Technical Drawing Series.

ELEMENTS

OF

MECHANICAL DRAWING.

USE OF INSTRUMENTS,

GEOMETRICAL PROBLEMS, AND PROJECTION.

BY

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PREFACE.

IN the preparation of this book, and others of the TECHNICAL DRAWING SERIES, it is the aim of the author to provide text-books rather than copy-books; treatises in which principles should be established, and methods suggested, but freedom permitted in their application. While the treatment of much of the subject is new, it has long since passed the experimental stage, and nothing is here published that practical experience in the class-room has not justified.

The THIRD ANGLE OF PROJECTION is used exclusively, as is required in all good practice, and the terms TOP and FRONT VIEW have been substituted for PLAN and ELEVATION by reason of the confusion arising from the use of the latter. The methods employed for the representation of objects oblique to the planes of projection have been found to give a clear and comprehensive understanding of a subject which is usually much encumbered with rules, soon to be forgotten. The introduction of a graphic statement of problems relieves the instructor of the mechanical, and frequently laborious, task of devising suitable problems, with proper dimensions, and enables the student to begin the drawing without delay. The treatise on Screw-Threads and Bolts is the embodiment of instruction and problems given to classes during the past six years, and is an introduction to the study of machine drawing, as treated in the second book of the series.

It is intended that the student should first thoroughly master the principles, and then, unaided, apply them to the solution of the problems, receiving such instruction as his special case may demand. By this means, individual instruction may be given to large classes, and the energy and talent of the teacher directed to the giving of instruction, instead of performing the petty details of devising problems, etc. It also enables the more competent student to advance, independent of the progress of those who may require more time.

This system has been successfully applied by the author and others to the teaching of evening drawing-schools, high and manual training schools, and college classes.

GARDNER C. ANTHONY.

TUFTS COLLEGE, MASS.,
Sept. 15, 1894.

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MECHANICAL DRAWING.

I.

THE OUTFIT.

THE following list comprises the smallest equipment of tools consistent with good work. Their selection should be intrusted to one experienced in their use.

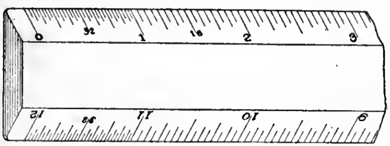
6 in. Compasses, joint in both legs, needle points, pencil and pen attachment.
5 in. Dividers. 3 in. Bow Pen. 3 in. Bow Pencil. 3 in. Bow Dividers.
5 in. Drawing Pen, provided with ebony handle, and no joint to blades.
12 in. Boxwood Scale, divided into 16th and 32nd as shown on page 10.
21 in. T Square. 7 in. 45° Triangle. 10 in. 60° Triangle. The T Square may be of pearwood and with a fixed head, but celluloid or hard rubber are more suitable for the Triangles.
HHHHHH Siberian Leads for use in compasses and bow pencil.
HHHH Siberian Lead Pencil and Medium Hard Pencil.
Hard Rubber for ink erasing. Soft Rubber for pencil erasing.
Pen Holder. Fine Pen. Pen Wiper.
Red, Blue, and Black Drawing Ink.
One-Ounce Tacks, either iron or copper.
Scrolls, three or four varieties having curves of long radii.
Pencil Sharpener. This is readily made by gluing No. 0 sandpaper on one side of a thin strip of wood 6 in. long and 1 in. wide.
16 x 21 in. Drawing Board.

THE USE OF INSTRUMENTS.

PREPARATION OF THE PAPER. — Place the drawing board with long edge next the body. Fasten the paper within about 3 in. of lower and left hand edges, observing that the lower edge of the paper should be square with the board. Use four tacks, one at each corner. The tack heads must be forced flush with surface of paper so as not to interfere with freedom in the use of the **T** square. See that the pencil is properly sharpened, as much of the accuracy of the work will be dependent on the care used in keeping it always in order.

To sharpen a pencil, remove the wood from both ends by means of a knife, exposing at least $\frac{3}{8}$ in. of lead. One end should then be filed to a round point, and the other flat or to a chisel edge, about $\frac{1}{32}$ in. wide, — a pencil sharpener or file being used for removing the lead. See Fig. 2, Pl. 1. The size of paper best adapted for the problems of this book is 11 x 15 in. It may be obtained from an imperial sheet, 22 x 30, by cutting the latter into four equal parts. The problems being designed to occupy a space not greater than 10 x 14 in., a margin of $\frac{1}{2}$ in. will remain.

Find the centre of the sheet by placing edge of **T** square to coincide with the opposite diagonal corners of the paper, and draw short fine lines which will intersect at the centre C, Fig. 1, Pl. 1. To the right and left of this point lay off 7 in., and above and below it lay off 5 in. In laying off dimensions from the scale, do it by means of the round point of the pencil, and make sure that the point be *exactly* opposite the required division on the scale. The best type of scale for general drawing is that shown by the accompanying figure. It should be made of boxwood, with a white edge, and being graduated as in the illustration may be conveniently used for scales of Full, Half, Quarter, and Eighth size. The scale should *never* be used as a ruler.



Next place the head of the T square against the left-hand edge of the board, which will be known as the working edge, and under no consideration should any other be employed. Only the upper edge of the blade should be used. This is designed for the drawing of all horizontal lines, and by sliding the T square head upon the working edge of the board, a series of parallel horizontal lines may be drawn. The T square should be moved by the head only.

Through the points previously laid off above and below the centre, draw horizontal lines, using the pencil against edge of T square as follows:—

Hold the pencil as nearly vertical as possible, with the flat side of the chisel point pressed lightly against the T square, and draw lines from left to right.

Vertical lines are to be drawn by means of the triangle used in connection with the T square. Fig. 1, Pl. 1.

With 60° triangle in the position shown in Fig. 1, draw vertical lines through the first-named points, using the same care in holding the pencil, and drawing the lines from bottom to top. If the triangle be too short to draw the entire line at one stroke, move the T square and triangle until the desired length is obtained. In general, the pencil should be moved from the body.

A STUDY OF STRAIGHT LINES.

Plate 2.

It should be observed that the only benefit to be derived from the making of this and the following drawings will be in gaining a knowledge of the use of instruments. If, however, the directions given for performing this work are not carefully observed, and the prescribed methods closely followed, much of the value of this study may be lost.

Having prepared the sheet as directed, divide it according to Pl. 2, making $3\frac{1}{2}$ in. squares, and the vertical distance between squares, $1\frac{1}{2}$ in.

Fig. 9. Divide A B and B D into quarter-inches by means of the scale, and through the divisions on A B draw horizontal lines by means of the T square. Through points on B D draw vertical lines by means of triangle on T square, as in Fig. 1, Pl. 1, observing carefully the instruction for use of pencil. Afterwards, measure the distance between the lines at their intersection with A C and C D. They should be exactly $\frac{1}{4}$ in. apart. Great care should be used in the pencilling of all drawings, since it is an almost universal experience that the inking of the drawing will be little or no better than the pencilling.

Fig. 10. Practice in the Use of Triangles. — Divide D F into quarter-inches, and from these points draw vertical lines as follows: Place the 60° triangle parallel to, and nearly coinciding with, D F, as in Fig. 3, Pl. 1. Now place the 45° triangle against short edge of 60° triangle, and on this edge slide the 60° triangle parallel to D F, until it is about $\frac{1}{4}$ in. distant from it; then holding the 60° triangle firmly, place the 45° triangle in position shown by the dotted lines in Fig. 3, and also by the accompanying figure. Use the 60° triangle as a base on which to slide the 45° triangle, and draw the required vertical lines. Draw lines parallel to D E with 45° triangle. Lines parallel to C F will be drawn by 45° triangle used on T square, as in Fig. 1. Draw all lines from left to right, when possible. Figs. 10, 12, 14, and 18 may to advantage be first made on a practice-sheet which may serve later for a trial-sheet in inking.

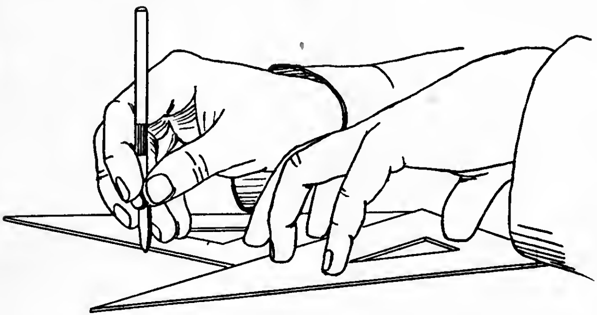


Fig. 11. Divide lines F E and F H into half-inches, and by means of T square and triangle

draw the small squares. Subdivide the divisions on E F and F H into eighth-inches and draw the intermediate lines. See that the lines start and stop exactly at the required points.

Fig. 12. Use the divisions already obtained on G H, and divide H L into half-inches: draw lines converging to a point as shown.

Fig. 13. Draw horizontal lines one-quarter inch apart, using care to hold the pencil vertically. This figure is designed chiefly for an inking practice.

Figs. 14 and 15. These squares are first to be subdivided into other squares as indicated by the dotted lines. This will facilitate the drawing of the figures by making the use of a scale unnecessary, for it will be observed that the outlines of the figures are drawn to the corners of the squares save in a few cases where the middle point of the line is used. The divisions of the squares may be made by the scale, and the lines drawn full instead of dotted. In pencilling, always use one width of line and let it be fine, since it is only in the inking of drawings that the distinction between light and shade lines is made. Do not pencil the lines drawn across the figures; these are known as section lines, and are to be drawn in ink only.

Figs. 16 and 17. Divide these figures into 25 squares each. As the divisions of the lines cannot readily be obtained by means of a scale, used in the ordinary manner, observe the following instructions:

Use of Dividers. — To divide a line into any number of equal parts, when the number of divisions required cannot be directly obtained from the scale, proceed as follows: Suppose it is desired to divide the line A B, Fig. 4, Pl. 1, into 5 equal parts. Open the dividers to a distance equal to A 1, about one-fifth of the required space, and holding them at the joint by the thumb, first, and second fingers, place one point at extremity A, of line to be divided, the other point being at 1. By rotating the instrument alternately in opposite directions, as if describing a series of semicircles (the path of the point being shown by the dotted lines in Fig. 4, Pl. 1), lay off divisions A-1, 1-2, 2-3, 3-4, 4-5. The point 5 being beyond the extremity of the line, the divisions are too great, and must be diminished by an amount equal to one-fifth of B 5. Make a second or third trial if necessary, so that the last division shall fall on point B. The following method may also be used. Suppose line A B, Fig. 4, to be $2\frac{3}{8}$ in. long, and it is required to be divided into 5 equal parts. Place zero of the scale to coincide with point A, and lay off by scale, five equal divisions, as $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$. From the last point C, by means of triangles, draw line B C, and through the remaining points, draw lines parallel to B C, intersecting A B; these will divide the line A B into the required number of equal parts. The dividers may also be used in place of scale to set off any equal divisions on A C. In using dividers, take care not to puncture the paper more than is necessary to make the point visible.

Fig. 18. Divide the left-hand vertical line into quarter-inches, and draw horizontal lines by means of **T** square. These are to be used as a special inking practice and may be pencilled by full lines.

Inking. — Before beginning to ink, see that the pen is clean and properly sharpened.[1] Having slightly opened the nibs, fill the pen by means of a quill, or writing pen, that may be inserted between the blades; and although this should not necessitate the wiping of the outside, care should be used to see that it is perfectly clean. Do not overload the pen with ink. Having filled the pen, nearly close the nibs and try the width of the line on a piece of paper, opening or closing the nibs by means of the screw to vary the width of the line. The pen should be held vertically by the thumb, first and second fingers, as shown by the figure on page 12, the thumb-screw being held from the body so as to be readily adjusted by means of thumb and second finger. Like the pencil, the pen should always be moved from left to right, and from bottom to top of board.

[1] Instruction concerning this should be given by the teacher.

When inking, both nibs must rest evenly on the paper, and the pen should be pressed lightly against the T square, so as not to close the nibs, and thus vary the width of the line. Never ink backwards on a line. In case the ink should not flow freely, touch the pen to the finger, and if it fails to flow, clean thoroughly and refill. Never refill the pen or lay it aside without cleaning.

Ink Figs. 9, 10, 11, and 12, using the same method as was employed in pencilling the lines. In Fig. 12, allow each line to dry before inking the following.

Corrections in inking should be made by means of a hard rubber, but never by scratching with a knife. In using an eraser of any kind, do not exert much pressure, for it does the work no more rapidly, and is liable to roughen the paper.

Never ink any portion of a drawing until the pencilling is completed.

In inking the long fine lines shown in Fig. 13, go over each line twice in succession, without moving the T square, endeavoring not to widen the line at the second inking. See that the pen always contains ink enough to finish a line, as it is difficult to continue with the same width of line after refilling.

The width of lines on different drawings may vary slightly, according to the character of the work, but guard against too fine a line, which is an error common to most students and many draughtsmen. Remember that a drawing is made to be read. The skill in inking does not depend on the fineness of the line, but on its clearness.

Two widths of lines are used in Figs. 14, 15, 16, and 17, the wider being known as a shade line and its use fully explained on page 55. First ink all fine lines and afterwards the shade lines, the latter being as wide as the widest in Fig. 18. The section lines are to be drawn last and always without pencilling, the space between the lines being dependent on the size of the figure. In this case they may be made about equal to the space between the vertical lines of Fig. 3, Pl. 1. Do not ink the divisions of the square shown by the dotted lines.

The lines in Fig. 18 should be inked in the manner shown. The first two being dotted lines, the third and fourth, broken lines, and the last, a very heavy line. Always use the greatest possible care in inking the dotted lines. Never make dots any longer than shown in Fig. 18, and see that they are equally spaced.

Lettering. — The subject of lettering is of such importance to the mechanical draughtsman, that he should early adopt some clear type and acquire proficiency in the freehand rendering of it. While it may be necessary at times to make use of instruments and mechanical aids for the construction of letters and figures, they must usually be written freehand and with fluency. Many good drawings have been disfigured by the lettering or figuring when a small amount of practice would have enabled the draughtsman to execute the same neatly and legibly. Plate 10 illustrates the two types of Alphabet recommended to students. These are such as will always be acceptable in the regular practice of draughting, and their study will afford not only an excellent freehand exercise, but skill in neatly figuring and lettering drawings. Both types should be written without the aid of instruments. The first, or Gothic, is the more simple, and should be used by most students. It can be quite easily written by means of a hard-wood stick, sharpened to a point like a pencil, the size of the point being varied according to the desired width of the line. In using this, care should be taken that the ink be black and slightly thick. The small letters should be about two-thirds of the height of the initial letters, capitals being used for both. The second type requires considerable practice, but being very clear and beautiful, should be mastered by every practising draughtsman. The student is recommended to practice the latter without the use of shade lines.

THE DRAWING OF CIRCLES AND CIRCULAR ARCS.

Plate 3.

To prepare the compasses for use, remove the movable leg and place in it a HHHHHH lead, which must be sharpened to a chisel-point according to directions given for the sharpening of lead pencils. Next fit the leg into the instrument, making sure to firmly clamp the same, then observe *first*, that the needle point and lead be of the same length; if not, adjust lead to needle point; *second*, that the point of the lead stands directly across the instrument, so that a line no wider than the point may be drawn. The lower part of the needle point leg should be kept nearly vertical, so that the point will not make a hole in the paper while revolving. It is not essential that the pencil point be kept vertical.

The compasses are to be held at the joint by means of the thumb, first, and second fingers, and the instrument always rotated from left to right, clockwise. In placing the needle point on a particular centre, the compasses may be steadied by lightly touching the needle point by a finger of the left hand. The line should be stopped as soon as the circumference is complete, otherwise it is liable to be widened. This is very important to observe when inking.

Fig. 19. Draw the fine lines passing through the centre of the figure, using full lines for this purpose. Their position is indicated by the dimension lines, which latter are not to be drawn. The number of lines required by the problems differ slightly from that shown in the figures. The diameter of each figure will be 4 in. Divide the diameter into quarter-inches, and draw a series of concentric circles, using care that they pass exactly through the quarter-inch divisions.

Fig. 20. A practice-sheet should first be made of Figs. 20, 21, and 24. Divide the horizontal diameter into half inches, and draw a series of circles through these points, tangent to the large circle at A. Use care to have each circle pass through A and the required points.

Fig. 21. Divide the horizontal diameter as before, and draw semi-circles in the following order, — A B, H K, A C, G K, A D, F K, A E, E K, A F, D K, etc. See that the circular arcs make continuous lines and pass through points A and K. Do not mind the position of the centre so long as it be upon the line A K, and enables you to draw a semi-circle through the required points.

Fig. 22. On the diameter E F lay off quarter-inches, and construct squares as follows: With 45° triangle, draw lines A C and B D, through the centre, and through points E, G, H, F, etc., draw perpendiculars. From their intersection with the diagonals, draw horizontals A B, K L, etc., making no square less than $1\frac{1}{2}$ in.

Next, set compasses to draw a circle of $\frac{3}{8}$ in. radius. Do this by laying off the amount on the paper, and set the compasses to it; but under no consideration take this directly from the scale by means of the instrument. In the corner of each square, draw a circular arc, just touching but not intersecting the sides of the square. To clearly indicate the position of a centre which is to be used a second time, lightly pencil a circle about it as at E, Fig. 24, but never put the point of a pencil in the centre to enlarge or blacken it.

Fig. 23. Divide the diameter into half-inches, and describe circles as in Fig. 19. Do not shade these lines in pencilling.

Fig. 24. Divide the diameter into half-inches and construct similar to Fig. 22.

Inking Circles and Circular Arcs. — *In inking with compasses*, use care to have the pen, as well as needle point, always vertical. The directions for cleaning and filling compass pen are the same as for ruling pen.

Ink figures 19, 20, and 21, but in the last two use care not to draw a second line until the first be dry, as a large number of lines pass through the point A, and would cause a blot. Alternate circles in Fig. 19 should be dotted.

Always ink small circles first. Circular arcs should always be inked before straight lines. Therefore in inking Figs. 22 and 24, put in circular arcs first, using care to draw no more than a quarter of a circle, and afterwards, ink straight lines by means of a ruling pen, joining the ends of the circular arcs.

In Figs. 23 and 24, the method of shading circles is shown. A and C in each figure represent parts of a solid; B and D the space. Do not ink the shaded circles without the following preliminary practice.

Having drawn a circle, immediately remove the point of the compasses from the centre to a position slightly below and to the right of the same, and with same radius, describe a circular arc through the outside or inside of the former circle, according to the desired position of the shaded line, as shown in the figure. All circles and circular arcs bounding the outside of a surface are shaded according to circle 1. All those bounding inside surfaces are shaded according to circle 2. In inking Fig. 24, observe all directions for inking figures 22 and 23.

Circles and circular arcs having radii less than $\frac{1}{2}$ in. should be inked by means of the bow compasses. Their use is similar to the large compasses, save in shading, when it is not necessary to remove the needle point from centre, but by slightly springing the same, the varying width of line may be obtained.

II.

GEOMETRICAL DEFINITIONS AND USEFUL PROPOSITIONS.

A *point* is used for marking positions only, and has no dimension.

A *line* is produced by the motion of a point, and has therefore length only.

A *surface* is produced by the motion of a line, and has length and breadth.

A *solid* is produced by the motion of a surface, and has therefore length, breadth, and thickness.

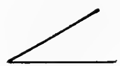
LINES.

Lines are either straight or curved. They may be horizontal, vertical, or oblique.

Parallel lines are everywhere equally distant.

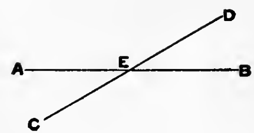
ANGLES.

RIGHT ANGLE.



ACUTE ANGLE.

OBTUSE ANGLE.



When two straight lines meet, they form an angle. There are three kinds of angles, *Right*, *Acute*, and *Obtuse* angles.

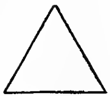
When one straight line meets another straight line, so that the angles on either side are equal, they are perpendicular to each other and the angles are *Right angles*.

An *Acute angle* is less than a right angle.

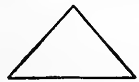
An *Obtuse angle* is greater than a right angle.

When one straight line crosses another straight line, the sum of the four angles is equal to four right angles; any two adjacent angles are equal to two right angles, and are said to be supplementary to each other. A E D + D E B is equal to two right angles, and D E B is the supplement of A E D.

TRIANGLES.



EQUILATERAL TRIANGLE.



ISOSCELES TRIANGLE.

SCALENE TRIANGLE.

A triangle is a figure enclosed by three straight lines. It has three sides and three angles. There are three kinds of triangles, named after their sides, viz:—*Equilateral*, *Isosceles*, and *Scalene*.

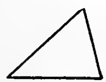
An *Equilateral triangle* has three of its sides equal.

An *Isosceles triangle* has two of its sides equal.

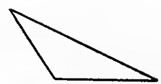
A *Scalene triangle* has none of its sides equal.

There are three kinds of triangles, named after their angles, viz:—*Right-angled triangle*, *Acute-angled triangle*, and *Obtuse-angled triangle*.

RIGHT-ANGLED TRIANGLE.



ACUTE-ANGLED TRIANGLE.



OBTUSE-ANGLED TRIANGLE.

A *Right-angled triangle* has one right angle.

An *Acute-angled triangle* has all its angles acute.

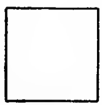
An *Obtuse-angled triangle* has one of its angles an obtuse angle.

The base of a triangle is its lowest side. The vertex of a triangle is the angle opposite the base.

The perpendicular height or altitude of a triangle is measured by a perpendicular line let fall from the vertex upon the base.

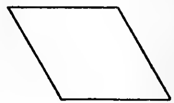
The sum of the three angles of any triangle is equal to two right angles, or 180°.

QUADRILATERALS.

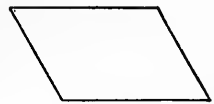


SQUARE.

RECTANGLE.



RHOMBUS.



RHOMBOID.

A figure of four sides is called a Quadrilateral. If the opposite sides are equal and parallel, it is a *Parallelogram.*

There are four kinds of parallelograms : —

The *Square*, which has its four sides equal, and all of its angles right angles.

The *Rectangle*, whose opposite sides only are equal, and all its angles are right angles.

The *Rhombus*, whose four sides are equal, and none of whose angles are right angles.

The *Rhomboid*, whose opposite sides only are equal, and none of whose angles are right angles.

The line that joins any two opposite angles of a quadrilateral is called its diagonal.

POLYGONS.

A plane figure bounded by straight lines is called a polygon. The term is usually applied to designate figures having more than four sides.

The Pentagon has five sides.
The Hexagon has six sides.
The Heptagon has seven sides.
The Octagon has eight sides.
The Nonagon has nine sides.
The Decagon has ten sides.

CIRCLES.

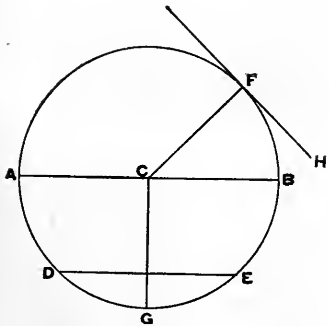


FIG. 1.

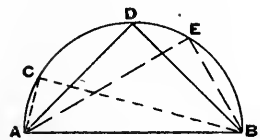


FIG. 2.

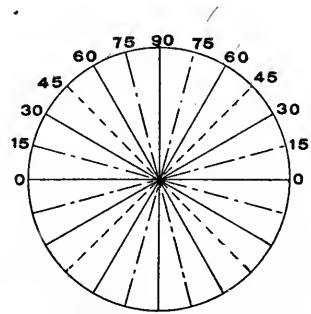


FIG. 3.

A *Circle* is a plane figure bounded by a curved line, called its *circumference*, all points of which are equally distant from a point called its centre.

A *Radius* is a straight line drawn from the centre to the circumference, as C F (Fig. 1). All radii of the same circle are equal.

The *Diameter* is a straight line drawn through the centre, terminating in the circumference, as A B.

An *Arc* of a circle is any portion of its circumference, as D G E or B E. An arc equal to one-half of the circumference is called a *semi-circumference*, as A F B.

A *Chord* is a straight line which has its extremities in the circumference, and joins the ends of an arc, as D E.

A *Tangent* is a straight line which touches the circumference, but does not intersect it, as F H. It is perpendicular to the radius at the point of tangency.

A *Segment* of a circle is a portion of a circle bounded by an arc and a chord, as the area D E G. A segment equal to one-half of the circle is called a *semi-circle*, as the area A B E G D.

A *Quadrant* is the fourth part of a circle, as B C G.

Every circle is supposed to be divided into 360 equal parts, called degrees, which are used as a measure of angles; therefore the arc of a semi-circle is equal to 180°. The arc of a quadrant is equal to 90°.

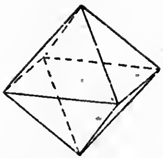
In Fig. 3, all angles indicated by the full lines may be drawn by a 60° triangle; all those indicated by the dotted lines may be drawn by a 45° triangle; those indicated by the broken lines may be drawn by the 45° and 60° triangle, as shown in Figs. 7 and 8, Pl. 1.

If from any point within a semi-circle lines be drawn to the extremities of the diameter, the included angle will be a right angle. In Fig. 2, A C B, A D B, and A E B are right angles.

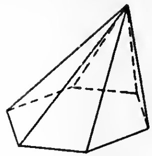
SOLIDS.

A *Polyhedron* is a solid bounded by planes. The polyhedron is named according to the number of these planes, or faces. One of four faces is called a tetrahedron, one of six faces a hexahedron, one of eight faces an octahedron, etc. They are also classified according to the shape and relation of these faces as follows: —

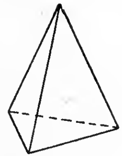
PYRAMIDS.



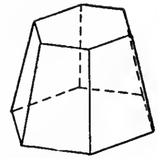
REGULAR OCTAHEDRON.



PYRAMID.



REGULAR PYRAMID.



FRUSTUM OF PYRAMID.

A pyramid is a polyhedron one face of which is a polygon and known as the base. The other faces are triangles having a common vertex known as the vertex of the pyramid.

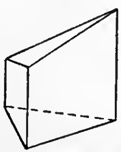
The *altitude* of a pyramid is the perpendicular distance from the vertex to the base.

A pyramid is *regular* if its base is a regular polygon whose centre lies in the perpendicular drawn from the vertex to the base.

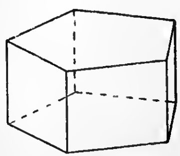
The *frustum* of a pyramid is that portion of a pyramid included between its base and a plane parallel to the base.

According to the character of their bases pyramids are called *triangular*, *rectangular*, *pentagonal*, etc.

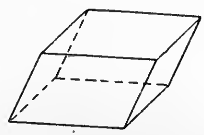
PRISMS.



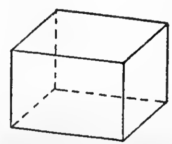
TRUNCATED PRISM.



RIGHT PRISM.



PARALLELOPIPED.



RIGHT PARALLELOPIPED.

A prism is a polyhedron of which two opposite faces, called bases, are equal and parallel polygons, and the other faces, called lateral faces, are parallelograms.

The *altitude* is the perpendicular distance between the bases.

A *truncated* prism is the part of a prism included between the base and a cutting plane inclined to the base.

A *right* prism is one whose lateral edges are perpendicular to the bases.

A *regular* prism is a right prism whose bases are regular polygons.

According to the character of their bases, prisms are called *triangular*, *quadrangular*, etc.

A prism whose bases are parallelograms is called a *parallelopiped*. If its lateral edges are perpendicular to the bases, it is called a *right parallelopiped*. If its six faces are all rectangles, it is called a *rectangular parallelopiped*. If its six faces are squares, it is called a cube.

CYLINDERS.

A cylindrical surface is a curved surface generated by the motion of a straight line which touches a given curve and continues parallel to itself.

A *cylinder* is a solid bounded by a cylindrical surface and two parallel planes intersecting this surface. The parallel faces are called bases.

The *altitude* of a cylinder is the perpendicular distance between the bases.

A *circular cylinder* is a cylinder whose base is a circle.

A cylinder of revolution is a cylinder generated by the revolution of a rectangle about one side as an axis.

CONES.

A *conical surface* is a curved surface generated by the motion of a straight line one end of which is fixed while the other is constrained to move in a curve.

A *cone* is a solid bounded by a conical surface and a cutting plane called the base.

The *altitude* of the cone is the perpendicular distance between the vertex, or fixed point, and the base.

A *circular cone* is a cone whose base is a circle.

A *right circular cone* is a circular cone whose axis is perpendicular to its base. It is also called a *cone of revolution.*

SPHERE.

A sphere is a solid generated by the revolution of a semicircle about its diameter.

GEOMETRICAL PROBLEMS.

GENERAL INSTRUCTION.

Divide the space within the margin line into twelve rectangles, and place one example in each rectangle. All work is to be performed with HHHH lead pencil. The greatest possible precision must be used. Construction lines are to be made very fine; given and required lines being made stronger by pencilling a second time. Where possible, avoid drawing the whole of a construction line, using only that portion of the line necessary. On the plates, all given and required lines are shown in full, and construction lines are dotted. When two methods are given, the problem will be constructed by the practical, or draughtsman's method, and tested by the geometrical.

The problems are not to be copied from the plates, but constructed from the data given in the examples on page 39. In doing this work it is advisable, first, to thoroughly master the problems relating to the drawing of perpendiculars, and then perform the examples involving these principles. Next, study those relating to angles, and similarly perform the dependent examples. Thus continue the subject according to the divisions indicated. Place the number of the example in the right hand lower corner of the square containing it.

PERPENDICULARS.

Plate 4.

Fig. 25. Problem 1. — *To bisect a straight line, A B.*

With centres A and B, and any radius greater than one-half of A B, describe arcs 1 and 2. Through the points of intersection of these arcs draw a line. Its intersection with the line A B, at C, will be the required point. The more practical method is to use dividers as explained on page 13.

Fig. 26. Problem 2. —*To bisect an arc of a circle.*

Proceed as in the preceding problem, the same lettering being used.

Problem 3. — *To draw a perpendicular to a straight line.*

Practical Method. — In all cases where the given line has been drawn by the **T** square, the perpendicular may be drawn by sliding the triangle along the edge of the square until it nearly coincides with the given point, when the required line may be drawn. If the given line makes an angle with the **T** square, the construction shown in Fig. 31 may be used.

Fig. 27. Case 1. — *When the point is on the line, and at, or near, the middle of the line.*

Let A B be the line, and C the point. From C, with any radius, draw arcs 1, and from the point of intersection of these arcs with A B, with any radius greater than arcs 1, draw arcs 2 and 3. The line drawn through the point of intersection of these arcs, and the given point C, will be the required line.

Fig. 28. Case 2. — *When the point is on the line, and at, or near, the extremity of the line.*

First Method. — Let A B be the given line, and A the given point. From A, with any radius, describe arc 1. With centre C, and same radius, describe arc 2. Through C, and the point of intersection of arcs 1 and 2, draw C E, and with same radius as before, from intersection of arcs

1 and 2, describe arc 3. A line drawn through the given point A, and the point of intersection of arc 3 and line C E, will be the required perpendicular.

Second Method. — From B, with any radius, describe arc 4. From point D, with same radius, describe arc 5. From the intersection of arcs 4 and 5 describe arc 6. From the intersection of arcs 4 and 6 describe arc 7. The line drawn through this last point of intersection F, and the given point B, will be the required perpendicular.

Fig. 29. Case 3. — *When the point is outside of, and opposite, or nearly opposite, the middle of the line.*

Let A B be the given line, C the given point. From C, with as great a radius as possible, describe arcs 1. From the point of intersection of these arcs with A B, with same radius, describe arcs 2 and 3. A line drawn through this point of intersection, and the given point C, will be the required line.

Fig. 30. Case 4. — *When the point is outside of, and at, or near, the extremity of the line.*

Let A B be the given line, and C the given point. From C draw any line C D. Find E, the centre of C D, by dividers or Problem 1. On C D as a diameter describe a semi-circle. Through the given point C and the intersection of semi-circle with A B, draw C F, which will be the required perpendicular.

ANGLES.

Plate 4.

Fig. 32. Problem 4. — *To bisect an angle A B C.*

From B, with any radius, describe arc 1. From its points of intersection with A B and C D describe arcs 2 and 3. The line drawn through this point of intersection, and B, will bisect the given angle.

Fig. 33. PROBLEM 5. — *To construct an angle F D E equal to a given angle A B C.*

Draw D E. From B and D, with the same radius, describe arcs 1 and 2. From E, with radius equal to chord A C, describe arc 3. Through D, and point of intersection of arcs 2 and 3, draw D F, the remaining side of the required triangle.

Fig. 34. PROBLEM 6. — *To construct an angle of* 45° *with A B at point A.*

Practical. — With **T** square draw A B, and with 45° triangle on **T** square draw A D.

Geometrical. — Through the given point A describe a semi-circle on A B; draw a perpendicular through the centre C. A line drawn through the point A, and intersection of perpendicular with semi-circle, will make the required angle with A B.

Fig. 35. PROBLEM 7. — *To construct angles of* 30° *and* 60° *with A B at point A.*

Practical. — With **T** square draw A B, and with 60° triangle on **T** square draw A D and A E.

Geometrical. — Through the given point A describe a semi-circle on A B. With same radius, from C describe arc 1. The line drawn through A, and this point of intersection, will make an angle of 30° with A B.

From the given point A as a centre, with any radius, describe arc 2. From B, with the same radius, describe arc 3. A line drawn through A, and this point of intersection, will make an angle of 60° with A B.

Fig. 36. PROBLEM 8. — *To construct an angle of* 15° *with A B at point A.*

Practical. — With **T** square draw A B, and with 45° and 60° triangle on **T** square, as in Fig. 7, Pl. 1, draw A C. In a similar manner an angle of 75° may be drawn. See dotted position in Fig. 7. See also Fig. 8.

Geometrical. — Construct an angle of 30° and bisect the same, as A C. It is not necessary to draw the 30° line.

TRIANGLES.

Plate 5.

Fig. 37. Problem 9. — *To construct an equilateral triangle, having given the side A B.*

Since the sides are equal, the angles will be equal, and therefore equal to 60°, since their sum is equal to 180°. (See page 22.)

Practical. — Through A and B, with 60° triangle, draw A C and B C.

Geometrical. — With A and B as centres, and radius A B, describe arcs 1 and 2. From point of intersection C, draw A C and B C.

Fig. 38. Problem 10. — *To construct an isosceles triangle, having given the base A B, and either the equal angles C A B and C B A, or equal sides A C and B C.*

If the angles be given, construct C A B and C B A equal to the given angle, and draw A C and B C (See Prob. 5).

If the sides be given, from centres A and B draw arcs 1 and 2 with radius equal to given sides. From point of intersection C draw A C and B C.

Fig. 39. Problem 11. — *To construct a scalene triangle, having given the sides A B, A C, and B C.*

Draw A B. With centres A and B, and radii equal to given sides, draw arcs 2 and 1. Draw A C and C B.

Fig. 40. Problem 12. — *To construct a right-angled triangle, having side A B and adjacent angle C A B given.*

Draw B C perpendicular to A B; make angle C A B equal to required angle.

TANGENTS.

Plate 5.

Problem 13. — *To draw a tangent to a circle.*

Fig. 41. Case 1. — *When the given point A is on the circle.*

Practical. — Place the triangle to coincide with centre C and given point A, as though to draw A C. By means of a second triangle used as a base, turn the first triangle into the position shown in dotted lines, and draw A B perpendicular to A C. A B is the required tangent.

Geometrical. — Draw A C and erect a perpendicular at A. (Prob. 3, Case 2.)

Fig. 42. Case 2. — *When the point is on an arc of the circle, and the centre not accessible.*

Practical. — From the given point A, with any radius, describe arcs 1. Place the edge of triangle to coincide with points B and D. Draw a parallel line through A. This will be the required tangent.

Fig. 43. Case 3. — *When the given point B is without the circle. In this case two tangents may be drawn.*

Practical. — From the given point B draw B A touching the circle. Through centre C draw perpendicular to A B. A will be the point of tangency. In like manner obtain B D.

Geometrical. — On B C as a diameter describe arc 1, its intersection with the circle at A and D will be the points of tangency. The angle B A C, inscribed in the semi-circle, will be a right-angle. (See page 24.)

Fig. 44. Problem 14. — *To draw a circle through a given point A, and tangent to given lines, A B and B D.*

Since the circle is to be tangent to A B and B D, its centre must lie upon the bisector of the angle D B A. Bisect the angle D B A, and through the point A draw A C perpendicular to A B.

C will be the centre of the circle, and A C its radius. The perpendicular may be obtained practically by triangles, or geometrically as shown.

Fig. 45. PROBLEM 15. — *To draw any number of circles tangent to each other and to two given lines A B and A D.*

Bisect angle D A B, and with any radius, H K, draw a circle tangent to A B and A D. From K draw K E perpendicular to A K, and with radius E K describe arc 4. Through F draw F C perpendicular to A B. C will be the centre, and F C the radius, of the second circle. Repeat the process.

Fig. 46. PROBLEM 16. — *To draw a tangent to two given circles A and C.*

Join A and C. From D lay off D H equal to A F. With centre C, and radius C H, draw arc 1. From A draw tangent to this arc. (Prob. 13, Case 3.) Through B, the point of tangency, draw C E, and through A draw A F parallel to C E. E and F will be the points of tangency, and E F the tangent.

Fig. 47. PROBLEM 17. — *To draw a circle of a given radius R, tangent to two given circles B and C.*

Draw indefinitely in any direction, lines C E and B F. Lay off H E and K F equal to given radius R, and through E and F describe arcs 1 and 2 intersecting at A, the required centre. These arcs will also intersect in a second centre.

Fig. 48. PROBLEM 18. — *Through three given points A, B, and D, not in the same straight line, to draw a circle.*

Bisect the imaginary chords A B and B D. The point of intersection C, of the bisecting lines, will be the required centre.

INSCRIBED AND CIRCUMSCRIBED POLYGONS.

Plate 6.

Fig. 49. Problem 19. — *To circumscribe a circle about a given triangle A B C.*

Bisect two of the sides, as A C and B C. The point of intersection of these lines will be the centre of the required circle. Draw circle through A, B, and C. (See Prob. 18.)

Fig. 50. Problem 20. — *To inscribe a circle within a given triangle A B C.*

Bisect two of the angles, as C A B and A B C. The point of intersection of these lines, D, will be the centre of the required circle. From this point draw a circle tangent to lines A C, C B, and A B.

Fig. 51. Problem 21. — *To inscribe an equilateral triangle within a circle.*

Practical. — Through centre D, with 60° triangle, draw A D, and with same triangle draw A C and C B. Draw A B.

Geometrical. — From any point in the circle, as C, draw arc 1 with a radius equal to that of the circle. From its intersection with the circle, and with the same radius, draw arc 2. With chord C B, and centre C, describe arc 3, and connect points A, B, and C, which will give the required triangle.

Fig. 52. Problem 22. — *To inscribe a square within a circle.*

Practical. — With 45° triangle draw two perpendicular diameters, A C and B D, and connect points A, B, C, and D, which will give the required square.

Geometrical. — Draw any diameter B D. Draw a second diameter A C perpendicular to it. Connect points A, B, C, and D as before.

Fig. 53. Problem 23. — *To inscribe a pentagon within a circle.*

Draw any diameter G F, and a radius A K perpendicular to it. Bisect K F, and with H as a centre, and radius A H, describe arc 3. With centre A, and radius A L, describe arc 4. A B is the side of a pentagon. Obtain the remaining points by describing arcs 5, 6, and 7 with same radius. Connect points A, B, C, D, and E to obtain the required pentagon.

CONSTRUCTION OF THE HEXAGON.

PLATE 6.

Fig. 54. PROBLEM 24.—*To inscribe a Hexagon within a circle.*

Practical. — Draw a horizontal diameter F C. With 60° triangle draw diameter E B. Draw A B and E D, and with triangle draw B C, F E, A F, and C D.

Geometrical. — Draw diameter F C. With centres F and C, and radius equal to that of the circle, draw arcs 1 and 2. Connect points of intersection A, B, C, D, E, and F, to obtain the required hexagon.

Observe that the angles at the centre, as B K C, are 60°. (See Prob. 7.)

Fig. 55. PROBLEM 25. — *To circumscribe a hexagon about a circle.*

Practical. — With 60° triangle draw diameters A D and E B, and with same triangle draw sides A B and E D, E F and B C, A F and C D, each tangent to the given circle.

Geometrical. — Draw any diameter A D. With H as centre, and radius equal to that of the circle, describe arc 1. Bisect the arc H L N, and through L draw A B parallel to H N. With centre K, and radius A K, describe circle A C E. In this inscribe a hexagon by Prob. 24.

Fig. 56. PROBLEM 26. — *To draw a hexagon, having given a long diameter A D.*

Practical. — With dividers find centre K. With 60° triangle, through A and D, draw A B and D E. With same triangle, through K draw B E, and through D and A draw C D and A F. Draw B C and F E, completing the required hexagon. The point B may be obtained without finding K, by drawing a line through D at an angle of 30° with A D. From its intersection with A B, B C may be drawn.

Geometrical. — Bisect A D. With K as centre describe circle A C E, and in this inscribe a hexagon by Prob. 24.

Fig. 57. PROBLEM 27. — *To draw a hexagon, having given the short diameter G H.*

Practical. — With dividers find centre K. With 60° triangle draw indefinitely F C; also through G and H draw perpendiculars F E and B C. Through centre K draw A D, and with triangle draw the remaining sides F A and D C, A B and D E.

Geometrical. — Bisect G H. From G draw perpendicular G F. From K draw F K C, making an angle of 30° with G K. Through point of intersection F, and with centre K, draw circle F B D, and inscribe a hexagon by Prob. 24.

Fig. 58. PROBLEM 28. — *To draw a hexagon, having given a side A B.*

Practical. — Through A and B, with 60° triangle, draw indefinitely A D and B E, and through their intersection K, draw F C. Draw F A, B C, F E, D C, and E D.

Geometrical. — With centres A and B, and radius A B, describe arcs 1 and 2. From point of intersection K, with same radius, describe circle A E C. Inscribe a hexagon by Prob. 24.

NOTE. — Observe that the above problems relating to the hexagon may be performed by use of 60° triangle and T square only. To attain the necessary familiarity with these problems they should be performed many times.

Fig. 59. PROBLEM 29.—*To inscribe an octagon within a given circle A C E G.*

Practical. — With 45° triangle and T square draw diameters A E, G C, F B, and H D. Connecting the points of intersection with the circle will give the inscribed octagon.

Geometrical. — Draw any diameter G C. At centre, and perpendicular to it, draw A E. Bisect A K G and A K C. Connect points of intersection with circle as before.

Fig. 60. Problem 30. — *To circumscribe an octagon about a circle A B C D.*

Practical. — With 45° triangle and T square draw tangents F E and L N, G H and P O. At 45°, draw also tangents F G, H L, etc., completing the octagon.

Geometrical. — Draw the perpendicular diameters A C and B D. With centres A, B, C, and D, and radius A K, describe arcs 1, 2, 3, 4. By connecting these points of intersection, a circumscribed square will be obtained. With the centres R, S, etc., and radius R K, describe arcs 5, 6, etc. to obtain the points G, H, L, N, etc., which, being connected, will complete the circumscribed octagon.

INSCRIBED AND CIRCUMSCRIBED CIRCLES.

Plate 7.

Fig. 61. Problem 31. — *Within an equilateral triangle A B C to draw three equal circles tangent to each other and one side of the triangle.*

Bisect the angles A, B, and C. Bisect the angle D C A. E is the centre of one of the required circles. With centre K, and radius K E, describe arc E F G. F and G will be the remaining centres. From these centres, with E L as radius, describe the required circles.

Fig. 62. Problem 32. — *Within an equilateral triangle A B C to draw three equal circles tangent to each other and two sides of the triangle.*

Bisect angles A, B, and C. Bisect angle D C A. E will be the centre of one of the circles. With K as centre, and radius K E, describe arc E F G to obtain the remaining centres. Draw circles tangent to sides A B, B C, and A C.

Note. — Instead of bisecting the angle D C A, to obtain one of the centres, describe a semicircumference A F C on one of the sides, as A C, thus obtaining centre F.

Fig. 63. PROBLEM 33. — *Within an equilateral triangle A B C to draw six equal circles which shall be tangent to each other and the sides of the triangle.*

Bisect angles and obtain E as in Prob. 31. Through E draw H N parallel to A C. Draw H M parallel to A B, and M N parallel to B C. With E, H, F, M, G, and N as centres, and with radius E L, describe the required circles.

Fig. 64. PROBLEM 34. — *Within a given circle A C E to draw three equal circles tangent to each other and the given circle.*

Divide the circle into six equal parts, by diameters A D, B E, C F. Produce A D indefinitely, and from E draw tangent E G. Bisect D G E. With K as centre, and radius H K, describe arc H L M, and with radius H E, from centres H, L, and M, describe the required circles.

Fig. 65. PROBLEM 35. — *Within a given circle to draw any number of equal circles tangent to each other and the given circle.*

General Method. — Divide the circle by diameters into twice as many equal parts as circles required; in this case, eight. Suppose the centre of one of these circles to lie on A K; then the circle must be tangent to both F K and K B. Draw tangent at A intersecting K B. Since the required circle must be tangent to the given circle at A, it will also be tangent to A B, and as it must now lie in the angles F K B and A B K, its centre must lie at D, the intersection of their bisectors. With centre K draw circle through D, and its intersection with E K, C K, etc., will give the required centres. From these centres, and with radius A D, describe the required circles.

Fig. 66. PROBLEM 36. — *About a given circle to circumscribe any number of equal circles tangent to each other and the given circle.*

Divide the circle by diameters into twice as many equal parts as circles required; in this case, six. From extremity A of any diameter, draw tangent A B. Produce K B, making B C equal to A B. At C, perpendicular to B C, draw C D, intersecting A K produced at D. This will be the centre, and A D will be the radius, of one of the required circles. With centre K, and radius D K, obtain other centres, and describe circles as in preceding problem.

EXAMPLES.

NOTE. — For Examples 1 to 40 inclusive, divide the sheet into 12 rectangles.
" " 41 " 46 " " " 6 "

PERPENDICULARS.

1. Bisect a line 3 in. long.
2. Bisect an arc of $2\frac{1}{2}$ in. radius and $2\frac{3}{8}$ in. chord.
3. Draw a line $2\frac{7}{8}$ in. long, and erect a perpendicular $1\frac{3}{8}$ in. from one end. Do not use **T** square.
4. Draw a line $2\frac{5}{16}$ in. long, and erect perpendiculars at the extremities. Use both methods.
5. From a point nearly over the centre of a line $3\frac{1}{4}$ in. long draw a perpendicular to the line.
6. From a point nearly over the extremity of a line $2\frac{7}{16}$ in. long, draw a perpendicular to the line. Do not use **T** square.

ANGLES.

7. Draw two lines intersecting and making any angle. Construct a similar angle, and bisect the same.
8. From one extremity of a line 3 in. long draw a second line making an angle of 45° with the first. Similarly construct an angle of 30° at the other end.
9. From one extremity of a line 3 in. long draw a second line making an angle of $22\frac{1}{2}$° with the first. Similarly construct an angle of 15° at the other end.

10. From one extremity of a line 3 in. long draw a second line making an angle of 60° with the first. Similarly construct an angle of 75° at the other end.

11. Describe a circle 3 in. in diameter, and divide it into angles of 15° by means of triangles and T square.

TRIANGLES.

12. Draw an equilateral triangle having a base $2\frac{11}{16}$ in.

13. Construct an isosceles triangle having a base 2 in. and the equal sides $2\frac{5}{8}$ in.

14. Construct an isosceles triangle having a base of $1\frac{1}{2}$ in. and the equal angles 75°.

15. Construct an isosceles triangle having a base of 3 in. and the angle at the vertex 150°.

16. Construct a scalene triangle having sides $2\frac{1}{4}$, $2\frac{3}{8}$, and $3\frac{13}{16}$ in.

17. Construct a right angle triangle having base $2\frac{3}{4}$ in. and one angle of 30°.

TANGENTS.

18. Draw a tangent to a circle $2\frac{1}{8}$ in. in diameter.

19. Draw a tangent to the middle point of the arc of a circle of $2\frac{1}{4}$ in. radius, having a chord of $2\frac{1}{2}$ in. Do not use the centre of circle.

20. Draw a tangent to a circle $2\frac{3}{8}$ in. in diameter from a point $2\frac{1}{4}$ in. from centre of circle.

21. Draw two lines making an angle of 45° with each other, and a circle tangent to these lines at a point $1\frac{3}{4}$ in. from vertex of the angle.

22. Draw two lines making an angle of 30° with each other, and two circles tangent to each other and these lines. The diameter of the smaller circle is $\frac{3}{4}$ in.

23. Draw a tangent to two circles having diameters of $1\frac{3}{4}$ and $\frac{7}{8}$ in. and their centres 2 in. apart.

24. Draw a circle having a diameter of $1\frac{1}{2}$ in. tangent to two circles whose diameters are $1\frac{3}{4}$ and $1\frac{1}{8}$ in. and whose centres are $1\frac{3}{4}$ in. apart.
25. Prescribe three points and draw a circle through them.

INSCRIBED AND CIRCUMSCRIBED POLYGONS.

26. Circumscribe a circle about a scalene triangle having sides $1\frac{1}{2}$ in., $2\frac{1}{8}$ in., and $2\frac{3}{4}$ in.
27. Inscribe a circle within an isosceles triangle whose base is $2\frac{3}{8}$ in. and equal sides $3\frac{1}{8}$ in.
28. Draw a right-angle triangle having a base of $2\frac{1}{2}$ in. and one of the oblique angles 30°. Circumscribe a circle about this triangle.
29. Within a circle 3 in. in diameter inscribe an equilateral triangle
30. Within a circle 3 in. in diameter inscribe a square.
31. Within a circle 3 in. in diameter inscribe a pentagon.
32. Within a circle 3 in. in diameter inscribe a hexagon.
33. About a circle $2\frac{3}{8}$ in. in diameter circumscribe a hexagon.

HEXAGONS.

34. Draw a hexagon having its long diameter $2\frac{7}{8}$ in.
35. Draw a hexagon having a short diameter $2\frac{3}{8}$ in.
36. Draw a hexagon having one side $1\frac{3}{8}$ in.
37. Draw a hexagon having one side $1\frac{1}{4}$ in. long and at an angle of 45° with the horizontal.
38. Draw a hexagon having its short diameter $2\frac{1}{4}$ in. and one side horizontal.
39. Within a circle 3 in. in diameter inscribe an octagon.
40. Circumscribe an octagon about a circle $2\frac{1}{2}$ in. in diameter

INSCRIBED AND CIRCUMSCRIBED CIRCLES.

41. Within an equilateral triangle having sides of 4 in. draw 3 equal circles touching each other and one side of the triangle.
42. Within an equilateral triangle having sides of 4 in. draw 3 equal circles touching each other and two sides of the triangle.
43. Within an equilateral triangle having sides of 4 in. draw 6 equal circles which shall be tangent to each other and the sides of the triangle.
44. Within a circle 4 in. in diameter draw 3 equal circles tangent to each other and the given circle.
45. Within a circle 4 in. in diameter inscribe 5 equal circles tangent to each other and the given circle.
46. About a circle $1\frac{3}{8}$ in. in diameter circumscribe 5 equal circles tangent to each other and the given circle.

III.

CONIC SECTIONS.

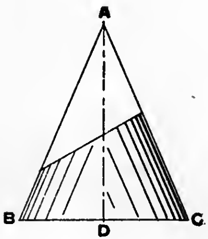


Fig. 1.

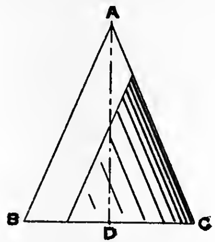


Fig. 2.

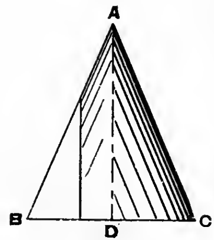


Fig. 3.

If a cone be cut by three planes, in the manner indicated in the figures, three important curves will be derived, viz: the *Ellipse*, *Parabola*, and the *Hyperbola*.

The *Ellipse* is obtained by a cutting plane oblique to the axis, and making a greater angle with it than D A B, Fig. 1. It is a closed or continuous curve, and is symmetrical with respect to two perpendicular axes. See Pl. 8.

The *Parabola* is obtained by a cutting plane parallel to A B, or making a smaller angle with the axis than A B, Fig. 2. It is not a closed curve. The branches of the curve continually approach parallelism but become parallel only at infinity. See Pl. 9, Figs. 71 and 72.

The *Hyperbola* is obtained by a cutting plane parallel to the axis of the cone. Fig. 3. In this curve the branches are always diverging. See Pl. 9, Figs. 73 and 74.

THE ELLIPSE.

Plate 8.

The Ellipse is a curve generated by a point moving in a plane so that the sum of the distances from this point to the two fixed points shall always be constant.

The fixed points, E and F, Fig. 67, are called the *foci*. They lie on the longest line that can be drawn, terminating in the curve of the ellipse. The line is known as the *major axis*, and the perpendicular to it at its middle point, also terminating in the ellipse, is the minor axis. Their intersection is called the *centre* of the ellipse, and lines drawn through this point, and terminating in the ellipse, are known as *diameters*. When two such diameters are so related that a tangent to the ellipse at the extremity of one is parallel to the second, they are called *conjugate diameters*. K L and M N are two such diameters.

In order to construct an ellipse, it is in general necessary that either of the following be given: the major and minor axes; either axis and the foci; two conjugate diameters; a chord and axis perpendicular to it.

Fig. 67. First Method. — By definition it may be seen that a series of points must be so chosen that the sum of the distances from either of them to the foci must equal the major axis. Thus H E + H F must equal C E + C F and K F + K E, each being equal to A B. If then the major axis and foci be given to draw the curve, points may be determined as follows: From E, with any radius greater than A E and less than E B, describe an arc. From F, with a radius equal to the difference between the major axis and the first radius, describe a second arc cutting the first. The points of intersection of these arcs will be points the sum of whose distances from the foci will equal the major axis, and therefore points of an ellipse. Similarly find as many points as may be necessary to enable the curve to be drawn freehand. Draw very lightly,

using a sharp point and describing an almost invisible curve. Mechanical methods should be used for inking only. Do not erase construction lines.

Having given the major and minor axes, we can find the foci by describing from C as centre an arc with a radius equal to one-half the major axis A B. The points of intersection with the major axis will be the foci; and this must be so, since the sum of these distances is equal to the major axis; and the point C being mid-way between A and B, the two lines C E and C F must be equal. Again, if we have the major axis and foci given, we may, with a radius equal to one-half this axis, describe arcs from the foci, cutting the perpendicular drawn at middle point of major axis, and thus obtain the minor axis. Having the two axes, proceed as before to describe the ellipse.

A tangent to an ellipse may be drawn at any point K, by producing K F, and bisecting the angle S K E; the bisecting line K T will be the required tangent.

Do not copy the problems from the plates, but construct from the data given in the examples on page 50.

Fig. 68. SECOND METHOD. — *To describe an Ellipse by Trammels.*

Let A B and C D be the major and minor axes of an ellipse. Lay off on a piece of paper, having a clean cut edge, the distance R T, equal to one-half the major axis, and R S, equal to one-half the minor axis. Now, if point T be placed upon the minor axis, and point S upon the major axis, and the paper constrained to move always under these conditions, the point R will describe an ellipse. If points be laid off upon the drawing to correspond with the different positions of R, a number of points may be obtained through which the required ellipse may be drawn. This is the best method for a draughtsman, since construction lines are not required.

Fig. 68. THIRD METHOD. — *To describe an approximate ellipse, the major and minor axes being given.*

For many purposes in drawing it is sufficiently accurate to describe the ellipse by means of four circular arcs of two different radii. The following is one of several methods employed for describing an ellipse from four centres.

On the minor axis lay off from the centre G F and G O equal to the difference between the major and minor axes (A B — C D). Next lay off points E and L on the major axis, their distance from the centre being equal to three-quarters of G F. Connect points F, E, O, and L, and produce the same. With centre E, and radius A E, describe arc L A H. With centre F, and radius F D, describe arc H D K. In similar manner describe the two remaining arcs, K B R and R C L.

Do not use this method when the major axis is twice as great as the minor axis.

Figs. 69 and 70. Fourth Method. — This is a very general method, and may be used when we have given, either the major and minor axes, one of the axes and a chord of the ellipse, or any two conjugate diameters. The number of construction lines required, make this method unsuitable for the draughtsman.

Fig. 69. Case 1. — *Having given the major and minor axes, to describe an ellipse.*

Draw B 6 parallel to the minor axis, and divide into any number of equal parts, in this case, six. Divide B G into the same number of equal parts. Through points 1, 2, 3, etc., on B 6, draw lines to extremity C, of minor axis. From the other extremity of the minor axis D draw lines through points 1, 2, 3, etc., on B G, intersecting the above lines in points which will lie in the required ellipse. Construct remainder of ellipse in the same manner.

Fig. 69. Case 2. — *Having given an axis C D, and chord F H.*

From F draw F 4 parallel to C D; divide it into any number of equal parts, in this case, four. Divide the half chord F E into the same number of equal parts; through these points and extremities of given axis draw intersecting lines as before, thereby obtaining the elliptical arc F D. Construct opposite side in the same manner.

CASE 3. — *Having given the conjugate diameters A B and C D.*

From A and B draw lines A 6 and B 6 parallel to the diameter C D. Divide these into any number of equal parts, and, having divided B G and A G into the same number of equal parts, draw lines from these points to the extremities of diameter C D, similar to Case 1. In the same manner describe opposite side.

THE INKING OF CURVES.

Use the following method for the inking of all curves other than circular arcs. Suppose it is required to ink the curve G A F H, Pl. 1, Fig. 5. Wherever possible use the compasses. In this case, a circular arc struck from the centre K will be seen to coincide with the required curve, through points G, A, B, and C. Just at the point C it will appear to leave the curve, and should therefore not be drawn up to that point, but the inked line should stop at the point B. Since the radius required to complete the curve would now be too large for the compasses, select such a *scroll* as will coincide with the largest portion of the curve, beginning at the point A, a little to the left of the line already inked. In the figure, this curve is seen to coincide with the line up to the point F, but the inked portion of this line should stop short of the point F, as at E. Now another portion of this, or some other scroll, should be made to coincide with the remaining portion of the required lines, beginning at least as far back as the point D, thus laying the scroll to coincide with a greater portion of the curve than is required to be inked at any one stroke.

In inking the ellipse, Pl. 8, Fig. 67, X B Y and W A V should first be inked, and by means of the compasses. Next, obtain, if possible, a scroll to coincide with arc C H X, and for a short distance on either side of points C and X, so as to insure a perfect copy of the curve. The line should now be drawn from the point C to the point X, but never should it pass to the left of the point C, with scroll in this position. Now, marking upon the scroll (which should be made of white wood), a point which coincides with the point C, and reproducing this point upon the opposite side of the scroll, reverse the scroll in order to draw lines to the left of C. Similarly ink lower half of ellipse.

The greatest possible care must be used in inking these curves, as no class of line work requires more skill and patience.

Construction lines should be left in pencil.

THE PARABOLA.

Plate 9.

The Parabola is a curve generated by a point moving in a plane so that its distance from a given point shall be constantly equal to its distance from a given right line.

The given point F, Fig. 71, is the *focus;* the given right line C D, the *directrix*. The line A B, perpendicular to C D, through F, is the *axis*, and V, its intersection with the curve, is the *vertex*.

Fig. 71. First Method. — *Having given the focus F, and the directrix C D, to describe a parabola.*

Bisect F A to find the vertex V. Through any point on the axis, as L, draw M N parallel to the directrix, and with radius L A, from focus F as centre, describe arcs 1, intersecting line M N at points M and N. Obtain other points in a similar manner, and through the points draw the curve freehand.

A tangent to the curve may be drawn at any point, as M, by drawing M O parallel to the axis and bisecting angle O M F. M T is a tangent to the curve at point M.

Fig. 72. Second Method. — *Having given the height V B, and the breadth G E.*

Draw A E and C G parallel and equal to V B. Divide A E and E B into the same number of equal parts. From the divisions on B E draw parallels to the axis, and from the divisions on A E draw lines converging to the vertex V. The intersection of these lines 1 and 1, 2 and 2, etc., will give points in the required curve. Produce opposite side in the same manner.

THE HYPERBOLA.

PLATE 9.

The Hyperbola is a curve generated by a point moving in a plane, so that the difference of its distances from two fixed points shall be constantly equal to a given line.

The two fixed points, F and F′, Fig. 73, are the *foci.* The given line, V V′, is the *transverse axis.*

Fig. 73. FIRST METHOD. — *Having given the axis V V′, and the foci F and F′, to describe an hyperbola.*

With any radius, from centres F and F′, describe arcs 1; from same centres, with radius increased by V V′, describe arcs 2 intersecting the arcs 1. These points of intersection will be points in the required curve.

Fig. 74. SECOND METHOD. — *Having given the axis V V′, the height of the curve V B, and its breadth A D.*

Draw A C and D E parallel to B V. Divide lines A B and A C into any number of equal parts, and from points on A C draw lines converging to the vertex V. From points on A B draw lines converging to the vertex V′. The points of intersection of these lines 1 and 1, 2 and 2, etc., will be points in the required hyperbola. Produce the remaining branches of the curve in the same manner.

The compasses should be used to ink a small portion of the curve on either side of the vertex, even if it is possible to ink no more than an eighth of an inch. The remainder of the curve requires the use of a scroll.

EXAMPLES.

1. Draw an ellipse having major axis 6 in. and minor axis 4 in. Use First Method. Draw a tangent to the curve at a point $2\frac{3}{8}$ in. from minor axis.
2. Draw an ellipse having major axis $5\frac{1}{4}$ in. and minor axis $4\frac{1}{4}$ in. Use Second Method. Try also Third Method.
3. Draw an ellipse having its minor axis 4 in. and the distance between foci 3 in. Use Third Method, and test by Second Method.
4. Draw an ellipse having major axis 6 in. and minor axis 4 in. Use Fourth Method.
5. Draw a segment of an ellipse having an axis of 4 in. and a chord of $5\frac{1}{2}$ in. which intersects the axis at $1\frac{1}{4}$ in. from its extremity. Use Fourth Method.
6. Draw an ellipse having conjugate diameters of 5 in. and $3\frac{1}{2}$ in., one being horizontal and the other making an angle of 60° with the horizontal.
7. Draw an ellipse having its major axis 6 in. and the distance between foci $4\frac{7}{8}$ in. Use Second Method, and test three points by Fourth Method.
8. Draw an ellipse having major axis 6 in. and minor axis 3 in. Use Third Method, and test by Second Method.
9. Draw a parabola having the focus $\frac{3}{4}$ in. from vertex. Draw a tangent to the curve.
10. Draw a parabola having the axis B V, Fig. 72, $3\frac{1}{2}$ in. and base E G, $5\frac{1}{2}$ in.
11. Draw an hyperbola having axis V V′, Fig. 73, $1\frac{1}{2}$ in. and distance between foci, $2\frac{1}{8}$ in.
12. Draw an hyperbola having axis V V′, Fig. 74, $1\frac{1}{2}$ in., B V, $1\frac{7}{8}$ in., and A D, $4\frac{3}{4}$ in.

IV.

PROJECTION.

Orthographic Projection, or Projection, as it is commonly termed, is the art of delineating an object on two or more planes, suitably chosen, and generally at right angles to each other, so as to exactly represent the form and dimensions of its lines and surfaces, and their relations to each other.

Suppose it is required to make the projection, or mechanical drawing, of a pyramid having a rectangular base, such as is shown in perspective by Fig. 1, Plate 11. Conceive the object as surrounded by transparent planes, called Planes of Projection, and shown in perspective in Fig. 2, by A D E C, A B G D, and A B H C. We may now have three representations of the object; one by looking through the front, one through the top, and one through the side plane of projection. These representations would be correct projections of the object if we imagine the eye as being directly opposite all of its points at the same time, so that the rays of light from the points to the eye be perpendicular to the plane of projection; or we may conceive the eye as at an infinite distance from the plane. Therefore; *if from each point of an object, perpendiculars be drawn to the planes of projection, the intersection of these perpendiculars with the planes will be the required projections of the points, and the lines joining these points will be the projections of the lines and surfaces of the object.*

Thus point 1, Fig. 2, is projected on the front plane A D E C at the point 1^F, on the top plane A B G D at 1^T, and on the side plane A B H C at 1^S. In like manner the points 2, 3, 4, and 5 are projected on the three planes. The small letters ^{F}, ^{T}, and ^{S}, above and to the right of the numbers, indicate the plane upon which the points lie, and when these letters are not affixed it signifies that the point or line itself is meant, and not its projection.

Since points 1 and 3 are projected on the front plane by the same perpendicular, it follows that they will have a common point for their projection, this being designated as the projection of two points by the figures 1^F 3^F, the first figure indicating the point nearer the plane. Observe similar cases on the side plane.

In order to represent these planes of projection upon a plane surface, as a sheet of drawing paper, it becomes necessary for us to revolve two of them into the plane of the other one, as shown in Fig. 3, which is a reproduction of Fig. 2 with the perspective of the pyramid omitted and the planes revolved. Thus the top plane A B G D is revolved about the line A D into the position A B″ G′ D, and the side plane is revolved about the line A C into the position A B′ H′ C. By this means we have obtained, upon a plane surface, three representations of the object as they would appear on planes at right-angles to each other. A good conception of the relation of the planes may be obtained by cutting a piece of paper in the form shown in Fig. 3 by G′ B″ A B′ H′ E G′, and then folding it on the lines A D and A C. This will represent three sides of a rectangular prism, as shown in the perspective view, which we may regard as the transparent planes behind which the object is supposed to be, and upon which it is to be projected. It should be observed that the top view is always above the front view, and the side view to the right or left of the front view, according as it may be a view of the right or left of the object.

The line A D is known as the FRONT AXIS OF PROJECTION. The line A B (shown in the revolved positions by A B″ and A B′), is designated the SIDE AXIS OF PROJECTION, and the line A C, the VERTICAL AXIS OF PROJECTION. For brevity, these lines are also designated as the Front, Side, and Vertical axes.

That representation which appears on the top plane of projection is called the TOP VIEW, that on the front plane, the FRONT VIEW, and that on the side plane, the SIDE VIEW. The view takes its name from the plane upon which the representation is made, and not from the face of the objects represented. For brevity these planes may be designated by the letters T, F, and S.

From the foregoing, the following laws are established: *The front and top views of any point lie in the same vertical line. The front and side views of any point lie in the same horizontal line. The top and side views of any point lie equally distant from the front and vertical axes of projection.*

Suppose it is required to obtain three views of a rectangular pyramid, as shown in Fig. 4, having given its dimensions. First draw the axes of projection D B′ and B″ C. The view included within the angle D A C will be the front view, that within the angle B″ A D, the top view, and that within the angle B′ A C, the side view. It is not necessary to limit these planes by drawing boundary lines, as in Fig. 3, since the planes are supposed to be indefinite in extent. In general, draw that view, first, about which most is known. The following order will be pursued in this problem: Front, Side, and Top view. At some convenient distance below A D and to the left of A C, draw the line 1^F 2^F, of the length required to represent the base of the pyramid. At its middle point erect a perpendicular equal to the required height, and connect points 1^F, 5^F, and 2^F. This will complete the front view. Since "The front and side views of any point lie in the same horizontal," it follows that the side view of the vertex, point 5, must lie in the horizontal projecting line drawn through the point 5^F, and at some convenient distance to the right of A C. The points of the base will be similarly projected, and since the pyramid is symmetrical, the points 2^S, 1^S, and 4^S, 3^S, will be equally distant from the vertical centre line drawn through 5^S, the line 2^S 4^S being made of the length required to represent the depth of the pyramid. To project points from front and side views to the top view, it is necessary to observe, first, that

"The front and top views of any point lie in the same vertical line;" second, that "The top and side views of any point lie equally distant from the front and vertical axes of projection." Therefore, to project any point, as 5, into the top view, a perpendicular to A D through 5^F must be drawn and the point will lie in this line, but its position may not be chosen as before, since two projections of a point being given, the third is fixed, and the distance of this point from the front plane of projection is determined by the side view. Draw a perpendicular through the point 5^S until it intersects the side axis A B′ at K, and remembering that the lines A B′ and A B″ are one, revolve the point K by describing the arc K K′ from the centre at A. From K′ draw the horizontal projecting line K′ 5^T, which will determine the top view of the point by its intersection with 5^F 5^T. In like manner the points of the base may be found, and since the vertex is connected with the four corners of the base, we shall have the lines 5^T 1^T, 5^T 2^T, 5^T 3^T, and 5^T 4^T to represent these edges.

Carefully observe the following: *The projection of a point is always a point. The projection of a line is either a point or a line. The projection of a surface is either a line or a surface.* To illustrate: The point 1, of the pyramid, is projected as a point on each of the planes, as shown at 1^F, 1^T, 1^S. The projection of the line 1 2 is a line on the front and top planes, and a point on the side plane. The surface 1 5 2 is projected on the front and top planes at 1^F 5^F 2^F and 1^T 5^T 2^T, but on the side plane by the line 1^S 5^S, or 2^S 5^S. In the same manner the front axis of projection A D, may be considered as the front view of the top plane of projection, or the top view of the front plane, and the vertical axis A C, as the front view of the side plane, or the side view of the front plane of projection.

GENERAL INSTRUCTION FOR DRAWING.

The dimension of the paper will be the same as for the GEOMETRICAL PROBLEMS, the margin line enclosing a space 10x14 in. Where several problems are designed for the same sheet, it is desirable to separate them by a fine inked line. Four sheets will, in general, be required for each plate of problems. All construction lines and lines of the object should be drawn very fine in pencil, and *no line that has been useful in the construction of the drawing should be erased.* Invisible lines of the object are better dotted in pencil, that no mistake be made in inking the same. Draw no dimension lines. Absolute accuracy must be used in the construction of all figures. Only lines of the object are to be inked, the visible lines being in full and the invisible in dotted lines. Shade lines are to be used only in those problems indicated, and never shown in pencil.

There is but one reason that can justify the use of the shade line, and that is, added clearness to the representation by indicating the relation of the surfaces to one another. It often adds much to the beauty of a drawing, and for that reason may sometimes be employed. By some draughtsmen the shade line is never used, while by others it is always used, and both are in error since no law can be established concerning it, there being times when it is a mistake not to use it, and others where it is equally wrong to use it. The following method is the only one that can consistently be used in practice :

Draw shade lines for all right-hand and lower edges in all views. Shade the lower right-hand quadrant of outside circles and the upper left-hand quadrant of inside circles. Never shade the intersecting line between visible planes. The additional width which is given to a shade line should not encroach on the surface which it bounds.

Use red ink for centre lines, making the lines full.

The following order should be pursued in the inking of all drawings: Small circles and circular arcs first, shading them at the same time. Next, the larger arcs, fine horizontal lines, fine vertical lines, other fine lines, and then all shade lines, pursuing the same order as in the inking of fine lines. Lastly, ink centre lines. Should section lines be used, they may be drawn before the centre lines, if dimension lines are not employed, otherwise they must be drawn last.

PROBLEMS.

Plate 12.

Problem 1. — Locate the axes according to the dimensions given. Draw front and top views, and from these obtain the side view. Draw the lines necessary for projecting the points, in pencil only. Having completed the projection of the object, it is an excellent practice to number in pencil the extremity of each line, as in Fig. 4, Pl. 11, and thus acquire familiarity with the different views of each surface, line, and point. It will also insure the representation of each line in every view.

Problem 2. — In this and the following problems, the student must use his own judgment in the location of the axes of projection. The front and side views being given, the top view will be drawn last. See that all the lines of the object are shown on this view.

Problem 3. — Note carefully the difference between the top view of this figure and that of the preceding.

Problem 4. — Draw the top view first, then front and side views. Remember to represent the invisible lines, since it is required in these problems to represent three views of every line of the object.

Problem 5. — Draw the top view without the aid of compasses, observing that it is an equilateral triangle.

PROBLEM 6. — This problem differs from the preceding in being a pyramid instead of a prism.

PROBLEM 7. — Having given the short diameter of a hexagon, the figure should be drawn without the aid of compasses.

PROBLEM 8. — Although the front view alone is given, it is better to draw the top view first.

PROBLEM 9. — This prism having a triangular hole from end to end, will necessitate the representation of several invisible lines.

PROBLEM 10. — It is required to represent the preceding object when turned around on its base. This, while changing the angle at which the top view is drawn, does not alter the relation of the lines to each other, and, therefore, the top view may be copied from Problem 9. It must also be observed that all points of the object retain their former height. Draw top view first, and then front and side views. Use care to represent the invisible lines of the object.

PROBLEM 11. — This object is similar to the preceding, save that the ends are bevelled, and the triangular space does not pass entirely through the prism. It must now be noticed that the drawing of the axes of projection is not a necessity, and might have been dispensed with earlier were it not useful in separating the planes of projection, and keeping clearly before the student the relation between the object and the planes. The top view of the pyramid in Fig. 4, Plate 11, could have been drawn from the front and side views, without the use of axes, as follows: Since the axes are supposed not to be drawn, we may draw the centre line $5^T K'$ at any convenient distance from the front view, and from this the lines $1^T 2^T$ and $3^T 4^T$ may be laid off on either side, making the distance between them equal to $2^s 4^s$ on the side view. This is equivalent to considering the centre lines of top and side views as the axes. The drawing of the projection lines may also be omitted, as in practice they would cause too much confusion on the drawing. In Problem 11, both the axes and projection lines are required to be omitted.

PROBLEM 12. — This problem is similar to Problem 10, save that the projection lines, but not the axes, are to be omitted.

OBJECTS OBLIQUE TO PLANES.

Plate 13.

To enable the drawing of an object to be made when the object is oblique to one or more of the planes of projection, it is first necessary to observe the changes which take place in its appearance when it is revolved in each of three ways: First, by turning it around on its base. Second, by revolving it forward or backward. Third, by revolving it to right or left. The first is a revolution about a *vertical axis* of projection, or any axis parallel to it. The second is a revolution about a *front axis* of projection, or any axis parallel to it. The third is a revolution about a *side axis* of projection, or any axis parallel to it.

From Problems 10 and 12 we have seen, that in revolving an object about a vertical axis, the top view remains unchanged, likewise the height of all points of the object is unaltered, and this knowledge was enough to complete the necessary views of the object. Similarly by revolving it about a front axis, the side view would be unchanged, and all dimensions parallel to the front axis, that is, breadths, would remain the same. Again, by revolving it about a side axis, the front view would be unchanged, and all dimensions parallel to the side axis, viz., the depths, likewise unchanged. By the terms height, breadth, and depth, is not necessarily meant the height, breadth, and depth of the object, but the distances of points of the object from the planes of projection, that is, distances parallel to the axes about which they are revolved. To illustrate, suppose the object as represented in Fig. 1, Plate 13, be revolved about a side axis, as shown in Fig. 2. The front view will be unchanged and may be copied from Fig. 1. To obtain the top view, it is only necessary for us to remember that during the revolution, the distance of each of the points from the front plane of projection remains the same; therefore, by projecting the points from the front view to the top, and making the distances from the front axis the same as in Fig. 1, all points will be determined. From these two, the side view may readily be drawn. Fig. 3 represents the pyramid revolved about a side and vertical axis, but since it is necessary to perform the problems separately, Fig. 2 must first be obtained and the pyramid revolved from this position about a vertical axis, as was done in Problems 10 and 12. Fig. 4 represents the pyramid revolved backward about a front axis. The term backward signifies a motion from the front plane. Fig. 5 is a revolution about a side and front axis. Fig. 6 is a revolution about a side, front, and vertical axis, the operations being shown by Figs. 1, 2, 5, and 6. Thus it is to be observed that there is always one view unchanged, and one set of dimensions unchanged. The following statement comprises all that has been said concerning the revolution of an object.

The unchanged view lies on that plane of projection which is perpendicular to the axis of revolution, and the unchanged dimensions are parallel to the axis of revolution.

In performing the problems, first determine and draw the unchanged view. Having a set of dimensions parallel to one of the axes, it is then possible to obtain a second view, and from these two, a third. Do not copy this plate, but observe the principles in performing the following problems.

PROBLEMS.

Plate 12.

Problem 13. — Fig. 1 represents a hollow triangular prism which is required to be shown in three positions. In Fig. 2 it is to be revolved about a side axis. The front view, being unchanged, is copied from Fig. 1, as shown. The points may now be projected into the top view, and since the depths remain the same, the points will have the same relative position with respect to the front plane of projection. In Fig. 3 the prism is revolved about a front axis, the side view being unchanged. Fig. 4 is a revolution about a vertical axis.

The different figures may be separated by pencil lines only, and the shade lines omitted in these problems.

Plate 14.

Problem 14. — This problem is similar to the preceding, but in this case the unchanged view is not shown on the plate. Use care to revolve the object in the direction prescribed, and through the proper number of degrees.

Problem 15. — Having drawn the pyramid, as shown in Fig. 1, proceed with the problems in the order indicated by the figures. Just here it is well to note that *lines of the object which are parallel, are always parallel in projection.* In many cases it will be found helpful to number the points, as in Plate 3, but should this be done, great care must be used to retain the same number for each point throughout the problem.

Problem 16. — This problem differs from the preceding not only in that the object to be revolved is a cone, but because of the absence of a view from which the necessary dimensions for the top view may be obtained. In this case the front view may be as readily drawn as though the cone was resting on its base, but in the top view, the depths are apparently missing. In order to find the curve of the base, in the top view and the several revolved positions, we must consider it as consisting of points, whose positions are to be determined in the same manner as the corners of the base of a pyramid. The points E^F, A^F, C^F, F^F, D^F, B^F, are points of the base whose position in the top view being found, the required curve can be drawn through them. Remembering the base of the cone to be a circle, imagine that half of it lying next the front plane of projection, to be revolved about the diameter of the base parallel to that plane, into the position $E^F A' C' F^F$. The point which was at A^F is now revolved to A', and the point C^F to C'. Next conceive a vertical plane as passing through the axis of the cone and this same diameter of the base. The top view of this plane would be the line $E^T G^T$, and the distance of the point A^T from this plane will equal $A^F A'$. The distance of the point B^T on the other side would also equal $A^F A'$. The points C^T and D^T would be at a distance from this plane equal to $C^F C'$, and the points E^T and F^T lie in the plane. In like manner obtain a sufficient number of points to enable the curve to be drawn. Since the vertex lies in the plane $E^T G^T$, its projection may be obtained at G^T, and by drawing tangent lines from this point to the curve of the base the figure is completed. Next revolve the cone as directed, determining the points in the same manner as though they were the corners of a polygon.

All the curves in this problem, being ellipses, might have been obtained by first finding the major and minor axes and constructing the curves from them. While this would be a shorter method, it would fail to give the practice most necessary at this stage, beside involving problems not yet explained.

Problem 17. — The revolution of a *surface* about the several axes of projection. Solve as in Problem 15.

SPECIAL METHODS FOR THE REVOLUTION OF LINES, SURFACES, AND SOLIDS.

Plate 15.

The methods used in the foregoing problems have been such as would best illustrate and explain the fundamental principles of projection, and their application has been chiefly applied to the representation of solids, as being more easily comprehended than that of lines and surfaces. Some of the shorter methods for the solution of those problems, together with the consideration of the projection of lines, their measurement, and the determining of the angles which they make with the planes of projection, are treated in the following:

First Method. — Let $A^F B^F$, $A^T B^T$, and $A^S B^S$, Fig. 1, Plate 15, represent three views of a line inclined to each of the planes of projections, which latter will for brevity be designated by F, T, and S. It is required to find the true length of this line, and the angles which it makes with the planes of projection. Since the line is inclined to each of these planes, its projection on them will be foreshortened views of the line. In order that the line shall be parallel to one of the planes, and thus seen in its true length on that plane, it will be necessary to revolve it about one of the axes according to the principles already established for the revolution of an object. It may be revolved about a vertical axis until it becomes parallel to either F or S; or about a front axis until parallel to either F or T; or about a side axis until parallel to S or T. Fig. 2 represents the line revolved about a vertical axis and parallel to F, the revolved position being shown by the broken line $A^F B'$. The top view of the line, which in the revolution about a vertical axis we know to be unchanged, is first drawn parallel with the front axis of projection, in which position it will be parallel to F. The height being unchanged, the front view may next be obtained, and the representation will be that of a line parallel to F, and therefore seen in its true length on that plane. This view also shows the correct angle R, which the line makes with a horizontal plane, since its position relative to that plane is unchanged by the revolution.

Similarly the line may be revolved about a side axis until it is parallel with T, as in Fig. 3, when its true length may be measured on the top view. This view also gives the angle which the line makes with F, or any plane parallel thereto.

Fig. 4 illustrates the revolution of the line about a vertical axis until parallel to S. This is seldom used, as it is much more simple to obtain the true length on F, as in Fig. 2, and avoid the drawing of an extra view.

Second Method. — If the quadrilateral formed by a line, the projecting lines of its extremities, and its projection, be revolved about the latter until the surface coincides with the plane of projection, the revolved position of the line itself will exactly represent the length of the line, and the angle which it makes with the plane of projection will be shown on the plane into which it has been revolved.

In Fig. 5 let $A^T B^T$ be the horizontal projection of a line, and $A^F C$, $B^F D$, the vertical projection of the projecting lines of its extremities. Since these last named lines are seen in their true length in this view, we have three sides of the quadrilateral, and the fourth will be the true length of the line. To obtain this result, draw $A^T C'$ and $B^T D'$ equal to $A^F C$ and $B^F D$, respectively, and perpendicular to $A^T B^T$. The points C', D', will be the extremities of the revolved position of the required line, and the angle which $C' D'$ makes with $A^T B^T$ is the angle which the given line makes with T, or any horizontal plane. The plane in which this quadrilateral lies is known as the horizontal projecting plane of the line, and $A^T B^T$ is called its horizontal trace, it being the line in which the projecting plane intersects the horizontal plane.

Similarly, in Fig. 6, the vertical projecting plane of the line has been used, and the revolution made into F, about its vertical trace, $A^F B^F$, which is the vertical projection of the given line. The true length of the line is $C' D'$, and the angle which this line makes with $A^F B^F$ is the angle which the given line makes with F.

An application of the foregoing principles is illustrated by Fig. 7. This represents a rectangular surface with its long edges making angles of 0°, 30°, 60°, and 90° with a vertical plane, and 35° with a horizontal plane. This surface might first have been drawn with its long edges horizontal and making the required angles with the vertical plane, after which it could have been revolved about a side axis until the edges made the required angle of 35° with the horizontal plane. This would have necessitated the drawing of two more views, which may be avoided in the following manner. Suppose the surface to coincide with F, as shown by $A^F B^F D^F C^F$, the long edges making the required angle with T. Next, conceive the surface as being revolved about one of its short edges, as $A^F C^F$, until the required angles with F are obtained. Suppose one such angle to be 30°, draw $A^F B'$ equal to $A^F B^F$, and making an angle of 30° with it. This will represent the position of the required line when revolved about the trace of its vertical projecting plane, precisely as was done in Fig. 6. Next, revolve the line about this trace and into the required position, obtaining thereby the points $B_{\prime\prime}^{F}$ and $D_{\prime\prime}^{F}$. The horizontal projection of these points may now be obtained, since their distance from F is determined, being equal to B', $B_{\prime\prime}^{F}$. In like manner the other required positions of the surface may be obtained.

Fig. 8 illustrates the case of a horizontal hexagonal prism, the lateral edges of which make an angle of 30° with F. In this problem the top view may readily be drawn, and from that the projection of the points in the front view determined, except as to height. If one of the bases of the prism be revolved into, or parallel to, T, the required measurement may be readily obtained. This is similar to Problem 16, Plate 14, and like it, the base might have been revolved about its horizontal diameter.

From the foregoing, the following principles may be established: —

The true length of a line can be seen only on that plane, or planes, to which the line is parallel.

If a right line is perpendicular to a plane of projection, its projection on that plane will be a point.

If a right line is parallel to a plane of projection, its projections on the other two planes will be either a point, or a line parallel to an axis of projection.

If a surface is parallel to a plane of projection, its projection on the other planes will be a line parallel to an axis of projection.

SPECIAL PROBLEMS IN PROJECTION.

Each problem will require a space of 7x5 in.

Three views are required, and all invisible as well as visible lines should be shown on each view.

Leave all construction lines in pencil.

All polygons referred to are regular polygons.

1. Draw the frustum of an octagonal pyramid having its base horizontal and two of its edges making an angle of 30° with F. The diameter of the circumscribing circle of its lower base is $1\frac{3}{4}$ in., and of the upper base, $1\frac{5}{8}$ in. The altitude is $1\frac{7}{8}$ in.
2. Revolve the pyramid as required in 1, 30° to right, about side axis.
3. Draw a pentagonal prism resting on one of its faces and having its lateral edges at an angle of $22\frac{1}{2}$° with F. Diameter of circumscribing circle of base, $1\frac{1}{2}$ in. Length of prism, $2\frac{1}{2}$ in.
4. Draw an equilateral triangular prism resting on one of its faces, and its lateral edges making an angle of 15° with F. The edges of the base are $1\frac{1}{2}$ in., and the length of the prism, $2\frac{1}{4}$ in. There is an equilateral triangular hole extending through the bases and making the thickness of the sides $\frac{1}{4}$ in.
5. Draw a cylinder with its axis parallel to F, and at an angle of 60° with T. The diameter of the base is $1\frac{3}{4}$ in., and the length of cylinder, $2\frac{1}{8}$ in. Obtain the ellipses by the method of trammels.
6. Draw an equilateral triangular pyramid having an altitude of $2\frac{1}{4}$ in., and the edges of the base, $1\frac{7}{8}$ in. The base makes an angle of 30° with T, and one of its edges is perpendicular to F.

7. Revolve the pyramid as required in 6, 45° forward.
8. Draw a box having the following outside dimensions. Length, 2 in., width, $1\frac{3}{4}$ in., depth, including cover, 1 in. Thickness of material, $\frac{1}{4}$ in. The long edges of the box are horizontal, and make an angle of 30° with F. The cover is hinged on long edge, and opened 30°.
9. Draw a pyramid formed of four equilateral triangles having $2\frac{1}{8}$ in. sides. The base is horizontal, and one of its edges makes an angle of 30° with F.
10. Draw a regular octahedron having edges $1\frac{5}{8}$ in. long. The square surface to be horizontal, with two of its edges making angles of 30° with F.
11. Draw a rectangular surface, $1\frac{1}{4}$x$2\frac{5}{8}$ in., in the following positions. The short edges horizontal, and making an angle of 75° with F; the long edges making angles of 15°, 30°, and 45° with T.
12. Revolve the surface from the positions required in 11, 15° forward.
13. Draw an isosceles triangle in three positions as follows. The base lying on F, and inclined at an angle of 30° with T. The altitude making angles of 90°, 30°, and 15° with F. The base of triangle is $1\frac{7}{8}$ in., and the altitude, $2\frac{1}{8}$ in.
14. Draw the same triangle revolved from the positions in 13, 30° in either direction about a vertical axis.
15. Draw an isosceles triangle in the following positions. The base horizontal, and making an angle of 60° with F. The altitude making angles of 45° and 60° with T. The base of triangle is 2 in., and the altitude, $2\frac{1}{4}$ in.
16. Revolve the same triangle from the positions in 15, 30° backward about front axis.
17. Draw an octagonal surface inclined at an angle of 60° with T, two of its edges being horizontal, and making angles of 15° with F. The diameter of circumscribing circle is $2\frac{1}{2}$ in.

18. Draw an hexagonal surface inclined at an angle of 45° with F, two of its edges being parallel to F, and making angles of 30° with T. The long diameter of hexagon is $2\frac{1}{2}$ in.
19. Draw the projections of a line located as follows. The left-hand extremity of the line is $\frac{1}{2}$ in. behind F, and $\frac{1}{4}$ in. below T. The right-hand extremity is $1\frac{1}{2}$ in. behind F, and $1\frac{3}{4}$ in. below T. The horizontal projection of the line makes an angle of 30° with F. Find the length of the given line by revolving it parallel to each of the planes of projection.
20. Draw the projections of a line of which the left-hand extremity is 1 in. behind F, and $1\frac{3}{8}$ in. below T. The right-hand extremity, $\frac{3}{8}$ in. behind F, and $\frac{3}{4}$ in. below T. The horizontal projection making an angle of 15° with F. Find the length of the line by revolving it into the planes of projection by the second method, page 62.

V.

THE DEVELOPMENT OF SURFACES.

PLATE 16.

IT is often required to illustrate the surfaces of an object in such a manner that a pattern being made from it, and properly folded or rolled, would exactly reproduce the object. In order to do this, an outline of each surface must be obtained, as it would appear on a plane of projection parallel to it, that is, there would be no foreshortening of the surface. Fig. 1 is the projection of a square pyramid, and it is required to produce the pattern, which if properly folded, would make a pyramid like the one in the drawing. This operation is called the development of the surface. Since four of the surfaces are triangles, it is possible to obtain their true area by finding the length of their sides. A line is seen in its true length on a plane when it is parallel to that plane. Thus the line D E may be measured on the front view, because the line of the pyramid which it represents is parallel to that plane, and we know that it is parallel to the plane because the top view of the line is parallel to the front axis of projection. Neither of the lines E A or E C may be measured from the drawing, but since the base of the pyramid is symmetrical with respect to the axis, we know these lines or edges to be of equal length with D E and E B. The only undetermined line of the surface A E D is A D, which may be measured on the top view, it being a horizontal line. Fig. 2 shows these lines in their proper lengths and relation to each other. The surfaces A E B, B E C, and C E D are of the same shape and size as A E D, and may be drawn in connection with it. The bottom surface alone remains to be drawn, and this being parallel to the top plane, is already seen in its true size and shape, and may, therefore, be copied directly. Having the length of one of the edges we might have used it for a radius to describe the arc D A B C D, and by spacing off the bottom edges D A, A B, etc., have attained the same result in an easier and more practical manner.

Fig. 3 illustrates a rectangular pyramid which it is required to develop, after removing such portion of the top as is indicated by the line F^F K^F. The operations are as follows: First obtain the top view, showing the top removed, and similarly the side view, if required. Secondly, obtain the development of the entire pyramid, disregarding the cutting plane. Finally, determine that portion of the developed surface not removed by the cutting plane, to which must be added the section cut by the plane. As the first operation is sufficiently well indicated by the drawing, we will consider only the development. None of the inclined edges being parallel to either of the planes of projection, it is necessary to revolve one of these edges until it shall become parallel to a plane, when it will be possible to measure it on that plane. Let A E be the line to be revolved. Since this is a revolution about a vertical axis, the top view of the line will be changed in position but not in length, and will be shown by A' E^T. The front view will then be A'' E^F which represents the true length of the line. Since all the inclined edges are of the same length, with radius equal to A'' E^F, describe an arc on which the chords A B, B C, C D, and D A may be drawn, as shown in Fig. 4, their lengths being obtained from the top view. The development of the base is obtained directly from the top view.

The surface B E C can be obtained in another manner, as follows: Since the true size of a surface is always to be found on a plane to which it is parallel, we have only to draw a plane, X Y, parallel to this surface, and project on to it in order to obtain the required surface. This new plane is perpendicular to the front plane, but not to the other planes. The line X Y may be regarded as the axis of the new plane, which, being revolved to coincide with the front plane, would be shown as in the figure, the distances of the points E′, B′, and C′ from the line X Y being their distances from the front plane of projection. As the centre line E′ S is known to be parallel to the front plane of projection, it is unnecessary to draw the axis X Y, since measurements may be made equally well from this centre line.

Finally, having obtained the development of the entire pyramid, it is required to find the length of the edges when cut off, and the section made by the cutting plane. Since we have found it possible to obtain the true length of the inclined edges, we may in like manner find that portion of them included between the base and cutting plane. As A″ E^F may be considered as the revolved position of any one of these lines, and since the heights remain unchanged by revolving about a vertical axis, the true length of A^F K^F and D^F H^F will be A″ N, and the length of B^F F^F and C^F G^F will be A″ O. (It is generally better to lay off these distances from the apex instead of the base.) The cut surface F G H K should be found in the same manner as the surface E′ B′ C′, observing that the lines F^T G^T and H^T K^T may be measured on the top view as these lines are horizontal. A copy of this surface should be drawn in connection with the developed surface as shown in Fig. 4.

PROBLEMS.

Plate 17.

In performing these problems, pursue the order specified in the instructions. Three views and the complete development will be required in each case. In general, begin to develop the surface on the shortest edge. It will assist the student to a better understanding of this subject if the developed surface be copied on to stiff paper and afterwards cut out and folded. If this be done, a small lap should be left on some of the edges, to fasten them together. Shade lines must not be used on the development.

Problem 18.— This is a prism having a square base. The vertical edges being parallel to the front plane, are seen in their true length on the front view. The length of the edges of the base can be obtained from the top view, and the true shape of the upper base will be found by projecting it on to an auxiliary plane, as shown in Plate 16.

Problem 19.— This differs from the preceding only in being an hexagonal prism.

Problem 20.— In this and the following problems, only those lines should be inked which lie below the cutting plane, the upper portion being supposed to be removed.

Problem 21.— This is a pyramid having a square base the long diameter of which is given.

Problem 22.— The cutting plane makes an angle of 50° in this case, and the auxiliary plane must be at the same angle, the projecting lines being drawn perpendicular to it.

Problem 23.— The surface cut by the plane will not be symmetrical with respect to the centre line, as in the previous problems. Care must, therefore, be used to take no dimensions from the top view that may not be represented by horizontal lines.

Problem 24.— This is similar to Problem 23, but one of the inclined edges will not appear in the finished development, as it is entirely removed by the cutting plane. Begin to develop the surface on this line.

Problem 25.— In the preceding problems, the inclined edges of the pyramids have been of the same length, and having determined the revolved position of one of them, it sufficed for the others. The form of this object makes it necessary to obtain them separately. One of these edges, A D, is shown in its revolved position at A D′. One-half of the development is also shown, and the method and order for drawing the lines indicated by the numbers. Thus, the line A B is drawn first, and then arcs 2 and 3 described from its extremities, A and B, with radii equal to the true lengths of A C and B C, which determines the point C. In like manner the remaining points and lines are found.

DEVELOPMENT OF SURFACES OF REVOLUTION.

A surface may be conceived to be generated by the motion of a line. Such a line is called the GENERATRIX, the path described by one of its points is called the DIRECTRIX, and the different positions of the GENERATRIX are called ELEMENTS.

Suppose a right line to have a motion about another right line, known as the AXIS, from which it is always equidistant, and to which it is parallel, then will the successive positions of this generating line constitute the surface of a cylinder. If one end of the generatrix were fixed to the axis, and the other free to describe a circular path, the surface generated would be that of a cone. These surfaces are called surfaces of revolution, and may be regarded as consisting of an infinite number of lines, or elements, which in the first case are parallel to, and in the second case intersect, the axis. This conception of the cylinder and cone is necessary to the study of the development and intersection of surfaces. (See page 26.)

We may also imagine the cylinder as generated by the motion of a circle whose directrix is a right line.

THE CYLINDER.

PLATE 18.

Three views of a cylinder are represented by Fig. 1, and from these it is required to develop the cylinder. Assume a number of elements, and, for convenience, they should be equidistant. These may be employed to obtain other views, sections, and development, in precisely the same manner as though they were the edges of a prism, save that instead of connecting their extremities by right lines, a curve must be drawn through them. It will be observed that in this problem the revolved section is an ellipse, the major axis of which is equal to A^F C^F, and minor axis equal to the diameter of the cylinder. From this data the curve might be drawn, but it is better to use this method as a test for the ellipse after having obtained the curve by means of the elements. To develop the cylinder [Fig. 2], first obtain the circumference of the base by computing the same from the diameter, and having laid it off, as would be done in the case of a prism, divide it into as many parts as there are elements. The perpendiculars to the base drawn through these points will be the required elements, and their lengths may be obtained directly from the front view, since they are parallel to the front plane. A free-hand curve should be carefully pencilled through these points, and afterwards neatly inked by the aid of compasses and scrolls.

THE CONE.

PLATE 18.

Fig. 3 illustrates a cone cut by vertical and oblique planes. Draw the requisite number of elements by dividing the circle of the base in the top view, and projecting these points into the front view, as at B^F, E^F, G^F, H^F, etc. These are then connected with the vertex A^F. It is now possible to obtain the top view of the section made by the cutting plane X Y, in the same manner as though the cone were a pyramid having these elements for its edges. A more simple and accurate method is as follows: Through the front view of any point N^F, lying on the surface of the cone, and also in the cutting plane, draw a horizontal line which will represent the front view of a circle of the cone, and shown on the top view by the fine dotted circle drawn through N^T. Since the assumed point must lie on this circle, as well as on its vertical projecting line, it must lie at their intersection N^T. Of course it is necessary to draw only small arcs to intersect the projecting line. In like manner any number of points may be obtained, and through them, the curve described. If the elements of the cone have already been drawn, as would be necessary if it is to be developed, the points assumed had better be at the intersection of the cutting plane and the elements. This curve, being an ellipse, may be obtained by finding the major and minor axes, and on these constructing the curve; but as in the case of the cylinder, this method should be used only as a test of the ellipse, until the problem is thoroughly understood. The side view illustrates the true section made by the vertical cutting plane, since it is parallel to the side plane of projection. The top view of this section is a straight line, but it is just as necessary to find the points by projection, as in the case of the ellipse, since their distances on either side of the axis must be known in order to obtain the side view. Thus the top view of point O must lie in the vertical projecting line drawn through O^F, and also in a circle, the diameter of which is equal to V M, therefore at O^T. From this, the side view of the point may be obtained, its distance, Q^S O^S, from the axis being equal to Q^T O^T.

Proceed with the development, as in the case of the pyramid, observing that because all the elements are equal in length, the development of the base will be a circular arc of length equal to the circumference of base, and radius equal to the length of an element. Having divided this arc into as many parts as there are elements, proceed to draw them, after which the true length of the cut portion may be found as follows: Having drawn any element, as A G, Fig. 4, it is required to find the points P and R. Since the line A G, upon which they lie, is not seen in its true length in any of the views, it must be revolved parallel to one of the planes. A^F B^F will represent its position when revolved parallel to the front plane. The point P^F will be seen at P′, and R^F at R′, and the lengths A^F P′, and P′ R′, may be laid off on the line A G of the developed surface. Similarly other points are found. In making a model, as was advised in the case of the prism and pyramid, it will of course be necessary to add the revolved sections, and the base, in order to complete the development.

PROBLEMS.

PLATE 19.

PROBLEM 26.—The development of a cylinder is required. Leave all elements in pencil, and use great care in the construction and inking of the curves.

PROBLEM 27.—The development of the cone will require even more care than that of the cylinder. Use the second and more simple method for determining the top view of points made by the cutting plane. Remember to revolve the elements before measuring.

PROBLEM 28.—This problem is introduced as a review of conic sections [page 43], and as a test of the student's ability to perform accurate work. Four cutting planes, C G, C B, C F, and C E, are shown on the front view, and the curves produced by them are to be drawn on the top view, they being projected without the use of elements of the cone. The revolved section of each of the curves is next to be found, and finally, they are to be tested by the following methods: The curve of the ellipse having been obtained, find its axes and foci, and eight points by the first method [page 44]. Test the curve by trammels also. The parabola and hyperbola are to be tested by the second methods only. In testing the hyperbola, it should be noted that the vertex of the conjugate hyperbola is as much above the vertex of the cone as the vertex of the given hyperbola is below it. Observe that the cutting plane for the parabola, C F, is drawn parallel to an element of the cone.

VI.

THE INTERSECTION OF SURFACES.

PLATE 20.

THREE views of two intersecting cylinders are shown in Fig. 1, and it is required to find their curve of intersection, and develop the cylinders. On the side view, assume an element of the small cylinder, as A^S B^S, and project this into the front and top views. A^T B^T is seen to pierce the large cylinder at the point C^T B^T, which is the top view of an element of this cylinder. Projecting this element, we shall obtain the front view of two intersecting elements, C^F B^F and A^F B^F. This point of intersection will therefore be common to both cylinders, and hence a point in the required curve of intersection. In like manner obtain a sufficient number of points to determine the curve. For convenience in the development of the small cylinder, it is desirable to have its elements equidistant.

In all problems relating to the development of surfaces, it is of first importance to determine those points of the curve, known as limiting points, at which the direction of curvature changes, or points of tangency occur. These define the general character of the curve, and often enable it to be drawn by the finding of a less number of points. 1^F, 2^F, 3^F, 4^F, 5^F, and 6^F are limiting points of this curve.

In developing the small cylinder, Fig. 2, open it on the element L 4, which will make it symmetrical with respect to the centre. The method of developing does not differ from Problem 26, and the length of the elements may be taken from the front or top views.

The development of the large cylinder, Fig. 3, will be a rectangle pierced by a hole which is symmetrical with respect to a horizontal centre line only. Having obtained the development of the cylinder, by opening it on G H, the element D E will be drawn in the centre of the surface, and from this the other elements obtained. On D E lay off the points 3 and 5, equidistant from the centre line, and equal to that portion of the element cut by the small cylinder, as shown on the front view. The distance between any of these elements, as D E and C N, may be found by measuring the circular arc D^T 2^T C^T, which should then be laid off to the left of E D, and through this point, C N may be drawn. Having the position of an element, immediately lay off the amount cut out by the second cylinder, as seen on the front view.

PROBLEMS.

Plate 19.

Problem 29.—This differs from the illustration in Plate 19 only in that the axes of the cylinders intersect.

Problem 30.— Obtain the limiting points first. Use great care in determining the curve, as the accuracy of the development is entirely dependent on it.

Problem 31, plate 21.— The axes of these cylinders intersect, but are not at right-angles as in the preceding case. The side view not being available for spacing the elements, the following method may be pursued: Draw any element of the small cylinder A^F B^F, and let X Y be a circle of the cylinder made by a cutting plane perpendicular to the axis. If this circle be revolved about X Y, parallel to the front plane, the point C^F, which is a point of both the element and the circle, will be revolved to C', and C^F C' will be the distance of the element from the vertical plane passing through the axis. This will determine the distance between the centre line, and the element in the top view, the latter piercing an element of the large cylinder at A^T. Since there is a second element, $E^F D^F$, of the small cylinder, lying in the same vertical plane as the first element, it will intersect the same element of the large cylinder at the point D^F. Thus find any number of points. As in the preceding cases, it will be convenient to have the elements of the small cylinder spaced equally, and this may be done by spacing them on the revolved position of the circle X Y, and through these points drawing the elements. In developing the small cylinder, it will be found necessary to assume some line of the surface which on being developed will be a straight line, for in this problem, the development of the cylinder ends will be curves. Such a line will lie in a plane perpendicular to the axis, and the section made by the plane is called a right section. The line X Y fulfils this condition, and may be used as a base line for obtaining the length of the elements in the development, and on it the distances between the elements may be measured.

USE OF AUXILIARY PLANES.

PLATE 22.

In determining the curve of intersection between two surfaces, it is customary to use a system of planes which shall cut either circles or straight lines from these surfaces. The intersection of these lines will be points of the curve. Let it be required to find the intersection of cylinders 1 and 2, Fig. 1. If we imagine them cut by a plane V W, parallel to the axes, the appearance will be as in Fig. 3. Two elements will have been cut from each cylinder, and since they lie in the same plane, their points of intersection will be common to both cylinders, and therefore in the required curve. Again, if we should employ a plane tangent to cylinder 2, it would cut that cylinder in a line, and the other, in two elements, as in Fig. 2. These last points, 7 and 8, would be the limiting points of the curve, and, ordinarily, the first to be determined.

To obtain the elements cut by these planes, proceed as follows: Revolve the bases of the cylinders, as in Problem 16, and having assumed a cutting plane, as V W, shown on the top view, lay off on the revolved bases, L M and N O, equal to the distance of the cutting plane from the plane of the axes. Through the points M and O, draw parallels to the bases, and these will indicate the amount cut from each cylinder. Next, revolve the points B, D, G, and K, back into the bases, and through them draw the elements. Their intersection at 3, 4, 5, and 6 will be the four points determined by the plane V W. In like manner the points 7 and 8 may be found by using a plane tangent to the small cylinder. The points in the top view are obtained by projecting them from the front view on to the plane in which they lie. Thus, the point 7 was found by means of the cutting plane X Y, and its top view 7^T must therefore lie on that line.

PROBLEMS.

Plate 21.

Problem 32.—Having obtained front and top views, proceed with the construction of the curve of intersection by first using a tangent plane to define the limits of the curves. In developing the large cylinder, which alone is required, open it on an element not cut by the second cylinder.

Problem 33.—It is required to find the intersection of a cylinder and prism without using a side view. Use cutting planes parallel with the vertical face of the prism. The base of the prism in the front view will have to be revolved in order to complete the top view and enable the intersection of the cutting planes and prism to be determined. Both surfaces are to be developed. A thorough understanding of previous examples should enable this and the following problems to be performed with little instruction.

Problem 34.— In all cases of intersection between prism and prism it is only necessary to find the point of intersection of each edge of both prisms with a face of the other prism. To avoid confusion, these should be taken consecutively as follows: Edge A B of the triangular prism with face A C of the hexagonal; edges C and D of the hexagonal prism with face E F B A of the triangular; then E F with face D G, etc. Much care will be required in developing these surfaces.

Problem 35.— The lines of intersection on the top view are to be completed, and the front view, with its lines of intersection, are required. Develop the prism only.

Problem 36.— The intersecting prisms being at an angle other than 90° makes this problem more difficult than the preceding. To obtain the point C, it will be necessary to first find the top view of the line of intersection of a plane coinciding with the upper face of the prism A, with the face B, of the second prism. The intersection of this line with the upper edge D C, will determine the required point. Develop the prism A.

VII.

SCREW-THREADS AND BOLT-HEADS.

SPIRALS.

PLATE 23.

The Spiral is a curve generated by a point moving in a plane about a centre, from which its distance is continually increasing.

Imagine a right line, A B, Fig. 1, free to revolve in the plane of the paper about one of its extremities, A, as an axis. Also conceive a point as free to move on this line. Three classes of lines may now be derived in the following manner: If the line be stationary and the point move, a straight line will be generated; if the point be stationary and the line revolve, a circle will be generated; if both the point and line move, a curve known as a SPIRAL will be generated. The character of this curve varies with the relative motions of point and line. If, during one uniform revolution of the line, the motion of the point be uniform, the SPIRAL OF ARCHIMEDES, or EQUABLE SPIRAL, will be generated. The line A B, Fig. 1, is the Radius Vector, and the radial distance traversed by the point during one revolution is called the PITCH. By varying the relative motions of the line and point, all classes of spirals may be generated.

Fig. 1 illustrates the equable spiral, and the method for drawing the same. Twelve successive positions of the radius vector being shown by A C, A D, A E, etc., the distance of the point from the centre will be increased by one-twelfth of the pitch during each twelfth of a revolution of the radius vector. Having found a sufficient number of points, lightly sketch the curve freehand.

INVOLUTES. — These curves are a class of spirals which may be generated by the unwinding of a perfectly flexible, but inextensible cord, from a polygon of any number of sides, the names of the involutes being derived from the polygons which determine their form. The curves consist of circular arcs, having for centres the vertices of the polygons, and radii increasing by an amount equal to the length of the sides. Fig. 2 is the involute of a square, drawn by describing the quadrant 4-5 from centre 1, with radius 1-4, then the quadrant 5-6 from centre 2, with radius 5-2, and so on, until the desired length of curve shall have been drawn. If two opposite sides of the square become infinitely small, the result will be a right line, the involute of which is shown in Fig. 3. Again, if the number of sides of the polygon become infinite, we shall obtain the involute of a circle.

THE HELIX.

PLATE 23.

If the line on which the generating point is supposed to move, be made to revolve about an axis with which it makes an angle of less than 90°, thus generating a cylinder or cone, a class of curves known as HELICES will be described. If the line A B, Fig. 4, be parallel to the axis about which it revolves, and the generating point move on this line, three classes of lines may be described, as in the case of spirals. A circle will be generated when the point is stationary and the line revolves; a right line is generated when the line is stationary and the point moves on the line; and since the circle and right line do not lie in the same plane, the result will be a HELIX when these motions take place simultaneously. The distance traversed by the generating point on the line A B, during one revolution, is called the PITCH. The motion of line and point being, in general, uniform, the curve is described as follows: Having assumed any desired number of equidistant positions of the generating element, A B, as 1, 2, 3, 4, etc., the pitch should be divided into the same number of parts, as shown by the horizontal lines drawn through 1′, 2′, 3′, 4′, etc. When the element shall have moved through one of its divisions to the position 1, which in this case is one-twelfth of a revolution, the generating point will have moved on the generating line through one-twelfth of the pitch, and the point K will be determined. Also when the element has made one-quarter of a revolution, the point will have traversed one-quarter of the pitch, and be at C. As the rate of curvature is most rapid at the points of tangency, A, D, and B, it is desirable to obtain a greater number of points by subdivision, as shown in the figure.

Fig. 5 illustrates the development of that portion of the cylinder lying below the helix, in which it will be seen that the development of the curve is a right line. If a triangle be constructed, having for its base the circumference of the cylinder, and for its perpendicular the pitch of the helix, the hypothenuse will be the development of the helix.

PROBLEMS.

Plate 24.

Problem 37. — In drawing the equable spiral, describe more than one revolution of the radius vector, and find at least twenty-four points in each revolution. Sketch the curve in pencil, and ink all save the outer portion by means of compasses.

The involutes, being composed of circular arcs, will be described entirely by means of compasses.

Problem 38. — In the first example observe that the pitch of the helix is but 2 inches, while the length of the cylinder is $2\frac{1}{2}$.

In Fig. 4, Plate 23, the helix is right-handed and single, but if it were required to be double, it would only be necessary to draw a parallel curve beginning at the point 6′, opposite D.

If the generating element shall describe a cone instead of a cylinder, the curve generated will be a conical helix, the top view of which is an equable spiral. The pitch is measured parallel to the axis, as in preceding cases.

In inking helices, always use the bow pen for a small part of the curve at the points of tangency, and since the curve is symmetrical on either side of the axis, be particular to use the same part of the scroll for the inking of each side, reversing the scroll in each case.

V AND SQUARE SCREW-THREADS.

PLATES 25 and 26.

In order to make the drawing of a screw-thread, it is necessary to know: The character of the thread, whether V (as in Plate 25), or square (as in Plate 26); whether single or double; the diameter, and the pitch. If the angle of the V thread is other than 60°, that also must be known.

If the thread to be drawn has a V section, as in Fig. 1, Plate 25, and the diameter and pitch are given, begin by drawing the section, as shown in dotted lines, throughout the length of the screw. Next construct a templet as follows: Draw on a rather light piece of card-board, two helices of the same pitch as the required thread, one being of a diameter equal to the outside of the thread, and the other equal to the diameter at the root of the thread. These may be drawn separately, or together, as in Fig. 2, and are to be carefully cut out so as to be used as a pattern in pencilling the curves. Having drawn the helices, mark the points of tangency B, G, and A, F, as well as the centres L and K, in the manner indicated. Also repeat these marks on the opposite side after cutting out. This will enable the templet to be used either side up, and be readily set to the drawing. Observe that the curve is not to be cut off abruptly at its termination, but continued a little beyond, so that in tracing the outline, the pencil point may not injure the extreme point of the templet curve. This may now be used for the drawing of all the helices on this screw, as from A to F, B to G, C to H, etc., Fig. 1. If the pitch were small in proportion to the diameter, the drawing of the screw might now be considered finished; but in order to correctly illustrate the projection of a V thread, we must consider the character of the surface, and apply a correction to the drawing. Therefore, imagine the line A B to revolve about the axis of the screw, and at the same time move in the direction of the axis so as to generate the upper half of the surface of the screw. Every point of the line will describe a helix, the diameters differing, but the pitch remaining constant. The helices generated by the extremities of the line have already been drawn, and the curves 1 2 3 and 4 5 6, described by two other points, 1 and 4, are shown by the fine dotted lines. The curved line M 5 2 N, drawn tangent to these helices, will be the visible outline of the surface, instead of the dotted line, which is concealed. As the labor of describing these helices would be great, it is customary to draw the outlines of the screw as follows: Having described the helices A F, B G, etc., reverse the templet and draw a small portion of the continuation of the helix on the opposite side of the screw, as at B P and C O. Then tangent to the two helices draw the line M N, and in the other direction the tangent O P, a portion of which is invisible. It will be observed that more than half the outer, and less than half the inner helix, is visible in each case.

Fig. 3 represents the nut for this thread, but as this is shown in section, the helices of the opposite side alone are visible. The outer helices are concealed at their extremities.

Plate 26 represents a single, and double, square thread and nut. These are described, as in the preceding case, by the making of a templet and afterwards using it to trace the curves, the section of the thread being first completed. The square section of the thread is equal to one-half the pitch in the single, and one-quarter in the double thread. The character of the latter will be more fully explained in Plate 27.

PROBLEM 39, Plate 24. — Having drawn the section of the screw and nut, prepare the templet, and since the diameter and pitch are the same in both examples, it may be used for the drawing of all the helices.

In the inking of the curves follow the directions given for Problem 38.

CONVENTIONAL THREADS.

PLATE 27.

In order to facilitate the drawing of screw-threads, it is customary to omit the drawing of the helix, substituting therefor a right line, as in Fig. 1. In most cases, however, even this would involve too much labor, as well as complication on the drawing, and the V's are likewise omitted, the representation shown in Fig. 6 being adopted. When this is done, no pains are taken to make the screw of the required pitch, and the spaces between the fine lines, which represent the outer helices, are estimated by the eye, as in section lining. But it is imperative, for the proper representation of a single thread, that the point C be over the middle of the space B D. Having drawn the line A B, make the space A C double that of E B, and draw parallels. Afterwards, draw the heavy lines to represent the root of the thread. As it is difficult to draw these of equal length without the aid of special lines, draughtsmen have come to adopt the method illustrated Fig. 7, which is the more practical representation, and the one usually employed.

THE DOUBLE THREAD. — As the use and character of the double thread is generally misunderstood, Figures 1, 2, and 3 have been drawn to more clearly explain this problem. Fig. 1 illustrates a screw, the diameter and pitch of which are supposed to have been given ; but as the pitch is excessive, for a screw of this diameter, the diameter at the root of the thread is small, and the screw proportionally weak. The only way to strengthen the thread at this point, without changing the angle of the V's, is by making a screw with the V's partly filled at the root, as shown by the dotted line in Fig. 1, and the complete screw in Fig. 2, thus increasing the diameter at the root. While overcoming one weakness we have introduced a second, by lessening the section of the thread, so that with a nut of a given length, the tendency of the thread to be stripped from the body is doubled. This last difficulty may be overcome by supposing an intermediate thread wound between the present threads, as shown in Fig. 2 by the dotted lines, and also by the completed drawing of Fig. 3, which is a representation of a double thread having the same diameter and pitch as the single thread of Fig. 1, but of increased strength. It must be noted that the full and dotted threads of Fig. 2 are entirely independent of each other, and that the point C of one, is diametrically opposite a point B in the parallel thread. This must be carefully observed in the practical representation of a double thread, as shown in Fig. 8.

Fig. 9 represents a left-hand single thread.

The conventional square thread is illustrated in Fig. 5, and this may also be simplified by omitting the lines representing the root of the thread.

A section of a U. S. Standard V Thread is shown in Fig. 4, and the proportions for the same are as follows: —

$D =$ Diameter of bolt.

$P =$ Pitch of thread $= 0.24 \sqrt{D + 0.625} - 0.175$.

$N =$ Number of threads per inch $= \frac{1}{P}$.

$S =$ Depth of thread $= 0.65\ P$. The angle of V's is always 60°.

Problem 40, Plate 24. The first eight examples are to be drawn by the conventional methods illustrated in Figs. 1, 3, and 5, of Plate 27, care being used in observing all the conditions prescribed. The last four examples are to be drawn by the methods illustrated in Figs. 6, 7, 8, and 9, of the same Plate. Make the pitch for single threads about equal to that shown in Fig. 1, Plate 29, estimating the spaces by the eye.

BOLTS.

SPHERE AND CUTTING PLANES.

PLATE 28.

If a sphere be cut by six planes equidistant from, and parallel to, a vertical axis, a representation will be obtained similar to that of a hexagonal bolt-head or nut, of which a careful study should be made.

Fig. 1 represents three views of a sphere cut by planes V W, X Y, etc., as prescribed above. Any cutting plane will intersect the sphere in a circle, and if the cutting plane is parallel to the plane of projection, this intersection will appear as a circle, but otherwise as an ellipse or straight line.

The plane VW intersects the sphere in a circle the diameter of which is E^T F^T, and shown on the front plane by E^F A^F B^F F^F. The plane X Y also intersects the sphere in a circle of equal diameter, being at the same distance from the axis; but this plane being inclined to the front plane, the circle will appear as an ellipse, having a major axis G^F H^F, equal to the diameter of the circle, and the minor axis K^F L^F, equal to the foreshortened diameter projected from the top view. The planes V W and X Y intersect in the line B^F O^F, thus cutting off a portion of the circle and ellipse. Similarly, the remaining curve of intersection, D^F A^F, may be found, and the side view also obtained.

Points in the curve may also be found by revolving the plane XY parallel to the top plane, thus obtaining the height of any point, as N, which may then be projected into the other views.

Fig. 2 is a similar example, in which four cutting planes are used, thus making it similar to a square-headed bolt or nut.

It is desirable that the details of these figures be carefully worked out by the student, treating this plate as a problem.

HEXAGONAL HEADS AND NUTS.

Plate 29.

Since we may regard the upper half of front and side views of Fig. 1, Plate 28, as a true representation of a hexagonal bolt-head, save that the proportion is not good, it is important to consider the salient points of this representation that they may be applied to the drawing of bolt-heads and nuts.

First: Three faces of the head are seen when it is shown "across corners," as in the front view, and one of these is double the width of the other two.

Second: The circular arc A^F M^F B^F is concentric with the circle of the sphere, and the points A^F and B^F are determined from the height of C^F, a point of the intersection of the two planes, or faces of the head, and the great circle of the sphere.

Third: The major axes of the ellipses and the diameters of the circles, made by the cutting planes, are equal, hence points G^F and M^F are at the same height.

Fourth: Two equal faces are seen when the head is shown "across flats," as in the side view.

Fifth: The points M^S and G^S are of equal height, as in the front view, and B^S must be obtained by projection from the front view, or in the same manner as C^F.

The proportion for a bolt-head or nut is shown in Fig. 1, Plate 29, which is an illustration of a U. S. Standard bolt. Fig. 5 is also a standard bolt, but with the ends chamfered instead of rounded. Either of these types may be used, the former being employed to represent a finished head only, and the latter for rough and finished heads.

Having considered both the representation and proportion of the head, we may proceed with the practical drawing of bolt-heads and nuts. Suppose it is required to draw a rounded head "across corners," as shown in Fig. 2, Plate 29. Having drawn the centre line, underside of head, and diameter, as indicated by lines 1, 2, 3, and 4, figure the short diameter of the hexagon, or distance "across flats." According to the proportion given, this is equal to the diameter of bolt, plus one-half the diameter, plus one-eighth of an inch. Lay off E F equal to one-half this amount, and draw the perpendicular F G and the 30° line E G, then will the triangle E F G represent one-twelfth of the top view of the head, and E G will be equal to half the long diameter required. Lay off this distance on either side of E, thus determining lines 8 and 9. Draw 10 and 11, remembering that these lines equally divide the spaces between 1 and 8, and 1 and 9, which spaces also equal twice F G. Next determine the thickness of head, and with a radius [1] equal to twice the diameter of the bolt, describe arc 12, which determines points D, C, A, and B. From the same centre describe arc 14. Arcs 15 and 16 should be drawn as circular arcs, their height being determined from 14.

If it is required to represent a bolt-head "across flats," as in Fig. 3, proceed as before, determining the short diameter, and drawing 5 and 6. Next figure the thickness of head and describe arc 7; this will determine E, and the height of arcs 12 and 13. Although these arcs are elliptical in theory, they should always be described as circular. To determine the point A, we must find the long diameter, as in the previous case, and so obtain line 10, the intersection of which with 7, will be at D, equal in height to A, B, and C. Finally draw arcs 12 and 13.

[1] There is no standard for this radius, but 2 D is recommended as being a convenient radius for draughtsmen.

Fig. 4 illustrates a rounded nut "across corners," the order for the drawing of the lines being indicated by the figures. It must be observed, however, that the thickness of a nut differs from that of the head, being equal to the diameter of the bolt; also that since the nut is pierced by a hole, the top will appear flat, and the arc 13 must, therefore, be struck from K, and with radius equal to 2 D.

If we substitute a cone for the sphere, in Plate 28, cutting it by the six vertical planes parallel to the axis, and then pass a seventh plane perpendicular to the axis, and tangent to the curves of intersection, a representation will be obtained similar to that of the head and nut in Fig. 5, Plate 29. This is called a chamfered bolt, and the curve of intersection is an hyperbola; but since these curves approximate circular arcs, the following concise method may be employed. If it is required to draw a chamfered head or nut across corners, construct a hexagonal prism of dimensions required for a standard bolt. From the centre line, with radius equal to diameter of bolt, describe the arc A B, tangent to end of nut, and from the same centre draw arcs D E, and C F, points D and C being determined by A and B. Finally draw arcs D A and B C, also tangent to E F. The method for drawing the chamfered head or nut "across flats" is apparent from the illustration of the bolt-head in Fig. 5.

Problem 41, Plate 29. — The diameters are given, and the sketch shows the character of the bolt, whether rounded or chamfered. Use care in observing every detail, and see that the dimensions are standard. It is better to draw first the rounded heads and nut, as shown at the left of the problem.

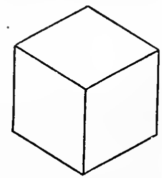
For the further consideration of this subject the student is referred to the chapter on Bolts and Screws, in Machine Drawing of this series.

VIII.

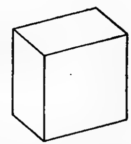
ISOMETRIC AND OBLIQUE PROJECTION.

It is frequently necessary to produce a pictorial effect in mechanical drawings, while also preserving the relative proportion of parts so that the drawings may be made to scale, and measurements readily taken therefrom.

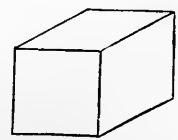
Three methods of accomplishing this are as follows:—



First: by revolving the object so that three of its faces, which are at right angles to each other, shall be equally inclined to a plane upon which it is orthographically projected. This is called the *Isometric Projection* of the object.



Second: by revolving the object into such a position, relative to the plane of projection, that two of its visible faces shall be foreshortened double that of the other one, and projecting as before. This is a modification of the preceding.



Third: by projecting the object by lines oblique to the plane of projection, one face of the object being parallel to the plane. This system is known as *Oblique Projection.*

It will be observed that in each case three faces of the object are visible, and the drawing is so made that measurements may be taken from either. One view is thus made to serve for two or more, and a perspective effect is also obtained. These methods are well adapted to the illustrating of much architectural work, the general views of Patent-Office drawings, the sketching of machinery, etc.

ISOMETRIC PROJECTION.

Plates 30, 31, and 32.

As a cube best illustrates the principles of isometric projection, conceive one as having been revolved about a vertical axis until the diagonals of its base are parallel to the vertical projecting planes. Fig. 1 represents the side view of the cube when so revolved. Next, incline it toward the front plane, as in Fig. 2, until the diagonal E C is horizontal, and draw the front view of the same. This view is shown by Fig. 3, which illustrates the object with all of its edges equally foreshortened, and capable of being measured by a special scale. Furthermore, it will be seen that all of the edges of the cube, except the vertical, make angles of 30° with the horizontal.

If a scale of equal parts were constructed for the measurement of Fig. 3, its units would be .81647 of an inch. The inconvenience due to the use of such a scale has caused the adoption of the full scale for all isometric representations. Fig. 6 illustrates the same cube drawn to the full scale, and this is distinguished from the preceding representation by being called the isometric *drawing*, instead of the isometric *projection*, of the object. It will be seen that this representation is larger than the object; but, it is more easily drawn, and serves equally well for an illustration.

The lines C D, C B, and C G are called *isometric axes;* and lines parallel to them are known as *isometric lines.* It is evident that only isometric lines may be measured, since they alone are equally foreshortened. Thus, the isometric of the diagonals of the square, A C and D B, are of unequal length although in the original we know them to be equal. Likewise, it is not possible to directly measure the angle between lines on an isometric drawing.

To make the isometric drawing of a cube, Fig. 6.—From the point C draw lines C B and C D at angles of 30°, and equal to the required length of the edges. Draw the vertical C G, of same length. As each edge of the cube is parallel to one or the other of these isometric lines, the drawing may be completed as shown. In shading an isometric drawing, it is customary to draw shade lines for the division between light and dark surfaces, the direction of the light being that of the diagonal D F. This makes the upper and left-hand vertical faces, light surfaces, all others being dark.

To inscribe a circle on the face of a cube, Figs. 5 *and* 6.—Let D′ C′ G′ H′, Fig. 5, be a view of the face on which the circle is inscribed. As the isometric drawing of every circle is an ellipse, it is only necessary to obtain the major and minor axes to enable the curve to be described by the method of trammels. These axes lie on the diagonals D G and H C, and their extremities may be determined as follows: The diagonals being non-isometric lines, the distance D K cannot be measured, and the point K must be determined by measurements parallel to the isometric axes, as P′ K′ and O′ K′, which equal P K and O K. Through K, an extremity of the major axis, draw K N parallel to D C, and its intersection with the second diagonal, at N, will determine one extremity on the minor axis. Having obtained M and N, as indicated, draw the ellipse by the method of trammels. As absolute accuracy is not always a requisite of isometric drawing, the labor of representing ellipses is often lessened by the following approximate method. Bisect the edges of the upper face, Fig. 6, and connect these points, S, T, R, V, with the vertices A and C. From their points of intersection, Y and Z, describe arcs R S and T V, and from centres C and A, describe arcs S T and R V.

To lay off any required angle, Figs. 6 *and* 7.—From C′, on C′ B′, Fig. 7, angles of 15°, 30°, 45°, 60°, and 75° have been drawn, and it is required to construct the isometric drawing of similar angles. As the points 1′, 2′, F′, etc., which determine the required angles, lie on isometric lines, it is only necessary to transfer them to the isometric drawing of the face, as at 1, 2, F, 3, etc., to enable the required line to be drawn. Points in the circular arc, as 5 and 6, may be obtained in the manner shown by the dotted lines.

To obtain the isometric drawing of an object similar to Fig. 4.—The rectangular plinth is drawn in like manner to the cube, A B, A D, and A C being isometric lines. This base is surmounted by the frustum of a pyramid, the upper base of which is a square. To determine E, one corner of this square, lay off F H equal to the perpendicular distance of E from face B A D, and G F equal to its distance from C A D; at their intersection erect the perpendicular F E equal to the required height of the frustum of the pyramid. Next construct the isometric of the square. A cylinder of height K L rests on this surface, and the circles of its bases are drawn by the approximate method. The cylinder is surmounted by a square abacus, the vertical faces of which are pyramids constructed as shown.

When a figure consists of lines which are mostly non-isometric, it may be referred to axes which are isometric, as in Fig. 9, Plate 31, which represents the isometric drawing of a regular pentagon. D E being chosen as an isometric line, or axis, a second axis Y Y is drawn isometrically perpendicular to it. Points C and F lie on this axis, and their position is readily determined. As line A B, drawn through C, is an isometric line, points A and B may be located at their proper distance to right and left of Y Y.

To make an isometric drawing of an oblique timber framed into a horizontal timber, Figs. 10 *and* 11.—Fig. 10 illustrates a side view of the timbers; and they being square, it supplies the necessary data for the drawing. After having drawn the lower piece, as in Fig. 11, the step for the oblique timber should be shown. The edges of this cut being non-isometric, they must be obtained by locating points C, D, and E, as in Fig. 11. Suppose the required pitch of the oblique timber to be two-thirds: that is, two vertical units, for every three horizontal units. From C,

Fig. 11, lay off on A B any three units, and erect a perpendicular equal to two of those units. The point found will determine the pitch of the oblique timber. Although H′ L′ is perpendicular to H′ C′, Fig. 10, it is not possible to make this measurement directly on the isometric drawing, as it is a non-isometric line; but if a perpendicular H′ K′ be drawn from H′, this distance may be laid off from H, Fig. 11, thereby determining a point K, on the lower side of the timber, through which a parallel to H C may be drawn. Finally, the point L may be obtained by laying off M N equal to M′ N′, and erecting a perpendicular to intersect lower edge of timber.

Plate 32 illustrates the application of the foregoing principles to the representation of architectural work. The figure on the right is the drawing from which the isometric is made; but the necessary dimensions for size of material, distance between joists, studding, etc., have been omitted in the cut. In representing incomplete work, as in this plate, it is necessary that the broken lines be isometric, as otherwise the parts will appear to lie in different planes. Observe that the relation between the different surfaces is suggested by the break at the extreme right of the isometric drawing, as well as by the isometric section.

Fig. 12, Plate 31, illustrates a cube drawn by a system which is a modification of isometric projection. The effect produced is more nearly that of a perspective drawing. While this representation involves a little more labor than the isometric drawing, it avoids much of the distortion which characterizes the latter. The system is particularly well adapted to illustrating groups of buildings where perspective cannot be used because of the increased difficulty of drawing, and the inability to measure the same. It is also suitable for Patent-Office work.

The theory upon which the representation is made, is as follows: A cube is revolved into such a position that two of the axes, or edges, as C G and C B, are equally inclined to the plane of projection, while the third axis, C D, is foreshortened one-half that of C B and C G. The angles which the edges make with a horizontal line are 7.2°, 41.4°, and 90°. This involves the use of two special triangles, one of which must have a right angle to enable the vertical lines to be drawn. Two scales are used: a full scale for dimensions parallel to C B and C G, and a half scale for those parallel to C D. As in isometric, all dimensions must be made parallel to one of the three axes, the ellipse of the front face, B C G F, may be described by obtaining the major and minor axes, as in the case of the isometric ellipse, and with centres on these axes, describe arcs tangent to the edges. The axes of the ellipse on face D C G H do not coincide with the diagonals; but a very close approximation to them may be obtained by drawing lines K L and M N, respectively perpendicular and parallel to C B. In unimportant work, the extremities of these axes may be estimated by the eye; but if accuracy is required, lines K L and M N must be drawn on a square of the given size, and their intersection with the circle found, as was done with the diagonals in isometric projection. Fig. 13 illustrates a chamfered bolt-head drawn by this method, and Fig. 14 represents the same by isometric projection.

OBLIQUE PROJECTION.

Plate 31.

In this system, one face of the object is drawn parallel to the plane of projection, and all edges perpendicular to this face are drawn as oblique lines. Only one scale is commonly used for the measurement of the different axes, as in isometric projection; but if it is desired to foreshorten faces perpendicular to the front face, a reduced scale may be employed. Fig. 15 illustrates a cube of equal size to that shown in Fig. 12, drawn in oblique projection. The axes of the ellipse on face B C G F may be obtained from the front face as shown by the dotted lines. The curve may be drawn by circular arcs tangent to the edges of the parallelogram. In this case the oblique lines are drawn at an angle of 30°; but 45°, or any other angle, might equally well have been used.

Fig. 16 illustrates the drawing of a cabinet, the oblique lines being drawn at an angle of 45°. As the top projects beyond the front face, it lessens the apparent height of that face. The point B on the upper surface of the top of the case is directly over A, the upper right-hand corner of the front face, and A B is the thickness of the top. B D and C D indicates the amount which the top overhangs the side and front, respectively. Oblique projection is well adapted to the representation of this class of work.

Plate 1.

PLATE 2.

PLATE 3.

PLATE 1

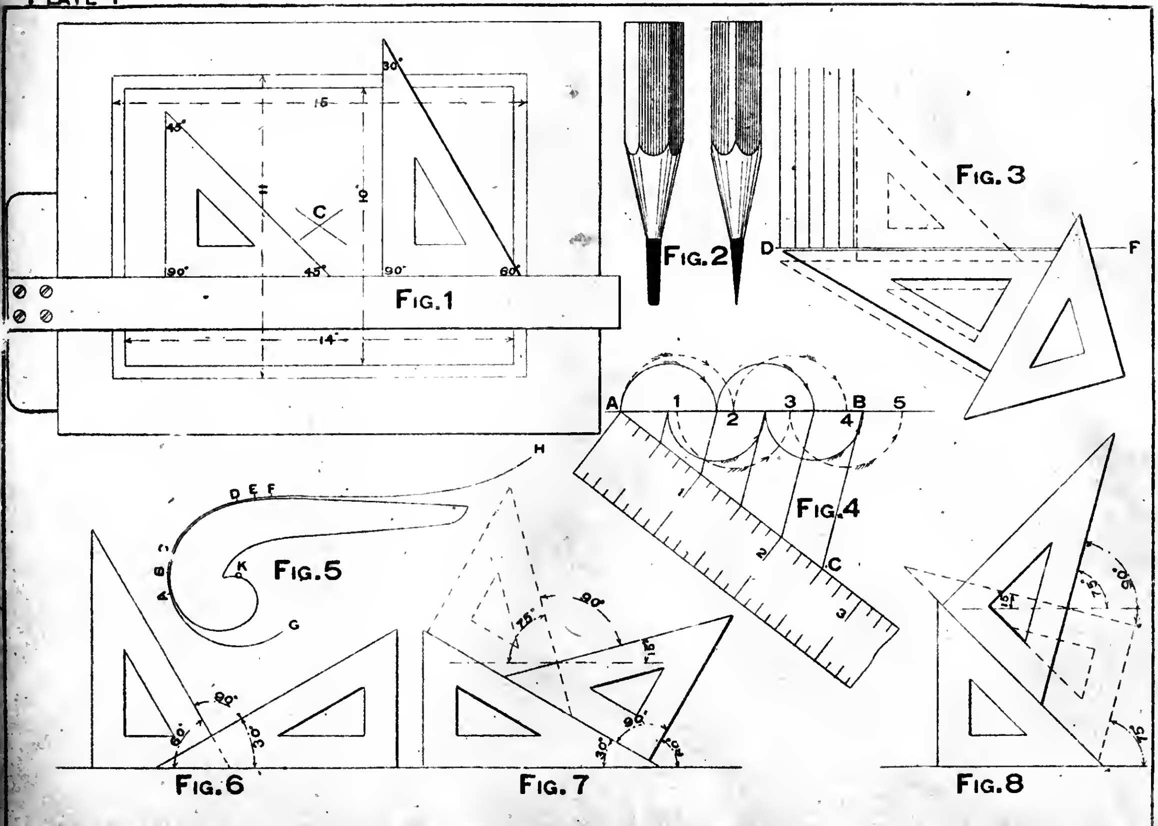


FIG. 1
15
14"
11
10"
C
30°
45°
45°
90°
90°
60°
FIG. 2
FIG. 3
D
F
FIG. 4
A
1
2
3
4
B
5
C
FIG. 5
D E F
H
A B C
K
G
FIG. 6
90°
60°
30°
FIG. 7
75°
90°
15°
90°
30°
60°
FIG. 8
15
75
90
75

PLATE 4

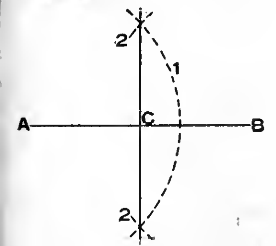


FIG. 25

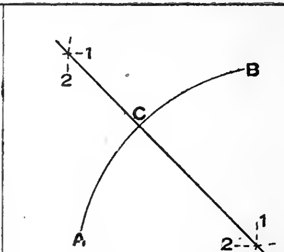


FIG. 26

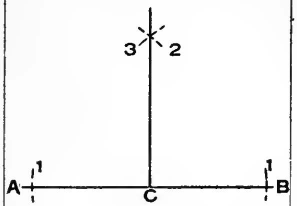


FIG. 27

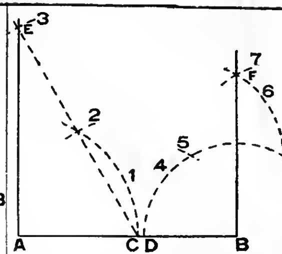


FIG. 28

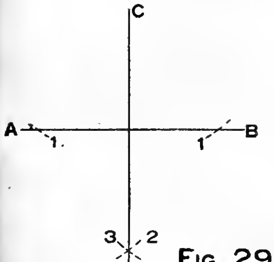


FIG. 29

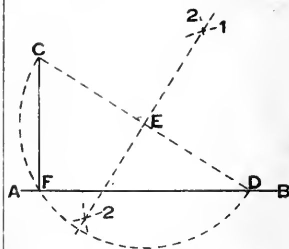


FIG. 30

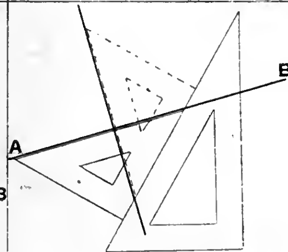


FIG. 31

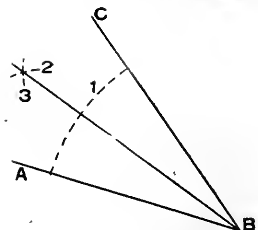


FIG. 32

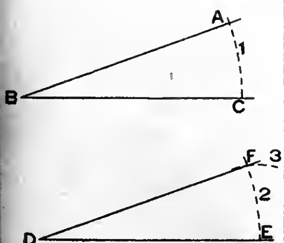


FIG. 33

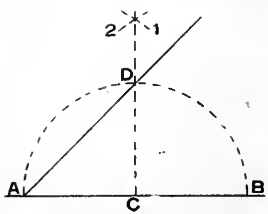


FIG. 34

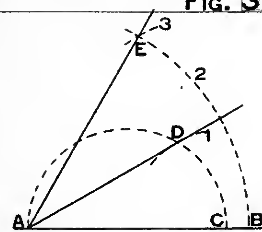


FIG. 35

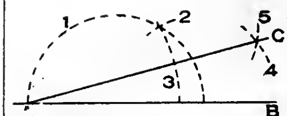


FIG. 36

PLATE 5

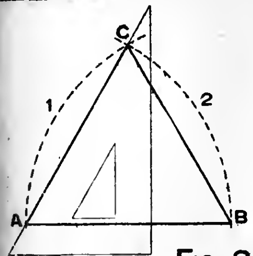


FIG. 37

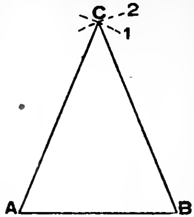


FIG. 38

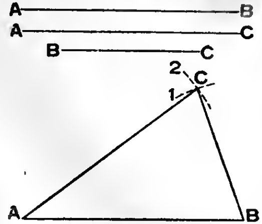


FIG. 39

FIG. 40

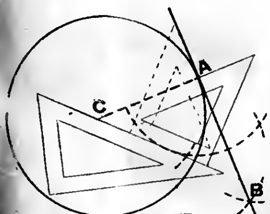


FIG. 41

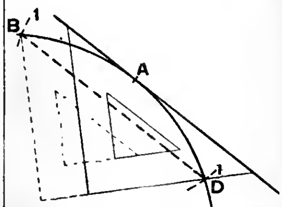


FIG. 42

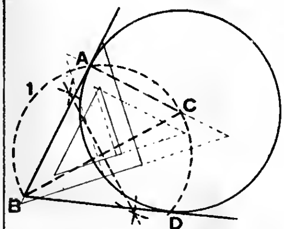


FIG. 43

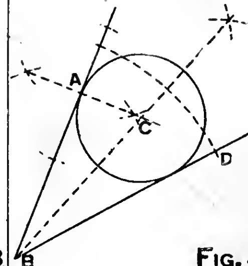


FIG. 44

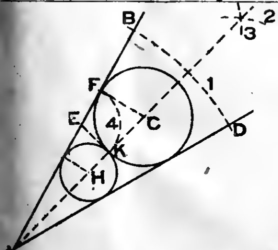


FIG. 45

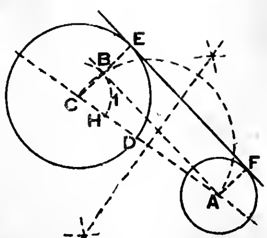


FIG. 46

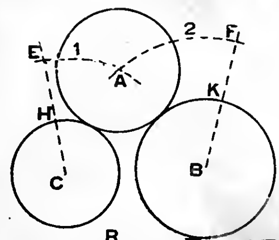


FIG. 47

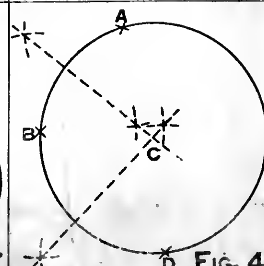


FIG. 48

PLATE 4.

PLATE 5.

PLATE 6

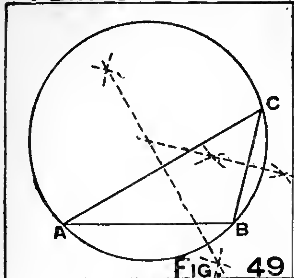


FIG. 49

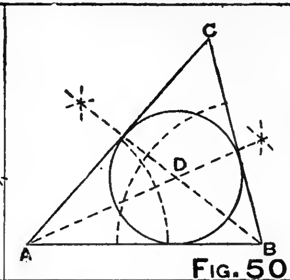


FIG. 50

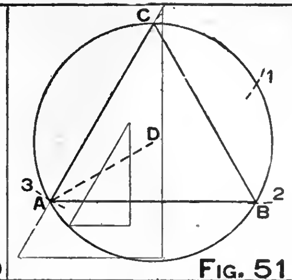


FIG. 51

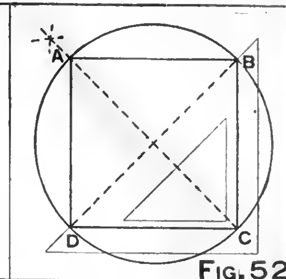


FIG. 52

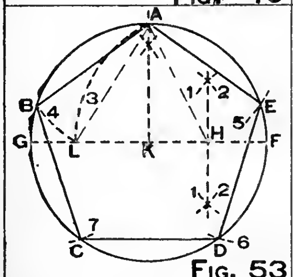


FIG. 53

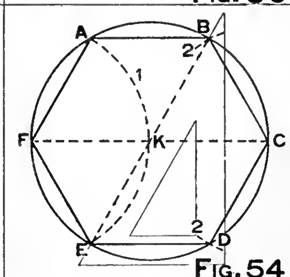


FIG. 54

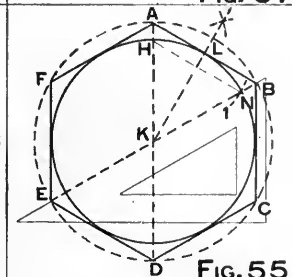


FIG. 55

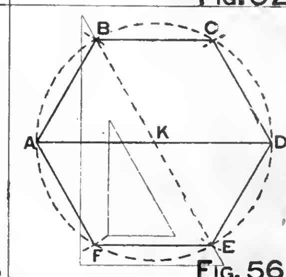


FIG. 56

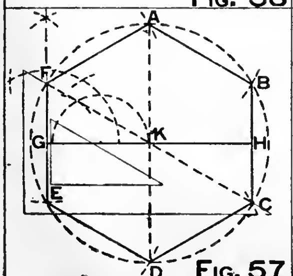


FIG. 57

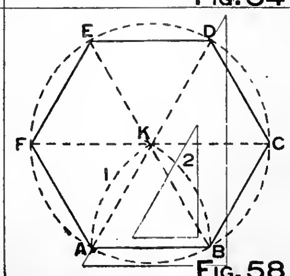


FIG. 58

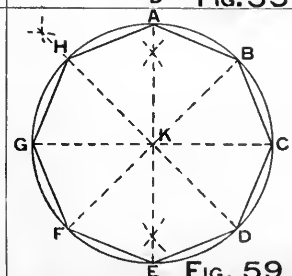


FIG. 59

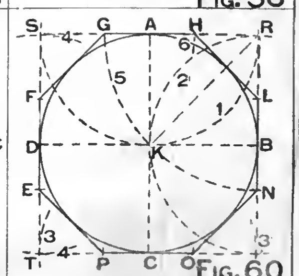


FIG. 60

PLATE 7

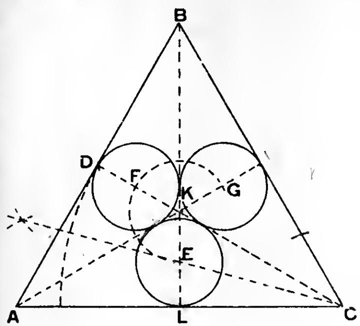


FIG. 61

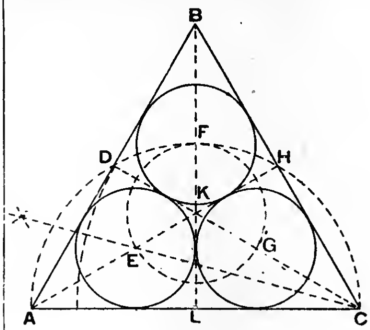


FIG. 62

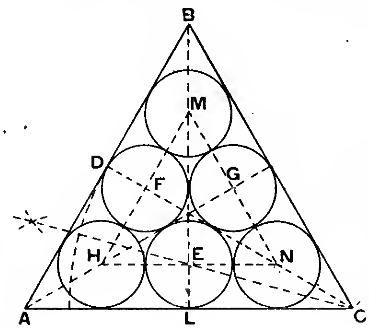


FIG. 63

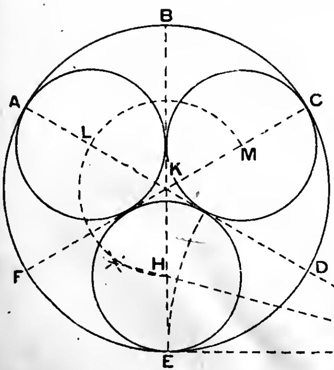


FIG. 64

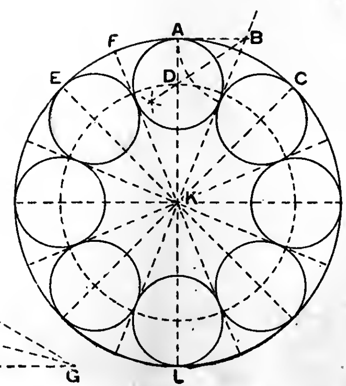


FIG. 65

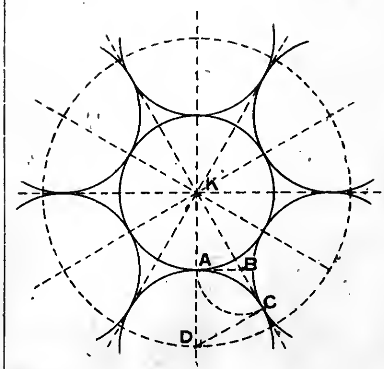


FIG. 66

PLATE 8.

PLATE 9.

Plate 9

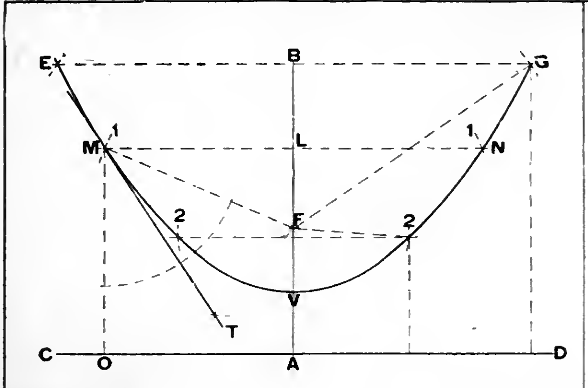


Fig. 71

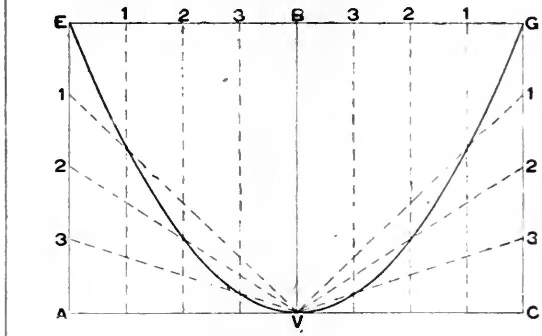


Fig. 72

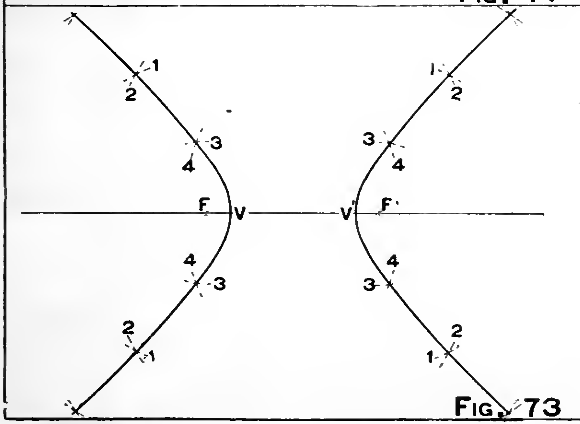


Fig. 73

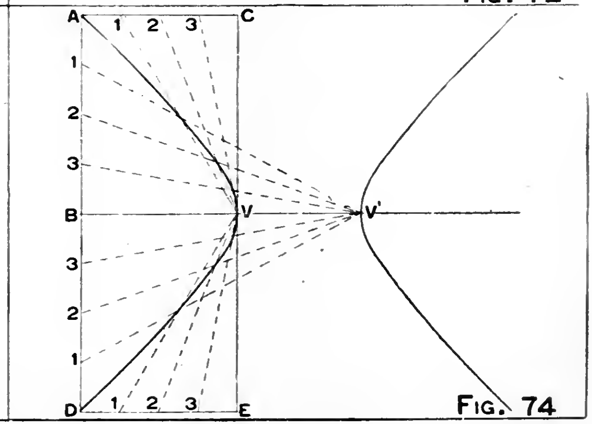


Fig. 74

PLATE 10.

PLATE 10

A B C D E F G H I J K L M N

O P Q R S T U V W X Y Z &

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n o p q r s t

u v w x y z

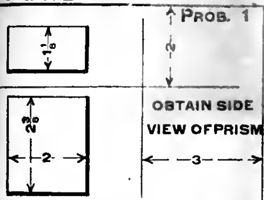
1 2 3 4 5 6 7 8 9 0 ½ 1 2 3 4 5 6 7 8 9 0 ½

PLATE 11.

PLATE 12.

PLATE 12

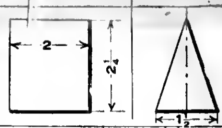
PROB. 1



OBTAIN SIDE VIEW OF PRISM

PROB. 2

OBTAIN TOP VIEW OF WEDGE.



PROB. 5

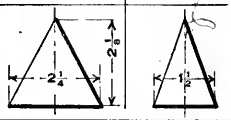
OBTAIN FRONT AND SIDE VIEWS OF EQUILATERAL TRIANGULAR PRISM $2\frac{1}{4}''$ HIGH.

PROB. 6

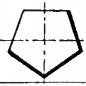
OBTAIN FRONT AND SIDE VIEWS OF EQUILATERAL TRIANGULAR PYRAMID $2''$ HIGH.

PROB. 3

OBTAIN TOP VIEW OF RECTANGULAR PYRAMID.



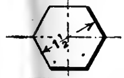
PROB. 4



DIAMETER OF CIRCUMSCRIBING CIRCLE $1\frac{3}{4}''$.

OBTAIN FRONT AND SIDE VIEW OF PENTAGONAL PRISM. $2\frac{1}{4}''$ HIGH

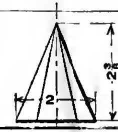
PROB. 7



OBTAIN FRONT AND SIDE VIEWS OF HEXAGONAL PRISM $2\frac{1}{4}''$ HIGH.

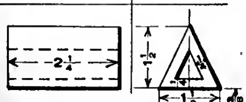
PROB. 8

OBTAIN TOP AND SIDE VIEWS OF HEXAGONAL PYRAMID.



PROB. 9

OBTAIN TOP VIEW.



PROB. 10

COPY TOP VIEW FROM PRECEDING PROBLEM BUT WITH THE LONG EDGES AT AN ANGLE OF 30° WITH THE FRONT AXIS. OBTAIN FRONT AND SIDE VIEWS.

PROB. 11

OBTAIN TOP VIEW WITHOUT THE USE OF AXES.

PROB. 12

COPY TOP VIEW FROM PRECEDING PROBLEM BUT WITH LONG EDGES MAKING AN ANGLE OF 30° WITH THE FRONT AXIS.

USE AXES BUT OMIT PROJECTION LINES.

OBTAIN FRONT AND SIDE VIEWS.

PROB. 13

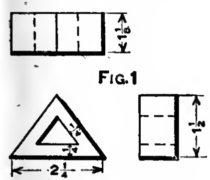


FIG. 1

OBTAIN TOP AND FRONT VIEWS.

OBTAIN TOP AND SIDE VIEW.

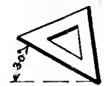


FIG. 2

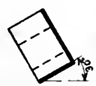


FIG. 3

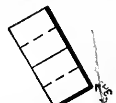


FIG. 4

OBTAIN FRONT AND SIDE VIEWS.

Plate 13.

PLATE 13

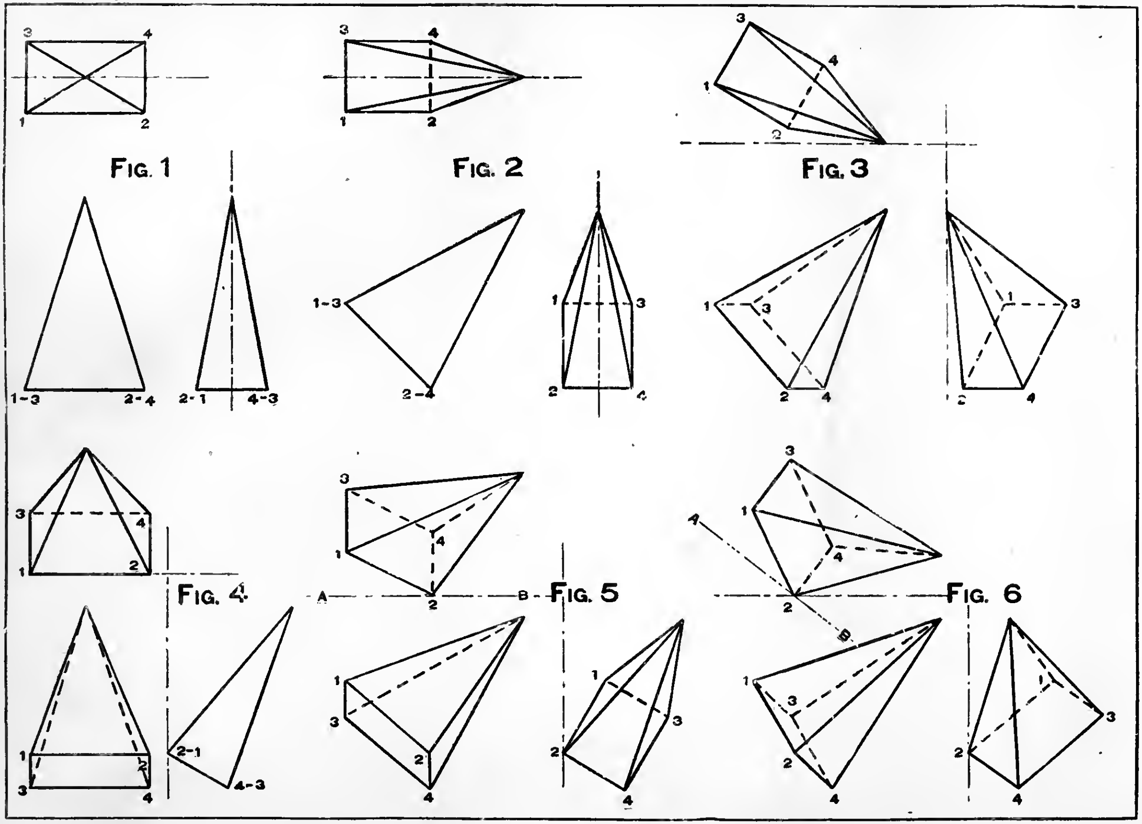


PLATE 14.

PLATE 14

PROB. 14

OBTAIN TOP VIEW.

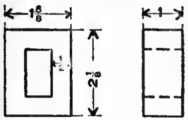


FIG. 1

REVOLVE THE OBJECT AS SHOWN IN FIG. 1 30° ABOUT SIDE AXIS.

FIG. 2

REVOLVE THE OBJECT AS SHOWN IN FIG. 1 30° ABOUT FRONT AXIS.

FIG. 3

REVOLVE THE OBJECT AS SHOWN IN FIG. 1 60° ABOUT VERTICAL AXIS.

FIG. 4

PROB. 15

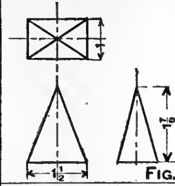


FIG. 1

REVOLVE THE OBJECT AS SHOWN IN FIG. 1 30° ABOUT SIDE AXIS TO LEFT.

FIG. 2

REVOLVE THE OBJECT AS SHOWN IN FIG. 2 25° ABOUT VERTICAL AXIS.

FIG. 3

REVOLVE THE OBJECT AS SHOWN IN FIG. 1 20° ABOUT FRONT AXIS FORWARD.

FIG. 4

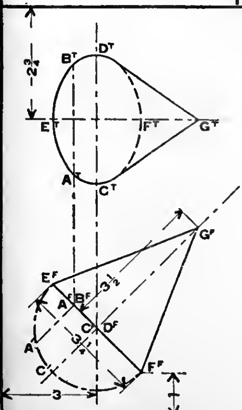
REVOLVE THE OBJECT AS SHOWN IN FIG. 2 15° ABOUT FRONT AXIS FORWARD.

FIG. 5

REVOLVE THE OBJECT AS SHOWN IN FIG. 5 35° ABOUT VERTICAL AXIS.

FIG. 6

PROB. 16



REVOLVE CONE 30° ABOUT VERTICAL AXIS. OBTAIN 3 VIEWS.

PROB. 17

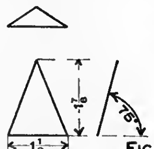


FIG. 1

REVOLVE THE SURFACE AS SHOWN IN FIG. 1 30° ABOUT SIDE AXIS TO LEFT.

FIG. 2

REVOLVE THE SURFACE AS SHOWN IN FIG. 2 25° ABOUT VERTICAL AXIS.

FIG. 3

REVOLVE THE SURFACE AS SHOWN IN FIG. 1 20° ABOUT FRONT AXIS FORWARD.

FIG. 4

REVOLVE THE SURFACE AS SHOWN IN FIG. 2 15° ABOUT FRONT AXIS.

FIG. 5

REVOLVE THE SURFACE AS SHOWN IN FIG. 5 35° ABOUT VERTICAL AXIS.

FIG. 6

PLATE 15.

PLATE 15

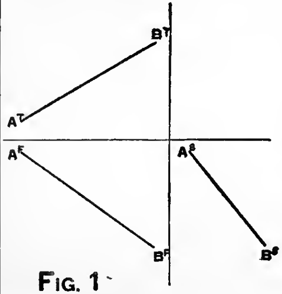


FIG. 1

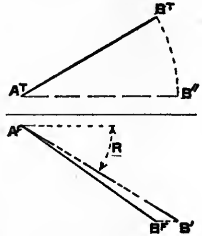


FIG. 2

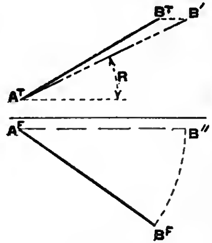


FIG. 3

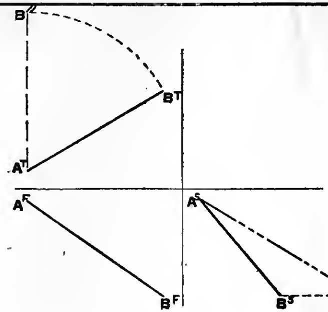


FIG. 4

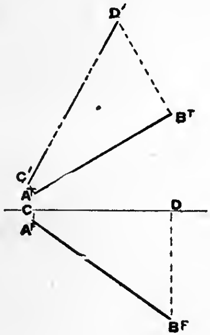


FIG. 5

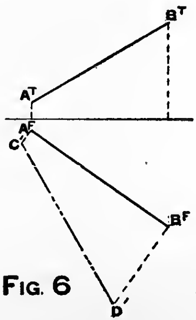


FIG. 6

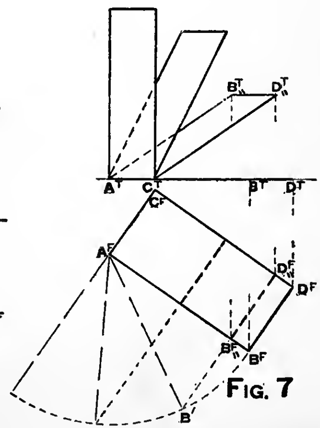


FIG. 7

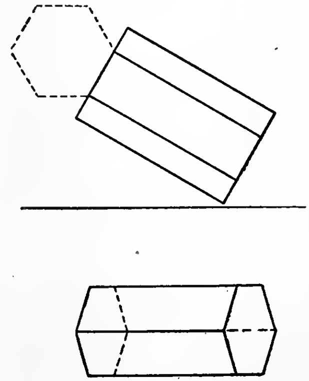


FIG. 8

Plate 16.

PLATE 16

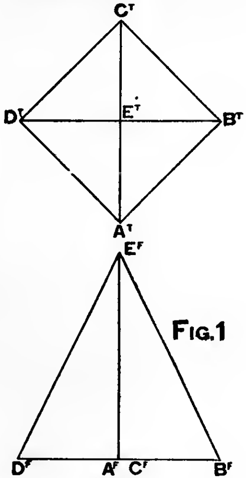


FIG.1

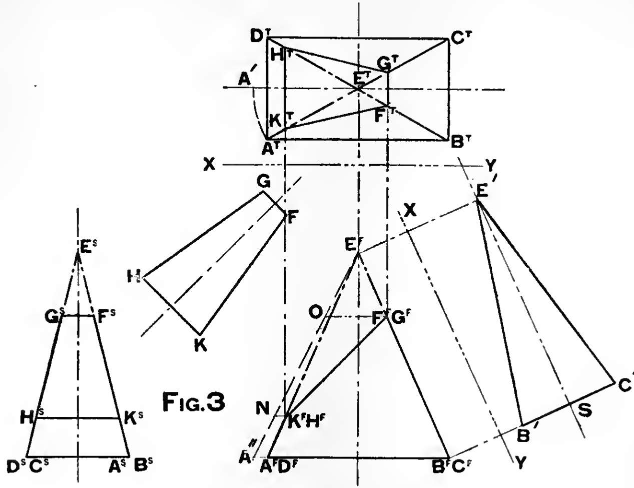


FIG.3

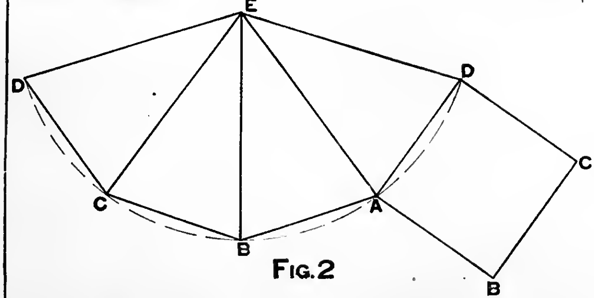


FIG.2

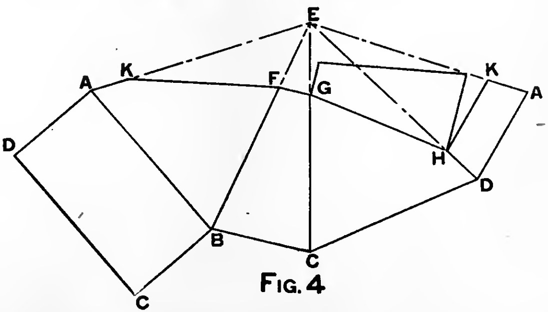


FIG.4

PLATE 17

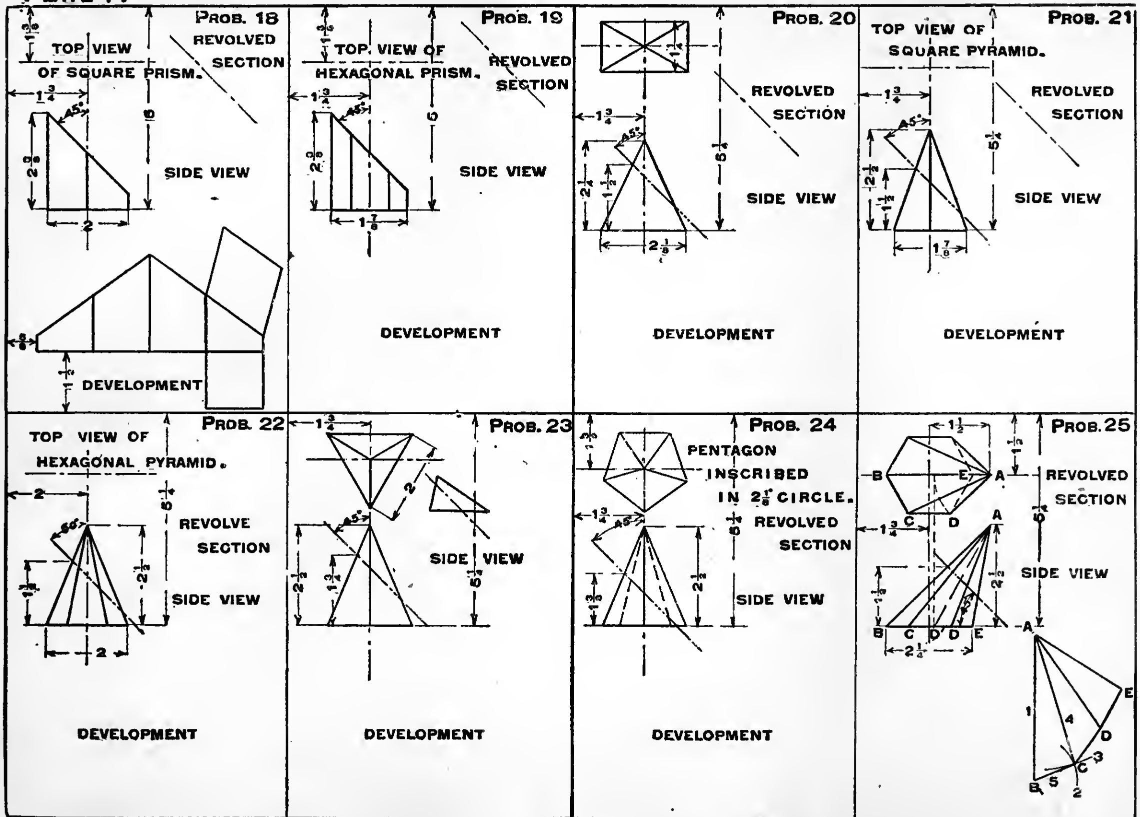

PROB. 18
TOP VIEW
OF SQUARE PRISM.
REVOLVED
SECTION
SIDE VIEW
DEVELOPMENT
PROB. 19
TOP. VIEW OF
HEXAGONAL PRISM.
REVOLVED
SECTION
SIDE VIEW
DEVELOPMENT
PROB. 20
REVOLVED
SECTION
SIDE VIEW
DEVELOPMENT
PROB. 21
TOP VIEW OF
SQUARE PYRAMID.
REVOLVED
SECTION
SIDE VIEW
DEVELOPMENT
PROB. 22
TOP VIEW OF
HEXAGONAL PYRAMID.
REVOLVE
SECTION
SIDE VIEW
DEVELOPMENT
PROB. 23
SIDE VIEW
DEVELOPMENT
PROB. 24
PENTAGON
INSCRIBED
IN 2 1/8" CIRCLE.
REVOLVED
SECTION
SIDE VIEW
DEVELOPMENT
PROB. 25
REVOLVED
SECTION
SIDE VIEW

Plate 18.

Plate 18

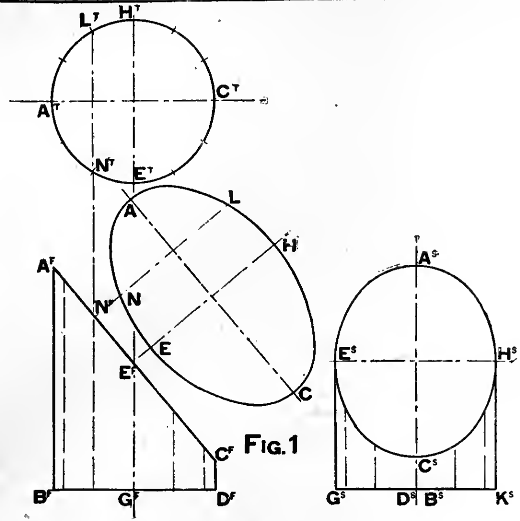


Fig. 1

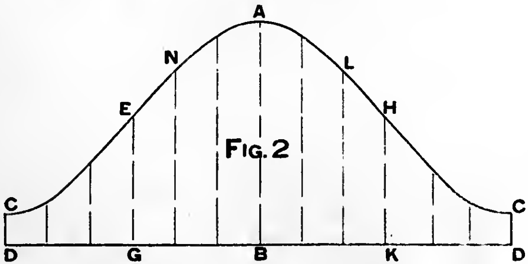


Fig. 2

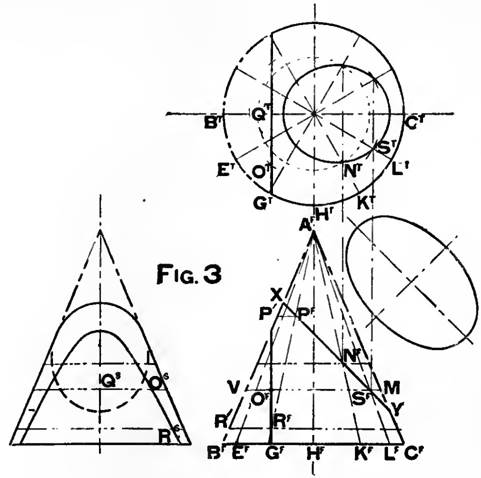


Fig. 3

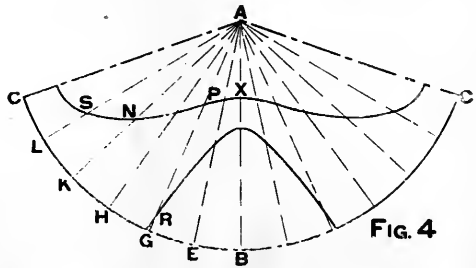
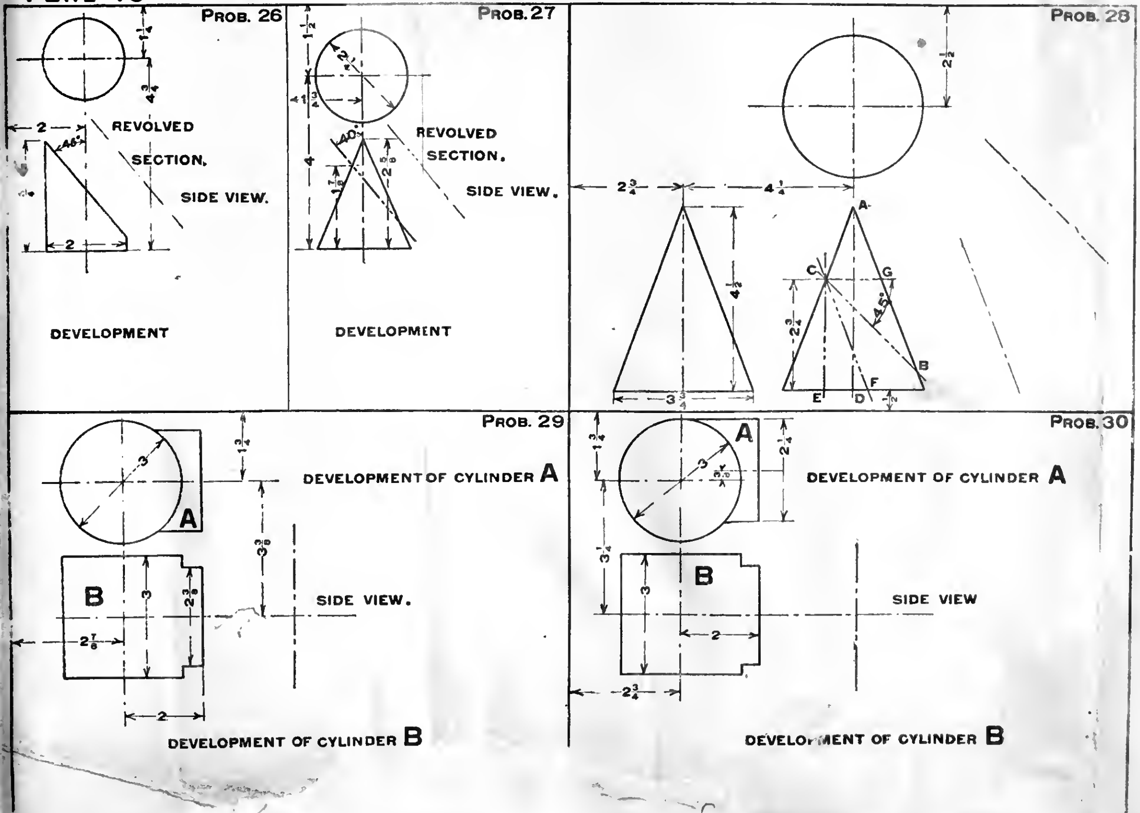


Fig. 4

PLATE 19.

PLATE 19



PROB. 26
REVOLVED SECTION.
SIDE VIEW.
DEVELOPMENT
PROB. 27
REVOLVED SECTION.
SIDE VIEW.
DEVELOPMENT
PROB. 28
A
B
C
D
E
F
G
45°
PROB. 29
DEVELOPMENT OF CYLINDER A
A
B
SIDE VIEW.
DEVELOPMENT OF CYLINDER B
PROB. 30
A
B
DEVELOPMENT OF CYLINDER A
SIDE VIEW
DEVELOPMENT OF CYLINDER B

PLATE 20.

PLATE 20

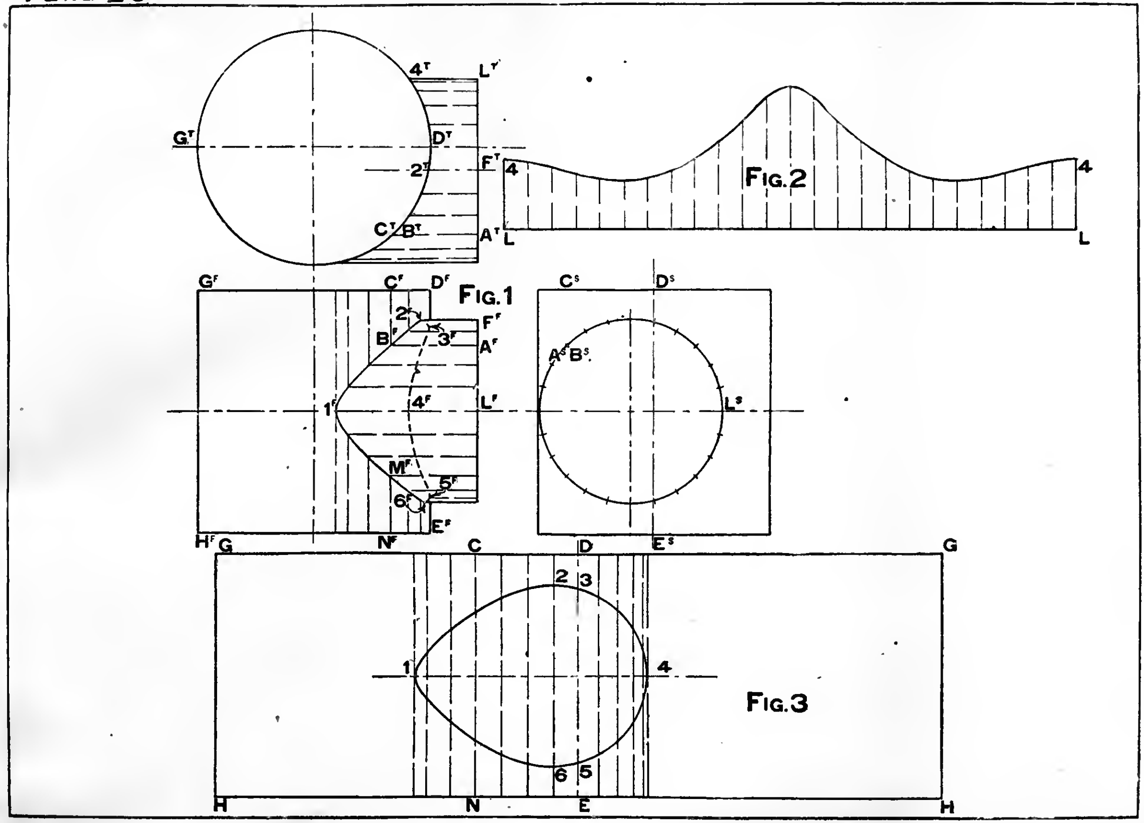


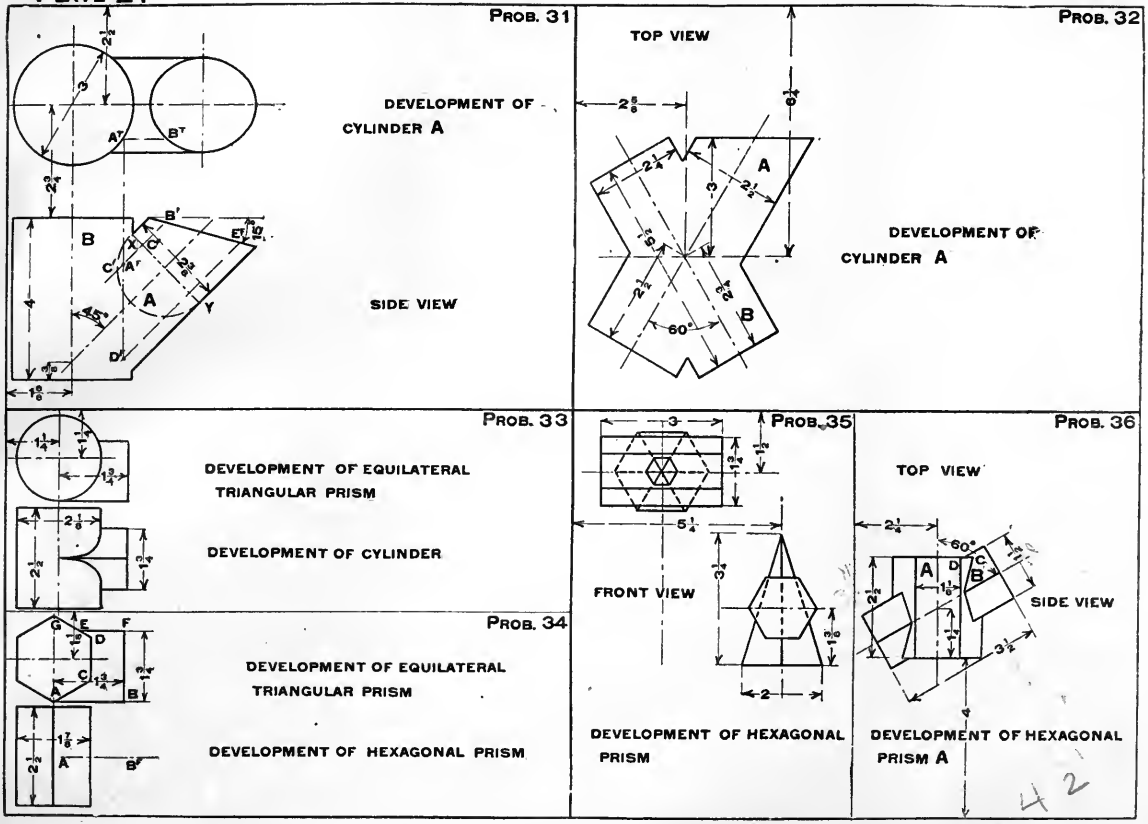
FIG. 1

FIG. 2

FIG. 3

PLATE 21

PLATE 21


PROB. 31
DEVELOPMENT OF CYLINDER A
SIDE VIEW
PROB. 32
TOP VIEW
DEVELOPMENT OF CYLINDER A
PROB. 33
DEVELOPMENT OF EQUILATERAL TRIANGULAR PRISM
DEVELOPMENT OF CYLINDER
PROB. 34
DEVELOPMENT OF EQUILATERAL TRIANGULAR PRISM
DEVELOPMENT OF HEXAGONAL PRISM
PROB. 35
FRONT VIEW
DEVELOPMENT OF HEXAGONAL PRISM
PROB. 36
TOP VIEW
SIDE VIEW
DEVELOPMENT OF HEXAGONAL PRISM A

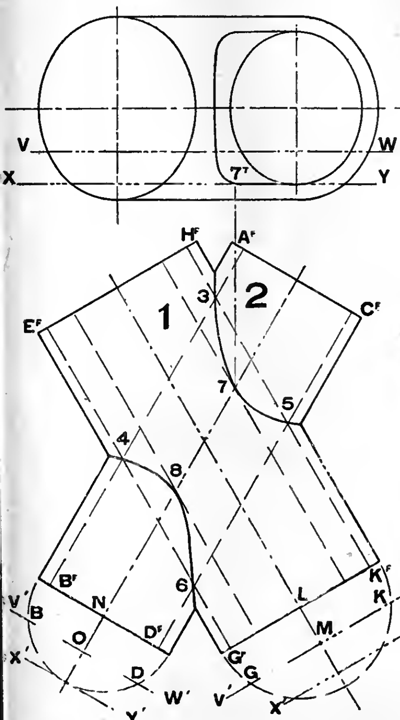


FIG. 1

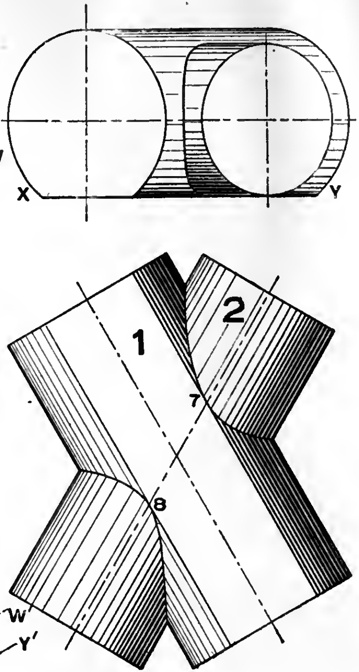


FIG. 2

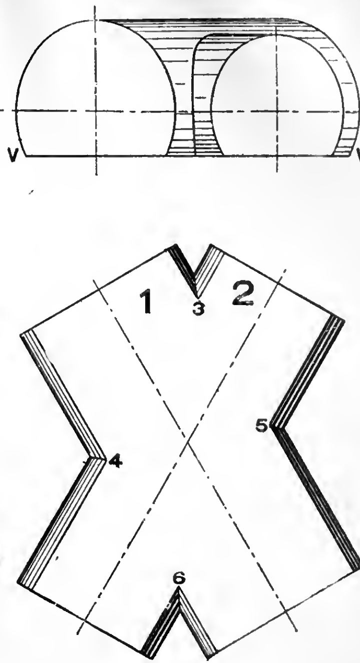


FIG. 3

Plate 22.

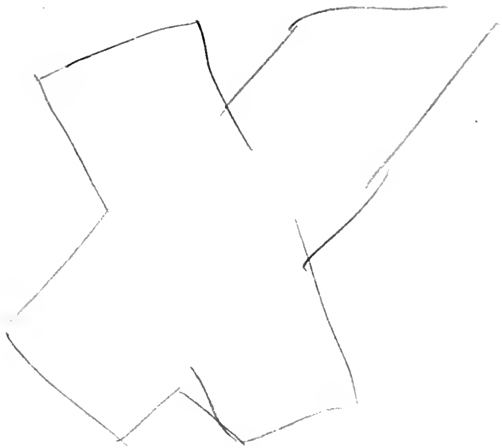


PLATE 23.

PLATE 24.

Plate 24

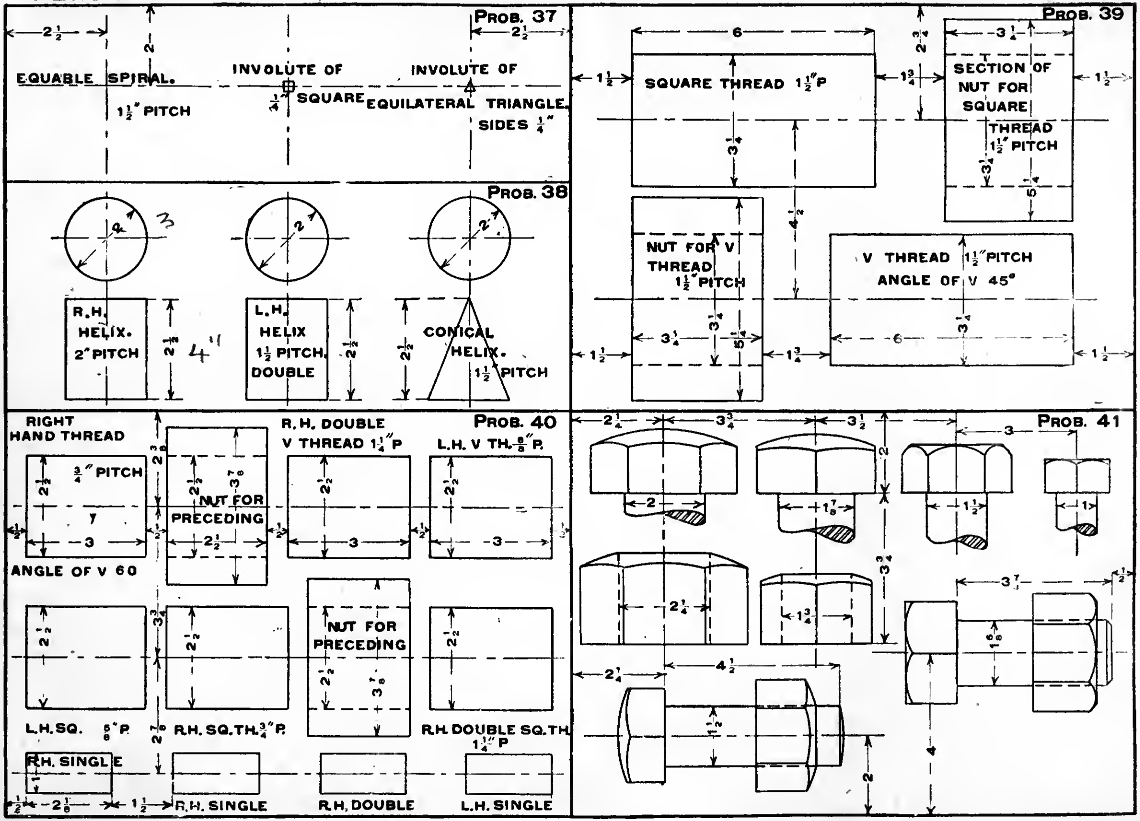

Prob. 37
EQUABLE SPIRAL.
1½" PITCH
INVOLUTE OF SQUARE
INVOLUTE OF EQUILATERAL TRIANGLE. SIDES ¼"
Prob. 38
R.H. HELIX. 2" PITCH
L.H. HELIX 1½ PITCH, DOUBLE
CONICAL HELIX. 1½" PITCH
Prob. 39
SQUARE THREAD 1½" P
SECTION OF NUT FOR SQUARE THREAD 1½" PITCH
NUT FOR V THREAD 1½" PITCH
V THREAD 1½" PITCH ANGLE OF V 45°
Prob. 40
RIGHT HAND THREAD
¾" PITCH
ANGLE OF V 60
NUT FOR PRECEDING
R.H. DOUBLE V THREAD 1¼" P
L.H. V TH. ⅝" P.
NUT FOR PRECEDING
L.H. SQ. ⅝" P.
R.H. SQ. TH. ¾" P.
R.H. DOUBLE SQ. TH 1¼" P
R.H. SINGLE
R.H. SINGLE
R.H. DOUBLE
L.H. SINGLE
Prob. 41

Plate 25.

PLATE 25

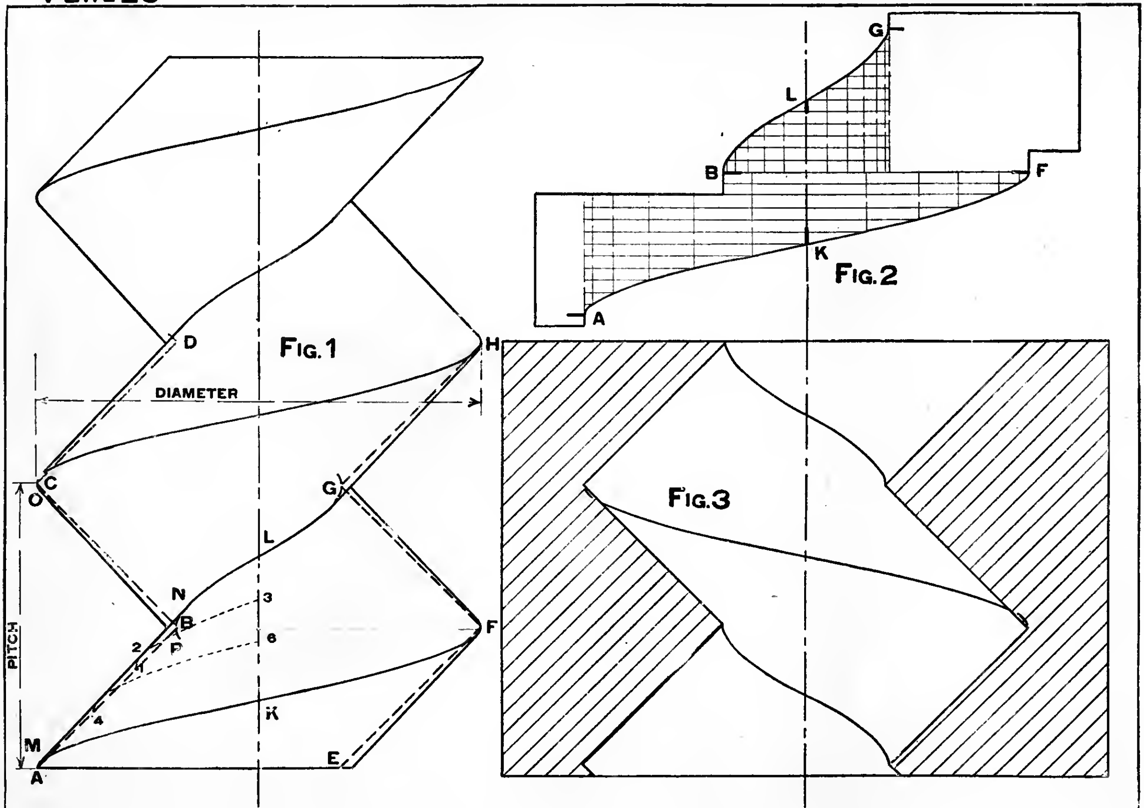


FIG. 1
DIAMETER
PITCH
A
B
C
D
E
F
G
H
K
L
M
N
O
P
1
2
3
4
6
FIG. 2
A
B
F
G
K
L
FIG. 3

PLATE 26.

PLATE 26

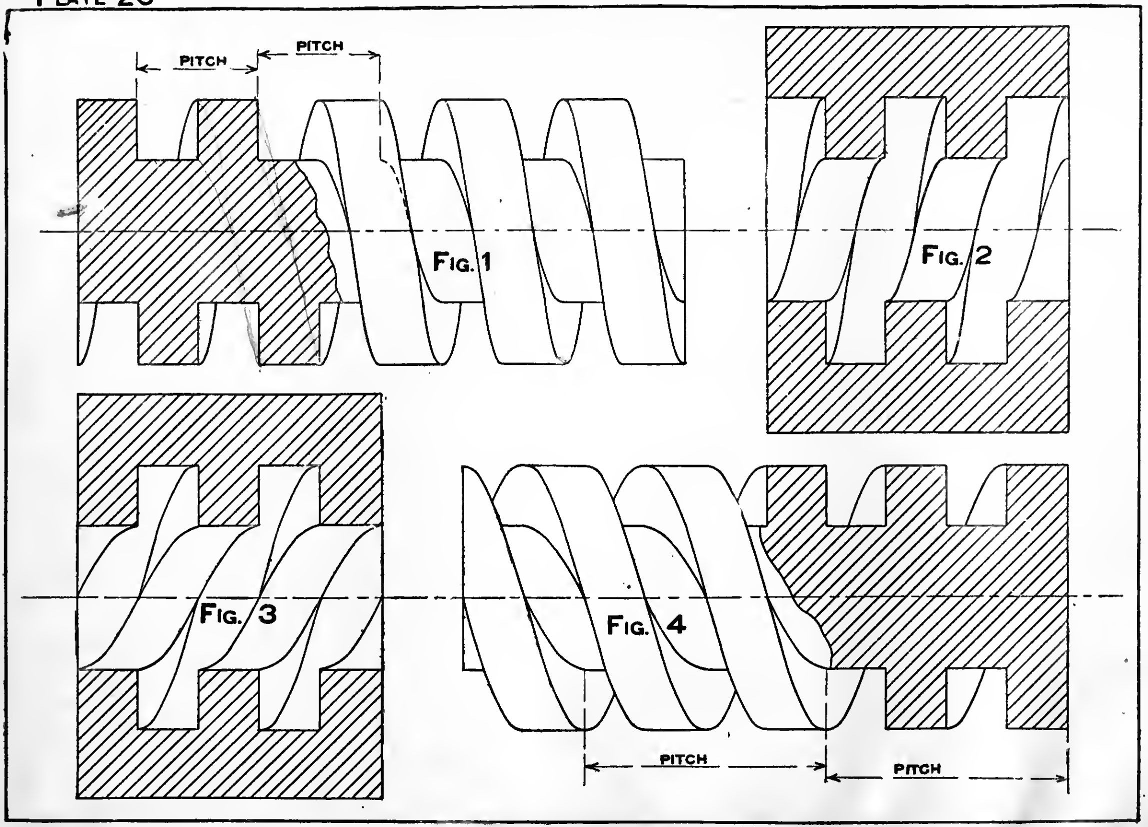


PLATE 27.

PLATE 27

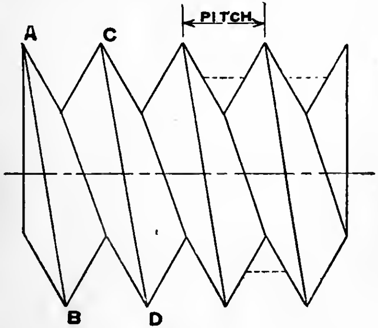


FIG. 1

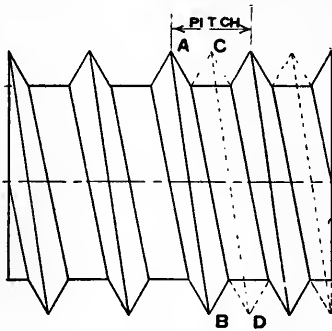


FIG. 2

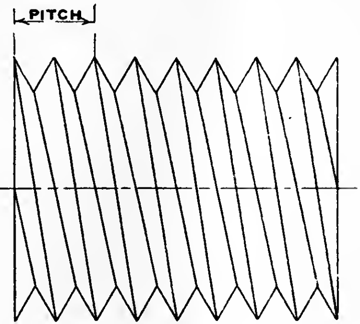


FIG. 3

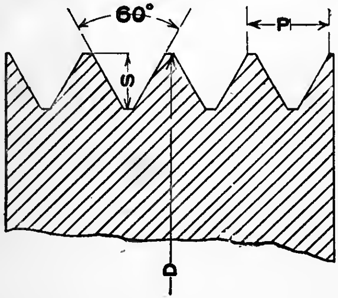


FIG. 4

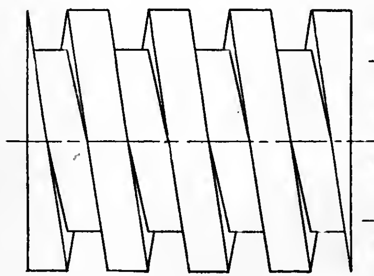


FIG. 5

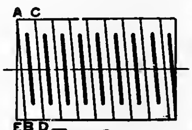


FIG. 6

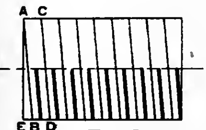


FIG. 7

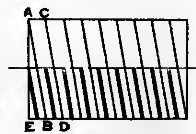


FIG. 8

FIG. 9

PLATE 28.

Plate 28

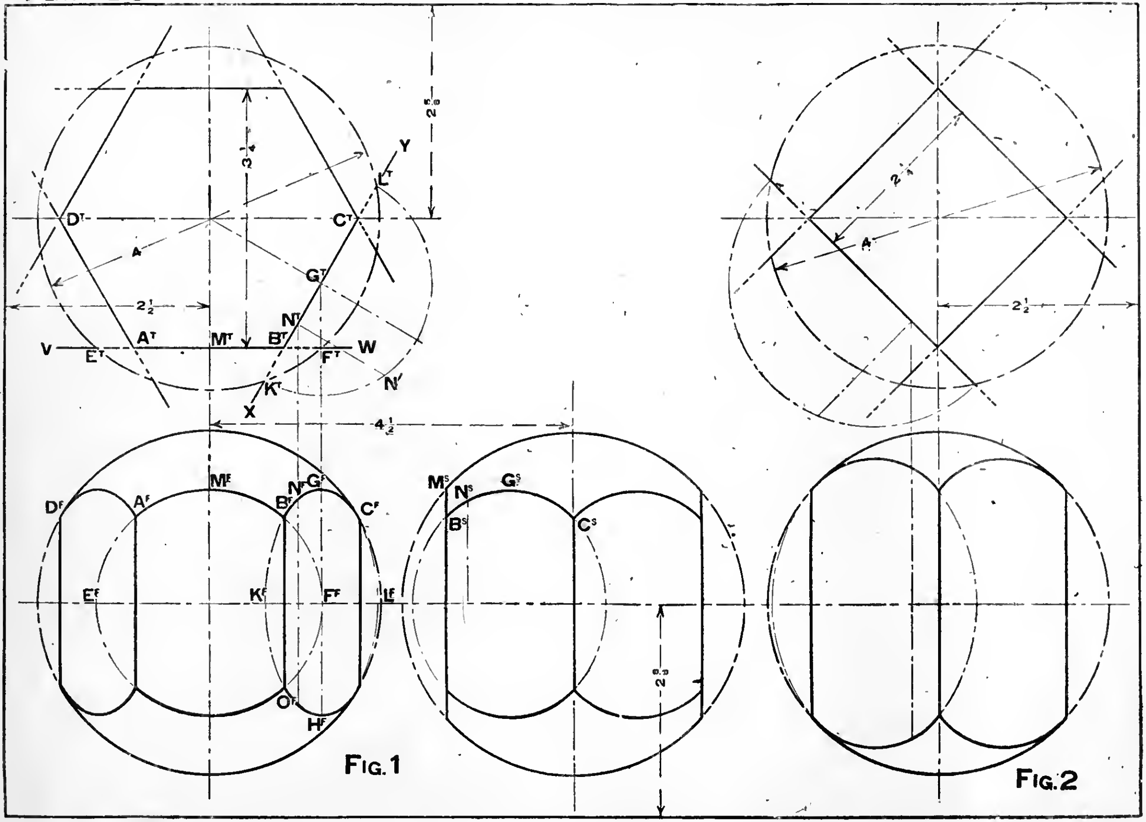


Fig. 1

Fig. 2

Plate 29.

PLATE 29

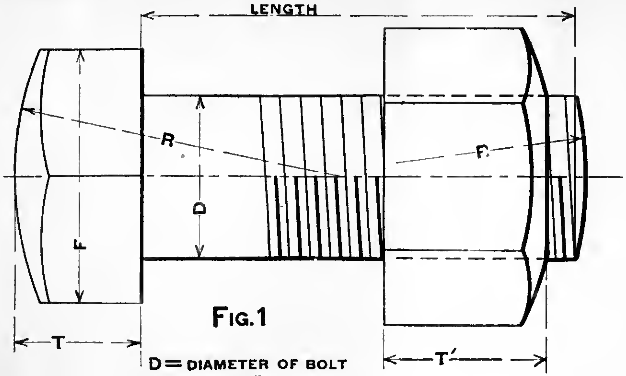


FIG. 1

$D =$ DIAMETER OF BOLT

$F = 1\frac{1}{2}D + \frac{1}{8}''$

$T = \frac{F}{2}$ $\quad T' = D$

$R = 2D$ OR $2\frac{1}{2}D$

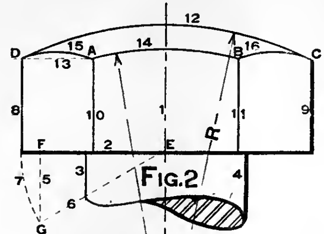


FIG. 2

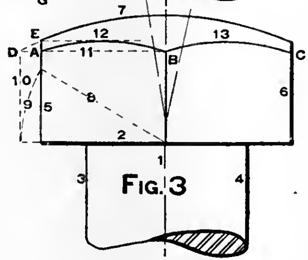


FIG. 3

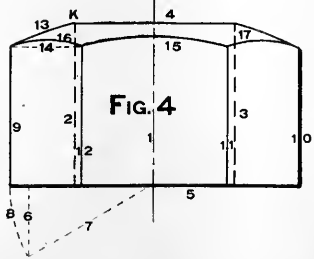


FIG. 4

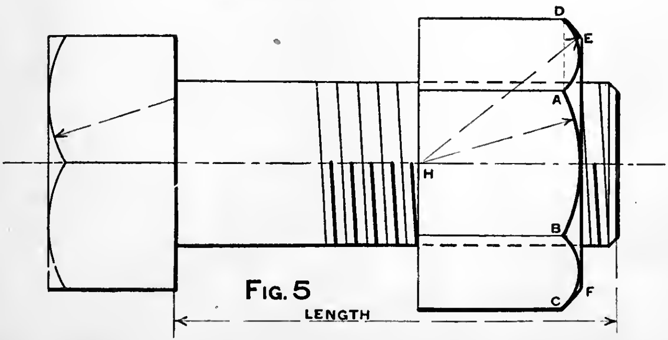


FIG. 5

Plate 30.

PLATE 30

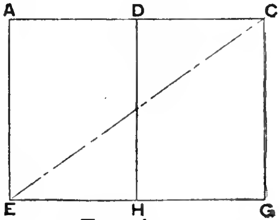


FIG. 1

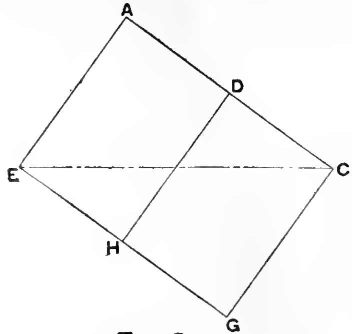


FIG. 2

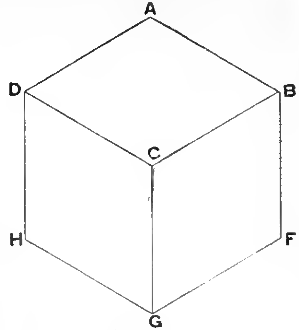


FIG. 3

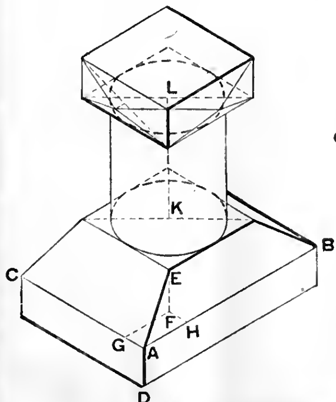


FIG. 4

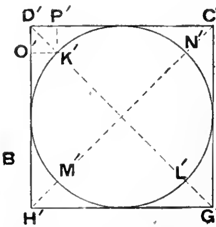


FIG. 5

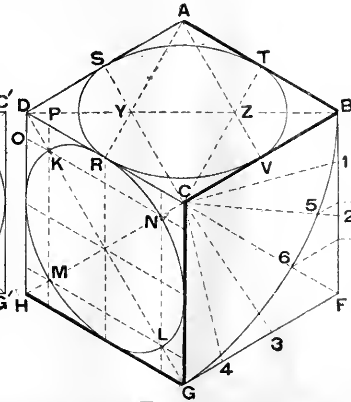


FIG. 6

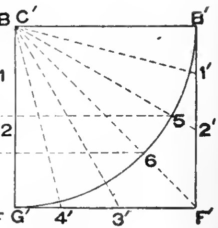


FIG. 7

PLATE 32

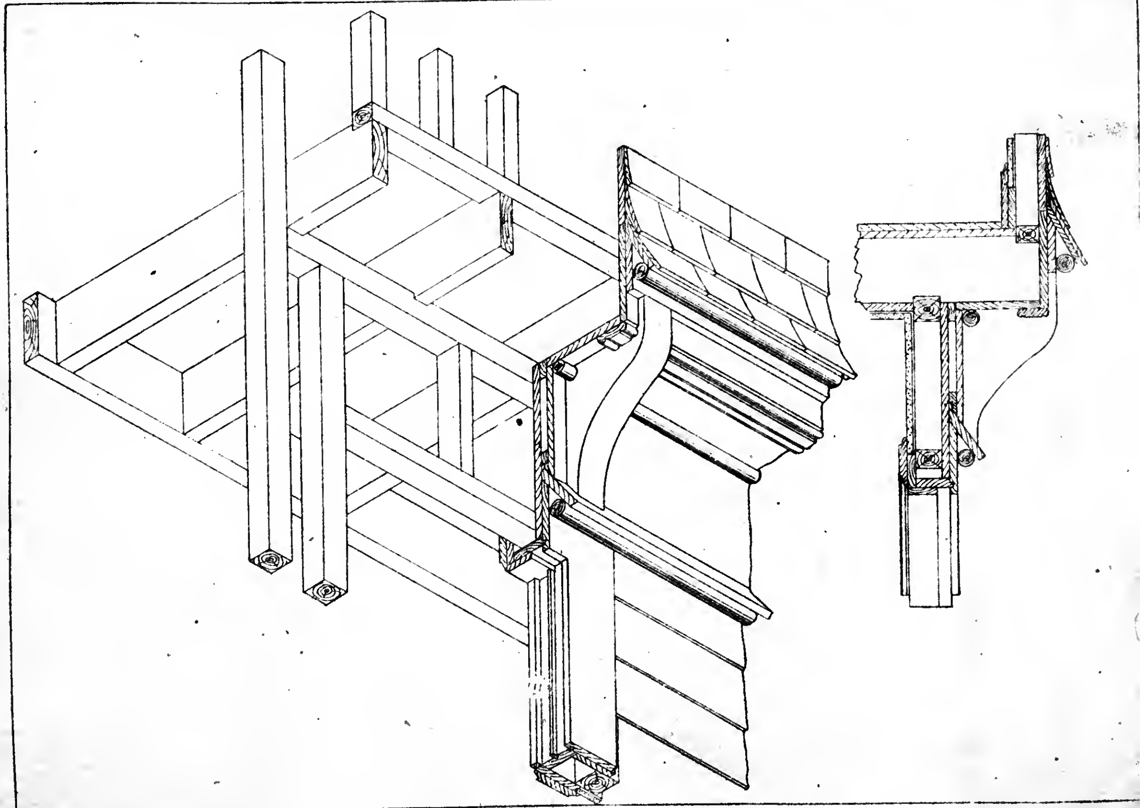


Plate 32.