantity of heat mentioned above, that is, $Q_H - X$, would be taken from the hot body **H** by the non-reversible engine **B** and, similarly, the quantity of heat represented by Q_H would be returned to the hot body **H** by the reverse action of the cycle of operations performed by **A.** This follows because the engine **A** is reversible and it returns, therefore, to **H,** when operating on the reverse cycle, the same amount of heat as it would take in from **H** when working on its direct cycle. By this arrangement the hot body **H** would be continually **receiving** heat, in the amount represented by X for each unit of work performed. At the same time the non-reversible engine **B** discharges to the refrigerator **R** a quantity of heat represented by $Q_R - X$, while the reversible engine **A** removes from the refrigerator **R** a quantity represented by Q_R. As a result of this last operation the cold body will be **losing** continually per unit of work performed a quantity of heat equal to X. The combined performances of the two engines, one working direct as a normal heat engine and the other, according to its reverse cycle, as a compressor or what might be called a "heat pump," gives a constant removal of heat from the refrigerator **R** to the hot body **H,** and as a result a degree of **infinite coldness** must be finally produced in the refrigerator.

If we assume that there is no mechanical friction, this combined machine, consisting of a normal heat engine and compressor, will require no power from outside the system. For this reason the assumption that the non-reversible engine **B** can be more efficient than the reversible engine **A** has brought us to a result which is impossible from the standpoint of experience as embodied in the statement of the "Second Law of Thermodynamics"; that is, it is impossible to have a self-acting engine capable of transferring heat, infinite in quantity, from a cold body to a hot body. We should, therefore, conclude that no non-reversible engine, as **B** for example, can be more efficient than a reversible engine **A** when both engines operate between the same temperature limits. More briefly, when the source of heat and the cold receiver are the same for both a reversible heat engine and any other engine, then the reversible engine must have a higher possible efficiency; and if both engines are reversible it follows that neither can be more efficient than the other.

A reversible engine is perfect from the viewpoint of efficiency; that is, its efficiency is the best obtainable. No other engine than a reversible engine which takes in and discharges heat at identical temperatures will transform into work a greater part of the heat which it takes in. Finally, it should be stated as regards this efficiency that the nature of the substance being expanded or compressed has absolutely no relation to the thermal efficiency as outlined above.

If an engine operating on Carnot's cycle is reversed in its action, so that the same indicator diagram shown in **Fig. 7** would be traced in the opposite direction, the reversed cycle, when beginning as before at point **a** with a perfect gas at the temperature T_1, will consist of the following stages:

(1) When the non-conductor **N** is applied and the piston is advanced toward the right by the source of power performing the reversed cycle, the gas will expand, tracing the adiabatic curve **ad**, with constant lowering of temperature which at the point **d** will be T_2.

(2) When the non-conductor **N** is now removed, the refrigerator **R** is applied, and the piston continues on its outward stroke. The gas will expand isothermally at the constant temperature T_2, tracing the curve **dc**. During this stage the gas is taking heat from the refrigerator **R**.

(3) When the refrigerator **R** is removed and the non-conductor **N** is again applied, which will be on the back stroke of the engine, the gas will be compressed, and on the indicator diagram another adiabatic curve **cb** will be traced. At the point **b** the temperature will be obviously T_1.

(4) When the non-conductor **N** is removed and the hot body **H** is again applied, with the compression continuing along the isothermal curve **ba,** heat will be discharged to the hot body **H,** while the temperature is maintained constant at T_1.

The cycle has now been traced in a reverse direction from the beginning back to the starting point at **a,** and is now complete. During this process an amount of work represented by the area of the indicator diagram, equivalent in foot-pounds to

$$R \log_e \frac{V_b}{V_a} (T_1 - T_2) \text{ (see equation 71)},$$

has been converted into heat. First, heat was **taken from** the refrigerator **R,** represented in amount by

$$RT_2 \log_e \frac{V_c}{V_d},$$

and second, heat was **rejected to** the hot body **H** in the amount

$$RT_1 \log_e \frac{V_a}{V_b} \quad \text{or} \quad -RT_1 \log_e \frac{V_b}{V_a}.$$

As in direct operation of Carnot's cycle no heat is given or lost in the first and third stages outlined above. The algebraic sum of these two quantities, remembering that $\frac{V_b}{V_a} = \frac{V_c}{V_d}$, gives the net amount of work done, W, on the gas, and, therefore, the net amount of heat (foot-pound units) transferred from the cold body **R** to the hot body **H** or,

$$W = RT_2 \log_e \frac{V_b}{V_a} - RT_1 \log_e \frac{V_b}{V_a} = -R \log_e \frac{V_b}{V_a} (T_1 - T_2). \quad (73)$$

Since the result is the same as given by equation (72), although opposite in sign on account of being work of compression, it will be observed that in the reverse cycle the same amount of heat is given to the hot body **H** as was taken from it in the direct operation of the same cycle, and that the same amount of heat is now taken from the refrigerator **R** as was in the other case given to it.

2. HOT-AIR ENGINE CYCLES

The hot-air engine is a non-explosive type of external combustion heat engine, the working substance being atmospheric air which undergoes no change in its physical state. This type of prime-mover was invented about 100 years ago. It is little used on account of the difficulty in transmitting heat through the metallic walls to the dry gas air. Then the hot-air engine is very bulky in proportion to its power. There are also difficulties in carrying out practically the theoretical cycles of operation, on account of the rapid deterioration of the heat-conducting surfaces. The mechanical efficiency of the hot-air engine is also low.

The advantages of the hot-air engine are ease of operation and safety. Also its speed being low, on account of the rate of heat transmission, it is well suited for the driving of small pumps and for other domestic uses where small powers are required and also where the fuel-economy is a matter of minor importance.

Hot-air engine cycles are divided into two groups: —

Group I. External-combustion hot-air engines with a closed cycle and constant-volume temperature changes.

Group II. External-combustion hot-air engines with an open cycle and constant-pressure temperature changes.

In the hot-air engines an attempt was made to put in practice a cycle of the ideal Carnot type with the addition of a regenerator. The regenerator was a storage-battery of heat, its function being to absorb, store and return heat rapidly, replacing the adiabatic expansion curves of the Carnot cycle by lines of constant volume in group I and by lines of constant pressure in group II. The regenerator consists of a chamber containing strips of metal, coils of wire, or any other heat-absorbing material arranged in such a manner as to present a very large surface to the air passing through it.

The Stirling Engine Cycle. The Stirling engine belongs to group I, the regenerator being so arranged that the pressure drops with the temperature at such a rate as to keep the volume constant. It is an external-combustion engine and its cycle of operation is closed, the same air is used over and over again, any loss by leakage being supplied by a small force-pump. The engine consists, essentially, of a pair of displacer cylinders, a double-acting working cylinder, a regenerator and a refrigerator. In the earlier forms the displacer cylinder had a double wall, the regenerator and refrigerator being placed in the annular space surrounding the displacer cylinder. In the latter forms the regenerator and refrigerator are arranged separately from the displacer cylinder but in direct communication with its top and bottom. A plunger works in the displacer cylinder, this plunger being filled with a non-conducting material such as brick-dust. The engine derives its heat by

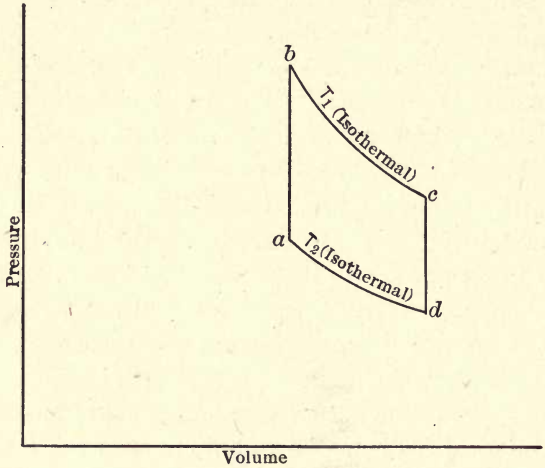

FIG. 8. — Stirling Engine Cycle.

conduction from a furnace which is placed beneath the displacing cylinder.

In the cycle represented by **Fig. 8,** the action of the engine is carried out as follows:

1. The substance, air, having a volume V_a, a pressure P_a and a temperature T_a receives heat at constant volume by passing through the regenerator, and receives also heat from the furnace. As a result of the heat addition, its pressure is increased to P_b, its temperature to T_b, this process being represented by the constant volume line *ab*, **Fig. 8.** During this event the plunger is moving up.

2. The pressure under the working piston increases as a result of the addition of heat and the expansion of the air follows, producing the working stroke. This expansion is represented by *bc* and is an isothermal, the air receiving heat from the furnace plates and the temperature remaining T_1.

3. The plunger descends, displacing the hot air and forcing the same through the regenerator and refrigerator. As a result of this, the temperature drops from T_c to T_d and the pressure is decreased from P_c to P_d, the volume remaining constant as shown by the line *cd*.

4. Due to this cooling effect the working piston descends compressing the air. This compression, represented by *da*, is isothermal, the air being cooled during the process by the regenerator and refrigerator to remain at the temperature T_2.

Referring to **Fig. 8** of the Stirling cycle, the useful work W is in foot-pounds,

$$W = \left(P_b V_b \log_e \frac{V_c}{V_b}\right) - \left(P_d V_d \log_e \frac{V_d}{V_a}\right). \qquad (74)$$

The mean effective pressure is in pounds per square foot,

$$\text{M.E.P.} = \frac{W}{V_d - V_a}. \qquad (75)$$

The horse power developed can be represented by

$$\text{H.P.} = \frac{W \times N}{33{,}000}. \qquad (76)$$

where N = revolutions (cycles) per minute.

Since the expansion and compression processes are isothermal, the efficiency of the Stirling cycle is:

$$E = \frac{P_b V_b \log_e \dfrac{V_c}{V_b} - P_d V_d \log_e \dfrac{V_d}{V_a}}{P_b V_b \log_e \dfrac{V_c}{V_b}}. \tag{77}$$

Since $V_a = V_b$ and $V_c = V_d$, also $RT_1 = P_b V_b$ and $RT_2 = P_d V_d$, equation (77) becomes,

$$E = \frac{RT_1 \log_e \dfrac{V_c}{V_b} - RT_2 \log_e \dfrac{V_c}{V_b}}{RT_1 \log_e \dfrac{V_c}{V_b}} = \frac{T_1 - T_2}{T_1}. \tag{78}$$

The Ericsson Engine Cycle. This engine belongs to group II, the regenerator changing the temperature at constant pressure. This engine consists of five parts: a compressing pump, a receiver, a regenerator, a refrigerator and a working cylinder. The ideal cycle for this engine is represented by **Fig. 9**, the order of events being as follows:

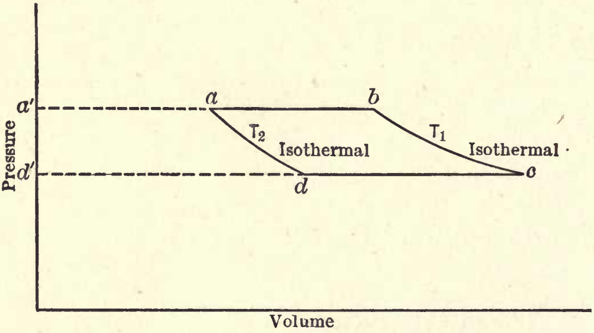

FIG. 9. — The Ericsson Engine Cycle.

Atmospheric air is drawn into the compressing pump as shown by *d'd*. This air is compressed isothermally to *a*, during the return stroke of the pump; it is then forced into a receiver as shown by *aa'*. Thus *d'daa'* represents the pump cycle.

As the working cylinder begins its forward or up stroke, the compressed air is admitted into the working cylinder at constant pressure. As this admission from a to b is through the regenerator, the entering air takes up heat increasing in temperature from T_2 to T_1. As soon as the supply of compressed air is cut off, the air expands isothermally along bc to atmospheric pressure, while heat is being supplied by a furnace at the bottom of the working cylinder. During the return stroke of the working piston, the air is discharged at constant pressure through the regenerator, giving up its heat and is cooled to the temperature T_2. In **Fig. 9**, $a'bcd'$ is the cycle of the working cylinder, the net work being represented by $abcd$.

The Ericsson cycle has the same efficiency as the Stirling cycle, the regenerator process being carried out at constant pressure instead of at constant volume.

3. INTERNAL-COMBUSTION ENGINE CYCLES

The internal-combustion engine utilizes as its working substance a mixture of air and gas or air and petroleum vapor. Combustion of the mixture takes place inside the engine cylinder, or in a communicating vessel, and the heat generated is converted into work. As the specific heat and the specific volume of the mixture do not differ much from that of air, the cyclical analysis and the theory of the internal-combustion engine are developed on the assumption that the working substance is air. The internal-combustion engine cycles can be divided into the following groups:

Group I. Engines without compression.

Group II. Compression engines.

Group I. Internal-Combustion Engines Without Compression

The Lenoir Engine. An engine of this group, invented by Pierre Lenoir in 1860, was the first successful practical gas engine. The action of this type of engine is as follows:

As the piston leaves the dead center it draws into the cylinder a charge of gas and air which is proportioned at the admission valve to form an explosive mixture. Admission is cut off somewhat before half-stroke and this is followed by the ignition of the mixture by means of an electric spark. The explosion produces a rapid rise in pressure above atmospheric, thus forcing the piston to the end of the stroke. The exhaust valve opens near the end of the stroke and the burnt products are expelled during the return stroke of the piston. A fly-wheel carries the piston over during the exhaust stroke as well as also during the suction stroke. The above series of operations takes place at each end of the piston, producing two impulses for each revolution.

Fig. 10 represents the cycle of operations of the Lenoir engine on P-V coördinates. The drawing in of the mixture of gas and air at atmospheric pressure is represented by ab. Ignition takes place at b and is followed by the constant volume combustion line bc. The working stroke, an adiabatic expansion, takes place to atmospheric pressure, as shown by cd, and the products of combustion are rejected during the return stroke of the piston da.

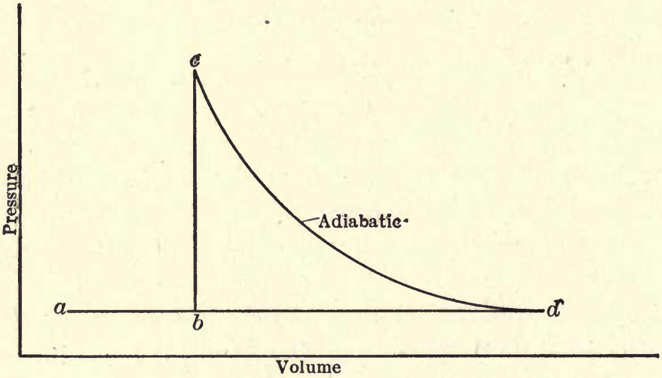

FIG. 10. — Ideal Lenoir Cycle.

Calling P, V, T the absolute pressure in pounds per square foot, the specific volume in cubic feet per pound of mixture, and the absolute temperature in degrees Fahrenheit respectively; also using subscripts b, c, d to designate the points at the corners bcd of **Fig. 10**, the following expressions will be obtained:

If the heat developed by the complete combustion of the mixture be designated by Q_1, then

$$Q_1 = wC_v(T_c - T_b). \tag{79}$$

Since the combustion of the mixture takes place at constant volume,

$$V_c = V_b$$

$$T_c = T_b + \frac{Q_1}{C_v}. \tag{80}$$

For unit weight,

$$P_c = P_b \frac{T_c}{T_b}. \tag{81}$$

The expansion *cd* being adiabatic ($n = \gamma$),

$$V_d = V_c \left(\frac{P_c}{P_d}\right)^{\frac{1}{n}}. \tag{82}$$

$$T_d = T_c \left(\frac{V_c}{V_d}\right)^{n-1}. \tag{83}$$

If Q_2 is the heat rejected by the engine after performing its cycle of operations,

$$Q_2 = wC_p(T_d - T_b). \tag{84}$$

The useful work available is:

$$W = J(Q_1 - Q_2). \tag{85}$$

The mean effective pressure is the average unbalanced pressure on the piston of the engine in pounds per unit area, and using symbols and units as on page 51,

$$\text{M.E.P.} = \frac{W}{V_d - V_b}. \tag{86}$$

The horse power developed would be found by equation

$$\text{H.P.} = \frac{W \times N}{33{,}000}, \tag{87}$$

where N designates the number of explosions per minute.

The cycle efficiency for the conditions of the problem is:

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \gamma \frac{T_d - T_b}{T_c - T_b}. \tag{88}$$

Group II. Compression Engines

The first compression engine was patented as early as 1799 by Philip Lebon. The thermodynamic advantages of compression before ignition will be evident from the demonstrations which follow. A mechanical advantage due to compression is the reduced shock due to the explosion and the resulting improved balance of parts.

1. **The Otto Cycle.** The Otto cycle, embodying the principles first proposed by Beau de Rochas in 1862, resulted in the most successful internal-combustion engine of to-day. This cycle was adopted by Dr. Otto in 1876, the operations being carried out in four consecutive equal strokes of the piston, requiring two complete revolutions of the engine crank-shaft. The ideal diagram for the Otto cycle takes the form shown by **Fig. 11**, the operation being as follows:

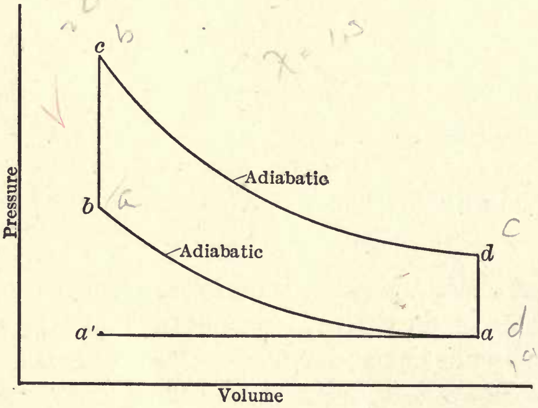

FIG. 11. — Otto Cycle.

A mixture of gas and air is drawn in during the complete forward stroke of the piston, as shown by *a′a*. The return of the piston compresses the mixture along the adiabatic curve *ab*. Explosion of the compressed charge takes place at *b*, with the consequent combustion at constant volume to *c*. *cd* is the adiabatic expansion producing the second forward stroke. The exhaust valve opens at d, cooling the gases to the exhaust pressure a, and rejecting them to the atmosphere.

The heat added during the combustion from b to c is

$$Q_1 = wC_v(T_c - T_b). \tag{89}$$

The heat rejected from d to a is

$$Q_2 = wC_v(T_d - T_a). \tag{90}$$

The cycle efficiency is

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{wC_v(T_c - T_b) - wC_v(T_d - T_a)}{wC_v(T_c - T_b)} = 1 - \frac{T_d - T_a}{T_c - T_b}. \tag{91}$$

Since the expansion and compression are adiabatic the following relations will hold:

$$T_aV_a^{n-1} = T_bV_b^{n-1}, \quad \text{and} \quad T_cV_c^{n-1} = T_dV_d^{n-1}.$$

Since $V_c = V_b$ and $V_a = V_d$,

$$\frac{T_d}{T_a} = \frac{T_c}{T_b}; \quad \text{also} \quad \frac{T_d - T_a}{T_c - T_b} = \frac{T_a}{T_b}.$$

Since $\frac{T_a}{T_b} = \left(\frac{V_b}{V_a}\right)^{n-1} = \left(\frac{P_a}{P_b}\right)^{\frac{n-1}{n}}$

$$E = 1 - \frac{T_a}{T_b} = 1 - \left(\frac{V_b}{V_a}\right)^{n-1} = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{n-1}{n}}. \tag{92}$$

Equation (92) shows that the efficiency of the Otto cycle depends on the amount of compression before explosion.

2. The Brayton Cycle. This cycle is often called the Joule cycle after its inventor, or the Brayton cycle after George B. Brayton, who in 1872 designed an engine with gradual constant pressure combustion. In this engine a mixture of gas and air is first compressed in a separate pump and forced into a receiver. On the way from the receiver to the engine cylinder the mixture is ignited by a gas jet which burns steadily without sudden explosion, producing temperature and volume changes at constant pressure. After expansion the piston drives out the products of combustion at atmospheric pressure.

The cycle of operations for the Brayton engine is represented by **Fig. 12.** $a'a$ represents the supply of the combustible mixture to the pump where it is compressed adiabatically to b and forced into a receiver. bc represents the burning of the compressed mixture at constant pressure. As the mixture enters the working cylinder, it expands adiabatically along cd. This is followed by the rejection of the burnt gases along the atmospheric line da.

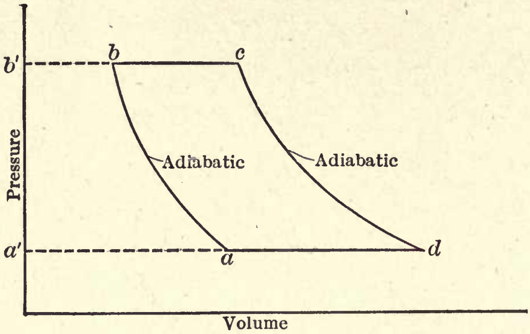

FIG. 12. — Brayton Cycle.

The heat added during the constant pressure combustion from b to c is

$$Q_1 = wC_p\,(T_c - T_b). \tag{93}$$

The heat rejected from d to a is

$$Q_2 = wC_p\,(T_d - T_a). \tag{94}$$

The cycle efficiency is

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{wC_p\,(T_c - T_b) - wC_p\,(T_d - T_a)}{wC_p\,(T_c - T_b)} = 1 - \frac{T_d - T_a}{T_c - T_b}. \tag{95}$$

Since the expansion and compression phases of the cycle are adiabatic and $P_c = P_b$; $P_d = P_a$, therefore,

$$\frac{T_d}{T_a} = \frac{T_c}{T_b}$$

and

$$\frac{T_d - T_a}{T_c - T_b} = \frac{T_a}{T_b} = 1 - \left(\frac{V_b}{V_a}\right)^{n-1} = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{n-1}{n}}. \tag{96}$$

Equation (96) is the same as equation (92) and shows that the cycle efficiencies of the Otto and Brayton cycles are the same, under the same initial conditions, and depend on the amount of compression of the charge before explosion.

3. **The Diesel Cycle.** This cycle is carried out in four strokes of equal length, as in the case of the Otto cycle. Atmospheric air is drawn in during the complete forward stroke of the piston as shown by *Ia* (**Fig. 13**). The return of the piston compresses the air adiabatically to *b*, a pressure of about 500 pounds per square inch. At the end of the compression stroke a charge of the liquid fuel is injected in a finely divided form by an auxiliary pump or compressor and burns nearly at constant pressure when it comes in contact with the highly compressed air. The supply and combustion of the fuel, as represented by the combustion line *bc*, is cut off at one-tenth to one-sixth the working stroke, depending upon the load. Expansion takes place during the balance of the stroke as shown by the adiabatic expansion line *cd*. Release occurs at *d* with the consequent drop in pressure, and the burnt gases are rejected during the fourth stroke of the cycle.

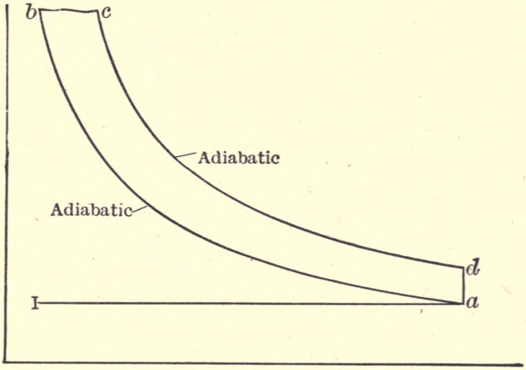

FIG. 13. — Diesel Engine Cycle.

The heat added (Q_1) during the combustion of the fuel is:

$$Q_1 = wC_p (T_c - T_b). \tag{97}$$

The heat rejected from d to a is

$$Q_2 = wC_v (T_d - T_a). \tag{98}$$

The cycle efficiency is then,

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{wC_p(T_c - T_b) - wC_v(T_d - T_a)}{wC_p(T_c - T_b)}$$

$$= 1 - \frac{C_v}{C_p}\left(\frac{T_d - T_a}{T_c - T_b}\right) = 1 - \frac{1}{\gamma}\left(\frac{T_d - T_a}{T_c - T_b}\right). \qquad (99)$$

PROBLEMS

1. A Carnot engine containing 10 lbs. of air has at the beginning of the expansion stroke a volume of 10 cu. ft. and a pressure of 200 lbs. per sq. in. absolute. The exhaust temperature is 0° F. If 10 B.t.u. of heat are added to the cycle, find

(a) Efficiency of the cycle.
(b) Work of the cycle.

2. A Carnot cycle has at the beginning of the expansion stroke a pressure of 75 lbs. per sq. in. absolute, a volume of 2 cu. ft. and a temperature of 200° F. The volume at the end of the isothermal expansion is 4 cu. ft. The exhaust temperature is 30° F. Find

(a) Heat added to cycle.
(b) Efficiency of cycle.
(c) Work of cycle.

3. A cycle made up of two isothermal and two adiabatic curves has a pressure of 100 lbs. per sq. in. absolute and a volume of 1 cu. ft. at the beginning of the isothermal expansion. At the end of the adiabatic expansion the pressure is 10 lbs. per sq. in. absolute and the volume is 8 cu. ft. Find

(a) Efficiency of cycle.
(b) Heat added to cycle.
(c) Net work of cycle.

4. In a Carnot cycle the heat is added at a temperature of 400° F. and rejected at 70° F. The working substance is 1 lb. of air which has a volume of 2 cu. ft. at the beginning and a volume of 4 cu. ft. at the end of the isothermal expansion. Find

(a) Volume at end of isothermal compression.
(b) Heat added to the cycle.
(c) Heat rejected from cycle.
(d) Net work of the cycle.

5. Air at a pressure of 100 lbs. per sq. in. absolute, having a volume of 1 cu. ft. and a temperature of 200° F., passes through the following operations:

1st. Heat is supplied to the gas while expansion takes place under constant pressure until the volume equals 2 cu. ft.

2d. It then expands adiabatically to 15 lbs. per sq. in. absolute pressure.

3d. Heat is then rejected while compression takes place at constant pressure.

4th. The gas is then compressed adiabatically to its original volume of 1 cu. ft.

Find (a) Pounds of air used.
(b) Temperature at end of constant pressure expansion.
(c) Heat added to the cycle.
(d) Net work of cycle.
(e) Efficiency of cycle.

6. A Stirling hot-air engine having a piston 16 inches in diameter and a stroke of 4 ft. makes 28 r.p.m. The upper temperature is 650° F., while the lower temperature is 150° F. Assuming that the volume of the working cylinder is one-half that of the displacer cylinder, calculate the pressures and volumes at each point of the cycle (Fig. 8); also calculate the mean effective pressure, the horse power developed, and the cycle efficiency.

7. Plot the Stirling engine cycle from the results of problem 6.

8. Compare the cycle efficiencies of internal-combustion engines working on the Otto cycle using the fuels and compression pressures as indicated below:

Gasoline, 75; producer gas, 150; and blast furnace gas, 200 lbs. per sq. in. gage pressure.

9. Calculate the theoretical pressures, volumes and temperatures at each point of an Otto cycle, as well as the mean effective pressure, horse power, and cycle efficiency for the following conditions of Fig. 11. Assume that the pressure after compression is 180 lbs. per sq. in. gage, that 80 B.t.u. are added during combustion and that n in $PV^n = 1.4$:

$$P_a = 14.7 \text{ lbs. per sq. in.}, \ V_a = 13.5 \text{ cu. ft.},$$
$$T_a = 70 + 459.5 = 529.5° \text{ F. absolute.}$$

10. Prove that cycle efficiency of the Diesel engine depends not only upon the compression pressure before ignition but also upon the point of cut-off C in Fig. 13.

11. In what respects do the actual cycles of internal-combustion engines differ from the theoretical Otto and Diesel cycles?

12. The demonstrations in the text for the various gas cycles were based on the assumption that the specific heats of gases are constant. Since the specific heats of gases vary at high temperatures, show the effect of this variability on the cyclic analysis. Refer to Lucke's *Engineering Thermodynamics*, Levin's *Modern Gas Engines and the Gas Producer*, and Clerk's *Gas, Petrol and Oil Engines*, Vol. II.

CHAPTER V

PROPERTIES OF VAPORS

Saturated and Superheated Vapors. As was explained in the chapter on the properties of perfect gases, a vapor can be liquified by pressure or temperature changes alone. At every pressure there is a fixed point, called the point of vaporization, at which a liquid can be changed into a vapor by the addition of heat. A vapor near the point of vaporization is called a saturated vapor and has a definite vaporization temperature for any given pressure. When a vapor is heated so that its temperature is greater than the vaporization temperature corresponding to a given pressure, it is said to be a superheated vapor. Superheated vapors only when far removed from the vaporization temperature approach nearly the laws of perfect gases.

Theory of Vaporization. When heat is added to a liquid its temperature will rise with a slight volume increase until the point of vaporization is reached. This is always a definite point for any given pressure and depends on the character of the liquid. Thus the point of vaporization of water at the atmospheric pressure of 14.7 pounds per square inch is 212° F., while that at a pressure of 150 pounds absolute is 358° F. On the other hand, the vaporization temperature of ammonia vapor at a pressure of 150 pounds absolute is 79° F. The increased temperature of vaporization with the pressure increase is due to the fact that the molecules being crowded, the velocity per molecule, or temperature, must be great enough to overcome not only molecular attraction but also external pressure before vaporization can take place.

When the point of vaporization is reached, any further addition of heat will not cause any temperature increase, but internal work accompanied by vaporization will be produced as well as an enormous increase in volume. The heat required to entirely vaporize a unit weight of a substance at a given pressure is termed the latent heat of vaporization.

The volume developed when one unit weight of the liquid has been completely evaporated is called the specific volume of the dry vapor, this volume depending on the pressure.

When vaporization is incomplete the vapor is termed a wet saturated vapor, which means that the vapor is in contact with the liquid from which it is formed. The percentage dryness in the vapor is called its quality. Thus the quality of steam is 0.97 when one pound of it consists of 97 per cent steam and 3 per cent water. While the volume of a vapor increases with the quality, the temperature is constant between the point of vaporization and the point of superheat for any given pressure. Thus the temperature of steam vapor corresponding to 150 pounds absolute is 358° F., no matter whether the quality is 0.10, 0.75 or 1.00.

Further addition of heat after complete vaporization will cause temperature as well as volume changes, the substance being in the superheated vapor condition.

Vapor Tables. The exact quantities of heat required to produce the above effects under various conditions, as well as the relations existing between pressure, volume, and temperature of saturated and superheated vapors have been determined exper-

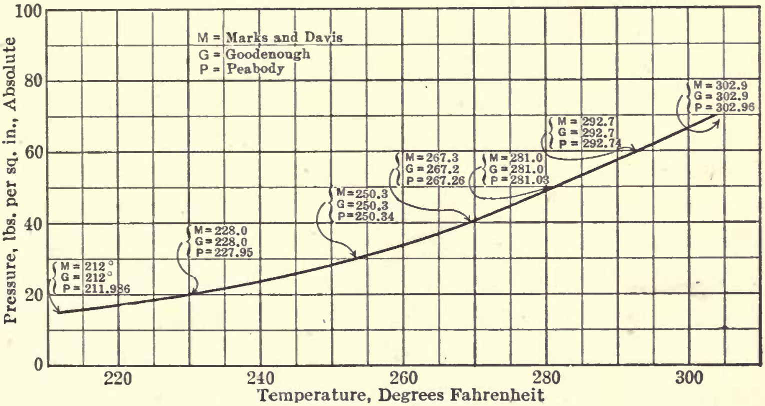

FIG. 14. — Pressure-Temperature Relations for Saturated Steam.

imentally. These experimental results have been given in the form of empirical equations from which vapor tables have been computed. Tables 3 and 4, showing the properties of dry saturated steam and of ammonia, are given in the appendix. This text should be supplemented by the more complete tables of Marks and Davis, Peabody or Goodenough.

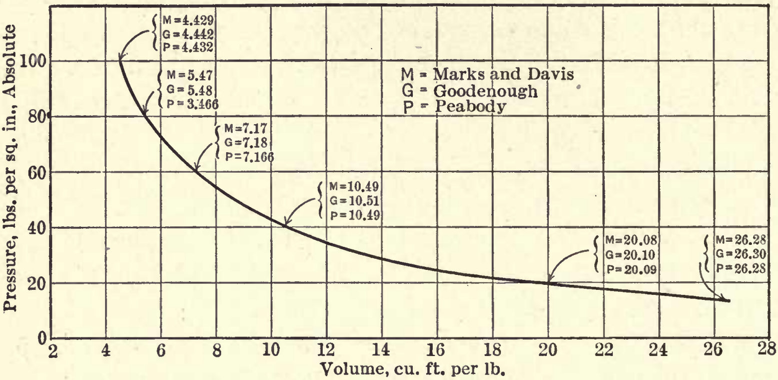

FIG. 15. — Pressure-Volume Relations for Saturated Steam.

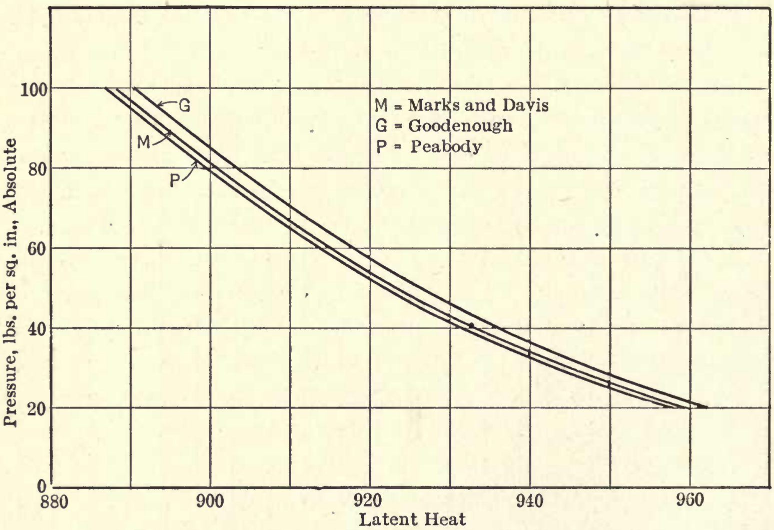

FIG. 16. — Pressure-Latent Heat Relations for Saturated Steam.

In **Figs. 14, 15** and **16** are plotted the several variables, as given in the above-mentioned tables, for various pressures.

In most vapor tables will be found, corresponding to the pressure of the vapor in pounds per square inch absolute (p), the vaporization temperature (t), the heat of the liquid (q, h or i'), the heat of vaporization (r or L), the specific volume (s or v), entropy of water (θ, s or n), entropy of vaporization $\left(\frac{r}{T} \text{ or } \frac{L}{T}\right)$, density pounds per cubic foot $\left(\frac{1}{s} \text{ or } \frac{1}{v}\right)$, internal latent heat ($\rho$, i or I). In some tables will also be found a column for the total heat of vapor ($H = q + r$), external latent heat (APu) and entropy.

Relation between Temperature, Pressure and Volume of Saturated Steam. The important relations of temperature, pressure and volume were first determined in a remarkable series of experiments conducted by a French engineer named Regnault, and it has been on the basis of his data, first published in 1847, that even our most modern steam tables were computed. Later experimenters have found, however, that these data were somewhat in error, especially for values near the dry saturated condition. These errors resulted because it was difficult in the original apparatus to obtain steam entirely free from moisture.

The pressure of saturated steam increases very rapidly as the temperature increases in the upper limits of the temperature scale. It is very interesting to examine a table of the properties of steam to observe how much more rapidly the pressure must be increased in the higher limits for a given range of temperature.

It should be observed that in most tables the pressure is almost invariably given in terms of pounds per **square inch,** while in nearly all our thermodynamic calculations the pressure must be used in pounds per **square foot.**

The specific volume of saturated steam (v or s) is equal to the sum of the volumes of water (σ) and of the increase in volume during vaporization (u), or

$$v = \sigma + u. \qquad (100)$$

Heat in the Liquid (Water) (h or q). The essentials of the process of making steam have been described in a general way. The relation of this process to the amount of heat required will now be explained. If a pound of water which is initially at some temperature t_0 is heated at a constant pressure **P** (pounds per square foot) to the boiling point corresponding to this pressure and then converted into steam, heat will first be absorbed in raising the temperature of the water from t_0 to t, and then in producing vaporization. During the first stage, while the temperature is rising, the amount of heat taken in is approximately $(t - t_0)$ heat units, that is, British thermal units (B.t.u.), because the specific heat of water is approximately unity and practically constant. This number of B.t.u. multiplied by 778 gives the equivalent number of foot-pounds of work. For the purpose of stating in steam tables the amount of heat required for this heating of water, the initial temperature t_0 must be taken at some definite value; for convenience in numerical calculations and also because of long usage, the temperature 32° F. is invariably used as an arbitrary starting point for calculating the amount of heat "taken in." The symbol **h** (or sometimes i' or **q**) is used to designate the heat required to raise one pound of water from 32° F. to the temperature at which it is vaporized into steam. In other words, "the heat of the liquid" (**h**) is the amount of heat in B.t.u. required to raise one pound of water from 32° F. to the boiling point, or:

$$h = \int_{t_0}^{t} C\,dt. \qquad (101)$$

C is the specific heat of water at constant pressure, t_0 is the freezing point of water, t is the temperature to which the water is raised.

As C is very nearly unity, the value of the heat absorbed by water in being raised to the steaming temperature (**h**) in B.t.u. can be expressed, approximately, by the formula

$$h = t - 32 \text{ (in B.t.u.).} \qquad (102)$$

Values of h, taking into consideration the variation in the specific heat of water, will be found in the usual steam tables (Table 3).

During this first stage, before any steaming has occurred, practically all the heat applied is used to increase the stock of internal energy. The amount of external work done by the expansion of water as a liquid is practically negligible.

Latent Heat of Evaporation (L or r). In the second stage of the formation of steam as described, the water at the temperature **t,** corresponding to the pressure, is changed into steam at that temperature. Although there is no rise in temperature, very much heat is nevertheless required to produce this evaporation or vaporization. The heat taken in during this stage is the **latent heat of steam.** In other words, the latent heat of steam may be defined as the amount of heat which is taken in by a pound of water while it is changed into steam at constant pressure, the water having been previously heated up to the temperature at which steam forms. The symbol **L** (also sometimes **r**) is used to designate this latent heat of steam. Its value varies with the particular pressure at which steaming occurs being somewhat smaller in value at high pressures than at low.

The latent heat (L or r) of saturated steam can be approximately calculated by the formula:

$$L = 970.4 - 0.655\,(t - 212) - 0.00045\,(t - 212)^2. \quad (103)$$

External Work of Evaporation (Apu). A part of the heat taken in during the "steaming" process is spent in doing external work. Only a small part of the heat taken in is represented by the external work done in making the steam in the boiler, and the remainder of the latent heat (**L**) goes to increase the internal energy of the steam. The amount of heat that goes into the performing of external work is equal to **P** (the pressure in pounds per **square foot**) times the change of volume (**u**) occurring when the water is changed into steam, or **Pu.**

Example. At the usual temperatures of the working fluid in steam engines the volume of a pound of water σ is about $\frac{1}{60}$ of a cubic foot. The external work done in making one pound of steam having finally a volume of **V** (cubic feet) at a constant pressure **P** (pounds per square foot) may be written in foot-pounds:

$$\text{External work} = Pu = P\left(V - \tfrac{1}{60}\right). \qquad (104)$$

This last equation can be expressed in British thermal units (B.t.u.) by APu where $A = \frac{1}{778}$. It is apparent also from this equation that the external work done in making steam is less at low pressure than at high,* because there is less resistance to overcome, or, in other words, **P** in equation (104) is less. The heat equivalent of the external work is, therefore, a smaller proportion of the heat added at low temperature than at high.

Total Heat of Steam (H). The heat added during the process represented by the first and second stages in the formation of a pound of steam, as already described, is called the total heat of saturated steam or, for short, **total heat of steam,** and is represented by the symbol **H.** Using the symbols already defined, we can write, per pound of steam,

$$\mathbf{H} = \mathbf{h} + \mathbf{L}\ \text{(B.t.u.)}. \qquad (105)$$

In other words, this total heat of steam is the amount of heat required to raise one pound of water from 32° F. to the temperature of vaporization and to vaporize it into dry steam at that temperature under a constant pressure.

Remembering that **h** for water is approximately equal to the temperature less 32 degrees corresponding to the pressure at which the steam is formed (**t**) the total heat of steam is approximately,

$$\mathbf{H} = (\mathbf{t} - 32) + \mathbf{L}. \qquad (106)$$

To illustrate the application of equation (106) calculate the total heat of steam being formed in a boiler at an absolute pressure of 115 pounds per square inch.

---

* Although at the lower pressure the volume of a given weight of steam is greater than at a higher pressure, the change of pressure is relatively so much greater in the process of steam formation that the product of pressure and change of volume, $P(V_2 - V_1)$, which represents the external work done, is less for low pressure steam than for high.

From the steam tables (Table 3) the temperature **t** of the steam at a pressure of 115 pounds per square inch absolute is 338° F., the latent heat of vaporization is 880 B.t.u. per pound and the total heat of the steam is 1189 B.t.u. per pound. To check these values with equation (106), by substituting values of **L** and **t**,

$$H = (338 - 32) + 880 = 1186 \text{ B.t.u. per pound.}$$

When steam is condensed under constant pressure, the process which was called the "second stage" is reversed and the amount of heat equal to the latent heat of evaporation (**L**) is given up during the change that occurs in the transformation from steam to water.

Internal Energy of Evaporation and of Steam. It was explained in a preceding paragraph that when steam is forming not all of the heat added goes into the **internal** or **"intrinsic" energy** of the steam, but that a part of it was spent in performing external work. If, then, the **internal energy** of evaporation is represented by the symbol $\mathbf{I}_L$,

$$\mathbf{I}_L = L - P\frac{(V - \frac{1}{60})}{778}. \qquad (107)$$

This equation represents the increase in internal energy which takes place in the changing of a pound of water at the temperature **t** into steam at the same temperature. In all the formulas dealing with steam the state of water at 32° F. has been adopted as the **arbitrary starting point** from which the taking in of heat was calculated. This same arbitrary starting point is used also in expressing the amount of internal energy in the steam. This is the excess of the heat taken in over the external work done in the process.

The **total internal energy** ($\mathbf{I}_H$) **of a pound of saturated steam** at a pressure **P** in pounds per square foot is equal to the total heat (**H**) less the heat equivalent of the external work done; thus,

$$\mathbf{I}_H = H - P\frac{(V - \frac{1}{60})}{778}. \qquad (108)$$

Such reference is made here to the internal energy of steam because it is very useful in calculating the heat taken in and rejected by steam during any stage of its expansion or compression.

Heat taken in = increase of internal energy + external work done.

When dealing with a compression instead of an expansion the last term above (external work) will be a negative value to indicate that work is done upon the steam instead of the steam doing work by expansion.

The following problem shows the calculation of internal energy and external work:

Example. A boiler is evaporating water into dry and saturated steam at a pressure of 300 pounds per square inch absolute. The feed water enters the boiler at a temperature of 145° F. The internal energy of evaporation per pound of steam is

$$I_L = L - \frac{P\left(V - \frac{1}{60}\right)}{778} = 811.3 - \frac{300 \times 144\left(1.551 - \frac{1}{60}\right)}{778}$$

$$= 811.3 - 85.3 = 726.0 \text{ B.t.u.},$$

or taken directly from saturated steam tables equals 726.8 B.t.u.

The total internal energy supplied above 32° F. per pound of steam is

$$I_H = H - \frac{P\left(V - \frac{1}{60}\right)}{778} = 1204.1 - 85.3 = 1118.8 \text{ B.t.u.},$$

or taken directly from saturated steam tables equals 1118.5 B.t.u.

External work done above 32° F. per pound of steam as calculated from steam tables is

$$H - I_H = 1204.1 - 1118.5 = 85.6 \text{ B.t.u.}$$

External work of evaporation per pound of steam as calculated from the steam tables is

$$L - I_L = 811.3 - 726.8 = 84.5 \text{ B.t.u.}$$

The external work done in raising the temperature of a pound of *water* from 32° F. to the boiling point is

$$85.6 - 84.5 = 1.1 \text{ B.t.u.}$$

The external work done in raising the temperature of a pound of water from 32° F. to 145° F. is

$$\frac{300 \times 144\,(0.0163 - 0.01602)}{778} = 0.01 \text{ B.t.u.}$$

The external work done in forming the steam from water at 145° F. is, then,

$$84.5 + 1.10 - 0.01 = 85.59 \text{ B.t.u.}$$

as compared with 85.3 B.t.u. as calculated from

$$\frac{P\,(V - \frac{1}{60})}{778}.$$

It is to be noticed that the external work done during the addition of heat when the liquid is raised to the vaporization temperature is small as compared with other values, and for most engineering work it is customary to assume that no external work is done during the addition of heat to the water. With this assumption

$$I_H = h + L - \frac{P\,(V - \frac{1}{60})}{778} = H - (L - I_L). \qquad (109)$$

Steam Formed at Constant Volume. When saturated steam is made in a boiler at **constant volume,** as, for example, when the piping connections from the boiler to the engines are closed, then no external work is done, and all the heat taken in is converted into and appears as internal energy I_H of the steam. This quantity is less than the total heat H of steam, representing its formation at constant pressure, by quantity $P\,(V - \frac{1}{60}) \div 778$, where P represents the absolute pressure at which the steam is formed in pounds per square foot and V is the volume of a pound of steam at this pressure in cubic feet.

Wet Steam. In all problems studied thus far, dealing with saturated steam, it has been assumed that the steaming process was complete and that the water had been completely converted into steam. In actual practice it is not at all unusual to have steam leaving boilers which is not perfectly and completely vaporized; in other words, the boilers are supplying to the engines a mixture of steam and water. This mixture we call **wet steam.** It is steam which carries actually in suspension minute particles of water, which remain thus in suspension almost indefinitely. The temperature of wet steam is always the same as that of dry saturated steam as given in the steam tables, so long as any steam remains uncondensed.

The ratio of the weight of moisture or water in a pound of **wet steam** to a pound of completely saturated steam is called the **degree of wetness;** and when this ratio is expressed as a per cent, it is called the **percentage of moisture** or "**per cent wet.**" If in a pound of wet steam there is 0.04 pound of water in suspension or entrained, the steam is four per cent wet.

Another term, called the **quality of steam,** which is usually expressed by the symbol **x,** is also frequently used to represent the condition of wet steam. Quality of steam may be defined as the proportion of the amount of dry or completely evaporated steam in a **pound of wet steam.** To illustrate with the example above, if there is 0.04 pound of water in a pound of wet steam; the **quality in this case would be 1 − 0.04 or 0.96.** In this case the quality of this steam is ninety-six one-hundredths.

With this understanding of the nature of wet steam it is obvious that **latent heat** of a pound of wet steam is **xL.** Similarly, the **total heat** of a pound of wet steam is $\mathbf{h + xL}$, and the volume of a pound of wet steam is $\mathbf{x}\ (\mathbf{V} - \frac{1}{60}) + \frac{1}{60} = \mathbf{xV} + \frac{1}{60}\ (1 - \mathbf{x})$, or approximately it is equal to **xV,** because the term $\frac{1}{60}\ (1 - \mathbf{x})$ is negligibly small except in cases where the steam is so wet as to consist mostly of water. Similarly, the internal energy in a pound of wet steam is

$$\mathbf{I}_H = h + x \left\{ L - \frac{P\,(V - \frac{1}{60})}{778} \right\}. \qquad (110)$$

Superheated Steam. When the temperature of steam is higher than that corresponding to saturation as taken from the steam tables, and is, therefore, higher than the standard temperature corresponding to the pressure, the steam is said to be **super-**

heated. In this condition steam begins to depart and differ from its properties in the saturated condition, and when superheated to a very high degree it begins to behave somewhat like a perfect gas.

There are tables of the properties of superheated steam just as there are tables of saturated steam. (See Marks and Davis', Goodenough's or Peabody's Steam Tables.) When dealing with saturated steam there is always only one possible temperature and only one specific volume to be considered. With superheated steam, on the other hand, for a given pressure there may be any temperature **above** that of saturated steam, and corresponding to each temperature there will be, of course, definite values for specific volume and total heat. Like a perfect gas the specific volume or the cubic feet per pound increases with the increase in temperature.

The total heat of superheated steam is obviously greater than the total heat of saturated steam.* Thus, since the total heat of dry saturated steam is, as before, $h + L$, the total heat of superheated steam with D degrees of superheat is

$$\mathbf{H}_s = h + L + C_p \times D; \tag{111}$$

or if the temperature of the superheated steam is $t_{\text{sup.}}$ and $t_{\text{sat.}}$ is the temperature of saturated steam corresponding to the pressure,

$$\mathbf{H}_s = h + L + C_p\,(t_{\text{sup.}} - t_{\text{sat.}}). \tag{112}$$

$\mathbf{H}_s$ (equations 111 and 112) is the total heat of superheated steam or is the amount of heat required to produce a pound of steam with the required degrees of superheat from water at 32° F.

To obtain the amount of internal energy of a pound of steam (superheated) corresponding to this total heat as stated above, the external work expended in the steaming process must be subtracted; that is,

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* In practice steam is called dry saturated when it is exactly saturated and has no moisture. It is the condition known simply as saturated steam as regards the properties given in the ordinary steam tables. The dry saturated condition is the boundary between wet steam and superheated steam.

$$\mathrm{I_H} = h + L - \frac{P\,(V_{\mathrm{sat.}} - \frac{1}{60})}{778} + C_p\,(t_{\mathrm{sup.}} - t_{\mathrm{sat.}}) - \frac{P\,(V_{\mathrm{sup.}} - V_{\mathrm{sat.}})}{778}$$

$$= h + L + C_p\,(t_{\mathrm{sup.}} - t_{\mathrm{sat.}}) - \frac{P\,(V_{\mathrm{sup.}} - \frac{1}{60})}{778}$$

$$= H_s - \frac{P\,(V_{\mathrm{sup.}} - \frac{1}{60})}{778}. \qquad (113)$$

The term $\frac{1}{60}$ **can be neglected** in the equations above for practically all engineering calculations as the maximum error from this is not likely to be more than one in one thousand or $\frac{1}{10}$ per cent.* The accuracy of our steam tables for values of latent and total heat is not established to any greater degree. Making this approximation, the equation above becomes

$$\mathrm{I}_H = h + L + C_p\,(t_{\mathrm{sup.}} - t_{\mathrm{sat.}}) - \left(\frac{P \times V_{\mathrm{sup.}}}{778}\right) = \mathrm{H}_s - \frac{P \times V_{\mathrm{sup.}}}{778}.$$

If examination is made of the properties of superheated steam as given in Marks and Davis' or in Goodenough's *Steam Tables* for superheated steam at 165 pounds per square inch absolute pressure and 150° F. superheat, the following results are secured:

	Marks & Davis	Goodenough
Temperature (t)	516.0	516.1
Volume (V, v)	3.43	3.41
Total Heat (H_s or h† or i)	1277.6	1280.8

The values of h and L for saturated steam at this pressure are respectively 338.2 and 856.8. From the curves given, **Fig. 17**,‡

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* The error in the value of total internal energy due to neglecting the term $\frac{1}{60}$ in the exercise worked out on page 70 is 0.85 B.t.u. per pound (one per cent of the external work), or an error of .076 per cent in the final result.

† In their tables Marks and Davis represent the total heat of superheated steam by h. In this book the symbol H_s is used for greater clearness and to be consistent with the symbols used in the preceding formulas. They use also v for specific volume in place of V as above.

‡ The curves for values of C_p, as given in **Fig. 17**, are for average and not for instantaneous values such as are given in **Fig. 18**, of Marks and Davis' *Tables and Diagrams*. Great caution must be observed in the use of curves of this kind. Those giving instantaneous values can only be used for the value of C_p in the formulas given for the total heat of superheated steam after the average value has been found by integrating the curve representing these values for a given pressure.

the specific heat of superheated steam at constant pressure (C_p) is 0.552. From these data,

$$H_s = h + L + C_p \times 150 = 338.2 + 856.8 + 0.552 \times 150 = 1277.7.$$

This value agrees very nearly with that given in the tables as previously indicated.

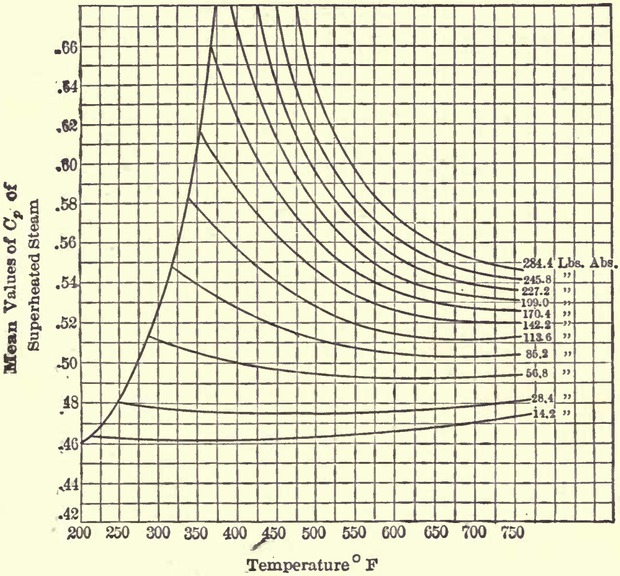

FIG. 17. — Mean Values of C_p Calculated by Integration from Knoblauch and Jakob's Data.

The specific volume (V) is calculated from the empirical formula derived from experimental results and is expressed as follows:

$$V = \left[0.5962\,T - p\,(1 + 0.0014\,p)\left(\frac{150{,}300{,}000}{T^3} - 0.0833\right)\right]\frac{1}{p}, \quad (114)$$

where p is in pounds per square inch, V is in cubic feet per pound and $T = t + 460$ is the absolute temperature on the Fahrenheit scale.*

Drying of Steam by Throttling or Wire-drawing. When steam expands by passing through a very small opening, as, for example,

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* The value of γ or $\frac{C_p}{C_v}$ of superheated steam used in ordinary engineering calculations is 1.3.

through a valve only partly open in a steam line, the pressure is considerably reduced. This effect is called **throttling or wire-drawing.** The result of expansion of this kind when the pressure is reduced and no work is done is that if the steam is initially wet it will be drier and if it is initially dry or superheated the degree of superheat will be increased. The reason for this is that the total heat required to form a pound of dry saturated steam (H) is considerably less at low pressure than at high, but obviously the total **quantity** of heat in a pound of steam must be

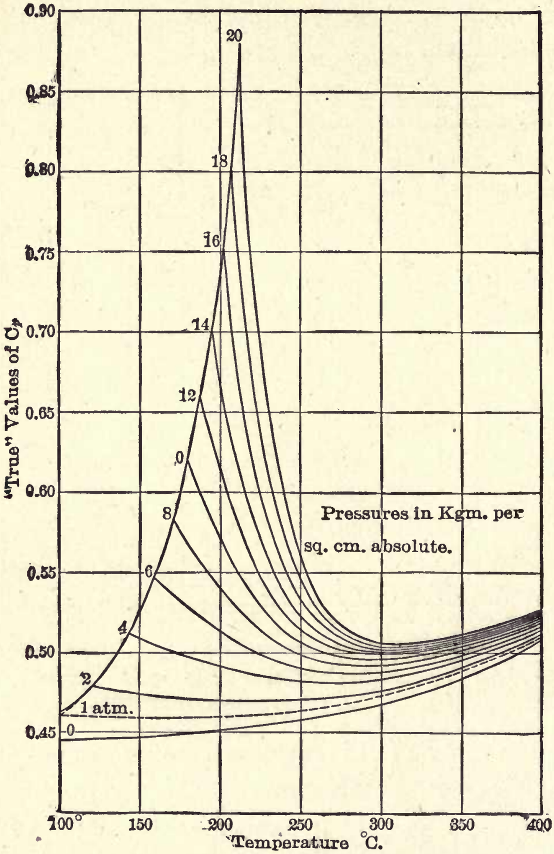

FIG. 18. — Values of the "True" Specific Heat of Superheated Steam.

the same after wire-drawing as it was before, neglecting radiation. Now if steam is initially wet and the quality is represented by x_1, then the total heat in the steam is represented by h_1 plus x_1L_1, in which h_1 and L_1 represent the heat of the liquid and the latent heat of the steam at the initial pressure. If, also, the quality, heat of liquid and the latent heat of the steam after wire-drawing are represented respectively by x_2, h_2 and L_2, then $h_1 + x_1L_1 = h_2 + x_2L_2$, or

$$x_2 = \frac{x_1L_1 + h_1 - h_2}{L_2}. \qquad (115)$$

This drying action of steam in passing through a small opening or an orifice is very well illustrated by steam discharging from a small leak in a high pressure boiler into the atmosphere. It will be observed that no moisture is visible in the steam a few inches from the leak but farther off it becomes condensed by loss of heat, becomes clouded and plainly visible. An important application of the throttling principle is also to be found in the throttling calorimeter (page 80).

Determination of the Moisture in Steam. Unless the steam used in the power plant is superheated it is said to be either dry or wet, depending on whether or not it contains water in suspension. The general types of steam calorimeters used to determine the amount of moisture in the steam may be classified under three heads:

1. Throttling or superheating calorimeters.
2. Separating calorimeters.
3. Condensing calorimeters.

Throttling or Superheating Calorimeters. The type of steam calorimeter used most in engineering practice operates by passing a sample of the steam through a very small orifice, in which it is superheated by throttling. A very satisfactory calorimeter of this kind can be made of pipe fittings as illustrated in **Fig. 19.** It consists of an orifice **O** discharging into a chamber **C,** into which a thermometer **T** is inserted, and a mercury manometer is usually attached to the cock V_3 for observing the pressure in the calorimeter.

It is most important that all parts of calorimeters of this type, as well as the connections leading to the main steam pipe, should be very thoroughly lagged by a covering of good insulating material. One of the best materials for this use is hair felt, and it is particularly well suited for covering the more or less temporary pipe fittings, valves and nipples through which steam is brought to the calorimeter. Throttling calorimeters have been found useless because the small pipes leading to the calorimeters were not properly lagged, so that there was too much radiation, producing, of course, condensation, and the calorimeter did not get a true sample. It is obvious that if the entering steam contains too much moisture the drying action due to the throttling in the orifice may not be sufficient to superheat. It may be stated in general that unless there are about 5° to 10° F. of superheat in the calorimeter, or, in other words, unless the temperature on the low pressure side of the orifice is at least about 5° to 10° F. higher than that corresponding to the pressure **in the calorimeter,** there may be some doubt as to the accuracy of results.* The working limits of throttling calorimeters vary with the initial pressure of the steam. For 35 pounds per square inch absolute pressure the calorimeter ceases to superheat when the percentage of moisture exceeds about 2 per cent; for 150 pounds absolute pressure when the moisture exceeds about 5 per cent; and for 250 pounds absolute pressure when it is in excess of about 7 per cent. For any given pressure the exact limit varies *slightly*, however, with the pressure in the calorimeter.

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* The same general statement may be made as regards determinations of superheat in engine and turbine tests. Experience has shown that tests made with from 0 to 10 degrees Fahrenheit superheat are not reliable, and that the steam consumption in many cases is not consistent when compared with results obtained with wet or more highly superheated steam. The errors mentioned when they occur are probably due to the fact that in steam, indicating less than 10 degrees Fahrenheit superheat, water in the liquid state may be taken up in "slugs" and carried along without being entirely evaporated.

In connection with a report on the standardizing of engine tests, the American Society of Mechanical Engineers * published instructions regarding the method to be used for obtaining a fair sample of the steam from the main pipes. It is recommended in this report that the calorimeter shall be connected with as short intermediate piping as possible with a so-called **calorimeter nipple** made of $\frac{1}{2}$-inch pipe and long enough to extend into the steam pipe to within $\frac{1}{2}$ inch of the opposite wall. The end of this nipple is to be plugged so that the steam must enter through not less than twenty $\frac{1}{8}$-inch holes drilled around and along its length. None of these holes shall be less than $\frac{1}{2}$ inch from the inner side of the steam pipe. The sample of steam should always be taken from a **vertical** pipe as near as possible to the engine, turbine or boiler being tested. A good example of a calorimeter nipple is illustrated in **Fig. 20.**

The discharge valve $\mathbf{V_2}$ should not be closed or adjusted without first closing the gage cock $\mathbf{V_3}$. Unless this precaution is taken, the pressure may be suddenly increased in chamber **C,** so that if a manometer is used the mercury will be blown out of it; and if, on the other hand, a low-pressure steam gage is used it may be ruined by exposing it to a pressure much beyond its scale.

Usually it is a safe rule to begin to take observations of temperature in calorimeters after the thermometer has indicated a maximum value and has again slightly receded from it. The quality or relative dryness of wet steam is easily calculated by the following method and symbols:

p_1 = steam pressure in main, pounds per square inch absolute,

p_2 = steam pressure in calorimeter, pounds per square inch absolute,

t_c = temperature in calorimeter, degrees Fahrenheit,

L_1 and h_1 = heat of vaporization and heat of liquid corresponding to pressure p_1, B.t.u.,

H_2 and t_2 = total heat (B.t.u.) and temperature (degrees Fahrenheit) corresponding to pressure p_2,

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* *Proceedings American Society of Mechanical Engineers*, vol. XXI.

C_p = specific heat of superheated steam. Assume 0.47 for low pressures existing in calorimeters,

x_1 = initial quality of steam.

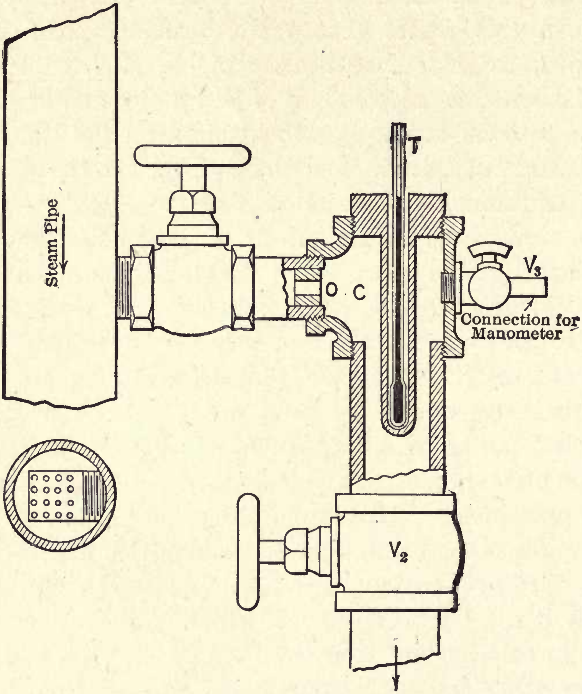

FIG. 19. — Simple Throttling Steam Calorimeter.

Total heat in a pound of wet steam flowing into orifice is

$$x_1L_1 + h_1,$$

and after expansion, assuming all the moisture is evaporated, the total heat of the same weight of steam is

$$H_2 + C_p (t_c - t_2).$$

Then assuming no heat losses and putting for C_p its value 0.47:

$$x_1L_1 + h_1 = H_2 + 0.47 (t_c - t_2) \tag{116}$$

$$x_1 = \frac{H_2 + 0.47 (t_c - t_2) - h_1}{L_1}. \tag{117}$$

The following example shows the calculations for finding the quality of steam from the observations taken with a throttling calorimeter:

Example. Steam at a pressure of 100 pounds per square inch absolute passes through a throttling calorimeter. In the calorimeter the temperature of the steam becomes 243° F. and the pressure 15 pounds per square inch absolute. Find the quality.

Solution. By taking values directly from tables of properties of superheated steam,* the total heat of the steam in the calorimeter at 15 pounds per square inch absolute pressure and 243° F. is 1164.8 B.t.u. per pound or can be calculated as follows:

$$\begin{aligned} H + C_p\,(t_c - t_2) &= 1150.7 + 0.47\,(243 - 213) \\ &= 1164.8 \text{ B.t.u. per pound.} \end{aligned}$$

(Note. t_c = temperature in calorimeter and t_2 = temperature corresponding to calorimeter pressure.)

The total heat of the steam before entering the calorimeter is $h + xL$. At 100 pounds per square inch absolute pressure, this is $298.3 + 888.0\,x$ in B.t.u. per pound. Since the heat in the steam per pound in the calorimeter is obviously the same as before it entered the instrument, we can equate as follows:

$$\begin{aligned} 298.3 + 888.0\,x &= 1164.8 \\ x &= 0.976 \end{aligned}$$

or the steam is 2.4 per cent wet.

Barrus Throttling Calorimeter. This is an important variation from the type of throttling calorimeter shown in **Fig. 19** and has been quite widely introduced by Mr. George H. Barrus. In this apparatus the temperature of the steam admitted to the calorimeter is observed instead of the pressure and a very free exhaust is provided so that the pressure in the calorimeter is atmospheric. This arrangement simplifies very much the observations to be taken, as the quality of the steam x_1 can be calculated by equation (117) by observing only the two temperatures

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* Marks and Davis' *Steam Tables and Diagrams* (1st ed.), page 24 or Goodenough's *Properties of Steam and Ammonia* (2d ed.), page 47.

t_1 and t_c, taken respectively on the high and low pressure sides of the orifice in the calorimeter. This calorimeter is illustrated in **Fig. 20.** The two thermometers required are shown in the figure. Arrows indicate the path of the steam.*

The orifice in such calorimeters is usually made about $\frac{3}{32}$ inch in diameter, and for this size of orifice the weight of steam † discharged per hour at 175 pounds per square inch absolute pressure is about 60 pounds. It is important that the orifice should always be kept clean, because if it becomes obstructed there will be a reduced quantity of steam passing through the instrument, making the error due to radiation relatively more important.

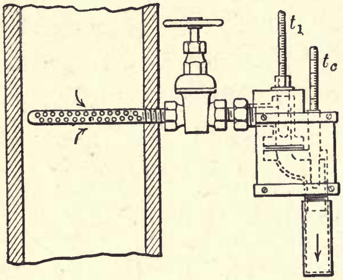

Fig. 20. — Barrus' Throttling Steam Calorimeter.

In order to free the orifice from dirt or other obstructions the connecting pipe to be used for attaching the calorimeter to the main steam pipe should be blown out thoroughly with steam before the calorimeter is put in place. The connecting pipe and valve should be covered with hair felting not less than $\frac{3}{4}$ inch thick. It is desirable also that there should be no leak at any point about the apparatus, as in the stuffing-box of the supply valve, the pipe joints or the union.

With the help of a diagram‡ giving the quality of steam directly the Barrus calorimeter is particularly well suited for use in power plants, where the quality of the steam is entered regularly on the log sheets. The percentage of moisture is obtained immediately from two observations without any calculations.

Separating Calorimeters. It was explained that throttling calorimeters cannot be used for the determination of the quality of steam when for comparatively low pressures the moisture

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* *Transactions of American Society of Mechanical Engineers*, vol. XI, page 790.

† In boiler tests corrections should be made for the steam discharged from the steam calorimeters.

‡ See page 113, and also Moyer's *Power Plant Testing*, (2d ed.), pages 58–60.

is in excess of 2 per cent, and when for boiler pressures commonly used it exceeds 5 per cent. For higher percentages of moisture than these low limits separating calorimeters are most generally used. In these instruments the water is removed from the sample of steam by **mechanical separation** just as it is done in the ordinary steam separator installed in the steam mains of a power plant. There is provided, of course, a device for determining, while the calorimeter is in operation, usually by means of a calibrated gage glass, the amount of moisture collected. This mechanical separation depends for its action on changing very abruptly the direction of flow and reducing the velocity of the wet steam. Then, since the moisture (water) is nearly 300 times as heavy as steam at the usual pressures delivered to the engine, the moisture will be deposited because of its greater inertia.

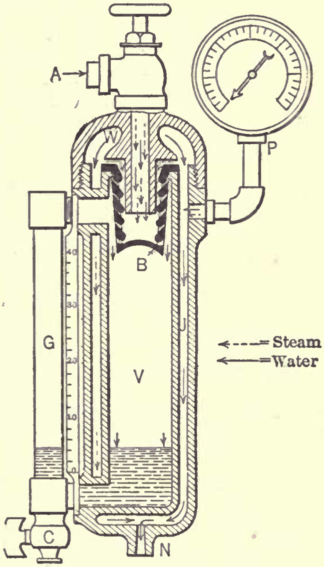

FIG. 21. — Separating Calorimeter.

Fig. 21 illustrates a form of separating calorimeter having a steam jacketing space which receives live steam at the same temperature as the sample. Steam is supplied through a pipe **A**, discharging into a cup **B**. Here the direction of the flow is changed through nearly 180 degrees, causing the moisture to be thrown outward through the meshes in the cup into the vessel **V**. The dry steam passes upward through the spaces between the webs **W**, into the top of the outside jacketing chamber **J**, and is finally discharged from the bottom of this steam jacket through the nozzle **N**. This nozzle is considerably smaller than any other section through which the steam flows, so that there is no appreciable difference between the pressures in the calorimeter proper and the jacket. The scale opposite the gage glass **G** is graduated to show, in hundredths of a pound, at the temperature corresponding to steam at ordinary working pressures, the variation of the level of the water accumulating. A steam pressure gage **P** indicates the pressure in the jacket **J**, and since the flow of steam through the nozzle **N** is roughly proportional to the pressure, another scale in addition to the one reading pressures is provided at the outer edge of the dial. A petcock **C** is used for draining the water from the instrument, and by weighing the water collected corresponding to a given difference in the level in the gage **G** the graduated scale can be readily calibrated. Too much reliance should not be placed on the readings for the flow of steam as indicated by the gage **P** unless it is frequently calibrated. Usually it is very little trouble to connect a tube to the nozzle **N** and condense the steam discharged in a large pail nearly filled with water. When a test for quality is to be made by this method the pail nearly filled with cold water is carefully weighed, and then at the moment when the level of the water in the water gage **G** has been observed the tube attached to the nozzle **N** is immediately placed under the surface of the water in the pail. The test should be stopped before the water gets so hot that some weight is lost by "steaming." The gage **P** is generally calibrated to read pounds of steam flowing in ten minutes. For the best accuracy it is desirable to use a pail with a tightly fitting cover into which a hole just the size of the tube has been cut.

If W is the weight of dry steam flowing through the orifice **N** and w is the weight of moisture separated, the **quality** of the steam is

$$\mathbf{x} = \frac{W}{W + w}. \qquad (118)$$

Condensing or Barrel Calorimeter. For steam having a large percentage of moisture (over 5 per cent) the condensing or barrel calorimeter will give fairly good results if properly used. In its simplest form it consists of a barrel placed on a platform scale and containing a known weight of cold water. The steam is introduced by a pipe reaching nearly to the bottom of the barrel. The condensation of the steam raises the temperature of the water, the loss of heat by the wet steam being equal to the gain of heat by the cold water. It will, therefore, be necessary to observe the initial and final weights, the initial and final temperatures of the water in the barrel and the temperature of the steam.

Let W = original weight of cold water, pounds.
w = weight of wet steam introduced, pounds.
t_1 = temperature of cold water, degrees Fahrenheit.
t_2 = temperature of water after introducing steam.
t_s = temperature of steam.
L = latent heat of steam at temperature t_s.
x = quality of steam.

Then,

Heat lost by wet steam = heat gained by water.

$$wxL + w(t_s - t_2) = W(t_2 - t_1)$$

$$x = \frac{W(t_2 - t_1) - w(t_s - t_2)}{wL}. \qquad (119)$$

The accuracy of the results depends obviously upon the accuracy of the observation, upon thorough stirring of the water so that a uniform temperature is obtained and upon the length of time required. The time should be just long enough to obtain accurate differences in weights and temperatures; otherwise, losses by radiation will make the results much too low.

Equivalent Evaporation and Factor of Evaporation. For the comparison of the total amounts of heat used for generating steam (saturated or superheated) under unlike conditions it is necessary to take into account the temperature t_0 at which the water is put into the boiler as well as also the pressure P at which the steam is formed.* These data are of much importance in comparing the results of steam boiler tests. The basis

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* As the pressure P increases the total heat of the steam also increases; but as the initial temperature of the water ("feed temperature") increases the value of the heat of the liquid decreases.

of this comparison is the condition of water initially at the boiling point for "atmospheric" pressure or at 14.7 pounds per square inch; that is, at 212° F. and with steaming taking place at the same temperature. For this standard condition, then, $h = 0$ and $H = L = 970.4$ B.t.u. per pound. Evaporation under these conditions is described as,

"**from** (a feed-water temperature of) **and at** (a pressure corresponding to the temperature of) **212° F.**"

To illustrate the application of a comparison with this standard condition, let it be required to compare it with the amount of heat required to generate steam at a pressure of 200 pounds per square inch absolute with the temperature of the water supplied (feed water) at 190° F.

For $P = 200$ pounds per square inch absolute the heat of the liquid $h = 354.9$ B.t.u. per pound and the heat of evaporation (L) is 843.2 B.t.u. per pound. For $t = 190°$ F. the heat of the liquid (h_0) is 157.9 B.t.u. per pound. The total heat actually required in generating steam at these conditions is, therefore,

$$843.2 + (354.9 - 157.9) = 1040.2 \text{ B.t.u. per pound.}$$

The ratio of the total heat actually used for evaporation to that necessary for the condition defined by "from and at 212° F." is called the **factor of evaporation.** In this case it is the value

$$1040.2 \div 970.4 = 1.07.$$

Calling F the factor of evaporation, h and L respectively the heats of the liquid and of evaporation corresponding to the steam pressure and h_0 the heat of the liquid corresponding to the temperature of feed water, then

$$\mathbf{F} = \frac{L + (h - h_0)}{970.4}. \tag{120}$$

The actual evaporation of a boiler (expressed usually in pounds of steam per hour) multiplied by the factor of evaporation is called the **equivalent evaporation.**

Vapors as Refrigerating Media. Ammonia is generally used as the refrigerating medium in connection with mechanical refrigeration. Carbon dioxide and sulphur dioxide are also used to a limited extent as refrigerating media.

The value of a substance as a refrigerating medium depends upon its latent heat, its vaporization temperature, its cost, and upon its chemical properties. The greater the latent heat of a refrigerating medium the more heat will it be capable of abstracting by evaporation. Upon the vaporization temperature of the refrigerant at different pressures depends the degree of cold it can produce as well as its practicability for use in hot climates where low temperature cooling water cannot be secured.

Goodenough's *Tables of Properties of Steam and Ammonia* give the various physical properties of saturated, superheated and liquid ammonia. Peabody's Tables give the properties of ammonia, sulphur dioxide, carbon dioxide and of other vapors used as refrigerating media.

Table 4 is an abridged table based upon Goodenough's values for saturated ammonia. Tables 5 and 6, based upon Peabody's *Tables*, show some of the properties of sulphur dioxide (SO_2) and of carbon dioxide (CO_2).

PROBLEMS

1. Dry and saturated steam has a pressure of 100 lbs. per sq. in. absolute. Calculate the temperature of the steam, the volume per pound, the heat of the liquid, the latent heat, and the total heat above 32° F. per pound of this steam.

2. Dry and saturated steam has a temperature of 300° F. What is its pressure, heat of the liquid, latent heat, and total heat?

3. A closed tank contains 9 cu. ft. of dry and saturated steam at a pressure of 150 lbs. per sq. in. absolute.
   (a) What is its temperature?
   (b) How many pounds of steam does the tank contain?

4. A boiler generates dry and saturated steam under a pressure of 200 lbs. per sq. in. absolute. The feed water enters the boiler at 60° F.
   (a) What is the temperature of the steam?
   (b) How many British thermal units are required to generate 1 lb. of this steam if this feed water is admitted at 32° F.?

(c) How many British thermal units are required to generate 1 lb. of this steam from feed water at 60° F. into the steam at the pressure stated at the beginning of this problem?

5. One pound of dry and saturated steam is at a pressure of 250 lbs. per sq. in. absolute.

(a) What is its internal energy of evaporation?
(b) What is its total internal energy above 32° F.?
(c) How much external work was done during its formation from 32° F.?
(d) How much external work was done during the evaporation?
(e) How much external work was done during the change in temperature of the water from 32 degrees to the boiling point corresponding to the pressure?

6. Dry and saturated steam is generated in a boiler and has a temperature of 400° F. The feed water enters the boiler at 200° F.

(a) What pressure is carried in the boiler?
(b) What is the total heat supplied to generate 1 lb. of this steam?
(c) How much external work was done during its formation?
(d) How much heat was used in increasing the internal energy? Check this by $(h_2 - h_1 + I_L)$ noting that this assumes no external work done in the heating of the liquid.

7. One pound of steam at a pressure of 100 lbs. per sq. in. absolute has a quality of 90 per cent dry. What is its temperature?

8. How many British thermal units would be required to raise the pound of steam in the above problem from 32° F. to the boiling point corresponding to the pressure stated?

9. What would be the volume of a pound of steam for the conditions stated in problem 7?

10. How many heat units (latent heat) are required to evaporate the steam in problem 7?

11. What is the amount of the total heat (above 32° F.) of the steam in problem 7?

12. What would be the external work of evaporation of the steam in problem 7?

13. How much external work (above 32° F.) is done in making steam as in problem 7?

14. What is the internal energy of evaporation of the steam in problem 7?

15. What is the total internal energy of the steam in problem 7?

16. A tank contains 9 cu. ft. of steam at 100 lbs. per sq. in. absolute pressure which has a quality of 0.95. How many pounds of steam does the tank contain?

17. Two pounds of steam have a volume of 8 cu. ft. at a pressure of 100 lbs. per sq. in. absolute. What is the quality?

Calculate its total heat above 32° F.

18. One pound of steam having a quality of 0.95 has a temperature of 325° F. What is the pressure?

19. One pound of steam at a pressure of 225 lbs. per sq. in. absolute has a temperature of 441.9° F.

(a) Is it superheated or saturated?
(b) What is the total heat required to generate such steam from water at 32° F.?
(c) What is its volume?
(d) How much external work was done (above 32° F.) in generating it?
(e) How much internal energy above 32° F. does it contain?

20. One pound of steam at a pressure of 300 lbs. per sq. in. absolute has a volume of 1.80 cu. ft.

(a) Is it saturated or superheated?
(b) What is its temperature?
(c) How much is its total heat above 32° F.?
(d) What is its total internal energy?

21. Steam in a steam pipe has pressure of 110.3 lbs. per sq. in. by the gage. A thermometer in the steam registers 385° F. Atmospheric pressure is 14 lbs. per sq. in. absolute. Is the steam superheated, and if superheated how many degrees?

22. Steam at a pressure of 200 lbs. per sq. in. absolute passes through a throttling calorimeter. After expansion into the calorimeter the temperature of this steam is 250° F. and the pressure 15 lbs. per sq. in. absolute. What is the quality?

23. Steam at a temperature of 325° F. passes through a throttling calorimeter. In the calorimeter the steam has a pressure of 16 lbs. per sq. in. absolute and a temperature of 236.3° F. What is the quality?

24. Steam at 150 lbs. per sq. in. absolute pressure passes through a throttling calorimeter. Assuming that the lowest condition in the calorimeter for measuring the quality is 10° F. superheat and the pressure in the calorimeter is 15 lbs. per sq. in. absolute, what is the largest percentage of wetness the calorimeter is capable of measuring under the above conditions?

25. Prepare a chart by means of which can be determined the largest percentages of wetness a throttling calorimeter will measure at all pressures from 50 to 300 lbs. per sq. in. absolute.

26. In a ten-minute test of a separating calorimeter the quantity of dry steam passing through the orifice was 9 pounds. The quantity of water separated was 1 pound. What was the quality?

27. A barrel contained 400 lbs. of water at a temperature of 50° F. Into this water steam at a pressure of 125 lbs. per sq. in. absolute was admitted until the temperature of the water and condensed steam in the barrel reached a temperature of 100° F. The weight of the water in the barrel was then 418.5 lbs. What was the quality?

28. Calculate the factor of evaporation and the equivalent evaporation per pound of coal for a boiler under the following conditions: Steam pressure, 190 lbs. per sq. in. gage; feed water temperature, 203° F.; steam apparently evaporated per pound of coal $7\frac{1}{2}$ pounds; steam 3 per cent wet.

29. Plot on one chart the pressure-temperature relations of ammonia, SO_2, and CO_2.

30. Plot on one chart the pressure-latent heat relations of ammonia, SO_2, and CO_2.

31. What conclusions do you reach from the graphic results in problems 29 and 30?

32. If cooling water for condensing ammonia cannot be secured at a temperature less than 90° F., what will be the pressure of the ammonia at the inlet to the condenser of the refrigerating system?

CHAPTER VI

ENTROPY

Pressure-volume diagrams are useful for determining the **work** (in foot-pounds), done during any process or a cycle, but they are of very limited use in analyzing the heat changes involved. It has, therefore, been found desirable to make use of a diagram which shows directly by an area the number of **heat units** (instead of foot-pounds) involved during any process. In order that an area shall represent this value the coördinates must be such that their product will give heat units. If the ordinates are in absolute temperature, the abscissas must be **heat units per degree of absolute temperature,** that is, $\frac{Q}{T}$, for then $T \times \frac{Q}{T} = Q$, the amount of heat added during the process.

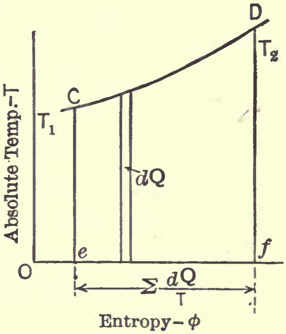

FIG. 22.—Analysis of Entropy Diagram.

Suppose that the heat Q, which is required to produce a process, such as an expansion or a compression, is divided up into a number of small increments dQ (**Fig. 22**) and that each small increment of heat is divided by the average absolute temperature at which the heat change occurs. There will then be a series of expressions $\frac{dQ}{T}$ which, when integrated, will give the total change in the abscissas. This quantity $\int \frac{dQ}{T}$ when multiplied by the average absolute temperature between **C** and **D** will give the total amount of heat involved during the process.

Mathematically expressed, the change in the abscissas is

$$d\phi = \frac{dQ}{T} \qquad \text{or } \phi = \int \frac{dQ}{T}. \tag{121}$$

The heat change involved is

$$dQ = T\,d\phi \quad \text{or} \quad Q = \int T\,d\phi. \tag{122}$$

The quantity ϕ in the equations is known as the **increase in entropy** of the substance, and may be defined as a quantity which, when multiplied by the average absolute temperature occurring during a process, will give the number of heat units (in B.t.u.) added or abstracted as heat during the process. The "increase in entropy" is employed rather than entropy itself, because only the differences in entropy are important.

This definition of entropy states that in a temperature-entropy diagram such as **Fig. 22,** where the ordinates are absolute temperatures, and the abscissas are entropies as calculated above some standard temperature, the area under any line **CD** gives the number of heat units added to the substance in passing from a temperature T_1 and entropy $\phi_1 = \mathbf{Oe}$ to a temperature T_2 and entropy $\phi_2 = \mathbf{Of}$ (or the number of heat units abstracted in passing from T_2 to T_1).

Entropy Changes During Constant Pressure Expansions of Gases. The heat supplied or abstracted during a constant pressure expansion or compression of a gas may be stated:

$$dQ = wC_p\,dt, \tag{123}$$

or

$$Q = wC_p\,(T_2 - T_1). \tag{124}$$

From equation (121) the change in entropy is

$$\phi = \int \frac{dQ}{T}. \tag{125}$$

Assuming the specific heat C_p constant, the change in entropy becomes:

$$\phi = wC_p \int_{T_1}^{T_2} \frac{dt}{T} \tag{126}$$

$$= wC_p\,(\log_e T_2 - \log_e T_1)$$

$$= wC_p \log_e \frac{T_2}{T_1}. \tag{127}$$

Entropy Changes of Gases at Constant Volume. The heat supplied or abstracted during a constant volume change of a gas may be stated:

$$dQ = wC_v\,dt, \tag{128}$$

or

$$Q = wC_v\,(T_2 - T_1). \tag{129}$$

Assuming, as before, the specific heat under constant volume (C_v) constant and substituting in equation (121), the change in entropy is

$$\phi = wC_v \int_{T_1}^{T_2} \frac{dt}{T} \tag{130}$$

$$= wC_v\,(\log_e T_2 - \log_e T_1) \tag{131}$$

$$= wC_v \log_e \frac{T_2}{T_1}. \tag{132}$$

Entropy Changes During Isothermal Processes of a Gas. During an isothermal change the temperature remains constant. The heat supplied or abstracted is consequently equal to the heat equivalent of the work done, which in terms of weight and temperature is

$$wRT \log_e \frac{V_2}{V_1}. \tag{133}$$

Since the temperature remains constant, the change in entropy is

$$\phi = \frac{wRT \log_e \dfrac{V_2}{V_1}}{T} \tag{134}$$

$$= wR \log_e \frac{V_2}{V_1}. \tag{135}$$

Entropy Changes During Reversible Adiabatic Processes of Gases. The heat supplied to or abstracted from an adiabatic expansion or compression is, by definition, zero. The entropy change in such a process is

$$\phi = \int_{T_1}^{T_2} \frac{dQ}{T} = 0. \tag{136}$$

Hence during an adiabatic process no change of entropy occurs.

It is possible for adiabatic processes to occur which are not reversible. These are processes in which, not only no heat is added or abstracted, but a portion or all the work of the process may reappear in the working medium as heat. In such a process the entropy does not remain constant. Reversible adiabatic processes are sometimes called **isoentropic** (equal entropy) to distinguish them from the irreversible adiabatic processes.

Entropy Changes During a Carnot Cycle. A pound of the working substance is first expanded **isothermally.** On a T-ϕ (temperature-entropy) diagram (**Fig.** 23), this process would be represented by line **AB,** where the temperature remains constant at T_1, and where the entropy increases from **Oe** to **Of,** because of the addition of heat that is required to keep the temperature constant.

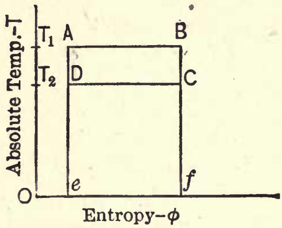

FIG. 23. — Entropy Diagram of Carnot Cycle.

The next process is **adiabatic** expansion from T_1 to T_2. Heat is neither added to nor abstracted from the substance during this expansion. Hence the entropy remains constant, as indicated by **BC.**

The substance is now isothermally compressed along **CD,** the temperature remaining constant at T_2 and the entropy decreasing because of the abstraction of heat.

The last process of the cycle is the adiabatic compression from **D** to **A,** no heat being added or abstracted, and the entropy, therefore, remaining constant.

The heat supplied during the cycle is represented by the area, $ABfe$ and equals

$$(\phi_f - \phi_e) T_1. \tag{137}$$

The heat abstracted during the cycle is represented by the area, $CDef$, and equals

$$(\phi_f - \phi_e) T_2. \tag{138}$$

Since the work of the cycle, in terms of heat, equals the heat added minus the heat rejected,

$$\begin{aligned}\text{Work} &= \text{Area } ABfe - \text{area } CDef \\ &= \text{Area } ABCD.\end{aligned}$$

In terms of entropy,

$$\begin{aligned}\text{Work} &= (\phi_f - \phi_e)\,T_1 - (\phi_f - \phi_e)\,T_2 \\ &= (T_1 - T_2)\,(\phi_f - \phi_e). \qquad (139)\end{aligned}$$

Efficiency of cycle is,

$$E = \frac{\text{Area } ABCD}{\text{Area } ABfe}$$

$$= \frac{(T_1 - T_2)\,(\phi_f - \phi_e)}{T_1\,(\phi_f - \phi_e)} = \frac{T_1 - T_2}{T_1}. \qquad (140)$$

From the foregoing discussion two important conclusions may be drawn in regard to the use of the T-ϕ diagram:

1. If any heat process be represented by a curve on a T-ϕ diagram, the **heat involved during the process** is equal to the area under the curve, that is, between the curve and the axis of absolute temperature.

2. If a cycle of heat processes be represented on a T-ϕ diagram by a **closed figure,** the net work done is equal to the enclosed area, that is, the enclosed area measures the amount of heat that was converted into work.

T-ϕ diagrams are useful for analyzing heat processes, and find particularly useful application in steam engineering, as indicated by the following:

1. Graphical analysis of heat transfers in a steam engine cylinder.
2. Determination of quality of steam during adiabatic expansion.
3. Calculations of steam engine cycle efficiencies.
4. Steam turbine calculations.

Temperature-Entropy Diagrams for Steam. A temperature-entropy diagram for steam is shown in **Fig. 24.** The various shaded areas represent the heats added to water at 32° F. to completely vaporize it at the pressure P_1. The area **ABCD** is the heat added to the water to bring it to the temperature of vaporization, or represents the heat of the liquid (h) for the pressure P_1. Further heating produces evaporation at the constant temperature T_1 corresponding to the pressure P_1, and is represented by the area under the line **CE.** When vaporization is complete, the latent heat, or the heat of vaporization (L), is the area **DCEF.** If, after all the water is vaporized, more heat is added, the steam becomes superheated, and the additional heat required would be represented by an area to the right of **E.**

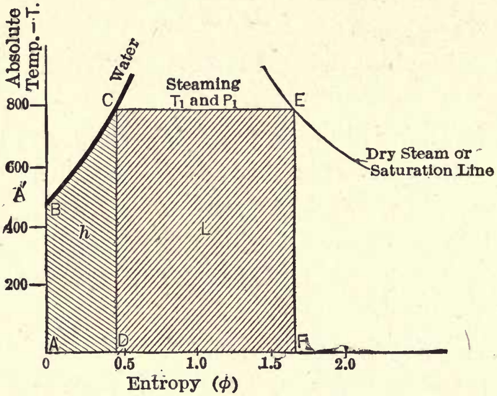

FIG. 24. — Temperature-entropy Diagram of Steam.

Calculation of Entropy for Steam. In order to lay off the increase in entropy as abscissas in the heat diagram for steam, it is necessary to determine various values. For convenience 32° F. has been adopted as the arbitrary starting point for calculating increase of entropy, as well as for the other thermal properties of steam. Referring to **Fig. 25** the entropy of water is seen to be 0 at 32° F. In order to raise the temperature from T_0 to T_1, h heat units are required, and by the equation (121), the entropy of one pound of the liquid θ will be

$$\theta = \int_{T_0}^{T_1} \frac{dh}{T}, \tag{141}$$

or, assuming the specific heat of water to be unity we can write

$$\begin{aligned} \theta &= \int_{T_0}^{T_1} \frac{dt}{T} \\ &= \log_e T_1 - \log_e T_0 = \log_e \frac{T_1}{T_0}. \end{aligned} \tag{142}$$

In order to evaporate the water into steam at the boiling point T_1, the latent heat, L, must be added at constant temperature T_1. The increase of entropy during the "steaming" process is represented by $\frac{L}{T}$ and is equal to, in this case,

$$\frac{L_1}{T_1}. \qquad (143)$$

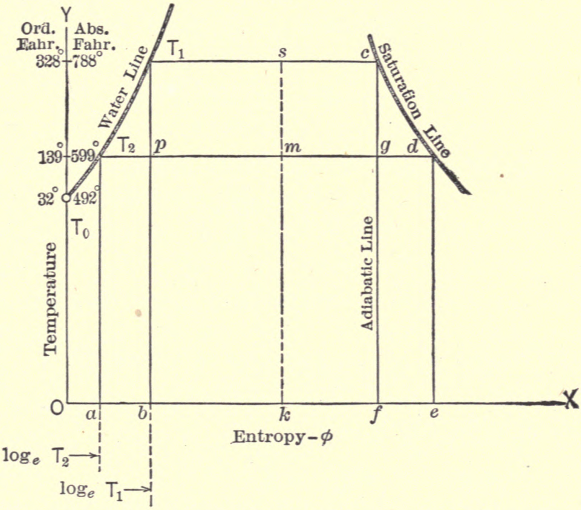

FIG. 25. — Diagram for Calculation of Entropy of Steam.

Further heating would produce superheated steam and the change in entropy would be

$$\text{Entropy of superheat} = \int_{T_{\text{sat.}}}^{T_{\text{sup.}}} \frac{C_p\,dt}{T} \qquad (144)$$

$$= C_p \log_e \frac{T_{\text{sup.}}}{T_{\text{sat.}}}, \qquad (145)$$

where C_p = specific heat of the superheated steam at constant pressure.

$T_{\text{sup.}}$ = absolute temperature of the superheated steam.

$T_{\text{sat.}}$ = absolute temperature of the saturated steam.

Total Entropy of steam, which is the sum of entropies, corresponding to the various increments making up the total heat of the steam, depends upon the quality of the steam.

For dry saturated steam the total entropy above 32° F. and temperature T_1 is equal to the sum of the entropy of the liquid and the entropy of evaporation for dry saturated steam. This may be written

$$\phi_{\text{sat.}} = \theta_1 + \frac{L_1}{T_1} \tag{146}$$

$$= \log_e \frac{T_1}{T_0} + \frac{L_1}{T_1}. \tag{147}$$

Similarly, for wet steam whose quality is x, the total entropy is

$$\phi_{\text{wet}} = \theta_1 + \frac{xL_1}{T_1} \tag{148}$$

$$= \log_e \frac{T_1}{T_0} + \frac{xL_1}{T_1}.$$

For superheated steam whose final temperature is $T_{\text{sup.}}$ and the temperature corresponding to saturation is T_1, the total entropy is, by equation (144),

$$\phi_{\text{sup.}} = \theta_1 + \frac{L_1}{T_1} + \int_{T_{\text{sat.}}}^{T_{\text{sup.}}} \frac{C_p\,dt}{T} \tag{149}$$

$$= \log_e \frac{T_1}{T_2} + \frac{L_1}{T_1} + C_p \log_e \frac{T_{\text{sup.}}}{T_{\text{sat.}}}. \tag{150}$$

Values of these entropies will be found in steam tables.

Referring to **Fig. 25**, the line **T$_1$c** represents the increase of entropy due to the latent heat added during the steaming process. If this steaming process had stopped at some point such that the steam was wet, having a quality x, this condition of the steam could be denoted by the point **s**, where

$$x = \frac{T_1s}{T_1c}.$$

This relation is obvious, for a distance along **T$_1$c** represents the entropy of steam, which is proportional to the latent heat added, which in turn is proportional to the amount of dry steam formed from one pound of water. In like manner, $\frac{T_2m}{T_2d}$ is the quality of steam at the point m that has been formed along T_2d at the temperature T_2.

It is thus apparent that any point on a T-ϕ diagram will give full information in regard to the steam. The proportional distances on a line drawn through the given point between the water and the dry steam lines and parallel to the X-axis give the quality of the steam as shown above. The ordinate of the point gives the temperature and corresponding pressure, while its position relative to other lines, such as constant volume and constant total heat lines which can be drawn on the same diagram, will give further important data.

The Mollier Chart. While the temperature-entropy diagram is extremely useful in analyzing various heat processes, its use in representing the various conditions of steam is not so convenient as that of the Mollier chart. This chart is illustrated in Plate 1 of the Appendix. The coördinates consist of the total heat of the steam above 32° F. and entropy. The saturation curve marks the boundary between the superheated and saturated regions. In the wet region lines of constant volume are drawn and in the superheated regions the lines of equal superheats appear. In both the superheated and saturated regions, lines of constant pressure are drawn and the absolute pressures of the various curves are labeled.

By means of the Mollier chart (Appendix) the curve for adiabatic expansion can be very readily drawn on a pressure-volume diagram when the initial quality of steam at cut-off is known. Assume that one pound of wet steam at a pressure of 150 pounds per square inch absolute and quality 0.965 dry is admitted to the engine cylinder per stroke, and that there is previously in the clearance space 0.2 cubic foot of steam (see **Fig. 26**), at **exactly** the same condition. The volume of a pound of dry saturated steam at this initial pressure of 150 pounds is, from the steam-tables, 3.012 cubic feet. At 0.965 quality it will be 1 (0.965 × 3.012) = 2.905 cubic feet. On the scale of abscissas this amount added to the 0.20 cubic foot in the clearance gives 3.105 cubic feet, the volume to be plotted at cut-off. Other points in the adiabatic expansion curve can be readily plotted after determining the quality by the method given on page 110.

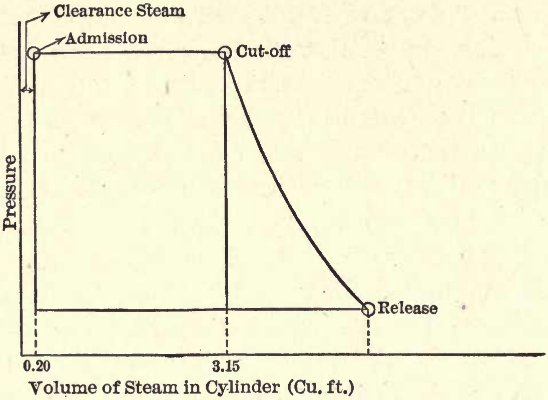

FIG. 26. — Illustrative Indicator Diagram of Engine using Steam with Expansion.

Temperature-entropy Diagram for the Steam Power Plant. Fig. 27 illustrates the heat process going on when feed water is received in the boilers of a power plant at 100° F., is heated and converted into steam at a temperature of 400° F., and then loses heat in doing work. When the feed water first enters the boiler its temperature must be raised from 100° to 400° F. before any "steaming" begins. The heat added to the liquid is the area **MNCD.** This area represents the difference between the heats of the liquid (374 – 68) or about 306 B.t.u. The horizontal or entropy scale shows that the difference in entropy between water at 100° and 400° F. is about 0.436.*

The curve **NC** is constructed by plotting from the steam tables the values of the entropy of the liquid for a number of different temperatures between 100° and 400° F.

If water at 400° F. is converted into steam at that temperature,

* As actually determined from Marks and Davis' *Steam Tables* (pages 9 and 15), the difference in entropy is 0.5663 – 0.1295 or 0.4368. Practically it is impossible to construct the scales in the figure very accurately.

the curve representing the change is necessarily a constant temperature line and therefore a horizontal, **CE**. Provided the evaporation has been complete, the heat added in the "steaming" process is the latent heat or the heat of evaporation of steam (*L*) at 400° F., which is approximately 827 B.t.u.

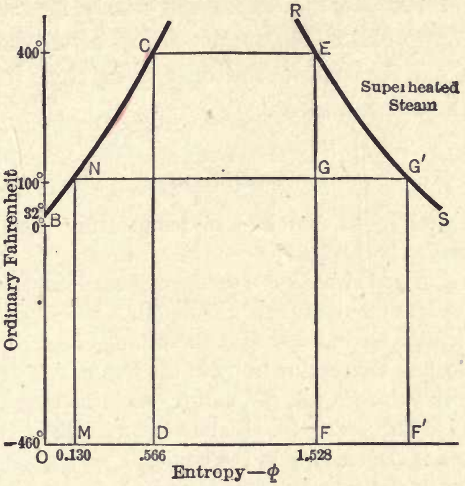

FIG. 27. — Temperature-entropy Diagram for the Steam Power Plant.

The change in entropy during evaporation is, then, the heat units added (827) divided by the absolute temperature at which the change occurs (400 + 460 = 860° F. absolute) or

$$\frac{L}{T} = \frac{827}{860} = 0.962.$$

The total entropy of steam completely evaporated at 400° F. is, therefore, 0.566 + 0.962, or 1.528.* To represent this final condition of the steam, the point **E** is plotted when entropy measured on the horizontal scale is 1.528 as shown in the figure. The point **E** is shown located on another curve **RS**, which is determined by plotting a series of points calculated the same as **E**, but for different pressures. The area **MNCEF** represents the total heat added to a pound of feed water at 100° F., to produce

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* Entropy, like the total heat (*H*) and the heat of the liquid (*h*), is measured above the condition of freezing water (32° F.).

steam at 400° F. Area **OBCEF** represents the total heat of steam (**H** in the steam tables) above 32° F. required to form one pound of steam at 400° F.

If the steam generated in the boiler is now allowed to expand adiabatically in an engine cylinder or in a turbine nozzle to a temperature of 100° F., this expansion will be represented by the line *EG*. *GN* represents the condensation of the exhaust steam. The area *NCEG* represents the energy in the steam available for work in the steam motor.

PROBLEMS

1. One pound of water is raised in temperature from 60° to 90° F. What is the increase in entropy?

2. Dry and saturated steam has a pressure of 100 lbs. per sq. in. absolute.
   (a) Calculate the entropy of the liquid.
   (b) Calculate the entropy of evaporation.
   (c) Calculate the total entropy of the steam.
   (d) Check values in (a), (b), and (c) with the steam tables.

3. Steam at 150 lbs. per sq. in. absolute has a quality of 0.90.
   (a) What is the entropy of the liquid?
   (b) What is the entropy of evaporation?
   (c) What is the total entropy of the steam?

4. Steam having a temperature of 300° F. has an entropy of evaporation of 1.1900. What is its quality?

5. Steam having a pressure of 200 lbs. per sq. in. absolute has a total entropy of 1.5400.
   (a) What is the total entropy of dry and saturated steam under the given pressure?
   (b) Is the steam wet or dry?
   (c) What is its quality?

6. Steam having a pressure of 125 lbs. per sq. in. absolute is superheated 100° F.
   (a) What is the total entropy of dry and saturated steam under the given pressure?
   (b) What is the entropy of the superheat?
   (c) What is its total entropy?

7. Steam having a pressure of 150 lbs. per sq. in. absolute has a total entropy of 1.6043.
   (a) What is the total entropy of dry and saturated steam under the given pressure?

(b) Is the above steam saturated or superheated? How can you tell?

(c) How much superheat has the steam?

8. A boiler generates steam of 0.90 quality at a temperature of 350° F. with the feed water admitted at 90° F. What is the increase in entropy?

9. Plot a temperature-entropy diagram for one pound of water vapor for the pressures of 15, 50, 100, 150, 200, 250. The diagram should show the liquid line, the 90 per cent quality line, the dry saturated line, and the 50° and 100° superheat curves for each of the given pressures.

10. What is the entropy of steam 92 per cent dry at a pressure of 15 lbs. per sq. in. absolute?

11. With a quality of 0.90, what is the entropy of evaporation of steam at a pressure of 25 lbs. per sq. in. absolute?

12. What is the total entropy of steam 94 per cent dry at a pressure of 100 lbs. per sq. in. absolute?

CHAPTER VII

EXPANSION AND COMPRESSION OF VAPORS

Vapors, like gases, can be expanded or compressed, but the laws governing their thermodynamic relations are more complex. In the heat changes of vapors the heat that is required to change the state of the substance must be accounted for, in addition to that necessary to do the external work and to produce the change in temperature of the substance.

Equation 12, showing the effect of the heat added or abstracted during an expansion or a compression, applies equally to vapors and gases.

The external work (equation 12) done during the expansion or compression of vapor is calculated, as in the case of gases, from the area under the expansion or compression curve. Work, being a product of pressure and volume, is independent of the working medium.

The internal energy changes (equation 12) involved during an expansion or compression of vapors are best measured by differences. The internal energy at the beginning and at the end of an expansion or compression may be determined by reference to the vapor tables. The values, as obtained from such tables (Tables 3, 4, 5, 6 in Appendix), are calculated from a standard datum temperature of 32° F., and their difference gives the change in internal energy involved in the process in question.

Thus, for any vapor the following general equation may be written:

Heat added = Internal energy at the end of the expansion or compression minus the internal energy at the beginning of the expansion or compression plus the heat equivalent of the external work done, or

$$Q = (I_2 - I_1) + W. \qquad (151)$$

The following problems show the application of equation (151) to various types of expansions.

1. Expansion of Wet Steam at Constant Volume. Assume that one pound of steam at a pressure of 15 pounds per square inch absolute, and 50 per cent dry, receives heat under constant volume raising the pressure to 30 pounds per square inch absolute.

Find: (a) the volume of the steam after the addition of the heat, (b) the quality of the steam, (c) the work done, (d) the heat added.

Solution: (a) Since the volume remains constant the final volume of the steam is

$$xV_{\text{sat.}} = 0.50 \times 26.27 = 13.13 \text{ cu. ft.},$$

in which x = the quality of the steam, V = volume of saturated steam at 15 pounds per square inch absolute pressure (26.27) as obtained from steam tables, in cubic feet per pound.

(b) From steam tables the volume of one pound of dry and saturated steam at the final condition of 30 pounds per square inch absolute is 13.74. Since the actual volume is less than that of the saturated steam, the steam in its final condition is wet and the quality is

$$xV_{\text{sat.}} = V_2$$

from which

$$x = \frac{13.13}{13.74} = 0.955 \text{ or } 95.5 \text{ per cent dry.}$$

(c) Since the volume is constant the work done is

$$W = 0.$$

(d) The heat added is

$$\begin{aligned} Q &= I_2 - I_1 + W \\ &= I_2 - I_1 + 0 \\ &= I_2 - I_1. \end{aligned}$$

The internal energy at the final condition is, from equation (110),

$$I_2 = h_2 + x_2 \left\{ L_2 - \frac{P_2 \left(V_2 - \frac{1}{60}\right)}{778} \right\} = h_2 + x_2 \rho_2$$

$$= (218.8 + 0.955 \times 869.0)$$
$$= 218.8 + 829.9 = 1048.7 \text{ B.t.u.},$$

where h_2 and ρ_2 are the heat of the liquid and internal latent heat respectively at 30 pounds per square inch absolute pressure.

The internal energy at the initial condition is

$$I_1 = h_1 + x_1\rho_1$$
$$= (181.0 + 0.50 \times 896.8)$$
$$= (181.0 + 448.4) = 629.4 \text{ B.t.u.}$$

Then,

$$Q = 1048.7 - 629.4$$
$$= 419.3 \text{ B.t.u.}$$

2. Expansion of Superheated Steam at Constant Volume. One pound of steam at 130 pounds per square inch absolute pressure and 50° F. superheat is heated under constant volume to a pressure of 180 pounds per square inch absolute.

Find (a) the final quality of the steam;
(b) the heat supplied.

Solution: (a) the initial volume of the steam is found by reference to the superheated steam tables to be 3.74 cubic feet. Since this equals the final volume of the steam, the final condition of the steam is 300° F. superheat, which from the steam tables is the condition when steam at 180 pounds per square inch absolute pressure has a volume of 3.74 cubic feet.

(b) The heat supplied equals

$$Q = I_2 - I_1 + W$$
$$= I_2 - I_1.$$

No values corresponding to the internal energy are available in the superheated tables and consequently these must be calculated.

$$I_2 = \left(H_{\text{sup.}} - \frac{P_2V_2}{778}\right), \qquad (152)$$

where

$H_{\text{sup.}}$ = the total heat of superheated steam as found in the superheat tables.

P_2 = pressure in pounds per square foot.
V_2 = volume of the superheated steam, cubic feet per pound.

Thus, for the conditions of the problem,

$$I_2 = \left(1353.9 - \frac{180 \times 144 \times 3.74}{778}\right)$$
$$= 1353.9 - 124.6 = 1229.3 \text{ B.t.u.}$$
$$I_1 = \left(1219.7 - \frac{130 \times 144 \times 3.74}{778}\right)$$
$$= 1219.7 - 89.5 = 1130.2 \text{ B.t.u.},$$

from which

$$Q = 1229.3 - 1130.2 = 99.1 \text{ B.t.u.}$$

3. Expansion at Constant Pressure. One pound of steam at a pressure of 150 pounds per square inch absolute and a volume of 1.506 cubic feet expands under constant pressure until it becomes dry and saturated.

1. What is the quality at the initial condition?

The volume of dry and saturated steam at the given pressure is 3.012 cubic feet per pound.

The quality then is $\dfrac{1.506}{3.012} = 0.50.$

2. What is the volume of the steam at the final condition?

The volume of a pound is 3.012 cubic feet since the steam is dry and saturated.

3. What is the work done during the expansion?

Work $= P_1 (V_2 - V_1) = 150 \times 144 (3.012 - 1.506) = 32{,}530$ foot-pounds.

4. How much heat is required? Several methods of reasoning may be used in calculating the values of the heat required and the work done during constant pressure expansion or compression.

$$Q = I_2 - I_1 + W$$
$$I_2 = h_2 + x_2 \rho_2$$
$$= 330.2 + 1 \times 780.4 = 1110.6$$

$$\begin{aligned} I_1 &= h_1 + x_1\rho_1 \\ &= (330.2 + 0.50 \times 780.4) \\ &= 720.4 \text{ B.t.u.} \end{aligned}$$

The heat equivalent of the external work during the expansion is

$$W = \frac{P(V_2 - V_1)}{778} \tag{153}$$

$$= \frac{150 \times 144 (1 \times 3.012 - 0.50 \times 3.012)}{778} = 41.8.$$

Substituting in equation (151),

$$\begin{aligned} Q &= 1110.6 - 720.4 + 41.8 \\ &= 432.0 \text{ B.t.u.} \end{aligned}$$

The amount of heat required in the case of constant pressure expansion may be calculated directly:

$$Q = (x_2 - x_1) L, \tag{154}$$

where x_2 and x_1 equal the qualities at the initial and final conditions of the expansion; L equals the latent heat of dry and saturated steam at the given pressure. Thus, for the above problem,

$$Q = (1.00 - 0.50)\, 863.2 = 431.6 \text{ B.t.u.}$$

Similarly, the heat supplied during constant pressure expansion may be expressed:

$$Q = H_2 - H_1,$$

where H_2 and H_1 are the total heat of the steam above 32° for the initial and final conditions respectively. Thus, for the problem in question,

$$\begin{aligned} H_2 &= 330.2 + 1.00 \times 863.2 \\ H_1 &= 330.2 + 0.50 \times 863.2 \\ Q &= (330.2 + 1.00 \times 863.2) - (330.2 + 0.50 \times 863.2) \\ &= 431.6 \text{ B.t.u.} \end{aligned}$$

Isothermal Lines for Steam. When the expansion of steam occurs at constant pressure as, for example, in the conversion of water into steam in a boiler when the engines are working, we have isothermal expansion. Isothermal lines for wet steam, which consists of a mixture of water and its vapor, are, therefore, straight lines of uniform pressure. On a pressure-volume diagram an isothermal line is consequently represented by a horizontal line parallel to the axis of abscissas. As steam becomes superheated the pressure decreases as the volume increases; for highly superheated vapors the isothermal curve approaches a rectangular hyperbola. On a T-ϕ diagram, the isothermal line is represented by a line of constant temperature, i.e., by a line parallel to the ϕ-axis.

Adiabatic Lines for Steam. Adiabatic lines will have different curvature depending upon the substances used. It will be remembered that the values of γ are different for the various gases and therefore the adiabatic line for each of these gases would have a different curvature. In the same way the curvature of adiabatic lines of steam will vary with the quality of the steam. Steam which is initially dry, if allowed to expand adiabatically, will become wet, the percentage of moisture which it will contain depending on the extent to which the expansion is carried. Also, on any T-ϕ diagram, an adiabatic (isentropic) line is represented by a line parallel to the T-axis, i.e., by a line of constant entropy. If steam is initially wet and is expanded adiabatically, it becomes wetter as a rule.*

In general, in any expansion, in order to keep steam at the same relative dryness as it was initially, while it is doing work, some heat must be supplied. If the expansion is adiabatic so that no heat is taken in, a part of the steam will be condensed and will form very small particles of water suspended in the steam, or it will be condensed as a sort of dew upon the surface of the enclosing vessel.

The relation between pressure and temperature as indicated by the steam-tables continues throughout an expansion, provided the steam is initially dry and saturated or wet.

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* When the percentage of water in steam is very great and the steam is expanded adiabatically there is a tendency for the steam to become drier. This is very evident from an inspection of diagrams like Fig. 25.

Adiabatic Curve for Steam. Whether steam is initially dry and saturated or wet, the adiabatic curve may be represented by the formula: $PV^n = \text{constant}$. The value of the index n depends on the initial dryness of the steam. Zeuner has determined the following relation:

$$n = 1.035 + \frac{x}{10}. \tag{155}$$

Solving this when $x =$ unity (dry and saturated steam) the value of n is 1.135, and when x is 0.75, n has the value 1.11.*

While the work during an adiabatic expansion may be calculated from the value of n as determined from Zeuner's relation (equation 155), the following method is more commonly used.

Since by definition the heat added or abstracted during an adiabatic process is zero, the heat equation becomes,

$$Q = I_2 - I_1 + W = 0,$$

or

$$W = I_1 - I_2 \tag{156}$$

The work done by an adiabatic process can then be determined by the difference between the internal energies at the beginning and end of the process.

In order to calculate the internal energies the qualities must be known and these may be found from the entropies.

Quality of Steam During Adiabatic Expansion. Since in an adiabatic expansion no heat transfer takes place, the entropy remains constant, and, therefore, on a T-ϕ diagram, this condition is represented by a straight vertical line as **cgf** (**Fig. 25**) or **smk.** Thus for **wet** steam:

$$\phi = \theta + x\frac{L}{T}, \tag{157}$$

where $x =$ the quality or dryness fraction of the steam, $\phi =$

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* Rankine gave the value of $n = \frac{10}{9}$, which obviously from the results given is much too low if the steam is at all near the dry and saturated condition. His value would be about right for the condition when $x = 0.75$. In an actual steam engine, the expansion of steam has, however, never a close approximation to the adiabatic condition, because there is always some heat being transferred to and from the steam and the metal of the cylinder and piston.

the total entropy of saturated steam, and θ = the entropy of the water.

Since in adiabatic expansion the entropy remains constant, the following equation can be written

$$\phi_1 = \phi_2 \text{ (total entropies)}$$

or

$$\theta_1 + x_1 \frac{L_1}{T_1} = \theta_2 + x_2 \frac{L_2}{T_2}. \tag{158}$$

Knowing the initial conditions of steam, the quality of the steam at any time during adiabatic expansion can be readily determined. Thus, suppose the initial pressure of dry saturated steam to be 100 pounds per square inch absolute, and the final pressure after adiabatic expansion 17 pounds per square inch.

From the steam tables we find that the total entropy ϕ_1, for dry steam at 100 pounds pressure, is 1.6020; that is

$$\phi_1 = 1.6020.$$

The entropies at 17 pounds pressure are also obtained from the tables,

$$1.6020 = 0.3229 + x_2\, 1.4215,$$

whence

$$x_2 = 0.899.$$

For a rapid and convenient means of checking the above result, the "**Total Heat-entropy**" diagram in the Appendix can be used. From the intersection of the 100-pound pressure line and that of unit quality ("saturation line") is dropped a vertical line (line of constant entropy = 1.602) to the 17-pound pressure line. This latter intersection is found to lie on the 0.90 quality line.

Example. One pound of steam having a quality of 0.95 at a pressure of 100 pounds per square inch absolute expands adiabatically to 15 pounds per square inch absolute.

1. What is the quality at the final condition?

The total entropy at the initial condition equals

$$\theta + x\frac{L}{T} = 0.4743 + 0.95 \times 1.1277 = 1.5456.$$

The total entropy at the end of the expansion equals

$$\theta + x\frac{L}{T} = 0.3133 + 1.4416\,x.$$

Since entropy is constant in adiabatic expansion

$$0.3133 + 1.4416\,x = 1.5456,$$

from which the final condition is: $x = 0.854$.

2. How much work is done during the expansion?

Since there is no heat added the work done equals the loss in internal energy.

The internal energy at the end of the expansion equals

$$h_2 + x_2\rho_2 = 181.0 + 0.854 \times 896.8 = 946.9 \text{ B.t.u.}$$

The internal energy at the initial condition equals

$$h_1 + x_1\rho_1 = 298.3 + 0.95 \times 806.6 = 1064.6 \text{ B.t.u.}$$

The work equals

$$1064.6 - 946.9 = 117.7 \text{ B.t.u. or } 91{,}576 \text{ foot-pounds.}$$

Polytropic Expansion ($n = 1$).

Example. One pound of steam has a pressure of 100 pounds per square inch absolute and a quality of 0.95. It expands along an $n = 1$ curve to 20 pounds per square inch absolute.

What is the quality at the end of the expansion?

Solution. The volume of the steam at the initial condition is $x \times V_{\text{sat.}}$

$$x \times V_{\text{sat.}} = 0.95 \times 4.429 = 4.207 \text{ cubic feet.}$$

Obviously, $P_1V_1^n = P_2V_2^n$

and since $n = 1$,

$$100 \times 4.207 = 20 \times V_2,$$

$$V_2 = 21.035 \text{ cubic feet.}$$

The volume of dry saturated steam at the end of the expansion or at 20 pounds per square inch is 20.08. Therefore the steam is superheated at end of expansion. How is this known?

Graphical Determination of Quality of Steam by Throttling Calorimeter and Total Heat-entropy Diagram. It will be remembered that the throttling calorimeter (pages 77 to 81) depends for its action upon the fact that the **total heat** of steam which expands without doing work remains the same, the heat in excess of that required to keep the steam dry and saturated going to superheat the steam. Suppose that steam enters the calorimeter at a pressure of 150 pounds per square inch absolute, and is throttled down to 17 pounds per square inch, the actual temperature being 240° F. Since the saturation temperature for steam at 17 pounds pressure is 219.4, the steam in the calorimeter is superheated 240° − 219.4° or 20.6 degrees. In order to find the quality of the live steam refer to the "**Mollier Diagram**" (Appendix or **Fig. 28**) and find the intersection of the 20.6 degrees superheat line with the 17-pound pressure line. From this point follow

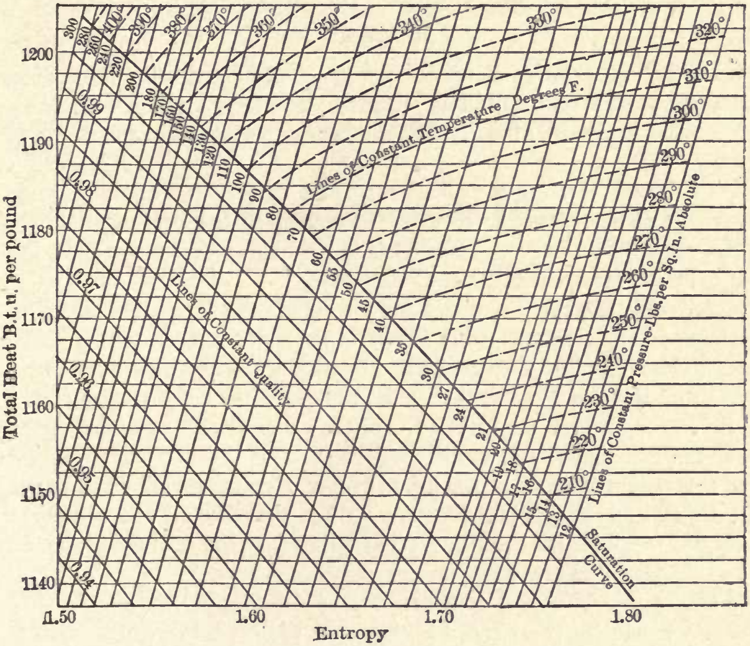

FIG. 28. — Mollier Diagram for Determining Quality of Steam.

a horizontal line (line of constant total heat) to the left until it intersects the 150-pound pressure line. This point of intersection is found to lie on the 0.96 quality line.

Formula 117 (page 80) gives the following result in close agreement with the diagram:

$$x_1 = \frac{1153.1 + 0.47\,(240 - 219.4) - 330.2}{863.2}$$
$$= 0.965.$$

PROBLEMS

1. Dry saturated steam at 100 lbs. per sq. in. absolute pressure contained in a closed tank is cooled until its pressure drops to 15 lbs. per sq. in. absolute. What is the final quality and the heat removed from each pound of steam?

2. One pound of steam at 15 lbs. per sq. in. absolute has a volume of 12.36 cu. ft. It is heated under constant volume until the pressure becomes 50 lbs. per sq. in. absolute. (a) What is the quality before and after the heating? (b) How much heat was supplied?

3. A closed tank containing dry and saturated steam at 15 lbs. per sq. in. absolute pressure is submerged in a body of water at a temperature of 59° F. What will be the ultimate pressure and quality of steam within the tank?

4. Prove that for a constant pressure expansion the heat supplied equals the difference between the total heats of the vapor at the beginning and at the end of the expansion.

5. One pound of steam at 100 lbs. per sq. in. absolute pressure and 50 per cent dry expands at constant pressure. What work is done and what heat is required to double the volume? What is the temperature at the beginning and at the end of the expansion?

6. One pound of steam at a pressure of 100 lbs. per sq. in. absolute and 50 per cent dry is expanded isothermally until it is dry and saturated. Find the heat supplied and the work done.

7. One pound of steam at a pressure of 150 lbs. per sq. in. absolute has a quality of 0.90. What work is done and what heat is required to double its volume at constant pressure?

8. If steam at 200 lbs. per sq. in. absolute, 95 per cent dry, is caused to expand adiabatically to 228° F., what are the properties of this steam at the lower point? (That is, final total entropy, entropy of evaporation, quality and volume.)

9. What will be the final total heat of dry saturated steam that is expanded adiabatically from 150 lbs. per sq. in. absolute down to 10 lbs. per sq. in. absolute?

10. Steam having a quality of 0.20 dry is compressed along an adiabatic curve from a pressure of 20 lbs. per sq. in. absolute to a pressure corresponding to a temperature of 293° F. What is the final quality?

11. Determine the final quality of the steam and find the quantity of work performed by 2 lbs. of steam in expanding adiabatically from 250 lbs. per sq. in. absolute pressure to 100 lbs. per sq. in. absolute, the steam being initially dry and saturated.

12. One pound of steam at 150 lbs. per sq. in. absolute, and 200° F. superheat, expands adiabatically. What is the pressure when the steam becomes dry and saturated? What is the work done during the expansion?

13. One pound of dry and saturated steam at 15 lbs per. sq. in. absolute pressure is compressed adiabatically to 100 lbs. per sq. in. absolute pressure. What is the quality at the end of the compression and the negative work done?

14. One pound of steam at 100 lbs. per sq. in. absolute pressure has a quality of 0.80. It expands along an $n = 1$ curve to a pressure of 15 lbs. per sq. in. absolute. (a) What is the volume at the beginning and at the end of the expansion, (b) the quality at the end of the expansion, (c) the work done during the expansion, (d) the heat supplied to produce the expansion?

15. One pound of steam at 150 lbs. per sq. in. absolute expands along an $n = 1$ curve to 15 lbs. per sq. in. absolute. The quality of the steam at 15 pounds is to be dry and saturated. (a) What must be the quality at the initial conditions? (b) What work will be done by the expansion? (c) What heat must be supplied?

16. Steam at a pressure of 100 lbs. per sq. in. absolute having a quality of 0.50 expands adiabatically to 15 lbs. per sq. in. absolute. What is the quality at the end of the expansion?

17. One pound of steam at a pressure of 100 lbs. per sq. in. absolute has a volume of 4 cu. ft. and expands adiabatically to 15 lbs. per sq. in. absolute.

(a) What is the quality at the initial and final conditions?
(b) What is the work done during the expansion?

18. One pound of steam having a pressure of 125 lbs. per sq. in absolute and volume of 4.17 cu. ft. expands adiabatically to 25 lbs. per sq. in. absolute.

(a) What is the quality at the initial and final conditions?
(b) What is the work of expansion?

19. Given the steam as stated in problem 18 but with expansion complete at 100 lbs. per sq. in. absolute. What is the quality at this pressure?

20. What would be the pressure if the steam in problem 18 were expanded adiabatically until it became dry and saturated?

21. One pound of steam at a temperature of 360° F. has a quality of 0.50, and expands under constant pressure to a volume of 3.4 cu. ft.

(a) What is the quality at the final condition?
(b) What is the work of the expansion?
(c) What heat is required?

22. Two pounds of steam at a pressure of 100 lbs. per sq. in. absolute have a volume of 4 cu. ft., and expand under constant temperature to a volume of 8 cu. ft.

(a) What is the quality at the initial and final conditions?
(b) What is the work of the expansion?
(c) How much heat is required?

CHAPTER VIII

CYCLES OF HEAT ENGINES USING VAPORS

Carnot Cycle. The Carnot cycle using a vapor employs the same apparatus as was explained on page 40. The cycle is made up of two isothermals and two adiabatics, but differs from the cycle using gas as the working medium in that the isothermal curves are lines of constant pressure.

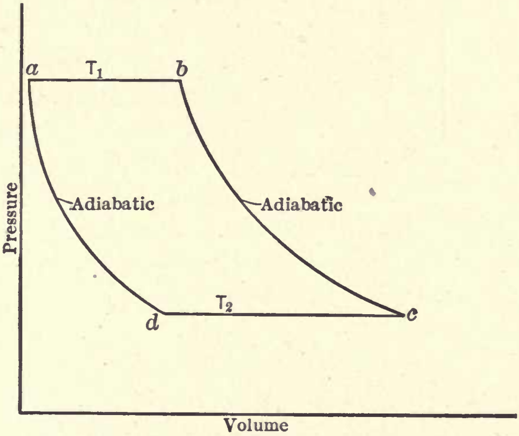

FIG. 29. — Carnot Cycle using Vapors.

As an illustration, assume that the vapor in the cylinder is in the liquid state and at a temperature T_1 when the heat is applied. The heating process continues until the vapor becomes dry and saturated. In order to maintain the temperature constant the heat must be supplied at such a rate as to maintain the pressure constant. The volume increases during this change from that of the specific volume of the liquid to the specific volume of the dry saturated vapor at the temperature T_1. This change is represented by the lines *ab* on the *PV* and temperature-entropy diagrams, **Figs.** 29 and 30.

The cylinder is then removed from the source of heat and the adiabatic expansion is produced. This is represented in **Figs. 29** and **30** by the curves *bc*.

The isothermal compression curve *cd* is produced when the cylinder is placed in communication with the refrigerator or condenser and heat is absorbed from the vapor. Since the cylinder contains a saturated vapor, the process results in a constant pressure curve at temperature, T_2.

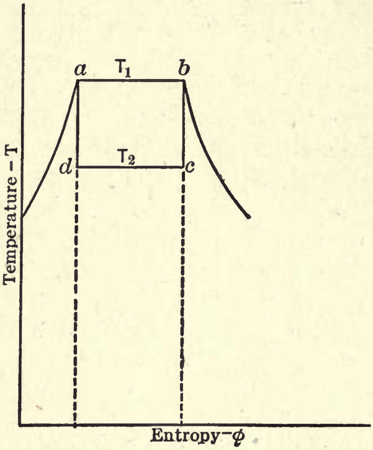

FIG. 30.—Temperature-entropy Diagram of Carnot Cycle using Vapors.

The adiabatic compression curve *da* follows when the cylinder is removed from the refrigerator. The vapor in its final condition is reduced to a liquid at the temperature T_1.

The heat added to the cycle is that required to produce the line *ab*, **Figs. 29** and **30**. For the assumed case this is:

$$Q_1 = H_b - h_a = L_1, \qquad (159)$$

where H_b = total heat of dry saturated steam at the pressure P_b,
h_a = heat of liquid above 32° F. for condition at *a*.

The heat rejected from the cycle is that absorbed by the refrigerator and for each pound of vapor it is equal to

$$Q_2 = (h_c + x_c L_c) - (h_d + x_d L_d). \qquad (160)$$

From the temperature-entropy diagram, **Fig. 30**, the heat added and heat rejected are

$$Q_1 = T_1(\phi_b - \phi_a), \qquad (161)$$

$$Q_2 = T_2(\phi_c - \phi_d). \qquad (162)$$

Since $(\phi_c - \phi_d) = (\phi_b - \phi_a)$,

$$Q_2 = T_2(\phi_b - \phi_a). \qquad (163)$$

The work of the cycle may be determined from the algebraic sums of the work under the individual curves, but can be more simply calculated from the temperature-entropy diagram, **Fig. 30.** The area *abcd* is proportional to the work **and since it is a rectangle,**

$$\text{work of cycle} = (T_2 - T_1)(\phi_b - \phi_a).$$

From equations (161) and (163), the efficiency of the cycle is

$$E = \frac{(T_2 - T_1)(\phi_b - \phi_a)}{T_1(\phi_b - \phi_a)}$$

$$= \frac{T_2 - T_1}{T_1}. \qquad (164)$$

Equation (164) shows that the efficiency of Carnot's cycle is not affected by the character of the working substance and is dependent only upon its initial and final temperatures.

The Rankine Cycle.* The Carnot cycle gives the maximum efficiency obtainable for a heat engine operating between given limits of temperature. In order that a steam engine may work on a Carnot cycle, the steam must be evaporated in the cylinder instead of in a separate boiler, and condensed in the cylinder instead of being rejected to the air or to a separate condenser. Such conditions are obviously impracticable, and it has, therefore, been found necessary to adopt some other cycle which conforms more with practical conditions. The most efficient practical steam cycle, and the one which has been adopted as the **standard** with which the efficiency of all steam engines may be compared is the **Rankine Cycle.** The pressure-volume diagram of this cycle is shown in **Fig. 31.** Steam is admitted at constant pressure and temperature along **ab** to a cylinder without clearance. At **b** cut-off occurs, and the steam expands adiabatically from **b** to **c,** some of **i** condensing during the process. The steam is then discharged at constant pressure and temperature along the back pressure line **cd.** Line **da** represents the rise in temperature and

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* Also known as the **Clausius Cycle,** having been published simultaneously and independently by Clausius.

pressure at constant volume when the condensed steam is converted into a vapor.

The four stages of the Rankine cycle may be stated as follows:

(1) Feed water raised from temperature of exhaust to temperature of admission steam. (Line **da.**)

(2) Evaporation at constant admission temperature. (Line **ab.**)

(3) Adiabatic expansion down to back pressure. (Line **bc.**)

(4) Rejection of steam at the constant temperature corresponding to the back pressure. (Line **cd.**)

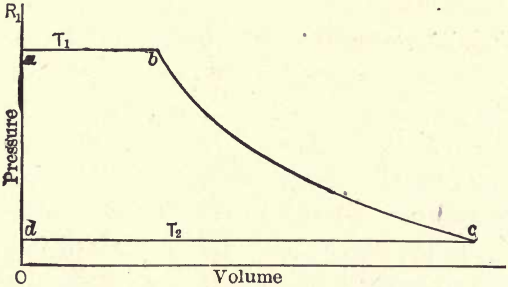

FIG. 31. — Indicator Diagram of Ideal Rankine Cycle.

The net work done in the cycle, assuming one pound of wet steam, is calculated as follows:

1. External work of evaporation (in B.t.u.),

$$(W_{ab}) = \frac{1}{778}(P_1 V_1 x_1). \tag{165}$$

2. Loss in internal energy,

$$(W_{bc}) = h_1 + x_1\rho_1 - (h_2 + x_2\rho_2) \text{ B.t.u.} \tag{166}$$

3. External work of evaporation at temperature of exhaust (B.t.u.),

$$W_{dc} = -\frac{1}{778}(P_c V_c) = -\frac{1}{778}(P_2 V_2 x_2). \tag{167}$$

4. $$W_{da} = 0. \tag{168}$$

Adding,

Net work of cycle,

$$W = \tfrac{1}{778} P_1 V_1 x_1 + h_1 + x_1 \rho_1 - h_2 - x_2 \rho_2 - \tfrac{1}{778} P_2 V_2 x_2,^*$$

but

$$\tfrac{1}{778} P_1 V_1 + \rho_1 = L_1.$$

Therefore,

$$W = h_1 + x_1 L_1 - (h_2 + x_2 L_2) \text{ (B.t.u.).} \qquad (169)$$

Thus the net work of the Rankine cycle is equal to the difference between the total heat of the steam admitted and the total heat of the steam exhausted. This statement applies whether the steam is initially wet, dry or superheated.

The heat added during the cycle is

$$Q_1 = h_a + x_b L_b - h_d. \qquad (170)$$

The heat rejected during the cycle is

$$Q_2 = x_c L_c. \qquad (171)$$

The efficiency of the Rankine cycle is

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{h_a + x_b L_b - h_d - x_c L_c}{h_a + x_b L_b - h_d}.$$

Since $h_a = h_b$ and $h_c = h_d$,

$$E = \frac{h_b + x_b L_b - h_c - x_c L_c}{h_b + x_b L_b - h_c}. \qquad (172)$$

Fig. 32 is the T-ϕ diagram for a Rankine cycle using dry saturated steam. The letters *abcd* refer to the corresponding points in the pressure-volume diagram **Fig. 31.** The net work of the cycle is the area **B + C,** which is the difference between the total heats at admission and exhaust. The heat added per cycle is represented by the areas **A + B + C + D** and the

$$\text{Thermal efficiency} = \frac{B + C}{A + B + C + D} \qquad (173)$$

$$= \frac{h_1 + x_1 L_1 - h_2 - x_2 L_2}{h_1 + x_1 L_1 - h_2}. \qquad (174)$$

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* The final quality can be determined by equation (158).

h_2 in the denominator must always be subtracted from $h_1 + x_1L_1$ (the total heat above 32° F.), in order to give the total heat above the temperature of feed water, which in engine tests is always assumed for the purpose of comparison to be the same as the exhaust temperature.

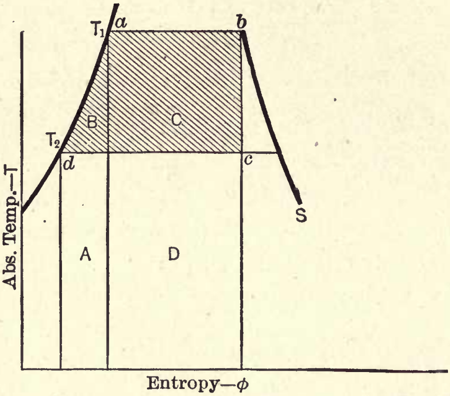

FIG. 32. — Temperature-entropy Diagram of Rankine Cycle.

Example. One pound of steam at a pressure of 160 pounds per square inch absolute and quality of 0.95 performs a Rankine cycle exhausting at 5 pounds per square inch absolute.

1. What is the quality of the exhaust?

The total entropy at the initial condition

$$= 0.5208 + 0.95 \times 1.0431.$$

The total entropy at the exhaust

$$= 0.2348 + 1.608\,x.$$

Then $\quad 0.5208 + 0.95 \times 1.0431 = 0.2348 + 1.6084\,x.$

From which $\quad x = 0.794.$

2. What is the net work of the cycle?

$$\text{Work} = H_1 - H_2 = 335.6 + 0.95 \times 858.8 - 130.1 - 0.794 \times 1000.3 = 227.2 \text{ B.t.u.}$$

3. What is the efficiency of the cycle?

$$\text{Efficiency} = \frac{227.2}{335.6 + 0.95 \times 858.8 - 130.1} = 0.222 \text{ or } 22.2 \text{ per cent.}$$

In the actual steam engine cycle it is impractical to expand the steam down to the back-pressure line.

The Rankine cycle for the actual steam engine is similar to that described, except that the adiabatic expansion terminates at a pressure higher than that of the back-pressure, that is, the expansion is incomplete.

The pressure-volume and T-ϕ diagrams of this modified Rankine cycle using dry saturated steam are shown in **Figs. 33** and **34**. The cylinder is without clearance. Steam is admitted at constant pressure and temperature along *ab*. Cut-off occurs at point *b* and the steam expands adiabatically to the terminal pressure *c*. Part of the steam is discharged at constant volume *cd*. The remainder is exhausted during the back-pressure stroke *de*. Line *ea* represents the rise in temperature of the feed water from T_2 to T_1.

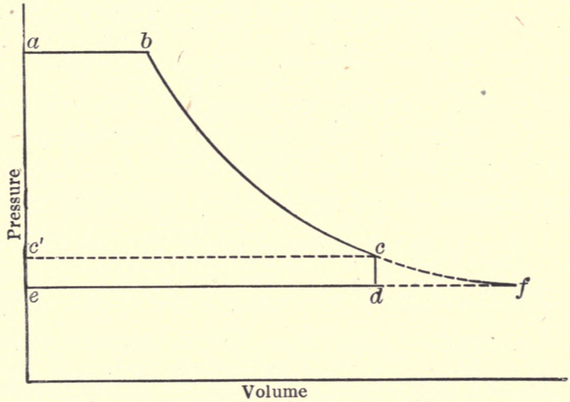

FIG. 33. — Rankine Cycle, Incomplete Expansion.

The net work of the cycle can best be calculated by dividing **Fig. 33** into the two areas *abcc'* and *c'cde*. The area *abcc'* is that of the theoretical or ideal Rankine cycle, while *c'cde* is that of the rectangle. The net work of the cycle is equal to the sum of these two areas.

The heat equivalent of the area *abcc′* is

$$h_b + x_b L_b - h_c - x_c L_c. \tag{175}$$

The area *c′cde* is

$$\frac{1}{778}(P_c - P_d)(V_d - 0). \tag{176}$$

The total heat equivalent of the work of the cycle is

$$h_b + x_b L_b - h_c - x_c L_c + \frac{(P_c - P_d)\,V_d}{778}. \tag{177}$$

The heat added to the cycle is

$$h_b + x_b L_b - h_d. \tag{178}$$

The efficiency of the cycle is

$$E = \frac{h_b x_b L_b - h_c - x_c L_c + \dfrac{(P_c - P_d)\,V_d}{778}}{h_b + x_b L_b - h_d}. \tag{179}$$

In the T-ϕ diagram, **Fig. 34**, the letters *abcde* refer to corresponding points in the pressure-volume diagram (**Fig. 33**). The net work of the cycle is represented by the area $B + C$. The heat added to the cycle is represented by the area $A + B + C + D$.

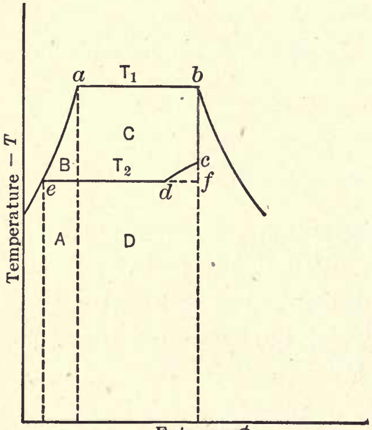

FIG. 34. — Rankine Cycle, Incomplete Expansion.

The incomplete Rankine cycle (**Figs. 33** and **34**) is less efficient than the ideal cycle (**Figs. 31** and **32**), because of failure to expand the steam completely. This loss is represented in **Figs. 33** and **34** graphically by the area *cdf*. This cycle is sometimes used as a standard in preference to the ideal Rankine cycle when the efficiencies of engines are compared.

Example. One pound of steam at a pressure of 160 pounds per square inch absolute and quality of 0.95 goes through a

Rankine cycle. The terminal pressure is 15 pounds per square inch absolute and the exhaust pressure 5 pounds per square inch absolute. What is (a) the work of the cycle, (b) the heat added to the cycle, and (c) the efficiency of the cycle?

Solution. The total entropy of the steam at the initial condition is

$$0.5208 + 0.95 \times 1.0431.$$

The total entropy of the steam at the terminal pressure is

$$0.3133 + x\,(1.4416).$$

Then $0.5208 + 0.95 \times 1.0431 = 0.3133 + x\,(1.4416)$

from which the quality at the terminal pressure is

$$x = 0.831.$$

The volume of the steam at the terminal pressure is

$$0.831 \times 26.27 = 21.83.$$

The total heat of the steam at the initial condition is

$$335.6 + 0.95 \times 858.8 = 1151.5.$$

The total heat of the steam at the terminal pressure is

$$181.0 + 0.831 \times 969.7 = 986.8.$$

The work of the cycle is then (in terms of heat units),

$$W = 1151.5 - 986.8 + \frac{(15 - 5) \times 144 \times 21.83}{778} = 205.1.$$

The heat added to the cycle is

$$335.6 + 0.95 \times 858.8 - 130.1 = 1021.4 \text{ B.t.u.}$$

The efficiency of the cycle is

$$\frac{205.1}{1021.4} = 20.08 \text{ per cent.}$$

The Practical or Actual Steam Engine Cycle. In the steam engine designed for practical operation it is impossible to expand the steam down to the back-pressure line; and, furthermore, it is evident that some mechanical clearance must be provided. The result is that in the indicator diagram from the actual steam engine, we have to deal with a clearance volume, and both incomplete expansion and incomplete compression as shown in **Fig. 35.** In order to calculate the theoretical efficiency of this practical cycle, it is necessary to assume that the expansion line **cd** and the compression line **fa** are adiabatic. Knowing then the cylinder feed of steam per stroke and the pressure and volume relations as determined from the indicator diagram, one can calculate the theoretical thermal efficiency by obtaining the net area of the diagram (expressed in B.t.u.) and dividing by the heat supplied per cycle. In order to obtain the net area of the diagram, the latter may be divided up into several simple parts as follows:

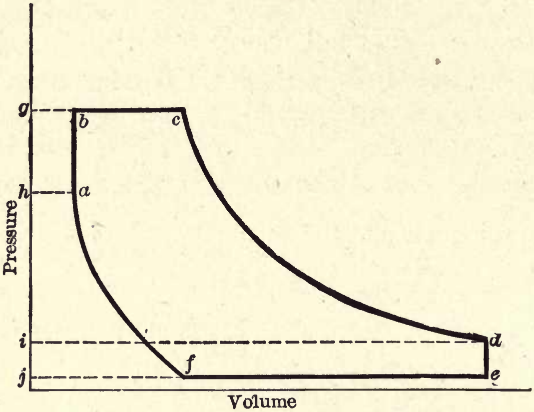

FIG. 35. — Indicator Diagram of Practical Engine Cycle.

gcdi — area equivalent to a Rankine cycle,
idej — a rectangle,
hafj — area equivalent to a Rankine cycle (negative),
gbah — a rectangle (negative).

The actual efficiency of the steam engine is usually determined by dividing the heat equivalent to a horse power by the heat in the steam required to produce a horse power. Since one horse power per hour is equal to $\frac{33{,}000 \times 60}{778}$ or 2545 B.t.u., the actual efficiency of a steam engine is

$$E = \frac{2545}{WR\ (H - h)}. \qquad (180)$$

In equation (180) WR is the water rate or the steam consumption per horse power per hour, H is the total heat in the steam at the initial pressure and quality as it enters the engine, h is the heat in the feed water corresponding to the exhaust pressure.

Efficiency of an Engine Using Steam Without Expansion. In the early history of the steam engine, nothing was known about the "expansive" power of steam. Up to the time of Watt in all steam engines the steam was admitted at full boiler pressure at the beginning of every stroke and the steam at that pressure carried the piston forward to the end of the stroke without any diminution of pressure. Under these circumstances the volume of steam used at each stroke at boiler pressure is equal to the volume swept through by the piston.

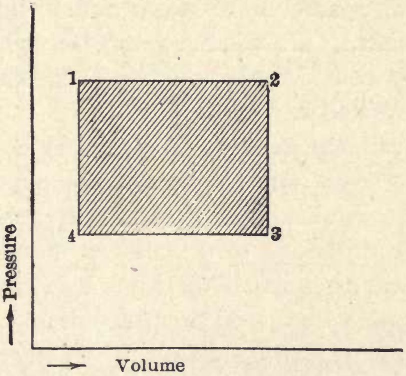

FIG. 36. — Indicator Diagram of an Engine using Steam without Expansion.

An indicator diagram representing the use of steam in an engine without expansion is shown in **Fig. 36.** This diagram represents steam being taken into the engine cylinder at 1 at the boiler pressure. It forces the piston out to the point 2 when the exhaust opens and the pressure drops rapidly from 2 to 3. On the back stroke from 3 to 4 steam is forced out of the cylinder into the exhaust pipe. At 4 the pressure rises rapidly to that at 1 due to the rapid admission of fresh steam into the cylinder. In this case the thermal efficiency (**E**) is represented by

$$E = \frac{\text{work done}}{\text{heat taken in}} = \frac{(P_1 - P_4)(V_2 - V_1)}{778\ (x_1L_1 + h_1 - h_3)}, \qquad (181)$$

where the denominator represents the amount of heat taken in, with the feedwater at temperature of exhaust, t_2. In actual practice the efficiency of engines using steam without expansion is about 0.06 to 0.07, when the temperature of condensation is about 100° F. When steam is used in an engine without expansion and also without the use of a condenser the value of this efficiency is still lower. It will be observed that under the most favorable conditions obtainable the efficiency of an engine without expansion cannot be made under normal conditions to exceed about 7 per cent.

In the actual Newcomen steam engines the efficiency was very much lower than any of the values given because at every stroke of the piston a very much larger amount of steam had to be taken in than that corresponding to the volume swept through by the piston on account of a considerable quantity of steam condensing on the walls of the cylinder.

Adiabatic Expansion and Available Energy. A practical example as to how the temperature-entropy diagram can be used to show how much work can be obtained by a theoretically perfect engine from the adiabatic expansion of a pound of steam will now be given. When steam expands adiabatically — without a gain or loss of heat by conduction — its temperature falls. Remembering that areas in the temperature-entropy diagram represent quantities of heat and that in this expansion there is no exchange of heat, it is obvious that the area under a curve of adiabatic expansion must be zero; this condition can be satisfied only by a vertical line which is a line of constant entropy.

The work done during an adiabatic process, while it cannot be obtained from a "heat diagram," can very readily be determined from the area under the adiabatic curve of a pressure-volume diagram, or better still by the use of steam-tables as follows: In an adiabatic expansion the amount of work done is the mechanical equivalent of the loss in internal energy as explained in Chapter III. Therefore, it is only necessary to determine the internal energy of the steam at the beginning and end of the adiabatic expansion.

$$I_1 = h_1 + x_1\rho_1, \tag{182}$$

$$I_2 = h_2 + x_2\rho_2. \tag{183}$$

Work during adiabatic expansion = loss in internal energy

$$W = (h_1 + x_1\rho_1 - h_2 - x_2\rho_2)\ 778 \text{ (foot-pounds)}. \tag{184}$$

Fig. 37 is a temperature-entropy diagram representing dry saturated steam which is expanded adiabatically from an initial temperature T_1 (corresponding to a pressure P_1) to a lower final temperature T_2 (corresponding to a pressure P_2). The initial and final conditions of total heat (H) and entropy (ϕ) are represented by the same subscripts 1 and 2. The available energy or the work that can be done by a perfect engine under these conditions is the area **NCEG**. It is now desired to obtain a simple equation expressing this available energy $\mathbf{E}_a$ in terms of total heat, absolute temperature and entropy.

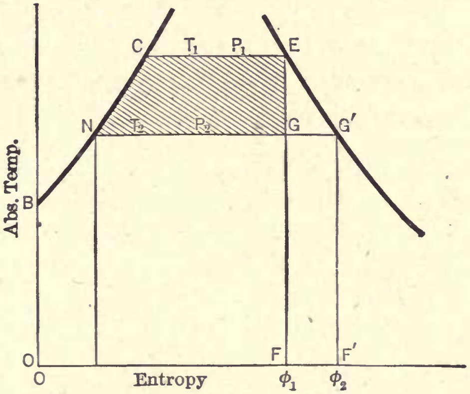

FIG. 37. — Temperature-entropy Diagram for Dry Saturated Steam Expanded Adiabatically.

H_1 = area **OBNCEF**,

H_2 = area **OBNG′F′**,

E_a = area (**OBNCEF** + **FGG′F′**) − **OBNG′F′**,

$$E_a = H_1 - H_2 + (\phi_2 - \phi_1)\,T_2.^* \tag{185}$$

---

* It should be observed that this form is for the case where the steam is **initially dry and saturated.** For the case of superheated steam a slightly different form is required.

An application of this equation will be made to determine the heat energy available from the adiabatic expansion of a pound of dry saturated steam from an initial pressure of 165 pounds per square inch absolute to a final pressure of 15 pounds per square inch absolute.

Example. $P_1 = 165$ $T_1 = 826$ degrees, from steam tables.
$P_2 = 15$ $T_2 = 673.0$ " "
$H_1 = 1195.0$ B.t.u. " "
$H_2 = 1150.7$ B.t.u. " "
$\phi_1 = 1.5615$ " "
$\phi_2 = 1.7549$ " "

Substituting these values in equation (185), we have

$$E_a = 1195.0 - 1150.7 + (1.7549 - 1.5615)\,673 = 174.5 \text{ B.t.u. per pound of steam.}$$

Now if in a suitable piece of apparatus like a **steam turbine nozzle,** all this energy that is theoretically available could be changed into velocity, then we have by the well-known formula in mechanics, for unit mass,*

$$\frac{V^2}{2g} = E_a \text{ (foot-pounds)} = E_a \text{ (B.t.u.)} \times 778,$$

$$V = \sqrt{778 \times 2\,gE_a} = 223.8\,\sqrt{E_a}, \qquad (186)$$

where V is the velocity of the jet and g is the acceleration due to gravity (32.2), both in feet per second.

Solving then for the theoretical velocity obtainable from the available energy we obtain the following:

$$V = 223.8\,\sqrt{174.5} = 223.8 \times 13.22 = 2956 \text{ feet per second.}$$

The important condition assumed as the basis for determining equation (185), that the steam is **initially dry and saturated,** must not be overlooked in its application. There are, therefore, two other cases to be considered:

(1) When the steam is initially wet,
(2) When the steam is initially superheated.

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* See Church's *Mechanics of Engineering*, page 672, or Jameson's *Applied Mechanics and Mechanical Engineering*, vol. I, page 47.

Available Energy of Wet Steam. The case of initially wet steam is easily treated in the same way as dry and saturated steam. **Fig. 38** is an example of the case in hand. At the initial pressure P_1 the total heat of a pound of wet steam $(h_1 + x_1L_1)$ is represented in this diagram by the area **OBNCE″F″**. The initial quality of the steam (x_1) is represented by the ratio of the lines $\frac{\mathbf{CE''}}{\mathbf{CE}}$. The available energy from adiabatic expansion from the initial temperature T_1 (corresponding to the pressure P_1) to the final temperature T_2 (corresponding to the pressure P_2) is the area **NCE″G″**. If we call this available energy E_{aw}, we have by manipulation of the areas,

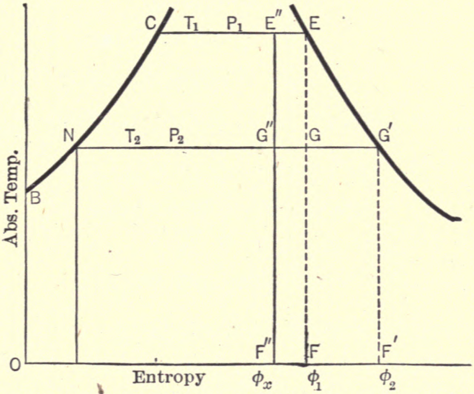

FIG. 38. — Temperature-entropy Diagram of Wet Steam Expanded Adiabatically.

$$E_{aw} = \text{area } \mathbf{OBNCEF} + \mathbf{FGG'F'} - \mathbf{OBNG'F'} - \mathbf{G''E''EG},$$

$$E_{aw} = H_1 - H_2 + (\phi_2 - \phi_1)\,T_2 - (\phi_1 - \phi_x)\,(T_1 - T_2)\,^{*} \quad (187)$$

or,

$$E_{aw} = H_1 - H_2 + (\phi_2 - \phi_1)\,T_2 - \frac{L_1}{T_1}(1 - x_1)\,(T_1 - T_2). \quad (188)$$

The velocity corresponding to this energy is found by substitution in equation (186), just as for the case when the steam was initially dry and saturated.

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* $\phi_1 = \frac{L_1}{T_1} + \theta_1, \quad \phi_x = x_1\frac{L_1}{T_1} + \theta_1, \quad \phi_1 - \phi_x = \frac{L_1}{T_1}(1 - x_1).$

Example. Calculations for the velocity resulting from adiabatic expansion for the same conditions given in the preceding example, except that the steam is initially 5 per cent wet, are given below.

$P_1 = 165$ lbs. absolute. $T_1 = 826$ degrees from tables
$P_2 = 15$ lbs. absolute. $T_2 = 673.0$ degrees " "
$H_1 = 1195.0$ B.t.u. " "
$H_2 = 1150.7$ " " "
$\phi_1 = 1.5615$ " "
$\phi_2 = 1.7549$ " "
$L_1 = 856.8$ B.t.u. " "
$x_1 = 1.00 - 0.05 = 0.95$

$$E_{aw} = (1195.0 - 1150.7) + (1.7549 - 1.5615)\,673 - \frac{856.8}{826}$$

$$\times (1 - 0.95)\,(826 - 673) = 44.3 + 130.2 - 7.93$$

$E_{aw} = 166.5$ B.t.u. per pound of wet steam

$$V = 223.8\,\sqrt{E_{aw}} = 223.8 \times \sqrt{166.5}$$

$$= 223.8 \times 12.9 = 2886 \text{ feet per second.}$$

This result can be checked very quickly by the "total heat-entropy" or "Mollier" diagram (Appendix). The intersection of the 0.95 quality line and 165 pounds pressure line is found to lie on the 1152 B.t.u. total heat line. Since the expansion is adiabatic, the entropy remains constant. Therefore, following the vertical or constant entropy line (entropy = about 1.507) down to its intersection with the 15 pounds pressure line, we find that the total heat at the end of adiabatic expansion is 985 B.t.u and

$$1152 - 985 = 167 \text{ B.t.u. available energy, as above.}$$

If the steam were initially superheated, the available energy during adiabatic expansion could be obtained in the same way by means of the diagram.

Available Energy of Superheated Steam. The amount of energy that becomes available in the adiabatic expansion of superheated steam is very easily expressed with the help of **Fig. 39.** Two conditions after expansion must be considered:

(1) When the steam in the final condition is superheated,

(2) When the steam in the final condition is wet (or dry saturated).

Using **Fig. 39** with the notation as before except $\mathbf{E}_{as}$ is the available energy from the adiabatic expansion of steam initially superheated in B.t.u. per pound, ϕ_s and $\mathbf{H}_s$ are respectively the total entropy and the total heat of the superheated steam at the initial condition, then from the diagram, when the steam is wet at the final condition,

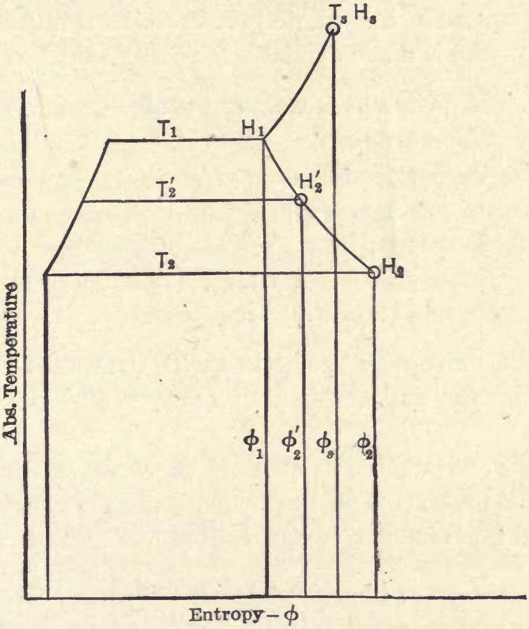

Fig. 39. — Temperature-entropy Diagram for Superheated Steam.

$$\mathbf{E}_{as} = H_s - H_2 + (\phi_2 - \phi_s)\, T_2. \qquad (189)$$

When the steam is superheated at the final condition,

$$\mathbf{E}_{as} = H_s - H_2' - (\phi_s - \phi_2')\, T_2'. \qquad (190)$$

It will be observed that these equations (189) and (190) are the same in form as (184), and that equation (190) differs only in having the terms H_s and ϕ_s in the place of H_1 and ϕ_1. In other words equation (184) can be used for superheated steam if the total heat and entropy are read from the steam tables for the required degrees of initial superheat.

The following examples illustrate the simplicity of calculations with these equations:

Example 1. Steam at 150 pounds per square inch absolute pressure and 300° F. superheat is expanded adiabatically to 1 pound per square inch absolute pressure. How much energy in B.t.u. per pound is made available for doing work?

Solution.
$$
\begin{aligned}
H_s &= 1348.8 \text{ B.t.u. per pound,}\\
H_2 &= 1103.6 \quad \text{"} \quad \text{"} \quad \text{"}\\
\phi_2 &= 1.980,\\
\phi_s &= 1.732,\\
T_2 &= 559.6^\circ \text{ F.,}\\
E_{as} &= 1348.8 - 1103.6 + (1.980 - 1.732)\, 559.6\\
&= 383.9 \text{ B.t.u. per pound.}
\end{aligned}
$$

The result above may be checked with the total heat-entropy chart (Appendix) and obtain thus (1349 − 967) or 382 B.t.u. per pound.

Example 2. Data the same as in preceding example except that the final pressure is now 35 pounds per square inch absolute. (Final condition of steam is superheated.) Calculate E_{as}.

Solution.
$$
\begin{aligned}
H_s &= 1348.8 \text{ B.t.u. per pound,}\\
H_2' &= 1166.8 \quad \text{"} \quad \text{"} \quad \text{"}\\
\phi_s &= 1.732,\\
\phi_2' &= 1.6868,\\
T_2' &= 718.9^\circ \text{ F.,}\\
E_{as} &= 1348.8 - 1166.8 - (1.7320 - 1.6868)\, 718.9\\
&= 149.5 \text{ B.t.u. per pound.}
\end{aligned}
$$

Application of Temperature-entropy Diagram to Analysis of Steam Engine. The working conditions of a steam engine, as stated before, can be shown not only by the indicator card, but also by the employment of what is known as a "temperature-entropy" diagram. This diagram represents graphically the amount of heat actually transformed into work, and in addition the distribution of losses, in the steam engine.

For illustration, a card was taken from a Corliss steam engine having a cylinder volume of 1.325 cubic feet, with a clearance volume of 7.74 per cent, or 0.103 cubic feet; the weight of steam in pounds per stroke (cylinder feed plus clearance) was 0.14664 pounds. Barometer registered atmospheric condition as 14.5 pounds per square inch. The scale of the indicator spring used in getting the card was 80 pounds to the inch. Steam chest pressure was taken as 153 pounds per square inch (absolute), and a calorimeter determination showed the steam to be dry and saturated.

The preliminary work in transferring the indicator card to a T-ϕ diagram, consists first in preparing the indicator card as follows: It was divided into horizontal strips at pressure intervals of 10 pounds with the absolute zero line taken as a reference; this line was laid off 14.5 pounds below atmospheric conditions. (See **Fig. 40.**) For reference, the saturation curve was drawn. Knowing the weight of steam consumed per stroke and the specific volume of the steam (from the Steam Tables), for various pressures taken from the card, the corresponding actual volumes could be obtained; this operation is, merely, weight of steam per stroke multiplied by specific volume for some pressure (0.14664 × column 5 in the table below), the resulting value being the volume in cubic feet for that condition. These pressures and volumes were plotted on the card and the points joined, resulting in the saturation curve, $2''$–$6''$–$8''$–$9''$.

The next step consisted in constructing the "transformation table" with the columns headed as shown. All the condensing and evaporation processes are assumed to take place in the cylinder and the T-ϕ diagram is then worked up for a total weight of **one pound** of steam as is customary. Column 1 shows the respective point numbers that were noted on the card; column 2, the absolute pressures for such points; column 3, the corresponding temperatures for such pressures; column 4, the vol-

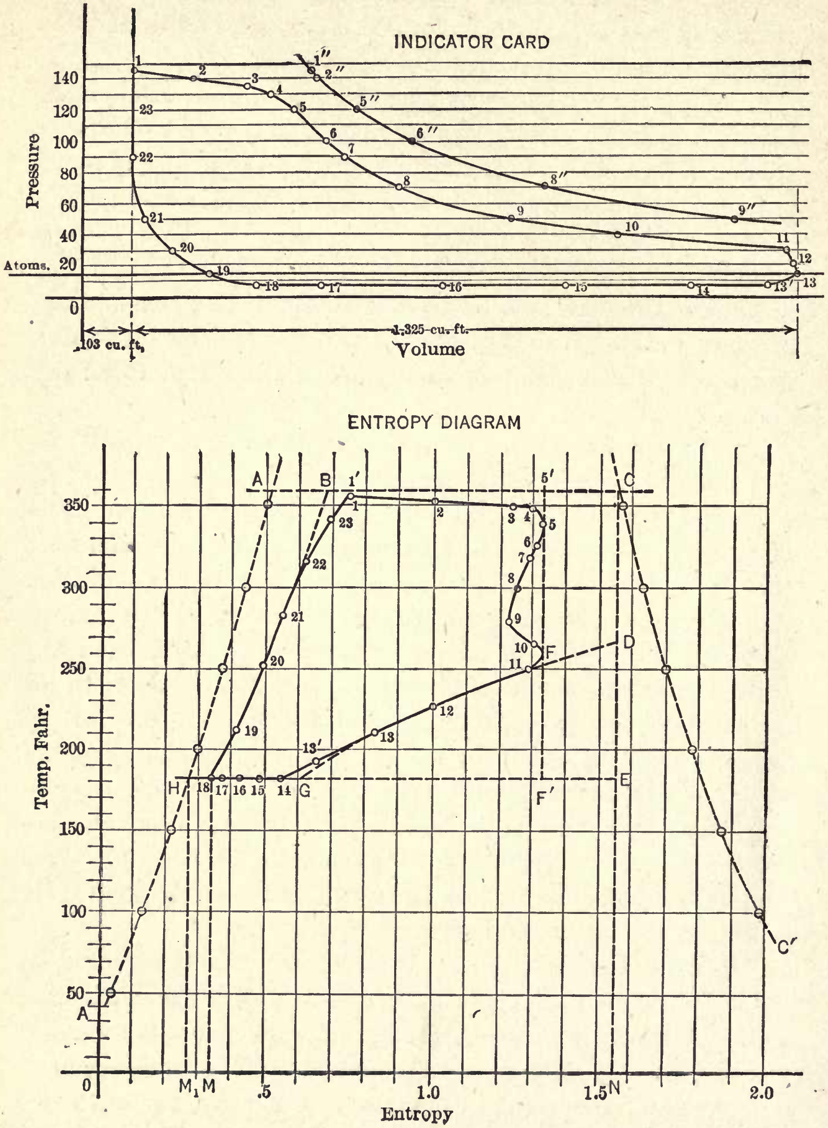

FIG. 40. — Temperature-entropy Diagram of Actual Steam Engine Indicator Diagram.

ume in cubic feet up to the particular point measured from the reference line of volumes; column 5, the specific volume of a pound of dry and saturated steam at the particular pressure (Steam Tables); column 6, the volume of actual steam per pound, obtained by dividing the volumes in column 4 by 0.14664 pound (total weight of steam in cylinder per stroke); column 7, the dryness fraction "x," found by dividing column 6 by column 5; column 8 is the entropy of evaporation $\left(\frac{L}{T}\right)$ for particular conditions (Steam Tables); column 9 is the product of column 7 and column 8; column 10 is the entropy of the liquid at various conditions as found in the Steam Tables; column 11 is the sum of column 9 and column 10, giving the total entropy.

TRANSFORMATION TABLE

1	2	3	4	5	6	7	8	9	10	11
Point number.	Pressure, lbs. per sq. in. abs.	Temperature. (Fahr.)	Volume to a certain point.	Sp. vol. per lb. dry and saturated steam.	Volume of actual steam per lb. (cu. ft.)	Dryness fraction (x).	Entropy of evaporation. L/T	$x\frac{L}{T}$	Entropy liquid. (θ)	Total entropy. (ϕ)
1	145	356	0.1050	3.11	0.716	0.2302	1.0612	0.2440	0.5107	0.7547
2	140	353	0.2250	3.22	1.536	0.4770	1.0675	0.5090	0.5072	1.0162
5	120	341	0.4230	3.73	2.890	0.7740	1.0954	0.8475	0.4919	1.3394
9	48	279	0.8500	8.84	5.810	0.6580	1.2536	0.8250	0.4077	1.2327
12	20	228	1.4225	20.08	9.700	0.4830	1.3965	0.6740	0.3355	1.0095
15	8	183	0.9758	47.27	6.660	0.1410	1.5380	0.2168	0.2673	0.4841
18	8	183	0.3500	47.27	2.390	0.0506	1.5380	0.0778	0.2637	0.3451
20	30	250	0.1825	13.74	1.245	0.0907	1.3311	0.1208	0.3680	0.4888

Above table is employed for transferring the P-V diagram to the T-ϕ diagram.

After this table was completed, columns 3 and 11 were plotted (**Fig. 40**). Convenient scales were selected, the ordinates as temperatures and the abscissas as entropies. The various points, properly designated, were connected as shown on the T-ϕ diagram, the closed diagram resulting. This area shows the amount of heat actually transformed into work. This diagram is the actual temperature-entropy diagram for the card taken and may be superimposed upon the Rankine cycle diagram in order to determine the amount and distribution of heat losses.

The water line, *A–A′*, and the dry steam line, *C–C′*, were drawn directly by the aid of Steam Table data, i.e., the entropy of the liquid and the entropy of the steam taken at various temperatures, and plotted accordingly.

Before the figure could be studied to any extent, the theoretical (Rankine) diagram had to be plotted, assuming that the steam reaches cut-off under steam-chest conditions; that it then expands adiabatically down to back pressure and finally exhausts at constant pressure to the end of the stroke without compression. This diagram is marked, *A–C–E–H*, on the T-ϕ plane. The steam-chest pressure of 153 pounds per square inch absolute fixes the point, *C*, when the temperature line cuts the steam line, *C–C′*. The rest of the cycle is self-evident.

Referring to the T-ϕ diagram, **Fig.** 40, *H–A–C–E* is the Rankine cycle with no clearance for one pound of working fluid. The amount of heat supplied is shown by the area, M_1–*H–A–C–N*, and of this quantity, the area M_1–*H–E–N* * would be lost in the exhaust while the remainder, *H–A–C–E*, would go into work. This is theoretical, but in practice there are losses, and for that reason, the Rankine cycle is used merely for comparison with the actual card as taken from a test. The enclosed irregular area, 1–2–3 . . . 22–23, is the amount of heat going into **actual work.** By observation, it is evident that a big area remains; this must represent losses of some sort or other. That quantity of work represented by the area, 1–5–5′–1′, is lost on account of wire-drawing; the area 5′–*C–D–F* shows a loss due to initial condensation; the loss due to early release is shown by the area *F*–12–14–*F′* for the real card, and by *D–G–E* for the modified Rankine cycle (such a loss, in other words, is due to incomplete expansion); that quantity represented by 22–*B*–1′–1 is lost on account of incomplete compression, and *H–A–B*–18 is

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* The areas $M_1\,HACN$ and $M_1\,HEN$ should have added to them, the rectangular area between M_1N, and the absolute zero line, which is not shown on the diagram (**Fig.** 40).

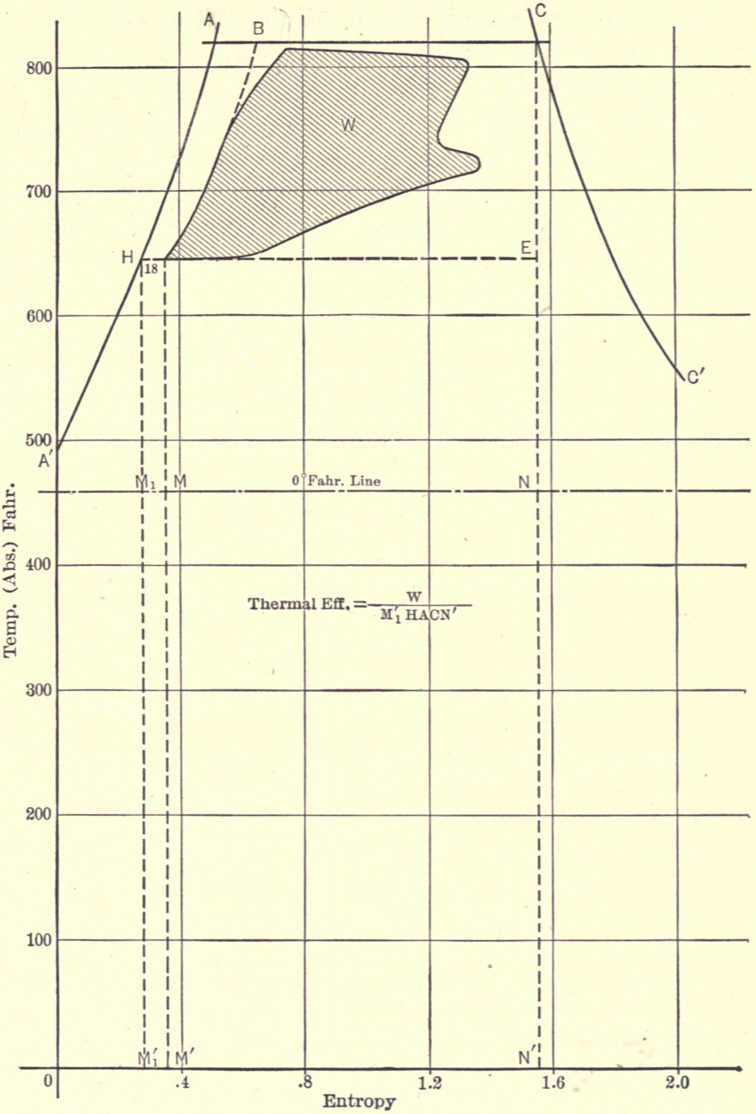

FIG. 41. — Temperature (absolute) -entropy Diagram of Actual Steam Engine Indicator Diagram.

the loss due to clearance. The expansion line from 5 on to 9 indicates that there is a loss of heat to the cylinder walls, causing a decrease of entropy; from 9 on to 10, re-evaporation is taking place (showing a gain of entropy).

All of the heat losses are not necessarily due to the transfer of heat to or from the steam, as there may be some loss of steam due to leakage. In general, however, the T-ϕ diagram is satisfactory in showing heat losses.

Fig. 41 was constructed for the purpose of showing how the **actual thermal efficiency** and the **theoretical thermal efficiency** (based on the Rankine cycle) can be obtained from the T-ϕ diagram. The letters in **Fig. 40** refer to the same points as in **Fig. 41**, the only difference between the two diagrams being the addition of the absolute zero temperature line to **Fig. 41.**

$$\text{Thermal efficiency} = \frac{\text{Work done}}{\text{Heat added}}.$$

$$\text{Actual thermal efficiency} = \frac{\text{Shaded area } W}{M_1'H\text{–}A\text{–}C\text{–}N'},$$

$$= \frac{2.50 \text{ sq. in.}}{26.00 \text{ sq. in.}} \text{ (Planimeter)}$$

$$= 0.096, \text{ or } 9.6 \text{ per cent.}^*$$

$$\text{Rankine cycle efficiency (theoretical thermal efficiency)}$$

$$= \frac{H\text{–}A\text{–}C\text{–}E}{M_1'\text{–}H\text{–}A\text{–}C\text{–}N'},$$

$$= \frac{5.29}{26.00} \text{ (by planimeter)}$$

$$= 0.203, \text{ or } 20.3 \text{ per cent.}$$

Combined Indicator Card of Compound Engine. The method of constructing indicator diagrams to a common scale of vol-

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* This value may be regarded as the actual thermal efficiency for *one stroke*. inasmuch as in the succeeding strokes the "cushion" steam will be used over and over again and hence will not constitute a heat loss. The actual thermal efficiency of a steam engine under running conditions would then be given by $\frac{W}{M'18\,BCN}$, where 18 BCE is a Rankine cycle with clearance and complete compression.

ume and pressure shows where the losses peculiar to a compound steam engine occur, and, to the same scale, the relative work areas.

As the first step divide the length of the original indicator diagrams into any number of equal parts (**Fig. 42**), erecting perpendiculars at the points of division.

In constructing a combined card, select a scale of absolute pressure for the ordinates and a scale of volumes in cubic feet for the abscissas. To the scale adopted draw in the atmospheric pressure ("**at.**") line, see **Fig. 43**.

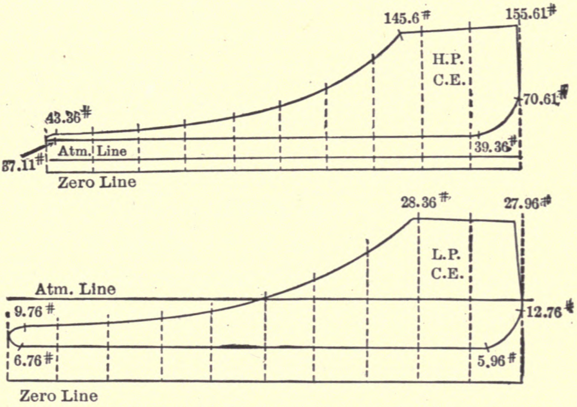

FIG. 42. — High and Low Pressure Indicator Diagram of Compound Steam Engine.

Lay off the low-pressure clearance volume on the x-axis to the scale selected. In like manner lay off the piston displacement of the low-pressure cylinder and divide this length into the same number of equal parts as the original indicator diagrams were divided. From the original low-pressure card (**Fig. 42**), determine the pressures at the points of intersection of the perpendiculars erected above the line of zero pressure, taking care that the proper indicator-spring scale is used. Lay off these pressures along the ordinates (**Fig. 43**), connect the points and the result will be the low-pressure diagram transferred to the new volume and pressure scales.

The high-pressure diagram is transferred to the new volume and pressure scale by exactly the same means as described for the low-pressure diagram.

The saturation curve is next drawn. This curve represents the curve of expansion which would be obtained if all the steam in the cylinder was dry and saturated. It is very probable that the saturation curve for each cylinder will not be continuous since the weight of the cushion steam in the low-pressure cylinder is usually not the same as that in the high-pressure cylinder. The saturation curve would be continuous for the two cards only when the weight of cushion steam in the high-pressure and low-pressure cylinders is the same (assuming no leakage or other losses).

On the assumption that the steam caught in the clearance spaces at the beginning of compression is dry and saturated, the weight of the cushion steam can be calculated from the pressure at the beginning or end of compression, the corresponding cylinder volumes and the specific volumes corresponding to the pressure (as obtained from steam-tables).

The total weight of steam in the cylinder is the weight of steam taken into the engine per stroke plus the weight of steam caught in the clearance space (cushion steam). The saturation curves may now be drawn by plotting the volumes which the total weight of steam will occupy at different pressures, assuming it to be dry throughout the stroke.

The quality curve (**Fig.** 43) shows the condition of the steam as the expansion goes on. At any given absolute pressure, the volume up to the expansion line shows the volume of the wet vapor, while the volume up to the saturation curve shows the volume that the weight of the wet vapor would have if it were dry. Thus at any given absolute pressure, the ratio of the volume of the wet vapor (as given by the expansion line of the indicator card) to the total volume of the dry vapor (as obtained from the saturation curve) is the measure of the quality of the steam.

Showing the quality by the use of the figure, we have *Vol.*

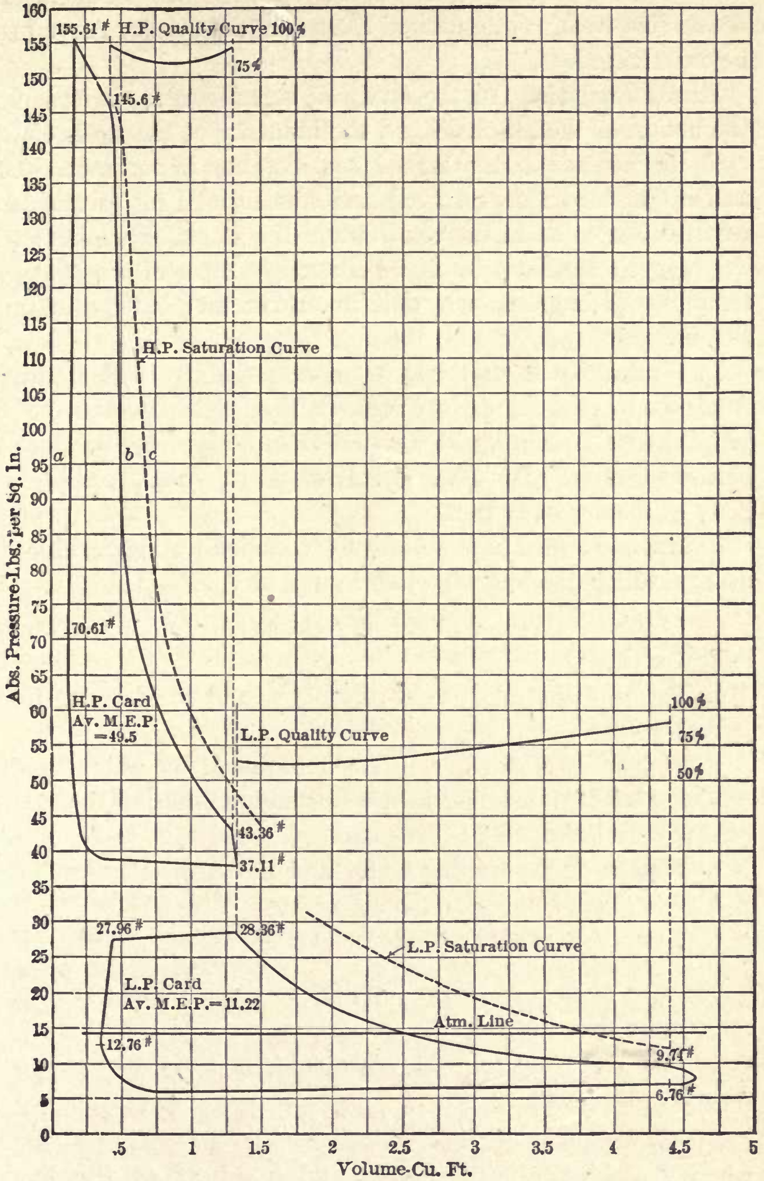

FIG. 43. — Combined Indicator Diagrams for Compound Engine.

$ab \div Vol.\ ac$ = quality. By laying off this ratio from a horizontal line to any scale desired as shown, the quality curve may be constructed.

Hirn's Analysis. In the study of steam-engine performance the action of the steam in and the influence of the walls of the cylinder become a matter of considerable importance. The amount of heat lost, restored and transformed into work as a result of variation in the condition of the steam throughout the engine-cycle can be determined either by means of the entropy-temperature diagram, or by calorimetric method based on Hirn's theory.

The **calorimetric method,** or more popularly called Hirn's Analysis, was developed by Professor V. Develshauvers-Devy of Liege. (See Table of the Properties of Steam by V. Develshauvers-Devy, *Trans. Am. Soc. Mech. Eng.*, Vol. XI; also Peabody's *Thermodynamics.*)

In order to apply Hirn's Analysis to engine testing besides the usual readings the following items must be determined:

1. Absolute pressures at cut-off, release and compression from indicator cards,
2. Per cent of stroke at cut-off, release and compression from indicator cards,
3. Weight of steam per stroke determined by weighing the condensed steam or boiler feed water and computing the steam used per stroke,
4. Weight of the cooling water and its temperature at inlet and outlet from condenser,
5. Temperature of the condensed steam.

Let w = pounds of steam supplied to the cylinder per stroke at pressure p and quality x.

Then the amount of heat brought in by the steam into the cylinder per stroke,

$$Q = w\,(h + xL), \tag{191}$$

where L = heat of vaporization and h = heat of the liquid above the freezing point.

At the end of compression a certain amount of steam is left in the clearance space and this steam mingles with the w pounds of steam at admission. Calling the weight of the steam caught in the clearance space w_1, the total amount of steam at admission is $w + w_1$.

According to thermodynamic laws the addition of heat to a substance produces in that substance internal and external changes, the internal changes being volume and temperature variations which are called intrinsic energy changes, while the external changes are changes in external potential energy or work done. Thus the steam brought into the cylinder per stroke on account of its available heat produces intrinsic energy changes and is capable of doing external work in overcoming resistance. On the other hand the steam w_1 caught in the clearance space at compression is able to produce only internal or intrinsic energy changes. The intrinsic energy changes can be calculated knowing the heat equivalent of internal work or internal latent heat (ρ) and the heat of the liquid (h). The external work can be determined from the indicator cards.

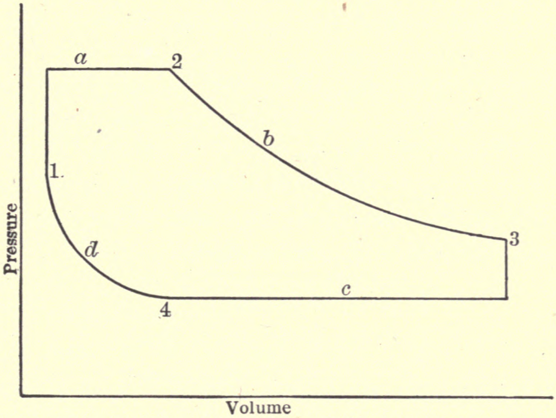

FIG. 44. — The Steam Engine Cycle.

Referring to **Fig. 44** and calling the heat absorbed by the cylinder walls during admission and expansion Q_a and Q_b; that restored during exhaust and compression Q_c and Q_d; the intrinsic energy at the points of admission, cut-off, release, and compression I_1, I_2, I_3, I_4; the external work in heat units AW_a, AW_b, AW_c, AW_d, during the events of the stroke, where $A = \frac{1}{778}$; also if the temperature of the condensed steam is t_s and that of the cooling water t_i and t_o at inlet and outlet, the total weight of cooling water used being G, we have the following relations:

$$Q_a = Q + I_1 - I_2 - AW_a. \tag{192}$$
$$Q_b = I_2 - I_3 - AW_b. \tag{193}$$
$$Q_c = I_3 - I_4 - wh_s - G\,(h_o - h_i) - AW_c. \tag{194}$$
$$Q_d = I_4 - I_1 + AW_d. \tag{195}$$

In equations (192), (193), (194), and (195) Q can be determined from $Q = w\,(h + xL)$. The values of G can be determined by weighing cooling water; h_o, h_i, and h_s by taking the temperature t_o, t_i, and t_s by thermometers and finding h_o, h_i, and h_s from steam tables. The intrinsic energy at the events of the stroke are calculated by means of the following equations:

$$I_1 = w_1\,(h_1 + x_1\rho_1), \tag{196}$$
$$I_2 = (w_1 + w)\,(h_2 + x_2\rho_2), \tag{197}$$
$$I_3 = (w_1 + w)\,(h_3 + x_3\rho_3), \tag{198}$$
$$I_4 = w_1\,(h_4 + x_4\rho_4), \tag{199}$$

in which

w_1 = weight of steam caught in clearance space.
w = weight of steam brought into the cylinder per stroke.
h_1, h_2, h_3, h_4 = heat of the liquid at events of stroke.
$\rho_1, \rho_2, \rho_3, \rho_4$ = internal latent heats at the events of stroke.
x_1, x_2, x_3, x_4 = quality of steam at events of stroke.

From the above it is evident that x_1, x_2, x_3, x_4, and w are unknown, these values being determined by the following procedure: The volume of w pounds of steam in cubic feet is

$$V = w\,(xu + \sigma). \tag{200}$$

x = quality of steam, u = the increase in volume produced by the vaporization of one pound of water of volume σ into dry steam of volume S, or $u = S - \sigma$. Calling the volume of the steam caught in the clearance space V_1 and that at cut-off, release and compression V_2, V_3, V_4,

$$V_1 = w_1(x_1u_1 + \sigma). \tag{201}$$

$$V_1 + V_2 = (w + w_1)(x_2u_2 + \sigma). \tag{202}$$

$$V_1 + V_3 = (w + w_1)(x_3u_3 + \sigma). \tag{203}$$

$$V_1 + V_4 = w_1(x_4u_4 + \sigma). \tag{204}$$

In the above equations the absolute pressures and volumes can be determined from the indicator cards, provided the dimensions of the engine and the clearance are known and it is assumed x_4 is unity. This assumption can be made without much error, as the steam at compression is very nearly dry. If x_4 is assumed equal to 1, then, from equation (204),

$$w_1 = \frac{V_1 + V_4}{u_1 + \sigma}. \tag{205}$$

The intrinsic energy at the events of the stroke is computed after finding x_1, x_2, and x_3 from equations (201), (202), and (203) by the aid of equation (205).*

Clayton's Analysis of Cylinder Performance. The indicator diagram has been used in determining the economy of the steam engine although inaccuracies enter because of cylinder condensation. The quantity of steam admitted to the engine per cycle can theoretically be determined from the difference between the weight of steam accounted for at the point of cut-off and point of compression. Little error is introduced in determining the weight of steam present in the cylinder at the point of compression, for the quality of the steam at that point can be fairly accurately estimated. To determine the weight of steam present at the point of cut-off the quality of the steam must be known. The quality of the steam at the point of cut-off is not the same as that at admission because of the condensation within the cylinder. Thus the study of cylinder performance from the indicator diagram is made inaccurate.

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* For log form of Hirn's Analysis see *Experimental Engineering* by Carpenter and Diederichs, Seventh Edition, pages 799–806.

Clayton's analysis was made to determine the relation between the quality of the steam at cut-off and other variables. From the results of a careful study of the forms of the expansion and compression curves which occur in indicator diagrams, it has been discovered that the value of n for the expansion curve bears a definite relation to the proportion of the total weight of steam mixture which was present at the point of cut-off.

To facilitate the analysis, the indicator diagram is transferred to logarithmic coördinates. The equation of the polytropic curve, $PV^n = C$, becomes a straight line when plotted upon logarithmic cross-section paper, the value of n becoming the slope of the curve. From the new diagram, the slope of the expansion and compression curves may be easily determined. The quality of the steam at cut-off may then be estimated, knowing the speed of the engine, the quality of the steam at admission, the pressure of the steam and other values that affect the slope of the curve.*

PROBLEMS

1. Assume that 1 lb. of steam of a pressure of 160 lbs. per sq. in. absolute and a quality of 0.95 performs an ideal Rankine cycle, being exhausted at a pressure of 5 lbs. per sq. in. absolute. Compute the quality of the steam exhausted, the efficiency of the cycle, and the final volume.

2. What is the work of an ideal Rankine cycle if the steam initially at 200 lbs. per sq. in. absolute pressure, superheated 200° F., goes through such a cycle with a back pressure of 1 lb. per sq. in. absolute?

3. One pound of steam at a pressure of 100 lbs. per sq. in. absolute with a quality of 0.90 performs an ideal Rankine cycle exhausting at a back pressure of 2 lbs. per sq. in. absolute. What is the net work and efficiency of the cycle?

4. Two pounds of steam at a pressure of 125 lbs. per sq. in. absolute and a volume of 8.34 cu. ft. perform an ideal Rankine cycle. The exhaust pressure is 25 lbs. per sq. in. absolute. What is the net work and efficiency of the cycle?

5. One pound of steam at a pressure of 160 lbs. per sq. in. absolute and a quality of 0.95 passes through a modified Rankine cycle. The terminal

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* For details concerning Clayton's "Analysis of the Cylinder Performance of Reciprocating Engines," see *Bulletin No. 58 of the Engineering Experiment Station of the University of Illinois.*

pressure is 11 lbs. per sq. in. absolute and the exhaust pressure 5 lbs. per sq. in. absolute. What is the efficiency of the cycle?

6. One pound of steam at a pressure of 200 lbs. per sq. in. absolute and 200° F. superheat passes through a modified Rankine cycle. The terminal pressure is 15 lbs. per sq. in. absolute and the exhaust pressure 10 lbs. per sq. in. absolute. What is the net work and efficiency of the cycle?

7. Two pounds of steam at a pressure of 125 lbs. per sq. in. absolute and volume of 8.34 cu. ft. pass through a modified Rankine cycle. The terminal pressure is 20 lbs. per sq. in. absolute and the exhaust pressure 15 lbs. per sq. in. absolute. What is the efficiency, net work, and heat added to the cycle?

8. A non-condensing steam engine receives steam of 0.95 quality and 100 lbs. per sq. in. absolute pressure. The exhaust pressure is 15 lbs. per sq. in. absolute and the steam consumption of the engine is 30 lbs. per indicated horse power per hour. What is the thermal efficiency of the engine? What is the efficiency of the ideal Rankine cycle operating under the same conditions? What is the efficiency of the Carnot cycle operating between the same temperature limits?

CHAPTER IX

FLOW OF FLUIDS

Flow through a Nozzle or Orifice. Thermodynamic problems embrace the measurement of the flow of air or of a mixture of a liquid and vapor through a nozzle or orifice. In the nozzle shown in **Fig. 45**, let **A** and **B** be two sections through which the substance passes. At **A** let a pressure of P_1 be maintained and at **B** a pressure of P_2. To maintain the constant pressure at **A** of P_1 let more substance be added, while at **B** allow enough of the substance to be discharged so that the constant pressure of P_2 is maintained. The quantity of energy and the mass passing into the section **A** must be accounted for at section **B** and the relation of these quantities will determine the change of the velocity of the substance.

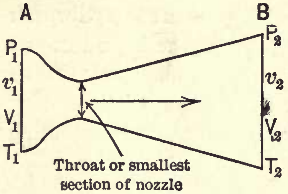

FIG. 45. — Typical Nozzle for Expanding Gases and Vapors.

After uniform conditions have been established in the nozzle, the same mass entering at **A** must be discharged at **B** during the same time. Thus any mass may be considered as a working basis, but as a rule **one pound** of the substance is used. All formulas refer to one pound, unless another mass is definitely stated.

The same quantity of energy discharged at **B** must enter at **A** unless heat is added to or taken from the substance between the sections **A** and **B**. Thus the general formula is derived:

Energy carried by substance at **B** = energy carried by substance at **A** + energy added between the sections **A** and **B**.

The energy carried by the substance at the entering or discharge end of the nozzle is made up of three quantities: (1) the amount of work necessary to maintain constant pressure at each end of the nozzle; (2) the internal energy of the substance, (3) the kinetic energy stored in the substance because of the velocity which it has when passing the section.

The amount of work necessary to maintain a constant pressure (pounds per square foot) of P_1 at **A** or of P_2 at **B** is $\frac{P_1 v_1}{778}$ or $\frac{P_2 v_2}{778}$, where v_1 and v_2 are the volumes of one pound of the substance at **A** and **B** respectively.

The **internal** or " intrinsic " energy **in B.t.u.** per pound (I_H) of the substance passing **A** or **B**, calculating **from 32° F.** is for:

Air or similar gases, $\frac{R \times (T - 492)^*}{778 \times 0.40}$, (206)

Liquid, h, (207)

Liquid and Vapor, $h + x\left(L - \frac{V^2}{2\,g \times 778}\right)$, (208)

Superheated Vapor, $h + \left(L - \frac{V^2}{2\,g \times 778}\right)$

$$+ C_p(T_{\text{sup.}} - T_{\text{sat.}}) - \frac{P(v_{\text{sup.}} - v_{\text{sat.}})}{778}, \quad (209)$$

where $\frac{V^2}{2\,g \times 778}$ is the kinetic energy in B.t.u. per pound of the substance as it passes a section, V = velocity in feet per second and $g = 32.2$ feet.

If Q represents in B.t.u. per pound the heat units added to the substance between the sections **A** and **B** (**Fig. 45**), the **energy equation for air** and similar gases can be found by equating the total heat energy put in at **A** plus the energy added between **A** and **B** to the energy discharged at **B**. This general formula reduces to

$$V_2^2 - V_1^2 = 2\,g\left[\frac{1.4}{0.4}(P_1 v_1 - P_2 v_2) + 778\,Q\right]$$

or

$$= 2\,g \times 778\,[C_p\,(T_1 - T_2) + Q]. \quad (210)$$

---

[*] See equation (61).

This thermodynamic equation is the usual form for the flow of air or similar gases.

The energy equation for superheated steam can be derived by the use of the same general formula stated above, on the assumption that the substance is superheated at **A** and **B.** This formula reduces to

$$\frac{V_2^2 - V_1^2}{2\,g \times 778} = [h_1 + L_1 + C_p\,(T_{\text{sup.}} - T_{\text{sat.}})_1] \\ - [h_2 + L_2 + C_p\,(T_{\text{sup.}} - T_{\text{sat.}})_2] + Q, \quad (211)$$

or
$$V_2^2 - V_1^2 = 2\,g \times 778\,[\textit{Tabular heat content}_1{}^{*} - \textit{Tabular heat content}_2 + Q]. \quad (212)$$

From a thermodynamic standpoint, the relation between the initial and final condition is that of **adiabatic expansion** when all the heat which disappears as such is used in changing the velocity, provided the nozzle is properly shaped and Q is zero. The diagram in **Fig. 46** represents this condition of affairs on a temperature-entropy diagram for air and similar substances. The area **acdf** is $C_p\,(T_2 - 492)$, and the area **abef** is $C_p\,(T_1 - 492)$. The quantity of heat energy changed into kinetic energy is therefore the area **bcde** and is the difference between the internal energy in the substance at the beginning and end of the operation, together with the excess of work done to maintain the pressure of P_1 at **A** over the pressure of P_2 at **B.** The line **cd** would incline to the right if heat were added in the nozzle, since the effect would be to increase the velocity or increase the area **cdeb.** The line **cd** would incline to the left from **c** if heat were lost, since the area **cdeb** would decrease.

In **Fig. 47** the diagram represents the conditions for superheated vapor. The area **aa′cdf** represents the heat required to raise the substance from a liquid at 32° F. to superheated vapor at the temperature of $T_{1\text{sup.}}$ The area **ab′bdf** is the heat content at **b,** the final condition. The area **a′cbb′** represents, therefore, the heat available for increasing the velocity. The areas

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* Tabular heat content means the total heat of superheated steam as read from tables of the properties of superheated steam.

representing the heat available for increasing the velocity in **Fig. 46** and **Fig. 47** are shown by the cross-hatched area in **Fig. 48** and are really the representation of the work done (theoretically) in an engine giving such an indicator diagram.

Evidently the greater the drop in pressure, the greater will be the cross-hatched area in **Fig. 48** and the greater will be the velocity, regardless of the substance. The line **ab** in **Fig. 49** represents the velocity curve with V as ordinates and $\frac{P_2}{P_1}$ as abscissas, but since with any substance expanding the weight of a cubic foot decreases as the pressure drops, the line **cd** will represent to some scale not here determined the weight of a cubic foot at any discharge pressure.

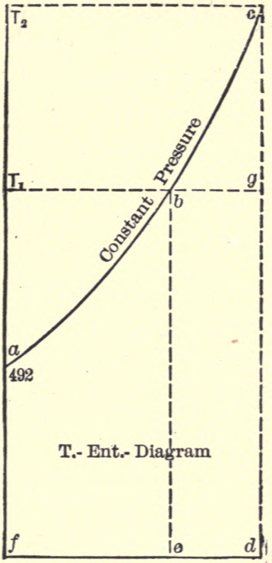

FIG. 46. — Temperature-entropy Diagram of Heat Available in Air.

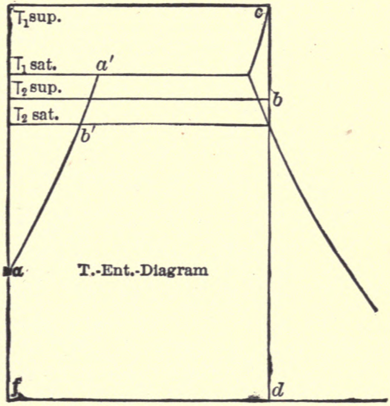

FIG. 47. — Temperature-entropy Diagram of the Heat Available in Superheated Steam for Increasing Velocity.

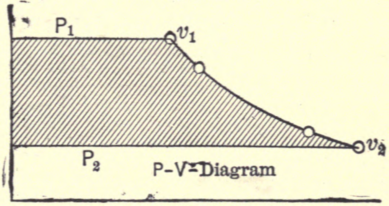

FIG. 48. — Heat (Work) Available for Increasing Velocity.

Since the product of the area through which the discharge takes place, the velocity and the weight per cubic foot of the substance is equivalent to the weight in pounds of the substance discharged per second, the product of the ordinates at any point of the curves **ab** and **cd** is proportional to the weight discharged from a pressure of P_1 to a condition where the pressure is P_2. The line **ceb** represents this product. Evidently there is some low pressure into which the weight discharged per square foot will be a maximum, and this will be the pressure corresponding to the high point **e** on the curve.

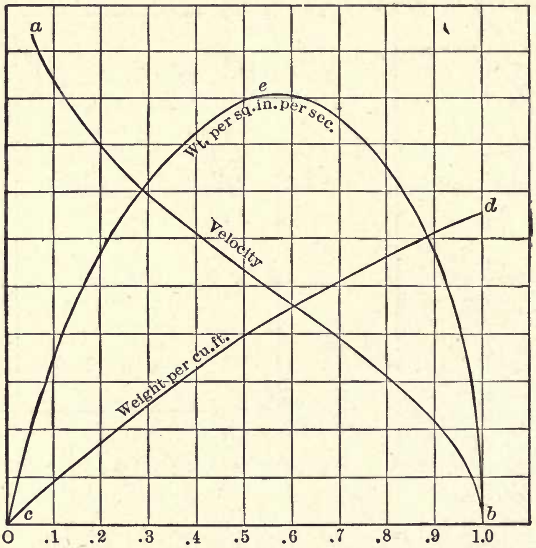

FIG. 49. — Illustrative Curves of Weight, Discharge and Velocity.

Weight per Cubic Foot. From the formula for adiabatic expansion the weight per cubic foot can be obtained if the substance is similar to air.* The general formula (applied to air) is

$$P_1 v_1^{1.40} = P_2 v_2^{1.40}, \tag{213}$$

which can be reduced to

$$\frac{1}{v_2} = \frac{P_2^{\frac{1}{1.4}} \times P_1}{P_1^{\frac{1}{1.4}} \times RT_1}, \text{ or } \quad \frac{1}{v_2} = \frac{P_1^{0.286}\, P_2^{0.714}}{RT_1}, \tag{214}$$

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* The exponent in the formula is the ratio of the specific heats (of air in this case).

which is the weight in pounds per cubic foot of discharge. Pressures are in pounds per square foot. If the supply to the nozzle is from a large reservoir so that V_1 can be taken as zero then the discharge velocity is

$$V_2 = \sqrt{2g \times \frac{1.4}{0.4}(P_1v_1 - P_2v_2)}$$

$$V_2 = \sqrt{2g \times \frac{1.4}{0.4} RT_1\left(1 - \frac{T_2}{T_1}\right)}$$

$$V_2 = 109.6\sqrt{T_1\left[1 - \left(\frac{P_2}{P_1}\right)^{0.286}\right]}. \qquad (215)$$

All quantities on the right-hand side of this equation must be obtained from the data of tests. Weight in pounds discharged through the area A (in square feet) is

$$w = A \times \frac{P_1^{0.286} P_2^{0.714}}{RT_1} \times 109.6\sqrt{T_1\left[1 - \left(\frac{P_2}{P_1}\right)^{0.286}\right]}. \qquad (216)$$

Maximum Discharge. This weight is a maximum when $\frac{dW}{dP_2} = 0$ or when $P_2 = 0.525\ P_1$. The maximum quantity of air will be discharged when the low pressure is 52.5 per cent of the high pressure.

Shape of Nozzle. See page 167, on the flow of steam.

Flow of Air through an Orifice. Air under comparatively high pressure is usually measured in practice by means of pressure and temperature observations made on the two sides of a sharp-edged orifice in a diaphragm. The method requires the use of two pressure gages on opposite sides of the orifice and a thermometer for obtaining the temperature t_1 at the initial or higher pressure p_1. The flow of air w, in pounds per second, may then be calculated by **Fliegner's formulas:**

$$w = 0.530 \times f \times a \frac{p_1}{\sqrt{T_1}} \text{ when } p_1 \text{ is greater than } 2\ p_2, \qquad (217)$$

$$w = 1.060 \times f \times a \sqrt{\frac{p_2(p_1 - p_2)}{T_1}} \text{ when } p_1 \text{ is less than } 2\ p_2, \qquad (218)$$

where **a** is the area of the orifice in **square inches, f** is a coefficient, T_1 is the absolute initial temperature in degrees Fahrenheit at the absolute pressure p_1 in the "reservoir or high-pressure side" and p_2 is the absolute discharge pressure, both in pounds per **square inch.** When the discharge from the orifice is directly into the atmosphere, p_2 is obviously the barometric pressure.

Westcott's and Weisbach's experiments show that the values of **f** are about 0.925 for equation (217) and about 0.63 for equation (218).

For small pressures it is often desirable to substitute manometers for pressure gages. One leg of a U-tube manometer can be connected to the high-pressure side of the orifice and the other leg to the low-pressure side. Valves or cocks are sometimes inserted between the manometer and the pipe in which the pressure is to be observed for the purpose of "dampening" oscillations. This practice is not to be recommended as there is always the possibility that the pressure is being throttled.* A better method is to use a U-tube made with a restricted area at the bend between the two legs. This will reduce oscillations and not affect the accuracy of the observations.

Discharge from compressors and the air supply for gas engines are frequently obtained by orifice methods.

When $p_1 - p_2$ is small compared with p_1, the simple law of discharge † of fluids can be used as follows:

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* Report of Power Test Committee, *Journal A.S.M.E.*, Nov., 1912, page 1695.

† If the density is fairly constant,

$$\frac{144\ p_1}{s} + \frac{v_1^2}{2\ g} = \frac{144\ p_2}{s} + \frac{v_0^2}{2\ g},$$

where v_1 is the velocity in feet per second in the "approach" to the orifice, and v_0 is the velocity in the orifice itself. Since v_1 should be very small compared with v_0,

$$\frac{v_0^2}{2\ g} = \frac{144\ (p_1 - p_2)}{s},$$

$$v_0 = \sqrt{\frac{2\ g \times 144\ (p_1 - p_2)}{s}}.$$

$$w = fav_0s = fas\sqrt{\frac{2\ g \times 144\ (p_1 - p_2)}{s}},$$

$$w = \frac{fa}{144}\sqrt{2\ g \times 144\ (p_1 - p_2)\ s}, \qquad (219)$$

where **f** is a coefficient from experiments, **g** is the acceleration due to gravity (32.2), and **s** is the unit weight of the gas measured, in pounds per cubic foot, for the average of the initial and final conditions of temperature and pressure. If the difference in pressure is measured in inches of water **h** with a manometer, then

$$144\ (p_1 - p_2) = \frac{62.4}{12} \times h \text{ (expressed in terms of pounds per square foot)},$$

$$w = \frac{fa}{144}\sqrt{2\ ghs \times \frac{62.4}{12}} \text{ (pounds per second)}, \qquad (220)$$

where 62.4 is the weight of a cubic foot of water (density) at usual "room" temperatures.

This equation can also be transformed so that a table of the weight of air is not needed, since by elementary thermodynamics $144\ pv = 53.3\ T$, where **v** is the volume in cubic feet of one pound and **T** is the absolute temperature in Fahrenheit. Since **v** is the reciprocal of **s**, then

$$s = 144\ p \div 53.3\ T,$$

and

$$w = 0.209\ fa \sqrt{\frac{hp}{T_1}}. \qquad (221)$$

Here **p** and **T** should be the values obtained by averaging the initial and final pressures and temperatures. Great care should be exercised in obtaining correct temperatures. For accurate work, corrections of **s** for humidity must be made.*

For measurements made with orifices with a well-rounded entrance and a smooth bore so that there is practically no con-

or $$w = fa\sqrt{2\ g \times 144\ (p_1 - p_2)s}.$$

Professor A. H. Westcott has computed from accurate experiments that the value of the coefficient **f** in these equations is approximately 0.60.

* Tables of the weight of air are given on page 181 and tables of humidity on page 368 in Moyer's *Power Plant Testing* (2d Edition).

traction of the jet the coefficient **f** in equations (217) and (218) may be taken as 0.98. In the rounding portion of the entrance to such a nozzle the largest diameter must be at least twice the diameter of the smallest section. For circular orifices with sharp corners Professor Dalby[*] stated that the coefficient for his sharp-edged orifices in a thin plate of various sizes from 1 inch to 5 inches in diameter was in all cases approximately 0.60; and these data agree very well with those published by Durley.[†]

When $p_2 \div p_1 = 0.99$ the values obtained with this coefficient are in error less than $\frac{1}{2}$ per cent; and when $p_2 \div p_1 = 0.93$ the error is less than 2 per cent.

Receiver Method of Measuring Air. None of the preceding methods are adaptable for measuring the volume of air at high pressures as in the case of measuring the discharge in tests of air compressors. Pumping air into a suitably strong receiver is a method often used. The compressor is made to pump against any desired pressure which is kept constant by a regulating valve on the discharge pipe:

P_1 and P_2 = absolute initial and final pressures for the test, pounds per square inch.

T_1 and T_2 = mean absolute initial and final temperatures, degrees Fahrenheit.

w_1 and w_2 = initial and final weight of air in the receiver, pounds.

V = volume of receiver, cubic feet.

$P_1V = wRT_1$, and $P_2V = w_2RT_2$, where R is the constant 53.3 for air, then weight of air pumped

$$w_2 - w_1 = \frac{V}{53.3}\left(\frac{P_2}{T_2} - \frac{P_1}{T_1}\right). \qquad (222)$$

In accurate laboratory tests the humidity of the air entering the compressor should be measured in order to reduce this

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[*] *Engineering* (London), Sept. 9, 1910, page 380, and Ashcroft in *Proc. Institution of Civil Engineers*, vol. 173, page 289.

[†] *Transactions American Society of Mechanical Engineers*, vol. 27 (1905), page 193.

weight of air to the corresponding equivalent volume at atmospheric pressure and temperature.

The principal error in this method is due to difficulty in measuring the average temperature in the receiver. Whenever practicable the final pressure should be maintained in the receiver at the end of the test until the final temperature is fairly constant.

Flow of Vapors. In **Fig. 45** suppose the sections **A** and **B** are so proportioned that the velocity of the substance passing section **A** is the same as that at section **B**. Such a condition might arise in a calorimeter or in the expansion of ammonia through a throttling or expansion valve, as in an ice machine. The pressure at **A** will be P_1 which is greater than the pressure at **B** of P_2. **Fig. 50** represents the entropy diagram for such a condition. As the pressure falls from P_1 to P_2 the maximum heat available to produce velocity through the nozzle is the area **acde**. The value of the quality, represented by the symbol x for the substance after leaving the nozzle corresponds to that of point **c** and the area **acde** is the excess kinetic energy represented by the increased velocity. This excess kinetic energy is destroyed by coming into contact with the more slowly moving particles at **B** and with the sides of the vessel. The area **acde** is equal to $(h_a + x_aL_a) - (h_c + x_cL_c)$ and the relation of x_a to x_c is obviously **adiabatic.** The area **ohdbf** equals area **oheag** (thus the heat content at **b** is the same as at **a**). The location of **b** can be found as follows:

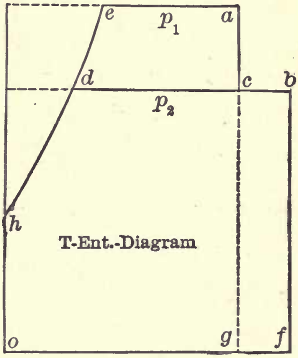

FIG. 50.—Diagram for no Velocity Change.

$$x_b = \frac{h_a + x_aL_a - h_b}{L_b}. \tag{223}$$

The curve shown in **Fig. 51** represents the discharge of a mixture of steam and water ($x = 0.6$ at 100 pounds per square inch absolute pressure) into a vessel having the pressures shown. The points on this curve cannot be determined by entropy tables. At 100 pounds per square inch pressure the total heat of the wet steam is $h + 0.6\,L$ or $298.5 + 0.6 \times 887.6 = 831.1$ B.t.u. per pound.

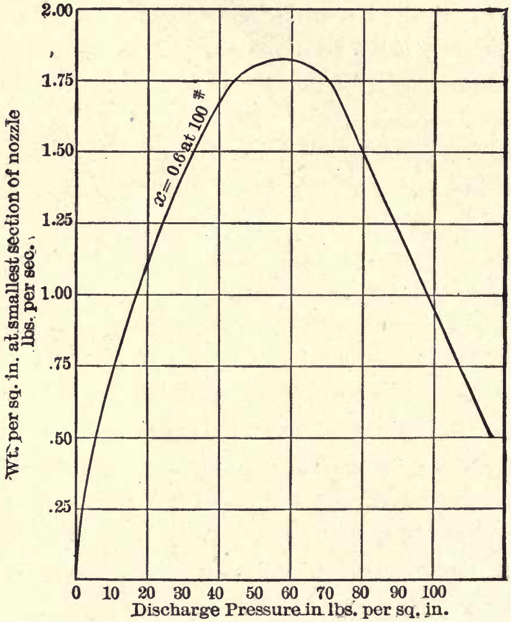

FIG. 51. — Discharge of Steam Under Various Pressures.

At 60 pounds per square inch absolute pressure the total heat may be found by the use of the entropy diagram shown in **Fig. 52.** The entropy values are taken directly from steam tables. The entropy for the initial point is, then,

$$\mathbf{ab} = 0.4748 + 0.6 \times 1.1273 = 1.1512.$$

The distance

$$\mathbf{dc} = \mathbf{ab} - 0.4279 = 0.7233,$$

and x at 60 pounds is:

$$\frac{\mathbf{dc}}{1.2154} = \frac{0.7233}{1.2154} = 0.595.$$

The total heat at 60 pounds pressure is

$$h + 0.595\,L = 262.4 + 0.595 \times 914.3 = 805.4.$$

The velocity of flow is

$$\sqrt{2 \times 32.2 \times 778\,(831.1 - 805.4)} = 1135 \text{ feet per second.}$$

The volume of one pound is

$$0.016\,(1.0 - 0.595) + 7.166 \times 0.595 = 4.27 \text{ cubic feet,}$$

and the weight per cubic foot is

$$\frac{1}{4.27} = 0.2342 \text{ pound.}$$

The weight discharged per square inch per second is

$$\frac{1135 \times 0.2342}{144} = 1.85 \text{ pounds.}$$

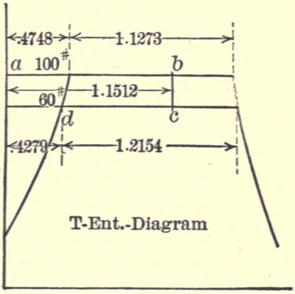

FIG. 52. — Temperature-entropy Diagram for Calculating the Weight of Discharge of Steam.

Velocity of Flow as Affected by Radiation. Fig. 53 shows the radiation losses. The condition at entrance is represented at **a** and the area **acde** represents the quantity of heat lost by radiation. Area **aefg** represents the velocity change while the point **e** represents the condition of the moving substance.

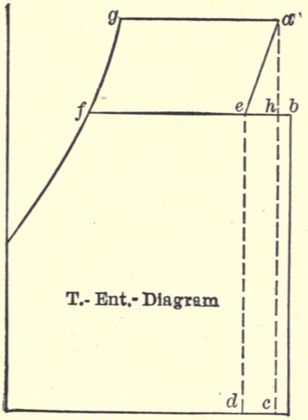

FIG. 53. — Diagram Illustrating Radiation Loss in Nozzle.

If, after passing through the nozzle, the velocity is reduced to that of entrance, a point located as at **b** will represent the condition of the substance. This point would be so located that

$$\mathbf{eb} = \frac{\text{area } \mathbf{aefg}}{L_f}. \qquad (224)$$

Friction Loss in a Nozzle. Fig. 54 shows the friction loss. The energy converted into heat by friction varies with the square of the velocity. In this figure, **a** is the initial condition and **acfg** is the energy available for change in velocity provided there is no frictional loss. The ratio of the areas **acde** to **acfg** is the proportional loss by friction. The point **c** represents the condition of the substance at the return of the friction heat to the substance. The heat is returned in exactly the same way as if it came from an outside source. The distance **ch** is the area **acde** ÷ L_c. The area **edfg** represents the energy expended in the velocity change and the point **h** represents the state of the substance at the point of discharge.

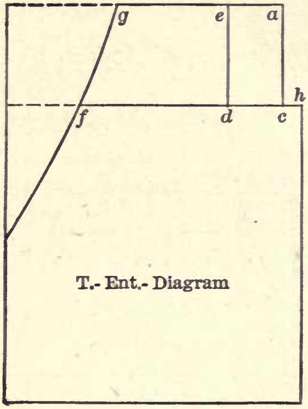

FIG. 54. — Diagram Illustrating Friction Loss in Nozzle.

This condition is found existing in the fixed nozzle of most turbines. Point **a** represents the condition on the high-pressure side of the nozzle and point **h**, the low-pressure side. The absolute velocity of discharge is really caused by the energy represented by the area **edfg.**

Impulse Nozzles. Suppose that the substance is discharged with an absolute velocity corresponding to the area **edfg** (**Fig. 54**), and that it passes into a moving nozzle, having the same pressure on the discharge as on the intake side. The energy represented by the area **edfg** would be used up in the following ways: (1) by the friction in moving nozzle; (2) residual absolute velocity; (3) and energy used in driving the moving nozzle against the resistance.

Fig. 55 shows the quantities used up by each. Point **a** represents the condition of the substance before passing into the fixed nozzle while point **h** shows its condition leaving the fixed nozzle, the velocity corresponding to the area **edfg.** Area **klde** represents the energy used up in friction in the moving nozzle; area **klnm** residual

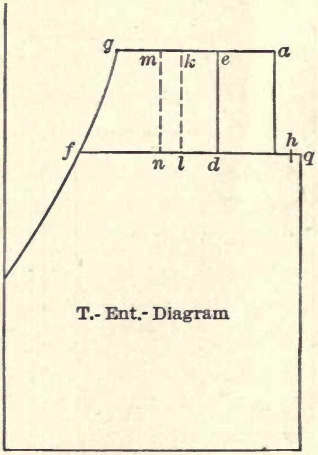

FIG. 55. — Diagram of Heat Losses in a Steam Nozzle (Impulse).

velocity after leaving moving nozzle and area **mnfg** represents useful work used in moving the nozzle against its resistance. The condition of the substance leaving the nozzle is shown at **q** and not at **h,** the distance **h** − **q** being the area **edlk** divided by L_h. The substance leaves the moving nozzle with a velocity corresponding to the area **klnm** and it will have done work corresponding to the area **mnfg.**

Turbine Losses. Fig. 56 is a simple velocity diagram showing, for an impulse nozzle such as occurs in many turbines, the relative value of those various losses. **A** is a stationary nozzle discharging against the movable blades **B.** The path of the steam is shown by the dotted line. The line **db** marked v represents the velocity of discharge of the stationary nozzle, which makes an angle α with the direction of motion of the moving blades. Call ϵv the velocity of the moving blades, then h is the amount and direction of the relative velocity of the steam over the surface of the moving blades. It loses a portion of this velocity as it passes over the surface of the blades and lh becomes the actual relative velocity of discharge. The direction of lh is determined by the discharge edge of the moving blades, the angles α and β being as shown. The residual absolute velocity is represented by r.

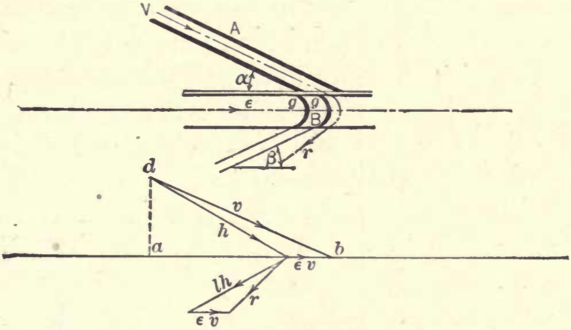

Fig. 56. — Impulse Nozzle and Velocity Diagrams.

The total energy equivalent of the velocity developed

$$\text{in B.t.u. per pound} = \frac{v^2}{2g}. \tag{225}$$

$$\text{The residual energy per pound} = \frac{r^2}{2g}. \qquad (226)$$

Reaction Nozzles. When the substance leaving the stationary nozzle passes into a moving nozzle having the pressure at the intake greater than at the discharge, the conditions differ from those just discussed. The velocity in this case is changed in passing through the moving nozzle. In the equations given in the previous discussion it was assumed that the moving nozzles were entirely filled with the substance, and when partly filled in the expanding portion, coefficients of correction were applied, but in this case the nozzles should be so designed that the substance entirely fills them, as the corrections are unknown.

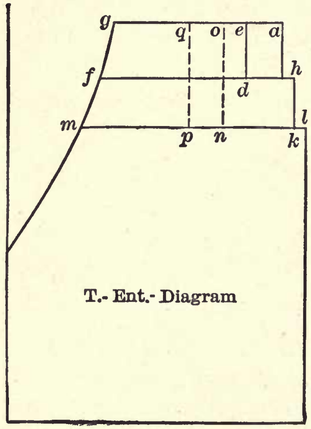

FIG. 57. — Diagram of Heat Losses in a Steam Nozzle (Reaction).

In **Fig.** 57 the lines of **Fig.** 55 are reproduced together with those relating directly to the reaction nozzle. Point **a** corresponds to the condition on entering the stationary nozzle, point **h** the condition on leaving it with a velocity corresponding to the area **edfg.** Point **k** represents the pressure at the discharge end of the moving nozzle, and if no friction losses or impact loss occur in the moving nozzle, point **k** would represent the condition of the discharged substance and the area **egmkhd** would be accounted for as useful work done and residual velocity. But since friction losses and impact loss do occur a portion of this area **edhkno** can be set aside to represent these losses, a portion **noqp** represents the residual velocity, while the remaining area **pqgm** represents the useful work done.

The condition of the substance leaving the moving nozzle is given by **k,** the distance **kl** being $\frac{\text{area } \mathbf{edhkno}}{L_k}$. Area **onpq** represents the residual velocity.

Coefficient of Flow. Few experiments have been carried on for determining the flow of steam in nozzles proportioned for maximum discharge. For a nozzle having a well rounded entrance and with the parallel portion of least diameter from 0.25 to 1.5 times the length of the converging entrance, the coefficient of discharge is about 1.05. For properly shaped entrances and for areas of orifices between 0.125 square inch and 0.75 square inch the coefficient of discharge varies from 0.94, the two pressures being nearly alike, to unity, the ratio of the pressures being 0.57. For an orifice through a thin plate the coefficient is about 0.82, the ratio of the pressures being 0.57.

Injectors. In an injector, steam enters at *A* in **Fig. 58** at the pressure of the supply. The quantity of water entering at *C*, the cross-section of the pipe, and the pressure of the water determine the pressure at *B*. At *D* the pressure should be zero (atmospheric) or equal to the pressure in the water supply pipe to which *D* may be connected. The total hydraulic head should exist as velocity head at this point. At *E* the pressure should be sufficient to raise the check valve into the boiler and the velocity sufficient to carry the intended supply into the discharge pipe.

FIG. 58. — Essential Parts of an Injector.

The shape of the nozzle from *A* to *B* should be such as to convert the energy in the steam at *A* into velocity at *B*. At *B* the water and steam meet, condensing the steam, heating the water and giving to the water a velocity sufficient to carry it through the nozzle *B–D*.

All the energy accounted for at *A* and *C* must be accounted for at *E*. The heat lost by radiation may be neglected. The velocity at any section of the nozzle equals

$$\frac{\text{volume passing in cubic feet}}{\text{area of section in square feet}},$$

or $V_A = \frac{aV}{a_A}$, where a = area at any section corresponding to velocity V and a_A = area at section A.

Weight of Feed Water Supplied by an Injector per Pound of Steam. Assuming the steam supply to be dry the heat units contained in the steam and feed water per pound and the heat in the mixture of steam and feed water per pound may be easily calculated.

Knowing the rise of temperature of the water passing through the injector and neglecting radiation losses, the pounds of feed water supplied per pound of steam used by the injector may be obtained. Thus:

Heat units lost by steam = Kinetic energy of jet + Heat units gained by feed water.

The kinetic energy of jet may be neglected since it is very small, then,

$$H - h_f = w\,(h_m - h_f)$$

$$w = \frac{H - h_f}{h_m - h_f}, \tag{227}$$

where w = the weight of feed water lifted per pound of steam.
h_m = heat of liquid of mixture of condensed steam and feed water.
h_f = heat of liquid of entering feed water.

Thermal Efficiency of Injector. The thermal efficiency of an injector neglecting radiation losses is unity. All the heat expended is restored either as work done or in heat returned to the boiler.

Mechanical Efficiency of Injector. The mechanical work performed by the injector consists in lifting the weight of feed water and delivering it into the boiler against the internal pressure. The efficiency, considering the injector as a pump, is

$$\frac{\text{Work done}}{\text{B.t.u. given up by steam to perform the work}},$$

or

$$E = \frac{U}{H - h_f}, \tag{228}$$

where $U = \left\{wl_s + (w + 1)\,l_p\right\} \div 778$ (in heat units).

l_p = pressure head corresponding to boiler gage pressure, in feet.
l_s = suction head in feet.
w = pounds of water delivered per pound of steam.

Orifice Measurements of the flow of steam are sometimes used for ascertaining the steam consumption of the "auxiliaries" in a power plant. This method commends itself particularly because of its simplicity and accuracy. It is best applied by inserting a plate $\frac{1}{8}$ inch thick with an orifice one inch in diameter, with square edges, at its center, between the two halves of a pair of flanges on the pipe through which the steam passes. Accurately calibrated steam gages are required on each side of the orifice to determine the loss of pressure. The weight of steam for the various differences of pressure may be determined by arranging the apparatus so that the steam passing through the orifice will be discharged into a tank of water placed on a platform scales. The flow through this orifice in pounds of dry saturated steam per hour when the discharge pressure at the orifice is 100 pounds by the gage is given by the following table:

Pressure drop, lbs. per sq. in.	Flow of dry steam per hour, lbs.	Pressure drop, lbs. per sq. in.	Flow of dry steam per hour, lbs.
$\frac{1}{2}$	430	5	1560
1	615	10	2180
2	930	15	2640
3	1200	20	3050
4	1400	....	....

Flow of Steam through Nozzles. The weight of steam discharged through any well-designed nozzle with a rounded inlet, similar to those illustrated in Figs. 59 and 60, **depends only on the initial absolute pressure (P_1), if the pressure against which the nozzle discharges (P_2) does not exceed 0.58 of the initial pressure.** This important statement is well illustrated by the following example. If steam at an initial pressure (P_1) of 100 pounds per square inch absolute is discharged from a nozzle, the weight of steam flowing in a given time is practically the same for all values of the pressure against which the steam is discharged (P_2), which are equal to or less than 58 pounds per square inch absolute.

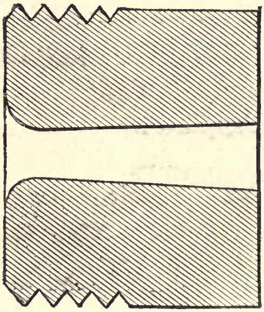

FIG. 59.—Example of a Well-designed Nozzle.

If, however, the **final pressure is more than 0.58 of the initial, the weight of steam discharged will be less, nearly in proportion as the difference between the initial and final pressures is reduced.**

The most satisfactory and accurate formula for the "constant flow" condition, meaning when the final pressure is 0.58 of the initial pressure or less, is the following, due to Grashof,* where w is the flow of steam † (initially dry saturated) in pounds **per second,** A_0 is the area of the smallest section of the nozzle in square inches, and P_1 is the initial **absolute** pressure of the steam in pounds per square inch,

$$w = \frac{A_0 P_1^{.97}}{60}, \tag{229}$$

or, in terms of the area,

$$A_0 = \frac{60\,w}{P_1^{.97}}. \tag{230}$$

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* Grashof, *Theoretische Maschinenlehre,* vol. 1, iii; *Hütte Taschenbuch,* vol. 1, page 333. Grashof states the formula,

$$w = 0.01654\, A_0 P_1^{.9696},$$

but the formula given in equation (229) is accurate enough for all practical uses.

† **Napier's formula** is very commonly used and is accurate enough for most calculations. It is usually stated in the form

$$w = \frac{A_0 P_1}{70},$$

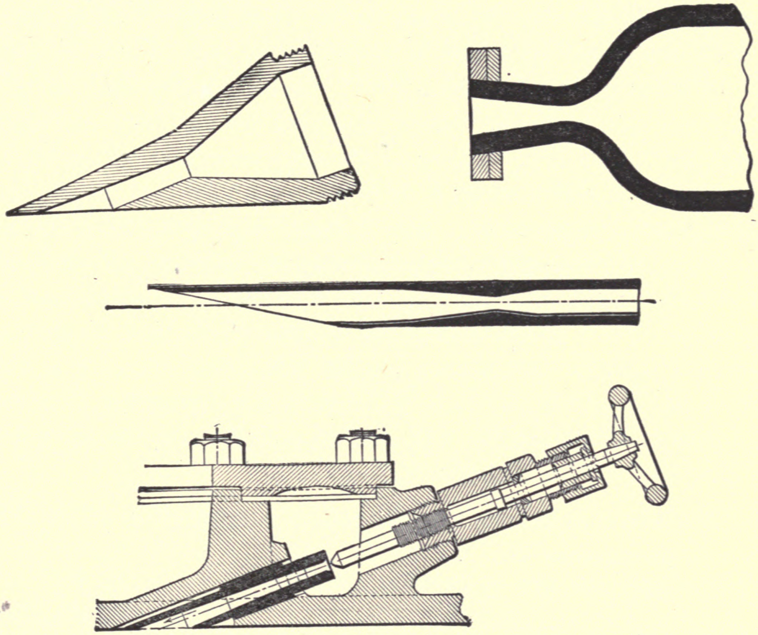
where w, P_1, and A_0 have the same significance as in *Grashof's* formula. The following **formula is given by Rateau,** but is too complicated for convenient use:

$$w = 0.001\, A_0 P_1\,[15.26 - 0.96\,(\log P_1 + \log 0.0703)].$$

Common or base 10 logarithms are to be used in this formula.

These formulas are for the flow of steam **initially dry and saturated.** An illustration of their applications is given by the following practical example.

Example. The area of the smallest section (A_0) of a suitably designed nozzle is 0.54 square inch. What is the weight of the flow (w) of dry saturated steam per second from this nozzle when the initial pressure (P_1) is 135 pounds per square inch absolute and the discharge pressure (P_2) is 15 pounds per square inch absolute?



DeLaval Type.

Curtis Type.

FIG. 60. — Examples of Standard Designs of Nozzles.

Here P_2 is less than 0.58 P_1 and Grashof's formula is applicable, or, $$w = \frac{0.54\,(135)^{.97}}{60},$$

$$w = \frac{0.54 \times 116.5^{*}}{60} = 1.049 \text{ pounds per second.}$$

When steam passes through a series of nozzles one after the other as is the case in many types of turbines, the pressure is reduced and the steam is condensed in each nozzle so that it becomes more moist each time. In the low-pressure nozzles of a turbine, therefore, the steam may be very wet although initially it was dry. Turbines are also sometimes designed to operate with **steam which is initially wet,** and this is usually the case when **low-pressure steam turbines** are operated with the exhaust from non-condensing reciprocating engines. In all these cases the nozzle area must be corrected for the moisture in the steam. For a given nozzle the weight discharged is greater for wet than for dry steam; but the percentage increase in the discharge is not nearly in proportion to the percentage of moisture as is often stated. The general equation for the theoretic discharge (w) from a nozzle is in the form †

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* The flow (w) calculated by Napier's formula for this example is $w = \frac{0.54 \times 135}{70}$ = 1.041 pounds per second.

† The general equation for the theoretic flow is

$$w = A_0 \sqrt{\frac{2\,gkP_1}{(k-1)\,v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]},$$

where the symbols w, A_0, P_1, and g are used as in equations (228) and (229). P_2 is the pressure at any section of the nozzle, v_1 is the volume of a pound of steam at the pressure P_1, and k is a constant. The flow, w, has its maximum value when

$$\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}$$

is a maximum. Differentiating and equating the first differential to zero gives

$$\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}.$$

P_2 is now the pressure at the *smallest section,* and writing for clearness P_0 for P_2, and substituting this last equation in the formula for flow (w) above, we have

$$w = K\sqrt{\frac{P_1}{v_1}}, \tag{231}$$

where P_1 is the initial absolute pressure and v_1 is the specific volume (cubic feet in a pound of steam at the pressure P_1). Now, neglecting the volume of the water in wet steam, which is a usual approximation, the volume of a pound of steam is proportional to the quality (x_1). For **wet steam** the equation above becomes then

$$w = K\sqrt{\frac{P_1}{x_1 v_1}}. \tag{232}$$

The equation shows, therefore, that the **flow of wet steam is inversely proportional to the square root of the quality** (x_1). Grashof's equations can be stated then more generally as

$$w = \frac{A_0 P_1^{.97}}{60\sqrt{x_1}}, \tag{233}$$

$$A_0 = \frac{60\, w\, \sqrt{x_1}}{P_1^{.97}}. \tag{234}$$

These equations become the same as (229) and (230) for the case where $x_1 = 1$.

Flow of Steam when the Final Pressure is more than 0.58 of the Initial Pressure. For this case the discharge depends upon the final pressure as well as upon the initial. No satisfactory formula can be given in simple terms, and the flow is most easily calculated with the aid of the curve in **Fig. 61,** due to Rateau.

$$w = A_0\sqrt{\frac{2\,gk}{k+1}\left(\frac{P_0}{P_1}\right)^{\frac{2}{k}}\left(\frac{P_1}{v_1}\right)}.$$

Now regardless of what the final pressure may be, the pressure (P_0) at the smallest section of a nozzle (A_0) is always nearly 0.58 P_1 for dry saturated steam. Making then in the last equation $P_0 = 0.58\,P_1$ and putting for k Zeuner's value of 1.135 for dry saturated steam, we may write in general terms the form stated above,

$$w = K\sqrt{\frac{P_1}{v_1}},$$

where K is another constant. See Zeuner's *Theorie der Turbinen*, page 268 (Ed. of 1899).

This curve is used by determining first the ratio of the final to the initial pressure $\frac{P_2}{P_1}$, and reading from the curve the corresponding coefficient showing the ratio of the required discharge to that calculated for the given conditions by either of the equations (229) or (233). The coefficient from the curve times the flow calculated from equations (229) or (233) is the required result. Obviously the discharge for this condition is always less than the discharge when the final pressure is equal to or less than 0.58 of the initial.

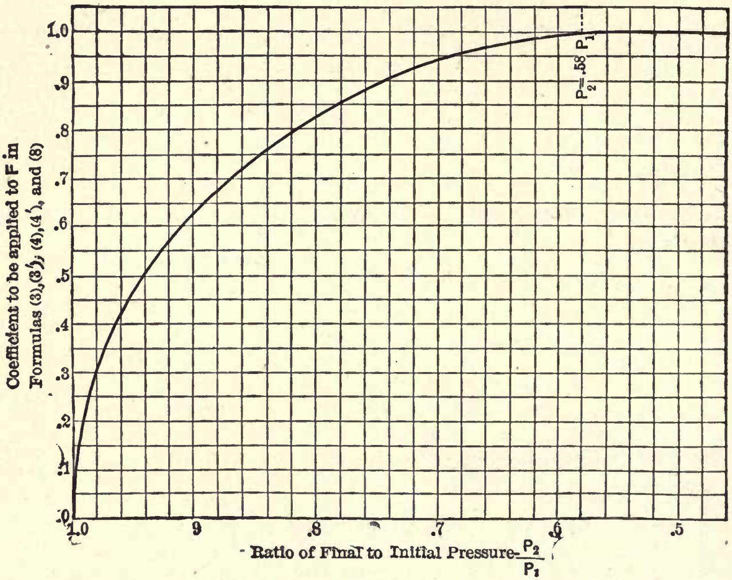

FIG. 61.— Coefficients of the Discharge of Steam when the Final Pressure is Greater than 0.58 of the Initial Pressure.

Length for Nozzles. The length of the nozzle is usually made to depend only on the initial pressure. In other words, the length of a nozzle for 150 pounds per square inch initial pressure is usually made the same for a given type regardless of the final pressure. And if it happens that there is crowding for space, one or more of the nozzles may be made a little shorter than the others.

Designers of De Laval nozzles follow practically the same "elastic" method. The divergence of the walls of non-condensing nozzles is about 3 degrees from the axis of the nozzle, and condensing nozzles for high vacuums may have a divergence of as much as 6 degrees[*] for the normal rated pressures of the turbine.

One of the authors has used successfully the following empirical formula to determine a suitable length, L, of the nozzle between the throat and the mouth (in inches):

$$L = \sqrt{15\,A_0},^{\dagger} \qquad (235)$$

where A_0 is the area at the throat in square inches.

Under- and Over-expansion. The best efficiency of a nozzle is obtained when the expansion required is that for which the nozzle was designed, or when the **expansion ratio** for the condition of the steam corresponds with the ratio of the areas of the mouth and throat of the nozzle. A little **under-expansion** is far better, however, than the same amount of **over-expansion,** meaning that a nozzle that is too small for the required expansion is more

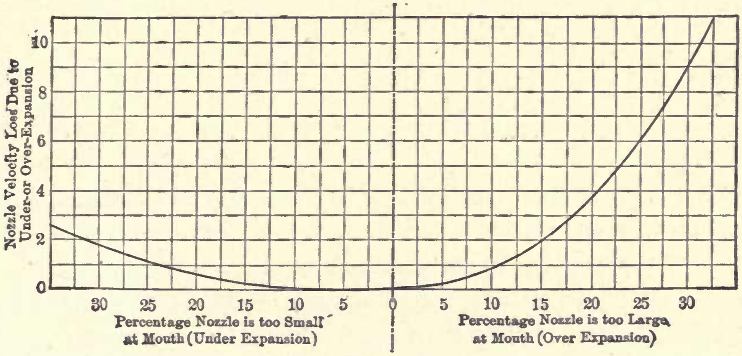

FIG. 62. — Curve of Nozzle Velocity Loss.

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[*] According to Dr. O. Recke, if the total divergence of a nozzle is more than 6 degrees, eddies will *begin* to form in the jet. There is no doubt that a too rapid divergence produces a velocity loss.

[†] Moyer's *Steam Turbines.*

efficient than one that is correspondingly too large.[*] **Fig. 62** shows a curve representing average values of nozzle loss[†] used to determine discharge velocities from nozzles under the conditions of **under-** or **over-expansion.**

Non-expanding Nozzles. All the nozzles of Rateau steam turbines and usually also those of the low-pressure stages of Curtis turbines are made non-expanding; meaning, that they have the same area at the throat as at the mouth. For such conditions it has been suggested that instead of a series of separate nozzles in a row a single long nozzle might be used of which the sides were arcs of circles corresponding to the inside and outside pitch diameters of the blades. Advantages would be secured both on account of cheapness of construction and because a large amount of friction against the sides of nozzles would be eliminated by omitting a number of nozzle walls. Such a construction has not proved desirable, because by this method no well-formed jets are secured and the loss from eddies is excessive. The general statement may be made that the **throat** of a well-designed nozzle should have a nearly symmetrical shape, as for example a circle, a square, etc., rather than such shapes as ellipses and long rect-

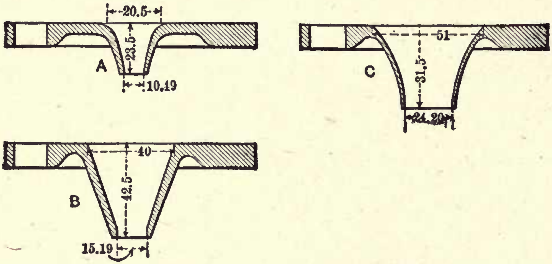

FIG. 63. — Non-expanding Nozzles.

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[*] It is a very good method to design nozzles so that at the rated capacity the nozzles under-expand at least 10 per cent, and maybe 20 per cent. The loss for these conditions is insignificant, and the nozzles can be run for a large overload (with increased pressures) in nearly all types without immediately reducing the efficiency very much.

[†] C. P. Steinmetz, *Proc. Am. Soc. Mech. Engineers*, May, 1908, page 628; J. A. Moyer, *Steam Turbines*.

angles. The shape of the mouth is not important. In Curtis turbines an approximately rectangular mouth is used because the nozzles are placed close together (usually in a nozzle plate like **Fig. 60**) in order to produce a continuous band of steam; and, of course, by using a section that is rectangular rather than circular or elliptical, a band of steam of more nearly uniform velocity and density is secured.

Fig. 63 shows a number of designs of non-expanding nozzles used by Professor Rateau. The length of such nozzles beyond the throat is practically negligible. Curtis non-expanding nozzles are usually made the same length as if expanding and the length is determined by the throat area.

Materials for Nozzles. Nozzles for saturated or slightly superheated steam are usually made of bronze. Gun metal, zinc alloys, and delta metal are also frequently used. All these metals have unusual resistance for erosion or corrosion from the use of wet steam. Because of this property as well as for the reason that they are easily worked with hand tools * they are very suitable materials for the manufacture of steam turbine nozzles. Superheated steam, however, rapidly erodes all these alloys and also greatly reduces the tensile strength. For nozzles to be used with highly superheated steam, cast iron is generally used, and except that it corrodes so readily is a very satisfactory material. Commercial copper (about 98 per cent) is said to have been used with a fair degree of success with high superheats; but for such conditions its tensile strength is very low. Steel and cupro-nickel (8 Cu + 2 Ni) are also suitable materials, and the latter has the advantage of being practically non-corrodible.

The most important part of the design of a nozzle is the determination of the areas of the various sections — especially the smallest section, if the nozzle is of an expanding or diverging type. In order to calculate the areas of nozzles we must know how to determine the quantity of steam (flow) per unit of time passing through a unit area. It is very essential that the

* Nozzles of irregular shapes are usually filed by hand to the exact size.

nozzle is well rounded on the " entrance " side and that sharp edges along the path of the steam are avoided. Otherwise it is not important whether the shape of the section is circular, elliptical, or rectangular with rounded corners.

Whether the nozzle section is throughout circular, square, or rectangular (if these last sections have rounded corners) the efficiency as measured by the **velocity** will be about 96 to 97 per cent, corresponding to an equivalent **energy efficiency** of 92 to 94 per cent.

PROBLEMS

1. Air at a temperature of 100° F. and pressure of 100 lbs. per sq. in. absolute flows through a nozzle against a back pressure of 20 lbs. per sq. in. absolute. Assuming the initial velocity to be zero, what will be the velocity of discharge?

2. If the area at the mouth of the above nozzle is 0.0025 sq. ft. and the coefficient of discharge is unity, how many pounds of air will be discharged per minute?

3. What will be the theoretical kinetic energy per minute of the above jet assuming no frictional losses?

4. Steam at a pressure of 150 lbs. per sq. in. absolute and 3 per cent moisture flows through a nozzle against a back pressure of 17 lbs. per sq. in. absolute. Calculate the velocity at the throat of the nozzle. What will be the velocity at the end of the nozzle?

5. Steam at a pressure of 200 lbs. per sq. in. absolute, temperature 530° F., expands in a nozzle to a vacuum of 28.5 ins. (30-in. barometer). Calculate the absolute velocity of the steam leaving the nozzle.

6. The area of a nozzle at its smallest section is 0.75 sq. in. It is supplied with dry saturated steam at 160 lbs. per sq. in. absolute pressure and the exhaust pressure is 2 lbs. per sq. in. absolute. How many pounds of steam per hour will the nozzle discharge?

7. A steam nozzle is to be designed that will discharge 500 lbs. of steam per hour. The pressure of the steam is 175 lbs. per sq. in. absolute and has 100° F. superheat. The exhaust pressure is 28 ins. vacuum (barometer 29.82). What will be the area of the nozzle at its smallest section and at its end?

8. A safety valve is to be designed for a 200 horse power boiler generating steam at 150 lbs. per sq. in. absolute pressure and 5 per cent moisture. Assuming that the safety valve should have a capacity such that it will release the boiler of all steam when generating double its rated capacity, what will be the smallest sectional area of the valve?

9. An injector supplies water to a boiler. The boiler pressure is 100 lbs. per sq. in. absolute and the steam generated is dry and saturated, the temperature of the entering feed water is 60° F. and the temperature of the discharged mixture of condensed steam and feed water is 180° F. The suction head is 3 ft. What is the weight of feed water supplied to the boiler per pound of steam? What is the mechanical efficiency of the injector?

10. Design a nozzle to deliver 400 lbs. of steam per hour, initial pressure 175 lbs. per sq. in. absolute, final pressure atmospheric (barometer 28.62 ins.), temperature of steam 600° F.

CHAPTER X

APPLICATIONS OF THERMODYNAMICS TO COMPRESSED AIR AND REFRIGERATING MACHINERY

COMPRESSED AIR

Air when compressed may be used as the working medium in an engine, in exactly the same way as steam. Furthermore, it is an agent for the transmission of power and can be distributed very easily from a central station for the purpose of driving engines, operating quarry drills and various other pneumatic tools.

Air Compressors. The type of machine used for the compression of air is that known as a piston-compressor and consists of a cylinder provided with valves and a piston.

The work performed in the air-cylinder of a compressor can best be studied successfully from an indicator diagram. If the compression is performed very slowly in a conducting cylinder, so that the air within may lose heat by conduction to the atmosphere as fast as heat is generated by compression, the process will in that case be isothermal, at the temperature of the atmosphere. Also if the compressed air is distributed to be used in compressed air motors * or engines without a change of temperature, and that the process of expansion in the compressed air motors or engines is also indefinitely slow and consequently isothermal, then (if we neglect the losses caused by friction in pipes) there would be no waste of power in the whole process of transmission. The indicator diagram would then be the same per pound of air in the compressor as in the air motor or engine, although the course of the cycle would be the reverse — that is, it would retrace itself.

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* Compressed air motors are similar to steam engines, but use compressed air instead of steam.

Adiabatic compression and expansion take place approximately if the expansion and compression are performed very quickly, or when the air is not cooled during compression, — in such a case the temperature of the air would rise. The theoretical indicator diagram of the compressor, **Fig. 64**, is **FCBE** and that of the air engine, **Fig. 65**, is **EADF.** **CB** and **AD** are both adiabatic lines. The change of volume of the compressed air from that of **EB** to **EA** occurs through its cooling in the distributing pipes, from the temperature produced by adiabatic compression down to the temperature of the atmosphere.

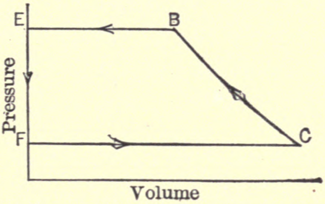

FIG. 64. — Diagram of Compressor.

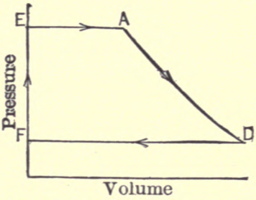

FIG. 65. — Diagram of Air Engine.

Suppose both diagrams of the compressor and of the air engine are superimposed as in **Fig. 66,** and then an imaginary isothermal line is drawn between the points **A** and **C.**

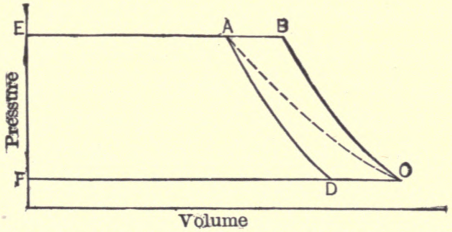

FIG. 66. — Superimposed Diagrams of Figs. 21 and 22.

It is then evident that the use of adiabatic compression causes a waste of power which is measured by the area **ABC,** while the use of the adiabatic expansion in the air engine involves a further waste, shown by the area **ACD.**

Work of Compression. Assuming no clearance in the compressor and isothermal compression, the pressure-volume diagram will be similar to **Fig. 64.** The work of the cycle will be

$$W = P_c V_c - P_c V_c \log_e \frac{V_c}{V_b} - P_b V_b, \qquad (236)$$

which becomes, since $P_c V_c = P_b V_b$,

$$W = -P_c V_c \log_e \frac{V_c}{V_b} = P_c V_c \log_e \frac{V_b}{V_c}. \qquad (237)$$

In practice the compression cannot be made strictly isothermal, as the operation of the piston would be too slow. The difference between isothermal and adiabatic compression (and expansion) can be shown graphically as in **Figs. 67** and **68.** In these illustrations the terminal points are correctly placed for a certain ratio for both compression and expansion. Note that in the compressing diagram (**Fig. 67**) the area between the two curves, **ABC,** represents the work lost in compressing due to heating, and the area between the two curves, **ACMNF** (in **Fig. 68**), shows the work lost by cooling during the expansion. The isothermal curve **AC** will be the same for both cases.

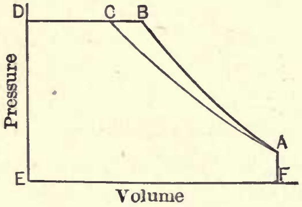

FIG. 67. — Compression Diagram.

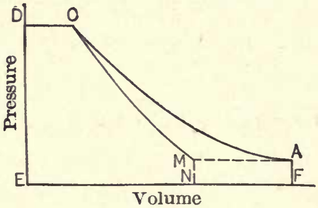

FIG. 68. — Expansion Diagram.

The temperature of the air is prevented as far as possible from rising during the compression by injecting water into the compressing cylinder, and in this way the compression curve will change. The curves which would have been PV = a constant, if isothermal, and $PV^{1.4}$ = a constant, if adiabatic, will be very much modified. In perfectly adiabatic conditions the exponent "n" = 1.40 for air, but in practice the compressor cylinders are water-jacketed, and thereby part of the heat of compression is conducted away, so that "n" becomes less than 1.40. This value of "n" varies with conditions; generally the value is between 1.2 and 1.3.

When the compression curve follows the law, PV^n equals a constant, work of compression (**Fig. 64**) is

$$W = P_cV_c + \frac{P_cV_c - P_bV_b}{n - 1} - P_bV_b$$

$$= \frac{n}{n - 1}(P_cV_c - P_bV_b), \qquad (238)$$

since $$P_cV_c^n = P_bV_b^n;\ V_b = V_c\left(\frac{P_c}{P_b}\right)^{\frac{1}{n}}. \qquad (239)$$

Combining equations (238) and (239),

$$W = \frac{n}{n - 1}P_cV_c\left(1 - \left(\frac{P_b}{P_c}\right)^{\frac{n-1}{n}}\right). \qquad (240)$$

The Effect of Clearance upon Volumetric Efficiency. It is impossible to construct a compressor without clearance, consequently the indicator diagram differs from the ideal case. At the end of the discharge stroke, the clearance volume is filled with compressed air. When the piston moves on its outward stroke, the clearance air expands and the suction valves of the compressor will be held shut until the piston has moved a sufficient distance to permit the entrapped air to expand to atmospheric pressure. When that point is reached any further movement of the piston opens the suction valves and external air is drawn into the cylinder. Thus the entire stroke of the compressor piston is not effective in pumping air. The ratio of the volume of air pumped to the volume swept by the piston, or piston displacement of the cylinder, is termed **volumetric efficiency.**

Fig. 69 illustrates an ideal compressor diagram with clearance. The air entrapped in the clearance space equals

$$v_3 = CV_s,$$

where V_s = volume swept or piston displacement of the cylinder.
C = percentage of clearance.

The air in **Fig. 69** expands to V_4 at which point the inlet valves open. The air drawn into the cylinder is represented by the difference in volume between V_1 and V_4. This air, as well as the clearance air, is compressed to point 2, while the compressed air is discharged from points 2 to 3. Knowing the per cent of clearance, the volume swept by the piston (V_s), and the initial and final pressure, the volumetric efficiency ($V_{ef.}$) may be determined from the following equations:

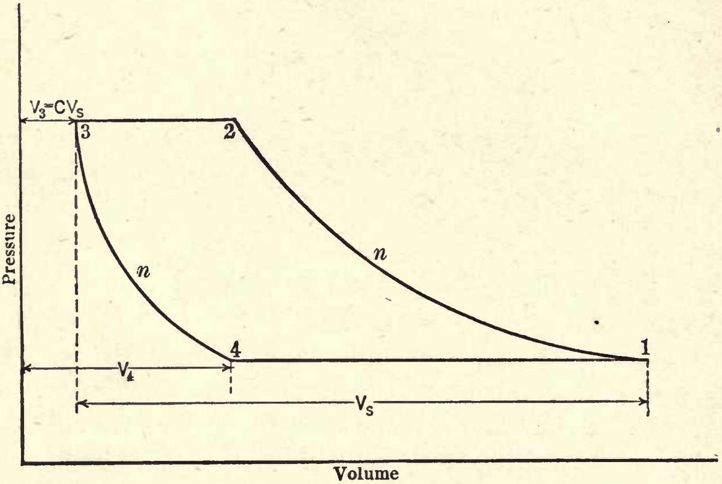

FIG. 69. — Ideal Air Compressor with Clearance.

$$V_{ef.} = \frac{V_1 - V_4}{V_s}.$$

$$P_3 (CV_s)^n = P_4 V_4^n.$$

$$V_4 = \left(\frac{P_3}{P_4}\right)^{\frac{1}{n}} CV_s,$$

since
$$V_1 = V_s + CV_s,$$

$$V_1 - V_4 = V_s + CV_s - \left(\frac{P_3}{P_4}\right)^{\frac{1}{n}} CV_s$$

$$= \left[1 + C\left(1 - \left(\frac{P_3}{P_4}\right)^{\frac{1}{n}}\right)\right] V_s.$$

Therefore volumetric efficiency is

$$V_{ef.} = \frac{\left[1 + C\left(1 - \left(\frac{P_3}{P_4}\right)^{\frac{1}{n}}\right)\right]V_s}{V_s}$$

$$= 1 + C\left[1 - \left(\frac{P_3}{P_4}\right)^{\frac{1}{n}}\right]. \qquad (241)$$

The volumetric efficiency also decreases with the altitude, as the weight of a cubic foot of air decreases as the altitude increases.

Two Stage Compression. The problem of economy, obviously, becomes one of abstracting the heat generated in the air during the process of compression. As previously mentioned, this is **partially** accomplished by water-jacketing the cylinders, and also by water injection. Nevertheless, owing to the short interval within which the compression takes place, and the comparatively small volume of air actually in contact with the cylinder walls, very little cooling really occurs. The practical impossibility of proper cooling to prevent waste of energy leads to the alternative of discharging air from one cylinder after partial compression has been effected, into a so-called **inter-cooler**, intended to absorb the heat generated during the first compression, and then compressing the air to the final pressure in another cylinder.

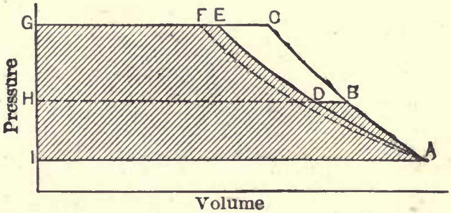

Fig. 70. — Indicator Diagram of Two-stage Air Compressor.

This operation is termed "**two-stage**" compression and when repeated one or more times for high pressures the term "**multi-stage**" compression applies.

Referring to **Fig. 70** and assuming the compression in a two-stage compressor to be adiabatic for each cylinder, the compression curve is represented by the broken line **ABDE**; the compression proceeds adiabatically in the first or low-pressure cylinder to **B**; the air is then taken to a cooler and cooled under practically constant pressure until its initial temperature is almost reached, and its volume reduced from **HB** to **HD**; it is then introduced to the second or high-pressure cylinder and compressed adiabatically along the line **DE** to the final pressure condition that was desired. It is seen that the compression curve approaches the isothermal line **FA**.* The isothermal condition is obviously desired and, in consequence, air-machines are built to approach that condition as nearly as possible.

Referring to **Fig. 70**, the work of each stage from equation (240) will be

$$W\ (\text{1st stage}) = \frac{n}{n-1} P_a V_a \left(1 - \left(\frac{P_b}{P_a} \right)^{\frac{n-1}{n}} \right).$$

$$W\ (\text{2d stage}) = \frac{n}{n-1} P_d V_d \left(1 - \left(\frac{P_e}{P_d} \right)^{\frac{n-1}{n}} \right).$$

The total work of compression is

$$W\ (\text{total}) = W\ (\text{1st stage}) + W\ (\text{2d stage})$$

$$= \frac{n}{n-1} \left\{ P_a V_a \left[1 - \left(\frac{P_b}{P_a} \right)^{\frac{n-1}{n}} \right] + P_d V_d \left[1 - \left(\frac{P_e}{P_d} \right)^{\frac{n-1}{n}} \right] \right\}.$$

With perfect cooling $P_a V_a = P_d V_d$; also $P_b = P_d$; $P_c = P_e$, then

$$W\ (\text{total}) = \frac{n}{n-1} P_a V_a \left[1 - \left(\frac{P_b}{P_a} \right)^{\frac{n-1}{n}} + \left(1 - \left(\frac{P_c}{P_b} \right)^{\frac{n-1}{n}} \right) \right]. \qquad (242)$$

The work of compression as expressed by equation (242) becomes a minimum when

$$\left(\frac{P_c}{P_b} \right)^{\frac{n-1}{n}} + \left(\frac{P_b}{P_a} \right)^{\frac{n-1}{n}} \qquad (243)$$

is a maximum. Since the initial and final pressures P_a and P_c are fixed, the pressure in the receiver, P_b, can be found by differ-

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* The line **FE** represents further cooling.

entiating equation (243) and equating this differential to zero, or

$$\frac{d}{dP_b}\left[\left(\frac{P_c}{P_b}\right)^{\frac{n-1}{n}}+\left(\frac{P_b}{P_a}\right)^{\frac{n-1}{n}}\right]=\frac{1-n}{n}\left(\frac{P_b^{\frac{1-2n}{n}}}{P_c^{\frac{1-n}{n}}}\right)+\frac{n-1}{n}\left(\frac{P_b^{-\frac{1}{n}}}{P_c^{\frac{1-n}{n}}}\right)=0,$$

or $$\frac{n-1}{n}\left(\frac{P_b^{-\frac{1}{n}}}{P_c^{\frac{1-n}{n}}}\right)=\frac{n-1}{n}\left(P_c^{\frac{n-1}{n}}P_b^{\frac{1-2n}{n}}\right),$$

$$\therefore \quad P_b^2 = P_aP_c. \tag{244}$$

For compressing air to high pressures three and four stage compressors are used.

REFRIGERATING MACHINERY

Refrigerating Machines or Heat Pumps. By a refrigerating machine or heat pump is meant a machine which will carry heat from a cold to a hotter body.* This, as the second law of thermodynamics asserts, cannot be done by a self-acting process, but it can be done by the expenditure of mechanical work. Any heat engine will serve as a refrigerating machine if it be forced to trace its indicator diagram backward, so that the area of the diagram represents work **spent on,** instead of **done by,** the working substance. Heat is then **taken in from the cold body** and heat is rejected to the hot body.

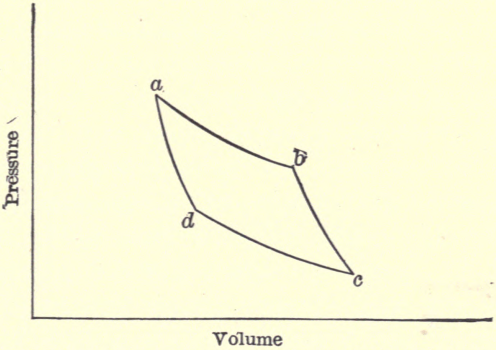

FIG. 71. — Pressure-volume Diagram of Carnot Cycle.

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* This statement is not at variance with our knowledge that heat does not *flow* of itself from a cold body to a hotter body.

Take the Carnot cycle, using air as working substance (**Fig. 71**), and let the cycle be performed in the order **dcba**, so that the area of the diagram is negative, and represents work spent upon the working substance. In the stage **dc**, which is isothermal expansion in contact with the cold body **R** (as in **Fig. 7**, page 41), the air takes in a quantity of heat from **R** equal to $wRT_2 \log_e r$ [equation (46)], and in stage **ba** it gives out to the hot body **H** a quantity of heat equal to $wRT_1 \log_e r$. There is no transfer of heat in stages **cb** and **ad**. Thus **R**, the cold body, is constantly being drawn upon for heat and can therefore be maintained at a temperature lower than its surroundings. In an actual refrigerating machine **operating with air**, the cold body **R** consists of a coil of pipe through which brine circulates while " working " air is brought into contact with the outside of the pipe. The brine is kept, by the action of the machine, at a temperature below 32° F. and is used in its turn to extract heat by conduction from the water which is to be frozen to make ice. The " cooler " **H**, which is the relatively hot body, is kept at as low a temperature as possible by means of circulating water, which absorbs the heat rejected to **H** by the " working " air.

The size of an air refrigerating machine is very large as compared with its performance. The use of a regenerator, as in Stirling's engine (**Fig. 8**), may be resorted to in place of the two adiabatic stages in the Carnot cycle, with the advantage of making the machine much less bulky. Refrigerating machines using air as working substance, with a regenerator, were introduced by Dr. A. C. Kirk and have been widely used.* The working air is completely enclosed, which allows it to be in a compressed state throughout, so that even its lowest pressure is much above that of the atmosphere. This makes a greater mass of air pass through the cycle in each revolution of the machine, and hence increases the performance of a machine of given size. In all air refrigerating machines the

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* See Kirk, On the Mechanical Production of Cold, *Proc. Inst. of C. E.*, vol. XXXVII, 1874. Also lectures on Heat and its Mechanical Applications, in the same proceedings for 1884.

temperature range must be high to produce a given refrigerating effect.

In another class of refrigerating machines the working substance, instead of being air, consists of a liquid and its vapor, and the action proceeds by alternate evaporation under a low pressure and condensation under a relatively high pressure. A liquid must be chosen which evaporates at the lower extreme of temperature under a pressure which is not so low as to make the bulk of the engine excessive. Ammonia, ether, sulphurous acid, and other volatile liquids have been used. Ether machines are inconveniently bulky and cannot be used to produce intense cold, for the pressure of that vapor is only about 1.3 pounds per square inch at 4° F., and to make it evaporate at any temperature nearly as low as this would require the cylinder to be excessively large in proportion to the performance. This would not only make the machine clumsy and costly, but would involve much waste of power in mechanical friction. The tendency of the air outside to leak into the machine is another practical objection to the use of so low a pressure. With ammonia a distinctly lower limit of temperature is practicable: the pressures are rather high and the apparatus is compact. Carbonic acid has been used as a refrigerant in small machines. The objection to carbonic acid is that the pressures are very high as compared with ammonia (see Appendix). The critical temperature of carbonic acid is less than 90° F.

Unit of Refrigeration. The capacity of a refrigerating machine is usually expressed in tons of refrigeration or ice melting effect per twenty-four hours. As the latent heat of fusion of ice is about 144 B.t.u., the heat units withdrawn per ton of refrigerating effect per twenty-four hours is

$$2000 \times 144 = 288{,}000 \text{ B.t.u.}$$

Systems of Mechanical Refrigeration. The standard systems of mechanical refrigeration are:*

(A) The dense-air system, so-called because the air which is

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* Lucke's *Engineering Thermodynamics*, page 1148.

the medium is never allowed to fall to atmospheric pressure, so as to reduce the size of the cylinders and pipes through which a given weight is circulating.

(B) The compression system, using ammonia, carbon dioxide or sulphur dioxide, and so-called to distinguish it from the third system, because a compressor is used to raise the pressure of the vapor and deliver it to the condenser after removing it from the evaporator.

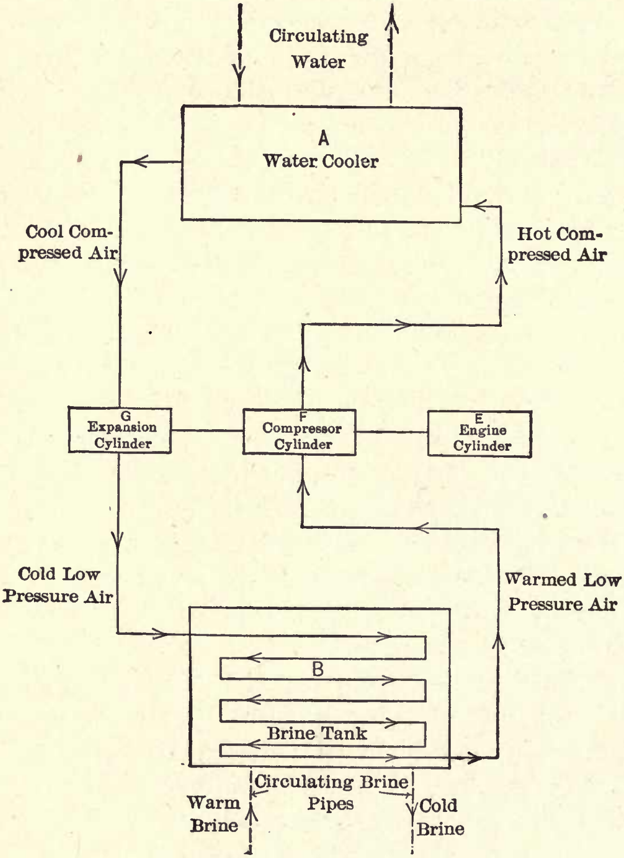

FIG. 72. — Dense Air System of Refrigeration.

(C) The absorption system, using ammonia, and so-called because a weak water solution removes vapor from the evaporator by **absorption,** the richer aqua ammonia so formed being pumped into a high-pressure chamber called a generator in communication with the condenser, where the ammonia is discharged from the liquid solution to the condenser by heating the generator, to which the solution is delivered by the pump.

No matter what system is used, circulating water is employed to receive the heat, the temperature of which limits the highest temperature allowable in the system and indirectly the highest pressure.

The Air System of Refrigeration. The **dense** or **closed air system** is illustrated in **Fig. 72,** in which air, previously freed of moisture, is continuously circulated. The engine cylinder **E** furnishes power * to drive the compressor cylinder **F.** This cylinder delivers hot-compressed air into a cooler **A** where it is cooled, and then passed on to the expansion cylinder **G** (tandem-connected to both, the compressor **F** and to the engine cylinder **E**), which in turn sends cold low-pressure air first through the refrigerating coils in the brine tank **B** and then back to the compressor cylinder **F**; thus the air cycle is completed. The courses of the circulating water and also of the brine are shown by the dotted lines.

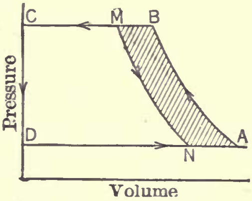

FIG. 73. — Pressure-volume Diagram of Dense Air Cycle of Refrigeration.

The dense-air cycle in a pressure-volume diagram is represented in **Fig. 73,** in which **BC** is the delivered volume of hot-compressed air; **CM** is the volume of cooled air admitted in the expansion cylinder; **MB** the reduction in volume due to the water cooler; **MN** the expansion; **NA** the refrigeration or heating of the air by the brine, and **AB** the compression. This operation is but a reproduction of that previously described.

The work W_c expended in the compression cylinder F is $DABC$

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* Since the work done by the expansion of the cool-compressed air is less than that necessary for the compressing of the air taken from the brine coils through the same pressure conditions, a means must be employed to make up for the difference, and for this purpose the engine cylinder is used.

(**Fig.** 73); that done by the expansion cylinder G is $W_e = DCMN$. The shaded area $MBAN$ represents the work which must be supplied by the engine E.

If w pounds of air are passing through the refrigerating machine per minute, the heat withdrawn from the cold room or absorbed by the brine along NA (**Fig.** 73) is

$$Q_{NA} = wC_p\,(t_a - t_n). \tag{245}$$

The work expended in compressing w pounds of air, assuming polytropic compression, is, by equation (240),

$$W_c = \frac{wnP_aV_a}{n-1}\left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{n-1}{n}}\right].$$

If the compression is adiabatic,

$$W_c = wP_aV_a\frac{\gamma}{\gamma-1}\left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{\gamma-1}{\gamma}}\right], \tag{246}$$

since $$\frac{T_b}{T_a} = \left(\frac{P_b}{P_a}\right)^{\frac{\gamma-1}{\gamma}} \text{ also } AR = C_p - C_v = C_p\left(\frac{\gamma-1}{\gamma}\right)$$

$$W_c = \frac{wC_p}{A}\,(t_b - t_a).$$

If the expansion is complete in the expansion cylinder, the work done in expansion is

$$W_e = \frac{wC_p}{A}\,(t_m - t_n). \tag{247}$$

The net work of the engine E required to produce refrigeration becomes

$$W = W_c - W_e = \frac{wC_p}{A}\,(t_b - t_a - t_m + t_n). \tag{248}$$

Since, from equation (245),

$$w = \frac{Q_{NA}}{C_p}\,(t_a - t_n), \text{ equation (248) becomes}$$

$$W = \frac{Q_{NA}}{A}\left(\frac{t_b - t_a - t_m + t_n}{t_a - t_n}\right). \tag{249}$$

The Vapor Compression System of Refrigeration. The **compression system** for ammonia or similar condensable vapors is shown in **Fig. 74.** The figure illustrates the essential members of a complete compression refrigerating system. **B** represents the direct-expansion coil in which the working medium is evaporated; **F,** the compressor or pump, for increasing the pressure of the gasified ammonia; **E,** the engine cylinder, — the source of power; **W,** the condenser, for cooling and liquefying the gasified ammonia; and **V,** a throttling valve, by which the flow of

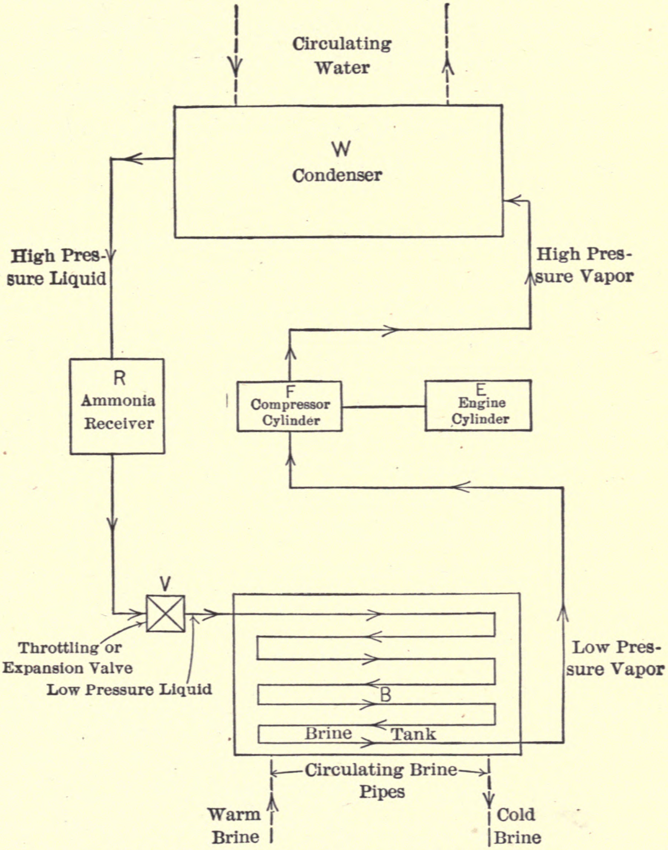

FIG. 74. — Compression System of Refrigeration.

liquefied ammonia under the condenser pressure is controlled as it flows from the receiver **R** to the expansion coils; **B**, the brine tank, in which a materially lower pressure is maintained by the pump or compressor in order that the working medium may boil at a sufficiently low temperature to take heat from and consequently refrigerate the brine which is already cooled.

The operation of the compressor F (**Fig. 74**) is theoretically a reversed Rankine cycle (**Figs. 31** and **32**). If one pound of vapor is passed through the system, the work (W_c) of the compressor is

$$W_c = (H_2 - H_1) \text{ (B.t.u.)},$$

where

H_1 = the total heat of the vapor at entrance to the compressor,

H_2 = the total heat of the vapor discharged from the compressor.

The vapor entering the compressor may be treated as though it were dry and saturated. The entropy-temperature diagram (**Fig. 75**) illustrates this case. The vapor discharged by the compressor will then be superheated, having a temperature t_s at a pressure P_2. The condition of the discharged vapor is determined by equating the entropies at the inlet and discharge pressures. The total heat (H_2) of the discharged vapor will be

$$H_2 = [h_2 + L_2 + C_p (t_s - t_2)]. \tag{250}$$

The horse power of the compressor is, if w pounds of vapor is circulated per minute:

$$\text{H.p.} = \frac{w (H_2 - H_1) 778}{33{,}000}. \tag{251}$$

The heat absorbed per minute by w pounds of vapor passing through the refrigerating or brine coils will be

$$\begin{aligned} Q_r &= w (h_1 + L_1 - h_0) \\ &= w (H_1 - h_0). \end{aligned} \tag{252}$$

h_o = the liquid heat of the vapor after leaving the condenser.

The heat absorbed per minute by the condenser (Q_c) is

$$w\,(H_2 - h_0)\ \text{(B.t.u.)}. \tag{253}$$

The heat absorbed by the condenser is theoretically equal to the sum of the heat absorbed in the refrigerator and the heat equivalent of the work of compression.

Ammonia compressors are operated either on the dry or on the wet system. In the dry system the ammonia entering the compressor is a dry vapor as illustrated in **Fig. 75.** In the wet system the ammonia enters the compressor in the wet state, the heat developed during the compression being used in evaporating the liquid ammonia into a vapor.

The Vapor Absorption System of Refrigeration. The absorption system depends upon the fact that anhydrous ammonia possesses the property of forming aqua ammonia. The amount of ammonia water will absorb depends upon the temperature of the water; the colder the water, the greater are its absorptive powers.

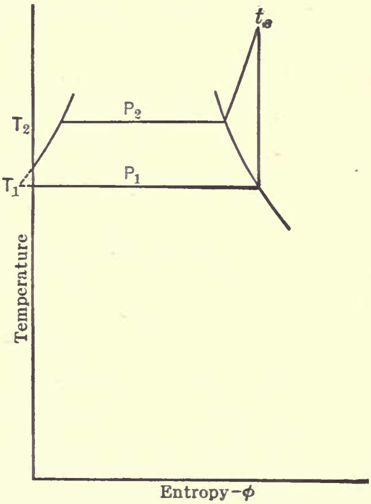

FIG. 75. — Entropy-Temperature Diagram for Ammonia.

The absorption system differs from the vapor compression system in that the absorption system replaces the compressor by an absorber, where the anhydrous ammonia is changed into aqua ammonia, a pump which transfers the ammonia from the absorber to the generator, and a generator, where the aqua ammonia is heated. Both systems have a condenser where the ammonia is cooled and liquefied and an expansion valve or throttling valve by means of which the flow of liquid ammonia to the expansion coils is controlled. In the absorption system the anhydrous ammonia vapor flows from the expansion coils to the absorber, in which the anhydrous vapor comes in contact with weak aqua ammonia. The weak ammonia absorbs the ammonia vapor. From the absorber the ammonia is pumped into the generator, where it is heated by steam coils. The vapor driven off in the generator passes to the condenser and from there through the expansion or throttle valve to the expansion coils which are located in the brine tank or in the refrigerating room.

The absorption system is usually provided with a rectifier to thoroughly dry the gas before it enters the condenser, an exchanger which heats the strong ammonia by cooling the weak ammonia, and with other auxiliary equipment to reduce the heat losses in the system.

COEFFICIENT OF PERFORMANCE OF REFRIGERATING MACHINES

$$\left.\begin{array}{c}\text{The Coefficient of Per-}\\ \text{formance}\end{array}\right\} = \frac{\text{Heat extracted from the cold body}}{\text{Work expended}}.$$

This ratio may be taken as a coefficient of performance in estimating the merits of a refrigerating machine. When the limits of temperature T_1 and T_2 are assigned it is very easy to show by a slight variation of the argument used in Chapter IV that no refrigerating machine can have a higher coefficient of performance than one which is reversible according to the Carnot method. For let a refrigerating machine **S** be driven by another **R** which is reversible and is used as a heat-engine in driving **S**. Then if **S** had a higher coefficient of performance than **R** it would take from the cold body more heat than **R** (working reversed) rejects to the cold body, and hence the double machine, although purely self-acting, would go on extracting heat from the cold body in violation of the Second Law of Thermodynamics. Reversibility, then, is the test of perfection in a refrigerating machine just as it is in a heat-engine.

When a reversible refrigerating machine takes in all its heat, namely Q_c at T_2, and rejects all, namely Q_a at T_1, and if we represent the heat equivalent of the work done by $W = Q_a - Q_c$, then the coefficient of performance is, as already defined,

$$\frac{Q_c}{W} = \frac{Q_c}{Q_a - Q_c} = \frac{T_2}{T_1 - T_2}. \tag{254}$$

Hence, the smaller the range of temperature, the better is the performance. To cool a large mass of any substance through a few degrees will require much less expenditure of energy than to cool one-fifth of the mass through five times as many degrees, although the amount of heat extracted is the same in both cases. If we wish to cool a large quantity of a substance it is better to do this by the direct action of a refrigerating machine working through the desired range of temperature, than to cool a portion through a wider range and then let this mix with the rest. This is only another instance of a wide, general principle, that any mixture of substances at different temperatures is thermodynamically wasteful because the interchange of heat between them is irreversible. An ice-making machine, for example, should have its lower limit of temperature only so much lower than 32° F. as will allow heat to be conducted to the working fluid with sufficient rapidity from the water that is to be frozen.

PROBLEMS

1. If 200 cu. ft. of free air per minute (sea-level) is compressed isothermally and then delivered into a receiver, the internal pressure of which is 102.9 lbs. per sq. in. absolute, find the theoretical horse power required.

2. What will be the net work done in foot-pounds per stroke by an air compressor displacing 3 cu. ft. per stroke and compressing air from atmospheric pressure to a gage pressure of 75 lbs.? (Isothermal compression.)

3. What horse power will be needed to compress adiabatically 1500 cu. ft. of free air per minute to a gage pressure of 58.8 lbs., when n equals 1.4?

4. A compressed-air motor without clearance takes air at a condition of 200 lbs. per sq. in. (gage) and operates under a cut-off at one-fourth stroke. What is the work in foot-pounds that can be obtained per cubic foot of compressed air, assuming free air pressure of 14.5 lbs. and n equal to 1.41?

5. Find the theoretical horse power developed by 3 cu. ft. of air per minute having a pressure of 200 lbs. per sq. in. absolute, if it is admitted and expanded in an air engine with one-fourth cut-off. The value of n is 1.2. (Neglect clearance.)

6. Compute the net saving in energy that is effected by compressing isothermally instead of adiabatically 50 cu. ft. of free air to a pressure of 200 lbs. per sq. in. gage. Barometer = 14 lbs. per sq. in., and a temperature of 70° F. What is the increase in intrinsic energy during each kind of compression? How much heat is lost to the jacket-water during each kind of compression?

7. Let a volume of 12 cu. ft. of free air be adiabatically compressed in a one stage air compressor from atmospheric pressure (15 lbs.) to 85 lbs. gage; the initial temperature of the air being 70° F.

- (a) What is the volume and temperature of the air after the compression?
- (b) Suppose this heated and compressed air is cooled to an initial temperature of 60° F. at constant volume, what is its pressure for that condition?
- (c) Now if the air occupies such a volume as found in (a) and at an absolute pressure as determined in (b), at a temperature of 60° F., and is then allowed to expand adiabatically down to atmospheric pressure (15 lbs.), what is the temperature and volume of the expanded air in Fahrenheit degrees?

8. Compare the work required to compress one pound of air from atmospheric pressure and a temperature of 60° F. to a pressure of 200 lbs. per sq. in. absolute in (1) a one-stage and (2) in a two-stage compressor. Assume $n = 1.25$.

9. Using equation 249 as a basis, deduce a formula for the horse power required to abstract a given number of heat units per minute by an air refrigerating machine.

10. Calculate the approximate dimensions of a ten-ton air refrigerating machine and the power required to drive it. Pressure in the cold chamber is atmospheric and the temperature 32° F. Pressure of air delivered by compressor is 90 lbs. per sq. in. gage. The temperature of the air coming from condenser is 85° F. and the machine operates at 60 r.p.m. Allow 12 lbs. drop in pressure between the compressor and the expanding cylinder.

11. Calculate the approximate dimensions of a ten-ton ammonia compression refrigerating machine and the power required to drive it. The temperature in the expansion coils is 15° F. and the temperature in the condenser is 85° F. The machine is double acting and operates at a speed of 80 r.p.m.

12. What is the ice-making capacity per twenty-four hours of the machine in problem 11? The temperature of the water to be frozen is 80° F.

APPENDIX

TABLE I

SPECIFIC HEAT OF GASES AND VAPORS

(Taken from Smithsonian Physical Tables)

Substance	Range of Temp.° C.	C_p	Authority	$\frac{C_p}{C_v}$	Authority	Calculated C_v
Air.........	−30–10	0.23771	Regnault			
	0–100	0.23741	"			
	0–200	0.23751	"			
	20–100	0.2389	Wiedemann			
	Mean	0.23788	....	1.4066	Various	0.1691
Alcohol, (ethyl)	108–220	0.4534	Regnault	1.136	Jaeger, Neyreneuf	0.3991
Alcohol, (methyl)	101–223	0.4580	Regnault			
Ammonia...	23–100	0.5202	Wiedemann			
	27–200	0.5356	"			
	24–216	0.5125	Regnault			
	Mean	0.5228	....	1.31	Cazin, Wüllner	0.3991
Benzene....	34–115	0.2990	Wiedemann			
	35–180	0.3325	"			
	116–218	0.3754	Regnault			
Carbon Dioxide	−28–77	0.1843	Regnault			
	15–100	0.2025	"			
	11–214	0.2169	"			
	Mean	0.2012	....	1.300	Wüllner Röntgen,	0.1548
Carbon Monoxide	23–99	0.2425	Wiedemann			
	26–198	0.2426	"	1.403	Cazin, Wüllner	0.1729
Ether.......	69–224	0.4797	Regnault			
	27–189	0.4618	Wiedemann			
	25–111	0.4280	"			
	Mean	0.4565	....	1.029	Müller	0.4436
Hydrogen...	−28–9	3.3996	Regnault			
	12–198	3.4090	"			
	21–100	3.4100	Wiedemann			
	Mean	3.4062	....	1.410	Cazin	2.419
Nitrogen	0–200	0.2438	Regnault	1.410	Cazin	0.1729
Sulphur Dioxide	16–202	0.1544	Regnault	1.26	Cazin Müller	0.1225
Water......	128–217	0.4805	Regnault			
	100–125	0.3787	Gray, Macfarlane,			
	Mean	0.4296	....	1.300	Various	0.3305

TABLE II

DENSITY OF GASES

(Taken from Smithsonian Physical Tables)

Gas	Specific gravity	Grams per cubic centimeter	Pounds per cubic foot
Air	1.000	0.001293	0.08071
Ammonia	0.597	0.000770	0.04807
Carbon dioxide	1.529	0.001974	0.12323
Carbon monoxide	0.967	0.001234	0.07704
Coal gas	0.320–0.740	0.000414–0.000957	0.02583–0.05973
Hydrogen	0.0696	0.000090	0.00562
Hydrogen sulphide	1.191	0.001476	0.09214
Marsh gas	0.559	0.000727	0.04538
Nitrogen	0.972	0.001257	0.07847
Oxygen	1.105	0.001430	0.08927
Sulphur dioxide	2.247	0.002785	0.17386

TABLE III. — PROPERTIES OF SATURATED STEAM.* (Marks and Davis.)

Absolute Pressure, Pounds per Square Inch.	Temperature, Degrees F.	Heat of the Liquid.	Heat of Vaporization.	Total Heat.	Heat Equivalent of Internal Work.	Heat Equivalent of External Work.	Entropy of the Liquid.	Entropy of the Vapor.	Total Entropy.	Specific Volume.	Density Weight per Cubic Foot, Pounds.
p	t	h	L	$H = h + L$	ρ	Apu	θ	$\frac{L}{T}$	$\theta + \frac{L}{T}$	v	w
†0.1	35.03	3.05	1071.7	1074.7	1017.3	54.4	0.0062	2.1666	2.1728	2935.0	0.000340
†0.2	53.15	21.23	1061.6	1082.8	1005.2	56.5	0.0423	2.0704	2.1127	1524.0	0.000656
†0.3	64.49	32.57	1055.3	1087.9	997.7	57.6	0.0640	2.0135	2.0775	1041.0	0.000961
0.4	72.91	40.95	1050.6	1091.6	992.4	58.5	0.0800	1.9730	2.0530	794.0	0.001259
0.5	79.68	47.71	1047.0	1094.6	987.6	59.3	0.0926	1.9413	2.0332	642.0	0.001555
0.6	85.32	53.34	1043.8	1097.1	983.9	59.9	0.1029	1.9155	2.0184	541.0	0.001850
0.7	90.18	58.18	1041.1	1099.3	980.7	60.4	0.1117	1.8936	2.0053	467.0	0.002143
0.8	94.46	62.45	1038.7	1101.2	977.8	61.0	0.1195	1.8747	1.9942	412.0	0.002431
0.9	98.33	66.31	1036.6	1102.9	975.2	61.4	0.1265	1.8578	1.9843	367.9	0.002719
1	101.83	69.8	1034.6	1104.4	972.9	61.7	0.1327	1.8427	1.9754	333.0	0.00300
2	126.15	94.0	1021.0	1115.0	956.7	64.3	0.1749	1.7431	1.9180	173.5	0.00576
3	141.52	109.4	1012.3	1121.6	946.4	65.8	0.2008	1.6840	1.8848	118.5	0.00845
4	153.01	120.9	1005.7	1126.5	938.6	67.0	0.2198	1.6416	1.8614	90.5	0.01107
5	162.28	130.1	1000.3	1130.5	932.4	68.0	0.2348	1.6084	1.8432	73.33	0.01364
6	170.06	137.9	995.8	1133.7	927.0	68.8	0.2571	1.5814	1.8285	71.89	0.01616
7	176.85	144.7	991.8	1136.5	922.4	69.4	0.2579	1.5582	1.8161	53.56	0.01867
8	182.86	150.8	988.2	1139.0	918.2	70.0	0.2673	1.5380	1.8053	47.27	0.02115
9	188.27	156.2	985.0	1141.1	914.4	70.6	0.2756	1.5202	1.7958	42.36	0.02361
10	193.22	161.1	982.0	1143.1	910.9	71.1	0.2832	1.5042	1.7874	38.38	0.02606
11	197.75	165.7	979.2	1144.9	907.8	71.5	0.2902	1.4895	1.7797	35.10	0.02849
12	201.96	169.9	976.6	1146.5	904.8	71.8	0.2967	1.4760	1.7727	32.36	0.03090

* Courtesy of the Publishers, Longmans, Green & Co.

† Interpolated.

TABLE III.—*Continued.* PROPERTIES OF SATURATED STEAM

Absolute Pressure, Pounds per Square Inch	Temperature, Degrees F.	Heat of the Liquid.	Heat of Vaporization.	Total Heat.	Heat Equivalent of Internal Work.	Heat Equivalent of External Work.	Entropy of the Liquid.	Entropy of the Vapor.	Total Entropy.	Specific Volume.	Density Weight per Cubic Foot, Pounds.
p	t	h	L	$H = h + L$	ρ	Apu	θ	$\frac{L}{T}$	$\theta + \frac{L}{T}$	v	w
13	205.87	173.8	974.2	1148.0	902.0	72.2	0.3025	1.4639	1.7664	30.03	0.03330
14	209.55	177.5	971.9	1149.4	899.3	72.6	0.3081	1.4523	1.7604	28.02	0.03569
14.7	212.00	180.0	970.4	1150.4	897.6	72.9	0.3118	1.4447	1.7565	26.79	0.03732
15	213.0	181.0	969.7	1150.7	896.8	72.9	0.3133	1.4416	1.7549	26.27	0.03806
20	228.0	196.1	960.0	1156.2	885.8	74.3	0.3355	1.3965	1.7320	20.08	0.04980
25	240.1	208.4	952.0	1160.4	876.8	75.3	0.3532	1.3604	1.7136	16.30	0.0614
30	250.3	218.8	945.1	1163.9	869.0	76.2	0.3680	1.3311	1.6991	13.74	0.0728
35	259.3	227.9	938.9	1166.8	862.1	76.9	0.3868	1.3060	1.6868	11.89	0.0841
40	267.3	236.1	933.3	1169.4	855.9	77.6	0.3920	1.2841	1.6761	10.49	0.0953
45	274.5	243.4	928.2	1171.6	850.3	78.1	0.4021	1.2644	1.6665	9.39	0.1065
50	281.0	250.1	923.5	1173.6	845.0	78.6	0.4113	1.2468	1.6581	8.51	0.1175
55	287.1	256.3	919.0	1175.4	840.2	78.9	0.4196	1.2309	1.6505	7.78	0.1285
60	292.7	262.1	914.9	1177.0	835.6	79.7	0.4272	1.2160	1.6432	7.17	0.1394
65	298.0	265.7	911.0	1178.5	831.4	79.8	0.4344	1.2034	1.6368	6.65	0.1503
70	302.9	272.6	907.2	1179.8	827.3	80.1	0.4411	1.1896	1.6307	6.20	0.1612
75	307.6	277.4	903.7	1181.8	823.5	80.5	0.4474	1.1778	1.6252	5.81	0.1721
80	312.0	282.0	900.3	1182.3	819.8	80.7	0.4535	1.1665	1.6200	5.47	0.1829
85	316.3	286.3	897.1	1183.4	816.3	81.0	0.4590	1.1561	1.6151	5.16	0.1937
90	320.3	290.5	893.9	1184.4	813.0	81.2	0.4644	1.1461	1.6105	4.89	0.2044
95	324.0	294.5	890.9	1185.4	809.7	81.5	0.4694	1.1367	1.6061	4.65	0.2151
100	327.8	298.3	888.0	1186.3	806.6	81.7	0.4743	1.1277	1.6020	4.429	0.2258

105	331.4	302.0	885.2	1187.2	803.6	81.9	0.4789	1.1191	1.5980	4.230	0.2635
110	334.8	305.5	882.5	1188.0	800.7	82.1	0.4834	1.1108	1.5942	4.047	0.2472
115	338.1	309.0	879.8	1188.8	797.9	82.3	0.4877	1.1030	1.5907	3.880	0.2577
120	341.3	312.3	877.2	1189.6	795.2	82.5	0.4919	1.0954	1.5873	3.726	0.2683
125	344.4	315.5	874.7	1190.3	792.6	82.6	0.4959	1.0880	1.5839	3.583	0.2791
130	347.4	318.6	872.3	1191.0	790.0	82.8	0.4998	1.0809	1.5807	3.452	0.2897
135	350.3	321.7	869.9	1191.6	787.5	82.9	0.5035	1.0742	1.5777	3.331	0.3002
140	353.1	324.6	867.6	1192.2	785.0	83.0	0.5072	1.0675	1.5747	3.219	0.3107
145	355.8	327.4	865.4	1192.8	782.7	83.2	0.5107	1.0612	1.5719	3.112	0.3213
150	358.5	330.2	863.2	1193.4	780.4	83.3	0.5142	1.0550	1.5692	3.012	0.3320
155	361.0	332.9	861.0	1194.0	778.1	83.5	0.5175	1.0489	1.5664	2.920	0.3425
160	363.6	335.6	858.8	1194.5	775.8	83.6	0.5208	1.0431	1.5639	2.834	0.3529
165	366.0	338.2	856.8	1195.0	773.6	83.7	0.5239	1.0376	1.5615	2.753	0.3633
170	368.5	340.7	854.7	1195.4	771.5	83.8	0.5269	1.0321	1.5590	2.675	0.3738
175	370.8	343.2	852.7	1195.9	769.4	83.9	0.5299	1.0268	1.5567	2.602	0.3843
180	373.1	345.6	850.8	1196.4	767.4	84.0	0.5328	1.0215	1.5543	2.533	0.3948
185	375.4	348.0	848.8	1196.8	765.4	84.1	0.5356	1.0164	1.5520	2.468	0.4052
190	377.6	350.4	846.9	1197.3	763.4	84.2	0.5384	1.0114	1.5498	2.406	0.4157
195	379.8	352.7	845.0	1197.7	761.4	84.3	0.5410	1.0066	1.5476	2.346	0.4262
200	381.9	354.9	843.2	1198.1	759.5	84.4	0.5437	1.0019	1.5456	2.290	0.437
205	384.0	357.1	841.4	1198.5	757.6	84.5	0.5463	0.9973	1.5436	2.237	0.447
210	386.0	359.2	839.6	1198.8	755.8	84.5	0.5488	0.9928	1.5416	2.187	0.457
215	388.0	361.4	837.9	1199.2	754.0	84.6	0.5513	0.9885	1.5398	2.138	0.468
220	389.9	363.4	836.2	1199.6	752.3	84.7	0.5538	0.9841	1.5379	2.091	0.478
225	391.9	365.5	834.4	1199.9	750.5	84.7	0.5562	0.9799	1.5361	2.046	0.489
230	393.8	367.5	832.8	1200.2	748.8	84.8	0.5586	0.9758	1.5341	2.004	0.499
240	397.4	371.4	829.5	1200.9	745.4	85.0	0.5633	0.9676	1.5309	1.924	0.520
250	401.1	375.2	826.3	1201.5	742.0	85.1	0.5676	0.9600	1.5276	1.850	0.541
275	409.5	384.2	818.6	1202.8	734.2	85.3	0.5780	0.9419	1.5199	1.686	0.593
300	417.5	392.7	811.3	1204.1	726.8	85.6	0.5878	0.9251	1.5129	1.551	0.645

TABLE IV. — PROPERTIES OF SUPERHEATED STEAM

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Pressure, Pounds Absolute.		Saturated Steam.	Degrees of Superheat.							Pressure, Pounds Absolute.
			50	100	150	200	250	300		
5	*t*	162.3	212.3	262.3	312.3	362.3	412.3	462.3	*t*	5
	v	73.3	79.7	85.7	91.8	97.8	103.8	109.8	*v*	
	h	1130.5	1153.5	1176.4	1199.5	1222.5	1245.6	1268.7	*h*	
10	*t*	193.2	243.2	293.2	343.2	393.2	443.2	493.2	*t*	10
	v	38.4	41.5	44.6	47.7	50.7	53.7	56.7	*v*	
	h	1143.1	1166.3	1189.5	1212.7	1236.0	1259.3	1282.5	*h*	
15	*t*	213.0	263.0	313.0	363.0	413.0	463.0	513.0	*t*	15
	v	26.27	28.40	30.46	32.50	34.53	36.56	38.58	*v*	
	h	1150.7	1174.2	1197.6	1221.0	1244.4	1267.7	1291.1	*h*	
20	*t*	228.0	278.0	328.0	378.0	428.0	478.0	528.0	*t*	20
	v	20.08	21.69	23.25	24.80	26.33	27.85	29.37	*v*	
	h	1156.2	1179.9	1203.5	1227.1	1250.6	1274.1	1297.6	*h*	
25	*t*	240.1	290.1	340.1	390.1	440.1	490.1	540.1	*t*	25
	v	16.30	17.60	18.86	20.10	21.32	22.55	23.77	*v*	
	h	1160.4	1184.4	1208.2	1231.9	1255.6	1279.2	1302.8	*h*	
30	*t*	250.4	300.4	350.4	400.4	450.4	500.4	550.4	*t*	30
	v	13.74	14.83	15.89	16.93	17.97	18.99	20.00	*v*	
	h	1163.9	1188.1	1212.1	1236.0	1259.7	1283.4	1307.1	*h*	
35	*t*	259.3	309.3	359.3	409.3	459.3	509.3	559.3	*t*	35
	v	11.89	12.85	13.75	14.65	15.54	16.42	17.30	*v*	
	h	1166.8	1191.3	1215.4	1239.4	1263.3	1287.1	1310.8	*h*	
40	*t*	267.3	317.3	367.2	417.3	467.3	517.3	567.3	*t*	40
	v	10.49	11.33	12.13	12.93	13.70	14.48	15.25	*v*	
	h	1169.4	1194.0	1218.4	1242.4	1266.4	1290.3	1314.1	*h*	
45	*t*	274.5	324.5	374.5	424.5	474.5	524.5	574.5	*t*	45
	v	9.39	10.14	10.86	11.57	12.27	12.96	13.65	*v*	
	h	1171.6	1196.6	1221.0	1245.2	1269.3	1293.2	1317.0	*h*	
50	*t*	281.0	331.0	381.0	431.0	481.0	531.0	581.0	*t*	50
	v	8.51	9.19	9.84	10.48	11.11	11.74	12.36	*v*	
	h	1173.6	1198.8	1223.4	1247.7	1271.8	1295.8	1319.7	*h*	
55	*t*	287.1	337.1	387.1	437.1	487.1	537.1	587.1	*t*	55
	v	7.78	8.40	9.00	9.59	10.16	10.73	11.30	*v*	
	h	1175.4	1200.8	1225.6	1250.0	1274.2	1298.1	1322.0	*h*	
60	*t*	292.7	342.7	392.7	442.7	492.7	542.7	592.7	*t*	60
	v	7.17	7.75	8.30	8.84	9.36	9.89	10.41	*v*	
	h	1177.0	1202.6	1227.6	1252.1	1276.4	1300.4	1324.3	*h*	
65	*t*	298.0	348.0	398.0	448.0	498.0	548.0	598.0	*t*	65
	v	6.65	7.20	7.70	8.20	8.69	9.17	9.65	*v*	
	h	1178.5	1204.4	1229.5	1254.0	1278.4	1302.4	1326.4	*h*	
70	*t*	302.9	352.9	402.9	452.9	502.9	552.9	602.9	*t*	70
	v	6.20	6.71	7.18	7.65	8.11	8.56	9.01	*v*	
	h	1179.8	1205.9	1231.2	1255.8	1280.2	1304.3	1328.3	*h*	
75	*t*	307.6	357.6	407.6	457.6	507.6	557.6	607.6	*t*	75
	v	5.81	6.28	6.73	7.17	7.60	8.02	8.44	*v*	
	h	1181.1	1207.5	1232.8	1257.5	1282.0	1306.1	1330.1	*h*	
80	*t*	312.0	362.0	412.0	462.0	512.0	562.0	612.0	*t*	80
	v	5.47	5.92	6.34	6.75	7.17	7.56	7.95	*v*	
	h	1182.3	1208.8	1234.3	1259.0	1283.6	1307.8	1331.9	*h*	
85	*t*	316.3	366.3	416.3	466.3	516.3	566.3	616.3	*t*	85
	v	5.16	5.59	6.99	6.38	6.76	7.14	7.51	*v*	
	h	1183.4	1210.2	1235.8	1260.6	1285.2	1309.4	1333.5	*h*	

t = Temperature, deg. Fahr. *v* = Specific volume, in cubic feet, per pound.
h = Total heat from water at 32 degrees, B.t.u.

TABLE IV. — *Continued*

Pressure, Pounds Absolute.		Saturated Steam.	Degrees of Superheat.							Pressure, Pounds Absolute.
			50	100	150	200	250	300		
90	*t*	320.3	370.3	420.3	470.3	520.3	570.3	620.3	*t*	90
	v	4.89	5.29	5.67	6.04	6.40	6.76	7.11	*v*	
	h	1184.4	1211.4	1237.2	1262.0	1286.6	1310.8	1334.9	*h*	
95	*t*	324.1	374.1	424.1	474.1	524.1	574.1	624.1	*t*	95
	v	4.65	5.03	5.39	5.74	6.09	6.43	6.76	*v*	
	h	1185.4	1212.6	1238.4	1263.4	1288.1	1312.3	1336.4	*h*	
100	*t*	327.8	377.8	427.8	477.8	527.8	577.8	627.8	*t*	100
	v	4.43	4.79	5.14	5.47	5.80	6.12	6.44	*v*	
	h	1186.3	1213.8	1239.7	1264.7	1289.4	1313.6	1337.8	*h*	
105	*t*	331.4	381.4	431.4	481.4	531.4	581.4	631.4	*t*	105
	v	4.23	4.58	4.91	5.23	5.54	5.85	6.15	*v*	
	h	1187.2	1214.9	1240.8	1265.9	1290.6	1314.9	1339.1	*h*	
110	*t*	334.8	384.8	434.8	484.8	534.8	584.8	634.8	*t*	110
	v	4.05	4.38	4.70	5.01	5.31	5.61	5.90	*v*	
	h	1188.0	1215.9	1242.0	1267.1	1291.9	1316.2	1340.4	*h*	
115	*t*	338.1	388.1	438.1	488.1	538.1	588.1	638.1	*t*	115
	v	3.88	4.20	4.51	4.81	5.09	5.38	5.66	*v*	
	h	1188.8	1216.9	1243.1	1268.2	1293.0	1317.3	1341.5	*h*	
120	*t*	341.3	391.3	441.3	491.3	541.3	591.3	641.3	*t*	120
	v	3.73	4.04	4.33	4.62	4.89	5.17	5.44	*v*	
	h	1189.6	1217.9	1244.1	1269.3	1294.1	1318.4	1342.7	*h*	
125	*t*	344.4	394.4	444.4	494.4	544.4	594.4	644.4	*t*	125
	v	3.58	3.88	4.17	4.45	4.71	4.97	5.23	*v*	
	h	1190.3	1218.8	1245.1	1270.4	1295.2	1319.5	1343.8	*h*	
130	*t*	347.4	397.4	447.4	497.4	547.4	597.4	647.4	*t*	130
	v	3.45	3.74	4.02	4.28	4.54	4.80	5.05	*v*	
	h	1191.0	1219.7	1246.1	1271.4	1296.2	1320.6	1344.9	*h*	
135	*t*	350.3	400.3	450.3	500.3	550.3	600.3	650.3	*t*	135
	v	3.33	3.61	3.88	4.14	4.38	4.63	4.87	*v*	
	h	1191.6	1220.6	1247.0	1272.3	1297.2	1321.6	1345.9	*h*	
140	*t*	353.1	403.1	453.1	503.1	553.1	603.1	653.1	*t*	140
	v	3.22	3.49	3.75	4.00	4.24	4.48	4.71	*v*	
	h	1192.2	1221.4	1248.0	1273.3	1298.2	1322.6	1346.9	*h*	
145	*t*	355.8	405.8	455.8	505.8	555.8	605.8	655.8	*t*	145
	v	3.12	3.38	3.63	3.87	4.10	4.33	4.56	*v*	
	h	1192.8	1222.2	1248.8	1274.2	1299.1	1323.6	1347.9	*h*	
150	*t*	358.5	408.5	458.5	508.5	558.5	608.5	658.5	*t*	150
	v	3.01	3.27	3.51	3.75	3.97	4.19	4.41	*v*	
	h	1193.4	1223.0	1249.6	1275.1	1300.0	1324.5	1348.8	*h*	
155	*t*	361.0	411.0	461.0	511.0	561.0	611.0	661.0	*t*	155
	v	2.92	3.17	3.41	3.63	3.85	4.06	4.28	*v*	
	h	1194.0	1223.6	1250.5	1276.0	1300.8	1325.3	1349.7	*h*	
160	*t*	363.6	413.6	463.6	513.6	563.6	613.6	663.6	*t*	160
	v	2.83	3.07	3.30	3.53	3.74	3.95	4.15	*v*	
	h	1194.5	1224.5	1251.3	1276.8	1301.7	1326.2	1350.6	*h*	
165	*t*	366.0	416.0	466.0	516.0	566.0	616.0	666.0	*t*	165
	v	2.75	2.99	3.21	3.43	3.64	3.84	4.04	*v*	
	h	1195.0	1225.2	1252.0	1277.6	1302.5	1327.1	1351.5	*h*	
170	*t*	368.5	418.5	468.5	518.5	568.5	618.5	668.5	*t*	170
	v	2.68	2.91	3.12	3.34	3.54	3.73	3.92	*v*	
	h	1195.4	1225.9	1252.8	1278.4	1303.3	1327.9	1352.3	*h*	

t = Temperature, deg. Fahr.
v = Specific volume, in cubic feet, per pound.
h = Total heat from water at 32 degrees, B.t.u.

Table IV. — *Continued*

Pressure, Pounds Absolute.		Saturated Steam.	Degrees of Superheat.							Pressure, Pounds Absolute
			50	100	150	200	250	300		
	t	370.8	420.8	470.8	520.8	570.8	620.8	670.8	*t*	
175	*v*	2.60	2.83	3.04	3.24	3.44	3.63	3.82	*v*	175
	h	1195.9	1226.6	1253.6	1279.1	1304.1	1328.7	1353.2	*h*	
	t	373.1	423.1	473.1	523.1	573.1	623.1	673.1	*t*	
180	*v*	2.53	2.75	2.96	3.16	3.35	3.54	3.72	*v*	180
	h	1196.4	1227.2	1254.3	1279.9	1304.8	1329.5	1353.9	*h*	
	t	375.4	425.4	475.4	525.4	575.4	625.4	675.4	*t*	
185	*v*	2.47	2.68	2.89	3.08	3.27	3.45	3.63	*v*	185
	h	1196.8	1227.9	1255.0	1280.6	1305.6	1330.2	1354.7	*h*	
	t	377.6	427.6	477.6	527.6	577.6	627.6	677.6	*t*	
190	*v*	2.41	2.62	2.81	3.00	3.19	3.37	3.55	*v*	190
	h	1197.3	1228.6	1255.7	1281.3	1306.3	1330.9	1355.5	*h*	
	t	379.8	429.8	479.8	529.8	579.8	629.8	679.8	*t*	
195	*v*	2.35	2.55	2.75	2.93	3.11	3.29	3.46	*v*	195
	h	1197.7	1229.2	1256.4	1282.0	1307.0	1331.6	1356.2	*h*	
	t	381.9	431.9	481.9	531.9	581.9	631.9	681.9	*t*	
200	*v*	2.29	2.49	2.68	2.86	3.04	3.21	3.38	*v*	200
	h	1198.1	1229.8	1257.1	1282.6	1307.7	1332.4	1357.0	*h*	
	t	384.0	434.0	484.0	534.0	584.0	634.0	684.0	*t*	
205	*v*	2.24	2.44	2.62	2.80	2.97	3.14	3.30	*v*	205
	h	1198.5	1230.4	1257.7	1283.3	1308.3	1333.0	1357.7	*h*	
	t	386.0	436.0	486.0	536.0	586.0	636.0	686.0	*t*	
210	*v*	2.19	2.38	2.56	2.74	2.91	3.07	3.23	*v*	210
	h	1198.8	1231.0	1258.4	1284.0	1309.0	1333.7	1358.4	*h*	
	t	388.0	438.0	488.0	538.0	588.0	638.0	688.0	*t*	
215	*v*	2.14	2.33	2.51	2.68	2.84	3.00	3.16	*v*	215
	h	1199.2	1231.6	1259.0	1284.6	1309.7	1334.4	1359.1	*h*	
	t	389.9	439.9	489.9	539.9	589.9	639.9	689.9	*t*	
220	*v*	2.09	2.28	2.45	2.62	2.78	2.94	3.10	*v*	220
	h	1199.6	1232.2	1259.6	1285.2	1310.3	1335.1	1359.8	*h*	
	t	391.9	441.9	491.9	541.9	591.9	641.9	691.9	*t*	
225	*v*	2.05	2.23	2.40	2.57	2.72	2.88	3.03	*v*	225
	h	1199.9	1232.7	1260.2	1285.9	1310.9	1335.7	1360.3	*h*	
	t	393.8	443.8	493.8	543.8	593.8	643.8	693.8	*t*	
230	*v*	2.00	2.18	2.35	2.51	2.67	2.82	2.97	*v*	230
	h	1200.2	1233.2	1260.7	1286.5	1311.6	1336.3	1361.0	*h*	
	t	395.6	445.6	495.6	545.6	595.6	645.6	695.6	*t*	
235	*v*	1.96	2.14	2.30	2.46	2.62	2.77	2.91	*v*	235
	h	1200.6	1233.8	1261.4	1287.1	1312.2	1337.0	1361.7	*h*	
	t	397.4	447.4	497.4	547.4	597.4	647.4	697.4	*t*	
240	*v*	1.92	2.09	2.26	2.42	2.57	2.71	2.85	*v*	240
	h	1200.9	1234.3	1261.9	1287.6	1312.8	1337.6	1362.3	*h*	
	t	399.3	449.3	499.3	549.3	599.3	649.3	699.3	*t*	
245	*v*	1.89	2.05	2.22	2.37	2.52	2.66	2.80	*v*	245
	h	1201.2	1234.8	1262.5	1288.2	1313.3	1338.2	1362.9	*h*	
	t	401.0	451.0	501.0	551.0	601.0	651.0	701.0	*t*	
250	*v*	1.85	2.02	2.17	2.33	2.47	2.61	2.75	*v*	250
	h	1201.5	1235.4	1263.0	1288.8	1313.9	1338.8	1363.5	*h*	
	t	402.8	452.8	502.8	552.8	602.8	652.8	702.8	*t*	
255	*v*	1.81	1.98	2.14	2.28	2.43	2.56	2.70	*v*	255
	h	1201.8	1235.9	1263.6	1289.3	1314.5	1339.3	1364.1	*h*	

t = Temperature, deg. Fahr. *v* = Specific volume, in cubic feet, per pound.
h = Total heat from water at 32 degrees, B.t.u.

TABLE V. — SATURATED VAPOR OF SULPHUR DIOXIDE

Reproduced by permission from Peabody's "Steam and Entropy Tables."

English Units.

Temperature, Degrees Fahrenheit. t	Pressure, Pounds per Square Inch. p	Heat of the Liquid. h	Total Heat. H	Heat of Vaporization. L	Heat Equivalent of Internal Work. ρ	Heat Equivalent of External Work. Apu	Entropy of the Liquid. θ	Specific Volume. s
−40	3.14	−29	166	195	182	13	−0.0632	23.0
−20	5.90	−21	169	190	176	14	−0.0447	12.7
0	10.35	−13	172	185	170	15	−0.0268	7.54
10	13.41	− 9	173	182	167	15	−0.0182	5.93
20	17.15	− 5	174	179	164	15	−0.0098	4.72
30	21.66	− 1	176	177	162	15	−0.0016	3.81
40	27.06	3	177	174	158	16	0.0064	3.10
50	33.45	7	178	171	155	16	0.0144	2.58
60	40.98	11	179	168	152	16	0.0221	2.11
70	49.75	15	181	166	150	16	0.0297	1.78

TABLE VI. — PROPERTIES OF CARBON DIOXIDE

Reproduced from Marks' "Mechanical Engineers' Handbook."

Temperature, Degrees Fahrenheit. t	Pressure, Pounds per Square Inch. p	Heat Content of Liquid. h	Heat Content of Vapor. H	Heat of Vaporization. L	Internal Energy of Vaporization. p	Heat Equivalent of External Work Apu	Specific Volume. s	Entropy of Liquid. θ
20	221.0	−24.75	100.50	125.25	109.0	16.3	0.4173	−0.0513
0	308.0	−16.00	101.00	117.00	101.3	15.7	0.2918	−0.0325
10	362.5	−11.36	100.89	112.35	96.9	15.4	0.2450	−0.0227
20	421.6	− 6.40	100.43	106.83	91.8	15.0	0.2060	−0.0126
30	488.8	− 1.04	99.43	100.47	86.7	14.2	0.1724	−0.0021
40	564.5	4.36	98.25	93.89	80.4	13.5	0.1444	0.0087
50	650.0	10.76	96.30	85.54	73.31	12.2	0.1205	0.0205
60	744.0	17.85	93.54	75.69	64.90	10.8	0.0986	0.0334
70	847.0	26.02	89.35	63.33	54.03	9.3	0.0816	0.0483

TABLE VII. — NAPERIAN LOGARITHMS

Reproduced by permission from Goodenough's "Properties of Steam and Ammonia."

$e = 2.7182818$ $\qquad \log e = 0.4342945 = M$

	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.00995	0.01980	0.02956	0.03922	0.04879	0.05827	0.06766	0.07696	0.08618
1.1	0.09531	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1739
1.2	0.1823	0.1906	0.1988	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4382	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6418	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7884	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7884	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387

TABLE VII. — *Continued.* NAPERIAN LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0668	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0881	2.0894	2.0906
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016
10.0	2.3026									

Table VIII. — LOGARITHMS

Reproduced by permission from Goodenough's "Properties of Steam and Ammonia."

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

TABLE VIII. — *Continued.* LOGARITHMS

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE IX

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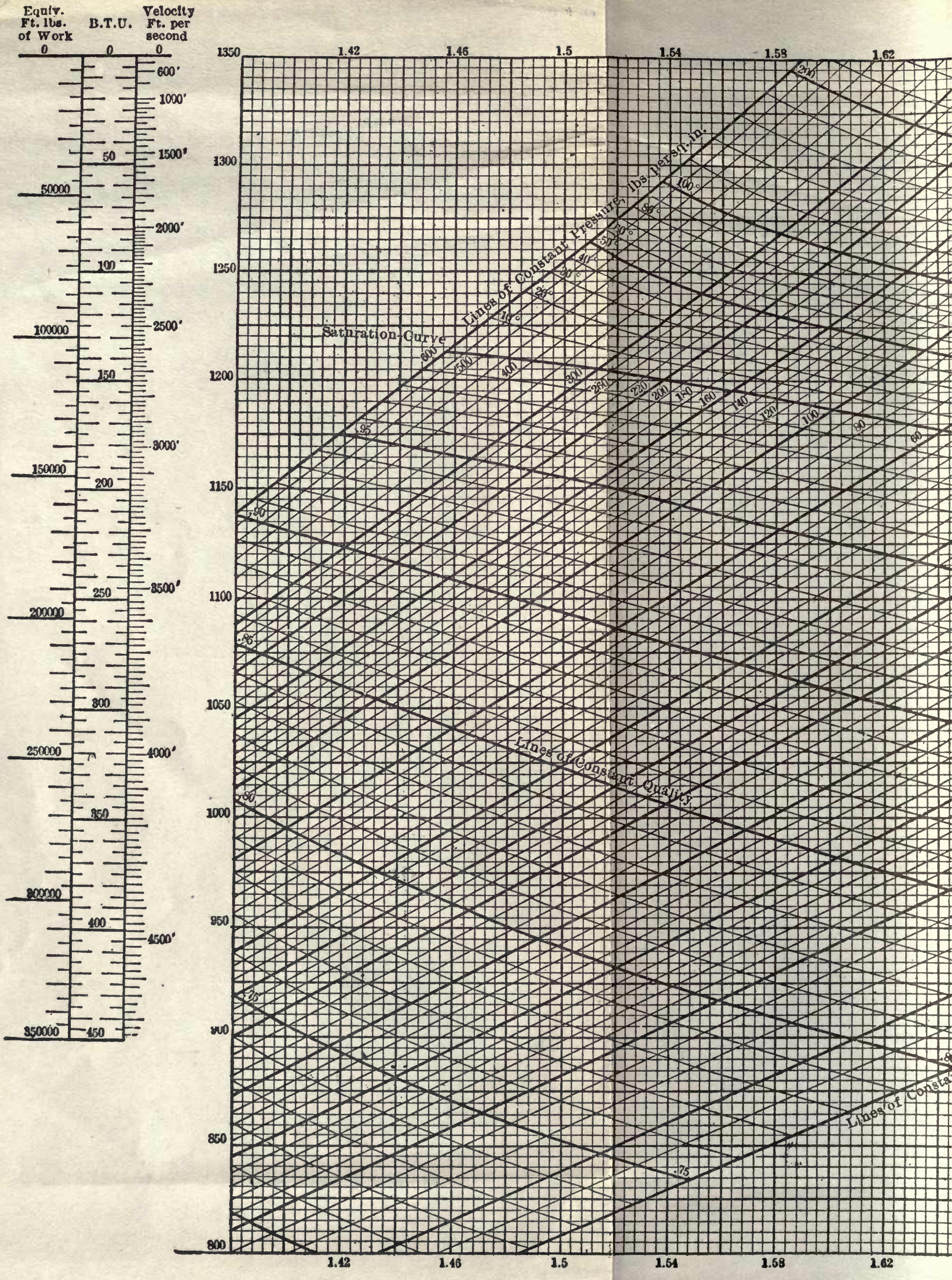

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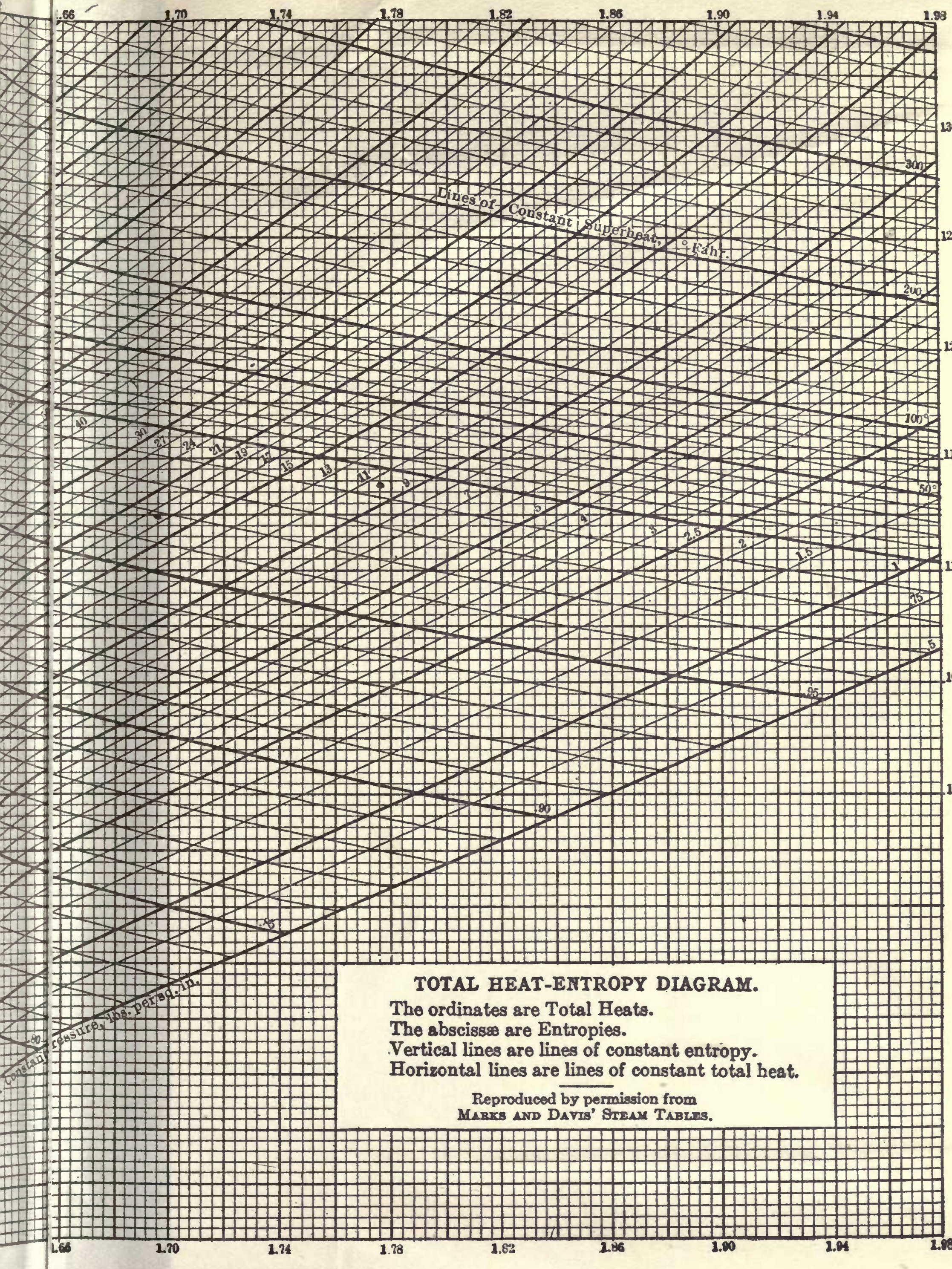

TOTAL HEAT-ENTROPY DIAGRAM.

The ordinates are Total Heats.
The abscissæ are Entropies.
Vertical lines are lines of constant entropy.
Horizontal lines are lines of constant total heat.

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